Partial Differential Equations

Introductions

A partial differential equation for $u(x, y, \ldots\ldots)$ is a relationship between u and its partial derivatives $u_x, u_y, u_{xy}, u_{yy}, \ldots$

It can be written as

$$F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}, \ldots\ldots) = 0$$

where F some is function, $x, y, \ldots\ldots$ are independent variables and $u(x, y, \ldots\ldots)$ is called a dependent variable.

Order of partial differential equation—Order of the partial differential equation is the order of the highest derivative appears in it.

Homogeneous and non-homogeneous partial differential equation—If each term of a partial differential equation contains either the dependent variable or its partial derivative, the equation is called homogeneous otherwise non-homogeneous.

First order partial differential equation—

$$F(x, y, u, u_x, u_y) = 0$$

Second order partial differential equation :

$$F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}) = 0$$

Linear partial differential equation— A partial differential equation is linear, if it is linear in the unknown function and all its derivatives with coefficients depends only on the independent variable.

Quasi-linear partial differential equation— It is quasi-linear, if it is linear in the highest order derivative of the unknown function.

Operator form—The partial differential equation can also be written in the operator form, as $L_x u(x) = f(x)$, where L_x is an operator.

(1) The operator L_x is linear operator, for any two function u and v and for any two constant a and b, if

$$L_x(au + bv) = a L_x u + b L_x v$$

(2) $L_x u(x) = f(x)$ is linear if L_x is a linear operator.

(3) $L_x u(x) = f(x)$ is in homogeneous (non-homogeneous) equation.

(4) An equation which is not linear is non-linear equation.

Solution of partial differential equation—A classical solution of partial differential equation $F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}, \ldots\ldots) = 0$ is an ordinary function $u \approx u(x, y, \ldots\ldots)$ defined in some domain D, which is continuously differentiable such that its all partial derivatives involved in the equation exist and satisfy $F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}, \ldots\ldots) = 0$ identically.

Weak (or generalised) solution—The solution $u = u(x, y, \ldots\ldots)$ is called a weak (or generalized) solution of $F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}, \ldots\ldots) = 0$ if u or its partial derivatives are discontinued in some or all points in D.

General solution—A general solution of partial differential equation is an expression that involves arbitrary functions.

For two independent variables x, y the solution $u = u(x, y)$ of partial differential equation $F(x, y, u, u_x, u_y) = 0$ is visualized geometrically as a surface and called an integral surface in (x, y, u) space.

Linear superposition principle—The general solution of linear homogeneous ordinary differential equation of order n is a linear combination of n linearly independent solutions with n arbitrary constants.

(1) If $u_1(x), u_2(x), \ldots\ldots u_n(x)$ are n independent (linear) solutions of nth order linear homogeneous ordinary differential equation $L_u(x) = 0$. Then for any arbitrary constants $c_1, c_2, \ldots\ldots, c_n$, $u(x) = \sum_{k=1}^{n} c_k u_k(x)$ represents the most general solution of $L_\mu(x) = 0$. This is called the linear superposition principle for ordinary differential

equation. The general solution of $L_\mu(x) = 0$ depends an exactly n-arbitrary constants.

(2) In linear homogeneous partial differential equation $L_x u(x) = 0$ the general solution depends on arbitrary function rather that arbitrary constants. So there are infinite many solution of $L_x u(x) = 0$. If $u_1(x)$, $u_2(x)$, ... are infinite solutions, then $u(x) = \sum_{k=1}^{\infty} c_k, u_k(x)$ where c_k are arbitrary constants, is a solution if the infinite series is convergent.

Boundary Conditions

(1) **Dirichlet conditions**—Where u is prescribed by each point of a boundary δD of a domain D. The problem of finding the solution of a given equation $L_x u(x) = 0$ inside D with prescribed values of u on δD is called the Dirichlet boundary value problem.

(2) **Neumann conditions**—Where value of normal derivative $\frac{\delta u}{\delta n}$, on the boundary δD are specified. The problem is called Neumann boundary value problem.

(3) **Robin condition**—Where $\left(\frac{\delta u}{\delta n} + au\right)$ is specified on δD. The problem is called Robin boundary value problem.

Well posed problems—A problem described by a partial differential equation in a given domain with a set of initial and/or boundary conditions (or other supplementary conditions) is said to be well posed (or properly posed) provided the following criteria are satisfied—

(1) **Existence**—There exists at least on solution of the problem.

(2) **Uniqueness**—There is at most one solution.

(3) **Stability**—The solution must be stable in the sense that it depends continuously on the data, *i.e.*, small change in the given data must produce a small change in the solution.

Classical linear model equations—

(1) **Wave equation**—$u_{tt} - c^2 \nabla^2 u = 0$,

where $\nabla^2 \equiv \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}$ and c is a constant.

(2) **Heat (Diffusion equation)**—$u_t - k \nabla^2 u = 0$, k is constant of diffusivity.

(3) **Laplace equation**—$\nabla^2 u = 0$

(4) **Poisson equation**—$\nabla^2 u = f(x, y, x)$ $\nabla^2 u = f(z, y, z)$

(5) **Helmholtz equation**—$\nabla^2 u + \lambda u = 0$, λ is constant.

(6) **Telegraph equation**—$u_{tt} + au_t + b u = u_{xx}$ a, b are constant.

(7) **Klien-Gordon equation**—

$$\psi + \left(\frac{mc^2}{h}\right) \psi = 0,$$

where $\equiv \frac{\delta^2}{\delta t^2} - c^2 \nabla^2$ is the *d'* Alembertian.

(8) **Korteweg-derives (or *k dv*) equation**—

$$u_t + a u_x + b u_{xx} = 0,$$

where a, b are constant.

(9) **Linear Boussinesq equation**—

$$u_{tt} - a^2 \nabla^2 u - b^2 \nabla^2 u_{tt} = 0,$$

where a, b are constant.

(10) **Bi-hormonic wave equation**—

$$u_{tt} + c^2 \nabla^4 u = 0,$$

where a is a constant.

Partial differential equation of first order

First order partial differential equation—A first order partial differential equation for $z(x, y)$ is a relationship between z and its partial derivatives $p = \frac{\delta z}{\delta x}$, $q = \frac{\delta z}{\delta y}$. It can be written as, $F(x, y, z, p, q) = 0$, where F some is function x, y are independent variable and $z(x, y)$ is called a dependent variable.

Formulation of partial differential equation—

(1) **Elimination of arbitrary constants**—Given $f(x, y, u, a, b) = 0$, where a, b arbitrary constants are. One can form $F(x, y, z, p, q) = 0$ be eliminating arbitrary constants a, b.

(2) **Elimination of arbitrary functions**—Given $\phi(u, v) = 0$ where $u = u(x, y, z)$ and $v = v(x, y, z)$. One can form a partial differential equation $Pp + Qq = R$, by eliminating arbitrary function $\phi(u, v)$. Here P, Q, R are the functions of x, y, z.

Solution to first order partial differential equation—F $(x, y, z, p, q) = 0$

(1) **Complete integral**—Any relation $f(x, y, u, a, b) = 0$ where a, b arbitrary constants, is called a complete integral.

(2) **Particular integral**—A solution obtained from the complete integral by assigning particular values to arbitrary constants is called a particular integral.

(3) **General integral**—In complete integral $f(x, y, u, a, b) = 0$ where $b = \phi(a)$, one have an envelope of the family of surface $f(x, y, u, a, \phi(a)) = 0$, then the solution containing an arbitrary function $\phi(a)$ is called the general integral.

(4) **Singular integral**—An envelope of the family of surface $f(x, y, u, a, b) = 0$, with parameters a, b, if exist is called a singular integral. Singular integral differs from the particular integral in the sense that they can not be obtained from complete integral by giving particular value to the constants.

Linear first order partial differential equation—If degree of p and q are unity, then first order partial differential equation is linear.

Lagrange's equation—(1) A first order linear partial differential equation of the form $Pp + Qq = R$.

(2) If P, Q, R are the functions of x, y, z, then it is called (quasi linear) Lagrange's equation.

(3) If P, Q, R are the functions of x and y only, then it is called (linear) Lagrange's equation.

(4) Equation $Pp + Qq = R$ gives subsidiary equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. Find two independent integrals of subsidiary equation (say) $u = a$ and $v = b$, the general integral of the equation is given by $\phi(u, v) = 0$ or $u = \phi(v)$, where ϕ is arbitrary function.

Non-linear first order partial differential equation—If degree of p and q are other than unity, then first order partial differential equation is non-linear.

(1) Given $F(p, q) = 0$, complete integral $z = ax + by + c$, where $F(a, b) = 0$.

(2) Given $F(z, p, q) = 0$, let $u = x + ay$, then $p = \frac{\delta z}{\delta u}$, $q = a\frac{\delta z}{\delta u} \Rightarrow F\left(z, \frac{\delta z}{\delta u}, a\frac{\delta z}{\delta u}\right)$.

This is the ordinary differential equation of first order $\frac{dz}{du} = \phi(z, a)$ integration gives a complete integral $f(z, a) = u + b$.

(3) Given $f(x, p) = F(y, q)$. Let $f(x, p) = F(y, q) = a$.

Solving for p, q gives $p = \phi(x)$, $q = \phi(y)$ and using the relation, $dz = p\,dx + q\,dy$, the complete integral is $z = ax + by + F(a, b)$.

(4) **Charpits method**—Given $F(x, y, z, p, q) = 0$. Subsidiary equations

$$\frac{dx}{-\frac{\delta F}{\delta p}} = \frac{dy}{-\frac{\delta F}{\delta q}} = \frac{dz}{-p\frac{\delta F}{\delta p} - q\frac{\delta F}{\delta q}} = \frac{dp}{\frac{\delta F}{\delta x} + p\frac{\delta F}{\delta z}}$$

$$= \frac{dq}{\frac{\delta F}{\delta y} + q\frac{\delta F}{\delta z}} = \frac{dF}{0}.$$

Solving for p, q and substituting in $dz = p\,dx + q\,dy$, a complete integral is $f(x, y, z, a, b) = 0$.

Partial differential equation with constant coefficients—Homogeneous linear partial differential equation of n order with constant coefficient :

$$\frac{\delta^n z}{\delta x^n} + A_1\frac{\delta^n z}{\delta x^{n-1}\,\delta y} + A_2\frac{\delta^n z}{\delta x^{n-2}\,\delta y^2} + A_3\frac{\delta^n z}{\delta x^{n-3}\,\delta y^3} + \ldots\ldots + A_n\frac{\delta^n z}{\delta y^n} = F(x, y),$$

where F some is function, x, y are independent variables, $z(x, y)$ is a dependent variable and $A_1, A_2, \ldots\ldots A_n$ are arbitrary constants.

(1) The expression is equivalent to

$$(D^n + A_1 D^{n-1} D^1 + A_2 D^{n-2} D^2 + A_3 D^{n-3} D^3 + \ldots\ldots + A^n D^m)\, z$$
$$= F(x, y) \Leftrightarrow f(D, D^1)\, z$$
$$= F(x, y),$$

where $D^r \equiv \frac{\delta}{\delta x^r}$, $D^{1r} \equiv \frac{\delta}{\delta y^r}$

(2) It is homogeneous because all terms contain derivatives of the same order. Solution is same as for ordinary differential equation with constant coefficients.

(3) Complete integral = complementary function + particular integral = $z_k + z_p$.

(4) **Complementary function**—$(D^n + A_1 D^{n-1} D' + A_2 D^{n-2} D'^2 + A_3 D^{n-3} D'^3 + \ldots\ldots + A_n D'^n)\, z = 0$.

Gives auxilliary equation—$m^n + A_1 m^{n-1} + A_2 m^{n-2} + A_3 m^{+ \ldots\ldots +} A_n = 0$.

(5) $m_1, m_2, m_3, \ldots\ldots m_n$ (distinct roots) the general solution is $z_k = f_1(y + m_1 x) + f_2(y + m_2 x) + \ldots\ldots + f_n(y + m_n x)$.

