

Vedic Maths: Addition, Subtraction, Multiplication and Squares

Answers and Explanations

1	d	2	d	3	b	4	b	5	d	6	c	7	a	8	d	9	b	10	a
11	d	12	a	13	b	14	b	15	a	16	d	17	b	18	d	19	a	20	a
21	b	22	c	23	d	24	c	25	b	26	c	27	c	28	a	29	c	30	c
31	d	32	b	33	d	34	a	35	a	36	a	37	d	38	b	39	b	40	a
41	c	42	a	43	b	44	a	45	b	46	b	47	b	48	b	49	c	50	d
51	d	52	c	53	b	54	a	55	c	56	b	57	d	58	c	59	c	60	b

1. d 2. d 3. b 4. b

5. d 6. c

$$\begin{aligned}
 7. a \quad & \frac{1}{9} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{50} + \frac{1}{72} \\
 &= \frac{1}{9} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \dots + \frac{1}{8 \times 9} \\
 &= \frac{1}{9} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{8} - \frac{1}{9} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 8. d \quad & \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \frac{1}{8 \times 9} + \frac{1}{9 \times 10} + \frac{1}{10 \times 11} \\
 &= \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + \frac{1}{7} - \frac{1}{8} + \frac{1}{8} - \frac{1}{9} + \frac{1}{9} - \frac{1}{10} + \frac{1}{10} - \frac{1}{11} \\
 &= \frac{1}{5} - \frac{1}{11} = \frac{11-5}{55} = \frac{6}{55}.
 \end{aligned}$$

$$\begin{aligned}
 9. b \quad & \text{Expression} \\
 &= 0.\overline{63} + 0.\overline{37} + 0.\overline{80} \\
 &= \frac{63}{99} + \frac{37}{99} + \frac{80}{99} \\
 &= \frac{63+37+80}{99} = \frac{180}{99} \\
 &= 1\frac{81}{99} = 1.\overline{81}
 \end{aligned}$$

10. a 11. d 12. a 13. b 14. b

15. a 16. d 17. b 18. d

$$\begin{aligned}
 19. a \quad ? &= 5 - [4 - \{3 - (3 - 3 - 6)\}] \\
 &= 5 - [4 - \{3 - (-6)\}] \\
 &= 5 - [4 - \{3 - 6\}] \\
 &= 5 - [4 - 9] \\
 &= 5 + 5 = 10
 \end{aligned}$$

$$\begin{aligned}
 20. a \quad ? &= 1 - [5 - \{2 + (-1)2\}] \\
 &= 1 - [5 - \{2 - 2\}] \\
 &= 1 - [5 - 0] \\
 &= 1 - 5 = -4
 \end{aligned}$$

$$\begin{aligned}
 21. b \quad & 243 \times 658 \\
 & \text{Step 1 : } 8 \times 3 = 24
 \end{aligned}$$

$$\begin{array}{ccccc}
 & 4 & & 3 & \\
 & \swarrow & & \searrow & \\
 & 5 & & 8 & \\
 \text{Step 2:} & & & & = 4 \times 8 + 5 \times 3
 \end{array}$$

$$= 32 + 15 = 47$$

$$\begin{array}{ccccc}
 & 2 & & 3 & \\
 & \swarrow & & \searrow & \\
 & 6 & & 5 & \\
 \text{Step 3:} & & & & = 2 \times 8 + 6 \times 3 + 4 \times 5
 \end{array}$$

$$= 16 + 18 + 20 = 54$$

$$\begin{array}{ccccc}
 & 2 & & 4 & \\
 & \swarrow & & \searrow & \\
 & 6 & & 5 & \\
 \text{Step 4:} & & & & = 2 \times 5 + 4 \times 6 = 10 + 24 = 34
 \end{array}$$

$$\text{Step 5: } 2 \times 6 = 12$$

$$\text{Now the answer is } 12|3^4|5^4|4^7|2^4 = 159894$$

Though here we have first done each step and then at the end write the answer, each step has to be done to keep getting each digit of the answer (starting from the units place simultaneously).

22. c 96×108

Base = 100, Deficit = $96 - 100 = -4$,
Surplus = $108 - 100 = 8$

$$\begin{array}{r} 96 \\ \times 108 \\ \hline (108-4) | -32 \end{array}$$

or
 $(96+8)$

$$\Rightarrow 104 | -32 \Rightarrow 103|100 - 32 \Rightarrow 103|68 = 10368$$

Right part will now be $(-4) \times 8$ i.e. -32 . To take care of the negative we will borrow 1 from the left part, which is equivalent to borrowing 100 (because we are borrowing from the hundredth place digit of the answer). Thus, this part will be $100 - 32 = 68$.

23. d $126 = 2 \times 3^2 \times 7$

Since 2 and 7 are not in pairs.

\therefore Required number to be multiplied = $2 \times 7 = 14$.

24. c 112×113

Base = 100, Surplus = 12 and 13

$$\begin{array}{r} 112 \\ \times 113 \\ \hline (112+13) | 156 \end{array}$$

$$\Rightarrow 125 + 1 | 56 = 12656.$$

25. b 92×97

Base = 100, Deficit = $92 - 100 = -8$

$= -8$ and $97 - 100 = -3$

$$\begin{array}{r} 92 | -8 \\ \times 97 | -3 \\ \hline (92-3) | +24 \end{array}$$

$$\Rightarrow 89 | 24$$

$$= 8924.$$

26. c 81×73

$$= 8 \times 7 | 8 \times 3 + 7 \times 1 | 3 \times 1$$

$$\begin{array}{r} 56 \\ +3 \\ \hline 59 \end{array}$$

$$= 5913.$$

27. c $63 \times 72 = 6 \times 7 | 6 \times 2 + 3 \times 7 | 3 \times 2$

$$= 42 | 33 | 6$$

$$= 42 + 3 | 36$$

$$= 4536.$$

$$\begin{array}{r} 6 \\ 7 \\ \hline 3 \\ 2 \end{array}$$

28. a $524 \times 368 = 5 \times 3 | 5 \times 6 + 3 \times 2 | 5 \times 8 + 2 \times 6 + 4 \times 3$
 $| 2 \times 8 + 6 \times 4 | 8 \times 4$
 $= 15 | 36 | 64 | 40 | 32$

$$\begin{array}{r} 5 \\ 2 \\ 4 \\ \hline 3 \\ 6 \\ 8 \end{array}$$

$$\begin{array}{r} = 15 | 36 | 64 | 40 + 3 (= 43) | 2 \\ = 15 | 36 | 64 + 4 (= 68) | 3 | 2 \\ = 15 | 36 + 6 (= 42) | 8 | 3 | 2 \\ = 15 + 4 | 2 | 8 | 3 | 2 = 192832. \end{array}$$

29. c $88 \times 93 = 8 \times 9 | 8 \times 3 + 8 \times 9 | 8 \times 3$
 $= 72 | 96 | 24$

$$\begin{array}{r} 8 \\ 8 \\ \hline 9 \\ 3 \end{array}$$

$$\begin{array}{r} = 72 + 9 (= 81) | 8 | 4 \\ = 8184. \end{array}$$

30. c Using the base of 100, the answer is
 $(106 + 45) \times 100 + 6 \times 45 = 15100 + 270 = 15370.$

31. d $426 \times 543 = 4 \times 5 | 4 \times 4 + 5 \times 2 | 4 \times 3 + 2 \times 4 + 6 \times 5$
 $| 2 \times 3 + 6 \times 4 | 6 \times 3$
 $= 20 | 26 | 50 | 30 | 18$
 $= 20 | 26 | 50 | 30 + 1 (= 31) | 8$
 $= 20 | 26 | 50 + 3 (= 53) | 1 | 8$
 $= 20 | 26 + 5 (= 31) | 3 | 1 | 8$
 $= 20 + 3 | 1 | 3 | 1 | 8 = 231318.$

32. b $52.5 \times 125 = 52.5 \times \frac{1000}{8} = \frac{52500}{8} = 6562.5.$

33. d $197 \times 203 = (200 - 3)(200 + 3) = 200^2 - 3^2$
 $202 \times 198 = (200 - 2)(200 + 2) = 200^2 - 2^2$
 $\Rightarrow (197 \times 203) + (202 \times 198)$
 $= 200^2 - 3^2 + 200^2 - 2^2$
 $= 2 \times 40000 - 13 = 79987.$

34. a It is obvious that one needs to do multiplication for solving the given equation. But observing the equation we can see that 121 is a multiple of 11 and 111 is a multiple of 3 and, among the given options only option (a) satisfies this condition.

35. a $375 \times 24 \times 11 = 125 \times 3 \times 8 \times 11$
 $= 9 \times 11 \times 1000 = 99000.$

Note: If you get multiples of 5 and even numbers, try to maximize the number of trailing zeroes and reduce the calculation.

36. a Largest number of 4 digits will be 9999

$$(99)^2 < 9999 < (100)^2$$

Hence, the required number will be $(99)^2 = 9801$.

37. d Here, base = 100
 Therefore, $(108)^2 = (100 + 2 \times 8) | 8^2$
 $= 116 | 64 = 11664.$

38. b Here, base = 100
 Therefore, $(92)^2 = 100 + 2 \times (-8) | (-8)^2$
 $= 84 | 64$
 $= 8464.$

39. b Here, base = 1000
 Therefore,
 $(1008)^2 = 1000 + 2 \times 8 | (8)^2 = 1016 | 064 = 1016064.$

40. a Here, base = 1000
 Therefore, $(993)^2 = 1000 - 2 \times 7 | (-7)^2 = 986 | 049$
 $= 986049.$

41. c Here, base = 1000
 Therefore, $(1012)^2 = 1000 + 2 \times 12 | (12)^2$
 $= 1024 | 144 = 1024144.$

42. a $116^2 = (100 + 16)^2 = 100^2 + 2 \times 16 \times 100 + 16^2$
 $= 100(100 + 32) + 256$
 $= 13200 + 256 = 13456.$

43. b $94^2 = (100 - 6)^2 = 100^2 - 2 \times 6 \times 100 + 6^2$
 $= 100(100 - 12) + 36$
 $= 8800 + 36 = 8836.$

44. a $54^2 = (50 + 4)^2 = 50^2 + 2 \times 50 \times 4 + 4^2$
 $= 2500 + 400 + 16 = 100(25 + 4) + 16$
 $= 2916.$

Alternate method of using (base 50):
 $54^2 = 25 + 4|4^2 = 2916.$

45. b The square of the number of the form 'a5'
 $= a \times (a + 1) | 5^2$
 Hence, $75^2 = 7 \times (7 + 1) | 5^2 = 5625.$
Note: Number 'a' can have more than one digit as well.

46. b $3034 - (1002 \div 20.04)$
 $= 3034 - \frac{1002}{20.04}$
 $= 3034 - \frac{1002}{2004} \times 100$
 $= 3034 - 50 = 2984$

47. b $\left[3 \frac{1}{4} \div \left\{ 1 \frac{1}{4} - \frac{1}{2} \left(1 \frac{1}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right]$
 $= \frac{17}{4} \left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{3}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right]$
 $= \frac{17}{4} - \left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \times \frac{6}{6} \right\} \right]$
 $= \frac{17}{4} - \left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \right\} \right]$
 $= \frac{17}{4} - \left[\frac{13}{4} \div \left\{ \frac{5-2}{4} \right\} \right]$
 $= \frac{17}{4} - \left[\frac{13}{4} \div \frac{3}{4} \right]$
 $= \frac{17}{2} - \left[\frac{13}{4} \times \frac{4}{3} \right] = \frac{17}{2} - \frac{13}{3}$
 $= \frac{51-26}{6} = \frac{25}{6} = 4 \frac{1}{6}$

48. b $0.008 \times 0.01 \times 0.072 \div (0.12 \times 0.0004)$
 $= 0.008 \times 0.01 \times 0.072 \div (0.000048)$
 $= 0.008 \times 0.01 \times \frac{0.072}{0.000048}$
 $= \frac{0.00000576}{0.000048} = 0.12$

49. c Expression
 $= 25 - 5[2 + 3 \{2 - 2(5 - 3) + 5\} - 10] \div 4$
 $= 25 - 5[2 + 3 \{2 - 2 \times 2 + 5\} - 10] \div 4$
 $= 25 - 5[2 + 9 - 10] \div 4$
 $= 25 - 5 \div 4 = 25 - \frac{5}{4}$
 $= \frac{100-5}{4} = \frac{95}{4} = 23.75$

50. d We have
 $\frac{5}{3} \div \frac{2}{7} \times \frac{*}{7} = \frac{5}{4} \times \frac{2}{3} \times 6$
 $\Rightarrow \frac{5}{3} \times \frac{7}{2} \times \frac{*}{7} = \frac{5 \times 2 \times 6}{4 \times 3}$
 $\therefore * = \frac{5 \times 2 \times 6 \times 3 \times 2 \times 7}{5 \times 7 \times 4 \times 3} = 6$

51. d $3.\overline{36} - 2.\overline{05} + 1.\overline{33}$

$$= 3\frac{36}{99} - 2\frac{05}{99} + 1\frac{33}{99}$$

$$= 3 + \frac{36}{99} - 2 - \frac{5}{99} + 1 + \frac{33}{99}$$

$$= (3 - 2 + 1) + \left(\frac{36}{99} - \frac{5}{99} + \frac{33}{99} \right)$$

$$= 2 + \left(\frac{36 - 5 + 33}{99} \right)$$

$$= 2 + \frac{64}{99} = 2\frac{64}{99} = 2.\overline{64}$$

52. c $[0.9 - \{2.3 - 3.2 - (7.1 - 8.9)\}]$

$$= [0.9 - \{2.3 - 3.2 + 1.8\}]$$

$$= [0.9 - 0.9] = 0$$

53. b $\left(\frac{5}{2} + \frac{3}{2} \right) \left(\frac{25}{4} - \frac{15}{4} + \frac{9}{4} \right)$

$$= 4 \times \frac{19}{4} = 19$$

54. a Expression = $\frac{(0.04 + 0.01)}{(0.01 + 0.02)}$

55. c Expression

$$= \frac{1}{2} + \left\{ \frac{19}{4} - \left(\frac{19}{6} - \frac{7}{3} \right) \right\}$$

$$= \frac{1}{2} + \left\{ \frac{19}{4} - \left(\frac{19 - 14}{6} \right) \right\}$$

$$= \frac{1}{2} + \left\{ \frac{19}{4} - \frac{5}{6} \right\}$$

$$= \frac{1}{2} + \frac{19}{4} - \frac{5}{6}$$

$$= \frac{6 + 57 - 10}{12} = \frac{53}{12} = 4\frac{5}{12}$$

56. b Expression

$$= \frac{17}{15} \times \frac{17}{15} + \frac{2}{15} \times \frac{2}{15} - 2 \times \frac{17}{15} \times \frac{2}{15}$$

$$= \left(\frac{17}{15} - \frac{2}{15} \right)^2$$

$$= \left(\frac{17 - 2}{15} \right)^2 = \left(\frac{15}{15} \right)^2 = 1$$

57. d Let $4\frac{11}{15} = a$ and $\frac{15}{71} = b$.

\therefore Expression

$$= (a + b)^2 - (a - b)^2$$

$$= (a^2 + b^2 + 2ab) - (a^2 + b^2 - 2ab) = 4ab$$

$$= 4 \times 4\frac{11}{15} \times \frac{15}{71} = 4 \times \frac{71}{15} \times \frac{15}{71} = 4$$

58.c $\left(\sqrt{2} + \frac{1}{\sqrt{2}} \right)^2$

$$= 2 + \frac{1}{2} + 2 \times \sqrt{2} \times \frac{1}{\sqrt{2}} = 4\frac{1}{2}$$

59. c Expression

$$= 71 \times 29 + 27 \times 15 + 8 \times 4$$

$$= 2059 + 405 + 32 = 2496$$

60. b Expression

$$= 0.05 \times 5 - 0.005 \times 5$$

$$= 0.25 - 0.025 = 0.225$$

**Vedic Maths : Square Roots, Cubes,
Cube Roots and Reciprocals**

Answers and Explanations

1	b	2	a	3	c	4	c	5	d	6	d	7	a	8	a	9	a	10	b
11	b	12	a	13	a	14	b	15	c	16	b	17	a	18	b	19	b	20	a
21	b	22	d	23	b	24	d	25	d	26	b	27	a	28	b	29	b	30	a
31	a	32	b	33	c	34	a	35	c	36	d	37	b	38	b	39	a	40	b
41	c	42	c	43	d	44	b	45	b	46	c	47	a	48	c	49	d	50	b
51	b	52	a	53	b	54	b	55	d	56	d	57	d	58	c	59	a	60	c

1. b $\sqrt{2304} = \sqrt{2 \times 2 \times 3 \times 3}$
 $= \sqrt{2^8 \times 3^2} = 2^4 \times 3$
 $= 48.$

2. a $7 + 4\sqrt{3}$
 $= 3 + 4 + 2.2\sqrt{3}$
 $= (\sqrt{3})^2 + (2)^2 + 2.2.\sqrt{3}$
 $= (2 + \sqrt{3})^2 \quad \{ \because a^2 + b^2 + 2ab = (a + b)^2 \}$
 $\therefore \text{Square root of } 7 + 4\sqrt{3} = 2 + \sqrt{3}.$

3. c $\sqrt[6]{\frac{30}{289}}$
 $= \sqrt[6]{\frac{1764}{289}} = \sqrt[6]{\frac{2 \times 2 \times 3 \times 3 \times 7 \times 7}{17 \times 17}} = \sqrt[6]{\frac{2^2 \times 3^2 \times 7^2}{17^2}}$
 $= \frac{2 \times 3 \times 7}{17} = \frac{42}{17} = 2\frac{8}{17}.$

4. c $\sqrt{156.25} + \sqrt{0.81} - \sqrt{3.61}$
 $= \sqrt{\frac{15625}{100}} + \sqrt{\frac{81}{100}} - \sqrt{\frac{361}{100}} = \sqrt{\frac{5^6}{10^2}} + \sqrt{\frac{9^2}{10^2}} - \sqrt{\frac{19^2}{10^2}}$
 $= \frac{125}{10} + \frac{9}{10} - \frac{19}{10} = 12.5 + 0.9 - 1.9 = 11.5.$

5. d The square root of $0.0036 \times 10^{-4} = \sqrt{36 \times 10^{-8}}$
 $= 6 \times 10^{-4}.$

6. d Cube root of $\frac{250}{686}$ will be equal to $\sqrt[3]{\frac{250}{686}}$
 $= \sqrt[3]{\frac{125}{343}} = \sqrt[3]{\frac{5 \times 5 \times 5}{7 \times 7 \times 7}} = \sqrt[3]{\frac{5^3}{7^3}} = \frac{5}{7}.$

7. a The number of boys in a row
 $= \sqrt{20736} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$
 $= \sqrt{2^8 \times 3^4} = 2^4 \times 3^2 = 144.$

8. a The last term of the given sequence of infinite terms
 $= \sqrt{3+2\sqrt{2}} = \sqrt{(1+\sqrt{2})^2} = 1+\sqrt{2}$
 On being added with the previous term i.e. $2+\sqrt{2}$
 it again becomes $3+2\sqrt{2}$
 This cycle repeats itself and finally the answer will be $\sqrt{2}+1.$

9. a $\sqrt{147} \times \sqrt{432} = \sqrt{3 \times 7 \times 7} \times \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$
 $= 4 \times 7 \times 9 = 252.$

10. b $614656 = 2 \times 7 \times 7 \times 7 \times 7$
 Hence, $\sqrt{614656} = 2 \times 2 \times 2 \times 2 \times 7 \times 7 = 784.$

11. b $x = 144 + 25 = 169$
 $\Rightarrow x = 13 \times 13$
 $\therefore \sqrt{x} = 13.$

12. a $\sqrt[2]{\frac{239}{361}} = \sqrt[2]{\frac{961}{361}} = \sqrt[2]{\frac{31 \times 31}{19 \times 19}}$
 $= \frac{31}{19} = 1\frac{12}{19}.$

13. a Let $0.2 = a$ and $0.08 = 2b$

\therefore Expression

$$= \sqrt[3]{\frac{a \times a \times a + b \times b \times b}{2a \times 2a \times 2a + 2b \times 2b \times 2b}} \\ = \sqrt[3]{\frac{a^3 + b^3}{8(a^3 + b^3)}} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2} = 0.5$$

14. b $\sqrt[3]{0.014 \times 0.14x}$

$$= 0.014 \times 0.14 \sqrt[3]{y}$$

On squaring both sides.

$$0.014 \times 0.014x$$

$$= (0.014)^2 \times 0.14^2 \times y$$

$$\therefore \frac{x}{y} = 0.014 \times 0.14 \times 0.00196$$

15. c $\sqrt{13} + \sqrt{1300} + \sqrt{0.013}$

$$= \sqrt{\frac{130}{10}} + 10\sqrt{13} + \sqrt{\frac{130}{10000}} \\ = \frac{1}{10}\sqrt{130} + 10\sqrt{13} + \frac{1}{100}\sqrt{130} \\ = \frac{11.40}{10} + 3.605 \times 10 + \frac{11.40}{100} \\ = 1.140 + 36.05 - 0.1140 \\ = 37.304$$

16. b Let $0.03 = x \Rightarrow 0.003 = \frac{x}{10}$

$$0.21 = y \Rightarrow 0.021 = \frac{y}{10}$$

$$\text{and } 0.065 = z \Rightarrow 0.0065 = \frac{z}{10}$$

\therefore Expression

$$= \sqrt{\frac{x^2 + y^2 + z^2}{\left(\frac{x}{10}\right)^2 + \left(\frac{y}{10}\right)^2 + \left(\frac{z}{10}\right)^2}} \\ = \sqrt{\frac{100(x^2 + y^2 + z^2)}{(x^2 + y^2 + z^2)}} \\ = \sqrt{100} = 10$$

17. a $\sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{29 + 7}}}}$

$$= \sqrt{5 + \sqrt{11 + \sqrt{19 + 6}}}$$

$$= \sqrt{5 + \sqrt{11 + \sqrt{25}}}$$

$$= \sqrt{5 + \sqrt{11 + 5}} = \sqrt{5 + 4}$$

$$= \sqrt{9} = 3$$

18. b $\sqrt{18225} = 135$

$$\therefore \sqrt{182.25} = 13.5;$$

$$\sqrt{1.8225} = 1.35,$$

$$\sqrt{0.018225} = 0.135$$

\therefore Expression

$$= 135 + 13.5 + 1.35 + 0.135$$

$$= 149.985$$

19. b Let the number of days of tour be x .

\therefore Total expenditure = x^2

$$\therefore x^2 = 361 \Rightarrow x = \sqrt{361} = 19$$

20. a Number of students in the last row = $\sqrt{1369} = 37$

21. b Base 100. Answer is $(100 + 36) | 3 \times 144 | 1728$
 $= 1404928$.

22. d $(30 + 1)^3 = (30)^3 + (1)^3 + 3 \cdot 30 \cdot 1(30 + 1)$
 $= 27000 + 1 + 2790 = 29791$.

23. b Base 1000. Answer is $(1000 + 15) | 3 \times 25 | 125$
 $= 1015075125$.

24. d Base 1000. Answer is $(1000 - 9) | 3 \times (-3)^2 | (-3)^3$
 $= 991 | 27 | -27$
 $= 991 | 026 | 1000 - 27 = 991026973$.

25. d Base 100. Answer is $(100 - 27) | 3 \times (-9)^2 | (-9)^3$
 $= 73 | 243 | -729$
 $= 73 | 235 | 800 - 729 \{ \text{Taking carry 8 from the other side} \}$
 $= 73 | 235 | 71 = 73 + 2(= 75) | 35 | 71 = 753571$.

26. b $1440 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 2^2 \times 3^2 \times 5$
To make 1440 a perfect cube, it must be multiplied by 2
 $\times 3 \times 5 \times 5 = 150$
The required sum = $1 + 5 + 0 = 6$

27. a $3000 = 3 \times 1000 = 3 \times 10^3$
Clearly, when we divide 3000 by natural number 3, the quotient is 1000 which is a perfect cube.

28. b $12 \times 12 \times 12 = 1728$
 \therefore Required number
 $= 1728 - 1720 = 8$

29. b $4320 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$
 $= 2^5 \times 3^3 \times 5$
 $\therefore \text{Required number} = 2 \times 5 \times 5 = 50$

30. a **Look at the pattern:**
 $1001 \times 1001 = 1002001$
 $1001 \times 1001 \times 1001 = 1003003001$

31. a $\sqrt[3]{1728} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$
 $= \sqrt[3]{2^6 \times 3^3} = 2^2 \times 3$
 $= 12.$

32. b $\sqrt[3]{144} \times \sqrt[3]{1500}$
 $= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 3 \times 3} \times \sqrt[3]{2 \times 2 \times 3 \times 5 \times 5 \times 5}$
 $= \sqrt[3]{2^4 \times 3^2} \times \sqrt[3]{2^2 \times 3 \times 5^3}$
 $= \sqrt[3]{2^6 \times 3^3 \times 5^3} = 2^2 \times 3 \times 5 = 60.$

33. c $\sqrt[3]{486 \sqrt{36 \sqrt[3]{64}}}$
 $= \sqrt[3]{486 \sqrt{36 \sqrt[3]{4^3}}} = \sqrt[3]{486 \sqrt{36 \times 4}} = \sqrt[3]{486 \sqrt{6^2 \times 2^2}}$
 $= \sqrt[3]{486 \times 6 \times 2} = \sqrt[3]{2^3 \times 3^6} = 2 \times 3^2 = 18.$

34. a $\sqrt[3]{0.000729} = \sqrt[3]{\frac{729}{1000000}}$
 $= \sqrt[3]{\frac{(9)^3}{(100)^3}} = \frac{9}{100} = 0.09.$

35. c $\sqrt[3]{32} \times \sqrt[3]{250} = \sqrt[3]{2^5} \times \sqrt[3]{2 \times 5^3}$
 $= \sqrt[3]{2^6 \times 5^3} = 2^2 \times 5 = 20.$

36. d $\sqrt[3]{12 \sqrt{5625 \sqrt{81 \times 10^4}}}$
 $= \sqrt[3]{4 \times 3 \sqrt{5 \times 5 \times 5 \times 5 \times 3 \times 3 \sqrt{900 \times 900}}}$
 $= \sqrt[3]{4 \times 3 \sqrt{5 \times 5 \times 5 \times 5 \times 3 \times 3 \times 900}}$
 $= \sqrt[3]{4 \times 3 \times 5 \times 5 \times 3 \times 30}$
 $= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5} = 2 \times 3 \times 5 = 30.$

37. b $\sqrt[3]{\frac{729}{9^2}} = \sqrt[3]{\frac{9 \times 9 \times 9}{(3^2)^2}} = \frac{9}{3} = 3.$

38. b $\sqrt[3]{24} \times \sqrt[3]{9} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} = 6.$

39. a $\sqrt[3]{\frac{512}{1000000}} = \sqrt[3]{\frac{8^3}{100^3}} = \frac{8}{100} = 0.08.$

40. b $\sqrt[3]{36 \times \sqrt{9 \times \sqrt{16}}} = \sqrt[3]{36 \times \sqrt{9 \times 4}} = \sqrt[3]{36 \times 6} = 6.$

41. c We know that
 $a^3 + b^3 + c^3 - 3abc$
 $= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
 $= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

$$\therefore \sqrt[3]{(333)^3 + (333)^3 + (334)^3 - 3 \times 333 \times 333 \times 334}$$

$$= \sqrt[3]{\frac{1}{2}(333 + 333 + 334)[(333 - 333)^2 + (333 - 334)^2 + (334 - 333)^2]}$$

$$= \sqrt[3]{\frac{1}{2} \times 1000 \times 2} = \sqrt[3]{1000} = \sqrt[3]{10 \times 10 \times 10} = 10$$

42. c Here, $\sqrt[3]{175616} = 56$
 $\therefore \sqrt[3]{175.616} = 5.6$
and $\sqrt[3]{0.175616} = 0.56$
 $\therefore \text{Required sum} = 5.6 + 0.56 + 0.056 = 6.216$

43. d $\sqrt[3]{\sqrt{0.000064}} = \sqrt[3]{0.008}$
 $= \sqrt[3]{0.2 \times 0.2 \times 0.2} = 0.2$

44. b Expression
 $= \sqrt[3]{15612 + \sqrt{154 + \sqrt{225}}}$
 $= \sqrt[3]{15612 + \sqrt{154 + 15}}$
 $= \sqrt[3]{15612 + 13}$
 $= \sqrt[3]{15625} = 25$

45. b Here, $\sqrt[3]{571787} = 83$
 $\therefore \sqrt[3]{571.787} = 8.3$
and $\sqrt[3]{0.571787} = 0.83$
 $\therefore \text{Required sum} = 8.3 + 0.83 + 0.083 = 9.213$

46. c Since $\frac{338}{473} \approx \frac{2}{3}$
 $\therefore \frac{338}{473} = \frac{338 + 27 \times \frac{2}{3}}{473 + 27} = \frac{356}{500} = 0.712 \text{ i.e. } 71.2\%.$

47. a Since $\frac{1}{7} = 14.28\%$

$$\therefore \frac{3}{7} = 14.28 \times 3 = 42.84\%.$$

48. c Since $\frac{443}{898} \approx \frac{1}{2}$

$$\text{Therefore, } \frac{443}{898} = \frac{443 + 102 \times \frac{1}{2}}{898 + 102} = \frac{494}{1000} = 0.494 \text{ i.e., } 49.4\%.$$

49. d Since $\frac{1}{19} \approx 5.26\%$

$$\therefore \frac{8}{19} = 8 \times 5.26 = 42.08\%.$$

50. b Since $\frac{547}{1973} \approx \frac{3}{11}$

$$\text{Therefore, } \frac{547}{1973} = \frac{547 + \frac{3}{11} \times 27}{1973 + 27} = \frac{547 + 7.36}{2000} = \frac{554.36}{2000} = 0.27718 \approx 27.72\%.$$

51. b $\sqrt{625} + \sqrt[3]{1331} = \sqrt{5 \times 5 \times 5 \times 5} + \sqrt[3]{11 \times 11 \times 11} = 5 \times 5 + 11 = 36.$

52. a $243 = 3 \times 3 \times 3 \times 3 \times 3$

Hence, the number is 3 and the resultant number will be $243 \times 3 = 729 = (27)^2 = (9)^3$.

53. b Let, $32 = a$

$$79 = b, -111 = c$$

$$\text{When } (a + b + c) = 0$$

$$\text{then } a^3 + b^3 + c^3 - 3abc = 0$$

$$\text{Here, } a + b + c = 32 + 79 - 111 = 0$$

$$\therefore (32)^3 + (79)^3 - (111)^3 + 3 \times 32 \times 79 \times 111 = 0$$

54. b If $a + b + c = 0$, then

$$a^3 + b^3 + c^3 = 3abc$$

$$\text{Here, } 0.111 + 0.222 + (-0.333) = 0$$

$$\therefore (0.111)^3 + (0.222)^3 + (-0.333)^3$$

$$= -3 \times 0.111 \times 0.222 \times 0.333$$

$$= -(0.333)^2 \times 0.222$$

$$\therefore \text{Expression}$$

$$= [-(0.333)^2 \times 0.222 + (0.333)^2 \times 0.222]^3 = 0$$

55. d Expression

$$\begin{aligned} &= (0.98)^3 + (0.02)^3 + 3 \times 0.98 \times 0.02 - 1 \\ &= (0.98)^3 + (0.02)^3 + 3 \times 0.98 \times 0.02 (0.98 + 0.02) - 1 \\ &= (0.98 + 0.02)^3 - 1 = 1 - 1 = 0 \end{aligned}$$

56. d Let $0.0347 = a$

and, $0.9653 = b$

$$\therefore \text{Expression} = \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$= \frac{(a+b)(a^2 - ab + b^2)}{a^2 - ab + b^2} = a + b$$

$$= 0.0347 + 0.9653 = 1$$

57. d Expression

$$= \frac{(3.2)^3 - (0.2)^3}{(3.2)^2 + 3.2 \times 0.2 + (0.2)^2}$$

Let $3.2 = a$ and $0.2 = b$

$$\therefore \text{Expression} = \frac{a^3 - b^3}{a^2 + ab + b^2}$$

$$= \frac{(a-b)(a^2 + ab + b^2)}{a^2 + ab + b^2} = a - b$$

$$= 3.2 - 0.2 = 3$$

58. c Expression

$$= \frac{(0.75)^3 + (1 - 0.75)[0.75^2 + 0.75 \times 1 + 1^2]}{1 - 0.75}$$

$$= \frac{(0.75)^3 + 1^3 - (0.75)^3}{0.25}$$

$$= \frac{1}{0.25} = \frac{100}{25} = 4$$

\therefore Required square root

$$= \sqrt{4} = 2$$

59. a Multiples of 11 whose square root are whole number

$$\text{First} = 11 \times 11 = 121$$

$$\text{Second} = 11 \times 11 \times 4 = 484$$

60. c Let the number be x . Then,

$$\begin{aligned} x^2 &= (75.15)^2 - (60.12)^2 \\ &= (75.15 + 60.12)(75.15 - 60.12) \\ &= 135.27 \times 15.03 \\ &= 2033.1081 \\ \Rightarrow x &= \sqrt{2033.1081} = 45.09 \end{aligned}$$

**Approximation and VBODMAS
Answers and Explanations**

1	b	2	c	3	a	4	d	5	b	6	a	7	c	8	b	9	c	10	d
11	b	12	a	13	c, d	14	c	15	b	16	c	17	b	18	d	19	d	20	b
21	c	22	c	23	a	24	d	25	c	26	a	27	a	28	c	29	a	30	d
31	d	32	d	33	b	34	a	35	b	36	b	37	d	38	a	39	a	40	b
41	c	42	b	43	b	44	d	45	a	46	b	47	d	48	b	49	a	50	d
51	d	52	a	53	d	54	c	55	a	56	c	57	b	58	d	59	b	60	c

1. b $0.2 + 2 \times (0.02)^2 - 3 \times (0.2)^3$
 $= 0.2 + 2 \times 0.0004 - 3 \times 0.008$
 $= 0.2 + 0.0008 - 0.024 = 0.2008 - 0.024 = 0.1768.$

2. c $5 \times 0.075 \div \left[\left(1\frac{1}{5} + 0.3 \right) \times 5 \right] - 4 \frac{1}{3} \times 6 \times 0.001$
 $= 5 \times 0.075 \div (1.5 \times 5) - \frac{13}{3} \times 6 \times 0.001$
 $= 5 \times 0.075 \div 7.5 - \frac{13}{3} \times 6 \times 0.001$
 $= 5 \times 0.01 - \frac{13}{3} \times 6 \times 0.001 = 0.05 - 0.026 = 0.024.$

3. a $\frac{\frac{1}{2} \times \frac{6}{5} \div \frac{3}{4} \text{ of } \frac{1}{2}}{\frac{3}{8} \div 3 \times \frac{24}{8} - 3} = \frac{\frac{1}{2} \times \frac{6}{5} \div \frac{3}{8}}{\frac{3}{8} \div 3 \times \frac{32}{8} - 3}$
 $= \frac{\frac{1}{2} \times \frac{16}{5}}{\frac{1}{8} \times \frac{32}{8} - 3} = \frac{8}{5} - 3 = \frac{8}{5}.$

4. d $11^2 + 11^3 \div 11^2 - 11 + 11 \text{ of } 11$
 $= 11^2 + 11^3 \div 11^2 - 11 + 11^2$
 $= 11^2 + 11 - 11 + 11^2$
 $= 2 \times 11^2 = 242.$

5. b $\frac{17}{20} - \frac{4}{5} + \frac{3}{20} \times \sqrt{7-2} \div 5$
 $= \frac{17}{20} - \frac{4}{5} + \frac{3}{20} \times 5 \div 5$
 $= \frac{17}{20} - \frac{4}{5} + \frac{3}{20} = \frac{17-16+3}{20} = \frac{4}{20} = \frac{1}{5}.$

6. a $36 \div 4 \times 8 \text{ of } 2 + 12 - 2 \text{ of } 3$
 $= 36 \div 4 \times 16 + 12 - 6$
 $= 9 \times 16 + 12 - 6$
 $= 144 + 12 - 6 = 150.$

7. c $8 \times [\overline{7-4+2} \div 5 \times (6+2) - 4 \text{ of } 2]$
 $= 8 \times [5 \div 5 \times 8 - 4 \text{ of } 2]$
 $= 8 \times [5 \div 5 \times 8 - 8]$
 $= 8 \times [1 \times 8 - 8] = 8 \times (8 - 8)$
 $= 8 \times 0 = 0.$

8. b $\frac{2 - \frac{4}{\frac{2-3}{2}}}{\frac{6}{\frac{17}{3}} - \frac{2 - \frac{4}{\frac{4-3}{3}}}{\frac{6-11}{3}}} = \frac{2 - \frac{4}{\frac{1}{2}}}{\frac{6}{\frac{24-17}{3}} - \frac{2 - \frac{4}{\frac{1}{3}}}{\frac{18-11}{3}}} = \frac{18}{7} - \frac{\frac{2-8}{1}}{3}$
 $= \frac{18}{7} - \frac{-18}{7} = \frac{18}{7} + \frac{18}{7} = \frac{36}{7}.$

9. c $3.2 \left[\frac{5.4}{1.8} - \frac{4.8}{2.4} \right] - 1.6 \times 5.7 = 3.2[3-2] - 1.6 \times 5.7$
 $= 3.2 - 1.6 \times 5.7$
 $= 1.6(2 - 5.7)$
 $= 1.6 \times (-3.7) = -5.92.$

10. d $\frac{18}{\sqrt{8}} \div \left(\frac{30}{\sqrt{8}} \div \frac{15}{\sqrt{27}} \times \frac{27}{\sqrt{54}} \right) = \frac{18}{2\sqrt{2}} \div \left(\frac{30}{2\sqrt{2}} \div \frac{15}{3\sqrt{3}} \times \frac{27}{3\sqrt{6}} \right)$
 $= \frac{9}{\sqrt{2}} \div \left(\frac{15}{\sqrt{2}} \div \frac{5}{\sqrt{3}} \times \frac{9}{\sqrt{6}} \right)$
 $= \frac{9}{\sqrt{2}} \div \left(\frac{15\sqrt{3}}{5\sqrt{2}} \times \frac{9}{\sqrt{6}} \right) = \frac{9}{\sqrt{2}} \div \frac{27}{2}$
 $= \frac{9}{\sqrt{2}} \times \frac{2}{27} = \frac{\sqrt{2}}{3}.$

11. b

$$\begin{aligned} \frac{1}{3} - \frac{5}{7} \text{ of } \frac{1}{4 - \frac{1}{1 - \frac{3}{5}}} &= \frac{1}{3} - \frac{5}{7} \text{ of } \frac{1}{4 - \frac{1}{5-3}} \\ &= \frac{1}{3} - \frac{5}{7} \text{ of } \frac{1}{4 - \frac{5}{2}} = \frac{1}{3} - \frac{5}{7} \text{ of } \frac{1}{\frac{8-5}{2}} \\ &= \frac{1}{3} - \frac{5}{7} \text{ of } \frac{2}{3} = \frac{1}{3} - \frac{10}{21} \\ &= \frac{7-10}{21} = \frac{-3}{21} = -\frac{1}{7}. \end{aligned}$$

12. a

$$\begin{aligned} &\left[\left(\frac{5}{2} + \frac{9}{4} - \frac{2}{3} \right) + 1\frac{1}{4} \right] \div \left(\frac{3}{5} - 2\frac{2}{5} - 1 \right) \\ &= \left[\left(\frac{30+27-8}{12} \right) + \frac{5}{4} \right] \div \left(\frac{3-12-5}{5} \right) \\ &= \left[\frac{49}{12} \times \frac{4}{5} \right] \div \left(-\frac{14}{5} \right) = -\frac{49}{15} \times \frac{5}{14} = -\frac{7}{6}. \end{aligned}$$

13. (i) c $8888 + 888 + 88 + 8 = 9872$.
(ii) d $11111 - 1111 + 111 - 11 + 1 = 10101$.

14. c

$$\begin{aligned} 5 \times \overline{6-3} \div 4 + 1 \div 4 + 9 &= 5 \times 3 \div 4 + 1 \div 4 + 9 \\ &= 5 \times \frac{3}{4} + \frac{1}{4} + 9 = \frac{15+1+36}{4} = \frac{52}{4} = 13. \end{aligned}$$

15. b

$$\begin{aligned} 3125 \div 25 \text{ of } 25 - 5^2 &= 3125 \div 625 - 5^2 = 5 - 25 = -20. \end{aligned}$$

16. c

$$\begin{aligned} \frac{9.5 \times 6.8 + 9.5 \times 3.2}{1.9 \times 5 - 1.9 \times 4} &= \frac{9.5(6.8 + 3.2)}{1.9(5 - 4)} = 5 \times 10 = 50. \end{aligned}$$

17. b

$$\frac{a^3 + b^3}{a^2 + b^2 - ab} = a + b = 0.73 + 0.27 = 1.$$

Here, $a = 0.73$ and $b = 0.27$.

18. d

$$\begin{aligned} \frac{5.76 \times 5.9 + 0.576 \times 41}{2.4 \times 3 - 24 \times 0.2} &= \frac{5.76(5.9 + 4.1)}{2.4(3 - 2)} = \frac{2.4 \times 10}{1} = 24. \end{aligned}$$

19. d

$$\begin{aligned} (a) 50^2 - 44^2 &= (50 - 44)(50 + 44) \\ &= 6 \times 94 \text{ (True)} \\ (b) (a + b + c)^2 &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ac \text{ (True)} \\ (c) (a + b)^2 - (a - b)^2 &= 4ab \text{ (True)} \\ (d) \left(2 + \frac{3}{2} \right)^2 &= 4 + \frac{9}{4} + 6 \text{ (False)} \end{aligned}$$

20. b

$$\begin{aligned} \frac{6^2 \times \frac{1}{3} \text{ of } \frac{1}{12} \div \frac{1}{18}}{\frac{1}{13} \text{ of } (216 \div 12 + 8) - 8} &= \frac{6^2 \times \frac{1}{3} \text{ of } \frac{1}{12} \div \frac{1}{18}}{\frac{1}{13} \text{ of } (18 + 8) - 8} \\ &= \frac{6^2 \times \frac{1}{3} \text{ of } \frac{1}{12} \div \frac{1}{18}}{\frac{1}{13} \text{ of } 26 - 8} = \frac{6^2 \times \frac{1}{36} \div \frac{1}{18}}{2 - 8} \\ &= \frac{36 \times \frac{1}{2}}{2 - 8} = \frac{18}{2 - 8} = \frac{18}{-6} = -3. \end{aligned}$$

21. c

$$\begin{aligned} \frac{\frac{1}{3} \div \frac{1}{18} \text{ of } 3 \times (?) + 3 - 5}{18 \div (2 + 7) \text{ of } \frac{1}{2} - 8} &= -4 \\ \frac{\frac{1}{3} \div \frac{1}{6} \times (?) + 3 - 5}{18 \div \frac{9}{2} - 8} &= -4 \\ \Rightarrow \frac{2 \times (?) + 3 - 5}{4 - 8} &= -4 \Rightarrow \frac{2(?) - 2}{-4} = -4 \\ \Rightarrow 2(?) &= 18 \\ \Rightarrow (?) &= 9. \end{aligned}$$

22. c

$$\begin{aligned} \frac{9 + 6 \div 18}{7 \div 14 \times 2} - \frac{7 \text{ of } \frac{1}{24} \div \frac{1}{8}}{\frac{1}{8} \div \frac{1}{24} \text{ of } \frac{1}{7}} &= \frac{9 + 6 \div 18}{7 \div 14 \times 2} - \frac{\frac{7}{24} \div \frac{1}{8}}{\frac{1}{8} \div \frac{1}{24} \times 7} \\ &= \frac{9 + \frac{1}{3}}{\frac{1}{2} \times 2} - \frac{\frac{7}{8}}{\frac{1}{24} \times 7} = 9 + \frac{1}{3} - \frac{7}{3} \times \frac{1}{21} \\ &= 9 + \frac{1}{3} - \frac{1}{9} = \frac{81+3-1}{9} = \frac{83}{9}. \end{aligned}$$

23. a

$$\begin{aligned} 0.66 \times 0.66 + 0.28 \times 0.28 + 0.06 \times 0.06 + 2(0.66 \times 0.28 &+ 0.66 \times 0.06 + 0.28 \times 0.06) \\ &= (0.66 + 0.28 + 0.06)^2 = (1.00)^2 = 1. \end{aligned}$$

24. d $\frac{343 \div (17 + 7 \times 3 - 24) \text{ of } 2 - 24}{45 \div (6 + 9) \text{ of } \frac{1}{4}}$

$$\begin{aligned} &= \frac{343 \div (38 - 24) \text{ of } 2 - 24}{45 \div 15 \text{ of } \frac{1}{4}} = \frac{343 \div 28 - 24}{45 \div \frac{15}{4}} = \frac{\frac{49}{4} - 24}{45 \times \frac{4}{15}} \\ &= \frac{\frac{49 - 96}{4}}{\frac{3 \times 4}{1}} = \frac{-\frac{47}{4}}{\frac{12}{1}} = -\frac{47}{48}. \end{aligned}$$

25. c $\because \frac{a^3 - b^3}{a^2 + ab + b^3} = a - b$

Here, $a = 0.03$ and $b = 0.01$
 $\therefore a - b = 0.03 - 0.01 = 0.02$
 $\therefore (?) = ab = (0.03)(0.01) = 0.0003.$

26. a $\frac{1}{2 - \frac{4}{2 - \frac{2}{2 - \frac{1}{3}}}} = \frac{1}{2 - \frac{4}{2 - \frac{2}{2 - \frac{6-1}{3}}}}$

$$\begin{aligned} &= \frac{1}{2 - \frac{4}{2 - \frac{6}{5}}} = \frac{1}{2 - \frac{4}{10-6}} = \frac{1}{2-5} = -\frac{1}{3}. \end{aligned}$$

For questions 27 to 30: After replacing the signs with respective meanings the expression becomes:

27. a $13 \times 26 \div 78 - 143 + 156$

$$= \frac{13 \times 26}{78} + 13 = \frac{13}{3} + 3 = \frac{52}{3}.$$

28. c $1001 \div 78 \times 36 \div 77 \times 3$

$$\begin{aligned} &= \frac{1001}{78} \times \frac{36}{77} \times 3 \\ &= \frac{7 \times 11 \times 13 \times 6 \times 6 \times 3}{13 \times 6 \times 11 \times 7} = 18. \end{aligned}$$

29. a $\frac{39}{8} \div \frac{13}{17} \times \frac{16}{34} + \frac{3}{5}$

$$= \frac{39}{8} \times \frac{17}{13} \times \frac{16}{34} + \frac{3}{5} = 3 + \frac{3}{5} = \frac{18}{5}.$$

30. d $(a + b)^2 - (a - b)^2 = 4ab.$

31. d $\left[\left(\frac{5}{2} + \frac{9}{4} - \frac{2}{3} \right) \div 1\frac{1}{4} \right] \div \left(-\frac{3}{5} - 2\frac{2}{5} - 1 \right)$

$$\begin{aligned} &= \left[\left(\frac{30+27-8}{12} \right) \div \frac{5}{4} \right] \div \left(-\frac{3}{5} - \frac{12}{5} - 1 \right) \\ &= \left[\frac{49}{12} \div \frac{5}{4} \right] \div \left(\frac{-3-12-5}{5} \right) \\ &= \left[\frac{49}{12} \times \frac{4}{5} \right] \div \left(\frac{-20}{5} \right) \\ &= \frac{49}{15} \div (-4) = -\frac{49}{15} \times \frac{1}{4} = -\frac{49}{60}. \end{aligned}$$

32. d $\frac{1}{2} \text{ of } \frac{1}{2} \text{ of } \left(\frac{3}{2} + \frac{1}{2} \text{ of } 13 \right) = \left(\frac{3}{4} + \frac{4}{3} \right) \text{ of } (?)$

$$\begin{aligned} &\Rightarrow \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } \left(\frac{3}{2} + \frac{13}{2} \right) = \left(\frac{9+16}{12} \right) \text{ of } (?) \\ &\Rightarrow \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } 8 = \frac{25}{12} \text{ of } (?) \\ &\Rightarrow \frac{1}{2} \times \frac{1}{2} \times 8 = \frac{25}{12} (?) \\ &\Rightarrow 2 \times 12 = 25 (?) \\ &\Rightarrow (?) = \frac{24}{25}. \end{aligned}$$

33. b $15^2 - 13^2 + 17^2 = (?)^2 - 11^2 - 110$

$$\begin{aligned} &\Rightarrow 225 - 169 + 289 = (?)^2 - 121 - 110 \\ &\Rightarrow (?)^2 = 225 + 289 + 121 + 110 - 169 \\ &\Rightarrow (?)^2 = 745 - 169 \\ &\Rightarrow (?)^2 = 576 = 24^2 \Rightarrow (?) = 24. \end{aligned}$$

34. a (a) $17^3 - 15^3 - 1535$

$$\begin{aligned} &= (17 - 15)(17^2 + 17 \times 15 + 15^2) - 1535 \\ &= 2[289 + 255 + 225] - 1535 \\ &= 1538 - 1535 = 3. \end{aligned}$$

