100 Years of QUANTUM MYSTERIES

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Planck Explains Blackbody Radiation (1900)

1900s

by Max Tegmark and John Archibald Wheeler

Quantum Electrodynamics

Schrödinger Equation; Copenhagen Interpretation (1926) Heisenberg Uncertainty Bohr's Theory of Principle (1927) Atomic Spectra Pauli Exclusion (1913)Principle (1925) Dirac Equation for the Electron (1928) Bose-Einstein Condensation Predicted Anti-Electron (1924)Discovered (1932)1910s 1920s 1930s

and Renormalization (1948)

Atomic Bomb (1945)

Einstein Explains Photoelectric Effect (1905)



Schrödinger's Cat Paper; Einstein-Podolsky-Rosen Paper about Local Realism (1935)

> Superfluidity Discovered (1938)



1940s

FOUNDATIONS of quantum mechanics were laid in the period 1900–1926, including seminal contributions from the seven physicists shown at the right. Over its century of development, quantum mechanics has not only profoundly advanced our understanding of nature but has also provided the basis of numerous technologies. Yet some fundamental enigmas of quantum theory remain unresolved.



MAX PLANCK (1858–1947)



ALBERT EINSTEIN (1879–1955)



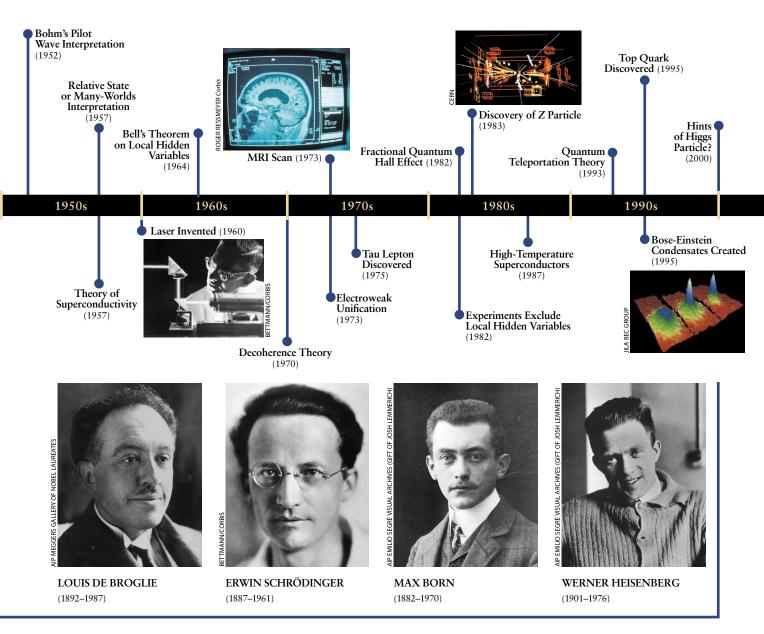
NIELS BOHR (1885–1962)

As quantum theory celebrates its 100th birthday, spectacular successes are mixed with persistent puzzles

n a few years, all the great physical constants will have been approximately estimated, and ... the only occupation which will then be left to the men of science will be to carry these measurements to another place of decimals." As we enter the 21st century amid much brouhaha about past achievements, this sentiment may sound familiar. Yet the quote is from James Clerk Maxwell and dates from his 1871 University of Cambridge inaugural lecture expressing the mood prevalent at the time (albeit a mood he disagreed with). Three decades later, on December 14, 1900, Max Planck announced his formula on the blackbody spectrum, the first shot of the quantum revolution.

This article reviews the first 100 years of quantum mechanics, with particular focus on its mysterious side, culminating in the ongoing debate about its consequences for issues ranging from quantum computation to consciousness, parallel universes and the very nature of physical reality. We virtually ignore the astonishing range of scientific and practical applications that quantum mechanics undergirds: today an estimated 30 percent of the U.S. gross national product is based on inventions made possible by quantum mechanics, from semiconductors in computer chips to lasers in compact-disc players, magnetic resonance imaging in hospitals, and much more.

