Disparate Vulnerability: on the Unfairness of Privacy Attacks Against Machine Learning

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Abstract

A membership inference attack (MIA) against a machine learning model enables an attacker to determine whether a given data record was part of the model's training data or not. The effectiveness of these attacks is reported using metrics computed across the whole population (e.g., average attack accuracy). In this paper, we show that the attack success varies across different subgroups of the data (e.g., race, gender), i.e., there is disparate vulnerability. Even if MIA's success looks no better than random guessing over the whole population, subgroups can still be vulnerable. We study the necessary and sufficient conditions for a classifier to exhibit disparate vulnerability, and we determine to what extent certain learning techniques (e.g., fairness constraints, differential privacy) can prevent it. Our work provides a theoretical framework for studying MIA attacks from a new perspective.

1 Introduction

Machine learning models are vulnerable to Membership Inference Attacks (MIAs). In these attacks, an adversary infers whether an example was part of the training dataset by using the outputs of the model. These attacks are particularly threatening when a model operates on sensitive domains [16, 21, 23]. Shokri et al. [28] observed that the attack's performance can differ across target classes, and later Long et al. [20] showed that MIAs can be successful against some individuals even if models are well-generalized. In this paper, we systematically analyze the phenomenon of disparate vulnerability: the fact that certain population subgroups are more vulnerable to MIAs than others. Typically, MIAs are evaluated through average success across examples in a dataset [22, 28]. Disparate vulnerability means that this can lead to overestimation of privacy for some individuals.

We develop a theoretical framework to model subgroup-based vulnerability to MIAs, applicable to any classifier. We show that vulnerability is caused by differences in model's behaviour on the training dataset and outside (more general than overfitting), and the main factors for its disparity are the size and the distribution of subgroups. As a result, disparate vulnerability mostly affects minorities who are less represented in the data. We show that this problem persists even if models are trained with fairness constraints (producing equal performance for all subgroups), and differential privacy (hiding the presence of individual examples in the training set), unless we sacrifice the accuracy of the classifier. These results complement the findings by Bagdasaryan et al. [4] and Pujol et al. [25] in terms of understanding the disparate impact on subgroups when using differentially-private models.

2 Membership Inference Attacks

Let Ω be a population of labeled examples, where each example $(x,y) \in \Omega$ represents an individual with x being a feature vector and $y \in \mathbb{Y} = \{1, \dots, p\}$ being a label. We assume that the population is partitioned in disjoint *subgroups* formed by examples with a given attribute, e.g., race or gender the way they are commonly codified in data: $G_z \subset \Omega$ for $z \in \mathbb{Z} = \{1, \dots, k\}$, with $\bigcup G_z = \Omega$.

We consider a classification setting in which a classifier takes as input a feature vector x, and returns a confidence prediction over the labels: $f(x) \mapsto [0,1]^p$. We train f(x) on the *training dataset* S sampled from the population Ω according to an unknown distribution. We denote as $M(X,Y) \triangleq \mathbbm{1}[(X,Y) \in S]$ the random variable indicating whether an example (X,Y) belongs to S, where $\mathbbm{1}$ is the indicator function. We let Z(X,Y) be the random variable indicating the subgroup to which (X,Y) belongs. We omit the arguments and use M and Z if no ambiguity arises. Finally, we let $\hat{Y} = f(X)$ be the random variable corresponding to classifier outputs.

MIA Formalization. The goal of a MIA is to predict whether an example $(x,y) \in \Omega$ is a member or a non-member of the training set of a target classifier f(x) (classifier when no ambiguity arises). The attack works as follows: given (x,y), the adversary queries the classifier with x to obtain a confidence prediction $\hat{y} = f(x)$. Based on \hat{y} and information about (x,y), the adversary attempts to predict whether (x,y) was in the training dataset or not. Similarly to previous works [28, 30], we assume that the example (x,y) given to the adversary is equally likely to be a member or a non-member:

Assumption 1.
$$\Pr[M(X,Y) = 0] = \Pr[M(X,Y) = 1] = \frac{1}{2}$$
.

This corresponds to the adversary not having any prior knowledge on the *membership* of examples in the training set. Hence, the distribution of examples given to the adversary (X,Y) is not the data distribution; instead, (X,Y) are uniformly sampled either from S or $\Omega \setminus S$ with ½ probability.

The success of the MIA adversary A is a measure of vulnerability of the target classifier to MIA:

Definition 1. We call MIA *vulnerability* the expected accuracy of the adversary A:

$$V^{\mathcal{A}} \triangleq \mathbb{E} \left[\mathbb{1}[\mathcal{A} = M(X, Y)] = \Pr[\mathcal{A} = M(X, Y)] \right]. \tag{1}$$

where A represents the adversary's prediction. Because our analysis focuses on the vulnerability for each population subgroup, we also define the success of adversary A against a subgroup z:

Definition 2. Let $z \in \mathbb{Z}$ be a subgroup of the population. We define the *subgroup vulnerability*:

$$V_z^{\mathcal{A}} \triangleq \mathbb{E}\left[\mathbb{1}[\mathcal{A} = M(X, Y)] \mid (X, Y) \in G_z\right]$$

Adversary model. We consider a regular adversary (\mathcal{A}^{R}) and a discriminating adversary (\mathcal{A}^{D}). The former is the standard MIA adversary [28], who uses only the true label y and the output of the classifier \hat{y} to predict the membership. We denote the vulnerability against this adversary as V^{R} .

The latter addresses the fact that an adversary can have additional information about the target example. We assume this knowledge is the subgroup z to which the example belongs (e.g., if the subgroup is a part of the feature vector x). Using this information gives the discriminating adversary an advantage over the regular adversary (See Proposition 1). We denote the vulnerability against this adversary as V^D . When measuring V^D we assume that M is jointly independent from Y and Z to avoid capturing the effect of sampling bias in the training dataset:

Assumption 2. $M \perp (Y, Z)$.

Violating any of our two assumptions can only increase vulnerability. If the adversary has additional prior knowledge (violating Assumption 1), the adversary's success can increase arbitrarily. If the adversary gets additional information on sampling bias of the dataset (violating Assumption 2), the adversary's success also increases. Therefore, relaxing these assumptions would only result in stronger evidence of disparate vulnerability, at the cost of increasing the analysis complexity.

Adversaries' optimality. In our analysis, we use optimal instances for both adversaries in order to give worst-case security guarantees. We assume these *Bayes adversaries* have perfect knowledge of the underlying probability distributions and can thus use Bayes-optimal classifiers [8, 26]:

$$\mathcal{A}^{\mathsf{R}}(y,\hat{y}) \triangleq \underset{m \in \{0,1\}}{\operatorname{arg\,max}} \Pr[m \mid y, \hat{y}], \quad \mathcal{A}^{\mathsf{D}}(y,\hat{y},z) \triangleq \underset{m \in \{0,1\}}{\operatorname{arg\,max}} \Pr[m \mid y, \hat{y}, z]. \tag{2}$$

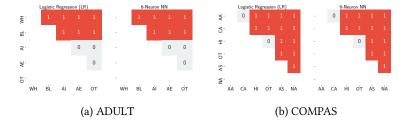


Figure 1: Statistical evidence of disparate vulnerability to the discriminating attacker. Each cell represents a pair of two subgroups (see Section A.2 for the subgroup-names key). "1" means there is evidence of disparate vulnerability between two subgroups, with significance level 0.005, "0" means no sufficient evidence.

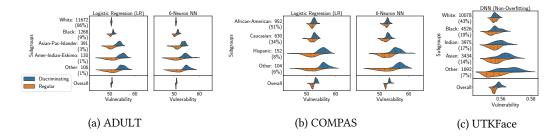


Figure 2: Subgroup vulnerability to discriminating and regular attacker. The distributions are over 35 different models trained on different train/test shuffles.

Because adding information cannot decrease the performance of a Bayes-optimal classifier, the success of the discriminating adversary is at least as high as that of the regular one:

Proposition 1. $V^{\mathsf{D}} \geq V^{\mathsf{R}}$.

The proof for this, and all the other formal statements in the rest of the paper, are in Appendix B.

3 Disparate Vulnerability

In this section, we introduce the measure of disparate vulnerability, experimental methods for estimating it, and present the first set of experimental results.

Definition 3. Disparity in vulnerability (disparity, for short) is the difference in vulnerability between two subgroups $z \neq z'$: $\Delta V_{z,z'} \triangleq |V_z - V_{z'}|$.