(b) $\frac{16^2}{6^6} \times 3^6$

$$= \frac{2^8}{2^6 \times 3^6} \times 3^6 = 4.$$

(c) $1889 - 5 \times 13 \times 29 = 1889 - 1885 = 4.$

(d) $3 - \frac{1}{2 - \frac{1}{1 - \frac{2}{3}}} = 3 - \frac{1}{2-3}$

$$= 3 + 1 = 4.$$

35. b

$$\begin{aligned}
 & \frac{(7+8) \div 5}{3 \times \frac{1}{3} \text{ of } 10} \times \frac{\frac{1}{10} \div \frac{1}{5} \text{ of } 10}{\frac{1}{10} \text{ of } \frac{1}{20} \div \frac{1}{10}} \\
 &= \frac{15 \div 5}{3 \times \frac{1}{3} \text{ of } 10} \times \frac{\frac{1}{10} \div \frac{1}{5} \text{ of } 10}{\frac{1}{10} \text{ of } \frac{1}{20} \div \frac{1}{10}} \\
 &= \frac{15 \div 5}{3 \times \frac{10}{3}} \times \frac{\frac{1}{10} \div 2}{\frac{1}{10} \times \frac{1}{20} \div \frac{1}{10}} = \frac{15 \times \frac{1}{5}}{3 \times \frac{10}{3}} \times \frac{\frac{1}{10} \times \frac{1}{2}}{\frac{1}{10} \times \frac{1}{20} \times 10} \\
 &= \frac{3}{10} \times \frac{1}{20} = \frac{3}{10} \times \frac{1}{20} \times \frac{20}{1} = 0.3.
 \end{aligned}$$

36. b

$$\begin{aligned}
 & 7 \times 0.35 \div \left[\frac{3}{3} \times 5 \right] - \frac{25}{3} \times 12 \times \frac{2}{1000} \\
 &= 7 \times 0.35 \times \frac{1}{5} - 0.2 \\
 &= 7 \times 0.07 - 0.2 \\
 &= 0.49 - 0.2 = 0.29.
 \end{aligned}$$

37. d

$$\begin{aligned}
 & \frac{3}{1 - \frac{1}{(?) + \frac{1}{1 + \frac{1}{5}}}} = \frac{69}{17} \\
 & \Rightarrow \frac{3}{1 - \frac{1}{(?) + \frac{5}{6}}} = \frac{69}{17} \\
 & \Rightarrow \frac{3}{1 - \frac{6}{6(?) + 5}} = \frac{69}{17} \\
 & \Rightarrow \frac{17}{23} = 1 - \frac{6}{6(?) + 5} \\
 & \Rightarrow \frac{6}{6(?) + 5} = \frac{6}{23} \\
 & \Rightarrow 6(?) + 5 = 23 \\
 & \Rightarrow 6(?) = 18 \\
 & \Rightarrow (?) = 3.
 \end{aligned}$$

38. a

$$\begin{aligned}
 & 1 + 0.1 + 0.01 + 0.001 = 2 + 0.2 \times x \\
 & \Rightarrow 1.111 = 2 + 0.2x \\
 & \Rightarrow 0.2x = -0.889 \\
 & \Rightarrow x = -\frac{0.889}{0.2} \Rightarrow x = -4.445 \\
 & \Rightarrow x \approx -4.5.
 \end{aligned}$$

39. a

$$\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b = 0.05 + 0.01 = 0.06.$$

40. b

$$\begin{aligned}
 & \frac{\sqrt{0.4} \times \sqrt{6.4} + \sqrt{0.0004}}{\sqrt{1\frac{11}{25}}} + \sqrt{0.0289} \\
 &= \frac{\sqrt{2.56} + \sqrt{0.0004}}{\sqrt{\frac{36}{25}}} + \sqrt{0.0289} \\
 &= \frac{1.6 + 0.02}{\frac{6}{5}} + 0.17 = \frac{1.62 \times 5}{6} + 0.17 \\
 &= \frac{8.10 + 1.02}{6} = \frac{9.12}{6} = 1.52.
 \end{aligned}$$

41. c

$$\begin{aligned}
 & 3 + \frac{1}{5} \text{ of } \left[20 - (11 + 9 - 5) + \frac{1}{3} \text{ of } 6 \right] - 7 \\
 &= 3 + \frac{1}{5} \text{ of } \left[20 - 15 + \frac{1}{3} \text{ of } 6 \right] - 7 \\
 &= 3 + \frac{1}{5} \text{ of } [20 - 15 + 2] - 7 = 3 + \frac{1}{5} \text{ of } 7 - 7 \\
 &= 3 + \frac{7}{5} - 7 = \frac{15 + 7 - 35}{5} = -\frac{13}{5} = -2.6.
 \end{aligned}$$

42. b

$$\begin{aligned}
 & 999 - \frac{729}{27} \times 48 - 27 = 999 - \frac{729}{9} \times 16 - 27 \\
 &= 999 - 81 \times 16 - 27 = 999 - 1296 - 27 \\
 &= 999 - 1323 = -324.
 \end{aligned}$$

43. b

$$\begin{aligned}
 & \frac{359}{1059} = x \\
 & \Rightarrow x \approx \frac{360}{1060} \\
 & \Rightarrow x \approx \frac{1}{3} \\
 & \Rightarrow x \approx 0.33.
 \end{aligned}$$

44. d

$$\begin{aligned}
 & x^2 + 0.1 = 0.776 \\
 & \Rightarrow x^2 = 0.676 \\
 & \Rightarrow x \approx 0.8.
 \end{aligned}$$

45. a

$$\begin{aligned}
 & 35.56 \times 41.29 = x \times 53.2 \\
 & \Rightarrow x = \frac{35 \times 41}{53} \text{ (By approximation)} \\
 & \Rightarrow x \approx 27.6.
 \end{aligned}$$

46. b $33.33 + 444.4 + 5.555 = 66.6 \times x$
 $\Rightarrow 66.6 \times x = 483.285$
 $\Rightarrow x = \frac{483.285}{66.6}$
 $\Rightarrow x \approx 7.25.$

47. d $\frac{987654321+123456789}{9876543210} = x$
 $\Rightarrow \frac{987654321}{9876543210} + \frac{123456789}{9876543210} = x$
 $\Rightarrow 0.1 + 0.01 \approx x \Rightarrow x \approx 0.11.$

48. b $x \times 0.0529 = \frac{0.0961}{x}$
 $\Rightarrow x^2 = \frac{0.0961}{0.0529} = \frac{961}{529}$
 $\Rightarrow x = \frac{31}{23} \Rightarrow x \approx 1.35.$

49. a $\frac{3.5}{2.4} \times \frac{x}{6.9} = 0.23$
 $\Rightarrow x = \frac{0.23 \times 6.9 \times 2.4}{3.5}$
 $\Rightarrow x = \frac{3.8088}{3.5} \Rightarrow x \approx 1.08.$

50. d $3569 \times 2387 \times 5389 = x$
 $\Rightarrow x \approx 3.6 \times 10^3 \times 2.4 \times 10^3 \times 5.4 \times 10^3$
 $\Rightarrow x \approx 46.656 \times 10^9$
 $\Rightarrow x \approx 5 \times 10^{10}.$

51. d $? = 1 + \frac{1}{1 + \frac{2}{1 + \frac{3}{2 + \frac{4}{1 + \frac{5}{}}}}}$

$$= 1 + \frac{1}{1 + \frac{2}{2 + \frac{3 \times 5}{5 + 4}}} = 1 + \frac{1}{1 + \frac{2}{2 + \frac{5}{3}}}$$

$$= 1 + \frac{1}{1 + \frac{2 \times 3}{6 + 5}} = 1 + \frac{1 \times 11}{11 + 6}$$

$$= 1 + \frac{11}{17} = 1\frac{11}{17}$$

52. a $? = 1 + \frac{2}{1 + \frac{3 \times 5}{9}} = 1 + \frac{2}{1 + \frac{5}{3}}$
 $= 1 + \frac{2 \times 3}{8} = \frac{7}{4}$

53. d $\frac{1}{3 + \frac{1}{2 - \frac{1}{\frac{7}{9}}}} + \frac{17}{22}$
 $= \frac{1}{3 + \frac{1}{2 - \frac{9}{7}}} + \frac{17}{22}$
 $= \frac{1}{3 + \frac{1}{\frac{14 - 9}{7}}} + \frac{17}{22}$
 $= \frac{1}{3 + \frac{1}{\frac{5}{7}}} + \frac{17}{22} = \frac{1}{3 + \frac{7}{5}} + \frac{17}{22}$
 $= \frac{1}{\frac{15 + 7}{5}} + \frac{17}{22}$
 $= \frac{5}{22} + \frac{17}{22} = \frac{22}{22} = 1$

54. c $x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$
 $= 1 + \frac{1}{1 + \frac{1}{1 + \frac{2}{1 + \frac{3}{3}}}} = 1 + \frac{1}{1 + \frac{1}{1 + \frac{3}{5}}}$
 $= 1 + \frac{1}{8} = 1 + \frac{5}{8} = \frac{13}{8}$
 $\therefore 2x + \frac{7}{4} = 2 \times \frac{13}{8} + \frac{7}{4}$
 $= \frac{13 + 7}{4} = \frac{20}{4} = 5$

55. a $\frac{19}{43} \div \frac{2}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4}}}}$

$$= \frac{19}{43} \div \frac{2}{2 + \frac{1}{\frac{4}{3 + \frac{4}{5}}}}$$

$$= \frac{19}{43} \div \frac{1}{2 + \frac{5}{19}} = \frac{19}{43} \div \frac{19}{43}$$

$$= \frac{19}{43} \times \frac{43}{19} = 1$$

56. c $\frac{5}{3 + \frac{3}{3 - \frac{2}{3}}} = \frac{5}{3 + \frac{3}{\frac{1}{3}}}$

$$= \frac{5}{3 + 3 \times 3} = \frac{5}{3 + 9} = \frac{5}{12}$$

57. b $2 = x + \frac{1}{1 + \frac{1}{3 + \frac{1}{4}}}$

$$\Rightarrow 2 = x + \frac{1}{1 + \frac{1}{1 + \frac{12+1}{4}}}$$

$$\Rightarrow 2 = x + \frac{1}{1 + \frac{4}{13}}$$

$$\Rightarrow 2 = x + \frac{1}{13 + 4}$$

$$\Rightarrow 2 = x + \frac{1}{17}$$

$$\Rightarrow 2 = x + \frac{13}{17} \Rightarrow x = 2 - \frac{13}{17}$$

$$= \frac{34 - 13}{17} = \frac{21}{17}$$

58. d $\frac{2}{1 + \frac{1}{\frac{1}{2}}} \times \frac{3}{\left(\frac{5}{6} \times \frac{3}{2}\right) \div \frac{5}{4}}$

$$= \frac{2}{1+2} \times \frac{3}{\frac{5}{4} \div \frac{5}{4}}$$

$$= \frac{2}{3} \times \frac{\frac{3}{5}}{\frac{4}{5} \times \frac{4}{5}} = \frac{2}{3} \times 3 = 2$$

59. b $1 + \frac{4}{2 + \frac{3}{\frac{10-1}{2}}} - \frac{1}{2} \times 5$

$$1 + \frac{4}{2 + \frac{6}{9}} - \frac{5}{2} = 1 + \frac{4}{2 + \frac{2}{3}} - \frac{5}{2}$$

$$= 1 + \frac{4}{8} - \frac{5}{2} = 1 + \frac{4 \times 3}{8} - \frac{5}{2}$$

$$= 1 + \frac{3}{2} - \frac{5}{2} = \frac{2+3-5}{2} = 0$$

60. c Suppose that

$$1 + \frac{1}{10 + \frac{1}{10}} = \frac{111}{101} = a$$

$$\text{and, } 1 - \frac{1}{10 + \frac{1}{10}} = \frac{91}{101} = b$$

$$= \frac{a^2 - b^2}{(a+b)} = \frac{(a+b)(a-b)}{(a+b)}$$

$$= (a - b)$$

$$= \frac{111}{101} - \frac{91}{101} = \frac{20}{101}$$

Numbers : Properties, Surds and Indices
Answers and Explanations

1	c	2	b	3	b	4	c	5	c	6	d	7	c	8	c	9	b	10	b
11	b	12	d	13	a	14	a	15	d	16	c	17	d	18	c	19	b	20	d
21	b	22	b	23	d	24	a	25	d	26	c	27	d	28	c	29	c	30	b
31	a	32	c	33	a	34	a	35	a	36	b	37	a	38	c	39	d	40	a
41	b	42	a	43	a	44	d	45	c	46	b	47	b	48	a	49	c	50	b
51	c	52	c	53	c	54	b	55	a	56	d	57	c	58	d	59	b	60	a
61	d	62	c	63	b	64	a	65	a	66	d	67	a	68	d	69	d	70	c

- | | |
|---|---|
| <p>1. c Prime numbers between 60 and 75 are 61, 67, 71 and 73.
 $\therefore \text{Sum} = 61 + 67 + 71 + 73 = 272.$</p> <p>2. b $\frac{6}{7} = 0.86$; $\frac{16}{19} = 0.84$;
 $\frac{19}{21} = 0.90$; $\frac{11}{14} = 0.78$ and $\frac{7}{10} = 0.7$.
 $\therefore \text{Descending order is } \frac{19}{21} > \frac{6}{7} > \frac{16}{19} > \frac{11}{14} > \frac{7}{10}$.</p> <p>3. b $\frac{78}{69} = 1.13$; $\frac{67}{58} = 1.15$;
 $\frac{25}{16} = 1.56$ and $\frac{41}{32} = 1.28$.
 $\therefore \text{Ascending order is } \frac{78}{69} < \frac{67}{58} < \frac{41}{32} < \frac{25}{16}$.</p> <p>4. c Sum of first n odd number = n^2.
 Hence, sum of first 38 odd number = $(38)^2 = 1444$.</p> <p>5. c Whole number.</p> <p>6. d $323 = 17 \times 19$
 Hence, it is a composite number.</p> <p>7. c 2 is the smallest prime number.</p> <p>8. c Since, the difference between the numerator and denominator of all the five fractions is a constant i.e. 13, then the value of the fraction with the biggest numerator will be the highest.
 $\therefore -\frac{121}{134} < -\frac{92}{105} < -\frac{87}{100} < -\frac{76}{89} < -\frac{51}{64}$.</p> | <p>9. b Dividing Numerator and Denominator by 73
 $\frac{1095}{1168} = \frac{15}{16}$.</p> <p>10. b $\sqrt{20} + \sqrt{80} = \sqrt{180}$, when we simplify LHS
 $\sqrt{20} + \sqrt{80} = \sqrt{20} + 2\sqrt{20} = 3\sqrt{20} = \sqrt{9 \times 20} = \sqrt{180}$.</p> <p>11. b Number = $35 \times 12 = 420$
 $\therefore \text{Correct quotient} = \frac{420}{21} = 20$.</p> <p>12. d $1 \times 2 \times 3 \times 4 = 24$; $2 \times 3 \times 4 \times 5 = 120$; $3 \times 4 \times 5 \times 6 = 360$ and so on.
 Hence, the required number = 24.</p> <p>13. a $\text{LCM of } \left(\frac{6}{7}, \frac{5}{14}, \frac{10}{21} \right) = \frac{\text{LCM of } (6, 5, 10)}{\text{HCF of } (7, 14, 21)} = \frac{30}{7}$.</p> <p>14. a $1000 + 1 = 1001 > 1000$
 Hence, number is 1.</p> <p>15. d Odd numbers = $\frac{71+1}{2} = 36$
 Even numbers = $\frac{71-1}{2} = 35$.</p> <p>16. c $F = 0.84\overline{181}$
 $= \frac{84181 - 841}{99000} = \frac{83340}{99000} = \frac{463}{550}$
 $\therefore \text{Denominator} - \text{Numerator} = 550 - 463 = 87$.</p> |
|---|---|

17. d Let $S = 0.5646464\dots$

$$10S = 5.646464\dots$$

$$1000S = 564.64646464\dots$$

$$\Rightarrow 1000S - 10S = 564 - 5$$

$$\Rightarrow 990S = 559$$

$$\Rightarrow S = \frac{559}{990}.$$

18. c (a) $0.\overline{37} = \frac{37}{99}$; (b) $0.\overline{053} = \frac{53}{999}$;

(c) $0.1\overline{254} = \frac{1254 - 12}{9900} = \frac{69}{550}$;

(d) $2.5\overline{36} = 2 + \frac{536 - 53}{900} = 2 + \frac{161}{300} = \frac{761}{300}$.

19. b Let $s = 0.0534534$

$$\Rightarrow 10s = 0.534534$$

$$\Rightarrow 10000s = 534.534$$

$$\Rightarrow 9990s = 534$$

$$\Rightarrow s = \frac{534}{9990}.$$

20. d $3.\overline{36} - 2.\overline{05} + 1.\overline{33}$

$$= 3 + \frac{36}{99} - 2 - \frac{5}{99} + 1 + \frac{33}{99}$$

$$= 2 + \frac{64}{99} = 2.\overline{64}.$$

21. b $8.\overline{31} + 0.\overline{6} + 0.0\overline{02} = 8.313131 + 0.666666 + 0.002222$
 $= 8.98\overline{20}$.

22. b Let $x = 0.\overline{423}$

$$\Rightarrow 10x = 4.2323\dots$$

$$\Rightarrow 1000x = 423.2323\dots$$

$$\Rightarrow 990x = 419 \Rightarrow x = \frac{419}{990}.$$

23. d $\frac{7}{6} = 1.166, \frac{7}{9} = 0.777$

$$\frac{4}{5} = 0.8 \text{ and } \frac{5}{7} = 0.714$$

Therefore, the smallest number is $\frac{5}{7}$

24. a $\frac{9}{13} = \frac{9 \times 4}{13 \times 4} = \frac{36}{52}$

$$\frac{17}{26} = \frac{17 \times 2}{26 \times 2} = \frac{34}{52}$$

$$\frac{33}{52} = \frac{33}{52}$$

Among these $\frac{33}{52}$ is the smallest

Again, $\frac{28}{29} = \frac{56}{58} > \frac{36}{52}$

25. d $\frac{8}{15}, \frac{14}{33}, \frac{7}{13}, \frac{11}{13}$

$$\frac{8}{15} = 0.533$$

$$\frac{14}{33} = 0.42$$

$$\frac{7}{13} = 0.538$$

$$\frac{11}{13} = 0.846$$

$$\therefore \frac{11}{13} > \frac{7}{13} > \frac{8}{15} > \frac{14}{33}$$

26. c $\frac{(1+\sqrt{2}) \times (\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3}) \times (\sqrt{5}-\sqrt{3})} + \frac{(1-\sqrt{2}) \times (\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3}) \times (\sqrt{5}+\sqrt{3})}$

$$= \frac{\sqrt{5}-\sqrt{3} + \sqrt{10}-\sqrt{6}}{5-3} + \frac{\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{5-3}$$

$$= \frac{2(\sqrt{5}-\sqrt{6})}{2}$$

$$= \sqrt{5} - \sqrt{6}.$$

27. d $3 - \frac{3+\sqrt{5}}{4} - \frac{1}{(3+\sqrt{5})} \times \frac{(3-\sqrt{5})}{(3-\sqrt{5})}$

$$= 3 - \frac{3+\sqrt{5}}{4} - \frac{3-\sqrt{5}}{9-5}$$

$$= 3 - \left[\frac{3+\sqrt{5}+3-\sqrt{5}}{4} \right]$$

$$= 3 - \frac{3}{2} = \frac{3}{2}.$$

28. c Let $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$

$$\Rightarrow x = \sqrt{6 + x}$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x-3)(x+2) = 0$$

$\therefore x = 3$. (As $x \neq -2$)

29. c $\sqrt{6} \times \sqrt{15} = \sqrt{2 \times 3 \times 3 \times 5}$

$$= 3\sqrt{10}$$

Comparing with $x\sqrt{10}$, we get $x = 3$.

30. b Re-write as,

$$\begin{aligned} & \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{4} + \sqrt{3}} + \frac{1}{\sqrt{5} + \sqrt{4}} + \frac{1}{\sqrt{6} + \sqrt{5}} \\ &= \frac{1}{(\sqrt{3} + \sqrt{2})} \times \frac{(\sqrt{3} - \sqrt{2})}{(\sqrt{3} - \sqrt{2})} + \frac{1(\sqrt{4} - \sqrt{3})}{(\sqrt{4} + \sqrt{3})(\sqrt{4} - \sqrt{3})} \\ &\quad + \frac{1(\sqrt{5} - \sqrt{4})}{(\sqrt{5} + \sqrt{4})(\sqrt{5} - \sqrt{4})} + \frac{1(\sqrt{6} - \sqrt{5})}{(\sqrt{6} + \sqrt{5})(\sqrt{6} - \sqrt{5})} \\ &= \frac{\sqrt{3} - \sqrt{2}}{3-2} + \frac{\sqrt{4} - \sqrt{3}}{4-3} + \frac{\sqrt{5} - \sqrt{4}}{5-4} + \frac{\sqrt{6} - \sqrt{5}}{6-5} \\ &= \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \sqrt{5} - \sqrt{4} + \sqrt{6} - \sqrt{5} \\ &= \sqrt{6} - \sqrt{2} \\ &= \sqrt{2}(\sqrt{3} - 1). \end{aligned}$$

31. a $\frac{7}{4 + \sqrt{2}} \times \frac{(4 - \sqrt{2})}{(4 - \sqrt{2})}$

$$= \frac{7(4 - \sqrt{2})}{16 - 2}$$

$$= \frac{7(4 - \sqrt{2})}{14}$$

$$= \frac{4 - \sqrt{2}}{2} = \frac{4 - 1.4142}{2} = \frac{2.5858}{2}$$

= 1.2929.

32. c $\frac{15}{\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{80} - \sqrt{5}}$

$$= \frac{15}{\sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - 4\sqrt{5} - \sqrt{5}}$$

$$= \frac{15}{3\sqrt{10} - 3\sqrt{5}} = \frac{15}{3\sqrt{5}[\sqrt{2} - 1]}$$

$$= \frac{\sqrt{5}}{\sqrt{2} - 1} \times \frac{(\sqrt{2} + 1)}{(\sqrt{2} + 1)}$$

$$= \frac{\sqrt{10} + \sqrt{5}}{2 - 1} = \sqrt{10} + \sqrt{5}.$$

33. a

$$\frac{1}{(\sqrt{16} - \sqrt{15})} \times \frac{(\sqrt{16} + \sqrt{15})}{(\sqrt{16} + \sqrt{15})} - \frac{1}{(\sqrt{15} - \sqrt{14})} \times \frac{(\sqrt{15} + \sqrt{14})}{(\sqrt{15} + \sqrt{14})}$$

$$+ \frac{1}{(\sqrt{14} - \sqrt{13})} \times \frac{(\sqrt{14} + \sqrt{13})}{(\sqrt{14} + \sqrt{13})} - \frac{1}{(\sqrt{13} - \sqrt{12})} \times \frac{(\sqrt{13} + \sqrt{12})}{(\sqrt{13} + \sqrt{12})}$$

$$+ \frac{1}{(\sqrt{12} - \sqrt{11})} \times \frac{(\sqrt{12} + \sqrt{11})}{(\sqrt{12} + \sqrt{11})} - \frac{1}{(\sqrt{11} - \sqrt{10})} \times \frac{(\sqrt{11} + \sqrt{10})}{(\sqrt{11} + \sqrt{10})}$$

$$+ \frac{\sqrt{10} + \sqrt{9}}{(\sqrt{10} - \sqrt{9})(\sqrt{10} + \sqrt{9})}$$

$$= \frac{\sqrt{16} + \sqrt{15}}{16 - 15} - \frac{\sqrt{15} + \sqrt{14}}{15 - 14} + \frac{\sqrt{14} + \sqrt{13}}{14 - 13} - \frac{\sqrt{13} + \sqrt{12}}{13 - 12}$$

$$+ \frac{\sqrt{12} + \sqrt{11}}{12 - 11} - \frac{\sqrt{11} + \sqrt{10}}{11 - 10} + \frac{\sqrt{10} + \sqrt{9}}{10 - 9}$$

$$= \sqrt{16} + \sqrt{15} - \sqrt{15} - \sqrt{14} + \sqrt{14} + \sqrt{13} - \sqrt{13} \\ - \sqrt{12} + \sqrt{12} + \sqrt{11} - \sqrt{11} - \sqrt{10} + \sqrt{10} + \sqrt{9}$$

$$= \sqrt{16} + \sqrt{9} = 4 + 3 = 7.$$

34. a Let $x = \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$

$$\Rightarrow x = \sqrt{-\sqrt{3} + \sqrt{3 + 8(2 + \sqrt{3})}}$$

$$\Rightarrow x = \sqrt{-\sqrt{3} + \sqrt{16 + 3 + 8\sqrt{3}}}$$

$$\Rightarrow x = \sqrt{-\sqrt{3} + (4 + \sqrt{3})}$$

$$\Rightarrow x = 2.$$

35. a
$$x = \frac{(\sqrt{5}+1) \times (\sqrt{5}+1)}{(\sqrt{5}-1) \times (\sqrt{5}+1)}$$

$$= \frac{5+1+2\sqrt{5}}{5-1} = \frac{3+\sqrt{5}}{2} \quad \dots \text{(i)}$$

How $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$
 $\Rightarrow \sqrt{5}x - x = \sqrt{5} + 1$
 $\Rightarrow \sqrt{5}x - \sqrt{5} = x + 1 \quad (\text{Squaring both sides})$
 $\Rightarrow 5x^2 + 5 - 10x = x^2 + 1 + 2x$
 $\Rightarrow 4x^2 - 12x + 4 = 0 \quad (\text{Divide by 4})$
 $\Rightarrow x^2 - 3x + 1 = 0 \quad (\text{Multiply by 5})$
 $\Rightarrow 5x^2 - 15x + 5 = 0$
 $\Rightarrow 5x^2 - 5x - 1 = 10x - 6 = 9 + 5\sqrt{5}. \quad [\text{from (i)}]$

36. b
$$\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{6-3} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{6-2} + \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{3-2}$$

$$= 2\sqrt{3} - \sqrt{6} - 3\sqrt{2} + \sqrt{6} + 3\sqrt{2} - 2\sqrt{3} = 0.$$

37. a
$$\frac{3+\sqrt{8}}{(3-\sqrt{8})(3+\sqrt{8})} - \frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8}-\sqrt{7})(\sqrt{8}+\sqrt{7})}$$

$$+ \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})} - \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})}$$

$$+ \frac{\sqrt{5}+2}{(\sqrt{5}-2)(\sqrt{5}+2)}$$

$$= \frac{3+\sqrt{8}}{9-8} - \frac{\sqrt{8}+\sqrt{7}}{8-7} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4}$$

$$= 3 + 2 = 5.$$

38. c
$$(\sqrt{2})^2 + (\sqrt{3})^2 + (-\sqrt{5})^2 + 2 \times \sqrt{2} \times \sqrt{3} + 2$$

$$\times \sqrt{3}(-\sqrt{5}) + 2\sqrt{2} \times (-\sqrt{5}) - 2\sqrt{6} + 2\sqrt{15} + 2\sqrt{10}$$

$$(\text{As } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca)$$

$$= 2 + 3 + 5 + 2\sqrt{6} - 2\sqrt{15} - 2\sqrt{10} - 2\sqrt{6} + 2\sqrt{15} + 2\sqrt{10} = 10.$$

39. d
$$x = \sqrt{11} + \sqrt{5} \approx 5.55$$

$$y = \sqrt{10} + \sqrt{6} \approx 5.61$$

$$z = \sqrt{13} + \sqrt{3} \approx 5.33$$

Hence, $y > x > z.$

40. a
$$\frac{1}{(a+b)} = \frac{a+b}{ab}$$

$$\Rightarrow (a+b)^2 - ab = 0$$

$$\Rightarrow a^2 + b^2 + ab = 0$$

Now, $a^3 - b^3 = (a-b)(a^2 + b^2 + ab) = 0.$
(As $a^2 + b^2 + ab = 0$)

41. b
$$9\sqrt{x} = 2\sqrt{3} + 7\sqrt{3}$$

$$\Rightarrow 9\sqrt{x} = 9\sqrt{3}$$

$$\Rightarrow x = 3.$$

42. a
$$\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{5}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$\frac{\sqrt{15}-\sqrt{10}}{3-2} = \sqrt{15} - \sqrt{10}$$

$$\frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} = \frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{3\sqrt{3}(\sqrt{5}-\sqrt{2})}{5-2} = \sqrt{15} - \sqrt{6}$$

$$= \frac{2\sqrt{2}}{\sqrt{5}-\sqrt{3}} = \frac{2\sqrt{2}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$= \frac{2\sqrt{2}(\sqrt{5}-\sqrt{3})}{5-3} = \sqrt{10} - \sqrt{6}$$

∴ Expression

$$= (\sqrt{15} - \sqrt{10}) - (\sqrt{15} - \sqrt{6}) + (\sqrt{10} - \sqrt{6})$$

$$= \sqrt{15} - \sqrt{10} - \sqrt{15} + \sqrt{6} + \sqrt{10} - \sqrt{6}$$

$$= 0$$

43. a
$$\frac{1}{\sqrt{3.25} + \sqrt{2.25}}$$

$$= \frac{1}{(\sqrt{3.25} + \sqrt{2.25})}$$

$$= \frac{1}{(\sqrt{3.25} + \sqrt{2.25})} \times \frac{\sqrt{3.25} - \sqrt{2.25}}{\sqrt{3.25} - \sqrt{2.25}}$$

$$= \frac{\sqrt{3.25} - \sqrt{2.25}}{3.25 - 2.25} = \sqrt{3.25} - \sqrt{2.25}$$

Similarly,

$$\frac{1}{\sqrt{4.25} + \sqrt{3.25}} = \sqrt{4.25} - \sqrt{3.25}$$

$$\frac{1}{\sqrt{5.25} + \sqrt{4.25}} = \sqrt{5.25} - \sqrt{4.25}$$

$$\frac{1}{\sqrt{6.25} + \sqrt{5.25}} = \sqrt{6.25} - \sqrt{5.25}$$

∴ Expression

$$\begin{aligned} &= \sqrt{3.25} - \sqrt{2.25} + \sqrt{4.25} - \sqrt{3.25} + \sqrt{5.25} \\ &\quad - \sqrt{4.25} + \sqrt{6.25} - \sqrt{5.25} \\ &= \sqrt{6.25} - \sqrt{2.25} = 2.5 - 1.5 = 1 \end{aligned}$$

$$44. d \quad \text{First term} = \frac{2}{\sqrt{7} + \sqrt{5}}$$

$$= \frac{2(\sqrt{7} - \sqrt{5})}{(\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5})}$$

$$= \frac{2(\sqrt{7} - \sqrt{5})}{7 - 5} = \sqrt{7} - \sqrt{5}$$

$$\text{Second term} = \frac{7}{\sqrt{12} - \sqrt{5}}$$

$$= \frac{7(\sqrt{12} + \sqrt{5})}{(\sqrt{12} - \sqrt{5})(\sqrt{12} + \sqrt{5})}$$

$$= \frac{7(\sqrt{12} + \sqrt{5})}{12 - 5}$$

$$= \frac{7(\sqrt{12} + \sqrt{5})}{7} = \sqrt{12} + \sqrt{5}$$

$$\text{Third term} = \frac{5}{\sqrt{12} - \sqrt{7}}$$

$$= \frac{5(\sqrt{12} + \sqrt{7})}{(\sqrt{12} - \sqrt{7})(\sqrt{12} + \sqrt{7})}$$

$$= \frac{5(\sqrt{12} + \sqrt{7})}{15 - 7} = \sqrt{12} + \sqrt{7}$$

∴ Expression

$$= (\sqrt{7} - \sqrt{5}) + (\sqrt{12} + \sqrt{5}) - (\sqrt{12} + \sqrt{7})$$

$$= \sqrt{7} - \sqrt{5} + \sqrt{12} + \sqrt{5} - \sqrt{12} - \sqrt{7} = 0$$

$$45. c \quad \frac{1}{\sqrt{3} + \sqrt{4}}$$

$$= \frac{1}{\sqrt{3} + \sqrt{4}} \times \frac{\sqrt{4} - \sqrt{3}}{\sqrt{4} - \sqrt{3}}$$

$$= \frac{\sqrt{4} - \sqrt{3}}{4 - 3} = \sqrt{4} - \sqrt{3}$$

Similarly,

$$\frac{1}{\sqrt{4} + \sqrt{5}} = \sqrt{5} - \sqrt{4} \dots \text{so on}$$

Expression

$$= \sqrt{4} - \sqrt{3} + \sqrt{5} - \sqrt{4} + \sqrt{6} - \sqrt{5}$$

$$+ \sqrt{7} - \sqrt{6} + \sqrt{8} - \sqrt{7} + \sqrt{9} - \sqrt{8}$$

$$= \sqrt{9} - \sqrt{3} = 3 - \sqrt{3}$$

$$46. b \quad \text{Expression}$$

$$= \frac{12}{3 + \sqrt{5} + 2\sqrt{2}}$$

$$= \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{[(3 + \sqrt{5}) + 2\sqrt{2}][(3 + \sqrt{5}) - 2\sqrt{2}]}$$

Rationalising the denominator

$$= \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{(3 + \sqrt{5})^2 - (2\sqrt{2})^2}$$

$$= \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{9 + 5 + 6\sqrt{5} - 8}$$

$$= \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{6\sqrt{5} + 6}$$

$$= \frac{2(3 + \sqrt{5} - 2\sqrt{2})}{\sqrt{5} + 1}$$

$$= \frac{2(3 + \sqrt{5} - 2\sqrt{2})(\sqrt{5} - 1)}{(\sqrt{5} + 1)(\sqrt{5} - 1)}$$

$$= \frac{2(3\sqrt{5} + 5 - 2\sqrt{10} - 3 - \sqrt{5} + 2\sqrt{2})}{5 - 1}$$

$$= \frac{2(2\sqrt{5} + 2\sqrt{2} - 2\sqrt{10} + 2)}{4}$$

$$= \frac{2 \times 2(\sqrt{5} + \sqrt{2} - \sqrt{10} + 1)}{4}$$

$$= 1 + \sqrt{5} + \sqrt{2} - \sqrt{10}$$

47. b Expression

$$\begin{aligned}
 &= 3 + \frac{1}{\sqrt{3}} + \frac{1}{3 + \sqrt{3}} + \frac{1}{\sqrt{3} - 3} \\
 &= 3 + \frac{1}{\sqrt{3}} + \frac{1}{3 + \sqrt{3}} - \frac{1}{3 - \sqrt{3}} \\
 &= 3 + \frac{1}{\sqrt{3}} + \left(\frac{3 - \sqrt{3} - 3 - \sqrt{3}}{(3 + \sqrt{3})(3 - \sqrt{3})} \right) \\
 &= 3 + \frac{1}{\sqrt{3}} + \frac{-2\sqrt{3}}{9 - 3} = 3 + \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{3} \\
 &= 3 + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} = 3
 \end{aligned}$$

48. a $= \frac{2}{\sqrt{5} + \sqrt{3}} = \frac{2(\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})}$
(Rationalising the denominator)

$$= \frac{2\sqrt{5} - \sqrt{3}}{5 - 3} = \sqrt{5} - \sqrt{3}$$

Similarly,

$$\frac{3}{\sqrt{6} - \sqrt{3}} = \frac{3(\sqrt{6} + \sqrt{3})}{6 - 3} = \sqrt{6} + \sqrt{3}$$

$$\frac{1}{\sqrt{6} + \sqrt{5}} = \frac{\sqrt{6} - \sqrt{5}}{6 - 5} = \sqrt{6} - \sqrt{5}$$

∴ Expression

$$= \sqrt{5} - \sqrt{3} + \sqrt{6} + \sqrt{3} + \sqrt{6} - \sqrt{5}$$

$$= 2\sqrt{6}$$

49. c $\frac{2}{3 - \sqrt{7}} - \frac{2}{\sqrt{7} - \sqrt{5}} + \frac{2}{\sqrt{5} - \sqrt{3}} - \frac{2}{\sqrt{3} - 1}$

$$= \frac{2(3 + \sqrt{7})}{9 - 7} - \frac{2(\sqrt{7} + \sqrt{5})}{7 - 5} + \frac{2(\sqrt{5} + \sqrt{3})}{5 - 2} - \frac{2(\sqrt{3} - 1)}{3 - 1}$$

$$= 3 + \sqrt{7} - \sqrt{7} - \sqrt{5} + \sqrt{5} + \sqrt{3} - \sqrt{3} - 1$$

$$= 3 - 1 = 2.$$

50. b $\frac{2\sqrt{3}(\sqrt{7} + \sqrt{5})}{7 - 5} - \frac{4\sqrt{3}(\sqrt{7} - \sqrt{5})}{7 - 3} - \frac{2\sqrt{3}(\sqrt{3} - \sqrt{5})}{3 - 5}$

$$= \sqrt{3}(\sqrt{7} + \sqrt{5}) - \sqrt{3}(\sqrt{7} - \sqrt{5}) + \sqrt{3}(\sqrt{3} - \sqrt{5})$$

$$= 3 + 3 = 6.$$

51. c Here, $2^{2x-y} = 2^4 \Rightarrow 2x - y = 4$

and $2^{x+y} = 2^5 \Rightarrow x + y = 5$

$$\Rightarrow x = 3 \text{ and } y = 2$$

Hence, $x \times y = 6$.

52. c Here, $3^{x+y} = 3^4 \Rightarrow x + y = 4$

$$3^{4(x-y)} = 3 \Rightarrow x - y = \frac{1}{4}$$

$$\Rightarrow x = \frac{17}{8} \text{ and } y = \frac{15}{8}.$$

53. c Here, $n^3 - n = n(n - 1)(n + 1)$
Product of three consecutive integers is always divisible by 6.

54. b Here, $5^{5x+5} = 5^\circ$
 $\Rightarrow 5x + 5 = 0$
 $\Rightarrow x = -1.$

55. a $a = (2^4)^{10} = (16)^{10},$
 $b = (3^3)^{10} = (27)^{10},$
 $c = (5^2)^{10} = (25)^{10}$
Hence, $a < c < b.$

56. d LCM of 4, 5, 10 and 2 is 20.

Hence, among $(13^5)^{\frac{1}{20}}, (16^4)^{\frac{1}{20}}, (41^2)^{\frac{1}{20}}$ and $(8^{10})^{\frac{1}{20}}$
 $(8^{10})^{\frac{1}{20}}$ or $\sqrt[4]{8}$ is greatest.

$$(256)^{\frac{1}{4}(-\frac{3}{2})} = (2^8)^{2(-\frac{3}{2})} = (2^8)^{\frac{1}{8}} = 2.$$

58. d LCM of 6, 3, 2 and 4 is 12.

Hence, among $(12^2)^{\frac{1}{12}}, (4^4)^{\frac{1}{12}}, (3^6)^{\frac{1}{12}}$ and $(5^3)^{\frac{1}{12}}$
Hence, $(5^3)^{\frac{1}{12}}$ or $\sqrt[4]{5}$ is smallest.

$$\begin{aligned}
 59. b \quad &(16)^{0.16} \times (16)^{0.04} \times (2)^{0.2} \\
 &= (2^4)^{0.16} \times (2^4)^{0.04} \times (2)^{0.2} \\
 &= 2^{0.64} \times 2^{0.16} \times 2^{0.2} \\
 &= (2)^{0.64+0.16+0.2} = 2
 \end{aligned}$$

$$\begin{aligned}
 60. a \quad &(64)^{\frac{-2}{3}} \times \left(\frac{1}{4}\right)^{-2} \\
 &= \frac{1}{(64)^{\frac{2}{3}}} \times (4)^2 \\
 &= \frac{1}{(4)^{\frac{3 \times 2}{3}}} \times 4^2 = \frac{1}{4^2} \times 4^2 = 1
 \end{aligned}$$

61. d Expression

$$= \left[8 - \left(\frac{\frac{9}{4^4} \sqrt{2 \times 2^2}}{2\sqrt{2^{-2}}} \right)^{\frac{1}{2}} \right]$$

$$= \left[8 - \left(\frac{(2)^{\frac{2 \times 9}{4} \times 2^2}}{2 \times (2^{-2})^{\frac{1}{2}}} \right)^{\frac{1}{2}} \right]$$

$$= \left[8 - \left(\frac{2^{\frac{9}{2}} \times 2^2}{2^1 \times 2^{-1}} \right)^{\frac{1}{2}} \right]$$

$$= \left[8 - \left(\frac{2^{\frac{9+3}{2}} \times 2^{\frac{3}{2}}}{2^{1-1}} \right)^{\frac{1}{2}} \right]$$

$$= \left[8 - (2^6)^{\frac{1}{2}} \right] = (8 - 2^3) = 8 - 8 = 0$$

62. c

$$\left[\left(\sqrt[5]{x^{-3/5}} \right)^{-5} \right]^5$$

$$= \left(x^{\frac{-3}{5}} \right)^{\frac{1}{5} \times -5 \times 5} = x^{\frac{-3 \times -5}{5}} = x.$$

63. b Expression

$$= \frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$$

$$= \frac{(3^5)^{\frac{n}{5}} \times 3^{2n+1}}{(3^2)^n \times 3^{n-1}} = \frac{(3)^{\frac{5n}{5}} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$$

$$= \frac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \frac{3^{3n+1}}{3^{3n-1}} = 3^{3n+1-3n+1} = 3^2 = 9.$$

$[a^m \times a^n = a^{m+n}, a^m \div a^n = a^{m-n}, (a^m)^n = a^{mn}]$

64. a The orders of the given surds are 3, 2, 4 and 6.

Their LCM = 12

Now we convert each surd into a surd of order 12.

$$\sqrt[3]{9} = (9)^{\frac{1}{3}} = (9^4)^{\frac{1}{12}}$$

$$= \sqrt[12]{6561}$$

Similarly,

$$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$$

$$\sqrt[4]{16} = \sqrt[12]{16^3} = \sqrt[12]{4096}$$

$$\sqrt[6]{80} = \sqrt[12]{80^2} = \sqrt[12]{6400}$$

$$\sqrt[12]{729} < \sqrt[12]{4096} < \sqrt[12]{6400} < \sqrt[12]{6561}$$

$\therefore \sqrt[3]{9}$ is the greatest number.

65. a LCM indices = LCM of 3, 6, 4 and 2 = 12

$$\therefore \sqrt[3]{4} = (4)^{\frac{1}{3}} = \sqrt[12]{4^4}$$

$$= \sqrt[12]{256}$$

$$\sqrt{2} = (2)^{\frac{1}{2}} = \sqrt[12]{2^6} = \sqrt[12]{64}$$

$$\sqrt[6]{3} = \sqrt[12]{3^2} = \sqrt[12]{9}$$

$$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$$

Clearly, $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$.

66. d $(36)^{\frac{1}{6}} = 6^{\frac{2 \times 1}{6}} = \sqrt[3]{6}.$

67. a $\left[\sqrt[3]{\sqrt[5]{5^9}} \right]^4 = \left[5^{\frac{9 \times 1 \times 1}{6 \times 3}} \right]^4$

$$= \left[5^{\frac{1 \times 4}{2}} \right] = 5^2$$

68. d $3^{x+8} = 3^{3(2x+1)}$
 $\Rightarrow x+8 = 6x+3$
 $\Rightarrow 5x = 5$
 $\therefore x = 1$

69. d $(125)^{2/3} \times (625)^{-1/4} = 5^x$

$$\Rightarrow 5^{\frac{3 \times 2}{3}} \times 5^{\frac{4 \times -1}{4}} = 5^x$$

$$\Rightarrow 5^2 \times 5^{-1} = 5^x$$

$$\Rightarrow 5 = 5^x \Rightarrow x = 1$$

70. c $\left(\frac{3}{4} \right)^3 \times \left(\frac{4}{3} \right)^{-7} = \left(\frac{3}{4} \right)^{2x}$

$$\Rightarrow \left(\frac{3}{4} \right)^3 \times \left(\frac{3}{4} \right)^7 = \left(\frac{3}{4} \right)^{2x}$$

$$\Rightarrow \left(\frac{3}{4} \right)^{10} = \left(\frac{3}{4} \right)^{2x}$$

$$\Rightarrow 2x = 10 \Rightarrow x = 5$$

**Numbers: HCF & LCM, Remainders
Answers and Explanations**

1	a	2	d	3	a	4	b	5	d	6	c	7	b	8	d	9	a	10	c
11	d	12	c	13	b	14	b	15	d	16	b	17	b	18	c	19	c	20	d
21	d	22	c	23	b	24	d	25	c	26	d	27	a	28	c	29	a	30	d
31	b	32	c	33	c	34	c	35	d	36	d	37	a	38	c	39	d	40	d
41	b	42	c	43	b	44	c	45	b	46	c	47	b	48	d	49	c	50	d

1. a $HCF \times LCM = \text{Product of two numbers}$
 $\Rightarrow 12 \times 240 = 48 \times \text{Second number}$
 $\Rightarrow \text{Second number} = \frac{12 \times 240}{48} = 60.$
2. d $513 = 3^3 \times 19$; $1134 = 2 \times 3^4 \times 7$ and $1215 = 3^5 \times 5$
HCF of (513, 1123, 1215) = $3^3 = 27$.
3. a $2436 = 2^2 \times 3 \times 7 \times 29$; $1001 = 7 \times 11 \times 13$ and $105 = 3 \times 5 \times 7$
Greatest number = HCF of (2436, 1001, 105) = 7.
4. b Since 8 is not the divisor of 60. Therefore, 60 cannot be LCM of two numbers.
5. d Product of two numbers = HCF \times LCM
 $16 \times \text{LCM} = 1536 \Rightarrow \text{LCM} = 96$
Now, let the numbers be $16x$ and $16y$.
 $16x \times 16y = 96 \times 16$
 $\Rightarrow xy = 6$
The pairs satisfying the given condition and prime to each other are (1, 6) and (2, 3).
Thus, the numbers are (16, 96) and (32, 48).
Therefore, 102 cannot be one of them.
6. c HCF of $(\frac{9}{10}, \frac{12}{25}, \frac{18}{35}, \text{ and } \frac{21}{40})$
 $= \frac{\text{HCF of } (9, 12, 18, \text{ and } 21)}{\text{LCM of } (10, 25, 35 \text{ and } 40)} = \frac{3}{1400}.$
7. b From options, only 192 and 144 are divisible by 48.
LCM of (96, 144 and 192) = 576
Hence, 192 can be value of N.
8. d $LCM \left(\frac{3}{13}, \frac{4}{39}, \frac{5}{182} \right) = \frac{[LCM (3, 4, 5)]}{[HCF (13, 39, 182)]} = \frac{60}{13}$
 $HCF \left(\frac{3}{13}, \frac{4}{39}, \frac{5}{182} \right) = \frac{[HCF (3, 4, 5)]}{[LCM (13, 39, 182)]} = \frac{1}{546}$

- Difference = $\frac{60}{13} - \frac{1}{546} = \frac{2520 - 1}{546} = \frac{2519}{546}.$
9. a $LCM \times HCF = N_1 \times N_2$ and $N_1 = 100$,
 $\therefore N_2 = \frac{500 \times 50}{100} = 250.$
10. c First number = $2 \times 50 = 100$
2nd number $\times 100 = 25 \times 500$
(Since product of two numbers = HCF \times LCM)
 \Rightarrow 2nd number = 125.
11. d Total number of trees are $48 + 60 + 96 = 204$.
Given that each row has the same number of trees and all trees in a row are of the same type. Since we need to minimise the number of rows, we need to maximise the number of trees in each row.
Number of trees in each row = HCF of 48, 60 and 96 = 12
 \therefore Minimum number of rows = $\frac{204}{12} = 17$.
12. c $LCM = \frac{2160}{6} = 360$
Required ratio = 6 : 360 = 1 : 60.
13. b Let the numbers be $2a$ and $2b$, where a and b are co-prime natural numbers.
Then, $2a \times 2b = 2 \times 84 \Rightarrow a \times b = 42$.
Possible pairs of a and b are (42, 1), (14, 3), (7, 6), (21, 2).
Minimum sum of numbers = $2(a + b) = 2(7 + 6) = 26$.

14. b HCF = $2^3 \cdot 3$.

15. d H.C.F. of the two 2-digit numbers = 16
Hence, the numbers can be expressed as $16x$ and $16y$, where x and y are prime to each other. Now,
First number \times second number
= H.C.F. \times L.C.M.
 $\Rightarrow 16x \times 16y = 16 \times 480$
 $\Rightarrow xy = \frac{16 \times 480}{16 \times 16} = 30$

The possible pairs of x and y , satisfying the condition $xy = 30$ are : (3, 10), (5, 6), (1, 30), (2, 15) Since the numbers are of 2-digits each.
Hence, admissible pair is (5, 6)
 \therefore Numbers are $16 \times 5 = 80$ and $16 \times 6 = 96$.

16. b Let LCM be L and HCF be H . Then,
 $L = 4H$
 $\therefore H + 4H = 125$
 $\Rightarrow 5H = 125$

$$\Rightarrow H = \frac{125}{5} = 25$$

$$\therefore L = 4 \times 25 = 100$$

\therefore Second number

$$= \frac{L \times H}{\text{First number}} = \frac{100 \times 25}{100} = 25.$$

17. b HCF = 13
Let the numbers be $13x$ and $13y$. Where x and y are co-prime.
 $\therefore LCM = 13 xy$
 $\Rightarrow 13 xy = 455$
 $\Rightarrow xy = \frac{455}{13} = 35 = 5 \times 7$
 \therefore Numbers are $13 \times 5 = 65$ and $13 \times 7 = 91$.

18. c First number = $2 \times 44 = 88$
 \therefore First number \times Second number
= H.C.F. \times L.C.M.
 $\Rightarrow 88 \times \text{Second number}$
 $= 44 \times 264$
 $\Rightarrow \text{Second number}$
 $= \frac{44 \times 264}{88} = 132.$

19. c L.C.M. of 4, 6, 8, 12 and 16 = 48
 \therefore Required number = $48 + 2 = 50$

20. d The greatest number of five digits is 99999.
LCM of 3, 5, 8 and 12

2		3, 5, 8, 12
2		3, 5, 4, 6
3		3, 5, 2, 3
		1, 5, 2, 1

$$\therefore LCM = 2 \times 2 \times 3 \times 5 \times 2 = 120$$

After dividing 99999 by 120, we get 39 as remainder
 $99999 - 39 = 99960$

$$= (833 \times 120)$$

99960 is the greatest five digit number divisible by the given divisors.

