Math Is Fun

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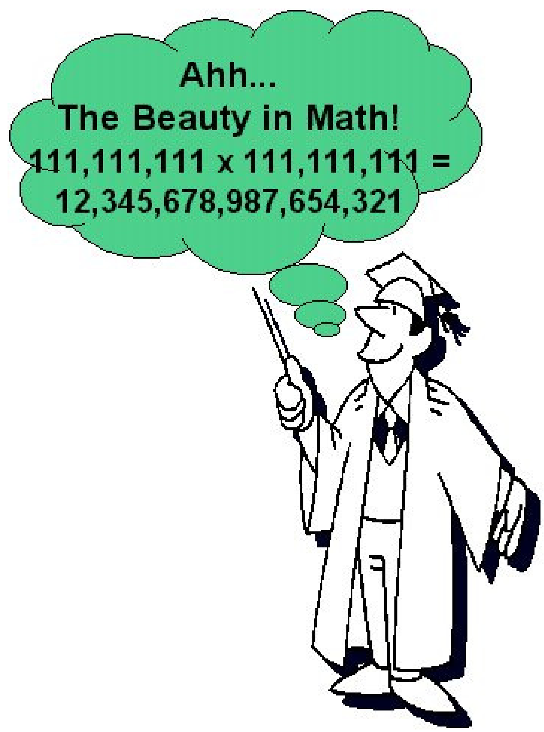

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Professor of Mathematics
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Professor of Mathematics
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Math Is Fun

Preface

We are told by a Bach aficionado that Johann Sebastian Bach prefaced his famous Well-Tempered Clavier work, the first difficult set of 24 preludes and fugues for keyboard, in a most modest way in 1722:
"For the use and profit of young musicians who are eager to learn, as well as for the entertainment of those who are already expert in the art."
Math Is Fun was written for the express purpose of encouraging students to explore more and more difficult math problems, which can be encountered in everyday life. Math is not an easy subject, yet it is our duty as teachers to make it seem easy and fun. To paraphrase Bach, Math Is Fun is for the use and profit of young students who are eager to learn, as well as for the entertainment of mathematics instructors in teaching the subject.
Every mathematics teacher would like to hear students graduating from high school College & Career Ready, with the world wide open before them, say to each other: "Math was challenging, but I had fun with it." That is the purpose of this booklet — make math fun.
This booklet contains a selected number of the fascinating problems, puzzles, we have compiled and created over the years, with lots of sweat, lots of tears, lots of love, and maybe even a little blood.
We hope you and your friends will enjoy working with these tantalizing motivators. This collection of MATH Tid-Bits has contributed to our success in teaching.

Vik Hovsepian - (...graduate of UCLA, taught grades 3 to College ... winner of numerous awards ➠ 1st Place Winner-Jaime Escalante Math Award - 1994, Finalist -Presidential Award of Excellence in Mathematics -1997, All-USA 1st Teacher Team - 1998). ✪ Former State of California Curriculum Commissioner ✪ One of the writers of the current k-12 California Mathematics Framework ✪ *Presently*: Professor of Mathematics @ Rio Hondo College ✪ Sr. National Math Consultant/Author for McGraw-Hill Education ✪ Math Content Review Panelist for the State of California ✪ California Standards Test Panelist ✪ Membership: NCTM and CMC.

Michael Hattar - (...taught mathematics grades 8 to College ... winner of numerous awards ➠ Finalist-Presidential Award of Excellence in Mathematics -1991, Teacher of the year-San Bernardino County-1995, Life-Time Achievement Award-Don Bosco Technical Institute -1998). ✪ *Presently*, Professor of Mathematics @ Mount San Antonio College and Rio Hondo College ✪ Presented Math Is Fun all over the world ✪ Membership: NCTM and CMC.

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Math Background

At A Glance

Appendix A

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- **Disclaimer: This Appendix, is by no means an exhaustive presentation. It is only intended for a quick glance at history of mathematics.**

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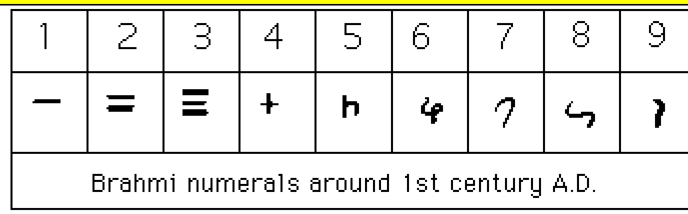
Earliest Uses of Various Mathematical Symbols

<table>
<tr><th>Date</th><th>Contribution</th></tr>
<tr><td>150 B.C.</td><td>Brahmi Numerals – Origin of Hindu – Arabic Numerals

</td></tr>
<tr><td>825 A.D.</td><td>Base 10 – numeration system – Arabia</td></tr>
<tr><td>1323-1382</td><td>Plus (+) and minus (-). Nicole d' Oresme may have used a figure which looks like a plus symbol as an abbreviation for the Latin et (meaning "and") in Algorismus proportionum, believed to have been written between 1356 and 1361. The symbol appears in a manuscript of this work believed to have been written in the fourteenth century, but perhaps by a copyist and not Nicole d' Oresme himself.

The plus symbol as an abbreviation for the Latin et, though appearing with the downward stroke not quite vertical, was found in a manuscript dated 1417.</td></tr>
<tr><td>1479</td><td>Hindu – Arabic digits as wee see them now
<table>
<tr><td>Arabic Numerals</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Arabic-Indic</td><td>٠</td><td>١</td><td>٢</td><td>٣</td><td>٤</td><td>٥</td><td>٦</td><td>٧</td><td>٨</td><td>٩</td></tr>
<tr><td>Eastern Arabic-Indic
(Persian and Urdu)</td><td>۰</td><td>۱</td><td>۲</td><td>۳</td><td>۴</td><td>۵</td><td>۶</td><td>۷</td><td>۸</td><td>۹</td></tr>
</table>
The Arabic numeral system is considered one of the most significant developments in mathematics. Most historians agree that it was first conceived of in India (particularly as Arabs themselves call the numerals they use “Indian numerals”, أرقام هندية, arqam hindiyyah), and was then transmitted to the Islamic world and thence, via North Africa and Spain, to Europe.

Somewhat speculatively, the origin of a base-10 positional number system used in India can possibly be traced to China. Because the Chinese Hua Ma system (see Chinese numerals) is also a positional base-10 system, Hua Ma numerals—or some numeral system similar to it—may have been the inspiration for the base-10 positional numeral system that evolved in India. This hypothesis is made stronger by the fact that years from 400 to 700, during which a positional base-10 system emerged in India, were also the period during which the number of Buddhist pilgrims traveling between China and India peaked. What is certain is that by the time of Bhaskara I (i.e., the seventh century AD) a base 10 numeral system with 9 glyphs was being used in India, and the concept of zero (represented by a dot) was known (see the Vāsavadattā of Subandhu, or the definition by Brahmagupta).

This numeral system had reached the Middle East by 670. Muslim mathematicians working in what is now Iraq, such as Al-Khwarizmi, were already familiar with the Babylonian numeral system, which used the zero digit between nonzero digits (although not after nonzero digits), so the more general system would not have been a difficult step. In the tenth century AD, Arab mathematicians extended the decimal numeral system to include fractions, as recorded in a treatise by Abu'l-Hasan al-Uqlidisi in 952-3.</td></tr>
</table>

Date	Contribution
1489	+ and – was introduced by **Johann Winmann**
1491	**Filippo Calandri** introduces the long division algorithm we use today.
1525	√ was introduced by **Christoff Rudolff**. Rudolff's book *Coss*, written in 1525, is the first German algebra book. The reason for the title is that cosa is *a thing* which was used for the unknown. Algebraists were called *cossists*, and algebra the *cossic art*, for many years. Rudolff calculated with polynomials with rational and irrational coefficients and was aware that $ax^2 + b = cx$ has 2 roots. He used √ for square roots (the first to use this notation) and ∛ for cube roots and ∜ for 4 th roots. He has the idea that $x^0 = 1$ which is important.
1545	Imaginary numbers were introduced by **Girolamo Gardano**.
1550 ⇩ **1593**	**Parentheses.** Parentheses ( ) are "found in rare instances as early as the sixteenth century" . 1556 **Brackets.** Brackets [ ] are found in the manuscript edition of *Algebra* by Rafael Bombelli. 1550 **Braces.** Braces { } are found in the 1593 edition of Francois Vieta's *Zetetica*
1557	= was used by **Robert Recorde**. Inventor of the equals sign and the “father of British mathematics”.
1583	**Christopher Clavins** used a dot for multiplication.
1591	**Francois Vieté** used letters to represent unknowns (variables).
1613	**William Oughtred** used the symbol ***X*** for multiplication.
1614	**John Napier** invented logarithms. John Napier would most commonly have been written Jhone Neper at that time. The only form of Napier that we are sure would not have been used in Napier's lifetime was the present modern spelling "Napier"!
1617	**John Napier** used the decimal point as we use it today.
1631	**Thomas Harriot** introduced the inequality symbols < and >. **Less than and greater than.** The symbols < and > first appear in The Analytical Arts Applied to Solving Algebraic Equations by Thomas Harriot (1560-1621), which was published posthumously in 1631: "Signum majoritatis ut a > b significet a majorem quam b" and "Signum minoritatis ut a < b significet a minorem quam b." While Harriot was surveying North America, he saw a native American with this symbol on his arm: . It is likely he developed the two symbols from this symbol. Harriot himself did not use the symbols which appear in the work, which was published after his death. He died of a cancerous ulcer of the left nostril, the 1st recorded case of death due to the use of tobacco.

Date	Contribution
1655	**John Wallis** introduced negative exponents and the symbol ∞ for infinity. In *Treatise on Algebra* Wallis accepts negative roots and complex roots. He shows that $a^3 - 7a = 6$ has exactly three roots and that they are all real. He also criticizes Descartes Rule of Signs stating, quite correctly, that the rule which determines the number of positive and the number of negative roots by inspection, is only valid if all the roots of the equation are real.
1659	**Johann Rahn** used the symbol $\div$ for division. He was the first to use the symbol $\div$ for division in his algebra book published in 1659. Rahn's book was written in German.
1706	**William Jones** used π to represent c/d. π became standard only when Euler used it in 1763.
1719 	**James Hodder** – The first math book printed in America. According to David Eugene Smith's "History of Mathematics" Vol.2, "...our earliest native American arithmetic, the Greenwood book of 1729,..." and "...the first in what is now the United States was a reprint of Hodder's English arithmetic, Boston, 1719." The full title of Hodder's book was *Arithmetick: or, That Necessary Art Made Most Easy*. The American Printing was a copy of the 25th edition in England.
1729	**Euler** - He was also the first to use the letter *e* for it in 1727 (the fact that it is the first letter of his surname is coincidental). As a result, sometimes *e* is called the Euler Number, the Eulerian Number, or Napier's Constant (but not Euler's Constant).
1729 	**Isaac Greenwood** - In 1729 the first arithmetic book published by a Native of Colonial America was published by Isaac Greenwood, a professor at Harvard. The title was Arithmetic, Vulgar (common) and decimal. Only one year later the second book published by a colonial native was published.
1734	**Less than or equal to, greater than or equal to. Pierre Bouguer** (1698-1758) used $\leqq$ and $\geqq$ in 1734. In **1670**, **John Wallis** used similar symbols each with a single horizontal bar, but the bar was above the < and > rather than below it.
1749	**Leonhard Euler** used i for $\sqrt{-1}$, *e* for the base of natural logarithms, and $\sum$ for summations. Euler was one the leading mathematicians of the 18th century. Although the majority of his work was in pure mathematics, he contributed to other disciplines, such as astronomy and physics, as well. In his lifetime he published more than 500 books and papers, and another 400 were published posthumously.

Date	Contribution
1832	**Carl F. Gauss** used the term “complex number”. In 1795 Gauss left Brunswick to study at Göttingen University. Gauss's teacher there was Kästner, whom Gauss often ridiculed. His only known friend amongst the students was Farkas Bolyai. They met in 1799 and corresponded with each other for many years. Gauss left Göttingen in 1798 without a diploma, but by this time he had made one of his most important discoveries - the construction of a regular 17-gon by ruler and compasses This was the most major advance in this field since the time of Greek mathematics and was published as Section VII of Gauss's famous work, *Disquisitiones Arithmeticae*.
1841	**Karl Weierstrass** introduced the absolute value symbol $\|n\|$. In his 1863/64 course on *The general theory of analytic functions* Weierstrass began to formulate his theory of the real numbers. In his 1863 lectures he proved that the complex numbers are the only commutative algebraic extension of the real numbers. He attended the University of Bonn to learn public administration, but he found that his passion was for *mathematics*. He read Laplace, Legendre, Jacobi, and Abel. He taught for 14 years at the secondary school level before he published 2 brilliant papers and received an offer to teach at the university level. He became a professor of mathematics at the University of Berlin. In his work to put mathematical analysis on a sound logical foundation, Weierstrass developed the modern definitions of limit (delta-epsilon) and continuity.

Page from al-Khwarizmi's Kitab al-Jabr wal-Muqabala, the oldest Arabic work on algebra
9th century

More math BACKGROUND

Math stuff	Explanation
e	***e* = 2.718281828… the Base of Natural Logarithms** ***e*** occurs almost as often in mathematics as π. Here is a way to remember ***e*** with class! President Andrew Jackson, who held office **2** times, was our **7th** president and was elected in **1828**. So **2.718281828** could be associated to Andrew Jackson's presidency. Fun!!! *e* is a real number constant that appears in mathematics problems. Examples of such problems are those involving growth or decay (including compound interest), the statistical "bell curve," the shape of a hanging cable (or the Gateway Arch in St. Louis), some problems of probability, some counting problems, and even the study of the distribution of prime numbers. It appears in Stirling's Formula for approximating factorials. It also shows up in calculus quite often, wherever you are dealing with either logarithmic or exponential functions. There is also a connection between *e* and complex numbers, via Euler's Equation. *e* is usually defined by the following equation: $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. Its value is approximately **2.718281828**... and has been calculated to 869,894,101 decimal places by Sebastian Wedeniwski (you'll find the first 50 digits in a Table of constants with 50 decimal places, from the Numbers, constants and computation site, by Xavier Gourdon and Pascal Sebah). The number *e* was first studied by the Swiss mathematician Leonhard Euler in the 1720s, although its existence was more or less implied in the work of John Napier. An effective way to calculate the value of *e* is not to use the defining equation above, but to use the following infinite sum: $e = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots$
$\sqrt[2]{3}$	**1.732 050 … Here is a way to remember $\sqrt[2]{3}$! President George Washington was born in 1732.**

A little Bit of Fermat

Fermat was not able to show that 4,294,967,297 was not a prime number. Euler later discovered that it is divisible by 641. In the 1800s there was a young American named [1]Warren Colburn who had a great capacity to do things in his head. He was shown this number and asked if it was prime. He said, "no, because 641 divides it." When asked how he knew that, he replied, "Oh, just felt it." He was unable to explain this great gift – he just had it.

[1] **Colburn, Warren** (1793-1833), American mathematician and author, capable of lecturing on mathematics and physics in simple, everyday language. Colburn...

The Chinese - Contributions To Mathematics

Several factors led to the development of mathematics in China being, for a long period, independent of developments in other civilizations. The geographical nature of the country meant that there were natural boundaries (mountains and seas) which isolated it. On the other hand, when the country was conquered by foreign invaders, they were assimilated into the Chinese culture rather than changing the culture to their own. As a consequence there was a continuous cultural development in China from around 1000 BC and it is fascinating to trace mathematical development within that culture. There are periods of rapid advance, periods when a certain level was maintained, and periods of decline.

The first thing to understand about ancient Chinese mathematics is the way in which it differs from Greek mathematics. Unlike Greek mathematics there is no axiomatic development of mathematics. The Chinese concept of mathematical proof is radically different from that of the Greeks, yet one must not in any sense think less of it because of this. Rather one must marvel at the Chinese approach to mathematics and the results to which it led.

Chinese mathematics was, like their language, very concise. It was very much problem based, motivated by problems of the calendar, trade, land measurement, architecture, government records and taxes. By the fourth century BC counting boards were used for calculating, which effectively meant that a decimal place valued number system was in use. It is worth noting that counting boards are uniquely Chinese, and do not appear to have been used by any other civilization.

Our knowledge of Chinese mathematics before 100 BC is very sketchy although in 1984 the *Suan shu shu* (A Book on Arithmetic) dating from around 180 BC was discovered. It is a book written on bamboo strips and was found near Jiangling in Hubei province. The next important books of which we have records are a sixteen chapter work *Suanshu* (Computational prescriptions) written by Du Zhong and a twenty-six chapter work *Xu Shang suanshu* (Computational prescriptions of Xu Shang) written by Xu Shang. Neither of these texts has survived and little is known of their content. The oldest complete surviving text is the *Zhoubi suanjing* (Zhou Shadow Gauge Manual) which was compiled between 100 BC and 100 AD. It is an astronomy text, showing how to measure the positions of the heavenly bodies using shadow gauges which are also called gnomons, but it contains important sections on mathematics. It gives a clear statement on the nature of Chinese mathematics in this period.

The *Zhoubi suanjing* contains a statement of the Gougu rule (the Chinese version of Pythagoras's theorem) and applies it to surveying, astronomy, and other topics. It is widely accepted that the work also contains a proof of Pythagoras's theorem.

In fact much Chinese mathematics from this period was produced because of the need to make calculations for constructing the calendar and predicting positions of the heavenly bodies. The Chinese word 'chouren' refers to both mathematicians and astronomers showing the close link between the two areas.

The Arabs - Contributions To Mathematics

Recent research paints a new picture of the debt that we owe to Arabic/Islamic mathematics. Certainly many of the ideas which were previously thought to have been brilliant new conceptions due to European mathematicians of the sixteenth, seventeenth and eighteenth centuries are now known to have been developed by Arabic/Islamic mathematicians around four centuries earlier. In many respects the mathematics studied today is far closer in style to that of the Arabic/Islamic contribution than to that of the Greeks.

There is a widely held view that, after a brilliant period for mathematics when the Greeks laid the foundations for modern mathematics, there was a period of stagnation before the Europeans took over where the Greeks left off at the beginning of the sixteenth century. The common perception of the period of 1000 years or so between the ancient Greeks and the European Renaissance is that little happened in the world of mathematics except that some Arabic translations of Greek texts were made which preserved the Greek learning so that it was available to the Europeans at the beginning of the sixteenth century.

... Arabic science only reproduced the teachings received from Greek science.

The regions from which the "Arab mathematicians" came was centered on Iran/Iraq but varied with military conquest during the period. At its greatest extent it stretched to the west through Turkey and North Africa to include most of Spain, and to the east as far as the borders of China.

The background to the mathematical developments which began in Baghdad around 800 is not well understood. Certainly there was an important influence which came from the Hindu mathematicians whose earlier development of the decimal system and numerals was important. There began a remarkable period of mathematical progress with al-Khwarizmi's work and the translations of Greek texts.

This period begins under the Caliph Harun al-Rashid, the fifth Caliph of the Abbasid dynasty, whose reign began in 786. He encouraged scholarship and the first translations of Greek texts into Arabic, such as Euclid's *Elements* by al-Hajjaj, were made during al-Rashid's reign. The next Caliph, al-Ma'mun, encouraged learning even more strongly than his father al-Rashid, and he set up the House of Wisdom in Baghdad which became the centre for both the work of translating and of research. Al-Kindi (born 801) and the three Banu Musa brothers worked there, as did the famous translator Hunayn ibn Ishaq.

We should emphasize that the translations into Arabic at this time were made by scientists and mathematicians such as those named above, not by language experts ignorant of mathematics, and the need for the translations was stimulated by the most advanced research of the time. It is important to realize that the translating was not done for its own sake, but was done as part of the current research effort.

Note: Saudi Arabia/2008 - McGraw-Hill Education and Obeikan Education
First Arab Country that has translated and localized a full K-12 United States math and science programs. These programs are fully utilized by the entire Kingdom of Saudi Arabia, Qatar and Bahrain.

Math Is Fun

Appendix B

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Interesting & Bazaar Mathematics

I. Distributing The Wealth

A rich Donkey Trader from the famous town of Glendale, dies and leaves 17 precious white donkeys to be divided amongst his three sons. The only condition was that the division be carried out as follows:

- Eldest son to have two-thirds of the donkeys
- Second son to have one-sixth of the donkeys
- Third son to have only one-ninth.

The sons tried all sorts of calculations, but there was no way it would work out. They were almost at the point of fighting, or selling the donkeys and dividing the money, when the eldest said "Let's ask Professor Hovsepian, he knows a thing or two."

What did Professor Hovsepian suggest?

Solution:

Professor Hovsepian offered the boys the loan of his beautiful black donkey, and told them to try the division again.

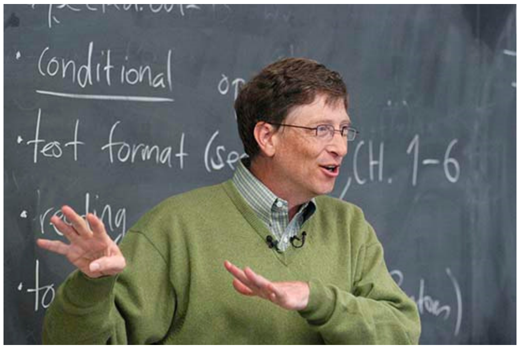
- Starting with **18** donkeys, two-thirds = **12** donkeys, which the eldest took.
- One-sixth = **3** donkeys, which the second son took.
- One-ninth = **2** donkeys, which the third son took.

Total number of donkeys distributed, 12 + 3 + 2 = 17 donkeys.

Professor Hovsepian then re-claimed his black donkey - proving that he did indeed 'know a thing or two'.

Of course, in this scenario, the eldest got the better end of the deal and the youngest lost out. Rounding would have given 11.333 or 11 donkeys to the eldest, 2.8333 or 3 to the middle son and 1.8888 or 2 donkeys to the youngest son. But remember! Better to have live donkeys than dead ones.

II. Rio Hondo's Board of Trustees invites Bill Gates to a Math Class



You're sitting in Prof Hovsepian's math class, engaged in a mathematical application, when in walks Philanthropist Bill Gates, Superintendent Dr. Ted Martinez and other dignitaries.

Philanthropist, Bill Gates has made it big, and now he has a job offer for two Rio Hondo students.

He doesn't give too many details, mumbles something in to Professor Hovsepian about the possibility of travel to Europe all expenses paid plus matching funds to Rio Hondo College. He's going to need two students who excel in mathematics for 30 days, and they have to miss school. Won't that just be too awful? But do you ever sit up at the next thing the billionaire says.

You'll have your choice of **two** payment options. Decision must be made in writing to Prof. Hovsepian, using mathematical analysis to maximize your **option earnings**. Immediately after Bill Gates completes his presentation you have 10 minutes to turn in your analysis. No collaboration allowed!

Option One: Bill Gates will pay you one cent on the first day, two cents on the second day, and double your salary every day thereafter for the thirty days; or

Option Two: Bill Gates will pay each student exactly $1,000,000. That's one million US dollars! + another $1,000,000 for Rio Hondo College.

Two students, Derek Ellis and Neil Tiwari were chosen. Both turned in detailed analysis and their choices were OPTION ONE. They packed their bags and left with Mr. Gate's staff on their mission.

So how smart were these students? Did they make the best choice? Why did they not choose option two? Were these students taught standards-based mathematics? What went wrong?

Solution: Break in Story

Let's investigate the first payment option the way any math teacher would encourage you to do. Complete a table like this for the first week's work.

Pay with First Option - Week 1		
Day No.	**Pay for that Day**	**Total Pay (In Dollars)**
1	.01	.01
2	.02	.03
3	.04	.07
4	.08	.15
5	.16	.31
6	.32	.63
7	.64	1.27

So you've worked a whole week and only made $1.27. That's pretty awful, all right. There's no way to make a million in a month at this rate. Right? Let's check out the second week. Fill out the second table.

Pay with First Option - Week 2		
Day No.	**Pay for that Day**	**Total Pay (In Dollars)**
8	1.28	2.55
9	2.56	5.11
10	5.12	10.23
11	10.24	20.47
12	20.48	40.95
13	40.96	81.91
14	81.92	163.83

Well, each will make a little more the second week, at least he's over $100. But there's still a big difference between $163.83 and $1,000,000. Want to see the third week?

Pay with First Option - Week 3		
Day No.	**Pay for that Day**	**Total Pay (In Dollars)**
15	**163.84**	**327.67**
16	**327.68**	**655.35**
17	**655.36**	**1310.71**
18	**1 310.72**	**2 621.43**
19	**2 621.44**	**5 242.87**
20	**5 242.88**	**10 485.75**
21	**10 485.76**	**20 971.51**

We're getting into some serious money here now, over $20,000, but still nowhere even close to a million. And there's only 10 days left. So it looks like the million dollars is the best deal. Of course, we suspected that all along.

Pay with First Option - Week 4		
Day No.	**Pay for that Day**	**Total Pay (In Dollars)**
22	**20 971.51**	**41 943.03**
23	**41 943.04**	**83 886.07**
24	**83 886.08**	**167 772.15**
25	**167 772.16**	**335 544.31**
26	**335 544.32**	**671 088.63**
27	**671 088.64**	**1 342 177.27**
28	**1 342 177.28**	**2 684 354.55**

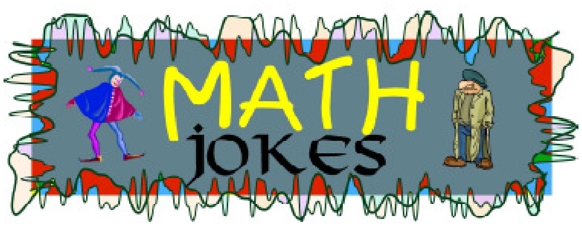
Hold it! Look what has happened. What's going on here? We went from $21 000 to over a million in 6 days. This can't be right. Let me check the calculations. No, I can't find any mistakes.

This is amazing. Look how fast this pay is growing. Let's keep going. I can't wait to see what the total will be.

Pay with First Option		
Day No.	**Pay for that Day**	**Total Pay (In Dollars)**
29	**2 684 354.56**	**5 368 709.11**
30	**5 368 709.12**	**10 737 418.23**

In 30 days, it increases from 1 penny to over 10 million dollars. That is absolutely amazing.

Moral Of The Story: Attend *Rio Hondo College*, they have an excellent Math Program.



The math professor’s six-year-old son knocks at the door of his father’s study. “Daddy,” he says. “I need help with a math problem I couldn’t do at school.” “Sure,” the father says and smiles. “Just tell me what’s bothering you.”
“Well, it’s a really hard problem: There are four ducks swimming in a pond when two more ducks come and join them. How many ducks are now swimming in the pond?” The professor stares at his son in disbelief. “You couldn’t do that?! All you need to know is that 4 + 2 = 6!”
“Do you think, I’m stupid?! Of course I know that 4 + 2 = 6. But what does this have to do with ducks!?

II. Very Very Interesting

1. If you multiply 1089×9 you get 9801. It's **<u>reversed</u>** itself! This also works with ☛ $10989 \times 9 = 98901$; ☛ $109989 \times 9 = 989901$; ☛ $1099989 \times 9 = 9899901$ and so on.

2. $19 = 1 \times 9 + 1 + 9$; $29 = 2 \times 9 + 2 + 9$.
   This also works for 39, 49, 59, 69, 79, 89 and 99.

3. **153**, **370**, **371** and **407** are all the "sum of the cubes of their digits".
   In other words $153 = 1^3 + 5^3 + 3^3$; $370 = 3^3 + 7^3 + 0^3$ etc

4. If you divide any square number by 8 you get a remainder of 0, 1 or 4.
   Examples: $81 \div 8 = 10\frac{1}{8}$; $36 \div 8 = 4\frac{4}{8}$; etc

5. Wrong Mathematics but correct answers.
   The students have committed major crimes and still have obtained the correct answers. Moral of the store, check student work !!!

$$\lim_{x \to 0} \frac{\sin 4x}{\sin 8x} = \lim_{x \to 0} \frac{\cancel{\sin}\,\cancel{4}\cancel{x}}{\cancel{\sin}\,\cancel{8}\cancel{x}} = \frac{1}{2}$$

$$\frac{49}{98} = \frac{\cancel{4}\cancel{9}}{\cancel{9}\cancel{8}} = \frac{1}{2}$$

6. If you multiply **21978** by **4** it turns backwards. $21978 \times 4 = 87912$

7. Sixty nine squared = 69^2 = 4761 and sixty nine cubed = 69^3 = 328509. These two answers use all the digits from 0 to 9 between them.

8. In the English language "forty" is the only number that has all its letters in alphabetical order.

9. 13^2 = 169 and if you write both numbers backwards you get 31^2 = 961.
   This also works with 12 because 12^2 = 144 and 21^2 =441.

10. The number **FOUR** is the only number in the English language that is written with the same number of letters as the number itself .

    1 = one, 2 = two, 3 = three, **FOUR = 4**, five = 5, etc

III. Observe the pattern and decipher the code !!!

4×6	$=$	24
14×16	$=$	224
24×26	$=$	624
34×36	$=$	1224
53×57	$=$	3021
42×48	$=$	2016
61×69	$=$	4209
75×75	$=$	5625
72×78	$=$	5616

Solution:

Multiply $4\times6=24$

$$14\times16=\ 224$$

It must ALWAYS Add up to 10

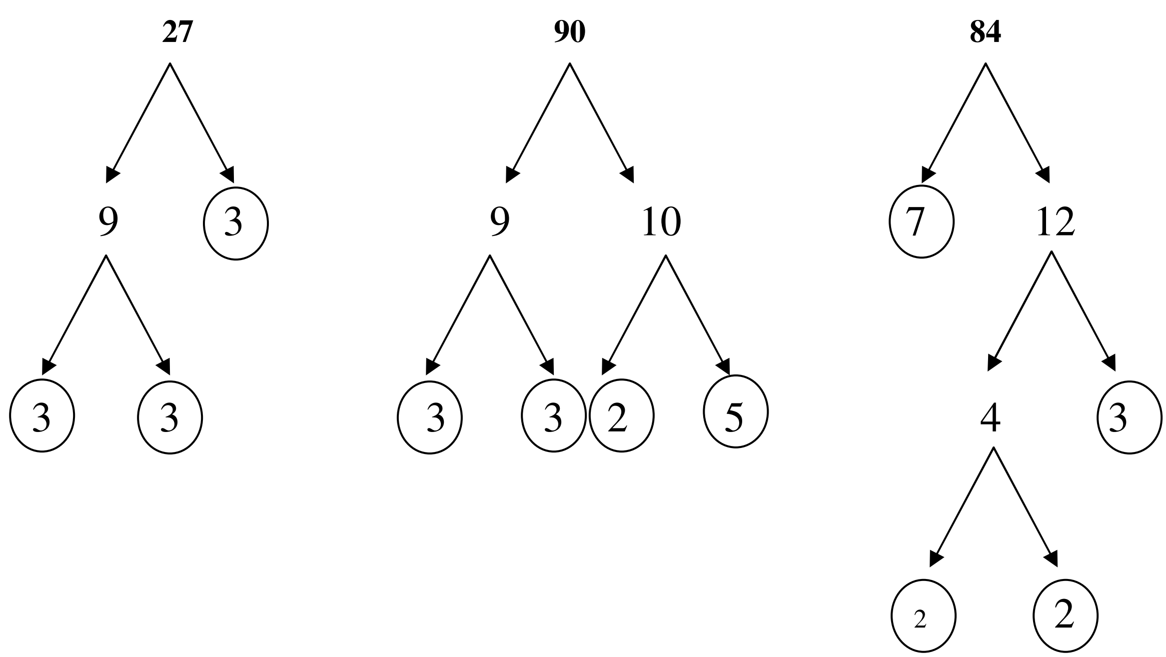
The 2 is obtained by multiplying

$$(1\times1)+1=2$$

IV. Lowest Common Multiple (LCM)

- Find the LCM of 27, 90, and 84.