To estimate vulnerability and its disparity in practice, we follow a procedure similar to the standard method by Chatzikokolakis et al. [6]. We plug in frequency-based estimates of $\Pr[M \mid Y, \hat{Y}]$ and $\Pr[M \mid Y, \hat{Y}, Z]$ into Equations (1) and (2), respectively. To obtain these estimates, we sample examples (x,y) with probability ½ from the model's training set S or from a hold-out set $\bar{S} \subset \Omega \setminus S$. We query the classifier with these examples and obtain the outputs $\hat{y} = f(x)$, which we discretize to be able to estimate the frequencies. As for the sets S and \bar{S} , we sample them such that they are equal-size and are stratified by (y,z), in order to satisfy Assumption 1 and 2. We denote the vulnerability and disparity estimates as \hat{V}_z and $\Delta \hat{V}_{z,z'}$. (More details in Section A.1.)

Datasets. We use three datasets: ADULT, based on the 1994 US Census [17] with income labels (more or less than \$50K), COMPAS with recidivism labels from the ProPublica's investigation [15], and UTKFace [31], a set of face images annotated with age, gender and ethnicity of the individuals. We discretize the UTKFace age attribute into 6 classes and use as labels for age classification. We employ standard data cleaning and pre-processing as detailed in Section A.2.

Models. We train and evaluate ML models with various architectures: logistic regression, single hidden-layer neural networks, and deep neural networks. For ADULT and COMPAS, we use logistic regression and a single-layer network with 6 hidden units. These classifiers do not overfit for neither dataset, achieving 85% accuracy for ADULT and 68% for COMPAS. We train a convolutional neural network on UTKFace for 100 epochs with dropout layers (Section A.3), obtaining 57% test accuracy (vs. 40% baseline). We avoid evaluating the vulnerability of pre-trained models on UTKFace, despite the potential gain in accuracy, because MIA is not well-defined for pre-trained models.

Statistical evidence of disparate vulnerability. To make sure that disparate vulnerability is not a random artifact, we train each model 35 times using stratified random shuffles of training and test sets (S and \bar{S}), and evaluate the vulnerability of each model. For every example in a shuffle we record a) whether the adversary has been correct in their inference, $\mathbb{1}(A=M)$, and b) the example's subgroup. Thus, we have measurements of vulnerability with a between-subject subgroup variable—many examples ("subjects") belong to the same subgroup,— and a within-subject shuffle variable—the same example appears in all shuffles. Such setup is known as a mixed-design experiment [3].

Because of this structure, our vulnerability measurements are not independent across shuffles and we cannot use the most common test statistics to establish disparate vulnerability. We employ the standard procedure for mixed-design setups: fit a linear mixed-effects model, and run an ANOVA test to tell whether vulnerability is significantly correlated with the subgroup [12]. If the test is conclusive, we conduct follow-up pairwise tests to identify particular subgroup pairs that exhibit disparity. We use the significance level of $\alpha=0.005$ with appropriate adjustments. This procedure provides high statistical power even when subgroups are small (e.g., in COMPAS), and accounts for the intra-shuffle and inter-shuffle dependencies in our measurement data. (More details in Section A.4.)

Figure 1 shows the results of disparity tests. A "1" indicates evidence of disparate vulnerability. The tests are inconclusive for the regular adversary, and against the well-generalized UTKFace model. However, we find clear evidence of disparity for the discriminating adversary on ADULT and COMPAS.

Disparate vulnerability effect size. Knowing whether a classifier exhibits disparate vulnerability is uninformative of its effect size. Figure 2 shows the distribution of vulnerability values, aggregated across shuffles. The overall vulnerability is close to the ½ baseline for most models, but we observe that vulnerability varies across subgroups. We study the underlying reasons in Section 4.

Definition 4. *Max-disparity* is the maximum *observed* pairwise disparity among any two subgroups: $\max_{z\neq z'} \Delta \hat{V}_{z,z'}$. It is measured in percentage points. We drop its units if it is clear from the context.

We measure max-disparity within each shuffle, and then average across the shuffles. On UTKFace, max-disparity of the non-overfitting deep neural network is the lowest: 0.83 (0.34) p.p. against the regular (discriminating) attacker. On ADULT, the logistic regression and the neural network have a low max-disparity: 3.27 (5.07) and 3.22 (5.04). In COMPAS, the max-disparity is much more pronounced, with 18.61 (29.03) and 15.93 (30.50), respectively. (See Appendix C for more details.)

Takeaways. From these experiments we conclude that *not being vulnerable to the regular adversary should not be interpreted as the model being private*: an adversary using subgroup information can have better performance overall, and also on particular subgroups. More worryingly, *subgroups with small representation are consistently more vulnerable* (see Figure 2). In contrast, a well-generalized neural network trained on the relatively large UTKFace did not exhibit significant disparate vulnerability, despite being vulnerable to MIA on average (see Appendix C).

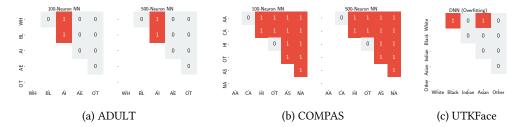


Figure 3: Overfitting effect on disparate vulnerability, discriminating adversary. Fig. 1 for plot details.

4 Effects of Subgroup Learnability

We study how learnability influences disparate vulnerability. In practice, two factors make certain subgroups harder to learn than others: a) how much data is available for the subgroup, and b) inherently, how hard is the subgroup's distribution to learn. For our evaluation, we use the UTKFace dataset, discarding the "Other" subgroup, which does not have enough examples for a meaningful analysis. "Asian" becomes the least populous subgroup with 3,434 examples.

Subgroup data distribution. We investigate whether the data distribution of certain subgroups makes them inherently more vulnerable to MIAs. In this experiment, at each step we take K examples from each of m subgroups, train a model on mK examples and measure subgroup and average overall vulnerability using mK train and mK test examples, satisfying Assumption 1).

Figure 4 (left) shows the result for the discriminating adversary with m=4 subgroups and at most K=1717 training examples per subgroup. The effect for the regular adversary is similar but less pronounced (Appendix C). The attacker performs similarly against "Indian", "Asian", and "White" subgroups and quite differently against "Black". The increase in vulnerability from K=300 to 700 examples is likely due to an increase in the model capacity for learning new visual features.

Subgroup data size. We measure how much the size of a single subgroup influences vulnerability. Intuitively, more examples improve the classifier generalization for the subgroup, reducing its vulnerability. We measure the subgroups vulnerability as we increase the number of available examples for a single (*target*) subgroup, keeping fixed the size of the other subgroups and sampling train and test datasets respecting Assumption 1 and 2.

In Figure 4 (right), we consider the "Asian" subgroup as target. We observe that when the target subgroup is small, its vulnerability is large. The vulnerability decreases rapidly as the subgroup size increases, improving generalization. Importantly, other subgroup vulnerabilities do not change. The results are similar for other subgroups (see Appendix C).

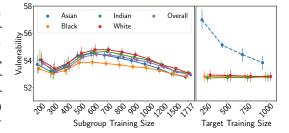


Figure 4: Vulnerability per subgroup for the discriminating adversary (error bars are 95% CI), when: i) each subgroup has the same amount of data (left), showing the impact of the difficulty in learning each group's distribution, ii) one subgroup's size ("Asian") is increased (right) showing how the amount of data per group affects disparity.

Takeaways. Our experiments show that a subgroup's *data distribution affects its vulnerability*, even if all subgroups were equally represented in the data. However, if the subgroup is *less represented in the data, it is more vulnerable.* On the positive side, improving the representation of a subgroup does not cause any privacy harm to others.

5 Effects of Overfitting

Overfitting in the sense of a difference between the loss on the training and test sets, is considered an important factor for MIA [28]. We study the effect of overfitting on both vulnerability and disparate vulnerability.

Standard overfitting. We train two overparametrized single-layer neural networks for ADULT and COMPAS datasets, one with 100 neurons and one with 500. These models use ReLU activation, and are trained for 200 epochs with the adam solver. For UTKFace, we use the same neural network as above, but without dropout. All these models significantly overfit.

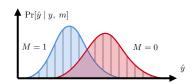


Figure 5: Distributional overfitting. The curves represent the distributions of a model's outputs on the training set (M=1) and outside (M=0) for a class y. The striped area shows the amount of distributional overfitting $\tau(y)$: total variation between model's outputs on training and outside.

We observe higher average vulnerability as compared to non-overfitting models, and statistical evidence of disparity for all datasets. Notably, on UTKFace overfitting marginally increases average vulnerability (1 p.p) with respect to the non-overfitting model, but now there is statistical evidence of disparity against *both* adversaries. In terms of effect size, in all datasets overfitting increases the max-disparity by 2–4 p.p. (Details in Appendix C.)

