

- L12 - Introduction to Protein Structure;  
Structure Comparison & Classification
- L13 - Predicting protein structure
- L14 - Predicting protein interactions
- L15 - Gene Regulatory Networks
- L16 - Protein Interaction Networks
- L17 - Computable Network Models

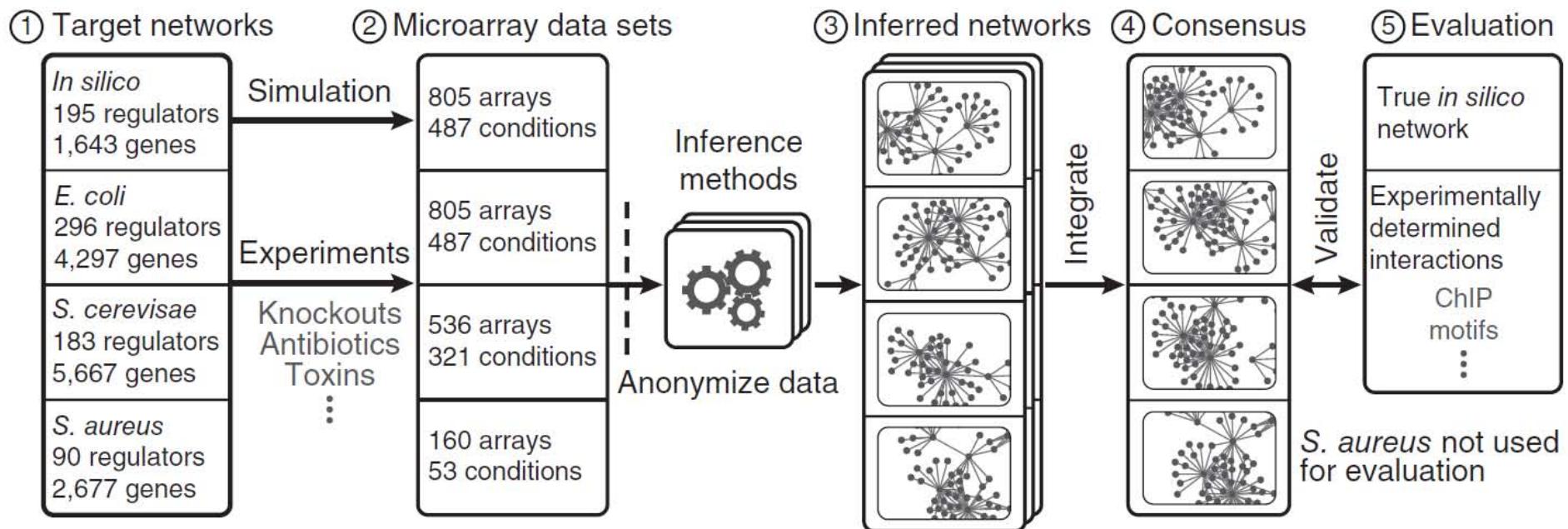
# Wisdom of crowds for robust gene network inference

Daniel Marbach, James C Costello, Robert Küffner, Nicole M Vega, Robert J Prill, Diogo M Camacho, Kyle R Allison, The DREAM5 Consortium, Manolis Kellis, James J Collins & Gustavo Stolovitzky

[Affiliations](#) | [Contributions](#) | [Corresponding author](#)

*Nature Methods* 9, 796–804 (2012) | doi:10.1038/nmeth.2016

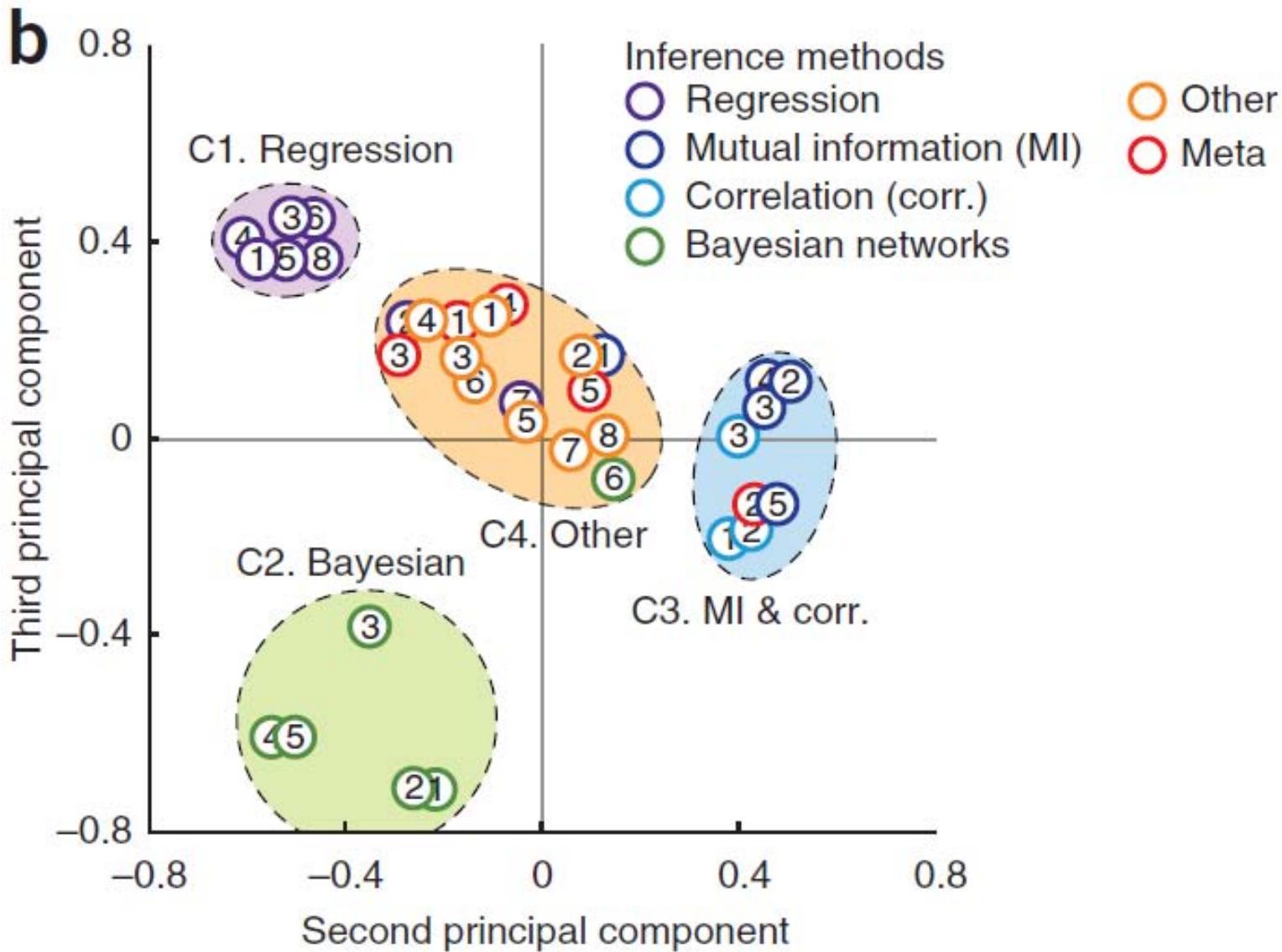
Received 31 October 2011 | Accepted 22 May 2012 | Published online 15 July 2012



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Source: Marbach, Daniel, James C. Costello, et al. "[Wisdom of Crowds for Robust Gene Network Inference](#)." *Nature Methods* 9, no. 8 (2012): 796-804.

**Wisdom of crowds for robust gene network inference**  
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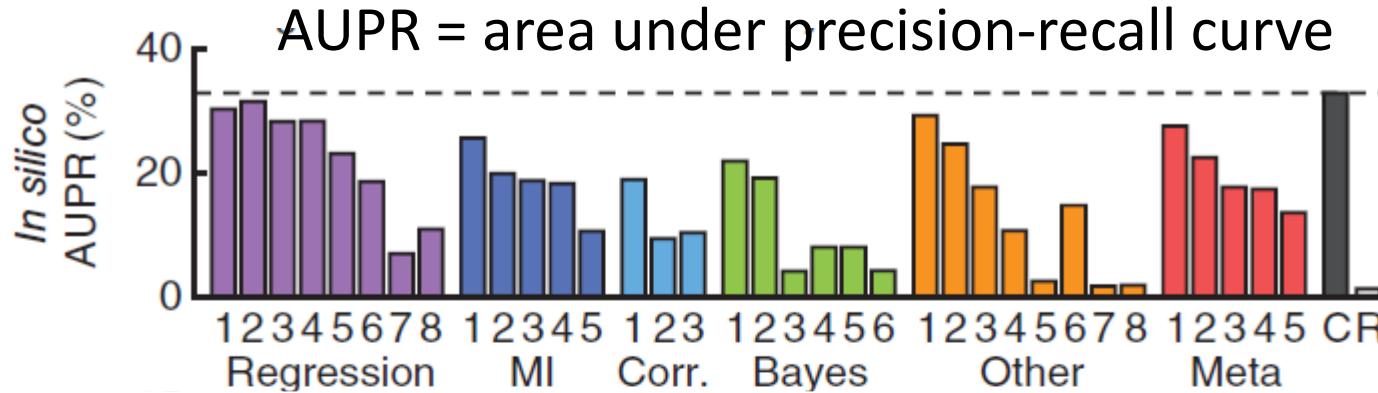
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Source: Marbach, Daniel, James C. Costello, et al. "Wisdom of Crowds for Robust Gene Network Inference." *Nature Methods* 9, no. 8 (2012): 796-804.

## Wisdom of crowds for robust gene network inference

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## Area under precision-recall curve



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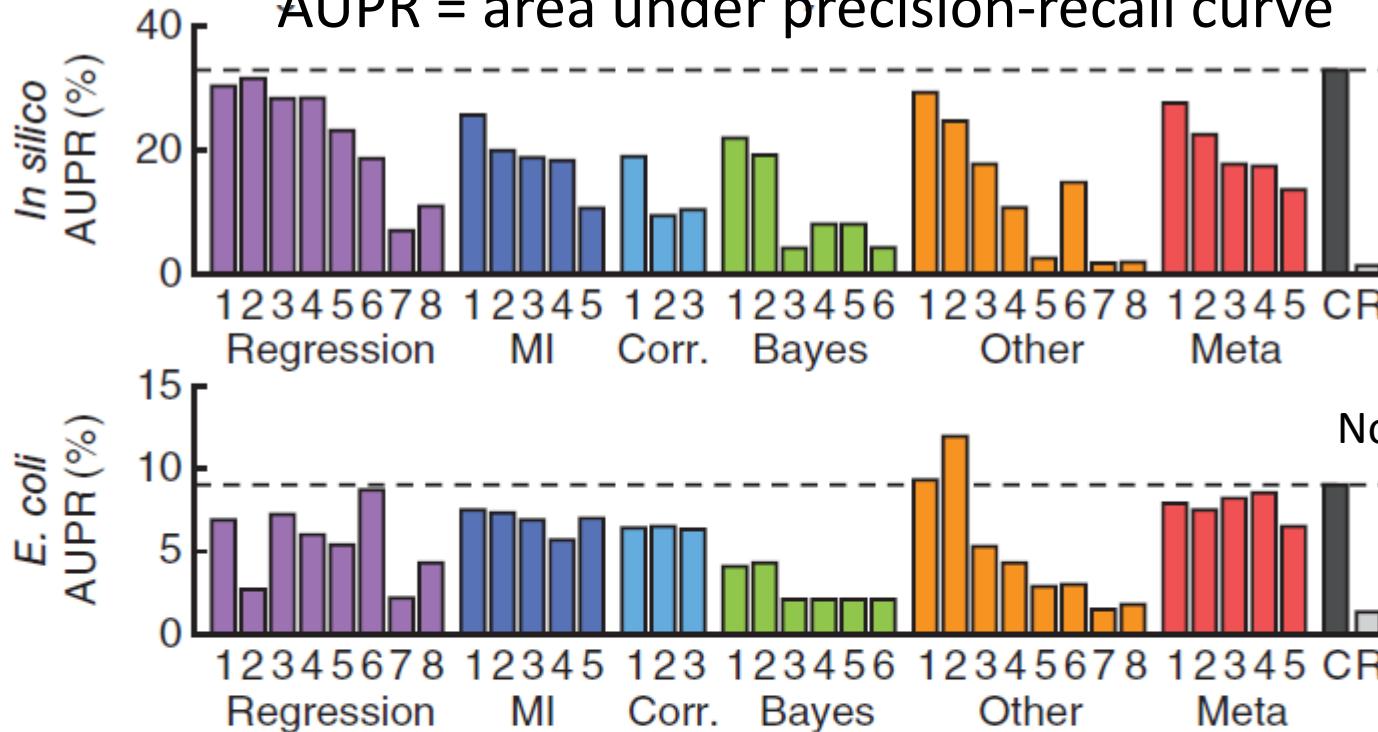
Source: Marbach, Daniel, James C. Costello, et al. "[Wisdom of Crowds for Robust Gene Network Inference](#)." *Nature Methods* 9, no. 8 (2012): 796-804.

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AUPR = area under precision-recall curve

Area under precision-recall curve

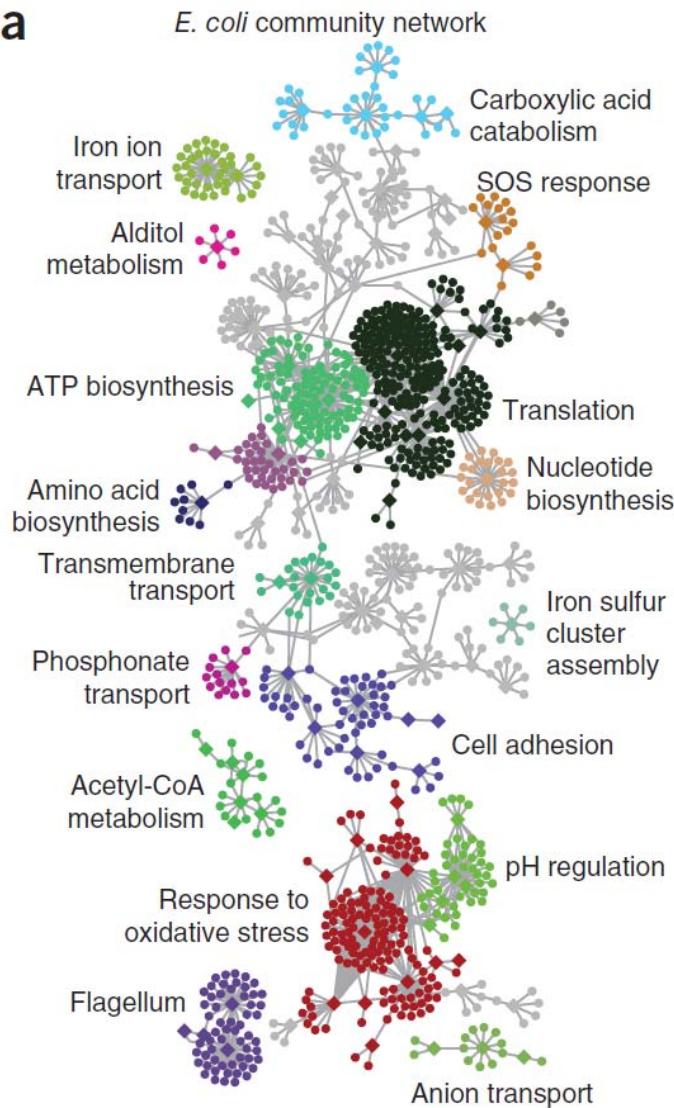


Note change of scale!

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Source: Marbach, Daniel, James C. Costello, et al. "[Wisdom of Crowds for Robust Gene Network Inference](#)." *Nature Methods* 9, no. 8 (2012): 796-804.

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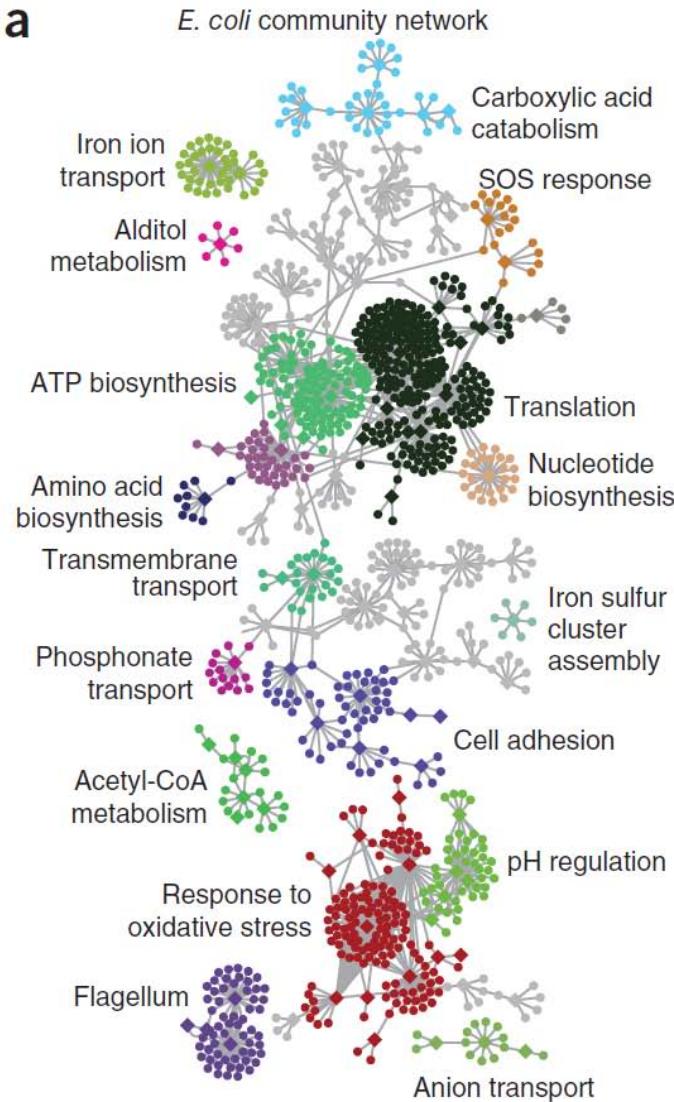
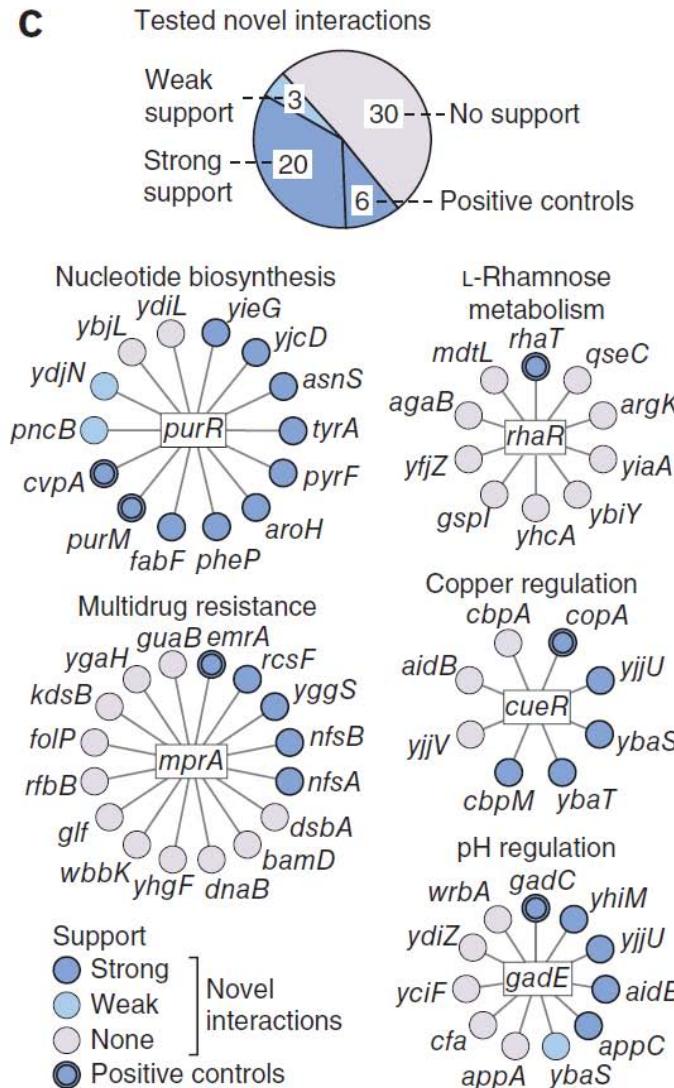
**a**

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Source: Marbach, Daniel, James C. Costello, et al. "[Wisdom of Crowds for Robust Gene Network Inference](#)." *Nature Methods* 9, no. 8 (2012): 796-804.

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**a****c**

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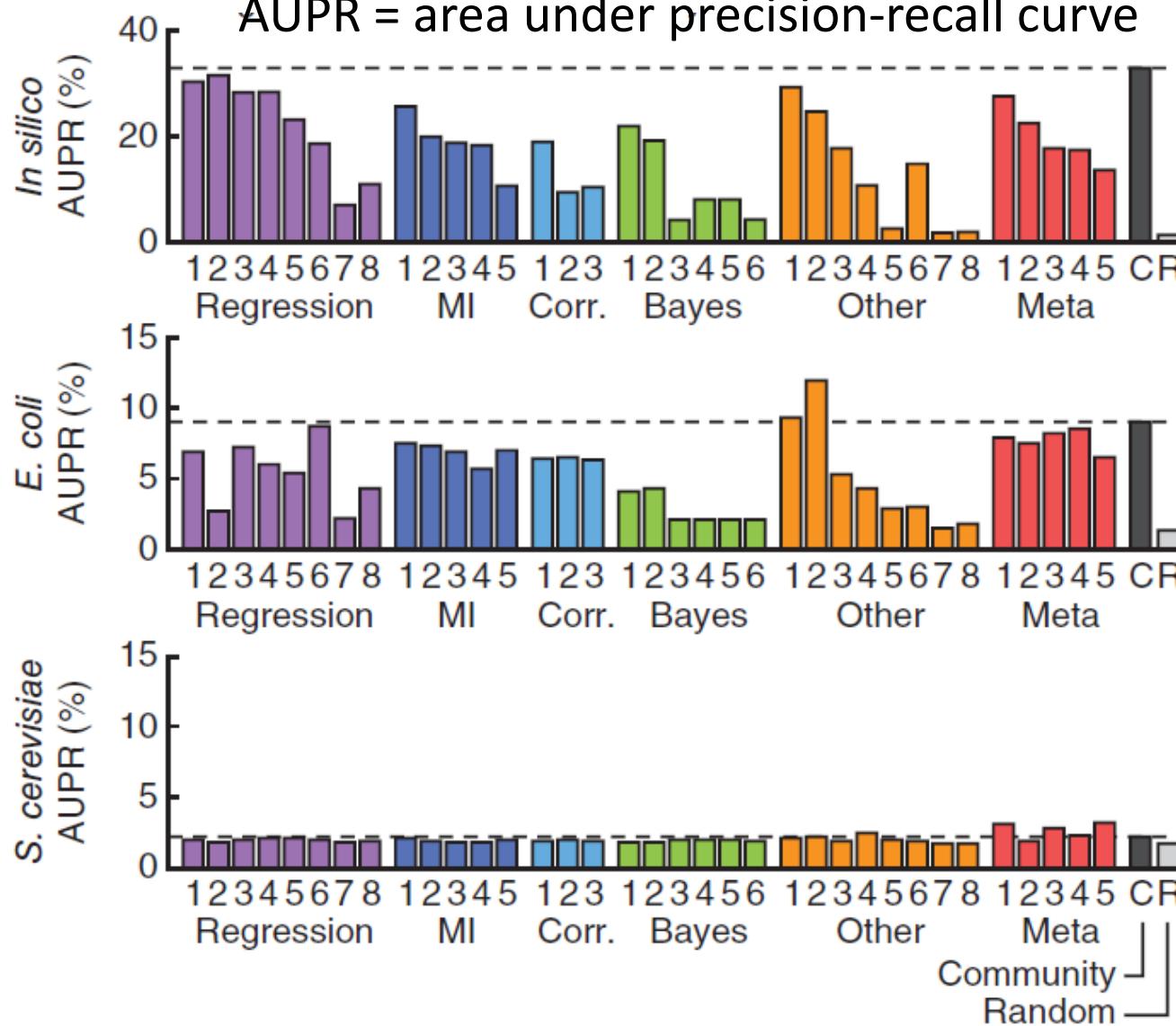
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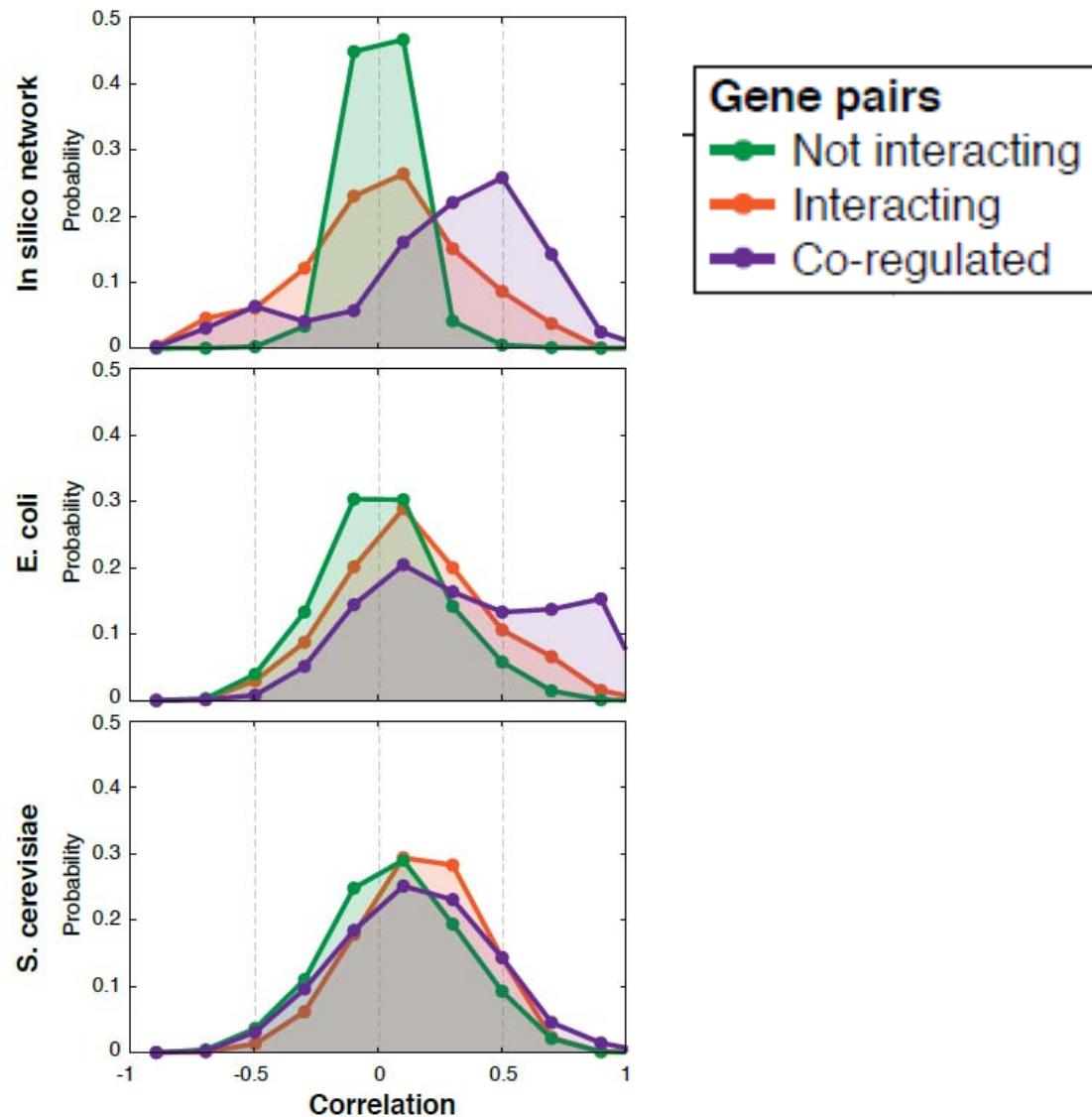


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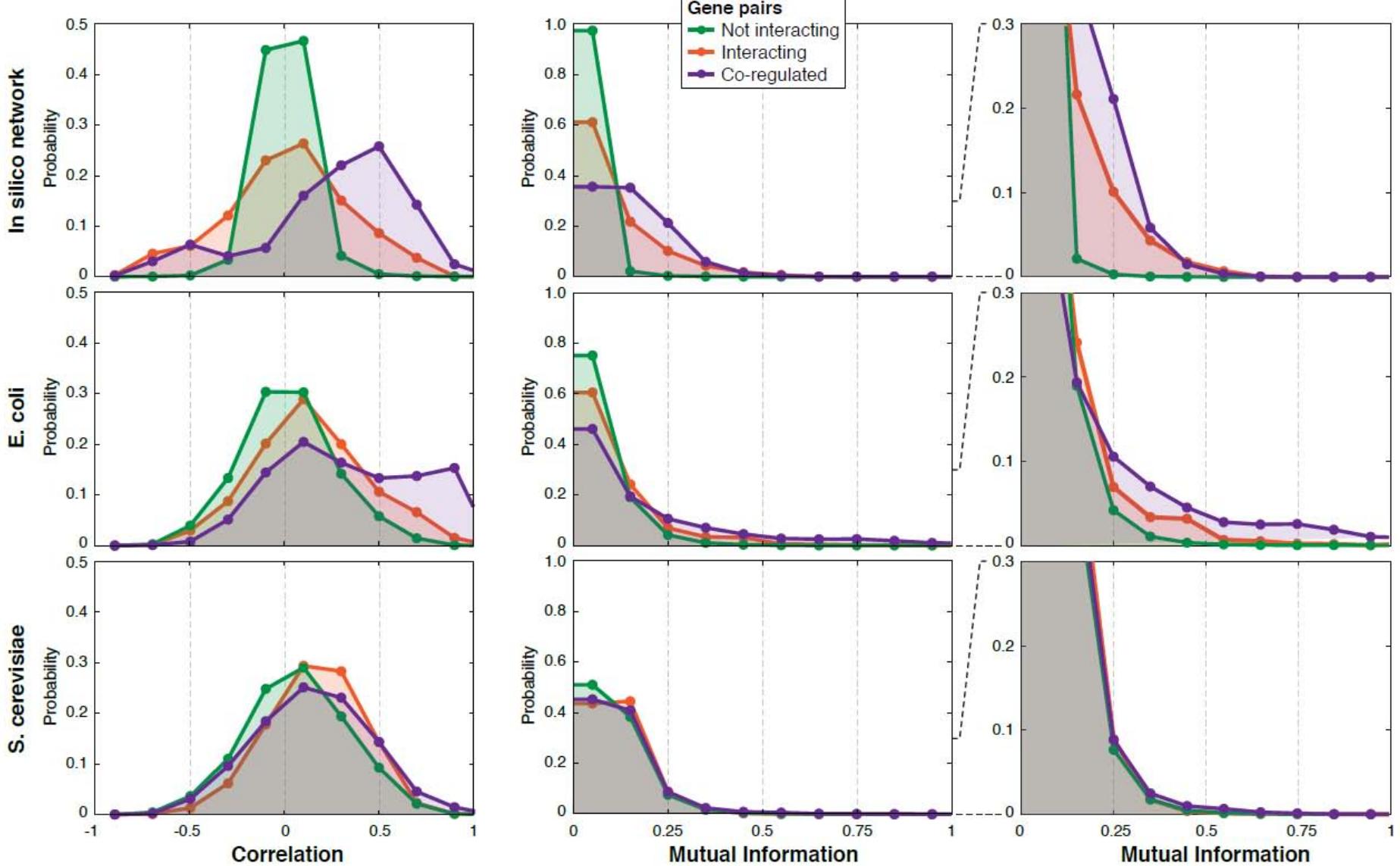
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Source: Marbach, Daniel, James C. Costello, et al. "[Wisdom of Crowds for](#)

[Robust Gene Network Inference.](#)" *Nature Methods* 9, no. 8 (2012): 796-804.

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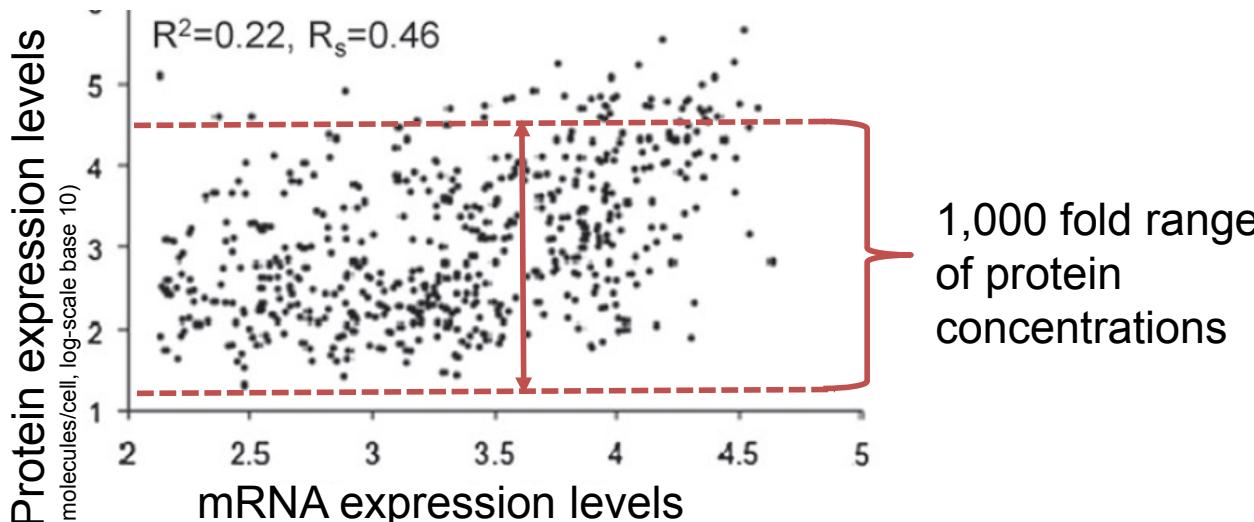
**Wisdom of crowds for robust gene network inference**  
*Nature Methods* 9, 796–804 (2012) doi:10.1038/nmeth.2016

# Thoughts on Gene Expression Data

- Useful for classification and clustering
- Not sufficient for reconstructing regulatory networks in yeast
- Can we infer levels of proteins from gene expression?

# Approach

## mRNA levels do not predict protein levels



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Source: de Sousa Abreu, Raquel, Luiz O. Penalva, et al. "Global Signatures of Protein and mRNA Expression Levels." *Molecular Biosystems* 5, no. 12 (2009): 1512-26.

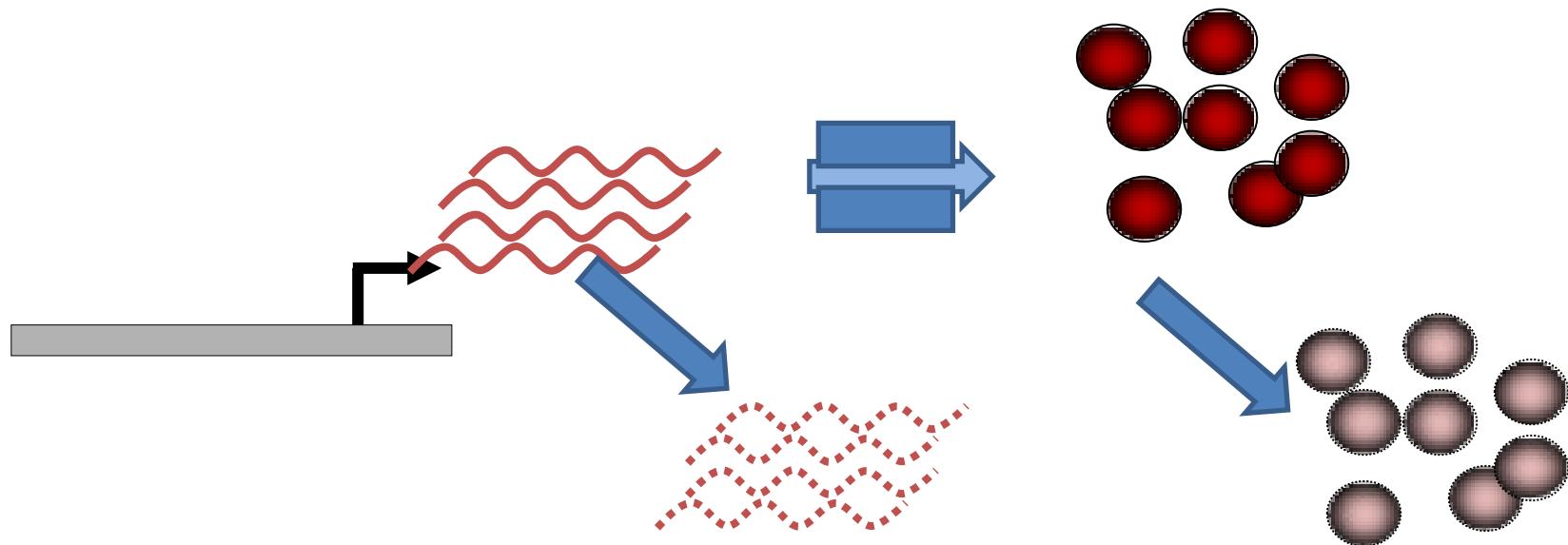
(arbitrary units, log-scale base 10)

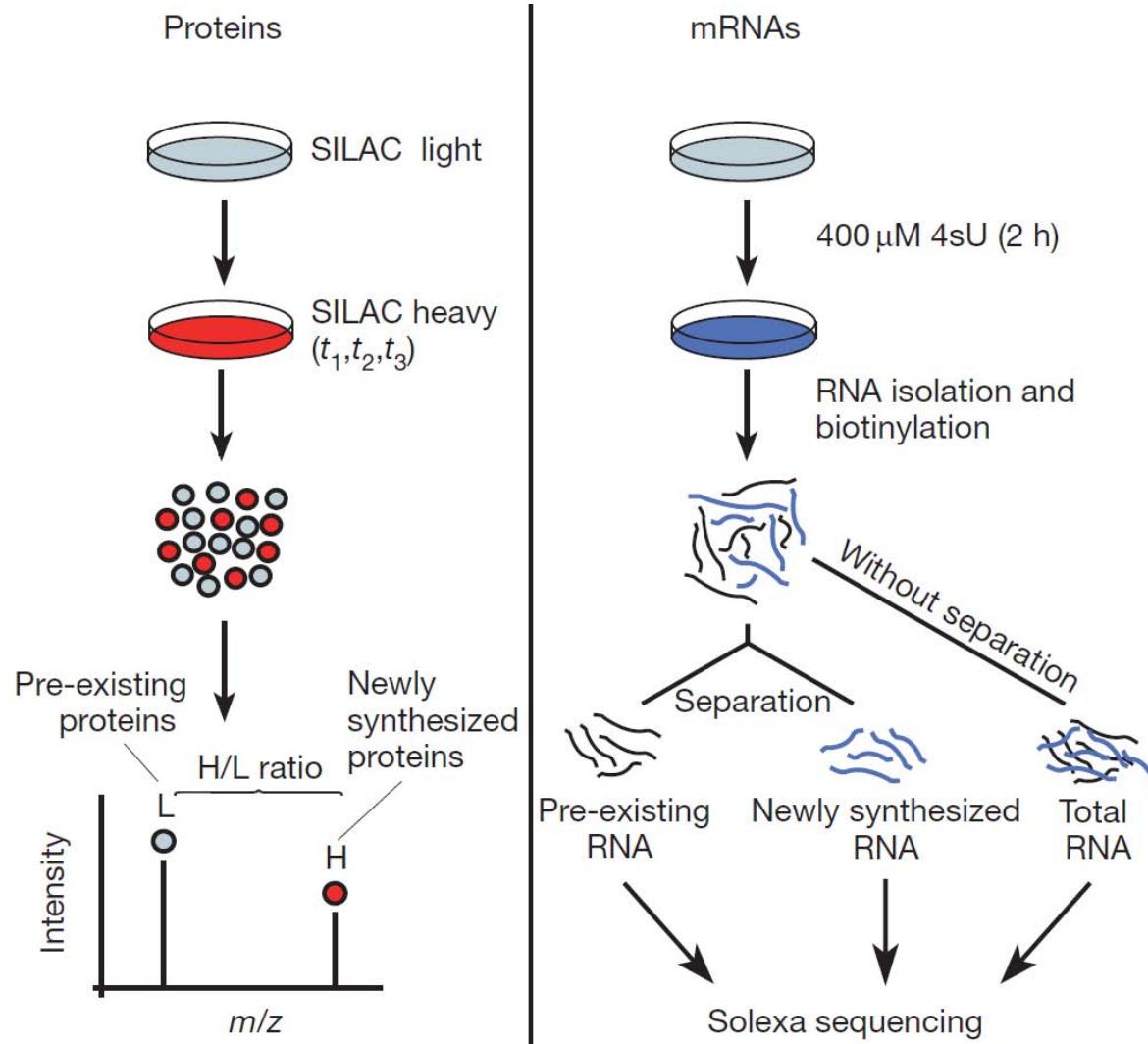
Raquel de Sousa Abreu, Luiz Penalva, Edward Marcotte and Christine Vogel, *Mol. BioSyst.*, 2009 DOI: [10.1039/b908315d](https://doi.org/10.1039/b908315d)

	<b>SpectrumMill</b>	<b>msInspect</b>	<b>msBID</b>	<b>NSAF</b>	<b>RPKM</b>	<b>Microarray</b>
<b>SpectrumMill</b>	-	0.91 (0.92)	0.91 (0.91)	0.90 (0.90)	0.49 (0.51)	0.36 (0.40)
<b>msInspect</b>	0.91 (0.92)	-	0.89 (0.91)	0.87 (0.88)	0.51 (0.53)	0.40 (0.44)
<b>msBID</b>	0.91 (0.91)	0.89 (0.91)	-	0.84 (0.89)	0.54 (0.54)	0.41 (0.42)
<b>NSAF</b>	0.90 (0.90)	0.87 (0.88)	0.84 (0.89)	-	0.51 (0.53)	0.42 (0.44)

Source: Ning, Kang, Damian Fermin, et al. "Comparative Analysis of Different Label-free Mass Spectrometry Based Protein Abundance Estimates and Their Correlation with RNA-Seq Gene Expression Data." *Journal of Proteome Research* 11, no. 4 (2012): 2261-71.

Kang Ning, Damian Fermin, and Alexey I. Nesvizhskii J Proteome Res. 2012 April 6; 11(4): 2261–2271.



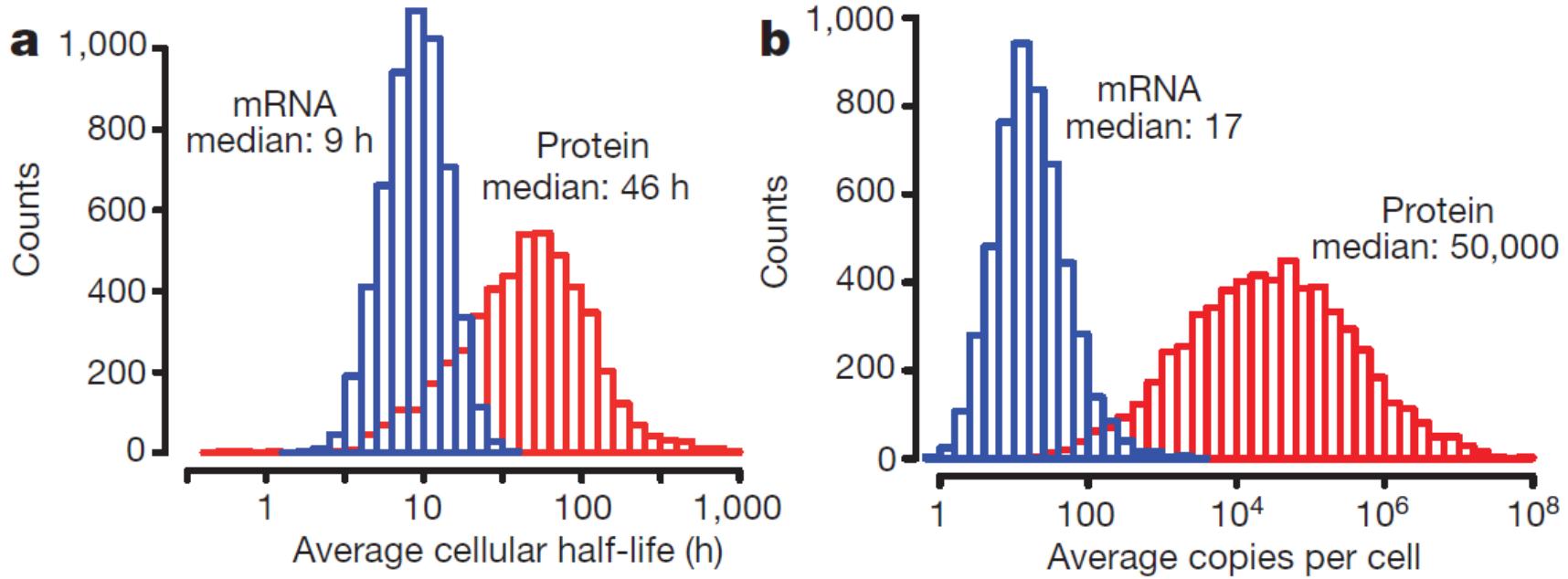


Courtesy of Macmillan Publishers Limited. Used with permission.  
 Source: Schwahnäusser, Björn, Dorothea Busse, et al. "Global Quantification of Mammalian Gene Expression Control." *Nature* 473, no. 7347 (2011): 337-42.

