# Topology

## Elements of Topological Space

**Topology**—A topology on a set X is a collection T of subsets of X having following properties—

(a) $\phi$ and X are in T

(b) The union of the elements of any subcollection of T is in T

(c) The intersection of the elements of any finite subcollection of T is in T

**Topology Space**—(X, T), A set X for which a topology T is defined.

**Open set**—(X, T) is a Topological space, U $\subset$ X is open set if U $\subset$ T.

**Finer and Strictly Finer Topology**—If T and T are two topologies on a set X, T is finer topology (then T) if T $\supset$ T. T is strictly finer if T properly contains T.

**Basis**—If X is a set, a basis for topology on X is a collection B of subsets of X called basis elements such that

(a) For each $x \in$ X, there is at least one basis element B containing $x$.

(b) If $x$ belongs to the intersection of two basis elements $B_1$ and $B_2$ then there is a basis element $B_3$ containing $x$ such that $B_3 \subset B_1 \cap B_2$.

**Topology Generated by B**—If B is a basis for a topology on X, the topology T generated by B is defined as : A subset U $\subset$ X is open in (X, T) if for each $x \in$ U, there is a basis element B $\in$ B such that $x \in$ B and B $\subset$ U.

**Standard Topology**—If the collection of all intervals in the real line $(a, b) = \{x|a < x < b|\}$, the topology generated by B is called the standard topology on the real line.

**Subbasis**—A subbasis $\xi$ for a topology on X is a collection of subsets of X whose union equals X.

**Topology Generated by the Subbasis**—Topology generated by the subbasis $\xi$ is defined as the collection T of all unions of finite intersections of elements of $\xi$.

**Order Topology**—Let X be a set with order relation. If B basis for X is a collection of all sets of the form—

(a) All open intervals $(a, b) \in$ X.

(b) All intervals of the form $[a_0, b]$, $a_0$ is the smallest element (if any) of X.

(c) All intervals of the form $[a_0, b_0]$, $b_0$ is the largest element (if any) of X.

The collection B is a basis for topology on X and is called order topology.

**Product Topology : X × Y**—If X and Y are topological spaces, the product topology on X × Y is the topology having as basis the collection B of all sets the form U × V, where U is an open subset of X and V is an open subset of Y.

**Subspace topology**—If X is a topological space with topology T and Y $\subset$ X, the collection $T_y = \{Y \cap U \mid U \in T\}$ is a topology on Y, called subspace topology. (Y, $T_Y$) is called subspace topology of (X, T).

**Closed sets**—A subset A of topological space X is closed if (X – A) is open.

**Interior set**—(X, T) is given a topological space, A $\subset$ X, then interior of A, A° is the union of all open sets contained in A.

**Closure set**—(X, T) is a given topological space A $\subset$ X, then closure of A, $\bar{A}$ is the intersection of all closed sets containing A.

**Neighbourhood of $x$**—An open set U containing $x$.

**Limit Point (cluster point)**—Given (X, T) a topological space and A $\subset$ X, a point $x \in$ X is a limit point of A if every neighborhood of $x$ intersects A in some point other than $x$.

**Hausdorff Space**—A topological space X is Hausdorff space if for each pair $x_1 + x_2 \in$ X, there exist neighbourhoods $U_1$ and $U_2$ of $x_1$ and $x_2$ respectively such that $U_1$ and $U_2$ are disjoint,

**Discrete Topology**—If X is any set, T is a collection of all subsets of X, then (X, T) is a discrete topology.

**Indiscrete Topology (trivial topology)**—If X is any set, T = {$\phi$, X}, then (X, T) is indiscrete topology.

## Theorems

1. Let X be a set; let B be a basis for a topology T on X, then T is equal to the collection of all unions of elements of B.
2. Let B and B′ be a basis for the topologies T and T′ respectively on X then following are equivalent.
   (a) T′ is finer than T.
   (b) For each $x \in$ X and each basis element B $\in$ B containing $x$, there is a basis element B′ $\in$ B′ such that $x \in$ B′ $\subset$ B.
3. Let X be a topological space. Suppose that collection C is a collection of open sets of X such that each $x \in$ X and each open set U of X, there is an element C $\in$ C such that $x \in$ C $\subset$ U. Then C is a basis for the topology of X.
4. The lower limit topology T′ an real line R is strictly finer than the standard topology T.
5. If B is a basis for the topology of X and C is a basis for the topology of Y, then the collection D = {B × C | B $\in$ B and C $\in$ C} is a basis for the topology of X × Y.
6. If B is a basis for the topology of X, then the collection $B_Y$ = {B $\cap$ Y | B $\in$ B} is a basis for the subspace topology on X.
7. Let Y be a subspace of X if U is open in Y and Y is open in X, then U is open in X.
8. If X is an ordered set in the order topology and if Y is an interval or a ray in X, then the subspace topology and order topology on Y are same.
9. If A is a subspace of X and B is a subspace of Y, then the product topology on A × B is the same as the topology A × B inherits as a subspace of X × Y.
10. If (X, T) is a topological space. Then following condition holds—
    (a) $\phi$ and X are closed.
    (b) Arbitrary intersections of closed sets are closed.
    (c) Finite unions of closed sets are closed.
11. If Y is a subspace of X then set A is closed in Y iff it is equal to the intersection of a closed set of X with Y.
12. If Y is a subspace of X. If A is closed in Y and Y is closed in X, then A is closed in X.
13. If Y is a subspace of X, A $\subset$ Y and $\bar{A}$ is a closure of A in X, then closure of A in Y is equal to A $\cap$ Y.
14. If A is a subset of topological space X.
15. $x \in \bar{A}$, closure of A in X, iff every open set U containing $x$, intersects A.
16. Suppose B is a basis for X, then $x \in \bar{A}$ iff every basis element B $\in$ B, containing $x$ intersects A.
17. Let A be a subset of topological space X and A′ be the set of all limit points of A. Then $\bar{A}$ = A $\cup$ A′.
18. A subset of a topological space is closed iff it contains all its limit points.
19. Any finite point set in a Hausdorff space X is closed.
20. Let X be a Hausdorff space, A $\subset$ X. Then a point $x$ is a limit point of A iff every neighbourhood of $x$ contains infinitely many points of A.
21. Every simple order set is a Hausdorff space in the order topology.
22. The product of two Hausdorff spaces is a Hausdorff space.
23. A subspace of a Hausdorff space is a Hausdorff space.

## Continuity

**Continuous Function**—Let X and Y are topological spaces. A function $f$ : X $\to$ Y is a continuous function if for each open subset V of Y, the set $f$ (V) is an open subset of X.

**Homeomorphism**—Let X and Y are topological spaces. Let $f$ : X $\to$ Y be a one-to-one function. If both the function $f$ and the inverse function $f'$ : Y $\to$ X are continuous, then $f$ is homeomorphism.

**Continuity at a point**—Let X and Y are topological spaces. A function $f : X \to Y$ is continuous at $x \in X$, if for every neighbourhood N of $f(x)$, there is a neighbourhood M of $x$ such that $f(N) \subset M$.

## Theorems

1. Let X and Y be topological spaces and $f : X \to Y$. Then the following are equivalent :
   (a) $f$ is continuous
   (b) For every $A \subset X, f(\bar{A}) \subset \overline{f(A)}$
   (c) For every closed set $B \subset Y$, set $f^{-1}(B)$ is closed in X
2. Let X, Y and Z be topological spaces :
   (a) The constant function $f : X \to Y$, *i.e.* $f(x) = Y_0$ for all $x \in X$ and single point $Y_0 \in Y$ is continuous function.
   (b) If A is a subspace of X, the inclusion function $f : A \to X$ is continuous.
   (c) If $f : X \to Y$ and $g : Y \to Z$ are continuous then composites $(g o f) : X \to Z$ is continuous.

## The Product Topology

**Boxtopology**—Let $\{X\}_{\alpha \in J}$ be an indexed family of topological spaces let us take a basis for a topology on the product space. $\pi_{\alpha \in J} X_\alpha$, the collection of all sets of the form, $\pi_{\alpha \in J} U_\alpha$, where $U_\alpha$ is open in $X_\alpha$ for each $\alpha \in T$. The topology generated by this basis is called boxtopology.

**Projection Mapping**—Let $\pi_\beta : \pi_{\alpha \in J} X_\alpha \to X_\beta$ be the function assigning to each element of the product space its $\beta$th coordinate, $\pi\beta\ [(x_\alpha)_{\alpha \in J}] = x_\beta$, it is called the projection mapping associated with the index $\beta$.

**Product Topology and Product Space**—Let $\delta_\beta = \{\pi_\beta^{-1}(U_\beta) | U_\beta \text{ open in } X_\beta]$ and $\delta = U_{\beta \in J}\ \delta_\beta$. The topology generated by the subbasis $\delta$ is called the product topology and $\pi_{\alpha \in J} X_\alpha$ is called product space.

## Theorems

1. **Comparison of the box and product topologies**—The box topology $n\pi X_\alpha$ has a basis all sets of the form $\pi U_\alpha$, where $U_\alpha$ is open in $X_\alpha$ for each $\theta$. The product topology on $\pi U_\alpha$ has a basis all sets of the form $\pi U_\alpha$ where $U_\alpha$ is open in $X_\alpha$ for each $\alpha$ and $U_\alpha$ equals $X_\alpha$ except for finetely many values of $\alpha$.
2. Suppose the topology on each space $X_\alpha$ is given by a basis $B_\alpha$. The collection of all sets of the form $\pi_{\alpha \in J} B_\alpha$, where $B_\alpha \in \beta_\alpha$ for each $\alpha$ will serve as a basis for the box topology on $\pi_{\alpha \in J} X_\alpha$.
3. The collection of all sets of the same form, where $B_\alpha \in B_\alpha$ for finitely many indices $\alpha$ and $B_\alpha = X_\alpha$ for all the remaining indices, will serve as a basis for the product topology on $\pi_{\alpha \in J} X_\alpha$.
4. Let $A_\alpha$ be a subspace of topological space $X_\alpha$ for each $\alpha \in T$. Then $\pi A_\alpha$ is a subspace of $\pi X_\alpha$ is both products are given the topology, or if both products are given the product topology.
5. If each space $X_\alpha$ is Hausdorff space, then $\pi X_\alpha$ is a Hausdorff space in both box and product topologies.
6. If $f : A \to \pi_{\alpha \in J} X_\alpha$ be given by the $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each $\alpha$. Let $\pi X_\alpha$ have the product topology, then the function $f$ is continuous iff each function $f_\alpha$ is continuous.

## Metric Topology

**Metric**—A metric an a set X is a function $d : X \times X \to R$ having the properties :

(a) $d(x, y) \geq 0$ for all $x, y \in X$ and $d(x, y) = 0$ iff $x =$

(b) $d(x, y) = d(y, x)$ for all $x, y \in X$

(c) $d(x, y) + d(y, z) = d(x, z)$, for all $x, y, z, \in X$ (triangular inequality) $\in$-bull centred at $x$ : $Bd(x, \in) = \{Y \mid d(x, y) < \in\}$.

**Metric Topology**—If $d$ is a metric on the set X, then the collection off all $\in$-balls $B_d(x, \in)$ for $x \in X$ and $\in > 0$, is a basis for a topology on X, called metric topology induced by $d$.

Or A set U is open in the metric topology induced by $d$ iff for each $Y \in U$, there is $\delta > 0$ : $t\, B_d(Y, \delta) \subset U$.

**Metrizable Space**—If X is a topological space, then X is metrizable iff there exist a metric $d$ on the set X that induces the topology on X.

A metric space is a metrizable space X together with a specific metric $d$ that gives the topology of X.

**Bounded Set**—Let X be a metric space with metric $d$. A subset A of X is said to be bounded if there is some number M such that $d(a_1, a_2) \leq M$ for each pair $a_1, a_2$ of points of A. If A is bounded, the diameter of A is diam A = lub $\{d(a_1, a_2) : a_1, a_2 \in A\}$.

**Euclidean Metric**—Given $\bar{x} \in R^n$, the norm of $\bar{x}$ is $|\bar{x}| = (x_1^2 + x_2^2 + \ldots + x_n^2)^{1/2}$

The Euclidean metric $d$ $R^n$ is

$$d(\bar{x}, \bar{y}) = \|\bar{x} - \bar{y}\| = [(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots + (x_n - y_n)^2]^{1/2}$$

**Square Metric**—Let $\bar{x} \in R^n$, the square metric P is

$$P(\bar{x}, \bar{y}) = \max\{|x_1 - y_1|, \ldots, |x_n - y_n|\}$$

**Standard Bounded Metric (corresponding to $d$)**—Let X be a metric space with metric $d$ and $\bar{d} : X \times X \to R$ such that $\bar{d}(x, y) = \min\{d(x, y), 1\}$ is called standard bounded metric corresponding to $d$.

**Uniform Metric**—Given an index set J and $\bar{x} = (x_2)_{\alpha \in J}$ and $\bar{y} = (y_2)_{\alpha \in J}$ of $R^J$. The uniform metric on $R^J$ is

$$\bar{P}(\bar{x}, \bar{y}) = \text{Qub}\{\bar{d}(x_2, y_2)\ \alpha \in J\}$$

where $\bar{d}(x_\alpha, y_\alpha) = \min\{d(x_\alpha, y_\alpha), 1,\}$ the standard bounded metric on R. The topology induced by uniform metric is called uniform topology.

**Convergent Sequence**—A sequence $\{x_m\}$ of points of X is convergences to a point $x \in X$ if for every neighbourhood U of $x$ there exists a positive integer N such that $x_n \in U$ for all $n \geq N$.

**Countable Basis at a Point**—A space X have a countable basis at the point $x$ if there is a countable collection $\{U_n\}_{n \in Z}$ of neighbourhood of $x$ such that any neighbourhood U of $x$ contains atleast one of the sets $U_n$.

**First Countable Axiom**—A space X that has a countable basis at each of its points is said to satisfy the first countability axiom.

## Some Important Theorems

1. Let $d$ and $d'$ be two metrics on the set X and let T and T′ be the topologies they induce respectively. Then T′ is finer than T iff for each $x \in X$ and each $\in > 0$, there exist $\delta \to 0$, such that $B_{d'}(x, \delta) \subset B_d(x, \in)$.
2. There topologies on $R^n$ induced by the euclidean metric $d$ and the square metric P are the same as the product topology on $R^n$.
3. The uniform topology on $R^J$ is finer than the product topology. They are different if J is infinite.
4. Let $\bar{d}(a, b) = \min\{|a - b|\}$ be the standard bounded metric on R. If $x$ and $y$ are two points of $R^w$, define
$$D(\bar{x}, \bar{y}) = \text{lub}\left\{\frac{\bar{d}(x_i, y_i)}{i}\right\}$$
Then D is a metric that induces the product topology on $R^w$.
5. Let $f : X \to Y$ also X and Y be metrizable with metrics $d_x$ and $d_y$ respectively. Then continuity of $f$ is equivalent, to given $x \in X$ and $\in > 0$, there exist $\delta > 0$ such that $d_x(x, y) < \delta \Rightarrow d_y(f(x) f(y)) < \in$.
6. **Sequence Lemma**—Let X be a topological space and $A \subset X$. If there is a sequence of points of A converging to $x$, then $x \in \bar{A}$; the converse holds if X is metrizable.
7. Let $f : X \to Y$ and X be a metrizable. The function $f$ is continuous iff for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ converges to $f(x)$.
8. The addition, subtraction and multiplication operations are continuous function from R × R into R and quatient operation is continuous function from $R \times (R - \in 0)$ into R.
9. If X is a topological space and if $f, g : X \to R$ are continuous functions, then $f + g$, $f - g$ and $f \cdot g$ are continuous functions. If $g(x) \neq 0$ for all $x$, then $f/g$ is continuous.
10. **Uniform Limit Theorem**—Let $f_n : X \to Y$ be a sequence of continuous functions from the topological space X into the metric space Y. If $\{f_n\}$ converges uniformely to $f$, then $f$ is continuous.
11. **Weierstrass M-test**—Let $f_n : X \to R$ be a sequence of functions from topological space X into R and $\delta_n(x) = \sum_{i=1}^{n} f_i(x)$. If $|f_i(x)| \leq b_i$ for all $x \in X$ and all $i = 1, \ldots, n$ and if the

series $\Sigma b_i$ is convergent, the sequence $\{\delta_n\}$ converges uniformly to a functions.

