# Protecting Decision Boundary of Machine Learning Model With Differentially Private Perturbation

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**Abstract**—Machine learning service API allows model owners to monetize proprietary models by offering prediction services to third-party users. However, existing literature shows that model parameters are vulnerable to extraction attacks which accumulate prediction queries and their responses to train a replica model. As countermeasures, researchers have proposed to reduce the rich API output, such as hiding the precise confidence. Nonetheless, even with response being only one bit, an adversary can still exploit fine-tuned queries with differential property to infer the decision boundary of the underlying model. In this paper, we propose boundary differential privacy (BDP) against such attacks by obfuscating the prediction responses with noises. BDP guarantees an adversary cannot learn the decision boundary of any two classes by a predefined precision no matter how many queries are issued to the prediction API. We first design a perturbation algorithm called boundary randomized response for a binary model. Then we prove it satisfies  $\epsilon$ -BDP, followed by a generalization of this algorithm to a multiclass model. Finally, we generalize a hard boundary to soft boundary and design an adaptive perturbation algorithm that can still work in the latter case. The effectiveness and high utility of our solution are verified by extensive experiments on both linear and non-linear models.

Index Terms—Model Defense, Boundary Differential Privacy, Model Extraction, Adversarial Machine Learning

# 1 Introduction

THE pervasive application of artificial intelligent has **I** encouraged the boosting business of machine learning services, such as Microsoft Azure Face API, Google Cloud Speech-to-Text, and Amazon Comprehend. To train these high-quality machine learning models, service providers need to spend intense human labor and computation resources to acquire a large well-labeled datasets and tune training process. However, a prediction API call, which consists of a query and its response, can be vulnerable to adversarial attacks that disclose the internal states of these models. Particularly, a model extraction attack [1] is able to restore important model parameters using the rich information (e.g., model type, prediction confidence) provided by the prediction API. Once the model is extracted, an adversary can further apply model inversion attack [2] to learn the proprietary training data, compromising the privacy of data contributors. Another follow-up attack on the extracted model is evasion attack [3], [4], which avoids a certain prediction result by modifying its query. For example, a hacker modifies the executable binaries of a malware or the contents of a phishing email in order not to be detected by an antivirus or spam email filter.

Countermeasures against *model extraction* attacks have received increased attention but are still inadequate. One of them is to restrict rich information in the prediction API,

for example, by rounding the prediction confidence value to a low granularity. However, even if the service provider completely eliminates this value in the prediction API, that is, to offer prediction label only, an adversary can still defeat this protection by issuing large number of fine-tuned queries and train a replica of the original model with great similarity [1], [3], [5]. The other countermeasure is to detect malicious extraction by monitoring feature coverage [6] or query distribution [7], and stop the service when a certain threshold is reached. However, since we cannot preclude user collusion, all queries and responses must be considered aggregately, which leads to significant false positive cases and eventually the early termination of service.

To address the disadvantages, in this paper we propose a new countermeasure that obfuscates the output label of a prediction response. There are three main concerns when designing this obfuscation mechanism. First, the accuracy of prediction API is highly correlated with the degree of obfuscation — if obfuscation needs to be applied to most queries, the utility of the machine learning service will degrade severely. Second, the obfuscation mechanism should be independent of the underlying machine learning models and can tackle a category of boundary-probing attacks. Third, the obfuscation mechanism should be customizable. That is, it should allow user-defined parameters that can trade utility for model privacy or vice versa.

Our key observation is that many model extraction attacks exploit fine-tuned queries near the decision boundary of a machine learning model achieve optimal extraction performance [8], [9]. We treat decision boundary probing as an abstract and necessary condition of high-quality extraction attacks. The responses of these queries disclose the details of

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model parameters and therefore should be obfuscated with priority. To this end, we propose a boundary differentially private layer (BDPL) for machine learning services. BDPL provides a parameterized approach to obfuscate responses whose queries fall in a predefined boundary-sensitive zone. The notion of differential privacy guarantees the responses of all queries in the boundary-sensitive zone are indistinguishable from one another. As such, adversary cannot learn the decision boundary no matter how many queries are issued to the prediction API. On the other hand, the majority perturbation falls within boundary-sensitive zone and out-of-zone query is less affected from obfuscation. In this way, we can make the best use of the obfuscation and retain high utility of the machine learning service. To summarize, our contributions in this paper are as follows.

- We propose a new protection mechanism, namely, boundary differential privacy, against model extraction with fine-tuned queries while balancing service utility and model protection level.
- We develop an efficient method to identify queries in the boundary-sensitive zone, and design a perturbation algorithm called boundary randomized response for binary model to guarantee boundary differential privacy.
- We generalize binary defense to multiclass model and develop corresponding perturbation algorithm in a pairwise manner.
- We design an alternative defense layer with soft margin to extend the scope of protection and implement an adaptive perturbation algorithm.
- We conduct extensive empirical study on both binary and multiclass, linear and non-linear machine learning models to evaluate the effectiveness of our solution.

The rest of the paper is organized as follows. Section 2 introduces the preliminaries for machine learning, model extraction and differential privacy. Section 3 elaborates on the threat model and problem definition with boundary-sensitive zone and boundary differential privacy. Section 4 presents the details of boundary differentially private layer. Section 5 introduces evaluation metrics and shows the experimental results of BDPL against model extractions. Section 7 reviews the related literature, and Section 8 concludes this paper and discusses future work.

#### 2 PRELIMINARIES

# 2.1 Supervised Machine Learning Model

A dataset  $\mathcal{X}$  contains samples in a d-dimensional feature space. Each sample has a membership in a set of predefined classes called *labels*. Supervised machine learning trains a statistical model by such sample-label pairs to make predictions of labels on unknown samples. In this paper we focus on classification models which have K possible outputs.

Formally, a classification model f produces a response y to a query sample x as follows.

$$y = f(x) = egin{cases} \mbox{"class 1" label} \mbox{"class 2" label} \mbox{...} \mbox{"class K" label} \end{cases}$$

Classification models have been widely adopted in many machine learning applications, such as activity categorization, face recognition and speaker identification. Depending on the nature of these applications, the model f can be either linear (e.g., logistic regression) or non-linear (e.g., neural network).

#### 2.2 Model Extraction With Only Labels

In a model extraction attack, a malicious party attempts to replicate a model from the original one by continuously exploiting the prediction API. Technically any queries can constitute such an attack. However, the more queries the more likely this malicious attack will be exposed. As such, in the literature most model extraction attacks fabricate *finetuned queries* by differential techniques such as line search [1], [5] and Jacobian augmentation [3]. These queries are carefully selected to capture the information about decision boundary where prediction results vary drastically.

Formally, a model extraction attack selects a set of finetuned queries  $\mathcal{X}_{diff}$  and obtains their responses  $\mathcal{Y}_{diff}$  to train a replica model f'.

$$\mathcal{X}_{diff} = \{ \boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_n \}, \quad \boldsymbol{x} \in \mathbb{R}^d,$$

$$\mathcal{Y}_{diff} = \{ y_1, y_2, \dots, y_n \}, \quad y \in \mathbb{R}^1,$$

$$\exists \boldsymbol{x}, \, \boldsymbol{x}' \in \mathcal{X}_{diff}, \, dist(\boldsymbol{x}, \boldsymbol{x}') = \delta \, \wedge \, y \neq y',$$

where  $dist(\cdot)^1$  measures the distance between two queries and  $\delta$  is the unit distance adopted in the differential techniques when searching for boundary, i.e., where two corresponding responses  $y \neq y'$ .

#### 2.3 Differential Privacy

Differential privacy [10] is proposed to bounds data sanitation with a measurable budget so that sensitive information can be released with a strong privacy guarantee. In centralized sanitation, all sensitive data are processed in one place and it is primarily defined in terms of adjacent datasets that differ on one data point with each other.

A perturbation algorithm  $A(\cdot)$  probabilistically modifies the original data to other values in the same domain. It achieves  $\epsilon$ -differential privacy, if and only if for any two adjacent datasets D,D' and any possible output  $\tau$  of the perturbation algorithm, the following inequality always holds.

$$e^{-\epsilon} \le \frac{Pr[A(D) = \tau]}{Pr[A(D') = \tau]} \le e^{\epsilon}$$

Intuitively, privacy budget  $\epsilon$  controls how close the sanitized data is to the original one. A larger privacy budget will induce a higher degree of similarity as well as utility.

1. In general, this notation can be any distance metrics (e.g., Manhattan distance, Euclidean distance). The implications of distance metrics to detailed algorithms will be discussed in Section 4.1.1.