(6) $m_1, \ldots\ldots m_1, m_{r+1}, \ldots\ldots m_n$ (r equal roots) then complementary function is

$$z_k = f_1(y + m_1 x) + x f_2(y + m_1 x) + \ldots\ldots + x^{r-1} f_r(y + m_1 x) f_{r+1}(y + m_{r+1} x) + f_n(y + m_n x)$$

Particular integral—

F (x, y)	Z_p	
e^{ax+by}	$\frac{1}{f(D, D')} e^{ax+by}$	Substitute $D = a, D' = b$
sin $(mx + ny)$	$\frac{1}{f(D^2, DD', D'^2)}$ sin $(mx + ny)$	Substitute $D^2 = m^2, D'^2 = -n^2$, $DD' = mn$
cos $(mx + ny)$	$\frac{1}{f(D^2, DD', D'^2)}$ cos $(mx + ny)$	Substitute $D^2 = m^2, D'^2 = -n^2$, $DD' = mn$
$x^m y^n$	$\frac{1}{f(D, D')} x^m y^n$	Expand $\frac{1}{f(D, D')}$ and operate on $x^m y^n$ term by term.

Non-Homogeneous linear partial differential equation of *n* order with constant coefficient—If in equation $f(D, D') z = F(x, y)$ the polynomial $f(D, D')$ is not homogeneous, then equation is called non-homogeneous equation. Complete integral = complementary function + particular integral = $Z_k + Z_p$.

(2) **Complementary function**—Factorized $f(D, D')$ into factors of the form

$$(D - mD' - c) z = 0$$

$$\Rightarrow \quad (P - mq) = cz$$

$$\Rightarrow \quad \frac{dz}{1} = \frac{dy}{-m} = \frac{dz}{cz}$$

Integration gives $y + mx = a$ and $z = be^{cx}$

Take $b = \phi(a)$ which gives $z = e^{cx} \phi(y + mx)$

Partial Differential Equation of Second Order

Second order partial differential equation—The general second order partial differential equation for $z(x, y)$ is given by

$$Rr + Ss + Tt + Pp + Qq + Zz = F,$$

where R, S, T, P, Q, Z, F are function of x, y or constant and

$$p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}, r = \frac{\partial^2 z}{\partial x^2} = \frac{\partial p}{\partial x}, s = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}, t = \frac{\partial^2 z}{\partial y^2} = \frac{\partial q}{\partial y}.$$

Monge's Method

(1) Given second order partial differential equation

$$Rr + Ss + Tt = F,$$

where R, S, T, F are function of x, y or constant.

Then Monge's subsidiary equation are

$$R\,dy^2 - S\,dx\,dy + T\,dx^2 = 0 \quad \ldots(1)$$

$$R\,dP\,dy + T\,dq\,dx - F\,dx\,dy = 0 \quad \ldots(2)$$

Since (2) is quadratic it gives two roots m_1, m_2 and one have $dy - m_1\,dx = 0$

and $\quad dy - m_2\,dx = 0$

or $\quad m_1 \neq m_2$

(1) and $dy - m_1\,dx = 0$ gives integral $u_1 = f(v_1)$

(2) (2) and $dy - m_2\,dx = 0$ gives integral $u_2 = f(v_2)$ which on solving gives general integral.

or $\quad m_1 = m_2$:

The equation reduces to Lagrange's equation $Pp + Qq = R$.

Classification of second order linear partial differential equations

Method of characteristics—The general second order linear differential equation in two independent variables x, y is given by

$$Rr + Ss + Tt + Pp + Qq + Zz = F,$$

where R, S, T, P, Q, Z F are function of x, y or constant. The given equation is

(1) **Hyperbolic equation**—$S^2 - 4RT > 0$

(2) **Parabolic equation**—$S^2 - 4RT = 0$

(3) **Elliptic equation**—$S^2 - 4RT < 0$

Characteristic equations—The characteristic equation of $Rr + Ss + Tt + Pp + Qq + Zz = F$ are

$$\frac{dy}{dx} = \frac{1}{2R}(S \pm \sqrt{S^2 - 4RT})$$

and their solutions are called the **characteristic curves** or simply the characteristic of equation

$$Rr + Ss + Tt + Pp + Qq + Zz = F$$

Characteristics curves—The solution of two ordinary differential equations

$$\frac{dy}{dx} = \frac{1}{2R}(S \pm \sqrt{S^2 - 4RT})$$

defines two distinct families of characteristic curves $\phi(x, y) = c_1$ and $\phi(x, y) = c_2$. There are three possible case to consider.

Hyperbolic equation : $S^2 - 4RT > 0$

Integrating $\frac{dy}{dx} = \frac{1}{2R}(S \pm \sqrt{S^2 - 4RT})$ gives two real and distinct famelies of characteristics curves $\phi(x, y) = C_1$ and $\phi(x, y) = c_2$ where c_1, c_2 are constants of intergration

Parabolic equation : $S^2 - 4RT = 0$

There is only one family of real characteristic curves whose slope, due to

$$\frac{dy}{dx} = \frac{1}{2R}(S \pm \sqrt{S^2 - 4RT}) \text{ is given by}$$

$$\frac{dy}{dx} = \frac{S}{2R}$$

Intergration of this equation gives $\phi(x, y) = \xi$ = constant or $\phi(x, y) = \eta$ = constant

Elliptic equation : $S^2 - 4RT < 0$

In this case equation $\frac{dy}{dx} = \frac{1}{2R}(S \pm \sqrt{S^2 - 4RT})$ have no real solutions; there are two families of complex characteristics.

Usually boundary value problems are associated with elliptic equations and initial-value problems arises in connection with heperbolic and parabolic equations.

Cononical Forms

Given general second order linear differential equation in two independent variables x, y is given by $Rr + Ss + Tt + Pp + Qq + Zz = F$ ...(1)

where R, S, T, P, Q, Z, F are function of x, y or constant.

Let $\xi = \xi(x, y)$ and $\eta(x, y)$ ...(2)

Then the transformation gives

$$\overline{R} z_{\xi\xi} + \overline{S} z_{\xi\eta} + \overline{T} z_{\eta\eta} + \overline{P} z_{\xi} + \overline{Q} z_{\eta} + \overline{Z} z = \overline{F} \quad \text{...(3)}$$

where

$$\overline{R} = R\xi^2_x + B\xi_x\xi_y + C\xi_y^2$$

$$\overline{S} = 2R\xi x\eta x + S(\xi x\eta y + \xi_y\eta_x) + 2T\xi y\eta y$$

$$\overline{T} = R\eta_x^2 + S\eta x\eta y + T\eta y^2$$

$$\overline{P} = R\xi xx + S\xi xy + T\xi yy + P\xi x + Q\xi y \quad \text{...(4)}$$

$$\overline{Q} = R\eta_{xx} + S\eta_{xy} + T\eta_{yy} + P\eta_x + Q\eta_y$$

$$\overline{Z} = Z$$

$$\overline{F} = F$$

Also since the classification of (1) depends only on R,S,T equation (1) can be reduced to

$Rr + Ss + Tt = H(x,y,z,p,q)$ ...(5)

which under the transformation of (2) gives there canonical forms,

Hyperbolic equation :

$Z_{\xi\xi} - Z_{\eta\eta} = \phi(\xi, \eta, Z, Z_\xi, Z_\eta)$

or $Z_{\xi\eta} = \phi(\xi, \eta, Z, Z_\xi, Z_\eta)$

Elliptic equation :

$Z\xi\xi + Z\eta\eta = \phi(\xi, \eta, Z, Z\xi, Z\eta)$

Parabolic equation :

$Z\xi\xi = \phi(\xi, \eta, Z, Z\xi, Z\eta)$

or $Z\eta\eta = \phi(\xi, \eta, Z, Z\xi, Z\eta)$

Laplace Equations :

$$\nabla^2 = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

Separation of Variable :

Given $\nabla^2 = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ ...(1)

Let $Z(x,y) = X(x)\,Y(y)$ be a solution of (1) substituting it in (1), one have

$$\frac{d^2X}{dx^2}Y + X\frac{d^2Y}{dy^2} = 0$$

and separating the variables gives

$$\frac{1}{X}\frac{d^2X}{dx^2} = \frac{-1}{Y}\frac{d^2y}{dy^2} = k \quad \text{...(2)}$$

Case I. $k = P^2$, P is real

$$\therefore \quad \frac{d^2X}{dx^2} - Px^2X = 0$$

and $$\frac{d^2y}{dy^2} + P^2y = 0$$

which gives $X = c_1e^{Px} + c_2e^{-Px}$

and $y = c_3\cos Py + c_4\sin Py$

$\therefore$ The solution is

$$z(x, y) = (c_1e^{Px} + c_2e^{-Px})(c_3\cos Py + c_4\sin Py)$$

Case II. $k = 0$

$$\frac{d^2x}{dx^2} = 0 \text{ and } \frac{d^2y}{dy^2} = 0$$

which gives $X = c_5x + c_6$ and $Y = c_7y + c_8$

The solution is

$$Z(x, y) = (c_5x + c_6)(c_7y + c_8)$$

Case III. $k = -P^2$, P is real

$$\therefore \quad \frac{d^2X}{dx^2} + P^2X = 0$$

and $$\frac{d^2y}{dy^2} - P^2Y = 0$$

which gives $X = c_q \cos px + c_{10} \sin px$ and $Y = c_{11} e^{Py} + c_{12} e^{-Py}$

$\therefore$ The solution is $Z(x, y) = (c_9 \cos Px + c_{10} \sin Px)(c_{11} e^{Py} + c_{12} e^{-Py})$

Here all c_i $(i = 1, 2, \ldots\ldots, 12)$ can be calculated by using boundary conditions.

Dirichlet Problem for Rectangle

Given partial differential equation :

$$\nabla^2 = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0, 0 \le x \le a, \quad 0 \le y \le b$$

Boundary conditions : $Z(x, b) = Z(a, y) = 0$,

$$Z(0, y) = 0, Z(x, 0) = f(x)$$

The possible solution is

$$Z(x, y) = (c_9 \cos Px + c_{10} \sin Px)(c_{11} e^{Py} + c_{12} e^{-Py})$$

and required solution is

$$Z(x, y) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{a}\right) \sin\left(n\pi \frac{y-b}{a}\right)$$

where $$A_n = \frac{2}{a \sin h\left(\frac{-n\pi b}{a}\right)} \int_0^a f(x) \sin\left(\frac{n\pi x}{a}\right) dx$$

Neumann Problem for a Rectangle

Given Partial differential equation :

$$\nabla^2 = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0, 0 \le x \le a, \quad 0 \le y \le b$$

Boundary Condition :

$$P(x, b) = q(a, y) = 0,$$
$$q(x, 0) = 0, q(x, b) = f(x)$$

The possible solution is :

$$Z(x, y) = (c_1 \cos Px + c_2 \sin Px)(c_3 e^{Py} + c_4 e^{-Py})$$

The required solution is :

$$Z(x, y) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{a}\right) \cosh\left(\frac{n\pi y}{a}\right)$$

where $$An = \frac{2}{a} \frac{1}{\sin h\left(\frac{n\pi b}{a}\right)} \int_0^a f(x) \cos\left(\frac{n\pi x}{a}\right) dx$$

Wave Equation

Wave equation is

$$\frac{\partial^2 z}{\partial y^2} = c^2 \frac{\partial^2 z}{\partial x^2}$$

Separation of varibles :

Given $$\frac{\partial^2 z}{\partial y^2} = c^2 \frac{\partial^2 z}{\partial x^2} \quad \ldots(1)$$

Let $Z(x, y) = X(x)\, Y(y)$ be a solution of (1) substituting it in (1) one have

$$\frac{d^2X}{dx^2} + kX = 0 \text{ and } \frac{d^2Y}{dy^2} - kc^2y = 0$$

Case I. $k = P^2$, P is real

$$X = c_1e^{Px} + c_2e^{-Px}$$

and $$Y = c_3e^{cPy} + c_4e^{-cPy}$$

$\therefore$ The solution is

$$Z(X, Y) = (c_1e^{Px} + c_2e^{-Px})(c_3e^{cPy} + c_4e^{-cPy})$$

Case II. $k = 0$

$$X = c_5x + c_6 \text{ and } Y = c_7y + c_8$$

$\therefore$ The solution is

$$Z(x, y) = (c_5x + c_6)(c_7y + c_8)$$

Case III. $k = -P^2$, P is real

$$X = c_9 \cos Px + c_{10} \sin Px$$
$$Y = c_{11} \cos cPy + c_{12} \sin cPy$$

$\therefore$ The solution is

$$Z(X, Y) = (c_9 \cos Px + c_{10} \sin Px)(c_{11} \cos cPy + c_{12} \sin cPy)$$

Here all c_i $(i = 1, 2, \ldots\ldots 12)$ can be calculated by using boundary condition.

