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WHY IS THE COSMIC MICROWAVE BACKGROUND FLUCTUATION LEVEL 10^{-5}?

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ABSTRACT

We explore the qualitative changes that would occur if the amplitude $Q \sim 10^{-5}$ of cosmological density fluctuations were different. If $Q \lesssim 10^{-6}$, the cosmological objects that form would have such low virial temperatures that they may be unable to cool and form stars, and they would be so loosely bound that even if they could produce a supernova explosion they might be unable to retain the heavy elements necessary for planetary life. If $Q \gtrsim 10^{-4}$, dense supermassive galaxies would form, and biological evolution could be marred by short disruption timescales for planetary orbits. If Q were still larger, most bound systems would collapse directly to supermassive black holes. These constraints on Q can be expressed in terms of fundamental constants alone and depend only on the electromagnetic and gravitational coupling constants, the electron-proton mass ratio, and the matter-to-photon ratio. We discuss the implications for inflation and defect models and note that the recent anthropic upper bounds on the cosmological constant Λ would be invalid if both Q and Λ could vary and there were no anthropic constraints on Q. The same applies to anthropic bounds on the curvature parameter Ω.

Subject headings: cosmology: theory — large-scale structure of universe

1. INTRODUCTION[4]

A key parameter in the standard, adiabatic, cold dark matter–based models of structure formation is the amplitude that fluctuations in the gravitational potential have when they enter the horizon. This number, which we will denote by Q, has been measured by the *COBE* satellite to be of order 10^{-5} (Smoot et al. 1992; Bennett et al. 1996) and is assumed to be virtually independent of scale in the most popular models. Why 10^{-5}? The answers proposed by theorists fall into two categories:

1. $Q \sim 10^{-5}$ can be computed from first principles using some (still undiscovered) fundamental theory.
2. $Q \sim 10^{-5}$ cannot be computed from first principles, since the correct fundamental theory merely predicts an ensemble of superhorizon-sized spatial regions with a wide range of Q, forcing us to treat Q as a random number, subject to various anthropic selection effects.

The purpose of this paper is to consider such selection effects by studying how the physical processes of structure formation depend on Q. Our motivation for this is threefold:

1. It affects which inflation/defect models should be considered natural, as opposed to fine tuned.
2. It is related to a crucial loophole in the recent arguments for an anthropic upper bound on Λ.
3. It poses useful test problems for comparing cosmological simulations.

The structure of our universe is fixed by a rather small number of physical parameters. The electron mass, the neutron mass, and the low-energy coupling constants of the four basic forces determine the physical properties of most objects on scales ranging from the atomic to the galactic (see, e.g., Carr & Rees 1979; Davies 1982; Barrow & Tipler 1986), and these parameters can in turn be computed from the roughly 20 free parameters of the standard model of particle physics. A number of additional parameters are often thought of as initial data laid down in the early universe: the baryon-to-photon ratio η, the relative abundances of various dark matter candidates, the vacuum density ρ_Λ contributed by a cosmological constant Λ, the spatial curvature (related to Ω), and the amplitude (Q) of cosmological density fluctuations; but it is not implausible that abundances such as η can ultimately be derived from other particle physics constants. Together with the basic laws, these parameters determine when cosmic structures first emerge and how they evolve. Although the detailed outcome in any one locality—and what complex systems evolve there—depends on local accidents, these parameters nonetheless determine the statistical properties.

Will it ever be possible to compute the values of all these parameters from first principles, within the framework of some yet-to-be-discovered fundamental theory? The answer is a resounding "no" within some variants of inflationary cosmology (e.g., Linde 1983, 1987, 1990, 1995; Linde & Zelnikov 1988; Coleman 1988; Albrecht 1994; Vilenkin 1995a, 1995b, 1996a, 1996b; Vilenkin & Winitzki 1997), where the spatial region that we conventionally call "our universe," itself perhaps extending far beyond the present observational horizon, is just one element in an ensemble whose members have widely disparate properties. Some physical parameters may take a range of different values throughout this ensemble of exponentially large and causally disconnected regions. The predictions of such theories therefore take the form of probability distributions for the parameters in question, and these must be computed in Bayesian fashion, taking into account the selection effect that observers are not equally likely to inhabit all parts of the ensemble. For instance, just as we expect low surface brightness galaxies to be underrepresented in many surveys, we might expect O stars to be underrepresented in solar systems containing planet-based extraterrestrial civilizations and, as we shall see, spacetime regions with $Q \sim 10^{-20}$ to be underrepresented in the set of regions that contain observers. The importance of such anthropic selection effects was stressed by Carter (1974) and is discussed in

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great detail in books by Davies (1982) and Barrow & Tipler (1986), for example. More recent reviews can be found in Balashov (1991) and Tegmark (1998), for example.

1.1. *Inflationary Predictions*

Many inflationary models predict an ensemble of exponentially large spacetime regions, each with a different value of Q (see, e.g., Linde 1990; Vilenkin 1995a, 1995b, 1996a, 1996b, and references therein). Although the cosmological literature abounds with remarks on the "unnaturally" flat potential required to produce $Q \sim 10^{-5}$ in our own Hubble volume, often as a motivation to study defect models, one can just as well argue that it is unnatural that the potential is not even flatter, since superflat potentials make inflation last longer and hence dominate the ensemble by volume (Vilenkin 1995a). This dispute cannot be resolved without taking the inevitable anthropic selection effects into account: if these turn out to place a firm *upper* limit on Q near the observed value, then inflation models predicting ensembles peaked at *high* Q clearly require no fine tuning to explain why we observe $Q \sim 10^{-5}$. Conversely, if these selection effects give a firm lower limit on Q near 10^{-5}, then inflation models predicting ensembles peaked at low Q require no fine tuning.