In order to get 2 as remainder in each case we will simply add 2 to 99960.

$$\therefore \text{Greatest number} = 99960 + 2 = 99962.$$

21. d Required time = LCM of 252, 308 and 198 seconds

2		252, 308, 198
2		126, 154, 99
7		63, 77, 99
9		9, 11, 99
11		1, 11, 11
		1, 1, 1

$$\therefore LCM = 2 \times 2 \times 7 \times 9 \times 11 = 2772 \text{ seconds}$$

= 46 minutes 12 seconds.

22. c LCM of 21, 36 and 66

$$\therefore LCM = 3 \times 2 \times 7 \times 6 \times 11 = 3 \times 3 \times 2 \times 2 \times 7 \times 11$$

\therefore Required number

$$= 3^2 \times 2^2 \times 7^2 \times 11^2 = 213444.$$

23. b LCM of 4, 6, 8, 9

2		4, 6, 8, 9
2		2, 3, 4, 9
3		1, 3, 2, 9
		1, 1, 2, 3

$$\therefore LCM = 2 \times 2 \times 3 \times 2 \times 3 = 72$$

\therefore Required number = 72, because it is exactly divisible by 4, 6, 8 and 9 and it leaves remainder 7 when divided by 13.

24. d LCM of 5, 6 and 8

$$5 = 5$$

$$6 = 3 \times 2$$

$$8 = 2^3$$

$$= 2^3 \times 3 \times 5 = 8 \times 15 = 120$$

$$\text{Required number} = 120k + 3$$

$$\text{when } k = 2, 120k + 3 = 120 \times 2 + 3 = 243$$

It is completely divisible by 9.

25. c LCM of 9, 10 and 15 = 90

\Rightarrow The multiple of 90 are also divisible by 9, 10 or 15.

$$\therefore 21 \times 90 = 1890 \text{ will be divisible by them.}$$

\therefore Now, 1897 will be the number that will give remainder 7.

$$\text{Required number} = 1936 - 1897 = 39.$$

26. d $1\frac{1}{2}$ hours = 90 minutes

1 hour and 45 minutes = 105 minutes

1 hour = 60 minutes

\therefore LCM of 30 minutes, 60 minutes, 90 minutes and 105 minutes

3	30, 60, 90, 105
5	10, 20, 30, 35
2	2, 4, 6 7
	1, 2, 3, 7

LCM = $3 \times 5 \times 2 \times 2 \times 3 \times 7 = 1260$ minutes

1260 minutes = $\frac{1260}{60} = 21$ hours

\therefore The bell will again ring simultaneously after 21 hours.

\therefore Time will be

= 12 noon + 21 hours = 9 a.m.

27. a The greatest number N = HCF of $(1305 - x)$, $(4665 - x)$ and $(6905 - x)$, where x is the remainder

= HCF of $(4665 - 1305)$, $(6905 - 4665)$ and $(6905 - 1305)$

= HCF of 3360, 2240 and 5600 = 1120

$\therefore N = 1120$

Sum of digits = $1 + 1 + 2 + 0 = 4$.

28. c Required maximum capacity of container

= HCF of 75 litres and 45 litres

Now, $75 = 5 \times 5 \times 3$

$45 = 5 \times 3 \times 3$

\therefore HCF = 15 litres.

29. a Let the numbers be $2x$, $3x$ and $4x$ respectively.

\therefore HCF = $x = 12$

\therefore Numbers are : $2 \times 12 = 24$

$3 \times 12 = 36$, $4 \times 12 = 48$

LCM of 24, 36, 48 = $2 \times 2 \times 2 \times 3 \times 3 \times 2 = 144$.

30. d Let HCF be h and LCM be ℓ .

Then, $\ell = 84h$ and $\ell + h = 680$

$\Rightarrow 84h + h = 680$

$\Rightarrow h = \frac{680}{85} = 8$

$\therefore \ell = 680 - 8 = 672$

\therefore Other number = $\frac{672 \times 8}{56} = 96$.

31. b $52563744 = 2 \times 3 \times 4 \times 2190156$

It is divisible by 24. Hence, remainder = 0.

32. c Divisor = $5 \times 46 = 230$

\therefore Quotient = $\frac{230}{10} = 23$

\therefore Dividend = $23 \times 230 + 46 = 5336$.

33. c $N = 6x + 1$ and $N = 7y + 1$

$\Rightarrow 6x + 1 = 7y + 1$

$\Rightarrow 6x - 7y = 0$

By hit and trial $x = 7$ and $y = 6$.

Therefore, $N = 6 \times 7 + 1 = 43$.

34. c A number is divisible by 9 if the sum of its digits is divisible by 9.

Sum of digits = $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 2 + 2 + 3 = 50$

When we divide 50 by 9, then remainder = 5.

35. d 3^5 when divided by 4 leaves a remainder of 3 whereas 5^6 leaves a remainder of 1 when divided by 4. So, the sum of these two numbers will be divisible by 4.

36. d Let N be the number. Then,

$N = 6x + 5$ and $N = 5y + 3$

$\therefore 6x + 5 = 5y + 3$

$\Rightarrow 5y - 6x = 2$

By hit and trial for smallest three digit number.

$x = 18$ and $y = 22$

Hence, $N = 6 \times 18 + 5 = 113$.

37. a Cyclicity of 2 is 4, then last digit of 2^{31} is $2 \times 2 \times 2 = 8$.

When we divide 2^{31} by 5, we get remainder = 3.

38. c When n is divided by 4, then remainder = 3

$$\therefore \frac{2n+1}{4} = \frac{n}{4} + \frac{n}{4} + \frac{1}{4} \Rightarrow \frac{3+3+1}{4}$$

\Rightarrow Remainder = 3.

39. d 9^2 when divided by 5 leaves a remainder of 1.

Therefore, 9^{83} will leave a remainder of 4 when divided by 5. 6 divided by 5 leaves a remainder of 1 therefore, 6^{53} will also leave a remainder of 1 when divided by 5. Total remainder will be $(4 \times 1) = 4$.

40. d Let N be the number.

Then, $N = 6x + 3$

$\therefore N^2 = 36x^2 + 9 + 36x$

$\Rightarrow N^2 = 6(6x^2 + 1 + 6x) + 3$

Hence, N^2 is divided by 6, remainder = 3.

41. b $353 = 44 \times 8 + 1 \Rightarrow a = 1$

and $353 = 27 \times 13 + 2 \Rightarrow b = 2$

$\therefore a - b = 1 - 2 = -1$.

42. c $\text{Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

$$= \frac{145227 - 159}{628} = 231.$$

43. b $6^{50} = (6^3)^{16} \times 6^2 = (216)^{16} \times 36 = (215 + 1)^{16} \times 36$
When this is divided by 215 remainder will be
 $(1)^{16} \times 36 = 36$.

44. c As $35 - 18 = 45 - 28 = 55 - 38 = 17$
Hence, required number = L.C.M (35, 45, 55) - 17
= 3448.

45. b When the second divisor is factor of first divisor, the second remainder is obtained by dividing the first remainder by the second divisor.
Hence, on dividing 29 by 8, the remainder is 5.

46. c The least number X in this case will be determined as follows :

4	X
5	Y - 1
	1 - 4

$$\begin{aligned} Y &= 5 \times 1 + 4 = 9 \\ X &= 4 \times Y + 1 = 4 \times 9 + 1 = 37 \end{aligned}$$

Now

5	37
4	7 - 2
	1 - 3

Hence, the respective remainders are 2, 3.

47. b By the Binomial expansion we have
 $(x + 1)^n = x^n + {}^nC_1 x^{n-1} + {}^nC_2 x^{n-2} + \dots + {}^nC_{n-1} x + 1$
Here, each term except the last term contains x.
Obviously, each term except the last term is exactly divisible by x.
Following the same logic,
 $7^{19} = (6 + 1)^{19}$ has each term except last term divisible by 6.
Hence, $7^{19} + 2$ when divided by 6 leaves remainder.
 $= 1 + 2 = 3$.

48. d Let the least number be x.

13	x	Remainder
5	y	1
	1	3

$$\begin{aligned} y &= 5 \times 1 + 3 = 8 \\ x &= 13 \times 8 + 1 = 105 \end{aligned}$$

On dividing 105 by 65, remainder = 40.

49. c $(17)^{200} = (18 - 1)^{200}$

We know that

$$\begin{aligned} \therefore x^n + nx^{n-1} \cdot a + \frac{n(n-1)}{1 \times 2} x^{n-2} a^2 \\ + \frac{n(n-1)(n-2)}{1 \times 2 \times 3} x^{n-3} a^3 + \dots + a^n \end{aligned}$$

We see that all the terms on the RHS, except a^n has x as one of its factor and hence are divisible by x. So, $(x + a)^n$ is divisible by x or not will be decided by a^n .

Let x = 18, a = -1
and n = 200

$\therefore (18 - 1)^{200}$ is divisible by 18 or not will depend on $(-1)^{200}$ as all other terms in its expansion will be divisible by 18 because each of them will have 18 as one of their factor.

$$(-1)^{200} = 1 (\because 200 \text{ is even})$$

1 is not divisible by 18 and is also less than 18.
 $\therefore 1$ is the remainder.

50. d If the remainder be x, then $(11284 - x) - (7655 - x)$

$= 3629$ is divisible by that number.

$$3629 = 19 \times 191$$

Hence, required number = 191

Sum of digits = $1 + 9 + 1 = 11$.

Numbers : Divisibility, Unit's Digit Cyclicity
Answers and Explanations

1	b	2	b	3	b	4	b	5	b	6	a	7	d	8	b	9	a	10	b
11	d	12	b	13	b	14	a	15	b	16	b	17	b	18	d	19	d	20	c
21	a	22	d	23	a	24	c	25	a	26	c	27	d	28	b	29	c	30	d
31	c	32	c	33	d	34	d	35	d	36	b	37	c	38	d	39	b	40	c
41	a	42	b	43	d	44	d	45	d	46	d	47	d	48	a	49	b	50	a

1. b Numbers divisible by 7 between 11 and 90 are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77 and 84.
Hence, 11 numbers are there.
2. b \therefore The given number is divisible by 5.
 $\therefore b = 0$ or 5.
 \therefore The given number is divisible by 3.
 $\therefore a = 2$ or 5; if $b = 0$
and $a = 3$ or 6 or 9 if $b = 5$
From options, $a = 6$ and $b = 5$.
3. b The sum of three consecutive odd numbers is always divisible by 3.
For example: $1 + 3 + 5 = 9$
It is divisible by 3.
4. b The number 311311 is divisible by 11 but not divisible by 3.
5. b We know that 143 is nothing but 11×13 . Thus, check the divisibility of the numbers given in the answer options by 11 and 13 both.
6. a For a number to be divisible by 132 it should be divisible by 11 and 12 or it should be divisible by 11, 3 and 4. Only 264, 396, 792 and 6336 are completely divisible by 132.
7. d If a number is divisible by 72, it has to be divisible by 8 and 9 both. For a number to be divisible by 8, the number formed by its last three digits must be divisible by 8, with that logic Y is definitely 2. Now for a number to be divisible by 9, sum of its digits must be divisible by 9. Sum of the digits of given number is $(22 + X + Y)$ that is $(24 + X)$, for this sum to be divisible by 9 'X' must be 3.
8. b $8961 = 106 \times 84 + 57$
 \therefore The least number must be added to $= 84 - 57 = 27$.
9. a Sum of the digits at odd places of the given number is $(4 + 6 + 0) = 10$ while the sum of the digits at even places of the given number is $(3 + 5 + 7) = 15$. Difference between the sums is -5 i.e. $(10 - 15)$. It simply means that if 5 will be added in this number or if 6 is subtracted from this number it will become a multiple of 11. Now $6 + 11 = 17$, means even after subtracting 17 from this number we will get a multiple of 11.
10. b $11158 = 144 \times 77 + 70$
 $\therefore 77 - 70 = 7$ be added to make it exactly divisible by 77.
11. d $9487 = 98 \times 96 + 79$
 $\therefore 98 - 79 = 19$ must be added to 9487 to make it exactly divisible by 98.
12. b Let the two-digit number be $(10x + y)$.
Then, $(10x + y) - (10y + x) = 7n$
 $\Rightarrow 9(x - y) = 7n$
If $9(x - y)$ is to be a multiple of 7, then, $(x - y)$ has to be equal to 7.
 \therefore For (x, y) , possible values are $(9, 2)$ and $(7, 0)$.
[Note: It is not given anywhere that 'M' is also a two digit number so 'y' can be equal to 0]
13. b $4^{31} + 4^{32} + 4^{33} + 4^{34} = 4^{31} (1 + 4 + 4^2 + 4^3)$
 $= 4^{31} \times 85$
 $= 4^{30} \times 4 \times 85$
 $= 4^{30} \times 340$
Which is clearly divisible by 10.
14. a $1000 = (45 \times 22) + 10$
 $\therefore 45 - 10 = 35$ to be added.
15. b Number = $xy\ xy\ xy$
 $= xy \times 10000 + xy \times 100 + xy$
 $= xy (10000 + 100 + 1)$
 $= xy \times 10101$

16. b Let the numbers be x and y .

$$\therefore xy = 11520$$

$$\text{and } \frac{x}{y} = \frac{9}{5}$$

$$\therefore xy \times \frac{x}{y} = 11520 \times \frac{x}{y}$$

$$\Rightarrow x^2 = 2304 \times 9$$

$$\Rightarrow x = \sqrt{2304 \times 9}$$

$$\Rightarrow 48 \times 3 = 144$$

$$\text{From } \frac{x}{y} = \frac{9}{5}$$

$$\Rightarrow y = \frac{5 \times 144}{9} = 80$$

$$\therefore \text{Required difference} = 144 - 80 = 64$$

17. b Let the given number be x .

Then,

$$\left(x \times \frac{3}{2} \right) - \left(x + \frac{3}{2} \right) = 10$$

$$\Rightarrow \frac{3}{2}x - \frac{2}{3}x = 10$$

$$\Rightarrow \frac{9x - 4x}{6} = 10$$

$$\Rightarrow 5x = 60 \Rightarrow x = 12$$

18. d Let the required number of persons be x .

According to the question,
 $2x^2 = 3042$

$$\Rightarrow x^2 = \frac{3042}{2} = 1521$$

$$\Rightarrow x = \sqrt{1521} = 39$$

19. d Sum of first 60 numbers

$$= \frac{60(60+1)}{2} = \frac{60 \times 61}{2} = 1830$$

The number 1830 is divisible by 61.

20. c Let the numbers be x and y and x is greater than y . Then,

$$xy = 9375 \quad \dots \text{(i)}$$

$$\frac{x}{y} = 15 \Rightarrow x = 15y$$

$$15y \times y = 9375 \quad [\text{From (i)}]$$

$$\Rightarrow y^2 = \frac{9375}{15} = 625$$

$$\Rightarrow y = \sqrt{625} = 25$$

$$\therefore x = 15y = 15 \times 25 = 375$$

$$\therefore x + y = 375 + 25 = 400.$$

21. a Let the number be $10x + y$.

After interchanging the digits, the number obtained

$$= 10y + x$$

Resulting number

$$= 10x + y + 10y + x$$

$$= 11x + 11y$$

$= 11(x + y)$ which is exactly divisible by 11.

$$2^{6n} - 4^{2n} = (2^6)^n - (4^2)^n = 64^n - 16^n$$

which is divisible by $64 - 16 = 48$.

$$4^{61} + 4^{62} + 4^{63}$$

$$= 4^{61}(1 + 4 + 4^2)$$

$= 4^{61} \times 21$ which is divisible by 3.

24. c Required Number

$$= 100x + 10y + z$$

$$\therefore 10y + z = 6m$$

\therefore Number $= 100x + 6m$, where m is a positive integer

$$= 2(50x + 3m).$$

25. a LCM of 4, 5 and 6 = 60

Quotient on dividing 800 by 60 = 13

Quotient on dividing 400 by 60 = 6

\therefore Required answer $= 13 - 6 = 7$

Alternate Method:

First number greater than 400 that is divisible by 60 = 420

Smaller number than 800 that is divisible by 60 = 780

It is an Arithmetic Progression with common difference = 60

$$\text{by } t_n = a + (n - 1)d$$

$$780 = 420 + (n - 1) \times 60$$

$$\Rightarrow (n - 1) \times 60 = 780 - 420 = 360$$

$$\Rightarrow (n - 1) = 360 + 60 = 6$$

$$\Rightarrow n = 6 + 1 = 7.$$

$$26. c 5^{71} + 5^{72} + 5^{73}$$

$$= 5^{71}(1 + 5 + 5^2)$$

$= 5^{71} \times 31$ which is exactly divisible by 155.

$$27. d 3^{25} + 3^{26} + 3^{27} + 3^{28}$$

$$= 3^{25}(1 + 3 + 3^2 + 3^3)$$

$$= 3^{25}(1 + 3 + 9 + 27)$$

$= 3^{25} \times 40$, which is clearly divisible by 30.

$$28. b n^3 - n = n(n^2 - 1)$$

$$= n(n + 1)(n - 1)$$

For $n = 2$, $n^3 - n = 6$

29. c Number $= 100x + 10y + z$

Sum of digits $= x + y + z$

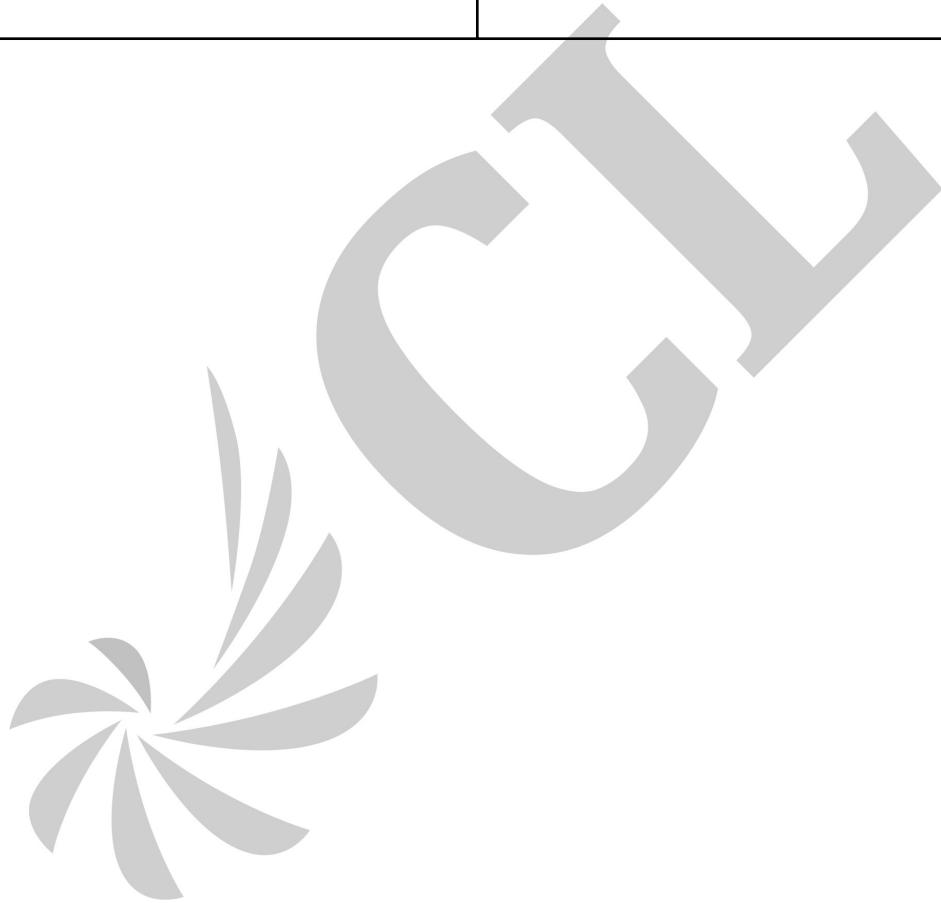
Difference $= 100x + 10y + z - x - y - z$

$$= 99x + 9y = 9(11x + y)$$

30. d For $n = 1$
 $n^4 + 6n^3 + 11n^2 + 6n + 24$
 $= 1 + 6 + 11 + 6 + 24 = 48$
For $n = 2$
 $n^4 + 6n^3 + 11n^2 + 6n + 24$
 $= 16 + 48 + 44 + 12 + 24$
 $= 144$ which is divisible by 48.
Clearly, 48 is the required number.
31. c The cyclicity of powers of 7 is 4 and the unit digit repeat in the order of (7, 9, 3, 1).
Now, the remainder when 173 is divided by 4 will be 1. Hence, its unit digit will be equal to the unit digit of 7 i.e. 7.
32. c The cyclicity of powers of 4 is 2. For every odd power, the unit digit is 4 and for every even power the unit digit is 6. For 424^{782} the power is even and thus its last digit will be 6.
Similarly, for every odd power of 9, the unit digit is 9 and for every even power, it is 1. Now, for 179^{137} , the power is odd and thus its last digit will be 9.
Adding, the last digit of the number, we get $6 + 9 = 15$. Hence, 5 will be at unit place of $424^{782} + 179^{137}$.
33. d Product = $(727)^{57} \times (621)^{23}$
 \Rightarrow Unit digit = (... 7) \times (... 1) = 7.
34. d Unit digit is governed by product of individual unit digit only.
Unit digit of $(9 \times 8 \times 7 \times 6) = 4$.
35. d Cyclicity of 7 and 3 is 4, and that of 6 is 1.
Hence, unit digit of the product = unit digit of $3 \times 6 \times 3 = 4$.
36. b Cyclicity of 3 and 7 is 4, and that of 1 is 1.
Hence, unit digit of the product = $1 \times 1 \times 3 = 3$.
37. c Cyclicity of 2 is 4.
Hence, unit digit of $(22)^{23}$ = unit digit of $(2)^3 = 8$.
38. d $(124)^{372}[1 + 124] = (124)^{372} \times 125$
Hence, unit digit is 0.
39. b Unit digit = unit digit of $[1 + 4 + 7 + 6 + 5 + 6] = 9$.
40. c Unit digit in $3^4 = 1$
So, unit digit in $3^{164} = 1$
Now, unit digit in $(2153)^{167}$
= unit digit in 3^{167}
= unit digit in $3^3 = 7$
41. a $(4)^{2m}$ gives 6 at unit digit.
 $(4)^{2m+1}$ gives 4 at unit digit.
 $(5)^n$ gives 5.
The same is the case with 1 $\Rightarrow 1^n = 1$.
 \therefore Required digit = Unit's digit in the product of $4 \times 5 \times 1 = 0$.
42. b $(251)^{98} = \dots 1$
 $(21)^{29} = \dots 1$
 $(106)^{100} = \dots 6$
 $(705)^{35} = \dots 5$
 $(16)^4 = \dots 6$
 $259 = \dots 9$
 \therefore Required answer = $1 + 1 - 6 + 5 - 6 + 9 = 16 - 12 = 4$.
43. d Expression = $(2137)^{754}$
Unit digit in 2137 = 7
Now, $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$, $7^5 = 16807$,
Clearly, after index 4, the unit digit follow the same order.
Dividing index 754 by 4 we get remainder = 2
 \therefore Unit digit in the expansion of $(2137)^{754}$ = Unit digit in the expansion of $(2137)^2 = 9$.
44. d $4^1 = 4$; $4^2 = 16$; $4^3 = 64$; $4^4 = 256$; $4^5 = 1024$
Remainder on dividing 372 by 4 = 0
Remainder on dividing 373 by 4 = 1
 \therefore Required unit digit
= Unit digit of the sum of 6 + 4 = 0.
45. d $7^1 = 7$; $7^2 = 49$; $7^3 = 343$; $7^4 = 2401$; $7^5 = 16807$
i.e., The unit digit repeats itself after power 4.
Remainder after we divide 245 by 4 = 1
 \therefore Unit digit in the product of $(4387)^{245} \times (621)^{75}$
= Unit digit in the product of $(4387)^1 \times (621)^{72}$
= $7 \times 1 = 7$
46. d $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$
 $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$
i.e., the digit at unit's place gets repeated after power 4. Unit place 6 remains same for any power.
 \therefore Required unit's place digit = Unit's place digit in the product of $7^3 \times 6 \times 3^1 = 4$.
47. d Unit's place digit in $3 \times 38 \times 537 \times 1256$
= Unit's place digit in $3 \times 8 \times 7 \times 6 = 4 \times 2 = 8$.
48. a Required unit's place digit
= Unit's place digit in the product of $7 \times 5 \times 8 \times 3 \times 9 = 0$.

49. b Let the number be 100
 $(2x) + 10y + x = 201x + 10y$... (i)
 $\therefore 2x + y + x = 18$
 $\Rightarrow 3x + y = 18$... (ii)
- When the digits are reversed,
number = 100 (x) + 10y + 2x
 $= 102x + 10y$... (iii)
 $\therefore 201x + 10y - 102x - 10y = 396$
 $\Rightarrow 99x = 396 \Rightarrow x = 4$
 \therefore From equation (i),
 $3 \times 4 + y = 18 \Rightarrow y = 18 - 12 = 6$
 \therefore Required difference = $2x - y = 2 \times 4 - 6 = 2$.

50. a Let the two digit number be $10y + x$ where $x > y$
 $\therefore 10x + y - 10y - x = 63$
 $\Rightarrow 9x - 9y = 63 \Rightarrow x - y = 7$
 $\therefore x = 7, 8, 9$ and $y = 0, 1, 2$



Percentage

Answers and Explanations

1	b	2	d	3	c	4	b	5	a	6	c	7	b	8	d	9	c	10	a
11	d	12	b	13	d	14	a	15	a	16	b	17	c	18	b	19	b	20	d
21	c	22	b	23	a	24	b	25	c	26	c	27	b	28	b	29	b	30	d
31	d	32	b	33	b	34	c	35	b	36	b	37	c	38	b	39	a	40	d
41	e	42	e	43	d	44	c	45	b	46	d	47	b	48	d	49	d	50	b

1. b $28\% \text{ of } 450 + 45\% \text{ of } 280$

$$= \frac{28}{100} \times 450 + \frac{45}{100} \times 280 \\ = \frac{28 \times 45 \times (10 + 10)}{100} = 28 \times 9 = 252.$$

2. d Let the third number be x . Then,

$$\text{First number} = 30\% \text{ of } x = \frac{3}{10}x$$

$$\text{and second number} = 40\% \text{ of } x = \frac{4}{10}x$$

$$\therefore \text{Percentage value} = \frac{\text{First number}}{\text{Second number}} \times 100$$

$$= \frac{\frac{3}{10}x}{\frac{4}{10}x} \times 100 = \frac{3}{4} \times 100 = 75\%.$$

Alternate:

$$\text{Percentage value} = \frac{30}{40} \times 120 = 75\%.$$

3. c Area of rectangle = $L \times B$; New length = $1.2 L$
 New area = $1.2 L \times B$
 Increase in area = 20% .

4. b (i) $20\% \text{ of } x = 10$

$$\Rightarrow \frac{20}{100} \times x = 10 \Rightarrow x = 50.$$

(ii) $8\% \text{ of } x = 90$

$$\Rightarrow \frac{8}{100} \times x = 90 \Rightarrow x = 1125$$

(iii) $15\% \text{ of } x = 15$

$$\Rightarrow \frac{15}{100} \times x = 15 \Rightarrow x = 100$$

(iv) $17.5\% \text{ of } x = 35$

$$\Rightarrow \frac{17.5}{100} \times x = 35 \Rightarrow x = 200.$$

5. a Let the initial money that Chandra had be x .
 Balance left with Chandra = $x - 20\% \text{ of } x - 10\% \text{ of } x - 9\% \text{ of } x - 7\% \text{ of } x = 2,700$
 $\Rightarrow 0.54x = 2700$
 $\Rightarrow x = ₹5,000.$

6. c $16\frac{2}{3}\% = \frac{50}{3} \times \frac{1}{100} = \frac{1}{6} = 0.1\bar{6}$

$$\frac{2}{15} = 0.1\bar{3}.$$

Hence, 0.17 is the greatest of amongst the given three.

7. b Let the total number of votes be x .
 Winner's percentage votes = $84\% \text{ of } x = 0.84x$
 Percentage votes of rest of candidates
 $= x - 0.84x = 0.16x$
 Majority = Difference of votes between winner and rest = $0.84x - 0.16x = 476$

$$\Rightarrow 0.68x = 476 \Rightarrow x = \frac{476 \times 100}{68} = 700.$$

8. d $20\% \text{ of total votes} = 20,000 \text{ votes}$
 $\Rightarrow \text{Total votes} = 2,00,000.$

9. c Runs by boundaries and sixes
 $= (3 \times 4) + (8 \times 6) = 60$
 Total number of runs scored = 110
 $\therefore \text{Runs scored by running between the wickets} = 110 - 60 = 50$
 Percentage of runs scored by running between the wickets
 $= \frac{50}{110} \times 100 = 45\frac{5}{11}\%.$

10. a Let the total number of births be 100. Then,
 Total number of twins = (5% of 100) \times 2 = 10
 Total number of single children = 95% of 100 = 95
 Total number of children = 10 + 95 = 105
- $$\text{Percentage of twins} = \frac{10}{105} \times 100 = 9.52\%.$$
11. d Let the fraction be $\frac{x}{y}$.
 Then, $\frac{x + 15\% \text{ of } x}{y - 8\% \text{ of } y} = \frac{15}{16}$

$$\Rightarrow \frac{1.15x}{0.92y} = \frac{15}{16} \Rightarrow \frac{x}{y} = \frac{15}{16} \times \frac{0.92}{1.15} = \frac{3}{4}.$$
12. b Let Q = x, then P = 6x.
 Percent of Q less than P = $\frac{P - Q}{P} \times 100$

$$= \frac{6x - x}{6x} \times 100 = 83\frac{1}{3}\%.$$
13. d Let 100 be the maximum marks for each subject. Then, from option (d),

$$\frac{90 + 95 + 95 + x}{4} = 96 \Rightarrow x = 104$$
 which is not possible.
 Hence, his average % marks cannot be 96%.
14. a $X = 37.5\% \text{ of } 20\% \text{ of } 48 = \frac{3}{8} \times \frac{1}{5} \times 48 = 3.6$

$$Y = 14.28\% \text{ of } 27.27\% \text{ of } 77 = \frac{1}{7} \times \frac{3}{11} \times 77 = 3$$

$$\therefore X > Y.$$
15. a Passing marks = $\frac{42}{100} \times 250 = 105$
 He is failed by $105 - 95 = 10$ marks.
16. b Let x be the number. Then,

$$\frac{74}{100} \times x = 555 \Rightarrow x = 750$$

$$\therefore 44\% \text{ of } 750 = \frac{44}{100} \times 750 = 330.$$
17. c Let C's salary be ₹100.
 $\Rightarrow B's \text{ salary} = 25\% \text{ of } C's \text{ salary}$
 $= 25\% \text{ of } 100 = ₹25$
 $A's \text{ salary} = 40\% \text{ of } B's \text{ salary}$
 $= \frac{40}{100} \times 25 = ₹10$
 Hence, A's salary is 10% of C's salary.
18. b Let maximum marks be x.
 Then, $\frac{20}{100}x + 10 = \frac{42}{100}x - 12 \Rightarrow x = 100.$
19. b A is 25% more than B,
 Take B = 100, then A = 125
 $\therefore B \text{ is less than } A \text{ by} = \frac{25}{125} \times 100 = 20\%.$
20. d 30% of M is same as 15% of 20% of N

$$\Rightarrow \frac{30}{100} \times M = \frac{15}{100} \times \frac{20}{100} \times N$$

$$\therefore \frac{N}{M} \times 100 = \frac{30}{100} \times \frac{100}{15} \times \frac{100}{20} \times 100 = 1000\%.$$
21. c Let the total wire bought be 'x' m. Then,
 Wire stolen = 10% of x = 0.1x.
 $\therefore \text{Wire left} = x - 0.1x = 0.9x.$
 Now 90% of remainder is used.
 $\therefore \text{Wire left is } 10\% \text{ of remainder}$
 $= 10\% \text{ of } 0.9x = 0.09x.$
 Now, $0.09x = 47.25$
 $\Rightarrow x = 525 \text{ m.}$
22. b Male population = $1,20,000 \times 0.55 = 66,000$
 $\therefore \text{Female population} = 1,20,000 - 66,000 = 54,000.$
 Number of male voters = $66,000 \times 0.48 = 31,680$
 Number of female voters = $54,000 \times 0.60 = 32,400$
 $\therefore \text{Total number of voters} = 32,400 + 31,680 = 64,080.$
23. a Let the cost price be 'x'.
 As, profit % = Cost Price

$$\therefore x = \frac{(S.P. - C.P.)}{C.P.} \times 100$$

$$\Rightarrow x = \left(\frac{24 - x}{x} \right) \times 100$$

$$\Rightarrow x = ₹20.$$
24. b Total votes = 1200
 Ram received = $0.30 \times 1200 = 360$
 Balu received = 720
 Kapil received = $1200 - (360 + 720) = 120$
 Percentages of votes which the winner got in comparison to his closest rival is given by

$$= \frac{720}{360} \times 100 = 200\%.$$
25. c Required percentage

$$= \frac{36}{14.4 \times 1000} \times 100 = \frac{36}{144} = \frac{1}{4} = 0.25\%.$$

26. c Let $X = 2x$, $Y = 3x$ and $Z = 5x$. Then,
 $3x = 9000 \Rightarrow x = ₹3,000$
 $\therefore X = ₹6,000$ and $Z = ₹15,000$
 \therefore Percentage value = $\frac{9000}{6000} \times 100 = 150\%$.
- Alternate:**
Percentage value = $\frac{5x - 2x}{2x} \times 100 = 150\%$
27. b Take initial value of business = ₹100
So, commission of dealer = 8% of 100 = ₹8
For next case also, commission for dealer = ₹8
Take final value of business = a
 $a \times 10\% = 8$
 $\Rightarrow a = ₹80$
Percentage change in value of business = $\frac{20}{100} \times 100 = 20\%$.
28. b Let the value of the machine two years ago be x .
Then, value of machine after one year
= $x - 10\% \text{ of } x = 0.9x$
Further value of machine after two years,
i.e., present value = $0.9x - 10\% \text{ of } 0.9x = 0.81x$
 \therefore Present value = $0.81x = 1,62,000$
 $\Rightarrow x = ₹2,00,000$.
29. b $410 = X\% \text{ of } 123$
 $\Rightarrow X = 333.33$
 $370 = Y\% \text{ of } 111$
 $\Rightarrow Y = 333.33$
 $\therefore X = Y$.
30. d Total cost of painting = Rate \times Surface area of sphere
As the radius increases by 10%.
Surface area will change by 21% [using formula
 $\frac{a + b + (a \times b)}{100}$]
Total change in cost of painting = $21 + 20 + \frac{20 \times 21}{100} = 45.2\%$.
31. d Let the number be x .
Correct answer = $\frac{x}{5}$.
Wrong answer = $5x$
Percentage error = $\frac{\left(5x - \frac{x}{5}\right)}{\frac{x}{5}} \times 100 = 2400\%$.
32. b Let the present value of machine be ₹100.
 \Rightarrow Value after one year = 80% of 100 = ₹80,
Value after two years = 80% of 80 = ₹64, and
value after three years = 80% of 64 = ₹51.20
- Depreciation = ₹100 – ₹51.20 = ₹48.80.
Percent depreciation = $\frac{\text{Depreciation}}{\text{Present value}} \times 100$
 $= \frac{48.80}{100} \times 100 = 48.80\%$.
33. b Let maximum marks be x .
 $\frac{30}{100}x + 14 = \frac{45}{100}x - 16 \Rightarrow x = 200$
Hence, passing marks = $\frac{30}{100} \times 200 + 14 = 74$.
34. c Area of square = side \times side
As the side increase by 20%, percentage increase in
area = $a + b + \frac{a \times b}{100} = 20 + 20 + \frac{20 \times 20}{100} = 44\%$.
35. b Given that $D = 300$
 $D = C \times 150\% \Rightarrow C = 200$
 $A = C \times 60\% \Rightarrow A = 120$
 $A = B \times 120\% \Rightarrow B = 100$.
36. b Let B's earning be x .
 \Rightarrow A's earning = $x + 33\frac{1}{3}\% \text{ of } x = \frac{4}{3}x$
 \Rightarrow B's earning less than A's = $\frac{\left(\frac{4}{3}x - x\right)}{\frac{4}{3}x} \times 100 = 25\%$.
37. c Required marks = 80%; Obtained marks = 1005;
Failed by 13%.
 $\therefore 1005 = (80 - 13)\% \text{ of maximum marks} = 67\% \text{ of maximum marks}$.
 \therefore Total marks = $1005 \times \frac{100}{67} = 1500$.
38. b Let x be the number. Then,
 $0.80x + 80 = x \Rightarrow 0.20x = 80 \Rightarrow x = 400$.
39. a Let x be the salary of Sandeep.
Then, savings = 30% of x
Expenses = 70% of x
LIC = 40% of 30% of x
Others = 25% of 30% of x
Difference = 40% of 30% of x – 25% of 30% of x
 $\Rightarrow 135 = 15\% \text{ of } 30\% \text{ of } x$
 $\Rightarrow x = ₹3,000$.
40. d Let number of students failed = 100
Number of students who passed = 120
Percentages by which number of students failed is
less than who have passed is given by
 $= \left(1 - \frac{100}{120}\right) \times 100 = 16.66\%$.

41. e Present value = ₹6

Inflation is 1000%

Value after one year = $6 + 1000\% \text{ of } 6$

$$= ₹6 + ₹60 = ₹66$$

Value after two years = $66 + 1000\% \text{ of } 66$

$$= ₹66 + ₹660 = ₹726.$$

42. e Earnings = ₹4,000/month.

Savings = 30% of 4000 = ₹1,200/month

Expenditure = Earning – Savings = ₹2,800/month

$$= 2800 \times 12 = ₹33,600/\text{year}.$$

43. d Base increases by 14.28% so to keep the area of triangle constant the height must be changed.

Let height change by x%.

$$\therefore \text{Area} = \frac{1}{2} \times b \times h$$

By using formula $a + b + \frac{a \times b}{100}$

$$\Rightarrow 14.28 + x + \frac{14.28 \times x}{100} = 0 \Rightarrow \frac{100}{7} + x + \frac{x}{7} = 0$$

$$\Rightarrow x = -12.5\%.$$

44. c Let the number of votes received by A be 'x'. Then,

B's vote count = 50% of votes of A = 50% of $x = \frac{x}{2}$

As, if A got 200 votes less, there would have been a tie. These 200 votes would have gone to B.

$$\Rightarrow x - 200 = \frac{x}{2} + 200 \Rightarrow \frac{x}{2} = 400 \Rightarrow x = 800$$

Total number of votes = votes of A + votes of B

$$= x + \frac{x}{2} = 800 + 400 = 1200.$$

45. b Let the price of the bicycle be ₹x. Then,

$$x \times \left(1 + \frac{9.09}{100}\right) \times \left(1 + \frac{8.33}{100}\right) \times \left(1 + \frac{7.7}{100}\right) = 1274$$

$$\Rightarrow x \times \left(1 + \frac{1}{11}\right) \times \left(1 + \frac{1}{12}\right) \times \left(1 + \frac{1}{13}\right) = 1274$$

$$\Rightarrow x \times \frac{12}{11} \times \frac{13}{12} \times \frac{14}{13} = 1274$$

$$\Rightarrow x = 91 \times 11 = ₹1,001.$$

46. d Let number of females be x.

Then, number of males = $15000 - x$

$$\therefore x \times \frac{10}{100} + (15000 - x) \times \frac{8}{100} = 16300 - 15000$$

$$\Rightarrow 10x + 120000 - 8x = 130000$$

$$\Rightarrow 2x = 130000 - 120000$$

$$\Rightarrow 2x = 10000$$

$$\Rightarrow x = 5000.$$

$$47. b \text{ Single discount} = 20 + 15 - \frac{20 \times 15}{100}$$

$$= 35 - 3 = 32\%.$$

$$48. d \frac{30}{100} \times A + \frac{40}{100} \times B = \frac{80}{100} \times B$$

$$\Rightarrow 30A + 40B = 80B$$

$$\Rightarrow 30A = 40B$$

$$\text{Percentage of A is B} = \frac{30}{40} \times 100 = 75\%.$$

$$49. d \frac{90}{100} \times A = \frac{30}{100} \times B$$

$$\Rightarrow \frac{B}{A} = \frac{90}{30} = 3$$

$$\frac{B}{A} = \frac{2x}{100}$$

$$\therefore 3 = \frac{2x}{100}$$

$$\Rightarrow 2x = 300 \Rightarrow x = 150.$$

50. b Percentage to reduce his consumption

$$= \frac{25}{125} \times 100 = 20\%.$$

Quantitative Aptitude - 8
Profit, Loss and Discount
Answers and Explanations

P-1 (BS)

1	b	2	d	3	a	4	c	5	b	6	c	7	d	8	b	9	c	10	a
11	a	12	d	13	d	14	c	15	c	16	a	17	a	18	a	19	c	20	b
21	c	22	b	23	c	24	d	25	c	26	c	27	c	28	c	29	d	30	d
31	c	32	a	33	d	34	b	35	e	36	e	37	e	38	a	39	e	40	b
41	b	42	d	43	e	44	b	45	d	46	d	47	a	48	a	49	c	50	b

1. b 10% stock at ₹96 = 10% of ₹96 = ₹9.60
 Total investment = ₹650 × 9.60
 = ₹6,240.
2. d Mark-up price = ₹2,000, S.P. = ₹1,500.
 Since cost price is not given we cannot find the profit or loss percentage. So, data is inadequate.
3. a Let the cost price be 'x'.
 Then, profit % = Cost price

$$\Rightarrow x = \frac{(SP - CP)}{CP} \times 100$$

$$\Rightarrow x = \left(\frac{24 - x}{x} \right) \times 100$$
 Hence, x = ₹20.
4. c SP = ₹y
 Discount = x%
 Let the list price be 'l'.

$$\Rightarrow x = \left(\frac{l - y}{l} \right) \times 100$$

$$\Rightarrow \frac{x}{100} = 1 - \frac{y}{l}$$

$$\Rightarrow \frac{y}{l} = \frac{100 - x}{100} \Rightarrow l = \frac{100y}{(100 - x)}$$
5. b Loss% = $\frac{CP - SP}{CP} \times 100$
 Let CP be x. Then,

$$\left(\frac{x - 816}{x} \right) \times 100 = 20$$

6. c Cost price of the pens = 25 × 60 = ₹1,500.
 Selling price of the pens = 30 × 0.88 × 60 = ₹1,584.

$$\therefore \text{Profit percentage} = \frac{1584 - 1500}{1500} \times 100 = 5.6\%$$
7. d Let the total investment = 36 × 30 = ₹1,080.
 CP of one book = ₹30, SP of one book = ₹36.

$$\text{Profit} = \frac{36 - 30}{30} \times 100 = \frac{6}{30} \times 100 = 20\%$$
- Alternative method:**

$$36 \times CP = 30 \times SP$$

$$\Rightarrow \frac{SP}{CP} = \frac{36}{30} = 120\%$$

$$\therefore \text{Profit} = 20\%$$
8. b Since selling price of 80 kg = CP of 80 kg – SP of 16 kg.
 (Assuming SP = ₹x per kg)

$$80x = CP - 16x$$

$$\therefore CP = (80 + 16)x = 96x \Rightarrow 96x = 384$$

$$\text{Hence, } x = \frac{384}{96} = 4. \text{ So, SP} = ₹4 \text{ per kg.}$$
9. c In fact, you get things worth ₹3,000, but you pay ₹2,000 only. So, you are getting a discount

$$= \frac{3000 - 2000}{3000} \times 100 = 33\frac{1}{3}\%$$

10. a Loss = CP – SP.
 1 dozen SP = 6 dozen CP – 6 dozen SP
 7 dozen SP = 6 dozen CP
- $$\text{Loss \%} = \frac{1}{7} \times 100 \approx 14.28\%.$$
11. a Let the CP of the article be x .
 $1.08x - 0.92x = 12$
 $\Rightarrow 0.16x = 12 \Rightarrow x = \frac{12}{0.16} = ₹75.$
12. d $(12.5\% \text{ of CP}) + \text{CP} = 540$
 $112.5\% \text{ of CP} = 540$
 Now $(25\% \text{ of CP}) + \text{CP} = 125\% \text{ of CP.}$
- $$\text{SP} = \frac{540}{112.5} \times 125 = ₹600 \text{ (earlier profit} = 12.5\%; \text{double the profit} = 25\%).$$
13. d Let SP be ₹100, then profit = ₹25;
 $\text{CP} = 100 - 25 = ₹75.$
 $\therefore \text{Profit percentage} = \frac{25}{75} \times 100 = 33.33\%.$
14. c Let price of furniture set be x . Then,
 $1.25x - 0.8x = 900$
 $\Rightarrow 0.45x = 900$
 $\Rightarrow x = \frac{900}{0.45} = ₹2,000$
 To make 20% profit he should sell it at $2000 \times 1.2 = ₹2,400.$
15. c $9.09\% = \frac{1}{11}\%$
 $\text{SP at } 9.09\% \text{ profit} = 1331 \times \frac{12}{11}$
 $= ₹1,452$
 If he don't offer any discount, then $\text{SP} = ₹1,552$
 His profit = $1,552 - 1,331 = ₹221.$
16. a
- | | X | Y |
|--------------|-----|-----|
| Par value | 10 | 10 |
| Market Price | 40 | 50 |
| Dividend | 20% | 40% |
| Investment | a | a |
- Therefore, number of shares each of them bought
 $= \frac{a}{40}, \frac{a}{50}$
(Dividend) Income
 $\Rightarrow 20\% \text{ of } 10 = 2$
 $\Rightarrow 40\% \text{ of } 10 = 4$

- Total return incomes of $X = \frac{a}{40} \times 2 \times 100 = \frac{100}{20} = 5$
Total return incomes of $Y = \frac{a}{50} \times 4 \times 100 = 4 \frac{100}{50} = 8$
 $\therefore \text{Ratio} = 5 : 8.$
17. a Let the price of one shirt be ₹100. Then,
 Price of 4 shirts = $4 \times 100 = ₹400.$
 Customer pays for 3 shirts i.e. ₹300.
 Discount = $400 - 300 = ₹100.$
- $$\text{Discount \%} = \frac{\text{Discount}}{\text{Total price}} \times 100 = \frac{100}{400} \times 100 = 25\%.$$
18. a Profit percentage = $\frac{\text{Profit}}{\text{CP}} \times 100 = \frac{40}{200} \times 100 = 20\%.$
19. c $\text{CP}_1 = \text{SP}_2$
 $\text{CP}_2 = \text{SP}_1$
 Hence, total $\text{CP} = \text{CP}_1 + \text{CP}_2 = \text{SP}_2 + \text{SP}_1 = \text{total SP}$
 So, he makes no profit, no loss.
20. b 20% profit on CP of ₹600 = ₹120.
 So, book seller has to sell the book at ₹720 after discount, to make 20% profit.
 Let marked price be x . Then,
 $0.9x = 720 \Rightarrow x = ₹800.$
21. c Let the cost price be ₹'C'.
 Then, $L = \left(\frac{C - X}{C} \right) \times 100$
 $\Rightarrow \frac{L}{100} = 1 - \frac{X}{C} \quad \dots (i)$
 and $P = \left(\frac{Y - C}{C} \right) \times 100$
 $\Rightarrow \frac{P}{100} = \frac{Y}{C} - 1 \quad \dots (ii)$
 Adding (i) and (ii), we get
 $\frac{L+P}{100} = \frac{Y-X}{C}$
 $\Rightarrow C = \frac{100(Y-X)}{(L+P)}.$
22. b Let 60 oranges of type-1 and 120 oranges of type-2 has been bought by vendor.
 Total cost of oranges = $3 \times 15 + 4 \times 20 = ₹125$
 Now selling price of 180 oranges = $\frac{18}{24} \times 180 = ₹135.$
 Hence, offered discount = $\frac{10}{135} \times 100 = 7.41\%$

23. c Let the marked price be x. Then,

$$SP = \frac{2}{5}x$$

$$\text{Loss\%} = \left(\frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \right) \times 100$$

$$\Rightarrow 25 = \left(1 - \frac{\frac{2}{5}x}{\text{C.P.}} \right) \times 100 \Rightarrow \text{C.P.} = \frac{8}{15}x$$

$$\therefore \text{Ratio of marked price : cost price} = x : \frac{8}{15}x \\ = 15 : 8.$$

24. d Effective % change of x% and y%

$$= \left(x + y + \frac{xy}{100} \right) \%$$

Now x = -20% and y = -20%

For two successive discounts the net discount

$$= (-20) + (-20) + \frac{(-20)(-20)}{100} = 36\%$$

Now let the amount of the bill be ₹x.

(Two successive 20% discounts on x) - 35% of x = ₹22

$$\Rightarrow 36\% \text{ of } x - 35\% \text{ of } x = 22$$

$$\Rightarrow 1\% \text{ of } x = 22$$

$$\Rightarrow \frac{x}{100} = 22 \Rightarrow x = ₹2,200.$$

25. c After the first discount of 10% the revised list price = 90% of ₹720 = ₹648

Now discount on revised list price = 648 - 550.80 = ₹97.20.

Now let the second discount be x%
⇒ x% of ₹648 = ₹97.20

$$\Rightarrow x \times \frac{648}{100} = 97.20 \Rightarrow x = 15\%.$$

26. c Let SP of one article be ₹1.

So, SP of 6 articles = ₹6.

CP of 6 articles = SP of 5 articles = ₹5.

Hence, profit = 6 - 5 = ₹1.

$$\therefore \text{Percentage profit} = \frac{1}{5} \times 100 = 20\%.$$

Alternative method:

Let the SP be x and CP be y.

$$\therefore 6y = 5x$$

$$\Rightarrow \frac{x}{y} = \frac{\text{SP}}{\text{CP}} = \frac{6}{5} = 120\%$$

$$\therefore \text{Profit} = 20\%$$

27. c Take LCM of 2, 3, 5, i.e. 30.