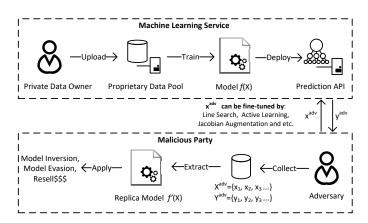


Fig. 1. Motivation and Threat Model

# 3 PROBLEM DEFINITION

#### 3.1 Motivation and Threat Model

A machine learning service provides a prediction result using a proprietary model as shown in Fig. 1. An adversary wants to produce a replica of this model by continuously querying it through the provided prediction API. We assume he can perform a typical two-stage extraction attack: 1) The adversary generates a set of fine-tuned real/synthetic queries normalized in [-1,1] and interacts with API under a large query budget. 2) He can store all responses, i.e., labels and reconstruct a replica model by training on the finetuned query-response pairs (e.g., minimize the empirical risk of an objective function). The success replication will results in intellectual property loss for the original provider and induce other attacks. The attack is semi-whitebox<sup>2</sup>, i.e., he can extract a replicated model using the same model type (e.g., convolutional neural network) and hyperparameters as the original one. Apart from public knowledge of model and defense settings, we assume the adversary has no apriori information of the decision boundary.

#### 3.2 Boundary-Sensitive Zone

Our problem is to protect against model extraction attacks by obfuscating query responses. Before we formally define the security model, we first introduce the notion of *decision boundary* and *boundary-sensitive zone*. For most supervised models, a decision boundary is a critical borderline in the feature space where labels are different on both sides. Fig. 2 illustrates the hypothetical decision boundaries in four combination cases in a 2D feature space. In a multi-dimensional feature space, a line boundary becomes a hyperplane, and a curve boundary becomes a hypersurface.

Our key idea is to protect the query responses near the decision boundary against most model extraction attacks. To this end, we introduce the notion of boundary-sensitive zone

**Definition 1.** (Boundary-Sensitive Zone) Given feature space Z, a model f and a parameter  $\Delta$  chosen by the model

2. The semi-whitebox assumption is based on the fact that state-ofthe-art models in specific application domains, such as image classification, are usually public knowledge. Nonetheless, our solution can also work against black-box attacks where such knowledge is proprietary. owner, all feature vectors adjacent to the decision boundary of f constitute a subspace  $Z_{\Delta}$  of Z, where

$$Z_{\Delta} = \{ \boldsymbol{x} \in \mathbb{R}^d \mid dist(\boldsymbol{x}, f) < \Delta \},\$$

where  $dist(\cdot)$  measures the distance between a feature vector x and the decision boundary of f. All queries in this zone  $Z_{\Delta}$  are considered particularly sensitive and have high risk of revealing the decision boundary of this model.

# 3.3 Boundary Differential Privacy

All queries in the boundary-sensitive zone need obfuscation, whose objective is to perturb the responses of any two sensitive queries so that they are indistinguishable for the adversary to determine the true decision boundary within this zone. To this end, we adopt the notion of differential privacy and formally define *boundary differential privacy* as follows.

**Definition 2.** (ε-Boundary Differential Privacy) A perturbation algorithm  $A(\cdot)$  achieves ε-boundary differential privacy, if and only if for any two queries  $x_1$ ,  $x_2$  in the boundary-sensitive zone  $Z_{\Delta}$ , the following inequality always holds for the true responses  $y_1$  and  $y_2$  and the perturbed ones  $A(y_1)$  and  $A(y_2)$ .

$$e^{-\epsilon} \le \frac{Pr[y_1 = y_2 | A(y_1), A(y_2)]}{Pr[y_1 \ne y_2 | A(y_1), A(y_2)]} \le e^{\epsilon}$$

The above inequality guarantees that an adversary cannot deduce whether two perturbed responses  $A(y_1)$  and  $A(y_2)$  originate from the same  $(y_1=y_2)$  or different labels  $(y_1\neq y_2)$  with high confidence (controlled by  $\epsilon$ ). As such, the adversary cannot use fine-tuned queries, no matter how many they are, to find the decision boundary within the granule of boundary-sensitive zone.

# 4 BOUNDARY DIFFERENTIALLY PRIVATE LAYER

In this section, we present our solution to protect against model extraction attacks with respect to  $\epsilon$ -boundary differential privacy ( $\epsilon$ -BDP) by appending a BDP layer to the model output. According to Definition 2, this layer consists of two major steps — identifying query sensitivity, and perturbing the responses of sensitive queries to satisfy BDP. In what follows, we first introduce the implement of binary defense with a technique to identify sensitive queries with the notion of corner points and a perturbation algorithm called boundary randomized response to guarantee  $\epsilon$ -BDP. Then we generalize it to multiclass model with a technique to identify counter class for developing its perturbation algorithm multiclass boundary randomized response. Finally, we introduce a zone-less variant with soft margin to globalize the defense while retaining majority of obfuscation inside boundary sensitive zone, where the perturbation is provided by adaptive boundary randomized response.

# 4.1 Binary Defense

We start from binary models which have only two labels — positive and negative, which are particularly popular in spam filtering, malware detection, and disease diagnosis.

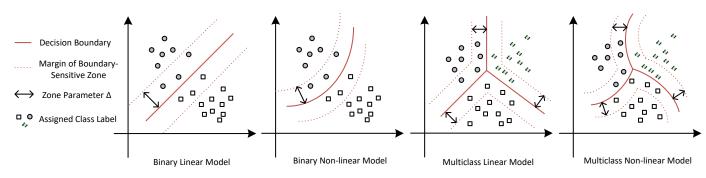


Fig. 2. Illustration of Hypothetical Decision Boundary and Boundary-Sensitive Zone in 2D

# 4.1.1 Identifying Sensitive Queries In Binary Model

A query is identified as sensitive if it falls in the boundary-sensitive zone according to Definition 1. However, in practice the decision boundary may not have a closed form (especially for complex models such as neural networks). In this subsection, we propose a method to determine if a query  $x_q$  is sensitive without deriving the boundary-sensitive zone. The idea is to test if a ball centered at  $x_q$  with radius  $\Delta$  intersects with the decision boundary<sup>3</sup>. In theory, this is equivalent to finding if there exists a flipping point x' in the ball that has a different label from that of the query point  $x_q$ . Formally,

**Definition 3.** (Query Sensitivity) A query  $x_q$  is sensitive, if and only if:

$$\exists x' \in B(x_q, \Delta), s.t., f(x') \neq f(x_q),$$

where  $B(\boldsymbol{x}_q, \Delta) = \{\boldsymbol{x} \in \mathbb{R}^d \ | dist(\boldsymbol{x}, \boldsymbol{x}_q) \leq \Delta \}$  is the ball centered at  $\boldsymbol{x}_q$  with radius  $\Delta$ .

The above definition needs to test infinite number of points in the ball, which is infeasible. Nonetheless, we observe that if the ball is convex and small enough, a sufficient condition of query  $x_q$  being sensitive is that at least one of the *corner points* in each dimension of this ball  $B(x_q, \Delta)$  is a flipping point. As such, the sensitivity of query  $x_q$  can be approximated by testing the labels of 2d corner points of  $x_q$  without false negatives. Furthermore, if the distance metric is the L1 distance (i.e., Manhattan distance), this is also a necessary condition, which means that testing corner points leads to the exact sensivitity. For example, given a two-dimensional query [a,b] and L1 radius rad

**Theorem 1.** (Flipping Corner Theorem) A sufficient condition of query  $x_q$  being sensitive is that,

$$\exists \ \Delta_i \in \Delta \cdot I, \ f(x_q \pm \Delta_i) \neq f(x_q),$$

where I is the identity matrix,  $\Delta_i$  is the projected interval on some dimension i, and  $x_q \pm \Delta_i$  denotes the two corner points in dimension i. If the distance metric is the L1 distance, this equation is also a necessary condition.

*Proof:* Let  $x_i$  be one of the corner points in dimension i.

• (Sufficient Condition) For any  $x_i$ , the decision boundary must exist between  $x_i$  and  $x_q$  where  $f(x_i) \neq f(x_q)$ . It intersects line  $x_i x_q$  at point  $b_i$ . As  $x_i$ ,  $x_q$  and  $b_i$  are on the same straight line, we have

$$dist(\boldsymbol{x}_i, \boldsymbol{b}_i) + dist(\boldsymbol{x}_q, \boldsymbol{b}_i) = dist(\boldsymbol{x}_i, \boldsymbol{x}_q) = \Delta.$$

Since  $dist(x_q, f)$  is the minimum distance between  $x_q$  and any point on the decision boundary, we have

$$dist(\boldsymbol{x}_q, f) \leq dist(\boldsymbol{x}_q, \boldsymbol{b}_i) = \Delta - dist(\boldsymbol{x}_i, \boldsymbol{b}_i) < \Delta.$$

According to Definition 1, query  $x_q$  is sensitive and this proves the sufficient condition.