Distributional Subgroup Overfitting. We showed experimentally that standard overfitting causes both vulnerability and disparate vulnerability. How can we reason about this systematically?

Recall that the regular adversary \mathcal{A}^{R} bases their decisions on the posterior distribution: $\Pr[M \mid \hat{Y}, Y]$. Because $\Pr[M]$ is uniform by Assumption 1, using $\Pr[\hat{Y}, Y \mid M]$ instead will result in the same decisions. Moreover, because $\Pr[Y \mid M] = \Pr[Y]$ by Assumption 2 we have the equivalent form:

$$\mathcal{A}^{\mathsf{R}}(\hat{y}, y) \triangleq \mathop{\arg\max}_{m \in \{0,1\}} \Pr[m \mid y, \hat{y}] = \mathop{\arg\max}_{m \in \{0,1\}} \Pr[\hat{y} \mid y, m].$$

In other words, to obtain the decisions, the adversary uses the difference in the *distributions of the* classifier's outputs \hat{y} between examples from the training dataset and outside of it.

Definition 5. We define the *distributional-overfitting distance* of a classifier for class y, as the total variation between the distributions of outputs of the model on the training and outside, given that the ground truth class is y (See Figure 5):

$$\tau(y) \triangleq \frac{1}{2} \sum_{\hat{y} \in \hat{\mathbb{Y}}} \left| \Pr[\hat{y} \mid y, M = 1] - \Pr[\hat{y} \mid y, M = 0] \right|.$$

For subgroups, distributional overfitting becomes:

Definition 6. We define the *subgroup distributional-overfitting distance* for a subgroup z and class y as the total variation between distributions of outputs of the model on the training dataset and outside, given that examples $(x, y) \in G_z$:

$$\tau_z(y) \triangleq \frac{1}{2} \sum_{\hat{y} \in \hat{\mathbb{Y}}} \left| \Pr[\hat{y} \mid y, z, M = 1] - \Pr[\hat{y} \mid y, z, M = 0] \right|.$$

We also define *class-bias*, the imbalance of classes across subgroups, as $\rho_z(y) \triangleq \Pr[y \mid z]$.

We can now show that vulnerability to both regular and discriminating adversaries depends on the distributional overfitting:

Theorem 1. The overall vulnerability values of the regular and discriminating adversaries are equal to the following averages of distributional-overfitting distances:

$$V^\mathsf{R} = \frac{1}{2} + \frac{1}{2} \sum_{y \in \mathbb{Y}} \Pr[y] \, \tau(y), \quad V^\mathsf{D} = \frac{1}{2} + \frac{1}{2} \sum_{y \in \mathbb{Y}} \sum_{z \in \mathbb{Z}} \Pr[z] \, \rho_z(y) \, \tau_z(y) \, .$$

As a consequence, the absence of standard overfitting does not prevent vulnerability: a model could have equal losses on the training and outside, and yet different output distributions.

Conditions for No Disparity.

Theorem 2 (Necessary and sufficient condition for no disparity). Let z and z' be two subgroups of the population. There is no disparate vulnerability between the two subgroups for a discriminating adversary (i.e., $\Delta V_{z,z'}^{\mathsf{D}} = 0$) iff:

$$\sum_{y \in \mathbb{Y}} \left(\rho_z(y) \, \tau_z(y) - \rho_{z'}(y) \, \tau_{z'}(y) \right) = 0. \tag{3}$$

The proof and equivalent necessary and sufficient condition for the regular adversary are in Section B.2. Based on Theorem 2, we observe the following *sufficient* condition to remove disparate vulnerability.

Corollary 1. Suppose a model does not overfit in the distributional sense for all subgroups z and for all classes y: $\tau_z(y) = 0$. Then the model does not exhibit disparate vulnerability for any pair of subgroups z and z': $\Delta V_{z,z'}^{\mathsf{R}} = \Delta V_{z,z'}^{\mathsf{D}} = 0$.

This condition prevents disparity against both adversaries: it ensures that the model's output has the same distribution on training and outside, for each subgroup.

Takeaways. We show a general result: without any parametric assumptions, and for any classifier, the vulnerability to the regular and discriminating adversaries exists if and only if the classifier exhibits *distributional overfitting*. This generalizes and complements the results of Yeom et al. and Sablayrolles et al.. Moreover, we show that disparate vulnerability arises if and only if distributional overfitting differs across subgroups. This gives a general guideline for privacy defences: *ensure that the model's behavior is consistent on the training dataset and outside, across population subgroups.*

6 Preventing Disparate Vulnerability

Unfortunately, the condition in Corollary 1 is hard to meet in practice. In this section, we evaluate whether techniques to enhance privacy and to prevent disparate effects can be effective mitigations.

6.1 Differential Privacy

First, we look at whether learning with differential privacy [11] helps in preventing disparity.

Definition 7. The training algorithm train(S) of a classifier satisfies ε -DP if for any two datasets S, S' differing by the records of one individual, for any set of classifiers A:

$$\Pr[\mathsf{train}(S) \in A] < \exp(\varepsilon) \Pr[\mathsf{train}(S') \in A]$$
.

Differential privacy (DP) limits the leakage of any individual record in the dataset and upperbounds the *average* vulnerability [10], and thus seems like a natural defence against MIAs.

Empirical Evaluation. To study how DP affects disparate vulnerability we train DP models with different privacy levels. As a target model, we use DP logistic regression with private empirical risk minimization [7], trained using the diffprivlib [14] implementation. We use a min-max scaler, and provide a maximum row norm equal to the square root of the number of features. We use privacy levels $\varepsilon = 1.0, 2.5, 5.0, 7.5$. We run this experiment on COMPAS, our smallest dataset, and ADULT, our largest dataset. All trained models beat the random accuracy baselines.

As with non-overfitting models (Section 3), we do not observe disparity for the regular adversary. For the discriminating adversary, we find that the highest privacy level in our experiments ($\varepsilon=1$) substantially mitigates disparity, yet does not remove it completely. As ε increases, the number of

pairs exhibiting statistical disparity increases too, with results comparable to non-private models when $\varepsilon=7.5$ (see Appendix C for details). In terms of effect size, compared to the non-overfitting models in Section 3, we observe a significant drop in max-disparity on COMPAS: from 29 (logistic regression) to 12 ($\varepsilon=1$) against the discriminating attacker, but only a small drop of 1 p.p. on ADULT. At the same time, DP has a significant impact on accuracy. The most private configuration results in about 10 p.p. accuracy drop on both ADULT and COMPAS.

6.2 Algorithmic Fairness

A technique that intuitively should prevent disparate vulnerability is fairness-constrained learning. Due to the dependency of disparate vulnerability on the disparate *behavior* of the model across subgroups (Figure 5), minimizing the discrepancy in classifier's performance across population subgroups [9] should also reduce disparate vulnerability.

We consider the *equality of odds* (EO) notion [13], which addresses difference in distributions of the values relevant to MIA adversaries: \hat{y} and y. A classifier satisfies EO if the following holds for any subgroups z and z', and any \hat{y}, y : $\Pr[\hat{y} \mid y, z] = \Pr[\hat{y} \mid y, z']$, where probabilities are over the data distribution.

Even if a classifier theoretically satisfies EO, the equality might not hold on a finite data sample S (i.e., $\Pr[\hat{y} \mid y, z, M = 1] \neq \Pr[\hat{y} \mid y, z', M = 1]$). Thus, there can still be disparity due to a possible difference between $\tau_z(y)$ and $\tau_{z'}(y)$. To avoid this, we strengthen this fairness definition:

Definition 8 (Generalized Equality of Odds). For a given training dataset, the classifier satisfies generalized equality of odds (GEO) if the following holds for any subgroups z and z', any \hat{y}, y , and any $m \in \{0, 1\}$:

$$\Pr[\hat{y} \mid y, z, m] = \Pr[\hat{y} \mid y, z', m]$$

Intuitively, this means that equality of odds holds both within the training dataset and outside. This formalization implies that the EO property generalizes beyond the training data.

The first implication of GEO is that a discriminating adversary has no advantage over a regular one.

Proposition 2. Suppose a classifier satisfies GEO. Then, $V_z^{\mathsf{R}} = V_z^{\mathsf{D}}$ for any subgroup z.

Moreover, in the case that when no class bias exists in the data (i.e., class distributions are equal across subgroups), GEO does completely prevent disparate vulnerability:

Proposition 3. Suppose a classifier satisfies GEO, and $\rho_z(y) = \rho_{z'}(y)$ holds for any y and for all z, z' (i.e., there is no class imbalance). Then, $\Delta V_{z,z'}^{\mathsf{R}} = 0$ and $\Delta V_{z,z'}^{\mathsf{D}} = 0$.

