Hack, Hack, Who's There? A gentle invitation to Model Theory

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Smashwords edition

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Introduction

The word “logic” still evokes the same concept of “thinking correctly” as in the time of Aristotle; introductions to logic for the lay person present the same type of elementary examples and puzzles as in the nineteenth century. Although these applications are not without their value, modern logic goes much further. It does not favor a single way of thinking; it is not about “how *should* you think”, but “how *do* you think”. It follows that in today’s world full of clashing cultures, such logic should be more widely understood. However, it is rare to find such applications in lay texts. Of course one can look in the Internet, but the explanations rapidly take refuge in mathematical notation, thereby relegating them to the same category as ancient cuneiforms for most lay persons.

One source of conflict lies in the different meanings attributed to the same concepts by different people. This is difficult to avoid. For example, you can probably give a pretty good definition of a chair, but what about the concepts of time, information, entity, love, liberty and happiness ? On the academic level, attempts to do so have led to revolutions in physics, computing, biology, politics, psychology and other fields; in discussions on the popular level, such concepts are “understood”, not considered in need of definitions. Another example is the very concept of meaning. Even popular books on psychology, philosophy or religion on “the meaning of life” still usually make no attempt to explain “meaning”.

This book selects that part of modern logic which occupies itself with the processes of ascribing meaning. It is known as “Model Theory”, despite the many meanings the word “model” has in other fields. Concepts from this theory are presented but also applied via a fictional story which reads like pedantic science fiction. The rationale behind the use of this genre is to show how Model Theory may help explain concepts out of physics which science fiction writers, for whom fidelity to scientific accuracy is of secondary concern, can misuse with impunity due to the lack of understandable explanations of them in the popular press. This is not to say that physicists do not try to explain black holes, wormholes, antimatter, teleportation, and similar concepts. However, when the writers wish to avoid assuming too much on the part of the reader, they start to use metaphors which are supposed to be clear, but often only muddy the explanation further. To cite a frequent example, the “fabric of space” usually shows up as a rubber sheet in explanations about gravity. This analogy quickly gets into problems, as any physicist will point out.

The concepts presented here are tips of the proverbial icebergs. For a reader interested in learning more, the traditional format would dictate a bibliography or list of recommended books. In the Internet Age, a much richer selection of sources is offered by a quick reference to the nearest computer. Were the themes here obscure or fringe topics which might make such a search difficult, the more traditional list of references would be more appropriate. However, the topics are all mainstream and good sources for all tastes and levels are readily accessible via your favorite search engine.

In order not to over-burden the text with technical terms that would then begin to look like the technobabble so beloved by science fiction writers and superficial philosophers, most such

terms have been paraphrased in the text. Nonetheless, enough terms have been left in to serve as keywords for anyone wishing to delve further. Even without the keywords, an Internet search with the paraphrasing will also usually turn up appropriate sites. If this does not suffice, the reader is invited to ask the author directly for a corresponding keyword or site. (The author's contact information is given at the end of the story.) Note, however, that not every sentence, paragraph or even section contains a mathematical idea; the supporting fictional story also takes up much of the text.

No prior mathematical knowledge is assumed beyond a vague recollection that the Pythagorean Theorem says something about the sides of a right triangle, that "inversely proportional" means one goes up as the other goes down, such that neither quantity can be zero, and the like. No diagrams are supplied, although the option is there to draw a few diagrams oneself, or even derive a couple of simple formulas by using basic algebra. But these are not essential to the main points and may easily be skipped.

Enjoy.

Chapter One

February 26, 2021

God, how he hated this job. Eight hours a day sitting at a computer....

Bill Wheaton knew that it was his own stupidity to blame. In the university he had only taken courses that had interested him, never heading towards any particular goal. He had assumed that, since his father was the CEO and majority stockholder in a major computer firm, there was no reason for a work ethic on his part. His father, however, thought otherwise, and had refused to give his son any further money after university. He did, however, use his influence to land Bill this job.

Hence Bill had ended up in a clerical job in San Francisco. The entries in the internal Ethernet constituted his universe. His superiors sent instructions as to how the internal memos should be archived. Bill created new folders, rearranged old ones, duplicated a number of them to go in multiple other folders; now and then he would try to put a folder into itself and was reprimanded by the computer. His life became nothing but folders. Therefore it was no surprise that he started dreaming of folders.

In one recurring dream, he was God's secretary. He was sure it was not any God of the major religions, because this God relied on a computer. A rather powerful computer, but a computer nonetheless. Bill's instructions started with "In the beginning was the Microsoft Word..." His further instructions were the same as at work, that of creating and rearranging folders. After a while, he was so befuddled that he got all this mixed up with the entirety of the physical universe, but this did not surprise him.... one is often not very clever in dreams. In the dream he had an assistant, Eugene, who told him, "We are God's department, after all. The collection of all the possible folders is the universe. Welcome to our reality, Bill."

Bill found Eugene scary. Bill could not say how, but Eugene was somehow the embodiment of the relationship of belonging. Bill looked for a way to escape. He found a button allowing him to fast-forward into the future. He pushed it. He heaved a sigh of relief in finding himself alone in the office, without Eugene. He sat down at his computer and opened it. He stared in horror at the screen. All the labels of all the folders had disappeared! Bill, or rather his dream avatar, started to panic, but Eugene appeared by his side, that all was well; his folders still obeyed the rules. Bill became unnerved by Eugene's appearances, but began to think of him as inevitable.

Nonetheless he tried again to escape him by again fast-forwarding. Again he had escaped Eugene, if only for a time. Bill looked at the folders on his desktop, and wondered whether there was some purpose in carefully guarding them all. He began opening folders, and the folders inside folders, and so forth. He was shocked to find out that the base folders were ...empty. He continued to search... everywhere there was no document, only folders, starting with empty folders. Relationships were the only thing that seemed to matter. But how could everything be based on emptiness? There had to be something hidden from him. He looked around. No Eugene in sight. In fact, no other workers at all around. He gathered up his courage, and snuck into the Head Office; he knew somehow that his boss, God, was off meditating at this time. Bill turned on the main computer. The labels were still all there. He moved the mouse on the screen and saw one folder marked "Bill Wheaton". He opened it, and found more folders in folders. He noticed an app that his computer didn't have. Being God got you the best computer, he guessed. The app let the unfolding of folders go to the end. Bill pressed on it and watched as all the folders went to the base, the essential meaning of everything. The result took a while, because there were a lot of folders to unfold, but then the icon appeared:"No items found." That is, he was founded on... nothingness. This did not cheer him up. He slinked out of the office and was startled to see Eugene standing there."What did you expect?" Eugene grinned.

Bill ran from Eugene, out of the building... and found that the building was supported by a cloud which, paradoxically, would not support Bill alone. Bill fell, and landed on Earth. He looked at his clothes. He was in a police uniform. He looked around; he was in a police station. Another police officer, who looked not at all like Eugene but rather a like a middle-aged tired office worker, handed him a paper. She told him, "Here is the search warrant you asked for. You can now try to get the information from the computer we confiscated from the drug dealer. We put it in that room over there, along with your laptop, and connected them. It's all yours. But be advised, we couldn't get the judge to grant full powers. The search warrant is limited."

Bill went to the room indicated, sat down, turned on the computer. Then he read the conditions of the warrant. From what he understood, he was allowed to look at the desktop. He saw an envelope labeled "Sabotage". He clicked on it, but "Sabotage" was locked. Bill stared at the screen for a moment, wondering what to do next. At this moment, another police officer stuck his head into the room and asked if Bill needed anything. Coffee? Tea? Bill told him,"No, thanks. But if there is some other computer software specialist that you could call, that would be appreciated. The officer replied, "I can have one here within a few minutes," and left. Bill continued to try to unlock the file, but true to his word, the other officer opened the door again after only ten minutes and said, "I have someone. Officer Wheaton, meet Computer Specialist Onegin." Something appeared ominous in this announcement; sure enough, in came Eugene.

Without further ado, Eugene managed to get past the lock, and initiate a command for all the folders inside the file to be copied into file marked "Evidence" in Bill's computer. That was done, but frustratingly enough, each of these files was locked. Eugene applied the same technique, getting more folders into "Evidence", but again these were locked. This continued, until there were empty folders, as in the earlier part of Bill's dream. In the end, Bill had a folder in which the contents of each folder were also contents of the main folder. The folders inside each subfolder were included in that folder. In that way, if one folder was in a second folder, which in its turn was in a third folder, then he would also be sure to find the first folder in the third folder. This started to confuse Bill. Eugene just shrugged."All this looks familiar. There are a lot of things where a relation is allowed to skip the middleman. I see a lot of them. Size

comparisons, for example. Before and after in time. Things like that. It looks like you have discovered a good standard basis for such comparisons."

But this abstraction started to make Bill's head spin.

Next his dream switched: he was now the criminal. Industrial espionage. He was to gain access to a competitor's computer, steal a file labeled "Ralph", and follow the instructions provided by his head office:

Open a new folder, label it "Betty"

Open Betty, copy Ralph into Betty

Open a new folder inside Betty, label it "Garry"

Copy Ralph into Garry

Close Garry and Betty

He did so, wondering whether he could keep his sanity doing this day in and day out. He was told that the process was useful: Betty contains Ralph and another folder with Ralph in it. This was called "adding one to Ralph."

Eugene was again by his side, saying, "Ah, you have a way to create a series of files like the ones that could do without the middleman. Congratulations."

This didn't seem like much consolation to Bill; he only wanted to flee. He fled into another compartment. He was now in Hades, and had taken over Sisyphus' position, except now Sisyphus was in an office in front of a computer. He started with nothing, taking the place of the Ralph of his previous dream. Then he kept making new folders, adding one each time. He was to keep going until he could not do it anymore.

That is, forever.

And Satan stood over him. Bill recognized him.

Eugene.

He would then wake up in a sweat, with a feeling of intense hatred towards folders. He wished he could quit. Maybe go teach English in China or something. But he never did.

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March 11, 2021

His father sent Bill to a psychologist, Dr. Margaret Schiller. She listened patiently listened to him recount his dream. Rather than ask Bill about his childhood, Dr. Schiller suggested that the roots of Bill's nightly excursions lay nearer to home. She asked Bill to send her his university transcript before the next session. Bill found the request odd, but figured that he wasn't the psychologist, he was the psycho.

The following week, Dr. Schiller showed Bill the transcript."Here, I found your problem," she told him."In your second year at university, you took three introductory courses. A little knowledge is ..."

"... a dangerous thing," Bill completed."But so is no knowledge. The courses were on Set Theory, Physics and Existentialism. I vaguely remember."

"Apparently your dream self remembers it more clearly. Your folders were the sets in the universe. You started generating sets that were perfectly analogous to the natural numbers, so that one set being a member of the next was analogous to one number being less than another. You had the operation that was analogous to the additional of one. You ..."

"Whoa. I vaguely remember, yes. But...are you using the expression 'natural numbers' the way they used it in high school, as the counting numbers excluding zero, or the way that was used in my university courses, which included zero?"

“I meant the non-negative integers, that is non-negative whole numbers, because it is natural to think of an empty folder as corresponding to zero. Most of the world, even in their secondary schools, include zero, but the US and England wait until university to get to zero. Funny, English speakers. Romeo and Juliet paid the mistake in ‘what’s in a name’ with their lives, but mathematicians are not quite so touchy as the Capulets and Montagues, so don’t worry about it. Best is for you to figure out from the context whether I am including zero. Or sometimes it doesn’t matter.”

“Maybe you can just stick with ‘counting numbers’ which starts with the number one.”

“Fine. The point is, whichever way you define them, physical laws are based on these numbers. Since these numbers describe physics, then generalizations about your folder universe, your universe of sets, would together seem to be enough to describe the physical universe.”

“Without any memos? Aren’t there elements that aren’t folders? What you are saying is that a human being is based on.... nothing? That was what my dream was supposed to tell me?”

“Perhaps.”

“But...everyday objects, say a chair or a table...”

“...are combined bunches of relations which we attribute to our eyes, sense of touch and the like. So you don’t need those memos. But if you like them, there are theories that have such elements that are not themselves folders. ‘Urelements’, they’re called. The choice is up to you, and some people insist on them. But generally they are not needed. Don’t worry about this; use your dreams as sources of insight to do something concrete. Record your dreams every time you have them, each time filling in details. Build scenarios with Lego blocks; I advised a patient of mine in Germany to do that, and she has told me that it works wonders. She still keeps her Lego kit handy in her office.”

Bill did not find it of much comfort to discover the origins of his problems. If you know your torturer, does that lessen the torture? But he didn’t want to get into an argument about the merits of psychoanalysis, so he signed the confidentiality form and left. He bought the Lego kit, and tried to find more insight into his problems, but this only deepened his sense of emptiness.

Depression begat pessimism, which not only made him despair of help by psychologists, but also made him see the world in dark colors, which deepened his depression. The vicious circle continued. , up to the point where Bill contemplated suicide. He went to a suicide forum, using an anonymous avatar. He wrote that he didn’t see any meaning in life, so why continue?

The answers were mostly predictable. Many were religious, but they found no echo in Bill’s feelings. Some told him to find his own purpose in life, but this seemed circular to Bill: what purpose was there to find a purpose? A few more philosophically minded urged Bill to seek out deeper truths, but this was too vague to be of any help. He was about to log out in order to figure out some spectacular way to die that would embarrass his father to the extreme, when he saw a message come in. He clicked it; the avatar “Weird_but_good” said nothing about the person, but the message was simple.”Meaningless? Can you tell me what meaning is? If you think it is in terms of truth or purpose, can you tell me what those are?”

Bill thought for a minute. No, he really couldn’t. He typed a reply.

“Can you?”

“Yes, Bill.”

“How do you know my name?”

“You don’t protect your computer very well. Want to talk more privately? Just say yes or no.”

“Yes.”

At that moment his Skype started up. The caller:"Weird_but_good"

He answered."How did you call me without my having put you on my contacts list?"

"Not hard. But I am not here to teach you my hacking tricks. You want to decide whether your life is meaningful without even knowing what that means. I'm Jutta, by the way."

"You are going to stop me from committing suicide by telling me that I have meaning in my life?"

"No, I am going to tell you what meaning is. There is no contradiction between having meaning in your life and committing suicide. I just thought it was silly to cite meaninglessness as a reason. But that's not uncommon. Even Searle didn't get it."

"Who?"

"Professor John Searle. You know, the guy who said computer consciousness wasn't possible because machines just shove around symbols. There are a lot of people who think like him, even if the terms are more complex, like, um, a 'self-organizing hierarchical statistical processes among pattern recognizers' or stuff like that."

"You mean that Chinese Room argument? A clerk who has a rulebook for rearranging Chinese hieroglyphs to respond to queries in Chinese, without having a clue what either the question or the answer meant."

"That's it."

"That's a weak one. Searle cheated on that one. He is making an analogy between the computer and his arrangement, but he conveniently leaves out the rulebook itself in this analogy. But the system to be considered should also include this rulebook, not just the clerk. Then it will be a different story."

"That was one weakness in his argument. But that is not enough. The idea that his mindless clerk was only one part is on the right track. His rules would be of different kinds, and he pretended that they were only one kind. He didn't think about meaning. He missed out on the whole idea of models."

"Model? What does fashion have to do with....?

"Not that kind of model. I'm going to have to give you a quick course in Model Theory."

"Why?"

"You want meaning? Don't commit suicide until after our next talk. Then you'll know enough to decide whether your life has meaning. OK?"

"Uh, I suppose."

"Right. I have to get back to work on the proverbial offer you can't refuse. That alone will be worth putting off your suicide. So, until next time."

"Wait, what do you mean....?" The Skype connection went dead.

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March 14, 2021

Jutta called again.

"I see you haven't committed suicide yet."

"I'm waiting to see if you have anything worthwhile to offer before deciding."

"OK, then I'll get straight to the point. Start with universe on one side,..."

"What does astronomy...."

"No, 'universe' as in the basic things which will be the basis of your meaning. Meaning is also called semantics, not to be confused with the everyday sense of trivial playing with words. Semantics might consist of perceptions being assigned to pictures, maybe thoughts being assigned to words. Some of these are going to be collections, or classes."

"Are we going back to school?"

"No, classes in the sense of collection. A class of wine, or something. Then your correspondence between those symbols..."

"Symbols? What symbols?"

"Like, say, an alphabet, or pictures, or something. Something that might mean nothing to an extraterrestrial. Symbols are something that can point to meaning. How they do it, that's the whole point. Some of those symbols are going to vary from one interpretation to another. They will point to collections of corresponding things in the universe. We are going to assume that one of the collections in the universe is simply empty, like your empty folder."

"How do you know about my dreams?"

"I blackmailed your shrink. Just listen, will you? You also have a bunch of truth values, arranged in a certain way. A formula..."

"No, no formulas, please. I was not good in calculating...."

"Not that kind of formula. No calculations. Sorry, there are only so many words in the English language. A formula here is a string of symbols that follows the grammar rules for making such strings. A sentence is a formula that can be given a truth value in some interpretation. 'It's red' without any indication as to what 'it' is, is a formula, but 'all cats are red' is a sentence. A false one in my experience of cats, but a sentence anyway."

"Oh. Will I have to remember the distinction?"

"No, I'm just explaining to you why I will use two different words, that's all. Each pair consisting of a sentence plus its interpretation is an instance of the interpretation relation. Each instance of the interpretation relation plus its truth value is an instance of the truth relation. OK so far?"

"More or less. Old wine in new bottles, it seems to me."

"No, new wine in old bottles. We use the same words as in everyday English, but with a bit more precision. So, for example, you perhaps know what a variable is."

"Vaguely; we used them in equations in school."

"You mean the variables you used for the model of arithmetic you were using. You were implicitly given to understand that the truth of your answers depended upon the correspondence between the model and the theory after what you called substitution. That is still correct in the general case. A variable will refer to a class of elements in your model, its domain. Many elements in the domain can be represented in your theory by a symbol. If there are elements that are not represented by a symbol, then you can try to make one. It is these symbols that you substitute in for the variables to see if the correspondence between theory and interpretation works without getting into contradiction. If you can always avoid contradiction, then the interpretation is called a model."

"Sounds a bit pedantic."

"That's to avoid getting mixed up in thinking that you substitute elements from the interpretation directly into the theory, which would be mixing apples and oranges. But now that I've said it once, I will assume you won't get mixed up between the model and the theory."

"I'll try. Go ahead."

"Let's start with a single model. Its universe is a collection, a class. Then you can have other collections which omit some of the elements of the collection. A collection of sentences is a theory."

"You said that you wouldn't be so explicit, always talking about the two levels, the syntactic and the semantic, the theory and the model."

“Sometimes it’s hard to avoid. Bear with me. Suppose we pick some of these sentences judiciously so that the domain of each variable has only elements from the universe in it. Needless to say, we would pick sentences that wouldn’t lead to a contradiction when in combination with the others.”

“I always found it puzzling that ‘needless to say’ is followed by the statement that one supposedly found needless to say.”

“Emphasis, I suppose. This is the bottom rung, so the variables are called ‘first-order’ variables. A sentence with only first-order variables in it is a first-order sentence, and a theory with only first-order sentences is called...”

“... a first-order theory. What else is there?”

“I’ll get to the next rung on the ladder, but I want to stay with first-order theories for a moment. There is a way to combine sentences to generate new sentences. One uses a basic rule for all theories using basic connectives like ‘and’ and ‘implies’. The rules for combining sentences will simultaneously determine the interpretation, including the truth values of the combination, from the interpretations and truth values of the individual sentences. Two sentences are, with respect to a model, independent if the truth value of one does not depend on the truth value of the other. A collection of sentences will generate a theory if that collection is all that is needed, with that rule I mentioned, to create all the sentences of the theory.”

“I think you are working up to telling me what axioms are, right?”

“Right. You tell me.”

“A collection of sentences that are independent of one another, and together generates the theory.”

“Right. So I can use the word. I am going to use another word that I want to sound like nonsense. Give me one.”

“Ah... Pif.”

“OK. You can imagine a theory that has two types of axioms. One, saying that something exists that satisfies a certain sentence, is a pif. Another type, saying that if something is a pif, then something else is also a pif. Together they generate all pifs.”

“Um. So?”

“First order theories that do this for the word ‘set’ are called Set Theories. The universe of a set theory can then be composed only of sets.”

“What else? But why do we need a new word for collection? You’ve been using the word ‘class’.”

“Not all classes will be sets. The axioms chosen must be chosen carefully to avoid contradictions. For example, the class of all these sets cannot be a set, because... well, think about having the set of all sets. Is it inside itself? Gets you into a mess, that.”

“Always?”

“Ah, no, you can actually create a theory that allows a set of all sets. But the theory has other annoyances. So, in our usual way of doing things, the class of all your sets, which will be part of your universe, cannot be a set with respect to that theory. So it’s a class that can’t be a set with respect to that theory.”

“I guess that makes sense. The truth values, they are true and false, no?”

“Not necessarily. But they should be made so that the connectives make sense, as far as the relations of the sentences going in tandem with the corresponding classes, both going in tandem with a truth value. To do that, you might have special rules. Like a model for probabilities...”

“Whoa! I thought that probabilities were fuzzy by definition.”

"You are getting your levels mixed up. The items about which the probabilities are about, they're fuzzy, because you can't always pin them down. But the probabilities themselves are definite. When you flip a fair coin, it is fuzzy which side you will get, but the 50-50 chances, that's definite. In general, a premise will imply a conclusion if the premise has at least as big a corresponding class as the conclusion does, and the truth value of the premise will be no bigger than the truth value of the conclusion. And so on."

"Where does true and false fit in with such fuzzy examples?"

"If you collect the classes that satisfy the completely true sentences, this collection will obviously not include the empty class, or the class satisfying the negation of a true sentence, but it would include classes satisfying sentences implied by true sentences, and so forth. The completely true and completely false sentences act pretty clearly. It is the others that are tricky. I'm getting to that."

"Before you get all fuzzy, I want to know where you are going with all this."

"You need to do some introspection, but unless you know what you are looking for, it won't do you any good," Jutta explained.

"Introspection? Are you saying that my mind is full of sets and classes and functions interacting to give a verdict of meaning of truth and meaning?"

"Or something that has a similar structure. For example, we usually view sets as primary, defining them first. Then we define functions in terms of sets, making them a secondary notion. However, you can also start with functions as a primary notion. There is a theory called Category Theory that does just that. That viewpoint has its uses. I have nothing against using this viewpoint, but since most people use the intuitive ideas like union and intersection, I will stick to using it."

But the more abstract stuff about sets aren't so intuitive. I majored in mathematics my first two years, and we got into some pretty weird stuff."

"I think that mathematics starts getting weird about the same time that reality does."

"What is that supposed to mean?"

"That our intuition evolved to deal with short-termed local events. But reality tends to be more complex than that. Look how many people don't understand the theory of evolution, for example. That doesn't mean that we didn't evolve, though, just because they don't understand it. To get you to understand reality, you necessarily are going to have to get into, well, weirdness."

"So you're saying that my head is a model for theories which also satisfy other models?"

"Something like that. What you have learned in your life, you learned one theory or one model at a time."

"Not only are there many models to each theory, but also there are many theories. Remember, your introspection would be covering an interval of time, no? You develop, don't you? A theory of one moment can morph into another theory. But you switch from one to the other, just as a child will switch from one language to another when his parents speak different languages with her. You may think that your head contains only one single set of rules that governs everything, and maybe it does. But consciously you concentrate on one at a time."

"I reserve the right to grumble."

"Be my guest. Anyway, if you will concentrate on the essence of these ideas, you will notice that we have been looking at a model and its theory from a God's eye view. That is, we have codified the model into symbols, and incorporated it into a new theory. We've made syntax out of semantics. The new theory must be more powerful than the old one. One nice result is that the interpretation relationship can be written as a first-order theory."

“Sounds like Hamlet examining Yorick’s skull.”

“In fact, it is like analyzing your brain, either from your thoughts’ point of view, or from a neurologist’s point of view. Emotions, empathy, even consciousness, expressed in terms of things like the amygdale, the caudate, the frontal gyrus, the claustrum, and all that. From one side of view, you have the mind, and from another, a bunch of electromagnetic patterns....”

“I read about that in a Philosophy book! It’s called the Mind-Body problem.”

“That’s what they used to call it. Today people just call it ‘the Hard Problem’. Something like the dispute between Materialists and Idealists.”

“You mean money-lovers versus unrealistic protestors?”

“No, I mean the philosophical versions of those words. But I don’t want to get into a philosophical discussion. The point is that we do this all the time. For a start, the theory may not contain all the true sentences of that model, and so in pursuing true sentences, one can enlarge one’s theory to include new truths... although this new theory again doesn’t always contain all the true sentences, and so forth. One might say that most thinking is nothing but trying to fill in the gaps to make it so, and in so doing just making new gaps. We go from model to model, adding things. But because we have a fixation on being the same person, it often appears as if we imagine some new possibility and, if it is useful but inaccessible from the mechanisms of your present model, we co-opt it for our model. This has the side effect that something that was a class in one model becomes a set in the next; useful as a set, so you stay in that universe. The new set, which used to be a proper class, if it also has the property that it is the same size if you take something away from it, is an infinite set. Or it might be something else that simply did not exist in your lower set either as a set or a class, but its presence is useful.”

“Like when we fill in the gaps in optical illusions.”

“Not a bad example. That is, we travel along a system of possible worlds, which are connected by accessibility relations.”

“This sounds like science fiction.”

“Oh, I think you are getting mixed up with the so-called parallel universes. I won’t get into that debate right now, but the idea isn’t as wild as it portrayed in science fiction, where they put in rather strange or simply unexplained accessibility relations. For the moment, I am just referring to mental constructions. Imagination.

We get new perceptions, expanding the model. We think, changing our theory or the interpretation function. We forget. We encode what we imagine to be others’ frameworks inside our own, creating a theory of mind. We modify our memories of the past, and create likely future scenarios. We solve our daily difficulties by choosing from possible imagined worlds. We invent new objects out of abstractions... reify, I think it’s called. We alter our memories to create false memories, and we invent scenarios for the future, letting these scenarios blend with the present models via wishful thinking. We regret and we hope. Our axioms change. We even hallucinate, or have multiple personalities. Interpretations and theories are in constant flux. We change the weights given to one interpretation over another; we learn to make certain accessibility relations easier or harder, changing the whole framework of our possible worlds. We flip from a God’s eye view, to more subjective viewpoints and back as if there were really no difference. This leads us to generalize, using expressions like ‘every’ and ‘all’.”

“I’ve heard of those expressions as quantifiers because they say something about the quantity of elements of the universe which satisfy a sentence.”

“Yes, but it goes even further. You have the modal quantifiers which say something about the quantity of models accessible from the present model.”

"In my English class, they told us that about modal verbs, like *must, shall, will, should, would, ought, can, could, may* and *might*."

"It takes a little rearranging to express quantifiers in logic to the form you use in natural language, but it's not that hard. It's all in the flexibility in the way you look at one's utterances. Which you lack. Which I have. Which is why you should hire me."

"You didn't answer my question."

"You didn't ask one."

"Apparently you also lack a way of thinking to interpret observations as questions. What does generalization have to do with these modal verbs?"

"You can trace it back when you start with 'it must be the case, given what I know now', or, as we usually say, 'it is necessary'. You can, for instance, make provability the same thing as necessity, and say that necessity is another way to say that the negation of something isn't possible. There you have the 'possible' in 'possible worlds'. 'Should' and so forth is a comparison of possible worlds. You work out the details."

"Flipping between models like that appears to be schizophrenic."

"I think you mean a multiple personality syndrome. Schizophrenia is something else. But you're right, people are sometimes not so good at switching, and end up in silly paradoxes, such as talking about the set of all sets."

"You mentioned the theories that talk about the elements of the universe, and now you have theories that talk about formulas."

"Yes, they are different, so you need a theory with higher-order variables. The second kind, the second order, is the next rung up the ladder, and no longer first-order. When we can, we deal with first-order, but that's not always practical."

"What is so great about first-order?"

"Making a theory about proof. Let me make a distinction between two terms that are often confused. I've mentioned that 'true' is relative to a model. 'Valid' is relative to a theory. A valid sentence is one that can be proven from the axioms, where the proof techniques are constructed so that a sentence can imply only sentences that are at least as true as itself."

"So, I get the idea that 'valid' is not the same as 'true', but at least, from what you said, for a given model, under the usual conditions, a valid system is true."

"Right. We usually assume this, as it makes sense to work with sound systems...."

"I like music too, but..."

"No, I mean systems where the valid sentences are true."

"Like a tautology, no?"

"A tautology is a trivial example, as it is satisfied by all models, no matter what the axioms. A valid sentence can be choosier. Be careful, though; you need to distinguish between something that is necessary and something that is a tautology."

"What's the difference?"

"Something can be necessary without being a tautology. That is, if you only have certain models available to you, and it is provably true in all those models, then it is necessary."

"You mentioned co-opting possible worlds into a single world. What do you get if you do that here?"

"Depends how you do it. A fuzzy logic, or a logic on its way to completion, or..."

"I read a little bit about this and got confused by the seemingly contradictory way that one talked about complete theories. This is important, since I was told that one's aim in life is often about completion."

“The word ‘complete’ is different according to context. A complete proof mechanism means that validity and provability are the same, but a complete theory with respect to a given model means that or that provability and truth are the same.”

“Ah, OK. So if a theory can formulate a sentence that it can neither prove nor disprove, then it is incomplete, right? Since such an undecidable sentence cannot be valid, as there will be some models that satisfy the given axioms in which the sentence is true, and others where it is false. Am I making sense?”

“I think you are. Do you think you are?”

“I’ll sleep on it. But you still haven’t answered my question as to why one likes first-order logics.”

“Because of the difficulty with proof checking. If validity and provability are the same for a higher order logic, then there is no systematic step-by-step method for checking whether a proof is correct.”

“But you’re saying higher-order logics are also nice.”

“Sometimes. For example, if we want to make generalizations not about individuals but about all sentences, we either need an infinite list of sentences in first-order logic...”.

“No one does that.”

“No? You are wrong. If we can restrict a higher-order system so that we only need to deal with first order, we do. Working with a sentence with only first-order variables may seem first-order, but sometimes one is cheating and the sentence is generic, not specific enough, so that one is really proving an infinite number of sentences at once. Since infinity is used as a jump to something that you can’t get in your system, an infinite amount of sentences in first-order logic can jump to a single statement in a higher-order logic.”

“Have we finished? You haven’t told me what the meaning or purpose is.”

“Haven’t you been listening? Your meaning is your interpretation, and the purpose is embodied in your accessibility relations between possible worlds which lead to that meaning. Whether you attain it or not is irrelevant; the road is not to the goal...”

“Yes, I read T-shirts too. In other words, I am looking for a workable possible worlds framework to lead me towards a model and a theory, so that I can have meaning?”

“If you consider it important to have meaning, then yes. Otherwise you are like Searle’s clerk. Searle forgot to include meaning, semantics. He stuck to the meaningless symbols, the syntax. So, that is all I have for you now. But I will be following your progress. Not altogether altruistically, by the way.”

Weird_but_good signed off.

As an after-thought, Bill glanced at his list of contacts. No Jutta, no Weird_but_good. He looked at the history of calls. None for that day. What did Jutta mean by “not altogether altruistically”?

Chapter Two

April 9, 2021

Bill’s mental state went from depression to brooding. His father continued to pay for sessions with Dr. Schiller, but Bill kept cancelling them, preferring long hours in the library. One day, as the library was closed due to a leaky water pipe, Bill finally went to the Dr. Schiller.

She told him,” Jutta is right. You need to do some introspection and manage your own mind. It seems that you see everything in black and white. There are grey areas. If you want something

cheap, it can get very stressful to decide under pressure where the exact cut-off between cheap and not cheap lies."

"I know, but fuzziness always gets me muddled. I prefer precision."

"Fine. Combine them, Make a precise theory of fuzziness. There's lot of ways to do it. Probability is one way. But I prefer the way of possible worlds. They often go in clusters, and the accessibility relations might be weighted, with some more accessible than others. So in a way, you have several truths at once. Only when the cluster comes into contact with something that forms another cluster, one of the models stands out, and that becomes your actual model. I think the physicists call this 'superposition'. Alternatively, you start with a fuzzy model, then you concentrate on the statements which can be syntactically proven. The theory containing only these statements form a new model. This model has an accessibility relation which is identified with proof in the theory. In the first model, you don't have the stressful law of the excluded middle."

"Which is what?"

"The principle of the excluded middle says that there are only two truth values, so the negation of something false is automatically true, and vice versa. That means that the truth value of negating a sentence twice gives you the same truth value as that of the original sentence. But for the intuitionists, a double negation is not the same thing as the original. For a double negation in this so-called 'intuitionist logic', you would have to prove that you cannot prove the negation of the original statement."

"Ooh, that's too many negatives in a sentence. So you have two types of model, one with fuzziness, one without. Which one is reality?"

"It depends on what you want to call reality. Physics still insist that only that which can be directly measured constitutes reality, and all other entities were simply mathematical conveniences. But the crux of the matter is the word 'directly'."

"How can such a word be, um, fuzzy?"

"You cannot measure a fuzzy state in a single measurement. But you can measure identical states many times, getting many individual concrete measurements. Some die-hards want to say that 'reality' consists simply of the many individual measurements. But others consider all the individual measurements as parts of a single measurement, so that this combined measured state is one which can give rise to the individual measured states. That is, that the fuzzy state is real."

"uh... the logic you said doesn't have double negation canceling out... can you give an example that doesn't have to do with this confusing physics?"

"I can try. Proving someone is not guilty is not always the same as proving them innocent."

"Why not?"

"In many countries, you must have a clear proof of someone's guilt in order to convict. So, suppose Mr. Brown is up on a charge of negligence, and has Mrs. Allenby as his defense attorney. The prosecuting attorney, Mr. Charleston, will try to show that Mr. Brown did not flip a certain switch on time."

"OK so far."

"Good. Mrs. Allenby only has to show a flaw in Mr. Charleston's proof that Mr. Brown didn't do that. That is, Mrs. Allenby shows that the statement 'Mr. Brown did not flip the switch' is not provable, that is, by the requirements of the legal system, that one cannot say that 'Mr. Brown did not flip the switch' is true. That is, 'Mr. Brown did not flip the switch' is considered false in this system. That is not the same as having a proof that Mr. Brown did flip the switch. At least, as far as the court is concerned."

“OK, I guess I can see where that might be useful. But I don’t see where this is getting me closer to a purpose.”

Dr. Schiller sighed.”A purpose is the attempt, successful or not, to make a possible world into an actual world. That is, to make possibility into necessity.”

“But I am not clear that I have the possibility. Events seem to govern my life; I am not sure I even have a choice.”

“The problem of free will versus determinism is a bickering over a point of view. There are usually things which are not decidable with the tools in the theory corresponding to your present world. So, from the point of view of that model and its theory, you have free will. From the point of view of a more powerful model, in which that is decided, you do not. There you have determinism for that process. So, with that in mind....when shall we say our next session is?”

Bill said that he would call to make an appointment. He really wasn’t sure he wanted to continue. The session had not raised his spirits. However, it gave him an idea that he needed to develop. For that, he had to get hold of Jutta.

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June 4, 2021

Bill’s attempt to get hold of Jutta had been fruitless. He told himself that he would just have to wait. However, he had barely put his hat up when his Skype beeped at him. The caller: “Weird_but_good”.

Without even waiting for a greeting, Bill asked.”What was that about altruism?

“I do research for a biotechnology firm in Frankfurt. I came up with something, but the company is taking it in a direction that will spell its ruin, despite my warnings. In order to convince you to take the project over... because I made sure I retained the rights to the main concept... I needed you to delve into this model theory stuff. Dr. Schiller is in on this.”

“No way. That would be against her professional ethics...”

“I don’t care. A little blackmail did wonders. Anyway, Wheaton Informatics could take this somewhere, I think, but I would give it to you only if I was convinced that you would take it in the right direction... with me at the head of your research department.”

“My research department? You’ve got your history wrong. Speak to my father.”

“I tried, but your father is presently against daring moves. Sorry, but I hacked into your father’s medical records. Your father is not telling you, but he is dying. You will soon take over.”

Jutta gave a moment’s pause to let this news sink in. After a pause, Bill ventured, “But...”

Jutta interrupted.”Listen to me. I know you want to hang up on me and go commiserate with your father. But maybe it would be best to respect his wish for privacy. The best you can do for him is to be ready to give a new direction to his company. I can help. I have been working on the idea of an improved human-computer interface for years. I got the idea after I developed the technique of transferring DNA and RNA information onto nanochips, using a process that made each nanochip as unique as the strand it was supposed to encode.”

“You’re the one who did that? I read about that.”

“I won’t get all pedantic on you. But in brief: you know that some genes are realized, some not. There are various mechanisms to turn them on or off. This is a bit reminiscent of a sentence, the genes, satisfying or not the model consisting of the organism. Don’t stretch the analogy too far, but it might help you to understand what I will outline. I will use a Lego kit to explain the basic idea.” Jutta took some Lego blocks and started assembling them.

Bill said, “Your shrink also told you to play with those?”

Jutta looked up from her construction.”Huh? Oh, that. No, I never consulted a psychologist. I use them as inspiration for constructing new things. The blocks don’t have the same complexity as the final product, but they help me get an overview. For example, let’s suppose that each DNA strand has only one gene, so in my explanation, I will get ‘DNA strand’ and ‘gene’ a bit mixed up for simplicity. The fact that in reality each DNA has many more genes just complicates the details, but the ideas will be the same.”

“Fair enough.”

“I have made some holes in some of the Lego blocks.”

She then made three constructions, without saying a word, for about five minutes. Then she looked at Bill and pointed to her first construction, which looked like a little robot.

“Bill, meet Fred. Fred is a DNA strand.” Jutta said.

“Hi, Fred,” Bill waved.

“One of Fred’s blocks is hollow, opening up to the outside. But I could still make a nanochip from it.”

“Nanochips from defective genes – what use is that?”

“I am going to show you. I have developed special nanochips that can go into that slot; with the special nanochip in there, the completion of Fred into a fully operational DNA.”

Jutta pointed to the second pile of blocks. There was a Lego dog made out of blocks. All but two blocks were blue. Of the two remaining one, one was white, the other one black. Next to it was something under a cloth.

“Some genes are turned off or blocked by various mechanisms. I have discovered how to take a DNA strand and reverse all the realizations of the genes on it. That is, suppose we have our dog, and we concentrate on a gene that will produce a white block unless the gene is blocked, whereupon it produces a black block. Now, my solution is....”

Jutta poured a pile of glitter over the dog, and when the glitter had settled, the black and white blocks had switched places.

Bob said, “Nice sleight of hand. Is that what keeps you occupied? Magic tricks?”

“No, be patient, and keep in mind this reversal-transformation.” She turned to the next construction.”I am presuming that we can duplicate the nanochips whenever we want. Now, Fred is looking for a nanochip to make him perfect. Suppose Fred either would produce a certain protein, or not.”

“Um, OK.”

“We can read off Fred and, from this knowledge, create a new DNA strand in this way”

She rearranged Lego blocks so quickly and deftly that Bill thought”If these were not Lego blocks I would swear she was doing origami.”

Jutta kept a running commentary as she was rearranging the blocks.

“First I have the robot Nathan who has a hollow. As soon as you put something in Nathan’s hollow, then you get his own chip implanted into his hollow. We now insert Nathan’s chip into Fred’s hollow, and make a chip from that. This chip we place in Nathan’s hollow. We get Sally.”

“I’m not sure I followed all that. But let it slide. What is important about Sally?”

“With respect to Fred, Sally is a special nanochip. When I put Sally’s nanochip into Fred’s missing slot...”

She did so.

“We shall call the result Fred-Sally. The trick is that Sally would produce the same proteins as Fred-Sally. A kind of fixed point.”

“Fixed point?”

“Yes, a fixed point with respect to some process is just any point that doesn’t change because of the process. Take a photocopy of a photo, and crumple it up, putting the whole crumpled photocopy on top of the original. Then there is at least one pixel in the photocopy that is directly above the other one. It’s a fixed point. Point not in the geometric sense of a dot, but just some collection in your universe.”

“So how is that here...”

“The idea is that Sally’s realization, in the model, does not change under the transformation, in the theory, of coding it and attaching it to Fred.”

“Cute. But...”

Jutta interrupted.”I can do this for any Fred. Call it Jutta’s Discovery.”

“What is all this leading to?”

“I told you my company wants to use this result. They want to use DNA into a computer ...”

“That’s been around. I think RNA is the stuff of choice, though.”

“Same principles. They already have system that says how the DNA can combine to give new DNA.”

“And?”

“But then, they are trying to include in each DNA a new gene which will give details about whether the genes in the DNA are to be realized. They call this project the Supervisor Project.”

“Good for them. What are you trying to do, sell me your company’s secrets?”

“No, nothing so direct. I am offering your company a chance to take advantage of my company’s inevitable failure, and to jump into the market with an improved version that will work.”

“And you think I will fall for some scam over the Internet. I am supposed to send you money, and...”

“No, I am not asking for any money that I do not earn. I am asking for a position in your company.”

“But I am not sure of the legality of this...”

“That has not stopped you in the past. Remember the conversations you had with the head of the Federal Reserve Board in the Petworth Neighborhood Library? That was in the year...”

Bill said, “So you are having me watched?”

“Not exactly. You are already being watched. There are CCTV cameras everywhere with pattern recognition systems in them, your library card puts the date you in the library into their computers, your credit card transactions, your phone record, your....”

“But all that’s confidential!”

“And most of it is accessible via a little hacking. But let’s not get sidetracked. Are you interested in my offer?”

“But I only have your word for it that this Supervisor Project will fail.”

“No, I will show you. But let me ask the question as a conditional. IF I convince you that it will fail, and IF I convince you that there is a viable alternative, will you hire me? I need to make plans.”

“All right. Go.”

“What, now?”

“If your argument is clear, you should be able to at least give me an outline. We can go over the details when we meet in person.”

“Fair enough. The basic idea of the Supervisor Project is like trying to put a function into a theory that could take the code of a sentence and tell whether the corresponding sentence was realized in a model, hence true, or not.”

“Wait a minute. I need to take notes to keep that straight.” A minute passed.”OK. Go on.”

“Suppose you could. Call this supposed universal truth-tester ‘the Pope’.”

“Um, I don’t think the Catholics claim that the Pope’s infallibility stretches to all statements.”

“It’s just a name. They don’t burn people at the stake anymore. At most their cook just burns steaks.”

“Try to avoid the lousy puns, please.”

“Sorry. I’m a programmer, not a comedian. Anyway, I analogize between the biological and the logical, by comparing the gene to a sentence, the nanochip to its codification, and the gene realization to the satisfaction in the model. That is, the Supervisor is a means of checking the truth of the sentence in the model.”

“Um, yea. That’s using the definition of truth as the successful modeling, is that it?”

“Yes. Then, the Pope would be given the nanochip of a gene, and declare, by itself producing a certain protein or not, whether the gene would produce a protein. Right?”

“Right.”

“Put the Pope through the reversal process I showed you. You get a new strand with a hollow part; call it the Antichrist. The Antichrist would do the opposite of what the Pope would do. When the Pope would produce a protein, the Antichrist wouldn’t, and vice versa.”

Bill said, “Stop. I am trying to get all this down in a reasonable form. Give me a few minutes.” Bill worked for about ten minutes, while Jutta waited patiently. Finally Bill said,”OK, go ahead with your story.”

Jutta said, “What? Oh, sorry, dozed off there for a minute. The time difference, you see, is....”

“Do you want to resume this later?”

“No, I’ll manage. I just had a late night last night. Anyway, where was I? Ah, right, we know that we can get a Sally for the Antichrist, as described by my Discovery. Take that chip of Sally’s, give it to the Antichrist. The very definition of this Sally is that Sally will produce the same proteins as the Antichrist. But by the definition of the Antichrist, he will produce the protein if Sally won’t. So you get a contradiction, and the process will not go through.”

“But...it seems to me that your analogy between gene realization and truth is maybe a bit stretched. This result, for example, would say that there is no way you could construct this, uh, Pope.”

“That’s correct. That’s exactly what I am saying. This is why the Supervisor Project is doomed.”

“But that would mean, in the analogy, that there would be no way to construct a truth-tester.”

“But we have one in our heads! We are able to judge the truth of statements, even if it takes us a bit of thinking and testing.”

“Yes and no. Our truth-testers are not universal ones, even if some people tend to forget this. The truth function in the theory is not a mirror of the one which looks at the overall model and theory. Your symbols in the theory will be in some system.... maybe numbers, words, whatever. You can make a coding for your sentences in that language, then you can define a function on the codes. The human brain does it by using a time-delay between your perception of something and your conscious awareness of it. The immediate past moment is codified and inserted into a

concept of the present. Since one is not aware of the time lapse, it looks as if one is actually thinking about the present, ending up with things like the concept of self."

"You don't like the famous 'I think therefore I am?'."

"Regardless of my view of whether consciousness represents a unified entity, this inference itself is a bit silly, because it assumes what it wants to prove. To say 'I think' I assume that there exists an 'I' that thinks. Anyway, to get to more serious problems, the problem in my story arose because this Pope's infallibility was supposedly universal. But if someone had said that the Pope's infallibility was restricted to, say, the best restaurant in the Vatican, then there would not be so much of a problem. The same goes for the Supervisor Project. It is simply too over-reaching; the inventors did not think to put in safeguards. This, however, your company can do. I can give you the exact timing of the launch of their product, and your lab will be ready to show that it doesn't work immediately afterwards. At the same time you can offer your alternative."

"On the face of it, you seemed to have proved your point, but I am still not convinced. After all, I was I was taught truth tables, and these did not depend on the interpretation.", Bill objected.

"I know where you got that impression. In school you were given examples that were true in every model, or false in every model, so they didn't bother to specify the model. But truth tables are about truth, right? And truth is dependent on the interpretation, remember?"

"But... well, there were those procedures for working things out. It was sort of automatic. I mean, suppose 'Alice' and 'Bob' are two strings of symbols. Then 'Bob' is a consequence of 'Alice' if, machine-like, the string 'Alice' can be turned into the string of symbols 'Bob' by using the string-transforming rules of a given system."

"Fine, but whether Bob is true depends on whether Alice is true, and for that you need the interpretation. Also, you have to interpret the string-transforming rules so that they make sense in the model. Implication, as a key player in this game, is a relationship in a theory, and as such has to do with the interpretation in the model."

"But you just said we are allowed to talk about truth, and if this word is part of my description, that means it is in the theory. That is, you have a symbol inside the theory that talked about the truth of a statement."

"Yes, but we also sometimes say, validly, that we don't know. That is, the truth function is not universal. In fact, it gets worse. For example, when there is someone who is not sure about a statement I want them to agree to, I use a trick. I don't use this to try to convince them of something that is blatantly false, because no argument I could use would do that. However, to make the method clear, I will convince you that Santa Claus exists."

"Of course he exists. As a child..."

"Oh, all right. Then let me convince you that Paris is the capital of Italy."

"The Romans are going to love you."

"Suppose I say, without asking for a commitment on your part to like the statement or not, that 'if this statement is true, then Paris is the capital of Italy.'"

"Go ahead."

"Now, we have the premise – 'this statement is true'—and the conclusion – 'Paris is the capital of Italy'. We don't know if this statement is true or not, so we will investigate the possibilities. Let's first see what happens when we assume that the premise is true."

"So far, no problem."

"If it is true, then the implication is also valid, because that is what the statement said."

"Um...yes. I sense trouble."

“Patience. Since the premise is true and the implication is valid, that means that the conclusion ‘Paris is the capital of Italy’ is also true.”

“That would be if the premise were true. Since I don’t agree with the statement, the premise must be false.”

“Indeed? But from a false statement you can deduce anything. So, the implication must be valid. Which is what the premise is saying. So the premise must be true.”

“And therefore the conclusion... Paris is the capital of Italy.”

“I guess the Romans had better start learning French.”

“Although I use the trick, I am not sure why it shouldn’t work.”

“It is not the self-reference and the use of truth per se that is the problem, but simply that one doesn’t have true self-reference. That is, look at the statement ‘this statement is true’. This is a statement in the theory. But this predicate...”

“Wait. I was never good at grammar. A predicate is....what?”

“If you look at a sentence as a function of the subject, then the function is the predicate. In other words, that part of a sentence that says something about the subject.”

“In this case, ‘is true’.”

“Right. So, as I was saying, this predicate is part of the theory and so has to be interpreted. We know that ‘is true’ is normally a function of the triple theory-interpretation function-model. That means we have to have this triple inside the theory.”

“Is that possible?”

“Sure. We have done this implicitly when we described the triple in words. It is like when you think about your mind. Your mind doesn’t really think about your mind, but rather about some codified version of your mind. In the same way, this triple which has been put into the language of the theory when you talk about truth. That is of course a different triple than the one that you use to model the whole theory, so let’s call it the minor triple.”

“Sounds like a new key in music. Why is it so obvious that it’s a different triple, other than being in codified form?”

“For one thing, the minor theory doesn’t contain that minor triple.”

“Ah. So where does this leave us?”

“Now we are ready to analyze the statement ‘this statement is true implies babble. .”

“Babble?”

“It takes too much time to keep saying ‘Paris is the capital of Italy.’”

“I just think you are afraid that you will start believing it after repeating it so much.”

“In the major model, you have something that will interpret ‘this statement is true’. When it judges whether that is true, you have different referents for ‘true’. So the reasoning you used to justify your argument falls through. In fact, the same is true for the liar paradox.” He paused for a moment.”Come to think of it, since sometimes negation of a statement is defined as that statement implying something false, your argument is basically the same as the liar paradox.”

“Which is what?”

“Variations on the sentence, ‘This sentence is false’. When you ask yourself whether this can be true or false, it appears that either answer leads you to a contradiction. The illusion of self-reference makes it look like the same sentence is being given two opposite truth values. Already the Greeks recognized this problem when they asked ‘who will guard the guards themselves?’.”

“So, introduce a third truth value. Indeterminate, or undecidable, or something. Sort of like in the physical world. In asking whether the measurement is a certain value, you can answer yes or no if you have measured it or at least measured things which imply that this would be the value.

Otherwise, you have to say that it is indeterminate. Then you could have a corresponding truth table... say, for implication. As usual, we could say that only the implications with the premise being false, or the conclusion being true, are true. All other implications involving indeterminate values are themselves indeterminate. All other statements are false."

"But this doesn't get you away from reference to the interpretation, because the measurement is an interpretation."

"Right, but at least we could talk about truth, without having the problem of the liar paradox, since 'This statement is false' could have an indeterminate truth value. Therefore, there would be no problem with whether it was true or false."

"Fine. But then you get the problem when asking about the truth value of 'This statement is either false or indeterminate'".

"Why?"

"Work it through. If you say it is true, then it would have to be either false or indeterminate. If it were false, it would have to be true, the only value left. If it were indeterminate, it would be true. So this doesn't get you out of the paradox, despite what you might read in the newspaper. No, the liar paradox cannot be solved that way."

"Spoil-sport. Maybe more truth values, some not comparable to others? We can make a sort of branching from the bottom, which represents falsity, but then where all the branches eventually join up again at the top, which represents truth. When the values are all lined up, and so all comparable to one another, then Alice implies Bob when Bob is at least as high up as Alice is. But if they aren't comparable, then no relation of implication can exist between them."

"How can they not be comparable?"

"Suppose each one gets a truth value of two cards, each card being Jack, Queen or King, where the Jack is less than the Queen or King, and a Queen is less than a king. Then what do you do when one player has a Queen of Hearts, and the other, a Queen of Clubs?"

"I see. They are not comparable. But how is this related to the liar paradox?"

"Since a truth value could be a pair, the liar paradox could be both true and false at the same time."

"Very nice, but then you have a contradiction. Your system is not consistent. So let me put it this way: you can't have a complete truth symbol and still have the system consistent, since you end up with the troublesome combination of total self-reference and truth."

"I thought negation was also necessary."

"Not really. Consider that paradox which made it look like I was proving Paris to be the capital of Italy. It didn't involve negation, and we only got into a problem when we assumed total self-reference."

"But even self-reference is not necessary, is it? For example, there is the similar problem of the infinite row of people, starting off at a first person standing on the left, each one saying that the person to her right is telling a lie. There's negation and truth involved, but no self-reference."

"You're wrong. There is self-reference. Each person in the row has to refer to herself to indicate which statements are false. Each one has to say, 'the persons who are to my right'. So there is self-reference, although rather subtle."

"OK, but why do you object to inconsistency so much?"

"From an inconsistency you can deduce anything."

"Only according to the classical simplistic way of thinking about implication. But people use contradictions all the time, and do not sink into babbling idiots."

“Actually, that’s not as true as many people claim. Just because someone says one thing one day, and its opposite the next day, does not mean that this is a contradiction. The model which the person is using will be different from day to day. One has experiences during the intervening time which are added to the model. Whichever construction you use in your possible worlds, one can move from one to the other, for example, by expanding your model with new constants and adding new axioms, or the reverse, contracting your model. To add new things to your universe... well, perception does that, but you can also do it by thinking; for example, you can reify things by taking something, maybe concentrating on a process defined by your theory, but which has no closure, no fixed point in your model, and add an axiom to your theory that says that this fixed point exists, and so add the constant to your model. You can do this by going to a nearby possible world, and figure out what you need to add. That is, you co-opt it from a more powerful model into your new model. That might be a hallucination, a fulfilled expectation, or a Muse.

Therefore, the same statement could be undecidable in yesterday’s model, true under today’s model and false under tomorrow’s model.”

“Right. But you must admit that there are cases of simple bad reasoning. Even such simple things as ignoring transitivity of implication.”

Bill stopped her.“Tell me again what ‘transitive’ means.”

“It means, for example, if one sentence implies another one, and this other one implies a third one, then the first one implies the third one.”

“That’s natural, isn’t it?”

“Not necessarily. It seems that many people sometimes invent other rules to override this one.”

“Well, apart from that....we still get the fact that people’s inner worlds change with time and experience. So does the world in general. A new experience requires either an extension of the interpretation relation or even a new axiom. That is, in your universe you get a new individual that satisfies the axiom, a moment of the past which forms the background for further happenings. You get a new possible world. In this way, bit by bit, you can jump from one possible world to another. An outsider can analyze the whole connected scenario of worlds which are accessible to one another, and even formalize the relation which describes how one is accessible from the other. That’s what life is all about, getting less fuzzy in one respect, but then a new fuzziness pops up. In sum, we get into a possible worlds scenario.”

“That always sounded sort of sci-fi to me.”

“No, every time you use some conditional thinking, wondering what would happen if....As well, whereas in mathematics we talk of a theory as being all the consequences of the axioms, humans don’t think out all of these at once. So, one builds the model up bit by bit, sometimes abandoning one way of thinking, backtracking to mental situations that are accessible to it, and so forth. In this case I have been speaking as though I am uniting all the worlds into a single theory, and you can understand that for humans this translates into a possible worlds scenario. Is that OK?”