## The Quotient Topology

**Saturated Set**—A subset C of a topological space X is saturated (with respect to the surjective map $P : X \to Y$) if C contains every set $P^{-1}(\{Y\})$ that it intersects.

**Quotient Map**—Let X and Y be topological spaces, $P : X \to Y$ be a surjective map the map P is said to be a quotient map. Provided a subset U $\subset$ Y is open in Y iff $P^{-1}$ (U) is open in X.

Equivalently P is quotient map, if P is continuous and P maps saturated open sets of X to open sets of Y (or saturated closed sets of X to closed sets of Y).

**Open Map**—A map $f$ : X is open map if for every open set U of X the set $f$(U) is open in Y.

**Closed Map**—A map $f : X \to Y$ is closed if for every closed set of X, the set $f$(A) is closed in Y.

**Quotient Topology**—If X is a topological space and A is a set if $P : X \to A$ is surjective map, then there exist exactly one topology T on A relative to which P is a quotient map and this topology is called quotient topology induced by P.

**Quotient Space (decomposition space)**—Let X be a topological space and X* be a partition of X into disjoint subsets whose union is X. Let $P : X \to X^*$ be the surjective map the carries each point of X to the element of X* containing it. In the quotient topology induced by P, the space X* is quotient space of X.

## Some Important Theorems

1. Let $P : X \to Y$ be a quotient map. Let Z be a space and let $g : X \to Z$ be a continuous map that is constant an each set $P^{-1}(\{Y\})$, for $Y \in y$. Then $g$ induces is continuous map $f : Y \to Z$ such that $f o P = g$.

$$X \xrightarrow{} \quad P \downarrow \quad Y$$

2. Let $g : X \to Z$ be a surjective continuous map. Let X* be the following collection of subsets of X : $X^* = \{g^{-1}(\{Z\}) \mid Z \in Z\}$ given the quotient topology.
   (a) If $z$ is Hausdorff so is X*
   (b) The map $g$ induces a bijective continuous map $f : X^* \to z$, which is a homeomorphism iff $g$ is a guotient map.
3. The product of two quotient map need not be a quotient map.

# Connectendness

## (1) Connected Topological Spaces

**Separation of Topological Space**—Let X be a topological space. A separation of X is a pair U, V of disjoint nonempty open subsets of X whose union is X.

**Connected Topological Space**—Let X be a topological space.

(i) The space X is called connected if there does not exist a separation of X.

(ii) A space X is connected iff the only subset of X that are both open and closed in X are empty set and X itself.

**Topology Disconnected Topological Space**—A topological space X is totally disconnected if its only connected subsets are one-point set.

## Some Important Theorems

1. If $y$ is a subspsace of topological space X, a separation of Y is a pair of disjoint nonempty sets A and B whose union is Y, and neither of which contains a limit point of the other.
2. If Y is a subspace of topological space X and Y is connected if there exist no separation of Y.
3. If sets C and D from a separation of topological space X and if Y is a connected subset of X, then Y lies entirely with in either C or D.
4. The union of a collection of connected sets that have a point in common is connected.
5. Let A be a connected subset of topological space X, if $A \subset B \subset \bar{A}$, then B is also connected.
6. The cartensian product of connected topological space is connected.

## Connected Sets in the Real Line

**Linear Continuum**—A simple ordered set L having more than one element is called linear continuum if.

(1) L has the least upper bounded property.

(2) If $x < y$, there exist $z$ such that $x < z < y$.

**Path**—Let X be a topological space and $x, y \in$ X. The path in X from $x$ to $y$ is a continuous map $f : [a, b] \to$ X of some closed interval $[a, b]$ in the real line into X, such that $f(a) = x$ and $f(b) = y$.

**Path Connected (topological) Space**— A topological space X is called path connected if each pair of points of X can be joined by a path in X.

**Comb (topological) Space**—Let

$$K = \left\{ \frac{1}{n} \;\middle|\; n \in Z_+ \right\}$$

and $C = ([0, 1] \times 0) \cup (K \times [0, 1]) \cup (0 \times [0, 1])$

Then C is comb space.

## Some Important Theorems

1. Let L is a linear continuum in theorder topology, then L is connected and so is very interval and ray in L.
2. The real line R is connected and so is every interval and ray in R.
3. **Intermediate value theorem**—Let $f : X \to Y$ be a continuous map of the connected topological space X into the ordered set Y, in the order topology.

   If $a, b \in x$ and if $r \in y$ such that $f(a)cr < f(b)$, then exist a point $c \in x$ such that $f(c) = r$.
4. Path connected topological space is connected but converse is not true.
5. The space I × I in the dictionary order topology is connected but not path connected.
6. Comb topological space is connected topological space but not path connected.

## Components and Path Components

**Components of Topological Space**—Let X be a topological space. Given an equivalence relation on X such that $x \sim y$ if there is a connected subset of X containing both $x$ and $y$. The equivalence classes are called the components (connected components) of X.

**Path Components of Topological Space**— Let X be a topological space. Given an equivalence relation on X such that $x \sim y$ if there is a path in X from $x$ to $y$. Then equivalence classes are called the path components of X.

## Some Important Theorems

1. The components of topological space X are connected disjoint subsets of X whose union is X such that each connected subsets of X intersects only one of them.
2. The path components of topological space X are path connected disjoint subsets of X whose union is X, such that each path connected subset of X intersects only one of them.

## Local Connectedness

**Local Connected Space**—A topological space X is locally connected at $x$ if for every neighbourhood U of $x$, there is a connected neighbourhood V of $x$, $V \subset U$. If X is locally connected at each $x \in$ X, then X is locally path connected space.

**Locally Path Connected Space**—A topological space X is locally path connected at $x$ iff for every neighbourhood U of $x$, there is a path connected neighbourhood V of $x$, U, $V \subset U$.

If X is locally path connected at each $x \in$ X, then X is locally path connected space.

**Connected in Kleinen at $x$**—A topological space X is connected in Kleinen at $x$, if for every neighbourhood U of $x$, there is a connected subset -Y of U that contains a neighbourhood of $x$.

## Some Important Theorems

1. A topological space X is locally connected iff for every open set U of X, each component of U is open in X.
2. A topological space X is locally path connected iff for every open set U of X each path component of U is open in X.
3. If X is a topological space then each path component of X lies in a component of X.
4. If X is a topological space and locally path connected then the components and the path components of X are the same.

# Compactness

## Compact Spaces

**Cover**—Let X be a topological space. A collection of subsets of X is said to cover X (covering of X), if the union of elements of $a$ is equal to X.

**Open Covering**—A covering of X is called open covering if its elements are open subsets of X.

**Compact Space**—A topological space X is compact if every open covering a of X contains a finite sub collection that coveres X.

**Finite Intersection Condition**—A collection G of subsets of topological space satisfies Finite Intersection Condition if for every finite subcollection $\{C_1, C_2, \ldots, C_n\}$ of G, the intersection $\bigcap_{i=1}^{n} G \neq \phi$.

## Some Important Theorems

1. Let $y$ be a subspace of topological space X then Y is compact iff every covering of Y be sets open in X contains a finite subcollection covering Y.
2. Every closed subset of a compact space is compact.
3. Every compact set of a Hausdorff space is closed.
4. If Y is a compact subset of the Hausdorff space X and $x_0 \notin Y$, then there exist disjoint open sets U and V of X, such that $x_0 \in U$ and $y \subset V$.
5. The image of a compact topological space under continuous map is continuous.
6. Let $f : X \to Y$ be bijective continuous function. If X is compact and Y is Hausdorff, then $f$ is homeomorphism.
7. The product of finitely many compact spaces is compact.
8. **Tube Lemma**—Consider the product space $X \times Y$, where Y is compact. If N is an open set of $X \times Y$ containing the 1 slice $x_0 \times Y$, of $X \times Y$, then N contains some tube $\omega \times Y$ about $x_0 \times Y$, where $\omega$ is a neighbourhood of $x_0 \in X$.
9. Let X be a topological space. Then X is compact iff for every collection G of closed sets in X satisfying finite intersection condition the intersection $\cup_{C \in G} C \neq \phi$.
10. The topological space X is compact iff for every collect a of subsets of X, satisfying the finite intersection condition, the intersect ion.

    $\cap_{A \in a} \overline{A} \neq \phi$, where $\overline{A}$ is the closure of $A \in a$.

## Compact Sets in the Real Line

1. Let X be a simply ordered set having the least upper bound property. In the order topology, each closed interval in X is compact.
2. Every closed interval in R is compact.
3. A subset $A \subset R^n$ is compact iff it is closed and bounded in the euclidean metric $d$ or square metric P.
4. **Maximum and Minimum Value Theorem**—Let $f : X \to Y$ be continuous, where Y is an ordered set in the order topology. If X is compact, then there C, $d \in X$ such that $f(C) \leq f(x) \leq f(d)$ for every $x \in X$.
5. Let X be a (non empty) compact Hausdorff space. If every point of X is a limit point of X, then X is uncountable.
6. Every closed interval R is uncountable.

## Limit Point Compactness

**Limit Point Compact Space**—A topological space X is limit point compact if every infinte subset of X has a limit point.

**Sequentially Compact**—If every sequence in a topological space has a convergent subsequence, then Y is sequentially compact.

**Countably Compact Space**—A topological space X is countably compact if every countable open covering of X contains a finite subcollection covering X.

## Some Important Theorems

1. Compactness implies limit point compactness (converge is not true).
2. **Lebesgue number lemma**—Let a be an open covering of metric space $(X, d)$. If X is compact, there is $\delta > 0$ such that each subset $(x - \delta, x + \delta) \subset X$, there exist $o \in a$ sch that $(x - \delta, x + \delta) \subset 0$.
3. **Uniform Continuity Theorem**—If $f : X \to Y$ is a continous map of the compact metric space $(X, d_x)$ to the metric space $(Y, d_Y)$. Then $f$ is uniformly continuous *i.e.* given $\in > 0$, there exist $\delta > 0$ such that $x, Y \in X$, $d_x(x, y) < \delta \Rightarrow d_y(f(x), f(y)) \subset \in$.
4. Let X be a metrizable space. Then following are equivalent—
   (i) X is compact
   (ii) X is limit point compact
   (iii) X is sequentially compact

5. If X is countably compact space then it is limit point compact space

## Local Compactness

**Locally Compact Space**—A topological space X is locally compact at X there is some compact subspace C of X that contains a neighbourhood of $x$.

If topological space X is locally compact at each $x \in X$, then X is locally compact.

**One Point Compactification**—Let X be a locally compact Hausdorff space. Let $Y = X \cup \{\infty\}$. The topology Y defines are collection of open sets in Y to be all sets such as—

(1) U–U where U is open subset of X.

(2) Y–C, where C is a compact subset of X.

Then topological space Y is one-point compactification of X.

## Some Important Theorems

1. Let X be a locally compact Hausdorff space which is not compact. Let Y be the one point compactification of X. Then (a) Y is compact Hausdorff space (b) X is subspace of Y (c) the set $Y - X$ consists of a single point (d) $\overline{X} = Y$.
2. Let X be a Hausdorff space. Then X is locally compact at $x$ iff for every neighbourhood U of $x$, there is neighbourhood V of $x$, such that $\overline{V}$ is compact and $\overline{V} \subset \overline{U}$.
3. Let X be a locally compact Hausdorff space and Y be a subspace of X. If Y is closed (or open) in X. Then Y is locally compact.
4. A space X is homeomorphic to an open subset of a compact Hausdorff space iff X is locally compact Hausdorff.
5. If $P : X \to Y$ is a quotient map and if Z is a locally compact Hausdorff space, then the map $\pi = P \times i_2 : X \times Z \to Y \times Z$ is a quotient map.
6. Let $P : A \to B$ and $g : C \to D$ be quotient maps. If B and C are locally compact Hausdorff spaces, then $P \times q : A \times C \to B \times D$ is a quotient map.

## The Countability Axioms

**Countable Basis at $x$**—A topological space X have a countable basis of $x$ if there is a countable collection B of neighbourhood $\delta$ of $x$, such that each neighbourhood of $x$ contains atleast one of the elements of B.

**First Countability Axiom**—A topological space has a countable basis at each of its points is called first coutability axiom.

**Second Countability Axiom**—A topological space X satisfies second countability axiom if X has a countable basis for its topology.

**Dense Set**—A subset Y of topological space X is dense in X if $\overline{Y} = X$.

**Lindel of Space**—A topological space for which every open covering contains a countable subcovering.

**Separable Space**—A topological space having a countable dense subset.

## Some Important Theorems

1. Let X be a topological space satisfying first countability axiom.
   (a) The point $x \in \overline{A}$, closure of $A \subset X$ iff there is a sequence of points of A converging to $x$.
   (b) The function $f : X \to Y$ is continuous iff for every convergent sequence $\{x_n\}$ in X, converging to $x$ the sequence $\{f(x_n)\}$ converges to $f(x)$.
2. A subspace of a first countable space is first countable and a countable product of first countable space is first countable.
3. A subspace of a second countable space is second countable and a product of second countable spaces is second countable.
4. If topological space has a countable basis then
   (a) Every open covering of X contains a countable subcollection covering X.
   (b) There exist a countable subset of X which is dense in X.
5. The product of two Lindel of spaces need not be lindel of.
6. A subspace of a space having a countable dense subset need not have a countable dense subset.

## The Separation Axioms

**Regular Space**—If one point sets are closed in X. Then $x$ is regular if for each pair consisting of a point $x$ and a closed set B disjoint from $x$, there exist disjoint open sets containing $x$ and B respectively.

**Normal Space**—A topological space X is normal if for each pair A, B, $A \cap B \neq \phi$ of closed sets of X, there exist disjoint open sets $\theta_1$ and $\theta_2$, $\theta_1 \cap \theta_2 \neq \phi$, such that $A \subset \theta_1$ and $B \subset \theta_2$.

**The Separation Axioms**—Given a topological space, the separation axioms states,

$T_1$ : Given two distinct points $x$ and $y$, there is an open set that contains $y$ but not $x$.

$T_2$ : Given two distinct points $x$ and $y$, there are disjoint open sets $\theta_1$ and $\theta_2$ such that $x \in \theta_1$ and $y_1 \in \theta_2$.

$T_3$ : In addition to $T_1$, given a closed set F and a point $x$ not in F, there are disjoint open sets $\theta_1$ and $\theta_2$ such that $x \in \theta_1$ and $F \subset \theta_2$.

$T_4$ : In addition to $T_1$, given two disjoint closed sets $F_1$ and $F_2$ there are disjoint open sets $\theta_1$ and $\theta_2$ such that $F_1 \subset \theta_1$ and $F_2 \subset \theta_2$.

> A topological space satisfying $T_1$ is called a Tychonoff space.
>
> A topological space which satisfies $T_2$ is called a Hausdorff space.
>
> A topological space which satisfies $T_3$ is called a regular space.
>
> A topological space which satisfies $T_4$ is called a normal space.

Here

(i) The condition $T_1$ is equivalent to the statement that each set consisting of single point is closed.

(ii) $T_4 \Rightarrow T_3 \Rightarrow T_2 \Rightarrow T_1$, *i.e.* every normal space is regular space, every regular space is Hausdorff space and every Hausdorff space is Tychonoff space.