Vibrating String :

Given $\frac{\delta^2 z}{\partial y^2} = c^2 \frac{\partial^2 z}{\partial x^2}; 0 \le x \le a, 0 > y$

Boundary Conditions :

$Z(0, y) = Z(a, y) = 0$

Initial condition

$Z(x, 0) = f(x), q(x, 0) = g(x)$

The possible solution is

$$Z(x, y) = (c_9 \cos px + c_{10} \sin px)(c_{11} \cos cpy + c_{12} \sin cpy)$$

The required solution is

$$Z(x, 0) = \sum_{n=1}^{\infty} A_n \cos\left(\frac{n \pi x}{a}\right)$$

where $A_n = \frac{2}{a}\int_0^a f(x) \sin\left(\frac{n\pi x}{a}\right)dx$

$$\frac{\partial z(x, 0)}{\partial y} = g(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{a}\right)$$

where $B_n = \frac{2}{n\pi c}\int_0^a g(x) \sin\frac{n\pi x}{a}dx$

Heat Equation

Wave equation is :

$$\frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2}$$

Separation of variable :

$$\text{Given } \frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2} \quad \ldots(1)$$

Let $Z(x, y) = X(x)\, Y(y)$ be a solution of (1) substituting it in (1) one have

$$\frac{d^2X}{dx^2} + kX = 0$$

$$\text{and } \frac{dY}{dy} - kc^2 y = 0 \quad \ldots(2)$$

Case I. $k = P^2$, P is real

$X = c_1 e^{Px} + c_2 e^{-Px}$ and $y = c_3 e^{c^2 p^2 y}$

$\therefore$ The solution is

$$Z(x, y) = (c_1 e^{Px} + c_2 e^{-Px}) c_3 e^{c^2 p^2 y}$$

Case II. $k = 0$

$X = c_4 x + c_5$ and $Y = c_6$

$\therefore$ The solution is $Z(x, y) = (c_4 x + c_5)c_6$

Case III. $k = -P^2$, P is real

$X = c_7 \cos Px + c_8 \sin Px$

and $Y = c_9 e^{-c^2 p^2 y}$

$\therefore$ The solution is

$$Z(x, y) = (c_9 \cos Px + c_{10} \sin Px)\, c_9 e^{-c^2 P^2 y}$$

Here all $c_i (i = 1, 2, \ldots\ldots, 12)$ can be calculated by using boundary conditions.

(B) Fourier Transform

(I) Fourier Series and Integral

Periodic functions—A function $f(x)$ is called periodic if it is defined for real $x \in R$ and if there is any positive number p, Such that $f(x + p) = f(x)$, the number P is called a period

e.g., $\sin x$, $\cos x$, $f(x) =$ constant

(1) If function $f(x)$ is periodic then $f(x + np) = f(x)$ where P is period and $n \in Z$ an integer.

(2) A function $h(x) = af(x) + bg(x)$ is a periodic function given $f(x)$ and $g(x)$ are periodic function of period of P.

(3) If a function $f(x)$ has a smallest period $P > 0$, then this is called fundamental period of $f(x)$

e.g. $\sin x$, $\cos x$ have fundamental period 2π. $\sin 2x$, $\cos 2x$ have fundamental period is π, $f(x) =$ constant has no fundamental period

(4) If $f(x)$ is a periodic function of x, of period P, $f(ax)$, $a \ne 0$ is a periodic function of x of period $\frac{P}{a}$ and $f\left(\frac{x}{b}\right)$, $b \ne 0$ is a periodic function of x of period bq.

5. Trigonometric Series

The series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, where $a_0, \ldots\ldots a_n$ and $b_1 \ldots\ldots b_n$ are coefficient of the series

Euler formula and Fourier Series

The trigonometric series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ with coefficient (called Euler formulas)

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx$$

is called Fourier series of $f(x)$

Orthogonality of the Trignometric Series

Real function f_m and f_n are orthogonal nality in the interval (a, b) if $\int_a^b f_m(x)\, f_n(x)\, dx = 0,\ m \ne n$ and $\int_a^b [f_n(x)]^2 = 1$, for all n.

The trigonometric system sin x, cos x, sin $2x$, cos $2x$, …… is orthogonal on interval $-\pi \le x \le \pi$

Convergence and Sum of Fourier Series

If a periodic function $f(x)$ with period 2π is piecewise continuous in the interval $-\pi \le x \le \pi$ and has a left hand derivative and right hand derivative at each point of that intervals, then the Fourier series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ of $f(x)$ with coefficient (Euler's formula)

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx$$

is convergent. Its sum is $f(x)$, expect at a point x_0 at which $f(x)$ is discontinuous and the sum of the series is the average of the left and righ hand limits of (fx) at x_0.

Functions of any Period P = 2L

If a function $f(x)$ of period P = 2L has fourier series then, the series

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

with the Fourier coefficients of $f(x)$ is given Euler formulas.

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\frac{\cos n\pi x}{L}dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\frac{\sin n\pi x}{L}dx$$

Even functions : A function $f(x)$ is even functions, if

$$f(-x) = f(x)$$

Odd function : A function $f(x)$ is odd function,

if $\quad f(-x) = -f(x)$

1. If $f(x)$ is even function, then

$$\int_{-L}^{L} f(x)dx = 2\int_{0}^{L} f(x)dx$$

2. If $f(x)$ is odd function, then

$$\int_{-L}^{L} f(x)dx = 0$$

3. The product of even and odd function is odd

4. The graph of even function $y = f(x)$ is symmetric about y-axis

Fourier Sine Series

The fourier series of an even function of period 2L is a fourier cosine series

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \frac{\cos n\pi x}{L},$$

where $\quad a_0 = \frac{1}{L}\int_{0}^{L} f(u)\, du$

and $\quad a_n = \frac{2}{L}\int_{0}^{L} f(x)\frac{\cos n\pi x}{L}dx.$

Fourier Cosine Series

The Fourier series of an odd function of period 2L is a Fourier sine series.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where $\quad b_0 = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$

1. The Fourier coefficients of a sum $f_1 + f_2$ are the sums of the corresponding fourier coefficient of f_1 and f_2
2. The Fourier coefficient of cf are c times the corresponding Fourier coefficients of f.

Complex Fourier Series

1. A function $f(x)$ with period 2π complex Fourier series

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$$

where $\quad c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, e^{-inx}dx,\ n = 1, 2$

2. A function $f(x)$ of period P = 2L has complex Fourier Series

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi x}{L}}$$

where $\quad c_n = \frac{1}{2L}\int_{-L}^{L} f(x)\, e^{\frac{in\pi x}{L}}\, dx,\ n = 1, 2$

Fourier Integral

If $f(x)$ is piecewise continuous, in every limit interval and has a right hand derivative and a left hand derivative at each point and if the intergal

$\int_{-\infty}^{\infty} |f(x)| dx$ exists, then $f(x)$ can be represented by a fourier integral

$$f(x) = \int_0^{\infty} [A(W) \cos Wx + B(W) \sin (W)x \, d\omega$$

where A(W) and B(W) are Fourier coefficients of Fourier series defined as

$$A(W) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(v) \cos Wv \, dV$$

$$B(W) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(v) \sin Wv \, dV$$

At the point where $f(x)$ is discountinuous the value of fourier integral equal the average of the left and right hand limits of $f(x)$ at that point.

Fourier sine and cosine integrals

1. If $f(x)$ is even function the $B(\omega) = 0$ and $A(\omega) = \frac{2}{\pi}\int_0^{\infty} f(v) \cos Wv \, dv$. Fourier cosine intgral is

$$f(x) = \int_0^{\infty} A(W) \cos Wx \, dW.$$

2. If $f(x)$ is odd function then $A(W) = 0$, $B(W) = \frac{2}{\pi}\int_0^{\infty} f(v) \sin (\omega v) dv$ Fourier sine integral is

$$f(x) = \int_0^{\infty} B(W) \sin Wx \, dW$$

(II) Fourier sine and cosine transforms

Fourier cosine transforms

1. Fourier cosine transforms of $f(x)$

$$F_c [f(x)] = \tilde{f}_c (W) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x) \cos Wx \, dx$$

2. Inverse Fourier cosine transforms of $\tilde{f}_c (W)$

$$f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} \tilde{f}(W) \cos Wx \, dx$$

Fourier sine Transforms

1. Fourier sine transform of $f(x)$

$$F_s [f(x)] = \tilde{f}_s (W) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x) \sin Wx \, dx$$

2. Inverse Fourier sine tranform of $\tilde{f}_s$ (W)

$$f_s (x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} \tilde{f} (W) \sin Wx \, dw$$

3. If $f(x)$ is absolutely integrable on positive x-axis and piecewise continuous on finite interval, then the Fourier cosine and sine transforms of $f(x)$ exist.

Fourier cosine and sine transform of derviatives—

If $f(x)$ is continuous and absolutely integrable on the x-axis, and $f'(x)$ is piecewise continuous on each interval, also $f(x) \to 0$ as $x \to \infty$, then

$$F_C \{f'(x)\} = WF_C \{f(x)\} - \sqrt{\frac{2}{\pi}} f(0)$$

$$F_S \{f'(x)\} = -WF_S \{f(x)\}$$

$$F_C \{f''(x)\} = -W^2 F_C \{f(x)\} - \sqrt{\frac{2}{\pi}} f'(0)$$

$$F_S \{f''(x)\} = -W^2 F_S \{f(x)\} + \sqrt{\frac{2}{\pi}} Wf(0)$$

Fourier sine transform

fx	$f_S \{f(x)\}$
$\begin{cases} 1, & 0 < x < a \\ 0, & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}}\left[\frac{1 - \cos a\omega}{W}\right]$
$e^{-ax} (a > 0)$	$\frac{W}{a^2 + W^2}$
$xe^{-ax^2} (a > 0)$	$\frac{W}{\sqrt{2a}} e^{-w^2/4a}$
x^{a-1} $0 < a < 1$	$\sqrt{\frac{2}{\pi}}\left[\Gamma(a) W^{-a} \sin \frac{a\pi}{2}\right]$
$\cos ax^2$	$\sqrt{\frac{\pi}{2a}}\left[\sin \frac{W^2}{4a} \cos\left(\frac{W}{\sqrt{2\pi a}}\right) - \cos \frac{W^2}{4a} \sin\left(\frac{W}{\sqrt{2\pi a}}\right)\right]$
$\sin ax^2$	$\sqrt{\frac{\pi}{2a}}\left[\cos \frac{W^2}{4a} \cos\left(\frac{W}{\sqrt{2\pi a}}\right) + \sin \frac{W^2}{4a} \sin\left(\frac{W}{\sqrt{2\pi a}}\right)\right]$

Fourier casine transform

fx	$fe\ [f(x)]$
$\begin{cases} 1, 0 < x < a \\ 0, \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{\sin aW}{W}$
$e^{-ax} (a > 0)$	$\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + w^2}$
$e^{-ax^2} (a > 0)$	$\frac{1}{\sqrt{2a}} e^{-w^2/4a}$
$\cos ax^2$	$\frac{1}{\sqrt{2\pi}} \cos\left(\frac{W^2}{4a} - \frac{\pi}{4}\right)$
$\sin ax^2$	$\frac{1}{\sqrt{2\pi}} \cos\left(\frac{W^2}{4a} + \frac{\pi}{4}\right)$
$x^{n-1} (0 < a < 1)$	$\Gamma(a)\ W^{-a} \cos \frac{a\pi}{2}$

Fourier Transform

1. Fourier transform of $f(x)$

$$F(w) = \int_{-\infty}^{\infty} f(x)\, e^{iwx}\, dx$$

2. Inverse Fourier transform of F(W)

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(W)\, e^{-iwx}\, dW$$

3. If $f(x)$ is absoluetly integrable on positive x-axis and piecewise continuous on finite interval, then the Fourier transform at $f(x)$ exists.

