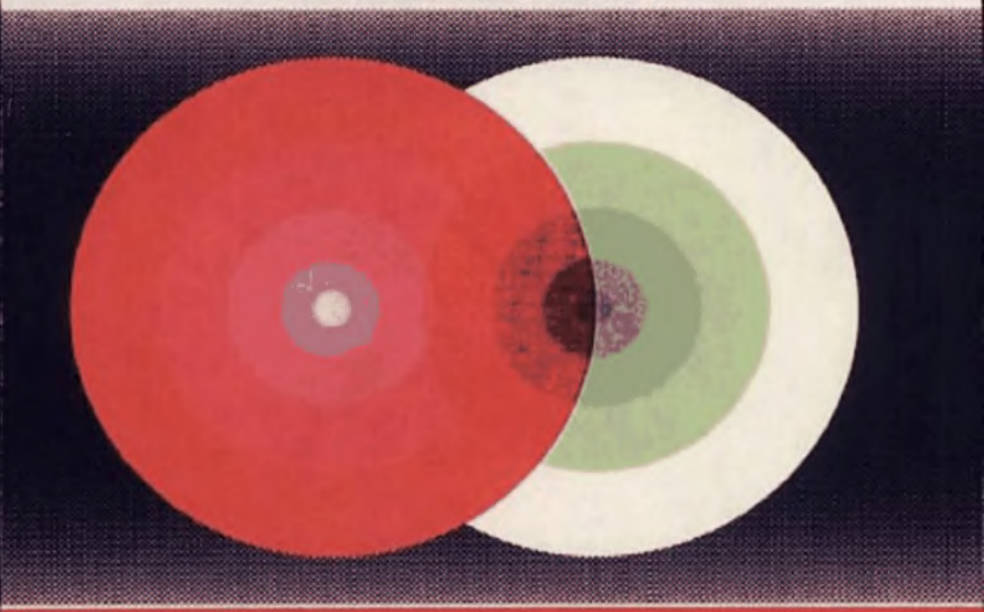
Quantum Mechanics from General Relativity

An Approximation for a Theory of Inertia

by

Mendel Sachs

D. Reidel Publishing Company



Fundamental Theories of Physics

Fundamental Theories of Physics

A New International Book Series on The Fundamental Theories of Physics: Their Clarification, Development and Application

Quantum Mechanics from General Relativity

An Approximation for a Theory of Inertia

by

Mendel Sachs

Department of Physics and Astronomy,
State University of New York at Buffalo, U.S.A.

D. Reidel Publishing Company

A MEMBER OF THE KLUWER ACADEMIC PUBLISHERS GROUP

Library of Congress Cataloging-in-Publication Data

Sachs, Mendel.
Quantum mechanics from general relativity.

(Fundamental theories of physics)
Bibliography: p.
Includes index.
1. Quantum theory. 2. Relativity (Physics) 3. Particles (Nuclear physics)
I. Title. II. Series.
QC174.12.S225 1986 530.1'2 86—17902
ISBN 90—277—2247—1

Published by D. Reidel Publishing Company,
P.O. Box 17, 3300 AA Dordrecht, Holland.

Sold and distributed in the U.S.A. and Canada
by Kluwer Academic Publishers,
101 Philip Drive, Assinippi Park, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers Group,
P.O. Box 322, 3300 AH Dordrecht, Holland.

DEDICATED TO
THE SIX MILLION MARTYRS

. . . , Avraham, Chaje Taube, Rubin, Cili, . . .

Contents

Preface

This monograph is a sequel to my earlier work, General Relativity and Matter [1], which will be referred to henceforth as GRM. The monograph, GRM, focuses on the full set of implications of General Relativity Theory, as a fundamental theory of matter in all domains, from elementary particle physics to cosmology. It is shown there to exhibit an explicit unification of the gravitational and electromagnetic fields of force with the inertial manifestations of matter, expressing the latter explicitly in terms of a covariant field theory within the structure of this general theory. This monograph will focus, primarily, on the special relativistic limit of the part of this general field theory of matter that deals with inertia, in the domain where quantum mechanics has been evoked in contemporary physics as a fundamental explanation for the behavior of elementary matter.

Many of the results presented in this book are based on earlier published works in the journals, which will be listed in the Bibliography. These results will be presented here in an expanded form, with more discussion on the motivation and explanation for the theoretical development of the subject than space would allow in normal journal articles, and they will be presented in one place where there would then be a more unified and coherent explication of the subject.

It goes without saying that Quantum Mechanics has been one of the outstanding successes of twentieth century physics — in its correctly predicting and representing many of the atomic, nuclear and elementary particle phenomena. From the point of view of the Philosophy of Science, it is indeed a *necessary condition* for any valid scientific theory to meet that it should accurately predict the empirical data relating to particular physical phenomena, if it is to claim to be a (scientifically) true explanation for these phenomena. Nevertheless, it is important to recognize that this requirement is not a *sufficient condition* to establish its scientific validity. For a valid theory in science must also be (1) logically and mathematically consistent, and (2) it should be successful in its full spectrum of potential predictions; that is to say, if some of its predictions should be verified and others not, the entire theory should then be subject to question.

In spite of the outstanding numerical successes of quantum mechanics in fitting the data of elementary matter experimentation, it has not been able to meet the criteria of consistency and completeness mentioned above, at least to this date. As we will discuss in Chapter 2, the extension of nonrelativistic quantum mechanics to the relativistic domain, that is a necessary extension for the logical consistency of the theory, on its own terms, entails a breakdown of the essential logical and mathematical ingredients of the quantum theory, and indeed yields a mathematical formalism that has no solutions. Since the quantum theory, if generally true as a theory of elementary matter, should apply equally to the relativistic region of elementary matter phenomena as to nonrelativistic phenomena, and since this has not been accomplished yet (for reasons that will be discussed in Chapter 2), in the form of a relativistically covariant 'quantum field theory' that would satisfy the requirements of both the quantum theory and the theory of relativity, simultaneously, it must be admitted by the objective scientist that the quantum theory has not yet established itself as a fundamental theory of elementary matter, even though it is an empirically correct description of atomic and elementary particle phenomena under particular experimental circumstances.

In addition to the empirical successes of low energy (nonrelativistic) quantum mechanics, over the past 60 years of physics research, there has been a great deal of success in phenomenological approaches to high energy elementary particle physics, though always in the context of the quantum theory. These discoveries have entailed new kinds of 'hidden symmetries' in the expanded spaces to describe the probability functions of elementary particles [2]. Further, to classify their species, proposals are made about (a) new types of particles involved in the classification of strongly interacting particles, that make up those particles, though 'confined' to their domains (the 'quarks'), (b) a generalization of quantum electrodynamics to incorporate the quarks, called 'quantum chromodynamics' [3], (c) generalizations of the gauge symmetry so as to unify the phenomenological description of the weak and electromagnetic interactions [4], etc. If a new theory of matter is to replace the quantum theory, it must still yield the correct empirical data as predictions, much of it thus far represented brilliantly by the present day phenomenological schemes in high energy particle physics. These current researches in theoretical particle physics must then serve as a guide toward the form of an underlying theory of elementary matter, at least insofar as they represent the empirical data.

The main aim of the research reported in this monograph is to present a fundamental theory of elementary matter, in terms of underlying principles, rather than taking a phenomenological approach. It will be shown that such a theory, based fundamentally on the starting ideas of the theory of general relativity, as a theory of matter, does indeed lead to the formal structure of quantum mechanics — as a linear approximation for the part of this field

theory of matter that is associated with the phenomenon of inertia. In this way, several of the features of matter in the microscopic domain are derived from first principles, rather than being imposed from the outset to fit the data.

In Chapter 2, after comparing the underlying concepts of the quantum and relativity theories, there will be a discussion of the critique of Einstein, Podolsky and Rosen, on the Copenhagen view of quantum mechanics, and thence to Bohr's rejoinder. This will then lead to a brief outline of the program of hidden variable theories, and separated from the resolution of this monograph, which is toward the underlying basis of general relativity (which entails 'exposed variables' instead, in the fashion of the Einstein field theory). Bell's inequalities will then be discussed in the context of their use as an asymptotic limit of the nonlinear field theory of matter implied by general relativity.

In Chapters 3, 4, and 5, the logical and mathematical development of general relativity, as a theory of elementary matter, will be presented, including the new consequences as a result of incorporating the *Mach principle.* This leads to the new idea, expressed as the *law of conservation of interaction* (to replace the *conservation of probability* of the standard quantum view), and the derivation of the nonlinear inertial field equations will be demonstrated, whose linear limit is the formal structure of quantum mechanics, then to the full expression of the electromagnetic field equations that fully exploits the Mach principle in general relativity.

It will be seen in Chapter 6 how this theory of inertia leads to the formal expression of quantum mechanics, as a low energy approximation. In the context of this field theory of inertia, the Pauli exclusion principle will be derived from first principles, from the exact form of the nonlinear, spinor matter field equations. It will then be shown, as a linear approximation, how this exact result yields the description of the many-particle system that incorporates the rules of Fermi-Dirac statistics.

Next, in Chapter 7, the matter field equations will be applied to the problem of the bound electron-positron system. An exact bound state solution will be demonstrated for the nonlinear, coupled field equations that exhibits all of the physical features that are normally attributed to the 'annihilation' of the pair, though, here, without any actual annihilation of matter. It is rather that the particle and antiparticle go into a state of binding so deep that they do not readily give up energy and momentum to their surroundings, i.e. they are 'invisible' to a detecting apparatus, such as a cloud chamber or a bubble chamber. However, when given enough energy to ionize this pair in their bound state, they become visible again — this is the data conventionally interpreted as 'pair creation'. It will then be demonstrated that a sea of such *real pairs*, when in thermodynamic equilibrium with the walls of a cavity, at some temperature, has a Planck distribution in regard to its energy density, as we normally associate with blackbody radiation. Thus, the

state of a single pair, identified experimentally with data associated with 'annihilation', and the blackbody radiation curve, are both associated with a system of matter that excludes the 'photon' concept altogether. It is shown in this monograph that the 'photon' concept may be abandoned, replacing it with matter fields alone, in a way similar to the 'delayed-action-at-a-distance' idea [69], though here there are no singular trajectories, only a closed system of matter fields, all mapped in the same space-time.

In Chapter 8 the theory is applied to the case of the *e-p* system: the bound states associated with hydrogenic atoms and the unbound *e-p* scattering problem. The complete hydrogen spectrum will be derived from the matter field equations, including the Lamb splitting. The latter is associated in quantum electrodynamics with radiative shifts of the (otherwise) accidentally degenerate states of hydrogen [5]; in this theory the Lamb splitting occurs for an entirely different reason — it is a consequence of a generalization of the electromagnetic interaction that appears in a natural way, that in turn lowers the symmetry of the ordinary Coulomb term in the Dirac Hamiltonian for hydrogen, causing thence a lifting of the accidental degeneracy in the states predicted by the Dirac theory of hydrogen. The renormalization numerical technique required by quantum electrodynamics (yielding a mathematically inconsistent scheme of prediction) is not encountered here, where everything is finite from the outset.

The electron-proton scattering process will then be analyzed in the light of the generalization of the matter field equations, including the generalized form of the electromagnetic interaction. It will be seen that, first without the background of electron-positron pairs, the Mott cross section for point particle scattering is modified — in the direction of the data (that is normally fully explained with the use of form factors of the proton, expressing the presence of a mesic cloud cover to give the 'observed' proton some structure). The *e-p* scattering problem will then be analyzed in the presence of a background of pairs, that this theory derived in the preceding chapter. The Coulomb potential then effectively modifies in two ways: one, due to the polarization of the medium introduces a Yukawa-Debye factor $\exp(-\mu r)$ and the other, due to the electromagnetic generalization, that introduces the factor $\exp(-b/r)$ so that the effetive *e-p* interaction takes the form $[\exp(-\mu r - b/r)]/r$. Note that the factor b is determined by the theory to be the order of 10^{-14} cm, so that when the momentum transfer is high enough that the effective *e-p* separation r is a parameter that decreases from this value toward zero, the effective *e-p* potential correspondingly tends to zero. There also appears in the analysis of the generalized electromagnetic interaction a factor that makes the sign of the *e-p* interaction change, from attractive to repulsive, at sufficiently high relative speed.

The results of the analysis in this monograph indicate hints that the weak interaction and the strong interaction between elementary matter fields are indeed manifestations of a generalized electromagnetic interaction, under the

proper circumstances of energy-momentum transfer and relative separation — i.e. it gives the hint of a unification of all elementary interactions, from the dynamical view. The gravitational interaction is also automatically incorporated by virtue of the global extension of this field theory to a curved space-time, where the metrical field plays the explicit role of the gravitational manifestation of interacting matter (as derived explicitly in GRM). It will also be demonstrated explicitly in this book that the 'quantization of electrical charge' may be derived from a local limit of the global representations of the symmetry group of general relativity theory (the 'Einstein group') — a result also demonstrated in GRM.

In Chapter 9 of this monograph, there will be presented an outline of some of the results of this theory that are concerned with high energy, elementary particle physics. The particular results of this chapter have largely to do with specific implications of the generalization of the electromagnetic interaction. This will be applied to the following problems: (1) the structure of the neutron, (2) the problem of the different masses for the charged versus the neutral pions, (3) the problem of CP violation in the decay of the long-lived, neutral kaon, (4) the admissibility of time-reversal noninvariance in nuclear forces, viewed as a component of the generalized electromagnetic interaction and (5) the general prediction of massive mediating composite states in weak interactions that could be associated with the recently observed $W^{\pm}$ particle resulting from high energy p-$\bar{p}$ interaction.

The results of the analysis in this monograph are not meant to indicate the completion of a theory of matter; they are rather meant to demonstrate the beginning of a theoretical investigation that has yielded a sufficient amount of results that, in my judgement, encourages further pursuit of the approach. For it seems evident to me, from this study, that when one takes the formal expression of quantum mechanics as a linear approximation for a generally covariant, nonlinear field theory of inertia in general relativity, new results follow from first principles that have never been derived from the basis of quantum mechanics itself. Some of the characteristics of the theory of matter that is presented that are in principle absent in the quantum theory, and are necessary here to arrive at correct predictions are: fundamental nonlinearity of a particular sort, the elementarity of the closed system (as expressed in terms of a generalized Mach principle) and the use of the (nonsingular) field concept as fundamental to a general theory of elementary matter.

Finally, the monograph will conclude with a brief Epilogue that focuses on two themes: one having to do with the question of determinism in physics, applied specifically to the question of time-irreversibility and the second law of thermodynamics, and the other dealing with the subject of scientific method. While these topics do not deal directly with the technical development in the text, they do treat important topics that are implicit in the philosophy of this work — on whether or not there is necessarily fundamental chaos and acausality in the laws of matter, and the idea that freedom

in the pursuit of scientific truth is a necessary ingredient in the method of science for there to be genuine progress.

The hope I have in writing this monograph is that the results will be encouraging enough to some of the readers to induce them to further pursue the point of view that is taken — a view in physics that was originally indicated by Einstein's intuition that the laws of elementary matter may in fact be based on a fully deterministic field theory, rooted in the axiomatic structure of general relativity, as a fundamental theory of matter — where the singular particles of the atomistic theories are replaced by the distinguishability of elementary relations of a nonsingular, continuous and unified field, covering all domains of interaction, from elementary particle physics to cosmology.

This monograph is addressed primarily to graduate students and other advanced researchers in theoretical physics and mathematics, and in the other sciences, concerned with the problems of elementary matter. It is assumed that the reader is acquainted with the concepts and mathematical formulations of nonrelativistic and relativistic quantum mechanics, as well as electromagnetic theory.

Early, crucial stages of some of the research results reported in this book were carried out in the late 1950's, in collaboration with my most respected colleague, Solomon L. Schwebel. The results of this collaboration are reported in the journal articles by both of us, listed in the Bibliography. I wish to express my most heartfelt gratitude to Sol for the opportunity to have a most fulfilling research collaboration with him, and for his advice that, in attacking the equations of theoretical physics, one should always try to achieve the discovery of exact solutions before resorting to approximation methods — whatever the odds may be against finding them!

Department of Physics and Astronomy Mendel Sachs
State University of New York at Buffalo, U.S.A.

Chapter 1

Fundamental Outlook

This is not a text to teach the rules of quantum mechanics. It is, rather, aimed at showing that a possible conceptual basis for the formal expression of quantum mechanics could be rooted in an approach entirely different from the present-day approach of the Copenhagen school or any of its theoretical modifications that have evolved over the years, that still maintain its essential probabilistic view [6].

The approach that is taken in this monograph is that of a fully relativistic field theory at the outset; that is, in conceptual and mathematical accord with Einstein's theory of general relativity, seen as a fundamental theory of matter. In explicit terms, it will be seen that quantum mechanics follows as *a linear approximation for a generally covariant field theory of inertia.* This field theory, which has been shown to be essential in leading to a unified field theory [1] wherein the force manifestations of matter must unify with its inertial manifestations, does not have any of the essential features of quantum mechanics, such as the form of a probability calculus expressed in terms of linear operators in a Hilbert space. But the field theory that explains the inertial properties of elementary matter, according to the theory developed in this book, does have, as a linear approximation, in the appropriate limit, the formal structure of the probability calculus of quantum mechanics.

The main thrust of this monograph is not purely speculative and philosophical, though the philosophical elements are necessarily present. Its aim is, rather, to present a genuine, rigorous alternative to quantum mechanics that is indeed different from the points of view of its conceptual basis as well as its general mathematical expression that follows, and to demonstrate that not only does this theory of inertia correctly predict all of the successful results of nonrelativistic quantum mechanics, but it also predicts new results in the high-energy domain that either are not predicted at all by conventional quantum mechanics or are not predicted by that theory in a mathematically satisfactory manner.

The history of science teaches us that the success of a particular scientific theory at a particular time, to predict particular phenomena, does not necessarily imply that the ideas that were assumed to underlie its mathe-

matical expression are the correct and unique *explanation* of the data at hand, even if there would be no obvious reasonable doubt about this explanation. But in twentieth-century physics there has been reasonable doubt that the usually accepted philosophy of quantum mechanics does *explain* atomic phenomena.

The trouble with quantum mechanics is revealed from a close study of the mathematical and conceptual structure of this theory when it is expressed in its full, unapproximated form, revealing intrinsic incompleteness and logical/mathematical inconsistencies. These difficulties will be discussed in detail in the next chapter, where it will be seen that the main trouble arises from the need to unify the quantum and relativity theories in the face of a demonstrable fundamental incompatibility of these two theories — thus resulting in the failure to unify them from the outset, when quantum mechanics was first discovered in the 1920s.

Of course, it is possible that these deficiencies will be resolved some day without giving up the essential part of the conceptual basis of quantum mechanics. But if this does happen, it would have to be at the expense of the abandonment of the essential aspects of the theory of relativity, since both theories under a single umbrella would contain logically dichotomous features, and thus would be logically inconsistent as a general theory. Equally, if the theory of relativity remains, it would have to be at the expense of abandoning the basis of the quantum theory, as a fundamental theory of matter. And of course one cannot reject outright the third alternative — that some day it may be found that both the quantum theory and the theory of relativity would have to be abandoned for some other fundamental theory of matter. But, as indicated above, it is the second alternative that will be explored in this monograph, fully exploiting the basis of the theory of relativity as a fundamental theory of matter, with the formalism of quantum mechanics serving as a low-energy approximation for the generally covariant field equations of inertia. Thus, all of the empirically correct predictions of nonrelativistic quantum mechanics will follow as predictions of this new theory. But extra predictions are made here, even in the nonrelativistic limit of the theory, that are out of the predictive domain of ordinary nonrelativistic quantum mechanics. Thus, this is a new theory of elementary matter that *supersedes* quantum mechanics, in accordance with the criteria of the philosophy of science.

The next question that arises is: What is the explicit physical meaning of the relativistic equations whose formal expression approaches that of nonrelativistic quantum mechanics in the low-energy limit? The answer is that the equations in their generally covariant form, called here the 'matter field equations', are the explicit laws of the inertial manifestations of elementary matter. For it will be argued in Chapter 3 (as also discussed in *GRM*, see reference [1]) that fully exploiting the principle of covariance implies that there must be, most primitively, a nonsingular field formalism that unifies all

possible force manifestations of matter with its inertial manifestations. The latter appears explicitly as generally covariant laws of inertia whereby the 'mass' of interacting matter is a particular sort of field whose features are in accord with the requirement of the Mach principle. That is, it relates to a dynamical coupling between the 'observed matter' and all of the other matter of a closed system that interacts with it.

The hint about the general structure of the matter field equations comes from the eigenfunction form of the equations of quantum mechanics. That is, the generally covariant matter field equations are a global extension of the equations of quantum mechanics, though in their general form they are not linear equations in a Hilbert space. Thus, they do not generally allow a probability interpretation, though in the appropriate limit, they do have the form of a probability calculus.

This is analogous to many such successions of ideas in the history of physics. For example, Einstein's tensor field formalism, which superseded Newton's equations for universal gravitation, are totally different than Newton's equations in their general form, and entail entirely different concepts, viz. Einstein's theory entails the idea of a finite propagation time of forces between interacting matter, replacing Newton's *action-at-a-distance*, and Einstein's theory is based on the field concept while Newton's theory is based on the concept of atomism. Einstein's field equations that relate to gravitation still *approach* the form of Newton's equations, asymptotically — thus predicting all of the correct results that Newton's theory predicts, in the appropriate limit. In the same way, it will be demonstrated that a generally covariant theory of inertia supersedes quantum mechanics, though asymptotically approaching the formal expression of the quantum theory in the appropriate limit.

In Chapter 5 it will be seen that the expression of the electromagnetic interaction also has a more general form than the conventional one, when based fully on the Mach principle and the symmetry requirements of relativity theory. In Chapter 6, the limit will be taken of the matter field equations that incorporates the generalized electromagnetic interaction, showing the emergence of quantum mechanics in the case of special relativity. This is an important limit, i.e. going from the generally covariant form of the matter field equations to its special relativistic form, since it highlights the feature of the matter field theory of inertia that yields the results conventionally associated with the form of Dirac's quantum mechanics in special relativity. It also introduces the notion, new in this theory, that I have called the 'interaction field amplitude'. This plays the role of a complex weighting amplitude for the interaction density throughout a closed system. It does not relate to the probability amplitude of quantum mechanics. Still, the interaction amplitude does reduce to a form that one may identify with a probability amplitude, in the linear limit of this theory, where its nonlinear formal expression approaches that of quantum mechanics.

The interaction amplitude relates to the elementarity of 'interaction' within a closed material system, just as the probability amplitude relates to the elementarity of the 'particle' in the open system, presupposed in the quantum theory.

In the special relativistic form of the (still nonlinear) matter field formalism, and with the use of the interaction field formalism, a proof will be demonstrated in Chapter 6 whose physical implications are identical with those of the *Pauli exclusion principle* of ordinary quantum mechanics. Yet the proof is based on features of this theory that are logically excluded from the basis of quantum mechanics! That is, it is claimed here that the Pauli exclusion principle, which must be imposed conventionally on the formal expression of quantum mechanics of a many particle system for empirical reasons (predicting, for example, the periodic chart) and has never been proven rigorously for a many-body system in quantum mechanics [7], with interactions on, is now proven from a set of axioms that are in logical opposition to the axiomatic basis of quantum mechanics!

With this result, as the nonrelativistic limit is approached, the interaction field amplitude for the (assumed) closed material system that satisfies the Pauli exclusion principle, approaches the Slater determinant (fully anti-symmetrized) form of the 'many-body' wave function in quantum mechanics. Thus, in the linear, nonrelativistic *approximation*, where it looks as though a closed material system of matter may be viewed as an ensemble of independent particles that interact with each other at a distance, we arrive at a description, asymptotically, that can be represented with Fermi—Dirac statistics. Still, it must be kept in mind that the limit of linearity may not be reached in reality, in principle, because of the elementarity of interaction in this theory — it cannot be turned off! And the nonrelativistic limit does not exist in principle, because the speed of propagation of the interaction between matter components is a finite number $c \neq \infty$, i.e. $v/c \neq 0$, no matter how small the relative speed between interacting matter components, v, becomes. Still, these limits may be used as an accurate approximation at low energy. We must then view Fermi—Dirac statistics of a system of particles as a useful mathematical approximation, but not as a fundamental statement about the system.

Fermi—Dirac statistics is not an elementary feature of a material system in this theory because, in principle, the system is not made out of *separable* particles (or fields). In its most general expression in general relativity, it is rather a continuum of interrelated, nonseparable modes of a unified field. It is only in a particular mathematical approximation where the system *appears* to be a sum of parts that are elementary particles of the particular type that have a spin one-half angular momentum — electrons, protons, muons, etc., and described with a generally covariant spinor field amplitude.

The next question that arises is: How does this generally covariant continuum field theory explain the empirical features of quantum mechanical

ensembles that conventionally evoke Bose—Einstein statistics? The answer to this question is in terms of ensembles of 'particle fields' that may each be broken down to composites of spinor (Fermi—Dirac) 'particles'. That is to say, the experimental facts that seem to imply the properties of an ensemble of Bose—Einstein particles (bosons), which are particles of integral spin quantum numbers, are in this view made up of fields described by spinor variables — i.e. characterized by the spin-$\frac{1}{2}$ quantum number. Recall that the direct product (Cartesian product) of two spin-$\frac{1}{2}$ fields is the sum of a spin-one field and a spin-zero field. The spin-one field is 'vector', represented for example by the 'photon' in quantum theory, which is a quantum of the electromagnetic (Maxwell) field, and the spin-zero is 'scalar', represented for example by the 'pion' in nuclear theory. The implication here is that the 'photon' and the 'pion' and all other bosons in nature are, in fact, composites of more elementary entities — spin-$\frac{1}{2}$ 'particle' fields, i.e. the photon and the pion and all other bosons are not entitled to the label 'elementary particle'!

If this is true, then how would this theory, without 'photons', predict the blackbody radiation curve — which is supposed to represent a cavity full of 'photons' in thermodynamic equilibrium with the walls, at temperature T? It will be shown in the text (Chapter 7) that the Planck distribution for blackbody radiation indeed follows from the properties of a 'gas' of electron—positron pairs (or pairs of any other particle—antiparticle, such as proton—antiproton), each in a particular state, that will be derived for the coupled, nonlinear matter equations for the pair, *without approximation.* In the context of this field theory, however, recall that these pairs are not truly pairs of separable particles; they are rather *distinguishable modes* of a field continuum.

In regard to the Planck blackbody radiation curve, it is instructive to recall that Planck himself did not use the ideas of quantum statistics to derive it, i.e. Planck did not assume the 'quantum view' that the 'particles' of radiation are indistinguishable, as one does in the derivation of Bose—Einstein statistics [8]. All that he did assume was that the energy associated with each of the vibrational modes of the radiation in the cavity must be linearly proportional to its frequency, that is, the 'quantum rule' that $E_\nu = h\nu$. But he 'tagged' each of these vibrations as distinguishable, as one does in the derivation of the classical Boltzmann statistics. In this way, Planck derived the blackbody radiation curve, which was precisely the same curve that was derived later by Einstein and Bose (independently) who used the quantum rule of indistinguishability, in their use of 'quantum statistics'.

It will be demonstrated in this text that the field theory of inertia derived gives the entire hydrogen spectrum, *including the Lamb splitting,* in numerical agreement with the data and with the preceding theories — agreeing with the Dirac theory of hydrogen in quantum mechanics for the entire spectrum except for the Lamb effect, and in numerical agreement with the prediction of quantum electrodynamics for the latter effect. But this

theory does not suffer from the theoretical deficiencies of the quantum theory, nor does it entail the need for renormalization and nonconvergent expansions to represent the solutions of the matter field equations — which in principle are necessary to predict *anything* about the atomic domain! It will also be demonstrated that (as a first step) the calculations following from this field theory give encouraging results for charged particle scattering and for the prediction of the anomalous magnetic moment of the electron.

Needless to say, there is no claim made here that the theory of inertia in general relativity that is presented answers all questions about microphysics. It is only that, starting with a quite different stand than that of quantum mechanics, it yields a theory with a different general expression that still reproduces all of the empirically correct results of quantum mechanics and some of the results of quantum electrodynamics, as well as making new predictions — enough so as to be encouraging to pursue this approach further. It is an avenue that is not really new, for it is based on fully exploiting the view of elementary matter that was proposed by Einstein, in his later period after he had come to the theory of general relativity. It is a view that Einstein argued for in his historic debates with Bohr [9], and which has been essentially untouched since then, while most physicists have fully accepted the Copenhagen view.

This monograph then attempts to fill this gap in the literature of theoretical physics, hoping to encourage further research on the subject of elementary matter along the lines of a continuum, deterministic field theory, as originally advocated by Albert Einstein.

Before going on to develop the generally covariant theory of inertia, that incorporates the formal expression of quantum mechanics as a mathematical approximation, it will be instructive to first justify the approach further (in the next chapter) by demonstrating a fundamental incompatibility between the quantum and relativity theories, in terms of a comparison of their respective axiomatic bases and ensuing mathematical expressions.

Chapter 2

On the Comparison of the Quantum and Relativity Theories

2.1. Competing Concepts

In this chapter we will present a critical comparison of the quantum and relativity theories, as elementary theories of matter. The headings of this comparison is shown in Table I, and will be discussed in detail in the following paragraphs.

A. In my view, the fundamental starting point of the philosophical basis of the quantum theory is Bohr's *principle of complementarity* [10]. This principle grew out of the idea of wave—particle dualism in physics. It was Einstein who first suggested the concept of wave—particle dualism, applied to the seemingly dualistic nature of electromagnetic radiation, whose quanta were called 'photons'. After de Broglie successfully extended the dualistic idea to material particles (such as 'electrons'), his speculation was confirmed in electron diffraction experiments.

Bohr then asserted the generality, in principle, of incorporating *opposing bases* that were to be separately true at separate times. That is to say, he proposed the idea that seemingly logically exclusive propositions can *both* be true, as complementary aspects of radiation or matter — in its most elementary description. Such a philosophy of physics is then pluralistic, whereby one assumes that at the outset there are simultaneous levels of explanation for the behavior of radiation and matter, even though when considered together these concepts would be logically dichotomous. This is an assertion of Bohr's principle of complementarity.

In opposition to this pluralistic view, the theory of relativity starts with a monistic approach. It is based on the idea that the laws of nature must be purely objective — *the principle of relativity* — thus claiming that their expression is independent of the reference frame that any observer may use to represent these relations (relative to his own reference frame). With this fundamental approach, 'observing' the effects of radiation, for example, as the effects of 'particles' or as the effects of 'waves', under correspondingly different sorts of experimental conditions, may be represented in terms of

TABLE I

Some opposing concepts of the quantum and relativity theories

	Quantum Theory	Relativity Theory
A.	Principle of Complementarity, pluralism	Principle of Relativity, monism
B.	Atomism → open system of separable 'things'. Elementarity of 'particle'	Continuity → closed system of distinguishable manifestations; no parts. Elementarity of interaction
C.	Logical Positivism	Abstract Realism
D.	Subjective → essential assertable features of matter depend on measurement by macroapparatus on micromatter, *not* vice versa.	Objective → features of matter independent of measurement. No essential difference between macro- and micromatter.
	Essential role of probability at level of explanation. Noncausal relation between observer and observed — asymmetric.	Probability descriptive, not explanatory. All relations causal. Symmetry between observer and observed.
	Macrovariables from rules of classical physics, microvariables from rules of quantum physics.	All variable solutions of basic covariant laws obey same rules.
E.	Nondeterministic → properties of elementary matter not predetermined.	Deterministic → all physical field variables predetermined.
F.	Linear, eigenfunction-type differential equations. Linear superposition principle Special reference frame for measuring apparatus.	Nonlinear, nonhomogeneous integro-differential equations. No linear superposition principle. No special reference frame.
	Separate space-time for each particle component of a system	One four-dimensional space-time for all field components (for their map).

comparisons of these phenomena from different possible reference frames of a single substantive system. For example, a physical criterion for viewing an 'elementary particle' as a discrete entity rather than a wave could change when transforming the mathematical description from one frame of reference to another, when frame-dependent parameters, such as wavelength, could change in a way that would alter the criterion that was originally used in the first frame, when expressed in other frames. That is to say, it may *appear* that the electron is a wave phenomenon when described in one Lorentz frame (say, in special relativity), as it may *appear* as a discrete particle phenomenon

in another, but these *appearances* are in fact based on a *single*, logically consistent law, independent of the labeling of the interacting components.

Implied in the relativistic approach to a fundamental theory of matter and radiation is that there is a single underlying order, as expressed in terms of the fully objective laws of nature. That is, it is assumed that *underlying* the behavior of elementary matter — in any domain, from elementary particle physics to cosmology — there must exist a single, logically ordered universe. There can be no conceptual lines of demarcation between one set of axioms, to underlie one sort of physical phenomenon, and other sets of axioms that underlie other phenomena. For example, in theories of matter, the conceptual notion of 'wave', which entails continuity and rules of combination that include 'interference' effects, is logically exclusive from the conceptual notion of 'particle', which in contrast entails discreteness (locality) and the ordinary arithmetic rule of combination. If there is to be a single conceptual basis that is self-consistent for the laws of matter, it cannot then include the logically dichotomous concepts of 'particle' *and* 'wave' in fundamental terms. It then follows that Bohr's concept of complementarity must be automatically rejected by the approach of self-consistency and wholeness implied by relativity theory, and vice versa. Rather it must be assumed in a theory of matter that is based fully on the principle of relativity that there is a single explanatory level for the workings of the universe, in any of its domains (from fermis and smaller to light years and greater). Such a philosophical view is then *monistic*.

B. With the quantum mechanical view of elementary matter, the world is fundamentally atomistic. As in Newton's approach in classical physics, it is assumed at the outset that the universe is a sum of parts. These are entities that, by definition, may be separated from the whole without in any way altering its fundamental characteristics. On the other hand, when exploiting the underlying symmetry requirement of the theory of relativity, it must be assumed at the outset that the universe, in any of its domains of description, is basically a continuum, representing a closed system — that is, a system that is truly without separable parts. This conclusion follows from the principle of relativity, which requires the laws of nature to be totally objective (i.e. covariant) with respect to continuous and continuously differentiable transformations, from the space-time language of one reference frame to the space-time languages of all other possible reference frames in which one may wish to compare the forms of the laws for any sort of phenomena.

Thus we see that the elements of matter that are supposed to be fundamental, according to the quantum mechanical view, are distinguishable 'parts' called 'elementary particles', while the fundamental concepts of a continuum representation of matter, according to the implications of the theory of relativity, are its (infinite variety of) distinguishable manifestations (modes) of a single system that is in fact without separable parts. The latter

may be characterized most basically in terms of the *elementarity of interaction* [11]. One may compare the latter view, metaphorically, with the distinguishable manifestations of a pond, called 'ripples'. These are indeed modes *of the entire pond*; they are not parts *in it.* The ripples *of* the pond may transfer energy and momentum between each other, scatter each other in different directions, etc., as though they were separable things, at first glance. Yet they are not truly separable from the pond as individual entities. They are not localizable, except to specify where they may 'peak', at one time or another.

In this regard, the full exploitation of the axiomatic basis of the theory of relativity in the problem of elementary matter implies that in fact there are no free, localizable, separable particles of matter, neither galaxies, planets and stars, people, rocks and trees, nor electrons, protons, mesons, etc., are in fact separable, individuated entities.

C. The epistemological approach of the quantum theory is essentially that of *logical positivism.* This is a philosophical approach to knowledge that is based on the assertion of the *principle of verifiability* [12]. The latter principle says that the only meaningful statements in science, whether expressed mathematically or in ordinary language, must be empirically verifiable.

We see that the notion of 'wave—particle dualism', and generally the principle of complementarity, are ideas that are consistent with the notion of logical positivism [13]. This is because in the latter view, one may assert the *truth* of logically opposing propositions, as long as at the different times when different sorts of experiments are done, they *empirically reveal* consistency with these separate opposing ideas. With this approach, one need not say that the 'particle' picture is the true one, and it *underlies* its wave aspects, or vice versa. Rather, at the different times when one should observe, say, an 'electron' as a particle, it is a particle, and at other times when one would observe it as a wave, it is a wave. According to this epistemological view, this is all that can be said *meaningfully* about the 'electron'.

On the other hand, the theory of relativity is based on the epistemological view of 'abstract realism'. This is an approach whereby it is assumed at the outset that there is an *underlying* real world, understood in theoretical science in terms of fundamental principles — whether or not some human observer or his instruments may be there to observe the implications of these principles. What we do see, then, is a sort of 'projection' of this underlying reality. I have called it 'abstract' realism because one does not directly observe the full set of principles that are to 'explain' the data. Rather, we arrive at scientific truths, according to this philosophy, in a hypothetico-deductive fashion, using our powers of reason as well as observation, to reach particulars (the theoretical predictions of experimental effects), from a

universal — the underlying law that is probed. Thus, the way that one arrives at the *alleged* universal at the outset — i.e. the hypothesized law of nature — is from hints that are received from the observable facts of nature, *as well as from our imaginations.* But one must still rely on the actual experimental facts to confirm these particulars, thence to verify the law of nature that is being investigated. One must always be ready to abandon an alleged law if its particulars do not stand the test of all possible experimental confirmations of its predictions.

D. The quantum theory entails an irreducible subjective element in its conceptual basis. In contrast, the theory of relativity when fully exploited, is based on a totally objective view. This very important difference between the approaches of the quantum and relativity theories has to do with the *fundamental* incorporation of the macroscopic measuring apparatus with the microscopic matter that it 'observes' by means of their mutual interaction. The postulate of the Copenhagen school is that there does not exist any underlying dynamical cause—effect relation for this interaction. It then follows that in this view there can be no certain outcome of a measurement of any property of micromatter — *in principle.* It is further contended that the outcome of the (asserted noncausal) couplings of the macro- and micromatter are the only meaningful statements that may be made about elementary matter.

It followed from this Copenhagen approach that at the basis of the physical laws about elementary matter there must be an *irreducible* probability calculus — i.e. a set of probability rules that are the limiting form of the law of nature pertaining to elementary matter. It is further asserted here that the rules for deriving the variables of the macroapparatus must be those of classical physics, while the rules for deriving the basic variables of the micromatter must be those of quantum mechanics. The latter are the rules that obey the properties of the complex probability amplitudes of a Hilbert space [14]. Thus we see that with the view of quantum mechanics, one must start at the outset with the coupled system [macroapparatus | micromatter], distinguishing the labels on the right and left in terms of precisely where the line of demarcation is placed. There is no strict rule about this (except for a statement about relative orders of magnitude of mechanical action, etc.) — the line could be moved arbitrarily to the right or left. But once the line is defined to be somewhere to describe some experimental arrangement, one thereby defines what is meant by the term 'apparatus' in this experiment. The remainder of the system, the micromatter, is then represented in terms of basic properties that are in accordance with *this arrangement,* and the ensuing statistical description involved in the calculus of quantum mechanics.

However, by moving the line of demarcation *arbitrarily,* the nature of the 'observer' changes, and the predicted properties of the 'observed matter' correspondingly changes. Since the basic properties of matter are, in this

view, taken to depend in part on the nature of the 'observer' — that is, the choice of precisely where one wishes to locate the line of demarcation between 'observer' and 'observed' — it must be concluded that as a fundamental theory of matter, quantum mechanics entails an irreducible element of subjectivity. This conclusion is consistent with the epistemological approach of logical positivism, as discussed above.

In contrast, the *principle of relativity* of Einstein's theory requires at the outset that the laws of elementary matter must be independent of the reference frame from which they are described — be this the frame of the 'observer' (*subject*) or that of the 'observed' (*object*). Since relativity theory requires a symmetry in the laws of nature with respect to the variables of the subject and those of the object of any subject—object interaction relation, it follows that (1) the laws of matter must be in terms of an entirely objective description, and (2) the variables that relate to the 'subject' and those that relate to the 'object' of an interaction relation *must both be covariant.* This is, the variables of the subject and object must *both* obey the transformation properties that maintain the covariance of the basic field equations.

We see, then, that the basic epistemological approach of the theory of relativity, as a fundamental theory of matter, is one of *realism* — asserting the existence of a real world that is independent of whether or not one may choose to make measurements, of one sort or another. Because some of the aspects of this reality are not directly observable, I have referred to this type of realism as 'abstract'.

It also follows that, in contrast with the probability calculus of quantum mechanics (embedded as a fundamental feature of matter) the probability function does not play any fundamental role in a theory of matter that would conform with the axiomatic basis of the theory of relativity. With the latter approach, the probability calculus could be useful as a 'tool' which an inquirer may utilize whenever he cannot determine the complete set of variables that underlie physical interactions. But when this is done, he is aware that there does exist a more complete description of matter that underlies his investigations. This is in the mode of thinking of Boltzmann's view, when he used statistics to describe a gas of molecules. Underlying the probabilistic description of Boltzmann's theory, he saw the existence of a complete description of these molecules, which was Newtonian dynamics. However, in contrast, the quantum theory asserts that the probability calculus it uses is at the limit of all possible knowledge about the material world of micromatter; that is, this is asserted to be as complete a description of the matter as there possibly can be. This is an approach that takes the laws of nature to be laws of chance!

E. The Copenhagen interpretation of quantum mechanics is *nondeterministic.* What this means is that the trajectories of the elementary particles of matter that comprise a system are *not predetermined*, independent of

measurement, as they would be in the classical, deterministic theories. With the quantum view, it is postulated that when one makes an observation of some physical property of micromatter (*necessarily* with a macroapparatus), then a group of possible states of the micromatter are 'projected out', some weighted more than others, generally. The assertion is that the more accurately that one attempts to ascertain a particular physical property of this matter, with particular types of measuring devices, the less accurately can some other ('canonically conjugate') physical property of the matter be simultaneously specified. This is a statement of the *Heisenberg uncertainty principle*, a crucial element in the philosophy of quantum theory.

According to this theory of elementary matter, the actual states that the macroapparatus respond to must be in the form of complex probability amplitudes in a Hilbert function space. Thus, the linear superposition of more than one such state function yields an interference pattern in the measured properties of the observed matter, no matter how rarefied this matter (or radiation) may be! — even if it seems that one is observing one particle at a time. According to this theory, it is impossible to reduce the interference effects altogether, because the measuring device in its interactions with any quantity of elementary matter in the measurement process, automatically generates a superposition of different states, thus yielding an interference pattern [15]. Such interference of probability waves was the interpretation given by the Copenhagen school for the wave aspect of particle behavior, such as the diffraction of 'electrons' when they scatter from a crystal lattice.

In contrast, the theory of relativity, as a basic theory of elementary matter, implies that the laws of nature are totally *deterministic*. In relativity this is not in terms of the predetermined trajectories of singular particles, as it is in the Newtonian view. It is rather in terms of predetermined field variables, mapped in a single four-dimensional space-time. This view is indeed in contrast with the elementary particle model of classical physics, where one must postulate a three-dimensional space for each of the constituent particles of a system, or the particle description of special relativity, where one must use a separate four-dimensional space-time for each of the constituent particles. Fully exploiting the axiomatic basis of relativity theory, whether in its special or general form, one is led to the elementarity of the field concept. The field theory in turn leads to a mathematical representation of an 'n-body system' in terms of n-coupled fields, each mapped in a single, common four-dimensional space-time.

Thus we see that 'determinism' in the sense of a relativistic field theory of matter does not single out the trajectories of spatial points, deterministically parametrized with the time measure for each material constituent of a system. In this theory, 'determinism' refers to all predetermined aspects of a closed system, coupled in a totally objective way that is independent of measurements that may or may not be carried out. This is not only in terms of a

'time' parametrization, but rather in terms of a complete set of logical/mathematical relations (geometric, algebraic and topological), between interdependent field components that underlie its basic description in the laws of matter.

F. Because of the conceptual basis of quantum mechanics, interpreted in terms of a particular sort of probability theory, this law of matter implies that the basic features of elementary particles must obey a property of the Hilbert space that is the *principle of linear superposition.* This principle says that any solution of the basic field equations for matter may generally be expressed as a linear sum of any number of other solutions of the same equations. This is the special property of this particular probability calculus having to do with the measurements of properties of elementary matter.

It is further asserted in this theory that the equations satisfied by the probability amplitudes must be *homogeneous* in these solutions, so that the matter field equations have the following form in terms of the probability amplitude state functions ψ_n:

$$\hat{O}\psi_n = o_n\psi_n.$$

The meaning of the terms in this 'eigenfunction' equation are as follows: The linear operator $\hat{O}$ represents the 'act' of making a particular sort of measurement. The linearity of this operator (i.e. its independence of ψ_n) follows from the assumption that the observed micromatter has no *dynamical* effect on the matter of the observing system. The measured value found in this observation is denoted by the number o_n, when the micromatter is in the state ψ_n, out of all possible other states $\{\psi_{m\neq n}\}$ of the Hilbert space.

As it was indicated above, it follows from the Copenhagen philosophy that one can never 'see' the micromatter in a pure state, such as ψ_n, because of the interference that is necessarily introduced by the measuring apparatus — though one may design an experiment that could come arbitrarily close to observing micromatter in a pure state. That is to say, one may in principle design an experimental set-up that could *approach* infinitely great resolution in the measurement of a particular property of micromatter, *but in principle one may never reach the actual limit of the pure state*! What this means, following the Copenhagen view to its logical extreme, is that it would be meaningless to even talk about the system in a pure state since the latter is an ideal case that is *independent of any actual measurement*!

It is interesting to recall Schrödinger's starting point for his formulation of wave mechanics — one of the two alternative expressions of quantum mechanics (the other being Heisenberg's matrix mechanics) [16]: Schrödinger started with the *nonlinear* Hamilton–Jacobi equation of classical mechanics. The solutions of the latter equation are the values of the mechanical 'action' for a considered system. Schrödinger's 'quantization' was then accomplished

by converting each term in the (nonlinear) Hamilton—Jacobi equation into an operator, acting on the imposed state functions of a linear Hilbert space of functions. These operators were then interpreted (later by Dirac) as the act of making a measurement. But the differential equation so constructed was then linear, and allowed one to use the principle of linear superposition in the description of the measurement.

To incorporate the coupling of matter with radiation that is transferred in an interaction process, one then proceeds to 'quantum field theory' [17]. This is accomplished by converting the Schrödinger state functions $\{\psi_n\}$ into operators, allowing them to 'act on' a second space of state functions (of a Hilbert space). This is called 'second quantization'. In this way, the 'nonlinear' interaction that depends on powers of ψ_n greater than one (arising, for example, from the radiation-matter coupling) become nonlinear *operators*, though operating in a linear Hilbert space, as in Schrödinger's original quantization scheme, applied to the Hamilton—Jacobi equation. The new state function operators then represent the act of 'creating' and 'annihilating' different numbers (and sorts) of elementary particles. In this way, the linearity of the Hilbert space is restored, allowing once again the use of a probability calculus.

In contrast with the fundamental linearity of the quantum theory, the elementarity of interaction in relativity theory implies a *fundamental nonlinearity* in the laws of elementary matter. Physically this is because the basic laws now represent a closed system that is not a sum of parts, as it would be in the conventional theory. With this type of formal structure of the field equations for matter, there is no longer the possibility of expressing the solutions of the basic matter field equations as a sum of other solutions of the same equations. In terms of the material components of the closed system that is now considered, should one of them be excluded at the outset, say for the purpose of mathematical approximation, then the general solutions of the 'reduced' system could not be obtained by a simple subtraction procedure from the original system. In principle, one would have to start over again to consider the new system on its own, in order to predict its physical properties.

Summing up, I have attempted to demonstrate here that the full conceptual structures of the quantum theory and the theory of relativity, each as a fundamental theory of elementary matter, is not compatible with the other. Thus, to accept the axiomatic basis of one of these theories of matter it would be necessary to reject the axiomatic basis of the other, if we are to formulate a logically consistent theory of elementary matter. On the other hand, there are domains of physics where the conditions that evoke each of these theories overlap, such as contemporary high-energy physics. This then leaves us with a major unresolved problem of contemporary physics; indeed, it is the main dilemma of physics in the current period. It must be resolved

before we can make any bona fide progress in our fundamental understanding of elementary matter.

In the next section I will discuss one of the important explicit reasons for the *necessity* of unifying the quantum theory and the theory of relativity, in leading toward a quantum field theory, and we will see why, thus far, no such theory has been formulated in a demonstrably consistent fashion.

2.2. Is the Quantum Jump Compatible with the Theory of Relativity?

One of the outstanding reasons for the *logical necessity* of expressing the formal description of quantum mechanics in a way that would be consistent with the symmetry requirements of the theory of relativity has to do with process of the 'quantum jump', which entails a relativistic quantum of energy, propagating from one quantity of quantized matter (called 'emitter') to another (called 'absorber'). Because the transmitted radiation is relativistic, the theory of relativity would require that the quantum mechanical equations must have a relativistic expression, i.e. an expression that would be in one-to-one correspondence in all inertial frames of reference (at least) [18]. If the rules of quantization are indeed scientifically valid, they must apply to all of the microscopic components of the system described — the emitting and absorbing matter *and* the (quantized) radiation that is created when a 'quantum jump' occurs between the energy levels of the emitting matter, and the annihilation of this quantized radiation when the absorbing matter later undergoes a 'quantum jump' upwards, when the radiation is absorbed.

The difficulty is the following. While the micromatter components of the coupled matter system (emitter and absorber) have a limiting nonrelativistic expression (derived from Schrödinger's wave mechanics or Heisenberg's matrix mechanics), the signal that is transferred between the emitter and absorber has no nonrelativistic limit, if it is electromagnetic (a 'photon'), since the latter propagates only at the speed of light. The latter may be explained in particle language by saying that the photon has no rest mass, thus it cannot be slowed down or speeded up by an external force, as one could do to a massive body, such as an electron. All that can be done to a photon is to 'annihilate' it (by absorbing it in matter), when the absorber simultaneously undergoes a 'quantum jump' of increased energy. [The same argument holds for other types of forces, such as the nuclear and weak forces in the nuclear domain, since the latter entail signals that are also relativistic because of the relative speeds that are involved in the transfer of the force between the emitter and absorber in these cases, even though when free, such 'signals', called 'mesons' may be brought to rest, i.e. they have nonzero rest masses.]

Since the signal component of the triad, *emitter—signal—absorber*, must have a relativistic description, the entire triad must be represented at the outset in a covariant manner, with all three components of the triad obeying

the rules of the quantum *and* relativity theories, simultaneously. Once such a formulation (called 'quantum field theory') has been formulated, one may then take the nonrelativistic limit of the matter components of the triad (as $v/c \rightarrow 0$), hoping then to recover the formalism of nonrelativistic quantum mechanics. The salient point here is that it is *logically necessary* to start out with a relativistic formulation at the outset that incorporates the 'quantum jump', in the form of a quantum field theory — a theory that incorporates the matter and the transferred radiation — in a logically and mathematically consistent fashion.

The well-known trouble that occurs when one attempts to extend ordinary nonrelativistic quantum mechanics to quantum field theory in relativity [19] is that the resulting formalism has no solutions! On the other hand, it is the set of solutions the quantum mechanical equations in relativity that are supposed to relate to the observable properties of micro-matter. The reason that there are no solutions is that the formalism of quantum field theory ('quantum electrodynamics, when applied to the electromagnetic force, where the signal is a 'photon') automatically entails infinities. This result then implies that all physical properties of micromatter have infinite magnitude! But in reality they are finite, such as the mass and charge of the electron, or its magnetic moment. Thus it must be admitted that at this stage of the quantum theory, in the late 1920s, the theory failed! For even nonrelativistic quantum mechanics can only claim to be a low-energy approximation for a relativistic quantum field theory, if the quantum jump is to be incorporated in the theory. But if a relativistic quantum field theory has not even been shown to exist, one may not claim that the basis of quantum mechanics has yet been verified since the 1920s! All that one has a right to claim in this regard is that, because of its empirical success, nonrelativistic quantum mechanics is a nonrelativistic approximation for *some* relativistic theory. But there is no guarantee that the latter theory is based on any of the concepts of the quantum theory (the left-hand column of Table I).

About twenty years after the discovery of the 'failure' of quantum mechanics, 'renormalization methods' were invented whereby numerical results were achieved by a particular subtraction procedure — infinite quantities were subtracted from the intrinsic infinities in quantum electrodynamics, so as to yield finite numbers to be compared with the data. In the case of quantum electrodynamics, this particular subtraction procedure yielded two remarkably good predictions that were not even predicted qualitatively from the quantum mechanics of Schrödinger or the relativistic wave mechanics of Dirac. These were (1) the Lamb shift in the fine structure in the spectra of hydrogenic atoms, and (2) the anomalous part of the magnetic moment of the electron. The former relates to extra energy levels in hydrogenic spectra, not predicted by Dirac's wave mechanics; the latter relates to a very small, but measurable extra contribution to the magnetic moment of the electron, to be added to the prediction that follows from

Dirac's wave mechanics. [It will be demonstrated in this monograph that the Lamb effect is predicted by the present theory, without the need to subtract infinities, i.e. it follows here from a finite theory of matter (Chapter 8).] In spite of the numerical successes of ordinary quantum electrodynamics, utilizing the method of renormalization, it is still in an unsatisfactory state since these results follow from a formalism that has never been shown to have bona fide solutions. The latter, of course, is because such a subtraction technique for removing infinities is not demonstrably mathematically consistent. The present theory of inertia does not suffer from this difficulty.

The latter point is pertinent in regard to the authenticity of quantum field theory, claiming to be a bona fide scientific theory, since it violates the presupposition in the philosophy of science, *per se*, that agreement between the predictions of a theory and the data is only a *necessary condition* for the acceptability of the theory, but it is *not sufficient*. For a theory in science to be a valid explanation of phenomena, it must also be logically consistent and it must predict unique answers for the questions about unique physical situations. In its present state, quantum electrodynamics (and generally, quantum field theory) has not satisfied these criteria. For example, because of the mathematical inconsistency in the latter scheme of computation, in terms of nonconvergent 'perturbation expansions', these expansions may be regrouped (in principle) without changing the physical conditions of the description, thereby yielding numbers that are different than the previously obtained numbers that were already in agreement with the data, for the same physical situations!

The present-day widespread feeling among physicists that quantum mechanics has been overwhelmingly successful is based on the large amount of empirical success for low-energy predictions, as well as the remarkable numerical success of the renormalization procedure in predicting such results as the Lamb shift and the anomalous magnetic moment of the electron. However, it must still be admitted that there is yet no theory! One must then not rule out the possibility that the general theory that we seek, to 'explain' the behavior of micromatter, under all conditions, may not be based on the concepts of the quantum theory at all (the left-hand column of Table I). The possibility must be considered that the general theory of matter that is sought could be based on the axioms of the theory of relativity, as a fundamental theory of matter (the right-hand column of Table I). In the latter case, it is the imposition of *the principle of correspondence* that requires that the nonlinear formal structure of the general field equations, representing in this theory the inertial manifestations of matter, must incorporate the formal structure of quantum mechanics as a *linear approximation*. It will be argued below that the general form of the field equations that explain inertia must, in fact, be generally covariant spinor equations in a curved space-time, where the curvature itself relates to the explicit measure of inertial mass. The latter result *predicts* that in the Newtonian limit of the theory, the gravitational

force can only have one sign, that it is only attractive. The theory also predicts a mass spectrum of elementary matter, in the linear limit, and the existence of mass doublets (which have been identified, e.g. in previous calculations, with the electron—muon doublet).

Finally, a further difficulty that implies a fundamental incompatibility between the quantum and relativity theories is that the quantum theory necessarily entails an absolute reference frame — that of the measuring macroapparatus. The eigenfunction formalism of quantum mechanics implies the 'absolute simultaneity' between the act of measurement (cause) and the revealed measured value (effect). This feature is incompatible with the relative simultaneity and finite speed of interaction that follows from the theory of special relativity — implying that there must be a finite time (no matter how short!) between the *cause* in the measurement process (the act of measuring some property of matter) and the *effect* (the revelation of the measured eigenvalue or distribution of eigenvalues). That is, with the standard Hamiltonian formulation of quantum mechanics, if the cause and effect are expressed simultaneously in one Lorentz frame, relativity theory predicts that they would not be simultaneous in their description in any other Lorentz frame. But this would then destroy the Hamiltonian formulation in quantum mechanics when expressed in arbitrary inertial frames of reference. Thus, because of the role of measurement in quantum mechanics and its expression with a Hamiltonian formalism, this theory is manifestly non-covariant (as soon as the interaction of apparatus and measured micromatter is fully expressed, including the signal that is transferred between the apparatus and the 'observed' micromatter.)

To sum up, then, it appears that the concept of the 'quantum jump' and its relation to a photon theory of light propagation is (1) logically incomplete and (2) logically inconsistent. The answer to the question that heads this section is then: No, the 'quantum jump' is incompatible with the mathematical and conceptual basis of the theory of relativity — whether expressed in the form of special or general relativity.

2.3. Is the Theory of Relativity Complete as a Theory of Matter?

When examining the full conceptual basis of the theory of relativity, as a fundamental theory of matter (the right-hand column in Table I) it is seen that there is still something missing from its present-day explicit form. This has to do with the need for an explicit covariant representation for the inertial manifestation of elementary matter.

According to the principle of relativity, one compares the expressions of the laws of nature in different reference frames, demanding that they should all be in one-to-one correspondence. But in order to make such comparisons, abstract spatial and temporal measures must be evoked to be correlated with

the real readings of measuring instruments — of any sort, whether it is the sophisticated instrumentation of modern physics or the ordinary 'rods' and 'clocks' that were originally referred to in the early writings in this theory. The point I wish to make here is Einstein's point that, after all, these instruments are material configurations and not theoretically self-sufficient entities without further need of explanation [20].

To express a basic theory of matter that would be self-consistent in accordance with the axiomatic basis of relativity theory, as indicated earlier there must be a symmetry with respect to the interchange of the field variables associated with the 'observer' and those associated with the 'observed'. It is then necessary in principle to express the matter field variables that underlie the behavior of the measuring instruments in a fully covariant manner. That is to say, the basic variables associated with the measuring instruments must also be continuous, analytic functions that solve objective laws of nature — field equations that relate to the mutual interactions of a closed system. The latter must be a generalization of the 'rod' and 'clock' of the earlier discussions of the meaning of the theory of relativity, at the beginning of this century. Such a view, which is basically dictated by the principle of relativity, then adheres to Bohr's insistence that the 'observer' must be incorporated with the 'observed' in a fundamental representation of matter. But in contrast with Bohr's philosophy and the ensuing mathematical expression of it, the theory of relativity introdues the 'observer' and 'observed' as a *symmetrical* interaction, in accordance with the spirit of Newton's third law of motion.

We see, then, that the conceptual requirement of symmetry between the 'observer' and the 'observed' in a covariant theory logically requires a continuous field theory that fuses the inertial manifestations of matter (represented covariantly by 'matter field equations') with its force manifestations (gravity, electromagnetism, nuclear and weak interactions, as well as any other interactions that may yet appear, say in hadron physics, etc.) in a self-consistent set of generally covariant field equations that represent the full set of manifestations of a *closed system* that is, in principle, without separable parts. Thus, the unified field theory that Einstein sought for the greatest part of his professional career was not only motivated by esthetic reasons, reasons of simplicity or for some not-understood intuitive feelings that he had. It was based on the recognition that such a unified field theory is *logically necessary*, if the theory of relativity is to claim its validity as the basis of a general theory of matter.

Summing up, it has been argued in this chapter that neither the quantum theory nor the theory of relativity are in themselves complete as fundamental theories of matter. The former is not complete because its nonrelativistic approximation has never been satisfactorily generalized relativistically, so as to properly accommodate the 'quantum jump', which is one of its own essential features. That is to say, it has still not been demonstrated that there

can be a mathematically/logically consistent relativistic quantum field theory. On the other hand, the theory of relativity is not complete in Einstein's original formulation because it did not yet incorporate the field variables of the 'observer' in a fully covariant 'observer—observed' relation, including the inertial features of matter.

I have discussed the reasons for the failure of the quantum theory in achieving such a complete expression. The completion of the theory of relativity does not suffer from the same conceptual and mathematical difficulties. But it is clear that to proceed from this viewpoint it is necessary to totally abandon the conceptual basis of the quantum theory, while keeping its linear, eigenfunction form, in terms of a particular sort of probability calculus, *serving as a nonrelativistic approximation* for an exact formalism that is not based on any of the philosophical requirements of the quantum theory.

If, in the long run, it turns out that the theory of relativity will indeed replace the quantum theory, as a fundamental theory of matter, then one might view the historical evolution of theories of matter as a progression from the idea of 'particle monism' (of the periods preceding the twentieth century) to 'wave—particle dualism' (of the period of the first half of the twentieth century) to (nonlinear) 'wave monism'. This would imply that the dualistic concept of continuous wave and discrete particle was not a permanent concept in our understanding of matter; it rather served the (very important) intermediate role of superposing the earlier atomistic concept onto the more abstract, wholistic, continuous field concept, in preparing the way in the history of ideas for the emergence of the latter approach to matter.

Such a replacement, while new in physics, is certainly not new in other domains in the history of ideas. The wholistic, continuum approach to the universe, as a truly closed system, may indeed be traced to ancient times in both the Western and Oriental cultures, thence reappearing in many philosophical works up to the present time.

My own research program, which will be developed in this monograph, takes the wholistic, continuum approach to matter, based on fully exploiting the theory of general relativity. From the results of these studies I believe that one may indeed satisfy Einstein's criteria when sufficient generalization has been implemented. One of these extensions is a generalization of the *Mach principle* to all manifestations of interacting matter, not only its inertial manifestations. In this way, the inertial features of elementary matter have been seen to fuse with its force manifestations in the form of a unified field theory that is nonsingular and generally covariant, and based fully on the field concept.

It was found in this research program that the generally covariant, nonlinear matter field equations, which explicitly express the inertial manifestations of elementary matter, incorporates the formalism of nonrelativistic,

linear quantum mechanics, as a low-energy approximation. Thus all of the successful results of empirically confirmed predictions of quantum mechanics, as well as the numerical result of quantum electrodynamics that gives the Lamb shift, are contained in the predictions of this theory, though without the dependence on any of the concepts of the quantum theory. The details of this relativistic theory of inertia will be developed in the remaining chapters of this monograph, and applied to problems of elementary particle and atomic physics.

Thus far we have discussed both the logical and mathematical dichotomies that are encountered in a general theory of matter that tries to fuse the quantum and relativity theories. In this regard, it is interesting to take note of Dirac's comment [21], in which he referred to 'Class One difficulties' as the logical sort and 'Class Two difficulties' as the mathematical sort, as follows:

> I have disposed of the Class One difficulties by saying that they are really not so important, that if one can make progress with them one can count oneself lucky, and if one cannot, it is nothing to be genuinely disturbed about. The Class Two difficulties are the really serious ones. They arise primarily from the fact that when we apply our quantum theory to fields in the way we have to if we are to make it agree with special relativity . . . we have equations that at first look all right. But when one tries to solve them, one finds that they do not have any solutions.

Dirac's comments about the Class One difficulties could be interpreted to mean that the argumentation that challenges the logical consistency of the Copenhagen interpretation, as we have discussed earlier in this chapter, is unimportant as long as quantitative predictions can be made in a mathematically consistent way. Still, any argumentation that relates to the logical consistency of the approach and also proposes a bona fide experiment to check the validity of its contentions must be taken into account. Indeed, it is one of the main purposes of this monograph to show that a deterministic field theory whose mathematical structure is generally different than that of the quantum theory and is interpreted differently, can resolve the Class Two difficulties of Dirac's discussion in a way that necessarily at the same time removes the Class One difficulties. A prime example of the latter, that will now be discussed, is the *Einstein—Podolsky—Rosen paradox*.

2.4. The Einstein—Podolsky—Rosen Paradox

In the historic paper of Einstein, Podolsky and Rosen in 1935 [22], they analyzed a *gedanken experiment* in which one measures the dynamical variables of one part (A) of an uncoupled two-part system (AB) by making measurements on B, which was previously bound to A and has since been separated by a mechanism that does not affect the correlation of the wave functions of the partial systems. They thereby demonstrated that an experimental situation may be created where one can determine to arbitrary

accuracy the dynamical variables of a microscopic entity A (or B) by measuring the properties of B (or A), i.e. without having the measuring instruments *disturb* the microsystem A (or B) in any way.

The EPR thought experiment was equivalent to the following: Consider a two-body system such as the hydrogen molecule H_2. While both hydrogen atoms are bound in this molecule they are correlated so as to yield a total angular momentum for their shared electrons equal to zero. That is, the spins of the shared electrons must be antiparallel. Suppose that a *spin-independent* external force is now applied to the H_2 molecule, such as a sufficiently strong collision with another spin-zero particle, so as to dissociate the molecule and remove each of the H atoms from the other by some great distance. Though they would be very far apart, the spin correlation of each of the H atoms relative to the other must still remain so as to conserve the total (zero) angular momentum of the original H_2 molecule.

The claim of the Copenhagen interpretation of quantum mechanics that the EPR thought experiment challenged is the following: All of the canonical variables of a microscopic quantity of matter are not precisely prescribed simultaneously because a measurement carried out by a macroapparatus to determine one of these variables automatically interferes with the knowable values of the variables that are canonically conjugate to this one. In the thought experiment mentioned above, the canonically conjugate variables are the spins and their orientations, for each of the two electrons (that were originally binding the molecule). According to the Heisenberg relations *for each* atomic electron,

$$\Delta S_1 \Delta \phi_1 \geqslant \tfrac{1}{2}\hbar, \qquad \Delta S_2 \Delta \phi_2 \geqslant \tfrac{1}{2}\hbar.$$

As explained by the conventional Copenhagen view, these inequalities are taken to mean that if one should measure the spin S_1 (*or* the spin S_2) arbitrarily precisely, so that the uncertainty in its measurement, ΔS_1 (or ΔS_2) would be correspondingly small, then the measurement of this property of the atom would interfere with the precision with which ϕ could be specified, so that $\Delta \phi_1$ (or $\Delta \phi_2$) would be correspondingly large, in accordance with the magnitude of $\hbar/\Delta S$.

Nevertheless, in the situation posed by the EPR thought experiment, a correlation does persist between the spins of the two atoms, as well as between their respective orientations, even though they may be very far apart and thus considered to be noninteracting. It then follows from this correlation that if one should measure the angular momentum of the first atom with infinitely great accuracy (i.e. with $\Delta S_1 = 0$) then while the uncertainty $\Delta \phi_1$ would be infinitely great, the persisting spin correlation between the two atoms would imply a precise value of the spin angular momentum, S_2, of the second atom. The point is that the latter information would be obtained without in any way causing an interference between a measuring apparatus and the second atom — since it was *deduced* from the known correlation

rather than from a direct measurement! Further, in a second experiment one could determine the spin orientation of the first atom, ϕ_1 with infinitely accurate precision (i.e. $\Delta\phi_1 = 0$); the correlation of the two atoms would then imply a knowledge of ϕ_2 (spin orientation of the second atom) with the same precision — again without in any way interfering with the second atom due to a direct measurement on it. Thus, Einstein, Podolsky and Rosen argued that one may indeed determine the canonically conjugate variables of elementary matter, all with arbitrary precision, *simultaneously*, without in any way interfering with this matter by the intrusion of a macroapparatus.

This thought experiment then led Einstein, Podolsky and Rosen to conclude that, in contrast with the opinion of the Copenhagen school, there must exist a *complete* dynamical description of the second atom in this experiment; that is, the one that was not directly measured at all. Thus, one may conclude that all of the elements of matter have such complete descriptions, independent of any measurement that may or may not be made by a macro-observer (or any other sort of observer). On the other hand, the quantum mechanical relations that lead to a comparison with a measured property of elementary matter entails a 'weighting' of a particular linear operator with respect to the state functions of this matter. The latter, in turn, is the solution of equations that represent a (particular sort of) probability calculus, as we have discussed earlier. Since the fundamental theoretical expression for the measured physical property of a single atom of matter entails a probability function *in its most basic mathematical description* (according to the Copenhagen view), this description of elementary matter is necessarily incomplete. Thus, in accordance with the EPR argument, as long as one should insist on interpreting quantum mechanics as the law of a single element of matter, one arrives at the paradox that this theory is both complete and incomplete. This is commonly referred to as the 'Einstein–Podolsky–Rosen paradox'. The essence of their conclusion is that quantum mechanics, as a fundamental mechanics of an atom of matter, is (at best) an incomplete theory.