“I guess. I reserve the right to object to it at any point in your further explanation, though.”

“That seems fair. But I am not going to let you off the hook so easily. There can be false statements entirely within a single one of those worlds. What happens when someone gives a mistaken figure for the mass of Jupiter? Could she could then say that Paris is the capital of Italy, and get away with it?”

“Well, um, ...”

“Or, hey, the contradictions even in mathematics that were not corrected until the twentieth century did not stop earlier mathematicians from eliminating other falsities and honing in on good theories. Science is full of false assertions, but this does not stop them from properly discarding rubbishy theories.. Data processing systems often have room for mistakes, to avoid the problems of more simplistic systems that froze every time a programmer made a typo. And...”

“You’ve made your point. What you are saying is that in practice, we allow non-explosive contradictions.”

“Explosions? Who is talking about bombs?”

“Not physical explosions. The explosions I mean are that from a contradiction, everything follows.”

“Why should that be the case?”

“In the usual way that one defines implication as being that either the premise is false or the conclusion is true. But we can get around that using some paraconsistent system.”

“Paranormal system? It’s true that the mystics don’t seem to let ordinary logic get in the way of their pronouncements...”

“No, no. Paraconsistent, that is, mostly consistent, with some contradictions isolated that then aren’t allowed to bother the rest of the system.”

“Like antibodies who surround a pathogen?”

“Not really. There are different ways. One way is to use relevance logics.”

“Relevant logics? To listen to mathematicians, all logics are relevant.”

“No, only the Australians say ‘relevant logics’; we call them ‘relevance logics’. That means that a conclusion can only follow from a premise if the two are related.”

“Everything is related, no?”

“Depends on your model. But OK, you can say that they are significantly related. That is, implication was allowed only between two statements whose models are accessible to one another in a possible worlds semantics. A statement that is a contradiction in classical logic might not be in a relevance logic.”

“Sort of the expression, ‘what does that have to do with the price of tea in China?’”

“Indeed. You have to get rid of ‘premise implies conclusion’ as having the only criterion ‘either the premise is true or the conclusion false’. You have to add, ‘and the conclusion is a relevant implication of the premise.’ For example, in classical logic the implication of a false statement to any statement is a valid implication. But that means that ‘Paris is the capital of Italy’ implies ‘the price of tea today is two yuan per kilo’. This seems a bit silly. So the two should have something to do with one another.”

“How much? Perhaps if they both talk at least about one thing?”

“That is not enough, because we would like that Alice implying Bob who implies Charlie can result in Alice implying Charlie. But if each one has two unknowns, where Alice talks about tea and honey, Bob about bread and honey, Charlie about bread and butter, then Alice is no longer related to Charlie. Of course, many people argue in this way, and their listeners often don’t notice the shift, but that is not a very good practice.”

“OK, how about that the conclusion doesn’t talk about any unknown that is not talked about by the premise?”

“That sounds reasonable until one considers that from a valid premise –conclusion implication, one wants also the negation of that conclusion to imply the negation of that premise.”

“Ah, that’s called the contranegative or something like that.”

“You seem to have a knack for messing words up. It’s called a contrapositive, and it wouldn’t always work with your idea.”

“Why would you want that principle of contrary positive?”

“Contrapositive. For example, in proofs by contradiction.”

“How does that go?”

“I assume something is true, I deduce a contradiction. I use this principle to show that the original assumption is false.”

“I read that not everyone accepts that principle.”

“Why not?”

“I think they say that something is true only if you can directly prove it. So proving that you cannot prove the contrary of a sentence does not mean that you can prove the sentence itself. You already gave some examples of that when we were talking about having more than just true or false. So, in this case maybe my system of variables would work.”

“I believe it might still get into difficulties. You see...”

“OK, if you want the usual contrapositive rule, then make it so that all the unknowns are the same,” Bill tried.

“That might work, even though it seems awfully restrictive. Then, for example, that today is a rainy Wednesday no longer would imply that today is rainy.”

“Do you have another suggestion?”

“One could look at it from the point of view of models or, if you want, from the point of view of worlds. A starting point would be to polish up the usual idea that says that the set satisfying the conclusion must be a subset set of the set satisfying the premise. With infinite sets, you could let the intersection of the premise-satisfying set and the conclusion-satisfying set to not be completely the same as the conclusion-satisfying set, but to say that the set of things that are in the conclusion and not in the premise is an insignificant set. If you arrange that significant sets always intersect to give significant sets, then that would satisfy the problems of transitivity and the contrapositive argument, as well as allowing a rainy Wednesday to be rainy, but allow a few insignificant contradictions to be satisfied without making the theory explode.”

“Hm. Such a construction... has it been worked out?”

“Yes, they are called ‘Ultrafilters’.”

“Sounds like a brand name of a water purification system.”

“Actually, I think it is, but the logicians had the name first. I won’t give you all the details, but it forms the basis for creating sets, although you do have to take on an extra assumption as to what is possible. However, if the base theory isn’t contradictory, then this extra assumption would not add any contradiction, so I see no reason not to adopt the assumption so that we can manage things like this. There are difficulties with the idea of truth...”

“So the philosopher’s search for truth...”

“I hate that definition of philosophy. There is no truth with a capital T, except maybe tautologies. You can do a lot with just tautologies, but not the grand sweeping statements of the philosophers who .... oh, don’t get me started. In any case, there is no reason to throw out the baby with the bath by abandoning the idea of truth the schemata for the biosensors A particular model and theory can have a truth function in the theory, as long as you remember its restrictions. That is where the engineers who designed the Supervisor Project were too bull-headed to listen to me, and why I would like to go over to your company, where I figure, once you take over from your father, I will get an open ear.”

“Assuming we agree on the conditions. Speaking of babies and baths, I presume that despite all their problems, the biosensors your company is developing has some useful aspects. Can you supply the schemata for them?”

“They are under lock and key, but if you let me choose my own team, I can work out the details.”

“Your demands are going up, I see.”

“Only within reason.” Jutta grinned, and cut the connection.

No record of Weird_but_good ever having connected.

Chapter Three

June 7, 2021

This time, Bill had been prepared. He had programmed his computer to follow the retreating signal. He had information, but he did not know how to interpret it. After a while he could get no further; his hacking skills were too limited. He phoned Alonzo, a friend who was a better hacker. The friend tried for a few minutes, and then phoned Bill back.

“It’s a tough one.”

“Why?”

“It is a bit like an HIV virus; the program mutates, and my data are also transformed. It won’t go further unless either the program stops or I say I give up; but every time I give up I am given less access. It as if it lives in a video game. So the best strategy is to predict whether a given random program with given random data will stop or not: if so, then simulate it running – even if it is very long, I would simply jump to its end – if not, then give up one of my lives. I can’t know for sure, so I would act on probabilities.”

“Can you do that?”

“It’s not that hard, on the face of it. A program is just a sort of yes/no game. Each move has two possibilities, so the total number of programs is the number of subsets of that length. Suppose you want to figure out the probability for the program to fail on all the pieces of data except a particular one; on that one it succeeds. This probability is then one out of the number of subsets.”

“Assuming all the subsets have the same probability.”

“Good point. Another simplification comes from the fact that not all programs are suited for my computer, and I can get rid of redundancies, so really there are only certain strings I have to consider. Anyway, if I add up the probabilities for all such lengths, I get the probability that if I set the computer to start randomly generating strings of yes’s and no’s, then at some point it will come up with a finite one where my computer can stop. It looks easy, and there is an answer, so my computer will start adding.”

“But there can be an infinite number of things to add up.”

“You apparently slept through a whole semester at school. There are lots of techniques to adding up an infinite number of things. Like.... you know the joke about there being a chocolate cake two meters from you, and to get there, you have to jump half the distance to the cake. Your jumps take you along always at the same speed, you can keep hopping without pausing, and let’s assume that you can be really skinny.”

“Which is why you need the cake.”

“Hum. You would never get to the cake, because there would always be half of the distance of your last jump separating you from the cake.”

“But you would get close enough for practical purposes. I know the joke in a slightly different version. But given this forever, you would move the whole two meters. That is, you are assumed to be able to make an infinite number of jumps in one big jump. Sort of like imagining a picture from just a few dots. Optical illusions. You know. I think you used that example with your shrink.”

“That is illegal, you know.”

Alonzo ignored his remark.”So the length adds up to one. But if you only jumped a third of the way every time, even after an infinity of jumps you would only get one and a half meters. No cake. Remember that sort of thing?”

“I probably had it in school, among other forgotten things.”

“So, back to my computer’s task. It thinks, yeah, I can do that... and gets bogged down. It gets slower and slower, and there is no way that the computer will be able to finish the calculation.”

“Why bogged down? Why don’t you just get your computer to look at the program, that is each one of those series of yes’s and no’s, and your input data and decide whether or not it is going to stop or not? A Decider, one that would print out ‘Eureka’ if it gets an answer, and ‘no way, folks’ if it decides that it won’t stop and should give up.”

“Sounds reasonable, but can’t do it. The key here is that the data and the program are both represented as a bunch of yes’s and no’s. If there were such a Decider, you could program a function Nasty that would say ‘no way, folks’ for a string if the Decider would take this string as both program and data and come up with ‘no way, folks’.”

“Ah, so you would have the yes/no string, duplicate it, and use it as both program and data. Cannibalistic. And if the Decider would come up with ‘Eureka’?”

“Then Nasty would go on calculating forever and never stop.”

“OK, so what?”

“You don’t see? That is impossible, because if Decider comes up with ‘no way’, then Nasty also does, but that is saying that Decider does come up with an answer, which is that it won’t stop. You can’t have it both ways: either Decider decides it can come up with an answer, or not.”

“What if Decider comes up with ‘Eureka’?”

“Then Nasty doesn’t come up with an answer, but since Nasty is just Decider on a special case, this means that Decider also can’t come up with an answer, so it couldn’t have said ‘Eureka’. This is called the Halting Problem.”

“Try it with a different computer.”

“Each computer has its own number like this, called a Chaitin constant.”

“But this sounds like it would cover an infinite number of possibilities. We are working with finite machines, so they don’t handle infinite processes...”

“You’re not quite correct there. I keep telling people that infinities are often just ways of jumping to a new ability, just like we jumped to getting to two meters instead of trying to get there by taking half-jumps. So it’s just as within the realms of computer possibility as within human ability. But that’s not the point. I think that we could perhaps find some fault with the program, but it would take too many resources. Maybe you can think of another way, theoretically, and I can put it into practice. Until then, I give up.”

Bill got more frustrated, and went to bury himself in the books at the library. He knew that everything was online, but sitting at a computer did not generate the same atmosphere of being surrounded by physical books. Nonetheless, he took his laptop along too.

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June 14, 2021

Jutta contacted Bill again."I see you have found a temporary purpose."

"I haven't cracked it, if that's what you're on about."

"I know; I explained the situation to your psych."

"Wait. This only concerns computers. What would she want to know?"

"For two reasons, you're wrong. First, the same reason she would be interested in the problems of the search for truth as last time. Didn't you notice the similarity between your hacker friend's description of the problem and my description in our previous Skype session? They are very similar. Secondly, you are bound to get some frustration out of all this. Your only advantage is that you are working with real computers, and the proofs dealt with hypothetical ones. The real ones can have places to crack that aren't there with the hypothetical ones."

"OK, so you have me here. What do you want?"

"Have you considered my offer?"

"I want to be convinced that this was not a one-shot deal, your only idea in your life."

"I thought as much. Here is a related project, one that I have withheld from my present company. The company's goal is to produce self-contained goo that would be used in gene therapy, in which the gene was not in the initial goo but which would evolve in it. The goo will be called Gene-all."

"I guess I can see how that could be useful. Is it going out onto the mass market anytime soon?"

"No, a gene will be realized in some patients, and in others it won't, so the goo will be made with respect to each patient. Like a sentence being true in one model and false in another. But that is not too tragic; cheap individual testers already are being developed. Then, in the goo made for this patient, any gene that will evolve in the goo will be realized in that patient."

"So where's the problem?"

"When a patient's goo comes off the production line, one has to test whether the desired gene will evolve in it. Otherwise the pharma companies won't touch it. It is even possible that a gene would evolve that would have opposite effects in the patient. So the company spent a year developing such a test for goos."

"So it has passed the hurdle of the pharma companies?"

"Yes and no. What the pharma industry would really like is that the goo would have its own tester – for example, once the doctor takes the goo out of its vacuum packing and touches the organ into which the gene is to be implanted, the goo turns blue if the desired gene will be realized, and red if the opposite effect will be realized. Something like that."

"Um, with the kind of analogy you made in our last session, that evolution would be sort of like a proof of the sentence asserting that the gene would be realized."

"More or less, right. The researchers in the company think that they have done it, and are about to announce the product. If, however, Wheaton Industry challenges the product as soon as it comes out, then this will force my company to retreat, losing patients' confidence, patients that can turn to the pharmaceutical division of Wheaton Informatics."

"And you are offering me the weapon to challenge the product?"

"Bravo. If you were really clever, then from my previous argument about truth, you could figure out this challenge all by yourself, but we may not have the time for that. After all, you know that there is no universal truth tester, and this is similar."

"Actually, I am not sure where the difference between Gene-all and the Supervisor is."

“To put it in terms of truth and proof, the Supervisor was to be a total truth indicator in the theory, which is not possible, whereas Gene-all will be equivalent to a sentence that you will show can be neither proven nor disproven. So to rubbish the Supervisor product, we show that the Supervisor is trying to do something that cannot be done, and to rubbish Gene-all, we show that there will be an end product but Gene-all, which is a means for exhibiting the evolution, that is, for proving that the end product will be produced, will not be able to do it. Gene-all will also not be able to validly show that it will not be produced, of course, especially since it will be produced. It just cannot show it.”

“But you are not saying that Wheaton Informatics test whether the gene, once evolved, would be realized?”

“No, just to show that Gene-all will not always work. You just have to show that there will be genes which would be realized in a patient, but not respond to Gene-all.”

“But I don’t see how we can convincingly do that.”

“If you had spent more time studying the information I gave you last week than trying to break my firewall, you could have figured it out yourself. In fact, if you had worked on the firewall more yourself, and made the connection between the firewall and the information I gave you last week, rather than giving it to Alonzo, you might ....”

“OK, I admit that I didn’t. What was so important?”

“Remember Jutta’s Discovery?”

“Only the temerity to name it after yourself. I found where you stole it from. I looked in a beginning logic book in the library. It is called the Diagonal Lemma, and it says.... wait, I have my notes here, I wanted to challenge you on your plagiarism.... it says that, for most interesting systems, that have the mechanism for coding and for proving, then... let’s see, here’s the important part..... for any formula that talks about codes, there is a fixed point, a sentence which will have the same truth value as that formula talking about that sentence’s code.”

“Right, I even gave you a way to construct the sentence, although we did it in terms of genes. So, now do you get it?”

“If you can cheat by going from the logical to the biological, so can I. I presume, since your company has developed a self-testing goo, then the testing mechanism , however you test it, will have some substance which will be a stand-in for the actual gene under question. That is like the coding. Am I on the right track?”

“Keep going.”

“You implied that there is a mechanism that gives some indication whether the gene would be realized ---like turning red, when the substance represents a gene that will not be realized. According to your so-called discovery, there will be some gene, some sentence, that will be realized or not exactly as the color-mechanism would be realized or not upon reading that gene’s coding substance. Is this what you meant by applying your so-called discovery?”

“Yes, and I admit I plagiarized. But that shouldn’t bother you. Your activity on the Internet has been less than kosher. Would you like a transcript sent to ....”

“You wouldn’t do that!”

“No, I guess not. It won’t be necessary, if you realize that Wheaton Pharmaceuticals can reverse-engineer the goo, find the mechanism, construct a gene according to the method I just outlined, and present it to him. The gene will turn the wrong color! In this way you will get a gene that will be realized, but there is no way that the tester will be able to show it. Confidence in the tester will be smashed.”

“It’s a bit unfair, though, since the gene that we will produce as counter-example will be a very artificial one that will not show up in most real situations.”

“You are partly right. That gene will just be for show, but there are lots of genes that would suffer the same fate if they are indeed producing a goo that has such prediction and coding capabilities.”

“Can they?”

“I actually don’t know. It’s theoretically possible, so this is something we should not try to challenge. So, take them at their word and give them enough rope to hang themselves.”

“Let me just make sure I have this right in nice simple terms. Something might be true but not provably so, as long as you are restricted to the tools of the system.”

“Assuming the whole system does not try to do opposite things at the same time. If it could, anything would be possible.”

“Could their goo indicate at least whether it is consistent in this way?”

“No, they would run into a similar predicament. In other words, the sentence that there is no contradiction would also be like this gene we are going to produce as counter-example. It may be true, but it cannot be proven to be so. So, are you convinced of my genius?”

“And your skill at hacking and plagiarism. If and when I am in a position to do so, I’ll hire you.”

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October 17, 2021

Soon after the funeral of Bill’s father, Silicon Valley prepared to metaphorically bury Wheaton Informatics as well; the new CEO could not stop the downward spiral of the company. To everyone’s surprise, Bill invoked his option as majority stockholder after his father’s death to take over the CEO seat. He re-organized the company, spreading it in different parts of the world, changing the way different departments communicated. Because the company’s reputation was still suffering, Bill allowed minor projects to be taken on, the type which his Artificial Intelligence Department had snubbed their noses at before. Since many of the head executives were wary of such minor projects, Bill himself headed the first such meeting with his aides. He ordered them to go through all the orders, no matter how trivial they might seem. He sat back and supervised the discussions.

“The Texans want to know what to do about Chicken.”

“What do I know about poultry?”

“No, the game called ‘Chicken’.” That’s where two thrill seekers, usually macho types, head towards each other, and the first one to swerve is called a coward.

“We can’t re-route people’s hormones, can we?”

“No, but all the cars on the road are self-driving cars. The kids hack into the system, disable the driver, and convert the car to an old-fashioned one where you manually drive it.”

“That’s insane. What does this Transport Department want us to do about it?”

“They want us to tell the car manufacturers how to rig their engines so that the kids can’t break in. Every time their programmers put up some sort of protection, the kids get past it.”

“Their programmers are simply approaching it wrong. We need to put something in that will convince the kids that they have succeeded, let them get in the car, then the car would lock them in and call the police. Arrest for tampering....”

“Sounds straightforward.”

“It’s not. We need to change their mentality, something that doesn’t occur in one lesson. The best approach, I think, is to teach them to play games in a systematic fashion.”

“I’m not sure that will go over well with the teachers.”

“We have to teach them as well. The first thing is to convince them that they already do that. They play games in their heads every time they make any choice, they just don’t call it a game. They go through back-and-forth procedures that are the selection of which universe and which interpretation they want. There are a lot of such bluffing games which end up unhappy for all concerned. Your management department already uses Game Theory...”

“My management plays games?” Bill started wondering about firing staff.

“Hm, maybe you need to be taught first. It is a remarkably effective approach in management to consider the agents and their possibilities in a sort of game format. The players are usually formulas, or sometimes we start with a formula with lots of variables and the players are expressions like ‘for every variable like this’ or ‘there is something which will satisfy this variable’. This can go in two directions: one, by taking such turns one can see what theory is needed to get to a given model. Another direction is to find out what model will result with restrictions on the formulas. Sometimes we come up with situations which may seem counter-intuitive until one realizes that the situations are quite common. For example, among the factors that an agent has to take into consideration in forming her interpretation is the interpretation that the other players are making of her. This might seem to end up in an infinite regress, but it usually settles into some sort of fixed point. A natural one is the equilibrium where each of the players gets into a position from which any change would make it worse for her. The dream of a purely competitive society. But you can arrive at the unfortunate result that in following such a goal, the resulting equilibrium turns out paradoxically to be the worst possible situation for both players. This happens a lot, and argues for a modicum of cooperation. There are lots of variations on these; they’re called topological games.”

“ Topographical games? Is that like that game GeoCash?”

“No, ‘topological’, referring to giving a useful structure to sets.”

“Sets again? Oh, I remember. There was an article that talked about the similarity of a cup to a donut example. Put me off donuts for weeks.”

“That’s an example of forgetting some aspects of each one, the cup and the donut, to get them down to the same form. You can put it another way. Take a common model for both cup and donut, in such a way that the common model can be restricted down each one until you get the donut and the coffee cup to have the same truth values, and hence can would be equivalent under a common interpretation.”

“Why express it in such a complicated form?”

“It is more complicated only for simple situations like the game of Chicken or the other examples we discussed earlier. But your management deals with much more complicated situations, with lots of agents, motives, and continuing situations. Even rats learn. But in order to learn, you also have to learn to forget. So this extension and restriction is constant. We live in a world of shadows.”

“Don’t go mystical on me.”

“Me? Not a chance. The human neocortex does this sort of thing all the time in order to recognize patterns. You do it all the time when you find similarities, similes, metaphors, symmetries, that sort of thing. Then you put them into a single model, and invent relations. Bigots try to avoid doing this, unfortunately.”

“Neo had a surname?”

“What are you talking about?”

“You’re talking about Neo from the film Matrix, right?”

"No. In fact, all this is symmetry, where something doesn't change under some sort of transformation. A fixed point. A fixed point is something in a collection which doesn't change when some change is applied to everything in that collection."

"The more things change, the more they stay the same."

"They say that about politics, but you're right, we could use it here. In our possible worlds semantics, we can expand and restrict models, which of course are accompanied by changes in the respective theories.

"Isn't that the principle of 'deep learning' in artificial intelligence?

"Yes. We will use that to program the car to have a double personality. One of the personalities is a model in which the kids are able to disable the driver, and responds to the kids in this fashion. The other one is an extension of that model, in which the driver is only partially disabled; the part that is not, like the other half of a dolphin's brain, notifies the police. What else do we have? The airport?"

"Um, face recognition. Old hat: just pattern recognition."

"Right. Next?"

"Next, the Department of Education wants to make the citizens more helping. Get rid of the Bystander Effect, where people all stand around not helping someone in need because they all assume the other people around will do it."

"Get the Virtual Reality people in on making a participant be a victim, and get a special offer out to schools..."

The humdrum projects were Bill's bread and butter, but his secret project was both his hobby and an obsession to be controlled. One deviation from good business practice that he allowed himself to indulge in was to hire a personal friend that he had met on his travels, Jutta Fischer. Jutta was made head of a separate R&D department that was responsible directly to Bill. This raised some eyebrows, but as Bill made the company more and more successful, and the salaries higher and higher, no one complained at a few quirks of the boss.

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July 8, 2041

Although most people think of Internet as the height of globalization, the long history of unbridled piracy in Russia had led to a different culture for hackers. Despite the speed that one could fly from Moscow to Vladivostok, the distance of the Russian Far East to the capital led to even less control than elsewhere. Hence it was only natural that one of the largest group of hackers in the world was concentrated in that area.

Although the train or plane were the most obvious options for traveling between Khabarovsk and Vladivostok, Natasha Petrova took the third option to rent a car instead. It was more expensive, but the annual hacker's summer conference in Khabarovsk was worth a splurge. Natasha was glad that her vacation from work overlapped with her academic vacation, so she could relax from both work and academic pressures. Of course, it was called the Far East Association for the Protection of the Long-tailed Goral; for added anonymity, anyone applying would have to get past a powerful firewall to submit the application, ensuring that only hackers, learning about it by word of mouth, would be able to join.

With only about two hours of travel to go, she was jolted out of her reverie by the sight of someone lying by the side of the road. She ordered to car to stop, and she took over manual control to back up to where she had seen the body. The woman lying there was unconscious and tied up; she would obviously die if left there. To call an ambulance would end up involving the police, who cooperated with the mafia. It was a safe bet that someone from the mafia had

probably tied the woman up to begin with. Since the woman was apparently only bruised, Natasha saw no other option but to take this person back to her apartment.

That evening, the woman regained consciousness. But she could remember nothing of her past except that she was called Julia. She had no identity papers, and had no idea why anyone would want to kill her. Natasha assured Julia that she could stay at her apartment long enough to recover and to give Natasha time to forge the necessary documents to get her a visa and passage to Japan, perhaps best by boat from Nakhodka to remain inconspicuous. Julia cleaned herself up a bit so that Natasha could take her photo to use on documents; and they agreed on the name Julia Antonovna Volkova.

Natasha apologized that she would be outside of the apartment from early morning until early afternoon at her work, and that even when at home, she would not be much company because she would have to be working on her doctoral thesis. However, Julia was free to use anything in the apartment. Natasha had stocked up well on food before she had left for Khabarovsk, so that would not be a problem. She would tell the neighbors that Julia was a friend from Khabarovsk who had come to visit her.

The next day, Natasha came rather tired back from work, had a bite to eat with Julia, and opened one of her computers to start working on her thesis. But after an hour she slumped over to the couch and fell asleep. Two hours later, she woke up, and saw Julia at her computer.

"What are you doing, Julia?"

Julia looked over."Don't worry, Natasha, I backed up all your documents. I was just reading your thesis, and found it interesting. Not having anything to do, I looked up a few sources for you, and made a couple of suggestions."

"Oh, that was... thoughtful of you. Do you know anything about the topic?"

"I didn't before researching it while you were asleep. The female reproductive system seems complicated... you expressed it quite nicely when you compared it to, um...." Julia peered at the screen, "um...here it is. 'Any comparison between the male hormone system and the female one is like comparing the transportation in a three-hut village to the Moscow Metro.' But I will let you..." Julia started to get up.

"No, stay seated, Julia. Just click on the button in the lower left where it says 'Projector', will you?"

A moment later, they were both looking at the wall where the computer screen appeared."Here are some sources you overlooked," Julia pointed. Some in Russian, some in English, some in Chinese."

"I don't read Chinese, Julia."

"I do. I can help you translate them."

"Perhaps you were a translator or interpreter before you got mixed up in... whatever it was. But I see a file with Professor Popov's name on it. I thought I had everything he published. He's my doctoral supervisor."

"You did have everything he published. What you didn't have was his private communications and his private files."

"What? Julia, how did you..."

"If I overstepped the bounds, I can erase the files."

"No, no... it's just that... well, I have been trying to hack his files for months. You are full of surprises, Julia. Tell me, what are the recommendations you mentioned?"

“Ah. I noted that you were trying to describe the female reproductive system in more efficient manner than has been done until now. I suggested a mathematical formulation that allows you a clear set of first-order variables....”

“Whoa. Julia, I don’t know what your background is, but it clearly included more mathematics than mine. What are first-order variables?”

“Variables that stand for individuals, not sets or formulas.”

“I lost you on what that has to do with my research.”

“I regard the woman as a collection of substances, along with relationships between them, bodily functions and special collections. This, together with an assignment between them and the symbols of my theory, make up my model. The variables in my theory have domains.... you know what the domain of a variable is, Natasha? And a subset?”

“I got that far in school. The domain of a variable is the set of possible values of the variable, and a subset of a set has some of the elements of the original set, and no others. Do I have that right?”

“That’ll do. So, if you look at the collection of bodily substances as a set, the domain of a first order set is a subset of that collection.”

“Don’t all variables just allow only individuals of whatever you are talking about? That’s all we used in school.”

“Explicitly. But your teachers probably used phrases like ‘any formula like this has that property.’”

“Yes, although I never found such statements very useful.”

“That is because, in school, they tend to only pick out obvious higher-order statements. That’s also why female biology is still difficult to manage. You’ll find some counter-intuitive statements in my theory using variables which don’t stick to first order. But that’s if you go into the justification. But if you just submit the first-order theory, then the results you can get in managing the female reproductive system will probably be sufficient justification for your professor. I hope this can help.”

“Oh, my God, Julia. ...and all that while I napped? What... what did you do while I was at work?”

“I hope you are not angry that I used your other computers.”

“My computers? But... all the others are locked by password.”

“Oh, you said I could use anything, so I quickly got past your passwords.”

“What did you do?” Natasha automatically started to get upset at this breach of her privacy.

“Oh, I upset you. I am so sorry. I just thought... well, I know that my stay here will cost you money, so I added some money to your bank account. I guarantee you that my activity cannot be traced to your computer. If anyone tries to track the activity, it will appear that the source is a computer in the Kremlin.”

Natasha experienced such a mixture of emotions that she did not know what to say. She herself had engaged in minor illegal hacking, but not to such a level. By the evening, though, her feelings were dominated by her desire to learn more from Julia.

By the time Natasha saw Julia off on a boat two weeks later, the two had struck up a friendship that they vowed to keep up via Skype. Neither one of them could foresee what this friendship would lead to.

Chapter Four

May 6, 2044

It had been 23 years since Bill had taken over the helm at Wheaton Informatics. The firm had grown many times in those years, but throughout that time Bill had always insisted that he be instructed on the basic principles of every new venture. When the Industry bought Steinway, Bill took piano lessons. When Wheaton bought Boeing, Bill took flying lessons. No one was quite sure why he took origami lessons, but the expenditure was approved without question. When the request came through to organize lessons in basic modern physics, this was perhaps the least surprising of all at first. Only when they discovered the reason were there a few eyebrows raised.

Andy waited for Bill in the auditorium. Andy was a programmer in Wheaton Informatics whose personal hobby was to investigate logical principles underlying physics, even though his exposure to physics was mostly autodidactic. He had been picked to brief Bill on the basis of modern physics. He was told that Bill was preparing to give a series of talks, and Bill had to have a basic idea of what he was talking about, even though he would be given notes, have a teleprompter, and a microphone to give him any answers to questions from the audience. The audience, Andy was told, would be a lay audience, but more details were not in the memo.

This whole idea irked Andy's personal ethics. He maintained a personal dislike of the ubiquitous practice of people who use their prominence in one field to gain credibility when they made pronouncements in another field. Wolfgang Pauli was a well-known example, but Andy could think of several others still alive. However, he did not want to bite the hand that fed him, so he swallowed his objections.

Bill came to the meeting with Andy, with a device in hand."Hi, Andy. I brought something we can use. Here, we place these devices strategically around the audience. From various directions, precisely aimed lasers shown on the throat of all those in the audience, giving a feedback to create sound waves which precisely interfered with any sound they make. Destructive interference. I will be able to speak and be heard, however."

"Bill, that's a teacher's dream, but this is a cultured audience. Put it away before just the sight of it gets some lawyer salivating at a possible lawsuit. Just take these index cards. They're your speech."

"You don't trust me to do this alone?"

"Tell me, Bill. How much mathematics and physics have you had?"

"Well, I learned the Pythagorean Theorem and the rough idea of a function before I my family had to flee. Since then I picked up a couple of things from newspapers."

Andy knew that Bill's real name was Melham, and he obviously had a Middle Eastern look about him. He also knew that of the interminable conflicts there. But it was legendary in the company that Bill's mood turned sour whenever someone tried to ask about Bill's past, so Andy decided not to press him for details. Instead, he said, "I will give you a lay person's run down on the principles that you will be talking about."

"You said that everything was on the note cards."

"Yes, but the audience will be allowed to ask questions. I will also be listening, and there will be a teleprompter in front of you that you can look at. As well, you will be wearing an earpiece that will allow me to tell you answers. But if you don't have any clue as to the general ideas, your delivery style will give it away to the audience. Therefore, I need to give you a crash course. But first, Bill, a question. I understand that the company wants to boost its image in having its head give these talks. But I was told that I was personally selected to brief you. Why me? There are more qualified people than me in physics and mathematics."

“First, of course, you are my employee with a contract that I can easily terminate, so that I have more assurance that you will not blab to the press about this pretence. But also, I find that a lot of articles and books supposedly for the lay person are written by specialists who have forgotten, or never knew, what a lay person really is. I know that you used to be a school teacher, and that you learned this stuff late in life. So I find you to be the right person to explain it to me.”

“OK, let’s start with the basic ideas today. Your talk is about unusual logics that are nonetheless applicable. For this, we will take examples from the physical world. We start with the idea of spacetime, which you can briefly introduce.”

“And why spacetime?”

“It’s like urelements...”

“Have you been speaking to Dr. Fischer? If so, I will have her hide for breaking doctor-patient confidentiality.”

“No, Bill. You yourself have recounted your dream at a party. You were quite loquacious about form giving rise to content, I think you expressed it as. So too here. Even space-time can be expressed as any four coordinates that obey certain properties. But it is very convenient to focus at first on a single point of spacetime. There are different aspects that you can measure at such a point. Each aspect has lots of possible values. For example, you might think of the way to identify humans: one aspect is hair color, and red as one of the possible values. Each combination, an aspect and a value, has a certain truth value for each point in spacetime. Each such triple: spacetime point, aspect and value, is represented by a circle. Some of the circles might be not altogether independent of one another: for example, skin color and hair color. Therefore human identity is often based on genes today.”

“Right. Wheaton Informatics is doing a lot of research on ....”

“I know, Bill. Let’s try to stick to the matter at hand, shall we? The total amount of information connected with a point or region in spacetime is called a state. It corresponds to a sentence. But it is a bit complex to try to figure out the entire sentence, so we break up the sentence into a combination of smaller sentences. We try to isolate one particular property, such as spin, charge, mass, and so forth. We can view it as a function from one part of the state to the rest of the states in spacetime. That is, one property, or aspect of the state, can usually be measured and put into numbers. Which values they are depend on the model used to satisfy it. Put another way, a sentence with unknown values for each one of the aspects; this would be satisfied by a measurement if –and this is a big ‘if’-- you could measure all the values at once. Such a measurement would be the assigning a value to each such unknown quantity. That is, the possible measurements for that state. But we can’t measure all of them at once, so we concentrate on one or two aspects at a time. For example, when you want to describe a person or to change her looks, you might first look at her head to see her hair color, and perhaps dye it. While doing that you don’t concentrate too much on her feet. Each aspect is associated with an operator, and the whole state is associated with an arrow.”

“So if I follow up on your imagery, the make-up of the arrow is only known through the actions of the operator on the individual aspects, just as your hair color is only known by the observations of people around you or with a mirror.”

“Well said. In fact, we could do everything starting only from the operators, if you know the model. For example, the operator will contain in itself all the information of the possible measurements in a given context, which means that you can apply the operator in such a way as to get a weighted average of the possible values, given you a value that you would roughly

expect. But it is easier to use states and so forth. Each of the aspects will be associated with a sphere, with the arrow beginning at the center of the sphere."

"That is, if I get what you are saying, the operator acts on the entire state, but in general we concentrate on the effect of the operator on selected parts of the state."

"Right. This operator, when applied to the state, that is to the sphere, makes the arrow move, or equivalently, makes the sphere move. I will keep mostly to the arrow-moving interpretation. There will be some shadow arrows that are rotated and enlarged in a plane. If you consider only the rotation, then each circle which a shadow arrow makes is associated with the transformation: the rotation and size-change. The circle then is associated with the value of the transformation. The ones that are only size-changes are the ones associated with possible measurements, and we shall ignore the others. These circles represent a sort of fixed point when you consider all the arrows of different sizes to be equivalent."

"Wait. Sorry for interrupting, but remind me what a fixed point is. I was told once, but I was told lots of things, and I don't remember all of them. My mind is not a Word document where I can just select 'Find'."

"No problem, ask as often as is necessary. A fixed point is something that doesn't change under a given transformation...in this case, the orientation of the shadow arrow inside the circle under the transformation of the moving of the arrow."

"OK, but..."

"The circle is also associated with the truth value given by the percentage of the radius of the original shadow. In sum, each operator will be associated with a set of circles; each one of these circles is associated with a possible measurement value and the truth value of that possible measurement value. These circles are the cross sections of the sphere."

"Seems a bit circular to me, if you'll pardon the pun. You started with the sphere, got to the cross-sections, and now you are saying that the cross-sections define the sphere."

"Bill, I think your attention is flagging. You need to get some sleep; I can see that you're tired. When we meet tomorrow, I hope this will be clearer. Same time tomorrow?"

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May 4, 2044

In the previous three years, Julia's career had shot upward. She had worked her way into a Tokyo firm which then sent her to their branch in Los Angeles. There she attended classes in the evening to keep up with the changes in Information Science. On the side, she teamed up with Natasha, communicating through the Internet. They became known to the world collectively only as"Kim the Ethical Hacker", since they refused jobs that they deemed harmful to decent people."Kim" had been picked for fun. This did not stop them from going beyond the law sometimes, so that they received and dispatched jobs through a website that no other hacker in the world had managed to break into.

One evening, Natasha called Julia.

"Hi, Julia. Are you up for a code-bashing session?"

"No, I'm exhausted. You have no idea what sort of work there is to get a degree at the institute."

"I never understood why you even try. We make enough money with hacking..."

"We've gone over this. The methods we have been using in the past are becoming outmoded. We can't rely on standard software anymore. We have to be a step ahead, designing our own tools."

"Yea, but we are both good programmers."

"Up till now. But now analyzing the programs you are trying to hack into gets more complicated. The old programs fit Searle's description to some extent of 'symbol-manipulators.'"

"But let's not get into that right now. Tell me, do we have any jobs pending for Kim?"

"What jobs do we have not yet completed?"

"Um, there's the one for the Texas Department of Transport, another one for the Danish Department of Education, another one from the Moscow airport authority, and one from an anonymous client."

"The anonymous one pays very handsomely, but the task is the hardest we've had. Bugging the Kremlin was easy in comparison. But there's no penalty for failing except for our expenses, so we should go for it."

"Don't build up the suspense, and tell me."

"Someone wants us to bug the *Scientific American* Cruise in September, and prevent other hackers from doing so."

"What's that?"

"It's a week or so on a cruise boat which goes somewhere nice, but instead of dances and stuff, there are lectures on interesting topics in science and mathematics."

"Sounds expensive."

"That's what is so special about this one. You don't sign up; you are invited, and the cruise is fully paid by Wheaton Informatics. Wheaton himself will be one of the main speakers."

"Wheaton, the CEO of the biggest informatics company in the world? Interesting, but where's the challenge? We bribe someone to put a few microphones in the right places, and..."

"No, I already tried. Unbribeable. But there is a simpler way. You go on the cruise. It starts from San Francisco anyway."

"Me? I need to be invited."

"That, dear girl, is not difficult. I foresaw this. A few key strokes, and you will be towards the top of the potential list of invitees. It will simply push a couple of people further down on the list."

"OK, that is solved."

"Not quite. I hacked into the least protected *Scientific American* branches and got a draft of the invitation." She handed her a sheet, which read,

"Dear .....

"We are happy to invite you to a special six-week-long *Scientific American*'s 2045 all-expenses-paid Insight Cruise, this year sponsored by Wheaton Informatics. The theme for this cruise will be 'Quirky logic is real'.

"We will sail from San Francisco on January 11, stop for a day in Hawaii on January 18 to 19, and have an interlude in Tahiti from February 1 to 6. From there the cruise will return to San Francisco, where it is previewed to arrive on February 26. Participants are asked to make allowances for delays in the cruise of up to three days, and not make any appointments until March.

"There is an unusual requirement for this year's cruise that the participants must take into consideration before accepting. There will be a complete ban on all transmitting devices, including telephones, during the part of the cruise up until Tahiti. For emergencies, there will be a fully equipped hospital on board with qualified personnel, and a helicopter will be standing by at nearby ports to transport anyone requiring more extensive medical attention. It is the

responsibility of each participant to make all necessary arrangements for being incommunicado for the corresponding period.

"If you would like to join us, please fill in the attached confidential form to inform us of your dietary and medical needs by October 3, 2044. If you have not responded by that date, we shall give this opportunity to another person.

"We hope you enjoy the cruise.

Scientific American and Wheaton Informatics."

Julia looked at Natasha. "That's going to be tough. But I don't have time to get the doctor's certificate."

"Julia, I never understood your attitude towards doctors."

"It's called iatrophobia."

"Giving it a name does not explain it."

"Consider it an axiom. I don't ask you to justify it, just to accept it.

Natasha sighed, and said, "All right, Julia. Give me about ten minutes." Natasha turned to her computer and started typing. Julia grabbed her latest novel, and sank into the couch. In less than the requested ten minutes, Julia's reading was interrupted by the sound of Natasha's printer.

"Here," Natasha said, handing Julia the print-out."A forged medical certificate, complete with stamp and signature. They never check these things."

"But I still need an invitation."

"Oh, that's a technicality. I've already hacked around a bit, and you will be one of the first to be invited. But maybe you should go to a doctor for your iatrophobia. Julia's looked at Natasha for a moment while a smile widened on her face; she then broke out laughing, leaving Natasha to figure out how silly her circular suggestion sounded.

Natasha fought back a smile as she tried to defend her position. She tried to convince Julia that some circularity was even useful if one could work out that it was the shadow of a spiral. Then you could replace the circle by a spiral. They discussed examples in which no one even bothered to try. Poor computer programs created loops, religious people told you that their holy book contains the truth because their holy book says that it does, a simplistic idea of sets could end you up by having the set of all sets which often leads to paradox, and so forth. This got them into the relations of self-reference to the self of psychology, and how self-reference was a kind of fixed point, which one could formulate as a kind of symmetry, and of course symmetry was nice. This led them to hours of discussion at nights, trying to formulate axioms that allowed for a useful symmetry without ending up with the pernicious symmetries.

After one such session, Natasha remarked that perhaps this method of working backwards from a theory to its axioms might be useful for Julia to find out about her past. Julia told Natasha with a sad sigh that she had tried, but so far had come up only with the sorts of axioms that at one time in the human evolutionary past had some sort of survival or reproductive use. Natasha consoled her, saying that most people never even got that far in their analysis.

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May 5, 2044

Silicon Valley was a place where secrets were meant to be guarded and rarely were. Andy discreetly asked around the company to find out what talk Bill was scheduled for. Even his personal secretary found that he had no talk scheduled. The only time in the next few months that was unaccounted for was Bill's month-long holiday in June. He had booked a cruise, but he had done that on his own. He had not yet made any appointments for after the holidays.

Andy began to suspect that Bill was simply pursuing his own personal interests on the company's time and payroll. On one side, it was not easy to hold a CEO of a big firm accountable for such a relatively small expense; the loss usually had to be in the billions before anyone paid much attention. Nonetheless, it made Andy uneasy. He would continue to investigate, but until he found out anything, he had little choice but to continue with the lessons.

Bill showed up punctually for the second lesson. Andy said, "Last time, you said we started with the idea of the sphere which was the stage where everything with the states was acted out. However, if you had been paying attention," Andy said with relish, switching the hierarchy from boss to worker to student and teacher, "You would have noticed that I started with the idea of information. I did not elaborate on it, but everything is based on the idea, so perhaps I should check that you know what information is."

"Um, news? Knowledge?"

"No. Roughly, it is that which distinguishes one thing from another. The word gets to be overloaded, referring to specific instances, or that which holds the information, or a part of that information, or its pattern, or a representation, or an interpretation, or the subject of the information, or the amount of the information, or..."

"I get the idea. So let's go on, leaving the idea of information intuitive, shall we? What good would it do to get into too many details?"

"At least that would have allowed me to have started this out with the operators."

"So perhaps you should explain operators a bit more if they are that important."

"Right you are, Bill. The operator is taking the sentence represented by the state, usually saying 'this possibility or that possibility or that possibility'. The operator it together with other sentences of the operator, saying 'this sentence and that sentence and that sentence...' to see if the result is still true, that is whether reality still satisfies it. Now and then, the operator sends sentences so that 'this and that' for most of the values form a falsity in the model, so that only one of the values remains possible. That is, the arrow gets in a position so that the truth values at all but one of the circles is totally false, but at that circle is totally true. That is, the probability for all other values becomes zero, and the probability for one value becomes certain – although only for that point in space-time, or at least for an interval long enough for it to be measured. The interaction of the operators are two way, though: that is, this will affect not only the truth values for the point or interval measured, but also the measuring apparatus, in case anyone measured the measuring apparatus. One says that the information is therefore transferred, and the amount of information remains the same when one takes a global view. They used to think the information maybe got lost because they were concentrating only on the measured object."

"Can you know the state so well?"

"Ah, not always, no. These are the basic, pure states that cannot be reduced to other states. However, realistically, it is possible that we don't know which state is there, but we have an idea which ones it could be, and the probabilities of each one."

"But then you can work backwards to figure out the pure states and their probabilities, no?"

"The tricky thing is that different mixtures of pure states can end up with the same mixed states. So we tend to be stuck with the mixed states. This ambiguous situation is what we usually end up referring to as the state of the region in space-time." .

"OK. So far it seems more or less clear, although I am still leagues away from seeing what good the construction is."

"Let's look at those circles again. They will be two-dimensional cross-sections of the spheres, each of which represents an aspect."

“Of course cross-sections of a sphere will be two-dimensional.”

“Not if the sphere has more than three dimensions. A cross section just means that you take at least one dimension and eliminate it by taking its proj... its shadows into other dimensions. So you can have a cross section of any number of dimensions less than the dimensions of your main object.”

“You surprise me.”

“Good. Live a hundred years, learn a hundred years, say the Russians. So as I was saying, we need more dimensions to indicate two aspects being independent. You do this by making the circles perpendicular to one another.”

“How can you get circles to be perpendicular to one another? Lines are perpendicular to one another.”

“One circle is perpendicular to another if all the lines in the first circle are perpendicular to all the lines in the other circle, and vice versa.”

“Ah, would that be, like, say slicing the earth at the equator, and slicing it from pole to pole?”

“No. Look at what you would have. A lot of lines in one wouldn’t be perpendicular to anything in the other. No, you would need four dimensions to do this for two circles. So the spheres will usually have more dimensions than the usual spatial three. Maybe four, maybe eight, maybe infinite.”

“Whoa. You just struck a shoal, captain. The ship won’t go any further in these waters. How can you get more dimensions? Are you talking about the four dimensions of spacetime, or the fancy theories which says that spacetime has eleven dimensions, or what?”

“No. Human imagination fails, because you are trying to imagine a sphere that you can see with your eyes. The surface of a sphere is the collection of points which all have the same distance from a given point. The ball we will be talking about, a solid sphere, is the collection of points which are at most a certain distance from the center. But distance does not have to be spatial distance. For example, computer people talk about distances between two sets of data.”

“So maybe I do not know what distance is.”

“Perhaps not. I shall give you a bit of homework.” Andy’s past as a teacher kicked in.”If you just look up ‘distance’, you are likely to be given the definition which really only refers to the everyday concept. So perhaps you should look up the more general concept, ‘metric’.”

“As in ‘the metric system’, with meters, grams, Celsius? I used that most of my life. This old Imperial system that you Americans stubbornly hang on to....”

“No, no. ‘Metric’ as in ‘means to measure something’. But you are trying to make me do your homework for you. Find the concept of metric for which distance is a special case, and then find out what a distance function is. Have your secretary arrange our next session.”

Chapter Five

May 6, 2044

In one of the most disputed little pieces of rock in the world, called by some Israel and others Palestine, the wars that occurred with unnerving regularity left their share of orphans. Moishe Cohen was a young rabbi on the staff of the Zion Orphanage in Jerusalem. He was given three duties: to oversee requests for donations, to accompany orphans on all field trips, and to check arrangements for the medical needs of the orphans. This last duty had become heavier recently, as a bomb had exploded into the nursery, leaving four of the youngest orphans, barely out of their mother’s wombs, with permanent brain damage. The same fate had been suffered by the

girl's orphanage, Girl's Town, leaving four young girls with brain damage. He worked together with the Rabbi from Girl's Town to look around for a means to launch an appeal for help, but the big newspapers were too expensive, and the small newspapers had too small an audience.

Walking down the corridor, Moishe noticed Joseph, one of the older boys, with his eyes glued to his laptop screen. Moishe knew that the orphanage had a strong filter against inappropriate sites, but he was always worried that this filter was not enough. Moishe sat down next to Joseph and asked him what was so engaging. Joseph said, enthusiastically, that there was a New York-based newspaper which published articles about religious foundational issues in modern terms. Moishe peered at the site, written in Hebrew. Moishe asked Joseph to tell him about it.

"The article says that the accusation against fundamentalism, that it cared so much about the letter of the law it often went against the spirit of the law, was silly, because the only way the theory could come into conflict with the interpretation was if the interpretation was not a model." Moishe let Joseph explain these concepts to him, then asked, "What sort of newspaper is it?"

Joseph said, "It is an international online paper called *The Foundation*. You can submit articles in any language, and they are translated into any written language you want to name. This article, for example, was written originally in the Tamil language."

"What, the paper hires so many translators?"

"No, it uses an online translator."

"Those are usually clumsy, no?"

"No, this uses Wheaton's Online Translator. No one knows how Bill Wheaton did it, but his translations are like a good human translator's. That kicked Google Translate, Babylon, and all the rest out of the running."

He asked Joseph, "Foundational issues, you say? Of Judaism, I presume."

"No. It's open to all those who defend a literal interpretation of their holy book, as long as they keep their articles to ideas which have broad support among a wide spectrum of fundamentalists. They figure that it is like the many mathematical theories which share certain basic rules like how premises fit together to give a conclusion, but nonetheless differ quite a bit in the details. So too the paper searches out issues that fundamentalists can agree upon, or at least find a common basis for discussion without anyone getting too upset."

Moishe did not like the term "fundamentalist", but he knew that it was common, so he let that pass. He understood that even a negative or positive connotation made for differences in models, and was amazed at how the artificial intelligence designers had managed to make their robots understand connotations like that.

"But is that possible?"

"Of course, Rabbi. Many of the kosher laws and the halal laws are similar. Or more extreme, don't you remember the story how, around the turn of the century, some rabbis went to meet Hamas, the sworn enemy of Israel, because both Hamas and the rabbis rejected the idea of the existence of the state of Israel?"

"For different reasons."

"That's right, but this shows that two theories which produce opposing models can nonetheless have some overlap. After all, there were Russian neo-Nazis who admired the man who was responsible for wiping out a quarter of the population of their country."

"I'm not sure that's overlap. I think that is the kind of rationalization that psychologists call 'cognitive dissonance'," Moishe remarked.

"And when two people reason together, the same thing, no?"Joseph asked.

"You mean when two people talk at cross purposes? When two points of view will be using different interpretation functions, so that the words can have different meanings, although the two speakers don't realize it. Think of words like 'love', or 'freedom'..."

"Or 'inappropriate'", mumbled Joseph.

"Hm?"

"Nothing, Rabbi. But I think the editor is not completely realistic."

"Oh, why is that?"

"His newspaper has become very popular. It beat out the *Wall Street Journal* last year. Yet he has not raised either the subscription price or the price of the ads you put in there since he started it. By the way, I checked a bit; the editor-in-chief appears to be Jewish."

That sparked Moishe's interest. With Joseph's help to navigate Wheaton's Translator, he sent his appeal for help with eight brain damaged orphans as an advertisement to be published in *The Foundation*. His first draft included that it was one of the 613 Commandments to lend to the poor and desperate, and not to press them to pay it back if you know they don't have it. If these two sentences were among the Commandments, then also their conjunction, joining them by 'and' was a commandment, just as in any good set of truth values. But Joseph censored this, saying that no one but Jews cared about the 613 Commandments. At first this rankled Moishe, because it had been one of his goals to fulfill as many of them as possible. But then, understanding that Joseph was right, Moishe drafted a text that would be wider reaching.

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May 6, 2044

Andy's attempts to find out about Bill's intentions had run into a dead end. He decided to try another tack. He needed to look in Bill's private affairs. He would start with his office. First, the only time that he would be sure that Bill would not be there would be while Bill was with him. Secondly, the only person who had the kind of access to do so, and also the person who would notice if anybody else tried to get this access, was George, Bill's personal secretary. He called George, and invited him to lunch at a nearby restaurant, which made a nice change from the company canteen.

"Hi, George. How do you like working for Mr. Wheaton?"

"Fine," said George, noncommittally as he picked up a menu."

"Tell me, George, what do you think of when you hear about a computer and a robot?"

"From the names, the computer thinks, the robot acts."

"Today we don't make this distinction. Thinking and acting are usually put together. George, didn't you ever read how this company grew so rapidly?"

"Um, it started off big, and then Mr. Wheaton improved the translation engine..."

"Right. But in order to do that, he came up with a completely different idea. Didn't you ever read the technical specs?"

"Not carefully. I'm a secretary, not an engineer."

"I'll put it simply. Each translator had a sort of controlled split personality, with each personality having access to the other one. A translator will interpret a text together with the axioms appropriate to that text and her experience to construct a structure of possible worlds. When the moment to translate comes, the axioms of the other language are used to interpret the actual model, the meaning of the text. This then becomes the translation. That is very rough, of course, but that gives you the idea. The axioms of the language must be tempered by experience in the culture to which the language belongs, which is why the firm of Wheaton Informatics sends out lots of its android translators to live in different parts of the world."

“But they are not human. How...”

“Legality? The spy films use forged passports. We select a much weaker link in the chain: birth certificates. They are forged to get passports, and these passports are used to get visas. But the technology used allows us to also take.... let’s take Bill as an example. Since our computer is a self-learning one...”

“They used to call them neural networks before everything except calculators went that way, didn’t they?”

“That was the most efficient way, if that’s what you mean. So, anyway, part of its learning capacity is to be able to extrapolate from Bill’s behavior, check to see if its extrapolations fit its further observations, and so forth...”

“The scientific method, basically.”

“Yes, you could say that. Anyway, once it is pretty sure of itself, it then extrapolates to Bill’s behavior outside of the data it has observed. Imagine Bill as a computer. He has a program, which we call his theory, and his actions are the interpretation, or model, of this program. So far, so good?”

George nodded, with a worried look.

“Good. The model-theory relationship is one that can be put down in symbols, and itself made into part of another theory. This new theory can be programmed into another computer, such as a human, and there can be actions based on this theory; a new model. This is what you do when you think about the motives of someone else. For example, you may know that Bill has a worldwide reputation of being the CEO who is the most accessible to the public of all the big firms.”

“Yea, the magazine *People* wrote that the other CEO’s are envious, and don’t know how he manages.”

“The secret is easy. We have a computer which that knows Bill’s style, and it is this computer that answers all the telephone calls, writes all the emails, Facebook posts, Twitter tweets, and personal snail mail.”

“How come no one has published that fact?”

“A few years ago we hired some professionals to post this theory in such a way that, from others on our payroll, it could be quickly made to look ridiculous. Since the public underestimates the capability of a computer to master such things as emotions, the myth of the CEO who doesn’t sleep is maintained. In fact, even for internal decisions we sometimes consult the computer. The quality of our coded version of Bill frees Bill from being bothered about every little technicality. Of course, we spot check on the correctness of our coded Bill, just like you check your concept of someone else... unless you have some kind of prejudice. Are you racist, sexist, homophobe, or anything like that?”

“Me? Of course not. Um, this lunch is on your personal account, right?”

“Sure, George. Feel free to order what you like. And you do not have any affairs that you are not telling your wife about?”

“I’ll have then the roast beef. Is this a lunch or an inquisition?”

“Calm down, this is not official. I just thought you should know that a rather meaty file has been building up on you in my department.”

“Your department? You are in charge of programming, not a moral police.”

“Precisely. You see, you know that everything you do inside the company is recorded.”