In 1871 scientists had good reason for their optimism. Classical mechanics and electrodynamics had powered the industrial revolution, and it appeared as though

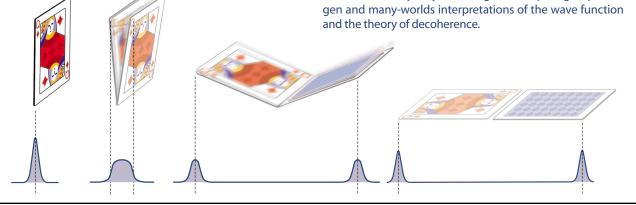


QUANTUM CARDS

A SIMPLE FALLING CARD IN PRINCIPLE LEADS TO A QUANTUM MYSTERY

According to quantum physics, an ideal card perfectly balanced on its edge will fall down in both directions at once, in what is known as a superposition. The card's quantum wave function (*blue*) changes smoothly and continuously from the balanced state (*left*) to the mysterious final state (*right*) that seems to have the card in two places at once. In

practice, this experiment is impossible with a real card, but the analogous situation has been demonstrated innumerable times with electrons, atoms and larger objects. Understanding the meaning of such superpositions, and why we never see them in the everyday world around us, has been an enduring mystery at the very heart of quantum mechanics. Over the decades, physicists have developed several ideas to resolve the mystery, including the competing Copenhagen and many-worlds interpretations of the wave function and the theory of decoherence.



their basic equations could describe essentially all physical systems. But a few annoying details tarnished this picture. For example, the calculated spectrum of light emitted by a glowing hot object did not come out right. In fact, the classical prediction was called the ultraviolet catastrophe, according to which intense ultraviolet radiation and x-rays should blind you when you look at the heating element on a stove.

The Hydrogen Disaster

n his 1900 paper Planck succeeded in deriving the correct spectrum. His derivation, however, involved an assumption so bizarre that he distanced himself from it for many years afterward: that energy was emitted only in certain finite chunks, or "quanta." Yet this strange assumption proved extremely successful. In 1905 Albert Einstein took the idea one step further. By assuming that radiation could transport energy only in such lumps, or "photons," he explained the photoelectric effect, which is related to the processes used in presentday solar cells and the image sensors used in digital cameras.

Physics faced another great embarrassment in 1911. Ernest Rutherford had convincingly argued that atoms consist of electrons orbiting a positively charged nucleus, much like a miniature solar system. Electromagnetic theory, though, predicted that orbiting electrons would continuously radiate away their energy and spiral into the nucleus in about a trillionth of a second. Of course, hydrogen atoms were known to be eminently stable. Indeed, this discrepancy was the worst quantitative failure in the history of physics—underpredicting the lifetime of hydrogen by some 40 orders of magnitude.

In 1913 Niels Bohr, who had come to the University of Manchester in England to work with Rutherford, provided an explanation that again used quanta. He postulated that the electrons' angular momentum came only in specific amounts, which would confine them to a discrete set of orbits. The electrons could radiate energy only by jumping from one such orbit to a lower one and sending off an individual photon. Because an electron in the innermost orbit had no orbits with less energy to jump to, it formed a stable atom.

Bohr's theory also explained many of hydrogen's spectral lines—the specific frequencies of light emitted by excited atoms. It worked for the helium atom as well, but only if the atom was deprived of one of its two electrons. Back in Copenhagen, Bohr got a letter from Rutherford telling him he had to publish his results. Bohr wrote back that nobody would believe him unless he ex-

plained the spectra of all the elements. Rutherford replied: Bohr, you explain hydrogen and you explain helium, and everyone will believe all the rest.

Despite the early successes of the quantum idea, physicists still did not know what to make of its strange and seemingly ad hoc rules. There appeared to be no guiding principle. In 1923 Louis de Broglie proposed an answer in his doctoral thesis: electrons and other particles act like standing waves. Such waves, like vibrations of a guitar string, can occur only with certain discrete (quantized) frequencies. The idea was so unusual that the examining committee went outside its circle for advice. Einstein, when queried, gave a favorable opinion, and the thesis was accepted.