Einstein recognized that one way (though not the only way) to get out of this trouble would be to reinterpret the wave function of quantum mechanics (or equivalently, the 'state function') as a distribution function for an entire ensemble of atoms of matter — to be used in a statistical analysis, in the same sense that Boltzmann's distribution function is used in the analysis of a gas as an ensemble of a large number of individual elements. With this interpretation, the 'paradox' above would naturally disappear, since the 'incompleteness' would be an expected feature of the statistical analysis. In this case, the quantum mechanical wave function would not be intended to replace the (more complete) underlying deterministic description of matter in terms of the individual trajectories of the constituent elements of matter of the system. It would rather be that quantum mechanics, *per se*, must be interpreted as a statistical theory that simply adds a different sort of information about a

material system, that could be useful in estimating the *averaged* physical properties of the (assumed) ensemble. But the theory would still be deterministic since the trajectories of the constituent elements of matter would still be predetermined, independent of any measurement — analogous to the deterministic Newtonian theory of the trajectories of the atoms of a gas that *also* has a statistical description using the Boltzmann statistical distribution function. A major difference between the statistical theory in describing a classical and a quantum mechanical gas would then be mathematical rather than conceptual — e.g. the quantum mechanical 'distribution function' is a complex function while in the classical theory it is a real number function. The latter difference, of course, takes care of the explanation of the observations of an ensemble of material particles in terms of interference effects. But it is still a feature of the statistics and not a nondeterministic feature of single particles of matter. I believe that this view comes closer to Schrödinger's interpretation of his own wave mechanics.

Summing up, Einstein concluded that the EPR paradox was indeed a bona fide *logical paradox* implicit in quantum mechanics, in the sense of revealing a fundamentally incomplete representation of an alleged complete theory of elementary matter. He concluded that the paradox could be removed, within a model of matter in terms of elementary particles, simply by reinterpreting the wave function as relating to a statistical distribution function for an ensemble of a very large number of atoms, rather than relating it to an intrinsic probability amplitude for a single atom of matter.

From what has just been said, it should be re-emphasized at this point that the latter interpretation of the wave function in terms of an ensemble of atoms' statistical distribution function is not the only possible interpretation that might yield a nonprobabilistic (deterministic) theory upon which the formal expression of quantum mechanics is based! An alternative interpretation is the one discussed earlier in the chapter, in terms of a continuum field theory, in line with the underlying postulates of the theory of relativity. The latter was the actual interpretation that Einstein chose, though he did not pursue it to the point of demonstrating a derivation of the formal expression of quantum mechanics from the continuum view in general relativity, as it is done in this monograph in the chapters that will follow.

However, if one should choose to investigate the interpretation of the wave function in terms of a statistical distribution function, which, as mentioned above, was implicit in Schrödinger's view, it would still be assumed that the individual atoms exist, *underlying* the statistical mechanical formalism of quantum mechanics. Thus it has been proposed by some investigators who wish to maintain the deterministic view of a system of real atoms that the incompleteness in the quantum mechanical (statistical) description alone should be complemented by introducing extra parameters, in addition to the space and time coordinates, as a full set of *independent* variables upon which the *dependent* variables of the theory (the wave

functions) must depend. In the latter studies it has been tacitly assumed that any actual variable that is observable does not relate directly to these extra independent variables. Thus, the latter are called 'hidden variables'. They are to complete the description of the (assumed predetermined) trajectories of the constituent elements of a material system. Thus, the attempt of the *hidden variable approach* is to remove the subjective aspects from the fundamental theory of elementary matter, while still maintaining outwardly the probability calculus of quantum mechanics. Before describing the hidden variable approach in more detail, the next paragraphs will review Bohr's reply to the argument of Einstein, Podolsky and Rosen.

2.4.1. *Bohr's Reply to Einstein, Podolsky and Rosen*

Bohr replied to the EPR argument [23] by saying that, in effect, it was fallacious because it was *out of context*! Bohr said that quantum mechanics is a theory of measurement, relating to the measurement of the properties of micromatter by a macroapparatus. Thus, this theory does nothing more than express the outcome of such a measurement (or series of measurements) by macroscopic apparatuses on micromatter — *when the measurements are carried out*! With this view, quantum mechanics does not deal with the history of the elements of matter, say from an earlier time when they were in a bound state to a later time when they are unbound. When one observes the components of a system in a bound state, or when they would be observed separately in unbound states, these would be different sorts of measurements — therefore they must be represented by different sorts of wave functions. Thus, Bohr rejected the EPR claim that quantum mechanics is incomplete because they were not interpreting the mathematical expression of this theory correctly. It was Bohr's claim that, based on his axiomatic starting point for this theory, quantum mechanics is as complete as it possibly can be!

At this stage of the dialogue (in 1935), Einstein realized that the debate had changed its character from questions about physics to questions about epistemology. He did not agree with the fundamental assertion of Bohr and Heisenberg that a probability calculus could express maximally complete knowledge about elementary matter (nor did Schrödinger agree with this assertion [24]). But this was because of his (and Schrödinger's) epistemological stand of realism, in contrast with the Copenhagen stand of logical positivism. Still, Bohr continually queried Einstein, asking for a replacement for quantum mechanics that could reproduce the empirical facts with as much success, if he wished to reject its basis. This was, of course, a legitimate request, and it is indeed the main aim of this monograph to provide such a replacement — based here on the main ideas of Einstein's theory of general relativity.

But even if an unacceptable theory would not be replaced, it is still

unacceptable if it lacks mathematical and logical consistency. That is, to achieve agreement with the empirical facts is a necessary requirement of a scientific theory to claim to be a valid law of nature, but it is not sufficient. For the theory must also be logically and mathematically consistent, predicting unique results for given physical situations.

Einstein also required that a theory should be simple — which he interpreted as whole and complete. He did not see that quantum mechanics satisfies this criterion. Further, when the attempt was made to fuse the quantum theory with the theory of relativity, fully, in the form of a relativistic quantum field theory, the mathematical consistency of the theory broke down, as Dirac commented (quoted in the preceding discussion). The attempt is made in this monograph to resolve this problem by going back to the full basis of the theory of relativity (in its general form), thus rejecting the basis of the quantum theory, though approaching its formal expression in a linear (asymptotic) limit.

Before going on to this development, we will conclude this chapter with a brief discussion of the hidden variable approach and an explanation of how recent analyses of *Bell's inequalities* fit into the context of a relativistic field theory of matter, in the sense of Einstein.

2.5. The Hidden Variable Approach

The activity in hidden variable theories in the early 1950s was largely motivated by an attempt to resolve the Einstein—Podolsky—Rosen paradox. This work was initiated in the 1920s (before the dialogues of Einstein and Bohr) by de Broglie's interpretation of wave mechanics in terms of real, singular particles with predetermined trajectories. But in the 1950s Bohm and his coworkers revived the approach [25].

One of the main aims was to answer Schrödinger's criticism [26] as well as to resolve the EPR paradox. What Schrödinger said was this: While one may associate a particle of matter with a highly localized wave, characterized by a single wavelength, this wave must eventually *disperse* into a family of waves, each having a different wavelength. Thus while a single particle may be identified initially with a single de Broglie wavelength, in due course this 'particle' would become many 'particles' — by virtue of the dispersion of the original wave. Then which one of the final 'particles' is to be associated with the initial 'particle'?

The dispersion that Schrödinger refers to occurs by virtue of the interaction that a single wave might have with any material object, such as *any sort of measuring apparatus*. Schrödinger then asserted that it would be impossible to objectively identify a single wave with a single particle of matter because the single particle *endures* while the single wave does not! He was then led to the same conclusion that was drawn in the EPR analysis: that

the quantum mechanical wave function must relate to an ensemble of elements of matter, rather than to a single element of matter.

Nevertheless, if one should insist on the theory in which the Schrödinger wave function is identified with a single particle, a resolution of Schrödinger's problem may be achieved if, in some way, it would be possible to construct a *dispersion-free* wave for the particle description. Can this be done within the formal structure of quantum mechanics? To answer this question, it was de Broglie's idea, in the 1920s, that a second (singular) solution of a *completed* form of quantum mechanics should play the role of holding the undispersed wave together, thereby yielding a dispersion-free wave [13]. The variables of the latter solution are then the hidden parameters that were sought. It is significant, however, that the full expression of such a second solution as de Broglie sought since the 1920s, or the later attempts such as those of Bohm and his collaborators who tried also to develop a hidden variable theory, have never been demonstrated to date. [It is noteworthy that a form of undispersed wave under recent investigation by theoretical physicists is a new look at a result of many years earlier in which it was discovered that there are solutions of the hydrodynamical problem in which propagating waves in shallow water are indeed undispersed by obstacles. The application of this result to particle physics brings in the so-called 'soliton'. However, it is not clear that there is any connection whatsoever between the hydrodynamical problem and that of the quantum theory of particles, e.g. the former is a highly nonlinear mathematical formalism while the latter is, by definition, a linear theory in terms of the Hilbert function space that necessarily underlies quantum mechanics. That is, there is no indication at this time that the soliton problem could in fact resolve Schrödinger's problem [27].] The lack of resolution of this problem may indeed be due to a result that was discovered early in the game by von Neumann [14]. It was that *as long as one maintains the Hilbert space structure of quantum mechanics*, hidden variables may be transformed away in any quantum mechanical formalism that attempts to incorporate them [28].

Von Neumann's proof has to do with the following ideas. If $\hat{A}$, $\hat{B}$ and $\hat{C}$ are three hermitian operators relating to physical observables, such that one of them may be expressed as a linear sum of the other two,

$$\hat{C} = \alpha\hat{A} + \beta\hat{B}$$

then the expectation values of these operators are similarly related, as

$$\langle\hat{C}\rangle = \alpha\langle\hat{A}\rangle + \beta\langle\hat{B}\rangle.$$

Now, for the hypothetical dispersion-free states, there is no distinction between the expectation value of an operator $\langle\hat{A}\rangle_n$ and its eigenvalue, A_n, when ψ_n is the dispersion-free state, i.e.

$$[\hat{A}\psi_n = A_n\psi_n] \equiv [\langle\hat{A}\rangle_n = A_n].$$

On the other hand, the eigenvalues of linear operators are not generally additive. Consider, for example, the linear operator that relates to the measurement of the coupling of the magnetic moment of an electron to an external magnetic field H_z, oriented in the z-direction. In this case, the 'interaction Hamiltonian' is:

$$\hat{\mathscr{H}}_{\text{mag}} = \beta\boldsymbol{\sigma} \cdot \mathbf{H} = \beta\sigma_z H_z,$$

where $\beta = -e\hbar/2mc$ is the value of the magnetic moment of the electron (it is the 'Bohr magneton'), and

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

is the component of the Pauli spin matrix that 'points' in the z-direction. The latter is the 'spin operator' in quantum mechanics that relates to the measure of spin angular momentum of an electron. The eigenvalues of the matrix σ_z are seen to be $+1$ and -1. Suppose, then, that we identify the two eigenfunctions (the distinct states) associated with these eigenvalues, with ψ_1 and ψ_{-1}.

Now suppose that we wish to measure the x and y components of the electron's magnetic moment. These would be determined by the Pauli spin matrices σ_x and σ_y. The former has the matrix form

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

As a next step, if one should 'operate on' an eigenstate of σ_z with σ_x, it would follow that this state would 'disperse'" into a linear combination of both states of σ_z, that is

$$\sigma_x\psi_1 = \alpha\psi_1 + \beta\psi_{-1},$$

where α and β are coefficients that measure the degree of dispersion from the original state ψ_1, relating to an electron with 'spin up', to a combination of states ψ_1 and ψ_{-1} that describe states with 'spin up' and 'spin down'.

We see, then, that the eigenvalues of σ_x and σ_z are *not additive* — because the operators that correspond to their measurement do not commute. According to the Copenhagen interpretation, this means that they are not simultaneously determinable with equal precision from measurements.

The hidden variable theory attempts to formulate a dispersion-free state by generalizing the space-time coordinate system to an expanded parameter space, i.e. the *independent* variables increase, as

$$\psi(x, t) \rightarrow \psi(x, t, \lambda), \tag{2.5.1}$$

where the set $\{\lambda\}$ are the hidden parameters. The rule is then imposed that the latter independent variables should satisfy the 'velocity relation':

$$d\lambda/dt = j_\psi(\lambda, t)/\rho_\psi(\lambda, t) \tag{2.5.2}$$

where j_ψ and ρ_ψ are the quantum mechanical current density (of probability) and the probability density, defined in terms of the wave function as:

$$j_\psi = (\hbar/2im)|(\psi^*\nabla\psi - \psi\nabla\psi^*)|_{r=\lambda} = (\hbar/2m)|\mathrm{Im}\ \psi\nabla\psi^*)|_{r=\lambda} \tag{2.5.3}$$

$$\rho_\psi = \psi^*\psi. \tag{2.5.4}$$

Next, it is assumed that the hidden variables are explicit functions of time, while the probability function ρ depends explicitly on the spatial coordinates, but only implicitly on the time coordinate, as $\rho[x, \lambda(t)]$. The variables $\lambda(t)$ are then supposed to denote the *real* positions of the particle at the time t. This is the variable that plays the role of the trajectory variable $x(t)$ of the classical Newtonian particle theory — though the latter variables are supposed to be directly observable while the parameters $\lambda(t)$ are not. Still, the generalization of the quantum mechanical formalism so as to include the hidden variables does yield a *deterministic* theory of matter, though expressed with the (generalized) quantum mechanical formalism of a probability calculus.

At this stage of the discussion it is important to note that the quantum statistical theory of measurement, that relates to the positions of a system of particles, depends on the probability density function ρ, which, in turn, is determined from the *deduced* distribution of positions of the system of particles. But other measurements, such as those that lead to statements about the 'spin coordinates' of an electron, also reduce to expressions in terms of spatial position measurements. Recall, also, that the positions of particles, *per se*, are not directly observed quantities either. Rather, these parameters are deduced from the direct measurements of energy and momentum transfer between the 'observed matter', such as the electrons, and the 'observing matter', such as a macroscopic apparatus. For example, to determine the spin of an electron in the x-direction, expressed in terms of the spin matrix σ_x, an electron beam may be made to pass through an inhomogeneous magnetic field (the 'Stern—Gerlach experiment'). One then observes that the electron beam is deflected only in part in the upward direction (with respect to its direction of motion) and in part in the downward direction. The latter 'observation of the spin direction' follows actually from the subsequent spatial position of the electron beam after it has passed through the inhomogeneous magnetic field. But this position measurement is the place where some film responds to a quantity of energy—momentum transfer from one ensemble of matter (the electron beam) to another ensemble of matter (the detecting film).

The preceding introduction of hidden variables in the formal expression of

quantum mechanics may be generalized from the description of a single particle trajectory to a family of trajectories for the many-particle system. The wave function generalizes by mapping now in a $3n$-dimensional space for the n-body system. The velocity variables for the hidden variables (2.5.2) are then generalized as follows, for the mth hidden variable trajectory:

$$d\lambda_m/dt = j_m(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n, t)/\rho(\lambda_1, \lambda_2, \ldots, \lambda_n, t)$$

where the current density and density variables here are the generalization of (2.5.3) and (2.5.4):

$$j_m(\lambda_1, \lambda_2, \ldots, \lambda_n, t) = (\hbar/2m)\mathrm{Im}\,|\psi^*\partial\psi/\partial\lambda_m|_{r=\lambda_m} \tag{2.5.5}$$

$$\rho(\lambda_1, \lambda_2, \ldots, \lambda_n, t) = |\psi(\lambda_1, \lambda_2, \ldots, \lambda_n, t)|^2. \tag{2.5.6}$$

The preceding is a sketch of the program of hidden variable theory in quantum mechanics. It must be investigated further to see if indeed it could lead to a resolution of the Einstein—Podolsky—Rosen paradox, as well as solving Schrödinger's problem of the requirement of a dispersion-free wave to describe an element of matter. However, at the present stage of its development, it does not yet appear to work! One of the serious troubles is that the many-body wave function depends on *all* of the hidden variables, that is, the trajectories of all of the particles of a material system, when supposedly describing the wave properties of a single one of them! The implication is then that the fundamental description of elementary matter is *nonlocal* — just as a continuum field theory that rejects the particle concept altogether would be. That is, with this description, the wave function that is supposed to accompany the trajectory of a single particle must still depend on the hidden variables of all of the other particles of a system, that are simultaneously at *other spatial positions.*

The question then arises: Can there be a hidden variable theory of an n-body elementary particle system that is local? The answer is that to this time, there have been several attempts that have failed in demonstrating a local hidden variable theory within the context of the Hilbert space formalism of quantum mechanics. But an answer might be that the conceptual approach of quantum mechanics is false in the first place; that it may only be a valid approximation when particular parameters are small. Indeed, should the conceptual approach to matter according to Einstein's theory of general relativity theory be valid, then the entire structure of quantum mechanics would be false in principle, with or without hidden variables! In this case, the formal structure of quantum mechanics appears only as a *linear approximation* for a nonlinear, deterministic field theory of matter, based axiomatically on the ideas of general relativity rather than those of the quantum theory, as illustrated earlier in Table I. Thus, with this view, the Hilbert space formalism of quantum mechanics (with or without hidden variables) would form a good approximation for an entirely different

mathematical formalism, which in turn follows from an entirely different conceptual basis — that of the theory of general relativity as a fundamental theory of matter, in all domains.

Before commencing with the development of the replacement of ordinary quantum mechanics, in the context of the theory of general relativity, it should be instructive to briefly discuss a current study of John Bell that poses a test for the formal predictions of the (Copenhagen view of the) quantum theory, in the context of the type of localized theory of matter that challenges the 'particle' aspect of quantum mechanics. The aim of the next section, then, is to analyse this in the light of the (nonparticle) continuous field view of matter — which, as we have seen, is neither an ordinary localized particle approach or the quantum mechanical particle view in terms of wave—particle dualism.

2.6. Bell's Inequalities and General Relativity

Bell's inequalities are concerned with a comparison of correlation functions of elementary particles (or any configuration that it supposed to conform with the rules of quantum mechanics) that are separated by *spacelike* distances, which were at earlier times bound with a particular correlation and then dissociated with a force that is independent of the earlier correlation variables. It is not the intention in this section to analyse the details of the derivation of Bell's inequalities. This is outlined in [29]. Rather, we will concentrate here on the meaning of these relations in the context of the general theory of relativity, as a nonatomistic approach to matter based entirely on the field concept, indicating possible experimental predictions of such a continuum view of matter, whereby the formalism of quantum mechanics plays the role of a linear, asymptotic limit of a nonlinear field theory of inertia — arising from a quite different conceptual view of matter than that of the quantum theory [30].

One of the assertions of the analysis that leads to Bell's inequalities is 'Einstein locality'. This is the idea that all of the units of elementary matter have their own sets of assigned, predetermined physical properties, independent of any measurements made by a macroapparatus. Such physical properties are then *localized* in space, where the particle is supposed to be. The concept was named after Einstein in view of the conclusion that he reached with Podolsky and Rosen, as we discussed in the preceding paragraphs. Nevertheless, it should be pointed out that while the EPR analysis started out with a model of matter in terms of localized particles, this was done to test the logical consistency of the Copenhagen interpretation of quantum mechanics, *on its own terms*. Still, Einstein did not accept this view of separability of matter as fundamental. His own view, rather, was based on the continuous field approach of the theory of general relativity.

As we will discuss in later chapters, Einstein's basic approach to the laws of matter in fact rejects the concept of 'Einstein locality', replacing the atomistic concept of an 'open system' with the continuum approach of a 'closed system', whereby the fundamental variables of matter (i.e. the solutions of the elementary laws of matter) are the regular (i.e. nonsingular) solutions of their corresponding covariant field equations. His view of the field concept also automatically rejected the nondeterministic, probabilistic measurement interpretation that is evoked by the Copenhagen view. Instead, the field view in general relativity leads to a deterministic, nonlinear field theory, expressed in terms of a set of coupled spinor variables *in one spacetime*, describing a *closed* material system at the outset, rather than representing separable parts, irrespective of whether or not their trajectories are predetermined [31].

2.6.1. *The State Vector and Bell's Inequalities*

In quantum mechanics, if two spin-$\frac{1}{2}$ particles are initially bound in an S-state, the corresponding (pure) state has the following form in terms of an antisymmetrized component of a Hilbert space:

$$|\Psi\rangle = 2^{-1/2}(|\mathbf{r}_1+\rangle|\mathbf{r}_2-\rangle - |\mathbf{r}_1-\rangle|\mathbf{r}_2+\rangle). \tag{2.6.1}$$

If, in accordance with the EPR *gedanken experiment* the particles are then removed from each other to *spacelike separations*, $|\mathbf{r}_1 - \mathbf{r}_2| > c|t_1 - t_2|$ (i.e. so that, according to special relativity, they no longer interact during the times that they are observed), the EPR conclusion implies that the new state functions for the 'unbound' particles (according to the notion of 'Einstein locality') must have a form that depends on spin vectors for each of the particles whose mutual orientations are arbitrary and predetermined, independent of measurements. If these particles were initially bound in an S-state, then the conservation of angular momentum would require that the total angular momentum of the separated system of two localized particles must still be zero.

A crucial point that separates the description that entails Einstein locality from the usual quantum mechanical description (according to Bohr) is the following: With Einstein locality, as one proceeds from the system bound at *timelike* separations, as described by the *pure state function*, (2.6.1), to the unbound state of *spacelike* separation, there is nothing to break the antisymmetric feature of this function or destroy its predetermined nature. Thus, even at spacelike separations, the state function that represents the matter field is still a 'pure state', with the antisymmetric form:

$$|\Phi\rangle = 2^{-1/2}(|\mathbf{r}_1, \boldsymbol{\sigma}_1^{\pm}\rangle|\mathbf{r}_2, \boldsymbol{\sigma}_2^{\mp}\rangle - |\mathbf{r}_1, \boldsymbol{\sigma}_2^{\mp}\rangle|\mathbf{r}_2, \boldsymbol{\sigma}_1^{\pm}\rangle), \tag{2.6.2}$$

where the variables $(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)$ have been inserted to specify that these *vectors* have precisely defined, independent status as the spin angular momenta of

particles 1 and 2, at either of the respective locations ($\mathbf{r}_1$, $\mathbf{r}_2$), and $\pm$ denote their respective axes of quantization — *which are not generally parallel* — that may be specified in particular experiments in which the two particles are observed in coincidence at their spacelike separation.

A special case of the form (2.6.2) is the state function (2.6.1), when the spin vectors would be parallel. But in the separated state, they do not have to be so, according to the view of 'Einstein locality', as long as their separation is spacelike. In this case, where the unbound particles described by the state function (2.6.2) are an S-state, $|\Phi\rangle$ is an eigenstate of $(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)^2$ and $(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)_3$, each with an eigenvalue equal to zero. But $|\Phi\rangle$ is not an eigenstate of $(\boldsymbol{\sigma}_1^2, \boldsymbol{\sigma}_{1_3})$ and $(\boldsymbol{\sigma}_2^2, \boldsymbol{\sigma}_{2_3})$ *separately*. That is,

$$\boldsymbol{\sigma}_1|\Phi\rangle \neq -\boldsymbol{\sigma}_2|\Phi\rangle .$$

With the view of Einstein locality, then, the predetermined spin vectors, $\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$, at either of the respective spatial locations, $\mathbf{r}_1$ or $\mathbf{r}_2$, may be oriented relative to each other in any predetermined direction. Thus, the expectation value of the scalar product of spin vectors, $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, *within the context of this probability calculus*, is an angle that is adjustable in coincidence experiments on correlated spin-$\frac{1}{2}$ objects, that are separated by spacelike distances.

Bell exploited this feature of a separated system that entails the notion of Einstein locality, in deriving his inequalities:

$$P(\mathbf{a}^+, \mathbf{b}^-) \leqslant P(\mathbf{a}^+, \mathbf{c}^+) + P(\mathbf{b}^-, \mathbf{c}^+) \tag{2.6.3}$$

where (**a**, **b**, **c**) are a set of spatial vectors, oriented relative to each other according to particular, predetermined choices of experimental arrangement for measurement of the spin orientations of (previously bound) spin-$\frac{1}{2}$ particles, in coincidence, when they have been separated by spacelike distances. The probability variable $P(\mathbf{a}^+, \mathbf{b}^-)$ denotes the correlation function relating to the probability that an experimental determination that one of the particles has spin 'up', relative to the orientation of the vector **a**, is coincident with an experimental determination that the other particle (previously bound to the first one) has spin 'down', relative to the direction vector **b**.

Bohr's interpretation [23] of the formal expression of quantum mechanics has to do with the outcome of a measurement (or series of measurements) by macroapparatuses on micromatter — *when the measurements are carried out.* Thus, quantum mechanics in this view does not deal with the history of the particles of matter, as we have emphasized earlier, say from when two particles are in a bound state to the later time when they are unbound, and separated widely. When one observes the component particles when they are bound, or when they are observed separately as unbound, according to the Copenhagen view these refer to different sorts of measurements — therefore, they must be represented by correspondingly different sorts of state functions, as we have indicated above was Bohr's reply to the EPR *gedanken experiment.*

In the Copenhagen view, then, if some previous measurement acted to prepare the state of a two-body system so as to have a total 3-component of its angular momentum equal to zero, there could be no direct knowledge before making a subsequent measurement on a property of a component part of the system, after it is separated, if it might have spin 'up' or 'down' relative to some new experimentally arranged direction (such as the polarization direction of an external magnetic field), and what would be the total angular momentum of the two-body *separated* system. Thus, in view of this interpretation of the state functions in terms of measurements, when they are carried out, Bohr would have predicted that the separated system must have the following two possible state functions:

$$|\mathbf{r}_1+\rangle|\mathbf{r}_2-\rangle \quad \textit{or} \quad |\mathbf{r}_1-\rangle|\mathbf{r}_2+\rangle. \tag{2.6.4}$$

The state function of the separated system, then, is not a pure state in Bohr's view, since the state functions in (2.6.4) are each combinations of a singlet state and a triplet state, with no predetermination for the degree of admixture. That is, it is Bohr's prediction that this is a 'mixed state', whereas the correlated state in (2.6.2) for the spacelike separated parts is a pure state. Since the pure state leads to Bell's inequalities (2.6.3), their experimental verification should provide a test of the validity of the interpretation of quantum mechanics in terms of the Einstein locality of 'particles' — the measurement interpretation of Bohr would then be automatically refuted!

If one should now hypothesize that there are only two mutually exclusive explanations for the results that come from tests of Bell's inequalities, experimentally, say B or C, then if it should be discovered that B is false, it must follow logically that C is necessarily true, where B may stand for a probability calculus that admits Bell's inequalities (based on 'Einstein locality') and C may stand for the probability calculus underlying the Bohr interpretation. However, it is clear that B and C are not the only possibilities to underlie the experimental facts of micromatter! To see this, note that both B and C are based on a particular sort of probability calculus — the formal structure of solutions in a Hilbert space, where the element of the Hilbert space for B has the form of (2.6.2) and the Hilbert space element for C has the form of (2.6.4), in the example chosen. On the other hand, the underlying laws of elementary matter may not depend on any sort of probability calculus in the first place — such as the continuous field approach to elementary matter implied by general relativity theory.

When fully exploiting the theory of general relativity as a fundamental basis for a theory of matter (which was Einstein's actual intention), one is necessarily led to a rejection of the (classical or quantum mechanical) atomistic models of matter — replacing them with the continuous field concept. In the latter view, one no longer has individuated, separable 'parts'. Matter is rather represented in this theory in terms of the modes of a *closed system*, described mathematically by the *regular* solutions of the field laws —

i.e. those solutions that are without any singularities throughout all of space-time. This would be a representation of matter that is fully deterministic and objective (covariant). It is a theory of matter that implies that there is a predetermined representation of matter that embeds all of the dynamical features of matter, in any domain, whether or not any measurements may be carried out. It is an objective theory of matter that would not distinguish between the laws of macromatter and those of micromatter, where there would be no fundamental difference betwen the 'observer' and the 'observed', except for convenience in deriving the predictions of the field equations. According to this view, the field solutions of the laws contain total information (in principle) about 'correlated' components of the closed system, *existentially*. Such a theoretical description of matter then yields knowledge about the system without the need to evoke the notion of spontaneous *action-at-a-distance*.

A specific deduction about a field variable of matter, somewhere, at some time, then implies a knowledge of this field variable everywhere else, at all other times — even at spacelike separations from the particular location and time determined. This is a feature of any *deterministic* representation of matter — that the implicit variables are predetermined, independent of any measurements that may or may not be carried out. In this case, the predetermined variables are the analytic field variables; in Newton's classical theory of matter, they are the spatial trajectories of the singular, interacting bits of matter.

The assumption that the system of matter has a predetermined, precise mathematical expression, everywhere and at all times, following from the laws of nature, does *not* logically imply that signals between distantly separated points occur spontaneously — that is that interactions propagate infinitely fast. Indeed, the *covariance* of the field equations incorporates the feature that interactions may not propagate faster than the speed of light. This implies, for example, that it would take an Earth-observer about four years to find out that an eruption had occurred on the Sun's nearest neighbor star. But this does not mean that the information we had about the physical past and future of that star, in accordance with the field laws that incorporate this information (as part of its deductions) had anything to do with spontaneous action-at-a-distance! Such an interpretation may be in accord with the Copenhagen view of the matter field as representing not more than our (subjective) knowledge of a microsystem, *by virtue of making a measurement on it* with a macroapparatus. But the matter field in the view of general relativity, as a fundamental theory of matter, does not have this interpretation; it rather has purely objective connotation.

An important feature of the matter field in general relativity, as we have emphasized earlier, is that it entails many coupled field variables, all mapped in a single space-time, x, rather than the particle views in which fusion with special relativity requires as many space-times as there are particles.

That is, in the latter view, there would be a set of n space-time coordinate systems, and n accompanying spin coordinates for the n-body system, $\{\sigma_1, x_1; \ldots \sigma_n, x_n\}$. The interactions would then correlate these coordinates, as one would in an n-body analysis, as in Bell's study or in the ordinary n-body quantum mechanical theory. Also, in the field theory of matter, according to general relativity, the set of spin coordinates $\{\sigma_i\}$ are associated with the distinguishable fields of the closed system, but not with individual objects, as in the particle theory in quantum mechanics. They appear here because of the covariance requirement of relativity theory that implies that the fundamental field variables must obey the algebra and calculus of spinors. Nevertheless, in the asymptotic limit where the mutual coupling is sufficiently weak, one may view each of the spinor solutions as uncoupled, as a first approximation. In the latter approximation, then, one may express each of the field solutions as mapped in its own space-time, thereby giving the appearance of a many-particle system, as in quantum mechanics.

Generally, the coupled spinor field equations for a closed material system are *nonlinear*, by virtue of their intrinsic coupling. But in the asymptotic limit of very small coupling, one approaches the linear limit of the formalism, where a linear mathematical approximation may be fairly accurate for the actually nonlinear solutions. It then follows that the linear Hilbert space formalism, leading to the state functions symbolized by (2.6.2) (and Bell's inequalities of the form (2.6.3)) *or* the mixed state function (2.6.4) as predicted by Bohr's interpretation of quantum mechanics, are *both* out of context in regard to the continuous field view of matter according to Einstein's general relativity, as a fundamental theory of matter.

Still, the experimental and theoretical investigations of Bell's inequalities are pertinent here because of the linear limit of the general nonlinear field theory, that could reveal an agreement with one of these approaches or the other. Thus, the current tests of Bell's inequalities could also be a bona fide test of the general field theory of matter developed in this monograph, that is, to test its (unique) asymptotic limits.

It will be seen in later chapters that this field theory predicts that a total connective field must exist, depending on all of the coupled spinor field solutions for the closed system, and that this connective field encompasses the *Pauli exclusion principle* as an *exact* feature of the field formalism. It will then be seen further that in the linear limit, this total connective field approaches the form of the totally antisymmetrized many-body function that underlies Fermi—Dirac statistics for the quantum mechanical system of fermions.

With this general feature of the field theory in mind, let us consider the physical conditions that underlie the experimental tests of Bell's inequalities. Within the context of this theory, if a pair of (asymptotic) spinor fields that were at one time correlated with a total spin $S = 0$, but are at the time of measurement apparently separated at a *spacelike* distance, then since there

can be no coupling between these fields at the respective locations $\mathbf{r}_1$ and $\mathbf{r}_2$, there can be no correlation regarding the latter spatial locations, in the limiting form of the matter field solution for the pair. Thus, in the limit of no coupling, the two-body matter field solution must be symmetric with respect to the interchange, $\mathbf{r}_1 \leftrightarrow \mathbf{r}_2$. However, the totally antisymmetrized form of the matter field solution that is *generally* predicted by this theory (as a consequence of its closed and nonlinear features) implies that this solution must then be antisymmetric with respect to the spin variables, $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$. This result then corresponds to a Hilbert space function of the type (2.6.2) — thus indicating a prediction that would be consistent with Bell's inequalities (2.6.3).

On the other hand, if the correlated spinor fields are considered to be separated by *timelike* distances, then the spatial coordinate-dependent interaction is transmitted between the points at $\mathbf{r}_1$ and $\mathbf{r}_2$ during the time of measurement of both locations in coincidence. In this case, the spatial coordinates would be correlated variables. This implies that the spatial part of the (asymptotic form of the) two-body matter field solution could be either symmetric or antisymmetric with respect to the interchange of spatial variables, $\mathbf{r}_1 \leftrightarrow \mathbf{r}_2$, corresponding to two possibilities for the matter field solution *for the interacting system.* The requirement of the general field theory that in the asymptotic limit the matter field must be totally antisymmetric then implies that the latter two possibilities would correspond, in terms of total spin, to the singlet and triplet states. This result would then predict a general type of matter field that would be a mixed state — in accordance with the Bohr prediction (2.6.4) in the context of his interpretation of quantum mechanics.

Summing up, the theory of elementary matter in general relativity appears to predict that there would be an agreement with the statistics that lead to Bell's inequalities when the correlated measurements (in the 'particle' approximation) correspond to two entities that are *spacelike* distances apart, and with the statistics in accordance with Bohr's interpretation of quantum mechanics when they are related to measurements of *timelike* separated microscopic entities. But *in principle*, both views of elementary matter are out of context in regard to the advocated theory of matter in general relativity, where indeed statistics and indeterminism play no basic role.

In the next chapter, we will commence the development of this theory, where it will be seen that the formal expression of quantum mechanics evolves as a linear limit of a generally covariant (nonlinear) field theory of inertia.

Chapter 3

Basis of a Matter Field Theory of Inertia — A Generalization of Quantum Mechanics

This theory is based on three essential axioms: the principle of general relativity, the generalized Mach principle, and the principle of correspondence. The approach is basically that of a deterministic, relativistic field theory which fully incorporates the idea that any realistic physical system is in fact a *closed system*, without separable parts. It will be shown that the most primitive form of this theory, following as a necessary consequence of its axioms, is in terms of a set of coupled, nonlinear spinor field equations, that has an asymptotic limit that corresponds to the many-particle quantum mechanical theory. The limit corresponds to the assumption of sufficiently small energy—momentum transfer among the component elements of the assumed closed system. Let us now discuss in more detail the implications of the three basic axioms indicated above.

AXIOM 1. *The principle of relativity* asserts that the laws of nature must be independent of the reference frame in which they are expressed. The 'frame of reference' will refer in particular to a system of space-time coordinates that is distinguishable from other space-time coordinates only in terms of their *relative motion*. When the relative motion happens to be characterized by constant rectilinear speed, the theory reduces to the special case called 'special relativity theory'. With any other (general) type of motion, it is called 'general relativity theory'.

AXIOM 2. *The generalized Mach principle* asserts that there are no intrinsic properties of 'free' matter — that *all* of the manifestations of any (apparently free) quantity of matter are in fact measures of the mutual dynamical coupling within the assumed closed material system.

Mach's original idea referred to the particular manifestation of matter associated with its inertia — the resistance with which matter opposes any change in its state of motion, due to the application of external forces. To review his argument, first consider the classical interpretation of inertia, according to Newton. According to his view, the inertial mass of matter is one of its *intrinsic* properties. It follows from the (empirically confirmed) law

that if F_1 and F_2 are the magnitudes of two different external forces that act on a *given* bit of matter, and if a_1 and a_2 are the magnitudes of the respective accelerations caused by these forces, then the following relation holds:

$$F_1/F_2 = a_1/a_2. \tag{3.1}$$

This (empirically confirmed) formula may be expressed as follows:

$$\mathbf{F} = m\mathbf{a}, \tag{3.2}$$

where m is the constant of proportionality between the *cause* of nonconstant motion, and a is the quantity of acceleration produced, the *effect*. The intrinsic property of matter, a quantification of its inertia, called 'inertial mass, m, then appears here by virtue of the imposition of an atomistic model.

In contrast with this model, Mach argued that it would still be consistent with the empirically verified law (3.2) to interpret it as pertaining to a linear relation between the ratio of forces that cause *different* bodies, with masses m_1 and m_2, to accelerate at the same rate a, and the ratio of these masses,

$$F_1/F_2 = m_1/m_2.$$

From this relation one may conclude the general relation between mass and force, analogous to going from (3.1) to (3.2), as follows:

$$m = kF, \tag{3.3}$$

where $k = m_2/F_2$ may be taken as a *standard*, for comparison with all other masses m. In this case, mass would be a relative (rather than an absolute) entity, i.e. from the conceptual view, one may interpret Equation (3.3) to mean that the source of the inertial mass m is rooted in the dynamical coupling between this body and all of the other bodies of a closed system, as expressed in terms of the total external force F that *acts on m*. Thus Mach's conclusion about the origin of the inertial manifestation of matter, as a *relative* feature of a closed system, is opposite to the interpretation of Newton. The latter view is atomistic, based on a property of a free, noninteracting bit of matter.

It is well known that Mach's view of inertia had a profound influence on Einstein in his development of the theory of relativity. He called this view 'Mach's principle'. It is the thesis of this monograph that a full exploitation of the relativistic view implies that not only the inertial manifestation of matter, but all of its manifestations must be related to the mutual dynamical coupling of the components of a *closed system*. This assertion will be referred to as the *generalized Mach principle*.

One of the important implications of the generalized Mach principle when it is incorporated with the theory of relativity is that the components of interacting system lose meaning as *things-in-themselves* — one loses all semblance of atomism! It is only the entire closed system that retains meaning as a 'thing-in-itself' — but this is, in principle, the universe.

However, one need not think that this means that one must be able to describe explicitly the entire universe before understanding any part of it! Fortunately, one may use mathematical approximations for many problems (though not all!) where it is as though one is (almost) describing the universe as a large number of uncoupled units. Then what is the smallest unit that would be consistent with the philosophical basis of this theory?

The answer is that the smallest unit must entail at least two components — because the entity that is elementary in this view is the mutual interaction, and not the thing, and there must be at least *two* components involved in an elementary interaction because of the empirical fact that (in particular limits) it looks as though there are at least two separate and distinguishable entities that are free of each other. However, because of the elementarity of the interaction, their mutual coupling may be arbitrarily weak, but it may never be 'off'. This is a feature of the theory that has more than philosophical significance. It also has mathematical consequences that differ from the mathematical description of a system of free things, though interacting. In the latter model, the basic variables would be 'free fields', which would then be perturbed with interactions. In the view of this theory, there are no free field variables in the first place. Thus at least some of the predictions of the two approaches would necessarily be different in regard to experimental predictions.

According to the generalized Mach principle, then, the 'observer—observed' relation is not to be taken as a coupling between independent things — 'observer' and 'observed' — since there is no meaning for the term 'observer' as a thing by itself, or the term 'observed' as a thing by itself. It is rather the *whole entity*, 'observer—observed', which in this theory is inseparable in principle that is a fundamental unit from which one must start to construct a general theory of matter.

It is interesting to note that, contrary to Mach's original positivistic view, 'observer' in the context of the relativistic theory discussed here does not necessarily refer to a human, or to his equipment. Neither does it necessarily refer to a macroscopic 'observer' whenever the 'observed' would be microscopic, as it is assumed in the quantum theory. Rather, 'observer—observed' refers to a closed system that is fundamentally one. It is sometimes necessary to identify a component of this closed system as a macroscopic measuring apparatus and the remainder of the system as an elementary particle or atom. However, the identifications can only come after the limiting form of the general system (that does not make such distinctions, in principle) has been taken. In other examples, one of the interacting components of the closed system may be an electron and the other a positron — here, there would be no 'macroscopic' component in the coupled system. The important requirement of the theory is that the overall description must be independent of which component of the closed system is called 'observer' and which is called 'observed'.

Since the 'observer' is the subject and the 'observed' is the object of any statement about a material system, and since the overall description is invariant with respect to an interchange of the object and subject, the entire description must then be totally objective. This implies that all of the features of the closed system are *predetermined.* As we have discussed in the preceding chapter, this approach is based on a philosophy of abstract realism. It is in contrast with the nondeterminism and logical positivism assumed by the Copenhagen approach in quantum mechanics. It is also in contrast with Mach's original positivistic interpretation of Newton's equations of motion.

AXIOM 3. *The correspondence principle* asserts that the expression of a new theory must *approach* the mathematical formalisms of the theories that it claims to supersede. Thus, to approach the quantum mechanical equations, the formal expression of the equations for inertia of this theory must be differential equations that must approach the usual Hamiltonian form, in the appropriate limit. This would correspond to the energy-momentum transfer between the interacting components of the assumed closed system becoming sufficiently small. This is the limit where the system appears to be in terms of distinguishable parts.

It will be found that the preceding three axioms lead uniquely to a generally covariant theory of inertia that is deterministic and has the form of a nonlinear spinor field theory — thus containing features that are not at all in correspondence with the present-day expression of high-energy physics in the elementary particle domain. However, this formalism, to be consistent with Axiom 3, does approach that of ordinary quantum mechanics and electromagnetic theory, when the energy-momentum transfer between coupled matter becomes sufficiently small. The theory of inertia presented attempts to contain all of the successful results of quantum mechanics and electromagnetic theory, in the low-energy region of measurements in the microscopic domain. However, in the high-energy domain, the theory makes predictions that either are not made at all in the conventional theory or are made in a mathematically unsatisfactory way, essentially because of the appearance of infinities there.

3.1. The General Mathematical Structure and Philosophical Implications

3.1.1. *The Symmetry Group from Axiom 1 and Fundamental Field Variables*

The principle of relativity implies a fundamental description of matter that necessarily entails *motion* — the only distinguishing feature between the different frames of reference in which one may wish to compare the laws of nature. Motion, in turn, is defined as a continuous entity — the continuous

differential changes of one set of space-time coordinates with respect to other sets of space-time coordinates. The transformation group that underlies this theory is then a continuous parameter group. Since the minimum number of coordinates required to express the laws of nature is four — usually identified with the three spatial coordinates and one temporal coordinate (or, sometimes with momentum-energy coordinates, as in scattering problems) — the number of essential parameters that characterizes the symmetry group in its most general form is $4 \times 4 = 16$. These might be represented in terms of the 16 derivatives of the coordinates of one reference frame with respect to those of any other, $\{\partial x^{\mu}/\partial x^{\nu'}\}$. In the general case, these derivatives are functions of where they are evaluated — such a space-time is *nonlinear*, and applies in general relativity. The space-time in special relativity is linear, for reasons that will be seen.

The 'symmetry group of general relativity' is defined in this monograph to be that set of continuous space and time transformations that leave invariant the squared differential interval

$$ds^2 = g^{\mu\nu}(x)\, dx_{\mu}\, dx_{\nu} \tag{3.1.1}$$

at each space-time point, excluding the reflections in space-time. That is, the expression of the invariance of ds^2 is a short-hand way of summarizing the group of transformations of the space and time coordinates that leave the laws of nature invariant in form (covariant), according to the theory of general relativity. This will be referred to as 'the Einstein group'.

The irreducible representations of the Einstein group are determined from a global extension of the irreducible representations of the group of special relativity theory (the Poincaré group). This is because ds^2 in general relativity is, by definition, a global extension of the (squared) Lorentz metric of the theory of special relativity:

$$ds_{sr}^2 = dx_0^2 - dr^2, \tag{3.1.2}$$

excluding its space and time reflections. This limit then corresponds to the required local limit of the metric tensor:

$$g^{\mu\nu}(x) \rightarrow (1 -1 -1 -1)\delta^{\mu\nu}. \tag{3.1.3}$$

The structuring of the irreducible representations of the Einstein group as a global extension of those of the Poincaré group was shown in GRM (Section 6.15), and in Section 4.6 of this book, to demonstrate the quantization of electrical change as a feature of the asymptotic properties of the irreducible representations of the Einstein group, as they approach those of the Poincaré group, to first order in e^2.

It should be noted at this stage of the discussion that though the representations of the Einstein group *approach* those of the Poincaré group, as closely as we please, the limit cannot be reached in principle, as long as we are describing a material system. That is to say, the Poincaré group is not, in

fact, a subgroup of the Einstein group, as we have defined the latter in terms of a *curved* space-time, relating to matter. It is only in the case of the vacuum (*everywhere*, in principle) where, in an exact sense, one must *replace* the Einstein group and its representations (i.e. invariance under the continuous transformations that leave ds^2 (Equation (3.1.1) unchanged) with the Poincaré group (invariance under the continuous transformations that leave ds^2_{sr} (Equation (3.1.2) unchanged).

Several of the analyses in this monograph deal with the special relativistic limit of this theory, though deriving new mathematical results within the context of the meaning of this formalism as based on the ideas of general relativity as an elementary theory of mater. Still, it is important for the reader to keep in mind the fact that this is meant only in the sense of an *asymptotic limit* that may not be reached, in principle, as long as we are discussing matter, *per se*. The underlying symmetry is still governed by the 16-parameter Einstein group, though in the special relativistic applications the irreducible representations of the underlying group of general relativity are *approximated* by the irreducible representations of the 10-parameter Poincaré group.

It follows from the principle of covariance of the theory of relativity that conservation laws must be incorporated in the local limit. An implication of this requirement is that the space-time transformations and the matter fields they apply to must be continuously differentiable functions to all orders, i.e. they must be analytic, everywhere. This is a consequence of the necessary and sufficient conditions for the existence of conservation laws in the local, flat-space limit, according to Noether's theorem [32]. The space-time coordinates themselves are not to be interpreted within this field approach as observable entities, as in the interpretation of the trajectories of point particles in the atomistic theories. They are rather chosen here as a convenient *language* — a set of parameters with a built-in logic (analogous to syntax) that is *used* to express the laws of nature. The latter, in turn, are in the form of field equations whose solutions are the *dependent* variables and relate directly to the predictions of observables. The algebraic symmetry group is, in fact, an expression of the algebraic part of the logic (syntax) of space-time in relativity theory. The other part of this logic is the set of relations between space-time points that are associated with geometry. The implication of the principle of relativity is then a formalism that is based on the concept of the *continuous field.*

Another explicit feature of a relativistic theory of matter that follows from its algebraic logic has to do with the most general type of field variable that would be consistent with the underlying symmetry group. It turned out to be the spinor variable. Historically, this conclusion was reached in the following way.

It was shortly after Dirac's discovery that a relativistic extension of the Schrödinger wave equation leads to the *necessity* to extend from a complex

scalar field description to a complex multi-component field description, whose minimum number of components is two, called the 'spinor', that Einstein and Mayer made a very important discovery about the significance of Dirac's result, in the context of relativity [34]. These authors addressed themselves to this question: Was the discovery of the spinor representation of the electron equation a consequence of quantum mechanics *per se*, or was it the result of forcing an equation (that just happened to be a formal expression of quantum mechanics) to be relativistically covariant?

To answer the question, Einstein and Mayer decided to study the form of the irreducible representations of the Poincaré group — since this is an aspect of the electron theory that is independent of the detailed structure of any mathematical formalism, as long as it would be covariant in special relativity. What they then found was that if only the *continuous transformations* that leave the (squared) metric, ds_{sr}^2 invariant would be maintained, i.e. leaving out reflection transformations such as $\mathbf{r} \rightarrow -\mathbf{r}$ and $t \rightarrow -t$, then the (real number) four-dimensional representations *decompose* into the direct sum of two two-dimensional (complex, hermitian) representations, which obey the algebra of quaternions. It then followed from the algebraic properties of these two-dimensional hermitian representations of the Poincaré group (which is actually a form of the irreducible representation of the group) that its basis functions transform as the two-component spinor variable that represents the electron in a special relativistic form of quantum mechanics.

When this theoretical expression is then (globally) extended to the representations of the Einstein group (in general relativity), the aspects of the geometrical part of the logic of space-time changes — in going from the flat space-time to the curved space-time. But the algebraic features do not change. Thus the two-dimensional hermitian representations of special relativity theory which obey the algebra of quaternions, having as basis functions spinor variables, are still quaternion representations whose basis functions are spinor variables.

Einstein and Mayer then made the very important discovery that the spinor variable is the most primitive type to underlie a theory that is consistent with the principle of (special or general) relativity — irrespective of whether the theory is quantized or not! This is the 'most primitive' type of variable in the sense that while spinors may be combined to make up scalars and vectors, which in turn may be combined to make up tensors (of any rank), no other type of covariant variable may be made up into a spinor. The implication in the context of the physical laws is that a spinor formalism could yield all of the physical predictions of scalar, vector, tensor, ... formalisms, but the spinor formalism could make *extra predictions* that have no counterpart in the other formalisms of covariant theories. A well-known example of this is Dirac's discovery of an energy term in the electron equation that entails a coupling between the electron spin (in terms of its

magnetic moment) and an external magnetic field. This prediction, in the Hamiltonian of the spinor electron theory, has no counterpart in the scalar, vector or tensor formulations — unless they would be inserted ad hoc, as Pauli did in the Schrödinger equation to fit the results of the anomalous Zeeman effect.

Thus we have seen that an important mathematical implication, with physical consequences! of the principle of relativity (Axiom 1) is that the most primitive sort of covariant theory must be in terms of a two-component spinor formulation, whose field variables are analytic *everywhere.*

3.1.2. *The Generalized Mach Principle*

According to this principle, there is no manifestation of matter that is not expressible in terms of the dynamical coupling between the matter components of a closed system. The two 'coupled' quantities refer to two aspects of a single closed system that only *appears* to be disconnected into separate parts in some asymptotic limit (of sufficiently small energy-momentum transfer between them).

The implication here is that the simplest expression of a theory that fully exploits this idea must be in terms of at least two coupled equations. Let us refer to them, symbolically at first, as follows:

$$\hat{O}(1,2)\psi^{(1)}(x) = 0, \tag{3.1.4a}$$

$$\hat{O}(2,1)\psi^{(2)}(x) = 0. \tag{3.1.4b}$$

According to our conclusion of the preceding section, one of these field variables, say $\psi^{(1)}$, may refer to 'observer' and the other, $\psi^{(2)}$, would refer to the 'observed'. However, we also concluded that with the generalized Mach principle in a relativistic field theory, it should make no difference as to which is called 'observer' and which is called 'observed'. Mathematically, this implies that the form of the coupled equations (3.1.4a, b) should remain unchanged under the interchange of these variables, i.e.

$$\psi^{(1)} \leftrightarrow \psi^{(2)} \Leftrightarrow \text{Equation (3.1.4a)} \leftrightarrow \text{Equation (3.1.4b)}$$

It then follows that the (generally integral-differential) operator, $\hat{O}(1,2)$ must depend on $\psi^{(2)}$ in precisely the same (functional) way that $\hat{O}(2,1)$ depends on $\psi^{(1)}$. Since the operator $\hat{O}(2,1)$ depends on $\psi^{(1)}$, the solution $\psi^{(2)}$ of (3.1.4b) must also depend on $\psi^{(1)}$. But since the operator $\hat{O}(1,2)$ in (3.1.4a) depends on $\psi^{(2)}$, which in turn depends on $\psi^{(1)}$ through (3.1.4b), it follows that the solution $\psi^{(1)}$ of (3.1.4a) depends on an operator $\hat{O}(1,2)$ that also depends on $\psi^{(1)}$. Thus the coupled equations (3.1.4) are *necessarily nonlinear.* The nonlinearity of the equations are, indeed, a consequence of the elementarity of the interaction rather than the free particle, according to the generalized Mach principle. This is a fundamental difference between the

theory of matter developed here and the basis of the atomistic theories, such as classical mechanics or quantum mechanics.

Thus far, we have concluded that a theory which fully exploits Axioms 1 and 2 must be in terms of at least two coupled, nonlinear, spinor field equations. It was also concluded that the space-time coordinates, the argument x of $\psi^{(1)}$ and $\psi^{(2)}$, are the same — that is, these coupled solutions of the matter field equations are mapped in the same space-time. It is only in a limit of sufficiently small energy-momentum transfer between components of the system that these fields seem to be uncoupled, so that as an approximation they may be treated *one at a time.* In the latter case one considers $\psi^{(1)}(x_1)$ and $\psi^{(2)}(x_2)$, where x_1 and x_2 are the space-times for the assumed separate parts of the system, whose respective descriptions are in terms of $\psi^{(1)}$ and $\psi^{(2)}$. In this *mathematical approximation* for the closed system under study, we can superpose the two uncoupled fields in an eight-dimensional space-time, spanned by x_1 and x_2. This would be in the linear limit of the nonlinear field equations (3.1.4a, b). The explicit structure of these field equations in describing electromagnetically coupled matter will be derived in later chapters, showing how the latter interaction, in a generalized form, has explicit influence on the inertial manifestation of this matter. First, however, we must discuss a new conservation law that is logically implied by this theory, called 'conservation of interaction'.

3.2. The Conservation of Interaction

One explicit way of expressing the idea of the elementarity of interaction, which is logically implied by the generalized Mach principle, is to assert this idea in terms of the *conservation of interaction* [11]. The explicit function that appears in the mathematical expression of this law is then to represent a connective relation between the component spinor field solutions $\{\psi^{(i)}\}$ of the coupled nonlinear equations of the closed system.

Taking the interaction field amplitude $\Psi(\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(n)})$, for an n-body coupled system, to also transform as a spinor variable, the differential form of the law of conservation of interaction may then be expressed in the form of the continuity equation, using the bispinor notation as follows:

$$\partial^{\mu}(\overline{\Psi}\gamma_{\mu}\Psi) = 0 \quad (\overline{\Psi} \equiv \Psi^{\dagger}\gamma_0). \tag{3.2.1}$$

This equation of continuity implies that *within any local observer's frame of reference*, the quantity represented by the integral

$$\int \Psi^{\dagger}\Psi \, \mathrm{d}\mathbf{r}$$

is constant with respect to the time measure *in this reference frame.*

With the normalization of the interaction field amplitude Ψ, the positive-definite function $\Psi^{\dagger}\Psi$ may then be considered to play the role of a *weighting function.* It is interpreted here as relating to a weighting of the total interaction within a closed system, as described in a single space-time. Note that the conservation of interaction weighting does not imply that it is necessarily uniform throughout space and/or time. It does mean that, given a closed material system, the mutual interaction has a generally 'flexible' mapping in space-time that persists for all times in a local observer's measurements. Any alteration of the environmental conditions in a local region that may be made in some experimental investigation, for example, would then give rise to a redistribution of this weighting within the entire system. But any such alteration *within a closed system* cannot cause the weighting function to vanish anywhere, at any time, even though it may become arbitrarily weak in particular regions of space and/or time.

To exemplify the role of the interaction field and its physical implications consider the commonly referred to events of pair annihilation and creation. If matter should indeed be annihilated and created at arbitrary times and places, as it is commonly assumed to happen in quantum field theory, then the weighting function, relating to the density of interaction conservation, would no longer relate to a conserved quantity, i.e. in this case it would no longer be true that $\int \Psi^{\dagger}\Psi \, \mathbf{dr}$ would be constant in time. Thus, the field theory discussed in this monograph must *predict* all of the experimental facts that are conventionally interpreted as pair annihilation and creation — but without actually creating or annihilating matter at any time. These results will be demonstrated in Chapter 7.

It is also interesting to examine the interpretation of the conventional description of the hydrogen atom in quantum mechanics (or, more generally, many-electron atoms) within the framework of this theory. In this approach, while the nonlinear field equations for the *e—p* system do approach the exact Schrödinger form for hydrogen, in the nonrelativistic limit, the properties of hydrogen (or more complex atoms) must still be interpreted differently. The important quantity here is the weighting of the interaction between the electron component and the proton component of the *closed system, electron—proton,* rather than considering hydrogen as two singular particles of matter, perturbing each other at a distance. According to the view presented, the presence of the electron and the proton in the universe must be accounted for in terms of a *continuous field* that *weights* their mutual interaction. It follows from the solutions of the matter field equations, that relate explicitly to the inertial feature of matter, that the electron—proton interaction is weighted most heavily in the region of space that is a sphere with a radius caleld the 'first Bohr orbit'. With this interpretation, then, no reference need be made to the electron and proton as isolated entities. In this way, the 'atom' can be represented with a formalism based entirely on the continuous field concept, and is in strict accord with Axioms 1 and 2 — the *principle of relativity* and the *generalized Mach principle.*

It should be emphasized at this point that the discreteness of physical observables, such as the atomic energy levels, is, within this theory, only an *apparent discreteness*. For it is only within an approximation for the exact nonlinear equations of the theory that one arrives at the linearized eigenvalue equations for the atomic system — thereby leading to the predicted (apparent) discreteness of atomic energy levels, asymptotically (in accordance with Axiom 3). Thus, the proposed theory predicts that these energy levels are not in principle discrete, but that they do indeed have finite width, arising from the physical coupling with the remainder of the closed system. Since, according to Axiom 2, this coupling can never be totally 'off', the line widths for the spectral distribution of atomic energy levels, for example, can never be zero. That is to say, the values for the properties of matter (of any quantity, in any domain, from micro- to macromatter) have a continuous set of values, in contrast with the discrete set of values of all physical properties of micromatter, according to the quantum theory.

To exemplify further the contrast between the aspects of continuity and discreteness in physically measured properties, consider the operation of a Geiger counter. At first sight, this device functions in a way that appears to entail the occurrence of *discrete energy bundles* that enter the window of the counter at *random intervals of time*. One then associates the 'clicks' of the counter with the existence of discrete things that appear in acausal fashion. But it is clear that the data does not compel one to assert the idea that discreteness and randomness are ingredients that must underlie these phenomena, in terms of a fundamental explanation. After all, the Geiger counter is no more than an electronic instrument that has a voltage bias which is set by the experimenter *at a convenient level*, in order to discharge electrical energy whenever its interaction with some electrical signal exceeds some predetermined threshold voltage. As the voltage bias (and therefore the threshold for a 'click' to occur) is lowered, more clicks would be heard in a fixed amount of time. In the limit where there would be no bias voltage, the discrete clicks would wash out into a steady background 'noise'. That is, in this limit the 'signal-to-noise' ratio would be reduced to unity. Now, to interpret this 'noise' as a random set of effects of uncoupled things is to assume an ideal limit that cannot be directly verified in an experiment; it can only be postulated! Indeed, this is a postulate of indeterministic atomism that underlies the quantum theory. Still, the actual data does not compel one to adopt this model as providing a unique explanation. The 'noise' in this experiment could also represent the peaks of a continuously connected curve, totally predetermined, though there is not enough resolution to see this in this particular experimentation.

As I have emphasized earlier, the property of discreteness in the atomistic models is *abstracted* from the measurements of continuous (though peaked) values for the conserved properties of the system — continuous in the sense that between any two measured values of some property, no matter how close they are, one can always measure another value of this property. In

quantum theory, one says that the latter statement is not true in the ideal limit, but it is true in actual experimentation on micromatter. The reason given is that there is a finite, irreducible line width associated with all measured (actually!) discrete values of properties of microscopic matter — because of the Heisenberg uncertainty principle. In the theory of matter following from general relativity, developed in this monograph, the ideal, discrete limit, postulated in the quantum theory, does not exist. The finite (irreducible) line width here has to do with the nonlinear coupling of the components of the closed system. The main point to be made here is that whether the underlying abstract idealization is based on a theory that matter is fundamentally discrete with discrete eigenvalues for its properties, or a theory of matter based on the continuous field concept, where there are no discrete values of the properties of matter in any limit, is something that must be tested indirectly. That is, these are theoretical abstractions that can only be postulated and then logically and mathematically tested; they are not directly observable assertions.

To sum up, the clicks of a Geiger counter, the optical spectrum of a radiating gas, the collision experiments of Franck and Hertz, etc., clearly indicate the 'peaked' nature of the *interaction weighting* of the corresponding coupled systems. But the results of these experiments do not necessarily require the *conceptual basis* of the quantum theory for an explanation. Indeed, the nonlinear field theory of inertia that fully exploits the principle of relativity by starting with the idea of the elementarity of interaction (the *closed system*) rather than the elementarity of the particle (the *thing-in-itself*) does describe the same data — this time in terms of a continuous, though peaked set of values.

These results will be derived in Chapter 6 from the invariance properties in the continuous field description of matter, and a use of Noether's theorem. This field theory, as explained earlier, is expressed in terms of a set of coupled, nonlinear field equations that do not generally have the eigenfunction structure of the quantum mechanical formalism. The peaked values of the predicted properties of micromatter follow here from the conservation laws in the low-energy limit, where the nonlinear equations approach the formal expression of quantum mechanics, *as a linear approximation.*

Finally, an important question that relates to the interaction field as a weighting function has to do with the interpretation of the *Pauli exclusion principle*, and its deviation from the theory of inertia discussed here. That is: How, within the framework of a single field approach, does one interpret the statement of this principle, which appears to entail the correlation of the positions and momenta of each of the particles of a many-particle system? To answer the question, it will be shown in Chapter 6 that as a consequence of the self-consistency and full nonlinearity of the field equations of inertia for the interacting particle fields, the connective relation that is the interaction field amplitude Ψ, satisfying the continuity equation (3.2.1), vanishes

identically when two of the coupled fields out of an n-field system (1) have the same inertial mass, (2) have a mutually repulsive interaction and (3) are each in precisely the same state of motion. In other words, the interaction between two identical particles, each in the same state of motion, makes no contribution to the total interaction weighting amplitude for the entire closed system (although their separate interactions with the rest of the closed system do contribute to the observables). The physical implications of this theoretical result are identical with those of the *Pauli exclusion principle*, as it is used in the quantum theory of many-particle systems.

In conclusion, it should be emphasized that the *law of conservation of interaction* is not an extra postulate of this theory. It is, rather, logically necessitated by Axioms 1 and 2 that underlie the theory.

3.3. Determinism

An important philosophical implication of the relativistic field theory of matter that incorporates the generalized Mach principle (Axioms 1 and 2) is that such a theory necessarily entails the concept of determinism. This underlying aspect of the theory is, as we have discussed earlier, in sharp contrast with a fundamental feature of the quantum theory which asserts that nature is intrinsically nondeterministic. Let us now briefly discuss what is meant by this term.

What is *not* meant by determinism is simply an ordering of events *in time* — although the latter is a special case of the general concept. In the latter case, one says that there is a law of nature, e.g. wave mechanics, that is based on an *equation of motion*,

$$H\psi = i\hbar \frac{\partial \psi}{\partial t}.$$

The term 'motion' is defined here only in terms of the change of the wave function with respect to the time variable, t. The integration of this equation and the knowledge of the solution ψ at some initial time then tells us how this function is ordered along the time coordinate.

Another well-known example of an equation of motion in time is Newton's second law of motion

$$m \frac{\mathrm{d}^2 \mathbf{r}}{\mathrm{d}t^2} = \mathbf{F}.$$

With the knowledge of two boundary conditions (initial position and velocity), one can integrate the equation to give a unique solution, $\mathbf{r}(t)$ — the trajectory of a particle with mass m, subject to the external force $\mathbf{F}$.

In our more general meaning of 'determinism', these are only special cases

of ordering. Generally, 'deterministic' refers to the feature of a law of matter that all of the variables that describe the considered system are predetermined. That is to say, it is the assertion of a deterministic theory that there exists a complete description of the entire system, precisely mapped out in the most convenient parameter space. The usual parameter space is the four-dimensional space-time coordinate system. Sometimes the energy-momentum coordinate system is more convenient (e.g. in scattering problems).

In quantum mechanics it is said that the states of motion of a material system are not predetermined — thus it is a nondeterministic theory of matter. It is true that the Schrödinger equation is a perfectly ordered description and does indeed have a predetermined set of solutions. But this is only meant in the sense that these are not more than the elements of a *language* that a (macroscopic) apparatus would use in reporting about the physical properties of some microscopic system that is under investigation — *a language that has to do only with probability statements.* The salient point here is the assertion of the quantum theory that the fundamental variables that relate to the states of motion of elementary particles have only to do with probability statements made by a particular measuring apparatus about a particularly prepared microscopic system. These conclusions then lead to the feature of the quantum theory (as discussed earlier in Chapter 2) that the basic properties of matter are not predetermined — that they depend instead on the nature of the measurement — and that all of the values of these properties are not determinable simultaneously, arbitrarily or precisely. It is concluded in the quantum theory that the accuracy to which some of these variables can be known depends inversely on the precision with which other properties can be specified *at the same time*, from a measurement carried out with a macroscopic-sized apparatus. This is the Heisenberg uncertainty principle. Thus we see that the quantum theory is fundamentally subjective in nature and is nondeterministic.

All other theories in physics (preceding and contemporary) are fundamentally objective, i.e. the properties of matter are taken to be independent of the conditions under which they are observed, and they are deterministic.

The important question here is the following: Is the nondeterministic interpretation of the equations of quantum mechanics (a formalism that is certainly *empirically* correct in the low-energy region) *necessary* in order to understand microscopic matter?

It will be shown in the following chapter that one can, in fact, arrive at the mathematical formalism of nonrelativistic quantum mechanics as an asymptotic approximation for a continuous field theory of inertia — a theory that, among other things, depends on the concept of determinism, in the broad sense of the term indicated above. Thus, nondeterminism is *not* an *a priori* necessary conceptual ingredient in the laws of nature.

Chapter 4

A Covariant Field Theory of Inertia

4.1. On the Origin of Inertial Mass

A central problem in the conventional elementary particle theory that is based on the formal structure of quantum mechanics, and in Einstein's original theory of gravitation, has to do with the origin of the inertial mass of matter. To resolve the problem, it would perhaps be significant to keep in view two outstanding empirical features of inertial mass. The first one, which has been known since the earliest observations of planetary motion, is that the masses of all interacting matter seem to have the same polarity. This is evidenced by the observation that the force of gravity has always been observed to be attractive. The second feature of inertial mass, which has only been known since the onset of high-energy elementary particle research, is that the inertial mass values in the microscopic domain lie in a sharply peaked distribution of values, not unlike the spectral distribution of frequencies of a radiating gas. It is suggested in the present analysis that a satisfactory understanding of the origin of inertial mass may not emerge until *both* of these observed properties can be derived from a common theoretical base.

According to the quantum approach to mass, the mass value assigned to an elementary particle must follow as an eigenvalue of the group representations that relate to the most general symmetry of the underlying description of matter. These 'mass eigenvalues', in turn, relate to the invariant magnitudes of the energy-momentum four-vector for a free elementary particle of one sort or another. This is analogous to the derivation of the angular momentum eigenvalues that are identified with the frequency spectrum of a radiating gas of atoms, following from the eigenvalues of the representations of the rotation group — which underlies the symmetry of the binding (Coulomb) force of the atom.