“Of course, that was part of my contract. But if my private life is being recorded, I’ll have the company in court...”

"No, no. But....ah, here's the waiter at last."

Andy ordered a modest meal, and George told the waiter, "I'll have the Chef's salad, the *soupe du jour*, and *Lauquen Artes* Mineral Water, please."

Andy said in surprise, "What, no roast beef?" as the waiter retreated to the kitchen.

"I am losing my appetite with this conversation. If there is no supervision of my personal life, then what..."

"The point I was making is that making our Big Brother for the employees is trivial compared to this. So we also have a file on you, and a computer which models your behavior."

"But its conclusions remain confidential."

"True. And this conversation in the restaurant has not taken place."

"Is that why you invited me to a restaurant outside of the company cameras and microphones?"

"You catch on fast, George. Here's your food. Enjoy your meal."

Andy waited until the waiter had retreated to continue. The company, of course, would never follow this up. But that does not stop me from checking on you myself. You seem to have links with a Miss Lawson, and you make anonymous contributions to a society in Russia that sends out hooligans to..."

"Hang on, where is all this leading? Blackmail? My salary isn't..."

"No, I don't want your salary. All I want is for you to look around Mr. Wheaton's office for anything that will indicate why he is taking my time for learning physics. while he is taking my lesson. It is for my own personal curiosity. I want to know whether I am wasting my time."

"If he found out..."

"He will not find out. I can arrange for the surveillance cameras to suffer a temporary blackout tomorrow. You will be able to continue your visits to Miss Lawson, although I would advise you to re-direct your charitable contributions."

"That's all?"

"That's all."

"That's a relief. I think my appetite is returning. I believe I will have some of that chocolate mousse."

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May 6 – 13, 2044

Although Arkansas had finally discarded the challenges to the theory of evolution from school at the beginning of this century, it was still possible to voice one's fundamentalist views about the origin of life at a party or at one's work place without meeting general derision. Not all fundamentalist ideas shared this fortune, however. Chaim and Daniela were both technicians for a small firm in Little Rock, but they also both were followers of the Gematria, the numerological part of the practice of the Kabbalah. They had submitted articles to Tal's newspaper under pen names, since they realized that their status in society might suffer if their belief were made known. But their efforts to publish had been in vain; *The Foundation* had rejected their article.

Chaim and Daniela had, again under their pen names, written a protest to the journal. They received a polite reply inviting them to come plead their case in front of the board of editors, and a form to fill in. Since neither of the siblings could get sufficient time off work to make the journey to New York, they asked a fellow practitioner of the Gematria who lived in New York, Avner Levi, to represent them.

The evening after the hearing, Avner called Chaim and Daniela on Skype.

"How did it go?" Daniela asked.

“Not well.”

“What happened?”

“I presented the many patterns and associations that are found in the Torah using Gematria.”

“And...” asked Chaim?

“I think I did a good job. But then the editor-in-chief, a Mr. Kishon, said that he would like to explain why the article had been rejected. The other editors let him do this.”

“What could he have said that did not indicate a blatant discrimination?”

“He didn’t actually testify. He called a mathematician up to do his dirty work for him.”

“A mathematician? But you could have presented the mathematics behind the correlation used to show how systematic it is. The formula is even in Wikipedia.”

“Yes, I know. But this was some other kind of mathematics that I am not familiar with. The mathematician quoted something called Ramsey’s Theorem.”

“I have heard of that in connection with the party puzzle,” interjected Daniela.

“The party puzzle?” Avner asked. Chaim also looked quizzically at her.

“Um, it puts the fictional situation of a party. A randomly picked pair of people there will either know each other, or they won’t.”

“That’s obvious. Is that the puzzle?”

“Hang on. The question is then to find the smallest number of people that need to be at the party so that there will be three people each of whom knows the other two, or three people none of whom know either of the other two.”

Chaim and Avner had a long friendly competition at finding puzzle solutions, so each one quickly started making sketches on paper to see who would get to the answer first. Daniela sighed, and said “Look, you can compete with each other later. I want to get to bed. The answer is six.”

“Chaim looked at her.”

“Party pooper.”

“You can try the similar puzzle for four people instead of three, if you want, after I go to bed. But I want to hear the end of Avner’s account.”

Chaim put away his pencil and said, “OK, Avner. What does this have to do with our case?”

“It appears that this puzzle is only a small application of the theorem. The fuller application, according to the mathematician, is that, given any finite set of data, a pattern can be found amongst them.”

“So what?” Chaim objected.

“Don’t you get it, Chaim?” Daniela asked, rhetorically.”The mathematician is basically saying that the patterns we found using Gematria were nothing special, that patterns could have also been found throwing dice or something.”

“What?” Chaim exploded.”That’s absurd!”

“I thought so too, but the mathematician claimed to have a proof of the theorem. So the board of editors ruled against you. Sorry, I couldn’t do anything.”

After signing out of Skype, Chaim paced up and down the room in a fury.

“This Cohen person has to be Haman in disguise.”

Daniela knew this to be an extremely harsh judgment, comparing the Jewish editor to one of the most reviled figures in Jewish history. She was also furious, but was not sure what Chaim meant to do about it. She asked him, and he replied, “If this Kishon suffered a major health problem, then that would be God’s will.” He silently took Daniela’s laptop from her, sat down

with it, typed in a few keywords, and then pointed to the screen."See, Daniela. It is written in the Torah. We must be the tools of the All High. Kishon must be suppressed."

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May 6, 2044

Back at Wheaton Informatics, Bills lessons continued. This time he came with his tablet. Andy asked him, "So, did you do your homework?"

Bill didn't seem to notice Andy's schoolteacher tone, and just read from his tablet,"A distance is a positive numerical result of comparing two points, as long as the comparison puts the distance from one point to itself as zero, the order of the two points is irrelevant, and figuring out the value by adding distances from the two points to an intermediary point does not diminish the value."

"OK, now in your own words, please. Don't read from your tablet."

"Fair enough. Otherwise put, the distance between two points is the shortest distance along any path. Physical distance that you are used to is of course one example, but by far not the only one."

"Ah. Well, it still is tricky to visualize."

"Don't try. The sphere is not in space-time; it indicates properties of that point in space time and its immediate neighborhood."

"Why should it have so many dimensions?"

"For several reasons. A dimension is a set which is independent of the other dimensions, and we will be talking about quantities that will be independent of one another. For example, we will have discs, solid circles, that are perpendicular to one another."

"Let's stick to saying circles instead of discs, and talk of the sphere, not a ball. I will keep in mind that you mean those that are filled in.""As you wish. Whatever works. I will have a bunch of circles, arranged in such a way that each circle will be perpendicular to all the others, and that are all cross sections of the same sphere. Now, we are going to stretch the imagination a bit further...."

"You've already got it to the snapping point."

"There is something called a projection from something in higher dimensions to lower dimensions. Think of a sundial at high noon, that is, when the sun is directly overhead. The shadow is the projection of the gnomon...."

"When did gnomes get into this?"

"No, the gnomon is the part of a sundial that casts a shadow. If you shine a light on the person from one angle, her shadow will fall entirely on the ground. That's the projection of the gnomon onto the ground. But if you shone a light directly in front of the gnomon, and the shadow fell on a vertical wall right behind it, that would be the projection of the gnomon onto the vertical wall. The idea of the projection is that the light will be shining directly along a line which is perpendicular to the wall upon which the shadow is cast."

"To remind me of that explanation, could you just call them 'shadows' instead of 'projections'?"

"I'm not sure that would make sense, because I will have to talk about shadows on all the circles. That would mean that the light would have to be shining at an infinite number of places without interfering with the other shadows."

"What little imagination you have! The lights are special lasers, or something. Since the picture is imaginary anyway, we can have imaginary technology."

“OK, but you’ll have to keep this in mind when I talk about shadows. No fair attacking me because of a problem with the representation. Agreed?”

“Agreed. I understand that all this is going to be an approximation anyway, but as long as I don’t intend to study the mathematics involved for a couple of decades to make it precise, I will go with the blemishes of the imagery. Go on with your explanation.”

“OK. We will shine the lights on the arrow to make shadows on each one of the circles.”

“How’s the arrow tilted with respect to the axes of the sphere?”

“That a crucial point that I will talk about later. But the basic idea is that this depends how the measurement is carried out. Another experiment of the same arrow might rotate the arrow and end up making different shadows.”

“OK, let me make a note to ask you about it later.” Bill scribbles on a notepad.”I’m not going to let you hand wave me on everything.”

“OK, where were we before I started explaining projections and circles?”

“You were going to tell me more about these spheres. You keep mentioning that the spheres represent a point or a region. Why the ambiguity? Why a region? Why not just the point?”

“Because some of the quantities concern how that point can change, so you need the infinitesimal region around a point in spacetime. As well, the limits of measurement often make measurement of a precise point impossible. I’ll explain more of that later.”

“You are building up quite a list of things to explain later. I am keeping a list, you know.”

“You sound like Ko-ko in ‘The Mikado’.”

“Are you calling me cuckoo?”

“Calm down. You’ve never seen *The Mikado* by Gilbert and Sullivan, have you?”

“Who?”

“Never mind.”

“OK, we have a ball representing some physical quantity. Then the perpendicular circular cross-sections. What’s next?”

“Each circle will represent one possible outcome of a measurement of that aspect at that point in spacetime. That is, each circle will be associated with some number. The sphere will have a radius that we will call one unit. In the sphere, there will be an arrow from the center to the surface. This arrow will represent the state of the point or region in spacetime. The axes of the sphere will be determined by the conditions of the intended measurement. Once those axes are set, also the axes of the individual circles are determined. Are you following?”

“Barely. I am cheating a little, doing as much visualizing in a normal sphere as possible. But do keep going, I will catch up.”

“OK. Now, according to the way the arrow is slanted, each circle will have a particular shadow.”

“Yes, although a shadow could disappear, like my shadow at high noon.”

“Right. Let’s still call that a shadow, even though it has no length. The maximum length of a shadow is the same as the radius of the sphere, one unit. Anyway, these shadows will also look like arrows, starting at the centers of the circles.”

“I gather that you are going to tell me why these shadow-arrows are so important.”

“Indeed, I am working up to the climax. The shadow-arrow in a circle is the truth-value of the statement that the measurement of the aspect of that point in spacetime would be the value associated with the circle. You can indicate the shadow-arrow by the position of its tip, for example. The length of that shadow would then be the probability of the measurement coming up with that value.”

“So which one is the truth value, the arrow or the corresponding probability?”

“From the truth values you can figure out corresponding probabilities. There might be the same probability for two different truth values, so they should not be confused with one another.”

Bill’s tablet peeped, and he said, “We’ll continue this as usual tomorrow. I’ve got to leave a bit early; I have an important meeting.

Andy panicked, as George would certainly still be investigating.”Ah, I have a break now. I want to visit someone on your floor. I’ll accompany you.”

“Only part-way. I am not going to my office; my meeting is another floor. But I can probably save you a trip; the whole administration will be at this floor. We’ll continue the lesson tomorrow.”

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May 7, 2044

Bill sat down in the lounge of a popular club with Jutta after work. Jutta had asked him to meet her there.

Jutta said, “Bill, I’ve received an invitation to a South Pacific cruise.... what’s that all about? The invitation states that it is co-sponsored by *Scientific American* and Wheaton Informatics.”

“That’s correct. It is good publicity.”

“So why did I get an invitation? In fact, why are there invitations at all? These cruises usually are quite costly.”

“Because, Jutta, and this is in strict confidence, I am going to stage something on the ship, and I want to choose my audience. The *Scientific American* editorial staff grumbled a bit, especially when they learned of the extra security arrangements for the trip. But when I offered to pick up the entire tab, and told them that there would be a lot of reporters among those invited, they reluctantly agreed to the arrangements.”

“Why was I invited?”

“I will be representing Wheaton Informatics, and you will be my personal advisor.”

“Nice. But that is certainly not all there is.”

“Why do you say that, Jutta?”

“One of your employees came to me yesterday. An Andrew Rant, or something like that.”

“Ah, yes. Head of programming. He’s also giving me lessons on Physics.”

“Yes, so he said. He did not want to say how he found out, but he says that you are doing some odd things with the company funds.”

“Like what?”

“He found out about this cruise thing, which is normal. But then he found out that the company made a significant contribution to a hospital....”

“We’re always contributing to schools and hospitals. Good for our image, good for the world.”

“Yes, but the hospital he was concerned about was in French Polynesia.”

“So? We are an international company.”

“Fine. But is it a coincidence that French Polynesia is also a major stop on the upcoming cruise?”

“Uh, no. I thought I would go out and see what is being done with our money.”

“That is a bit odd, but then our amateur Sherlock came up with a doozy.”

“Jutta, I don’t know where you learned your English, but...”

“Stop trying to stall, Bill. Andy found out that you have been registered as a patient at the same time that the cruise gets to the island. The psychiatric ward, no less.”

“Best way to check on something is from the inside, wouldn’t you say? Any undercover agent will tell you that.”

“Bill, tell me seriously. Is it tied in with your increasing forgetfulness? Even Dr. Schiller wrote in her report about it.”

“Don’t knock forgetfulness. It can be a good thing if it’s directed. .”

“You mean, it gives personal secretaries something to do.”

“No, Jutta. I meant...well, for example, space-time. We forget the structure of space-time in order to concentrate only on space, or only on time. But we need to keep in mind what we are forgetting, because when you use an analogy and forget to forget, you are pushing the analogy too far. A perfect analogy is called an isomorphism, or ‘same-form’ in Greek. If we push the analogy just a little beyond, we make imperfect analogies. We constantly shift how much we forget.”

“Quit trying to dodge the question. Leave all the imperfect analogies, metaphors, similes, pretense, suspension of disbelief, and so forth to your literature teachers, and answer my question.”

Bill heaved a big sigh.”I suppose it’s only a matter of time before you or Andy hack into my file at Mayo Clinic. If you do, you will find out that my brain is, well, deteriorating.”

“What is the diagnosis? Early Alzheimer’s?”

“Not exactly. I forget the medical term, but it is something that is driving me slowly but inexorably towards paranoid schizophrenia.”

“When did you know?”

“Ten years ago.”

“What? And you’ve never told me?”

“The progress is so slow that I have had time to be working on a solution. In fact, you might say everything I’ve done in the company was to get the nuts and bolts of the solution. And I have a solution. However, it would be considered unethical in most countries. But with the government in a mess in the French Polynesian archipelago soon after it got its independence from France....”

“That was about five years ago, as I recall.”

“Six, actually. Anyway, the government was willing to give me assurances of autonomy, in exchange for the exorbitant price I paid for a fifty-year lease on one of the islands that Papeete doesn’t consider important. Don’t worry, that’s out of my funds, not the firm’s.”

“Who’s Papeete?”

“That’s the capital of French Polynesia. So I’ve been building a self-contained town centered around something I call an infospital, a combination of a research facility and a hospital. The town will have all the comforts, but of course it will be rather isolated from friends and family.”

“I don’t have any. All dead.”

“Ah. Sorry. We’ve never talked about your family, Jutta.”

“Nor will we. Keep going.”

“There will be a large ward designed primarily for me, and most all my funds will go into a trust for the infospital. You’ll be the head, if you will accept. I’ll set up a trust with a guarantee that the center will continue after my death, with your position inviolable.”

“Where will all this money come from?”

“My fortune is more considerable than even *Forbes* guesses.”

“This doesn’t explain the physics lessons.”

“Haven’t you heard, use it or lose it? I need to exercise my brain, and the stuff that Andy is teaching me definitely does that. Even if I forget it.”

“So the story about the lecture is a fiction?”

“No. You know me, if I don’t have a goal, I will slack off. *Scientific American* agreed to let me give one of the lectures on the cruise.”

“But... normally the speakers on those Insight Cruises are highly qualified. You’re not. How did you convince them?”

“Ah, you know I spent my youth wandering around the Middle East, but no one has found out the precise details, because of the wars. It was not difficult to forge some university diplomas. *Scientific American* assumed that someone who has founded a technology giant must have some qualifications, and it would not enter their heads that such a respected person as myself would have the chutzpah to lie about it.”

“Chutzpah?”

“Yiddish, I think. Audacity, either positive or negative. You pick up all sorts of things in the Middle East. Anyway, to use mathematical terminology, I like to think of my action as closure.”

“Closure? In psychology that means a desire for a firm answer to a question. What question are you...”.

No, in the mathematical sense, that is, taking an unfinished opportunity to its logical end. I have given the impression to the public that I singlehandedly came up with everything we do here.”

“Hm. In reality, you give a few vague ideas to us at R&D, and expect us to work out the details.”

“So?”

“So, the obvious extension of this for public relations is to give these lectures so as to appear the originator of the Grand Ideas, and then I let the plebs do the menial work. But it’s not something I can whip up the night before. So I will have my team there, including Andy, you, and others. As a first step, I need to present the outline of some counterintuitive ideas from the physics that you guys in R&D use for all these nanoelectronics used in our robot; a few weird but true ideas and a couple of terms I don’t have to go too far, since the audience will be hand-picked, and they will not include any specialists in physics. I will also dazzle with some visual imagery. A Power Point is not enough. People expect holographic images nowadays. Andy will guide me through my movements during your talk.”

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May 8, 2044

When Bill returned to the lesson, neither Andy nor he let on that they knew anything new about each other. Bill just started as normal, “Let me see if I remember from yesterday. If the sphere represented the mass at a point on the moon yesterday at midnight, and I measured it using a laser, then the arrow would be tilted a certain way, perhaps different to the way it would be tilted if I had gone to the moon and used a scale. So far, so good?”

“Yes. Keep going.”

“OK, now suppose that the outcome could be anything. I look at a circle in the sphere representing the statement that the result of the measurement will be seven grams.”

“Fine.”

“The circle has two axes, which I can label horizontal and vertical. Since I intend to use a laser, the arrow will have a shadow at a certain slant in that circle, say measured as going six

percent of the way along the horizontal, and say eight percent along the vertical. That would be the truth value of the statement that this point was associated with a mass of seven grams. The probability that my measurement would come up with seven grams would be about ten percent, because the arrow was a tenth of a unit long. Otherwise put, if I perform the measurement a thousand times, about a hundred of the measurements would come up with seven grams."

"Well done."

"That cost some effort, you know."

"Good. Keeps you fit. Now you will be ready for the next stage."

"There's more?"

"Sure."

"But wait. Before you go on, there is something that bothers me."

"Yes?"

"To get the probability, it did not matter where in the circle my shadow-arrow was, as long as it was a tenth of the radius. So it could have been on one of the axes. We could do that in each of the circles. Then we could do without the rest of the circles altogether. You could have gotten the probability by just using normal lines that were perpendicular to each other, and skip the circles. Your sphere could have half the number of axes, it would be easier to visualize, and even in two or three dimensions, you could have examples. Why have these truth values at all? Why not just have the probabilities as your truth values? Your set-up seems too complicated."

"Calm down, Bill. You are right that we wouldn't need the circles if we only had one measurement of one aspect of one point in spacetime and all we needed were the probabilities. But we use these truth values to be able to calculate more easily the probabilities when we are dealing with more than one aspect or more than one point in spacetime or more than a single measurement. When we make combinations, the truth values add up similar to the addition of the arrows, and from the result you get the corresponding probability. You can't always get it by using the probabilities alone. If we were to be precise, we would give a few more conditions and label these arrows 'vectors'."

"Vectors? You mean the organisms that are carriers of a disease or something? Like mosquitoes and..."

"No, that's another meaning of the word. But forget I mentioned it. I'll still call them 'arrows'."

But that will become clearer as I go."

"You're adding to the list."

"Yes, Ko-ko."

"Stop calling me that. I have the suspicion that you are insulting me. I'm going to look up that play."

"Good. Even if you don't learn much mathematics or physics, you might get a little culture under your belt. You've spent too much of your life only with your business."

"Just keep explaining, will you?"

"OK. I will stay with the same point in spacetime for a while. Different spheres for different aspects. We could then make quite a bit of effort to combine them all into one sphere, so that there would be a single state-arrow, whereby each aspect-sphere and each aspect-arrow would be the shadows of these. But then we would get too tied up in our imagery and lose track of the main ideas. So, for now, just imagine ... not visualize, imagine... that all the spheres are interconnected, so that the state is a big billiard game."

"Go ahead. I won't object."

“I mentioned that the observations are the interactions of the point with other points in spacetime. We can put this together to also talk about going from one point to another point. For example, keeping the spatial coordinates the same but changing the time coordinates, we talk about motion of a particle.”

“Ah, here’s something I often see. One talks about the paradox of something being both a wave and a particle.”

“No paradox. The truth values we have talked about have patterns. Because the variation of these properties can be reduced to continuous patterns, they are more properly called waves. A volume of spacetime in which the sum of the probabilities for each of a certain collection of properties adds up to close to certainty, one hundred percent, is called a particle. It’s really waves, but particles remain intuitive. When they lack certain of these properties, we say the particle does not exist at that point.”

“Seems a bit abstract. I liked it better when you were using arrows.”“Fine, I will use that. The arrow rotates under the effect of the operators, to end up in a different state. This should set you thinking in terms of waves...”

Bill did not let him get further.”Um, no, it doesn’t. Why should it?”

“These arrows are rotating.”

“Hold on, you are going too fast. You were talking about waves, now you are talking about rotations. That’s a jump that I don’t follow.”

“Have you ever thought about what a wave is?”

“Um, those things in the sea...”

“Those are only one example. But there are also radio waves, sound waves, and other waves. What do they all have in common?”

“Um, they all have repeating patterns of something?”

“Bingo. Now, what happens when you go around in a circle?”

“You come back to the place you started.”

“Do you start to see the connection? Moving around in a circle, not necessarily uniformly during each rotation, is the same as a wave. You can describe waves in terms of rotation, or rotation in terms of waves.”

“I’ll work on that. In the meantime, what would make these arrows rotate?”

“What would make them rotate?”

“Bill, pay attention. I mentioned that time intervals are operators, so as time goes by, the arrows rotate. Looked at another way, energy is an operator moving an arrow. The arrow keeps changing, so the energy operator keeps making it rotate. If you want to look that up, there are equations that do all this.”

“The whole thing sounds awfully ... well, non-deterministic.”

“Looked at one way, yes, but looked at in the way that the movements of these arrows are what underlies reality, or perhaps better to say that the movements of these arrows, or what they represent, are reality, then the whole thing is quite deterministic, because we have ways to predict the motions of these arrows.”

“I’ve seen explanations with little springs.”

“Right, same thing. I’ll use the spring image and hook it up with waves and circles. Use a very good spring, put a weight on it, put a light on the weight, and turn off the lights. Pull down the weight a bit, and let go. Watch the light as it goes up and down. Notice how it speeds up around the middle, and slows down at the ends. After that, do two further experiments. First take a guitar string and pluck it. It eventually makes waves that will have a certain height above the

middle, its amplitude. Have the spring horizontal across the screen. Make it big. You will see that there are some fixed points, some nodes, where the guitar string apparently is not moving. Half-way between two nodes, put a little light, and now turn off the room light. Compare the movement to the spring one. Same thing, no?"

Bill tried to imagine the situation, and then grunted, "I suppose."

Andy continued."Second experiment. Place a wheel so that you do not see its width. That is, as if the wheel were ready to travel directly away from you in the direction you are looking. Now put a light on one point on its rim, so that you can see it no matter how the wheel rotates. Turn off the room light so that only this little light is visible. Again you will see the same as with the spring."

Bill tried to derail Andy, saying "I would like to have a comparison to a wave that is travelling, like a water wave, not like a guitar wave that doesn't go anywhere."

Andy shrugged."The movement of the guitar string is going somewhere; it's just that it returns, so interferes with itself. But OK, try this. Take your wheel with a light on, but now have it travel alone a line perpendicular to the one it was traveling on before. You should be a little distance from its path. Turn off the room light again. Have your contraption start some distance to your left, and travel from left to right, passing you and going some further distance. What does the path of the light look like?"

Bill traced the idea out with his finger on the dusty table top. Then he admitted, "Ah, a wave!"

"Right. So, there you have it. A wave just means this sort of movement. The arrow is something that will be composed of several different waves, so it may be more of an orchestra than a solo, but both are possible. So, the idea is that these arrows, these truth values, also change according to a periodic wave-like function, and so can be called waves. Therefore they can interfere with one another."

"Um, I think I need time to digest all this."

"That's why I asked to see you well before the lecture."

Chapter Six

May 9, 2044

Further south, in San Diego, lay another confrontation in the making. Tal Kishon, the editor of *The Foundation*, did not hide his religious orientation from his friends, but did not let it be known in his publications. He was indeed Jewish, which generally gave no problem, but the fact that he was a Reform Jew could have, given that the paper gained many of its readers from Orthodox Judaism, who viewed Reform Judaism as having erred from the proper ways with its self-applied label of "reasonable Judaism". So he merely put down that he was Jewish, and let his readers apply their own presuppositions. He did not think of this as hypocritical, only realistic. He saw no reason to lose readers, but also no reason to have to share the views of his newspaper contributors. He did not empathize with them, but he considered that he could understand them. In fact, he considered it an advantage.

He explained it to a drinking companion in his favorite bar that evening.

"You know the model and theory stuff, right?"

"It's in the school thingy....curriculum."

"Doesn't mean you learned it."

After his companion tried to playfully hit Tal in response and missed, Tal said,

“Look, you’ve got a model, and a theory. If you are looking at it from a possible worlds perspective, a possible world can contain the universe of another one as a set, and as such, it can be represented by a symbol. Also the interpretation function will be, in this enlarged view, a matching of sets, which can again be represented by a symbol. In other words, I can reduce the model-theory structure down to a sentence with normal variables.”

“Whady’a mean, normal? You saying I’m not normal?” His companion was already losing it from the alcohol.

Tal considered explaining the difference between different levels of variables, but decided that this would only get him sidetracked. He continued in the style of all pedants who don’t notice they are boring their audience.”So what you can have is a model of a theory that includes a representation of another model and its theory.”

“So you figure you’re higher up on the ladder of possible worlds than these fundamentalists?”

“Something like that, yea. So I can ... Adam? Adam, get off the floor, I’m trying to explain my objectivity. That’s why I’ve been invited on a cruise.”

Adam said from the floor, “Tal, you told me yesterday that you were chosen because of your subscription numbers. Load of codswallop, this objective stuff. You’re about as objective as ....” and he fell asleep.

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May 10, 2044

Bill had not noticed any threat from Andy, so he approached this lesson more relaxedly. “Yesterday you have only talked about waves. Where do particles come into the picture?”

“Usually a particle is identified with a contiguous volume of space where the truth values add up almost to certainty. For example, a proton is considered to have a size. Or the orbitals of an electron around an atom. So one associates a particle to a point or region in spacetime where a single measurement has an almost certain probability of giving one of the possible values for each of the aspects associated with the particle. It is common to say that the particle has a certain state, even though it is a bit redundant.”

“Ah. So basically there is no question of something being a particle or a wave; a particle is just one kind of wave.”

“Right; so don’t worry about the supposed paradox of the wave versus particle interpretations. There’s no paradox. If it’s any help, you can refer to your dream the whole universe could be based on nothing except the forms provided by sets? Essentially that’s analogous to what is going on here. Just as you can do without urelements in mathematics, so too you can do without particles in physics.”

“But I read about the experiment where a beam of electrons is scattered differently by a barrier with two slits, depending on whether the slit are illuminated or not. The pattern of the scattering when it is not illuminated is that of a wave that is split into two smaller waves which then interfere with one another. But the pattern when it is illuminated, so that one can see an electron exit at one or the other slits, is that of a bunch of particles.”

“Yes, that’s a classic. But the key is examining the differences in the two situations. In both cases, the electron is just a bunch of variations in truth values in a way that is periodic like a wave. So, when you have two slits, the information is restricted by the two slits, and then comes back together as the truth values add. The result is reminiscent of two water waves meeting. However, when you introduce a third element where information is exchanged, say, with a photon that meets the electron , this is putting another truth value there, one which changes the

balance of truth values in the whole region. That is, in the area of the photon the truth value becomes higher, and hence in the other areas, lower. That is, the information balance is more lopsided, and so the pattern on the screen is one that is more localized. Although, of course, it isn't really completely localized, just disbalanced. That behavior is what one associates with a particle. In either case, the electron is a particular variation in the truth values connected with points of space-time. So particles become simply special cases of waves. There is no dual nature involved."

"So why do you object to the inclusion of particles?"

"Because particles then are too often thought of as being completely bounded in a little interval, and so things like the two-slit experiment, or non-locality, are thought of as paradoxical, when they aren't."

"Non-locality?"

"That's the fact that two electrons can affect each other even though they do not transmit a signal between them. As long as the electrons are considered to be only within a little space, this phenomenon presents difficulties. But when you consider that both electrons are simply part of a wave that pervades all of spacetime, then such problems disappear."

"But if things are so spread out, why do we have this fragmentation into things that appear to be particles?

"Energy acts on a space-time interval; you can break up the state into component waves, sort of like you can break up a sound into composite notes. Each component wave can have a small spread of possible measurements, therefore looking as if it is localized. It will be considered localized discrete, quantized. The spreading out of the field is seen as scattering of particles. Although there is really no real need for the concept of particles, it is more intuitive to a species that evolved dealing with rocks to throw at other simians."

"It is probably better for school children, who don't learn about things like non-locality or the two-slit experiment until later."

"You are wrong. The so-called 'dual nature of light' is already in school curricula. But even those curricula that do not get past Newton present the elementary formulas for forces like gravity or electric charge, where two particles are supposed to act on each other, such that the closer one gets to the particle, the stronger the force."

"Yea, so?"

"OK, most school kids swallow that whole, especially as they are usually more interested in their neighbor than in the teacher. But now and then a thinking kid notices that you have divide by the distance to the center of the particle, and asks himself what happens when you consider the particle acting on itself. Then the distance is zero, but their mathematics teacher told them that they can't divide by zero. The teacher then usually mumbles something which doesn't decide the problem, and then assigns homework."

"Self-reference seems to be notorious for creating problems. But what answer do the physicists themselves give?"

"In the particle theory they 'solve' it by having so-called virtual particles which pop in and out of existence around the particle, preventing any possible interaction with the center. This is a clumsy construction. There's a more elegant solution when you only have fields"

"It seems you don't like the particle idea."

"You're right. Particles aren't even more intuitive in all instances. For example, antimatter seems more intuitive when you consider it as an inverse wave to matter, doesn't it?"

"I don't know. What's antimatter?"

“In particle language, that is a particle with the right mixture of identical and opposite qualities to a regular particle, so that when the matter and antimatter particles come together, they turn into the equivalent amount of energy. Unfortunately, it’s said that they annihilate each other, which sounds like they disappear altogether, which is not really correct. Anyway, as a wave one can see how some properties like charge can add up to zero, when two waves are mirror images of each other. It’s less clear when you talk about particles. Or, to take another example, when a piece of light is split into two and, by making the path of one half longer than the other, to get the two halves to interfere with each other.... that is, the photon interferes with itself...”

“Weird sounding experiment.”

“Not at all. Interferometers are standard tools for lots of research. Anyway, the point is that to say that something is sometimes particle, sometimes wave, sounds mysterious, but this is just a magician’s sleight of hand. Better to say that it is a wave.... that is, it is interpreted as a model using waves... or that sometimes, by restricting the wave model, we end up with a model in which there are particles.”

“OK. But you will allow me to talk about particles?”

“Sure, go ahead. Just make sure that you don’t think of them as totally isolated little pieces.”

“Are you saying that they are continuous? I thought there was something called quanta, that is, little isolated pieces.”

“Yes and no. The quanta just indicate that the possible values in the measurements are distinct, not continuous. Energy, for example, is only measured in certain amounts. But the probabilities remain continuous.”

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May 11, 2044

Moishe was glad that his answer from Wheaton Informatics had not had to depend on the variable service given by the Israeli Post Office for an answer. Moishe let out an uncharacteristic whoop of joy when he read the positive answer per email. It was stated that all eight orphans would be taken to a special center for treatment. Furthermore, Wheaton Informatics would do its best to ensure that the orphans landed in good families after their treatment. They would be brought to the island, with all their belongings, in August. There would be a fully qualified set of neurologists on board the ship taking them there, and the CEO himself would accompany them. The orphanage was free to send a Rabbi on the cruise, which would be fully paid by Wheaton Informatics. Would he please respond within the week?

After the initial euphoria came suspicion. Moishe checked on the background of Wheaton Informatics, including its CEO. He found very little on Bill Wheaton himself, so he checked on his parents. He had a friend help him hack into medical records, genealogy records, and so forth. What he found astounded him. Such thoroughness would not have been possible in the early twenty-first century, but recent advances allowed incredibly precise tracing of bloodlines, in this case all the way back to ancient Israel. There was a semi-nomadic tribe called Amalekites, and this tribe was an enemy of the Israelites at the time. In the Torah it is said that God ordered the complete extermination of all Amalekites, present and future. Therefore the killing of every Amalekite, or at least every one that a Jew came across, made it into the list of 613 Commandments that Moishe had attempted all his life to fulfill. Because Jews lost the ability, and hence the tradition, of distinguishing Amalekites from other people a long time ago, all Jews, including Moishe, ignored this particular commandment today. But new evidence from magnificent archaeological finds in the Middle East had put into question the idea that the

Amalekites had disappeared already in ancient times. A bit of computer work, not all of it legal, helped Moishe discover the bloodline of Bill's parents. They, and hence of course Bill, appeared to be Amalekites! Bill's parents were both deceased. Moishe checked for other relatives, but it appeared that Bill was the only one of his family still alive.

In Moishe's acceptance of the invitation, he did not mention his goal of changing that situation. Bill Wheaton must die.

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May 12, 2044

Bill, unaware that his life was now in danger, calmly started another lesson with Andy."OK. But tell me, if you have movement in spacetime, there must be some sort of a fixed set of coordinates for it, no? How?"

"Spacetime is a bit of a fiction. When we say that we have changed spacetime positions, we mean that there is an interval .... well, technically not an interval of spacetime, since an interval of spacetime assumes distances, and the definition of spacetime does not give us distances in spacetime. We have intervals of time, and intervals of space. We can measure distances. We get... well, not directly distances in spacetime intervals, but we get distances in the square of spacetime intervals."

"Don't bother with the technicalities. I get the idea. So there is no time-operator, I suppose."

"No, at most something that can be construed as a time-interval operator. Whatever, a change in spacetime can bring about a change in state. The chance of that happening is given by one kind of interaction. Sort of like the angle between two state arrows. Then you have another kind of interaction, in which every aspect interacts with every other aspect in a kind of round robin, although all at once. Simultaneous chess. The interactions are a transformation from one interpretation function in our model to another."

"These operators... the name sounds like a deliberate process. Could one also argue for the different states all existing and the transformations just being the relation between one state and another? or one region of spacetime with another?"

"Yes, and many of the interactions are handled in this way, but the vocabulary is a result of our machine-age thinking. Like the reason that predestination tends to be a minority view in the Western religions."

"Let's not get into religion. There is already too much trouble on the ship with all the sects which are arising."

"OK, to recap a little. An operator finds a value for an aspect when it finds that fixed point, and among all these fixed points, when there is a point when it is a hundred percent for one of these values and zero for the others, then that's a measurement, or an event, depending if you want a more active or a more passive vocabulary. The interactions continue, and in surrounding instants of spacetime, the chances of that value go back to less than certainty."

"The measurements... do they take place instantaneously?"

"No. Remember that the measurements result from an interaction among different spacetime points, so although this happens pretty quickly, it is not necessarily instantaneously as they used to think."

"And if we have two operators? Can you measure them together?"

"What do you mean, measured together? The same particle measured with two different operators simultaneously?"

"That would be tricky, I suppose."

“Right, so one does something equivalent. You take a bunch of particles, all in the same state, and you take half of them and measure them with one operator, and the other half and measure them with the other operator.”

“Why a bunch? Why not just two?”

“Classically, all the measurements for one aspect would always be the same, give or take measurement errors and approximations, but today we know that this need not be the case. So you have to take a lot of measurements to be able to figure out the probabilities for each value for each aspect.”

“Theoretically, though, it could happen that there is only one value possible, so that the measurement for one aspect would always be the same.”

“Yes, and that is what would be meant by a precise measurement.”

“And then theoretically it could happen for two different operators.”

“You might think so, but this is not always the case. It depends on the operators involved. There are some aspects where this is the case. For example, change of distance and change of time. So we can measure velocities pretty well. But for other pairs of operators... Remember that the operator acts on the whole-state arrow, even if we usually concentrate only on one part. Starting from the same arrow for each operator, each of the two operators would have to have the same transformation for at least one of its fixed points. That is, each of the two operators would have to line the arrow and the axes up to get the certainty for both aspects. This would mean that the operators could be applied to the state in whichever order. If not, then it won’t work. An analogy would be a person standing on the street with that two different lights would each have to make your shadow line up with the same light post. It doesn’t always work. For example, the momentum associated with a state, and the measurement of the position with respect to something fixed. Or the spin as measured with the measuring magnets lined up along one direction, and the spin measured with the magnets lined up in a direction perpendicular to the first one. For example, on that Uncertainty Principle: another way to put it for two incompatible quantities is that if you have two incompatible operators, each one separately will have a model, but together, there is no model, among the models possible.”

“Is it like the two ways you see an optical illusion? For example, you can see the duck and the rabbit, but not both together. Or the old lady and the young woman? Or...”

“I get the idea. I’ve seen a lot of them in Picasso’s painting, not to mention Escher’s lithographs.”

“Anyway, it still sounds like a contradiction,” .

“No, a contradiction would be if all models were accessible. Then incompatible sentences would also be inconsistent. But we don’t have all models possible. The physical world imposes limitations on your possible worlds semantics.”

“Like what?”

“From the point of view of truth values, the operators can represent models, and there are pairs of models that do not share any possible measurements for a common arrow, so that you can’t get both measurements precise. You can’t get both statements completely true at the same time, although you can get both of them to have the fuzzy truth values. The exact numerical relation is referred to as an uncertainty principle, although the name in English is a bit stupid. A better translation would have been the ‘Fuzziness Principle’. Uncertainty sounds too much like there is something definite that we humans are just are not sure about, instead of being the fuzziness inherent in nature itself.”

"Oh, is that the effect I read about where measuring one quality of a particle disturbs it so that you are not able to then measure another quality of the particle?"

"No! That certainly happens, but that is a classical effect that is definitely not the content of the uncertainty principle, which has to do with inherent probability. That is, the classical uncertainty is epistemological uncertainty, meaning that something is either totally true or it is totally false, just not known which by the theory. But the theory could be expanded to be able to determine which it is. The inherent probability, the ontological uncertainty, on the other hand, is not totally true or totally false before measuring, and so in that case no theory can precisely predict it beforehand, no matter how much you expand it."

"Ah, so the electron has more free will than I do?"

"You could say that. There is no more powerful system to determine ahead of time what the electron will 'choose'. Hindsight, however, is a different matter, of course, so if you considered all of time as a constantly progressing system of possible worlds, then you can get a better handle on what time is. Time is inextricably linked with the idea of inherent probability in fuzzy logic. That is, an ontological uncertainty, not the probability of cards, where the fact is definitive but simply one's knowledge is incomplete; this is an epistemological uncertainty. The two kinds of probability will mostly obey the same laws, which made the debate about whether quantum phenomena were one or the other rather tricky. But through the truth value set-up, one could finally decide that there is indeed the inherent probability. Reality really is uncertain."

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May 15, 2044

The security firm hired for the cruise had the reputation to always use technology that worked like the proverbial clockwork. Their secret was to check first that the physics behind it was sound. For the technology used in the upcoming cruise, the references led to a professor of the Physics Faculty at Heidelberg University in Germany. Thankfully Wheaton Informatics had a branch in Germany, Wheaton Informatik. Thus it was that Frau Joan Michelson was ten minutes early for her appointment with Professor Hans Landsman. After the preliminary pleasantries, Joan started by saying, "Professor, Wheaton Informatics is willing to go to great expense to ensure that its unusual requirements for this year's *Scientific American* cruise be fulfilled."

"I have heard of this veil of silence. May I ask the reason for these requirements?"

"No. But the point is that the skills of many hackers have grown enormously with the recent introduction of accessible quantum computing into the cloud. So it will become known that the stakes are high; we suppose then that quite a bit of expense will be put into breaking through our security requirements."

"Why do you wish to consult with me?"

"Although our devices are cleverer every year, they are based mostly on the physics of the end of the nineteenth century, after Maxwell and his famous equations. The principles of relativity with the GPS added a little bit, of course. But there are many principles which are still unfamiliar to us. I'm not asking for an entire course in physics, but at least some indication as to the direction we should start looking."

The professor leaned back in his chair and thought for a minute. Then he said, "OK, it's about lunchtime. How would you like to join me in the Mensa?"

"With all due respect, sir, I am not here to engage in quizzes...."

"What? No, not the high-IQ society *Mensa International*. A Mensa in Germany is the student cafeteria. The students complain about it, but I quite like it. Cheap, too."

Over a plate with food that vaguely resembled chicken, the professor started, “Let’s start with the basics. The espionage that you are worried about has the task to get information past your barriers without your knowledge, is this a fair appraisal?”

“Yes.” Joan was concentrating on balancing the mound of salad that she had heaped on much too small a plate.

“What would you say that information is?”

“Um, that which can distinguish one thing from another , the number of yes/no questions you need to get answered to fully specify the system, a measure of one's freedom of choice when one selects a message, the number of combinations of component parts that are available to be chosen arbitrarily...”

“All very good definitions. To put them into practice on the quantum scale, you need to distinguish two states.”

“So if you get an atom, you measure which state it’s in...”

“No, that is often impossible. It’s like trying to figure out the range of a variable in a model if you only know one interpretation.”

“Could you please be less mathematical, professor?”

“Sorry. Suppose your friend says, ‘Every time I go to the Shire, I go to a house of a friend.’ You know she went one time to Bilbo’s house. That will not tell you all the houses she might go to. No, you will have to measure the same state many times in order to find out the information that it can produce.”

“Excuse me for interrupting, but it seems we have gotten off the topic. We were talking about security.”

“So was I. The idea is that the sketches that you have sent me are based on an old idea of information transfer. Your idea is that the information that you can intercept will be the same information that is being sent. But you don’t take into account that the information could be sent in two parts, one of which would be the same for the interceptor and the receiver, but in such an innocuous form that the interceptor may not even recognize it as a message, and the second part which would be using quantum information which would be destroyed if intercepted.”

“That exists?”

“Yes, and that is one of the possibilities that you must guard against. We could help you; this will depend on our budget. You see, we are dependent upon contributions, and...”

“Speak no further, Professor. Wheaton Informatics will be glad to help the University.”

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May 16, 2044

Bill started his next session with Andy with a question.”So far you have just talked about points or tiny regions in spacetime. But what about particles that go somewhere? You mentioned velocity.”

“That’s when there is continuity, more or less, of changes of state along a path in spacetime. After all, every state is connected to all the other states.”

“So changing one changes the other?”

“Not necessarily. Some of them are not directly connected, as changing the state of one may leave another one unchanged. The state of one point in spacetime depends on the states of many, or even all other points in spacetime. So of course the change of one state at one point will imply a change of state of other points. Some pairs stand out, so that any change in one changes the other.

But you have to distinguish between implication and causation. A change in one particle may logically imply the change in another particle, because really one can see the two states as part of a single state. This can happen without causation, and the two particles are called entangled. Since entanglement can happen over large stretches of space, it seems magic."

"If I were a mystic, I would cite this as the unity of..."

"Don't, Bill. I do not want to hear any silly Chopra idiocy. Anyway, causation is more familiar to most people. It requires a chain of implications along a continuous path in spacetime, each implication requiring time."

"How?"

"The way you get this continuity is by these interactions between the points in spacetime that I mentioned. There may be entanglement involved, but it can take time for two separate particles to become entangled, or two entangled particles to become separate. As I said, these interactions are not instantaneous, and the probability is high that the continuity for one path will be no faster than the speed of light."

"Wait. I have always heard that the information can definitely not travel faster than the speed of light."

"Information, in the sense of measurement results, no. But each path has a truth value that is obtained by combining the truth values of each point along the way – which is a bit like adding arrows—and then the truth values of each path are combined to give the most probable final path. So, for example the straight line of a beam of light comes from the combination of many possible paths. The starting and finishing points of each path are what are measured, and which give you information."

"But... here is something that bothers me. Suppose you want to find the probability of an unmeasured particle following a path. Then you split up the path into an infinite number of infinitesimal sections. Then the probability of the particle following the path would be the multiplication of all the probabilities. But if there are an infinite number of these probabilities, each one corresponding to an infinitesimal interval, then unless most of these probabilities are a hundred percent, then the multiplication will end up being zero, no?"

"There are two objections to your reasoning. First, of course, my caveat about not combining the probabilities directly, but first combining the truth values, and then the probabilities calculated from there. However, that wouldn't necessarily be a killer argument, because we still want the laws of probability to be valid. So, here I need to point out the difference between infinitesimal and zero, as well as the difference between an infinitesimal interval and a point. The multiplication you mentioned would get close to zero, and would certainly equal zero in some theories. But as long as you are going to allow division of a line into infinite pieces, you also have to allow for addition, and if you don't make some allowance for such things, then you get something like the Tortoise and Achilles paradox."

"That's when boastful but lazy Achilles and a resolute tortoise has a race, and overconfident Achilles naps partway so that the tortoise..."

"No, that's Aesop's hare and the tortoise. This one is from another Greek. Achilles would have won any Olympics race, so he gives a tortoise a hundred meter head start in a two hundred meter race."

"Still not fair."

"That's the idea. The tortoise only goes one percent of Achilles' speed. But when Achilles gets to where the tortoise was a moment ago, that moment is enough to put the tortoise ahead. In

this way, every time Achilles advances to where the tortoise was, the tortoise is still ahead, so it seems that Achilles never passes the tortoise, which is of course nonsense."

"Ah, yea, but when you get down to quantum level.."

"Leave that aside, and suppose that the time can be infinitely divided up. The key is the conjunction of the concepts 'never' and 'infinitely divided up.' If you can divide infinitely, you can also add infinitely. Remember that infinity is what can't be reached by normal means, so you have a jump. That is, the 'never' is jumped over, and an infinite addition of infinitely small pieces can still add up to something finite, and so Achilles passes the tortoise after all."

"That would be a relief for Achilles, I guess."

"If Achilles had seen the paradox before the race. Anyway, yes, one path of the particle when not measured can have an infinitesimal but non-zero probability. But if we were to then find the probability of it having one of those paths, you would add up all the infinite little probabilities of all the paths in that region, and you can thus get a finite number, just like Achilles passing the tortoise. If you include all of spacetime, then that would add up to certainty. In considering causation, where the implication that one state change causes the other state change by doing so along a path, with each infinitesimal section implying the change of the next infinitesimal section and so on down the line, thus limiting the speed with which causation can occur, you'll get the same result if you just consider paths, and ignore the rest of the collections, which will end up being insignificant."

"I would have to work that out to be sure of what you say, but for the moment I will provisionally take your word for it. Andy. I am bushed. I see that this could go on forever. As you say, you'll be in the background ready to answer questions. You'll be watching me from the lighting box, right?"

"No, I can't handle the lighting and be ready to intervene at the same time. We were already underway when I received the order for briefing you. I have asked for a couple of technicians to be sent out . I was given rather short notice of this arrangement, so I had to hire two lighting technicians rather quickly to replace me."

"Two?"

"I don't trust only one person to do it. I know my own reliability, but when someone works under me, I need time to get to know what their letters of reference are hiding. I took a brother/sister team who had some excellent qualifications and references, and hope that one of them will back up the other if there is any problem. Chaim and Daniela Levi. They will be flying out from Arkansas."

"OK, fair enough. I read the draft for the program which the audience will get. It says that each person invited will get a little toy. What's that all about?"

"It is a translucent sphere that illustrates some of the principles I just explained to you. You will have a corresponding hologram on stage that you can direct by indicating with your hand. You will be able to either move the arrow while the sphere remains fixed in place, or move the sphere itself, and keeping the arrow fixed, as you wish. You should do both at least once to show that the two viewpoints are equivalent. You will be wired up before your interview."

"Should I read up on other explanations on the Net?"

"Not for now; they would only confuse you. Of course there are the mathematical versions which are pretty uniform, but there are a lot of different ways to explain this stuff to the lay person. For example, a lot of explanations you see will use little clocks instead of circles, but we already are using metaphors about arrows, so we decided not to confuse the issue already with little clocks."

“I need to motivate my audience. Why should they care about this stuff?”

“Because they might notice that more and more things around them on the everyday scale of things get explained in the daily press by referring to processes on the super-tiny, quantum, level, but that these explanations just jump over basic concepts with hand-waving expressions like ‘somehow’. There are good explanations in the Internet, but people who try to figure out what that ‘somehow’ means are too often discouraged from looking further after they open a Wikipedia article which is full of formulas which require a doctorate in quantum physics to understand. You are to be filling in some of these ‘somehows’. Not all, of course, but it will be a start.”

“The players can experiment with their own operators, but beginners should try at first to stick to the preprogrammed list of operators, with names like Energy, Momentum, Time Intervals, and so forth. The names the physicists use, like ‘Hamiltonian’, would be too confusing.”

“OK, but this toy: seems a bit expensive for a single lecture. They will just throw it away after the lecture.”

“No, the challenge for someone playing with the toy will be to find cross-sections by looking for the fixed points.”

“What good is that? They already have games on their phones...”

“Remember, you prohibited phones for the cruise.”

“You know about the cruise?”(before?) I don’t know why, but a nice side effect is that this will be the only game they will have access to for the first day of the trip.”

“OK, but ...”

“It is not so easy, and it is somewhat simplified to get it down to three dimensions, but when someone can get a fixed point, the toy registers points. If you get all the fixed points, then the player can turn in the sphere, or mail it, and she wins a trip for two to the Bahamas. The toy concentrates on the spin of an electron. After all, we are trying to illustrate the logic, not explain all of physics, and the eventual choice of two values is the easiest to comprehend.”

“Easiest does not mean easy. How am I going to get infinite-dimensional spaces in a holograph?”

“For God’s sake, Bill, you can’t go around talking about infinite dimensions to a lay audience. You can’t even mention five-dimensional spaces. People start to try to imagine five spatial dimensions, even if you warn them not to, and from then on you’ve lost them.”

“Funny; when you mention all the points in a line, they seem to get that.”

“Or pretend to, because this is so common, they usually haven’t gone very deeply into that either.”

“OK, how complicated can the formulas I use be?”

“You aren’t paying attention, Bill. No symbols. If you put so much as a plus sign up there, half of your audience will turn off immediately. In fact, you’ll have to constantly remind your audience that the way you use some of the everyday words may have a special meaning. For example, a point. A point is a piece of data, and one example is the geometric point. Another example: a collection of these points is a space, one example is the space of space-time. The space which forms the basis of your model is the universe, one example of which is that of cosmology. And so forth. So don’t forget. Stick to the notes.”

“Send them to me.”

“You forgot. You forbad Internet access from the ship. I printed them up for you . You should have time to go through them and ask me anything that still confuses you.”

For the next couple of days, Bill worked through the notes until he was confident enough to give the lecture. But there was one thing he would use that he did not tell Andy about until the last minute. Bill had been so proud of his silencer, that he could not resist closing the lecture with a demonstration of his own. He finished the last few minutes of his lecture as follows.

"To get the idea of interference, I will close today's lecture with a little demonstration. I am going to turn on a silencer for two minutes, and you may shout your hearts out without offending your neighbors. Have fun, and I hope to see you at the next lecture."

For the next few minutes the audience had lots of fun, although there were a few worried looks here and there. If they remember nothing else from this lecture, Bill thought, at least they will remember this.

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January 4, 2045

Wheaton Informatics had informed Moishe that it would pay for a flight to and from San Francisco. Moishe had then asked whether the flight from Tel Aviv to San Francisco could include a stop-over in New York, as he wished to visit some relatives. Wheaton had kindly obliged, giving him three days in the Big Apple.

Moishe had some relatives in the Queens borough, but after the first obligatory greetings and excess food, Moishe told his relatives that he had an errand in Brighton Beach. He did not tell them the nature of the errand, which had to do with his intention to rid the world of the last Amalekite. Poison seemed the most reasonable option, as Moishe had no intention of spending years in a US prison. But there was one complication. In order for the poison to fit Moishe's requirements, a 'designer poison' had to be created. Moishe had found the address of a very clever Jewish chemist in Brighton Beach who worked for the Abergil family, a family with a reputation for having its own peculiar outlook on the laws of the land.

Moishe arrived at the address given. The chemist tried to greet Moishe in Hebrew, but Moishe assured him that they could speak in English. The chemist then explained to Moishe that such a specific poison could be designed precisely for Bill, but it would need two things: a bit of Bill's DNA, and time. The DNA would be no problem, since Bill had a full head of hair, and therefore a barber somewhere, who could be found and either bribed, threatened, or both. But again, this would take time. Then the DNA analysis, and finally the formulation of the poison. However, the chemist said that the compounds needed to make the poison would be among six compounds, and would be one of five hundred different recipes for making it, and that he would know which one in about a month.

"But I will be out to sea! I will be almost to Tahiti by then," Moishe had objected.

"No problem," the chemist told him."This is not the first time such a difficulty has arisen." He handed Moishe a flash drive."If you open the only document which is on there, you will find a book which listed all the possible recipes, numbered from one to five hundred. All I have to do is to phone you the correct number, and you can look it up from there."

Moishe explained that all communication would be cut while he was on the ship, and in fact all electronic devices. The chemist considered; he then asked Moishe to leave the flash drive with him, and to come back the next day.

Moishe came, and the chemist greeted him with a smile."I have found a solution." He handed Moishe a book and a booklet."The book is the same one as on the flash drive. Destroy it after using it. The booklet has all the numbers from one to 720, with a six-letter word after it. Each word is composed of one of the permutations of the letters r,o,y,g,b and v. They stand for the colors red, orange, yellow, green, blue and violet. They are the colors of the LBGT movement

flag. There will be a large gay pride festival in Tahiti the day before you arrive there. To announce it, there will be a plane which will trail the flag in colored smoke behind it. That will be visible from your ship. There is a fixed order, but few people pay attention to it. Read the colors as if the plane were at the top of a page."

"You are dealing with homosexuals? But the Torah says that homosexuality is abominable."

"Sure, and the Valkyries on Viagra,..."

"Don't make blasphemous puns like that. The book of Leviticus is called Vayiqra in Hebrew. Anyway, the Valkyries were female."

"Whatever. Didn't mean to offend. Yea, I know the passage. It also recommends the death penalty for homosexuality. But the Sanhedrin, the ones who are supposed to do the executing, don't exist, and in general the world of 2700 years ago doesn't exist anymore. Even the Orthodox don't go around stoning adulterers. Lots of Jews today accept or even support the LGBT community,, at least the Reform and Reconstructionist ones. But I know you're Orthodox, so if it's any comfort, Rabbi, I am straight. That passage in Leviticus doesn't say that approval of homosexuality is abominable, now does it?"