In November 1925 Erwin Schrödinger gave a seminar on de Broglie's work in Zurich. When he was finished, Peter Debye asked, You speak about waves, but where is the wave equation? Schrödinger went on to produce his equation, the master key for so much of modern physics. An equivalent formulation using matrices was provided by Max Born, Pascual Jordan and Werner Heisenberg around the same time. With this powerful mathematical underpinning, quantum theory made explosive progress. Within a few years, physicists had explained a host of measurements, including spectra of more complicated atoms

But what did it all mean? What was this quantity, the "wave function," that Schrödinger's equation described? This central puzzle of quantum mechanics remains a potent and controversial issue to this day.

Born had the insight that the wave function should be interpreted in terms of probabilities. When experimenters measure the location of an electron, the probability of finding it in each region depends on the magnitude of its wave function there. This interpretation suggested that a fundamental randomness was built into the laws of nature. Einstein was deeply unhappy with this conclusion and expressed his preference for a deterministic universe with the oft-quoted remark, "I can't believe that God plays dice."

Curious Cats and Quantum Cards

C chrödinger was also uneasy. Wave functions could describe combinations of different states, so-called superpositions. For example, an electron could be in a superposition of several different locations. Schrödinger pointed out that if microscopic objects such as atoms could be in strange superpositions, so could macroscopic objects, because they are made of atoms. As a baroque example, he described the well-known thought experiment in which a nasty contraption kills a cat if a radioactive atom decays. Because the radioactive atom enters a superposition of decayed and not decayed, it produces a cat that is both dead and alive in superposition.

The illustration on the opposite page shows a simpler variant of this thought experiment. You take a card with a perfectly sharp edge and balance it on its edge on a table. According to classical physics, it will in principle stay balanced forever. According to the Schrödinger equation, the card will fall down in a few seconds even if you do the best possible job of balancing it, and it will fall down in both directions—to the left and the right—in superposition.

If you could perform this idealized thought experiment with an actual card, you would undoubtedly find that classical physics is wrong and that the card falls down. But you would always see it fall down to the left or to the right, seemingly at random, never to the left and to the right simultaneously, as the Schrödinger equation might have you believe. This seeming contradiction goes to the

very heart of one of the original and enduring mysteries of quantum mechanics.

The Copenhagen interpretation of quantum mechanics, which grew from discussions between Bohr and Heisenberg in the late 1920s, addresses the mystery by asserting that observations, or measurements, are special. So long as the balanced card is unobserved, its wave function evolves by obeying the Schrödinger equation—a continuous and smooth evolution that is called "unitary" in mathematics and has several very attractive properties. Unitary evolution produces the superposition in which the card has fallen down both to the left and to the right. The act of observing the card, however, triggers an abrupt change in its wave function, commonly called a collapse: the observer sees the card in one definite classical state (face up or face down), and from then onward only that part of the wave function survives. Nature supposedly selects one state at random, with the probabilities determined by the wave function.

The Copenhagen interpretation provided a strikingly successful recipe for doing calculations that accurately described the outcomes of experiments, but the suspicion lingered that some equation ought to describe when and how this collapse occurred. Many physicists took this lack of an equation to mean that something was intrinsically wrong with quantum mechanics and that it would soon be replaced by a

more fundamental theory that would provide such an equation. So rather than dwell on ontological implications of the equations, most physicists forged ahead to work out their many exciting applications and to tackle pressing unsolved problems of nuclear physics.

That pragmatic approach proved stunningly successful. Quantum mechanics was instrumental in predicting antimatter, understanding radioactivity (leading to nuclear power), accounting for the behavior of materials such as semiconductors, explaining superconductivity, and describing interactions such as those between light and matter (leading to the invention of the laser) and of radio waves and nuclei (leading to magnetic resonance imaging). Many successes of quantum mechanics involve its extension, quantum field theory, which forms the foundations of elementary particle physics all the way to the present-day experimental frontiers of neutrino oscillations and the search for the Higgs particle and supersymmetry.