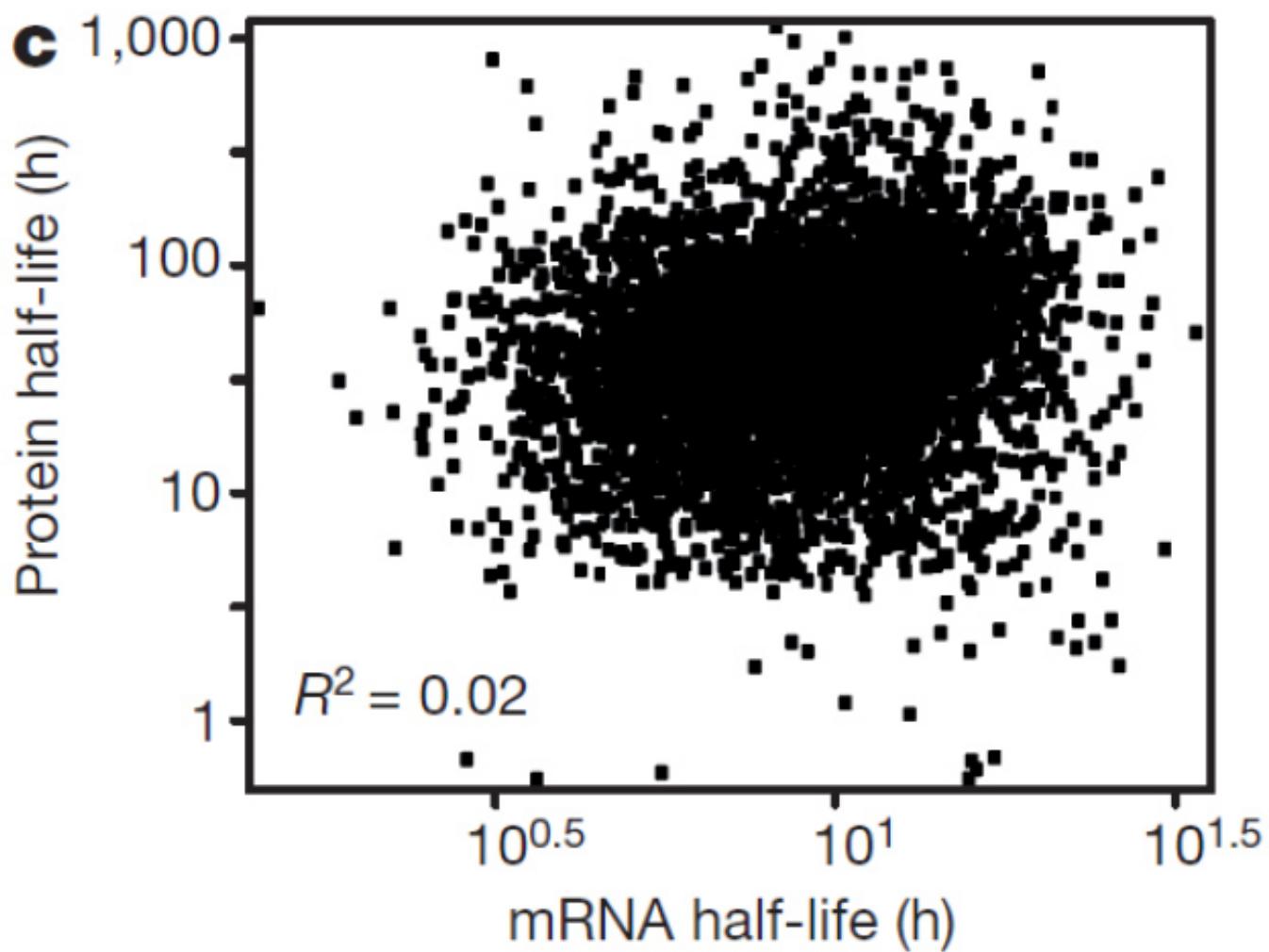
Nature. 2011 May 19;473(7347):337-42. doi: 10.1038/nature10098.

Global quantification of mammalian gene expression control.

Schwahnäusser B1, Busse D, Li N, Dittmar G, Schuchhardt J, Wolf J, Chen W, Selbach M.



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 Source: Schwahnäusser, Björn, Dorothea Busse, et al. "[Global Quantification of Mammalian Gene Expression Control](#)." *Nature* 473, no. 7347 (2011): 337-42.



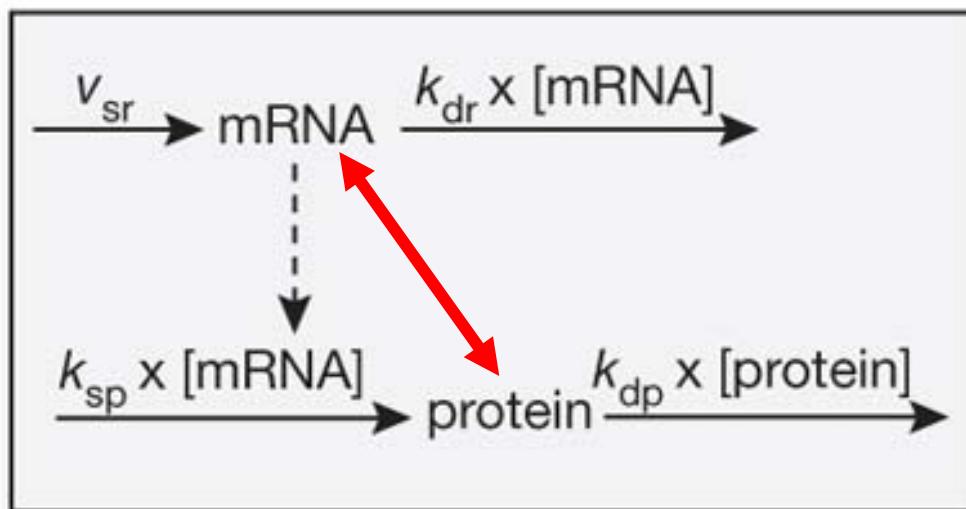
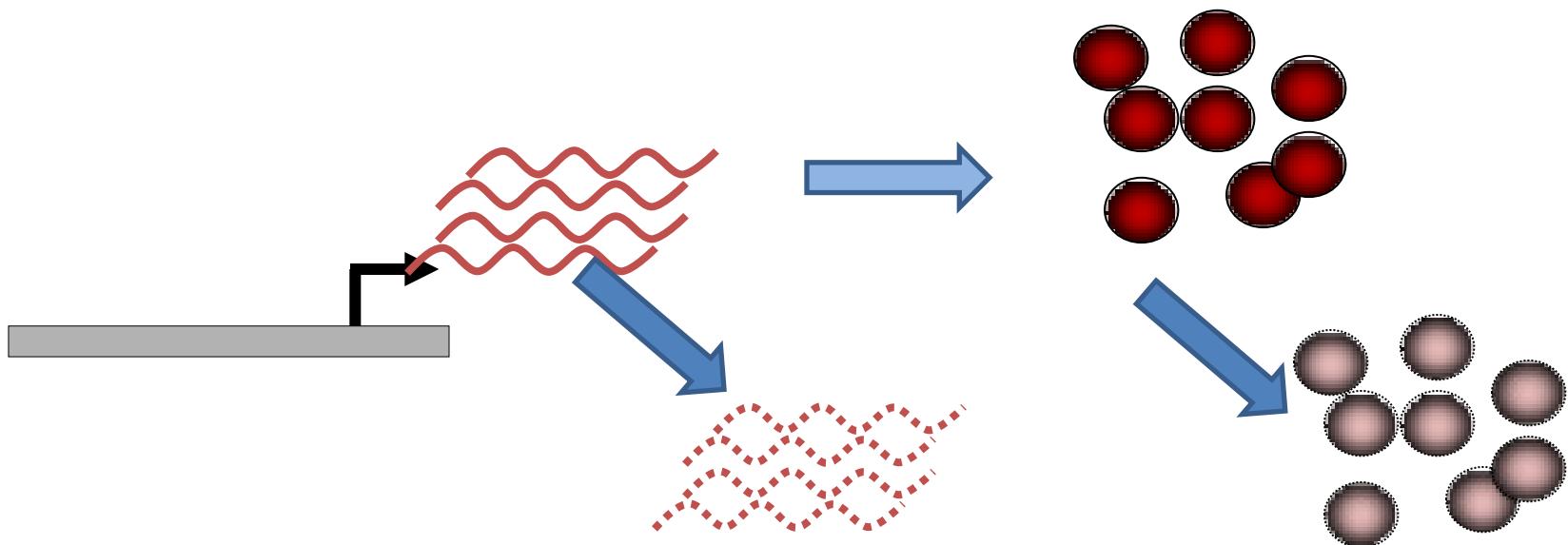
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Source: Schwahnhäuser, Björn, Dorothea Busse, et al. "Global Quantification of Mammalian Gene Expression Control." *Nature* 473, no. 7347 (2011): 337-42.

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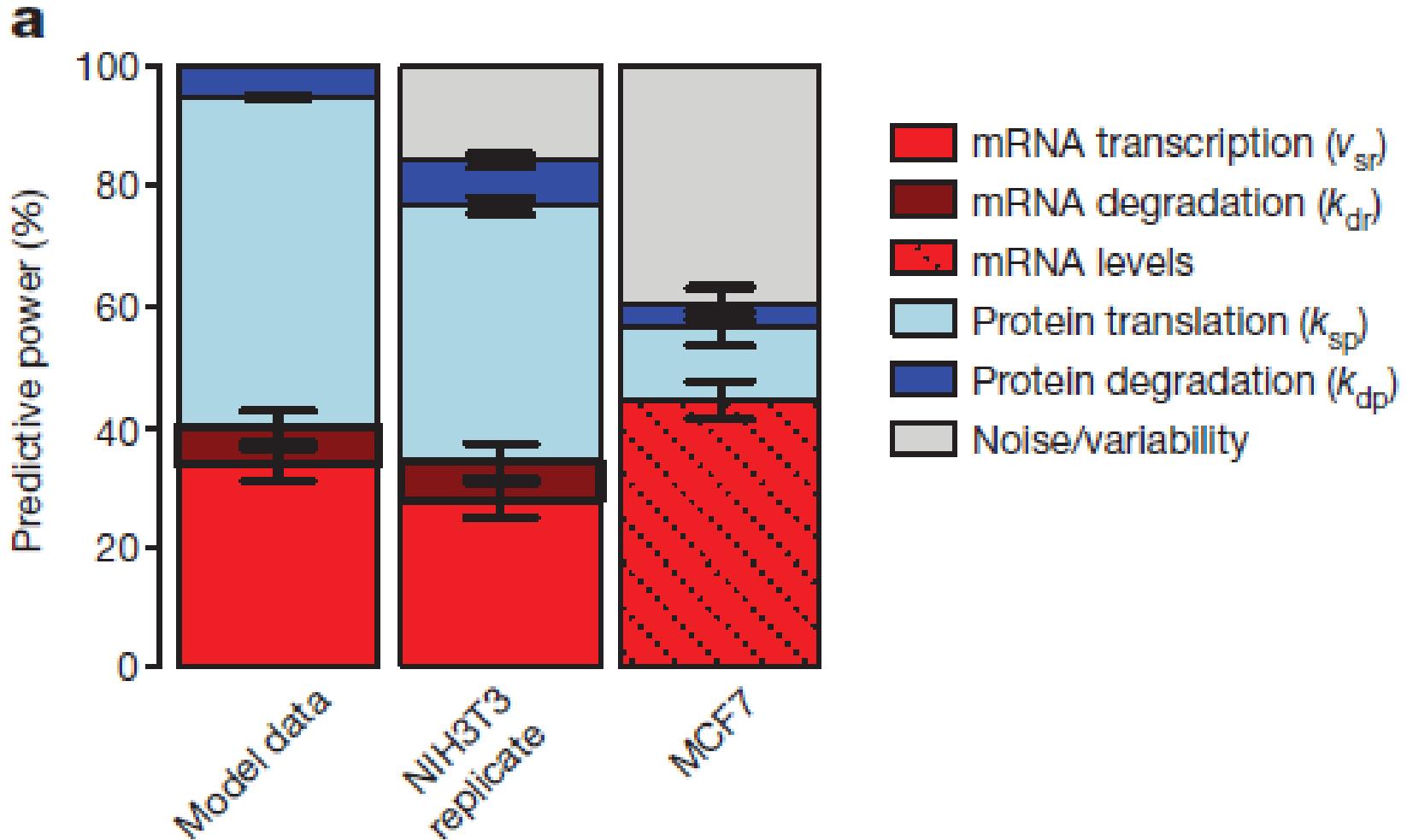
$$\frac{dR}{dt} = v_{sr} - k_{dr}R$$

$$\frac{dP}{dt} = k_{sp}R - k_{dp}P$$

Nature. 2011 May 19;473(7347):337-42. doi: 10.1038/nature10098.

Global quantification of mammalian gene expression control.

Schwanhäusser B<sup>1</sup>, Busse D, Li N, Dittmar G, Schuchhardt J, Wolf J, Chen W, Selbach M.



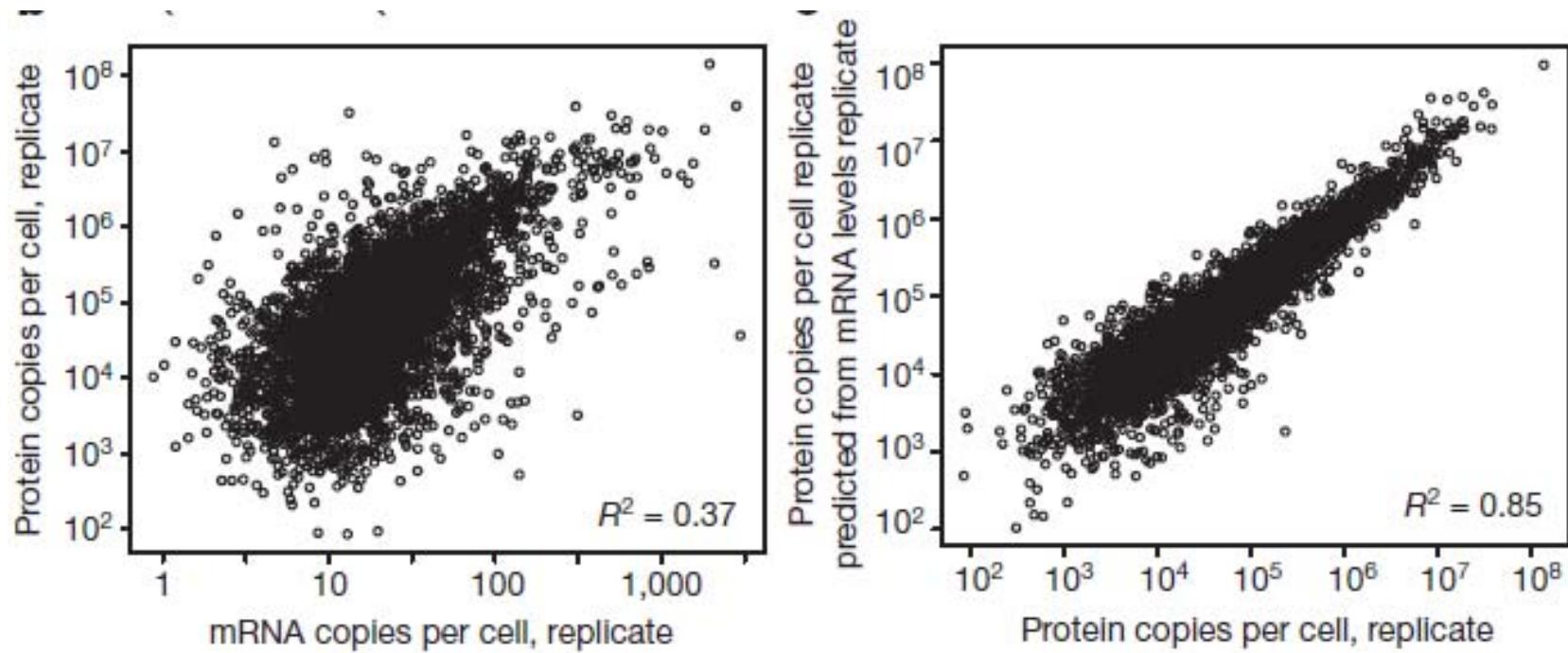
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Source: Schwanhäusser, Björn, Dorothea Busse, et al. "Global Quantification of Mammalian Gene Expression Control." *Nature* 473, no. 7347 (2011): 337-42.

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Source: Schwahnhäuser, Björn, Dorothea Busse, et al. "[Global Quantification of Mammalian Gene Expression Control](#)." *Nature* 473, no. 7347 (2011): 337-42.

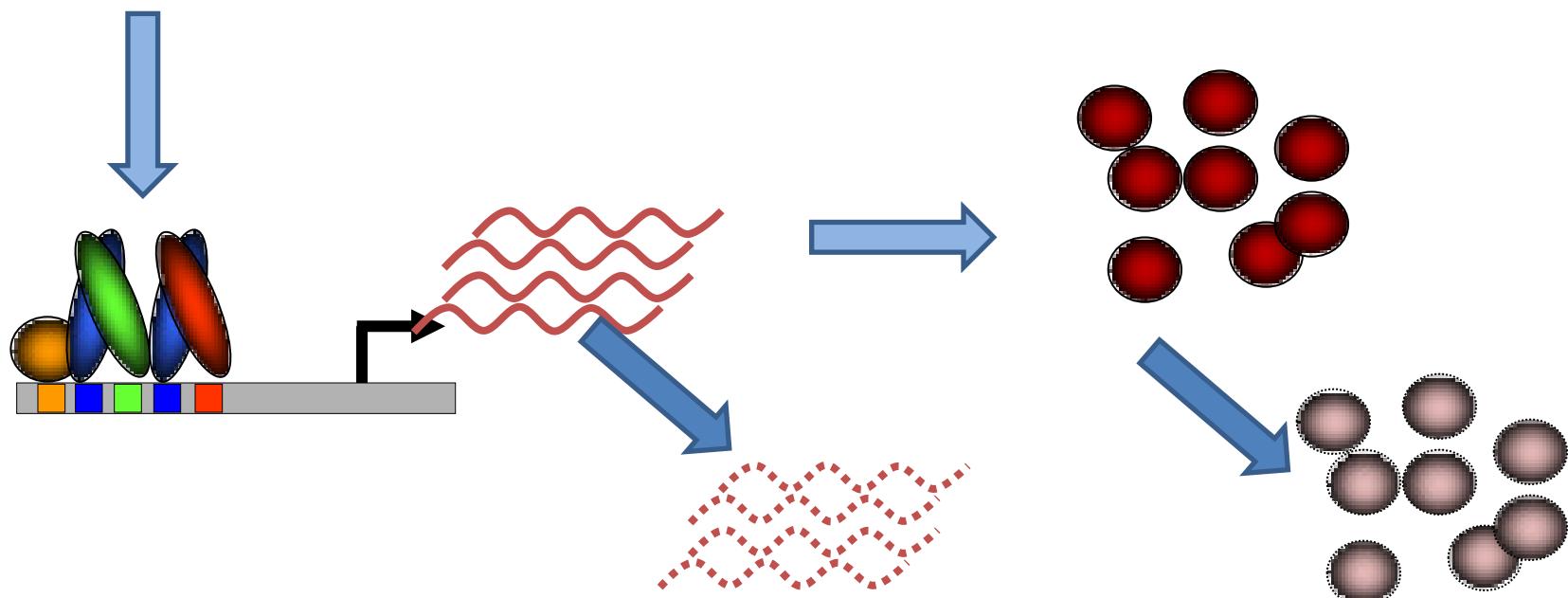
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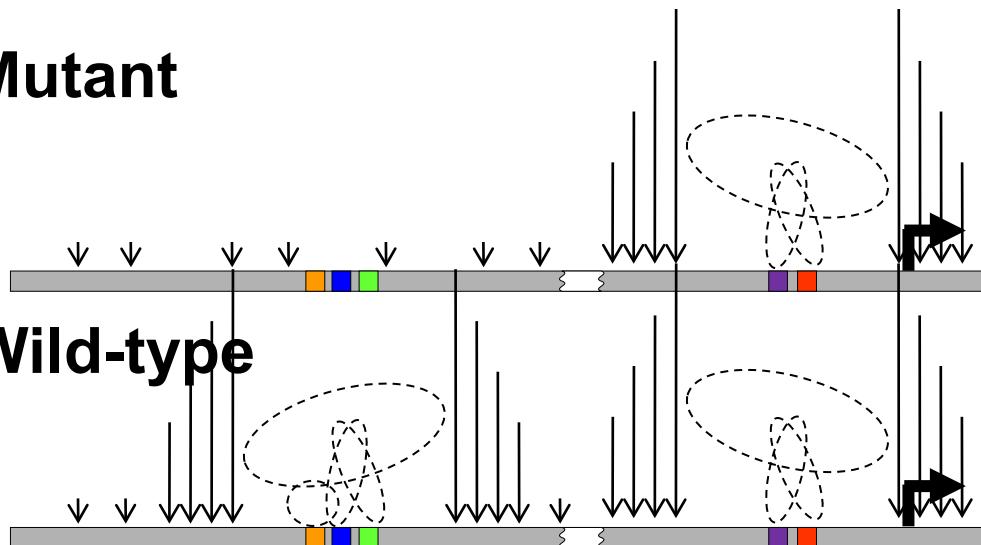
# Strategies:

1. Use expression to infer upstream events
2. Explicitly model downstream steps



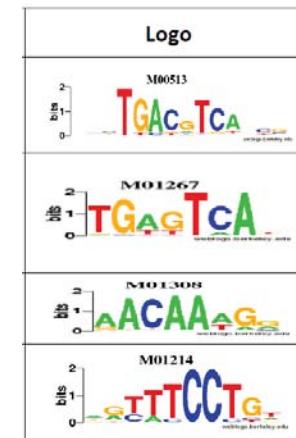
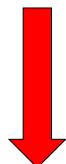
# L18 Chromatin and DNase-seq Analysis

**Mutant**



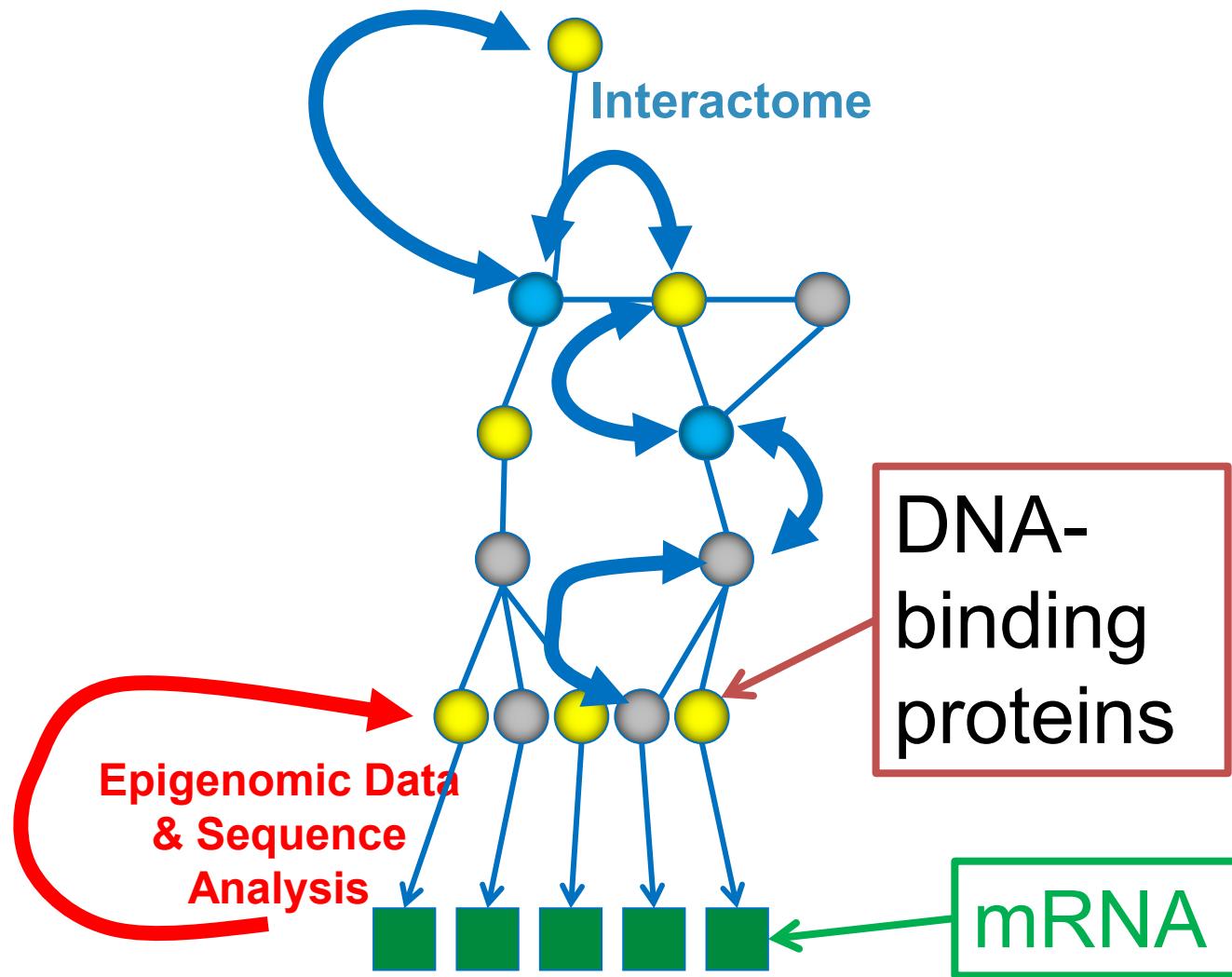
**Wild-type**

Sequence  
Analysis



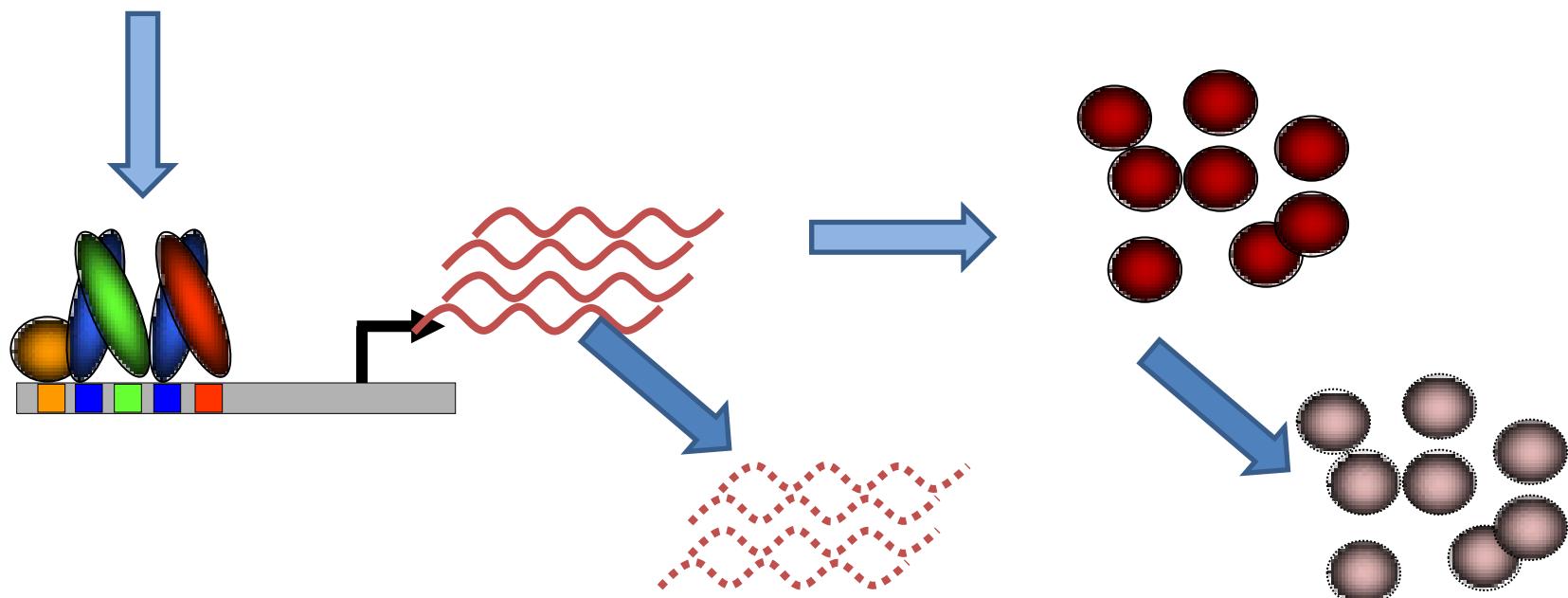
# Move upstream of transcription

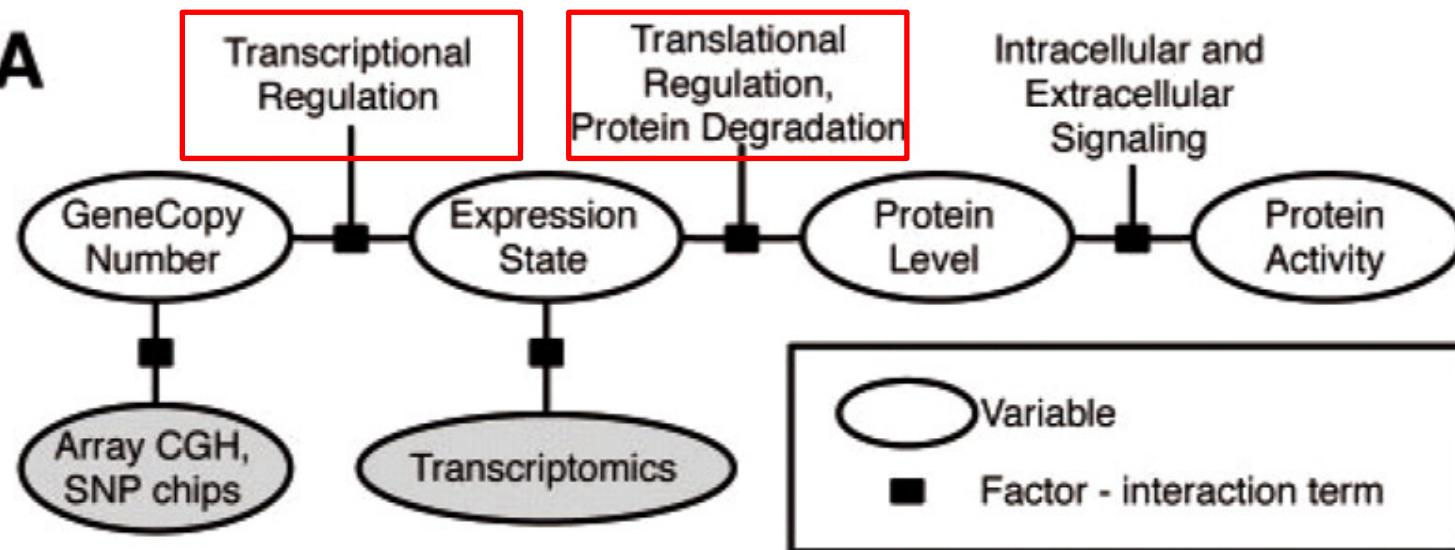
Network  
integration



# Strategies:

1. Use expression to infer upstream events
2. Explicitly model downstream steps



**A**

Courtesy of Vaske et al. License: CC-BY.

Source: Vaske, Charles J., Stephen C. Benz, et al. "Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM." *Bioinformatics* 26, no. 12 (2010): i237-i45.

Vaske C J et al. *Bioinformatics* 2010;26:i237-i245

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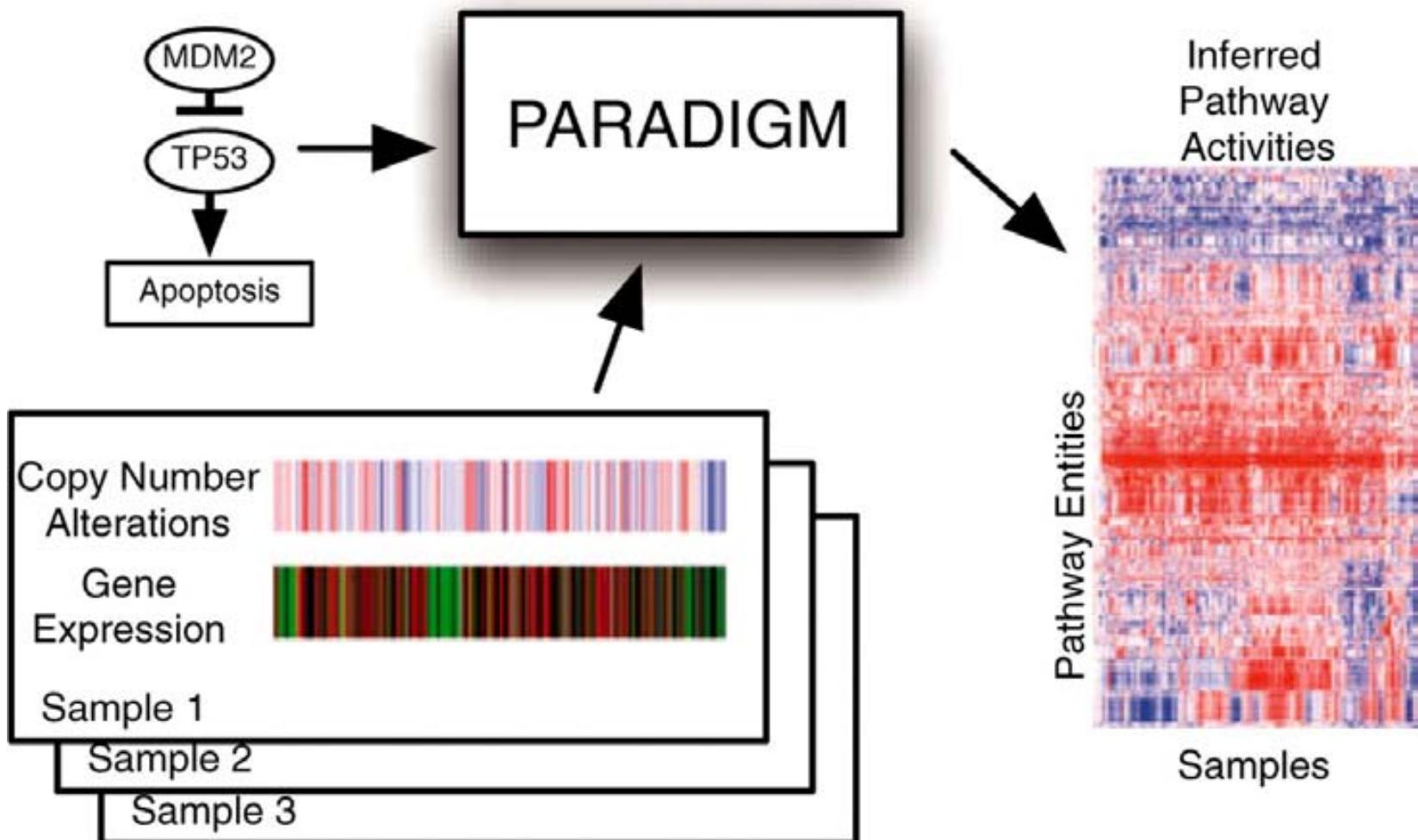
## Inference of patient-specific pathway activities from multi-dimensional cancer genomics data using PARADIGM

Charles J. Vaske<sup>1,†</sup>, Stephen C. Benz<sup>2,†</sup>, J. Zachary Sanborn<sup>2</sup>, Dent Earl<sup>2</sup>, Christopher Szeto<sup>2</sup>, Jingchun Zhu<sup>2</sup>, David Haussler<sup>1,2</sup> and Joshua M. Stuart<sup>2,\*</sup>

<sup>1</sup>Howard Hughes Medical Institute and <sup>2</sup>Department of Biomolecular Engineering and Center for Biomolecular Science and Engineering, UC Santa Cruz, CA, USA

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## Overview of the PARADIGM method.

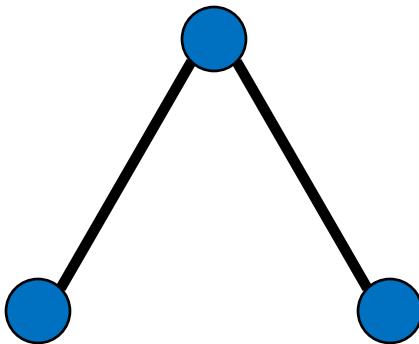


Vaske C J et al. *Bioinformatics* 2010;26:i237-i245

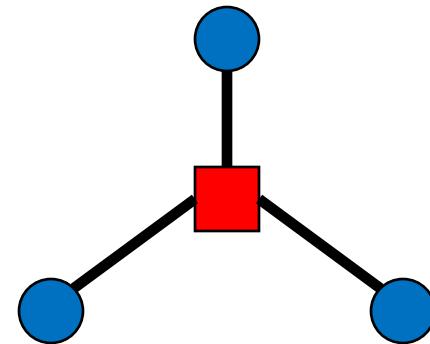
Courtesy of Vaske et al. License: CC-BY.

Source: Vaske, Charles J., Stephen C. Benz, et al. "Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM." *Bioinformatics* 26, no. 12 (2010): i237-i45.