"Er..." Moishe realized that he was talking with someone who would not stop at murder, and so his talk of sin was a bit superfluous. Instead, he looked at the booklet and objected, "How am I supposed to find that word among all those words? If this were on a computer, I could just use 'find', but it's on paper."

"Not to worry. The permutations are listed in an order that is easy to understand. You have a couple weeks in your cruise; that will give you ample time to understand the system. Then it will be easy."

Moishe had his doubts, but he figured he would manage. He took the pills containing the compounds, the forged prescriptions, the book and the booklet. Moishe asked,"How much do I owe you?"

The chemist looked at him and said, "Rabbi! How could I charge you? Just pray for me, will you?"

Chapter Seven

January 11, 2045

The stage was set, and the day of sailing had come. Most of the guests came a little early to wander a bit through Pier 39, a popular tourist attraction nearby. Knowing the tendency of those new to the area to get a bit lost, the sailing was actually set for a bit later that the time announced to allow for latecomers.

After the ship left the moorings and an hour was given for everyone to get settled in their cabins and to get to know their way around the ship. a general reception was held in the impressive main hall. As first impressions were important, Bill had emphasized that the lighting had to be meticulously planned. Daniela and Chaim were ready in the lighting box.

Daniela looked down at the audience and asked her brother, "Which one?"

"Tal is the tall one. No pun intended."

"It is too bad, he's sort of handsome." Upon a dirty look from Chaim, she said, "I'm joking, Chaim. Do you think we can do it so that he will repent before dying? We must worry about his soul as well as those whom his hate sheet corrupts."

"Daniela, we may be able to disable him without killing him. The goal, remember, is to remove his influence, not him."

"But if that's the only way?"

"Then so be it."

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January 12, 2045

Bill's lecture took place the next evening; it went more or less successfully. Contrary to Andy's fears, Bill's silencing mechanism did not produce any angry outbursts or threats of legal action.

After the lecture, a reception with refreshments took place.

Moishe knew that he would be searched, and anyway Bill Wheaton's bodyguards would never be very far. He carried no gun or knife in his baggage; the only unusual things that were in his baggage were a large assortment of pills, which however he had the prescriptions for, compliments of the Abergil family in Brighton Beach.

Moishe listened to the lectures. At first he had had no intention of paying any attention to the content of the lectures; he would only take the opportunity to study his intended victim. However, there seemed to be little that Moishe could learn about Bill that would change his plans, so Moishe, who hated to waste time, found himself trying to follow the lectures.

Tal tagged along with Moishe. At the reception after the conference, Julia also wanted to listen to Bill, in case her bugged mikes missed something over the hubbub. But she also was interested in this tall man, obviously only a friend of the other one, since the other one was wearing a kippah. So, Tal, Moishe and Julia came to Bill Wheaton and introduced themselves. At first there was a bit of awkwardness as to who should get Bill's attention, but Bill graciously invited them all to a small table that seated four.

Moishe sat silently, observing. Julia broke the silence."Mr. Wheaton, I was wondering why we weren't allowed to bring our phones."

Bill smiled and said, "This cruise is not only a *Scientific American* cruise. It is also mine. I have two purposes in mind. One, I invited reporters from all over the world, to whom ... and yes, I still use 'whom', I read a lot of classical literature ... anyway, to whom I will make an announcement. But the news shall be released when I want it, thus the prohibition on phones."

"And the other purpose?" Tal asked.

"The other? Oh, yes. It is also a recruiting cruise. I invited not only reporters, but people whom I thought I could use to work for me."

"At Wheaton Informatics?"

"No. I will be retiring soon, but I have founded a hospital on the island we are sailing towards. A bit isolated, but in this era of ubiquitous computers, it seems that this doesn't matter much anymore. I invited a number of journalists to see which ones might be eligible for a public relations position. Mr. Kishon, you are among the candidates because of your skill in balancing different opinions in your paper, despite not sharing them; that is, to asking your readers to submit restrictions of their theories, and then you find a theory that will subsume both. ... Yes, Mr. Kishon, I know your stance as editor of and contributing author to *The Foundation*. You write quite well. If you tell me you would accept such a position, I will subject you to certain tests during this cruise. Otherwise I will leave you alone and hope that you enjoy the lectures. You will find a list of the conditions in your mailbox; I can tell you now that they are very generous. If you refuse, then you will be given the same amenities as the other guests. If you accept, then I will inform you whether I am offering you the position. Then you will continue with me on the boat to another destination. You will not need to return to the US; I will arrange to have your belongings shipped, your apartment subleased, and so forth."

Bill turned to Moishe.”Mr. Cohen, you are aware of the circumstances of your invitation. You will be asked also to preside over some circumcisions. I will explain that later, but I need you as a rabbi. I have asked the orphanage whether they can spare you for a bit longer than previewed. In fact, perhaps you would like to be the first rabbi for our community. We do have some Jewish members on our research team. Mr. Kishon, here, I believe would welcome...”

Tal coughed.”Um, Mr. Wheaton, do you know the difference between the different Jews?”

“There’s a difference?”

“That is correct, sir. It is a bit like entropy, sir.”

“I don’t see the connection.”

“In the concept of entropy, as I understand it, there is some relation which is taken to consider things equivalent, and then all the things which are equivalent are put into the same class. That is, according to one relationship, you can’t tell the difference among the members of one class. But according to another relation, you can. So, the definition of entropy uses both relationships: the measure of the entropy with regard to these two relationships is the number of relationships there are according to the more discerning relationships of an equivalence class based on the less discerning relationship.”

“That was quite a mouthful, but what does that have to do with...?

“You are using the relationship between two people as ‘are both Jews’. This puts everyone into one big set in your model, and that can work. But there are other relations that will make other classes of all those who, for example, are Ultra-Orthodox, Orthodox, Reform, Conservative, secular, ... or you can also divide them into Ashkenazi, Sephardim, and so forth. Or you can divide them up into ....”

“I get the idea, Mr. Kishon. You are not the same kind of Jew as Rabbi Cohen. Will this be a problem?”

The two looked at each other with suspicion, but Tal said, “No, I think we can coexist.”

Bill nodded and said,” “That’s settled, then? Good.”

He turned to Julia.“As for you, Miss Volkova. I did not invite you, but you got on the list. I know the kind of protection our websites put up, and I am impressed that you were able to circumvent them. In a word, you are an able hacker. This is a skill that I will need occasionally, although less publicized as Mr. Kishon’s and Mr. Cohen’s roles. In other words, I am asking whether you would like to be a candidate for a well-paid job on the island. The same conditions apply as I just explained to the other two, with an additional requirement, that if you get the position, you restrict your personal hacking to your own time and responsibility.”

Julia nodded. She had yet one more worry.

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January 16, 2045

Julia went back to her cabin and put the “Do Not Disturb” sign outside the door. She had no intention to let her candidacy for a job under Mr. Wheaton stop her duty to finish the bugging job. In fact, her candidacy gave her a personal reason to find out what her possible future employer was planning.

She and Natasha had put their plan into effect; it was now up to her to make sure it ran smoothly, but also to stop the other industrial spies. Natasha had hacked the list of invited guests, and by cross-checking figured out which ones were most likely industrial spies, not being invited for any other reason. To stop them, she needed to know their plans. The ship had not allowed internet connections, but they had allowed a ship’s Ethernet on the computers which were part of each cabin’s accessories. That was all Julia needed. She admired Natasha’s foresight in arranging

her to be in a private room; most guests were paired up in their cabins. She turned all the microphones on in all the other cabins so that they worked even when the computer was off.

She assumed that the spies would look for any hole in the veil of silence to take advantage of. Naturally, they might look at communications inside the ship that the crew and security staff depended . These would be purposely made too weak to be captured from a distance far from the ship, the outer walls and hull of the ship would provide substantial interference. But they would provide a cover in case any spy wished to bug a room. That, however, was probably not high priority, first, because the measures for the trip indicated that the important news would be given publicly; the only concern was to get this information off the ship before docking. As well, the rooms where Mr. Wheaton would hold his private conferences would almost certainly be searched for bugs. Julia wasn't sure how the other spies would do it, but she was determined to find out.

Julia's first eavesdropping session was on the security planning room. There was a discussion about the fact that the only other ships that were in the area were one a short distance ahead of the cruise ship, and another one which was a short distance behind them. The fact that they kept the same route as the ship was not surprising, since this was the most efficient route to Tahiti. But their proximity caused some concern, and it was decided to put boats full of electronic detection between the cruise ship and these other two ships. Julia found this quite amusing for reasons that no one on the boat would understand until it was too late.

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January 16, 2045. Evening

Natasha had explained to Julia that each industrial spy team consisted of a traditional spy and a science consultant. This was convenient, because the science consultants constantly needed to explain their steps to the suspicious traditional spies, in layperson's terms. In Room 354, such a discussion was going on. Julia listened in.

"Of course you haven't heard of it, Jake. It's cutting edge. The general principles were known already last century, and they could even do it at short distances at the beginning of this century, but how to extend it to these distances... well, that was only figured out recently."

"But Paul, how does it work? I'm not going along with any scheme that I don't have at least a general idea of."

"The details are way too complex."

"OK, but can you at least give me an idea of the principles at short range?"

"Well, there's no activities on tonight, so I might as well. I'll start from the ideas presented in the lecture."

"I wasn't paying a lot of attention."

"But maybe something seeped in. If not, you can re-run the lecture on the ship's Ethernet. As you should know, the particles are just convenient ways to talk about certain localized variations in the truth values. You heard about the entanglement and the difference between changing states and measurement. So, we split the changing of state part into two parts – part of it will accompany the measurement, but the other part does not. If you only have one of the parts, you do not get information transfer. So, the measurement part, the classical, can be captured by the censors, but since they don't have the other part, they do not have all the information. On the other hand, if you only have the non-measured part, you also don't get the information. So, you have to wait to get the classical message through regular causal paths, which of course takes time."

“We needed entangled pairs. That was that satchel that looked like a bunch of magazines that we brought on board.”

“And where does one get such things?

“They are actually quite common. For example, two electron orbitals in the same orbital around an atom are entangled; there is no model where they are independent. There are ways, using magnets and stuff, to entangle two particles, used in quantum computing. Thankfully, our contacts have their own quantum computer which they used to produce them. Stabilizing them was the major problem...I mentioned that already, I believe. Anyway, we brought one half of them in our luggage. We record the information with a classical bug, code it into states of a bunch of electrons. Then every time we want to transmit some information, we ‘infect’ some of the particles whose entangled partners are located with our partners at headquarters.”

“That’s a long way away.”

“No signal is passed. In the model which satisfies all this so far, there is no role for space-time, so the space-time separation doesn’t matter. This ‘infection’ and a few other manipulations will then affect the partner electrons where you are. This will then affect their partners, located wherever you are keeping them, without having any signal passed. We then measure the original piece of information, and send headquarters this measurement by classical means. Those at Headquarters do some manipulations of their particles, which after our measurement are independent of our particles, based on this information. Then they measure them, and they get the original state. With enough of them, they can now get the information that we are sending.”

“I see a problem. The sending of the classical signal.... that does require a signal. But they are all blocked, except intership communication. And that is filtered strongly, with a program that takes the waves apart by something called Fourier analysis, and analyzes them to see if there are any hidden messages mixed in, perhaps masquerading as noise. That’s hopeless.”

“Agreed. But they don’t filter their GPS signals. Headquarters infiltrated into the firm that outfitted this ship with its GPS, and I have hacked into the ship’s navigation system. Headquarters also has its own satellite orbiting close to the BPS satellites, and it will pick up the signals, which will be mixed with a certain amount of noise, which will be the information that we are getting paid to send. Are you satisfied?”

“Why didn’t we just send the information directly with the GPS signals, then, instead of all this fooling about?”

“We have to be ready for the possibility that it is intercepted and decoded. If the entire message is intercepted, they would know what we are hacking. This way, the message will seem harmless. Put another way, we cut the syntax down so drastically that they cannot reconstruct a reasonable interpretation function.”

The last remark, Julia thought, was pretty superfluous, but apparently all the hackers in the world were undergoing retraining that included Model Theory, so its vocabulary was seeping into their language.

Julia was not interested in the rest of their conversation. Natasha had shown her how to insert false memos into the communications between the different ship’s compartments, so she inserted one recommending that the same filters which analyzed inter-ship communication also be adjusted to be able to analyze the GPS signals, The details were for the engineers to work out. In this way, Julia crossed Paul and Jake off her worry list. She had one more pair of spies still on her list; she would wait.

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January 17, 2045

Julia didn't have to wait long. The couple, supposedly a Mr. and Mrs. Jones, were discussing, as Julia listened in. Mrs. Jones was explaining, "It's a kind of beaming, so we don't have to worry about any measures that the ship uses. It's called quantum tunneling. That comes from the fact that there are operators which do not have common models."

"What does this common model stuff have to do with anything?"

"Think about it. If you have the limits on the momentum, this will automatically adjust the position limits. The main arrow here is momentum-position, and the changing of the possibilities of one changes the possibilities of the other. So, what you can do is to first change the momentum possibilities, using lasers and whatnot, by giving the position a wide spread, and then, take away limits of the momentum so that your position has to be more precise. So it will pop up somewhere in the original limits. It does not mean that it will pop up each time where you want it, but statistically, some of them will.... and you would know how many of them will. Now, you have a sort of virtual bar, a dipolariton they call it, something like a bar magnet. Now make another one of these virtual bars on your side, so that one end of yours will interact with the end of the one from the ship. Then you can get the sum of the probabilities to add up at the place you want them to appear. It's not exactly that the particle has disappeared in the ship and reappeared in your outpost, skipping the censors. No, the particle was in the whole region between the ship and the outpost, but unless the censors know what to do to capture that particular particle at that point in space-time – which is unlikely, given the precision needed – then it does appear that way. The particle is in the whole region, but it is measured in the ship and measured in your outpost, without being measured by the censors."

Julia worried about this. But she redid the calculations... and she hacked into the navigation system again. She noticed that the Jones' were using an app on the computer to do the calculations every time they needed to send a message. So, Julia hooked up that app directly to the ship's gyroscope system, and the ship would tumble every time the app was in action. That would deprive the Jones' of the stability they needed for the beaming. She crossed them off her worry list. She was glad to have done this before they were to dock at Honolulu Harbor. Obviously, whatever was being protected would not be publicly known yet, since from Honolulu there were lots of opportunities to send information. The next week should see something interesting, and she would be the only hacker to have access to it.

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January 20, 2045

At Honolulu, other speakers boarded. Bill was glad to have finished his round of lectures. His more important task, for which he was better prepared, was to be initiated at the dance that evening.

At the dance, Bill stepped up onto the stage."Ladies and Gentlemen. Oh, and you others as well."

There was the expected polite titter, although no one found it funny. Bill continued,

"First, I would like to announce that I am disbanding the so-called Chinese Room."

Everyone in the room knew the rumors. On one side, he had apparently stuck to the market demands of specialized artificial intelligence. He had made the best machine translators available, and then had surprised everyone by buying up a dying pharmaceutical technology firm and making it a success. All thoughts of making a system with over-all intelligence seemed to be definitively dead. Yet there had been the rumors of a third part of his empire, a firm that pursued this goal, taking some of the profits of the other two. This mysterious third firm was dubbed "the

Chinese Room." The books of both firms had been closely audited, and no irregularities were found, so legally the matter rested there. But the legend stayed on like an old Elvis legend.

"Not because it is a dead end, but because it was deemed to be unprofitable, and the only way to counter these arguments was with hard cash. The concept has proven profitable, and this evening I am going to reveal the proof."

The reporters were frustrated. They had anticipated some sort of announcement, and had pleaded, in vain, with the crew to let them use the ship's phones. Now all they could do is take notes.

Bill went on, "Ladies and Gentlemen, I was not joking when I addressed the 'others'. Among the guests are a mixture of humans of my confidence as well as..." he paused for dramatic effect,"...fully operational androids who have been my translators. You see, it was often said that a true translation could only be given by someone who had experienced the complexities of human life. Therefore I have made androids about whom I have no doubt in my mind that they are just as conscious as any of you.

The *Wall Street Journal* reporter asked the inevitable question, "Regardless of whether they are conscious.... how do you intend to market them?"

"Market them? Would you market your friends? A theatrical pause, and then he said, "Well, maybe some of your editors would." Mild laughter. "But these should be given a chance to exercise their free will..."

The reporter from the *Christian Science Monitor* interrupted."How can you talk of free will?"

Bill sighed."I don't want to get into a philosophical discussion, but..."

"But you're going to give me the usual bunk about some as-yet-discovered perfect laws of physics that can act as a theory for everything we can think, aren't you?"

"No, although neurobiology is marching in that direction. No, I was just going to point out that, whatever it is that is making your decisions, it is either subject to some logic – even if it isn't the logic of physics, although I don't see why not – or it is random, in which case your decision-maker is random, which doesn't seem to be the case. If it is not random, then your will is subject to this logic and hence not completely free."

"That's sophism, that is. After all, your particles managed to have a free will. That is, there are undecidable statements which are nonetheless decided by the march of time and space."

"I will agree that electrons and so forth have a sort of freedom, but I think the word 'will' would be misplaced. The electron doesn't will, it just is. But I must apologize, we could go on with this discussion for days. The man in the blue suit, please?"

The reporter for *Le Monde* asked, "Monsieur, how did you instill consciousness?"

"That remains my trade secret. Therefore I leave you to try to figure out, between here and Tahiti. Try to find out who they are, without any tricks such as 'accidentally' cutting a guest. Such tricksters will be immediately thrown out of the gathering, thereby missing a newsworthy second announcement," he said."Oh, and don't trust the Three Laws of Robotics from science fiction, they aren't programmed with them."

The reporter from the *Dallas Morning News* asked his neighbor, "What is he talking about?"

"You know, the Asimov books," his neighbor told him. The first one being not to harm humans, the second one to obey humans unless it goes against the first one, and only then the third one, with self-preservation if it doesn't get in the way of the other two. Amusing axiom system, but many of Asimov's robot stories are telling why his Three Laws run into difficulties. After all, axioms are supposed to be independent, and the Three Laws are clearly interdependent."

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January 20, 2045

After he left the dancers to be looking at their partners with suspicion, Bill went into a neighboring room, where Jutta was waiting for him.

"So, ready for the island?"

"Yes, but are you sure that the paparazzi won't be following us?"

"Why? I will be retired. Anyway, remember Jim?"

"Yea, you helped him through his exams.... Whatever became of him?"

"He's a reporter for the *New York Times*. He's here tonight. He owes me, and he knows it. He'll be the first reporter with the story about the location of my retirement, and about the infospital. He'll write a story that I, well-known to be eccentric, gave away most of my money to the hospital, which will not only serve the residents of the island, but also have a large pediatric section devoted to providing free care to infant orphans with brain damage. I will live in a modest house near the beach. All true, and will be presented to show that I am now a very uninteresting person. In the article there will be no mention of my special privileges in the hospital. Perfectly legal, by the way, since it is my money."

"Why infants?"

"Their brain is more plastic, increasing the chances of success."

"Why orphans?"

"Less limelight. No parents wanting to know every detail."

"Why Israel?"

"Why not? They were among the possibilities from all the hospitals, and they were the most flexible in giving permission for treatment."

"But there will be cruise ships, tourists, the occasional reporter doing a human interest story..."

"I will hire a public relations officer, teachers, that sort of thing. Oh, the time. I've got to go; I've got a couple of bombshells to drop. I invited several reporters besides Jim."

The last day of the cruise

He asked for silence.

"Ladies and gentlemen... has anyone figured out anything conclusive? Yes, Professor Sanchez?"

"So, you did it. We can't tell the difference. Your androids pass the Turing Test. However, that alone doesn't prove consciousness."

"No, I know the Turing test has long dropped out of the running for this. This was just a cute demonstration, not the real proof. But it doesn't matter what you think about their consciousness. I am convinced of it, and my company is the one paying them on that basis. Legally, it was a bit tricky, but de facto they are under contracts with almost all the rights and obligations of a human employee, including access to funds. Wheaton Corporation has obligated itself to continue this policy, even after my resignation, which the board is considering at this very moment. We will dock soon, and you will be free to get back to your papers."

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January 26, 2045

The evening before their scheduled docking in Tahiti, Bill summoned those who had accepted the positions he offered them.

He first summoned Julia.

“Miss Volkava, I am still considering positions. I just have a few more questions to ask you. It is possible that you have heard these before, since they are standard fare in Philosophy faculties. But bear with me.”

Julia nodded politely, waiting for Bill to continue.

“Suppose you were to be able to code yourself and transfer everything about you into another substrate, another body, in such a way that the original was destroyed. Would that consciousness still be you?”

“I know where that is going. On one side, you can use the grain of sand argument.... starting with the transplants and prostheses used today... you can slowly replace cells in the brain,... and so forth. In the end, it is still the same person. On the other hand, you can do this all at once instead of bit by bit. Am I going right?”

“I see you’ve been to coffee shops. Continue.”

“On the other hand, now you repeat the experiment, except this time you do it in such a way that the original is not destroyed. The standard answer in science fiction is that the original one is the same person. But in what way is the duplicate this time different to the first one?”

“Can you answer that?”

“I know the standard cop-out. Every moment is a different you, and there is no original at a given moment. So in the first case you are creating a new individual in the same way that living in time does. In the second case, you are creating two different individuals instead of one. They are different individuals. In fact, they are not identical even at the time of creation.... they occupy different points in space, for example. There are some other reasons that they aren’t the same, but they have to do with some quantum stuff.”

“We needn’t go into that now. I have decided to offer you the position. Do you accept?”

“With pleasure. Was that my test?”

“Oh, no, that was just a detail. You underwent the test earlier, with flying colors. You were asked to bug this cruise, and thwart the efforts of your fellow hackers. You did a magnificent job of it. I still am not completely sure how you did it.”

“But... *you’re* the mysterious client? Telling me to spy on yourself? That’s sounds almost narcissistic.”

“So you’re not going to tell me how you did it? Professional secret?”

“A professional secret is for those who repeat themselves. I don’t intend to use this trick again, so I might as well tell you. Fish.”

“Come again?”

“Fish. Your security checked for transmitting devices. They did not care that I brought a lot of my own spices. Or what looked like spices, but really was a composition of nanochips. I downloaded the day’s information onto them every evening....

“Ah, not only the public announcements....”

“No, sir. Your hearing aid was bugged when you went swimming. That transmitted a weak signal that my receivers could pick up. The information went into the nanochips, that went into the food I didn’t eat, that was then put into the waste that was thrown overboard despite the filters, and was eaten by fish which were bred to go for the waste, and themselves be especially appetizing to dolphins. The fish and, by an appropriate time lapse, trained dolphins who had a mania for these fish, are released at the right times by the boat in front of us, and the dolphins, after eating their fill of their favorite fish, swam to the boat in back of us. The chips, by the way, could be retrieved without hurting the dolphins.”

“Good God. Quite elaborate, but no quantum principles involved.”

"No, sir, if you don't include the quirks of the nanochips, where the dimensions are so tiny that the electrons can tunnel, and things like that."

"Ah, well, I look forward to working with you... Miss Kim," he added, making a gesture that the conversation was finished."

The interviews with the other candidates amused Bill a lot less. Among others, Tal and Moishe also accepted the proffered positions. Now the hard part, he thought.

Chapter Eight

February 1, 2045

The cruise ship didn't actually dock in Tahiti. The ship informed the passengers that small boats would pick up the passengers from the boat and take them into the island. The ship itself would make a side trip, but would return to pick everyone up as announced on the schedule, in five days. Lectures would continue on the island. Accommodation and food in a five-star hotel in Tahiti were included in the price of the cruise.

The reason for this arrangement was not given. These stemmed from Bill. To avoid all the hassles of disembarking and re-embarking the children, Bill had arranged to have them stay on the ship, and the ship would then sail for the small island which Bill had christened after himself, at least for the duration of the lease. As well, a number of the staff that had been hired to make up the infrastructure of the island had flown directly to Tahiti; they would then embark on the ship and continue to Wheaton Island with Bill, the children, and the passengers whom Bill had recruited during the voyage. There the ship would leave them, continuing the cruise without them.

Among the new arrivals on the ship was a team, jocularly called the Nightmare Team, consisting of physicists, programmers, mathematicians, biologists and linguists working on the program for the computer. Bill had to apologize to Jutta for misleading her with the impression that the programming was complete. It was not. It had been arranged that there would be a meeting immediately after the team embarked to brief Bill and Jutta on their progress, as well as to consult with Bill and Jutta on a couple of non-scientific details.

The chairwoman began the meeting by saying, "This is a good moment to figure out an appropriate name for the project, rather just calling it 'The Project'. As this is not a matter of scientific expertise, our distinguished patron," she motioned towards Bill, "is invited to take part."

For a half an hour, various names were advanced, but the name "Paralife" won the day.

Before recording this as official, the chairwoman did a quick check on her tablet; Internet access had been restored to the ship. She remarked, "The name has been taken. Perhaps we need to check the copyright...."

Bill remarked, "When you want points of legality that are worse than dubious in this project, Madam Chairperson, I can give you a long list. A greater concern would be that going into the computer like that seems to me to be claustrophobic."

"You shouldn't worry, Mr. Wheaton," a biologist countered."Your brain is just...a bunch of nerves firing in a tiny skull. You know the famous Hamlet quote.... 'O God, I could be bounded in a nutshell and count myself a king of infinite space...'"

"Yea," Bill replied, "but I know how that ends, too, '...were it not that I have bad dreams.'"

Jutta intervened."You'd have bad dreams in or out of the nutshell, Bill. What I don't understand is the two main guidelines. One, it is supposed to be science fiction, but two, it

doesn't violate physical laws. How do you reconcile the two? Doesn't science fiction always involve a spaceship going at superluminal speed or through some other spacetime and stuff like that to get places? The galaxy is too big to get anywhere at a normal speed."

"Paralife will be on a spaceship that will have become the permanent home for the passengers."

"So, their minds – or maybe just the patterns of their minds – come into existence complete with this spaceship and..."

"Not exactly. The program will already have been running for a while when Mr. Wheaton and the orphans go in."

"And how do they get back? They say 'end program', or get killed, or..."

"Neither one. Normally, if a player is killed in Paralife, the computer then will transfer her mind to one of the bodies in the ship which had been put under a kind of hibernation, which can keep the bodies at the same biological age. They become a new character in the simulated reality, and they keep going."

"So they have to know the background of each character? That's a bit much."

"I have an idea. We put in the program that the ship has been underway for a few years, and along the way there was an accident that wiped out all the biographical data of those under hibernation. Let's call them, uh... 'Frozens'. Then they can make up their own history."

"Excellent. Put that in."

"But you still haven't told me how they are supposed to get back, if death doesn't do it."

"Death will, but only together with a block of the transfer to a new Frozen. That will be controlled by us, because we will know when they are ready to be transferred to their new, real, body, not them."

"Why not just stop the program?"

"First of all, we want to keep the program running for other patients. Anyway, stopping the program with them in it would wipe out their minds, essentially killing them for real."

"OK, that's settled. Then this is transferred back to an almost virgin brain, give or take the basic bodily functions which will be kept going. Will they remember anything of it when they come back?"

"Yes, indeed. The transformation into the computer, then the transformations inside the computer, and finally the transformations back to reality, end up not the same as transforming to and then from the computer. Doing it right, the difference between the two will correspond, as far as memories, to the difference inside Paralife."

One of the physicists looked at the scribble sheet of a programmer.

"You're going to save yourself a lot of trouble if you will put that stuff in space-time coordinates, instead of doing space and time separately."

"Why, what difference will it make?"

"If you use space-time coordinates, all your laws of physics will be objective, in that they will be the same for everyone. For example, the measure of the interval of spacetime will be the same between two events if you only move or rotate the system of axes that you use to measure the events, so causation order remains the same from all viewpoints, and the speed of light and other physical constants are like tautologies, being the same in all the models."

"That sounds rather stilted. Why not just say that the spacetime distance remains the same?"

"Because it's not a distance in the usual sense."

"Space-time coordinates are just space coordinates plus one, right?"

"Yes and no. It depends what you envision as an extra coordinate. The relationship of the time coordinate to the space coordinates is not the same as the relationship of one space coordinate to another."

"Why, what sort of relationship is there in space-time?"

"Although space and time are actually shadows of space-time, it is easier to describe it as if we already had a measure for spatial differences, and already had a measure for time differences."

"OK, I'll attack you on that later."

"We start with one where the space part is flat. That is, where the Pythagorean Theorem works. Take two events. You have two things you can directly measure: One, the spatial distance between two events. Two, the minimum time it would take for light to pass from one to the other. To put the two together, in order not to be comparing apples and oranges, we reduce them both to the same thing. We can pick, for example, spatial distances. Since the speed of light is constant, we can use it as an independent arbiter. With that, we can turn express time in terms of spatial distance by making the light-distance be the distance that light would cover in that time."

"Could we also do it all in terms of time instead?"

"You could, by expressing the spatial distance in terms of the minimum amount of time light would take to traverse that distance. But I prefer to do it with spatial distances, if that is all right with you. I'll call the original spatial distance between the two events the space-distance, and I'll say, rather clumsily, the 'time-distance' for the spatial distance that the light would cover in the time difference between the two events."

"OK, I'll forgive you. With both time and space differences expressed in terms of spatial distances, do you just measure the spacetime distance as if time were a fourth spatial dimension?"

"No, no. That's what makes it so special. You can use a right triangle like you do in spatial distance, but in this case you arrange things a bit differently. We have two cases. One is when the events are close enough that the space-distance is less than the time-distance. That is, there is the possibility that one event is the cause of the other. In this case, you can make a right triangle with the space-distance being one of the legs of a right triangle, and the time-distance would be the hypotenuse. That's the longest side, in case you've forgotten. Then we look for the distance of the other leg of that triangle."

"So, do you call it a spacetime distance? Wouldn't that get mixed up with spatial distance?"

"Yea, the word 'distance' does get misunderstood by people, but everyone in this room knows what it is. Would you prefer me to use the word 'metric' all the time?"

"No, that's OK. As you say, the spacetime distance is not really a metric, or distance. Separate events get zero measure, and even that bit about inserting another point on your path not diminishing the measure, even that does not always hold. It looks like it."

"What happens when the events are further apart, so that the space-distance is more than the time-distance? That is, the two events are causally disconnected. It doesn't fit into the right triangle anymore."

"Another one said, "No problem. Then we can rearrange it. This time make the time-distance be one of the legs of a right triangle, and the space-distance be the hypotenuse."

"Then make the other leg be your spacetime measure."

"Not quite, because then you would get it mixed up with the other situation. To distinguish it, you take the negative of the length of that other leg as your measure of the spacetime difference."

“Ah. I see why you couldn’t call it a distance. In fact, it’s a bit weird. Obviously the measure would end up with zero if the two events were at the same time and place, but also if they were separated in both time and space by the right amounts.”

“Right, so if one event emits light, and that light gets just in time to the other event, they are in a way equivalent.”

“Hm. I will have to think about that. But, um, what if I wanted to do it the other way, and make the cases where they causally disconnected to be measured with a positive number, and where they are possibly causally connected to have the negative sign?”

“Easy enough. No problem. In fact, some people like one way, and some people like another way. It’s up to you, as long as you mention somewhere early on in the discussion which you are using, so that you don’t end up talking at cross-purposes with others. Anyway, one takes the square of this measure and calls it a spacetime interval to talk about the quantity that doesn’t change between two events, so the difference in notation doesn’t usually bother.”

“Ladies and gentlemen, I think that this is enough for now; I have an appointment before dinner. May we resume this in the morning?”

&&&

February 1, 2045

Guests on the cruise had three options for eating: they could come to the common eating hall, they could order food over an intercom from their room, and the food would be delivered via a kind of sophisticated dumbwaiter to their rooms, or they could prepare their own food in a mini-kitchen in their rooms, with ingredients freely available through the same ordering system. The plates, glasses, silverware, cooking utensils and so forth would be returned to the kitchen by the reverse route.

Although Bill went to the main eating hall for the festive occasions, he preferred to eat in his cabin, ordering prepared dishes through the intercom. Moishe had bugged Bill’s intercom, and had also learned where he could intercept the dumbwaiter. Throughout the trip he had practiced cooking the less complicated dishes which Bill preferred. His plan was to substitute his preparation for the kitchen’s in one of Bill’s meals towards the end of the trip, to which Moishe would include a tasteless slow-acting poison which would make Bill’s death appear a natural one. Just in case, he would also switch Bill’s plate back before it got to the kitchen.

But the success of the venture would depend upon the correct mixture of the poison; that in turn would depend on finding the right formula, which in its turn would depend on Moishe understanding the pattern in the booklet that the chemist had given him. The chemist, Moishe reflected, would not have made a very good teacher. A common mistake of many teachers, and people in general, is to assume that something easy for them is easy for others.

Moishe sighed, and once more bent over the booklet to understand the system.

As he was sunk in this study, he noticed that someone was reading over his shoulder. He looked over to see a girl of about 10 years old looking at the book. She said, “I see how those are arranged. Do the letters mean anything?”

“Who are you, young girl?”

“My name is Dora. I am the chief steward’s daughter.”

“Pleased to meet you Dora. It’s just an exercise.”

“What do you have to do?”

“There are different arrangements of the same six letters. I have to figure out what order they were put into.”

“Oh, that’s easy. We have to do that in mathematics class.” Dora quickly explained the system. Then she asked, “What do the letters stand for?”

“Oh, just letters.”

“Shame. I thought the letters might stand for the traditional colors of the rainbow, but I see that indigo is missing.”

Moishe wanted to divert Dora’s attention from the booklet; the best reply he could come up with was, “Why traditional?”

“We learned about rainbows in school. Our teacher said that Newton made up seven because he wanted it to be the holy number. Rainbows don’t always have seven colors.”

“Ah, that’s interesting. Do you know why seven is a holy number?”

“Because it says in the Bible that God took a relaxing six days to make the world, and chilled out on the seventh.”

“I’m not sure that’s the way it was stated, but essentially you are correct.”

“But our teacher says that the Bible just took the number of days in a seven day week.”

“You don’t think it was the other way around?”

“Naw. The Romans used to have both eight day and seven day weeks so that they could add up to a lunar month. If there had been eight days in a week, God would have devoted one day to something else. Like, maybe devoted an entire day to mushrooms. Or maybe done a better job in creating humans.”

“I think I should have a word with your teacher.”

“Anyway, I think it’s holy because it’s a prime number. Do you know what a prime number is?”

“Yes, Dora. I did not have much mathematics, but a number bigger than one which can only be factored into the two positive whole numbers, itself and one...” Moishe stopped himself.”But no, Dora. The number seven comes up all over the place.”

“My teacher says that this is like the Internet: even if the original post is debunked, a false idea can spread to so many other sites that the original source is no longer looked at. Like the seven planets making seven special.”

“It comes up in the Torah more than other numbers.”

“That was intentional. So that’s not fair. It’s like the seven metals of alchemy.”

“Well, then music. Do you study music, Dora?”

“Yes we do. So what?”

“You have seven white keys in an octave, and seven semitones in a fifth.” Moishe did not study music, and knew only these two facts from a game of Trivial Pursuit he had played.

“That is not two facts. That is one fact.”

“Huh? Look, Dora, you perhaps know more than me about music.”

“In Western music,” Dora said, proud to show off,”one tries to compose by the same way one creates novels. An interplay between conflict and resolution. That started with Pythagoras...”

“the one who made that theorem about the right triangles?”

“Aren’t you silly. Of course he didn’t make the theorem. It was known before him.”

“But they named it after him.”

“Because the followers of his philosophy were called Pythagoreans, and they showed that they knew about it. So it really could be called something like ‘the theorem used by Pythagoreans.’ Anyway, Pythagoras apparently noticed that when you pluck a string on a musical stringed instruments to get a note, there were some fixed points that occurred either only at the ends, or also half-way down the string, or a third of the way, or a quarter of the way, or a

fifth of the way, and so forth. Then he played around with these fractions and noted that when two of these were played with the same endpoints, the sounds didn't interfere with each other in an annoying way like the sounds did sometimes when the string lengths were different."

"Did he really?"

"I don't know. I think there's a lot of stuff made up because it sounds nice. Anyway, the facts about the strings are true. They called the pleasing non-interference 'harmony', so they called the sequence of fractions the harmonic sequence. I like that, because it's used in mathematics. I like my mathematics class. I will talk about chords made of combinations of two notes, because I haven't gotten very far in my lesson about chords. Also I will only talk about Western music, because we haven't learned about other forms yet."

"That's fine," Moishe said non-commitally.

"I want to play the sitar, but I was told to wait until I first understood Western music. I think that's stupid."

"I'm not the one to say, Dora. I've never been a parent. Please go on with your description."

"I thought you and Mr. Kishon adopted children. Aren't you married to Mr. Kishon?"

"No, Dora. The easiest division was into two parts, so they just divided the wavelength of a sound into a half, then a half of that half, and a half of a half of a half, and so forth. Or, in terms of how often the air made a cycle of pushing and pulling the air molecules, the frequencies were twice, then four times, then eight times, and so forth, of the original pitch. My math teacher said this was expon.... expon..."

"Exponential. Like the world population. Or the Richter scale for earthquakes. Or the multiplication of rabbits. Or..." Moishe stopped, realizing that all these examples were meaningless to Dora, who had grown up on this ship."Um, maybe a better example..."

"That's OK, Rabbi. I understand. I just can't pronounce the word very well. Anyway, so that had this interval of sounds between one frequency and double that frequency. They called it an octave. That is from a word for eight."

"I know. But I don't know what an octave is eight of. Do I have to?"

"I have to present my essay to the class, and I want to practice on you. May I?"

Moishe was not sure he wanted to listen to it, but this was the steward's daughter... perhaps he could be patient for a while, and maybe even learn something."

"I suppose," Moishe told her."Are you going to tell me the history of it?"

"Not really, I want to more or less put it in logical order. So I'm going to call the octave a double-interval until I explain why it's called an octave, OK?"

Moishe opened his mouth to object, but then shrugged and said, "Why not?"

She went on."So they had these chords separated by these double-intervals for the peaceful part. Consonance, they called it. But other musicians looked for chords that would give the most conflict, the dissonance. After a bit of back-and-forth, they decided to also make the smaller intervals also expon...expon..."

"Exponential."

"That's right.

They found an interval which made chords have an unpleasant sound to the Western ear, which I will call the dissonance-interval for now."

"Why not use the standard name?"

"I will, but I want to get to the name. Let me tell it in my way."

Moishe patiently nodded. She continued.”They decided that would be their smallest interval, and figured out about what the factor, the number that would be multiplied each time would be. So they made the intervals one after another, each one a certain factor times the last one.”

“Exponential, you said.”

They found it best to make the factor always the same for each of the smallest intervals, because they found that to be what was important. That made it like... I think there was a word that meant the same form, in Greek or something...”

“Isomorphic? It means analogous, as far as I know.”

“That’s it. Anyway, it was like a certain kind of structure with rules. A band, or a gang, or something like that.”

“A genus? A family? A group? A ring?”

“Ah, both the ring and the group, they said. But they talked mostly about the group.”

“A group is just a collection...”

“They said that in mathematics it means not only a collection. There’s a collection, which is the universe of a model, but there is the rest of the model which then obeys certain rules. So any model which obeys the theory of a group is a group, for example. They said the groups are important because you can find a simple theory, and a simple model that then is like lots of other models that look more complicated. That is, the group is an example of a structure that one finds all over the place. I asked them for an example, and they told me that in cryptography, the kind of group that the doubling-interval makes out of those dissonance-steps is a special kind of group that made the cryptography from the classical computers possible.”

“Classical computers?”

“The kind that were around until the quantum computers made them obsolete for secure Internet connections. But I still have one in my room. I use it to type my homework and to play games on.”

“So music is connected to cryptography? That is sounding like the conclusion of a Spielberg film about aliens.”

“What’s a Spielberg film?”

“The ship’s library has them. It’s not important. Go on.”

“They also found that the chord of two notes separated by two of these intervals was also dissonant, although not as much as the chord with the smaller interval. But all this dissonance stuff is a theory that depends on one’s model in one’s head, formed by culture and stuff like that. In any case, they seemed not to give so much importance to the smaller interval until later, so the two-interval got the name ‘tone’, and the smaller one the subsidiary name of the ‘semitone’. One of those things where notation sticks, despite the logic changing. My science teacher says that the name for electric current....”

“Dora, can you stick to music? I would like to take a rest from physics.”

“OK. Anyway, they found out that also a chord of a note and another note seven semitones down was also pleasant to a lot of people.”

“Aha!” exclaimed Moishe.”Seven, you see?”

“I doubt if that is where the holiness came from. I think it’s a coincidence. After all, a chord with a separation of three semitones is also not bad. Anyway, they also noticed that after seven intervals, the frequency was about one and a half times the original one, and after twelve intervals you had the doubling. I thought that was pretty cool, because I saw a pattern with the harmonic sequence. They called a beginning frequency the fundamental frequency, and then the frequencies that corresponded to the harmonic sequence were one, two, three, and so forth times

the fundamental frequency. So, comparing the first two, you get the doubling interval of twelve semitones, and comparing the second and third, you had this one and a half ratio, and comparing the third and the fourth you got pretty close to the separation of five semitones, and comparing the fourth and the fifth you get close to four semitones. Then it jumps a little, you compare the ninth with the eighth, you get close to two overtones."

"Isn't it impressive how these nice simple ratios...."

"...are an oversimplification, like many of these 'isn't it amazing', or 'that can't be coincidence that...' that people use. These ratios are only approximate, because there are other things happening with the waves. It's nice to make things simple, but one shouldn't throw out data to do so. I think Einstein said something like that. We studied Einstein in our science class. Did you know Einstein, Rabbi Cohen?"

"I'm not that old, Dora. But I heard of him. Anyway, the fractions thing is cute. Is that what musicians do?"

"That's part of their whole theory. But they work a lot with the one and a half and the doubling. The espontspal... esponshal... exponential."

"You're getting it. But all this with the fractions of whole numbers, and the exponentials... are they compatible? After all, I think most of those ratios of the exponential variety cannot be expressed as such fractions."

"Only if you allow a bit of fuzziness at each number. That is, the real tuning starts there and makes trial-and-error adjustments. But they still would like to make the two bases somehow work together. They try to do this with things like the Pimple function."

"Uh, the Pimple function? Are you sure you have the name right?"

"Something like that. The Zit function, maybe. They also said something about a belt. Can belts have zits, Rabbi?"

"The Zit.... Dora, maybe you heard the zeta function? The Riemann Zeta function?"

"Oh, yea. That's what I said."

"I understand the zit and zeta confusion. But why belt?"

"Oh, I think they told me that right after my Russian lesson. Riemann means belt, I think."

"I learned a little Russian. I think it's more like 'remen'. Riemann was a man, and he wasn't Russian."

"Anyway, they played around with this stuff, and they make models of how the human brain models these combinations into emotions and associations. But my music teacher and my mathematics teacher did some team teaching the other day. My mathematics teacher said that, like all interpretations, the model will depend on the universe, so the model that people will make for music will differ from culture to culture, from epoch to epoch. My music teacher gave some examples, but I don't remember them all. Anyway, there aren't many cultures on the ship. My music teacher took examples from the history teacher's videos."

"I know some good examples, Dora. But I am not a good musician. Sometime I will introduce you to Middle Eastern music styles.

"Oh, that would be nice. I think. Although I guess I can't say that until I've heard them."

"Fair enough. But let's go on with Western classical music."

"OK. So they lined up these seven-semitone intervals, one after another, and also the double-intervals. When they started both at the same note, they mostly didn't end up at the same place. Only after seven double-intervals and twelve seven-semitone intervals did they finally more or less overlap."

"Aha! Seven again!"

“Our psychology teacher calls your behavior an obsession. Do you know what an obsession is?”

“Yes, Dora. Go on with your music theory.”

“Anyway, that’s why you have seventy-seven semitone-keys on a piano today. But then notes that were a twelve-semitone interval apart got the same names, sort of an equivalence class.”

“An equivalence class? Is that music terminology?”

“I learned that today in my mathematics lesson. Let’s see if I remember. It’s a collection of things that are considered equivalent under some relationship, because the relationship acts a bit like equality. That is, it is being reversible, and something in that class has that relation to itself. Also, if you have two such relationships with a common thing in each relationship, then the other two things also have that relationship. I think that’s how it goes. I just thought of it for the notes just now. I have to think of how to phrase the equivalence relationship, I think ‘is a multiple-of-twelve-semitone interval apart from’. Or something. I think it could be better.”

“That’s OK.”

“Anyway, they figured that any note from one equivalence class should have the same name. Of course they have different names when you change the relationship by adding which double-interval it’s in, so that it is no longer an equivalence relationship.”

“What, how can something be equivalent and not equivalent?”

“It can be equivalent according to one relation, and not to another. For example, I have heard that you and Mr. Kishon are equivalent in both being Jews, but that you aren’t when one is talking about the direction of Judaism you are in. Right?”

“You have big ears, young lady.”

“They are hidden by my hair. How can you...”

“It’s an expression; keep going.”

“Anyway, then each double-interval had exactly one representative from each equivalence class.”

“That would make sense. So those are the notes on a piano. Why are some white, and some black?”

“That having that relation to its they figured that two notes that were pleasing together, even if they were in different octaves, should also be pleasing together when they were in the same octave. Basically, they had the keys lined up in semitones, and then they found a model that would satisfy the following sentence: ‘if a key is a white key, then so are the keys seven steps further in either direction, and twelve steps in either direction, but not the key one step further in either direction.’. In other words, it was periodic.”

“Dora, you are taking a long time to tell me where the eight comes from for the octave.”

“Here, I will draw part of a piano keyboard on the deck.”

“I don’t think that is allowed, Dora.”

“It’s just chalk. It will wash off. I do this all the time. I’ll start with this white key in the middle, and then the double interval is” she drew a rough rendition of part of a piano keyboard, speaking as she drew, “white, black, white, black, white, white, black, white, black, white, black, white, and back to white. The last white is the same note as the first, but about double in frequency.”

“Yes, and?”

“If we look only at the white keys, we have eight white keys. The doubling occurs from the first to the last. So we need to count the intervals between the white keys.”

“I count only seven intervals.”

That’s because you are looking at it wrong. Remember I told you about that group structure?”

“So the idea is that you have one generator, which is that constant factor for the smallest dissonance chord, and any note. Then the doubling is produced from this pair, the selected note and the factor, by multiplying that note’s frequency times the different powers of the factor until it starts to repeat. So, to get that first note, you have to multiply by one, which is the factor to the power of zero. Then you keep going, so your group has the factor of exponents from zero to twelve. So you have a null interval. In the octave, you are counting intervals which are not as regular, so it doesn’t form as nice a structure, but if use my example of the eight white keys from, um,...” she pointed to her drawing on the ground, “...white, black, white, black, white, white, black, white, black, white, black, white, which correspond to the factor to the powers of...” she wrote in red chalk under each key, “...zero, one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve”. She stepped back, put the red chalk back, and took a piece of blue chalk. Then she started to circle the numbers under the white keys and cross out the numbers under the black keys. As she did this, she read out, “So if we just take the white keys then you have to multiply by the factors with exponents of zero, two, four, five, seven, nine, eleven, and twelve. That’s...” she counted, “...eight factors. That is why it’s called an octave.” Dora stepped back, spread her arms, and said “Ta-da. Trumpet fare. Wasn’t that great?”

“I am sure your presentation would be a hit in a university, Dora, but it’s a bit long for primary school. Why don’t you take the cheap way out and do like most teachers do, and ....”

“No. I don’t like that.”

“OK, but I would perhaps drop the mathematical explanation. Most of your classmates probably don’t know what exponents are, and anyway, I doubt that this is the way it happened historically. The name of octave has been around a lot longer than the idea of groups.”

Dora pouted for a moment. Then she said, “OK, historically if you like.” She thought for a moment. How’s this for an opening line? What do you think was the first musical instrument in human history? Or prehistory, I should say.”

“Um, the human voice?”

“Indeed. So, think of the possible distances that two voices could be if they could only sing these white keys. That is, only do, re, mi, fa, so, la, si, do. Then there are possible distances between, say, do to each one are zero through seven. Those are your intervals. Eight of them. Is that better?”

“Yes. But maybe you could do a bit of research to clean it up. The ship’s library...”

Dora pointed to something in back of Moishe.”Oh. Ooh, look, Rabbi.

Moishe thought that this was just a diversion away from the discussion about homework, but he turned anyway. He then saw the plane that he had been expecting, trailing plumes of six colors. He jotted down the order of the colors, and left Dora to watch the show in the sky.”Just in time,” he thought.”Bill will be eating in another couple of hours.”

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Same day, evening

That evening, as the boat moored off of Tahiti, Moishe stared at his notes. He didn’t understand it. He had followed the recipe, and the plate has come back with only a couple of crumbs from his desert left on it. Yet Bill had gotten nothing more than a mild stomach ache.

Moishe was not happy. His only consolation was that the Internet ban would no longer apply when they got to their destination, and he would be able to email the chemist.

Chapter Nine

January 28, 2045

In a conference room of the Infospital, or the 'Wheaton Island Infospital' as it had been officially named, the chairman of the team opened up the meeting. Looking at her notes, she said, "We adjourned saying that any two events which are along the path of a photon in vacuum have space-time distance zero, whichever convention you use."

Another team member asked, "I read that space-time isn't flat. I don't even understand what flatness has to do here."

"Flat, no curvature. That is, how far off something is from the usual use of the Pythagorean Theorem. When spacetime isn't flat, you have forces, like gravity. In the constructions I just outlined, it is clear that the Pythagorean Theorem does not work in spacetime. But it is a good starting point to be able to measure the curvature."

"What about just space? Can space-time be bent, but space flat?"

"Sure. If we take observable space as a whole, galaxies tend to be extremely spread out, so gravity becomes almost negligible, and the bending of space-time comes primarily from the expansion of space over time. Taking only a snapshot of the observable universe to get only space. However, once we consider smaller hunks of space where gravity is important, space also is bent. That is, the Pythagorean Theorem doesn't work near mass."

"Now I remember. I saw an analogy of space bending around a mass. It was a rubber sheet with a mass weighing it down, so that other masses would roll towards it."

"The thing that a lot of people don't understand of that analogy is that it is supposed to show the Pythagorean Theorem no longer working, but this is rarely explained."

An image of a typical ball in the middle of a rubber sheet was shown. A psychologist would have been able to read off the faces of the audience that they had seen this diagram before, but were now attempting to look at it in a new light. The head of the team told them, "You will find this model in each of your cabins; the cleaning personnel are setting them up now. You can play with it this evening. But now we will go on. The idea is that, according to how you look at the situation, the time between two events, or the distance between two events, will change, but the space-time will stay the same, no matter which way your spaceship is going, as long as it is going at constant velocity. That is, you can have one model for space, another model for time, and they can disagree, but then you can have a model for space-time that subsumes both of these. That is what we mean by saying that space and time are shadows of space-time. The idea that space-time distance is a fixed point is the basic idea of relativity: physical laws should look the same from everyone's viewpoint. Special relativity is a theory that is the restriction to what happens when each of two observers are going at a constant velocity, although each one may be going at a different constant velocity to the other one. The basic idea of special relativity is that although space-time between events stays the same when the measuring system stays the same, this does not mean that the two shadows stay the same, as you can see if you regard your own shadow."

"So what happens when this way of measuring, this metric of space-time, changes?"

"This depends on your point of view. Let's look at it from our possible worlds perspective. In each one of them, you are going at a constant velocity. But it is a slightly different constant velocity from one possible world to the other. In each possible world, you seem to yourself the same... because your perception changes along with everything else. So, if you weigh seventy

kilograms in one possible world, measured with the tools of that world, then you will weigh seventy kilograms in the next world, if you measure yourself with the tools of that world. But interpreting the representation of one model with another model's tools is going to give you a different result than these. So you may weigh more in another interpretation, that's all."

"Are you saying there are no forces?"

"Not exactly. I am saying that forces are simply a measure of how we judge one quantity with another model."

"It is sort of not fair, like judging the ethics of the ancients with our modern ethics."

"Um, yes, in history that is true. But for us... well, we don't have any other choice, really. We could do that in another way, by referring to the way that straight lines and angles associate differently with one another according to the operators you are using to describe things. That's what they mean by saying that the geometry of space replaces the concept of forces."

"Examples, please. That's getting too abstract."

"In free fall you don't feel any forces on you; but if you hit the ground, and have then to describe your resulting state by the model used at the ground, then this is a different matter. It's all according to your point of view."

"So, if forces are seen in this way, how do you see energy?"

"Bending space is potential energy, and kinetic energy is the energy from the resulting bending. That is, energy per se is an operator, and this is applied in every possible world; however, going from one possible world to another, that is potential energy. Since we subsume the whole structure under one structure, this dual being-and-becoming together is your total energy."

"The physical laws here...."

"If you are looking at the whole structure of physical laws, then some of them talk about several possible worlds. A law inside one possible world might just talk about members of that universe, and the physical laws talk about what must go on in all the possible worlds connected to one another. For example, the changing way of measuring things can then be classified as to how far away it is from the usual way, where the Pythagorean Theorem is valid. If, in the space portion of space-time, the hypotenuse is less than given by the Pythagorean Theorem, or, alternatively stated, if the angles of a triangle add up to less than a half-circle, then we have a positive curvature. This is associated with repulsive forces. The negative curvature is the opposite."

"Hm. Well, if we are not allowed to change the physical laws, we can at least change some of the viewpoints."

"We can go further than that. We are not allowed to change the physical laws, but we can change the galaxy. There's nothing saying that reality couldn't be in a galaxy where interesting things were closer to Earth than in our galaxy."

The mathematician spoke up:"I was wondering.... there is all this computing power here. Why not kill two birds with one stone and insert in some open problems that the computer could be working on?"

"Hm, I think that would be allowed, said the project manager, only if the problem was seen as relevant to the story. At that, I could allow maximum of one problem. Go to it."

The programmers then programmed the computer with the basic physics. In order to get all the details, they debated the risks of connecting it to the Internet. One of the programmers said that they had figured this out. He had gotten ideas from the best hackers in the world by posting Hackers' War Games, harvesting the results and letting Paralife itself figure out its own defenses.

The model which Paralife would develop by adding and testing new axioms would be greater than all possible hacking efforts. The protection in place, the programmers made filters for Paralife to pick out useful information while rejecting the majority of silly sites.

They then faced the problem that no one had hired a good storywriter. This did not worry them; they had made Paralife not only a learning program, but had given it creativity as well. They instructed Paralife to scan the files of the participants to form a profile of their emotions, then scan Internet first to understand human emotions, then, in light of this understanding, search for inspiration for an entertaining story.

