Ajay Kumar Tamrakar
Dineshkumar Harursampath

2015

maestro
A Wiley Brand

WILEY ACING THE GATE

MECHANICAL ENGINEERING

- Content by eminent authors with excellent credentials
- Systematic approach covering complete spectrum of the discipline at both macro (chapters) and micro (sections) levels
- Problem solving skills built through solved and unsolved questions
  - covering all topics
  - organized in the order of difficulty
  - with complete solutions and detailed explanations

FREE

Online resources available at **www.wileymaestro.com/GATE**

- Mock test papers for practice
- Forum where aspirants can query author/experts
- Interaction with author/experts via e-mail
- Solution of the latest examination paper

WILEY ACING THE GATE

MECHANICAL ENGINEERING

maestro
A Wiley Brand

WILEY ACING THE GATE

MECHANICAL ENGINEERING

Ajay Kumar Tamrakar
B.E. (GEC, Rewa), M.Tech (IISc)
Senior Divisional Mechanical Engineer
West Central Railway, Jabalpur, MP

Dineshkumar Harursampath
B.Tech (IITM), M.E. (IISc), Ph.D. (GeorgiaTech)
Assistant Professor and Head, NMCAD Lab
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

WILEY

WILEY ACING THE GATE

MECHANICAL ENGINEERING

Other Wiley Editorial Offices:

John Wiley & Sons, Inc. 111 River Street, Hoboken, NJ 07030, USA

Wiley-VCH Verlag GmbH, Pappellaee 3, D-69469 Weinheim, Germany

John Wiley & Sons Australia Ltd, 42 McDougall Street, Milton, Queensland 4064, Australia

John Wiley & Sons (Asia) Pte Ltd, 1 Fusionpolis Walk #07-01 Solaris, South Tower Singapore 138628

John Wiley & Sons Canada Ltd, 22 Worcester Road, Etobicoke, Ontario, Canada, M9W 1L1

First Edition: 2014

ISBN: 978-81-265-4542-1

ISBN: 978-81-265-8155-9 (ebk)

www.wileyindia.com

PREFACE

The Graduate Aptitude Test in Engineering (GATE) essentially examines the aptitude of an engineering graduate. It differs from other examinations as it calls for an aptitude based learning and approach on engineering topics. However, the aspirants face difficulties in preparation for this examination due to various complications, such as huge syllabus, unavailability of standard books and trusted texts, existing incongruity with syllabus and approach of the competition, lack of competent teachers. To qualify this exam, an aspirant should have perfect understanding on the fundamentals concepts, and requires ability to interpret the fundamental quantities and their relationships. Purpose of this book is to present the subjects of mechanical engineering in a systematic, integrated and precise manner, primarily to meet the requirements of the aspirants of GATE (ME) and to truly help them qualify the examination without unreasonable strains. The book is a self-explanatory and fully-competent text which is equally useful for similar competitions and undergraduate courses in mechanical engineering.

The book starts with two introductory articles, "Methodological Concepts in Engineering Studies" and "Strategy for success in GATE". Each chapter in the manuscript begins with an overview of the content and introduces topics from a level that an average student can follow comfortably. Using easily perceptible language, standard symbols, figures and tables, the book describes various concepts, definitions, and derivations, in a much elucidated text, like a tutor. The book contains almost every mathematical aspect of the subjects with the help of illustrations and by highlighting the implicit path, thus, stimulating curiosity and interest. A summary of important formulas is placed after the theoretical contents in every chapter to provide a quick review of the useful derivations. Solved examples are included to amply demonstrate the theoretical concepts. Each chapter is provided with solutions of previous 11 years' GATE questions (2003–2013). Each chapter also contains rightly answered and explicitly explained Multiple Choice Questions (MCQ's) and Numerical Answer Questions (NAQ's), which are specifically designed to meet the latest pattern and standard of GATE. These questions cover the topics from simplest case and add complexities gradually. The explanations of the questions amply illustrate the topics in a very appealing way. A solved question bank with mixed topics is also placed at the end of the book.

The book incorporates every sentence of importance and avoids undesirable repetition. Throughout the content, an intuitive and systematic approach is used to reinforce the fundamental understandings. The emphasis on fundamental concepts unveils the inbuilt opportunities for developing the aptitude, the vital element for success in GATE.

Despite our best efforts, some errors may have inadvertently crept in the book, we solicit your feedback and suggestions for improvement.

Ajay Kumar Tamrakar

Dineshkumar Harursampath

ACKNOWLEDGMENTS

To a mother, who is but a manifestation of the Divine virtues of the Earth, this book is one petite offering. Thank you Amma for being my mother! Thank you Papa for all the liberty, confidence and the discipline showered on me. Thanks to Kiran and Gudiya (my sisters), Amit (my brother), Akash (Kiran's son) for all the care, love and pride. Thanks to my wife, Deepti, for unflinching support to all my endeavors. Her parents, and Divya, Deepika, Disha (sisters-in-law) own equal gratitude from my side for all their pride for me and for the implicit encouragement.

I always seem to get inspiration and renewed vitality by contact with great novel land of my alma mater, Indian Institute of Science, Bangalore, where I enjoyed the privilege of studying with as Dr. Dineshkumar Harursampath, the co-author of this book. It is my proud privilege to express my deep sense of gratitude to him for constant encouragements and invaluable contributions in this book. This project worked as the golden way through which I could return myself to such a venerable teacher.

I must also submit my gratitude to Wiley India for considerately accepting the initial proposal of the book which was totally different in pattern and orientation. In spite of busy schedule, Ms. Anjali Chadha and Ms. Sruthi Guru studied the script meticulously and persistently provided support, feedback and inquiries to finally convert the proposal into present book. I am indebted to Mr. Paras Bansal of Wiley India for giving me the opportunity of becoming an author of the publication of such a high repute.

Last and not least, I beg forgiveness of all those who have been with me over the course of the years and whose names I have failed to mention.

Ajay Kumar Tamrakar

ABOUT THE AUTHORS

Ajay Kumar Tamrakar is currently serving Indian Railways as Senior Divisional Mechanical Engineer under the prestigious cadre of Indian Railway's Services of Mechanical Engineers (IRSME). He received his B.E. (Mechanical Engineering) degree from Government Engineering College, Rewa in 2001. After scoring All India 2nd rank with 99.98 percentile in GATE-2001, he obtained is M.Tech. degree from Indian Institute of Science (IISc), Bangalore in 2004. During his tenure at IISc, his research interests were smart materials, multi-functional composites, and multi-body dynamics and he presented several research papers in national and international conferences. Further, he qualified the Combined Engineering Services Examination (CESE) in 2002 based on which he served Central Electricity Authority (Ministry of Power) as Assistant Director. Later on, he qualified CESE again in 2005, and joined the IRSME in 2006.

Dineshkumar Harursampath is the Founder-Head, Nonlinear Multifunctional Composites Analysis & Design (NMCAD) Lab and has been an aerospace engineering faculty member at IISc, Bangalore since 2001. He is also the visiting professor at TU, Delft, The Netherlands; ENS de Cachan, France; PoliMi and PoliTo, Italy. His educational qualifications include B.Tech. from IIT, Madras; M.E. from IISc, Bangalore and Ph.D. from Georgia Tech, Atlanta. In addition to his vast teaching experience, he has guided about fifty post-graduate student theses. With over a hundred book chapters and articles in international journals and conference proceedings, his primary research interests are in geometric and material non-linearities as well as evolutionary interaction. He has pioneered multiple collaborative training, research and development projects with counterparts within India, USA, France, Italy and Canada. Collaborators include Boeing, Bell Helicopters, Ameron International, LM Glasfiber, DRDO, RWRDC, HAL, ARDB and DST. He is a Senior Member, AIAA, AeSI, ISSS; Founder Charter Member, AHMEC; Governing Board Member, ISAMPE. He is also an Editor of the International Journal of Mechanical Engineering and Materials Science as well as Advances in Aircraft and Spacecraft Science. He has served on International Scientific Committees and chaired sessions at international conferences conducted by SPIE, ICCM, ISTAM, ISAMPE etc.

ABOUT GATE

Please refer to website of the current organizing institute for detailed and updated information about GATE.

INTRODUCTION

Graduate Aptitude Test in Engineering (GATE) is an all India examination that primarily tests the comprehensive understanding of various undergraduate subjects in Engineering and Technology. The GATE score of a candidate reflects a relative performance of a candidate. The score is used for admissions to post-graduate engineering programs (e.g. M.E., M.Tech, direct Ph.D.) in Indian higher education institutes with financial assistance provided by MHRD and other Government organizations. The score may also used by Public sector units for employment screening purposes.

FINANCIAL ASSISTANCE

A valid GATE score is essential for obtaining financial assistance (scholarship) during Masters programs and in some cases during direct Doctoral program in Engineering/Technology/Architecture, and Doctoral programs in relevant branches of Science in an Institution supported by the MHRD or other Government organizations. To avail the financial assistance, the candidate must first secure admission to a program in these institutes, by a procedure that could vary for different institutes. Qualification in GATE is also an essential requirement to apply for various fellowships awarded by many Government organizations.

STRUCTURE OF GATE

A candidate can apply for only one of the 22 papers listed in the GATE brochure. The syllabus for each of the papers is given separately. Opting for the appropriate paper while filling GATE application is the responsibility of the candidate.

The candidate is expected to appear in a paper appropriate to the discipline of his/her qualifying degree. The candidate is, however, free to choose any paper according to his/her admission plan, keeping in mind the eligibility criteria of the institutions in which he/she wishes to seek admission.

General Aptitude Questions

All the papers will contain few questions that test the General Aptitude (Language and Analytical Skills), apart from the core subject of the paper.

Duration and Examination Pattern

The GATE examination consists of a single paper of 3-hour duration that contains 65 questions carrying a maximum of 100 marks. The question paper will consist of both multiple choice questions (MCQ) and numerical answer type questions.

In all the papers, there will be a total of 65 questions carrying 100 marks, out of which 10 questions carrying a total of 15 marks are in General Aptitude (GA). In Mechanical Engineering (ME) paper, the Engineering Mathematics will carry around 13% of the total marks, the General Aptitude section will carry 15% of the total marks and the remaining percentage of the total marks is devoted to the subject of the paper.

GATE 2014 would contain questions of two different types in various papers:

1. Multiple Choice Questions (MCQ) carrying 1 or 2 marks each in all papers and sections. These questions are objective in nature, and each will have a choice of four answers, out of which the candidate has to mark the correct answer.

2. Numerical Answer Questions (NAQ) of 1 or 2 marks each in all papers and sections. For these questions the answer is a real number, to be entered by the candidate using the virtual keypad. No choices will be shown for this type of questions.

Mode of Examination

The examination for all the papers will be carried out in an ONLINE Computer Based Test (CBT) mode where the candidates will be shown the questions in a random sequence on a computer screen. The candidates are required to either select the answer (for MCQ type) or enter the answer for numerical answer type question using a mouse on a virtual keyboard (keyboard of the computer will be disabled). Candidates will be provided with blank paper sheets for rough work and these have to be returned back after the examination. At the end of the 3-hour window, the computer will automatically close the screen from further actions.

Marking Scheme

For a wrong answer chosen for the multiple choice questions, there would be negative marking. For 1-mark multiple choice questions, 1/3 mark will be deducted for a wrong answer. Likewise, for 2-mark multiple choice questions, 2/3 mark will be deducted for a wrong answer. However, there is no negative marking for a wrong answer in numerical answer type questions.

SYLLABUS OF GATE (MECHANICAL ENGINEERING)

GENERAL APTITUDE

Verbal Ability English grammar, sentence completion, verbal analogies, word groups, instructions, critical reasoning and verbal deduction.

Numerical Ability Numerical computation, numerical estimation, numerical reasoning and data interpretation.

ENGINEERING MATHEMATICS

Linear Algebra Matrix algebra, Systems of linear equations, Eigen values and eigen vectors.

Calculus Functions of single variable, Limit, continuity and differentiability, Mean value theorems, Evaluation of definite and improper integrals, Partial derivatives, Total derivative, Maxima and minima, Gradient, divergence and curl, Vector identities, Directional derivatives, Line, surface and volume integrals, Stokes, Gauss and Green's theorems.

Differential Equations First order equations (linear and non-linear), Higher order linear differential equations with constant coefficients, Cauchy's and Euler's equations, Initial and boundary value problems, Laplace transforms, Solutions of one-dimensional heat and wave equations and Laplace equation.

Complex Variables Analytic functions, Cauchy's integral theorem, Taylor and Laurent series.

Probability and Statistics Definitions of probability and sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Poisson, normal and binomial distributions.

Numerical Methods Numerical solutions of linear and non-linear algebraic equations Integration by trapezoidal and Simpson's rule, Single and multi-step methods for differential equations.

APPLIED MECHANICS & DESIGN

Engineering Mechanics Free body diagrams and equilibrium, Trusses and frames, Virtual work, Kinematics and dynamics of particles and of rigid bodies in plane motion, including impulse and momentum (linear and angular) and energy formulations, impact.

Strength of Materials Stress and strain, stress-strain relationship and elastic constants, Mohr's circle for plane stress and plane strain, Thin cylinders, Shear force and bending moment diagrams, Bending and shear stresses, Deflection of beams, Torsion of circular shafts, Euler's theory of columns, Strain energy methods, Thermal stresses.

Theory of Machines Displacement, velocity and acceleration analysis of plane mechanisms, Dynamic analysis of slider-crank mechanism, Gear trains, Flywheels.

Vibrations Free and forced vibration of single degree of freedom systems, Effect of damping, vibration isolation, Resonance, Critical speeds of shafts.

Design Design for static and dynamic loading, Failure theories, Fatigue strength and the S-N diagram, Principles of the design of machine elements such as bolted, riveted and welded joints, Shafts, Spur gears, Rolling and sliding contact bearings, Brakes and clutches.

FLUID MECHANICS & THERMAL SCIENCES

Fluid Mechanics Fluid properties, Fluid statics, manometry, buoyancy, Control volume analysis of mass, momentum and energy, Fluid acceleration, Differential equations of continuity and momentum, Bernoulli's equation, Viscous flow of incompressible fluids, Boundary layer, Elementary turbulent flow, Flow through pipes, Head losses in pipes, bends, etc.

Heat-Transfer Modes of heat transfer, One-dimensional heat conduction, Resistance concept, Electrical analogy, Unsteady heat conduction, Fins, Dimensionless parameters in free and forced convective heat transfer, Various correlations for heat transfer in flow over flat plates and through pipes, Thermal boundary layer, Effect of turbulence, Radiative heat transfer, Black and gray surfaces, Shape factors, Network analysis, Heat exchanger performance, LMTD and NTU methods.

Thermodynamics Zeroth, first and second laws of thermodynamics, Thermodynamic system and processes, Carnot cycle. Irreversibility and availability, Behavior of ideal and real gases, Properties of pure substances, Calculation of work and heat in ideal processes, Analysis of thermodynamic cycles related to energy conversion.

Applications *Power engineering:* Steam tables, Rankine, Brayton cycles with regeneration and reheat.
I.C. engines: air-standard Otto, Diesel cycles.
Refrigeration and air-conditioning: Vapor refrigeration cycle, Heat pumps, Gas refrigeration, Reverse Brayton cycle, Moist air, psychrometric chart, basic psychrometric processes.
Turbomachinery: Pelton-wheel, Francis and Kaplan turbines' impulse and reaction principles, Velocity diagrams.

MANUFACTURING & INDUSTRIAL ENGINEERING

Engineering Materials Structure and properties of engineering materials, Heat treatment, Stress-strain diagrams for engineering materials.

Metal Casting Design of patterns, Molds and cores, Solidification and cooling, Riser and gating design, Design considerations.

Forming Plastic deformation and yield criteria, Fundamentals of hot and cold working processes, Load estimation for bulk (forging, rolling, extrusion, drawing) and sheet (shearing, deep drawing, bending), Metal forming processes, Principles of powder metallurgy.

Joining Physics of welding, Brazing and soldering, Adhesive bonding, Design considerations in welding.

Machining and Machine Tool Operations Mechanics of machining, Single and multi-point cutting tools, Tool geometry and materials, Tool life and wear, Economics of machining, Principles of non-traditional machining processes, Principles of work holding, Principles of design of jigs and fixtures.

Metrology and Inspection Limits, fits and tolerances, Linear and angular measurements, Comparators, Gauge design, Interferometry, Form and finish measurement, Alignment and testing methods, Tolerance analysis in manufacturing and assembly.

Computer Integrated Manufacturing Basic concepts of CAD/CAM and their integration tools.

Production Planning and Control Forecasting models, Aggregate production planning, Scheduling, Materials requirement planning.

Inventory Control Deterministic and probabilistic models, Safety stock inventory control systems.

Operations Research Linear programming, Simplex and duplex method, Transportation, Assignment, Network flow models, Simple queuing models, PERT and CPM.

STRATEGY FOR SUCCESS IN GATE

The relative performance reflected in the GATE score of a candidate is used for admission to post-graduate engineering programs with financial assistance and for employment screening purposes in public-sector units. Success in GATE needs motivation, confidence, determination, time, books, energy and hard work. For preparing well, the following points of strategy are recommended:

1. One must be totally motivated and be full of natural curiosity. For an aspirant, qualifying a competition is a long-term goal. The sacrifices for a competition would be evident in the whole life of the candidate, his family, and generations. The expectations of family and society are actually the major sources of motivation. Their words of appreciation or criticism are always beneficial to the aspirant.
2. Self-confidence is the faith in one's capabilities. The best policy to generate and maintain self-confidence is to get tested. This also helps in knowing the status of preparedness and in knowing the unknown and weak topics for further inputs. The conversation with classmates instigates self-confidence and maintains the spirit of competitiveness.
3. The aspirant must be absolutely determined to give the best possible through the rigorous efforts. The initial hesitation for study can be avoided by discovering the long-term objective of success in GATE and ultimate aim in life.
4. Unworthy activities and out-of-context books should be avoided to save priceless time and resources. Instead of trying hard every time, difficult topics should be discussed with teachers and classmates without any reluctance or feeling of inferiority. Priority of coverage should be given to frequently asked topics based on the pattern and intensity of previous years' questions. Important facts and formulas should also be summarized to work on least asked topics. The candidate must broaden the span of his/her mind to capture the variety of topics in regular rotation.
5. Selection of right books is a very essential aspect to ensure success. A small mistake of choosing the wrong book can nullify all the efforts. Books must be selected only after prior review and recommendation of the able audience. The books must be from only reputed publication and authored by competent engineers or faculty of reputed institutions.

Success in GATE involves significant sacrifices on the part of the individual, but it brings immense glory and career to the individuals.

Deepti Tamrakar, M.Tech.

Research Scholar
Indian Institute of Information Technology, Design and Manufacturing
Jabalpur, Madhya Pradesh

METHODOLOGICAL CONCEPTS OF ENGINEERING STUDIES

Great minds discuss ideas. Average minds discuss events. Small minds discuss people.

—Eleanor Roosevelt

This article is a rudimentary effort to explicitly and succinctly identify several simple and methodological concepts, generally related to engineering studies, and subjectively concerned with the mechanical engineering discipline. The motivation behind this article is to enhance the quality of scientific communication from fundamental engineering texts to an aspirant of an aptitude-based competitive examination, like GATE.

The term "engineering" is derived from the word "engineer." The word "engine" is derived from the Latin "ingenium," meaning innate quality, especially intellectual power. In the modern era, the broad term "mechanical engineering" is used to describe a large number of classical and applied branches of basic engineering, such as civil, automobile, aerospace, naval architecture, materials and chemical engineering. Mechanical engineering is arguably one of the most exciting and fascinating disciplines. It possesses the evergreen charm of career-making because of its importance in all fields of engineering applications. The discipline explores engineers to the conception, design, implementation and operation of mechanical systems in various aspects of life. Simply phrased, these can range from bicycles to space shuttles, and beyond in both directions. Mechanical engineers work in the design, manufacturing and operation of innumerable industries, including aerospace, automotive, bio-mechanics, energy and power conversion, robotics, manufacturing, food processing, pulp and paper, textiles, heavy machinery and household appliances.

Science, engineering and technology are three interlinked streams of scientific studies which differ in approaches based on attitudes of students. Science is concerned with basic theories and exact calculations, while engineering aspires to serve humanity with creations, based on simplified scientific foundations, that can survive most types of perceived loads. An engineer adapts approximations, numerical analyses and empirical data sheets/tables to make designs realistically tractable. Scientists are predominantly oriented towards discoveries, not necessarily motivated by the application aspects. Engineers innovatively convert the scientific discoveries into useful applications. Engineering is a technical application of scientific phenomenon, and is instrumental in creating new technologies.

Hypotheses, theories and laws drive scientific progress by which scientists, collectively and over time, endeavor to construct accurate representations of the world. A "hypothesis" refers to the state of knowledge proposed for explanation of an observable phenomenon. This also helps in defining the parameters of the research. A key component of a formal scientific hypothesis is that it is testable and falsifiable. The word "model" is reserved for situations when it is known that the hypothesis has at least limited validity.

Engineering concepts are explained scientific theories which represent hypotheses confirmed through repeated experimental tests. A theory delves into underlying processes so as to understand the systematic reasons for a particular occurrence or non-occurrence of a phenomenon. It is usually laced with a set of convincing and logically

interconnected arguments. In simple words, a good theory explains, predicts, and delights. A phenomenon can be explained by more than one theory.

Theories and laws are used interchangeably but formally differ from each other. Theories are often formulated in terms of a few concepts and equations, which are identified with “laws of nature,” suggesting their universal applicability, such as Newton’s first law of motion. Thus, laws describe phenomena, often mathematically. Accepted scientific theories and laws become part of our understanding of the universe and the basis for explaining and exploring less well-understood areas of knowledge.

The terms “rule” and “principle” are derived from “laws”. A “theorem” is basically derived from a theory and, in general, is conditional and can be easily proven in a reverse manner. Axioms or postulates are the propositions which are assumed to be true and cannot be deduced from other assumptions. While explaining a phenomenon, some practical observations are also embedded into the respective equations. All such derivations and formulas are required to satisfy the theories and practical observations.

The knowledge of the objective(s) in every stage of learning is essential to appreciate any study and to rigorously focus available energies in the appropriate directions. This approach enables efficient utilization of resources. This is possible through hierarchy and parallelism of study which simplify the complex details of learning. This results in a systematically organized body of knowledge, easy to grasp and communicate to others. Therefore, every text tries to provide a hierarchical system of learning, which is evident in the form of chapters, sections, sub-sections, paragraph, and enumerations. In the present stage of engineering, a doubtful inquiry on the established body of knowledge may be unworthy, with notable exceptions, but curiosity on the fundamental concepts should be appreciated.

Mathematical derivations in engineering studies involve determination of the design dimensions or specification of the phenomenon. This accumulates all the relevant governing aspects (such as theories, laws, rules and principles) and transforms them into mathematical equations for explaining the phenomenon. Every step in a derivation accommodates one or more properties or features of the problem. To reduce the uncertainty and various other intractable factors in real problems, some assumptions are generally introduced in any study. Such approximations simplify the study considerably without violating the suitability for practical applications. A physical quantity involved in a phenomenon benefits from a standard symbol and conventions (e.g., positive direction for a vector). Such a quantity can be a scalar or vector, constant or variable, real or imaginary, and can have complicated interpretations. Therefore, before analyzing any phenomenon or topic, one must clearly define all the assumptions, symbols and sign conventions and appreciate the physical interpretations for all quantities involved. These approximations and conventions need attention in application of the derivations to problem-solving. Practicing the derivations always helps in understanding concepts from their root-level and clarifying the applicability of the expressions.

Dineshkumar Harursampath, Ph.D.

Assistant Professor and Head, NMCAD Lab
Department of Aerospace Engineering
Indian Institute of Science
Bangalore

CONTENTS

PART I

APPLIED MECHANICS & DESIGN

CHAPTER 1

ENGINEERING MECHANICS

Engineering mechanics is the branch of scientific analysis that describes and predicts the behavior of a stationary or moving body under the action of forces. In general, the subject is subdivided into three branches: rigid-body mechanics, deformable-body mechanics, and fluid mechanics. Present context is concerned with the mechanics of rigid-bodies that forms the foundation of mechanical devices. It is based on the assumption that the bodies are perfectly rigid. This is studied in two parts:

1. Statics *Statics* is concerned with mechanics of stationary systems. It requires the study of equilibrium stationary structures under forces and torque systems.
2. Dynamics *Dynamics* is concerned with the systems variant with time, and deals with the motion resulting from unbalanced force or torque systems. Dynamics is subdivided into two branches:
   (a) Kinematics *Kinematics* describes the motion of bodies without reference to the forces which either cause the motion or are generated as a result of motion. The subject is also referred to as the '*geometry of motion*'.
   (b) Kinetics *Kinetics* deals with the motion of rigid bodies under the action of forces.

1.1 FORCE

Force is a vector quantity which tends to change the state of a body. It means force is capable to bring a static body into motion or a moving body into static position. The study of mechanics encounters various types of force systems. The forces meeting at one point constitute a *concurrent force* system. The forces lying in one plane constitute a *coplanar force* system.

The SI unit of force is Newton (N), defined as the force acting on mass of 1 kg which produces an acceleration of 1 m/s^2.

1.1.1 Characteristics of a Force

Let a force $\vec{F}$ acts on a rigid body placed on a rough horizontal plane. Depending upon the *magnitude* of $\vec{F}$, the body can start moving in a straight line, if the *line of action* of $\vec{F}$ passes through the *center of gravity* of the body. This motion is called *translation.* If line of action does not pass through the center of gravity of the body,

the force will also result into *rotation* of the body. Thus, a force is characterized by its magnitude, line of action, direction and point of application.

1.1.2 Resolutions of a Force

Force is a vector quantity, therefore, has its resolved components in given directions, which are called *resolutions*. Engineering problems frequently need resolution of a force in orthogonal directions. Consider a force F in a x–y plane at an angle θ with the x-axis [Fig. 1.1].

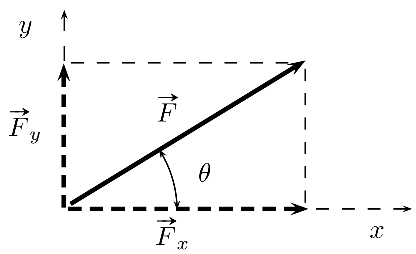


Figure 1.1 | Resolution of a force.

Magnitudes of the resolved parts of force F along x and y directions are the given by

$$F_x = F\cos\theta$$
$$F_y = F\cos\left(\frac{\pi}{2} - \theta\right)$$

Thus, the resolved part of a given force in a given direction is equal to the magnitude of the force multiplied with the cosine of the angle between the line of action of the force and the direction.

Using the resolved parts, a force can be presented in vector form:

$$\vec{F} = F_x\hat{i} + F_y\hat{j}$$

where $\hat{i}$ and $\hat{j}$ are the unit vectors in x and y directions, respectively. Magnitude of force $\vec{F}$ can be found as

$$F = \sqrt{F_x^2 + F_y^2}$$

The concept of resolved components is used to add two or more forces by summing their x and y components:

$$R_x = \sum F_x$$
$$R_y = \sum F_y$$

where R_x and R_y are the resolved components of the resultant force expressed as

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

1.2 MOMENT OF A FORCE

1.2.1 Definition

Moment of a force about a point or axis is the measure of the tendency of the force to cause a body to rotate about the point or axis. It is quantified by the product of the force and the perpendicular distance of its line of action from the point. This perpendicular distance is called *arm of the moment*.

Consider a force $\vec{F}$ acting on a rigid point. Moment of this force can be determined about a point O situated at distance $\vec{r}$ from line of action [Fig. 1.2].

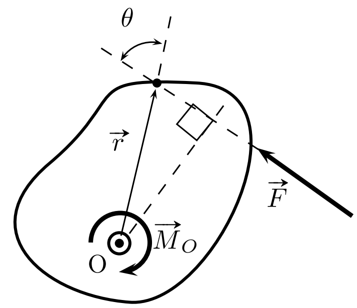


Figure 1.2 | Moment of a force.

The moment $\vec{M}_O$ of the force about point O is defined as the cross product of force vector and distance vector:

$$\vec{M}_O = \vec{r} \times \vec{F} \tag{1.1}$$

The direction of $\vec{M}_O$ is determined by the right hand rule. Magnitude of the moment is given by

$$M_O = rF\sin\theta \tag{1.2}$$

where θ is the angle between $\vec{r}$ and $\vec{F}$.

The curl or sense of rotation can always be determined by observing in which direction the force would "orbit" about the fulcrum point O. The point is referred only for a two dimensional case, however, the moment always acts about an axis perpendicular to the plane containing $\vec{F}$ and $\vec{r}$, and this axis intersects the plane at the point O.

Eq. (1.2) indicates that a force will not contribute a moment about a specified axis if line of action of the force is parallel to the axis ($\theta = 0$).

1.2.2 Resultant Moment of a System of Forces

Let a system of forces acts upon a rigid body. Resultant moment of the forces about a point is determined by

the vector addition of the moments of individual forces about that point:

$$\vec{M}_R = \sum(\vec{r} \times \vec{F})$$

1.2.3 Varignon's Theorem

According to *Varignon's theorem*[1], the algebraic sum of moments of several concurrent forces about any point is equal to the moments of their resultant about the same point.

Varignon's theorem can be stated alternatively as the moment of a force about any point equal to the sum of moments of its components about that point.

For the system of coplanar concurrent forces shown in Fig. 1.3, the Varignon's theorem is written as

$$Fr = F_x x + F_y y$$

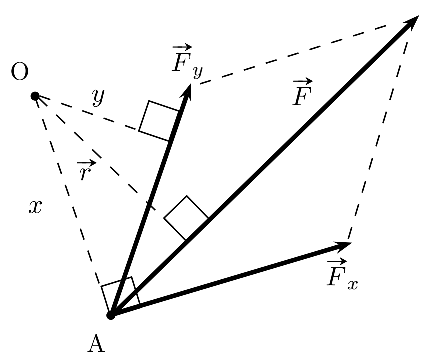


Figure 1.3 Varignon's theorem.

1.2.4 Principle of Moments

The *principle of moments* is a corollary derived from the Varignon's theorem, which states that if a system of coplanar forces is in equilibrium, then the algebraic sum of their moments about any point in their plane is zero.

1.2.5 Moment of a Couple

Two equal but opposite parallel forces having different lines of action form a *couple*[2]. The resultant force of the two forces in any direction is zero. However, the only effect is to produce a tendency to rotate a body upon which the couple act because sum of the moments of the two forces about a given point is not zero.

Let two forces $\vec{F}$ and $-\vec{F}$ be situated at distances $\vec{r}_1$ and $\vec{r}_2$ from a point O [Fig. 1.4].

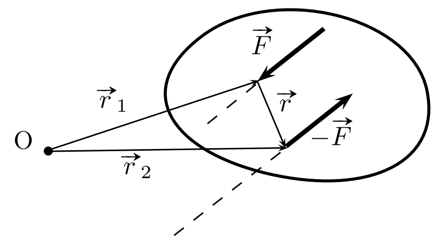


Figure 1.4 Moment of a couple.

The moment of the couple $\vec{M}$ is given by

$$\begin{aligned}\vec{M} &= \vec{r}_1 \times \vec{F} + \vec{r}_2 \times \left(-\vec{F}\right) \\ &= (\vec{r}_1 - \vec{r}_2) \times \vec{F} \\ &= \vec{r} \times \vec{F} \qquad (1.3)\end{aligned}$$

where $\vec{r}$ $(= \vec{r}_1 - \vec{r}_2)$ is the distance vector between the lines of action of the parallel forces. This vector is called *arm of the couple*. Direction of $\vec{M}$ is a vector perpendicular to $\vec{r}$ and force $\vec{F}$.

Equation (1.3) shows that *moment of the couple* is equal to the vector product of either force of the couple with the arm of the couple. The moment of a couple is independent of r_1 or r_2 vectors, therefore, point O can be chosen arbitrarily. It means that moment of a couple is a *free vector*, unlike the moment of a force which requires a definite axis.

1.3 EQUIVALENT SYSTEM OF A FORCE

A force tends to cause translation and rotation of a body. This depends upon the magnitude, direction, and line of action of the force with respect to the center of gravity of the body. A body can be subjected to a system of forces. The problem is generally simplified by determining an equivalent system of resultant force and moment that can produce the same effect of translation and rotation with respect to any point on the body.

Consider a body subjected to a force $\vec{F}$ at point P. The force is to be moved to another point O without changing the effect on the body [Fig. 1.5].