# Factor graphs generalize Bayesian networks



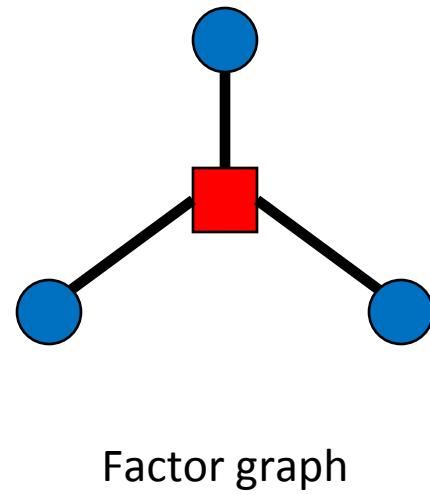
**Bayesian network**



**Factor graph**

# Factor graphs

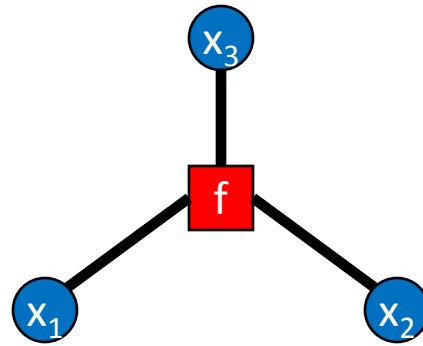
- Bipartite graph  
(means there are two types of nodes)
- Describes how a global function can be factored into a product of local functions
- Bayesian networks are a type of factor graph



# Factor graphs

Global function of the variables :  $g(x_1, x_2, x_3) = \prod_{j \in J} f_j(X_j)$

- Variable node,  $x$
- Factor node,  $f$
- Edge exists  
iff  $x$  is an argument of  $f$

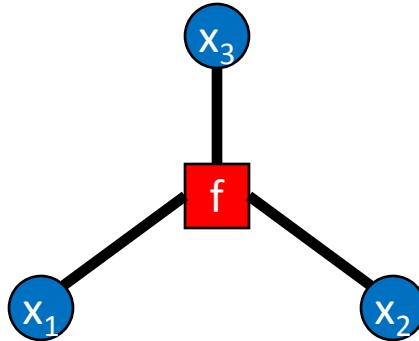


Factor graph

# Factor graphs

- A node for:
  - $x_1$  – every variable and
  - $f$  – every function  $f_j(X_j)$
- Node  $x_i$  is connected to factor  $f_j$  iff  
the variable  $x_i$  appears as a term in  $f_j$

$$g(x_1, x_2, x_3) = \prod_{j \in J} f_j(X_j)$$



Factor graph

# In our setting

Joint probability function :  $P(x_1, x_2, x_3) = \prod_{j \in J} f_j(X_j)$



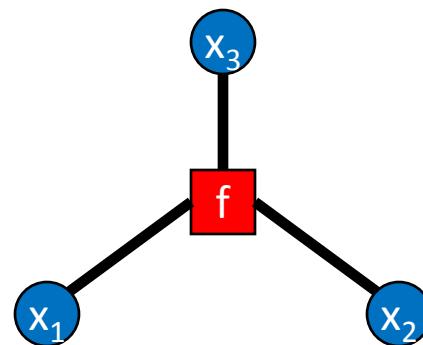
Variable node,  $x$  = state of gene/protein/pathway



Factor node,  $f$  describes relationships



Edge exists iff  $x$  is an argument of  $f$

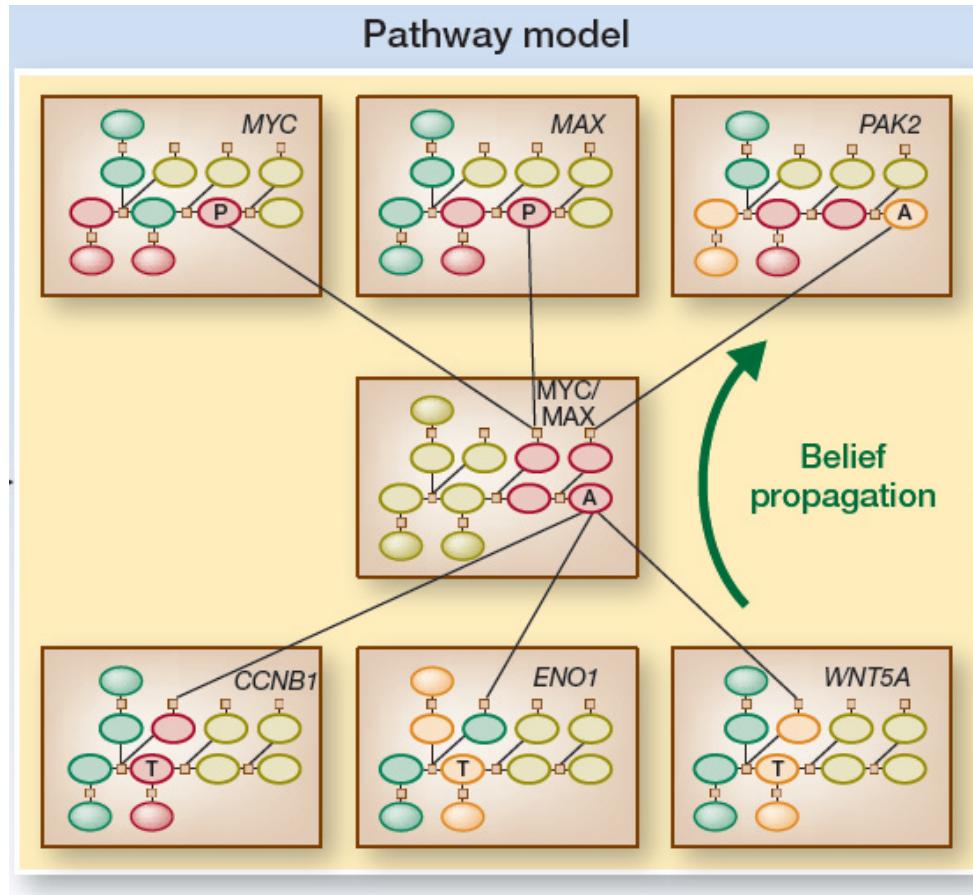


Factor graph

Global function:  $g(x_1, x_2, x_3, x_4, x_5)$

Marginal  $g_i(a)$  : sum  $g(x_1, x_2, x_3, x_4, x_5)$

over all configurations of the variables with  $x_i=a$



What is the probability that MYC/MAX is active?  
 $P(x_i=\text{active})$

Factor graphs provide a method to compute such marginals

## Global function:

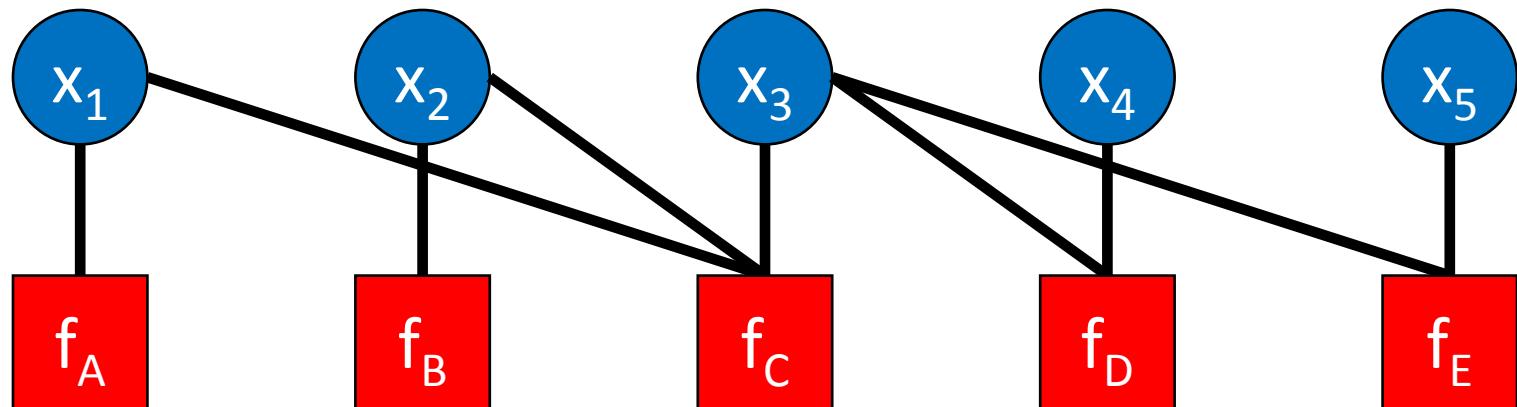
$$g(x_1, x_2, x_3, x_4, x_5) = f_A(x_1)f_B(x_2)f_C(x_1, x_2, x_3)f_D(x_3, x_4)f_E(x_3, x_5)$$

Marginal  $g_i(a)$  : sum  $g(x_1, x_2, x_3, x_4, x_5)$

over all configurations of the variables with  $x_i=a$

$$g_1(x_1) = f_A(x_1) \times$$

$$\left( \sum_{x_2} f_B(x_2) \left( \sum_{x_3} f_C(x_1, x_2, x_3) \left( \sum_{x_4} f_D(x_3, x_4) \left( \sum_{x_5} f_E(x_3, x_5) \right) \right) \right) \right)$$



## Global function:

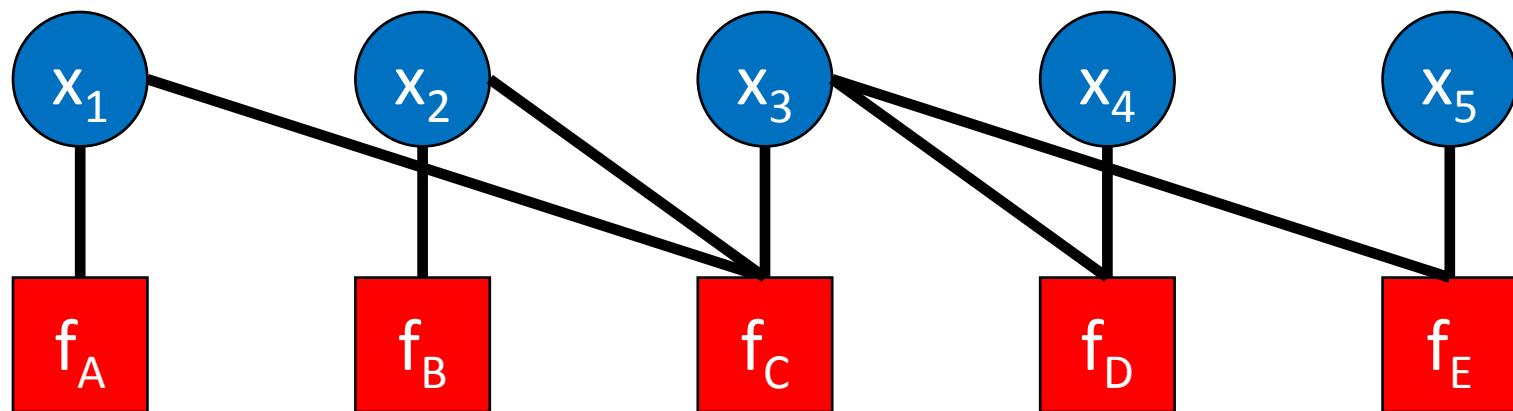
$$g(x_1, x_2, x_3, x_4, x_5) = f_A(x_1)f_B(x_2)f_C(x_1, x_2, x_3)f_D(x_3, x_4)f_E(x_3, x_5)$$

Marginal  $g_i(a)$  : sum  $g(x_1, x_2, x_3, x_4, x_5)$

over all configurations of the variables with  $x_i=a$

$$g_i(x_i) = \sum_{\sim\{x_i\}} g(x_1, x_2, x_3, x_4, x_5)$$

“not-sum” or summary  
over all values of  $x_{j \neq i}$



## Global function:

$$g(x_1, x_2, x_3, x_4, x_5) = f_A(x_1) f_B(x_2) f_C(x_1, x_2, x_3) f_D(x_3, x_4) f_E(x_3, x_5)$$

Marginal  $g_i(a)$  : sum  $g(x_1, x_2, x_3, x_4, x_5)$

over all configurations of the variables with  $x_i=a$

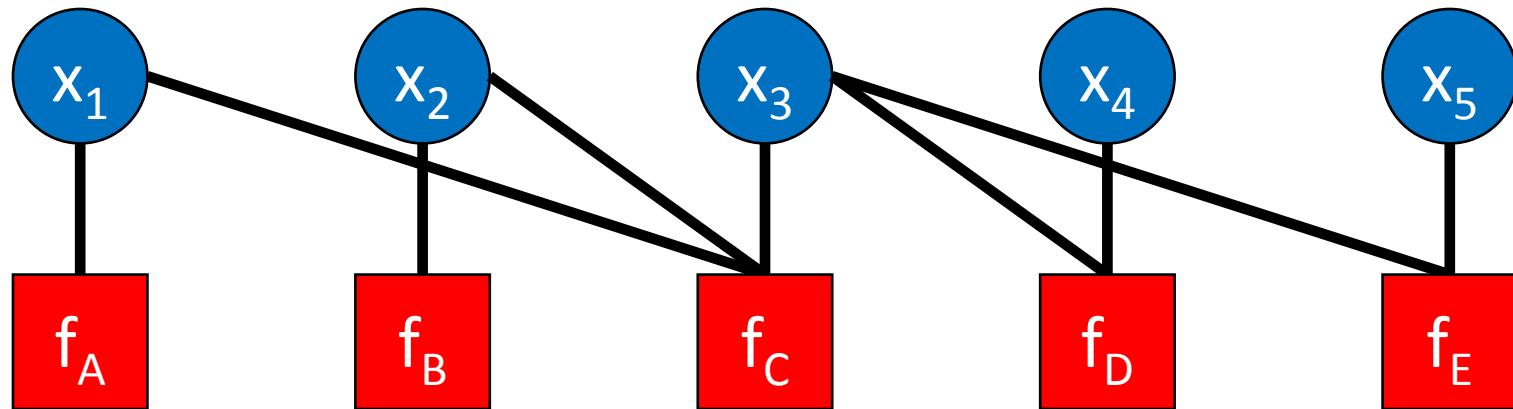
$$g_1(x_1) = f_A(x_1) \times \\ \left( \sum_{x_2} f_B(x_2) \left( \sum_{x_3} f_C(x_1, x_2, x_3) \left( \sum_{x_4} f_D(x_3, x_4) \right) \left( \sum_{x_5} f_E(x_3, x_5) \right) \right) \right)$$

$$g_1(x_1) = f_A(x_1) \times \\ \sum_{\sim\{x_1\}} \left( f_B(x_2) f_C(x_1, x_2, x_3) \left( \sum_{\sim\{x_3\}} f_D(x_3, x_4) \right) \left( \sum_{\sim\{x_3\}} f_E(x_3, x_5) \right) \right)$$

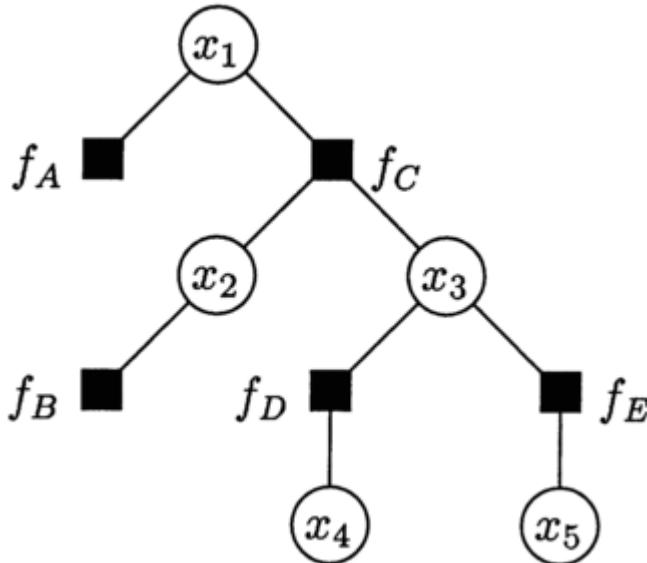
## Global function:

$$g(x_1, x_2, x_3, x_4, x_5) = f_A(x_1)f_B(x_2)f_C(x_1, x_2, x_3)f_D(x_3, x_4)f_E(x_3, x_5)$$

How do we find the marginal for any factor graph?

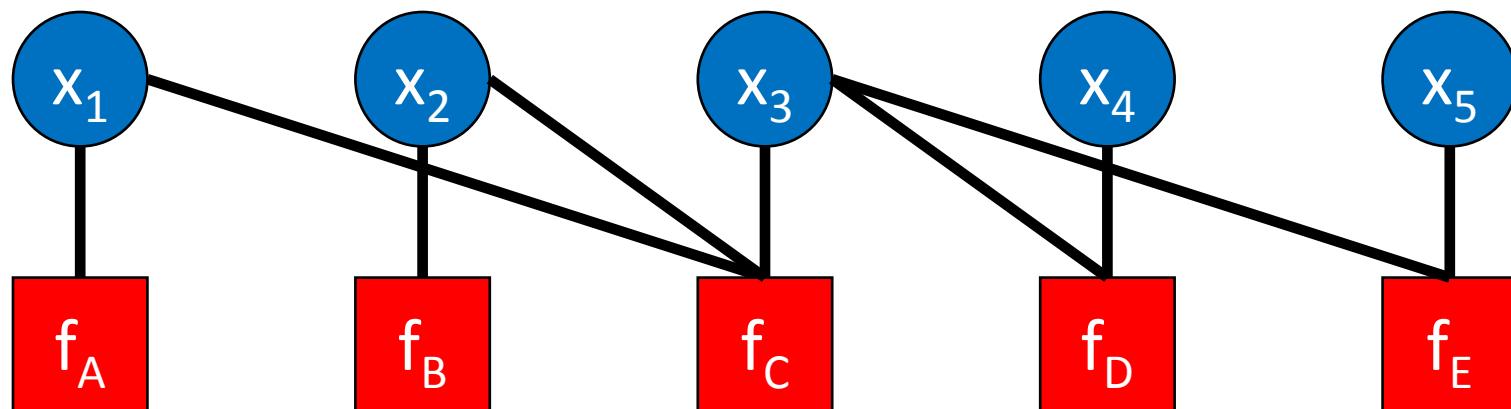


To compute the marginal with respect to variable  $x_i$ :  
draw the factor graph as a tree with root  $x_i$

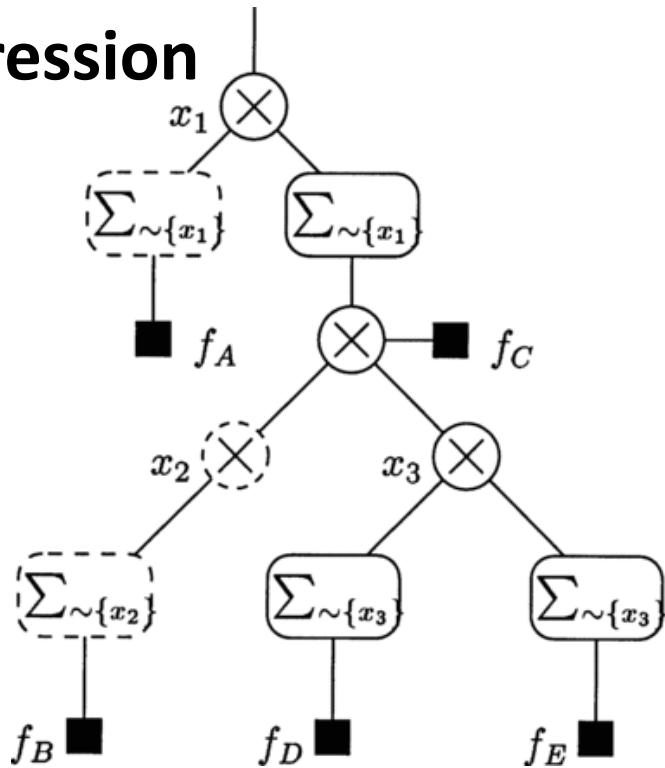


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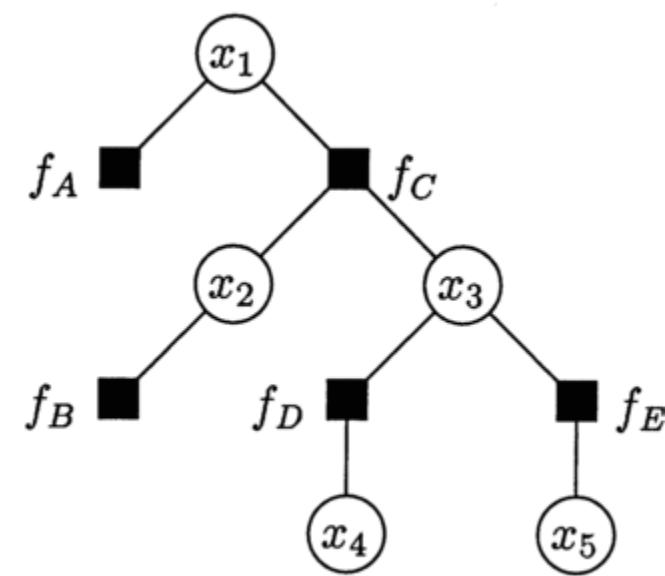
Source: Kschischang, Frank R., Brendan J. Frey, et al. "Factor Graphs and the Sum-product Algorithm." *Information Theory, IEEE Transactions on* 47, no. 2 (2001): 498-519.



# Expression Tree



# Factor Graph



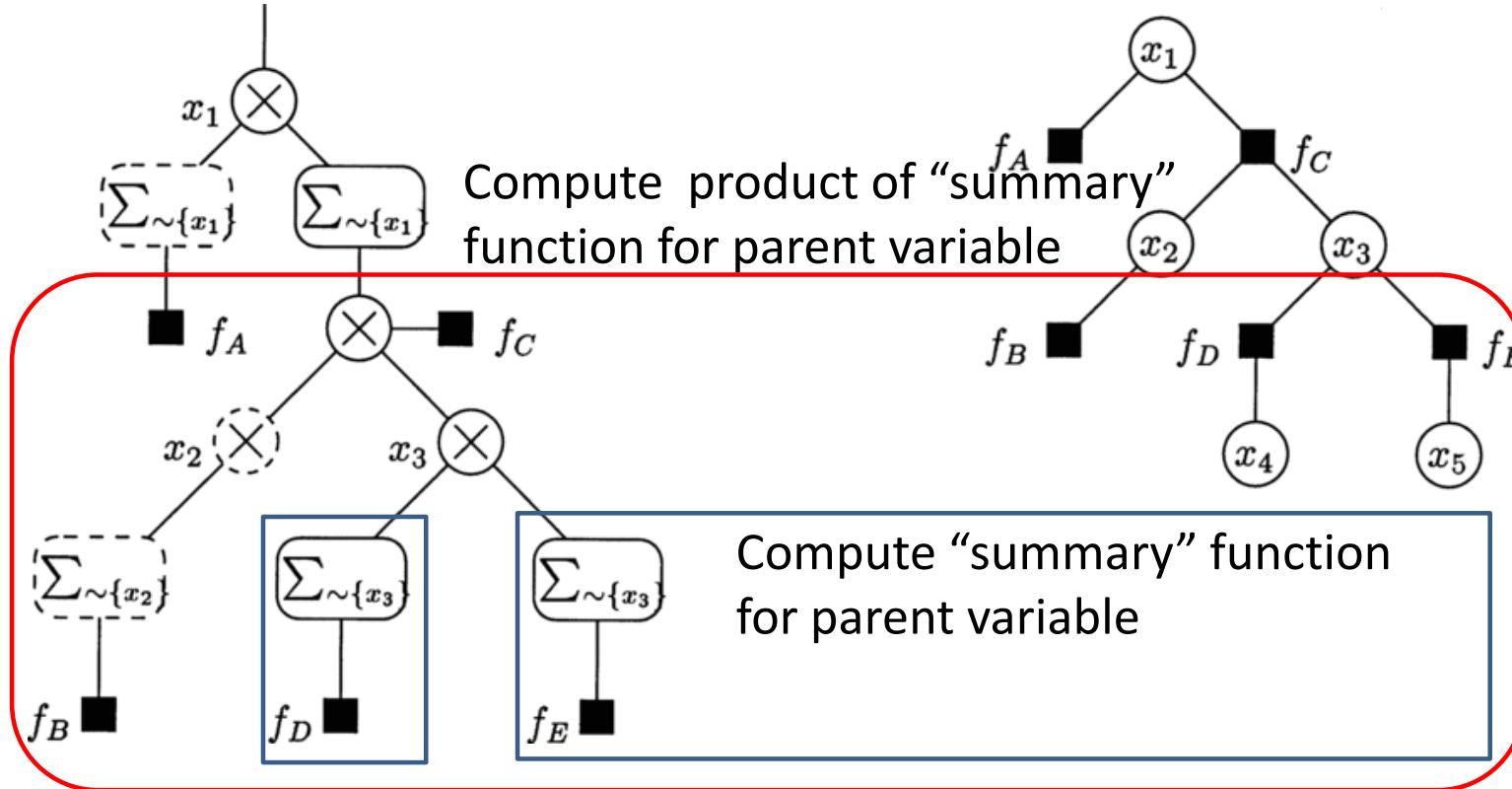
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Source: Kschischang, Frank R., Brendan J. Frey, et al. "Factor Graphs and the Sum-product Algorithm." *Information Theory, IEEE Transactions on* 47, no. 2 (2001): 498-519.

## Marginal:

$$g_1(x_1) = f_A(x_1) \times$$

$$\sum_{\sim\{x_1\}} \left( f_B(x_2) f_C(x_1, x_2, x_3) \left( \sum_{\sim\{x_3\}} f_D(x_3, x_4) \right) \left( \sum_{\sim\{x_3\}} f_E(x_3, x_5) \right) \right)$$



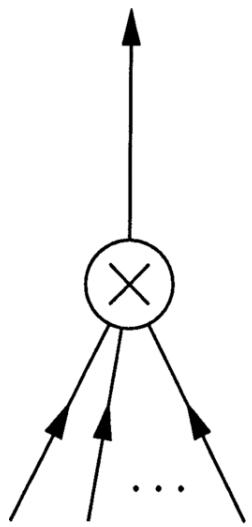
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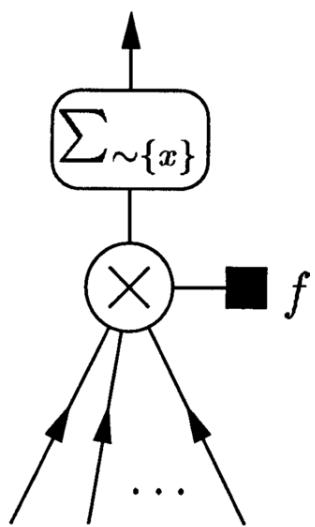
## Marginal:

$$g_1(x_1) = f_A(x_1) \times$$

$$\sum_{\sim\{x_1\}} \left( f_B(x_2) f_C(x_1, x_2, x_3) \left( \sum_{\sim\{x_3\}} f_D(x_3, x_4) \right) \left( \sum_{\sim\{x_3\}} f_E(x_3, x_5) \right) \right)$$



(a)



(b)

Messages flow up from leaves:

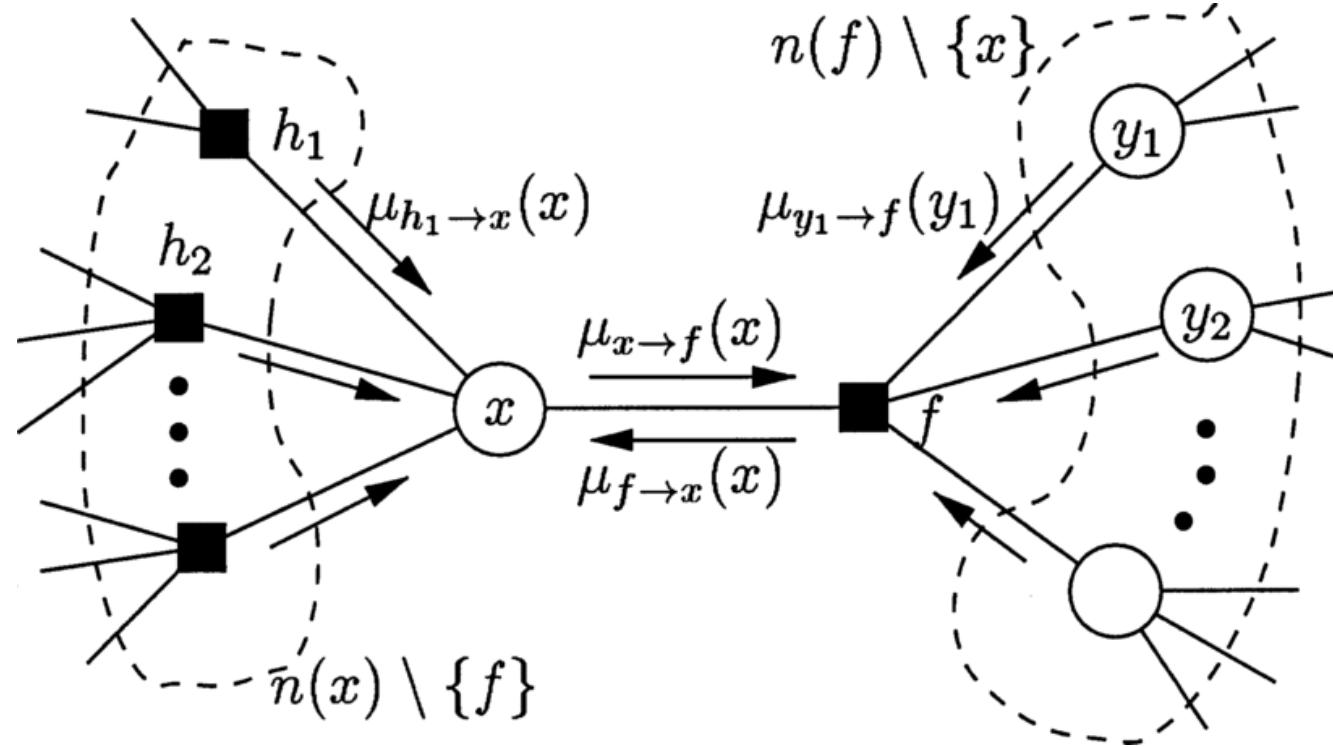
- Each vertex waits for messages from all children before computing message to send to parents
- Variable nodes send product of messages from children
- Factor nodes with parent  $x$  send the “summary” for  $x$  of the product of the children’s functions.

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 Source: Kschischang, Frank R., Brendan J. Frey, et al. "Factor Graphs and the Sum-product Algorithm." *Information Theory, IEEE Transactions on* 47, no. 2 (2001): 498-519.

# Belief propagation:

An algorithm known as “Sum-Product” can be used to simultaneously compute all marginals!

See citation for details

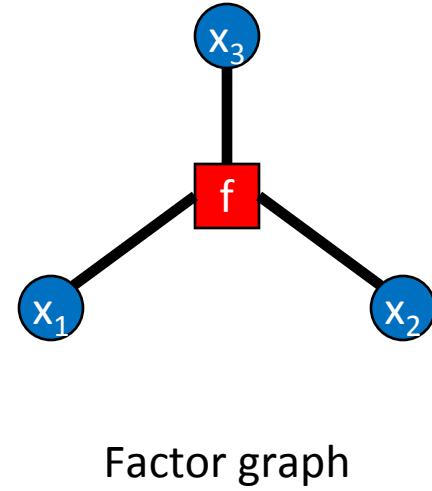


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Source: Kschischang, Frank R., Brendan J. Frey, et al. "Factor Graphs and the Sum-product Algorithm." *Information Theory, IEEE Transactions on* 47, no. 2 (2001): 498-519.

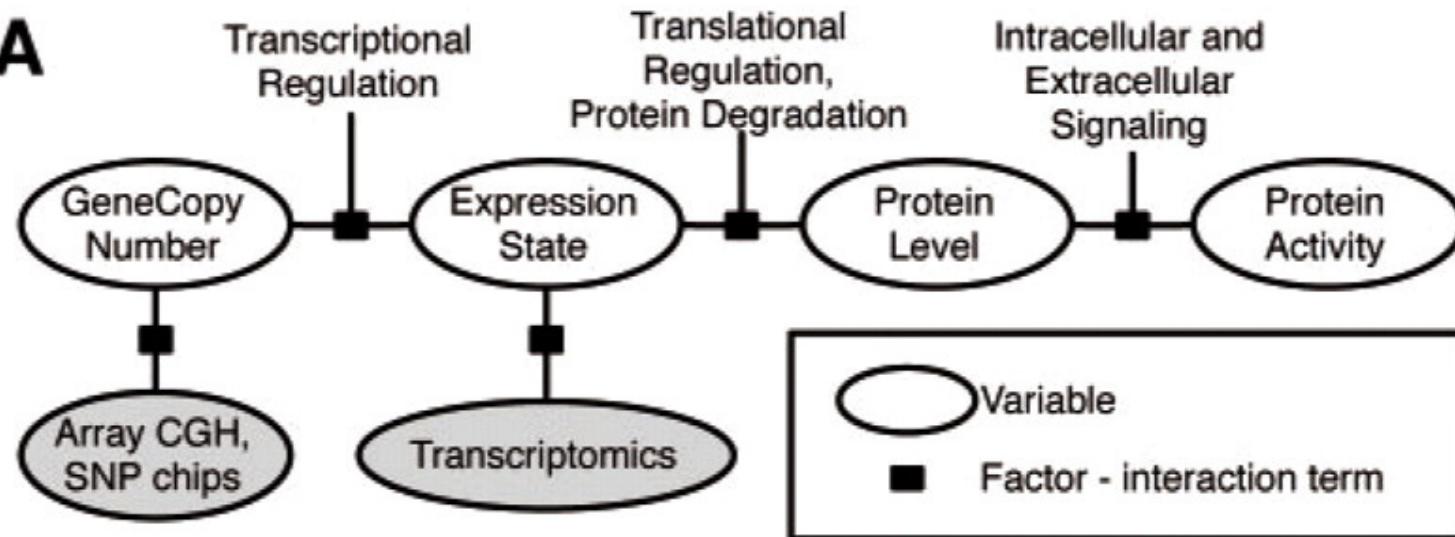
Kschischang, F.R.; Frey, B.J.; Loeliger, H.-A., "Factor graphs and the sum-product algorithm," 2001 <http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=910572&isnumber=19638>

# Factor graphs in PARADIGM

- Variable node,  $x$ :  
three states:
  - 1 activated
  - 0 nominal
  - 1 deactivated



- Factor node,  $f$
- Edge exists iff  $x$  is an argument of  $f$

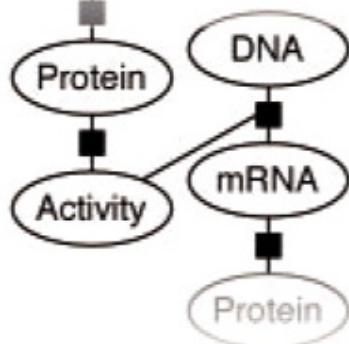
**A**

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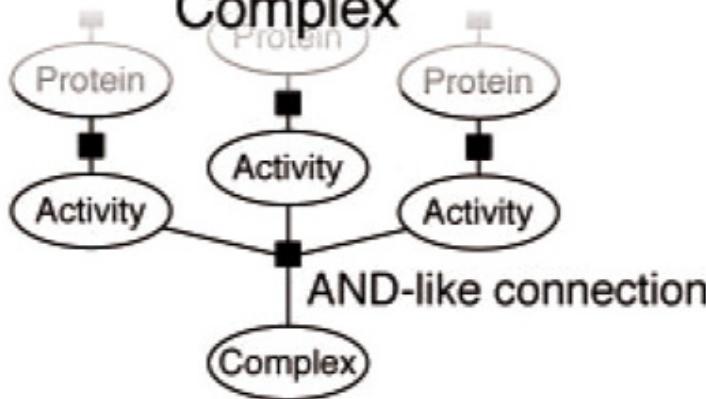
Source: Vaske, Charles J., Stephen C. Benz, et al. "[Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM](#)." *Bioinformatics* 26, no. 12 (2010): i237-i45.

Vaske C J et al. *Bioinformatics* 2010;26:i237-i245

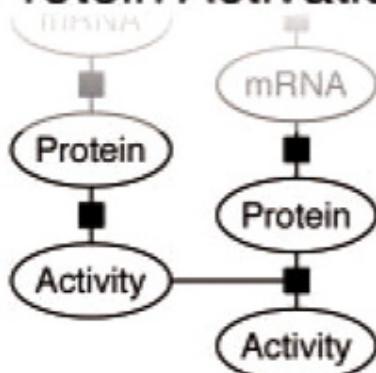
## Transcriptional Regulation



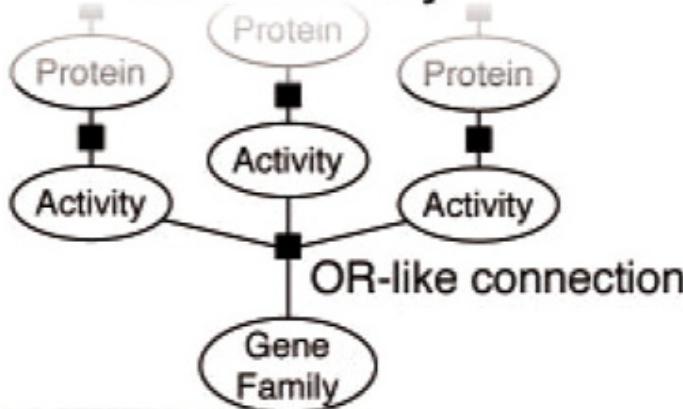
## Formation of Complex



## Protein Activation



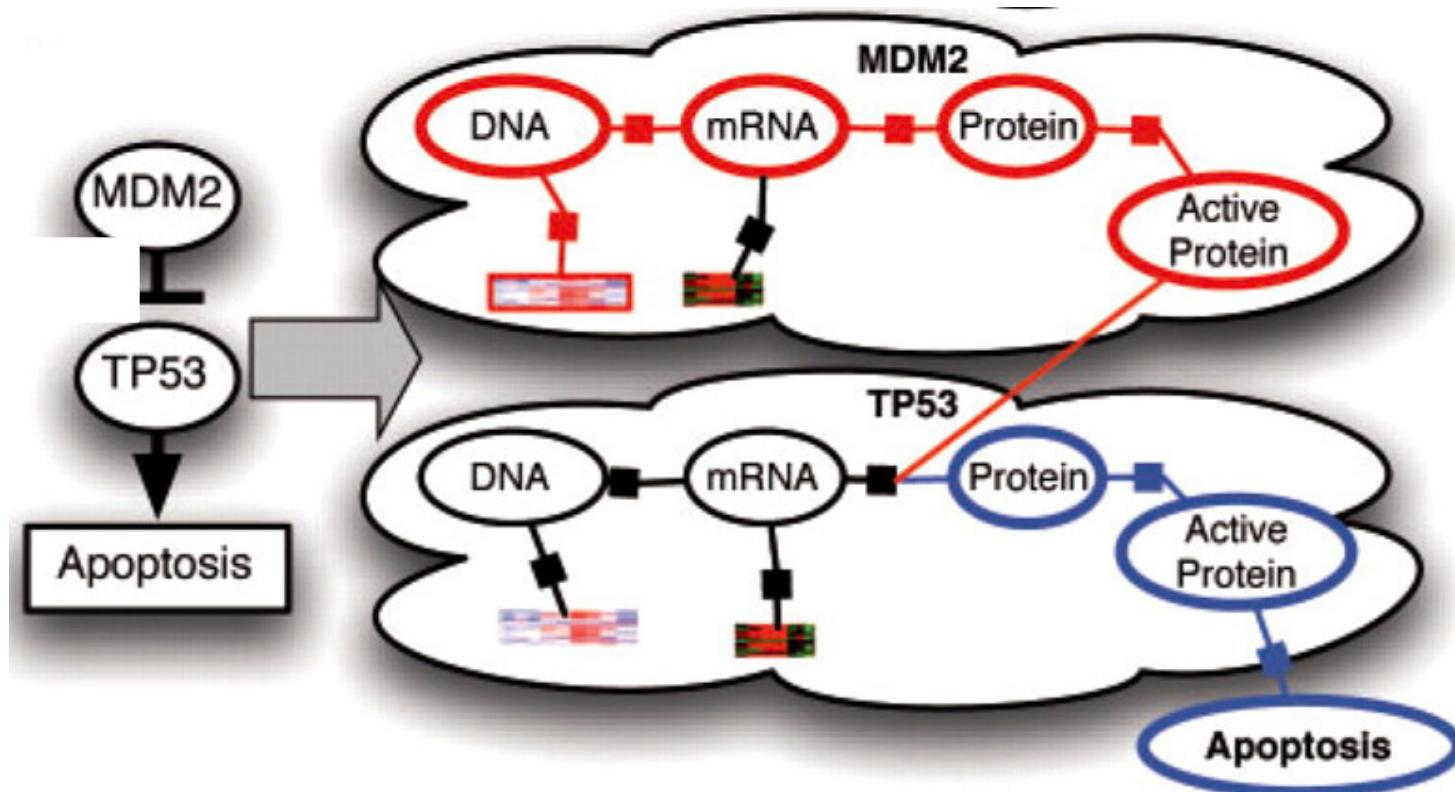
## Gene Family



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Source: Vaske, Charles J., Stephen C. Benz, et al. "Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM." *Bioinformatics* 26, no. 12 (2010): i237-i45.

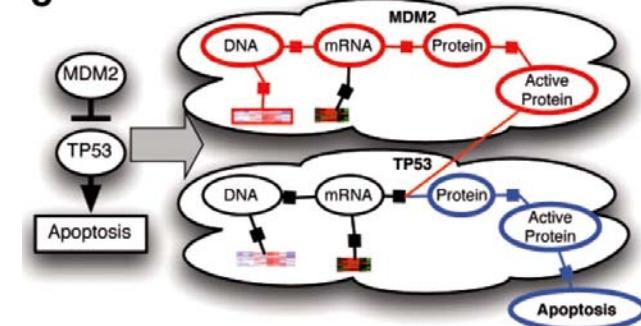
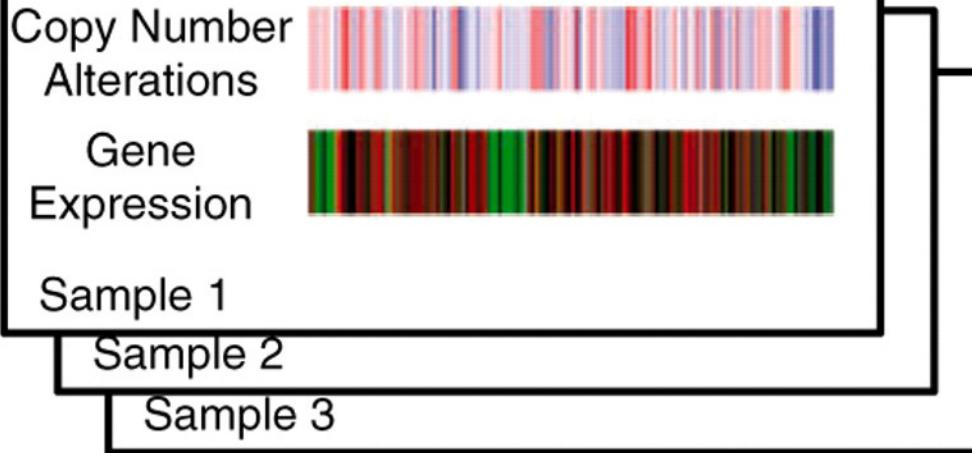
Vaske C J et al. *Bioinformatics* 2010;26:i237-i245



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Source: Vaske, Charles J., Stephen C. Benz, et al. "[Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM](#)." *Bioinformatics* 26, no. 12 (2010): i237-i45.

Vaske C J et al. *Bioinformatics* 2010;26:i237-i245



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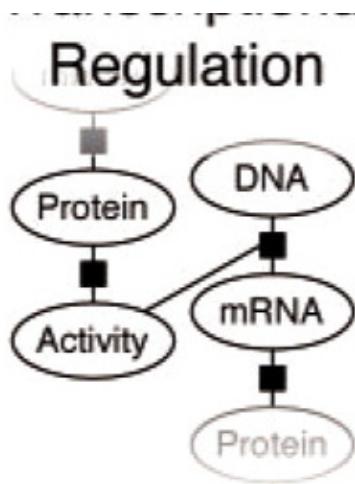
Source: Vaske, Charles J., Stephen C. Benz, et al. "Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM." *Bioinformatics* 26, no. 12 (2010): i237-i45.

- Goal:
  - Estimate probability that pathways are active
  - Use log likelihood ratio

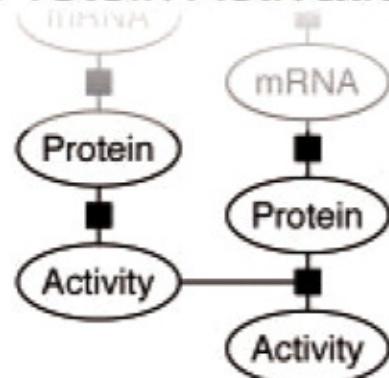
$$\begin{aligned}
 L(i, a) &= \log\left(\frac{P(D, x_i=a|\Phi)}{P(D, x_i \neq a|\Phi)}\right) - \log\left(\frac{P(x_i=a|\Phi)}{P(x_i \neq a|\Phi)}\right) \\
 &= \log\left(\frac{P(D|x_i=a, \Phi)}{P(D|x_i \neq a, \Phi)}\right).
 \end{aligned}$$

Parameters estimated by EM from experimental data

# Manually constructed



## Protein Activation

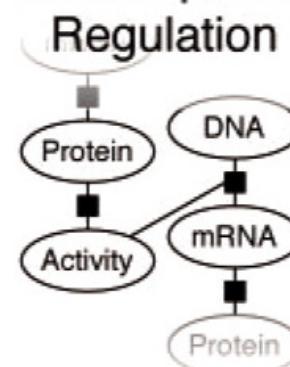


Known pathways:

- Convert to a directed graph
- Each edge is labeled as either positive or negative based on influence
- Define joint probability

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Source: Vaske, Charles J., Stephen C. Benz, et al. "[Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM](#)." *Bioinformatics* 26, no. 12 (2010): i237-i45.

# Defining joint probability



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Source: Vaske, Charles J., Stephen C. Benz, et al. "[Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM](#)." *Bioinformatics* 26, no. 12 (2010): i237-i45.

## Expected state:

- Majority vote of parent variables
- If a parent is connected by a positive edge it contributes a vote of +1 times its own state to the value of the factor.
- If the parent is connected by a negative edge, then the variable votes -1 times its own state.

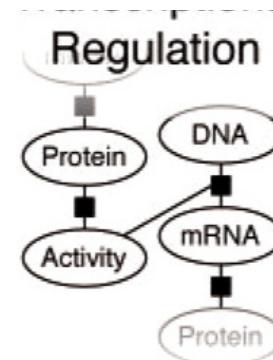
$$\phi_i(x_i, \text{Parents}(x_i)) = \begin{cases} 1 - \epsilon & x_i \text{ is the expected state from } \text{Parents}(x_i) \\ \frac{\epsilon}{2} & \text{otherwise.} \end{cases}$$

$\epsilon$  was set to 0.001

## Defining factors manually

$$\phi_i(x_i, \text{Parents}(x_i)) = \begin{cases} 1 - \epsilon & x_i \text{ is the expected state from Parents}(x_i) \\ \frac{\epsilon}{2} & \text{otherwise.} \end{cases}$$

$\epsilon$  was set to 0.001



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Source: Vaske, Charles J., Stephen C. Benz, et al. "[Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM](#)." *Bioinformatics* 26, no. 12 (2010): i237-i45.

## Logic:

- AND: The variables connected to  $x_i$  by an edge labeled ‘minimum’ get a single vote, and that vote’s value is the minimum value of these variables
- OR: The variables connected to  $x_i$  by an edge labeled ‘maximum’ get a single vote, and that vote’s value is the maximum value of these variables, creating an OR-like connection.
- Votes of zero are treated as abstained votes.
- If there are no votes the expected state is zero. Otherwise, the majority vote is the expected state, and a tie between 1 and -1 results in an expected state of -1 to give more importance to repressors and deletions.

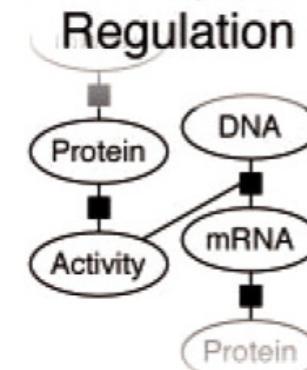
## Defining factors manually

$$\phi_i(x_i, \text{Parents}(x_i)) = \begin{cases} 1 - \epsilon & x_i \text{ is the expected state from } \text{Parents}(x_i) \\ \frac{\epsilon}{2} & \text{otherwise.} \end{cases}$$

$\epsilon$  was set to 0.001

Logic:

- AND: The variables connected to  $x_i$  by an edge labeled 'minimum' get a single vote, and that vote's value is the minimum value of these variables
- OR: The variables connected to  $x_i$  by an edge labeled 'maximum' get a single vote, and that vote's value is the maximum value of these variables, creating an OR-like connection.



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Source: Vaske, Charles J., Stephen C. Benz, et al.  
["Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM."](#) *Bioinformatics* 26, no. 12 (2010): i237-i45.