✪ **The Most Accepted Practice is by using factor tree.**

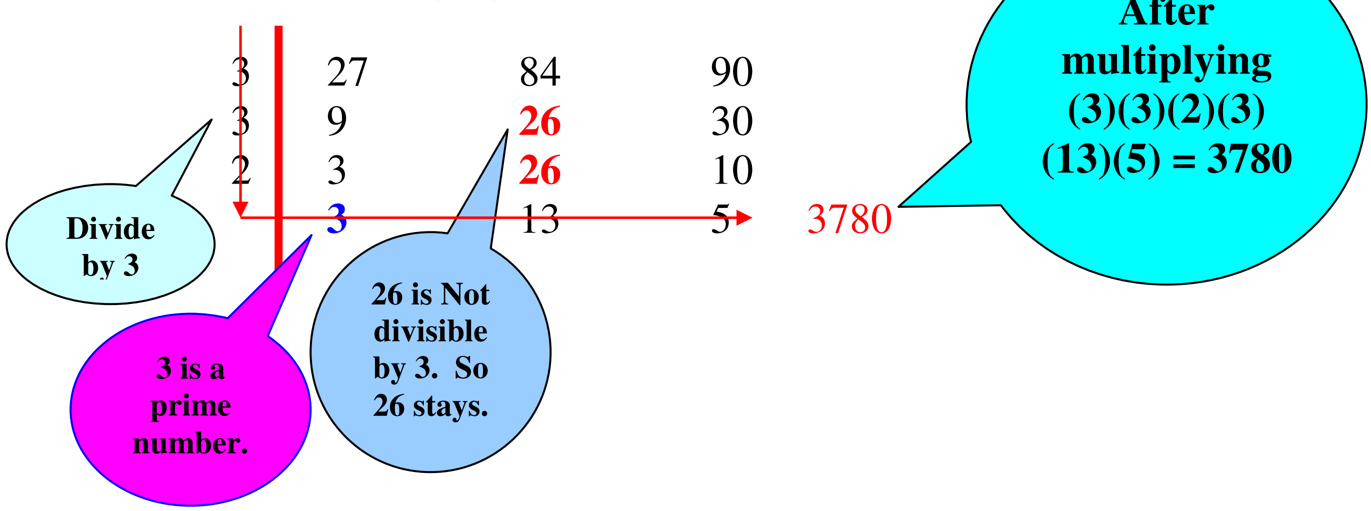


(3)(3)(3) (2)(3)(3)(5) (2)(2)(3)(7)

Therefore, the LCM of 27, 90, and 84 is:

(2)(2)(3)(3)(3)(5)(7) = 3,780

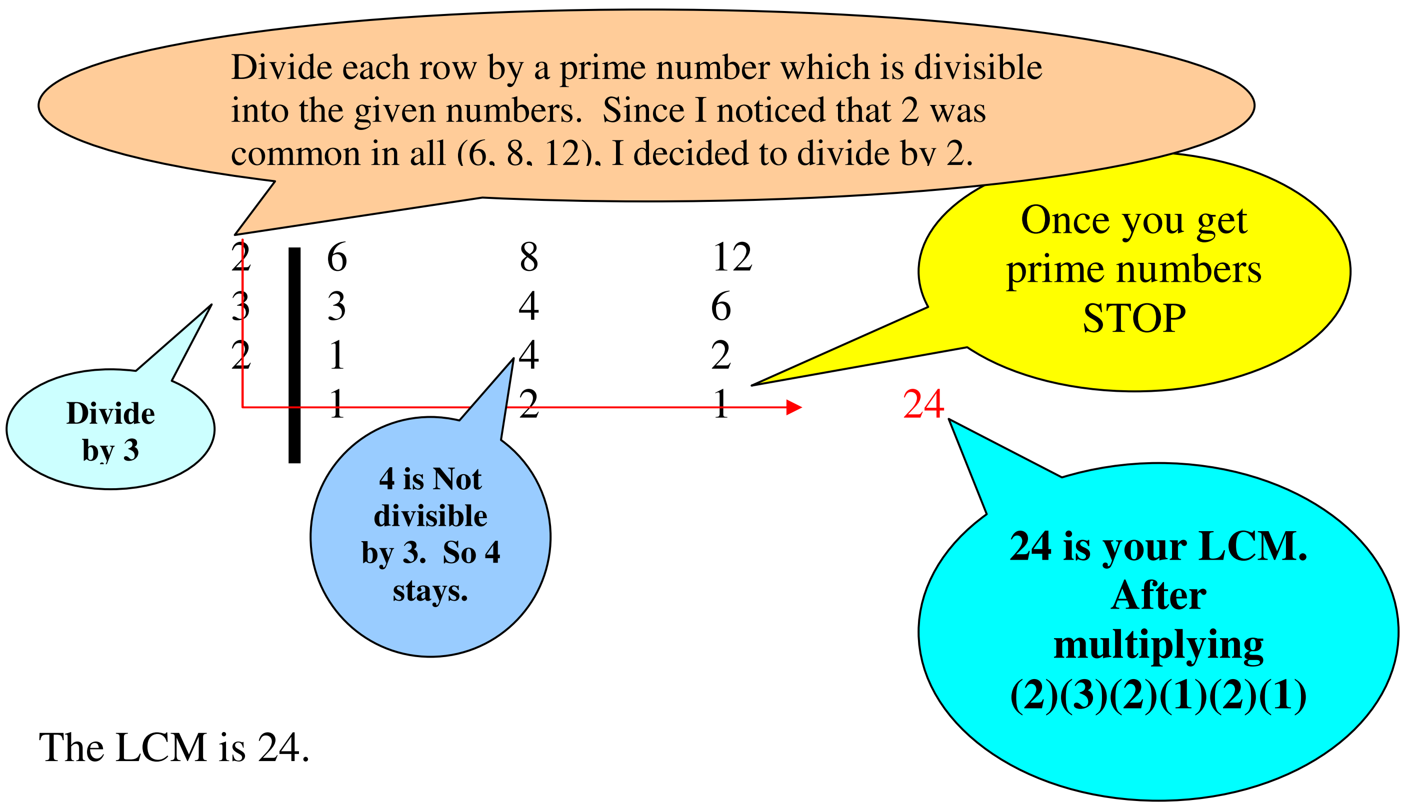
- Find the LCM of 27, 90, and 84. ➲➲➲



A Real World Connection using LCM.

My LCM story: (solution provided)

Nikkia's mom loves to throw pool parties and barbeques. She serves hamburgers, hamburger buns, and coca colas. Hamburgers are sold in packs of 6, hamburger buns in packs of 8, and coca-colas in packs of 12. What is the least number of the items Nikkia's mom must purchase?



The LCM is 24.

So Nikkia's mom must purchase:

1. 4 packs of Hamburgers
2. 3 packs Hamburger Buns
3. 2 packs of Coca-Colas

Find the **LCM** of 80 and 100

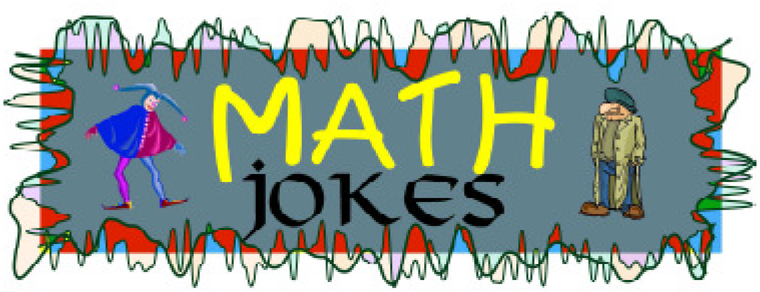
$$\begin{array}{c|cc} 4 & 80 & 100 \\ 5 & 20 & 25 \\ & 4 & 5 \\ \end{array}$$

$$\begin{array}{cc} 400 & \mathbf{400} \end{array}$$

The LCM is 400.

Let's say we wanted to add 1/80 + 1/100 ?

$\frac{1}{80}+\frac{1}{100}=\frac{1}{80}\cdot\frac{5}{5}+\frac{1}{100}\cdot\frac{4}{4}$ ……… do you see the beauty of the approach ?



Question: What caused the big bang?
Answer : God divided by zero. Oops!

V. Infallible I. Q. Test

Match the definitions in **Column I** with the terms in **Column II**

Column I			Column II	
1	That which Noah built		A	Hypotenuse
2	An article for serving ice cream		B	Polygon
3	What a bloodhound does in chasing a woman		C	Inscribe
4	An expression to represent the loss of a parrot		D	Geometry
5	An appropriate title for a knight named Koll		E	Unit
6	A sunburned man		F	Center
7	A tall coffee pot perking		G	Decagon
8	What one does when it rains		H	Arc
9	Young dog sitting in a refrigerator		I	Circle
10	A vocal sound of sadness		J	Axiom
11	What you call a person who writes for an INN		K	Cone
12	What the captain said when his boat was bombed		L	Coincide
13	What the little acorn says when he grows up		M	Cosecant
14	What you do if you have yarn and needles		N	Tangent
15	What one does to trees that are in the way		O	Loci
16	Why your friend does not help you with these answers		P	Perpendicular

Answers: Infallible I. Q. Test

Column I	Column II
1	H
2	K
3	F
4	B
5	I
6	N
7	A
8	L
9	P
10	O
11	C
12	G
13	D
14	E
15	J
16	M

VI. Amazing !!!

The following was discovered by accident, can you find more !!!

$$1!+4!+5!=145$$

$$5^1+9^2+8^3=598$$

$$1^4+6^4+3^4+4^4=1634$$

$$4^5+1^5+5^5+0^5=4150$$

$$1^1+3^2+5^3=135$$

$$12^2+33^2=1233$$

$$4^5+1^5+5^5+1^5=4151$$

VII. Chick Hearn's Theorem

Chick Hearn called his first Lakers game in March 1961. His last game was June 12, when the Lakers beat the New Jersey Nets to complete a sweep of the NBA Finals

Chick Hearn set the standard for NBA play-by-play announcers.

Chick Hearn, who made "slam dunk" and "air ball" common basketball expressions during his 42-year broadcasting career with the Los Angeles Lakers, died on Aug. 5, 2002. He was 85.

The below mathematical discovery is dedicated to Chick. A True story (Michael & I were at one of the Laker's games in the Coliseum back in the mid eighties…]

The Discovery: The Iron man Chick Hearn, the Announcer for the Lakers for over 3000 consecutive games, gave a score during a game to be 97 to 79, and announced that the Lakers were ahead by 18.

We observed when subtracting 7 from 9 and multiply by 9 the answer is 18. Then we wondered if this was true in all cases when subtracting two reversed integers!!! After several tries, we found this to be true, as you can see in the following examples and a simple proof.

Examples:

$$95-59=4(9)=36$$
$$97-79=2(9)=18$$
$$52-25=3(9)=27$$
$$62-26=4(9)=36$$
$$91-19=8(9)=72$$
$$82-28=6(9)=54$$
$$73-37=4(9)=36$$
$$64-46=2(9)=18$$
$$55-55=0(9)=0$$

The Chick Hearn's Theorem

By Hattar & Hovsepian

If x and y are any two single digit integers and $x > y$, then $xy - yx = 9(x - y)$.

Proof:

Let $xy = 10x + y$, and $yx = 10y + x$, then

$$\begin{aligned} xy - yx &= (10x + y) - (10y + x) \\ &= 10x + y - 10y - x \\ &= 9x - 9y \\ &= 9(x - y) \end{aligned}$$

VIII. Rahm's proof that Infinity = -1

First *Rahm Emanuel,* the Chief of Staff of President Obama, found the sum of the infinite series

$1 + a + a^2 + a^3 + a^4 + a^5 + \ldots = C$
(which he called **C** for Chicago).

He very cleverly factored an 'a' out

$1 + a(1 + a + a^2 + a^3 + a^4 + a^5 \ldots) = C$

Dr. Nasser and I, warned him to be careful!!! He ignored our input!!!

Note: What's in parentheses is C

So $1 + a(C) = C$
Then he proceeded in solving for C

$$aC - C = -1$$
$$C(a - 1) = -1$$
$$C = -1/(a-1)$$

So $\mathbf{1 + a + a^2 + a^3 + a^4 + a^5 + \ldots = 1/(1-a)}$

Now Rahm replaced 2 in for **a** and got

$$1 + 2 + 4 + 8 + 16 + 32 \ldots = 1/(1-2) = -1$$

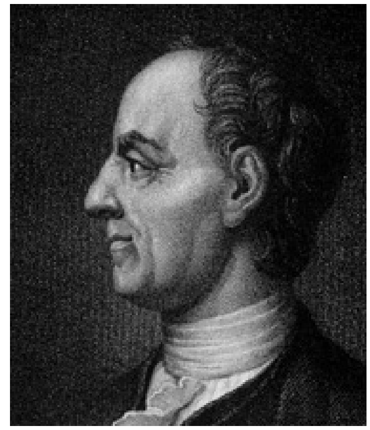
Since the left side goes to infinity, Rahm concluded that

Infinity = -1

Sir Isaac Newton

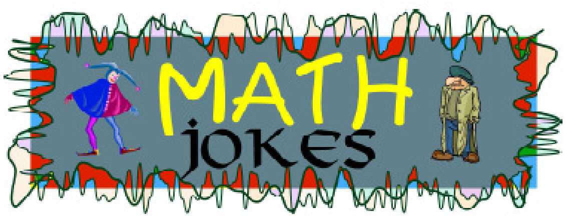
What's wrong with Rahm's argument?

[2]Sir Isaac Newton, [3]Leonard Euler and other mathematicians had made similar mistakes with infinite series!



Leonard Euler

The problem is that the infinite series converges only when the absolute value of a is less than 1. So it's OK to make mistakes … if you start early!



A visitor to the Geological Museum in Yerevan, Armenia asks a museum employee: "How old is the prehistoric elephant discovered in a sand pit in Gyumri.?" "Precisely 60 million and three years, two months, and 12 days."

"How can you know that with such precision?"

"That's easy. When I was appointed working here by the Armenian Revolutionary Federation, a sign said that the skeleton was 60 million years old. And that was three years, two months, and 12 days ago. . . "

[2] Sir Isaac Newton, one of the greatest scientists (1642 – 1727).

[3] Leonard Euler, one of the greatest mathematicians (1707 – 1783).

A collection of DAILY motivators designed to raise the interest of students in appreciating the beauty of mathematics. Allocate no more than 15 - 20 minutes per MOTIVATOR at most and/or assign them as take home CHALLENGE work for discussion at the next class meeting.

Fun

With Mathematics

Appendix C

The Collection

"Math Motivators"

Fun With Mathematics

The following pages are some of the fascinating problems,
puzzles, we have [4]compiled over the years,
with lots of sweat,
lots of tears, lots of love,
and maybe even a little blood.
We hope you and your
friends will enjoy working with these
tantalizing motivators.

Hovsepian, Viken "Vik"
Professor of Mathematics
Rio Hondo College

Hattar, Michael
Professor of Mathematics
Mount San Antonio College

Special Note:

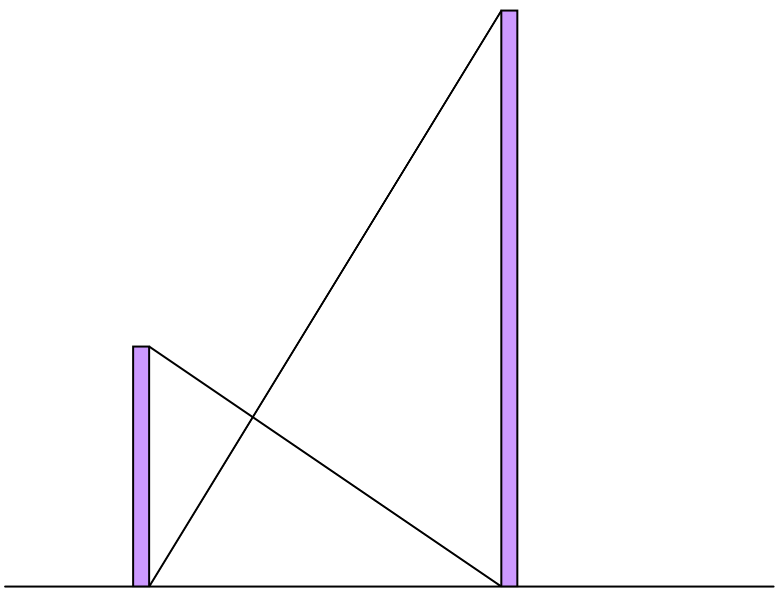
The math level of each MOTIVATOR is indicated in Appendix D-Solutions

E-Mail us at: vhovsepian@riohondo.edu, mhattar@mtsac.edu

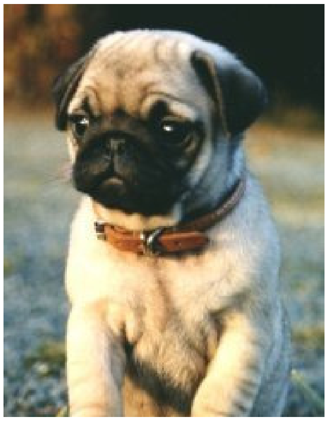
[4] We have been using them since 1971.

✪ Motivator #1 ✪

Two poles are 30 meters and 110 meters high. Find the height of the point of intersection of the lines joining the top of each pole to the foot of the other. The distance between the buildings is 100 feet.



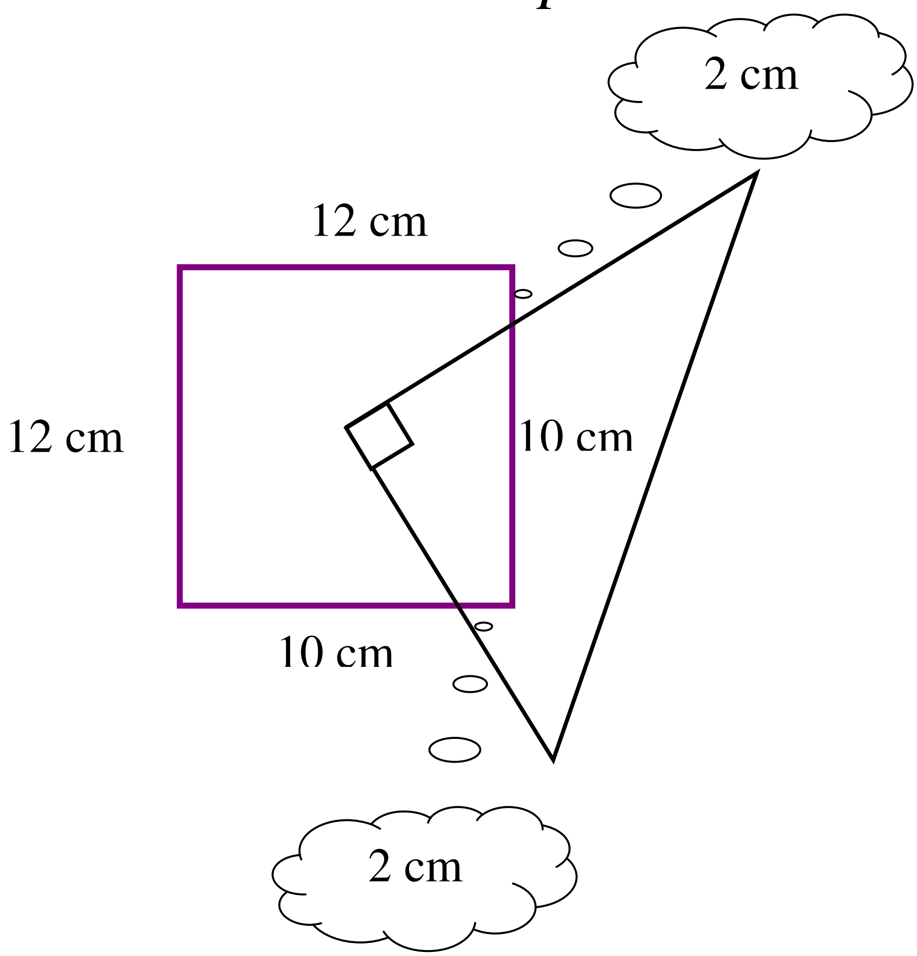
✪ Motivator #2 ✪



Lucie & Vik bought three puppies of equal value for $1,500. Lucie contributes $840 and Vik contributed $660. By the end of the California Puppy Fair in Palm Springs, they had sold one puppy for $1,500 and decided to end their partnership. How should they divide the cash fairly if Lucie keeps the two puppies ?

✪ Motivator #3 ✪

A right triangle, whose legs are greater than or equal to $6\sqrt{2}$ and a square with sides 12 cm overlap as shown. The right angle of the triangle is in the center of the square. What is the area of the *overlap* ?

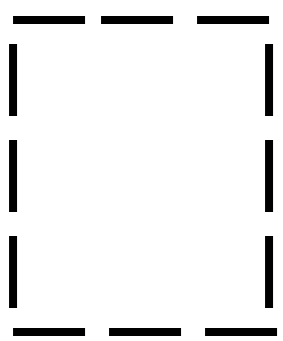


✪ Motivator #4 ✪

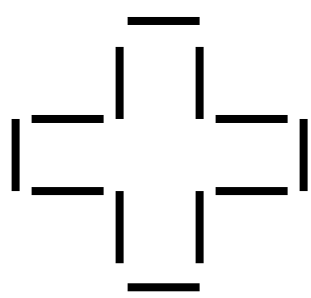
Given that a *stick* or a *match* has a length of 1 unit, it is possible to place the 12 sticks on a plane in various ways to form [5]polygons with **different areas** keeping however the **same perimeter**

Examples:

Perimeter 12 units and area 9 $units^2$



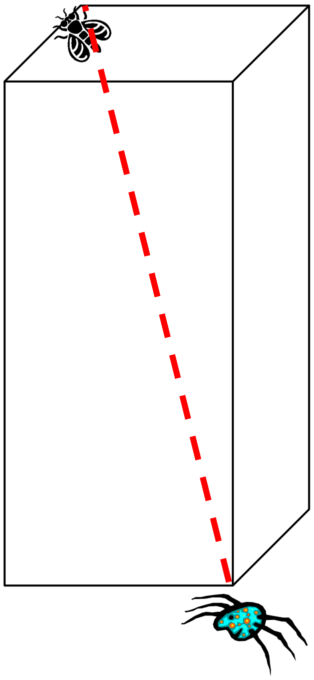
Perimeter 12 units and area 5 $units^2$



The Question: Can you use all 12 sticks to form the perimeter of a polygon with an area of EXACTLY 4 $units^2$

[5] A polygon is a closed figure made by joining line segments, where each line segment intersects exactly two others.

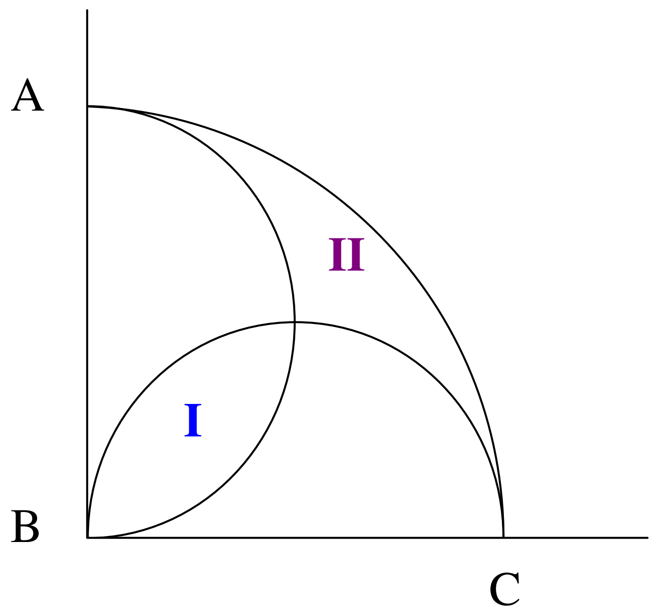
✪ Motivator #5 ✪



A storage room is 20 feet by 30 feet by 120 feet. What is the shortest distance a spider must **crawl** to go from one lower corner of the room to the opposite upper corner to catch the fly (assuming the fly does not fly) ?

✪ Motivator #6 ✪

The figure *ABC* is a quadrant of a circle and semi-circles are drawn on *AB* and *BC*. Region **I** and Region **II** are indicated. Find the ratio of area **I** to area **II**.



✪ Motivator #7 ✪

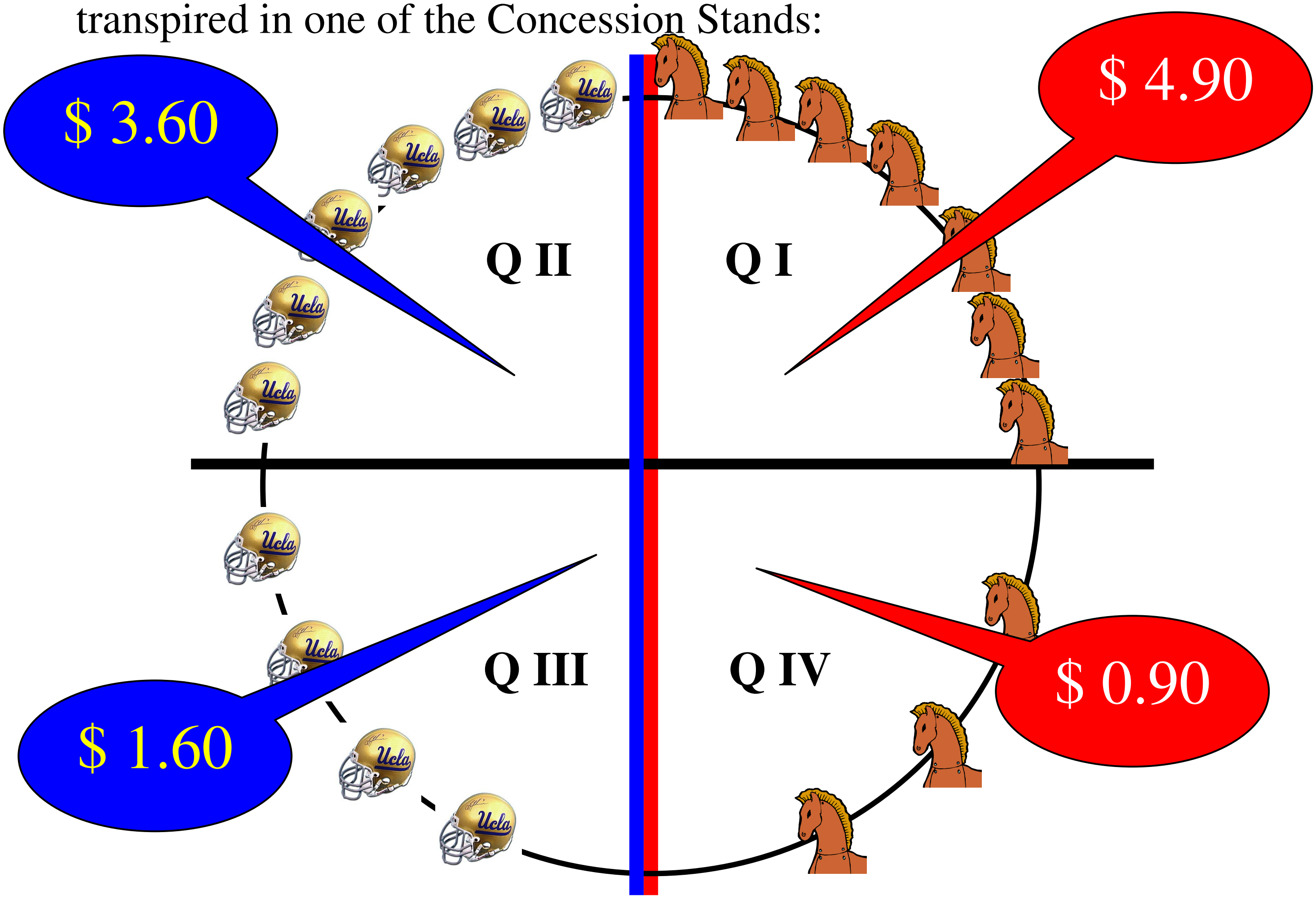
In the array of numbers given below, determine the column in which the number 1000 appears.

Columns				
one	two	three	four	five
	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
	18	19	20	21
25	24	23	22	
	…	…	…	
…	…	…	…	…
	…	…	…	
…	…	…	…	…
	…	…	…	

✪ Motivator #8 ✪

Find the mathematical error and explain the moral of the story !!!

During the rivalry football game between the U$C Trojans and the UCLA Bruins held at the Rose Bowl, the following scenario transpired in one of the Concession Stands:



✪ 1st Quarter: UCLA scores a touchdown and secures the point after and the lead Bruin in QII purchases a round of Pepsi to all his fellow Bruins in QII including himself and immediately after that the process is repeated by the rest of the Bruins in QII. That means every Bruin buys the other Bruin Pepsi. If Pepsi costs 10 cents per can, then $3.60 was spent by the Bruins in Q II.

✪ 2nd Quarter: The Trojans score and they secure the point after, and the lead Trojan in QI purchases a round of Pepsi to all his fellow Trojans in QII including himself and immediately after that the process is repeated by the rest of the Trojans in QI, that means every Trojan buys the other Trojan Pepsi. If Pepsi costs 10 cents per can, then $4.90 was spent by the Trojans in Q I.

✪ 3rd Quarter: The Trojans score but they miss the point after, and the lead Trojan in QIV purchases a round of Pepsi to all his fellow Trojans in QIV including himself and immediately after that the process is repeated by the rest of the Trojans in QIV. That means every Trojan buys the other Trojan Pepsi. If Pepsi costs 10 cents per can, then $0.90 was spent by the Trojans in QIV.

✪ 4th Quarter: The Bruins score and they secure the point after, and the lead Bruin in QIII purchases a round of Pepsi to all his fellow Bruins in QIII including himself and immediately after that the process is repeated by the rest of the Bruins in QIII. That means every Bruin buys the other Bruin Pepsi. If Pepsi costs 10 cents per can, then $1.60 was spent by the Bruins in QIII.

Final Score: **UCLA** 14, **U$C** 13

The Mathematics:

✪ The total amount of money spent by the 10 Trojans buying each other Pepsi is $5.80

✪ The total amount of money spent by the 10 Bruins buying each other Pepsi is $5.20

Therefore, the moral of the story is it costs too much to attend U$C and therefore do not send your kids to U$C.

Your Charge: Explain the outcome.

✪ Motivator #9 ✪

Find the error in the following proof that $1 = 2$

Prove: $\quad$ *If* $a = b,\ a > 0,$ *and* $b > 0,$ *then* $1 = 2$

Proof	
Statements	**Reasons**
1. $a, b > 0$	Given
2. $a = b$	Given
3. $ab = b^2$	Multiply both sides by b
4. $ab - a^2 = b^2 - a^2$	Subtract a^2 from both sides
5. $a(b-a) = (b+a)(b-a)$	Factor
6. $a = (b+a)$	Divide both sides by $(b-a)$
7. $a = a + a$	Substitute a for b
8. $a = 2a$	Simplify
9. $1 = 2$	Divide both sides by a
Q.E.D.	

✪ Motivator #10 ✪

The numbers [0 through 9] need to be arranged in a unique way. Some of the numbers have been arranged for you. Find the proper sequence of the Missing Numbers by breaking the code.

8, 5, 4, 9, 1, …, … , … , … , …

✪ Motivator #11 ✪

These numbers are American.

16325, 34721, 12347,
52163, 90341, 50381

These numbers are NOT American.

2564, 12345, 854,
12635, 34325, 45026

Which of these numbers are

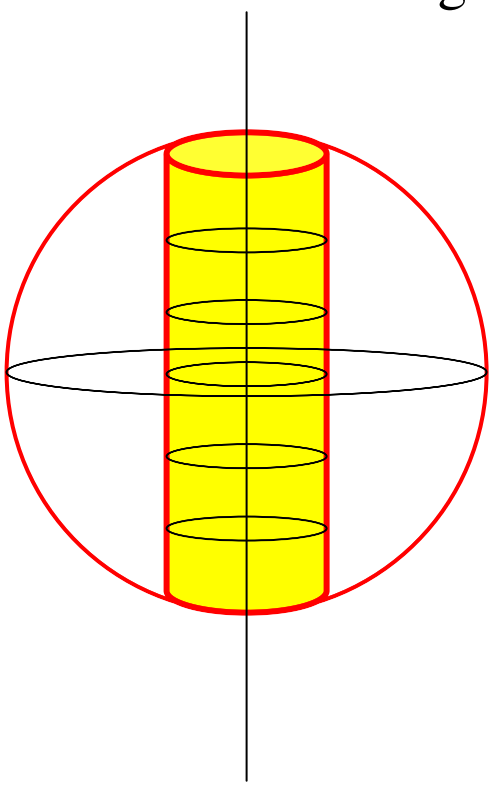
American ?

72521, 72341, 70523,
4562, 13562, 52703

✪ Motivator #12 ✪

Have I supplied enough information to solve the problem below?

A cylindrical hole six inches long is drilled straight through the center of a solid sphere. What is the volume remaining in the sphere ?



✪ Motivator #13 ✪

Professor Hattar, very much involved with numbers, was asked by his students how old he was. He agreed to tell them, but not without throwing in the following *mini* challenge:

"My son is 24 years younger than I am. He, in turn, is 25 years older than my grandson. My grandson and I together are 73 years old."

How old is Professor Hattar?

✪ Motivator #14 ✪

Using exactly four 4s and the operations $+$, $-$, $\times$, and $\div$, make equations that equal to 0, 1, 2, 3, 4, 6, 7, 8, and 9.

a) $4 \quad 4 \quad 4 \quad 4 \quad = \quad 0$

b) $4 \quad 4 \quad 4 \quad 4 \quad = \quad 1$

c) $4 \quad 4 \quad 4 \quad 4 \quad = \quad 2$

d) $4 \quad 4 \quad 4 \quad 4 \quad = \quad 3$

e) $4 \quad 4 \quad 4 \quad 4 \quad = \quad 4$

f) $4 \quad 4 \quad 4 \quad 4 \quad = \quad 6$

g) $4 \quad 4 \quad 4 \quad 4 \quad = \quad 7$

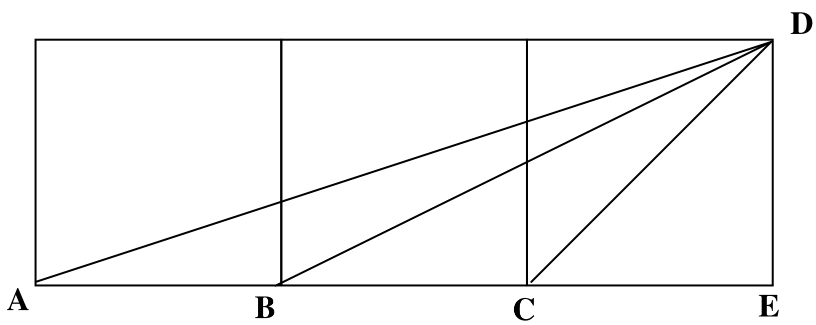
h) $4 \quad 4 \quad 4 \quad 4 \quad = \quad 8$

i) $4 \quad 4 \quad 4 \quad 4 \quad = \quad 9$

✪ Motivator #15 ✪

Using ONLY elementary geometry, NOT even trigonometry, prove that angle C equals the sum of angles A and B.

There are many ways to prove that angle DCE in the figure provided is the sum of angles DAB and DBC.



Given: Three congruent squares as shown.
Show That: $m\angle A + m\angle B = m\angle C$

Notes: Charles Trigg published 54 different proofs in the Journal of recreational mathematics, Vol. 4, April 1971, pages 90 – 99.

A proof using paper cutting, by Ali R. Amir-Moez, appeared in the same journal, Vol. 5, Winter 1973, pages 8 – 9.

For other proofs, see Roger North's contribution to The Mathematical Gazette, December 1973, pages 334-336, and its continuation in the same journal, October 1974, pages 212-215.

For GENERALIZATION of the problem to a row of n squares, see Trigg's "Geometrical Proof of a Result of Lehmer's," in the Fibonacci Quarterly, Vol. 11, December 1973, pages 539-540.

✪ Motivator #16 ✪

Using exactly four 5s and the operations $+$, $-$, $\times$, and $\div$, **to make equations that equal to 3, 5, 6, 11, 24, 25, 30, and 120.**

a) 5 5 5 5 = 3

b) 5 5 5 5 = 5

c) 5 5 5 5 = 6

d) 5 5 5 5 = 11

e) 5 5 5 5 = 24

f) 5 5 5 5 = 25

g) 5 5 5 5 = 30

h) 5 5 5 5 = 120

✪ Motivator #17 ✪

Dr. Nasser's Mini-Challenge !!!

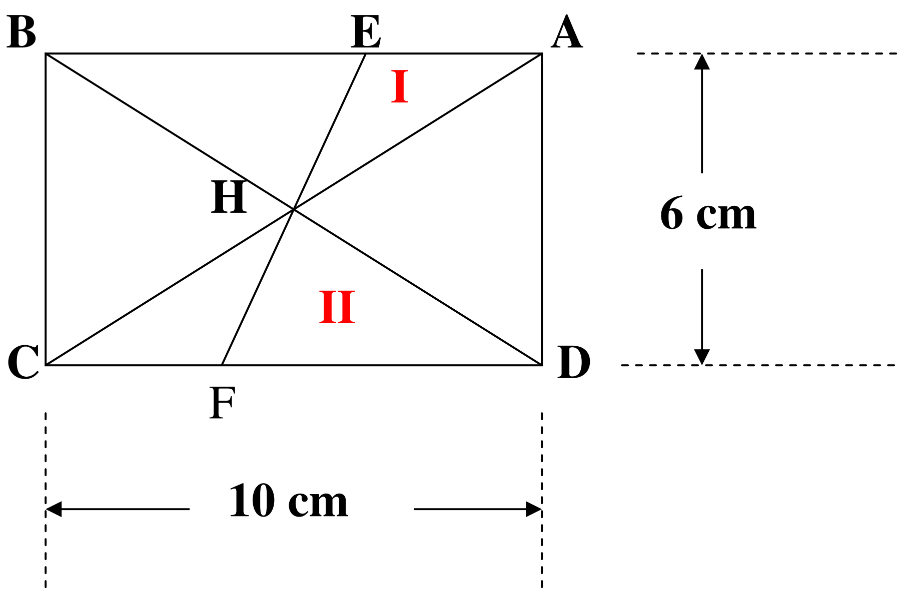
Dr. Nasser (far left) loves teaching mathematics. To show his devotion towards mathematics challenged his two friends Vik and Ahmed a round trip ticket to New York. All expenses would be paid provided Vik and Ahmed solve the below equation for x within 10 minutes max.