This can be done by applying equal and opposite forces $\vec{F}$ and $-\vec{F}$ at point O. Thus, the original force $\vec{F}$

[1]Pierre Varignon (1654–1722) was a French mathematician. He was a friend of Newton, Leibniz, and the Bernoulli family. His principal contributions were to graphic statics and mechanics. Varignon's theorem is a statement in Euclidean geometry by him that was first published in 1731.

[2]Moment is created by single force, while a couple is created by equal and opposite forces. Interestingly, a single force acting on a body creates a reaction in opposite direction from the body, thus constitutes a couple.

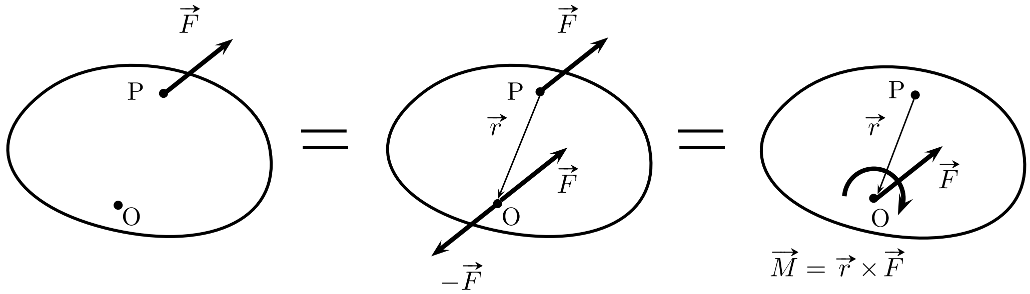


Figure 1.5 Equivalent system of a force.

at point A and force $-\vec{F}$ at point O form a couple whose moment depends upon the distance vector $\vec{r}$ between point O and P, like a *free vector*. Thus, an equivalent system of a force $\vec{F}$ acting at point A is found at point O.

1.4 SINGLE RESULTANT FORCE

Consider a situation when a rigid body is subjected to a system of forces and couple moments [Fig. 1.6]. The system of forces and moments can be reduced to a resultant force $\vec{F}_R$ acting at point O, and resultant moment $\vec{M}_R$ by the vector sum of the respective quantities:

$$\vec{F}_R = \sum \vec{F}$$
$$\vec{M}_R = \sum \vec{M}$$

As a special case, if $\vec{M}_R$ and $\vec{F}_R$ are perpendicular to each other, the situation can be further simplified by replacing $\vec{F}_R$ and $\vec{M}_R$ at point O by a single force $\vec{F}_R$ acting at a distance d from point O. The distance d is given by following expression:

$$d = \frac{M_R}{F_R}$$

This effect is the reverse of determining an equivalent force of a system. This observation can be applied in the following special cases:

1. *Concurrent Force Systems* The forces meeting at one point constitute a *concurrent force* system. Thus, there is no resultant couple moment, and the resultant force acts at a specific point O only.

2. *Coplanar Force Systems* The forces lying in one plane constitute a *coplanar force* system. Such a system can be replaced by the resultant coplanar force F acting at a point O, and the resultant moment M_R along an axis passing through point O and normal to the plane of forces. This can be further simplified by a resultant force F_R acting at a distance $d = M_R/F_R$ from point O.

1.5 EQUILIBRIUM OF RIGID BODIES

The concept of equilibrium of rigid bodies is derived from the *Newton's first law of motion*, which states that if the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion). Thus, a body is considered in *equilibrium* when its condition (motion or rest) is unaffected by the forces acting on it. For example, a body moving with a constant acceleration caused by applied force is said to be in equilibrium.

1.5.1 General Condition

The necessary and sufficient conditions for complete equilibrium of a rigid body under a force system are as follows:

1. For any system of forces keeping a body in equilibrium, the algebraic sum of forces, in any direction is zero:
$$\sum \vec{F} = 0 \tag{1.4}$$

2. For any system of forces keeping a body in equilibrium, the algebraic sum of the moments of all the forces about any point in their plane is zero:
$$\sum \vec{M} = 0 \tag{1.5}$$

These are the fundamental equations of statics, which are essentially used in determining the unknown forces and reactions acting on a body under equilibrium. In this reference, a problem is called *statically determinate* if the number of unknown reactions is equal to the

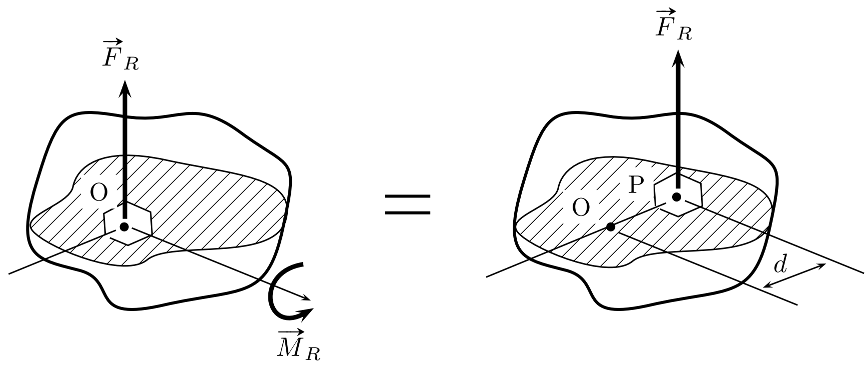


Figure 1.6 Single resultant force.

number of equations of equilibrium. The problem is *statically indeterminate* if the number of unknown reactions is less than the number of equations of equilibrium.

1.5.2 Free Body Diagrams

Two bodies in contact exert forces on the other. One of these force is called *action*, and the other is called *reaction*. The concept of free body is derived from the *Newton's third law of motion* which states that action and reaction are always equal and opposite, and when bodies are smooth, they are normal to the surfaces in contact.

Equilibrium of a body can be examined using Eqs. (1.4)–(1.5). This requires knowledge of all the forces acting on a body. This is achieved by drawing the body's free body diagram. A diagram showing the forces acting on a body, together with reactions at the supports but not showing the supports is called a *free body diagram*. A body so isolated from its supports or surrounding is called a *free body*. Thus, a free body diagram shows all active and reactive forces acting on the body.

For example, consider a body resting on a surface [Fig. 1.7]. Its weight W acts downward which creates a normal reaction $\vec{R}_n$ at the surface. If a force $\vec{F}$ is applied to move the body in the horizontal direction, the surface exerts a friction force $\vec{F}'$ that acts opposite to it. The resultant of $\vec{R}_n$ and $\vec{F}'$ is R.

In the free body diagram, the weight W and force $\vec{F}$ are to be included along with the resultant reaction $\vec{R}$.

Internal forces of a body always occur in equal but opposite collinear pairs, therefore, their net effect on the body is zero. Thus, internal forces are not drawn in free body diagrams.

Weight of a body is the resultant of the gravity forces acting on the particles that constitute the body. The point of application of this resultant force is known as

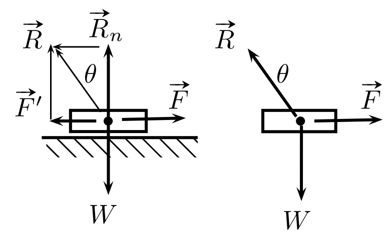


Figure 1.7 Free body diagram.

the center of gravity. Weight is an external force, thus, it is included in free body diagrams.

1.5.3 Support Reactions

Knowledge of support reactions is necessary for drawing free body diagram of a body to examine its equilibrium using Eqs. (1.4)–(1.5). As a general rule, a support can prevent translation of a body in the given direction by exerting a reaction force on the body in the opposite direction. Likewise, a support can prevent the rotation of a body in a given direction by exerting a couple moment on the body in the opposite reaction. The force and couple moment are the reactions exerted by a support on a supported body.

The following are the three kinds of supports that offer different types of reactions [Fig. 1.8]:

1. *Roller Support* A roller support prevents the body from translation in the vertical direction because the roller can exert a reaction force along the common normal at tangent point.

2. *Hinged Support* A hinged or pin support does not offer resistance against rotation. Thus, it offers both horizontal and vertical reactions, but does not exert a couple moment on the supported body.

3. *Fixed Support* The most restrictive way to support a body is using a fixed support because it prevents both translation and rotation of a supported body. Thus, a fixed support offers all the three elements of reactions; horizontal and vertical reactions and moment.

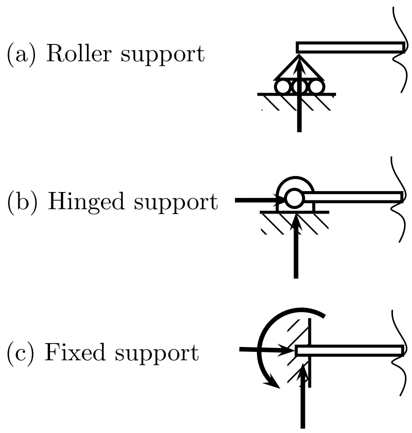


Figure 1.8 | Support reactions.

1.5.4 Equilibrium of Three Coplanar Forces

Using the general condition of equilibrium, the condition of equilibrium of three coplanar forces can be stated as follows:

1. If three coplanar forces acting upon a rigid body under equilibrium, they must either meet in a point or be all parallel.

2. If three forces are in equilibrium, they must be coplanar.

1.5.5 Triangle Law of Forces

The problem of equilibrium of three coplanar forces can be represented in triangular fashion. This is known as the *law of triangle of forces* which states that if three forces acting upon a particle can be presented in the magnitude and direction by the sides of a triangle taken in order, the forces will be in equilibrium.

In converse way, if three forces acting upon a particle in equilibrium, they can be represented in magnitude and direction by the sides of any triangle which is drawn so as to have its sides respectively parallel to the directions of the forces.

Consider three forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ acting on a particle or rigid body in equilibrium. The law of triangle of force is represented in Fig. 1.9 where these forces form a triangle.

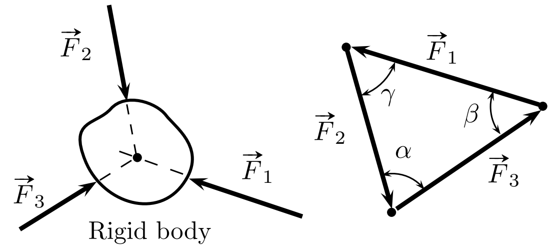


Figure 1.9 | Law of triangle of forces.

The law of triangle of forces is equivalent to the vector sum of the forces; the net force and moments acting on a particle is zero, therefore, particle is in equilibrium.

1.5.6 Lami's Theorem

In statics, *Lami's theorem*[3] is an equation that relates the magnitudes of three coplanar, concurrent and non-collinear forces, that keep a body in static equilibrium. The theorem states that if three forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two forces.

Consider three forces F_1, F_2, F_3 acting on a particle or rigid body making angles α, β, γ with each other [Fig. 1.10].

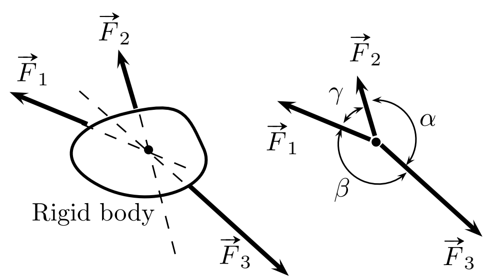


Figure 1.10 | Lami's theorem.

According to Lami's theorem, the particle shall be in equilibrium if

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma} \tag{1.6}$$

The angle between the force vectors is taken when all the three vectors are emerging from the particle.

[3]Bernard Lamy (1640-1715) was a French Oratorian mathematician and theologian. His best known work is the parallelogram of forces (1679).

1.6 STRUCTURAL ANALYSIS

1.6.1 Trusses and Frames

A structure can consists two types of members:

1. *Trusses* A *truss* is an articulated structure composed of straight members arranged and connected in a such a way that they transmit primarily axial forces. For this, a truss is made up of several slender bars, called *members*, joined together at their ends by hinges or rivets. The bar members, therefore, act as two-force members which can either be in tension or in compression; there can be no transverse force in a member of a truss [Fig. 1.11].

   For the purpose of calculations, the joints (nodes) are supposed to be hinged or pin-jointed. A truss is designed to carry loads at the nodes, otherwise, truss members can be subjected to lateral loads.

   A perfect truss is composed of least number of members to prevent distortion of its shape when loaded. If the number of nodes in a perfect truss is n, then the minimum number of members is $2n-3$.

2. *Frame* A *frame* consists of members which can be subjected to a transverse load in addition to the axial load. Thus, members carry loads at points other than nodes. If load is applied at a point other than a joint, the member is subjected to bending also; and in such a case, the force in the member is not purely axial. To find the forces in the members subjected to bending, the equilibrium of each member is considered separately by constructing its free body diagram.

1.6.2 Assumptions

To determine the axial forces developed in the truss members, following assumptions are made:

1. Each truss is composed of rigid members, all lying in one plane.
2. Forces are transmitted from one member to another through smooth pins fitted in the members.
3. All the loads are applied at the joints.
4. Weight of the members are neglected because they are small in comparison with the loads.

The effect of axial forces acting at the joints of a member is shown by marking arrows over the member, according to the direction of the forces [Fig. 1.11].

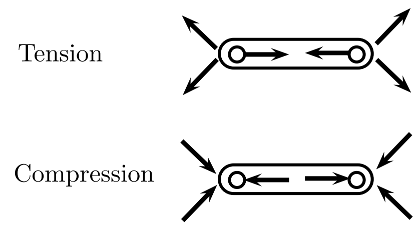


Figure 1.11 Sign of forces in members.

A member can be subjected to two types of axial forces: tension (arrow directed away from joint) or compression (arrow directed toward joint).

1.6.3 Method of Joints

A plane truss or frame can be subjected only to a coplanar force system. Therefore, any point on a member of the plane truss can be subjected to coplanar and concurrent systems only. This condition of concurrency of force system follows from the equilibrium of forces at a given point on a truss. The *method of joints* is based on these observations. This method takes one point at a time and analyzes it for equilibrium. At every node, the forces must be along the members at that joint and must satisfy the necessary conditions of equilibrium:

$$\sum \vec{F} = 0$$
$$\sum \vec{M} = 0$$

The sum of moments can be examined only at the points of application of the support reactions.

1.6.4 Zero-Force Members

Truss analysis using the method of joints is greatly simplified with the knowledge of zero-force members in the truss. Special situations of forces in truss members are explained as follows [Fig. 1.12]:

(a) If two members not in the same straight line meet at a point which does not carry any load, the force in each member is zero.

(b) If two members in the same straight line meet at a point, they carry equal and opposite forces.

(c) If three members meet at a joint which does not carry any load, and two members are in same line, then the force in third member will be zero.

(d) If four members meet at a point at which there is no load with two of the members in straight lines, then forces in members aligned in the same lines will be equal.

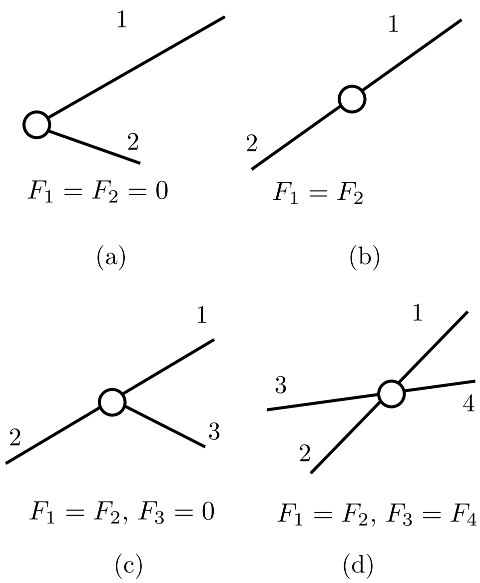


Figure 1.12 Forces in truss members.

These points are useful in predicting the forces in truss members without the actual calculations.

1.6.5 Method of Sections

Method of joints is used in determining the unknown forces on each member of a truss while method of sections is preferred in determining axial forces in only few members. In this method, the truss is cut at a section such that most of the members of unknown forces are covered. The equations of equilibrium are then applied to determine the unknown forces.

1.7 RECTILINEAR KINEMATICS

Rectilinear kinematics deals with the motion of a particle in rectilinear or straight line path. It is characterized by particle's position, velocity, and acceleration, at any given instant of time. Curvilinear motion occurs when motion of a particle follows a curved path. *Speed* is the rate of change of distance irrespective of the direction of motion of the body; *Velocity* is a vector quantity of magnitude equal to speed. *Acceleration* is the rate of change of velocity with respect to time.

Consider a particle, moving in a straight line, changes its position from $\vec{x}$ to $\vec{x}+\delta\vec{x}$ in time δt [Fig. 1.13]. Velocity and acceleration of the particle at any instant of time are defined as follows:

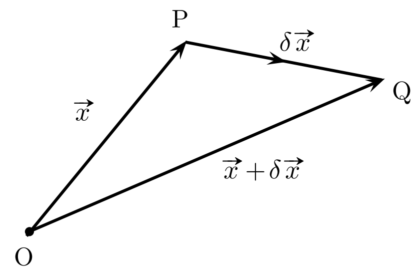


Figure 1.13 Linear displacement of a particle.

1. *Velocity* Instantaneous velocity of the particle is defined as

$$\vec{v} = \lim_{\delta t \to 0} \frac{\delta\vec{x}}{\delta t} = \frac{d\vec{x}}{dt} \tag{1.7}$$

2. *Acceleration* Instantaneous acceleration of the particle is defined as

$$\vec{a} = \frac{d}{dt}\vec{v} = \frac{d^2\vec{x}}{dt^2} = v\frac{d\vec{x}}{dt} \tag{1.8}$$

Velocity and acceleration are vector quantities. If two velocities (say $\vec{u}$ and $\vec{v}$ at angle α) are represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant ($\vec{w}$) will be represented in magnitude and direction by the diagonal of the parallelogram. This is called the *law of parallelogram* [Fig. 1.14].

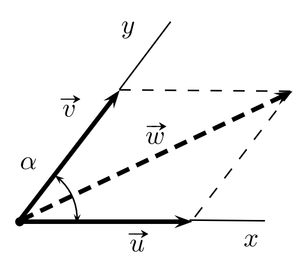


Figure 1.14 Law of parallelogram.

The magnitude of resultant velocity of u and v at angle α is given by

$$w = \sqrt{u^2+v^2+2uv\cos\alpha} \tag{1.9}$$

The law of parallelogram is based on the algebra of vectors. Therefore, it is equally valid for the forces also.

Let a particle moves at initial velocity u. In the time interval of t, it traces a distance s reaching its final velocity v. Using Eq. (1.8), the differential equation for acceleration a as

$$\frac{d^2 s}{dt^2} = a$$

Integrating w.r.t. time (t),

$$\frac{ds}{dt} = at + c_1 \tag{1.10}$$

where c_1 is a constant. When $t = 0$, $ds/dt = u$, so,

$$c_1 = u$$

Therefore, Eq. (1.10) becomes

$$v = u + at \tag{1.11}$$

Integration of Eq. (1.10) gives

$$s = \frac{1}{2}at^2 + ut + c_2 \tag{1.12}$$

where c_2 is constant. When $t = 0$, $s = 0$ so $c_2 = 0$, therefore,

$$s = ut + \frac{1}{2}at^2 \tag{1.13}$$

Using Eqs. (1.11) and (1.13),

$$v^2 = u^2 + 2as \tag{1.14}$$

Equations (1.11)–(1.14) are called *equations of linear motion*.

1.8 ANGULAR MOTION

Let a particle moving in a circle travels angle $\delta\theta$ in time δt. *Angular velocity* of the particle is defined as

$$\begin{aligned} \vec{\omega} &= \lim_{\delta t \to 0} \frac{\delta \vec{\theta}}{\delta t} \\ &= \frac{d\vec{\theta}}{dt} \end{aligned} \tag{1.15}$$

Angular acceleration of the particle is defined as

$$\vec{\alpha} = \frac{d}{dt}\vec{\omega} = \frac{d^2\vec{\theta}}{dt^2} \tag{1.16}$$

Let a particle follow a circular path of radius r with constant linear speed v at angular velocity ω. So in a unit time, it runs an arc of $r\omega$, which is of the same length equal to v. Therefore,

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Differentiating w.r.t. time,

$$\begin{aligned} \frac{d\vec{v}}{dt} &= r\frac{d\vec{\omega}}{dt} \\ \vec{a} &= \vec{\alpha} \times \vec{r} \end{aligned}$$

1.9 MOTION UNDER GRAVITY

Motion under gravity is described in the following subsections.

1.9.1 Universal Gravitation

Newton's law of *universal gravitation* states that any two particles or point masses attract each other along the line connecting them with a mutual force whose magnitude is directly proportional to the product of the masses and inversely proportional to the square of the distance between the particles.

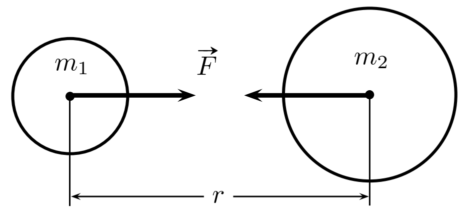


Figure 1.15 Gravitational force.

For the configuration shown in Fig. 1.15, the gravitational force F between two masses m_1 and m_2 at distance r is given by

$$F = G \times \frac{m_1 m_2}{r^2} \tag{1.17}$$

where G is the *universal constant of gravitation*, equal to 6.67×10^{11} Nm^2/kg^2.

1.9.2 Earth's Gravity

Assuming earth to be stationary and spherical body of radius R, the gravitational force of the earth (due to its mass M) acting on a body of mass m, placed at a height h above the surface of the earth, is given by

$$F = G\frac{Mm}{(R+h)^2}$$

This force is the weight of the body equal to mg. Therefore, acceleration due to gravity g is derived as

$$g = \frac{GM}{(R+h)^2} \tag{1.18}$$

The value of g is approximately 9.81 m/s^2. In engineering applications, g is usually considered as a constant and the weight force is assumed to be directly perpendicular to the earth's surface.

When a particle is projected vertically upward, there is a retardation upon it due to earth's attraction. This

retardation is denoted by $-g$. When a particles falls down under gravity, it possesses an acceleration equal to g.

1.9.3 Projectile

The particle projected under gravity other than vertical is called a *projectile*. The *angle of projection* is the angle of initial velocity with horizontal plane. The path described by the particle is called *trajectory*. The *range of projectile* is the distance between the point of projection and the point where trajectory meets any horizontal plane through the projection.

Let a particle is projected upward at an angle α from horizontal at initial velocity of u [Fig. 1.16].

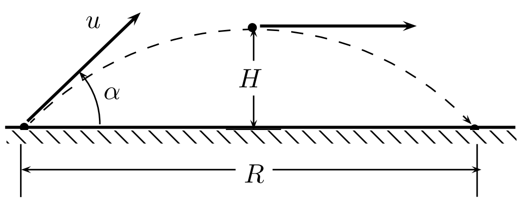


Figure 1.16 Projectile.

The following are the features of a projectile:

$$\text{Flight time } t = 2 \times \frac{u \sin\alpha}{g}$$
$$\text{Maximum height } H = \frac{u^2 \sin 2\alpha}{2g}$$
$$\text{Range } R = \frac{u^2 \sin 2\alpha}{g}$$

The range is maximum if $\alpha = \pi/4$.

1.9.4 Vertical Projection

Consider a particle of is projected vertically upward ($\alpha = \pi/2$) at initial velocity u, and let it reaches upto height h where velocity v becomes zero. Using Eq. (1.14),

$$0^2 = u^2 + 2(-g)H$$
$$H = \frac{u^2}{2g}$$

Using Eq. (1.11), the time t taken in the reaching to the height h is determined as

$$0 = u - gt_{1/2}$$
$$t = \frac{u}{g}$$

1.10 DEPENDENT MOTION OF PARTICLES

In some types of engineering applications, motion of particles is dependent upon others. Two blocks interconnected with an inextensible spring over a pulley represent the most simple situation of dependent motion [Fig. 1.17].

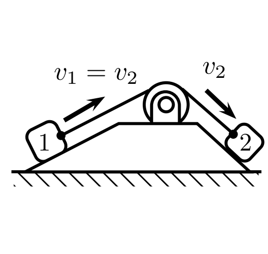


Figure 1.17 Dependent motion of two particles.

The relationship between dependent velocities can be found using constant length of the inextensible string.

1.11 NEWTON'S LAWS OF MOTION

The problems of mechanics can be solved by applying *Newton's laws of motion*[4], described as follows:

1. First Law of Motion *Newton's first law of motion* states that every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force.

   The first law of motion is normally taken as the definition of *inertia*. If there is no net force acting on an object (if all the external forces cancel each other out) then the object will maintain a constant velocity. If that velocity is zero, then the object remains at rest. If an external force is applied, the velocity will change because of the force.

2. Second Law of Motion *Newton's second law of motion* states that if the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force. The law explains how the velocity of an

[4]Sir Isaac Newton was one of the greatest scientists and mathematicians that ever lived. He was born in England on December 25, 1643. He was born the same year that Galileo died. He was the first to formalize these laws and published them in 1986.

object changes when it is subjected to an external force. The law defines a force to be equal to change in momentum (mass times velocity) per unit time.

For an object with a constant mass m, *Newton's second law of motion* states that the force $\vec{F}$ is the product of an object's mass and its acceleration $\vec{a}$:

$$\vec{F} = m \times \vec{a}$$

For an externally applied force, the change in velocity depends on the mass of the object. A force will cause a change in velocity; and likewise, a change in velocity will generate a force. The above equation works in both ways.

3. Third Law of Motion *Newton's third law of motion* states that for every action (force) in nature there is an equal and opposite reaction. In other words, if object A exerts a force on object B, then object B also exerts an equal force on object A.

   The third law of motion can be used to explain the generation of lift by a wing and the production of thrust by a jet engine.

1.12 WORK AND ENERGY

1.12.1 Energy

The *energy* of a body is its capacity of doing work. Energy is possessed by a body, while the work is done by force on a body when it has a displacement in the direction of the force. If position vector is denoted by $\vec{r}$, then work dW, a scalar quantity, is defined as the dot product of force $\vec{F}$ and displacement vector $d\vec{r}$:

$$dW = \vec{F} \cdot d\vec{r}$$

1.12.2 Modes of Mechanical Energy

Energy can be in several forms like mechanical energy, electrical energy, heat, light, sound, pressure. The present context is of *mechanical energy*, the energy possessed by a body due to its position or motion. Hence, mechanical energy can be of two types: potential energy and kinetic energy, described as follows:

1. Potential Energy The energy which a body possesses by virtue of its position or configuration is called *potential energy.* Few examples to clarify the concept of potential energy are following:

   (a) If a body of mass m is raised through a height h above a datum[5] level, then the work done on it by the gravitational force is written as

   $$W = mgh$$

   This energy is stored in the body as potential energy. In coming down to the original position, the body is capable of doing work equal to mgh.

   (b) If a spring is twisted through an angle θ by application of a torque varying from zero in the beginning to T in the end, the work done by the average torque $T/2$ is written as

   $$\begin{aligned} W &= \frac{0+T}{2} \times \theta \\ &= \frac{1}{2} T\theta \end{aligned}$$

   This is the potential energy of the spring due to its configuration.

   (c) If a spring of stiffness k is stretched, the force F acting on it does not remain constant, but increases with displacement x undergone by the spring. At any time,

   $$F = kx$$

   Therefore, average force acting on the spring is

   $$\begin{aligned} \bar{F} &= k \times \frac{0+x}{2} \\ &= k\frac{x}{2} \end{aligned}$$

   Hence, the work done by the average force $\bar{F}$ for displacement x of the spring is written as

   $$W = k\frac{x^2}{2}$$

   This is the potential energy of the spring due to its configuration.

   Gravity force, elastic spring, and torsional spring are examples of *conservative forces.* Thus, *potential energy* is the measure of the amount of work done by a conservative force in moving a body from one position to another.

2. Kinetic Energy The energy which a body possesses by virtue of its motion is called *kinetic energy.* It is measured by the amount of work required to bring the body to rest.

   Let a body of mass m moving with velocity v be brought to rest by the application of a constant force $\vec{F}$ which causes a retardation $-\vec{a}$. If s is the distance through which the body moves in this period, the kinetic energy is given by the work done by force $\vec{F}$ on the body. Using third equation of linear motion,

   $$0^2 - v^2 = 2(-a)s$$

[5] A surface to which elevations, heights, or depths on a map or chart are related.

Therefore, kinetic energy is determined as

$$\begin{aligned} U &= ma \times s \\ &= \frac{1}{2} mv^2 \end{aligned}$$

A system of particles or body can have both forms of mechanical energy. During motion or change in the amount or direction forces, one form of energy gets converted into another form.

1.12.3 Principle of Work and Energy

The principle of work and energy states that the work done by all of the external forces and couples as a rigid body moves from position 1 to position 2 is equal to the change in the potential energy of the body:

$$\sum U_{1-2} = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$$

1.12.4 Principle of Conservation of Energy

The *principle of conservation of energy* states that the total amount of energy in the universe is constant; energy can neither be created nor destroyed although it can be converted into various forms.

The principle of conservation of energy can be appropriately stated as when a particle moves under the action of conservative forces, the sum of the kinetic energy and potential energy of the particle remains constant. If potential energy and kinetic energy are denoted by U and T, respectively, the principle can be stated for a system between two instances 1 and 2 as

$$T_1 + U_1 = T_2 + U_2$$

The principle of conservation of energy is generally applied to solve the problems involving forces, displacements and velocities. The principle can be applied to each element of a structure or body separately. The problems involving energy dissipation through friction and damping can be solved by considering suitable sign of the energy component of the system.

1.13 D'ALEMBERT'S PRINCIPLE

According to the *d'Alembert's principle*[6], the external forces acting on a body and the resultant *inertia* forces on it are in equilibrium. D'Alembert's principle is, indeed, a restatement of *Newton's second law of motion* but it suggests that the term $(-ma)$ can be considered as a fictitious force, often called *d'Alembert's force* or the *inertia force*. Accordingly, the net external force $\vec{F}$ actually acting on the body and the inertia force $\vec{F_i}$ together keep the body in a state of *fictitious equilibrium*:

$$\vec{F} + \vec{F_i} = 0$$

The d'Alembert's principle tends to give the solution procedure of a dynamic problem, an appearance like that of a static problem, and the above equation becomes equation of *dynamic equilibrium*.

[6]D'Alembert's principle is named after its discoverer, the French physicist and mathematician Jean le Rond d'Alembert.

1.14 IMPULSE AND MOMENTUM

1.14.1 Linear Momentum

Momentum $(\vec{p})$ is a measure of the tendency of an object to keep moving once it is set in motion. Let a particle of mass m move with a velocity $\vec{v}$ and acceleration $\vec{a}$. Using *Newton's law of motion*, the force acting on the body is given by

$$\vec{F} = m\vec{a}$$

The rate of change of momentum is

$$\begin{aligned} \frac{d\vec{p}}{dt} &= \frac{d}{dt} m\vec{v} \\ &= m\frac{d\vec{v}}{dt} \\ &= m\vec{a} \\ &= \vec{F} \end{aligned}$$

This equation states that the rate of change of momentum is equal to the applied force. This statement is known as the principle of linear momentum. The law is also known as *Euler's first law*. If there are no forces applied to a system, the total momentum of the system remains constant; the law in this case is known as the *law of conservation of momentum*.

1.14.2 Angular Momentum

Angular momentum $(\vec{h})$ is the moment of momentum about an axis; it is the product of the linear momentum of the particle and the perpendicular distance from the axis of its line of action. Consider a particle of mass m moving with a velocity $\vec{v}$ and acceleration $\vec{a}$ [Fig. 1.18].

The angular momentum about an axis passing through point O at distance $\vec{r}$ is given by

$$\vec{h} = \vec{r} \times m\vec{v}$$

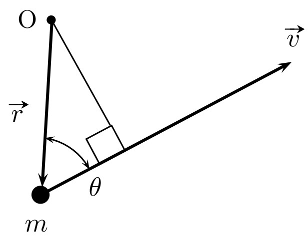


Figure 1.18 Angular momentum.

The rate of change of angular momentum can be determined as

$$\begin{aligned}\frac{d\vec{h}}{dt} &= \frac{d}{dt}(\vec{r}\times m\vec{v})\\ &= \vec{r}\times m\frac{d\vec{v}}{dt}\\ &= \vec{r}\times\vec{F}\end{aligned}$$

This equation is known as the principle of angular momentum. It states that the resultant moment of the external forces ($\vec{F}$) acting on the system of a particle equals the rate of change of the total angular momentum of the particles. The law is also known as *Euler's second law*.

1.14.3 Impulse–Momentum Principle

If a constant force $\vec{F}$ acts for time t on a body, the product $\vec{F}\times t$ is called the *impulse* of the given force. Similarly, if a torque $\vec{T}$ acts on a body for time t, then the angular impulse is $\vec{T}\times t$.