It is well known that the representations of the symmetry group of special relativity theory (the Poincaré group) do not in themselves lead to a mass spectrum. Thus a great deal of contemporary research has been devoted to extending the space-time symmetry of relativity theory so as to include 'internal spaces' adjoined to the ordinary Euclidean space-time of the theory

of special relativity. Thus far, these theories have had limited success insofar as they have provided a phenomenological description that yields formulas that do give the masses of some of the elementary particles, e.g. the 'Okubo formula' [35]. However, difficulties still remain in this approach, the main one, in my view, being that as soon as one includes the Poincaré group with the new internal symmetry groups, in a single covering group, the mass spectrum derived previously disappears [36]. In addition, there does not seem to be a way of going from the theory of internal symmetries to the fact that the masses of all interacting particles have the same polarity — i.e. they all interact gravitationally with an attractive force, at least in the Newtonian limit of the gravitational theory.

The latter theoretical explanation of inertial mass, of course, is tied to Einstein's original conclusion from special relativity theory that 'the inertial mass of a body is identically a measure of its energy content', as shown in the derived formula, $E = mc^2$. I do not believe that it is valid, logically, to conclude from the latter formula that indeed 'mass is *equivalent* to energy', if this is intended to infer that this formula is an 'if-and-only-if' relation. My reason for this statement is that inertia *per se*, and energy *per se*, refer to conceptually *distinct features* of matter, in both the classical and Einstein theories [37].

On the one hand, inertia, *per se*, relates to the resistance to the change of state of motion of a body, or a matter field, as would be 'caused' according to particle physics, by an external force that acts on the given matter (i.e. a force that is rooted in *other matter*). In general relativity theory, with the incorporation of the Mach principle, the inertial mass of any quantity of matter is a measure of its dynamical coupling (as a component of a closed system) with all of the other matter components of that system. With this view, that I have argued in previous chapters must be incorporated in a general theory of matter based on general relativity, the inertial mass of matter is a *global property* of a component of a material (closed) system.

On the other hand, energy *per se* is defined as the work that a given quantity of matter is capable of doing. It is expressed in terms of the solutions of a particular sort of conservation law — a law that follows from the relativistic requirement that the totality of laws of matter must be covariant with respect to arbitrary continuous shifts of the origin of the time axis. This type of physical law refers only to 'local time' — *the time measure of a local observer.* In the global expression of the general form of general relativity theory, it is well known that there are no conservation laws. Thus, unlike the inertial mass concept, 'energy' is an undefined concept in global terms. The role of the energy concept in physics is that *energy differences* (rather than absolute quantities of energy) are among the theoretical predictions that are to be identified with the actual measured values for the properties of matter *in any local observer's frame of reference.* In general relativity theory, global tensors (i.e. generally covariant mathematical entities)

necessarily appear in the formal expressions of all of the laws of nature, that relate to energy, *per se, only asymptotically.* That is to say, these mathematical quantities do not become 'energy' until the proper limit is taken, *in an approximation* to a global expression, that corresponds to the special relativistic (i.e. local) limit of the more general expressions.

Thus we see that energy and inertial mass are conceptually different entities that are complementary features of matter, according to the full form of the theory of relativity. But this does not mean that the formula $E = mc^2$ is a scientifically false statement! It only means that, in the language of formal logic, it is an 'if—then' relation, rather than an 'if-and-only-if' relation. Thus, in principle, one must first derive the inertial mass of a quantity of matter from the global properties of a closed system. Once this is done, its local asymptotic limit is taken, expressing it as an averaged mass field, called m, which is then inserted into the formula $E = mc^2$ in order to derive the corresponding quantity of rest energy — i.e. the capability for that particular matter to do work, as expressed in a local observer's frame of reference. However, one may not proceed in the opposite direction — that of deriving the *global* feature of inertia from a *local* feature, energy. This would be to make the logically fallacious move of claiming to be able to derive a unique universal from one (or any number of) particular(s).

The approach to the origin of inertial mass, as a relative quality of a closed system of mass points, that was originally proposed by Mach, led to the *Mach principle* — an assertion that the origin of inertia is the dynamical coupling of a given quantity of matter with all of the other matter of a closed system [38]. This principle, when incorporated with Einstein's theory of general relativity, implies that the inertial mass of any interacting matter (whether it be an elementary particle or a galaxy of stars) must relate to the geometrical field properties of an underlying Riemannian space-time — in terms of a *continuous field* of mass, rather than the discrete mass eigenvalues of the quantum approach. Further, since the *conserved properties* of a physical system are not defined globally, the inertial mass in the quantum description, as the magnitude of the (locally) conserved energy—momentum of an elementary particle, is only a special (limiting) property of matter in the local domain. On the other hand, the Mach approach to inertial mass relates it to a coupling within the entire closed system, defining it only in a global sense. Whatever properties the inertial mass has in the local domain, according to our measurements, must then necessarily follow only as a *limit* of the more general global description in general relativity, according to this view.

It will be shown in this chapter that the variable that plays the role of inertial mass in the matter field equations, whose most general form corresponds to a two-component spinor equation in a curved space-time that goes into the Dirac form, locally, is a field that is *positive definite.* This result then implies that gravitational forces *in the Newtonian limit,* where Einstein's

field equations approach the form of Newton's equations for universal gravitation, are only attractive — in agreement with all of the experimental facts. It will be demonstrated further that when *gauge covariance* in imposed on the spinor matter field equations, a *nonpositive-definite* vector coupling term *necessarily* appears. That is to say, gauge covariance would be maintained in the matter field equations only if this vector coupling term, that is nonpositive definite, is present from the outset. The absence of the latter vector coupling term in the matter field equations, which would correspond to the 'free particle' equation, is then not compatible with the general structure of this theory of matter. When the *required* nonpositive-definite term is identified with electromagnetic coupling, we can conclude that electromagnetic forces, in contrast with gravitational forces, can be either attractive or repulsive, in agreement with the experimental facts, and that the theory excludes charge-neutral elementary matter.

Finally, it should be added that the requirement of the gauge covariance of the formal structure of the matter equations is the requirement of the theory that it include a law of conservation of interaction — an essential ingredient in the formal structure of this theory of inertia, because of the postulated elementarity of 'interaction' (rather than the elementarity of 'particle', as in the quantum theory), and the requirement that the theory contain expressions for the conservation of the sources of interactions.

From what has been said so far (to be proven below) the generally covariant field theory of inertia that is in terms of a two-component spinor formalism in a Riemannian space-time, already contains the predictions of two important experimental facts about the inertial and electromagnetic features of elementary interactions — that the inertial mass of interacting matter is always positive in the Newtonian limit of the theory, implying that gravitational forces in this limit can only be attractive, and that the electromagnetic force can be either attractive or repulsive. Let us now derive the explicit form of the inertial mass field from the postulates of this theory, by starting with the form of the spinor matter equations in special relativity and then globally extending to a curved space-time.

4.2. The Spinor Formalism in Special Relativity

The *irreducible* form of the first-order differential equations that lead to the form of quantum mechanics, in a linear limit, as expressed in terms of the Dirac equations, is the following (Majorana) form in terms of coupled two-component spinor equations:

$$(\sigma_\mu \partial^\mu + \mathscr{I})\eta = -\lambda_a \chi \tag{4.2.1a}$$

$$(\tilde{\sigma}_\mu \partial^\mu + \tau \mathscr{I})\chi = -\lambda_a \eta, \tag{4.2.1b}$$

where $\mathcal{J}$ is a functional that depends on the variables that couple to the matter spinor field (η, χ), representing the interaction between the latter matter field and those of all of the other interacting components of the physical system. For example, in the Dirac theory of charged particles in an electromagnetic field, the effect of all other coupled matter fields is summarized in the form of the electromagnetic potential A^μ, and the functional in the matter field equations is $\mathcal{J} = ie\sigma_\mu A^\mu$.

The time-reversals of the functions $\mathcal{J}$ and η are

$$\tau\mathcal{J} = \varepsilon\mathcal{J}^*\varepsilon, \qquad \tau\eta = \chi = \varepsilon\eta^*, \tag{4.2.1c}$$

where

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

is the two-dimensional Levi-Civita symbol. It has the property that its square is the negative of the unit two-dimensional matrix, $\varepsilon^2 = -\sigma_0$, where

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{4.2.2a}$$

The two-dimensional matrices

$$\sigma_\mu = (\sigma_0; \sigma_k), \qquad \tilde{\sigma}_\mu = (-\sigma_0; \sigma_k) \quad (k = 1, 2, 3)$$

are the basis elements of the quaternion first-order differential operator, $\sigma_\mu\partial^\mu$ and its conjugate operator, $\tilde{\sigma}_\mu\partial^\mu$, respectively. The three two-dimensional quaternion basis elements σ_k are the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4.2.2b}$$

The usual summation convention

$$A^\mu B_\mu = A^0 B_0 - \mathbf{A} \cdot \mathbf{B}$$

will be used henceforth.

The *parameter* λ_a is conventionally inserted into the matter equations (4.2.1), with a value that would fit the observed inertial mass value that is identified with the coupled spinor fields (η, χ). From a *topological* point of view, this parameter may also be interpreted in terms of a measure in the *mapping* (4.2.1) of the spinor field η to its time-reversed field χ. Since such a mapping must generally entail *continuous variables* in relativity theory, in accordance with the underlying symmetry group of this theory (as discussed above), the parameter λ_a could not be derived from special relativity, where it rather appears as a fixed number, for a given spinor matter field (η, χ).

The aim of the analysis of this chapter is then to *derive* the inertial mass term λ_a from the mapping between the time-reversed spinor matter field variables η and χ, when they are mapped in a Riemannian space-time. The *limiting form* of the general spinor matter field equations should then give the special relativistic form (4.2.1) of the matter field equations. It will be shown that such a form necessarily leads to the positivity of inertial mass parameter and the consequent prediction that *in the Newtonian limit*, the force of gravity can only be attractive.

4.3. The Spinor Variables in General Relativity

Just as the mapping of a vector field in a Riemannian space-time requires the introduction of a fundamental second-rank tensor field, $g_{\mu\nu}(x)$, so the mapping of a spinor field in a Riemannian space-time requires the introduction of a fundamental second-rank spinor field, $q^{\mu}(x)$. Its form is a consequence of the structure of the irreducible representations of the *Einstein group* — the group of continuous, analytic transformations that leave invariant the differential metric

$$ds^2 = g_{\mu\nu}(x)\,dx^{\mu}\,dx^{\nu}. \tag{4.3.1}$$

From a generalization of the quaternion number field, $\sigma^{\nu}x_{\nu}$, it follows from this group that the second-rank spinor fields, which have the algebraic structure of a quaternion field, have the following irreducible form in terms of two-dimensional Hermitian matrices:

$$q^{\mu}(x) = {}_{\nu}v^{\mu}(x)\sigma^{\nu} = \begin{pmatrix} {}_0v^{\mu} + {}_3v^{\mu} & -({}_1v^{\mu} - i\,{}_2v^{\mu}) \\ -({}_1v^{\mu} + i\,{}_2v^{\mu}) & {}_0v^{\mu} - {}_3v^{\mu} \end{pmatrix}. \tag{4.3.2a}$$

[The algebra and calculus of quaternions and spinors are developed in *GRM*, Ch. 3.] The quaternion field that is conjugate to this, corresponding to its time-reversal, is

$$\tilde{q}^{\mu}(x) = {}_{\nu}v^{\mu}(x)\tilde{\sigma}^{\nu}. \tag{4.3.2b}$$

The superscript indices, μ, on the quaternion variables refer to their behavior as (contravariant) four-vectors under the transformations of general relativity. In addition to their vector properties, the quaternions transform as second-rank spinors, of the type $\eta_{\alpha}\eta^{*}_{\dot{\beta}}$, where α and $\beta = 1, 2$ denote the spinor components.

Under time reversal, the quaternion variables map into their conjugates, since the time components change sign under conjugation, ${}_0v^{\mu}$, ${}_0v_{\mu} \rightarrow -{}_0v^{\mu}$, $-{}_0v_{\mu}$; it also follows from the property that q^{μ} are second-rank spinors of the

type $\eta \times \eta^*$ and from the form of the time-reversal transformations (4.2.1c) that

$$\tau q^\mu = \varepsilon(q^\mu)^* \varepsilon = \tilde{q}^\mu \tag{4.3.3}$$

and similarly for the conjugate covariant vector quaternions, q_μ.

Just as the affine connection $\{\Gamma^\rho_{\mu\nu}\}$ must be introduced in general relativity in order to define the derivative of a vector (or higher tensor) field covariantly in a Riemannian space-time, so the spin-affine connection $\{\Omega_\mu\}$ must be introduced in order to define the derivative of a spinor field in a Riemannian space-time, covariantly, expressing the 'covariant derivative' of the spinor variable as follows:

$$\eta_{;\mu} = \partial_\mu \eta + \Omega_\mu \eta. \tag{4.3.4}$$

The form of the spin affine connection variables follows from the vanishing of the covariant derivatives of the fundamental metric quaternion variables, q_μ, just as the components of the ordinary affine connection $\{\Gamma^\rho_{\mu\nu}\}$ follow in general relativity from the vanishing of the covariant derivatives of the metric tensor field $g_{\mu\nu}$. In this way we find that the spin affine connection has the following explicit form in terms of the quaternion metric field variables [see *GRM*, Ch. 3]:

$$\Omega_\mu = \tfrac{1}{4}(\partial_\mu \tilde{q}^\rho + \Gamma^\rho_{\tau\mu} \tilde{q}^\tau) q_\rho = -\tfrac{1}{4} \tilde{q}_\rho (\partial_\mu q^\rho + \Gamma^\rho_{\tau\mu} q^\tau). \tag{4.3.5}$$

Combining Equations (4.3.3) and (4.3.5) we find the following relation for the time-reversed spin-affine connection field:

$$\tau \Omega_\mu = -\Omega^\dagger_\mu, \tag{4.3.6}$$

where the 'dagger' denotes the hermitian conjugate of the field.

Consider now the following hermitian and antihermitian matrix fields:

$$\Lambda_\pm = q^\mu \Omega_\mu \pm \text{h.c.} \tag{4.3.7}$$

It follows from Equations (4.3.3) and (4.3.6) that the time-reversals of the preceding matrix fields are:

$$\tau \Lambda_\pm = \pm \varepsilon \Lambda^*_\pm \varepsilon. \tag{4.3.8}$$

Finally, the following identities are readily verified:

$$[(\tau \Lambda_\pm)\Lambda_\pm] = \mp (\det \Lambda_\pm)\sigma_0 = \pm |\det \Lambda_\pm| e^{i\delta} \sigma_0. \tag{4.3.9}$$

The *continuous set of values*, $\det \Lambda_\pm$, are real variables (mapped in a Riemannian space-time), $\delta = 0$ when $\det \Lambda_\pm < 0$ and $\delta = \pi$ when $\det \Lambda_\pm > 0$. From Equation (4.3.9) we may set up the following matrix equation:

$$[(\tau \Lambda_\pm)\Lambda_\pm]\eta = (2a_\pm)^2 \eta \, e^{i\delta} \tag{4.3.9'}$$

which, in turn, may be *factorized* as follows [39]:

$$\Lambda_+\left(\eta \exp\left[-\frac{i\delta}{2}\right]\right) = 2\alpha_+(\chi \exp[i\delta_+]) \tag{4.3.10a}$$

$$\tau\Lambda_+(\chi \exp[i\delta_+]) = 2\alpha_+\left(\eta \exp\left[\frac{i\delta}{2}\right]\right) \tag{4.3.10b}$$

$$\Lambda_-\left(\eta \exp\left[-\frac{i\delta}{2}\right]\right) = 2i\alpha_-(\chi \exp[i\delta_-]) \tag{4.3.10c}$$

$$\tau\Lambda_-(\chi \exp[i\delta_-]) = 2i\alpha_-\left(\eta \exp\left[\frac{i\delta}{2}\right]\right) \tag{4.3.10d}$$

where

$$(2\alpha_\pm)^2 = |\det \Lambda_\pm| \tag{4.3.11}$$

and η and χ are the time-reversed two-component spinor variables, related according to Equation (4.2.1c).

The factorization (4.3.10) of the matrix equation (4.3.9′) is unique only up to the arbitrary relative phase between η and χ. Thus, the decompositions above are independent of the phase factors $\exp[i\delta_-]$ and $\exp[i\delta_+]$ that are incorporated in χ in the factorized equations. Since these phases may be adjusted continuously without altering the form of the factorization, the equations (4.3.10) may be grouped so that the relative phases between η and χ in (4.3.10a, b) are the same as they are in (4.3.10c, d). In this way, Equation (4.3.10a) (or (4.3.10b)) may be added to Equation (4.3.10c) (or (4.3.10d)) without the need of explicitly specifying the relative phases.

Combining the definition of the matrix fields $\Lambda_\pm$ (Equation (4.3.7)) with the sum of Equations (4.3.10a) and (4.3.10c), the following relation between the time-reversed spinor variables is obtained:

$$q^\mu \Omega_\mu \eta = (\alpha_+ + i\alpha_-)\chi = \lambda\, e^{i\gamma}\chi \tag{4.3.12}$$

where

$$\lambda = \mathrm{mod}(\alpha_+ + i\alpha_-) = \tfrac{1}{2}[|\det \Lambda_+| + |\det \Lambda_-|]^{\frac{1}{2}} \tag{4.3.13}$$

and the phase $\gamma(x)$ in Equation (4.3.12) relates to the geometrical field for the entire closed system in accordance with the relation,

$$\gamma(x) = \tan^{-1}|\det \Lambda_-/\det \Lambda_+|^{\frac{1}{2}}. \tag{4.3.14}$$

The geometrical mapping between the time-reversed spinor fields that was sought is then given in Equations (4.3.12)—(4.3.14).

Finally, the time-reversed equation that accompanies (4.3.12) is obtained by taking the complex conjugate of the latter equation, and applying the Levi-Civita matrix as follows:

$$\varepsilon(q^\mu)^*\varepsilon\varepsilon\Omega_\mu^*\varepsilon\varepsilon\eta^* = \lambda\, e^{-i\gamma}\varepsilon\chi^*.$$

Combining this result with Equations (4.2.1c) and (4.3.5), it follows that

$$-\tilde{q}^\mu\Omega_\mu^\dagger\chi = \lambda\, e^{-i\gamma}\eta. \tag{4.3.15}$$

4.4. The Spinor Matter Field Equations in General Relativity

The generalization of the spinor matter field equations to their expression in a Riemannian space-time, which relates the two-component spinor variable to the first derivatives of its time-reversed field — a particular mapping that does not introduce any mass parameter at the outset — is obtained by considering the transformation properties of the term that is scalar in coordinate space, but a third-rank spinor:

$$q^\mu\eta_{;\mu}$$

Defining the interaction term $\mathscr{I}$ according to the equality,

$$q^\mu\eta_{;\mu} = -\mathscr{I}\eta \tag{4.4.1}$$

it follows from (4.3.4) and (4.3.12) that Equation (4.4.1) has the equivalent form:

$$q^\mu\partial_\mu\eta + \lambda\, e^{i\gamma}\chi = -\mathscr{I}\eta. \tag{4.4.2}$$

The time-reversal of this equation is:

$$\tilde{q}^\mu\partial_\mu\chi + \lambda\, e^{-i\gamma}\eta = -(\tau\mathscr{I})\chi, \tag{4.4.3}$$

where

$$\tau\mathscr{I} = \varepsilon\mathscr{I}^*\varepsilon$$

is the time-reversal of the interaction operator in (4.4.2).

Equations (4.4.2, 3) have the precise form of the Dirac two-component spinor equation (the Majorana equation), (4.2.1a, b), except for the presence of the phase factor, $e^{\pm i\gamma}$. In the next section it will be seen that if we impose gauge invariance, this factor is automatically transformed away. The *fundamental* reason for the requirement of gauge invariance is that, in accordance with Noether's theorem, it is a *necessary and sufficient* condition for the inclusion in the field theory of laws of conservation. If the latter inclusion is a requirement of the theory from the outset, because of the empirical facts in the local domain, then any field theory must necessarily be gauge invariant.

4.4.1. *Gauge Invariance*

Using the product rule for covariant differentiation and ordinary differentiation, we have,

$$\left(\eta \exp\left[-\frac{i\gamma}{2}\right]\right)_{;\mu} = \eta_{;\mu} \exp\left[-\frac{i\gamma}{2}\right] - \left(\frac{i}{2}\right)\eta \exp\left[-\frac{i\gamma}{2}\right]\gamma_{;\mu} \tag{4.4.4a}$$

and

$$\partial_\mu\left(\eta \exp\left[-\frac{i\gamma}{2}\right]\right) = (\partial_\mu\eta)\exp\left[-\frac{i\gamma}{2}\right] - \left(\frac{i}{2}\right)\eta \exp\left[-\frac{i\gamma}{2}\right]\partial_\mu\gamma. \tag{4.4.4b}$$

Multiplying Equation (4.4.4a) on the left by $\exp[-i\gamma/2]$, using the above two equations and subtracting the term $\frac{1}{2}iq^\mu(\partial_\mu\gamma)\eta \exp[-i\gamma/2]$, the following result is obtained:

$$q^\mu\left(\eta \exp\left[-\frac{i\gamma}{2}\right]\right)_{;\mu} + \left(\frac{i}{2}\right)q^\mu\eta \exp\left[-\frac{i\gamma}{2}\right](\gamma_{;\mu} - \partial_\mu\gamma) =$$

$$= q^\mu\partial_\mu\left(\eta \exp\left[-\frac{i\gamma}{2}\right]\right) + \lambda \exp\left[\frac{i\gamma}{2}\right]\chi =$$

$$= -\left(\not{\partial}\eta \exp\left[-\frac{i\gamma}{2}\right] + \left(\frac{i}{2}\right)q^\mu(\partial_\mu\gamma)\eta \exp\left[-\frac{i\gamma}{2}\right]\right).$$

Since γ is a scalar field, it follows that

$$\gamma_{;\mu} = \partial_\mu\gamma$$

so that the preceding relations take the form

$$q^\mu\eta'_{;\mu} = q^\mu\partial_\mu\eta' + \lambda\chi' = -\not{\partial}'\eta' \tag{4.4.5}$$

where

$$\eta' = \eta \exp\left[-\frac{i\gamma}{2}\right], \qquad \chi' = \chi \exp\left[\frac{i\gamma}{2}\right] \tag{4.4.6a}$$

and

$$\mathscr{S}' = \mathscr{S} + \tfrac{1}{2} i q^{\mu} \partial_{\mu} \gamma. \tag{4.4.6b}$$

Equations (4.4.6a) denote gauge transformations *of the first kind* and Equation (4.4.6b) is a gauge transformation *of the second kind.* Together they denote the group of transformations that underlie the *gauge invariance* of the matter field equations (4.4.5). A similar result is obtained for the gauge invariance of the time-reversed field equations (4.4.3). The final result, after applying the required gauge invariance of the matter field theory, are the spinor matter field equation and its conjugate equation:

$$\begin{aligned} (q^{\mu} \partial_{\mu} + \mathscr{S}) \eta &= -\lambda \chi \\ (\tilde{q}^{\mu} \partial_{\mu} + \tau \mathscr{S}) \chi &= -\lambda \eta. \end{aligned} \tag{4.4.7}$$

The field equations (4.4.7) are precisely in the form of the two-component spinor Dirac equation (the Majorana equation), *with interaction.* They are generally covariant under continuous coordinate transformations in a Riemannian space-time, and they are covariant with respect to the group of gauge transformations defined in Equation (4.4.6). Thus we have seen that the spinor matter field equations of the Dirac form are derived from a consideration of a particular mapping of time-reversed two-component spinor variables in a Riemannian space-time. The geometrical relation was seen to be in terms of a *positive-definite* field variable, λ, since it is the *modulus of a complex function.* The implication of this result in general relativity theory, together with the principle of equivalence, is that gravitational forces, in the Newtonian limit of Einstein's field theory applied to gravity, can only be attractive — thereby accounting for the empirical facts, in the Newtonian limit.

In the local limit of the generally covariant matter field equations (4.4.7), the quaternion fields have the limiting form of the unit two-dimensional matrix and the three Pauli matrices, $q^{\mu} \rightarrow \sigma^{\mu}$. In the exact limit of the flat space-time, the mass term goes to zero, since the spin-affine connection fields $\Omega_{\mu} \rightarrow 0$ in this limit. But as we asymptotically approach this limit, but do not reach it precisely, there is still a finite inertial mass λ, and in the special relativistic form of the matter field equations, this will be taken as a constant number, though assumed to be an averaged mass field. [This is discussed in more detail in Chapter 5, and in *GRM*, Ch. 3.] Thus, in the limit in which we *approach* special relativity, the matter field equations (4.4.7) take the two-component spinor (Majorana) form of the quantum mechanical equations (4.2.1).

The latter form of the formalism of quantum mechanics is covariant *only* with respect to the continuous space-time transformations of special relativity theory (*GRM*, Ch. 3). If one should now wish to recover reflection symmetry in space or time, Equations (4.2.1a, b) may be combined to yield the single

bispinor equation for quantum mechanics — called the Dirac equation. That is, with the following four-component 'bispinor',

$$\psi \leftrightarrow \begin{pmatrix} \eta + \chi \\ \eta - \chi \end{pmatrix},$$

Equations (4.2.1a, b) may be expressed in the (more restricted) form of the Dirac bispinor equation in special relativity:

$$(\gamma^\mu \partial_\mu + \mathscr{I})\psi = -\lambda_a \psi$$

where

$$\gamma^0 = \begin{pmatrix} \sigma^0 & 0 \\ 0 & -\sigma^0 \end{pmatrix}, \qquad \gamma^k = -i \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}, \quad (k = 1, 2, 3)$$

are the 'Dirac matrices'. When the interaction is such that $\mathscr{I} = \tau\mathscr{I}$, the Dirac equation above is covariant with respect to space or time reflections as well as with respect to the continuous transformations of special relativity theory.

It should be noted at this point that while Equations (4.4.7) are generally covariant (since they are equivalent to Equation (4.4.1) and its accompanying time-reversed equation) the inertial mass term λ does not appear as a constant parameter. It is rather in terms of a field variable, depending on the spin-affine connection field Ω_μ. The latter matrix field transforms covariantly as a four-vector, under the transformations in coordinate space of general relativity. However, Ω_μ does not transform covariantly with respect to its spin degrees of freedom; it rather transforms in the following fashion [see *GRM*, Ch. 3]:

$$\Omega_\mu \rightarrow \Omega'_\mu = S\Omega_\mu S^{-1} - (\partial_\mu S)S^{-1}$$

where $\{S\}$ are the group representations for the two-component spinor variables η, in general relativity, i.e. the irreducible representations of the *Einstein group*. It then follows that Ω_μ, of one reference frame, could be a null matrix, while in some other reference frame, Ω'_μ would not be a null matrix as long as we are in the curved space-time (i.e. where the group representations S are functions of space-time coordinates).

It is readily verified that with the transformation in spinor space of Ω_μ, indicated above, as well as its behavior as a covariant four-vector, and the behavior of the quaternion field q^μ as a contravariant four-vector, as well as transforming as a second-rank spinor of the type $\eta \times \eta^*$, that the Lagrangian density that leads to the field equations of the form (4.4.7)

$$\mathscr{L} \sim \eta^\dagger q^\mu \eta_{;\mu} + \text{h.c.}$$

is scalar. Indeed, the incorporation of the λ-term in the spin-affine connec-

tion with the quaternion variable (contracted), as $q^{\mu}\Omega_{\mu}$, as shown above, then means that the inertial mass term is a *necessary* addition to the Lagrangian density for the matter field equations, in order to ensure the relativistic covariance of the latter equations in general relativity.

To sum up, we see an important point of departure of the role of inertial mass here from its role in the quantum approach to elementary particle theory. For in the latter theory, the inertial mass parameter is taken to be related to the constant magnitude of the energy-momentum four-vector for a free elementary particle, but in the present field theory the inertial mass parameter relates to a *noninvariant* geometrical field, which has to do with the way in which the entire closed system of matter is coupled together. In the quantum theory, the inertial mass relates to a constant intrinsic *conserved* property of an elementary particle. But since the conservation laws are only defined in the local domain, in general relativity, inertial mass, in this view, would not be a general property of matter, but it would rather be a special feature that appears only in the local domain. On the other hand, the inertial mass, as it is defined in the spinor field theory discussed here, is explicitly a *global* property of matter, and thus has to do with the general features of the entire system. This is in accordance with the statement of the *Mach principle.*

To indicate further the mathematical connection of this theory of inertia with the Mach principle, it is noted that if the local limit is taken to be a flat space-time, precisely, corresponding in general relativity to a matterless universe — the true vacuum — then in this limit the spin-affine connection fields Ω_{μ} would be null matrices and, according to Equation (4.3.13), the inertial mass of the matter considered would be identically zero. Of course, this is because the inertial mass of matter relates to its coupling to *other* matter, which in turn is expressed mathematically in terms of the curvature of space-time. The vanishing of the spin-affine connection then corresponds to the curved space-time becoming a flat space-time, which is a representation in terms of geometry of the vacuum state. Still, it is valid to use the local (special relativistic) form (4.2.1) of the matter field equations when the physical conditions are appropriate, *as a mathematical approximation*, as it is indicated by the empirical facts. In these applications, the mass parameter λ_a, for the (almost) uncoupled sets of matter field equations (i.e. for those cases where the coupling between the interacting components of a closed material system is *asymptotically* weak) would actually represent an averaged mass field.

4.4.2. *Electromagnetic Coupling*

The imposed gauge invariance (4.4.6) of the spinor matter field equations (4.4.7) is necessitated by the field theory in order to incorporate an equation of continuity (and conservation laws).

It is apparent from the form of the gauge transformations (4.4.6b) that the

interaction term $\mathscr{I}$ must contain a term that has a vector coupling form. As in the Dirac theory, an identification of this term with *electromagnetic coupling* leads to predictions that are in agreement with properties of interacting, electrically charged matter. It should be noted, however, that the necessary presence of a vector coupling term does not exclude the natural appearance in electromagnetic forces of nonvector coupling (i.e. forces that also depend on the constant of coupling, e^2). Such terms will be discussed in the next chapter.

The application of the gauge transformations to the spinor matter field equations (4.4.7) gives the term

$$\tfrac{1}{2} iq^{\mu}(\partial_{\mu}\gamma),$$

where γ is the field (4.3.14) whose origin lies in the geometrical variables of the underlying Riemannian space-time. This result then implies that the spinor field matter equations will be gauge covariant if, and only if, there would be a term in the coupling field $\mathscr{I}\eta$ to cancel the latter vector term. Since the electromagnetic coupling (due to interacting matter) can become arbitrarily weak, due to imposed conditions, it follows that if the expression above is nonpositive definite, then the electromagentic coupling term in the generally covariant spinor matter equations (4.4.7) is also nonpositive definite.

The explicit form of $\partial_{\mu}\gamma$ is as follows:

$$\partial_{\mu}\gamma = \partial_{\mu} \tan^{-1} \left| \frac{\det \Lambda_-}{\det \Lambda_+} \right|^{\frac{1}{2}}$$

$$= \frac{[|\det \Lambda_+|\partial_{\mu}|\det \Lambda_-| - |\det \Lambda_-|\partial_{\mu}|\det \Lambda_+|]}{8\lambda^2|\det \Lambda_- \det \Lambda_+|}. \tag{4.4.8}$$

It is clear that the right-hand side of this equation can be positive, negative or zero; i.e. $\partial_{\mu}\gamma$ is nonpositive definite. In accordance with the preceding discussion, this result then implies that electromagnetic forces can be attractive, repulsive or null. This conclusion, which is in agreement with the empirical facts, is based on the assumption that the basic matter fields must be expressed most primitively in terms of spinor field solutions of the equations of the form (4.4.7).

The special case of *charge-neutral* matter would correspond in this theory to the situation in which one sets the coupling constant $e^2 = 0$, when representing electromagnetic coupling to be absent, i.e. $\mathscr{I} = 0$. But if this situation could be realized then there would be no term in the interaction functional $\mathscr{I}$ (proportional to e^2) to subtract from in order to form the gauge transformed functional $\mathscr{I}'$, as shown in Equation (4.4.6b). That is, in the case of charge-neutral matter, within the context of this field theory, there could not be gauge invariance of the second kind. It would then follow that

there could not be gauge invariance of the first kind (4.4.6a). On the other hand, the covariance with respect to gauge transformations of the first kind is required, as discussed previously, in order to maintain an incorporation in the field theory the laws that correspond to the continuity equation and conservation. Since the latter is a conceptual and an empirical requirement of the theory from the outset, it must then be concluded that within the context of this theory of inertia, there cannot be charge-neutral matter, as elementary components of a system. The experimental data that is normally interpreted as charge-neutral elementary matter (such as the neutron, the neutrino, π^0, K^0, etc.) must in fact be composites of component charged matter fields, whose net charge adds up to zero.

4.5. Matter and Antimatter

Empirically, there is a symmetry in the universe between oppositely interacting charged matter, such as electrons and protons and their respective antimatter relatives, positron and antiproton. The members of the particle–antiparticle pairs: electron—positron, proton—antiproton, etc., are equally massive particles, but they interact electromagnetically with a given electrical charge oppositely. That is, if the electron is attracted to some charged matter, the positron would be repelled by it, etc. From these data, it is conventionally concluded that the electron has a negative elementary charge, $-e$, while the positron has a positive elementary charge $+e$ (the proton has a positive elementary charge $+e$ while the antiproton has the negative charge $-e$). In this section it will be proven that within the context of this field theory of inertia, the equality of masses of the particle and antiparticle necessarily implies that they would interact oppositely with given charged matter, thus demonstrating the proof of a force symmetry exerted by equally massive components of matter, between repulsive and attractive forces. While this result does not entail the assumption of opposite polarities assigned to matter and antimatter particles, as free entities, it nevertheless implies all of the physical consequences of the latter particle model.

The analysis of the particle—antiparticle coupling that corresponds conventionally to the model that assumes 'pair annihilation' and 'pair creation' will be shown in Chapter 7, demonstrating a *derivation* of all of the experimental facts associated with these phenomena, though without actual 'annihilation' or 'creation' of matter.

4.5.1. *Proof of Force Symmetry of Matter and Antimatter* [40]

The difference between the electron and positron (or any other matter and antimatter) interactions with electromagnetically coupled matter can be understood in this theory in terms of a combination of Equation (4.3.13) for

the inertial mass of charged matter (or antimatter) and Equation (4.4.8), which determines the sign of the electromagnetic interaction with other charged matter. That is, if the function $\partial_\mu \gamma$ is positive (or negative) then the electromagnetic interaction would be attractive (or repulsive).

Since the electron (matter) and the positron (antimatter) have equal inertial masses, by definition, then

$$\lambda_{e^-} = \tfrac{1}{2}[|\det \Lambda_+| + |\det \Lambda_-|]^{\frac{1}{4}}_{e^-}$$
$$= \lambda_{e^+} = \tfrac{1}{2}[|\det \Lambda_+| + |\det \Lambda_-|]^{\frac{1}{4}}_{e^+}. \tag{4.5.1}$$

For this equality to hold, either

$$|\det \Lambda_\pm|_{e^-} = |\det \Lambda_\pm|_{e^+} \tag{4.5.2a}$$

or

$$|\det \Lambda_\pm|_{e^-} = |\det \Lambda_\mp|_{e^+}. \tag{4.5.2b}$$

The first case, (4.5.2a), is inadmissible since it would imply that $\partial_\mu \gamma$, Equation (4.4.8), is identical for the electron and positron interaction with an electron, in contradiction with the assumption that the electron and positron are distinguishable elementary components of matter. Thus, the only acceptable solution is Equation (4.5.2b). According to Equation (4.4.8) this result implies that

$$(\partial_\mu \gamma)_{e^-} = -(\partial_\mu \gamma)_{e^+}, \tag{4.5.3}$$

thereby predicting that if there are two equally massive spinor particle fields, one of them could interact attractively and the other repulsively with a given charge. This result corresponds to the assertion that there is a symmetry in the universe between matter and antimatter components of an interacting system — a symmetry between the existence of electrons and positrons, between protons and antiprotons, etc. This assertion is in agreement with the experimental outcome predicted by the standard elementary particle theory of matter and antimatter as being attractive and repulsive to other electrically charged matter. But in the standard theory, one must postulate the latter result, while it was derived above from first principles.

4.6. On the Quantization of Electrical Charge from General Relativity [41]

One of the outstanding features of elementary matter that must be explained by any theory that claims to correctly relate to microscopic matter in the quantum domain is the apparent quantization of electrical charge. That is, there are no elementary particles that have a charge other than $\pm e$, and the

nuclei of the atomic species have the discrete charge values e, $2e$, $3e$, . . . , Ze, where Z is an integer (equal to the number of electron charges surrounding the atomic nucleus in the charge-neutral atom).

Noting that that the actual observations are not precisely this, but rather that the values of the electromagnetic coupling constant appears in a discrete set, e^2, $2e^2$, . . . , Ze^2, . . . , it will be shown in this section how this result follows from the irreducible representations of the symmetry group that underlies general relativity theory — the 'Einstein group' — as they asymptotically approach those of the symmetry group of special relativity theory — the Poincaré group — as the curved space-time of general relativity approaches the flat space-time of special relativity.

One of the important features of the Einstein group is that in the limit, as the curvature of space-time approaches zero — corresponding physically to an approach to the 'vacuum situation' — its representations correspondingly approach those of the Poincaré group of special relativity $\{P\}$, where these satisfy the equation

$$P^{\dagger}\sigma_{\mu}P = (\partial x^{\mu'}/\partial x^{\nu})\sigma_{\nu}. \tag{4.6.1}$$

The solutions of this equation are found to be the double-valued functions of the parameters $\theta_0^{\mu\nu}$ that characterize the space-time transformations of special relativity. They are:

$$P(\mu\nu, 0) = \exp[\tfrac{1}{2}\,\sigma_{\mu}\sigma_{\nu}\theta_0^{\mu\nu}]. \tag{4.6.2}$$

The notation used above, with '0' in the argument of P, denotes that these irreducible representations are the same at all space-time points — since the transformation coefficients, $(\partial x^{\mu'}/\partial x^{\nu})$, which are the transformations that leave the differential of special relativity,

$$ds^2 = dx_0^2 - dr^2$$

invariant, are linear transformations. The indices in Equation (4.6.2), '$\mu\nu$', are not summed over.

The representations of the Einstein group, underlying the invariance of the Riemannian differential

$$ds^2 = g^{\mu\nu}(x)\,dx_{\mu}\,dx_{\nu},$$

are the *global extension* from the linear transformations of the Poincaré group to the nonlinear transformations of the Einstein group. Since the flat space limit of the quaternion metrical field variable, $q_{\mu}(x)$ (as discussed in the preceding section on the inertial mass derivation) is σ_{μ}, and the global extension of the space-time independent parameters, $\theta_0^{\mu\nu}$, are the space-time dependent parameters $\theta^{\mu\nu}(x)$, it follows that the global extension of the representations (4.6.2) are the Einstein group representations:

$$P(\mu\nu, 0) \rightarrow E(\mu\nu, x) = \exp[\tfrac{1}{2}\,q_{\mu}(x)q_{\nu}(x)\theta^{\mu\nu}(x)]. \tag{4.6.3}$$

The factorization of Einstein's field equations that has been demonstrated in the research program on elementary particle physics from general relativity yields the following quaternion metrical field equations, relating the geometry of space-time to the interactions of matter [*GRM*, Ch. 6]:

$$\tfrac{1}{4}(K_{\mu\lambda}q^{\lambda} + q^{\lambda}K^{\dagger}_{\mu\lambda}) + \tfrac{1}{8}Rq_{\mu} = k\mathcal{T}_{\mu} \tag{4.6.4}$$

where

$$K_{\mu\lambda} = (\partial_{\mu}\Omega_{\lambda} + \Omega_{\mu}\Omega_{\lambda} - \partial_{\lambda}\Omega_{\mu} - \Omega_{\lambda}\Omega_{\mu}) \tag{4.6.5}$$

is the spin curvature, defined in terms of the second covariant derivatives of a spinor field,

$$\eta_{;\lambda;\mu} - \eta_{;\mu;\lambda} = K_{\mu\lambda}\eta, \tag{4.6.6}$$

R is the Riemann scalar, which is the following in terms of the quaternion metrical field variables:

$$R = \tfrac{1}{2}\,\mathrm{Tr}[\tilde{q}^{\mu}K_{\mu\lambda}q^{\lambda} + q^{\lambda}K^{\dagger}_{\mu\lambda}\tilde{q}^{\mu}], \tag{4.6.7}$$

where 'Tr' denotes the trace, and $\mathcal{T}_{\mu}$ on the right-hand side of the metrical field equations (4.6.4) is the matter field source of the metrical field, as determined from the variational derivatives of the matter field Lagrangian density with respect to the quaternion variables.

When the matter content of a physically closed system is sufficiently rarefied, so as to allow the use of a 'particle approximation' to this field theory of matter, then the quaternion variables may be expressed in terms of the linear approximation,

$$q_{\mu}(x) \simeq \sigma_{\mu} + \varepsilon\Lambda_{\mu}(x) \tag{4.6.8}$$

and the transformation parameters may be expressed as

$$\theta^{\mu\nu} \simeq \theta^{\mu\nu}_{0} + \varepsilon\theta^{\mu\nu}$$

where the expansion parameter ε, which is a measure of the curvature of space-time in the vicinity of the 'observed matter', depends on the coupling to this matter from the other matter of a closed system; when these forces would be electromagnetic in origin, $\varepsilon \propto e^2$.

With the linear approximation (4.6.8) in Equation (4.6.3) for the irreducible representations of the Einstein group, we have the following form for its asymptotic limit in first order in ε:

$$E(\mu\nu, x)_{a} = P(\mu\nu, 0)P(\mu\nu, \varepsilon)\,[1 + \varepsilon(\sigma_{\mu}\Lambda_{\nu}(x) + \Lambda_{\mu}(x)\sigma_{\nu})\theta^{\mu\nu}_{0}] \tag{4.6.9}$$

where $P(\mu\nu, 0)$ are the representations (4.6.2) of the Poincaré group of special relativity, and

$$P(\mu\nu, \varepsilon) = \exp[\tfrac{1}{2}\varepsilon\sigma_{\mu}\sigma_{\nu}\theta^{\mu\nu}(x)]$$

are the infinitesimally displaced representations of the Poincaré group.

The main focus of attention here is on the part of the representation E_a that has the product form:

$$P(\mu\nu, 0)P(\mu\nu, \varepsilon)\,[\sigma_\mu\Lambda_\nu(x) + \Lambda_\mu(x)\sigma_\nu]\theta_0^{\mu\nu}. \tag{4.6.10}$$

It is in this term where the parameters that characterize the *interaction* between 'observed' elementary matter and its environment of 'other matter' appears. The quaternion $\Lambda_\mu(x)$ is the solution of the linearized form of the field equations (4.6.4). That is, the curvature parameter ε, in first order, is proportional to the electromagnetic coupling constant (the 'fine structure constant'), e^2, when the quaternion factorization $\mathscr{T}_\mu$ of the energy–momentum tensor source of Einstein's field equations represents electromagnetic coupling alone.

We see, then, that while the eigenvalues of the first part of E_a are the 'spin' quantum numbers — which may be generated to other half-integral values and to integral values, to arbitrary magnitude in accordance with the Lie algebra that accompanies this group — so the eigenvalues of the matrix representation (4.6.10) of the Einstein group may be seen as proportional to a set of discrete *integral numbers* that multiply the constant e^2. The form $\Lambda_\mu\sigma_\nu$ is a second-rank quaternion whose basis functions transform as those of the direct product representation, $\mathscr{D}^{\frac{1}{2}} \times \mathscr{D}^{\frac{1}{2}}$, thereby implying that the quantum numbers multiplying e^2 may only be generated in *integral steps*, in accord with the Clebsch–Gordan series for this product of spin-$\frac{1}{2}$ representations. This is the prediction that, of course, is in agreement with the empirical facts about the charge quantization of elementary matter.

The idea suggested here is that the origin of the quantization of the electric charge of elementary matter is in the features of the symmetry group that prescribes the general covariance of the laws of nature in a curved space-time. Implications of this result are that (1) the 'noninteracting limit' when, in principle, elementary matter is in a true vacuum, it has no associated electric charge, and (2) the actual discrete (eigenvalue) nature of the electrical charge of elementary matter is not in principle real; it is rather a weak coupling approximation for a continuously distributed interaction field — even though it does become arbitrarily sharply peaked as the mutual interaction becomes correspondingly weak. But it is important to note that the actual limit of zero coupling (that is, going *exactly* from general relativity to special relativity, from the Einstein symmetry group of a curved space-time to the Poincaré symmetry group of a flat space-time) corresponds to a limit of zero electrical charge, in contrast with the conventional elementary particle theories in quantum mechanics, where the electrical charge e is an intrinsic constant that does *not* vanish in this limit (or in any other limit). This difference is indeed crucial in providing a test that could differentiate between the implications of the conventional quantum approach to elementary matter and the theory of elementary interactions expressed here, based on a continuum, nonlinear field theory in general relativity.

4.7. Conclusions

It has been shown in this chapter that a continuous mapping of a particular sort (in terms of the first derivatives of a two-component spinor variable) to its time-reflected spinor (or, equivalently, to its space-reflected spinor), in a Riemannian space-time, *necessarily* leads to the generally covariant form of the two-component spinor Dirac equation (4.4.7) — which in the special relativistic limit takes the form of the quantum mechanical equations. A *field*, λ, Equation (4.3.13) is *derived* from this mapping, and appears where the mass parameter (divided by $\hbar c$) is normally inserted into the matter field equations. The derivation reveals the mass field as the modulus of a complex variable, thus λ is found to be a positive-definite function. In the Newtonian limit of general relativity theory, where the gravitational force appears, it depends on the product of the interacting masses, $M_1 M_2$; the positivity of the mass of micromatter, λ, and the composition of any macroscopic quantity of mass, M, in terms of a large sum of λ's, then predicts that the force of gravity *in the Newtonian limit* can have only one sign, i.e. the force of gravity, under this condition, is always attractive or it is always repulsive. Observing it to be attractive in only one case, such as the attraction of our bodies to the Earth's surface to give us 'weight', then predicts from this theory that in this same (Newtonian) limit, the force of gravity must always be attractive. This prediction is in agreement with all of the known experimental facts. It is a result of a derivation of this theory that is not proven from first principles from Newton's original theory of universal gravitation or from Einstein's tensor formulation of general relativity. In the latter theories, the attractiveness of the gravitational force is inserted, rather than derived, in order to fit the known data.

The second significant result of this analysis is the *necessary* appearance of a *nonpositive-definite* vector coupling term in the field equations of inertia — the matter field equations in their (irreducible) spinor form. Because the logical structure of this field theory of inertia requires an incorporation of a continuity equation (a form of laws of conservation), and because gauge invariance of this formalism is a necessary and sufficient condition for the incorporation of a continuity equation (following from Noether's theorem), it follows that gauge invariance must be imposed on this field theory of inertia. From this requirement a vector coupling term must appear and this may be identified with electromagnetic interaction. With this identification and the proof that the vector coupling term is nonpositive definite it follows that electromagnetic forces may be attractive or repulsive, depending on features of the geometrical field of interacting matter. Similar to the result in regard to the pure attractiveness of gravitational force in the Newtonian limit, the *derived* result that electromagnetic forces can be attractive or repulsive followed from first principles in this theory whereas it must be inserted in the conventional theories in order to match the empirical data. It also followed

from the same field theory of inertia that there cannot be any charge-neutral elementary matter, thus predicting that any seemingly neutral matter, such as neutrons and neutral pions and kaons, must be defined here in terms of composites of charged matter fields whose total charge adds up to zero.

A third important result found here was that the inertial mass of any 'observed matter' relates directly to all of the matter content of a physically closed system, in terms of the curvature fields of the system. In the limit, then, where all of the 'other' matter of the system, except for the 'observed matter' would vanish, the curvature of space-time at the site of the observed matter would correspondingly vanish, and the inertial mass of the latter would go to zero. That is, it is shown in this theory of inertia, in explicit fashion, how the mass of matter is dynamically related to all of the other matter of a system that it couples to — in accordance with the general statement of the *Mach principle* about the origin of inertial mass.

Finally, two more significant *derived* results from this theory are (1) that there is a symmetry in nature between matter and antimatter — i.e. elementary matter fields with numerically equal inertial masses but oppositely polarized electrical coupling with other charged matter, and (2) the quantization of the electromagnetic coupling of elementary matter, as e^2, $2e^2$, $3e^2$, . . . , ne^2, ... The latter result was found to follow from the asymptotic properties of the irreducible representations of the Einstein group of general relativity, as they approach the representations of special relativity, to first order in the constant e^2.

In the next chapter, the full, explicit form of the electromagnetic interaction functional $\mathscr{I}$ will be derived, in accordance with fully exploiting the symmetry of this theory of inertia in general relativity. We will then be in a position to derive the full form of the field equations of inertia whose limit yields the formal expression of quantum mechanics, as a linear approximation. These results will then be applied, specifically, to the case of the hydrogen atom and to an exact solution for the particle—antiparticle pair that yields all of the physical results normally interpreted in terms of pair annihilation and creation — though without actually annihilating or creating matter according to this field theory.

Chapter 5

The Electromagnetic Interaction

5.1. On the Meaning of the Electromagnetic Field Equations

The formulation of a covariant field theory of inertia that gives quantum mechanics as a linear approximation has been shown in the preceding chapter to necessarily entail an incorporation of the electromagnetic force (minimally) as well as gravity. The gravitational manifestations of interacting matter are expressed in this theory in terms of the quaternion metrical field, which was derived and discussed in detail in *General Relativity and Matter* [*GRM*, Ch. 6]. The general form of the electromagnetic field theory, according to general relativity, was also examined in detail in *GRM* (in Chapter 5). In this chapter we will briefly review the latter development of electromagnetism for the purpose of its necessary incorporation in the matter field equations, in the following chapters. The latter incorporation is a crucial ingredient in the derivation of important features of the field equations of inertia, such as the derivation from first principles of the Pauli exclusion principle (in Chapter 6).

In view of the logical implication of the *generalized Mach principle*, regarding the elementarity of the interaction, rather than the free particle of matter, there follows an interpretation of the Maxwell field equations of electromagnetism that differs from the usual one (though coming closer to Faraday's original meaning of the field). The interaction is described here in terms of the coupling of field variables associated with the components of a *closed system*. Electromagnetic phenomena are expressed in terms of two types of field variables. One set relates to the field intensity, conventionally associated with the electric and magnetic *force potential* of 'influencing' electrical matter; the other set of variables relates to the 'source fields' conventionally associated with the charge density and its motion of the 'influenced' electrical matter. According to the interpretation of the electromagnetic field equations advocated here, they are no more than a (covariant) prescription for determining one of these types of field variables in terms of the other. Thus, the electromagnetic field equations are interpreted as *an identity*. The actual physical observables are not described until the variables

representing one of the interacting components of electrical matter are coupled to the variables of the other interacting components of the physical system, that is *closed* through mutual forces.

With this interpretation it follows that for each component of the closed system there is a separate set of electromagnetic field equations. It also follows that a matter component of the closed system cannot be allowed to interact with its own force field, since such a description would be redundant within this interpretation. The disappearance of these *self-energy* terms automatically removes divergent quantities that appear in the conventional quantum field theory, having their roots in the classical Lorentz theory of electrodynamics. Thus, these divergences do not have to be removed, according to this theory they are not present from the outset!

Two other important deviations from the conventional theory of electrodynamics are (1) if the source fields should vanish, then the field intensities associated with them must correspondingly vanish, and vice versa, and (2) the vector representation of the electromagnetic field theory may be reduced to the two-component spinor representation, as an irreducible representation of the symmetry group of relativity theory. We will now discuss the implications of these points in more detail.

The first deviation above implies that the homogeneous solutions of the electromagnetic equations are to be rejected as unphysical — these are the 'radiation' solutions of the equations with no source fields. Thus, the idea of a source-free radiation field, *and the photon*, as an elementary particle, must be abandoned here. The photon concept is replaced in this theory with the *process* of energy-momentum transfer between systems of charged matter, that may be identified with 'emitter' and 'absorber' — which are in terms of fundamental matter fields. It follows from this conclusion that an atom in an excited state could not decay spontaneously, with the emission of a 'free photon'. That is to say, the theory logically implies that an excited atom will only emit a signal (a quantity of energy-momentum transfer) if there is present in the given system another atom to absorb that signal, at the later time when it gets to the place where it is. This concept is referred to in the literature as *delayed-action-at-a-distance*; this is because in this view an atomic system would not emit a signal in the first place if there would not be an absorber somewhere else, at the later time when the signal would get there. But note that the major difference between the previous delayed-action-at-a-distance theories and the field theory proposed here is that the former are particle theories, in which one describes n discrete charged matter trajectories in a $4n$-dimensional space-time, [69] while this theory is based on the continuous field concept in which one may have n coupled fields, all mapped in the same four-dimensional space-time.

It might be remarked at this point that one motivation for the rejection of the photon, as a bona fide interacting particle, is the fact that the interpretation of most experimental phenomena that are conventionally attributed to

photons can be equally explained in terms of the transfer of continuum electromagnetic fields of force between quantized matter that interacts over a large distance. This applies to the Compton effect, the photoelectric effect, bremsstrahlung, etc. But there are two sets of experimental data conventionally attributed to the manifestations of free photons in which there is no matter around to play a role, at least for a part of the time. One of these is the blackbody radiation spectrum. The other is the data conventionally attributed to the annihilation of a particle—antiparticle pair, with the simultaneous creation of two photons. It will be seen in Chapter 7 how both of these effects are predicted from a particular bound state of the particle—antiparticle pair. Thus, according to this theory, matter is not annihilated (nor created), nor are photons created from matter to explain the data relating to the 'annihilation phenomenon'.

The deviation from the standard vector representation of the electromagnetic field theory, in terms of Maxwell's equations, by factorization to the two-component spinor representation leads to a true generalization in the sense that all of the physical predictions of the usual Maxwell theory follow here, but there are additional physical predictions that follow naturally in the spinor formalism that have no counterpart in the standard formalism — analogous to Dirac's prediction of the coupling of the electron spin to an external magnetic field — a generalization that has no counterpart in the Klein—Gordon equation (whose factorization leads to the Dirac equation). Indeed, it is the set of extra predictions that follow in this way from the spinor expression of the electromagnetic interaction that lead to new, testable physical predictions about the behavior of elementary matter at high energy-momentum transfer, that either are not predicted at all by the standard quantum theory, or are predicted in the standard theory in a mathematically unsatisfactory way. In Chapter 8 it will be seen that an example of the latter is the prediction here, in a mathematically satisfactory way, of the experimental data associated with the Lamb shift.

5.2. Generalization of the Elementary Interaction Formalism

According to the interpretation of the Maxwell equations as identities, it follows in accordance with our preceding discussion that these equations must be labeled for each of the interacting components of a closed system,

$$\begin{aligned} &\nabla \times \mathbf{E}^{(u)} + \partial^0 \mathbf{H}^{(u)} = 0, \quad \nabla \cdot \mathbf{H}^{(u)} = 0 \\ &\nabla \times \mathbf{H}^{(u)} - \partial^0 \mathbf{E}^{(u)} = 4\pi \mathbf{j}^{(u)}, \quad \nabla \cdot \mathbf{E}^{(u)} = 4\pi \rho^{(u)} \end{aligned} \tag{5.2.1}$$

where (u) stands for the uth interacting field component of the physically closed system.

The conclusion above, that it would be logically inconsistent within this

theory to allow a field to interact with itself implies that the conservation equations that accompany the field equations (5.2.1) must be generalized in the following way, for the respective conservation laws of the energy and momentum in the electromagnetic field:

$$\left(\frac{1}{8\pi}\right)\partial^0 \sum_{u \neq v} (\mathbf{E}^{(u)} \cdot \mathbf{E}^{(v)} + \mathbf{H}^{(u)} \cdot \mathbf{H}^{(v)}) + \left(\frac{1}{4\pi}\right)\nabla \cdot \sum_{u \neq v} (\mathbf{E}^{(u)} \times \mathbf{H}^{(v)})$$

$$= - \sum_{u \neq v} \mathbf{E}^{(u)} \cdot \mathbf{j}^{(v)}$$

$$\left(\frac{1}{4\pi}\right)\partial^0 \sum_{u \neq v} (\mathbf{E}^{(u)} \times \mathbf{H}^{(v)}) = \sum_{u \neq v} (\rho^{(u)}\mathbf{E}^{(v)} + \mathbf{j}^{(u)} \times \mathbf{H}^{(v)}),$$

where, again, all fields are mapped in the same space-time coordinate system.

Clearly, as the number of interacting field components increases indefinitely, the corresponding macroconservation laws (5.2.1′) start to lose sight of the underlying grid (labeled by (u) and (v)) and the resulting equations subsequently 'blur' into the standard form of the conservation equations (where these labels do not appear). In the latter form, the electromagnetic field intensities, **E** and **H**, stand for the sums over such variables with the (u), (v) indices. Thus, the conservation equations (5.2.1′) do not differ in their predictions from the standard equations of the Maxwell field theory applied to low-energy, macroscopic phenomena, such as Ohm's law, the reception and transmission of radio communication, the scattering of radio waves from metallic or dielectric substances, shaped in one way or another, etc. On the other hand, these macroscopic phenomena would have to be interpreted in a different way, in principle. For example, in the conventional interpretation of the Maxwell theory, one would assert that a radio transmitting antenna emits a signal at some time t, the signal then proceeds *on its own*, and at the later time, $t + R/c$, another antenna, R meters away, absorbs this signal. The later event of absorption is said to be independent of the emitting antenna.

In contrast, the elementary interaction approach, in principle, considers the emitting antenna and the absorbing antenna *together*, in terms of their *mutual influence* — i.e. one may not reject the reaction of the emitting antenna to its coupling to the absorbing antenna (or to a city full of absorbing radio antennas). In practice, of course, this coupling is certainly sufficiently weak that one may consider that the approximation of *uncoupling* is accurate, then allowing one to use the solutions of the uncoupled equations. This case corresponds to the *limit* of sufficiently small energy-momentum transfer, in which the predictions of the standard electromagnetic conservation equations and the generalized equations (5.2.1′) would merge. The latter corresponds to Faraday's original conception of the field (for the 'emitter')

and the uncoupled test charge (the 'absorber'). But the general consideration in terms of an unbreakable mutual coupling, *in principle*, is in accord with the spirit of the dynamical claim of Newton's third law of motion. It is also the basis of a fundamental nonlinearity of the field equations that unify the electromagnetic equations with the dynamics of the matter fields of electrically charged substance.

5.3. A Spinor Formulation of Electromagnetism

The Maxwell formulation of the electromagnetic field equations was the first discovered law of physics that was found to be covariant with respect to the transformations of special relativity theory (the Poincaré group). The form of these equations, in the context of group representations, is that of a vector representation. But this is not an irreducible representation of the Poincaré group.

It was found in 1932, by Einstein and Mayer, that the irreducible form of the representations of the group of special relativity theory must be in terms of two-dimensional complex, hermitian representations, whose basis functions are the two-component spinor variables [34]. This followed when they recognized that the reflections in space and time, that are symmetry elements that underlie the invariance of the *squared* differential metric, $c^2\,\mathrm{d}t^2 - \mathrm{d}r^2$, are not required by the symmetry requirements of the theory of relativity *per se*. Thus, when removing these (discrete) symmetry elements, leaving only the continuous transformations in space and time (which are the required transformations of special relativity), the four-dimensional representations of the group that leaves the squared metric (above) invariant decomposes (factorizes) into the direct sum of two two-dimensional representations. The basis functions of the (reducible) four-dimensional representations are the four-vector variables; the basis functions of the (irreducible) two-dimensional representations of the group of special relativity are the spinor variables.

One additional assumption made in elementary particle theory that requires the maintainance of the four-dimensional representations as primitive is that the photon — a 'quantum' of the *vector* field equations — is an elementary particle. However, as it has been discussed above, the photon is rejected here as a fundamental entity in electromagnetic theory. It then follows that it should be in order to factorize the Maxwell field formalism, in accordance with the reduction of the four-dimensional group representations to the two-dimensional representations; that is, the irreducible representations whose basis functions are the two-component spinor variables.

To demonstrate such a factorization of the Maxwell formalism, let us make an initial identification between the *real number* components of the vector-tensor formalism, **E**, **H**, and the *complex number* components of the spinor variables. To do this, consider the complex vector whose spatial and

temporal components are as follows:

$$G_k = (H_k + iE_k), \qquad G_0 = 0 \quad (k = 1, 2, 3) \tag{5.3.1}$$

and let the structuring of the two-component spinor variables be guided by the correspondence between the features of the matrix representations of the Poincaré group — the hermitian components of a quaternion — and those of a four-vector, as follows

$$\sigma^\mu x_\mu \equiv \sigma^0 x_0 - \boldsymbol{\sigma} \cdot \mathbf{r} = \begin{pmatrix} x_0 - x_3 & -(x_1 - ix_2) \\ -(x_1 + ix_2) & x_0 + x_3 \end{pmatrix} \Leftrightarrow \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

where the quaternion basis elements σ^μ are defined in Equations (4.2.2a, b). With this correspondence, an initial guess at the spinor structure of the electromagnetic variables in the language of the standard variables, *in a particular Lorentz frame*, is as follows

$$\phi_1 = \begin{pmatrix} G_3 \\ G_1 + iG_2 \end{pmatrix} \qquad \mathrm{T}_1 = -4\pi i \begin{pmatrix} \rho + j_3 \\ j_1 + ij_2 \end{pmatrix}$$

$$\phi_2 = \begin{pmatrix} G_1 - iG_2 \\ -G_3 \end{pmatrix} \qquad \mathrm{T}_2 = -4\pi i \begin{pmatrix} j_1 - ij_2 \\ \rho - j_3 \end{pmatrix}. \tag{5.3.2}$$

It is readily verified with direct substitution, using the quaternion differential operator

$$\sigma^\mu \partial_\mu \equiv \sigma^0 \partial_0 - \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \tag{5.3.3}$$

that the two two-component spinor field equations

$$\sigma^\mu \partial_\mu \phi_\alpha = \mathrm{T}_\alpha \quad (\alpha = 1, 2) \tag{5.3.4}$$

are two equations that are in one-to-one correspondence with the set of differential equations associated with the standard form of Maxwell's equations, shown in (5.2.1) — but the two forms are not identical. For with the applications of the transformations of the Poincaré group of special relativity, the spinor transforms in a way where there would be no form-invariance with regard to the transformed variables **E** and **H**, i.e. under the space-time transformations of special relativity theory,

$$\phi_\alpha(\mathbf{E}, \mathbf{H}) \xrightarrow{x \to x'} \phi'_\alpha(\mathbf{E}', \mathbf{H}')$$

This is so because these are inequivalent representations of the Poincaré group; (**E**, **H**) are the solutions of equations that are a four-dimensional

representation of the group, while ϕ_a are the basis functions of the two-dimensional representations of the same group.

But the *physical requirement* of this field theory does not require such form-invariant correspondence. It only requires a form-invariant correspondence in the invariants and the conservation equations of the theory. This is because it is the latter, and not the field equations themselves, that are directly related to the observables. In view of the empirical validity of Maxwell's field equations and the rules for determining the observables from the solutions, it must be required that all of the invariants and conservation equations of the vector representation of the theory must correspond to at least some of the invariants and conservation equations of the (more general) spinor formulation of electromagnetism. It will be seen below that indeed the spinor formulation (5.3.4) predicts all of the physical consequences of the usual (vector) formulation, but in addition there are physical predictions that have no counterpart in the vector formulation. Thus, the two-component spinor formulation (5.3.4) of electromagnetic field theory is a true generalization of the vector formalism (5.2.1).

5.3.1. *Invariants and Conservation Equations*

As we discussed in *GRM* (Ch. 5, §6) the two-component spinor equations (5.3.4) would be relativistically covariant if and only if the spinors that appear in them transform as follows:

$$\phi_a(x) \xrightarrow{x \to x'} \phi'_a(x') = S\phi_a(x)$$

$$\mathrm{T}_a(x) \xrightarrow{x \to x'} \mathrm{T}'_a(x') = (S^\dagger)^{-1}\mathrm{T}_a(x) \tag{5.3.5}$$

where $\{S\}$ are the irreducible representations of the Poincaré group [referred to in the preceding chapter as $P(\mu\nu, 0)$], which solve the matrix equations (4.6.1):

$$S^\dagger \sigma^\mu S = \alpha^\mu_\nu \sigma^\nu$$

with solutions

$$S = \exp[\tfrac{1}{2}\sigma^\mu \sigma^\nu \theta_{\mu\nu}]$$

where $\theta_{\mu\nu}$ are the parameters that characterize the vector transformations, symbolized by $\alpha^\mu_\nu = \partial x^{\mu'}/\partial x^\nu$.

It then follows from the algebraic properties of the spinors and their transformations that the following are invariants in the spinor formulation of electromagnetism:

$$\tilde{\phi}_1 \varepsilon \phi_2 = J_1 \quad \text{and} \quad \tilde{\mathrm{T}}_1 \varepsilon \mathrm{T}_2 = J_2$$

where 'tilda' denotes the 'transpose', and

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Substituting into these invariants the identification with the conventional electromagnetic variables, (5.3.2), the following one-to-one correspondence between the invariants of the respective vector and spinor formalisms is found:

$$J_1 \leftrightarrow (E^2 - H^2) + 2i\mathbf{E} \cdot \mathbf{H}$$

$$J_2 \leftrightarrow j^2 - \rho^2.$$

The complex invariant J_1 of the spinor formalism then corresponds to two real invariants, which are the invariants of the standard vector formulation. The second invariant, J_2, corresponds to the invariant modulus of the four-vector j^μ.

In addition to the invariants J_1 and J_2, it follows from the transformation properties (5.3.5) of the spinor solutions of the field equations (5.3.4) that

$$I_{\alpha\beta} = \phi_\alpha^\dagger \mathrm{T}_\beta \quad (\alpha, \beta = 1, 2) \tag{5.3.6}$$

are four additional complex number invariants (thus, eight real number invariants) associated with this formal expression of electromagnetic field theory. These are to be compared with the single invariant, $j^\mu A_\mu$, describing the electromagnetic coupling in the vector form of the theory. The extra invariants of Equation (5.3.6) have no counterparts among the invariants of the standard theory of electromagnetism.

Thus we have established a one-to-one correspondence between some of the invariants of the spinor formulation of electromagnetic theory and all of the invariants of the standard formulation. It now remains to be shown that this generalization carries over to the conservation equations.

Multiplying Equation (5.3.4) on the left with the hermitian adjoint of ϕ_β, we have

$$\phi_\beta^\dagger \sigma^\mu \partial_\mu \phi_\alpha = \phi_\beta^\dagger \mathrm{T}_\alpha$$

Adding to this equation its hermitian adjoint, with interchanged labels α and β, we obtain the following *conservation equations*:

$$\partial_\mu (\phi_\beta^\dagger \sigma^\mu \phi_\alpha) = (\phi_\beta^\dagger \mathrm{T}_\alpha + \mathrm{T}_\beta^\dagger \phi_\alpha) \tag{5.3.7}$$

These conservation equations, in turn, correspond to eight real number equations, as contrasted with the single real number conservation equation with the form (5.2.1′), which has the equivalent form in tensor notation:

$$\partial_\mu T_\nu^\mu = K_\nu \tag{5.3.8}$$

where T^{μ}_{ν} is the electromagnetic energy-momentum tensor and

$$K_{\nu} = \{(\rho \mathbf{E} + \mathbf{j} \times \mathbf{H}); -\mathbf{j} \cdot \mathbf{E}\} \tag{5.3.9}$$

is the Lorentz force (four-vector) density.