The mini challenge:

Given $\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}}}=3$, Solve for x.

✪ Motivator #18 ✪

Find the area of regions I & II of rectangle ABCD if the length is 10 cm and the width is 6 cm.

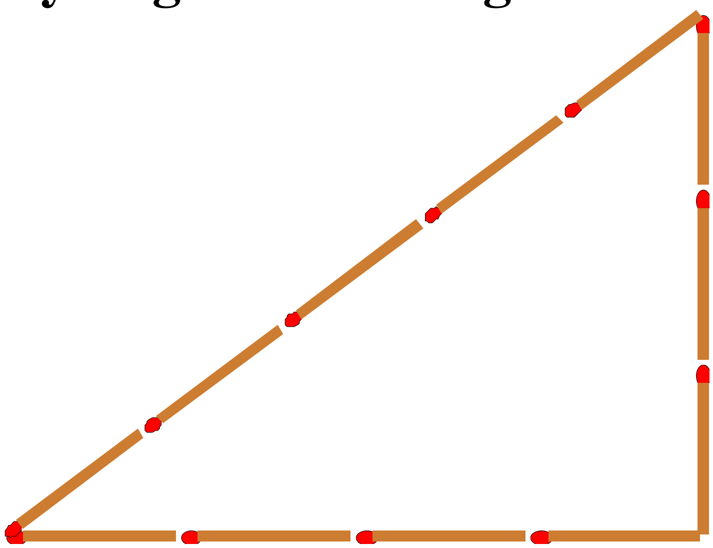


✪ Motivator #19 ✪

Looking out into the backyard of the Annita Hovsepian pre-school playground in Gharabakh, I saw a mix of girls and cats playing. I counted 17 heads and 44 feet. How many girls were there and how many cats ?

✪ Motivator #20 ➲ A ✪

Pythagorean Triangles



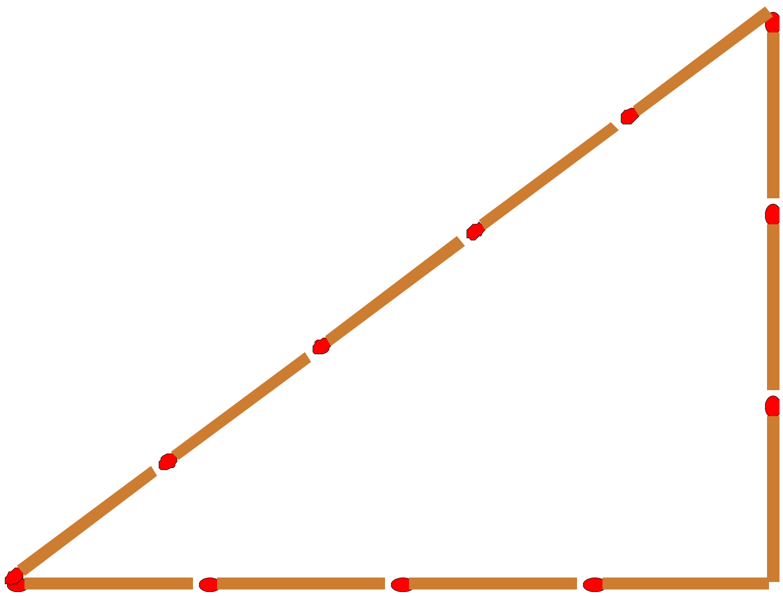
With 12 matches a perfect right angle can be built.

Puzzle 1

Add two matches and divide this traingle into two equal areas. It can be done exactly!

✪ Motivator #20 ➲ B ✪

Pythagorean Triangles

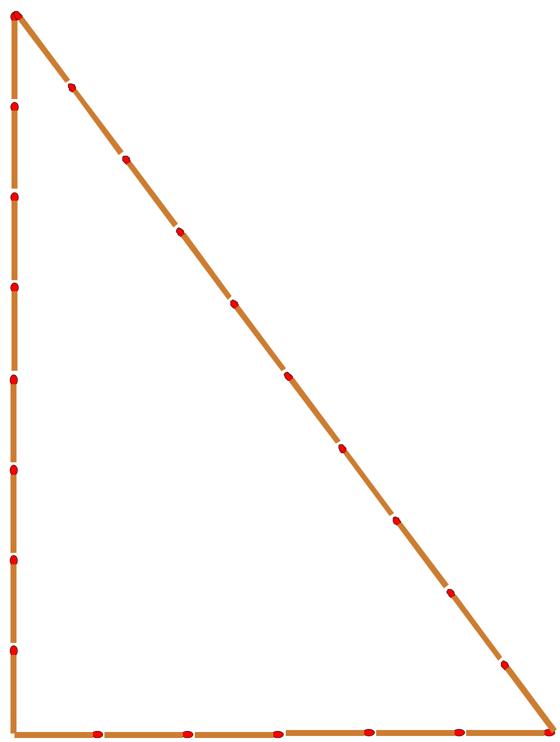


Puzzle 2

Now divide the triangle into 3 equal areas using 4 matches

✪ Motivator #20 ➲ C ✪
2 Parts

Divide the right triangle with sides 6, 8, and 10 into 4, 6, equal areas using 10, and 15 matches respectively.



Puzzle 3

Divide it into **4 equal** areas using **10** matches.

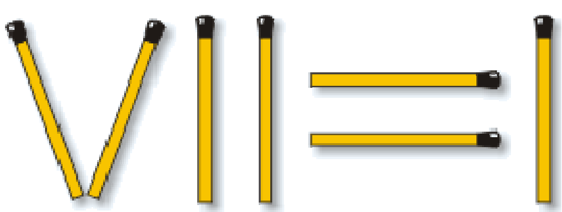
Puzzle 4

Divide it into **6 equal** areas using **15** matches.

✪ Motivator #21✪

2 Puzzles

Arrange seven matches into the equation shown in the illustration. It can be seen that the equation itself is not correct.



Puzzle 1

Move one match to a new position in order to make this equation correct.

Puzzle 2

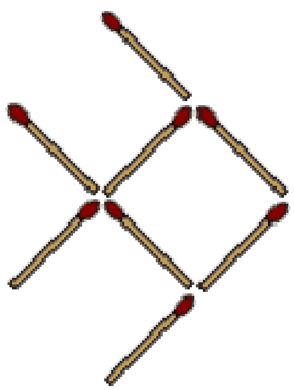
Move three matches to new positions to get a correct equation. This puzzle can be solved in two different ways.

Note: In both puzzles it is not allowed to break matches and an equation sign has to remain in the final expression.

✪ Motivator #22✪

Arrange 8 matchsticks to form a fish swimming as shown below.

The object of the puzzle is to move 3 matches to make the fish swimming in the opposite direction.

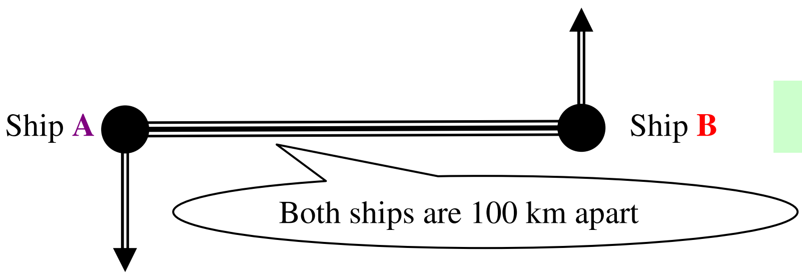


✪ Motivator #23 ✪

A single railroad track is laid one mile over level ground. It is firmly secured at the ends so that they cannot move. If in the heat of the day, the track expands one inch over its length, and the track arcs above the ground, then how high is the arc at the center?

✪ Motivator #24 ✪

At noon, Ship **A** is 100 km west of ship **B**. Ship **A** is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 P.M. ?

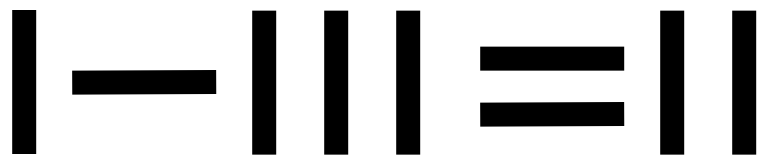


At NOON

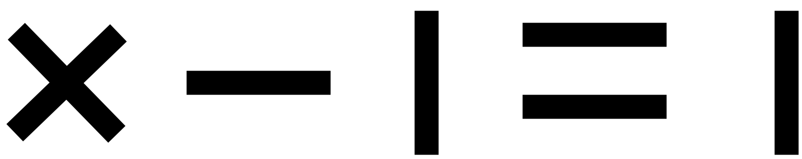
✪ Motivator #25 ✪

Change the position of **ONE** match or stick to make each statement true.

1.



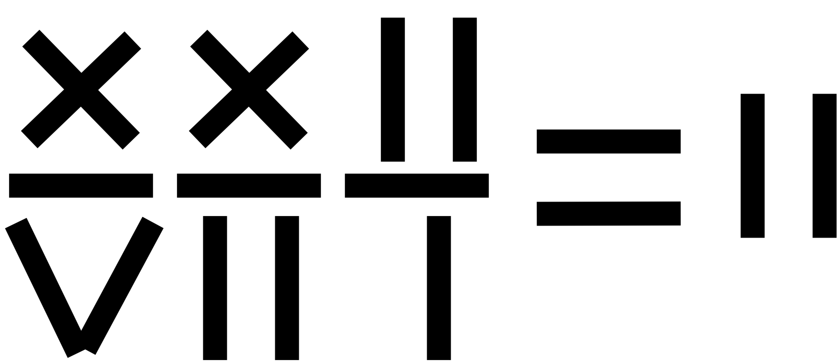
2.



3.

4.

5.



✪ Motivator #26 ✪

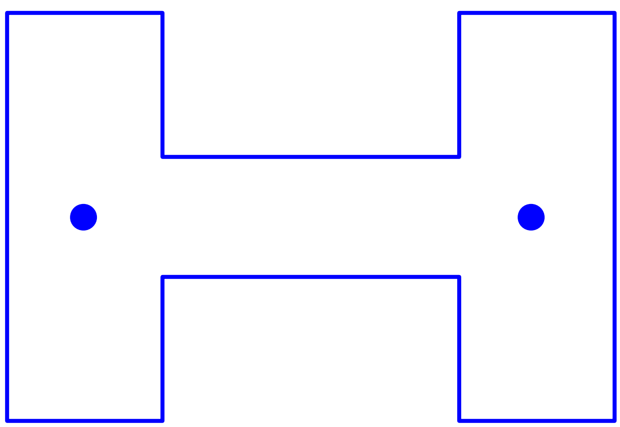
The amount of a monthly payment on a loan is frequently calculated by banks using the formula

$$P = \frac{\frac{r}{12}l}{1-\left(1+\frac{r}{12}\right)^{-m}}$$

Where P is the monthly payment, r is the interest rate per year, m is the total number of monthly payments, and l is the amount of the loan. Suppose Nicole borrows \$250,000 at an annual rate of 10% to be paid back in 30 years. What is Nicole's monthly payment ?

✪ Motivator #27 ✪

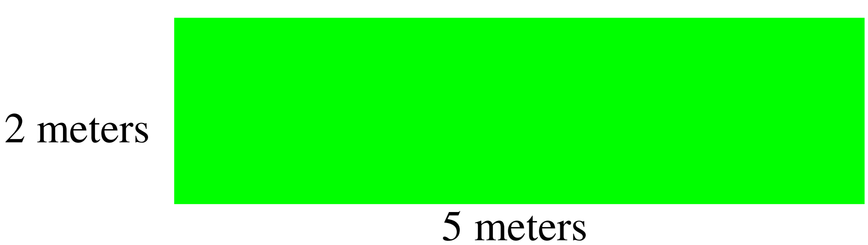
Can you cut the letter "H" shown below into four identical pieces (same shape and area) and with all the cutting lines passing through the two points shown ?



Motivator #28

Muna Hattar was always looking for ways to save money. While in the remnant shop, she came across just the material she wanted to make a tablecloth. Unfortunately, the piece of material was in the form of a 2 meters by 5 meters rectangle and her table was 3 meters square. She bought it, however, having decided that the area was more than enough to cover the table.

When Muna got home she decided she had been a fool because she couldn't see how to cut up the material to make a square. But just as she despaired, she had a notion, and with three straight cuts, in no time at all, she had 5 pieces which fit together in a symmetrical pattern to form a square using all the material. How did Muna do it ?



✪ Motivator #29 ✪

The starting five of a famous NBA basketball team shakes hands with every other member of the starting five.

How many handshakes will there be?

✪ Motivator #30 ✪

A cat is at the bottom of an 18-foot well.

Each day the cat climbs up 3

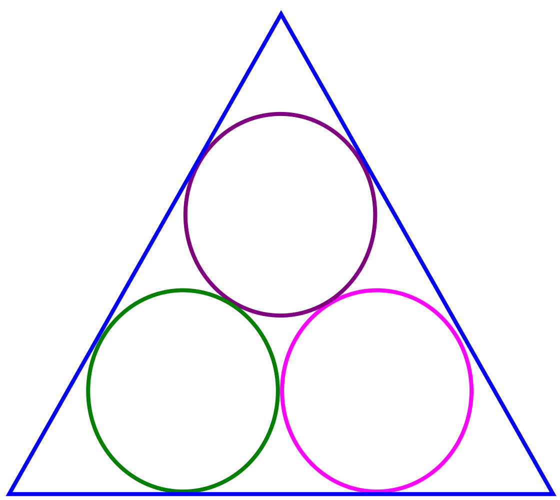
feet; each night it slides back 2 feet. How long will it take the cat to get out of the well ?

✪ Motivator #31 ✪

Three logs with diameter of exactly 10 feet each, one tangent to the other two. Find the distance from the ground to the top of the highest log. Expand the problem using triangular number of logs.

✪ Motivator #32 ✪

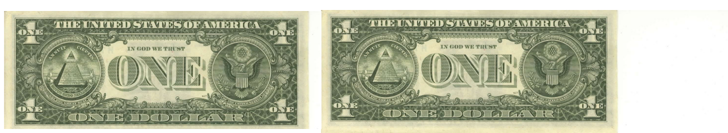
In the figure below, each of the 3 circles is tangent to the other 2, and each side of the equilateral triangle is tangent to 2 of the circles. If the length of one side of the triangle is ***w***, what is the radius, in terms of ***w***, of one of the circles ?



✪ Motivator #33 ✪

Estimate, **investigate**, then **calculate**:

A. How long would one billion dollars be, laid out end to end.



B. How high would a stack of one billion pennies be?

✪ Motivator #34 ✪

On a table top arrange four spheres each of radius 1 inch into two layers, with three spheres on the lower layer and one sphere on the top layer. Find the distance between the highest point of the top sphere and the table top.

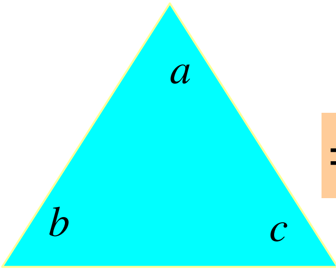
✪ Motivator #35 ✪

Fun with digits

Start with the sequence of non-zero digits 123456789. The problem is to place plus or minus signs between them so that the result of thus described arithmetic operation will be 100.

Motivator #36

Apply the following definition for any numbers $a,\ b,\ c,$

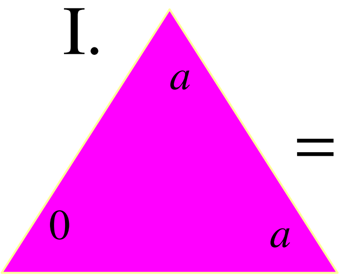


$= abc - (a + b + c)$

and answer the below question.

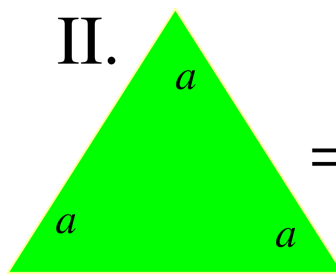
Q: For which of the following equations is it true that there is exactly one positive integer that satisfies it ?

I.



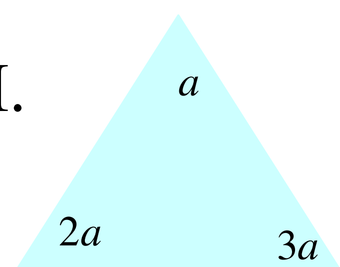
$= 0$

II.



$= 0$

III.



$= 0$

(A) none (B) I only (C) III only
(D) I and III (E) I, II, and III

✪ Motivator #37 ✪

Assuming that the **binomial theorem** continues to hold when ***n*** is a positive rational number that is not an integer. In general, the coefficient in the expansion will not be integers, and instead of terminating, the expansion will be an infinite series:

$$(a+b)^n = a^n + \frac{n}{1}a^{n-1}b + \frac{n(n-1)}{1\cdot 2}a^{n-2}b^2 + \ldots$$

a) Write the first four terms in the expansion of $(1+x)^{\frac{1}{2}}$

b) Use your answer in (a) with $x=1$ to find a numerical estimate of $\sqrt{2}$.

✪ Motivator #38 ✪

Find the number of ways the letters of each word can be *arranged*.

a) ESCALANTE
b) AMERICAN
c) MISSISSIPPI
d) SCHWARZENEGGER
e) HOVSEPAIN
f) ANNETTE
g) HATTAR
h) SHAMONEH
i) FOOTBALL

Fun

With Mathematics

Appendix D

The Collection

"Math Motivators"

SOLUTIONS

Attached you will find the solutions for the 38 motivators from Appendix C

Fun With Mathematics

SOLUTIONS To Motivators

The following pages are some of the fascinating problems, puzzles, we have [6]compiled over the years, with lots of sweat, lots of tears, lots of love, and maybe even a little blood. We hope you and your friends will enjoy working with these tantalizing motivators.

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Professor of Mathematics
Rio Hondo College

Hattar, Michael
Professor of Mathematics
Mount San Antonio College

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[6] We have been using them since 1971.

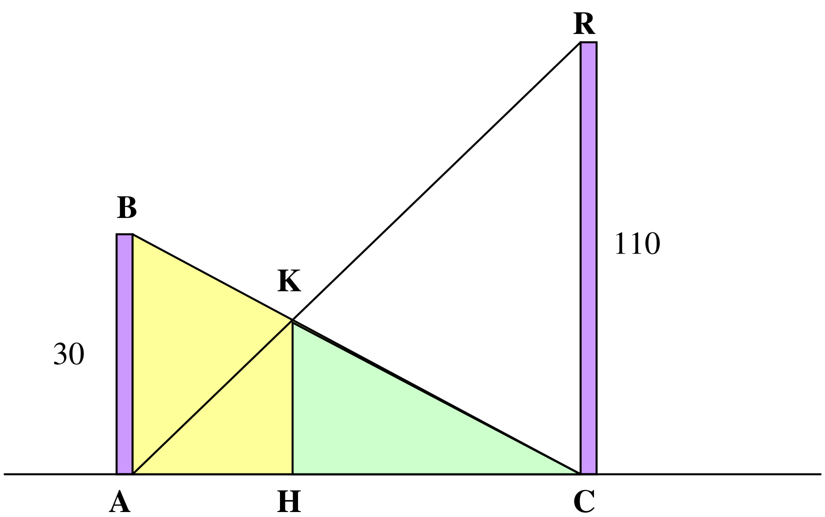
Solution Motivator #1

Algebra, Geometry

Answer: $23\frac{4}{7}$ meters

<u>Two Solutions</u> <u>have been supplied:</u>

Solution One: A Geometric approach using ***similar triangles***



$\triangle ABC \sim \triangle HKC$ and $\triangle ARC \sim \triangle AKH$

$$\frac{KH}{30} = \frac{HC}{AC} \quad ; \quad \frac{KH}{110} = \frac{AH}{AC}$$

Now, add these two equations: $\frac{KH}{30} + \frac{KH}{110} = \frac{HC}{AC} + \frac{AH}{AC}$

$$(11)KH + (3)KH = 330\left[\frac{HC + AH}{AC}\right]$$

Note: $\frac{HC + AH}{AC} = \frac{AC}{AC} = 1$

$$\therefore \; 14KH = 330$$

$$KH = \frac{330}{14} = 23\frac{4}{7} \text{ meters}$$

Please note: The distance between the buildings makes no difference.

Solution Two: Using the Lens Formula or better known as the Gaussian lens formula, which was discovered by *Edmund Halley* in 1693.

Background: **Halley, Edmund [1656 – 1742]**

English astronomer who established the first observatory in the southern hemisphere on the island of St. Helena. He became good friends with Newton and convinced him to publish the *Philosophiae Naturalis Principia Mathematica.* In his "Ode to Newton," with which he prefaced the *Principia,* he wrote "Nearer the gods no mortal may approach" (Westfall 1988). After studying comets, he noticed that the path of the comets of 1456, 1531, and 1607 were surprisingly similar. He surmised that these three sightings were different apparitions of a single comet, which he predicted would return again around 1758. He died before his prediction was tested, but the comet indeed returned and has been known as Halley's Comet ever since.

Introduction: The Lens Formula

There are three variables to consider

1. The object distance from the **Object** to the lens. Denote this by **O**.
2. The image distance from the **image** which is formed to the lens. Denote this by **i**. If this number is positive then the image is real. If it is negative, then the image is virtual.
3. The **focal** length of the lens. Denote this by **f**. For a converging lens the focal length is positive, and for a diverging lens the focal length is negative.

With these three variables defined, they are related to each other by the **lens formula**

$$\frac{1}{O}+\frac{1}{i}=\frac{1}{f}$$

Surprisingly, this same formula also holds for mirrors, even though mirrors work on the totally different principle of reflection.

The **Solution Using the Lens Formula**: $\frac{1}{110}+\frac{1}{30}=\frac{1}{x}$ and solve for x.

$$(330x)\left[\frac{1}{110}+\frac{1}{30}=\frac{1}{x}\right]$$
$$14x=330$$
$$\therefore x=23\frac{4}{7} \text{ meters}$$

Please note: The distance between the buildings makes no difference.

Solution Motivator #2

Pre-Algebra, Algebra

Basic Answer: Lucie keeps \$ 400, and Vik keeps \$1,100 [no partnership agreement exists]
Please Note: Solutions may vary pending upon partnership agreements.

Solution:

The **Venture** between Lucie & Vik: \$1,500 (initial Investment)

\$840 by Lucie   \$660 by Vik

Available assets after the sale of 1 puppy

Cash ………………………………………..	\$ 1,500
2 puppies worth ……(original)………	\$ 1,000
	\$ 2,500

The venture profit:

Lucie's Worth $\Rightarrow \frac{840}{1500}\cdot(\$2,500)=\$1,400$

Vik's Worth $\Rightarrow \frac{660}{1500}\cdot(\$2,500)=\$1,100$

Ending the venture:

Lucie will keep the 2 puppies $\Rightarrow$ worth \$ 1,000 + \$400 CASH = \$1,400
Vik will take only the cash $\Rightarrow$ \$1,100

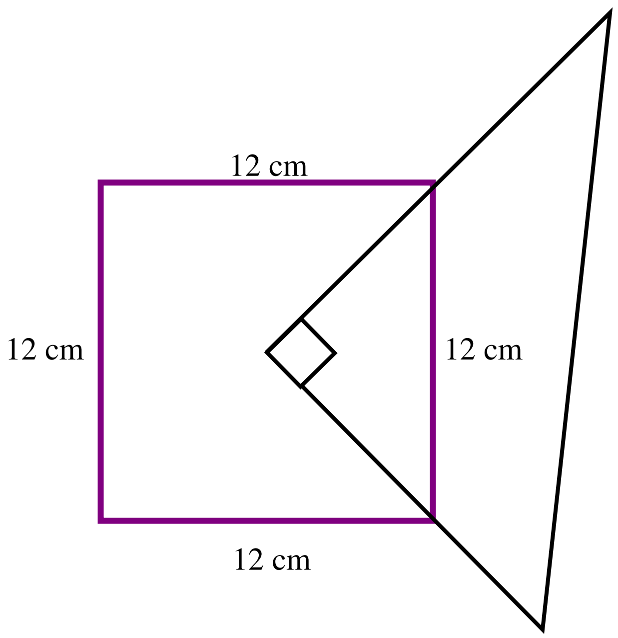
Bottom Line: The \$1,500 profit they made will be distributed in the following way

- Lucie gets \$ 400
- Vik gets \$ 1,100

Solution ✪ Motivator #3 ✪
Geometry

Answer: 36 cm^2

Solution:



We know that any two perpendicular lines that intersect at the center of the square divide the square into 4 congruent regions.

Now tilt the triangle, as shown above and you will see what I mean.

Area of the square $=(12)(12) \Rightarrow 144\ cm^2$

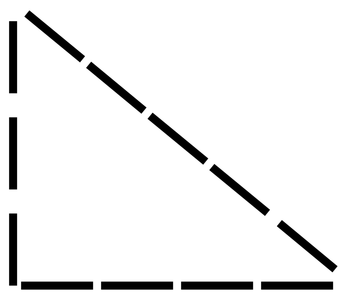
Divide the area by 4 $= 36\ cm^2$

Solution ✪ **Motivator #4** ✪

Pre-Algebra, Algebra, geometry

Solution:

Very interesting, since it uses the famous Pythagorean Theorem in its solution.



$$A = \frac{1}{2}(4)(3) = 6 \text{ units}^2$$

Figure 1 (shows Perimeter 12 and Area of $6\,\text{units}^2$)

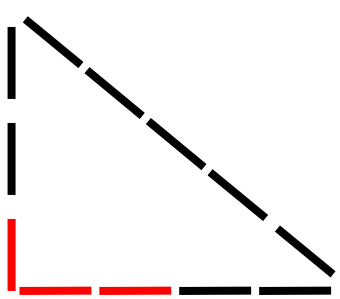


Figure 2

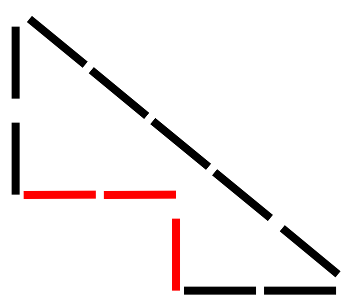
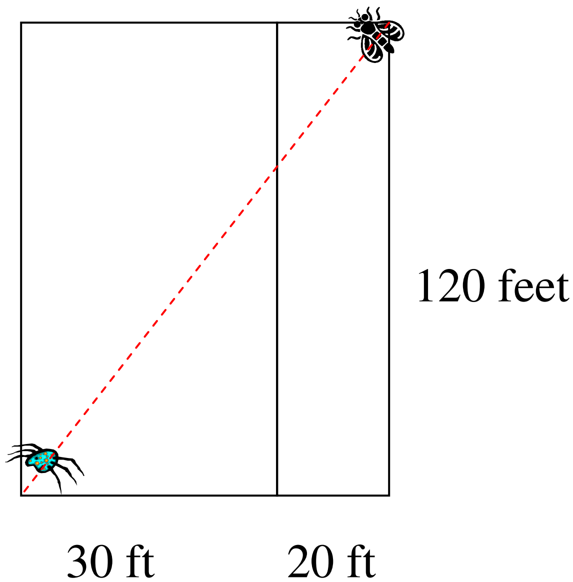


Figure 2 A (The solution 6 squares – 2 squares) = 4 squares

Solution ✪ Motivator #5 ✪

Pre-Algebra, Algebra, geometry

Answer: 130 feet



Best way is to imagine that the room is a box. Then, fold-out the box to see the path of the spider. The path then becomes the diagonal of a rectangle with sides measuring 50 ft and 120 ft.

Let $P =$ the path of the spider

$P^2 = 50^2 + 120^2$ .... This is a Pythagorean triple

$\therefore\ P = 130$ feet

Solution ✪ Motivator #6 ✪

Geometry

Answer: 1

Solution:

Let $r =$ radius of each semicircle

$\therefore\ 2r =$ radius of the quadrant

Hence, $A_{\text{Quadrant}} = A_{\text{semicirle 1}} + A_{\text{semicirle 2}} + \text{II} - \text{I}$

$$\frac{1}{4}\pi\left(2r\right)^2 = \frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2 + \text{II} - \text{I}$$

$$\therefore\ \pi r^2 = \pi r^2 + \text{II} - \text{I}$$

$$\therefore\ \text{II} = \text{I}$$

$$\therefore\ \frac{\text{I}}{\text{II}} = 1$$

Solution ✪ Motivator #7 ✪
Algebra, Geometry, Algebra II

Answer: Column 2

Solution:

Factors of $1000 = (2)(2)(2)(5)(5)(5)$

The number 1000 is a multiple of 8. Multiples of 8 appear in column #2

Solution ✪ Motivator #8 ✪

Pre-Algebra, Algebra I, Geometry, Algebra II

Solution: (I am a UCLA Bruin, so I have to give my bias version)

❶ The arithmetic is correct. The logic in deriving the answer is not correct. The USC Trojans had more Pepsi cans, that is why they spent more money. The UCLA Bruins were smarter in distributing their workers in 6 and 5 configuration rather that 7 and 3 configuration.

❷ The moral of the story could be still correct, since we know it costs more to attend U$C.

Note: I use "Do Now #8" as a start in problem solving. It reinforces the concept that we have to compare quantities which are the same and the rules must be the same for all situations. It is fun and if you are not careful you will get into a trap!!!

Solution ✪ Motivator #9 ✪

Algebra I, Geometry, Algebra II

Solution:

When we divided both sides by $(b-a)$ we divided by 0 because $a=b$.

Solution ✪ Motivator #10 ✪

Pre-Algebra, Algebra, Geometry, Algebra II

Solution:

8, 5, 4, 9, 1, 7, 6 , 3 , 2 , 0

8	5	4	9	1	7	6	3	2	0
eight	five	four	nine	one	seven	six	three	two	zero

The numbers have been arranged alphabetically.

Solution ✪ Motivator #11 ✪
Geometry, Algebra II, Math Analysis

Solution: **72341, 70523, 52703**

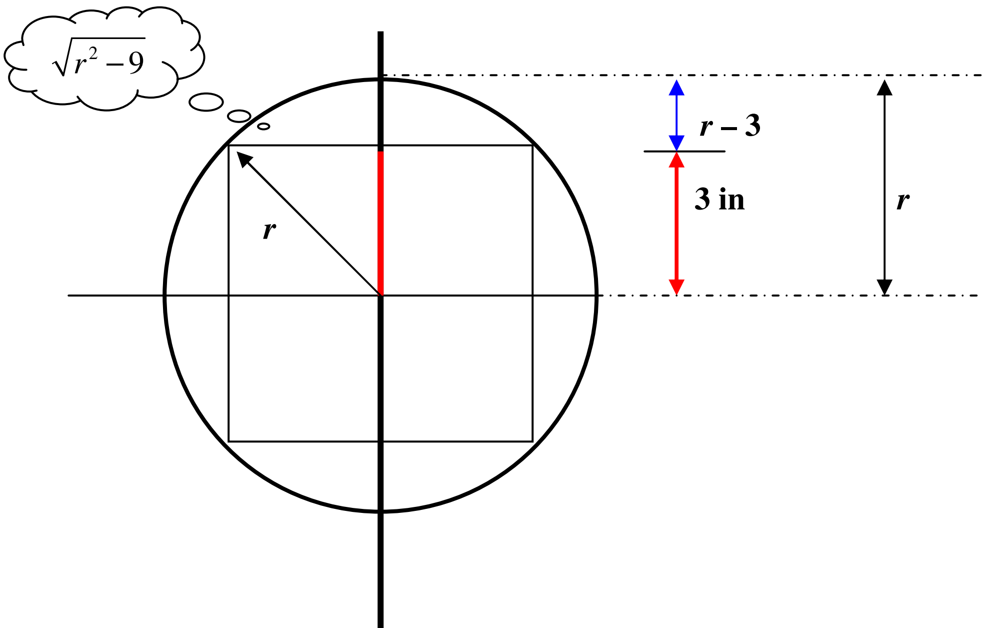
American numbers are

- ◎ **five-digit odd numbers**
- ◎ **digits alternate odd even**
- ◎ **the sum of the digits is always 17**
- ◎ **the digits never repeat**

Solution ✪ Motivator #12 ✪
Algebra II, Math Analysis

Solution:

Let us draw a cross-section of the sphere:



✪ **The radius of the cylindrical hole** $=\left(\sqrt{r^2-9}\right)$

✪ **r – 3, the altitude of the spherical caps**

✪ $V_{Cap}=\dfrac{\pi}{6}\left(4r^3-18r^2+54\right)$

✪ $V_{Cylinder}=6\pi\left(r^2-9\right)$

✪ $V_{Sphere}=\dfrac{4}{3}\pi r^3$

After simplifying

✪ $V_{desired}=\dfrac{4}{3}\pi r^3-2\left(\dfrac{\pi}{6}\left(4r^3-18r^2+54\right)\right)-6\pi\left(r^2-9\right)$

✪ **Answer:** $36\pi\ \text{in}^3$

[….the residue remains a CONSTANT regardless of the hole's diameter or the size of the sphere…]

Solution ✪ Motivator #13 ✪
Pre-Algebra, Algebra I

Solution:

Professor Hattar's Age is 61.