1.2. *The Cosmological Constant Puzzle*

Another hotly debated parameter is Λ, the cosmological constant. Although one might expect the most "natural" value of the vacuum density ρ_Λ to be of the order of the Planck density, the observational upper limits on $|\rho_\Lambda|$ are a striking factor of 10^{123} smaller. This has led to fine-tuning criticism of cosmological models with $\Lambda \neq 0$, the argument being that they were ruled out at high confidence, since such a small value of Λ was extremely unlikely (see Dolgov 1998 for an up-to-date review). As was pointed out by Barrow & Tipler (1986), Weinberg (1987, 1989), and Efstathiou (1995), there is a flaw in this argument, since it neglects a powerful anthropic selection effect. If Λ is too large, then the universe becomes vacuum dominated before the density fluctuations have grown enough to form nonlinear structures. Hence, the fluctuations stop growing, and neither galaxies nor observers will ever form. It is therefore no surprise that we find ourselves in a region where Λ is small. A calculation of the probability distribution for ρ_Λ, given our existence, shows that values of the order of the current limits are in fact rather typical (Efstathiou 1995), and more accurate calculations (Weinberg 1996; Martel, Shapiro, & Weinberg 1998) have confirmed this conclusion.

Unfortunately, there is a loophole in this argument (Rees 1997). As described in more detail in § 5, increasing Λ by some factor f can be completely offset by increasing Q by a factor of $f^{1/3}$, as far as this argument is concerned. Whether this is really a loophole thus depends crucially on the topic of the present paper: specifically, on whether observers could exist if $Q \gg 10^{-5}$. The analogous potential loophole exists for anthropic lower bounds on Ω (see Barrow 1982; Vilenkin & Winitzki 1997).

1.3. *Simulation Testing*

A third and entirely different motivation for exploring counterfactual values of parameters such as Q is that it provides a challenging and bias-free test of cosmological simulation techniques. State-of-the-art simulations, including hydrodynamics (which breaks the degeneracy between Q and t in pure gravity simulations), gas chemistry, and star formation, often achieve a good fit to our actual universe (see, e.g., Kang et al. 1994 and references therein) but only after tweaking a number of parameters empirically. It is therefore unclear to what extent the agreement between different groups is due to realistic modeling, as opposed to simply living in (and parameter-fitting to) the same universe. It would be far more convincing if two groups could obtain indistinguishable results for hypothetical universes with other values of Q, where the answer would not be known beforehand.

In § 2, we outline how Q affects structure formation in a universe with $\Omega = 1$ and $\Lambda = 0$. We discuss the effects of lowering and raising Q in §§ 3 and 4, respectively, and the effects of changing Ω and Λ in § 5.

2. GALAXY FORMATION AND COOLING

2.1. *Notation*

We will find it convenient to work in Planck units, where $\hbar = c = G = k = 1$, and the fundamental units of length, time, mass, and temperature are $r_{\rm Pl} \equiv (\hbar G/c^3)^{1/2} \approx 2 \times 10^{-35}$ m, $t_{\rm Pl} \equiv (\hbar G/c^5)^{1/2} \approx 5 \times 10^{-44}$ s, $m_{\rm Pl} \equiv (\hbar c/G)^{1/2} \approx 2 \times 10^{-8}$ kg, and $T_{\rm Pl} \equiv (\hbar c^5/G)^{1/2}/k \approx 1 \times 10^{32}$ K, respectively. Important dimensionless constants that will recur frequently are the electromagnetic coupling constant $\alpha \equiv e^2 \approx 1/137$, the gravitational coupling constant $\alpha_g \equiv m_p^2 \approx 6 \times 10^{-39}$, the electron-proton mass ratio $\beta \equiv m_e/m_p \approx 1/1836$, the baryon-to-photon ratio $\eta \sim 10^{-9}$, the baryon fraction $\Omega_b/\Omega \sim 10^{-1}$ of the nonrelativistic matter density (which we take to equal the critical value that gives a spatially flat universe), and the matter-to-photon ratio

$$\xi \equiv m_p \eta \frac{\Omega}{\Omega_b} = \alpha_g^{1/2} \eta \frac{\Omega}{\Omega_b} \sim 10^{-27} . \quad (1)$$

This constant ξ is simply the amount of nonrelativistic matter per photon, ρ_m/n_γ, measured in Planck masses. Since our goal is to highlight the main physical effects rather than to make detailed numerical calculations, we will frequently use the symbol $\sim$, which we take to mean that numerical factors of other unity (π and the like) have been omitted. For instance, the hydrogen binding energy (1 rydberg), the Bohr radius, and the Thomson cross section are given by ryd $\sim \alpha^2 \alpha_g^{1/2} \beta$, $a_0 = \alpha^{-1} \alpha_g^{-1/2} \beta^{-1}$, and $\sigma_t \sim \alpha^2 \alpha_g^{-1} \beta^{-2}$, respectively.