However, if class bias exists, this strengthened notion of equality of odds is not sufficient to prevent disparate vulnerability.

Empirical evaluation. We use a logistic regression classifier, post-processed to satisfy vanilla EO, based on the *fairlearn* library [2]. We evaluate it on the COMPAS and ADULT datasets.

Even though the theoretical results implying EO is not sufficient to prevent disparity, in practice on ADULT EO decreases it in both statistical sense (Figure 6), and in terms of max-disparity, lowering it from 5 to 3 against the discriminating adversary. On COMPAS, it lowers max-disparity from 30 to 17, but the disparity is still statistically significant. On the negative side, EO *increases* the average vulnerability to the regular adversary and, on COMPAS, even introduces the disparity in the statistical sense. Thus, EO makes regular and discriminating vulnerabilities closer (see Proposition 2), but this happens at the cost of an increase of regular vulnerability. (Details in Appendix C.)



Figure 6: Disparity of a model satisfying equality of odds on ADULT. See Figure 1 for details.

6.3 Takeaways

DP provides an upper bound on the vulnerability of all individuals. Because DP guarantees are often at odds with accuracy, in practical applications ε is usually set high, allowing for a lot of variation; we observe this variation in our experiments. As for models satisfying EO, they decrease max-disparity, but still exhibit statistical evidence of disparity against both adversaries.

We observe that although DP and EO models can yield similar results in terms of statistical evidence of disparity, they offer different trade-offs in terms of max-disparity and model accuracy. E.g., the $\varepsilon=2.5$ DP model on COMPAS exhibits about 9 p.p. lower max-disparity than EO, but EO model is 5 p.p. more accurate (See Appendix C.) On the downside, EO does not provide any means to control the vulnerability-accuracy trade-off as DP does.

These results show that existing techniques, applied directly, cannot guarantee the absence of disparate vulnerability. We conclude that designing a privacy defense with equal effect on all individuals without a significant penalty in accuracy needs additional investigation.

7 Conclusions

Our findings reveal that Membership Inference Attacks can *disparately target population subgroups*. We provide new insights into why and when membership inference is possible and why and when these attacks have disparate impact. We demonstrate that current defenses do not provide equal protection to all individuals represented in datasets, and we provide the community with a new set of evaluation criteria to reason about the privacy of models.

Our results surface a more general problem of *aggregate privacy measures* with profound societal implications: they can overestimate the protection for certain subgroups, in particular for the most sensitive individuals. Models that are considered privacy-preserving might not only offer worse results to minorities [4] but also leave their privacy unprotected.

Broader Impact

The study of privacy of ML models is essential to ensure they can be safely employed to critical applications and data (e.g., diagnosis and prognosis, census data); this entails a complete understanding of several aspects of the problem, pertaining to how data is collected, where it is processed, and what information the model itself reveals once deployed. In this context, our work extends on our knowledge (as a community) about the leakage of the ML model outputs in the context of MIA.

One of the possible societal impacts of our work is to encourage further regulations concerning the application of ML in critical settings. In particular, we show that it is not sufficient to measure privacy on average: doing so can discriminate certain subgroups. Veale et al. have argued that vulnerability of models to MIA can qualify them as private data under data protection law [29]. The fact that attacks can be more effective against under-represented individuals might change the conditions under which a model is deemed private. We further give tools to systematically analyze the actual vulnerability of ML models in this context.

This line of research has an unfortunate potential consequence: it uncovers new attack strategies that can be misused by malicious entities. Our work in particular is unable to show any definite solution against disparate vulnerability; conversely, it shows that standard defenses can fail. Nevertheless, the field of security has for long acknowledged that it is equally important to expose attacks as it is to design defenses. Indeed, if no defense could be designed for preventing an attack against a system, the knowledge of this fact would enable us to limit the system's use in critical applications.

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Table 1: Summary of the datasets, together with the name of the attribute used to define the subgroups, and the name of each subgroup.

Dataset	Details	Subgroup Attribute	Subgroups			
ADULT	48,842 examples, 14 features, binary classification	Race	White (WH), Black (BL), Asian-Pac-Islander (AI), Amer-Indian-Eskimo (AE), Other (OT)			
COMPAS	6,172 examples, 15 features, binary classification	Race	African-American (AA), Caucasian (CA), Hispanic (HI), Native American (NA), Other (OT)			
UTKFace	23,705 face images, $ \mathbb{Y} = 6$	Ethnicity	White, Asian, Indian, Black, Other			

A Experiment details

A.1 Vulnerability Estimation

Because we define vulnerability as the accuracy of Bayes adversaries, it is not possible to compute $\Delta V_{z,z'} = |V_z - V_{z'}|$ exactly. Instead, we estimate the vulnerability of a classifier f, trained on a dataset S, against estimates of our adversaries (Equation (2)) as follows. We construct an evaluation set E which contains an equal number of "in" samples S, and "out" samples \bar{S} from $\Omega \setminus S$, i.e., such that $\Pr[M(X,Y) = 0 \mid (X,Y) \in E] = 1/2$, which satisfies Assumption 1. To satisfy Assumption 2, when sampling S and \bar{S} examples we stratify them by values of (y,z), i.e., ensuring that $\Pr[Y,Z \mid M=1] = \Pr[Y,Z \mid M=0]$.

We produce the confidence outputs observed by an adversary by querying the classifier with all the examples $(x,y) \in E$, and obtaining $\hat{y} = f(x)$, which we discretize into 10 uniformly spaced bins. Then we approximate the probability distributions for the regular and discriminating adversaries, respectively $\Pr[M \mid Y, \hat{Y}]$ and $\Pr[M \mid Y, \hat{Y}, Z]$, by counting the relative frequencies of confidence outputs \hat{y} . Based on these distributions, we estimate the success of each adversary (Equation (2)) as their average accuracy on the same set E.

In our experiments, for each model and each dataset we construct 35 random training/test shuffles, meaning 35 different sets E.

A.2 Details on Datasets and Pre-processing

ADULT or Census Income [17]. Contains 48,842 examples from the 1994 Census database¹. The prediction task is to determine if a yearly salary is over/under \$50K. It contains attributes such as age, sex education, race, native-country, etc. After one-hot-encoding all such features, the dataset contains 91 features. We use "race" as the subgroup attribute z, defining the following groups: White (WH), Black (BL), Asian-Pac-Islander (AI), Amer-Indian-Eskimo (AE), and Other (OT).

COMPAS [15]. Contains data from the ProPublica's investigation, concerning arrests in Florida and information on new crimes over the next two years. It also includes COMPAS recidivism risk scores, which we do not use. We follow ProPublica's pre-processing strategy: we drop examples with missing entries for charge degree and recidivism status, and those with a charge date later than 30 days from the arrest. We use the *2-year-recidivist* feature as the target label *y*. After one-hot-coding, the dataset contains 6,172 examples and 15 features. We use "race" as the subgroup *z*, which has the following categories: African-American (AA), Caucasian (CA), Hispanic (HI), Native American (NA), and Other (OT).

UTKFace [31]. Contains over 20,000 face images labeled by the age of the person, and annotated with "ethnicity". We use the ethnicity attribute as the subgroup attribute z, taking values: White, Asian, Indian, Black, and Other. The classification task is age estimation where age is an integer from 1 to 106. We discretize the age attribute into 6 bins: 0–16, 17–25, 26–40, 41–55, 56–65, and above 65. We use this new 6-class categorical feature as our y labels for age estimation.

Table 1 presents a summary of the relevant characteristics of our datasets.

Inclusion of the sensitive attribute for training. We assume both target and attacker classifiers are well aware of the sensitive attribute. This is because, in most cases, explicit removal of the sensitive attribute is either undesirable or not entirely possible due to the existence of proxy features. Therefore, the training datasets for both ADULT and COMPAS, explicitly include the sensitive attribute.

¹https://archive.ics.uci.edu/ml/datasets/adult

Table 2: Model hyper-parameters

Model	Hyper-parameters									
LR	$C = 0.01, l_2$ penalty, solver = "lbfgs"									
NN-6	$\alpha = 0.01$, solver = "adam"									
	solver = "adam", 200 epochs									
NN-500	solver = "adam", 200 epochs									

Table 3: UTKFace model architecture

Layer	Output Shape	# Parameters
Conv2D	(32, 32, 64)	832
MaxPooling2D	(16, 16, 64)	0
Dropout	(16, 16, 64)	0
Conv2D	(16, 16, 32)	8224
MaxPooling2D	(8, 8, 32)	0
Dropout	(8, 8, 32)	0
Flatten	(2048,)	0
Dense	(256,)	524544
Dropout	(256,)	0
Dense	(6,)	1542

A.3 Model Details

For all models other the convolutional neural net for UTKFace, we use *scikit-learn* [24]. Table 2 details the hyper-parameters used for each model type.