• (Necessary Condition for L1 Distance) If  $x_q$  is a sensitive query, an L1-ball centered at  $x_q$  with radius  $\Delta$  will be given by

$$B(\boldsymbol{x}_q, \Delta) = \{ \boldsymbol{x} \in \mathbb{R}^d \mid dist_{L1}(\boldsymbol{x}, \boldsymbol{x}_q) \le \Delta \}.$$
 (1)

Let  $b_m$  be the point which is the closest to  $x_q$  on the decision boundary of f. According to Definition 3, we have

$$dist_{L1}(\boldsymbol{x}_q, \boldsymbol{b}_m) = dist_{L1}(\boldsymbol{x}_q, f) < \Delta.$$

Since  $x_q$  is sensitive,  $b_m$  must be inside this L1-ball:

$$\boldsymbol{b}_m \in B(\boldsymbol{x}_q, \Delta).$$

This means that the decision boundary must intersect the ball at  $b_m$ . As such, at least one convex vertex of the ball is on a different side of the decision boundary than point  $x_q$ . Since the convex vertices of an L1-ball are exactly those corner points, there exists at least one corner point  $x_i$  such that  $f(x_i) \neq f(x_q)$ . And this proves the necessary condition.

Therefore, flipping corner point is a sufficient condition for query  $x_q$  being sensitive and a necessary condition under the L1 distance metric.

# 4.1.2 Perturbation Algorithm: Boundary Randomized Response

Randomized response [11] is a privacy-preserving survey technique developed for surveying sensitive questions. A randomized boolean value is given to the answer and provides plausible deniability. As the perturbation algorithm defined in boundary differential privacy has exactly two

<sup>3.</sup> The case of tangency is rarely reached in real life given that the feature space is usually continuous. For simplicity, we mainly consider intersection

<sup>4.</sup> If  $\Delta$  is small, the decision boundary near the ball can be treated as a hyperplane.

**Algorithm 1** Boundary Differentially Private Layer For Binary Model

```
Input:
                  Boundary-Sensitive Zone Parameter \Delta
                  Boundary Privacy Budget \epsilon
                  Query \boldsymbol{x}_q \in R^d
                  Model f
 Output:
                  Original Response y_q or Perturbed Response y'_q
 Procedure:
 1: if x_q is not cached then
        y_q = f(\boldsymbol{x}_q)
        CornerPoints = getCornerPoints(\Delta, x_q)
 3:
 4:
        for x_i in CornerPoints do
 5:
            if x_i is a flipping point then
 6:
                y_q' = BRR(y_q, \, \epsilon)
 7:
                Cache(\boldsymbol{x}_q, y_q')
 8:
                return y'_a
        return y_q
 9.
10: else
        return qetCached(x_q)
11:
```

output choices, we design the following BRR algorithm based on randomized response to satisfy  $\epsilon$ -BDP.

**Definition 4.** (Boundary Randomized Response, BRR) Given query sample  $x_q$  and its true response  $y_q \in \{0,1\}$ , the boundary randomized response algorithm  $A(y_q)$  perturbs  $y_q$  by the following:

$$A(y_q) = \begin{cases} y_q, & \text{w.p.} \quad \frac{1}{2} + \frac{\sqrt{e^{2\epsilon} - 1}}{2 + 2e^{\epsilon}} \\ \\ 1 - y_q, & \text{w.p.} \quad \frac{1}{2} - \frac{\sqrt{e^{2\epsilon} - 1}}{2 + 2e^{\epsilon}} \end{cases}$$

**Theorem 2.** The boundary randomized response algorithm  $A(y_q)$  satisfies  $\epsilon$ -BDP.

# 4.1.3 Summary for Binary Defense

Algorithm 1 summarizes the detailed procedures of BDP layer that can be tapped to the output of any binary machine learning model f. When a new query  $x_q$  arrives, if it has already been queried before, the layer directly returns the cached response  $y'_q$  to prevent attacker from learning multiple perturbed responses of the same query response, which can lead to a less private BDP. Otherwise, the layer first obtains the real result  $y_q$  from model f. Then it determines whether  $x_q$  is in the boundary-sensitive zone by checking all corner points. As long as one corner point is as a flipping point, the query is identified as sensitive, and the boundary randomized response algorithm  $BRR(\cdot)$ with privacy budget  $\boldsymbol{\epsilon}$  will be invoked. The layer will thus return the perturbed result  $y_q'$  and cache it for future use. Otherwise, if  $x_q$  is not sensitive after checking all corner points, the real result  $y_q$  will be returned. As for time complexity for identifying sensitive queries if they are not cached, since we only need to check two corner points in each dimension for an m-dimensional query, the upper bound time complexity will be O(mT) where T is the time cost of each model prediction. Nonetheless, the average time cost can be much smaller as the process can terminate early as long as one flipping corner is found and we also propose n-shot sampling to speed up this process. As for T, in our evaluation environment we find the average of T

over a variety of models is  $70-90\mu s$  for logistic regression,  $400-500\mu s$  for shallow neural network, and  $600-700\mu s$  for convolutional neural network.

#### 4.2 Generalization to Multiclass Model

We now consider the case where the prediction domain of a model has more than two classes. To adapt the current algorithm to multiclass model and retain  $\epsilon$ -BDP guarantee, we observe that the decision boundary in a multiclass model is essentially a union of binary boundaries, each of which separates two classes. As such, we can extend the definition of sensitive query in a multiclass model in terms of its nearby decision boundary. Formally,

**Definition 5.** (Multiclass Query Sensitivity) A query  $x_q$  is sensitive to the decision boundary of classes  $u,v\in Domain(f)$ , if and only if:

$$\exists x' \in B(x_q, \Delta), s.t., f(x') = u, f(x_q) = v, u \neq v,$$

where u is called the *counter class* to the true response v and Domain(f) contains all possible output classes.

In this way, one class (i.e., true response) can be treated as the "positive" label and the other as the "negative" one (i.e., the counter class). The multiclass case is thus reduced to the same problem of protecting binary decision boundaries except that there are boundaries for each pair of classes.

# 4.2.1 Identifying Counter Class

The key idea of avoiding multiple decision boundaries from a variety of candidate counter classes for u is to only associate sensitive query with its nearest decision boundary and identify the corresponding class on the other side as the counter class. To identify this class, we use the majority vote from all flipping corner points. However, a full scan of all these points is not practical particularly in a high dimensional dataset with hundreds or even thousands of corner points. To strike a balance between accuracy and efficiency, we perform an N-shot sampling over the flipping corners. Formally,

**Definition 6.** (N-shot Flipping Corner) An estimate of query  $x_q$  being sensitive to the decision boundary of classes  $u, v \in Domain(f)$  is that,

$$\forall \Delta_{i \in N} \in \Delta \cdot I, \ V(f(x_q \pm \Delta_i) \neq f(x_q)) = u,$$

where  $\Delta_{i \in N}$  denotes flipping corner points in N sampling dimensions with flipping corners and  $V(\cdot)$  finds the class with the highest flipping rate from the comparison results of provided corner points.

#### 4.2.2 Multiclass Boundary Randomize Response

Now that we can detect a sensitive query in the multiclass case, given its true response and counter class, we use the following Multiclass Boundary Randomized Response (MBRR) algorithm to achieve  $\epsilon$ -BDP.

**Definition 7.** (Multiclass Boundary Randomized Response, MBRR) Given a query sample  $x_q$ , its true response  $y_q$  and counter class  $\overline{y_q}$  ( $y_q$ ,  $\overline{y_q} \in \{u,v\}$ ), the multiclass

# **Algorithm 2** Boundary Differentially Private Layer For Multiclass Model

```
Input:
                  Flipping Corner Shot N
                 Boundary-Sensitive Zone Parameter \Delta
                 Boundary Privacy Budget \epsilon
                 Query \boldsymbol{x}_q \in R^d
                 Model f
 Output:
                 Original Response y_q or Perturbed Response y'_q
 Procedure:
 1: if x_q is not cached then
       y_q = f(\boldsymbol{x}_q)
        cPoints = getCornerPoints(\Delta, \boldsymbol{x}_q, N)
3:
       for x_i in CornerPoints do
4:
5:
            if x_i is a flipping point then
                CounterClass = getCounterClass(cPoints)
 6:
 7:
                y_q' = MBRR(y_q, \epsilon, CounterClass)
 8:
                Cache(\boldsymbol{x}_q, y_q')
9:
               return y'_q
10:
       return y_q
11: else
       return getCached(x_q)
12:
```

boundary randomized response algorithm  $A(y_q)$  perturbs  $y_q$  by the following:

$$A(y_q) = \begin{cases} y_q, & \text{w.p.} \quad \frac{1}{2} + \frac{\sqrt{e^{2\epsilon} - 1}}{2 + 2e^{\epsilon}} \\ \\ \overline{y_q}, & \text{w.p.} \quad \frac{1}{2} - \frac{\sqrt{e^{2\epsilon} - 1}}{2 + 2e^{\epsilon}} \end{cases}$$

**Theorem 3.** The multiclass boundary randomized response algorithm  $A(y_q)$  provides  $\epsilon$ -BDP to each binary decision boundary  $f_{u,v}$  in the model.

The proof is similar to that of Theorem 2 and is thus omitted.