The electromagnetic field equations (5.2.4) and the accompanying conservation equations (5.2.7) *in general relativity* are expressed by making the following global extensions of these equations

$$\sigma^{\mu} \rightarrow q^{\mu}(x) \qquad \partial_{\mu}\phi_{\alpha} \rightarrow \phi_{\alpha;\mu} \equiv (\partial_{\mu} + \Omega_{\mu})\phi_{\alpha}$$

where $q^{\mu}(x)$ is the quaternion metrical field, defining the invariant (quaternion) metric

$$ds = q^{\mu}(x)\, dx_{\mu}$$

of the curved space-time. Together with the standard form of the Riemannian metric, it follows that the metric tensor of Einstein's formalism relates to the quaternion metric as follows:

$$-\tfrac{1}{2}(q^{\mu}\tilde{q}^{\nu} + q^{\nu}\tilde{q}^{\mu}) \leftrightarrow g^{\mu\nu}.$$

The quaternion metric field solves the field equations (4.6.4), obtained from a factorization of Einstein's symmetric tensor field equations, as discussed in detail in *GRM* (Ch. 6). The term Ω_{μ}, defined above in the covariant derivative of the spinor field, $\phi_{\alpha;\mu}$, is the spin-affine connection, as defined earlier (Equation (4.3.5)).

Henceforth in this text, we will focus on the special relativistic limit of the field equations. Thus, the matter field equations (4.4.1) (or, equivalently, (4.4.7)) will be expressed in special relativity, where the nonzero inertial mass field λ will be taken as a pure number — though following from an averaged field. Asymptotically, the latter have the form of a discrete set of values,

$$\lambda_s = |\langle \eta_s | (-\tilde{q}^{\mu}\Omega^{\dagger}_{\mu})_{\alpha}(q^{\mu}\Omega_{\mu})_{\alpha} | \eta_s \rangle|^{\frac{1}{2}}, \tag{5.3.10}$$

which follow from the combination of the time-reversed mass field eigenfunction equations (4.3.12) and (4.3.15). In the expression above, λ_s is a mass eigenvalue associated with the matter field eigenfunction η_s, which is an element of the Hilbert space that is the *asymptotic form* of the function space spanned by the matter field solutions of the generally covariant equations (4.4.7), in the linear, special relativistic limit.

It was already proved in Chapter 4 that the mass field λ is a positive-definite function, since it is the modulus of a complex function; it then follows that the asymptotic values of the eigenvalues (5.3.10) must all be positive numbers. It should also be noted from this equation that if all of the matter of a closed system, except the bit of matter with the mass λ_s, should vanish, the spin-affine connection field Ω_{μ} associated with the matter would correspondingly vanish and thus, the mass of the observed matter, λ_s, would

correspondingly vanish. This result, of course, is in accordance with the requirement of the *Mach principle.*

Secondly, it should be noted that the expression (5.3.10) implies that the inertial mass of matter in the microdomain lies in a discrete spectrum, when there is sufficiently small coupling to the remainder of the matter of the closed system. This result is in agreement with the experimental facts having to do with a discrete mass spectrum of elementary matter — except for one essential difference. In the *actual limit* of a bit of matter *in a vacuum*, the inertial mass of the matter would be identically equal to zero, as indicated above, whereas in the elementary particle theory, the inertial mass of the particle in this limit would be a finite number. In the field theory discussed here, it also follows that as the matter of the closed system is continuously depleted, the 'line width' in the mass measurement of the observed 'particle' would correspondingly get continuously sharper — *approaching* an actual discrete eigenvalue. But the line width, in this theory, cannot go exactly to zero because it represents the coupling to other matter that is giving rise to the observed mass in the first place. In mathematical terms, the line width is due to the nonlinear features of the theory, which in principle cannot turn off, though they can come arbitrarily close to zero.

Finally, it should be noted that because the mass field in (4.3.12) is a two-dimensional matrix field in the spinor theory, the eigenfunctions $\{\eta\}$ associated with it *each* correspond to two mass eigenvalues. That is, the spinor character of this theory (in general) predicts mass doublets. In previous publications (reviewed in *GRM*, Ch. 6), the mass values of the electron and muon were explained in this way, and their values were calculated in the context of this field theory.

It follows from the new invariants (5.3.6) that the eight real number force density terms on the right-hand side of the conservation equations (5.3.7) are separately scalar fields. Thus, Equations (5.3.7) are four complex number *scalar* conservation equations, rather than the single *vector* conservation equation (5.3.8) of the standard Maxwell formalism.

To exhibit the correspondence of the conservation equations in the two formalisms, consider the sum of Equation (5.3.7) with $\alpha = \beta = 1$ and this equation with $\alpha = \beta = 2$. Substituting the identification (5.3.2), it is readily verified that this sum of equations corresponds to the usual conservation equation with the form (5.2.1′).

5.4. The Interaction Lagrangian

The principle of least action yields the factorized spinor equations (5.3.4) when the Lagrangian density has the form:

$$\mathscr{L}_M = ig_M \sum_{\alpha=1}^{2} (-1)^{\alpha} \phi_\alpha^\dagger (\sigma^\mu \partial_\mu \phi_\alpha - 2T_\alpha) + \text{h.c.} \tag{5.4.1}$$

In this functional form, we have chosen to write the (otherwise arbitrary) set of two coefficients for each of the spinor variables, ϕ_1 and ϕ_2, as $(-1)^\alpha$ because of a later application (in Chapter 7) to the particle—antiparticle bound system, where it will be seen that such a choice leads to a unique set of predictions for this two-particle system corresponding to all of the observations associated with pair annihilation and creation.

When the Lagrangian density (5.4.1) is varied with respect to the spinor variables ϕ_1 and ϕ_2, we obtain the two spinor equations (5.3.4). However, when this Lagrangian is varied with respect to the matter field variables (that are implicit in the source fields, T_α) we obtain an interaction that corresponds to the term:

$$\mathscr{L}_M^{(\text{int})} = 2ig_M \sum_{\alpha=1}^{2} (-1)^\alpha \phi_\alpha^\dagger T_\alpha + \text{h.c.} \tag{5.4.2}$$

This electromagnetic interaction has no counterpart in the usual vector formulation of the Maxwell theory. This extra interaction will be shown (in Chapter 7) to yield an important contribution to the fine structure of hydrogen, that is normally attributed to quantum electrodynamics — the 'Lamb splitting' — a prediction that is in very good numerical agreement with the data.

The constant in the Lagrangian interaction (5.4.2), g_M, has the dimension of a length (since the field term it multiplies has the dimension of energy density per length). It will be found that the analysis of the hydrogen spectrum requires that this constant is of order 2×10^{-14} cm. The new fundamental constant, with the dimension of length, with this magnitude, then appears in the theory of electrodynamics as a consequence of the factorization of the vector formalism for electromagnetism into a two-component spinor formalism — a generalization that follows from fully exploiting the symmetry requirements of the theory of relativity.

The next generalization of this theory, that would fully incorporate the Mach principle, implies that for each matter field component of the closed system considered, there is a separate set of spinor field equations of the form (5.3.4),

$$\sigma^\mu \partial_\mu \phi_\alpha^{(u)}(x) = T_\alpha^{(u)}(x) \tag{5.4.3}$$

along with the corresponding set of conservation equations:

$$\partial_\mu \sum_{u \neq v} (\phi_\beta^{(u)\dagger} \sigma^\mu \phi_\alpha^{(v)}) = \sum_{u \neq v} (\phi_\beta^{(u)\dagger} T_\alpha^{(v)} + T_\beta^{(u)\dagger} \phi_\alpha^{(v)}). \tag{5.4.4}$$

5.4.1. *The Electromagnetic Four-Potential*

We have seen in the preceding chapter that gauge invariance leads to the requirement that an interaction term must appear in the matter field

equations for inertia that entails the coupling of a current density, for one interacting component of a system, to the equivalent of a four-potential, representing the influence of the remaining part of the closed system. In terms of the spinor matter variables, the four-current density in the case of electromagnetic forces, has the usual forms (for the uth matter field):

$$j_\mu^{(u)} = \pm e\bar{\psi}^{(u)}\gamma_\mu\psi^{(u)} \quad \text{or} \quad \pm e\eta^{(u)\dagger}\sigma_\mu\eta^{(u)} \tag{5.4.5}$$

for the four-component Dirac bispinor formalism or the two-component spinor formalism (the former incorporating reflection symmetry, with $\bar{\psi} \equiv \psi^\dagger\gamma_0$). The vector four-potential solves the field equation

$$\Box A_\mu^{(u)} = 4\pi j_\mu^{(u)} \tag{5.4.6}$$

where $\Box$ is the d'Alembertian operator $\partial_0^2 - \nabla^2$. The *particular* solutions of (5.4.6) are:

$$A_\mu^{(u)} = \pm e \int j_\mu^{(u)} S(x - x')\, \mathrm{d}^4x'. \tag{5.4.7}$$

These are the only acceptable solutions of (5.4.6) since the homogeneous (radiation) solutions are automatically rejected by the interpretation of the electromagnetic field, in accordance with our preceding discussion. Also, the only acceptable Green's function, $S(x - x')$, in accordance with this theory, is the symmetrical one:

$$S(x - x') = \left(\frac{1}{4\pi}\right)^3 \int \left(\frac{1}{k_\rho k^\rho}\right) \exp[ik^\mu(x_\mu - x'_\mu)]\, \mathrm{d}^4k$$

$$= \left(\frac{1}{2|\mathbf{r} - \mathbf{r}'|}\right) \{\delta[(t - t') - |\mathbf{r} - \mathbf{r}'|] + \delta[(t - t') + |\mathbf{r} - \mathbf{r}'|]\}. \tag{5.4.8}$$

This Green's function in the solution (5.4.7) corresponds to an average of the retarded and the advanced potential. This expression of the potential is the unique one for this theory because it puts the 'emitter' and the 'absorber' on an equal footing — the theory is covariant with respect to the interchange of the interacting components of the system of matter components.

Recall that the retarded potential alone is the one that is used in the usual expression of electrodynamics. It is based on consistency with the particle theory in which one considers one charged particle to emit a signal at some initial time, t_0, and the second charged particle to absorb this signal at the later time, $t_0 + R/c$, when the particles are separated by R meters. One does not consider the advanced potential there since it seems to imply that an effect precedes a cause, i.e. implying a violation of causality. But in the field theory considered in this book, 'emitter' and 'absorber' are only names that

are assigned for convenience, according to boundary conditions *as viewed from a particular frame.* That is, the 'observer' would call the interacting component in his frame 'emitter', seeing it send a signal to another interacting component somewhere else (the 'absorber'); but there could be an 'observer' in the other interacting component's frame of reference (that he would call 'emitter' instead of the name 'absorber'). Objectively, then, signals (interactions) are emitted simultaneously from each of the interacting components to the other. Only in this description of the interaction would it have a completely *covariant* representation. The term 'causality', in this view, is then not tied to an absolute emitter and an absolute 'time order'; causality in this view is meant only in the Spinozist sense of logical connection — that all 'physical effects' must be logically related to 'physical causes'; but this connection is not unique in terms of an absolute time order.

From the mathematical side, this view must be expressed in terms of a symmetry between the advanced and the retarded terms of the potential, i.e. with the use of the symmetric Green's function (5.4.8) for the differential equation of electromagnetism, (5.4.6). This is a consequence of strict adherence to the covariance requirement of relativity theory.

Finally, the interaction functional $\mathscr{I}$ that appears in the matter field equations (4.4.7), or its special relativity limit, or in their expression in terms of the four-component bispinor of the Dirac theory, (the latter two forms to be discussed in more detail in the next chapter), is in two parts, one following from the variation of the interaction Lagrangian $\mathscr{L}_{\mathrm{D}}^{(\mathrm{int})}$ with respect to the matter field spinor variables, where

$$\mathscr{L}_{\mathrm{D}}^{(\mathrm{int})} = \sum_{u \neq v} j_{\mu}^{(u)} A^{\mu(v)}$$

and the other follows from the variation of the interaction Lagrangian $\mathscr{L}_{M}^{(\mathrm{int})}$ (5.4.2) with respect to the matter variables, which are implicit in the source fields T_{α}.

Expressing the matter fields in terms of the bispinor variables, the general electromagnetic interaction then has the following form in special relativity, for the operator that appears in the uth coupled matter field equation:

$$\mathscr{I}_{u} = \delta(\mathscr{L}_{\mathrm{D}}^{(\mathrm{int})} + \mathscr{L}_{M}^{(\mathrm{int})})/\delta\bar{\psi}^{(u)}$$

giving:

$$\mathscr{I}_{u} = \pm e^{2}\gamma_{\mu} \sum_{\substack{v=1 \\ (v \neq u)}}^{n} \left\{ \int \bar{\psi}^{(v)}\gamma_{\mu}\psi^{(v)} S(x-x')\,\mathrm{d}^{4}x' \right.$$

$$\left. + \left(\frac{ig_{M}}{|e|} \right) \sum_{\alpha=1}^{2} (-1)^{\alpha} [\phi_{\alpha}^{(v)\dagger}(x) \cdot \Gamma_{\alpha} - (\gamma_{0}\Gamma_{\alpha}^{\dagger}\gamma_{0}) \cdot \phi_{\alpha}(x)] \right\} \qquad (5.4.9)$$

where the following notation is used above:

$$\phi_\alpha^\dagger \cdot \Gamma_\alpha \equiv \phi_\alpha^*(1)\Gamma_\alpha(1) + \phi_\alpha^*(2)\Gamma_\alpha(2).$$

It should be re-emphasized at this stage that *in its general form*, all coupled matter fields $\psi^{(\nu)}$ are mapped in the same space-time x. The matrices Γ_α are the combinations of Dirac matrices in the spinor source terms of the electromagnetic equations, as indicated in the combination of Equations (5.3.2) and (5.4.5).

With the generalized form of the electromagnetic interaction functional (5.4.9) we are now in a position to formulate an explicit form of the covariant matter field equations, expressing the inertial manifestation of elementary matter, under the assumption that the general form of the interaction within the closed system is electromagnetic, expressed in its most general form that is consistent with the Mach principle and the symmetry requirements of the theory of relativity. This formulation will be derived in the next chapter, where it will be shown to lead, precisely, to the form of quantum mechanics of a many-particle system, in the linear limit — *including the prediction of all of the manifestations of the Pauli exclusion principle.*

Chapter 6

Quantum Mechanics from the Matter Field Equations and Derivation of the Pauli Exclusion Principle

There were four notable opponents to the *Copenhagen school* in the formative years of quantum mechanics (who never did change their opposition!) — Planck, de Broglie, Schrödinger and Einstein. All four were also instrumental in the initial formation of quantum mechanics! In the arguments with these scholars, or anyone else who had serious, technical criticisms of the quantum theory, it is challenged by the supporters of the Copenhagen school (or its variants) that while the criticisms may be technically valid, one should also ask: Where is a candidate for a different theory of micromatter that could reproduce all of the successful mathematical predictions of quantum mechanics? The assertion is that until a satisfactory replacement appears on the scene, physicists should keep to the usual quantum mechanical scheme of calculation, in spite of its weaknesses, since it is, at least, an empirically successful theory.

In this chapter I wish to demonstrate such a new theory of micromatter, to be considered by the defenders of the Copenhagen school. I will demonstrate a theory of matter that (a) duplicates all of the successful mathematical results of quantum mechanics — the nonrelativistic Schrödinger version, and the semirelativistic version of Dirac (i.e. the version of a spinor theory that treats the interacting radiation classically), and (b) a theory of micromatter that makes new predictions in the general expression of the theory that either are not derived at all in the usual quantum mechanical theories or are not derived by those theories in a mathematically acceptable manner (the latter refers primarily to the divergent results of quantum field theory that then require the method of renormalization for calculation — an admittedly mathematically nonconsistent scheme).

The new theory to be presented is based, principally, on the approach to a basic theory of matter that was taken by Einstein, in terms of fully exploiting the axiomatic basis of the theory of general relativity. As we have seen from the discussion in Chapter 2, such a starting point automatically rejects the conceptual basis of quantum mechanics, according to the Copenhagen view. Instead, it takes the formal expression of nonrelativistic quantum mechanics as not more than a linear approximation for a generally covariant field theory

of matter, expressing explicitly its inertial manifestations. While the four scholars, Planck, de Broglie, Schrödinger and Einstein, disagreed with the views of the Copenhagen school, they also disagreed with each other on the path toward a resolution of the problem of micromatter; it is Einstein's view, particularly, that will be fully exploited here.

6.1. Approximations to Quantum Mechanics

Generally, for the closed system that would entail n-coupled matter fields, expressed in terms of the n-coupled two-component spinor variables, the latter solve the set of coupled differential equations that have the form (4.4.7) *in special relativity*, that we have seen derives, in turn, from a generally covariant formalism, as a local limit. In the latter limit,

$$\begin{aligned} &\vdots \\ &\sigma^{\mu}\partial_{\mu}\eta^{(u)}(x) + \lambda^{(u)}\chi^{(u)}(x) = -\mathscr{I}_{u}\eta^{(u)}(x) \quad (u = 1, 2, \ldots, n) \\ &\tilde{\sigma}^{\mu}\partial_{\mu}\chi^{(u)}(x) + \lambda^{(u)}\eta^{(u)}(x) = -\tau\mathscr{I}_{u}\chi^{(u)}(x), \\ &\vdots \end{aligned} \tag{6.1.1}$$

where the interaction functional

$$\mathscr{I}_{u} = \mathscr{I}_{u}(\eta^{(1)}, \eta^{(2)}, \ldots, \eta^{(u-1)}, \eta^{(u+1)}, \ldots, \eta^{(n)})$$

plays the role of the (irreducible) coupling that underlies the inertial manifestation of matter, that is in accordance with the postulation of the generalized Mach principle, advocated with this theory. The argument of the functions above, (x), is inserted only to emphasize that each of the pairs of spinor matter fields, $(\eta^{(u)}, \chi^{(u)})$ is generally expressed as functions of the same space-time coordinate system. Note also that the equations above in χ are not independent of the equations in η; they are the time-(or space-)reversals of each other.

If one wishes to discuss matter field equations that are more amenable to interaction descriptions that are space- and time-reflection symmetric, it is more convenient to use the four-component Dirac *bispinor* equations:

$$\begin{aligned} &\vdots \\ &(\gamma^{\mu}\partial_{\mu} + \lambda^{(u)})\psi^{(u)}(x) = -\mathscr{I}_{u}\psi^{(u)} \quad (u = 1, 2, \ldots, n), \\ &\vdots \end{aligned} \tag{6.1.1′}$$

where the Dirac matrices, γ^{μ}, are defined in Equation (4.4.7′). In the latter expression of the matter field equations in special relativity, the explicit form of the interaction functional for the case of electrodynamical coupling, in its most general form, is shown in Equation (5.4.9).

The uth matter field equation (6.1.1) (or (6.1.1′)) is fundamentally *nonlinear* because of the necessary appearance (in principle) of the interaction functional $\mathscr{I}_{u}$, that depends on all of the other coupled fields, $\psi^{(v \neq u)}$.

These *determine* the uth matter field, for the latter matter fields $\{\psi^{(v)}\}$ depend on the uth matter field in the same functional way — thus, the operator that determines the uth matter field solution depends *implicitly* on this field solution itself; the equations are then basically *nonlinear*. The matter field equations in this theory, which are an explicit field representation of the inertia of matter, are fundamentally nonlinear — in contrast with the general form of the quantum mechanical equations, which are fundamentally linear (so as to fulfill the requirement of the principle of linear superposition of quantum mechanics).

The nonlinearity of the matter field equations of this theory cannot go to zero, in principle, because the coupling $\mathscr{I}_u$ is responsible for the inertial manifestation of matter, which these equations are to represent, and the latter interaction that cannot then become zero (in principle) implies that the equations are nonlinear. Nevertheless, in particular limits one may accurately approximate the fundamentally nonlinear matter field equations by linear differential equations. It is in the latter approximation where the formal expression of this theory goes into the formal expression of quantum mechanics. This approximation should be fairly accurate when the energy-momentum transfer between the interacting matter components of the considered closed system is sufficiently weak — corresponding to the nonrelativistic limit or to a relativistic approximation that may be identified with spinor matter fields in a classical radiation field (the Dirac theory). In the latter limit of small energy-momentum transfer, the dependence of the interaction functional $\mathscr{I}_u$ on the coupled matter fields themselves smooths out to an approximation in which one describes an average background potential for each of the matter components, separately, i.e.

$$\mathscr{I}_u(\psi^{(1)}(x), \psi^{(2)}(x), \ldots, \psi^{(u-1)}(x), \psi^{(u+1)}(x), \ldots, \psi^{(n)}(x)) \to \mathscr{I}_u(x) \qquad (6.1.2)$$

(in the language of bispinor matter fields $\psi^{(u)}$). The limit (6.1.2) corresponds to an approximation used in nuclear theory that is called 'the individual particle model', where one considers each constituent nucleon in a composite nucleus one at a time in an averaged background potential field.

6.1.1. *The Free Field Limit*

Finally, in the asymptotic limit, where

$$\mathscr{I}_u \to 0 \qquad (6.1.3)$$

we have the 'free field approximation'. It should be kept in mind, when using this approximation, that the actual limit of a truly 'free' matter component does not exist in principle, according to this theory of matter, though the limit may be *approached* arbitrarily closely. In the limit of the free field, where there would be total uncoupling, it would be possible to consider the matter free field solutions $\{\psi^{(u)}\}$ *one at a time*. In this case, the coupled

equations (6.1.1′) would *uncouple* and the solutions may then each be represented in their own space-times, x_u, i.e. in this limit,

$$(\gamma^\mu \partial_\mu^{(u)} + \lambda^{(u)})\psi^{(u)}(x_u) = 0 \quad (u = 1, 2, 3, \ldots, n) \tag{6.1.4}$$

and $\partial_\mu^{(u)} \equiv \partial/\partial x^\mu_{(u)}$ are the derivatives with respect to the uth four-vector components.

Since each of the n spinor equations in the approximation (6.1.4) is a linear differential equation, the sum of all n of these equations may be taken, yielding the following single field equation:

$$\left[\sum_{u=1}^{n} \gamma^\mu \partial_\mu^{(u)} + \lambda^{(u)} \right] \Psi(\psi^{(1)}(x_1), \psi^{(2)}(x_2), \ldots, \psi^{(n)}(x_n)) = 0. \tag{6.1.5}$$

The solution of this equation, Ψ, is a limiting form of a functional which, generally, represents a connective relation between the elements of the set of solutions $\{\psi^{(u)}\}$ of the coupled nonlinear equations (6.1.1′). In this linear limit, it has the form of the product function

$$\Psi = \prod_{u=1}^{n} \psi^{(u)}(x_u), \tag{6.1.6}$$

as well as $n! - 1$ other equivalent solutions, since this is independent of the order of the factors in the product solution, and there are $n!$ permutations of this order.

Because the n-coupled differential equations for the matter system uncouple to n independent equations (6.1.4) which are then functionally identical, the complete set of solutions of each of these equations must cover the same function space. Further, the requirement of the square integrability of the solutions, implied by the conservation of interaction (described in Section 3.2) implies that these solutions have the same eigenfunction character as the solutions of ordinary quantum mechanics. It should be noted that the condition of square integrability, i.e. that for each solution of the matter field equations (u),

$$\int \psi^{(u)\dagger}\psi^{(u)}\, \mathrm{d}\mathbf{r} = \text{Finite number}$$

is generally required by this theory, independent of any approximation, because of the imposition of the concept of *conservation of interaction.*

With this feature of square integrability of the solutions, the matter field equations in the 'free field approximation' (6.1.4) are the eigenfunction type, with the prediction of discreteness of all of the physical properties of matter that follow. With this limiting form of the equations for inertia, having the same mathematical structure as the probability calculus (in the Dirac form),

the indices above, (u), for each of the field solutions, stand for a set of 'quantum numbers, $\{m_u\}$, that relate to the Hilbert space of functions for the solutions of each of these equations.

If we should take account of the fact that the orders of products in the solution (6.1.6) does not affect the predicted properties of the system (when it is accurate to use this approximation for the actually nonlinear theory), we may use the fact that the approximation is linear and take a linear combination of all permutations of such product functions, assuming that each of the terms in such combinations has equal weighting, in representing the total interaction of the closed system that is described. The general solution of (6.1.5) would then take the general form:

$$\psi = \left(\frac{1}{n!}\right)^{\frac{1}{2}} \sum_P \exp(i\alpha_P) \prod_u \psi^{(m_u)}(x_u), \tag{6.1.7}$$

where α_P is a phase associated with the Pth permutation in the order of these product functions. Its value will be determined later on, from the exact form of the function that fully exploits the nonlinear structure of the (unapproximated) equations.

6.1.2. *Coupling to an External Potential*

The 'free field limit', discussed above, was based on the assumption that each link, $\mathscr{I}_u\psi^{(u)}$, can be neglected in each of the coupled matter field equations (6.1.1′), compared with the influence on the solutions of the other terms. This was equivalent to the requirement that the energy-momentum transfer within the interacting components of the system is small compared with the 'intrinsic energy' associated with each of these components. The latter is in terms of the effect of the operator $\gamma^\mu\partial_\mu$ and the term λ (representing the 'rest energy') compared with the negligible effect of the interaction term in the matter field equations, to achieve an uncoupling of the full set of equations (6.1.1′). Still, the limit $\mathscr{I}_u\psi^{(u)} \to 0$ is not really required to achieve this uncoupling. It is only necessary that the following type of approximation should be reached:

$$\mathscr{I}_u\psi^{(u)} \to f(x_u)\psi^{(u)}$$

where $f(x_u)$ is an integrable function of the coordinates, expressible in a form that is not explicitly a function of the field solutions that couple to the solution $\psi^{(u)}$.

It should be emphasized here that even with the linear approximation that we have discussed, the manifestations of the actual nonlinearity of the theory still appear in the predictions. For example, the *finite width* for the measured value of *any* observable (rather than the zero width predicted, ideally, from the eigenvalue theory) is, here, a manifestation of the nonlinear features of

the equations that entail coupling. The fact that the measured values of all of the properties of elementary matter are peaked, but do have 'width', is, of course, in agreement with the experimental facts that indicate that there is always an *irreducible* line width. The quantum theory interprets this 'natural width' in terms of argumentation based on the Heisenberg uncertainty principle. Thus the two theories agree, qualitatively, on this fact from experimental physics. Where the two theories should differ most strikingly is in the high-energy limit, since the quantum theory rests on the feature of linearity, with the Hilbert space of functions as an elementary starting point, while the present theory of matter is fundamentally nonlinear and at high energy-momentum transfer, where one may not generally use the linear approximation, different predictions would follow than from the (linear) quantum theory.

6.2. The Pauli Exclusion Principle — a Derivation

In comparing the proposed field theory of matter, representing a closed system, with the standard quantum mechanical approach to a 'many-body' system, the following question naturally arises: How, within the framework of a purely field description of a closed system, does one interpret the *Pauli exclusion principle*, which appears to entail correlations of the positions and momenta of different *particles*?

To answer the question, consider the physical implications of this principle. It asserts that two equivalent spin-$\frac{1}{2}$ particles cannot simultaneously be in the same state of motion — i.e. they cannot simultaneously be in the same location with the same constants of motion. Thus, the conventional form of quantum mechanics places a restriction on the separate probability amplitudes $\psi^{(m_u)}(x_u)$, $\psi^{(m_v)}(x_v)$ of the trajectories of the uth and the vth particles. On the other hand, according to the field theory of inertia proposed, there is only one space-time, x, and an interaction field amplitude $\Psi(x)$ that is mapped onto it. As it was indicated in the previous section, the interaction field amplitudeΨ is a connective relation between the component matter fields, $\{\psi^{(u)}(x)\}$, which are the solutions of the coupled nonlinear equations (6.1.1′) that we start with at the outset.

One of the restraints imposed on the interaction field amplitude $\Psi(x)$ is that it must express a law of *conservation of interaction* [42]. When this function has the form of the four-component Dirac bispinor, the conservation law would follow from the equation of continuity:

$$\partial_\mu(\bar{\Psi}\gamma^\mu\Psi) = 0 \tag{6.2.1}$$

The time-component of the four-vector that this entails,

$$\bar{\Psi}\gamma^0\Psi(x) = \Psi^\dagger\Psi(x),$$

is a measure of the *weighting* of the mutual influence of all n-components of

the closed system, at the space-time point x. Within this interpretation it follows that if the physical situation should be approached that would correspond to an identical vanishing of the field amplitude Ψ at all space-time points x, it would have to be concluded that such a physical situation does not relate to any observable.

It will now be proven that if any two components of a physical system, identified with the indices (u) and (v), out of an n-component *closed* system, should have the general electrodynamical coupling, and

(1) a repulsive interaction, i.e. if $e^{(u)}e^{(v)} = +e^2$,
(2) the same inertial mass, i.e. $\lambda^{(u)} = \lambda^{(v)}$,
(3) the same state of motion, i.e.

$$\bar{\psi}^{(u)}\gamma^{\mu}\psi^{(u)} = \bar{\psi}^{(v)}\gamma^{\mu}\psi^{(v)} \text{ for all } x, \tag{6.2.2}$$

then the contribution of the mutual coupling of (u) and (v) to the interaction weighting for the entire closed system must be identically equal to zero.

The latter result is *physically* equivalent to the implications of the Pauli exclusion principle. It will be shown that when this *exact* feature of the theory is incorporated with the nonrelativistic approximation for the many-particle interaction field amplitude, the latter function must then vanish identically in the $4n$-dimensional space-time of the many-particle description. In particular, the latter asymptotic form of the interaction field amplitude will be found to be the totally antisymmetrized Schrödinger wave function for a many-fermion system.

To proceed with the derivation then, let Ψ_{uv} denote the interaction weighting amplitude for a system with coupled components, not counting the interaction of the components labeled by u and v. The remaining amplitude will be denoted by ψ_{uv}. To clarify this denotation further, Ψ_{uv} can depend on the separate amplitudes $\psi^{(u)}$ and $\psi^{(v)}$, but only with respect to their separate couplings to other particle fields of the system. The amplitude Ψ_{uv} then omits the contribution of the *mutual coupling* of (u) and (v). If, then, we can find a field amplitude ψ_{uv} that vanishes under the three conditions mentioned above, it would follow that $\Psi \equiv \Psi_{uv}$, and the physical consequences of the Pauli principle would follow.

At this stage, we are only *assuming* that such a field ψ_{uv} exists. If no such two-body field would exist, then the total interaction field amplitude Ψ could not be separated into the two parts, Ψ_{uv} and ψ_{uv}. Nevertheless, it will now be shown that such a field does indeed exist as a feature of the *exact form* of the coupled spinor matter field equations.

To determine the form of ψ_{uv} for the special case under study, note first that if $\bar{\psi}_{uv}\gamma_0\psi_{uv} \equiv \psi^{\dagger}_{uv}\psi_{uv}$ represents an *additive contribution* to the weighting function for the closed system, then, by itself, it must satisfy an equation of continuity: (The Dirac matrices γ^{μ} are defined in Section 4.4).

$$\partial_{\mu}(\bar{\psi}_{uv}\gamma^{\mu}\psi_{uv}) = 0. \tag{6.2.3}$$

We must then seek such a solution ψ_{uv} that depends on the two matter field solutions, $\psi^{(u)}$ and $\psi^{(v)}$ of the coupled set of nonlinear equations (6.1.1′), and satisfies the continuity equation (6.2.3), and is a solution that vanishes identically under the three conditions mentioned above.

Multiplying the uth equation in the set (6.1.1′) by the (conjugated) solution $\bar{\psi}^{(v)}$ of the vth equation, and multiplying the hermitian conjugate of the vth equation by $\gamma_0\psi^{(u)}$ (on the right), subtracting these two equations and then repeating the operation with (u) and (v) interchanged, the sum of the resulting equations is as follows:

$$\partial_\nu(\bar{\psi}^{(v)}\gamma^\nu\psi^{(u)} + \bar{\psi}^{(u)}\gamma^\nu\psi^{(v)}) = (\bar{\psi}^{(u)}\gamma^\nu\psi^{(v)} - \bar{\psi}^{(v)}\gamma^\nu\psi^{(u)})R_\nu -$$

$$- ig_M \left\{ \sum_{\alpha=1}^{2} \Phi_\alpha(\bar{\psi}^{(v)}\Gamma_\alpha\psi^{(u)} - \bar{\psi}^{(u)}\Gamma_\alpha\psi^{(v)}) - \text{h.c.} \right\} +$$

$$+ (\lambda^{(v)} - \lambda^{(u)})(\bar{\psi}^{(v)}\psi^{(u)} - \bar{\psi}^{(u)}\psi^{(v)}) \tag{6.2.4}$$

where

$$R_\nu = e^{(u)}e^{(v)} \int (\bar{\psi}^{(v)}\gamma_\nu\psi^{(v)} - \bar{\psi}^{(u)}\gamma_\nu\psi^{(u)})S(x-x')\,\mathrm{d}^4x' +$$

$$+ (e^{(u)} - e^{(v)}) \sum_{w \neq u,\, v} e^{(w)} \left\{ \int \bar{\psi}^{(w)}\gamma_\nu\psi^{(w)}S(x-x')\,\mathrm{d}^4x' \right\}, \tag{6.2.5}$$

$e^{(u)}e^{(v)} = +$ (or $-$) e^2 for a repulsive (or attractive) interaction, and

$$\Phi_\alpha = (-1)^\alpha\{e^{(u)}\phi_\alpha^{(v)\dagger} - e^{(v)}\phi_\alpha^{(u)\dagger}\} + (e^{(u)} - e^{(v)}) \sum_{w \neq u,\, v} \phi_\alpha^{(w)\dagger}. \tag{6.2.6}$$

The right-hand side of Equation (6.2.4) contains three terms, each of them vanishing identically under special conditions. First, if the uth and vth interacting components are each related to the same state of motion, i.e. under the special condition when $\bar{\psi}^{(u)}\gamma^\mu\psi^{(u)} = \bar{\psi}^{(v)}\gamma^\mu\psi^{(v)}$ (Condition (3) above), it also follows that

$$\bar{\psi}^{(u)}\Gamma_\alpha\psi^{(u)} = \bar{\psi}^{(v)}\Gamma_\alpha\psi^{(v)} \tag{6.2.7}$$

since the matrices Γ_α are linear combinations of the Dirac matrices γ^μ. With the same equation of motion for the uth and vth matter fields, i.e. with (Equation (6.2.2)), the first term in R_ν above (in Equation (6.2.5)) vanishes.

The spinor form of the electromagnetic field equations, (5.4.3), with the inserted matter field expressions for the sources, are

$$\sigma^\mu\partial_\mu\phi_\alpha^{(u)} = e^{(u)}\bar{\psi}^{(u)}\Gamma_\alpha\psi^{(u)}.$$

It then follows that

$$\sigma^{\mu}\partial_{\mu}(e^{(v)}\phi_{\alpha}^{(u)} - e^{(u)}\phi_{\alpha}^{(v)}) = e^{(u)}e^{(v)}(\bar{\psi}^{(v)}\Gamma_{\alpha}\psi^{(v)} - \bar{\psi}^{(u)}\Gamma_{\alpha}\psi^{(u)}). \tag{6.2.8}$$

Since, in the conceptual framework of this theory, we only accept the particular solutions of the electromagnetic equations (see Chapter 5), the substitution of Equation (6.2.7) into the right-hand side of Equation (6.2.8) implies that the only solution must correspond to

$$e^{(v)}\phi_{\alpha}^{(u)} - e^{(u)}\phi_{\alpha}^{(v)} = 0. \tag{6.2.9}$$

It then follows that with the uth and the vth matter fields corresponding to the same state of motion, the first part of Φ_{α} (Equation (6.2.6)) vanishes.

Second, if the inertial mass parameters for the uth and vth fields are the same $\lambda^{(u)} = \lambda^{(v)}$, so that the last term on the right-hand side of Equation (6.2.4) vanishes.

Finally, the second term on the right-hand side of Equation (6.2.5) and the second term on the right-hand side of Equation (6.2.6) $[\phi_{\alpha}^{(w)} \sim e^{(w)}]$ entails a coupling of the uth and the vth field components, *separately*, to all of the other interacting components of the system. If the *mutual* coupling between the uth and vth fields is repulsive, i.e. if $e^{(u)}e^{(v)} = +e^2$, then it follows that their separate coupling to other fields would have the same sign, i.e. $e^{(u)}e^{(w)} = e^{(v)}e^{(w)}$. Under these circumstances, the last terms on the right-hand sides of Equations (6.2.5) and (6.2.6) vanish identically.

Summarizing, when any two, out of an n-component closed system, are (1) in the same state of motion, (2) have the same inertial mass and (3) have a repulsive interaction, then R_{ν}, Φ_{α} and $(\lambda^{(u)} - \lambda^{(v)})$ all vanish identically — therefore, the entire right-hand side of Equation (6.2.4) vanishes under these conditions. It then follows that

$$\partial_{\nu}(\bar{\psi}^{(v)}\gamma^{\nu}\psi^{(u)} + \bar{\psi}^{(u)}\gamma^{\nu}\psi^{(v)}) = 0. \tag{6.2.10}$$

Combining this result with the continuity equation for each of the separate matter fields (which is, fundamentally, a consequence of gauge invariance, as we have discussed in Chapter 4), i.e.

$$\partial_{\mu}(\bar{\psi}^{(u)}\gamma^{\mu}\psi^{(u)}) = \partial_{\mu}(\bar{\psi}^{(v)}\gamma^{\mu}\psi^{(v)}) = 0 \tag{6.2.11}$$

and the requirement of this theory that the description must, generally, be symmetric with respect to the interchange

$$\psi^{(u)}(x) \leftrightarrow \psi^{(v)}(x),$$

under all conditions and independent of approximation, it follows that

$$\partial_{\mu}[\overline{(\psi^{(u)} \pm \psi^{(v)})}\gamma^{\mu}(\psi^{(u)} \pm \psi^{(v)})] = 0.$$

from (6.2.10) and (6.2.11). Thus, the solution ψ_{uv} of Equation (6.2.3) is

$$\psi_{uv}(\pm) = \psi^{(u)} \pm \psi^{(v)}. \tag{6.2.12}$$

The only remaining ambiguity is in the + or − sign. If the interaction field is, generally, unique for the considered situation, then only one of these signs is valid, for all x. We will now determine the correct sign. Since both the uth and the vth interacting fields correspond in this case to the same state of motion, and have the same separate interactions with the rest of the closed system of matter field components, it follows that since they also have identical boundary conditions imposed by the remainder of the closed system, in the special case where (6.2.12) is valid each of the matter fields $\psi^{(u)}$ and $\psi^{(v)}$ solve identical differential equations, with identical boundary conditions. Thus, in this case, they *map onto each other*, i.e.

$$\psi^{(u)}(x) = \psi^{(v)}(x) \tag{6.2.13}$$

It is important that there are not different valued constants multiplying each side of this equation because of the special conditions imposed on these solutions, in this particular case, and the feature that these solutions solve nonlinear differential equations.

Now if the plus sign in (6.2.12) is the correct one, then we must next solve for the matter field solution $\psi_{uv} = 2\psi^{(u)}$, to determine the interaction field amplitude. However, in this case this would be a solution of the field equation (6.1.1′) in which the interaction term is

$$\mathscr{I}_u(\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(v=u)}, \ldots, \psi^{(n)})\psi^{(u)}.$$

But this would then describe a matter field $\psi^{(u)}$ interacting with itself (as well as with the other components of the considered closed system). The acceptance of the solution for the 'uv' interaction amplitude, $\psi_{uv}(+)$, is then unacceptable because it is logically incompatible with the initial premise of the theory that automatically rules out the appearance of any self-energy terms. To be logically consistent with the theory, it then follows that the only solution in (6.2.12) permissible is

$$\psi_{uv}(-) = \psi^{(u)} - \psi^{(v)} = 0, \tag{6.2.14}$$

where the zero on the right follows from the special conditions that led to Equation (6.2.13) — conditions that are equivalent to the axioms imposed in the conventional quantum mechanical theories leading to the statement of the Pauli exclusion principle.

Summing up, what has been seen thus far is that a *necessary consequence* of the three conditions imposed on a system of n mutually interacting spinor field components, that two of the interacting components, (u) and (v), are (1) in the same state of motion, (2) have a repulsive mutual interaction and (3) have equal inertial masses, is that the total interaction field amplitude for the closed system, $\Psi(x)$ then reduces to the amplitude $\Psi_{uv}(x)$ that excludes the mutual coupling of (u) and (v). This means, within the context of this theory of matter, that no possible measurement could relate to the mutual coupling of the matter field components (u) and (v). This *derived* result then proves

the necessity of the three conditions mentioned above for the *Pauli exclusion principle*, though coming from a totally different theory of matter than the quantum theory, since this theory is based on a continuum, nonlinear, deterministic, nonsingular field approach, rather than the probabilistic, singular particle approach of quantum mechanics. Indeed, it was the explicit features of this theory that are not allowed, in principle, in the quantum theory, that led to the desired proof!

6.2.1. *Sufficiency of the Three Conditions for Proof of the Pauli Principle*

What has been proven thus far is that the three conditions mentioned above are a *necessary condition* for the Pauli principle to operate. But it can also be shown that they are a *sufficient condition*, within this nonlinear field theory of a closed system.

Sufficiency may be proven by starting with the assumption that the two matter fields, $\psi^{(u)}$ and $\psi^{(v)}$, solve the same differential equation (the uth and vth equations of the set of spinor field equations (6.1.1′)) and then proving that the three conditions above necessarily follow. Noting that, in this field theory, the solutions $\psi^{(u)}$ and $\psi^{(v)}$ are *defined* to be mapped in the same space-time x, it follows that since they solve the same differential equations and have the same boundary conditions (imposed by the remainder of the closed system),

$$\psi_{uv}(-) = \psi^{(u)}(x) - \psi^{(v)}(x) = 0$$

at all space-time x. Thus, it follows that the derivatives of $\psi_{uv}(-)$ must also be zero everywhere, so that the following continuity equation is automatically obeyed, everywhere:

$$\partial_\mu[\bar{\psi}_{uv}(-)\gamma^\mu\psi_{uv}(-)] = 0. \tag{6.2.15}$$

Combining this equation with the vanishing terms of the continuity equations for the separate matter components (u) and (v), Equation (6.2.11), it follows that (6.2.10) is true. Now with the latter, the left-hand side of Equation (6.2.4) vanishes. Because of the independence of each of the terms on the right-side of (6.2.4), we have:

$$R_\nu = 0, \qquad \Phi_\alpha = 0, \qquad \lambda^{(u)} = \lambda^{(v)}. \tag{6.2.16}$$

Because of the independence of the two terms in R_ν (Equation (6.2.5)), it *follows* from its vanishing that the matter fields for (u) and (v) correspond to the same state of motion, i.e. that Equation (6.2.2) is true, and that the mutual interaction of (u) and (v) is repulsive, i.e. that $e^{(u)} = e^{(v)}$.

With the latter result and the fact that $\Phi_\alpha = 0$, it follows that

$$e^{(u)}\phi_\alpha^{(v)} = e^{(v)}\phi_\alpha^{(u)}.$$

This relation is equivalent to the assertion that the uth and vth matter fields

are equivalent, since the electromagnetic spinor solutions ϕ_α are the particular solutions of the electromagnetic equations (5.4.3), which may be expressed in the form (6.2.8). That is, with equivalent spinor matter fields, the right-hand side of (6.2.8) automatically vanishes, so that the particular solutions of these equations correspondingly vanish.

Thus we have seen that, starting with the equivalence of the spinor matter fields, $\psi^{(u)} = \psi^{(v)}$, at all space-time points x, if the contribution to the total interaction field amplitude from the (u)—(v) coupled nonlinear components should vanish, then it *follows* that the three conditions mentioned earlier are derived, i.e. it follows that (1) the mutual (u)—(v) interaction is repulsive, (2) their inertial masses are the same and (3) they are in the same state of motion. Thus it has been demonstrated that the conditions (1), (2) and (3) are both *necessary and sufficient* for the physical conditions of the Pauli exclusion principle to follow, thereby completing the proof of the principle from this field theory of matter.

6.2.2. *Fermi—Dirac Statistics from the Nonrelativistic Approximation for* Ψ

Let us now return to the nonrelativistic approximation (6.1.7) for the interaction field amplitude. In this limit, the time coordinate becomes a common parameter for all field components and any two factors in the products expressed in this equation, when they correspond to equivalent states of motion, contribute the product

$$\psi_{uv}^{(0)} = \psi^{(m_u)}(\mathbf{r}_u)\psi^{(m_v = m_u)}(\mathbf{r}_v = \mathbf{r}_u). \tag{6.2.17}$$

If (u) and (v) represent interaction matter fields with equal mass and mutually repelling force, then the *exact* result derived in the preceding section implies that the product above, Equation (6.2.17), *is only an approximation* for the exact solution $\psi_{uv}(-)$. But since the latter was found to be identically equal to zero, in all space-time, the function $\psi_{uv}^{(0)}$, in Equation (6.2.17), must actually be a nonzero approximation for zero! Finally, since the product function in Equation (6.2.17) is a factor that *multiplies* the product of all other matter field solutions in the general form in the nonrelativistic approximation shown in Equation (6.1.7), it follows that the actual vanishing of $\psi_{uv}^{(0)}$ causes the total interaction field amplitude to vanish, under the same conditions that leads to the former vanishing.

To incorporate this result into the asymptotic field amplitude for a system of equivalent 'particles', we must choose the phase that appears in the solution (6.1.7) as follows:

$$\alpha_P = \pm \pi P \rightarrow \exp(i\alpha_P) = (-1)^P.$$

In this case, then, an equivalent way to express the nonrelativistic approximation for the total interaction field amplitude in in terms of the Slater-

determinant wave function for the many-body system of 'fermions', as follows:

$$\Psi = \left(\frac{1}{n!}\right)^{\frac{1}{2}} \sum_{P} (-1)^{P} \prod_{u=1}^{n} \psi^{m_u}(r_u) = \left(\frac{1}{n!}\right)^{\frac{1}{2}} \begin{vmatrix} \psi^{m_1}(r_1) \dots \psi^{m_1}(r_n) \\ \vdots \qquad\qquad \vdots \\ \psi^{m_n}(r_1) \dots \psi^{m_n}(r_n) \end{vmatrix} . \qquad (6.2.18)$$

This is the totally antisymmetrized wave function for the many-particle system (of spin-$\frac{1}{2}$ entities) that is used in the description of Fermi—Dirac statistics, for an ensemble of indistinguishable objects. Of course, it is this type of statistics of a particle system that has led to the empirically correct results having to do with the classification of the Periodic Chart, the properties of metals, etc.

Even though the interaction field amplitude (6.2.18) is identical with the many-body wave function in quantum mechanics, for a system of noninteracting fermions, it should be noted that the interpretation here is not in terms of particles (i.e. a collection of separate, singular 'things'). The statistical aspect appears in this theory only because we are using a limiting form in which the coupled fields that describe interacting matter, appear to be uncoupled *in a first approximation.* The Pauli exclusion principle, on the other hand, deals with features of a system of matter implied by a particular state of mutual interaction — independent of approximation.

Within the context of quantum field theory it is interesting to note that the Pauli exclusion principle has never been proven from first principles, for the realistic system of *interacting* fermions [7]. On bases of the quantum theory, the principle automatically entails the use of Fermi—Dirac statistics, in an exact sense. In the present covariant field theory of inertia, whose formal expression gives back quantum mechanics as a linear approximation (for sufficiently small energy-momentum transfer between interacting matter components), the individual particle aspects are only a manifestation of an asymptotic approximation for a *closed system*, corresponding to the (actually unattainable) limit of no interaction between the component matter fields. Thus, Fermi—Dirac statistics appears within this theory only as an approximation to describe a closed system in terms of (spinor) fields that are in a state of weak coupling. The use of this approximation should be accurate only when the quantities of energy-momentum transfer are nonrelativistic. If we should now combine this requirement on the interaction field amplitude in the linear approximation, as expressed with the Slater determinant form (6.2.18), with the Hartree approximation for the quantum mechanical description of the many-body system, we are led to the 'Hartree—Fock formalism'. Let us now examine this limit in more detail, within the context of the theory discussed here. We will start with a discussion of the Hartree theory itself.

6.3. The Hartree Approximation from the Matter Field Equations

Taking the nonrelativistic approximation for the uncoupled spinor field equations (6.1.4), the time part of each of the particle four-vectors, x_u, collapses to a common time parameter, i.e.

$$v/c \to 0 \Rightarrow x_u \equiv (\mathbf{r}_u, t_u) \to (\mathbf{r}_u, t)$$

and, at the same time, the bispinor solutions $\psi^{(u)}$ of these equations approach the solutions ψ_S of Schrödinger's nonrelativistic wave mechanics.

If one now wishes to improve the solutions of the uncoupled equations (6.1.4) when they are expressed in the nonrelativistic approximation, the interaction functional may be re-introduced by inserting the nonrelativistic approximation for $\mathscr{I}_u$ into Equation (6.1.1′) — recalling that in this limit all field solutions are described in their own spaces, and with a common time parameter, i.e. $\psi^{(u)}(x) \to \psi(\mathbf{r}_u, t)$. The latter limiting form for the interaction functional may then be treated as a perturbation on the solutions $\psi(\mathbf{r}_u, t)$ of the (otherwise) uncoupled equations (6.1.4). In this way, the nonlinear, relativistically covariant formalism that we started with (in the form of Equation (6.1.1′)) reduces to the usual Hartree formalism. This result will now be demonstrated.

The nonrelativistic limit of the Dirac bispinor solutions are the Schrödinger solutions ψ_S. The added (generalized) part of the electromagnetic interaction functional (5.4.9) entails a mixing of the four components of the bispinor $\psi^{(u)}$. [This will be shown in an explicit example in the analysis of the hydrogen spectrum in Chapter 8.] That is, the latter interaction functional is in terms of a nondiagonal operator. It then follows that in the Schrödinger limit, where three out of the four components of the Dirac bispinor solution may be neglected, the latter 'mixing' interaction may be ignored.

Thus the only part of the generalized electrodynamic interaction that remains to perturb the (otherwise uncoupled) matter fields, in the Schrödinger limit, is the following nonrelativistic limit of the first part of the interaction functional $\mathscr{I}_u$:

$$\mathscr{I}_u(\mathbf{r}_u, t) = e^{(u)} \sum_{v \neq u} e^{(v)} \int \psi_S^*(\mathbf{r}_v, t)\psi_S(\mathbf{r}_v, t)S(\mathbf{r}_u - \mathbf{r}_v)\,\mathrm{d}\mathbf{r}_v. \tag{6.3.1}$$

When all time coordinates are equated, the Green's function (5.4.8) becomes:

$$S(\mathbf{r}_u - \mathbf{r}_v) = \frac{1}{[4\pi|\mathbf{r}_u - \mathbf{r}_v|]}. \tag{6.3.2}$$

With this substitution, the nonrelativistic limit of the matter field equation (6.1.1′) takes the following form:

$$i\partial^{t}\psi_S(\mathbf{r}_u, t) = \left\{ -\left(\frac{1}{2\lambda^{(u)}} \right) \nabla_u^2 + \right.$$

$$\left. + \sum_{\substack{v=1 \\ (\neq u)}}^{n} e^{(v)} \int \frac{\psi_S^*(\mathbf{r}_v, t)\psi_S(\mathbf{r}_v, t)}{4\pi|\mathbf{r}_v - \mathbf{r}_u|} \, d\mathbf{r}_v \right\} \psi_S(\mathbf{r}_u, t) \qquad (6.3.3)$$

where $u = 1, 2, \ldots, n$, for an 'n-body' system, and where $e^{(u)}e^{(v)} = \pm e^2$.

In the usual application of wave equation (6.3.3) to the description of a many-electron atom, the solutions describe the 'stationary states'. In this case, the solutions factor into a product of a space-dependent part and a time-dependent part as follows:

$$\psi_S(\mathbf{r}_v, t) \rightarrow U_S(\mathbf{r}_v)\exp(-iE_v t).$$

With this substitution, Equation (6.3.3) takes the following form:

$$\left\{ \left(-\frac{1}{2\lambda^{(u)}} \right) \nabla_u^2 + e^{(u)} \sum_{\substack{v=1 \\ (\neq u)}}^{n} e^{(v)} \int \frac{U_S^*(\mathbf{r}_v)U_S(\mathbf{r}_v)}{4\pi|\mathbf{r}_u - \mathbf{r}_v|} \, d\mathbf{r}_v \right\} U_S(\mathbf{r}_u)$$

$$= E_u U_S(\mathbf{r}_u). \qquad (6.3.3')$$

These are a set of n-coupled differential equations for the n-particle system. Each set of solutions (there is an infinite number of such sets $[U(r_1), U(r_2), \ldots, U(r_n)]_q$) for an n-body system, correspond to the energy eigenvalues $[E_q, \ldots]$ of the total system. The formal set of coupled differential equations (6.3.3′) expresses the *Hartree theory* for a many-body system interacting electrodynamically, such as the example of the many-electron atom. It has been applied quite successfully to the predictions of properties of complex atoms. In particular, the method of determining the energy eigenvalues of these equations with the Hylleraas variational method, especially in the case of helium, has yielded very close agreement with the observed energy level spectrum of helium.

In the terminology of quantum mechanics, the operator in the brackets, $\{\}$, on the left side of the wave equation (6.3.3′), is the time-independent Hamiltonian for the many-body system, interacting electrodynamically and nonrelativistically. The physical system that is described by this Hamiltonian contains the Coulomb interaction operator that couples the nucleus to the uth electron field, with (attractive) interaction constant $-Ze^2$, as well as all of the other (repulsive) Coulomb couplings with the remaining electrons of the atomic system.

Clearly, the eigenfunctions of the Hamiltonian operator must depend on all n coordinate vectors of the interacting electrons and nucleus, $\{\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_{u-1}, \mathbf{r}_{u+1}, \ldots, \mathbf{r}_n\}$ as well as the coordinate vector $\mathbf{r}_u$ of the uth electron. The energy eigenvalues of this Hamiltonian may then be labeled $E_{u;1,2,3,\ldots,u-1,u+1,\ldots,n}$.

To determine the form of the many-particle eigenfunction solutions of the wave equation (6.3.3′), the next step follows from the fact that each of the electron solutions (associated with each of the indices u) solves an eigenfunction equation that is functionally identical to each of the other eigenfunction equations. It then follows that the eigenfunctions of the operator which is the sum of all such individual operators, must satisfy the continuity equation:

$$\partial^t |U_S|^2 + (\hbar/2i\lambda)\nabla \cdot (U_S^* \nabla U_S - U_S \nabla U_S^*) = 0.$$

We may then take the eigenfunctions of the sum of operators, as appears in Equation (6.3.3′), as an approximation for the weighting functions in the nonrelativistic description of the n-electron atom. In the structuring of these weighting functions, all of the electrons are treated on an equal footing. Only the binding to the nuclear Coulomb field is different. Assuming now that the atomic nucleus is sufficiently inertial (relative to the electrons' inertial property), one may set up a preferred reference frame (in this nonrelativistic approximation) in which the origin of the spatial coordinates is the fixed center of mass of the atomic nucleus. This corresponds to the assumption that the nucleus is stationary relative to the electrons' motion, i.e. that the nucleus does not recoil under the influence of the electrons' interactions with it. Mathematically one would then represent the nuclear wave function in terms of the Dirac delta function:

$$|U_S(\mathbf{r}_N)|^2 \sim \delta(\mathbf{r}_N).$$

One may then treat the nuclear coordinates $\mathbf{r}_N$ as a constant parameter, standing for the vector that locates the origin of the coordinate system, which is the center of mass of the nucleus.

6.3.1. *Another Approximation for the Many-Electron Atom*

In the previous example, as in the example to be discussed below, the atomic nucleus is considered to be dynamically uncoupled from the binding electrons, because of the much larger mass of the nucleus than the electron mass. But since all of the electrons each have the same mass, they may not be uncoupled from each other in this way. It then follows that the weighting function that considers the electrons one at a time, is not too accurate a description for the true state of affairs for the electron system, even in the

nonrelativistic approximation. For the many-electron atom, we may consider a better approximation by going to a multiple-coordinate space, as follows:

$$\{U_S(\mathbf{r}_1), U_S(\mathbf{r}_2), \ldots, U_S(\mathbf{r}_n)\} \rightarrow U_S(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n)$$

To arrive at the n-electron wave function on the right, we recognize that the Coulomb function, $1/|\mathbf{r}_v - \mathbf{r}_u|$ in Equation (6.3.3′) is a much more slowly varying function of $\mathbf{r}_v$ than is the (squared) amplitude $|U_S(\mathbf{r}_v)|^2$. Thus, we may use the following approximation without too much loss of accuracy:

$$\int \frac{|U_S(\mathbf{r}_v)|^2}{4\pi|\mathbf{r}_v - \mathbf{r}_u|}\, d\mathbf{r}_v \rightarrow \frac{1}{|\mathbf{r}_v - \mathbf{r}_u|} \int \frac{|U_S(\mathbf{r}_v)|^2}{4\pi}\, d\mathbf{r}_v = \frac{1}{|\mathbf{r}_v - \mathbf{r}_u|}$$

where we have assumed that $\{U_S(\mathbf{r}_v)\}$ is a set of normalized functions. It should be noted that there is no rigorous justification for this approximation; its only justification at this stage is in the success of the predictions of the Hartree theory with the approximation.

With the preceding replacement, then, the wave equation (6.3.3′) takes the following form:

$$\left\{\left(-\frac{1}{2\lambda^{(u)}}\right)\nabla_u^2 - \frac{Ze^2}{|\mathbf{r}_N - \mathbf{r}_u|} + e^2 \sum_{\substack{v=1 \\ (\neq u)}}^{n} \frac{1}{|\mathbf{r}_v - \mathbf{r}_u|}\right\} U_S(\mathbf{r}_u) = E_u U_S(\mathbf{r}_u).$$

This is a vast simplification of the wave equation (6.3.3′) since it is no longer a differential-integral equation, but is only a (partial) differential equation. It is this form that has been successful in predicting the properties of many-electron atoms, especially the lighter ones. An example of a very successful calculation from this equation was the variational method applied to it by Pekeris [43], to determine the energy levels of neutral helium. But even so, the calculations did reveal a small deviation from the measured values of the helium energy levels, large enough to be out of the domain of experimental error. In later years, these deviations were explained in terms of radiative effects, that appear in quantum electrodynamics — though with the usual trouble of divergences that must then be subtracted away with the method of renormalization. However, the field theory of matter propounded in this monograph was applied to the problem of helium, explaining these corrections to the energy levels of helium from the predictions of a finite (convergent) calculation [44].

6.4. Scattering Cross Section

Since the 'free particle' is not defined within the relativistic interaction theory of matter discussed here, the definition of an experimentally observed entity,

called 'cross section', must be redefined, since it is usually interpreted in terms of *particles* scattering from *particles.*

In this theory, the 'cross section' is defined in terms of the weighting of the interaction between the 'projectile field', ψ^P, the 'target field', ψ^T, and the rest of the scattering environment of the scattering entities. The terms 'target' and 'projectile' are used here, not to signify separate, singular entities, but rather they refer to *apparent* separate entities that asymptotically look that way, when the intrinsic coupling is sufficiently weak. This would be analogous to the scattering of moving ripples from each other in a pond — these ripples are not in reality separate entities; they are rather all modes of the same continuum.

The 'cross section' is formally defined as the ratio of the total flux flowing out of some domain of interaction (units s^{-1}) to incident flux (units $cm^{-2}\ s^{-1}$) flowing into it. But 'flux' in this theory refers to the flow of interaction weighting, rather than the flow of actual free particles of matter. Defining the interaction current density as $\mathbf{j} = \bar{\Psi}\boldsymbol{\gamma}\Psi$, where Ψ is the interaction field amplitude discussed earlier, the mathematical expression for the cross section is as follows:

$$\sigma = \left[\int_\Sigma \mathbf{j} \cdot \mathbf{n}\, d\Sigma\right] \Big/ j_{\text{inc}} \tag{6.4.1}$$

where the integration is over a 'surface integral' of domain Σ, that encloses some volume V. The field current density of incident flux is

$$j_{\text{inc}} = f^T \bar{\psi}_0^P \gamma_k \psi_0^P \tag{6.4.2}$$

where k denotes the spatial direction of the incident beam of interacting matter, ψ_0^P is the asymptotic limit of the projectile matter field, when it is very weakly coupled to the target matter field, defined to be *outside of* an interaction volume V, surrounded by the surface Σ, and f^T are the (asymptotic) target variables. It then follows that the scattering cross section (6.4.1) may be expressed in the following form:

$$\sigma = \left| \left[\partial_t \int \Psi^\dagger \Psi\, d^3x\right] \Big/ f^T(\bar{\psi}_0^P \gamma_k \psi_0^P) \right|. \tag{6.4.3}$$

The numerator in this relation followed from the use of the Gauss theorem applied to the continuity equation, i.e.

$$-\partial_t \int_V \Psi^\dagger \Psi\, d^3x = \int_V \partial^j(\bar{\Psi}\gamma_j\Psi)\, d^3x = \int_\Sigma (\bar{\Psi}\gamma_j\Psi) n^j\, d\Sigma.$$

The general expression for the cross section is then given by Equation (6.4.3).

With the use of the results of the preceding section, in the proof of the Pauli exclusion principle, it follows that whenever the two scattering particle fields would be such that they satisfy the three conditions, (1) in the same state of motion, (2) having a repulsive electrodynamic interaction and (3) have equal inertial masses, the cross section reduces to the form in which the interaction field amplitude Ψ in the numerator of Equation (6.4.3) is replaced with the interaction field amplitude, Ψ_{uv} — representing the total interaction field amplitude of the closed system excluding the interaction of the two matter fields that obey the restriction of the Pauli principle. In this case, the numerator in (6.4.3) would refer, physically, to the background field that the scattered electrons or protons, etc., move through — such as the set of electron—positron pairs that are present in the background of any observed matter, according to the general results of this field theory (as we will see in the next chapter), or according to the model in quantum field theory.

In the case in which the nonlinear coupling between the projectile and target matter components would be sufficiently weak, the target variables in the interaction field of the numerator of (6.4.3) would factor out, canceling the target variable f^T in the denominator. The remaining rate of transition from the unscattered beam to the scattered beam, expressed in the numerator, would then only entail the coupling of the projectile matter and the environment of the target. It will be seen, in Chapter 7, that this theory of matter, that is an explicit expression of its inertial manifestation, entails a 'physical vacuum' of a type different from the model of the vacuum of quantum field theory. Here, any 'observed' matter must be in an environment of a (very dense) sea of particle—antiparticle pairs, each in its state that corresponds to minimum energy and momentum, which turns out to be null energy and null momentum. This result will be found precisely, from an exact solution of the bound particle—antiparticle pair, in the next chapter. These pairs in the environment of any observed matter are then a *countable set* of matter fields, as in the case of a gas of molecules, such as the model of air. It is a model in contrast with the model of quantum electrodynamics, where the 'physical vacuum' is a *noncountable set* of pairs, supposedly annihilating and being created from a vacuum, spontaneously and randomly.

The problem of analysing the scattering of fermion particles may then entail the interaction of the projectile and a countable set of pair fields, that belong to the 'physical vacuum', according to this theory. The source of the interaction is here a predetermined, nonlinear coupling. In this regard, an important problem for future investigation concerns the implications of the Pauli principle in high-energy, charged matter scattering, within the context of this deterministic, nonlinear field theory of matter and inertia.

In the next two chapters, the general theory of matter developed thus far will be applied to two cases of two-particle systems — the system of particle and antiparticle and the system of electron—proton. In Chapter 7, an *exact*

solution for the coupled matter—antimatter field equations will be demonstrated, revealing predictions of all of the empirical results normally attributed to pair annihilation and creation, though from a basis in which matter is not actually annihilated or created. The analysis will then be applied to a derivation of the spectrum of blackbody radiation, without the use of the photon concept, and other phenomena that are conventionally interpreted in terms of photon interactions, such as the Compton effect, will be discussed in the same context. In Chapter 8, the energy level spectrum of hydrogen will be derived, including a correct prediction of the Lamb shift (normally attributed to the model in quantum electrodynamics). In conclusion, analysis of the e—p system, unbound, will be applied to their scattering at high energy, particularly focusing on the added effects of the generalized electromagnetic theory.

Chapter 7

The Particle—Antiparticle Pair Without Annihilation Creation

Throughout the preceding theoretical development of a basis for the behavior of matter in the microscopic domain of atoms, nuclei and elementary particles, we have concentrated on the features of a formalism for either an entire closed system of many interacting component matter fields, or the one-body approximation, where one considers, as an approximation, the coupling of a single matter component to an averaged background field, representing the remainder of the assumed closed system. In this chapter, and in the following chapter, we will investigate two very important examples of 'two-body' systems, within the context of this theory — assuming again that the 'rest' of the closed system may either be neglected altogether or that it may be represented by an averaged background matter field.

With the assumed uncoupling of the two-body system from the rest of its environment, (which is nevertheless *there*!) we will consider the coupled spinor field Equations (6.1.1′) applied to the particle—antiparticle pair, determining an *exact* solution that has particularly important physical consequences in modern-day physics. It should be noted here that we *define* the term 'antiparticle' as being associated with a matter field that has the same inertial mass as a 'particle' matter field, though with opposite polarity in its interaction with a third charged matter field. That is, if a 'particle' field should interact with the third charged matter field attractively, the 'antiparticle' field would interact with it repulsively. Thus, the 'particle' and 'antiparticle' fields would interact with each other always attractively.

The *exact* solution to be demonstrated below, for a particular bound state of the particle—antiparticle two-body system will be shown to relate to *all* of the experimental facts that are conventionally interpreted as 'pair annihilation', including the observations normally interpreted as the simultaneous production of a pair of photons — though here without the need to introduce the photon as an elementary concept.

In Chapter 8 we will analyse the electron—proton system in its bound states, revealing the entire hydrogen spectrum, including the Lamb splittings. The latter effect, which is predicted in the conventional theory of quantum electrodynamics with renormalization calculational methods has been

claimed by the adherents of the Copenhagen school and the formal expression of the quantum theory, as one of the giant successes of that theory — even though its only method of calculation is not demonstrably mathematically consistent. But the same experimental data will be found to follow from this theory, *for entirely different physical reasons*, from a mathematically consistent formalism. Combining these results with the results of this chapter, it will then be seen in Chapter 8 that the correct 'lifetimes' of the excited states of hydrogen must also follow.

7.1. The Field Equations for the Particle—Antiparticle Pair

In the formulation that follows, there will be a change of representations for the spinor electromagnetic Equations (5.3.4). Rather than the identification with the standard electromagnetic variables (**E**, **H**, ρ and **j**) according to Eq. (5.3.2), the following *equivalent* correspondences will be used (in any particular frame), with $\bar{G}_k = H_k - iE_k$, for $\sigma^\mu \partial_\mu \phi_\alpha = \mathrm{T}_\alpha$:

$$\phi_1 = \begin{pmatrix} \bar{G}_3 \\ \bar{G}_1 + i\bar{G}_2 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} -\bar{G}_1 + i\bar{G}_2 \\ \bar{G}_3 \end{pmatrix} \tag{7.1.1a}$$

$$\mathrm{T}_1 = 4\pi i \begin{pmatrix} -\rho + j_3 \\ j_1 + ij_2 \end{pmatrix} \equiv e\bar{\psi}\Gamma_1\psi = e4\pi i \begin{pmatrix} \bar{\psi}(-\gamma_0 + i\gamma_3)\psi \\ \bar{\psi}(i\gamma_1 - \gamma_2)\psi \end{pmatrix} \tag{7.1.1b}$$

$$\mathrm{T}_2 = 4\pi i \begin{pmatrix} -j_1 + ij_2 \\ \rho + j_3 \end{pmatrix} \equiv e\bar{\psi}\Gamma_2\psi = e4\pi i \begin{pmatrix} \bar{\psi}(-i\gamma_1 - \gamma_2)\psi \\ \bar{\psi}(\gamma_0 + i\gamma_3)\psi \end{pmatrix}$$

where $j_\mu = (\rho; j) = ie\bar{\psi}\gamma_\mu\psi$, $\bar{\psi} = \psi^\dagger\gamma_0$, $\partial_0 = -i\partial_t$, $\sigma^\mu\partial_\mu \equiv \sigma^0\partial_0 + \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}$ and the Dirac matrices have the following representation:

$$\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma_k = -i\begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} \quad (k = 1, 2, 3)$$

$\sigma^0 = iI$, I is the unit two-dimensional matrix and σ_k are the Pauli matrices, (4.2.2b).