Let H = Professor Hattar's Age

Let G = grandson's age

$H + G = 73$ ………. Eq. #1

$H - G = 49$ ……..... Eq. #2

$2H = 122$

$H = 61$ and $G = 12$

Solution ✪ Motivator #14 ✪
Pre-Algebra

Solution: (answers may vary)

$$4+4-4-4=0$$

$$(4\div 4)+(4-4)=1$$

$$(4\div 4)+(4\div 4)=2$$

$$(4+4+4)\div 4=3$$

$$(4-4)\times 4+4=4$$

$$\left[(4+4)\div 4\right]+4=6$$

$$4+4-(4\div 4)=7$$

$$(4\times 4\div 4)+4=8$$

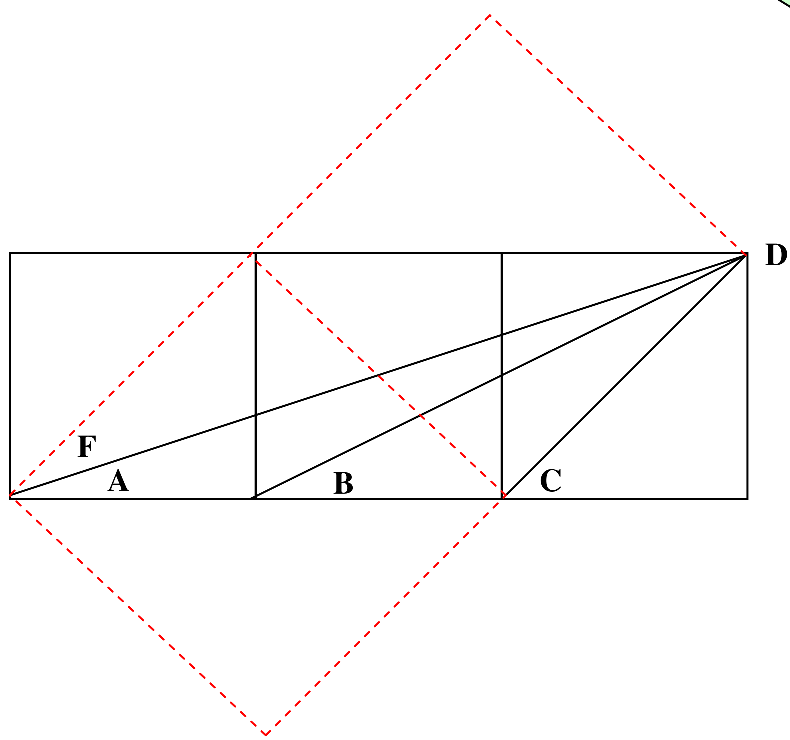
$$4+(4\div 4)+4=9$$

Solution ✪ Motivator #15 ✪

Geometry

The PROOF presented here is the most INTERESTING one .

Show That: $m\angle A + m\angle B = m\angle C$



Statement	Reason
1. Construct the squares indicated by the dash lines.	By Construction
2. $m\angle B = m\angle F$	Corresponding angles of similar right triangles
3. Since, $m\angle A + m\angle F = m\angle C$	
4. $m\angle B$ can be substituted for $m\angle F$	
It follows immediately that $m\angle A + m\angle B = m\angle C$	

Q.E.D.

Solution ✪ Motivator #16 ✪

Pre-Algebra, Algebra

Answers may vary

a) $5-\left[\left(5+5\right)\div 5\right]=3$

b) $5-\left[\left(5-5\right)\cdot 5\right]=5$

c) $\left[\left(5\cdot 5\right)+5\right]\div 5=6$

d) $\left(5\div 5\right)+5+5=11$

e) $\left(5\cdot 5\right)-\left(5\div 5\right)=24$

f) $\left[\left(5\cdot 5\right)+5\right]-5=25$

g) $\left[5+5\div 5\right]\cdot 5=30$

h) $5\cdot 5\cdot 5-5=120$

Solution ✪ Motivator #17 ✪
Algebra

[7]Vik and Ahmed's elegant Solution:

Vik and Ahmed argued, what's under the biggest square root is 9, because $\sqrt{9}$ is 3.
They cleverly set what's under the biggest radical then, equal to 3.

$$\left[\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}}}\right]^2 = [3]^2$$

$$x+\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}} = 9$$

$$x+\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}} = 9$$

But she pointed out that

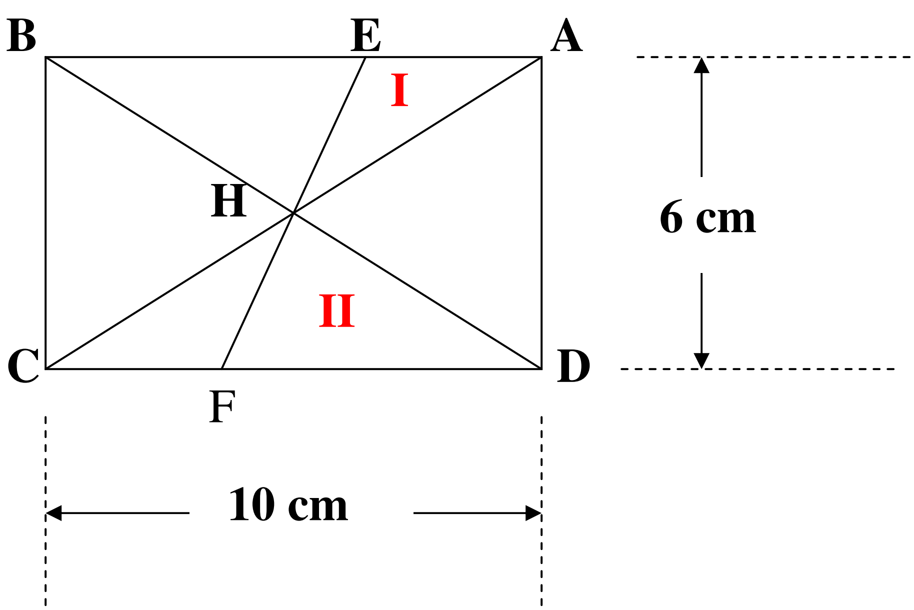
$$x+\underbrace{\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}}}_{\Uparrow} = 9$$

But this much is the original problem ! and it equals to 3

Hence, $x+3=9$
And $x=6$ the final answer.

[7] Dr. Nasser Al-Uwashig is an educator in Saudi Arabia and in charge of the first US K-12 Math Program translation in the world.

Solution ✪ Motivator #18 ✪
Geometry



Solution: The Area of $\triangle BHE$ = Area $\triangle DHF$

The Area of $\triangle AHE$ = Area $\triangle CHF$

The Area of $\triangle BHA = \frac{1}{4}(10)(6) = 15$

$\therefore$ Area I + Area II $= 15 \text{ cm}^2$

Note: Students need to show that the triangles are congruent first and then proceed in making the connection.

Solution ✪ Motivator #19 ✪
Algebra

Let G = # of Girls
Let C = # of Cats

$$G + C = 17$$
$$2G + 4C = 44$$

Therefore, G = 12; C = 5

Check ☑ (Heads)

12 Girls = 12 Heads
5 Cats = 5 Heads

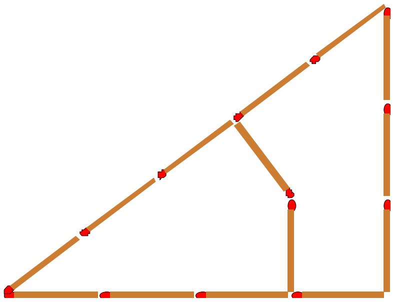
Check ☑ (Feet)

12 Girls = (12)(2) = 24 Feet
5 Cats = (5)(4) = 20 Feet

Solution ✪ Motivator #20 ➲ A ✪

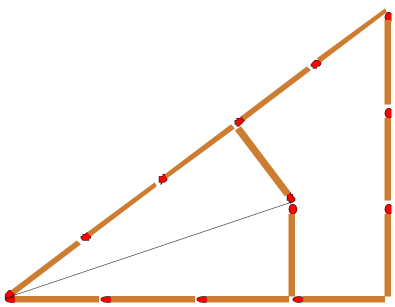
Pre-Algebra, Geometry

Solution:



Puzzle 1

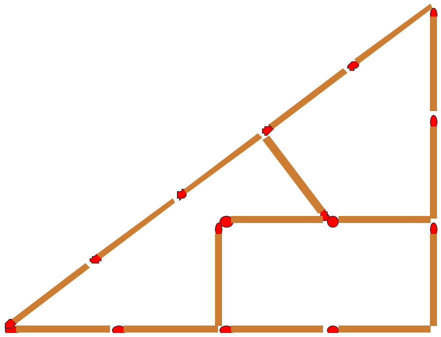
Two matches fit exactly as shown.

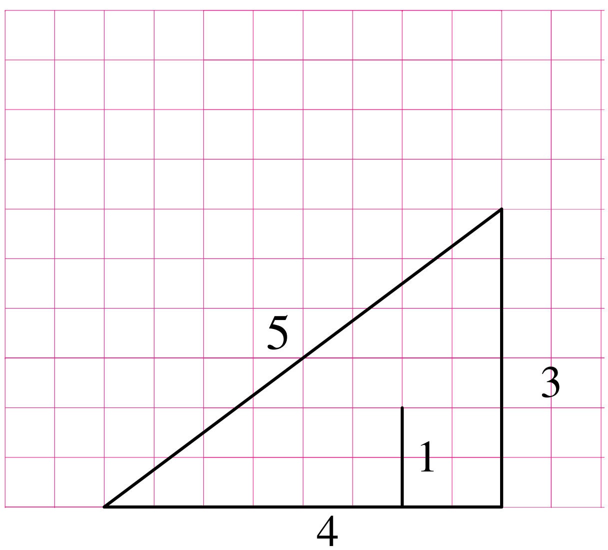


Each space has an area of 3 units.
Look at the kite which is made up of two triangles each of area 1 ½ units.

Solution ✪ Motivator #20 ➲ B ✪

Solution:





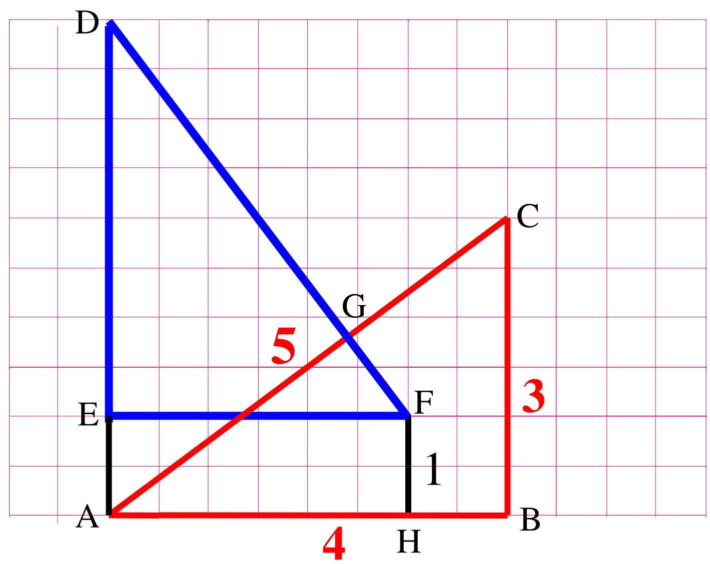
Here is an explanation of the way the matches perfectly divide the space;

The triangle is a 3 4 5 as shown. A line of 1 unit (one match) is added 3 units along the base.

Construct a congruent right triangle **DEF** at the point **F** then **DF** will be perpendicular to **AC**,

DF = 5 units
DE = 4 units
EF = 3 units

Extending **DE** to **A** creates a third congruent right triangle ADG in which **AD** = 5, **DG** = 4, **AG** = 3 units. Do you see the beauty!!!



It is now clear that GF = DF – DG = 5 – 4 = 1 unit **AG** = 3 units and **GC** = 2 units.

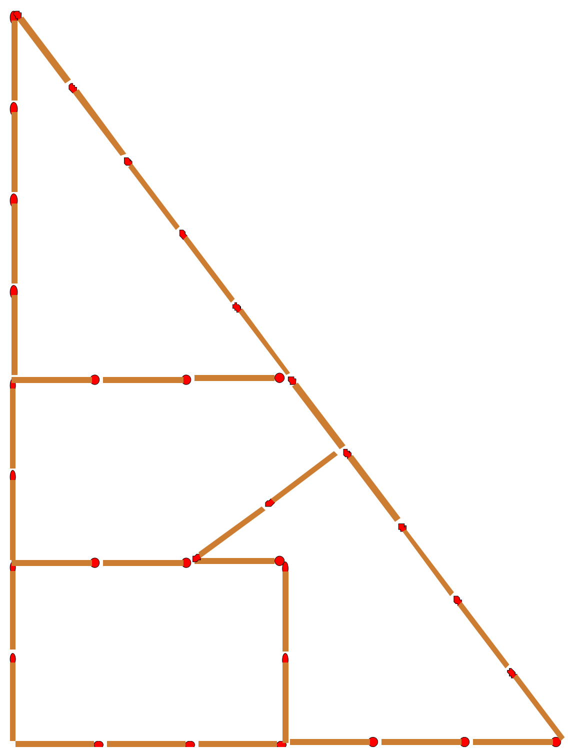
Solutions ✪ Motivator #20 ➲ C ✪

To 2 Parts (Puzzles 3 & 4)

A construction allows us to see what happens in the 6, 8, 10 triangle when three matches are placed perpendicular to the base at the 7 cm point; the match of 1 unit length once again fits perfectly.

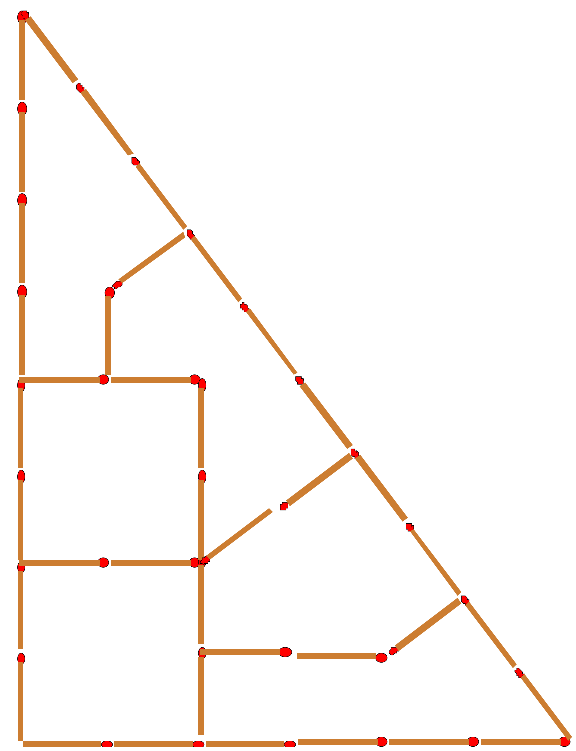
Solutions

Puzzle 3



Using 10 matches

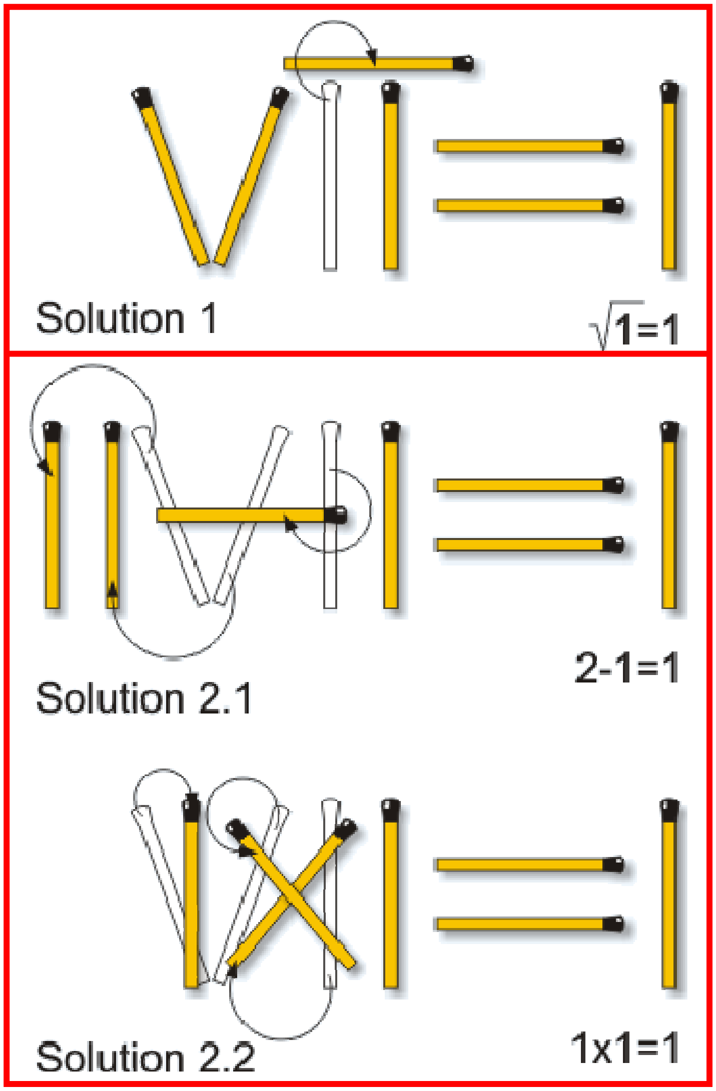
Puzzle 4



Using 15 matches

Solutions ✪ Motivator #21 ✪

Puzzles 1 & 2

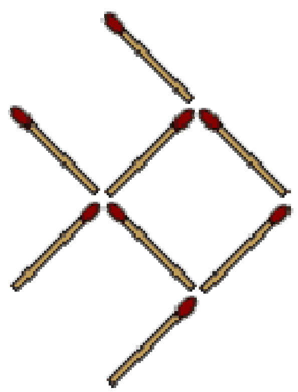


Puzzle 1

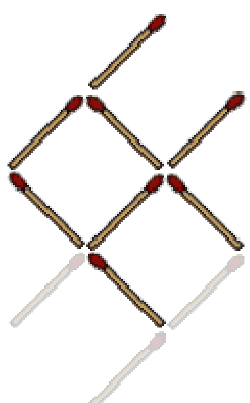
Puzzle 2

Solution ✪ Motivator #22✪
Pre-Algebra

Solution:



Original



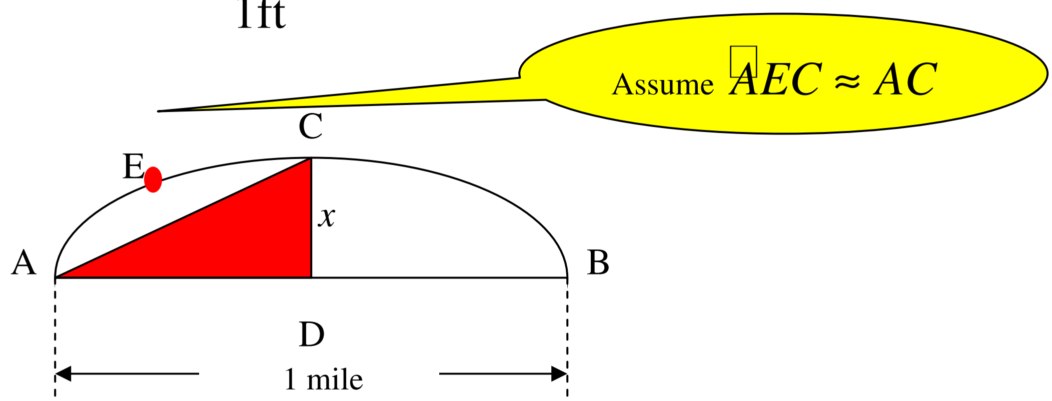
Solution

Solution ✪ Motivator #23 ✪

Geometry, Algebra II

Solution: **Answer** ≈ 13.34 **feet**

$$1 \text{ mile} = 5280 \text{ feet} \cdot \frac{12\,\text{in}}{1\,\text{ft}} = 63{,}360 \text{ in}$$



$$AD = \frac{63360}{2} = 31680 \text{ in}$$

$$AC = 31680 + .5 = 31680.5 \text{ in}$$

$$(AD)^2 + (DC)^2 = (AC)^2$$

$$(31680)^2 + x^2 = (31680.5)^2$$

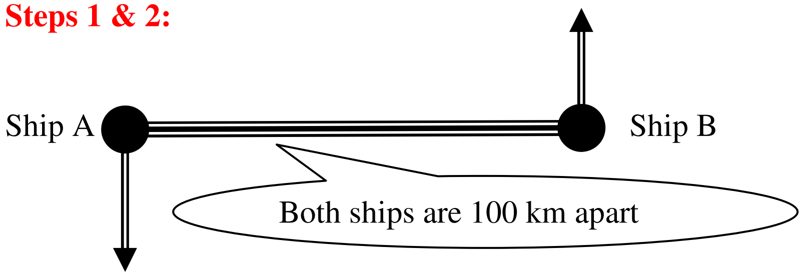
$$x^2 \approx 31680.25, \quad x \Rightarrow 177.9896 \text{ in}$$

$$\frac{177.9896}{12} \approx 13.34 \text{ feet}$$

Solution ✪ Motivator #24 ✪

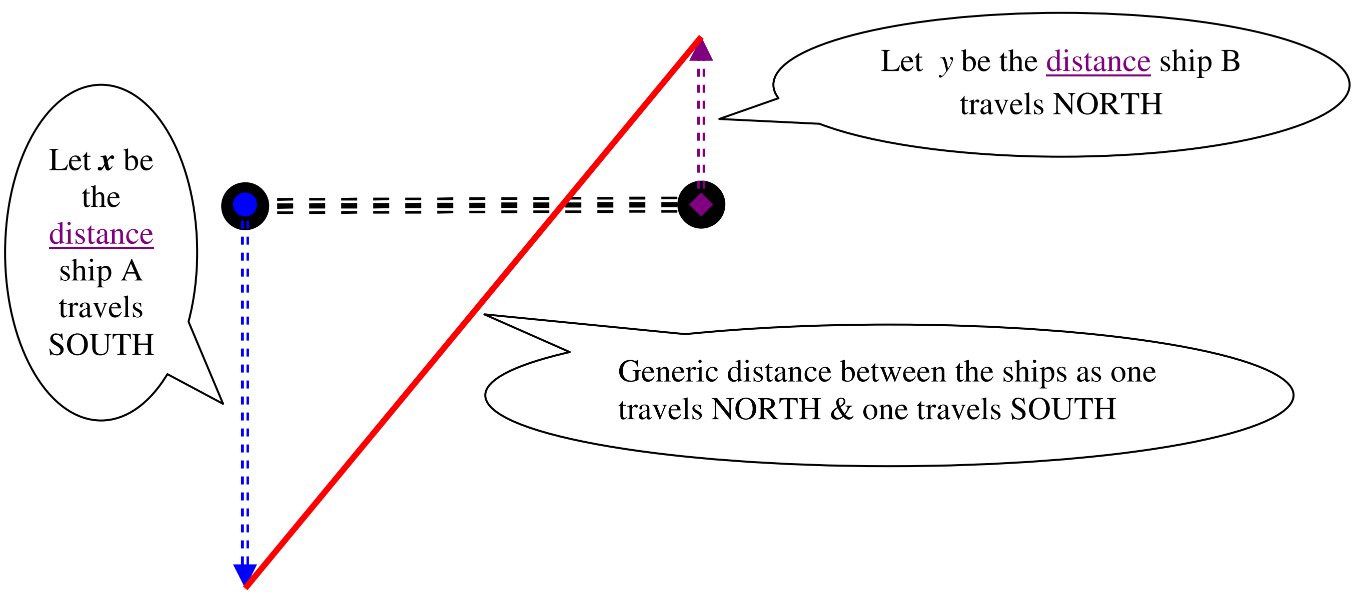
Calculus

Steps 1 & 2:

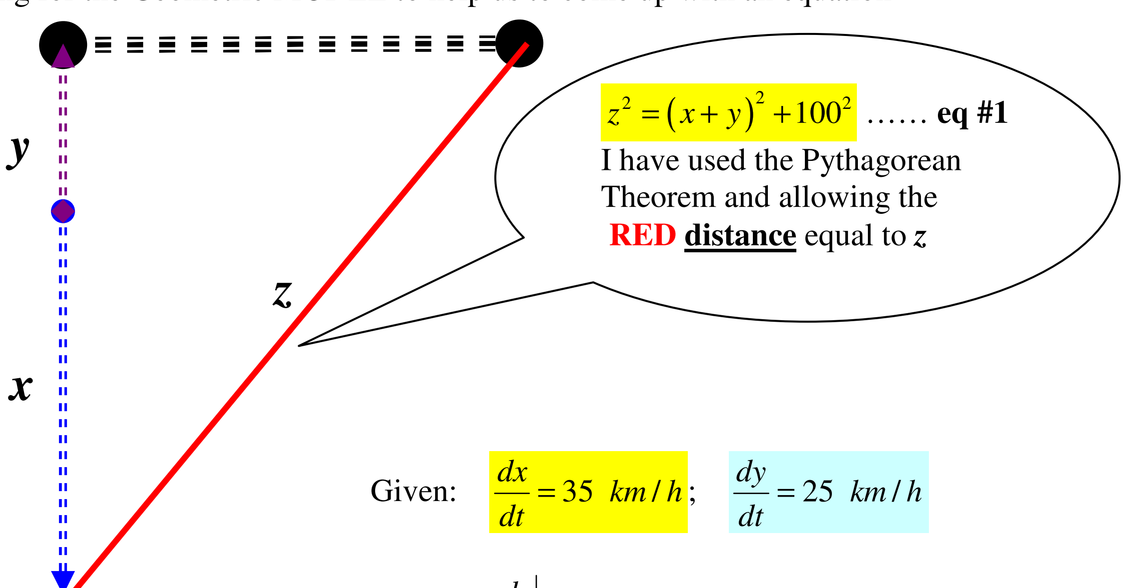


At NOON

After some time we have the following **generic** situation:



Step 3: Searching for the Geometric MODEL to help us to come up with an equation



Given: $\dfrac{dx}{dt} = 35 \ \ km/h$; $\dfrac{dy}{dt} = 25 \ \ km/h$

Find: $\left.\dfrac{dz}{dt}\right|_{at\ 4{:}00\ pm}$

Continued

Solution (continued) ✪ Motivator #24 ✪
Calculus

Here comes the most important connection at 4:00 PM !!!

Distance = (Rate)(Time) …………………………….. remember back in Algebra ?

Now you need to write ***x***, ***y***, and ***z*** in terms of distance

$x = 4(35) = 140$ km

$y = 4(25) = 100$ km

differentiate

$z = \sqrt{(140+100)^2 + 100^2} = 260$ km

Step 4: Now you are ready to GIVE LIFE to $z^2 = (x+y)^2 + 100^2$ …………….. equation #1

$$\frac{d}{dt}\left[z^2 = (x+y)^2 + 100^2\right]$$

$$2z\frac{dz}{dt} = 2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

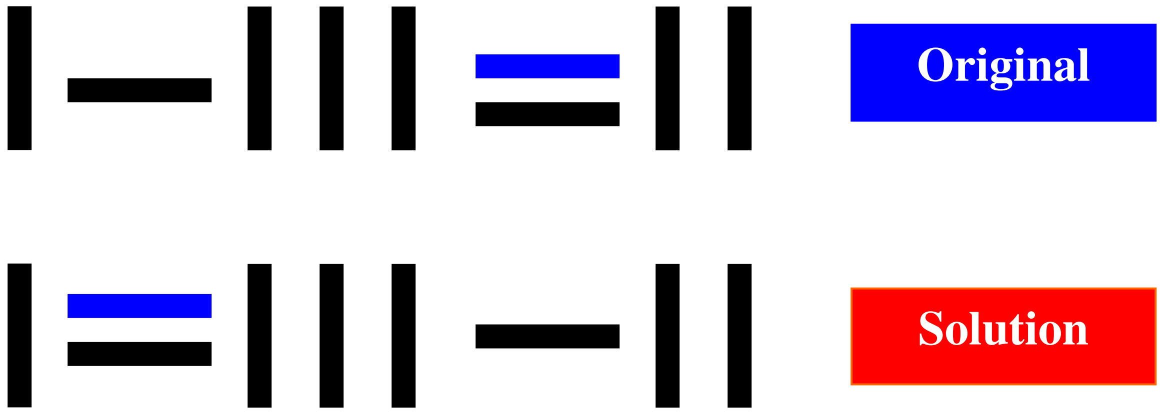
Step 5: *Substitute* all the values and you will get the answer for $\left.\frac{dz}{dt}\right|_{at\ 4:00\ pm}$

$$= \frac{720}{13} \approx 55.4 \quad km/h$$

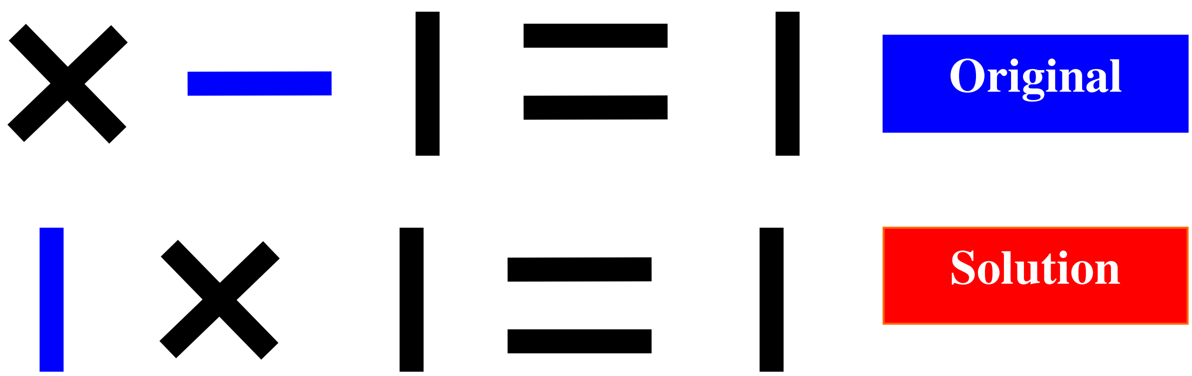
Now write a sentence explaining what the answer means: The distance between the ships is changing at the rate of 55.4 *km/h*.

Solution ✪ Motivator #25 ✪
Pre-Algebra, Algebra

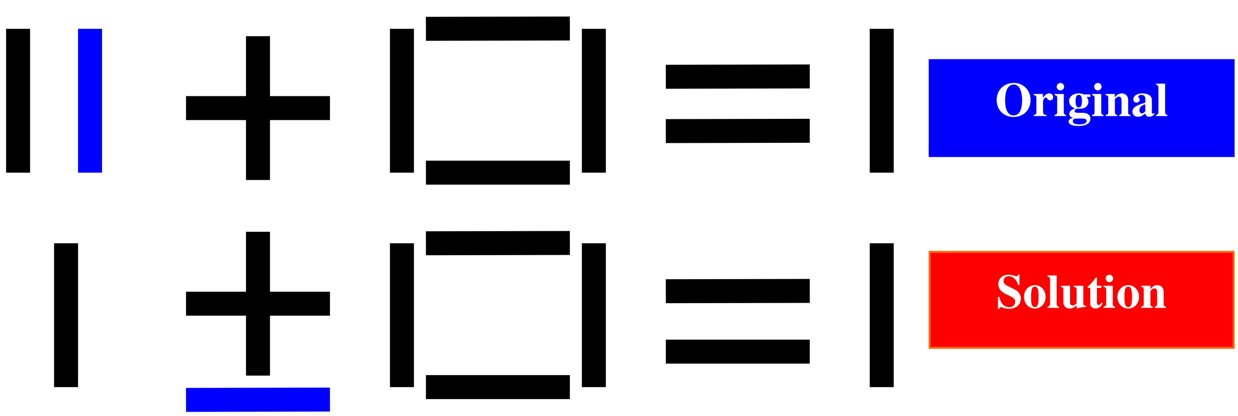
1.



2.



3.

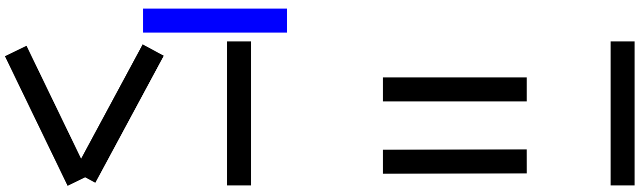


Solution ✪ Motivator #25 ✪

Pre-Algebra, Algebra

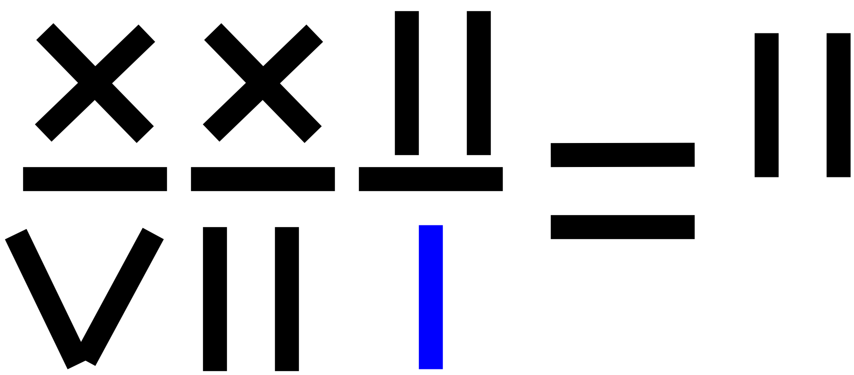
4.

Original

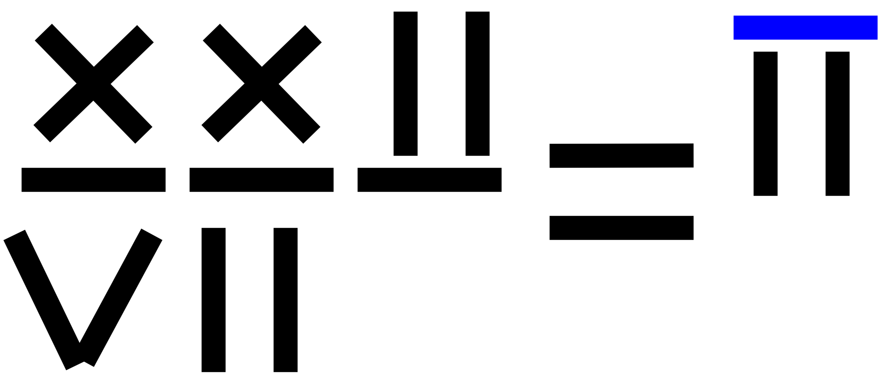


Solution

5.