# 4.2.3 Summary for Multiclass Defense

Algorithm 2 summarizes the detailed procedures of BDP layer for a multiclass machine learning model f. Similar to the binary model, the layer directly returns a cached response to retain privacy guarantee if query  $x_q$  has been processed before. In addition to zone parameter  $\Delta$  and privacy budget  $\epsilon$ , the number of samples N to determine the counter class can be tuned between efficiency and accuracy. After the counter class is determined, the multiclass boundary randomized response algorithm  $MBRR(\cdot)$  with privacy budget  $\epsilon$  is invoked. The layer then returns the perturbed result  $y_q'$  and caches it for future use.

# 4.3 Zone-less Boundary Differentially Private Layer

In the previous section, the decision boundary is formulated as a zone with a hard margin controlled by  $\Delta$ — a query is either inside this zone (i.e., sensitive) or outside of it (i.e., non-sensitive).  $\epsilon\text{-BDP}$  can be achieved in the former case but no privacy is provided in the latter case. This makes the choice of  $\Delta$  a crucial and challenging task for the user—a small value leaves some decision boundary unprotected and yet a large value introduces unnecessary noise to non-boundary area where no protection is needed. To make  $\Delta$  less influential, in this section we propose a soft margin approach as an alternative to the hard margin.

# 4.3.1 Soft Query Sensitivity

The soft margin is essentially defined through the notion of soft query sensitivity, in which a query is no longer a hard "0" (non-sensitive) or "1" (sensitive). Instead, it is 1 on the soft margin and is larger than 1 when inside the margin. Then the degree of perturbation depends on the query sensitivity. The rationale behind a soft sensitivity of a query is three-folded. First, it must have a negative correlation with its distance to the nearest decision boundary, because query results reveal more information about the boundary and thus are more sensitive when they are closer to it. Second, how much the sensitivity depends on the distance should be controlled by the model owner. Third, the soft and hard sensitivity should be a unified notion. That is, the perturbation protocol for the hard sensitivity, Boundary Randomized Response (BRR), must still work with minimum adaptation. The following is a definition that satisfies all three rationales.

**Definition 8.** (Soft Query Sensitivity) Given a model f and a zone parameter  $\Delta$  chosen by the model owner, the sensitivity of a query  $x_q$  is a fractional function as:

$$s(\boldsymbol{x_q}) = \frac{\Delta}{dist(\boldsymbol{x_q}, f_{u,v})},$$
 (2)

where  $dist(x_q, f_{u,v})$  measures the distance between query  $x_q$  and the nearest decision boundary  $f_{u,v}$ .

To derive the distance without a closed form of the decision boundary, we can still adopt the flipping-corner-point method. The idea is to perform a binary search with an initial distance guess. If flipping corner points occur, the distance must be smaller, so we reduce the current guess to half and repeat the search; otherwise we double the guess. The final distance is obtained when a precision threshold or a maximum number of iterations is reached.

#### 4.3.2 Adaptive Boundary Randomized Response

Given the above definition of query sensitivity, the perturbation algorithm, Adaptive Boundary Randomized Response (ABRR), is exactly the same as BRR, except for the exponents. In BRR, the exponent is  $\epsilon$  which is implicitly  $\frac{\epsilon}{s(\boldsymbol{x_q})}$  where  $s(\boldsymbol{x_q})$  is always 1. ABRR uses the same formulae where  $s(\boldsymbol{x_q})$  is defined in Eqn. 2.

**Definition 9.** (Adaptive Boundary Randomized Response, ABRR) Given query sample  $x_q$  normalized to [-1,1], query sensitivity  $s(x_q)$ , its true response  $y_q$  and counter class  $\overline{y_q}$  ( $y_q$ ,  $\overline{y_q} \in \{u,v\}$ ), the adaptive boundary randomized response algorithm  $A(y_q)$  perturbs  $y_q$  by the following:

$$A(y_q) = \begin{cases} y_q, & \text{w.p.} \quad \frac{1}{2} + \frac{\sqrt{e^{2\psi} - 1}}{2 + 2e^{\psi}} \\ \\ \overline{y_q}, & \text{w.p.} \quad \frac{1}{2} - \frac{\sqrt{e^{2\psi} - 1}}{2 + 2e^{\psi}} \end{cases}$$

where  $\psi = \frac{\epsilon}{s(\boldsymbol{x_q})}$ .

**Theorem 4.** The adaptive boundary randomized response algorithm  $A(y_q)$  satisfies  $\psi$ -BDP to each binary decision boundary  $f_{u,v}$ , where  $\psi \leq \frac{2}{\Delta} \epsilon$ .

*Proof:* We assume  $p_1, p_2$  are the probabilities of retaining true responses for two queries  $x_1, x_2$ . According to ABRR, for

Input:

any two responses  $y_1,y_2\in\{u,v\}$ , the four possible cases to derive the BDP inequality are:

$$\frac{Pr[y_1 = y_2 | A(y_1) = u, A(y_2) = u]}{Pr[y_1 \neq y_2 | A(y_1) = u, A(y_2) = u]}, or$$

$$\frac{Pr[y_1 = y_2 | A(y_1) = v, A(y_2) = v]}{Pr[y_1 \neq y_2 | A(y_1) = v, A(y_2) = v]} = \frac{p_1 p_2 + (1 - p_1)(1 - p_2)}{p_1(1 - p_2) + p_2(1 - p_1)}$$

and

$$\begin{split} \frac{Pr[y_1 = y_2|A(y_1) = u, A(y_2) = v]}{Pr[y_1 \neq y_2|A(y_1) = u, A(y_2) = v]}, or \\ \frac{Pr[y_1 = y_2|A(y_1) = v, A(y_2) = u]}{Pr[y_1 \neq y_2|A(y_1) = v, A(y_2) = u]} = \frac{p_1(1 - p_2) + p_2(1 - p_1)}{p_1p_2 + (1 - p_1)(1 - p_2)}. \end{split}$$

Since  $p_1, p_2 \in [\frac{1}{2}, 1)$ , the partial derivatives to  $p_1$  and  $p_2$  of the right-hand side term in the former two cases are

$$\frac{\partial \frac{p_1 p_2 + (1 - p_1)(1 - p_2)}{p_1(1 - p_2) + p_2(1 - p_1)}}{\partial p_1} = \frac{2p_1 - 1}{(-2p_1 p_2 + p_1 + p_2)^2} \ge 0,$$

$$\frac{\partial \frac{p_1p_2 + (1 - p_1)(1 - p_2)}{p_1(1 - p_2) + p_2(1 - p_1)}}{\partial p_2} = \frac{2p_2 - 1}{(-2p_1p_2 + p_1 + p_2)^2} \ge 0.$$

As such, the right-hand side term in the former two cases are monotonically increasing. Similarly, that term in the latter two cases are monotonically decreasing. Let  $p_{max} = \max\{p_1, p_2\}$ . Then the two terms are bounded as follows.

$$\frac{p_1 p_2 + (1 - p_1)(1 - p_2)}{p_1 (1 - p_2) + p_2 (1 - p_1)} \le \frac{p_{max}^2 + (1 - p_{max})^2}{2p_{max} (1 - p_{max})}, \quad (3)$$

$$\frac{p_1(1-p_2)+p_2(1-p_1)}{p_1p_2+(1-p_1)(1-p_2)} \le 1.$$
(4)

Furthermore, since  $p_1, p_2 \in [\frac{1}{2}, 1)$ , the right-hand side term of Eqn. 3 also serves as the upper bound of the right-hand side term of Eqn. 4. That is,

$$\frac{p_{max}^2 + (1 - p_{max})^2}{2p_{max}(1 - p_{max})} \ge 1.$$

According to ABRR, we can derive  $p_{max}$  as

$$p_{max} = \frac{\sqrt{e^{2\psi_{max}} - 1}}{2 + 2e^{\psi_{max}}}.$$

By replacing  $p_{max}$  in the right term of Eqn. 3 with it, we have

$$\frac{{p_{max}}^2 + (1 - p_{max})^2}{2p_{max}(1 - p_{max})} = e^{\psi_{max}}.$$

Finally, since the sensitivity  $s(x_q)$  is in the range  $[\frac{2}{\Delta}, +\infty]$ , we derive the bound of  $\psi_{max}$  as

$$\psi_{max} \le \frac{\epsilon}{s(\boldsymbol{x}_{\boldsymbol{\alpha}})} \le \frac{\Delta}{2} \epsilon.$$
(5)

Due to the monotonicity of exponential function, we have

$$e^{\psi_{max}} < e^{\frac{2}{\Delta}\epsilon}$$
 (6)

Therefore, for any two queries, the algorithm satisfies  $\psi$ -BDP where  $\psi \leq \frac{2}{\Delta}\epsilon$ .