The only profile that was more complex than the mixture of emotions being developed by the children was that of Bill. Through an analysis of Bill's private memos, Paralife came to the conclusion that Bill was constantly fighting his own regret. Then Paralife scanned for sites about regret. For some reason, it seemed to like the song by a pop star from a century before named Cher on this topic, but Paralife was puzzled by the lyrics which indicated that reaching the stars was in some way equivalent to finding a way to turn back time. Paralife decided to investigate this question by having the story work it out. The story would begin as soon as the first person was sent in; for the characters created by the computer, they will come complete with a history. Sort of like the dinosaur bones of the creationists. This would include already being a decade in space to eliminate any complications of communication with a fictitious Earth : the date in the far future, chosen from a science fiction novel by the previous year's sci-fi winner Eli Levinsky, would allow for great advances in technology from the present day. For character names Paralife searched through its archives of novels as well as the information on the participants. The author Paralife started spinning the story, based on the restrictions already given by Paralife's "editors", its programmers.

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February 3, 2045

Natasha had fulfilled all of her end of the Wheaton bugging task. She had received her doctorate, and already patents based on it were being filed. Her job as a secretary at the Vladivostok Information and Computing Center had been upgraded, but she only kept it because it gave the access she needed for her hacking to the supercomputer which was only a couple of years old. She had been offered an assistant lecturer position in Novosibirsk, but she did not want to leave the Vladivostok region. There would be an opening in a year and a half at the Far Eastern National University in Vladivostok. Her hacking activity allowed her to wait that long.

Meanwhile, she wanted to test the hypotheses of her doctorate on computer models. Julia had helped her to translate the mathematics into computer code, and so Natasha looked around at sites which offered realistic models of the female physiology, so that she could play with it to test her speculations. So far, no suitable sites had come up; she would perhaps have to go through hours of creating it herself. But there would be time for that after the annual Hackers Reunion. This year it was being held in the winter, but Natasha had asked permission from her employer to take her vacation earlier this year.

At this year's convention, everyone was talking about the upcoming so-called Hackers' War. No one could figure out who the anonymous host was, which led the hackers to believe that whoever was capable of putting up such strong defenses would certainly mount an interesting game. The game had cash prizes, but more important would be the prestige of winning. The application for joining followed a similar strategy to the one that the Far East Association for the Protection of the Long-tailed Goral adopted. The game itself would require the hackers to defend 'castles' and attack the 'castles' of other hackers. It was to begin the following week.

Natasha thought she would try it for fun. But it was not the actual Wars that interested her. She eyed a more formidable opponent: the host of the War. She wanted to cheat, not to win the war, but to show that she could get past the host's defenses, succeeding where other hackers had failed.

After returning to Vladivostok, she successfully submitted her application and got it accepted. In her application, she planted a virus that would search for the file which provided the details for the Hacker's War program, including the defenses against intruders. Others had certainly done the same; however, Natasha's virus was slightly different. It would pass to the computer processing the application without having any measureable effect on any program, and so would go undetected. It was a telltale marker that would only be noticeable on Natasha's computer. She had developed her virus after studying the attacks of other hackers. The simpler attacks tried to search for programs written in the same style and format as the programs written for the public site. But if you asked the better hackers what strategy their searches took, they could not tell you in detail. They had set up learning program, where each search learned from the successes and failures of the preceding search, what they called a recursive procedure. At any moment, the resulting complex program was known only to the computer; the human programmer long having lost any possibility of knowing it. A bit like the human brain.

Natasha went one step further. Her computer would learn to write its own learning program. If the other programs were like a human brain, Natasha's program was a brain which was quickly evolving beyond human. One of the differences was that Natasha's program no longer was looking for a program which contained elements of a Hacker's War. No, since hackers had determined that the Hacker's War program was the most heavily guarded file on the Internet, Natasha had limited the instructions to her computer to find simply the most heavily guarded file, whether in the Internet or, more diabolically, stored on private computers. Many people ignored the indirect access to private files on their computers, often even on computers without direct Internet access.

Such a search would have taken much too long on a normal computer, but Natasha set up the search on the university's newly acquired Super Quantum computer. Even then, the search took quite a while of continuous running. Eventually there was one very large private file that matched. The location of the base computer of the file was hidden, but the file itself looked much more interesting than the Hackers' War.

Natasha had stumbled onto the file for Paralife.

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March 4, 2045

Jutta, who had been appointed chief supervisor of the Mind Transfer Project, asked to see Bill at quarter to nine in the morning."OK, Bill. The technology is as good as it is going to get. We will send the children in at the same time as you. But you must be sure you want to go through with it? It has never been done before, and if it fails, the result is death."

"Are you sure all the technicians are thoroughly familiar with ...."

"I am sure they are all excellent, but they will be taking their orders directly from me. We have a new technician coming in at nine. I am going to brief him, and you can sit in the other room and watch the interview on closed-circuit TV. This is why I wanted you here at this time."

The new technician, Dmitri, came in. Jutta started, "The aim of this wing is to take terminally ill patients with irreversible brain damage, who have given up all hope. Our treatment is unusual, and some would say unethical, but it gives the patients a chance they would otherwise not have. Regardless of whether we succeed or fail, you are bound to silence. Do you want to go on?"

"Yes, I understood all that."

"Four of our patients are, for now, orphan babies, since their brains are simpler and so give us a greater chance of success. But we have one adult patient already. Dr. Schultz, meet William Wheaton."

"Mr. Wheaton, the billionaire?"

"Yes. Although he is no longer a billionaire. Most of his money is in a trust for this infospital. The infospital, in return, provides him with free medical care. The procedure can be simplified as follows: the patient's mind is put inside the computer, which then reloads the mind into a new body. We need to do it now for Wheaton because his brain will soon start to deteriorate."

Dmitri was silent for a moment, and then asked, "How can a mind be stored in the computer?"

"Why not? Your nervous system is composed of a set of ion exchanges, and that is your mind. We can recreate them in the computer."

"I know, but.... it's creepy. First, there is the nagging suspicion that there might be something that we are missing..."

"There's no way that one can convince you, unless you yourself experienced it."

"Even if you did create consciousness, it would seem to me that all one would be doing is creating a second person, and the first person would be dead with her body."

"That's for the philosophers. After all, you replace most of your cells over the course of your life; are you a different person? Don't answer that, I don't want to get into a philosophical argument. Let's put it this way: we are doing the best we can for these terminally ill patients."

"The central problem is that the scan is invasive, and the patient's real nervous system will be destroyed by the procedure, so we have to be very confident that it will work. Bill trusts my judgment, and so I am taking all responsibility. You will take all orders directly from me."

After Dmitri had left, Bill turned to Jutta."The operation is invasive and destroys the brain. You didn't tell me that."

"I had hoped to get it to a non-invasive procedure, and I still think it is possible, but we don't have time for me to complete that. So this is the best I can do. But it doesn't make any difference to you; your disease would destroy your brain anyway."

Bill was silent for a while.

"The new nervous systems will be partly organic, to allow them to have their DNA which identifies them according to the law, and a number of inorganic circuits which will allow the computer to re-insert the data, the mind. This procedure will also destroy the corresponding data in the computer."

"Um, a bit of Matrix, a bit of Borg?"

"I see you know your twentieth century film history."

"Mind Transfer is all over science fiction books and films."

"Yes, but usually by some telepathy. This isn't telepathy. The mind in the computer continues functioning, and at a rate many times the speed of the brain in the organic brain. Unfortunately, the process can take hours or even days, during which time all the bodies, old and new, are in life support systems. In fact, the only thing limiting the number of people who can be in the computer at one time is the number of life-support systems we have. We have now not only those for the five first patients, but also several more ready so that, if tomorrow another patient appears, we will be ready. In addition, the minds are not going to be dormant. It would be

a waste to let the minds get bored during this time, so we provide them with a world to live in while they are there."

"You put them in a virtual reality?"

"No, better. A simulated reality. We call it Paralife."

"That's not the same thing?"

"No. In a virtual reality, you can tell the difference between reality and the virtual world. In the simulated reality, you cannot tell that you are not in the real world."

"The monsters and so forth might give it away."

"Paralife is not a cheap one-person video game. There are no monsters. The players – our name for the patients, it's less depressing for them-- have no super-powers. The universe will obey the same physical laws, as far as we know them. The universe will be a bit different, though. You see, the reality will take place on a space ship."

"Why? The babies won't care."

"I told you, there might be more patients. We intend to expand the use of the system. In fact, our eventual goal is to make the procedure to be completely non-invasive, it could be introduced in schools as educational. So, are you ready to join our team?"

"Yes, I find it challenging. As far as the philosophical points, it nonetheless sounds like the best shot the patients have."

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March 4, 2045

Moishe was brooding. He had emailed the chemist in Brighton Beach, and after a couple of email exchanges, Moishe's mistake was clear. In the sky, in the absence of any such alphabet to guide him, he had switched automatically to his default right-to-left reading mode from Hebrew. But in writing it down, the necessity to use Latin letters led his brain to switch to the left-to-right writing mode. So, instead of looking for bgorvy, he should have looked for yvrogb. He mused that this was a new twist on the Homeric proverb, "There's many a slip twixt cup and lip." He continued kicking himself, looking at the ground as he walked in the park outside the infospital. He did not notice Chaim and Daniela who approached him.

Chris had never learned anything about Judaism before embarking on this project, but he had convinced Daniela to learn a few basics together. Chris and Daniela made as if their well-researched encounter was an accident. They brought the conversation around to their work, and offered to show Moishe around the electronics workshop where, they claimed, they worked.

While they were in the workshop, Chris said, as if offhand, "Mr. Cohen, I was reading a bit of Hassidic lore. Perhaps you could explain to me what it says about telepathy."

Moishe was always happy to show off his knowledge of Judaic writings."Telepathy was a gift of the tsadik, the righteous ones. The righteous person, by negating himself, in a spirit connected to prayer, transcends the limits of space and time to communicate with another Jewish soul when he needs help. Like the prophet Elisha, or Job's friends, or the great righteous ones....but are you interested for a particular reason?"

Neither Chaim nor Daniela were really interested in telepathy, but the important thing to them was that Moishe was. Daniela smiled and said, "Wheaton and I are making a communication between humans and the computer." Daniela's inclusion of herself to the Bill's project was purely fictional, but Moishe could not know that."The signals can pass from the human brain to the computer, and from the computer enter another human brain. That is, it should be able to skip the middleman there, going from one human brain to another. We could

do this in two directions, and establish a connection between ourselves. This will make us telepathic, and therefore righteous."

"Um, there's a slight flaw in your logic. The righteous could be telepathic. But that doesn't mean that someone who is telepathic is righteous."

"Ah, but if it is only the righteous who are telepathic? Then if we are telepathic, we must be righteous, no? So, are you willing to try to become righteous?"

Moishe considered. He had never learned to distinguish a good argument from sophism, and he was attracted to Daniela's argument."

"All right. What would this entail?"

"Is there anyone on this boat that you would consider becoming telepathic with?"

Chaim and Daniela, of course, knew Moishe's answer before Moishe even thought about it. There was no other adult Orthodox Jew on the trip. Tal was Jewish, even though a Reform Jew, but he supposed that would still be acceptable.

"I suppose Mr. Tal Kishon."

"Fine, fine. We can perform the transfer whenever you and Mr. Kishon are ready."

Moishe realized he would have to convince Tal. He was not sure how he would do that, since Tal did not adhere to Hassidic teachings.

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March 7, 2045

The *Scientific American* Insights Cruise continued on its way without the passengers who had disembarked on Wheaton Island, but the organizers had cleverly arranged for new clients to be picked up for a one-way trip to the continental United States.

Tal was told that the infrastructure for the public relations post was not completely ready. He was, however, on contract from the moment he stepped foot on the island, so he would receive his pay and housing. But he had a few days in which to just enjoy the island.

Tal was not much of an outdoors person, and he wanted to do something other than read. He saw an ad for a temporary position at a restaurant; he decided that this would be a good place to get a feel of the people around, and earn a few extra meals. The job was not difficult; it consisted in taking orders from the tables at the cafe "New Horizons". He did not actually have to go to the tables; the clients would flip a switch on their tables, activating a transmitter at their tables. Tal just had to sit at the receiver in the kitchen and jot down the orders.

One day a group had reserved a room; one of the group had come and asked for the use of two trolleys upon which he would arrange a surprise for the group. He asked the restaurant staff not to lift the cover that would be on the trolleys. The group filed in on time, and sat down first to order. The dishes they asked for would take a while to prepare; this was no fast-food restaurant, so that it was frequented by people who enjoyed discussions while munching on the appetizers which the restaurant was famous for. After they ordered, they apparently forgot to turn the microphone off. Possibly the light indicator had burned out; they really should invest in better circuits, Tal thought. As no other orders were coming in, and he didn't have anything else to do, he listened to what this group was talking about.

"'Popping' is not the right word," a programmer was telling his companion."We are not going to show any blatant violation of the conservation of mass-energy,"

"It's not true for the entire universe..."

"But for a closed system it should be."

"Wait. Why is this so crucial? So a bit of mass gets added?"

“The fact that there are ways to transform the path ends up with different fixed points. Each one then corresponds to some physical fixed point, that is, some quantity that stays conserved throughout the transformation.”

“You’re not being very clear.”

“I thought I would need to be more concrete. So, wait for a minute.” He jumped up and went to a neighboring room. Then he wheeled in two tables, each covered by a cloth. He made a flourish, and then whipped the covers away as if he were a magician uncovering some astonishing spectacle.

The others were not infected by his enthusiasm. They looked at him, then at the tables which appeared to have two identical mazes with rats running around them, then back at him, waiting for an explanation. He pointed to each, “Note that they are not normal mazes, but branching systems, with a circle with water and food in a circle at each branching point. The circles are possible worlds, and the paths are the accessibility relations from one to another. Note that some of the paths have little doors in them to signify that the path is one-way. But not all of them. The rats are twins, with one twin in each maze. I run the rats through the mazes a lot of times, and record the percentage that one set of decisions was made as opposed to another.”

“Cute,” said another researcher, “but why two of them?”

“One of them represents the situation in the present, and another in the future. Here’s where we get a bit abstract: there is a transformation from one to the other. Despite this transformation, the collections of decisions will be the same in both. That is, there will be a quality of the maze that stays the same when this transfer through time occurs. This is a fixed point for this transformation.”

“Oh, I read about that. When you have a fixed point of certain types of equations, you get some quantity of the system conserved. Emmy’s Theorem, wasn’t it?”

“Close. Noether’s Theorem. Emmy Noether. Anyway, in this case, that fixed point is the mass-energy of the system involved.”

“Ah, your point being...”

“That is what happens in any observable closed system in our universe. And it will play havoc with the rest of the physical laws in Paralife if you change that. Ergo...”

“We get it, so we will keep the mass-energy the same, but allow us to cheat a little, in rearranging the brain of the avatar. Agreed?”

Tal did not completely understand all that, or what Paralife was. Some physicist lingo, he assumed. He thought no more about it, as other orders started coming in.

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March 9, 2045. Early morning

Moishe managed to convince Tal, although not with the arguments of becoming a tsadik. Tal’s interest in the mentalities of different belief groups had guided him in the balancing act he had as an editor of *The Foundation*. He liked to think of himself as possessing a model which was able to encode the other models.

Chaim and Daniela led the pair in their uneasy alliance down to the lab when Chaim was certain that no one else would be there. Tal didn’t know much electronics, but he had taught at schools with Smartboards. He read that there was a calibration necessary in every such machine, so he looked at the simplified circuit diagram that Chaim had given him. He started to tell Chaim, “Ah, I see. I will just have to calibrate this electron beam here. I just have to place the target so that the beam, going through this magnetic field, will hit it. I took some physics in high school and learned how to predict the path of an electron. I nearly bent my hand out of shape

because there were these rules with our hands and stuff, but the calculations were pretty straightforward. Here, I'll put the target.... there. OK, it's ready. I'll just make a few trials..."

"If your calculations are correct, why try it?"

"Just in case. Anyway, let me try it a few times."

Chaim stood patiently by as Tal became frustrated."Some of the beams are going a bit to one side, and some to the other. I don't see any pattern...."

Daniela told him in her most soothing voice, "Our body's electrical charge is probably affecting it."

"I'm charged? I always wondered what those mystical photos of the body aura were."

"That, or your body heat. Anyway, when we do the actual run, you two will be in these protected pods, and Chaim and I will be out of the room. So I'm sure it will be OK."

"Well, ...I guess you know this stuff better than I do. Let's go ahead with it."

Chaim set the timer, and Moishe and Tal lay down to go to sleep in the pods.

They did not wake up as planned.

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March 9, 2045. Late morning

The technicians came down at their regular times.

"What are these two doing in the pods?"

"They are connected to Paralife! The readings indicate that they have activated the Paralife program, and completed a Mind Transfer."

The doctor examined the two."But they did it rather clumsily. It looks like their brains have been damaged. We can't bring them out immediately."

"Why did they do this?"

One of the technicians found Tal's notes."They were trying a sort of Mind Meld. They might have even succeeded if they had succeeded in aligning that beam that they were concentrating on. But the one named Tal didn't include the fact that spin will alter the path in only a probabilistic fashion. In fact, from his calculations, apparently he didn't know about spin at all."

While the technicians were figuring out what to do next, the linguist cornered a physicist.

"What, the electrons are spinning?" asked the linguist.

"No, but they deviate that extra little bit, as if they were a spinning magnet."

"Spin? Where does that come from?"

"It is necessary, or the universe wouldn't function, given all the rest of the laws. We know about the effect of the operators of electromagnetic charge of an electron. But we also know that two observers should predict the same change of state. To do this, there has to be another innate quantity besides charge and mass-energy; that extra innate property is called spin. Er, actually the word is ambiguous. On one side, one uses it to indicate what sort of domain the interpretation function might have, and on the other side, it is used to indicate individual values inside that domain. For example, an electron's spin value will be one of two possible values, which we can label, for example, yin and yang, and their associated truth values. For a photon, there are three different possible values, for a Higgs boson, only one."

"It's spinning?"

"You haven't heard a word I have said, have you? I told you it's not spinning. It just makes things react as if there were a spinning magnets there. The 'as if' is not the same as 'is'."

"Is it healthy to use analogies which are so limited? They are so easily misinterpreted. Perhaps one should stick to the worlds that are possible."

“We do this constantly: not all our possible worlds have to have the same universe. For example, here we have a possible world where the universe is restricted to the influence of magnets on the electron, and which overlaps with the regular model. The accessibility relation here is two-way.”

“Why have it at all? Why not use fairies?”

“Sometimes fairies can also be useful. Slowly expanding and contracting models is what thinking by analysis and synthesis is all about.”

The technicians finally said,

“Nothing for it. They don’t seem to have damaged the system; we can go ahead with the others as planned, but for these two we will have to make new bodies. They are going to face a whopping bill when we get them back.”

Chapter Ten

January 1, 5815

Moishe woke up first. He panicked as he saw himself in a hospital bed. Had something gone wrong?

A nurse shouted to someone outside of Moishe’s view.”Number 314A is coming around, doctor.”

The nurse then asked Moishe, “How do you feel?”

Moishe asked, “Have I been put under arrest?”

“Is that what you call your induced hibernation? Arrest? Yes, of course you were put into a state of biological arrest. That is why you are now in the ship’s Reanimation Center. Do you remember your name?”

“Yes, I am Moishe Cohen. I was attempting to connect to Paralife...”

“Paralife?” The nurse turned to the doctor, who had come with his notepad. The doctor told the nurse, “‘RV Paralife’ was an early proposal for the ship’s name. But then they changed it to ‘RV Serendipity’ when it was discovered how nicely the cosmic elements happened to be lined up to allow us to perform our mission. This patient was put into stasis before the name change.” The doctor turned to Moishe.”Your mind may be a little foggy today, but that will clear up by tomorrow. You have been in stasis for a decade, so it is natural that it will take some time to get your bearings. When you have recovered, we will have some questions for you, but for now just lay back and relax.”

“Years? What year is it?”

“5815, ship time. We keep the same calendar as on Earth, with the month names and all that, so that new arrivals will not have to adapt. It is New Year’s Day, 5815. I’ll come around later; if you need anything, just ask the attending nurse.”

After the doctor left, Moishe considered the little information he had to go on. That didn’t make sense. If he was only a decade asleep, it should be the year 2055, not 5807. He took a better look around the hospital ward. He did not recognize the place. Perhaps he and Tal had died in the attempt to get telepathy and had become reincarnated. What did the kabala say about reincarnation? OK, it wasn’t the same as the Hindu variety, but there was some kind of reincarnation done. He had been taught that every Jew must fulfill all 613 Commandments, and if he doesn't succeed in one lifetime, he comes back again and again until he finishes. After all, he thought, I didn’t fulfill my obligation to destroy the descendants of the Amalekites. Tal, of course, would have a lot more of the commandments to fulfill. But he had been taught that

heaven maintained the ability to exercise complete freedom of choice by not allowing us conscious knowledge of previous incarnations. Was he an exception?

Moishe heard another patient moan. He looked over automatically, but the patient was hidden by a curtain. However, he could hear the nurse and the doctor give the unseen patient the same spiels as they had given him. The patient gave his name. Tal Kishon.

Moishe continued his train of thought. Tal was there, and also remembered. Therefore, he reasoned, this could not be reincarnation of that sort.

5815... maybe... He did a small calculation. Eureka! That was... 2055, according to the Jewish calendar. These spaceship people were enlightened enough to have abandoned the year of the Gregorian calendar, although not enlightened enough to adopt the Jewish months and days. New Year's Day, indeed. No matter. But how did he get onto a spaceship? He tried to read Tal's mind, and could not. So the attempt had failed, he and Tal had been unconscious, and Wheaton's people had taken advantage to kidnap them.

The new patient was left alone. Moishe got out of bed to go to him. He pulled aside the curtain and said, "Tal?" But then he saw that Tal's face was not the same."Oh, sorry, I thought...um, it's a common name you have and ..."

The patient Tal Kishon turned to him and asked, "Who are you?"

"Ah, I'm Moishe Cohen. I have a friend with your name."

"That's a coincidence, I have a friend with your name. But as you say, it's a common name."

They fell into conversation and compared notes until they realized that they were indeed the Tal and Moishe they had thought. Moishe then added to his list of accusations against Wheaton's gangsters that of forced plastic surgery.

Tal proposed another possibility. He remembered the discussion in the restaurant. He remembered that someone had talked about the avatars having minds. On the other hand, he felt perfectly conscious and he did not think that an avatar was supposed to be conscious.

Tal asked Moishe if he thought that they could be in a computer program.

"Of course not! Just think, if it were, the creationist beliefs that you reject would be valid, no?"

Tal was taken aback. His thinking was still a bit fuzzy, and at the moment Moishe's reasoning appeared valid. So he decided that perhaps Moishe's kidnapping story did hold more substance. The absurd possibility of them being conscious avatars was thrown out of his mind.

Both of them voiced their accusations when the nurses came back. The nurses had been warned that such behavior would happen, that perhaps not some of the Frozens would regret their decisions and therefore claim to have been taken against their will. However, after some bouts of shouting, the two newcomers realized that continued objections would not only be useless but also would be counterproductive. They knew that the use of psychiatric wards to silence dissidents was an old practice. Moishe said he would accept God's will in putting him there, and Tal didn't see any alternative.

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February 10, 5815. (Paralife time)

Upon waking up in Paralife, Bill received a briefing from the attendants in the Freezing station, as it was informally called, although the official name was Crew Reanimation Section.

"Hello. How are you feeling after your long sleep?"

"Um, groggy," he replied.

"That is normal. Can you remember where you were last?"

"On Earth?" Bill remembered that the simulated reality would be in a space ship.

“Good, good. We must apologize. You were told upon being put into stasis on Earth that we would have your complete file, but the details of the identities of the Frozens, as well as the schedule of the unfreezing, were destroyed in a glitch soon after take-off from Earth. The main computer has the schedule, but it is buried in its program, inaccessible to us. So we just let it reanimate crew members at regular intervals, according to the need of the voyage. So we must ask you your name, age, profession, and other details. We will also give you a complete medical check-up and get you set up with living quarters and a job.”

“I need to work?”

“Of course. Your salary will be a bit low until you get adjusted, I’m afraid, but we subsidize you. You will report to the Absorption Department. I will give you their card. Now, if you will excuse me, I must attend to the children...”

“Children?”

“Yes, yesterday we had some children be unfrozen.”

Bill thought of the fact that, although he had come in only a couple of hours after the children, the difference in time meant that this had been a couple of days for the orphans.

The nurse continued, “Usually, the parents are unfrozen about the same time. However, the glitch means that this is not always the case, so we unfortunately had to begin an orphanage. Since we do not know who the parents are, all we can do is ask each unfrozen whether they are a parent. Are you, by any chance?”

Bill shook his head.

He learned that two other adults had recently been unfrozen by the names Moishe Cohen and Tal Kishon. They both had asked what sort of hospital it was, but this was a normal first reaction after being unfrozen. They had looked in a mirror. The one who asked for a kippah, Moishe, was a bit more hysterical; the other one, Tal, was a bit disoriented as well, but only tried to calm the other one down. Moishe accused the hospital of kidnapping and of plastic surgery.

Bill considered his options. A menial job? Hm. What had they told him about dying? He then took a poison before he became too involved in this world, but first arranged it to look like a natural death. He published in the ship news site not only the prediction of his death, but more so, the prediction of his reincarnation, in which he would write down a long number in front of witnesses who would not see the number; then seal it and keep it in a bank vault so that no one would see it until someone came to give the password at the bank, which the bank’s computers produced randomly. With this double protection, he came back in a new body, gave the password and, in front of the TV cameras, dictated the number. It was shown to be the same. Most people still dismissed it as some trick, but there were enough to give him a following. He became a guru, supported by his followers. Bill thought he was going to enjoy this stay. He wondered how long it would be. They had told him that it would take about three days, that is two and a half months in Paralife, to be able to bring him back. He intended to have fun in that time, not work some menial job. He was able to retain his name.

When Moishe found out that Bill had become a guru, he also dismissed it as a trick. But it did bring to his mind that he was to put false prophets to death when possible. Bill is now a guru, posing as a prophet, and definitely a false one. All the more reason to continue my quest to kill him.

Despite being a guru, the regulations said that Bill would have to report to the Absorption Committee once a week for counseling. Bill had no objection.

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February 17, 5815

At Bill's first session at the Absorption Committee, the Absorption counselor told Bill:"You have been unfrozen for a week now. You probably have many questions about what you have observed. Why don't we start with your questions."

Bill thought for a moment, and then asked, "How many people are in this ship?"

"About ten thousand people unfrozen, but there are about as many Frozens."

"What? On a single space ship?"

"Oh, no. The population increase was not foreseen. Population control was abandoned as people realized that they would not be returning to Earth. A mistake, in my opinion, but I was not asked. So the ship was able to build other modules along the way."

"What, it stopped on planets?"

"No, no. We have been accelerating steadily since we left Earth. You can only accelerate so much, you know."

"Why, not enough fuel?"

"Oh, we have enormous reserves of fuel."

Bill wondered how the programmers had managed that, but before he could ask, she continued."We accelerate at about the same rate that something would fall on the Earth. In that way, if you are standing in the same direction as the direction of travel, then you feel as if you were under Earth's gravity. The physicists tell me it is actually the same thing, but I have never absorbed this fact. Something about both being the bending of space-time, so that they are basically two theories for the same model. In any case, if we accelerated much more, our bodies could not take it."

"Standing in the direction of travel must be inconvenient. I mean, here is the rocket ship...." Bill took the writing pad from the counselor's desk and drew a typical cigar-shaped rocket ship.

She laughed."Why the dickens would you go in the direction limiting your standing space like that? Oh, dear, I know. You are used to the ship going through air, so it had to be streamlined, aerodynamic, or whatever the engineers called it. But in space this isn't necessary. Here we do have levels, and we have continued to build levels as the population has increased."

"Why not have a ring rotating around the ship? Most science fiction stories I see use that."

"Did you ever stop to consider how big the ring would have to be? But you should concern yourself with other matters. Security, for one."

"What, is there much violent crime here?"

"No, but the eco-terrorists do a lot of material damage."

"You have eco-terrorists board? Against what?"

"Oh, a myriad of imaginary conspiracies."

"I would imagine that they are easy to catch, given that this is on a ship."

"No so easy. They are very cautious, not leaving anything behind that our experts are able to use to identify them. We have posted information on the Ethernet in order to get people to come forward with information, but that has not led to anything worthwhile."

"You have Internet?"

"What's that? An Earth thing, no doubt. All our information was stored on Earth or developed since then, and the computers are in the ship itself. Anyway, you have yet other issues. Are you aware that even religious leaders must pay taxes on this ship?"

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March 7, 5820

It had been four years since Moishe and Tal had been assigned the care of the children who had been unfrozen shortly after Moishe and Tal had been. The Orphanage was small, since only

those four children were in it. Moishe had determined that the two boys had been circumcised, and he assumed that these four children had been kidnapped from Jewish families. Hence they should be raised as their lineage dictated. Moishe and Tal were not in accord as to whether they should be raised as Orthodox or as Reform Jews, but since Tal was less zealous in the execution of his duties, Moishe was able to inculcate Orthodox principles.

One day an inspector came and told them that certain arrangements would have to be made for "Switch Day", which was the following week.

"Switch Day?"

The inspector looked at his documents."Ah, unfrozen already five years, and you haven't read all the government documents sent to you?"

"Um, they were rather a lot."

"No excuse. This one is important. You may have lost sight of the fact that this ship has an ultimate purpose for which it must be on the other side of the galaxy to witness a certain event."

"What event?"

"For that information, refer to ...."

"...the government document. Yes, we understand. But what does that have to do with our day-to-day running of the Orphanage?"

"We have been accelerating at a constant and comfortable rate for the last twelve and a half years. This allows the daily gravity, with 'up' being the same as the direction of travel. However, we need to slow down for the remaining twelve and a half years, so we are going to start decelerating at the same constant and comfortable rate, with 'down' being the same as the direction of travel. The switch will be gradual, for a whole day, allowing furniture to be transferred effortlessly from the old floor to the new floor...that is, what is now the ceiling. Electrical fixtures will be transferred easily, since this switch was foreseen. But that means that you will need to set aside Switch Day for the transition. Afterwards, you will need to apply some transformation to your thinking, since you might have to turn right when you used to turn left in getting around the station."

"That will take some doing."

"Why? Oh, I forgot. You never attended school on the ship. Here the children learn that things like 'greater than' or 'less than' are relative, so that with a simple transformation, they can be switched. We make sure the kids do not get the silly ideas of absolute superiority that the Earth kids used to imbibe a millennia ago."

"Why, what was wrong with the Earth way of thinking?" Moishe bristled. He liked tradition.

"I've read that people used to label their race, country, religion or species 'superior' without specifying the order on which this superiority was based. Didn't really make any sense, of course, since 'superior' can become 'inferior' very simply with a change in criteria. The people were never very flexible. But I ramble. Sign these forms to say that you have been informed, and that you are responsible for any objects falling on the orphans' heads."

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November 19, 5821

On the ship, there were no religions that Tal and Moishe were used to, much to their surprise. But there were religions. The religious sought out a source of truth, and the ship's newspaper published debates on the nature of truth. Despite this, the only religious figure that had arisen was Bill's. Originally this was just to avoid work, but Bill knew he had to have some sort of story. He decided to kill two birds with one stone, being a guru and warning the children, who were getting old enough to be confused when the download back to their real bodies took place.

The time in Paralife was not supposed to be so long, but Bill assumed, or hoped, that the technicians in the real world would eventually solve whatever was delaying the download.

Bill explained to the children, and anyone else who would listen, that this world would end, and that eventually those with a high enough spiritual level would return to their true bodies in a world of which this universe was only a part. Some took his words as metaphors, but some took them more literally. Bill himself let it remain ambiguous. Of course, as with all religions, the founder's words were twisted in many different ways, and sects started to grow. In the end some children decided that the spaceship was heading towards a portal, a word they got from science-fiction, to a paradise one didn't have to die for. This worried both Moishe and Tal, as the children could come under the influence of some sect that would divert them from Judaism. They redoubled their religious instruction.

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October 2, 5826

Moishe and Tal had no proper birth records for the children, so they had to content themselves with the pediatrician's best estimates for their biological ages. They assumed that they were roughly the same age, about twelve. This determination was important so that one could hold the ceremonies to mark the crucial stages in the children's lives at the appropriate times. The next stage would be to mark the children's status of religious adulthood. That itself took a lot of wrangling, given the disagreements between Tal and Moishe about the age that the girls would undergo the ceremony, but finally a compromise was reached. These ceremonies, the bat mitzvah and the bar mitzvah, took place with a larger than expected attendance, as a number of people were curious about these customs. After the ceremonies, a party was held in the children's honor in the evening.

At the party Sheila chatted with Tal, who was explaining the origins of the terms of 'bat' for 'daughter' and 'bar' for 'son'. He told her, "I know that a lot of people find the terms confusing. There was even a play on this among some people in California ...."

"Oh, I read some novels that used that name. I never knew if it was real." Sheila interrupted.

"Er, you're right, it was never very clear. Anyway, they had a bark mitzvah for their dog at thirteen months."

"Muzzle tov!" deadpanned Sheila.

Moishe came over to find out why Tal was laughing. "What's the joke?"

Sheila started to explain. But Tal held up his hand to stop her." I don't think the Rabbi will find this funny."

"Oh, excellent!"

"What?"

"That is precisely what I want to find out, why one person finds something funny, and the other not."

Moishe said, "Will someone tell me the joke, and let me decide?"

Tal obliged. Moishe did not even smile. Sheila jotted down something in her book.

Tal looked at Sheila's notes and asked, "What are all these circles and lines? If you don't mind me playing the part of Professor Higgins' friend... I forget his name."

"Pickering? I am impressed. Not many on this ship have read Shaw."

"Er, I saw the movie version..."

"But the analogy is not bad. I am trying to teach not a flower girl, but a computer, not how to speak, but to have a sense of humor."

"What?"

She explained, “Humor is basically a result of the way humans imagine a framework of possible worlds, assigning each one a truth value and striving by a judicious selection to complete the model and accompanying theory. Quickly dealing with a threatening situation by being able to consider unusual actions such as fight, flight or other possible solution has evolutionarily been rewarded by survival, but to keep up this training in non-threatening situations, or even avoiding despair in threatening insolvable situations, or even to remember our past successes, is rewarded by the joy of humor. For example, in some jokes one part of the brain considers the usual possible world with a high truth value created by a theory based on genes, culture, personal logic and/or experience, while another part considers other possible worlds when there is time and ability to do so. When the final solution had to jump quickly from a low to a high truth value, then it has the possibility to be funny. Put more simply, expectation plus incongruity when action is not called for can, although not must, result in humor. The theory is more complicated, of course, and even the more complicated version needs polishing, so this is what I am doing.”

Tal asked her, “So Rabbi Cohen doesn’t find our joke funny because he anticipated something like it?”

“Perhaps, although, without getting Freudian, perhaps he feels threatened by its frivolity, and feels he can defend himself or others of his group.”

Moishe said, “Interesting theory, miss, and I wish you luck in polishing it. However, if you will excuse us, I need to settle a different sort of question with Mr. Kishon.”

After Sheila had left, Tal said, “Why did you chase her away? I thought you and I solved our differences about the children.”

“It’s life, Tal. One problem solved, another one pops up. Sort of like climbing that framework that your lady friend was describing. “ Moishe explained that the children’s sexual maturity was linked to the question of eventual marriage. On Earth, early marriages were no longer customary, but the need to extend the Jewish lineage was an extenuating circumstance. Tal said that he was hoping for conversions over to the Jewish faith, but Moishe pointed out that neither one of them liked proselytizing, so this simply was not realistic. So after another hour of discussion, they agreed that the children could be married at sixteen.

But to whom? Again, the religious backgrounds of Tal and Moishe clashed. Moishe thought that, given the few choices, an arranged marriage might be the most suitable. Tal was against the idea of an arranged marriage. However, Moishe pointed out that the children obviously had an This brought up the issue of their ancestry. Were they related? The four did not resemble one another, and were all approximately the same age, and so Moishe decided that the Orthodox prohibition for them to marry non-Jews was stronger than the remote possibility that they were siblings. When Tal and Moishe were discussing whether one could make such a conclusion, Moishe made use of his training to twist Talmudic text every which way. He asserted that one could define implication in order to allow this.

Tal said, “You’re going a bit far, aren’t you? Implication is implication, no?”

Moishe pulled out his tablet and pointed to a site from the ship’s library which indicated how one could, in fact, define a structure that allowed one to say, “it is true in all cases except some insignificant ones”. The trick was to define what insignificant was in such a way that the logic still worked, but it seemed possible. Tal thought that Moishe had not read very far in the site to find out that this was done with infinite sets, so that doing it with the finite sets that Moishe was talking about didn’t work. But Tal paused to consider how to make this clear to Moishe. Moishe interpreted Tal’s silence as Tal’s inability to find a counterargument, and Moishe considered that

a generalization was the same as the absence of a counterexample. Moishe felt that his point of view then was fully justified, and he said so. Tal then abandoned his argument about exceptions in order to make the more general point about Moishe's manner of reasoning. Tal stated the maxim that an absence of proof is not a proof of absence. Moishe countered that proof was a relative concept and he would go with his convictions. Somehow a lot of their arguments ended this way, with Moishe convinced by a logic that was unclear to Tal. Tal had to admit that there were many different ways to interpret a model, and hence did not have the strength of conviction that Moishe had, so Tal let Moishe have his way.

A stronger factor than Moishe's rationalizations led to Tal's acquiescence There were so few children of their age on the ship.

After much wrangling, the wedding was set for the year 5830.

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April 2, 2045

Despite the rather loose ethics of many of the Vladivostok city administration departments, the local department that granted research permits were rather strict about medical experiments involving human subjects. Therefore Natasha had the idea to combine her hobby with her profession, and run simulated tests on the complex program that she had stumbled onto and was following with interest. In that program she found two female characters with sufficiently similar backgrounds that they would serve as an excellent study pair. She decided to experiment with three aspects at once: fertility, gender selection and birth control. She would manipulate their cells and their anatomy already before conception. The two girls would ovulate two eggs, each with a slightly different coating: one to select for X sperm, the other, for Y sperm. As well, the birth would then render the girls infertile. She had not even tried to get approval for this experiment on human subjects, but she was confident the computer models would give the same results as in the real case. She was however not sure whether she could convince a research grant committee to see this if she did not reveal the source of the computer program used. Therefore she would have to write her own, which would take time. She decided to run the test on this Paralife program while she was writing her own, creating a theory that would have a submodel of that presented by Paralife. Then the results would carry over, and she could claim that she had carried out the test on a program written by herself.

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April 16, 5831

It came as no surprise when the two girls gave birth within a month of each other. What was surprising was that both gave birth to twins.

This coincidence awakened interest in the ship's newspaper, and an investigation was launched when it was found that they had all unfrozen soon one after another. The Reanimation Center was investigated; as there were no faults found, it was assumed that the babies had been exposed to some conditions or chemicals before being frozen on Earth. Or maybe it was just a coincidence. Sometimes a cigar is just a cigar. The case, and interest in it, quickly dropped.

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April 17, 5832

Bill's role as a guru allowed him a soft life. But Bill remembered that he had come here because his brain had deteriorated, and with it his mind. Now he had a different brain, but his mind could still deteriorate. He resolved to try to remember the topics that he had given the lectures on during the cruise.

Bill had dinner with the chief engineer. After they had ordered their food, Bill asked,"I understand that you people try to work with things which I call operators. Or maybe you have a different name for them? I mean functions that assign one physical state to another physical state."

"As opposed to where? Of course we call them operators."

"Good. As far as I understood it, there is something weird about them. Since there are energy operators are effective everywhere, and since energy and time intervals are fuzzy, that means that energy will vary in the vacuum. And since mass and energy can convert to one another, you would have particles appearing and disappearing."

"Yes, nothing magic about that if you keep in mind that the particles are just one state of the field. But OK, we can speak in terms of particles. In order to keep everything else associated with mass balanced, they appear in pairs. Opposite charge, opposite spin, opposite momentum. And to keep it that way, they are entangled, so you can consider them as one wave exhibiting itself in two places for a short time. For the charged particles, the opposite charges attracting each other is associated with the brevity of their separation, and thus with the amount of time a bit of energy can vary from its zero point. These time intervals are, in general, so short that they are insignificant when considered individually. That is, we can't measure them, but when there are enough of them, which there appears to be, the addition of those existing at that time adds up to something significant. They are popularly called virtual particles. Silly name, in my opinion."

"Ah, so that's why you use that theory of the hierarchy of truth values, leading to that big infinity?"

"Indeed. Or, if you look at it from the point of view of the particles at the moment they are existing, the energy is what comes back out after they get together again. It is said that they are created and annihilated, and so you can even break these up into two separate operators, which are just operators moving mass-energy levels. Space seems to be a constant battle between them. But these operators are not going to give the same thing if you first annihilate, and then create, as vice versa."

"That's counter-intuitive."

"Not so much. Suppose you and a friend were staying in Quito, Ecuador, and you got two separate messages. One was to travel sixteen hundred kilometers east, and another was to travel twelve hundred kilometers south. Naively thinking that you would then end up two thousand kilometers southeast of Quito, you and your friend decide to take two different routes and meet up together to compare experiences. You go due east, then due kilometers south; your friend does the reverse. You both forgot that you are on the surface of the Earth, where the Pythagorean Theorem does not work. So, you and your friend end up in two different spots in the Amazon. It is typical of operators which are not compatible that they do not reverse well. That is, they are not independent; they are both shadows of the same thing. Since they cannot have the same possible values, their spreads will be dependent on each other. In general, when you look at one value, you will always have a certain amount to get to the next value, because if you had the two values be continuous, then their possible values would overlap. They have to leap frog each other. Therefore when you restrict one, then the other one will also only come restricted."

Bill started making sketches on a spare bit of paper to try to follow the explanation. The engineer continued,

"However, if there is absolutely no restriction on one, for example, if an electron is in space and it could be anywhere in space, you can get one shadow to be exact, wherever you want. Since there is no value for one shadow, then there is no problem of overlapping, and the other

one can take on any value. However, once you restrict one, say, by the space it can have, then that restricts the others, and it will only take on certain values. Therefore, for example, around an atom, the electrons will only take on certain energy values, while in free space they can take any. This is relevant in our project, where there is more possibilities for the energies of the fluctuations, called virtual particles, in unrestricted space, than in the restricted space of the plates. Therefore the vacuum energy outside can add up to more than in the inside, and this means that the plates are pushed together. Casimir effect, we call it."

"Cashmere, like the sweaters?"

"No, Casimir, like the scientist who thought of it. Anyway, the point is that the field outside has more energy than inside, so the plates shift towards each other."

"A, but I thought we took the vacuum was the zero point..."

"Right, just like you take the freezing of water as the zero point on your Celsius thermometer. That doesn't mean that nothing is there. So, energies less than this are termed negative energy."

"Oh, so you don't mean that there is somehow less than no energy."

"No, that's not what negative energy means. Anyway, you know how we explained that mass-energy will distort space? The default is flat, as far as we can make out. This means that in space the Pythagorean Theorem will work. But then with increasing mass-energy, the Pythagorean Theorem no longer works . So, with negative energy, the relations are past the default, so we say that space is bent in the other direction. Whereas the curvature of space produced by the attractive gravitational field of ordinary matter acts like a converging lens, negative energy acts like a diverging lens."

"Vacuum energy... who would have thought.... but you said that the vacuum energy is seen in this effect. If that's the only place you see it, maybe the effect is due to something else, like the attraction of the molecules in the plates for each other?"

"You could argue that, but we see the vacuum energy elsewhere."

"On some other tiny level?"

"Quite the contrary. We guess that this is the reason that the universe is expanding at an accelerating pace. There are some difficulties in our calculations on that, but....ah, the desert. Let us continue this conversation another time. Or you are welcome to come to the engineering section."

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May 18, 5832

On Earth, Moishe and Tal would have avoided one another, sticking to their respective congregations, Orthodox verses Reform. Here, however, they thrown together, as each one wanted to discuss topics that no one else was interested in. Thus they were usually to be seen together involved in one of their eternal discussions. As the saying goes: One God, Two Jews, Three Opinions.

Moishe had not been taught much math at the yeshiva. What mathematics was taught was very traditional, avoiding all modern things such as set theory and model theory. These theories spoke about there being more than one infinity, but everyone knew that there is only one infinity, God. Model Theory spoke of multiple truths, which also went against common sense. Moishe believed that divine truth had precedence over silly human truth, but on the other side, he liked a good discussion.

Tal told him, "In terms of models, an infinity is something that allows a jump to a truth that you can't get to without it. God fits this description. Like a fractal."

“First time I’ve hear God compared to a fractal.”

“Ah, the idea is not mine. After all, the key feature of a fractal is a sort of self-reference, also called self-similarity. According to some formulations, God is the fixed point of the relation ‘is caused by’; in another formulation the universe is God looking at herself, in another...”

“What does that have to do with God’s infinite wisdom?”

“Each one of the descriptions of a fractal is incomplete, and you have to jump, as if there were an infinite number of steps, to the final result.”

“Infinity is jumping?”

“Um, that is not the only condition to be infinite. If you have a part of a collection that can be put into a one-to-one matching with the original set, then that collection and its part are infinite. But it would take me some thinking to figure out how I could apply that to God. So maybe it is simply wrong to use the word ‘infinity’ for God.”

“Oh, I don’t know. After all, it was a Catholic medieval monk who said that if you are thinking about God, then really you are thinking about one of God’s creatures.”

“Those monks were definitely ahead of their time. In modern parlance, if you have a statement in a higher model which is based on an infinity which is powerful enough, and the statement talks about the universe of a lower model, then there is something in that lower model which also satisfies that statement, if you restrict things down to the universe of the lower model.”

“You lost me there.”

“In other words, given a powerful enough model, then there is something similar on a smaller scale, and so one cannot say that any statement completely characterizes that powerful model.”

“What do you want to say with all that?”

“That is, maybe God created infinity, but God himself would not be infinite. We just lack words for what God is.”

Moishe liked this resolution. But Tal was hard to stop once he got going. He elaborated, “You can just think of the infinities as a license to be able to do certain things. Adopting an assumption like this allows us to go to another possible world that contains more items. Of course, we have to be careful. We don’t want things to seem possible when they produce results that are contrary to our measurements.”

“Let’s see if I get this straight. You add new axioms in order to create a new possible world which can imply more than your present one without contradicting the old conclusions. Can you always do that, or isn’t there some sort of limit”

“No, no limit.”

“But then that means that whenever you are in one model you know that there is another one outside of your reach. That seems a way to create your own frustration.”

“From one point of view, yes. From another point of view, it is encouraging. If we assume the right kind of axiom, we can be pretty sure that it will be possible to understand at least everything which can be expressed as a theory that uses only variables referring to individuals from the universe.”

“What do you mean, pretty sure?”

“The first requirement will be to be sure that our theory is consistent. But if the theory includes basic generalization that allows you to do simple arithmetic, then there is no way that we can prove mathematically that some contradiction will not come out. But with arithmetic, we are pretty confident nonetheless, as long as we are careful with our axioms. So, when we want to

assume a new axiom because it will be useful, often the best we can do is to check that, if arithmetic is consistent, then so is our new axiom. That's what I mean by 'pretty confident'"

"Oh."

"So, for example, we are used to dealing with possible worlds. And we like to put possible worlds all together in the past. So, we like to have the possibility to regard different possible results of different theories. We would like, in other words, to be able to look at a collection of all sets that could be formed from what used to be a universe, so that we are looking at the erstwhile universe as a set. For example, if we want to be able to count in our theory, we need a universe that includes all the counting numbers. But you can have other classes which are just as big as the universe... for example, the class of odd numbers. Such classes can satisfy sentences, and you would want to be able to combine such sentences. So it would be handy for you to make a new possible world, in which this universe becomes a set. That is, you want a set in your universe that cannot be reached by simple counting starting from any set below it."

"But that is limited. In order to be able to do other things that we want, for example be in a world where quantum effects exist, then it is convenient to accept the existence of other infinities."

"That's what you Reform people are all about, accepting the convenient."

"And you Orthodox find that tradition is more important than the individual."

The argument continued to deteriorate, but they always knew when to break it off before it became acrimonious. After all, with whom could they discuss if they became enemies?

Chapter Eleven

May 20, 5832

Bill actually found the chief engineer quite agreeable, and so he took every excuse to talk to her. He quickly found out that Sheila, as she was called, was not interested in small talk, so he continued to consult her on technical matters, even if he had to make them up. He sat down next to her in the Cafeteria."Remember our conversation about the Casimir project?"

"Yes. Why?"

"I thought... why can't there be regions like that in space, with bit of energy that was less than the regular vacuum energy?"

"There are, of course, since energy fluctuates. But remember that energy fluctuations and time interval fluctuations are connected, so the interval of time that these negative energy...."

"You call it negative energy?"

"I know, that makes it sound like it is less than nothing. But you have, in the temperature scales of Celsius and Fahrenheit, negative temperatures, when they are simply less than a convenient reference temperature. So, the negative energy regions generally are quite small. They would bend space-time in the opposite way that, say, gravity does.

"So that you go from one point in space to another faster than you can ordinarily do. It sounds like going faster than the speed of light."

"That's a non-starter. You know that the speed of something is with relation to its immediate surroundings, not to your starting and destination points. The speed of light is not attained with respect to the surroundings, because except for the entrance and exit areas, there are two paths connecting these two areas. It's as if you compare the distance of a human taxi driver and her colleague who is however a ghost. To get from the intersection of Second Avenue and First Street to the intersection of Eighth Avenue and Ninth Street, the human taxi driver would have to

reckon on a distance of fourteen blocks, but her ghostly counterpart could go through the buildings, and would only need ten blocks."

"So it's a tunnel through space?"

"Um, more precisely, a tunnel through spacetime."

"That gives an interesting possibility, if a spaceship could go through it."

"That a big 'if'. We figured this would not be possible. We still aren't sure, but it seemed like a question we would never be able to investigate experimentally. We were astounded when we spotted some stable, large, long-lived such pockets. The question is, as you say, whether it would stay open if a spaceship went through it. Generally, to keep a wormhole open, you need a lot of negative energy, the amount to form a black hole if it were positive, normal energy. The bigger the mouth, the more energy, so just getting a mouth about a meter wide would normally take converting the mass of Jupiter into energy. Therefore it seemed unlikely that the distance function would settle on precisely an arrangement that would allow a mass to go through it without collapsing, since the mass will change the distance function, so the distance function has to be very robust. But we seemed to have sighted one. However, from such a long distance, we can't be sure. Therefore, that is, in fact, where we are heading. Our astronomers think they have sighted a few possible wormholes. There is one in particular that will present a highly interesting phenomenon as a black hole will be passing by the path of one mouth, and will be heading for another mouth. When they get close to one another, we will be right behind, watching."

"Why?"

"Because we are not sure what is going to happen. I'll outline the problem. On one side, the laws of physics say you could build a time machine with it. But on the other side, a time machine usually brings paradoxes. So, together that means that the laws of physics bring us to a paradox. Therefore, we need to change the laws of physics so we don't get a paradox, just as the mathematicians changed the axioms about sets to avoid paradoxes. However, unlike mathematicians, we have a model that we have to find the theory for, whereas mathematicians just make lots of theory-model combinations, most of which are not real. So, we will, in a way, try to build the time machine, or rather let the black hole make it for us."

"Whoa. Roughly what would that look like?"

"Come see me tomorrow, and I will have an explanation, with diagrams, ready."

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May 21, 5832

At dinner time when Moishe and Tal sat down at the same table, one could not have guessed that they had had a break in their conversation. No greeting, no preamble. Moishe started speaking as if Tal had only spoken to him a second ago."In fact, in general, we only have a finite mind, so of course ...."

"We manage. Sometimes we just restrict ourselves to finite models, like a lot of computer programs do. Rather than make a bunch of possible worlds each finite, but with ever bigger universes, leading to a potential infinity, it is often advantageous to assume a single model whose universe contains at least all the counting numbers.

Moishe said,"I think you are using words in a different sense than the usual ones."

"More or less, but at least I am telling you which senses I am using them in. That is more than you get from most theological discussions. Look, let me ask you a question. When you talk about He Who Must Not Be Named..."

"Voldemort?"

“If I had made that joke, you would be at my throat. No, I mean your refusal to say the entire name of God. But when you describe him, that’s a name, isn’t it?”

“No, it’s more a title. My own last name, Cohen, was a title, but became a name.”

“Right, it transforms. Just like the transformation of the action of loving to the noun ‘love’. Reification, or in the case of God, the opposite. Or, we identify a place with the result of applying a process, like going north and east to get there from a fixed reference point.”

“That may be interesting, but we were talking about...”

“Infinity”

“I think of infinity as something you can’t get to.”

“Remember how models can change? If you have a theory that includes the process of counting, then it has to take the individuals out of your universe. So the entire process is in the universe. However, it is possible that the collection itself is not one of the subsets of the universe that your variables in the theory can access. So it is inaccessible to your theory. Not an end of counting. But you now change your model by including that subset of your old universe in your new universe, and add a symbol in your theory for it. This is like reification of the process of love into an abstract noun.”

“So you reify counting, you have one kind of infinite. This infinity may not exist as an object in your theory. So it is unattainable with your theory, which is why it is called infinite. But now make a new theory by adding the assumption that this new object exists just like other abstract objects that you assume exist. You assume ‘love’ as an abstract noun exists, even though you only experience the process.”

“I can do this with other processes that are unattainable in my theory, and co-opt them...”

“Co-opting is called closure; this is what a fractal, when seen as a fixed point, achieves. There are various ways to make fractals, but a common one is to have a step-by-step process that depends on the previous results – a recursive process -- keeps going – like, shrink a shape, and add it to, or subtract it from, the last shape. Or rotation and dilation, shift by a fixed amount, repeat. There are lots of variations, including many with colors. The end result cannot be attained by this process, so you jump ahead to another figure that you call the fractal.”

“For instance, when people add up an infinite number of things that keep getting relatively smaller, then they of course never finish, but that whole thing must be in the model. The fixed point exists in the model, even if the theory doesn’t manage to get to it. Not everything true can be proven, you know. Anyway, I am going to bed. Good night.”

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May 22, 5832

Bill showed up at the time Sheila had set. She gave him some tea, and pointed to a set of drawings she had made. She pointed to the first one.”Suppose that Alice is sitting on the Earth at noon on New Year’s Day, 3050. Standing near her is her very sturdy and intelligent robot Bob. Between them there is a short wormhole. Each of Bob and Alice are next to one opening, and they stay next to that opening. The openings are called mouths. I will say ‘Alice’s mouth’ to mean, in this context, that mouth of the wormhole next to which Alice is standing, and ‘Bob’s mouth’ to have the corresponding meaning. So far, so good?”

“OK. Does the wormhole stretch out as Alice and Bob move around in normal space?”

“No, it stays the same. If you want, you can imagine them holding hands through the wormhole all during my little fable.”

“OK. go slowly; I am going to make drawings as you go.”