Many Worlds

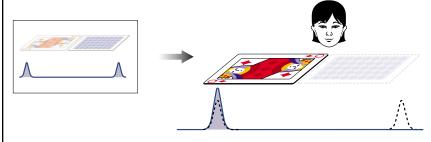
y the 1950s this ongoing parade of successes had made it abundantly clear that quantum theory was far more than a short-lived temporary fix. And so, in the mid-1950s, a Princeton University student named Hugh Everett III decided to revisit the collapse postulate in his doctoral thesis. Everett pushed the

COPENHAGEN INTERPRETATION

IDEA: Observers see a random outcome; probability given by the wave function. **ADVANTAGE:** A single outcome occurs, matching what we observe.

PROBLEM: Requires wave functions to "collapse," but no equation specifies when.

When a quantum superposition is observed or measured, we see one or the other of the alternatives of the distributions. other of the alternatives at random, with probabilities controlled by the wave function. If a person has bet that the card will fall face up, when she first looks at the card she has a 50 percent chance of happily seeing that she has won her bet. This interpretation has long been pragmatically accepted by physicists even though it requires the wave function to change abruptly, or collapse, in violation of the Schrödinger equation.



MANY-WORLDS INTERPRETATION One Alternative **IDEA:** Superpositions will seem like alternative parallel worlds to their inhabitants. Parallel World ADVANTAGE: The Schrödinger equation always works: wave functions never collapse. **PROBLEMS:** The bizarreness of the idea. Some technical puzzles remain. f wave functions never collapse, the Schrödinger equation predicts that the person looking at the card's superposition will herself enter a superposition of two possible outcomes: happily winning the bet or sadly losing. These two parts of the total wave function (of person plus card) carry on completely independently, like two parallel worlds. If the experiment is re-4/4 1/4 0/4 peated many times, people in most of the parallel worlds will see the card falling face up about half the time. Stacked cards SUCCESS RATE AFTER DROPPING FOUR CARDS (right) show 16 worlds that result when a card is dropped four times.

quantum idea to its extreme by asking the following question: What if the time evolution of the entire universe is always unitary? After all, if quantum mechanics suffices to describe the universe, then the present state of the universe is described by a wave function (an extraordinarily complicated one). In Everett's scenario, that wave function would always evolve in a deterministic way, leaving no room for mysterious nonunitary collapse or God playing dice.

Instead of being collapsed by measurements, microscopic superpositions would rapidly get amplified into byzantine macroscopic superpositions. Our quantum card would really be in two places at once. Moreover, a person looking at the card would enter a superposition of two different mental states, each perceiving one of the two outcomes. If you had bet money on the queen's landing face up, you would end up in a superposition of smiling and frowning. Everett's brilliant insight was that the observers in such a deterministic but schizophrenic quantum world could perceive the plain old reality that we are familiar with. Most important, they could perceive an apparent randomness obeying the correct probability rules [see illustration above].

Everett's viewpoint, formally called the relative-state formulation, became popularly known as the many-worlds interpretation of quantum mechanics, because each component of one's superposition perceives its own world. This viewpoint simplifies the underlying theory by removing the collapse postulate. But the price it pays for this simplicity is the conclusion that these parallel perceptions of reality are all equally real.

Everett's work was largely disregarded for about two decades. Many physicists still hoped that a deeper theory would be discovered, showing that the world was in some sense classical after all, free from oddities like big objects being in two places at once. But such hopes were shattered by a series of new experiments.

Could the seeming quantum randomness be replaced by some kind of unknown quantity carried about inside particles—so-called hidden variables? CERN theorist John S. Bell showed that in this case quantities that could be measured in certain difficult experiments would inevitably disagree with the standard quantum predictions. After many years, technology allowed researchers to conduct the experiments and to eliminate hidden variables as a possibility.

A "delayed choice" experiment proposed by one of us (Wheeler) in 1978 was successfully carried out in 1984, showing another quantum feature of

the world that defies classical descriptions: not only can a photon be in two places at once, but experimenters can choose, after the fact, whether the photon was in both places or just one.

The simple double-slit interference experiment, in which light or electrons pass through two slits and produce an interference pattern, hailed by Richard Feynman as the mother of all quantum effects, was successfully repeated for ever larger objects: atoms, small molecules and, most recently, 60-atom buck-yballs. After this last feat, Anton Zeilinger's group in Vienna even started discussing conducting the experiment with a virus. In short, the experimental verdict is in: the weirdness of the quantum world is real, whether we like it or not.