So, cost of 30 pencils at the rate of 2 per rupee = ₹15 and cost of 30 pencils at the rate of 3 per ₹2 = ₹20.

Hence, total cost of 60 pencils = 15 + 20 = ₹35.
Selling price of 60 pencils at the rate of 5 per ₹3 = ₹36.

$$\text{Hence, profit percentage} = \frac{1}{35} \times 100 = \frac{20}{7} = 2\frac{6}{7}\%.$$

Alternative method:

The easier way is to assume that he has one pencil of each kind.

$$\frac{\text{SP}}{\text{CP}} = \frac{\left(\frac{3}{5} \times 2 \right)}{\left(\frac{1}{2} + \frac{2}{3} \right)} = \frac{1.2}{1.16}$$

Now as $\frac{\text{SP}}{\text{CP}} > 1$. It is a gain and also the gain is less

than 3% as $\frac{1.2}{1.16} - 1 \approx \frac{.03}{1.16}$ and

as 3% of 1.16 ≈ 0.034 > 0.03 .

28. c Spring balance shows 800 gm for 1 kg
His loss = 200 gm per kg.

$$\text{His loss percent} = \frac{200}{1000} \times 100 = 20\% \text{ loss.}$$

29. d Let the list price of the shirt be ₹100.

$$\text{Price for which the shirt is bought (CP)} = \frac{3}{4} \text{ of } 100 \\ = ₹75$$

$$\text{Price for which shirt is sold (SP)} = 100 + 50\% \text{ of } 100 \\ = ₹150.$$

$$\text{Profit} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100 = \frac{150 - 75}{75} \times 100 = 100\%.$$

30. d Cost of 10 kg of oranges = ₹405.

It is to be noted here that after 1 kg rotten oranges is found, he sells only 9 kg, but the cost to him will remain ₹405.

For 10% profit, he should sell the oranges at $(1.1 \times 405) = ₹445.50$

So, SP of 9 kg = ₹445.50

$$\text{Hence, SP per kg} = \frac{445.50}{9} = ₹49.50.$$

31. c Let the cost price be x. Then,

$$\text{Gain\%} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100$$

$$\Rightarrow 16 = \frac{40.60 - x}{x} \times 100$$

$$\Rightarrow x = ₹35.$$

32. a Let the selling price of 1 m of cloth is ₹1.
 \therefore Selling price of 33 m of cloth = ₹33
 \therefore Profit = ₹11
Cost price = S.P. – Profit = ₹22
- $$\therefore \text{Gain\%} = \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{11}{22} \times 100 = 50\%.$$
33. d Let the listed price be ₹100.
Then, net selling price = 95% of 90% of 80% of ₹100
 $= \frac{95}{100} \times \frac{90}{100} \times \frac{80}{100} \times 100 = ₹68.40$
 \therefore Required discount = $(100 - 68.40)\% = 31.6\%$.
34. b Selling price = ₹100. Gain = ₹20
 \therefore CP = SP – gain = ₹100 – ₹20 = ₹80
 \therefore Gain percentage = $\frac{20}{80} \times 100 = 25\%.$
35. e Let cost price of each pen be ₹1. Then,
CP of 12 pens = ₹12
Also, SP of 8 pens = ₹12
 \therefore SP of 1 pen = $\frac{12}{8} = ₹1.50$
 $\therefore \text{Gain\%} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100$
 $= \frac{1.50 - 1}{1} \times 100 = 50\%.$
36. e Let the cost price of 1 cm of cloth be ₹1. Then,
CP of 1 m of cloth = SP of 1 m of cloth = ₹100
Also, CP of 90 cm of cloth = ₹90
Gain on selling 1 m of cloth = ₹100 – ₹90 = ₹10.
 $\text{Gain\%} = \frac{\text{Gain}}{\text{CP of 90cm of cloth}} \times 100$
 $= \frac{10}{90} \times 100 = 11.11\%.$
37. e Let the cost price of the washing machine be ₹100 and list price be x.
Then, $x - x \times \frac{20}{100} = 112$
 $\Rightarrow 0.8x = 112 \Rightarrow x = ₹140$
When the shopkeeper gives a discount of 25%, then
 $\text{selling price} = 140 - 140 \times \frac{25}{100} = 140 - 35 = ₹105$
Hence, there will be a profit of 5%.

38. a Let price of milk be ₹10 per L
9 litre of milk costs = ₹90
(9 + 2) litre milk will fetch him = $1.1 (90 + 20) = ₹121$
His net profit = $\frac{121 - 90}{90} \times 100 = \frac{31}{90} \times 100 = 34.4\%.$
39. e Successive discount of 20% and 15%
 $= -20\% - 15\% + \frac{20 \times 15}{100}$
 $= -35 + 3\% = -32\%$
 \Rightarrow Successive discount of 20% and 15% = discount of 32%.
40. b Cost price = $\frac{\text{Selling price} \times 100}{100 - \text{loss\%}} = \frac{3520 \times 100}{100 - 12}$
 $= \frac{3520 \times 100}{88} = ₹4,000$
Now, selling price = $\left(\frac{100 + \text{profit\%}}{100} \right) \times \text{Cost price}$
 $= \left(\frac{100 + 12}{100} \right) \times 4000 = ₹4,480.$
41. b Let the cost price of goods be ₹100. Then,
Marked price = 150% of 100 = ₹150.
Discount = 25% of ₹150 = ₹37.50
 \therefore Selling price = Marked price – Discount
 $= 150 - 37.50 = ₹112.50$
 $\text{Gain\%} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100 = \frac{112.50 - 100}{100} \times 100$
 $= 12.5\%.$
42. d CP = ₹1,200
Profit = 25%
 \therefore SP = 125% of 1200 = ₹1,500
 $\text{Discount \%} = \left(\frac{\text{Marked Price} - \text{S.P.}}{\text{Marked Price}} \right) \times 100$
 $\Rightarrow 20\% = \left(\frac{\text{Marked Price} - 1500}{\text{Marked Price}} \right) \times 100$
 $\Rightarrow \text{Marked Price} = ₹1,875.$
43. e Selling price of 17 balls = ₹720
Let cost price of each ball be x. Then,
cost price of 17 balls = $17x$
and cost price of 5 balls = $5x$
Now, loss = cost price – selling price
 $\Rightarrow \text{loss} = 17x - 720 = \text{cost price of 5 balls}$
 $\Rightarrow 17x - 720 = 5x$
 $\Rightarrow 12x = 720$
 $\Rightarrow x = 60$
 \therefore Cost price of one ball is ₹60.

44. b Let Mr. Alphonso buys mangoes for ₹150 [LCM of 50, 30]

$$\text{First type he buys } \frac{150}{30} = 5 \text{ dozen}$$

$$\text{Second type he buys } \frac{150}{50} = 3 \text{ dozen}$$

In total he spent ₹300 for 8 dozen mangoes

He sells all mangoes at ₹40 per dozen

He earns $8 \times 40 = ₹320$

$$\text{His profit percentage} = \frac{320 - 300}{300} \times 100 = 6\frac{2}{3}\%$$

45. d Let CP of 1 gm sugar be ₹1.

Then, SP of 1 gm sugar will be ₹1.

The seller uses a weight of 800 gm in place of 1000 gm.

Impurities in 800 gm = $800 \times 0.2 = 160$ gm

\therefore CP of sugar = $800 - 160 = ₹640$ which is sold in ₹1,000.

Hence, profit percentage

$$= \frac{1000 - 640}{640} \times 100 = 56.25\%.$$

$$\begin{aligned} 46. d \quad \text{Percentage gain} &= 30 - 10 - \frac{30 \times 10}{100} \\ &= 20 - 3 = 17\%. \end{aligned}$$

47. a Marked price of article

$$= 450 \times \frac{100}{90} \times \frac{120}{100} = ₹600.$$

48. a SP of 10 articles = CP of 11 articles

$$\therefore \text{Gain percentage} = \frac{11 - 10}{10} \times 100 = 10\%.$$

49. c Let the marked price of watch be ₹x.

$$\therefore x \times \frac{95}{100} - x \times \frac{94}{100} = 15$$

$$\Rightarrow \frac{x}{100} = 15$$

$$\Rightarrow x = ₹1,500.$$

50. b Let the cost price of first horse be ₹x.

Then, cost price of second horse = ₹(19500 - x)

$$\therefore x \times \frac{80}{100} = (19500 - x) \times \frac{115}{100}$$

$$\Rightarrow 80x = 19500 \times 115 - 115x$$

$$\Rightarrow 80x + 115x = 19500 \times 115$$

$$\Rightarrow 195x = 19500 \times 115$$

$$\Rightarrow x = \frac{19500 \times 115}{195} = ₹11,500$$

Cost price of second horse

$$= 19500 - 11500 = ₹8,000.$$

Alternate:

Let the selling price of each horse be 100y

Then, cost price of first horse = 125y

$$\text{and cost price of second horse} = \frac{2000}{23}y$$

$$\therefore 125y + \frac{2000}{23}y = 19,500$$

$$\Rightarrow y = 92$$

Hence, cost price of horses are ₹11,500 and ₹8,000.

Simple Interest and Compound Interest Answers and Explanations

1	b	2	c	3	d	4	b	5	c	6	a	7	c	8	d	9	a	10	a
11	d	12	a	13	a	14	d	15	a	16	b	17	b	18	b	19	b	20	b
21	c	22	d	23	c	24	b	25	c	26	c	27	b	28	b	29	d	30	b
31	a	32	d	33	c	34	d	35	b	36	e	37	d	38	c	39	d	40	c
41	b	42	a	43	e	44	e	45	a	46	b	47	a	48	b	49	c	50	c

1. b Let R be the rate percentage. Then,

$$SI = \frac{1}{9} \times P$$

$$\Rightarrow \frac{1}{9}P = P \times \frac{R}{100} \times R \quad (\text{No. of years} = R)$$

$$\Rightarrow \frac{R^2}{100} = \frac{1}{9} \Rightarrow R^2 = \frac{100}{9} \Rightarrow R = \frac{10}{3} = 3\frac{1}{3}\%.$$

2. c $SI = P \times \frac{5}{100} \times 2 = \frac{10}{100}P$ (where P = Sum of money)

$$\text{Amount} = P \left(1 + \frac{r}{100}\right)^n$$

$$CI = P \left(1 + \frac{5}{100}\right)^2 - P = \frac{11025P}{10000} - P = \left(\frac{1025}{10000}\right)P$$

$$\text{Difference} = \frac{1025}{10000}P - \frac{10}{100}P = 1.50.$$

$$\Rightarrow \frac{25}{10000}P = 1.5 \Rightarrow P = ₹600.$$

Short cut:

$$\text{Difference in interest} = CI_2 - SI_2 = \left(\frac{r}{100}\right)^2 P$$

(Only when difference of CI and SI for 2 years)

$$\text{Sum} = \frac{1.50 \times 100 \times 100}{5 \times 5} = ₹600.$$

3. d Amount = ₹8,820 and the principal = ₹8,000.

$$\frac{8820}{8000} = \left(1 + \frac{5}{100}\right)^n \Rightarrow \left(\frac{21}{20}\right)^2 = \left(\frac{21}{20}\right)^n \Rightarrow n = 2 \text{ years.}$$

4. b Let the principal amount be P. Then,

$$SI \text{ in 4 years} = P \times \frac{4}{100} \times 4$$

$$SI \text{ in 3 years} = P \times \frac{5}{100} \times 3$$

Difference = ₹80

$$\Rightarrow \frac{16}{100}P - \frac{15}{100}P = 80$$

$$\Rightarrow P = ₹8,000.$$

$$5. c \quad SI = \frac{500 \times 5 \times 10}{100} = ₹250.$$

6. a Let T be the time. Then,

$$\text{Interest} = \frac{P \times R \times T}{100}$$

$$\Rightarrow T = \frac{100 \times \text{Interest}}{P \times R} = \frac{100 \times 500}{2000 \times 10} = 2.5 \text{ years.}$$

7. c Let sum be ₹x, then $SI = \frac{r}{100}(648 - x)$

$$\therefore \frac{x \times 4 \times 5}{100} = 648 - x$$

$$\Rightarrow x = ₹540$$

Now, $P = ₹540$, $R = 10\%$, $T = 2$ years

$$SI = \left(\frac{540 \times 10}{100} \times 2 \right) = ₹108$$

$$\therefore \text{Amount} = 540 + 108 = ₹648.$$

8. d Let the sum be ₹P. Then,

$$P \left(1 + \frac{R}{100} \right)^3 = 1200 \quad \dots (i)$$

$$P \left(1 + \frac{R}{100} \right)^6 = 1800 \quad \dots (ii)$$

$$\text{On dividing, we get } \left(1 + \frac{R}{100} \right)^3 = \frac{3}{2}$$

$$\text{Substituting this value in (i), we get } P \times \frac{3}{2} = 1200$$

$$\Rightarrow P = 1200 \times \frac{2}{3} = ₹800.$$

9. a $P = ₹5,000$ $r = 3\%$ p.a. $n = 10$ years

$$\therefore SI = \frac{P \times n \times r}{100} = \frac{5000 \times 10 \times 3}{100} = ₹1,500.$$

10. a **Case 1:** $SI = A - P = 2P - P = P \Rightarrow P = \frac{5PR}{100}$

$$\Rightarrow R = 20\%$$

Case 2: $SI = A - P = 3P - P = 2P \Rightarrow 2P = \frac{12PR}{100}$

$$\Rightarrow R = 16.66\%$$

Hence, rate of interest is better in case-1.

11. d Difference of sum after 2 and 3 years = $1,728 - 1,440 = ₹288$

₹288 is the simple interest on ₹1,440 for one year.

$$288 = \frac{1440 \times r \times 1}{100} \Rightarrow r = 20\% .$$

$$12. a \quad 4840 = P \left(1 + \frac{r}{100} \right)^2 \quad \dots (i)$$

$$5324 = P \left(1 + \frac{r}{100} \right)^3 \quad \dots (ii)$$

Dividing (ii) by (i), we get

$$\left(1 + \frac{r}{100} \right) = \frac{5324}{4840} \Rightarrow \frac{r}{100} = \frac{484}{4840}$$

$$\Rightarrow r = 10\%.$$

13. a Let the principal amount be p.

For it to become four times, interest should be 3p.

$$\therefore 3p = p \times \frac{5}{100} \times n$$

$$\Rightarrow n = \frac{p \times 300}{5 \times p} = 60 \text{ years.}$$

$$14. d \quad SI = \frac{P \times R \times n}{100} = \frac{982 \times 6 \times 8}{100 \times 12} = ₹39.28.$$

$$15. a \quad 3P = P \left(1 + \frac{r}{100} \right)^4$$

$$\Rightarrow \left(1 + \frac{r}{100} \right) = 3^{\frac{1}{4}}$$

$$\text{Now, } 81P = P \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow 3^4 = 3^{\frac{n}{4}}$$

$$\Rightarrow n = 16 \text{ years.}$$

$$16. b \quad SI = \frac{P \times R \times T}{100}$$

$$\Rightarrow P = \frac{100 \times SI}{R \times T} = \frac{100 \times 150}{2.5 \times 5} = ₹1,200.$$

17. b SI for 1 year = $1111 - 1045 = ₹66$

SI for 5 years = $66 \times 5 = ₹330$

∴ Principal = $1045 - 330 = ₹715$.

18. b $SI = 1,380 - 1,200 = ₹180$, $P = ₹1,200$,
 $T = 3$ years

$$\therefore R = \left(\frac{100 \times 180}{1200 \times 3} \right) \% = 5\%,$$

$$\text{New rate} = (5 + 3) = 8\%$$

$$\text{New } SI = \frac{1200 \times 8 \times 3}{100} = ₹288$$

$$\therefore \text{New amount} = 1,200 + 288 = ₹1,488.$$

19. b Let two parts be ₹x and ₹(1,105 – x). Then,

$$x \left(1 + \frac{10}{100}\right)^5 = (1105 - x) \left(1 + \frac{10}{100}\right)^7$$

$$\Rightarrow \frac{x}{1105 - x} = \left(1 + \frac{10}{100}\right)^2 = \frac{11}{10} \times \frac{11}{10}$$

$$\Rightarrow x = ₹605.$$

So, the two parts are ₹605 and $(1105 - 605) = ₹500$.

20. b First part = ₹x, $R = 6\%$, $N = 2$ years

Second part = $3,650 - x$, $R = 4\%$, $N = 3$ Years

According to the given condition,

$$\Rightarrow \frac{x \times 6 \times 2}{100} = \frac{(3650 - x) \times 4 \times 3}{100} \Rightarrow 2x = 3650$$

$$\Rightarrow x = ₹1,825$$

$$\text{Second part} = 3,650 - 1,825 = ₹1,825.$$

21. c Since the rate is compounded half-yearly, time period

for $1\frac{1}{2}$ years = 3.

$$\text{Population after } 1\frac{1}{2} \text{ years} = P \left(1 + \frac{r}{100}\right)^3.$$

$$= 1000 \left(1 + \frac{10}{100}\right)^3 = 1000 \times 1.10 \times 1.10 \times 1.10 = 1331.$$

22. d $\left[P \left(1 + \frac{5}{100}\right)^3 - P \right] - \left[P \left(1 + \frac{5}{100}\right)^2 - P \right] = 110.25$

$$\Rightarrow (1.05)^3 P - (1.05)^2 P = 110.25$$

$$\Rightarrow P[(1.05)^3 - (1.05)^2] = 110.25$$

$$\Rightarrow P(1.05)^2 (0.05) = 110.25$$

$$\Rightarrow P \frac{(11025)}{10000} \times \frac{5}{100} = \frac{11025}{100}$$

$$\Rightarrow P = ₹2,000.$$

23. c Since rate of interest is half yearly i.e. $\frac{5}{2} = 2.5\%$

$$\text{Amount on June 30} = 1600 \left(1 + \frac{2.5}{100}\right)^1 = ₹1,640.$$

$$\therefore \text{Principle on July 1} = 1640 + 1600 = ₹3,240.$$

$$\text{Amount on December 31} = 3240 \left(1 + \frac{2.5}{100}\right)^1 = ₹3,321$$

$$\text{Total CI} = 3,321 - 1,600 - 1,600 = ₹121.$$

24. b Let the rate be $R\%$ p.a., then

$$\left[1500 \left(1 + \frac{R}{100}\right)^2 - 1500 \right] - \left[\frac{1500 \times R \times 2}{100} \right] = 15$$

$$\Rightarrow 1500 \left[\frac{(100+R)^2}{10000} - 1 - \frac{2R}{100} \right] = 15$$

$$\Rightarrow R = 10\% \text{ p.a.}$$

Alternate method:

$$\frac{PR^2}{100^2} = 15 \Rightarrow \frac{1500 \times R^2}{100^2} = 15$$

$$\Rightarrow R^2 = 100 \Rightarrow R = 10\%.$$

25. c Let deposited amount be ₹x.

$$\text{Then, } \frac{x \times 2 \times 6}{100} + \frac{x \times 3 \times 16}{100} + \frac{x \times 1 \times 20}{100} = 6080$$

$$\Rightarrow x = ₹7,600.$$

26. c Let the value of each installment be x , Then,

$$\left(1 + \frac{10}{100}\right) + \left(1 + \frac{10}{100}\right)^2 = 6720$$

$$\Rightarrow \frac{10x}{11} + \frac{100x}{121} = 6720$$

$$\Rightarrow 210x = 6720 \times 121 \Rightarrow x = ₹3872$$

So, each installment = ₹3,872.

27. b Remaining part = $1 - \left(\frac{1}{3} + \frac{1}{6}\right) = \frac{1}{2}$

Average rate percentage per annum on the total sum

$$= \left(\frac{1}{3} \times 3\right) + \left(\frac{1}{6} \times 6\right) + \left(\frac{1}{2} \times 8\right) = 6\%$$

$$\therefore P = \frac{100 \times SI}{R \times T} = \frac{100 \times 600}{6 \times 2} = ₹5,000$$

The original sum is ₹5,000.

28. b $SI = P \times \frac{r}{100} \times 2 = 40$

$$\Rightarrow \frac{r}{100} = \frac{20}{P}$$

... (i)

$$\text{Amount} = P \left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow (P + I) = P \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow (P + 40.80) = P \left(1 + \frac{20}{P}\right)^2 \quad (\text{from (i)})$$

$$\Rightarrow (P + 40.80) = P \left(1 + \frac{400}{P^2} + \frac{40}{P}\right)$$

$$\Rightarrow P + 40.80 = P + \frac{400}{P} + 40$$

$$\Rightarrow (40.80 - 40)P = 400$$

$$\Rightarrow P = \frac{400}{0.8} = ₹500$$

$$\text{Thus, } r = \frac{20}{P} \times 100 = \frac{20}{500} \times \frac{100}{1} = 4\% .$$

29. d $P \left(1 + \frac{R}{100}\right)^{10} = 2P, \left(1 + \frac{R}{100}\right)^{10} = 2 \quad \dots (i)$

$$\text{Let } P \left(1 + \frac{R}{100}\right)^n = 16P, \left(1 + \frac{R}{100}\right)^n = 16 = 2^4$$

Using (i)

$$\left(1 + \frac{R}{100}\right)^n = \left[\left(1 + \frac{R}{100}\right)^{10}\right]^4 = \left(1 + \frac{R}{100}\right)^{40}$$

$$\Rightarrow n = 40$$

So, the required time = 40 years.

30. b Principal = $\frac{1440}{\left(1 + \frac{20}{100}\right)} + \frac{1440}{\left(1 + \frac{20}{100}\right)^2}$

$$= \frac{1440 \times 5}{6} + \frac{1440 \times 25}{36}$$

$$= ₹2,200.$$

31. a $P = ₹72,000, R = 12\frac{1}{2}\% = \frac{25}{2}\% \text{ p.a.}$

$$T = \frac{8}{12} = \frac{2}{3} \text{ years.}$$

$$\therefore SI = \frac{P \times R \times T}{100} = 72000 \times \frac{25}{2} \times \frac{2}{3} \times \frac{1}{100} \\ = ₹6,000.$$

32. d Principal amount = $\frac{3600 \times 100}{9 \times 5}$
= ₹8,000.

33. c Amount = $12500 \times \left(1 + \frac{6}{100}\right)^2 = ₹14,045$

$$\therefore CI = 14,045 - 12,500 = ₹1,545.$$

34. d Rate = 10% p.a., time = 2 years, S.I. = ₹1,000

$$\text{Principal} = \frac{100 \times 1000}{10 \times 2} = ₹5,000$$

$$\text{Amount at CI} = 5000 \times \left(1 + \frac{10}{100}\right)^2 = 5000 \times \frac{11}{10} \times \frac{11}{10}$$

$$= ₹6,050$$

$$\text{CI} = 6,050 - 5000 = ₹1,050.$$

35. b Let the value of each installment be ₹x. Then,

$$\frac{x}{\left(1 + \frac{8}{100}\right)} + \frac{x}{\left(1 + \frac{8}{100}\right)^2} = 6,500$$

$$\Rightarrow \frac{25x}{27} + \frac{625x}{729} = 6500$$

$$\Rightarrow 1300x = 6500 \times 729$$

$$\Rightarrow x = 3,645$$

So, each installment = ₹3,645.

36. e Let the sum be x. Then,

$$\left(\frac{x \times 10 \times 11}{100 \times 2}\right) - \left(\frac{x \times 12 \times 9}{100 \times 2}\right) = 30$$

$$\Rightarrow x = ₹3,000.$$

37. d P = ₹2,000, Rate = 2% per half year,
Time = 2 years = 4 half year

$$\therefore \text{Amount} = 2,000 \times \left(1 + \frac{2}{100}\right)^4 = ₹2,164.86$$

$$\therefore \text{CI} = 2,164.86 - 2,000 = ₹164.86.$$

38. c P = ₹400, Amount = ₹441, time = 2 years
Let the rate be R% p.a. Then,

$$400 \left(1 + \frac{R}{100}\right)^2 = 441 \Rightarrow \left(1 + \frac{R}{100}\right)^2 = \frac{441}{400}$$

$$\Rightarrow 1 + \frac{R}{100} = \frac{21}{20} \Rightarrow R = 5\% \text{ p.a.}$$

39. d $P \left(1 + \frac{R}{100}\right)^8 = 2P, \quad \left(1 + \frac{R}{100}\right)^8 = 2 \quad \dots(i)$

$$\text{Let } P \left(1 + \frac{R}{100}\right)^n = 8P, \quad \left(1 + \frac{R}{100}\right)^n = 8 = 2^3$$

Using (i), we get

$$\left(1 + \frac{R}{100}\right)^n = \left[\left(1 + \frac{R}{100}\right)^8\right]^3 = \left(1 + \frac{R}{100}\right)^{24}$$

$$\Rightarrow n = 24$$

So, the required time = 24 years.

$$40. c \quad \text{Principal} = \frac{1694}{\left(1 + \frac{10}{100}\right)} + \frac{1694}{\left(1 + \frac{10}{100}\right)^2}$$

$$= \frac{1694 \times 10}{11} + \frac{1694 \times 100}{121}$$

$$= 1,540 + 1,400 = ₹2,940.$$

41. b P = ₹4,500, Time = 9 months = 3 quarters
R = 16% p.a. = 4% per quarter

$$\therefore \text{Amount} = 4500 \times \left(1 + \frac{4}{100}\right)^3$$

$$= 4500 \times \left(\frac{26}{25}\right)^3 = ₹5061.88$$

$$\therefore \text{C.I.} = 5,061.88 - 4,500 = ₹561.88.$$

42. a Let the sum be x. Then,

$$\text{CI} = x \left(1 + \frac{10}{100}\right)^2 - x = \frac{21x}{100}$$

$$\text{SI} = \frac{x \times 10 \times 2}{100} = \frac{x}{5}$$

$$\therefore (\text{CI}) - (\text{SI}) = \frac{21x}{100} - \frac{x}{5} = 105$$

$$\Rightarrow \frac{x}{100} = 105 \Rightarrow x = ₹10,500.$$

Alternate Method:

$$\Rightarrow \frac{x \times R^2}{100^2} = 105 \Rightarrow \frac{x \times 10^2}{100^2} = 105 \Rightarrow x = ₹10,500.$$

43. e Let sum be x . Then, $SI = \frac{9x}{16}$

Let rate = $R\%$ and time = R year. Then,

$$\frac{x \times R \times R}{100} = \frac{9x}{16}$$

$$\Rightarrow R^2 = \frac{9 \times 100}{16} = \frac{9 \times 25}{4}$$

$$\Rightarrow R = \frac{3 \times 5}{2} = \frac{15}{2} = 7\frac{1}{2}$$

\therefore Rate = $7\frac{1}{2}\%$ and time = $7\frac{1}{2}$ years.

44. e $P = ₹100$, Amount = $₹121$, Rate = 10% p.a.
Let the time be n years. Then,

$$100 \left(1 + \frac{10}{100}\right)^n = 121$$

$$\Rightarrow \left(1 + \frac{1}{10}\right)^n = \frac{121}{100} \Rightarrow \left(\frac{11}{10}\right)^n = \left(\frac{11}{10}\right)^2$$

$$\Rightarrow n = 2 \text{ years.}$$

45. a Let amount after 4 years be A . Then,

$$A = 5000 \left[1 + \frac{2}{100}\right] \left[1 + \frac{4}{100}\right] \left[1 + \frac{8}{100}\right] \left[1 + \frac{12}{100}\right]$$

$$\Rightarrow A = ₹6,415$$

$$\therefore CI = 6,415 - 5,000 = ₹1,415.$$

46. b Difference = $\frac{PR^2}{(100)^2}$

$$= \frac{4000 \times 25}{100 \times 100} = ₹10.$$

47. a Let sum be $₹P$.

Then interest = $₹2P$.

$$\therefore 2P = \frac{P \times R \times 25}{100}$$

$$\Rightarrow R = 8\%.$$

48. b Interest = $5700 - 5000 = ₹700$

$$\text{Rate} = \frac{700 \times 100}{5000 \times 1} = 14\%$$

$$\text{Now, interest} = \frac{7000 \times 5 \times 14}{100} = ₹4,900$$

$$\text{Total amount} = 7000 + 4900 = ₹11,900.$$

49. c Let $₹x$ doubles itself in 4 years.

$2x$ become $4x$ in next 4 years,
therefore, it takes 8 years for amount to be four times.

50. c $SI = (956 - 800) = 156 = \frac{800 \times 3 \times R}{100}$

$$\Rightarrow R = \frac{156}{24} = \frac{13}{2}\%$$

$$\text{New rate} = 4 + \frac{13}{2} = \frac{21}{2}\%$$

$$\text{New SI} = \frac{800 \times 21}{100 \times 2} \times 3 = ₹252$$

$$\text{Total amount} = 800 + 252 = ₹1,052.$$

**Ratio, Proportion and Partnership
Answers and Explanations**

1	b	2	c	3	a	4	b	5	d	6	b	7	c	8	a	9	d	10	b
11	c	12	a	13	d	14	a	15	d	16	c	17	c	18	c	19	b	20	a
21	d	22	b	23	a	24	d	25	a	26	a	27	d	28	a	29	a	30	b
31	b	32	a	33	c	34	e	35	b	36	d	37	b	38	c	39	c	40	a
41	a	42	c	43	d	44	d	45	c	46	d	47	e	48	c	49	b	50	d

1. b $\frac{28}{x} = \frac{x}{7} \Rightarrow x^2 = 28 \times 7 \Rightarrow x = 14.$

2. c Given ratio = $\frac{1}{2} : \frac{1}{3} : \frac{1}{4} = \frac{6}{12} : \frac{4}{12} : \frac{3}{12}$ i.e. $6 : 4 : 3$

$$\therefore \text{Largest part} = 195 \times \frac{6}{(6+4+3)} = 195 \times \frac{6}{13} = ₹90.$$

3. a Let $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-3b+5c}{k} = \lambda$
 $\therefore a = 2\lambda, b = 3\lambda, c = 4\lambda$
 $\therefore \frac{2a-3b+5c}{k} = \lambda$
 $\Rightarrow \frac{2 \times 2\lambda - 3 \times 3\lambda + 5 \times 4\lambda}{k} = \lambda$
 $\Rightarrow \frac{15\lambda}{k} = \lambda \Rightarrow k = 15.$

4. b A : B : C = $\frac{1}{10} : \frac{1}{9} : \frac{1}{4} = \frac{18}{180} : \frac{20}{180} : \frac{45}{180} = 18 : 20 : 45$
 $\therefore \text{B's share} = \frac{20}{83} \times 4980 = ₹1,200.$

5. d Given ratio = $5 : 4 : 9 : 7$
Sum of ratio terms = 25
Largest part = $125 \times \frac{9}{25} = ₹45.$

6. b Total age of 3 children = $10 \times 3 = 30$ years
Ratio of their age = $2 : 3 : 5$
Age of middle child = $\frac{3}{10} \times 30 = 9$ years.

7. c A : B = 4 : 5 and B : C = 3 : 5
 $\Rightarrow A : B : C = 4 \times 3 : 5 \times 3 : 5 \times 5 = 12 : 15 : 25$
A : B : C = 12 : 15 : 25 and C : D = 3 : 2
 $\Rightarrow A : B : C : D = 12 \times 3 : 15 \times 3 : 25 \times 3 : 25 \times 2 = 36 : 45 : 75 : 50$
Hence, C gets the maximum amount.

Alternative method:

A : B is $4 : 5 \Rightarrow B$ is greater than A.
B : C is $3 : 5 \Rightarrow C$ is greater than B.
C : D is $3 : 2 \Rightarrow C$ is greater than D.

8. a Let father's age be $4x$ and son's age be x . Then,
 $\frac{4x+5}{x+5} = \frac{3}{1}$
 $\Rightarrow 4x+5 = 3x+15$
 $\Rightarrow x = 10$
Hence, the present age of father = $4x = 4 \times 10 = 40$ years.

9. d A = C + 1200, B = C + 500
 $\Rightarrow C + 1200 + C + 500 + C = 4700$
 $\Rightarrow 3C = 3000 \Rightarrow C = 1000$
Hence, A : B : C = 22 : 15 : 10
A's share = $\frac{22}{47} \times 423 = ₹198.$

10. b $\frac{x^2}{y^2} = \frac{a^2}{b^2} = \frac{9}{4} \Rightarrow \frac{x^2 + a^2}{y^2 + b^2} = \frac{9}{4}.$

11. c Given ratio = $\frac{1}{2} : \frac{1}{3} : \frac{1}{5} = \frac{15}{30} : \frac{10}{30} : \frac{6}{30}$ i.e. $15 : 10 : 6$
 \therefore The difference between highest and lowest shares = $465 \times \frac{(15-6)}{31} = ₹135.$

12. a A : B = 5 : 4, B : C = 8 : 9
 $\therefore A : B : C = 40 : 32 : 36 = 10 : 8 : 9$
 C's share is $= \frac{9}{27} \times 3600 = ₹1,200.$
13. d Let $4x$, $5x$ and $6x$ be the number of coins of one rupee, 50-paisa and 25-paisa respectively.
 $\therefore 4x + \frac{5x}{2} + \frac{6x}{4} = 32$
 $\Rightarrow \frac{16x + 10x + 6x}{4} = 32 \Rightarrow 32x = 128 \Rightarrow x = 4$
 \therefore Number of coins are 16, 20 and 24.
14. a Let 'f' be the age of father and 's' be the age of son.
 $f - 5 = 5(s - 5) \Rightarrow f - 5s = -20 \quad \dots(i)$
 and $f + 2 = 3(s + 2) \Rightarrow f - 3s = 4 \quad \dots(ii)$
 Subtracting (i) from (ii), we get
 $2s = 24 \Rightarrow s = 12$ years
 $\therefore f = 40$ years
 \therefore Required ratio $= \frac{40}{12} = \frac{10}{3}$ i.e. 10 : 3.
15. d Either incomes or expenditure must be given. Hence, it cannot be determined.
16. c $\frac{A}{5} = \frac{B}{7} = \frac{C}{6} = k$ (Let)
 $\Rightarrow A = 5k, B = 7k$ and $C = 6k$
 $\therefore \frac{A+B+C}{C} = \frac{5k+7k+6k}{6k} = \frac{18k}{6k} = 3.$
17. c 20% of B $= \frac{1}{3}G$
 $\Rightarrow \frac{20}{100}B = \frac{1}{3}G \Rightarrow \frac{B}{G} = \frac{5}{3}, B : G = 5 : 3.$
18. c Let B joined after x months, then B's money was invested for $(12 - x)$ months.
 $\therefore \frac{4500 \times 12}{3000 \times (12-x)} = \frac{3}{1}$
 $\Rightarrow x = 6$ months.
19. b Let the shares of A, B and C after diminishing the respective amounts from their shares be $8x$, $15x$ and $20x$ respectively. Then
 $(8x + 25) + (15x + 28) + (20x + 52) = 3115$
 $\Rightarrow 43x = 3010 \Rightarrow x = 70$
 A's share $= 8 \times 70 + 25 = ₹585$
 C's share $= 20 \times 70 + 52 = ₹1,452$
 Required difference $= 1,452 - 585 = ₹867.$
20. a A : B : C $= 800 \times 4 : 1200 \times 8 : 1400 \times 10$
 $= 3200 : 9600 : 14000 = 32 : 96 : 140$
 $= 8 : 24 : 35$
 \therefore A's share $= \frac{8}{67} \times 1340 = ₹160.$
21. d Ratio of their shares $= (35000 \times 8) : (42000 \times 10) = 2 : 3$
 \therefore Pawan's share $= 31,570 \times \frac{2}{5} = ₹12,628.$
22. b Let x , $2x$ and $3x$ be the number of coins of 25 paisa, 10 paisa and 5 paisa respectively.
 Then, $\frac{25}{100} \times x + \frac{10}{100} \times 2x + \frac{5}{100} \times 3x = 45$
 $\Rightarrow 25x + 20x + 15x = 4500 \Rightarrow 60x = 4500 \Rightarrow x = 75$
 \therefore Number of 10 paisa coins $= 2 \times 75 = 150.$
23. a $B + C = 100, C + A = 150, A = 2B$
 $C + A = C + 2B = 150 (\because A = 2B)$
 $\Rightarrow (B + C) + B = 150 \Rightarrow 100 + B = 150$
 $B = 150 - 100 = ₹50$
 $\therefore A$ get ₹100.
 $\therefore A + B + C = ₹200.$
24. d $\frac{5m - 4n}{5m + 4n} = \frac{1}{4}$
 $\Rightarrow \frac{5m - 4n + 5m + 4n}{5m - 4n - 5m - 4n} = \frac{1+4}{1-4}$
 $\Rightarrow \frac{10m}{-8n} = \frac{5}{-3} \Rightarrow \frac{5m}{4n} = \frac{5}{3} \Rightarrow m = \frac{4}{3}n$
 $\therefore 3m + 2n = 24$
 $\Rightarrow 3 \times \frac{4}{3}n + 2n = 24 \Rightarrow 6n = 24 \Rightarrow n = 4$
 $\therefore m = \frac{4}{3} \times 4 = \frac{16}{3}.$
25. a Let $A = k, B = 2k, C = 5k$
 A's new salary $= \frac{120}{100} \text{ of } k = \frac{120}{100} \times k = \frac{6}{5}k$
 B's new salary $= \frac{115}{100} \text{ of } 2k = \frac{115}{100} \times 2k = \frac{23}{10}k$
 C's new salary $= \frac{110}{100} \text{ of } 5k = \frac{110}{100} \times 5k = \frac{11}{2}k$
 \therefore New ratio $= \frac{6}{5}k : \frac{23}{10}k : \frac{11}{2}k = 12 : 23 : 55.$
26. a A : B : C $= 6000 : 7000 : 8000 = 6 : 7 : 8$
 Share of A $= 3360 \times \frac{6}{21} = ₹960$
 Share of B $= 3360 \times \frac{7}{21} = ₹1,120$
 Share of C $= 3360 \times \frac{8}{21} = ₹1,280.$

27. d Let C's capital be ₹x. Then,

$$\text{B's capital} = \text{₹} \frac{3}{4} x$$

$$\text{A's capital} = 4 \times \frac{3}{4} x = \text{₹} 3x$$

$$\therefore \text{Ratio of their capitals} = 3x : \frac{3}{4} x : x = 12 : 3 : 4$$

$$\therefore \text{A's share} = \frac{12}{19} \times 7,220 = \text{₹} 4,560.$$

$$28. \text{ a } x : y = 1 : 2 \Rightarrow \frac{x}{y} = \frac{1}{2}$$

$$\therefore \frac{4x+3y}{7x-3y} = \frac{y\left(\frac{4x}{y} + 3\right)}{y\left(\frac{7x}{y} - 3\right)} = \frac{4 \times \frac{1}{2} + 3}{7 \times \frac{1}{2} - 3}$$

$$= \frac{5 \times 2}{7 - 6} = \frac{10}{1} \text{ i.e. } 10 : 1.$$

$$29. \text{ a } \text{Compound ratio} = \frac{2}{3} \times \frac{6}{7} \times \frac{3}{4} = \frac{3}{7} = 3 : 7.$$

30. b Let 7x and 4x be the numbers. Then,

$$7x - 4x = 81$$

$$\Rightarrow 3x = 81 \Rightarrow x = 27$$

\therefore Numbers are 189 and 108.

$$31. \text{ b } \frac{x}{y} = \frac{z}{w} = \frac{p}{q} = 1.25$$

$$\Rightarrow \frac{2x+3z+4p}{2y+3w+4q} = 1.25. \quad \left[\because \frac{x}{y} = \frac{z}{w} = \frac{p}{q} = \frac{ax+bx+cp}{ay+bw+cq} \right]$$

32. a Let Kaushal invested for x months.

$$\text{Then, } 3000 \times 8 : 4000 \times x = 2 : 1$$

$$\Rightarrow \frac{3000 \times 8}{4000 \times x} = \frac{2}{1}$$

$$\Rightarrow x = 3 \text{ months.}$$

33. c Product of amount and duration will give the ratio in which the profit is divided.

$$5x : 6y : 8z = 5 : 3 : 1$$

$$\Rightarrow x = 1, y = \frac{1}{2} \text{ and } z = \frac{1}{8}$$

\therefore Ratio of duration is $x : y : z = 1 : \frac{1}{2} : \frac{1}{8}$ i.e. $8 : 4 : 1$.

$$34. \text{ e } \text{Seeta : Geeta} = (4200 \times 36) : (5400 \times 30) \\ = 14 : 15$$

$$\therefore \text{Seeta's share} = \frac{14}{(14+15)} \times 3190 = \text{₹} 1,540.$$

$$35. \text{ b } \text{Number of boys} = \frac{11}{(11+9)} \times 360 = \frac{11}{20} \times 360$$

$$= 198.$$

$$\text{Number of girls} = \frac{9}{(11+9)} \times 360 = \frac{9}{20} \times 360$$

$$= 162.$$

$$36. \text{ d } \text{Let fourth proportional be } x. \text{ Then,} \\ 7 : 9 = 14 : x$$

$$\therefore x = \frac{14 \times 9}{7} = 18.$$

$$37. \text{ b } \text{Let } 2A = 3B = 6C = 6. \text{ Then,} \\ \text{ratio of shares A, B and C is } 3 : 2 : 1 \\ \therefore \text{The difference between A's share and C's share}$$

$$= 600 \times \frac{3}{6} - 600 \times \frac{1}{6} = 300 - 100 = \text{₹} 200.$$

38. c Wealth of saif is ₹30 crores. Then,

$$\text{Amrita gets } 30 \times \frac{1}{3} = \text{₹} 10 \text{ crores,}$$

$$\text{Kareena gets } 20 \times \frac{1}{4} = \text{₹} 5 \text{ crores,}$$

$$\text{Parikrma gets } 15 \times \frac{1}{5} = \text{₹} 3 \text{ crores,}$$

$$\text{Sara gets } \frac{12}{2} = \text{₹} 6 \text{ crores and Ibrahim gets } \text{₹} 6 \text{ crores.}$$

$$\therefore \text{Difference between the wealth of Amrita and Sara} \\ = 10 - 6 = \text{₹} 4 \text{ crores.}$$

$$39. \text{ c } \text{Number of coins} = \frac{\text{Amount in rupees}}{\text{Value of coins in rupees}}$$

Let the value of one rupee, fifty paise and twenty five paise coins be $2x$, $5x$ and $7x$ respectively.

$$\therefore \text{Number of 1 rupee coin} = \frac{2x}{1}$$

Number of 50 paise coin = $\frac{5x}{\frac{1}{2}} = 10x$

Number of 25 paise coin = $\frac{7x}{\frac{1}{4}} = 28x$

$$\therefore 2x + 10x + 28x = 400$$

$$\Rightarrow x = 10$$

$$\therefore \text{Value of one rupee coin} = 2 \times 10 = ₹20$$

$$\text{Value of 50 paise coin} = 5 \times 10 = ₹50$$

$$\text{Value of 25 paise coin} = 7 \times 10 = ₹70$$

$$\text{Value of total amount} = ₹140.$$

40. a Third proportion of 3 and 4 = $\frac{4^2}{3} = \frac{16}{3}$

Fourth proportion of 6, 8 and 10 = $\frac{8 \times 10}{6} = \frac{40}{3}$

$$\text{Required ratio} = \frac{16}{3} : \frac{40}{3} = 2 : 5.$$

41. a Ratio of respective number of coins = 5 : 6 : 4

$$\text{Number of ₹1 coins} = \frac{5}{15} \times 465 = 155$$

$$\text{Number of 50 paise coins} = \frac{6}{15} \times 465 = 186$$

$$\text{Number of 25 paise coins} = \frac{4}{15} \times 465 = 124.$$

42. c Let amount invested by B be ₹x, then

$$\frac{12 \times 7000}{7 \times x} = \frac{2}{3}$$

$$\Rightarrow 12000 \times 3 = 2x$$

$$\Rightarrow x = \frac{12000 \times 3}{2} = ₹18,000.$$

43. d Let weight of a piece of diamond be 6y.

$$\text{Let original price of one piece be ₹x}$$

$$\text{So, } x(6y)^2 = 5184$$

$$\text{New price} = x(y^2 + 4y^2 + 9y^2)$$

$$\Rightarrow 14xy^2 = \frac{14 \times 5184}{36} = ₹2,016$$

$$\text{Loss} = 5184 - 2016 = ₹3,168.$$

44. d Let total capital be 12x and total time be 12t.

$$\text{Then, ratio of investment} = 4xt : 9xt : 84xt$$

$$= 4 : 9 : 84$$

$$\text{Hence, part of B} = \frac{9}{97} \times 19400 = ₹1,800.$$

45. c Let D's share be 27x.

$$\text{Then, C's share} = 9x, \text{B's share} = 3x \text{ and A's share} = x$$

Then, $x + 3x + 9x + 27x = 4,00,000 \Rightarrow x = 10,000$
Hence, C's share = $9x = ₹90,000$.

46. d Let x be subtracted. Then,

$$\frac{17-x}{24-x} = \frac{1}{2}$$

$$\Rightarrow 34 - 2x = 24 - x$$

$$\Rightarrow -2x + x = 24 - 34 \Rightarrow x = 10$$

∴ 10 is subtracted from each term to get ratio 1 : 2.

47. e Let income of A and B be 9x and 7x, and expenditure of A and B be 4y and 3y.

$$\text{Then, } 9x - 4y = 200$$

$$\text{and } 7x - 3y = 200$$

$$\Rightarrow x = 200 \text{ and } y = 400$$

$$\text{Hence, sum of incomes} = 9x + 7x = 16x = ₹3,200.$$

48. c Let the number be 3x, 4x and 5x

$$\text{Then, } 3x + 5x = 4x + 52$$

$$\Rightarrow 4x = 52 \Rightarrow x = 13$$

$$\therefore \text{The smallest number} = 3 \times 13 = 39.$$

49. b Let the present ratio of their ages be $\frac{x}{y}$.

Before 10 years

$$\frac{x-10}{y-10} = \frac{1}{3}$$

$$\Rightarrow 3x - y = 20$$

...(i)

After 5 years

$$\frac{x+5}{y+5} = \frac{2}{3}$$

$$\Rightarrow 3x + 15 = 2y + 10$$

$$\Rightarrow 3x - 2y = -5$$

...(ii)

Solving (i) and (ii), we get

$$x = 15 \text{ and } y = 25$$

$$\text{Hence, required ratio} = \frac{15}{25} = 3 : 5.$$

50. d Let the income of A, B and C be 7x, 9x and 12x and spending of A, B and C be 8y, 9y and 15y

$$\text{Then, } 7x - 8y = \frac{1}{4} \times 7x$$

$$\Rightarrow 7x - \frac{7}{4}x = 8y$$

$$\Rightarrow 21x = 32y$$

$$\frac{x}{y} = \frac{\text{Total salary}}{\text{Spending}} = \frac{32}{21}$$

Ratio of savings of A, B and C

$$= (7x - 8y) : (9x - 9y) : (2x - 5y)$$

$$= (7 \times 32 - 8 \times 21) : (9 \times 32 - 9 \times 21) : (12 \times 32 - 15 \times 21)$$

$$= 56 : 99 : 69.$$

Average

Answers and Explanations

1	b	2	a	3	b	4	d	5	c	6	d	7	b	8	a	9	b	10	c
11	c	12	c	13	b	14	c	15	c	16	d	17	c	18	b	19	c	20	b
21	d	22	c	23	b	24	c	25	d	26	b	27	e	28	e	29	c	30	c
31	b	32	a	33	c	34	a	35	c	36	b	37	a	38	c	39	d	40	a
41	c	42	b	43	d	44	b	45	d	46	b	47	d	48	a	49	c	50	b

1. b Average = $\frac{70 + 63 + 72 + 81 + 74}{5} = \frac{360}{5} = 72$ marks.

2. a There are four prime numbers between 20 and 40.

$$\text{Required average} = \frac{23 + 29 + 31 + 37}{4} = \frac{120}{4} = 30.$$

3. b Let x be the first number, then 2x and 4x will be second and third respectively.

$$\text{Average} = \frac{x + 2x + 4x}{3} = 7$$

$$\Rightarrow 7x = 21$$

$$\Rightarrow x = 3$$

Smallest number is 3.

4. d Average = $\frac{15 + x + 17 + 19}{4} = 18$

$$\Rightarrow \frac{51 + x}{4} = 18$$

$$\Rightarrow x = 72 - 51$$

$$\Rightarrow x = 21.$$

5. c Let x kg be the weight of new person. Then,

$$\frac{10 \times 50 + x}{11} = 51$$

$$\Rightarrow 500 + x = 561$$

$$\Rightarrow x = 561 - 500$$

$$\Rightarrow x = 61 \text{ kg.}$$

6. d Let x be the next test score. Then,

$$\text{The average} = \frac{78 + 92 + 83 + 99 + x}{5} = 90$$

$$\Rightarrow \frac{352 + x}{5} = 90$$

$$\Rightarrow x = 98.$$

7. b Total weight of students = $25 \times 38 + 32 \times 35 = 2070$ kg

$$\text{Average weight of the whole class} = \frac{2070}{57} = 36.3 \text{ kg.}$$

8. a A + B + C = $40 \times 3 = 120$ kg

Weight of A and B = $35 \times 2 = 70$ kg

Weight of B and C = $36 \times 2 = 72$ kg

$$\therefore \text{Weight of B} = (A + B) + (B + C) - (A + B + C) = 70 + 72 - 120 = 22 \text{ kg.}$$

9. b As 69 is misread as 96, we have taken extra 27 value in account which is supposed to be taken away from the total of 20. So, the new average will be

$$63 - \frac{27}{20} = 63 - 1.35 = 61.65.$$

10. c Sum of ages of 9 students = $25 \times 15 - 16 \times 13 = 167$ years

$$\text{Required average} = \frac{167}{9} = 18.5 \text{ years.}$$

11. c Mean of 50 numbers = 30

Sum of 50 numbers = $30 \times 50 = 1500$

Wrongly entered numbers are 82 and 13

$$\therefore 1500 - (82 + 13) = 1500 - 95 = 1405$$

$$\text{Correctly entered numbers} = 1405 + (28 + 31) = 1405 + 59 = 1464$$

$$\text{Required mean} = \frac{1464}{50} = 29.28.$$

12. c Let the four numbers be a, b, c and d.

$$\therefore \frac{a+b+c+d}{4} = 60$$

$$\Rightarrow a + b + c + d = 240 \text{ and } a = \frac{b+c+d}{4}$$

$$\Rightarrow a = \frac{240 - a}{4} \Rightarrow \frac{5a}{4} = 60 \Rightarrow a = 60 \times \frac{4}{5} = 48.$$

13. b Total = $13 \times 25 = 325$
According to the question,
Total = $320 - 48 + 73 = 350$

$$\text{Average of 25 numbers} = \frac{350}{25} = 14.$$

14. c Let the average age of team be x years.
Total age = $11x$

$$\text{Required average} = \frac{11\left(x + \frac{1}{6}\right) - (112 - 18 - 20)}{2}$$

= 19 years 11 months.

