

# KEY CONCEPTS (COMPOUND ANGLES)

## 1. BASIC TRIGONOMETRIC IDENTITIES :

- (a)  $\sin^2\theta + \cos^2\theta = 1$  ;  $-1 \leq \sin \theta \leq 1$  ;  $-1 \leq \cos \theta \leq 1 \quad \forall \theta \in \mathbb{R}$   
 (b)  $\sec^2\theta - \tan^2\theta = 1$  ;  $|\sec \theta| \geq 1 \quad \forall \theta \in \mathbb{R}$   
 (c)  $\operatorname{cosec}^2\theta - \cot^2\theta = 1$  ;  $|\operatorname{cosec} \theta| \geq 1 \quad \forall \theta \in \mathbb{R}$

## 2. IMPORTANT T' RATIOS:

- (a)  $\sin n\pi = 0$  ;  $\cos n\pi = (-1)^n$  ;  $\tan n\pi = 0$  where  $n \in \mathbb{I}$   
 (b)  $\sin \frac{(2n+1)\pi}{2} = (-1)^n$  &  $\cos \frac{(2n+1)\pi}{2} = 0$  where  $n \in \mathbb{I}$   
 (c)  $\sin 15^\circ$  or  $\sin \frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$  or  $\cos \frac{5\pi}{12}$  ;  
 $\cos 15^\circ$  or  $\cos \frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$  or  $\sin \frac{5\pi}{12}$  ;  
 $\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75^\circ$  ;  $\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15^\circ$   
 (d)  $\sin \frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$  ;  $\cos \frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$  ;  $\tan \frac{\pi}{8} = \sqrt{2}-1$  ;  $\tan \frac{3\pi}{8} = \sqrt{2}+1$   
 (e)  $\sin \frac{\pi}{10}$  or  $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$  &  $\cos 36^\circ$  or  $\cos \frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$

## 3. TRIGONOMETRIC FUNCTIONS OF ALLIED ANGLES :

If  $\theta$  is any angle, then  $-\theta$ ,  $90^\circ \pm \theta$ ,  $180^\circ \pm \theta$ ,  $270^\circ \pm \theta$ ,  $360^\circ \pm \theta$  etc. are called **ALLIED ANGLES**.

- (a)  $\sin(-\theta) = -\sin \theta$  ;  $\cos(-\theta) = \cos \theta$   
 (b)  $\sin(90^\circ - \theta) = \cos \theta$  ;  $\cos(90^\circ - \theta) = \sin \theta$   
 (c)  $\sin(90^\circ + \theta) = \cos \theta$  ;  $\cos(90^\circ + \theta) = -\sin \theta$   
 (d)  $\sin(180^\circ - \theta) = \sin \theta$  ;  $\cos(180^\circ - \theta) = -\cos \theta$   
 (e)  $\sin(180^\circ + \theta) = -\sin \theta$  ;  $\cos(180^\circ + \theta) = -\cos \theta$   
 (f)  $\sin(270^\circ - \theta) = -\cos \theta$  ;  $\cos(270^\circ - \theta) = -\sin \theta$   
 (g)  $\sin(270^\circ + \theta) = -\cos \theta$  ;  $\cos(270^\circ + \theta) = \sin \theta$

sine & cosec only +ve	All +ve
tan & cot only +ve	cos & sec only +ve

## 4. TRIGONOMETRIC FUNCTIONS OF SUM OR DIFFERENCE OF TWO ANGLES :

- (a)  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$   
 (b)  $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$   
 (c)  $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B) \cdot \sin(A-B)$   
 (d)  $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B) \cdot \cos(A-B)$   
 (e)  $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$  (f)  $\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

## 5. FACTORISATION OF THE SUM OR DIFFERENCE OF TWO SINES OR COSINES :

- (a)  $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$  (b)  $\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$   
 (c)  $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$  (d)  $\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$

## 6. TRANSFORMATION OF PRODUCTS INTO SUM OR DIFFERENCE OF SINES & COSINES :

- (a)  $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$  (b)  $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$   
 (c)  $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$  (d)  $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$