For the UTKFace model, we use *keras* [1]. Table 3 details the architecture of the model. We train it using the Adam optimizer with batch size of 64 for 100 epochs. For the non-overfitting version, we keep the best model in terms of validation loss across the 100 epochs, where the validation set is a randomly chosen subset of the training set with 2000 examples.

A.4 Statistical Testing for Disparate Vulnerability

Outline. We fit a linear mixed-effects model (LMM) to model vulnerability with the subgroup as a fixed effect, and example-and shuffle-level crossed random effects [12]. Using this model, we conduct an ANOVA test to identify whether any two subgroups have different mean vulnerability. If the test is conclusive at the level $\alpha=0.005$, we conduct follow-up tests using the estimated marginal means method [27] to identify which particular subgroup pairs exhibit disparity, at level $\alpha=0.005$ with Bonferroni adjustment for multiple testing.

LMM specification. Let i be an index of an example from a dataset, j be the index of a shuffle (corresponding to one evaluation dataset E), and s_i be the one-hot encoded subgroup of the example i. Consider a Bernoulli random variable V_{ij} showing whether the adversary succeeded in inferring membership of the example i against the model trained on shuffle j. We have two types of dependencies among all $\{V_{ij}\}$: within-shuffle measurements are dependent because they all relate to the same target model, and between-shuffle measurements are dependent because same examples appear in all shuffles. We thus use the following LMM to model V_{ij} :

$$V_{ij} = \beta \cdot s_i + \mu + \alpha_i + \alpha_j + \varepsilon_{ij},$$

where β is a global vector of coefficients corresponding to the fixed effects of the subgroup, μ is the global intercept, α_i is the random intercept of the example i, α_j is the random intercept of the shuffle j, and ε_{ij} is the independent noise term. We choose the linear model as opposed to a mixed-effect logistic regression as we are only interested in average effects.

Software. We use R *lme4* [5] package to fit the LMM, *lmerTest* [18] package to conduct the ANOVA tests, and *emmeans* [19] package to conduct pairwise comparisons.

B Proofs

In this section we provide the proofs of the formal statements made in the main paper. For clarity, we use in and out to mean outcomes 1 and 0 of the random variable M. In a slight abuse of notation we also use in and out to denote events M=1 and M=0 as a shorthand.

B.1 Regular vs. Discriminating Vulnerability

Proposition 1. $V^{D} \geq V^{R}$.

Proof of Proposition 1. Recall that the Bayes adversary uses a Bayes-optimal classifier that maximizes the success probability (i.e., vulnerability) among all the possible classifiers. That is, for the regular and discriminating adversaries, we have respectively:

$$\begin{split} V^{\mathsf{R}} &= \max_{g: \mathbb{Y} \times \mathbb{Y} \mapsto \{0,1\}} \Pr[g(Y,\hat{Y}) = M] \\ V^{\mathsf{D}} &= \max_{g: \mathbb{Y} \times \hat{\mathbb{Y}} \times \mathbb{Z} \mapsto \{0,1\}} \Pr[g(Y,\hat{Y},Z) = M] \,. \end{split}$$

Let $F = \{f \mid f = g \circ h, h(y, \hat{y}, z) = (y, \hat{y}), g : \mathbb{Y} \times \hat{\mathbb{Y}} \mapsto \{0, 1\}\}$; that is, F is the set of functions $f : \mathbb{Y} \times \hat{\mathbb{Y}} \times \mathbb{Z} \mapsto \{0, 1\}$ that first reduce the vector (y, \hat{y}, z) to (y, \hat{y}) and then apply a function g to the remaining input. Clearly, $F \subset \{g \mid g : \mathbb{Y} \times \hat{\mathbb{Y}} \times \mathbb{Z} \mapsto \{0, 1\}\}$.

Then, to prove this proposition it suffices to observe that the regular adversary is equivalent to a discriminating one restricted to the set of functions F.

$$\begin{split} V^{\mathsf{D}} &= \max_{g: \mathbb{Y} \times \hat{\mathbb{Y}} \times \mathbb{Z} \mapsto \{0,1\}} \Pr[g(Y,\hat{Y},Z) = M] \\ &\geq \max_{f \in F} \Pr[f(Y,\hat{Y},Z) = M] \\ &= \max_{g: \mathbb{Y} \times \mathbb{Y} \mapsto \{0,1\}} \Pr[g(Y,\hat{Y}) = M] \\ &= V^{\mathsf{R}} \,. \end{split}$$

B.2 Vulnerability and Distributional Overfitting (Section 5)

B.2.1 Overall Vulnerability

In this section, we prove Theorem 1, which establishes a connection of total variation between model in/out output distributions and overall vulnerability.

In order to make this and further proofs simpler, we first prove a series of simpler statements, and introduce some helper notions. For convenience, let us define the following additive components of the total variation between *in* and *out* output distributions:

Definition 9. We define *model-output gaps* as follows:

$$\begin{split} \gamma(\hat{y}, y) &\triangleq \Pr[\hat{y} \mid y, in] - \Pr[\hat{y} \mid y, out] \\ \gamma_z(\hat{y}, y) &\triangleq \Pr[\hat{y} \mid y, z, in] - \Pr[\hat{y} \mid y, z, out] \,. \end{split}$$

Note that $\tau(y)=\frac{1}{2}\sum_{\hat{y}}|\gamma(\hat{y},y)|$ and $\tau_z(y)=\frac{1}{2}\sum_{\hat{y}}|\gamma_z(\hat{y},y)|$ (see Figure 5).

Suppose an adversary makes a prediction \hat{m} for the membership m of an example. We measure their gain as the negative 0-1 loss:

Definition 10.

$$v(\hat{m}, m) \triangleq \mathbb{1}[\hat{m} = m]. \tag{4}$$

Using the model-output gaps we can reformulate the adversary's gain:

Proposition 4. Let $A(\hat{y}, y)$ be the regular adversary. Then,

$$\begin{split} v(\mathcal{A}(\hat{y},y),in) &= \mathbb{1}[\gamma(\hat{y},y) > 0] \\ v(\mathcal{A}(\hat{y},y),out) &= \mathbb{1}[\gamma(\hat{y},y) \leq 0] \,. \end{split}$$

Proof of Proposition 4. Recall that we define v as the negative zero-one loss. Hence,

$$\begin{split} v(\mathcal{A}(\hat{y},y),in) &= \mathbbm{1}[\mathcal{A}(\hat{y},y) = in] \\ &= \mathbbm{1}[\underset{m' \in \{in,out\}}{\arg\max} \Pr[m' \mid \hat{y},y] = in] \\ &= \mathbbm{1}[\Pr[in \mid \hat{y},y] > \Pr[out \mid \hat{y},y]] \,. \end{split}$$

As the prior $\Pr[M] = 1/2$ is uniform, a max-aposteriori decision is equivalent to the max-likelihood decision:

$$\begin{split} &\mathbb{1}[\Pr[in \mid \hat{y}, y] > \Pr[out \mid \hat{y}, y]] \\ =& \mathbb{1}[\Pr[\hat{y}, y \mid in] > \Pr[\hat{y}, y \mid out]] \\ =& \mathbb{1}[\Pr[\hat{y}, y \mid in] - \Pr[\hat{y}, y \mid out] > 0] \,. \end{split}$$

By $Y \perp M$ assumption (consequence of Assumption 2), the last form is equivalent to:

$$\mathbb{1}[\Pr[\hat{y} \mid y, in] - \Pr[\hat{y} \mid y, out] > 0] = \mathbb{1}[\gamma(\hat{y}, y) > 0].$$

 $v(\mathcal{A}(\hat{y}, y), out)$ can be obtained as $1 - v(\mathcal{A}(\hat{y}, y), in)$.