Let a constant force F acts on a body of mass m for time t and changes its velocity from u to v under acceleration a. Then, impulse ($\vec{J}$) is given by

$$\begin{aligned}\vec{J} &= \vec{F}\times t\\ &= m\vec{a}\times t\\ &= m\times\frac{\vec{v}-\vec{u}}{t}\times t\\ &= m(\vec{v}-\vec{u})\\ &= m\vec{v}-m\vec{u}\end{aligned}$$

Therefore, *impulse–momentum principle* states that the component of resultant linear impulse along any direction is equal to change in the component of momentum in that direction.

1.15 LAW OF RESTITUTION

Impact is the collision of two particles for a very short period of time that results into relatively large impulsive forces exerted between the particles. An impact is called *central or line impact* when direction of motion of the mass centers of the two colliding particles is in a single line, otherwise, it is called *oblique impact*.

The *law of restitution* states that the velocity of separation of two moving bodies which collide with each other bears a constant ratio with their velocity of approach. The constant of proportionality is called the *coefficient of restitution*, denoted by e. This property, first discovered by Newton, is known as the *Newtons law of restitution*.

Consider two particles moving with initial velocities u_1 and u_2 towards each other. These particles collide on center-line (center impact), and after impact, their respective velocities become v_1 and v_2 [Fig. 1.19].

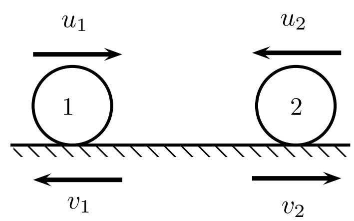


Figure 1.19 Coefficient of restitution.

The coefficient of restitution (e) is expressed as the ratio of relative velocities of the particles' separation just after impact ($v_2 - v_1$) to the relative velocity of the particles' approach just before impact ($u_1 - u_2$):

$$e = \frac{v_2 - v_1}{u_2 - u_1}$$

Experiments show that e varies appreciably with impact velocity as well as with the size and shape of the colliding particles, ranging from 0 to 1. The value of the coefficient of restitution has got physical meaning. Based on the limiting values of e, the collision can be classified into two types:

1. <u>Elastic Collision</u> A *perfectly elastic collision* occurs without loss of kinetic energy of the particles. Thus, for elastic collisions, $e = 1$.
2. <u>Inelastic Collision</u> A *inelastic collision* or *plastic collision* is one in which part of the kinetic energy is changed to some other form of energy in the collision.

   Momentum is conserved in inelastic collisions, however, the kinetic energy in the the collision is converted into other forms of energy. For inelastic collisions, $e = 0$.

The principle of work and energy cannot be used for the analysis of impact problems because it is impossible to

know the variation in the internal forces of deformation and restitution during the collision. The energy loss can be calculated as the change in kinetic energy of the particles.

1.16 PRINCIPLE OF VIRTUAL WORK

When the point of application of a force is imagined to be displaced through a differential distance in the direction of the force, the imaginary work done by the force is called *virtual work*.

The *principle of virtual work* states that the work done on a rigid body or a system of rigid bodies in equilibrium is zero for any virtual displacement compatible with the constraints on the system. Conversely, if the virtual work for all such displacements is zero, then the body is in equilibrium.

Virtual displacement is an imaginary infinitesimal displacement. A differential virtual displacement is denoted by δ to distinguish it from a differential displacement generally denoted by d.

The method of virtual work is explained by two examples:

1. Consider a rod AB which can rotate about a fulcrum O [Fig. 1.20]. A vertical load F_1 is applied at end A. It is required to calculate the a vertical force F_2 to be applied at end B to keep the rod in current position. In virtual work method, the body

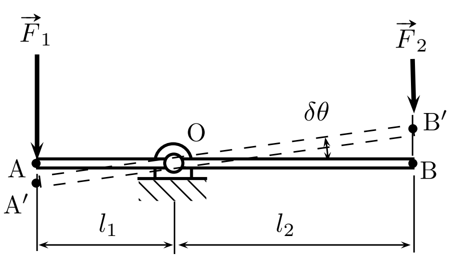


Figure 1.20 Virtual displacement.

is assumed to be virtually displaced. For present case, let the rod undergo a virtual rotation through angle $\delta\theta$ about the fulcrum O to assume the new position A′B′. The total virtual work during this rotation is given by

$$\delta W = F_1 \times l_1 \delta\theta - F_2 \times l_2 \delta\theta$$

According to the principle of virtual work, total virtual work must be zero, therefore,

$$F_1 \times l_1 \delta\theta - F_2 \times l_2 \delta\theta = 0$$
$$F_2 = F_1 \frac{l_1}{l_2}$$

2. Consider a lazy tong mechanism [Fig. 1.21]. The joint A has a pin which is free to slide inside the vertical groove provided in the frame. The joint E has a torsional spring to keep the mechanism in equilibrium under the external force $\vec{F}$ applied at the hinge joint E.

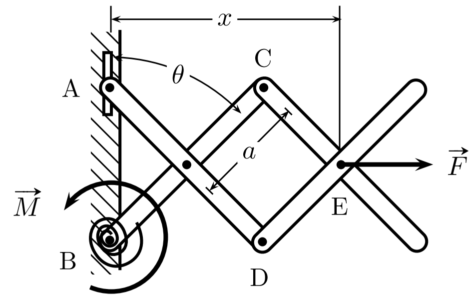


Figure 1.21 Lazy tong mechanism.

The magnitude of the moment M required to keep the mechanism in equilibrium can be determined using the method of virtual work. The horizontal distance x of joint E from the AB is

$$x = 3a\cos\theta$$

The virtual rotation of link BC for virtual displacement of δx of the joint E is given by

$$\delta x = -3a\sin\theta \times \delta\theta$$

The reactions at the joints A and B will not cause any work, the total virtual work done by moment M and external force F must be zero:

$$M\delta\theta + F\delta x = 0$$
$$M\delta\theta - F \times 3a\sin\theta \times \delta\theta = 0$$
$$M = 3Fa\sin\theta$$

In applying the method of virtual work, it is necessary to only calculate the displacements of the points of application of the forces, and hence a problem of equilibrium is converted into one of geometry, which is usually easier to solve. Also, forces whose points of application are not displaced, or the displacement is perpendicular to the force, need not be considered. The superiority of the method of virtual work is that the method eliminates all unknown reactions.

There is no added advantage of applying the principle of virtual work in equilibrium problems. Each application of the virtual work equation, leaves an equation that could have been directly obtained by simply applying the equation of equilibrium.

IMPORTANT FORMULAS

Force Resolution of a force

$$F_x = F\cos\theta$$

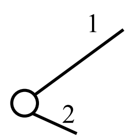
Moment

1. Moment of a force
$$\vec{M}_O = \vec{r}\times\vec{F}$$
$$M_O = rF\sin\theta$$
2. Varignon's theorem
$$\vec{M}_R = \sum\left(\vec{r}\times\vec{F}\right)$$
3. Two equal but opposite parallel forces having different lines of action form a couple.

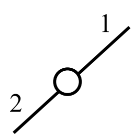
Equilibrium

1. Condition of equilibrium
$$\sum F = 0$$
$$\sum M = 0$$
2. Lami's theorem
$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

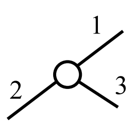
Structural Analysis



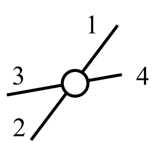
$F_1 = F_2 = 0$



$F_1 = F_2$



$F_1 = F_2$
$F_3 = 0$



$F_1 = F_2$
$F_3 = F_4$

Kinematics

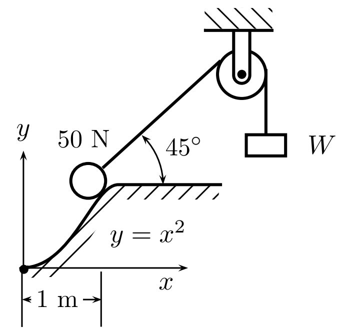
1. Velocity and acceleration
$$\vec{v} = \frac{d\vec{x}}{dt}, \quad \vec{a} = \frac{d^2\vec{x}}{dt^2}$$
2. Resultant velocity
$$w = \sqrt{u^2+v^2+2uv\cos\alpha}$$
3. Equations of linear motion
$$v = u+at$$
$$s = ut+\frac{1}{2}at^2$$
$$v^2 = u^2+2as$$
4. Angular motion
$$\omega = \frac{d\theta}{dt}$$
$$\alpha = \frac{d}{dt}\omega$$
$$\vec{v} = \vec{\omega}\times\vec{r}, \quad \vec{a} = \vec{\alpha}\times\vec{r}$$
5. Projectile:
$$t = 2\times\frac{u\sin\alpha}{g}$$
$$H = \frac{u^2\sin 2\alpha}{2g}$$
$$R = \frac{u^2\sin 2\alpha}{g}$$
R is maximum when $\alpha = \pi/4$.

Dynamics

1. Newton's laws of motion
   (a) First law of motion states that every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force.
   (b) For an object with a constant mass m, Newton's second law of motion states that the force $\vec{F}$ is the product of an object's mass and its acceleration $\vec{a}$:
   $$\vec{F} = m\times\vec{a}$$
   (c) Newton's third law of motion states that for every action (force) in nature there is an equal and opposite reaction.
2. Potential energy
   (a) Gravity
   $$W = mgh$$
   (b) Spring
   $$W = k\frac{x^2}{2}$$
   (c) Torsional spring
   $$W = \frac{1}{2}T\theta$$
3. Kinetic energy
$$T = \frac{1}{2}mv^2$$
4. Principle of work and energy
$$\sum U_{1-2} = \frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2$$
5. Conservation of energy
$$T_1+U_1 = T_2+U_2$$
6. D'Alembert's principle states that the net external force $\vec{F}$ actually acting on the body and the inertia force $\vec{F}_i$ together keep the body in a state of fictitious equilibrium:
$$\vec{F}+\vec{F}_i = 0$$
7. Impulse–momentum principle states that the component of resultant linear impulse along any direction is equal to change in the component of momentum in that direction.
8. Law of conservation of momentum states that the total momentum of a system of bodies remains unaltered by mutual action between them.
9. Coefficient of restitution
$$e = \frac{v_2-v_1}{u_2-u_1}$$
For elastic collisions, $e = 1$.
10. The principle of virtual work states that the work done on a rigid body or a system of rigid bodies in equilibrium is zero for any virtual displacements compatible with the constraints on the system.

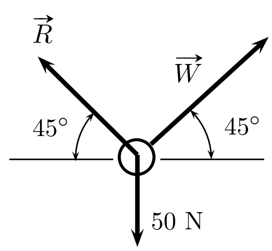
SOLVED EXAMPLES

1. A 50 N cylinder rests on the frictionless parabolic surface under support on an inextensible string over a pulley holding a weight W:



Determine the normal force that the weight exerts on the surface and the required weight W in Newton.

Solution. The cylinder is supported where $x = y = 1$, the parabolic surface makes angle 45° from horizontal. The reaction R is at angle 45°. Free body diagram of the cylinder is shown below:



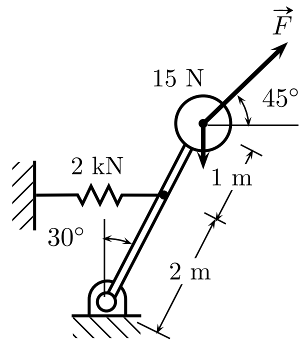
Taking horizontal and vertical equilibrium of the body

$$\begin{aligned} W\cos 45^\circ - R\cos 45^\circ &= 0 \\ W\sin 45^\circ + R\sin 45^\circ &= 50 \end{aligned}$$

Therefore,

$$R = W = 35.35 \text{ N}$$

2. Determine the force F required to maintain a spherical body of 15 N attached to a weightless beam which is supported by a spring of stiffness 2 kN/m and hinged at another end. The beam was initially vertical and the spring was unstretched.



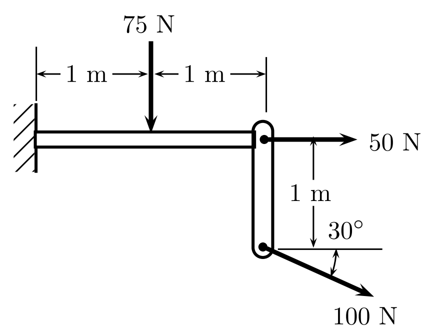
Solution. Taking moments of forces about the pin:

$$\begin{aligned} 3\sin 30^\circ \times (15 - F\sin 45^\circ) + 3\cos 30^\circ \times F\cos 45^\circ \\ = 200 \times 2\sin 30^\circ \times 2\cos 30^\circ \end{aligned}$$

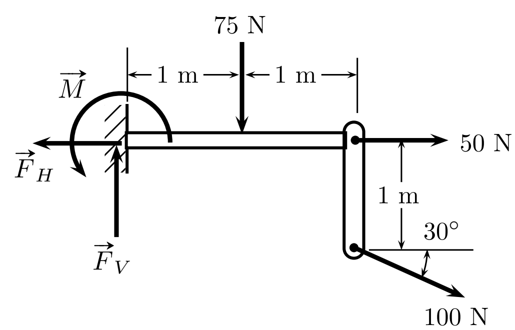
Solving for F,

$$\begin{aligned} 22.5 + 0.77 \times F &= 201.73 \\ F &= 232 \text{ N} \end{aligned}$$

3. Determine the horizontal and vertical components of the resultant forces and the resultant moment transferred to the support of the bracket shown in the figure:



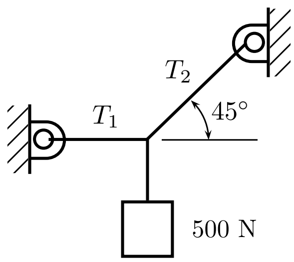
Solution. The reactions on the support are shown as follows:



The force components and the resultant moment are

$$
\begin{aligned}
F_H &= 50+100\cos 30^\circ \\
&= 136.6 \text{ N} \\
F_V &= 75+100\sin 30^\circ \\
&= 125 \text{ N} \\
M &= 75\times 1+50\times 0+100\sin 30^\circ\times 2-100\cos 30^\circ\times 1 \\
&= 88.39 \text{ Nm}
\end{aligned}
$$

4. Determine the tension in the cables, T_1 and T_2, for equilibrium of the 500 N body shown in the figure:



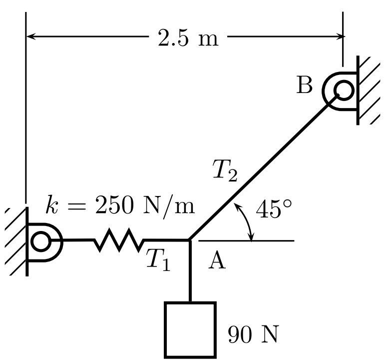
Solution. Applying Lami's theorem,

$$\frac{T_1}{\sin 135^\circ} = \frac{T_2}{\sin 90^\circ} = \frac{500}{\sin 135^\circ}$$

Therefore,

$$
\begin{aligned}
T_1 &= 500 \text{ N} \\
T_2 &= 707 \text{ N}
\end{aligned}
$$

5. Determine the length of cable AB to support the body of 90 N along with an spring of stiffness 250 N/m and unstretched length 0.8 m, as shown in the figure:



Solution. Given that $k = 250$ N/m. Applying Lami's theorem,

$$\frac{T_1}{\sin 135^\circ} = \frac{T_2}{\sin 90^\circ} = \frac{90}{\sin 135^\circ}$$

Therefore,

$$T_1 = 90 \text{ N}, \quad T_2 = 127.28 \text{ N}$$

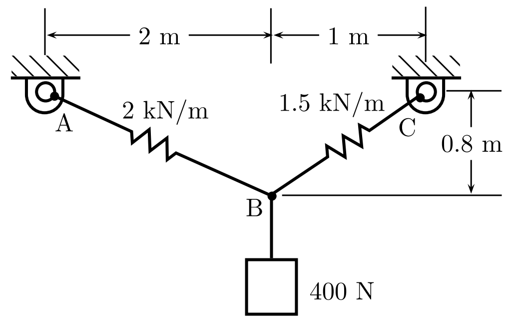
Extension in the spring is

$$
\begin{aligned}
\delta x &= \frac{T_1}{k} \\
&= 0.36 \text{ m}
\end{aligned}
$$

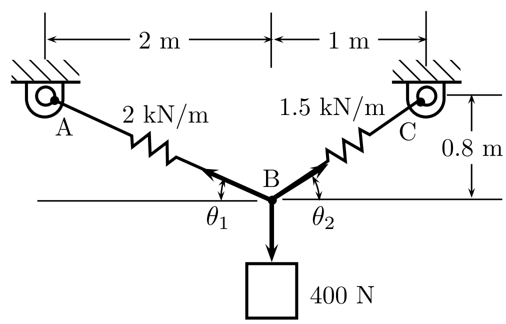
Therefore, the length of the cable AB is

$$
\begin{aligned}
AB\cos 45^\circ+0.8+0.36 &= 2.5 \\
AB &= 1.89 \text{ m}
\end{aligned}
$$

6. Determine the unstretched lengths of the two springs supporting a 400 N block between two supports at a span length of 2 m, as shown in the figure:



Solution. The configuration is redrawn, as shown:



The unknown angles, θ_1 and θ_2, can be determined as

$$
\begin{aligned}
\tan\theta_1 &= 0.8/2 \\
\theta_1 &= 21.80^\circ \\
\tan\theta_2 &= 0.8/1 \\
\theta_2 &= 38.65^\circ
\end{aligned}
$$

Applying Lami's theorem:

$$
\begin{aligned}
\frac{T_{AB}}{\sin(90^\circ+\theta_2)} &= \frac{T_{BC}}{\sin(90^\circ+\theta_1)} \\
&= \frac{400}{\sin(180^\circ-\theta_1-\theta_2)}
\end{aligned}
$$

$$\frac{T_{AB}}{\sin 128.65^\circ} = \frac{T_{BC}}{\sin 111.8^\circ} = \frac{400}{\sin 119.55^\circ}$$

Therefore,

$$T_{AB} = 359 \text{ N}$$
$$T_{BC} = 426.9 \text{ N}$$

Lengths of the stretched springs are

$$l_{AB} = 2/\cos\theta_1 = 2.15 \text{ m}$$
$$l_{BC} = 1/\cos\theta_2 = 1.28 \text{ m}$$

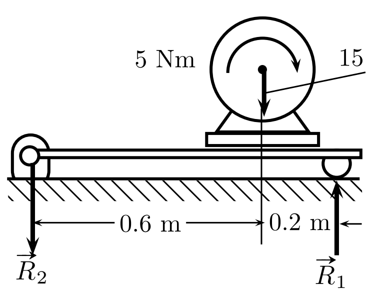
The extension in the springs are

$$\delta x_{AB} = \frac{T_{AB}}{k_{AB}} = \frac{359}{2\times 10^3} = 0.1795 \text{ m}$$
$$\delta x_{BC} = \frac{T_{BC}}{k_{BC}} = \frac{426.9}{1.5\times 10^3} = 0.2846 \text{ m}$$

Therefore, unstretched lengths of the springs are

$$l'_{AB} = 2.15 - 0.1795 = 1.97 \text{ m}$$
$$l'_{BC} = 1.28 - 0.2846 = 0.995 \text{ m}$$

7. A motor weighing 15 N is supported by a frame hinged at one and roller support at another end. The motor exerts a moment of 5 Nm in clock-wise direction:



Determine the support reactions $\vec{R}_1$ and $\vec{R}_2$ as shown in the figure.

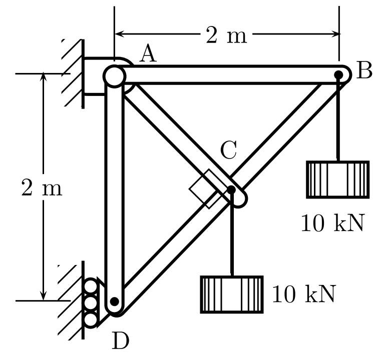
Solution. Taking moments about the hinged support,

$$R_1 \times 0.8 = 15\times 0.6 + 5$$
$$R_1 = 17.5 \text{ N}$$

Therefore, equilibrium in vertical direction is

$$R_1 - R_2 = 15$$
$$R_2 = 2.5 \text{ N}$$

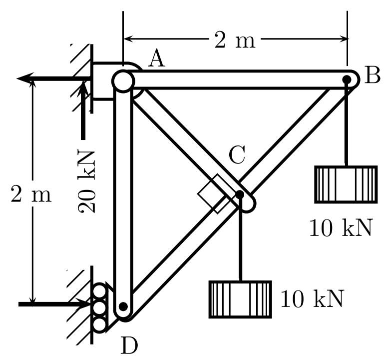
8. Determine the force in each member of the truss:



Solution. The reaction at roller support is determined by taking moments about the hinged point:

$$2\times R_D = 10\times 1 + 10\times 2$$
$$R_D = 15 \text{ kN}$$

Horizontal reaction at the hinged point will be $-R_D = -15$ kN, while the vertical reaction will be $10+10 = 20$ kN. The forces are shown in the figure:



Using method of joints,

(a) At point D

$$F_{BD}\sin 45^\circ = 15$$
$$F_{BD} = 21.21 \text{ kN (Compressive)}$$
$$F_{AD} = F_{BD}\cos 45^\circ = 15 \text{ kN (Tension)}$$

(b) At point B

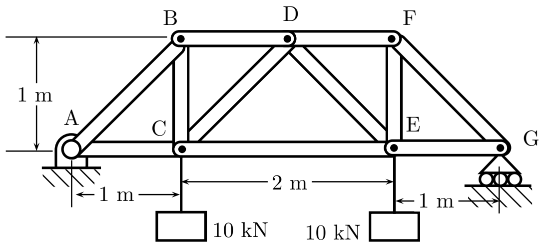
$$F_{BC}\sin 45^\circ = 10$$
$$F_{BC} = 14.14 \text{ kN (Compressive)}$$
$$F_{AB} = F_{BC}\cos 45^\circ = 10 \text{ kN (Compressive)}$$

(c) At point A

$$F_{AC}\cos 45^\circ + F_{AD} = 20$$
$$F_{AC} = 7.07 \text{ kN (Tensile)}$$

Thus, forces in all the members are determined.

9. Determine the force in each member of the truss:



Solution. The truss and loads are symmetric w.r.t. central axis. Therefore, forces on both sides of the central axis will be equal. Vertical reactions at supports A and G will be equal to 10 kN. Now,using method of joints,

(a) At point A

$$F_{AB}\cos 45^\circ = 10$$
$$F_{AB} = 14.14 \text{ kN (Compressive)}$$
$$F_{AC} = F_{AB}\cos 45^\circ$$
$$= 10 \text{ kN (Tensile)}$$

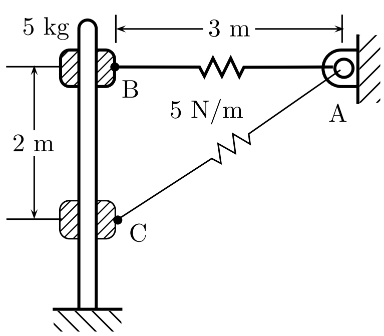
(b) At point B

$$F_{BC} = F_{AB}\cos 45^\circ$$
$$F_{BC} = 10 \text{ kN (Compressive)}$$
$$F_{BD} = F_{AB}\cos 45^\circ$$
$$= 10 \text{ kN (Tensile)}$$

(c) At point C

$$F_{CD}\cos 45^\circ = F_{BC} + 10$$
$$F_{CD} = 28.28 \text{ kN (Tensile)}$$
$$F_{BE} = F_{CD}\cos 45^\circ - F_{AC}$$
$$= 10 \text{ kN (Compressive)}$$

10. A frictionless 5 kg cast iron collar is attached to a spring having a stiffness 5 N/m and unstretched length $AB = 3$ m. The collar is released from point A and slides down on a stainless steel rod. What will be acceleration of the collar when it passes a point B?



Solution. Mass of the collar is $m = 5$ kg. The extension in the spring is

$$\delta = \sqrt{AB^2 + BC^2} - AB$$
$$= 0.6 \text{ m}$$

At point B, the spring exerts a force

$$F_s = 5 \times 0.6$$
$$= 3 \text{ N}$$

Inclination (θ) of CA given by

$$\theta = \tan^{-1}\left(\frac{2}{3}\right)$$
$$= 33.69^\circ$$

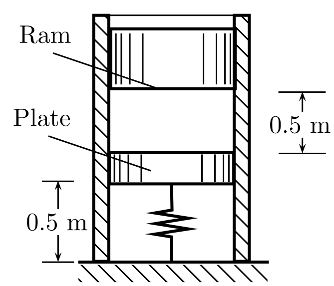
The net vertical force on the collar is

$$F = 5 \times 9.8 - 3\sin 33.69^\circ$$
$$= 47.33 \text{ N}$$

The acceleration of the collar is

$$a = \frac{F}{m}$$
$$= \frac{47.33}{5}$$
$$= 9.47 \text{ m/s}^2$$

11. A ram of 150 kg is released from rest 0.5 m over a plate of 5 kg which is supported by a spring of stiffness $k = 15$ kN/m. The spring is initially stretched by 98.1 mm. Determine the compression in the spring.

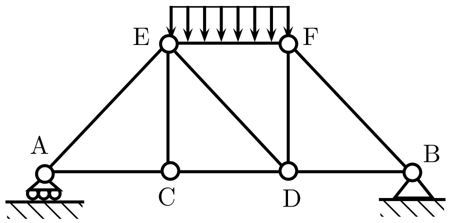


Solution. Using the principle of energy conservation

$$150 \times 9.81\,(0.5 + \delta) = \frac{15 \times 10^3}{2} \times \left[(0.0981 + \delta)^2 - 0.0981^2\right]$$
$$0.0981 + 0.1962\delta = \delta^2 + 2 \times 0.0981\delta$$
$$\delta = 0.31 \text{ m}$$

GATE PREVIOUS YEARS' QUESTIONS

1. A truss consists of horizontal members (AC, CD, DB and EF) and vertical members (CE and DE) having length l each. The members AE, DE and BF are inclined at 45° to the horizontal.

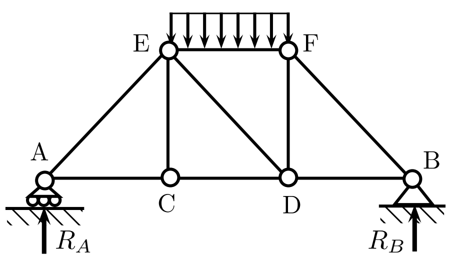


For a uniformly distributed load p per unit length on the member EF of the truss shown in figure, the force in the member CD is

(a) $pl/2$ (b) pl
(c) 0 (d) $2pl/3$

(GATE 2003)

Solution. Let R_A and R_B be the respective support reactions, as shown in the figure.



Using equilibrium equations,

$$R_A + R_B = p \times l$$
$$R_B \times 3l - pl \times \frac{3l}{2} = 0$$

Therefore,

$$R_A = \frac{pl}{2}$$

At point A, only vertical reaction is permitted:

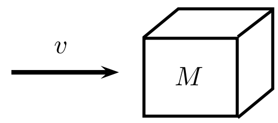
$$F_{AE} = \frac{R_A}{\sin 45°}$$
$$F_{AC} = -F_{AE}\cos 45° = -pl$$

As CE ⊥ AD, therefore,

$$F_{CE} = 0$$
$$F_{CD} = F_{AC} = -pl$$

Ans. (b)

2. A bullet of mass m travels at a very high velocity v (as shown in the figure) and gets embedded inside a block of mass M initially at rest on a rough horizontal floor. The block with the bullet is seen to move a distance s along the floor.



Assuming μ to be the coefficient to kinetic friction between the block and the floor and g the acceleration due to gravity, what is the velocity v of the bullet?

(a) $\dfrac{M+m}{m}\sqrt{2\mu g s}$ (b) $\dfrac{M-m}{m}\sqrt{2\mu g s}$

(c) $\mu\dfrac{M+m}{m}\sqrt{2gs}$ (d) $\dfrac{M}{m}\sqrt{2\mu g s}$

(GATE 2003)

Solution. Let u be the initial velocity of combined mass $(M+m)$. Using the principle of conservation of linear momentum,

$$mv = (M+m)\,u$$
$$u = \frac{m}{M+m}v$$

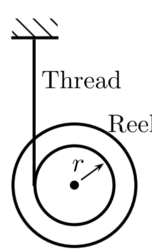
The frictional deceleration on the combined mass will be μg in horizontal direction, therefore,

$$0 - u^2 = -2\mu g s$$
$$u = \sqrt{2\mu g s}$$
$$v = \frac{M+m}{m}\sqrt{2\mu g s}$$

Ans. (a)

Linked Answer Questions

A reel of mass m and radius of gyration k is rolling down smoothly from rest with one end of the thread wound on it held in the ceiling as depicted in the figure. Consider the thickness of the thread and its mass negligible in comparison to radius r of the hub and the reel mass m. Symbol g represents the acceleration due to gravity.



3. The linear acceleration of the reel is

(a) $gr^2/\left(r^2+k^2\right)$ (b) $gk^2/\left(r^2+k^2\right)$
(c) $grk/\left(r^2+k^2\right)$ (d) $mgr^2/\left(r^2+k^2\right)$

(GATE 2003)

Solution. The motion consists of two parts. The first is translational, for which

$$mg-T=ma$$

The second is rotary, for which

$$T\times r=mk^2\times\frac{a}{r}$$

Eliminating T from above equations,

$$mg-\frac{mk^2a}{r^2}=ma$$
$$a=\frac{gr^2}{r^2+k^2}$$

Ans. (a)

4. The tension in the thread is

(a) $mgr^2/\left(r^2+k^2\right)$ (b) $mgrk/\left(r^2+k^2\right)$
(c) $mgk^2/\left(r^2+k^2\right)$ (d) $mg/\left(r^2+k^2\right)$

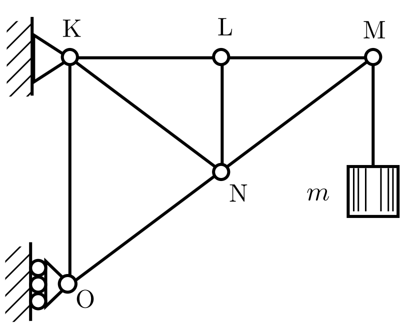
(GATE 2003)

Solution. Using d'Alembert's principle for the torsional equilibrium of the reel,

$$T\times r=mk^2\times\frac{a}{r}$$
$$T=\frac{mgk^2}{r^2+k^2}$$

Ans. (c)

5. The figure shows a pin-jointed plane truss loaded at the point M by hanging a mass of 100 kg.



The member LN of the truss is subjected to a load of

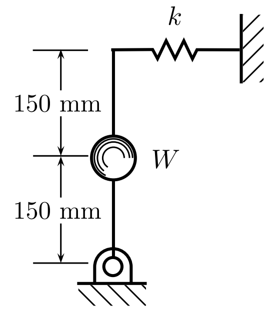
(a) 0 Newton
(b) 490 Newtons in compression
(c) 981 Newtons in compression
(d) 981 Newtons in tension

(GATE 2004)

Solution. There will be zero force in the perpendicular member.

Ans. (a)

6. A uniform stiff rod of length 300 mm and having a weight of 300 N is pivoted at one end and connected to a spring at the other end.



For keeping the rod vertical in a stable position, the minimum value of spring constant k needed is

(a) 300 N/m (b) 400 N/m
(c) 500 N/m (d) 1000 N/m

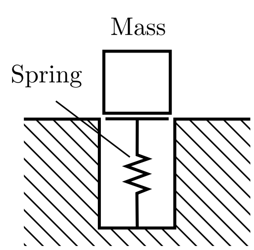
(GATE 2004)

Solution. For angular displacement of θ in anticlockwise direction, the equilibrium equation is

$$300\theta\times0.150=k\times0.3\theta\times0.3$$
$$k=\frac{300\times0.150}{0.3\times0.3}$$
$$=500\text{ N/m}$$

Ans. (c)

7. An ejector mechanism consists of a helical compression spring having a spring constant of $k=9.81\times10^3$ N/m. It is pre-compressed by 100 mm from its free state.



If it is used to eject a mass of 100 kg held on it, the mass will move up through a distance of

(a) 100 mm (b) 500 mm
(c) 981 mm (d) 1000 mm

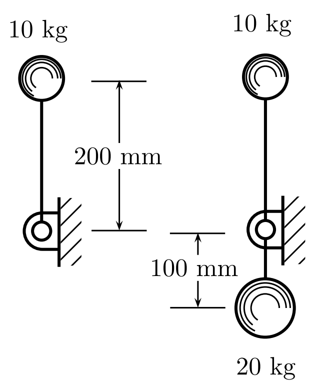
(GATE 2004)

Solution. To lift the mass (m) 100 kg by the spring of stiffness $k = 9.81\times10^3$ N/m, the compression (δ) in the spring will be

$$\begin{aligned}\delta &= \frac{mg}{k}\\ &= \frac{100\times9.81}{9.81\times10^3}\\ &= 0.1 \text{ m}\\ &= 100 \text{ mm}\end{aligned}$$

Ans. (a)

8. A rigid body shown in the Fig.(a) has a mass of 10 kg. It rotates with a uniform angular velocity ω. A balancing mass of 20 kg is attached as shown in Fig. (b).