Quantum Censorship—Decoherence

The experimental progress of the past few decades was paralleled by great advances in theoretical understanding. Everett's work had left two crucial questions unanswered. First, if the world actually contains bizarre macroscopic superpositions, why don't we perceive them?

The answer came in 1970 with a seminal paper by H. Dieter Zeh of the University of Heidelberg, who showed that the Schrödinger equation itself gives rise

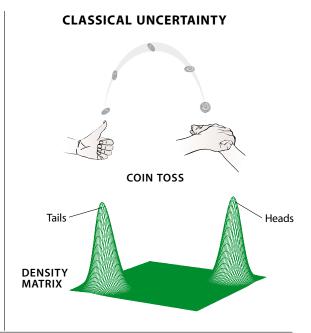
DECOHERENCE: HOW THE QUANTUM GETS CLASSICAL

IDEA: Tiny interactions with the surrounding environment rapidly dissipate the peculiar quantumness of superpositions. **ADVANTAGES:** Experimentally testable. Explains why the everyday world looks "classical" instead of quantum. **CAVEAT:** Decoherence does not completely eliminate the need for an interpretation such as many-worlds or Copenhagen.

The uncertainty of a quantum superposition (*left*) is different from the uncertainty of classical probability, as occurs after a coin toss (*right*). A mathematical object called a density matrix illustrates the distinction. The wave function of the quantum card corresponds to a density matrix with four peaks. Two of these peaks represent the 50 percent probability of

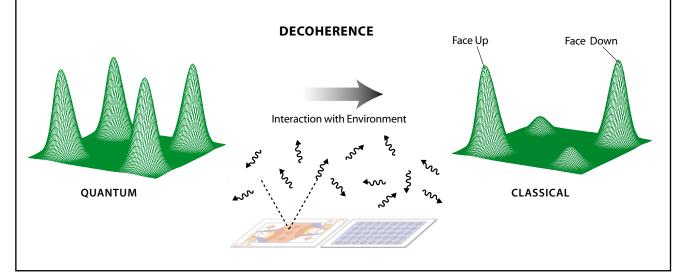
each outcome, face up or face down. The other two indicate that these two outcomes can still, in principle, interfere with each other. The quantum state is still "coherent." The density matrix of a coin toss has only the first two peaks, which conventionally means that the coin is really either face up or face down but that we just haven't looked at it yet.

COHERENT SUPERPOSITION Interference Face Up DENSITY MATRIX



Decoherence theory reveals that the tiniest interaction with the environment, such as a single photon or gas molecule bouncing off the fallen card, transforms a coherent den-

sity matrix very rapidly into one that, for all practical purposes, represents classical probabilities such as those in a coin toss. The Schrödinger equation controls the entire process.



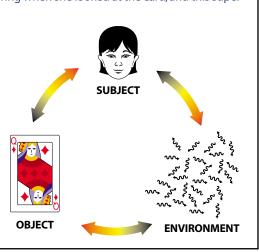
SPLITTING REALITY

t is instructive to split the universe into three parts: the object under consideration, the environment, and the quantum state of the observer, or subject. The Schrödinger equation that governs the universe as a whole can be divided into terms that describe the internal dynamics of each of these three subsystems and terms that describe interactions among them. These terms have qualitatively very different effects.

The term giving the object's dynamics is typically the most important one, so to figure out what the object will do, theorists can usually begin by ignoring all the other terms. For our quantum card, its dynamics predict that it will fall both left and right in superposition. When our observer looks at the card, the subject-object interaction extends the superposition to her mental state, producing a superposition of joy and disappointment over winning and losing her bet. She can never perceive this superposition, however, because the interaction between the object and the environment (such as air molecules and photons bouncing off the card) causes rapid decoherence that makes this superposition unobservable.