Compared to Bayesian networks, factor graphs provide an more intuitive way to represent these regulatory steps

# Joint probability of graph

$$\phi_i(x_i, \text{Parents}(x_i)) = \begin{cases} 1 - \epsilon & x_i \text{ is the expected state from } \text{Parents}(x_i) \\ \frac{\epsilon}{2} & \text{otherwise.} \end{cases}$$

$$P(X) = \frac{1}{Z} \prod_{j=1}^m \phi_j(X_j),$$

 Product over all  $m$  factors  $\phi_j$

$$Z = \prod_j \sum_{\mathbf{S} \sqsubset X_j} \phi_j(\mathbf{S})$$

$\mathbf{S} \sqsubset X$       Setting of variables = possible values

## Marginal

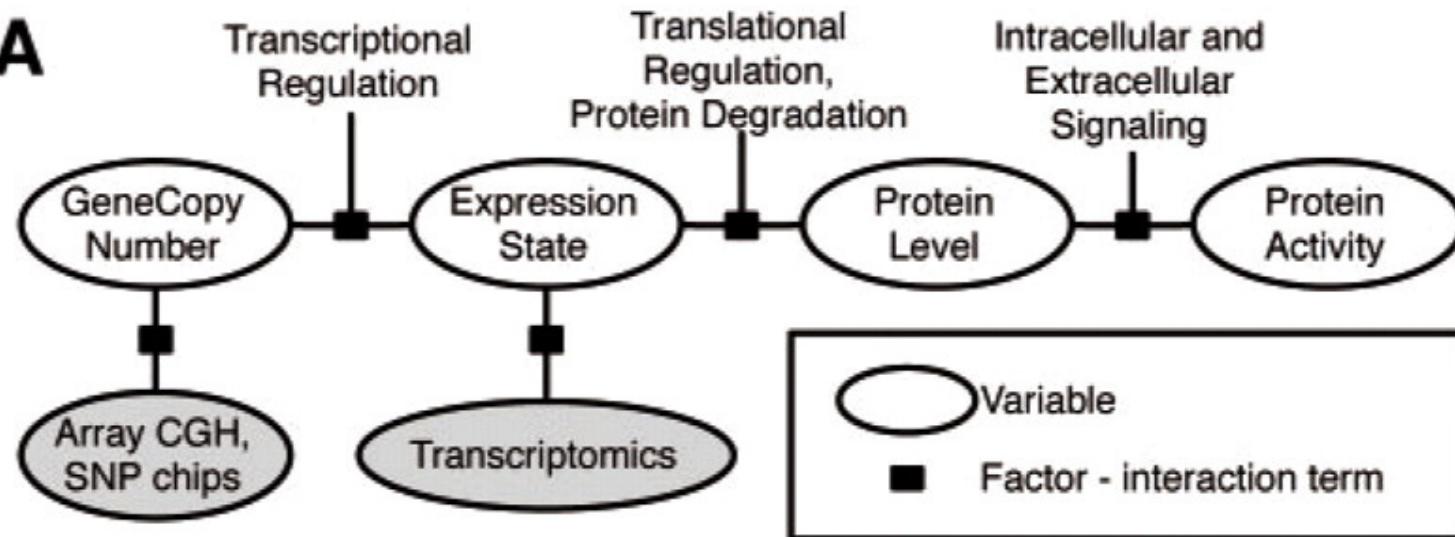
$$P(x_i = a | \Phi) = \frac{1}{Z} \prod_{j=1}^m \sum_{\mathbf{S} \sqsubset_{A_i(a)} X_j} \phi_j(\mathbf{S})$$

$\{\mathbf{S} \sqsubset_D X\}$  Set of all possible assignments to the variables X consistent with data D

$A_i(a)$  represents the singleton assignment set  $\{x_i = a\}$   
 $\Phi$  Full specified factor graph

## Likelihood

$$P(x_i = a, D | \Phi) = \frac{1}{Z} \prod_{j=1}^m \sum_{\mathbf{S} \sqsubset_{A_i(a) \cup D} X_j} \phi_j(\mathbf{S})$$

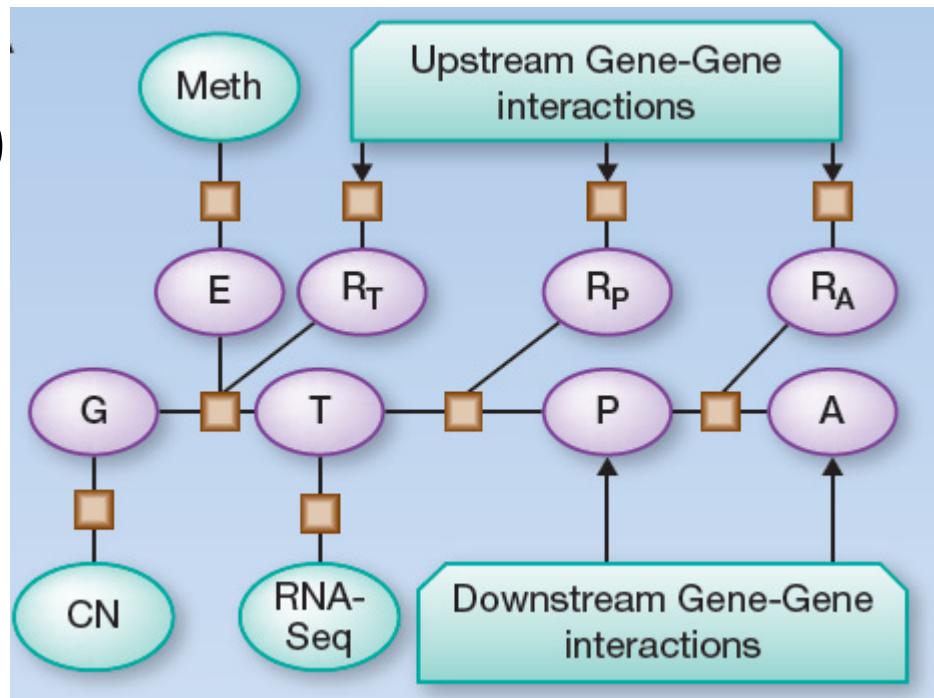
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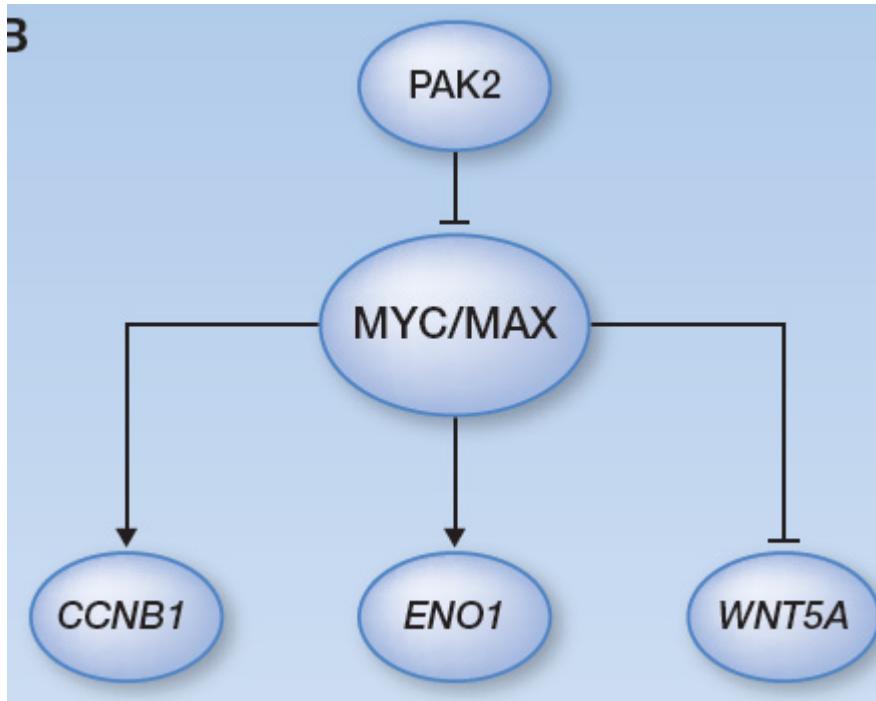
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Vaske C J et al. Bioinformatics 2010;26:i237-i245

- genomic copies (G)
- epigenetic promoter state (E)
- mRNA transcripts (T)
- peptide (P)
- active protein (A).
- Regulation gene expression
  - transcriptional (RT)
  - translational (RP)
  - post-translational (RA)

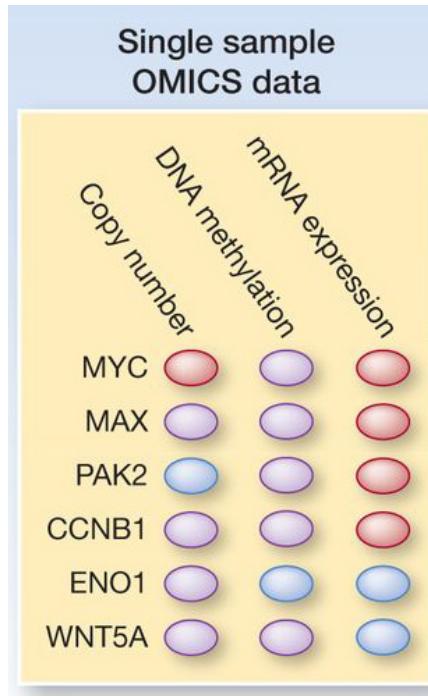
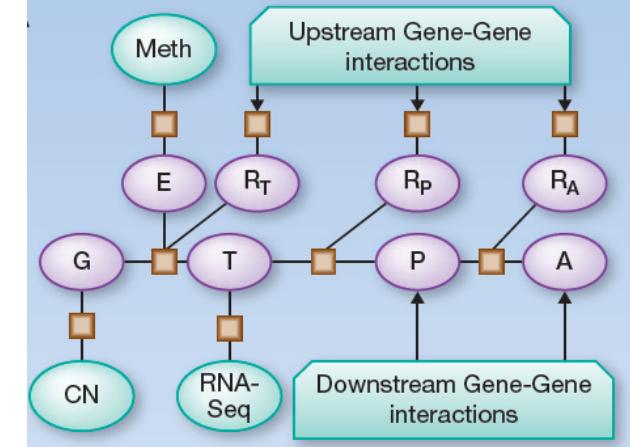
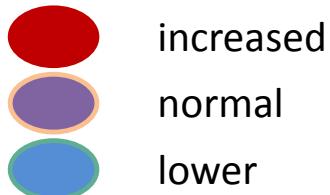
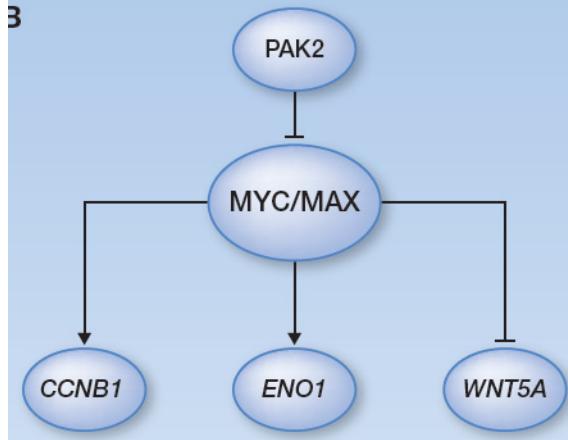


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 Source: Goldstein, Theodore C., Evan O. Paull, et al. "Molecular Pathways: Extracting Medical Knowledge from High-throughput Genomic Data." *Clinical Cancer Research* 19, no. 12 (2013): 3114-20.

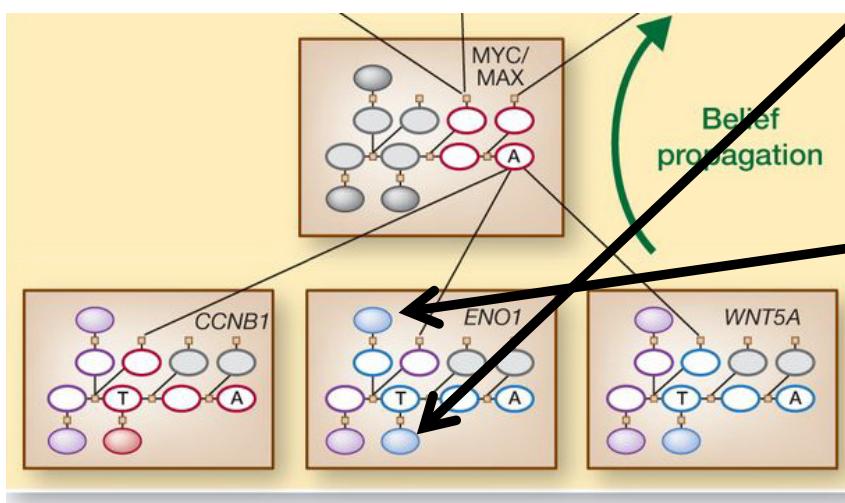


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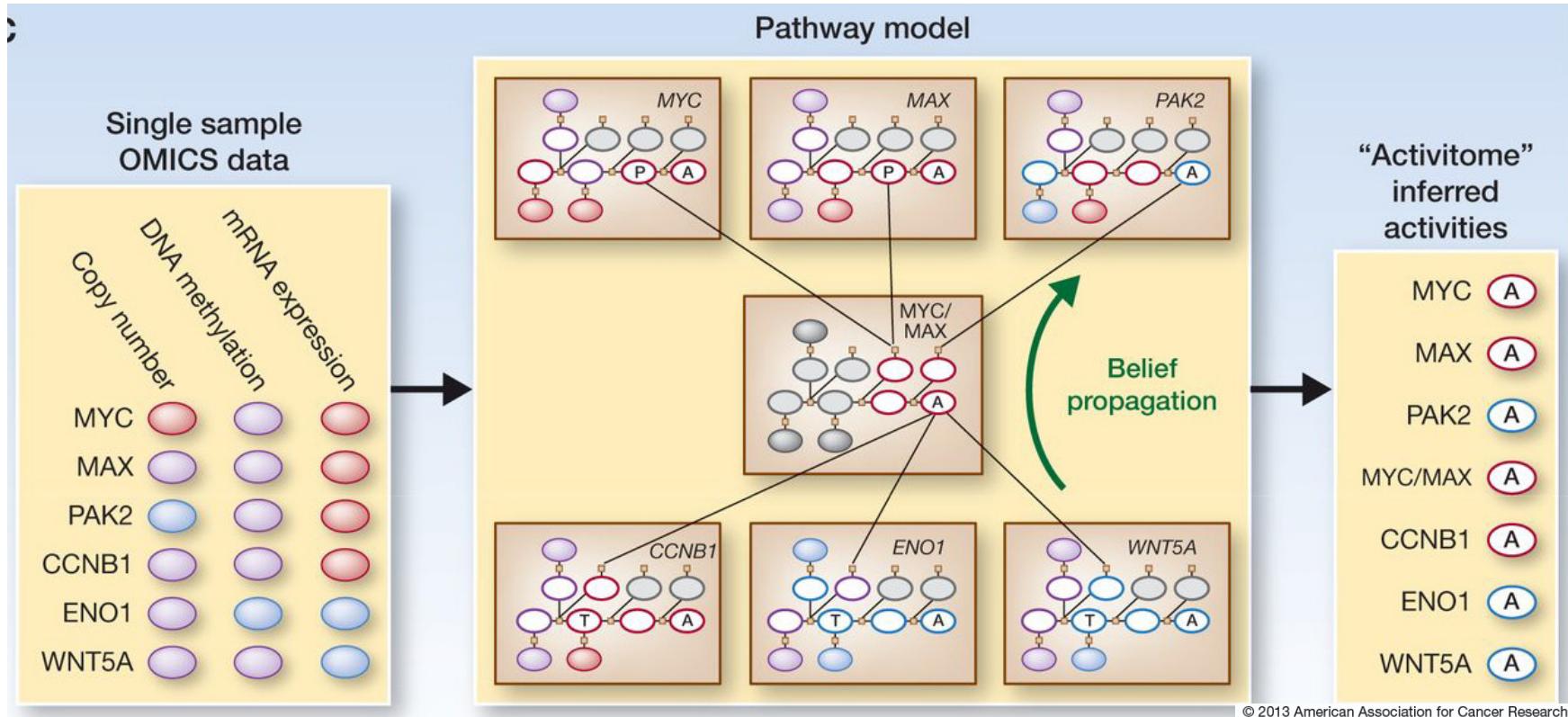


"MYC/MAX ... is active because one of its known activated targets (CCNB1) is highly expressed while one of its repressed targets (WNT5A) has lower expression"



What about ENO1, which should be increasing?

Note lack of epigenetic change

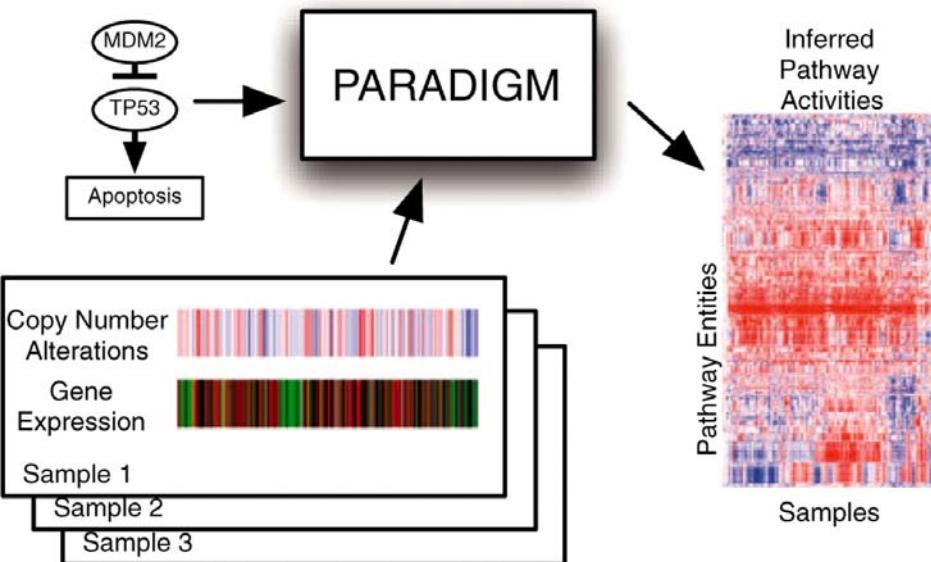


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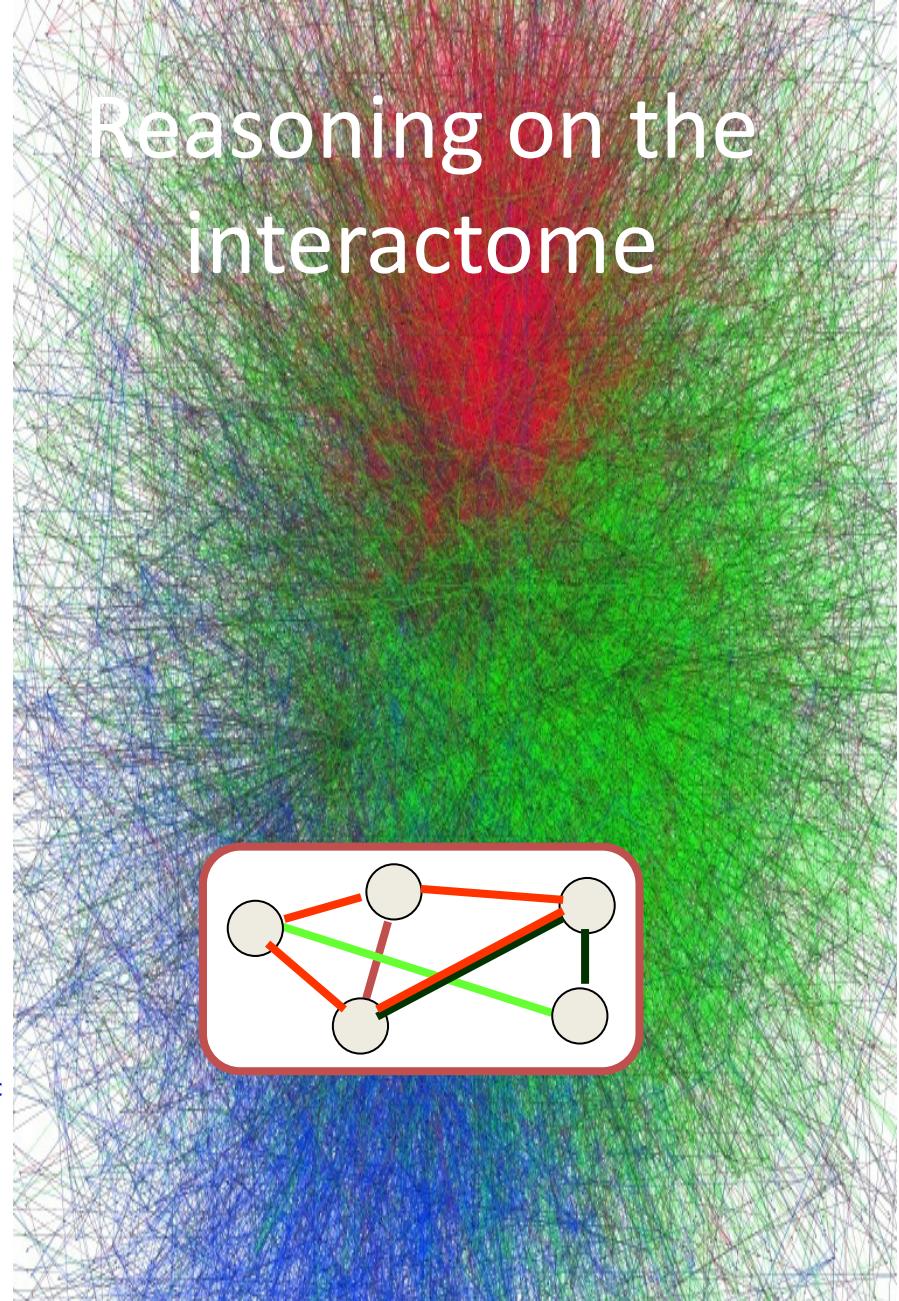
# Reasoning on curated pathways



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Source: Vaske, Charles J., Stephen C. Benz, et al. "Inference of Patient-specific Pathway Activities from Multi-dimensional Cancer Genomics Data Using PARADIGM." *Bioinformatics* 26, no. 12 (2010): i237-i45.

# Reasoning on the interactome



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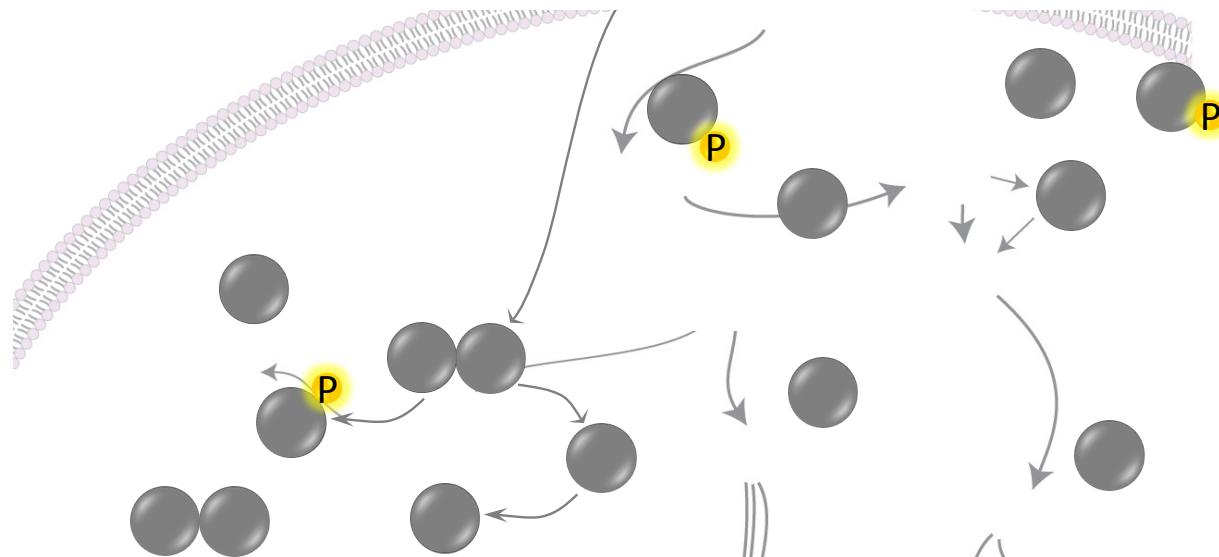
# Network Models

- Structure of network
  - Coexpression
  - Mutual information
  - Physical/genetic interactions
- Analysis of network
  - Ad hoc
  - Shortest path
  - Clustering
  - Optimization

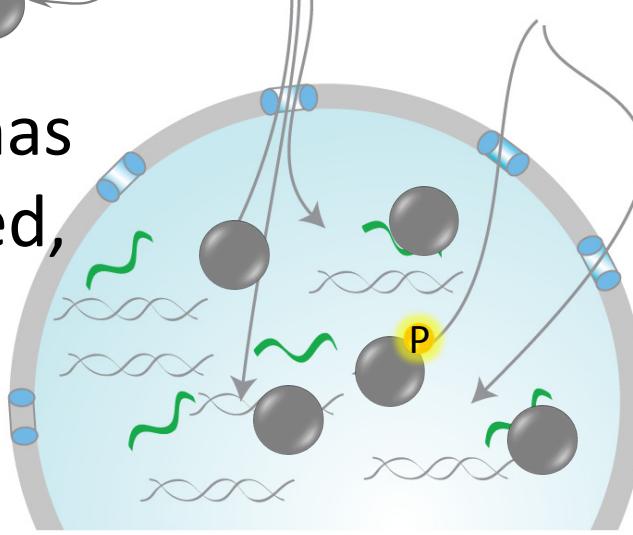
# Graph Algorithms for Interaction Networks

- Rich area of computer science
- Applications to Interaction Networks:
  - Distances:
    - Finding kinase substrates
  - Clustering
    - PPI->Protein complexes, functional annotation
    - Coexpression -> Modules
    - Blast ->Protein families
  - Active subnetworks
    - Finding hidden components of processes

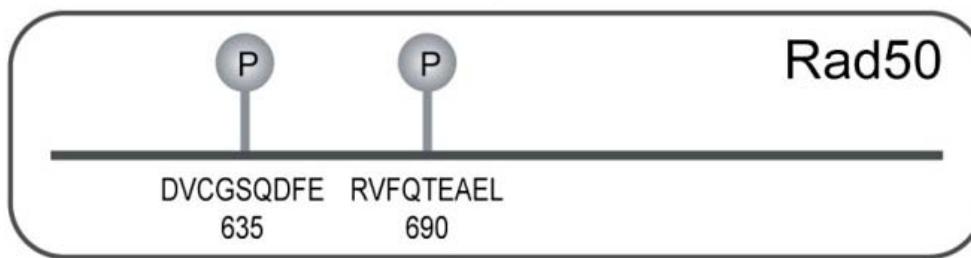
# Networkin



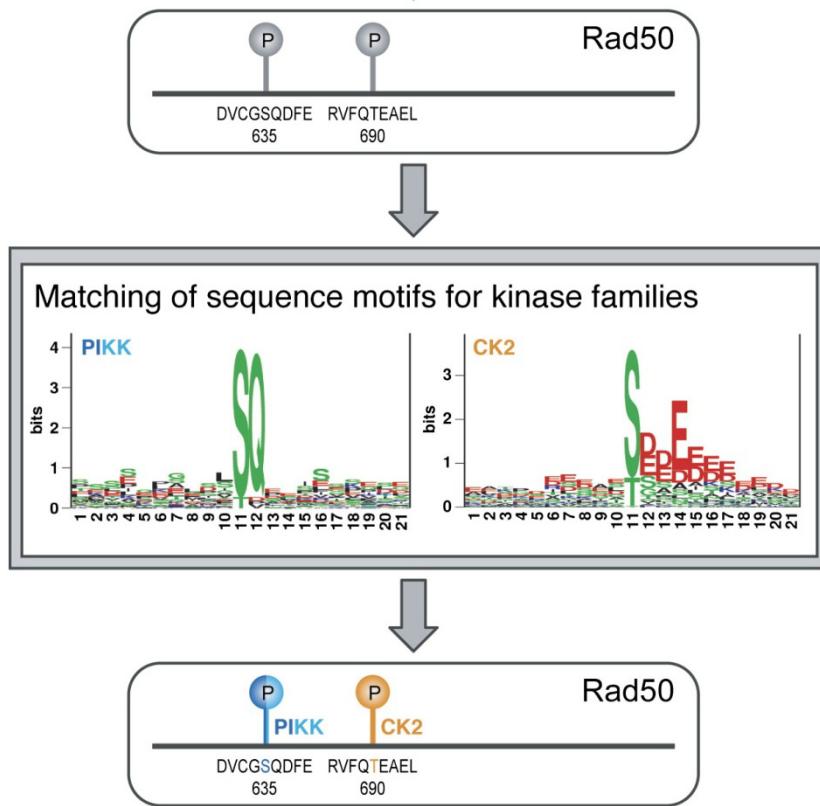
If I know a protein has been phosphorylated, can I determine the kinase?



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Source: Yeger-Lotem, Esti, Laura Riva, et al. "Bridging High-throughput Genetic and Transcriptional Data Reveals Cellular Responses to Alpha-synuclein Toxicity." *Nature Genetics* 41, no. 3 (2009): 316-23.

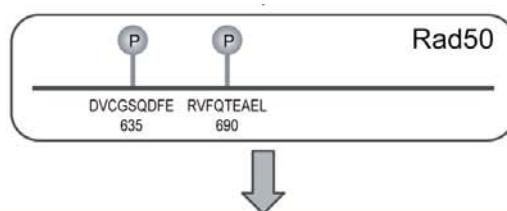


Courtesy of Elsevier, Inc., <http://www.sciencedirect.com>. Used with permission.  
Source: Linding, Rune, Lars Juhl Jensen, et al. "[Systematic Discovery of in Vivo Phosphorylation Networks](#)." *Cell* 129, no. 7 (2007): 1415-26.

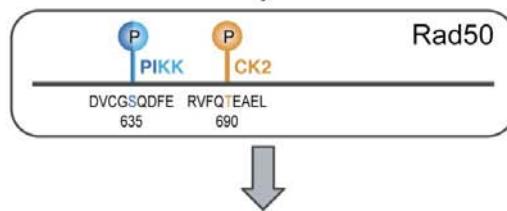
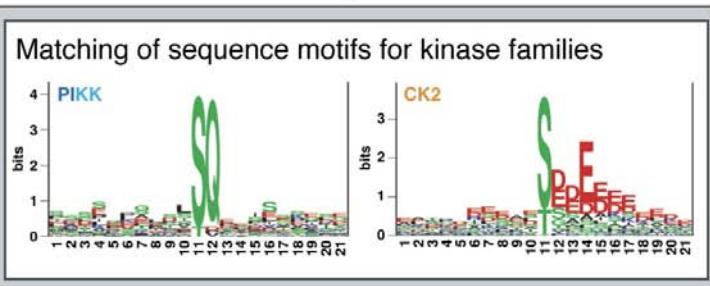


Step 1: Use sequence motifs to determine family of kinase

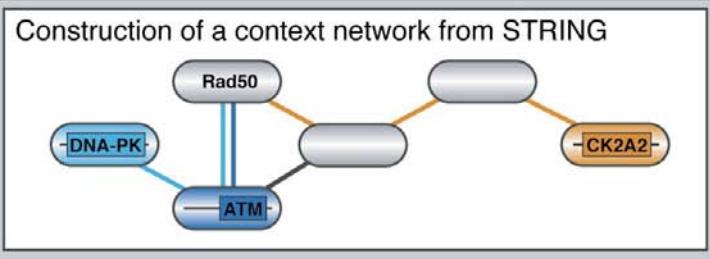
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Source: Linding, Rune, Lars Juhl Jensen, et al. "Systematic Discovery of in Vivo Phosphorylation Networks." *Cell* 129, no. 7 (2007): 1415-26.



# Step 1: Use sequence motifs to determine family of kinase



# Step 2: Use Interactome data to find most likely family member

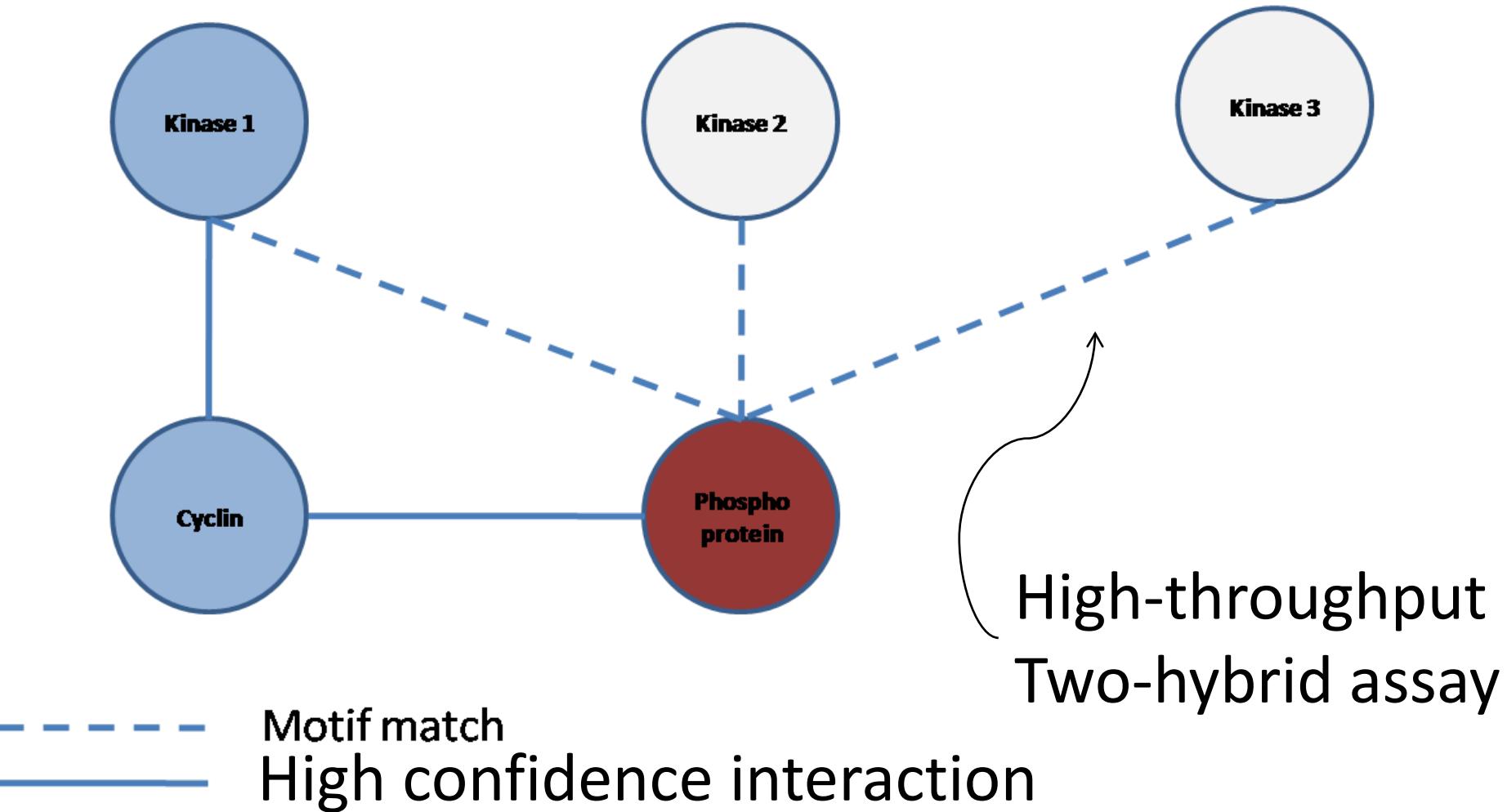


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Source: Linding, Rune, Lars Juhl Jensen, et al. "Systematic Discovery of in Vivo Phosphorylation Networks." *Cell* 129, no. 7 (2007): 1415-26.

Linding et al. (2007) Cell. doi:10.1016/j.cell.2007.05.052

# Which is best?



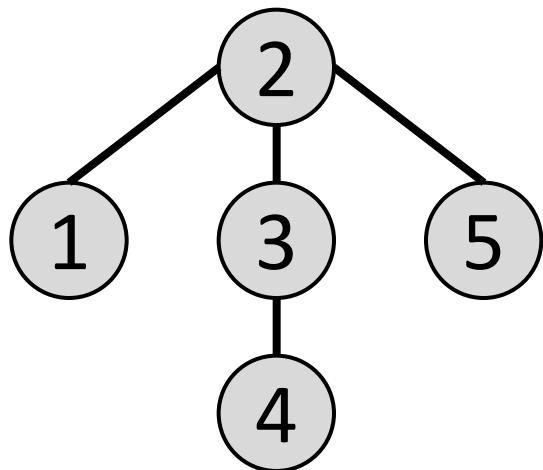
# How do we find the closest kinase?

- Many efficient algorithms exist once we treat our problem as one in Graph Theory.

# Graph Terminology

- $G=(V,E)$
- Undirected vs. directed
- Weights – numbers assigned to each edge
- $\text{Degree}(v)$  – number of edges incident on  $v$ 
  - In-degree and out-degree
- Path from  $a$  to  $b$  is a series of vertices  $\langle a, v_0, \dots, b \rangle$  where edges exist between sequential vertices
- Path length = sum of edges weights (or number of edges) on path.

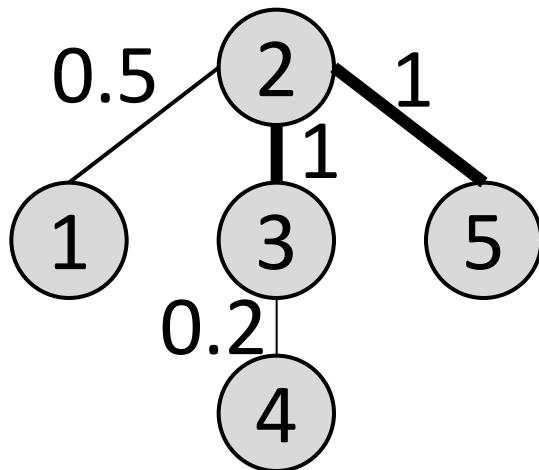
# Data Structure



Adjacency Matrix

	1	2	3	4	5
1	0	1	0	0	0
2	1	0	1	0	1
3	0	1	0	1	0
4	0	0	1	0	0
5	0	1	0	0	0

# Data Structure



Weights can represent our confidence in the link

Adjacency Matrix

	1	2	3	4	5
1	0	.5	0	0	0
2	.5	0	1	0	1
3	0	1	0	.2	0
4	0	0	.2	0	0
5	0	1	0	0	0

Weighted graph:

$a_{ij} = w_{ij}$  if edge exists; 0 otherwise

# Shortest Path Algorithms

- Efficient Algorithms for
  - single pair  $(u,v)$
  - single source/destination to all other nodes
  - all-pairs

# Reliability of edges

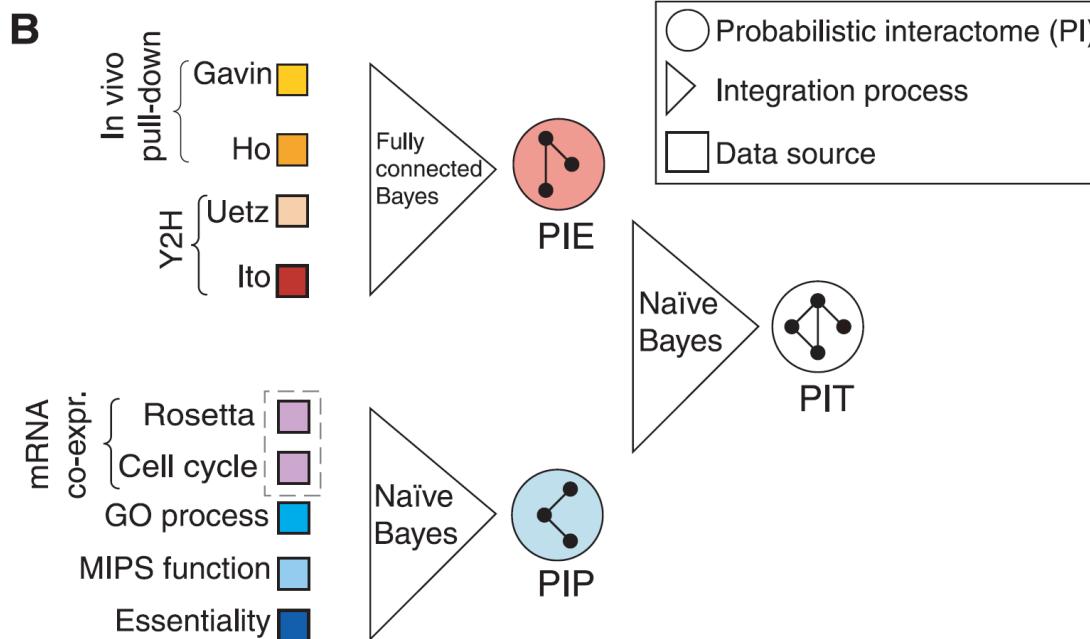
- Assign weight to each edge based on reliability.
- Total distance in network = sum of edge weights
- If  $\text{weight}_{ij} = -\log(P_{ij})$ :  
$$\min \sum w_{ij} = \min(-\log \prod P_{ij})$$
$$= \max(\text{joint probability})$$
$$= \text{most probable path}$$

# Interaction Weights

- How do we assign reliability of edges?

# A Bayesian Networks Approach for Predicting Protein-Protein Interactions from Genomic Data

Ronald Jansen,<sup>1,\*</sup> Haiyuan Yu,<sup>1</sup> Dov Greenbaum,<sup>1</sup> Yuval Kluger,<sup>1</sup>  
Nevan J. Krogan,<sup>4</sup> Sambath Chung,<sup>1,2</sup> Andrew Emili,<sup>4</sup>  
Michael Snyder,<sup>2</sup> Jack F. Greenblatt,<sup>4</sup> Mark Gerstein<sup>1,3†</sup>

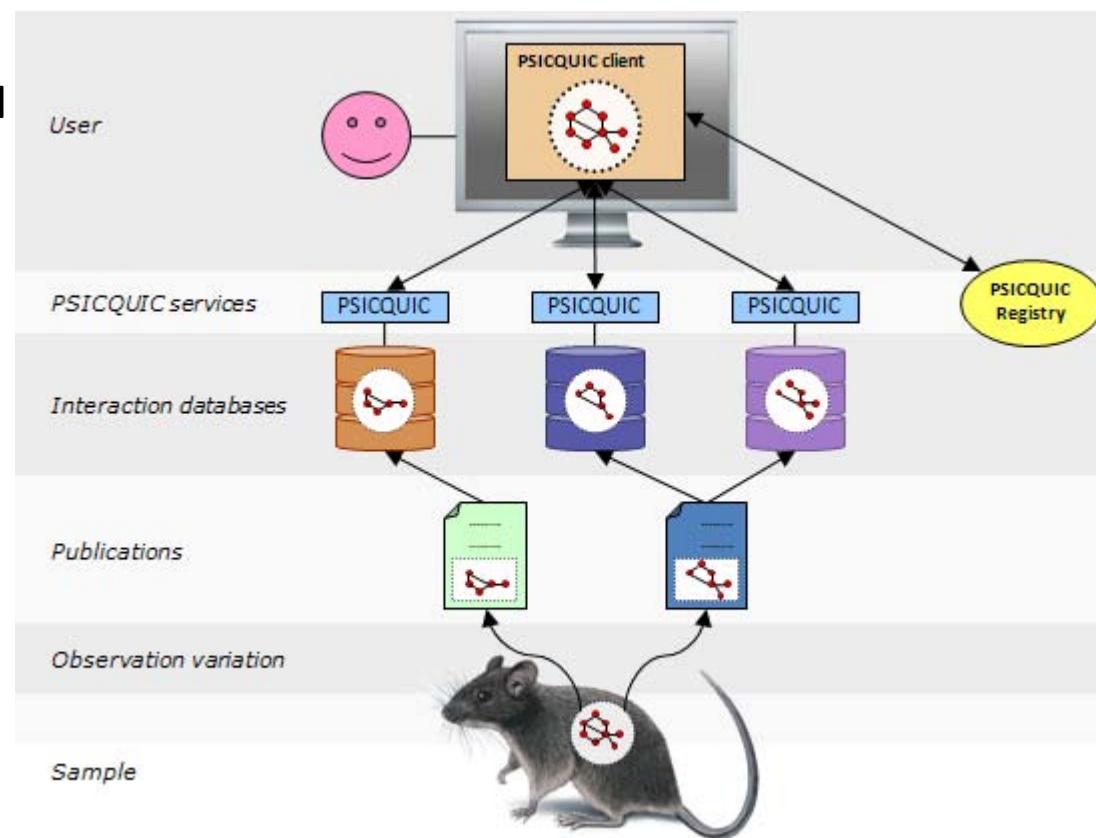


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# PSICQUIC and PSISCORE: accessing and scoring molecular interactions

Nature Methods 8, 528–529 (2011)

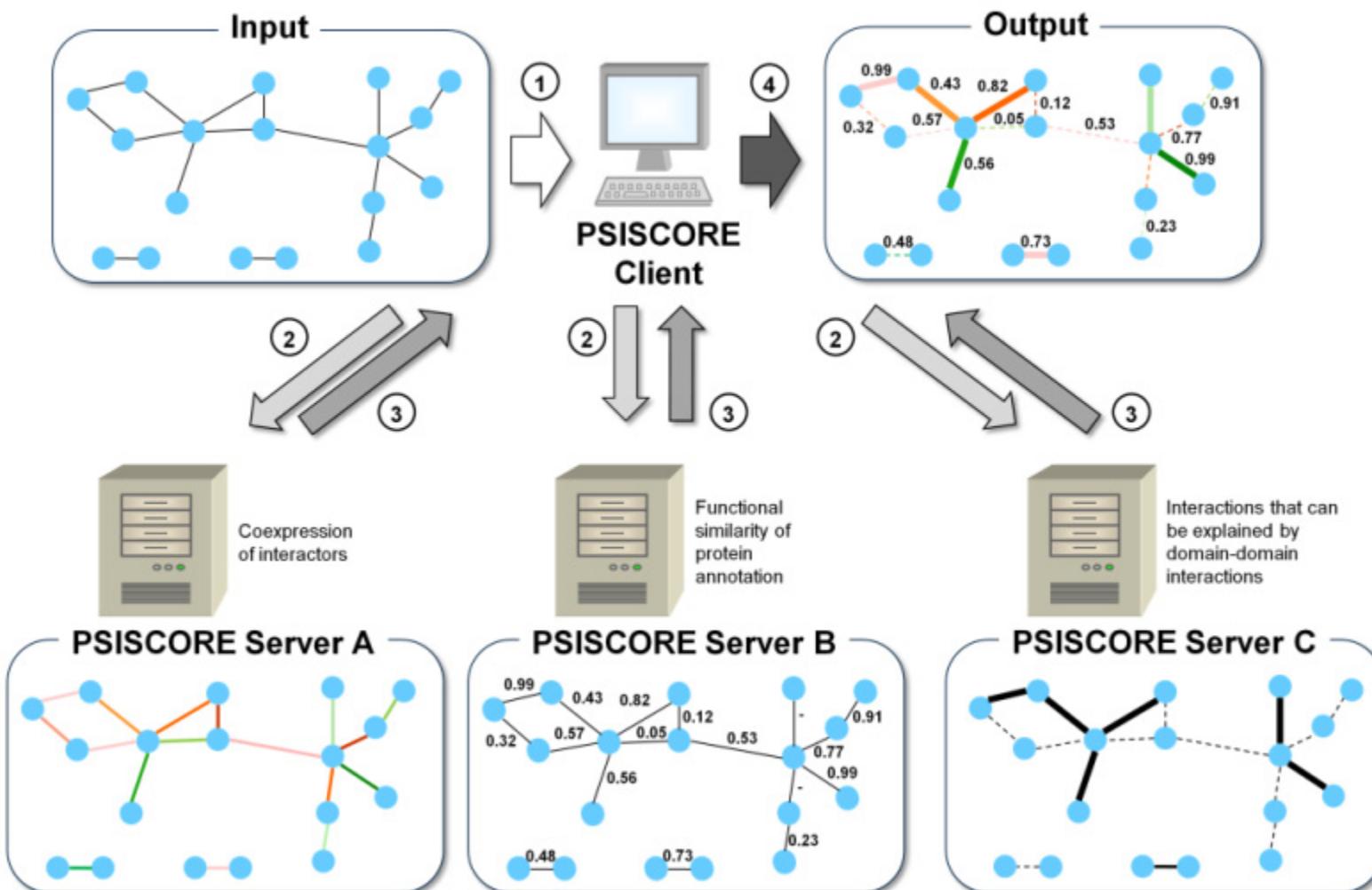
doi:10.1038/nmeth.1637



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Source: Aranda, Bruno, Hagen Blankenburg, et al. "PSICQUIC and PSISCORE: Accessing and Scoring Molecular Interactions." *Nature Methods* 8, no. 7 (2011): 528-9.