Notably, for queries inside the margin of  $\Delta$ , sensitivity is equal or greater than 1. As a result, we can prove that a minimum of  $\epsilon$ -BDP is always achieved, same as the requirement

**Algorithm 3** Zone-less Boundary Differentially Private Layer for Multiclass Model

Query  $x_a \in R^d$ 

```
Model f
Soft Margin \Delta
Boundary Privacy Budget \epsilon

Output: Perturbed Response y_q'

Procedure:

1: if x_q is not cached then

2: y_q = f(x_q)
3: s_q = getSensitivity(f, x_q, \Delta)
4: CounterClass = getCounterClass(f, x_q)
5: y_q' = ABRR(y_q, \epsilon, s_q, CounterClass)
6: Cache(x_q, y_q')
7: return y_q'
8: else
9: return getCached(x_q)
```

for boundary-sensitive zone in BDPL. In other words, ABRR essentially provides stronger non-uniform  $\epsilon\text{-BDP}$  than BRR in  $\Delta$  boundary-sensitive zone. We summarize this property as follows.

Corollary 4.1. For any query  $x_q$  inside the margin of  $\Delta$ , the adaptive boundary randomized response algorithm  $A(y_q)$  satisfies  $\psi$ -BDP where  $\psi \leq \epsilon$ 

*Proof:* Since query  $x_q$  now has  $s(x_q)$  in the range  $[1, +\infty]$ , we can prove the following by Eqn. 5 and 6.

$$e^{\psi_{max}} \le e^{\epsilon}$$

# 4.3.3 Summary for Zone-less Defense

Algorithm 3 summarizes the detailed procedures of zoneless BDP layer with soft margin. Caching policy is still carried out for any historical query. If  $x_q$  is a new query, the sensitivity of  $x_q$  to nearest decision boundary is first measured using a binary search with the corner-point technique. Then the counter class is calculated for a multiclass model. Finally, the adaptive boundary randomized response algorithm  $ABRR(\cdot)$  is invoked to perform perturbation with BDP protection.

# **5** EXPERIMENTS

In this section, we evaluate the effectiveness of boundary differentially private layer (BDPL) against model extraction attacks. Specifically, we implement those motivating extraction attacks using fine-tuned queries as in [1], [5] and compare the success rates of these attacks with and without BDPL.

#### 5.1 Setup

# 5.1.1 Datasets and Machine Learning Models

We evaluate three datasets and two models used in the literature [1] — a Botany dataset Mushrooms (113 attributes, 8124 records), a census dataset Adult (109 attributes, 48842 records) and a general social survey dataset GSS (101 attributes, 16127 records). The former two datasets are obtained from UCI machine learning repository [12] while the last one is from NORC [13]. All categorical items are

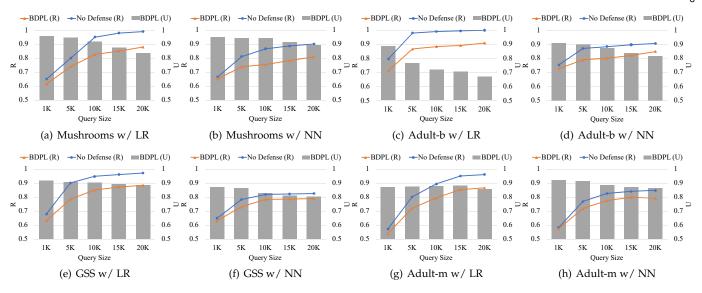


Fig. 3. Overall Protection Effect by BDPL: Extraction Rate and Utility

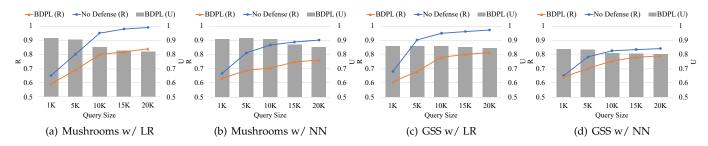


Fig. 4. Overall Protection Effect by Zone-less BDPL: Extraction Rate and Utility

processed by *one-hot-encoding* [14] and missing values are replaced with the mean value of this attribute. We adopt *min-max normalization* to unify all feature domains into [-1,1]. Data augmentation is not used for all the experiments, in accordance with the configuration of the original attacks.

For the evaluation of binary defense, *Mushrooms* dataset is trained on the label that shows whether a mushroom is poisonous or edible, and *Adult* dataset is trained on the label that shows whether the annual income of an adult exceeds 50K (Adult-b). As for multiclass defense, *GSS* dataset is used to train a model to predict level of happiness while *Adult* dataset is used to predict the race of the participants (Adult-m).

We train both a linear model, namely, logistic regression (LR), and a non-linear model, namely, 3-layer neural network (NN), to predict unknown labels on the above datasets. Logistic regression is implemented using *crossentropy* loss with L2 regularizer. Neural network is implemented using TensorFlow r1.12 [15]. The hidden layer contains 20 neurons with tanh activation. The output layer is implemented with a sigmoid function for binary prediction and a softmax function for multiclass prediction.

#### 5.1.2 Attack and Evaluation Metrics

We implement the extraction attack defined in Section 2 using original attack code of line-search technique in [1]. Specifically, the attacker first creates a seed set of pairwise queries with opposite or different response labels, and then searches for new samples that lie on the line segment

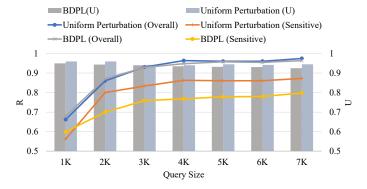


Fig. 5. BDPL vs. Uniform Perturbation

connecting this pair to approach the decision boundary. This process is repeated until either a searching threshold or query limit is reached. The size of a seed set is 4 and the searching threshold is 0.05. It is a white-box attack which produces an extracted model f' with the same hyperparameters and architectures as the original model f. To compare f and f', we adopt extraction rate [1], [6] to measure the proportion of matching predictions (i.e., both f and f' predict the same label) in an evaluation query set. Formally,

• Extraction Rate (R). Given an evaluation query set  $\mathcal{X}_e$ , the extraction rate

$$R = \frac{1}{|\mathcal{X}_e|} \sum_{\boldsymbol{x}_i \in \mathcal{X}_e} \mathbb{1}(f(\boldsymbol{x}_i) = f'(\boldsymbol{x}_i)),$$

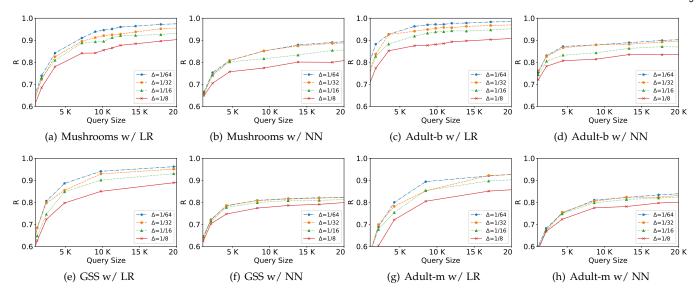


Fig. 6. Impact of Varying  $\Delta$  in BDPL with  $\epsilon$  = 0.01

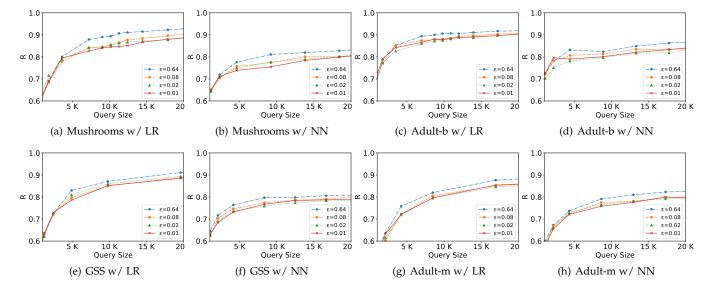


Fig. 7. Impact of Varying  $\epsilon$  in BDPL with  $\Delta$  = 1/8

where  $\mathbb{1}(\cdot)$  is an indicator function that outputs 1 if the input condition holds and 0 otherwise. The extraction rate essentially measures the similarity of model outputs given the same inputs.

• Utility (U). This second metric is evaluated on the test data points and measures the proportion of responses that are perturbed (i.e., flipped) by BDPL. It indicates how useful these responses are from a normal user's perspective. Formally, given the entire queries  $\mathcal{X}_q$  issued from test set by clients, and the set of (perturbed) responses  $\mathcal{Y}_q$  from the service provider,

$$U = \frac{1}{|\mathcal{X}_q|} \sum_{\boldsymbol{x}_i \in \mathcal{X}_q, y_i \in \mathcal{Y}_q} \mathbb{1}(f(\boldsymbol{x}_i) = y_i).$$

We follow the same evaluation setting as the original works. In the classic attack (Tramer's [1]) of Section 5.2-5.5, training set sample are used for the construction of victim model, whereas test set samples and uniformly sampled

points are applied in the evaluation of utility and extraction rate respectively. In the advanced attack (ActiveThief [8]) of Section 5.6, the configuration is similar, except that the extraction rate is evaluated against test set samples.

# 5.2 Overall Evaluation

To evaluate how well the decision boundary can be protected by our solution, we launch extraction attacks on a number of model/dataset combinations and plot the extraction rate R of sensitive queries in Figs. 3 and 4 in terms of the number of queries.