Let us now apply the field Equations (6.1.1′) to the bound electron—positron pair, *neglecting their coupling to the rest of the universe*. [The same analysis applies to any other particle—antiparticle pair in a bound state.] Since, *by definition*, each of these matter components has the same value of inertial mass, the field equations for the bound pair have the following form:

$$\{\gamma^\mu\partial_\mu - \mathscr{A}(e^+) + \lambda\}\psi^{(e^-)} = 0 \tag{7.1.1c}$$

$$\{\gamma^\mu\partial_\mu - \mathscr{A}(e^-) + \lambda\}\psi^{(e^+)} = 0, \tag{7.1.1d}$$

where the charge conjugated bispinor solutions relate to each other as follows:

$$\psi^{(e^+)} = C\psi^{(e^-)}, \quad \text{where} \quad C = \gamma^2 K_0 = \begin{pmatrix} & & & -1 \\ & & 1 & \\ & 1 & & \\ -1 & & & \end{pmatrix} K_0$$

and K_0 is the operator corresponding to taking the complex conjugate of the function to which it applies.

As we have seen in Chapter 6, the interaction coupling in electrodynamics has two terms

$$\mathscr{I} = \mathscr{I}_1 + \mathscr{I}_2$$

where

$$\mathscr{I}_1(e^\pm)\psi^{(e^\pm)} = [e^\pm e^\mp \gamma^\nu \int \bar{\psi}^{(e^\pm)}\gamma_\nu \psi^{(e^\pm)} S(x - x')\, \mathrm{d}^4x']\,\psi^{(e^\mp)} \tag{7.1.2a}$$

$$\mathscr{I}_2(e^\pm)\psi^{(e^\pm)} = -ig_M e^\mp \sum_{a=1}^{2} a_a(\phi_a^{(e^\pm)} \cdot \Gamma_a - \gamma_0\Gamma_a^\dagger\gamma_0 \cdot \phi_a^{(e^\pm)})\psi^{(e^\mp)}. \tag{7.1.2b}$$

[Normally, the part of the generalized interaction called $\mathscr{I}_2$ would violate charge conjugation symmetry. But with this particular situation, *with the solution to be demonstrated*, C-symmetry is not violated because it turns out that $\mathscr{I}\psi = 0$ here. The rest of the equation above does obey C-symmetry.] It is clear that the coupled Equations (7.1.1c,d) satisfy the requirement of this theory that the form of the field equations is unaltered under the exchange of the matter field variables, $\psi^{(e^-)}$ and $\psi^{(e^+)}$, as well as satisfying the requirements of special relativity theory, i.e. covariance with respect to the Poincaré group.

Consider now the special case in which the particle and antiparticle fields are in the same state of motion. In this case, the source fields in the electromagnetic field equations for the particle and antiparticle must be the same; that is, they must be characterized by the same constants of the motion. Denoting the latter index by n, the spinor form of the electromagnetic equations (as discussed in Chapters 5 and 6) for the electron and positron fields are, respectively:

$$\sigma^\mu \partial_\mu \phi_a^{(e^-)} = -e\bar{\psi}_n^{(e^-)}\Gamma_a \psi_n^{(e^-)} \tag{7.1.3a}$$

$$\sigma^\mu \partial_\mu \phi_a^{(e^+)} = e\bar{\psi}_n^{(e^+)}\Gamma_a \psi_n^{(e^+)}, \tag{7.1.3b}$$

where we have used the notation

$$e^+ = -e^- = e. \tag{7.1.3c}$$

Substituting $C\psi_n^{(e^-)}$ for $\psi^{(e^+)}$ in Equation (7.1.3b), we have, after some mathematical manipulation,

$$\sigma^\mu \partial_\mu \phi_\alpha^{(e^+)} = e\bar{\psi}_n^{(e^-)} \Gamma_\alpha \psi_n^{(e^-)}. \tag{7.1.3d}$$

It then follows that when the electron and positron fields correspond to the same state of motion, i.e. with Equation (6.2.2), and $u = e^-$, $v = e^+$, the *sum* of Equations (7.1.3a) and (7.1.3b) yields the following electromagnetic field equations *for the pair*:

$$\sigma^\mu \partial_\mu \phi_\alpha^{(\text{pair})} = 0, \tag{7.1.4}$$

where, by definition, the pair field is the sum of spinor fields:

$$\phi_\alpha^{(\text{pair})} = \phi_\alpha^{(e^-)} + \phi_\alpha^{(e^+)}. \tag{7.1.5}$$

The pair field, $\phi_\alpha^{(\text{pair})}$ for the electron and positron *in this particular state of motion*, represents the electromagnetic field of influence that is exerted by the pair, *as a unit*, on other charged matter. But according to the interpretation of the electromagnetic equations, as an identity, the vanishing source field on the right-hand side of Equation (7.1.4) must correspond to a solution $\phi_\alpha^{(\text{pair})}$ that is identically zero. It is then concluded that when the interacting components of the particle—antiparticle pair are in the same state of motion, this bound two-particle system will not couple, *as a unit*, to other charged matter. Nevertheless, it is still true that each constituent interacting component of the pair couples separately to other charged matter, since each, on its own, does have a nonzero field value in space-time. It also follows that the electron and positron field components separately maintain their inertial property, with mass value m. Consequently, the field Equation (7.1.4) does incorporate a substructure that would not allow its solutions to be interpreted in terms of source-free electromagnetic radiation.

Finally, it is to be noted that when the particle and antiparticle are in the same state of motion, Equation (7.1.3d) in Equation (7.1.2) implies that (under these special circumstances),

$$\mathscr{I}(e^-)\psi^{(e^+)} = \mathscr{I}(e^+)\psi^{(e^-)}. \tag{7.1.6}$$

This is an important result, to be used later on.

7.2. An Exact Bound State Solution for the Particle—Antiparticle Pair

It will now be demonstrated that with the choice $a_1 = -a_2$ in the coupling term $\mathscr{I}_2$ [Equation (7.1.2b)] and with the electron and positron fields in the

same state of motion, an exact solution of the field Equation (7.1.1c) is the following:

$$\psi^{(e^+)} = -\psi^{(e^-)} = \begin{pmatrix} \exp(-i\lambda t) \\ 0 \\ 0 \\ \exp(i\lambda t) \end{pmatrix} \equiv \psi. \tag{7.2.1}$$

Substituting this function into the electromagnetic field Equations (7.1.1b) and (7.1.3), we have

$$\sigma^\mu \partial_\mu \phi_1^{(e^+)} = e\bar{\psi}\Gamma_1\psi = -8\pi ie \begin{pmatrix} 1 \\ -\exp(2i\lambda t) \end{pmatrix} \tag{7.2.2a}$$

$$\sigma^\mu \partial_\mu \phi_2^{(e^+)} = e\bar{\psi}\Gamma_2\psi = 8\pi ie \begin{pmatrix} -\exp(-2i\lambda t) \\ 1 \end{pmatrix}. \tag{7.2.2b}$$

Using the integral representation of the solutions of (7.2.2) [as derived in *GRM*, Equation (5.30)] the following result is obtained [47]:

$$\phi_1(x)^{(e^+)} = \frac{8\pi ie}{(2\pi)^4} \int \exp[ik^\mu(x_\mu - x'_\mu)] \frac{-i\bar{\sigma}^\beta k_\beta}{k^\alpha k_\alpha} \times$$

$$\times \begin{pmatrix} 1 \\ -\exp(2i\lambda t') \end{pmatrix} d^4k\, d^4x', \tag{7.2.3a}$$

$$\phi_2(x)^{(e^+)} = \frac{8\pi ie}{(2\pi)^4} \int \exp[ik^\mu(x_\mu - x'_\mu)] \frac{-i\bar{\sigma}^\beta k_\beta}{k^\alpha k_\alpha} \times$$

$$\times \begin{pmatrix} -\exp(-2i\lambda t') \\ 1 \end{pmatrix} d^4k\, d^4x', \tag{7.2.3b}$$

where $\bar{\sigma}^\beta \equiv (\sigma^0; -\sigma^k)$ are the (space-)conjugated quaternion basis elements.

Carrying out the integrations indicated above, we have the solutions

$$\phi_1^{(e^+)}(x) = \left(\frac{4\pi e}{\lambda}\right) \begin{pmatrix} 0 \\ \exp(2i\lambda t) \end{pmatrix}, \tag{7.2.4a}$$

$$\phi_2^{(e^+)}(x) = \left(\frac{4\pi e}{\lambda}\right) \begin{pmatrix} \exp(-2i\lambda t) \\ 0 \end{pmatrix}. \tag{7.2.4b}$$

The spinor field solutions (7.2.3) satisfy the differential Equations (7.2.2), while the integrated functions (7.2.4) do not. The apparent difficulty is associated with the treatment of the constant part of the source fields in the integral form of the solutions (7.2.3), and with the associated breakdown of the conditions for the existence of a Fourier transform. The solution (7.2.4) only reveals a part of the actual solution — the part that is coordinate-dependent. To complete the solution, one may add the spinors in (7.2.4) to the constant spinors,

$$s_1 = \left(\frac{4\pi e}{\lambda}\right)\begin{pmatrix}1\\0\end{pmatrix} \quad \text{and} \quad s_2 = \left(\frac{4\pi e}{\lambda}\right)\begin{pmatrix}0\\1\end{pmatrix} \tag{7.2.5}$$

respectively, yielding the following particular solutions of the electromagnetic field Equations (7.2.2):

$$\phi_1(x) = \left(\frac{4\pi e}{\lambda}\right)\begin{pmatrix}1\\ \exp(2i\lambda t)\end{pmatrix}, \tag{7.2.6a}$$

$$\phi_2(x) = \left(\frac{4\pi e}{\lambda}\right)\begin{pmatrix}\exp(-2i\lambda t)\\ 1\end{pmatrix}. \tag{7.2.6b}$$

These are, then, the unique electromagnetic spinor field solutions to be inserted into the coupling terms (Equation (7.1.2b)), which in turn are inserted into the field Equations (7.1.1c,d).

A similar difficulty as the above, concerned with the constant term, occurs in the coupling term $\mathscr{I}_1$ (Equation (7.1.2a)). The integral appearing in this term stands for the particular solution A_μ of the equation

$$\Box A_\mu = 4\pi e\bar{\psi}\gamma_\mu\psi. \tag{7.2.7}$$

As we have seen earlier, the d'Alembertian operator $\Box$ is a product of conjugated first-order quaternion differential operators, i.e. a fundamental expression of Equation (7.2.7) is

$$\sigma^\nu\partial_\nu(\bar{\sigma}^\nu\partial_\nu A_\mu) = 4\pi\sigma^0(e\bar{\psi}\gamma_\mu\psi), \tag{7.2.7'}$$

which is equivalent to the pair of first-order differential equations:

$$\begin{aligned}\sigma^\mu\partial_\mu\Sigma_\nu &= \sigma^0(e\bar{\psi}\gamma_\nu\psi)\\ \Sigma_\nu &= \bar{\sigma}^\mu\partial_\mu A_\nu.\end{aligned} \tag{7.2.7''}$$

It follows that the same technique that was used to solve the spinor Equations (7.2.2) may be used to solve (7.2.7). The solutions Σ_ν are given by

the form in Equation (7.2.3), with the substitutions:

$$\phi_a \to \bar{\sigma}^\mu \partial_\mu A_\nu$$
$$\bar{\psi}\Gamma_a\psi \to (e\bar{\psi}\gamma_\nu\psi)\sigma^0. \tag{7.2.8}$$

Also, as in the preceding analysis of the spinor electromagnetic equations, the constant spinors proportional to $\binom{1}{0}$ and $\binom{0}{1}$ must be added to the integral form of the solutions whenever a source field might be constant. As in the previous case, this addition completes the solution of the (first iteration of the) differential Equation (7.2.7′), and gives a null solution for a null source, as it is required by the conceptual basis of this theory.

Once we have solved for $\Sigma_\nu = \bar{\sigma}^\mu \partial_\mu A_\nu$ in this way, the operation must be repeated once more in order to solve for A_ν (from the second of the spinor Equations (7.2.7″)). The form of the solution of the latter equation is identical with the first (iterated) form, except for the conjugation of all quaternions that appear in (7.2.3).

Carrying out this procedure for the specific solution (7.2.1), the following result is obtained for the interaction functional $\mathscr{I}_1$ (Equation (7.1.2a))

$$\mathscr{I}_1 = \left(\frac{4\pi e^2 i}{2\lambda^2}\right)(\gamma_0 - \gamma_1 \cos 2\lambda t - \gamma_2 \sin 2\lambda t). \tag{7.2.9}$$

Since the solution (7.2.1) in the function $\bar{\psi}\gamma_0\psi = \psi^\dagger\psi$ gives a constant — which in turn yields a null value for the integral

$$\int \psi^\dagger \psi S(x - x')\, \mathrm{d}^4 x',$$

the first term on the right-hand side of Equation (7.2.9) had to be derived from the insertion of the constant contributions to the solutions of Equation (7.2.7′), as specified above. The second and third terms on the right side of (7.2.9) follow from integrating the right-hand side of Equation (7.1.2b), with the use of the symmetric Green's function (5.4.8) for $S(x - x')$.

It is readily verified, with the solution (7.2.1) for the electron and the positron field, that the following is true:

$$(\gamma_1 \cos 2\lambda t + \gamma_2 \sin 2\lambda t)\psi = \gamma_0 \psi. \tag{7.2.10}$$

It then follows from (7.2.9) and (7.2.10) that for the first part of the electromagnetic interaction functional,

$$\mathscr{I}_1(e)\psi^{(e)} = 0. \tag{7.2.11}$$

To evaluate the second part of the generalized interaction functional [Equation (7.1.2b)], we substitute the spinor field solution (7.2.6), for the electromagnetic field intensity, and (7.2.1) for the matter field of the electron or positron. Also using the relationship (7.2.10) we find that

$$\mathscr{J}_2(e)\psi^{(e)} = -32\pi^2(e/\lambda)g_M(a_1 + a_2)\gamma_3\psi^{(e)}. \tag{7.2.12}$$

It is at this stage where we set $a_1 = -a_2 = 1$, thereby fixing the form of the electrodynamic functional added by the generalization to the spinor formalism, for all future applications. It is for this reason that in earlier chapters the phases were specified to be $a_\alpha = (-1)^\alpha$, (see the corresponding Lagrangian (5.4.1)). With this specification of the unique form of this functional, we have the following result for the case of the solution (7.2.1):

$$\mathscr{J}_2\psi^{(e)} = 0. \tag{7.2.11'}$$

Thus we see that for the total electromagnetic interaction functional, for the bound electron—positron (or any other particle—antiparticle) system in this particular state,

$$(\mathscr{J}_1 + \mathscr{J}_2)\psi^{(e)} = \mathscr{J}\psi^{(e)} = 0. \tag{7.2.13}$$

At this stage it is important to note that while the operation of the functional $\mathscr{J}$ on the bound state function $\psi^{(e)}$ yields a zero value, the coupling functional $\mathscr{J}$ itself is not zero. The result (7.2.13) is a consequence of the nonlinear features of the coupled matter equations for the pair — under the special set of conditions that have been specified in this analysis. It should also be noted that the solution (7.2.1) is expressed in the proper frame of the particle—antiparticle pair. In any other Lorentz frame, the argument of the exponential would, of course, generalize to the scalar $k^\mu x_\mu = \mathbf{k}\cdot\mathbf{r} - \lambda t$, where the vector $\mathbf{k}$ takes account of the motion of the pair relative to an observer.

With the result (7.2.13) in the matter field Equations (7.1.1), it follows that the function shown in Equation (7.2.1) is indeed an *exact* solution of these nonlinear field equations, when they represent the special case in which the particle and antiparticle, while bound, are each in the same state of motion.

7.3. The Energy and Momentum of the Bound Particle—Antiparticle in its Ground State

It follows from Noether's theorem [32] that the covariance of a field theory with respect to infinitesimal translations in time and space imply, respectively, the conservation of energy, P_0, and the conservation of the three

components of momentum, P_k, defined as follows:

$$P_0 = \int \sum_{i=1}^{n} \left[\left(\frac{\partial \mathscr{L}}{\partial(\partial_0 \Lambda_\xi^{(i)})} \partial_0 \Lambda_\xi^{(i)} \right) - \mathscr{L} \right] \mathrm{d}\mathbf{r} \tag{7.3.1a}$$

$$P_k = \int \sum_{i=1}^{n} \left(\frac{\partial \mathscr{L}}{\partial(\partial_0 \Lambda_\xi^{(i)})} \right) (\partial_k \Lambda_\xi^{(i)}) \, \mathrm{d}\mathbf{r} \quad (k = 1, 2, 3) \tag{7.3.1b}$$

The summations in these integrals are taken over all fields $\{\Lambda_\xi^{(i)}\}$, where ξ denotes the field components. In the special case of the bound electron–positron pair that we have been discussing, there are twelve such field variables

$$\{\psi^{(e^-)}, \bar{\psi}^{(e^-)}, \psi^{(e^+)}, \bar{\psi}^{(e^+)}, \phi_\alpha^{(e^-)}, \phi_\alpha^{(e^-)\dagger}, \phi_\alpha^{(e^+)}, \phi_\alpha^{(e^+)\dagger}\} \tag{7.3.1c}$$

where $\alpha = 1, 2$ denotes the two separate electromagnetic field spinors.

Taking the Lagrangian density $\mathscr{L}$ in Equation (7.3.1) to be the sum of the integrands in the contributions to the action functionals,

$$\mathscr{L} = \mathscr{L}_M + \mathscr{L}_D$$

we obtain the energy and momenta for the pair in the particular bound state (7.2.1), in accordance with the expressions (7.3.1). The two parts of the Lagrangian density for the pair are the following explicit forms:

$$\mathscr{L}_M = ig_M \sum_{u \neq v=1}^{2} \sum_{\alpha=1}^{2} (-1)^\alpha \phi_\alpha^{(u)\dagger} (\sigma^\mu \partial_\mu \phi_\alpha^{(v)} - 2e^{(v)} \bar{\psi}^{(v)} \Gamma_\alpha \psi^{(v)}) + \text{h.c.} \tag{7.3.2a}$$

which follows from (5.4.1), where 'h.c.' is the hermitian conjugate of the term that precedes this symbol, (u) and (v) stand for the electron and positron fields, and where

$$\mathscr{L}_D = \hbar c \sum_{u=1}^{2} \{\bar{\psi}^{(u)} (\gamma^\mu \partial_\mu + \mathscr{A}_1(u) + \lambda(e)) \psi^{(u)} + \text{h.c.}\}. \tag{7.3.2b}$$

The constant $\hbar c$ is inserted above in $\mathscr{L}_D$ to demonstrate that this is the point where Planck's constant appears in the theory (rather than appearing most fundamentally in commutation relations between the position and momentum operators, as in quantum mechanics), where, as usual, $h = 2\pi\hbar$ is Planck's constant. This constant will not appear normally in the theoretical development only because units are used with $\hbar = c = 1$. In $\mathscr{L}_D$, $\lambda(e)$ is the inertial mass (divided by $\hbar c$) for the electron or positron (or for proton or antiproton, etc.).

With the fields for the pair, indicated in (7.3.1c), and the solutions for this particular bound state, indicated in (7.2.1) and (7.2.6), it is readily shown from the expressions (7.3.1) that the conserved energy and momenta of a

pair, in this bound state, are:

$$P_0 = P_1 = P_2 = P_3 = 0. \tag{7.3.3}$$

That is, the conserved energy-momentum of the pair in this state is a null vector, with each component equal to zero. Because of the latter feature, that all four components of P_ν are separately zero in some Lorentz frame, they must all be zero in any other Lorentz frame. Thus, the result derived for the *ground state* of the pair — the state of minimum energy and momentum being zero — is *Lorentz-invariant.*

The reason that the zero-energy for this problem is indeed the *ground state* is due to the feature of any 'classical field theory' that would not allow *both* negative and positive energy values in the total spectrum. This follows from the definition of 'energy' in terms of a set of generators of continuous transformations in the function space that represents the physical system. That is, the energy spectrum is related to the invariance with respect to continuous transformations in the direction of the time coordinate axis, the momentum spectrum is related to invariance with respect to continuous transformations in the direction of the three spatial coordinate axes. Now the transition from positive to negative energy values (or vice versa) requires a discontinuous jump. This would be allowed in the quantum theory, according to its basis, but it is not allowed in a 'classical' field theory. Thus, starting from a zero value of energy, all other energy values can be either only positive or only negative, but there cannot be a mixture of positive and negative energy values in the spectrum. The convention that will be chosen here, as in the usual 'classical' field theories, is that of a positive energy spectrum. The reason that it would not have mattered had we chosen the negative energy spectrum instead is that all measurements of these quantities always entail energy differences — the difference between two negative energy values would be the same as the difference between two corresponding positive energy values. But if there would be both positive and negative energy values present in the spectrum, as in the quantum theory, extra transitions would be allowed between the positive and negative energy values, that would not appear in the predictions of the 'classical' field theory.

It also follows from the general form of the Lagrangian density above, due to its invariance with respect to rotations of the space-time coordinate system (a subclass of the transformations of the symmetry group of special relativity), Noether's theorem predicts a particular set of components of a tensor, called 'angular momentum', that is conserved. With the field solutions of this problem, it is then readily shown that the conserved angular momentum for the electron—positron pair, in this particular bound state, is identically zero (in all of its tensor components, separately). Thus, when the pair is in this ground state of null energy and null momentum, this would correspond, spectroscopically, to the pair being in the $^1S_0$ state.

7.4. The Free Particle Limit and Pair Creation

When the electromagnetic coupling functional $\mathscr{I}$ in Equation (7.1.1), *in itself*, approaches zero (though not the case, as above, where $\mathscr{I} \neq 0$, but $\mathscr{I}\psi = 0$, to arrive at the physical equivalent of 'pair annihilation'), then the coupled matter field equations approach the form of the 'free field' equations for the electron and positron, separately. The solutions of these equations are the conventional ones, that are the plane waves appearing in the Dirac theory [6]. With these solutions in the two-particle (uncoupled) system Lagrangian, the conserved energy and momentum for the electron and positron are the components of the energy-momentum four-vector

$$p_\mu^{(e^-)} + p_\mu^{(e^+)} = \{\mathbf{p}^{(e^-)} + \mathbf{p}^{(e^+)};\, 2\lambda\}, \tag{7.4.1}$$

where $\mathbf{p}^{(e^-)}$ and $\mathbf{p}^{(e^+)}$ are the continuum values of the 3-momenta associated with the limiting 'free particle' and 'free antiparticle' matter fields. The conserved energy, $2\lambda = 2mc^2$, where m is the electron mass, follow from the expression (7.3.1a) when the plane wave Dirac solutions are inserted into the Lagrangian, and the corresponding spinor solutions ϕ_a would be used for the free particles, called 'electron' and 'positron'.

To sum up, we have seen that the *general form* of the matter field equations for an electron–positron pair (or any other pair, with the appropriate insertion of a different mass value) predicts two limiting solutions that correspond to the extrema of the energy spectrum for this two-body system. The *ground state* of null energy and null momentum, corresponding to the exact solution (7.2.1) of the coupled matter field Equations (7.1.1) represents the *maximum binding* for the pair. This state corresponds to the physical phenomena associated with 'pair annihilation' — though matter is not in fact annihilated here; it is rather that the pair is 'invisible' because of being in a state of such high binding that it does not readily give up energy and momentum to its surroundings, that would make it visible, for example as 'tracks' in a bubble chamber. On the other hand, the asymptotic 'free particle' solutions of Equation (7.1.1), corresponding, physically, to the interaction functional being arbitrarily close to zero (to describe the *appearance* of two free particles, electron and positron, that are effectively an infinite distance apart) represents the limit of *no binding*. Note once again that this limit is not physically realizable in practice or in principle, even though it may be approached arbitrarily closely.

We have seen, then, that this theory predicts that in any given 'rest frame', the range of energy that is available for the mutual interaction of the particle–antiparticle pair is

$$\Delta E = (2\lambda - 0) = 2\lambda = 2mc^2.$$

For the electron–positron pair, this is the order of 1 MeV, for the proton–antiproton pair it is the order of 2 GeV. The physical meaning of this *exact*

result from the nonlinear field equations for matter—antimatter coupling is that it should take such quantities of energy-momentum transfer to the pair in its ground state to excite it into a state in which the constituent particles of the pair appear to be free of each other. These events, of course, are observed in high-energy physics experimentation, and they are interpreted as 'pair creation'. Similarly, when a particle and antiparticle field, that appear to be free of each other, are in such a state that they are capable of giving up such quantities of energy to their surroundings, then the maximum energy that can be transferred is equal to $2mc^2$. The latter occurs when the pair goes into its ground state discussed above, that is conventionally interpreted as 'pair annihilation'. But this event, as observed in terms of the vanishing of converging tracks at a vertex, does not correspond, here, to an actual annihilation; it is rather the trapping of a particle and antiparticle in their field of mutual influence, causing them to go into their state of maximum binding.

The processes of pair annihilation and pair creation, according to this theory of matter (based on the theory of general relativity) predicts that these events are not intrinsically statistical, as it is asserted by quantum field theory, and there is no need to invent a mechanism that would play the role of actually annihilating matter (or creating it) since the experimental observations follow from this theory without in any way altering the actual quantity of matter that makes up the closed system.

7.5. The Continuity of Energy Values

The general expression for the energy and momentum that is associated with the field description in given in Equation (7.3.1). Since the total Lagrangian density $\mathscr{L}$ is a function that is continuous with respect to the differential changes of the parameters that appear in its argument, and since the field variables that appear in this functional are continuously distributed in their function space (and are not, generally, the solutions of eigenfunction-type equations) it follows that the appropriate values of energy and momentum of the pair, according to Equation (7.3.1), are *continuously distributed* from 0 to 2λ.

On the other hand, as we have seen earlier (in Chapter 6), the coupling of the matter fields can approach (though not reach) zero, in which case the matter field Equations (7.1.1) would correspondingly approach the linear eigenfunction form of quantum mechanics, in this limit. In the latter limit of weak coupling, the interaction weighting (that relates in this theory to the observed energy spectrum) approaches a *peaked distribution* of values. For example, the observed ^3S and ^1S states of the usual 'positronium' bound states, do indicate the peaked distribution of values for the energy and angular momentum when these fields are very weakly coupled compared with the

magnitude of their maximum binding, i.e. these states of 'positronium' are only a few electron-volts below the 'ionization' state where the electron and positron would appear to be free of each other, whereas the maximum binding of the bound pair, as we have determined above, are the order of a million electron-volts below the 'free' state. Even so, the *observed* ^3S and ^1S states of positronium have a *finite width*. According to the present theory, the source of the finiteness of this measured width is the nonlinear coupling between the electron and positron components of the pair. Since the latter coupling can never 'turn off', in principle, the *actual limit* of discreteness of the energy levels does not exist, though it can be approached arbitrarily closely.

The prediction then follows from the theory that as the particle—anti-particle pair energy spectrum is viewed from minimum binding (i.e. maximum total energy = $2mc^2$) to the maximum binding (i.e. minimum total energy = 0), the peaks on the energy spectrum of the pair become less and less sharp, until they wash out altogether, when the relative energy of coupling is great enough.

7.5.1. *Rejection of the Photon Model in 'Pair Annihilation'*

From the experimental data that imply that 2λ units of energy are transferred to an apparatus when a pair annihilates, it is also concluded that this energy is distributed equally between two 'photons', $\gamma(\pm\mathbf{k})$, that are said to be emitted simultaneously, propagating in opposite directions. The conventional interpretation of the 'annihilation' process then asserts that there must be photons in existence at times when matter does not exist. This interpretation is in contradiction with the conclusion of this theory (as we have discussed earlier) that 'photons' do not exist as fundamental entities in the electrodynamic theory of matter. Thus it is incumbent on this theory to explain all of the experimental facts having to do with 'pair annihilation', usually understood in terms of the participation of 'photons', without the need to introduce photons at all.

To start out, then, we note that one does not directly *see* the photons $\gamma(\pm\mathbf{k})$ with instrumentation — they are rather *inferred* from the observed response of the charged matter in the detector (e.g. a Geiger counter or a bubble chamber) to other (electromagnetically coupled) charged matter of the source of the interaction. Might it then not be possible that the actual observations can be explained in terms of *direct* current—current coupling, without the need to introduce the *intermediate* photons? We will now see that this is indeed the case.

We have seen above that when an electron and positron go into their ground state of maximum binding, the maximum quantity of energy-momentum that could be transferred to a detecting apparatus — say two Geiger counters, one on each side of the pair — would be 2λ. This prediction is in

agreement with the experimental facts. To complete the comparison with experimentation on this phenomenon, however, it is further required to show that when these two Geiger counters are equidistant, along a common axis on each side of a positron—electron source, they would respond simultaneously (i.e. in coincidence), each absorbing a quantity of energy equal to λ.

The simultaneity of the responses of the two counters in this problem follows automatically from the prediction of the theory since here one does not have separate space-time coordinate systems at the outset. The fields representing the interaction between the pair and the detecting apparatus are all mapped in the same space-time; that is to say, the interactions that are described by all of the field variables of this theory depend on only one time parameter. Further, since the solution (7.2.1) (in its particular Lorentz frame) does not single out any spatial orientation, it follows from the isotropy of the description and the feature of this theory that energy is not transferred into free space, that in the macroscopic measurement where two Geiger counters respond to an ensemble of particle—antiparticle pairs, each counter should absorb (on the average) half of the total transferred energy, as the pairs go into their state of null energy. Thus, in agreement with experiment, the theory predicts that there should be a correlation between a 'pair annihilation' process and the *coincident* transferral of λ units of energy (on the average) to each of the two detecting apparatuses.

7.6. Dynamical Properties of the Pair in its Ground State

With the idea that the electromagnetic source fields serve only as factors in the description of the coupling of a pair to the detecting apparatus, it would be more instructive at this point to express these variables in terms of the usual charge and current densities of the standard Maxwell notation — as directly detectable variables of the electron—positron pair.

The proper identification was indicated in Equation (7.1.1b), with the source terms of the Maxwell formalism expressed in the source terms of the spinor form of the electromagnetic equations (in a particular Lorentz frame) as follows:

$$e^{\pm}\bar{\psi}^{(e^{\pm})}\Gamma_1\psi^{(e^{\pm})} = 4\pi i \begin{pmatrix} -\rho + j_3 \\ j_1 + ij_2 \end{pmatrix},$$

$$e^{\pm}\bar{\psi}^{(e^{\pm})}\Gamma_2\psi^{(e^{\pm})} = 4\pi i \begin{pmatrix} -(j_1 - ij_2) \\ \rho + j_3 \end{pmatrix}. \tag{7.6.1}$$

Comparing the right-hand sides of Equations (7.2.2a,b) with the right-hand sides of Equation (7.6.1), the source fields (for $\alpha = 1, 2$) for the pair in its

ground state may be expressed as follows with the usual variables:

$$\alpha = 1: \quad \rho = 2e^{\pm}, \quad j_3 = 0, \quad j_1 + ij_2 = 2e^{\pm} \exp(2i\lambda t), \tag{7.6.2a}$$

$$\alpha = 2: \quad \rho = 2e^{\pm}, \quad j_3 = 0, \quad j_1 - ij_2 = 2e^{\pm} \exp(-2i\lambda t), \tag{7.6.2b}$$

It is clear from this form that the ground state of the pair corresponds to two oppositely polarized currents that are mutually transverse with respect to the x_3 direction. Thus, the coincident response of two equidistant counters, placed on opposite sides of the pair (in its ground state), at time t, would be to two spatially transverse currents that are 90° out of phase. These responses of the counters would be in terms of a *direct coupling* with two *distinguishable currents*. The latter assertion will now be demonstrated.

To derive the effect of the currents, $j_{\pm} = j_1 \pm ij_2$ (associated with a pair at the origin of our coordinate system) on the two counters, equidistant at $\pm r$, it will be necessary to calculate the electric field intensities $E_{\pm}$ at the locations of the counters. This is because it is the electric field intensity that determines the motion of a test charge in the detecting apparatus.

In the Lorentz frame of the detecting device, the electromagnetic vector that corresponds to the oppositely polarized current densities, $j_{\pm}$, follows from the particular solutions of d'Alembert's equation,

$$\Box A_{\pm}(\mathbf{r}', t') = 4\pi j_{\pm}(\mathbf{r}', t'), \tag{7.6.3}$$

where $(\mathbf{r}', t')$ are the coordinates of the test charge of the apparatus, while $(\mathbf{r} = 0, t)$ are the coordinates of the emitting source — the pair itself. The solution of Equation (7.6.3) is:

$$A_{\pm}(r', t') = \int j_{\pm}(t) S(x - x') \, \mathrm{d}^4 x, \tag{7.6.4}$$

where $S(x - x')$ is the symmetric Green's function (5.4.8). Though this corresponds to a symmetry between the retarded and the advanced potentials, it should be noted that because of the symmetry of the experimental set-up in this particular problem, the result to be derived regarding the responses of the two apparatuses (on opposite sides of the pair, along a common axis and equidistant from the plane of polarization of the pair currents $j_{\pm}$) is insensitive to the appearance or lack of appearance of the advanced term in the Green's function.

Since $j_3 = 0$, substitution of the Green's function (5.4.8) into Equation (7.6.4) (with $\mathbf{r} = 0$) yields the following solution:

$$A_3 = \int j_3(t) S(x - x') \, \mathrm{d}^4 x = 0,$$

$$\mathbf{A}_{\pm}(r', t') = \left(\frac{2e^{\pm}}{2r'} \right) \{\exp[\pm 2i\lambda(t' + r')] + \exp[\pm 2i\lambda(t' - r')]\} \hat{\mathbf{e}}_{\pm} \tag{7.6.5}$$

where $\hat{\mathbf{e}}_{\pm} = \hat{\mathbf{e}}_1 \pm i\hat{\mathbf{e}}_2$ and $\hat{\mathbf{e}}_j$ is a unit vector in the jth direction.

It then follows from Equation (7.6.5) that the electric field intensities $\mathbf{E}_{\pm}(r', t')$ at the sides of the counters (representing the effects of the polarized currents $j_{\pm}$ of the particle—antiparticle source) have the following form:

$$E_{\pm}(r', t') = -\frac{\partial A_{\pm}}{\partial t'}$$

$$= \mp\left(\frac{2i\lambda e^{\pm}}{r'}\right) \times$$

$$\times \{\exp[\pm 2i\lambda(t' + r')] + \exp[\pm 2i\lambda(t' - r')]\}\hat{\mathbf{e}}_{\pm}. \qquad (7.6.6)$$

Thus we see that $\mathbf{E}_{\pm}$ describes the wave motion of an oscillating charge, whose angular frequency is $\omega = 2\lambda = 2mc^2/\hbar$ with a wave vector with magnitude ω/c. It follows then that when the phase of the current density has some fixed value at $t = 0$, say zero, then the phase of the electric field oscillation at $r' \neq 0$ that is produced by this current would not become zero until the later time $t' = r'/c$ in the retarded solution, and at the time $t' = -r'/c$ in the advanced solution. Thus, the *magnitude* of time taken for the propagation of the electromagnetic interaction between the pair and each of the detectors (each located a distance r' from it) is r'/c.

A salient point here is that for the solution $\mathbf{E}_+$, the sign of the propagation vector is positive in the retarded term and the sign of this vector is negative in the advanced term. The oppositely polarized current density j_- gives rise to the electric field intensity $\mathbf{E}_-$ with the same functional form as $\mathbf{E}_+$, except that the propagation vector in this case is negative in the retarded term and positive in the advanced term.

Thus we see that the oppositely polarized currents, $j_{\pm}$, at the common spatial location ($\mathbf{r} = 0$) gives rise to oppositely polarized electric field vectors that propagate in opposite directions — such that when each counter is an equal distance r' on each side of the pair, along a common axis, they will simultaneously detect oppositely polarized currents at the time $t' = r'/c$, each absorbing energy mc^2.

This *derived* result agrees with the experimental facts and with the law of energy conservation. It also agrees with the prediction of the model proposed in quantum field theory asserting that two photons, correlated with a 90° phase difference, are simultaneously created, each with energy equal to mc^2, when the pair annihilates. However, the present theory does not require the introduction of photons nor that matter should really 'annihilate'. An important difference in the two theories lies in the *deterministic* field approach to the experimental facts in all domains (of micro- as well as macrophysics), contrasted with the intrinsically statistical approach of the quantum theory, as we have discussed in detail in Chapter 2.

7.7. The Compton Effect

Considering the Compton effect, in which an electron—positron pair (rather than a photon) is scattered by an electron, this theory sees the process in terms of the tightly bound pair (the 'projectile') coupled weakly to the third matter field component (the 'target' electron), all considered, in principle, from the view of a *closed system*. The mathematical description, then is in terms of a set of three coupled spinor matter equations, of the type (6.1.1′), where one of these equations may be assumed to be approximately uncoupled from the other two. In this approximation, the uncoupled equation takes the form of the free particle Dirac equation, as an approximation. The pair, in turn, is represented the way it was in the preceding paragraphs, with the exact solution (7.2.1).

To derive the expression for the Compton cross section, so as to compare this theory with experiment [the meaning of 'cross section', in the context of this theory, was discussed in Section 6.4], one must introduce a small coupling between the projectile pair and the target electron, treating the latter as a perturbation on the free particle Dirac solutions of the target matter field.

We have seen that when the particle—antiparticle pair is in its ground state of null energy and null momentum, it behaves dynamically as a pair of oppositely polarized current densities, whose phases are correlated with a 90° difference; and that in this state, the pair has the same dynamical features as the pair of photons that are conventionally evoked to explain the data interpreted as 'pair annihilation'. The electromagnetic potential that corresponds to the current densities (7.6.2) and solves Equation (7.6.3) has the same time behavior of the current density sources. If one should make a Lorentz transformation to the rest frame of the (assumed uncoupled) target electron, then the effective vector potential of the Maxwell field for the pair, that acts on the target electron (rather than on an apparatus, as in the preceding example) takes the following form (in a unit volume):

$$A_3 = 0, \qquad A_\pm \propto \exp[\pm i(\omega t - \mathbf{k}\cdot\mathbf{r})] \tag{7.7.1}$$

$$\omega = \left(\frac{2mc^2}{\hbar}\right)\left[\frac{1-\beta}{1+\beta}\right]^{1/2}, \quad \mathbf{k} = \frac{\omega\boldsymbol{\beta}}{c\beta}$$

and β is the relative velocity of the target and projectile matter fields (in units of c).

With the exact solution (7.2.1) for the pair in its ground state, and the approximation of weak coupling between the pair and the target electron, the usual formal perturbation technique may be applied to determine the scattering cross section for this process of the Compton effect. To first order, the perturbing interaction is given by the standard electromagnetic form,

$e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$, where ψ is the free field Dirac solution for the target electron and A_{μ} is the electromagnetic four-potential, which in this problem is the space-part given in (7.7.1). The time-part of this potential in this problem, A_0, is zero for the field of the electron—positron pair, as a unit; i.e. away from the pair, its Coulomb potential acting on a third charge is zero.

The interaction density $e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$, for the coupled particle—antiparticle and electron (as projectile and target, respectively) is identical with the standard interaction form between the photon field and the target electron, in the conventional description of the Compton effect. It then follows that the application of the usual perturbation theory to this problem yields the same result as is obtained from the quantum theory (the 'Klein—Nishina formula' for the Compton scattering cross section [45]). The predictions of this theory are also identical with the standard prediction of the correlations of the polarizations of the scattered electron and 'photon' fields.

It should be noted at this point, however, that if the coupling between the pair and the target electron should become increasingly great, the tendency would be to excite the pair into a different state that would no longer display the dynamical properties that are identical with those of the photons created in 'pair annihilation'. It rather might indicate a behavior that appears as 'pair creation'. To describe the latter quantitatively would then require the study of the *three* coupled nonlinear matter equations, for the 'projectile' pair and the target electron, *all strongly coupled.*

7.8. Blackbody Radiation — a Derivation of Planck's Law

In this section of Chapter 7, it will be shown that the experimental results that are used to deduce the properties of a 'photon gas' from the spectral distribution of blackbody radiation are equally explained in terms of a 'gas' of particle—antiparticle pairs, rather than photons, when the pairs, when unperturbed, are in their ground states of null energy and null momentum, as derived above.

A cavity is maintained at a constant temperature so that the 'radiation' within its walls can come to thermodynamic equilibrium with the matter of the walls. The energy density within the cavity is then measured as a function of frequency of internal radiation, from the response of the charged matter of a detecting apparatus that couples to the inside of the cavity through a filter placed in a small window in the cavity wall. By changing the filters for a spectrum of frequencies, one may then measure the intensity of radiation in the cavity as a function of its frequency. This is the spectral distribution curve.

The first remarkable feature that came from the data of these experiments was the insensitivity of the resulting spectral distribution to the type of

constituent material of the cavity wall, i.e. its atomic make-up. Indeed, it appeared from these experiments that the radiation in the cavity was not at all dynamically coupled to the states of motion of the charged matter constituting the walls, but rather, once emitted by the walls, was an entity independent of the material walls. This result then seemed to imply that instead of being a representation of moving electrically charged matter, radiation is a thing by itself, once it is emitted by the matter. This result seems to defy the original interpretation of electromagnetic radiation that was propounded by Faraday and advocated in the fundamental theory of matter from general relativity proposed in this monograph, saying that radiation is not a thing by itself. We will see below that this problem is circumvented in the present theory, returning to the Faraday view of radiation, because here there is no radiation (free photons) in the first place, in the cavity. Instead, it is postulated that any region of space must be populated by some very dense (yet to be determined) ensemble of pairs, weakly coupled to each other, and each in their ground states of null energy-momentum, as we have derived above. When one observes the inside of the cavity, that is in thermodynamic equilibrium with the 'radiation' inside of it, as in these experiments, one is describing (in this theory) the direct electromagnetic coupling of the charged matter of the apparatus to a distribution of pair currents, $j_{\pm}$, inside of the cavity. As we have seen in the preceding discussion, the latter have the same mathematical representation as that of an ensemble of photons; in both models, the 'currents' inside of the cavity have a distribution of possible frequencies, and they are in thermodynamic equilibrium with the walls of the cavity, at a given temperature T.

Experimentally, it was found that when kept in thermodynamic equilibrium with the cavity walls, at the temperature T, the spectral distribution of the radiation observed through a filter for each frequency component fits the Planck formula:

$$\frac{du}{d\nu} = \left(\frac{8\pi h\nu^3}{c^3}\right)\left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]^{-1}, \tag{7.8.1}$$

where du is the density of radiation energy in the frequency range between ν and $\nu + d\nu$ at the temperature T. This behavior of the energy density with frequency and temperature, as well as the insensitivity of the spectral distribution to the material of the cavity, are the empirical facts about blackbody radiation that are to be explained here from a deterministic field theory of matter, rooted in the axiomatic basis of general relativity, rather than in terms of statistical concepts of the quantum theory.

Assuming now that we have a cavity populated with particle—antiparticle pairs, coupled to the walls, the full set of field equations for this close system

may be expressed in the form (6.1.1′), as follows:

$$\begin{aligned}
&\hat{O}(1^+, 1^-; 2^+, 2^-; \ldots, n^+, n^-; f)\psi^{(1^+)}(x) = 0\\
&\hat{O}(1^-, 1^+; 2^+, 2^-; \ldots, n^+, n^-; f)\psi^{(1^-)}(x) = 0\\
&\qquad\qquad\vdots\\
&\hat{O}(n^-, n^+; 1^+, 1^-; \ldots, (n-1)^+, (n-1)^-; f)\psi^{(n^-)}(x) = 0\\
&\hat{O}(f; 1^+, 1^-; \ldots, n^+, n^-)\psi^{(f)}(x) = 0
\end{aligned} \tag{7.8.2}$$

where the operator in these equations, $\hat{O}$, has the explicit form according to the following ordering of its argument:

$$\hat{O}(1^+, 1^-; \ldots, n^+, n^-; f) = \left\{ \gamma^\mu \partial_\mu - \lambda - \mathscr{J}(e^{1^-}) + \sum_{k \neq 1^-, 1^+} \mathscr{J}(k) \right\}.$$

The labels $n^\pm$ refer to the particular $(e^+—e^-)$ pair of the system of tightly bound pairs. The solutions $\psi^{(n^+)}$ and $\psi^{(n^-)}$ that are denoted this way are related according to the charge conjugation transformation indicated in Equation (7.1.1). The label f refers to the field variables associated with the interacting matter fields of the cavity walls and the detecting apparatus.

In the asymptotic limit, when the interaction coupling term $\mathscr{J}(k)$, with $k \neq 1^+, 1^-$, can be assumed to have a negligibly small effect on the solution $\psi^{(1^+)}$, compared with the effect of $\mathscr{J}(1^-)$ on this solution, the field Equations (7.8.2) reduce to $n/2$ uncoupled pairs of equations for the bound particle—antiparticle units of the inside of the cavity (as well as the field equations that describe the interacting charges of the cavity walls and the detecting apparatus). In the consideration of this asymptotic limit, note that the formalism reduces to the description of $n/2$ *distinguishable interactions.*

It was shown earlier, from the features of the ground state solution for the pair, (7.2.1), that the sum of the electromagnetic field equations for the electron and positron fields, when each are in the same state of motion, gives Equation (7.1.4) — that is source-free. Recalling that the physically admissible solutions of these equations, because they are interpreted in this theory as identities, are the particular solutions only, the electromagnetic field intensities $\phi_\alpha^{(\text{pair})}$ for the pair, *as a unit*, are identically zero. It then follows that the system of pairs in the cavity that we are considering do not couple electromagnetically to the walls — they would then appear to be dynamically uncoupled from the moving charged matter of the atomic constituents of the walls of the cavity. This prediction, that the spectral distribution of the pair currents in the cavity is independent of the material of the walls, corresponds to the facts regarding blackbody radiation, as we have discussed above.

Recall also that each of the two components of the pairs can couple,

separately, to other charged matter. Thus, when in its ground state of null energy-momentum, each component of each pair is capable of absorbing electromagnetic energy and momentum from the matter of the cavity walls — from its direct electromagnetic coupling to the currents in the walls (which would occur from the view of the observations in this experiment, incoherently) — thereby decreasing the relative binding of the pair by the corresponding amount of energy transfer. The pair would eventually convert this absorbed energy into increased kinetic energy, before giving this energy back to the charged matter of the walls and returning to its ground state of null energy. Such is the energy exchange process that maintains this system of pairs at a constant temperature T with respect to the walls and the heat bath that are maintained at this temperature.

The energy density that is detected in the cavity is predicted from the expressions that have the form of Equation (7.3.1a) for the conserved energy, where the sum indicated there is taken over all electron and positron solutions for the 'gas' of pairs, the solutions corresponding to the matter of the cavity walls and the detecting apparatus. With the result that the energy of mutual coupling of each of the pairs is exactly zero, when they are in their ground states, the only nonzero contribution to the energy term (7.3.1a) must come from the coupling of the individual components of the pairs to the charged matter of the detecting apparatus. The corresponding matter field components are the stationary states with frequency ω_0, the latter is a frequency controlled in the experimental arrangement (e.g. the frequency controlled by placing a fixed frequency filter in the window of the wall of the cavity). Thus, the fields that contribute to the observed energy density have the form

$$\psi^{(0)} \propto V^{-1/2} \exp(-i\omega_0 t).$$

It follows that the corresponding expression for the energy density contributions reduce to the form

$$P^{(\omega_0)} = \int \Theta_{00}\, d\mathbf{r} = \int i\hbar\psi^{(0)\dagger}\partial_t\psi^{(0)}\, d\mathbf{r} = \hbar\omega_0 \tag{7.8.3}$$

for each of the frequencies ω_0 of the detecting apparatus.

Using the effective vector potential associated with each of the pairs in the cavity (Equation (7.7.1)) and the continuous distribution of frequencies relative to a given frame of reference, as indicated in Equation (7.7.1), it follows that for any detector frequency ω_0, the apparatus will respond to the interior of the cavity (through a small window in its wall) at frequencies that are integral multiples of this driving frequency. This follows from the finiteness of the cavity and the boundary conditions associated with retricting the interaction field solutions to vanish at the walls. For a given detector frequency ω_0, the energy of the mth mode of interaction between the detector

and the system of pairs (in a volume V) is then

$$(V\Theta_{00})_m = m\hbar\omega_0 \quad (m = 1, 2, 3, \ldots). \tag{7.8.4}$$

We see here that the response of the detecting apparatus to the individual pairs in the cavity will display a set of *distinguishable interactions*, each labelled with the particular mode of oscillation, m.

With the establishment of a constant temperature T in the system of pair currents in the cavity, Maxwell—Boltzmann statistics may then be applied to the system of *distinguishable interactions* to determine the statistically averaged energy. Thus,

$$V\langle\Theta_{00}\rangle = \frac{\Sigma_{m=0}^{\infty}(\exp[-m\hbar\omega_0/kT])(m\hbar\omega_0)}{\Sigma_{m=0}^{\infty}\exp[-m\hbar\omega_0/kT]} = \frac{\hbar\omega_0}{\exp(\hbar\omega_0/kT) - 1} \tag{7.8.5}$$

where k is 'Boltzmann's constant, and T is the equilibrium temperature in degrees Kelvin.

Finally, dividing this expression by the volume of the cavity and then weighting the averaged energy density with the differential increment of detecting frequency modes per frequency interval, we arrive at the expression for the energy density per frequency interval — the quantity that is to be compared with the experimental data on blackbody radiation.

The density of modes is derived in the usual way from the volume of wave-vector space, as follows:

$$\frac{dg}{d\omega_0} = \frac{d}{d\omega_0}\frac{2}{2\pi^3}\int d\mathbf{k}_0 = \frac{\omega_0^2}{\pi^2 c^3}. \tag{7.8.6}$$

The factor 2 appears above because the detector, with a given frequency, can respond to two source fields associated with either of the currents of a pair in the cavity.

The product of the right-hand sides of Equations (7.8.5) and (7.8.6) is the density of interaction energy per interval of the detectable frequency spectrum. The result,

$$\frac{du}{d\omega_0} = \langle\Theta_{00}\rangle\left(\frac{dg}{d\omega_0}\right) = \left(\frac{\hbar\omega_0^3}{\pi^2 c^3}\right)\left[\exp\left(\frac{\hbar\omega_0}{kT}\right) - 1\right]^{-1} \tag{7.8.7}$$

is the required distribution function, originally discovered by Planck and found to be in agreement with the shape of the blackbody radiation distribution curves.

With this result and the preceding analysis that indicated that the observations interpreted as 'pair annihilation' and 'pair creation' can be derived from the features of a bound state of the particle—antiparticle pair — without the need to introduce photons — it has been demonstrated that the version of

electrodynamics discussed here, that is a nonsingular, totally field theoretic approach to a 'delayed-action-at-a-distance' theory, can indeed totally dispense with the photon concept.

In the following chapter, the two-body system, electron—proton, will be studied, regarding the bound states of hydrogen, in view of the generalized form of the electrodynamical interaction. It will be found there that the entire energy level spectrum of hydrogenic atoms is predicted, including the extra contribution associated with the experimentally found Lamb shift. But the latter appears here as a result of the generalization of the electromagnetic interaction, in going to the spinor formulation, rather than from the radiative effects that are evoked in quantum electrodynamics, to explain these data. In the next chapter we will also study the generalization encountered in the analysis of high-energy electron—proton scattering.

7.9. The Anomalous Magnetic Moment of the Electron

Before proceeding to Chapter 8, on the analysis of the electron—proton system in the context of this field theory, this last section of Chapter 7 will demonstrate that the system of particle—antiparticle pairs derived above as relating to the empirical observations of the spectrum of blackbody radiation, also reveals a deviation of the predicted value of the magnetic moment of the electron, in such a medium, to be the same order of magnitude as the observed deviation from the Dirac value.

In the context of the standard quantum theory, one of the striking successes of quantum electrodynamics, that should be matched by the matter theory from general relativity of this monograph, is the prediction of the anomalous part of the magnetic moment of the electron. This is the part in addition to the Dirac prediction, $\mu = e\hbar/2mc$, with value, $\delta\mu/\mu = (\gamma/2\pi) + O(\gamma^2) = 0.00116942$, where $\gamma = e^2/\hbar c$ is the fine structure constant. The total electron magnetic moment is $\mu + \delta\mu$. It turned out, experimentally, that the value of the anomalous part of the magnetic moment of the electron is $\delta\mu = 0.001169644(7)\mu$ — a value that agrees with the theoretical prediction of quantum electrodynamics to one part in a billion! This is probably the most accurately measured number in physics today — thus giving further credence to the view of the quantum theory, and quantum electrodynamics in particular. Nevertheless, the difficulty of mathematical inconsistency remains in the structure of the theory, implying that other views should be explored simultaneously, as we have discussed earlier.

My investigations of quantum mechanics as a linear limit of a generally covariant field theory of inertia also predicts that there must be an anomalous part to the magnetic moment of the electron. But the physical mechanism responsible for this is not the same as that which is responsible for the Lamb shift, according to this theory (the latter will be derived in Chapter 8). As we

have discussed above, the source of this anomaly in the present field theory is the effect of the background 'gas' of (countable) particle—antiparticle pairs, each in their ground states of null energy and null momentum — the medium we have already seen could account for the spectrum of blackbody radiation (without invoking the 'photon' concept). The explicit reason for the anomaly in the magnetic moment of the electron is that an externally applied magnetic field must induce a small diamagnetic contribution [46] in the pairs that permeate the region of the 'observed' electron field.

To determine the magnitude of this effect, consider the quantum mechanical limit of this nonlinear field theory, where the Hamiltonian for the magnetic energy of the electron in the external magnetic field is

$$\mathscr{H}_{\text{mag}} = -\hat{\boldsymbol{\mu}} \cdot \mathbf{H} + \mathscr{H}_D, \tag{7.9.1}$$

where $\mathbf{H}$ is the magnetic field intensity applied to the electron in the background of pairs, and

$$\mathscr{H}_D = \left(\frac{e^2}{8mc^2} \right) r^2 H^2 \tag{7.9.2}$$

is the diamagnetic contribution. The magnetic moment contribution that is diamagnetically induced by the background of pairs is then

$$\mu_D = -\left(\frac{\partial \mathscr{H}_D}{\partial H} \right) = -\left(\frac{e^2}{4mc^2} \right) r^2 H = -\frac{r_0 r^2 H}{4}, \tag{7.9.3}$$

where the parameter $r_0 = e^2/mc^2$ is called the 'electron radius'.

The effective magnetic field intensity associated with this magnetic moment is

$$H_D = \left(\frac{r_0 H}{4r} \right) - \left(\frac{3r_0 r(\mathbf{H} \cdot \mathbf{r})}{4r^3} \right) = \left(\frac{Hr_0}{4r} \right) (1 - 3\cos^2\theta), \tag{7.9.4}$$

where θ is defined to be the orientation with respect to the direction of the external magnetic field intensity $\mathbf{H}$.

The total magnetic coupling energy of the observed electron to the magnetic field is then

$$\mathscr{H}_{\text{mag}} = -\hat{\boldsymbol{\mu}} \cdot (\mathbf{H} + \mathbf{H}_D) = -\hat{\mu} H \left[1 + \left(\frac{r_0}{4r} \right) (1 - 3\cos^2\theta) \right]. \tag{7.9.5}$$

Thus we see that the usual Dirac magnetic moment operator $\hat{\mu}$ is replaced by $\hat{\mu}[1 + (r_0/r)(1 - 3\cos^2\theta)]$. The measured enhancement of the electron

magnetic moment is then the expectation value

$$\frac{\delta\mu}{\mu} = \left(\frac{r_0}{4}\right)\left\langle\frac{1-3\cos^2\theta}{r}\right\rangle . \tag{7.9.6}$$

The experimental result for this quantity, which is in very close agreement with the prediction of quantum electrodynamics, is to order γ,

$$\frac{\delta\mu}{\mu} = \frac{\gamma}{2\pi} . \tag{7.9.7}$$

The comparison of this field theory's prediction of the anomalous magnetic moment with that of quantum field theory is then a comparison of Equations (7.9.6) and (7.9.7).

The expectation value in (7.9.6) depends on the matter field solutions for the free electron in the domain of the electron—positron pairs of the background, according to the analysis discussed above. With the observed electron in the vicinity of a given pair, the mutual energy for this pair should be close to zero, since the magnetic energy aligning the 'electron's magnetic moment' raises the null energy somewhat. Still, it is important here that the magnitude of the electron's energy relative to a positron of the background is much less than mc^2, and close to zero.

If we consider the electron—positron $1/r$ potential in the Dirac Hamiltonian (for the pair), the radial part of the wave solution is

$$\psi(r) \simeq \exp\left(-\frac{r}{a}\right),$$

where $a = \hbar c/[(mc^2)^2 - E^2]^{1/2}$. With $E \ll mc^2$, it follows that $a \simeq \hbar/mc = \lambda\!\!\!^{-}$ — the reduced Compton wavelength of the electron. The radial part of the electron wave function then depends on $\exp(-r/\lambda\!\!\!^{-})$. Thus, the radial part of the expectation value in Equation (7.9.6) is

$$\left\langle\frac{1}{r}\right\rangle = \left(\frac{1}{\lambda\!\!\!^{-}}\right)\left[\frac{\int_0^\infty \exp(-2u)u\,du}{\int_0^\infty \exp(-2u)u^2\,du}\right] = \frac{1}{\lambda\!\!\!^{-}} .$$

Thus Equation (7.9.6) reduces to

$$\frac{\delta\mu}{\mu} = \left(\frac{r_0}{4\lambda\!\!\!^{-}}\right)\langle 1-3\cos^2\theta\rangle = \left(\frac{\gamma}{4}\right)\langle 1-3\cos^2\theta\rangle. \tag{7.9.8}$$

This result must then be compared with the experimental result, $\delta\mu/\mu = \gamma/2\pi$.

At this stage of the analysis it may be said that the present theory at least predicts the correct order of magnitude for the anomalous part of the

electron's magnetic moment. In pursuing this analysis further, it should be noted that if the symmetry in the low-lying energy states of the bound electron—positron system, subjected to an external magnetic field, were purely that of an S-state, then $\langle \cos^2 \theta \rangle = \frac{1}{3}$, implying thereby that $\delta\mu/\mu = 0$, according to Equation (7.9.8). However, the field solutions in reality solve nonlinear equations for the bound pair, as we have discussed earlier in this chapter, rather than the eigenfunction equations of the quantum mechanical formalism. In the nonlinear field theory of the approach from general relativity, the bound state of the pair is not represented by precisely discrete states of a Hilbert space. Thus the energy spectrum is smeared out, continuously, rather than being a discrete set. According to the formal structure of the theory, which we have indicated incorporates the 'correspondence principle' as one of its underlying axioms, the electron—positron energy spectrum approaches discreteness in the asymptotic limit, as the binding may be considered sufficiently weak, so that as an approximation it may be assumed to be zero — this is the state of 'ionization' of the pair. The energy levels of the pair *near* this asymptotic state, that are conventionally identified with the ^1S and ^3S bound states of positronium, then also correspond in this theory to (almost) discrete energy levels. However, in the more deeply bound energy states, the feature of discreteness disappears altogether.

A closer determination of $\delta\mu$ will require a better knowledge of $\langle \cos^2 \theta \rangle$ with respect to the *nonlinear* low-lying states of the electron—positron system — i.e. the states that are close to the actual ground state that corresponds to a null mutual energy for the bound pair, relative to the other end of the spectrum of (near) ionization, where the relative energy would be $2mc^2$.

Note further that the empirical result (and that of quantum electrodynamics) that $\delta\mu/\mu = \gamma/2\pi$ would correspond here to nonlinear solutions that yield the expectation value

$$\langle \cos^2 \theta \rangle = \langle \cos^2 \theta \rangle_S - \frac{2}{3\pi} .$$

Thus, continuation on this problem will attempt to determine better nonlinear solutions for the bound electron—positron system, especially near the maximum binding that corresponds to $E = 0$.

Chapter 8

The Electron—Proton System

In this chapter we will analyze the bound electron—proton system called 'hydrogen'. Recall that this atom, the lightest one in the periodic chart, was the first to test Bohr's quantum hypothesis — his assertion that the angular momenta of the atomic orbital electrons are quantized in units of $\hbar = h/2\pi$, thereby leading to the atomic energy levels and the idea of the quantum jump, to explain the emission and absorption spectra — in agreement with a great deal of empirical data.

It was Bohr's new rule of quantization together with his rejection of the result of the Maxwell theory, that the acceleration of a charged body, such as a rotating electron, must radiate energy away, that led him to his notion of 'stationary states', and the subsequent description of light propagation in terms of a source that entails 'quantum jumps'. However, as we have discussed in Chapter 2, the idea of the 'quantum jump' led to apparent inconsistencies, especially when attempting to unify this theory of micro-matter and the symmetry requirements of the theory of special relativity. The latter, of course, is a *necessary* requirement in view of the fact that the quantum emitted when a quantum jump in the atom occurs is indeed only relativistic, i.e. it is radiant energy that has no nonrelativistic limit!

In spite of this difficulty, Bohr's formulation of these quantum processes, used in the formalism of nonrelativistic quantum mechanics (in Schrödinger's version of wave mechanics or Heisenberg's matrix mechanics) did have enormous success in fitting the data of atomic and molecular spectroscopy. Then how could a continuum field theory of matter, rooted in the underlying notions of the theory of general relativity, duplicate all of these successful results of nonrelativistic quantum mechanics, including the apparent need for quantized angular momenta and energy levels of atoms? and an apparent need for the process of the discrete quantum jump? The answer to this question, as we have discussed earlier, is that the quantization we use in the nonrelativistic description of atomic phenomena is a low-energy, linear approximation for a nonlinear matter field theory for a closed system — this time where the system is the coupled *electron—proton*. It is the principle of correspondence, imposed in the axiomatic basis of this theory, that requires

that the limit of the linear quantum formalism must be contained in the formal structure of the matter field equations, though the interpretation of the equations is different. In this theory, as we have discussed in Chapter 4, the matter field equations are an explicit representation for the inertial manifestation of matter.

But even in the linear, low-energy limit, there is a change in the 'Hamiltonian' of the hydrogen equation, compared with its expression in the Dirac theory. This is due to the generalization of the formal expression of the electromagnetic field equations, as a factorization of the Maxwell vector form into a pair of two-component spinor equations (as in Chapter 5). We will see in this chapter how this generalization, in the linear limit, leads to the correct prediction of the entire hydrogen spectrum, including the Lamb shift. It will then be shown how the enlargement to include a background gas to electron—positron pairs (as we discussed in the preceding chapter) with an 'observed' hydrogen atom leads to the correct lifetimes of the excited states of hydrogen. Finally, the high energy e—p scattering will be analyzed in this context.

8.1. Linearization of the Hydrogen Field Equations

Our determination of the exact solution (7.2.1) for the ground state of the electron—positron system was facilitated by the symmetry in the nonlinear field equations for this particular case. Specifically, the symmetric features that led to the solution were (1) the inertial mass parameter for each of the coupled fields was the same and (2) the electron and positron were each taken to be in the same state of motion, to demonstrate the solution. In this chapter, we will treat a two-body system that has neither of these symmetries. The mass parameters, m and M, for the electron and proton, respectively, differ by several orders of magnitude. Further, their respective states of motion are quite different — it will be assumed that the proton is relatively stationary while the electron field corresponds to an orbiting motion.

At the present stage, no exact solution has been found for the e—p system, as it was in the case of electron—positron. The solutions to be studied follow from an approximation scheme in which the field equations are linearized. This is justified in view of the fact that the ratio of masses, m/M, is sufficiently small so that the actual momentum transfer to the proton field from the electron field in the bound system may be assumed to be negligible. This is physically equivalent to the assumption that for the binding energy of the electron and proton that is considered, one may neglect the recoil of the proton field under the influence of the electron. Thus we will assume that the proton field represents a source of binding at a spatial point, that is, it is at a stationary location, relative to the coordinates of the electron field. This is equivalent to using Dirac's assumption in the structuring of

his hydrogen equation, when he took the *covariant* interaction term, $\bar{\psi}^{(e)}\gamma^{\mu}\psi^{(e)}A_{\mu}^{(p)}$, and approximated it by the noncovariant Coulomb term, $\psi^{(e)\dagger}\psi^{(e)}A_0^{(p)}$, where $A_0^{(p)} = ie/r$, and $r = 0$ locates the stationary proton.

Starting with the matter field equations (6.1.1′) and (5.4.9), applied to the electron—proton system, we will consider the solutions of the coupled equations:

$$(\gamma^{\mu}\partial_{\mu} - e\mathscr{A}(p) + \lambda)\psi^{(e)} = 0 \tag{8.1.1a}$$

$$(\gamma^{\mu}\partial_{\mu} + e\mathscr{A}(e) + \Lambda)\psi^{(p)} = 0, \tag{8.1.1b}$$

where the mass parameters in these equations are $\lambda = mc/\hbar$ and $\Lambda = Mc/\hbar$, which are, of course, the inverse Compton wavelengths of the electron and the proton, respectively.

We will now consider the proton solution to have the following form of a stationary point field, in its *proper* Lorentz frame:

$$\psi^{(p)} = [\exp(-i\Lambda t)]f(\mathbf{r})s, \tag{8.1.2}$$

where s is a *constant four-component bispinor* whose only restriction is that it must be normalized, i.e. $s^{\dagger}s = 1$.

Dividing Equation (8.1.1b) by Λ, we have the equivalent equation:

$$\left[\left(\frac{1}{\Lambda}\right)\gamma^0\partial_0 - \left(\frac{1}{\Lambda}\right)\gamma^k\partial_k + \left(\frac{e}{\Lambda}\right)\mathscr{A}(e) + 1\right]\psi^{(p)} = 0. \tag{8.1.3}$$

The operator, $(1/\Lambda)\gamma^k\partial_k$ corresponds to the ratio of the kinetic energy of the proton to its rest energy. Since it is assumed that the proton's motion (relative to the electron field) — i.e. its recoil energy — is negligibly small compared with its rest energy ($\sim$1 GeV), we may neglect this term in (8.1.3). Further, the operator $e\mathscr{A}(e)/\Lambda$ represents the ratio of electron—proton binding ($\sim$eV) to the proton's rest energy. This may also be assumed to be small enough to neglect in the problem at hand. Neglecting the latter two terms in the proton Equation (8.1.3), and choosing $s^{\dagger} = (1, 0, 0, 0)$ (in the proton Lorentz frame), Equation (8.1.3) takes the form:

$$\left[\left(\frac{1}{\Lambda}\right)\gamma^0\partial_0 + 1\right][\exp(-i\Lambda t)]f(\mathbf{r})s = 0. \tag{8.1.4}$$

That is, with these approximations, the function (8.1.2) correspondingly approaches an exact solution of the proton Equation (8.1.1b).

Next, it is observed that the space-dependent part of the proton function, $f(\mathbf{r})$, may be chosen arbitrarily without altering the validity of Equation (8.1.4). Appealing once again to the physical argument that led to the assumption that the proton field is at rest at the origin (relative to the electron field), we may take $f(\mathbf{r})$ such that its square is the three-dimensional

Dirac delta function,

$$|f(\mathbf{r})|^2 = \delta(\mathbf{r}). \tag{8.1.5}$$

This substitution in (8.1.2), and the subsequent insertion of this solution in the interaction functional $\mathscr{I}(p)$ in the electron Equation (8.1.1a), gives the explicit form of the operator that determines the electron states of hydrogenic atoms.