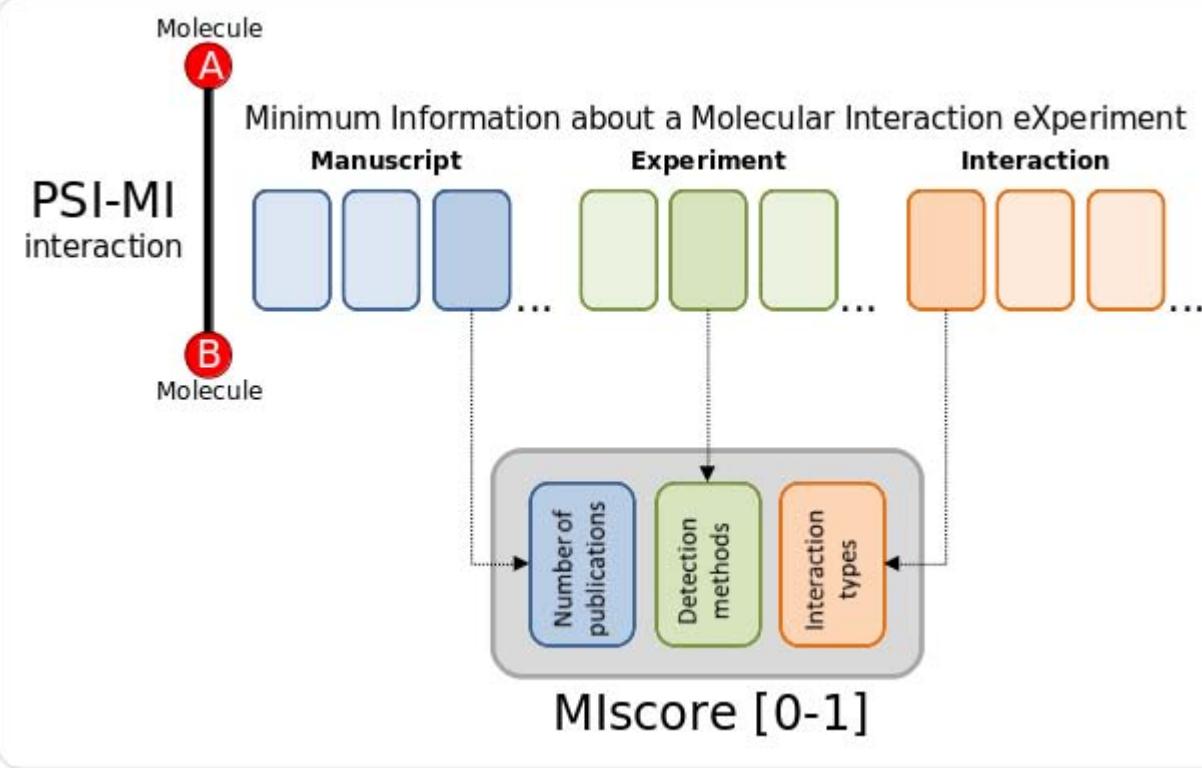
Human Proteome Organization Proteomics Standards Initiative (HUPO-PSI) released the PSI molecular interaction (MI) XML format

PSI common query interface (PSICQUIC), a community standard for computational access to molecular-interaction data resources.



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# Miscore algorithm



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Miscore is a normalized score between 0 and 1 that takes into account several variables:

- Number of publications
- Experimental detection methods found for the interaction
- Interaction types found for the interaction

Each of these variables is also represented by a score between 0 and 1. The importance of each variable in the main equation can be adjusted using a weight factor.

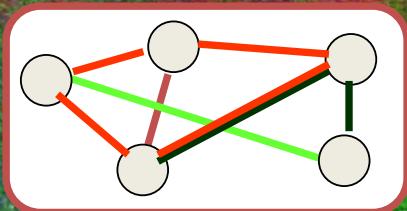
# Miscore algorithm

$$S_{MI} = \frac{K_p \times S_p(n) + K_m \times S_m(cv) + K_t \times S_t(cv)}{K_p + K_m + K_t}$$

Depends on

- Number of publications
- Experimental method (biophys.; imaging; genetic)
- Annotation of interaction type (physical, genetic)

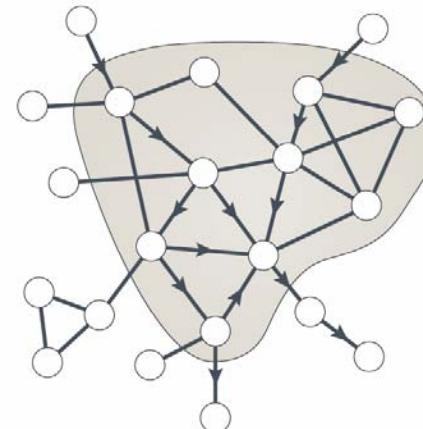
# Weighted Interactome



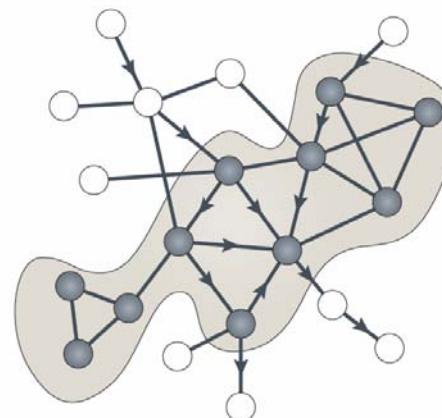
# Finding Modules

- Topological module:
  - locally dense
  - more connections among nodes in module than with nodes outside module
- Functional module:
  - high density of functionally related nodes

a Topological module

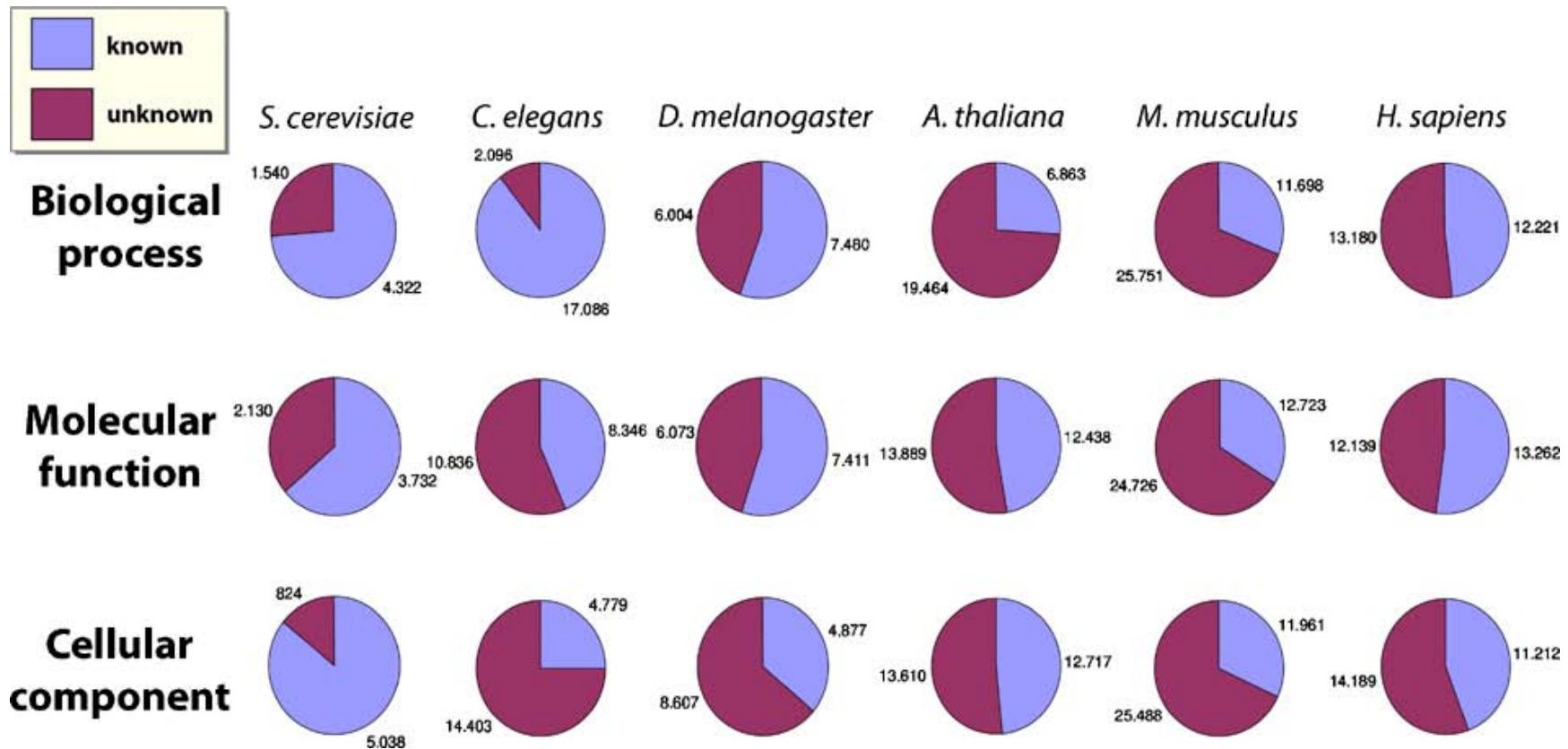


b Functional module



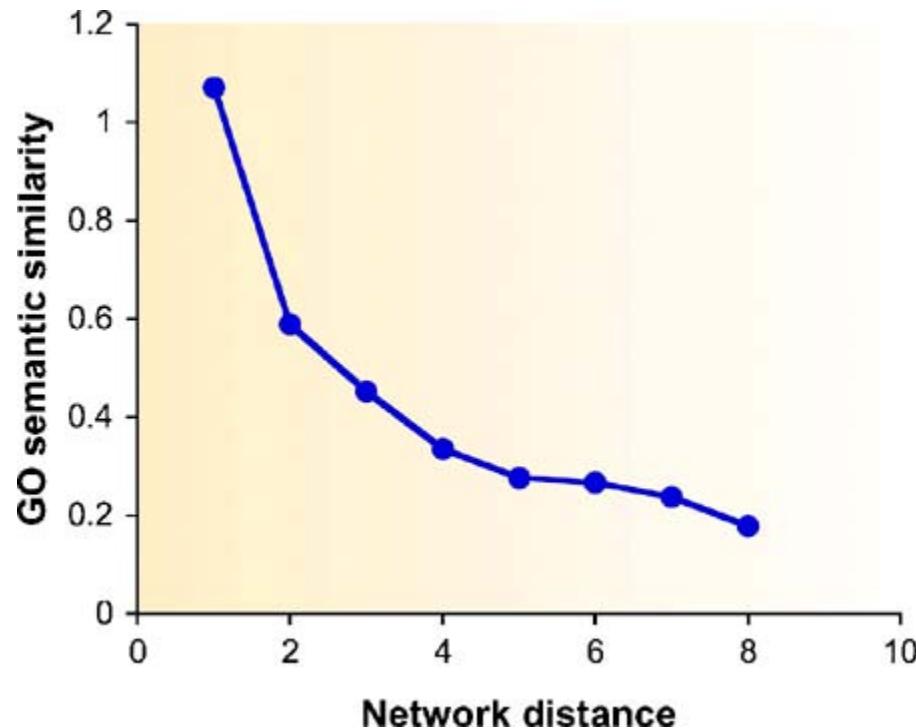
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Source: Barabási, Albert-László, Natali Gulbahce, et al. "[Network Medicine: A Network-based Approach to Human Disease.](#)" *Nature Reviews Genetics* 12, no. 1 (2011): 56-68.

# Can we use networks to predict function



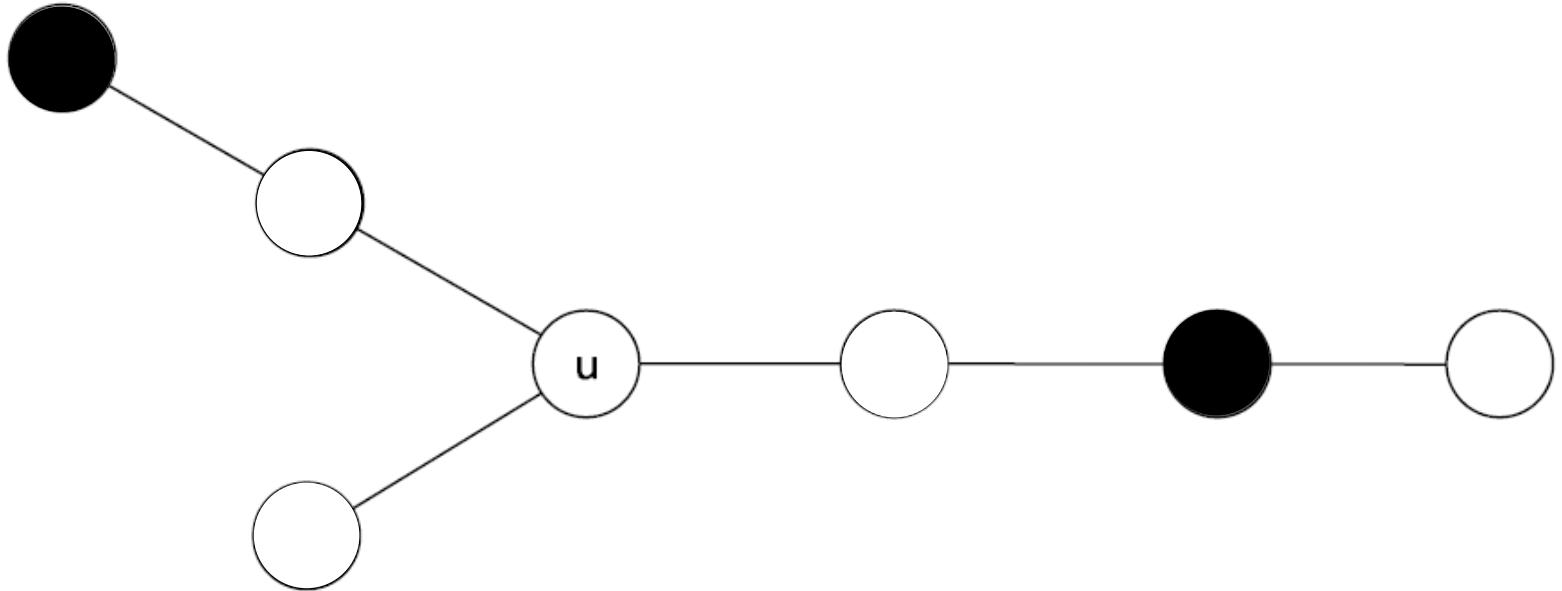
Courtesy of EMBO. Used with permission.  
Source: Sharan, Roded, Igor Ulitsky, et al. "Network-based Prediction of Protein Function." *Molecular Systems Biology* 3, no. 1 (2007).

# Can we use networks to predict function

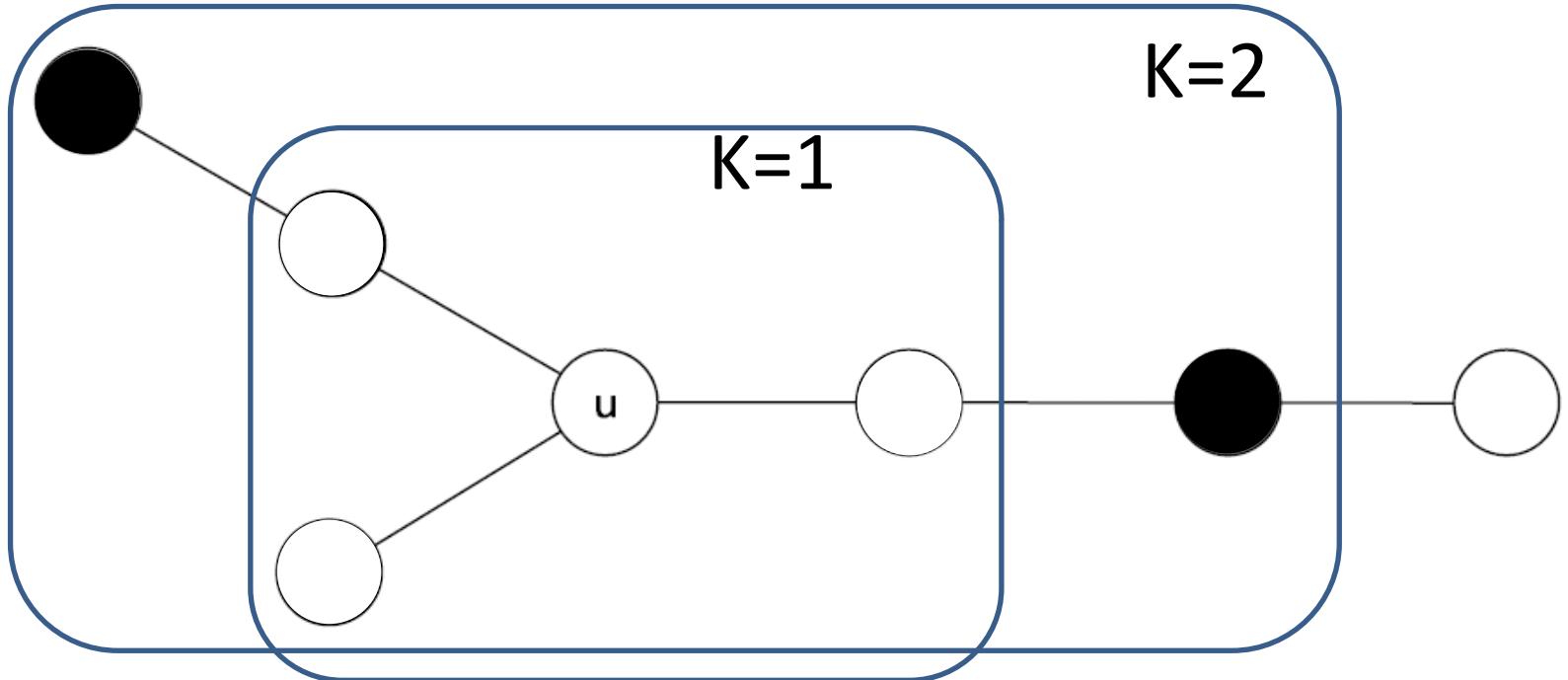


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Source: Sharan, Roded, Igor Ulitsky, et al. "[Network-based Prediction of Protein Function](#)." *Molecular Systems Biology* 3, no. 1 (2007).

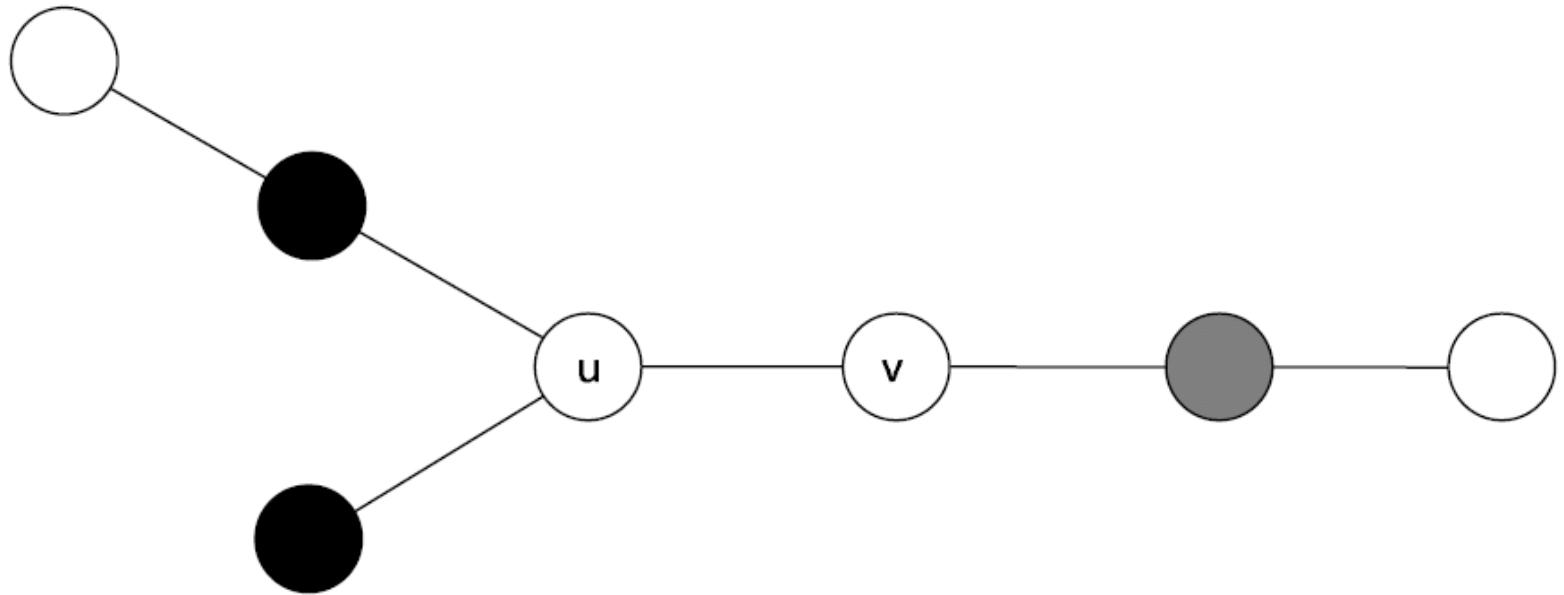


Systematically deduce the annotation of unknown nodes  $u$  from the known (filled) nodes

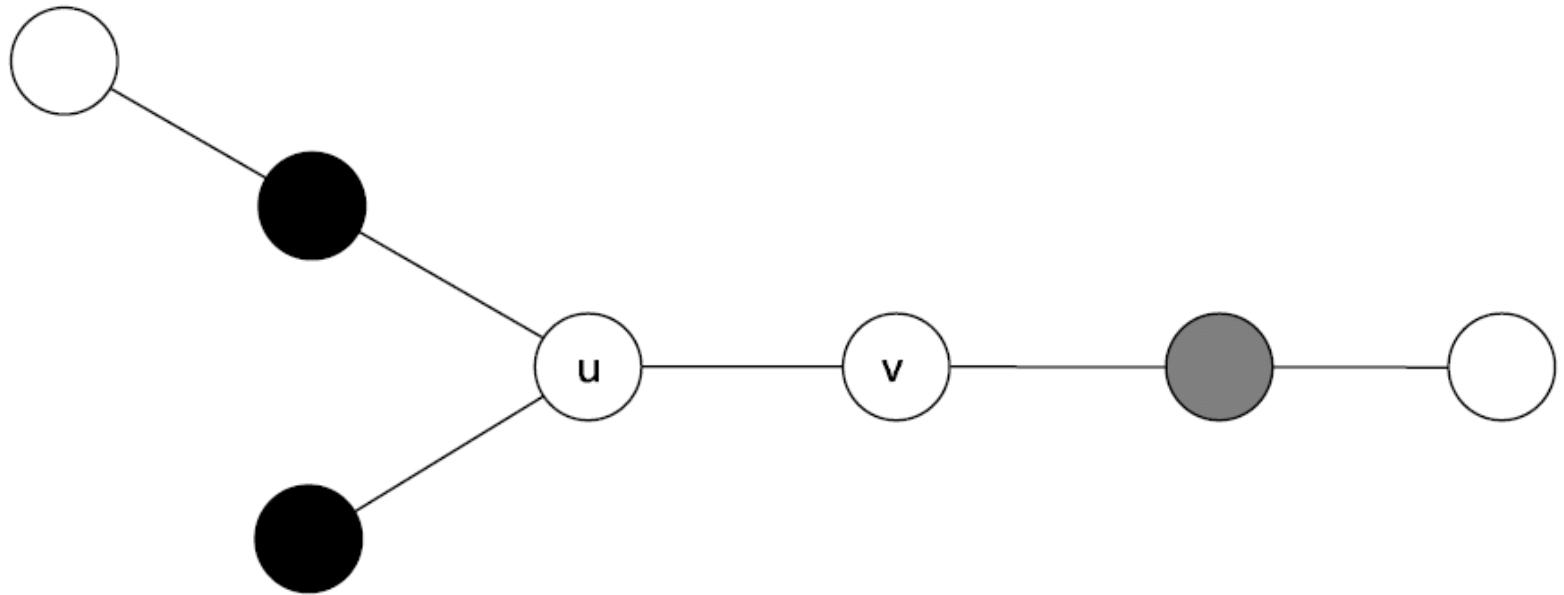


“Direct” method for gene annotation

- K-nearest neighbors
  - assume that a node has the same function as its neighbors



Should  $u$  and  $v$  have the same annotation?



Advantages of kNN approach:

very easy to compute

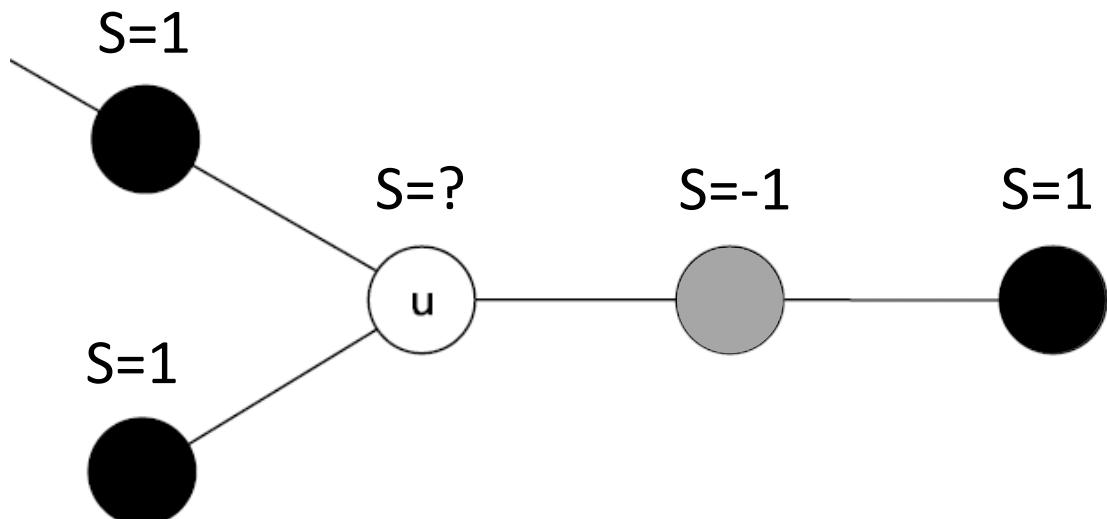
Disadvantages:

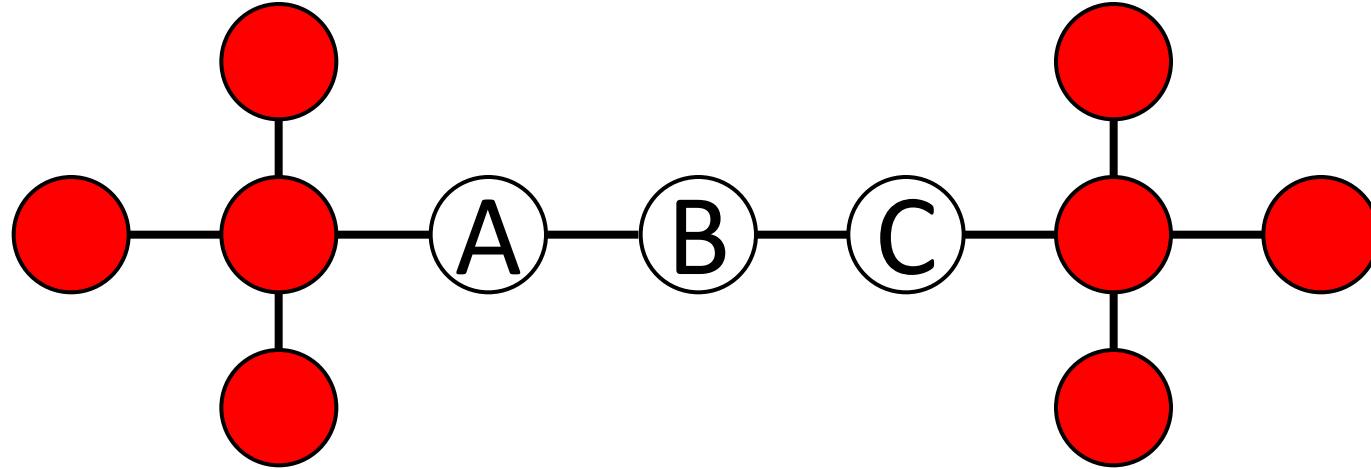
how do you choose the best annotation?

# “Direct”

Local search (Karaoz[2004]):

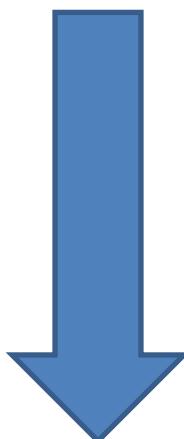
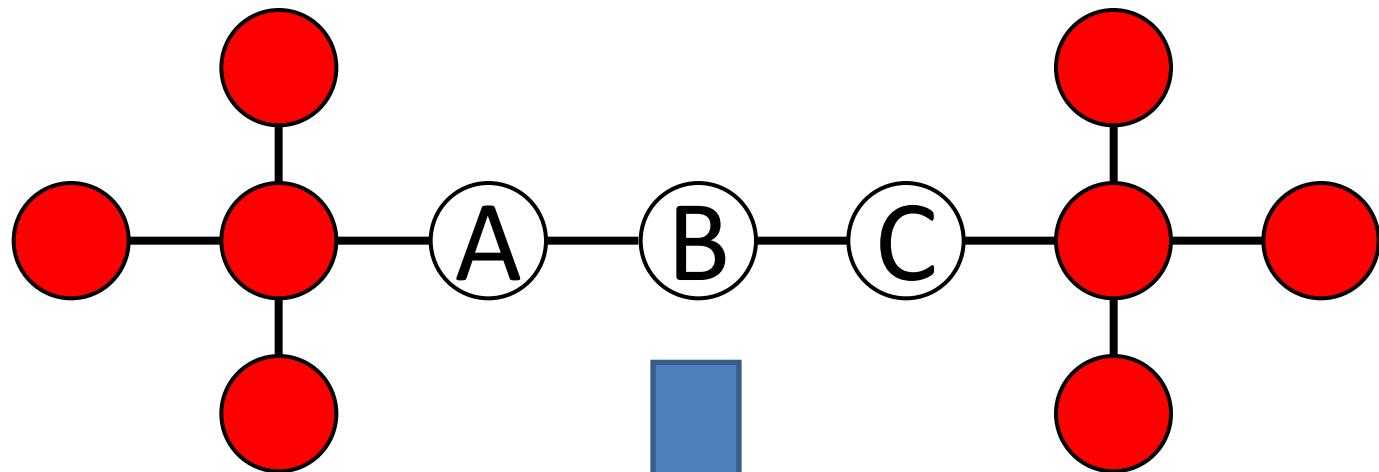
- For each annotation:
  - $S_v = 1$  if  $v$  has the annotation, -1 otherwise
  - Procedure: for each unassigned node  $u$ , set  $S_u$  maximize  $\sum S_u S_v$  for all edges  $(u,v)$
  - iterate until convergence



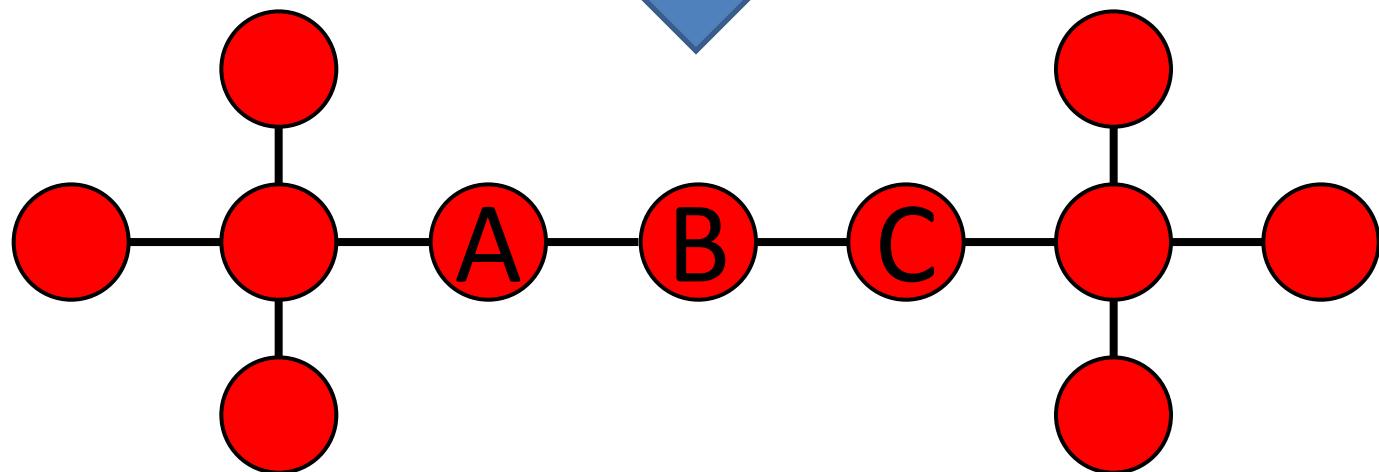


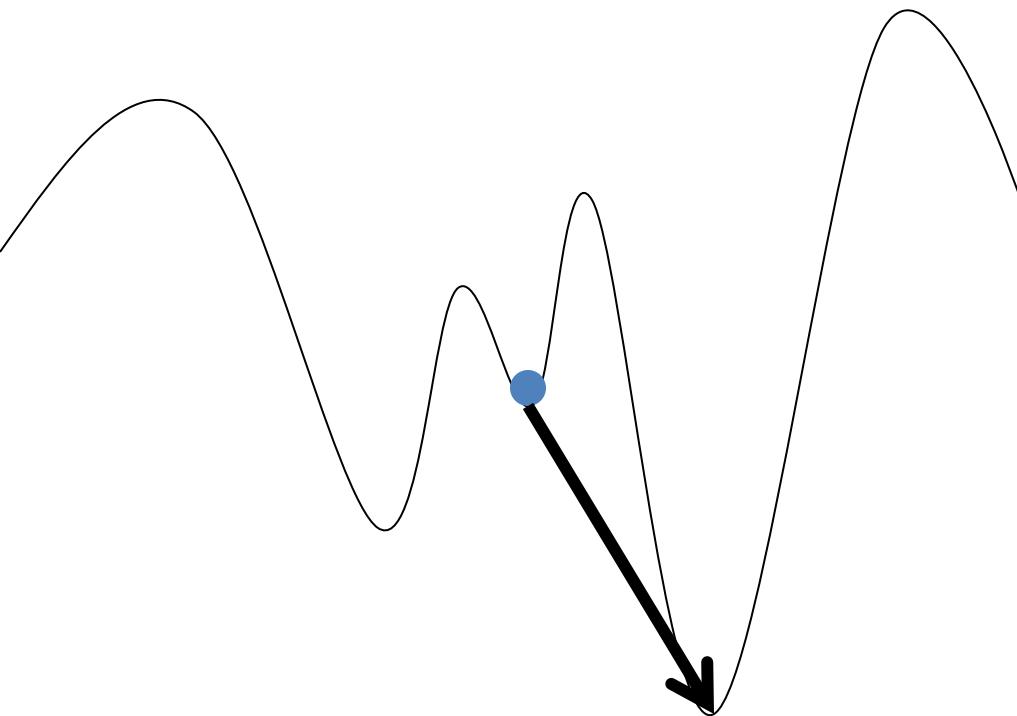
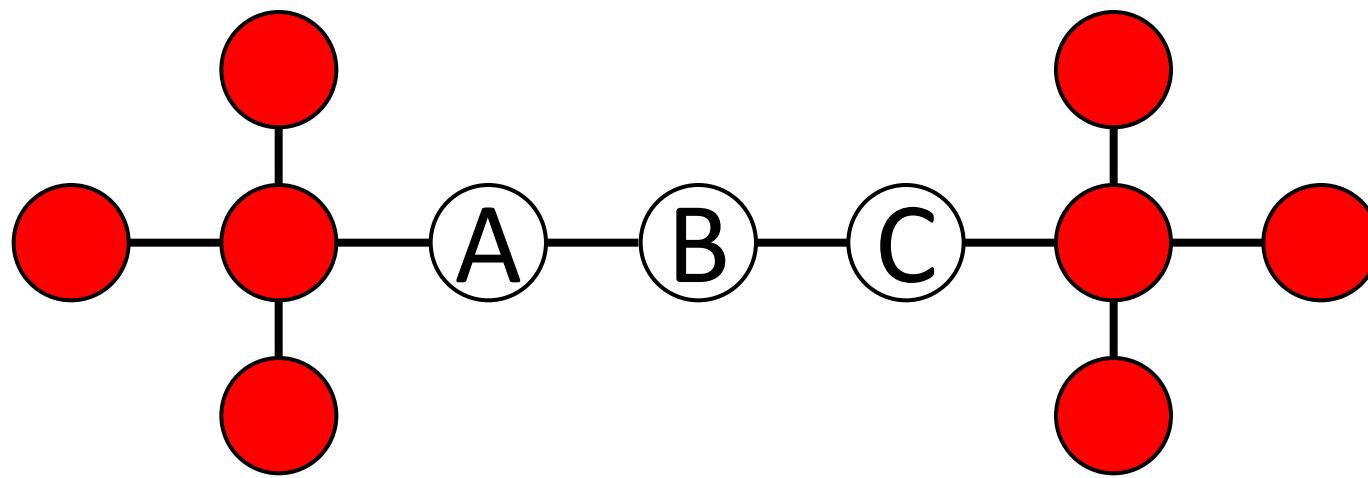
Local search may not find some good solutions.

$\sum S_u S_v$  does not improve if I only change A or C. Changing only B makes the score worse.

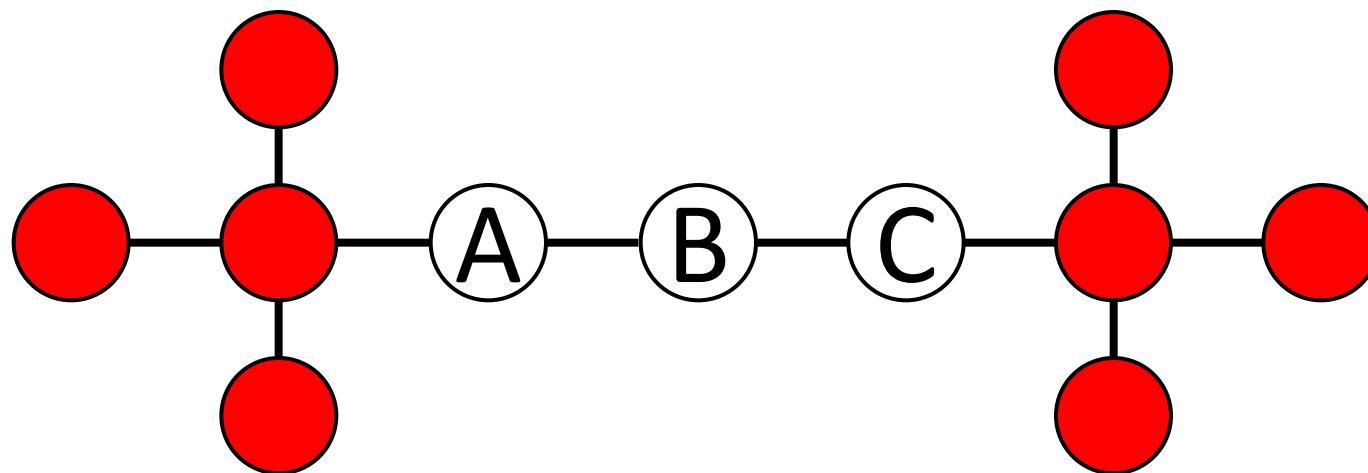


Can't get there  
by a local optimization





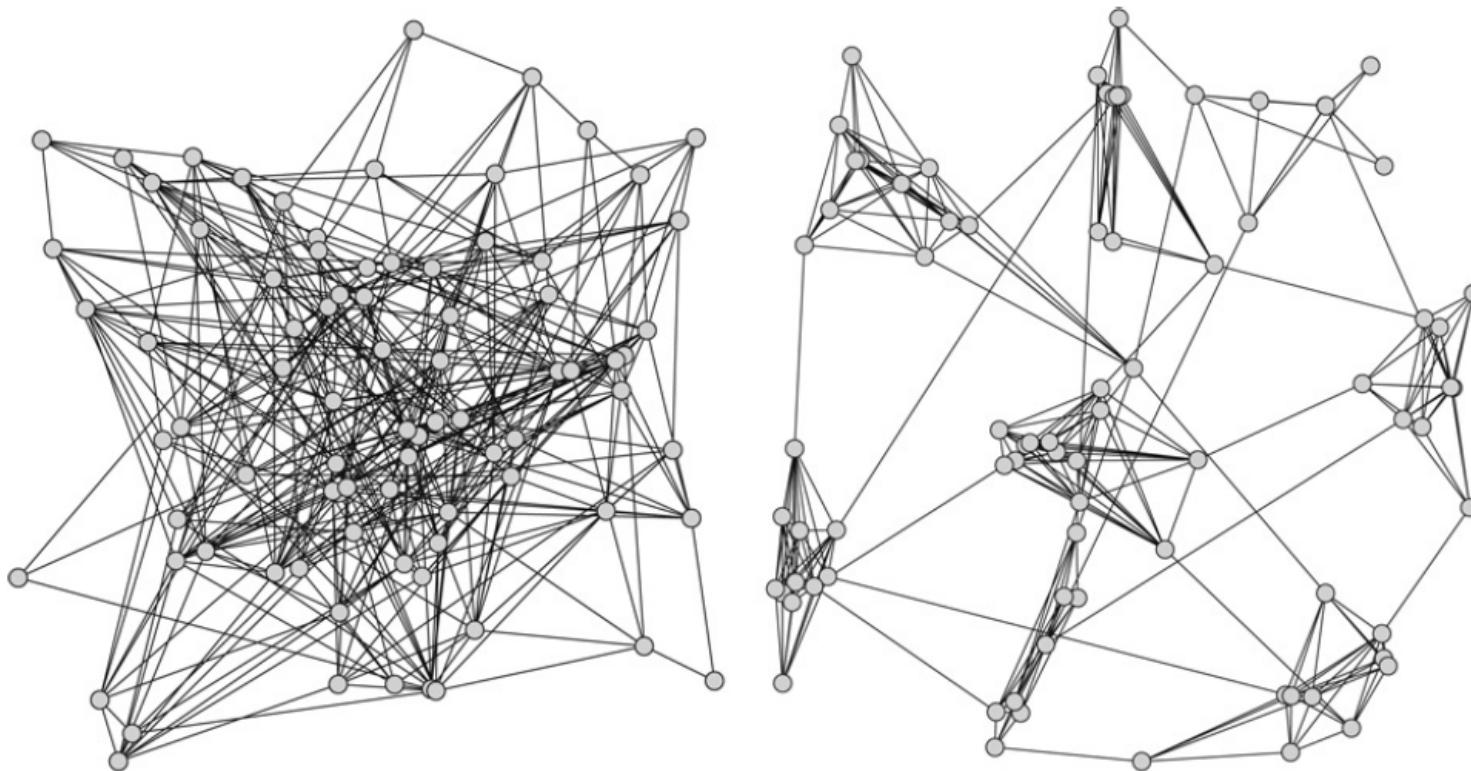
How can we  
move away from  
a locally optimal  
solution?