**Evaluation of BDPL.** In this experiment, we set  $\Delta=1/8$ , and  $\epsilon=0.01$  for all models. As shown in Fig. 3, except for the initial extraction stage (query size less than 5K), BDPL exhibits a significant protection effect for all 8 combinations — up to 12% drop on R — compared with no defense. The drops in  $GSS\ w/NN$  and Adult- $m\ w/NN$  are smaller (around 5%-6%) because these two models are the most compli-

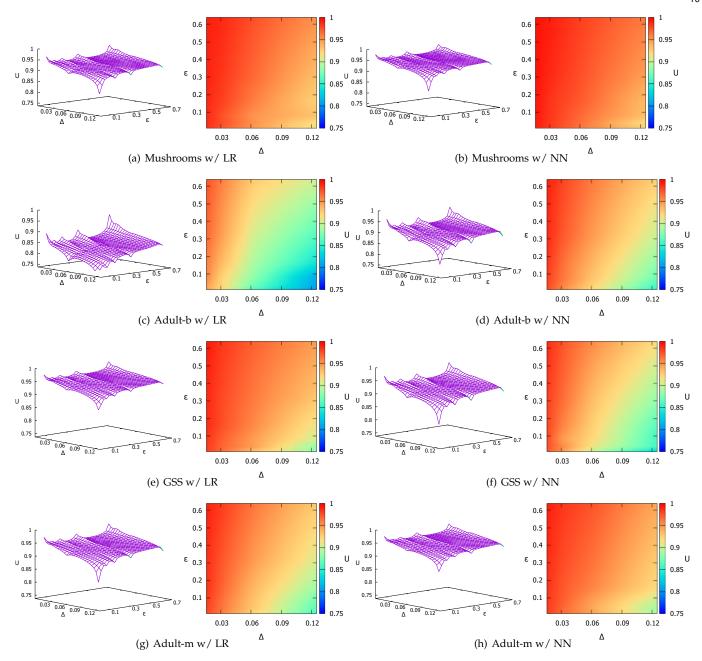


Fig. 8. Utility vs.  $\Delta$  and  $\epsilon$  in BDPL

cated (multiclass neural networks) and the least vulnerable to label perturbation.

The secondary axis of Fig. 3 also plots the utility of BDPL using bar chart. We observe that the utility saturates at over 80% after 20K queries in all combinations (among which 4 can achieve nearly 90% utility) except for *Adult w/LR*. This model has the fewest parameters and feature inputs, so BDPL has to perturb more sensitive queries to retain the same BDP level as the others. The impact on utility by  $\Delta$  and  $\epsilon$  will be further discussed in Section 5.4.

It is noteworthy that 1% reduction of extraction rate is more significant in later attack stage than earlier stage where the attackers need to increase the amounts of queries tremendously. For example, in Fig.6(e), the adversary spends 15K queries to improve extraction rate from 90% to 97%, which is canceled off by BDPL using after 15K

queries with only 11% utility loss. Obviously, 7% drop on extraction rate is more significant than an 11% utility loss because the former costs 15K queries whereas the latter costs only about 1.5K queries.

Evaluation of Zone-less BDPL. In this experiment, we set  $\Delta=1/8$  and increase  $\epsilon$  to 0.16 so that the overall utility is similar to the previous experiment (i.e., over 80%). The experiment results on Mushrooms and GSS are plotted in Fig. 4. Zone-less BDPL provides even better protection in all 4 combinations than BDPL, particularly at the initial stage (query size less than 5K) with a much lower extraction rate. Furthermore, all extraction rates saturate even earlier (after 10k of queries) than BDPL. Overall, we observe that zone-less BDPL performs particularly well with logistic regression models, where we witness an extra drop of 4% on R compared with BDPL. The impact on R and U by  $\Delta$ 

and  $\epsilon$  will be shown in Section 5.5.

#### 5.3 BDPL vs. Uniform Perturbation

In this experiment, we compare BDPL on binary model (single decision boundary) with a uniform perturbation mechanism that randomly flips the response label by a certain probability, whether the query is sensitive or not. To have a fair comparison, we use trial-and-error<sup>5</sup> to find this probability so that the overall extraction rates of both mechanisms are almost the same. In Fig. 5, we plot the extraction rates of both mechanisms for Mushrooms w/LR with  $\Delta = 1/8$  and  $\epsilon = 0.01$ . We observe that BDPL outperforms uniform perturbation by 5%-7% extraction rate, which is very significant as this leads to an increase of misclassification rate by 30%-50%. As such, we can conclude that BDPL is very effective in protecting the decision boundary by differentiating sensitive queries from non-sensitive ones, and therefore it retains high utility for query samples that are faraway from the boundary.

# 5.4 Impact of $\epsilon$ and $\Delta$ in BDPL

In this subsection, we evaluate BDPL performance with respect to zone parameter  $\Delta$  and privacy budget  $\epsilon$ . In Fig. 6, we fix  $\epsilon$  and vary  $\Delta$  from 1/64 to 1/8 for all 8 model/dataset combinations. In Fig. 7, we fix  $\Delta$  and vary  $\epsilon$  from 0.01 to 0.64 for all 8 model/dataset combinations.

Impact on Extraction Rate When  $\Delta$  increases from 1/64 to 1/8, the extraction rate is significantly reduced in both logistic regression (up to 12% drop) and neural network (up to 10% drop). Nonetheless, for neural networks, the extract rate does not change much when  $\Delta$  increases from 1/64 to 1/32, which indicates that if the boundary-sensitive zone is too small, BDPL may not provide effective protection, especially when the decision boundary is non-linear. As for privacy budget  $\epsilon$ , its impact is not as significant as  $\Delta$ . We only observe up to 4% drop of extraction rate when  $\epsilon$  decreases from 0.64 to 0.01 for all 8 model/dataset combinations.

Last but not the least, the extraction rates under all these settings saturate as the query size increases. In most cases, they start to saturate before 5K queries, and in the worst case, they saturate at 15K or 20K. This indicates that BDPL imposes a theoretical upper bound on the extraction rate no matter how many queries are issued.

Impact on Utility In this part, we evaluate BDPL performance regarding utility under similar varying settings. In Fig. 8, we plot the final utility with respect to  $\Delta$  and  $\epsilon$  after 20K queries for all model/dataset combinations. Except for Adult w/LR, all utilities are higher than 80% and most of them are above 90%, which means that BDPL does not severely sacrifice the accuracy of a machine learning service. As expected, the utility reaches peak when  $\Delta=1/64$  (the smallest zone size) and  $\epsilon=0.64$  (the least probability of perturbation). Furthermore, as is coincided with the extraction rate, the utility is more sensitive to  $\Delta$  than to  $\epsilon$ . For example, an increase of  $\Delta$  from 0.01 to 0.1 leads to a drop of utility by

5. To do this, we start with 1 random flip out of all responses and measure its overall extraction rate. We then repeatedly increment this number by 1 until the overall extraction rate is very close to that of BDPL.

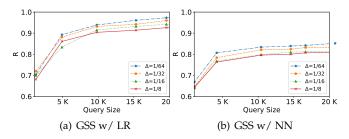


Fig. 9. Impact of Varying  $\Delta$  in Zone-less BDPL

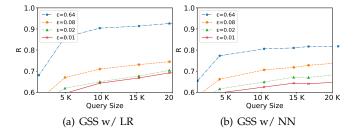


Fig. 10. Impact of Varying  $\epsilon$  in Zone-less BDPL

10%, whereas a decrease of  $\epsilon$  from 0.1 to 0.01 leads to only 5% drop.

To conclude, BDPL permanently protects decision boundary of both linear and non-linear models with moderate utility loss. The changes of  $\Delta$  and  $\epsilon$  (particularly the former) have modest impact on the extraction rate and utility.

# 5.5 Impact of $\epsilon$ and $\Delta$ in Zone-less BDPL

In this subsection, we evaluate zone-less BDPL performance with respect to  $\Delta$  and  $\epsilon.$  In Fig. 9, we fix  $\epsilon$  and vary  $\Delta$  from 1/64 to 1/8. In Fig. 10, we fix  $\Delta$  and vary  $\epsilon$  from 0.01 to 0.64. Due to space limitation, we only plot the results on 2 dataset/model combinations, i.e., GSS w/ LR and GSS w/ NN.