A similar result holds for the discriminating adversary:

Proposition 5. Let $A^{D}(\hat{y}, y, z)$ be the discriminating adversary. Then,

$$v(\mathcal{A}^{\mathsf{D}}(\hat{y}, y, z), in) = \mathbb{1}[\gamma_z(\hat{y}, y) > 0]$$
$$v(\mathcal{A}^{\mathsf{D}}(\hat{y}, y, z), out) = \mathbb{1}[\gamma_z(\hat{y}, y) \leq 0].$$

Proof of Proposition 5.

$$\begin{split} v(\mathcal{A}^{\mathsf{D}}(\hat{y},y,z),in) &= \mathbbm{1}[\mathcal{A}(\hat{y},y,z) = in] \\ &= \mathbbm{1}[\underset{m' \in \{in,out\}}{\arg\max} \Pr[m' \mid \hat{y},y,z] = in] \\ &= \mathbbm{1}[\Pr[in \mid \hat{y},y,z] > \Pr[out \mid \hat{y},y,z]] \,. \end{split}$$

As before, because the prior $\Pr[M] = 1/2$ is uniform, a max-a posteriori decision is equivalent to the max-likelihood decision:

$$\begin{split} &\mathbb{1}[\Pr[in \mid \hat{y}, y, z] > \Pr[out \mid \hat{y}, y, z]] \\ =& \mathbb{1}[\Pr[\hat{y}, y, z \mid in] > \Pr[\hat{y}, y, z \mid out]] \,. \end{split}$$

By $Z \perp M$ assumption (consequence of Assumption 2):

$$\begin{split} &\mathbb{1}[\Pr[\hat{y}, y, z \mid in] > \Pr[\hat{y}, y, z \mid out]] \\ =& \mathbb{1}[\Pr[\hat{y}, y \mid z, in] > \Pr[\hat{y}, y \mid z, out]] \\ =& \mathbb{1}[\Pr[\hat{y}, y \mid z, in] - \Pr[\hat{y}, y \mid z, out] > 0] \,. \end{split}$$

Observe that by $(Y, Z) \perp M$ (Assumption 2),

$$\Pr[Y \mid Z, M] = \frac{\Pr[Y, Z, M]}{\Pr[Z, M]} = \frac{\Pr[Y, Z] \Pr[M]}{\Pr[Z] \Pr[M]} = \Pr[Y \mid Z].$$

Hence the expression is equivalent to:

$$\mathbb{1}[\Pr[\hat{y} \mid y, z, in] - \Pr[\hat{y} \mid y, z, out] > 0] = \mathbb{1}[\gamma_z(\hat{y}, y) > 0].$$

Proposition 6. The distributional-overfitting distances have the following equivalent forms:

$$\tau(y) = \sum_{\hat{y} \in \hat{\mathbb{Y}}} \mathbb{1}[\gamma(\hat{y}, y) > 0] \gamma(\hat{y}, y)$$

$$\tau_z(y) = \sum_{\hat{y} \in \hat{\mathbb{Y}}} \mathbb{1}[\gamma_z(\hat{y}, y) > 0] \gamma_z(\hat{y}, y).$$
(5)

Proof of Proposition 6. Observe that $\tau(y)$ and $\tau_z(y)$ by definitions (see Figure 5) equal to the following:

$$\tau(y) = \frac{1}{2} \sum_{\hat{y} \in \mathbb{Y}} |\gamma(\hat{y}, y)| = \frac{1}{2} \sum_{\hat{y} \in \mathbb{Y}} [\mathbb{1}[\gamma(\hat{y}, y) > 0] \gamma(\hat{y}, y) - \mathbb{1}[\gamma(\hat{y}, y) \le 0] \gamma(\hat{y}, y)]$$

$$\tau_z(y) = \frac{1}{2} \sum_{\hat{y} \in \mathbb{Y}} |\gamma_z(\hat{y}, y)| = \frac{1}{2} \sum_{\hat{y} \in \mathbb{Y}} [\mathbb{1}[\gamma_z(\hat{y}, y) > 0] \gamma_z(\hat{y}, y) - \mathbb{1}[\gamma_z(\hat{y}, y) \le 0] \gamma_z(\hat{y}, y)].$$

Using simple algebraic manipulations, we can obtain the following form:

$$\begin{split} \tau(y) &= \frac{1}{2} \sum_{\hat{y} \in \mathbb{Y}} \left[\mathbbm{1}[\gamma(\hat{y}, y) > 0] \, \gamma(\hat{y}, y) - \mathbbm{1}[\gamma(\hat{y}, y) \leq 0] \, \gamma(\hat{y}, y) \right] \\ &= \frac{1}{2} \sum_{\hat{y} \in \mathbb{Y}} \left[\mathbbm{1}[\gamma(\hat{y}, y) > 0] \, \gamma(\hat{y}, y) - (1 - \mathbbm{1}[\gamma(\hat{y}, y) > 0]) \, \gamma(\hat{y}, y) \right] \\ &= \sum_{\hat{y} \in \mathbb{Y}} \left[\mathbbm{1}[\gamma(\hat{y}, y) > 0] \, \gamma(\hat{y}, y) - \frac{1}{2} \gamma(\hat{y}, y) \right], \end{split}$$

and analogously:

$$\tau_z(y) = \sum_{\hat{y} \in \hat{\mathbb{Y}}} \left[\mathbb{1} \left[\gamma_z(\hat{y}, y) > 0 \right] \gamma_z(\hat{y}, y) - \frac{1}{2} \gamma_z(\hat{y}, y) \right] \,.$$

Then, noting the following property of the output gaps:

$$\begin{split} & \sum_{\hat{y} \in \mathbb{Y}} \gamma(\hat{y}, y) = \sum_{\hat{y} \in \mathbb{Y}} \Pr[\hat{y} \mid y, in] - \sum_{\hat{y} \in \mathbb{Y}} \Pr[\hat{y} \mid y, out] = 0 \\ & \sum_{\hat{y} \in \mathbb{Y}} \gamma_z(\hat{y}, y) = \sum_{\hat{y} \in \mathbb{Y}} \Pr[\hat{y} \mid y, z, in] - \sum_{\hat{y} \in \mathbb{Y}} \Pr[\hat{y} \mid y, z, out] = 0, \end{split}$$

we obtain the forms in Equation (5).

Finally, we can prove the theorem.

Proof of Theorem 1. By the law of total expectation we have:

$$\begin{split} V^{\mathsf{R}} &= \mathbb{E}[v(\mathcal{A}(\hat{Y},Y),M)] \\ &= \frac{1}{2} \sum_{y \in \mathbb{Y}} \sum_{\hat{y} \in \mathbb{Y}} v(\mathcal{A}(\hat{y},y),in) \Pr[\hat{y},y \mid in] \\ &+ \frac{1}{2} \sum_{y \in \mathbb{Y}} \sum_{\hat{y} \in \mathbb{Y}} v(\mathcal{A}(\hat{y},y),out) \Pr[\hat{y},y \mid out] \,. \end{split}$$

Because $v(\cdot, in) = 1 - v(\cdot, out)$, this is equal to:

$$V^{\mathsf{R}} = \frac{1}{2} + \frac{1}{2} \sum_{y \in \mathbb{Y}} \sum_{\hat{y} \in \mathbb{Y}} v(\mathcal{A}(\hat{y}, y), in) (\Pr[\hat{y}, y \mid in] - \Pr[\hat{y}, y \mid out]).$$

By $Y \perp M$ assumption (consequence of Assumption 2), we have that $\Pr[\hat{Y}, Y \mid M] = \Pr[\hat{Y} \mid Y, M] \Pr[Y]$. Using this fact and Proposition 4 we have:

$$V^{\mathsf{R}} = \frac{1}{2} + \frac{1}{2} \sum_{y \in \mathbb{Y}} \Pr[y] \sum_{\hat{y} \in \hat{\mathbb{Y}}} \mathbb{1}[\gamma(\hat{y}, y) > 0] \, \gamma(\hat{y}, y) \,.$$

From Proposition 6 we immediately obtain the sought expression for the regular adversary:

$$V^{\mathsf{R}} = \frac{1}{2} + \frac{1}{2} \sum_{y \in \mathbb{Y}} \Pr[y] \tau(y).$$

Analogously, for the discriminating adversary we have:

$$\begin{split} V^{\mathsf{D}} &= \mathbb{E}[v(\mathcal{A}(\hat{Y},Y,Z),M)] \\ &= \frac{1}{2} \sum_{z \in \mathbb{Z}} \sum_{y \in \mathbb{Y}} \sum_{\hat{y} \in \mathbb{Y}} v(\mathcal{A}(\hat{y},y,z),in) \Pr[\hat{y},y,z \mid in] \\ &+ \frac{1}{2} \sum_{z \in \mathbb{Z}} \sum_{y \in \mathbb{Y}} \sum_{\hat{y} \in \mathbb{Y}} v(\mathcal{A}(\hat{y},y,z),out) \Pr[\hat{y},y,z \mid out] \\ &= \frac{1}{2} + \frac{1}{2} \sum_{z \in \mathbb{Z}} \sum_{y \in \mathbb{Y}} \sum_{\hat{y} \in \mathbb{Y}} v(\mathcal{A}(\hat{y},y,z,in)) (\Pr[\hat{y},y,z \mid in] - \Pr[\hat{y},y,z \mid out]) \,. \end{split}$$