## Some Important Theorems

1. Let X be a topological space. Let one point sets in X be closed.
   (a) X is regular iff given a point $x \in X$ and neighbourhood U of $x$, there is a neighbourhood V of $x$ such that $\overline{V} \subset U$.
   (b) X is normal iff given a closed set A and open set U, $A \subset V$, there is an open set V, $A \subset U$ such that $\overline{V} \subset U$.
2. (a) A subspace of Hausdorff space is Hausdorff and product of Hausdorff space is Housdorff.
   (b) A subspace of regular space is regular and product of regular spaces is regular.
   (c) A subspace of normal space need not be normal and product of normal spaces need not be normal.
3. Every metrizable space is normal.
4. Every compact Hausdorff space is normal.
5. Every regular space with countable basis is normal.
6. Every well ordered set X is normal in the order topology.

## The Urysohn Lemma

**Sets Separated by Continuous Function**—Let X be a topological space A, $B \subset X$, $f : X \rightarrow [0, 1]$ a continuous function such that $f(A) = \{0\}$ and $f(B) = \{1\}$, then sets A and B are separated by continuous function.

**Completely regular space**—A topological space X is completely regular if one point sets are closed in X and if every $a \in X$, every closed set B, $a \in B$, there exists a continuous function $f : X \rightarrow [0, 1]$ such that $f(a) = 0$ and $f(B) = \{1\}$.

## Some Important Theorems

1. **Urysohn Lemma**—Let X be a normal space; A and B are disjoint closed subsets of X and $[a, b]$ closed interval of R. Then there exist a continuous map $f : X \rightarrow [a, b]$ such that for every $x \in A$, $f(x) = a$ and for every $x \in B$, $f(x) = b$.
2. **Tietze Extension Theorem**—Let X be a normal space. Let A be closed subset of X.
   (a) Any continuous map of A into the closed interval $[a, b]$ of R may be extended to a continuous map of all of X into $[a, b]$.
   (b) Any continuous map of A into R may be extended to a continuous map of X into R.
3. **Strong form of Urysohn Lemma**—Let A and B closed disjoint subsets of the normal space X. There exist a continuous function $f : X \rightarrow [0, 1\}$ such that $f^{-1}\{0\} = A$ and $f(B) = \{1\}$ iff A is a $G_\delta$ set in X. A is a $G_\delta$ set in X if A is the intersection of a countable collection of open sets of X.
   (1) **Urysohn Metrization Theorem**—Every regular space X with countable basis is metrizable.

(2) **Imbedding Theorem**—Let X be Hausdorff space $\{f\}_{\alpha \in J}$ is a collection of continuous function $f_\alpha : X \to R$ satisfying for each $x_0 \in X$ and each neighbourhood U of $x_0$, there is an index $\alpha$ such that $f_\alpha$ is positive at $x_0$ and vanishes outside U. Then function $F : X \to R^J$ defined by $F(x) = [f_\alpha(x)]_{\alpha \in J}$ is an imbedding of X in $R^J$.

## Partitions of Unity

**Support of $\phi$**—If $\phi : X \to R$, the support of $\phi$ is the closure of set $\phi$ (R–{0}) *i.e.*, If $x$ lies outside the support of $\phi$, there is some neighbourhood of $x$ on which $\phi$ vanishes.

**Partition of Unity**—Let $\{U_1, U_2, \ldots U_n\}$ be finite indexed open covering of the space X. An indexed family of countable functions.

$\phi_i : X \to [0, 1]$, $i = 1, 2, \ldots, n$ is called partition of unity dominated by $\{U_i\}$ if :

(a) (support) $\subset U_i$ for each $i$.

(b) $\sum_{i=1}^{n} \phi_i (x) = 1$, for each $x$.

***m*-manifold**—An $m$-manifold is a Hausdorff space with a countable basis that each point $x \in X$, has a neighbourhood that is homeomorphic with an open subset of $R^m$.

**Point-Finite Indexed**—An indexed collection $\{A_\alpha\}$ of subsets of X is point-finite indexed family if each $x \in X \Rightarrow x \in A_\alpha$ for only finitely many values of $\alpha$.

**Locally Finite Indexed**—Let X be a topological space. An indexed family $\{A_\alpha\}_{\alpha \in j}$ of subsets of X is locally finite indexed family if each point of X has a neighbourhood that intersects $A_\alpha$ for only finitely many values of $\alpha$.

## Some Important Theorems

1. **Existence of Finite Partitions of Unity**—Let $\{U_1, \ldots\ldots, U_n\}$ be a finite open covering of the normal space X. Then there exists a partition of unity dominated by $\{U_i\}$.
2. If X is compact $m$-manifold, then X can be imbedded in $R^N$ for some positive integer N.

## Tychonoff Theorem

(1) **Tychonoff Theorem**—Any arbitrary product of compact spaces is compact in the product topology.

(2) **Completely Regular Spaces**—A space X is completely regular if one-point sets are closed in X and for each $x_0 \in X$ and each closed set A, $x_0 \notin A$, there is a continuous function $f : X \to [0, 1]$ such that $f(x_0) = 1$ and $f(A) = \{0\}$.

## Some Important Theorems

1. A subspace of a completely regular space is completely regular and the product of completely regular spaces is completely regular.
2. If X is completely regular, then X can be imbedded in $[0, 1]^J$ for some J.
3. Let X be a topological space. Then following are equivalent :
   (a) X is completely regular.
   (b) X is homeomorphic to a subspace of compact Hausdorff space.
   (c) X is homeomorphic to a subspace of a normal space.
4. Every Hausdorff topological group is completely regular.

## The Stone-Cech Compactification

**Compactification of Space**—A compactification of a space X is compact Hausdorff space Y containing X such that X is dense in Y. (*i.e.*, $\overline{X} = Y$).

**Equivalent Compactification**—Compactification $y_1$ and $y_2$ of X are equivalent if there is a homeomorphism $h : Y_1 \to Y_2$ such that $h(x) = x$ for every $x \in X$.

Compactification Y of X, induced by $h$ : If $h : x \to z$ is an imbedding of X in the compact Housdorff space $z$, then $h$ inducess a compactification $y$ of $x$.

**Stone-Cech Compactification**—The compactification of X induced by $h$ is called the stone-cech compactification of X, $\beta$ (X).

## Some Important Theorems

1. Let X be completely regular $\beta(X)$ be its stone-Cech compactification. Then every bounded continuous real valued function of X can be uniquely extended to continuous real valued function of $\beta(X)$.
2. Let $A \subset X$; $f : A \to Z$ be continuous map of A into Hausdorff space Z. There is at most one extension of $f$ to a continuous function.

$$g : \overline{A} \to Z$$

# Metrization Theorems

## Local Finiteness

**Locally Finite Collection**—Let X be a topological space. A collection $a$ of subsets of X are locally finite if every point of X has a neighbourhood that intersect only finitely many elements of $a$.

**Countably Locally Finite**—A collection B of subsets of X is countably locally finite if B can be written as countable union of collections $B_n$, each of which is locally finite.

## Some Important Theorems

1. Let a be a locally finite collection of subsets of X. Then
   (a) Any subcollection of a is locally finite.
   (b) The collection $B = \{\overrightarrow{A}\}_{A \in a}$ of closures of the elements of a is locally finite.
   (c) $\overline{U_{A \in a} A} = U_{A \in a} \overline{A}$

**The Nagata Smirnov Metrization Theorem**

**$G_\delta$ set**—A subset A of topological space X is $G_\delta$ set in X if it is equals to the intersection of a countable collection of open subsets of X.

## Some Important Theorems

1. Let X be a regular space with a basis B that is countably locally finite. Then X is normal and every closed set of X is $G_\delta$ set in X.
2. Let X be a regular space with basis B that is countably locally finite. Then X is metrizable.

## The Nagata–Smirnov Theorem

**Refinement**—Let a be a collection of subsets of the space X. A collection B of subset of X is called refinement of a (refine a) if for each $B \in B$, there is $A \in a$ such that $B \subset A$.

If $B \in B$ are open sets, then B is an open refinement of $a$.

If $B \in B$ are closed sets, then B is a closed refinement of $a$.

**Locally Discrete Collection**—A collection $a$ of subsets of X is locally discrete if each point of X has neighbourhood that intersects at most one element of $a$.

**Countably locally discrete collection**— A collection B is countably locally discrete ($a$-locally discrete) if it is equal to a countable union of locally discrete collections.

## Some Important Theorems

1. Let X be a metrizable space. If $a$ is an open covering of X, then there is a collection D of subsets of X such that,
   (a) D is an open covering of X.
   (b) D is a refinement of $a$.
   (c) D is countably locally finite.
2. Let X be metrizable space. Then X has a basis that is countable locally finite.
3. **Bing metrization theorem**—A space X is metrizable iff it is regular and has a basis that is countably locally discrete.

## Para Compactness

**Para Compact Space**—A space X is para compact if it is Housdorff and every open covering $a$ of X has a locally finite open refinement B that covers X.

## Some Important Theorems

1. Every para compact space X is normal.
2. Every closed subspace of a para compact space is para compact.
3. An arbitrary subspace of a para compact space and product of para compact spaces need not be para compact.
4. **Stone's theorem**—Every metrizable space is para compact.
5. Let $x$ be a regular space. Then following are equivalent. Every open covering of $x$ has a refinement. That is—
   (a) An open covering of $x$ and countably locality finite.
   (b) A covering of $x$ and locally finite.
   (c) A closed covering of $x$ and locally finite.
   (d) An open covering of $x$ and locally finite.

## Smirnov Metrization Theorems

**Locally Metriable Theorem**—A space X is locally metrizable if every $x \in X$ has a nighbourhood U that is metrizable in the subspace topology.

**Smirnov Metrization Theorem**—A space X is metrizable iff it is para compact and locally metrizable.

## Homotopy of Path

**Homotopy**—If $f$ and $f'$ and continuous map of the space X into space Y, then $f$ is homotopic to $f'$ ($f = f'$) if there is one continuous map $F : X \times [0,$

1] $\rightarrow$ Y such that $F(x, 0) = f(x)$ and $F(x, 1) = f'(x)$ for each $x \in X$.

The map F is called a homotopy between $f$ and $f'$.

**Path Homotopic**—Two paths $f$ and $f'$, mapping the interval [0, 1] into X are said to be path homotopic $f =_p f'$ if they have the same intial $x_0$ and same final path $x_1$ and if there is continuous map. $F[0, 1] \times [0, 1] \rightarrow X$ such that

$$\begin{aligned} F(s, 0) &= f(s) \\ \text{and} \quad F(s, 1) &= f'(s) \\ F(0, t) &= x_0 \\ \text{and} \quad F(1, t) &= x_1, \end{aligned}$$

for each $s \in$ I and each $t \in [0, 1]$. The map F is called path homotopy between $f$ and $f'$.

**Straight Line Homotopy**—If $f, g : X \rightarrow R^2$ such that the may $F(x, t) = (1 - t) f(x) + tg(x)$, is a homotopy between $f$ and $g$. Then F is called straight line homotopy.

**Composition**—If $f$ is a path in X from $x_0$ to $x_1$ and $g$ is a path in X from $x_1$ to $x_2$ then composition $f * g$ of $f$ and $g$ is a path $h$ given by.

$$h(s) = \begin{cases} f(2s) & \text{if } S\,\Sigma \left[0, \frac{1}{2}\right] \\ g(2s-1) & \text{if } S\,\Sigma \left[\frac{1}{2}, 1\right] \end{cases}$$

**Contractiable**—A space is contractible if the identity map $ix : X \rightarrow X$ is homotopic to a constant map.

## Fundamental Group

Fundamental group (First homotopy group) : Let X be a topological space; let $x_0 \in X$. The path in X, that beings and ends at $x_0$ is called a loop based at $x_0$, with the operation * is called fundamental group $\Pi_1(X, x_0)$ of X relative to the base point $x_0$.

**Simply Connected Space**—In simply connected space X and two paths having the same initial and final points are path homotopic.

**Homeomorphism induced by $h$**—

Let $h : (X, x_0) \rightarrow (Y, y_0)$ be a continuous map and $h^* : \pi_1(X, x_0) \rightarrow \pi_1(Y, y_0)$ is such that $h^*([f]) = [h, \text{o}f]$.

The map $h^*$ is called homeomorphism induced by $h$, relative to the base point $x_0$ zero homeomorphism : Let $A \subset R^n$, $h : (A, a_0) \rightarrow (Y, y_0)$. If $h$ is extandable to a continuous map of $R^n$ into $y$. Then $h^*$ is zero homeomorphism.

## Some Important Theorems

1. If X is path connected and $x_0, x_1 \in X$ then $\pi$, $(X, x_0)$ is isomorphic to $\pi$, $(X, x)$.
2. In a simply connected space X, any two paths having the same initial and final points are path homotopic.
3. If $h : (X, x_0) \rightarrow (Y, y_0)$, $K : (Y, y_0) \rightarrow (Z, z_0)$, then$(Koh)^* = K^* \, o \, h^*$.
4. If $i : (X, x_0) \rightarrow (X, x_0)$ is identity map, then $i^*$ is the identity homeomorphism.
5. If $h : (X, x_0) \rightarrow (Y, y_0)$ is homeomorphism of X with Y, then $h^*$ is anisomorphism of $\pi$, $(X, x_0)$ with $\pi$, $(Y, y_0)$.

## Covering Spaces

**Evenly Covered Set and Slice**—Let $P : E \rightarrow B$ be continuous surjective map. The open set $U \subset B$ is said to be evenly convered by $p$ if the inverse $p^{-1}(U)$ can be written as union of disjoint sets $V_2 \subset E$ such that for each 2, the restriction of P to $V_2$ is a homeomorphism of $V_2$ onto U. Each of the sets $V_2$ is called slice.

**Covering map and Convering Space**—Let $p : E \rightarrow B$ be continuous and surjective. If every point $b \in B$ has neighbourhood U that is evenly covered by $p$, then $p$ is called a covering map and E a covering sapce of B.

## Some Important Theorems

1. If $p : E \rightarrow B$ is a covering map, then for each $b \in B$, the subject $p^{-1}(b) \subset E$ has the discrete topology.
2. The map $P : R \rightarrow S^1$ given by $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map, where $S^1$ is the unit sphere in $R^2$.
3. If $p : E \rightarrow B$ is a covering map then P is an open map.

## Fundamental Group of the Circle

Lifting of $f$ : Let $p : E \rightarrow B$ be a map. If $f$ is continuous mapping of some spaces X into B, a lifting of $f$ is a map $f : X \rightarrow E$ such that $p \, o \, f = f$.

Universal covering space of B : If E is a simple connected space and if $p : E \rightarrow B$ is a covering map, then E is a universal covering space of B.

## Some Important Theorems

1. If $p : E \rightarrow B$ be covering map, let $p(e_0) = b_0$. Any path $f : [0, 1] \rightarrow B$ beginning at $b_0$ has a unique lifting to a path. $f$ in E beginning at $e_0$.
2. Let $p : E \rightarrow B$ be a covering map, let $p(e_0) = b_0$ and let $F : [0, 1] \rightarrow B$ be continuous map, with $F(0, 0) = b_0$. There is a lifting of F to continuous map $F : [0, 1] \rightarrow E$ such that $F(0, 0) = e_0$. If F is a path homotopy, then F is a path homotopy.
3. Let $p : E \rightarrow B$ be a covering map; let $p(e_0) = b_0$ let $f$ and $g$ be two paths in B from $b_0$ to $b'_1$, let $f$ and $g$ be their respective lifting to paths in E beginning at $e_0$. If $f$ and $g$ are path homotopic, then $f$ and $g$ end at the same point of E and are path homotopic.
4. The fundamental group of the cricle is finite cyclic.
5. Let $p : (E, e_0) \rightarrow (B, b_0)$be a covering map. If E is path connected, then there is surjection $\phi : \pi, (B, b_0) \rightarrow p^{-1}(b_0)$. If E is simply connected, $\phi$ is bijection.