Using (8.1.5) in (8.1.2),

$$\bar{\psi}^{(p)}\gamma^k\psi^{(p)} = 0, \qquad \bar{\psi}^{(p)}\gamma^0\psi^{(p)} = \delta(\mathbf{r}), \tag{8.1.6}$$

where $k = 1, 2, 3$. According to the form of Equation (5.4.9), the interaction coupling term $\mathscr{I}(p)$ has two parts. Inserting (8.1.6) into the first part, it is the ordinary Coulomb term that appears in Dirac's theory of hydrogen:

$$\mathscr{I}(p)_1 = \frac{\gamma^0 e}{r}. \tag{8.1.7}$$

Before the second part of the interaction functional, $\mathscr{I}(p)_2$ can be determined, explicitly, we must know the form of the spinor solutions of the electromagnetic field equations for the proton field, $\phi_\alpha^{(p)}$. These are the two solutions of the spinor Equations (7.1.3), (8.1.6),

$$\sigma^\mu\partial_\mu\phi_1^{(p)} = -4\pi ie\delta(\mathbf{r})\begin{pmatrix}1\\0\end{pmatrix}$$

$$\sigma^\mu\partial_\mu\phi_2^{(p)} = 4\pi ie\delta(\mathbf{r})\begin{pmatrix}0\\1\end{pmatrix}. \tag{8.1.8}$$

The solutions of these electromagnetic spinor equations for the point charge were derived in *GRM* (§5.9), where it was shown that the particular solutions of this equation are as follows:

$$\phi_1 = -\left(\frac{ie}{r^3}\right)\begin{pmatrix}x_3\\x_1 + ix_2\end{pmatrix}$$

$$\phi_2 = \left(\frac{ie}{r^3}\right)\begin{pmatrix}x_1 - ix_2\\-x_3\end{pmatrix} \tag{8.1.9}$$

Finally, substituting the solutions (8.1.9) into the form of $\mathscr{I}(p)$ and using the result of the preceding chapter on the electron—positron solution, deter-

mining uniqely that $a_\alpha = (-1)^\alpha$ in the second part of the interaction functional, the following explicit form results:

$$\mathcal{J}(p)_2 = 16\pi i g_M \left(\frac{e}{r^3}\right)(\mathbf{r}\times\boldsymbol{\gamma})_3. \tag{8.1.10}$$

The noncovariant 'look' of this functional, in terms of the 'third component' of a vector (which relates to the 'axis of quantization' direction) follows here as a consequence of the approximation in which the electromagnetic vector potential A_μ in the electron equation is replaced with the single term A_0, for the Coulomb potential of the proton, in acting on the electron matter field. Note, however, that the *exact* form of this interaction functional is one of the relativistic invariants of the theory, denoted earlier by the invariant $\phi_\alpha^\dagger \mathrm{T}_\alpha$ (Equation (5.2.6)). The latter are invariants of the spinor form of the electromagnetic field theory that we have seen do not have any counterparts in the standard vector representation of the theory.

Combining Equations (8.1.7) and (8.1.10), the *linearized* field equation following from an approximation for Equation (8.1.1a) for hydrogen may then be expressed in the form [47]:

$$\left\{-\alpha_r p_r - i\left(\frac{\alpha_r}{r}\right)\beta\hat{K} + \frac{e^2}{r} + (16\pi g_M e^2)r^{-3}(\mathbf{r}\times\boldsymbol{\alpha})_3 - \beta\lambda + E\right\}\psi^{(e)} = 0, \tag{8.1.11}$$

where the following notation is used:

$$\beta = \gamma^0, \qquad \boldsymbol{\alpha} = i\gamma^0\boldsymbol{\gamma}, \qquad \hat{K} = \beta(\boldsymbol{\sigma}\cdot\mathbf{L} + 1),$$
$$p_r = -i\left(\frac{\partial}{\partial r} + \frac{1}{r}\right), \qquad \alpha_r = \frac{(\boldsymbol{\alpha}\cdot\mathbf{r})}{r} \tag{8.1.12}$$

The Dirac matrix representation for γ^μ is defined in §7.1 and $\mathbf{L} = -i\mathbf{r}\times\nabla$ is the usual angular momentum operator.

Following the usual procedure that is used in quantum mechanics [48], we introduce the change of variables:

$$\rho = \eta r$$

with

$$\eta = (\eta_1\eta_2)^{1/2}, \qquad \eta_1 = \lambda + E, \qquad \eta_2 = \lambda - E.$$

The wave Equation (8.1.11) then takes the form:

$$\hat{\mathcal{H}}\psi = (\hat{\mathcal{H}}_0 + \hat{V})\psi = -i\left(\frac{E}{\eta}\right)\psi, \tag{8.1.13}$$

where

$$i\eta\hat{\mathscr{H}}_0 = i\left\{\alpha_\rho\left(\frac{\partial}{\partial\rho}+\frac{1}{\rho}\right)-\frac{\alpha_\rho\beta\hat{K}}{\rho}-i\left(\frac{\gamma}{\rho}+\frac{\beta\lambda}{\eta}\right)\right\}\eta. \tag{8.1.14}$$

$\hat{\mathscr{H}}_0$ is the unperturbed (generalized) Dirac Hamiltonian for the hydrogenic electron, and γ stands for the fine structure constant $e^2/\hbar c$.

If we should now make use of the well-known derivation of the eigenfunctions and eigenvalues of the Dirac unperturbed Hamiltonian for hydrogen, the extra term, $\hat{V}$, in Equation (8.1.13) may then be treated as a perturbation on these eigenfunctions, and the corresponding hydrogenic Dirac eigenvalues for the energy levels would follow. With this procedure we then first determine the unperturbed Dirac eigenvalues for $i\hat{\mathscr{H}}_0$, finding the result:

$$\frac{E_{Jn}}{\eta}=\left(\frac{\lambda}{\eta}\right)\left[\frac{1+\gamma^2}{(s+n)^2}\right]^{-1/2}, \tag{8.1.15}$$

where the quantum numbers indicated above are as follows:

$$s^2 = K^2 - \gamma^2, \qquad K^2 = (J+\tfrac{1}{2})^2$$

and J is the total angular momentum quantum number and n is any positive integer or zero (0, 1, 2, . . .). In the usual spectroscopic notation, $(n+1)$ is called the 'principal quantum number'.

The energy eigenvalues shown in Equation (8.1.15) are precisely those that are predicted by the Dirac theory of hydrogen. Except for the Lamb splitting, these values are in quite close agreement with the energy level spectrum of hydrogen. Without the term $\hat{V}$ in the total Hamiltonian, the present theory, with the linear approximation used in this section, is in exact agreement with the predictions for hydrogen that are made by the Dirac theory. The extra interaction term, $\hat{V}$, comes from the part of the interaction functional $\mathscr{I}(e)_2$ that results from the generalization from the vector to the spinor formulation of the electromagnetic field equations. We will now investigate the effect of this added interaction, showing that indeed it predicts the Lamb shift in the spectrum of hydrogen, from both the qualitative and quantitative sides.

8.2. The Lamb Splitting

The feature of the predicted energy level spectrum (8.1.15) that does not agree with the actual data for hydrogen is the *accidental degeneracy* that appears in the Dirac hydrogenic states, where the principal quantum numbers are greater than (their minimum value of) unity. For the Dirac states with

the pair of eigenvalues $\pm K$ (of the operator $\hat{K}$) correspond to the same energy eigenvalues E_{Jn}. For example, when $J = \frac{1}{2}$, the states $(n + 1)S_{1/2}$ and $(n + 1)P_{1/2}$ of the Dirac theory are degenerate, occurring at the same value of energy. However, contrary to this prediction of the Dirac theory, Lamb and his coworkers discovered that the energy of the $2S_{1/2}$ state and that of the $2P_{1/2}$ state are different, with the energy of the former higher than that of the latter; similarly it was found experimentally that the energy for the $3S_{1/2}$ state is greater than that of its accidentally degenerate counterpart, the $3P_{1/2}$ state. This deviation from the Dirac theory is accounted for in the quantum theory — quantum electrodynamics — by models and numerical methods that are in question, according to the discussion in Chapter 2. If these empirical data on the Lamb splittings are to be explained by the theory of inertia advocated in this monograph, it must come from the additional interaction $\hat{V}$ that appears in the completed Hamiltonian (8.1.13). It will now be shown that this is indeed the case.

The perturbing potential

$$i\hat{V} = \left(\frac{\kappa}{\rho^2} \right) \left[\frac{(\boldsymbol{\rho} \times \boldsymbol{\alpha})_3}{\rho} \right] \tag{8.2.1}$$

in the matter field Equation (8.1.13) can be seen to lift the *accidental degeneracy* of the eigenstates of the 'unperturbed Hamiltonian $i\hat{\mathscr{H}}_0$ due to its lack of reflection symmetry in both space and time. The strength of this interaction is measured by the constant

$$\kappa = 16\pi g_M \gamma \eta = 16\pi[(s + n)^2 + \gamma^2]^{-1/2} \left(\frac{g_M}{\lambda_C} \right) \gamma^2 \tag{8.2.2}$$

where λ_C is the reduced Compton wavelength of the electron matter field, numerically equal to $\hbar/mc$. This is the reciprocal of the mass parameter that appears in the matter field equation for the electron, that we have denoted by λ.

Our perturbation on the eigenstates of the Dirac hydrogenic Hamiltonian diverges at the origin as ρ^{-2}. In order to ensure that the solutions of the unperturbed problem have a behavior at the origin that would lead to a rapidly convergent perturbation series of functions, that successively depend on ρ^{-n} $(n \geqslant 2)$, we will redefine the unperturbed and perturbing energy operators in the wave Equation (8.1.13) by adding and subtracting the term $i\kappa\alpha_\rho/\rho^2$. The latter expression is chosen not only because it depends on the radial coordinate the same way as $\hat{V}$ does, but also because it depends on the Dirac matrix $\boldsymbol{\alpha}$, which mixes the large and small components of the unperturbed Dirac solutions (thus corresponding to a 'velocity-dependent' potential), just as $\hat{V}$ does.

Thus, let us consider

$$i\hat{\mathscr{H}}'_0 = i\left(\hat{\mathscr{H}}_0 - \frac{\kappa\alpha_\rho}{\rho^2}\right) \tag{8.2.3}$$

as the unperturbed Hamiltonian, and the remaining portion,

$$i\hat{V}' = \left(\frac{\kappa}{\rho^2}\right)\left[\frac{(\boldsymbol{\rho}\times\boldsymbol{\alpha})_3}{\rho} + i\alpha_\rho\right] \tag{8.2.4}$$

as the perturbation. Clearly the problem has not been altered, since the total Hamiltonian is the same, i.e.

$$\hat{\mathscr{H}} = \hat{\mathscr{H}}_0 + \hat{V} = \hat{\mathscr{H}}'_0 + \hat{V}'.$$

It turns out that the electron equation with the new unperturbed Hamiltonian

$$\hat{\mathscr{H}}'_0\psi = -\left(\frac{i}{\eta}\right)E\psi$$

can be solved *exactly*, giving the solutions

$$\psi = \psi_D \exp\left(-\frac{\kappa}{\rho}\right), \tag{8.2.5}$$

where ψ_D are the exact Dirac eigenfunctions of the hydrogen Equation (8.1.14). Note that the new exact solutions ψ converge at the origin more rapidly than any polynomial in $1/\rho$ would diverge there.

The accidental degeneracy in the first excited state of hydrogen is described by the two orthogonal wave functions, $|+K\rangle$ and $|-K\rangle$, corresponding to the same energy eigenvalue in accordance with Equation (8.1.15). The eigensolutions that are characterized by the angular momentum quantum numbers, $J = \frac{1}{2}$, $J_3 = +\frac{1}{2}$, are as follows

$$|-K\rangle_n = \begin{pmatrix} F_-(n) \\ 0 \\ -i\cos\theta\ G_-(n) \\ -i\sin\theta\ \exp(i\phi)G_-(n) \end{pmatrix} \exp\left(-\frac{\kappa}{\rho}\right) \tag{8.2.6}$$

$$|+K\rangle_n = \begin{pmatrix} \cos\theta\ F_+(n) \\ \sin\theta\ \exp(i\phi)F_+(n) \\ -iG_+(n) \\ 0 \end{pmatrix} \exp\left(-\frac{\kappa}{\rho}\right), \tag{8.2.7}$$

where $|K| = 1$. The terms $\eta F_\pm/\rho$ and $\eta G_\pm/\rho$ are the respective large and small components of the radial solutions of the Dirac equation for hydrogen, with the values of the quantum number $K = \pm 1$.

It is readily verified that after integration over the angular variables has been carried out, the pertinent matrix elements in the contributions of the perturbation expansion of the electron energy levels are as follows:

$$\langle \pm K | \, i\hat{V}' \, | \mp K \rangle = 0 \tag{8.2.8}$$

$$\langle \pm K | \, i\hat{V}' \, | \pm K \rangle = \left(\pm \frac{4\tau}{3\eta} \right) \int_0^\infty \left(\frac{F_\pm G_\pm}{\rho^2} \right) \exp\left(-\frac{2\kappa}{\rho} \right) d\rho, \tag{8.2.9}$$

where (inserting $\hbar$ and c) τ depends on the quantum numbers n, with values

$$\tau_n = \eta\kappa\hbar c = \left(\frac{16\pi}{(s+n)^2 + \gamma^2} \right) \left(\frac{g_M}{\lambda_C} \right) \gamma^3 (mc^2) \tag{8.2.10}$$

It then follows that the difference between the energy values associated with the (formerly accidentally degenerate) states, called $(n+1)S_{1/2}$ and $(n+1)P_{1/2}$ is the *Lamb splitting*,

$$E[(n+1)S_{1/2}] - E[(n+1)P_{1/2}] = \tfrac{4}{3}\,\tau_n |\, I_+ + I_- |_n \equiv$$

$$\equiv \Delta_n = \left(\frac{64\pi}{3[(s+n)^2 + \gamma^2]} \right) \left(\frac{g_M}{\lambda_C} \right) \gamma^4 \, |\, I_+ + I_- |_n (mc^2), \tag{8.2.11}$$

where we have defined

$$|\, I_\pm |_n = (\eta\gamma)^{-1} \int_0^\infty \left(\frac{F_\pm(n) G_\pm(n)}{\rho^2} \right) \exp\left(-\frac{2\kappa}{\rho} \right) d\rho. \tag{8.2.12}$$

These integrals are evaluated explicitly in Appendix A, where it is found that to leading order in the fine structure constant γ,

$$|\, I_+ + I_- |_1 = \tfrac{1}{3} + O(\gamma^2) \tag{8.2.13a}$$

$$|\, I_+ + I_- |_2 = \tfrac{2}{9} + O(\gamma^2) \tag{8.2.13b}$$

$$|\, I_+ + I_- |_3 = \tfrac{1}{6} + O(\gamma^2). \tag{8.2.13c}$$

Combining Equations (8.2.10), (8.2.11) and (8.1.13), it is found that to order γ^4, the Lamb splitting in the first excited state of hydrogen, with principal quantum number $(n+1) = 2$, is

$$\Delta_1 = \left(\frac{16\pi}{9} \right) \gamma^4 \left(\frac{g_M}{\lambda_C} \right) (mc^2). \tag{8.2.14}$$

The Lamb splitting in the next higher principal quantum number state, Δ_2, i.e. with $(n+1)=3$, divided by Δ_1 is then found to be the ratio

$$\Delta_2/\Delta_1 = 0.2965\ldots \tag{8.2.15}$$

The significance of the theoretical *ratio* of Lamb splittings is that to the accuracy that is required to make a comparison with the experimental data, *its magnitude is independent of the extra fundamental constant in this theory*, g_M. It is only after a favorable comparison has been made with the *ratio* of experimental Lamb splittings that we will use the most accurately determined Lamb splitting from experiment to determine the magnitude of this constant. For the *ratio*, to the accuracy required for comparison with experiment, is independent of any new constant to adjust!

The experimental values for the Lamb splittings are, according to the definition of Δ_n in Equation (8.2.11), [49]

$$(\Delta_1)_{\text{exp}}/h = 1057.8514 \pm 0.0019 \text{ MHz} \tag{8.2.15'a}$$

$$(\Delta_2)_{\text{exp}}/h = 314.819 \pm 0.048 \text{ MHz} \tag{8.2.15'b}$$

$$(\Delta_3)_{\text{exp}}/h = 132.53\ {}^{+0.58}_{-0.78} \text{ MHz}. \tag{8.2.15'c}$$

Thus, the experimental ratio to be compared with the theoretical prediction, (8.2.15) is

$$(\Delta_2/\Delta_1)_{\text{exp}} = 0.2976 \pm 0.0003.$$

The comparison of this experimental ratio with the theoretical ratio (8.2.15), according to this theory of elementary matter, is then only a difference that is the order of 0.2%. In view of the approximations that have been used in the theoretical analysis, it may be concluded that this theory has been successful in its comparison with the data regarding the energy level spectrum of hydrogen, in totality.

With this result, then, it has been demonstrated that the matter field theory of inertia of this view (that stems from the concepts of general relativity theory, implying a factorization of the vector formalism for electromagnetic field theory, to a spinor formalism) predicts the observed energy level spectrum of hydrogen, including the Lamb splittings. In contrast with the conventional model of the quantum theory that leads to the Lamb splitting from an appeal to an enlargement of the two-body electron—proton system so as to include an infinite sea of radiation and pairs of particle—antiparticle units, truly annihilating and being created at random, but on the average causing the proper fluctuations in the spectrum of hydrogen to give the effect, the present theory derives the hydrogen spectrum *including the Lamb effect*, as a property of the *closed electron—proton system alone*. Recall that it follows here from a generalization of the Coulomb potential in the Dirac equation for hydrogen as natural consequence of a factorization of the Maxwell formalism. The extra term that predicts the Lamb effect has no

counterpart in the standard theory of electrodynamics. In this way, the Coulomb interaction, e^2/r in the hydrogen wave equation is generalized to a term that is less symmetric, such that it causes a lifting of the accidental degeneracy in the Dirac states for hydrogen. As we have seen above, this calculation is entirely finite — there are no infinities at any stage, so that there is no need to introduce a renormalization procedure to explain the data. Thus, in contrast with quantum electrodynamics to explain the Lamb shift, the explanation with this theory of inertia is mathematically consistent — the underlying formal expression of the theory is in terms of field equations with bona fide nonsingular field solutions.

Using the experimental value for the Lamb splitting Δ_1 which is the most accurately measured Lamb shift for the different states of hydrogen, we find from the theoretical expression for this splitting shown in Equation (8.2.4) that the new fundamental constant of the theory has the value

$$g_M = (2.087 \pm 0.001) \times 10^{-14} \text{ cm}. \tag{8.2.16}$$

This is the extra fundamental constant that appears in the presented theory of inertia. We have seen that in the particular application to the hydrogen spectrum, it is in the domains whose radii are this order of magnitude where the conventional Coulomb interaction must be modified. This appears most strikingly in the expression for the wave function (Equation (8.2.5)) for the unperturbed hydrogenic electron. It is seen from this form that when the electron–proton separation is the order of g_M, the modified wave function is significantly different that the ordinary Dirac solution for hydrogen. But for distances that are large compared with the order of g_M, the new solution effectively reduces to that of Dirac.

It is perhaps significant that (a) the magnitude of g_M is close to the Compton wavelength of the proton (i.e. relating to the proton mass) and (b) the constant that is the product of this length and the fine structure constant, i.e. $g_M\gamma$, is the same order of magnitude as the intermediate boson Compton wavelength that relates in field theory to the weak interaction coupling — a 'particle' that has recently been identified in high-energy physics experimentation, and named the '*W*-particle' [3, 4]. The hint here is that perhaps weak interactions are not more than a manifestation of the electromagnetic interaction at sufficiently small distances. This is also strongly supported by the fact that the two-component spinor form of the electromagnetic equations that we have been using in this monograph is not covariant under temporal or spatial reflections. Such a general formalism, of course, is a natural one with which to describe the weak interaction, in view of the violation of spatial reflection symmetry (parity) there.

Thus far we have deduced that the two measured Lamb splittings in hydrogen, for the states with principal quantum numbers $n = 2$ and 3, are correctly predicted by the matter theory of inertia that has been developed. The Lamb splitting Δ_3 (for $n = 4$) is, from Equation (8.2.13c) and the value

of g_M in Equation (8.2.16) as follows:

$$\Delta_3 \equiv \frac{E(4S_{1/2} - E(4P_{1/2})}{h} = 132.22 \text{ MHz}.$$

The experimentally determined value for this splitting of $132.53\,^{+0.58}_{-0.78}$ MHz (shown in Equation (8.2.15'c)) is then, within the experimental accuracy, in agreement with the prediction of this theory.

It is interesting to note that the Lamb splitting Δ_3 ($n = 4$) is calculated in quantum electrodynamics to be 133.084 ± 0.001 MHz (Brown and Pipkin [49]). The value given by the theory of this monograph is then somewhat better than the conventional theoretical result.

Experimental investigations of the Lamb splittings in the higher states of hydrogen should become quite significant in regard to a test of the predictability of this field theory of matter, compared with the results of quantum electrodynamics.

Aside from these differences, however, it is interesting to note that while the two theories are so different in respect to both formalism and interpretation, they still give results that are so close, numerically.

8.3. Deuterium and He^+

In view of the form of the field equations for hydrogen, the Lamb splittings of hydrogenic states in which the atomic nuclei are D and He should be determined, respectively, from sets of 3- and 5-coupled nonlinear equations because of the respective structures of these nuclei in terms of their constituent nucleons. On the other hand, to treat these hydrogenic atoms in the same order of perturbation that we treated hydrogen above, would be to assume that it is accurate to use the linear approximation with increased mass, for D, and increased mass as well as increased charge for He. It is not clear at the present state of this research if these would be accurate assumptions. Nevertheless, should such assumptions be made, the linearization acquired by assuming an infinite nuclear mass (as we said above) would imply that the Lamb splitting for D should be the same as for H. The actual experimental ratio of Lamb splittings for the excited state with principal quantum number 2, D::H is 1.0012. Thus, the error in neglecting the two-body nucleus of D is of order 0.1%.

The error is greater in ionized helium, He^+. In this case, the continued assumption of an infinite nuclear mass, but a doubled nuclear charge, leads to a replacement of the fine structure constant γ in (8.2.11) by the constant $Z\gamma = 2\gamma$. Since the leading term in the derived expression for the Lamb splitting, following from the generalization of the electromagnetic interaction in the spinor formalism for this theory, depends on γ^4 for H, it follows that the Lamb splitting for He^+ should be $2^4 = 16$ times greater. The experi-

mental ratio of Lamb splittings in the first excited state of He^+ compared with that in hydrogen is the order of 14. It may be that the factor of '16' above may be reduced toward '14' after the structure of the helium nucleus is taken into account, by considering better approximations for the solutions of the 5-coupled nonlinear field equations for this atomic system.

8.4. The Lifetimes of Atomic Excited States

Within the matter theory discussed in this monograph, *spontaneous emission* does not occur. The lifetime of an atom in any state is therefore predicted to be infinitely long — that is, if this atom is the entire closed system. How then does this theory predict the experimental observations of the lifetimes of the excited atomic states? The answer to this question lies in the theoretical results of our analysis (in Chapter 7) of the electron—positron pairs. Recall that it was found that a bound state solution for the pair corresponds to all of the experimental facts that are normally attributed to 'pair annihilation'. But matter is not annihilated here (nor created), at arbitrary places and times, *nor at all*! Since matter *persists*, according to this theory, we were led to the conclusion that any arbitrary region of space must be populated with some *definite number* of such pairs in their ground states of null energy and null momentum. Recall that this model of the 'physical vacuum' led to a correct derivation of the spectral distribution of blackbody radiation.

With this model of the 'physical vacuum' in view, it follows that any container that may contain a gas of hydrogen, sufficiently rarefied that one may consider one of its atoms at a time, must still entail the electromagnetic coupling of these 'observed' atoms to their environment of the background pairs, as well as the coupling to the walls of the container. Indeed, it is just the former coupling that predicts the decay of the excited atom, at a particular rate. On the other hand, the walls of the container normally induce a much smaller rate of decay than would be observable in the laboratory. This follows, empirically, from the observation that the measured atomic lifetimes seem to be independent of the sizes and shapes of their containers — be they laboratory sized or the size of a star! This observation is due to the fact that the matter components of the container that induce the decay of the excited atom are, on the average, much further than the neighboring pairs, thus the magnitude of the interaction with the pairs is much greater, on the average, than the interaction with the walls of the container. It is also noted that the lifetimes indicate that the interactions that cause the decay of the excited atoms are isotropic, consistent with the isotropic distribution of the surrounding pairs, while the matter of the container are not generally distributed isotropically. Thus we conclude that the decay of excited atoms in a container are primarily due to their interaction with the background electron—positron (and any other matter—antimatter) pairs.

The general expression for the latter coupling follows from the coupled field Equations (6.1.1′). Two of these coupled equations, in the application to, say, hydrogen, must be taken to have the form given in Equation (8.1.1a,b) for hydrogen, except for the inclusion here of contributions to the interaction functional $\mathscr{I}^{(k)}$ that relate to the electromagnetic coupling of the hydrogenic electron and proton to the background matter field of pairs. Similarly, the matter field equations for the pairs (7.1.1c,d) must now also include the bound electron—proton system in the interaction functionals that appear — that is, the solutions of Equation (8.1.1) must be included in the pair functionals.

Finally, to consider the excited atoms of a hydrogen gas, one at a time, including their interaction with a background matter field of pairs — as distinguishable interactions — we may assume that the coupling is sufficiently weak between all of the constituent two-body systems so that the interaction functionals $\mathscr{I}^{(k)}$ may be accurately represented in terms of an *averaged background potential*, as we have discussed in Chapter 6.

We have seen in the preceding chapter that each of the background pairs, when it is in its ground state of null energy and null momentum, does not interact electromagnetically *as a unit*. Nevertheless, we also saw that the separate bound matter components of the pairs do couple electromagnetically to other mutually interacting charged matter. According to the result that was obtained, the action of each pair on an atomic electron is that it is equivalent to the effect of two oppositely polarized electrical currents, directly coupled to the atomic electron. It was further shown that in the frame of the 'target', in this case the excited atom, the effective potential that represents the action of the pair has the form of a plane polarized potential field, with associated frequencies between 0 and 2λ, running over a continuum of values — because of the continuum of relative velocities between a given pair and the atom.

Taking account of the fact that the effect of the interaction $\mathscr{I}_2$ is negligable in comparison with the effect of $\mathscr{I}_1$ (in this particular problem of inducing the decay of an excited atom), we will ignore the coupling term (8.1.10) and only consider the coupling to the proton field (8.1.7), in addition to the interaction with the pairs, $\gamma^\nu A_\nu$, where A_ν is the vector four-potential of a pair at the site of an atomic electron. We have seen earlier that this has the form

$$\hat{e}_k |\mathbf{A}| \exp[\pm i(\omega t - \mathbf{k}\cdot\mathbf{r})], \quad 0 \leqslant \omega \leqslant 2\lambda.$$

The polarization direction of the vector potential, denoted by the unit vector $\hat{e}_k$, is perpendicular to the direction of motion, $\mathbf{k}$, of the propagating interaction between the electron and the pair, where the magnitude of $\mathbf{k}$ is ω/c. The electron coordinate, $\mathbf{r}$, denotes its position relative to its parent nucleus. The average value of $\mathbf{r}$ is then much smaller than the wavelength $2\pi/k$ for the nonrelativistic hydrogen gas. Thus, with the inequality $\mathbf{k}\cdot\mathbf{r} \ll 1$, the factor

in the potential, $\exp(\pm i\mathbf{k}\cdot\mathbf{r})$ may be approximated by unity. This is the 'dipole approximation'. It corresponds to taking the effective potential at the site of the electron in hydrogen to have the following form:

$$\hat{e}_k(A_1 \pm iA_2) = \hat{e}_k|\mathbf{A}|\exp(\pm i\omega t) \tag{8.4.1}$$

where the other component of the vector potential is taken to be zero.

Using the nonrelativistic approximation for the coupled matter field equations, (8.1.1), and treating the effect of the pair potential as a small perturbation on the atomic states, the usual expression for the transition probability is obtained from time-dependent perturbation theory, as it is in ordinary quantum mechanics. Such a reduction of this formalism clearly follows from the discussion in Chapter 6, where it was demonstrated how the exact nonlinear matter field formalism of this theory reduces to the formalism of quantum mechanics in the limit of sufficiently small energy-momentum transfer between the interacting components of the (actually) physically closed system.

In the latter limit, the interaction functional $\mathscr{I}_1 \propto \gamma^k A_k$ (in the Dirac form of the theory) reduces to the Schrödinger form $-iA_k\partial^k$. The vector potential A_k for the pair matter field, in turn, is given in Equation (7.6.4). Thus, the coupling of the pair matter fields to the bound hydrogenic electron of a constituent atom of a hydrogen gas yields the identical formal expression for the probability of transition from one atomic state $|n\rangle$ to another $|n'\rangle$, utilizing the formal expression of time-dependent perturbation theory in quantum mechanics. With the electric dipole approximation that we have discussed above, this expression has the usual form [50]:

$$W_{nn'} = \left(\frac{E_{nn'}^3}{2\pi}\right)|\hat{e}_k \cdot d_{nn'}|^2 \ \mathrm{s}^{-1} \tag{8.4.2}$$

where

$$E_{nn'} = E_n - E_{n'} = \omega \tag{8.4.3}$$

is the quantity of energy transfer when a transition occurs, and the electric dipole moment 'matrix element' for this transition is:

$$\mathbf{d}_{nn'} = e\sum_j \langle n'|\,-i\nabla_j\,|n\rangle, \tag{8.4.4}$$

where the summation over j refers to the constituent atomic components of the gas, and $|n\rangle$, $|n'\rangle$ are the limiting Schrödinger states of the bound electrons. Note that the summation in the electric dipole matrix element above refers to the effect of one electron—positron pair of the background on all of the hydrogen atoms of the gas, that are radiating within some container.

Since the only index that distinguishes the different pairs of the back-

ground pair matter field from each other are the *randomly oriented* polarization vectors $\hat{e}_k$, the total effect of all of the pairs in the container on the radiating gas may be taken into account by integrating Equation (8.4.2) with respect to the angular variables. After doing this, it is found that the total transition probability that connects the excited state $|n\rangle$ to all other states $|n'\rangle$ has the following form:

$$T_n^{-1} = \left(\frac{4\omega^3}{3} \right) \sum_{n'} |d_{nn'}|^2 \, \mathrm{s}^{-1}. \tag{8.4.5}$$

T_n is the 'lifetime' of the excited state denoted by $|n\rangle$. Its expression in (8.4.5) is identical with the formula obtained from quantum electrodynamics, using the electric dipole approximation. It is also the same expression obtained from the nonrelativistic Schrödinger formalism for quantum mechanics. The successful predictions of the lifetimes of excited atomic states is then not an absolute test of quantum electrodynamics nor is it absolute test of nonrelativistic quantum mechanics, nor the present deterministic matter theory of inertia. The reason, of course, is that at nonrelativistic energy-momentum transfer all three formalisms may be approximated by the same theoretical formulas for the lifetimes of atomic states.

The salient point in regard to the matter field theory discussed here is that a hydrogen gas 'radiates' only by virtue of the transfer of energy and momentum from the excited atoms of the gas to the background of particle—antiparticle pairs, in their particular state of null energy and null momentum, i.e. their true ground states. The reason that the predicted lifetimes are the same here as in the quantum theory is that the vector potential that represents the electromagnetic effect of a pair, in its particular state of null energy-momentum, that acts on the constituent hydrogen atoms of the gas, is *formally* identical with the potential field that would describe a background radiation field of free 'photons', that is postulated in the quantum theory, though there are no 'photons' in this theory of matter.

In view of the constancy of the measured lifetimes of atomic states, it may be concluded within the context of this theory that the density of pairs that form the background 'absorber' of the radiation from the de-exciting hydrogen atoms is necessarily sufficiently great everywhere in the universe that no appreciable effect on the measured lifetimes, due to fluctuations in the density of the background pairs could have been observed. Nevertheless, the present analysis does suggest that if the measurements of spectral line shapes (which, in turn, depend on the lifetimes of the excited states) could be carried out with sufficiently high resolution, under more varied conditions of rarefaction and condensation of the 'observed excited gas', differences in the lifetimes should be seen.

8.5. Atomic Helium

The problem of the Lamb shift does not arise in the case of atomic helium because there is no accidental degeneracy predicted by the Dirac theory for the two-electron atom. Still, the conventional atomic structure calculations, using Dirac theory for the two-electron atom, and very accurate variational methods of calculation (such as the work of Pekeris [43]) do not yield values for the lower energy levels that exactly agree with the data.

Within the conventional quantum approach, this discrepancy between the calculated energy levels of atomic helium and the measured values is resolved by appealing to the radiative effects of quantum electrodynamics — the same physical source as that which is used to explain the Lamb splitting in hydrogenic atoms, though there is no degeneracy to be lifted in this case. And indeed the conventional field theoretic calculations in quantum electrodynamics, utilizing the renormalization technique, does yield radiative shifts of the low-lying energy levels of atomic helium that are the correct order of magnitude (of order 1 cm^{-1}) to account for the required corrections.

However, in the spirit of this monograph, where we are seeking an alternative explanation for the results of quantum electrodynamics, where one would not encounter divergent integrals in the predictions of physical observables, or the need for renormalization, it is the extra electromagnetic interaction $\mathscr{I}_2$ of this theory that one would expect to provide the required corrections to the energy level positions of the helium atom. It was found (by Yu and Sachs [44]) that this is the case. The calculations within the theoretical basis of this theory of matter, following from the generalization of the electromagnetic interaction functional, $\mathscr{I}_1 \rightarrow \mathscr{I}_1 + \mathscr{I}_2$, following from the spinor factorization of the Maxwell formalism, indeed yielded the same corrections to the energy levels of helium, that were obtained in quantum electrodynamics. This result was, of course, dependent on the magnitude of the new fundamental constant in this theory, $g_M = (2.087 \pm 0.001) \times 10^{-14}$ cm, already determined from the analysis of the Lamb shift in hydrogen (Equation (8.2.16)).

8.6. Electron–Proton Scattering in a Vacuum

Thus far, in this chapter, we have discussed the bound states of the electron–proton system, and some other bound state systems (deuterium and helium). We will now briefly outline some of the analysis of this research program that considers the scattering of electrons by protons. In this section it will be assumed to take place in a vacuum; in the next section we will consider the background of pairs, discussed above, to mediate the scattering process, thereby demonstrating some surprising predictions of this theory at small distances, i.e. at large energy-momentum transfer.

In the analysis on e—p scattering that follows, it is seen that there are no essential differences predicted from the standard point particle scattering process, as exhibited in terms of the expression of the Mott cross section for elastic scattering, at sufficiently large mutual separation, i.e. small momentum transfer. But in the exact expression of this theory, in relativity, and even assuming a linearization of the field equations by virtue of the small value for the ratio of electron mass to proton mass, m/M, the generalization of electromagnetic theory that occurs here indicates marked differences with the Mott cross section, and in a direction toward the actual data on e—p scattering. These deviations are conventionally accounted for by assuming that while the electron is a point particle, the proton is 'clothed' with mesic fields, giving it a finite radius, expressed in terms of form factors. The present analysis yields results that, at least, account for a portion of the phenomenologically imposed form factors of the porton, in the standard analyses.

The first step in this analysis of e—p scattering, then, is to start with the modified Dirac function for the electron, (8.2.5), that follows from the generalized spinor electromagnetic interaction that includes $\mathscr{I}_2$, together with the assumption that $m/M \ll 1$, where m is the electron mass and M is the proton mass. In terms of the radial variable, the modified Dirac function has the form

$$\psi = \psi_D \exp\left(-\frac{b}{2r}\right), \tag{8.6.1}$$

where, in accordance with the earlier determined value of g_M, the constant b is

$$b = 32\pi g_M \gamma = 0.158 \times 10^{-13}\,\text{cm}$$

and $\gamma = e^2$ is the fine structure constant (in units with $\hbar = c = 1$).

Following the standard quantum mechanical procedure, the *first Born approximation* [51] gives the following expression for the differential cross section:

$$\begin{aligned}\sigma(\theta) &= \sigma_M(\theta)\left|\langle\psi|\,\frac{e^2}{r}\,|\psi\rangle\right|^2 \Big/ \left|\langle\psi_D|\,\frac{e^2}{r}\,|\psi_D\rangle\right|^2 \\ &= \varepsilon(bq)\sigma_M(\theta)\end{aligned} \tag{8.6.2}$$

where

$$\varepsilon(bq) = \left|\int_0^\infty \left[\exp\left(-\frac{bq}{u}\right)\right]\sin u\,du\right|^2 \tag{8.6.3}$$

is a correction factor resulting from the modification (8.6.1) of the electron

wave function,

$$\sigma_M(\theta) = \left(\frac{e^2}{2E} \right)^2 \frac{1 - \beta^2 \sin^2 \frac{1}{2}\theta}{\sin^4 \frac{1}{2}\theta} \tag{8.6.4}$$

is the Mott cross section, β is the electron velocity (in units of c) and

$$q = |\mathbf{p}_f - \mathbf{p}_i| \tag{8.6.5}$$

is the momentum transfer from the initial electron beam (projectile) to the final scattered electron beam, at an angle θ (in the rest frame of the proton).

The integration indicated in the correction factor, in Equation (8.6.3), is carried out (in Appendix B), giving:

$$\varepsilon(bq) = 4bq \, |\mathrm{ker}'(2\sqrt{bq})|^2 \tag{8.6.6}$$

where ker(x) is the real part of the modified, zeroth-order Bessel function of the second kind, with argument $x\sqrt{i}$, i.e. [52],

$$K_0(x\sqrt{i}) = \mathrm{ker}(x) + i \, \mathrm{kei}(x).$$

The prime in (8.6.6) refers to the derivative of this function with respect to its argument.

In the limit in which we may assume that $m/M = 0$, the momentum transfer q becomes:

$$q = 2E \sin \tfrac{1}{2}\theta \tag{8.6.5'}$$

where here (as above) E is the (initial and final) electron energy, and θ is the angle of scattering in the rest frame of the proton. Finally, substitution of (8.6.5') into the cross section (8.6.2) and the correction factor (8.6.3) yields the expression for the differential scattering cross section of the e—p system, as a function of the electron's initial (and final) energy E and its direction of scattering θ.

It is important to note that the limiting values of the correction factor $\varepsilon(bq)$ (and therefore the limiting values of the e—p cross section itself) are the classical values when $q \to 0$, where $\varepsilon(bq) \to 1$ and $\sigma \to \sigma_M$; and at the extreme energy-momentum transfer, where $q \to \infty$, $\varepsilon(bq) \to 0$ and $\sigma \to 0$. Since $b \sim 0.1$ fm, these results indicate that when the effective mutual e—p separation is large compared with 0.1 fm ($= 10^{-14}$ cm), the scattering cross section is essentially the standard Mott cross section (8.6.4). But when the mutual e—p separation is of order 0.1 fm or less (high momentum transfer), the cross section decreases toward zero, as momentum transfer q approaches infinity (corresponding to the approach toward zero mutual separation). A decrease of the cross section from the Mott cross section, as momentum transfer increases in this way is what is observed in experimentation, and conventionally attributed to the mesic field cover of the scattering proton.

In view of the interpretation of the cross section, in this field theory

of matter, as a *weighting* of the mutual interaction, the preceding result indicates that as the effective mutual separation between the electron and proton becomes small, the measured interaction should become correspondingly weak. Thus, the infinitely massive (target) proton appears to the high-energy electron as a 'hole', with a radius that is of the order of 0.1 fm. That is to say, at separations of this order of magnitude or smaller, the scattering cross section tends to vanish, as though the e—p interaction were 'turning off' rapidly — leading to zero interaction at the limiting value of zero separation. (Incidentally, this is the same sort of asymptotic behavior, *in this limit*, as the present-day proponents of the quark theory of nucleons are stating, for the quark interactions in nucleons, in the name of a 'confinement theory'.)

Consider the preceding analysis of e—p scattering one step further by taking into account of the proton recoil [53]. That is, consider the situation in which $m/M \neq 0$. If E_f is the energy of the scattered electron beam, then it relates to its initial energy E according to:

$$E_f = \zeta E \tag{8.6.8}$$

where

$$\zeta = \left[1 + \left(\frac{2E}{M} \right) \sin^2 \tfrac{1}{2}\theta \right]^{-1}. \tag{8.6.9}$$

Using the relativistic momentum-energy relation,

$$p = \beta E \tag{8.6.10}$$

and the approximation at high energy that $\beta_i \sim \beta_f \sim 1$, the momentum transfer (8.6.5) becomes

$$q = E[1 + \zeta^2 - 2\zeta \cos \theta]^{1/2}. \tag{8.6.11}$$

Finally, using (8.6.9) in (8.6.11), we have the following corrected expression for the momentum transfer, taking into account the finite mass of the proton (i.e. taking account of the recoil of the proton):

$$q = 2\xi E \sin \tfrac{1}{2}\theta \tag{8.6.12}$$

where the 'recoil factor' ξ has the following form

$$\xi = \frac{[1 + (2E/M + E^2/M^2) \sin^2 \tfrac{1}{2}\theta]^{\frac{1}{2}}}{1 + (2E/M) \sin^2 \tfrac{1}{2}\theta}. \tag{8.6.13}$$

Note that if we should allow $M \to \infty$, then $\xi \to 1$, correspondingly, as it should, i.e. reducing q to the earlier value (8.6.5′).

With (8.6.12) and (8.6.13) in the correction factor (8.6.6), we have the same expression for $\varepsilon(bq)$, except that its argument may now be expressed in the form

$$4bq = [0.655\bar{E} \sin \tfrac{1}{2}\theta] \frac{[1 + (0.213\bar{E} + 0.0113\bar{E}^2) \sin^2 \tfrac{1}{2}\theta]^{1/2}}{1 + 0.213\bar{E} \sin^2 \tfrac{1}{2}\theta}$$

where $\bar{E}$ is the electron's initial energy in units of 0.1 GeV. With this definition of the argument $4bq$ in the correction factor $\varepsilon(bq)$, in Equation (8.6.6), we obtain the correction to the Mott cross section, in (8.6.2), that takes account (in a particular approximation) of the proton's recoil during the scattering event. (Curves that show a comparison of this theoretical result with the data and with the Mott cross section are shown in Sachs and Schwebel [53].)

It is emphasized at this point that the analysis of e—p scattering, thus far, is not meant to be an exact prediction of high-energy e—p scattering. It is only meant to be an initial step, to see if the correction to the cross section that results from the generalized electromagnetic interaction will correct the conventional Mott cross section *in the direction of the actual data*. The result we have demonstrated is that indeed the matter field theory developed does lead in the right direction. The implication is that the modification of the electromagnetic coupling that has been discovered here must account for at least some of the phenomenologically introduced 'form factor' of the proton, that is conventionally attributed fully to the mesic fields in the model of the 'clothed' proton of the standard quantum mechanical theory [54].

It should be further noted that when the proton's 'recoil' is properly taken into account in the context of this theory of matter, it would be necessary to use the *nonlinear* coupled matter field equations, without the linear approximation that we considered above. On the other hand, it is felt that the procedure followed above, which handles the proton recoil with linearized field equations, does lead to a result that is not far different from the predictions of the solutions of the nonlinear equations.

The determination of better solutions of the nonlinear matter field equations should give, to higher precision, the contribution of the electromagnetic generalized interaction in e—p scattering, at high energy. The higher the quantity of energy-momentum transfer, the more important should be the nonlinear features of the theory. After such calculation, the remaining deviation from the data would be due to the coupling with mesic fields. Thus, within the framework of this theory, it is anticipated that the higher order calculations of electrodynamical scattering may give a clue as to the nature of that portion of the Lagrangian that represents the meson—nucleon interaction.

8.7. Electron—Proton Scattering in a Background of Pairs

The modification of the electron matter field (8.6.1) corresponds to a 'weakening' of the effective Coulomb interaction, as

$$\left\langle \frac{1}{r} \right\rangle \rightarrow \left\langle \frac{\exp(-b/r)}{r} \right\rangle , \tag{8.7.1}$$

demonstrating most dramatically the approach of this interaction toward a zero limit, rather than toward an infinite limit, as $r \rightarrow 0$. But this corresponds to the 'bare interaction' of electron and proton, leaving out the background of pairs in their ground states of null energy and null momentum, that we have indicated earlier must permeate any observed matter. To analyze the e—p interaction with this background, though at small distances and relativistic quantities of energy-momentum transfer, the effective magnitude of the interaction $\mathscr{I}'_1$ diminishes compared with the effect of $\mathscr{I}'_2$. For the latter reason, we will start this analysis with 'free' Dirac electron states, rather than the Coulomb wave functions, as it was done in the case of determining the bound states of hydrogen. We will then consider the perturbation of these states by the 'short range' interaction $\mathscr{I}'_2$.

The free Dirac electron states were found by Darwin [55]; he also determined their exact form in the presence of a potential. [See Mott and Massey, *ibid.*, p. 67.] The Darwin solution for the free electron states is as follows:

$$\psi_1 = -\left\{ \frac{N_1 p_3 + N_2(p_1 - ip_2)}{E + m} \right\} S, \qquad \psi_3 = N_1 S$$

$$\psi_2 = -\left\{ \frac{N_1(p_1 + ip_2) - N_2 p_3}{E + m} \right\} S, \qquad \psi_4 = N_2 S \tag{8.7.2}$$

which are the components of the four-component Dirac bispinor

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} .$$

In this expression, E is the total electron energy, m is its rest mass and

$$S = \exp\left[\frac{i(\mathbf{p} \cdot \mathbf{r} - Et)}{\hbar} \right], \qquad \frac{N_2}{N_1} = -\operatorname{ctg} \tfrac{1}{2}\theta \exp(i\Phi).$$

The constants N_1, N_2 relate to the normalization that defines the density of interacting electrons as follows:

$$\frac{(|N_1|^2 + |N_2|^2)2E}{E + m} = |N_1|^2 \operatorname{cosec}^2 \tfrac{1}{2}\theta \left[\frac{2E}{E + m} \right] \equiv \rho_e,$$

where ρ_e is defined to be the number of interacting electrons/cm^3. The latter may be expressed in terms of the electron current density j_e as follows:

$$e\rho_e = \frac{j_e}{v} = j_e \left(\frac{\lambda}{\lambda_C} \right) \Gamma^{-1},$$

where Γ is the factor from special relativity, $[1 - v^2]^{1/2}$, λ is the de Broglie wavelength of the electron, and λ_C is its Compton wavelength ($= \hbar/mc$). We then have the relation

$$|N_1|^2 \operatorname{cosec}^2 \tfrac{1}{2}\theta \left[\frac{2E}{E + m} \right] = j_e \left(\frac{\lambda}{\lambda_C} \right) \Gamma^{-1} \tag{8.7.3}$$

to express the normalization of the electron wave function.

8.7.1. *The Screening Effect of the Background Pairs on the e—p Interaction*

The electron—positron pairs in the domain of the interacting electron and proton will polarize, with the antiparticle components oriented toward the scattering electron and the particle components of the pairs oriented toward the target proton. The net effect of the polarization of the medium is to screen the e—p mutual interaction.

To determine the explicit screening effect, let us consider at first the classical equation for the electrostatic potential A_0:

$$\nabla^2 A_0 = -4\pi\rho \tag{8.7.4}$$

where the right-hand side of this equation, the 'source', expresses the electric charge density as the combination of densities of positive and negative charge densities as follows:

$$\rho = e^+ n^+ + e^- n^- = e(n^+ - n^-) = e\ \delta n,$$

where n^+ and n^- are respectively the number densities of positive and negative charges (positrons and electrons) of the pair background. Clearly if there would be no polarization in the vicinity of the scattering electron or target proton, the number densities of electrons and positrons in their vicility would be the same and in this case $\rho = 0$. But with the polarization effect due to the presence of the e—p system, there would be an imbalance of charge density in their vicinity, so that there would be some number density δn acting on the scattering electron. This imbalance is created by the

electrostatic potential that acts on the observed electron in a way that tends to *decrease* the charge density in its vicinity. Thus, we may take

$$-\delta n = aA_0$$

where the constant of proportionality, a, is the 'polarizability' of the medium of background pairs. The electrostatic Equation (8.7.4) then takes the form in which the source term is proportional to the solution, i.e.

$$\nabla^2 A_0 = \mu^2 A_0,$$

where $\mu^2 = 4\pi ea$. The solution of this equation has the 'Yukawa' form:

$$A_0 = \frac{K[\exp(-\mu r)]}{r}$$

that also appears in the description of the short-range nuclear force. However, in this solution μ plays the role of a screening constant (with dimension of inverse length). The constant K is to be determined from the boundary conditions imposed on A_0.

The derivation above is similar to the Debye—Hückel derivation of the screening effect of a plasma on one of its constituent charges [56]. However, in the case of e—p scattering in a background of electron—positron pairs, the cause of the effect is different. In the Debye—Hückel analysis, the polarization effect is caused by temperature fluctuations, whereas in this case it is simply due to the polarizability of the medium of pairs, independent of thermodynamic considerations. In the analysis to follow, we will treat the polarizability constant a as a phenomenological parameter, to be determined precisely from dynamical studies of this medium of pairs in future studies.

To determine the value of the integration constant K, we note that if the observed electron should come arbitrarily close to a single positron of a pair of the background medium, the screening effect of the background would vanish in this case. This is equivalent to the limit in which $\mu \to 0$, so that $A_0 \to \rho/r$ (in accordance with classical electrostatics). Thus we see that the integration constant K must be equal to the charge density ρ, thereby giving the Debye—Yukawa form for the potential

$$A_0 = \left(\frac{\rho}{r}\right) \exp(-\mu r). \tag{8.7.5}$$

We have seen, then, that the effective charge density corresponding to a charge imbalance of the background pairs in the vicinity of an interacting e—p system is altered, due to a polarization of the medium, in accordance with the change

$$\rho \to \rho \exp[-\mu r].$$

Since the charge density relates to the matter fields as $\rho \propto \psi^{\dagger}\psi$, the screening effect on the projectile electron in its interaction with the target proton may be taken into account by altering the matter field as

$$\psi \rightarrow \psi \exp[-\tfrac{1}{2}\mu r].$$

Combining this result with the modification (8.6.1), due to the generalization of the electromagnetic interaction in this theory, the electron matter field spinor may then be expressed as the following form, that incorporates both alterations (at small distances):

$$\psi = \psi_e \exp\left[-\frac{1}{2}\left(\frac{b}{r} + \mu r\right)\right], \tag{8.7.6}$$

where ψ_e is the 'free electron' Dirac field, given in (8.7.2).

By using the de Broglie relation $p = \hbar k$ in (8.7.2) and substituting (8.7.6) into Equation (5.3.4) for the spinor expression of the electromagnetic equations, with the source terms given by

$$T_a = e\bar{\psi}\Gamma_a\psi,$$

where the four-dimensional matrices Γ_a are defined in (7.1.1b), it follows that the spinor source fields of the electromagnetic field equations are:

$$T_1(x') = \left[\frac{8\pi |N_1|^2 e\hbar}{E+m}\right] \operatorname{cosec}^2 \tfrac{1}{2}\theta \exp\left(-\frac{b}{r'} - \mu r'\right) \times$$

$$\times \begin{pmatrix} k_3 + \tfrac{1}{2}i(E-m) \\ -(k_1 + ik_2) \end{pmatrix} \tag{8.7.7a}$$

$$T_2(x') = \left[\frac{8\pi |N_1|^2 e\hbar}{E+m}\right] \operatorname{cosec}^2 \tfrac{1}{2}\theta \exp\left(-\frac{b}{r'} - \mu r'\right) \times$$

$$\times \begin{pmatrix} k_1 - ik_2 \\ k_3 - \tfrac{1}{2}i(E-m) \end{pmatrix}. \tag{8.7.7b}$$

The spinor solutions ϕ_σ of (5.3.4) are then obtained by substituting the sources (8.7.1a, b) into the integral form of the solutions [as shown in [1], *GRM*, Ch. 5, §8].

$$\phi_a = \int \left[-\frac{i\sigma^{\nu}k_{\nu}}{(2\pi)^4 k_{\rho}k^{\rho}}\right] \exp[ik_{\mu}(x^{\mu} - x^{\mu'})]\, d^4k\, T_a(x')\, d^4x' \tag{8.7.8}$$

to give their explicit forms.

Since the source fields T_a are time-independent, in this particular problem, the integration over the $x^{0\prime}$ coordinate yields the Dirac delta-function as follows:

$$\int \exp[-ik_0' x^{0\prime}]\, dx^{0\prime} = (2\pi)^{1/2}\delta(k_0').$$

Substitution into Equation (8.7.8) and the subsequent integration over k_0' gives:

$$\phi_a = K_a \left[\frac{(2\pi)^{1/2} |N_1|^2 e\hbar}{(2\pi)^3 (E+m)} \right] \operatorname{cosec}^2 \tfrac{1}{2}\theta \int \left[\frac{\sigma \cdot \mathbf{k}'}{(-k')^2} \right] \times$$

$$\int \exp[-i\mathbf{k}' \cdot (\mathbf{r} - \mathbf{r}')] \exp\left[-\frac{b}{r'} - \mu r' \right] d\mathbf{r}'\, d\mathbf{k}', \qquad (8.7.9)$$

where

$$K_1 = \begin{pmatrix} k_3 + \frac{1}{2}i(E-m) \\ -(k_1 + ik_2) \end{pmatrix}, \qquad K_2 = \begin{pmatrix} k_1 - ik_2 \\ k_3 - \frac{1}{2}i(E-m) \end{pmatrix}.$$

With the integration

$$\int \exp[-i\mathbf{k}' \cdot (\mathbf{r} - \mathbf{r}')] \left(\frac{i\sigma \cdot \mathbf{k}'}{-k'^2} \right) d\mathbf{k}' = \frac{2\pi^2 i \sigma \cdot (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$

in the solutions (8.7.8), and using the fact that the integral remaining over $\sigma \cdot \mathbf{r}'$ vanishes because of its odd parity, we are left with the solution

$$\phi_a = K_a \left[\frac{(2/\pi)^{1/2} |N_1|^2 e\hbar}{E+m} \right] \operatorname{cosec}^2 \tfrac{1}{2}\theta (\sigma \cdot \mathbf{r}) I, \qquad (8.7.10)$$

where I is the integral

$$I = \int\int_0^\infty \left[\exp\left(-\frac{b}{r'} - \mu r' \right) \right] \left(\frac{r'^2}{|\mathbf{r} - \mathbf{r}'|^3} \right) d\mathbf{r}'\, d\Omega. \qquad (8.7.11)$$

With the identity, $|\mathbf{r} - \mathbf{r}'|^3 = [r^2 + r'^2 - 2rr' \cos\theta]^{3/2}$, the evaluation of the angular part of the integral in I yields:

$$I = \left(\frac{4\pi}{r} \right) \int_0^\infty \left[\exp\left(-\frac{b}{r'} - \mu r' \right) \right] \left[\frac{r'^2}{r^2 - r'^2} \right] dr'. \qquad (8.7.12)$$

Using the following integral transform:

$$\exp\left[-\frac{b}{r'}\right] = r' \int_0^\infty [\exp[-sr']J_0[2(bs)^{1/2}]\,ds, \tag{8.7.13}$$

where J_0 is the zeroth-order Bessel function, it follows that I may be expressed in the form

$$I = \left(\frac{4\pi}{r}\right) \int_0^\infty \exp[-(\mu + s)r'] \left[\frac{r'^3}{r^2 - r'^2}\right] dr' \int_0^\infty J_0[2(bs)^{1/2}]\,ds. \tag{8.7.14}$$

Note that the function $rI(r)$ entails the dependence of the generalized electromagnetic interaction $\mathscr{I}_2$ on r, b and μ, parameters that are crucial to the role of the added part of this interaction at high energy-momentum transfer between the electron and proton.

To determine the explicit form of I from Equation (8.7.14), we will expand the denominator in the integrand in a binomial series:

$$(r^2 - r'^2)^{-1} = r^{-2} \sum_{n=0}^{\infty} \left(\frac{r'}{r}\right)^{2n}.$$

Substitution into (8.7.14) and integration over the r' variable of the series yields

$$\int_0^\infty \exp[-(\mu + s)r'] \left[\frac{r'^3}{r^2 - r'^2}\right] dr' = r^{-2} \sum_{n=0}^{\infty} \frac{r^{-2n}(2n+3)!}{(\mu + s)^{2n+4}}. \tag{8.7.15}$$

By using the change of variables, $y = 2(bs)^{1/2}$,

$$J_0[2(bs)^{1/2}]\,ds = \left(\frac{1}{2b}\right) J_0(y)y\,dy$$

so that the integral I may be expressed in the form:

$$I = 8\pi \left(\frac{4b}{r}\right)^3 \sum_{n=0}^{\infty} (2n+3)! \left(\frac{4b}{r}\right)^{2n} \int_0^\infty \left[\frac{J_0(y)y}{(y^2 + 4b\mu)^{2n+4}}\right] dy.$$

The remaining integration is determined in (Watson [52], pp. 434, 737, 739),

where it is shown that

$$\int_0^\infty \left[\frac{J_0(y)y}{(y + 4b\mu)^{2n+4}} \right] dy = \frac{K_{2n+3}(2\sqrt{b\mu})}{2^{4n+6}(b\mu)^{n+3/2}(2n+3)!},$$

where K_N is the special Bessel function of imaginary argument and order N, as discussed earlier [it is defined explicitly in Watson, *ibid*, p. 78]. With this result we may then express the required integral (8.7.14) as follows:

$$I = 8\pi \left(\frac{b}{r} \right)^3 (b\mu)^{-3/2} \sum_{n=0}^{\infty} \left(\frac{b}{r} \right)^{2n} (b\mu)^{-n} K_{2n+3}(2\sqrt{b\mu}). \tag{8.7.16}$$

With this expression for I in Equation (8.7.10), we have the explicit form for the electromagnetic spinor solutions ϕ_α, as a power series expansion in (b/r). By expressing everything in spherical-polar coordinates in (8.7.10), we may substitute there

$$k_3 = k \cos\theta, \qquad k_1 \pm ik_2 = k \sin\theta \exp[\pm i\phi],$$

$$(\boldsymbol{\sigma} \cdot \mathbf{r}) = r \begin{pmatrix} \cos\theta & \sin\theta \exp[-i\phi] \\ \sin\theta \exp[i\phi] & -\cos\theta \end{pmatrix}. \tag{8.7.17}$$

With the definition of the wavenumber as $k = E/\hbar c$, and with $mc/\hbar k = \lambda/\lambda_C$, where λ is the de Broglie wavelength and λ_C is the Compton wavelength of the projectile electron, substitution of (8.7.17) into (8.7.10) yields the following explicit form of the electromagnetic spinor solutions:

$$\phi_1 = \begin{pmatrix} \cos 2\theta + \tfrac{1}{2}i\left(1 - \dfrac{\lambda}{\lambda_C}\right)\cos\theta \\ \sin 2\theta \exp[i\phi] + \tfrac{1}{2}i\left(1 - \dfrac{\lambda}{\lambda_C}\right)\sin\theta \exp[i\phi] \end{pmatrix} \times$$

$$\times 4(2\pi)^{1/2} \left(\frac{j_e}{\Gamma} \right) \left(\frac{\lambda}{\lambda_C} \right) f(r, b, \mu)$$

$$\phi_2 = \begin{pmatrix} \sin 2\theta \exp[-i\phi] - \tfrac{1}{2}i\left(1 - \dfrac{\lambda}{\lambda_C}\right)\sin\theta \exp[-i\phi] \\ -\cos 2\theta + \tfrac{1}{2}i\left(1 - \dfrac{\lambda}{\lambda_C}\right)\cos\theta \end{pmatrix} \times$$

$$\times 4(2\pi)^{1/2} \left(\frac{j_e}{\Gamma} \right) \left(\frac{\lambda}{\lambda_C} \right) f(r, b, \mu), \tag{8.7.18}$$

where (r, θ, ϕ) are the electron coordinates relative to the interacting proton target, and

$$f(r, b, \mu) = \left(\frac{b}{r}\right)^3 r(b\mu)^{-3/2} \sum_{n=0} \left(\frac{b}{r}\right)^{2n} (b\mu)^{-n} K_{2n+3}(2\sqrt{b\mu}) \quad (8.7.19)$$

is the function in this theory that gives the dependence of the generalized electromagnetic interaction functional $\mathscr{I}_2'$ on the parameters b (associated with a weakening of the electromagnetic force at distances of order $b \sim 10^{-14}$ cm, before the background of pairs is taken into account), μ (arising from the mediating effect of the electron—positron pairs on the e—p interaction at small distances, and perhaps associated with the (strong) Yukawa interaction when the proton—antiproton pairs are taken into account), and the electron—proton separation r. In the spinor solutions (8.7.18), j_e is the projectile current density.

Note that the expression (8.7.19) for $f(r, b, \mu)$ should be useful if $r > b$ ($\sim$0.1 fm) since it would in this case not entail too many terms of the expansion. Since the sum of this expression derives from a finite integral (8.7.11), it is finite; however, for $r < b$, as $r \to 0$, successively more terms in the sum would have to be used for an accurate evaluation. It is also important to note that for $r < b$, the magnitude of the function f, and therefore the amplitude of the spinor solutions ϕ_α, increases greatly, compared with the amplitude of this field intensity for distances where $r \gg b$. This behavior hints at a possible unification of the electromagnetic and the (strong) nuclear interaction at small distances, also keeping in view the Yukawa behavior of the fields with the mediating pairs present.

8.7.2. *The Generalized Electromagnetic Interaction*

With the electromagnetic spinor solution (8.7.18), we are in a position to evaluate the expectation value $\langle \mathscr{I}_2' \rangle$ at small distances. In accordance with the spinor equations for electromagnetism (5.3.4), we have

$$\langle \psi^{(p)} | \mathscr{I}_2' | \psi^{(p)} \rangle = E_1 + E_2, \quad (8.7.20a)$$

where $\psi^{(p)}$ is the Dirac proton field, and

$$E_1 = 2eg_M \operatorname{Im} \int \sum_{\alpha=1}^{2} (-1)^\alpha \tilde{\psi}^{(p)} \phi_\alpha^\dagger \cdot \Gamma_\alpha \psi^{(p)} \, \mathrm{d}\mathbf{r} \quad (8.7.20b)$$

$$E_2 = be^2 \int \tilde{\psi}^{(p)} \left(\frac{\gamma_r}{r^2}\right) \psi^{(p)} \, \mathrm{d}\mathbf{r}. \quad (8.7.20c)$$

With the definition of the matrices Γ_α given in Equation (7.1.1b) in

Equation (8.7.20b), E_1 has the following explicit form:

$$E_1 = \left[\frac{8(2\pi)^{1/2}}{\Gamma} \right] j_e \left(\frac{\lambda}{\lambda_C} \right) e g_M \left\{ \int \psi^{(p)\dagger} f(r, b, \mu) \times \right.$$

$$\times \left[2 \cos 2\theta \, \alpha_3 + \left(1 - \frac{\lambda}{\lambda_C} \right) \cos \theta + 2 \sin 2\theta \cos \phi (\alpha_1 - \alpha_2) + \right.$$

$$\left. \left. + \left(1 - \frac{\lambda}{\lambda_C} \right) \sin \theta \sin \phi \, \alpha_2 \right] \psi^{(p)} r^2 \, dr \, d\Omega \right\}, \tag{8.7.21a}$$

where α_k ($k = 1, 2, 3$) are the Dirac matrices

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}$$

that play the role of 'velocity operators' in the Dirac wave mechanics.

The second part of the interaction above may be expressed in the following form:

$$E_2 = 32\pi e^2 g_M \int \psi^{(p)\dagger} \left(\frac{\alpha_r}{r^2} \right) \psi^{(p)} \, dr, \tag{8.7.21b}$$

where $\alpha_r = \boldsymbol{\alpha} \cdot \mathbf{r}/r$.

The terms in E_1 that depend on the 'velocity operator' α_k, as well as the interaction E_2, may be viewed as recoil terms in the proton's reaction to its coupling to the projectile electron. The interaction term E_1 also depends on the electron current density j_e, thus demonstrating its velocity dependence on the projectile electron's state of motion.

In the approximation whereby one may neglect the recoil terms, when the electron—proton energy-momentum transfer is small compared with the proton's rest energy, the interaction $\langle \mathscr{H}'_2 \rangle$ becomes:

$$\langle \mathscr{H}'_2 \rangle \to 16\pi(2\pi)^{1/2} \left(\frac{j_e}{\Gamma} \right) \left(\frac{\lambda}{\lambda_C} \right) \left(1 - \frac{\lambda}{\lambda_C} \right) \left(\frac{b}{\mu} \right)^{3/2} e g_M \cdot \cos \theta \times$$

$$\times \sum_{n=0}^{\infty} \left(\frac{b}{\mu} \right)^n K_{2n+3}(2\sqrt{b\mu}) \langle r^{-2(n+1)} \rangle, \tag{8.7.22}$$

where $\langle r^{-2(n+1)} \rangle$ are the expectation values of the inverse $2(n+1)$ powers of the e—p separation (when they are interacting over the order of nucleon dimensions).

8.7.3. *Concluding Remarks*

There are a few significant features of $\langle \mathscr{I}'_2 \rangle$ that should be noted:

(1) In the low energy-momentum limit, i.e. as $v/c \to 0$, $j_e \to 0$ and $\mathscr{I}'_2 \to 0$.

(2) In the high energy-momentum limit, i.e. as $v/c \to 1$, it follows that $\Gamma^{-1} \equiv [1 - (v/c)^2]^{-1/2} \to \infty$. The latter limit corresponds to an effective change at high energy-momentum transfer in the electromagnetic coupling as: $e^2 \to e^2/\Gamma$.

(3) As the screening length μ would be allowed to become indefinitely large, the interaction $\langle \mathscr{I}'_2 \rangle \to 0$. But as the screening effect should be allowed to vanish, i.e. with $\mu = 0$, the interaction $\langle \mathscr{I}'_2 \rangle$ would diverge. This limit is consistent with the property that the integral I (in ϕ_a, Equation (8.7.10)) diverges with $\mu = 0$.

(4) At sufficiently high energy—momentum transfer, in the e—p interaction, when the electron de Broglie wavelength λ exceeds the electron's Compton wavelength $\lambda_C \simeq 10^{-11}$ cm, the sign of the interaction $\langle \mathscr{I}'_2 \rangle$ changes, altering its polarity from attractive to repulsive, and vanishing when $\lambda = \lambda_C$. The latter corresponds to an electron speed that is the order of $0.7c$. Thus the prediction follows from the form of this interaction that when the speed of the projectile electron exceeds the order of $0.7c$, in the vicinity of the target proton, the generalized electromagnetic interaction of the e—p system changes from attractive to repulsive, thence increasing the strength of the interaction as $[1 - (v/c)^2]^{-1/2}$ as v increases further toward the magnitude of the speed of light c. In the range, $c > v > 0.7c$, this interaction would compete with the (strong) Yukawa interaction for the case of interacting protons.

8.8. Summary

In this chapter we have examined the implications of the field theory of inertia from general relativity, in the (low energy) special relativity limit, in the cases of the bound state and scattered electron—proton (or other nucleus) system. The bound system, called the 'hydrogenic atom', is the simplest example of the atomic species of Mendeleev's Periodic Chart. The anticipation for future applications is that if this theory of matter would be successful in the hydrogenic problem, it might be reasonable to expect its continued success in application to the other atomic species — the same assumption that is made in the application of ordinary quantum mechanics to the atomic many-body systems. It was seen here that indeed, the entire energy spectrum of hydrogenic atoms is predicted correctly, including the Lamb shift. As first approximations, it was also seen that the Lamb shifts in other hydrogenic atoms where the nucleus is more than the proton: the

bound systems e-deuteron and the ionized helium atom, He^+, also give reasonable agreement with the empirical data. It was also indicated how this theory gives good results for the shifts of the low-lying energy levels of atomic helium, He.