By $(Y,Z)\perp M$ assumption (Assumption 2) we have that $\Pr[\hat{Y},Y,Z\mid M]=\Pr[\hat{Y}\mid Y,Z,M]\Pr[Y,Z]$. Using this fact and Proposition 5 we obtain:

$$\begin{split} V^\mathsf{D} &= \frac{1}{2} + \frac{1}{2} \sum_{z \in \mathbb{Z}} \sum_{y \in \mathbb{Y}} \Pr[y, z] \sum_{\hat{y} \in \mathbb{Y}} \mathbb{1}[\gamma_z(\hat{y}, y) > 0] \, \gamma_z(\hat{y}, y) \\ &= \frac{1}{2} + \frac{1}{2} \sum_{z \in \mathbb{Z}} \sum_{y \in \mathbb{Y}} \Pr[z] \rho_z(y) \sum_{\hat{y} \in \mathbb{Y}} \mathbb{1}[\gamma_z(\hat{y}, y) > 0] \, \gamma_z(\hat{y}, y) \,. \end{split}$$

Finally, from Proposition 6 we immediately obtain the sought expression for the discriminating adversary:

$$V^{\mathsf{D}} = \frac{1}{2} + \frac{1}{2} \sum_{z \in \mathbb{Z}} \sum_{y \in \mathbb{Y}} \Pr[z] \, \rho_z(y) \, \tau_z(y) \,.$$

B.2.2 Subgroup Vulnerability

Previously, we showed how overall vulnerability relates to total variation. In this section, we show how subgroup vulnerability relates to total variation between *in* and *out* output distributions on *subgroups*. We use these forms to show the necessary and sufficient conditions for the absence of disparate vulnerability (see Theorem 5).

Lemma 1. The subgroup vulnerability to the discriminating adversary can be expressed as follows:

$$V_z^{\mathsf{D}} = \frac{1}{2} + \frac{1}{2} \sum_{y \in \mathbb{Y}} \rho_z(y) \, \tau_z(y) \,. \tag{6}$$

Proof of Lemma 1. By the law of total expectation:

$$\begin{split} V_z^{\mathsf{D}} &= \mathbb{E}[v(\mathcal{A}(\hat{Y}, Y, z), M) \mid z] \\ &= \frac{1}{2} \sum_{y \in \mathbb{Y}} \sum_{y \in \mathbb{Y}} v(\mathcal{A}(\hat{y}, y, z), in) \Pr[\hat{y}, y \mid z, in] \\ &+ \frac{1}{2} \sum_{y \in \mathbb{Y}} \sum_{y \in \mathbb{Y}} v(\mathcal{A}(\hat{y}, y, z), out) \Pr[\hat{y}, y \mid z, out] \,. \end{split}$$

Because $v(\cdot, in) = 1 - v(\cdot, out)$, this is equal to

$$\frac{1}{2} + \frac{1}{2} \sum_{y \in \mathbb{Y}} \sum_{y \in \mathbb{Y}} v(\mathcal{A}(\hat{y}, y, z), in) (\Pr[\hat{y}, y \mid z, in] - \Pr[\hat{y}, y \mid z, out]).$$

As previously, by $(Y, Z) \perp M$ (Assumption 2), we have that $\Pr[\hat{Y}, Y \mid Z, M] = \Pr[\hat{Y} \mid Y, Z, M] \Pr[Y \mid Z]$. Using this and applying Proposition 5 we get:

$$V_z^{\mathsf{D}} = \frac{1}{2} + \frac{1}{2} \sum_{y \in \mathbb{Y}} \left[\rho_z(y) \sum_{\hat{y} \in \mathbb{Y}} \mathbb{1} [\gamma_z(\hat{y}, y) > 0] \gamma_z(\hat{y}, y) \right]. \tag{7}$$

By Proposition 6 this is equal to the sought form

The analogous form for the subgroup vulnerability to the regular vulnerability is different:

Lemma 2. The subgroup vulnerability to the regular adversary can be expressed as follows:

$$V_z^{\mathsf{R}} = \frac{1}{2} + \frac{1}{2} \sum_{y \in \mathbb{Y}} \left[\rho_z(y) \sum_{\hat{y} \in \mathbb{Y}} \mathbb{1}[\gamma(\hat{y}, y) > 0] \gamma_z(\hat{y}, y) \right]. \tag{8}$$

Proof of Lemma 2. By the law of total expectation we have

$$\begin{split} V_z^{\mathsf{R}} &= \mathbb{E}[v(\mathcal{A}(\hat{Y},Y),M) \mid z] \\ &= \frac{1}{2} \sum_{y \in \mathbb{Y}} \sum_{\hat{y} \in \mathbb{Y}} v(\mathcal{A}(\hat{y},y),in) \Pr[\hat{y},y \mid z,in] \\ &+ \frac{1}{2} \sum_{y \in \mathbb{Y}} \sum_{\hat{y} \in \mathbb{Y}} v(\mathcal{A}(\hat{y},y),out) \Pr[\hat{y},y \mid z,out] \,. \end{split}$$

Because $v(\cdot,in)=1-v(\cdot,out)$, this is equal to

$$\frac{1}{2} + \frac{1}{2} \sum_{y \in \mathbb{Y}} \sum_{\hat{y} \in \tilde{\mathbb{Y}}} v(\mathcal{A}(\hat{y}, y), in) (\Pr[\hat{y}, y \mid z, in] - \Pr[\hat{y}, y \mid z, out]).$$

By $(Y,Z)\perp M$ (Assumption 2), we have that $\Pr[\hat{Y},Y\mid Z,M]=\Pr[\hat{Y}\mid Y,Z,M]\Pr[Y\mid Z]$. Using this and applying Proposition 4 we obtain the sought form.

Note the difference in the vulnerability-gain expression (uses γ) and probability difference part (uses γ_z). This is because we are conditioning a regular adversary (hence the gain expression uses γ) on a subgroup z (hence the probabilities are conditioned on z through γ_z). In the case of discriminating adversary, both parts use γ_z , which simplifies the expression.

B.2.3 Disparate Vulnerability

In this section, we (re)state the necessary and sufficient conditions for the absence of disparate vulnerability (see Theorem 5). Having the equivalent subgroup vulnerability forms from Lemmas 1 and 2 in the previous section, the proofs are straightforward.

Theorem 2. Let G_z and $G_{z'}$ be two subgroups of the population Ω . There is no disparate vulnerability for a **discriminating** adversary between the two subgroups (i.e., $\Delta V_{z,z'}^D = 0$) iff:

$$\sum_{y \in \mathbb{Y}} \left(\rho_z(y) \, \tau_z(y) - \rho_{z'}(y) \, \tau_{z'}(y) \right) = 0.$$

Proof of Theorem 2. Follows directly from Lemma 1 as $\Delta V_{z,z'}^{\rm D} = |V_z^{\rm D} - V_z^{\rm D}|.$

Theorem 3. Let G_z and $G_{z'}$ be two subgroups of the population Ω . There is no disparate vulnerability for a **regular** adversary between the two subgroups (i.e., $\Delta V_{z,z'}^{\mathsf{D}} = 0$) iff:

$$\sum_{y \in \mathbb{Y}, \hat{y} \in \hat{\mathbb{Y}}} \mathbb{1}[\gamma(\hat{y}, y) > 0] \left(\rho_z(y) \gamma_z(\hat{y}, y) - \rho_{z'}(y) \gamma_{z'}(\hat{y}, y) \right) = 0.$$

$$(9)$$

Proof of Theorem 3. Analogously, follows directly from Lemma 2 as $\Delta V_{z,z'}^{\mathsf{R}} = |V_z^{\mathsf{R}} - V_{z'}^{\mathsf{R}}|$.

B.3 Disparate Vulnerability and Equality of Odds (Section 6.2)

In this section, we prove the statements concerning connections of disparate vulnerability and the GEO property.

Proposition 2. Suppose a classifier satisfies GEO. Then, $V_z^{\mathsf{R}} = V_z^{\mathsf{D}}$ for any subgroup z.

Proof of Proposition 2. By the theorem statement, for any m, \hat{y}, y , the value $p \triangleq \Pr[\hat{y} \mid y, z, m]$ is equal across all subgroups $z \in \mathbb{Z}$. This implies that for any z:

$$\Pr[\hat{y}\mid y,m] = \sum_{z\in\mathbb{Z}} \Pr[z\mid y,m] \Pr[\hat{y}\mid y,z,m] = p \sum_{z\in\mathbb{Z}} \Pr[z\mid y,m] = p = \Pr[\hat{y}\mid y,z,m] \,.$$

Therefore, $\gamma(\hat{y}, y) = \gamma_z(\hat{y}, y)$. Recalling equations 8 and 7, this implies $V_z^{\mathsf{R}} = V_z^{\mathsf{D}}$.