## The Fundamental Group of Punctured Plane

**Strong Deformation Retration**—Let A be a subspace of X. Then A is said to be a strong deformation retract of X if three is continuous map $H : X \times [0, 1] \rightarrow X$ such that—

$$H(x, 0) = x \text{ for } x \in X$$
$$H(x, 1) \in A \text{ for } x \in X.$$
$$H(a, 1) = a$$

for $a \in A$ and $t \in [0, 1]$

The map H is called strong deformation retraction.

## Some Important Theorems

1. Let $x_0 \in S^1$; the unit sphere in $R^2$ the inclusion mapping $J : (S^1, x_0) \rightarrow (R_2 - 0, x_0)$ induces an isomorphism of fundamental groups.
2. Let $x_0 \in S^{n-1}$, the unit sphere in $R^n$ the inclusions : $(A, a_0) \rightarrow (X, a_0)$ induces an isomorphism of fundamental groups.
3. Let A be a strong deformation retract of X. Let $a_0 \in A$. Then the inclusion maps : $(A, a_0) \rightarrow (X, a_0)$ induces an isomorphism of fundamental groups.

## Fundamental Group of $S^n$

1. **Special Van Kampen Theorem**—Let $X = U \cup V$, U and V are open in X and $U \cap V$ is path connected. Let $x_0 \in U \cap V$. If both inclusions $i : (U, x_0) \rightarrow (X, x_0)$ and $J : (V, x_0) \rightarrow (X, x_0)$, induces zero homeomorphism of fundamental groups, then $\pi, (X, x_0) = 0$.
2. For $n \geq 2$, the $n$–sphere $S^n$ is simply connected.
3. $R^n - 0$ is simply connected if $n > 2$.
4. $R^n$ and $R^2$ are not homeomorphism for $n > 2$.

**Fundamental Group of Surfaces** :

**Projective Plane**—The projective plane $p^2$ is the space obtained from $S^2$ by identifying each $x \in S^2$ with its antipodal point $-x$.

## Some Important Theorems

1. $\pi, (x \times y, x_0 \times y_0)$ is isomorphism with $\pi, (X, x_0) \times \pi, (y, y_0)$.
2. The fundamental group of the torus $T = S' \times S'$ is isomorphic to the group $Z \times Z$.
3. The projective plane $P^2$ is a surface and the map $P : S^2 \rightarrow P^2$ is a covering map.
4. $\pi, (p^2, y)$ is a group of order 2.
5. The fundamental group of the double torus $T_2$ is not Abelian.

## Essential and Inessential Maps :

**Essential and Inessential map**—A map $hx \rightarrow y$ is inessential map if $h$ is homotopic to a constant map, otherwise essential.

## Some Important Theorems

1. Let $h : S' \rightarrow y$. Then following are equivalent :
   (i) $h$ is inessential
   (ii) $h$ can be extended to a continuous map $g : B^2 \rightarrow y$.
2. Let $h : X \rightarrow y$, If $h$ in essential, then $h^*$ is the zero homomorphism.
3. Let T be a closed triangular region $R^2$. Let $BdT$ de note the union of the edges of T. There is no continuous map $f : T \rightarrow BdT$ that maps each edge of T into itself.

## Homotopy Type

**Homotopy Equivalence and Inverse**—A continuous map $f : X \rightarrow Y$ is homotopy equivalence, if there is a continuous map $g : y \rightarrow$

X such that $gof$ is homotopic to the identity map $i_x$ of X and $fog$ is homotopic to the identity map $i_y$ of $y$. The map $g$ is homotopy inverse for the map $f$.

## Theorems

1. Let $h$, K : X $\to y$, $h(x_0) = y_0$ and $K(x_0) = y_1$. If $h$ and K are homotopic then there is a path $d \in y$ from $y_0$ to $y_1$ such that $K = \overrightarrow{d}\ oh_*$. If $y_0 = y_1$ and if the base point to remains fixed during the homotopy, then $h_* = k_*$.
2. Let $h$, $k$ : X $\to y$ and $h(x_0) = y_0$, $k(x_0) = y_1$. Suppose that $h$ and Kare homotopic. If $K_*$ is injective (or surjective or zero homomorphism), then so is $h_*$. If $h$ is homotopic to a constant map, then $h_*$ is the zero homeomorphism.
3. Let $f$ : X $\to y$ be continuous, $f(x_0) = y_0$. If $f$ is homotopy equivalence, then $f_* : \pi_1 (X, x_0) \to \pi_i (Y, y_0)$ is an isomorphism.

   Jordan separation and curvetheorem :

   Arc : An arc is a space homeomorphic to the unit interval [0, 1]. Simple closed curve : A simple closed curve is a space homeomorphic to the circle S′.

## Some Important Theorems

1. Let $a$, $b \, \Sigma \, S^2$ and A be a compact space. Let $f : A \to S^2 - a - b$ be continuous map. If $a$ and $b$ lie in same component of $S^2 - f(A)$, then $f$ is inessential.
2. **Jordan Separation Theorem**—Let C be a simple closed curve in $S^2$. Then $S^2 - C$ is not connected.
3. Let A and B be closed connected subsets of $S^2$ whose intersection consists of precisely two points then $A \cup B$ separates $S^2$.
4. **Homotopy existence lemma**—Let X and $X_*$ [0, 1] be normal, Let A be a closed subset of X. If $f : A \to R^2 - 0$ is continuous map and $f$ is inessential, then $f$ can be extended to a continuous may $g : X \to R^2 - 0$.
5. **Borsuk Theorem**—Let $x$ be a compact subset of $R^2$. If 0 lies in a bounded component of $R^2 - X$, then the inclusion map $j : X \to R^2 - 0$ is essential (and conversely).
6. **Non-Separable Theorem**—No arc separates $R^2$, no space homeomorphic to ball $B^2$ separates $R^2$.
7. **Brouwer theorem on invarience of domain for $R^2$**—If U is an open subset of $R^2$ and $f$ : U $\to R^2$ is contiuous and injective, then $f(U)$ is open in $R^2$ and $f$ is an imbedding.
8. Let X be the union of two open sets U and V suppoose that $U \cap V$ can be written as the union of two disjoint open sets A and B. Let $a \in A$ and $b \in B$ and $a$ and $b$ are joined by paths in U and in V. then $\pi_1 (X, a) \neq 0$.
9. Let $X = U \cup V$, where U and V are open sets and $U \cap V = A \cup A' \cup B$, where A, B, A′ are disjoint open sets. Let $a \in A$, $a' \in A'$ and $b \in B$. Suppose that $a$, $a'$ and $b$ are joined by paths in U and V-then $\pi$; $(X, a)$ is not infinite cyclic.
10. **Non-Separation Theorem**—Let A be an arc in $S^2$. Then $S^2 - A$ is connected.
11. **Jordan Curve Theorem**—Let C be a simple closed curve in $S^2$. Then $S^2 - C$ has precisely two components $W_1$ and $W_2$ of which C is the common boundary.

## OBJECTIVE TYPE QUESTIONS

1. Let X be a set for which a topology T is defined, then—
   (A) Only X is in T (B) Only X is not in T
   (C) $\phi$ and X are in T (D) None of these
2. Let X be a set for which a topology T is defined, then—
   (A) The union of the elements of finite subcollection of T is in T
   (B) The union of the elements of any subcollection of T is in T
   (C) The union of the elements of any subcollection of T is not in T
3. Let X be a set for which a topology T is defined, then—
   (A) The intersection of the elements of any finite subcollection of T is in T
   (B) The intersection of the elements of any subcollection of T is in T
   (C) The intersection of the elements of any finite subcollection of T is not in T
   (D) None of these

4. Let X be a set for which a topology T is defined then following is true—
   (A) $\phi$ and X are in T
   (B) The union of the elements of any subcollection of T is in T
   (C) The intersection of the elements of any finite subcollection of T is in T
   (D) None of these

5. Let (X, T) is a topological space, $U \subset X$ is open set—
   (A) U is a subset of T
   (B) U is an element of T
   (C) U is a superset of T
   (D) None of these

6. If B is a basis for a topology on X, the topology T generated by B is defined as—
   (A) Subset $U \subset X$ is closed in (X, T) if for each $x \in U$, there is a basis element $B \in B$ such that $x \in B$ and $B \subset U$
   (B) Subset $U \subset X$ is open in (X, T) if for each $x \in U$, there is a basis element $B \in B$ such that $x \in B$ and $B \subset U$
   (C) Subset $U \subset X$ is open in (X, T) if for some $x \in U$, there is a basis element $B \in B$ such that $x \in B$ and $B = U$
   (D) None of these

7. If B is the collection of all intervals in the real line $(a, b) = \{x \mid a < x < b|\}$, the topology generated—
   (A) Standard topology on the real line
   (B) Discrete topology on real line
   (C) Hausdorff topology on real line
   (D) None of these

8. A subbasis $\xi$ for a topology on X is a collection of subsets of X—
   (A) Whose union equals X
   (B) Where union is subset of X
   (C) Whose union superset of X
   (D) None of these

9. Topology generated by the subbasis $\xi$ is defined as—
   (A) The collection T of all unions of any intersections of elements of $\xi$
   (B) The collection T of all unions of finite intersections of elements of $\xi$
   (C) The collection T of all intersection of any intersections of elements of $\xi$
   (D) None of these

10. Let X be a set with order relation. The collection B is a basis for topology on X and is called order topology. If B basis for X is a collection of all sets of the form—
    (A) All open intervals $(a, b) \in X$
    (B) All intervals of the form $[a_{01}, b]$, $a_0$ is the smallest element (if any) of X
    (C) All intervals of the form $[a, b_0]$, $b_0$ is the largest element [if any] of X
    (D) All of these

11. If X and Y are topological spaces. The product topology on $X \times Y$ is—
    (A) Topology having as basis the collection B of all sets of the form $U \times V$, where U is a closed subset of X and V is a closed subset of Y.
    (B) Topology having as basis the collection B of all sets of the form $U \times V$, where U is an open subset of X and V is an open subset of Y.
    (C) Topology having as basis the collection B of all sets of the form $U \times V$, where U and V are nulsets.
    (D) None of these

12. If X is a topological space with topology T and $Y \subset X$, the collection $T_Y = \{Y \cap U \mid U \in T]$ is a topology on Y. Then $(Y, T_Y)$ is called ......... of (X, T).
    (A) Subspace topology
    (B) Super spacetopology
    (C) Discreate topology
    (D) None of these

13. A subset A of topological space X is closed set if—
    (A) (X–A) is open (B) A is open
    (C) A is closed (D) None of these

14. Let (X, T) is given topological space, $A \subset X$, then interior of A—
    (A) Is the union of all open sets contained in A
    (B) Is the intersection of all open sets contained in A

(C) Is the union of all closed sets contained in A

(D) None of these

15. Let (X, T) is given topological space, A $\subset$ X, then closure of A.

(A) Is the intersection of all closed sets containing A

(B) Is the union of all closed sets containing A

(C) Is the intersection of all open sets contianing A

(D) None of these

16. Neighbourhood of $x$ is—

(A) An open set U containing $x$

(B) An closed set U contianing $x$

(C) A null set

(D) None of these

17. Given (X, T) a topological space and A $\subset$ X, a point $x \in$ X is a limit point of A—

(A) If every neighbourhood of $x$ intersects A in some point other than $x$

(B) If every neighbourhood of $x$ intersects A in all point other than $x$

(C) If every neighbourhood of $x$ intersects A in some point including $x$

(D) None of these

18. A topological space X is Hausdorff space if for each pair $x_1 \neq x_2 \in$ X.

(A) There exist neighbourhood $U_1$ and $U_2$ of $x_1$ and $x_2$ respectively such that $U_1$ and $U_2$ are disjoint

(B) There exist neighbourhood $U_1$ and $U_2$ of $x_1$ and $x_2$ respectively such that $U_1$ and $U_2$ are not disjoint

(C) There exist no neighbourhood $U_1$ and $U_2$ of $x_1$ and $x_2$ respectively such that $U_1$ and $U_2$ are not disjoint

(D) None of these

19. If X is any set, T is a collection of all subsets of X then topology (X, T) is—

(A) A discrete topology

(B) Indiscrete topology

(C) Trivial topology

(D) None of these

20. If X is any set, T = {$\phi$, X}, then topology (X, T).

(A) A discrete topology

(B) Indiscrete topology

(C) Trivial topology

(D) Both (A) and (B)

21. Let X be a set; let B be a basis for a topology T on X. Then T is equal to—

(A) The collection of all intersection of elements of B

(B) The collection of all unions of elements of B

(C) The collection of all sum of elements of B

(D) None of these

22. Let B and B′ be basis for the topologies T and T′ respectively on X. Then T′ is finer than T is equivalent to—

(A) For each $x \in$ X and each basis element B $\in$ B containing $x$, there is a basis element B′ $\in$ B′ such that $x \in$ B′ $\subset$ B.

(B) For some $x \in$ X and some basis element B $\in$ B contianing $x$, there is a basis element B′ $\in$ B′ such that $x \in$ B′ $\subset$ B.

(C) For $x \in$ X and basis element B $\in$ B containing $x$, there is a basis element B′ $\in$ B′ such that $x \in$ B′ $\subset$ B.

(D) None of these

23. Let X be a topological space suppose that collection C is a collection of open sets of X such that each $x \in$ X and each open set U of X, there is an element C$\in$ C such that $x \in$ C $\subset$ U.

(A) Then C is not a basis for the topology of X

(B) Then C is a basis for the topology of X

(C) Then C is sub basis for the topology of X

(D) None of these

24. The lower limit topology T′ an real line R—

(A) Is strictly finer than the standard topology T

(B) Is inferior than the standard topology T

(C) Is finer than the standard topology T

(D) None of these

25. If B is a basis for the topology of X and C is a bais for the topology of Y, then—
    (A) The collection D = {B × C | B∈B and C ∈ C} is a basis for the topology of X × Y
    (B) The collection D = {B × C | B∈B and C∈C} is not a basis for the topology of X × Y
    (C) The collection D = {B × C | B∈B and C∈C} is not a basis for the topology of X × X
    (D) None of these

26. If B is a basis for the topology of X, then—
    (A) The collection $B_Y$ = {B∩Y | B∈B} is not a basis for the subspace topology on X
    (B) The collection $B_Y$ = {B∩Y | B∈B| is a basis for the subspace topology on X
    (C) The collection $B_Y$ = {B∩Y | B∈B| is a basis for the topology on X
    (D) None of these

27. Let Y be a subspace of X. If U is open in Y and Y is open in X—
    (A) Then U is open in X
    (B) Then U is closed in X
    (C) Then U is null set in X
    (D) None of these

28. If X is an ordered set in the order topology and if Y is an interval or a ray in X—
    (A) Then the subspace topology and order topology on Y are same
    (B) Then the subspace topology and order topology on Y are different
    (C) Then the subspace topology and order topology on Y are open
    (D) None of these

29. If A is a subspace of X and B is a subspace of Y, then—
    (A) Product topology on A × B is the same as the topology A × B inherits as a subspace of X × Y
    (B) Product topology on A × B is different as the topology A × B inherits as a subspace of X × Y
    (C) Product topology on A × B is the same as the topology A × B
    (D) None of these

30. If (X, T) is a topological space. Then—
    (A) $\phi$ is open (B) $\phi$ is closed
    (C) $\phi$ is discrete (D) None of these

31. If (X, T) is a topological space. Then—
    (A) X is open (B) X is closed
    (C) X is discrete (D) None of these

32. If (X, T) is a topological space. Then—
    (A) Arbitrary intersection of closed sets are open
    (B) Arbitrary intersection of closed sets are closed
    (C) Arbitrary intersection of open sets are closed
    (D) None of these

33. If (X, T) is a topological space. Then—
    (A) Finite unions of closed sets are closed
    (B) Finite unions of closed sets are open
    (C) Finite unions of open sets are closed
    (D) None of these

34. If Y is a subspace of X. Then set A is closed in Y iff—
    (A) It is equal to the intersection of a open set of X with Y
    (B) It is equal to the unions of a closed set of X with Y
    (C) It is equal to the intersection of a closed set of X with Y
    (D) None of these

35. If Y is a subspace of X. If A is closed in Y and Y is closed in X.
    (A) Then a is semi-closed in X
    (B) Then A is open in X
    (C) Then A is closed in X
    (D) None of these

36. If Y is a subspace of X. A ⊂ Y and $\bar{A}$ is a closure of A in X. Then closure of A in Y—
    (A) Is equal to A ∩ Y
    (B) Is equal to A ∪ Y
    (C) Is equal to Y
    (D) None of these

37. If A is a subset of topological space, X, Let $x \in$ A, closure of A in X iff—
    (A) Every open set U containing $x$, intersects A

(B) Every open set U that does not contain $x$, intersects A

(C) Every closed set U containing $x$, intersects A

(D) None of these

38. If A is a subset of topological space X. Suppose $\mathcal{B}$ is a basis for X, then $x \in \bar{A}$, iff—

(A) Every basis element $B \in \mathcal{B}$, containing $x$ does not intersects A.