The reader may find it unfamiliar to see almost no reference below to familiar quantities such as the redshift z, the current cosmic microwave background (CMB) temperature $T_0 \approx 2.728$ K, the current Hubble constant H_0, and the current density parameter Ω_0. This is because we strive to highlight how structure formation depends on fundamental parameters, and these quantities are not fundamental, since they have meaning only once the epoch at which we happen to be living has been specified. Indeed, for the open universe case, T_0, H_0, and Ω_0 can be thought of as merely alternative time variables, since they all decrease monotonically with t. For instance, we are not interested in examining what Q-values allow galaxies to form by the present epoch $t_0 \sim 10^{10}$ yr, but what Q-values allow them to form at all.

2.2. *When Nonlinear Structures Form*

The rising curves in Figure 1 show when different mass scales go nonlinear, defined as the time when linear perturbation theory predicts an overdensity of 1.69 in a top-hat

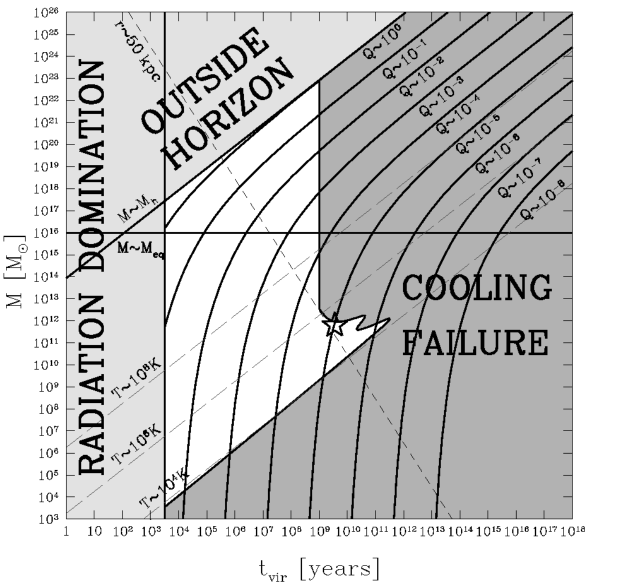

FIG. 1.—The nine rising curves show the largest virialized mass scale as a function of time for different values of Q. Structures with $M \lesssim M_{\rm eq}$ (*horizontal line*) are seen to all virialize about a factor $Q^{-3/2}$ after the end of the radiation-dominated epoch (*shaded*, *left*), whereas for later times, the virialized mass scale asymptotes to about $Q^{3/2}$ times the horizon mass (*shaded*, *upper left*). Cooling is inefficient in the remaining shaded region (*right*). The star corresponds to the Milky Way.

sphere containing the mass M (Press & Schechter 1974). The curves were computed for the cold dark matter (CDM) power spectrum fit of Bond & Efstathiou (1984), with $h = 0.5$, "shape parameter" $\Gamma = 0.25$, and an 8 h^{-1} Mpc normalization $\sigma_8 = 0.7 \times (Q/10^{-5})$. We assume a standard, spatially flat universe ($\Omega = 1$, $\Lambda = 0$) everywhere in this paper[5] except in § 5. Since fluctuations cannot grow before the matter-radiation equality epoch[6] $t_{\rm eq} \sim \xi^{-2} \sim 10^{11}$ s (the vertical line in Fig. 1), all scales below the horizon mass at this epoch,

$$M_{\rm eq} \sim \rho_{\rm eq} t_{\rm eq}^3 \sim \xi^{-2} \sim \alpha_g \xi^{-2}\, M_\odot \sim 10^{16}\, M_\odot\,, \quad (2)$$

have similar fluctuation levels and are seen to virialize roughly simultaneously (up to a logarithmic factor), at

$$t_{\rm vir} \sim t_{\rm eq} Q^{-3/2} f_{\rm vir} \sim \xi^{-2} Q^{-3/2} f_{\rm vir}\,. \quad (3)$$

(The origin of the "3/2" is that, during the matter-dominated epoch, fluctuations grow as the scale factor a and $a \propto t^{2/3}$.) Since Figure 1 shows that the actual curves approach vertical only for very small mass scales, we have included a factor $f_{\rm vir}$ in equation (3) which depends weakly on mass. The value of $f_{\rm vir}$ is of order 1 for $M \sim M_{\rm eq}$, with the value for typical galactic scales $M \sim 10^{12}\, M_\odot$ being $f_{\rm vir} \sim 0.03$. Far above this mass scale, $P(k) \overset{\propto}{\sim} k$ (we assume the standard spectral index $n = 1$), which means that $M \sim M_{\rm hor} Q^{3/2}$, where the horizon mass is $M_{\rm hor} \equiv t$ (*straight solid line*). Thus the curves all have the same shape, and their left and right asymptotes lie about a factor of $Q^{-3/2}$ to the right of the two heavy straight lines in Figure 1, giving the following broad-brush picture. Mass scales $M \lesssim M_{\rm eq}$ virialize roughly simultaneously, at $t \sim t_{\rm vir}$. As time progresses, ever larger scales keep virializing, the nonlinear mass scale always being a fraction $Q^{3/2}$ of the horizon mass scale (a fraction $Q^{1/2}$ in radius). Note that the number 10^{16} occurring in this crucial mass $M_{\rm eq}$ is simply α_g/ξ^2 ; the well-known result that a stellar mass $M_\odot \sim \alpha_g^{-1}$ (Dyson 1971) was used in equation (2).