15. c Let number of student in section A and B be a and b respectively.

$$\therefore \frac{77.5a + 70b}{a+b} = 74$$

$$\Rightarrow 77.5a + 70b = 74a + 74b$$

$$\Rightarrow \frac{a}{b} = \frac{4}{3.5} = \frac{8}{7} \text{ i.e., } 8 : 7.$$

16. d Total weight of 20 boys = $89.4 \times 20 = 1788$ kg
Now, $1788 - 78 + 87 = 1797$ kg

$$\text{The correct average weight} = \frac{1797}{20} = 89.85 \text{ kg.}$$

17. c Let consecutive odd numbers be x, x + 2, x + 4.

$$\frac{x+x+2+x+4}{3} = 12 + \frac{x}{3}$$

$$\Rightarrow \frac{3x+6}{3} = \frac{36+x}{3}$$

$$\Rightarrow 3x + 6 = 36 + x$$

$$\Rightarrow 2x = 30$$

$$\Rightarrow x = 15$$

$$\therefore \text{Largest number} = x + 4 = 15 + 4 = 19.$$

18. b Average of 18 observations = 124
 $\therefore \text{Sum of 18 observations} = 124 \times 18 = 2232$
 Difference between wrong entry
 $= (46 + 82) - (64 + 28) = 36$
 $\therefore \text{New sum of 18 observations after correction} = 2232 - 36 = 2196$

$$\therefore \text{New average} = \frac{2196}{18} = 122.$$

$$19. c 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\therefore \text{Average of these numbers} = \frac{n+1}{2}$$

$$\therefore \text{Required average} = \frac{100+1}{2} = 50.5.$$

20. b Total age of father and mother = $2 \times 35 = 70$ years
 Total age of father, mother and son = $27 \times 3 = 81$ years
 \therefore Son's age = $81 - 70 = 11$ years.

21. d Sum of 30 numbers = $30 \times 12 = 360$
 Sum of first 20 numbers = $20 \times 11 = 220$
 Sum of next 9 numbers = $9 \times 10 = 90$
 \therefore Last number = $360 - (220 + 90) = 50$.

22. c Total age of 4 members = $4 \times 36 = 144$ years
 Total age of 3 members before 12 years
 $= 144 - 4 \times 12 = 96$ years

$$\therefore \text{Required average age} = \frac{96}{3} = 32 \text{ years.}$$

23. b Let the 6th number be x.

$$\therefore 8 \times 20 = 31 + \frac{64}{3} \times 3 + x + x + 4 + x + 7$$

$$\Rightarrow 160 = 31 + 64 + x + x + 4 + x + 7 = 3x + 106$$

$$\Rightarrow x = \frac{160 - 106}{3} = 18$$

$$\therefore 8^{\text{th}} \text{ number} = 18 + 7 = 25.$$

24. c Let the number of wickets taken before last match by the cricketer be x. Then,
 $12.4x + 26 = 12.2(x + 5)$
 $\Rightarrow 0.2x = 61 - 26$
 $\Rightarrow x = \frac{35}{0.2} = 175.$

25. d Fifth number = $5 \times 46 - 4 \times 45 = 230 - 180 = 50$.

26. b The average weight of the class is

$$\frac{12 \times 50 + 24 \times 40}{36} = \frac{50 + 2 \times 40}{3} = 43.33 \text{ kg.}$$

27. e The rain fall from Monday to Saturday = $5 \times 6 = 30$ inches.
 The rain fall for the whole week = $7 \times 8 = 56$ inches
 \therefore The rain fall on Sunday = $56 - 30 = 26$ inches.

28. e Let x cm be the height of the person. Then,

$$\begin{aligned}\frac{5 \times 162 - x}{4} &= 161 \\ \Rightarrow 810 - x &= 644 \\ \Rightarrow x &= 810 - 644 \\ \Rightarrow x &= 166 \text{ cm.}\end{aligned}$$

29. c Total age of Ram's family = $40 \times 4 = 160$ years
Three years later, the total age of Ram's family = $160 + 3 \times 4 = 172$ years.
But now Ram dies at the age of 53 years while a new member is born. Hence, overall the total member remains same but the total age of family decreases by 53 years

$$\text{New average of the family } = \frac{172 - 53}{4} = \frac{119}{4} = 29.75 \text{ years.}$$

30. c Average of ' n ' numbers = x
 \therefore Sum of ' n ' numbers = $n \times x = nx$

$$\begin{aligned}\therefore \frac{nx - 36 - 36}{n} &= x - 8 \\ \Rightarrow x - \frac{72}{n} &= x - 8 \\ \Rightarrow \frac{72}{n} &= 8 \\ \Rightarrow n &= \frac{72}{8} = 9.\end{aligned}$$

31. b Let present ages of A, B and C be x years, y years and z years respectively.
 $\therefore (x - 4) + (y - 4) + (z - 4) = 3 \times 25 = 75$
 $\Rightarrow x + y + z = 75 + 12 = 87 \quad \dots(i)$
Again, $(y - 5) + (z - 5) = 2 \times 20 = 40$
 $\Rightarrow y + z = 40 + 10 = 50 \quad \dots(ii)$
 \therefore Present age of A = $(x + y + z) - (y + z) = 87 - 50 = 37$ years.

32. a Let the distance of complete journey be x km.

$$\begin{aligned}\therefore \text{Average speed} &= \frac{\text{Total distance}}{\text{Total time}} \\ &= \frac{2x}{\left(\frac{x}{20} + \frac{x}{12}\right)} \\ &= \frac{2 \times 20 \times 12}{(12 + 20)} \\ &= \frac{40 \times 12}{32} = 15 \text{ km/hr.}\end{aligned}$$

33. c Average age of the boys

$$\begin{aligned}&= \frac{20 \times 12 + 5 \times 7}{(20 + 5)} = \frac{(240 + 35)}{25} \\ &= \frac{275}{25} = 11 \text{ years.}\end{aligned}$$

34. a Let the first number be x . Then, the second and third number will be

$$\frac{x}{2} \text{ and } \frac{x}{3} \text{ respectively.}$$

$$\begin{aligned}\therefore \frac{x + \frac{x}{2} + \frac{x}{3}}{3} &= 49.5 \\ \Rightarrow \frac{6x + 3x + 2x}{6} &= 49.5 \times 3 \\ \Rightarrow \frac{11}{6}x &= 49.5 \times 3 \\ \Rightarrow x &= \frac{49.5 \times 3 \times 6}{11} = 81\end{aligned}$$

Therefore, first number is 81 and third number is 27
Difference between first and third number = $81 - 27 = 54$.

35. c Let the ages of 3 girls be $7x$, $5x$ and $3x$ years respectively.

$$\therefore 7x + 5x + 3x = 3 \times 20 = 60$$

$$\Rightarrow x = \frac{60}{15} = 4$$

\therefore Age of the youngest girl = $3x = 3 \times 4 = 12$ years.

36. b Let the age of teacher be x years.
Then, $27 \times 17.5 + x = 28 \times 18.5$

$$\begin{aligned}\Rightarrow x &= 518 - 472.5 \\ \Rightarrow x &= 45.5 \text{ years.}\end{aligned}$$

37. a Let the earlier average be x runs per match.
Then, $25x + 98 = 26(x + 2)$
 $\Rightarrow x = 46$.

38. c Let average marks of the girls be $x\%$. Then,

$$\text{Number of boys} = \frac{4}{7} \times 70 = 40$$

$$\text{Number of girls} = \frac{3}{7} \times 70 = 30$$

$$\text{As, } \frac{40 \times 86 + 30 \times x}{70} = 80 \Rightarrow x = 72.$$

39. d Average price of mixture (₹ per kg) = $\frac{5 \times 45 + 7 \times 42}{12}$
 $= \frac{519}{12} = 43.25.$

40. a Let the average monthly salary of remaining employees be ₹x. Then,

$$\frac{12 \times 75,000 + 78 \times x}{90} = 32,000$$

$$\Rightarrow x \approx 25,400.$$

41. c Let the sales in the 6th month be ₹x. Then,
 $54,000 + 62,000 + 55,000 + 40,000 + 72,000 + x = 6 \times 60,000$
 $\Rightarrow x = ₹77,000.$

42. b Let first number be x.
 Then, 7th number will be $x + 6$, and 14th number will be $x + 13$.

Average of first seven numbers

$$= K = \frac{x + x + 1 + x + 2 + x + 3 + x + 4 + x + 5 + x + 6}{7}$$

$$\Rightarrow K = \frac{7x + 21}{7} = x + 3$$

Average of next seven numbers

$$= \frac{x + 7 + x + 8 + x + 9 + x + 10 + x + 11 + x + 12 + x + 13}{7}$$

$$= \frac{7x + 70}{7} = x + 10 = K + 7.$$

43. d Total time taken in Journey

$$= \frac{250}{75} + \frac{350}{70} + \frac{200}{30} = 15 \text{ hr}$$

∴ The average speed

$$= \frac{250 + 350 + 220}{15} = 53.33 \text{ km/hr.}$$

44. b Sum of A, B, C and D = $79 \times 4 = 316$
 Sum of B, C, D and E = $63 \times 4 = 252$
 $\text{So, } A - E = 316 - 252 = 64$
 $\therefore E = 23, \text{ then } A = 64 + 23 = 87.$

45. d Average of numbers will also be increased by same proportion.
 $\therefore \text{New average} = 31 \times 3 + 5 = 98.$

46. b Sum of first 'n' odd natural numbers = n^2
 Hence, average of first 'n' odd natural numbers = n
 Hence, answer is 13.

47. d Let the fifth number be x. Then
 $4 \times 17 + x + 4 \times 20 = 9 \times 18$
 $\Rightarrow x = 14.$

48. a Two years ago, the average age of the family = 43 years
 Sum of ages of all members at present
 $= (43 + 2) \times 5 + 2 = 227 \text{ years}$
 Average of ages of the family members

$$= \frac{227}{6} = 37 \frac{5}{6} \text{ years.}$$

49. c Average weight of the group

$$= \frac{72 \times 18 - 81 + 63}{18} = \frac{72 \times 18 + 63}{18} = 71 \text{ kg.}$$

50. b Average speed of the train

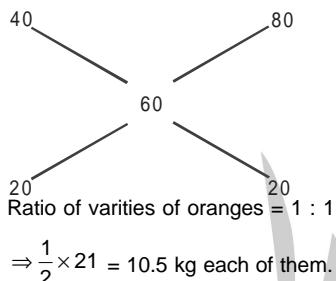
$$= \frac{63 \times 5 + 58 \times 3 + 72 \times 2}{10} = \frac{633}{10} = 63.3 \text{ km/hr.}$$

Mixtures and Alligations

Answers and Explanations

1	c	2	b	3	b	4	c	5	b	6	a	7	c	8	c	9	d	10	a
11	b	12	b	13	a	14	c	15	b	16	d	17	b	18	d	19	c	20	a
21	b	22	a	23	c	24	b	25	c	26	a	27	d	28	b	29	c	30	a
31	d	32	a	33	a	34	d	35	c	36	c	37	c	38	b	39	b	40	e
41	d	42	a	43	b	44	d	45	c	46	b	47	d	48	e	49	c	50	c

1. c



2. b

Initial quantity of milk in solution = $\frac{4}{5} \times 35 = 28$ litres
and, initial quantity of water in solution = $35 - 28 = 7$ litres
Hence, required ratio = $28 : (7 + 7) = 2 : 1$.

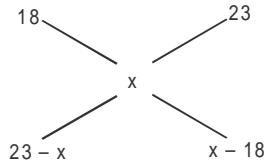
3. b

Milk in first mixture $M_1 = \frac{3}{5} \times 400 = 240$ ml
Water in first mixture $W_1 = 400 - 240 = 160$ ml
Milk in second mixture $M_2 = \frac{4}{5} \times 1000 = 800$ ml
Water in second mixture $W_2 = 1000 - 800 = 200$ ml

Hence, total milk = $240 + 800 = 1040$ ml and
Total water = $160 + 200 = 360$ ml
Hence, milk : water = $1040 : 360 = 26 : 9$.

4. c

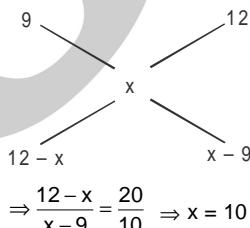
Let the price of mixed variety be ₹x per kg.



$$\therefore \frac{23-x}{x-18} = \frac{2}{3} \Rightarrow 69 - 3x = 2x - 36 \\ \Rightarrow x = \frac{105}{5} = 21.$$

5. b

Let cost price of the mixture be ₹x per kg.
Then,



$$\Rightarrow \frac{12-x}{x-9} = \frac{20}{10} \Rightarrow x = 10$$

Selling price of mixture = $10 \times \frac{120}{100} = ₹12$ per kg.

6. a

Quantity of Spirit = $\frac{5}{7} \times 70 = 50$ litres

Quantity of Alcohol = $\frac{2}{7} \times 70 = 20$ litres

Hence, 30 litres alcohol has to be mix to make quantity of both equal.

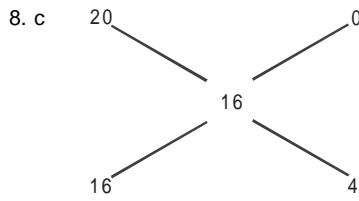
7. c

Remaining mixture = $80 - 16 = 64$ litres

In 64L mixture, milk = $64 \times \frac{5}{8} = 40$ litres

and water = $64 - 40 = 24$ litres

$$\therefore \text{Required ratio} = \frac{40+16}{24} = \frac{56}{24} \text{ i.e. } 7 : 3.$$



$$\text{Milk : water} = 16 : 4 = 4 : 1$$

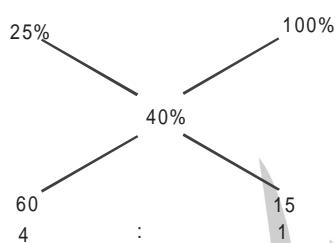
$$\therefore \text{Quantity of milk in the mixture} = 4 \times 10 = 40 \text{ litres.}$$

9. d Percentage of water in the original mixture

$$= \frac{1}{4} \times 100 = 25\%$$

Percentage of water in the resulting mixture

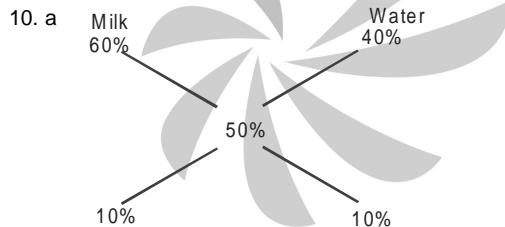
$$= \frac{2}{5} \times 100 = 40\%$$



Therefore, the ratio in which the mixture and water are to be added is 4 : 1.

Then quantity of water to be added

$$= \frac{600 \times 1}{4} = 150 \text{ litres.}$$

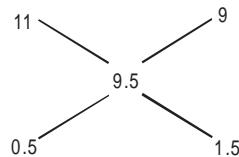


Ratio of milk to water = 10 : 10 = 1 : 1

$$\therefore \text{Quantity of milk} = \frac{90 \times 1}{2} = 45 \text{ litres.}$$

11. b SP of 1 kg of mixture = ₹10.45, gain = 10%

$$\therefore \text{CP of 1 kg of mixture} = \frac{100}{110} \times 10.45 = ₹9.50$$



\therefore Ratio of quantity of 1st and 2nd type = 1 : 3
Let x be the sugar of 1st type be mixed with 21 kg of 2nd type.

$$\therefore 1 : 3 = x : 21$$

$$\Rightarrow x = \frac{21}{3} = 7 \text{ kg.}$$

12. b If there is 1 unit of wine, wine part = $\frac{3}{4}$ unit and

water part = $\frac{1}{4}$ unit.

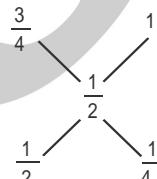
Let x be the amount of solution withdrawn. Then,

$$\begin{aligned} \frac{3}{4} - \frac{3}{4}x &= \frac{1}{4} - \frac{1}{4}x + x \Rightarrow \frac{3}{4} - \frac{1}{4} = \frac{3x}{4} - \frac{1}{4}x + x \\ \Rightarrow \frac{1}{2} &= \frac{1}{2}x + x \Rightarrow \frac{1}{2} = \frac{3}{2}x \Rightarrow x = \frac{1}{3}. \end{aligned}$$

Hence, $\frac{1}{3}$ rd part must be replaced.

Alternative method:

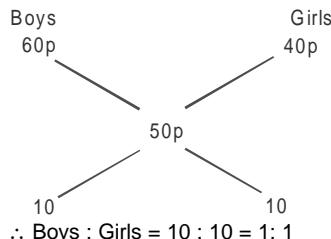
Wine is $\frac{3}{4}$ th part of solution.



Ratio of wine solution and replaced water = 2 : 1.

Hence, $\frac{1}{3}$ rd part of solution must be replaced with water.

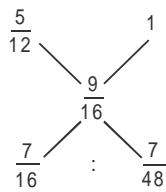
13. a Mean value of money per student = $\frac{2400}{48} = 50\text{p}$



$$\therefore \text{Boys : Girls} = 10 : 10 = 1 : 1$$

Number of boys = $\frac{1}{2} \times 48 = 24$,
and number of girls = 24.

14. c Using alligation with liquid - B



$$\Rightarrow 3:1$$

Hence, total quantity of mixture = $4 \times 9 = 36$ litres

Hence, quantity of liquid A = $\frac{7}{12} \times 36 = 21$ litres.

15. b Initially quantities of copper, zinc and nickle in 100 kg alloy is 50 kg, 30 kg and 20 kg respectively.
Let x kg nickel be added.

$$\text{Then, } \frac{30}{20+x} = \frac{3}{3} \Rightarrow x = 10\text{kg.}$$

Hence, 10 kg nickle must be added.

16. d Let capacity of each vessel be x litres.

$$\text{Then, amount of water and milk in vessel one} = \frac{3}{7}x$$

$$\text{and } \frac{4}{7}x$$

$$\text{Amount of water and milk in vessel two} = \frac{5}{8}x$$

$$\text{and } \frac{3}{8}x.$$

$$\text{Total amount of water} = \frac{3}{7}x + \frac{5}{8}x = \frac{24+35}{56}x$$

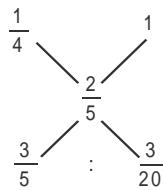
$$= \frac{59}{56}x.$$

Similarly,

$$\text{total amount of milk} = \frac{4}{7}x + \frac{3}{8}x = \frac{32+21}{56}x = \frac{53}{56}x.$$

$$\therefore \text{Ratio of water and milk} = \frac{\frac{59}{56}x}{\frac{53}{56}x} = \frac{59}{53} \text{ i.e. } 59 : 53.$$

17. b



\Rightarrow Ratio = 4 : 1

Hence, water should be added to the mixture.

$$= \frac{240}{4} = 60\text{cc.}$$

18. d Let quantity of acid be x litres

$$\therefore \text{Quantity of water} = 3x \text{ litres}$$

$$\therefore \frac{x+5}{3x} = \frac{1}{2}$$

$$\Rightarrow 2x + 10 = 3x$$

$$\Rightarrow x = 10$$

$$\therefore \text{Quantity of new mixture} = x + 3x + 5 = 10 + 3 \times 10 + 5 = 45 \text{ litres.}$$

19. c By observation, we see that relative percentage of milk is minimum in third container.

20. a Let the required ratio be 1 : x.

\therefore Cost price of the mixture = ₹(192 × 1 + 150 × x) and Selling price of the mixture = ₹194.40 × (1 + x)

$$\therefore 194.40 (1+x) = (192 + 150x) \times \frac{120}{100}$$

$$\Rightarrow 19440 (1+x) = (192 + 150x) \times 120$$

$$\Rightarrow 19440 + 19440x = 23040 + 18000x$$

$$\Rightarrow 19440x - 18000x = 23040 - 19440$$

$$\Rightarrow 1440x = 3600$$

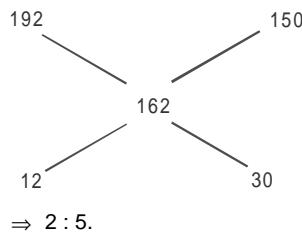
$$\Rightarrow x = \frac{3600}{1440} = \frac{5}{2}$$

\therefore Required ratio = 2 : 5.

Alternative method:

$$\text{Cost price of mixture} = \frac{100}{120} \times 194.4 = ₹162.$$

Using Alligation:



21. b Let each of the two alloys was mixed with equal quantity of 1 kg.

There is no lead in the first alloy.

$$\therefore \text{Quantity of lead in second alloy} = \frac{3}{5+4+3} = \frac{1}{4}\text{kg}$$

$$\therefore \text{Quantity of lead in final mixture} = \frac{1}{2} = \frac{1}{8}\text{kg.}$$

22. a Let the amount of water to be added be x litres.
Quantity of water in 1st mixture

$$= \frac{2}{7+2} \times 729 = 162 \text{ litres}$$

and quantity of milk in 1st mixture

$$= \frac{7}{7+2} \times 729 = 567 \text{ litres}$$

$$\therefore \frac{162+x}{567} = \frac{3}{7}$$

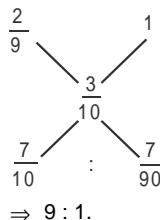
$$\Rightarrow 1134 + 7x = 1701$$

$$\Rightarrow 7x = 1701 - 1134$$

$$\Rightarrow x = \frac{567}{7} = 81 \text{ litres.}$$

Alternative method:

Using Alligation:



$$\Rightarrow 9 : 1.$$

$$\text{Hence, water to be added} = 729 \times \frac{1}{9} = 81 \text{ litres.}$$

23. c Let the water to be added be x litres

$$\text{Then, quantity of milk in 1st mixture} = \frac{7}{8} \times 40 = 35 \text{ litres}$$

and quantity of water in 1st mixture

$$= \frac{1}{8} \times 40 = 5 \text{ litres}$$

$$\therefore \frac{35}{5+x} = \frac{3}{1}$$

$$\Rightarrow 15 + 3x = 35$$

$$\Rightarrow x = \frac{20}{3} = 6\frac{2}{3} \text{ litres.}$$

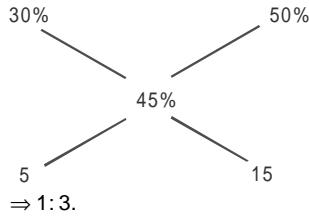
24. b Let the ratio be $1 : x$

$$\text{Then, } 30 \times 1 + 50x = (1+x)45 = 45 + 45x$$

$$\Rightarrow x = 3$$

$$\therefore \text{Required ratio} = 1 : 3.$$

Alternative method:



$$\Rightarrow 1 : 3.$$

25. c Let quantity of second variety of rice be x kg.

$$\therefore \text{CP of the mixture} = 10 \times 10 + 15 \times x$$

$$= ₹(100 + 15x)$$

$$\text{SP of the mixture} = (10 + x) \times 14 = ₹(140 + 14x)$$

$$\text{Then, } 140 + 14x = (100 + 15x) \times \frac{(100+5)}{100}$$

$$\Rightarrow 140 + 14x = (100 + 15x) \times \frac{21}{20}$$

$$\Rightarrow 2800 + 280x = 2100 + 315x$$

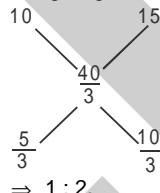
$$\Rightarrow (315 - 280)x = 2800 - 2100$$

$$\Rightarrow x = \frac{700}{35} = 20 \text{ kg.}$$

Alternative method:

$$\text{Cost price of final mixture} = \frac{100}{105} \times 14 = ₹\frac{40}{3}$$

Using Alligation:



$$\Rightarrow 1 : 2.$$

Hence, quantity of second variety = 20 kg.

26. a Amount of sugar in solution = 20% of 100 litres
= 20 litres

As after evaporation, sugar will remain constant
Hence, water present in solution after evaporation

$$= \frac{100}{80} \times 20 - 20 = 5 \text{ litres}$$

Hence, evaporated water = 80 - 5 = 75 litres.

27. d Let 24 units of each type be taken. Then,

$$\text{Milk content in 1st type} = \frac{1}{6} \times 24 = 4 \text{ unit}$$

$$\text{Water content in 1st type} = 24 - 4 = 20 \text{ unit}$$

$$\text{Milk content in 2nd type} = \frac{3}{8} \times 24 = 9 \text{ unit}$$

$$\text{Water content in 2nd type} = 24 - 9 = 15 \text{ unit}$$

Hence, required ratio = $(15 + 20) : (4 + 9) = 35 : 13$.

28. b In 66L adulterated milk, milk = $66 \times \frac{5}{6} = 55$ litres

and water = $66 - 55 = 11$ litres

Let 'x' be the quantity of water be added.

$$\therefore \frac{55}{11+x} = \frac{5}{3}$$

$$\Rightarrow 55 + 5x = 165$$

$$\Rightarrow 5x = 165 - 55 = 110$$

$$\Rightarrow x = 22 \text{ litres.}$$

29. c Juice content in 1st type = $\frac{1}{3} \times 3 = 1$ litres

Water content in 1st type = $3 - 1 = 2$ litres

Juice content in 2nd type = $\frac{5}{8} \times 2 = \frac{5}{4}$ litres

Water content in 2nd type = $2 - \frac{5}{4} = \frac{3}{4}$ litres

Hence, juice percentage in mixture

$$= \frac{1 + \frac{5}{4}}{3 + 2} \times 100 = \frac{9}{20} \times 100 = 45\%.$$

30. a Initially,
Alcohol = 50 litre, water = 150 litre

Alcohol taken out = $\frac{50}{4} = 12.5$ litre

New alcohol content = $50 - 12.5 + 50 = 87.5$ litre

Final alcohol percentage = $\frac{87.5}{200} \times 100 = 43.75\%$.

31. d As each mixture is of equal quantity. The ratio of milk and water in the mixtures will be 24 : 4, 20 : 8 and 21 : 7.

In the final mixture,

Required ratio = $\frac{24+20+21}{4+8+7} = 65:19$.

32. a Use the formula $x \left(1 - \frac{y}{x}\right)^n$, where x is the original quantity, y is the quantity taken out n times.
 \therefore Milk left after three operations

$$= 80 \left(1 - \frac{8}{80}\right)^3 = 58.32 \text{ litres.}$$

Alternative method:

8 litres as compared to 80 litres is $\frac{1}{10}$ of the total volume.

So, if $\frac{1}{10}$ is removed the fraction leftover is $\frac{9}{10}$.

Hence, the final milk quantity is

$$= \left(\frac{9}{10}\right)^3 \times 80 = 58.32 \text{ litres.}$$

33. a Let 104 unit of each type be melted together.

Copper content in first type alloy = $\frac{5}{13} \times 104 = 40$ unit.

Iron content in first type alloy = $\frac{8}{13} \times 104 = 64$ unit.

Copper content in second type alloy

$$= \frac{5}{8} \times 104 = 65 \text{ unit.}$$

Iron content in second type alloy = $\frac{3}{8} \times 104 = 39$ unit.

Required ratio = $(40 + 65) : (64 + 39) = 105 : 103$.

34. d Let amount of water added to the mixture be x litres.
 Amount of milk in the original mixture

$$= \frac{(100 - 10)}{100} \times 100 = 90 \text{ litres}$$

Then, $\frac{90}{(100+x)} = \frac{50}{100} = \frac{1}{2}$

$\Rightarrow 180 = 100 + x$

$\Rightarrow x = 180 - 100 = 80$ litres.

35. c Let 15 unit of each type be mixed together.
 Total milk content in final mixture

$$= \frac{1}{3} \times 15 + \frac{2}{5} \times 15 + \frac{3}{5} \times 15 + \frac{7}{15} \times 15$$

$= 5 + 6 + 9 + 7 = 27$ unit

Total water content in final mixture

$= 4 \times 15 - 27 = 33$ unit

Hence, required ratio = 9 : 11.

36. c Quantity of chemical A in 21kg fertilizer

$$= \frac{2}{(2+5)} \times 21 = 6 \text{ kg}$$

Quantity of chemical B in 21 kg fertilizer

$$= \frac{5}{(2+5)} \times 21 = 15 \text{ kg}$$

After adding 3 kg, chemical A in the fertilizer,

Required ratio = $\frac{6+3}{15} = \frac{9}{15} = 3:5$.

37. c Final concentration of alcohol in container

$$= 60 \times \left(1 - \frac{20}{100}\right) \left(1 - \frac{10}{100}\right)\%$$

$$= 60 \times \frac{80}{100} \times \frac{90}{100}\% = 43.2\%$$

38. b Let price of third quality rice be ₹x per kg.

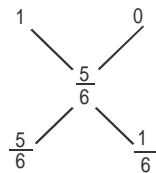
Then, $\frac{120 \times 2 + 132 \times 1 + 3 \times x}{2+1+3} = 135$

$\Rightarrow x = 146$.

39. b Let selling price of 1 liter milk be ₹1.
Then, cost price of pure milk in 1 liter adulterated milk

$$= \frac{100}{120} \times 1 = \frac{5}{6}$$

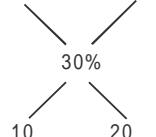
Using Alligation:



Ratio of milk and water = 5 : 1

$$\text{Hence, percentage of water} = \frac{1}{6} \times 100 = 16\frac{2}{3}\%$$

40. e 50% 20%



Ratio of both type = 1 : 2.

$$\text{Hence, quantity replaced} = \frac{2}{3} \text{ rd.}$$

41. d Let quantity of milk initially be x litres. Then, after 3 operations

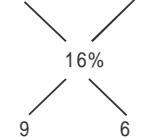
$$x \times \left(1 - \frac{6}{x}\right)^3 : x = 8 : 125$$

$$\Rightarrow \left(1 - \frac{6}{x}\right)^3 = \left(\frac{2}{5}\right)^3$$

$$\Rightarrow 1 - \frac{6}{x} = \frac{2}{5}$$

$$\Rightarrow x = 10 \text{ litres.}$$

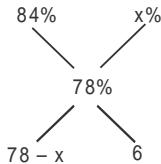
42. a 10% 25%



Ratio of quantities of rice sold at 10% profit and 25% profit = 3 : 2

$$\text{Hence, quantity sold at 10% profit} = \frac{3}{5} \times 50 = 30 \text{ kg.}$$

43. b Let average marks of boys were $x\%$.
Using Alligation



$$\Rightarrow \frac{78-x}{6} = \frac{7}{9}$$

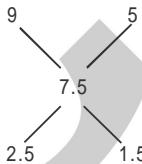
$$\Rightarrow x = 73.33\%.$$

44. d Let 12 unit of both the alloys be mixed.
Then, tin in final alloy = $4 + 7 = 11$ unit
aluminum in final alloy = 2 unit
Hence, required ratio = 11 : 2.

45. c Effective interest rate per annum

$$r = \frac{1350 \times 100}{6000 \times 3} = 7.5\% \text{ per annum}$$

Using Alligation on rate percentage

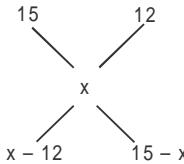


Ratio of amount lent on 9% and 5% per annum = 5 : 3

Hence, amount on 9% per annum

$$= \frac{5}{8} \times 6000 = ₹3,750.$$

46. b Let mean price of mixture be ₹ x per kg.
Then,



$$\Rightarrow \frac{x-12}{15-x} = \frac{12}{18}$$

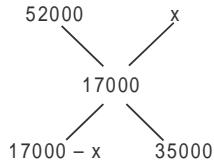
$$\Rightarrow x = 13.2$$

$$\therefore \text{Total percentage gain} = \frac{16.5 - 13.2}{13.2} \times 100$$

$$= \frac{3.3}{13.2} \times 100 = 25\%.$$

47. d Let x litres of water was present initially.
 Then, $x + x + 12 = 72$
 $\Rightarrow x = 30$
 After adding 8 litre milk, quantity of milk become
 $= 42 + 8 = 50$ litres
 Required ratio = $50 : 30 = 5 : 3$.

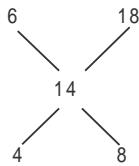
48. e Let average monthly salary of executive be ₹ x .



$$\Rightarrow \frac{17000 - x}{35000} = \frac{3}{8}$$

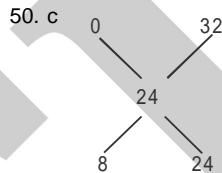
$$\Rightarrow x = 3,875.$$

49. c Average speed = $\frac{70}{5} = 14$ km/hr



\Rightarrow Ratio of time taken on foot and on bicycle = $1 : 2$.

Hence, distance traveled on foot = $\frac{1}{3} \times 5 \times 6 = 10$ km.



$\Rightarrow 1 : 3$

Hence, ratio of water and milk must be $1 : 3$.

Time, Speed and Distance : Relative Motion, Trains
Answers and Explanations

1	b	2	a	3	b	4	c	5	d	6	b	7	c	8	a	9	c	10	b
11	c	12	c	13	d	14	c	15	a	16	b	17	c	18	d	19	a	20	a
21	c	22	c	23	b	24	c	25	b	26	a	27	b	28	b	29	b	30	a
31	c	32	b	33	c	34	d	35	a	36	c	37	b	38	c	39	c	40	d
41	c	42	d	43	a	44	d	45	d	46	a	47	c	48	a	49	c	50	d

1. b B covers some distance in 48 min. As the ratio of speed of A and B is 3 : 4, the time taken will be in the ratio 4 : 3. (Reverse of speed)

∴ Time taken by A to cover the same distance is =

$$48 \times \frac{4}{3} = 64 \text{ min.}$$

2. a $\frac{3}{5}$ th of the trip is equal to 1200 km

$$\therefore \text{Total distance} = \frac{1200 \times 5}{3} = 2000 \text{ km.}$$

3. b Let the normal speed of the train be x and actual time taken be t .

$$\text{Total distance} = x t$$

$$\frac{\frac{xt}{2}}{\frac{x}{3}} = t + \frac{1}{2} \Rightarrow t = 1 \text{ hr.}$$

4. c Let the distance between the starting point and turning point be x . Then,

$$\frac{x}{4} + \frac{x}{3} = \frac{7}{2} \Rightarrow x = 6 \text{ km.}$$

5. d Speed of train relative to man = $(x + 8)$ kmph

$$= (x + 8) \times \frac{5}{18} \text{ m / sec.}$$

$$\therefore \frac{150}{(x + 8) \times \frac{5}{18}} = 9$$

$$\Rightarrow x + 8 = 60 \Rightarrow x = 52 \text{ km / hr.}$$

6. b Let the length of the train be L .

$$\text{Speed of the train is } \frac{L}{15} = \frac{L + 100}{30}$$

$$\Rightarrow 2L = L + 100$$

$$\Rightarrow L = 100 \text{ m}$$

∴ The length of the train is 100 m.

7. c Speed = $\frac{\text{distance}}{\text{time}} = \frac{520}{4} = 130 \text{ km/hr}$

Now this is increased by 20 km/hr. Hence, speed is 150 km/hr. At this speed time taken by the train to

$$\text{cover } 900 \text{ km} = \frac{900}{150} = 6 \text{ hours.}$$

8. a Average speed = $\frac{2 \times 15 \times 5}{15 + 5} = \frac{150}{20} = 7 \frac{1}{2} \text{ kmph}$

Distance travelled in 2 hours 40 minutes i.e.,

$$2 \frac{2}{3} \text{ hrs.} = \frac{8}{3} \times \frac{15}{2} = 20 \text{ km.}$$

$$\text{Distance of the school from house} = \frac{20}{2} = 10 \text{ km.}$$

9. c Let the length of the train be x .

$$\text{Speed of the train is } \frac{x}{8} = \frac{x + 250}{24}$$

$$\Rightarrow 3x = x + 250 \Rightarrow x = 125 \text{ m.}$$

10. b The speed of the man is $6 \times \frac{5}{18} = \frac{5}{3} \text{ m/sec}$

Let the speed of the train be S .

$$\text{Given } S + \frac{5}{3} = \frac{180}{6} \Rightarrow S = \frac{85}{3} \times \frac{18}{5} = 102 \text{ km/hr.}$$

11. c Total distance covered = $10 + 12 = 22 \text{ km}$

$$\text{Total time taken} = \frac{10}{12} + \frac{12}{10} \text{ hr}$$

$$\text{Average speed} = \frac{22}{\frac{10}{12} + \frac{12}{10}} = 10.8 \text{ km/hr.}$$

12. c The guard can never catch the thief as the speed of the thief is more than that of the guard.

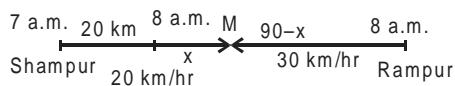
13. d Let the length of train be 'x'
 \therefore Total distance while covering platform = $x + 240$
 Total distance while passing man = x
 Since the speed of the train remains the same

$$\frac{x+240}{25} = \frac{x}{5} \Rightarrow 5x + (240 \times 5) = 25x$$

$$\Rightarrow x = \frac{240 \times 5}{20} = 60 \text{ m}; \text{ Speed} = \frac{60}{5} = 12 \text{ m/sec}$$

For questions 14 and 15:

First train covers 20 km in one hour from 7 a.m. to 8 a.m.
 Therefore, distance between two trains at 8 a.m. is $(110 - 20) = 90 \text{ km}$.



14. c Both trains are running in opposite direction and hence relative speed is $(20 + 30) = 50 \text{ km/hr}$.

Thus, they will meet after $\frac{90}{50} = 1.8 \text{ hrs}$ after 8 a.m.

\therefore Two trains will meet at 9:48 a.m.

$$15. a \quad \frac{x}{20} = \frac{90-x}{30}$$

$$\Rightarrow x = 36 \text{ km}$$

Distance from Shampur = $20 + 36 = 56 \text{ km}$

16. b Let speed of the car be $x \text{ km/hr}$.

Then, time taken to cover 300 km = $\frac{300}{x}$.

$$\therefore \frac{300}{x} - \frac{300}{x+5} = 2 \Rightarrow x = 25 \text{ km/hr}$$

17. c Let A and B meet after t hour from 11. 30 a.m.

Then, distance covered by A in $(t + 2\frac{1}{2})$ hours

= Distance covered by B in t hours

$$\Rightarrow 4\left(t + \frac{5}{2}\right) = 9t \Rightarrow t = 2 \text{ hr}$$

\therefore They will meet at 11.30 a.m. + 2hr = 1.30 p.m.

18. d Speed of train relative to man = $64 + 8 = 72 \text{ km/hr}$.

$$\text{Then, } 72 \times \frac{5}{18} = 20 \text{ m/sec.}$$

Time taken by the train to cross the man
 = time taken by it to cover 200 m at 20 m / sec

$$= \frac{200}{20} = 10 \text{ sec.}$$

19. a Let the required distance be $x \text{ km}$.

Then,

$$\frac{x}{3} + \frac{x}{2} = 5$$

$$\Rightarrow \frac{2x + 3x}{6} = 5$$

$$\Rightarrow 5x = 6 \times 5$$

$$\therefore x = \frac{6 \times 5}{5} = 6 \text{ km}$$

20. a Samit runs 660 m in 88 sec.

$$\therefore \text{Samit runs } (660 - 60) \text{ m in } \frac{88}{660} \times 600 \text{ sec}$$

i.e. 80 sec.

But Sanjay runs 660 m in 81 sec
 So, Samit wins by $(81 - 80)$ sec, i.e. 1 sec.

21. c Let the length of the train be x metres.

When a train crosses a platform it covers a distance equal to the sum of lengths of train and platform. Also, the speed of train is same.

$$\therefore \frac{x+162}{18} = \frac{x+120}{15}$$

$$\Rightarrow 6x + 720 = 5x + 810$$

$$\Rightarrow 6x - 5x = 810 - 720$$

$$\Rightarrow x = 90$$

\therefore The length of the train = 90 m.

22. c Let the speed of train be x kmph and its length be y km.
 When the train crosses a man, it covers its own length
 According to the question.

$$\frac{y}{(x-3) \times \frac{5}{18}} = 10$$

$$\Rightarrow 18y = 10 \times 5(x-3)$$

$$\Rightarrow 18y = 50x - 150$$

... (i)

and $\frac{y}{(x-5) \times \frac{5}{18}} = 11$

$$\Rightarrow 18y = 55(x-5)$$

$$\Rightarrow 18y = 55x - 275 \quad \dots \text{(ii)}$$

From (i) and (ii),
 $55x - 275 = 50x - 150$
 $\Rightarrow 55x - 50x = 275 - 150$
 $\Rightarrow 5x = 125$
 $\Rightarrow x = \frac{125}{5} = 25$
 $\therefore \text{Speed of the train} = 25 \text{ kmph.}$

23. b Clearly, A beats B by 20 sec.

Distance covered by B in 20 sec = $\frac{2000}{400} \times 20 = 100 \text{ m.}$
 $\therefore \text{A beats B by } 100 \text{ m.}$

24. c If the speed of train be x kmph.
Then, its relative speed = $(x + 3)$ kmph
 $\therefore \text{Time} = \frac{\text{Length of the train}}{\text{Relative speed}}$
 $\Rightarrow \frac{10}{3600} = \frac{1000}{(x+3)} = \frac{240}{1000(x+3)}$
 $\Rightarrow x + 3 = 86.4$
 $\Rightarrow x = 83.4 \text{ kmph}$

25. b Let two trains meet after t hours when the train from town A leaves at 8 AM.
 $\therefore \text{Distance covered in } t \text{ hours at } 70 \text{ kmph} + \text{Distance covered in } (t-2) \text{ hours at } 110 \text{ kmph} = 500 \text{ km}$
 $\therefore 70t + 110(t-2) = 500$
 $\Rightarrow 70t + 110t - 220 = 500$
 $\Rightarrow 180t = 500 + 220 = 720$
 $\Rightarrow t = \frac{720}{180} = 4 \text{ hours}$

Hence, the trains will meet at 12 noon.

26. a Total distance travelled = $20 \times 3 + 25 \times 2 = 60 + 50 = 110 \text{ km}$
Total time taken = $3 + 2 = 5 \text{ hr.}$
 $\therefore \text{Average speed} = \frac{110}{5} = 22 \text{ km/hr.}$

27. b Distance travelled by bus at the rate of $60 \text{ km/hr} = 300 - 30 \times 2 - 40 \times 3 = 120 \text{ km}$
Time taken to cover 120 km at the speed of

$60 \text{ km/hr} = 2 \text{ hr}$
Hence, total time taken = $2 + 3 + 2 = 7 \text{ hr.}$

28. b If one travels more than the other, then it is with the higher speed since it travelled 180 km extra distance i.e. $180 = (95 - 80)t$

$$\Rightarrow t = 12 \text{ hr}$$

Since they are travelling towards each other,
Total distance = Distance travelled by train 1 + Distance travelled by train 2
 $= 80 \times 12 + 95 \times 12$
 $= 12(95 + 80) = 2100 \text{ km}$

29. b Let the distance be x . Then,

$$\frac{x}{40} - \frac{x}{50} = \frac{8}{60} \Rightarrow x = 26.6 \text{ km.}$$

30. a Speed of the car = $\frac{120}{8} = 15 \text{ mph}$

Changed speed = $\frac{3}{2} \times 15 = \frac{45}{2} \text{ mph}$

Time required to cover $450 \text{ miles} = \frac{450 \times 2}{45} = 20 \text{ hr.}$

31. c Distance = 100 m
Let speed of the bike be x .
 $\therefore \text{Net speed} = (x - 3) \text{ km/hr}$

Time = $\frac{4}{60} \text{ hr}$

$\frac{\text{Distance}}{\text{Speed}} = \text{Time}$

$$\Rightarrow \frac{0.1}{x-3} = \frac{4}{60} \Rightarrow x = \frac{9}{2} = 4\frac{1}{2} \text{ km/hr.}$$

32. b Distance travelled by Sam in first 5 minutes = $20 \times 5 = 100 \text{ m}$
Relative speed between Sam and Sandra = $25 - 20 = 5 \text{ m/min}$

So, Sandra will reach Sam in $\frac{100}{5} = 20 \text{ min}$

Speed of Sandra's dog = 30 m/min

$\therefore \text{Distance travelled by the dog} = 30 \times 20 = 600 \text{ m.}$

33. c $d = s \times t$
 where d = distance traveled
 s = speed
 t = time taken
 According to given condition

$$d = \left(\frac{3}{4}s\right) \times t'$$

$$\Rightarrow t' = \frac{4}{3}t = t + \frac{t}{3}$$

$$\therefore \frac{t}{3} = 10 \text{ minutes}$$

$$\Rightarrow t = 30 \text{ minutes.}$$

Alternate:

$$S \propto \frac{1}{T}$$

Hence if the speed is reduced to $\frac{3}{4}$ of usual speed

then time taken is increased by $\frac{4}{3}$ of usual time.

$$\therefore \frac{4}{3} \text{ of usual time} - \text{usual time} = 10 \text{ min.}$$

$$\Rightarrow \text{Usual time} = 30 \text{ min.}$$



$$\frac{x}{6} = \frac{x+20}{8} \quad [\because \text{Time is same}]$$

$$\Rightarrow 8x = 6x + 120$$

$$\Rightarrow 2x = 120 \Rightarrow x = 60 \text{ m}$$

$$\therefore \text{Distance between A and B} = 60 + 60 + 20 = 140 \text{ m.}$$

$$35. a \quad \text{Speed of train} = 108 \times \frac{5}{18} = 30 \text{ m/sec.}$$

$$\text{Distance covered in passing the platform} = (90 + 120) = 210 \text{ m.}$$

$$\therefore \text{Time taken} = \frac{210}{30} = 7 \text{ sec.}$$

36. c Total distance travelled = (400 + 600 + 500 + 150)
= 1650 km.

$$\text{Total time taken} = \frac{400}{80} + \frac{600}{30} + \frac{500}{250} + \frac{150}{50}$$

$$= 5 + 20 + 2 + 3 = 30 \text{ hours}$$

$$\therefore \text{Average speed} = \frac{1650}{30} = 55 \text{ km/hr.}$$

37. b Let trains' speed be x and y m/sec.
 Total distance covered = Sum of the length of two trains = 120 + 180 = 300 m.
 When they run in same direction, relative speed ($x - y$) is given by:-

$$x - y = \frac{300}{15} = 20 \quad \dots(i)$$

When they run in opposite directions relative speed ($x + y$) is given by :-

$$x + y = \frac{300}{5} = 60 \quad \dots(ii)$$

Solving (i) and (ii), we get

$$\Rightarrow x = 40 \text{ m/sec} = 144 \text{ km/hr.}$$

$$\Rightarrow y = 20 \text{ m/sec} = 72 \text{ km/hr.}$$

38. c Let the speed of the man when rowing upstream be x kmph, then his speed downstream = $3x$ kmph.

$$\therefore \text{Rate in still water} = \frac{1}{2}(3x + x) = 2x \text{ kmph}$$

$$\text{So, } 2x = 22 \Rightarrow x = 11$$

$$\therefore \text{Rate upstream} = 11 \text{ km/hr.}$$

$$\text{Rate downstream} = 33 \text{ km/hr.}$$

$$\text{Hence, the rate of stream} = \frac{1}{2}(33 - 11) = 11 \text{ km/hr.}$$

39. c Speed downstream = $7 + 2 = 9$ kmph.

Speed upstream = $7 - 2 = 5$ kmph.

Let the distance be x . Then,

$$\frac{x}{9} + \frac{x}{5} = \frac{56}{60}$$

$$\Rightarrow 14x = \frac{56 \times 45}{60}$$

$$\Rightarrow x = 3 \text{ km.}$$

40. d Clearly, Nikhil covers 20 m in 5 sec.

$$\therefore \text{Nikhil's time over the course} = \frac{5}{20} \times 1000$$

$$= 250 \text{ sec}$$

$$\therefore \text{Alok's time over the course} = (250 - 5) = 245 \text{ sec}$$

$$= 4 \text{ min 5 sec}$$

41. c Let the distance be x km.

$$\text{Time taken by A} = \frac{x}{40} \text{ hrs.}$$

$$\text{Time taken by B} = \frac{x}{50} \text{ hrs}$$

$$\text{Now, } \frac{x}{40} - \frac{x}{50} = \frac{15}{60}$$

$$\Rightarrow \frac{5x - 4x}{200} = \frac{15}{60}$$

$$\Rightarrow x = \frac{15}{60} \times 200 = 50 \text{ km}$$

Method 2:

$$= \frac{\text{Product of speed}}{\text{Diff. of speed}} \times \text{Diff. in time}$$

$$= \frac{40 \times 50}{50 - 40} \times \frac{15}{60} = 50 \text{ km.}$$

42. d Suppose. time taken in walking be x hours and time taken in riding be y hours
 \therefore According to question,

$$x + y = 4 \frac{1}{2} \text{ hours}$$

Then, $2y = 3$ hours

$$\Rightarrow y = 1 \frac{1}{2} \text{ hours}$$

$$\therefore x = 4 \frac{1}{2} - 1 \frac{1}{2} = 3 \text{ hours}$$

Hence, time required to walk both way = 6 hours.