(B) Every basis element $B \in \mathcal{B}$, containing $x$ intersects A.

(C) Every basis element $B \in \mathcal{B}$, containing $x$ is disjoint to A

(D) None of these

39. Let A be a subset of topological space X and A′ be the set of all limit points of A. Then closure of A—

(A) $\bar{A} = A \cup A'$ (B) $\bar{A} = A - A'$

(C) $\bar{A} = A \cap A'$ (D) None of these

40. A subset of a topological space is closed iff—

(A) It contains none of its limit points

(B) It contains all its limit points

(C) It contains some of its limit points

(D) None of these

41. Let X be a Hausdorff space, $A \subset X$. Then a point $x$ is a limit point of A, iff—

(A) Every neighbourhood of $x$ contains infinitely many points of A

(B) Every neighbourhood of $x$ contains finitely many points of A

(C) Every neighbourhood of $x$ contains no points of A

(D) None of these

42. Every simple ordered set is a Hausdorff space in the—

(A) Order topology

(B) Non-order topology

(C) Discrete topology

(D) None of these

43. The product of two Hausdorff space is a—

(A) Hausdorff space (B) Discrete space

(C) Closed set (D) None of these

44. A subspace of a Hausdorff space is a—

(A) Hausdorff space (B) Discrete space

(C) Closed set (D) None of these

45. Let X and Y are topological spaces. A function $f : X \to Y$ is a continuous function—

(A) If for each open set V of Y, the set $f^{-1}$ (V) is an closed subset of X

(B) If for each closed subset V of Y, the set $f^{-1}$ (V) is an open subset of X

(C) If for each open subset V of Y, the set $f^{-1}$ (V) is an open subset of X

(D) None of these

46. Let X and Y are topological spaces. Function $f$ is homeomorphism if—

(A) Function $f : X \to Y$ be a one-to-one function

(B) Function $f$ is continuous

(C) Inverse function $f^{-1} : Y \to X$ is continuous

(D) All the above

47. Let X and Y be topological spaces. A map $f : X \to Y$ is open map if—

(A) For every open set U of X, the set $f$(U) is open in Y

(B) For every closed set U of X, the set $f$(U) is open in Y

(C) For every open set U of X, the set $f$(U) is closed in Y

(D) None of these

48. Let X and Y be topological spaces. A map $f : X \to Y$ is closed map if—

(A) It for every closed set A of X, the set $f$(A) is closed in Y

(B) For every closed set U of X, the set $f$(U) is open in Y

(C) For every open set U of X, the set $f$(U) is closed in Y

(D) None of these

49. If X is a topological space and A is $x$ set if $P : X \to A$ is surjective map then there exists exactly one topology T on A relative to which P is a quotient map and this topology is called—

(A) Quotient topology induced by P

(B) Hausdorff topology

(C) Discrete topology

(D) None of these

50. Let X and Y are topological spaces. A function $f : X \to Y$ is continuous at $x \in X$—
   (A) If for every neighbourhood N of $f(x)$, there is a neighbourhood M of $x$ such that $f(N) = M$
   (B) If for every neighbourhood N of $f(x)$, there is a neighbourhood M of $x$ such that $f(N) \subset M$
   (C) If for some neighborhood N of $f(x)$, there is a neighborhood M of $x$ such that $f(N) = M$
   (D) None of these

51. Let X and Y be topological spaces and $f : X \to Y$. If $f$ is continuous, then—
   (A) For every closed set $B \subset Y$, set $f^{-1}(B)$ is closed in X
   (B) For every open set $B \subset Y$, set $f^{-1}(B)$ is closed in X
   (C) For every closed set $B \subset Y$, set $f^{-1}(B)$ is open in X
   (D) None of these

52. Let X, Y and Z be topological spaces. If $f : X \to Y$ and $g : Y \to Z$ are continuous, then—
   (A) Composite $(gof) : x \to Z$ is continuous
   (B) Composite $(gof) : x \to Z$ is discontinuous
   (C) Composite $(fog) : x \to Z$ is continuous
   (D) None of these

53. Let X and Y be topological spaces the constant function $f : X \to Y$ is—
   (A) Continuous function
   (B) Discontinuous function
   (C) Inverse function
   (D) None of these

54. If A is a subspace of topological space X, the inclusion function $J : A \to X$ is—
   (A) Continuous function
   (B) Discontinuous function
   (C) Inverse function
   (D) None of these

55. If Y is a subspace of topological space X, a separation of Y is a pair of disjoint non-empty set A and B whose union is Y, and—
   (A) Neither of which contains a limit point of the other
   (B) Each of which contains a limit point of the other
   (C) One of which contains a limit point of the other
   (D) None of these

56. If Y is a subspace of topological space X—
   (A) Y is connected if there exist no separation of Y
   (B) Y is connected if there exist at least one separation of Y
   (C) Y is connected if there exist separation of Y
   (D) None of these

57. If sets C and D from a separation of topological space X and if Y is a connected subset of X, then—
   (A) Y lies entirely with in C
   (B) Y lies entirely with in D
   (C) Y lies entirely with in either C or D
   (D) Y lies entirely with in C and D

58. The union of a collection of connected sets that have a point in common is—
   (A) Connected (B) Separable
   (C) Disconnected (D) None of these

59. Let A be a connected subset of topological space X. If $A \subset B \subset \overline{A}$, then B is—
   (A) Connected (B) Separable
   (C) Disconnected (D) None of these

60. If set B is formed by adjoining to the connected set A of topological space X some or all its limit points, then B is—
   (A) Connected (B) Separable
   (C) Disconnected (D) None of these

61. Let Y be a subspace of topological space X. Then Y is compact, iff—
   (A) Every covering of Y by sets open in X contains a finite subcollection covering Y.
   (B) Every covering of Y by sets closed in X contains a finite subcollection covering Y
   (C) Every covering of Y by sets open in X contains a infinite subcollection covering Y
   (D) None of these

62. Every closed subset of a compact space is—
(A) Compact space (B) Open set
(C) Null set (D) None of these

63. Every compact subset of a Hausdorff space is—
(A) Closed set (B) Open set
(C) Null set (D) None of these

64. If Y is a compact subset of the Hausdorff space X and $x_0 \notin$ Y, then—
(A) There exist disjoint open sets U and V of X, such that $x_0 \in$ U and Y$\subset$ V
(B) There exist overlapping open sets U and V of X, such that $x_0 \in$ U and Y $\subset$ V.
(C) There exist disjoint closed sets U and V of X, such that $x_0 \in$ U and Y $\subset$ V
(D) None of these

65. A topological space X is paracompact space if it is Hausdorff space and—
(A) Every open covering a of X has a locally in finite open refinement B that covers X
(B) Every open covering a of X has a locally finite open refinement B that covers X
(C) Every closed covering a of X has a locally infinite open refinement B that covers X
(D) None of these

66. The image of a compact topological space under continuous map is—
(A) Continuous (B) Discontinuous
(C) Constant (D) None of these

67. Let $f$ : X $\rightarrow$ Y be bijective continuous function. If X is compact and Y is Hausdorff, then—
(A) $f$ is automorphism
(B) $f$ is isomorphism
(C) $f$ is homeomorphism
(D) None of these

68. The product of finitely many compact spaces is—
(A) Compact space (B) Open set
(C) Null set (D) None of these

69. Let X be a topological space. If every collection G of closed sets in X, satisfy finite intersection condition the intersection $\cap_{C \in G} \subset \neq \phi$. Then X is—
(A) Compact space (B) Hausdorff space
(C) Null set (D) None of these

70. Let X be a simply ordered set having the least upper bound property. In the order topology each closed interval in X is—
(A) Compact space (B) Hausdorff space
(C) Null set (D) None of these

71. Every closed interval in real line P is—
(A) Compact space (B) Hausdorff space
(C) Null set (D) None of these

72. A subset A$\subset$ $R^n$ is compact iff—
(A) It is closed in the Euclidean metric $d$ or square metric P
(B) In bounded in the Euclidean metric $d$ or square metric P
(C) Both (A) and (B)
(D) None of these

73. Maximum and minimum value theorem states—
(A) Let $f$ : X $\rightarrow$ Y be continuous, where $y$ is an ordered set in the order topology. If X is compact. Then there C, $d \in$ X such that $f(c) \leq f(x) \leq f$(D) for every $x \in$ X
(B) Let $f$ : X $\rightarrow$ Y be continuous, where Y is an ordered set in the order topology. If X is compact then there C, $d \in$ X such that $f(c) = f(x) = f$(D) for every $x \in$ X.
(C) Let $f$ : X $\rightarrow$ Y be discontinuous, where Y is unordered set in the order topology. If X is compact, then there C, $d \in$ X such that $f(c) \leq f(x) \leq f$(D) for every $x \in$ X.
(D) None of these

74. Let X be a (Non empty) compact Hausdorff space. If every point of X is a limit point of X, then—
(A) X is uncountable
(B) X is countable
(C) X is disjoint
(D) None of these

75. Let X be a topological space. Let one point sets in X be closed X is ......... if given a point $x \in$ X and neighbourhood U of $x$, there is a neighbourhood V of $x$ such that $\overline{V} \subset$ U—

(A) Regular (B) Normal
(C) Disjoint (D0 None of these

76. Let X be a topological space. Let one point sets in X be closed. X is ……… iff given a closed set A and open set U, A⊂ U there is an open set V, A⊂ V such that $\overline{V}$⊂ U—
(A) Regular (B) Normal
(C) Disjoint (D) None of these

77. A subspace of Hausdorff space is—
(A) Hausdorff (B) Normal
(C) Regular (D) None of these

78. A topological space X is completely regular—
(A) If one point sets are open in X and if every $a \in X$, every open set B, $a \in B$ there exists a continuous function $f : X \to [0, 1]$ such that $f(a) = f(B) = \{1\}$
(B) If one point sets are closed in X and if every $a \in X$, every closed set B, $a \in B$, there exists a continuous function $f : X \to [0, 1]$ such that $f(a) = 0$ and $f(B) = \{1\}$
(C) If one point sets are closed in X and if every $a \in X$, every closed set B, $a \in B$ there exists a discontinuous function $f : X \to [0, 1]$ usch that $f(a) = f(B)$
(D) None of these

79. Product of Hausdorff spaces is—
(A) Hausdorff (B) Normal
(C) Disjoint (D) None of these

80. A subspace of regular space is—
(A) Hausdorff (B) Regular
(C) Disjoint (D) None of these

81. A product of regular spaces is—
(A) Hausdorff (B) Regular
(C) Disjoint (D) None of these

82. A subspace of normal space is—
(A) Need not normal
(B) Normal
(C) Hausdorff
(D) Need not Hausdorff

83. Product of normal spaces is—
(A) Need not normal
(B) Normal
(C) Hausdorff
(D) Need not Hausdorff

84. Every metrizable space is—
(A) Hausdorff (B) Normal
(C) Disjoint (D) None of these

85. Every compact Hausdorff space is—
(A) Hausdorff (B) Normal
(C) Disjoint (D) None of these

86. Every regular space with countable basis is—
(A) Hausdorff (B) Normal
(C) Disjoint (D) None of these

87. Every well ordered set X is normal in the—
(A) Order topology
(B) Non order topology
(C) Discrete topology
(D) None of these

88. Let X be a topological space A, B ⊂ X, $f : X \to [0, 1]$ a continuous function such that $f(A) = \{0\}$ and $f(B) = \{1\}$ then—
(A) Sets A and B are separated by continuous function
(B) Sets A and B are separated by discontinuous function
(C) Set A and B are separted by step function
(D) None of these

89. A topological space X is completely regular—
(A) If one point sets are open in X and if every $a \in X$, every open set B, $a \in B$, there exists a continuous function $f : X \to [0, 1]$ such that $f(a) = 0$ and $f(B) = \{1\}$
(B) If one point sets are closed in X and if every $a \in X$, every closed set B, $a \in B$, there exists a continuous function $f : X \to [0, 1]$ such that $f(a) = 0$ and $f(B) = \{1\}$
(C) If one point sets are closed in X and if every $a \in X$, every set B $a \in B$, there exists a discontinous function $f : X \to [0, 1]$ such that $f(a) = 0$ and $f(B) = \{1\}$
(D) None of these

90. Urysohn lemma states—
(A) Let X be a normal space. A and B are disjoint closed subsets of X and $[a, b]$ closed interval of R. Then there exists a continuous map $f : X \to [a, b]$ such that for every $x \in A$, $f(x) = a$ and for every $x \in B$, $f(x) = b$

(B) Let X be a normal space; A and B are open subsets of X and $[a, b]$ closed interval of R. Then there exist a discontinuous map $f : X \to [a, b]$ such that for every $x \in A$, $f(x) = a$ and for every $x \in B$, $f(x) = b$

(C) Let X be a normal space; A and B are disjoint open subsets of X and $[a, b]$ closed interval of R, then there exist a discontinuous map $f : X \to [a, b]$ such that for every $x \in A$, $f(x) = a$ and for every $x \in B$, $f(x) = b$

(D) None of these

91. Tietze extension theorem states : Let X be a normal space. Let A be closed subset of X—

(A) Any continuous map of A into the closed interval $[a, b]$ of R may be extended to a continuous map of all of X into $[a, b]$

(B) Any continuous map of A into R may be extended to a continuous map of X into R

(C) Both (A) and (B)

(D) None of these

92. Set A is a $G_\delta$ set in X if—

(A) Set A is the intersection of a countable collection of open sets of X

(B) Set A is the union of a countable collection of open sets of X

(C) Set A is the intersection of a uncountable collection of closed sets of X

(D) Set A is the union of a countable collection of closed sets of X

93. Strong form of Urysohn lemma states—

(A) Let A and B closed disjoint subsets of the normal space X. There exists a continuous function $f : X \to [0, 1]$ such that $f^{-1}\{0\} = A$ and $f(B) = \{1\}$ iff A is a $G_\delta$ set in X

(B) Any continuous map of A into the closed interval $[a, b]$ of R may be extended to a continuous map of all of X into $[a, b]$

(C) Every regular space X with countable basis is metrizable

(D) None of these

94. Urysohn metrization theorem states—

(A) Let A and B closed disjoint subsets of the normal space X. There exists a continuous function $f : X \to [0, 1]$ such that $f^{-1}\{0\} = A$ and $f(B) = \{1\}$ iff A is $a$ $G_\delta$ set in X

(B) Any continuous map of A into the closed interval $[a, b]$ of R may be extended to a continuous map of all of X into $[a, b]$