2.3. *Their Virial Temperature*

When an overdensity has collapsed, the resulting virial halo will have a typical density that exceeds the background density by a collapse factor $f_\rho \sim 18\pi^2$, i.e.,

$$\rho_{\rm vir} \sim \rho_{\rm eq}\left(\frac{t_{\rm vir}}{t_{\rm eq}}\right)^{-2} f_\rho \sim \xi^4 f_{\rm vir}^{-2} f_\rho Q^3\,. \quad (4)$$

For a CDM halo of mass M, this corresponds to a characteristic size $R \sim (M/\rho_{\rm vir})^{1/3}$, velocity $v_{\rm vir} \sim (MG/R)^{1/2} \sim (M^2 \rho_{\rm vir} G^3)^{1/6}$, and virial temperature

$$T_{\rm vir} \sim m_p v_{\rm vir}^2 \sim \alpha_g^{1/2} \xi^{4/3} f_{\rm vir}^{-2/3} f_\rho^{1/3} M^{2/3} Q\,. \quad (5)$$

A number of isotherms are plotted in Figure 1, and we see that as time progresses and ever larger halos form, the virial temperature stops increasing around the characteristic time $t \sim t_{\rm vir} \propto Q^{-3/2}$ and approaches a maximum value $T_{\rm max} \sim m_p c^2 Q$, corresponding to a maximum virial velocity $v \sim Q^{1/2} c$. Thus for our $Q \sim 10^{-5}$ universe, typical cluster temperatures are $\sim$10 keV, about 10^{-5} times the proton rest energy, and characteristic cluster velocity dispersions are 1000 km s^{-1}, about $10^{-5/2}$ times the speed of light.

3. WHAT IF $Q \ll 10^{-5}$?

This direct link between Q and halo temperatures immediately indicates why lowering Q can cause qualitatively different structure formation scenarios. Unless $m_p c^2 Q$ exceeds typical atomic energy scales $\sim$1 ryd, which corresponds to

$$Q \gtrsim \frac{\rm ryd}{kT_{\rm max}} \sim \frac{m_e c^2 \alpha^2}{m_p c^2} = \alpha^2 \beta \sim 10^{-8}\,, \quad (6)$$

it will be difficult for the gas in these halos to dissipate their energy to collapse and form stars. Hydrogen-line cooling freezes out at about ryd/15 $\sim 10^4$ K, for instance, corresponding to $Q \sim 10^{-9}$. We will now discuss cooling constraints in more detail and see that these cause qualitative changes even for much smaller departures from $Q \sim 10^{-5}$.

The fate of the baryons in a virialized halo depends crucially on the ratio of the cooling timescale $\tau_{\rm cool} \equiv T/\dot{T}$ to the gravitational collapse timescale $\tau_{\rm grav} \sim (\rho_{\rm vir} G)^{-1/2}$ (see, e.g., Binney 1977; Rees & Ostriker 1977; Silk 1977; White & Rees 1978). If M and t are such that $\tau_{\rm cool} \gtrsim \tau_{\rm grav}$ (the dark-shaded region in Fig. 1), the cloud cannot promptly commence free-fall collapse and fragment into stars but will remain pressure supported for at least a local Hubble time. For the halo-formation curve corresponding to $Q \sim 10^{-5}$, the part of the $\tau_{\rm cool} = \tau_{\rm grav}$ curve setting the upper limit on galaxy mass is seen to have a logarithmic slope around -2 (because bremsstrahlung, with $\tau_{\rm cool} \propto T^{1/2}/\rho$, is the dominant cooling process), corresponding to $M \propto \rho \propto t^{-2}$ and a constant radius $R \sim \alpha^3 \alpha_g^{-3/2} \beta^{-3/2} \sim 50$ kpc (Carr & Rees 1979). The corresponding mass scale is seen to be $M \sim 10^{12}\, M_\odot$. For slightly lower Q, the upper limit is dominated by line cooling in neutral hydrogen (*rightmost bump*), helium

[5] We assume a standard scale-invariant Harrison-Zeldovich primordial power spectrum throughout this paper. More general primordial spectra would correspond to a scale-dependent Q, thus requiring more than a single number for their parameterization.

[6] At $t_{\rm eq}$, the radiation energy per proton, $T_{\rm eq}/\eta$, equals the dark matter energy per proton, $m_p \Omega/\Omega_b$, so $T_{\rm eq} \sim m_p \eta \Omega/\Omega_b = \xi$. Since the energy density is $\rho_{\rm eq} \sim T_{\rm eq}^4$, the Friedmann equation gives the Hubble expansion rate $H \sim \rho^{1/2} \sim T_{\rm eq}^2$, and so the age of universe at this time is $t_{\rm eq} \sim H^{-1} \sim T_{\rm eq}^{-2} \sim \xi^{-2}$.

(*second bump*), and any heavier elements released by early stars (not included here). The lower mass limit is set by the $T \sim 10^4$ K isotherm, below which there are essentially no free electrons and both line cooling and bremsstrahlung become ineffective. Molecular cooling can potentially lower this mass limit slightly (see, e.g., Haiman, Thoul, & Loeb 1996; Abel et al. 1997; Gnedin & Ostriker 1997; Tegmark et al. 1997) but is ignored in Figure 1 for the same reason as heavy elements: it is irrelevant to our Q constraints, which depend only on how far the cooling region extends to the right, not on the vertical extent.