Even if she could completely isolate the card from the environment (for example, by doing the experiment in a dark vacuum chamber at absolute zero), it would not make any difference. At least one neuron in her optical nerves would enter a superposition of firing and not firing when she looked at the card, and this super-

position would decohere in about 10⁻²⁰ second, according to recent calculations. If the complex patterns of neuron firing in our brains have anything to do with consciousness and how we form our thoughts and perceptions, then decoherence of our neurons ensures that we never perceive quantum superpositions of mental states. In essence, our brains inextricably interweave the subject and the environment, forcing decoherence on us. —M.T. and J.A.W.



to a type of censorship. This effect became known as decoherence, because an ideal pristine superposition is said to be coherent. Decoherence was worked out in great detail by Los Alamos scientist Wojciech H. Zurek, Zeh and others over the following decades. They found that coherent superpositions persist only as long as they remain secret from the rest of the world. Our fallen quantum card is constantly bumped by snooping air molecules and photons, which thereby find out whether it has fallen to the left or to the right, destroying ("decohering") the superposition and making it unobservable [see box on preceding page].

It is almost as if the environment acts as an observer, collapsing the wave function. Suppose that your friend looked at the card without telling you the outcome. According to the Copenhagen interpretation, her measurement collapses the superposition into a definite out-

come, and your best description of the card changes from a quantum superposition to a classical representation of your ignorance of what she saw. Loosely speaking, decoherence calculations show that you do not need a human observer (or explicit wave-function collapse) to get much the same effect—even an air molecule bouncing off the fallen card will suffice. That tiny interaction rapidly changes the superposition to a classical situation for all practical purposes.

Decoherence explains why we do not routinely see quantum superpositions in the world around us. It is not because quantum mechanics intrinsically stops working for objects larger than some magic size. Rather macroscopic objects such as cats and cards are almost impossible to keep isolated to the extent needed to prevent decoherence. Microscopic objects, in contrast, are more easily isolated from their surroundings so

that they retain their quantum behavior.

The second unanswered question in the Everett picture was more subtle but equally important: What mechanism picks out the classical states—face up and face down for our card—as special? Considered as abstract quantum states, there is nothing special about these states as compared to the innumerable possible superpositions of up and down in various proportions. Why do the many worlds split strictly along the up/down lines that we are familiar with and never any of the other alternatives? Decoherence answered this question as well. The calculations showed that classical states such as face up and face down were precisely the ones that are robust against decoherence. That is, interactions with the surrounding environment would leave face-up and face-down cards unharmed but would drive any superposition of up and down into classical faceup/face-down alternatives.

Decoherence and the Brain

hysicists have a tradition of analyzing the universe by splitting it into two parts. For example, in thermodynamics, theorists may separate a body of matter from everything else around it (the "environment"), which may supply prevailing conditions of temperature and pressure. Quantum physics traditionally separates the quantum system from the classical measuring apparatus. If unitarity and decoherence are taken seriously, then it is instructive to split the universe into three parts, each described by quantum states: the object under consideration, the environment, and the observer, or subject [see box at left].

Decoherence caused by the environment interacting with the object or the subject ensures that we never perceive quantum superpositions of mental states. Furthermore, our brains are inextricably interwoven with the environment, and decoherence of our firing neurons is unavoidable and essentially instantaneous. As Zeh has emphasized, these conclusions justify the long tradition of using the textbook postulate of wavefunction collapse as a pragmatic "shut up and calculate" recipe: compute probabilities as if the wave function collapses when the object is observed. Even though in the Everett view the wave function technically never collapses, decoherence researchers generally agree that decoherence produces an effect that looks and smells like a collapse.

The discovery of decoherence, combined with the ever more elaborate experimental demonstrations of quantum weirdness, has caused a noticeable shift in the views of physicists. The main motivation for introducing the notion of wave-function collapse had been to explain why experiments produced specific outcomes and not strange superpositions of outcomes. Now much of that motivation is gone. Moreover, it is embarrassing that nobody has provided a testable deterministic equation specifying precisely when the mysterious collapse is supposed to occur.