Original



Solution

Solution ✪ Motivator #26 ✪
Algebra II

Solution: Answer $2,193.93

$$P=\frac{\frac{r}{12}l}{1-\left(1+\frac{r}{12}\right)^{-m}}$$

$$P=\frac{\frac{r}{12}l}{1-\left(1+\frac{r}{12}\right)^{-m}}=\frac{\frac{.10}{12}(250{,}000)}{1-\left(1+\frac{.10}{12}\right)^{-36}}$$

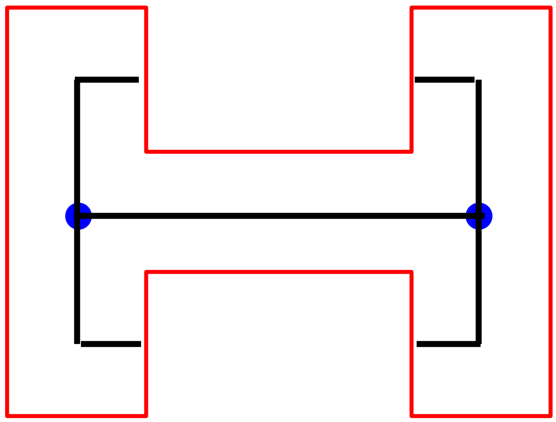
$P=\$2193.93$ monthly payment

$2193.93\times 360=\$789{,}814.41$ Total

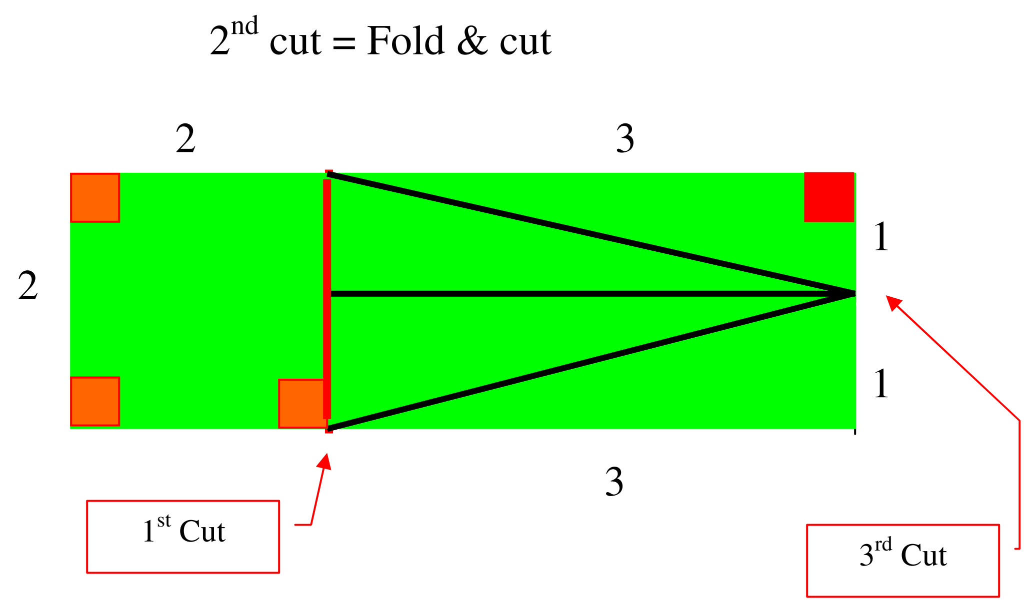
$789{,}814.41-\$250{,}000.00=\$539{,}814.41$ interest

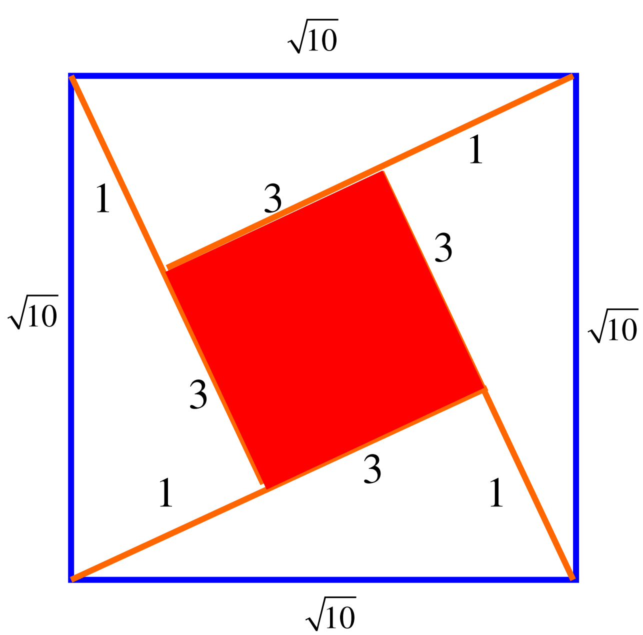
Solution ✪ Motivator #27 ✪

Geometry



Solution ✪ Motivator #28 ✪
Geometry





Note: Figures NOT to scale

Solution ✪ Motivator #29 ✪
Algebra II

Solution: $4 + 3 + 2 + 1 = 10$

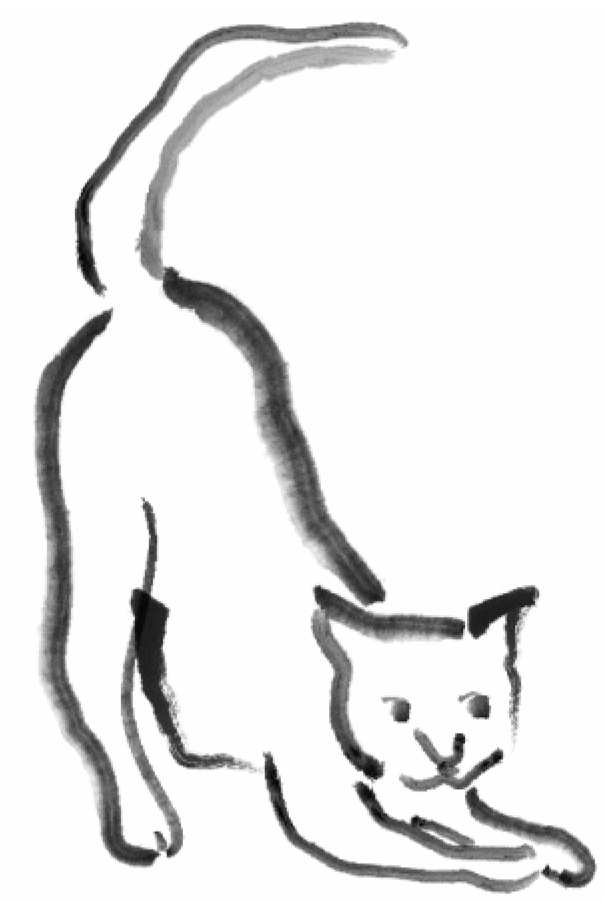
Or

$$_5C_2 = \frac{5!}{2!(5-2)!} = 10$$

Solution ✪ Motivator #30 ✪

Pre-Algebra

<u>Solution</u>: Answer 16 days



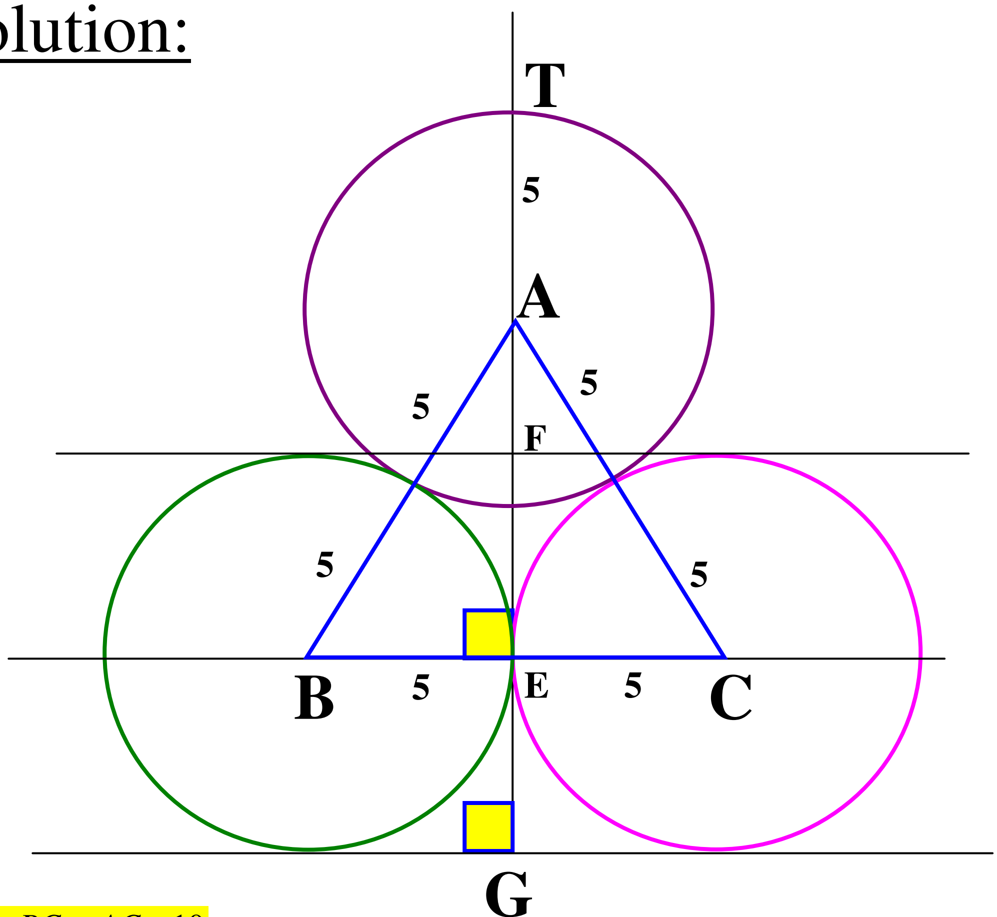
After 15 days the cat still has 3 feet to go. The cat does this the next day and is at the top of the well after 16 days.

Solution ✪ Motivator #31 ✪

Geometry

Answer: 18.66 feet

Solution:



$AB = BC = AC = 10$

$m\angle B = 60^0$, $m\angle BAE = 30^0$

$AE = 5\sqrt{3}$; $AF = AE - FE$; $AF = 5\sqrt{3} - 5$

$TG = GF + FA + AT$

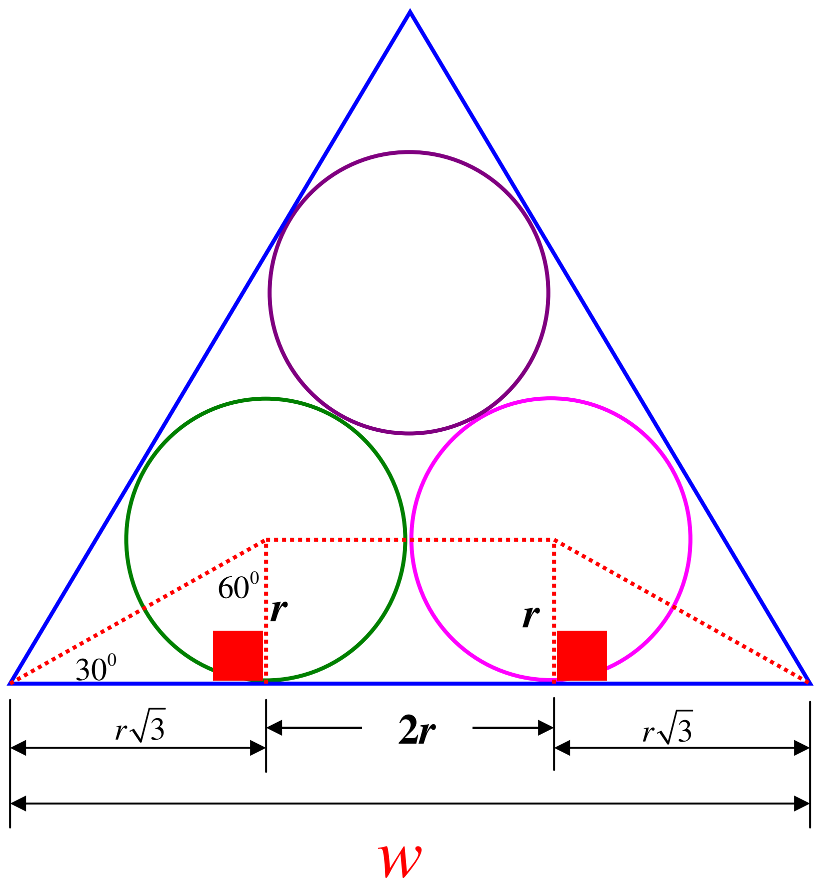
$= 10 + 5\sqrt{3} - 5 + 5$

$= 10 + 5\sqrt{3}$

$TG \approx 18.66$ feet

Solution ✪ Motivator #32 ✪
Geometry

Solution: Answer $r = \frac{w}{2+2\sqrt{3}}$



$w = r\sqrt{3} + 2r + r\sqrt{3}$

$w = 2r + 2r\sqrt{3}$

Isolate ***r*,** $r = \frac{w}{2+2\sqrt{3}}$

Solution ✪ Motivator #33 ✪
Algebra 1

Solution:

1 dollar = 15.5 cm or $6\frac{1}{8}$ inch

a) $6.125 \times 1{,}000{,}000{,}000 = 6{,}125{,}000{,}000$ inches

$$(6{,}125{,}000{,}000\ \ in)\left(\frac{1\ foot}{12\ in}\right)\left(\frac{1\ mile}{5280\ \ foot}\right) = 96{,}669.82\ \ miles$$

b) The <u>diameter</u> of the penny can be easily obtained by using a ruler. We measured it as .75 inches or 19 Millimeters. (Don't take our word for it though – try these measurements yourself.)

Determining the <u>thickness</u> is somewhat tougher isn't it? Instead of trying to measure 1 penny's thickness why not several stacked together? (We measured 30 pennies as 43 millimeters, making each penny 1.43 mm thick). A more precise method would be to use something not as common as a ruler – a micrometer. We used one and obtained a value of 1.27 mm (or .05 inches) for the thickness.

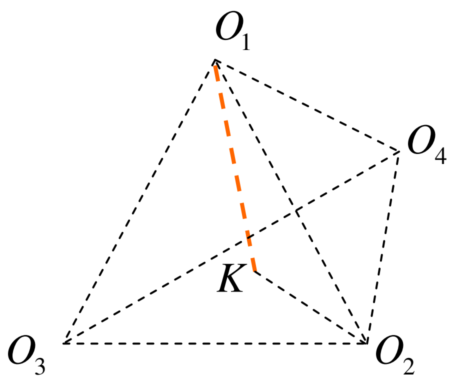
Official Measurements:

Value	**One cent**
Pictures on both side of the coins	Lincoln and Lincoln memorial
Standard weight	2.5 grams
Standard diameter	0.750 in. or 19.05 mm
Thickness	1.55 mm
Legal/official (exact) definitions of inch-pound units as set by U.S. law: • 1 inch = 25.4 millimeters	

$$(1{,}000{,}000{,}000\ \ cents)\left(\frac{1.55\ mm}{1\ cent}\right)\left(\frac{1\ inch}{25.4\ mm}\right)\left(\frac{1\ foot}{12\ inches}\right)\left(\frac{1\ mile}{5280\ \ feet}\right) = 963.125348\ miles$$

Solution ✪ Motivator #34 ✪
Geometry

Spheres are tangent



$\therefore \quad O_1O_2 = O_1O_3 = O_1O_4 = O_3O_4$

$O_1O_2O_3O_4$ is a regular tetrahedron

K is the center of $\Delta O_3O_4O_2$

$$O_2K = \frac{2}{3}\sqrt{3}$$

$O_1K =$ altitude

$$(O_1K)^2 + (O_2K)^2 = (O_1O_2)^2$$

$$(O_1K)^2 = 2^2 - \left(\frac{2}{3}\sqrt{3}\right)^2 = \frac{8}{3}$$

$$O_1K = \sqrt{\frac{8}{3}} = \frac{2}{3}\sqrt{6}$$

Distance = $1 + \frac{2}{3}\sqrt{6} + 1 = 2 + \frac{2}{3}\sqrt{6} + 1 = 2 + \frac{2}{3}\sqrt{6} \approx 3.633$ inches

Solution ✪ Motivator #35 ✪
Pre-Algebra

1+2+34-5+67-8+9 = 100
12+3-4+5+67+8+9 = 100
123 - 4 - 5 - 6 -7+8 – 9 =100
123+4-5+67-89 =100
123+45-67+8-9 =100
123-45-67+89 = 100
12-3-4+5-6+7+89 = 100
12+3+4+5-6-7+89 = 100
1+23-4+5+6+78-9 = 100
1+23-4+56+7+8+9 = 100
1+2+3-4+5+6+78+9 =100

If we put a "-" before 1, we have one more solution:

-1+2-3+4+5+6+78+9=100

Using the "." decimal separation we found another solution:

1 + 2.3 – 4 + 5 + 6.7 + 89 = 100

What about 987654321 ?

98-76+54+3+21=100
9-8+76+54-32+1=100
98+7+6-5-4-3+2-1=100
98-7-6-5-4+3+21=100
9-8+76-5+4+3+21=100
98-7+6+5+4-3-2-1=100
98+7-6+5-4+3-2-1=100

98+7-6+5-4-3+2+1=100
98-7+6+5-4+3-2+1=100
98-7+6-5+4+3+2-1=100
98+7-6-5+4+3-2-1=100
98-7-6+5+4+3+2+1=100
9+8+76+5+4-3+2-1=100
9+8+76+5-4+3+2+1=100
9-8+7+65-4+32-1=100

Write the sign "-", three solutions:

Solution ✪ Motivator #35 ✪
Pre-Algebra

Write the sign "-", three solutions:

-9+8+76+5-4+3+21=100
-9+8+7+65-4+32+1=100
-9 - 8+76 - 5+43+2+1=100

With the decimal point:

9 + 87.6 + 5.4 – 3 + 2 – 1 = 100

If I "shuffle" the digits there are many solutions. I found some when I was young, for example:

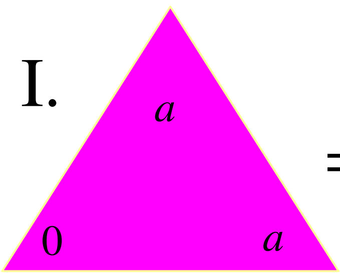
91 + 7.68 + 5.32 – 4 = 100
98.3 + 6.4 - 5.7 + 2 – 1 =100
538 + 7 – 429 – 13 = 100
(8 x 9.125)+37 – 6 – 4 = 100 etc etc etc ....

Solution ✪ Motivator #36 ✪

Geometry, Algebra 2

The answer is III.

I.



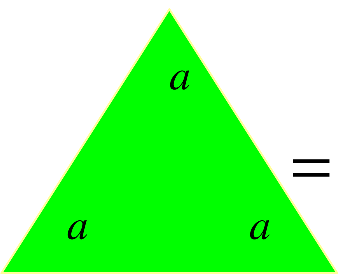
$= 0$

$$(a)(0)(a)-(a+0+a)=0$$

$$-2a=0$$

Hence, $a=0$ (not a + positive integer)

II.



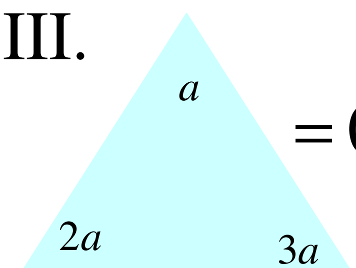
$= 0$

$$(a)(a)(a)-(a+a+a)=0$$

$$a^3-3a=0;\ a(a^2-3)=0$$

$$a=0,\ \ a=\pm\sqrt{3}$$ (not a + positive integer)

III.



$= 0$

$$(a)(2a)(3a)-(a+2a+3a)=0$$

$$6a^3-6a=0$$

$$6a\left(a^2-1\right)=0$$

$$a=0;\ \ a=\pm 1$$

Hence $a=1$ (is a positive integer)

Solution ✪ Motivator #37 ✪
Algebra 2, Math Analysis

Solution:

a)

$$1^{\frac{1}{2}}+\frac{1}{2}\left(1^{-\frac{1}{2}}\right)x+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1\cdot 2}\left(1^{-\frac{3}{2}}\right)x^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\cdot 2\cdot 3}\left(1^{-\frac{5}{2}}\right)x^3=$$

$$1+\frac{x}{2}-\frac{1}{8}x^2+\frac{x^3}{16}$$

b) $\sqrt{2}=(1+1)^{\frac{1}{2}}=1+\frac{1}{2}-\frac{1}{8}+\frac{1}{16}\approx 1.4375$

Solution ✪ Motivator #38 ✪
Algebra 2, Math Analysis

If a set of *n* elements has n_1 elements of one kind alike, n_2 of another kind alike, and so on, then the number of permutations, P, of the n elements taken n at a time is given by

$$P = \frac{n!}{n_1!n_2!\cdots}$$

j) **EACALANTE** $\frac{9!}{2!3!}$

k) **AMERICAN** $\frac{8!}{2!}$

l) **MISSISSIPPI** $\frac{11!}{4!4!2!}$

m) **SCHWARZENEGGER** $\frac{14!}{3!2!2!}$

n) **HOVSEPIAN** $\frac{9!}{1!}$

o) **ANNETTE** $\frac{7!}{2!2!2!}$

p) **HATTAR** $\frac{6!}{2!2!} = 180$

q) **SHAMONEH** $\frac{8!}{2!}$

r) **FOOTBALL** $\frac{8!}{2!2!}$

Math Is Fun

Appendix E

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Magic In Multiplication

Magic 1: Multiplying any single digit by 9

- Method One ✪ Page 124
- Method Two ✪ Page 125

Magic 2: Multiplying any 2 digits ✪ Page 126

Magic 3: Multiplying two digits by 9

- **Case A:** ✪ Page 127
  When the second digit is **LARGER** than the first digit
- **Case B:** ✪ Page 128
  When the second digit is **SMALLER** than the first

Trivia: From a million to googolplex ✪ Page 129

Magic 4: Multiplying two digit numbers ✪ Page 130

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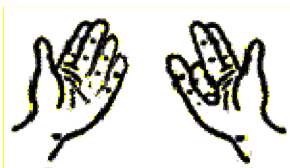
Magic 1: Multiplying any single digit by 9
Method One

- **Multiply 9 by 7**
  - Spread your two hands in front of you ..with the fingers spread Up
  - From left to right count 7 fingers, Bend down the last finger (the seventh)
  - What you will see (from left to right): six fingers up, one finger down, and three up
  - Consider the finger that is bent down is a separator
  - The answer is 6 (the first six fingers) ⇨ (6)(10) = 60 and 3 (the following 3 fingers
  - Therefore the answer is 60 + 3 = 63

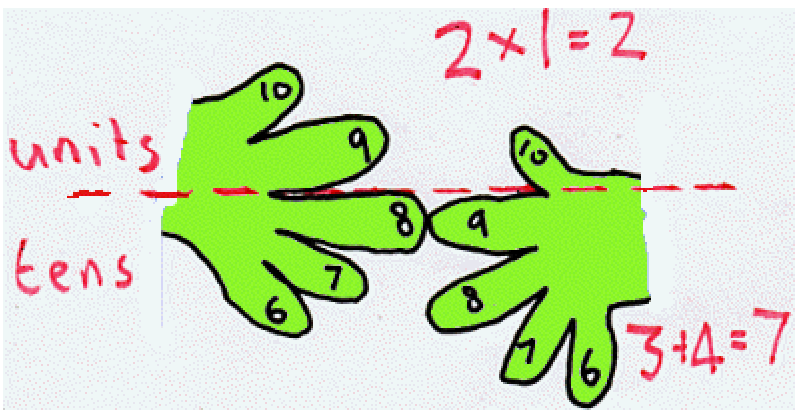


- **Multiply 9 by 3**
  - Spread your two hands in front of you ..with the fingers spread Up
  - From left to right count 3 fingers, Bend down the last finger (the third)
  - What you will see (from left to right): Two fingers up, one finger down, and Seven up
  - Consider the finger that is bent down is a separator
  - The answer is 2 (the first two fingers) ⇨ (2)(10) = 20 and 7 (the following 7 fingers)
  - Therefore the answer is 20 + 7 = 27

Magic 1: Multiplying any single digit by 9

Method Two

- First you need to imagine your fingers on each hand are numbered like below.

- Now suppose you want to multiply 8 x 9, you touch together the 8 finger from one hand and the 9 finger of the other hand.
- 

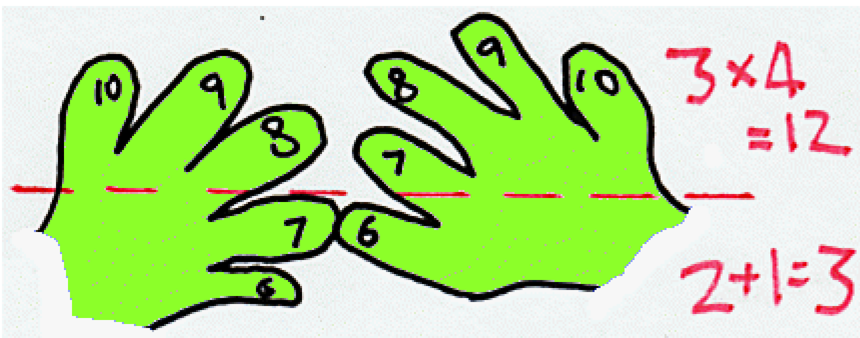
- You work out the "tens" and "units" separately:
- To get the "tens" you count up the fingers that are touching and all the fingers underneath. Here you'll see there are 7 fingers that count altogether, so that gives us 7 tens or 70.
- To get the "units" you look at the fingers on each hand above the two touching fingers, and multiply these numbers together. Here you'll see 2 fingers on one hand and one finger on the other, so you get 2x1= 2 units
- That gives us a final answer of 70 + 2 = 72.

It works! Because 8 x 9 DOES make 72!

Magic 2: Multiplying any 2 digits

Multiply 7 x 6 = 42

- Now suppose you want to multiply 7 x 6, you touch together the 7 finger from one hand and the 6 finger of the other hand. (see diagram below)

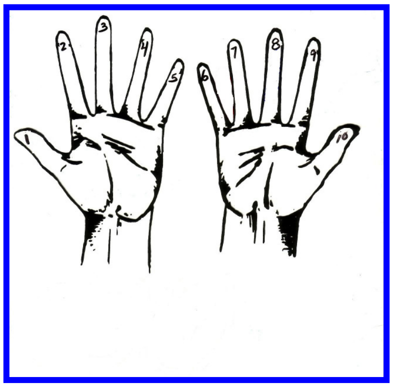


- You work out the "tens" and "units" separately.
- To get the "tens" you count up the fingers that are touching and all the fingers underneath. Here you'll see there are 3 fingers that count altogether, so that gives us 3 tens or 30.
- To get the "units" you look at the fingers on each hand above the two touching fingers, and multiply these numbers together. Here you'll see 3 fingers on one hand and four fingers on the other, so you get 3 x 4= 12 units
- That gives us a final answer of 30 + 12 = 42.

It works again! Because 7 x 6 DOES make 42

Magic 3: Multiplying 2 digits by 9 (up to 99)

Case A: When the second digit is **LARGER** than the first digit



The algorithm used is similar to the following:

- To multiply a two-digit number by 9,
- ENTER the multiplicand by forming a between the tens-digit finger and the next finger to its right (SPREAD).
- ENTER the units digit by FOLDING the units-digit finger.

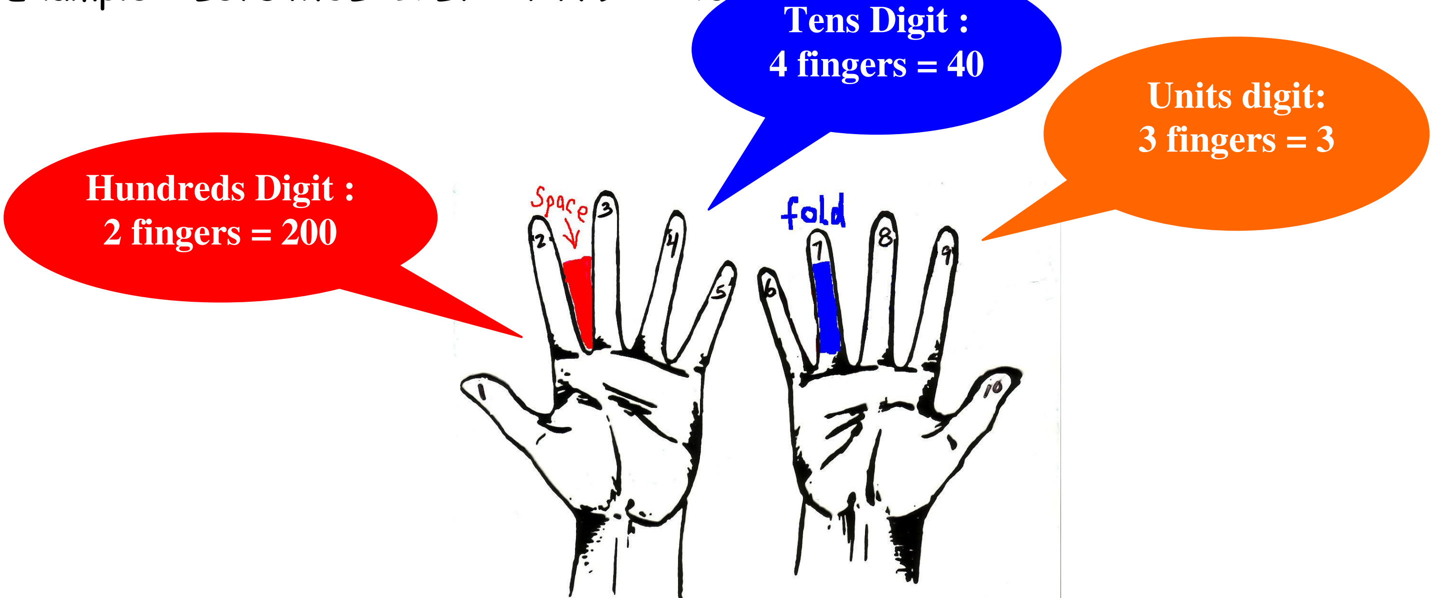
To read the product

Hundreds digit: Count the standing fingers before the SPREAD

Tens digit: Moving from left to right, count the fingers from the SPREAD to the FOLD; wrap around if necessary.

Units digit: Count the standing fingers to the right of the FOLD.

Example: Let's MULTIPLY 27 X 9 = 243

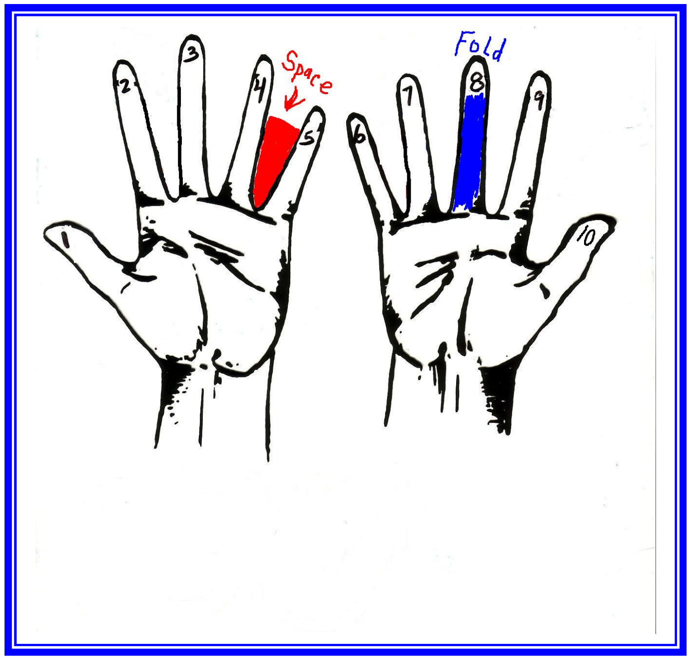


As you can see:
200 + 40 + 3 = 243

Additional Example:

48 x 9

Enter (1 2 3 4 SPACE 5 6 7 F 9 0)



Read:

Hundreds (1 2 3 4) 4

Tens (5 6 7) 3

Units (9 0) 2

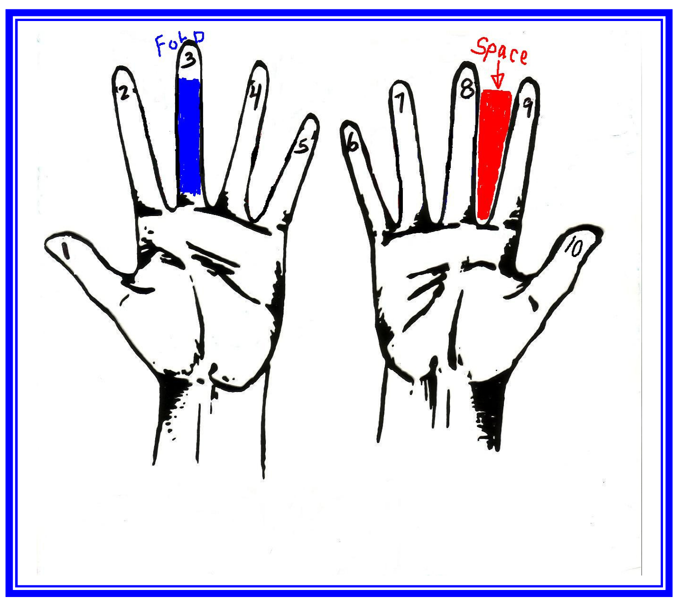
Answer: 432

Magic 3: Multiplying 2 digits by 9 (up to 99)

Case B: When the second digit is SMALLER than the first digit

Example: **83 x 9**

Enter (1 2 F 4 5 6 7 8 SPACE 9 0)



Read:

Hundreds (1 2 4 5 6 7 8) 7

Tens (9 0 1 2) 4 (wraps around)

Units (4 5 6 7 8 9 0) 7

Answer: 7 4 7

Trivia: Do you know the academic vocabulary of some of the numbers?

Here they are:

million	10^6
billion	10^9
trillion	10^{12}
quadrillion	10^{15}
quintillion	10^{18}
sextillion	10^{21}
septillion	10^{24}
octillion	10^{27}
nonillion	10^{30}
decillion	10^{33}
undecillion	10^{36}
duodecillion	10^{39}
tredecillion	10^{42}
quatuordecillion	10^{45}
quindecillion	10^{48}
sexdecillion	10^{48}
septendecillion	10^{54}
octodecillion	10^{57}
novemdecillion	10^{60}
vigintillion	10^{63}
googol	10^{100}
googolplex	$10^{googol} = 10^{10100}$

Multiplying 2 digit numbers

Step 1: Multiply the ones digit together

Step 2: Cross-multiply and add

Step 3: Multiply the tens digit together

$$\begin{array}{r} 1\,2 \\ \times\,2\,3 \\ \hline 6 \end{array}$$

(2x3)

$$\begin{array}{r} 1\,2 \\ \times\,2\,3 \\ \hline 7\,6 \end{array}$$

(1x3)+(2x2)

$$\begin{array}{r} 1\,2 \\ \times\,2\,3 \\ \hline 2\,7\,6 \end{array}$$

(1x2)

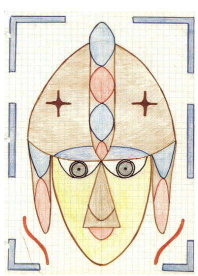
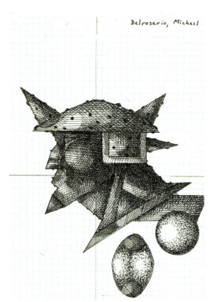
Answer is 276

Fun

With Mathematics

Appendix F

Projects

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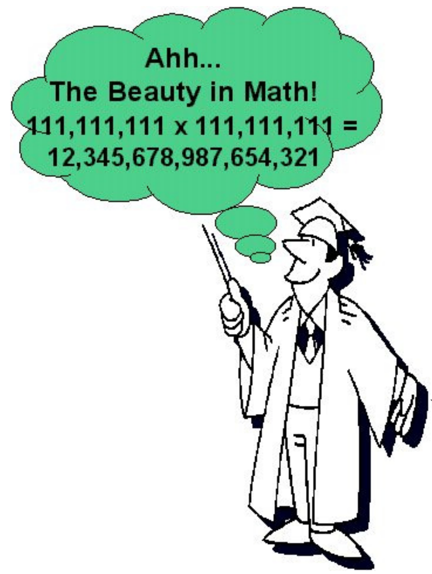
Fun With Mathematics

The following pages are some of the fascinating projects we have been using over the years.
We hope you and your friends will enjoy them.