(C) Every regular space X with countable basis is metrizable

(D) None of these

95. Every closed interval of real line R is—

(A) Uncountable (B) Countable

(C) Disjoint (D) None of these

96. The cartesian product of connected topological space is—

(A) Connected (B) Separable

(C) Disconnected (D) None of these

97. Let X be a topological space. A covering of X is called open covering if—

(A) Its elements are open subsets of X

(B) Its elements are open subsets of X

(C) Its elements are null sets

(D) None of these

98. A topological space X is compact if every open covering a of X contains—

(A) A finite subcollection that covers X

(B) A infinite subcollection that covers X

(C) A finite subcollection that does not covers X

(D) None of these

99. A collection G of subjects of topological space satisfies finite intersection condition if for every finite subcollection $\{C_1, C_2 \dots C_n\}$ of G, the intersection—

(A) Is a null set (B) Is not a null set

(C) Is a set G (D) None of these

100. Let $d$ and $d'$ be two metrics on the set X and let T and T′ be the topologies they induce respectively. Then T′ is … than T iff for each $x \in X$ and $\varepsilon > 0$, there exist $\delta > 0$, such that $B_d(x\ \delta) \subset B_d(x, \varepsilon)$—

(A) Finer (B) Strictly finer

(C) Not finer (D) None of these

101. The topologies on $R^n$ induced by the Euclidean metric $d$ and the square metric pare …… as the product topology on $R^n$—

(A) Same (B) Dissimilar
(C) Different (D) None of these

102. The uniform topology on $R^3$ is …… than the product topology. They are different is S is infinite.
(A) Finer (B) Strictly finer
(C) Not finer (D) None of these

103. An arc is—
(A) Space homeomorphic to the unit interval [0, 1]
(B) Not space homeomorphic to the unit interval [0, 1]
(C) Both (A) and (B)
(D) None of these

104. A simple closed curve is—
(A) A space homeomorphic to the circle $\delta'$
(B) Not space homeomorphic to the circle $\delta'$
(C) Both (A) and (B)
(D) None of these

105. Let $a, b \in S^2$ and A be a compact space. Let $f^1$. $A \to S^2 - a - b$ be continuous map. Then $f$ is in essential—
(A) If $a$ and $b$ lie in same component of $S^2 - f(A)$
(B) If $a$ and $b$ lie in different component of $S^2 - f(A)$
(C) If $a$ and $b$ does not lie in same component of $S^2 - f(A)$
(D) None of these

106. Brouwer theorem on invariance of domain for $R^2$ states—
(A) If U is an closed subset of $R^2$ and $f : U \to R^2$ is continuous and injective, then $f(U)$ is open in $R^2$ and $f$ is an imbedding
(B) If U is an open subset of $R^2$ and $f : U \to R^2$ is discontinuous and injective, then $f$ (U) is open in $R^2$ and $f$ is an imbedding
(C) If U is an open subset of $R^2$ and $f : U \to R^2$ is continuous and injective, then $f$ (U) is open in $R^2$ and $f$ is an imbedding
(D) None of these

107. Jordan separation theorem states—
(A) Let C be a simple closed curve in $S^2$. Then $S^2 - C$ is not connected
(B) Let C be a simple closed curve in $S^2$. Then $S^2 - C$ is connected
(C) Let C be a simple open curve in $S^2$. Then $S^2 - C$ is not connected
(D) None of these

108. Let A and B be closed connected subset of $S^2$ whose intersection consists of precisely two points—
(A) Then $A \cup B$ separates $S^2$
(B) Then $A \cap B$ separates $S^2$
(C) Then $A - B$ separates $S^2$
(D) None of these

109. Homotopy existence lemma states—
(A) Let X and X + [0, 1] be normal, let A be a closed subset of X. If $f : A \to R^2 - 0$ is continuous map and $f$ is in essential, then $f$ can be extended to a continuous map $g : X \to R^2 - 0$
(B) Let X and X + [0, 1] be normal, let A be a open subset of X. If $f : A \to R^2 - 0$ is discontinuous map and $f$ is in essential. Then $f$ can be extended to a continuous map $g : X \to R^2 - 0$
(C) Let X and X + [0, 1] be normal, let A be a closed subset of X. If $f : A \to R^2 - 0$ is continuous map and $f$ is in essential. Then $f$ cannot be extended to a continuous map $g : X \to R^2 - 0$
(D) None of these

110. Borsuk theorem states—
(A) Let X be a compact subset of $R^2$. If 0 does not lies in a bounded component C of $R^2 - X$, then the inclusion map $J : X \to R^2 - 0$ is essential (and conversely)
(B) Let X be a compact subject of $R^2$. If 0 lies in a bounded component C of $R^2 - X$, then the inclusion map $J : X \to R^2 - 0$ is essential (and conversely)
(C) Let X be a compact subset of $R^2$. If 0 lies in a bounded component C of $R^2 - X$, then the inclusion map $J : X \to R^2 - 0$ is not essential (and conversely)
(D) None of these

111. Non-separable theorem states—
(A) No arc separates $R^2$
(B) No space homeomorphism to ball $B^2$ separates $R^2$
(C) Both (A) and (B)
(D) None of these

112. Brouwer theorem on invariance of domain for $R^2$ states—
   (A) If U is an closed subset of $R^2$ and $f$ : U $\rightarrow R^2$ is continuous and injective, then $f$ (U) is open in $R^2$
   (B) If U is an open subset of $R^2$ and $f$: U $\rightarrow R^2$ is discontinuous and injective, then $f$ (U) is open in $R^2$
   (C) If U is open subset of $R^2$ and $f$: U $\rightarrow R^2$ is continuous and injective then $f$ (U) is open in $R^2$
   (D) None of these

113. Let X be the union of two open sets U and V. Suppose that U $\cap$ V can be written as the union of two disjoint open sets A and B. Let $a \in$ A and $b \in$ B and $a$ and $b$ are joined by paths in U and in V, then—
   (A) $\pi$; (X, $a$) is not zero
   (B) $\pi$; (X, $a$) is zero
   (C) $\pi$; (X, $a$) is positive
   (D) None of these

114. Let X = U $\cup$ V, where U and V are open sets and U $\cap$ V = A $\cup$ A 'U where A, B, A' are disjoint open sets. Let $a \in$ A, $a' \in$ A' and $b \in$ B, suppose that $a$, $a'$ and $b$ are joined by paths is U and in V. Then—
   (A) $\pi$; (X, $a$) is not cyclic
   (B) $\pi$; (X, $a$) is not infinite cyclic
   (C) $\pi$; (X, $a$) is infinite cyclic
   (D) None of these

115. Non-separation theorem states—
   (A) Let A be an arc in $S^2$. Then $S^2$ – A is disconnected
   (B) Let A be an arc in $S^2$. Then $S^2$ – A is connected
   (C) Let A be an arc in $S^2$. Then $S^2$ – A is separable
   (D) None of these

116. Jordan curve theorem states : Let $c$ be a simple closed curve in $S^2$—
   (A) Then $S^2 - c$ has precisely two components $w_1$ and $w_2$ of which $c$ is not the common boundary
   (B) Then $S^2 - c$ has precisely two components $w_1$ and $w_2$ of which $c$ is the common boundary
   (C) Then $S^2 - c$ has no components $w_1$ and $w_2$ of which $c$ is the common boundary
   (D) None of these

117. Brouwer theorem on in variance of domain for $R^2$ states—
   (A) If U is an closed subset of $R^2$ and $f$ : U $\rightarrow R^2$ is continuous and injective, then $f$ is an imbedding
   (B) If U is an open subset of $R^2$ and $f$ : U $\rightarrow R^2$ is discontinuous and injective, then $f$ is an imbedding
   (C) If U is an open subset of $R^2$ and $f$ : U $\rightarrow R^2$ is continuous and injective, then $f$ is an imbedding
   (D) None of these

118. A continuous map $f$ : X $\rightarrow$ Y is homotopy equivalence—
   (A) If there is a continuous map $g : y \rightarrow X$ such that $gof$ is homotopic to the identity map $i_x$ of X and $fog$ is homotopic to the identity map $i_y$ of Y
   (B) If there is a discontinuous map $g$ : Y $\rightarrow$ X such that $gof$ is homotopic to the identity map $i_x$ of X and $fog$ is homotopic to the identity map $i_y$ of Y
   (C) If there is a constant map $g$ : Y $\rightarrow$ X such that $gof$ is homotopic to the identity map $i_x$ of X and $fog$ is homotopic to the identity map $i_y$ of Y
   (D) None of these

119. Let continuous map $f$ : X $\rightarrow$ Y is homotopy equivalence and if there is a continuous map $g$ : Y $\rightarrow$ X such that gof is homotopic to the identity map $i_x$ of X and fog is homotopic to the identity map $i_y$ of $y$—
   (A) The map $g$ is homotopy inverse for the map $f$
   (B) The map $g$ is not homotopy inverse for the map $f$
   (C) The map $g$ is equal to map $f$
   (D) None of these

120. Let $h$, K : X $\rightarrow$ Y, $h(x_0) = y_0$ and $K(x_0) = y_1$. If $h$ and K are homotopic then there is a path $\alpha \in y$ from $y_0$ to $y_1$ such that $K = \hat{\alpha}oh$. If $y_0 = y_1$ and if the base point $x_0$ remains fixed during the homotopy. Then—

(A) $h$ is equal to K
(B) $h$ is not equal to K
(C) Both (A) and (B)
(D) None of these

121. $h$, K : X $\to$ Y and $h$ $(x_0) = y_0$, $k$ $(x_0) = y_1$. Suppose that $h$ and $k$ are homotopic. If $k$ is …… then so is $h$—
(A) Injective
(B) Surjective
(C) Zero homomorphism
(D) All the above

122. Let $h$ : X $\to$ Y and $h$ $(x_0) = y_0$. If $h$ is homotopic to a constant map, then—
(A) $h$ is the zero homeomorphism
(B) $h$ is the isomorphism
(C) $h$ is the homeomorphism
(D) None of these

123. Let $f$ : X $\to$ Y be continuous, $f(x_0) = y_0$. If $f$ is Homotopy equivalence, then $f : \pi : (X, x_0) \to \pi : (Y, y_0)$ is—
(A) Zero homeomorphism
(B) Isomorphism
(C) Homeomorphism
(D) None of these

124. Let X be a topological space. A separation of X—
(A) Is a pair U, V of disjoint non empty open subsets of X whose union is X
(B) Is a pair U, V of non disjoint empty open subsets of X whose union is X
(C) Is a pair U, V of disjoint empty closed subsets of X whose union is X
(D) Is a pair U, V of non empty closed subsets of X whose intersection is X

125. Let X be a topological space. The space X is called connected—
(A) If there does not exist a separation of X
(B) If there exist a separation of X
(C) If there exist some separation of X
(D) None of these

126. Let X be a topological space. A space X is connected, iff—
(A) The only subset of X that are both open and closed in X are empty set and X itself
(B) The only subset of X that are both open but not closed in X are empty set and X itself
(C) The only subset of X that are closed in X are empty set only
(D) The only subset of X that are both open and closed in X are X itself only

127. A topological space X is totally disconnected if—
(A) Its only connected subsets are open-point set
(B) Its only connected subsets are empty set
(C) Its only connected subsets are set X itself
(D) Its only connected subsets are set X itself

128. A simple ordered set L having more than one element is called linear continum if—
(A) L has the least upper bound property
(B) If $x < y$, there exist $z$ such that $x < z < y$
(C) Both (A) and (B)
(D) None of these

129. Given $\overline{x} \in R^n$, the norm of $\overline{x}$ is—
(A) $|\overline{x}| = (x_1^2 + x_2^2 + \ldots\ldots + x^2_n)^{1/2}$
(B) $|\overline{x}| = (x_1^2 + x_2^2 + \ldots\ldots + x^2_n)^2$
(C) $|\overline{x}| = (x_1^2 + x_2^2 + \ldots\ldots + x^2_n)$
(D) None of these

130. The Euclidean metric $d$ on $R^n$ is defined as—
(A) $d(\overline{x}, \overline{y}) = [(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots\ldots + (x_n - y_n)^2]$
(B) $d(\overline{x}, \overline{y}) = [(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots\ldots + (x_n - y_n)^2]^2$
(C) $d(\overline{x}, \overline{y}) = [(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots\ldots + (x_n - y_n)^2]^{1/2}$
(D) None of these

131. Let $\overline{x} \in R^n$, the square metric P is defined as—
(A) $p(\overline{x}, \overline{y}) = \min. \{|x_1 - y_1|, \ldots, |x_n - y_n|\}$
(B) $p(\overline{x}, \overline{y}) = \max. \{|x_1 - y_1|, \ldots, |x_n - y_n|\}$
(C) $p(\overline{x}, \overline{y}) = \{|x_1 - y_1|, \ldots, |x_n - y_n|\}$
(D) None of these

132. Standard bounded metric (corresponding to $d$) is defined as—

(A) Let X be a metric space with metric $d$ and $\overline{d} : X \times X \to R$ such that $\overline{d}\,(x, y)$ = max. $\{d\,(x, y), 1\}$

(B) Let X be a metric space with metric $d$ and $\overline{d} : X \times X \to R$ such that $\overline{d}\,(x, y)$ = $\{d\,(x, y), 1\}$

(C) Let X be metric space with metric $d$ and $\overline{d} : X \times X \to R$ such that $\overline{d}\,(x, y)$ = min. $\{d\,(x, y), 1\}$

(D) None of these

133. Given an index set J, and $\overline{x}\,(x_\alpha)_{\alpha \in j}$ and $\overline{y}$ = $(y_\alpha)_{\alpha \in j}$ of $R^j$, $\overline{y}\,(x_\alpha, y_\alpha)$ = min. $\{d\,(x_\alpha, y_\alpha), 1\}$ the standard bounded metric on R. The uniform metric on $R^j$ is defined as—

(A) $\overline{P}(\overline{x}, \overline{y}) = \{\overline{d}\,(x_\alpha, y_\alpha)_{, \alpha \in J}\}$

(B) $\overline{P}(\overline{x}, \overline{y})$ = lub $\{\overline{d}\,(x_\alpha, y_\alpha), {}_{\alpha \in}\}$

(C) $\overline{P}(\overline{x}, \overline{y})$ = glb$\cdot$ $\{\overline{d}\,(x_\alpha, y_\alpha), {}_{\alpha \in j}\}$

(D) None of these

134. A sequence $\{x_m\}$ of points of X is convergence to a point $x \in$ X—

(A) If for every neighbourhood U of $x$ there exists positive integer N such that $x_n \in$ U for all $n \geq$ N

(B) If for every neighbourhood U of $x$ there exists positive integer N such that $x_n \in$ U for all $n <$ N

(C) If for every neighbourhood U of $x$ there exists positive integer N such that $x_n \in$ U for all $n$ = N

(D) None of these

135. A topological space X have a countable basis at the point $x$—

(A) If there is a countable collection $\{U_n\}_{n \in z}$, of neighbourhood of $x$ such that any neighborhood U of $x$ contains atleast one of the sets $U_n$

(B) If there is a uncountable collection $\{U_n\}_{n \in z}$, of neighbourhood of $x$ such that any neighbourhood U of $x$ contains none of the sets $U_n$

(C) If there is a countable collection $\{U_n\}_{n \in z}$, of neighbourhood of $x$ such that any neighbourhood of $x$ contains more of the sets $U_n$

(D) None of these

136. A topological space X that has a countable basis at each of its points is said to satisfy—

(A) The first countability axiom

(B) The second countability axiom

(C) The third countability axiom

(D) The fourth countability axiom

137. Let $f$ : X $\to$ Y also X and Y be metrizable with metrics $d_x$ and $d_y$ respectively. If given $x \in$ X, and $\in\, > 0$, there exist $\delta > 0$ such that $d_x\,(x, y) < \delta \Rightarrow d_y\,[f\,(x), f\,(y)] < \in$ then—