## Simulated Annealing Solution:

- Initialize T and subgraph Gn with score Sn
- Repeat while
  - Pick a neighboring node v to add to the subgraph
  - Score new subgraph -> Stest
  - If  $S_n < S_{test}$ : keep new subgraph
  - Else keep new subgraph with
$$P = \exp[-(S_{test} - S_n)/T]$$
- Modify T according to “cooling schedule.”

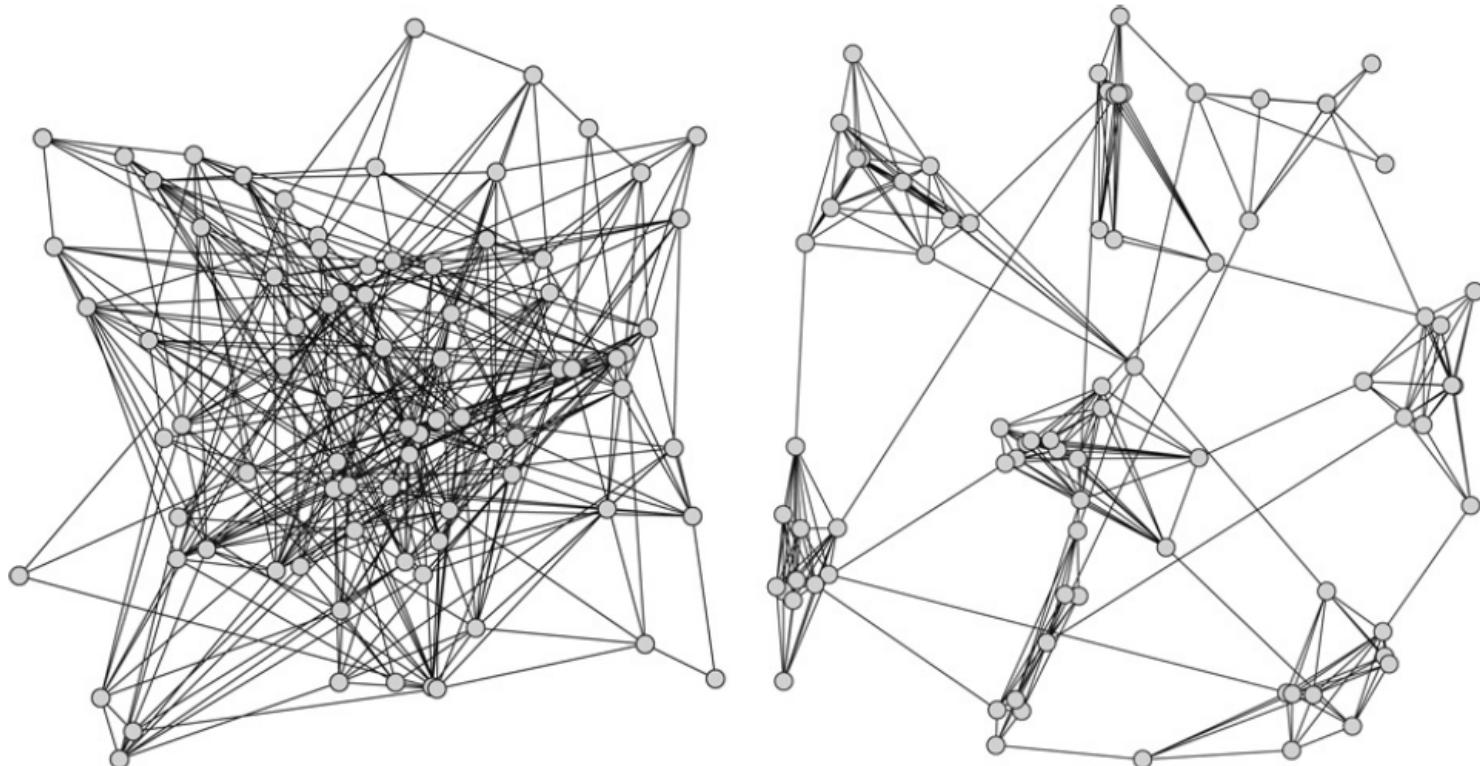
# Clustering Graphs



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Source: Schaeffer, Satu Elisa. "Graph Clustering." *Computer Science Review* 1, no. 1 (2007): 27-64.

Goal: divide the graph into subgraphs each of which has lots of internal connections and few connections to the rest of the graph

# Clustering Graphs

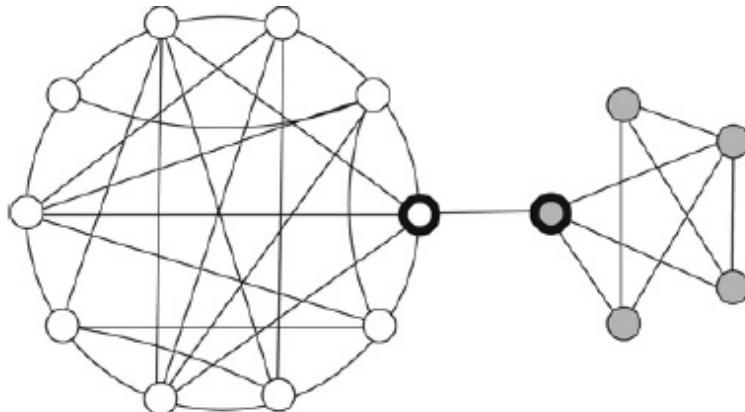


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Two algorithms:  
edge betweenness  
markov clustering

# Betweenness clustering

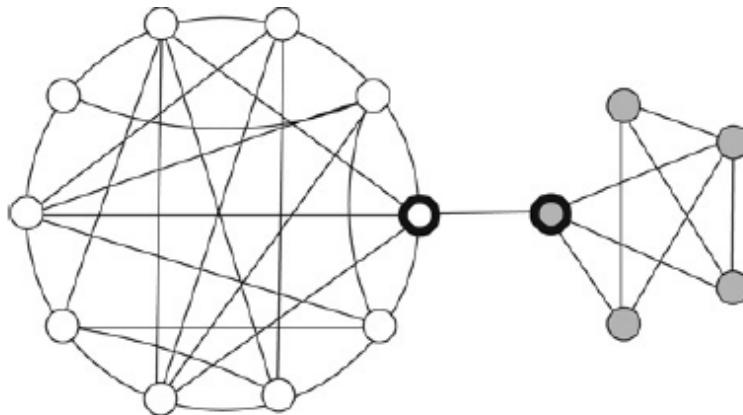
- Edge betweenness = number (or summed weight) of shortest paths between all pairs of vertices that pass through the edge.
  - Take a weighted average if there are >1 shortest paths for the same pair of nodes.



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# Betweenness clustering

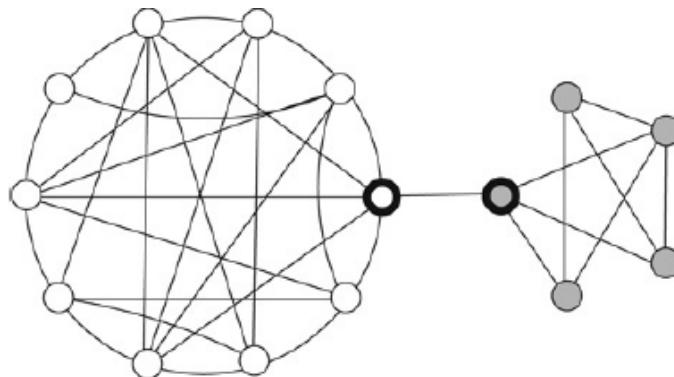
- Repeat until  $\max(\text{betweeness}) < \text{threshold}$ :
  - Compute betweeness
  - Remove edge with highest betweeness



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Source: Schaeffer, Satu Elisa. "Graph Clustering." *Computer Science Review* 1, no. 1 (2007): 27-64.

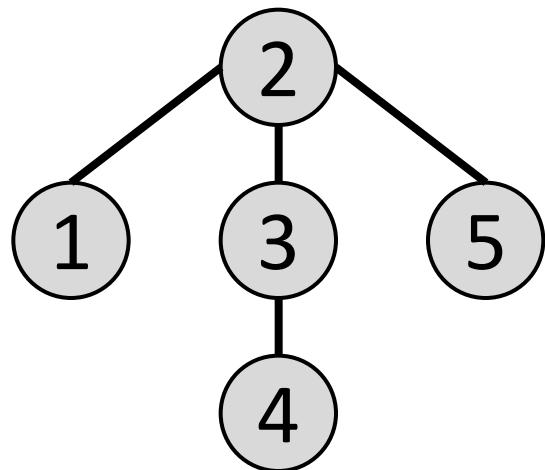
# Markov clustering (MCL)

- Goal: produce sharp partitions
- Intuition: A random walk will spend more time within a cluster than passing between clusters.
- Concisely explained here: Enright *et al.* NAR (2002) <http://www.ncbi.nlm.nih.gov/pmc/articles/PMC101833>



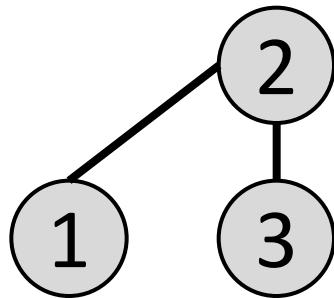
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Source: Schaeffer, Satu Elisa. "Graph Clustering." *Computer Science Review* 1, no. 1 (2007): 27-64.

# Adjacency Matrix



	1	2	3	4	5
1	0	1	0	0	0
2	1	0	1	0	1
3	0	1	0	1	0
4	0	0	1	0	0
5	0	1	0	0	0

# Adjacency Matrix



	1	2	3
1	0	1	0
2	1	0	1
3	0	1	0

×

	1	2	3
1	0	1	0
2	1	0	1
3	0	1	0

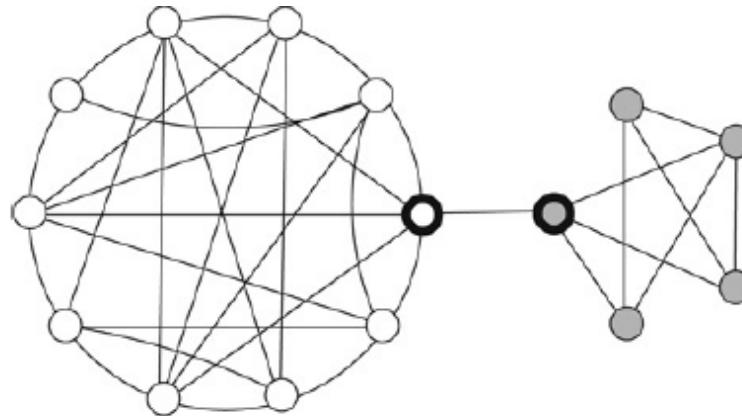
=

	1	2	3
1	1	0	1
2	0	2	0
3	1	0	1

$A^N: a_{ij} = m$  iff there exist exactly  $m$  paths of length  $N$  between  $i$  and  $j$ .

# MCL clustering

- Stochastic Matrix: each element  $M_{ij}$  represents a probability of moving from  $i$  to  $j$  (this is a “Column Stochastic Matrix”).

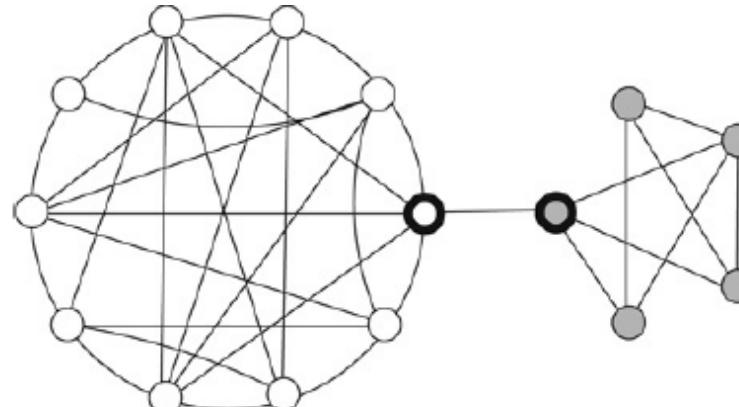


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Source: Schaeffer, Satu Elisa. "Graph Clustering." *Computer Science Review* 1, no. 1 (2007): 27-64.

# MCL clustering

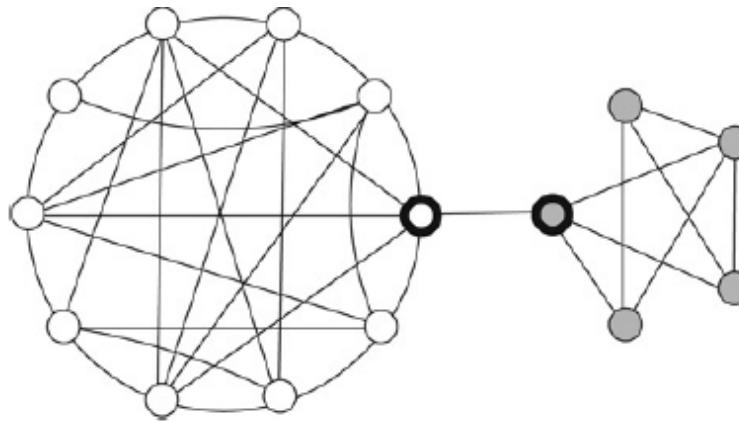
- Stochastic Matrix: each element  $M_{ij}$  represents a probability of moving from  $i$  to  $j$  (this is a “Column Stochastic Matrix”).
- Therefore,  $\sum_j p_{ij} = 1$
- The probability of moving from  $i$  to  $j$  in two steps is given by

$$(M^2)_{ij} = \sum_k p_{ik} p_{kj}$$



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Source: Schaeffer, Satu Elisa. "Graph Clustering." *Computer Science Review* 1, no. 1 (2007): 27-64.

- If we keep multiplying the stochastic matrix by itself, we compute the probabilities of longer and longer walks – we expect that the transitions will occur more frequently within a natural cluster than between them.



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Source: Schaeffer, Satu Elisa. "Graph Clustering." *Computer Science Review* 1, no. 1 (2007): 27-64.

- This procedure won't produce discrete clusters, so the algorithm includes an “inflation” step that exaggerates these effects: raise each element of the matrix to the power  $r$  and renormalize.

$$p_A = 0.9$$

$$p_B = 0.1$$

$$p_A \rightarrow \frac{.81}{.81 + .01} = .99$$

$$p_B \rightarrow \frac{.01}{.81 + .01} = .01$$

$$(\Gamma_r M)_{pq} = (M_{pq})^r \left/ \sum_{i=1}^k (M_{iq})^r \right..$$

**G** is a graph

add loops to **G** # needed for a prob. of no transition

set  $\Gamma$  to some value # affects granularity

set **M\_1** to be the matrix of random walks on **G**

while (change) {

**M\_2** = **M\_1** \* **M\_1** # expansion

**M\_1** =  $\Gamma(\mathbf{M}_2)$  # inflation

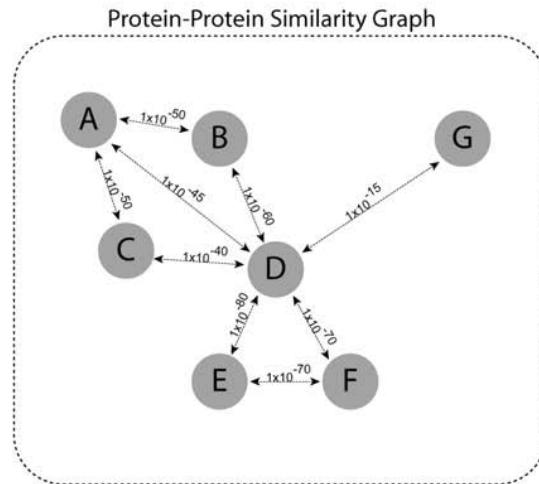
    change = difference(**M\_1**, **M\_2**)

}

set CLUSTERING as the components of **M\_1**

# Example

- Identifying protein families
- BLAST will identify proteins with shared domains, but these might not be very similar otherwise (eg: SH2, SH3 domains)

**A**

Generate weighted transition matrix using BLAST E-Values as weights (-logE)

**B**

Weighted Transition Matrix

	A	B	C	D	E	F	G
A	100	50	50	45	0	0	0
B	50	100	0	60	0	0	0
C	50	0	100	40	0	0	0
D	45	60	40	100	80	70	15
E	0	0	0	80	100	70	0
F	0	0	0	70	70	100	0
G	0	0	0	15	0	0	100

Transform weights into column-wise transition probabilities

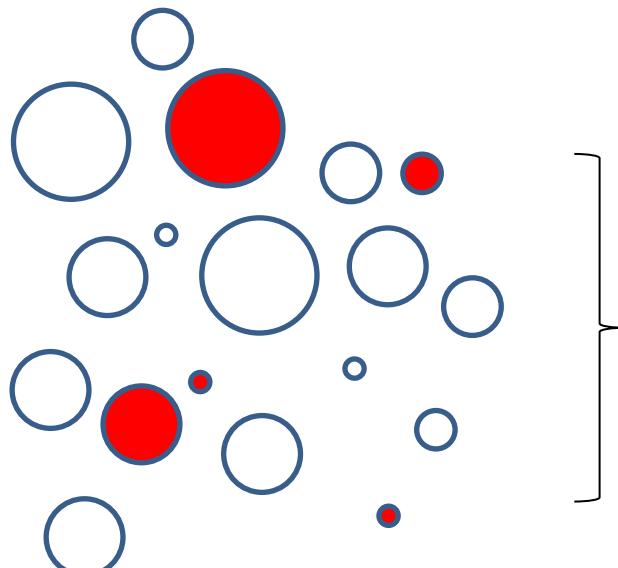
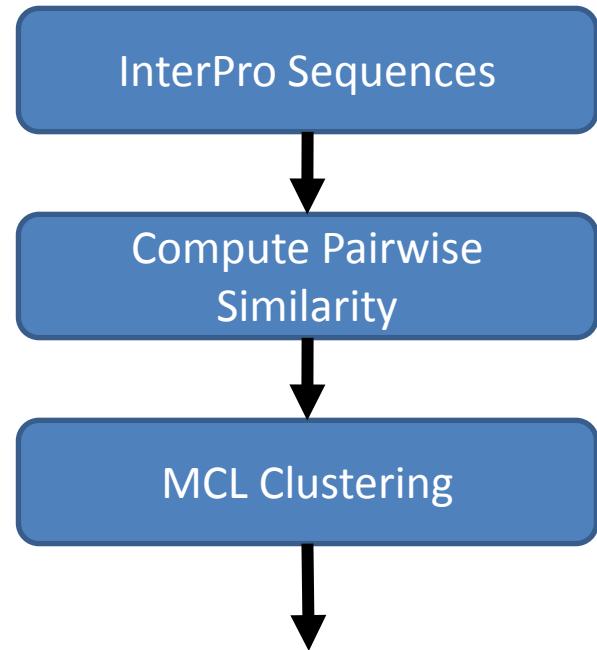
Markov Matrix

	A	B	C	D	E	F	G
A	0.42	0.24	0.20	0.11	0.00	0.00	0.00
B	0.20	0.48	0.24	0.15	0.00	0.00	0.00
C	0.20	0.00	0.40	0.10	0.00	0.00	0.00
D	0.18	0.28	0.16	0.24	0.32	0.29	0.13
E	0.00	0.00	0.00	0.19	0.40	0.29	0.00
F	0.00	0.00	0.00	0.17	0.28	0.42	0.00
G	0.00	0.00	0.00	0.04	0.00	0.00	0.87

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Source: Enright, Anton J., Stijn Van Dongen, et al. "An Efficient Algorithm for Large-scale Detection of Protein Families." *Nucleic Acids Research* 30, no. 7 (2002): 1575-84.

Extremely fast, since it  
only requires matrix  
operations



InterPro ID	No. of families	Domain description
IPR001064	141	Crystallin
IPR000504	110	RNA-binding region RNP-1 (RNA recognition motif)
IPR003006	107	Immunoglobulin and major histocompatibility complex domain
IPR000531	97	TonB-dependent receptor protein
IPR003015	96	Myc-type, helix-loop-helix dimerisation domain
IPR001680	76	G-protein β WD-40 repeats
IPR000561	73	EGF-like domain
IPR000169	72	Eukaryotic thiol (cysteine) proteases active sites
IPR001777	42	Fibronectin type III domain

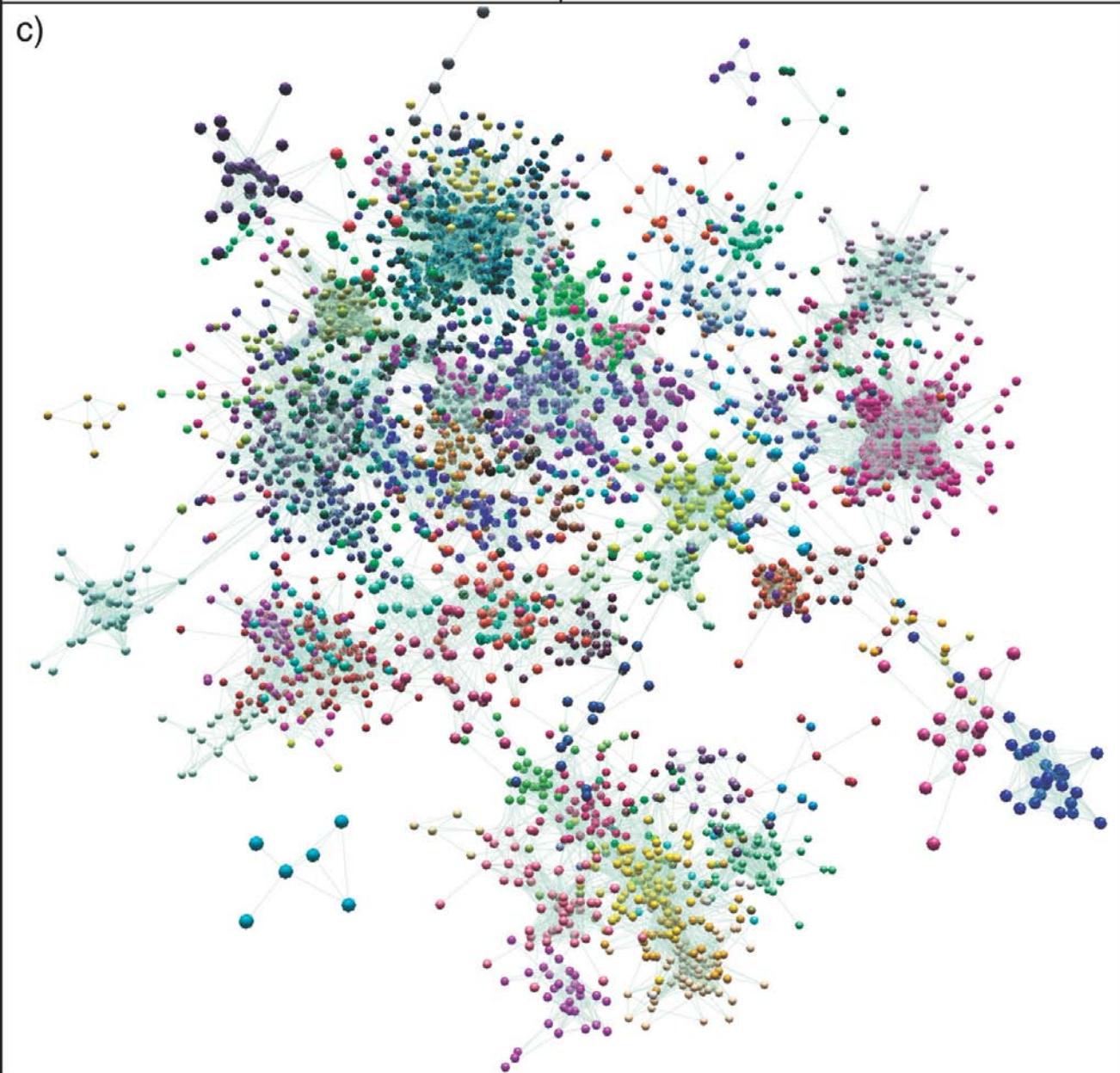
Distinct clusters identified by MCL can still share a common domain

# Example

- Clustering expression data for 61 mouse tissues
- Nodes = genes
- Edges = Pearson correlation coefficient > threshold
- Network gives an overview of connections not obvious from hierarchical clustering

Nodes=genes  
Edges=pearson  
correlation of  
expression in  
mouse tissues  
Clustered by  
MCL

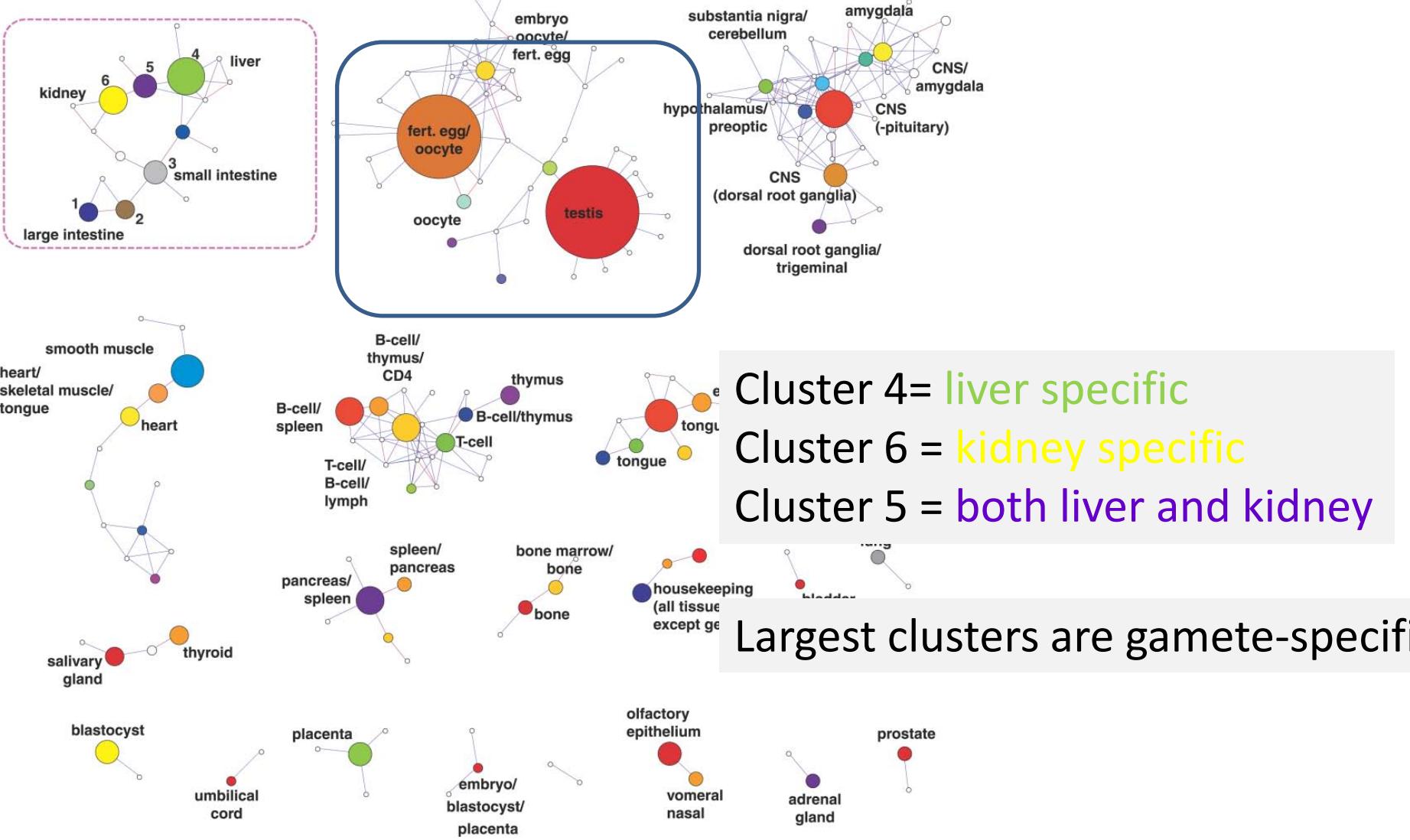
Freeman, et  
al.(2007) PLoS  
Comput Biol  
3(10): e206.  
doi:10.1371/journ  
al.pcbi.0030206



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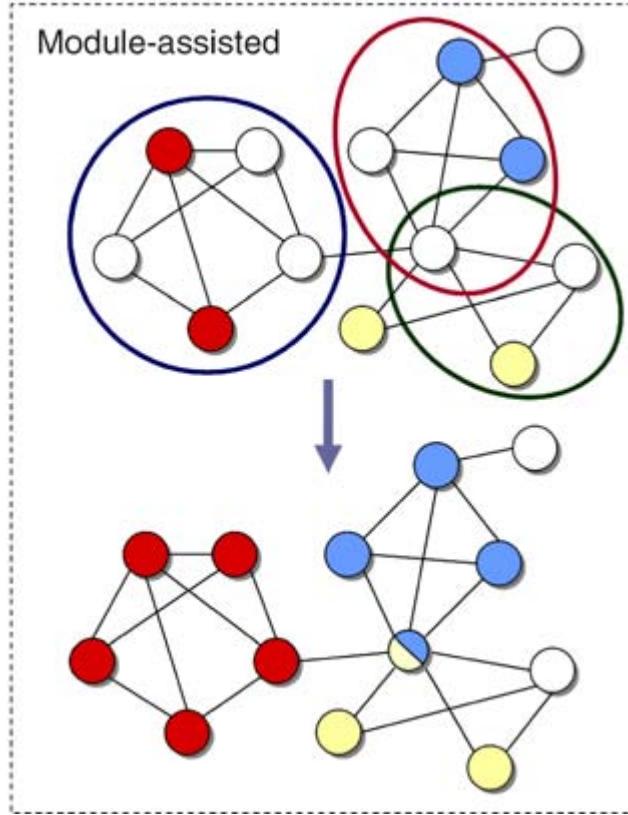
Source: Freeman, Tom C., Leon Goldovsky, et al. "Construction, Visualisation, and Clustering of Transcription Networks from Microarray Expression Data." *PLoS Computational Biology* 3, no. 10 (2007): e206.

c)



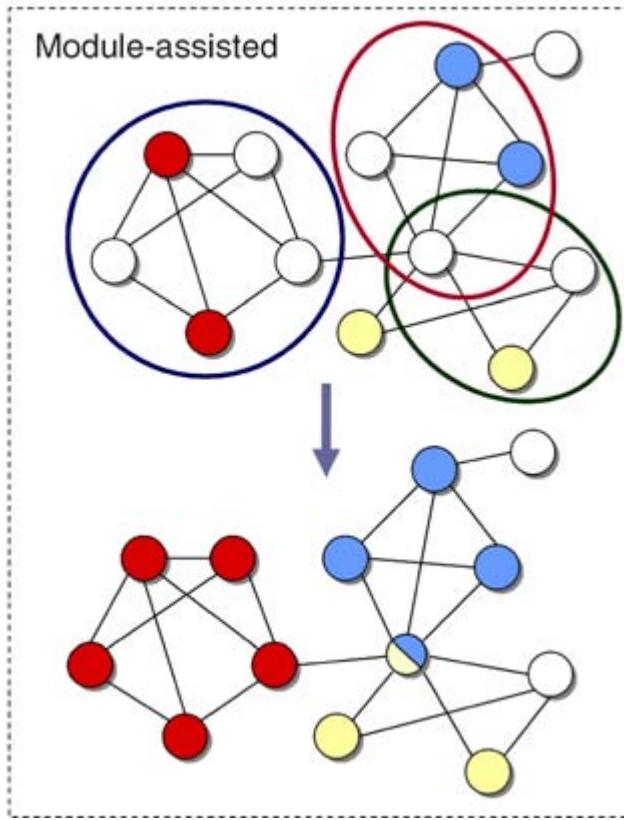
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Source: Freeman, Tom C., Leon Goldovsky, et al. "Construction, Visualisation, and Clustering of Transcription Networks from Microarray Expression Data." *PLoS Computational Biology* 3, no. 10 (2007): e206.



How do we decide  
which function to  
assign to members  
of a cluster?

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Source: Sharan, Roded, Igor Ulitsky, et al. "[Network-based Prediction of Protein Function](#)." *Molecular Systems Biology* 3, no. 1 (2007).



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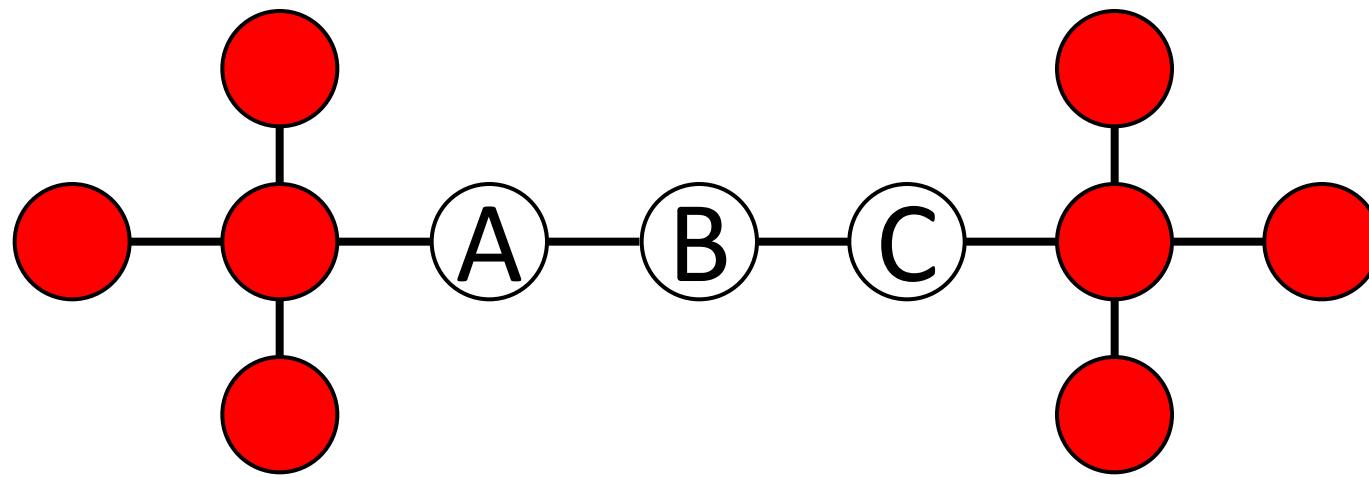
Source: Sharan, Roded, Igor Ulitsky, et al. "[Network-based Prediction of Protein Function](#)." *Molecular Systems Biology* 3, no. 1 (2007).

How do we decide which function to assign to members of a cluster?

- Consensus
- Significant by hypergeometric

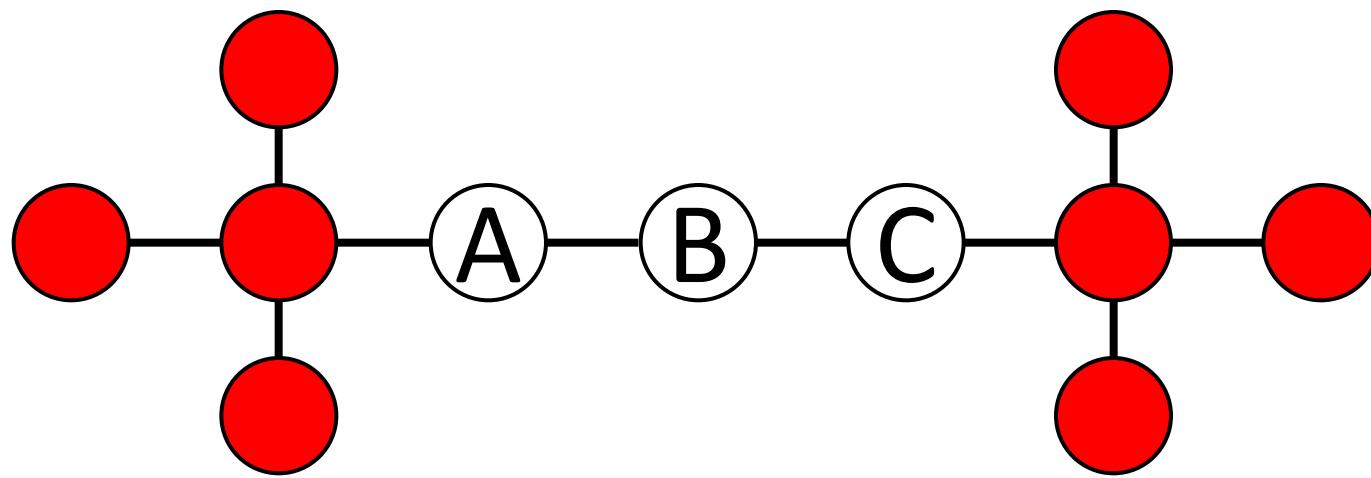
# Network Models

- Structure of network
  - Coexpression
  - Mutual information
  - Physical/genetic interactions
- Analysis of network
  - Ad hoc
  - Shortest path
  - Clustering
  - Optimization



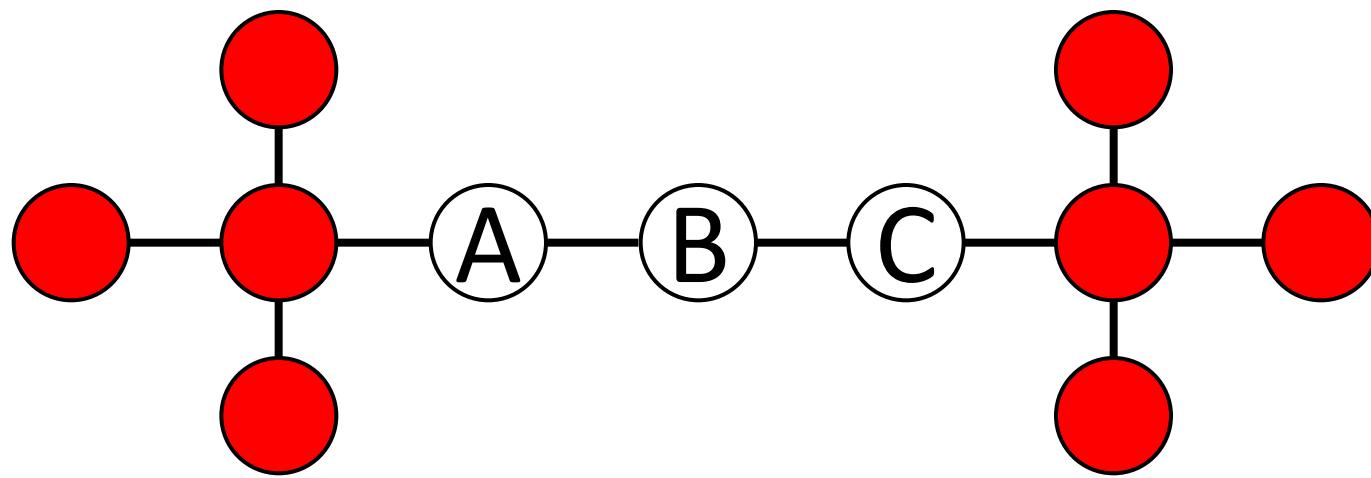
How do we find modules associated with specific data?

Example: paint a PPI network with expression data. Try to find connected components that have overall high expression. (Example: Ideker et al. (2002) Bioinformatics).

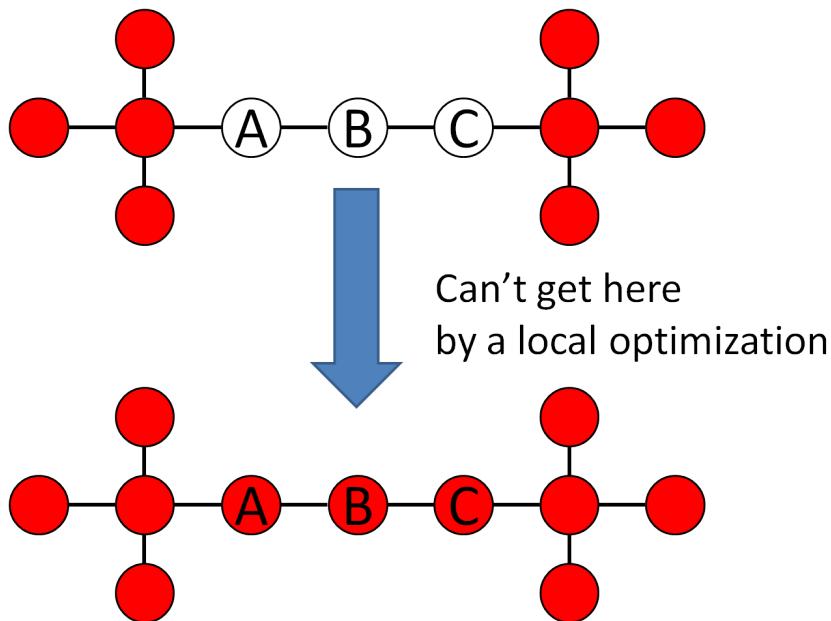


Active subgraph problem:

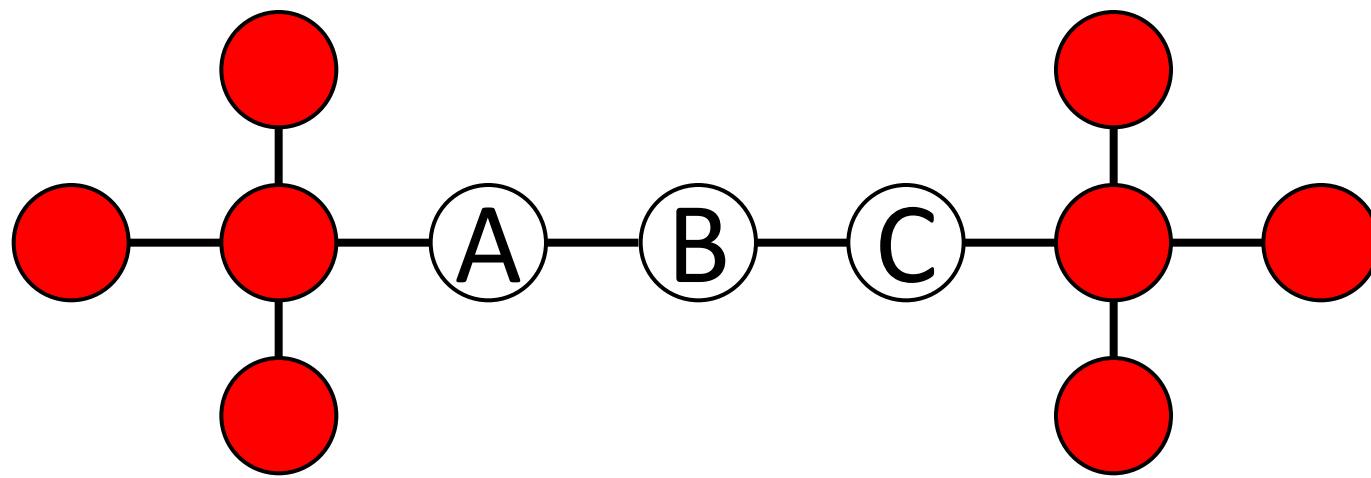
Can reveal hidden components of a biological response.