(a) (b)

The percentage increase in mass moment of inertia as a result of this addition is

(a) 25% (b) 50%
(c) 100% (d) 200%

(GATE 2004)

Solution. Moment of inertia of a spherical solid body of mass m and radius r is given by

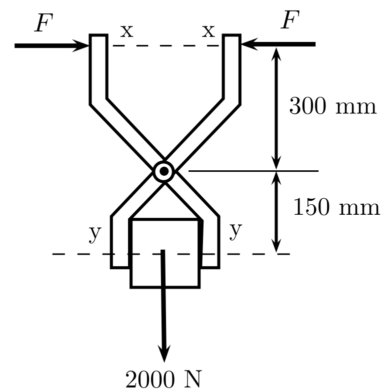
$$I = mr^2$$

Moment of inertia increases from 10×0.2^2 by 20×0.1^2. Therefore, percentage increase is

$$\begin{aligned}\frac{\Delta I}{I} &= \frac{20\times0.1^2}{10\times0.2^2}\times100\\ &= 50\%\end{aligned}$$

Ans. (b)

9. The figure shows a pair of pin-jointed gripper tongs holding an object weighing 2000 N. the coefficient of friction (μ) at the gripping surface is 0.1. xx is the line of action of the input force and yy is the line of application of gripping force.



If the pin joint is assumed to be frictionless, the magnitude of force F required to hold the weight is

(a) 1000 N (b) 2000 N
(c) 2500 N (d) 5000 N

(GATE 2004)

Solution. This reaction on both arms shall generate the net friction force to hold the weight 2000 N. Therefore,

$$\begin{aligned}2\times\mu R_y &= 2000\\ R_y &= \frac{3000}{2\times0.1}\\ &= 10000 \text{ N}\end{aligned}$$

The force F required at xx is

$$\begin{aligned}R_y &= F\times\frac{0.30}{0.15}\\ F &= \frac{R_y}{2}\\ &= 5000 \text{ N}\end{aligned}$$

Ans. (d)

10. The time variation of the position of a particle in rectilinear motion is given by $x = 2t^3+t^2+2t$. If v is the velocity and a is the acceleration of the particle in consistent units, the motion started with

(a) $v = 0$, $a = 0$
(b) $v = 2$, $a = 0$
(c) $v = 0$, $a = 2$
(d) $v = 2$, $a = 2$

(GATE 2005)

Solution. Given that

$$x = 2t^3 + t^2 + 2t$$

Velocity (v) and acceleration (a) are determined as

$$\begin{aligned} v &= \frac{dx}{dt} \\ &= 6t^2 + 2t + 2 \\ a &= \frac{dv}{dt} \\ &= 12t + 2 \end{aligned}$$

At the start ($t = 0$),

$$\begin{aligned} v &= 2 \\ a &= 2 \end{aligned}$$

Ans. (d)

11. Two books of mass 1 kg each are kept on a table, one over the other. The coefficient of friction on every pair of contacting surfaces is 0.3. the lower book is pulled with a horizontal force F. The minimum value of F for which slip occurs between the two books is

(a) zero (b) 1.06 N
(c) 5.74 N (d) 8.83 N

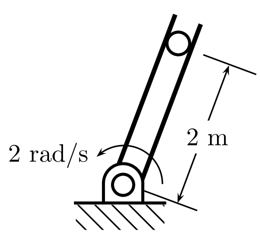
(GATE 2005)

Solution. The force must be more than the friction force exerted at top and bottom both, which is given by

$$\begin{aligned} F &= (1+2) \times 9.81 \times 0.3 \\ &= 8.829 \text{ N} \end{aligned}$$

Ans. (d)

12. A shell is fired from a cannon. At the instant the shell is just about to leave the barrel, its velocity relative to the barrel is 3 m/s, while the barrel is swinging upwards with a constant angular velocity of 2 rad/s.



The magnitude of the absolute velocity of the shell is

(a) 3 m/s (b) 4 m/s
(c) 5 m/s (d) 7 m/s

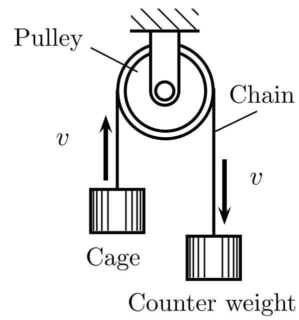
(GATE 2005)

Solution. The absolute velocity of shell is composed of its velocity relative to the cannon and absolute velocity of the end of the cannon. Given that the length of cannon is 2 m. Therefore, the magnitude of absolute velocity (m/s) will be

$$\begin{aligned} V &= \sqrt{3^2 + (2 \times 2)^2} \\ &= 5 \text{ m/s} \end{aligned}$$

Ans. (c)

13. An elevator (lift) consists of the elevator cage and a counter weight, of mass m each. The cage and the counterweight are connected by a chain that passes over a pulley. The pulley is coupled to a motor. It is desired that the elevator should have a maximum stopping time of t seconds from a peak speed v.



If the inertia of the pulley and the chain are neglected, the minimum power that the motor must have is

(a) $\dfrac{mv^2}{2}$ (b) $\dfrac{mv^2}{2t}$

(c) $\dfrac{mv^2}{t}$ (d) $\dfrac{2mv^2}{t}$

(GATE 2005)

Solution. The total kinetic energy of the mass system is

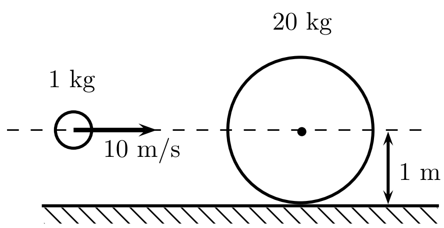
$$T = 2 \times \frac{mv^2}{2} = mv^2$$

Hence, the power requirements (in t seconds) is

$$\begin{aligned} P &= \frac{T}{t} \\ &= \frac{mv^2}{t} \end{aligned}$$

Ans. (c)

14. A 1 kg mass of clay, moving with a velocity of 10 m/s, strikes a stationary wheel and sticks to it. The solid wheel has a mass of 20 kg and a radius of 1 m.



Assuming that the wheel and the ground are both rigid and that the wheel is set into pure rolling motion, the angular velocity of the wheel immediately after the impact is, approximately,

(a) zero (b) 1/3 rad/s
(c) $\sqrt{10/3}$ rad/s (d) 10/3 rad/s

(GATE 2005)

Solution. The moment of inertia of wheel of mass m and radius r w.r.t. a normal axis passing through circumference is

$$I = \frac{mr^2}{2} + mr^2 = \frac{3}{2}mr^2$$

If ω is the velocity of pure rolling of wheel after the impact, total kinetic energy of both the masses before and after the impact will be equal, therefore,

$$\frac{1}{2} \times 1 \times 10^2 = \frac{1}{2}\left(\frac{3}{2} \times 20 \times 1^2\right)\omega^2$$
$$\omega = \sqrt{\frac{10}{3}}$$

Ans. (c)

15. A weighing machine consists of a 2 kg pan resting on a spring. In this condition, the length of the spring is 200 mm. When a mass of 20 kg is placed on the pan, the length of the spring becomes 100 mm. For the spring, the undeformed length l_0 and the spring constant k (stiffness) are

(a) $l_0 = 220$ mm, $k = 1862$ N/m
(b) $l_0 = 210$ mm, $k = 1960$ N/m
(c) $l_0 = 200$ mm, $k = 1862$ N/m
(d) $l_0 = 200$ mm, $k = 2156$ N/m

(GATE 2005)

Solution. For the load of 20 kg, the net deflection is $0.2 - 0.1 = 0.1$ m, therefore,

$$k = \frac{20 \times 9.18}{0.1} = 1962 \text{ N/m}$$

Also, if l_0 is the original length of the spring, for the second situation of $20 + 2 = 22$ kg load,

$$22 \times 9.81 = k\,(l_0 - 0.1)$$
$$l_0 = 0.210 \text{ m} = 210 \text{ mm}$$

Ans. (b)

16. If a system is in equilibrium and the position of the system depends upon many independent variables, the principle of virtual work states that the partial derivatives of its total potential energy with respect to each of the independent variables must be

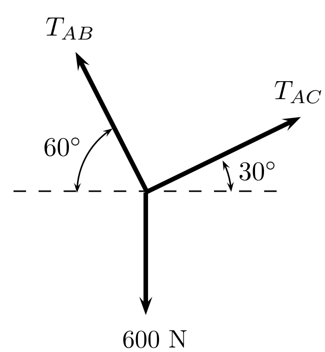
(a) -1.0 (b) 0
(c) 1.0 (d) ∞

(GATE 2006)

Solution. The principle of virtual work states that the work done on a rigid body or a system of rigid bodies in equilibrium is zero for any virtual displacements compatible with the constraints on the system. Conversely, if the virtual work for all such displacements is zero, then the body is in equilibrium.

Ans. (b)

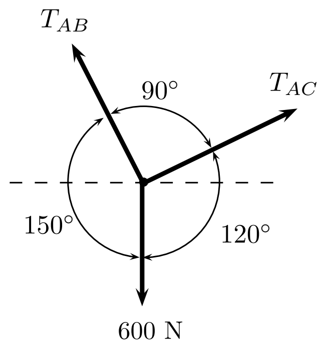
17. If point A is in equilibrium under the action of the applied forces, the values of tensions T_{AB} and T_{AC} are, respectively,



(a) 520 N and 300 N (b) 300 N and 520 N
(c) 450 N and 150 N (d) 150 N and 450 N

(GATE 2006)

Solution. The angles between the forces is shown in the figure.



Using Lami's theorem,

$$\frac{T_{AB}}{\sin(120^\circ)} = \frac{T_{AC}}{\sin(150^\circ)} = \frac{600}{\sin 90^\circ}$$

Therefore,

$$\begin{aligned} T_{AB} &= \frac{600}{\sin 90^\circ} \times \sin(120^\circ) \\ &= 520 \text{ N} \\ T_{AC} &= \frac{600}{\sin 90^\circ} \times \sin(150^\circ) \\ &= 300 \text{ N} \end{aligned}$$

Ans. (a)

18. During inelastic collision of two particles, which one of the following is conserved?

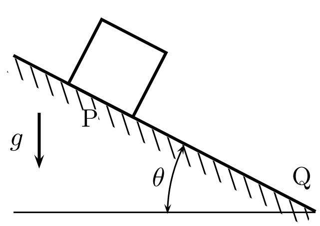
(a) total linear momentum only
(b) total kinetic energy only
(c) both linear momentum and kinetic energy
(d) neither linear momentum nor kinetic energy

(GATE 2007)

Solution. A perfectly elastic collision is defined as one in which there is no loss of kinetic energy in the collision. An inelastic collision is one in which part of the kinetic energy is changed to some other form of energy in the collision. Momentum is conserved in inelastic collisions, but one cannot track the kinetic energy through the collision since some of it is converted into other forms of energy.

Ans. (a)

19. A block of mass M is released from point P on a rough inclined plane with inclination angle θ, as shown in the figure. The coefficient of friction is μ.



If $\mu < \tan\theta$, then the time taken by the block to reach another point Q on the inclined plane, where PQ $= s$, is

(a) $\sqrt{2s/\left[g\cos\theta\,(\tan\theta - \mu)\right]}$
(b) $\sqrt{2s/\left[g\cos\theta\,(\tan\theta + \mu)\right]}$
(c) $\sqrt{2s/\left[g\sin\theta\,(\tan\theta - \mu)\right]}$
(d) $\sqrt{2s/\left[g\sin\theta\,(\tan\theta + \mu)\right]}$

(GATE 2007)

Solution. Using d'Alembert's principle along the inclined plane, for the equilibrium the net force on the mass should be zero, hence

$$\begin{aligned} Ma &= Mg\sin\theta - \mu Mg\cos\theta \\ a &= g\sin\theta - \mu g\cos\theta \\ &= g\cos\theta\,(\tan\theta - \mu) \end{aligned}$$

If t is time in seconds required to travel distance s, the second equation of motion provides for $u = 0$:

$$\begin{aligned} s &= \frac{1}{2}at^2 \\ t &= \sqrt{\frac{2s}{a}} \\ &= \sqrt{\frac{2s}{g\cos\theta\,(\tan\theta - \mu)}} \end{aligned}$$

Ans. (a)

20. A straight rod of length $L(t)$, hinged at one end and freely extensible at the other end, rotates through an angle $\theta(t)$ about the hinge. At time t, $L(t) = 1$ m, $\dot{L}(t) = 1$ m/s, $\theta(t) = \pi/4$ rad and $\dot{\theta}(t) = 1$ rad/s. The magnitude of the velocity at the other end of the rod is

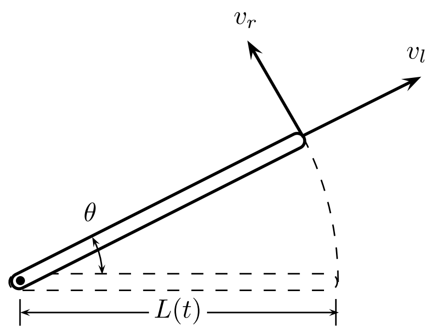
(a) 1 m/s
(b) $\sqrt{2}$ m/s
(c) $\sqrt{3}$ m/s
(d) 2 m/s

(GATE 2008)

Solution. The longitudinal and radial velocities at the free end at time t are, respectively,

$$\begin{aligned} v_l &= \dot{L}(t) \\ &= 1 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v_r &= L(t)\dot{\theta}(t) \\ &= 1 \times 1 \\ &= 1 \text{ m/s} \end{aligned}$$

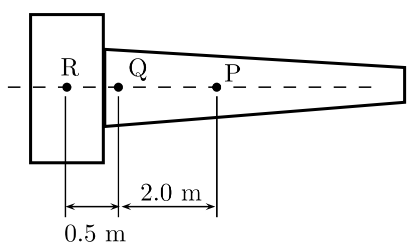


Net velocity at the free end is

$$\begin{aligned} v &= \sqrt{{v_r}^2 + {v_l}^2} \\ &= \sqrt{1^2 + 1^2} \\ &= \sqrt{2} \text{ m/s} \end{aligned}$$

Ans. (b)

21. A cantilever type gate hinged at Q is shown in the figure. P and R are the centers of gravity of the cantilever part and the counterweight respectively. The mass of the cantilever part is 75 kg.



The mass of the counterweight, for static balance, is

(a) 75 kg (b) 150 kg
(c) 225 kg (d) 300 kg

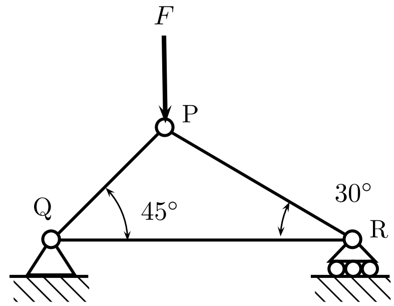
(GATE 2008)

Solution. For equilibrium, moments of forces about Q will be zero. Therefore, the mass of counter weight m_R is given by

$$\begin{aligned} m_R \times 0.5 &= 75 \times 2 \\ m_R &= 300 \text{ kg} \end{aligned}$$

Ans. (d)

22. Consider a truss PQR loaded at P with a force F as shown in the figure.



The tension in the member QR is

(a) $0.5F$ (b) $0.63F$
(c) $0.73F$ (d) $0.87F$

(GATE 2008)

Solution. For equilibrium at P in horizontal and vertical directions,

$$\begin{aligned} T_{PQ} \cos 45^\circ + T_{PR} \cos 60^\circ + F &= 0 \\ T_{PQ} \sin 45^\circ - T_{PR} \sin 60^\circ &= 0 \end{aligned}$$

Solving the above equations,

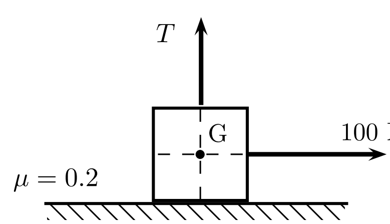
$$\begin{aligned} T_{PQ} &= 1.2247 \times T_{PR} \\ 1.36 \times T_{PR} &= -F \\ T_{PR} &= -F \times 0.7338 \end{aligned}$$

Similarly, at Q

$$\begin{aligned} T_{QR} &= T_{PQ} \times \cos 45^\circ \\ &= 1.2247 \times T_{PR} \times \cos 45^\circ \\ &= 1.2247 \times F \times 0.7338 \times \cos 45^\circ \\ &= 0.6354F \end{aligned}$$

Ans. (b)

23. A block weighing 981 N is resting on a horizontal surface. The coefficient of friction between the block and the horizontal surface is $\mu = 0.2$. A vertical cable attached to the block provides partial support as shown. A man can pull horizontally with a force of 100 N.



What will be the tension, T (in N) in the cable if the man is just able to move the block to the right?

(a) 176.2 (b) 196.0
(c) 481.0 (d) 981.0

(GATE 2009)

Solution. Given that

$$W = 981 \text{ N}$$
$$\mu = 0.2$$
$$F = 100 \text{ N}$$

Normal reaction at the contact surface is

$$R_n = W - T$$

For the equilibrium in horizontal direction,

$$\begin{aligned} F &= \mu R_n \\ &= \mu (W - T) \\ T &= W - \frac{F}{\mu} \\ &= 981 - \frac{100}{0.2} \\ &= 481.0 \text{ N} \end{aligned}$$

Ans. (c)

24. The coefficient of restitution of a perfectly inelastic (plastic) impact is

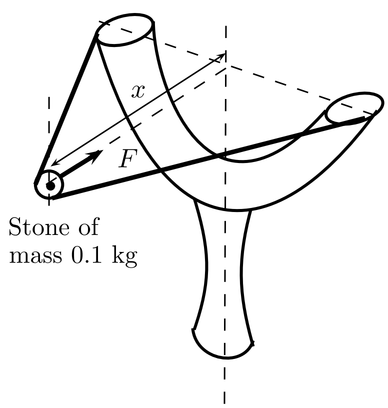
(a) 0 (b) 1
(c) 2 (d) ∞

(GATE 2011)

Solution. Coefficient of restitution is the ratio of relative velocities of separation and approach. It is zero for perfectly plastic impact.

Ans. (a)

25. A stone with mass of 0.1 kg is catapulted as shown in the figure. The total force F_x (in N) exerted by the rubber band as a function of distance x (in m) is given by $F_x = 300x^2$.



If the stone is displaced by 0.1 m from the unstretched position ($x = 0$) of the rubber band, the energy stored in the rubber band is

(a) 0.01 J (b) 0.1 J
(c) 1 J (d) 10 J

(GATE 2011)

Solution. The energy stored (E) in the rubber band for small stretching of δx is given by

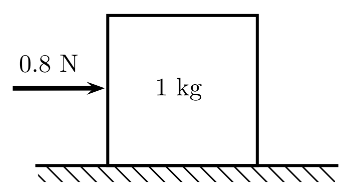
$$\delta E = F_x \delta x$$

Therefore, energy for stretching of 0.1 m is

$$\begin{aligned} E &= \int_0^{0.1} 300x^2 dx \\ &= \left[100x^3\right]_0^{0.1} \\ &= 0.1 \text{ J} \end{aligned}$$

Ans. (b)

26. A 1 kg block is resting on a surface with coefficient of friction $\mu = 0.1$. A force of 0.8 N is applied to the block, as shown in the figure.



The friction force is

(a) 0 (b) 0.8 N
(c) 0.98 N (d) 1.2 N

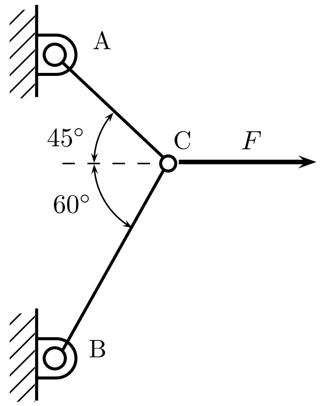
(GATE 2011)

Solution. The friction force will be equal to applied force of 0.8 N because the applied force is less than the limiting friction force is $1 \times 9.81 \times 0.1 = 0.98$ N.

Ans. (b)

Linked Answer Questions

Two steel truss members AC and BC, each having cross-sectional area of 100 mm^2, are subjected to be horizontal force F as shown in the figure. All joints are hinged.



27. The maximum force F in kN that can be applied at C such that the axial stress in any of the truss members DOES NOT exceed 100 MPa is

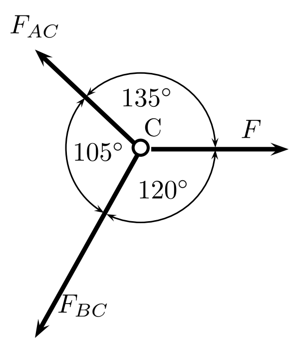
(a) 8.17 (b) 11.15
(c) 14.14 (d) 22.30

(GATE 2012)

Solution. Due to force F, maximum force will be generated in AC because it is less inclined to F than BC. Maximum allowable force in AC is

$$F_{AC} = 100 \times 100 = 10000 \text{ N}$$

The inclination of forces is shown in the figure.



Therefore, using triangle of forces in equilibrium,

$$\frac{F}{\sin 105°} = \frac{F_{AC}}{\sin 120°}$$

$$F = F_{AC} \times \frac{\sin 105°}{\sin 120°} = 11.1536 \text{ kN}$$

Ans. (b)

28. If $F = 1$ kN, the magnitude of the vertical reaction force developed at the point B in kN is

(a) 0.63 (b) 0.32
(c) 1.26 (d) 1.46

(GATE 2012)

Solution. Given that

$$F = 1 \text{ kN}$$

Using Lami's equation,

$$\frac{F_{BC}}{\sin 135°} = \frac{F}{\sin 105°}$$

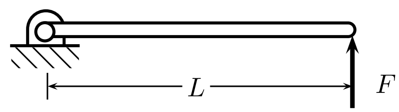
$$F_{BC} = 0.732 \text{ kN}$$

The vertical reaction at B is given by

$$R_B \uparrow = F_{BC} \cos 30° = 0.633975 \text{ kN}$$

Ans. (a)

29. A pin-jointed uniform rigid rod of weight W and length L is supported horizontally by an external force F as shown in the figure.



The force F is suddenly removed. At the instance force removal, the magnitude of vertical reaction developed at the support is

(a) zero (b) $W/4$
(c) $W/2$ (d) W

(GATE 2013)

Solution. Moment of inertia of the rod about the support is

$$I = \frac{mL^2}{3}$$

When support is suddenly removed, weight W $(= mg)$ will exert a torque about the pivot which will be balanced by the inertia I and angular acceleration α:

$$W \times \frac{L}{2} = I \times \alpha$$

$$\alpha = \frac{WL}{2I} = \frac{3mgL}{2 \times mL^2} = \frac{3g}{2L}$$

Using d'Alembert's principle, if R is the vertical reaction at pivot, for equilibrium

$$mg = R + \alpha \times \frac{L}{2} \times m$$

$$R = mg - \frac{3g}{2L} \times \frac{L}{2} \times m = mg\left(1 - \frac{3}{4}\right) = \frac{W}{4}$$

Ans. (b)

MULTIPLE CHOICE QUESTIONS

1. According to Varignon's theorem, the algebraic sum of moments of the two forces about any point in their plane is equal to

(a) zero
(b) the moments of the maximum force about the same point
(c) the moments of the minimum force about the same point
(d) the moments of their resultant about the same point

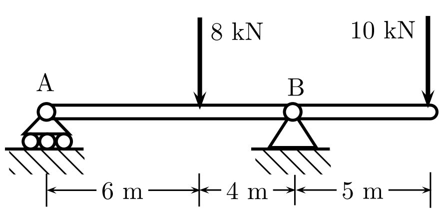
2. According to the principle of moments, if a system of coplanar forces be in equilibrium, then the algebraic sum of their moments about any point in their plane is

(a) zero
(b) the moments of the maximum force about the same point
(c) the moments of the minimum force about the same point
(d) indeterminate

3. Two forces act at a point. The first force has x and y components of 3 N and -5 N, respectively. The resultant of these forces falls on the x-axis and has a magnitude of -4 N. The x and y components of the second force are

(a) -7, 5 (b) -7, -5
(c) -7, 0 (d) 7, 0

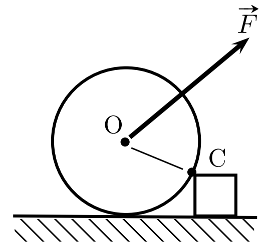
4. A beam is loaded as shown in the figure.



The reaction in kN at the support A is (upward +ve)

(a) 18 (b) -1.8
(c) 1.8 (d) -0.8

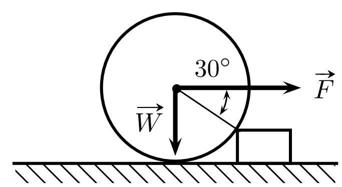
5. The road roller shown in the given figure is being moved over an obstacle by a pull F.



The magnitude of $\vec{F}$ required will be the minimum when it is

(a) horizontal
(b) vertical
(c) at 45° to the horizontal
(d) perpendicular to the line CO

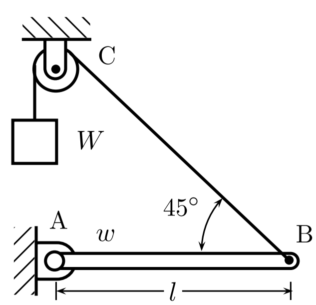
6. A roller of weight W is rolled over the wooden block shown in the given figure.



The pull F required to just cause the said motion is

(a) $W/2$ (b) W
(c) $\sqrt{3}W$ (d) $2W$

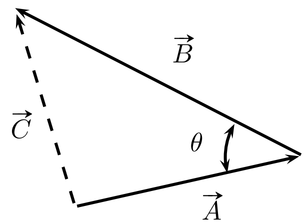
7. A uniform heavy rod AB of length l and weight w is hinged at A and tied to a weight W by a string at B. The massless string passes over a frictionless and small pulley at C.



If the rod is in equilibrium at horizontal configuration, then W will be equal to

(a) w (b) $w/2$
(c) $\sqrt{2}w$ (d) $w/\sqrt{2}$

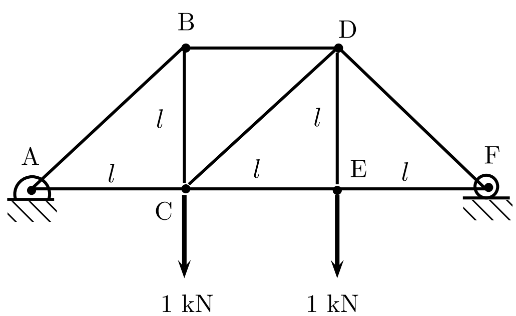
8. The vectors of two forces $\vec{A}$ and $\vec{B}$ are at angle θ as shown in the figure.



The magnitude of the resultant force $\vec{C}$ of these two forces will be given by

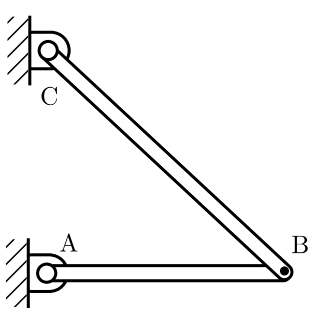
(a) $C^2 = A^2 + B^2 + 2AB\cos\theta$
(b) $C^2 = A^2 + B^2 - 2AB\cos\theta$
(c) $C^2 = A^2 + B^2 + 2AB\sin\theta$
(d) $C^2 = A^2 + B^2 - 2AB\sin\theta$

9. For the loading on truss shown in the figure, the force in member CD is



(a) 0 kN (b) 1 kN
(c) $\sqrt{2}$ kN (d) $1/\sqrt{2}$ kN

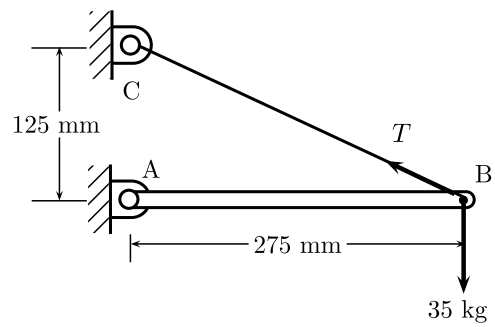
10. Bars AB and BC, each of negligible mass, support load P as shown in the figure.



In this arrangement,

(a) bar AB is subjected to bending but bar BC is not subjected to bending.
(b) bar AB is not subjected to bending but bar BC is subjected to bending
(c) neither bar AB nor bar BC is subjected to bending
(d) both bars AB and BC are subjected to bending

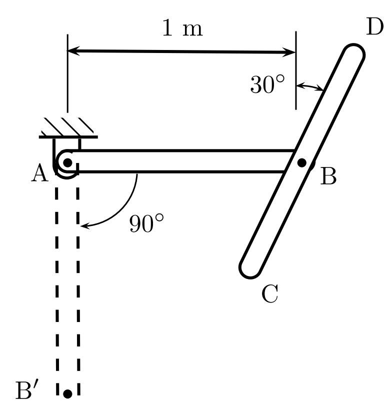
11. A mass of 35 kg is suspended from a weightless bar AC, which is supported by a cable CB and a pin at A as shown in the figure.



The pin reactions at A on the bar AB are

(a) $R_x = 343.4$ N, $R_y = 755.4$ N
(b) $R_x = 343.4$ N, $R_y = 0$ N
(c) $R_x = 755.4$ N, $R_y = 343.4$ N
(d) $R_x = 755.4$ N, $R_y = 0$ N

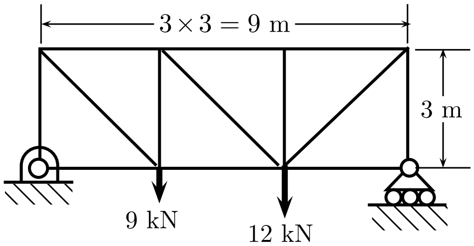
12. AB and CD two uniform and identical bars of mass 10 kg each, as shown in the figure. The hinged joint B is the centroid of the bar CD. The hinges at A and B are frictionless. The assembly is released from rest and motion occurs in the vertical plane.



At the instant that the hinge B passes the point B′, the angle between the two bars will be

(a) 60° (b) 37.4°
(c) 30° (d) 45°

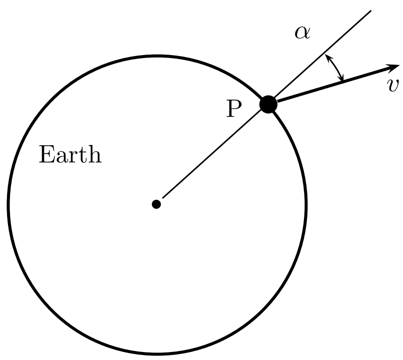
13. A truss of span length 9 m is loaded as shown in the figure.



The number of the force members carrying zero force is

(a) 1 (b) 2
(c) 3 (d) 4

14. A particle P is projected from the earth's surface at latitude 45° with escape velocity $v = 11.19$ m/s. The velocity vector makes an angle α with the local vertical.



The particle will escape the earth's gravitational field

(a) only when $\alpha = 0°$
(b) only when $\alpha = 45°$
(c) only when $\alpha = 90°$
(d) irrespective of value of α

15. A car moving with uniform acceleration covers 450 m in a 5 second interval, and covers 700 m in the next 5 second interval. The acceleration of the car is

(a) 7 m/s^2 (b) 10 m/s^2
(c) 25 m/s^2 (d) 50 m/s^2

16. A person carrying on his hand a jewelery box of weight w jumped down from the third floor of a building. Before touching the ground, he would feel a load of magnitude

(a) zero (b) $w/2$
(c) w (d) infinity

17. If a body is moving with initial velocity u and uniform acceleration a, then the distance traveled by the body in tth second is given by

(a) $(u+a)(1-2t)/2$
(b) $(u+a)(t-2)/2$
(c) $u+a(2t-1)/2$
(d) $u+a(t-1)/2$

18. The equation of motion of a body is given by

$$s = 2t^3 + 3t^2 + 7$$

where s is in meter and t is in seconds. Starting from rest, it will travel in 2 s a distance of

(a) 35 m (b) 28 m
(c) 27 m (d) 20 m

19. The motion of a particle (distance in meters and time in seconds) is given by the equation

$$s = 2t^3 + 3t$$

Starting from $t = 0$, to attain a velocity of 9 m/s, the particle will have to travel a distance of

(a) 5 m (b) 10 m
(c) 15 m (d) 20 m

20. A bullet is fired vertically upward from a rifle, with a velocity of 110 m/s from the top of a 115 m high tower. If $g = 10$ m/s^2, the velocity with which the bullet will strike the ground is

(a) 220 m/s (b) 175 m/s
(c) 120 m/s (d) 115 m/s

21. A ball is projected vertically upward with a certain velocity. It takes 40 s for its upward journey. The time taken for its downward journey is

(a) 10 s (b) 20 s
(c) 30 s (d) 40 s

22. An electric lift is moving downwards with an acceleration of $g/3$. The vertical force between the passenger in the lift and its floor is equal to

(a) 2/3 of the passenger's weight
(b) 3/4 of the passenger's weight
(c) the passenger's weight
(d) 4/3 of the passenger's weight

23. The position of a particle at any instant t is given by

$$r = A\left(e^{\alpha t}\hat{i} + e^{-\alpha t}\hat{j}\right)$$

where α is a constant. What will be the velocity of the particle at this instant?