43. a When A covers 200 m, B covers 190 m and C covers 195 m.

i.e. when C covers 195 m, B covers = 190 m, and

$$\text{when C covers 200 m, B covers} = \frac{190}{195} \times 200$$

$$= 194.87 \text{ m}$$

Hence, C would beat B by $200 - 194.87 = 5.13$ m in a 200 m race.

Short cut:

In a race of L length, if 1st beats 3rd by x_1 distance, 1st beats 2nd by x_2 distance and 2nd beats 3rd by x_3 distance, then their relation is given by

$$(L - x_2)x_3 = L(x_1 - x_2).$$

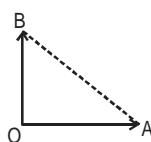
Using this, we get

$$(200 - 5)x_3 = 200(10 - 5)$$

$$\Rightarrow 195x_3 = 200 \times 5$$

$$\Rightarrow x_3 = 5.13 \text{ m.}$$

44. d



Let O be the starting point. The car running at 36 kmph is moving along OB and that at 48 kmph moving along OA. Also let they reach at B and A respectively after 15 seconds.

$$\therefore OA = 48 \times \frac{3}{18} \times 15 = 200 \text{ m}$$

$$\text{and } OB = 36 \times \frac{5}{18} \times 15 = 150 \text{ m}$$

\therefore Required distance = AB

$$= \sqrt{(200)^2 + (150)^2}$$

$$= \sqrt{40000 + 22500}$$

$$= \sqrt{62500} = 250 \text{ m.}$$

45. d Let the length of train be x metres.

$$\text{Then, speed of train when it passes a post} = \frac{x}{8} \text{ m/sec.}$$

$$\& \text{ speed of train, when it passes the bridge} = \frac{x + 264}{20}$$

Clearly,

$$\frac{x}{8} = \frac{x + 264}{20} \Rightarrow \frac{x}{2} = \frac{x + 264}{5}$$

$$\Rightarrow 5x = 2x + 528 \Rightarrow 3x = 528 \Rightarrow x = \frac{528}{3} = 176 \text{ m}$$

$$\therefore \text{Speed of train} = \frac{176}{8} = 22 \text{ m/sec} = 22 \times \frac{18}{5} \text{ kmph} \\ = 79.2 \text{ kmph.}$$

46. a Let the length of train be x metres.

When the train crosses the standing man, its speed

$$= \frac{x}{9}$$

When the train crosses the platform of length 84 m, its

$$\text{speed} = \frac{x + 84}{21}$$

$$\text{Obviously, } \frac{x}{9} = \frac{x + 84}{21}$$

$$\Rightarrow 21x - 9x = 9 \times 84$$

$$\Rightarrow 12x = 9 \times 84$$

$$\Rightarrow x = \frac{9 \times 84}{12} = 63 \text{ m.}$$

$$\therefore \text{Required speed} = \frac{63}{9} \text{ m/sec}$$

$$= \frac{63}{9} \times \frac{18}{5} \text{ kmph} = 25.2 \text{ kmph.}$$

47. c Ratio of the rates of Raj and Ravi = $\frac{5}{3} : 1 = 5 : 3$

So, in a race of 5 m, Raj gains 2 m over Ravi.

\therefore 80 m are gained by Raj in a race of

$$\left(\frac{5}{2} \times 80\right) = 200 \text{ m.}$$

\therefore Winning post must be 200 m away from the starting point.

48. a Distance covered by A = $(500 - 170) = 330$ m, while A covers 2 m, B covers 3 m.

$$\text{While A covers } 330 \text{ m, B covers } \left(\frac{3}{2} \times 330\right) \text{ m} \\ = 495 \text{ m}$$

\therefore When A reaches the winning post, B covers 495 m and therefore is 5 m behind.

\therefore A wins by 5 m.

49. c Let x km be the required distance.
Difference in time = $2.5 + 5 = 7.5$ minutes

$$= \frac{7.5}{60} \text{ hrs.} = \frac{1}{8} \text{ hrs.}$$

$$\text{Now, } \frac{x}{8} - \frac{x}{10} = \frac{1}{8}$$

$$\Rightarrow \frac{5x - 4x}{40} = \frac{1}{8}$$

$$\Rightarrow x = \frac{40}{8} = 5 \text{ km.}$$

50. d Let the distance be x km and initial speed be y kmph.
According to question,

$$\frac{x}{y} - \frac{x}{y+3} = \frac{40}{60} \quad \dots (\text{i})$$

and

$$\frac{x}{y-2} - \frac{x}{y} = \frac{40}{60} \quad \dots (\text{ii})$$

From (i) and (ii),

$$\frac{x}{y} - \frac{x}{y+3} - \frac{x}{y-2} - \frac{x}{y}$$

$$\Rightarrow \frac{1}{y} - \frac{1}{y+3} = \frac{1}{y-2} - \frac{1}{y}$$

$$\Rightarrow \frac{y+3-y}{y(y+3)} = \frac{y-y+2}{y(y-2)}$$

$$\Rightarrow 3(y-2) = 2(y+3)$$

$$\Rightarrow 3y - 6 = 2y + 6$$

$$\Rightarrow y = 12$$

From (i),

$$\frac{x}{12} - \frac{x}{15} = \frac{40}{60} \Rightarrow \frac{5x - 4x}{60} = \frac{2}{3}$$

$$\Rightarrow x = \frac{2}{3} \times 60 = 40$$

Hence, distance = 40 km.

**Time, Speed and Distance : Boats & Streams, Circular Motion
Answers and Explanations**

1	c	2	a	3	c	4	a	5	b	6	a	7	c	8	b	9	b	10	c
11	d	12	c	13	d	14	c	15	d	16	a	17	a	18	c	19	b	20	b
21	c	22	d	23	a	24	b	25	d	26	b	27	c	28	a	29	b	30	b
31	c	32	b	33	c	34	c	35	c	36	b	37	b	38	c	39	c	40	c
41	a	42	b	43	a	44	a	45	a	46	d	47	a	48	b	49	a	50	a

1. c Downstream speed = $\frac{21}{\frac{1}{4}} = \frac{21 \times 4}{21} = 4 \text{ km/hr.}$

Upstream speed = $\frac{6}{2\frac{1}{4}} = \frac{6 \times 4}{9} = \frac{8}{3} \text{ km/hr.}$

\therefore Speed of current = $\frac{1}{2} \left(4 - \frac{8}{3} \right) = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3} \text{ km/hr.}$

2. a Speed downstream = $\frac{18}{4} \text{ km/hr}$

Speed upstream = $\frac{18}{12} \text{ km/hr}$

Speed of the current

= $\frac{1}{2} (\text{Speed downstream} - \text{Speed upstream})$

= $\frac{1}{2} \left(\frac{18}{4} - \frac{18}{12} \right) = 1.5 \text{ km/hr.}$

3. c Upstream speed = $\frac{35}{7} = 5 \text{ kmph.}$

Downstream speed = $\frac{30}{5} = 6 \text{ kmph.}$

Speed in still water = $\frac{5+6}{2} = 5.5 \text{ kmph.}$

4. a Let the speed of man in still water be V and the speed of stream be S.

Therefore, $x = V + S$; $y = V - S$

$\therefore x + y = 2V \Rightarrow V = 0.5(x + y).$

5. b Upstream speed = $\frac{\frac{3}{8}}{\frac{30}{60}} = \frac{3}{4} \text{ kmph}$

Downstream speed = $\frac{\frac{3}{8}}{\frac{12}{60}} = \frac{15}{8} \text{ kmph}$

\therefore Speed in still water = $\frac{\frac{3}{4} + \frac{15}{8}}{2} = \frac{21}{16} \text{ km/hr.}$

6. a Let x be the speed of the boat.

	Upstream	Down stream
Speeds	$x - 2$	$x + 2$
Time(Mins)	10	5
Time(Ratio)	2	1
Speed(Ratio)	1	2

$\therefore \frac{x+2}{x-2} = \frac{2}{1}$

$\Rightarrow 2x - 4 = x + 2 \Rightarrow x = 6 \text{ m/sec.}$

7. c Speed downstream = $12 + 3 = 15 \text{ kmph.}$
Speed upstream = $12 - 3 = 9 \text{ kmph.}$

Let distance between P and Q be x km.

Then, $\frac{x}{15} + \frac{2}{9} = 22 \Rightarrow \frac{x}{15} + \frac{x}{18} = 22$
 $\Rightarrow x = 180 \text{ km.}$

8. b Rate upstream = 4 kmph
Rate downstream = 5 kmph

\therefore Speed of boat in still water $\frac{1}{2}(4+5) = 4.5 \text{ kmph}$

\therefore Speed of current = $\frac{1}{2}(5-4) \text{ kmph} = 0.5 \text{ kmph.}$

9. b Rate downstream = 10 kmph
Rate upstream = 5 kmph

$$\therefore \text{Speed of current} = \frac{1}{2}(10 - 5) = 2.5 \text{ kmph.}$$

10. c Let the speed of boat in still water be x kmph and that of current be y kmph. Then

$$\frac{12}{x-y} + \frac{18}{x+y} = 3 \quad \dots(\text{i})$$

$$\frac{36}{x-y} + \frac{24}{x+y} = \frac{13}{2} \quad \dots(\text{ii})$$

By (i) $\times 3$ – (ii),

$$\begin{aligned} \frac{54}{x+y} - \frac{24}{x+y} &= 9 - \frac{13}{2} \\ \Rightarrow \frac{30}{x+y} &= \frac{5}{2} \Rightarrow x+y = 12 \quad \dots(\text{iii}) \end{aligned}$$

From (i),

$$\begin{aligned} \frac{12}{x-y} + \frac{18}{12} &= 3 \\ \Rightarrow \frac{12}{x-y} &= 3 - \frac{3}{2} = \frac{3}{2} \\ \Rightarrow x-y &= \frac{12 \times 2}{3} = 8 \quad \dots(\text{iv}) \end{aligned}$$

$$\therefore \text{Speed of current} = \frac{1}{2}(12-8) = 2 \text{ kmph.}$$

11. d Rate downstream
= $10 + 5 = 15$ kmph
Rate upstream = $10 - 5 = 5$ kmph
Time taken in swimming 60 km downstream
 $= \frac{60}{15} = 4$ hours
Time taken in swimming 60 upstream
 $= \frac{60}{5} = 12$ hours
From, given options, boy can swim 60 km downstream in 4 hours.

12. c Rate upstream = $4 - 2 = 2$ kmph
Hence, required time = $\frac{10}{2} = 5$ hours.

13. d Speed of current
 $= \frac{1}{2}(\text{Rate downstream} - \text{Rate upstream})$
 $= \frac{1}{2}(12-8) = 2$ kmph.

14. c Rate downstream = 5 kmph
Rate upstream = 1 kmph

$$\text{Hence, required time} = \frac{10}{5} + \frac{10}{1} = 12 \text{ hours.}$$

15. d The distance covered upstream = AC = d
 $AB = 100$

$$BC = 100 + d$$

$$\text{Rate upstream} = (x - y) \text{ m/minute}$$

$$\text{Rate downstream} = (x + y) \text{ m/minute}$$

$$\therefore \frac{d}{x-y} = 5$$

$$\Rightarrow d = 5(x-y) \quad \dots(\text{i})$$

Again,

$$\frac{100+d}{x+y} = 5$$

$$\Rightarrow \frac{100+5(x-y)}{x+y} = 5 \quad (\text{By (i)})$$

$$\Rightarrow 100 + 5x - 5y = 5x + 5y$$

$$\Rightarrow 10y = 100$$

$$\Rightarrow y = 10 \text{ m/minute} = \frac{10}{1000} \times 60 \text{ kmph} = 0.6 \text{ kmph.}$$

16. a Speed in still water = x km/h
Speed of current = y km/h

$$\therefore x+y = \frac{1}{\frac{4}{60}} = 15$$

$$x-y = \frac{1}{\frac{10}{60}} = 6$$

$$\therefore \text{Speed of current} = \frac{1}{2}[(x+y)-(x-y)]$$

$$= \frac{1}{2}(15-6) = \frac{9}{2} = 4.5 \text{ km/h.}$$

17. a Let the speed of boat in still water be x kmph and the rate of stream be y kmph.
 \therefore Downstream rate = $(x + y)$ kmph
and upstream rate = $(x - y)$ kmph
Now,

$$\frac{20}{x+y} = 1 \Rightarrow x+y = 20 \quad \dots(\text{i})$$

$$\text{and } \frac{20}{x-y} = 2 \Rightarrow x-y = 10 \quad \dots(\text{ii})$$

From (i) and (ii), we have
 $x = 15$ kmph.

18. c Let the speed of boat in still water be x km/hr and speed of current be y km/hr.

$$\therefore \frac{24}{x-y} + \frac{36}{x+y} = 6 \quad \dots \text{(i)}$$

$$\frac{36}{x-y} + \frac{24}{x+y} = \frac{13}{2}$$

$$\left[\text{Equation (i)} \times \frac{3}{2} \right] - \left[\text{Equation (ii)} \right]$$

$$\frac{30}{x+y} = \frac{5}{2} \Rightarrow x+y = 12$$

$$\text{From equation (i), } \frac{24}{x-y} = 3 \Rightarrow x-y = 8$$

$$\therefore \text{speed of current} = \frac{1}{2}(12-8) = 2 \text{ km/hr.}$$

19. b Speed upstream = $\frac{40}{8} = 5$ kmph

$$\text{Speed downstream} = \frac{36}{6} = 6 \text{ kmph}$$

\therefore Speed of boat in still water

$$= \frac{1}{2}(5+6) = 5.5 \text{ kmph.}$$

20. b Let the speed of boat in still water be x kmph. Then,

$$\frac{12}{x+3} + \frac{12}{x-3} = 3$$

$$\Rightarrow 12 \left(\frac{x-3+x+3}{(x+3)(x-3)} \right) = 3$$

$$\Rightarrow 4 \times 2x = x^2 - 9$$

$$\Rightarrow x^2 - 8x - 9 = 0$$

$$\Rightarrow x^2 - 9x - x - 9 = 0$$

$$\Rightarrow x(x-9) + 1(x-9) = 0$$

$$\Rightarrow (x-9)(x+1) = 0$$

$$\Rightarrow x = 9 \text{ because } x \neq -1$$

\therefore Speed cannot be negative.

Hence, speed of boat in still water = 9 kmph.

21. c Let the speed of motorboat in still water be x kmph.

$$\therefore \frac{10}{x-5} + \frac{10}{x+5} = \frac{50}{60}$$

$$\Rightarrow 10 \left(\frac{x+5+x-5}{(x+5)(x-5)} \right) = \frac{5}{6}$$

$$\Rightarrow 20x \times 6 = (x^2 - 25) \times 5$$

$$\Rightarrow x^2 - 24x - 25 = 0$$

$$\Rightarrow x^2 - 25x + x - 25 = 0$$

$$\Rightarrow x(x-25) + 1(x-25) = 0$$

$$\Rightarrow (x-25)(x+1) = 0$$

$$\Rightarrow x = 25 \text{ because } x \neq -1$$

Speed of motorboat in still water = 25 kmph

22. d Speed of boat in still water

$$\begin{aligned} &= \frac{1}{2} [\text{Rate downstream} + \text{Rate upstream}] = \frac{1}{2} (15 + 9) \\ &= \frac{1}{2} \times 24 = 12 \text{ kmph.} \end{aligned}$$

23. a Let the speed of sailor in still water be x kmph and speed of current = y kmph

$$\therefore x+y = \frac{12}{\frac{48}{60}} = \frac{12}{\frac{48}{60}} \times 60 = 15 \text{ kmph}$$

$$\text{and } x-y = \frac{12}{\frac{80}{60}} = \frac{12 \times 60}{80} = 9 \text{ kmph}$$

Adding these equations,

$$2x = 15 + 9 = 24$$

$$\Rightarrow x = 12 \text{ kmph.}$$

24. b Let the speed of boat in still water be x kmph.

$$\therefore \frac{6}{x+4} + \frac{6}{x-4} = 2$$

$$\Rightarrow 6 \left(\frac{x-4+x+4}{(x+4)(x-4)} \right) = 2$$

$$\Rightarrow 6x = x^2 - 16$$

$$\Rightarrow x^2 - 6x - 16 = 0$$

$$\Rightarrow x^2 - 8x + 2x - 16 = 0$$

$$\Rightarrow x(x-8) + 2(x-8) = 0$$

$$\Rightarrow (x+2)(x-8) = 0$$

$$\Rightarrow x = 8 \text{ kmph and } x \neq -2 \text{ kmph.}$$

25. d Let the speed of man in still water be x kmph.

$$\therefore \frac{15}{x+5} = 1$$

$$\Rightarrow x + 5 = 15 \Rightarrow x = 10 \text{ kmph}$$

\therefore Time taken in swimming upstream

$$= \frac{15}{10-5} = 3 \text{ hours.}$$

26. b Time = $\frac{\text{Distance}}{\text{Rate downstream}}$
 $= \frac{26}{5+3} = \frac{13}{4} = 3\frac{1}{4}$ hours.

27. c Let speed in still water = x kmph and speed of current = y kmph

$$\therefore x+y = \frac{15}{3} = 5 \text{ kmph}$$

$$\text{and } x-y = \frac{15}{\frac{15}{2}} = 2 \text{ kmph}$$

On adding,

$$2x = 7 \Rightarrow x = \frac{7}{2} = 3.5 \text{ kmph.}$$

28. a Let the speed of the current be x kmph.
 According to the question, time taken to go upstream is twice, so speed will be half

$$\Rightarrow \frac{6}{2} = 3$$

$$\therefore \frac{6}{6-x} = 3$$

$$\Rightarrow 18 - 3x = 6 \Rightarrow 3x = 18 - 6$$

$$\Rightarrow x = \frac{12}{3} = 4 \text{ kmph.}$$

29. b Let the rate of swimming in still water be x kmph.
 \therefore Rate downstream = $(x + 3)$ kmph
 \therefore Rate upstream = $(x - 3)$ kmph
 According to the question,
 $(x + 3)t = 2(x - 3) \times t$
 $\Rightarrow x + 3 = 2x - 6$
 $\Rightarrow x = 9 \text{ kmph.}$

30. b Let the speed of stream be x kmph, then speed of boat in still water = $4x$ kmph.
 \therefore Rate downstream = $4x + x = 5x$ kmph
 Rate upstream = $4x - x = 3x$ kmph
 $\therefore \frac{30}{3x} + \frac{30}{5x} - 8 \Rightarrow \frac{10}{x} + \frac{6}{x} = 8$
 $\Rightarrow \frac{16}{x} = 8 \Rightarrow x = \frac{16}{8} = 2 \text{ kmph.}$

31. c Let the speed of current be x kmph.
 $\therefore 2\left(\frac{15}{2} - x\right) = \frac{15}{2} + x$
 $\Rightarrow 15 - 2x = \frac{15}{2} + x \Rightarrow x = \frac{5}{2} = 2\frac{1}{2} \text{ kmph.}$

32. b Let the speed of stream be x kmph.

$$\therefore \text{Rate upstream} = \frac{9}{2} - x$$

$$\text{Rate downstream} = \frac{9}{2} + x$$

$$\text{Then, } \frac{2}{\frac{9}{2} + x} = \frac{1}{\frac{9}{2} - x}$$

$$\Rightarrow 9 - 2x = \frac{9}{2} + x \Rightarrow 3x = 9 - \frac{9}{2} = \frac{9}{2}$$

$$\Rightarrow x = \frac{9}{2 \times 3} = \frac{3}{2} = 1.5 \text{ kmph.}$$

33. c Let the distance be x km.

$$\text{Speed upstream} = 5 - 1 = 4 \text{ kmph}$$

$$\text{Speed downstream} = 5 + 1 = 6 \text{ kmph}$$

$$\therefore \frac{x}{6} + \frac{x}{4} = 1$$

$$\Rightarrow \frac{2x + 3x}{12} = 1 \Rightarrow 5x = 12$$

$$\Rightarrow x = \frac{12}{5} = 2.4 \text{ km.}$$

34. c Let the speed of motor boat be $36x$ kmph and speed of current = $5x$ kmph.

The boat goes along with the current in 5 hours 10 minutes i.e. $\frac{31}{6}$ hours.

$$\therefore \text{Distance} = \frac{31}{6} \times (36x + 5x) = \frac{41x \times 31}{6} \text{ km}$$

$$\text{Rate upstream} = 36x - 5x = 31x \text{ kmph}$$

$$\therefore \text{Time taken} = \frac{41x \times \frac{31}{6}}{31x} = \frac{41}{6} \text{ hours}$$

or 6 hours 50 minutes.

35. c Let the distance of the destination from the starting point be x km.

$$\text{Rate downstream} = (10 + 4) \text{ kmph} = 14 \text{ kmph}$$

$$\text{Rate upstream} = (10 - 4) \text{ kmph} = 6 \text{ kmph}$$

According to the question,

$$\frac{x}{14} + \frac{x}{6} = 5$$

$$\Rightarrow \frac{3x + 7x}{42} = 5$$

$$\Rightarrow 10x = 42 \times 5$$

$$\Rightarrow x = \frac{42 \times 5}{10} = 21 \text{ km.}$$

36. b Let the speed of the stream be x kmph and let both the boats meet after t hours.

According to the question,

$$(12 + x) t + (15 - x) t = 108$$

$$\Rightarrow 12t + 15t = 108$$

$$\Rightarrow 27t = 108$$

$$\Rightarrow t = \frac{108}{27} = 4 \text{ hours.}$$

37. b Let the required distance be x km.

$$\therefore \frac{x}{6-2} - \frac{x}{6+2} = 3$$

$$\Rightarrow \frac{x}{4} - \frac{x}{8} = 3 \Rightarrow \frac{2x-x}{8} = 3$$

$$\Rightarrow x = 3 \times 8 = 24 \text{ km.}$$

38. c Let the speed of the current be x kmph.

$$\therefore \text{Rate downstream} = (x + 45) \text{ kmph.}$$

According to the question,

$$\frac{80}{x+45} = 1 \text{ hour } 20 \text{ minutes} = \frac{4}{3} \text{ hours}$$

$$\Rightarrow 4x + 180 = 240$$

$$\Rightarrow 4x = 240 - 180 = 60$$

$$\Rightarrow x = \frac{60}{4} \text{ kmph} = 15 \text{ kmph}$$

$$\text{Rate upstream} = 45 - 15 = 30 \text{ kmph}$$

$$\therefore \text{Required time} = \frac{80}{30} \text{ hours}$$

$$= \frac{8}{3} = 2 \text{ hours } 40 \text{ minutes.}$$

39. c Let the required distance be x km. Then,

$$\frac{x}{5+3} + \frac{x}{5-3} = 3$$

$$\Rightarrow \frac{x}{8} + \frac{x}{2} = 3$$

$$\Rightarrow \frac{x+4x}{8} = 3$$

$$\Rightarrow 5x = 24$$

$$\Rightarrow x = \frac{24}{5} = 4.8 \text{ km.}$$

40. c Let the speed of boat in still water be x kmph and that of current be y kmph. Then,

$$x + y = 12$$

$$x - y = 8$$

$$\Rightarrow 2x = 20$$

$$\Rightarrow x = 10 \text{ kmph}$$

$$\text{Hence, required time} = \frac{24}{10} = 2.4 \text{ hours.}$$

41. a Speed of current

$$= \frac{1}{2} (\text{Rate downstream} - \text{Rate upstream})$$

$$= \frac{1}{2} (12 - 6) = 3 \text{ kmph}$$

$$[\text{Rate downstream} = \frac{1}{5} \times 60 = 12 \text{ kmph}]$$

42. b Speed upstream = $\frac{13}{5}$

$$\text{Speed downstream} = \frac{28}{5}$$

$$\therefore \text{Speed of stream} = \frac{1}{2} \left(\frac{28}{5} - \frac{13}{5} \right) = \frac{1}{2} \left(\frac{15}{5} \right) = \frac{3}{2} = 1.5 \text{ km/hr.}$$

43. a Upstream speed of boat

$$= \frac{\text{Distance}}{\text{Time}} = \frac{45}{6} = \frac{15}{2} = 7.5 \text{ kmph}$$

$$\therefore \text{Speed of current} = 10 - 7.5 = 2.5 \text{ kmph.}$$

44. a Speed of stream

$$= \frac{1}{2} \left(\frac{36}{6} - \frac{40}{8} \right) = \frac{1}{2} = 0.5 \text{ kmph.}$$

45. a Let speed of Sham and Ram be 5 m/sec and 3 m/sec. To overtake Ram, Sham has to cover 1500 m extra.

$$\text{Time taken by Sham to overtake Ram, } T = \frac{1500}{2}$$

$$= 750 \text{ sec. So, distance covered by Sham, } 750 \times 5 = 3,750 \text{ m.}$$

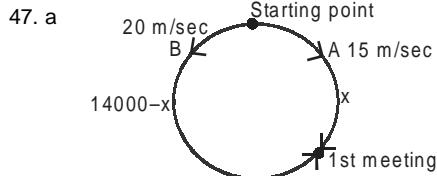
$(2 \times 1,500 + 750)$ i.e. 750 m away from starting point (diametrically opposite point). Next time they will meet at starting point, so there are two points.

46. d Length of track = 700 m

$$P's \text{ speed} = 18 \times \frac{5}{18} = 5 \text{ m/sec}$$

$$Q's \text{ speed} = 27 \times \frac{5}{18} = 7.5 \text{ m/sec}$$

$$\therefore \text{Time} = \frac{700}{5 + 7.5} = \frac{700}{12.5} = 56 \text{ sec.}$$



Let x be the distance covered by A.

$$\therefore \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\Rightarrow \frac{x}{15} = \frac{14000 - x}{20}$$

$$\Rightarrow 35x = 14000 \times 15$$

$$\Rightarrow x = 6000 \text{ m} = 6 \text{ km.}$$

48. b Length = 900 m

$$A's \text{ speed} = 27 \times \frac{5}{18} = 7.5 \text{ m/sec.}$$

$$B's \text{ speed} = 36 \times \frac{5}{18} = 10 \text{ m/sec.}$$

Time taken by A to complete one round

$$= \frac{900}{7.5} = 120 \text{ sec.}$$

Time taken by B to complete one round

$$= \frac{900}{10} = 90 \text{ sec}$$

They will meet at starting point at LCM of (120, 90) = 360 sec = 6 min.

49. a Length (L) = 1200 m.

$$\text{Speed of X (x)} = 18 \times \frac{5}{18} = 5 \text{ m/sec.}$$

$$\text{Speed of Y (y)} = 27 \times \frac{5}{18} = 7.5 \text{ m/sec.}$$

$$\text{Speed of Z (z)} = 45 \times \frac{5}{18} = 12.5 \text{ m/sec.}$$

They will meet for first time on starting point in the

$$\text{LCM of } \left[\frac{L}{x}, \frac{L}{y}, \text{ and } \frac{L}{z} \right].$$

$$\frac{L}{x} = \frac{1200}{5} = 240 \text{ sec, } \frac{L}{y} = \frac{1200}{7.5} = 160 \text{ sec}$$

$$\frac{L}{z} = \frac{1200}{12.5} = 96 \text{ sec}$$

LCM of (240, 160 and 96) is 480 sec.
i.e. 8 minutes.

50. a Required time = LCM of 252, 308 and 198 seconds.

Now,

$$252 = 2 \times 3 \times 7$$

$$308 = 2 \times 2 \times 7 \times 11$$

$$198 = 2 \times 3 \times 3 \times 11$$

$$\therefore \text{LCM} = 2 \times 2 \times 3 \times 3 \times 7 \times 11$$

$$= 36 \times 77 \text{ seconds}$$

$$= \frac{36 \times 77}{60} \text{ minutes}$$

$$= \frac{231}{5} = 46 \text{ minutes } 12 \text{ seconds.}$$

Time and Work, Pipes and Cisterns
Answers and Explanations

1	b	2	d	3	c	4	a	5	c	6	a	7	a	8	b	9	a	10	d
11	a	12	b	13	d	14	a	15	b	16	d	17	c	18	c	19	c	20	a
21	b	22	c	23	c	24	d	25	a	26	a	27	d	28	c	29	d	30	c
31	c	32	a	33	c	34	a	35	d	36	c	37	c	38	a	39	b	40	c
41	a	42	d	43	c	44	b	45	c	46	c	47	c	48	a	49	a	50	a

1. b A can copy 100 pages in 5 hr.
 \therefore A can copy 20 pages in 1 hr.

$$\text{Given } 20 + B = \frac{100}{4}.$$

$$\Rightarrow 20 + B = 25 \Rightarrow B = 5 \text{ pages per hour}$$

\therefore For 20 pages, B needs 4 hr

2. d Smith's 2 hr work = $\frac{2}{x}$

$$\text{Work left} = 1 - \frac{2}{x} = \frac{x-2}{x}$$

3. c Let the work done by a man and a boy in one hour be 'm' and 'b' respectively.

$$\therefore 5m + 2b = 4(m + b) = 4m + 4b$$

$$\Rightarrow 1m = 2b$$

Therefore, work done by man : work done by a boy = 2 : 1.

4. a Work done by the waste pipe in 1 min.

$$= \frac{1}{15} - \left(\frac{1}{18} + \frac{1}{36} \right)$$

$$= -\frac{1}{60} \text{ (minus sign means emptying)}$$

$$\therefore \text{Volume of } \frac{1}{60} \text{ part} = 2 \text{ gallons}$$

$$\therefore \text{Capacity of the tank} = 2 \times 60 = 120 \text{ gallons.}$$

5. c A's 1 day's work = $\frac{1}{18}$,

$$\text{B's 1 day's work} = \frac{1}{27}$$

$$(A + B)'s 1 \text{ day's work} = \frac{1}{18} + \frac{1}{27} = \frac{5}{54}$$

\therefore Both A and B will finish the work in $\frac{54}{5} = 10\frac{4}{5}$ days.

6. a $(A + B)'s 1 \text{ day's work} = \frac{1}{15}$

$$A's 1 \text{ day's work} = \frac{1}{21}$$

$$B's 1 \text{ day's work} = \frac{1}{15} - \frac{1}{21} = \frac{2}{105}$$

\therefore B alone can finish the work in $\frac{105}{2} = 52\frac{1}{2}$ days.

7. a Ravi's 1 day's work = $\frac{1}{21}$ and

$$\text{Prem's 1 day's work} = \frac{1}{10\frac{1}{2}} = \frac{2}{21}$$

$\therefore (Ravi + Prem)'s 1 \text{ day work}$

$$= \frac{1}{21} + \frac{2}{21} = \frac{3}{21} = \frac{1}{7} \text{ th part of the work.}$$

8. b Let initial number of men be x.

Number of days is 10, so number of man days is 10x.
If there were 2 more men i.e. (x + 2), it can be completed in 8 days.

$$\text{So, } 10x = (x + 2)8$$

$$\Rightarrow x = 8.$$

9. a Part of the tank filled by A in 1 minute = $\frac{1}{10}$
Part of the tank filled by B in 1 minute = $\frac{1}{15}$
Part of the tank filled by both the pipes in one minute
 $= \frac{1}{10} + \frac{1}{15} = \frac{1}{6}$
∴ The tank can be filled in 6 minutes.
10. d Work done by the 3 pipes together in 1 minute
 $= \frac{1}{18} + \frac{1}{27} - \frac{1}{36} = \frac{5}{54} - \frac{1}{36} = \frac{7}{108}$
So, the empty tank will be filled in $\frac{108}{7} = 15\frac{3}{7}$ minutes.
11. a Time taken by A, B and C to complete a work = $1x$, $2x$ and $3x$.
 $3x = 12 \Rightarrow x = 4$
Time taken by B to complete the work = 8 hr
∴ In 1 hr, B will complete $\frac{1}{8}$ th of the work.
Alternate method:
According to the question,
 $A = 2B$ and $A = 3C$
Thus $2B = 3C \Rightarrow B = \frac{3}{2}C$
Now work completed by C in 1 hr = $\frac{1}{12}$
∴ Work completed by B in 1 hr = $\frac{3}{2} \times \frac{1}{12} = \frac{1}{8}$ th part.
12. b Let x hr be the required time to fill the tank.
 $\therefore \left(\frac{1}{10} - \frac{1}{20} \right) x = 1$
 $\Rightarrow \frac{x}{20} = 1 \Rightarrow x = 20$
∴ The tank can be filled in 20 hr.
13. d Let original labourers = x .
 $10 \times x = (x - 5) \times 12 \Rightarrow x = 30$
14. a Let X, Y and Z be the three persons and they complete the work in a , b and c days respectively.
 $\frac{1}{a} + \frac{1}{c} = \frac{3}{b}$... (i)
 $\frac{1}{a} + \frac{1}{b} = \frac{2}{c}$... (ii)

- $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{10}$... (iii)
Subtracting (i) from (iii), we get $b = 40$ days.
Subtracting (ii) from (iii), we get $c = 30$ days.
Substituting the values of b and c in (iii), we get $a = 24$.
∴ X, Y and Z complete the work in 24, 40 and 30 days respectively.
15. b One man can complete $\frac{1}{5}$ th part of the work in one day.
Also, 1 woman can complete $\frac{1}{12}$ th part of the work in one day.
Therefore, part of the work completed by a man and 2 women $= \frac{1}{5} + \frac{2}{12} = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$
Hence, the number of days required for completing the piece of work $= \frac{30}{11} = 2\frac{8}{11}$ days.
16. d $(A + B)$'s 1 day's work $= \frac{1}{8}$;
 $(A + B)$'s 4 day's work $= \frac{1}{2}$; Remaining work $= \frac{1}{2}$
Work completed by A in 1 day $= \frac{1}{8} - \frac{1}{12} = \frac{1}{24}$
∴ A alone takes 24 days to complete the job.
Since only $\frac{1}{2}$ of the work is left, A will complete the remaining work in $\frac{1}{2} \times 24 = 12$ days.
17. c Amount of work completed by 10 men in 1 day is $\frac{1}{18}$ th.
Therefore, amount of work completed by 10 men in 6 days
 $= \frac{6}{18} = \frac{1}{3}$
Amount of work left $= 1 - \frac{1}{3} = \frac{2}{3}$
Since 10 men require 12 days to complete the remaining $\frac{2}{3}$ work, time required by 1 man is $12 \times 10 = 120$ days.
Therefore, time required for $(10 + 5) = 15$ men
 $= \frac{120}{15} = 8$ days.

18. c Time required to fill the cistern = $\frac{1}{2 - \frac{5}{4}} = \frac{1}{\frac{3}{4}}$
 $= \frac{4}{3}$ hr = $1\frac{1}{3}$ hr

19. c $\therefore A = 3B$
 Let A takes x days to complete the work alone.
 $\therefore B$ takes $(x + 60)$ days to complete the work alone.
 A's one day's work = $\frac{1}{x}$
 B's one day's work = $\frac{1}{x+60}$
 $\therefore \frac{1}{x} = \frac{3}{x+60}$
 $\Rightarrow x + 60 = 3x \Rightarrow 2x = 60 \Rightarrow x = 30$ days
 A can complete the work in 30 days.
 B can complete the work in 90 days.

In 1 day $\frac{1}{30} + \frac{1}{90} = \frac{4}{90} = \frac{2}{45}$ work is completed.
 \therefore The work will take $\frac{45}{2}$ days = 22.5 days.

20. a $(A + B + C)$'s 1 hour's work = $\frac{1}{6}$
 Remaining part of tank = $1 - \frac{1}{6} = \frac{5}{6}$
 Time taken by $(A + B)$ to fill this $\frac{5}{6}$ of the tank = 8 hours.
 $\Rightarrow A$ and B together fill the tank is $\frac{48}{5}$ hours
 Now, we know $A + B + C = 6$ hours
 $A + B = \frac{48}{5}$ hours
 $\therefore C = \frac{1}{6} - \frac{5}{48} = \frac{1}{16}$

Hence, C alone can fill the tank in 16 hours.

21. b Ratio in number of days taken by Tejas:Aniket:Ketan = 12 : 15 : 25
 Let us assume that Ketan can complete the work in x days. Then,
 $\frac{30}{x} = \frac{12}{25}$
 $\Rightarrow x = 62.5$ days.

22. c Rahul's 1 day's work = $\frac{1}{16}$
 Number of days Rahul worked = $4 + 4 = 8$

\therefore Total work done by Rahul = $8 \times \frac{1}{16} = \frac{1}{2}$

The remaining $\frac{1}{2}$ of the work is done by Rohit in 4 days.
 \therefore Complete work will be done by Rohit in
 $4 \times \frac{2}{1} = 8$ days.

23. c Whole work completed by A = $12 \times 6 = 72$ days.

Whole work completed by B = $\left(40 \times \frac{100}{40}\right) = 100$ days.

Whole work completed by C = $14 \times 5 = 70$ days.
 \therefore C will complete the work first.

24. d Work done by the leak in 1 hour = $\frac{1}{6} - \frac{1}{6\frac{1}{2}}$

$$= \frac{1}{6} - \frac{2}{13} = \frac{1}{78}$$

\therefore The leak will empty the tank in 78 hours.

25. a Let B closed after x minutes. Then, part filled by $(A + B)$ in x min. + part filled by A in $(9 - x)$ min. = 1.

$$\Rightarrow x\left(\frac{1}{12} + \frac{1}{16}\right) + (9-x) \times \frac{1}{12} = 1,$$

$$\Rightarrow \frac{7x}{48} + \frac{(9-x)}{12} = 1 \Rightarrow x = 4 \text{ min.}$$

Hence, B must be closed after 4 min.

26. a A in 1 day can complete $\frac{1}{8}$ th of work.

B in 1 day can complete $\frac{1}{16}$ th of work.

C in 1 day can complete $\frac{1}{24}$ th of work.

Let the work completed in x days.

Then, $\frac{1}{8}x + \frac{1}{16}[x-1] + \frac{1}{24}[x-2] = 1$

$$\Rightarrow \frac{6x + 3x - 3 + 2x - 4}{48} = 1 \Rightarrow 11x - 7 = 48$$

$$\Rightarrow 11x = 55 \Rightarrow x = 5 \text{ days.}$$

27. d Ratios of one day's work of A, B, C and D

$$= \frac{1}{32} : \frac{1}{20} : \frac{1}{12} : \frac{1}{24} = 15 : 24 : 40 : 20$$

Total amount = ₹99

$$\text{A's share} = \frac{15}{99} \times 99 = ₹15$$

$$\text{B's share} = \frac{24}{99} \times 99 = ₹24$$

$$\text{C's share} = \frac{40}{99} \times 99 = ₹40$$

$$\text{D's share} = \frac{20}{99} \times 99 = ₹20$$

28. c Rate of pipe A and the leakage is same i.e. 15 min. Thus, when only 'A' is opened, by the end of 5 min, the tank would still be empty.

When A, B and leak work together,
Construction by pipe A = destruction by the leak in the tank. Hence, only pipe B will help in filling the tank.
Hence, it takes 20 min to fill the tank.

29. d A can complete the work in $4 \times 9 = 36$ hours
B can complete the work in $6 \times 7 = 42$ hours

$$\therefore (\text{A} + \text{B})\text{'s 1 hour's work} = \frac{1}{36} + \frac{1}{42} = \frac{13}{252}$$

$$\therefore \text{Both will finish the work in } \frac{252}{13} \text{ hrs.}$$

$$\begin{aligned} \text{Number of days of } 8\frac{2}{5} \text{ hrs. each} &= \frac{252}{13} \times \frac{5}{42} \\ &= \frac{30}{13} = 2\frac{4}{13} \text{ days.} \end{aligned}$$

30. c A's work for 6 days = $6 \times \frac{1}{12}$ work = $\frac{\text{work}}{2}$

The remaining $\frac{1}{2}$ work was completed by A and B together.

$$\text{Work completed by A and B in a day} = \frac{1}{12} + \frac{1}{18} = \frac{5}{36}$$

\therefore Number of days they worked together

$$\begin{aligned} &= \frac{1}{\frac{5}{36}} = \frac{36}{5} = \frac{18}{5} = 3\frac{3}{5} \text{ days.} \end{aligned}$$

Hence, B left after $3\frac{3}{5}$ days from the start of the work.

31. c $(20 \text{ M} + 30 \text{ W}) \times 5 = (6 \text{ M} + 5 \text{ W}) \times 20$

$$\Rightarrow 20 \text{ M} + 30 \text{ W} = 24 \text{ M} + 20 \text{ W}$$

$$\Rightarrow 1 \text{ W} = \frac{2}{5} \text{ M}$$

$$\therefore 30 \text{ W} = 12 \text{ M}$$

Now, we have $20 \text{ M} + 30 \text{ W}$ takes 5 days
or $(20 + 12)$ men can complete the work in 5 days

$$40 \text{ men will complete the work in } = \frac{32 \times 5}{40} = 4 \text{ days.}$$

$$32. \text{ a } \left(\frac{1}{\text{A}} + \frac{1}{\text{B}} \right) = \frac{2}{7}$$

$$\left(\frac{1}{\text{B}} + \frac{1}{\text{C}} \right) = \frac{1}{5}$$

$$\left(\frac{1}{\text{A}} + \frac{1}{\text{C}} \right) = \frac{1}{4} \Rightarrow 2 \left(\frac{1}{\text{A}} + \frac{1}{\text{B}} + \frac{1}{\text{C}} \right) = \frac{2}{7} + \frac{1}{5} + \frac{1}{4} = \frac{103}{140}$$

$$\Rightarrow \left(\frac{1}{\text{A}} + \frac{1}{\text{B}} + \frac{1}{\text{C}} \right) = \frac{103}{280}$$

A, B and C together can complete the work in

$$\frac{280}{103} = 2\frac{74}{103} \text{ days.}$$

33. c Earning of men, women, boys and girls are

$$1 \text{ man earning} = \frac{8}{6} \text{ woman earning}$$

$$1 \text{ woman earning} = \frac{3}{2} \text{ boy earning}$$

$$1 \text{ boy earning} = \frac{5}{4} \text{ girl earning} = \frac{5}{4} \times 50$$

$$\therefore 1 \text{ man earning} = \frac{8}{6} \times \frac{3}{2} \times \frac{5}{4} \times 50 = ₹125.$$

Alternate method:

Initials used

Man : M, Woman : W, Girl : G, Boy : B

Now according to the question

$$6M = 8W$$

$$2W = 3B$$

$$4B = 5G$$

$$\text{Thus, } M = \frac{8}{6} \times \frac{3}{2} \times \frac{5}{4} \times G = \frac{5}{2} \times 50 = ₹125.$$

34. a Given Harish, Ramesh and Suresh can finish a project in 22, 33 and 44 days respectively. Let them take x days to complete the job while working together.

$$\left(\frac{1}{22} + \frac{1}{33} + \frac{1}{44} \right)x = 1 \Rightarrow \left(\frac{6+4+3}{132} \right)x = 1$$

$$\Rightarrow x = \frac{132}{13} \Rightarrow x = 10\frac{2}{13} \text{ days.}$$

35. d As A, B and C together require 15 days to complete the job, they can complete $\frac{1}{3}$ rd of the job in 5 days. Therefore, amount of work left when A leaves is

$$1 - \frac{1}{3} = \frac{2}{3}. \text{ It is given that B and C together take } 20$$

days for $\frac{2}{3}$ rd work. Therefore, B and C need 30 days to complete the job.

A's 1 day's work

$$= (A + B + C)'s \text{ 1 day's work} - (B + C)'s \text{ 1 day's work}$$

$$= \frac{1}{15} - \frac{1}{30} = \frac{1}{30}$$

A can finish the work in 30 days.

36. c Let total work be 60 units. 1st tap fills 3 units per minute, 2nd tap fills 4 units per minute and waste pipe empties 5 units per minute. So, when opened together all three will fill $(3 + 4 - 5) = 2$ units per min. Hence, in first 9 min, 18 units of tank is filled. Since the waste pipe is closed ∴ The remaining 42 units will require

$$\frac{42}{3+4} = 6 \text{ min to fill.}$$

37. c From the given data men in the first group are two-third as efficient as men from the second group. Hence, 27

$$\text{men of the first group are equivalent to } \left(\frac{2}{3}\right) \times 27 \\ = 18 \text{ men of second group. Further more} \\ (\text{Men} \times \text{hours per day} \times \text{number of days})$$

= constant

$$\frac{\text{Work done}}{18 \times 6 \times 21} = \frac{18 \times 9 \times D}{x \times 2x} \Rightarrow D = 28 \text{ days}$$

38. a $(A + B + C)'s \text{ 1 day's work} = \frac{1}{8}$

Work completed by A, B and C in 6 days'

$$= 6 \times \frac{1}{8} = \frac{3}{4}$$

$$\therefore \text{Remaining work} = 1 - \frac{3}{4} = \frac{1}{4}$$

∴ B and C can finish the remaining $\frac{1}{4}$ th of the work in

$$\frac{1}{4} \times 12 = 3 \text{ days.}$$

39. b (20×12) men can complete the work in 1 day.

$$\therefore 1 \text{ man's 1 day's work} = \frac{1}{240}$$

$$20 \text{ men 4 day's work} = \frac{1}{12} \times 4 = \frac{1}{3}$$

$$\text{Remaining work} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$32 \text{ men 1 day's work} = \frac{32}{240} = \frac{2}{15}$$

Now, $\frac{2}{15}$ work is completed by them in 1 day.

$$\therefore \frac{2}{3} \text{ work is completed by them in } \frac{15}{2} \times \frac{2}{3} = 5 \text{ days.}$$

40. c Work completed by two pipes in 1 hour = $\frac{1}{12} + \frac{1}{18} = \frac{5}{36}$

∴ Time taken by these pipes to fill the tank

$$= \frac{36}{5} = 7 \frac{1}{5} \text{ hours} = 7 \text{ hr } 12 \text{ min.}$$

Due to leakage, time taken = 7 hr. 12 min. + 48 min. = 8 hours.

∴ Work completed by (two pipes + leak) in an hour

$$= \frac{1}{8}$$

$$\text{Work completed by the leak in 1 hour} = \frac{5}{36} - \frac{1}{8} = \frac{1}{72}$$

∴ Leak will empty the full cistern in 72 hours.

41. a Let the slower pipe takes x hr to fill the reservoir. Then,

$$\frac{1}{x} + \frac{1}{(x+5)} = \frac{1}{6}$$

$$\Rightarrow 12x + 30 = x^2 + 5x \Rightarrow x^2 - 7x - 30 = 0$$

$$\Rightarrow x(x-10) + 3(x-10) = 0$$

$$\Rightarrow (x-10)(x+3) = 0, \text{ i.e } x = 10 \text{ hr. } (\because x \neq -3)$$

42. d In 30 days, $\frac{1}{2}$ of the work was done by 45 people.

So, 45 people require 60 days to complete the work.
1 person requires 60×45 days.

∴ To complete the entire job in 15 days,

$$\frac{60 \times 45}{15} = 180 \text{ people are required.}$$

Only half the work is left, $\frac{180}{2} = 90$ people are

required. As there are already 45 people, 45 more people are required to complete the job.

43. c If A and B work together, then they can lay 30 + 40 = 70 bricks per minute.

= $(70 \times 15 \text{ cm} \times 5 \text{ cm})$ of the wall in 1 minute

(Note → here width of brick = width of wall)

So, 'w' of brick is not required)

Dimensions of the wall = 14 m × 6 m

$$\therefore \text{Total time} = \frac{14 \times 6 \times 100 \times 100}{70 \times 15 \times 5} \text{ (convert cm to m)}$$

= 160 minutes = 2 hrs and 40 mins.

44. b Let us consider that the total work = 360 units

A	B	C	D
10	12	15	18
36	30	24	20

... (minutes)

... (units/minutes)

Let us consider that it takes 'x' minutes to empty the completely full tank.

∴ The outlet C does $(x - 5) \times 24$ units

The outlet D does $x \times 20$ units.

$$\therefore 36 \times 1 + 30 \times 2 + (x - 5) \times 24 + 20x = 360$$

$$\Rightarrow x = 8\frac{8}{11} \text{ min.}$$

45. c $(A + B)$'s 2 day's work $= \frac{1}{6} + \frac{1}{8} = \frac{7}{24}$

Work completed in 3 pairs of days $= 3 \times \frac{7}{24} = \frac{21}{24} = \frac{7}{8}$,

Remaining work $= 1 - \frac{7}{8} = \frac{1}{8}$,

On 7th day, it is A's turn $\frac{1}{6}$ work is completed by him in 1 day.

$\frac{1}{8}$ work is completed by him in $\frac{1}{8} \times 6 = \frac{3}{4}$ day.

$$\therefore \text{Total time taken} = 6 + \frac{3}{4} = 6\frac{3}{4} \text{ days.}$$

46. c If time taken by C to complete the work be x days then

$$\frac{10}{20} + \frac{6}{15} + \frac{4}{x} = 1$$

$$\Rightarrow \frac{1}{2} + \frac{2}{5} + \frac{4}{x} = 1$$

$$\Rightarrow \frac{5+4}{10} + \frac{4}{x} = 1$$

$$\Rightarrow \frac{4}{x} = 1 - \frac{9}{10} = \frac{1}{10}$$

$$\therefore x = 10 \times 4 = 40 \text{ days.}$$

47. c A + B complete a work in 8 days

B + C complete a work in 12 days

C + A complete a work in 8 days

(A + B + C) can complete $\left(\frac{1}{8} + \frac{1}{12} + \frac{1}{8}\right)$ work in one day.

$$2(A + B + C) = \frac{3+2+3}{24}$$

$$A + B + C = \frac{8}{24 \times 2}$$

$$A + B + C = \frac{1}{6}$$

A + B + C complete a work in = 6 days.

48. a X is 3 times as fast as Y. Therefore, if x complete a work in 1 day y take 3 days to complete it.

Therefore, the difference between the working days is 2.