The extra interaction in this theory that accounts for the effects that are generally attributable to the Lamb shift arises from the generalization of the form of the electromagnetic field theory whereby the standard vector form of the Maxwell equations is factorized to a pair of two-component spinor equations — which is the most general form that is consistent with the irreducible representations of the Poincaré group of special relativity (or the Einstein group of general relativity). It was shown in Chapter 5 that the latter generalization gives back all of the standard predictions of the Maxwell formalism, as well as new predictions that have no counterpart in the vector equations. It is some of the latter new predictions that accounted for the Lamb effect in hydrogenic atoms, because of a reduction of the symmetry of the Coulomb interaction in the Dirac Hamiltonian, in the added interaction.

The latter additional interaction also predicted new results in the problem of e—p scattering, indicating better agreement with the empirical results than is predicted by the usual Mott analysis of point particle scattering, yet without the need to add any phenomenological models, such as the form factors of mesic cloud covers for the proton. In addition, new generalization in the e—p scattering interaction appears when taking account of the sea of electron—positron pairs that this theory indicates must mediate all electromagnetic interactions of 'observable matter'. It is seen that the new electromagnetic interaction that appears in the generalization, when taking account of the mediating pairs, implies a Yukawa—Debye type of interaction, as well as a weakening of the interaction when the mutual separation becomes less than the order of $b \sim 10^{-14}$ cm, with an effective potential that behaves as

$$\frac{e^{-b/r - \mu r}}{r},$$

where the parameter μ relates to the polarizability of the medium of electron—positron (and all other particle—antiparticle) pairs.

With this effective interaction between elementary particles at small distances, there is a hint, *from a dynamical view*, that the weak and the strong nuclear interactions are indeed manifestations of the electromagnetic interaction, at sufficiently high energy-momentum transfer. Of course, there is a great deal more research to be carried out on this idea of *dynamical unification* of the elementary forces, following from the full use of the continuous field concept in general relativity. But there is indeed a very strong hint from the analysis of this monograph that such a unification does indeed occur in nature. It would be a unification that is in full accord with Einstein's meaning of general relativity as a fundamental theory of matter, in all domains, where 'quantum mechanics' is not more than a mathematical

formalism that is a linear approximation for a truly nonlinear, fully covariant field theory of matter, that dynamically incorporates the inertial manifestations of matter with its force manifestations, in all domains. As we have discussed in the initial chapters of this book, this would be a replacement of the indeterministic view of matter according to the quantum theory, where probability plays an elementary role, with the view of matter according to the philosophy of general relativity, that is fully deterministic (in the generalized sense of the underlying reality that gives us an elementary understanding of matter), where probability per se does not play any fundamental role. This view of matter, in all domains, would then reinforce Einstein's belief that rejected the idea that God was playing with dice when He created the universe!

Chapter 9

Elementary Particle Physics

In this final chapter, the theory of inertia from general relativity that has been developed will be applied to special outstanding problems in the area of elementary particle physics. It is not intended here to claim conclusive derivations for qualitative and quantitative features of pions, kaons and neutrons, that will be explored. The purpose here is only to show that the presented theory of matter, based fundamentally on the approach of general relativity, does yield results regarding the empirical evidences of some interesting (yet unsolved) problems in elementary particle physics that are qualitatively correct and, quantitatively, yield correct orders of magnitude.

The specific problems that will be considered here, within the context of a field theory of inertia from general relativity, are: (1) the structure of the neutron, (2) the mass of the pion, (3) the ratio of the lifetimes of the charged pions to the neutral pion, (4) the origin of the CP violation in long-lived neutral kaon decay, (5) the question of time reversal noninvariance in nuclear forces, and (6) the explanation of proton—antiproton collision yielding an (apparent) intermediate boson (the $W^{\pm}$-particle) from the prediction in general relativity of a spectrum of such mediating bosons, that are in fact composites of spinor field 'particles' in the context of general relativity.

What it is that all of the above problems have in common in the context of the theory presented in this monograph is their dependence on the composite elementary matter structure, and the important role played by the spinor formulation of electromagnetic field theory (as we have seen in Chapter 5 follows from fully exploiting the basis of general relativity theory, leading to a factorization of the Maxwell field formalism).

As we have seen earlier, this field theory implies that the model of matter entails no separable, singular elementary particles of matter. But there are, instead, modes of behavior of a continuous matter field that make it appear, in particular approximations, that there are elementary particles that interact with each other in particular ways. Thus far in this theory, the elementary matter fields we have considered are those associated with the 'electron' and the 'proton', and their respective antimatter fields (the positron and the antiproton). The muon (and antimuon) have been seen earlier, within the

context of this theory [57] (as derived in *GRM*) to be a 'heavy' electron (and 'heavy' positron), by virtue of the geometrical field of excited pairs in the vicinity of an 'observed' electron. The muon appears in this theory as a member of an elementary mass doublet (because the geometrical field that determines inertial mass in the two-component spinor formalism in general relativity is minimally a two-dimensional matrix field). The same theory of inertia predicts that the proton also has a heavier sister, that also is the order of 200 times more massive than its companion — i.e. the prediction of a proton with a mass that is the order of 200 GeV [58]. In the case of the heavy electron from general relativity (the 'muon') it was found to have a lifetime (by virtue of the de-excitation of the surrounding pairs) that is the correct order of magnitude (of order 10^{-6} s) [59]. The determination of the lifetime of the heavy proton is the subject of future analyses.

Let us then commence the present survey of particular features of elementary particle physics by starting with the 'neutron' — the first new matter field discovered beyond the electron and proton. The idea is to see if it is even feasible to reduce the neutron to a composite that entails protons and electrons and their antimatter components.

9.1. The Neutron

After Chadwick's discovery of the neutron in 1932, it was claimed to be an essential building block of the atomic nucleus and perhaps understanding it in fundamental terms would be a key to our understanding of the nuclear force, from a dynamical point of view.

Before the discovery of the neutron, it was assumed that the proton and the electron are the only massive elementary particles. Thus it was conjectured after the discovery of the massive particle called 'neutron' that perhaps it isn't elementary, but rather is a composite of a proton and electron, bound in a domain that is the order of nuclear dimensions, that is, the order of 10^{-12} cm. But this idea had to be rejected for two primary reasons. The first reason is that with our prior knowledge of the fermion characteristic of the electron and of the proton, such a model would mean that the neutron must be a boson. But the neutron was seen, empirically, to also be a fermion. The second reason for rejecting this composite model of the neutron was that if the electron would be bound to a proton in a complex nucleus, with a radius of order 10^{-12} cm, the Heisenberg uncertainty relations would imply that the binding of the electron to the proton must be the order of

$$(\Delta p)c \sim hc/(10^{-12}\,\text{cm}) \simeq 20\ \text{MeV}.$$

This magnitude would then indicate that electrons interact with the protons of a nucleus in accordance with the strength of the nuclear forces that bind

neutrons and protons. Since there was no empirical evidence that indeed electrons couple to nuclei with the strength of the nuclear force, this model of the neutron as an electron—proton composite had to be rejected.

The nonlinear spinor field theory of matter developed in this monograph does give a possible explanation of the neutron matter field in terms of an electron—proton composite, though modified by the incorporation of the particle—antiparticle pairs of the background of any observed matter, in a particular dynamical state. This state was derived in the preceding chapter from an exact solution of the particle—antiparticle coupled spinor field equations, in its (unapproximated) nonlinear form. From the exact solution for the pair in the expression for the conserved properties from Noether's theorem, the true energy of the pair in this state is null, and its true momentum is null — thus corresponding to the actual ground state of the pair, according to this theory of matter.

9.1.1. *Polarization of the Pair Participation in the Neutron State*

It followed from the solutions (7.2.6) for the pair in its ground state that in this case the electromagnetic spinor solution for the pair is null, that is, for this particular (ground) state of the pair,

$$\phi_\alpha^{(e^+)} + \phi_\alpha^{(e^-)} = \phi_\alpha^{(\text{pair})} = 0. \tag{9.1.1}$$

In accordance with the meaning of this solution in this theory, as a field of influence of the pair on other charged matter, this null solution means that the pair, in this particular dynamical state, could have no influence on the proton and electron components of the composite neutron that is postulated. Nevertheless, the pair in the vicinity of the electron—proton would have an influence because the domain of the electron—proton would polarize. Thus consider the electron—proton in the vicinity of a proton—antiproton pair in its ground state. Placing the origin of the Lorentz frame of the pair at the $\bar{p}$-component, the p-component of the pair would be in the displaced Lorentz frame, with the phase in the solution (7.2.6) replaced as follows: (λ is the proton inertial mass, $m_p c^2$, with $c = 1$)

$$2\lambda t \to 2\lambda t - \mathbf{k} \cdot \mathbf{r},$$

where $|\mathbf{k}| = 2\lambda$. In this case, the spinor electromagnetic field $\phi_\alpha^{(p)}$ would not cancel the field $\phi_\alpha^{(\bar{p})}$ in the sum in (9.1.1), which would then represent the polarized electromagnetic spinor field for the pair. The matter field for the composite neutron would then have a form that indicates the binding of the proton and electron spinor fields with the spinor electromagnetic field of the polarized pair, symbolized as $n \equiv e$—pair—p. The negatively charged component of the polarized pair (now *almost* in its ground state) would have the form of $\phi_\alpha^{(e^-)}$ in Equation (7.2.6), replacing the denotation e^- with $\bar{p}$. But the

positively charged component of the proton—antiproton pair would have the spinor electromagnetic field solution,

$$\phi_1^{(\bar{p})} = \frac{4\pi e}{m}\begin{pmatrix} 1 \\ \exp[i(2\lambda t - \mathbf{k}\cdot\mathbf{r})] \end{pmatrix},$$

$$\phi_2^{(\bar{p})} = \frac{4\pi e}{m}\begin{pmatrix} \exp[-i(2\lambda t - \mathbf{k}\cdot\mathbf{r})] \\ \cdot \qquad 1 \end{pmatrix}. \tag{9.1.2}$$

With this model of the neutron, it is then a composite of three spinor fields of the first rank — thus it is a fermion, as required by the empirical facts.

9.1.2. *The Binding Energy of the Neutron*

The idea presented for the model of a neutron is that a constituent proton—antiproton of the 'physical vacuum' (of the type we discussed in Chapter 7, as populated with real particle—antiparticle pairs in their true ground states of null energy and momentum) may draw a proton and an electron together, forming a bound system of proton—pair (polarized)—electron, predominantly in the nuclear force domain of fermis (10^{-13} cm). The binding energy of this 'neutron' would then be due, primarily, to the electromagnetic Coulomb energy that is the order of $2e^2/\langle r\rangle$, where $\langle r\rangle$ is the order of the neutron radius. The latter parameter may be expressed, phenomenologically, as the order of the potential well width that is the range of the influence of the neutron, as a whole, on another nucleon. This is equivalent to replacing the Yukawa potential, $[\exp(-\mu r)]/r$ with a square well with a width of order μ^{-1}. This would be the order of magnitude of the deuteron well width, ~4 fermi. According to this model, then, the neutron binding energy should be the order of

$$B(n) \sim \frac{2e^2}{4 \times 10^{-13}\,\text{cm}} \sim 0.72\,\text{MeV}.$$

The empirical results for the binding energy of the neutron in terms of this composite model, give

$$B(n) = m_n - (m_p + m_e) = [939.5527 - (938.2592 + 0.5110)]$$
$$= 0.7825\,\text{MeV}.$$

[The masses of the components of the pair (proton—antiproton) are not included in this calculation since the state of the pair that leads to (9.1.2) corresponds (according to Noether's theorem) to a null total energy, except for the relatively small polarization energy mentioned above. At this stage of the analysis, the mediating pair for the neutron could still be either an electron—positron pair or a proton—antiproton pair.]

If the neutron, according to this composite model, should decay to its lower energy state, it would be as follows:

$$n \rightarrow p + e + \phi_{\alpha}^{(\text{pair})}, \tag{9.1.3}$$

where the spinor field $\phi_{\alpha}^{(\text{pair})}$ plays the role of the neutrino in the conventional interpretation of neutron decay. While the electromagnetic spinor for the pair in its ground state solves the same field equation as the neutrino field (i.e the Weyl equation), it represents something here quite different than the neutrino — an inertialess elementary particle! Rather, in this case it represents the transferred electromagnetic energy when a neutron decay occurs, from the composite neutron to its surroundings, which is a sea of particle—antiparticle pairs of the background 'physical vacuum'. The determination of the rate of neutron decay, in this model, has yet to be carried out within the context of the present nonlinear spinor matter field theory. The mechanism that is the cause of decay is the extra part of the electromagnetic interaction discussed in the two preceding chapters, the violates the discrete symmetries, such as space-reflection and time reversal. In future studies, an attempt will be made to see if the latter may be identified with the weak interaction, dynamically, so that the total electromagnetic interaction in the generalized (spinor) version of the Maxwell, factorized (irreducible) formalism would naturally unify the ordinary electromagnetic and the weak interaction force fields. If this should turn out to be successful, it would put the weak interaction on the same platform as the electromagnetic interaction as following from the same set of field equations, though becoming important only in the small domain of fermis.

9.1.3. *The Magnetic Moment of the Neutron*

A test of the validity of the presented composite model of the neutron relates to its predictions of the different magnitudes and opposite polarizations of the magnetic moments of the neutron and the proton, which empirically are as follows:

$$\mu_n = -1.9103 \left(\frac{e\hbar}{2m_p c} \right), \qquad \mu_p = 2.7896 \left(\frac{e\hbar}{2m_p c} \right). \tag{9.1.4}$$

The reason for the opposite polarizations of the magnetic moments of the neutron and proton could lie in the presently proposed composite model of the neutron, while the proton is considered as an elementary matter field. Because of the latter, the sign of the proton's magnetic moment should be that of the positive charge of the elementary proton. But the magnetic moment of the neutron, as a composite of bound positively and negatively charged bodies is dominated by the negatively charged $\bar{p}$ in its orbit about

the proton, within the domain of the neutron as a whole, that is the order of 4×10^{-13} cm.

A feature of the neutron that must be addressed is that it is charge-neutral, but at the same time it has a magnetic moment, negatively polarized, with the same order of magnitude as that of the (positively polarized) proton magnetic moment. In the preceding paragraph we saw how the binding energy of the composite neutron (with this model) has the right order of magnitude. It was important that this calculation was independent of whether the spinor electromagnetic field for the mediating pair was due to an e—e^+ pair or a p—$\bar{p}$ pair. However, the magnitude of the negative polarization of the magnetic moment of the neutron, viewed as a composite p—pair—e is sensitive to the inertial mass of the particles of the pair. For if the predominant contribution to μ_n is due to rotation about the 'observed' proton component of the antiproton part of the (slightly polarized) p—$\bar{p}$ pair, that mediates the binding of the electron and proton in the neutron state, then the predicted sign of the magnetic moment μ_n must be negative, and its magnitude would be proportional to e/m_p, in accordance with the empirical evidence.

To sum up, we have seen that the composite model of the neutron,

$$n \equiv p\text{—}\phi_a(\bar{p} - p)\text{—}e \tag{9.1.5}$$

is at least compatible with the neutron's empirical binding energy (as would be measured in its beta-decay), the sign and order of magnitude of the neutron's magnetic moment, and it could explain the products of neutron decay, identifying the (anti)neutrino $\bar{\nu}$ with the transferred electromagnetic energy, in terms of the (factorized) spinor electromagnetic field for the proton pair, $\phi_a(\bar{p} - p)$. The more detailed mathematical investigations regarding the lifetime of the neutron and the magnitudes of its inertial mass and magnetic moment are within the domain of this field theory and will be an important part of its program of study. But one important feature of the generalized electromagnetic interaction, in the factorized, spinor form, is its accommodation of a pair that lacks reflection symmetry (parity), that we know from the empirical facts about weak interactions must be present. Thus, there is the possibility here that the generalized electromagnetic interaction in general relativity that has been demonstrated does indeed unify, *in a dynamical way*, the weak interaction with the electromagnetic and the gravitational interactions.

Another of the matter fields in elementary particle physics that decays by the weak force and is also intimately related to the strong (nuclear) force is that of the pion. In the next section, then, we will examine some of the facts about pions and see if they may be derived within the context of this field theory.

9.2. The Pion

Thus far, the analysis of a generally covariant field theory of elementary matter indicates that the most primitive (irreducible) field variables in general relativity are the two-component spinor functions, mapped in a curved space-time. Thus how does one explain the results of high-energy physics that relate to boson fields with inertia, such as the pion? According to the logical stand of this theory, 'bosons' are not elementary; i.e. there are no boson matter fields that are irreducible. In earlier chapters, it was shown that all experiments that are supposed to entail photons — the massless boson — may be explained without an elementary photon field, such as the explanation of the blackbody radiation curve, thereby eliminating the need for an elementary photon. Next, the massive bosons, such as the charged and neutral pions (and the kaons, to be discussed in the next section) must relate to composites of two-component spinor fields. In the case of the pions, this indicates that, mathematically, an iteration of the spinor field equations (4.4.7) should yield the form of the relation that gives the inertial mass of the pion.

Substituting one of Equations (4.4.7) into the other, we have the following eigenfunction equation

$$[\tilde{\mathscr{I}}(\mathscr{I} + q^\nu\partial_\nu) + \tilde{q}^\mu\partial_\mu(\mathscr{I} + q^\nu\partial_\nu)]\eta = \lambda^2\eta.$$

The equation may then be expressed in the form

$$[I + \Box]\eta = \lambda^2\eta, \tag{9.2.1}$$

where

$$I = \tilde{\mathscr{I}}\mathscr{I} + (\tilde{\mathscr{I}}q^\mu + \tilde{q}^\mu\mathscr{I} + q^\nu\partial_\nu\tilde{q}^\mu)\partial_\mu + \tilde{q}^\mu(\partial_\mu\mathscr{I}) \tag{9.2.1'}$$

is a doubly iterated (squared) energy operator, of the form $\tilde{\xi}\zeta$, and

$$\Box = q^\mu\tilde{q}^\nu\partial_\mu\partial_\nu = \tfrac{1}{2}(q^\mu\tilde{q}^\nu + q^\nu\tilde{q}^\mu)\partial_\mu\partial_\nu \equiv -g^{\mu\nu}\sigma^0\partial_\mu\partial_\nu = -\sigma^0(\partial^\nu\partial_\nu)$$

is the d'Alembertian operator in a curved space-time.

Since Equation (9.2.1) is a reducible form of the original spinor Equation (4.2.1), it may always be rewritten by multiplying the right-hand side by the product of spinor functions that describe a composite (rather than an elementary) scalar or pseudoscalar field solution of the same operator. That is to say, since (9.2.1) is homogeneous in η (in contrast with Equation (4.2.1), which entails both η and its reflected spinor χ), (9.2.1) may also be expressed as

$$[I + \Box]\Phi_\pm = \lambda^2\Phi_\pm, \tag{9.2.2}$$

where

$$\Phi_\pm \propto \eta^\dagger\chi \pm \chi^\dagger\eta \tag{9.2.3}$$

are scalar (+) and pseudoscalar (−) field variables.

Empirically, we know that the pion matter field transforms as a pseudoscalar. Thus, only the solution Φ_- is pertinent to the charged and neutral pion fields. These will be referred to henceforth as $\Phi(\pi)$.

Because of the imposition of the correspondence principle in the axiomatic structure of this field theory, whereby the nonlinear field Equations (4.2.1) must asymptotically approach the linear operator equations in a Hilbert space, as in ordinary quantum mechanics, in the limit in which the energy-momentum transfer between interacting elementary matter becomes sufficiently small, the limiting forms of the solutions of (9.2.2) are the eigenfunctions $|\Phi(\pi)\rangle$, which are the elements of a Hilbert space. In this case, the (squared) mass eigenvalues of the pions are given by the expectation values

$$\lambda^2 = \langle \Phi(\pi)| \, I + \Box \, |\Phi(\pi)\rangle. \tag{9.2.4}$$

9.2.1. *The Mass Ratio of Neutral to Charged Pions* [60]

The interesting experimental fact about pions that is pertinent to the present study of the origin of inertial mass is that the mass of the π^0 is not the same as that of the $\pi^{\pm}$. Of course, it is expected that such a difference should occur, even in the conventional elementary particle theory, where an intrinsic charge or lack of charge would contribute differently to the net rest energies of the respective particles. But the quantitative ratio of these masses has never been derived from quantum field theory, from first principles. Thus, the aim in this section is to see if the present alternative theory could at least give the correct order of magnitude of this ratio.

According to this theory, the charge-neutral, pseudoscalar pion is a composite of the proton and antiproton matter fields, as follows:

$$|\Phi(\pi^0)\rangle \propto |\eta_{\bar{p}}^{\dagger}\chi_p - \chi_{\bar{p}}^{\dagger}\eta_p\rangle. \tag{9.2.5}$$

The charged pion is more complicated. Because the neutron is not an elementary matter field, as we have discussed in the preceding section, one may not view the $\pi^{\pm}$ as a composite of a proton or antiproton with a neutron. Instead, here it must be viewed as a composite of a proton or antiproton with the electromagnetic spinor field solution, of the form (9.1.5). Thus,

$$|\Phi(\pi^{\pm})\rangle \propto |\eta_{\pm}^{\dagger}\phi_a(\bar{p} - p) - \chi_{\pm}^{\dagger}\tilde{\phi}_a(\bar{p} - p)\rangle, \tag{9.2.6}$$

where $(\eta_{\pm}, \chi_{\pm})$ refer to the matter spinor fields for p or for $\bar{p}$. This eigenfunction describes the charged pion as a composite of three nucleon fields. The spinor form of the electromagnetic coupling for the pair then yields a pseudoscalar pion from a three-nucleon system. The field $\tilde{\phi}_a$ in (9.2.6) then represents the time-(or space-)reflected spinor with respect to ϕ_a (just as the matter field spinor χ is the reflection of η).

The pion mass formula (9.2.4) indicates that in the case of the π^0 the expectation value of the (squared) interaction term, I, which is the electromagnetic coupling to its environment of proton—antiproton pairs of the background, should be much smaller than the rest of the expectation value, i.e.

$$\langle \Phi(\pi^0)|\, I\, |\Phi(\pi^0)\rangle \ll \langle \Phi(\pi^0)|\, \Box\, |\Phi(\pi^0)\rangle. \tag{9.2.7}$$

The reason for the inequality, of course, is that the neutral pion contribution from the interaction operator I is a measure of its coupling to particles and antiparticles of the pairs of the background 'physical vacuum'. As a composite of proton and antiproton, the neutral pion appears to a third charged particle (of the physical vacuum) as electrically neutral — until very small impact parameters have been reached, corresponding to very high energy-momentum transfer to the pairs. If we assume that the 'kinetic energy' terms are roughly the same to the neutral and charged pions, then

$$\langle \Phi(\pi^0)|\, \Box\, |\Phi(\pi^0)\rangle \simeq \langle \Phi(\pi^\pm)|\, \Box\, |\Phi(\pi^\pm)\rangle.$$

It then follows from (9.2.4) (with the relation $\lambda = mc/\hbar$) that

$$\left[\frac{m(\pi^0)}{m(\pi^\pm)}\right]^2 = (1+R)^{-1}, \tag{9.2.8}$$

where

$$R = \frac{\langle \Phi(\pi^\pm)|\, I\, |\Phi(\pi^\pm)\rangle}{\langle \Phi(\pi^0)|\, \Box\, |\Phi(\pi^0)\rangle}. \tag{9.2.9}$$

To evaluate the matrix elements in R, we note that I is a doubly iterated operator with the dimension of a squared energy. If we take $I = \bar{\xi}\zeta$, the numerator in R may be expressed in the form

$$\langle \Phi(\pi^\pm)|\, I\, |\Phi(\pi^\pm)\rangle = \langle \Phi(\pi^\pm)|\, \bar{\xi}\, |p^\pm, p^+, p^-\rangle\langle p^\pm, p^+, p^-|\, \zeta\, |\Phi(\pi^\pm)\rangle,$$

where the notation is used above where $p^+ = p$, $p^- = \bar{p}$.

The factor operators $\bar{\xi}$ and ζ act separately to project Hilbert space eigenfunctions of the three free protons $|p^\pm, p^+, p^-\rangle$ onto their binding state $|\Phi(\pi^\pm)\rangle$. To estimate the magnitude of the matrix element, we may use the law of energy conservation. If ω is the energy required to move a free proton or antiproton to a state of binding with the p^+ and p^-, to form the charged pion, $\pi^\pm$ in the pseudoscalar field structure, (9.2.6), then each of the factor matrix elements above is the order of 3ω. This follows from the earlier result of the nonlinear spinor field theory, applied to the particle—antiparticle pair, in which the ground state of zero energy — that is $2m$ below the state at which the particle and antiparticle would be free of each other — followed

from the uniquely *nonlinear* features of this theory. In the case of the pion, the energy ω of each nucleon must similarly be close to the mass of the nucleon and close to the ground state of null energy for the three-nucleon system.

Thus it follows that the (squared) energy operator I has the expectation value

$$\langle \Phi(\pi^{\pm}) |\, I \,| \Phi(\pi^{\pm}) \rangle \simeq 9\omega^2. \tag{9.2.10}$$

To evaluate the denominator in R, we may use the following identity for the d'Alembertian operator:

$$\Box \equiv -\partial^{\nu}\partial_{\nu} = (\nabla - i\partial^0)\cdot(\nabla + i\partial^0).$$

It then follows that

$$\begin{aligned}\langle \Phi(\pi^0) |\, \Box \,| \Phi(\pi^0) \rangle &= \langle \Phi(\pi^0) |\, \nabla - i\partial^0 \,| p^+, p^- \rangle \langle p^+, p^- |\, \nabla + i\partial^0 \,| \Phi(\pi^0) \rangle \\ &= |\langle \Phi(\pi^0) |\, \nabla - i\partial^0 \,| p^+, p^- \rangle|^2. \end{aligned} \tag{9.2.11}$$

Multiplying the eigenfunction (9.2.5) for the neutral pion by $1/\sqrt{2}$ for normalization, it follows that in the rest frame of the free proton and antiproton,

$$\begin{aligned}\langle \Phi(\pi^0) |\, \nabla - i\partial^0 \,| p^+, p^- \rangle &= \left(\frac{1}{\sqrt{2}}\right) \langle \eta_+^{\dagger}\chi_- - \chi_+^{\dagger}\eta_- |\, \nabla \,| \exp[i\mathbf{k}\cdot\mathbf{r}] s_i \rangle \\ &= \left(\frac{1}{\sqrt{2}}\right) \int \exp[-i\mathbf{k}'\cdot\mathbf{r}] k \exp[i\mathbf{k}\cdot\mathbf{r}]\, \mathrm{d}^3x \langle s_f \,|\, s_i \rangle \\ &= \left(\frac{\mathbf{k}}{\sqrt{2}}\right) \sqrt{(2\pi)^3}\, \delta(\mathbf{k} - \mathbf{k}')\delta(s_f - s_i),\end{aligned}$$

where k' is the energy of the bound system, π^0, and s_i, s_f are the respective initial and final spin states of the proton—antiproton system.

Integrating over the free particle states of motion $|k\rangle$, we obtain the following result:

$$\begin{aligned}|\langle \Phi(\pi^0) |\, \nabla - i\partial^0 \,| p^+, p^- \rangle| &\to \left| \int \left(\frac{k}{\sqrt{2}}\right) \sqrt{(2\pi)^3}\, \delta(\mathbf{k} - \mathbf{k}')\, \mathrm{d}^3x \right| \\ &= \left(\frac{k'}{\sqrt{2}}\right) \sqrt{(2\pi)^3}.\end{aligned}$$

Thus we obtain the final result

$$\langle \Phi(\pi^0)| \Box |\Phi(\pi^0)\rangle = |\langle \Phi(\pi^0)| \nabla - i\partial^0 |p^+, p^-\rangle|^2$$
$$= \left(\frac{k'^2}{2}\right)(2\pi)^3 = \left(\frac{\omega^2}{2}\right)(2\pi)^3, \qquad (9.2.12)$$

where, again, ω is the energy required to bind both nucleons to form the π^0 from a free proton and a free antiproton.

Substitution of (9.2.12) and (9.2.10) into (9.2.9) then yields the ratio

$$R = \frac{9\omega^2}{[(2\pi)^3\omega^2/2]} = \frac{18}{(2\pi)^3}.$$

According to Equation (9.2.8), the squared mass ratio is then equal to

$$\left[\frac{m(\pi^0)}{m(\pi^\pm)}\right]^2 = \left[1 + \frac{18}{(2\pi)^3}\right]^{-1}.$$

Using the value $\pi = 3.142$, the predicted ratio of pion masses is then, to this number of significant figures,

$$\left[\frac{m(\pi^0)}{m(\pi^\pm)}\right]_{\text{theor}} = 0.966.$$

The corresponding experimental ratio is:

$$\left[\frac{m(\pi^0)}{m(\pi^\pm)}\right]_{\text{exp}} = \frac{134.9645}{139.5688} = 0.967.$$

The difference between the theoretical and experimental values for this ratio of pion masses is then the order of only 0.1%.

9.2.2. *The Ratio of Neutral to Charged Pion Lifetimes* [61]

The π^0 decay process is conventionally analyzed in high-energy physics in terms of the transition to two gammas,

$$\pi^0 \to 2\gamma \quad (\tau = 0.828 \times 10^{-16}\,\text{s}) \qquad (9.2.13)$$

while the charged pion decay is analyzed in terms of a transition to a muon and a neutrino,

$$\pi^\pm \to \mu^\pm + \nu \quad (\tau = 2.603 \times 10^{-8}\,\text{s}). \qquad (9.2.14)$$

It is interesting to note the very large difference between these lifetimes of the neutral and charged pions, and, in fact, that the lifetime of the π^0 is the

order of the square of that of the $\pi^{\pm}$. The attempt will be made now, within the context of this field theory of matter, in which pions are viewed as particular composites of protons and antiprotons, to determine, phenomenologically, an estimate of the ratio of lifetimes of the neutral to charged pion matter fields.

Within this theory, the primary decay mode of the π^0, as a bound $p\bar{p}$, is to the minimum energy state of this system, which is the state of null energy and null momentum, as determined (for the ground state of any pair) in Chapter 7. That is,

$$\pi^0 \equiv (\bar{p} - p)_- \rightarrow (\bar{p} - p)_+ \equiv 2\gamma,$$

where the subscript $(-)$ denotes the antisymmetric (pseudoscalar) state of the pair, (9.2.5), that is the neutral pion mode of this combination of spinor fields, and the subscript $(+)$ denotes the symmetric (scalar) state of the pair, that was found in Chapter 7 to be equivalent to the dynamics of two correlated photons, and at an energy that is $2m$ below that of the free particle and antiparticle, i.e. this is the ground state of the pair, in this theory, in accordance with an exact solution derived from the coupled, nonlinear spinor equations for the pair. (The analysis in Chapter 7 was for the electron—positron, but recall that the solution was indeed insensitive to their mass; that is, it was true for any spinor particle—antiparticle pair.)

The π^0 decay process is rapid because it depends on the ratio of the electromagnetic coupling of the pair, before their binding occurs, to the total binding when they are in the zero-energy state — a total binding equal to $2M \simeq 2$ GeV, which is the order of energy required to remove the proton from its bound antiproton, when in this ground state.

The charged pion in this theory is a proton or antiproton coupled to the spinor electromagnetic field of a pair. Accordingly, the decay process (9.2.14) is identified here as follows:

$$\pi^{\pm} \equiv p^{\pm} - \phi_a(p^+ - p^-) \rightarrow (e^{\pm})^* + \phi_a(p^+ - p^-) \equiv \mu^{\pm} + \nu,$$

where $(e^{\pm})^*$, called conventionally $\mu^{\pm}$, is the 'heavy electron' predicted here, due to the excitation of pairs of the background 'physical vacuum', that cause an alteration of the geometrical fields (spin affine connection) that relates to the inertial mass of the observed particle (Chapter 4) [57]. Note that the proton itself does not transform into a muon, in this theory. What happens is this: the proton component of the pion transfers energy to a pair of the physical vacuum, causing its excitation and the subsequent alteration of the geometrical field in the vicinity of an electron in the background, thereby causing its mass increase. The original $p^{\pm}$ fuses into the 'vacuum'. $\phi_a(p^+ - p^-)$ is the electromagnetic spinor field for the proton—antiproton pair (in its particular state of null energy—momentum) that as we have seen earlier plays the role of the neutrino because, in this particular state of the proton fields, it solves the Weyl equation, as does the neutrino field (by definition). The

lifetime of the charged pion is expected to be longer than that of the neutral pion because it entails the coupling of a proton to a tightly bound proton–antiproton pair, breaking that bond only when a sufficient amount of electromagnetic energy is supplied.

The transition rates for these processes, in the limit where the coupling is sufficiently weak that the eigenfunction formalism in a Hilbert space is a good approximation for the nonlinear solutions of the field equations, has the usual form [48] as in time-dependent perturbation theory of quantum mechanics:

$$\tau^{-1} = W_{\Omega f} = \left(\frac{2\pi}{\hbar}\right) |\langle f| \, S \, |\Omega\rangle|^2 \rho_f \, \mathrm{s}^{-1}, \tag{9.2.15}$$

where $|\Omega\rangle$ is the state of binding of the nucleons in the pion state, $|f\rangle$ is the state in which the nucleons would be freed, S is the interaction operator that in principle entails the weak force field, and ρ_f is the density of final states.

In Fermi's multiple meson production theory [62], the foregoing transition rate may be expressed phenomenologically as follows:

$$\tau^{-1} = \left(\frac{2\pi}{\hbar}\right) D^2 \left(\frac{\Omega}{V}\right)^{n-1} \rho_f, \tag{9.2.16}$$

where V is an arbitrary volume which entails the process of break-up into n-particle fields, and Ω is the 'interaction volume' that is defined to be the space in which the interaction predominates to bind the composite system, before the break-up happens. The equivalent potential well-depth is D, corresponding to the intrinsic interaction of the initial composite system, which in this study is the pion.

According to an earlier analysis of multiple break-up reactions [63] if the decay consisted in n-parts, with masses m_i and spins I_i, then the density of final states is as follows:

$$\rho_f = \frac{V^{n-1}\Pi(2I_i + 1)(2\pi)^{3(n-1)/2}\Pi(m_i)^{3/2}\, W^{(3n-5)/2}}{(2\pi\hbar)^{3(n-1)}\Gamma[3(n-1)/2](\Sigma m_i)^{3/2}}, \tag{9.2.17}$$

where the products and summation above are taken over $i = 1$ to n and $\Gamma(x)$ is the gamma function, with $\Gamma(n) = (n-1)!$, $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. The total energy of the constituents of the composite system is W. In the problem of the composite pion, in its rest frame, $W = m_\pi$.

Using Fermi's phenomenological expression (9.2.16), it will be assumed that the potential well depth D for the break-up of the neutral pion is approximately the same as it is for the charged pions. Assuming also that the interaction volumes for the neutral and charged pions are about the same, i.e. that $\Omega_{\pi^0} = \Omega_{\pi^\pm} = \Omega_\pi$, it follows from the above expressions that the ratio of

the lifetimes of the neutral to charged pions is

$$\frac{\tau(\pi^0)}{\tau(\pi^\pm)} = \left((m_p m_\pi)^{3/2} \frac{c^3 \Omega_\pi}{48\sqrt{3}} \right) \pi\hbar^3. \tag{9.2.18}$$

The only unknown quantity in this ratio is the magnitude of the interaction volume Ω_π. In this analysis the radius of the interaction volume is the effective range of binding force of the proton—antiproton pair, that in accordance with this theory gives rise to the inertial mass of the pion via the space-time curvature it creates. In the previous analysis that led to the mass of the electron and muon (as derived in *GRM*), the interaction volume of the electron—positron pair was found to be of order 10^{-45} cm^3. Since the latter analysis was based on a physical vacuum of pairs whose ground state is insensitive to the actual mass of the components of the pairs, the interaction volume of the proton pairs should be the same order of magnitude as that of the electron pairs. Thus, taking $\Omega_\pi \cong 10^{-45}$ cm^3, it follows from Equation (9.2.18) that

$$\frac{\tau(\pi^0)}{\tau(\pi^\pm)} \simeq 2.3 \times 10^{-8}.$$

The experimental value for this ratio is

$$\left[\frac{\tau(\pi^0)}{\tau(\pi^\pm)} \right]_{\text{exp}} = \frac{0.83 \times 10^{-16}}{2.6 \times 10^{-8}} = 0.32 \times 10^{-8}.$$

Since the theoretical value for this ratio is within a factor of 10 of the experimental result, and in view of the approximation of the model that was used here, phenomenologically, it is surprising that the theoretical and experimental results are so close. But it does seem apparent that the composite model of the pions that was used in coming to the prediction is indeed compatible with the experimental facts.

The second important 'boson' of elementary particle physics that should have properties that are sensitive to its composite structure in terms of spinor field components is the kaon. One of these properties, that has never been satisfactorily predicted in the conventional quantum theory, will be explored in the next section.

9.3. On the Possible Origin of CP-Violation in Neutral Kaon Decay

The investigations of discrete symmetries in elementary particle physics since the middle of the twentieth century have been of utmost significance and they have led to some very profound experimental and theoretical discoveries. Particularly, C. S. Wu's experimental discovery [64] of parity violation in

weak interactions (signifying a lack of invariance of the weak ineraction with respect to an inversion of the spatial coordinate system), and the subsequent realization that accompanying this P-violation there is also a violation of the weak interaction with respect to charge conjugation ($e \rightarrow -e$), C, while the combined inversion, CP, was conserved [64], led physicists to believe in the conservation of time-reversal symmetry, T, in weak interactions, because of their belief in the CPT theorem.

The subsequent experimental investigations by Cronin [65] and Fitch [66] and their collaborators on neutral kaon decay, indicating a violation of CP-symmetry in the weak interaction, then implied that T may be violated also, based on the validity of the CPT-theorem. However, there is little direct evidence of the violation of time-reversal invariance in weak interactions. And, of course, a problem arose because the weak interaction studied by Wu indicated conservation with respect to the combined CP-symmetry, for the same type of interaction.

It should be pointed out, parenthetically, that many of the conclusions that have been drawn from the foregoing experimental studies are indeed *model-dependent*. Firstly, they often rely on the validity of the CPT-theorem, which in turn is based not only on the assumption of Lorentz covariance of special relativity, which is accepted here only as an asymptotic limit of general (Einstein) covariance, but it is also dependent on several other assumptions that underlie quantum field theory [67]. Some of these assumptions are: gauge covariance, microcausality (relating to the postulated commutation or anticommutation of the boson or fermion 'field operators'), the assumption that the vacuum state in an assumed Hilbert space corresponds to minimal energy (rather than proving this claim from first principles), and the assumption of the principle of linear superposition. In view of the precarious state that still persists in the claim of rigorous validity of quantum field theory, as an elementary theory of matter, because of the logical and mathematical difficulties yet unresolved (as we have discussed in Chapter 2), it must be recognized that some (or possibly all) of the assumptions that underlie the validity of the CPT-theorem (and quantum field theory) are far from established as truths of nature.

Setting aside the CPT-theorem, one may still make statements about the individual discrete symmetries: P, C, T, or any combinations of them two or three at a time, within the context of any relativistically covariant field theory (whether or not it may contain added axioms, such as those of the quantum theory). For it follows from the underlying symmetry group of relativity theory, which expresses the algebraic logic of this theory, that *none* of the discrete symmetries are contained; that is to say, the discrete symmetries, such as P, C, T, CP, etc., are not even defined within the context of this theory, since the underlying assumption implies covariance only within the context of a continuous (analytic) group of transformations — *a Lie group*. One should then not be surprised to discover in experimentation that indeed

there are violations of *any* of the discrete symmetries, such as CP- or T-symmetry, as will be discussed below.

9.3.1. *Neutral Kaon Decay*

An important experimental discovery of the violation of a discrete symmetry was the result of Cronin [65] and Fitch [66], demonstrating a violation of CP symmetry in neutral kaon decay. Before their investigation, it was known that there are two distinct neutral kaons — a short-lived one that decays into two pions:

$$K_S^0 \to \pi^+ + \pi^- \quad (\tau = 0.86 \times 10^{-10}\,\text{s}) \tag{9.3.1}$$

and a long-lived neutral kaon that decays into three pions:

$$K_L^0 \to \pi^+ + \pi^- + \pi^0 \quad (\tau = 5.38 \times 10^{-8}\,\text{s}). \tag{9.3.2}$$

After predicting these two branches of neutral kaon decay, based on an application of the principle of linear superposition and the additional classification category of 'strangeness' of Gell-Mann and Pais [68], Cronin and Fitch discovered that a small fraction of the K_L^0 decays (around 2×10^{-3}) went to two pions rather than three! Thus, the earlier distinguishability of the two and three decays, between K_S^0 and K_L^0, was broken. The combination of the added decay

$$K_L^0 \to \pi^+ + \pi^- \tag{9.3.3}$$

and the more frequent three-pion decay (9.3.2) from the K_L^0 was then direct evidence for a violation of CP-symmetry in the mechanism for the decay of this particle (the weak interaction). This, result was in contradiction with the earlier observation that the (weak) beta interaction seemed to conserve CP, while violating C and P separately. To this date, there has been no satisfactory explanation provided from quantum field theory for this violation, nor for the quantitative magnitude of the fraction of CP-violating decays (of order 2×10^{-3})

The main purpose of this section of the monograph is to suggest that the approach to elementary particle physics from general relativity, developed here — particularly its prediction of a generalized form of the electromagnetic interaction that could incorporate the weak interaction in a unified dynamical scheme — does predict the CP-breaking interaction, and an argument will be presented that leads to the correct order of magnitude of this breaking.

9.3.2. *The Irreducible Spinor Matter Field Equations and CP-Violation*

The start of this investigation is the recognition that the irreducible representations of the symmetry group of the theory of relativity (the Einstein

group of general relativity or the Poincaré group of special relativity) obey the algebra of quaternions, and have their structure built up from two-component spinor variables. (I have spelled out the details of these representations in my earlier book [1], Ch. 3).

Since the two-component spinor formalism is only covariant with respect to continuous transformations [*GRM*, Ch. 3], all discrete transformations are undefined within the context of this theory of matter. Consider the example of the discrete transformation CP, applied to the two-component spinor matter field Equations (4.2.1) in special relativity. It follows from the covariance of these matter field equations that under the transformations of the Poincaré group, the covariance is prescribed by:

$$x \to x' = \alpha x \Rightarrow \begin{cases} \eta(x) \to \eta'(x') = S\eta(x) \\ \chi(x) \to \chi'(x') = S^{\dagger -1}\chi(x) \end{cases} \tag{9.3.4}$$

where the spinor representations $\{S\}$ of the group relate to the representation $\{\alpha\}$ of the vector transformations according to:

$$S^{\dagger}\sigma^{\mu}S = \alpha^{\mu}_{\nu}\sigma^{\nu}, \tag{9.3.5}$$

where σ^{μ} are the (complex) quaternion basis elements for the irreducible representations of the Poincaré group — they are the unit matrix σ^0 and the three Pauli matrices, σ^k, and the coefficients α are the elements of the transformation matrices applied to the space-time coordinates, as well as applied to the charge parameter e.

The discrete transformation CP applied to the matter field spinor is defined as follows:

$$(\mathrm{CP})\eta(\mathbf{r}, e) = \eta(-\mathbf{r}, -e) \tag{9.3.6a}$$

(suppressing the time variable in the argument of η, since it is unaffected by CP). Now if CP should exist in the covariance group of the matter field Equation (4.2.1), then according to the second of Equations (9.3.4), it must follow that

$$\chi(-\mathbf{r}, -e) = (\mathrm{CP})^{\dagger -1}\chi(\mathbf{r}, e) \tag{9.3.6b}$$

Applying the discrete transformation to the space coordinates and the charge parameter, $\mathbf{r} \to -\mathbf{r}$, $e \to -e$ in the matter field Equations (4.2.1), and using (9.3.6a,b), we have:

$$\begin{aligned} &[\sigma^0\partial_0 - \boldsymbol{\sigma}\cdot\boldsymbol{\nabla} - ie\mathscr{A}(\mathbf{r})]\eta(\mathbf{r}, e) \\ &\quad = -\lambda\chi(\mathbf{r}, e) \to (\mathrm{CP})^{\dagger}[\sigma^0\partial_0 + \boldsymbol{\sigma}\cdot\boldsymbol{\nabla} + ie\mathscr{A}(-\mathbf{r})](\mathrm{CP})\eta(\mathbf{r}, e) \\ &\quad = -\lambda\chi(\mathbf{r}, e). \end{aligned} \tag{9.3.7}$$

The relativistic covariance of this equation, maintaining it should be the same as the untransformed equation, then requires that the following three

relations must be simultaneously valid:

$$(CP)^{\dagger}\sigma^{0}(CP) = \sigma^{0} \Rightarrow (CP)^{\dagger} = (CP)^{-1} \tag{9.3.8a}$$

$$(CP)^{-1}\sigma^{k}(CP) = -\sigma^{k} \Rightarrow \sigma^{k}(CP) + (CP)\sigma^{k} = 0 \quad (k = 1, 2, 3) \tag{9.3.8b}$$

$$(CP)^{-1}\mathcal{J}(-\mathbf{r})(CP) = -\mathcal{J}(\mathbf{r}) \Rightarrow \mathcal{J}(-\mathbf{r})(CP) + (CP)\mathcal{J}(\mathbf{r}) = 0. \tag{9.3.8c}$$

Clearly, whatever the explicit form of the interaction functional $\mathcal{J}$ is, Equation (9.3.8b) indicates that within this formalism, the operator (CP) does not exist, because there is no two-dimensional matrix that *simultaneously* anticommutes with all three Pauli matrices — in accordance with the fundamental structure of the quaternion algebra.

The latter algebraic result might be compared with the algebra of the four-component Dirac *bispinor* formalism, which, by construction, is built up from the two-component spinor formalism with the added constraint that it is to be covariant with respect to the discrete transformations, as well as the continuous transformations, provided that one does not add, ad hoc, reflection-nonsymmetric parts to its interaction functional. In this formal structure, the four Dirac matrices anticommute, $\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 0$ $(\mu \neq \nu = 0, 1, 2, 3)$ as well as anticommuting with a fifth four-dimensional matrix, which is the product of the other four, i.e. $\gamma^{5}\gamma^{\mu} + \gamma^{\mu}\gamma^{5} = 0$, where $\gamma^{5} = \gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}$. Within this four-component Dirac bispinor formalism the discrete transformations, C, P, CP, T, etc., do exist, as long as there are no discrete transformation violating parts in the interaction $\mathcal{J}$.

Thus, in the case of the most general expression of the matter field equations in special relativity, i.e. Equation (4.2.1), all of the discrete transformation elements, including (CP), do not exist. The other such elements (P, T, etc.) all lead to similarly inconsistent relations. In the case of violation of CP, the functional $\mathcal{J}$ may then generally have terms in it that are even and those that are odd under CP. Of course it may happen in nature that the numerical values of the effects induced by some of these terms (say the odd ones) are very small compared with the effects of the other (even) terms. In the latter cases, one may then have the illusion that there is an intrinsic reflection symmetry in the interaction under study (in addition to the covariance with respect to the continuous transformations). In the latter cases, it may be more practical to express the theory in terms of the Dirac four-component bispinor formalism — as it has been done predominantly in physics until the discovery of parity violation in weak interactions. But the theory of relativity, in itself, does not require this. Thus, if one should postulate that the theory of relativity is the sole basis for the underlying covariance of the laws of matter, experimentation should seek out extra predicted components of a general interaction that would be odd with respect to *any* of the reflection transformations — C, P, T, CP, CT, PT or CPT.

9.3.3. *The Generalized Electromagnetic Interaction*

In view of our conclusion that all relativistically covariant field theories of matter must have *irreducible* forms that have the spinor-quaternion structure, the generalized electromagnetic interaction, that we first discussed in Chapter 5, resulting from a factorization of the Maxwell formalism to a spinor form, entails two parts — one that is even and the other that is odd with respect to discrete transformations. One of these parts comes from the portion of the Lagrangian density that yields the spinor field equations of electromagnetism, which is the global extension of (5.3.4). This Lagrangian has the form

$$\mathscr{L}_M = ig_M \sum_{\alpha=1} (-1)^\alpha \phi_\alpha^\dagger (q^\mu \phi_{\alpha;\mu} - 2\mathrm{T}_\alpha) + \text{h.c.} \tag{9.3.9}$$

(where h.c. stands for the hermitian conjugate of the preceding terms).

Varying $\mathscr{L}_M$ with respect to the matter field variables (that are implicit in the source field T_α) then yields a contribution to the interaction functional in the matter field equations that is not symmetric under CP.

As we have discussed in earlier chapters, the principle of correspondence, which is one of the underlying axioms of this covariant field theory of inertia, requires the inclusion in the total Lagrangian of a term associated with the usual form of the current–current coupling (that gives rise to the Lorentz force). The latter contribution is even with respect to the discrete transformations, while the former is odd, so that in the jth matter field equation (of form (4.2.1), we have the total interaction functional as the sum of an even (+) and an odd (−) term:

$$\mathscr{I}^{(j)} = \mathscr{I}_+^{(j)} + \mathscr{I}_-^{(j)}. \tag{9.3.10}$$

The even term has the form

$$\mathscr{I}_+^{(j)} = 2e^2 \sigma^\mu \sum_{i \neq j} \int \eta^{(i)\dagger} \sigma_\mu \eta^{(i)} S(x - x')\, \mathrm{d}^4 x', \tag{9.3.11}$$

where $S(x - x')$ is the Green's function of d'Alembert's equation, corresponding in this theory, *uniquely*, to the average of the advanced and retarded terms: (as we have discussed in Chapter 5, leading to Equation (5.4.8))

$$S(x - x') = [\tfrac{1}{2}|\mathbf{r} - \mathbf{r}'|^{-1}][\delta(x^0 - x^{0\prime} - |\mathbf{r} - \mathbf{r}'|) + \delta(x^0 - x^{0\prime} + |\mathbf{r} - \mathbf{r}'|)].$$

The factor 2 appears in the interaction (9.3.11) because there are equal contributions from the spinor field currents that are the conjugates,

$$\eta^{(i)\dagger} \sigma_\mu \eta^{(i)} \quad \text{and} \quad \chi^{(i)\dagger} \tilde{\sigma}_\mu \chi^{(i)}$$

in the total current that couples to the matter field $\eta^{(j)}$ (the tilda over $\tilde{\sigma}_\mu$ denotes the conjugated quaternion basis elements, with $\sigma^0 \to -\sigma^0$ and $\sigma^k \to \sigma^k$).

The interaction (9.3.11) is the same as the conventional one that yields the Lorentz force density (and the Coulomb interaction) except for (1) the rejection of the self-energy terms ($i = j$) and (2) the unique expression above for the Green's function $S(x - x')$, as an average of retarded and advanced terms — because of the interpretation here of the electromagnetic field equations with the concept of (delayed)-action-at-a-distance, as we have discussed in Chapters 2 and 5, and in *GRM*. Recall that there is a conceptual difference from the Wheeler—Feynman action-at-a-distance approach [69] in that theirs entails all coupled (singular) particles, in symmetrical interaction, and no fields, while this theory entails, *conceptually*, all coupled (non-singular) fields, and no singular particles; the former 'particle approach' must use a $4n$-dimensional space-time for n-particles, while the field theory of this book must use n-coupled fields, all mapped in a single four-dimensional space-time.

The second part of the interaction functional in (9.3.10) comes from the variation of the Lagrangian (9.3.9) with respect to the matter field variables $\eta^{(j)}$, yielding the following form in the special relativistic limit:

$$\mathscr{I}_{-}^{(j)} = 32\pi g_M e \sum_{i \neq j} \{\phi_1^{(i)}(1)^*(-\sigma^0 + \sigma^3) + \phi_1^{(i)}(2)^*(\sigma^1 + i\sigma^2) - \\ - \phi_2^{(i)}(1)^*(-\sigma^1 + i\sigma^2) - \phi_2^{(i)}(2)(\sigma^0 + \sigma^3)\} + \text{h.c.} \qquad (9.3.12)$$

where the summation over i is taken over all n matter fields, except (j). This part of the interaction violates CP and will be exploited as the mechanism that underlies the experimental observation of the violation of CP in K_L^0 decay.

9.3.4. *CP-Violation in Kaon Decay*

Since there is a part of the generalized electromagnetic interaction that is not reflection symmetric, given in (9.3.12), then if there is an asymptotic state that may be associated with a unique mass of the K^0 meson, there must be two such components of K^0 — one that is scalar and the other pseudoscalar. As in the usual elementary particle description, these two states should be distinguishable in terms of their distinct lifetimes — the shorter-lived one, which will be due to the scalar particle state $|K_S^0\rangle$ decay, and the longer-lived one, which is due to the pseudoscalar particle state $|K_L^0\rangle$ decay. It is a subject of future investigations to explain within the context of this field theory of inertia why the scalar kaon decays faster than the pseudoscalar kaon, *and not vice versa*, and why the order of magnitude of the ratio $\tau(K_S^0)/\tau(K_L^0)$ is the order of 1/100.

The implication of the theory discussed here is that of the total interaction (9.3.10), there is a scalar part, $\mathscr{I}_+$, that induces no change in parity for the transition $|K_L^0\rangle \to |3\pi\rangle$, and there is the pseudoscalar part, $\mathscr{I}_-$, that induces

a change in partity in the transition $|K_L^0\rangle \rightarrow |2\pi\rangle$, given that the $|K_L^0\rangle$ state is intrinsically pseudoscalar.

The comparison of the measured ratio

$$R = \frac{\tau^{-1}(K_L^0 \rightarrow \pi^+ + \pi^-)}{\tau^{-1}(K_L^0 \rightarrow \pi^+ + \pi^- + \pi^0)} \approx (2.0 \pm 0.4) \times 10^{-3} \tag{9.3.13}$$

with the prediction of this theory follows only after defining the matter states that are signified by the kaon and pion elementary particle labels. Before this is done, however, it is seen that indeed $R \neq 0$ because the nonzero value of the numerator is provided in principle by the mechanism of the pseudoscalar part of the total interaction, $\mathscr{I}_-$, according to this theory, as the mechanism for the denominator is provided by the scalar part, $\mathscr{I}_+$.

Similar to the composite structure of the pion states, discussed in the preceding section, the long-lived, pseudoscalar kaon is represented as a composite of the proton and antiproton states. The pions, that are themselves composites, also form a composite to structure the higher composite kaon state,

$$|K_L^0\rangle \equiv |\pi^+\pi^-\pi^0\rangle. \tag{9.3.14}$$

The branch of the K_L^0 state that *appears to* decay to two pions (π^+, π^-), corresponds here to a decay of the π^0 component in K_L^0 to a proton–antiproton bound pair of minimal energy-momentum (i.e. where the components of its momentum-energy in this state are zero, as we discussed previously in this chapter and in the preceding chapter, following from an exact solution of the nonlinear, coupled spinor field equations for the pair). That is, the antisymmetric proton pair, corresponding to the neutral pion with the composite state shown in (9.2.5), transforms to the symmetric state of the pair (which is its true ground state),

$$\begin{aligned}\langle\Phi(\pi^0)\rangle &= |\eta_p^\dagger\chi_p - \chi_p^\dagger\eta_p\rangle \equiv (\bar{p} - p)_a^* \rightarrow (\bar{p} - p)_s \\ &\equiv |\eta_p^\dagger\chi_p + \chi_p^\dagger\eta_p\rangle\end{aligned} \tag{9.3.15}$$

thereby transferring the excess energy-momentum to the other pairs of the 'physical vacuum' background. The subscripts above, '*a*' and '*s*', refer respectively to the antisymmetric and symmetric states of the proton–antiproton pairs.

In the view of this theory, then, the number of composite particle states that make up the original kaon, K_L^0, does not change when it transforms to two pions or to three pions — in the former case, it is rather that the neutral pion component that was there in the initial (composite) kaon becomes invisible in the final state, i.e. it no longer transfers energy-momentum to its surroundings when it is in this state of minimal (zero) energy and momentum. Symbolically, then, there are two branches of the K_L^0 decay, explaining the violation of CP-symmetry as appeared in the Cronin–Fitch experiment, as

follows when we break down the pions into their proton components, in accordance with the preceding section:

$$|K_L^0\rangle \equiv |(p\phi_a(p-\bar{p}))_a(\bar{p}\phi_a(p-\bar{p}))_a(p-\bar{p})_a^*\rangle \rightarrow$$

$$\rightarrow \begin{cases} |p\phi_a(p-\bar{p})\rangle_a + |\bar{p}\phi_a(p-\bar{p})\rangle_a + |(p-\bar{p})\rangle_a \equiv \pi^+ + \pi^- + \pi^0 \\ |p\phi_a(p-\bar{p})\rangle_a + |\bar{p}\phi_a(p-\bar{p})\rangle_a + |(p-\bar{p})\rangle_s \equiv \pi^+ + \pi^- + (p-\bar{p})_s \end{cases} \tag{9.3.16}$$

which are the two branches of neutral, long-lived kaon decay, identified here with the observations of Cronin and Fitch.

The short-lived neutral kaon, which is the scalar part of the scalar–pseudoscalar pair of fields predicted from the lack of reflection symmetry in the theory, does decay to a final state of two pions at the outset,

$$|K_S^0\rangle \equiv |(p\phi_a(p-\bar{p})_a(\bar{p}\phi_a(p-\bar{p}))\rangle_a \rightarrow$$
$$\rightarrow |p\phi_a(p-\bar{p})\rangle_a + |\bar{p}\phi_a(p-\bar{p})\rangle_a \equiv \pi^+ + \pi^-. \tag{9.3.17}$$

Note that the plus signs between the states on the right-hand sides of Equations (9.3.16) and (9.3.17) symbolize (asymptotically) separated fields that appear as pions — they do not symbolize correlated states. The subscripts 'a' and 's' above denote antisymmetric and symmetric states, respectively, regarding the elements of the composite states referred to.

9.3.5. *Estimate of The Magnitude of CP-Violation in K_L^0 Decay*

Noting that the number of asymptotically free states, within the dynamical scheme proposed, is the same in both branches in Equation (9.3.16), time-dependent perturbation theory applied to the linear limit of the matter field equations yields the ratio of decay rates for the two branches to be the ratio of the absolute values of the squares of the matrix elements for the respective parts of the generalized electromagnetic interaction:

$$R \simeq \frac{|\langle 2\pi|\, \mathscr{I}_-\, |K_L^0\rangle|^2}{\langle 3\pi|\, \mathscr{I}_+\, |K_L^0\rangle|^2}, \tag{9.3.18}$$

where the final states in this ratio, $|2\pi\rangle$ and $|3\pi\rangle$ denote the states indicated in the branches in Equation (9.3.16).

While a detailed evaluation of the theoretical value of this ratio, within the context of this theory, has not yet been carried out, it is possible at this stage to estimate its order of magnitude from the dependence of the parts of the interactions $\mathscr{I}_+$ and $\mathscr{I}_-$ on the r-coordinate. The electromagnetic spinor field ϕ_a is an electromagnetic field intensity, depending on e/r^2, in the proper frame of the kaon matter field. In accordance with the expressions for

this interaction, given in Equations (9.3.11) and (9.3.12), it follows that $\mathscr{I}_- \sim 32\pi g_M e^2 \langle 1/r^2 \rangle$, while $\mathscr{I}_+ \sim 2e^2 \langle 1/r \rangle$. The ratio above should then be the order of

$$R \sim \left| 16\pi g_M \frac{\langle 1/r^2 \rangle}{\langle 1/r \rangle} \right|^2$$

The value of g_M was already determined (in Chapter 8), with the result

$$g_M = (2.087 \pm 0.001) \times 10^{-14}\ \text{cm}.$$

The values of the expectation values $\langle 1/r^2 \rangle$ and $\langle 1/r \rangle$ in R depend, of course, on the details of the (asymptotic) form of the matter field variables for the domain of the neutral, long-lived kaon. Analogous to Yukawa's reasoning in his original prediction of the pion field, as mediating the nucleon—nucleon interaction, the extension of the domain of the kaon will be assumed to be the order of magnitude of the *electromagnetic force* in this bound system of protons and antiprotons as shown in Equation (9.3.16). Thus one may estimate (as an intuitive guess at this stage) that this is the order of magnitude of the electron Compton wavelength. The reasoning is as follows: The domain of the kaon (its 'form factor region') must be greater than the proton or pion Compton wavelengths, as these (much smaller) domains would entail the respective internal structures of the nucleon and the pion fields in themselves. In the other extreme, the domain of the kaon could not be as great as the (infinite) range that is implicit in photon exchange. The selection of the electron Compton wavelength, $\lambda_e = \hbar/m_e$, then assumes that when it is studied in detail, the components of the composite that we call K_L^0 will, in its fundamental description, entail transformations to electron—positron pairs. This is because the kaon is a composite of pion states which, in turn, are composites of proton—antiproton states — which eventually transform $e^{\pm}$ to $\mu^{\pm}$, and at the end of the chain, the muons decay to electron pairs. Thus the *maximum domain* in which the many-body interactions of the composite of kaon take place may be assumed to be the order of magnitude of the electron Compton wavelength — i.e. assuming there is a Yukawa—Debye type cut-off for the kaon at λ_e, as we have discussed in Section 8.7 in the case of electron—proton in a background of pairs.

What we are saying here is that there is a screening action of the 'plasma field' of the composite protons and antiprotons that make up the neutral kaon, in acting on any particular charged component in the domain.

With the Yukawa—Debye type factor present, the ratio of the reciprocal radii in Equation (9.3.19) should then be of order

$$\frac{\langle 1/r^2 \rangle}{\langle 1/r \rangle} \simeq \frac{\int_0^\infty [(\exp[-r/\lambda_e])/r^2] r^2\, dr}{\int_0^\infty [(\exp[-r/\lambda_e])/r] r^2\, dr} = \frac{1}{\lambda_e}. \tag{9.3.20}$$

With this method of estimation, it follows that the ratio in (9.3.19) is

$$R \simeq \left| 16\pi \left(\frac{g_M}{\lambda_e} \right) \right|^2 = \left| \frac{16\pi(2.09 \times 10^{-14})}{3.86 \times 10^{-11}} \right|^2 = 0.74 \times 10^{-3}.$$

Thus we see that the theoretical value of R — measuring the amount of CP-violation in K_L^0 decay, estimated in this way — is the same order of magnitude as the experimental value, $\sim 2 \times 10^{-3}$, discovered by Cronin and Fitch. This result, while not rigorous, does encourage further investigation of this problem and, generally, other features of elementary particle physics from the basis of general relativity.

9.4. On Time Reversal Noninvariance in Nuclear Forces — a Magnetic Resonance Experimental Test

In the light of what was said in the preceding section regarding a violation of the discrete symmetries, according to the underlying group of relativity theory, and in view of some recent experimental discoveries that reveal a possibility that indeed time reversal symmetry may be violated in the nuclear force [70], it is of interest to examine the violation of T in nuclear forces in the context of the theory of this monograph, to see if there may be some further experimental implications.

Consider an atomic system whose nucleus has a large quadrupole moment but whose ground state of bound atomic electrons is one of zero angular momentum (i.e. an S-state). Combining the electronic and the nuclear angular momenta, the total angular momentum of this atom must then be the angular momentum of its nucleus, which we may call I. If one should embed such atoms at random lattice sites of a noncubic, nonmagnetic crystal, the resulting configuration would be paramagnetic because of the nonzero magnetic moments of the embedded nuclei, which would align themselves parallel to the C-axis of the crystal.

This is similar to the arrangement used by Wu *et al.* [64] who embedded radioactive ^{60}Co nuclei to observe an asymmetry in their emitted beta rays, relative to the plane that is perpendicular to the C-axis of the crystal. In such an arrangement, the method of nuclear alignment was proposed earlier, independently by Gorter [71] and Rose [72] — it is called 'the Rose—Gorter effect'. An early experiment on nuclear alignment in this way was carried out by Ambler *et al.* [73]. It was shown in these researches that, for typical cases, to depopulate the higher nuclear magnetic states to see the effect of the net magnetization of the crystal, it would require temperatures in the range from 0.1 to 0.01 K.

The nuclear alignment entails a small distortion of the embedded atom's electronic configuration, yielding a nonzero electronic angular momentum,

though close to zero, called 'effective spin', $\mathbf{S}_{\text{eff}}$. The subsequent nuclear alignment is due to the hyperfine coupling, that is proportional to $\mathbf{S}_{\text{eff}} \cdot \mathbf{I}$. With the magnetic nuclei aligned in this way in the crystal, if its net spin is a half integer value (i.e. if it corresponds to an odd number of fermion particles), the magnetic states must still maintain a two-fold degeneracy (when maximum degeneracy would be lifted), corresponding to the states with $I_z = \pm I, \pm(I-1), \ldots, \ldots \pm\frac{1}{2}$, where z denotes the axis of quantization, which in this case is the direction of the C-axis of the crystal. Thus, if the nuclear spin would be $I = \frac{5}{2}$, the maximum lifting of degeneracy in this state must leave the magnetic substates $|I_z\rangle = |\pm\frac{5}{2}\rangle, |\pm\frac{3}{2}\rangle, |\pm\frac{1}{2}\rangle$ energetically distinct. Their separation in energy would be caused by the electromagnetic coupling of the host crystal to the effective spin of the electronic cloud surrounding the magnetic nuclei, which in turn couple magnetically to the nuclei. For example, if the latter coupling is predominantly quadrupolar, its matrix elements would be proportional to the 'effective spin Hamiltonian', $3I_z^2 - I(I+1)$, which, of course, would be numerically different for each value of $|I_z|$.

The main point here is that the basic reason for the maintainance of the double degeneracy of the states with $+I_z$ and $-I_z$ is that the intrinsic forces (in this case, nuclear) maintain time reversal invariance. This general implication for a fermion system is the 'Kramers theorem'. In addition to Kramers' original proof of this theorem [74], I have demonstrated a proof based on considerations of group theory and a use of the Wigner—Eckart theorem, [75].

Thus we see that if, indeed, there would be a small component in the nuclear force field that violates T-invariance — a violation we argued in the preceding section to be perfectly valid from the view of general relativity as an underlying theory of elementary matter — one should then observe a lifting of Kramers' degeneracy in the doubly degenerate magnetic states of aligned nuclei. Of course, any extraneous magnetic field would also lift this degeneracy, without T-violation. To investigate T-violation in such an experiment would then require a careful elimination of all extraneous magnetic fields from external sources as well as from the constituency of the crystal itself.

If there should be a component of the nuclear force field that violates T-invariance, it might generally be expressed in terms of an operator that is an odd function of I_z. In first order, one may then express the T-violating interaction Hamiltonian as

$$H' = AI_z, \tag{9.4.1}$$

where A entails a (time-reversible) operator associated with the nuclear field, not dependent on any fields external to the nucleus, and it is independent of I_z. This form for H' is indicated by the statement of the Wigner—Eckart theorem [76].

H' is clearly odd under time-reversal and it does predict a lifting of Kramers' degeneracy in a paramagnetic crystal, as we have discussed above.

If one should consider a nucleus with spin angular momentum quantum number, $I = \frac{5}{2}$, then with Equation (9.4.1), the magnitude of the splitting of the Kramers' degenerate states due to the presence of H' would be as follows:

$$\delta_5 = \langle \tfrac{5}{2} | H' | \tfrac{5}{2} \rangle - \langle -\tfrac{5}{2} | H' | -\tfrac{5}{2} \rangle = 5\langle A \rangle$$

$$\delta_3 = \langle \tfrac{3}{2} | H' | \tfrac{3}{2} \rangle - \langle -\tfrac{3}{2} | H' | -\tfrac{3}{2} \rangle = 3\langle A \rangle$$

$$\delta_1 = \langle \tfrac{1}{2} | H' | \tfrac{1}{2} \rangle - \langle -\tfrac{1}{2} | H' | -\tfrac{1}{2} \rangle = \langle A \rangle.$$

In a magnetic resonance experiment, where the selection rule is $|\Delta I_z| = 1$, it then follows that the energies of absorbed resonant frequencies should be as follows:

$$\Delta_5 = E_{5/2} - E_{3/2} = (E_{5/2} - E_{-3/2}) + (E_{-3/2} - E_{3/2})$$
$$= (E_{5/2} - E_{-3/2}) - \delta_3$$

$$\Delta_{-5} = E_{-5/2} - E_{-3/2} = (E_{5/2} - E_{-3/2}) - \delta_5$$

etc., so that

$$\Delta_5 - \Delta_{-5} = \delta_5 - \delta_3 = 2\langle A \rangle \neq 0.$$

If it should then be discovered in experimentation that $\Delta_5 \neq \Delta_{-5}$, this would indicate there is a lifting of Kramers' degeneracy and thus a violation of time-reversal symmetry.