(A) $f$ is continuous (B) $f$ is discontinuous

(C) $f$ is constant (D) None of these

138. Let X and Y are topological space. A map $h$ : X $\to$ Y is in essential map—

(A) If $h$ is homotopic to a constant map

(B) If $h$ is homotopic to a non constant map

(C) If $h$ is not homotopic to a constant map

(D) None of these

139. The uniform topology on $R^J$ is finer than the product topology and—

(A) They are same if J is infinite

(B) They are different if J is infinite

(C) They are different if J is finite

(D) None of these

140. Let $\overline{d}\,(a, b)$ = min. $\{|a - b|\}$ be the standard bounded metric on R. If $x$ and $y$ are two points of $R^w$, define

$$D(\overline{x}, \overline{y}) = \text{lub}\left\{\frac{\overline{d}\,(x_i, h_i)}{i}\right\}, \text{ then—}$$

(A) D is a not a metric that induces the product topology on $R^w$

(B) D is a metric that induces the product topology on $R^w$

(C) D is a metric that does not induces the product topology on $R^w$

(D) None of these

141. Let $f$ : X $\to$ Y also X and Y be metrizable with metric $d_x$ and $d_y$ respectively. Then $f$ is continuous if—

(A) To given $x \in X$ and $\in > 0$, there exist $\delta > 0$ such that $d_x(x, y) < \delta \Rightarrow d_y[f(x), (y)] < \in$

(B) To given $x \in X$ and $\in > 0$, there does not exist $\delta > 0$ such that $d_x(x, y) < \delta \Rightarrow d_y[f(x), (y)] < \in$

(C) To given $x \in X$ and $\in > 0$, there exist $\delta > 0$ such that $d_x(x, y) = \delta \Rightarrow d_y[f(x), f(y)] = \in$

(D) None of these

142. A topological space X is locally connected at $x$ is—

(A) For every neighbourhood U of $x$, there is a connected neighbourhood V of $x$, $V \subset U$

(B) For every neighbourhood U of $x$, there is a connected neighbourhood V of $x$, $V = U$

(C) For every neighbourhood U of $x$, there is a disconnected neighbourhood V of $x$

(D) None of these

143. Topological space X is locally path connected space—

(A) If X is locally connected at each $x \in X$

(B) If X is locally connected at some $x \in X$

(C) If X is locally connected at each $x \notin X$

(D) None of these

144. A topological space X is locally path connected at $x$, if—

(A) For every neighbourhood U of $x$, there is a path connected neighbourhood V of $x$, $V \subset U$

(B) For every neighbourhood U of $x$, there is a path connected neighbourhood V of $x$, $V = U$

(C) For every neighbourhood U of $x$, there is a disconnected neighbourhood V of $x$

(D) None of these

145. A topological space X is connected in $k$ leinen at $x$, if—

(A) For every neighbourhood U of $x$, there is a disconnected subset $y$ of U that contains a neighbourhood of $x$

(B) For every neighbourhood U of $x$, there is a connected subset $y$ of U that contains a neighbourhood of $x$

(C) For every neighbourhood U of $x$, there is a connected subset $y$ of U that does not contains a neighbourhood of $x$

(D) None of these

146. If one point sets are closed in topological space X. Then X is regular, if—

(A) For each pair consisting of a point $x$ and a closed set B disjoint from $x$, there exist disjoint open sets containing $x$ and B respectively

(B) For each pair consisting of a point $x$ and a open set B disjoint from $x$, there exist disjoint closed sets containing $x$ and B respectively

(C) For each pair consisting of a point $x$ and a open set B disjoint from $x$, there exist disjoint closed sets not containing $x$ and B respectively

(D) None of these

147. A topological space X is normal—

(A) If for each A, B, $A \cap B = \phi$ of open sets of X, there exist disjoint open sets $O_1$ and $O_2$, $O_1 \cap O_2 \neq \phi$, such that $A \subset O_1$ and $B \subset O_2$

(B) If for each pair A, B, $A \cap B \neq \phi$ of closed sets of X, there exist disjoint open sets $O_1$ and $O_2$, $O_1 \cap O_2 \neq \phi$, such that $A \subset O_1$ and $B \subset O_2$

(C) If for each pair A, B, $A \cap B = \phi$ of closed sets of X, there exist non disjoint open sets $O_1$ and $O_2$, $O_1 \cap O_2 = \phi$, such that $A \subset O_1$ and $B \subset O_2$

(D) None of these

148. A topological space X is locally connected iff—

(A) For every open set U of X, each component of U is closed in X

(B) For every closed set U of X, each component of U is open in X

(C) For every open set U of X, each component of U is open in X

(D) None of these

149. A topological space X is locally path connected iff—

(A) For every closed set U of X, each path component of U is open in X

(B) For every open set U of X, each path component of U is open in X
(C) For every open set U of X, each path component of U is closed in X
(D) None of these

150. A subspace of a completely regular space is—
(A) Normal
(B) Regular
(C) Completely regular
(D) None of these

151. Let X be a topological space given an equivalence relation on X such that $x \sim y$ if there is connected subset of X containing both $x$ and $y$—
(A) Then equivalence classes are called the connected components of X
(B) Then equivalence classes are called the separation of X
(C) Then equivalence classes are called the compact of X
(D) None of these

152. The product of completely regular spaces is—
(A) Normal
(B) Regular
(C) Completely regular
(D) None of these

153. If X is completely regular, then—
(A) X cannot be imbedded in $[0, 1]^J$ for some J
(B) X can be imbedded in $[0, 1]^J$ for some J
(C) Both (A) and (B)
(D) None of these

154. Let X be a topological space X is completely regular if—
(A) X is homeomorphic to a subspace of a compact Hausdorff space
(B) X is homeomorphic to a subspace of a normal space
(C) Both (A) and (B)
(D) None of these

155. Let X be a locally compact Housdorff space which is not compact. Let Y be the one point compactification of X, then—
(A) X is subspace of Y
(B) The set Y – X consists of a single point
(C) $\overline{X} = Y$
(D) All the above

156. Let X be a topological space satisfy first countability axiom if—
(A) The point $x \in \overline{A}$, closure of $A \subset X$ iff there is a sequence of point of A converging to $x$
(B) The point $x \in \overline{A}$, closure of $A \subset X$ iff there is a sequence of point of A diverging to $x$
(C) The point $x \in \overline{A}$, closure of $A \subset X$ iff there is a sequence of points of A converging to o
(D) None of these

157. Let X be a topological space satisfy first countability axiom if—
(A) The function $f : X \rightarrow Y$ is continuous iff for every diverting sequence $\{x_n\}$ in X, converging to $x$ the sequence $\{f(x_n)\}$ converges to $f(x)$
(B) The function $f : X \rightarrow Y$ is continuous iff for every convergent sequence $\{x_n\}$ in X, converging to $x$ the sequence $\{f(x_n)\}$ converges to $f(x)$
(C) The function $f : X \rightarrow Y$ is continuous iff for every convergent sequence $\{x_n\}$ in X, converging to $x$ the sequence $\{f(x_n)\}$ diverges to $f(x)$
(D) None of these

158. A subspace of a first countable space is—
(A) First countable
(B) Second countable
(C) Third countable
(D) Fourth countable

159. Every Hausdorff topological group is—
(A) Normal
(B) Regular
(C) Completely regular
(D) None of these

160. A compactification of a space X—
(A) Is Hausdorff space Y containing X such that X is dense in Y

(B) Is compact Hausdorff space Y containing X such that X is dense in Y
(C) Is compact Hausdorff space Y not containing X
(D) None of these

161. Let X be a topological space. Given an equivalence relation on X such that $x \sim y$ if there is a path in X from $x$ to $y$—
(A) Then equivalence classes are called the path components of X
(B) Then equivalence classes are called the separation of X
(C) Then equivalence classes are called the compact of X
(D) None of these

162. Let X be a topological space. A collection a of subset of X are locally finite—
(A) If every point of X has a neighbourhood that intersect only finitely many elements of a
(B) If some point of X has a neighbourhood that intersect infinitely many elements of a
(C) If every point of X has a neighbourhood that intersect infinitely many elements of a
(D) None of these

163. A collection B of subsets of topological space X is countably locally finite—
(A) If B can be written as uncountable union of collections $B_n$, each of which is locally finite
(B) If B can be written as countable intersection of collections $B_n$, each of which is locally finite
(C) If B can be written as countable union of collection $B_n$ each of which is locally finite
(D) None of these

164. Let $a$ be a locally finite collection of subsets of X. Then—
(A) Any subcollection of $a$ is locally finite
(B) The collection $B = \{\overline{A}\}_{A \in a}$ of closures of the elements of a is locally finite
(C) $\overline{U_{A \in a} A} = U_{A \in a} \overline{A}$
(D) All of above

165. A countable product of first countable spaces is—
(A) First countable
(B) Second countable
(C) Third countable
(D) Fourth countable

166. Nagata—Smirnov metrization theorem of $G_\delta$ set states : A subset A of topological space X is $G_\gamma$ set in X—
(A) If it is equals to the union of a countable collection of open subset of X
(B) It is equal to the intersection of a countable collection of open subsets of X
(C) If it is equal to the union of a uncountable collection of open subsets of X
(D) None of these

167. Let X be a regular space with a basis B that is countably locally finite. Then—
(A) X is normal
(B) Every closed set of X is $G_\delta$ set in X
(C) Both (A) and (B)
(D) None of these

168. Let X be a regular space with basis B that is countably locally finite. Then—
(A) X is metrizable only
(B) X is normal only
(C) X is metrizable and normal both
(D) None of these

169. Let $a$ be a collection of subsets of the space X, A collection B of subsets of X is called refinement of a (refine $a$)—
(A) If for some $B \in B$, there is $A \in a$ such that $B = A$
(B) If for each $B \in B$, there is $A \in a$ such that $B \subset A$
(C) If for each $B \in B$, there is $A \in a$ such that $B = \subset A$
(D) None of these

170. Sequence lemma states; Let X be a topological space and $A \subset X$—
(A) If there is a sequence of points of A converging to $x$, then $x \in \overline{A}$
(B) The converse holds of X is metrizable
(C) Both (A) and (B)
(D) None of these

171. If X and Y are topological space. Let $f$ : X $\rightarrow$ Y and X be metrizable. The function $f$ is continuous, iff—

(A) For every divergent sequence $x_n \rightarrow x$ in X, the sequence $f(x_n)$ divergent to $f(x)$

(B) For every divergent sequence $x_n \rightarrow x$ in X, the sequence $f(x_n)$ converges to $f(x)$

(C) For every convergent sequence $x_n \rightarrow x$ in X, the sequence $f(x_n)$ converges to $f(x)$

(D) For every convergent sequence $x_n \rightarrow x$ in X, the sequence $f(x_n)$ diverges to $f(x)$

172. The … operations are continuous function from R × R into R and quotient operation is continuous function from R × (R – {0}) into R—

(A) Addition (B) Subtraction

(C) Multiplication (D) All of these

173. If X is a topological space and if $f, g$ : X $\rightarrow$ R are continuous function, then … are continuous functions.

(A) $f + g$ (B) $f - g$

(C) $fg$ (D) All of the above

174. If X is a topological space and if $f, g$ : X $\rightarrow$ R are continuous function, then—

(A) $f/g$ is continuous

(B) If $g(x) \neq 0$ for all $x$, then $f/g$ is continuous

(C) If $g(x) = 0$ for all $x$, then $f/g$ is continuous

(D) None of these

175. Uniform limit theorem states. Let $f_n$ : X $\rightarrow$ Y be a sequence of continuous functions from the topological space X into the metric space Y—

(A) If $\{f_n\}$ converges uniformly to $f$, then $f$ is continuous

(B) If $\{f_n\}$ diverges uniformly to $f$, then $f$ is continuous

(C) If $\{f_n\}$ converges uniformly to $f$, then $f$ is discontinuous

(D) None of these

176. An $m$ – manifold is—

(A) Hausdorff space with a uncountable basis such that some point $x \in$ X, has a neigbhourbood that is homeomorphic with an open subset of $R^m$

(B) Hausdorft space with a uncountable basis such that each point $x \in$ X, has a neigbhourbood that is homeomorphic with an closed subset of $R^m$

(C) Hausdorff space with a countable basis such that each point $x \in$ X, has a neigbhourbood that is homeomorphic with an open subset of $R^m$

(D) Hausdorff space with a countable basis such that some point $x \in$ X, has a neigbhourbood that is homeomorphic with an closed subset of $R^m$

177. An indexed collection $\{A_\alpha)$ of subsets of X is point – finite indexed family—

(A) If some $x \in X \rightarrow x \in A_\alpha$ for infinitely many values of $\alpha$

(B) If each $x \in X \rightarrow x \in A_\alpha$ for only finitely many values of $\alpha$

(C) If each $x \in X \rightarrow x \in A_\alpha$ for infinitely many values of $\alpha$

(D) If some $x \in X \rightarrow x \in A_\alpha$ for only finitely many values of $\alpha$

178. Let X be topological space. An idexed family $\{A_\alpha\}_{\alpha \in J}$ of subset of X is locally finite indexed family—

(A) If some point of X has a neighbourhood that intersects $A_\alpha$ for infintely many values of $\alpha$

(B) If each point of X has a neighbourhood that intersects $A_\alpha$ for infintely many values of $\alpha$

(C) If each point of X has a neighbourhood that intersects $A_\alpha$ for only finitely many values of $\alpha$

(D) None of these

179. Following is false—

(A) Every para compact space X is normal

(B) Every closed subspace of a para compact space is para compact

(C) An arbitary subspace of a para compact space and product of para compact spaces need not be para compact

(D) Every metrizable space need not be para compact

180. Let X be a regular space. Every open covering of X has a refinement. If—
   (A) An open covering of X and countably locally finite
   (B) A covering of X and locally finite
   (C) A closed covering of X and locally finite
   (D) All the above

181. Let X be a regular space. Every open covering of X has a refinement. If—
   (A) An open covering of X and countably locally finite
   (B) An open covering of X and locally finite
   (C) A closed covering of X and locally finite
   (D) All the above

182. A space X is locally metrizable if every $x \in$ X has a neighbourhood $\cup$ that is metrizable in the subspace topology—
   (A) Locally metrizable theorem
   (B) Smirnov metrizable theorem
   (C) Special Van Kampen theorem
   (D) None of these

183. Weierstrass M–test : Let $f_n : X \to R$ be a sequence of functions from topological space X into R and

$$S_n(x) = \sum_{i=1}^{N} f_i(x). \text{ The sequence } \{S_n\}$$

converges uniformly to a function S if—
   (A) $|f_i(x)| \le b_i$ for all $x \in X$ and all $i = 1, \ldots, n$
   (B) The series $\Sigma b_i$ is convergent
   (C) Both (A) and (B)
   (D) None of these

184. If X is a topological space, then—
   (A) Each path component of X lies in a component of X
   (B) Some path component of X lies in a component of X
   (C) Each path component of X does not lies in a component of X
   (D) None of these

185. If X is a topological space and locally path connected then—
   (A) The components and the path components of X are different
   (B) The components and the path components of X are the same
   (C) The components and the path components of X are the disjoint
   (D) None of these

186. Let X be a topological space. Given an equivalence relation on X such that $x \sim y$, if there is a connected subset of X containing both $x$ and $y$—
   (A) Then equivalence classes are called the components of X
   (B) Then equivalence classes are called the separation of X
   (C) Then equivalence classes are called the compact of X
   (D) None of these

187. The components of topological space X are connected disjoint subsets of X whose—
   (A) Intersection is X such that each connected subset of X intersects only one of them
   (B) Union is X such that each connected subset of X intersects all of them
   (C) Union is X such that each connected subset of X intersects only one of them
   (D) None of these