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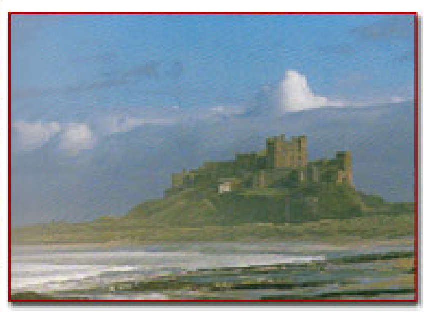
Three of the biggest challenges facing teachers are adequate planning, classroom management and a BAG of Math Is Fun motivators.
Gaining confidence in teaching takes time and requires goal setting and reflection.
Novice teachers might make their initial goals becoming familiar with teacher standards and subject matter standards plus benchmark indicators at the state and national levels.
Another goal might be to investigate assessment methods and how they might be incorporated into lesson plans. Traditional assessments include multiple choice, true/false, and matching, for example. However, consider alternative assessments as short answer questions, essays, portfolios, journal writing, oral presentations, demonstrations, creation of a product, student self-assessment, or performance tasks that are assessed by predetermined criteria.
This collection of MATH Tid-Bits has made Michael Hattar and Vik Hovsepian survive the classroom pressures.

Conic Pictures Project	Pages 134 - 137
• Sample Student Work	Pages 138 - 142
Linear Picture Project	Pages 143 - 144
• Sample Student Work	
Linear Programming Project	Pages 145 - 146
A Bobo Project [King Arthur's Round Table]	Page 147
• Sample Student Work	Pages 148 - 159
Monsters & Mini- Monsters Projects	Pages 160 - 169



Bamburgh Castle

The marriage of King Arthur ❤ Princess Guinevere

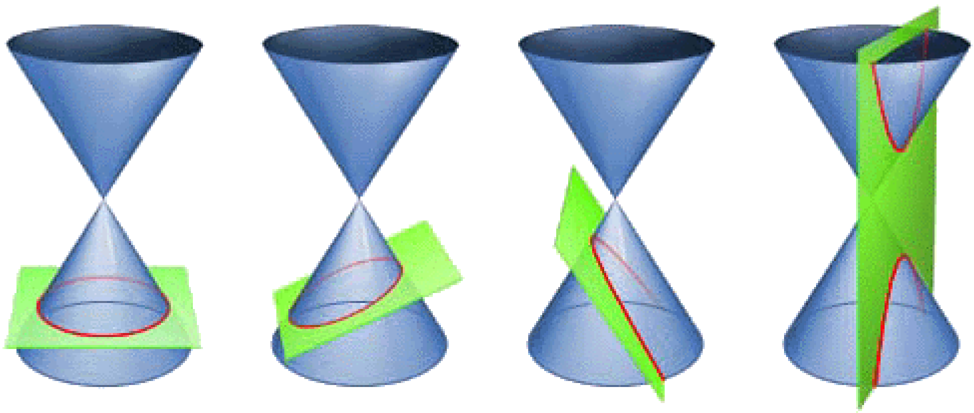
✪ Conic Pictures Project ✪

Math Is Fun

Concept Focus: **Conics**
Standards: Algebra I - 6, 22, 25
Algebra II – 1, 2, 10, 17

Skills: Graphing on the coordinate plane, making mathematical connections with art.

Materials: Graph paper, straightedge, pencils, markers, construction paper.



Description: This special project, which is submitted as a portfolio, allows students to use their knowledge of the conic sections to create pictures or designs that are based on conic sections.

Background and Rationale:
Learning conic sections seems to be a difficult concept that has kept many high school students from taking advanced mathematics classes past second year algebra.

This project allows students to use their creativity and design their own "piece of art." After specific modeling and guided practice, they build a portfolio for their own picture or design. They use equations of conic sections combined with linear functions and appropriate restrictions on the domain and/or range to describe their picture design. The project takes two or three days of class time, and the remainder of the work is done on the student's own time. The project is usually done over a total of two weeks with occasional progress checks to make sure that students are on task. This is provided they have completed the conic section in Algebra II. If not, an additional 2 weeks need to be added.

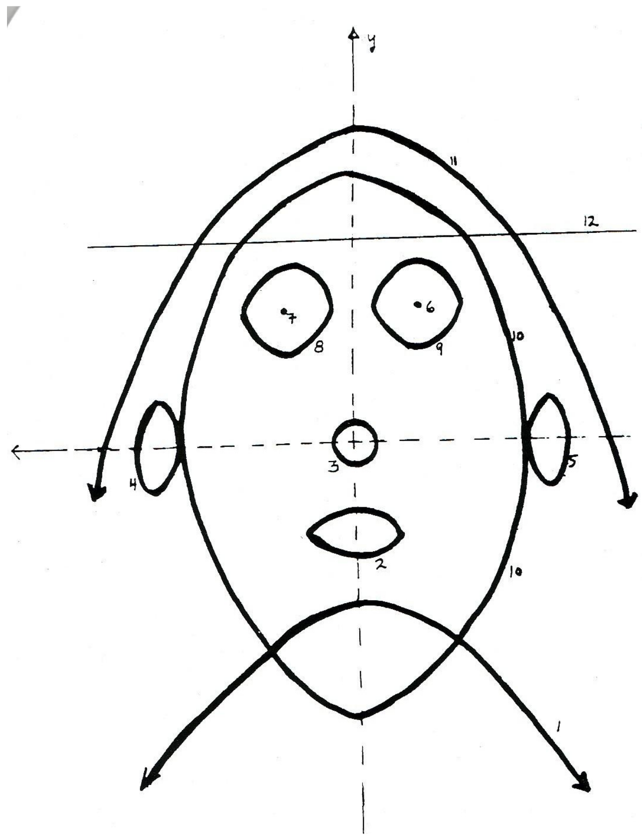
The students become involved in combining art and mathematics, often using tens of equations. The cover of the portfolio is an artistic rendition of the picture inside. The pictures range from cars to airplanes, from Mickey Mouse to Ninja Turtles. Students realize that they can have fun using mathematics, and their creativity in designing their projects is amazing. Every student's portfolio is on display for fellow students as well as community members to see.

Day One:
Review graphing techniques of conic sections. Discuss restrictions can be placed on the domain and/or range of conic sections so that only desired portion of the conic remains. Guided practice in small groups is recommended.

Day two
Students work in small groups to graph the 12 equations below which produces a face.

On one set of axes, graph the following:

1. $x^2 = -12(y+7)$
2. $x^2 + 4(y+4)^2 = 4$
3. $x^2 + y^2 = 1$
4. $4(x+9)^2 + y^2 = 4$
5. $4(x-9)^2 + y^2 = 4$
6. $(x-3)^2 + (y-6)^2 = 0$
7. $(x+3)^2 + (y-6)^2 = 0$
8. $(x+3)^2 + (y-6)^2 = 4$
9. $(x-3)^2 + (y-6)^2 = 4$
10. $yx^2 + 4y^2 = 576$
11. $x^2 = 10(14-y)$
12. $y = 9$



Supply students graph paper and instruct to make one large coordinate system on which they will graph all 12 equations. The resulting "conic face" graph contains many conic sections and makes use of the degenerate forms for conics (circles with radius 0 for pupils of eyes, etc.) Encourage students to come up with restrictions on the graphs, so that various final pictures are derived from the same basic set of equations. Students share and discuss their graphs with the class.

Some discussion might be, "Should equations 11 and 12 be used to form a hat or headband and hair ?" or "How could we put restrictions on equation 1 so that it appears to be a chin ? a pair of shoulders ?"

Project Description (See Activity Sheet on page 125 – ***what you give to the students***)
Now students are ready to start their project. Give them the following project description and also grading standards and due dates. Encourage students to include more than the minimum amount of work in their portfolio. I also share projects from past years with the class and put them on display as students work on their own projects.

Additional class days can be used for working on the project and checking student progress. I am also available to help students before and after school, and during lunch.

All conic pictures must conform to accepted standards of decency. (The accepted standards are MY standards which are not necessarily the same as yours.) See me for clarification if needed.

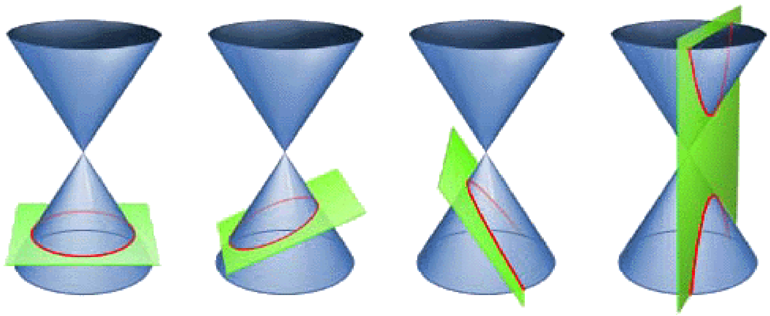
EXTENSION: Students print out their equations using a graphing utility.
[…computer generated pictures …]

Check the next page for
the HANDOUT given to the students

Project-based learning is a terrific way to link your
curriculum with real world events and applications of concepts that
your students are learning.

Conic Pictures Project

Directions: This special project, to be submitted as a portfolio, will allow you to use your knowledge of conic sections and your creativity to make a picture or design that is based on conics.



To receive FULL credit, your picture or design must contain at least one of each of the conics we have studied, i.e., circle, ellipse, parabola, and hyperbola. You may, of course, have more than one of any conic section, and you may include as many degenerate conics (lines and points) as you need to complete your picture.

Along with your final "visually pleasing" project, you need to submit a working copy of the picture with each graph statement numbered. The numbers will be used to check your equation. These numbers will also be used to identify any restrictions you wish to put on the domain and/or the range of the graph segments. These restrictions should be provided on a separate piece of paper.

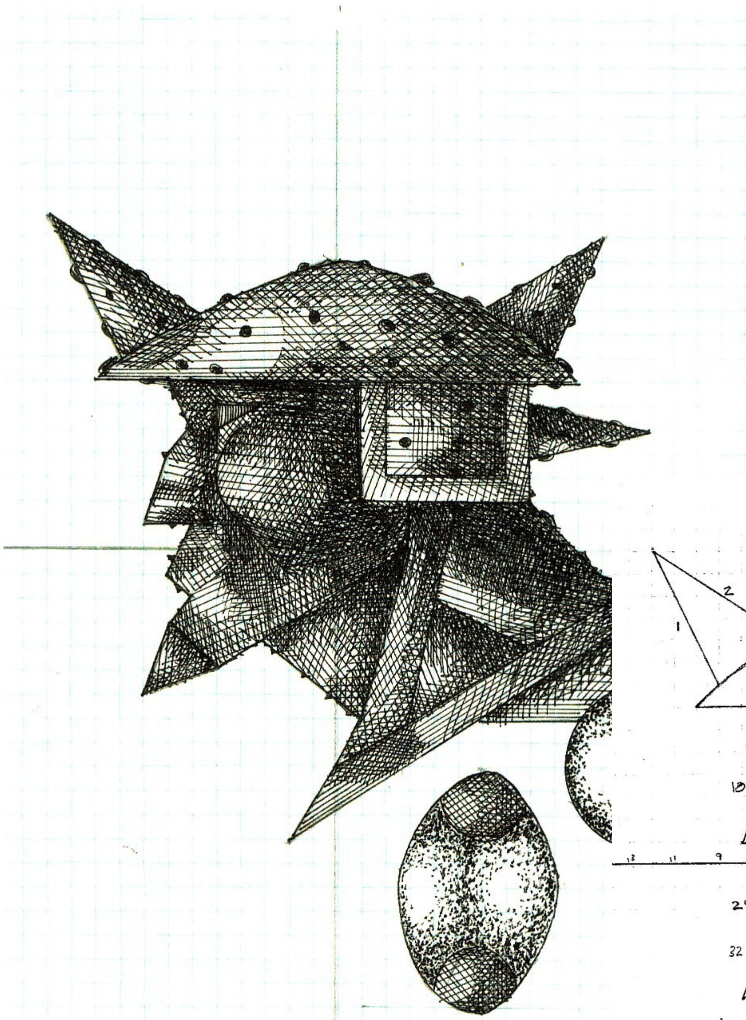
The cover of the portfolio should be an artistic rendition of the graphics picture inside.

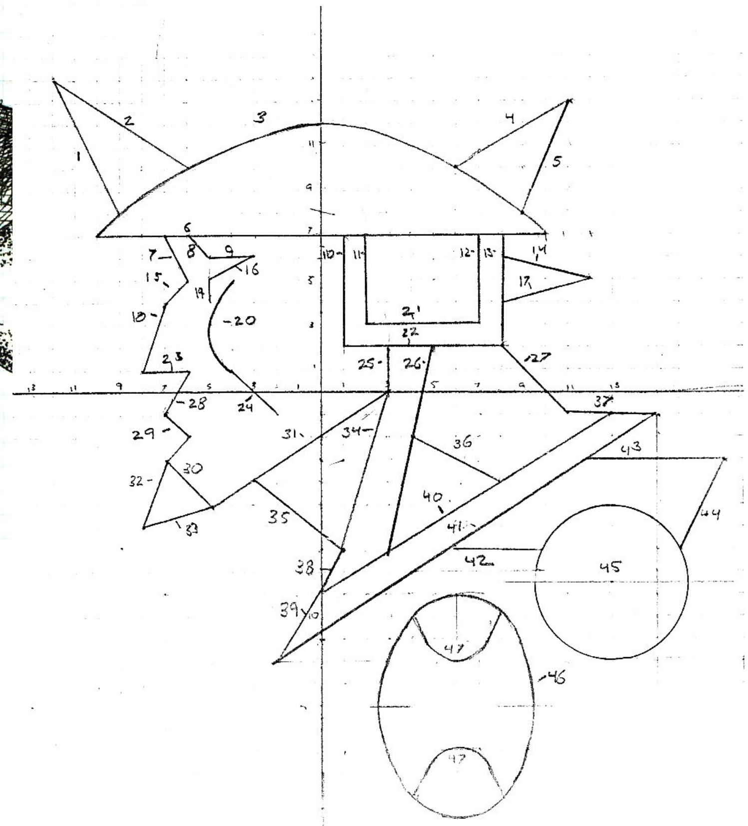
Remember that in grading the Conic Section, I will be looking for:

- ⊙ Inclusion of the necessary conic sections,
- ⊙ Accuracy of your equations, including the restrictions,
- ⊙ Neatness,
- ⊙ Originality of design,
- ⊙ Creativity.

All conic pictures must conform to accepted standards.

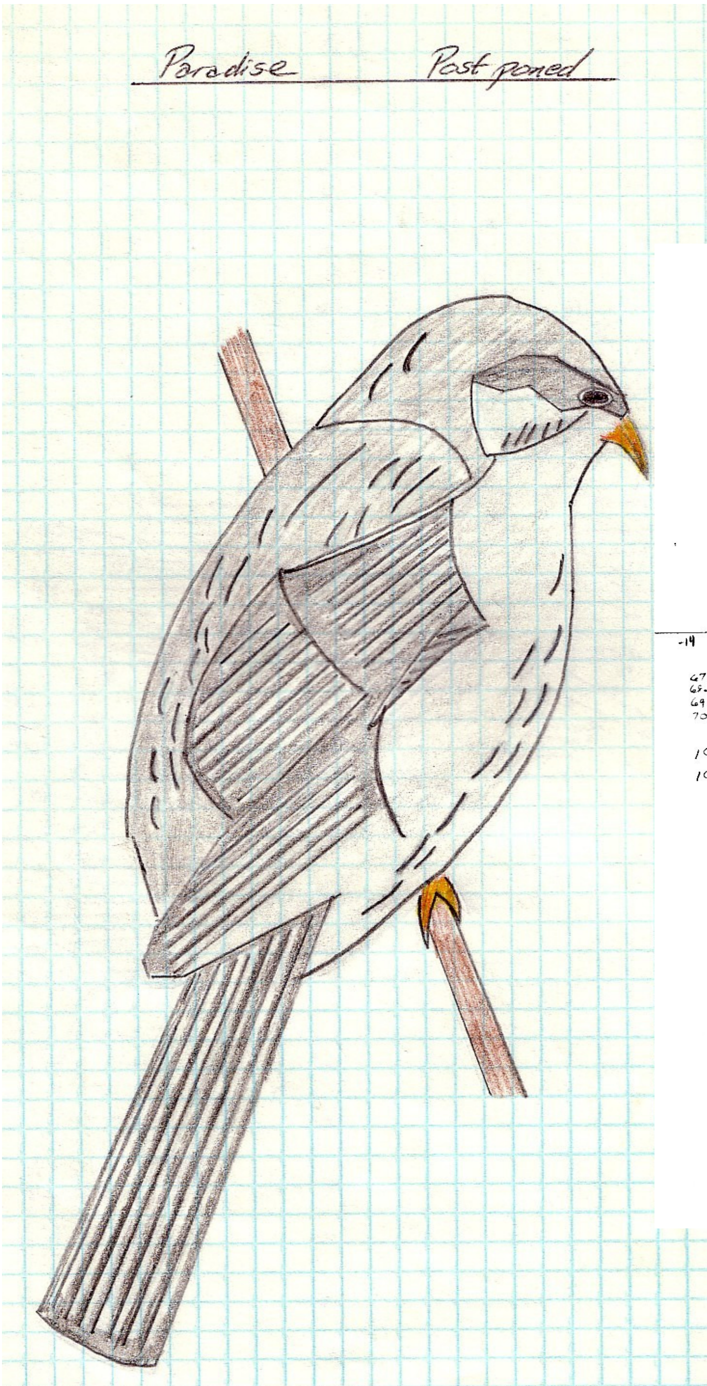
Sample Student Work ☞ #1

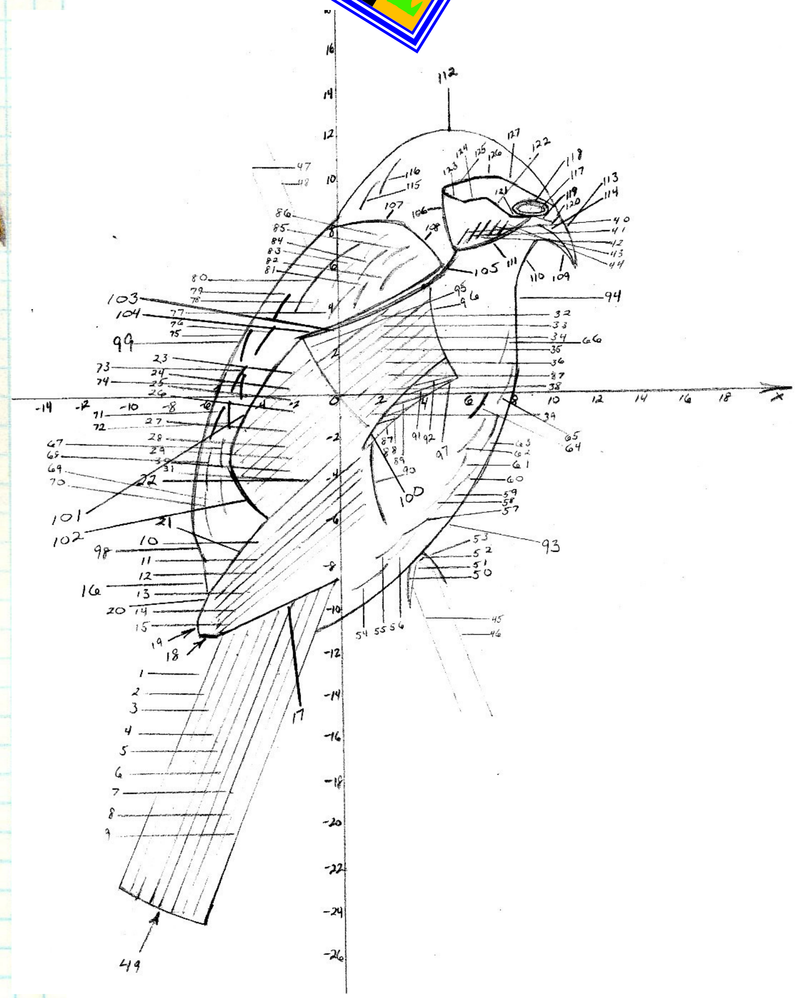




A total of 47 **EQUATIONS** were used

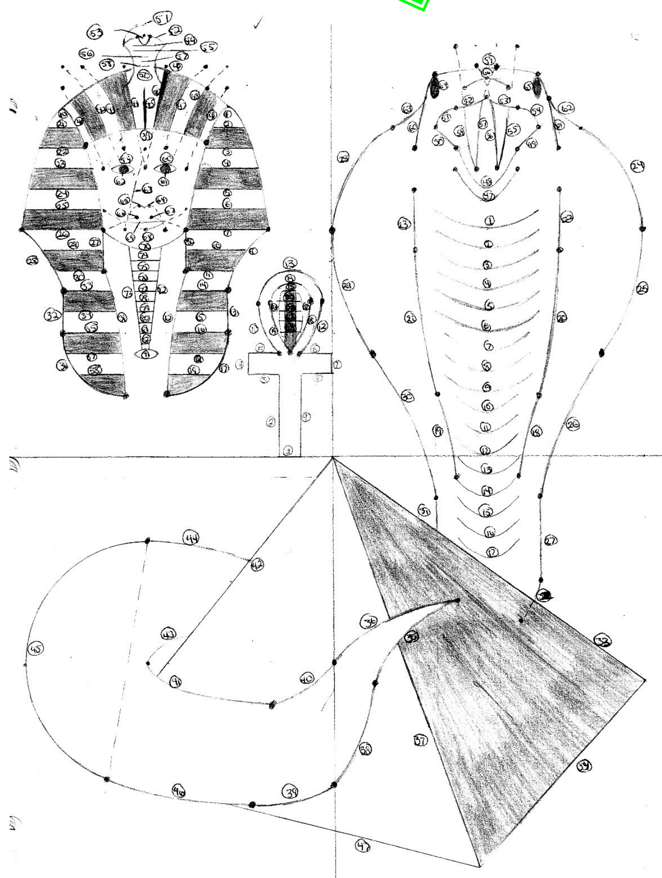
Sample Student Work ☞ #2





A total of 111 **EQUATIONS** were used.

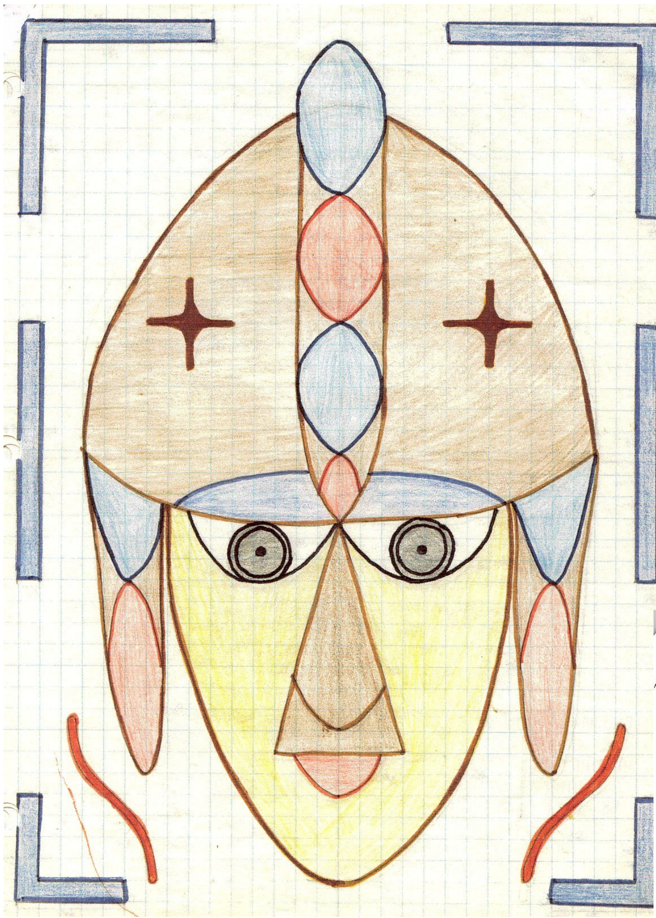
Sample Student Work ☞ #3



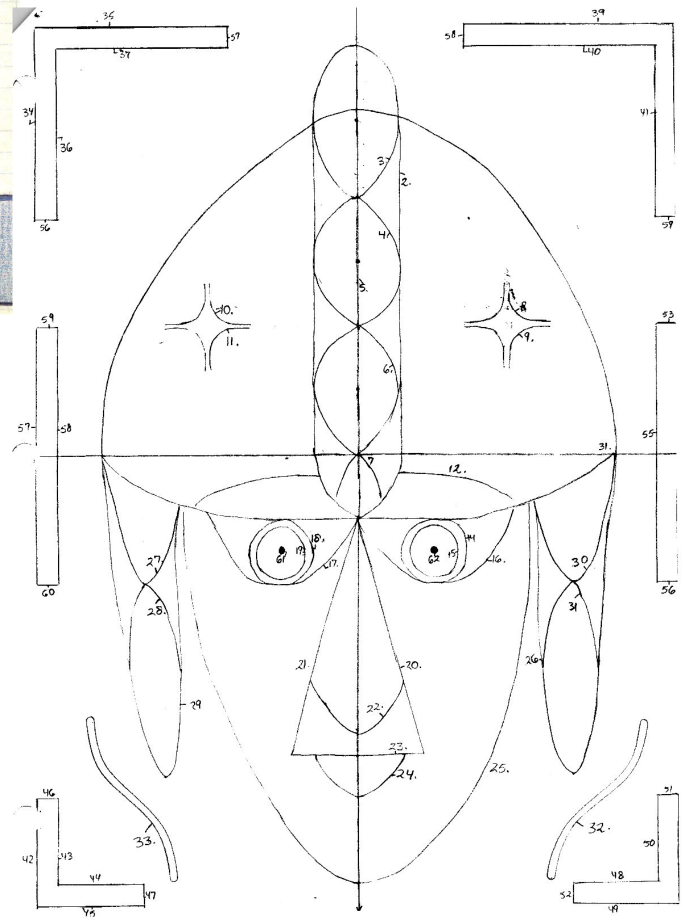
The **EQUATIONS** were broken down into the following:

- THE ANKH .... 23 equations
- KING TUTANKHAMUN MASK .... 82 equations
- EGYPTIAN COBRA GUARDING PYRAMID .... 66 equations

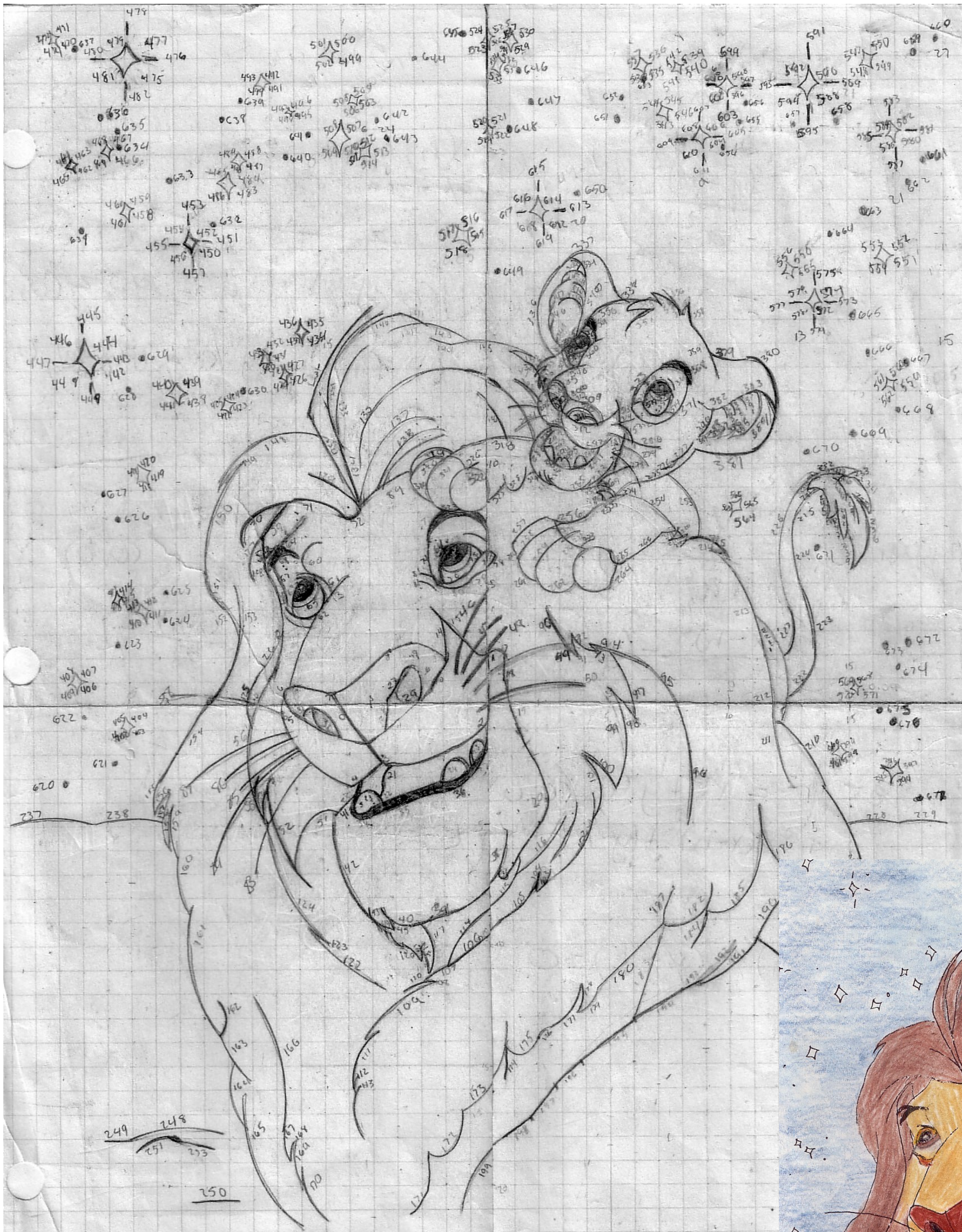
Sample Student Work ☞ #4



A TOTAL of 62 EQUATIONS were used.



Sample Student Work ☞ #5



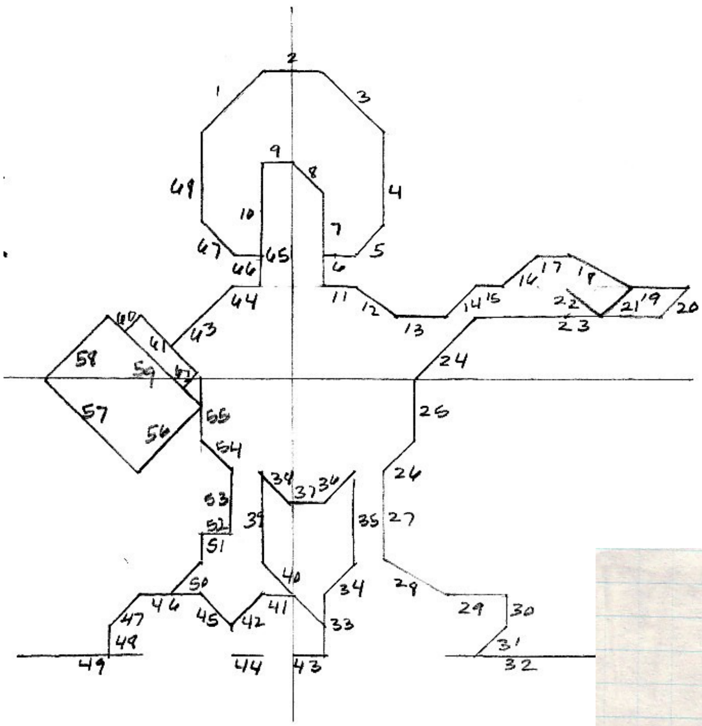
A TOTAL of

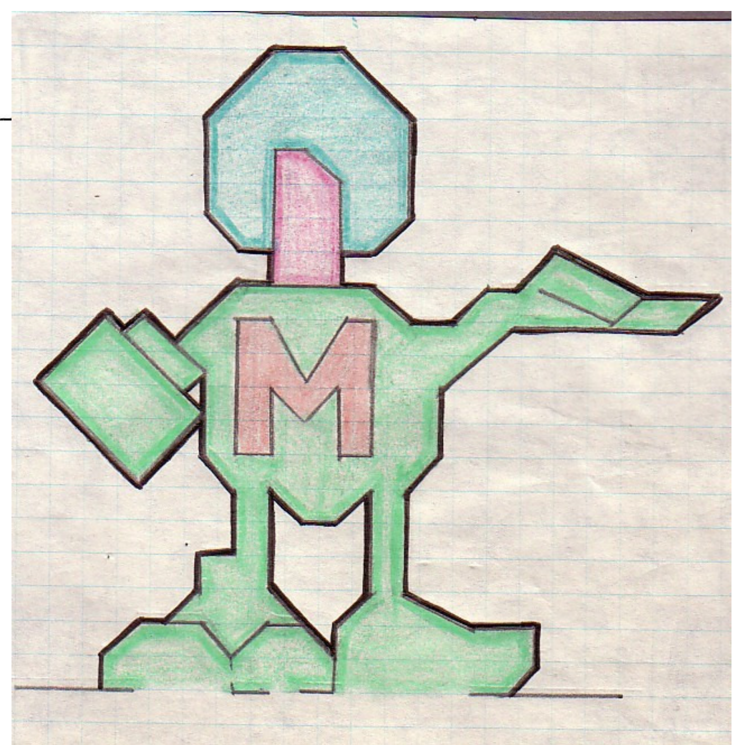
677

equations

to accomplish this
EXTRAORDINARY work.
WOW !!!

Sample Student Work ☞ #6
[Linear Equations]





A TOTAL of 68 equations were used
to complete the picture

.....see equations along with the restrictions on the next page ......