What happens if we start lowering Q? The first change is that the upper limit becomes set not by bremsstrahlung but by line cooling. Figure 1 indicates that as we keep lowering Q, the range of galactic masses narrows down and finally vanishes completely for $Q < Q_{\min} \sim 10^{-6}$. Let us express this critical value $Q_{\min}$ in fundamental constants. The figure shows that it is determined by the "hydrogen bump" in the cooling function, which is caused by free electrons collisionally exciting neutral hydrogen atoms into their first excited state, which is immediately followed by emission of a Lyα photon. This gives a cooling timescale (e.g., Dalgarno & McCray 1972)

$$t_{\text{cool}} \sim \left(\frac{m_e^2 c\alpha}{\hbar^2}\right)\frac{\gamma^{-3/2}e^{(3/4)\gamma}}{x(1-x)n} , \tag{7}$$

where $\gamma \equiv \text{ryd}/kT$, n is the total (bound and free) proton number density, and x is the ionization fraction. In thermal equilibrium, this is given by (see, e.g., Tegmark, Silk, & Evrard 1993)

$$x \sim (1 + \alpha^3\gamma^{7/6}e^{\gamma})^{-1} . \tag{8}$$

Substituting equation (8) into equation (7) gives

$$t_{\text{cool}} \sim \left(\frac{m_e^2 c}{\hbar^2\alpha^2 n}\right)[\gamma^{-8/3}e^{-\gamma/4}(1 + \alpha^3\gamma^{7/6}e^{\gamma})^2] , \tag{9}$$

where the dimensionless quantity in square brackets is minimized for $\gamma \sim \ln(\alpha^{-2}) \sim 10$, corresponding to $T \sim \text{ryd}/10 \sim 15{,}000$ K. This minimum value is $\sim \gamma^{-8/3}e^{-\gamma/4} \sim \alpha^{1/2}\ln(\alpha^{-2})^{-8/3} \sim 1/5000$. Equating this minimal cooling time-scale with $t \sim (G\rho)^{-1/2}$, using $n = \rho\Omega_b/m_p$, finally tells us that the latest time at which line cooling can be efficient is

$$t_{\max} \sim \alpha^{3/2}\ln(\alpha^{-2})^{8/3}\alpha_g^{-3/2}\beta^{-2}\Omega_b \sim 10^{19} \text{ s} . \tag{10}$$

Equating this with t_{vir} from equation (3) thus tells us that efficient cooling occurs when

$$Q \gtrsim \alpha^{-1}\ln(\alpha^{-2})^{-16/9}\alpha_g\beta^{4/3}\xi^{-4/3}f_{\text{vir}}^{2/3}\Omega_b^{-2/3} \sim 10^{-6} . \tag{11}$$

If $Q \ll 10^{-6}$, then what is the ultimate fate of the quasi-static pressure-supported gas clouds? It is plausible that they will become increasingly rarefied as their dark matter halos eventually merge into larger (and less dense) halos, thereby never entering a phase of runaway cooling, fragmentation, and star formation. However, even in the arguably contrived case where such a cloud escaped any further collisions and eventually managed to cool (perhaps through some exotic mechanism such as 21 cm cooling) after a (perhaps exponentially) long time and developed a dense, self-gravitating core that fragmented into stars, there would still be reason to doubt whether it could produce intelligent observers. Since the binding energy of the halo is so low (of order T_{vir}), the first supernova explosion might well eject all the gas from the halo, thereby precluding the production of Population II stars and planets containing heavy elements.

4. WHAT IF $Q \gg 10^{-5}$?

What happens if we start increasing Q instead? The allowed mass range for galaxies keeps broadening at a steady rate until Compton cooling suddenly eliminates the upper mass limit altogether. This is because the timescale on which CMB photons at temperature T_γ cool an ionized plasma,

$$\tau_{\text{comp}} \sim \frac{\hbar^3 c^4 m_e}{\sigma_t(kT_\gamma)^4 x} , \tag{12}$$

is independent of both its density and temperature (assuming that $T \gtrsim 15{,}000$ K, so that $x \sim 1$). Since $T_\gamma \sim T_{\text{eq}}(t/t_{\text{eq}})^{-2/3} \sim \xi^{-1/3}t^{-2/3}$, this timescale τ_{comp} equals the age of the universe t at a characteristic time

$$t_{\text{comp}} \sim \alpha^{6/5}\alpha_g^{-13/10}\beta^{-9/5}\xi^{-4/5} \sim 10^{16} \text{ s} . \tag{13}$$

Setting $t_{\text{comp}} = t_{\text{vir}}$, we find that the upper limit to galaxy masses persists only for

$$Q \lesssim \alpha^{-4/5}\alpha_g^{3/5}\beta^{6/5}\xi^{-4/5}f_{\text{vir}}^{2/3} \sim 10^{-4.5} . \tag{14}$$

For larger Q-values, all mass scales can cool efficiently, so the characteristic mass for the first generation of galaxies will simply be $M_{\text{eq}} \sim 10^{16}\ M_\odot$, given by equation (2). This corresponds to a characteristic size $R \sim t_{\text{eq}}/Q \propto t_{\text{eq}}\,t^{2/3}$ for newly formed galaxies, which is constant in *comoving* coordinates (rather than in absolute coordinates like the aforementioned cooling scale $R \sim 50$ kpc). It would plainly need detailed simulations to determine the mix of disks and spheroids and the effects of subsequent mergers. However, the galaxies could well have a broader luminosity function than in our actual universe (as well as a much higher characteristic mass); clustering would also extend up to a larger fraction of the Hubble radius.