Where did we see something similar?



- The annotation problem attempts to label the entire graph.
- The active subnet problem searches for a part of the graph that is enriched in a label.



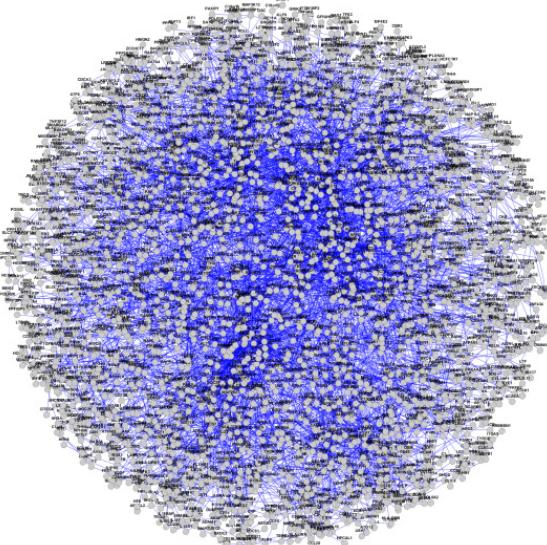
- **Steiner Tree Problem:** Find the smallest tree connecting all the vertices of in a set of interest (terminals).
- Downside: will include all terminals, including false positives.

# Experimental hits

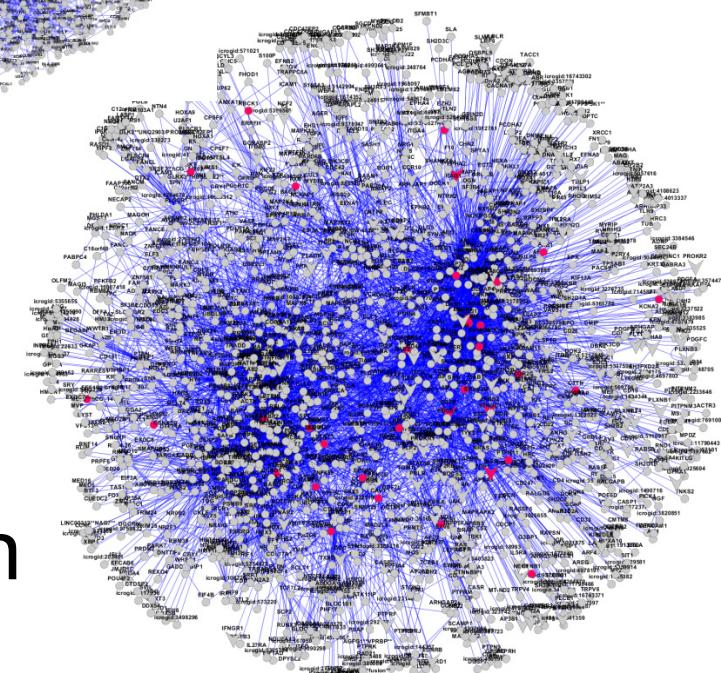
PXN	ENO1	FRK	INSR	CTTN	MAPK1	MAPK3	EFNB1
RBCK1	GIT1	BCAR1	ACP1	CCDC50	TNS3	PIK3R1	STAM2
STAM	PTPRA	PTK2	CBL	EGFR	EPS15	EPHB1	TNK2
PLEKHA5	PTPN11	ANXA2	PTPN18	SKT	GSK3B	INPPL1	SHC1
STAT3	ERBB2	CTNND1	PLCG1	ARHGEF5	AHCYL1	CAV1	PKP3
PRPF4B	RIN1						



# Interactome



# Naïve methods

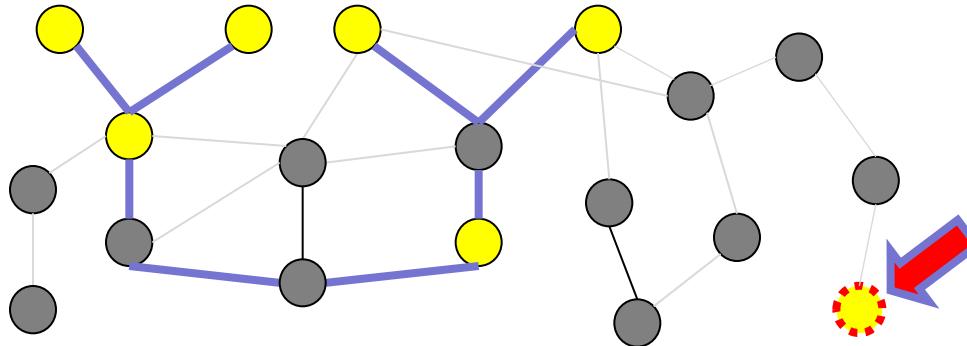


- Not all hits are real
- Not all edges are real
- Not all edges are known

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# Avoiding False Positives

- terminals
- no data



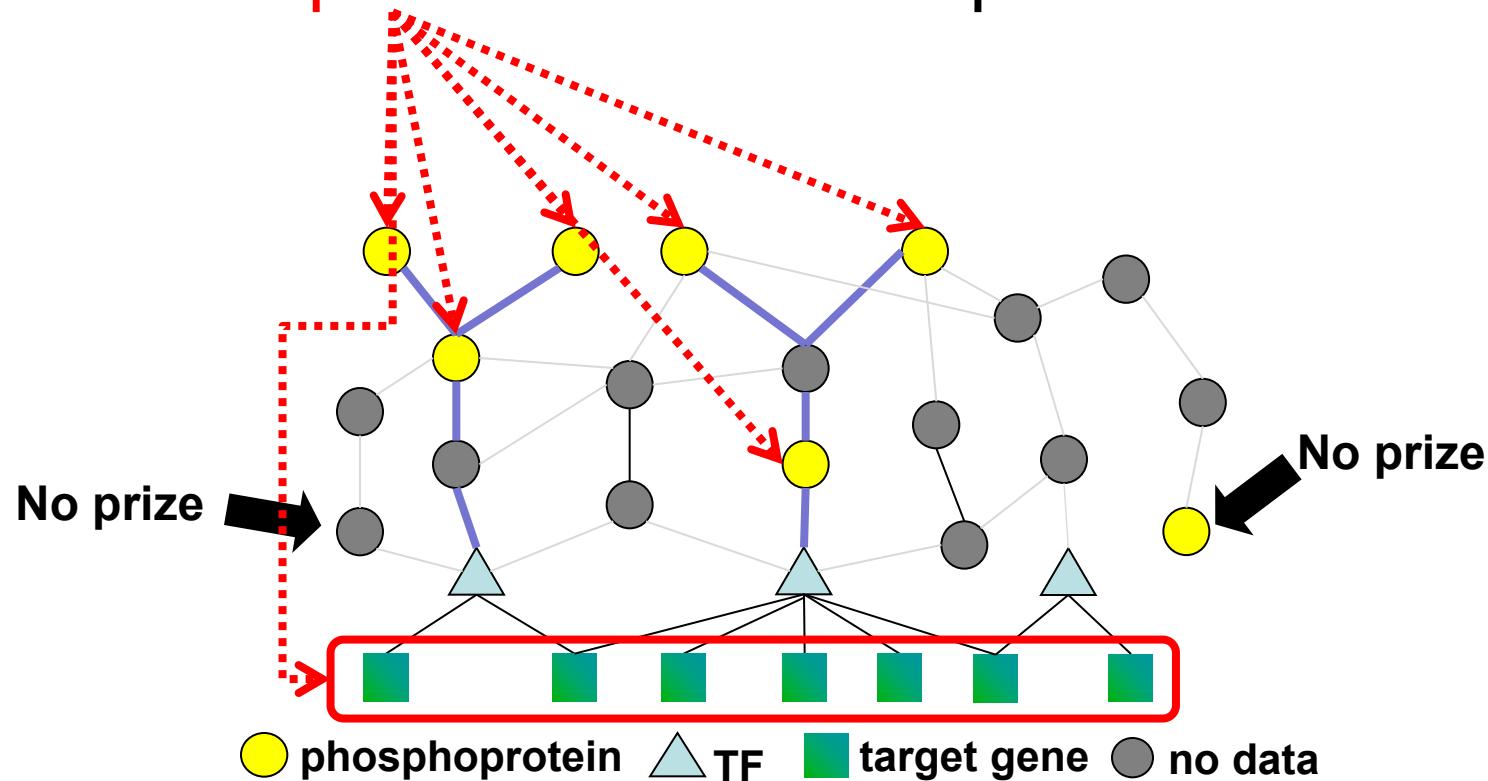
Steiner tree  
is forced to  
include this  
node

# Network Models

- Structure of network
  - Coexpression
  - Mutual information
  - Physical/genetic interactions
- Analysis of network
  - Ad hoc
  - Shortest path
  - Clustering
  - Optimization

# Prize Collecting Steiner Tree

- Collect a **prize** for each data point included

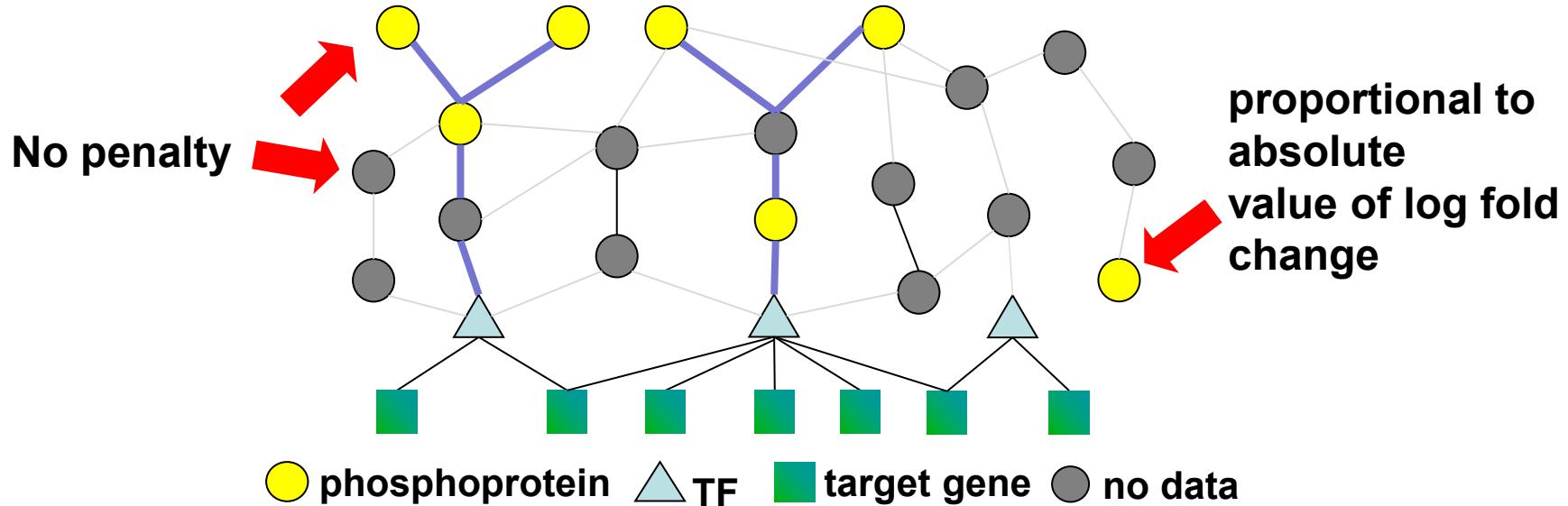


Courtesy of Huang et al. Used with permission.

Source: Huang, Shao-shan Carol, David C. Clarke, et al. "Linking Proteomic and Transcriptional Data through the Interactome and Epigenome Reveals a Map of Oncogene-induced Signaling." *PLoS Computational Biology* 9, no. 2 (2013): e1002887.

# Don't Include All Data

- Pay a **penalty** for **excluding** nodes



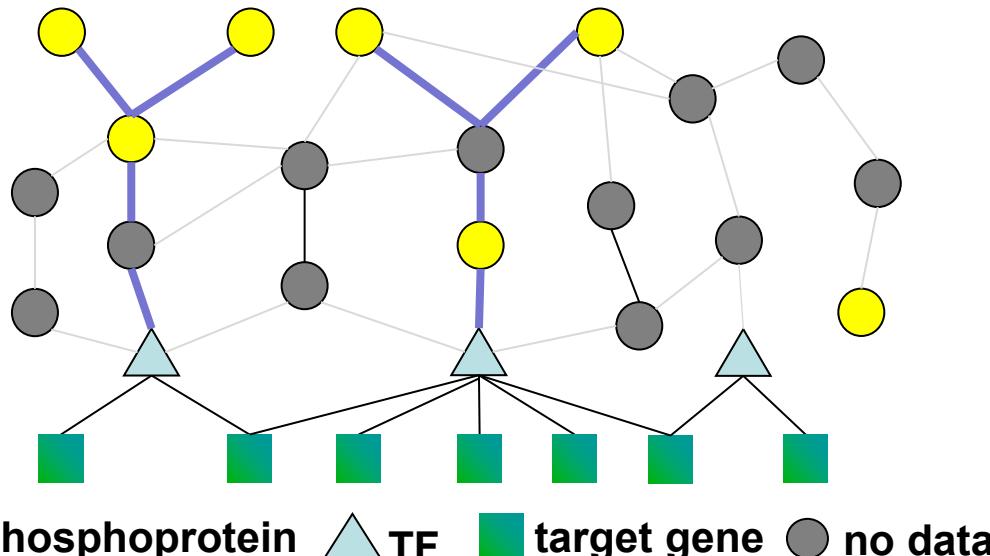
Courtesy of Huang et al. Used with permission.

Source: Huang, Shao-shan Carol, David C. Clarke, et al. "Linking Proteomic and Transcriptional Data through the Interactome and Epigenome Reveals a Map of Oncogene-induced Signaling." *PLoS Computational Biology* 9, no. 2 (2013): e1002887.

$$\sum_{v \text{ not in } T} \beta \text{penalty}(v) + \sum_{e \text{ in } T} \text{cost}(e)$$

# Avoid Unlikely Interactions

- Pay a **cost** for including edges based on probability



Courtesy of Huang et al. Used with permission.

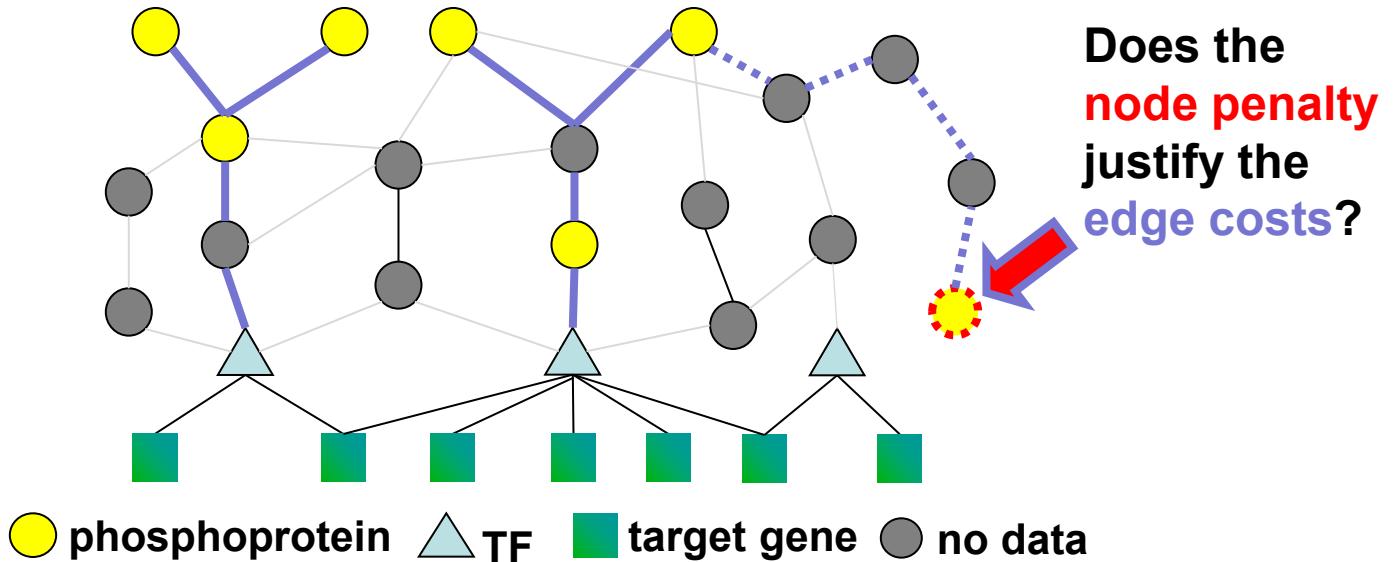
Source: Huang, Shao-shan Carol, David C. Clarke, et al. "Linking Proteomic and Transcriptional

Data through the Interactome and Epigenome Reveals a Map of Oncogene-induced Signaling."

*PLoS Computational Biology* 9, no. 2 (2013): e1002887.

$$\sum_{v \text{ not in } T} \beta \text{penalty}(v) + \boxed{\sum_{e \text{ in } T} \text{cost}(e)}$$

# Balanced Objective Function



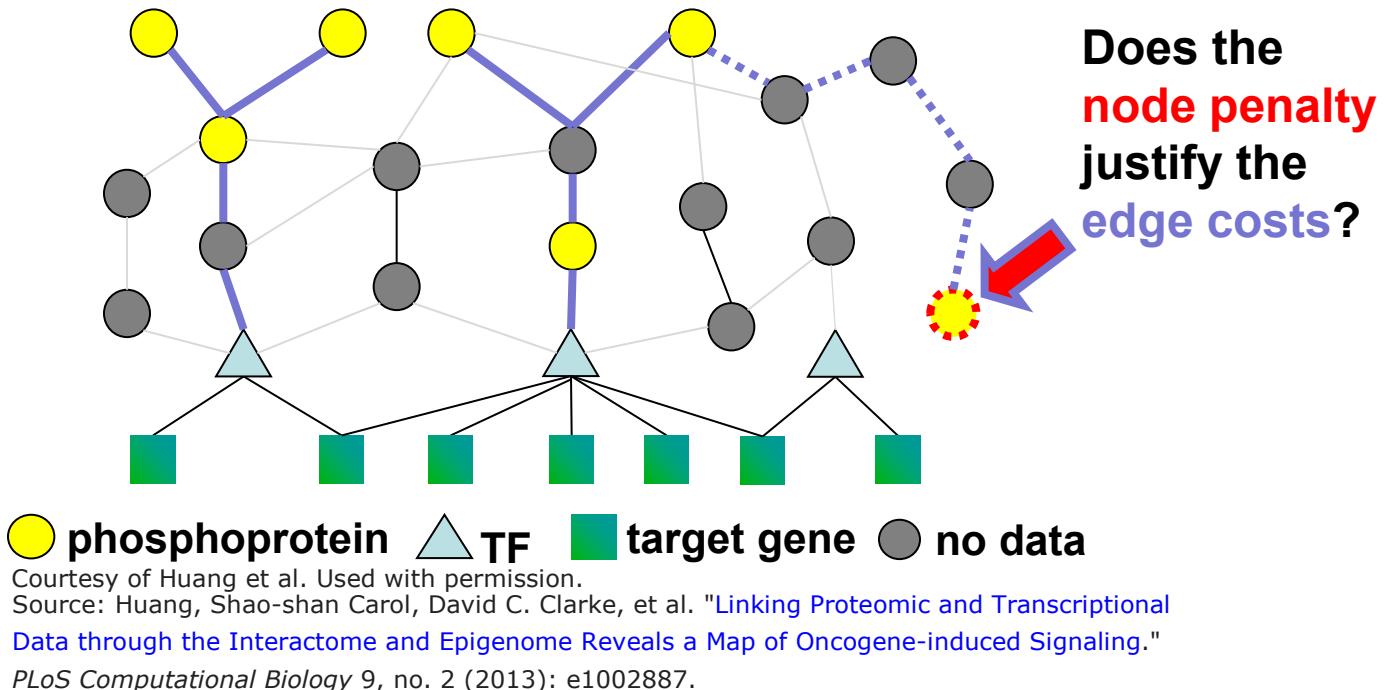
Courtesy of Huang et al. Used with permission.

Source: Huang, Shao-shan Carol, David C. Clarke, et al. "Linking Proteomic and Transcriptional Data through the Interactome and Epigenome Reveals a Map of Oncogene-induced Signaling." *PLoS Computational Biology* 9, no. 2 (2013): e1002887.

$$\sum_{v \text{ not in } T} \beta \text{penalty}(v) + \sum_{e \text{ in } T} \text{cost}(e)$$

# Optimization methods:

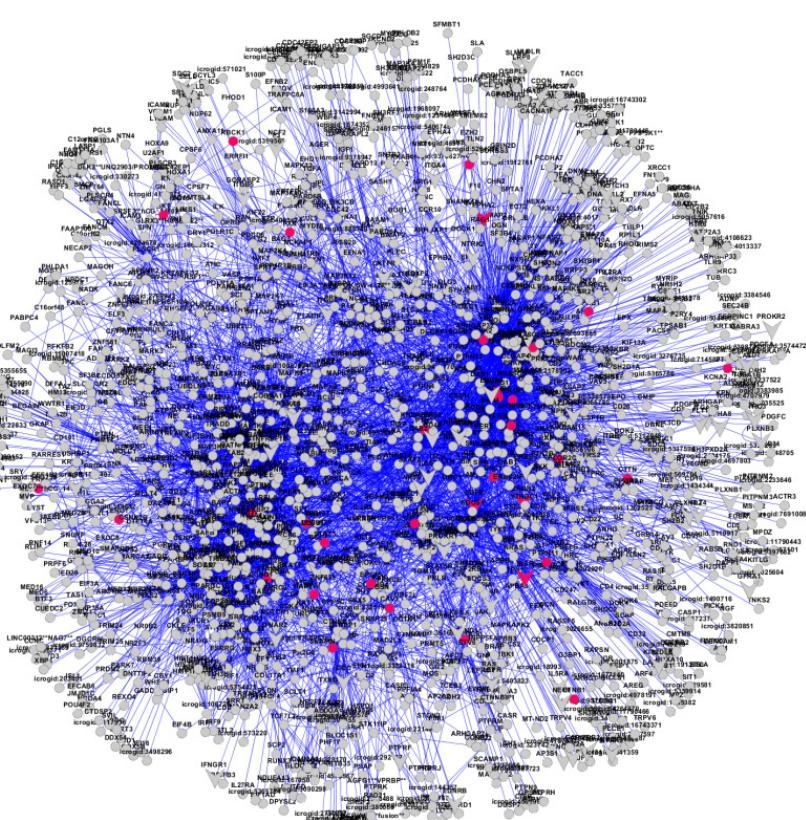
- Biazzo I, Braunstein A, Zecchina R.  
Phys Rev E Stat Nonlin Soft Matter Phys. 2012 Aug;86(2 Pt 2):026706.
- I. Ljubic, R. Weiskircher, U. Pferschy, G. Klau, P. Mutzel, and M. Fischetti:  
Mathematical Programming, Series B, 105(2-3):427-449, 2006.



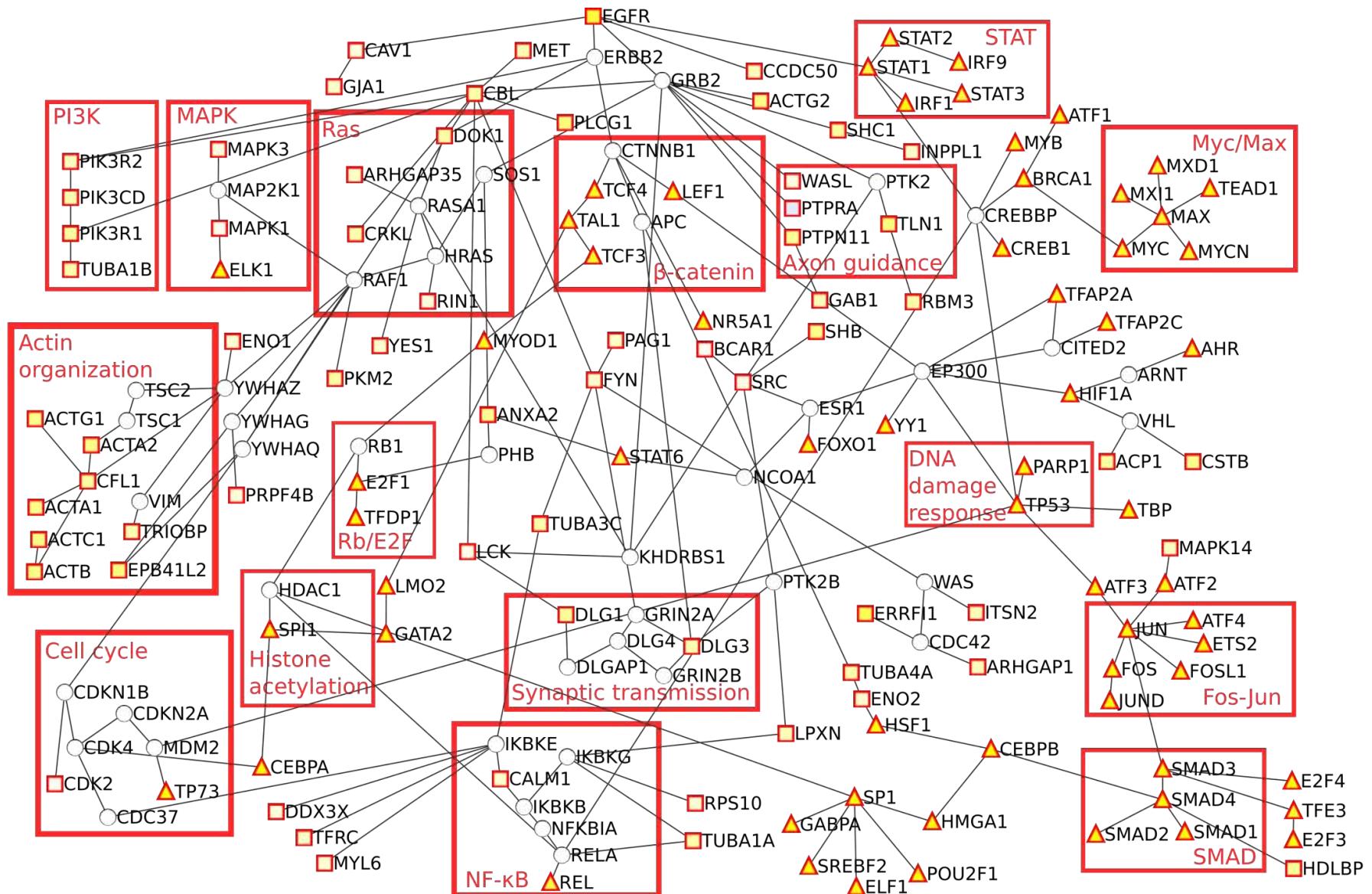
$$\sum_{v \text{ not in } T} \beta \text{penalty}(v) + \sum_{e \text{ in } T} \text{cost}(e)$$

# Naïve Methods

- >2,500 nearest neighbors of phosphoproteins
  - >4,500 nearest neighbors of phosphoproteins +transcription factors



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Courtesy of Huang et al. Used with permission.

Source: Huang, Shao-shan Carol, David C. Clarke, et al. "Linking Proteomic and Transcriptional Data through the Interactome and Epigenome Reveals a Map of Oncogene-induced Signaling."

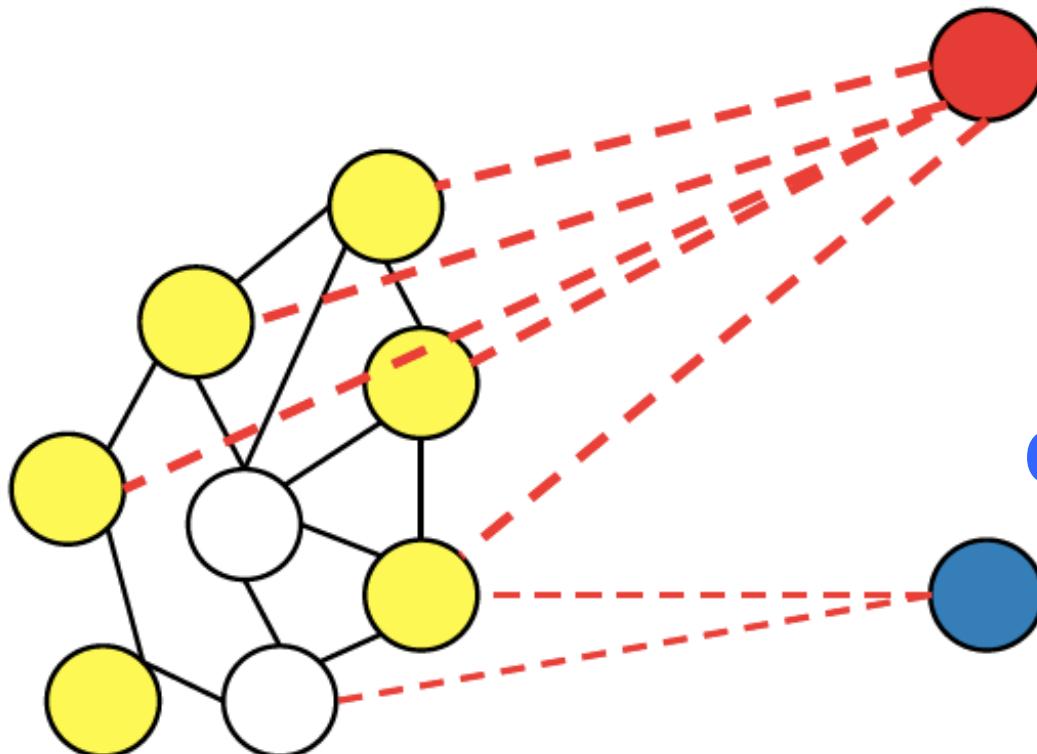
*PLoS Computational Biology* 9, no. 2 (2013): e1002887.

# Can we find drug targets?

Rank every node by  
weighted distance to all  
prize-collecting Steiner tree  
nodes

**High rank targets**

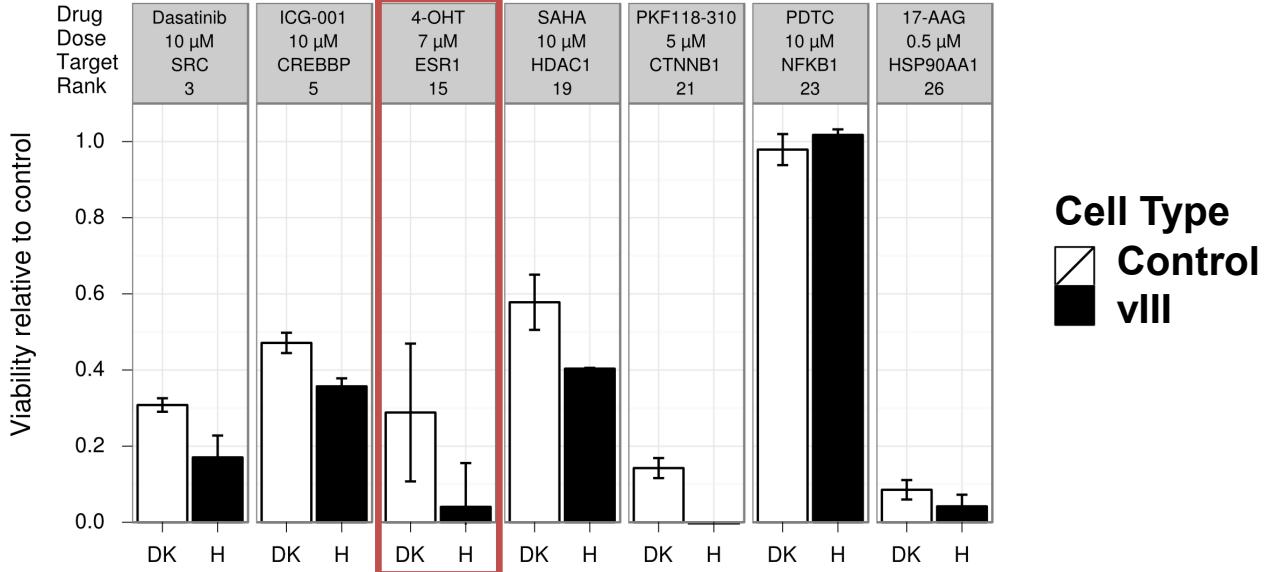
**Steiner Tree**



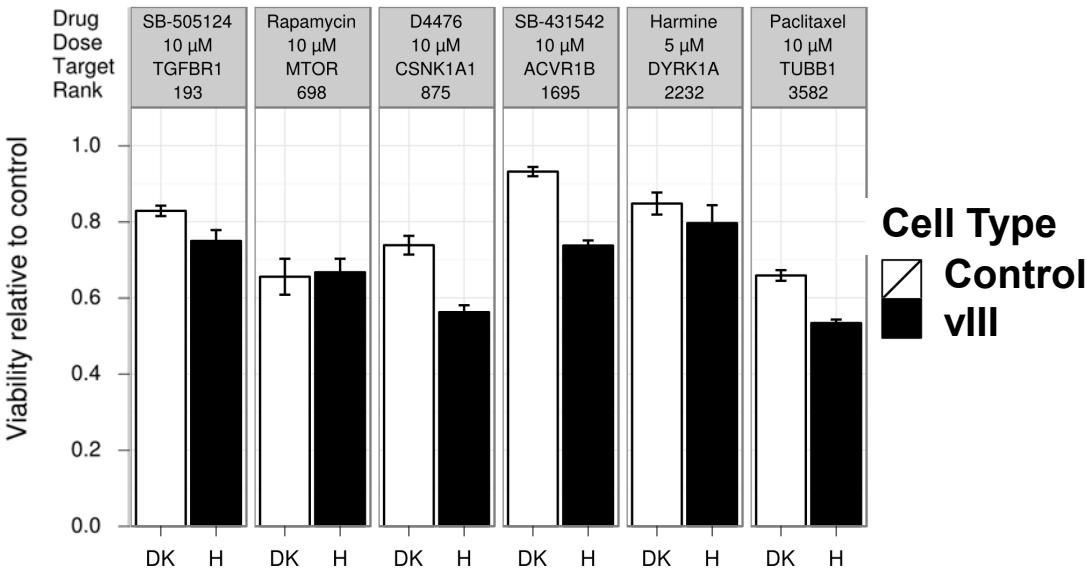
**Control targets**

Courtesy of Huang et al. Used with permission.  
Source: Huang, Shao-shan Carol, David C. Clarke, et al. "Linking Proteomic and Transcriptional Data through the Interactome and Epigenome Reveals a Map of Oncogene-induced Signaling." *PLoS Computational Biology* 9, no. 2 (2013): e1002887.

**Rank**  
**<27  
out of  
11,637**



**Lower  
Rank  
Targets  
193 to  
3,582  
out of  
11,637**



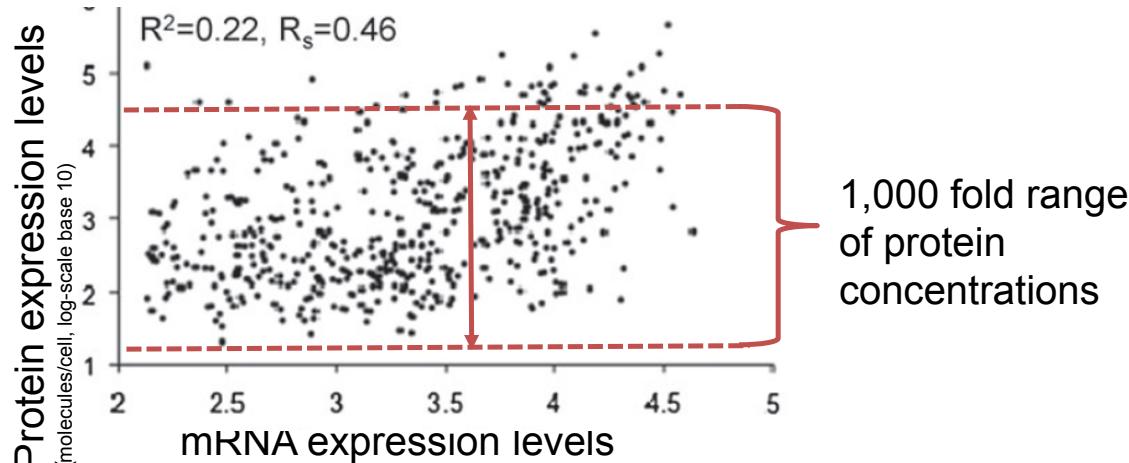
Courtesy of Huang et al. Used with permission.

Source: Huang, Shao-shan Carol, David C. Clarke, et al. "Linking Proteomic and Transcriptional Data through the Interactome and Epigenome Reveals a Map of Oncogene-induced Signaling." *PLoS Computational Biology* 9, no. 2 (2013): e1002887.

# Data Integration

# Approach

## mRNA levels do not predict protein levels



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Source: de Sousa Abreu, Raquel, Luiz O. Penalva, et al. "Global Signatures of Protein and mRNA Expression Levels." *Molecular Biosystems* 5, no. 12 (2009): 1512-26.

(arbitrary units, log-scale base 10)

Raquel de Sousa Abreu, Luiz Penalva, Edward Marcotte and Christine Vogel, *Mol. BioSyst.*, 2009 DOI: [10.1039/b908315d](https://doi.org/10.1039/b908315d)

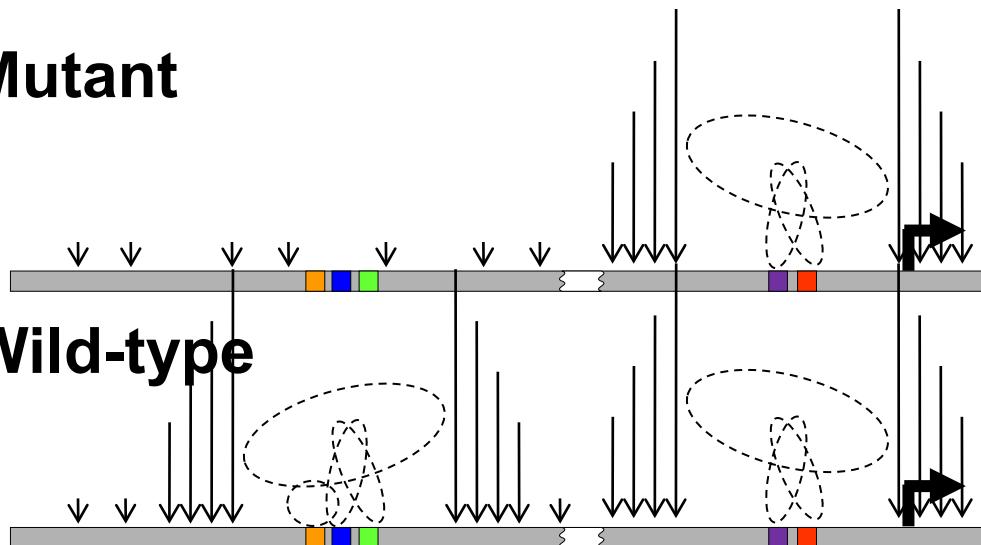
	<b>SpectrumMill</b>	<b>msInspect</b>	<b>msBID</b>	<b>NSAF</b>	<b>RPKM</b>	<b>Microarray</b>
<b>SpectrumMill</b>	-	0.91 (0.92)	0.91 (0.91)	0.90 (0.90)	0.49 (0.51)	0.36 (0.40)
<b>msInspect</b>	0.91 (0.92)	-	0.89 (0.91)	0.87 (0.88)	0.51 (0.53)	0.40 (0.44)
<b>msBID</b>	0.91 (0.91)	0.89 (0.91)	-	0.84 (0.89)	0.54 (0.54)	0.41 (0.42)
<b>NSAF</b>	0.90 (0.90)	0.87 (0.88)	0.84 (0.89)	-	0.51 (0.53)	0.42 (0.44)

Source: Ning, Kang, Damian Fermin, et al. "Comparative Analysis of Different Label-free Mass Spectrometry Based Protein Abundance Estimates and Their Correlation with RNA-Seq Gene Expression Data." *Journal of Proteome Research* 11, no. 4 (2012): 2261-71.

Kang Ning, Damian Fermin, and Alexey I. Nesvizhskii J Proteome Res. 2012 April 6; 11(4): 2261–2271.