Proposition 3. Suppose a classifier satisfies GEO, and $\rho_z(y) = \rho_{z'}(y)$ holds for any y and for all z, z'. Then, $\Delta V_{z,z'}^{\mathsf{R}} = 0$ and $\Delta V_{z,z'}^{\mathsf{D}} = 0$.

Proof of Proposition 3. Fix \hat{y}, y . Let $p_{in} \triangleq \Pr[\hat{y} \mid y, in]$. Define p_{out} analogously.

A model-output gap γ can be expressed as follows: $\gamma(\hat{y},y) = p_{in} - p_{out}$, and it is equal for all subgroups z.

As mentioned in the proof of Proposition 2, GEO implies $\gamma(\hat{y},y)=\gamma_z(\hat{y},y)$. Hence, the necessary and sufficient condition for the absence of disparate vulnerability w.r.t groups z,z' against both attackers becomes:

$$0 = \sum_{y \in \mathbb{Y}, \hat{y} \in \hat{\mathbb{Y}}} \mathbb{1}[\gamma(\hat{y}, y) > 0] \left(\rho_{z}(y) \gamma(\hat{y}, y) - \rho_{z'}(y) \gamma(\hat{y}, y)\right)$$

$$= \sum_{y \in \mathbb{Y}, \hat{y} \in \hat{\mathbb{Y}}} \mathbb{1}[\gamma(\hat{y}, y) > 0] \gamma(\hat{y}, y) \left(\rho_{z}(y) - \rho_{z'}(y)\right).$$

$$(10)$$

Because $\rho_z(y) = \rho_{z'}(y)$ for every y and pair z, z' by the theorem statement, every term in the summation is zero. This implies no disparate vulnerability against both adversaries.

C Additional Figures and Tables

This section contains additional figures and tables for our experiments.

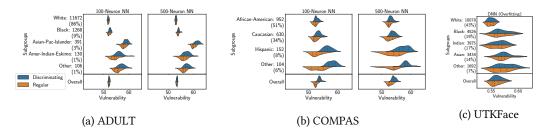


Figure 7: Effect of overfitting on subgroup vulnerability. See Figure 2 for context.

Table 4: Vulnerability and maximum vulnerability disparity of different models. The columns are test accuracy, overfitting, vulnerability, and maximum pairwise vulnerability disparity (max-disparity), with their mean and standard deviation (std) values over N=35 training/test set shuffles on each dataset. Numbers are percentage points. Negative overfitting is an artifact due to a large number of duplicate feature vectors.

(a) COMPAS

	Test Acc. mean std		Regular Overfitting Max-Disparity				Vulner	ability	Discriminating Max-Disparity		Vulnerability	
			mean std		mean	std	mean	std	mean	std	mean	std
Logistic Regression (LR)	68.15	2.42	0.73	2.00	18.61	8.39	52.09	0.87	29.03	6.84	60.10	10.83
6-Neuron NN	68.75	3.12	1.81	3.22	15.93	8.19	51.72	0.50	30.50	6.71	60.70	11.42
100-Neuron NN	67.61	4.58	8.22	11.21	18.25	10.51	53.77	2.37	32.40	8.84	61.72	12.16
500-Neuron NN	67.29	4.24	9.27	10.60	16.65	10.35	54.95	4.03	35.02	7.78	62.36	12.78
DP LR, $\varepsilon = 1$	59.41	5.15	-0.34	0.85	9.66	5.54	51.78	0.39	12.01	9.74	55.23	2.27
DP LR, $\varepsilon = 2.5$	60.65	4.99	-0.26	0.66	7.12	2.41	51.46	0.70	7.74	4.76	54.66	1.55
DP LR, $\varepsilon = 5$	64.99	4.94	0.12	0.91	11.30	6.56	51.51	0.63	19.44	8.63	57.23	5.37
DP LR, $\varepsilon = 7.5$	66.50	3.19	-0.79	1.59	14.99	7.49	51.48	0.77	22.19	8.31	58.23	7.34
Fair LR (Equalized Odds)	64.10	1.47	1.88	2.17	13.63	7.43	53.44	3.64	16.91	6.05	55.37	6.07

(b) ADULT

	Regular								Discriminating					
	Test Acc.		Overfitting		Max-Disparity		Vulnerability		Max-Disparity		Vulnerability			
	mean std		mean	std	mean	std	mean	std	mean	std	mean	std		
Logistic Regression (LR)	87.22	3.93	0.49	0.55	3.27	1.32	50.55	0.10	5.07	0.86	52.83	2.05		
6-Neuron NN	87.23	3.86	0.87	0.76	3.22	1.39	50.73	0.23	5.04	1.23	52.82	2.01		
100-Neuron NN	85.98	4.05	7.13	4.83	6.12	0.83	53.92	2.40	7.31	0.76	54.83	3.05		
500-Neuron NN	85.40	3.95	9.95	5.03	7.01	0.67	55.44	2.63	8.45	0.86	56.20	3.25		
DP LR, $\varepsilon = 1$	77.07	6.59	-0.14	0.35	2.37	1.51	50.41	0.25	4.27	1.65	51.89	1.04		
DP LR, $\varepsilon = 2.5$	76.10	7.47	0.21	0.44	2.17	1.38	50.33	0.18	3.65	1.15	51.65	0.95		
DP LR, $\varepsilon = 5$	84.27	5.27	0.29	0.36	3.14	1.73	50.50	0.25	5.06	1.38	52.61	1.73		
DP LR, $\varepsilon=7.5$	85.33	4.69	0.37	0.45	2.75	1.12	50.47	0.25	4.88	1.16	52.68	1.87		
Fair LR (Equalized Odds)	85.43	2.64	0.79	0.65	2.30	1.07	50.79	0.56	2.90	0.93	51.26	1.03		

(c) UTKFace

					Discriminating							
	Test Acc.		Overfitting		Max-Disparity		Vulnerability		Max-Disparity		Vulnerability	
	mean	std	mean	std	mean	std	mean	std	mean	std	mean	std
DNN (Non-Overfitting)	57.18	4.89	3.13	0.80	0.83	0.34	55.54	0.06	1.21	0.44	55.99	0.34
DNN (Overfitting)	55.93	4.62	24.47	2.92	2.78	0.87	56.22	0.80	3.06	0.92	56.68	0.98

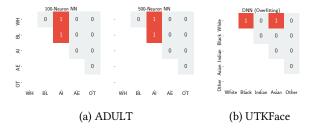


Figure 8: Effect of overfitting on disparate vulnerability (regular adversary.) See Figure 1 for context.

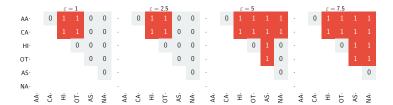


Figure 9: Statistical evidence of disparate vulnerability of ε -DP LR models trained on COMPAS (discriminating addversary.)

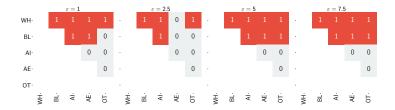


Figure 10: Statistical evidence of disparate vulnerability of ε -DP LR models trained on ADULT (discriminating adversary.)

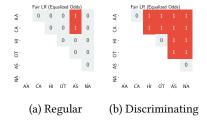


Figure 11: Effect of EO on disparate vulnerability on COMPAS

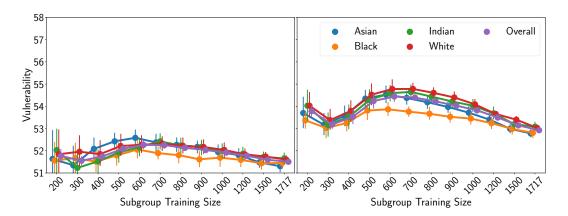


Figure 12: The effect of subgroup distribution. Data for the all subgroups are gradually increased, and subgroup and overall vulnerability are measured against the regular (left) and discriminating (right) attackers. See Figure 4 (left) for context.

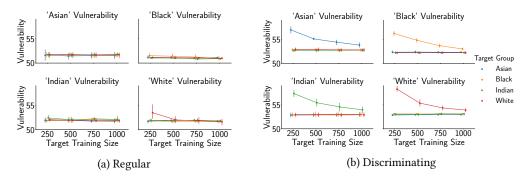


Figure 13: The effect of data volume. Data for the 'target' subgroup is gradually increased and subgroup vulnerability is measured for all subgroups, against the regular (left) and discriminating (right) attackers. See Figure 4 (right) for context.