But in this case the difference is 40 days.

Therefore, A completes a work in 20 days and B completes the work in 60 days.

A and B together completes a work

$$= \frac{1}{20} + \frac{1}{60} = \frac{3+1}{60} = \frac{1}{15} = 15 \text{ days.}$$

49. a A completes a work in 12 days

$$\text{Work done by A in one day} = \frac{1}{12}$$

$$\text{Work done by A in 3 day} = \frac{3}{12} = \frac{1}{4}$$

$$\text{Remaining work} = 1 - \frac{1}{4} = \frac{3}{4}$$

A + B completes $\frac{3}{4}$ work in 3 days

$$A + B \text{ complete work in } = 3 \times \frac{4}{3} = 4 \text{ days}$$

$$B's \text{ one day work} = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$$

∴ B will take 6 days to complete the work.

50. a A complete half as much work as B in three-fourth of the time.

∴ A can complete whole work as B in one and a half times.

Let the number of days taken by A and B to complete the same piece of work be 3x and 2x respectively.

A and B completes $\left(\frac{1}{3x} + \frac{1}{2x}\right)$ work in 1 day.

$$\therefore \frac{1}{3x} + \frac{1}{2x} = \frac{1}{18}$$

$$\Rightarrow x = 15$$

∴ B completes the work in 2x days = 30 days.

Permutations and Combinations

Answers and Explanations

1	c	2	c	3	b	4	b	5	b	6	c	7	d	8	a	9	d	10	a
11	d	12	a	13	d	14	c	15	a	16	b	17	c	18	b	19	d	20	d
21	b	22	c	23	d	24	d	25	c	26	c	27	b	28	c	29	a	30	d
31	e	32	e	33	b	34	e	35	c	36	a	37	e	38	b	39	d	40	c
41	c	42	b	43	a	44	a	45	c	46	c	47	d	48	b	49	a	50	c

1. c

1	2	3	4	5	6	7	8	9

Now 4 men can be played at 2nd, 4th, 6th, 8th positions whereas 5 women can be played at 1st, 3rd, 5th, 7th and 9th positions.

So, 4 men can be arranged in $4!$ ways whereas 5 women can be arranged in $5!$ ways.

So, the total number of arrangements = $4! \times 5!$.

2. c
$${}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5$$

$$= \frac{5!}{4!} + \frac{5!}{3!} + \frac{5!}{2!} + \frac{5!}{1!} + \frac{5!}{0!}$$

$$= 5 + 20 + 60 + 120 + 120 = 325.$$

3. b Total number of ways = $7!$.

4. b Men are in majority, therefore, we have two cases:

(i) when all are 4 men or

(ii) when there are 3 men and 1 woman.

∴ Required number of ways

$$= {}^5C_4 + {}^5C_3 \times {}^6C_1 = 5 + 10 \times 6 = 65.$$

5. b First letter can be posted in any of the 4 boxes. Second can also be posted in any of the 4 boxes. Similarly, all 7 can be posted in any of 4 boxes. So, total number of ways

$$= 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^7.$$

6. c Number of letters in the word 'LAUNCHER' = 8 Since repetition is allowed, therefore number of 3-letter words = $8 \times 8 \times 8 = 8^3$.

7. d Number of ways = ${}^{11}P_2 = 11 \times 10 = 110$.

8. a Total number of persons = 3(men) + 4(women) = 7
Hence, required number of ways

$$= {}^7C_4 = \frac{7!}{4! \times 3!} = 35.$$

9. d There are 4 kings and remaining cards = $52 - 4 = 48$. Therefore, required number of ways
 $= {}^{48}C_3 \times {}^4C_1 = 69184$.

10. a Since in forming a necklace clockwise and anticlockwise arrangements are not different. Therefore, required number of arrangements

$$= \frac{(11 - 1)!}{2} = \frac{10!}{2}.$$

11. d Standard form is

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$$

For selecting at least 1 out of them

$${}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n - 1$$

In this case ${}^{11}C_1 + {}^{11}C_2 + \dots + {}^{11}C_{11} = 2^{11} - 1$.

12. a As zero cannot come in the ten thousand's place, so it can be filled in 9 ways, while all other places can be filled in 10 ways.

So, the total number of ways = $9 \times 10 \times 10 \times 10 \times 10 = 90000$.

13. d Number of ways of selecting 3 boys out of 5 boys
 $= {}^5C_3$

Number of ways of selecting a girl out of 2 girls = 2C_1

$$\therefore \text{Total number of ways} = {}^5C_3 \times {}^2C_1 = \frac{5!}{2!3!} \times \frac{2!}{1!1!} = 10 \times 2 = 20.$$

14. c Number of ways = ${}^9C_5 = \frac{9!}{5! \times 4!} = \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 1 \times 5!} = 126$.

15. a Total letters in the word 'OPTIONS' = 7
There are 3 vowels in the given word, which are 2 O's and 1 I.
Assuming these 3 letters as 1 letter. This 1 letter and remaining 4 letters can be arranged in 5! ways.

$$2 \text{ O's and } 1 \text{ I can be arranged in } \frac{3!}{2!}.$$

$$\text{Therefore, required arrangements} = 5! \times \frac{3!}{2!}.$$

16. b Required number of ways = ${}^5C_2 \times {}^4C_2 \times {}^4C_2$
= $10 \times 6 \times 6 = 360$.

17. c The team can consist of (i) 1 boy and 3 girls (ii) 2 boys and 2 girls (iii) 3 boys and 1 girl
Required number of ways = ${}^5C_1 \times {}^4C_3 + {}^5C_2 \times {}^4C_2 + {}^5C_3 \times {}^4C_1$
= $5 \times 4 + 10 \times 6 + 10 \times 4 = 20 + 60 + 40 = 120$.

18. b $57 \times \times \times \times \times$
Since two digits 5 and 7 are fixed. Therefore, remaining digits = 8.
Since digits are not repeated. Therefore, remaining places can be filled in $8 \times 7 \times 6 \times 5 \times 4 = 6720$ ways.

19. d There are 4 suits: diamond, club, spade, heart and there are 13 cards of each suit.
The required number of ways = ${}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = (13)^4$.

20. d Each person will get 4 things.
Therefore, required number of ways
= ${}^{16}C_4 \times {}^{12}C_4 \times {}^8C_4 \times {}^4C_4$
= $\frac{16!}{4! \times 12!} \times \frac{12!}{4! \times 8!} \times \frac{8!}{4! \times 4!} \times 1 = \frac{16!}{(4!)^4}$.

21. b Among 5 letters of word, 2 letters are T's and remaining 3 letters can be selected in 4C_3 ways.
Now, total number of words = ${}^4C_3 \times \frac{5!}{2!} = 240$.

22. c SIMULTANEOUS is a 12 letter word in which there are 2S and 2U.
 \therefore Total number of words = $\frac{12!}{2!2!}$.

23. d Total number of balls = $3 + 4 = 7$
 \therefore Required number of ways = ${}^7C_4 = 35$.

24. d For table having 8 chairs, number of ways of selecting 8 persons is ${}^{14}C_8 = \frac{14!}{8!6!}$.

\therefore Total number of ways for 14 persons to be seated
= $\frac{14!}{8!6!} \times 7! \times 5!$.

25. c As the table is square and there are 4 chairs placed equidistant along each side, hence, the square table is similar to a circular table.
 \therefore Number of ways for 16 persons to be seated = $(16 - 1)! = 15!$.

26. c Number of letters in the word 'ARRANGEMENT' = 11
So, the total number of ways = $\frac{11!}{2! \times 2! \times 2! \times 2!}$ as there are 2 A's, 2 R's, 2 N's, 2 E's.

27. b The possible arrangement of boys and girls is B G B G B G B G or G B G B G B G B G B = $5! \times 5! \times 2$.

28. c One handshake involves 2 persons. So, if there are 66 handshakes in total, then the total number of persons in the room will be ${}^nC_2 = 66$

$$\Rightarrow \frac{n!}{2!(n-2)!} = 66 \Rightarrow \frac{n(n-1)}{2} = 66$$

$$\Rightarrow n^2 - n = 132 \Rightarrow n^2 - n - 132 = 0$$

$$\Rightarrow (n-12)(n+11) = 0 \Rightarrow n = 12$$

29. a Since the chairs are numbered, this is taken as a row arrangement, as there is no need to fix a position. Hence, the number of ways is 8!.

30. d Total number of balloons = 5(blue) + 4(pink) + 2(white) = 11
Therefore, the number of arrangements
= $\frac{11!}{5! \times 4! \times 2!} = 6930$.

31. e Required number of ways = $8 \times 7 \times 6 \times 5 \times 4 \times 3 = 20160$.

32. e 6 boys can be arranged in a row in $6!$ ways.
 $\times B \times B \times B \times B \times B \times B$
There are 7 cross marked places and 4 girls can be seated in 7P_4 ways.
Therefore, required number of ways = $6! \times {}^7P_4$.

For questions 33 to 35:

Total number of persons = 6(men) + 5(women) + 4(children) = 15

33. b Number of ways = ${}^5C_3 = 10$.

34. e Number of ways = ${}^6C_2 \times {}^4C_1 = 15 \times 4 = 60$.

35. c Number of ways = ${}^{15}C_4 = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365$.

36. a There are 5 such odd integers, viz. 1, 3, 5, 7 and 9. So, the total numbers that can be formed is $5! = 120$.

37. e In the given word there are 5 vowels and 4 different consonants. Considering the 5 vowels as one unit, total number of permutations is $5!$. For each of these arrangements, the vowels can be arranged among themselves in $5!$ ways. Therefore, total number of different words = $5! \times 5!$.

38. b Three programmes for first day can be selected in 6C_3 ways. Now 3 programmes can be arranged in $3!$ ways and for the other day, they can be arranged in $3!$ ways. So, required number of ways = ${}^6C_3 \times 3! \times 3!$.

39. d A particular official never included.
Required number of ways = ${}^{12-1}C_5 = {}^{11}C_5 = 462$.

40. c The total number of possible arrangements are

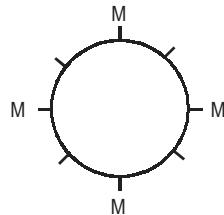
$$\begin{aligned} & {}^5C_3 \times {}^5C_1 + {}^5C_2 \times {}^5C_2 + {}^5C_1 \times {}^5C_3 \\ &= 50 + 100 + 50 = 200. \end{aligned}$$

41. c In this question we must bear in mind that we have to only form committees. We are not concerned with the arrangement of officials or non-officials.
Total: 8 officials, 4 non-officials.
3 officials out of 8 can be selected in ${}^8C_3 = 56$ ways.
2 non-officials out of 4 can be selected in ${}^4C_2 = 6$ ways
 \therefore The number of ways in which the committee can be formed is = $56 \times 6 = 336$.

42. b The five boys can be arranged along a circle in $4!$ ways. The five girls can then be arranged in the 5 places in $5!$ ways.
Therefore, the required number of ways = $4! \times 5!$.

43. a Unit's place digit can be filled by 0, 2, 4 and 6 i.e. 4 ways. Since digits can be repeated. Therefore, ten's place digit can be filled by 1, 2, 3, 4, 5, 6 and 7 i.e. 7 ways.
Therefore, required numbers = $4 \times 7 = 28$.

44. a 4 men can be seated along a circular table in $(4 - 1)!$ i.e. $3!$ ways.



There are 4 places for 4 women. Therefore, 4 women can be seated in $4!$ ways.
Hence, required number of ways = $3! \times 4! = 144$.

45. c Number of straight line = ${}^7C_2 - {}^3C_2 + 1 = 21 - 3 + 1 = 19$.

46. c Total number of mathematics teachers = $5 + 3 = 8$
 \therefore Required number of ways = ${}^8C_5 = \frac{8!}{3! \times 5!} = 56$.

47. d Total number of men teachers = 12 and total number of women teachers = 8.
 \therefore Required number of ways = ${}^{12}C_4 \times {}^8C_4$.

48. b Number of men chemistry teachers = 3 and number of women chemistry teachers = 3
At least a woman chemistry teacher i.e. 3 men chemistry teachers and a woman chemistry teacher or 2 men chemistry teachers and 2 women chemistry teachers or 1 man chemistry teacher and 3 women chemistry teachers.
 \therefore Required number of ways

$$\begin{aligned} & {}^3C_3 \times {}^3C_1 + {}^3C_2 \times {}^3C_2 + {}^3C_1 \times {}^3C_3 \\ &= 1 \times 3 + 3 \times 3 + 3 \times 1 = 15. \end{aligned}$$

49. a Mathematics teachers = 8, Physics teachers = 6 and Chemistry teachers = 6
 \therefore Required number of ways

$$= {}^8C_3 \times {}^6C_2 \times {}^6C_1 = 56 \times 15 \times 6 = 5040$$
.

50. c Total number of men physics teachers = 4 and total number of women physics teachers = 2
 \therefore Required number of ways

$$= {}^4C_2 \times {}^2C_1 = 6 \times 2 = 12$$
.

Probability and Set Theory

Answers and Explanations

1	c	2	b	3	d	4	c	5	a	6	b	7	a	8	c	9	b	10	b
11	c	12	d	13	b	14	a	15	b	16	d	17	b	18	c	19	c	20	c
21	a	22	b	23	d	24	a	25	a	26	d	27	a	28	b	29	d	30	a
31	a	32	b	33	c	34	d	35	c	36	a	37	b	38	c	39	b	40	c
41	b	42	b	43	c	44	b	45	c	46	d	47	a	48	c	49	d	50	b

1. c Here we can have four cases:
 I. a is even, b is even.
 II. a is odd, b is even.
 III. a is even, b is odd.
 IV. a is odd, b is odd.
 Out of these four cases, in cases (II) and (III) the sum will be odd. So, the required probability = $\frac{2}{4} = \frac{1}{2}$.
2. b Total number of events (sample space) = $6 \times 6 = 36$. Favourable events are (5, 6) and (6, 5).
 So, the required probability = $\frac{2}{36} = \frac{1}{18}$.
3. d Total number of red cards = 26.
 Ways of getting red card from a single draw = ${}^{26}C_1$
 Total ways = ${}^{52}C_1$
 \therefore Required probability = $\frac{{}^{26}C_1}{{}^{52}C_1} = \frac{26}{52} = \frac{1}{2}$.
4. c Total cases = $(6 + 7)$ people, so ${}^{13}C_2$ ways.
 Favourable cases = ${}^6C_1 \times {}^7C_1$
 \therefore Probability = $\frac{{}^6C_1 \times {}^7C_1}{{}^{13}C_2}$.
5. a From the decade 1991-2000, there are three (1992, 1996 and 2000) leap years.
 \therefore Probability = $\frac{3}{10}$.
6. b A leap year contains 366 days and therefore, 52 weeks and 2 days. Clearly, there are 52 Sundays in 52 weeks. The remaining 2 days maybe:
 (i) Sunday and Monday (ii) Monday and Tuesday
 (iii) Tuesday and Wednesday (iv) Wednesday and Thursday
 (v) Thursday and Friday (vi) Friday and Saturday
 (vii) Saturday and Sunday.
 Now, for having 53 Sundays in the year, one of the above 2 days must be Sunday. Thus, out of above 7 possibilities, 2 favour the required event.
 \therefore The required probability = $\frac{2}{7}$.
7. a Multiples of 6 from 1 to 50 = 6, 12, 18, 24, 30, 36, 42, 48
 Multiples of 9 from 1 to 50 = 9, 18, 27, 36, 45
 Either multiples of 6 or 9 = $M(6) + M(9) - M(6 \text{ and } 9)$
 $= 8 + 5 - 2 = 11$
 [Multiples of 6 and 9 = $M(6 \cap 9) = 2$]
 \therefore Required probability = $\frac{11}{50}$.
8. c Favourable ways of selecting 5 non-defective bulbs = ${}^{90}C_5$
 Total number of ways of selection = ${}^{100}C_5$
 $\therefore P(\text{non-defective bulbs}) = \frac{{}^{90}C_5}{{}^{100}C_5}$.
9. b There are 26 red cards.
 There are 2 kings that are black.
 There are 2 kings that are red.
 Probability of getting a red card or a king in a single draw is $P(R \text{ or } K) = P(R) + P(K) - P(R \cap K)$
 $= \frac{{}^{26}C_1}{{}^{52}C_1} + \frac{{}^4C_1}{{}^{52}C_1} - \frac{{}^2C_1}{{}^{52}C_1} = \frac{7}{13}$.

10. b

1st Pick	2nd Pick	3rd Pick
K	Q	J
K	J	Q
Q	K	J
Q	J	K
J	Q	K
J	K	Q

$$\therefore \text{Required probability} = \left(\frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} \right) \times 6 = \frac{16}{5525}.$$

11. c There are 6 favorable cases HHT, HTH, HTT, THT, TTH, THH out of total 8 cases.

$$\therefore \text{Required probability} = \frac{6}{8} = \frac{3}{4}.$$

12. d Total tickets = 100

The number of multiples of 8 or 9 = (8, 9, 16, 18, 24, 27, 32, 36, 40, 45, 48, 54, 56, 63, 64, 72, 80, 81, 88, 90, 96 and 99) = 22

$$\therefore \text{Required probability} = \frac{22}{100} = \frac{11}{50}.$$

13. b When 3 coins are tossed once, then total number of outcomes

$$= (\text{HHH}, \text{HHT}, \text{HTH}, \text{THH}, \text{HTT}, \text{THT}, \text{TTT}) = 8$$

Favourable number of outcomes (getting at least a tail) = (HHT, HTH, THH, HTT, THT, TTH, TTT) = 7

$$\therefore \text{Required probability} = \frac{7}{8}.$$

14. a Total number of outcomes = $6 \times 6 = 36$

The favourable number of outcomes = (2, 6), (3, 5), (4, 4), (5, 3) and (6, 2) = 5

$$\therefore P(\text{a sum of } 8) = \frac{5}{36}.$$

15. b **Case:**

K_1	Q	K_2
K_1	K_2	Q
Q	K_1	K_2

$$\text{1st case} \rightarrow \frac{4}{52} \times \frac{4}{51} \times \frac{3}{50} = \frac{48}{52 \times 51 \times 50}$$

$$\text{2nd case} \rightarrow \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} = \frac{48}{52 \times 51 \times 50}$$

$$\text{3rd case} \rightarrow \frac{4}{52} \times \frac{4}{51} \times \frac{3}{50} = \frac{48}{52 \times 51 \times 50}$$

$$\text{Total cases} = \frac{48 \times 3}{52 \times 51 \times 50}.$$

16. d Total number of balls = 2(white) + 3(green) + 4(red)

= 9

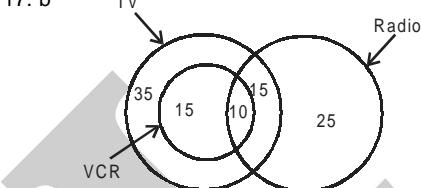
Total number of ways = 9C_5

Number of ways of drawing 2 red balls, 2 green balls and a white ball = ${}^4C_2 \times {}^3C_2 \times {}^2C_1$

Therefore, required probability

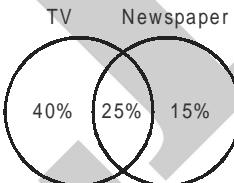
$$= \frac{{}^4C_2 \times {}^3C_2 \times {}^2C_1}{{}^9C_5} = \frac{6 \times 3 \times 2}{126} = \frac{36}{126} = \frac{2}{7}.$$

17. b



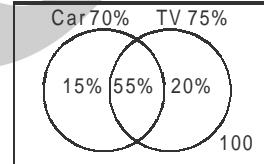
$$\therefore \text{Families having TV only} = 100 - (15 + 10 + 15 + 25) = 35.$$

18. c



$$\text{Percentage of people surveyed neither watch news on TV nor read a newspaper} = 100 - (40 + 25 + 15) = 100 - 80 = 20\%.$$

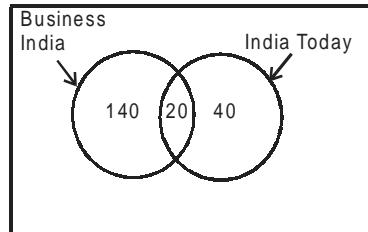
19. c



$$\therefore \text{Percentage of those surveyed did not own either a car or a TV} = 100 - (70 + 75 - 55) = 100 - (145 - 55) = 100 - 90 = 10\%.$$

For questions 20 to 23:

From the given information we can draw the following Venn diagram.

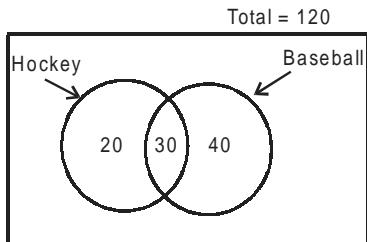


$$\text{Total} = 250$$

20. c 40 residents read only "India Today".
21. a 140 residents read only "Business India".
22. b $(140 + 40) = 180$ residents read exactly one magazine.
23. d $250 - (140 + 20 + 40) = 50$ residents read neither 'India Today' nor 'Business India'.

For questions 24 and 25:

From the given data, we can draw the following Venn-diagram.



24. a 20 students like only Hockey.
25. a 90 students like atleast one game.
26. d Probability of atleast one being alive
= 1 – Probability when all are dead

$$= 1 - \left[\left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \right]$$

$$= 1 - \left[\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \right] = 1 - \frac{1}{4} = \frac{3}{4}.$$

27. a Probability of getting a head = $\frac{1}{2}$
- Probability of even number = $\frac{3}{6} = \frac{1}{2}$
- [Even numbers are 2, 4 and 6].
- \therefore Probability that we will get a head and an even number = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

For questions 28 to 30:

- Total number of balls = 8(white) + 4(red) + 6(blue) = 18
- Total number of ways of drawing 3 balls out of 18 balls = ${}^{18}C_3$
28. b Number of ways of drawing 3 white balls out of 8 white balls = 8C_3
- Therefore, required probability = $\frac{{}^8C_3}{{}^{18}C_3} = \frac{56}{816} = \frac{7}{102}$.
29. d Number of ways of drawing 2 blue and 1 red ball
 $= {}^6C_2 \times {}^4C_1$
- Therefore, required probability
- $$= \frac{{}^6C_2 \times {}^4C_1}{{}^{18}C_3} = \frac{15 \times 4}{816} = \frac{5}{68}.$$

30. a Number of ways of drawing 1 of each colour = ${}^8C_1 \times {}^4C_1 \times {}^6C_1$
- Therefore, required probability
- $$= \frac{{}^8C_1 \times {}^4C_1 \times {}^6C_1}{{}^{18}C_3} = \frac{8 \times 4 \times 6}{816} = \frac{4}{17}.$$

For questions 31 to 34:

- A : Husband selected. B : Wife selected.
A': Husband not selected B': Wife not selected

$$P(A) = \frac{1}{7}, \quad P(B) = \frac{1}{5},$$

$$P(A') = 1 - \frac{1}{7} = \frac{6}{7} \quad P(B') = 1 - \frac{1}{5} = \frac{4}{5}.$$

31. a Only one of them will be selected
= $P(A) \times P(B') + P(B) \times P(A')$
= $\frac{1}{7} \times \frac{4}{5} + \frac{1}{5} \times \frac{6}{7} = \frac{2}{7}$.
32. b Both selected = $P(A) \times P(B) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$.
33. c None of them selected = $P(A') \times P(B')$
= $\frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$.
34. d At least one of them selected = $1 - \frac{24}{35} = \frac{11}{35}$.
35. c Possible outcomes = $6 \times 6 \times 6 = 216$
Outcomes of obtaining a sum of 16 are
(6, 6, 4), (6, 4, 6), (4, 6, 6), (5, 5, 6), (5, 6, 5) and
(6, 5, 5).

$$\therefore P(\text{a sum of } 16) = \frac{6}{216} = \frac{1}{36}.$$

36. a Total number of ways of selecting 2 cards from a complete pack of cards = ${}^{52}C_2$
The number of ways of selecting a king and a queen
 $= {}^4C_1 \times {}^4C_1 = 16$

$$\therefore \text{Required probability} = \frac{16}{{}^{52}C_2}.$$

37. b Total number of five-digit numbers = $5! = 120$. Now to be a multiple of 4, the last 2 digits of the number has to be divisible by 4, i.e. they must be 12, 24, 32, or 52. Corresponding to each of these ways there are 3!, i.e. 6 ways of filling the remaining 3 places.

$$\therefore \text{The required probability} = \frac{4 \times 6}{120} = \frac{1}{5}.$$

For questions 38 and 39:

$$P(A) = \frac{3}{5}, P(A') = \frac{2}{5}, P(B) = \frac{1}{3}, P(B') = \frac{2}{3}$$

$$P(C) = \frac{4}{7}, P(C') = \frac{3}{7}$$

38. c Required probability =
 $P(A) \times P(B') \times P(C') + P(A') \times P(B) \times P(C') + P(A') \times P(B') \times P(C)$
 $= \frac{3}{5} \times \frac{2}{3} \times \frac{3}{7} + \frac{2}{5} \times \frac{1}{3} \times \frac{3}{7} + \frac{2}{5} \times \frac{2}{3} \times \frac{4}{7}$
 $= \frac{6}{35} + \frac{2}{35} + \frac{16}{105} = \frac{40}{105} = \frac{8}{21}$.

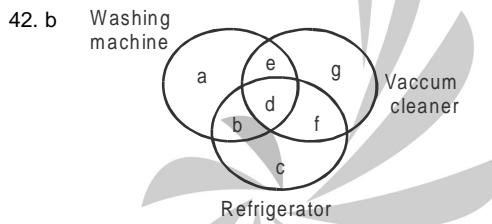
39. b Required probability =
 $P(A) \times P(B) \times P(C') + P(A) \times P(B') \times P(C) + P(A') \times P(B) \times P(C)$
 $= \frac{3}{5} \times \frac{1}{3} \times \frac{3}{7} + \frac{3}{5} \times \frac{2}{3} \times \frac{4}{7} + \frac{2}{5} \times \frac{1}{3} \times \frac{4}{7}$
 $= \frac{3}{35} + \frac{8}{35} + \frac{8}{105} = \frac{41}{105}$.

For questions 40 and 41:

Total number of balls = 8(red) + 6(blue) = 14.

40. c Probability that both balls are red = $\frac{8}{14} \times \frac{8}{14} = \frac{16}{49}$.

41. b Probability that one of them is blue and other is red
 $= \frac{6}{14} \times \frac{8}{14} + \frac{8}{14} \times \frac{6}{14} = \frac{24}{49}$.



$$a + b + c + d + e + f + g = 1000$$

$$a + e + g = 400$$

$$a + b + c = 380$$

$$c + f + g = 542$$

$$d + e = 294$$

$$d + f = 277$$

$$d + b = 120$$

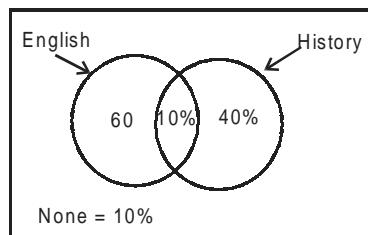
After solving, $g = 62$.

Alternative Method:

$$\begin{aligned} n(W \cup V \cup R) &= n(W) + n(V) + n(R) - n(W \cap V) \\ &\quad - n(W \cap R) - n(V \cap R) + n(W \cap V \cap R) \\ \Rightarrow 1000 &= (1000 - 400) + (1000 - 380) + (1000 - 542) \\ &\quad - 294 - 277 - 120 + n(W \cap V \cap R) \\ \Rightarrow n(W \cap V \cap R) &= 13 \text{ (This value is basically the 'd' of the first method). Now from the figure, all the values can be determined one by one.} \end{aligned}$$

For questions 43 to 46:

From the given information we can draw the following Venn diagram.



$$100\% = 60 + 10\% + 40\% + 10\%$$

$$\Rightarrow 40\% = 60$$

40% of the total number of students = 60

Total students = 150.

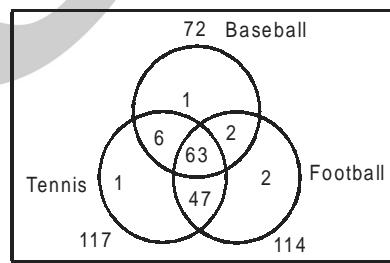
43. c There are 150 students in the class.

44. b 10% of the students do not like any of the two subjects.
 $10\% \text{ of } 150 = 15$.

45. c 40% of the total students like only History.
 $40\% \text{ of } 150 = 60$.

46. d $60 + 40\% \text{ of } 150 = 60 + 60 = 120$
120 students like exactly one subject.

For questions 47 to 50: The given information can be shown as:



47. a 4 players play only one sport.

48. c 118 players play at least two sports.

49. d 59 players play at most two sports.

50. b Ratio is 72 : 63 i.e. 8 : 7.

Algebra : Basics of Linear & Quadratic Equations and Ages

Answers and Explanations

1	b	2	c	3	a	4	b	5	d	6	a	7	d	8	c	9	d	10	c
11	a	12	a	13	c	14	d	15	d	16	a	17	b	18	d	19	a	20	d
21	b	22	a	23	c	24	b	25	c	26	d	27	c	28	c	29	a	30	d
31	b	32	a	33	a	34	a	35	b	36	c	37	b	38	d	39	b	40	b
41	b	42	a	43	d	44	b	45	c	46	c	47	c	48	b	49	c	50	a
51	b	52	d	53	a	54	d	55	c	56	c	57	e	58	e	59	c	60	a

- | | | | |
|------|--|-------|--|
| 1. b | $5x - 3 = 2x + 9$
$\Rightarrow 5x - 2x = 9 + 3 \Rightarrow 3x = 12 \Rightarrow x = 4.$ | 8. c | $x + 12x = 234 \Rightarrow 13x = 234 \Rightarrow x = 18.$ |
| 2. c | $\frac{3}{5}x - \frac{1}{3}x = 44$

$\Rightarrow \frac{9x - 5x}{15} = 44 \Rightarrow \frac{4x}{15} = 44 \Rightarrow x = 165.$ | 9. d | Let daughter's present age be x years. Then,
$36 = 3x \Rightarrow x = 12$ years
Daughter's age after 5 years = $12 + 5 = 17$ years |
| 3. a | Let $p(x) = 2x^2 + kx + 3$, then

$p(-1) = 0 \Rightarrow 2(-1)^2 + k(-1) + 3 = 0 \Rightarrow k = 5.$ | 10. c | $\frac{3}{7}x = 54 \Rightarrow x = 126$

$\therefore \frac{1}{2}$ of 126 = 63. |
| 4. b | Let 'x' be the number. Then,

$\frac{1}{3} \times \frac{1}{4} \times \frac{1}{5}x = 2 \Rightarrow x = 120$

\therefore Three times of the number = $3(120) = 360.$ | 11. a | Let the present age of Anil be x years. Then,
$x + 8 = 3(x - 8)$
$\Rightarrow 2x = 32 \Rightarrow x = 16$ years. |
| 5. d | $\frac{4}{5}x - 36 = 36 \Rightarrow \frac{4}{5}x = 72 \Rightarrow x = 90$

$\therefore \frac{1}{2}$ of 90 = 45. | 12. a | $\because p(-5) = 0$
$\therefore x + 5$ is the factor of $p(x).$ |
| 6. a | $2x + 5y = 9$... (i)
and $2x + 3y = 7$... (ii)
Subtracting (ii) from (i), we get
$2y = 2 \Rightarrow y = 1$
$\therefore x = 2.$ | 13. c | Let B's age be x years. Then, A's age = $x + 7.$
B's age 15 years ago was $x - 15.$
A's age 15 years ago was $x + 7 - 15 = x - 8$

$\therefore x - 15 = \frac{3}{4}(x - 8) \Rightarrow 4x - 60 = 3x - 24$
$\Rightarrow x = 36$
\therefore B's age = 36 years and A's age = 43 years. |
| 7. d | Let $p(x) = 4x^3 - 3x^2 + 2x - 4$
$\therefore p(-4) = 4(-4)^3 - 3(-4)^2 + 2(-4) - 4$
$= -256 - 48 - 8 - 4 = -316.$ | 14. d | Let $f(x) = x^3 + px^2 + 3x - 9$
$\because (x + 3)$ is a factor of $f(x)$, then $f(-3) = 0$
$\Rightarrow (-3)^3 + p(-3)^2 + 3(-3) - 9 = 0$
$\Rightarrow p = 5.$ |

15. d $x - \frac{1}{x} = \frac{3}{2} \Rightarrow \frac{x^2 - 1}{x} = \frac{3}{2} \Rightarrow 2x^2 - 2 = 3x$

$$\Rightarrow (x-2)(2x+1) = 0 \Rightarrow x = 2 \text{ or } x = -\frac{1}{2}.$$

16. a $\frac{40}{100}x + 120 = x \Rightarrow 120 = \frac{60x}{100} \Rightarrow x = 200.$

17. b Let $p(x) = x^3 - (k^2 - 1)x + 2$

$$\therefore p(k) = 0 \Rightarrow k^3 - (k^2 - 1)k + 2 = 0$$

$$\Rightarrow k^3 - k^3 + k + 2 = 0 \Rightarrow k = -2.$$

18. d After t years, let boy's age = $3a$

$$\text{Then, } a + t = 3a \Rightarrow t = 2a$$

$$\text{Father's age after } t \text{ years} = 5a + t = 5a + 2a = 7a \text{ years}$$

$$\text{Father's age when the son was born} = 5a - a = 4a \text{ years.}$$

19. a Let the cost of a book and a pencil be B and P respectively.

$$\text{Then, } 6B + 4P = 34 \text{ and } 5B + 5P = 30$$

Solving them, we get $B = ₹5$ and $P = ₹1$.

20. d $3x + 12 = 4x - 7 \Rightarrow x = 19.$

21. b Let the number of people be x . Then, each person gets $\frac{180}{x}$.

$$\frac{180}{x} = \frac{180}{x-40} - 6 \Rightarrow x^2 - 40x - 1200 = 0$$

$$\Rightarrow x = 60$$

$$\therefore \text{Each person gets } \frac{180}{60} = ₹3.$$

22. a Let the number be x .

$$\text{Then, } \frac{x}{7} - \frac{x}{11} = 100$$

$$\Rightarrow \frac{11x - 7x}{77} = 100$$

$$\Rightarrow x = \frac{100 \times 77}{4} = 1925.$$

23. c Let the fraction be $\frac{x}{y}$.

$$\therefore y = x + 4 \quad \dots \text{(i)}$$

and $y - 1 = 3x - 3 \quad \dots \text{(ii)}$

Solving equation (i) and (ii), we get

$$x = 3 \text{ and } y = 7$$

$$\Rightarrow \text{The fraction is } \frac{3}{7}.$$

24. b Let the number of one rupee coins be ' x ' and number of five rupee coins be ' y '

$$\therefore x + y = 26 \quad \dots \text{(i)}$$

$$\text{and } x + 5y = 50 \quad \dots \text{(ii)}$$

Solving (i) and (ii), we get

$$y = 6$$

Hence, number of five rupee coins is 6.

25. c Let the digit at unit's place be ' x ' and digit at hundred's place be ' y '.

Then, the number is, $y \times 100 + 5 \times 10 + x \times 1$

$$\therefore \text{Sum of digits } y + 5 + x = 14 \Rightarrow x + y = 9 \quad \dots \text{(i)}$$

$$\text{and } (100y + 10 \times 5 + x) - (100x + 10 \times 5 + y) = 297 \quad \dots \text{(ii)}$$

Solving (i) and (ii), we get

$$y = 6 \text{ and } x = 3.$$

Hence, the number is, $6 \times 100 + 5 \times 10 + 3 \times 1 = 653$.

26. d Let x be the number of 25-paisa coins and y be the number of 10-paisa coins. Then,

$$0.25x + 0.10y = 8.25 \text{ and } x = \frac{1}{3}y$$

Solving, we get $x = 15$ and $y = 45$

\therefore Total number of coins = $15 + 45 = 60$.

27. c Let the present ages of Father and Son be x and y respectively. Then,

$$x + y = 45 \Rightarrow y = 45 - x$$

Five years ago, age of father = $(x - 5)$

Five years ago, age of son = $(45 - x) - 5 = (40 - x)$

$$\therefore (x - 5)(40 - x) = 4(x - 5)$$

$$\Rightarrow 40 - x = 4$$

$$\Rightarrow x = 36 \text{ years.}$$

28. c Let the total number of students be xy .

$$\therefore (x + 2)(y - 4) = (x - 3)(y + 12) = xy$$

Using first and 3rd equations, we get

$$2x + 4 = y \quad \dots \text{(i)}$$

Using last two equations, we get

$$4x - y = 12 \quad \dots \text{(ii)}$$

On solving equations (i) and (ii), we get

$$x = 8 \text{ and } y = 20$$

\therefore Total number of students = $xy = 20 \times 8 = 160$.

29. a If larger and smaller integers be x and y respectively.

$$\therefore x - y = 5 \text{ and } x^2 - y^2 = 65 \quad \dots \text{(i)}$$

$$\text{or, } (x - y)(x + y) = 65$$

$$\Rightarrow x + y = 13 \quad \dots \text{(ii)}$$

Solving the two equations, we get $x = 9$ and $y = 4$

\therefore Larger integer = 9.

30. d Let $f(x) = x^2 + 3qx - 2q$
 $\therefore f(2) = 0 \Rightarrow 4 + 6q - 2q = 0$
 $\Rightarrow q = -1.$
31. b $x^2 - 3x - 10 = x^2 - 5x + 2x - 10 = (x - 5)(x + 2)$
 $\therefore k = -2.$
32. a Use remainder theorem
put $x = 1$ in $x^3 - 6x + 7$.
The remainder is 2.
33. a Let the number of two legged, three legged and four legged chairs be x, y and z respectively.
 $\therefore x + y + z = 27 \dots (i)$
and $2x + 3y + 4z = 78 \dots (ii)$
On solving equations (ii) and (i), we get
 $y + 2z = 24$
 $\therefore y : z = 2 : 3$
 $\therefore y + \frac{3y}{2} \times 2 = 24 \Rightarrow y = 6$ and $z = 9$
 \therefore Number of two legged chairs = $27 - (6 + 9) = 12.$
34. a For a given system of equations having infinite solutions
 $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
 $\Rightarrow \frac{7}{5} = \frac{8}{p} = \frac{35}{25}$
 $\Rightarrow p = \frac{40}{7}.$
35. b Let father's present age be 'x' years and son's present age be 'y' years.
 $\therefore x - 10 = 3(y - 10) \dots (i)$
and $x + 10 = 2(y + 10) \dots (ii)$
Solving (i) and (ii), we get
 $x = 70$ and $y = 30$
Hence, $x : y = 7 : 3.$
36. c Let's Rajan's present age be 'x' years.
Then, $x = \frac{6}{5}(x - 8) \Rightarrow x = 48$
Now Rajan's sister is 10 years younger to him.
 \Rightarrow Rajan's sisters age is 38 years.
37. b Let the digit at unit's place be x and digit at ten's place be y .
 $\therefore x + y = 10 \dots (i)$
 $(10y + x) - 10 = 2(10x + y) + 16 \dots (ii)$
Solving equation (i) and (ii), we get
 $x = 2$ and $y = 8$
Hence, the number is $8 \times 10 + 2 = 82.$
38. d Let the present ages of A and B be x and y years respectively.
 $\therefore \frac{x - 7}{y - 7} = \frac{3}{4}$ and $\frac{x + 9}{y + 9} = \frac{7}{8}$
Solving them, we get $x = 19$ and $y = 23.$
 \therefore B's age at present = 23 years.
39. b Let the present age of father and son be x and y years respectively.
 $\therefore x = 4y \dots (i)$
Also, $x + 20 = 2(y + 20) \dots (ii)$
Solving equations (i) and (ii), we get
 $x = 40$ and $y = 10.$
 \therefore The sum of present ages of father and son
 $= x + y = 50$ years.
40. b Let the present ages of Ajay and Vijay be $4x$ and $3x$ years respectively.
Then, $4x + 6 = 26 \Rightarrow 4x = 20 \Rightarrow x = 5.$
 \therefore Vijay's age = $3x = 15$ years.
41. b Let the age of the younger person be x years..
Then, elder person's age is $(x + 16).$
6 years before, $3(x - 6) = (x + 16 - 6)$
 $\Rightarrow 3x - 18 = x + 10$
 $\Rightarrow x = 14$ years
So, other person's age = $x + 16 = 30$ years.
42. a Let the number be $xy.$
Then, $x + 6 = y \dots (i)$
and $4(10x + y) + 3 = (10y + x)$
 $\Rightarrow 39x - 6y + 3 = 0$
 $\Rightarrow 13x - 2y + 1 = 0 \dots (ii)$
From (i) and (ii),
 $11x = 11 \Rightarrow x = 1$ and $y = 7$
 \therefore Number = 17.
43. d Let the boy have x ₹1 coins then, ₹2 coins will be $30 - x.$
 $\therefore x \times 1 + (30 - x) \times 2 = 48$
 $\Rightarrow x = 12.$
 \therefore Number of coins of ₹1 is 12.
44. b The remainder when $f(x)$ is divided by $(x + 2)$ is $f(-2)$
 $\therefore f(-2) = (-2)^3 - 5(-2)^2 + 3(-2) - 11$
 $= -8 - 20 - 6 - 11 = -45.$
 \therefore The remainder when $x^3 - 5x^2 + 3x - 11$ is divided by $(x + 2)$ is -45.
45. c Let $f(x) = 2x^2 + kx - 12$
 $\therefore f(x)$ is when divided by $(x + 3)$ gives remainder -3
 $\therefore f(-3) = -3 \Rightarrow 2(-3)^2 + k(-3) - 12 = -3$
 $\Rightarrow 18 - 3k - 12 = -3 \Rightarrow 3k = 9$
 $\Rightarrow k = 3.$

46. c Let $f(x) = x^3 + 5x^2 + px + q$
 $\therefore x^2 + 2x - 3 = (x + 3)(x - 1)$
 $\therefore x^2 + 2x - 3$ is a factor of $x^3 + 5x^2 + px + q$
 $\therefore f(-3) = 0$ and $f(1) = 0$
 $\therefore f(-3) : (-3)^3 + 5(-3)^2 + p(-3) + q = 0$
 $\Rightarrow 3p - q = 18 \quad \dots(i)$
and $f(1) : (1)^3 + 5(1)^2 + p(1) + q = 0$
 $\Rightarrow p + q = -6 \quad \dots(ii)$
From (i) & (ii), $p = 3, q = -9$
 $\therefore p + q = -6.$

47. c The given polynomials are
 $16x^3y^2(x^2 - y^2)$ and $4x^2y^3(x + y)$
HCF = $4x^2y^2(x + y)$
LCM = $16x^3y^3(x^2 - y^2).$

48. b For infinite solutions.
 $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{m}{2} = \frac{4}{2} = \frac{8}{8-m}$
 $\Rightarrow m = 4.$

49. c For no solution
 $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
 $\frac{b}{3} = \frac{4}{b} \neq \frac{7}{8} \Rightarrow b^2 = 12$
 $\Rightarrow b = \pm 2\sqrt{3}.$

50. a For unique solution
 $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{2}{5} \neq \frac{3}{k}$
 $\Rightarrow k \neq \frac{15}{2}.$

51. b $2x^2 + x - 6 = 0$
 $\Rightarrow (x+2)(2x-3) = 0 \Rightarrow x = -2$ or $x = \frac{3}{2}.$

52. d $x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$
i.e. $x^2 - (2-3)x + 2(-3) = 0$
 $\Rightarrow x^2 + x - 6 = 0.$

53. a Let number be x and y , then
 $x + y = 9$ and $xy = 20$
 $\therefore (x+y)^2 = x^2 + y^2 + 2xy$
 $\Rightarrow 9^2 = x^2 + y^2 + 2 \times 20 \Rightarrow x^2 + y^2 = 81 - 40 = 41.$

54. d Since -3 is a root of $2x^2 - kx - 3 = 0$.
 $\therefore 2(-3)^2 - k(-3) - 3 = 0$
 $\Rightarrow 18 + 3k - 3 = 0$
 $\Rightarrow k = -5.$

55. c Let $x = \frac{6}{1 + \frac{6}{1 + \frac{6}{1 \dots}}}$
 $\Rightarrow x = \frac{6}{1+x} \Rightarrow x^2 + x = 6$
 $\Rightarrow x^2 + x - 6 = 0 \Rightarrow (x+3)(x-2) = 0$
 $\Rightarrow x = 2 \text{ or } -3$
 $\therefore x \neq -3 \text{ because } x \text{ is positive.}$
 $\therefore x = 2.$

56. c If $(2 + \sqrt{3})$ is one root, then
 $(2 - \sqrt{3})$ will be the other root (Irrational roots are in pairs).
The equation would be
 $\therefore x^2 - (2 + \sqrt{3} + 2 - \sqrt{3})x + (2 + \sqrt{3})(2 - \sqrt{3}) = 0$
 $\Rightarrow x^2 - 4x + 1 = 0.$

57. e $\alpha + \beta = -\frac{b}{a} = -\frac{34}{17} = -2, \alpha \times \beta = \frac{c}{a} = \frac{68}{17} = 4.$
 $\therefore 2 \times (\alpha + \beta) = -4$ and $2\alpha \times 2\beta = 16$
Hence, new equation will have $-\frac{b}{a} = -4$ and $\frac{c}{a} = 16.$
Both (a) and (c) satisfy the conditions.

58. e α, β, γ are the roots of $x^3 + ax^2 + bx + c = 0.$
 $\therefore \alpha + \beta + \gamma = -a, \alpha \times \beta \times \gamma = -c$
and α, β are the roots of the equation $x^2 + ax + b = 0.$
 $\therefore \alpha + \beta = -a$
So, $\gamma = 0 \Rightarrow c = 0.$

59. c Quadratic equation $x^2 - px + 4 = 0$
 \therefore Roots are imaginary,
Discriminant is less than 0.
 $\therefore (-p)^2 - 4 \times 4 < 0$
 $\Rightarrow p^2 - 16 < 0$
 $\Rightarrow (p-4)(p+4) < 0$
 $\Rightarrow -4 < p < 4.$

60. a $\sqrt{1 + \frac{x}{961}} = \frac{32}{31}$
Squaring both sides, we get
 $1 + \frac{x}{961} = \left(\frac{32}{31}\right)^2$
 $\Rightarrow \frac{961+x}{961} = \frac{1024}{961}$
 $\Rightarrow 961+x = 1024$
 $\Rightarrow x = 1024 - 961 \Rightarrow x = 63.$

Mensuration - 2D (Plane Figures)

Answers and Explanations

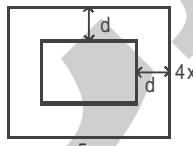
1	d	2	c	3	c	4	a	5	a	6	d	7	d	8	d	9	c	10	b
11	d	12	d	13	b	14	b	15	b	16	c	17	b	18	b	19	c	20	a
21	b	22	b	23	b	24	b	25	a	26	c	27	c	28	d	29	c	30	a
31	a	32	b	33	b	34	b	35	b	36	c	37	c	38	c	39	d	40	c
41	b	42	a	43	a	44	d	45	c	46	c	47	a	48	c	49	c	50	a

1. d $2(l + b) = 30$
 $\Rightarrow 2(4b + b) = 30 \Rightarrow 10b = 30 \Rightarrow b = 3$ m and
 $l = 12$ m.
So, area of the field = $l \times b = 3 \times 12 = 36$ m².
2. c To calculate the area, first of all calculate the length of the side of the square. First of all put 4 poles on 4 corners and now we are left with 16 poles. Hence, 4 poles each between 2 corner poles. Hence, each side of the square contains 6 poles. So, distance between 2 extreme poles on one side of the square will be $5 \times 5 = 25$ m. So, area = $(25)^2 = 625$ m².
3. c Perimeter of the rectangle = Circumference of the circular wire = $2 \times \frac{22}{7} \times 42$ cm = 264 cm.
Let the dimensions of the rectangle be $6x$ and $5x$ respectively.
 $\therefore 2(6x + 5x) = 264 \Rightarrow x = 12$
 \therefore Smaller side = $5x = 60$ cm.
4. a Radius of the circle = 14 cm
Angle of the sector = 36°
 \therefore Area of the sector = $\frac{\theta}{360^\circ} \times \pi r^2$
 \therefore Area of the sector = $\frac{36}{360^\circ} \times \frac{22}{7} \times 14 \times 14 = 61.6$ cm².
5. a Area per rotation = $(2\pi R) \times L = 1.32$ m²
 \Rightarrow Total area = $1.32 \times 400 = 528$ m²
 \Rightarrow Total cost = ₹5,280.
6. d $\angle ADC$ is a right angle (Angle in a semicircle)
So, $BD^2 = AB \times BC \Rightarrow 16 \times 3 = 4 \times BC$
 $\Rightarrow BC = 12$ cm

Now shaded area = Area of bigger semicircle – Areas of smaller semicircles

$$= \frac{1}{2} \pi (8)^2 - \frac{1}{2} \pi (2)^2 - \frac{1}{2} \pi (6)^2 \\ = 32\pi - 2\pi - 18\pi = 12\pi \text{ cm}^2.$$

7. d



Let $5x$ and $4x$ be the length and breadth of the garden. Then, $5x \times 4x = 2000 \Rightarrow x^2 = 100 \Rightarrow x = 10$
 \therefore Length = 50 m and breadth = 40 m
Let 'd' be the width of the road. Then,
 $(50 - 2d)(40 - 2d) = 2000 - 344$
 $\Rightarrow d = 2$ m.