“Fine. Now, Bob has a very powerful spaceship, which can not only accelerate quickly, but also take Bob’s mouth with it.”

“Ah, the strong acceleration is the reason you want Bob to be a sturdy robot instead of a human, is that it?”

“That’s right. Bob then takes off, in normal space of course, while Alice stays at home. Bob makes a big circle with his spaceship, coming back to Earth after a while.”

“OK, wait a minute while I draw that.”

“Good. While you are drawing, then I will ask you to draw the situation graphically through time, instead of in space.”

“From the point of view of which one?”

“Each one from her own proper time, that is, from their own point of view. The diagram will not be exact, but just to give the idea. Draw Alice’s path through time as a hill, sort of a semi-circle, and Bob’s path as a tunnel going through the hill. To make it clearer, let us say that at every point in such a path, the time since the initial separation between the two is proportional to the length of the path in the drawing.”

“Oof, that sounds like an equation.”

“Not really, because my numbers are not going to be exact, because Bob’s speed keeps changing. This will just be intuitive. It just means that the longer the path since the beginning point, the longer the time it took to get there.”

“OK, I’ll try. But I’m no artist.”

After Bob finished, Sheila continued.”Since Bob traveled close to the speed of light, compared to Alice, then the laws of relativity start being significant. That is, during this speedy part of his voyage, time proceeds more slowly for Bob than for Alice. Otherwise put, if Alice observes Bob’s heartbeat through normal space, she will see that the time for one heartbeat, by her watch, will be longer than for hers.”

“Is there some kind of simple way to tell how much?”

“Yes, but you don’t really need it to follow this story. Show me your diagram.”

Sheila looked at Bill’s diagram.”Too small. Make it larger. A whole sheet of paper. You will need more room in the middle.”

Bill did as she instructed. She looked at the new drawing, and said, “Good. Label the beginning point where they are together START, and the finish point, where they come back together FINISH.”

He did this. Sheila continued, “Um, your diagram is in blue ink. Do you have a different color?”

“I have red.”

“Fine. In red, make a little tunnel between a point on Alice’s path to a point on Bob’s path, so that the distance from the beginning of the paths is the same for both of them. Say, half way along Bob’s path, and about a third of the way along Alice’s path.”

Bob did so. Alice nodded her approval.

“Well done. The blue paths are the times in normal spacetime, and that red tunnel is an example of the wormhole that connects them. One detail for which the diagram is not faithful is the fact that the wormhole does not change length. So, in this drawing, the length of the red tunnel has no meaning. Can you follow that?”

“It’s a little confusing, but I will try to stay with you.”

“Good. Now, this red path is just an example. For every point in their paths, the wormhole still connects them. Clear?”

"Go on."

"Since the time according to the two characters will be different, I will be a little tiresome in adding whose watch a particular time interval is being measured by."

"Understood."

"All that is the background for my little story. The first two years, by Bob's watch, that Bob is absent, Alice looks through the wormhole. They are holding hands, such that their watches are next to each other, so she agrees with him that two years have passed before she sees him land on her front lawn on Earth. Then she finally looks around her, checks that her Earth calendar says that it is New Year's Day, 3052. However, when she looks out at her front lawn, Bob is not there."

"Slow down a little. How would I represent that on my drawing?"

"The point on Bob's path is at FINISH, but the corresponding point on Alice's path is only about two-thirds of her path."

"Ah, where Alice's path is the same distance as the length of Bob's path?"

"Right. Go ahead and draw it. If your first drawing of a wormhole clutters up the drawing, make a new drawing on another sheet of paper with only this latest wormhole drawn."

"Yes, I think that would be better. Wait for a minute, then, will you?"

"Fine. I'll get myself some coffee while I'm waiting. Would you like some?"

"Ah, no, but if there is herbal tea, then could you bring a cup? Thanks."

"OK. While I am gone, make another diagram without the wormhole, just the two paths. Do you have enough paper?"

"No problem. I am using the back of committee reports."

A few minutes later, Bill showed her the diagram."

"Fine. Now, Alice waits until she arrives to the same point as Bob. To make the fact that Bob's path in time is continuing although his path in space is not continuing, I should now try to make Bob's path longer, yet come to FINISH, but that would complicate the diagram. I could do it in three dimensions, but..."

"That's not necessary. I'll just pretend that Bob is treading water. Sort of. Go on with the story. Alice is waiting..."

"Right. Finally, in the afternoon of the twentieth of February 3053, she sees Bob's spaceship land on her lawn. For the first time in this story, she leaves her mouth, and goes over to Bob's. Bob still has his hand in the wormhole. Alice looks through the wormhole, and sees the moment that he landed from his point of view. Remember, at that moment, Bob was holding hands with the Alice of New Year's Day, 3052. Alice climbs through the wormhole, ending up in 3052. Since she started in 3053, this becomes a time machine."

"Sounds fascinating. When do we begin to build ours?"

"There's a couple of problems here. A big one is that the effect of bringing the two holes closer will make the build-up of positive energy in the wormhole cancel out the negative energy."

"Run that by me more slowly."

"Suppose that a traversable wormhole exists. Then imagine bringing the two mouths of the wormhole close together as in the story. Notice that in the story, on New Year's Day there would have been two Alice's. One was older than the other, so they are not really the same being, and in the story we consider that there is a possibility that both exist simultaneously. But this is not so clear for, say, a photon. A bit of light. Basically, a photon could act like one of those children at the swimming pool who go back to the line for the slide each time they slide down into the

water. Imagine if the child went back to the same point in time each time, and both children now went down the slide together. Then again, with four children. And so forth. You would have an exponential build-up of children all going down the slide simultaneously, or trying to."

Bill thought about his earlier trips to the pool, as the screaming of the children made it seem like this must have already happened. But he did not express this aloud, since then he would have to explain about Earth swimming pools, which were different to those on the ship. For that he would have to explain Earth. He let Sheila continue her explanation.

"That is, a piece of radiation could keep duplicating; making the build-up of radiation going through the wormhole enormous. Because it would be negative energy keeping the wormhole open, this amount of positive energy would likely collapse the wormhole even before anyone could pass through it."

"I have a little difficulty imagining negative energy."

"Try thinking of it in terms of the bending of space. If positive energy bends space such that the hypotenuse of a right triangle is bigger than predicted by the Pythagorean Theorem, then negative energy bends space such that the hypotenuse is smaller than predicted. If these two bendings come together, you can end up with no bending at all."

"And if this doesn't happen? Would you try to put the spaceship through it?"

"Not at first. The ship would bring energy that would bend the space-time of the wormhole a bit; that could be critical, and it is likely the wormhole would simply collapse, leaving you with only the original path."

"What would happen to the spaceship if it did that?"

"Have you ever watched someone kneading dough?"

"Oh. Um, can I think about that, and see you again another day?"

"Sure. You can come during the work day; I start my lunch break at twelve thirty. If you want to come after work, just give me a call beforehand to see if I am available."

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May 23, 5832

The next day was a holiday, so Moishe and Tal decided to go bowling. On the way to the Game Salon, they continued their discussion of the previous evening. Moishe started,"But this infinity you mentioned.... infinity is infinity, no?"

"Well, no. You might want to first understand what numbers and sets have to do with each other."

"Well, there's sets of numbers, I guess."

"That's true, but what is a number?"

"Um, something you count with?"

"So what would an uncountable number be?"

"An oxymoron."

"No, because your definition of number is too narrow. Remember the way you used the ordinals to make the counting numbers, starting with zero? Not only the numbers themselves, with each ordinal associated with the number of elements in it. But also the relationship between them, with one ordinal being a member of another matches with the associated element of the first one being less than the associated element of the second one."

"Right. So what?"

"But an ordinal can be made into a one-to-one matching with its successor."

"I've heard of that, where you have a story about a hotel with the infinite number of guests. To fit one more guest in, the manager just has all the guests move up to the room which is

numbered one higher than their present rooms. So the infinite ordinal plus one can be matched up with the ordinal, something you can't do if it was finite."

"That's spot on. You can use what you just said to define an infinite set, a completed infinity, by saying that it is infinite if you can match up a subset of it with itself."

"But I thought that a matching was a way to say that two things are equal in number."

"They are, in one way. But not in another way."

"So we make the ordinals which cannot be put into a matching with one of its elements, a cardinal."

"Slow down. A cardinal is one type of ordinal, isn't it?"

"Sort of. Basically, take a set whose elements are such that that you can make a given comparison between any two of them, and such that for any subset, there is always some element below which the comparison cannot go any further. Like having a bunch of people having their heights compared, and for every bunch of people, there is always a shortest one."

"That's always be the case, wouldn't it?"

"Of course not. Only for the relationships which your axioms would outlaw. But you wouldn't want to outlaw infinite regress for every such 'smaller than'. For example, infinite regress is OK for the integers with the usual 'smaller than relationship."

"Ah, then I mean that it would always be the case in comparing sets, no?"

"Naw. Suppose you defined 'smaller than' in one system to be the same as 'larger than' in the usual system?"

"That's cheating. I mean a 'smaller than' that would seem natural."

"Natural? That's an ambiguous word, but I'll go with it. How's this? Consider saying that one set Bob is smaller than another set Alice if Alice has all the elements of Bob, but Bob is missing at least one element which Alice has. Or, to put it another way, their intersection is Alice and not Bob."

"That's called a proper subset, if I remember correctly."

"Something like that. Alice would not necessarily be smaller than Bob by using the comparison of ordinals or of cardinals, but nonetheless we could call her 'smaller than' him if we remembered what we mean by it, and this seems pretty natural, wouldn't you agree?"

"I guess so."

"This type of 'smaller than' would allow infinite regress. For example, suppose Alice is the set of all counting numbers bigger than one. Then say her mother is the set of all counting numbers bigger than two, her grandmother is the set of all counting numbers bigger than three, her great-grandmother ...."

"I get the idea. OK, but that is OK; it is not the 'smaller than' that gives problems. It's making infinite regress with the relationship of membership that gives problems. The old definition of led to annoying paradoxes, such as allowing a special set, let's call it Roland. Roland is defined as the collection of all sets which are defined by the property that they do not belong to themselves."

"I know that one, where the killer question is whether Roland is an element of itself. You get a contradiction, no matter what you answer."

"Right. So it was replaced by a notion of set which basically says: form any new set in levels, where each level is created by collecting all sets whose elements are at earlier levels. To be able to put this into effect, you have to start from somewhere. So the infinite regress with membership wouldn't allow you to, so we outlaw it."

"Would it lead to contradictions?"

“Not necessarily, but it could. To construct a theory allowing it is possible, but such theories can’t do much. So it is easiest just to do without it.”

“OK, we do without it. What does this have to do with anything?”

“Remember that we are using the ideas of sets because we maintain that a lot of stuff is analogous to something using sets which follow this assumption that sets don’t allow infinite membership regression.”

“A tall order.”

“One that I think is justifiable. Having the universe of a model be a collection be seen as a set from the point of view of another universe, and having the only relationship be that of set membership, has been pretty successful at modeling about anything one can think of.”

“If you only need the membership relationship, why are other relations usually included in models?”

“For convenience. For example, it would be possible to build all the houses making the bricks from scratch at every house, but that would not efficient. However, if a house collapses and the architecture was OK, then one can also examine the make-up of the building material.”

“But these models must be awfully complicated.”

“Not really. For example, a lot of relationships are similar to the membership relation in various aspects, including this lack of infinite regression for any of its parts. What’s more, a lot of them have the additional property that there are no incomparable pairs. That is, any two elements can be compared using the relationship. In other words, of any pair of distinct elements in the collection, one of them has that relation to the other one.

“OK, but what is your point?”

“A first step in analyzing a new structure or idea is figuring out what to make an analogy with.”

“That’s a pretty good general principle.”

“One of the ways to make an analogy for one of these relationships with the lack of infinite regression and the total comparability of all of its elements is to say that two such relationships are similar if they both line things up similarly. That is, one looks like the other if all you were concerned about was the order that the relationship imposes on its elements.”

“An order imposed on its elements? That sounds like a despotic ruler imposing his idea of a social hierarchy on his subjects.”

“Sort of. Anyway, any two such orders can be compared. To do that, you first say what it is for them to be equivalent in this regard. I’ll give you an example. Suppose you are building an artificial body that exactly copies the functioning of the real body, but with different materials. Let’s simplify and say protoplasm and silicon. The constructor takes a simplified approach. She checks that she can make a one-on-one matching between the particles of protoplasm that she uses to the particles of silicon. Then, she checks the important relations between the particles in each case, that is, a protoplasm-relationship between the particles of protoplasm, and a silicon-relationship between the particles of silicon. She does this by checking that, if the two particles of protoplasm are related by the protoplasm-relationship, then the corresponding particles of silicon are related by the silicon-relationship.”

“That won’t be very efficient, because...”

“I know. This is just to get the idea that one can compare collections according to the way they are ordered by their respective relationships.”

“You should have said so.”

“Hm. If I had started with it, you would have asked for an example, no?”

“Maybe. But so what?”

“Hang on, I’m not done. Rather than compare all possible collections and their relationships to one another, one can take one kind of collection, and one relation among its members, as a reference.”

“You’re going to tell me that the relationship of set membership among sets is a good reference, right?”

“Good guess. What you can do is to take them from a collection of sets which is transitive with respect to this relationship....”

“I always forget what transitive means. Something about transit areas at the airport?”

“Similar idea. That you can always cut out the middleman. In this case, suppose that a bunch of collections were each named after persons, and these collections joined an association so that if Tom, Dick and Harry are members of that club, and Tom is a member of Dick, and Dick is a member of Harry, then Tom is a member of Harry. So the club would be transitive.”

“So you are talking about three properties now: no infinite regress, no incomparable elements, and transitivity.”

“Right. The first two properties will be part of the collection you are trying to find a comparison for, and the comparison among the sets will have as a bonus that they are transitive. In fact, all members of a transitive set are also transitive.”

“Keep to the point. You wanted to talk about them being a representative of all the possible ways to put an order to a set which has those two properties.”

“Right, that representative is called an ordinal. An ordinal is a representative of the similarly ordered sets.”

“*A* representative? Why not *the* representative?

“You can often get two different ordinals which both could represent the same order.¨

“Ah, so you can’t use the ordinals to talk about the size of the set?”

“No, for that you use a cardinal number. The cardinal number of a set is the smallest ordinal number that can be put into one-to-one matching with that set. The cardinal doesn’t say anything about the orders of the ordinal and the set, so it is not always the same thing. Anyway, when I say the size of a set, that’s what I mean.”

“So ‘smaller than’ means one thing when comparing two ordinals, and another thing when comparing two cardinals. For instance, you can’t make an infinite cardinal bigger just by adding an element, although you can do it for an ordinal.”

“Oh, that’s confusing: two different orders: ordinal ‘smaller’ from the idea of membership, and cardinal ‘smaller’, from the idea of matching. But what makes up a bigger cardinal? Taking also all the negative numbers?

“No, there are the same number of whole numbers, including the negatives, as the number of positive whole numbers.

“That’s absurd.”

“No, it isn’t. Let’s take all possible prices, as given in cents. They are all the non-negative whole numbers.”

“Non-negative? Sounds like a double negative.”

“I mean either zero or bigger.”

“Why not just say ‘positive’?”

“That doesn’t include zero. But OK, it doesn’t make any difference, as I pointed out in the example of the extra room at the hotel a few minutes ago.”

“OK, go ahead.”

“First step, you have a huge sale. All prices cut in half. But your price-cutting allows some half-cents to be in the sale price. If the sale price is a whole number of cents, then turn that price into the corresponding negative numbers. For example, make ‘eighty cents’ into ‘negative eighty cents.”

“Quirky, but OK. Then?”

“Round up the prices that aren’t whole numbers. Eighty and a half cents becomes eighty-one cents.”

“OK, this is what stores do anyway.”

“Now, match each old price with its new sales price. There is then a one-to-one matching between the collection of original prices and the collection of sale prices. That is, the collection of all non-negative whole numbers is then matched to the collection of all whole numbers. That is, each collection has the same number of elements as the other one.”

“Let me try again. What if I find all pairs of integers? That would be like looking the set of all the fractions? Wouldn’t that be bigger than just the set of the counting numbers?”

“No. Since putting in negatives doesn’t change the amounts, let’s take positive whole numbers. Let’s go bowling. I will show you what I mean while we are there.”

In the ship’s bowling alley, Tal pointed to the bowling pins.”

“But I want to continue making an extension of this bowling pin arrangement. A triangle, except that it will keep going beyond 10 pins. I am going to have each row be connected to a pair. The pins have rows, right? In each row, you have the first number of the pair starting on the left hand side of the row with one. Then you count up, one, two, three, until you run out of pins in the row.”

“Easy enough.”

“Now you have someone who speaks a Semitic language, say, Arabic, numbering the pins. They put the numbers 1, 2, 3 and so forth, but they start from the other end, on the right. The Arabic person will number the pins, putting them as the second number.”

“OK, you have an infinite number of such rows. I will guarantee you that all pairs of counting numbers are there.”

“Ah?”

“But now there is the owner. He just counts how many pins he has, by starting with the first pin, and going along, not starting over at each row. He keeps going, counting. That is a perfect match-up of all the pairs of counting numbers to all the counting numbers.”

“So you can use that to show that there are just as many counting numbers as fractions with whole number top and bottom. Weird. This battle is lost, but the war goes on.”

Chapter Twelve

May 24, 5832

Bill showed up again at Sheila’s office. “What are you now working on?”

“The next step in the Casimir Project. After we have accomplished our first research goals, perhaps the next step would be to send a probe into a wormhole. At the same time we are trying to find a theory where backward time-travel scenario would be a model to fulfill it. Then the question will be, even if we have a theory, does it match reality? And can we even get a theory?”

“Time travel? Isn’t that... I mean, time is something that is used up, gone, finished. It isn’t recyclable.”

“Depends on your definition of time. Tricky little concept. Remember, it really should be in terms of something we can measure.”

“Wouldn’t any definition forbid a backward movement?”

“No, the one-direction propensity of large-scale time has always been problematic. Try it. What is your definition of time?”

“I associate time with ... what are they called? ...ordinal numbers, where one moment is contained in the following moment, and the moments string together. With time travel you are likely to come up with a set being a member of itself, which is a no-no, according to my world view. Or another idea is that time is the stage for causation. That is, causation is implication that is not reversible, and the order that this imposes on events, we call time.”

“Those are nice attempts, but it is a bit trickier than that. Time as we know it is actually a mixture of two different concepts. The trickiest part is to figure out how these two concepts are tied in with one another. On the micro level, time is not an operator, but intervals of time can be associated with one. Under certain conditions, the operators can reverse. This is, in a sense, going back in time. Do you want some tea?”

“No, thanks.”

“I will just make myself some, if you don’t mind.” She got up to turn on an electric kettle, which seemed out of place among all the electronic devices in her office. As she was looking through her selection of teas, she continued to explain, “On the macro scale, however, it gets trickier. First, there is the difference between measuring time between two events. To make it simpler, suppose the two events take place at the same place. For example, the time between two successive heartbeats of Alice. Time is measured in comparison with some repetitive event which we call the workings of a clock. However, this may be different, according to whether the clock is moving at the same velocity as the event, or not. So time is a fluid concept at the macro level.”

Sheila paused to sip her tea. Then she continued, “One more tricky part is to decide what changes we should label positive, and which way, negative. The first step is to see where the directions make a difference. On the micro scale, the operators are symmetric, but on the macro scale, there is a direction that seems not to reverse itself. This apparently has to do with the way the energy can spread itself out. That is, it is associated with increase in the number of possibilities available. This is popularly referred to as an increase in disorder. That is, if you take the number of possibilities which a given number of things have to interact with one another, then you get the set of all subsets, that is, the power set. As we know, the power set always has more elements than the original set. Of course, in the physical universe there are constraints, such as the speed of light limiting causation relationships, but even you are generally going to have an increase in a closed system. That is, a system where mass-energy is neither added nor subtracted. That would be cheating.”

“Generally? Are you talking about exceptions like crystals or life?”

“No, those are not closed systems. For example, yes, the human body does create order out of disorder, but look at all the energy that it consumes in order to do so. If you took everything into consideration, the human body is more destructive than constructive.”

“You could say that in several senses. But you haven’t explained what you mean by ‘generally’. Are there exceptions?”

“Yes, just as there is a chance that you will come up with a lucky streak in gambling. But in the long run, an amateur gambler will lose. Las Vegas lives on this idea.”

She took another sip before continuing.

“This redistribution of energy in its turn is associated with the way of measuring; specifically, the expansion of space-time. The curvature of space-time is changing, and that is tied up with entropy. Time becomes the ultimate trapdoor – easy in one direction, hard in the other.”

“In science fiction...”

“The science fiction version is that the entire universe but you goes back in time. It is never considered what might happen at the boundary between the parts going backward in time, whatever that means, and the parts going forward.”

“What about the conservation of matter?”

“Conservation of mass-energy works when the same sentences will still be valid if a sentence talking about a certain time interval will have the same truth value in another possible world of a different time. That is hard to tell, since although time intervals are operators, time itself is not, so it could well be that a shift in the past, when space-time was differently curved, mass-energy conservation may not necessarily be the case. I’m afraid you can't use conservation of energy as an argument to affirm that time travel is impossible.”

“But if it were possible, wouldn’t that mean that you could have an effect before a cause? Like, where you go back in time to kill your grandmother at birth, making your existence impossible, making your action impossible, meaning that your grandmother lived, meaning that your action is possible, making...”

“Maybe not. Paradoxes in logic are avoided by examining closely the concepts of set and model. Perhaps Mother Nature avoids the grandmother paradox also via a model theoretic notion, that of possible worlds.”

“You mean the science fiction idea of parallel universes?”

“No, not exactly. The problem is that that expression includes a lot of different ideas, and the notion I am thinking about is only one of them. Whichever concept you deal with, it would not contain the contradictions that are usual in science fiction. For example, one idea is based on the fact that we only have contact with the observable universe. Beyond that observable universe, the physical universe could contain other universes observable to other species but not to us. They might not even have the same physical laws.”

“That makes sense.”

“Um, yes, except the problem is that this set-up, as well as a lot of the parallel universe ideas, seem to be by definition non-testable.”

“So which one are you hawking?”

“I’m not hawking it. I am using it hypothetically, as in ‘what would the world be like with computer governments?’”

“OK, which one?”

“It has the moniker ‘Many Worlds Interpretation’.”

“Why is it called an interpretation, and not a theory?”

“Matter of emphasis. We have a model, physical reality. We have various theories for reality. But between theories and models there are the third and fourth components, that of interpretation, and that of truth value assignment. For most theories, we have an idea how to assign truth values, even if the details are sketchy. There are several interpretations where it is tricky to figure out the truth value assignments. This is one of them.”

“Maybe it is analogous to the statements in logic that are unprovable yet true.”

“Not an altogether bad analogy when you realize what you have said. The truth of a sentence which is unprovable by a theory is decided by a higher model or by an expansion of the theory.

The new model would include the truth determination based on the lower model and the original theory. In the new situation, the statement is no longer undecidable. The phrase ‘undecidable but true’ cheats a bit, in that it is undecidable using one set of tools, and true by another.”

“Yes, I know all that.”

“Not bad to be reminded of it. My point is that there will be another expansion of the model or theory in which the statement is false.”

“Give me a starting point, here, Sheila.”

“Bill, you already know a lot of it. You yourself explained the theory which had the possibilities, some of which became observations, or physical events.”

“Oh, that.”

“Yes, that. Your analogy about undecidability being akin to indeterminateness is flawed, in that undecidability has an eventual model that can decide the theory, whereas indeterminateness has none. However, for the sake of the analogy, we will take a sentence ‘this electron has spin up’, first in the context of the indeterminate state. Let’s say that a moment later we have a new model which consists of the previous model plus a restriction of the domain of the spin from ‘up, down’ to a domain that has only has the domain ‘up’, that is the model that then satisfies the expanded theory consisting of the old theory plus the sentence ‘this electron has spin up’, decides the question.”

“That is, an event decides the question, if only for a point in spacetime.”

“Yes. The second model will be a possible world in the previous moment, and it becomes necessary one you have moved up the possible worlds ladder. But before then, if the passage in spacetime had chosen another branch, that other interpretation could have chosen the domain ‘down’, that is added the sentence ‘the spin of that electron is down’ , and the interpretation would have been just as valid. The question is: does this ‘down’ possibility branch ever get a chance to be used in some physical sense? I don’t know the answer to that question, and possibly no one ever will, but the idea does not appear to be self-contradictory. That is, it could correspond to a model. If this model can satisfy a universe with time travel, then time travel would not be self-contradictory, unless we get new data that will make it so.”

“OK, that was an elaborate preamble. So how would you use it to resolve the grandmother paradox of time travel?”

“If there are different realities, perhaps you don’t actually go back in the past of your present self but the past of another self in a parallel universe.”

“Ah, I sense something foul here. It might seem to you that it is another time, but you are not actually traveling back into your own time, but into another time. So you’ll have to come up with something better.”

“I will admit, the assumption is that nature has some rule that we haven’t discovered yet that will make time-travel inconsistent, that is, for making the accessibility relations between possible worlds going in only one direction.”

“Maybe it’s possible but just doesn’t happen?”

“The theoreticians don’t like that answer. The only way to know that it doesn’t happen is to show that it cannot happen.”

“But wait a minute. I remember that a when you determine a particle’s path by measurement, the result is really a combination of the different possible paths of its undetermined state.”

“What does that have to do with time travel?”

“Don’t throw me off. I am still formulating my idea. Um.... suppose there is a star that sends a particle of light towards the earth. But there is a galaxy in the way between the star and the earth.”

“So the light doesn’t get to us, being absorbed by the galaxy?”

“That’s what Newton would have believed. But it turns out that it can get to us. On its way, not being determined, it will take lots of paths, but basically you can split all these paths up into two parts, one which adds up to a path on one side of the galaxy, and one which adds up to a path on the other side of the galaxy. So we talk of two possible paths, even though this is a bit of a simplification.”

“OK, two paths. Do we see both of them?”

“No, they come back together in the final path, but the fact that it took the two paths means that each partial path interferes with the other one. An astronomer can notice when she analyzes the light getting to her.”

“I still don’t see where you are going with this.”

“Wait, wait. Suppose the astronomer who is measuring all this doesn’t point her telescope so that she can get the light coming from both. Rather, she points her telescope to only measure whether there is light coming on one side of the galaxy, and ignores the other side. Then the path has to collapse to one or the other side.”

“More or less, but...”

“Wait. If you interrupt, I may lose my train of thought. Now, the question is: the light will be going on both paths at the time it goes around the star, but then years later, when the earth astronomer observes it, it is only going around one path. This is the future determining the past, no?”

“Are you finished?”

“Yes. What did you want to say?”

“That you made a key mistake in the middle. That is, the photon, the piece of light, still goes around both sides, no matter how the photon will be observed on the earth. In both cases, the photon is undetermined until it is measured on the earth. The whole path is determined when it is measured by the astronomer, no matter what she does. The measurement affects the photon’s history, because the photon is a wave, and that wave goes through an interval of spacetime. We are not shooting marbles here. You have the paths on all sides of the galaxy, and the measurement just assigns the truth values. If you want to call this time travel, go ahead, but that would be deviating from the usual definitions of time, including your own, that include a transfer of information. No information is transferred in your example until the photon is measured. Since, in your set-up, the photon remains unmeasured in the past, and only is measured in the astronomer’s present, this means there is no transfer of information from the present to the past. So no time travel. Nice try, though. There are lots of experiments which try variations on this, with set-ups which erase information to see how it affects the interpretation of the data, but none of them show time travel, even though the results are often counter-intuitive.”

“I read some of those erasing experiments. I saw some videos on the Ethernet which say that, because the measurements affect reality in such a way, and because it is conscious beings doing the measurements, that our consciousness shapes reality.”

“Oh, ugh. Deepak Chopra’s rubbish. Look, there is a huge mistake there. The word ‘measurement’ is misinterpreted as being from a conscious observer. It’s not. Measurement just means an exchange of information, and information just consists of having two states be

independent of one another. So whereas it is true that human consciousness is affecting the universe, it is also true that that the color of a rock on Jupiter is also affecting it."

"Human consciousness is a rock?"

"That's not what I said. But human consciousness comes from certain brain activity, and that is just as much a physical process as the changing of the wavelength of the rock as light hits it."

"Ah. But it's a harmless little bit of New Age..."

"No, it isn't harmless. He gets people to think they can change their cancer cells by just being optimistic. It doesn't work that way, and this is a dangerous idea. But let's not get started on him. He's not on this ship."

"Some of his followers are, and his videos are in the Ethernet."

"What can I say? The best I can do is to try to spread a bit better understanding of this stuff, so people would be less likely to be convinced by such superficial interpretations. Let's get back to the subject of time travel. Do you have any other ideas?"

"You're the expert. Or do you just enjoy showing me that I'm wrong."

"Yes, I'm the expert, but I just told you that I think it is important that non-experts get a basic idea of this stuff. Also, often experts can get ideas from non-experts. You can be my muse while you are learning something. So... any other ideas about time travel?"

"Yea, actually. In that telescope example, the two parts of the paths are clearly entangled when considering each path as the path of a particle. So this brings me to entanglement between two particles which can be more clearly be measured as two separate particles."

"What did you have in mind?"

"Entanglement, I remember, will be between two particles, two points in spacetime. Not just space. That means that a particle in the future can flip to affect a particle in the past."

"My comments to your previous example are applicable here too. The counter-intuitive part comes when you consider the two undetermined particles as separate. They aren't. They are one entity. Wouldn't you say that what something is in the future is intimately linked with what it is in its past? But that is not time travel."

"But... couldn't you, I don't know, have two entangled particles, one in the past of the other. Call them today's particle and yesterday's particle. Yesterday, today's particle is indeterminate, which means that yesterday's also is. And yesterday's was indeterminate all day yesterday. But then, today, I measure it's entangled particle, and yesterday's is now determined as well. But this doesn't work, because... I'm confused. How could you go about showing that particles are entangled across time?"

"I see that you are confused. That is natural. The arrangement to show entanglement across time is not evident. Even when the way to do it has been worked out, the technical realization can be even further down the road. You have to be cleverer. I will outline the general idea of one such experiment. Take two pairs of entangled particles, let's call them Pair One and Pair Two. From each pair, one particle is sent to Alice and one is sent to Bob. The remaining ones are kept by Victor."

"Don't go too fast. I'm going to write this down. Alice-One is entangled with Victor-One, and Bob-Two is entangled with Victor-Two."

"Good. Then Victor has a choice. He can either entangle Victor-One and Victor-Two, or he can measure both of them."

"You're leaving out the possibility that he measures one but not the other."

"That's right, he's not given that choice. If he entangles his particles, then Alice's particle becomes entangled with Bob's."

"Yea?"

"Yea. And if he puts his particles in a determinate state, then this will have the same effect on Alice's and Bob's."

"So?"

"But we can measure Alice's and Bob's photons to see if they were entangled."

"I don't see how you do that. If you measure them, and one then is measured with an opposite spin, this could be due to entanglement, or this could be chance."

"When I talk about measuring a single particle in such a situation, I really mean we measure many particles in the same state. That way we can decide if it was by chance or not."

"Ah, why didn't you say so?"

"Sorry. Can I go on?"

"Please."

"The quirky thing is that we can do the measurements in the following order: first measure Alice's and Bob's particles. Then, later measure Victor's. Only then figure out whether Alice's and Bob's particles were entangled or not when they were measured."

"But, between the measurements of Alice and Bob, and Victor's measurements, you would have some sort of pattern available, no? These cannot just disappear."

"No, but it is set up so that the interpretation of the pattern will depend on the outcome of Victor's measurements."

"Ah, but that would mean that Victor's measurements affected the past."

"In a way, yes, but again, this is not time travel, just like I mentioned for the astronomy example. The way to do the experiment is rather complicated, otherwise someone would have done it in the twentieth century instead of the twenty-first. But you can look up the details if you are interested. They are in the ship's library."

"Thanks. You have just given me something to do for the next couple of centuries."

"Any other ideas?"

"Yea, one more. I read that antimatter is matter going backwards in time. Is this something like time travel?"

"Ah, that's one way to look at it. This is looking at time in the sense of transformations. Matter, as well as antimatter, can be seen as a collection of waves. The waves for the antimatter are then essentially the same as for the corresponding matter, except opposite. For many of these waves, when they come together, they just add up to zero, but for energy, the wave turns out to be the same as its opposite. So they don't add up to zero. That, in essence, is why matter and antimatter come together in a flash of light. So-called annihilation, although they are not really annihilated, just transformed."

"So how is that moving backward in time?"

"In some representations it is convenient. For example, if you measured the change in electrical current of a bunch of electrons moving from left to right, you would get the same result if you measured the change in current of the same number of positrons moving from right to left. So you can loosely talk about the positrons as if they were electrons moving in the opposite manner, so backwards in time in a way. But this gets difficult when you extend the analogy. The laws of nature are not completely time-symmetric, so 'moving backwards in time' is hard to even define."

"But can we reverse everything by replacing everything with antimatter?"

"You would have to do more than that. You would have to if you express spacetime relationships not in terms of the four positive axes as we do now, but in terms of their negatives

axes. Also change all the charges to their opposites. Then you would have converted from a ‘matter’ system to an ‘antimatter’ system whose motions are a mirror opposite of the ‘matter’ system. But the forces, for example, would stay the same; gravity and opposite charges would still attract and all that. To go backwards in time like the science fiction re-wind of a film, you would have to have repulsive gravity.”

“I didn’t know that gravity entered beauty contests.”

“No, I mean that the sun would push the earth away from it, not attract it.”

“I read that there is repulsive gravity.”

“That is speculation to try to account for the accelerating expansion of the universe. That’s not what I am talking about. My point is that the switching around of things to make your theory hang together does not look like going back in time.”

“OK, I guess that was a lead balloon. Let’s get back to what all this has to do with your Casimir experiment.”

“To find out whether we have yet another scenario to figure out how to explain apparent time travel, or if time travel can really exist so that we have to tweak our conception of time, all this constitutes our motive for this attempt to observe a wormhole where the ends have been brought together. Offhand, I can think of at least two reasons why that scenario for a time machine might not work. Both are based on the fact that... well, imagine that you are in a lottery, where every person has her own personal winning lottery number for February 4. Inside, they don’t check off your number, because you have to claim the prize personally, with your DNA, and they are confident that the line outside is too long: you could circle around and get in line again, but by the time you reached the door, it would be February 5. However, your identical twin is also in the line, way in front of you. Your twin learns the number, circles back, and informs you. This is what might happen with the photons. They would travel back in time through one path, and forward through another path, doubling up at one point. You get so much energy that the wormhole collapses.”

She clapped her hands to give a bit of imagery before continuing.

“Alternatively, also charged particles can do the same thing, and you might get so much charge at the entrances that they repel each other when you get close enough to make the time travel possible.”

“But how are you going to make the wormhole mouths meet?”

“The whole idea was a wonderful bit of serendipity. Two groups of astronomers, one at Stanford and one at Caltech, were mapping out the sky. The Stanford group mapped out black holes. There were a lot of them.”

More, thought Bill, than in the real universe.

“The Caltech group was mapping out wormholes.”

There appeared to definitely be more wormholes here than there were in the real universe, thought Bill. Aloud, he said,

“I thought wormholes were just a kind of science fiction concept. Nice for the story, but physically impossible.”

Sheila told him, “Well, tiny ones are possible, and even probable, because the variations of the mass-energy in space also mean variations of the bending of space, which means that the means of measurement, the metric, would vary, so that now and then you would get a metric of a wormhole. However, you know that these variations change quickly, so the probability is that the metric would most likely quickly go back to normal. But larger, more stable wormholes are

theoretically possible. Just no one thought that they would be found. But they were. A lot of them."

"Hm," thought Bill."Large stable wormholes might not be possible in the real world, but no one has proven that in this universe, they don't seem to cause any contradiction... yet. Could Paralife crash, taking them all with it? It would not even occur to Sheila that her reality could engender a contradiction and collapse, just as we don't in the real world. But Paralife could be doomed. If I don't get out of here soon...." He took leave of Sheila, visibly worried.

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May 25, 5832

Moishe came back to Tal to continue his discussion from last time."I give up. So you have convinced me that there is nothing bigger than the counting numbers," said Moishe.

"Sure there is." countered Tal."I never said that there wasn't. You just never came up with a good counter-example. But I can. There are counter-examples all around you, in time and space."

"But we can measure only things that can be expressed as fractions of whole numbers. That's why they're called rational. So, you yourself have shown me that the set of all these rational numbers is only countable."He stopped to consider."I guess, if you stretched a point, and considered indirect measurement, then we can measure things like the square root of two by measuring the two sides of a right triangle, or pi by measuring the diameter of a circle, and that sort of thing. Is that what you mean to say?"

"No, that's not going to work. Even if we take all of that sort of thing, we have a set that is still countable. But you learned that possibilities are part of reality. So, for example, we have infinite possible positions that something can have its center. Or you have infinite probabilities..."

"Yea, but they are probably also only countable, using some cute trick."

"Actually, not if you stick to one system. That is, if you make something be the set of real numbers by a very limited system, and then view it from a wider theorem, it becomes countable. But sticking to one system, one can show that the reals are not countable. Or even if you don't worry about real numbers, possibilities are a source of uncountability, assuming you accept the idea that you can collect all subfolders possible of a given folder, and putting them all into a new folder?"

"Sounds like a nightmare. But I'll accept it. Kind of hard not to, when you think about it."

"Then you can figure out how many subfolder possibilities there would be. For finite sets, we use exponents. Powers. Raise two to the power of the number of the number of elements."

"You lost me there."

"Suppose three friends are deciding what to do one evening. One idea is to go to the cinema. Maybe everyone wants to go. Maybe no one wants to go. Maybe only a pair, but there are three possible pairs. And so forth. Eight possible collections of people wanting to go, eight possible worlds, or, when all in one frame, eight possible sets. Two to the power of three. The set of all these sets is called the power set of the set of three friends."

"Wow, that's using the word 'set' four times in one phrase."

"Sorry. But do you get what I mean?"

"But why should I worry about sets when I am talking about logic? Why can't I worry about individual elements?"

"It's sort of like the transformation that society has gone through since the introduction of Facebook. Friendship has switched from a concept of individuals to a concept of sets."

“I’m not convinced, but I get the idea; in the power pet of a set of five elements, I would have... what’s two to the power of five?”

“Thirty two. Now think about applying this to an infinite set.”

“I would guess that there would then be the same number of elements, like when you added elements.”

“You would be wrong. The Power Set is always bigger than the original.”

“Naw, you must have some clever way of matching, like you did with the others.”

“No, no. For example, suppose you are a con-man. You are going to convince each member of a set of victims that she has been made the president of a club. Just in case the victims communicate with each other, the clubs should have different members, although all the members must be taken from the group of victims. So each victim is associated with one of the subsets. But you decide that all possible clubs should be represented. However, an aide points out that in this way there might be members which are not members of the club they are supposed to represent. OK, put them aside, you tell him, and form a new club. But then, the aide objects, this club won’t have a president. So you will have failed...”

“What was the point of that story?”

Tal sighed. He tried again.”If there were a matching between the members of a set and its subsets, you could form the set, say Alfred, of all members of the set which are not members of their associated subset. Question: since Alfred is one of the subsets, is there any member of the set which is associated with it?”

“Ah, I get it. So you are saying that the power set of a set is always larger than the original set, right?”

“That’s it.”

“Fine. But you have just skipped over the idea of real numbers. Don’t think I don’t notice these things.”

“No, I am not so presumptuous. I am just getting there a bit slowly. Let’s just take the real number between zero and one, in fact. Then express them all in binary. That is, only with ones and zeros.”

“A bit harder, but my computer can convert.”

“Now line up one of those numbers. Say, the first place after the decimal, line up with one. The second place after the decimal, line up with two. And so forth. Each counting number will have either a zero or a one matched to it. Make a special set, call it Special. If a counting number is matched up with a one, it is in Special. If it is matched up with zero, then it isn’t. In this way, you get every real number between zero and one to be matched up with the set of subsets of the counting numbers, or any other countable set.”

“Is that all there is to it?”

“You have to tweak it a little, because some numbers have two ways to write them in binary. But when that happens, you only take one. Anyway, in this way you get a perfect match between the set of real numbers between zero and one, and the power set of the counting numbers. And so since the power set is always bigger than the original set, that means that there are more elements in the set of real numbers between zero and one, than in the set of counting numbers.”

“So the set of real numbers would be even greater? After all, the whole infinite line is bigger than that little interval.”

“No, it’s not bigger. I will take a line, and bend it into a circle, so that the ends of the line touch each other. I haven’t added any points so far, have I?”

“You sound like a magician. Roll up your sleeves.”

“Hm. I’ll put this circle on the paper, and draw a horizontal line under it” He drew.”Start at the uppermost point on the circle. Call it the North Pole, so that the line running through the North and South poles is perpendicular to the line.”

“OK, so?”

“For every other point on the circle, draw a straight line from the North Pole to that point and keep going until you hit the line.”

Moishe does that a few times.

“OK, with a little imagination, you can see that you will have matched all the points in your circle to all your points on the line. So, no, extending the line doesn’t add points either.”

So, you’re saying that the set of real numbers, like on a line, is bigger than the set of the integers. I can accept that. But you still haven’t got to the lowest infinity that God can be, since he is in all of time and space.”

“Oh, I already got there. Adding dimensions doesn’t increase the number of members. You already agreed to that.”

“When?”

“Remember when we matched up pairs of numbers? Let the first number in each pair be, say, going east, and the second one go east. Then....”

“That argument was for counting numbers!”

“Same argument. Then it can be extended to three, or four, or more dimensions. There you have your space-time.”

“OK, I didn’t say that God was only as small as space-time. Are you working up to a definitive description of the infinity of God?”

“No, it’s tricky enough finding a definitive description of an uncountable set in first order. It’s almost as bad as trying to find ‘the set not describable by less than ten words’.” The problem is that, sticking to first-order stuff, our formulas are finite, our alphabet is countable, so you will end up with a countable number of formulas. So even if the standard interpretation of the theory is that there is a uncountable set, there will be a countable set that will fulfill it as well. So it would be hard to point to the theory as a definitive description of uncountability.”

“But if the second theory is completely off-the-wall, ...”

“That’s not the point, but even if it were, you can do this with a model that is not so artificial. The idea comes out of the objection some people have about how loosely you can use words like ‘all’. The spirit of the axiomatic method is to define things in terms of things that had been defined before, but defining the word ‘define’ gets tricky. It’s a bit like the difference between truth and provability. You know the idea of a formula, and let’s take the standard first-order theory that one uses to make mathematics.”

“I’ve pretty well assumed this anyway.”

“To distinguish two numbers, you need two independent formulas to describe them, just like we need two independent concepts’ to distinguish two objects.”

“I remember something about that in the lectures in the cruise. Since there are an uncountable number of real numbers, and only a countable number of formulas, that means that there are subsets of the counting numbers that have no first-order sentence from our theory to describe them. From our association between real numbers and these subsets, we can then say that there are real numbers that cannot be described by our first-order formulas.”

“Why would we need them? After all, we can’t measure more than a finite amount of accuracy.”

"In the measurement, you are right, but in the theory that allows you to figure out the measurements, there are. For instance, we assume that we live in a continuum of space-time. Be that as it may, some people are quite willing to restrict themselves to only using the techniques of defining sets with explicit formulas which talk about sets with previously defined explicit formulas."

"Like what?" Moishe asked.

"Like forming the subsets of the counting numbers that could be described with such a first-order formula, and also using only functions for matching that can be constructed in this way. With only these tools, a countable set can still not be matched with the collection of subsets which they say are constructible; the same proof as before still works," Tal answered. He rolled his bowling ball down to get his first strike of the evening. When he came back, he continued, "That is, from the point of view of the unrestricted, normal use of the power set, the set of constructible sets is even countable, including the reals. But from the point of view of the model in which the axioms for a constructible universe are the minimum, there is no function from the natural numbers to the constructible reals, so it is uncountable. With this understood, we can say that we have a countable model for the sentence that there exists an uncountable set."

"A countable model of an uncountable set. You want to win the argument by making me crazy, don't you?"

Tal shrugged."Keep in mind that this is a bit loose; a sentence doesn't 'say' anything by itself without a model. I got that phrasing past you for fun, but it is rather sloppy. So, really, in this situation we could better say that we have a sentence, which in one model is interpreted with a set which is uncountable from the point of view of a model using a constructible universe, and countable from the point of view of the models that we usually use."

"Is the set of the reals the first uncountable infinity?"

"Depends on your assumptions. If you use this constructible universe, then yes. But if not, then it depends on what other axioms you use. By adding axioms correctly, we can in fact make an infinity that is between the counting number set size and the size of its power set, which is the same size as the real numbers in the unrestricted use of 'all'."

"But hang on. In either universe, you would have to have all these power sets in them, so be bigger than them. Doesn't your argument for accepting the existence of a countably infinite set as the reification of counting.... shouldn't you be able to reify this Power-Set-forming operation? That is, have an axiom that says that there is a set which is bigger than anything you can get by making the Power Set?"

"Indeed, the argument is good . Not everyone likes it, but it is true that it is an assumption that is usually added in order to get a model of most of the mathematics that engineers and physicists use."

"But no one can actually envision an infinity of things at once."

"You can't envision even 50 million people at once, yet you accept the existence of Mexico City. You can also make statements, like Mexico City has a larger population than Berlin. Infinity, like other numbers, is a tool, a reification, a jump...."

Chapter Thirteen

May 27, 5832

Bill again showed up at Sheila's, thinking to finally try to explain to her about Earth. But he needed to go about it in an indirect fashion. He began, "If I understand correctly, you don't

actually have to do anything except be at the right place at the right time. One black hole is in a large orbit around the huge black hole, and one mouth of the wormhole is being dragged along by the black hole. The other mouth is closer to the Earth, and that is your ultimate goal. Compared to the Earth, the time in that other mouth is slower than the time on Earth. It is such an amazing bit of timing... doesn't it sound almost orchestrated?" he ventured.

Sheila just kept on with her adjustments."I was warned that you might get religious on me. That line of argument has been used by theologians for centuries. I can make it up myself. Take any phenomena. Then trace its causes. Say that this could not have been accidental. Had, of course, the phenomena turned out otherwise, the same argument could have been used. We are more Buddhist in our approach. Whatever happens, happens, and we try to appreciate that we are part of it, in harmony with it."

"Uh...harmony?"

"Sure. The two mouths could be close to one another, and we could have just said, 'Ah. The mouths are close to one another.' Instead, we observe that there will be another black hole which will veer close to the path of that further wormhole, where time is slower. It will break the hold of the first black hole on the mouth, and the mouth will then get in the field of gravitational influence of the second black hole. However, although it will be tugged by the black hole, the dynamics of the arrangement will mean that the hold will get ever weaker, and the mouth will slow down and get further away from that second black hole. That second black hole will be heading towards the general area of the first mouth, and the second mouth will almost stop within a short distance from the first mouth."

"And then?"

"The stage will be set. There are several possibilities for what will happen next, and our mission is to see which one."

Bill resolved, once he returned to reality, to fire the programmer who had set up wormholes and black holes like this.

"One thing that many armchair theorists forget is that there are fewer possible physical worlds in reality than in mathematics, so just because a theory seems to avoid self-contradiction is not enough for it to be right."

"You seem to have covered all the bases."

"All but one. I explained some of these complexities to the local newspaper, which of course oversimplified them. Some people understood that on one side time travel might be possible, but on the other side the laws of matter would get in the way, not to mention the annoying traits of self-reference which could interfere with causation. So they put the two concepts clumsily together and came up with the idea of a time travel that was not material, and above causation. That is, their souls could experience this self-reference, so that a wormhole would be a way to attain higher consciousness."

"Wild. But what's the harm?"

"The harm is that they want us to get closer to the wormhole than we feel is safe. Of course we have ruled against it, but many of the followers of this idea are among our technicians. This is an added danger."

"The black hole... isn't it dangerous?"

"No, we know where it is going, and we are careful to stay out of the way of the radiation caused by its rotation. We also like to study it. We know how much it bends space. We are especially interested in the extreme bending which changes the shadows of energy-mass to mass.

If you have a strong enough bending, energy can convert to mass. We use this fact, albeit indirectly, for some of our fuel needs."

This perked up Bill's ears. Fuel was always a good starting point for explosions, and perhaps .... He noted that Sheila was starting to look at the clock, so he just resolved to find out more about this when he saw her again.

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May 30, 5832

Moishe had encouraged the four original orphans, now adults with their own young children, to consult the ship's computer as much as they could, since the ship had no sites that would be deemed inappropriate, and plenty of sites on Judaism.

Sara was the one that took this advice most to heart. She studied music at the ship's Conservatorium, but in the evening she was more often on the mathematics and physics sites. She had sometimes listened respectfully to the discussions between Tal and Moishe, but had never taken part. However, from the occasional smile on Sara's face at a particularly well-made point on either side, Tal guessed that she had a gift for analytic thinking.

One day, Moishe was occupied putting in order the little synagogue he had created. Tal felt that the discussions with Moishe were getting dangerously close to areas about the physical world which were unfamiliar to Tal, and that he would not be able to hold his own. He sent to the Conservatorium when he knew Sara had a long break between classes, and found her in the library. He sat down next to her and quietly asked her if she had ideas about the connection of infinity to the work that was done on the ship.

Sara said, "Sure. The full quantities of reality are continuous, but we often deal with shadows, and they can get discontinuous. It's as if you had continuous waves, but a flying fish is only conscious of the water that it encounters. To take a well-known example, mass-energy truth values fluctuate in a continuous manner, but if we now measure energies as we go through time, then the precision in time will be greater as the precision in Energy is less, so your energy measurements will jump around, and we can get what appears to be discontinuities. Even though these discontinuities might be small, they still disrupt the standard theory which is based on complete continuity. To handle this annoyance, we assume the existence of new infinities."

Tal asked her, "It seems that these possible worlds, with higher and higher infinities and more and more powerful theories, it seems that we are teetering on the brink of a crash, no?"

Sara laughed."That's why, in the introduction of any new infinite set axiom, we first have to check that the new axiom does not introduce a new contradiction."

"You mean you have to prove that the theory with the new axiom is not contradictory?"

"No, unfortunately we can't do that for the interesting theories. But we can prove that, if our previous theory was not contradictory, then adding the new axiom does not make it contradictory either. If you trace it back far enough, it is then enough to prove that if arithmetic with the new axiom and arithmetic without the new axiom both would stand or fall together."

"You are getting off the topic. We were talking about the probabilities..."

"Right. Sorry. I just wanted to make this idea of equiconsistency clear.

"Equestrianism? isn't that to do with horses?"

"No, equiconsistency, equal consistency, ..."

"It's true that I always have problems getting my batter to be the same consistency from one batch to the other, but what do my crepes have to do with ...."

"No, consistency as in non-self-contradictoriness. That is, when we have two systems, and we don't know whether they are consistent, but we know that each one is self-contradictory if,

and only if, the other one is. For example, we usually assume that infinite sets exist when we do mathematics, but you can also work in a system that does not assume it. Neither system can prove its own consistency, but the systems are equiconsistent. This is one example where I said that two systems stand or fall together."

Tal wondered if he should ask about systems of religious belief, but Sara did not give him a chance, as she continued, "But don't worry, I am coming back to the concept of probability. I needed this idea of different infinities first, and our ability to add assumptions."

"OK, that is clear, I think."

"Consider the set of the counting numbers. The hotel example showed that taking a single number out of it does not change the size of the set. Then, taking another one also doesn't. And so forth. Taking a million numbers, or any finite number of elements from an infinite set does not change the size of the set. So, in measuring the size of an infinite set, a finite subset is insignificant. In a similar way, with respect to an uncountable set, a countable set is insignificant. In general, any subset which has a smaller cardinal size than the original set is, with respect to this bigger set, insignificant."

"Where is this leading?"

"Classic logic tells us that if we are generalizing about a statement that can be about any element in an infinite set, then if that statement is not true for any number of elements in that set, then we cannot say that it is true for all members of that set."

"Of course not."

"But if it is true for all but an insignificant number of exceptions, we say that it is true 'almost everywhere', or 'almost always' true. So there are statements about events which are almost everywhere true."

"Why do you sometimes say 'events', and sometimes 'measurements'? From the contexts, it seems that they are the same thing."

"They are. Exchange of information, interpretation....but there is a connotation difference, where 'measurement' makes one think of a conscious intension, whereas an 'event' does not. An event is something that happens, that's all."

"Sorry for the interruption. Go ahead." Tal found the roles reversed; he had been Sara's teacher before.

"No problem. The things popping in and out of existence, the discontinuities, should not be considered significant until they get past a certain amount."

"I'm not sure I'm following."

"That is, we need to be able to deal with probability on a larger set. For example, I started to explain the idea of significance in measurements. This can be made more precise by showing how significant or insignificant events combine; for example, that two of them happen together, or that one happens and the other not, and so forth. You can figure out whether the results will be significant or not by figuring out how the corresponding sizes of the sets combine in corresponding ways This works. It is not hard when you have a finite number of such events. But if you are dealing with an infinite number of possible events...."

"How can you have that?"

"Well, for example the number of points in spacetime that something could occur."

"Ah."

"An infinite number of events, each with an insignificant probability, can combine together to give a significant probability. That's what happens with the energy in the vacuum. Each one is insignificant, but when we get a lot of them, they get significant."

“I guess that makes sense; something like that would be an alternative answer to that so-called paradox about the heap of sand, I suppose.”

“What is that?”

“That if you start with a bit of sand, which you don’t consider a heap, then adding a grain of sand doesn’t make it a heap, but if you add a lot of them, it becomes a heap. So there must be some border where a heap becomes a non-heap, but we just said that a single grain does not do it.”

“Um, yes, you could adopt the significance idea, but... you said that this would be an alternative. To what?”

“The usual explanation is just saying that we suppress a lot when we use an ambiguous word such as ‘heap’. The idea is a bit like that quantum example, where we had the truth values connected to the framework of possible worlds. In this case, we have the possible worlds being different people, or the same person at different times, or something like that. Then you have the power set, the set of all subsets of the worlds, so that you can talk about the probability of a world referring to a certain number of grains as a ‘heap’. That is, the probability that someone would refer to ten grains of sand is small, and the probability that one would refer to a million grains of sand is large. So the word always remains ambiguous, but very low and very high probabilities are mistakenly referred to as ‘definitely not’, or ‘certainly’. The probabilities in the middle...”

“OK, I get the idea. Not the most interesting paradox, I must admit.”

“Sorry. Do you want to get back to your idea of significance?”

“Maybe another day. My next class is starting in five minutes. But you know where and when to find me if you want.”

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June 2, 5832

Bill’s visits to Sheila were becoming more and more frequent. He found her, as usual, working at her desk, and he always timed his visits to coincide with the beginning of her lunch break. They usually picked a restaurant nearby. After they had ordered and the usual small talk, Bill asked, “What did you mean about the bending of spacetime being a source of energy?”