Fourier trnasform of derivatives

If $f(x)$ is continuous on the x-axis and $f(x) \to 0$ as $|x| \to \infty$ $f'(x)$ is absolutely integrable on then axis.

$$F\{f'(x)\}\} = iWF\{f(x)\}$$

$$F\{f''(x)\} = -W^2F\{f(x)\}$$

$$F\left\{\frac{d^nf}{dx^n}\right\} = (-iW)^n F\{f(x)\}$$

Relation between Fourier Transforms

If F(W) is the Fourier transform of $f(x)$, $-\infty < x < \infty$ then

$f(x)$ is even $\Rightarrow F(W) = 2F_C(W)$

$f(x)$ is odd $\Rightarrow F(W) = 2i\, F_S(W)$

Fourier Transforms and Initial Bondary-Value Problem

1. The Fourier transform of $u(x, t)$, $x \in$ R is denoted by $F\{u(x, t)\} = U(w, t)$ and is defined by the integral

$$F\{\mu(x, t)\} = U(w, t)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-iwx}\, u(x, t) dx$$

where w is real and is called the transform variable.

2. The inverse Fourier transform is

$$f\{U(w,t)\} = \mu(x, t)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{iwx} U(w, t) dw$$

3. If $\mu(x, t) \to 0$ as $|x| \to \infty$, then

$$f\left\{\frac{\partial u}{\partial x}\right\} = iw\, U(w, t)$$

4. If $\mu(x, t)$ is continuously n-time differentiable and $\frac{\partial^k \mu}{\partial u^k} \to 0$, as $|x| \to \infty$ for $k = 1, 2, 3, \ldots,$ $(n-1)$, then

$$F\left\{\frac{\partial^n u}{\partial x^n}\right\} = (-iw)^n$$

$$F\{u(x,t)\} = (-iw)^n\, U(w, t)$$

$$F\left\{\frac{\partial u}{\partial t}\right\} = \frac{dU}{dt}$$

$$F\left\{\frac{\partial u}{\partial t}\right. = \frac{\partial^2 U}{\partial t^2}, \ldots F\left\{\frac{\partial^n u}{\partial t^n} = \frac{\delta^n U}{\partial t^n}\right.$$

5. A sufficient condition for $\mu(x, t)$ to have a fourier transform is that $\mu(x, t)$ is absoluetly integrable in $-\infty < x < \infty$

1. Convolution Theorem—If

$$F\{f(x)\} = F(W)$$

and $F\{g(x)\} = G(W)$ then

$$F\{(x)^* g(W)\} = \sqrt{2\pi}\, F(W)\, G(W)$$

2. $\int_{-\infty}^{\infty} f(t - \tau) g(\tau) d\tau = \int_{-\infty}^{\infty} e^{iwx}$ $F(W)G(W)dW$

3. $f(x)^* \sqrt{2\pi}\, \delta(x) = f(x) = \sqrt{2\pi}\, \delta(x)^* f(x)$ (identity)

4. The fourier transform are useful in solving a wide variety of initial boundary value problem goverened by linear differential equations.

Some important Fourier transforms

$f(t)$	F(W)	
$f(t)$	$F(W) = \int_{-\infty}^{\infty} f(t) e^{-iWt} dt$	
$\frac{1}{2\pi}\int_{-\infty}^{\infty} f(W)e^{iWt} dW$	F(W)	
$af(t) + bg(t)$, a, b constant	$aF(t) + bG(t)$	Lineartiy Property
$f(at)$	$\frac{1}{\lvert a\rvert} F\left(\frac{W}{a}\right)$	
$f(-t)$	$F(-W)$	
$f(t-T)$, Tisreal	$e^{-iWt} F(W)$	
$e^{irt} f(t)$ r is real	$F(W-r)$	
Ft	$2\pi f(-W)$	
$\left(\frac{d}{dt}\right)^n f(t)$	$(iW)^n F(W)$	
$(-it)^n f(t)$	$\left(\frac{d}{dW}\right)^n F(W)$	
$\int_{-\infty}^{\infty} f(\tau)d\tau$	$\frac{F(W)}{iW} + \pi F(0) \delta(W)$	
$f(t)^* g(t) = \int_{-\infty}^{\infty} f(t-\tau) g(\tau)d\tau$	$\sqrt{2\pi}\, F(W)G(W)$	Convolution
$f(t)\, g(t)$	$\frac{1}{2\pi} F(W)^* G(W)$	
$\delta(t)$	1	
$\delta^{(n)}(t)$	$(iW)^n$	
$\delta^{(n)}(t-T)$	$(iW)^n e^{-iWt}$ $(n = 0, 1, 2, \ldots\ldots)$	
1	$2\pi\, \delta(W)$	
t^n	$2\pi i^n\, \delta^{(n)}(W)$ $(n = 0, 1, 2, \ldots)$	
t^{-n}	$\frac{\pi(-i)^n}{(n-1)!} W^{n-1}$ δgnW $(n = 1, 2, 3)$	
$\theta(t) = H(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$	$\frac{1}{iW} + \pi\, \delta(W)$	
$t^n\theta(t)$	$\frac{n!}{(iW)^{n+1}} + \pi i^n \delta^{(n)}(W)$ $n = 1,$	

OBJECTIVE TYPE QUESTIONS

1. Given wave equation $\frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2}$. By separation of variable let $z(x, y) = X(x)Y(y)$ be a solution. Substituting it in $\frac{\partial z}{\partial y} = c^2\frac{\partial^2 z}{\partial x^2}$ one have $\frac{d^2X}{dx^2} + kX = 0$ and $\frac{dY}{dy} - kc^2y = 0$ if $k = -p^2$, p is real then the solution is (c's are constant)
   (A) $z(x, y) = (c_9 \cos px)c_9\, c^{-c^2p^2y}$
   (B) $z(x, y) = (c_9 \cos px + c_{10} \sin px)c_9\, c^{-c^2p^2y}$
   (C) $z(x, y) = e^{-c^2p^2y}$
   (D) $z(x, y) = (c_{10} \sin px)c_9\, c^{-c^2p^2y}$

2. Given wave equation $\frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2}$. By separation of variable, Let $z(x, y) = X(x)\, Y(y)$ be a solution. Subsitituting it in $\frac{\partial z}{\partial y} = c^2\frac{\partial^2 z}{\partial x^2}$ are have $\frac{d^2X}{dx^2} + kX = 0$ and $\frac{dY}{dy} - kc^2Y = 0$. If then the solution is (c's are constant)—
   (A) $z(x, y) = c_6$
   (B) $z(x, y) = (c_4x + c_5)$
   (C) $z(x, y) = (c_4x)$
   (D) $z(x, y) = (c_4x + c_5)c_6$

3. Given wave equation : $\frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2}$. By separation of variable let $Z(x, y) = X(x), x\, Y(y)$ be a solution substituting it in $\frac{\partial z}{\partial y} = c^2\frac{\partial^2 z}{\partial x^2}$ are have $\frac{d^2X}{dx^2} + kX = 0$ and $\frac{dY}{dy} -$

$kc^2Y = 0$. If $k = P^2$, p is real, then the solution is (c's are constant)—

(A) $Z(x, y) = (c_1 + c_2\, e^{-px})e^{c^2p^2y}$

(B) $Z(x, y) = e^{c^2p^2y}$

(C) $Z(x, y) = (c_1\, e^{px} + c_2\, e^{-px})c_3\, e^{c^2p^2y}$

(D) $Z(x, y) = (c_1e^{px} + c_2\, e^{-px})$

4. Given $\dfrac{\partial^2 z}{\partial y^2} = c^2 \dfrac{\partial^2 z}{\partial x^2}$ $0 \le x \le a$, $0 > y$ with boundary conditions $Z(0, y) = Z = (a, y) = 0$ and initial conditions :

$Z(x, 0) = f(x)$, $q(x, 0) = g(x)$.

If the possible solution is

$Z(x, y) = (c_9 \cos px + c_{10} \sin px)$
$(c_{11} \cos cpy + c_{12} \sin cpy)$

then x required solution is given by.

(A) $Z(x, 0) = \sum\limits_{n=1}^{\infty} A_n \cos\left(\dfrac{n\pi x}{a}\right)$

where $A_n = \dfrac{2}{a}\displaystyle\int_0^a f(x) \sin\left(\dfrac{m\pi x}{a}\right) dx$

(B) $\dfrac{\partial z}{\partial y}(x, 0) = g(x) = \sum\limits_{n=1}^{\infty} B_n \sin\left(\dfrac{n\pi x}{a}\right)$,

where $B_n = \dfrac{2}{n\pi c}\displaystyle\int_0^a g(x) \sin\dfrac{n\pi x}{a} dx$

(C) Both (A) and (B) a bove

(D) None of these

5. Given the wave equation $\dfrac{\partial^2 z}{\partial y^2} = c^2 \dfrac{\partial^2 z}{\partial x^2}$ · By separation of variable Let $Z(x, y) = X(x)\, Y(y)$ be a solution of wave equation subsitiuting it in wave equation, one have $\dfrac{d^2X}{dx^2} + kX = 0$ and $\dfrac{d^2Y}{dy^2} - Kc^2Y = 0$. If $k = -P^2$, P is real then the solution is (c's are constant)

(A) $Z(x, y) = (c_9 \cos Px + c_{10} \sin Px)$

(B) $Z(x, y) = (c_9 \cos Px + c_{10} \sin Px)$
$+ (c_{11} \cos cPy + c_{12} \sin cPy)$

(C) $Z(x, y) = (c_9 \cos Px - c_{10} \cos Px)$
$(c_{11} \cos cPy + c_{12} \cos Py)$

(D) $Z(x, y) = (c_9 \cos Px + c_{10} \sin Px)$
$(c_{11} \cos cPy + c_{12} \sin cPy)^*$

6. Given the wave equation $\dfrac{\partial^2 z}{\partial y^2} = c^2 \dfrac{\partial^2 z}{\partial x^2}$ · By separation of variable. Let $Z(x, y) = X(x)\, Y(y)$ be a solution of wave equation. Substituting it in wave equation, one have $\dfrac{d^2X}{dx^2} + kX = 0$ and $\dfrac{d^2Y}{dy^2} - kc^2Y = 0$. If $k = 0$ then the solution is (c's are constant)

(A) $Z(x, y) = (c_5x + c_6)$

(B) $Z(x, y) = (c_5\, x)$

(C) $Z(x, y) = (c_5x + c_6) + (c_7x + c_8)$

(D) $Z(x, y) = (c_5x + c_6)\, (c_7y + c_8)$

7. Given the wave equation $\dfrac{\partial^2 z}{\partial y^2} = c^2 \dfrac{\partial^2 z}{\partial x^2}$ · By separation of variable let $Z(x, y) = X(x)\, Y(y)$ be a solution of wave equation substituting it in wave equation one have $\dfrac{d^2X}{dx^2} + kX = 0$ and $\dfrac{d^2Y}{dy^2} - kc^2Y = 0$. If $k = P^2$, P is real, then the solution is (c's are constant)—

(A) $Z(x, y) = (c_1 + c_2\, e^{-Px})$

(B) $Z(x, y) = (c_1\, e^{Px})\, (c_2\, e^{cPy})$

(C) $Z(x, y) = (c_1\, e^{Px} + c_2\, e^{-Px})$

(D) $Z(x, y) = (c_1\, e^{Px} + c_2\, e^{-Px})$
$(c_3\, e^{cPy} + c_4\, e^{-cPy})$

8. Given Laplace equation $\nabla^2 = \dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

Let $Z(x, y) = X(x)\, Y(y)$ be a solution of given equation. Substituting it in given equation one have $\dfrac{d^2X}{dx^2}\, Y + X\dfrac{d^2Y}{dy^2} = 0$ and separating the variable gives $\dfrac{1}{X}\dfrac{d^2X}{dx^2} = \dfrac{-1}{Y}\dfrac{d^2Y}{dx^2} = k$