Sample Student Work ☞ The equations for #6

1 $y=1x+11; -3\le x\le -1$

2 $y=10=-1\le x\le 1$

3 $y=-1x+11; 1\le x\le 3$

4 $x=4; 5\le y\le 8$

5 $y=1x+2; 2\le x\le 3$

6 $y=4; 1\le x\le 2$

7 $x=1; 3\le y\le 6$

8 $y=-1x+7; 0\le x\le 1$

9 $y=7; -1\le x\le 0$

10 $x=-1; 3\le y\le 7$

11 $y=3; 1\le x\le 2$

12 $y=-1x+5; 2\le x\le 3$

13 $y=2; 3\le x\le 4$

14 $y=1x+-3; 4\le x\le 5$

15 $y=3; 5\le x\le 6$

16 $y=1x+-4; 6\le x\le 7$

17 $y=4; 7\le x\le 8$

18 $y=-1/2x+8; 8\le x\le 10$

19 $y=3; 10\le x\le 12$

20 $y=-1x+-9; 11\le x\le 12$

21 $y=-1x+-7; 9\le x\le 10$

22 $y=1/2x+7; 7\le x\le 9$

23 $y=2; 6\le x\le 11$

24 $y=1x+-4; 4\le x\le 6$

25 $x=4; 0\le y\le 2$

26 $y=1x+-6; 3\le x\le 4$

27 $x=3; -3\le y\le -6$

28 $y=1/2x+-5; 3\le x\le 5$

29 $y=-7; 5\le x\le 7$

30 $x=7; -7\le y\le -8$

31 $y=1x+-16; 6\le x\le 7$

32 $y=-9; 9\le x\le 10$

33 $x=1; -7\le y\le -9$

34 $y=1x+-8; 1\le x\le 2$

35 $x=2; -3\le y\le -6$

36 $y=1x+-5; 1\le x\le 2$

37 $y=-4; 0\le x\le 1$

38 $y=-1x+-4; -1\le x\le 0$

39 $x=-1; -3\le y\le -6$

40

41 $y=-7; 1\le x\le 0$

42 $y=1x+-6; -2\le x\le -1$

43 $y=-9; 0\le x\le 1$

44 $y=-9; 2\le x\le 1$

45 $y=-1x+-1; -3\le x\le -2$

46 $y=-7; 5\le x\le 3$

47 $y=1x+-3; 6\le x\le 5$

48 $x=-6; -8\le y\le -9$

49 $y=-9; -10\le x\le -5$

50 $y=1x+-2; -4\le x\le 3$

51 $x=-3; -5\le y\le -6$

52 $y=-5; -3\le x\le -2$

53 $x=-2; -5\le y\le -3$

54 $y=-1x+-5; -3\le x\le -2$

55 $x=-3; 0\le y\le -2$

56 $y=1x+2; -5\le x\le -3$

57 $y=1x+-8; -9\le x\le -5$

58 $y=1x+8; -8\le x\le -6$

59 $y=1x+8; -6\le x\le -3$

60 $y=1/2x+7; -6\le x\le -5$

61 $y=1x+-2; -5\le x\le -3$

62 $y=1/2x+3; -3.5\le x\le 3$

63 $y=1x+5; -4\le x\le -2$

64 $y=3; -2\le x\le 1$

65 $x=1; 3\le y\le 7$

66 $y=4; -2\le x\le -1$

67 $y=1x+2; -3\le x\le -2$

68 $x=-3; 5\le y\le 8$

✪ Linear Programming Application ✪

Math Is Fun

Concept Focus: **Linear Programming, like many applications of math, was developed and used in defense in the beginning in the 1940s. It is now used in many fields especially in areas of business.**

Standards: Algebra I - 2, 4, 5, 6, 7, 9, 10, 12, 13
Algebra II – 1, 2

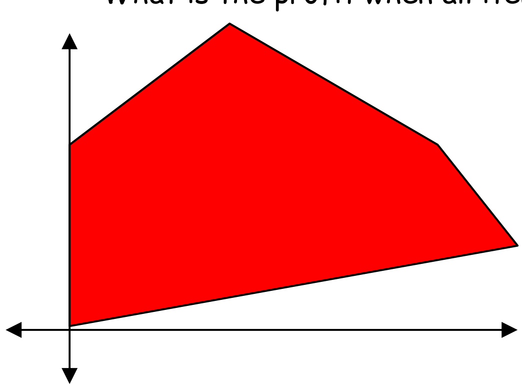
Your Task: You are the assistant manager of an appliance store in Los Angeles. The manager has asked you to do a *cost analysis* to figure out what stereo systems the store should order.

Next month you will order two types of stereo systems, a less expensive **Model A** and a more expensive **Model B**. As assistant manager you must figure out how much of each model to order to minimize costs. You expect to sell at least 100 units some **Model A** and some **Model B**.

Model A leaves a $40 profit for the store. **Model B** leaves a $60 dollar profit for the store. Total profits must be at least $4800. The wholesale cost of Model A is $250 dollars. The wholesale cost of **Model B** is $400. As a store you buy at the wholesale cost.

Questions you need to respond to:

1. What does a point in the solution region represent? How does it compare to a point not in your solution region?
2. Find the minimum and maximum costs (if they exist).
3. How many of each model should you order to minimize your costs?
4. What is the profit when all items are sold ?
5. After reviewing your report, the manager decides that the store must order at least 30 **Model B** stereos even if it increases the minimum cost. You go back to the drawing board to revise your analysis.
   - Now how many of each model should be ordered to minimize costs
   - What is the profit when all items are sold?



Skills included or Needed for this project

- Students need an understanding of systems of equations and systems of inequalities.
- Students must be able to solve a system of equations.
- Students must be able to work with equations.
- Students must know how to isolate a variable.
- Students must be able to graph lines.
- Students need an understanding of solution area for systems of inequalities.
- Students must be able to translate from written sentences to algebraic expressions.

Remember that in grading this Project, I will be looking for:

⊙ Your clear explanation to the questions asked.
⊙ Neatness in your presentation.
⊙ Ability to present your findings to the class.

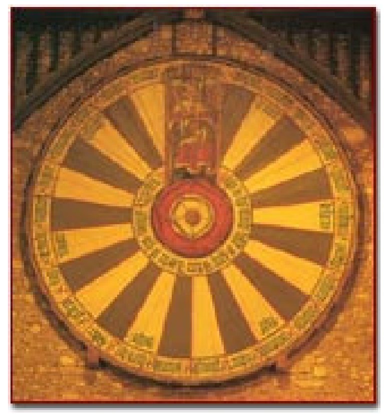
Project-based learning is a terrific way to link your curriculum with real world events and applications of concepts that your students are learning.

Project Bobo

King Arthur's The Round Table

"Helping Bobo Where To Sit"

<u>The Problem</u> Bobo's curiosity made him participate for big prizes at King Arthur's Round Table contest. After paying the entrance fee Bobo was asked to sit at the **<u>only</u>** remaining seat at this huge round table. No one new what the game was all about. However, the interesting thing was that all the other eighteen people had already been seated by some unknown fashion and the only seat that was not occupied was seat #7. So Bobo sat in seat #7.



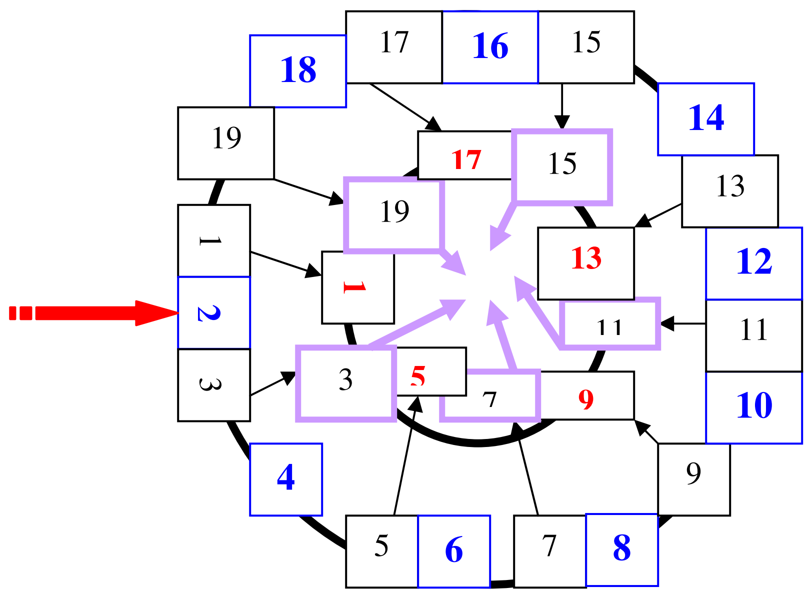
Then someone dressed as King Arthur went around the table ELIMINATING people from winning any prizes. The way it was done was he started from seat #1 and eliminated every second person **<u>in order</u>** until one remained. The first one he eliminated was the person seated in seat #2.

Now there was a second contest for a even bigger prize! Because Bobo had won the last round he got to pick which seat to sit in first. There was only one problem. Bobo doesn't have a clue to know where to sit if the pattern was continued. Bobo now needs to decide where to sit in, but he can't do it alone. He needs our help.

A Student's Explanation:

Explaining how I proceeded to solve this problem

I plan to solve this problem by creating several diagrams or similar examples to find a pattern that works for everything. I first will check to see if the pattern in the first contest that **Bobo** was in works. Then I will create smaller examples starting from the minimum number of players needed to play which is 2. I will work my way up and take all the answers. Then I will try to find a pattern to see if the pattern works for every answer.



To begin - the chairs where numbered *consecutively* from 1 through 19 and were organized in a circular manner around the huge round table.

- During the first round of elimination, the following chairs numbered **2, 4, 6, 8, 10, 12, 14, 16, 18** were eliminated. So the following remained **1, 3, 5, 7, 9, 11, 13, 15, 17, 19**
- During the second round of elimination, the following chairs numbered **1, 5, 9, 13, 17** were eliminated. So the following remained **3, 7, 11, 15, 19**

- During the third round of elimination, the following chairs numbered 3, 11, 19 were eliminated. So the following remained 7, and 15.
- During the last round of elimination between Chair #7 & Chair #15. Chair #15 gets eliminated since Chair #7 is considered Chair number 1.

What was I thinking and what did I do before I began to solve the problem ?

I had the following concerns regarding the Bobo puzzle:

1. What if the numbers were not consecutive integers. In other words the numbers were attached to the contestants as they entered the arena and they sat randomly.
2. When the man was eliminating every second person **in order** until one is remaining confused me at first. I understood what it was meant after I actually had to read the problem over and over again and began to draw a circle and place the 18 other contestants and Bobo in an orderly manner (as shown above) to really understand how it worked. Chair # 19 becomes Chair # 1 after the first round of elimination. If you miss this one you miss the entire problem. So thanks to the tip given in the problem where it is stated that the eliminated numbers are 2, 4, 6, 8, 10, 12, 16, 18, and then comes 1, 5, 9, 13, 17, and then 3, 11, 19, and lastly 15. This information was very valuable for me to decipher the pattern.
3. I was also concerned about drawing a circle with 100 consecutive positive integers and then going over the process of eliminating every 2^{nd} one – a long process that takes time and also the chance of making a mistake was great since I had NOT yet discovered the pattern. I was determined to find a pattern with SMALLER numbers and then showing that it works for any POSITIVE integers greater than 19. I did require some help from my Uncle. He explained to me what Arithmetic Sequences are all about.

   Example: Given the following Arithmetic Sequence to find the 5^{th} term
   7, 9, 11, 13, …

   Solution:
   We know the answer is 15 because the sequence is increasing by 2.
   What my uncle provided me was a powerful FORMULA $a_n = a_1 + (n-1)d$.

   He explained to me that $a_n =$ the last term desired; $a_1 =$ the first term in the sequence; $n =$ number of terms; $d =$ the difference or pattern between the consecutive sequence. Anyway, I used this formula to come up with the answer.

The Work

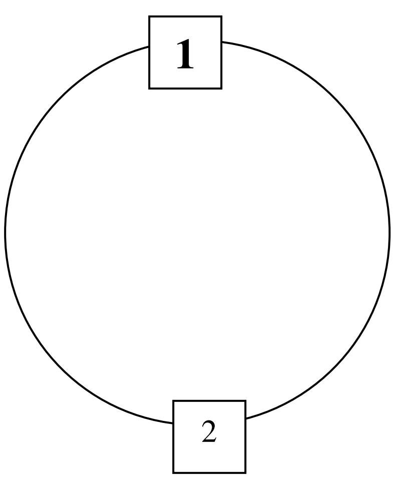
I started by first proving that Bobo, who was seated at Chair #7 was really the winner among the19 contestants seated in chairs placed in a circular manner.

Elimination Round Number	19 Contestants Seated in a circular manner starting with #1	Seats Eliminated	Pivot Number (person situated in seat #1)	Comments
ONE [19 Contestants Seated in a circular manner starting with #1]	1, 2, 3, 4, 5, 6, 7, 8, 9,10, 11, 12, 13, 14, 15, 16, 17, 18, 19	2, 4, 6, 8, 10, 12, 14, 16, 18,	1	
TWO [10 Contestants Seated in a circular manner starting with # 19]	19, 1, 3, 5, 7, 9, 11, 13, 15, 17	1, 5, 9, 13, 17	19	Important to place # 19 in position #1.
THREE [5 Contestants Seated in a circular manner starting with # 19]	**19**, **3**, 7, **11**, 15 [Note: 3 gets eliminated first and then 11 and last 19]	3, 11, 19	19	Important to place # 19 in position #1.
FOUR [2 contestants Seated in a circular manner starting with #7]	7, **15**	15	7	Important to place # 7 in position #1.

So after FOUR rounds the contestant who is seated in Seat #7 is the WINNER. Bobo did win.

So I started by taking a SMALLER sample to see if a pattern is developing.

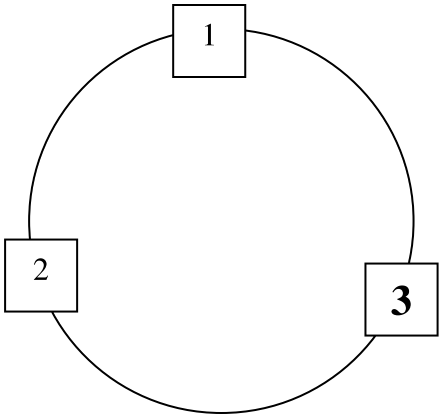
CASE 1
2 contestants



It is OBVIOUS that contestant # 2 will be the looser.

# of Contestants	Winning Seat	Explanation
2	1	This is very obvious

CASE 2
3 contestants

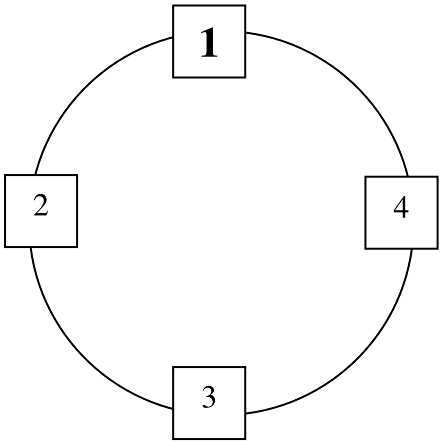


It is OBVIOUS that contestant # 3 will be the winner.

Elimination Round Number	3 Contestants Seated in a circular manner starting with #1	Seats Eliminated	Pivot Number (person situated in seat #1)	Comments
ONE [3 Contestants Seated in a circular manner starting with #1]	1, **2**, 3	2	1	
TWO [2 Contestants Seated in a circular manner starting with # 3]	3, **1**	1	3	Important to place # 3 in position #1.

Winning Seat: # 3

CASE 3
4 contestants

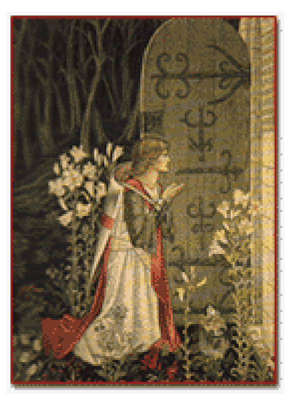


Elimination Round Number	3 Contestants Seated in a circular manner starting with #1	Seats Eliminated	Pivot Number (person situated in seat #1)	Comments
ONE [4 Contestants Seated in a circular manner starting with #1]	1, **2**, 3, **4**	2, 4	1	
TWO [2 Contestants Seated in a circular manner starting with # 3]	1, **3**	3	1	Important to place # 1 in position #1.

Winning Seat: # 1

King Arthur

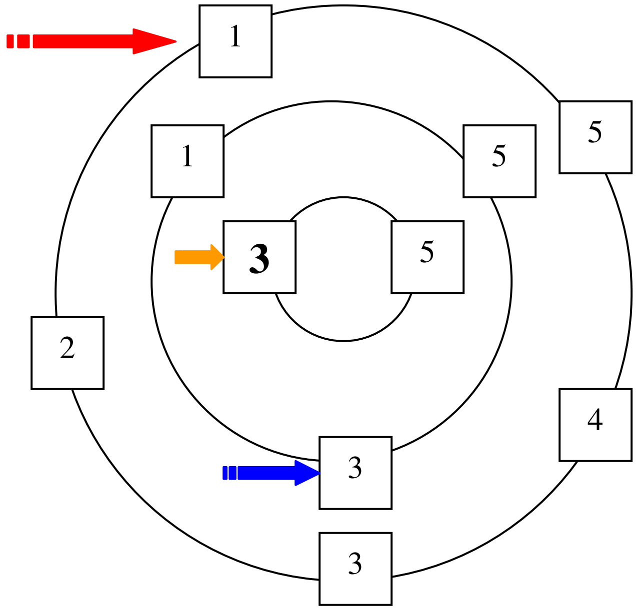
Sir Lancelot



Sir Galahad

Student Work

CASE 4
5 contestants



Elimination Round Number	**5 Contestants Seated in a circular manner starting with #1**	**Seats Eliminated**	**Pivot Number (person situated in seat #1)**	**Comments**
ONE [5 Contestants Seated in a circular manner starting with #1]	1, **2**, 3, **4**, 5	2, 4	1	
TWO [3 Contestants Seated in a circular manner starting with # 3]	**1**, 3, 5	1	3	Important to place # 3 in position #1.
THREE [2 Contestants Seated in a circular manner starting with # 3]	3, **5**	5	3	

Winning Seat: # 3

The Work (Pattern Discovered)

Observations that helped me come to a FINAL conclusion:

I was now able to see some OBVIOUS observations:

1. Similarities of Answers:

- All the answers are odd.
- Numbers sometimes repeat themselves. (see chart #1)
- Most of the answers are prime.

2. Important Similarities

- All the numbers skip by two

3. Important Non-Similarities

- While the numbers are skipping by two the numbers come back to1 then start over.

4. Important Questions Raised

- Why does the numbers go back to 1 and repeat itself?
- Is there a pattern that I can see ?

When the number moves back to 1, the number of people participating are always 2 to the something power. (see chart #1)

Chart #1

Number of Contestants	Winning Number	Mathematical Formula	Comments Observations
			Note that 2^n ALWAYS gave me the PIVOT or STARTING point. This ALWAYS gave me the winning number 1.
2	**1**	2^1	
3	3		
4	**1**	2^2	
5	3		
6	5		
7	7		
8	**1**	2^3	
9	3		
10	5		
11	7		
12	9		
13	11		
14	13		
15	15		
16	**1**	2^4	
17	3		
18	5		
19	7		
20	9		

Student Work

Chart #1 (cont)

Number of Contestants	Winning Number	Mathematical Formula	Comments Observations
21	11		
22	13		
23	15		
24	17		
25	19		
26	21		
27	23		
28	25		
29	27		
30	29		
31	31		
32	**1**	2^5	Note that 2^n ALWAYS gave me the PIVOT or STARTING point. This ALWAYS gave me the winning number 1.
33	3		
34	5		
35	7		
36	9		
37	11		
38	13		
39	15		
40	17		
41	19		
42	21		
43	23		
44	25		
45	27		
46	29		
47	31		
48	33		
49	35		
50	37		
51	39		
52	41		
53	43		
54	45		
55	47		
56	49		
57	51		
58	53		
59	55		
60	57		

Student Work

Chart #1 (cont)

Number of Contestants	Winning Number	Mathematical Formula	Comments Observations
61	59		
62	61		
63	63		
64	**1**	2^6	
65	3		
66	5		
67	7		Note that 2^n ALWAYS gave me the PIVOT or STARTING point. This ALWAYS gave me the winning number 1.
68	9		
69	11		
70	13		
71	15		
72	17		
73	19		
74	21		
75	23		
76	25		
77	27		
78	29		
79	31		
80	33		
81	35		
82	37		
83	39		
84	41		
85	43		
86	45		
87	47		
88	49		
89	51		
90	53		
91	55		
92	57		
93	59		
94	61		
95	63		
96	65		
97	67		
98	69		
99	71		Chair #73 is the winner
100	73		

The Answer (Pattern Discovered)

- Give the answer in full sentence form. Explain your answer clearly.
- Answer(s) must be correct. Could there be any other answers ?
- For extra points, include an extra example which extends the original problem. Include all the above information in the extra example

The answer to our Bobo And The Round Table is Chair Number 73. Chart #1 clearly indicates the pattern that was discovered – that is:

Number of Contestants	Winning Number	Mathematical Formula	Comment
2	1	2^1	Bobo always needs to find the PIVOT first, make a simple calculation before he decides where to sit – given he has the first choice to sit. He will always win.
4	1	2^2	
8	1	2^3	
16	1	2^4	
32	1	2^5	
64	1	2^6	
128	1	2^7	
256	1	2^8	
512	1	2^9	
1024	1	2^{10}	

Solution:

$W = 1 + (\mathbf{n} - 1)\, d$ …………….. Arithmetic Sequence will lead into the Winning Chair Numbers

Always a 1. Since all PIVOT numbers gives 1 as your winning

n stands for the INCLUSIVE numbers between the desired number of contestants and the PIVOT number.

d is ALWAYS a 2. We discovered it by the early experimentation. All the numbers skip by two

So, $W = 1 + (37 - 1)2 = 73$

$W = 73$, which is our DESIRED answer. This was clearly indicated by the chart.

Let us say you want to find the winning chair number for 40 contestants.

Solution:

- 1^{st} find the closest power of base two **involved** lower than 40. Because this will be your PIVOT number. In this case it would be 32. which is = 2^5
- Now subtract 40 – 32 = 8. What this means that there are 8 consecutive numbers **between** 32 and 40. Which translates to 9 INCLUSIVE numbers
  1, 3, 5, 7, 9, 11, 13, 15, 17 (see chart below)
  32, 33, 34, 35, 36, 37, 38, 39, 40 (see chart below)
- Now, using the Arithmetic Sequence $\mathbf{W = 1 + (n - 1)}\, \boldsymbol{d}$, where W = Winning Seat, n = number of odd numbers between the pivot and the term needed, and $\boldsymbol{d} = 2$.

  $W = 1 + (9 - 1)2$

  $W = 17$ → is the winning Chair Number

Number of Contestants	Winning Number	Mathematical Formula
32	**1**	2^5
33	3	
34	5	
35	7	
36	9	
37	11	
38	13	
39	15	
40	17	

Conclusion: The winning Chair Number is 17.

Let us say you want to find the winning chair number for 67 contestants.

Solution:

- 1^{st} find the closest power of base two **involved** lower than 67. Because this will be your PIVOT number. In this case it would be 64. Which is = 2^6
- Now subtract 67 – 64 = 3. What this means that there are 3 consecutive numbers **between** 64 and 67. Which translates to 4 INCLUSIVE numbers
  1, 3, 5, 7 (see chart below)
  64, 65, 66, 67 (see chart below)
- Now, using the Arithmetic Sequence $\mathbf{W = 1 + (n - 1)}\, \boldsymbol{d}$, where W = Winning Seat, n = number of odd numbers between the pivot and the term needed, and $\boldsymbol{d} = 2$.

Number of Contestants	Winning Number	Mathematical Formula
64	**1**	2^6
65	3	
66	5	
67	7	

$W = 1 + (4 - 1)2$

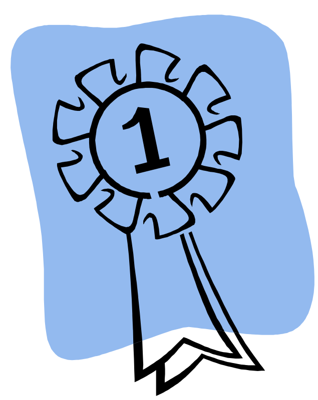
$W = 7$ → is the winning Chair Number

Final Analysis:

Bobo will always win this GAME if he is given the opportunity to choose first. **Bobo** knows the formula for SUCCESS. He knows mathematics.

No matter how many chairs are placed around the Round Table, he needs to follow the following steps:

1. Choose the PIVOT number 1st. The PIVOT number is that magic number represented by 2^n. **Bobo** must be careful and choose the PIVOT number which smaller than the number of chairs involved in the game.
2. **Bobo** KNOWS that all PIVOT numbers have a winning Chair number of 1.
3. **Bobo** then needs to know how many INCLUSIVE numbers are involved between the number of chairs and the base PIVOT number. If he subtracts these numbers, he then need to add one.
4. **Bobo** further knows that ***d*** in the formula is ALWAYS a 2.
5. **Bobo** knows that the numbers generated are done through the Arithmetic Sequence Formula.
6. He goes ahead and cranks the numbers in the formula and obtains the WINNING Chair number.



Monsters
And
Mini – Monsters

Projects

♥

Fun With Rational Expressions

Strengthening Basic Arithmetic Skills
And
Basic Conceptual Understanding

Purpose: Students by the time they are Algebra ready should be able to demonstrate their deductive reasoning abilities in breaking down simple rational arithmetic expressions into simpler and simpler parts **without** the use of calculators.

Objective: Problems Illustrating a combination of order of operations, basic Arithmetic Conceptual Understanding depending on the complexity. By the end of Semester 2, students should be able with ease to solve *rational expressions* of the form

$$\frac{8-2(5-8)-2^3}{14-\frac{5}{8}}-\frac{4\frac{1}{2}+1\frac{1}{3}}{10-3^0+\frac{1}{2}}+\frac{3\frac{1}{3}\div 1\frac{1}{3}}{\left(2\frac{1}{3}\right)\left(3\frac{1}{2}\right)}=$$

Solution:

My Plan is $\frac{A}{B}-\frac{C}{D}+\frac{E}{F}=$

- Students then will take each component and solve it explicitly. They must show work in detail. Partial points are given appropriately.

$A: \quad 8-2(5-8)-2^3 =$

$8-2(-3)-8=$ ☞ 6 answer

$8+6-8=6$

- $B: \ 14-\frac{5}{8}=$ student work ☞ $13\frac{3}{8}$ or $\frac{107}{8}$ answer
- $C: \ 4\frac{1}{2}+1\frac{1}{3}=$ student work ☞ $5\frac{5}{6}$ or $\frac{35}{6}$ answer
- $D: \ 10-3^0+\frac{1}{2}=$ student work ☞ $9\frac{1}{2}$ or $\frac{19}{2}$ answer
- $E: \ 3\frac{1}{3}\div 1\frac{1}{3}=$ student work ☞ $\frac{5}{2}$ or $2\frac{1}{2}$ answer
- $F: \ \left(2\frac{1}{3}\right)\left(3\frac{1}{2}\right)=$ student work ☞ $\frac{49}{6}$ or $8\frac{1}{6}$ answer

My Plan **now** is $\frac{A}{B}-\frac{C}{D}+\frac{E}{F}=\frac{6}{\frac{107}{8}}-\frac{\frac{35}{6}}{\frac{19}{2}}+\frac{\frac{5}{2}}{\frac{49}{6}}=$

$\frac{A}{B}\rightarrow \quad \frac{6}{\frac{107}{8}}\Rightarrow 6\div\frac{107}{8}\Rightarrow 6\cdot\frac{8}{107}=\frac{48}{107}$ Answer

$$\frac{C}{D} \rightarrow \qquad \frac{\frac{35}{6}}{\frac{19}{2}} \Rightarrow \frac{35}{6} \div \frac{19}{2} \Rightarrow \frac{35}{6} \cdot \frac{2}{19} = \frac{35}{57}$$ Answer

$$\frac{E}{F} \rightarrow \qquad \frac{\frac{5}{2}}{\frac{49}{6}} \Rightarrow \frac{5}{2} \div \frac{49}{6} \Rightarrow \frac{5}{2} \cdot \frac{6}{49} = \frac{15}{49}$$ Answer

Therefore, $\frac{A}{B} - \frac{C}{D} + \frac{E}{F} = \frac{48}{107} - \frac{35}{57} + \frac{15}{49}$ Full Credit

By the <u>end of Chapter 2</u> they should be able to simplify the Rational Complex Fraction into its simpler components, with No Calculators, as shown above.

After Chapter 2 has been completed. Students should be able to apply over and over the concept of the identity member of multiplication in completing the final solution. At this time and only at this time calculators will be permitted since the numbers are getting large.

Using calculators $\Rightarrow \frac{48}{107} - \frac{35}{57} + \frac{15}{49} \Rightarrow \frac{48}{107} \cdot \frac{57}{57} \cdot \frac{49}{49} - \frac{35}{57} \cdot \frac{107}{107} \cdot \frac{49}{49} + \frac{15}{49} \cdot \frac{107}{107} \cdot \frac{57}{57}$

$$\Rightarrow \frac{134,064}{298,851} - \frac{183,505}{298,851} + \frac{91,485}{298,851} =$$

$$\Rightarrow \frac{134,064 - 183,505 + 91,485}{298,851} =$$

$$\Rightarrow \frac{42,044}{298,851}$$ Final Answer

Please Note: Calculators should be avoided as much as possible. However, after students have mastered the concepts, a 4 operation calculator is encouraged in obtaining the final answer.

Fractional Expressions

Additional Exercises

Please use the “My Plan” approach in completing these Rational Expression problems. The purpose is to show organization in attacking these problems. These problems will be assigned through out the semester.

Avoid Calculators.

1. $$\frac{5-2(5-8)^2}{8-10}+2=$$

2. $$\frac{5-2}{7-13}+\frac{2^2}{8^1}-1^7=$$

3. $$\frac{3-1(5-6)-3^2}{18-3(7-8)+(-3)^2}=$$

4. $$\frac{3-8}{12+2(6-8)}+\frac{1+4\frac{1}{2}}{-5-4(5-8)}-\frac{10+11+1^0}{3^3+1^1}=$$

5. $$\frac{\left(3\frac{1}{3}\right)\left(5\frac{2}{3}\right)\left(1\frac{1}{3}\right)\div 56\frac{2}{3}}{\left(\frac{1}{5}-\frac{1}{3}\right)+\frac{1}{2}}-\frac{3^2}{\frac{121}{11}}=$$

6. $$\frac{\left(-2\frac{1}{3}\right)\left(3\frac{4}{5}\right)\left(2\frac{1}{5}\right)\div 1463}{\left(\frac{1}{5}-\frac{1}{3}\right)-\frac{1}{2}}-\frac{-3}{15}=$$

7. $$\frac{\left(3\frac{1}{5}\right)\left(4\frac{1}{4}\right)+\frac{-3}{20}-\frac{9}{180}}{\left(\frac{3}{6}\right)(8)\div(7)\left(\frac{5}{6}\right)-1}-\frac{1430}{110}=$$

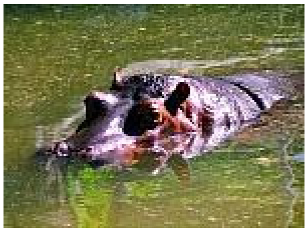
8. $$\frac{6-2\left(7-9\right)^2}{7-2(7-9)}-\frac{12-3(8-9)^3}{(-2)^3}=$$

9. $$\frac{7-7\left(7-8\right)^3-14}{2-2(6-9)}-\frac{11-2(8-9)^3+21}{(-3)^3}=$$

10. $$\frac{3-\frac{1}{9}}{8-\frac{4}{7}}-\frac{4^3}{(-1)^3}=$$

11. $$\frac{7-2\left(11-9\right)^2}{7-2(7-9)}-\frac{11-3(8-10)^3}{(-3)^2-3}=$$

12. $$\frac{\frac{3}{5}\cdot\frac{10}{3}}{12-\frac{1}{4}}+\frac{3-3(3-6)(2)-4^2}{\left(3\frac{1}{2}\right)\left(2\frac{2}{3}\right)}=$$



13. $$\frac{\frac{7}{5}\cdot\frac{8}{7}}{11-\frac{1}{5}}+\frac{4-3(3-7)(4)-2^2}{\left(3\frac{1}{4}\right)\left(2\frac{3}{4}\right)}=$$

14. $$\frac{\frac{3}{10}\cdot\frac{20}{3}}{7-\frac{1}{5}}+\frac{5-2(3-7)(3)-5^2}{\left(3\frac{1}{5}\right)\left(2\frac{2}{5}\right)}=$$

Be carefool !!!