(a) $A\alpha\left(e^{-\alpha t}\hat{i}+e^{\alpha t}\hat{j}\right)$

(b) $A\alpha\left(e^{-\alpha t}\hat{i}-e^{\alpha t}\hat{j}\right)$

(c) $A\alpha\left(e^{\alpha t}\hat{i}+e^{-\alpha t}\hat{j}\right)$

(d) $A\alpha\left(e^{\alpha t}\hat{i}-e^{-\alpha t}\hat{j}\right)$

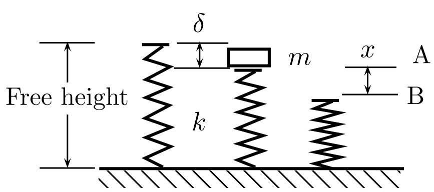
24. Two balls of the same size but different weight fall from the same height. Neglecting the air resistance, which ball reaches the earth's surface first?

(a) lighter one
(b) heavier one
(c) both at same time
(d) cannot be determined

25. According to the principle of virtual work, , the work done on a rigid body or a system of rigid bodies for any virtual displacements compatible with the constraints on the system in equilibrium is

(a) zero
(b) maximum
(c) minimum
(d) none of the above

26. In the figure shown, the spring deflects by δ to position A (the equilibrium position) when a mass m is kept on it.



During free vibration, the mass is at position B at some instant. The change in potential energy of the spring-mass system from position A to B is

(a) $kx^2/2$ (b) $kx^2/2-mgx$
(c) $k(x+\delta)^2/2$ (d) $kx^2/2+mgx$

27. Steel wheel of 600 mm diameter rolls on a horizontal steel rail. It carries a load of 500 N. The coefficient of rolling resistance is 0.3. The force in N, necessary to roll the wheel along the rail is

(a) 0.5 (b) 5
(c) 15 (d) 150

28. A spring of stiffness k is extended from a displacement x_1 to x_2. The work done by the spring is

(a) $k\left(x_1^2-x_2^2\right)/2$ (b) $k\left(x_1-x_2\right)^2/2$
(c) $k\left(x_1+x_2\right)^2/2$ (d) $k\left(x_1-x_2\right)^2/4$

29. In the case of a flywheel of mass moment of inertia I, rotating at an angular velocity ω, the expression $I\omega^2/2$ represents the

(a) centrifugal force
(b) angular momentum
(c) torque
(d) kinetic energy

30. The power required by a machine having an efficiency of 80% for raising a load of 24 N through a distance of 36 m in 1 min is

(a) 12 W (b) 18 W
(c) 50 W (d) 450 W

31. A mechanical system can be said to be conservative if the

(a) potential energy of the system remains constant
(b) kinetic energy of the system remains constant
(c) sum of the kinetic and potential energies remains constant
(d) linear momentum remains constant

32. The moment of inertia of flywheel is 2000 kg.m^2. Starting from rest, it is moving with a uniform acceleration of 0.5 rad/s^2. After 10 s from the start, its kinetic energy will be

(a) 250 Nm (b) 500 Nm
(c) 5000 Nm (d) 25000 Nm

33. A man is drawing water from a well with the help of a bucket which leaks uniformly. The bucket weighs 200 N when full, and 100 N when water leaks out by the time is arrives at the top. Water is available in the well at a depth of 10 m. The work done by in drawing the water from the well is

(a) 1000 J (b) 1500 J
(c) 2000 J (d) 3000 J

34. The springs of a chest expander are 60 cm long when unstretched. Their stiffness is 10 N/mm. The work done in stretching them to 100 cm is

(a) 600 Nm (b) 800 Nm
(c) 1000 Nm (d) 1600 Nm

35. A spring of stiffness 1000 N/m is stretched initially by 10 cm from the undeformed position. The work required to stretch it by another 10 cm is

(a) 5 Nm (b) 7 Nm
(c) 10 Nm (d) 15 Nm

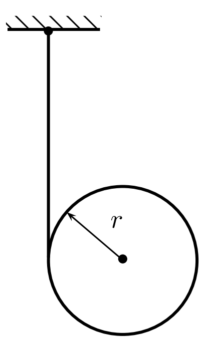
36. A truck weighing 150 kN and traveling at 2 m/s collides with a buffer spring which compresses 1.25 cm per 10 kN. The maximum compression in the spring is

(a) 20.00 cm (b) 22.85 cm
(c) 27.66 cm (d) 30.00 cm

37. A wheel of centroidal radius of gyration k is rolling on a horizontal surface with constant velocity. It comes across an obstruction of height h. Because of its rolling speed (v), it just overcomes the obstruction. To determine v, one should use the principle of conservation of

(a) energy
(b) linear momentum
(c) energy and linear momentum
(d) energy and angular momentum

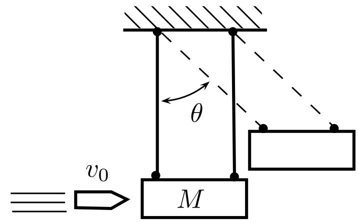
38. A cord is wrapped around a cylinder of radius r and mass m as shown in the figure.



If the cylinder is released from rest, the velocity of the cylinder after it has moved through a distance h will be

(a) $\sqrt{2gh}$ (b) $\sqrt{gh}$
(c) $\sqrt{4gh/3}$ (d) $\sqrt{gh/3}$

39. A simple way to measure the speed of a bullet is with a ballistic pendulum as illustrated in the figure. It consists of a wooden block of mass M into which the bullet is shot.



The block is suspended from cables of length l and the impact of the bullet causes it to swing through a maximum angle θ. The initial speed of the bullet can be expressed as

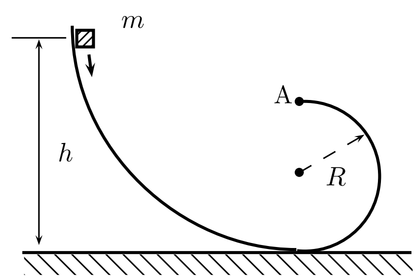
(a) $\sqrt{2\left(\dfrac{M+m}{m}\right) g\,(1-\sin\theta)\,l}$

(b) $\sqrt{2\left(\dfrac{M+m}{m}\right) g\,(1+\sin\theta)\,l}$

(c) $\sqrt{2\left(\dfrac{M+m}{m}\right) g\,(1-\cos\theta)\,l}$

(d) $\sqrt{2\left(\dfrac{M+m}{m}\right) g\,(1+\cos\theta)\,l}$

40. A small block of mass m starts from rest and slides along a frictionless loop as shown in the figure.



What should be the initial height h so that the mass pushes against the top of the track (point A) with a force equal to its weight.

(a) R (b) $2R$
(c) $3R$ (d) $4R$

41. According to d'Alembert's principle, the external forces acting on a body is equilibrium with

(a) the resultant inertia forces on the body
(b) the weight of the body
(c) both (a) and (b)
(d) none of the above

42. If a constant force F acts on a body of mass m for time t and changes its velocity from u to v under

an acceleration a, all in the same direction, then for the equilibrium of the body

(a) $F = mu/t$
(b) $F = mv/t$
(c) $F = m(v-u)/t$
(d) none of the above

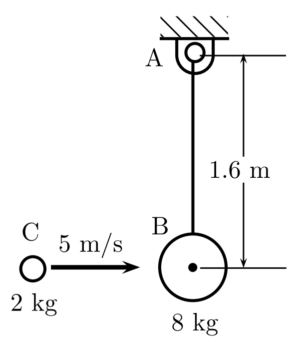
43. A perfectly elastic collision is defined as one in which

(a) there is no loss of kinetic energy
(b) coefficient of restitution is zero
(c) coefficient of restitution is more than unity
(d) none of the above

44. A plastic or inelastic collision is one in which

(a) there is no loss of kinetic energy
(b) coefficient of restitution is zero
(c) coefficient of restitution is unity
(d) a part of the kinetic energy is changed to some other form of energy in the collision

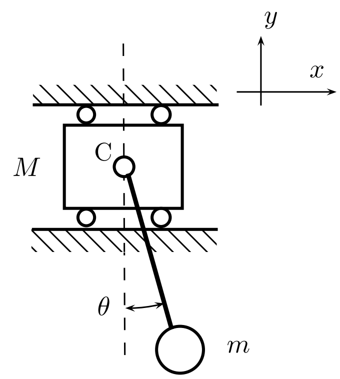
45. The 2 kg mass C moving horizontally to the right, with a velocity of 5 m/s, strikes the 8 kg mass B at the lower end of the rigid massless rod AB. The rod is suspended from a frictionless hinge at A and is initially at rest.



If the coefficient of restitution between mass C and mass B is one, determine the angular velocity of the rod AB immediately after impact.

(a) 1.0 rad/s (b) 1.25 rad/s
(c) 2.0 rad/s (d) 2.15 rad/s

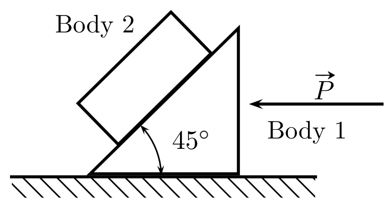
46. Mass M slides in a frictionless slot in the horizontal direction and the bob of mass m is hinged to mass M at C, through a rigid massless rod. This system is released from rest with $\theta = 30°$.



At the instant when $\theta = 0°$, the velocities of m and M can be determined using the fact that, for the system (i.e., m and M together),

(a) the linear momentum in x and y directions are conserved but the energy is not conserved
(b) the linear momentum in x and y directions are conserved and the energy is also conserved
(c) the linear momentum in x direction is conserved and the energy is also conserved
(d) the linear momentum in y direction is conserved and the energy is also conserved

47. Two bodies 1 and 2 shown in the figure have equal mass m. All the surfaces are smooth.



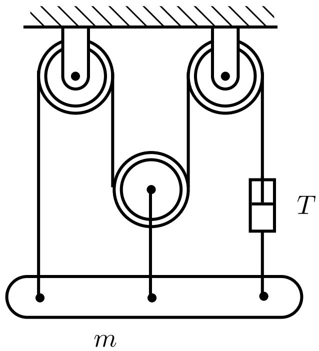
The value of force P required to prevent sliding of body 2 on body 1 is

(a) $2mg$ (b) $\sqrt{2}mg$
(c) $2\sqrt{2}mg$ (d) mg

48. A ball A of mass m falls under gravity from a height h and strikes another ball B of mass m which is supported at rest on a spring of stiffness k. Assume perfectly elastic impact. Immediately after the impact,

(a) the velocity of ball A is $0.5\sqrt{2gh}$
(b) the velocity of ball A is zero
(c) the velocity of both ball is $0.5\sqrt{2gh}$
(d) None of the above

49. A spring scale indicates a tension T in the right hand cable of the pulley system shown in the figure. Neglecting the mass of the pulleys and ignoring friction between the cable and pulley, the mass m is



(a) $2T/g$
(b) $T\left(1+e^{4\pi}\right)/g$
(c) $4T/g$
(d) None of the above

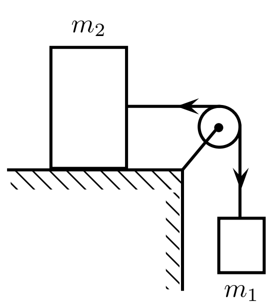
50. A stone of mass m at the end of a string of length l is whirled in a vertical circle at a constant speed. The tension in the string will be maximum when the stone is

(a) at the top of the circle
(b) half-way down from the top
(c) quarter was down from the top
(d) at the bottom of the circle

51. A shell is fired from a cannon with a speed v at an angle θ with the horizontal direction. At the highest point in its path, it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon. The speed of other piece immediately after explosion is

(a) $3v\cos\theta$ (b) $2v\cos\theta$
(c) $3v\cos\theta/2$ (d) $\sqrt{3/2}\times v\cos\theta$

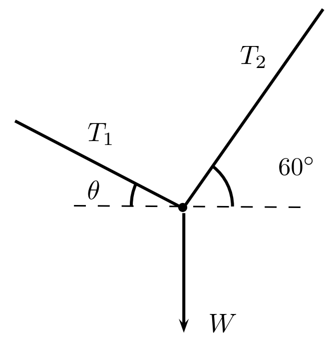
52. In the given figure, two bodies of masses m_1 and m_2 are connected by a light inextensible string passing over a smooth pulley. Mass m_2 lies on a smooth horizontal plane.



When mass m_1 moves downwards, the acceleration of the two bodies in m/s^2 is equal to

(a) $m_1g/(m_1+m_2)$ (b) $m_2g/(m_1-m_2)$
(c) $m_2g/(m_1+m_2)$ (d) $m_1g/(m_1-m_2)$

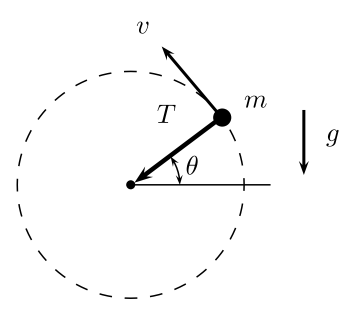
53. A weight W is supported by two cables as shown in the given figure.



The tension in the cable making angle θ will be the minimum when the value of θ is
(a) 0° (b) 30°
(c) 45° (d) 60°

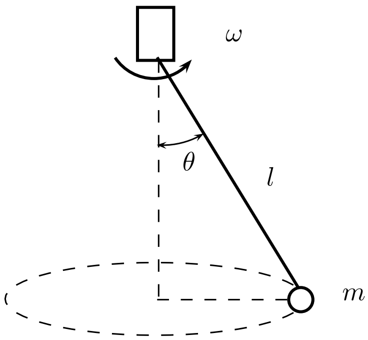
54. An elevator weighing 10000 N attains an upward velocity of 4 m/s in 2 seconds with uniform acceleration. Then what is the tension in the wire rope?
(a) 8000 N (b) 5000 N
(c) 2500 N (d) 12000 N

55. Mass m is whirled on the end of a string of length l. The motion is in a vertical plane in the gravitational field of the earth as shown in the figure.



What will be tension in the string if it is making angle θ from horizontal?
(a) $m\left(v^2/l-g\sin\theta\right)$ (b) $m\left(v^2/l+g\sin\theta\right)$
(c) $m\left(v^2/l-g\cos\theta\right)$ (d) $m\left(v^2/l+g\cos\theta\right)$

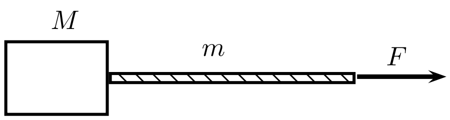
56. Mass m hangs by a massless rod of length l which rotates at constant angular frequency ω as shown in the figure.



The mass moves with steady speed in a circular path of constant radius. What is the angle θ that the string makes with vertical?

(a) $\cos^{-1}\left(g/\omega^2 l\right)$ (b) $\cos^{-1}\left(\omega^2 l/g\right)$
(c) $\sin^{-1}\left(g/\omega^2 l\right)$ (d) $\sin^{-1}\left(\omega^2 l/g\right)$

57. A block of mass M in free space is being pulled by a force F through a string of mass m. Block and string both are in dynamic state in the free space.

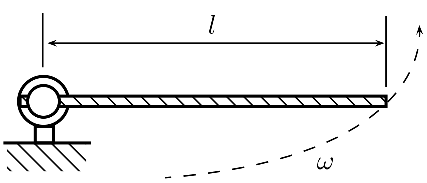


What is the actual force that the string transmits to the block?

(a) $\dfrac{FM}{M+m}$ (b) $\dfrac{Fm}{M+m}$

(c) $\dfrac{F(M+m)}{M}$ (d) $\dfrac{F(M+m)}{m}$

58. A uniform rope of mass m and length l is pivoted at one end and whirls with uniform angular velocity ω in a horizontal plane as shown in the figure.

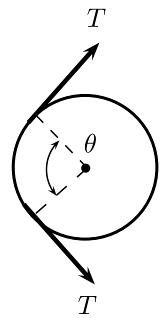


What is the tension in the rope at distance r from the pivot?

(a) $\dfrac{m\omega^2}{l-r}$ (b) $\dfrac{m\omega^2(l-r)}{2}$

(c) $\dfrac{m\omega^2\left(l^2-r^2\right)}{2l}$ (d) $\dfrac{m\omega^2\left(l^2-r^2\right)}{2r}$

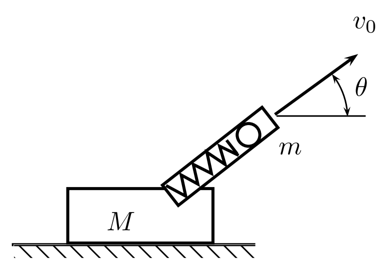
59. A string with constant tension T is deflected through angle θ by a smooth fixed pulley as shown in the figure.



What is the net force on the pulley?

(a) $2T\sin\theta$ (b) $2T\cos\theta$
(c) $2T\sin(\theta/2)$ (d) $2T\cos(\theta/2)$

60. A spring loaded gun of mass M, initially at rest on a horizontal frictionless surface firs a marble of mass m at angle of elevation θ and initial velocity v_0 as shown in the figure.



What is the final velocity of the gun on surface?

(a) $\dfrac{mv_0\sin\theta}{M+m}$ (b) $\dfrac{mv_0\sin\theta}{M}$

(c) $\dfrac{mv_0\cos\theta}{M+m}$ (d) $\dfrac{mv_0\cos\theta}{M}$

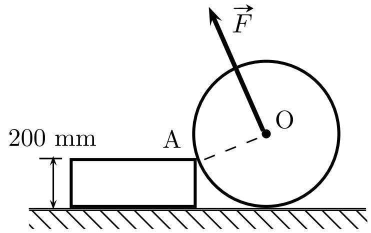
61. A ball of mass m moving with initial velocity u_1 collides with another ball of mass m at rest with coefficient of restitution e and conservation of momentum. The ratio of velocities after the collision is given by

(a) $\dfrac{1-e}{1+e}$ (b) $\dfrac{1+e}{1-e}$

(c) $\dfrac{1-e}{e}$ (d) $\dfrac{1+e}{e}$

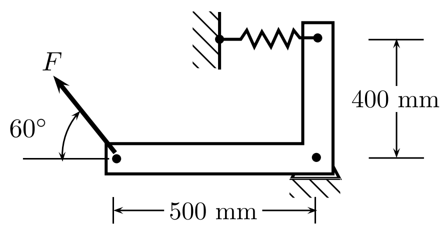
NUMERICAL ANSWER QUESTIONS

1. A wheel of 500 mm diameter, weighing 10 kN rests against a rigid rectangular block of 150 mm height as shown in figure.



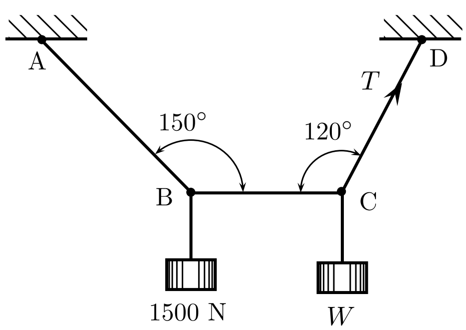
What is the least pull, through the center of the wheel, required just to turn the wheel over the corner of the block. Also calculate the reaction on the block.

2. A crank lever along with spring is shown in the figure. The arms of lever of 500 mm and 400 mm. The lever weights 0.1 N/mm.



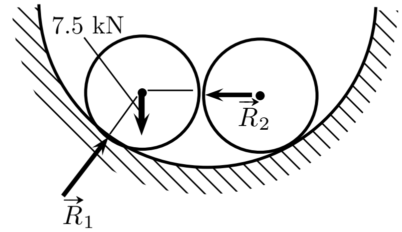
A force F of 100 N is applied on horizontal arm at 60°. Assuming negligible deflection, calculate the tension in the spring. If the spring stiffness is 2 N/mm, calculate the extension in the spring.

3. A string ABCD is attached to roof at two fixed points A and D. Other two points B and C are loaded with weights of 1500 N and W N respectively. The inclination of the three parts of the string is shown in the figure:



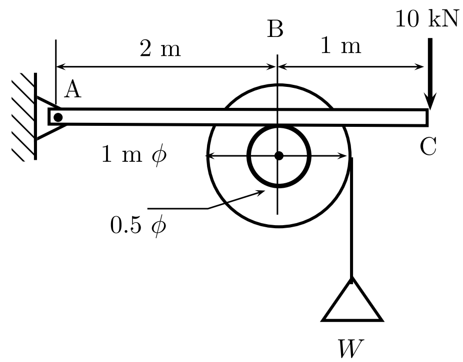
Calculate the value of weight W at point C and the tension in the string part CD.

4. Two spheres of equal diameter 150 mm diameter and weighing 7.5 kN are in equilibrium within a smooth cup of 450 mm radius.



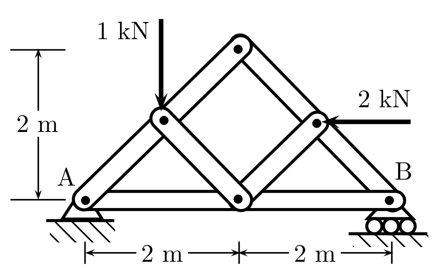
Calculate the reaction (R_1) at the surface of the cup, and the reaction (R_2) between the two spheres.

5. A weightless beam ABC is hinged at A. A weight W hangs on a pulley of diameter 1 m and a holding force of 10 kN is applied at point C to hold the weight through friction wheel of diameter 0.5 m attached to the pulley. A holding force 10 kN is applied at free end C. The coefficient of friction between the beam and friction wheel is 0.3.



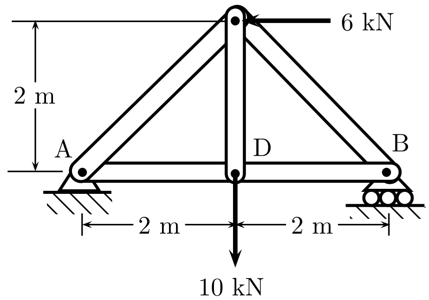
Calculate the horizontal reaction at point B and the maximum value of weight W.

6. A truss of 4 m span length and 2 m height supports loads of 2 kN and 1 kN, as shown in the figure.



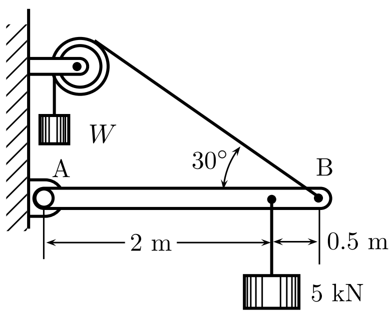
Calculate the horizontal reaction at hinged joint, and the vertical reaction at roller joint.

7. A frame of 4 m span and 2 m height is subjected to loads of 6 kN at its apex and 10 kN at its mid-span on lower arm as shown in the figure:



Calculate the force on two members, BC and AD.

8. A weightless beam AB of length 1.5 m is held in equilibrium by the application of a load W on a rope through a pulley to hold a weight of 5 kN as shown in the figure.



Calculate the load W to hold the 5 kN weight. Also calculate the corresponding reaction at hinged joint A.

9. A particle of 20 g mass starts moving from rest in a straight line with following equation of motion

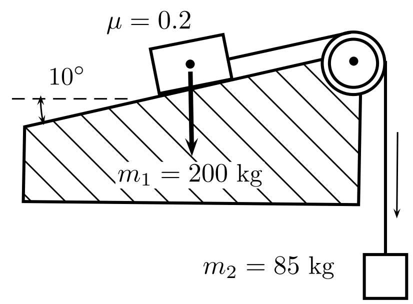
$$x = 4t^3 - 2t^2 + 5$$

where x in m and t in seconds. Calculate the acceleration of the particle after 5 s. Also determine the value of driving force on the particle.

10. A bullet of 25 g is fired horizontally with a velocity of 350 m/s from a gun carried in a carriage. The total mass of carriage and gun is 125 kg which rests on a surface which resists its movement by a force of 25 N via a horizontal spring. What is the velocity of recoil of the gun? What is the distance moved by the gun before it comes to rest?

11. A car of mass 1500 kg is running down over a gradient plane of 1:80 at a speed of 12 m/s. The effective coefficient of friction is 0.002. Calculate the acceleration of the car. Also calculate velocity of the car after 75 s.

12. A body of mass $m_1 = 200$ kg is pulled up from rest upon a rough plane inclined at 10° to the horizontal by means of a flexible rope running parallel to the plane. The portion of the rope, beyond the pulley, hangs vertically downward and carries a mass of $m_2 = 85$ kg at the end.

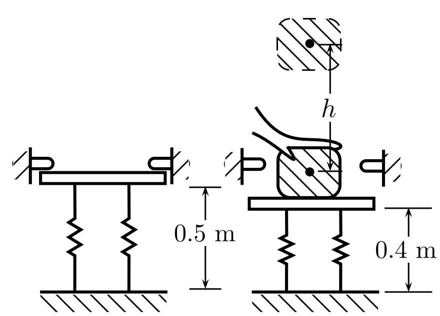


The coefficient of friction between the plane and body is 0.2. The acceleration due to gravity is 9.81 m/s^2. Estimate the acceleration of the body on the plane, and the corresponding tension in the rope.

13. A ball of mass 2 kg moving with a velocity of 4 m/s impinges directly on a ball of mass 5 kg at rest. After impinging, the first ball comes to rest. Calculate the velocity of the second ball after the impact? What is the coefficient of restitution?

14. A sphere of mass 5 kg, moving at 15 m/s overtakes another sphere of mass 25 kg in the same line at 3 m/s. The coefficient of restitution is 0.75. Calculate the velocity of the spheres after the impact.

15. A platform is supported by two springs of stiffness 100 N/m and a stopper to compressed length of 0.5 m from unstretched length of 1 m. A block of 3 kg is placed on the platform that further compresses the spring to 0.4 m. Calculate the height h the block rises after its release.



ANSWERS

Multiple Choice Questions

1. (d)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (c)
11. (b)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (c)
21. (d)	**22.** (a)	**23.** (d)	**24.** (c)	**25.** (a)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (d)	**30.** (b)
31. (c)	**32.** (d)	**33.** (b)	**34.** (b)	**35.** (d)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (c)	**40.** (c)
41. (a)	**42.** (c)	**43.** (a)	**44.** (d)	**45.** (b)	**46.** (c)	**47.** (d)	**48.** (d)	**49.** (c)	**50.** (d)
51. (a)	**52.** (a)	**53.** (b)	**54.** (d)	**55.** (a)	**56.** (a)	**57.** (a)	**58.** (c)	**59.** (c)	**60.** (c)
61. (a)									

Numerical Answer Questions

1. 9.81 kN, 2 kN	**2.** 77 N, 38.5 mm	**3.** 4500 N, 5196.15 N
4. 8.66 kN, 2.99 kN	**5.** 4.5 kN, 2.25 kN	**6.** 2 kN, 0.75 kN
7. -1.414 kN, 3 kN	**8.** 8 kN, 6.99 kN	**9.** 118 m/s^2, 2.36 N
10. 0.07 m/s, 12.25 mm	**11.** 0.103 m/s^2, 19.72 m/s	**12.** 0.374 m/s^2, 801.176 N
13. 1.6 m/s, 0.4	**14.** 6.5 m/s, -2.5 m/s	**15.** 0.37 m

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (d) According to Varignon's theorem, the algebraic sum of moments of two forces about any point in their plane is equal to the moments of their resultant about the same.

2. (a) If a system of coplanar forces be in equilibrium, then the algebraic sum of their moments about any point in their plane is zero. This is called principle of moments, which is derived from Varignon's theorem.

3. (a) The x and y components of the second force will be given by

$$\begin{aligned} F_x &= -4-3 \\ &= -7 \text{ N} \\ F_y &= 0-(-5) \\ &= 5 \text{ N} \end{aligned}$$

4. (b) The vertical reaction at A will be given by taking moment about point B

$$\begin{aligned} R_A \times 10 &= -10\times 5+8\times 4 \\ R_A &= -1.8 \text{ kN} \end{aligned}$$

5. (d) The required pull will be minimum when it is perpendicular to the line CO.

6. (c) Taking moments about the contact point gives

$$\begin{aligned} F\times R\sin 30^\circ &= WR\cos 30^\circ \\ F &= W\cot 30^\circ \\ &= \sqrt{3}W \end{aligned}$$

7. (d) The angle between BC and AB is 45°. The tension in BC will be equal to W which will balance the load moment at A caused by load w at center of AB. Thus,

$$\begin{aligned} W\sin 45^\circ \times l &= w\frac{l}{2} \\ W &= \frac{w}{\sqrt{2}} \end{aligned}$$

8. (b) The actual angle between the two forces is $\pi-\theta$ and $\cos(\pi-\theta) = -\cos\theta$, therefore, resultant is given by

$$C = \sqrt{A^2+B^2-2AB\cos\theta}$$

9. (a) The horizontal reactions at A and F will be zero, hence

$$F_{AC} = 0$$
$$F_{EF} = 0$$

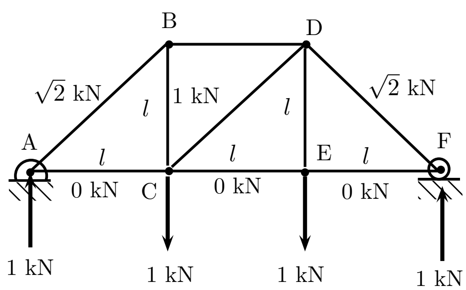
Hence, $F_{CE} = 0$. The vertical reactions at A and F are determined as

$$R_E \times 3l = 1\times 2l + 1\times l$$
$$R_E = 1 \text{ kN}$$
$$R_A \times 3l = 2-1$$
$$R_E = 1 \text{ kN}$$

The force on the members is determined as

$$F_{AB} = \frac{1}{\cos 45^\circ} = \sqrt{2} \text{ kN}$$

The forces are shown in the figure.



Thus, the force in CD is determined as

$$F_{CD}\cos 45^\circ = 1-1$$
$$F_{CD} = 0$$

10. (c) Hinged bars bars cannot be retain bending moment.

11. (b) Angle between bar and cable CB is

$$\theta = \tan^{-1}\left(\frac{125}{275}\right) = 24.44^\circ$$

Taking moments about point A:

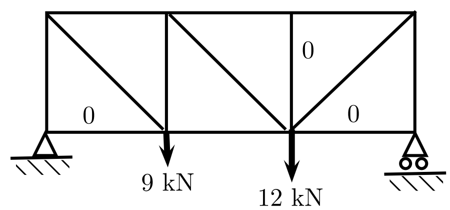
$$T\sin 24.44^\circ = 35$$
$$T = 84.59 \text{ kg}$$
$$= 829.83 \text{ N}$$

The vertical reaction at A will be zero. The horizontal reaction is given by

$$R = T\cos 24.44^\circ$$
$$= 77.01 \text{ kg}$$
$$= 755.47 \text{ N}$$

12. (c) As the hinged joint B is friction less and it is also the centroid of the bar CD, hence, there is no unbalanced torque to act upon the bar CD, thus it will remain in same orientation, making 30° angle with vertical.

13. (c) Zero-force members are shown in the figure.



For example, roller support (right side) can exert a reaction in vertical direction only. Therefore, the horizontal member joined with roller support will have zero force.

14. (d) If the kinetic energy of an object launched from the earth is equal in magnitude to the potential energy, then in the absence of frictional resistance it could escape from the earth. Thus, escape velocity is independent of the angle α.

15. (b) The following equations can be formed from the given data

$$450 = u\times 5 + \frac{1}{2}a\times 5^2$$
$$700 = (u + a\times 5) + \frac{1}{2}a\times 5^2$$

Solving these equations, one obtains

$$a = 10 \text{ m/s}^2$$

16. (a) Since both the person and body will fall with the same speed, therefore, the load will not be felt.