If $k = -P^2$, P is real the solution is (c's are constant)

(A) $Z(x, y) = (c_9 \cos Px + c_{10} \sin Px)$

(B) $Z(x, y) = (c_9 \cos Px + c_{10} \sin Px)$
$(c_{11}\, e^{Py} + c_{12}\, e^{-Py})$

(C) $Z(x, y) = (c_9 \cos Px \sin Px)$

(D) $Z(x, y) = (\cos Px.\ \sin Px)\, (e^{Py}\, e^{-Py})$

9. Given Laplace equation $\nabla^2 = \dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$.

Let $Z(x, y) = X(x)\, Y(y)$ be a solution of given equation. Substituting it in given equation one have $\dfrac{d^2X}{dx^2}\, Y + X\dfrac{d^2Y}{dy^2} = 0$ and separating the variables given $\frac{1}{X}\frac{d^2X}{dx^2} = \frac{-1}{Y}\frac{d^2Y}{dy^2} = k$. It $k = 0$ the solution is (c's are constant)

(A) $Z(x, y) = (c_5 x + c_6)$

(B) $Z(x, y) = (c_5\, xy)$

(C) $Z(x, y) = (c_5 x + c_6)(c_7 y + c_8)^*$

(D) None of these

10. Given Laplace equation $\nabla^2 = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$. Let $Z(x, y) = X(x)\,Y(y)$ be a solution of given equation. Substituting it in given equation one have $\frac{d^2X}{dx^2}Y + X\frac{d^2Y}{dy^2} = 0$ and separating the variables gives $\frac{1}{X}\frac{d^2X}{dx^2} = \frac{-1}{Y}\frac{d^2Y}{dy^2} = K$. If $K = P^2$, P is real, then solution is (c's are constant)

(A) $Z(x, y) = (c_1 e^{Px} + c_2 e^{-Px}) + (c_3 \cos Py + c_4 \sin Py)$

(B) $Z(x, y) = \dfrac{(c_1 e^{Px} + c_2 e^{-Px})}{(c_3 \cos Py + c_4 \sin Py)}$

(C) $Z(x, y) = (c_1 e^{Px} + c_2 e^{-Px})$

(D) $Z(x, y) = (c_1 e^{Px} + c_2 e^{-Px})(c_3 \cos Py + c_4 \sin Py)$

11. Given second order partial differential equation $Rr + Ss + Tt = F$. Where R, S, T, F are functions of x, y or constant. Then Monge's subsidiary equations are—

(A) $Rdy + Sdxdy + Tdx = 0$ and $Rdpdy + Tdqdu - Fdxdy = 0$

(B) $Rdy - Sdxdy - Tdx = 0$ and $Rdp + Tdq - Fdx = 0$

(C) $Rdy^2 - Sdxdy + Tdx^2 = 0$ and $Rdpdy + Tdqdx - Fdxdy = 0$

(D) $Rdy\, Sdx + Tdx^2 = 0$ and $Rdy + Tdx - F = 0$

12. A first order linear partial differential equation of the form $F(x, y, z, p, q) = 0$ solved by Charpit's method have subsidiary equation—

(A) $$\frac{dx}{\frac{\partial F}{\partial p}} = \frac{dy}{\frac{\partial F}{\partial q}} = \frac{dz}{p\frac{\partial F}{\partial q} - q\frac{\partial F}{\partial q}} = \frac{dp}{\frac{\partial F}{\partial x} + p\frac{\partial F}{\partial z}} = \frac{dq}{\frac{\partial F}{\partial y} + q\frac{\partial F}{\partial z}} = \frac{dF}{0}$$

(B) $$\frac{dx}{-\frac{\partial F}{\partial p}} = \frac{dy}{-\frac{\partial F}{\partial q}} = \frac{dz}{-p\frac{\partial F}{\partial p} - q\frac{\partial F}{\partial q}} = \frac{dp}{\frac{\partial F}{\partial x} + p\frac{\partial F}{\partial z}} = \frac{dq}{\frac{\partial F}{\partial y} + q\frac{\partial F}{\partial z}} = \frac{dF}{0}$$

(C) $$\frac{dx}{\frac{dF}{dp}} = \frac{dy}{\frac{\partial F}{\partial q}} = \frac{dz}{p\frac{\partial F}{\partial p} + q\frac{\partial F}{\partial p}} = \frac{dp}{\frac{\partial F}{\partial x} + p\frac{\partial F}{\partial z}} = \frac{dq}{\frac{\partial F}{\partial y} + q\frac{\partial F}{\partial z}} = \frac{dF}{0}$$

(D) None of these

13. A First order linear partial differential equation of the form $Pp + Qq = R$ called (linear) Lagrange's equation has a subsidiary equation—

(A) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ (B) $\frac{dx}{P} = \frac{dy}{q} = \frac{dz}{r}$

(C) $\frac{1}{P} = \frac{1}{q} = \frac{1}{R}$ (D) None of these

14. A first order linear partial differential equation of the form $Pp + Qq = R$ is called (linear) Lagrange's equation—

(A) If P, Q, R are the functions of x, y, z

(B) If P, Q, R are the functions of x and y only

(C) If P, Q, R are the function of y and z only

(D) None of these

15. A first order linear partial differential equation of form $Pp + Qq = R$ is called (Quasi-Linear) Lagrange's equation—

(A) If P, Q, R are the function of x, y, z

(B) If P, Q, R are the function of x and y only

(C) If P, Q, R are the function of x and z only

(D) None of these

16. In first order partial differential equation $F(x, y, z, p, q) = 0$ if degree of p and q are …, then first order partial differential equation is linear—

(A) One (B) Two

(C) Three (D) Four

17. In first order partial differential euqation $F(x, y, z, p, q) = 0$. General integral is—

(A) A solution obtained from the complete integral $f(x, y, u, a, b) = 0$ by assigning particular values to arbitrary constants a, b

(B) In complete integral $f(x, y, u, a, b) = 0$ where $b = \phi(a)$ one have an envelope of the family of surface $f(x, y, u, a, \phi(a)) = 0$

(C) In complete integral $f(x, y, \mu, a, b) = 0$ is only constant

(D) None of these

18. In first order partial differiential equation, $F(x, y, z, p, q) = 0$. Particular integral is—

(A) A solution obtained from the complete integral $f(x, y, u, a, b) = 0$ by assinging particular ... values to arbitrary constant a, b

(B) In complete integral $f(x, y, u, a, b) = 0$ where $b = \phi(a)$ one have an envelope of the family of surface $f(x, y, z, p, q) = 0$

(C) In complete integral $f(x, y, z, P, Q) = 0$ is constant

(D) None of these

19. For the first order partial differiential equation $F(x, y, z, p, q) = 0$.
Complete integral is—

(A) Any relation $f(x, y, u, a, b) = 0$ where a, b arbitary constant

(B) An envelope of the family surface $f(x, y, u, a, b) = 0$ with parameters a, b if exist

(C) Any relation $f(x, y, z, p, q) = 0$ is only constant

(D) None of these

20. Bi-harmonic wave equation is—

(A) $u_t + au_x + bu_{xx} = 0$, where a, b are constant

(B) $u_{tt} - a^2\nabla^2 u - b^2\nabla^2 u_{tt} = 0$, where a, b are constant

(C) $u_{tt} + c^2\nabla^4 u = 0$, where a is constant

(D) None of these

21. Linear Boussinesq equation is—

(A $u_t + au_x + bu_{xx} = 0$, where a, b are constant

(B) $u_{tt} - a^2\nabla^2 u - b^2\nabla^2 u_{tt} = 0$ where a, b are constants

(C) $u_{tt} + c^2\nabla^4 u = 0$, where a is constant

(D) None of these

22. Korteweg derives (or kdV) equation is—

(A) $u_t + au_x + bu_{xx} = 0$ where a, b are constants

(B) $u_{tt} - a^2\nabla^2 u - b^2\nabla^2 u_{tt} = 0$, where a, b are constants

(C) $u_{tt} + c^2\nabla^4 u = 0$, where a is constant

(D) None of these

23. Klien-Gordon equation is represented by—

(A) $\nabla^2 u + \lambda u = 0$; where λ is constant

(B) $u_{tt} + au_t + bu = u_{xx}$, where a, b are constants

(C) $\psi + \left(\frac{mc^2}{h}\right)\psi = 0$, where $\frac{\partial^2}{\partial t^2} - c^2\nabla^2$ is the d' Alembertian

(D) None of these

24. Telegraph equation is represented by—

(A) $\nabla^2 u + \lambda u = 0$ where λ is constant

(B) $u_{tt} + au_t + bu = u_{xx}$ where a, b are constant

(C) $\psi\left(\frac{mc^2}{h}\right)\psi = 0$, where $\frac{\partial^2}{\partial t^2} - c^2\nabla^2$ is the d' Alembertian

(D) None of these

25. Helmholtz equation is represented by—

(A) $\nabla^2 u + \lambda u = 0$. where λ is constant

(B) $u_{tt} + au_t + bu = u_{xx}$ where a, b are constants

(C) $\psi\left(\frac{mc^2}{h}\right)\psi = 0$. where $= \frac{\partial^2}{\partial t^2} - c^2\nabla^2$ is the d' Alembertian

(D) None of these

26. Poisson equation is represented by—

(A) $u_{tt} - c^2\nabla^2 u = 0$, c is constant

(B) $u_t - k\nabla^2 u = 0.$,k is constant

(C) $\nabla^2 u = 0$

(D) $\nabla^2 u = f(x, y, z)$

27. Laplace equation is represented by—

(A) $u_{tt} - c^2\nabla^2 u = 0$, c is constant

(B) $u_t - k\nabla^2 u = 0$, k is constant

(C) $\nabla^2 u = 0$

(D) $\nabla^2 u = f(x, y, z)$

28. Heat (Diffusion equation) is represented by—

(A) $u_{tt} - c^2\nabla^2 u = 0$, where c is a constant

(B) $u_t - k\nabla^2 u = 0$, where k is constant

(C) $\nabla^2 u = 0$

(D) $\nabla^2 u = f(x, y, z)$

29. Wave equation is represented by—
    (A) $u_{tt} - c^2\nabla^2 u = 0$, where c is constant
    (B) $u_t - k\nabla^2 u = 0$, where k is constant
    (C) $\nabla^2 u = 0$
    (D) $\nabla^2 u = f(x, y, z)$

30. A problem described by a partial differential equation in a given domain with a set of intial and/or boundry conditions (or other supplementry conditions is said to be well posed (or properly posed) provided which of the following criteria are satisfied ?
    (A) Existence : There exists at least one solution of the problem
    (B) Uniquences : There is at most one solution.
    (C) Stability : The solution must be stable in the sense that it depends continuously on the data
    (D) All the above

31. In Robin conditions—
    (A) u is prescribed by each point of a boundary ∂D of a domain D
    (B) Where value of normal derivative $\frac{\partial \mu}{\partial n}$ on the boundry ∂D are sepcified
    (C) $\left(\frac{\partial u}{\partial n} + au\right)$ is specified on ∂D
    (D) None of these

32. In Neumann condition—
    (A) μ is prescribed by each point of boundary ∂D of a domain D
    (B) Where value of normal derivative $\frac{\partial u}{\partial n}$ on the boundary ∂D are specified*
    (C) $\left(\frac{\partial u}{\partial n} + au\right)$ is specified ∂D
    (D) None of these

33. In Dirichlet conditions—
    (A) U is prescribed by each point of a boundary ∂D of a domain D
    (B) Where value of normal derivative $\frac{\partial u}{\partial n}$ on the boundary ∂D are specified
    (C) $\left(\frac{\partial u}{\partial n} + au\right)$ is specified on ∂D
    (D) None of these

34. Following expression $F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}) = 0$ represents [where $u = u(x, y)$]—
    (A) First order partial differential equation
    (B) Second order partial defferential equation*
    (C) Third order partial differential equation
    (D) None of these