8. d

Let side of rectangle be $2x$ and x units.
and side of square = y units

$$\therefore 4y = 6x \Rightarrow \frac{x}{y} = \frac{4}{6} = \frac{2}{3}$$

$$\therefore \frac{2x \times x}{y^2} = \frac{2x^2}{y^2} = \frac{2 \times 4}{9} = 8 : 9.$$

9. c

Area of square = $a^2 = 81$ cm²
 $\Rightarrow a = 9$ cm

Perimeter of square = $4 \times a = 4 \times 9 = 36$ cm

Perimeter of semi circle = $\pi r + 2r$

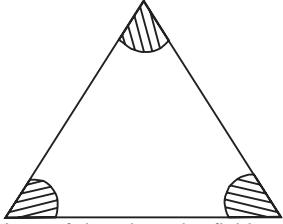
$$\Rightarrow 36 = r(\pi + 2)$$

$$\Rightarrow 36 = r \left(\frac{36}{7} \right) \Rightarrow r = 36 \times \frac{7}{36} = 7$$

$$\text{Area of semicircle} = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2.$$

10. b



Area of the triangular field

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$\left[\because s = \frac{26+28+30}{2} = 42 \text{ cm} \right]$$

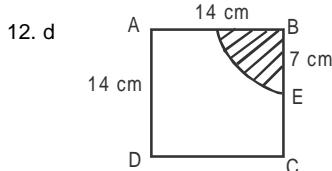
$$= \sqrt{42(42-26)(42-28)(42-30)} = 336$$

$$\text{Area grazed by cows} = \frac{\pi r^2}{2} = \frac{22}{7} \times \frac{7 \times 7}{2} = 77 \text{ m}^2$$

$$\text{Hence, ungrazed area} = 336 - 77 = 259 \text{ m}^2.$$

11. d Number of rotations = $\frac{\text{Total distance}}{\text{Circumference of the wheel}}$

$$= \frac{88 \times 100000}{2 \times \frac{22}{7} \times 56} = 25000.$$



$$\text{Area of the square of side } 14 \text{ cm} = (14)^2 = 196 \text{ cm}^2$$

Area of the field grazed by the cow

$$= \text{Area of shaded region} = \pi r^2 \times \left(\frac{90^\circ}{360^\circ} \right)$$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{1}{4} = 38.5 \text{ cm}^2$$

Percentage of the area grazed by the cow

$$= \frac{38.5}{196} \times 100 = 19.6\%.$$

13. b Perimeter of rectangular wire = $2(10 + 4) = 28 \text{ cm}$ Perimeter of circular wire = $2\pi r = 28$

$$\Rightarrow 2r = \frac{28}{22} \times 7 \approx 9 \text{ cm.}$$

14. b Let the length of the rope be r .Horse can graze an area equal to area of the circle of radius r .

$$\text{Then, } \pi r^2 = 154 \Rightarrow r = 7 \text{ m.}$$

15. b Radius of the outer circle = $28 + 14 = 42 \text{ m}$

Area of the path

$$= \text{Area of outer circle} - \text{Area of inner circle} \\ = \pi (42)^2 - \pi (28)^2 = 3080 \text{ m}^2.$$

16. c Let r be the radius of the circular plot. Then,
 $\pi r^2 = 154 \Rightarrow r = 7 \text{ m}$

$$\text{Circumference of the plot} = 2 \times \frac{22}{7} \times 7 = 44 \text{ m}$$

Cost of fencing the plot = ₹44 × 2.75 = ₹121.

17. b Side of the square = $\frac{32}{4} = 8 \text{ m.}$

This implies that each side has 9 poles.

$$\text{Total poles} = 9 + 8 + 8 + 7 = 32.$$

18. b Side of the square = $\sqrt{484} = 22 \text{ cm}$
Perimeter of the square = $4 \times 22 = 88 \text{ cm}$
 \therefore Perimeter of the circle = $2\pi \times \text{radius} = 88 \text{ cm}$
 \Rightarrow Radius = 14 cm
Area of the circle = $\pi \times (14)^2$

$$= \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2.$$

19. c Area of equilateral triangle = $\frac{\sqrt{3}}{4} a^2$ [where a is side of triangle]

$$\therefore \frac{\sqrt{3}}{4} a^2 = 121\sqrt{3}$$

$$\Rightarrow a = 22 \text{ cm.}$$

$$\text{Perimeter of triangle} = 22 \times 3 = 66 \text{ cm}$$

$$\text{Perimeter of circle} = 2\pi r$$

$$\text{i.e. } 2\pi r = 66$$

$$\Rightarrow r = 66 \times \frac{7}{22 \times 2} = \frac{21}{2} \text{ cm}$$

Now area of triangle = πr^2 .

$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 346.5 \text{ cm}^2.$$

20. a Area of the rectangular garden = $12 \times 5 = 60 \text{ m}^2$

= Area of the square garden

$$\therefore \text{Side of the square garden} = \sqrt{60} \text{ m}$$

$$\therefore \text{Diagonal of the square garden} = \sqrt{2} \times \text{side}$$

$$= \sqrt{2} \times \sqrt{60} = \sqrt{120} = \sqrt{4 \times 30} = 2\sqrt{30} \text{ m.}$$

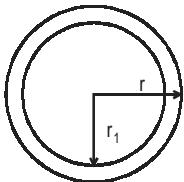
21. b Let d_1 , d_2 be the diagonals of a rhombus.

$$\text{Area} = \frac{1}{2} d_1 \cdot d_2$$

$$\Rightarrow 150 = \frac{1}{2} \times 10 \times d_2$$

$$\Rightarrow d_2 = \frac{150}{5} = 30 \text{ cm.}$$

22. b



Circumference of outer circle = $2\pi r = 132 \text{ cm}$

$$\Rightarrow r = \frac{132}{2 \times 22} \times 7 = 21 \text{ cm}$$

Circumference of inner circle = $2\pi r_1 = 88 \text{ cm}$

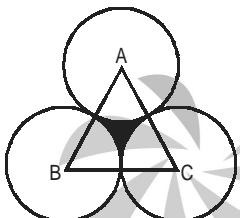
$$\Rightarrow r_1 = \frac{88}{2 \times 22} \times 7 = 14 \text{ cm}$$

$$\therefore \text{Area of outer circle} = \pi r^2 = \frac{22}{7} \times 21 \times 21 = 1386 \text{ cm}^2$$

$$\& \text{area of inner circle} = \pi r_1^2 = \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2$$

Hence, area of ring = $1386 - 616 = 770 \text{ cm}^2$

23. b



Radius of each circle = 3.5 cm

From the figure,

$\triangle ABC$ will be an equilateral triangle of side 7 cm each.

Now, the required area

= Area of $\triangle ABC$ - $3x(\text{Area of a sector of angle } 60^\circ \text{ in a circle of radius } 3.5 \text{ cm})$

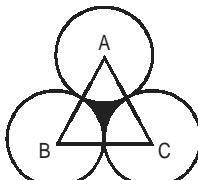
$$= \frac{\sqrt{3}}{4} \times (7)^2 - 3 \left[\frac{60}{360} \times \frac{22}{7} \times (3.5)^2 \right]$$

$$= \frac{49\sqrt{3}}{4} - 19.25$$

$$= 21.217 - 19.25$$

$$= 1.967 \text{ cm}^2$$

24. b



Obviously, the triangle ABC will be equilateral.
 $AB = BC = CA = 2 \text{ cm.}$

Area of $\triangle ABC$

$$= \frac{\sqrt{3}}{4} \times 2 \times 2 = \sqrt{3} \text{ cm}^2$$

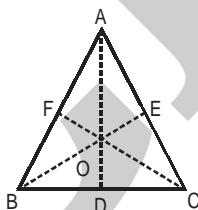
Then, area 'A' of the three sectors each of angle 60° in a circle of radius 1 cm.

$$A = 3 \times \frac{60}{360} \times \pi \times 1 = \frac{\pi}{2}$$

\therefore Area of the shaded portion

$$= \left(\sqrt{3} - \frac{\pi}{2} \right) \text{ cm}^2.$$

25. a



Let the side of the equilateral triangle be $x \text{ cm.}$

$\therefore \Delta AOB + \Delta BOC + \Delta COA = \Delta ABC$

$$\Rightarrow \frac{1}{2} x \times 3 + \frac{1}{2} \times x \times 4 + \frac{1}{2} \times x \times 5 = \frac{\sqrt{3}}{4} x^2$$

$$\Rightarrow 6 = \frac{\sqrt{3}}{4} x \Rightarrow x = \frac{24}{\sqrt{3}} = 8\sqrt{3}$$

$$\therefore \text{Area of } \triangle ABC = \frac{\sqrt{3}}{4} \times \text{side}^2$$

$$= \frac{\sqrt{3}}{4} \times 8\sqrt{3} \times 8\sqrt{3} = 48\sqrt{3} \text{ cm}^2.$$

26. c

Area of original rectangle = xy

Area of new rectangle

$$= 1.25x \times 0.80y = xy = \text{original area}$$

$$\therefore \text{Effective change} = \left(25 - 20 - \frac{25 \times 20}{100} \right)\% = 0\%$$

Hence, the area of the rectangle remains unchanged.

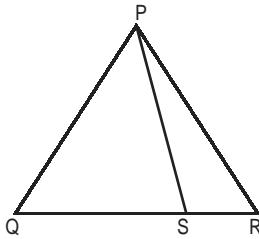
27. c

$$\text{Required net effect} = \left(x + y + \frac{xy}{100} \right)\%$$

Negative sign shows decrease

$$= \left(5 - 2 - \frac{5 \times 2}{100} \right)\% = 2.9\%.$$

28. d



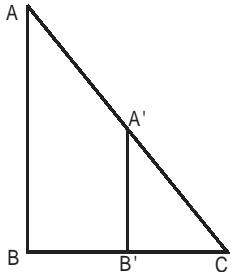
$$QR = 4.5 \text{ cm}$$

$$SR = 1.5 \text{ cm}$$

$$\therefore QS = 4.5 - 1.5 = 3 \text{ cm}$$

$$\frac{\Delta PQS}{\Delta PSR} = \frac{\frac{1}{2} \times h \times QS}{\frac{1}{2} \times h \times SR} = \frac{3}{1.5} = 2 : 1.$$

29. c



In $\triangle ABC$ and $\triangle A'B'C$,
 $A'B' \parallel AB$

$$\angle B' = \angle B, \angle A' = \angle A$$

$$\therefore \triangle ABC \sim \triangle A'B'C$$

By Midpoint theorem

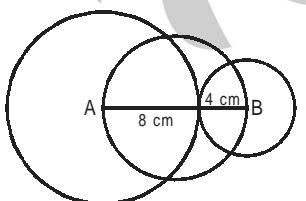
$$A'B' = \frac{1}{2} AB \text{ and } B'C = \frac{1}{2} BC$$

$$\therefore \text{Area of } \triangle A'B'C = \frac{1}{2} \times B'C \times A'B'$$

$$= \frac{1}{2} \times \frac{1}{2} BC \times \frac{1}{2} AB = \frac{1}{4} \left(\frac{1}{2} \times BC \times AB \right)$$

$$= \frac{1}{4} \times \text{Area of } \triangle ABC.$$

30. a



$$\text{Diameter} = AB = 8 + 4 = 12 \text{ units}$$

$$\text{Radius} = \frac{12}{2} = 6 \text{ units}$$

$$\therefore \text{Area of circle} = \pi r^2 = \pi \times 6^2 = 36\pi \text{ sq. units.}$$

31. a Area of triangle = $\frac{1}{2} \times \text{side} \times \text{height}$
Given that value of area = height

$$\therefore \frac{1}{2} \times \text{side} \times h = h$$

$$\Rightarrow \text{side} = 2 \text{ units.}$$

32. b Radius of circle = r units

According to question,

Area of circle = Circumference of circle

$$\Rightarrow \pi r^2 = 2\pi r$$

$$\Rightarrow r = 2 \text{ units}$$

$$\therefore \text{Area of circle} = \pi r^2 = 4\pi \text{ sq. units.}$$

33. b Ratio of the lengths of sides = 5 : 6 : 7

$$\text{Sum of ratios} = 5 + 6 + 7 = 18$$

$$\therefore \text{Sides} \Rightarrow \frac{5}{18} \times 54 = 15 \text{ metre;}$$

$$\frac{6}{18} \times 54 = 18 \text{ metre; } \frac{7}{18} \times 54 = 21 \text{ metre;}$$

$$\text{Semi-perimeter, } s = \frac{15+18+21}{2} = \frac{54}{2} = 27 \text{ m}$$

$$\therefore \text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{27(27-15)(27-18)(27-21)}$$

$$= \sqrt{27 \times 12 \times 9 \times 6}$$

$$= \sqrt{3 \times 3 \times 3 \times 2 \times 2 \times 3 \times 3 \times 3 \times 2 \times 3}$$

$$= 3 \times 3 \times 3 \times 2\sqrt{6} = 54\sqrt{6} \text{ sq. metre.}$$

34. b Side of square, with perimeter 24 cm = $\frac{24}{4} = 6 \text{ cm}$
So, area of the square = $6^2 = 36 \text{ cm}^2$

Side of square, with perimeter 32 cm = $\frac{32}{4} = 8 \text{ cm}$

$$\text{So, area of this square} = 8^2 = 64 \text{ cm}^2$$

$$\text{Area of new square} = 64 + 36 = 100 \text{ cm}^2$$

$$\therefore \text{Side of the new square} = \sqrt{100} = 10 \text{ cm}$$

Hence, perimeter of new square = $10 \times 4 = 40 \text{ cm.}$

35. b Let the length = l m and breadth = b m.

$$\therefore 2(l+b) = 28$$

$$\Rightarrow l+b = 14 \quad \dots \text{(i)}$$

$$lb = 48 \quad \dots \text{(ii)}$$

$$\text{Now, } (l-b)^2 = (l+b)^2 - 4lb$$

$$= (14)^2 - 4 \times 48 \quad [\text{From (i) \& (ii)}]$$

$$= 196 - 192 = 4$$

$$\Rightarrow l-b = 2 \quad \dots \text{(iii)}$$

$$\therefore l = 8, b = 6$$

$$\therefore \text{Diagonal} = \sqrt{8^2 + 6^2} = 10 \text{ m.}$$

36. c Ratio of sides of triangle

$$= \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$$

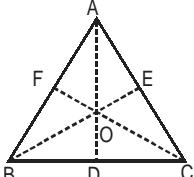
Now, $6x + 4x + 3x = 52$

$$\Rightarrow 13x = 52$$

$$\Rightarrow x = 4 \text{ cm}$$

\therefore Length of smallest side = $3x = 4 \times 3 = 12 \text{ cm.}$

37. c



Let ABC be an equilateral triangle of side x cm.

Also, Let $OD = \sqrt{3} \text{ cm}$,

$OE = 2\sqrt{3} \text{ cm}$ and $OF = 5\sqrt{3} \text{ cm.}$

From the figure,

area. $\triangle BOC$ + area. $\triangle AOC$ + area. $\triangle AOB$ = area. $\triangle ABC$
or,

$$\Rightarrow \frac{1}{2} \times x \times \sqrt{3} + \frac{1}{2} \times x \times 2\sqrt{3} + \frac{1}{2} \times x \times 5\sqrt{3} = \frac{\sqrt{3}}{4} x^2$$

or,

$$\Rightarrow x(2\sqrt{3} + 4\sqrt{3} + 10\sqrt{3}) = \sqrt{3}x^2$$

$$\Rightarrow x = 2 + 4 + 10 = 16$$

\therefore Perimeter of the triangle = $3x = 3 \times 16 = 48 \text{ cm.}$

38. c $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$

$$= \frac{1}{3} \times 60 : \frac{1}{4} \times 60 : \frac{1}{5} \times 60$$

$$= 20 : 15 : 12$$

$$\therefore 20x + 15x + 12x = 94$$

$$\Rightarrow 47x = 94 \Rightarrow x = \frac{94}{47} = 2$$

\therefore The smallest side = $12x$
 $= 12 \times 2 = 24 \text{ cm.}$

39. d Ratio of the sides of triangle

$$= \frac{1}{4} : \frac{1}{6} : \frac{1}{8}$$

$$= \frac{1}{4} \times 24 : \frac{1}{6} \times 24 : \frac{1}{8} \times 24$$

[LCM of 4, 6, 8 = 24]

$$= 6 : 4 : 3$$

$$\therefore 6x + 4x + 3x = 91$$

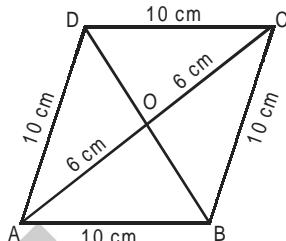
$$\Rightarrow 13x = 91$$

$$\Rightarrow x = \frac{91}{13} = 7$$

\therefore Required difference = $6x - 3x = 3x = 3 \times 7 = 21 \text{ cm.}$

40. c $4 \times \text{side} = 40 \text{ cm}$

$$\Rightarrow \text{Side} = \frac{40}{4} = 10 \text{ cm.}$$



Diagonals of a rhombus bisect each other at right angles.

$$\therefore OB = \sqrt{(10)^2 - (6)^2} = 8 \text{ cm}$$

$$\therefore \text{Diagonal } BD = 8 \times 2 = 16 \text{ cm.}$$

41. b Diameter of the wheel = 3 metres

\therefore Circumference = $\pi \times \text{diameter}$

$$= \frac{22}{7} \times 3 = \frac{66}{7} \text{ metres}$$

Since a wheel covers a distance equal to its circumference in one revolution, therefore, distance

$$\text{covered in 28 revolutions} = 28 \times \frac{66}{7} = 264 \text{ metres}$$

Now, 264 metres distance is covered in 1 minute.

$$\therefore 5280 \text{ metres distance will be covered in} = \frac{5280}{264} = 20 \text{ minutes.}$$

42. a Distance covered in 1 revolution

= Circumference of wheel

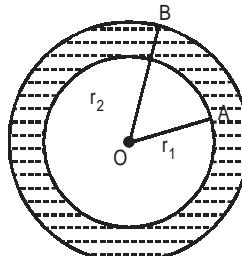
$$= 2\pi r = 2 \times \frac{22}{7} \times 20 \text{ cm}$$

Total distance = 176 m = 17600 cm

$$\therefore \text{Number of revolutions} = \frac{17600}{2 \times \frac{22}{7} \times 20}$$

$$= \frac{17600 \times 7}{2 \times 22 \times 20} = 140 .$$

43. a



Breadth of road = $r_2 - r_1$

$$\begin{aligned}\Rightarrow 2\pi r_2 - 2\pi r_1 &= 66 \\ \Rightarrow 2\pi(r_2 - r_1) &= 66 \\ \Rightarrow r_2 - r_1 = \frac{66}{2\pi} &= \frac{66 \times 7}{2 \times 22} = 10.5 \text{ metres.}\end{aligned}$$

44. d Let the radius of circular field be r metre. Then,

$$\begin{aligned}\frac{2\pi r}{30} - \frac{2r}{30} &= \frac{30}{60} \Rightarrow \frac{\pi r}{15} - \frac{r}{15} = \frac{1}{2} \\ \Rightarrow \pi r - r &= \frac{15}{2} \Rightarrow r(\pi - 1) = \frac{15}{2} \\ \Rightarrow r\left(\frac{22}{7} - 1\right) &= \frac{15}{2} \Rightarrow r \times \frac{15}{7} = \frac{15}{2} \\ \Rightarrow r &= \frac{7}{2} = 3.5 \text{ metre.}\end{aligned}$$

45. c Let the internal radius of the park be r and the external radius (with the path) be R .

The difference between the internal and external circumferences is 132 m.

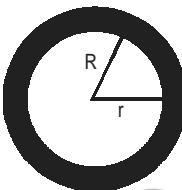
$$\text{i.e. } 2\pi R - 2\pi r = 132$$

$$\Rightarrow 2\pi(R - r) = 132$$

$$\Rightarrow R - r = \frac{132}{2\pi} = \frac{132 \times 7}{2 \times 22} = 21$$

Hence, the width of path = 21 metres.

46. c



Let the shaded portion be the circular path.
Let the inner radius be r metres.

$$\therefore \text{Outer radius } R = (r + 5) \text{ metres.}$$

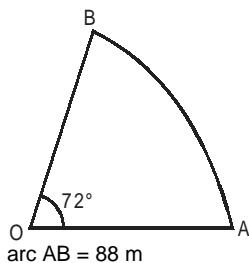
$$\therefore \frac{2\pi R}{2\pi r} = \frac{23}{22} \Rightarrow \frac{R}{r} = \frac{23}{22}$$

$$\Rightarrow \frac{r+5}{r} = \frac{23}{22} \Rightarrow 23r = 22r + 110$$

$$\Rightarrow r = 110 \text{ metres}$$

$$\therefore \text{Diameter} = 2 \times 110 = 220 \text{ metres.}$$

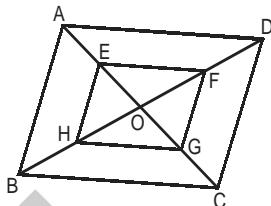
47. a



$$\Rightarrow 2\pi r \times \frac{72}{360} = 88$$

$\Rightarrow r = 70$ m which is the length of the rope.

48. c



Since E, F, G and H are midpoints of AO, DO, CO and BO.

\therefore By Midpoint theorem EH, EF, FG and GH are $\frac{1}{2}$ of AB, AD, DC and BC respectively.

$$\therefore EH + HG + FG + EF = \frac{1}{2}(AB + BC + CD + AD)$$

$$\Rightarrow \text{Perimeter of } EFGH = \frac{1}{2} \times \text{Perimeter of } ABCD$$

$$\therefore \text{Required ratio} = 1 : 2.$$

49. c Perimeter of equilateral triangle = 3 × side

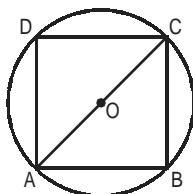
$$\Rightarrow 3 \times \text{side} = 18$$

$$\Rightarrow \text{Side} = \frac{18}{3} = 6 \text{ cm.}$$

$$\therefore \text{Length of median} = \frac{\sqrt{3}}{2} \times \text{side}$$

$$= \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3} \text{ cm.}$$

50. a



Side of a square

$$= AB = \sqrt{2} a \text{ units}$$

$$\therefore AC = \text{Diagonal} = \sqrt{2} \times \sqrt{2}a = 2a \text{ units}$$

= Diameter (d) of circle

$$\therefore \text{Circumference of circle} = \pi \times d$$

$$= \pi \times 2a = 2\pi a \text{ units.}$$

Properties of Triangles

Answers and Explanations

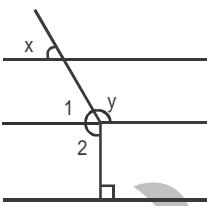
1	d	2	c	3	c	4	a	5	d	6	b	7	d	8	c	9	a	10	c
11	d	12	b	13	a	14	b	15	e	16	e	17	d	18	c	19	b	20	d
21	c	22	d	23	d	24	d	25	c	26	d	27	c	28	d	29	b	30	b
31	b	32	b	33	d	34	c	35	c	36	d	37	c	38	a	39	b	40	b
41	c	42	c	43	b	44	a	45	a	46	c	47	c	48	b	49	b	50	a

1. d $2x = 180^\circ - 2y$ (Interior angles) $\Rightarrow 2x + 2y = 180^\circ$
 $\Rightarrow x + y = 90^\circ$

In $\triangle ABC$, $\angle ACB + x + y = 180^\circ$
 $\Rightarrow \angle ACB = 180^\circ - (x + y) = 90^\circ$.

2. c Assume $\angle AFG = a$
 $\angle ABC = \angle BFE = 128^\circ$
 (Corresponding angles)
 $a = 180^\circ - 128^\circ = 52^\circ$.
 In $\triangle AGF$, $\angle AFG + 20^\circ + x = 180^\circ \Rightarrow x = 108^\circ$.

3. c



$\angle 2 = 90^\circ$ (Alternate angles),
 $\angle 1 = x$ (Corresponding angles)
 $\angle 1 + \angle 2 = 3x + 10^\circ$
 $\Rightarrow x + 90^\circ = 3x + 10^\circ$
 $\Rightarrow 2x = 80^\circ \Rightarrow x = 40^\circ$
 $\Rightarrow y = 180^\circ - \angle 1$ (Adjacent angles)
 $\Rightarrow y = 180^\circ - 40^\circ = 140^\circ$.

4. a Let the side of the square be a . Its diagonal is $a\sqrt{2}$.

Area of the first equilateral triangle $= \frac{\sqrt{3}}{4} \times a^2$

Area of the second equilateral triangle $= \frac{\sqrt{3}}{4} \times (a\sqrt{2})^2$

Required ratio $= 1 : 2$.

5. d Use the formula for the length of median
 (Apollonius theorem)

$$\Rightarrow 2(AD)^2 + 2(BD)^2 = AB^2 + AC^2$$

$$\Rightarrow 2 \times 100 + 2BD^2 = 200$$

$$\Rightarrow BD^2 = 0$$

$\Rightarrow BD = 0$
 \Rightarrow The triangle does not exist under the given conditions.

6. b $\angle PBA = 180^\circ - 95^\circ = 85^\circ$
 $\angle QPR = 180^\circ - (85^\circ + 65^\circ) = 30^\circ$
 \therefore In $\triangle PAB$, $\angle PAB = 180^\circ - (85^\circ + 30^\circ) = 65^\circ$
 Thus, $\triangle PAB$ and $\triangle PRQ$ are similar.

$$\therefore \frac{PQ}{PB} = \frac{QR}{AB}$$

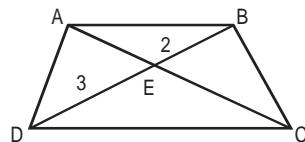
Now $QR = 1.4 AB$

$$\therefore \frac{PQ}{PB} = 1.4$$

$$\Rightarrow PQ = 1.4 \times 10 = 14 \text{ cm.}$$

7. d Since $AB = AC$, $\angle B = \angle C$.
 In $\triangle XBC$ and $\triangle YBC$, $BX = CY$ and $\angle XBC = \angle YCB$.
 BC is the common side.
 Hence, $\triangle XBC$ and $\triangle YBC$ are congruent.

$$\therefore XC = YB \text{ and } \frac{XC}{YB} = 1.$$



$BE : DE = 2 : 3$ and $AE : EC = 2 : 3$ (By property)

$\triangle AEB$ and $\triangle CED$ are similar.

The ratio of the proportional sides is $2 : 3$.

$$\therefore \frac{\text{Area } (\triangle AEB)}{\text{Area } (\triangle CED)} = \frac{4}{9}.$$

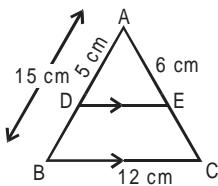
9. a $\triangle ADB \sim \triangle BDC$

$$\therefore \frac{AD}{BD} = \frac{BD}{DC}$$

$$\Rightarrow BD^2 = AD \times DC = 8 \times 2 \Rightarrow BD^2 = 16$$

$$\Rightarrow BD = 4 \text{ cm.}$$

10. c



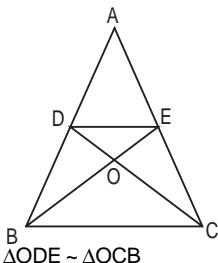
Since $DE \parallel BC$,

$\therefore \triangle ADE \sim \triangle ABC$

$$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\Rightarrow AC = 6 \times \frac{15}{5} = 18 \text{ cm and } DE = \frac{5}{15} \times 12 = 4 \text{ cm.}$$

11. d



$\triangle ODE \sim \triangle OCB$

$$\frac{\text{Area of } \triangle ODE}{\text{Area of } \triangle OBC} = \frac{DE^2}{BC^2} = \frac{1}{4}$$

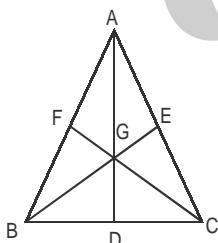
$$\text{Area of } \triangle ODE = \frac{1}{4} \text{ Area of } \triangle OBC$$

$$= \frac{1}{4} \times \frac{1}{3} \text{ Area of } \triangle ABC$$

{ \because Area of $\triangle OBC = \frac{1}{3}$ of area of $\triangle ABC$ }

$$= \frac{1}{12} \text{ Area of } \triangle ABC.$$

12. b



$$\text{Required area} = \frac{1}{3} \times 60 = 20 \text{ sq.cm. } \{ \because \text{Centroid of a triangle divides the triangle in three equal areas.} \}$$

13. a Let the sides of the triangle be $3x$, $4x$ and $5x$. Then,

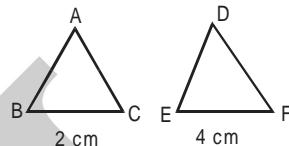
$$5x - 3x = 4 \Rightarrow x = 2$$

\therefore The sides of the triangle = 6 cm, 8 cm, 10 cm which is a Pythagorean triplet.

$$\therefore \text{Area of the triangle} = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2.$$

14. b Since $\triangle ABC$ is similar to $\triangle DEF$,

$$\frac{BC}{EF} = \frac{AB}{DE} \text{ (Corresponding sides in proportion)}$$



$$\Rightarrow \frac{2}{4} = \frac{3}{DE} \Rightarrow DE = 6 \text{ cm}$$

$$\text{Similarly, } \frac{BC}{EF} = \frac{AC}{DF} \Rightarrow \frac{2}{4} = \frac{2.5}{DF}$$

$$\Rightarrow DF = 5 \text{ cm}$$

$DE = 6 \text{ cm and } DF = 5 \text{ cm}$

$$\therefore \text{The perimeter of } \triangle DEF = DE + EF + DF = 6 + 4 + 5 = 15 \text{ cm.}$$

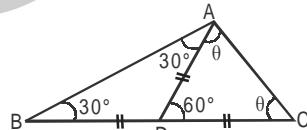
15. e $\triangle ADC \sim \triangle AEB$

Let x be the length of the side CE .

$$\therefore \frac{AD}{AC} = \frac{AE}{AB} \Rightarrow \frac{14}{4} = \frac{4+x}{2}$$

$$\Rightarrow 7 = 4 + x \Rightarrow x = 3 \text{ cm.}$$

16. e



In $\triangle ABD$,

$AD = BD$

$$\Rightarrow \angle ABD = \angle DAB = 30^\circ$$

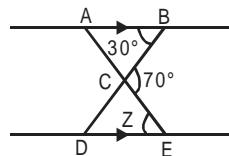
$$\therefore \angle ADB = 180^\circ - (\angle ABD + \angle DAB) = 120^\circ$$

$$\Rightarrow \angle ADC = 180^\circ - \angle ADB = 60^\circ$$

Let $\angle DAC = \angle DCA = \theta$

$$\therefore 60^\circ + 2\theta = 180^\circ \Rightarrow \theta = 60^\circ.$$

17. d



$\angle BAC = z$

$$y = x + z$$

$$\therefore 70 = 30 + z$$

$$\Rightarrow z = 40$$

[External angle of $\triangle ABC$]

18. c Sum of the angles of a triangle is 180° .

In $\triangle ABC$

$$\therefore 2a + 2b + 30^\circ = 180^\circ$$

$$\Rightarrow 2(a + b) = 150^\circ$$

$$\Rightarrow a + b = \frac{150^\circ}{2} = 75^\circ$$

and in $\triangle ACD$

$$a + b + d = 180^\circ$$

$$\Rightarrow d = 180^\circ - 75^\circ = 105^\circ.$$

19. b Let $\angle B$ and $\angle C$ be $3x$ and $5x$ respectively.

Then, $3x + 5x = 120^\circ$ (Exterior angle is equal to the sum of the interior opposite angles.)

$$\Rightarrow 8x = 120^\circ$$

$$\Rightarrow x = 15^\circ$$

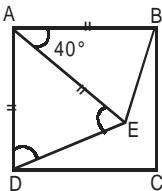
$$\therefore \angle B = 3 \times 15^\circ = 45^\circ \text{ and } \angle C = 5 \times 15^\circ = 75^\circ.$$

20. d In $\triangle ABC$, $\angle NBA = 135^\circ$ by exterior angle property.

Since $LM \parallel NC$

$$\therefore \angle x = \angle NBA = 135^\circ.$$

21. c



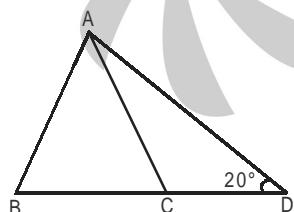
$$\angle BAE = 40^\circ \Rightarrow \angle EAD = 90^\circ - 40^\circ = 50^\circ$$

$$AB = AE = AD$$

$\therefore \triangle AED$ is an isosceles triangle.

$$\therefore \angle AED = \angle ADE = \frac{180^\circ - 50^\circ}{2} = 65^\circ.$$

22. d



$$CD = CA$$

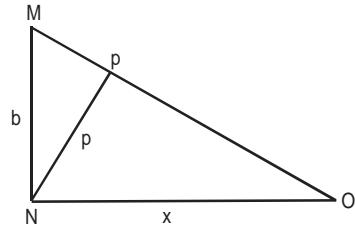
$$\angle ACB = 40^\circ$$

$$\angle ABC = \angle ACB \quad (\text{exterior angle})$$

$$\angle ABC = \angle ACB \quad [\because AB = AC]$$

$$\therefore \angle ACD = 180^\circ - 40^\circ = 140^\circ$$

23. d Let length of one side and the altitude on the hypotenuse be x and p respectively.



$$\text{Area of } \triangle MNO = A = \frac{1}{2} \times b \times x$$

$$= \frac{1}{2} \times p \times MO$$

$$\Rightarrow x = \frac{2A}{b} \text{ and } p = \frac{2A}{MO}$$

$$\text{But, } (MO)^2 = x^2 + b^2$$

$$\Rightarrow (MO) = \sqrt{x^2 + b^2}$$

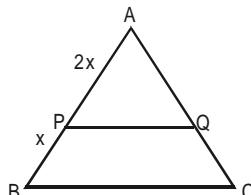
$$= \sqrt{\left(\frac{2A}{b}\right)^2 + b^2}$$

$$\text{Now, } p = \frac{2A}{\sqrt{x^2 + b^2}}$$

$$= \frac{2A}{\sqrt{\left(\frac{2A}{b}\right)^2 + b^2}} = \frac{2A}{\sqrt{\frac{1}{b^2}(4A^2 + b^4)}}$$

$$= \frac{2A.b}{\sqrt{b^4 + 4A^2}}.$$

24. d



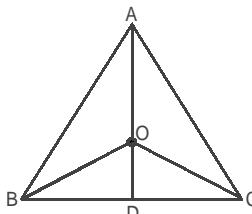
In $\triangle ABC$, $PQ \parallel BC$

$\Rightarrow APQ \sim ABC$

$$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} = \frac{2x}{3x} = \frac{2}{3} = 2:3$$

[Let $PB = x \Rightarrow AB = 3x$ and $AP = 3x - x = 2x]$

25. c



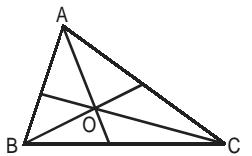
BO is the internal bisector of $\angle B$

$$\angle ODB = 90^\circ; \angle BOD = 15^\circ$$

$$\therefore \angle OBD = 180^\circ - 90^\circ - 15^\circ = 75^\circ$$

$$\Rightarrow \angle ABC = 2 \times 75^\circ = 150^\circ.$$

26. d



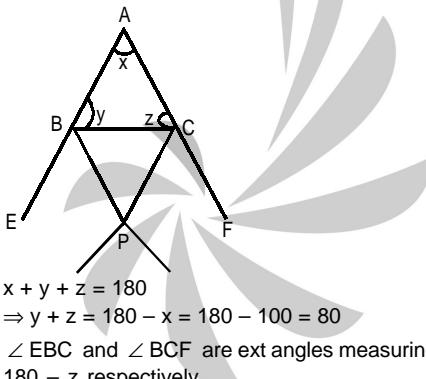
The point of intersection of internal bisectors of a triangle is called in-centre.

$$\angle BOC = 90^\circ + \frac{\angle A}{2} \Rightarrow 116^\circ = 90^\circ + \frac{\angle A}{2}$$

$$\Rightarrow \frac{\angle A}{2} = 116^\circ - 90^\circ = 26^\circ$$

$$\therefore \angle A = 26^\circ \times 2 = 52^\circ.$$

27. c



$$x + y + z = 180^\circ$$

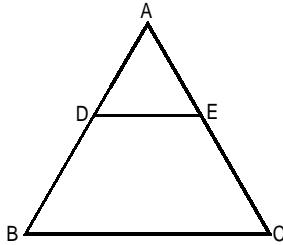
$$\Rightarrow y + z = 180^\circ - x = 180^\circ - 100^\circ = 80^\circ$$

$\angle EBC$ and $\angle BCF$ are ext angles measuring $180^\circ - y$, $180^\circ - z$ respectively.

$$\therefore \text{In } \triangle BPC, \angle BPC = 180^\circ - \left[\frac{(180^\circ - y)}{2} + \frac{(180^\circ - z)}{2} \right]$$

$$= \frac{y+z}{2} = \frac{80^\circ}{2} = 40^\circ.$$

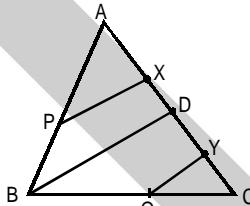
28. d



$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$$

$$\therefore \frac{DE}{BC} = \frac{1}{3} \Rightarrow DE = \frac{15}{3} = 5 \text{ cm}$$

29. b

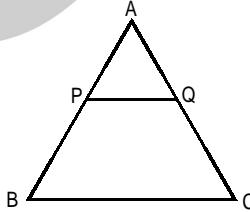


$$PX \parallel BD, PX = \frac{1}{2}BD$$

$$QY \parallel BD \text{ and } QY = \frac{1}{2}BD$$

$$\therefore PX : QY = 1 : 1$$

30. b



$$\frac{AP}{PB} = \frac{AQ}{QC} = \frac{1}{2}$$

$$\Rightarrow \frac{QC}{AQ} = \frac{2}{1} \Rightarrow \frac{QC + AQ}{AQ} = \frac{3}{1}$$

$$\Rightarrow AC = 3AQ = 9 \text{ cm.}$$

31. b

$$AB + BC = 12$$

$$BC + CA = 14$$

$$CA + AB = 18$$

$$\therefore 2(AB + BC + CA)$$

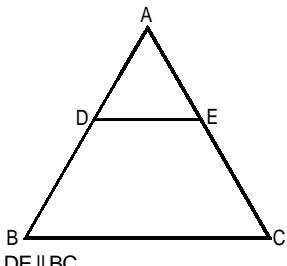
$$= 12 + 14 + 18 = 44$$

$$\Rightarrow AB + BC + CA = 22$$

$$\therefore 2\pi r = 22$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{7}{2} \text{ cm.}$$

32. b



$$DE \parallel BC$$

$$\angle ADE = \angle ABC$$

$$\angle AEE = \angle ACB$$

$\therefore \triangle ADE \sim \triangle ABC$

Now, $\frac{\square BDEC}{\Delta ADE} = \frac{1}{1}$ [DE divides \triangle into two equal parts]

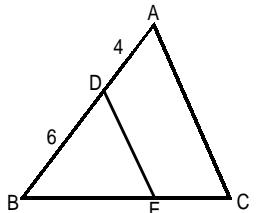
$$\frac{\square BDEC}{\Delta ADE} = 1:1$$

$$\Rightarrow \frac{\Delta ABC}{\Delta ADE} = \frac{2}{1} = \frac{AB^2}{AD^2} \Rightarrow \frac{AB}{AD} = \sqrt{2}$$

$$\Rightarrow \frac{AB}{AD} - 1 = \sqrt{2} - 1 \Rightarrow \frac{BD}{AD} = \sqrt{2} - 1$$

$$\Rightarrow \frac{AD}{BD} = \frac{1}{\sqrt{2}-1} \text{ or } 1 : \sqrt{2}-1.$$

33. d



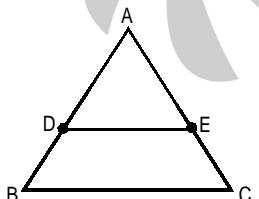
$$DE \parallel AC$$

$$\triangle BDE \sim \triangle BAC$$

$$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC} \Rightarrow \frac{6}{4} = \frac{BE}{EC}$$

$$\Rightarrow BE : CE = 3 : 2.$$

34. c

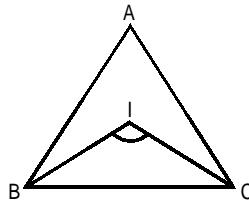


$$\frac{AD}{AE} = \frac{AB}{AC}$$

$$\triangle ADE \sim \triangle ABC$$

$$\therefore DE = \frac{1}{4} BC = \frac{1}{4} \times 12 = 3 \text{ cm.}$$

35. c

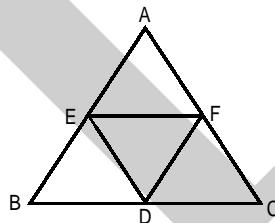


$$\angle IBC = \frac{70^\circ}{2} = 35^\circ$$

$$\angle ICB = \frac{50^\circ}{2} = 25^\circ$$

$$\therefore \angle BIC = 180^\circ - 35^\circ - 25^\circ = 180^\circ - 60^\circ = 120^\circ.$$

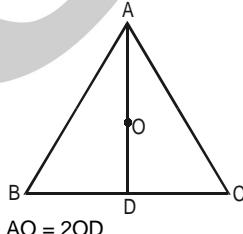
36. d



$$\triangle DEF = \frac{1}{4} \triangle ABC$$

$$= \frac{1}{4} \times 24 = 6 \text{ sq. units.}$$

37. c



$$AO = 2OD$$

$$\Rightarrow OD = \frac{AO}{2} = \frac{10}{2} = 5 \text{ cm.}$$

38. a The sum of angles in a triangle is 180° .

$$\angle A = 2x^\circ, \angle B = 3x^\circ, \angle C = x^\circ$$

$$\Rightarrow 2x^\circ + 3x^\circ + x^\circ = 180^\circ$$

$$\Rightarrow 6x^\circ = 180^\circ$$

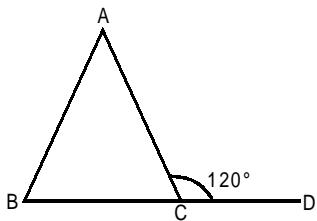
$$\Rightarrow x = \frac{180}{6} = 30$$

$$\therefore \angle A = 2 \times 30^\circ = 60^\circ$$

$$\angle B = 3x \times 30^\circ = 90^\circ$$

$$\angle C = x = 30^\circ.$$

39. b



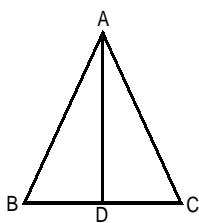
$$\angle ACB = 180^\circ - 120^\circ = 60^\circ$$

$$AB = BC$$

$$\therefore \angle ABC = \angle ACB = 60^\circ$$

$$\therefore \angle BAC = 60^\circ.$$

40. b

In $\triangle ABD$,

$$AB^2 = AD^2 + BD^2$$

In $\triangle ADC$,

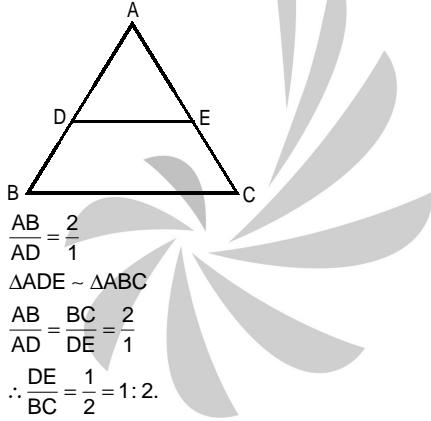
$$AC^2 = AD^2 + CD^2$$

$$\therefore AB^2 + CD^2 = AD^2 + BD^2 + CD^2$$

$$= (AD^2 + CD^2) + BD^2$$

$$= AC^2 + BD^2.$$

41. c



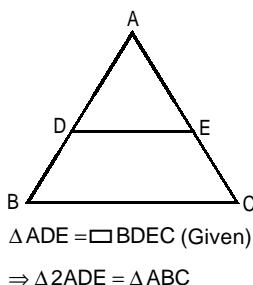
$$\frac{AB}{AD} = \frac{2}{1}$$

 $\triangle ADE \sim \triangle ABC$

$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{2}{1}$$

$$\therefore \frac{DE}{BC} = \frac{1}{2} = 1:2.$$

42. c

 $\triangle ADE = \square BDEC$ (Given)

$$\Rightarrow \triangle ADE \sim \triangle ABC$$

$$DE \parallel BC$$

 $\therefore \triangle ADE \sim \triangle ABC$

$$\therefore \frac{\triangle ADE}{\triangle ABC} = \frac{1}{2} = \frac{AD^2}{AB^2} \Rightarrow \frac{AB}{AD} = \sqrt{2}$$

$$\frac{AB}{AD} - 1 = \sqrt{2} - 1 = \frac{AB - AD}{AD} = \sqrt{2} - 1$$

$$\Rightarrow \frac{BD}{AD} = \sqrt{2} - 1$$

$$\therefore \frac{BD}{AB} = \frac{BD}{AD} \times \frac{AD}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}.$$

43. b In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + \angle B = 70^\circ$$

$$\angle B + \angle C = 130^\circ$$

$$\therefore \angle A = (\angle A + \angle B + \angle C) - (\angle B + \angle C) \\ = 180^\circ - 130^\circ = 50^\circ.$$

44. a $2\angle A = 3\angle B = 6\angle C$

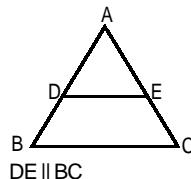
$$\Rightarrow \frac{2\angle A}{6} = \frac{3\angle B}{6} = \frac{6\angle C}{6}$$

$$\Rightarrow \frac{\angle A}{3} = \frac{\angle B}{2} = \frac{\angle C}{1}$$

$$\Rightarrow \angle A : \angle B : \angle C = 3 : 2 : 1$$

$$\therefore \angle B = \left(\frac{2}{1+2+3} \right) \times 180^\circ = \frac{2}{6} \times 180^\circ = 60^\circ.$$

45. a

 $DE \parallel BC$

$$\therefore \frac{AD}{AB} = \frac{AE}{AC}$$

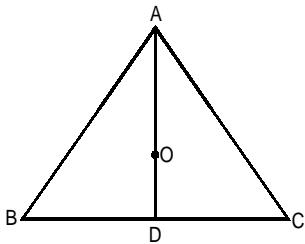
$$\frac{AD}{BD} = \frac{3}{5} \Rightarrow \frac{BD}{AD} = \frac{5}{3} \Rightarrow \frac{BD}{AD} + 1 = \frac{5}{3} + 1$$

$$\Rightarrow \frac{BD + AD}{AD} = \frac{5+3}{3} \Rightarrow \frac{AB}{AD} = \frac{8}{3} \Rightarrow \frac{AD}{AB} = \frac{3}{8}$$

$$\therefore \frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{3}{8} = \frac{AE}{4}$$

$$\Rightarrow AE = \frac{3 \times 4}{8} = 1.5 \text{ cm.}$$

46. c



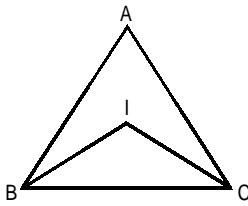
Point 'O' is centroid and AD is median.

$$\therefore AO = \frac{2}{3} AD$$

$$\Rightarrow 10 = \frac{2}{3} AD \Rightarrow AD = \frac{10 \times 3}{2} = 15 \text{ cm}$$

$$\therefore OD = \frac{1}{3} AD = \frac{15}{3} = 5 \text{ cm.}$$

47. c



In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\therefore \angle B + \angle C = 180^\circ - \angle A$$

$$\therefore \frac{1}{2}(\angle B + \angle C) = 90^\circ - \frac{\angle A}{2}$$

In $\triangle BIC$,

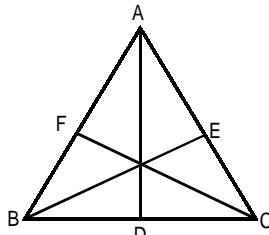
$$\frac{\angle B}{2} + \frac{\angle C}{2} + \angle BIC = 180^\circ$$

$$\therefore 90^\circ - \frac{\angle A}{2} + \angle BIC = 180^\circ$$

$$\Rightarrow \angle BIC = 180^\circ - 90^\circ + \frac{\angle A}{2} = 90^\circ + \frac{\angle A}{2}$$

$$\therefore X = 90^\circ.$$

48. b



Since sum of 2 sides of a triangle > third side

$$AD + BD > AB$$

$$AD + DC > AC$$

$$BE + AE > AB$$

$$BE + CE > BC$$

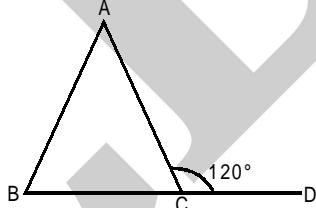
$$CF + FB > BC$$

$$CF + AF > AC$$

$$\therefore 2(AD + BE + CF) + (AB + BC + CA) > 2(AB + BC + CA)$$

$$\Rightarrow 2(AD + BE + CF) > AB + BC + CA$$

49. b



$$\angle CAB = 2 \angle ABC$$

$$\angle ACB + \angle ACD = 180^\circ$$

$$\Rightarrow \angle ACB + 120^\circ = 180^\circ$$

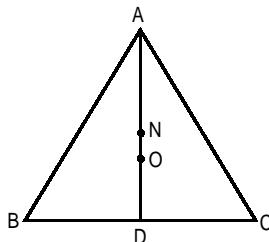
$$\Rightarrow \angle ACB = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore \angle A + \angle B = 180^\circ - 60^\circ = 120^\circ$$

$$\Rightarrow 2 \angle B + \angle A = 120^\circ$$

$$\Rightarrow \angle B = \frac{120^\circ}{3} = 40^\circ$$

50. a



$$AD = 27 \text{ cm}$$

Centroid = O

$$\therefore OD = \frac{1}{3} AD = \frac{1}{3} \times 27 = 9 \text{ cm}$$

$$ND = 12 \text{ cm}$$

$$\therefore ON = DN - OD = 12 - 9 = 3 \text{ cm.}$$