17. (c) The distances traveled in $t-1$ and t seconds are

$$s_{t-1} = u\,(t-1) + \frac{1}{2}a\,(t-1)^2$$
$$s_t = ut + \frac{1}{2}at^2$$

Thus, the distance traveled in t^{th} second is

$$= s_t - s_{t-1}$$
$$= u + \frac{1}{2}a\left(t^2 - t^2 - 1 + 2t\right)$$
$$= u + \frac{1}{2}a\,(2t-1)$$

18. (b) At the rest, when starting, the location of the body is $s = 7$. The location after $t = 2$ seconds will be determined as

$$\begin{aligned} s &= 2\times 2^3 + 3\times 2^2 + 7 \\ &= 35 \text{ m} \end{aligned}$$

The distance traveled in 2 seconds is

$$\begin{aligned} &= 35 - 7 \\ &= 28 \text{ m} \end{aligned}$$

19. (a) The velocity at time t is determined as

$$\begin{aligned} v &= \frac{ds}{dt} \\ 9 &= 2\times 3t^2 + 3 \\ t &= 1 \end{aligned}$$

Thus, the distance traveled is

$$\begin{aligned} s &= 2\times 1^3 + 3\times 1 \\ &= 5 \text{ m} \end{aligned}$$

20. (c) In the downward motion, the initial velocity at the top of the tower will be 110 m/s, and acceleration will be uniform 10 m/s^2. Thus, the velocity after traveling a height $s = 115$ m will be given by

$$\begin{aligned} v^2 - 110^2 &= 2\times 10\times 115 \\ v &= 120 \text{ m/s} \end{aligned}$$

21. (d) The time taken in the upward journey and that in the downward journey will be equal.

22. (a) Net gravity will be

$$\begin{aligned} g' &= g - \frac{g}{3} \\ &= \frac{2g}{3} \end{aligned}$$

Hence, the force on the floor of the lift will be 2/3 times the passenger's weight.

23. (d) The velocity is related to position vector as

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} \\ &= A\alpha\left(e^{\alpha t}\hat{i} - e^{-\alpha t}\hat{j}\right) \end{aligned}$$

24. (c) Both the balls will experience the same acceleration g, therefore, will take equal time for the same height.

25. (a) Principle of virtual work states that the work done on a rigid body or a system of rigid bodies in equilibrium is zero for any virtual displacements compatible with the constraints on the system.

26. (b) Net change in the potential energy from A to B is

$$\text{PE} = \frac{1}{2}kx^2 - mgx$$

27. (d) The rolling frictional force is

$$\begin{aligned} F &= \mu R \\ &= 0.3\times 500 \\ &= 150 \text{ N} \end{aligned}$$

28. (a) Work done by the spring during extension from x_1 to x_2 is given by

$$W = \frac{1}{2}\left(x_2^2 - x_1^2\right)$$

29. (d) Kinetic energy of flywheel is written as

$$T = \frac{1}{2}I\omega^2$$

30. (b) The power requirement is given by

$$\begin{aligned} P &= \frac{1}{0.8}\times\frac{24\times 36}{60} \\ &= 18 \text{ W} \end{aligned}$$

31. (c) For a conservative mechanical system, the sum of the kinetic energy and potential energy is constant.

32. (d) Speed of the flywheel after 10 s will be

$$\begin{aligned} \omega &= 0.5\times 10 \\ &= 5 \text{ rad/s} \end{aligned}$$

Kinetic energy of the flywheel will be

$$\begin{aligned} T &= \frac{1}{2}\times 2000\times 5^2 \\ &= 25000 \text{ Nm} \end{aligned}$$

33. (b) The remaining load at top is 100 N, while initial load is 200 N. Thus, the work done in traveling 10 m distance is

$$\begin{aligned} W &= \frac{200+100}{2}\times 10 \\ &= 1500 \text{ J} \end{aligned}$$

34. (b) Given $k = 10\times 10^3$ N/m. The spring is stretched by

$$\begin{aligned} x &= 100 - 60 \\ &= 40 \text{ cm} \end{aligned}$$

The work done in stretching is

$$\begin{aligned} W &= \frac{1}{2} \times 10 \times 10^3 \times 0.40^2 \\ &= 800 \text{ Nm} \end{aligned}$$

35. (d) The work required for addition 0.1 m stretch is

$$\begin{aligned} W &= \frac{1}{2} \times 1000 \times \left(0.2^2 - 0.1^2\right) \\ &= 15 \text{ Nm} \end{aligned}$$

36. (c) The kinetic energy of the truck will be stored as potential energy of the spring. Thus, the maximum compression in the spring is given by

$$\begin{aligned} \frac{1}{2} \times \frac{10}{0.0125} x^2 &= \frac{1}{2} \times \frac{150}{9.81} \times 2^2 \\ x &= 0.2765 \text{ m} \\ &= 27.65 \text{ cm} \end{aligned}$$

37. (a) The principle of conservation of energy will be used to determined the relationship between v and h. Let m be the mass of the wheel and ω be the angular velocity of rolling. To over come the obstruction, the center of the wheel will be shifted to height h. Thus, using the principle of conservation of energy

$$\frac{1}{2} m k^2 \omega^2 + \frac{1}{2} m v^2 = mgh$$

38. (b) The potential energy of the cylinder will be converted into kinetic energy, therefore

$$\begin{aligned} \frac{mv^2}{2} + \frac{1}{2} m r^2 \times \left(\frac{v}{r}\right)^2 &= mgh \\ mv^2 &= mgh \\ v &= \sqrt{gh} \end{aligned}$$

39. (c) Let v_0 is the initial speed of the bullet. Using the principle of conservation of energy,

$$\begin{aligned} \frac{1}{2} m v_0^2 &= (M+m)\, g\,(1-\cos\theta)\, l \\ v_0 &= \sqrt{2\left(\frac{M+m}{m}\right) g\,(1-\cos\theta)\, l} \end{aligned}$$

40. (c) It means the acceleration of mass at point A should be g. The vertical distance traveled by the mass in half loop is $2R$. The mass has to hit the top of the track with a force $F = mg$, for which the centrifugal force should provide the F and weight $W = mg$ of the mass. Thus, if v is the velocity of the mass at A,

$$\begin{aligned} \frac{v^2}{R} &= F + W \\ &= mg + mg \\ &= 2mg \\ v^2 &= 2Rmg \end{aligned}$$

Using the principle of conservation of energy

$$\begin{aligned} mgh &= mg2R + \frac{1}{2} m v^2 \\ &= mg2R + \frac{1}{2} 2Rmg \\ h &= 3R \end{aligned}$$

41. (a) According to d'Alembert's principle, the external forces acting on a body and the resultant inertia forces on it are in equilibrium.

42. (c) The impulse (Ft) of the force is related as

$$Ft = m\,(v-u)$$

43. (a) A perfectly elastic collision occurs without loss of kinetic energy in the collision $(e = 1)$.

44. (d) A inelastic or plastic collision is one in which part of the kinetic energy is changed to some other form of energy in the collision.

45. (b) Given that

$$\begin{aligned} e &= 1 \\ m_1 &= 2 \text{ kg} \\ m_2 &= 8 \text{ kg} \\ u_1 &= 5 \text{ m/s} \\ u_2 &= 0 \text{ m/s} \\ l &= 1.6 \text{ m} \end{aligned}$$

Using the principle of conservation of momentum,

$$\begin{aligned} m_1 u_1 + m_2 u_2 &= m_1 v_1 + m_2 v_2 \\ 2 \times 5 &= 2v_1 + 8v_2 \\ v_1 + 4v_2 &= 5 \end{aligned}$$

Using the definition of the coefficient of restitution,

$$\begin{aligned} e\,(u_1 - 0) &= v_2 - v_1 \\ v_2 - v_1 &= 5 \end{aligned}$$

Solving the above two equations,

$$\begin{aligned} v_2 &= 2 \text{ m/s} \\ v_1 &= -3 \text{ m/s} \end{aligned}$$

The angular velocity of the rod is

$$\begin{aligned} \omega &= \frac{v_2}{l} \\ &= \frac{2}{1.6} \\ &= 1.25 \text{ rad/s} \end{aligned}$$

46. (c) When $\theta = 0°$, there will be no movement in y direction. Hence, conservation of energy and momentum in x direction will suffice to determine the velocities of m and M.

47. (d) Using equilibrium of the body

$$mg\sin 45° = P\cos 45°$$
$$P = mg$$

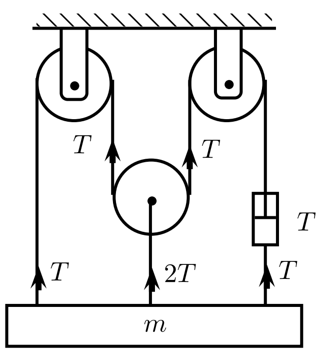
48. (d) Given that

$$v_1 = 0$$
$$e = 1$$

Applying the principle of conservation of momentum,

$$\sqrt{2gh} - 0 = v_2 - v_1$$
$$m\left\{\sqrt{2gh} - 0\right\} = m\left(v_2 - v_1\right)$$
$$v_2 = \sqrt{2gh}$$

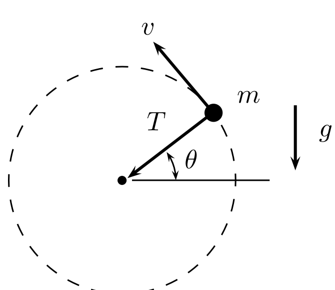
49. (c) The tension on the cable at different location is shown in the figure.



It is seen that the tension T is equal through the length of the cable. Thus, weight mg is supported by 4T. Thus, value of mass is

$$m = \frac{4T}{g}$$

50. (d) The configuration is shown in the figure.



The tension in the spring is given by

$$T = m\left(\frac{v^2}{l} - g\sin\theta\right)$$

which will be maximum when

$$\sin\theta = -1$$
$$\theta = 270°$$

This value of θ occurs when the mass is at the bottom of the circle. Additionally, the string can pull but not push, so T cannot be negative:

$$m\frac{v^2}{l} \geq mg\sin\theta$$

The maximum value of $\sin\theta$ occurs when $\theta = \pi/2$, that is, when the mass is vertically up, and then the tension in the string will be minimum.

51. (a) If v' is the velocity of second piece after the explosion, the conservation of momentum is written as

$$v\cos\theta = -\frac{m}{2}v\cos\theta + \frac{m}{2}v'$$
$$v' = 3v\cos\theta$$

52. (a) If T is the tension in the string, the equilibrium equations will be written for the two masses as

$$T = -m_1a + m_1g$$
$$= m_2a$$

Therefore,

$$m_2a = -m_1a + m_1g$$
$$a = \frac{m_1g}{m_1 + m_2}$$

53. (b) Using Lami's theorem,

$$\frac{T_1}{150} = \frac{W}{\sin\left(180 - \theta - 60°\right)}$$
$$T_1 = \frac{W}{\sin\left(\theta + 60°\right)}$$

Thus, T_1 will be minimum when

$$\theta = 30°$$

54. (d) Given that the mass and acceleration of the elevator are, respectively,

$$m = \frac{10^4}{9.81}$$
$$a = \frac{4}{2}$$
$$= 2 \text{ m/s}^2$$

Equilibrium of the mass is given by

$$\begin{aligned}T &= W + ma \\ &= m(a+g) \\ &= \frac{10\times10^3}{9.81}(2+9.81) \\ &= 12038.7 \text{ N}\end{aligned}$$

55. (a) For the equilibrium of the mass in radial direction,

$$\begin{aligned}T &= m\frac{v^2}{l} - mg\sin\theta \\ &= m\left(\frac{v^2}{l} - g\sin\theta\right)\end{aligned}$$

56. (a) If T is the tension in the string, for the equilibrium of mass in horizontal direction

$$\begin{aligned}T\sin\theta &= m\omega^2 l\sin\theta \\ T &= m\omega^2 l\end{aligned}$$

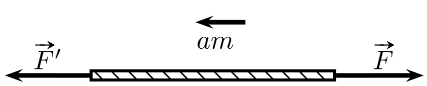
For the equilibrium of mass in vertical direction,

$$\begin{aligned}T\cos\theta &= mg \\ \cos\theta &= \frac{mg}{T} \\ &= \frac{g}{\omega^2 l}\end{aligned}$$

57. (a) The block and the string are in dynamic state under the external force F:

$$\begin{aligned}F &= a(M+m) \\ a &= \frac{F}{M+m}\end{aligned}$$

Let F' is the force transmitted by the string to the block, as shown in figure.



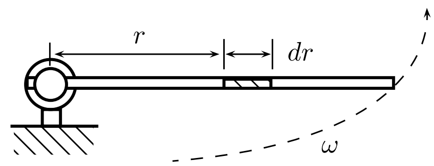
For the equilibrium of string,

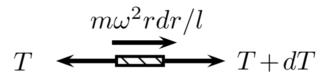
$$\begin{aligned}F &= am + F' \\ F' &= F - am \\ &= a(M+m) - am \\ &= aM\end{aligned}$$

Using the value of a, determined earlier,

$$= \frac{F}{M+m}M$$

58. (c) Let T_r be the tension in the string at a distance r. The mass per unit length of the string is m/l. Consider an element of rope of length dr at a distance r from the pivot where tension in the string is T.





If dT is the difference in tension across this element, for its equilibrium

$$\begin{aligned}T + dT - T &= -\frac{m}{l}dr\times\omega^2 r \\ dT &= -\frac{m\omega^2 r}{l}dr\end{aligned}$$

Thus, integrating from $r = 0$ to r,

$$T_r = T_0 - \frac{m}{l}\times\omega^2\frac{r^2}{2}$$

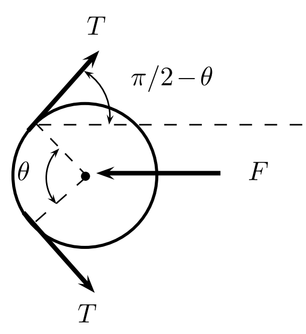
At $r = l$, $T_r = 0$, so

$$T_0 = \frac{m}{l}\times\omega^2\frac{l^2}{2}$$

Thus,

$$T_r = \frac{m\omega^2\left(l^2 - r^2\right)}{2l}$$

59. (c) The forces acting on the pulley are shown in the figure.



The pulley is subjected to equal and opposite forces, $T\cos(\theta/2)$, in the vertical direction; there is no force in vertical direction. However, the net force on the pulley in horizontal direction is

$$F = 2\times T\sin\frac{\theta}{2}$$

60. (c) Let v_f be the final velocity of the gun. Using conservation of horizontal momentum, one gets

$$Mv_f = m(v_0 \cos\theta - v_f)$$
$$v_f = \frac{mv_0 \cos\theta}{M+m}$$

61. (a) Using the principle of conservation of momentum,

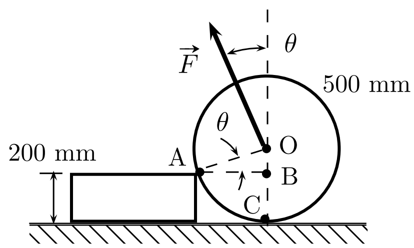
$$mu_1 - 0 = m(v_2 + v_1)$$
$$u_1 = v_2 + v_1$$

Using the definition of the coefficient of restitution,

$$e(u_1 - 0) = v_2 - v_1$$
$$e(v_2 + v_1) = v_2 - v_1$$
$$v_1(1+e) = v_2(1-e)$$
$$\frac{v_1}{v_2} = \frac{1-e}{1+e}$$

Numerical Answer Questions

1. The pull will be minimum when it is normal to the line joining corner of the block and center of the wheel AO.



The inclination (θ) of the line AO from the horizontal is

$$\sin\theta = \frac{\text{OC} - \text{OB}}{\text{OA}}$$
$$\theta = \sin^{-1}\left(\frac{500/2 - 200}{500/2}\right)$$
$$= 11.54^\circ$$

The weight of the wheel (say W) acts at point O. For equilibrium, moment of the pull F and weight (about point A) should be zero:

$$F \times AO - W \times AB = 0$$
$$F \times \frac{500}{2} - 10 \times \frac{500\cos\theta}{2} = 0$$
$$F = 9.8 \text{ kN}$$

The reaction R at the corner of the block can be determined by considering equilibrium of the body in horizontal direction:

$$F\sin\theta = R\cos\theta$$
$$R = F\tan\theta$$
$$= 2 \text{ kN}$$

2. Considering the weights acting at center of span of each arm, the moments of forces about the pivot is

$$T \times 400 = 100\sin 60^\circ \times 500 - 0.1 \times \frac{500^2}{2}$$
$$T = 77 \text{ N}$$

Given that

$$k = 2 \text{ N/mm}$$

The extension in the spring is calculated as

$$\delta = \frac{T}{k}$$
$$= \frac{77}{2}$$
$$= 38.5 \text{ mm}$$

3. The equilibrium of forces at point B is given by

$$\frac{F_{BC}}{\sin 120^\circ} = \frac{1500}{\sin 150^\circ}$$
$$F_{BC} = 2598.07 \text{ N}$$

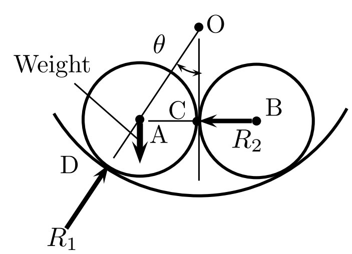
The equilibrium of forces at point C is given by

$$\frac{W}{\sin 120^\circ} = \frac{F_{BC}}{\sin 150^\circ}$$
$$W = 4500 \text{ N}$$

The equilibrium of forces at point C is given by

$$\frac{T}{\sin 90^\circ} = \frac{W}{\sin 120^\circ}$$
$$T = 5196.15 \text{ N}$$

4. The reactions are shown in the figure:

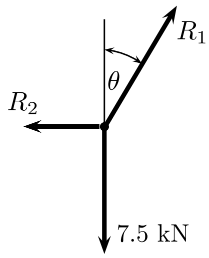


The angle θ is calculated as

$$\sin\theta = \frac{\mathrm{AC}}{\mathrm{AO}} = \frac{\mathrm{AC}}{\mathrm{OD}-\mathrm{AD}}$$

$$\theta = \sin^{-1}\left(\frac{150/2}{450/2-150/2}\right) = 30^\circ$$

The equilibrium of forces at center of any sphere is shown in the following figure



Using the law of triangle of forces,

$$\frac{R_1}{\sin 90^\circ} = \frac{7.5}{\sin 120^\circ}$$
$$R_1 = 8.66 \text{ kN}$$

The equilibrium of forces at center of any sphere also gives

$$\frac{R_2}{\sin 150^\circ} = \frac{7.5}{\sin 120^\circ}$$
$$R_2 = 12.99 \text{ kN}$$

5. The horizontal reaction R is calculated by taking moments about point A, as

$$\frac{R}{\mu}\times 2 = 10\times 3$$
$$R = 4.5 \text{ kN}$$

The weight W is obtained by the equilibrium of the friction wheel

$$W\times 0.5 = 4.5\times 0.25$$
$$W = 2.25 \text{ kN}$$

6. The reaction at point A will be given by the equilibrium of horizontal forces acting on the truss as a free body. Thus,

$$R_A = 2 \text{ kN}$$

The vertical reaction at point B will be given by moments of forces about point A, thus

$$R_B\times 4 = 2\times 1 + 1\times 1$$
$$R_B = 0.75 \text{ kN}$$

7. The forces on AD and DB will be equal. The force on CD will be equal to 10 kN. Taking moments about hinged joint A,

$$R_B\times 4 = 10\times 2 - 6\times 2$$
$$R_B = 1 \text{ kN}$$

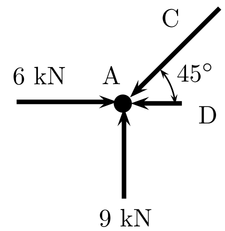
Considering the equilibrium of joint B

$$F_{BC} = \frac{R_B}{\sin 45^\circ} = 1.414 \text{ kN (compressive)}$$

The vertical reaction at point A will be

$$10 - 1 = 9 \text{ kN}$$

Similarly, the horizontal reaction at point A will be equal to 6 kN (towards right). These forces are shown below:



Hence, force in member AC will be given by

$$F_{AC} = \frac{9}{\sin 45^\circ}$$
$$F_{AC} = 12.727 \text{ kN (compressive)}$$

The force in AD will be given by

$$F_{AD} = F_{AC}\cos 45^\circ - 6 = 3 \text{ kN (tension)}$$

8. The value of W is given by taking moments about the point A

$$W\sin 30^\circ \times 2.5 = 5\times 2$$
$$W = 8 \text{ kN}$$

Taking the beam as free body, the sum of horizontal forces should be zero. Therefore, horizontal reaction at A is

$$W\cos 30^\circ = 6.928 \text{ kN}$$

Vertical reaction at A is

$$5 - W\sin 30^\circ = 1 \text{ kN}$$

The net reaction at A is

$$R = \sqrt{1^2 + 6.928^2} = 6.99 \text{ kN}$$

9. Given that $m = 0.02$ kg. The acceleration is defined as

$$\begin{aligned} a &= \frac{d^2x}{dt^2} \\ &= 12 \times 2t - 2 \end{aligned}$$

Using this, acceleration at $t = 5$ is

$$a = 118 \text{ m/s}^2$$

Therefore, the driving force is

$$\begin{aligned} F &= ma \\ &= 2.36 \text{ N} \end{aligned}$$

10. The recoil velocity of the gun will be given by

$$\begin{aligned} 125 \times v &= 0.025 \times 350 \\ v &= 0.07 \text{ m/s} \end{aligned}$$

The resistance to the movement of carriage is

$$R = 25 \text{ N}$$

Mass of the carriage is

$$M = 125 \text{ kg}$$

Therefore, desired deceleration of the carriage is

$$\begin{aligned} a &= \frac{F}{M} \\ &= 0.2 \text{ m/s}^2 \end{aligned}$$

The distance traveled s is given by

$$\begin{aligned} 2as &= v^2 - u^2 \\ 2 \times -0.2 \times s &= 0^2 - 0.07^2 \\ s &= 12.25 \text{ mm} \end{aligned}$$

11. The angle of inclination of the plane α is given by

$$\begin{aligned} \sin\alpha &= \frac{1}{80} \\ &= 0.0125 \end{aligned}$$

The acceleration of the car will be given by

$$\begin{aligned} ma &= mg\sin\alpha - \mu mg\cos\alpha \\ a &= g(\sin\alpha - \mu\cos\alpha) \\ &= 0.103 \text{ m/s}^2 \end{aligned}$$

The initial speed $u = 12$ m/s, therefore, final velocity v of the car after $t = 75$ s is

$$\begin{aligned} v &= u + at \\ &= 19.725 \text{ m/s} \end{aligned}$$

12. Given that

$$\begin{aligned} \mu &= 0.2 \\ g &= 9.81 \text{ m/s}^2 \\ \alpha &= 10^\circ \end{aligned}$$

The system is governed by following two equations of equilibrium of the two masses

$$\begin{aligned} T + m_2a &= m_2g \\ T &= m_1g(\sin\alpha + \mu\cos\alpha) + m_1a \end{aligned}$$

Eliminating T from the above two equations, one gets

$$\begin{aligned} a &= \frac{m_2 - m_1(\sin\alpha + \mu\cos\alpha)}{m_1 + m_2} g \\ &= 0.3744 \text{ m/s}^2 \end{aligned}$$

The tension in the rope is given by

$$\begin{aligned} T &= m_2(g - a) \\ &= 801.176 \text{ N} \end{aligned}$$

13. The velocity v_2 of the second ball shall be given by

$$\begin{aligned} 2 \times 0 + 5 \times v_2 &= 2 \times 4 + 5 \times 0 \\ v_2 &= 1.6 \text{ m/s} \end{aligned}$$

The coefficient of restitution is given by

$$\begin{aligned} e &= \frac{v_2 - v_1}{u_1 - u_2} \\ &= \frac{1.6 - 0}{4 - 0} \\ &= 0.4 \end{aligned}$$

14. Given that

$$\begin{aligned} m_1 &= 5 \text{ kg} \\ m_2 &= 25 \text{ kg} \\ u_1 &= 15 \text{ m/s} \\ u_2 &= 3 \text{ m/s} \\ e &= 0.75 \end{aligned}$$

For the conservation of momentum,

$$\begin{aligned} m_1v_1 + m_2v_2 &= m_1u_1 + m_2u_2 \\ v_1 + 5v_2 &= 30 \\ v_1 &= 30 - 5v_2 \end{aligned}$$

The expression for the coefficient of restitution is

$$\begin{aligned} v_2 - v_1 &= e(u_1 - u_2) \\ v_2 - v_1 &= 9 \end{aligned}$$

Replacing the value of v_1,

$$v_2 - 30 + 5v_2 = 9$$

Solving, one obtains

$$\begin{aligned} v_2 &= 6.5 \text{ m/s} \\ v_1 &= -2.5 \text{ m/s} \end{aligned}$$

15. Effective stiffness of the springs in parallel is

$$k_e = k_1 + k_2 = 200 \text{ N/m}$$

Initial compression of the spring is

$$\begin{aligned} x_1 &= 1 - 0.5 \\ &= 0.5 \text{ m} \end{aligned}$$

Final compression is

$$\begin{aligned} x_2 &= 1.0 - 0.4 \\ &= 0.6 \text{ m} \end{aligned}$$

Using the principle of work and energy:

$$\begin{aligned} mgh &= \frac{1}{2} k_e \left(x_2^2 - x_1^2\right) \\ 3 \times 9.81 \times h &= \frac{1}{2} \times 200 \left(0.6^2 - 0.5^2\right) \\ h &= 0.37 \text{ m} \end{aligned}$$

CHAPTER 2

STRENGTH OF MATERIALS

Strength of materials is the study of the effects of forces in causing changes in the size and shape of the bodies by dealing with their strength, stability and rigidity. It includes the study of elastic properties of materials and their behavior during interaction with external forces. The particles of a rigid body are connected together by the force of cohesion, and this force must be completely overcome before the body can be ruptured. A force acting upon a rigid body, first tends to change the relative position of its particles, and finally separates them from each other. Such forces are generally applied to bodies slowly, and the changes in size and shape occur while the forces are increasing up to their final values. *Elasticity* is the property which bodies possess by taking a new form under the action of a force and also of resuming their original form when the force is withdrawn. All bodies possess elasticity in a greater or less degree.

2.1 STRESS AND STRAIN

The response of a material to external loads is evident in two forms:

1. ***Stress*** *Stress* is an *internal force* that resists the change in shape or size of material elements, and when the applied forces have reached their final values, the internal stresses hold them in equilibrium. In other words, stress is the response of a material in the form of forces generated in material points to simultaneously combat the external forces trying to deform the material. Stress is expressed in terms of force per unit area of response.

2. ***Strain*** *Strain* is the response of a material in the form of *deformation* generated in material points to simultaneously combat the external forces. It is expressed in terms of deformation of the dimension per unit original dimension.

Different forms of stresses and strains are elaborated under the following headings.

2.1.1 Types of Stresses

Depending upon the mode of applied force, the stress can be classified as follows.

1. ***Normal Stress*** The first evident effect of an *axial force* (F) is to change the length of the body upon which it acts. The stress generated by such an axial load is measured per unit area, which is called *normal stress* (σ), or simply *stress* [Fig. 2.1]:

$$\sigma = \frac{F}{A}$$

where A is the area where axial load F acts orthogonally.

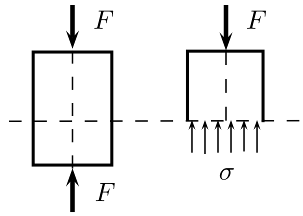


Figure 2.1 | Compressive stress.

When external forces act upon the ends of a material element in outward direction, they are called *tensile forces*; when they act inwards, they are called *compressive forces*. A *pull* is a tensile force, and *push* is a compressive force, and these two cases are frequently called *tension* and *compression*. The resisting stresses receive similar designations; a *tensile stress* is that which resists tensile forces; a *compressive stress* is that which resists compressive forces.

2. Shearing Stress When two equal and opposite forces act at right angles to a body and are very close to each other, they are called *shear forces*, because they tend to cut or shear the body. The action of the forces is similar to those in a pair of shears, and from this analogy the name is derived. The resisting stresses are called *shear stresses*, or simply, shear (τ) [Fig. 2.2].

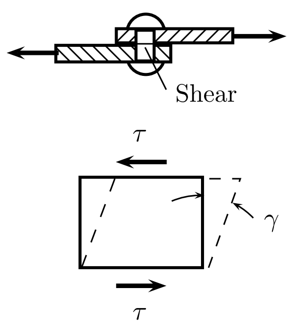


Figure 2.2 | Shear stress.

If force F acts parallel to area A, the shear stress (τ) is given by

$$\tau = \frac{F}{A}$$

Tension and compression cause stresses which act normally to a section area, but shear causes stresses which act parallel with and along the section area.

3. Volumetric Stress Three mutually perpendicular like direct stresses of the same intensity produced in a body constitute a *volumetric stress*. The volumetric stress produces a change in the volume of the body without producing any distortion to its shape.

2.1.2 Types of Strains

The strains can be of three types:

1. Longitudinal Strain *Longitudinal strain* (ε) is the change in length per unit length of the element in a particular direction [Fig. 2.3]. If a body of original length l is elongated by δl, then

$$\varepsilon = \frac{\delta l}{l} \tag{2.1}$$

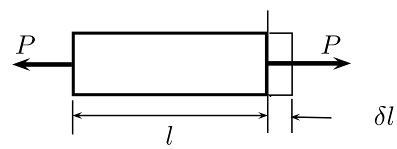


Figure 2.3 | Tensile strain.

The *deformation* continues until the resisting stresses have attained such magnitudes that bring the material under equilibrium with the applied forces. The deformation of a bar which occurs in tension is called *elongation*, and that which occurs in compression is called *shortening*.

2. Shear Strain *Shear strain* (γ) is the measure of distortion in terms of angle rotated by the plane normal to shear force [Fig. 2.2].

3. Volumetric Strain *Volumetric strain* (ε_v) is the change in volume per unit initial volume of the material element, and is expressed as

$$\varepsilon_v = \frac{\delta V}{V} \tag{2.2}$$

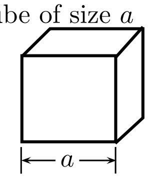
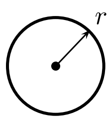
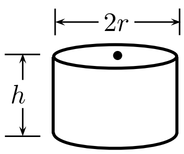
Expressions of volumetric strains are summarized in Table 2.1.

2.1.3 Saint Venant's Principle

Saint Venant's principle[1] states that the stresses and strains in a body at points that are at sufficient distance

[1]Saint Venant's principle was published in French by Saint-Venant in 1855 and later described by von Mises in 1945.

Table 2.1 | Expressions of volumetric strains

Object	**Volume**	ε_v
Cube of size a 	a^3	$3\dfrac{da}{a}$
Sphere of radius r 	$\dfrac{4\pi}{3}r^3$	$\dfrac{4}{3}\dfrac{dr}{r}$
Cylinder of radius r, height h 	$\pi r^2 h$	$2\dfrac{dr}{r}+\dfrac{dh}{h}$

from points of application of load depends only on the static resultant of the loads, and not on the distribution of loads. This principle is used to replace complicated stress distributions or weak boundary conditions into ones that are easier to solve, as long as that boundary is geometrically short.

2.1.4 Hooke's Law

There is corresponding deformation of body with respect of applied force. The relationship between stress and strain is simply described by Hooke's law in the form of constitutive relations[2].

According to *Hooke's law*[3], within its elastic limits, strain developed in a material is proportional to applied stress [Fig. 2.4]. Materials for which Hooke's law is a useful approximation are known as *linear-elastic* or *Hookean materials*.

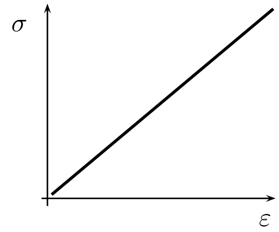


Figure 2.4 | Hooke's law.

The relations based on Hooke's law for the three sets of stress–strain give three types of elastic constants: modulus of elasticity, modulus of rigidity and bulk modulus of elasticity. These constants represent the rigidity of the material element against the external forces trying to deform the material. The higher the value of the constants, the lesser is the deformation possible. All these constants have relevance within elastic limits only. These are described as follows:

1. Modulus of Elasticity For a material element subjected to longitudinal stress σ, the respective strain ε is given by [Fig. 2.4],

$$\varepsilon \propto \sigma$$

Elasticity of a material is described by the constant of proportionality of the above expression. The Hooke's law is written in appropriate form as

$$E = \frac{\sigma}{\varepsilon} \tag{2.3}$$

where constant E is the known as *modulus of elasticity* or *Young's modulus*[4].