35. In linear homogeneous partial differential equation, the general solution depends on—
    (A) arbitrary function
    (B) Arbitrary constants
    (C) Only constant
    (D) None of these

36. Linear superposition principle states that—
    (A) The general solution of a linear homogeneous ordinary differential equation of order n is linear combination of n dependent solutions
    (B) The general solution of linear homogeneous ordinary differential equation of order n is a linear combination of n linearly independent solutions with n arbitrary constants
    (C) The general solution of linear homogeneous ordinary differential equation of order n is a linear combination of n linearly in dependent solutions with no arbitrary constants
    (D) None of these

37. Non-homogeneous partial differential equation is—
    (A) If each term as a partial differential equation contains either the dependent variable or its partial derivative
    (B) If each term of a partial differential equation does not contains either the dependent variable or its partial derivative
    (C) If each term of a partial differential equation contains the dependent variable but not its partial derivative
    (D) None of these

38. For the independent variables x, y, the solution $\mu = \mu(x, y)$ of partial differential equation $F(x, y, u, \mu_x, \mu_y) = 0$ is visualized geometrically as a surface and called—
    (A) An integral surface in (x, y, u) space

(B) Differential surface in (x, y, u) space
(C) An solid surface in (x, y, u) space
(D) None of these

39. A general solution of a partial differential equation is an expression—
(A) That involves arbitary functions*
(B) That does not involves arbitary functions
(C) Both (A) and (B)
(D) None of these

40. The solution $u = u(x, y, \ldots)$ is called a weak (or generalized) solution of $F(x, y, u, u_x, \mu_y, \mu_{yy}, u_{xx}, u_{xy}, u_{yy} \ldots) = 0$. if—
(A) u or its partial derivatives are discontinued in some or all points in D
(B) u or its partial derivative are continued in some or all points in D
(C) u or its partial derivatives do not exist in some or all point in D
(D) None of these

41. A classical solution (solution, of partial defferential equation $F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy} \ldots) = 0$ is an ordinary function $u = \mu(x, y, \ldots)$ defined in some domain D, which—
(A) Is countinuously differentiable such that its all partial derivatives involved in the equation exist
(B) Satisfies $F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy} \ldots) = 0$ identically
(C) Both (A) and (B) above
(D) None of these

42. Quasi-linear partial differential equation is—
(A) If it is linear in the lowast order derivative of the known function
(B) If it is linear in the any derivative of the unknown function
(C) If it is linear in the hightest order derivative of the unknown function
(D) None of these

43. A partial differential equation is linear—
(A) If it is linear in the known function and all its derivatives with coefficients depends only on the dependent variable
(B) If it is linear in the unknown function and all its derivatives with coefficients depends only on the independent variable
(C) if it is linear in the known function and all its derivatives with coefficients depends only on the dependent variable
(D) None of these

44. Following expression $F(x, y, u, u_x, u_y) = 0$ represents where $u = u(x, y)$—
(A) First order partial differential equation
(B) Second order partial differential equation
(C) Third order parital differential equation
(D) None of these

45. Homogeneous partial differential equation is—
(A) If each term of a partial differential equation contains either the dependent variable or its partial derivatives
(B) If each term of a partial differential equation does not contains either the dependent variable or its partial derivative
(C) If each term of partial differential equation contains the dependent variable but not its partial derivative
(D) None of these

46. The general second order linear differential equation in two independent variable x, y is given by $Au_{xx} + Bu_{xy} + Cu_{yy} + D\mu_x + Eu_y + Fu = G$. where A, B, C, D, E and F are given functions of x and y or constant. The characteristic equation is given by—
(A) $\frac{dy}{dx} = (B \pm \sqrt{B^2 - 4AC})$
(B) $\sqrt{B^2 - 4AC}$
(C) $\frac{dy}{dx} = \frac{1}{2A}(B \pm \sqrt{B^2 - 4AC})$
(D) None of these

47. The general second order linear differential in two independent variable x, y is given by $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$, where A, B, C, D, E and F are given functions of x and y or constant. The equation is elliptic equation if —
(A) $B^2 - 4AC > 0$ (B) $4AC = 0$
(C) $B^2 - 4AC < 0$ (D) None of these

48. The general second order linear differential equation in two independent variable x, y is

given by $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$, where A, B, C, D, E and F are given functions of x and y or constant. The equation is parabolic equation if—

(A) $B^2 - 4AC > 0$ (B) $4AC = 0$
(C) $B^2 - 4AC < 0$ (D) None of these

49. The general second order linear differential equation in two independent variable x, y is given by $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$. Where A, B, C, D, E and F are given functions of x and y or constant. The equation in Hyperbolic equation if—

(A) $B^2 - 4AC > 0$ (B) $B^2 - 4AC = 0$
(C) $B^2 - 4AC < 0$ (D) None of these

50. Order of the partial differential equation is—
(A) The order of the highest derivative appears in it
(B) The order of the lowest derivative appears in it
(C) The order of any derivative appears in it
(D) None of these

51. A function $f(x)$ is called periodic if it is defined for real $x \in R$ and—
(A) if there is any positive number p, such that $f(x+p) = f(x) + f(p)$
(B) if there is any positive number p, such that $f(x+p) > f(x)$
(C) if there is any positive number p, such that $f(x+p) = f(x)$
(D) None of these

52. Which of the following function is a periodic function ?
(A) $\sin x$ (B) $\cos x$
(C) $f(x)$ = constant (D) All of these

53. If a function $f(x)$ is periodic, then for p period and $n \in z$.
(A) $f(x + np) = f(x + n)$
(B) $f(x + np) = f(x)$
(C) $f(x + np) = f(x + p)$
(D) None of these

54. The fundamantal period of $f(x)$ is—
(A) Smallest period $p > 0$
(B) Largest period $p > 0$
(C) Any period $p > 0$
(D) None of these

55. The trigonometric function $\cos x$ have fundamental period—
(A) π (B) 2π
(C) 3π (D) 4π

56. The trigonometric function $\sin x$ have fundamental period—
(A) π (B) 2π
(C) 3π (D) 4π

57. The trigonometric function $\cos 2x$ have fundametal period—
(A) π (B) 2π
(C) 3π (D) 4π

58. The trigonometric function $\sin 2x$ have fundametal period—
(A) π (B) 2π
(C) 3π (D) 4π

59. The function $f(x)$ = constant—
(A) have no fundamental period
(B) have fundamental period π
(C) have fundamental period 2π
(D) have fundamental period 3π

60. If $f(x)$ is a periodic function of x of period p, then—
(A) $f(ax)$, $a \neq 0$ is a periodic function of x of period p/a
(B) $f(ax)$, is a periodic function of x of period p
(C) $f(ax)$, $a \neq 0$ is a periodic function of x of period $1/a$
(D) None of these

61. If $f(x)$ is a periodic function of x of period p, then—
(A) $f\left(\frac{x}{b}\right)$, $b \neq 0$ is a periodic function of x of period bp
(B) $f\left(\frac{x}{b}\right)$, is a periodic function of x of period p
(C) $f\left(\frac{x}{b}\right)$, $b \neq 0$, is a periodic function of x of period b
(D) None of these

62. In Fourior series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, Euler formula is—

(A) $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx$

(B) $a_0 = \int_{-\pi}^{\pi} f(x)\, dx$

(C) $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) \sin x\, dx$

(D) None of these

63. In Fourier series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, Euler formula is—

(A) $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$

(B) $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx$

(C) $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx$

(D) None of these

64. In Fourier series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, Euler formula is—

(A) $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx$

(B) $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx$

(C) $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$

(D) None of these

65. Real function f_m and f_n are orthogonal in the interval (a, b)—

(A) if $\int_a^b f_m(x) f_n(x)\, dx = 0$ and for all m, n

(B) if $\int_a^b f_m(x) f_n(x)\, dx = 0$, $m \neq n$ and for all $n \int_a^b [f_n(x)]^2 = 0$

(C) if $\int_a^b fm(x) f_n(x)\, dx = 1$, $m \neq n$ and for all $n, \int_a^b [f_n(x)]^2 = 0$

(D) None of these

66. The trigonometric system $\sin x, \cos x, \sin 2x, \cos 2x \ldots$ is—

(A) Orthogonal an interval $-\pi \leq x \leq \pi$

(B) Not orthogonal on interval $-\pi \leq x \leq \pi$

(C) Both (A) and (B)

(D) None of these

67. If a periodic function $f(x)$ with period 2π is piecewise continuous in the interval $-\pi \leq x \leq \pi$ and has a left hand derivative and right hand derivative at each point of that interval, then the Fourier series is—

(A) Convergent (B) Divergent

(C) Constant (D) None of these

68. If a function $f(x)$ of a period $p = 2L$ has Fourier series, then in series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ the Fourier coefficients a_0 is given by—

(A) $\frac{1}{2L}\int_{-L}^{L} f(x)\, dx$ (B) $\int_{-L}^{L} f(x)\, dx$

(C) $\frac{1}{2}\int_{-L}^{L} f(x)\, dx$ (D) None of these

69. If a function $f(x)$ of period $P = 2L$ has Fourier series, then in series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficients a_n is given by—

(A) $\int_{-L}^{L} f(x) \frac{\sin n\pi x}{L}\, dx$

(B) $\frac{1}{L}\int_{-L}^{L} f(x) \frac{\cos n\pi x}{L}\, dx$

(C) $\int_{-L}^{L} f(x) \frac{\cos n\pi x}{L}\, dx$

(D) None of these

70. If a function $f(x)$ of period $P = 2L$ has Fourier series, then in series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, the Fourier coefficients b_n is given by—

(A) $\frac{1}{L}\int_{-L}^{L} f(x) \frac{\sin n\pi x}{L}\, dx$

(B) $\frac{1}{L}\int_{-L}^{L} f(x) \frac{\cos n\pi x}{L}\, dx$

(C) $\int_{-L}^{L} f(x) \frac{\sin n\pi x}{L}\, dx$

(D) $\int_{-L}^{L} f(x) \frac{\cos n\pi x}{L}\, dx$

71. A function $f(x)$ is even function, if—
    (A) $f(-x) = f(x)$
    (B) $f(-x) = -f(x)$
    (C) $f(-x) = \text{constant}$
    (D) None of these

72. A function $f(x)$ is odd function, if—
    (A) $f(-x) = f(x)$
    (B) $f(-x) = -f(x)$
    (C) $f(-x) = \text{constant}$
    (D) None of these

73. If $f(x)$ is even function, then—
    (A) $\int_{-L}^{L} f(x)\, dx = 2\int_{0}^{L} f(x)\, dx$
    (B) $\int_{-L}^{L} f(x)\, dx = 0$
    (C) $\int_{-L}^{L} f(x)\, dx = \int_{0}^{L} f(x) dx$
    (D) None of these

74. If $f(x)$ is odd function, then—
    (A) $\int_{-L}^{L} f(x) dx = 2\int_{0}^{L} f(x) dx$
    (B) $\int_{-L}^{L} f(x) dx = 0$
    (C) $\int_{-L}^{L} f(x) dx = \int_{0}^{L} f(x) dx$
    (D) None of these

75. The product of even and odd function is—
    (A) Odd (B) Even
    (C) Constant (D) None of these

76. The graph of even function $y = f(x)$ is symmetric about—
    (A) x-axis
    (B) y-axis
    (C) Both (A) and (B) above
    (D) None of these

77. The Fourier series of an even function of period 2L is a—
    (A) Fourier cosine series
    (B) Fourier sine series
    (C) Fourier complex series
    (D) None of these

78. The Fourier series of an odd function of period 2L is a—
    (A) Fourier cosine series
    (B) Fourier sine series
    (C) Fourier complex series
    (D) None of these

79. The Fourier series of an even function of period 2L is given by—
    (A) $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \frac{\cos n\pi x}{L}$, where $a_0 = \frac{1}{L}\int_0^L f(x) dx$, $a_n = \frac{2}{L}\int_0^L f(x) \frac{\cos n\pi x}{L} dx$
    (B) $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$, where $b_0 = \frac{2}{L}\int_0^L f(x) \frac{\sin n\pi x}{L} dx$
    (C) $f(x) = \sum_{n=-\infty}^{\infty} e_n e^{inx}$, where $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx} dx$, $n = 1, 2, \ldots\ldots$
    (D) None of these