4.1. *Disruption of Planetary Orbits*

Would this qualitative change affect the number of habitable planets produced? Let us first consider the stability of planetary orbits.[7]

Lightman (1984) has shown that if the planetary surface temperature is to be compatible with organic life, the orbit around the central star should be fairly circular and have a radius of order

$$r_{\text{au}} \sim \alpha^{-5}\alpha_g^{-3/4}\beta^{-2} \sim 10^{11} \text{ m} , \tag{15}$$

roughly our terrestrial "astronomical unit," precessing one radian in its orbit on a timescale

$$t_{\text{orb}} \sim \alpha^{-15/2}\alpha_g^{-5/8}\beta^{-3} \sim 0.1 \text{ yr} . \tag{16}$$

An encounter with another star with impact parameter $r \lesssim r_{\text{au}}$ has the potential to throw the planet into a highly eccentric orbit or even unbind it from its parent star. This happens on a timescale $\tau_{\text{enc}} \sim 1/n_* v r_{\text{au}}^2$, where n_* and $v \sim v_{\text{vir}}$ denote the typical stellar density and stellar velocity in a galaxy, respectively. Writing $n_* \sim f_*\rho_{\text{vir}}/M_*$, where $M_* \sim \alpha_g^{-1}$ and f_* is the additional factor by which the dissipating baryons collapse relative to the dark matter before fragmenting into stars, the Milky Way is empirically fitted by $f_* \sim 10^1$. For Earth, this gives $\tau_{\text{enc}} \sim 10^{22}$ s, orders of magnitude above its present age. Moreover, the distant encounters that

[7] After submission, it was brought to the authors' attention that A. Vilenkin (1997, private communication) discussed orbit disruption caused by high Q-values at the 1995 Tokyo RESCEU symposium in Tokyo. Orbit disruption constraints on galaxy densities have also been discussed in the context of axion physics (Linde 1998).

we have experienced in the past have had a completely negligible effect, since they were adiabatic. This means that the impact duration $r/v \gg t_{\rm orb}$, so that the solar system returned to its unperturbed state once the encounter was over. For hypothetical galaxies forming before $t_{\rm comp}$, on the other hand, $M \sim M_{\rm eq}$, so the time between nonadiabatic encounters is

$$\tau_{\rm adiab} \sim \frac{1}{n_* v^3 t_{\rm orb}^2} \sim \frac{\alpha^{15}\alpha_g^{1/4}\beta^6 f_{\rm vir}^3}{\xi^4 Q^{9/2} f_\rho^{3/2} f_*} \sim 10^5 \text{ yr} \quad (17)$$

for $Q = 10^{-4}$, and dropping as $Q^{-9/2}$ if we increase Q further. In other words, nonadiabatic encounters are frequent events for $Q \gtrsim 10^{-4}$, occurring often during the geological timescales required for a planet to form, cool, and ultimately evolve life. In the conservative approximation of ignoring gravitational focusing (assuming that the flyby speed v exceeds the orbital speed), the typical time interval between $r < r_{\rm au}$ encounters is

$$\tau_{\rm enc} \sim \frac{1}{n_* v r_{\rm au}^2} \sim \frac{\alpha^{10}\alpha_g^{1/2}\beta^4 f_{\rm vir}^{7/3}}{\xi^4 f_\rho^{7/6} f_* Q^{7/2}} \sim 10^7 \text{ yr} \quad (18)$$

for $Q = 10^{-3}$. Requiring this to exceed some geological or evolutionary timescale $t_{\rm min}$ thus gives an upper limit $Q \propto t_{\rm min}^{-2/7}$. Although it is far from clear what is an appropriate $t_{\rm min}$ to use, the smallness of the exponent $\frac{2}{7}$ implies that it makes only a minimal difference whether we choose 10^6 or 10^{10} yr. Taking $t_{\rm min} \sim 10^9$ yr $\sim \alpha^2\alpha_g^{-3/2}\beta^{-2}$, the lifetime of a bright star (Carr & Rees 1979), we obtain the limit

$$Q \lesssim \alpha^{16/7}\alpha_g^{4/7}\beta^{12/7}\xi^{-8/7} f_{\rm vir}^{2/3} f_\rho^{-1/3} f_*^{-2/7} \sim 10^{-4} . \quad (19)$$

This upper limit appears more uncertain than the lower limit from cooling. The momentum kick given to the planet scales as v^{-2}, so an impact with $r \ll r_{\rm au}$ would not necessarily cause a catastrophic disturbance of the planetary orbit; the event rate for this grows only as $Q^{5/2}$, or as Q^3 if the galactic stars settle into a disk where v is roughly independent of Q. On the other hand, a very close encounter (especially with an O star) might cause disastrous heating of the planet. In view of this uncertainty, as well as the uncertainty regarding f_* and $f_{\rm min}$, we now consider two additional effects of raising Q.