She replied, “Remember that we need our laws to look the same from different viewpoints? For example, if we look at the electromagnetic field as it changes around the strong bending around the black holes. Very, very roughly, these changes should look the same from the viewpoint of the past or the future. Both will see the electromagnetic field as positive. But the two possible worlds are linked by the accessibility relation.”

“Let me see if I understand where this is going. What then corresponds, in the future, to those waves with positive frequency as seen from the past? The opposite frequency, or a negative frequency. Therefore, from the future, the field has two parts under the influence of the strong gravity: one a positive frequency, one a negative frequency. That is, one with a positive energy, which makes the probability for the mass to be further away from the black hole: that is, it can be seen shooting away from the black hole. The other half is one a negative energy, which means that it will decrease the probability for the field to be away from the black hole: speaking in terms of particles, the particle is absorbed by the black hole.”

“Um... and interesting formulation. It is of course a bit more complicated, but you are getting the general idea. The mass of the ingoing particle makes the mass-energy of the black hole increase, but the greater negative energy makes the black hole decrease. In sum, it looks as if the black hole is radiating energy, and losing mass-energy.”

Bill said, “I read something about that.”

Sheila said, “Yes, I daresay you have. There are lots of pop explanations for this, because negative energy sounds too weird. All that stuff about operators, infinities, and so forth, are often the accessibility relations from one possible world composed of measurements to another such possible world. There are different reactions to this. Some people say only the possible worlds are real, and not the accessibility relations. Others include the accessibility relations in reality. But some try to make the accessibility relations consist also of observables. That’s why you get various explanations of this radiation; one of the most popular is that a virtual particle pair gets just too close to the event horizon, so that one falls in. This is more acceptable to a lot of people.”

“So is the explanation false?”

“Mm... I don’t want to condemn the explanation altogether. Sometimes such explanations work, but you need to interpret the words a bit differently. Unfortunately, often no one tells the reader that. For example, a common explanation for the appearance and disappearance of mass-energy is that the particles ‘borrow energy from the future for a short time.’ What this is trying to say is that if you take a number of identical states that all evolve to the same state, and for half of them you measure the amount of time it takes for the state to change, and for the other half you measure the amount of energy involved in this evolution, then the minimal spreads will depend on one another in that one will go up when the other goes down. This is not a causal dependence; if the spreads were particles I might talk of entanglement. But I am afraid someone might try to stretch the analogy beyond the intended comparison, and in so doing pretend to be proving my original point wrong. Of course, some people try the reverse and try to prove a point by proving one part of an analogy right and then pretending that they can push the analogy. Such people would agree that just because someone is in the same family as me does not mean that they are the same gender as me, and vice-versa, but then they go ahead and do the same thing in their arguments.”

“You mentioned entanglement. All you stated sounded like a logical argument, not a causal one, so are the two parts of the field entangled?”

“Yes, they are. People like to talk of two particles, but since one part of the field is immediately absorbed by the black hole, you have to assume the extension of the model in order to talk about what happens inside a black hole. Until that moment, we can perfectly talk about the black hole as if it had no interior. However, if you do assume that the black hole has an interior, then since the emission and absorption are done together, one can use another theory which is compatible with the other one as far as the exterior is concerned.”

“Fine, but the question I wanted to ask: if they are entangled, are they antimatter particles of one another?”

“Yes, that would follow.”

“So does the antimatter one go into the black hole, and the matter one escape?”

“Why would you think that? You’ll see more matter particles, that is true, but only because an antimatter one will generally encounter a particle of matter between its emission and its being observed, so that only energy is observed. But they will both be emitted.”

“So the black hole emits energy and has a temperature.

“Yes, they are not so black. But for the big ones, the temperature is really tiny, even less than the surrounding space. The temperature goes down as the mass goes up. The reverse, the temperature goes up as mass gets smaller, is actually a help.”

“Ah, but wait, what would happen if the mass got to zero? You would be dividing by zero?”

"This never presents a problem of dividing by zero, because the radius never gets to zero: the black hole stops existing at that point, and the temperature is very high, so a lot of mass-energy is emitted. In a word, it explodes. These big black holes were formed sometime during the history of the universe by a lot of mass getting together. But there is another way for them to have formed, when you consider them as extreme curvature. There is the possibility that some could have formed at the beginning of the universe, and are about at that –off point. We thought that they were not so probable, but we were proved wrong."

"Huh?" Bill caught himself; he knew that primordial black holes were only a conjecture in the real universe, but here perhaps they had been discovered.

"You know....the primordial black holes that would have started off small but expanded along with the expansion of space-time would be at their last point of evaporation right about now if they had been about the mass of an oil tanker when they started at the beginning of our universe.

They are all over the place, and we just have to calculate the time and place of their explosions, and use the energy. The universe seems to have been made just for us."

Bill thought, "You don't know how right you are."

Aloud, he asked, "Ah, since you are saying that gravitation and acceleration is the same that should mean that with enough acceleration you would get massive particles to appear out of the vacuum energy?

"Theoretically yes, but to get anything measurable the ship would have to go from zero to almost light speed in a fraction of a second, so you aren't going to be able to test that experimentally. I've got to get back to work; enjoy your lunch."

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June 4, 5832

The Galaxy Snack Bar was not the best source of good food on the ship, but it had turned out to be the favorite of the younger members of the Destiny's population. As happened almost every week, a number of young people were discussing their disappointment with the sect that centered around the possibility of a wormhole spiritual time machine. The perfect alignment of the path of the supermassive black hole to bring the two mouths of the wormhole together seemed too much a coincidence. They resolved to return to the Guru's teachings, which concentrated on the black hole as a portal.

They reviewed their position. The standard explanation was that black holes are nothing but regions of space where the probability of a particle being in a neighborhood of another particle of other space-points in a region close to one: if it is in one point, then the probability of being in a different space-time point will be such and such. For a black hole, the probability is close to one. As close as anything will be to one, given the uncertainty which allowed tunneling.

However, there were also the ideas that the Guru had told them about. On one side, the Guru said that the edge of the black hole was the border of our knowledge, which is the same thing he said about the universe of a model. You could treat this epistemologically, saying there is stuff on the other side, or ontologically, which seemed more reasonable, saying that there was nothing on the other side... in our universe. But that there could be an extension of the universe on the other side seemed reasonable. They had been told by the Guru that there was another universe where they would one day go. Not an afterlife. The universe that they inhabited here was part of that other, more real, universe. His other explanations were not too clear, but this part did indicate that the other universe must be an accessible extension of ours. He even speculated that this extension was perhaps inside a black hole. This did, of course, present a difficulty, namely

that the strength of gravity between two objects depends on both the distance between them and the mass. So, there will be a difference between the gravitational force on a person's head and her feet. For a planet's mass, this difference is not significant. Even if the mass is huge but the distance is also great, such as at the event horizon of a black hole, the difference is not great, since the mass is at the center of the black hole, and the event horizon is just the places where light cannot escape from the black hole except for the occasional quantum effect. But the event horizon of a medium-size black hole is another kettle of fish. The distance to the center from near the event horizon balances out with the mass of that center to create an unpleasant situation for a human: his body would be torn apart by the great difference in gravitational attraction affecting his head and the one affecting his feet. In fact, this would happen long before the event horizon, so one had no hope of getting past the event horizon of a medium size black hole alive.

Hence they wished to head for the supermassive black hole, about a million solar masses, that they thought would be a portal to the other universe. True, this difference in forces, called the tidal forces because the Earth tides from the moon are explained with them, would eventually kill them as they got closer to the center. But they were confident that they would enter into the other universe before that happened, and would be safe.

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October 7, 2045

In Vladivostok, Natasha monitored her two virtual test subjects. Paralife was proceeding much faster than she was, but she brought supercomputers and the quantum computers from all over the world online to work on this. Dialogue and events were filtered and presented in summary form. By analyzing the developments as she would to find out the consequences of a set of axioms, that is to see where a theory might lead, she understood what direction the lives of her test subjects were heading. This led her to notice the other characters who were associated with the test subjects; to understand these secondary characters it was necessary to check the characters with whom they had in contact; the tertiary characters led to the quaternary, and so forth. This process started to appear like reverse engineering; the results started to interest her more than her principal goal.

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June 5, 5832

Moishe paid the owner of the Galaxy Snack Bar and received the transcript of the discussions of the night before, recorded at every table. Moishe started to get interested in the different sects on the ship, as they were influencing his children. He asked a physicist about the discussions about the black hole. Moishe did not understand everything the physicist told him, but he understood that the information comprising anything falling into the black hole would be scattered on the event horizon. The event horizon got bigger when something entered it, and the information of the black hole was reflected, or as far as outside observers were concerned. The black hole would be the ultimate trapdoor function, almost a one-way relation between possible worlds. Except tunneling out, of course. But Moishe only imperfectly understood tunneling, so we did not ask further about that. On the other hand, he asked whether the gravity would crush an infalling human. No, the physicist replied. After all, the gravity and acceleration would balance each other out so, in the language of forces, there would be no net force acting on a free-falling human. However, they pointed out that the difference in gravity between one's head and one's feet could be large enough to tear a human apart before reaching the event horizon; however, a supermassive black hole had a large enough size to that this would not happen outside of the black hole. What would happen inside, if there was an inside.... Moishe wondered

about this last remark, but let it pass.... would be that eventually, yes, the human would die before getting to the center, but could live for a little while before hitting the center.

Moishe did not believe in the portal as some sects called it, but he thought he might use this idea.

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October 7, 2045

The rest of the population of Wheaton Island had noticed that some of the staff at the Infospital were coming home from work late with dazed expressions. But the staff did not divulge the reasons for this, making up various excuses. The normal patients would have had perhaps less confidence in their medical treatment if they got wind of the fate of the special patients trapped in Paralife. Jutta was a frequent visitor to the regular meetings of the technicians. In one of them, she exhorted the technicians, "You have to change tactics. You have been working all year to repair the problem, without success."

One of the technicians spoke up, saying "Um, Jutta, we have a new problem. The children's minds have certainly outgrown their brains' capacities by then, given the difference in speed. We have started thinking about making older bodies. Can you find the funds for this?"

"If necessary. What do we do with the babies' bodies?"

"Keep them on line; since they will have the same DNA as the returning children, they will be ideal donors if necessary."

"There was one possibility, that of sending someone in to kill all the players and destroy all Frozens. But no one would want to go in and have her brain destroyed, especially with the risk of not coming back."

"Wait. The reason the brains are destroyed is that they are biological. What if we send in an android?"

"It will still wipe their brain clean."

"But they would get their mind reinserted."

"Yes, if they succeed. Are you sure that we can find a competent android?"

"Bill says that they got a lot of experience.... but I don't want the responsibility. Bill left all responsibility with Jutta; submit the plan to her. Let her choose and convince an android."

Jutta found the plan good, but was unsure which android to pick. He was about to ask George to review all the androids, when George came in himself and said, "Jutta, I think you had better see this." He handed Bill a file marked "Julia Volkova".

Jutta looked at the application. Birthplace:"Novosibirsk". In red:"No record".

She flipped to the next page. The health report. In red:"No such doctor".

She flipped to the third page. BB, the initials of "Big Brother", the surveillance system."Not human."

"What does this mean?"

"We checked in our records of androids you had sent abroad. Even those who stopped working for us kept in touch, but some were killed. Some went missing. The physical appearance of one of them, Julia Norton, matches Ms. Volkova's. She had been sent to the region of Russian Far East and Northwestern China to increase her language skills; that's the last we heard of her."

"So... send for her, would you?"

Jutta said, "Have a seat, Miss Volkova."

"Thank you, Dr. Fischer. I have been able to turn away three cruise ships this morning and..."

"I saw your report. Good work. We have another job for you. Tell me, does the name Norton mean anything to you?"

"Norton? There is an anti-virus app..."

"No, that's not it. Tell me about your childhood, could you?"

"I'm afraid I lost all memory of my childhood when I was, um, in an accident in Russia."

Bill studied Julia's expression. True, among her skills was that of a superb liar, but the total lack of reaction reassured him.

"Hm. Ms. Volkova, perhaps you have heard that we need an android to be sent into Paralife."

"No, I've been rather busy. Would you like me to ask around? But now that it will be hard to get back..."

"I was thinking of someone with some, uh, empathy towards one of the persons who are stuck in Paralife."

"Everyone found the children cute, but..."

"I was thinking of one of the young men."

"Oh. I am not much on gossip, sir. Um, Moishe was friends with the android Sara, who is our Hebrew translator."

"I have noticed that you are rather fond of Tal Kishon."

Julia reddened. Jutta was impressed that the programmers had thought of this detail.

But Julia rebounded, saying, "But I am not an android."

Jutta passed her the result of the Big Brother scans. This time, the blood drained from Julia's face. Jutta was again impressed. She just said, "I think this may help explain your aversion to medical tests."

Julia just stared at her.

"So, think it over. You apparently need time to reflect."

Julia walked out of the office, a zombie.

Alone in her room, she brooded. After about an hour, she started to wonder why she was brooding. Why did she accept the prejudice that it was better to be a robot than a human? It was just the humans wanting to be superior. Galileo and Darwin successfully demolished other attempts to put man at the center of creation. It was clear that soon all measures of human intelligence would be surpassed by robots. So the humans were trying to prove that robots couldn't be conscious. John Searle's idea was no longer considered a proof. But every so often a misunderstanding of the mathematical result about the inevitability of undecidable statements in complex systems led to a flawed "proof" that humans had some sort of non-algorithmic part of the brain that is able to decide these statements that are undecidable for an algorithmic computer. Completely foolish, of course, she decided. She had even read that the great physicist Roger Penrose fell into this trap. His argument, as she remembered it, was that a computer would hit some sort of limit that a human could go beyond. He had concluded that humans were therefore superior, forgetting that humans also have limits. He forgot that humans would face the same problems. True, humans evolved to worry more about survival and reproduction than long-term results, so they developed different theories that allowed them to either ignore such problems or update their models and theories. Possible worlds semantics and all that. Natasha did not worry about which sort of formalism would be involved; she just knew that machines need not be any different than humans when faced with the type of problem that Penrose posed.

Natasha knew that she should not have walked out of Jutta's office like a zombie. There are those that believed the "zombie argument"; after all, if there was a zombie that did a perfect imitation of a human, we could not know that it is not conscious. Silly, of course, in that this is the old trap of assuming what you want to prove. The same fallacy that even Descartes used in

his famous “Cogito ergo sum”. Come to think of it, a lot of philosophers did this, thought Julia with a smile.

Still, absence of proof is not proof of absence, as they say. On either side. But the opponents of her having consciousness seem to be getting closer and closer to the old fallback, religion. She would ignore them. She had no reason to be depressed.

The next morning, she was back in Jutta’s office. “Right. I’ll do it.”

Julia went through a thorough physical examination, albeit this time administered by a team of programmers and doctors.

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June 14, 5832

Julia came in with a simple plan: blow the ship up.

Once unfrozen, she explained that she was a specialist in security matters. Fuel sources were especially vulnerable. She would aid the head of security. This sounded reasonable. Sheila, the head of security, welcomed the chance to have an aide; she felt that security was given too little attention on the ship.

Julia asked, “Do you have some specific complaint?”

Sheila said that yes, the eco-terrorists had become more bothersome.”They keep trying to monkey with the engineering, but that is only a minor annoyance. They don’t seem to be either very imaginative or very well organized, except for their ability to escape detection. But their disruptions are less than our usual glitches. We take them in stride.”

“Nonetheless, I would like to investigate it. Any idea who they might be?”

“No, given their attempts, I would say that it does not include anyone with much technical background. We do have some CCTV tapes of them, but there’s not much on them. I haven’t had any luck getting any clues out of it. Perhaps you can have better luck. I’ll send them to your address so you can work quietly in your quarters.”

Most of the time, the eco-terrorists blacked out the camera of the CCTV. What they overlooked was the sound recording. True, they did not speak to each other, perhaps communicating by unseen hand gestures. But Julia listened closely, turning up the volume. She then realized that one of the eco-terrorists was lightly humming a tune.”Sevivon”, a Jewish children’s Hanukah song. That indicated either Moishe or Tal. Possibly the children, if Moishe or Tal had taught them. In any case, she would try to see what they were doing. If she approached them in her capacity of Security Aide, she might scare them away.

There was a small synagogue on the ship. On Shabbat, there were always some who joined the service out of curiosity. Julia dressed modestly, and took her place among the women. Two of them had babies; Julia guessed that they were Sara and Abby, the two orphans who were among the few real people in Paralife. After the service, she hung around the women, specifically following these two women. She was surprised to find that they did not go straight home, but rather went to a park. There they sat on benches and talked. Julia managed to sit down on a bench alone with Abby. She asked Abby casually whether she didn’t miss Earth. Abby told her that she had not known Earth; the ship was her home. Julia apologized, saying that she had only recently been unfrozen. She was interested in the fate of the ship... they were to observe some phenomenon that she, Julia, did not understand....but whatever it was, was that the purpose of their lives? Abby said that no, the purpose was to go the other life.”After death?”, Julia asked. Abby said no, perhaps one could reach it by the s black hole portal. Not any black hole, mind you. A supermassive black hole.

Julia stopped herself from grinning at the thought that she was finally onto a way to destroy the ship. She cautiously asked Abby why she thought this way. Was it part of Judaism?

Abby told her no, but Sara could explain it better. She called Sara over.

Sara asked her, "You know what a black hole is?"

Julia remembered her school lessons, and recited, "A black hole is a region of spacetime where something inside the event horizon had a very, very low probability of getting out."

"Why not zero?" Sara probed to see how far Julia actually understood what she had parroted.

"Well, no," Julia said more freely, "the fuzziness means that something can come out of it. It's called tunneling, I think, when the uncertainty makes something go where classical physics says it can't. Something like that. I slept through most of my classes."

Sara told her, "You are talking about an interior of the black hole as if you knew that it exists. From our world, the interior of the black hole simply does not exist, since we don't get any data from it."

"Maybe you could get something out of it if something, uh, tunnels out of it."

Sara laughed."Ah, but that can be seen as simply the spontaneous changing of the surface of the event horizon. The information is spread out on the event horizon, which gets bigger when information is added to it and smaller when it is not."

Abby asked, "I thought it was entropy that changed."

Sara told them, "Yes, where entropy is defined in terms of the area of the event horizon. That means that the black hole is a unity according to one point of view, one relation that says that if it has any elements in its interior, they form one single equivalence class."

"And equivalence classes are?"

"Collections whose members are related to each other by an equivalence relation."

"You'd make a lousy dictionary. You are borrowing from Peter to pay Paul, to borrow a Christian expression. What is an equivalence relation?"

"It says that the two things that it labels equivalent get the same truth value under an interpretation where that relation makes sense. Not quite like equality, because under a different relation, the elements of the equivalence class may no longer be equivalent. So which relation you pick is relevant. For example, if you have a sibling, then the two of you are equivalent under the relationship 'is a sibling of", but unless you are twins, you are not the same under the relationship 'is the same age as'.

"All this is nice to show you have studied mathematics, but what does it have to do with the black hole, Sara?" Abby asked.

"I'm getting to that. If we extend our model by assuming that the black hole has an interior that means that there are other units which might not be equivalent. In fact, we can pick usual relationships among particles that we use for the rest of the universe. According to this relation, you could arrange the particles in another bunch of equivalence classes. There are many different relations of the particles inside a black hole that would each give the same outward result. It is this number of relationships upon which the amount of entropy depends."

Abby said, "That's entropy and all that. But the main point, uh... I didn't get your name?"

Julia proffered her hand."Julia."

"Anyway, Julia, this entropy changes, and we believe that there is an interior that is changing, even if you can't see it. Just as there are many spiritual entities that you cannot observe. So we believe in this extended model. True, the other one works, but it doesn't feel right. Our theory describes a model which is an extension of the other one, and so more encompassing. Closer to the Truth."

Julia did not object, but just said, “Why a supermassive one? Why a black hole at all?”

“When we were children, Guru Wheaton told us that the universe here is only a part of an extended universe, and we came from the other part. The restricted model has a reasonable theory, but this would mean that out universe ends at the event horizons. But so, in the extension, the rest of the universe must be on the other side. If the black hole is too small, the difference between the change of measurements.... forces, that is.... of our feet and our heads would tear us apart, and we would die before getting into the portal. So we need a supermassive one, where that will not happen.”

“The Guru told you this?”

“Well, no, we figured it out ourselves.”

“We?”

“The Outerworlders.”

Chapter Fourteen

December 2, 5832

Several months went by without anything new. But that morning, a sensation occurred to alleviate the drudgery of the voyage. As part of her security clearance, Julia was on the bridge that morning. An officer yelled, “There’s a ship coming up from the rear.”

“A ship? How is that possible?” the captain asked.

“Another ship from Earth, I suppose.”

“Warn everyone that we will be cutting acceleration in thirty seconds. Report.”

“The other ship is going at constant velocity, at sixty percent of the speed of light compared to us. It will pass us in 40 seconds.”

“Open all observing stations. Report all data.”

They watched as the ship came close to them.

“Report!”

“Readings, length in direction of travel, twenty five meters; mass twelve and a half tons, heartbeat of sole occupant, one and a quarter seconds.”

As it passed without doing any damage, the captain breathed a sigh of relief.”Ah, right. A standard one-person shuttle. Twenty meters long, twenty tons, and an athletic occupant with an even second’s heartbeat. Going at about eighty times the speed of light with respect to the Earth.”

“Ma’am, I think you got that mixed up. The data...”

“I did not, lieutenant. We are only going at about sixteen percent of the speed of light.”

Julia had come up behind the captain and blurted out, “You can’t go any faster?”

The captain turned around at this unexpected interruption. As she understood that there was nothing more for her to do, she decided not to object to this breach of protocol.”Hello, I recognize you. Am I right in saying that you were unfrozen only a few months ago?”

Julia was complimented that the captain would take notice of her. She nodded. The captain continued, “So you were not here for Switch Day twelve years and a half years ago. We are actually decelerating. Partway through our trip, we had to move everybody to ...well, their ceilings. Since we didn’t have an Earth-sized mass to travel with, we had to find another solution to keep everyone comfortable. After all, the effects of acceleration and gravity are equivalent. They both bend space-time.”

Julia did a double-take on the natural way that the captain referred to bending space-time. On Earth this phrase would not have been used in casual conversation outside of Physics departments, but she then reflected that such terminology had been a natural part of the education on the spaceship. After all, she considered, the way that Earth schoolchildren refer to satellites and such things would have appeared astounding to anyone from the eighteenth century.

The captain did not notice Julia's reaction, but continued her explanation. "This has its own headaches, and now and then we have to warn the crew that we are abandoning acceleration, so people have to be uncomfortable for a while. But the occasional float does everyone some good. We don't do it long enough to cause long-term health problems. Our goal was across the galaxy from Earth, so we accelerated at the same speed as an object would fall on the Earth for half of the trip, and we are decelerating the other half. We will be close to our destination soon."

"How long has that been?"

"On-board time? About 23 years."

"Earth time?"

"Close to a hundred thousand years. Forces you to think about the accessibility relations between possible worlds. Remember the time slows down from another perspective, although from our perspective time is normal, it is distances that get smaller. The problem is the fuel we need. If I made a fuel to ship mass ratio, I would get over 10 billion to one. No good. But somehow the universe has been constructed so that we have been able to get around this. I can imagine the universe not being so nicely made, making such travel impossible, but why it is so nicely made is one of those mysteries."

"While everyone else in the galaxy explains the remarkably short travel time in terms of the people on the ship moving slower through time, the people on the ship attribute it to the distance being shorter."

Julia stopped to think. She remembered that the ratio of reality to Paralife was the time the people on the ship experienced, not the time experienced by people on Earth in Paralife. Assuming they even existed. Julia did not know everything about the Paralife program.

"We are far away from any star at the moment, so it is not too hard to roughly envision what is happening. In fact, it's taught in school here. I'll let my twelve-year old daughter explain it. She motioned to a girl, who was taking apart an old rusty bicycle. Amazing, Julia thought to himself. She is not playing electronic games! The captain called, "Naiya?"

The girl came over."I heard you, Mother. Can I show her how to envision it using things from my collection?"

The captain explained to Julia, "Naiya collects parts of old machines. Like old bicycle parts, and stuff like that." She turned to Naiya."Of course, dear, anything that works."

Naiya grabbed Julia's hand and took her to her pile of bicycle pieces. She directed Julia, "OK, Miss Julia, grab those three old bicycle wheels in the corner, and give them to me. Please," she added, noticing her mother's stern look as she headed off to her other duties.

Naiya then fiddled with the wheels for a minute. Then she told Julia, "OK, I have arranged the wheels. Here's the story."OK, I've marked them. To make it easier, we are dealing with a situation in which one person, say Alice, is measuring some of the life signs of her friend, say Bob, who is in a different spacecraft, in such a way that the three points: Bob's head, Bob's feet, and Alice's telescope all make a straight line. Bob is traveling in the same direction as his body is pointing. Here, I'll draw it for you." She made an elementary drawing of a spaceship and two stick people. She showed it to Julia, then continued, "Alice has a list of Bob's life signs she has

to measure, and she must cross check her measurements with each other. She had measured his height, his heartbeat time, and his mass, when he was going the same speed as Alice."

"Um, usually the nurses measure my weight, not my mass. I'm not Catholic."

"Mass...oh, I see, it's a joke. Mother told me to laugh at jokes unless they were not politically correct. Is your joke politically correct?"

"Skip it, Naiya. Where did you learn to say 'OK' so much?"

"Sorry. Mother always scolds me for that. But it's better than putting 'like' between every three words, like my older sister does."

"OK." Julia grinned.

"Since Bob is not accelerating when she measures this, and he is nowhere near a significant source of gravity, then his weight will be zero. That would not be a big help. However, I will have to say something about mass later. First I will deal with the other two."

"Height and heartbeat."

"Uh-huh. We'll call those measurements his rest height, his rest-heartbeat, and his rest mass. He was resting, you see, when they were measured."

"I see, Naiya."

"Right. The first thing she measures when Bob has stopped accelerating will be the percentage of the speed of light that Bob is traveling. She measures the spoke lengths, and then takes that percentage of the spoke length. So if Bob is going half the speed of light, and the spoke is thirty centimeters, then she takes fifteen centimeters. Alternatively, we could do everything in percentages."

"Let's continue with your thirty centimeter wheel."

"OK, then she selects the spoke that touches the rim at that distance away, when measured horizontally, from the vertical spoke. That is going to represent Bob." Naiya picked a spoke at was about sixty degrees from the horizontal."Then she checks the height of the Bob-point. For reference, she also marks that distance on the horizontal spoke. Do you see a marker lying around?"

"No... wait, there's one. Will red do?"

"OK." She marked the spoke red, and then continued."She will also mark the height that that Bob-point makes to the horizontal, on the vertical spoke. Can you give me the blue marker there?"

"Where?"

"On the couch."

"Here you are."

"Thanks." She colored about twenty six centimeters of the vertical spoke.

"OK, now I am going to re-label everything. I will now label the vertical height, the blue part, the 'rest height'. So, say Bob was a meter and eighty centimeters. Then I figure out what the whole length of the spoke would represent. Um, let's see, that would be... can you give me my calculator?"

"Where?"

"On my desk. My phone. Thanks." She typed a little, and said,"So she would now measure him as being two meters, eight centimeters."

"Oh, now I am confused. He was a meter eighty. How does he suddenly qualify for a basketball team?"

"Height is something you measure, right? It's not something absolute. The measurements change when you are not going the same velocity. Anyway, he would not yet qualify for the

basketball team, because with respect to himself, or other players on the same court in his spaceship, he would still be a meter eighty."

"Ah, it's an optical illusion?"

"No. He really is two meters and eight centimeters, at least from Alice's viewpoint."

"It will take me a while to digest that, but go ahead with your example. What about his heartbeat?"

"Same idea. If his rest heartbeat was a second per heartbeat, then his heartbeat on the ship would be about a hundred and fifty milliseconds longer."

"What, he gets excited?"

"No, I am assuming that his heartbeat, with respect to himself, was constant. That was just an example. In general, time slows down for him, from Alice's point of view, although it stays normal for him."

"Oof. OK, what about the mass?"

Naiya's mother came by and listened for a moment. Then she warned Naiya, "Young woman, be careful at this point. You know what your Uncle Herman thinks of the word."

"Oh, stodgy old Uncle Herman. You mean I should distinguish between the mass due to gravity and the mass as a measure of how it resists acceleration?"

"No, of course not, you silly. Those are equivalent. But rather, you have to consider that many people want to use the word 'mass' only to refer to the rest mass."

"But mother, what is wrong with talking about the mass I measure when a body is not at rest with respect to me? Should I say 'relativistic mass'?"

"No, many people, like your uncle, don't like that expression, because mass is supposed to be an intrinsic quality. Just don't refer to the mass at all except for the rest mass. Talk about the momentum which changes in accordance to velocity."

"But that's just the mass times the velocity. Uncle Herman is just being..."

"No, the size of the momentum is defined by your bicycle tire. Leave it at that, or..."

Julia intervened in order to stop a family feud."So a piece of light doesn't have momentum? Then how can one move things with lasers?"

Naiya said, "Of course it has momentum, silly."

Her mother intervened, "Now, Naiya, you mustn't call people silly."

"You just called me silly."

"I'm your mother."

Julia decided to intervene again to stop this exchange from deteriorating. She asked, 'But it has no mass. So..."

The captain moved away, figuring that she had admonished her daughter enough for one day. Naiya told Julia,

"Ah, mass and energy are both different shadows of mass-energy. They are equivalent. So the momentum of a photon, a piece of light as you called it, is just its energy expressed in terms of units of the speed of light."

Julia looked at her watch."Um, sorry, so what does Uncle Herman say about measuring Bob's mass?"

"He says that the rest mass is always the same, and the momentum is really what Alice would be measuring as different. He doesn't like the term 'relativistic mass'. But anyway, you get the idea. The momentum increases. If you consider the word 'mass' to refer to an intrinsic property of an object, then that cannot change with velocity. So this intrinsic mass is the same as the rest

mass. So what Uncle Herman would say is that with increase of the velocity of an object you need more and more energy in order to make it move faster."

Julia wondered whether Uncle Hermann was pedantic or Naiya was being impatient with details. She was a child, after all. Aloud, she said, "I have to be going, Naiya. It was very nice talking to you. Maybe I'll see you again."

"Oh, I would like that, Miss Julia."

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December 5, 5832

"Hi, Sara. Do you have to prepare for your next class?"

"Hi, Moishe. No, I am just staring at the wall. What have you been up to?"

"I was thinking about our last discussion."

"I forget. What was it about? That heap problem?"

"No, your idea about significance. In theory it sounds nice, but I haven't been able to envision what that would look like in practice."

"Ah. For a good example, the first thing we do is to have infinitesimals in general."

"Those are absurd. There's supposed to be smaller than any fraction, yet positive and non-zero. No real number matches that."

"Who said infinitesimals were real? We can make an expanded model with the reals but the infinitesimals in the expanded universe, solve problems, and then restrict the model back down to the universe with only reals in it. In this way it is often easier than trying to stay in the same model all the time. Haven't you ever had an inspiration to solve a real problem by imagining something wild?"

"Sounds suspicious to me."

"First, consider the following. Suppose we have a model whose theory has an infinite bunch of first-order sentences. If that infinite bunch is consistent, then there is a model. If there is no model for them , then there is an inconsistency, but that only requires two sentences, not the infinite bunch. So, if it is consistent, then no finite bunch of them is inconsistent. That is: if every finite bunch of sentences is consistent, then the whole lot together is consistent. So, if each finite bunch of sentences has a model, then the whole bunch of them has a model."

"OK."

"So, you have a model for 'there's something smaller than one unit.'"

"Yes."

"And for 'there's something smaller than half a unit.'"

"Er, I guess. A third, for example."

"And any sentence is true having 'there's something positive bigger than' any number you want to name, right?"

"Yea. The line is a model of each one of them."

"But not of all of them together, since among real numbers there is no positive number that is smaller than all the other real numbers. That would be one of those contradictions. But that's not the point. The point is that there is some other model for all of them together. The things that would satisfy all of them together are infinitesimally small. Just like with infinity, infinitesimals offer a shortcut: you can solve a problem by adding the infinitesimals to your problem, and then getting rid of them at the end."

"So, can you take any sum?"

"Naw. You have to alter your theory a bit to eliminate really weird sets, and you can then add up everything of interest. Like the probabilities over an infinite number of points."

“That sounds tricky.”

“Not so much. It is like the Achilles and Tortoise. It helps to look at the universe of a model from a higher universe, so that you can get that universe to be a set. You can resume looking at either model as the top model.”

“What, the ultimate in fashion?”

“Let’s not get carried away with the stupid jokes. If you want, you can stop at one set and set it up to be the universe that isn’t a set, as long as you remember what you are doing.”

“Try not to confuse me too much. Stick to sets. Let’s just talk about sets; I’ll remember we are looking at them from the outside.”

“Fine. Going from the base up, you want your model to be ready to accept theories. Let’s start with a universe, seen as a set, as a basis. Take subsets of that universe.”

“Subset? A set under water?”

“No, a set that does not contain anything that the original set doesn’t have,” Sara said patiently.

“But don’t you deal with individualistic points?”

“Indirectly, just as sociology does not only deal in individual persons. Or I suppose a more modern example is the fact that those raised on Facebook no longer communicate with individual friends, but rather with collections of friends.”

“Ah...”

“So, as I was trying to say, we take the set of all subsets of our universe.”

“That’s called the power set of the universe, right?”

“Ah, so you’ve had some set theory. Good. Then I want to select some of these subsets that could correspond to the true sentences of a theory. For example, you would not pick a set which would satisfy a contradiction, would you?”

“I suppose not.”

“Or even a sentence that indirectly implied a contradiction.”

“No.”

“So, it would seem reasonable that if you have two formula that you wanted to include in your theory, and you wanted them to have a significant probability of occurring together, then you would want the intersection of the corresponding sets in the model to be big enough to be significant.”

“If I get the idea,” Moishe said, “you are trying to make a universe that corresponds to statements in your theory, by connecting building blocks of theories with building blocks of sets.”

“I try to make a consistent picture of the world, but every time I think I understand the world, someone sends me something from the Inter... um, the Ethernet that contradicts it.”

“You have to build your theory to be able to separate the wheat from the chaff. Deal with the exceptions, learn to distinguish silly from good. Not to mention double-check everything. Construct your generalizations in such a way that insignificant exceptions are possible,” Sara said.” There are all sorts of theories on the Ethernet. A few are easily spotted as being contradictory. Others seem pretty self-contained, but are dismissed as silly by scientists. It is sometimes tricky to separate out silliness.”

Tal thought, “I know that in statistics, they have ways to measure how spread out the occurrence of something is, but they say that both genius and insanity are on the outer reaches of these distributions.”

“Right, so that is not what I am talking about, although it is related. Let’s take a trivial example. One can say that there is a planet made of green cheese in the Orion Galaxy. Fine, but this would be silly. This shouldn’t have to fit into your picture of the world, which will have room for small exceptions. Statements that you can say are true almost everywhere.”

“Hm. There are a lot of details that would have to be ironed out for me to get this into my head. Can I see you then another day?” Moishe asked sheepishly.

Sara shrugged.”Why not? I come here almost every day. They have a good menu.”

Chapter Fifteen

December 7, 5832

Julia received a message from the captain.

“Julia, could I ask you to do me a favor? I promised to take Naiya to the art museum today, but an emergency has arisen. She got on so well with you. I can sign a chit relieving you of your duties today if you would be so kind as to accompany Naiya.”

Julia agreed.

Naiya, however, was no ordinary child. It was Naiya that broached the matter of relativity. After a few minutes of Naiya talking about it, Julia asked her,”If I understand correctly... and I don’t guarantee that ... you could switch perspectives back and forth, so you end up with Alice seeing the Bob as longer and slower than her, but Bob seeing Alice as longer and slower than him?”

“Sure. That’s why it’s called relativity We switch models so that the same sentence, for example that Bob is slower than Alice, is true under one model and false under a different model If we kept to a single model, this wouldn’t make sense, as we would have one slower than the other slower than the first.”

“So a third observer...”

“...would use a model that was a third model. But the laws of physics are a model that includes all these models.”

She looked at the construction, and asked, “Well, let’s see, couldn’t you have something go on the other side, like the mirror image?”

The guard laughed.”Don’t get her started on tachyons.”

Naiya protested.”But, Ed....”

Julia asked the guard, “You know this child?”

“Naiya? Sure. Everyone knows her. Our Wunderkind. She gets around.” Ed turned to Naiya, “Naiya, you know better. Besides the lack of experimental evidence...”

“That didn’t stop hypothesizing for things which were not testable at the time. The Higgs Boson, Relativity, ...”

“Right, but they were in a possible model in a physicist’s imagination, whereby the accessibility relations were made up of continuing the applications of the physical laws known to the physicist. In other words, at least they were testable in theory. It is not certain that tachyons are testable even in theory. If not, they aren’t physics.”

“But one assumes a lot of theories which are yet well established.”

“Yes, provisionally, since it is useful. Assuming the existence of tachyons wouldn’t contribute any explanatory power to other parts of physics. If you remember the way we that we treated silliness with those structures, you will recognize the spirit. Show this young women things that you know have been tested.”

“What, show me?” Julia was surprised.”We are in an art museum, not a science museum!”

“The science museum is next door. Maybe when you have had your fill of paintings, you would like to step in there. It’s on the same ticket.”

Naiya turned to Julia.”Miss Julia, can we? They have some new hands-on exhibits that I have been wanting to try.”

Ed smiled.”I am sure her mother would approve.”

Julia gave in. Oh, all right.”

When they got to the science museum, she walked directly over to what appeared to Julia to be another pile of old stuff. But apparently it was an exhibit; it had explanations, with the entreaty at the end in capital letters, “Try it!”

Naiya quickly scanned the explanations, nodding now and then. Then she asked Julia,

“You wanted to know about velocity. That requires these two old aquariums. Here, grab those old aquariums. Um, please.”

Julia complied.

“The two aquariums have the same height --- the two velocities added together, like you were doing. But the bigger one has a square base: each side, which I will let stand for the speed of light. The smaller one has a rectangular base: one side if one of the velocities which you are combining and the other side is the other velocity. Clear so far?”

“As mud...” Julia mumbled, but Naiya didn’t pay attention to her.

“Before I fill them with water, I will insert a pipe with a valve between them so that water can flow from one to the other. OK? Now I fill the larger one with water. Then I join them together.” She fiddled some more.

“Now watch how far down the water sinks. It will of course end up on the same level in both containers. The new height is going to be the combined velocity, which will be lower than if you just added the two velocities.”

“This combined model...” Julia started.

“That’s called special relativity. I haven’t learned the equations for general relativity yet. But I’m told that special relativity is just a special case of general relativity. Oh, look, there is something going on Outside the museum!” She ran over to a window.

Julia joined her. This window offered a clear view of the ship’s docking facilities. was a second spaceship coming upon them at a much slower velocity. This one was able to dock. The landing was a special event. No one had landed from another ship. The amateur linguists in the crowd debated whether this should be called “landing”;”shipping” did not work. Even Julia showed up, but she could not see the face of the person, who had anyway been in a spacesuit and was whisked off to quarantine. Julia found this interesting, but nothing to do with her goal, so she kept looking.

In quarantine, a person, apparently a human female, stepped out of the spacesuit. She explained to the technical personnel that the other spaceship had been a decoy. The human presence and heartbeat? No, there was no human aboard, but the new arrival knew what their scanners would pick up, and had programmed a unit inside the spaceship to give precisely the right outputs. She had picked up the idea from the earliest attempts to pass the Turing Test, the newcomer explained.

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December 9, 5832

Moishe came to the cafe, hoping to see Sara. She was there, at her favorite table. She looked up as he approached.

“Hi, Moishe.”

“Shalom, Sara. I have been thinking about what you told me. I would like to use this ultrafilter thingy, but I have the whole lifetime of experiences to deal with.”

“That can be dealt with. A universe can be shrunk down. Questions are about something. Those ‘somethings’ can be organized into a structure. I won’t burden you with an exact definition. It essentially mirrors the sort of logic one would expect for a model that was complete. If you want to look it up, it’s called an ultrafilter.”

“Sounds like a new water purification system.”

“It’s a way of building models that can handle theories that contain exceptions. I will just give you the general idea. Consider the statements in the theory to be questions which are posed to the model, which then presents sets of possible answers taken from a class of possibilities. Maybe from the Ethernet, or whatever source you agree upon. That is, regarding this reference collection as a set, then we make a universe that will be part of the power set of that reference collection.”

“I forget what a power set is. Is that an electricity generator for your ultrafilter?”

“No, it is the set of all subsets of the original set.”

“Ah, now I remember.”

“But we limit which sets we take.

“This is possible, with something called ultrafilters.”

“What, you pour the sentences down a funnel, and it filters out the insignificant ones?”

“No, it’s not like that....In a nutshell, remember that collection of truth values should have a hierarchy? There are several ways to do this, but one way is to start with everything that you definitely want in your model, that is, minimal satisfaction sets of the axioms. Then see what sort of structure you can build that makes things like implication follow their rules. There may be sentences which are not in the structure. With respect to this attempt, those sentences are not important. They are silly, or not significant. These are the unnecessary hypotheses of Ockham’s razor.”

“I am having problems visualizing this.”

“Suppose you take all the sets which you could form by taking out a finite set of numbers.”

Tal took a moment to work that out, and then said, “OK, so what?”

“Then, consider the finite sets, which are outside the structure. How much would each one be in comparison to one of the sets inside?”

“Ah... insignificant. Is that what you’re getting at?”

“I think that is the way my professor described it. I didn’t get all the details, but the general idea is all you need. With these principles, you start constructing the ultrafilters in the expected manner.”

“And what would one expect?”

“For example, that if a smaller set is part of this collection of true sets, then so is another set that contains it. Or that a set which is the complement of a second one...”

“I didn’t know sets complimented each other. ‘Gentlesets, you look splendid today. I like your tie.’ Something like that?”

“Complement, with two ‘e’’s. What’s left from your universe after you take the original set members out of it.”

“Ah. Let me see if I get it. The complement of a set corresponds to the negation of the formula of the original set.”

“You’re getting the spirit. So obviously you don’t want both to be considered true, so you pick at most one.”

“Since the intersection in the model would be those elements which are in both sets, and therefore satisfying both corresponding sentences, intersection would correspond to the connective ‘and’ in the theory. And similarly for other correspondences between set operations and connections between sentences in the theory. I get the reasoning. So I have a possible world. Where does the idea of silliness fit in here?”

“The silly statements correspond to insignificant sets in your universe. For this not to make haywire in your head, you have to make sure that the significant and insignificant sets combine in a sensible way. For example, by taking only sets which are significant, and in addition whose intersections with other sets in the ultrafilter are also significant. Sets containing the same information as an accepted answer are also taken into the ultrafilter; after all, it may be an answer to a question not yet posed. In fact, ideally every question will have either a yes or no answer among the sets in the ultrafilter. A few more tweaks like that, and you have a model.”

“But mathematicians are always building castles in the air. Will it work for theories of physical phenomena?”

“You can tweak it a bit to do so. For example, we like a theory of spacetime in which a state contains, at least implicitly, information from the past or from its surroundings, so that the past of the present moment will be part of the past of future moments. So, you can tweak your model to do use ordinal numbers, that is, sets which work like this using the membership relation as ‘before’.”

“But if you took the universe of all possible sets, along with membership between them, then this would also work as a model, no?”

“A big and clumsy one, yes. With this method you can get models that are much smaller and that will fulfill the same first-order theories.”

“Is there a zero-order theory?”

“Sure, the kind in which you can’t make generalizations. The quantities are known. The first order one allows generalizations, but the things that are being generalized about are single individuals, not collections or functions.”

“So what sort of questions can you answer?”

“Lots of the normal ones which can’t be answered when there are exceptions, even though the exceptions are generally unimportant. In the classic theory, if you wanted to find an infinite sum of infinitesimal quantities, whether directly or through approaching it by making large sums of tiny quantities both as large as you wish and as tiny as you wish, respectively, together, the whole thing collapsed when there was any little interruption, any discontinuity. Today our theories need to take these interruptions into account, and with an ultrafilter and its idea of significance, this works. There are some other conditions that will allow insignificant exceptions, which is what we started with. But in constructing this, there was something that can lead us to understand even more of the physical world. If your model is complete, then given any two complementary sets, you have to pick one.”

“I thought that this was not possible. Undecidable sentences, and all that.”

“You are getting ‘true’ and ‘provable’ mixed up. The undecidable sentence may not be provable, but it can nonetheless be true. So you can have a complete model, even though your theory is incomplete.”

“But what about the indeterminate states in physics? If the model is complete, then everything is true or false in an objective sense, and the indeterminateness is just undecidability,

which is epistemological uncertainty. But I thought this quantum indeterminateness was ontological."

"Yes, yes, I understand the confusion. Again a description in terms of the models here may help. The key is the contact with the environment. The models for the two situations – observed and unobserved – can be both seen as ultrafilters which can tell the difference between significant and insignificant. These ultrafilters are based on selected subsets of certain large infinities that appear not to add any new contradiction if we assume them."

"Large? In terms of the way you measured the size, what is it called again?"

"Cardinalities. So these infinite sets are called large cardinals."

"What, do they become fat popes? Or maybe they are related to Big Bird in Sesame Street?"

"Do you want the explanation or not?"

"Sorry. Go ahead. So there are two directions, unobserved to observed, or vice-versa."

"Start with unobserved."

"OK, if we start with an ultrafilter which is an extension of regular probability functions up into an infinite number of points, so that we get real numbers between zero and one as probabilities. But then... first, let me make something clear. Suppose you say that you will be going to the cinema or that you will go to the library, and then someone tells you that you cannot go to the cinema. Putting them together you have 'Either I will go to the library, or I will go and I will not go to the cinema.' The latter is a contradiction, so you are left with the first one."

"Rather drawn-out way to put it."

"To make a point. The interaction of the environment with the possibilities prunes down the model based on the first ultrafilter in a similar way, even if not quite so crudely. The state that is subjected to measurement, and the measurement tools, are each one or more waves, and they interfere with one another. But it leaves all the same ordinal numbers that were in the original model, so that you can still have spacetime for both, and other important features satisfied by both models. The resulting ultrafilter still can be seen as a probability measure, but only insofar as it only admits two possibilities, certainty and impossibility, except for insignificant exceptions."

"Ah, so.... but the other direction?"

"Did you hear that attempt to find a set that was bigger than the set of counting numbers and the set of real numbers?"

"No, what was that all that about?"

"The attempt was based on starting with a minimal model, and adding additional assumptions that would force the model to have more elements."

"The universe in a grain of sand?"

"Setting the poetry aside, one side effect is that -- remember that you said that we could adopt the idea that the axioms that all sets were constructible, that is, definable in the restricted sense? You talked about this with the Reb. But if we adopt the assumption that there this big infinite number, a number that allows us to do all this, then we can no longer assume that all the numbers are constructible. We may therefore, with this assumption, construct models in which there is a set whose size is between that of the set of counting numbers and that of the set of real numbers."

"OK, that sounds like a curiosity, but I still don't see..."

"What the person was trying to show was not realistic, but the technique allows us to understand the transition from a fuzzy state to a definite measurement and back."

“I remember that, in the explanation about waves and particles being the same, it was mentioned that the particle is just an excited section of a wave, but...so, in sum, a measured object still comes into contact with the environment, but in such a way so that the contact adds new parts to the minimal model, and forces new elements, until you end up with the ultrafilter which can give out any of the probabilities, not just yes and no. Have I summed that up correctly?”

“More or less. This explains a bit more the argument that particles are just certain kinds of the waves that we call fields. Even the ‘ultra’ in the name gives a hint about the fashion in which the transitions around measurements take place.”

“I thought it was just added to make it sound glamorous, like in advertising.”

“No. Again, part of the construction was, when faced with two complements, you take at most one of them. But until you take exactly one of them, the process is not complete. The model with all choices made is the last step of a process... and this process can take time. It doesn’t just suddenly collapse, it sort of slowly collapses. It comes unstuck, loses the coherence it had. Perhaps a more intuitive image is from acoustics: imagine a room full of musicians all playing the same note, but now and then one of the instruments is a little out of tune. You get an annoying beat from the interference. This beat is a bit like the particle. That’s a very rough image, but perhaps it can help. In any case, none of this is a full explanation of measurement, but at least we have one less problem in describing it.”

“Is all this possible with the usual assumptions?”

“No, you do need to add an extra assumption to your theory. But this assumption is pretty harmless, since it would not add any contradiction to the usual theory. If you take the assumption for the yes-no type, which fits the measurement then you do not have to assume anything more for the probabilistic type, so the idea of states is not as wild as one might think.”

“Hm. To fulfill the conditions to construct an ultrafilter. Many fanatics base their idea on a very small collection of ideas. Could you maybe pick a set with one element, and picking all the sets of which that set was a subset? That would fulfill these requirements.”

“Except that the individual points should be insignificant, so that sort of thing will be ruled out. In fact, you can’t base it on a finite set, for the same reason. No fanaticism.”

“Why?”

“Consider. You have a hectare of land. Someone takes a mathematical point out of it. How much land do you have?”

“I get the point.”

“Lousy pun.”

“So I’ve been told.”

“In fact, you would need anything smaller than the infinity of the universe to be insignificant.”

“How is this tied in?”

“This would keep us going forever. Let’s stop here, it’s getting late,” Sara said, hiding her uncertainty.

“OK. Anyway, I need time to digest all this. Um, could we meet next week?”

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December 14, 5832

The following week, Bill met Sheila again. He told her, “I read this quotation, ‘Mass tells space-time how to curve, and space-time tells mass how to move,’ by an Earth physicist named Wheeler. Seemed a bit circular to me.”

Sara told him, “It isn’t. Energy is a shadow of energy-momentum, and that warps space-time. Bending space-time means changing the measurements between points, so that the variation from the inverted triangle of space-time, which is the variation on Pythagoras in space, changes so that these triangles no longer work, although they do in infinitesimal neighborhoods of each point, from the point of view of the person measuring it. But if you are changing possible worlds, this means that when you measure, your neighborhood expands so that you have to compare two different worlds, and you will see that the triangles no longer work.”

Bill could only nod.

Sheila continued, “How much they change when you stray from a single world will also change. The amount of warping expressed as a matching of a beginning state and an end state amount could be expressed as a force. Alternatively it could be expressed using the truth functions regarding position. Assuming that a path was connected, this still permitted an uncountable number of possible worlds. They formed a hierarchy where the truth values for them would be structured so that their interaction, their interference, would make sense in the model used as a reference point. There would be a distance function defined for any two points based on the truth values, so that the smallest distance between two points, the minimum of energy-time intervals, the path the most probable for light as the truth values interacted in a wavy fashion -- from that point of view, its end path would be labeled a straight line, even if another point of view, another model with another theory, would not consider it a straight line. A bit like someone judging the behavior of someone three thousand years ago by today’s mores.”

“Keep the preaching down, please,” Bill joked.

“Sorry. So, in sum, that sentence means that associated with mass-energy will be operators, such as the gravitational energy, which are associated with the distance function, which then dictates how mass will move as it seeks out the straight lines.”

“It sounds catchier in his formulation, though.”

“True. The straight line could be considered a sort of game-type equilibrium, where any deviation by any of the participants would result in a less favorable, or probable, situation.”

“Never thought of reality as a game like that. I thought physicists thought in terms of geometry.”

“Whatever works, Bill. Or in terms of models. Each model assigns every point in space-time a value; this assignment is called a field, and, as models changed in the possible worlds hierarchy, fields change. The amount they change changes the straight lines and their relationships; this bending is named ‘warping’ by the fans of science fiction. In other words, the field that assigns a function for the space-time distance function is called force, although it could also just be called the geometry of space-time. On an infinitesimally small volume around each space-time point, the usual triangles are valid. But this little infinitesimal volume is like a car whose wheels followed the path of a hilly road. The car remains straight, but its gradually changes which direction it is pointed in. At each such point, you have a road sign indicating how steep the incline is, and another road sign indicating how much change this incline is about to make. The first road sign can be compared to the kinetic energy, and the second one, to the potential energy. Of course each one depends on your viewpoint, that is, what you considered to be horizontal. That is, how the potential and kinetic energy get distributed will depend on how you slice your spacetime.

“Ah, I guess so...”

“One could sum that up in saying that the shadows of mass-energy will follow a curve based on this hilliness. Does all that help, Bill?”

“I’m getting dizzy from your hills. But thanks, Sheila, I am glad that you understand it. Perhaps I will take you along to my discussions with the others.”

“No thanks. I am more a one-on-one person. I don’t do well in crowds.”

“If you are going to become a teacher, you had best get prepared for it.”

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December 17, 5832

While Moishe, Tal and Sara were involved in their endless discussions, Julia was trying to find out more about the Casimir project, to find out whether it could fit into her plans of destruction. She appeared concerned for the safety of the ship as she asked Sheila, “I heard that you had some problems in figuring out the vacuum energy?”

“For that Casimir project, we have everything well calculated. The vacuum energy is well measured. However, the explanation, attributing it to the fluctuations in the probabilities, ended up with infinity. We got rid of that.”

“How?”

“The problem in the calculation for the vacuum energy is that, although we have the difference in the energies for our project well calculated, and we seem to be able to measure the vacuum energy pretty well, when we calculated it according to the mechanism of these fluctuations, these virtual particles, we get a different answer. In fact, at first we got an infinity, but we figured out what to do with that.”

“It sounds like self-reference. Doesn’t that lead to paradoxes?”

“Not always. It can even be helpful. We looked at self-reference in nature in a sort of fractal effect, where smaller scales meant smaller copies of the larger scales. But this is true only up to a point. The smaller scales will differ in more than size. Our first mistake was to assume that conditions were always the same. Our theory was too simple to be able to describe the model. We had continued to think of metal as a good conductor, but at the higher frequencies it isn’t. We had to think more carefully about the density of the energy right outside the plates, and that energy can tunnel through the plates.”

“Tunnel?”

“That is, where classical physics says that the energy should not go, in modern physics there is a chance that it will. The longer the time or the greater the energy, the better the chance. The build-up of energy seems to go off to infinity on both sides of the plate, but this build-up is exactly the same on both sides, so it doesn’t end up contributing to the push and pull. This is what people mean when they use the very awkward phrase to say that the infinities subtract.”

“But something does contribute.”

“Yes, and so we had to find that fine point in the high frequencies where the tunneling doesn’t become so significant that the plates themselves become insignificant. This calculation has to be finely tuned, a bit like finding the dimensions of a fractal.”

“I never understood how something could have a fractional dimension. I always think of dimensions as positive whole numbers.”

“The dimensions you are used to are scaling factors, where if you change the length of a figure with that number of dimensions, you change the content of the figure by that exponent.”

“Huh?”