9.4.1. *A Possible Source of T-Violation in Nuclear Forces*

Thus far, in this discussion, the T-violating part of the nuclear force field was introduced phenomenologically. Then what is a possible source of this contribution? Perhaps a hint to the answer lies in the generalization of the electromagnetic interaction at small distances that we discussed in the last chapter (Section 8.7). It was seen there that in the range of nucleon dimensions at high momentum transfer, the extra part of the electromagnetic interaction $\mathscr{I}_2$ that comes from the factorization of the Maxwell formalism, automatically violates both parity conservation and time reversal conservation. The important point here is that the latter generalization should be important at small distances, but insignificant at large distances (sufficiently small energy-momentum transfer between interaction matter), where the ordinary electromagnetic coupling, $\mathscr{I}_1$ should dominate.

We saw that in the range of nucleon dimensions $\mathscr{I}_2$ becomes important because of the following r-dependence

$$\mathscr{I}_2 \propto \frac{\exp(-\mu r - b/r)}{r}, \tag{9.4.2}$$

where μ is the order of the inverse Compton wavelength of the pion ($\sim 10^{-12}$ cm)$^{-1}$ and b is the order of the new universal constant length of this theory, $g_M \sim 10^{-14}$ cm. The r-dependence of this coupling term (in the approximation that makes such a view of the covariant coupling expressible in terms of an r-dependent 'potential') then appears as a superposition of a Yukawa nuclear potential $[\exp(-\mu r)]/r$ and a factor, $\exp(-b/r)$ that weakens the potential as r decreases (momentum transfer correspondingly increasing) from orders of magnitude of 10^{-14} cm toward zero (i.e. toward infinite momentum transfer).

Summing up, it is speculated here that if magnetic resonance experimentation should indeed reveal that T-invariance is violated in nuclear forces, so that $\langle A \rangle \neq 0$, further investigations may reveal the direct relation between the phenomenological parameter $\langle A \rangle$ and the expectation value of the added part of the generalized electromagnetic interaction $\mathscr{L}_2$ in the domain of nuclear forces. The nonzero value of $\langle A \rangle$ would then indicate a further step toward a fundamental goal of the covariant field theory of inertia advocated in this monograph — which is the goal of dynamical unification of the electromagnetic, weak and nuclear force manifestations, together with that of its gravitational manifestation, in a unified field theory in general relativity that incorporates the inertial feature of matter.

In the next (final) section of this chapter, the goal of unification of the fundamental forces in the elementary particle domain will be discussed further. The formal expression in this theory for the composite field structures that form the *boson* fields transferred in interaction between fermion matter components will be explored. This idea will be applied to recent high-energy physics experimentation on p—$\bar{p}$ scattering that has revealed the possibility of the formation of an intermediate boson, called the $W^{\pm}$-particle. This particle, and its mass, have been predicted within the context of quantum field theory by the Glashow—Salam—Weinberg theory of electro-weak interaction [77]. It will be demonstrated how the same result *could* follow from the present theory of elementary matter from general relativity. A difference appears, however, in this theory's prediction of a whole spectrum of such intermediate bosons (and, of course, that in this theory they are not elementary, but rather are composites of the elementary spinor matter fields, which are the protons, electrons and their antiparticles).

9.5. Proton—Antiproton Collisions and the $W^{\pm}$-Particle from General Relativity

The very interesting and important recent result in elementary particle physics has been the observation that revealed that at sufficiently high energy, a $p\bar{p}$ collision yields single electrons (and single positrons) and neutrinos, with the signature of a two-body decay that corresponds to an intermediate particle of mass that is of order 80 GeV. This value

comes close to the value that was predicted by the GSW model of the electroweak interaction — which is framed within the formal structure of the quantum field approach, and was thus hailed as a success for the quantum view.

The actual (model-independent) observations from the $p\bar{p}$ collision experiment [UA1 Collaboration, CERN, *Phys. Lett. B* **122**, 103 (1983); UA2 Collaboration, *Phys. Lett. B* **122**, 476 (1983)], were as follows:

$$p + \bar{p} \rightarrow (\text{Intermediate energy} \sim 80\ \text{GeV}) \rightarrow e^{\pm} + \nu, \tag{9.5.1}$$

where $e^{\pm}$ is the isolated electron (or positron) reaction product in the transverse plane and ν stands for the *missing* transverse energy from the original $p\bar{p}$ energy transfer; thus it is identified with a neutrino product of the reaction, in accordance with the rules of conservation of energy and angular momentum.

According to the GSW model of the electroweak interaction (which, of course, is expressed in the context of quantum field theory), the $p\bar{p}$ interaction in this experiment should entail the formation of massive intermediate vector bosons to mediate the weak interaction between fermions — these are the $W^{\pm}$-particles [77].

The $W^{\pm}$-particles subsequently decay to electrons and neutrinos, distributing their (predicted) mass energy of 80 GeV equally between them. Their theory anticipates the intermediate vector bosons to form from the interaction of quark and antiquark constituents of the colliding proton and antiproton, so that the right-hand side of (9.5.1) comes from the fundamental reaction (in their theory) as follows:

$$\begin{array}{l} \text{Quark} + \text{Antiquark} \rightarrow W^{\pm} + \text{Something else} \\ \qquad\qquad\qquad\qquad\quad \searrow \\ \qquad\qquad\qquad\qquad\qquad e^{\pm} + \nu. \end{array} \tag{9.5.2}$$

The purpose of the following discussion is to indicate that perhaps other theoretical models that do not entail the quark concept or the renormalization methods of quantum field theory (that are necessary there to come to the numerical result for the mass of the intermediate vector boson) [78], could lead to predictions of the same observations.

To demonstrate this, at least qualitatively, let us now re-analyze the electromagnetic field Equations (5.3.4), and substitute the electromagnetic spinor solution ϕ_a for the matter field solution η of Equation (4.4.7), also substituting the reflected electromagnetic spinor solution, $\tilde{\phi}_a \equiv \varepsilon\phi_a^*$ for the reflected matter field spinor variable, $\chi \equiv \varepsilon\eta^*$ (where, as usual, ε is the two-dimensional Levi-Civita symbol and the asterisk denotes complex conjugation). Such a substitution is formally permissible because the spinor fields ϕ_a and η transform alike in accordance with the underlying symmetry group of general relativity (or in the limit, special relativity). With such a substitution, the generalization of the electromagnetic (spinor) formalism implies that

there can be an inertial field associated with the electromagnetic force. This result also implies that, asymptotically, as the matter and electromagnetic field formalisms approach those of a Hilbert space representation (which is required according to the correspondence principle of this theory), the inertial mass associated with the electromagnetic field in this way correspondingly approaches a discrete spectrum of mass values. This result is similar to the GSW result, though it occurs for different fundamental reasons than it does in their theory, and there is no prediction at this stage precisely where the mass values lie, in mediating the weak interaction, nor how many such masses would be required.

Recall that in the GSW theory, one generalizes the gauge, thus resulting in a generalized electromagnetic (quantized) vector field operator, A_μ, which may then incorporate a discrete set of mass values for the intermediate vector bosons — the photon having zero mass and the $W^\pm$ and Z^0 particles having finite mass, of order 10^2 GeV. On the other hand, the result of massive bosons to mediate interactions between fermions in the elementary particle domain does *not entail any extension* of the electromagnetic potential; it rather arises fundamentally from a *factorization* of it to an irreducible spinor form (in accordance with the irreducible representations of the symmetry group of relativity theory!).

In my approach, then, the formal relations in general relativity, (4.3.12) and (4.3.15), applied to the reflected electromagnetic spinor variables, results in the corresponding mappings between the reflected spinors as follows:

$$(q_\mu \Omega_\mu)\phi_\alpha = \Gamma[\exp(i\gamma)]\tilde{\phi}_\alpha. \tag{9.5.3}$$

The spinor form of the electromagnetic field equations (5.3.4) may then take the following form in general relativity:

$$q^\mu \partial_\mu \phi_\alpha + \Gamma[\exp(i\gamma)]\tilde{\phi}_\alpha = \mathrm{T}_\alpha. \tag{9.5.6a}$$

Combining this equation with its reflected spinor equation

$$\tilde{q}^\mu \partial_\mu \tilde{\phi}_\alpha + \Gamma[\exp(-i\gamma)]\phi_\alpha = \tilde{\mathrm{T}}_\alpha \tag{9.5.6b}$$

the following field equation in the electromagnetic spinor variable is obtained in terms of the 'Klein–Gordon operator' in a curved space:

$$(\Box - \Gamma^2)\phi_\alpha = \tilde{q}^\mu \partial_\mu \mathrm{T}_\alpha - \Gamma[\exp(i\gamma)]\tilde{\mathrm{T}}_\alpha, \tag{9.5.7}$$

where

$$\Box \equiv (\tilde{q}^\mu \partial_\mu)(q^\mu \partial_\mu) = (-\partial_t^2 + \partial_k^2)\sigma_0$$

is the d'Alembertian operator and Γ is a field in general relativity that plays the role of the inertial mass associated with the electromagnetic spinor field ϕ_α, were it not for the appearance of the source field on the right-hand side of Equation (9.5.7).

Recalling that within the context of this field theory, the electromagnetic

field equations are interpreted as identities, one must look on Equation (9.5.7) as a way of representing the field intensity variables, ϕ_α in terms of the charged matter source fields T_α, *or vice versa*. Thus, in contrast with the matter field equations in (η, χ), which are implicitly nonlinear because of the dependence of the interaction functional $\mathscr{I}$ in (4.4.7) on the other matter fields of a closed system that forms the environment of the 'observed' field, *and vice versa*, the interpretation of the electromagnetic field equations as identities yields them to be linear in ϕ_α. Their solutions are then the sum of the homogeneous solutions (solving the source-free equation),

$$(\Box - \Gamma^2)\phi_\alpha^{(h)} = 0 \tag{9.5.8}$$

and the particular solutions, which in the special relativity limit have the form

$$\phi_\alpha^{(p)}(x) = \int S(x - x')(\sigma^\mu \partial_\mu T_\alpha - \langle\Gamma\rangle[\exp(i\gamma)]\tilde{T}_\alpha)\, d^4x', \tag{9.5.8'}$$

where $\langle\Gamma\rangle$ are the expectation values of the inertial mass associated with the electromagnetic mass field in the local (flat space-time) approximation. In the latter limit, $S(x - x')$ above represents the Green's function for the Klein–Gordon equation.

In accordance with Equations (9.5.6a,b), Γ^2 are the eigenvalues (generally continuously distributed) of the iterated operator, with respect to the variables of the electromagnetic spinor solutions, as basis functions. That is to say, analogous to the mass eigenvalue equation that followed from the analysis in Chapter 4,

$$(-\tilde{q}_\mu \Omega_\mu^\dagger)(q^\mu \Omega_\mu)\,|\,\eta\rangle = \lambda^2\,|\,\eta\rangle,$$

where

$$\lambda = \tfrac{1}{2}\{|\det \Lambda_+| + |\det \Lambda_-|\}^{1/2}$$

is the inertial mass field (appearing as the (positive-definite) modulus of a complex function), and where $\Lambda_\pm = q^\mu \Omega_\mu \pm$ h.c., it also follows formally that

$$(-\tilde{q}^\mu \Omega_\mu^\dagger)(q^\mu \Omega_\mu)\,|\,\phi_\alpha\rangle = \Gamma^2\,|\,\phi_\alpha\rangle. \tag{9.5.9}$$

As in the case of the matter field equations, in the asymptotic limit of a flat space-time, the electromagnetic spinor solutions approach the elements of a Hilbert space, $\{|\phi_\alpha\rangle_n\}$ and the squared mass values correspondingly approach the spectrum of values:

$$\langle\Gamma^2\rangle_n = \frac{\langle\phi_{an}|\,(-\tilde{q}^\mu \Omega_\mu^\dagger)_a (q^\mu \Omega_\mu)_a\,|\,\phi_{an}\rangle}{\langle\phi_{an}\,|\,\phi_{an}\rangle}, \tag{9.5.10}$$

where 'a' denotes the asymptotic values of the operators they modify and

$|\phi_{an}\rangle$ denote the Hilbert space limit of the spinor form of the electromagnetic solutions, corresponding to the nth (squared) mass eigenvalue — for the transferred energy in the form of a Klein—Gordon *boson* — with value $\langle\Gamma^2\rangle_n$.

Note that when the *actual* flat space-time limit is reached, all of the mass eigenvalues in Equation (9.5.10) automatically vanish, since in this limit the spin-affine connection field Ω_μ is null, identically. As in the derivation of the matter field inertia, the very existence of a nonzero value for this term, i.e. $\langle\Gamma^2\rangle_n \neq 0$, signifies (in the context of this field theory of matter), an observed feature of the curved space-time in general relativity *per se*, in the elementary particle domain.

The homogeneous solutions of Equation (9.5.8) then relate to the spectrum of (squared) mass values of the Klein—Gordon operator, and correspond to the (approximately) transverse part of the electromagnetic field. The particular solutions (9.5.8′) (approximately) correspond to the longitudinal solutions. We are concerned here only with the transverse part of the electromagnetic field that is transferred between fermion matter field components.

The experimental facts about the CERN experiment that revealed the possibility of interpretation of a $W^{\pm}$-particle to mediate the weak interaction are that the proton and antiproton beam (colliding from parallel rings) interact with a momentum transfer above 15 GeV/c, and several events reveal large missing transverse energy that is correlated with the high-energy electrons in the transverse plane. The latter missing energy was then identified with the neutrino components that are conventionally anticipated in the reaction products.

Recall that in the theory of elementary matter proposed in this monograph, all observed particles are continuum fields embedded in a sea of real (i.e. countable) particle—antiparticle pairs, e^-—e^+, p^-—p^+, ..., whose respective ground states (an exact derived feature of this theory, as demonstrated in Chapter 8) are at *zero* energy and momentum. This is a value relative to the energy-momentum of the particle—antiparticle when they would be (asymptotically) free of each other, which would be the energies $\geqslant 2m_ic^2$, where m_i are the inertial masses of the respective members of the pairs.

In this field theory, there are no neutrinos *per se*, though in the special dynamical situation whereby a pair is in its true ground state of null energy and null momentum, the spinor form of the electromagnetic field for the pair, in the special relativity limit, satisfies the Weyl equation, as we discussed in the preceding sections, that is, the transferred electromagnetic field in this state for the pair obeys the same field law as that of the neutrino. But still, the neutrino does not exist as a thing-in-itself, in this view. This is a feature of the theory that allowed a possible explanation for the absence of an abundance of neutrinos from the Sun [79]. Thus, this is how one must envision the

'missing energy' in the transverse plane of the $p\bar{p}$ collision of the CERN experiment, according to this theory.

If this field theory of matter in general relativity does indeed explain the colliding $p\bar{p}$ experiment, then each of the proton and antiproton pairs, when they interact, must go into their ground states of null energy and null momentum, thereby giving up their binding energy (near 2 GeV) plus the total kinetic energy (which is most of the transferred energy to the real pairs of the background physical vacuum). Thus, pairs in the background are excited, thereby converting constituent electrons and positrons to muons, unbound (according to the mechanism discussed earlier, whereby the excited pairs in the vicinity of an 'observed' electron alter the geometry in terms of the spin-affine connection field, so as to generate the higher mass member of the mass doublet that the theory predicts). These 'muons' (i.e. heavy electrons) then decay to ordinary electrons (in the order of 10^{-6} s) by virtue of the de-excitation of the surrounding pairs to their ground states, thereby restoring the normal (minimal energy) geometrical field of the background physical vacuum of pairs [57]. Thus we have the sequence

$$p^+ + p^- \rightarrow \mu^+ + \mu^-$$

$$\mu^\pm \rightarrow e^\pm + \phi_\alpha(e^+e^-) + \tilde{\phi}_\alpha(e^+e^-),$$

where ϕ_α, $\tilde{\phi}_\alpha$, which play the role of the neutrinos of opposite helicity, ν and $\tilde{\nu}$, carry the transferred electromagnetic energy when the reactions occur. (The + and − notation is used above for particle and antiparticle states of the protons and muons.)

Each of the electromagnetic spinor field solutions, playing the role of neutrinos above, correspond to homogeneous solutions of Equation (9.5.2) after iteration of the fundamental equations. Thus they each entail a mass term Γ of the (mathematically) corresponding boson field representation — a term in this theory that is to incorporate a feature of the nonzero curvature of space-time. In accordance with Noether's theorem, the asymptotic limit of this formalism, in the limit of a flat space-time, implies intrinsic energy in this field with magnitude Γc^2. In accordance with the mass spectrum formula (9.5.10), this would then correspond in the local limit to the spectrum of 'composite boson' mass values, $\{\Gamma_n c^2\}$.

If this theory does indeed *explain* the colliding $p\bar{p}$ experiment at CERN, one of the latter mass terms, of the entire spectrum, must correspond, numerically, to a value that is of order 80 GeV/c^2. The demonstration of this quantitative result from this field theory of inertia of elementary matter will depend on a study of the coupling of the metrical field to the matter field and electromagnetic field equations, in order to determine the explicit effect of the space-time curvature on the *apparent* inertial field Γ, which arises when the physical vacuum of real pairs is excited by the impinging $p\bar{p}$ colliding beams. It should also be noted that this theory predicts further, that there is

more than one such intermediate boson; there is a whole spectrum of values of Γ that mediate the coupling of spinor matter fields at high energy. Their numerical determination must also await further theoretical and experimental investigations.

9.6. Concluding Remarks

The qualitative and quantitative results about elementary particle physics that were demonstrated in this chapter were meant only to present the reader with *feasibility arguments* for the possibility that the explanation of empirical facts, especially some of the outstanding, thus far unexplained problems, could follow from the continuum field approach of general relativity, rather than from the particle approach of the quantum theory, which has thus far been unsuccessful in providing a satisfactory explanation for some of these phenomena.

Of course, there is a great deal more to do with this theory in order to establish some sort of conclusiveness in its results; in this chapter I have tried to point the way toward such further analyses. But at the present stage I believe it is at least possible to say that many of the empirical results of elementary particle physics do not necessarily come from the basis of quantum mechanics, but they could, alternatively, come from an underlying field theory that is based fully on Einstein's theory of general relativity when it is fully exploited in the problem of matter.

Epilogue

I should like to express some concluding remarks in this Epilogue that deal more with the philosophy of physics and scientific method than they are concerned with the details of the program in theoretical physics that has been developed in this monograph. This first subject questions present-day explanations for the empirical facts about irreversible processes in nature, in terms of arguments about intrinsic statistics and probabilities of complex material systems; the second subject deals with the methods of scientific investigation that I believe are in the best interests of progress in science.

In the latter decades of the nineteenth century, with the advent of statistical mechanics and the kinetic theory of gases to underlie the laws of thermodynamics and equations of state of many-body systems, ideas were beginning in infuse into physics, later to come to fruition with the discovery of quantum mechanics and its interpretation according to the Copenhagen school, that statistical laws and laws of probabilities form an indispensable *explanatory basis* for the fundamental laws of matter. This was the beginning of a truly revolutionary stance in science, for before this development of ideas, statistics and rules of probability were only considered to be a useful tool for the researcher, as in analyzing the errors in the experimental results, or in learning how to estimate an *average* over a very large number of variables, without the need to have at hand the explicit variables themselves. But it was believed in the earlier time of science that beneath the statistics and probability analyses there is a precisely ordered system, regarding all of its intrinsic components, that is independent of measurements or averaging estimates.

One of the very striking empirical results that statistical mechanics attempted to explain, before the appearance of quantum mechanics, was concerned with the time-irreversible character of the evolution of complex systems, from some nonequilibrium configuration to its equilibrium configuration. The second law of thermodynamics deals specifically with this physical situation, asserting that the entropy of a complex system, which is a measure of its degree of disorder, must be less than its maximum value in the nonequilibrium state, but then it must proceed *irreversibly in time* toward its maximum value which is achieved when equilibrium is reached.

Similarly in quantum mechanics, it is asserted that there is a time-irreversible aspect to the fundamental description of micromatter. This is that part of the theory concerned with the measurement process. It is said here that before a measurement is made (necessarily by a macroapparatus) the 'observed' micromatter is in an indefinite number of possible states (characterized by the solutions of the equations of quantum mechanics). But when the measurement is carried out, a small (packet) of these states project out; the observable property measured in then 'weighted over' these states in order to determine the value of the observable. At that stage, there can be no reversal to the previous wide distribution of possible states of the micro-matter. And there is no dynamics in the theory that predicts the coupling between the apparatus and the observed matter, thus probability enters here in a fundamental way.

Because of this similarity in the appearance of irreversible processes, in the second law of thermodynamics *vis-à-vis* the measurement process in quantum mechanics, some authors have theorized that there must be a profound fundamental relation between the entropy concept in thermo-dynamics and the roots of the quantum theory [80]. This may indeed be the case. However, in the context of the deterministic field theory of inertia developed in this monograph, which is based on the ideas of general relativity theory as a fundamental theory of matter, though a covariant formalism that asymptotically approaches the formal structure of the prob-ability calculus of quantum mechanics, in the low energy limit, there is no fundamental role played by 'disorder' and 'chaos' here, and the empirical facts of the irreversibility of physical systems is understandable in terms of causal dynamics alone.

I fully accept the second law of thermodynamics as a correct aspect of the *descriptive* part of a complex system; however, I wish to question its use as *explanatory*. It is my contention that indeed statements about probabilities and order—disorder processes in nature are subjective, based on observers' particular knowledge of a complex system, and not objective statements about the underlying causes for the empirical facts about irreversibility.

The use of the entropy concept to 'explain' time-irreversible processes in nature seems to me similar to the idea of 'explanation' in Aristotle's assertion that the reason that a heavy object falls to the ground level is that this is a more natural place for it to be! But it was a giant step when Galileo and Newton did indeed *explain* the fall of a material object in terms of precise, objective laws, relating to the motion of free fall as due to a particular cause — the gravitational interaction with other matter, leading to the discovery of the 'force of gravity'.

What would Newton have answered if asked this question: How is it that, on the one hand, when a complex system is left on its own in a non-equilibrium state, it always is seen to evolve time-irreversibly toward equi-librium, while on the other hand, the dynamical laws of motion of this matter are time-reversible? [81] I don't believe that he would have evoked any

subjective explanation, regarding an observer's particular knowledge about the complex system and the use of probabilities. I believe that he would have said that the disorder we talk about in this problem is only in regard to the observer's lack of knowledge about each of the constituent elements of the complex system, which is quite apart from the mechanical forces that give rise to their predetermined paths, irrespective of what the observer happens to know or not know! Of course, it is also well known that Einstein, who was aware of the later developments concerning order and disorder and the use of probabilities in physics, also asserted a basic belief in a predetermined system of matter, irrespective of what a 'knower' may or may not be aware of.

The reason that a ball rolls down a hill to the bottom, and does not roll up again to the top, even though there is such a time-reversed solution of Newton's equations of motion, is that it would take work from the outside (against the action of gravity) to establish the precise boundary conditions that would correspond to projecting the ball up the hill back to its initial point at the top. Most physicists would certainly agree with this conclusion.

Consider now some examples of complex systems that evolve irreversibly in time, with the view that the *explanation* for their irreversible behavior may not at all be based on ideas of order and disorder, but rather are similarly rooted in predetermined dynamical processes, not out of step with the rest of 'deterministic physics'. The order—disorder processes, as the use of the entropy concept, are considered here in an entirely different context — one that is descriptive rather than explanatory.

Example 1. Given the initial conditions of position, velocity and torque of a flipped coin, its complete interaction with the gravitational field of the Earth and the (10^{24}) molecules it encounters as it flip-flops through the air, up to some maximum height and then down again, there is indeed a predetermined path for this coin and a precise answer to the question: Will it land heads? *But in another context*, one may ask: What is the probability that the coin will land heads? The answer to this question is obtained without the need to solve any equations of motion. It is $\frac{1}{2}$ — because we know in advance that there are two (equally weighted) possibilities. But the theory of probability that gives this answer does not at all explain the behavior of the coin! That is, it is not a fundamental theory of the temporal behavior of the coin's motion. It is only a descriptive element that is useful to the observer — it is subjective while the dynamics of the system (of coin plus Earth plus atmosphere) is objective.

Example 2. Consider the following well-known example that does not entail any gravitational force. A dark blue ink drop is released into a colorless liquid, in a beaker in outer space. One should then observe the diffusion of the molecules of the ink drop into the liquid, until the drop, as

such, would disappear from view and the entire liquid in the beaker has turned a pale blue, uniformly. The question asked is: Why do we not ever see all of the ink molecules in the beaker move back along the original paths to form the initial concentrated, dark blue drop of ink, in view of the time-reversed solutions for the paths of the individual molecules? I believe that the answer is: They do not do so because it would take work from the outside to set up the precise boundary conditions on each of the ink molecules in the beaker (at each collision), to force them to retrace their original trajectories from the time that the ink drop broke up.

The reason that the ink drop broke up in the first place did not have anything to do with fundamental order and disorder, or probabilities, e.g. it was not that it would be more probable for the ink molecules to be found in the larger volume of the beaker than in the initial confining space of the ink drop. The reason for its break-up, rather, was that the forces acting on the drop by the surrounding liquid caused it to be in an unstable state, just as the Earth's gravity field caused the ball at the top of the hill to be in an unstable state. In both cases, the inserted matter did work on the host system, thereby evolving to the stable configuration (of minimum intrinsic energy).

To restore the initial unstable configuration would then require external work to be done, in order to set up the initial conditions on each of the elements of the original ink drop, leading to their time-reversed trajectories. That is to say, the *reason* for the fact that we do not observe a restoration of the original ink drop is one of dynamics and not in terms of fundamental processes that convert order to disorder or vice versa.

This dynamical explanation is quite aside from the fact that there is also a probability that the ink molecules that have diffused evenly throughout the liquid, by chance collisions may cause each other to return to the original confinement of the ink drop — though the probability for this to happen is close to zero. The point is that the latter probability statement is not an explanation for the empirical fact that the ink drop is not seen to reconstitute itself.

Example 3. Consider the example of radioactive decay of some macro-quantity of radium. Does this not necessarily entail a fundamental process in which order changes to disorder, acausally? Empirically, one does observe a seemingly random emission (in time) from the radium, causing an apparently random distribution of pulses in some detecting device, such as a Geiger counter, as: . . . | | | | | . . . . But, is there any guarantee that these signals are not in actual fact connected (below the experimenter's arbitrarily set voltage bias), as the peaks of a coherently connected curve?

If we should consider the *closed system* of the radium and its decay products, as well as the entire environment that they interact with, an objective theory might prescribe that a predetermination of their respective trajectories in space and time follows. Again, disorder in this example would

only be a matter of a 'knower's' lack of awareness of the exact state of the entire closed system, at any time. But in this view, there is still a fundamental order underlying the system of radium and its products and environment. To restore the original radium nuclei (the unstable state) external work would then have to be done on the stable system (that is, when all of the products are in equilibrium) to establish the proper boundary conditions on the elements so that they may retrace their original paths back to the original unstable radium nucleus. What I am saying here is that the empirical facts do not compel us to *explain* radioactive decay in terms of fundamental disorder and probabilities, though a *description* with these concepts may certainly be useful for the scientist who observed such irreversible processes.

Example 4. What about the irreversible ageing of a human body? I believe that this is identical, in principle, with the preceding examples. Our aging is a matter of cellular decay in which our biological molecular structures change, apparently irreversibly. However, in principle, by supplying the proper external work to return the atomic constituents to their previous configurations (that made up the youthful body!), we should be able to recover the earlier states of our bodies. This does not happen in practice because it would be extremely difficult to become aware of, and then establish all of the boundary conditions that would lead to such a reversal of ageing. The idea is that our bodies do not naturally return to their youthful states because such boundary conditions are not there, because the necessary external work that must be done to establish them is not provided. The reason for irreversible ageing is, then, not a matter of a fundamental explanation in terms of the second law of thermodynamics. That is to say, it is the same reason that the ball at the bottom of the hill does not choose to roll up the hill again, to the place where it was released, even though there is such a (time-reversed) solution of Newton's equations of motion for this system.

These examples of irreversible ageing and that of the diffusion of an ink drop into a clear liquid are analogous to the situation in which a pool player breaks up an arrangement of balls in a triangular configuration, at one end of the pool table, scattering them in all directions due to a collision with the cue ball. It is very improbable that a second collision would be able to scatter the balls back into the triangular configuration, because such processes would not normally set up the boundary conditions to time-reverse all of the trajectories of the pool balls on the table. To accomplish the latter would take external work — because it would correspond to increasing the total (configuration) energy of the system. It is not a matter of fundamental order and disorder. In principle, such a reverse process could be carried out without violating any of the fundamental dynamical laws of physics.

A third example, similar to these, is the following. One starts initially with an unshuffled deck of cards, arranged according to suits and in sequence of numbers, ace to ten, Jack, Queen, King. A thorough shuffling would then destroy all of that order. To restore it by reshuffling would then require a

certain amount of external work to set up the proper boundary conditions that would send each of the fifty two cards to the proper place in the deck. That is, I see this example as no different from the preceding ones, whereby the *explanation* for the configurations of the cards has to do with predetermined trajectories, not with order and disorder. The latter concepts are only useful *descriptive* elements for the complex system, in the same sense that we *use* probabilities to conclude that the chance is $\frac{1}{2}$ that the coin will land heads.

Example 5. Finally, consider the cosmological problem. It is claimed in contemporary astrophysics that the universe is expanding without limit, starting at the initial big bang, about 15 billion years ago. At the time there was maximum order (minimum entropy) and the subsequent expansion of the universe entails an evolution toward maximum entropy at the equilibrium state, when all of the stars will have burned out and the universe would be a homogeneous sea of interstellar dust. Whether or not this is a physically valid model of the universe, the explanation for the process of an expanding universe must be in objective terms, rather than the subjective aspects of the process of entropy change. When the universe should become more and more populated with dust, and less and less containing individual stars, we would have less awareness of the locations, speeds, etc., of the constituent elements of the universe. But the *explanation* for these events are still the fundamental forces that control the whereabouts of the constituent elements of the universe, whether or not human consciousnesses are aware of them! I believe that Newton would have replied in this way and that present-day cosmologists who rely on the law of entropy to *explain* the universe as a whole would not have made their case with Newton.

On this point, I believe that Einstein was in full accord with Newton, because they both saw the universe in terms of an underlying order, in any domain, from the world of elementary matter to that of the universe as a whole. This seems to me to then reduce to the main question in the historic debates between Bohr and Einstein, as to whether the laws of nature must entail some irreducible subjectivity, or whether there is a fully objective universe with accompanying laws of nature that are independent of whether or not we are there to observe its manifestations. This is the well-known debate in epistemology between whether our knowledge of the universe, in any of its domains, large or small, must be governed by the stand of positivism or that of realism. The conventional stand on the meaning of quantum mechanics today is based on Bohr's positivistic view; the theory presented in this monograph that attempts to explain the formal expression of quantum mechanics, is based on Einstein's realistic view. By going further than ordinary, linear, quantum mechanics does in several applications and proofs, as indicated in this book, I believe that Einstein's realistic approach is a very strong contender for the physics of the future.

An overview of the history of science reveals impressively that indeed

a most healthy attitude toward the preservation of progress of our fundamental understanding in science is in the nurturing of opposing views toward common problems. No single point of view should ever be claimed as *the end — the final understanding.* Of course, this is because the actual amount of understanding there is to be had of all of the manifestations of the universe, from the domain of elementary particle physics to that of cosmology, is most likely infinite, while we, as human beings, are certainly finite in all respects, including our intellectual capacity to understand. Thus, we are bound to err along the path toward increased understanding. It seems to me that the best way to recognize such errors, that are bound to happen, is to take seriously and encourage critical dialogue. It is with this thought in mind that the Middle Ages philosopher and theologian, Moses Maimonides, wrote the following prayer for the 'scientists' of his day [82]:

"Grant me strength, time and opportunity to correct what I have acquired, always to extend its domain; for knowledge is immense and the spirit of man can extend infinitely to enrich itself daily with new requirements. Today he can discover his errors of yesterday and tomorrow he may obtain a new light on what he thinks himself sure of today."

It was in this spirit of Maimonides that the foregoing monograph was written, pursuing a new explanation for the empirically correct formalism of nonrelativistic quantum mechanics, to follow from the deterministic, continuum notions of the field theory of matter according to the basis of general relativity theory, rather than from the nondeterministic, singular particle notions of the present-day conventional interpretation. It has been demonstrated in this book that such a turn of interpretation, viewing quantum mechanics as a linear approximation for matter field equations that are a generally covariant set of laws of the inertial manifestation of matter, in all domains, has led to new results, as well as overlapping the empirically correct results of the older quantum mechanical theory that was viewed strictly in terms of a probability calculus.

Whether or not these studies will continue to be successful in providing further explanation for the basic nature of the physical world in any of its domains of behavior, it would at least be a healthy attitude for physics itself in taking an alternative view to its present-day stand. It would also be satisfying a need in theoretical physics to investigate the intuitive expectations about quantum mechanics of one of the greatest physicists since the times of Galileo and Newton, Albert Einstein. Whether or not he turns out to be right after all, his intuition about this question will at least have been given the chance it deserves.

Appendix A

Computation of the Lamb Splitting

The Lamb splitting was determined in the text [Equation (8.2.11)] to be proportional in first order to the number, $|I_+ + I_-|$, where

$$I_\pm = (\eta\gamma)^{-1} \int_0^\infty \frac{F_\pm G_\pm}{\rho^2} \exp\left(-\frac{2\kappa}{\rho}\right) d\rho \tag{A.1}$$

$\eta^2 = \lambda^2 - E^2$, $\kappa = 16\pi g_M \gamma\eta$ and $F_\pm$, $G_\pm$ are the respective (unperturbed) 'large' and 'small' components of the radial Dirac solutions for hydrogen; the plus and minus signs refer respectively to the states with $K = +1$ and $K = -1$.

The general expressions for the integrals $I_\pm$ will be derived below in terms of a (finite) power series expansion in the fine structure constant γ. While only the first parts of these expansions will be utilized as present, because of the limits of accuracy in the experimental Lamb splittings to be compared with the theory, the derived expansions may be evaluated to any desired accuracy. These expansions in γ are demonstrably convergent since they represent an analytic function in closed form, integrated over all space. The latter is the conserved energy $\int \theta_{00}\, d^3x$ that follows from the Lagrangian formalism as a consequence of its invariance with respect to time translations [Equation (7.1.1) with the exact hydrogenic solutions of Equations (8.1.1) inserted]. It should be noted that not only the integrals themselves, but the integrands are finite everywhere. This is because the factor $\exp(-2\kappa/\rho)$ approaches zero faster than any polynomial in $1/\rho$ approaches infinity at the origin.

Consider the Lamb splitting in the hydrogenic state with $n = 1$. This corresponds, in the conventional spectroscopic notation, to the $2S_{1/2} - 2P_{1/2}$ energy separation. In this case, the radial wave functions have the following form in terms of a power series expansion in ρ:

$$\begin{pmatrix} F_\pm \\ G_\pm \end{pmatrix} = \rho^s \sum_{\nu=0}^{\infty} \begin{pmatrix} a_\nu(\pm) \\ b_\nu(\pm) \end{pmatrix} \rho^\nu e^{-\rho} \tag{A.2}$$

Since $K^2 = 1$,

$$s = [1 - \gamma^2]^{1/2} = 1 - \tfrac{1}{2}\gamma^2 + O(\gamma^4) \tag{A.3}$$

With the approximation $\rho^s \simeq \rho$ it follows that

$$I_\pm = (\eta\gamma)^{-1}[a_0 b_0 L_0 + (a_0 b_1 + a_1 b_0)L_1 + a_1 b_1 L_2] \tag{A.4}$$

where

$$L_m = (\kappa)^{(m+1)/2} J_m \tag{A.5}$$

and

$$J_m = \int_0^\infty u^m \exp\left[-2\sqrt{(\kappa)}\left(u + \frac{1}{u}\right)\right] \mathrm{d}u \tag{A.6}$$

These integrals will be evaluated in the last section of this appendix.

The wave functions (A.2) are determined by the normalization condition and the recursion relations (Schiff [48], p. 324) for the coefficients a_ν, b_ν. The normalization condition

$$\eta^{-1} \int_0^\infty (F^2 + G^2) \exp\left(-\frac{2\kappa}{\rho}\right) \mathrm{d}\rho = 1$$

gives, for $n = 1$,

$$\eta = [(a_0^2 + b_0^2)L_2 + 2(a_0 a_1 + b_0 b_1)L_3 + (a_1^2 + b_1^2)L_4] \tag{A.7}$$

and the recursion relations give

$$\begin{aligned} \frac{a_1}{b_1} &= -\frac{\eta}{\eta_1}, \qquad \frac{a_0}{b_0} = \frac{(s+K)}{\gamma} \\ \frac{b_0}{b_1} &= -\frac{2s+1}{2} \frac{\gamma\eta + (s-K)\eta_1}{\gamma\eta + (s-K+1)\eta_1} \end{aligned} \tag{A.8}$$

To order γ^2, Equation (A.8) reduces to

$$\frac{a_1}{b_1} = -\frac{\gamma}{4}$$

$$\left.\begin{matrix} \frac{a_0}{b_0} = 2/\gamma \\ -\gamma/2 \end{matrix}\right\} \quad K = \left.\begin{matrix} +1 \\ -1 \end{matrix}\right\} \quad \frac{b_0}{b_1} = \left\{\begin{matrix} 3\gamma^2/8 \\ -(1 - 3\gamma^2/8) \end{matrix}\right. \tag{A.9}$$

To the same accuracy,

$$\eta = \frac{1}{2r_0} \tag{A.10}$$

where $r_0 = \hbar^2/me^2$ is the 'first Bohr radius'.

We will see below that to this same order of approximation the integrals L_m have the following values (up to $m = 8$):

$$L_0 \cong \tfrac{1}{2} \qquad L_1 \cong L_2 \cong \tfrac{1}{4} \qquad L_3 \cong \tfrac{3}{8} \qquad L_4 \cong \tfrac{3}{4}$$
$$L_5 \cong \tfrac{15}{8} \qquad L_6 \cong \tfrac{45}{8} \qquad L_7 \cong \tfrac{315}{16} \qquad L_8 \cong \tfrac{315}{4} \tag{A.11}$$

With Equations (A.11), (A.10), (A.9) and (A.7), the following result, to leading order in γ, is obtained:

$$K = -1 \begin{cases} a_0 = \gamma\sqrt{\dfrac{1}{2r_0}} & b_0 = -\sqrt{\dfrac{2}{r_0}} \\ a_1 = -\gamma\sqrt{\dfrac{1}{8r_0}} & b_1 = \sqrt{\dfrac{2}{r_0}} \end{cases},$$

$$K = 1 \begin{cases} a_0 = \gamma\sqrt{\dfrac{3}{8r_0}} & b_0 = \gamma^2\sqrt{\dfrac{2}{32r_0}} \\ a_1 = -\dfrac{\gamma}{4}\sqrt{\dfrac{2}{3r_0}} & b_1 = \sqrt{\dfrac{2}{3r_0}} \end{cases} \tag{A.12}$$

Inserting equations (A.12) and (A.11) into (A.4), we finally obtain the following result:

$$I_+ = \tfrac{1}{6} + O(\gamma^2) \qquad I_- = -\tfrac{1}{2} + O(\gamma^2)$$

Thus, for the quantum number $n = 1$,

$$|I_+ + I_-|_1 = \tfrac{1}{3} + O(\gamma^2) \tag{A.13}$$

In a similar way, the $n = 2$ solution is characterized by the radial wave function

$$\begin{pmatrix} F_\pm \\ G_\pm \end{pmatrix} \cong \rho \sum_{\nu=0}^{2} \begin{pmatrix} a_\nu \\ b_\nu \end{pmatrix} \rho^\nu e^{-\rho}$$

so that, for this case,

$$I_{\pm} = (\gamma\eta)^{-1}[a_0 b_0 L_0 + (a_0 b_1 + a_1 b_0)L_1 + (a_0 b_2 + a_2 b_0 + a_1 b_1)L_2 + \\ + (a_1 b_2 + a_2 b_1)L_3 + a_2 b_2 L_4] \tag{A.14}$$

The normalization condition gives:

$$\eta = \{(a_0^2 + b_0^2)L_2 + 2(a_0 a_1 + b_0 b_1)L_3 + [2(a_0 a_2 + b_0 b_2) + (a_1^2 + b_1^2)]L_4 + \\ + 2(a_1 a_2 + b_1 b_2)L_5 + (a_2^2 + b_2^2)L_6\} \tag{A.15}$$

For this state, $\eta \simeq 1/3r_0$. Using this approximation together with the recursion relations, the following coefficients are obtained for the corresponding wave function, to leading order in γ:

$$K = -1 \begin{cases} a_0 = -\gamma\sqrt{\dfrac{1}{3r_0}} & b_0 = 2\sqrt{\dfrac{1}{3r_0}} \\ a_1 = \dfrac{10\gamma}{9}\sqrt{\dfrac{1}{3r_0}} & b_1 = -4\sqrt{\dfrac{1}{3r_0}} \\ a_2 = \dfrac{-2\gamma}{9}\sqrt{\dfrac{1}{3r_0}} & b_2 = \dfrac{4}{3}\sqrt{\dfrac{1}{3r_0}} \end{cases},$$

$$K = 1 \begin{cases} a_0 = \dfrac{-2\gamma}{3}\sqrt{\dfrac{2}{3r_0}} & b_0 = \dfrac{\gamma^2}{3}\sqrt{\dfrac{2}{3r_0}} \\ a_1 = \dfrac{2\gamma}{3}\sqrt{\dfrac{2}{3r_0}} & b_1 = -\dfrac{4}{3}\sqrt{\dfrac{2}{3r_0}} \\ a_2 = \dfrac{-\gamma}{9}\sqrt{\dfrac{2}{3r_0}} & b_2 = \dfrac{2}{3}\sqrt{\dfrac{2}{3r_0}} \end{cases} \tag{A.16}$$

Inserting Equation (A.16) into Equation (A.14), we find that for $n = 2$,

$$|I_+ + I_-|_2 = |\tfrac{1}{9} - \tfrac{1}{3}| = \tfrac{2}{9} + O(\gamma^2) \tag{A.17}$$

The radial wave function for the $n = 3$ state is expressed as the following sum

$$\begin{pmatrix} F_{\pm} \\ G_{\pm} \end{pmatrix} \simeq \rho \sum_{\nu=0}^{3} \begin{pmatrix} a_\nu \\ b_\nu \end{pmatrix} \rho^\nu e^{-\rho}$$

and in this case,

$$I_{\pm} = (\gamma\eta)^{-1}[a_0 b_0 L_0 + (a_0 b_1 + a_1 b_0)L_1 + (a_0 b_2 + a_2 b_0 + a_1 b_1)L_2 + \\ + (a_0 b_3 + a_3 b_0 + a_1 b_2 + a_2 b_1)L_3 + (a_1 b_3 + a_3 b_1 + a_2 b_2)L_4 + \\ + (a_2 b_3 + a_3 b_2)L_5 + a_3 b_3 L_6] \tag{A.18}$$

The normalization condition for this solution gives the relation:

$$\eta = \{(a_0^2 + b_0^2)L_2 + 2(a_0 a_1 + b_0 b_1)L_3 + [2(a_0 a_2 + b_0 b_2) + \\ + (a_1^2 + b_1^2)]L_4 + 2(a_1 a_2 + b_1 b_2 + a_0 a_3 + b_0 b_3)L_5 + \\ + [2(a_1 a_3 + b_1 b_3) + (a_2^2 + b_2^2)]L_6 + 2(a_2 a_3 + b_2 b_3)L_7 + \\ + (a_3^2 + b_3^2)L_8\} \tag{A.19}$$

In this state, $\eta \sim 1/4r_0$ so that with the recursion relations, the following coefficients are obtained for the radial wave function, to leading order in γ:

$$K = 1 \begin{cases} \dfrac{a_0}{b_3} = \dfrac{15\gamma}{8} & \dfrac{b_0}{b_3} = \dfrac{15\gamma^2}{16} \\ \dfrac{a_1}{b_3} = \dfrac{-25\gamma}{8} & \dfrac{b_1}{b_3} = 5 \\ \dfrac{a_2}{b_3} = \dfrac{5\gamma}{4} & \dfrac{b_2}{b_3} = -5 \\ \dfrac{a_3}{b_3} = \dfrac{-\gamma}{8} & b_3 = \dfrac{1}{15r_0} \end{cases}$$

$$K = -1 \begin{cases} \dfrac{a_0}{b_3} = \dfrac{3\gamma}{2} & \dfrac{b_0}{b_3} = -3 \\ \dfrac{a_1}{b_3} = \dfrac{-21\gamma}{8} & \dfrac{b_1}{b_3} = 9 \\ \dfrac{a_2}{b_3} = \dfrac{9\gamma}{8} & \dfrac{b_2}{b_3} = -6 \\ \dfrac{a_3}{b_3} = \dfrac{-\gamma}{8} & b_3 = \dfrac{1}{3r_0} \end{cases} \tag{A.20}$$

Inserting Equation (A.20) into (A.18), we find that

$$|I_+ + I_-|_3 \cong |\tfrac{1}{12} - \tfrac{1}{4}| = \tfrac{1}{6} + O(\gamma^2) \tag{A.21}$$

Evaluation of the Integrals J_m

The terms L_m that appear in the terms of the preceding equations depend on the integrals J_m [Equation (A.6)] according to the proportionality shown in Equation (A.5). Let us now proceed to compute the terms J_m.

It is readily verified that

$$J_{-1} \equiv \int_0^\infty \frac{\exp[-2\sqrt{(\kappa)}(u + 1/u)]}{u}\, du = 2K_0(4\sqrt{\kappa}) \tag{A.22}$$

where K_0 is the zeroth-order modified Bessel function of the second kind (Watson [52]). Taking derivatives of J_{-1} with respect to $2\sqrt{\kappa}$ will then generate the other integrals J_m in terms of these derivatives. If we denote the nth derivative of K_0 with respect to its argument by $K_0^{(n)}$, the following relationships are obtained:

$$\begin{aligned}
J_0 &= -2K_0^{(1)} \\
J_1 &= 4K_0^{(2)} - 2K_0 \\
J_2 &= -8K_0^{(3)} + 6K_0^{(1)} \\
J_3 &= 16K_0^{(4)} - 16K_0^{(2)} + 2K_0 \\
J_4 &= -32K_0^{(5)} + 40K_0^{(3)} - 10K_0^{(1)} \\
J_5 &= 2(32K_0^{(6)} - 48K_0^{(4)} + 18K_0^{(2)} - K_0) \\
J_6 &= 2(-64K_0^{(7)} + 112K_0^{(5)} - 56K_0^{(3)} + 7K_0^{(1)}) \\
J_7 &= 2(128K_0^{(8)} - 256K_0^{(6)} + 160K_0^{(4)} - 32K_0^{(2)} + K_0) \\
J_8 &= -16(32K_0^{(9)} - 72K_0^{(7)} + 54K_0^{(5)} - 15K_0^{(3)} + K_0^{(1)})
\end{aligned} \tag{A.23}$$

The exact forms for the derivatives in the preceding equation are as follows:

$$K_0^{(1)}(z) = -K_1(z)$$

$$K_0^{(2)}(z) = K_0(z) + \frac{K_1(z)}{z}$$

$$K_0^{(3)}(z) = -\left\{\left(1 + \frac{2}{z^2}\right) K_1(z) + \frac{K_0(z)}{z}\right\}$$

$$K_0^{(4)}(z) = \left(\frac{3}{z^2} + 1\right)\left(K_0(z) + \frac{2K_1(z)}{z}\right)$$

$$K_0^{(5)}(z) = -\left\{2\left(1 + \frac{6}{z^2}\right)\frac{K_0(z)}{z} + \left(\frac{24}{z^4} + \frac{7}{z^2} + 1\right) K_1(z)\right\}$$

$$K_0^{(6)}(z) = \left(1 + \frac{9}{z^2} + \frac{60}{z^4}\right) K_0(z) + 3\left(1 + \frac{11}{z^2} + \frac{40}{z^4}\right)\frac{K_1(z)}{z}$$

$$K_0^{(7)}(z) = -\left\{3\left(1 + \frac{120}{z^2} + \frac{17}{z^4}\right)\frac{K_0(z)}{z} + \right.$$

$$\left. + \left(1 + \frac{15}{z^2} + \frac{192}{z^4} + \frac{720}{z^6}\right) K_1(z)\right\}$$

$$K_0^{(8)}(z) = \left(1 + \frac{18}{z^2} + \frac{1272}{z^4} + \frac{975}{z^6}\right) K_0(z) +$$

$$+ \left(4 + \frac{405}{z^2} + \frac{1011}{z^4} + \frac{5040}{z^6}\right)\frac{K_1(z)}{z} \qquad \text{(A.24)}$$

In the problem to which these functions are applied in the text, the argument of the Bessel functions $z = 4\kappa < 10^{-5}$. Thus, to approximate the integrals J_m to one part in 10^5 we will only keep terms to the highest power in $1/z$.

According to the power series expansions for the zeroth and first-order Bessel functions, K_0 and K_1, the leading terms are (Watson [52]).

$$K_0(z) = -\ln\left(\frac{z}{2} + E\right) + O\left(\frac{1}{z^2}\right) \qquad (E = \text{Euler's constant})$$

$$K_1(z) = \frac{1}{z} + O\left(\frac{1}{z^2}\right) \tag{A.25}$$

With Equation (A.21) (keeping terms to the highest power in $1/z$) and Equation (A.25) in Equation (A.24), we obtain the approximations for the terms L_m that are given in Equation (A.11).

Appendix B

Evaluation of the Scattering Correction Factor $\varepsilon(bq)$

The scattering correction factor has the form

$$\varepsilon(a) = |I(a)|^2 \qquad (a \equiv bq), \tag{B.1}$$

where

$$I(a) = \int_0^\infty e^{-a/u} \sin u \, du. \tag{B.2}$$

To integrate (B.2) we take the limit

$$I(a) = \lim_{\mathscr{E} \to 0} I(a, \mathscr{E})$$

$$= \lim_{\mathscr{E} \to 0} \int_0^\infty e^{-a/u - \mathscr{E}u} \sin u \, du. \tag{B.3}$$

The integration is carried out by first determining

$$\lim_{\mathscr{E} \to 0} \frac{\partial I(a, \mathscr{E})}{\partial a} = \lim_{\mathscr{E} \to 0} \left[- \int e^{-a/u - \mathscr{E}u} \frac{\sin u}{u} \, du \right]. \tag{B.4}$$

Using the Laplace transform [83]

$$\frac{e^{-a/u}}{u} = \int_0^\infty J_0(2\sqrt{at}) \, e^{-ut} \, dt,$$

Equation (B.4) becomes

$$\frac{\partial I(a,\mathscr{E})}{\partial a} = -\int_0^\infty \int \mathrm{e}^{-\mathscr{E}u - ut} J_0(2\sqrt{at}) \sin u \, \mathrm{d}u \, \mathrm{d}t$$

$$= -\int_0^\infty J_0(2\sqrt{at}) \, \mathrm{d}t \int_0^\infty \mathrm{e}^{-(t+\mathscr{E})u} \sin u \, \mathrm{d}u$$

$$= -\int_0^\infty \frac{J_0(2\sqrt{at}) \, \mathrm{d}t}{(t+\mathscr{E})^2 + 1},$$

$$\lim_{\mathscr{E} \to 0} \frac{\partial I(a,\mathscr{E})}{\partial a} = \frac{\partial I(a)}{\partial a} = -\int_0^\infty \frac{J_0(2\sqrt{at})}{t^2+1} \, \mathrm{d}t, \tag{B.5}$$

where J_0 is the zero order Bessel function.

Changing the variables

$$x = 2\sqrt{at} \text{ with } \alpha = 2\sqrt{a},$$

Equation (B.5) takes the form

$$\frac{\partial I(a)}{\partial a} = -8a \int_0^\infty \frac{x^{1/2} J_0(x)}{x^4 + \alpha^4} x^{1/2} \, \mathrm{d}x. \tag{B.6}$$

The right-hand side of (B.6) is a Hankel transform [84]

$$\frac{\partial I(a)}{\partial a} = 8a\alpha^{-2} \operatorname{kei}(\alpha) = 2 \operatorname{kei}(2\sqrt{a}), \tag{B.7}$$

where

$$K_0(x\sqrt{i}) = \operatorname{ker}(x) + i \operatorname{kei}(x)$$

defines the ker and kei functions and K_0 is the zero order modified Bessel function of the second kind [52].

Integrating (B.7) we have

$$I(a) = 2 \int_0^a \operatorname{kei}(2\sqrt{a}) \, \mathrm{d}a + I(0),$$

where

$$I(0) = \lim_{\mathscr{E} \to 0} \int_0^\infty \sin u \, \mathrm{e}^{-\mathscr{E}u} \, \mathrm{d}u = \lim_{\mathscr{E} \to 0} [\mathscr{E}^2 + 1]^{-1} = 1.$$

Once again, changing variables according to

$$x = 2\sqrt{a}, \qquad z = x\sqrt{i},$$

we have

$$I(a) = \int_0^{2\sqrt{a}} x \,\mathrm{kei}(x)\,\mathrm{d}x + 1$$

$$= \frac{1}{2i}\int_0^{2\sqrt{a}} x[K_0(x\sqrt{i}) - (K_0(x\sqrt{i}))^*]\,\mathrm{d}x + 1$$

$$= -\frac{1}{2}\left[\int_0^{z_0 = 2\sqrt{ai}} zK_0(z)\,\mathrm{d}z + \text{c.c.}\right] + 1 = -\mathrm{Re}\int_0^{z_0} zK_0(z)\,\mathrm{d}z + 1. \quad \text{(B.8)}$$

Using the relations [52]:

$$K_0'(z) = -K_1(z), \qquad -zK_0(z) = zK_1'(z) + K_1(z)$$

and integrating (B.8) by parts, the following result is obtained:

$$I(a) = \mathrm{Re}\{zK_1(z)\}_0^{z_0} + 1 = \mathrm{Re}\{z_0K_1(z_0)\}. \quad \text{(B.9)}$$

The last part of (B.9) follows from the relation

$$\lim_{z \to 0} zK_1(z) = 1.$$

Since $\sqrt{i} = \mathrm{e}^{\frac{1}{4}i\pi}$ and

$$K_1(z_0) = -\frac{\mathrm{d}}{\mathrm{d}z}K_0(z)\bigg|_{z=z_0} = \mathrm{e}^{-\frac{1}{4}i\pi}\frac{\mathrm{d}}{\mathrm{d}x}K_0(x\sqrt{i}),$$

we have

$$z_0K_1(z_0) = -x\,\mathrm{e}^{\frac{1}{4}i\pi}\mathrm{e}^{-\frac{1}{4}i\pi}\frac{\mathrm{d}}{\mathrm{d}x}K_0(x\sqrt{i})$$

$$= -x\frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{ker}(x) + i\,\mathrm{kei}(x)]. \quad \text{(B.10)}$$

Thus, from (B.9) and (B.10)

$$I(a) = \mathrm{Re}\{z_0K_1(z_0)\} = -x\,\mathrm{ker}'(x) = -2\sqrt{a}\,\mathrm{ker}'(2\sqrt{a}). \quad \text{(B.11)}$$

Finally with (B.11) in (B.1), we have

$$\varepsilon(a) = 4a\,|\mathrm{ker}'(2\sqrt{a})|^2$$

as is shown in Equation (8.6.6).

Bibliography

1. M. Sachs, *General Relativity and Matter* (Reidel, 1982).
2. F. Dyson, *Symmetry Groups in Nuclear and Particle Physics* (Princeton, 1966).
3. K. Huang, *Quarks, Leptons and Gauge Fields* (World Scientific, 1982).
4. E. Leader and E. Predazzi, *An Introduction to Gauge Theories and the New Physics* (Cambridge, 1982).
5. A collection of the key papers in this field is: J. Schwinger, *Quantum Electrodynamics* (Dover, 1958).
6. A classic book on the explication and mathematical development of Quantum Mechanics is: P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford, 1958), Fourth Edition. Seminal papers in this field are reprinted in: B. L. van der Waerden, *Sources of Quantum Mechanics* (Dover, 1967). A recent interesting modification of the standard view of quantum mechanics is: E. Prugovečki, *Stochastic Quantum Mechanics and Quantum Spacetime* (Reidel, 1984).
7. Pauli completed a proof of the exclusion principle, though restricted to noninteracting particles only, in: W. Pauli, *Phys. Rev.* **58**, 716 (1940).
8. For example, see: R. C. Tolman, *The Principles of Statistical Mechanics* (Oxford, 1950), pp. 378, 382.
9. For a summary of their debate, see Bohr's article and Einstein's reply, in: P. A. Schilpp, editor, *Albert Einstein — Philosopher-Scientist* (Library of Living Philosophers, 1949), pp. 201, 666.
10. A clear history of Bohr's development of his complementarity principle is in: M. Jammer, *The Conceptual Development of Quantum Mechanics* (McGraw-Hill, 1966), Sec. 7.2.
11. I have elaborated on the 'elementary interaction' concept in: M. Sachs, *Brit. J. Phil. Sci.* **15**, 213 (1964).
12. A. J. Ayer, *Language, Truth and Logic* (Dover, 1952), p. 5.
13. An exception to the Copenhagen interpretation of 'wave—particle dualism' is that of L. de Broglie — who was one of its principle architects! For his views, see: L. de Broglie, *Recherches d'Un Demi-Siècle* (Albin Michel, 1976); L. de Broglie, *The Current Interpretation of Wave Mechanics* (Elsevier, 1964). For a scholarly discussion of a spectrum of views of the concept of wave—particle dualism, see: S. Diner, D. Fargue, G. Lochak and F. Selleri (eds), *The Wave Particle Dualism* (Reidel, 1984).
14. For a clear exposition of the relation of the formal expression of quantum mechanics to Hilbert space, see: J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton, 1955).
15. An important experimental verification of the prediction of an irreducible interference effect was: R. L. Pfleegor and L. Mandel, *Phys. Rev.* **159**, 1084 (1967). An interesting response was given by: L. de Broglie and J. A. Silva, *Phys. Rev.* **172**, 1284 (1968).
16. Schrödinger's analysis is outlined in: M. Sachs, *Ideas of Matter* (University Press of America, 1981), Chs 6, 7.

17. P. A. M. Dirac, *Lectures on Quantum Field Theory* (Yeshiva, 1966).
18. The question of the logical consistency of the concept of the quantum jump was also discussed (from a different view) by: E. Schrödinger, *Brit. J. Phil. Sci.* **3**, 3 (1952).
19. These difficulties in quantum field theory have been discussed in the literature by many authors, including some of the most creative leaders of developments of this subject. For example, see: P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford, 1958), Fourth edition, Sec. 81; R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and the Path Integral* (McGraw-Hill, 1970), Ch. 9 and p. 260.
20. A. Einstein, 'Autobiographical Notes', in P. A. Schilpp (ed.), *Albert Einstein — Philosopher-Scientist* (Library of Living Philosophers, 1949), p. 59.
21. P. A. M. Dirac, *Scientific American* **208**, 45 (1963).
22. A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.* **47**, 777 (1935) (referred to henceforth as EPR).
23. N. Bohr, *Phys. Rev.* **48**, 696 (1935).
24. See, for example, the following article by E. Schrödinger: "The Meaning of Wave Mechanics", in *Louis de Broglie, Physicien et Penseur* (Albin Michel, 1952).
25. D. Bohm, *Phys. Rev.* **85**, 166; 180 (1952).
26. E. Schrödinger, 'What is an Elementary Particle?', *Smithsonian Institution Report for 1950* (Publication # 4028), pp. 183–196.
27. A. C. Scott, F. Y. F. Chu and D. W. McLaughlin, *Proc. IEEE* **61**, 1443 (1973); N. H. Christ and T. D. Lee, *Phys. Rev. D* **12**, 1606 (1975).
28. For an overview of hidden variable theories, see: F. Belinfante, *Survey of Hidden Variable Theories* (Pergamon, 1975).
29. J. S. Bell, *Physics* **1**, 195 (1965); E. P. Wigner, *Amer. J. Phys.* **38**, 1005 (1970); J. F. Clauser and A. Shimony, *Rep. Prog. Phys.* **41**, 1881 (1978); B. d'Espagnat, *Phys. Rev. D* **11**, 1424 (1975); **18**, 349 (1978); F. Selleri, *Found. Phys.* **8**, 103 (1978).
30. M. Sachs, *Nuovo Cimento* **58A**, 1 (1980).
31. I have analyzed the EPR paradox in the context of a continuum field theory in: M. Sachs, *Int. J. Theor. Phys.* **1**, 387 (1968).
32. For example, see: N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience, 1980), Third Edition, Sec. 1.2.
33. L. P. Eisenhart, *Continuous Groups of Transformations* (Dover, 1961).
34. A. Einstein and W. Mayer, *Preuss. Akad. Wiss. Phys. Math. Klass. Sitz.* (1932), p. 522.
35. S. Okubo, *Prog. Theor. Phys.* **27**, 949 (1962).
36. L. O'Raifeartaigh, *Phys. Rev.* **139**, 258 (1965).
37. M. Sachs, *Int. J. Theor. Phys.* **8**, 377 (1973).
38. E. Mach introduced his interpretation of inertial mass in his treatise: E. Mach, *The Science of Mechanics* (Open Court, 1964). It was named 'the Mach principle' by Einstein. I have discussed this interpretation and its generalization in my book, *GRM* (ref. 1).
39. M. Sachs, *Nuovo Cimento* **53B**, 398 (1968).
40. M. Sachs, *Lettere al Nuovo Cimento* **5**, 947 (1972).
41. M. Sachs, *Lettere al Nuovo Cimento* **21**, 123 (1978).
42. M. Sachs, *Nuovo Cimento* **27**, 977 (1965); **44B**, 289 (1978).
43. C. L. Pekeris, *Phys. Rev.* **115**, 1216 (1959); **126**, 1470 (1962); **127**, 509 (1962).
44. H. Yu and M. Sachs, *Int. J. Theor. Phys.* **13**, 73 (1975).
45. See, for example, W. Heitler, *The Quantum Theory of Radiation* (Oxford, 1944), Second Edition, p. 154.
46. J. H. van Vleck, *Theory of Electric and Magnetic Susceptibilities* (Oxford, 1932), p. 178.
47. M. Sachs and S. L. Schwebel, *Nuovo Cimento Suppl.* **21**, 197 (1961).
48. See, for example, L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, 1949), Ch. 12.
49. The measured Lamb shift Δ_1 ($n = 2$) is from: V. G. Pal'chikov, Yu. L. Sokolov and V. P. Yakovlev, *JETP Lett.* **38**, 418 (1983); Δ_2 ($n = 3$) is from: C. W. Fabjan and F. M.

Pipkin, *Phys. Rev. A* **6**, 556 (1972); and Δ_3 ($n = 4$) is from: R. A. Brown and F. M. Pipkin, *Ann. Phys. (NY)* **80**, 479 (1973).
50. See, for example, H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, 1957).
51. N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford, 1949), Second Edition.
52. G. N. Watson, *Theory of Bessel Functions* (Cambridge, 1945).
53. M. Sachs and S. L. Schwebel, *Nuclear Physics* **43**, 204 (1963).
54. This analysis has also been applied to high-energy proton-nucleus scattering, where 'dips' in the cross section were predicted with locations depending on a mixing of the generalized electromagnetic interaction parameters, e^2, g_M and the nuclear parameter μ of the Yukawa potential. See M. Sachs, *Lettere al Nuovo Cimento* **27**, 586 (1980). Similar 'dips' were predicted in the e—He^4 scattering cross section in: M. Sachs, *Nuovo Cimento* **53A**, 561 (1968).
55. C. G. Darwin, *Proc. Roy. Soc.* **A118**, 654 (1928).
56. P. P. Debye and E. Hückel, *Zeit. Physik* **24**, 185 (1923).
57. M. Sachs, *Nuovo Cimento* **7B**, 247 (1972).
58. M. Sachs, *Nuovo Cimento* **59A**, 113 (1980).
59. M. Sachs, *Lettere al Nuovo Cimento* **10B**, 339 (1972).
60. M. Sachs, *Nuovo Cimento* **43A**, 74 (1978).
61. M. Sachs, *Lettere al Nuovo Cimento* **24**, 577 (1979).
62. E. Fermi, *Prog. Theor. Phys.* **5**, 570 (1950).
63. M. Sachs, *Phys. Rev.* **103**, 671 (1956).
64. C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes and R. P. Hudson, *Phys. Rev.* **105**, 1413 (1957).
65. J. W. Cronin, *Rev. Mod. Phys.* **53**, 373 (1981).
66. V. L. Fitch, *Rev. Mod. Phys.* **53**, 367 (1981).
67. See, for example, G. Lüders, *Ann. Phys (NY)* **2**, 1 (1957).
68. M. Gell-Mann and A. Pais, *Phys. Rev.* **97**, 1387 (1955).
69. J. A. Wheeler and R. P. Feynman, *Rev. Mod. Phys.* **17**, 157 (1945).
70. R. J. Slobodrian, C. Rioux, R. Roy, H. E. Conzett, P. von Rossen and F. Hinterberger, *Phys. Rev. Lett.* **47**, 1803 (1981).
71. C. J. Gorter, *Physica (the Hague)* **14**, 504 (1948).
72. M. E. Rose, *Phys. Rev.* **75**, 213 (1949).
73. E. Ambler, M. A. Grace, H. Halban, N. Kurti, N. Durand, C. E. Johnson and H. R. Lemmer, *Phil. Mag.* **44**, 216 (1953).
74. H. A. Kramers, *Proc. Acad. Amsterdam* **33**, 959 (1930).
75. M. Sachs, *Solid State Theory* (McGraw-Hill, 1963, Dover reprinting, 1972), Appendix A.
76. E. P. Wigner, *Zeit. Physik* **43**, 624 (1927); C. Eckart, *Rev. Mod. Phys.* **2**, 305 (1930).
77. A survey of the key articles on the Glashow—Salam—Weinberg theory is reproduced in: C. H. Lai, editor, *Gauge Theory of Weak and Electromagnetic Interactions* (World Scientific, 1981). (Referred to hereafter as GSW).
78. A. Sirlin, *Phys. Rev. D* **22**, 971 (1980); W. J. Marciano and A. Sirlin, *Phys. Rev. D* **22**, 2695 (1980).
79. M. Sachs, *Lettere al Nuovo Cimento* **32**, 307 (1981).
80. Two notable scholars who have taken this view are: L. de Broglie, *Recherches d'un Demi-Siècle* (Albin Michel, 1976), p. 15 and I. Prigogine, *From Being to Becoming* (Freeman, 1980).
81. Well-known 19th-century theoreticians who were concerned with this problem were: J. Loschmidt, *Wien. Sitzungsber.* **73**, 139 (1876); *ibid.* **75**, 67 (1877) and E. Zermelo, *Ann. Physik* **57**, 585 (1896); *ibid.* **59**, 793 (1896).

82. For the full text of this prayer, see: J. S. Minkin, *The World of Moses Maimonides* (Yoseloff, 1957), p. 149.
83. H. Batėman, *Tables of Integral Transforms* (McGraw-Hill, 1954), Vol. I, p. 185.
84. H. Bateman, *Tables of Integral Transforms* (McGraw-Hill, 1954), Vol. II, p. 7 (transform no. 8).

Index