Hoop Stress (σ_h) is the stress developed in a cylinder or a ring along the periphery (i.e., perpendicular to radius) due to any type of force (pressure, temperature). If the diameter of a ring changes from d_i to d_f then the hoop stress is determined as

$$\sigma_h = \frac{d_f - d_i}{d_i}E$$

2. Modulus of Rigidity The relation between shear strain (γ) and shear stress (τ) for a material is expressed as

$$G = \frac{\tau}{\gamma} \tag{2.4}$$

where G is called *modulus of rigidity*.

3. Bulk Modulus of Elasticity In case the material is subjected to volumetric stress σ (pressure),

[2]In physics, a constitutive equation is a relation between two physical quantities that is specific to a material or substance, and approximates the response of that material to external forces. For example, in structural analysis, constitutive relations connect applied stresses or forces to strains or deformations.

[3]Hooke's law is named after the 17th century British physicist Robert Hooke. He first stated this law in 1660, and published its solution in 1678 as "As the extension, so the force".

[4]Young's modulus is named after Thomas Young, the 19th century British scientist, but the concept was developed in 1727 by Leonardo Euler.

the volumetric strain (ε_v) is related to stress by Hooke's law as

$$K = \frac{\sigma}{\varepsilon_v} \tag{2.5}$$

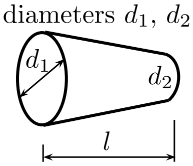
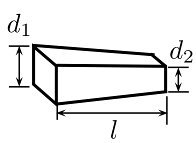
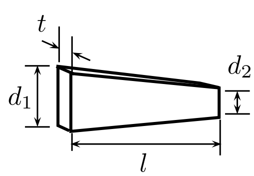
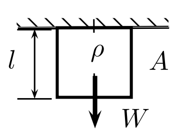
where K is known as the *bulk modulus of elasticity*.

The speed of acoustics (a) in a given material is related to its modulus of rigidity K and density ρ as

$$a = \sqrt{\frac{K}{\rho}}$$

Some examples of longitudinal deformation are shown in Table 2.2.

Table 2.2 Deformation Formulas

Object	δ_l
1. Circular taper of length l and diameters d_1, d_2 	$\dfrac{4Pl}{\pi d_1 d_2 E}$
2. Square taper of length l and widths d_1, d_2 	$\dfrac{Pl}{d_1 d_2 E}$
3. Simple taper of length l, thickness t and widths d_1, d_2 	$\dfrac{Pl \ln(d_1/d_2)}{Et(d_1 - d_2)}$
4. Deformation due to self weight W in a bar of length l, and uniform area A 	$\dfrac{1}{2}\left(\dfrac{Wl}{AE}\right)$

2.1.5 Bar of Uniform Strength

Consider a bar of varying cross-sectional area (A) loaded by its self-weight [Fig. 2.5]. The bar is said to have uniform strength when the stress at any point in the bar is uniform.

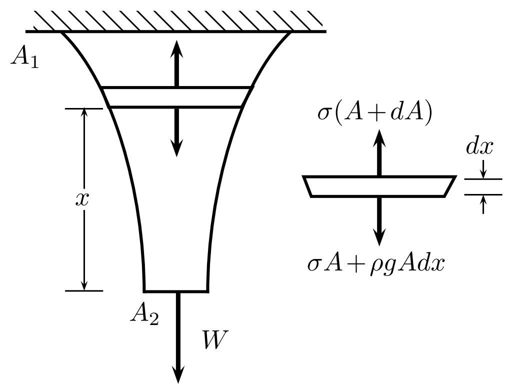


Figure 2.5 Bar of uniform strength.

The density of material is ρ. Let σ be the uniform tensile stress at any section. For an infinitesimal element of length dx at distance x from the free end, the equilibrium is given by

$$\begin{aligned} \sigma(A+dA) &= \sigma A + \rho g A dx \\ \sigma dA &= \rho g A dx \\ \frac{dA}{A} &= \frac{\rho g}{\sigma} dx \\ \ln A &= \frac{\rho g}{\sigma} x + c \end{aligned}$$

The value of constant c can be found using the boundary condition $x = 0$, $A = A_2$ for which

$$c = \ln A_2$$

Therefore,

$$\begin{aligned} \ln\left(\frac{A}{A_2}\right) &= \frac{\rho g}{\sigma} x \\ A &= A_2 \exp\left(\frac{\rho g}{\sigma} x\right) \end{aligned}$$

This is the expression for the exponential profile of cross-sectional area for the beam of uniform strength σ and density ρ.

2.1.6 True Stress and True Strain

Consider a general case of a bar of original length l_0 and cross-sectional area A_0, subjected to axial force F. Normal tensile stress at any point is $\sigma_0 = F/A_0$ and normal strain is $\varepsilon_0 = \delta l / l_0$. Therefore,

$$\delta l = \frac{P l_0}{A_0 E}$$

By definition of strain, the length of the bar after deformation is

$$\begin{aligned} l &= l_0 + \delta l \\ &= l_0 + \varepsilon_0 l_0 \\ &= l_0 (1 + \varepsilon_0) \end{aligned}$$

For volume constancy during the deformation,

$$A_0 l_0 = Al$$
$$\frac{A}{A_0} = \frac{l_0}{l}$$

Therefore,

$$\frac{l}{l_0} = \frac{A_0}{A} = (1 + \varepsilon_0) \tag{2.6}$$

Thus, the true values of stress and strain, measured with true dimensions of the deformed body based on volume constancy, are different from engineering stress and strain based on original dimensions. The true values are related engineering values as follows:

1. True Stress *True stress* (σ) is the instantaneous stress at a given instant:

$$\text{True stress} = \frac{\text{Load}}{\text{Actual cross-section}}$$
$$\begin{aligned} \sigma &= \sigma_0 \times \frac{A_0}{A} \\ &= \sigma_0 \times \frac{l}{l_0} \end{aligned}$$

Using Eq. (2.6), σ and σ_0 are related as

$$\sigma = \sigma_0 (1 + \varepsilon_0) \tag{2.7}$$

2. True Strain Similarly, *true strain* (ε) is defined as the deformation per unit instantaneous length during the deformation of a given material at a given instant:

$$\text{True strain} = \frac{\text{Deformation}}{\text{Instantaneous dimension}}$$
$$\begin{aligned} \varepsilon &= \int_{l_0}^{l} \frac{dl}{l} \\ &= \ln\left(\frac{l}{l_0}\right) \\ &= \ln\left(\frac{A_0}{A}\right) \end{aligned}$$

Using Eq. (2.6), ε and ε_0 are related as

$$\varepsilon = \ln (1 + \varepsilon_0) \tag{2.8}$$

2.1.7 Poisson Ratio

When a material is subjected to stress in a direction, due to continuity or molecular bonding, the material gets deformed in the other directions also. In other way, when the bonds elongate in the direction of load, they shorten in the other directions. This behavior multiplied millions of times throughout the material lattice is what drives the phenomenon. Therefore, deformation in one direction is related to deformation in the other direction also. This relation is expressed by *Poisson's ratio*[5] (μ), defined as the ratio of lateral strain and axial strain:

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Axial strain}}$$

Some materials, known as *auxetic materials*, display a negative Poisson's ratio. For example, if a paper is stretched in an in-plane direction, it expands in its thickness direction due to its network structure.

The relevance and role of Poisson's ratio can be seen in following cases:

1. Bi-Axial Stress System When a material element is subjected to a bi-axial stress system [Fig. 2.6], the strain sin x and y directions are written as

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \mu\sigma_y)$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \mu\sigma_x)$$

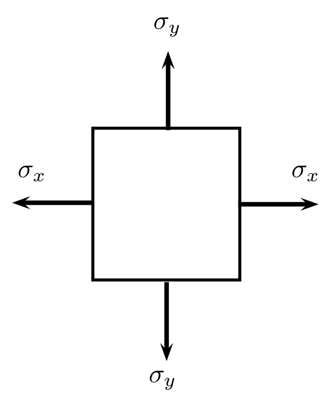


Figure 2.6 Bi-axial stress system.

The effect of Poisson's ratio μ is incorporated in lateral strains through lateral stress σ_y.

[5] Poisson's ratio is named after Simeon Denis Poisson (1781-1840), a French mathematician, geometer, and physicist, who first observed this relationship and found its value for many isotropic materials.

2. *Tri-Axial Stress System* When a material element is subjected to tri-axial stress system [Fig. 2.7], the strains in x, y and z directions are written as

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \mu\left(\sigma_y + \sigma_z\right)\right]$$
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \mu\left(\sigma_z + \sigma_x\right)\right]$$
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu\left(\sigma_x + \sigma_y\right)\right]$$

The effect of Poisson's ratio μ is incorporated in lateral strains through lateral stresses.

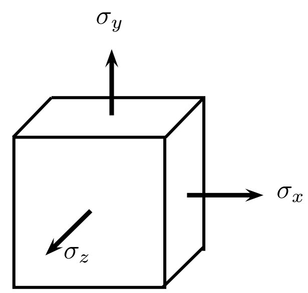


Figure 2.7 Tri-axial stress system.

One area in which Poisson's effect has a considerable influence is in pressurized pipe flow. When the fluid inside a pipe is highly pressurized, it exerts a uniform force on the inside of the pipe, resulting in a radial stress within the pipe material. Due to Poisson's effect, this radial stress will result into hoop and longitudinal stresses which cause the pipe to slightly increase in diameter and decrease in length.

2.1.8 Elastic Constants

Due to the effect of Poisson's ratio (μ), strain in one direction develops the corresponding strains in lateral directions. This concept can be used to relate three of elastic constants (E, G, and K) by incorporating μ, as explained in following sections.

2.1.8.1 Relation of E, G and μ Consider a rectangular block of size a subjected shear stress τ which results shear strain $\gamma = \tau/G$ (G is modulus of rigidity of the material) [Fig. 2.8]. Due to this, the top face of the block is shifted by $a\gamma$. The elongation of the diagonal AB is found to be equal to $a\gamma/\sqrt{2}$.

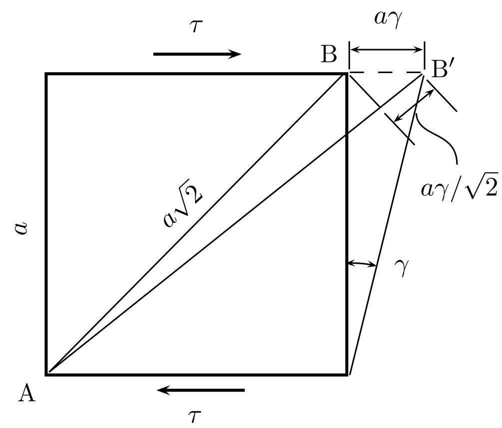


Figure 2.8 Relation between E and G.

Longitudinal strain along the diagonal is determined as

$$\begin{aligned}\varepsilon &= \frac{a\gamma/\sqrt{2}}{a\sqrt{2}}\\ &= \frac{\gamma}{2}\\ &= \frac{\tau}{2G}\end{aligned} \tag{2.9}$$

Normal stress along the diagonal is τ. Therefore, normal strain along the diagonal can also be determined by incorporating Poisson ratio μ as

$$\begin{aligned}\varepsilon &= \frac{1}{E}\left(\tau + \mu\tau\right)\\ &= \frac{\tau}{E}\left(1 + \mu\right)\end{aligned} \tag{2.10}$$

The strains represented by Eqs. (2.9) and (2.10) are the same, therefore

$$\begin{aligned}\frac{\tau}{2G} &= \frac{\tau}{E}\left(1 + \mu\right)\\ G &= \frac{E}{2\left(1 + \mu\right)}\end{aligned} \tag{2.11}$$

This equation represents the relationship among E, G, and μ.

2.1.8.2 Relation of E, K and μ Isotropic materials are characterized by properties which are independent of direction in space. When an isotropic[6] material is

[6] In practical, materials are not isotropic. General case is for anisotropic materials for which each material property is dependent upon the direction. In an anisotropic material, due to symmetry, number independent elastic constants is 21 while for an isotropic material these are only 2. That means, if two of E, G, K, μ are known then other can be calculated based on compatibility equations.

subjected to uniform tri-axial stresses σ in all three directions, the stress σ is called *spherical stress* [Fig. 2.9]. The strain in any direction (say i = 1, 2, 3) is written as

$$\varepsilon_i = \sigma \frac{1-2\mu}{E}$$

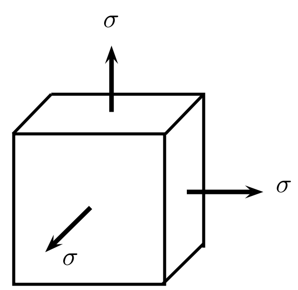


Figure 2.9 Relation between E and K.

Taking the element as cubic in dimension, the volumetric strain is written as

$$\varepsilon_v = 3\varepsilon_i$$
$$= 3\sigma \frac{1-2\mu}{E}$$

Hence, by the definition of bulk modulus of elasticity,

$$K = \frac{\sigma}{\varepsilon_v}$$
$$K = \frac{E}{3(1-2\mu)} \tag{2.12}$$

This expression represents the relationship among E, K, and μ.

2.1.8.3 Relation of E, K and G Eliminating μ from Eqs. (2.11)–(2.12),

$$E = \frac{9KG}{3K+G} \tag{2.13}$$

Eqs. (2.11)–(2.13) can be used to deduce the following points:

1. Each elastic constant is related to the other constant through μ. Therefore, μ along with any of the three elastic constants can be used to represent elastic properties of the material. It means, for an isotropic material, there are only two independent elastic constants.
2. Maximum value of Poisson ratio can be found as $(1-2\mu) > 0$, therefore, $\mu < 0.5$. Similarly, minimum value of Poisson ratio can be found as $(1+\mu) > 0$, therefore, $\mu > -1.0$. Therefore,
$$-1.0 < \mu < 0.5$$
3. When $\mu = 0$, maximum values of G and minimum value of K are found as
$$G_{max} = \frac{E}{2}$$
$$K_{min} = \frac{E}{3}$$

An important parameter known as *Lame's constant*[7] (in plane strain condition [*Section* 2.3.2]) is defined as

$$\lambda = \frac{E\mu}{(1+\mu)(1-2\mu)} \tag{2.14}$$

2.1.9 Principle of Superposition

According to the principle of *superposition of forces*, when a Hookean body is acted upon by a number of external forces on various points, then the resultant effect on the body is the vector sum of the effects caused by each of the loads acting independently on the respective points of the body.

The principle is equally applicable in measuring the effects in terms of stress, strain and deflection. However, it is not applicable to materials with non-linear characteristics.

2.1.10 Thermal Loads

Thermal expansion is the tendency of matter to change volume in response to a change in temperature. When a substance is heated, its particles begin moving more rapidly and thus usually maintain a greater average separation, and the material responds to changes in its temperature by expansion or contraction.

In the analysis of thermal loads, therefore, the *coefficient of thermal expansion* (α) becomes a very important property of the material which measures the fractional change in size per degree change in temperature at a constant pressure.

Consider a material element, having modulus of elasticity E and coefficient of thermal expansion α, subjected to a temperature change ΔT. If the material is not free; the expansion or contraction is restricted, then, the stresses are analyzed through corresponding imaginary *thermal strain* (ε_t) and *thermal stress* (σ_t), as follows:

1. *Thermal Strain* The imaginary thermal strain is determined as
$$\varepsilon_t = \frac{\alpha \Delta T l}{l}$$
$$= \alpha \Delta T$$

[7]Lame's constant is named after Gabriel Lame (1795-1870), a French mathematician.

2. *Thermal Stress* Using Hooke's law, the corresponding thermal stress is written as

$$\begin{aligned}\sigma_t &= E\varepsilon_t \\ &= E\alpha\Delta T\end{aligned}$$

In case the expansion is not restricted, only thermal strain would exist in the material.

2.2 DYNAMIC LOADING

A *static load* is slowly applied, gradually increasing from zero to its maximum value, thereafter, the load remains constant. Generally, when strength of the machine elements is considered, it is assumed that the loading is static.

Consider a material element having cross-sectional area A and length l. A static load W, applied gradually, produces a normal stress:

$$\sigma_{st} = \frac{W}{A} \tag{2.15}$$

For modulus of elasticity E, the material gets elongated by

$$\delta_{st} = \frac{Wl}{AE}$$

In many practical applications, the loads are dynamic in character (i.e., they vary with time). When loads are applied suddenly and as *impact loads*, the resulting transient stresses (and deformations) induced in the machine elements are much higher than if the loads are applied gradually. Impact loads result in shock waves propagating through the elements with possible serious consequences. Therefore, it is essential to predict the response of materials against dynamic loads also.

The two categories of dynamic loads are impact loads and shock loads. It is possible to complete a relatively simply stress evaluation for suddenly applied and impact loads by using the principle of conservation of energy and the condition that the materials considered are operating within their elastic regions.

2.2.1 Impact Load

Consider a loading regime with a weight W being dropped through a height h onto the cross-section of a vertical bar, which behaves as a spring of stiffness k. The support bar has a length l, an area A and modulus of elasticity E [Fig. 2.10].

The weight would impact onto the bar which would elastically deform until all of the potential energy has been absorbed. The bar would then contract initiating damped oscillations until the system assumes a static position. The principle of work and energy in form of following equation determines the initial maximum deformation δ_{max}, the highly stressed condition:

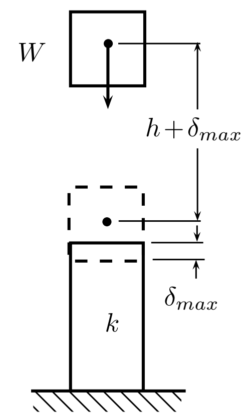


Figure 2.10 | Impact load.

$$W\left(h+\delta_{max}\right) = \left(\frac{1}{2}k\delta_{max}\right)\delta_{max}$$

Solution of this quadratic equation[8] gives

$$\delta_{max} = \frac{W}{k} + \sqrt{\left(\frac{W}{k}\right)^2 + 2\frac{W}{k}h} \tag{2.16}$$

When the load is applied gradually, the material get elongated by

$$\begin{aligned}\delta_{st} &= \frac{W}{k} \\ &= \frac{Wl}{AE}\end{aligned}$$

Substituting this into Eq. (2.16),

$$\delta_{max} = \delta_{st}\left\{1+\sqrt{1+\frac{2h}{\delta_{st}}}\right\} \tag{2.17}$$

The ratio of maximum deflection (δ_{max}) and static deflection (δ_{st}) is called *impact factor*, which can be written as

$$\frac{\delta_{max}}{\delta_{st}} = 1+\sqrt{1+\frac{2h}{\delta_{st}}}$$

[8]Solution of a quadratic equation, $ax^2+bx+c=0$, is written as

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

2.2.2 Shock Load

Impact loads discussed earlier are applied through a height (potential energy), but shock or suddenly applied loads are without potential energy. For suddenly applied loads, $h = 0$ in Eq. (2.17),

$$\delta_{max} = 2\delta_{st} \tag{2.18}$$

This indicates that shock loads of equal values result in the deflection that is two times of the static deflection.

2.3 BI-AXIAL STRESS AND STRAIN

The choice of material and dimensions is based on the the magnitude and direction of maximum and minimum stresses. *Tensor* is a quantity that is used to express many dimensions, like a vector. For a material point subjected to three-dimensional stresses, tensors are used to express all components of stresses and strains respectively as

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_{yy} & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \varepsilon_{zz} \end{bmatrix}$$

Engineering shear strain (γ_{xy}) is related to *tensor shear strain* (ε_{xy}) as

$$\gamma_{xy} = 2\varepsilon_{xy}$$
$$\gamma_{yz} = 2\varepsilon_{yz}$$
$$\gamma_{zx} = 2\varepsilon_{zx}$$

The material behavior is generally represented in the form of constitutive equation:

$$\sigma = C\varepsilon$$

where C is the matrix of elastic constants.

The state of stress at a point in the body is defined by all the stress vectors associated with all planes (infinite in number) that pass through that point. However, according to *Cauchy's stress theorem*, merely by knowing the stress vectors on three mutually perpendicular planes, the stress vector on any other plane passing through that point can be found through coordinate transformation equations.

The stress analysis is simplified by treating the structure as one-dimensional or two-dimensional based on the physical dimensions and the distribution of loads. Cauchy's stress theorem is investigated for two types of bi-axial systems: plane stress and plane strain conditions.

2.3.1 Plane Stress

Plane stress is defined to be a state of stress in which the normal stress σ_z, and the shear stresses, σ_{xz} and σ_{yz}, directed perpendicular to the x-y plane are assumed to be zero. In case of plane stress ($\sigma_z = 0$), the strains in x and y directions are written as

$$\varepsilon_x = \frac{\sigma_x - \mu\sigma_y}{E}$$
$$\varepsilon_y = \frac{\sigma_y - \mu\sigma_x}{E}$$

Solving these equations for σ_x and σ_y,

$$\sigma_x = \frac{E(\varepsilon_x + \mu\varepsilon_y)}{1-\mu^2}$$
$$\sigma_y = \frac{E(\mu\varepsilon_x + \varepsilon_y)}{1-\mu^2}$$

Using Eq. (2.9), the shear stress in x-y plane is given by

$$\begin{aligned}\tau_{xy} &= \gamma_{xy}G \\ &= \frac{E\gamma_{xy}}{2(1+\mu)}\end{aligned}$$

Therefore, the constitutive relation for plane stress condition is represented as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\mu^2}\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

The plane stress condition is the simplest form of behavior for continuum structure and represents situations frequently encountered in practice. For example, when loads are applied uniformly over the thickness of a plate; acting in the plane of the plate.

Fig. 2.11 shows an infinitesimal material of dimensions δx, δy and δz, subjected to stresses σ_x and σ_y along x and y axes, respectively, and shear stress τ_{xy} working downward when seen at x-axis accompanied by shear stresses τ_{xy} to balance the moments. Consider a material plane having its normal at angle θ from x-axis. The equilibrium of a part either left or right of the plane can be examined as follows:

1. Along σ_θ Equilibrium of forces along the normal of the new plane can be written as

$$\begin{aligned}\sigma_\theta \times \delta s = {} & \sigma_x \delta s \cos\theta \times \cos\theta + \sigma_y \delta s \sin\theta \times \sin\theta \\ & -\tau_{xy}\delta s \sin\theta \times \cos\theta - \tau_{xy}\delta s \cos\theta \times \sin\theta\end{aligned}$$

Hence,

$$\sigma_\theta = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta - \tau_{xy}\sin 2\theta \tag{2.19}$$

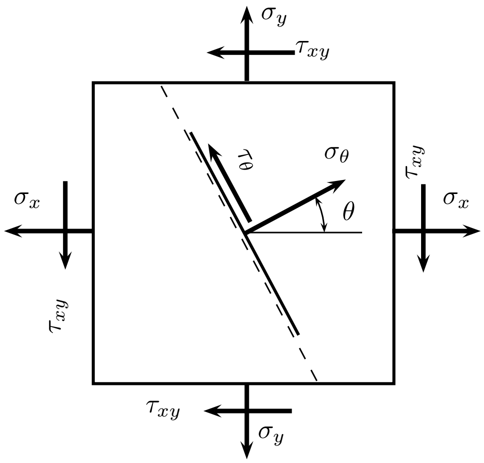


Figure 2.11 Bi-axial stresses.

For $\theta \to (\theta+\pi/2)$ (orthogonal plane),

$$\sigma_{(\theta+\pi/2)} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau_{xy}\sin 2\theta \tag{2.20}$$

2. *Along τ_θ* Equilibrium of forces along the tangent of the new plane can be written as

$$\tau_\theta = \sigma_x \delta s \cos\theta \times \sin\theta - \sigma_y \delta s \sin\theta \times \cos\theta + \tau_{xy}\delta s \sin\theta \times \cos\theta + \tau_{xy}\delta s \cos\theta \times \sin\theta$$

Therefore,

$$\tau_\theta = \sigma_x \frac{\sin 2\theta}{2} - \sigma_y \frac{\sin 2\theta}{2} + \tau_{xy}\cos 2\theta \tag{2.21}$$

Using Eqs. (2.19)–(2.21), the transformation equation in matrix form is written as

$$\begin{Bmatrix} \sigma_\theta \\ \sigma_{\left(\theta+\frac{\pi}{2}\right)} \\ \tau_\theta \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -\sin 2\theta \\ \sin^2\theta & \cos^2\theta & \sin 2\theta \\ \frac{\sin 2\theta}{2} & -\frac{\sin 2\theta}{2} & \cos 2\theta \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

This equation can be used to calculate the stresses in any arbitrary plane at angle θ in a material element under plane stress condition.

2.3.2 Plane Strain

Plane strain is a condition of a body in which the displacements of all points in the body are parallel to a given plane, and the values of these displacements do not depend on the distance perpendicular to the plane. In other words, plane strain is defined to be a state of strain in which the strain normal to the x-y plane, ε_z, and the shear strains, γ_{xz} and γ_{yz}, are assumed to be zero.

The constitutive relation for plane strain condition can be derived by taking $\sigma_z = 0$ and

$$\sigma_z = \frac{\sigma_z - \mu(\sigma_x + \sigma_y)}{E}$$
$$\varepsilon_x = \frac{\sigma_x - \mu(\sigma_y + \sigma_z)}{E}$$
$$\varepsilon_y = \frac{\sigma_y - \mu(\sigma_z + \sigma_x)}{E}$$

Using Eq. (2.9), the shear stress in x-y plane is given by

$$\tau_{xy} = \gamma_{xy}G = \frac{E\gamma_{xy}}{2(1+\mu)}$$

Solution of these equations gives following equation:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \times \begin{bmatrix} 1-\mu & \mu & 0 \\ \mu & 1-\mu & 0 \\ 0 & 0 & \frac{1}{2}-\mu \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2.22}$$

Plane strain is considered for the situations in which the dimension of the structure is one directional; that is, one dimension (say z) is very large in comparison with the other dimensions. For example, in the analysis of dams, tunnels, and other geotechnical works. Plain strain condition is also encountered in small-scale problems, such as bars and rollers subjected to forces normal to their cross-section.

Like bi-axial stress system, strains also have their components in three-dimensions. Similar to Eq. (2.22), strain components in two-dimensional case can be related as

$$\begin{Bmatrix} \varepsilon_a \\ \varepsilon_b \\ \gamma_{ab} \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -\frac{\sin 2\theta}{2} \\ \sin^2\theta & \cos^2\theta & \frac{\sin 2\theta}{2} \\ \frac{\sin 2\theta}{2} & -\frac{\sin 2\theta}{2} & \frac{\cos 2\theta}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2.23}$$

2.4 MOHR'S CIRCLE

Mohr's circle[9] is a graphical method of representing stresses and strains. This enables calculation of components of stresses and strains in an arbitrary oblique plane.

[9]Mohr's circle is named after Christian Otto Mohr (1835–1918), a German civil engineer, developed in 1982.

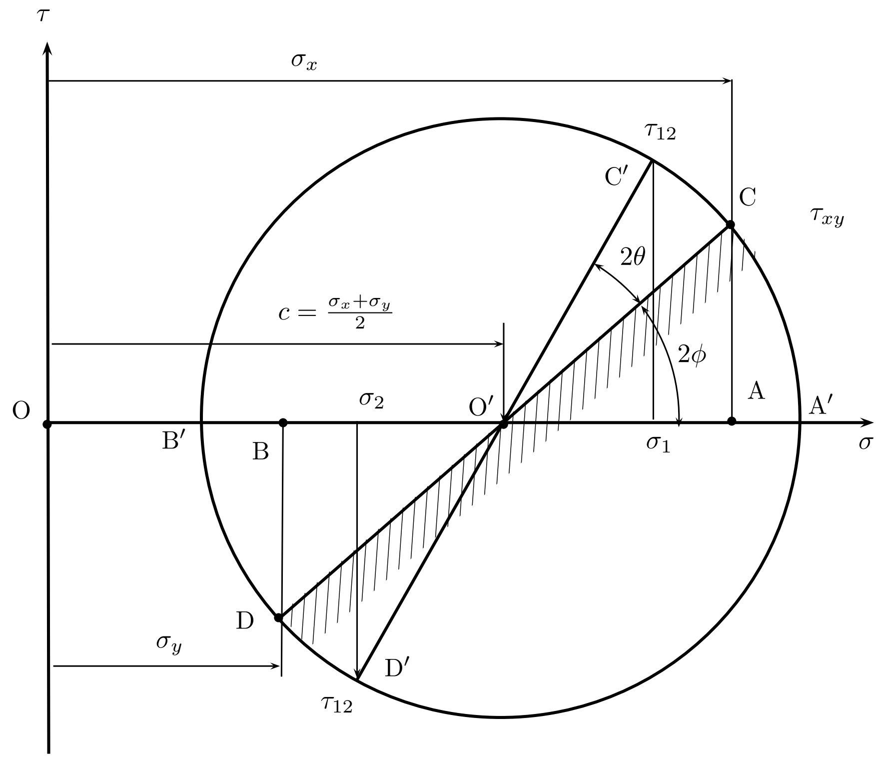


Figure 2.12 Mohr's circle of stresses.

2.4.1 Mohr's Circle for Plane Stress

Eq. (2.22) represents the manner in which σ_θ and τ_{xy} vary w.r.t. θ in plane stress condition. It depicts a circular fashion and offers an alternative way to calculate stresses in oblique plane by drawing *Mohr's circle for stresses* having longitudinal and shear stresses on x and y axes, respectively [Fig. 2.12].

The Mohr's circle for stresses is drawn in the following simple steps:

1. Take origin O as the reference point.
2. Find points A and B by mapping σ_x and σ_y on the x-axis.
3. Find point C and D by mapping τ_{xy} along y-axis from points A and B in opposite directions respectively. Points C and D denote the stresses on the x and y planes, respectively.
4. Draw a full circle on CD as the diameter; point O′ on x-axis as the center.
5. To evaluate stresses on an oblique plane having its normal at angle θ from x-axis in anti-clockwise direction [Fig. 2.11], find point C_θ by rotating diameter CD with angle 2θ anti-clockwise to find C′D′.

A circle is defined by the coordinate of center point and its radius. This can be examined in Mohr's circle for stresses as follows:

1. *Radius* Radius of Mohr's circle is found as

$$R_\sigma = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

   This is also the value of maximum shear stress possible.

2. *Coordinates of Center* The center of Mohr's circle is always on x-axis, therefore, x-coordinate is given by

$$\mathrm{OO'} = \frac{\sigma_x + \sigma_y}{2}$$

Following points can be deduced from Mohr's circle of stresses:

1. *Principal Planes* The extreme points of the circle on x-axis, A′ and B′, show the maximum and minimum possible normal stresses on two orthogonal planes where shear stress is zero. These planes are known as *principal planes* located at an angle ϕ with x-axis in clockwise direction. The value of ϕ can be calculated by solving

$$\tau_\theta = 0$$

For $\theta \to -\phi$,

$$\sigma_x \frac{\sin 2\phi}{2} - \sigma_y \frac{\sin 2\phi}{2} = \tau_{xy} \cos 2\phi$$

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

This is the expression for orientation of principle plane, where shear stress is zero. The principal stresses can be determined as

$$\sigma_{A'} = \mathrm{OO}' + R_\sigma$$
$$\sigma_{B'} = \mathrm{OO}' - R_\sigma$$

2. *Maximum Shear Stress* The maximum shear stress occurs at the diameter orthogonal to AB. Therefore, the plane of maximum in-plane shearing stress are inclined at $90°/2 = 45°$ with the principal planes. The value of maximum shear stress is equal to the radius of the Mohr's circle.

2.4.2 Mohr's Circle for Plane Strain

Strains are also presented by drawing *Mohr's circle for strains* in which normal strains (ε) are represented on x-axis, and half of shear strains ($\gamma/2$) are represented on y-axis. In fact, Mohr's circle for strains is based on tensor strains [Fig. 2.13].

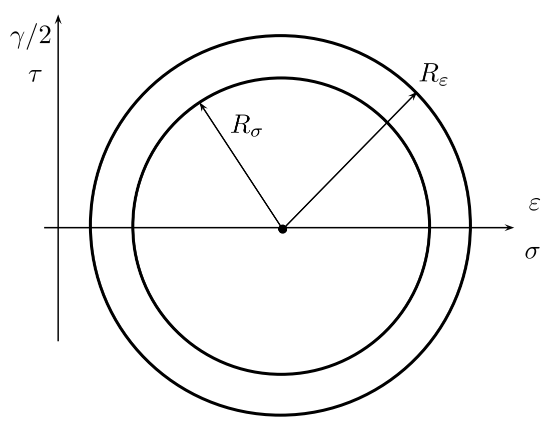


Figure 2.13 | Mohr's circle for strain.