15. $$\frac{\left(3\frac{1}{3}\right)\left(4\frac{1}{2}\right)+\frac{-3}{6}-\frac{9}{24}}{\left(\frac{5}{6}\right)(12)\div(14)\left(\frac{5}{6}\right)-1}-\frac{3}{8}=$$

16. $$\frac{13x+2x-12x}{4x-3x+9x}-\frac{2}{5}=$$

17. $$\frac{\frac{3}{7}+\frac{2}{7}-\frac{1}{7}+\frac{3}{7}}{\frac{2}{5}+\frac{3}{5}}-10=$$

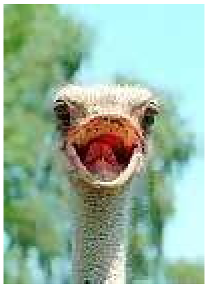
18. $$\frac{\frac{3}{9}+\frac{2}{9}-\frac{1}{9}+\frac{3}{9}}{\frac{4}{9}+\frac{3}{9}}-10=$$

19. $$\frac{17x+5x-12x}{12x-6x+9x}-\frac{2}{5}=$$

20. $$\frac{8x+15x-12x}{9x-6x+7x}-\frac{2}{3}=$$

Answers: (answers to the 1st 10 have been provided)

1. Plan: $\boxed{\dfrac{A}{B}+C}$



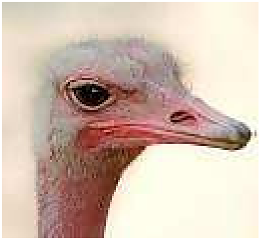
$A=-13,\quad B=-2,\quad C=2,\quad \dfrac{A}{B}=\dfrac{13}{2}$

$\text{Final Answer}=8\dfrac{1}{2}$

2. Plan: $\boxed{\dfrac{A}{B}+\dfrac{C}{D}-E}$

$A=3,\ B=-6,\ C=4,\ D=8,\ E=-1$

$\text{Final Answer}=-1$



3. Plan: $\boxed{\dfrac{A}{B}}$

$A=-5,\quad B=30$

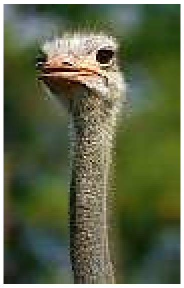
$\text{Final Answer}=\dfrac{-1}{6}$

4. Plan: $\boxed{\dfrac{A}{B}+\dfrac{C}{D}-\dfrac{E}{F}}$

$A=-5,\quad B=8,\quad C=\dfrac{11}{2},\quad D=7,\quad E=22,\quad F=28$

$\dfrac{A}{B}=\dfrac{-5}{8},\quad \dfrac{C}{D}=\dfrac{11}{14},\quad \dfrac{E}{F}=\dfrac{11}{14}$

$\text{Final Answer}=\dfrac{-5}{8}$



5. Plan: $\boxed{\dfrac{A}{B}-\dfrac{C}{D}}$

$A=\dfrac{4}{9},\quad B=\dfrac{11}{30},\quad C=9,\quad D=11$

$\dfrac{A}{B}=\dfrac{40}{33},\quad \dfrac{C}{D}=\dfrac{9}{11}$

$\text{Final Answer}=\dfrac{13}{33}$

6. Plan: $\boxed{\frac{A}{B}-\frac{C}{D}}$

$A=\frac{1}{75}$, $B=\frac{-19}{30}$, $C=-3$, $D=15$

$\frac{A}{B}=\frac{-2}{45}$, $\frac{C}{D}=\frac{-1}{5}$

Final Answer $=\frac{7}{45}$

7. Plan: $\boxed{\frac{A}{B}-\frac{C}{D}}$

$A=\frac{67}{5}$, $B=\frac{1}{35}$, $C=1430$, $D=110$

$\frac{A}{B}=469$, $\frac{C}{D}=13$

Final Answer $=456$

8. Plan: $\boxed{\frac{A}{B}-\frac{C}{D}}$

$A=-2$, $B=11$, $C=15$, $D=-8$

$\frac{A}{B}=\frac{-2}{11}$, $\frac{C}{D}=\frac{15}{-8}=\frac{-15}{8}$

Final Answer $=\frac{149}{88}=1\frac{61}{88}$

9. Plan: $\boxed{\frac{A}{B}-\frac{C}{D}}$

$A=0$, $B=8$, $C=34$, $D=-27$

$\frac{A}{B}=0$, $\frac{C}{D}=\frac{34}{-27}=\frac{-34}{27}$

Final Answer $=0-\frac{-34}{27}=\frac{34}{27}=1\frac{7}{27}$

10. Plan: $\boxed{\frac{A}{B}-\frac{C}{D}}$

$A=\frac{26}{9}$, $B=\frac{52}{7}$, $C=64$, $D=-1$

$\frac{A}{B}=\frac{7}{18}$, $\frac{C}{D}=\frac{64}{-1}=-64$

Final Answer $=\frac{7}{18}-(-64)=\frac{7}{18}+64=64\frac{7}{18}$

Mathematics with Hovsepian

Name:______________________

** Group Work/Group Grade **
Mini-Monster #2 Form 8

Dear Cousins! Part I of this "Mini-Monster" project is due 10 minutes before the class ends and is worth 100 points. Each group got a different form of the monster and each member must do their own work. You should consult with your group members to work out your strategy as you work on Part I. This part includes your strategy and the work on the problem. As usual, I will guide you as I visit your groups periodically. Remember, you are getting a Group Grade and becoming mathematically powerful.

Cousins! After I have given you the **green light** to proceed because your solution is correct (see the "Feedback/Assessment" box), then and ONLY then may you continue with Part II, the FINAL version of your revision. Part II is due on Thursday the latest and will be worth 400 points. Explain your process and the mathematical concepts you've used to arrive at your solution in writing bubbles on construction paper with the necessary LOVE and affection. (Take PRIDE in your work !!)

For now, keep your work in the Classwork Section of your notebook. You will transfer the final version to the PORTFOLIO section after I have graded it.
Remember! All work with LOVE.

$$\frac{\left[\left(3\frac{1}{3}\right)\left(2\frac{1}{6}\right)-\frac{\frac{1}{3}}{7}\right]\cdot\frac{126}{2}}{426+1\left[16-{}^{-}2+2(7-9)^2\right]}+\frac{\frac{\frac{1}{3}}{2\frac{2}{5}}}{\frac{1}{10-2(3-5)^3}}+\frac{\frac{1}{3}-\frac{1}{6}}{\left[\left(\frac{6}{7}\right)\cdot\left(\frac{-7}{6}\right)\div\left(\frac{2}{3}\right)\cdot\left(\frac{3}{2}\right)\right]}=$$

Feedback/Assessment From Mr. Hovsepian

- ✪ Your group STRATEGY write-up was satisfactory.
- ✪ *Redo* the circled portions on your WORK (☑ check work).
  - ○ your answer was not correct
  - ○ your answer was correct but it lacked elegance (math power)
- ✪ Observation of Group chemistry (holistic scoring)
  - ○ excellent
  - ○ satisfactory
  - ○ revise your work to meet our standards
- ✪ Your Paper has met my requirements. Proceed to Final Version (Part II), keeping the cover sheet along with your work in your Portfolio Section
- ✪ Place this project into your Classwork/Homework Section - Revision is needed. You still have time to make the necessary improvements.

Dear Cousins! Each group received a different form of the "Mini-Monster" project and each member must do all the work. You must consult with your group members as you work out your strategy. This monster project consists of two parts:

Part I: As a group, work out the problem on regular paper nicely and with LOVE. This part of the mini-monster is due by the end of class.

When you receive your work back, refer to the "Feedback/Assessment" box below for the criteria you need to consider as you revise and correct your work. You must keep improving your work until you meet my standards. After I give you the green light, you may proceed to Part II.

Group Leader: ....

Table # ---- : ....; ....

..................; ....

75 pt + 25 pt (math power) = 100 pt

Impress Me !!!

<u>Part II</u>: You are now ready to complete a final version of the mini-monster. Include the KEY mathematical concepts (AS MUCH AS POSSIBLE) you've used to arrive at your solution in writing bubbles on construction paper. Take pride in your work - do it with LOVE. Remember you are getting a Group Grade and becoming more mathematically powerful.

Bottom Line:

The above project should prepare you for the EXAM. You should be able to work these mini-monster problems out WITHOUT the use of calculators UNDER 22 minutes.

✂ ---

Feedback/Assessment From Mr. Hovsepian

Table #: .... Leader: ....

- ✪ Your group STRATEGY write-up was satisfactory.
- ✪ *Redo* the circled portions on your WORK (☑ check work).
  - ○ your answer was not correct
  - ○ your answer was correct but it lacked elegance (math power)
- ✪ Observation of Group chemistry (holistic scoring)
  - ○ excellent
  - ○ satisfactory
  - ○ revise your work to meet our standards
- ✪ Your Paper has met my requirements. Proceed to Final Version (Part II), keeping the cover sheet along with your work in your Portfolio Section
- ✪ Place this project into your Classwork/Homework Section - Revision is needed. You still have time (one day)to make the necessary improvements.

Math Is Fun

Appendix G

Mastering
Order Of Operations

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Please Note: Pages 5 & 6 of this document are the slides I use in my classes.

Unit on Positive and Negative Numbers

AND

Mastering Order Of Operations

Note: This unit is 24 pages long. Only sample questions have been included.

Audience: These classes contain mostly juniors and seniors who have flunked math once, twice, or even more. They have turned off to math and fail to see its potential for fun. Because many of them are affiliated with gangs, creating a family feeling within the class is important. Therefore, students are addressed as "**cousins**".

Unit on **Positive** and **Negative** Numbers in motion

Dear Cousins: This is what you will learn in the next few weeks (✪ = new concept)

✪ A) Simplifying expressions involving integers (Boys vs Girls; Guards/Houses/Bedrooms/Mansions/Castles,...) - (Humanizing Mathematics)

B) Mr. Hovsepian's GROUND RULE regarding order of operations. Math is BINARY. What is a mathematical term and how do we deal with it. How to avoid committing mathematical CRIMES! REVIEW

✪ C) To add, subtract, multiply, and divide positive and negative integers

✪ D) To find powers of positive and negative numbers

E) To compare integers (revisited)

F) Simplifying expressions involving radicals (we have covered this) - REVIEW

✪ G) Become mathematically powerful in dissecting and simplifying COMPLEX expressions MONSTER Problems using "The Plan" with PATIENCE.

✪ H) To use positive and negative integers in analyzing word problems.

A) The contract! Girls Rule !!!!! ❤❤❤

Numbers have Gender. (Look to each number's left side.)
Our objective is to be able to find whether a number is + or -.

(+) are boys ❤
(-) are girls ❤
1. Boys get together.
2. Girls get together.
3. Boys and girls pair up.

(+)(-) = ...
(-)(+) = ...

Remember, GIRLS RULE !

(+)(+) = ...
(-)(-) = ...
Whenever girls fight it takes a boy to separate them!

parenthesis ( ) is a HOUSE
bracket [ ] is a MANSION
brace { } is a CASTLE

{ [( )] }

Everyone must get outside their houses, mansions, and castles using the "Ground Rules". (Smaller dwelling places first.)
Homeless people have no dwelling place and are treated like those who are outside their homes.

Sample Questions

SQ #1- How do we determine if a number is POSITIVE or NEGATIVE? Our Rule!
SQ #2- Cousins! Every House/Mansion/Castle in mathematics MUST have a guard. Sometimes more than one guard is present. *Explain* this using examples your group can agree to. All work with LOVE.
SQ #3- What do you see in the problem below? Please use words in describing the expression .

$$4-\left({}^{-}5-{}^{+}7\right)-6^{2}$$

SQ #4- Translate into a mathematical EXPRESSION.
I see a house with four bedrooms. In the first bedroom there are seven girls. In the second bedroom there are four boys. In the third bedroom there are five girls. In the fourth bedroom there are eight girls. Outside the house there is a female guard.

SQ #5- Describe what you see in this situation.

$-5^2 + (-5)^2$

SQ #6- Use your imagination and simplify the below situations?

$5\left[37 + 2\left(^{-}10 - ^{-}8 + 4\right) - (9 - 12)\right] - 6^2 - 1^{109}$

SQ #7- Cousins:
- ✪ Write a positive number or a negative number to represent each situation.
- ✪ Compute the sum of the numbers (same thing as Boys vs Girls....)
- ✪ Answer each question

➲ On October 16, 2003, the Hoover High Drama Club treasury contained $1,234. The club then presented a play on November 3, 2003, for which they paid a royalty of $280. Scenery and costumes cost $1,876, and programs and other expenses totaled $1,355. The sale of tickets brought in $3900. How much money was in the treasury after the play was presented?

➲ The volume of water in a tank **during** a six-day period changed as follows: up 568 Liters, down 340 L, up 93 L, down 158 L, up 34 L, and down 39 L. What was the volume of water in the tank at the **beginning** of the six-day period if the final volume was 54 L ?

B) Hovsepian's Ground Rule
DO NOT CROSS THE FORBIDDEN ZONE until everything on the left is on the ground with multiplication and everything on the right is on the ground with addition. Then you may cross.

◆ radicals ◆ exponential #	
multiplication (x)	addition (+) ⇦ *Ground Level*
◆ division	◆ subtraction

Numbers with exponents are referred to as "dope addicts". They must be brought back down to ground level (because they are high) by eliminating the exponent before determining their gender. $-3^2 = -1 \cdot 9 = {}^{-}9$

Dope houses must also be brought down to ground level. Because dope confuses the brain, determining gender comes last. $\left(^{-}3\right)^2 = 9$

Remember how to deal with your dwelling places first.!!!

Simplify: ✪ $\frac{\sqrt{3}}{-4} \cdot \frac{4}{\sqrt{3}} \div \frac{12}{7} \cdot \frac{-7}{12} =$ ✪ $\left(\frac{\sqrt{3}}{-4} \cdot \frac{4}{\sqrt{3}}\right) \div \left(\frac{12}{7} \cdot \frac{-7}{12}\right) =$

Cousins! Do you see/understand the difference between the above two ?

SQ #8- Please be EXTRA neat! Simplify the monster problem. All work on separate papers and with LOVE. Group leaders check with Mr. Hovsepian for specific instructions.

$$\frac{\left[\left(3\frac{1}{3}\right)\left(2\frac{1}{6}\right)-\frac{\frac{1}{3}}{7}\right]\cdot\frac{126}{2}}{426+1\left[16-{}^{-}2+2(7-9)^{2}\right]}+\frac{\frac{\frac{1}{3}}{2\frac{2}{5}}}{\frac{1}{10-2(3-5)^{3}}}+\frac{\frac{1}{3}-\frac{1}{6}}{\left[\left(\frac{6}{7}\right)\cdot\left(3\frac{2}{3}\right)\right]\div\left[\left(\frac{2}{3}\right)\cdot\left(\frac{6}{7}\right)\right]}=$$

"The Plan" Dear Mr. H, our plan is $\frac{A}{B}+\frac{C}{D}+\frac{E}{F}$

SQ #9- Please be EXTRA neat! Simplify the monster problem. All <u>work</u> on separate papers and with LOVE. Group leaders check with Mr. Hovsepian for specific instructions.

$$\frac{\left[\left(-3\frac{1}{5}\right)\left(4\frac{2}{6}\right)-\frac{\frac{4}{3}}{10}\right]\cdot\frac{12}{428}}{\frac{(4442)^{0}}{\left\{4\left[10-2\left(12+{}^{-}15\right)^{3}\right]-215\right\}}}+\frac{5\frac{1}{3}-14\frac{3}{6}}{3\frac{2}{3}-\frac{111}{888}-\frac{25}{24}}=$$

The Plan Is:

$\frac{A}{B}+\frac{C}{D}$

(you know the requirements! ....all work on separate papers ...)

Answers:

A = ...10 P

B= .... 10 P $\quad \frac{A}{B}$ = ...10

C= .... 10 P $\quad \frac{C}{D}$ = ...10

D= .... 10 P

Final Answer: $\boxed{\quad}$ 30 P

Cousins, remember to tell me when you see another way to do a process. More ways means you've done more thinking which means that you receive more math power points!

Target Group: Those who are struggling in [8]Mathematics !!!
My Take On How To DEAL with
POSITIVE and NEGATIVE
Numbers
Order Of Operations

1st place the next two questions on the board and ask the students to find the solution without the use of any calculators.

$$8-10\left(6-{}^{-}11-3\right)-4^2=$$

$$12+\left({}^{-}3\right)^2-\left(17-{}^{-}4\right)\cdot(-2)-5^2=$$

You will find out that most students will get the wrong answers. As a matter of fact you will get as many different answers as you have students in the class ! (a slight exaggeration, but you get the point)

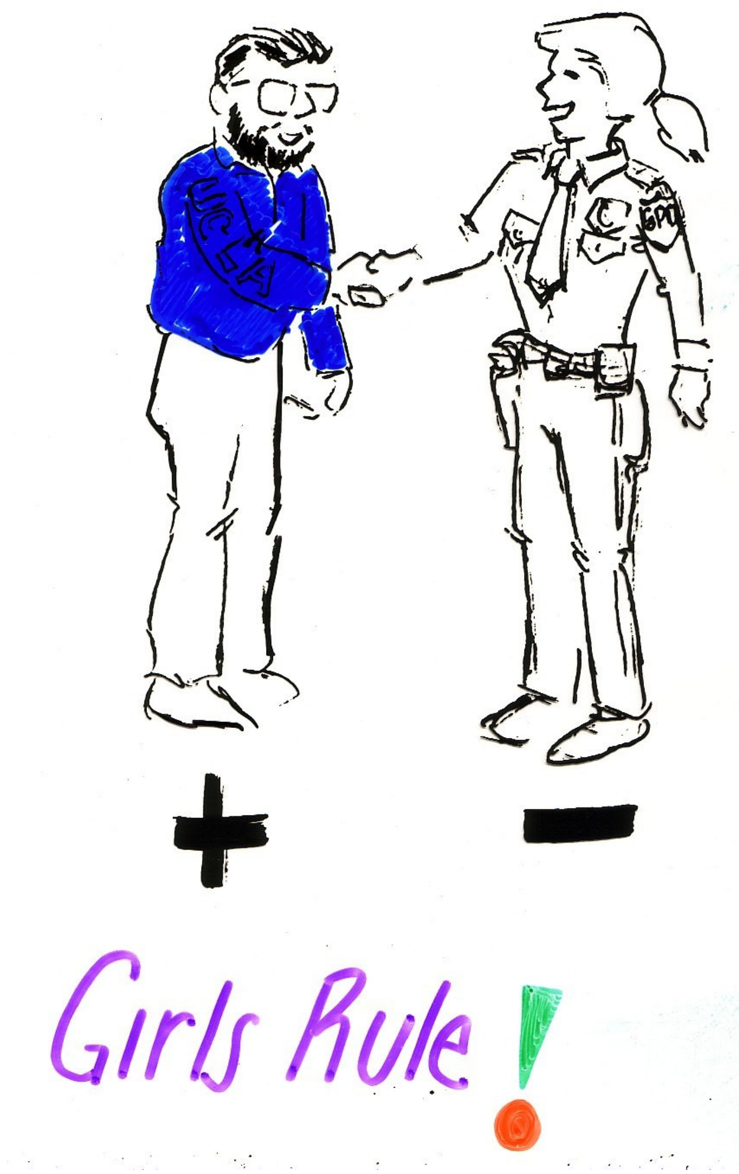
Bottom line the students are CONFUSED as how to attack the problem.

Lead the discussion that you believe that numbers must have gender!!! Place slide #1 on the screen.

Agree at the end that positive numbers are BOYS and negative numbers are GIRLS. This will get there attention , trust me. Especially when you say what happens when we multiply (+) number with a (-) number ? Negative numbers win !!!! and GIRLS win!!!!

“make sure you emphasize--- this is strictly for fun and will help them get the answers correct” Now place slide #2.

[8] As of September 2004, over 50% of our students are NOT Algebra Ready as they enter High Schools. Furthermore, a significant number of High School students are still in Algebra !!!



This slide shows the agreement that BOYS represent POSITIVE numbers and GIRLS represent negative numbers. Also, it indicates that GIRLS RULE !

Because

$$(+)(-)=(-)$$

Math Is Fun

Happy Holidays

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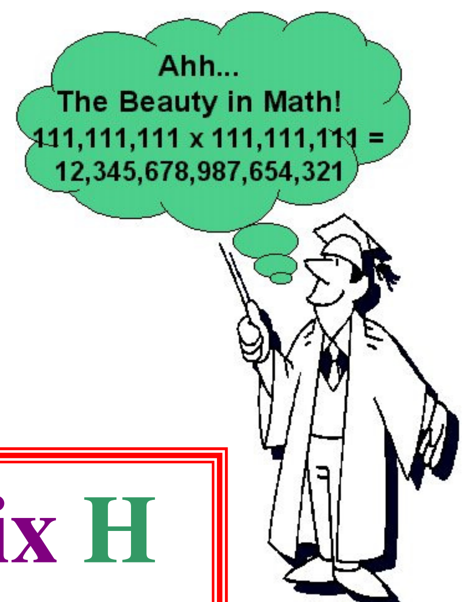

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Appendix H

Holiday Problems

Do your magic! If the problem involves letters, rearrange the letters **of the answer** to form a word related to the holidays.

1. **Simplify:** $Xas\left[\frac{1}{as}-\frac{m}{X}\right]$

2. **Simplify:** $\frac{4x^5p^3}{3x^2a^2}\cdot\frac{6ca^3e^2}{8p^2x^3}$

3. **Simplify:** $\frac{(ry)^3}{3x^2a^2}\cdot\left[\frac{y}{a}+\frac{r}{e}\right]\cdot\frac{(ae)^2}{(ry)^2}$

4. **Simplify:** $\left[2^2(5)(4)+3\right]\cdot\sqrt{5(10)^2+2\left[\frac{100-10}{3}+2^3\right]}+\ln e^{18}$

5. **Simplify:** $\left(\ln e^2\right)\left(\log 10^7\right)\left(\frac{1331}{11^2}\right)\sqrt{169}+\lfloor 7.9 \rfloor$

6. **Simplify:** $\left[\frac{Sn}{a+S}+\frac{1}{a-S}-\frac{S-S^2n}{a^2-S^2}\right]\div\frac{1}{a^2-S^2}$

7. **Find** $f(7)$, if $f(x)=6x^3-x^2-x+9$

8. **Simplify:** $\left[\frac{N}{E}\cdot\frac{L}{O}\right]\cdot\left(O^2E^2\right)$

9. **Simplify:** $\frac{15(ry)^3}{ae}\cdot\left[\frac{3}{5}+\frac{1}{15}\right]\cdot\frac{199a^2e^2}{y^2r^2}+21eray$

10. **Subtract.** Each letter represents a different number:

$$\begin{array}{r} SANTA \\ -\ CLAUS \\ \hline XMAS \end{array}$$

11. **Solve:** $x^2 + 37x = 2010$

12. Factor **2011** over the set of prime numbers.

13. **Evaluate:** $abc^2 + ab^2 - d$, when $a = 4$, $b = 5$, $c = 10$, and $d = 89$.

14. **Find** $f(44)$, if $f(x) = x^2 + 2x - 14$

15. **Solve:** $x^2 - 2010 = 0$

16. **Simplify:** $2011^2 - 2010^2$

17. **Simplify:** $2010^2 - 2000^2$

18. **Find** the first year after 2010 which has the prime number 67 as a factor.

19. **Solve:** $5\left(x - \frac{83}{5}\right) + 3x = 16005$

20. **Solve:** $\frac{x}{2009} + \frac{x}{2010} = 1$

21. **Solve:** $x^2 - 4021x = 4042110$

22. **Simplify:** $\dfrac{p^3x^5}{x^2a^2}\cdot\dfrac{ca^3x^{-3}}{p^2e^{-2}}$

23. **Simplify:** $(O+n)\div\left(\dfrac{1}{O}+\dfrac{1}{n}\right)$

24. **Simplify:** $\left[\dfrac{TH}{\dfrac{1}{E}+\dfrac{1}{A}+\dfrac{1}{R}}\right]\cdot(AR+ER+EA)$

25. **Simplify:** $\dfrac{1}{4}\left[\dfrac{H+A}{H-A}-\dfrac{H-A}{H+A}\right]\cdot\left[YP\cdot\left(\dfrac{P}{Y}+\dfrac{1}{P}\right)\cdot\left(\dfrac{(H^2-A^2)P^2Y}{P^2+Y}\right)\right]$

26. **Simplify:** $\dfrac{3N^2E^2}{5}\div\dfrac{9N}{W}\cdot\dfrac{15}{E}$

27. **Simplify:** $\{M,e,r,r,y,X,M,a,s\}\cap\{H,a,p,p,y,N,e,w,Y,e,a,r\}$

28. **Simplify:** $\dfrac{76(San)+76a}{n}\div\dfrac{152}{2n}$

29. **Simplify:** Solve for C: $\dfrac{OC}{L}+\dfrac{LO}{O^{-1}}=\dfrac{EO^2V^2}{V}+LO^2$

30. **Find** all “prime years” of this century.

31. **Simplify:** $\sqrt{\dfrac{x^2G^3d^5}{O^{-2}Gx^2d^3}}$

32. **Simplify:** $UV^2\left(\dfrac{LS}{V}+\dfrac{1}{V^2}\right)$

33. **Simplify:** $a^2\left(\frac{nS}{a}+\frac{1}{a}\right)$

34. **Simplify:** $\sqrt{S^2}\sqrt{-1}$

35. **Simplify:** $\sqrt{-1}\cdot\sqrt{\frac{(CO)^2 n^4 m^2}{n^2 g^{-2}}}$

36. **Simplify:** $-2(i)^{2002}$

37. **Simplify:** $(nt)^2\left(\frac{O}{t}+\frac{w}{n}\right)\left(\frac{Ow}{On+tw}\right)$

38. **Evaluate:** $\int_{-a}^{San} dx$

39. **Simplify:** $\sqrt{\frac{R^2 a^7 m^2}{an^{-2}d^{-2}}}$

40. **Simplify:** $\left(\sqrt{\frac{m^2 o^2}{y^{-2}}}\right)\left(\sqrt{-1}\right)\left(\sqrt{\frac{u^2 p^4}{(kr)^{-2}}}\right)$

41. **Simplify:**

$$\left\{\left[M(X+A)(X-A)+A^2M\right](E)\right\}\cdot$$
$$\left\{\left[\frac{\frac{R+X}{X}+\frac{X}{R-X}}{\frac{R}{R-X}}\right]^2-\left[\frac{4AS}{3x\left[(E+Y)^2-(E-Y)^2\right]}+\frac{4AS}{3E\left[(Y+X)^2-(Y-X)^2\right]}+\frac{4AS}{3Y\left[(X+E)^2-(X-E)^2\right]}\right]\right\}\cdot Y$$

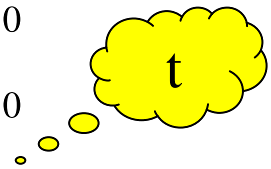
Answers:

1. $X - mas$

2. peace

3. year

4. 2010



5. 2010

6. San + a

7. 2009

8. NOEL

9. 2011 year

10.

SANTA	***2 4 9 7 4***
- *CLAUS*	***1 8 4 3 2***
XMAS	***6 5 4 2***

11. $S = \{30, -67\}$

12. 2011 is a prime number.

13. 2011

14. 2010

15. $x = \pm\sqrt{2010} \approx \pm 44.83$

16. 4021

17. 4019

18. 2077

19. 2011

20. 1004.75

21. $S = \{2010, 2011\}$

22. peace

23. On

24. THEAR = EARTH

25. HAPPY

26. NEW

27. year

"Of course he hasn't left you anything, that's because you've been a very naughty boy, Charlie"

28. San + a

29. LOVE

30. 2003, 2011, 2017, 2027, 2029, 2039, 2053, 2063, 2069, 2081, 2083, 2087, 2089, 2099

31. God

32. LUVS + U

33. San + a

34. is

35. coming

36. 2

37. town

38. San + a

39. **Ramadan**

40. **Yom Kippur**

41. ***MERRY* - *XMAS***

$$\left\{\left[M\left(X+A\right)\left(X-A\right)+A^{2}M\right]\ \left(E\right)\right\} = \boxed{MX^{2}E}$$

$$\left[\frac{\dfrac{R+X}{X}+\dfrac{X}{R-X}}{\dfrac{R}{R-X}}\right]^{2} = \boxed{\frac{R^{2}}{X^{2}}}$$

$$-\left[\frac{4AS}{3X\left[\left(E+Y\right)^{2}-\left(E-Y\right)^{2}\right]}+\frac{4AS}{3E\left[\left(Y+X\right)^{2}-\left(Y-X\right)^{2}\right]}+\frac{4AS}{3Y\left[\left(X+E\right)^{2}-\left(X-E\right)^{2}\right]}\right]y$$

$$=$$

$$MXAS$$

Conclusion: $\boxed{MX^{2}E\left[\dfrac{R^{2}}{X^{2}}-\dfrac{AS}{EXY}\right]\cdot y}$ = ***MERRY* - *XMAS***

Test Your Christmas Trivia !!!

1. How to build a goat

In Sweden, a common Christmas decoration is the Julbukk, a small figurine of a goat. Of what material is it usually made?

A) Candy
B) Straw
C) Uranium
D) Fir wood

2. Feeding the wren

What is the Irish custom of "feeding the wren" or "hunting the wren" on December 26?

A) Taking one's in-laws out to dinner
B) Carrying a wren door to door, to collect money for charity
C) Leaving a basket of cakes at the door for passers-by
D) Putting out suet and seeds for the wild birds

3. The Nutcracker's enemy

In Tchaikovsky's ballet "The Nutcracker", who is the nutcracker's main enemy?

A) A girl called Clara

B) The King of the Mice

C) Dr. Almond

D) Drosselmeyer the magician

4. The which of endor

At lavish Christmas feasts in the Middle Ages, swans and peacocks were sometimes served "endored". What does that mean?

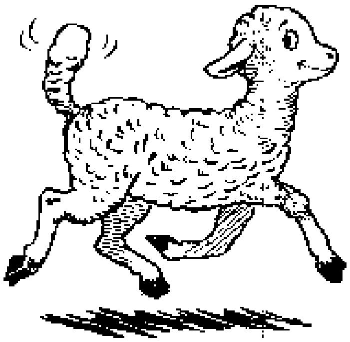
A) The feet and beaks were coated with gold

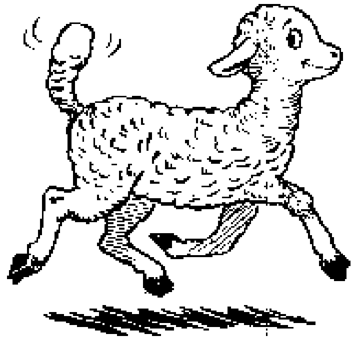
B) The guests knelt in adoration as the birds were brought in

C) The birds had been raised on grain soaked in brandy

D) The flesh was painted with saffron dissolved in melted butter

5. Christmas shear

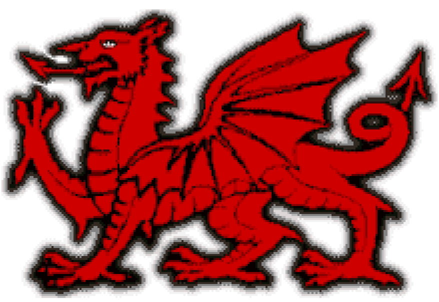




All through the Christmas season in old England, "lambswool" could be found in the houses of the well-to-do. What was it?

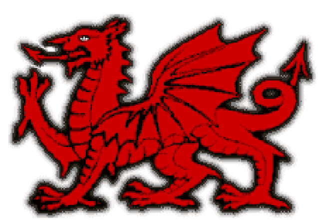
A) Imitation snow used in decorations
B) A brew of hot ale with roast apples floating in it
C) The material used for knitting Christmas gifts
D) A fluffy confection made from almonds and sugar

6. Snip, snap, dragon!



The ancient game of Snapdragon has been part of English Christmases for over 300 years. Players are egged on by a chant, part of which goes, "Take care you don't take too much, Be not greedy in your clutch, Snip, snap, dragon!" What is "the dragon" in this game?

A) A costumed child
B) Flames of burning brandy
C) The oldest male in the room
D) A "snapper" made from fireplace tongs



Answers:

1. **B**

Scandinavian Christmas festivities feature a variety of straw decorations in the form of stars, angels, hearts and other shapes, as well as the Julbukk.

2. **B**

One explanation for this St. Stephen's day custom refers to a legend in which the saint was given away by a chattering wren while hiding from his enemies. Children cage the wren to help it do penance for this misdeed. Often the children carry a long pole with a holly bush at the top - which is *supposed* to hide a captured wren. An artificial wren may also be used.

3. **B**

The King of the Mice, usually represented with seven heads, leads his troops against the nutcracker's toy soldiers. He loses the battle when Clara, the heroine, stuns him with a shoe

4. **D**

In addition to their painted flesh, "endored" birds were served wrapped in their own skin and feathers, which had been removed and set aside prior to roasting.

5. **B**

"Lambswool" was the drink that filled the wassail bowl. Sugar, eggs and spices were added to the ale, and toast floated on top with the apples. Poor people would bring their mugs to the door hoping for a share of the steaming drink.

6. **B**

When the room is dark, a bowl of raisins soaked in brandy is lit. Who will be brave enough to claim the prize from the fierce dragon flames?

Math Is Fun

Appendix I

A Determinant Moment

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Determinants in Analytic Geometry

Objective: How determinants may be used to answer certain geometrical questions and to find equations for geometrical objects.

One must consider determinants of matrices whose entries are variables or algebraic expressions. In this case the determinant will itself be an algebraic expression.

For example, consider the equation

$$\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & -5 & 1 \end{vmatrix} = 0.$$

Since x and y are variables, this equation may be true for some points (x, y) in the $x-y$ plane, but untrue for other points $(1,2)$ and $(3,-5)$. Notice that this equation is clearly true for the points and since plugging those points in produces a matrix with two identical rows, whose determinant must be zero. *Expanding* the determinant along the first row gives

$$\begin{vmatrix} 2 & 1 \\ -5 & 1 \end{vmatrix} x - \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} y + \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} 1 = 0.$$

Or $7x+2y-11=0$. This is clearly the equation for a line, and since the points $(1,2)$ and $(3,-5)$ satisfy this equation, it is an equation for the line passing through $(1,2)$ and $(3,-5)$. Thus there is a way to use a <u>determinant</u> to express the <u>equation</u> of a line through two given points.

Or use the below technique to accomplish the same answer faster!!!:

$$\begin{vmatrix} x & y \\ 1 & 2 \\ 3 & -5 \\ x & y \end{vmatrix} \qquad 2x+(1)(-5)+(3)(y) \quad \ldots \#1$$

$$\begin{vmatrix} x & y \\ 1 & 2 \\ 3 & -5 \\ x & y \end{vmatrix} \qquad -(1)(y)-(2)(3)-(-5)(x) \quad \ldots \#2$$

Simplifying expressions #1 & #2 gives: $2x-5+3y-y-6+5x=7x+2y-11$

The question remains of how to discover this "determinant form" of an algebraic equation.

☑ Consider the problem of finding the equation of a circle through three given points. The standard equation for a circle is $(x-h)^2+(y-k)^2=r^2$, which may be expanded and regrouped into the form $a(x^2+y^2)+bx+cy+d=0$

Math Is Fun

Appendix J

Selected Quotes

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Selected Math Quotes

Imagination is more important than knowledge.	Albert Einstein
The primary question was not what do we know, but how do we know it.	Aristotle
Perfect numbers like perfect men are very rare…	Rene Descartes
The things of this world cannot be made known without a knowledge of mathematics.	Roger Bacon
I am interested in mathematics only as a creative art.	Godfrey Hardy
Mathematics is an independent world Created out of pure intelligence.	William Wordsworth
Mathematics consists of proving the most obvious thing in the least obvious way.	George Polya
The primary question is not What do we know but How do we know it.	Aristotle
A mathematician, like a painter or poet, is a maker of patterns … with ideas.	Godfrey H. Hardy
Mathematics consists of proving the most obvious thing in the least obvious way.	Henri Poincare

With me everything turns into mathematics.	Rene Descartes
Mathematics is the cheapest science. Unlike physics or chemistry, it does not require any expensive equipment. All one needs for mathematics is a pencil and paper.	George Polya
Imagination is more important than knowledge	Albert Einstein
If I were to awaken after having slept for a thousand years, my first question would be: Has the Riemann hypothesis been proven?	David Hilbert
If others would but reflect on mathematical truths as deeply and as continuously as I have, they would make my discoveries.	Karl Gauss
Music is the pleasure the human soul experiences from counting without being aware it is counting.	Gottfried Leibniz