4.2. *Black Hole Domination*

For much greater Q-values, of order unity, typical fluctuations would be of black hole magnitude already by the time they entered the horizon, converting some substantial fraction f of the radiation energy into black holes already shortly after $t_{\rm infl}$, the end of the inflationary era. At $t_{\rm eq}$, the universe has expanded by a factor $a \sim (t_{\rm eq}/t_{\rm infl})^{1/2}$, and the energy densities in black holes and photons have dropped by factors of a^3 and a^4, respectively. The black hole density will therefore completely dwarf the density of cold dark matter and baryons if $f \gg a^{-1}$. Thus, even if $Q \ll 1$, extremely rare fluctuations that are Q^{-1} standard deviations out in the Gaussian tail can cause black hole domination if $\Phi(Q^{-1}) \sim a^{-1}$, where $\Phi(x) \equiv (2\pi)^{-1/2}\int_x^\infty \exp(-u^2/2)\,du$. This gives the upper limit

$$Q \equiv \Phi^{-1}(\xi) \sim \Phi^{-1}(10^{-27}) \sim 10^{-1} , \quad (20)$$

where we have simply assumed that $t_{\rm infl}$ is within a few orders of magnitude of unity (the Planck time), since this affects the result only logarithmically. As opposed to the previous constraints, this one depends strongly on whether the power spectrum is strictly scale invariant or not; increasing the spectral index n from its scale-invariant value $n = 1$ to $n = 1.3$ causes primordial black hole domination, even if Q is as low as 10^{-5} (Green, Liddle, & Riotto 1997).

Even if Q were low enough to avoid black hole formation in the early radiation-dominated phase (say in the range 10^{-3} to 10^{-2}), rampant black hole formation may still occur in the matter-dominated era. At times of order 10^6 yr, i.e., shortly after recombination, clumps of order $M_{\rm eq}$ will collapse. If dissipation leads to enough reionization to make their Thomson optical depth larger than c/v (itself of order $Q^{-1/2}$), then they will trap the background radiation and collapse like supermassive stars without being able to fragment. The dominant structures in such a universe would then be supermassive black holes, and it is unclear whether any galaxies and stars would be able to form. Even if they could, they would be hurtling around at speeds of order a tenth of the speed of light, and it is far from clear how anthropically favorable such an environment would be!

5. WHAT IF Λ AND Ω WERE DIFFERENT?

Our discussion above applied to a flat FRW universe with $\Omega = 1$ and $\Lambda = 0$. As we will now describe, anthropic limits on these two parameters are intimately linked with Q. In Planck units, the Friedmann equation that governs the time evolution of the radius of curvature of the universe, a, is conveniently written as

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}(\rho_\gamma + \rho_m + \rho_c + \rho_\Lambda) , \quad (21)$$

where ρ_γ, ρ_m and ρ_Λ are the energy densities corresponding to radiation, nonrelativistic matter, and vacuum energy (a cosmological constant), respectively; $\rho_c \equiv \pm 3/8\pi a^2$ is the contribution from spatial curvature (the sign is positive if $\Omega < 1$ and negative if $\Omega > 1$; for the flat case $\Omega = 1$, the radius of curvature is infinite and a must be redefined). The first three of these densities evolve as

$$\rho_\gamma \sim \rho_\Omega^{-2} a^{-4} , \quad (22)$$

$$\rho_m \sim \xi \rho_\Omega^{-3/2} a^{-3} , \quad (23)$$

$$\rho_c \sim a^{-2} , \quad (24)$$

and ρ_Λ does not evolve at all. The constant ρ_Ω is defined as the curvature that the universe *would* have had at the Planck time if there was no inflationary epoch, and can be evaluated at any time in the postinflationary radiation-dominated epoch as $\rho_\Omega = \rho_c t \sim t/a^2$, during which this quantity is time independent. We have introduced ρ_Ω simply because we need a constant that quantifies the curvature, and the more familiar Ω is unusable since it changes with time. The epochs of matter domination $a_{\rm md}$, curvature domination $a_{\rm cd}$, and vacuum domination $a_{\rm vd}$ are given by $\rho_\gamma \sim \rho_m$, $\rho_c \sim \rho_m$, and $\rho_\Lambda \sim \rho_m$, respectively; i.e.,

$$a_{\rm md} \sim \xi^{-1}\rho_\Omega^{-1/2} , \quad (25)$$

$$a_{\rm cd} \sim \xi \rho_\Omega^{-3/2} , \quad (26)$$

$$a_{\rm vd} \sim \xi^{1/3}\rho_\Omega^{-1/2}\rho_\Lambda^{-1/3} . \quad (27)$$

It is well known that subhorizon fluctuations can only grow during the matter-dominated epoch, where they grow at the same rate as the scale factor a. As we saw in § 2.2, the first nonlinear structures therefore form at $a_{\rm vir} \sim a_{\rm md} Q^{-1}$, provided that the universe remains matter-dominated until this epoch ($a_{\rm cd} \gtrsim a_{\rm vir}$ and $a_{\rm vd} \gtrsim a_{\rm vir}$); otherwise, no nonlinear structures will ever form. We thus obtain the two anthropic constraints

$$\rho_\Omega \lesssim \xi^2 Q \sim 10^{-59}\,, \tag{28}$$

$$\rho_\Lambda \lesssim \xi^4 Q^3 \sim 10^{-124}\,. \tag{29}$$

Although we tacitly assumed that $\Omega < 1$ here, the closed case gives essentially the same constraints—indeed, if no nonlinear structures have formed at the epoch $a_{\rm cd}$ in a closed universe, time is literally running out for not yet evolved life forms, since the Big Crunch is imminent!