188. A topological space X is limit point compact—
   (A) If every disjoint subset of X has a limit point
   (B) If every infinite subset of X has a limit point
   (C) If every finite subset of X has a limit point
   (D) None of these

189. A topological space X is sequentially compact—
   (A) If every sequence in a topological space has a convergent subsequence
   (B) If some sequence in a topological space has a convergent subsequence
   (C) If no sequence in a topological space has a convergent subsequence
   (D) None of these

190. A topological space X is countably compact—

(A) If every countable open covering of X contains a infinite subcollection covering X

(B) If every countable open covering of X contains a finite subcollection covering X

(C) If every countable open covering of X contains no subcollection covering X

(D) None of these

191. One of the following statement is true—

(A) Compactness implies limit point compactness

(B) Limit point compactness implies compactness

(C) Both (A) and (B)

(D) None of these

192. Uniform continuity theorem states : $f$ is uniformly continuous on topological space X and Y—

(A) If $f : X \to Y$ is a continuous map of the compact metric space $(X, d_x)$ to the metric space $(Y, d_y)$

(B) If $f : X \to Y$ is a discontinuous map of the compact metric space $(X, d_x)$ to the metric space $(Y, d_y)$

(C) If $f : X \to Y$ is a continuous map of the non-compact metric space $(X, d_x)$ to the metric space $(Y, d_y)$

(D) None of these

193. Let X be a metrizable space X is limit point compact, then—

(A) X is sequentially compact

(B) X is not sequentially compact

(C) Both (A) and (B)

(D) None of these

194. Let X be a locally compact Hausdorff space which is not compact. Let Y be the one point compactification of X. Then—

(A) Y is compact Hausdorff space

(B) The set Y – X consists of a single point

(C) $\overline{X} = Y$

(D) All the above

195. One of the following statement is true—

(A) If X is countably compact space then it is limit point compact space

(B) If X is countably compact space then it is not limit point compact space

(C) Both (A) and (B)

(D) None of these

196. A topological space X is locally compact at $x$—

(A) If there is some compact subspace. C of X that contains a neighbourhood of $x$

(B) If there is some compact subspace. C of X that does not contains a neighbourhood of $x$

(C) If there is no compact subspace, C of X that does not contains a neighbourhood of $x$

(D) None of these

197. Tychonoff theorem states—

(A) Any arbitrary product of compact spaces is compact in the product topology

(B) A topological space forwhich every open covering contains a countable subcovering

(C) Path connected topological space is connected but converse is not true

(D) None of these

198. A space X is completely regular if—

(A) One-point sets are closed in X

(B) For each $x_0 \in X$ and each closed set A, $x_0 \notin A$, there is a continuous function $f : X \to [0, 1]$

(C) Both (A) and (B)

(D) Either (A) or (B)

199. Compactification $Y_1$ and $Y_2$ of topological space X are equivalent—

(A) If there is a homeomorphism $h : Y_1 \to Y_2$ such that $h(x) = x$ for every $x \in X$

(B) If there is a homeomorphism $h : Y_1 \to Y_2$ such that $h(x) = x$ for some $x \in X$

(C) If there is a homeomorphism $h : Y_1 \to Y_2$ such that $h(x)$ = constant $x$ for every $x \in X$

(D) None of these

200. Let X be completely regular, $\beta(X)$ be its Stone – Cech compactification—

(A) Then every continuous real valued function of X can be uniquely extended to real valued function of $\beta(X)$

(B) Then every bounded real valued function of X can be uniquely extended to real valued function of $\beta(X)$

(C) Then every bounded continuous real valued function of X can be uniquely extended to continuous real valued function of $\beta(X)$

(D) None of these

201. Let $A \subset X$ a topological space and $f : A \rightarrow Z$ be continuous map of A into Hausdorff space $z$—

(A) There is at most one extension of $f$ to a continuous function $g : \overline{A} \rightarrow z$

(B) There is at no extension of $f$ to a continuous function $g : \overline{A} \rightarrow z$

(C) Then every continuous real valued function of X can be uniquely extended to real valued function of $\beta$ (X)

(D) Then every bounded real valued function of X can be uniquely extended to real valued function of $\beta$ (X)

202. A collection of a subsets of topological space X is locally discrete—

(A) If each point X has neighbourhood that intersects at most one element of $a$

(B) If each point X has neighbourhood that interesects at all element of $a$

(C) If each point of X has neighbourhood that intersects no one element of $a$

(D) None of these

203. A collection B is countably locally discrete ($\sigma$ – locally discrete) if—

(A) It is equal to a countable intersection of locally discrete collections

(B) It is equal to a countable union of locally discrete collections

(C) It is equal to uncountable union of locally discrete collections

(D) None of these

204. Let X be a metrizable space. If $a$ is open covering of X, then there is a collection D of subsets of X such that—

(A) D is an open covering of X

(B) D is a refinment of $a$

(C) D is countably locally finite

(D) All the above

205. Let X be metrizable space. Then X has a basis that is—

(A) Countable locally finite

(B) Countable locally infinite

(C) Uncountable locally finite

(D) Uncountable locally infinite

206. Bing metrization theorem states—

(A) A space X is metrizable iff it is regular and has a basis that is countably locally discrete

(B) A space X is metrizable iff it is non-regular and has a basis that is not countably locally discrete

(C) Every para compact space X is normal

(D) Every closed subspace of a para compact space is para compact

207. X is locally compact if—

(A) It topological space X is locally compact at each $x \in X$

(B) It topological space X is locally compact at some $x \in X$

(C) It topological space X is locally compact at none $x \in X$

(D) None of these

208. The path components of topological space X are path connected disjoint subsets of X whose—

(A) Intersection is X, such that each path connected subset of X intersects only one of them

(B) Union is X, such that each path connected of X intersects all of them

(C) Union is X, such that each path connected subset of X intersects only one of them

(D) None of these

209. Let X be a locally compact Hausdarff space which is not compact. Let Y be the one point compactification of X. Then—

(A) Y is compact hausdorff space

(B) X is subspace of Y

(C) The set Y – X consists of a single point

(D) All of these

210. Let X be a hausdorff space. Then X is locally compact at $x$, iff—

(A) For every neighbourhood U of $x$, there is a neighbourhood V of $x$, such that $\overline{V}$ is compact and $\overline{V} \subset \overline{U}$*

(B) For some neighbourhood U of $x$, there is a neighbourhood V of $x$, such that $\overline{V}$ is compact and $\overline{V} = \overline{U}$

(C) For every neighbourhood U of $x$, there is a neighbourhood V of $x$, such that V is compact and $V \subset U$

(D) None of these

211. Let X be a locally compact Hausdorff space and Y be a subspace of X. Y is locally compact—

(A) If Y is closed in X

(B) If Y is open in X

(C) Both (A) and (B)

(D) (A) or (B)

212. A topological space X have a countable basis at $x$—

(A) If there is a countable collection B of neighbourhood of $x$, such that each neighbourhood of $x$ contains atleast one of the elements of B

(B) If there is a collections B of neighbourhood of $x$, such that some neighbourhood of $x$ contains all the elements of B

(C) If there is a countable collections B of neighbourhood of $x$, such that each neighbourhood of $x$ contains all of the elements of B

(D) None of these

213. A topological space has a countable basis at each of its points is called …. countability axiom—

(A) First (B) Second

(C) Third (D) Fourth

214. A topological space X satisfies … countability axiom if X has a countable basis for its topology—

(A) First (B) Second

(C) Third (D) Fourth

215. A subset Y of topological space X is dense in X, if—

(A) $\overline{Y} = X$ (B) $Y = X$

(C) $\overline{Y} \not\subset X$ (D) None of these

216. Separable space—

(A) A topological space having a countable dense subset

(B) A topological space having a countable non dense subset

(C) A topological space having a uncountable dense subset

(D) None of these

217. If topological space has a countable basis. Then—

(A) Every open covering of X contains a countable subcollection covering X

(B) There exist a countable subset of X which is dense in X

(C) Both (A) and (B)

(D) Either (A) or (B)

218. One of the statement is true—

(A) A subspace of a topological space having acountable dense subset need not have a countable dense subset

(B) A subspace of a topological space having a countable dense subset have a countable dense subset

(C) Both (A) and (B)

(D) None of these

219. X is locally compact Hausdorff iff—

(A) A space X is homeomorphic to an open subset of a compact Hausdorff space

(B) A space X is homeomorphic to a closed subset of a compact Hausdarff space

(C) A space X is homeomorphic to a null subset of a compact Hausdorff space

(D) None of these

220. A subset C of a topological space X is saturated (with respect to the surjective map $P : X \to Y$), if—

(A) C contains every set $P^{-1}$ {Y} that it intersects

(B) C contains some set $P^{-1}$ {Y} that it intersects

(C) C contains every set $P^{-1}$ {Y} that it does not intersects

(D) None of these

221. Let X and Y be topological spaces, $P: X \to Y$ be a surjective map. The map P is said to be a quotient map—

(A) Provided a subset U⊂Y is closed in Y iff $P^{-1}$ (U) is open in X
(B) Provided a subset U⊂Y is open in Y iff $P^{-1}$ (U) is open in X
(C) Provided a subset U⊂Y is open in Y iff $P^{-1}$ (U) is open in X
(D) None of these

222. Following statement is true—
(A) The product of two quotient map need not be a quotient map
(B) The product of two quotient map is a quotient map
(C) Both (A) and (B)
(D) None of these

223. Let X be a topological space and $x, y \in$ X. The path in X from $x$ to $y$ is—
(A) A continuous map $f : [a, b] \to$ X of some open interval $[a, b]$ in the real line into X, such that $f(a) = x$ and $f(b) = y$
(B) A discontinuous map $f : [a, b] \to$ X of some closed interval $[a, b]$ in there of line into X, such that $f(a) = f(b)$
(C) A continuous map $f : [a, b] \to$ X of some closed interval $[a, b]$ in the real line into X, such that $f(a) = x$ and $f(b) = y$
(D) A continuous map $f : [a, b] \to$ X of some open interval $[a, b]$ in the real line into X, such that $f(a) = x$ and $f(b) = y$

224. A topological space X is called path connected if—
(A) Each pair of points of X can be joined by a path in X
(B) Some pair of points of X can be joined by a path in X
(C) Some pair of points of X cannot be joined by a path in X
(D) None of these

225. Following is true—
(A) If P : X → Y is a quotient map and if $z$ is a locally compact Hausdorff space, then the map $\pi = P \times i_2 : X \times Z \to Y \times Z$ is a quotient map
(B) Let P : A → B and $q$ : C → D be quotient maps. If B and C are locally compact Hausdorff spaces, then $P \times q : A \times C \to B \times D$ is a quotient map
(C) Both (A) and (B)
(D) None of these

226. A space X is metrizable iff it is para compact and locally metrizable—
(A) Locally metrizable theorem
(B) Smirnov metrizable theorem
(C) Special van kampen theorem
(D) None of these

227. Let X = U∪V, U and V are open in X and U∩V is path connected. Let $x_0 \in$ U∩V. If both inclusions $i : (U, x_0) \to (X, x_0)$ and $j : (V, x_0) \to (X, x_0)$ induce zero homeomorphisms of fundamental groups, then $\pi_1 (X, x_0) = 0$—
(A) Locally metrizable theorem
(B) Smirnov metrization theorem
(C) Special van kampen theorem
(D) None of these

228. If $f$ and $f^1$ are continuous map of the space X into space Y, then $f$ is ... to $f^1$ $(f = f^1)$ if there is one continuous map F : X × [0, 1] → Y such that F $(x, 0) = f(x)$ and F $(x, 1) = f^1(x)$ for each $x \in$ X—
(A) Homotopic (B) Path homotopic
(C) Homomorphic (D) None of these

## Answers

| | | | | |
|---|---|---|---|---|
| 1. (C) | 2. (B) | 3. (A) | 4. (A) | 5. (A) |
| 6. (B) | 7. (A) | 8. (A) | 9. (B) | 10. (D) |
| 11. (B) | 12. (A) | 13. (A) | 14. (A) | 15. (A) |
| 16. (A) | 17. (A) | 18. (A) | 19. (A) | 20. (B) |
| 21. (B) | 22. (A) | 23. (B) | 24. (A) | 25. (A) |
| 26. (B) | 27. (A) | 28. (A) | 29. (A) | 30. (B) |
| 31. (B) | 32. (B) | 33. (A) | 34. (C) | 35. (C) |
| 36. (A) | 37. (A) | 38. (B) | 39. (A) | 40. (B) |
| 41. (A) | 42. (A) | 43. (A) | 44. (A) | 45. (C) |
| 46. (C) | 47. (A) | 48. (A) | 49. (A) | 50. (B) |
| 51. (A) | 52. (A) | 53. (A) | 54. (A) | 55. (A) |
| 56. (A) | 57. (C) | 58. (A) | 59. (A) | 60. (A) |
| 61. (A) | 62. (A) | 63. (A) | 64. (A) | 65. (B) |
| 66. (A) | 67. (C) | 68. (A) | 69. (A) | 70. (A) |
| 71. (A) | 72. (C) | 73. (A) | 74. (A) | 75. (A) |
| 76. (B) | 77. (A) | 78. (B) | 79. (A) | 80. (B) |
| 81. (B) | 82. (A) | 83. (A) | 84. (B) | 85. (B) |
| 86. (B) | 87. (A) | 88. (A) | 89. (B) | 90. (A) |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 91. (C) | 92. (A) | 93. (A) | 94. (C | 95. (A) | 161. (A) | 162. (A) | 163. (C) | 164. (D) | 165. (A) |
| 96. (A) | 97. (B) | 98. (A) | 99. (B) | 100. (A) | 166. (B) | 167. (B) | 168. (C) | 169. (B) | 170. (C) |
| 101. (A) | 102. (A) | 103. (A) | 104. (A) | 105. (A) | 171. (C) | 172. (D) | 173. (D) | 174. (B) | 175. (A) |
| 106. (C) | 107. (A) | 108. (A) | 109. (A | 110. (B) | 176. (C) | 177. (B) | 178. (C) | 179. (D) | 180. (D) |
| 111. (B) | 112. (C) | 113. (A) | 114. (B) | 115. (B) | 181. (D) | 182. (A) | 183. (C) | 184. (C) | 185. (B) |
| 116. (B) | 117. (C) | 118. (A) | 119. (A) | 120. (B) | 186. (A) | 187. (C) | 188. (B) | 189. (A) | 190. (B) |
| 121. (A) | 122. (A) | 123. (C) | 124. (A) | 125. (A) | 191. (A) | 192. (A) | 193. (A) | 194. (D) | 195. (A) |
| 126. (A) | 127. (A) | 128. (A) | 129. (A) | 130. (C) | 196. (A) | 197. (A) | 198. (C) | 199. (A) | 200. (C) |
| 131. (B) | 132. (C) | 133. (B) | 134. (A) | 135. (A) | 201. (A) | 202. (A) | 203. (B) | 204. (D) | 205. (A) |
| 136. (A) | 137. (A) | 138. (A) | 139. (B) | 140. (A) | 206. (A) | 207. (A) | 208. (C) | 209. (D) | 210. (A) |
| 141. (A) | 142. (A) | 143. (A) | 144. (A) | 145. (B) | 211. (D) | 212. (A) | 213. (A) | 214. (B) | 215. (A) |
| 146. (A) | 147. (B) | 148. (C) | 149. (B) | 150. (C) | 216. (A) | 217. (C) | 218. (A) | 219. (A) | 220. (A) |
| 151. (A) | 152. (C) | 153. (B) | 154. (B) | 155. (D) | 221. (C) | 222. (A) | 223. (C) | 224. (A) | 225. (C) |
| 156. (A) | 157. (B) | 158. (A) | 159. (C) | 160. (B) | 226. (B) | 227. (C) | 228. (A) | | |