80. The Fourier series of an odd function of period 2L is given by—
    (A) $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \frac{\cos n\pi x}{L}$, where $a_0 = \frac{1}{L}\int_0^L f(x) dx$, $a_n = \frac{2}{L}\int_0^L f(x) \frac{\cos n\pi x}{L} dx$.
    (B) $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$, where $b_0 = \frac{2}{L}\int_0^L f(x) \sin \frac{n\pi x}{L} dx$
    (C) $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$, where $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx} dx$, $n = 1, 2, \ldots\ldots$
    (D) None of these

81. The Fourier cosine series of function of period 2L is given by—
    (A) $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \frac{\cos n\pi x}{L}$, where $a_0 = \frac{1}{L}\int_0^L f(x) dx$, $a_n = \frac{2}{L}\int_0^L f(x) \frac{\cos n\pi x}{L} dx$
    (B) $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$, where $b_0 = \frac{2}{L}\int_0^L f(x) \sin \frac{n\pi x}{L} dx$

(C) $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$, where $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx}dx$, $n = 1, 2, \ldots\ldots$

(D) None of these

82. The Fourier sine series of a function of period 2L is given by—

(A) $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \frac{\cos n\pi x}{L}$, where $a_0 = \frac{1}{L}\int_0^L f(x)dx$, $a_n = \frac{2}{L}\int_0^L f(x)\frac{\cos n\pi x}{L}dx$

(B) $f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$, where $b_0 = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

(C) $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$, where $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx$, $n = 1, 2, \ldots\ldots$

(D) None of these

83. The Fourier complex series of a function of period 2L is given by—

(A) $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \frac{\cos n\pi x}{L}$, where $a_0 = \frac{1}{L}\int_0^L f(x)dx$, $a_n = \frac{2}{L}\int_0^L f(x)\frac{\cos n\pi x}{L}dx$

(B) $f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$, where $b_0 = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

(C) $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$, where $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx}dx$, $n = 1, 2, \ldots\ldots$

(D) None of these

84. A function $f(x)$ with period 2π have a complex Fourier series—

(A) $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$, where $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx$, $n = 1, 2, \ldots\ldots$

(B) $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi x}{L}}$, where $c_n = \frac{1}{2L}\int_{-L}^{L} f(x)\, e^{\frac{in\pi x}{L}} dx$, $n = 1, 2, \ldots\ldots$

(C) $f(x) = \int_0^{\infty} [A(\omega)\cos\omega x + B(\omega)\sin\omega x]d\omega$

(D) None of these

85. $f(x)$ can be represented by a Fourier integral if $f(x)$—

(A) is piecewise continuous, in every finite integral

(B) has a right hand derivative and a left hand derivative at each point in a finite integral

(C) in the integral $\int_{-\infty}^{\infty} |f(x)|dx$ exists

(D) All the above

86. If $f(x)$ represented by Fourier integral $f(x) = \int_0^{\infty} [A(\omega)\cos\omega x + B(\omega)\sin\omega x]\, d\omega_y$ then $A(w)$ is defined as—

(A) $\frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\cos\omega v\, dv$

(B) $\frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\sin\omega v\, dv$

(C) $\int_{-\infty}^{\infty} f(v)\cos\omega v\, dv$

(D) $\int_{-\infty}^{\infty} f(v)\sin\omega v\, dv$

87. If $f(x)$ represented by Fourier integral $f(x) = \int_0^{\infty} [A(\omega)\cos\omega x + B(\omega)\sin\omega x]d\omega$, then $B(\omega)$ is defined as—

(A) $\frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\cos\omega v\, dv$

(B) $\frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\sin\omega v\, dv$

(C) $\int_{-\infty}^{\infty} f(v)\cos\omega v\, dv$

(D) $\int_{-\infty}^{\infty} f(v)\sin\omega v.dv$

88. At the point where $f(x)$ is discontinuous, the value of the Fourier integral equals—

(A) Average of the left and right hand limits of $f(x)$ at that point

(B) Sum of the left and right hand limits of $f(x)$ at that point

(C) Product of the left and right hand limits of $f(x)$ at that point

(D) Difference of the left and right hand limits of $f(x)$ is even function

89. If $f(x)$ represented by Fourier integral and if $f(x)$ is odd funtion, then $f(x)$ is—
(A) Fourier cosine integral
(B) Fourier sine integral
(C) Both (A) and (B) above
(D) None of these

90. If $f(x)$ represented by Fourier integral and if $f(x)$ is odd function, then $f(x)$ is—
(A) Fourier cosine integral
(B) Fourier sine integral
(C) Fourier tan integral
(D) None of these

91. If $f(x)$ represented by Fourier integral and if $f(x)$ is even function, then $f(x)$ is—
(A) $f(x) = \int_0^\infty n(\omega) \cos \omega x \, d\omega$
(B) $f(x) = \int_0^\infty B(\omega) \sin \omega x \, d\omega$
(C) Both (A) and (B) above
(D) None of these

92. If $f(x)$ represented by Fourier integral and if $f(x)$ is odd function, then $f(x)$ is—
(A) $f(x) = \int_0^\infty A(\omega) \cos \omega x \, d\omega$
(B) $f(x) = \int_0^\infty B(\omega) \sin \omega x \, d\omega$
(C) If $f(x)$ is even function, then $B(\omega) = 0$ and
(D) If $f(x)$ is odd function, then and $B(\omega) = \frac{2}{\pi} \int_0^\infty f(v) \sin \omega v \, dv$

93. If $f(x)$ represented by Fourier integral $f(x) = \int_0^\infty [A(\omega) \cos \omega x + B(\omega) \sin \omega x] d\omega$ and $f(x)$ is even function, then—
(A) $B(\omega) = \frac{2}{\pi} \int_0^\infty f(v) \sin \omega v \, dv$
(B) $B(\omega) = 0$
(C) Both (A) and (B)
(D) None of these

94. If $f(x)$ represented by Fourier integral $f(x) = \int_0^\infty [A(\omega) \cos \omega x + B(\omega) \sin \omega x] d\omega$ and $f(x)$ is odd function, then—
(A) $B(\omega) = \frac{2}{\pi} \int_0^\infty f(v) \sin \omega v \, dv$
(B) $B(\omega) = 0$
(C) Both (A) and (B)
(D) None of these

95. If $f(x)$ represented by Fourier integral $f(x) = \int_0^\infty [A(\omega) \cos \omega x + B(\omega) \sin \omega x] d\omega$ and $f(x)$ is odd function, then—
(A) $A(\omega) = 0$
(B) $A(\omega) = \frac{2}{\pi} \int_0^\infty f(v) \cos \omega v \, dv$
(C) Both (A) and (B) above
(D) None of these

96. If $f(x)$ represented by Fourier integral $f(x) = \int_0^\infty A(\omega) \cos \omega x + B(\omega) \sin \omega x] d\omega$ and $f(x)$ is odd function, then—
(A) $A(\omega) = 0$
(B) $A(\omega) = \frac{2}{\pi} \int_0^\infty f(v) \cos \omega v \, dv$
(C) Both (A) and (B) above
(D) None of these

97. Fourier cosine transform of $f(x)$ is—
(A) $\sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos \omega x \, dx$
(B) $\sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin \omega x \, dx$
(C) Both (A) and (B) above
(D) None of these

98. Fourier sine transform of $f(x)$ is—
(A) $\sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos \omega x \, dx$
(B) $\sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin \omega x \, dx$
(C) Both (A) and (B) above
(D) None of these

99. The Fourier cosine and sine tranforms of $f(x)$ exist if—
(A) $f(x)$ is absolutely integrable on positive x-axis
(B) $f(x)$ piecewise continuous on finite interval
(C) Both (A) and (B)
(D) None of these

100. If $f(x)$ is continuous and absolutely integrable on the x-axis, and $f'(x)$ is piecewise continuous an each interval, also $f(x) \to 0$ as $x \to \infty$. Then Fourier cosine transform—

(A) $F_C\{f''(x)\} = -\omega^2 F_c\{f(x)\} - \sqrt{\frac{2}{\pi}} f'(0)$

(B) $F_C\{f''(x)\} = f(x) - \sqrt{\frac{2}{\pi}} f'(0)$

(C) $F_C\{f''(x)\} = -\omega^2 f(x) - \sqrt{\frac{2}{\pi}} f(0)$

(D) None of these

101. If $f(x)$ is continuous and absolutely integrable on the x-axis and $f'(x)$ is piecewise continuous on each interval, also $f(x) \to 0$ as $x \to \infty$. Then Fourier cosine transform—

(A) $F_C\{f'(x) = \omega F_C\{f(x)\} - \sqrt{\frac{2}{\pi}} f(0)$

(B) $F_C\{f'(x) = \omega F_C\{f(x)\}$

(C) $F_C\{f'(x)\} = \omega\{f(x)\} - f(0)$

(D) None of these

102. If $f(x)$ is continuous and absolutely integrable on the x-axis, and $f'(x)$ is piecewise continuous an each interval, also $f(x) \to 0$ as $x \to \infty$. Then Fourier sine transoform—

(A) $F_S\{f'(x)\} = w f(x)$

(B) $F_S\{f'(x)\} = F_S\{f(x)\}$

(C) $F_S\{f'(x)\} = -wF_S\{f(x)\}$

(D) None of these

103. If $f(x)$ is continuous and absolutely integrable on the x-axis, and $f'(x)$ is piecewise continuous on each interval, also $f(x) \to 0$ as $x \to \infty$. Then Fourier sine transform—

(A) $F_S\{f''(x)\} = \sqrt{\frac{2}{\pi}} \omega f(0)$

(B) $F_S\{f''(x)\} = -\omega^2 F_S\{f(x)\} + \sqrt{\frac{2}{\pi}} \omega f(0)$

(C) $F_S\{f''(x)\} = \omega^2 F_S\{f(x)\} + \sqrt{\frac{2}{\pi}} f(0)$

(D) None of these

104. Fourier sine transform of e^{-ax} $(a > 0)$ is—

(A) $\frac{\omega}{a^2 + \omega^2}$

(B) $\frac{\omega}{\sqrt{2a}} e^{-\omega^2/4a}$

(C) $\sqrt{\frac{2}{\pi}} \left(\Gamma(a)\omega^{-a} \sin \frac{a\pi}{2}\right)$

(D) None of these

105. Fourier sine transform of xe^{-ax^2} $(a > 0)$ is—

(A) $\frac{\omega}{a^2 + \omega^2}$

(B) $\frac{\omega}{\sqrt{2a}} e^{-\omega^2/4a}$

(C) $\sqrt{\frac{2}{\pi}} \left(\Gamma(a)\omega^{-a} \sin \frac{a\pi}{2}\right)$

(D) None of these

106. Fourier sine transform of x^{a-1} $a < 0 < 1$ is—

(A) $\frac{\omega}{a^2 + \omega^2}$

(B) $\frac{\omega}{\sqrt{2a}} e^{-\omega^2/4a}$

(C) $\sqrt{\frac{2}{\pi}} \left(\Gamma(a)\omega^{-a} \sin \frac{a\pi}{2}\right)$

(D) None of these

107. Fourier sine transform of $\cos ax^2$ is—

(A) $\sqrt{\frac{\pi}{2a}} \left[\sin \frac{\omega^2}{4a} \cos\left(\frac{\omega}{\sqrt{2\pi a}}\right) - \cos \frac{\omega^2}{4a} \sin\left(\frac{\omega}{\sqrt{2\pi a}}\right) \right]$

(B) $\sqrt{\frac{\pi}{2a}} \left[\cos \frac{\omega^2}{4a} \cos\left(\frac{\omega}{\sqrt{2\pi a}}\right) + \sin \frac{\omega^2}{4a} \sin\left(\frac{\omega}{\sqrt{2\pi a}}\right) \right]$