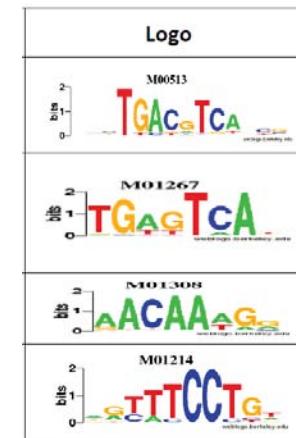
# L18 Chromatin and DNase-seq Analysis

**Mutant**



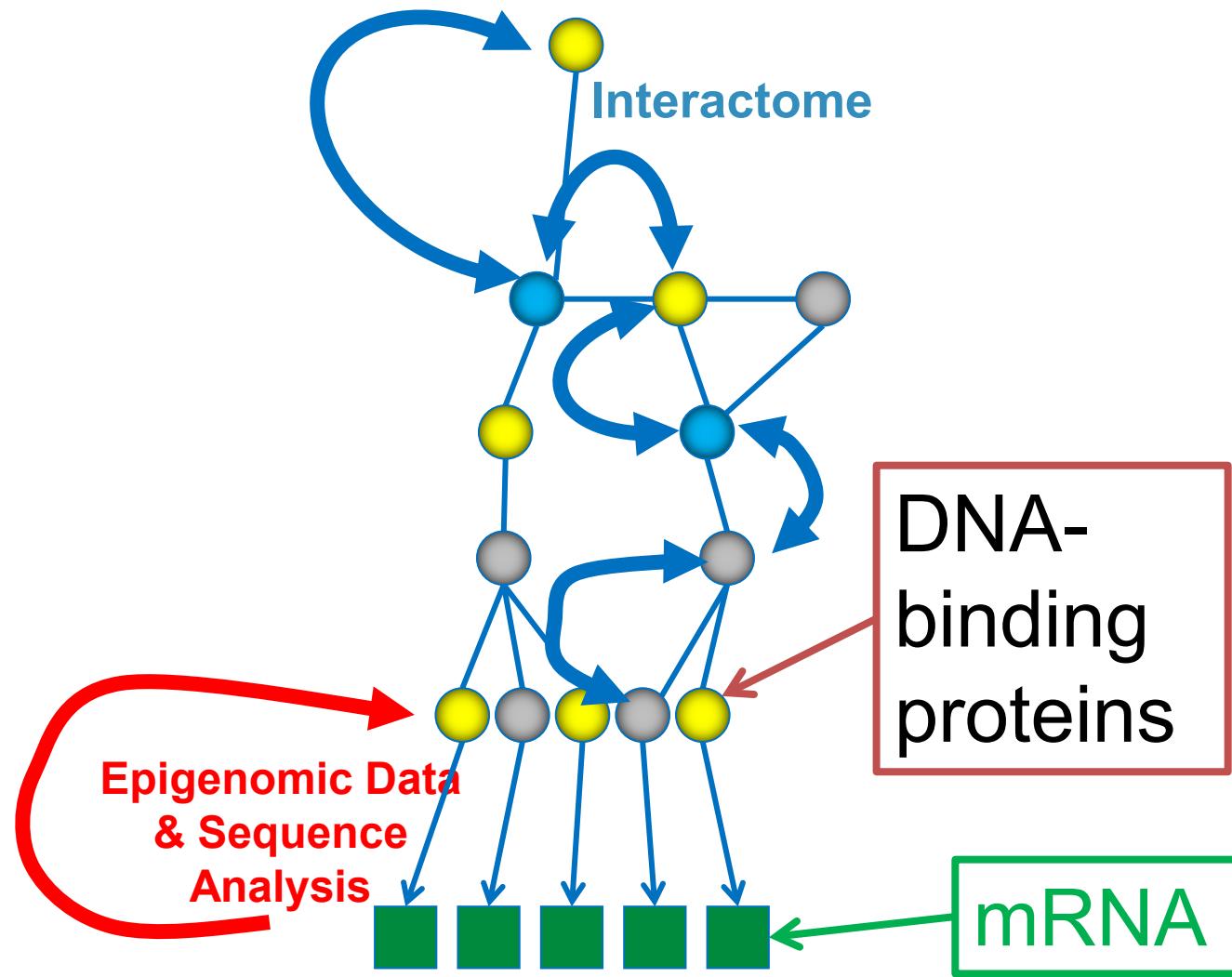
**Wild-type**

Sequence  
Analysis



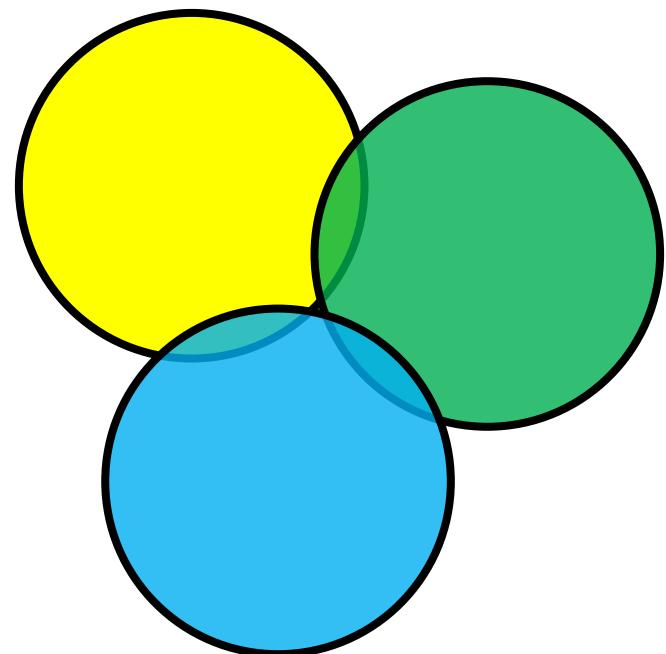
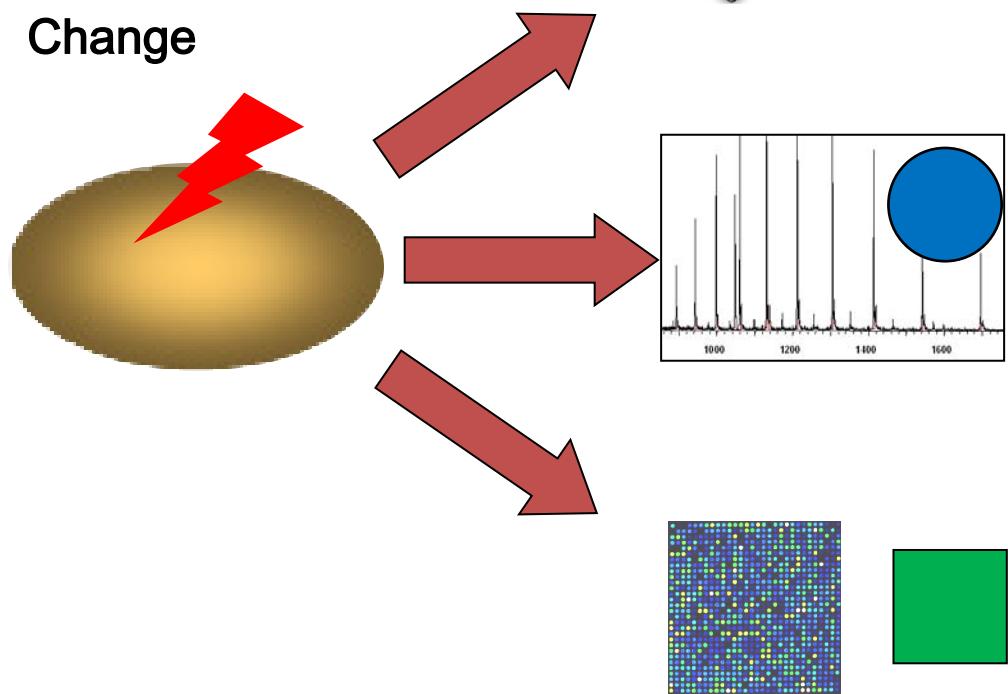
# Move upstream of transcription

Network  
integration

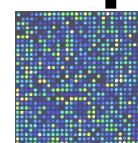


# 'Omic data don't agree

Toxic  
Compound,  
Mutation,  
Environmental  
Change



# Genetic vs. Expression Data



Perturbation	Differentially expressed genes	Genetic hits	Number of overlapping genes
Growth arrest (Hydroxyurea)	59	86	0
DNA damage (MMS)	198	1448	43
Protein biosynthesis block (Cycloheximide)	20	164	0
ER stress (Tunicamycin)	200	127	5
ATP synthesis block (Arsenic)	828	50	9
Fatty acid metabolism (oleate)	269	103	9
Gene inactivation (24 datasets, median shown)	27	130	0

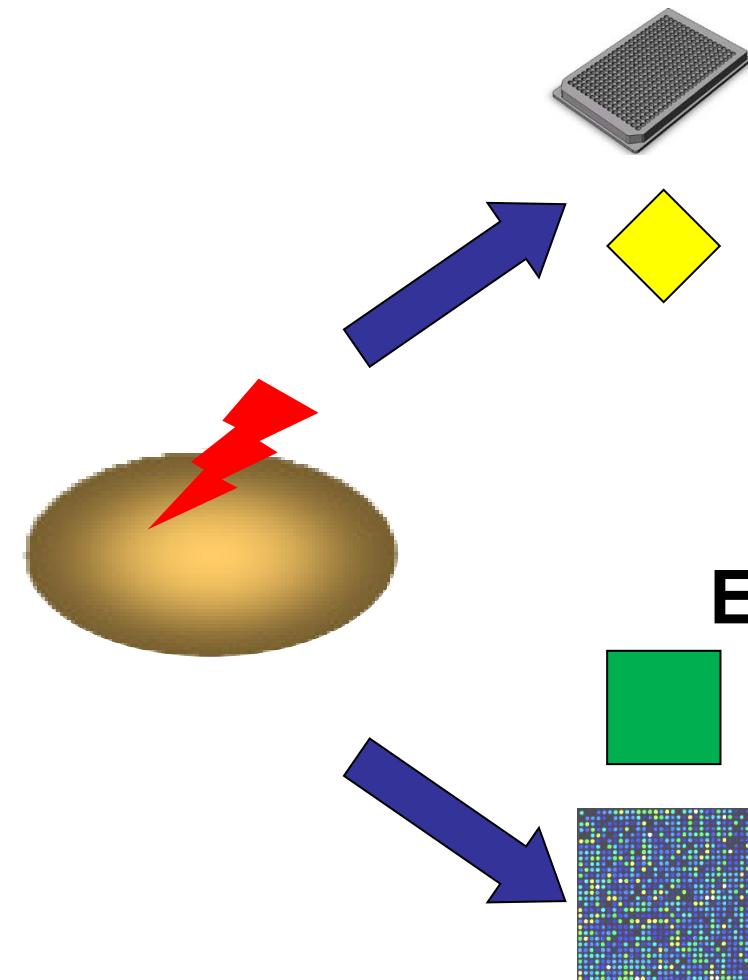
Bridging high-throughput genetic and transcriptional data reveals cellular responses to alpha-synuclein toxicity

Nature Genetics Published online: 22 February 2009

# For 156 perturbations:

## Genetic Data Enriched for:

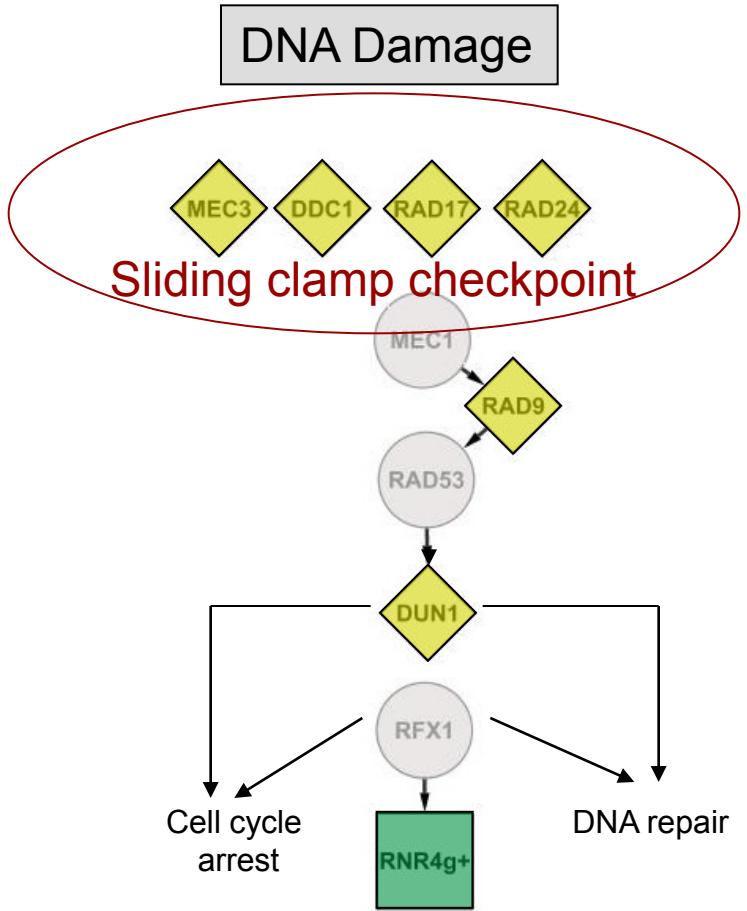
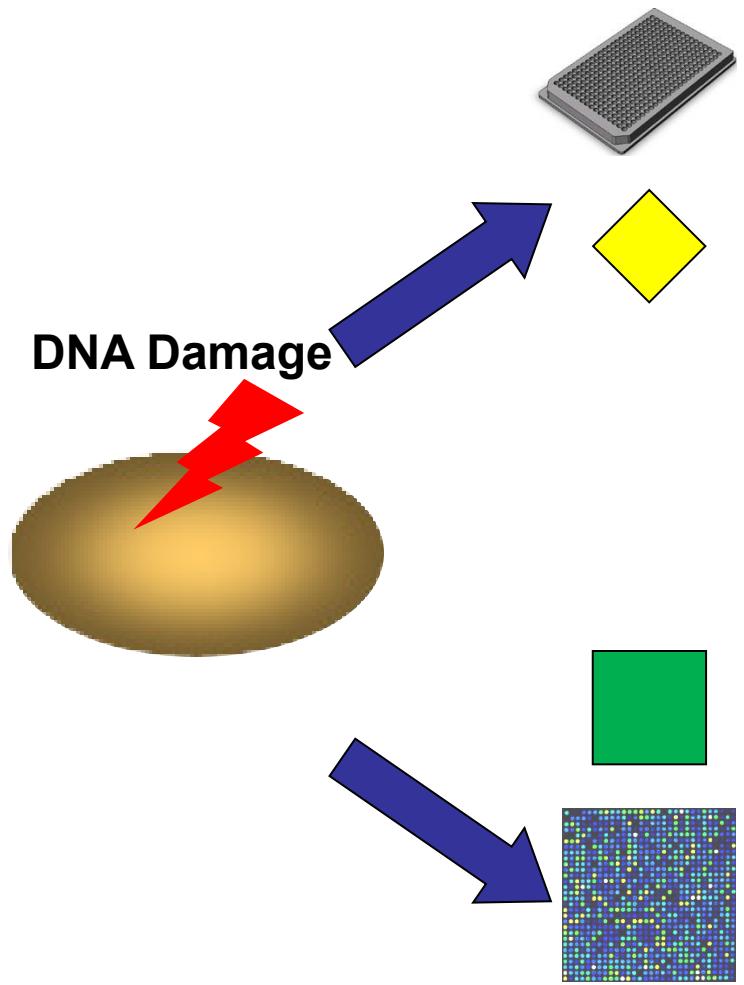
- Transcriptional regulation
- Signal transduction

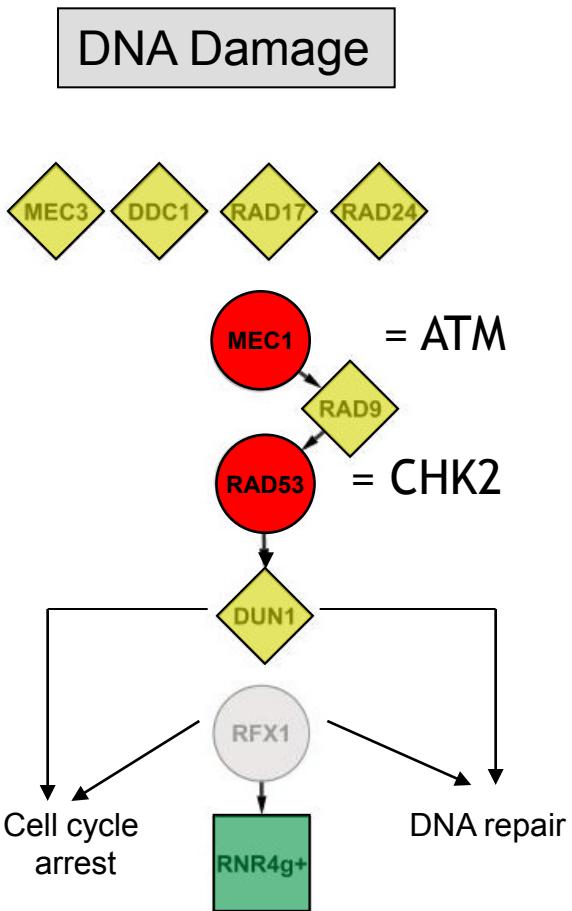
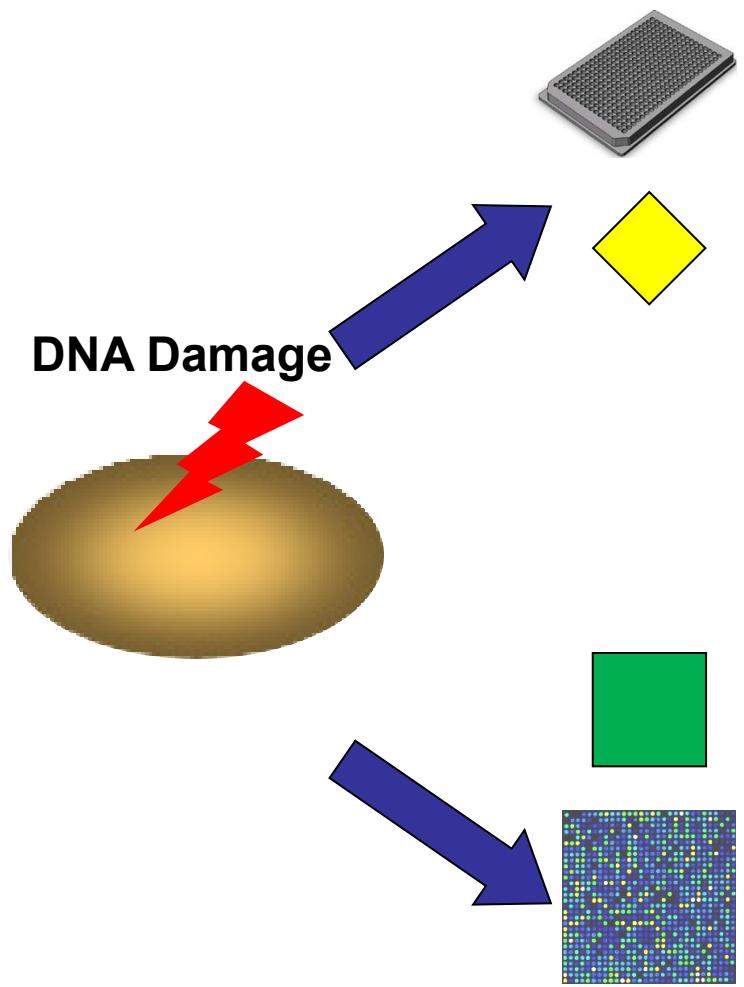


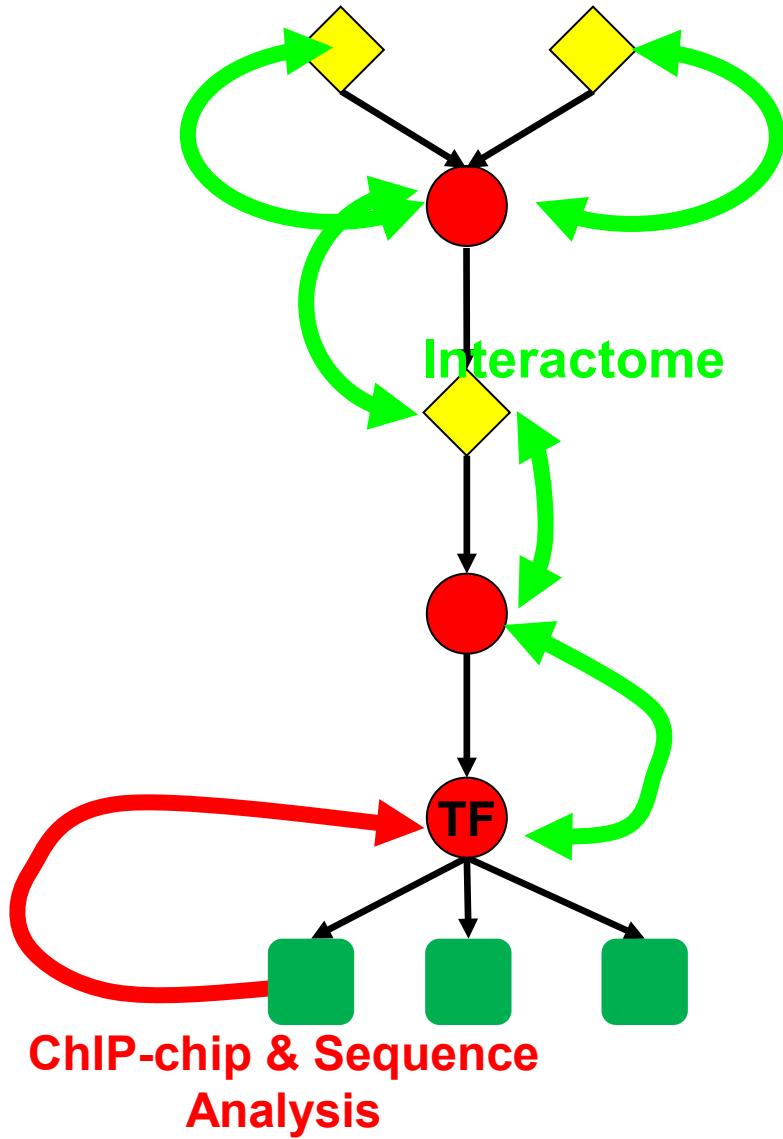
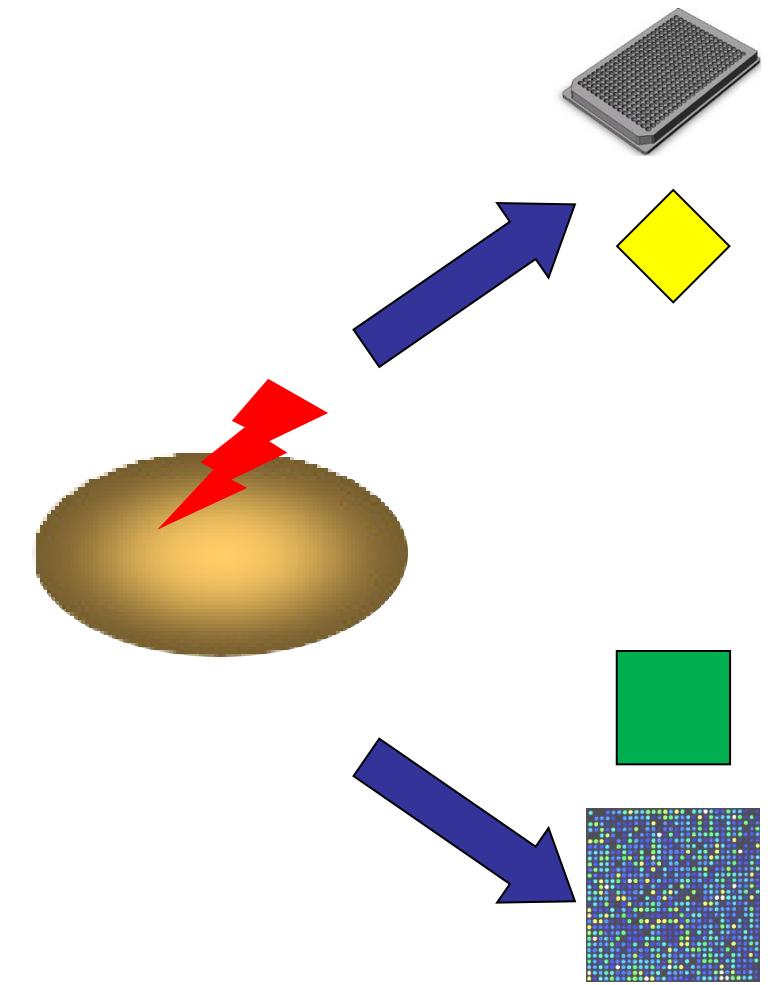
## Expression Data Enriched for:

### Metabolic Processes

e.g., organic acid  
metabolic process,  
oxidoreducatse activities





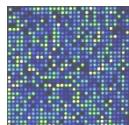


# Test case: Perturbing pheromone response pathway

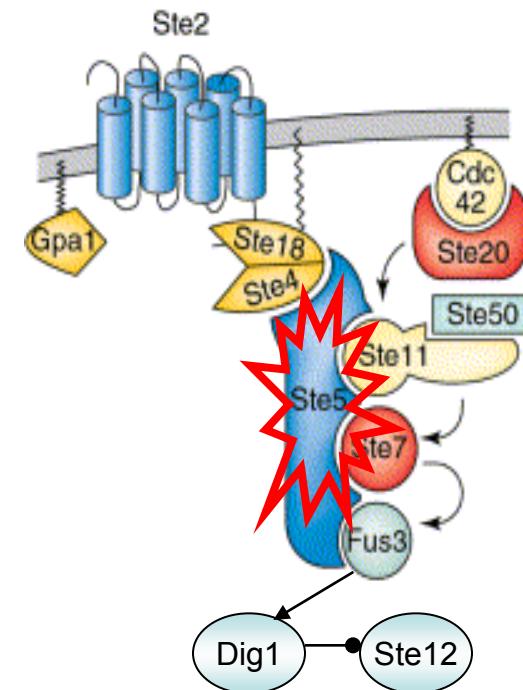
## Perturbing Ste5



20 genes rescue mating phenotype (SGD)



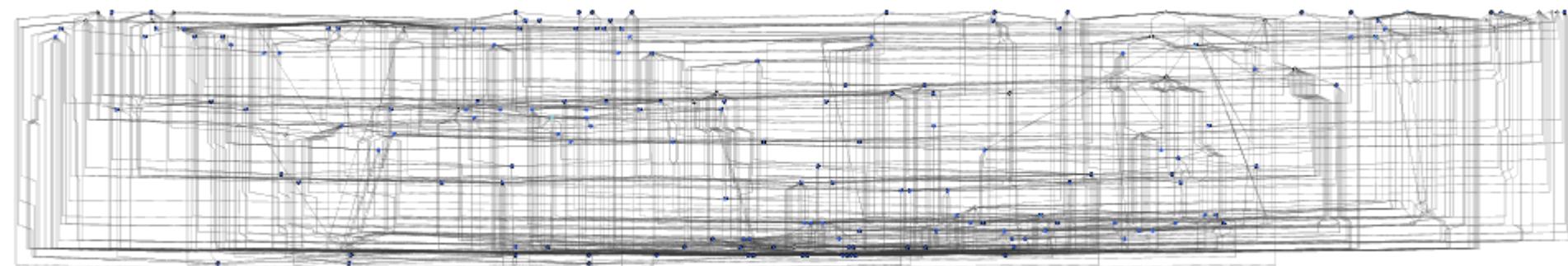
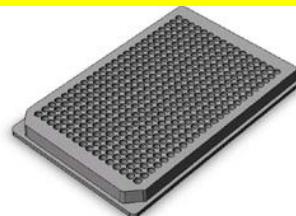
12 genes differentially expressed  
(Rosetta compendium)



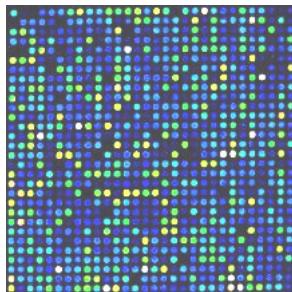
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# $\Delta$ ste5: Naïve approach Paths limited to length 3

Genetic Data



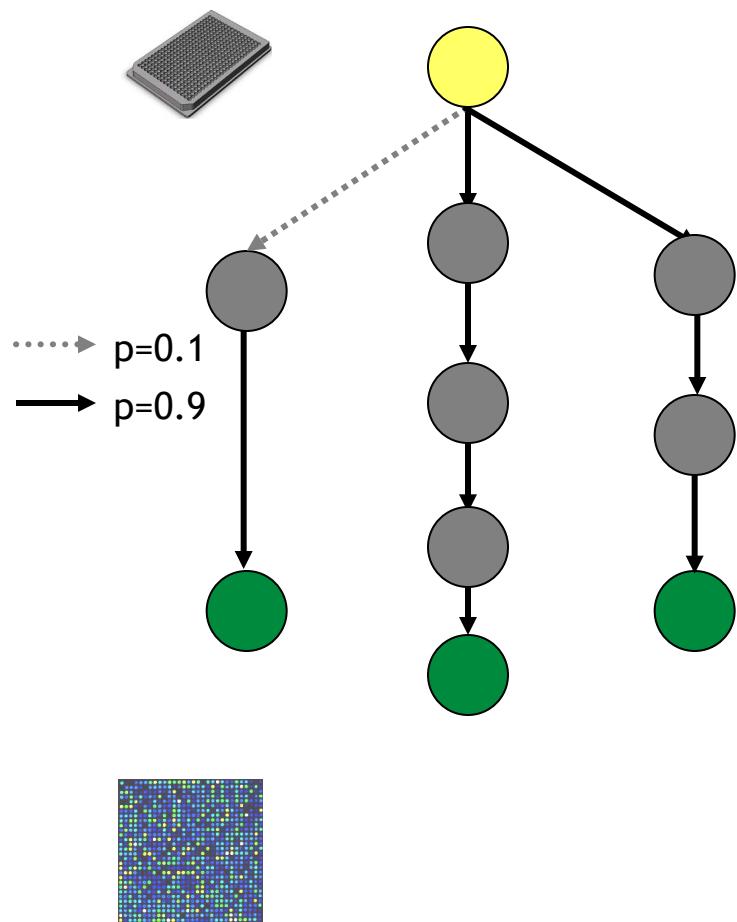
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Expression Data

193 nodes, 778 edges

# Maximize the connectivity via reliable paths



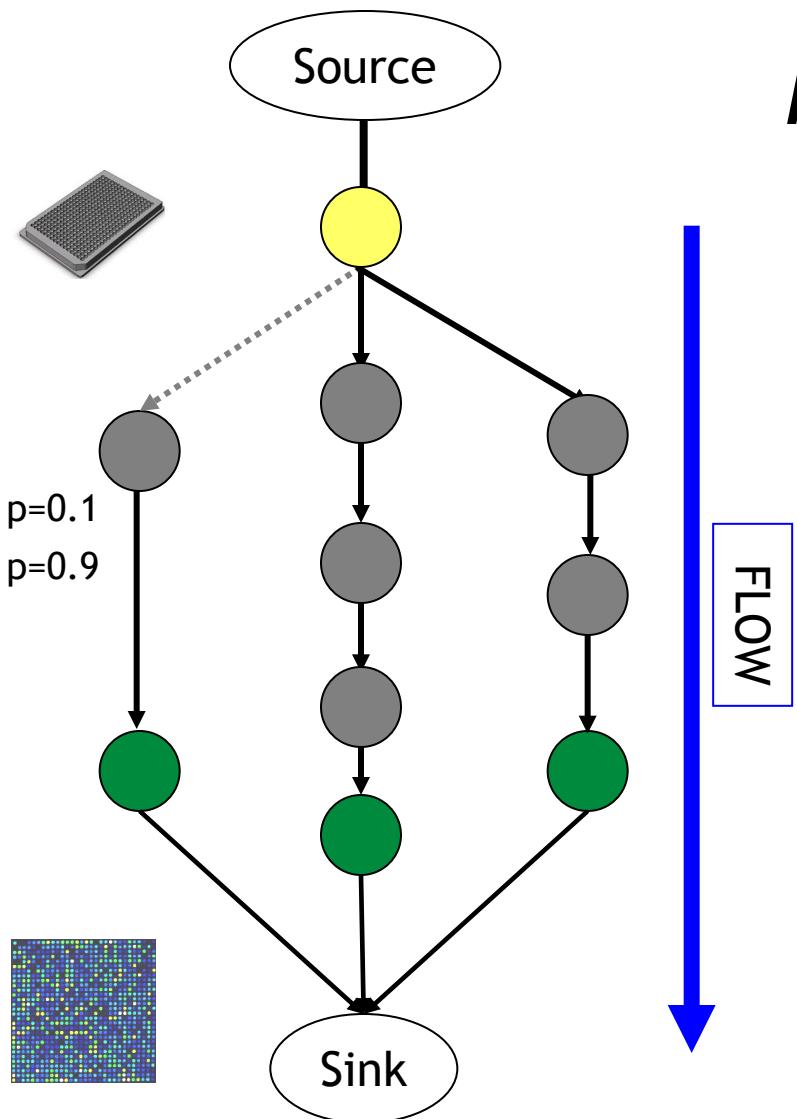
Goal: find paths that maximize product of  $P_{ij}$

Assign probabilities using a Bayesian approach based on reliability of underlying data type:

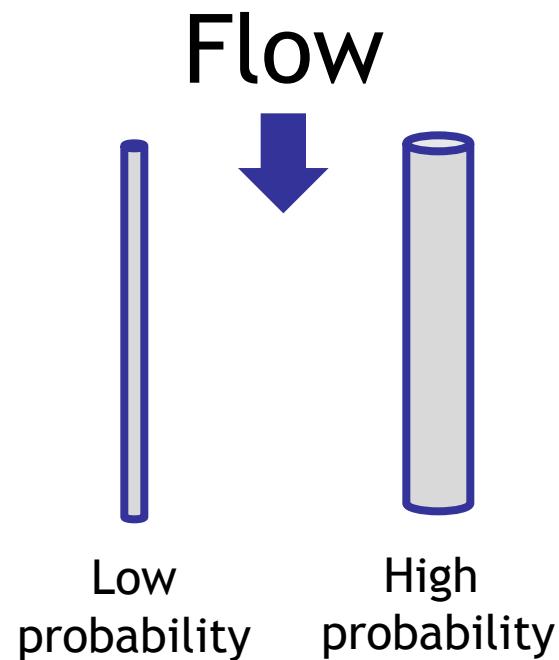
Myers, C.L. et al. Genome Biology (2005).

Jansen, R. et al. Science (2003).

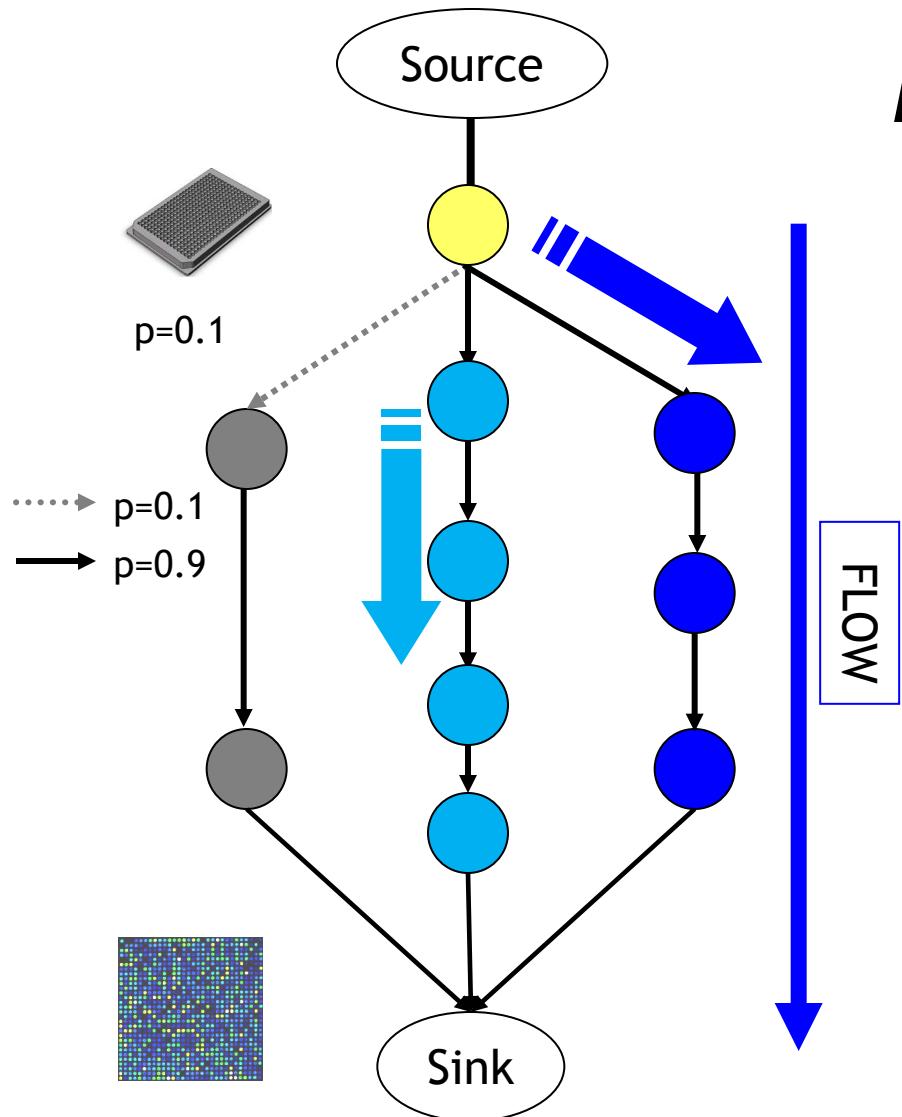
# Maximize the connectivity via reliable paths



## Minimum cost flow problem

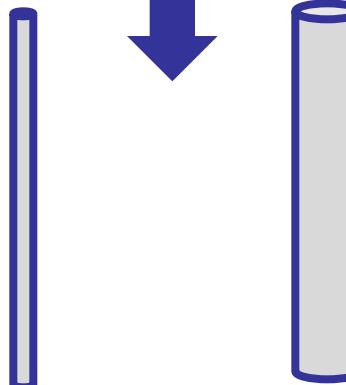


# Maximize the connectivity via reliable paths



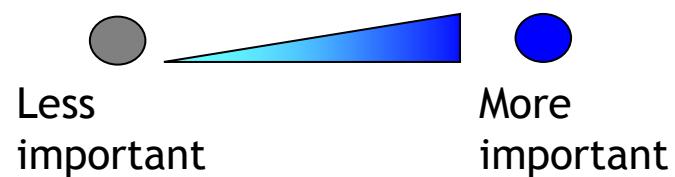
## Minimum cost flow problem

Flow

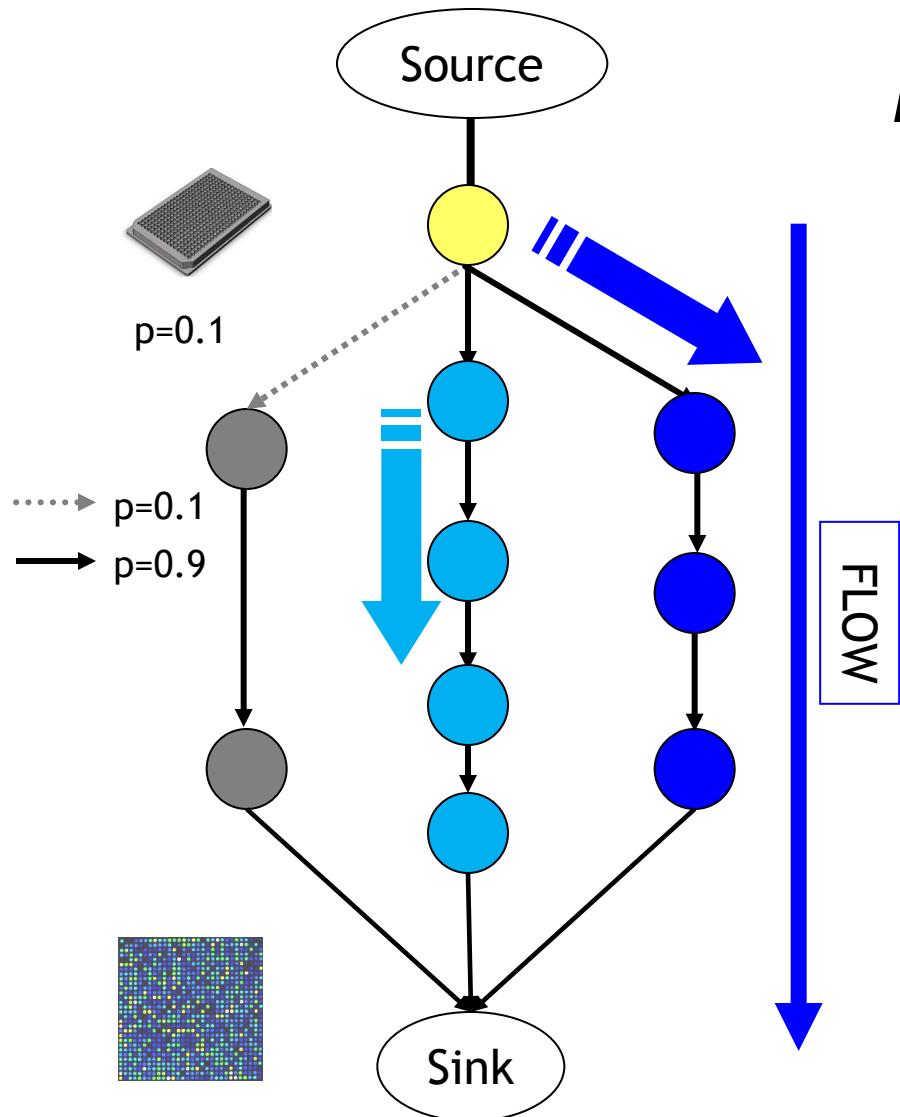


Low probability      High probability

Proteins ranked by their incoming flow:



# Maximize the connectivity via reliable paths



## Minimum cost flow problem

Maximize flow: source to sink

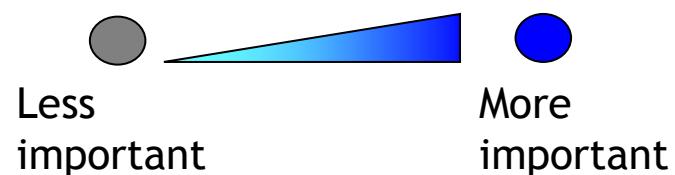
Minimize cost ( $e_{ij}$ ) =  $f_{ij} * (-\log P_{ij})$

$\min (\sum \text{cost}(e_{ij}) - \gamma * \sum f_{Sj})$

$f_{ij}$  = flow through  $e_{ij}$

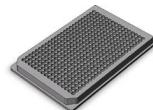
$c_{ij}$  = capacity of  $e_{ij}$  = 1 for all  $e_{ij}$

Proteins ranked by their incoming flow:

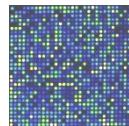


# Test case: Perturbing pheromone response pathway

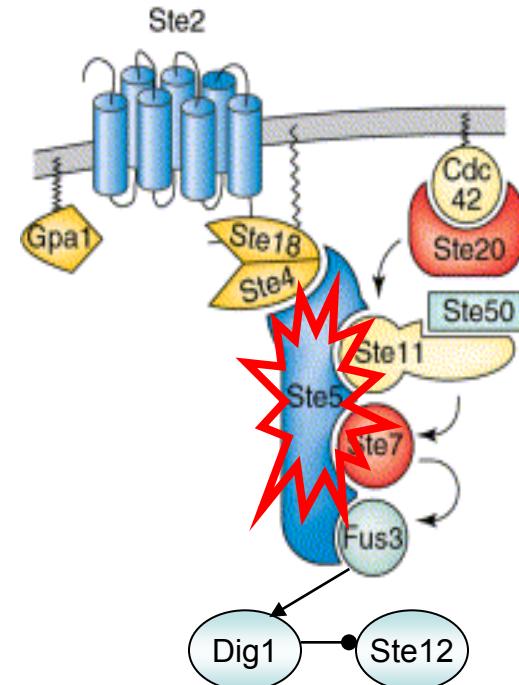
## Perturbing Ste5



20 genes rescue mating phenotype (SGD)

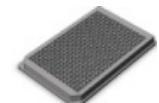


12 genes differentially expressed  
(Rosetta compendium)

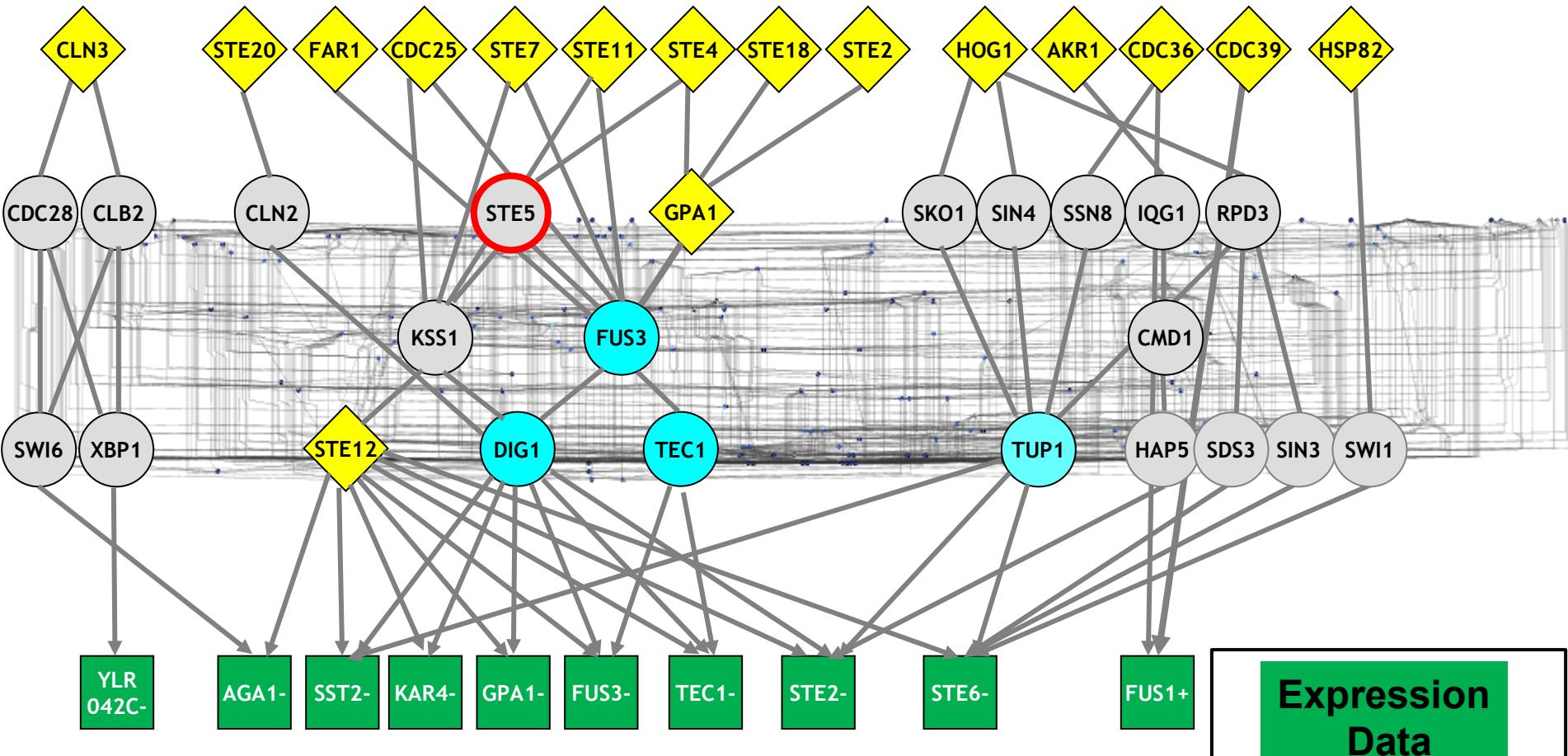


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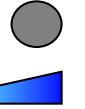
## Genetic Data



Enriched for pheromone response  $p < 10^{-18}$



49 nodes, 96 edges

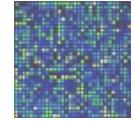


**Predicted genes**



**Importance**

## Expression Data



# Network Models

- Structure of network
  - Coexpression
  - Mutual information
  - Physical/genetic interactions
- Analysis of network
  - Ad hoc
  - Shortest path
  - Clustering
  - Optimization

Physical  
Relationships

Known  
Components

Unknown  
Components

Differential  
equations

Boolean logic,  
decision trees

Bayesian  
networks

mutual  
information

regression,  
clustering

Statistical  
Relationships

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