An informal poll taken in July 1999 at a conference on quantum computation at the Isaac Newton Institute in Cambridge, England, suggests that the prevailing viewpoint is shifting. Out of 90 physicists polled, only eight declared that their view involved explicit wavefunction collapse. Thirty chose "many worlds or consistent histories (with no collapse)." (Roughly speaking, the consistent-histories approach analyzes sequences of measurements and collects together bundles of alternative results that would form a consistent "history" to an observer.)

But the picture is not clear: 50 of the researchers chose "none of the above or undecided." Rampant linguistic confusion may contribute to that large number. It is not uncommon for two physicists who say that they subscribe to the Copenhagen interpretation, for example, to find themselves disagreeing about what they mean.

This said, the poll clearly suggests that it is time to update the quantum text-books: although these books, in an early chapter, infallibly list explicit nonunitary collapse as a fundamental postulate, the poll indicates that today many physicists—at least in the burgeoning

field of quantum computation—do not take this seriously. The notion of collapse will undoubtedly retain great utility as a calculational recipe, but an added caveat clarifying that it is probably not a fundamental process violating the Schrödinger equation could save astute students many hours of confusion.

Looking Ahead

fter 100 years of quantum ideas, what lies ahead? What mysteries remain? How come the quantum? Although basic issues of ontology and the ultimate nature of reality often crop up in discussions about how to interpret quantum mechanics, the theory is probably just a piece in a larger puzzle. Theories can be crudely organized in a family tree where each might, at least in principle, be derived from more fundamental ones above it. Almost at the top of the tree lie general relativity and quantum field theory. The first level of descendants includes special relativity and quantum mechanics, which in turn spawn electromagnetism, classical mechanics, atomic physics, and so on. Disciplines such as computer science, psychology and medicine appear far down in the lineage.

All these theories have two components: mathematical equations and words that explain how the equations are connected to what is observed in experiments. Quantum mechanics as usually presented in textbooks has both components: some equations and three fundamental postulates written out in plain English. At each level in the hierarchy of theories, new concepts (for example, protons, atoms, cells, organisms, cultures) are introduced because they are convenient, capturing the essence of what is going on without recourse to the theories above it. Crudely speaking,

the ratio of equations to words decreases as one moves down the tree, dropping near zero for very applied fields such as medicine and sociology. In contrast, theories near the top are highly mathematical, and physicists are still struggling to comprehend the concepts that are encoded in the mathematics.

The ultimate goal of physics is to find what is jocularly referred to as a theory of everything, from which all else can be derived. If such a theory exists, it would take the top spot in the family tree, indicating that both general relativity and quantum field theory could be derived from it. Physicists know something is missing at the top of the tree, because we lack a consistent theory that includes both gravity and quantum mechanics, yet the universe contains both phenomena.

A theory of everything would probably have to contain no concepts at all. Otherwise one would very likely seek an explanation of its concepts in terms of a still more fundamental theory, and so on in an infinite regress. In other words, the theory would have to be purely mathematical, with no explanations or postulates. Rather an infinitely intelligent mathematician should be able to derive the entire theory tree from the equations alone, by deriving the properties of the universe that they describe and the properties of its inhabitants and their perceptions of the world.

The first 100 years of quantum mechanics have provided powerful technologies and answered many questions. But physics has raised new questions that are just as important as those outstanding at the time of Maxwell's inaugural speech—questions regarding both quantum gravity and the ultimate nature of reality. If history is anything to go by, the coming century should be full of exciting surprises.

The Authors

MAX TEGMARK and JOHN ARCHIBALD WHEELER discussed quantum mechanics extensively during Tegmark's three and a half years as a postdoc at the Institute for Advanced Studies in Princeton, N.J. Tegmark is now an assistant professor of physics at the University of Pennsylvania. Wheeler is professor emeritus of physics at Princeton, where his graduate students included Richard Feynman and Hugh Everett III (inventor of the many-worlds interpretation). He received the 1997 Wolf Prize in physics for his work on nuclear reactions, quantum mechanics and black holes. In 1934 and 1935 Wheeler had the privilege of working on nuclear physics in Niels Bohr's group in Copenhagen. On arrival at the institute he asked a workman who was trimming vines running up a wall where he could find Bohr. "I'm Niels Bohr," the man replied.

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Further Information

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