In comparison, the current observational limits are (very conservatively) $\rho_\Lambda \lesssim \rho_m \sim 10^{-123}$ and $0.1 \lesssim \Omega \lesssim 2$, which correspond to $a_{\rm vd} \gtrsim 10^3 a_{\rm md}$ and $\rho_\Omega \lesssim 10^{-57}$. The conclusion is that, although the anthropic upper limits superficially appear quite strong on both curvature and vacuum density, these constraints are only strong if the two variables on the right-hand side (ξ and Q) are independently constrained, which was one of our motivations for studying upper limits on Q.

The parameter ξ probably deserves more attention than it has received in this context so far (e.g., Rees 1979), and the effects of varying the baryon/photon ratio and introducing a nonzero neutrino mass would also warrant further study. We note in passing that we can obtain crude Q-independent limits on ξ by requiring that our lower limits on Q not exceed our upper limits. For instance, the virialization epoch of equation (3) will occur too late for cooling to be efficient (after $t_{\rm max}$ of eq. [10]) unless $\xi \gtrsim 10^{-32} Q^{-3/4}$. Thus the white region in Figure 1 disappears completely if $\xi \lesssim 10^{-32}$, and the conservative limit $Q \lesssim 10^{-3}$ gives the (rather weak) constraint $\xi \gtrsim 10^{-30}$. Conversely, the planetary disruption constraint of equation (19) gets stronger if we *increase* ξ and conflicts with the ξ-independent limit of equation (6) unless $\xi \lesssim 10^{-23}$. In addition, there are, of course, separate limits on the baryon fraction Ω_b, in that if there are too few baryons, the cooling becomes less efficient; see equation (10). Lowering Ω_b may also impede galaxy and star formation, since a gas cloud must collapse by a larger factor before it becomes self-gravitating.

For the reader preferring to think in terms of Ω_0 and redshift z, the above argument can be reexpressed as follows: If the current matter density is ρ_m, then vacuum domination occurs at the epoch $(1 + z_{\rm vd}) = (\rho_\Lambda/\rho_m)^{1/3}$. If $\Omega_0 \ll 1$, then the universe became curvature-dominated at a redshift given by $(1 + z_{\rm cd}) \sim \Omega_0^{-1}$. Since the first structures form at an epoch $(1 + z_{\rm vir}) \propto Q$, the upper limits on Λ and Ω_0^{-1} thus scale as $\Lambda \propto Q^3$ and $\Omega_0 \propto Q^{-1}$ for the $\Omega_0 \ll 1$ case. For instance, maintaining spatial flatness but making Λ a million times larger than the current observational limits could correspond to $Q \sim (10^6)^{1/3} \times 10^{-5} = 10^{-3}$, with galaxy formation about 10 expansion times after recombination. When the universe had reached its current age of $\sim 10^{10}$ yr, it would have expanded by a further factor of $\sim e^{100}$, and ours would be the only galaxy in the local Hubble volume—alas, a drab and dreary place for extragalactic astronomers but not ruled out by the aforementioned Λ-arguments alone, although perhaps by the Q-arguments that we have presented.

6. DISCUSSION

We have explored counterfactual cosmological scenarios with Q shifted away from its observed value $\sim 10^{-5}$. We found that qualitative changes occur if we either increase or decrease Q by about an order of magnitude. If $Q \lesssim 10^{-6}$, efficient cooling becomes impossible for gas in virialized halos. If $Q \gtrsim 10^{-4}$, Compton scattering against CMB photons enables efficient cooling in arbitrarily massive halos, and the higher stellar densities and velocities may lead to planetary orbits being disrupted before observers have had time to evolve.

Needless to say, this does not preclude the evolution of some form of life in a universe with a more extreme Q-value as a result of lucky circumstances: for instance, around a field star that was ejected from its giant host galaxy in a $Q \sim 10^{-3}$ scenario. However, as stressed by Vilenkin (1995a), the key feature of anthropic selection effects is not what the rock-solid extreme limits are on a parameter, but which is *the most favorable value* for producing observers. This point is also emphasized by García-Bellido & Linde (1995), for example. To predict a probability distribution for the observed value of Q from some inflationary model (to potentially rule the model out), its a priori probability distribution for Q (of quantum origin, say) must be multiplied by some Bayesian selection function, such as the number of observers or civilizations corresponding to each Q-value. It seems plausible that many more stars with habitable planets are formed for $Q \sim 10^{-5}$ (where perhaps 1%–10% of all baryons are in stars) than in a $Q \sim 10^{-6}$ universe (where 1000 times lower densities make cooling difficult). Likewise, it appears likely that $Q \sim 10^{-4}$ gives fewer planets in favorable stable orbits than $Q \sim 10^{-5}$, where close encounters are completely negligible for most stars. In conclusion, it is possible that the anthropic selection function peaks at $Q \sim 10^{-5}$. If this is the case, then what Vilenkin terms "the principle of mediocrity" would imply that since we are most likely to be a typical civilization, this is what we should expect to observe.

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