“For example, if you have a figure that is two dimensional, and you triple the length of the sides, you will increase its content, in this case called area, by nine times. Three to the power of two. Or if you make the sides half as long, your area changes by a quarter – a half to the power of two.

“So if you were making a model for a statue, and the statue’s height will be five times the height of the height of your model, then you will have to use... what’s five to the power of three?”

“One hundred and twenty five.”

“Right. You will need one hundred and twenty five times as much clay, since the content, called the volume, will change by the exponent of three, because it’s three dimensional. In the case of a fractal, the content is achieved by an infinite process, and if you, say, double the lengths of the sides, then the content will increase by using an exponent. If you make this exponent too small, the result goes down to zero. If you make this exponent too big, the result goes off to infinity. It is only at the Goldilocks point of ‘just right’ that the result will be a non-zero finite number. That’s the dimension of the fractal, and there is no reason the exponent has to be a whole number, although it can be.”

“Ah, so what does this have to do with tunneling?”

“Nothing; I was just using an example of fine-tuning. You get me off into these side issues. Although, come to think of it, it is true that self-reference keeps popping up in looking at fields. So one could probably push the analogy, but let’s not, OK?”

“You just mentioned fields. I presume you do not mean fields of grain. Is that like force fields in science fiction?”

“That’s one kind, but there are lots of kinds. A field is something that gives a value to all points in space time. If the value is just a number, then the field is like a description. But the values can also be specifying how these conditions are about to change.”

“Can these changes affect themselves to get self-reference?”

“You are obsessed with this idea, aren’t you? Why?”

“In school I often wondered at the formulas for force that require you to divide by the distance between two things. Or the distance squared, same idea: the question that the teacher always waved away was: what about the effect of something on itself? That distance is zero, but we were taught that you cannot divide by zero.”

“So how did you resolve this in school?”

“We didn’t. The teacher told us it wouldn’t be on the test, so we forgot about it.”

“Ah, the wonders of modern education. That is the problem when you have two kinds of entities, particles and fields, in your theory. You have particles producing fields which affect the particles which affect the fields... and you get your infinite regress. If you work only with fields, some of these problems go away.”

“Some? Not all?”

“Er, no. But you can approach it differently. With fields, you assume that you have some sort of continuity. Continuity requires that you look what happens when you get close to annoying values. As the precision of your measurement goes up, the spread of possible distances gets smaller. Therefore, the possible spread of the energy measurements must get bigger. There is only so much that you can do with this, so there must be the cut-off point, like we encountered with the Casimir effect. That is, a field spreads out, but the relationships are a bit different around the middle of it, so that you don’t get something like a division by zero. That no-no-point of division by zero never appears.”

“So you are saying that you have to be careful with generalizations, just like when you analyze paradoxes.”

“You are determined to link this to paradoxes, aren’t you?”

“I like paradoxes, you don’t like particles. Each to his own taste. But you did say that particles are sometimes useful, if only as a shorthand way of speaking. I read explanations talk about the particle pulling virtual particles out of the vacuum around it. That seemed pretty useful.”

“If you insist on particles, I guess that’s the best you can do. But you can do it with fields using all this stuff about a cut-off. This idea led us to the realization that when we measured things like charge and mass, we were doing so from a couple of centimeters away. This was fine for a lot of calculations. But some calculations required the value of the space-time point, or the particle if you like, to be the value before the loops of self-reference took effect.”But if we want to figure out what would happen when these things affected the center, we can’t just extrapolate. At the center, an extrapolation would mean that each time we apply some transformation of the state, then it changes, giving way to a further change of the transformation again, and so forth off to infinity. But maybe there is a fixed point, where the transformation does not change the state, but leaves it, so that no matter how many times the transformation is applied, it remains stable. So no infinity at the center. Maybe. I don’t know.”

“If I follow what you want to say, you want to find the value of the particle without the virtual particles.”

“More or less. So, we had to go backwards to figure out what this ‘bare’ value was, because we couldn’t measure it directly. Once all these loops were then taken care of, weird sums added up to more regular values. One of them gave an expression that looked familiar. Since you like self-reference, this should please you. You see, there’s a famous function called the Riemann zeta function. It’s mostly famous for its connection with prime numbers, which seems pretty abstract.”

“I think I read about that somewhere.”

“Yea, well, you want to be careful. There’s a function which makes sense only for certain values, and you have to tweak it to make it make sense for most of the other values. Some sites call the untweaked function the zeta function, and some are referring to the tweaked version.”

“Which one do you mean?”

“The tweaked version.”

“OK, so what was so interesting about this zika function?”

“Zeta. No one gets it right. The Greek letter zeta. Zika was a disease on Earth.”

“Whatever. What was so interesting about this Greek-letter function?”

“Lots of things. For you, fixated on self-reference, the function would be interesting because a certain type of curve can be approximated by somewhere on the curve of the zeta function, and the zeta function curve itself is this type. That is, the zeta function curve is a kind of fractal.”

“Is that why you were interested in it?”

“No. When we were doing the calculations for this bare value, the value for this zeta function for negative three came out.”

“Maybe it was a coincidence?”

“No, the way it was calculated indicated clearly that one was talking about the zeta function. That set me to looking at the times when this function added up to zero.”

“Why? What’s so special about zero?”

“You know how the word association game that psychologists, linguists and drunks like to play? That’s what happened to me, although I was not drunk. See if you can follow this. You can only measure things that have some kind of change, and so generally our formulas are about change. Fields, for example, are functions on all of space-time, there are a lot of fixed points.

The way the fixed points are distributed seemed important, such as the possible measurements of energy. For example, the energy levels of electrons around atoms are fixed points, where any energy less than a certain amount will not change the energy levels. The electrons stay put with too little energy. All of chemistry, for example, is based on the way electrons change energy levels."

"What does this have to do with your yeti function?"

"Zeta. We don't have a function for an abominable snowman. What I was trying to say is that if you know the places where the function is zero, then you know about fixed points, and you know a lot about the field. So I started looking into it. I found some more patterns connecting it to physics. However, I have to go to work now, so try asking Sara about it."

"Sara? She's a music teacher, isn't she?"

"She says that music is her gateway to mathematics. How did she put it? 'Only music and mathematics get their beauty from form rather than content.'"

Chapter Sixteen

June 8, 5832

Julia found Sara in her apartment near the Conservatorium. Sara listened patiently while Julia explained her question. Sara gave her tea, and showed her a slow-motion picture of a plucked guitar string."You will notice that it starts off like a wave traveling in only one direction, like a water wave, alternating up and down. Now, when it hits the other side, it bounces off, as if it the wave decided to keep going but in the other direction. Then it is like the first rush of holiday shoppers who have finished their shopping and have to get through the second crowd of those who haven't started. They interfere with one another. This comedy happens at the other end, until the interference stabilizes into a fixed pattern. Then you have a wave that looks like it's only bobbing up and down, alternating between a hill shape and a valley shape. That pushes and pulls air; since one complete sound wave is a cycle of push and pull, you end up with a note with a wavelength twice as long as the string. So far, so good?"

"I had something about this forty years ago in my science class, but...."

"OK, the idea is that you have half a wavelength connected with the fundamental note of your string. However, the wave will not usually be quite so simple. The pattern also has this wave combined with waves that, if you isolated them, would not have only one hill-valley wave, but two, or three, or four, and so forth. These are the overtones. If you start listing the fractions of the string that one of these hill-valleys takes up, you would get a sequence of fractions: one, a half, a third, and so forth."

"What about the loudness?"

"In general, the amplitudes of the overtones get progressively weaker. They do add together, though, and how they get together is the study of harmony. So the different wave patterns are called the harmonics. That's enough for your first lesson."

Julia went home and decided to imprint these basic ideas on her memory. She used a technique that she had learned when she had taken a short stint as a school teacher. She made up a silly story and would find someone to listen to it. It would help Julia, if not the listener.

She thought about using elves and gnomes. She realized, although schoolchildren had had no objection, most adults would not want to listen to her story. At least no one sober. So, she would try it out on the only place she knew where people lacked this qualification: the local bar. Since

the inebriated were not particularly religious, she could use angels and devils without risk of offending anyone's religious sensibilities.

She found Paul in the bar, who had consumed enough to stay conscious, but enough not to remember much afterwards. A perfect listener.

She started by describing an orchestra in Hell."Every devil was given a tuning fork and a one-note horn. The horns would all play the same note, and all the devil could do is to vary the volume he played the horn with."

"Seems pretty useless, the tuning fork."

"The devils used the fraction of the tuning fork note to know how to modify the volume of their contribution. They would blow harder or softer, according to the fundamental harmonic of the conductor, who blew on a ship's horn as a reference. All harmonics were represented."

"What about the acoustics?"

"Acoustics? They were in hell, not in some hall. Devils and their instruments don't have size, so they were all giving out their music from the same point."

"But if they had size..."

"OK, say they did. Each devils had a rank. Each devil plus instrument was half the size of the one who had the next highest rank, with the first violin having the highest rank, so that they all together added up to only a couple times the size of the director."

"You pulled that out of your hat, didn't you?"

"In the grand tradition of most theologies. You bring up silly objections, I bring up silly answers."

"No, I mean you added up all those infinite sizes?"

"Oh, that I did in one step. You can often substitute a finite process for an infinite one, as long as it makes the same jump. Anyway, don't push me on the analogy, it's taking me away from the main point.

"So no hall. So they didn't get the annoyance that happens when a note bounces off a wall and interferes with another note?"

"Of course, the volumes added up. When they all blew their horns at the volumes indicated by their tuning forks, it added up to an infernal din, since the total volume was infinite. But infinity was OK in the afterlife. Satan decided to send his orchestra on tour, starting with Heaven."

"Nice guy."

"Yea. Have another beer. In Heaven, Saint Peter found out what the devils intended to do. Saint Peter preferred quiet. He got the devil conductor, Beelzebub, into his office, and explained that heaven's atmosphere was different to hell's, the horns would not work here. But Saint Peter would issue more appropriate instruments."

"Harps?" Paul asked between slurps.

"No, not harps, since Heaven was in the midst of a music synthesizer craze. So each devil's horn was replaced by a synthesizer which could play one note at a time, but the note had a divine and an infernal aspect. Also each one of these two aspects had an up and down aspect, which was referred to as positive and negative because of the ways they could add together, even ending up in silence sometimes." .

"Ah, so you do have built in acoustics."

"Sort of. But let me get on with the story. Beelzebub was shown to the podium, and gave him a telescoping conductor's baton. The baton could not only be extended out as far as he wanted from the handle, but it could even shrink into the handle and come out the other end."

“I won’t make any silly analogies.”

“Right. Don’t. The forward part was blue, and the backward part, also called negative, was green. Because Beelzebub was color-blind, they called them positive and negative.

Next, he was shown a swinging conductor’s podium that he could rotate on. It didn’t matter whether he rotated clockwise or counterclockwise, they both gave the same result. When the conductor, that is Beelzebub, would start facing east and then turn around on the podium, and extend or shrink his baton as he liked, those two actions would affect all the synthesizers in the orchestra, based on the individual devil’s identification number.

The rehearsal began. When Beelzebub did not do anything with his baton and did not turn, the synthesizers gave off only the infernal sounds from before. Saint Peter encouraged Beelzebub to be more active. This time, whenever Beelzebub extended his baton at all, the din became finite, which was a relief. But when he shrunk his baton smaller than the default length, or kept going out the other side of the baton, the infernal din again became infinitely loud.”

“You expect me to keep this straight?”

“No. You just have to get the idea that Beelzebub’s conducting could change the total sound.”

“I can do that.”

“Good. Saint Peter summoned his arch-engineers Gabriel and Michael. They discussed the problem.”Maybe not weighting them by simply multiplying, but by like raising each one to the same exponent...”

“That will work... for bigger exponents than 1. But for one or less, it still adds up to infinity.”

“Well... maybe this part is only part of a theory which wasn’t defined well enough on the other exponents. So let’s start by restricting ourselves to the finite part, and then extend the domain.”

“That might work. Where did you get that idea?”

“We do that sort of thing all the time in going from one possible model to another. Of course, sometimes we are a bit quick on the trigger with the extensions, so we make unfair arguments, pushing analogies too far, that sort of thing. We take two possible worlds with partially overlapping universes, so that the same sentences are satisfied from both theories on this overlap. Then the theory of the first one will be the continuation of the theory of the second one on the universe of the first one.”

“Wha... I lost you there. You’re getting too general. Talk about the case at hand.”

“What I mean is that we can adjust the synthesizers so that the output will always be finite for both divine and infernal parts of the music, except when Beelzebub doesn’t change the size of the baton from its initial size. But the devils should not be able to tell the difference for the result of the new sound to the old sound when the old sound had been finite. That is, first the model would be reduced to those settings that gave a finite result, then the model would be extended in another direction than the original model, giving only finite results, except at the original baton setting without swiveling... there seems no way around that. There were some other desired qualities of the new adjustment, such as that the small adjustments in the baton will change the sounds only slightly, and the like. Here, I’ll show you....”

Julia pulled out a little working model, which Paul tried to focus on. His efforts were not completely successful, but Julia tried to pretend that she was talking to someone sober. She continued, “The archangels were left to experiment. Since the notes were all the same, they were most interested in just getting the orchestra to either stop playing or, failing that, so that their sounds would interfere with one another perfectly as to render it completely silent. Given the

infinite contributions of the orchestra, at first that seemed too much to hope for. However, happy day! It was discovered that when Beelzebub extended his baton out a negative even number of baton lengths, and kept facing east, that complete silence resulted. But now that Beelzebub had discovered how much fun it was to swivel about, he decided to see what possibilities included swiveling. The only ones he found were all when the baton was shrunk to half of its size, on the positive side. It didn't work all the time: it depended on how many times Beelzebub swung around. At one time, Saint Peter asked Beelzebub to swing around and to extend his baton different lengths; they seemed to only occur, besides those others on the negative side, when the baton was that half-length. But that was a guess, which Saint Peter called the Riemann Hypothesis, RH for short. He asked Beelzebub once to keep his baton at half-length, and swivel around and around as much as he could. Beelzebub swiveled around a few billion times before he got tired of it; Saint Peter found some patterns in the amounts he swiveled. Those patterns, although only conjecture, were interesting in themselves. For example....Paul? Paul, could you please get up off the floor? Paul?"

Julia asked the bartender to arrange transport home for Paul, and she went home to try to think whether the analogy would be appropriate for a less inebriated audience.

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December 19, 5832

The next morning, Julia blearily looked at the news on the Ethernet. Something about the guest that had arrived by spaceship becoming a new guru. Normally Julia didn't care about such things. But she had learned that gurus could often help convince people when reasonable arguments would not. Religions are like that, she mused. She decided to call on the new guru. She went to the Ethernet site indicated in the news article, and made an appointment for the same day.

When she showed up to the address indicated on the site, she was ushered into the guru's office. The guru smiled at her. Julia was startled to hear,"Hey, Julia. I wondered how long it would take for you to figure out that I was here. Ready for some code bashing?"

Julia stared. The face was different, of course, but... Finally, she said,"Natasha?"

"In the flesh. Er, well, not exactly."

"What ..?"

"I came to rescue my partner."

"How...?"

"I noticed what a mess this program is in. I was already working on a solution when I noticed you had come into Paralife. I figured you were also after some solution. I am not sure what it is, but I thought you might need some help. So I decided to perfect what Wheaton was trying to do. I asked Paralife to come up with some improvements, and so I could come in, like you. It took me a lot of hacking to set up the necessary brain scans.... it turns out that the scans used are OK, but they need to be used in another conjunction... oh, I'll explain all that later. What that means is that my body is sitting in a system similar to the one on that island, except a lot better. The trickiest was actually to get it all into the Internet to feed into this computer."

"But that's a huge amount of money!"

"Yes, well, I skimmed some mafia accounts and left a false trail to someone who died in a fire."

"You're just like me here?"

"Not exactly. Different programming. For a start, of course, I'm not an android. But that's irrelevant. The point is, you were programmed as a member of this universe. I programmed

myself one step up, in a larger universe. I did that by bringing some more quantum computers online."

"And that helped? I heard it does lots of calculations at once."

"Sort of. It manipulates states, and the states interfere with themselves. This is easiest when imagining the states as waves. Have you seen the programs that breaks an orchestral piece and breaks it down into its individual notes? We can do something similar to the waves which represent the states. Then you can more easily find the period of each wave, and its amplitude. Finally you can put them back together, writing the combined wave as a combination of its parts. Among other things, you can find how often the wave repeats itself, its period. Of course each time you measure, you will only have one possibility, so you do enough repetitions to find that period with a high probability. Then..."

"Stop with all the technicalities. Fast-forward to your conclusion."

"OK, I do some calculations in higher-order logic, dealing with possibilities that you do not see, and then observation shrinks the possibilities down to actualities, giving me normal first-degree logic which is what I can present to you."

"Are you saying you have see things that I don't?"

"Yes. By the standards of this universe, I have super-human powers. So there are a few little things – not much, unfortunately –that I will be able to reprogram."

"That's how you got away with being a guru?"

"Right. Very useful. Now, what is your plan?"

"To destroy the ship."

"I won't ask the reason, I assume you know what you are doing. From what I've seen, the religious fanatics offer the best hope. They are always the most destructive. But you always have problems accepting help. I think it's time to include someone else. For a start, you might want to improve that story you made up last night."

"That? How did you know about that?"

"One of the ways I keep up my status as a guru is to do a bit of spying, so that I know things that people assume I don't. I spring stuff on them at the right moment, and..."

"Wonderful. But the story was just something I read. It is not really important. We have more important things..."

"Don't forget, I have a clearer view, and I say that your story has an importance that you are not yet aware of. So work on that. I have to go work a few miracles; in the meantime, figure out where that devil story of yours was leading. I'll be back when I can."

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December 23, 5832

The next morning, with a clearer head, Julia tried to figure out how she could improve the story. She had found where she had got most of the stuff that she had told Paul, but she wasn't sure where she got that remark at the end of the conversation, before Paul lost consciousness, about the patterns that she had told Paul about of Beelzebub's podium swivels. Had she really somehow unconsciously derived them, or maybe she had read about them, or maybe she had made them up on the spur of the moment? Of course Paul would not remember the conversation clearly enough to ask for details, but a proper audience might. She couldn't find the Ethernet site that she had consulted. She really must remember to bookmark such sites.

She went to see Sara.

Sara knew what Julia was referring to and reassured her that there did seem to be a pattern. She explained, "Take the swivel angles, and find the distance between any two. For each

distance, we count how many pairs of answers are separated by that distance. We get a trend opposite to that of the primes: primes get rarer as they get larger, the swivel silence-points crowd together with increasing height, which we can approximate better and better as the numbers get bigger. This matching is similar to what happens when you take a random operator, oh, say for example the energy levels around a uranium-238 atom or something else with a total spin of zero."

Julia tried to recall what she had heard about operators during the cruise, as Sara continued, "Then all its possible measurements appear to follow the same pattern. Put another way, the two models seem to have the same theory. So, you could work in one theory or the other, and get the answers for one situation."

"Sounds useful."

"There's still guesswork to establish this. In particular, there is no natural element with this spread of possible measurements. For fun, we put out a fictional document saying that we had discovered an element with exactly the same operator as this function which is at the heart of RH to get that pattern."

Sheila was at the door."Yes, you did, Sara. That was a mistake. Unfortunately, the eco-terrorists apparently take it seriously. We would like to convince them otherwise, but we don't know who they are. Getting the newspapers or magazines or even a TV spot about it is difficult, and there would be no guarantee that the eco-terrorists would even notice them. Now and then there is another break-in to the lab; since we don't really have anything valuable there, security is low. The terrorists are apparently searching for this element, thinking that it has some powers. When they don't find it, they leave notes accusing us of covering up the existence of this element. We came up with a strategy, inspired by the hullabaloo which is made in the press over major mathematical discoveries, even though the hullabaloo is always short-lived and the whole thing quickly forgotten – only mathematicians are likely to remember the furor when major theorems were proven, like Fermat's last theorem or the four-color theorem. But the results did have their days of glory, and perhaps so our best shot is to prove this guess. We didn't get permission to have the main ship's computers to work on it, but we do have a shuttle, the shuttle 'RH' which is rarely used, but which has a good computer. We put that computer to work on the problem. If we could present the solution of the hypothesis, it would become the talk of the whole ship, and the eco-terrorists would be sure to hear about it, and realize that the element is fictional."

"Could the eco-terrorists think that you are keeping the element in the shuttle?"

"Possibly, but we're not worried about that. The security guarding the shuttle is better than that guarding our lab."

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December 27, 5832

In the bar, the dock workers were discussing various topics. They had all been given Model Theory in primary school, although some teachers had misgivings about how well the children would understand such concepts. The worry was justified, but it did have the strange effect of having dockworkers who used the vocabulary of mathematics.

"If you have no contradictions in your theory, then you can get a model, even if you have to make it up. Now, suppose you have an infinite model...."

"Infinite model? What kind of silliness is that?"

"Well, how many points are there in your hand"

“That’s cheating: you are assuming that I can get infinitely small, so you are assuming what you want to prove.”

“OK, I’ll take another tack. If my assumption about infinity does not lead to a contradiction, then it must have a model, right?”

“Um.... OK, but I’m not convinced that it will have anything to do with the physical world. There is nothing infinite that you can measure in nature...”.”

“That you can measure. But you do want to jump from one event to another event. Infinities are what allow you to do this.”

“Fun, but why are we doing it?”

“It has something to do with a wormhole we are heading to.”

“That can’t be it. I think they want to build a wormhole.”

“Except that wouldn’t work, would it? No, I think the idea is to surround the ship with a negative energy closure in such a way that there can be a smooth transition.... and then inside the shell we can go under the speed of light, but the spaceship’s bubble would contract in front and contract in back, so that, although we would be going under the speed of light locally, we would be getting places faster than light could go in ordinary space.”

“That’s a wacky idea.”

“There’s nothing that contradicts our present knowledge of physics, and in fact is derived from a solution of formulas upon which our knowledge of the structure of the curvature of the universe is based. It’s just that to say some of the details are missing is an understatement...not to mention the huge amount of energy that it would apparently need.”

“Ah, based on equations...”

“Er, there is a caveat. These formulas have a bunch of solutions in the theory. However, we have not yet finished constructing the model which would satisfy this theory...”

“What’s the hold-up?”

“There are some other formulas about the very tiny which the model will also have to satisfy. It is possible that, once a model has been constructed, the elements of the model universe may not contain the values which would satisfy the formulas for the Alcubierre Drive,

“Albuquerque Drive? Is that in New Mexico? Ah, no, isn’t that a street in the town of Virginia Beach? Is that where the inventor got his way-out idea?”

“So way-out it isn’t. And that’s Alcubierre. He lives in Mexico.”

“What inspired him?”

“How should I know? Maybe he heard of warp drive in science fiction, associated the warp with the curvature of space, and thought about the age of the observable universe...”

“What does the age of the universe have to do with it?”

“Haven’t you ever wondered how you can get a universe of fourteen billion years to have stars which have moved in that time more than 46 light-years?”

“That is a bit strange, yes.”

“The stars didn’t move faster than light, but the substratum with respect to which they have gone less than the speed of light, has moved. A bit like you funning on a treadmill inside a truck. You don’t run so fast, but the treadmill takes you along with it.”

“This Casimir thingy...it’s a balancing act, y’see? The density of energy in space is usually at a certain level, or above, but then when you get less inside the plates, then since there is less inside, it all balances out by having less outside, and they press together. Balance, you see?”

“Huh? It’s vacuum inside and outside.”

“Yes, but a vacuum has energy, remember? It’s from these energy waves popping into existence for a while.”

“But there are only a countable number of wavelengths that it can be inside...“

“Why?”

“Ah, well, it’s like the fixed points in a guitar string. The more nodes for a given space, the more energy. You get only one, two, three .... half-wavelengths in order for there to be resonance. But they can be like that because their position is restricted. These restrictions don’t hold outside, the wavelength can be any length.”

“Uh, wait a minute. You would quickly get an infinite amount of energy that way, no?”

“Not necessarily. You can get an infinite number of possibilities, but their differences can add up to something finite. And anyway, not all of them are popping into being at the same time, so they sort of average out. But when they average out, the size of the infinity makes a difference.”

“What’s the difference? Infinity is infinity.”

Moishe spoke up, explaining that no, infinities differ.

One of the workers had sat next to Moishe and Tal in the ship’s dining room, so that some of their discussions rubbed off on him. He explained as best he could about the different infinities.

“So why are there different sizes inside and outside?”

“There is only a certain amount of space there, so that the only waves that can get in there are like the ones in guitar strings, like the counting numbers. But the waves outside can be any length , so there are more of them. So, the number of waves outside is like the real numbers, and inside like the counting numbers. Reals bigger than countables, so they push in.”

“That’s silly. The countables are insignificant compared to the reals, and so they would simply go ‘whoomp’.” He clapped his hands together.

“Whoomp?”

“Valid scientific term, that.” The speaker didn’t really think that whoomp was a scientific term, but it sounded just as good as some of the other real scientific terms that he didn’t understand.

“Maybe the elimination of the infinities gets rid of that whoomp.”

“Yea, so why do they come out with too big an amount in space? They can’t even calculate that right! Gotta depend on measuring it.”

“Maybe the amount is somewhere between the naturals and the continuum.”

“Is that possible?”

“Possible, depending on which axioms are valid for the real world.”

“Maybe you need a model of infinity where the number of possible wavelengths is bigger than the set of counting numbers, but smaller than the set of real numbers.”

“What silliness.”

“Depends on your assumptions.”

“Are you telling me that with one set of assumptions, there is nothing between the size of the counting numbers and the set of the real numbers?”

“Exactly.”

“But one set of assumptions must be more natural than the other.”

“No, not really.”

“That can’t be. Tell me the two assumptions, so that I can pick.”

“That will take a few minutes.”

“Go ahead.”

"OK, for both of them, we take the basic assumptions that one uses to construct sets without getting into paradoxes. Like, the empty set and the set of counting numbers exist, an axiom to insure against infinite regression using set membership, and axioms that say that from existing sets you can make unions, intersections, pairs, subsets by taking those elements from a given set that you can describe with your theory, the set of all subsets of a given set, and so on, that sort of thing."

"'That sort of thing'. When I studied, that was called hand-waving."

"Let him go on. He gets too pedantic as it is. Continue, kid."

"Um, kid?"

"Sounds nice. Continue."

"OK, OK. But that little word 'all' in the axiom on the power set is the main difference. 'All' means 'all the ones of the sets in the universe of the model.' So, a model can have other sets than the ones dictated by the theory, as long as their existence does not contradict any of the theory's axioms. Therefore a model can include more sets than are absolutely called for by the axioms. But if you don't do that, and only include the smallest model which will satisfy the standard axioms, then you will notice that the only axiom that allows for making an infinite set bigger is the power set axiom. That would mean that the next biggest set after the set of counting numbers would be the power set of that set. Nothing in between. Therefore, it is possible that there is nothing in between. That is one model. But there are other models, that allow one to have something in-between."

"How would you do that?"

"You start with one of those minimal models. Such a minimal model will be countable when seen from another, more powerful model, but from its own point of view, it will contain infinite sets that are bigger. For example, when we begin with the minimal model, then the power set of the set of counting numbers will be the next biggest, then take the power set of that, and you get the third on in the row, and so forth. In general, any model satisfying the basic axioms will have the infinite sets getting bigger, one after another. Let's say that you have some sort of countable model. For convenience, we will take an ordinal as the universe of the model. the set of natural numbers is named Alf, and his son is the next biggest, Alf Junior, and the grandson is Alf the Third. Then remember that a real number is a member of the power set of Alf, that is, where one chooses for each counting number whether it is in the set or not. So let's say we have a city where streets all start either along a southernmost line, or from a westernmost line. The streets going eastward are named after the members of Alf the Third, and the streets going northward are named after the counting numbers. People live at the intersections of these streets, so every address is simply the name of an Alf the Third street together with a counting number.

"OK, although there are many uninhabited intersections. On a map, someone puts an X where someone inhabits the intersection, and an O otherwise. The arrangement is such that no two north-south streets have the same pattern of X's and O's. I'll come back to that point."

He paused to see if anyone was taking notes, in case he forgot to come back to that point. No one was. He continued.

"A member of Alf Junior is just such a street: where the list of inhabitants is listed by their addresses. Since each street is different, each such street represents a different real number, then there are Alf-the-Third number of such streets. Alf-the-Third number of real numbers."

"Wait a minute. How can you boldly say that there are that many of them? Maybe you run out of them?"

"No, remember that Alf the Third, although uncountable with respect to this countable model, is with respect to a larger model uncountable."

"Maybe there is no such larger model that fits."

"You have a point. We actually have to add an assumption, an axiom, to insure that there must be other models. Remember when we were talking about probabilities that would be significant or not? To be able to do that, we need to assume that a large enough infinite set exists."

"What, bigger than the ones you can get to by the usual axioms?"

"Roughly. That is, a large cardinal is an infinite cardinal number whose existence can neither be proved nor disproved using the normal axioms, but the existence of which can, if added to the usual assumptions, help build an interesting model. So, for example, here we look to an extension of the usual probability property of probabilities that the probability of that number of sets is the same as the infinite sum of the individual probabilities. That is, suppose we are dealing with a measurement for Cathy. That is, we are measuring the subsets of Cathy. We do that using an ultrafilter which is formed from subsets of Cathy. Remember that in the ultrafilter, the intersection of two members is also a member. Well, we assume that the intersection of any number of members, as long as that number is smaller than Cathy. Then, the measure is saying that if the subset is in the ultrafilter, then the measure is one, that is, it is certain, whereas if not, then it is impossible, with probability zero. However, you can also get the results to be the probabilities between zero and one as well in doing it slightly differently. So it would depend whether you are looking at the indeterminate states, or only the determinate ones, but in both you can use an ultrafilter to talk about the probability. But you cannot prove that the set for the ultrafilter exists using the standard axioms, so you have to assume it. But if you then assume it, you can then prove that there are sets which are not constructible using only the standard axioms."

"So you are saying that there are sets that are not constructible by the usual axioms. Or, to put it another way, that there are real numbers that you cannot construct. Can you give me an example?"

"Tricky to do it explicitly, since we don't really need these numbers in ordinary practice. But, well, let's see. Suppose you took all the constructible sets of numbers, and codified them all into counting numbers. Then, the set containing all these codes could not be one of the constructible sets."

"Why not?"

"Think about the code of that set. It would have to correspond to a constructible set that existed before you started to code. That, of course, it cannot do. That set, then, is a subset of the counting numbers, so it corresponds to a real number. So, it is a real number that you cannot construct."

"Awesome. But you were going to tell me how you could make sure that each street was a different arrangement."

"Because the housing commission assigns houses one street at a time, so that each street is designed while looking at the others which have already been designed. Recursive planning, sort of. You are making sure that you end up with an ultrafilter as you go, making sure each set corresponds to the conditions for an ultrafilter. You do this delicately, so as not to upset the order of the Alf family. That is, that Alf still is the size of the set of the counting numbers, that Alf Junior is still the next biggest, and that Alf the Third is bigger than Alf Junior. Alf the Third will be the set of all the possible subsets in the model, and therefore the power set, which corresponds

to the real numbers. Hence you get a model in which there is something in between, namely Alf Junior."

"But Alf the Third need not be as big as the full set of real numbers. It could even still be countable! This is then a toy situation."

"Yes, but one can show that if there is a lower-cardinal model for a statement, then there is a model for any cardinality. So, even though I may not be able to explicitly show it, I can show that a model which allows for an intermediate between the usual real numbers and the naturals does exist.

In the opposite direction, we know that if there is a model of any cardinality, there is a countable model. So, this is another way to say that there is a countable model for the statement that there exists an uncountable model."

"That's all very well and good. But if there are more of these energy fluctuations in space than there are points in the continuum, it would still be a bigger infinity than the counting numbers, and if the interpretation is correct that there are only the countable fluctuations between the plate, then they would still be insignificant and the Whoomp effect would still make the plates collapse much faster than they are observed to do. So, while that was a nice piece of mathematics, it doesn't solve our problem in the physical world."

Chapter Seventeen

January 3, 5833

Natasha and Julia found a time in Bill's schedule of consultations at his "Heavenly consultation room". When they were finally alone, Julia remarked, "Corny name you picked."

Bill shrugged."It works. How can I help you?"

Natasha and Julia explained the situation to Bill. After a bit of discussion, they agreed that the three of them should infiltrate the sects. There were hidden cameras and microphones, but those were not enough. Of the three main religious sects, one was the Outerworld sect, the other was the Higher Consciousness sect, and finally there was Judaism. Bill was supposedly the founder of the Outerworld sect, and was even referred to as "The Founder" by its adepts. Therefore he would infiltrate that sect . Julia would infiltrate the Higher Consciousness sect, and Natasha would approach Moishe to start the process of conversion to Judaism.

Julia asked Bill whether he thought that also Moishe and Tal could be of help. Bill said no. The Higher Consciousness sect was headed by Tal, so he was not to be trusted. Bill was not sure about Moishe. Moishe had founded a new sect that he claimed was consistent with the ideas of all three sects. In so doing, Moishe had tried to be a mediator between the other two sects, saying that one could obtain the Higher Consciousness before then going through the Portal. When asked to justify his position, he quoted Genesis, Job, Deuteronomy, and whatever else would stop him from putting together a cogent argument. Natasha suggested that perhaps this was Moishe's attempt to make sure that his small Jewish congregation did not completely reject Judaism. There had already tensions that were beginning to show during Moishe's weekly meetings with the eight orphans, whom he affectionately called The Chosen Ones. The four wives turned out to be adepts of the Outerworld sect, while the husbands belonged to the Higher Consciousness sect. The legendary submissiveness of Hassidic wives apparently no longer was absolute, but Moishe's solution seemed to reduce these tensions. Nonetheless, Moishe's motives were too unclear to be trusted.

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January 5, 5833

Moishe had formulated his conciliatory tone not out of conviction, but with another plan in mind. He was still convinced that he had been kidnapped, and that the black hole meant only death. He didn't trust the wormhole. But he could then get the support of all the four orphans, as well as the two technicians Julia and Natasha, for the part of his plan that he intended to make public.

His first step was to get a number of sugar pills and capsules containing only gelatin, of various colors. He ostentatiously took these pills at every opportunity where people might notice him doing so. When asked what they were for, he mumbled something about emphysema. The true reason for this subterfuge was not shared with anyone.

A further step in his plans involved influencing the orders which went back and forth between different administrative officials. He needed authorization to head for the region between the wormhole and the black hole. The excuse would be the path of the ship that Natasha had used as a decoy by passing by at great velocity. Earlier, the data for the wormhole had indicated that it was stable. However, the decoy ship had continued its path, and eventually entered the wormhole. That would change things. The earlier data and calculations about the wormhole had indicated that it was stable. However, the entrance by the ship might have destabilized it. If it became so unstable as to make the wormhole collapse, a large amount of energy would be given off, endangering the ship. A decision had to be made whether one could still approach it. Moishe manipulated the data coming from the wormhole so as to give a slightly ambiguous picture. Namely, it appeared that although the entrance of the spaceship had made the wormhole a bit less stable, it nonetheless would not cause collapse. To be sure, though, more data needed to be gathered at closer range. The conclusion was that a shuttle could approach the area, gather more data, and return to the spaceship; if there was any evidence that the area was dangerous, the shuttle could more easily turn around and vacate the area than the huge spaceship. The crew would consist of twenty one persons. Each name was justified; the fact that eight of the names belonged to babies was something that Moishe counted on being overlooked. The list consisted of Tal, Natasha, Julia, Bill and Moishe as head of the expedition, and the four families, including their children. Moishe gathered them all together to inform them that they would be heading for the Portal and, if all went well, head off enlightened to a supermassive black hole.

The next steps in Moishe's plans were even trickier. He had obtained the design sketches for the RH shuttle, which was the one they were to take, explaining that as head of the mission, he needed to know about it. He also gained permission to gain access to the shuttle for preliminary checks.

After several hours searching through the ship's reference books, Moishe found a gas that could conceivable be accidentally synthesized from a brief burst of intense heat applied to the chemical fuel in the shuttle, and that would knock humans unconscious for a while without killing them. The same burst could then conceivably make a hole in the fuel compartment, leaking the newly synthesized gas into the living compartments of the ship. An unforeseen meteor hitting the ship in the right place would be enough to trigger this chain of events. A freak accident, but freak accidents do occur. Moishe carefully programmed the ship's records to record such a meteor hit, replacing the real event of a hidden bomb carefully placed there by Moishe. In addition, Moishe hacked into the ship's medical log to show that he was under medication; coincidentally, this medication provided a partial antidote to the effects of the gas. In reality, Moishe had obtained a full antidote that would allow him to maintain full consciousness the entire time, although he would feign unconsciousness after completing his plan but before the

others regained consciousness. But in the story that Moishe would tell afterwards, he would have passed out like everyone else, but not immediately. Moishe would explain that he had felt dizzy, and as he had seen everyone else pass out around him, had not known how long he himself would maintain consciousness. He had not known what was happening, and he had feared that there might be no chance for everyone to go through with their plan of becoming enlightened by the wormhole. However, he did know that The Founder claimed already to be enlightened, and that The Founder was a fervent Outerworlder. Therefore Moishe, his story would go, had made the snap decision to at least honor the Founder's wish, sending him to the black hole. The decision may or may not have been correct, he would apologize, but under the effects of the gas, his powers of judgment had not been at their best. Therefore, Moishe's story would continue, he had to send The Founder via the probe to the black hole. Since the others would be convinced that Moishe was acting out of religious conviction, known to justify many even more absurd things, his story would be credible.

Moishe did not believe that the black hole offered a portal to another world. His true purpose of sending Bill to the black hole was to kill Bill. Moishe could of course could simply kill Bill by throwing him out into space. But Bill would then just reincarnate. However, if Bill were to die inside a black hole, the information of Bill's death could not reach the computer in the ship, and Bill would be totally and finally dead.

The plan went flawlessly. After all the others had passed out, Moishe put Bill into a spacesuit with enough oxygen to last him until well after the event horizon of the black hole. To avoid Bill having a heart attack upon waking up in his helpless and hopeless position, Moishe gave his a heavy dose of sleeping medicine. Then he sent the probe, along with Bill, towards the supermassive black hole. Meanwhile, the ship had automatically sent a robot out to repair the fuel leak, and to refresh the air inside the living area. When everyone was again conscious, the ship's computer informed them that their remaining fuel did not give them enough time to get sufficiently close to the wormhole to take the necessary data. The ship, the computer advised, had enough fuel to return to the ship.

One of the wives quickly did a few calculations, and announced that they would have enough fuel to proceed to the black hole. This would mean that they would have to give up on their chance to become enlightened via the wormhole, but since they may not get another chance anyway at the wormhole, they might as well proceed to the Portal. The husbands were disappointed, but apparently Moishe's reconciliation efforts had had their effect, and they admitted the logic of the argument. They could work on enlightenment through the Torah in their new Outerworld home.

This was an unexpected turn of events for Moishe. He said that he did not find that a good idea, but he failed to think of an argument against it that would not reveal his hypocrisy.

Tal was not sure what to think, as he had long doubted Moishe's version of kidnapping. He had been an agnostic about the Portal, though. He urged caution, asking how much time they had before a final decision had to be reached.

Julia was more certain. She felt that avoiding certain death via the black hole overruled any other objections, and acted quickly. She reasoned that just as the black hole would block Bill's data, it would also block hers and Natasha's. She grabbed three stunners, and threw one to Natasha and one to Moishe."Natasha, aim at the wives. I'll aim at the husbands. Moishe, make sure that no one else moves. We can steer the shuttle back to safety if we act now!"

Moishe looked at the stunner."How do I put it on stun?"

Natasha grabbed the stunner, and pointed it at Julia.

“What the f...” yelled Julia, before Natasha fired at her. Julia sank to the ground.

Moishe hesitated. He understood that Natasha, for whatever reason, intended to head towards the black hole. The Commandments strictly forbad him from harming other Jews, but it said nothing about the others. Bill had not been Jewish, so he had had no qualms in killing him. But Natasha? She was not Jewish, but she was on her way. He had no idea how to adjust the firing power of his weapon, and feared that he could kill her if he fired at her. He thought of the fact that many commentators had interpreted the Commandments to say that one had to sacrifice oneself rather than violate a prohibition of the Commandments. But his self-preservation instincts were putting up a bold fight.

All this thinking took its toll on Moishe’s reaction time. Natasha made a quick decision without being hampered by these details. That did not mean that she did not consider the options. But she, like many higher animals, had developed jumps such as infinities; reasoning something out to the end could mean the difference to an animal between having lunch and being lunch. In a word, this result of evolution kicked in for Natasha faster than for Moishe, and she zapped him.

Natasha quickly convinced the others that they did not have time to come to a consensus if they wanted to continue to the black hole. They tied Julie, Tal and Moishe up, despite Tal’s protests that he had not been against the others.”Ah, but he who is not with us is against us,” quoted one of the Chosen Ones.

Tal yelled, “That’s a zero-one logic. Too classic. Reality is fuzzy.”

Another Chosen One countered with “Right, if it is fuzzy in the same sense as the quantum fuzziness, then you are unpredictable; there is a probability that you would be against us.”

Tal kept trying.”Why can’t I just be neutral? That’s another kind of fuzzy logic. Three-valued logic, for instance, where...”

“Oh, shut up. You are not needed, and we don’t know which logic you are following.”

With that, the ship headed towards the supermassive black hole.

At that moment, the shuttle informed them that it had found the solution to the RH. Sara, as a pure mathematician, felt that this portal exit should be delayed until the answer was made public. This meant aborting this portal attempt and returning to the ship. She grabbed a stun gun in order to disable the others. Her reactions illustrated another fact. There is pure mathematics, which deals with the abstract. There is applied mathematics, which is more concrete. Sara had been more interested in pure mathematics, and therefore was always very practical. She did not take off the safety, giving the others time to subdue her.

She loudly complained, “But you can’t let the RH die! Going through the portal, we will be transported, but not the shuttle!”

“What’s so great about the RH?” asked Paul, one of the four orphans.

Not being sure she could explain a complicated problem in thirty seconds, she replied,”All sorts of papers are already based on the assumption that it is true...”

“That’s weak. Try again. Why should I care? I’m in cryptography.”

“Uh, quantum computing is powerful, but there are still a lot of classical computers around that depend on the cryptography based on factoring into primes. And a lot of other things are based on primes. If you understand the RH, you can understand the primes better. Also....”

“Stick to this point, it interests me. What’s the connection between the RH and primes? I heard you describing the RH in terms of a devil’s orchestra, but I was a little tipsy, and I may have missed the connection..”

“The part of the zeta function which is finite before the extension is finite can be rewritten as a formula which involves primes.”

“What good is that?”

“It would be very useful to you if you could figure out how many primes were below a given number, wouldn’t it? That would be enough to crack most of the safety codes used today.”

“Can you show me? Let’s do a trial run. But how will I know if your answers are correct? I won’t have time to go through all your reasoning.”

“Well, you could at least be pretty sure. There is an old formula that will give a pretty good approximation”

“How approximate?”

“For that, there is another formula based on those swivel points of Beelzebub that gave him silence.”

“Why don’t we just combine the formulas? That would give an even better approximation, wouldn’t it?”

“Yes, of course we can do that. So if you allow me, I can show you.” Sara was lying; she knew that an understanding of the RH could lead to such methods, but as she had not seen the computer’s proof, she had no idea whether she would be able to derive the necessary formula. It was doubtful that she could do it quickly. But she could buy a little time, and figure out something.

“OK, enough, I guess you made your point.” He turned to the others.”Can we delay a few minutes?”

He was cut short as a warning of the shuttle to get into the pods was broadcast throughout the ship. It was too late.

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January 5, 5833

The ship’s observation deck was rather fuller than usual this afternoon, as word hat gotten around that a kidnapped shuttle was heading for a black hole. From the point of view of the spectators on the ship of this grim show, the shuttle never successfully entered the event horizon. The spectators were a mixture of those who understood, and those who didn’t. Even the captain was there. It is said that death levels all, and this was true even when death had become a spectacle. The cook turned to the captain and asked,

“What is happening? Are they going to stretch this out forever?”

“From our interpretation, we will not see them entering, but rather their information slowly being spread out onto the event horizon. They are, after all, information...”

“What? Are you one of the Outerworlders, Captain?”

“No, not at all. The laws of physics can be restated in terms of information and its interpretations. Here, for example, it depends on the model that is being used. The idea is that information should not be lost. Fine. But the event horizon splits our universe into two possible worlds with only a very thin accessibility relationship between them.”

“But you are in this model; how can you talk about their model?”

“We look at the models from a more powerful theory, with a larger universe. For example, when you think of a geometry, what do you get when you have two lines which are both perpendicular to a third line?”

“Two parallel lines that never meet, of course.”

“Not so much ‘of course’. You are automatically thinking in two dimensions. But what is the solution to that matchstick puzzle, you know, where you have to make four triangles with six matchsticks, without breaking them?”

“You’re saying that you never told me to keep to two dimensions. So, for example, the longitudes on the Earth are parallel, yet meet. To envision this, I can either make a complicated geometry, or introduce a third dimension. With the third dimension, I can look at a couple of different kinds of two-dimensional spaces, whereby each one of them would contradict the other if we put them both into the same two-dimensional space, but they can both exist without a problem in the three-dimensional case.”

“What is all this getting to?”

“Ah, I almost forgot. I do get off on tangents. We can have two theories which seem to contradict one another , yet each one is valid, as each one has a model which satisfies it.”

“I’ve heard of that. Someone pointed out that the theory of two-dimensional geometry that we learn in school is contradicted by the theory that one uses for airline pilots on the surface of the earth, but that both are valid.”

“Yes, if you’ve noticed, we haven’t been using school geometry when we have been talking about a lot of these cosmic phenomena. But that is not my point. Here we have two situations. On one side, we have the information representing our friends stays on this side of the event horizon, albeit too jumbled to allow for life; this is in the model that we share. On the other hand, the information representing our friends goes into the other side of the event horizon, with them still alive, at least until they hit the center. Somewhere around a minute, their time.”

“You mean their mass-energy would exist on both sides of the event horizon at the same time?”

“Um, the two are linked. Again, we have the two interpretations, but there is that small accessibility relation I told you about. Not enough for us to peer inside the black hole, or them to escape, but at least to link their two-dimensional existence in one model to their three-dimensional existence in the other model. A bit like a hologram, which is two-dimensional from one point of view and three-dimensional from another. But in any case, I’m afraid that all this will not change the fact that we should hold a funeral service for them.”

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November 15, 2045

Most of the crew of the RH awoke, back in reality. Moishe and Tal were the first to return, and the eight parents came next. Finally, Natasha woke up.

The technicians and nurses soothed everyone, without understanding their questions.”What happened? Why aren’t we dead?” asked Julia, Tal and Moishe.

Some of them asked, “This is the Outerworld?”

“Where’s Natasha?” asked Julie.

Two questions the nurses did understand, but could not answer, were, “Where is Mr. Wheaton?” and “Where’s Bill?”

There was one question that the nurses quickly were able to figure out. The nurses gave the task to imparting the sad answer to the group of psychologists who were the first step for the orphans’ reentry into the real world. The explanation that the children had not entered and therefore, as virtual characters, could not exit, met will a total lack of comprehension by the parents. However, before the parents could arrange a proper ceremony for their dead children, one of the technicians yelled, “The baby bodies! Look!”

Everyone rushed over to see the bodies, which were still on-line for, as the nurses said, “spare parts”, started moving. And crying.

There was confusion as the doctors checked the health of the new arrivals. It was decided that the baby bodies of the eight orphans, now adults, would be given the minds of their children

born in Paralife. Identity problems could be sorted out later. Since the new bodies had been grown from DNA samples of the old bodies, any future gene tests would confirm the kinship.

The parents had been overjoyed to see their babies; they overlooked the fact that the babies had the physical features of only one parent. They were not taught much genetics; in fact, only Sara had managed to break out of Moishe's restrictive academic program.

Moishe and Tal at first were still under the impression that they had been kidnapped and given plastic surgery. They began to doubt this version as they were informed that they could return home as soon as a few formalities were cleared up, and as they became familiar with the extent that their new bodies differed from the old ones. Both objected to the fact that their new bodies had not been circumcised. The doctors told them this would be easy, if slightly painful, to fix. However, Moishe and Tal would have to stay on the island for pre- and postoperative observation for the coming week.

Bill had set aside a special stipendium for the orphans' education when he believed that they would be coming right back from Paralife. This money would help the new parents establish a new life, although they would still need to find work. Jutta offered to find them work on the island; once they had a bit more retraining and a few forged documents, they could decide whether to return to Israel.

After a week, Sara finally got the courage to ask a question that had been bothering her, but had not been brought up as the issue of the loss of the children had taken precedence.

"Was the RH Shuttle saved? The proof of the Riemann Hypothesis?"

The blank looks of the technicians gave her the answer. The only good news there, Sara thought, was that at least it is a solvable problem, not one of those nasty undecidable ones. That is, if the RV Serendipity RH Shuttle, now forever out of reach of peer review, had not made a mistake.

As time went on and there was still no signal from Paralife to restore Bill's mind to a real body, Moishe was secretly very satisfied. It appears that he had finally been successful in ridding the world of the last Amalekite.

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November 29, 2045

Moishe and Tal, having booked their tickets home, were about to board the ferry to Tahiti. They was stopped by a police officer.

"What is it?"

"There is a legal order to detain both of you."

Moishe was taken aback. He looked at Tal; Tal shook his head. But who else? Wheaton and the girl Natasha had died, and no one else was on the ship.

When he got back, Bill, Julia and Natasha all stood there. At first Moishe did not recognize Bill, due to his new body. After the introductions passed, Moishe and Tal kept a cautious silence. Natasha was given the task to explain the situation, since she was the main agent in this part of the events.

Natasha, before diving into Paralife, had found it probable that the group that needed saving would end up diving into the black hole. She made a few tweaks that would not disrupt the program but would increase the probability of this happening. The universe as it was set up in Paralife did indeed finish at the event horizon; she extended the universe to Paralife-2, which included Paralife, but also the interior of the supermassive black hole. In the extension, within a few seconds of entering the event horizon, the people would exist in the two copies, one copy

spread out on the event horizon, and the other one inside. The information from the inside copy then would be transferred to several computers to be then fed back to the original computer.

Natasha was very much interested in getting the four children back ... out of humanitarian grounds, she told everyone, but in secret she was mainly interested in following up the progress of the new children to see, if her techniques had damaged their viability. To get the children back, she did some more tweaking, so that the Paralife computer did not need a perfect match of the DNA. About half of it, including the sex chromosomes, was all that was required. The four babies were then perfectly matched to the four baby bodies still on life-support, but because of this extra subprogram that Natasha had installed, it had taken a bit longer.

When Natasha had found out that Julia had entered the program, she resolved to go into Paralife herself. Of course she was concerned for Julia as well as for the success of her plan, but she also knew that only by undergoing this brain-in-a-vat experience would she know for certain that it was possible for a computer subprogram to have consciousness. For this, she set up her own life support system. It had been meant for long space voyages, but she lived not far from Russia's Far Eastern Spaceport, where she had, um, friends. She did not give more details about this.

She explained further that the return of Bill's mind was problematic. Due to his age, his brain was not quite as plastic as that of the younger participants, but that was not a major hurdle. It did lead, however, to a slightly longer examination time of his mind. This had been found to be in a lamentable state due to the high dose of the drug which Moishe had given him. If his mind had been sent immediately back to his body, it would have risked permanent damage. Therefore the computers had sent him back and forth in a process to insure the stability of his mind before finally sending him back. He had awoken soon after Moishe and Tal had headed for the airport, and Natasha had arrived in a flight that morning. Both had given damaging testimony concerning the behavior of Mr. Cohen.

The accompanying police officer interjected, "However, you and Mr. Kishon were being held on other charges."

Moishe did not let the officer finish. He countered, "That was all in a simulated reality. There is nothing in the law that will make me responsible for the action of an avatar. In fact, you will have difficulty convincing a jury that we, that is our avatars, were conscious in there, or..."

"We know. We are not pressing charges for attempted murder, although we would like to. But we can press you on willful destruction of private property. You broke into the computer room to try your telepathy trick, remember?"

"Breaking and entering? Oh, a misdemeanor."

"You were to do something that could have, and indeed did, endanger the lives of several people. That's breaking and entering with intent to commit a felony. Anyway, remember that you are in a country where the laws are a bit different. We might even be able to make the attempted murder charge stick."

Tal was taken aback."You aren't serious?"

"Very serious. So we are going to explain what we would like you to do. We realize that we won't have the same leverage once you get out of our jurisdiction, which is the same problem we have with Chaim and Daniela Reed. And in fact, we shouldn't really care if the Reeds want to try again in New York. But a criminal investigation might lead to us, and we would prefer to remain out of the public eye until the procedures here have been accepted. Therefore we would like to kill several birds with one stone, if you'll pardon the image. Once you get back to New York,

you will find that much of your staff has quit, and some advertisers have canceled their contracts, mostly in protest at the change of your editorial policy."

"What new editorial policy? I haven't sent any communication...."

"Hack, hack. Who's there?" Julia said, grinning.

The technician continued."But you will also find a set of candidates for your new staff on the web site, just waiting to have dates for their interviews set. As well, there are some new advertisers whose businesses are connected in some way to your new editorial policy."

"You haven't told me what this so-called new editorial policy is."

"True. You can continue to aim for an audience among the religious. However, as you can imagine, many of the objections to the world view of your individuality will come from deep-rooted religious feelings, ideas of the soul, and so forth. It will take your skill to publish articles which will lead your readers to eventually accept the sort of thing we are doing here... without, of course, mentioning that it is a reality. Yet."

"And if I don't agree?"

Natasha waved at him, and said, "Hack, hack, who's there?"

Bill turned to Moishe."Finally, Rabbi Cohen. We understood that your intention is to kill me."

"You have no proof..."

"We are not even going to try to press charges or put pressure on you in any way. I just wish to point out one fact you overlooked when you formed your intention. I myself did not know this fact until now. My parents, you see, hired the best computer hackers to cover it up."

"Not very well. It was clear that they were Amalekite."

"Correct. They were. I am not. I was adopted."

Moishe paused. All he found to say was, "Oops".

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Thanks for reading!

About the author

As a mathematics and physics teacher who has been wandering, studying and working throughout all the habitable continents most of his adult life, David Reid has been learning about the wide variability of mentalities among different cultures and ages, and in turn brought about both the necessity to explain complex concepts to lay audiences, as well as his enjoyment in doing so.

This is the author's first book. If you feel that you learned something from this book, or even just possibly enjoyed parts of it, then he would highly appreciate a positive review on his Smashwords author site. You will find the link to the site directly below the choices you were given for the format in which you wished to download the book. If you wish to contact him directly for references or keywords, he will be glad to hear from you via email: reidnomad [at] gmail [dot] com.