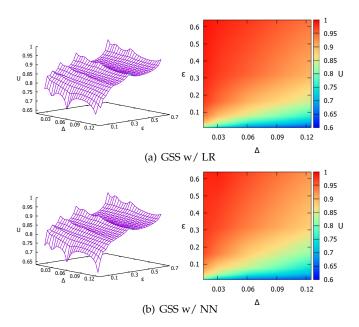
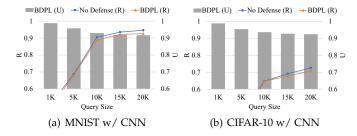


Fig. 11. Utility vs.  $\Delta$  and  $\epsilon$  in Zone-less BDPL

TABLE 1							
Recent Advances in Model Extraction Attacks							

Features		Attacks				
		Papernot et al. [3]	Juuti et al. [7]	Orekondy et al. [9]	Yu et al. [16]	Pal et al. [8]
Victim Model	CNN	✓	✓	✓	✓	✓
	RNN					✓
Adversary Knowledge	Problem Data	✓	✓			
	Non-problem Data			✓		
	Non-problem Data without Labels				✓	✓
Main Query Strategy	Passive Query			<b>√</b>	✓	✓
	Reinforce-based Probing			✓		
	Adversarial-based Probing	√	√		✓	✓
API Output Level	Probability		<b>√</b>	<b>√</b>	<b>√</b>	<b>√</b>
	Label	✓	✓		✓	✓



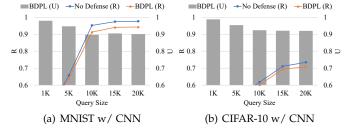


Fig. 12. Overall Protection Effect by BDPL on ATRS: Extraction Rate and Utility

Fig. 13. Overall Protection Effect by BDPL on ATHS: Extraction Rate and Utility

Impact on Extraction Rate. Both parameters maintain effectiveness in protecting decision boundary and saturating the extraction rate. Particularly, compared to the hard margin solution, when  $\epsilon$  decreases from 0.64 to 0.01, zoneless BDPL draws significant drop over extraction rate (up to 25% drop in logistic regression and 15% in neural network). This coincides with Corollary 4.1 in Section 4.3 that zoneless BDPL achieves better  $\epsilon\text{-BDP}$  protection than BDPL. Meanwhile, varying zone parameter  $\Delta$  has less eminent effect than in the hard margin case. This coincides with our zone-less design to protect the decision boundary with a soft margin.

Impact on Utility. We evaluate zone-less BDPL in terms of utility after 20K queries. In Fig. 11, we observe that the change of  $\epsilon$  leads to 35% change of utility while the change of  $\Delta$  only leads to 10%. This can be explained by the fact that zone-less BDPL adopts ABRR which obfuscates results in the global feature space . In addition, utility is still independent of model types and remains over 80% when  $\epsilon$  is greater than 0.15. As expected, utility reaches peak when  $\Delta=1/64$  (when soft margin is the most concentrated near a decision boundary) and  $\epsilon=0.64$  (the least probability of perturbation).

To conclude, zone-less BDPL provides strong protection for decision boundary in the global feature space. Privacy budget  $\epsilon$  brings more control over the extraction rate than  $\Delta$ .

# 5.6 Evaluation of BDPL on Advanced Attack

Recent study has shown that the extraction attacks are becoming threatening on complex models such as convolutional neural network. In this subsection, we turn to these emerging attacks which substantially scale both the input dimensions and model complexity. We expect these to be bigger challenges for BDPL as these attacks allow attackers to draw natural data as query from the same domain such as images.

In Table.1, we review those high-quality extraction attacks on complex model from peer-reviewed papers. To precisely address the feature of recent attacks, we list out whether the attacks support two of the most predominant advanced models, i.e., convolutional neural network (CNN) and recurrent neural network (RNN). Adversary knowledge is leveraged to illustrate adversary capability on data acquisition, specifically whether they can access any problem domain dataset. They are divided into three levels with an increasingly stringent requirement on dataset knowledge. We also categorize query strategy based on the nature of query such as reinforce-based probing (e.g., reinforce learning) and adversarial-based probing (e.g., adversarial samples). Detailed techniques can be found in the related works.

Due to space limitation, we select state-of-the-art ActiveThief [8] as the attack scheme for evaluation because it is the most recent and advanced attack. Activethief is a model extraction framework for neural networks using non-problem domain datasets and pool-based active learning strategies. We adopt the same configuration of the original paper and implement it on a convolutional neural network with various image datasets.

**Datasets and Machine Learning Models**. We evaluate two datasets for training victim models — a hand-written digits image dataset MNIST (28 \* 28 resolution, 1 channel, 60k records) and a colorful general objects dataset CIFAR10

(32\*32 resolution, 3 channels, 60k records). The two datasets are obtained from their official repository respectively [17] [18]. Compared to previous evaluation, feature size has scaled to 7x and 30x respectively. As for adversary query database, we use the downsampled and unannotated subset of the ILSVRC2012-14 dataset from ImageNet [19]. In each experiment, images from ImageNet are resized to fit the input size of the victim model.

With regards to the model architecture, we adopt the same CNN design in [8] for evaluation, which has 3 blocks of convolution. In each block, there are 2 repeated units of 2 convolution layers using a  $3 \times 3$  kernel, followed by 1 pooling layer using  $2 \times 2$  kernel. The stride length is 1 and 2 for convolution kernel and pooling kernel respectively. ReLU is adopted as the activation function for each convolution layer. The last pooling layer is attached to a fully connected layer which produces a final prediction using softmax function.

Attack and Evaluation Metrics. The attack is performed as follows. A random subset of initial seed images are selected from the adversary database. Then the attacker queries these images against the victim model and obtains a set of responses. A basic replica model is developed by training on these query-response pairs. The attacker then queries the remaining images using a designated strategy.

We evaluate one non-probing and one probing strategy regarding decision boundary. The non-probing strategy is ActiveThief Random Strategy (ATRS) where a subset of images are selected uniformly at random. The probing strategy is ActiveThief Hybrid Strategy (ATHS) where k-center and DeeplFool are combined for subset selection and it is the strongest attack in ActiveThief. This process is repeated for a fixed number of iterations. In each iteration, the replica model is retrained from all accumulated query-response pairs. Strategy hyperparameters such as number of seed samples and iteration numbers are the same in [8]. Previous evaluation metrics are adopted, that is, extraction rate R and utility U. To be consistent with original attack, the extraction rate is evaluated against test set samples in the following experiments.

#### 5.6.1 Effectiveness of BDPL on Advanced Attack

We launch two extraction attacks (ATRS and ATHS) on 2 models and plot the extraction rate R in Figs. 12 and 13 in terms of the budget of queries. BDPL parameters  $\Delta=1/7$  and  $\epsilon=0.01$  are set for all models in this experiment.

Effectiveness on ATRS. Fig. 12 presents the evaluation results on the non-probing attack. Despite the significant growth of attack complexity, our defense still draws a 1.5% drop over extraction rate on both models. The decrease is small because BDPL is focused on queries neighboring decision boundary whereas ATRS draws queries uniformly from normal image distribution and ratios of sensitive queries may be low. The mismatch leads to smaller perturbation as expected. Nevertheless, BDPL still maintains a decreased and saturated upper bound for model extraction.

From the secondary axis of Fig. 12, we observe that the utility trend stabilizes at over 90% after 20K queries for both models. Particularly, the utility remains over 95% before 10K for CIFAR-10 w/CNN. This is consistent with the small drop of extraction rate given that perturbation is light.

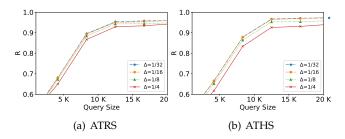


Fig. 14. Impact of Varying  $\Delta$  in BDPL on MNIST w/ CNN

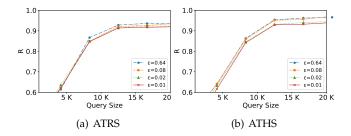


Fig. 15. Impact of Varying  $\epsilon$  in BDPL on MNIST w/ CNN

The impact on utility by  $\Delta$  and  $\epsilon$  will be further discussed in Section 5.6.2.

Effectiveness on ATHS. Fig. 13 presents the evaluation results on the probing attack. Notably, the extraction is reduced by 4% and 2.7% respectively for two models, which is more significant compared to that in ATRS. This corresponds to the nature of probing strategy that leverages k-center (diversifying classes) and DeepFool (approaching decision boundary). BDPL demonstrates stronger capability against such attack. The drop of extraction rate is smaller in CIFAR-10 w/ CNN given that it has great complexity (over 3000 input dimensions) and low risk in current extraction attack.

As for the utility trend, both models are saturated at over 90%. The perturbation stays obviously light which inhibits further drop of extraction rate. We conjecture that current defense is not entirely on its optimal performance due to high-dimensionality and great model complexity. Section 6 will further identify the limitations of BDPL and areas of improvement.

#### 5.6.2 Impact of $\epsilon$ and $\Delta$ on Complex Model

In this subsection, we evaluate BDPL performance with respect to zone parameter  $\Delta$  and privacy budget  $\epsilon$ . In Fig. 14, we fix  $\epsilon$  and vary  $\Delta$  from 1/32 to 1/4 in BDPL. In Fig. 15, we fix  $\Delta$  and vary  $\epsilon$  from 0.01 to 0.64 in BDPL. We mainly plot the results on *MNIST w/ CNN* under non-probing and probing attacks.