(C) $\sqrt{\frac{2}{\pi}} \frac{\sin a\omega}{\omega}$

(D) None of these

108. Fourier sine transform of $\sin ax^2$ is—

(A) $\sqrt{\frac{\pi}{2a}} \left[\sin \frac{\omega^2}{4a} \cos\left(\frac{\omega}{\sqrt{2\pi a}}\right) - \cos \frac{\omega^2}{4a} \sin\left(\frac{\omega}{\sqrt{2\pi a}}\right) \right]$

(B) $\sqrt{\frac{\pi}{2a}}\left[\cos\frac{\omega^2}{4a}\cos\left(\frac{\omega}{\sqrt{2\pi a}}\right) + \sin\frac{\omega^2}{4a}\sin\left(\frac{\omega}{\sqrt{2\pi a}}\right)\right]$

(C) $\sqrt{\frac{2}{\pi}}\left[\frac{1-\cos a\omega}{\omega}\right]$

(D) $\sqrt{\frac{2}{\pi}}\frac{\sin a\omega}{\omega}$

109. Fourier sine transform of $\begin{cases} 1, 0 < x < a \\ 0, \text{otherwise} \end{cases}$ is—

(A) $\sqrt{\frac{\pi}{2a}}\left[\sin\frac{\omega^2}{4a}\cos\left(\frac{\omega}{\sqrt{2\pi a}}\right) - \cos\frac{\omega^2}{4a}\sin\left(\frac{\omega}{\sqrt{2\pi a}}\right)\right]$

(B) $\sqrt{\frac{\pi}{2a}}\left[\cos\frac{\omega^2}{4a}\cos\left(\frac{\omega}{\sqrt{2\pi a}}\right) + \sin\frac{\omega^2}{4a}\sin\left(\frac{\omega}{\sqrt{2\pi a}}\right)\right]$

(C) $\sqrt{\frac{2}{\pi}}\left[\frac{1-\cos a\omega}{\omega}\right]$

(D) $\sqrt{\frac{2}{\pi}}\frac{\sin a\omega}{\omega}$

110. Fourier cosine transform of $\begin{cases} 1, 0 < x < a \\ 0, \text{otherwise} \end{cases}$ is—

(A) $\sqrt{\frac{\pi}{a}}\left[\sin\frac{\omega^2}{4a}\cos\left(\frac{\omega}{\sqrt{2\pi a}}\right) - \cos\frac{\omega^2}{4a}\sin\left(\frac{\omega}{\sqrt{2\pi a}}\right)\right]$

(B) $\sqrt{\frac{\pi}{2a}}\left[\cos\frac{\omega^2}{4a}\cos\left(\frac{\omega}{\sqrt{2\pi a}}\right) + \sin\frac{\omega^2}{4a}\sin\left(\frac{\omega}{\sqrt{2\pi a}}\right)\right]$

(C) $\sqrt{\frac{2}{\pi}}\left[\frac{1-\cos a\omega}{\omega}\right]$

(D) $\sqrt{\frac{2}{\pi}}\frac{\sin a\omega}{\omega}$

111. Fourier Cosine transform of e^{-ax} $(a > 0)$ is—

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\omega^2}$

(B) $\frac{1}{\sqrt{2a}}e^{-\omega^2/4a}$

(C) $\frac{1}{\sqrt{2\pi}}\cos\left(\frac{\omega^2}{4a}-\frac{\pi}{4}\right)$

(D) $\frac{1}{\sqrt{2\pi}}\cos\left(\frac{\omega^2}{4a}+\frac{\pi}{4}\right)$

112. Fourier cosine transform of e^{-ax^2} $(a > 0)$ is—

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\omega^2}$

(B) $\frac{1}{\sqrt{2a}}e^{-\omega^2/4a}$

(C) $\frac{1}{\sqrt{2\pi}}\cos\left(\frac{\omega^2}{4a}-\frac{\pi}{4}\right)$

(D) $\frac{1}{\sqrt{2\pi}}\cos\left(\frac{\omega^2}{4a}+\frac{\pi}{4}\right)$

113. Fourier Cosine transform of $\cos ax^2$ is—

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\omega^2}$

(B) $\frac{1}{\sqrt{2a}}e^{-\omega^2/4a}$

(C) $\frac{1}{\sqrt{2\pi}}\cos\left(\frac{\omega^2}{4a}-\frac{\pi}{4}\right)$

(D) $\frac{1}{\sqrt{2\pi}}\cos\left(\frac{\omega^2}{4a}+\frac{\pi}{4}\right)$

114. Fourier cosine transform of $\sin ax^2$ is—

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\omega^2}$

(B) $\frac{1}{\sqrt{2a}}e^{-\omega^2/4a}$

(C) $\frac{1}{\sqrt{2\pi}}\cos\left(\frac{\omega^2}{4a}-\frac{\pi}{4}\right)$

(D) $\frac{1}{\sqrt{2\pi}}\cos\left(\frac{\omega^2}{4a}+\frac{\pi}{4}\right)$

115. Fourier cosine transform of x^{n-1} $(0 < a < 1)$ is—

(A) $\Gamma(a)\,\omega^{-a}\cos\frac{a\pi}{2}$

(B) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\omega^2}$

(C) $\frac{1}{\sqrt{2a}}e^{-\omega^2/4a}$

(D) $\frac{1}{2\pi}\cos\left(\frac{\omega^2}{4a}-\frac{\pi}{4}\right)$

116. Fourier transform of $f(x)$ is—

(A) $F(\omega)=\int_{-\infty}^{\infty} f(x)\,e^{i\omega x}dx$

(B) $F(\omega)=\int_{-\infty}^{\infty} f(x)\,dx$

(C) $F(\omega)=\int_{-\infty}^{\infty} e^{i\omega x}\,dx$

(D) None of these

117. Inverse Fourier transform of $F(\omega)$—

(A) $f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} \omega e^{i\omega x}d\omega$

(B) $f(x)=\int_{-\infty}^{\infty} F(w)\,e^{-iwx}dw$

(C) $f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)e^{-iwx}d\omega$

(D) None of these

118. Fourier transforms of $f(x)$ exist if—

(A) If $f(x)$ is absolutels integrable on positive x-axis

(B) If $f(x)$ is piecewise continuous on finite interval

(C) Both A and B

(D) None of these

119. If $f(x)$ is continuous on the x-axis and $f(x)\to 0$ as $|x|\to\infty$. $f'(x)$ is absolutely integrable on the x-axis. Then Fourier transform of derivative—

(A) $F\{f'(x)\}=i\,\omega\,F\{f(x)\}$

(B) $F\{f'(x)\}=F\{f(x)\}$

(C) $F\{f'(x)\}=\omega\,F\{f(x)\}$

(D) None of these

120. If $f(x)$ is continuous on the x-axis, and $f(x)\to 0$ as $|x|\to\infty$. $f'(x)$ is absolutely integrable on the x-axis. Then Fourier transform of derivative—

(A) $F\{f''(x)\}=-w^2\,F\{f(x)\}$

(B) $F\{f''(x)\}=w\,F\{f(x)\}$

(C) $F\{f''(x)\}=F\{f(x)\}$

(D) None of these

121. If $f(x)$ is continuous an the x-axis and $f(x)\to 0$ as $|x|\to\infty$. $f'(x)$ is absolutely integrable on the x-axis. Then Fourier transform of derivative—

(A) $F\left\{\frac{d^nf}{dx^n}\right\}=(-i)^n\,F\{f(x)\}$

(B) $F\left\{\frac{d^nf}{dx^n}\right\}=(\omega)^n\,F\{f(x)\}$

(C) $F\left\{\frac{d^nf}{dx^n}\right\}=(-i\omega)^n\,F(f(x)\}$

(D) $F\left\{\frac{d^nf}{dx^n}\right\}=F\{f(x)\}$

122. If $F(w)$ is the Fourier transform of $f(x)$, $-\infty < x < \infty$, then—

(A) $f(x)$ is even $\Rightarrow F(\omega)=F_C(\omega)$

(B) $f(x)$ is even $\Rightarrow F(\omega)=2F_C(\omega)$

(C) $f(x)$ is even $\Rightarrow F(\omega)=4F_C(\omega)$

(D) None of these

123. If $F(w)$ is the Fourier transform of $f(x)$, $-\infty < x < \infty$, then—

(A) $f(x)$ is odd $\Rightarrow F(\omega)=F_S(\omega)$

(B) $f(x)$ is odd $\Rightarrow F(\omega)=2F_S(\omega)$

(C) $f(x)$ is odd $\Rightarrow F(\omega)=2iF_S(\omega)$

(D) None of these

124. The Fourier transform of $\mu(x,t)$, $x\in R$ is defined by the integral—

(A) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-iwx}\,\mu(x,t)dx$

(B) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-iwx}dx$

(C) $\int_{-\infty}^{\infty} e^{-iwx}\,\mu(x,t)dx$

(D) None of these

125. If $F(w)$ is the Fourier transform of $f(x)$, then Fourier transform of $f(t)$ is—

(A) $\frac{1}{|a|}F\left(\frac{\omega}{a}\right)$ (B) $F(-\omega)$

(C) $e^{-iwt}F(\omega)$ (D) $F(\omega-r)$

126. If $F(w)$ is the Fourier transform of $f(x)$. Then Fourier transform of $f(at)$ is—

(A) $\frac{1}{|a|}F\left(\frac{\omega}{a}\right)$ (B) $F(-\omega)$

(C) $e^{-iwt}F(\omega)$ (D) $F(\omega-r)$

127. If $f(w)$ is the Fourier transform of $f(x)$, then Fourier transform of $f(t - \mathrm{T})$, T is real is—

(A) $\frac{1}{|a|}\mathrm{F}\left(\frac{\omega}{a}\right)$ (B) $\mathrm{F}(-\omega)$

(C) $e^{-iwt}\,\mathrm{F}(\omega)$ (D) $\mathrm{F}(\omega - r)$

128. If $\mathrm{F}(w)$ is the Fourier transform of $f(x)$, then Fourier transform of $e^{irt} f(t)$, r is real is—

(A) $\frac{1}{|a|}\mathrm{F}\left(\frac{\omega}{a}\right)$ (B) $\mathrm{F}(-\omega)$

(C) $e^{-iwt}\,\mathrm{F}(\omega)$ (D) $\mathrm{F}(\omega - r)$

Answers

1. (B)	2. (D)	3. (C)	4. (C)	5. (D)
6. (D)	7. (A,D)	8. (B)	9. (C)	10. (D)
11. (C)	12. (B)	13. (A)	14. (B)	15. (A)
16. (A)	17. (B)	18. (A)	19. (A)	20. (C)
21. (B)	22. (A)	23. (C)	24. (B)	25. (A)
26. (A)	27. (A)	28. (B)	29. (A)	30. (A)
31. (C)	32. (B)	33. (A)	34. (B)	35. (A)
36. (B)	37. (B)	38. (A)	39. (A)	40. (A)
41. (C)	42. (C)	43. (B)	44. (A)	45. (A)
46. (C)	47. (B)	48. (B)	49. (A)	50. (A)
51. (C)	52. (D)	53. (B)	54. (A)	55. (B)
56. (B)	57. (A)	58. (A)	59. (A)	60. (A)
61. (A)	62. (A)	63. (C)	64. (A)	65. (B)
66. (A)	67. (A)	68. (A)	69. (B)	70. (A)
71. (A)	72. (B)	73. (A)	74. (B)	75. (A)
76. (B)	77. (A)	78. (B)	79. (A)	80. (B)
81. (A)	82. (B)	83. (C)	84. (A)	85. (D)
86. (A)	87. (B)	88. (A)	89. (A)	90. (B)
91. (A)	92. (B)	93. (B)	94. (A)	95. (A)
96. (B)	97. (A)	98. (B)	99. (C)	100. (A)
101. (A)	102. (C)	103. (B)	104. (A)	105. (B)
106. (C)	107. (A)	108. (B)	109. (C)	110. (D)
111. (A)	112. (B)	113. (C)	114. (D)	115. (A)
116. (A)	117. (C)	118. (C)	119. (A)	120. (A)
121. (C)	122. (B)	123. (C)	124. (A)	125. (B)
126. (A)	127. (A)	128. (D)		