The extraction rate is reduced more significantly on ATHS when  $\Delta$  increases from 1/32 to 1/4. As coincided with the design of BDPL, it has bigger impact on probing strategy. Moreover, the drop of extraction rate is obviously greater on  $\Delta=1/4$  than the other 3 parameters, which indicates that a large  $\Delta$  is required for effective defense on the current model. As for  $\epsilon$ , when decreasing to 0.01, the change is less significant compared to that in  $\Delta$ . This indicates that complex model is less sensitive against the change of perturbation (privacy budget). Overall, the extraction rates

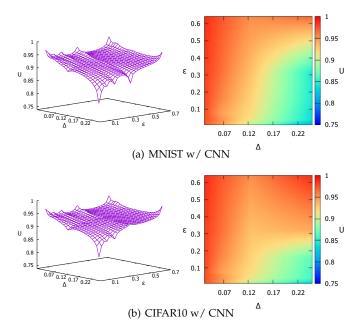


Fig. 16. Utility vs.  $\Delta$  and  $\epsilon$  On Complex Models

are consistently bounded and saturate as the query size increases.  $\epsilon$ -BDPL still imposes a theoretical upper bound on the extraction rate and prevent full extraction. Furthermore, BDPL displays flexible control over the extraction in probing strategy.

We also evaluate the utility distribution on the same varying settings on both models. The results are reported in Fig.16 after 20K. The utility is saturated at over 80% under both models, which shows that BDPL has not severely degraded service quality given the significant growth of model complexity. Consistently, the peak utility is achieved when  $\Delta=1/32$  (the smallest zone size) and  $\epsilon=0.64$  (the least probability of perturbation).

To conclude, BDPL alleviates the threat of extraction rate in spite of the significant growth of victim complexity and strong attack. The change of  $\Delta$  takes dominant control over the extraction rate. We will discuss the limitations and improvements in the following section.

# 6 Discussion

In this section, we review the evaluation of BDPL and identify areas of future improvement.

First, for attacks that only support probability-level extraction, our BDPL, which is at label-level, cannot protect against it. On the other hand, we'd argue that BDPL can still protect against attacks extraction using natural data. First, normal and natural data can also be close to decision boundary, although the ratio of sensitive queries in these attacks is lower than that in the fine-tuned ones. Second, the optimal strategy in recent studies [7], [8] leverages adversarial techniques which implicitly perform fine-tuned probing on the decision boundary. This means our BDPL can effectively protect against them, as indicated in our new experimental results against [8] in Section 5.6. Nonetheless, extraction using natural data is limited on specific model types, such as images and text, where same domain data can be easily obtained and used as query set. If the victim

is a genetic model, it would be difficult to perform such extraction since genetic data is usually proprietary.

Second, we identify two core components that can be further improved for better performance in complex model. One is distance metrics and the other is perturbation mechanism.

**Distance Metrics.** In the current version, we adopt flipping-corner-point technique on two essential assumptions: modular design and generality. The first one allows plug-in feature for practical deployment, where service providers can tap in BDPL with the same API for users. The second one ensures compatibility with both parametric (e.g. neural network) and non-parametric models (e.g., decision tree). Unlike the classical models, deep neural networks such as image models may have high model complexity and thousands of input dimensions. Our intuition for flipping-corner technique, which comes from the *L*-norm ball, may have unexpected behavior in such space [20].

To improve the scalability, we can start by relaxing the first assumption and access the internals of provider's model. As such, a straightforward remedy becomes feasible by performing flipping-corner detection in the middle part of model, such as a bottleneck layer [21]. The dimensionality is greatly reduced after the intermediate representation compared to the raw input. Apart from the high-dimensionality, the manifold assumption discussed in adversarial robustness of complex model may also render flipping-corner-point technique unstable. Corner points may not flip properly when samples fall out of the manifold. As a result, L-norm distance metrics may degrade the accuracy of detection process. By relaxing the second assumption, we can leverage the gradient from parametric model to propose gradient-based distance metrics, which is more suitable under the manifold assumption. We believe it is a viable and practical solution for future improvement as complex parametric models are prevailing in machine learning services.

**Perturbation Mechanism.** As motivated by binary defense, randomized response is adopted as the basic framework for multiclass defense. We perform a one-vs-one approximation for each class and treat multiclass defense as a combination of binary defense. This assumption may incur imbalanced noises since only one counter class is considered. Perturbation may be concentrated on specific pairs of classes. To resolve this, a natural framework capable of categorical-value perturbation, such as k-ary randomized response [22], can be adopted for multiclass defense. Furthermore, if we extend the mechanism to be numeric-value suitable, probability-level defense becomes feasible. Nonetheless, these existing mechanisms will still need significant adaptation to satisfy  $\epsilon$ -BDP before applying it to our defense. We plan to implement them in future work.

# 7 RELATED WORKS

There are three streams of related works, namely, machine learning model extraction, defense, and differential privacy.

**Model Extraction.** Machine-learning-as-a-service (MLaaS) has furnished model extraction attacks through the rich information available from prediction API. Tramer *et al.* [1] proposed extraction methods that leveraged

the confidence information in the API and managed to extract the full set of model parameters using equationsolving. Papernot [3] et al. introduced a Jacobian-based data augmentation technique to extend queries and to train a substitute DNN. Similarly, Juuti et al. [7] leveraged both optimal hyperparameters and Jacobian-based data augmentation to extract models under their generalized framework. Orekondy et al. [9] proposed a knockoff model to steal the functionality of an image classifier and developed reinforce-based query strategy using multiarmed bandits problem. Pal et al. [8] further improved the extraction attack on image model without annotated data. Their hybrid strategy combines greedy clustering with adversarial samples. Yu et al. [16] proposed new adversarial samples generation technquie named FeatureFool using feature-based optimization algorithm and performed model extraction on MLaaS platform. Besides extracting model parameters, Wang et al. [23] also extracted the hyperparamters of a fully trained model by utilizing the zero gradient technique. Oh et al. [24] developed a modelof-model to infer internal information of a neural network such as layer type and kernel sizes.

Model extraction without confidence is similar to *learning with membership query* [25], [26], which learns a concept through querying membership on an oracle. This technique has been exploited by Lowd *et al.* to extract binary classifiers [5]. They used line search to produce optimized queries for linear model extraction. This technique was extended by Tramer *et al.* [1] to non-linear models such as a polynomial kernel support vector machine. They adopted adaptive techniques such as active learning to synthesize fine-tuned queries and to approximate the decision boundary of a model. Pal *et al.* [8] further improved the extraction attack using only top-1 label on image model.

Model Extraction Defense. Confidence rounding and ensemble model were shown effective against equationsolving extractions in [1]. Lee et al. [27] proposed perturbations using the mechanism of reverse sigmoid to inject deceptive noises to output confidence, which preserved the validity of top and bottom rank labels. Kesarwani et al. [6] monitored user-server streams to evaluate the threat level of model extraction with two strategies based on entropy and compact model summaries. The former derived information gain with a decision tree while the latter measured feature coverage of the input space partitioned by source model, both of which were highly correlated to extraction level. Juuti et al. [7] adopted a different approach to monitor consecutive queries based on the uniqueness of extraction behavior. A warning would be generated when queries deviated from a benign distribution due to malicious probing. Quiring et al. [28] adopted the notion of closeness-to-theboundary in digital watermarking and applied it to protect against extraction attacks on decision trees. The defense strategy was devised from protection of watermark detector and it monitored the number of queries that fell into security

**Differential Privacy.** Differential privacy (DP) was first proposed by Dwork [10] to guarantee the privacy of a centralized dataset with standardized mathematical notation. Duchi *et al.* [29] extended this notation to local differential privacy (LDP) for distributed data sources. Randomized

response proposed by Warner *et al.* [11] is the baseline perturbation algorithm for LDP, which protects binary answers of individuals. Although differential privacy has not been used in model extraction and defense, it has been applied in several adversarial machine learning tasks. For example, Abadi *et al.* [30] introduced differentially private stochastic gradient descent to deep learning, which can preserve private information of the training set. Lee *et al.* [31] further improved its effectiveness using an adaptive privacy budget. Their approaches are shown effective against model inversion attack [2] or membership inference attack [32].

#### 8 Conclusion and Future Work

In this paper, we propose boundary differentially private layer to defend machine learning models against extraction attacks by obfuscating the query responses. This layer guarantees boundary differential privacy in a user-specified boundary-sensitive zone. To identify sensitive queries that fall in a zone, we develop an efficient approach that uses corner points as indicators. We design boundary randomized response as the building block for perturbation algorithm, followed by a generalization to multiclass model and an adaptive version that can protect a soft margin of decision boundary. We prove such perturbation algorithm satisfies  $\epsilon$ -BDP. Through extensive experimental results, we demonstrate the effectiveness and flexibility of our defense layer on protecting decision boundary while retaining high utility of the machine learning service.

For future work, we plan to propose defense more suitable for complex model and consider strong adversary with evasion. We also plan to extend our defense layer to protect against other machine learning attacks such as model evasion and inversion.

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