The functions of the transformation matrix of Eq. (2.23), involving rotation of shear strains are found to be half of those in Eq. (2.22). Therefore, Mohr's circle for strains can be drawn in the same manner as that for stresses, except taking shear strain in tensor form. Following is the relationship between radii of these circles:

$$R_\sigma = R_\varepsilon \times \frac{E}{1+\mu}$$

where R_σ and R_ε are the radii of Mohr's circles for stress and strain, respectively, E and μ are Young's modulus and Poisson's ratio of the material, respectively.

2.5 BEAM LOADING

In most of the structures, the members must resist forces applied laterally or transversely to their axes. Such members are called *beams*, such as an axle of a car, shafts of machinery. A beam can be subjected to axial force, shear force and bending moment.

Different types of cross-sections of beams are shown in Fig. 2.14.

2.5.1 Types of Beams

Based on the number reactions w.r.t. equilibrium equations, the beams are classified into two types:

1. *Statically Determinate Beams* *Statically determinate beams* are those for which equilibrium conditions are sufficient to compute all the reactions and draw free body diagrams.

2. *Statically Indeterminate Beams* *Statically indeterminate beams* are those for which static equilibrium conditions are insufficient to compute all the reactions and draw free body diagrams. The number of reactions in excess of the number of equilibrium equations is called the *degree of static indeterminacy*, which is zero for statically determinate beams.

   Additional information regarding deflection (*compatibility conditions*) is essential for analyzing statically indeterminate beams.

Present study concerns with statically determinate systems, such as a simply supported beam (pinned at one end and roller supported at the other end), a cantilever (fixed at one end and free at the other end). Fixed beams have both the ends fixed; the ends have neither deflection nor slope.

2.5.2 Transverse Loads

Different types of mechanical components and bodies are subjected various types of loads in their applications. For analysis, these loads are categorized into following [Fig. 2.15]:

1. *Point Load* A point load acts on a point on the span of a beam structure. It results in a moment at another point, equal to the force multiplied by its distance of a given point. The moment of a force F acting at mid span w.r.t. any end point [Fig. 2.15] is

$$M = F \times \frac{l}{2}$$

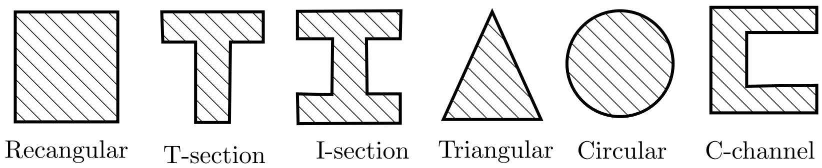


Figure 2.14 Types of cross-section.

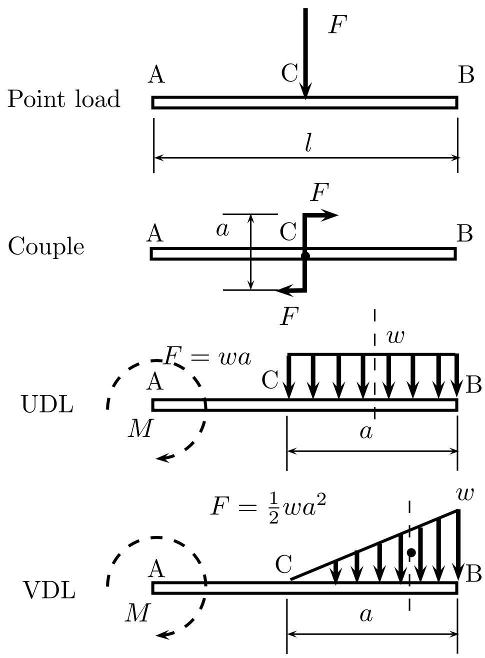


Figure 2.15 Transverse loads.

2. *Couple* A couple consists of two equal and opposite forces at equal distance from the given point. The net force acting on the beam is zero. Moment of a couple constituted by two equal and opposite forces of magnitude F acting at mid span [Fig. 2.15] is

$$M = F \times a$$

3. *Uniformly Distributed Load* A *uniformly distributed load* (UDL) consists of several equally distributed point loads. It is represented by its constant intensity (w) (load per unit length). The equivalent to this is a point load equal to the intensity multiplied by its span over which it is distributed. This force acts at the centroid (i.e., mid span of UDL). The resulting moment at a point is calculated by integral sum of intensity multiplied by the distance of this point. For the UDL [Fig. 2.15], the total vertical load, and moment about left end are given by

$$F = wa$$
$$M = wa\left(l - \frac{a}{2}\right)$$

4. *Uniformly Varying Load* A *uniformly varying load* (UVL) consists of several distributed point loads with varying intensity. It is represented by its peak intensity w. The equivalent to this is a point load equal to integral sum of the intensity multiplied by its span over which it is distributed. This point load acts at the centroid of the UVL. The resulting moment at a point is calculated by integral sum of varying intensity multiplied by the distance of the point.

For the UVL shown in Fig. 2.15, the intensity of UVL at distance x from pont C is given by

$$w_x = \frac{w}{a}x$$

Therefore, the total vertical load of UVL is calculated as

$$F = \int_0^a w_x dx$$

$$= \int_0^a \left(\frac{w}{a}x\right) dx$$
$$= \frac{wa^2}{2}$$

This load acts at center of gravity (CG) of UVL (i.e., at distance $a/3$ from B). Therefore, moment about end A is found as

$$M = \frac{wa^2}{2}\left(l + a - \frac{a}{3}\right)$$
$$= \frac{wa^2}{2}\left(l + \frac{2a}{3}\right)$$

2.5.3 Support Reactions

When all the forces are applied in a single plane (x, y), the following three equations of static equilibrium are

employed for the analysis:

$$\sum F_x = 0 \tag{2.24}$$

$$\sum F_y = 0 \tag{2.25}$$

$$\sum M_z = 0 \tag{2.26}$$

Using the above equations, all the support reactions can be calculated for statistically determinate beams.

Different types of supports are provided to get specific type of reaction components, as described in Fig. 2.16. For example, a fixed support offers both horizontal and vertical reactions along with moment. A hinged support does not offer moment but only horizontal and vertical reactions. A roller support or a simple support offers only a vertical reaction.

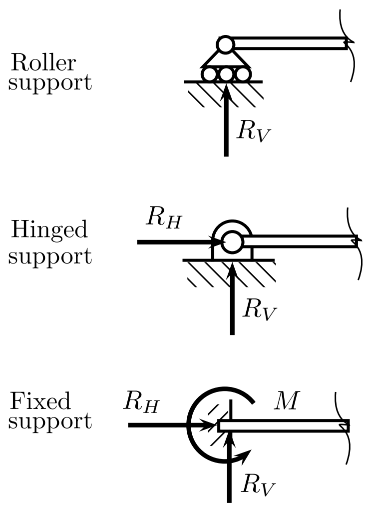


Figure 2.16 Support reactions.

2.5.4 SF and BM Diagrams

A beam can be loaded with different types of loads but their effect at a given section of the beam is examined in two forms of loads:

1. Shear Force *Shear force* (SF) is defined for a given section as the sum of all the forces acting on either side trying to shear the sectional area.

2. Bending Moment *Bending moment* (BM) is defined for a given section as the sum of moments of all forces and couples acting on either side trying to bend the material body at the section.

The knowledge of shear forces and bending moments is essential to determine the resulting shear stresses and bending stresses in the beams.

Figure 2.17 show the sign conventions for shear forces and bending moments adopted in this study as in most of the literature.

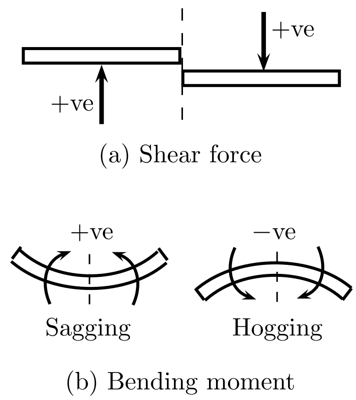


Figure 2.17 Sign conventions for SF and BM.

The sign of shear force and bending moment is decided based on the direction of forces w.r.t. the section (right or left) considered for the analysis:

1. Sign of Shear Force Upward forces on left section and downward forces or right section are taken positive.

2. Sign of Bending Moment Sagging moments are taken positive and hogging moments are taken negative.

The profiles of shear force and bending moment are represented graphically by plotting their values against span length (say x). These diagrams graphically show the variation of the SF and BM and highlight the significant points, such as of maximum SF and BM that enables the design process.

The procedure of drawing SF and BM diagrams starts with visualizing the beam as free body. Using the loads acting on it, the reactions at support must be determined first by incorporating equations of equilibrium [*Section* 2.5.3]. The sections between loads are considered one by one, proceeding continuously along the length of beam from right to left, or left to right directions. Shear force and bending moments are calculated at each section according to the definitions and sign conventions of these quantities.

Consider a section at distance x over the span of a beam. The force at this section is represented by a discrete or continuous function $f(x)$. By definitions [*Section* 2.5.4], the shear force $F(x)$ and bending

moment $M(x)$ at the section are found as

$$F(x) = \sum_0^{x_s} f(x) = \frac{1}{x_s}\int_0^{x_s} f(x)dx$$

$$M(x) = \sum_0^{x_s} f(x)x = \int_0^{x_s} f(x)dx$$

Thus, shear force at any location x can be written as

$$F(x) = \frac{dM(x)}{dx} = \text{Slope of BM diagram} \tag{2.27}$$

Similarly, bending moment can be written as

$$M(x) = \int F(x)dx = \text{Area of SF diagram} \tag{2.28}$$

The intensity (w) of loading (i.e., load per unit length of the beam) can be determined as

$$w(x) = \frac{dF(x)}{dx} = \text{Slope of SF diagram}$$

The following points can be deduced from above discussion:

1. Eq. (2.27) indicates that a constant shear force produces a uniform change or slope in the bending moment, resulting in straight line in the moment diagram. Zero shear force between any two points indicates no change in bending moment between those points; the slope of BM diagram is zero.

2. The bending moment can have the maximum or minimum value where the shear is zero (i.e., where SF changes its sign).

3. Using Eq. (2.28), the area under the SF diagram between x_1 and x_2 is given by

$$\int_{x_1}^{x_2} F(x)dx = M(x_2) - M(x_1)$$

4. Using Eqs. (2.27) and (2.28), the relationship between signs of SF and BM can be predicted [Fig. 2.18].

5. The *point of inflexion* (or *inflection*) of a curve is a point at which the concavity or curvature changes its sign, while *point of contraflexure* is the point where bending moment changes its sign.

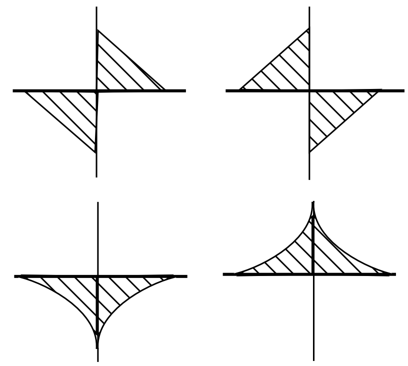


Figure 2.18 Relationship between SF and BM.

2.6 THEORY OF BENDING

When a material element of a body is subjected to external loads, resulting into shear forces and bending moments, it gets stressed and strained accordingly. The resulting stresses in axial directions are called *bending stresses* which vary according to the induced bending moments from point to point in a given section. The axial strains are different in different lamina of the same section due to different values of bending stresses. This results into *shear stresses* between lamina in the longitudinal direction. These stresses are described under following subsections.

2.6.1 Bending Stresses

Consider an initially straight body (say, beam) made of lamina of isotropic material, subjected to bending moment M due to which the body bends to the curvature ρ [Fig. 2.19].

An element, subtending an angle $d\theta$ at center of curvature O, is considered for analysis. On this element, deformation of a lamina situated at a distance y from central axis is

$$\delta = y d\theta$$

Strain in this particular lamina is

$$\varepsilon_y = \frac{\delta}{\rho d\theta} = \frac{y}{\rho}$$

Using Hooke's law [Eq. (2.3)], the bending stress at this point is written as

$$\sigma_y = E\varepsilon_y = E\frac{y}{\rho} \tag{2.29}$$

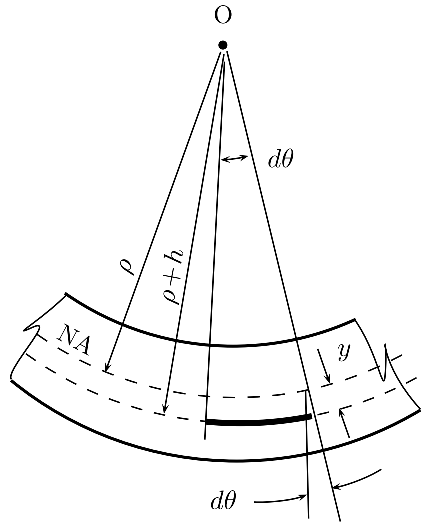


Figure 2.19 Bending stress in a lamina.

Two essential conditions for the equilibrium of this section are as follows.

1. The sum of forces along x-axis must be zero:

$$\int \sigma_y dA = 0$$
$$\frac{E}{\rho}\int y dA = 0$$
$$\int y dA = 0 \tag{2.30}$$

Therefore, y is to be measured from an axis such that $\int y dA = 0$. This axis is called *neutral axis* (NA). In other words, the natural axis is the transverse line in the given section about which sum of moments of area is zero.

2. Moment of all the forces about an axis must be zero:

$$\begin{aligned} M &= \int \sigma_y y dA \\ &= \int E\frac{y}{\rho} y dA \\ &= \frac{E}{\rho}\int y^2 dA \\ &= \frac{E}{\rho} I \end{aligned} \tag{2.31}$$

where I is called *area moment of inertia.*

Combining Eqs. (2.29) and (2.31),

$$\frac{\sigma_y}{y} = \frac{M}{I} = \frac{E}{\rho} \tag{2.32}$$

This equation is known as *flexure formula* (elastic-line equation) which can be used to determine the value of bending stress (σ_y) at the lamina located at distance y from the natural axis. The maximum value of bending stress occurs at the outer-most lamina (at a distance y_{max} from the natural axis), and is given by

$$\begin{aligned} \sigma_y^{max} &= \frac{M}{I} y_{max} \\ &= \frac{M}{Z} \end{aligned}$$

where Z ($= I/y_{max}$) is an important property of the cross-section, known as *section modulus.* Ends of a semi-elliptical leaf spring are made triangular in order to make M/I constant throughout the length of the spring.

2.6.2 Shear Stresses

Laminae at different positions from the natural axis bear different values of normal stresses, therefore, the strain in these laminae is also different resulting into shear stresses in the beam. Let τ_y be the shear stress induced in a lamina, having width b and elemental length dx, situated at y distance from natural axis [Fig. 2.20].

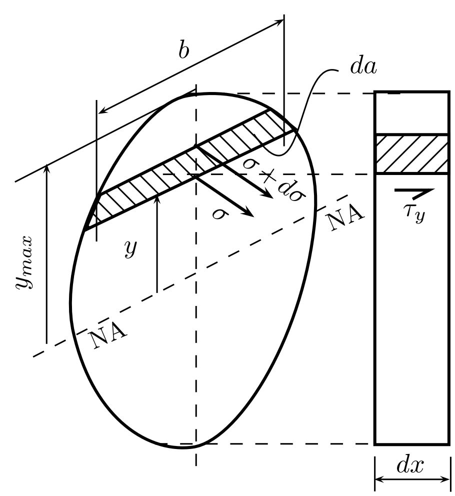


Figure 2.20 Shear stress in a lamina.

For equilibrium, the forces due to normal bending stresses along the length of the laminae and forces due to shear stresses in the interface of the laminae should be equal and opposite to each other:

$$\tau_y b dx = \int_y^{y_{max}} (d\sigma)\, da$$

where da is the area over which the stress variation is $d\sigma$. Thus,

$$\begin{aligned}\tau_y b dx &= \int_y^{y_{max}} \left(d\frac{M}{I}y\right) da \\ &= \frac{dM}{I}\int_y^{y_{max}} yda \\ \tau_y &= \frac{dM}{dx}\frac{1}{Ib}\int_y^{y_{max}} yda \\ &= \frac{F_s}{Ib}A\bar{y} \qquad (2.33)\end{aligned}$$

where F_s is the shear force at the section, $A\bar{y}$ is moment of area above the lamina.

The shear stress due to shear force at the lamina, above which area is A and its center of gravity is situated at distance $\bar{y}$ from the natural axis, is given by

$$\tau_y = \frac{F_s}{Ib}A\bar{y}$$

The shear stress at the lamina farthest from natural axis would be zero.

2.6.3 Area Moment of Inertia

The formula from the above derivations can be summarized as follows:

1. An axis through the cross-section is the natural axis of bending, if

$$\int ydA = 0$$

2. The flexure formula relates bending stress, bending moment, and curvature:

$$\frac{\sigma_y}{y} = \frac{M}{I} = \frac{E}{\rho}$$

The following are the two theorems useful in determining moment of inertia of an area w.r.t. parallel or perpendicular axes:

1. *Parallel Axis Theorem* Area moment of inertia (I_{xx}) about any axis (xx) of a section of area A is related to the area moment of inertia about a parallel axis through the centroid (I) and the perpendicular distance between the axes d as [Fig. 2.21],

$$I_{xx} = I + A \times d^2$$

This equation is known as the *parallel axis theorem.*

2. *Perpendicular Axis Theorem* According to *perpendicular axis theorem*, the moment of inertia of an planar area about an axis perpendicular to the plane is the sum of the moments of inertia of two perpendicular axes through the same point in the plane of the area. Using this, the moment of inertia about z-axis is related to that about x- and y- axes as

$$I_z = I_x + I_y$$

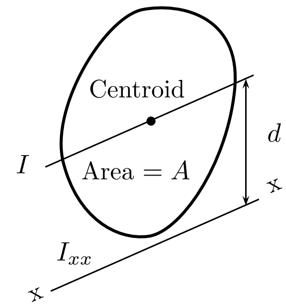


Figure 2.21 Parallel axis theorem.

Based on the above equation, the location of natural axis, expressions of moment of inertia and section modulus are derived for different types of sections.

2.6.3.1 Rectangular Section

Consider a rectangular section of height h and width b. The natural axis is at the middle of height h [Fig. 2.22]. Therefore,

$$A = bh$$

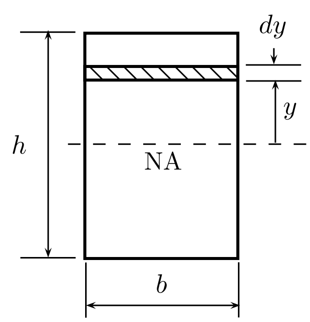


Figure 2.22 Rectangular section.

Taking an infinitesimal area of height dy at distance y from natural axis,

$$\begin{aligned} I &= \int_{-h/2}^{h/2} bdyy \\ &= \frac{bh^3}{12} \\ Z &= \frac{I}{y_{max}} \\ &= \frac{I}{h/2} \\ &= \frac{bh^2}{6} \end{aligned}$$

It is interesting to observe that the moment of inertia of an square section of size a about any axis passing through its centroid is equal to

$$I = \frac{a^4}{12}$$

2.6.3.2 Box Section Consider a box section height h_2 and width b_2, minus a rectangular section of height h_1 and width b_1. The natural axis is at the middle of height h_2 or h_1. Therefore,

$$\begin{aligned} A &= b_2h_2 - b_1h_1 \\ I &= \frac{b_2{h_2}^3 - b_1{h_1}^3}{12} \\ Z &= \frac{I}{y_{max}} \\ &= \frac{I}{h_2/2} \end{aligned}$$

2.6.3.3 Circular Section Consider a circular section of diameter d. The natural axis is at the horizontal diameter $x-x$ [Fig. 2.23]. Therefore,

$$A = \frac{\pi d^2}{4}$$

Consider a horizontal element at angle θ from $x-x$. Its distance from natural axis is $y = (d/2)\sin\theta$ and width $b = (d/2)\cos\theta$. Therefore,

$$\begin{aligned} I &= 4\int_0^{d/2} bdyy^2 \\ &= 4\int_0^{\pi/4} \left(\frac{d}{2}\right)^4 \cos^2\theta \sin^2\theta d\theta \\ &= \frac{\pi d^4}{64} \end{aligned}$$

Moment of inertia represents the distribution of area or mass of the cross-section. Moments of inertia for equal

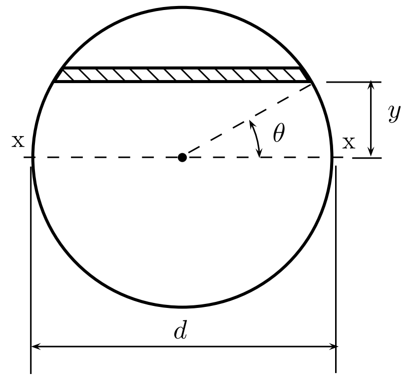


Figure 2.23 Circular section.

area of two cross-sections, square of size a and circle of diameter d, can be examined as follows:

$$\begin{aligned} a^2 &= \frac{\pi d^2}{4} \\ a &= \frac{\sqrt{\pi}}{2}d \end{aligned}$$

Thus, the ratio of moment of inertia of square and circular cross-sections is found as

$$\begin{aligned} \frac{I_{square}}{I_{circle}} &= \frac{64 \times (\sqrt{\pi}d/2)^4}{12 \times \pi d^4} \\ &= \frac{\pi}{3} \\ &= 1.047 \end{aligned}$$

This indicates that for the same area, square cross-section has moment of inertia more than that of circular cross-section.

2.6.3.4 Cylindrical Section Consider a cylindrical section having inner and outer diameters d_1 and d_2, respectively. The natural axis is at horizontal diameter $x-x$. For this section,

$$\begin{aligned} A &= \frac{\pi}{4}\left({d_2}^2 - {d_1}^2\right) \\ I &= \frac{\pi}{64}\left({d_2}^4 - {d_1}^4\right) \\ Z &= \frac{I}{y_{max}} \\ &= \frac{I}{d_2/2} \\ &= \frac{\pi}{32}\frac{\left({d_2}^4 - {d_1}^4\right)}{d_2} \end{aligned}$$

2.6.3.5 Half-Circular Section Consider a half circle of diameter d, or radius r. Its cross-sectional area is $\pi r^2/2$. Since the section is not symmetrical in the

vertical direction, let the natural axis be at distance y_0 from the base $x-x$ [Fig. 2.24]. Consider a horizontal element at angle θ from the axis $x-x$. The distance of this element from the natural axis is $y = r\sin\theta$. Width of this element is $b = r\cos\theta$.

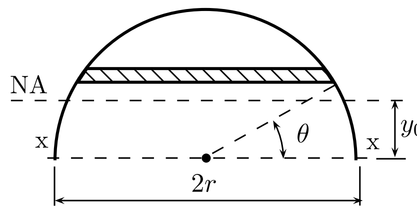


Figure 2.24 Half-circular section.

The location of natural axis (y_0) is determined as

$$y_0 \times \frac{\pi r^2}{2} = 2\int_0^{\pi/2} r\cos\theta r\sin\theta d\theta r\sin\theta$$

Solving with the help of Gamma function[10],

$$\begin{aligned} y_0 &= \frac{2}{3}r^3 \\ &= \frac{4r}{3\pi} \end{aligned}$$

Using parallel axis theorem, the area moment of inertia with respect to natural axis is found as

$$I_{NA} = I_{xx} + {y_0}^2 \times \frac{\pi r^2}{2}$$

2.6.3.6 Triangular Section Consider a triangular section of width b and height h. Its area is $bh/2$. Since the section is not symmetrical in the vertical direction, let the natural axis is at distance y_0 from the apex point [Fig. 2.25].

The location of natural axis (y_0) is determined as

$$\begin{aligned} \frac{bh}{2}\times y_0 &= \int_0^h \frac{b\times y}{h} dy \times y \\ &= \frac{bh^2}{3} \\ y_0 &= \frac{2}{3}h \end{aligned}$$

Moment of inertia about the natural axis can be determined by taking an element of depth dx at distance

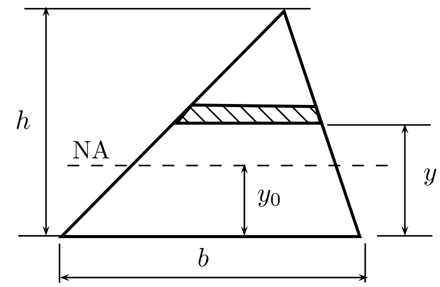


Figure 2.25 Triangular section.

x from apex of the triangle. Width of the element would be bx/h. Therefore,

$$\begin{aligned} I &= \int_0^h \left(\frac{2h}{3} - x\right)^2 \frac{bx}{h} dx \\ &= \int_0^h \left(\frac{4h^2}{9} + x^2 - \frac{4hx}{3}\right)\frac{bx}{h} dx \\ &= \int_0^h \left(\frac{4bh}{9}x + \frac{b}{h}x^3 - \frac{4bx^2}{3}\right) dx \\ &= \left[\frac{4bh}{18}x^2 + \frac{b}{4h}x^4 - \frac{4bx^3}{9}\right]_0^h \\ &= \frac{4bh^3}{18} + \frac{bh^3}{4} - \frac{4bh^3}{9} \\ &= \frac{bh^3}{36} \end{aligned}$$

2.6.4 Expressions for Shear Stress

For a section having moment of inertia I, subjected to shear force F_s, the shear stress on a lamina at distance y from natural axis of the section is given by Eq. (2.33) as

$$\tau_y = \frac{F_s}{Ib} A\bar{y} \tag{2.34}$$

where F_s is the shear force at the section, $A\bar{y}$ is the moment of area above the lamina. Shear stress τ_y, being a function of y, varies in the section.

The following derivations of the expressions of τ_y for different types of sections will strengthen the concepts [Table 2.4].

2.6.4.1 Rectangular Section Consider a rectangular section of height h and width b. Its cross-sectional area is bh. The average value of shear stress over the section is

$$\tau_{avg} = \frac{F_s}{bh}$$

Using Eq. (2.33),

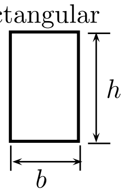
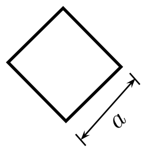
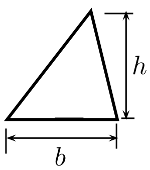
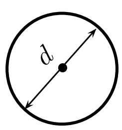
$$\tau_y = \frac{F_s}{b\times bh^3/12} \times b\left(\frac{h}{2} - y\right)\left(\frac{h/2 - y}{2} + y\right)$$

[10]Gamma function $\Gamma(x)$ is used to evaluate the integrals as

$$\int_0^{\pi/2} \sin^m\theta\cos^n\theta d\theta = \frac{\Gamma((m+1)/2)\,\Gamma((n+1)/2)}{2\Gamma((m+n+2)/2)}$$

Here, $\Gamma(1) = 1$, $\Gamma(n+1) = n\Gamma(n) = n!$, $\Gamma(2) = 1\Gamma(1) = 1$, $\Gamma(3) = 2\Gamma(2) = 2$, $\Gamma(1/2) = \sqrt{\pi}$.

Table 2.3 Properties of cross-sections

Cross-section	A	I	Z
Rectangular 	bh	$\frac{bh^3}{12}$	$\frac{bh^2}{6}$
Square 	a^2	$\frac{a^4}{12}$	$\frac{a^3}{6\sqrt{2}}$
Triangular 	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^2}{24}$
Circular 	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	$\frac{\pi d^3}{32}$

This distribution is shown in Fig. 2.26 where maximum shear stress is found at $y = 0$; therefore,

$$\tau_{max} = \frac{3}{2}\tau_{avg}$$

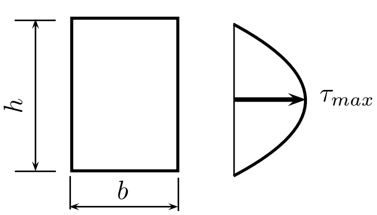


Figure 2.26 Shear stress in rectangular section.

2.6.4.2 Solid Circular Consider a solid circular section of diameter d. Its cross-sectional area is $\pi d^2/4$. The average value of shear stress over the section is

$$\tau_{avg} = \frac{F_s}{\pi d^2/4}$$

This distribution is shown in Fig. 2.27 where the shear stress is found maximum at the natural axis.

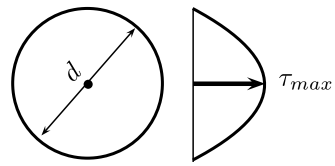


Figure 2.27 Shear stress in circular section.

Using Eq. (2.33), maximum shear stress at $y = 0$ is given as

$$\begin{aligned}\tau_{max} &= \frac{F_s\left[\pi d^2/(2\times 4)\right]\times 2d/(3\pi)}{(\pi d^4/64)\,d}\\ &= \frac{4}{3}\times\frac{F_s}{\pi d^2/4}\\ &= \frac{4}{3}\tau_{avg}\end{aligned}$$

2.6.4.3 Triangular Section Consider a triangular section of height h and width b. Its cross-sectional area is $bh/2$. The average value of shear stress over the section is

$$\tau_{avg} = \frac{F_s}{bh/2}$$

The expression for shear stress distribution for a lamina at a distance y from apex can be written as

$$\tau_y = \frac{F_s\left[(y\times by/h)/2\right](2h/3-2y/3)}{(bh^3/36)(by/h)}$$

This distribution is shown in Fig. 2.28. Using this expression, the maximum shear stress is found at $y = 0$, and is given by

$$\begin{aligned}\tau_{max} &= \frac{3}{2}\frac{F_s}{bh/2}\\ &= \frac{3}{2}\tau_{avg}\end{aligned}$$

Shear stress at natural axis ($y = 2h/3$) is determined using Eq. (2.33):

$$\begin{aligned}\tau_{2h/3} &= \frac{F_s\left[(1/2)(2b/3)(2h/3)\right](1/3\times 2h/3)}{(bh^3/36)\times(2b/3)}\\ &= \frac{8F_s}{3bh}\\ &= \frac{4}{3}\frac{F_s}{bh/2}\\ &= \frac{4}{3}\tau_{avg}\end{aligned}$$

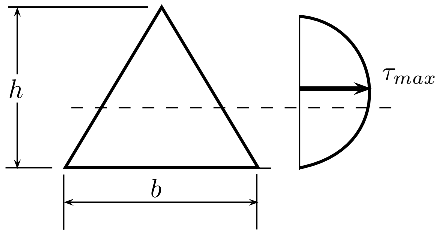


Figure 2.28 Shear stress in triangular section.

The bending stresses for different cross-sections is given in Table 2.4.

Table 2.4 Bending shear stresses

Cross-section	τ_{avg}	τ_{max}
1. Rectangular section of width b, height h	$\dfrac{F_s}{bh}$	$\dfrac{3}{2}\tau_{avg}$
2. Solid circular of diameter d	$\dfrac{F_s}{\pi d^2/4}$	$\dfrac{4}{3}\tau_{avg}$
3. Triangular section width b, height h	$\dfrac{F_s}{bh/2}$	$\dfrac{3}{2}\tau_{avg}$

2.7 BEAM DEFLECTION

After having the knowledge of bending stresses, it is interesting to know their effects in terms of beam deflection. This section represents the basic equations and methods used to determine beam deflections[11].

An *elastic curve* is the curve that shows the deflected or distorted positions of beam section points [Fig. 2.29]. The *curvature* (ρ) of elastic curve on $x-y$ plane is given by

$$\rho = \frac{1}{d^2y/dx^2} \tag{2.35}$$

[11]This is done without determining the bending stresses, because their effects are included in terms of bending moment directly.

The flexure formula [Eq. (2.32)] relates this *curvature* (ρ) with the bending moment (M):

$$\frac{\sigma_y}{y} = \frac{M}{I} = \frac{E}{\rho}$$

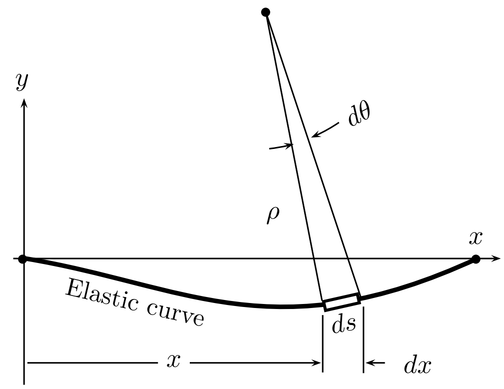


Figure 2.29 Elastic curve.

Flexure formula offers the governing differential equation representing the elastic curve, and can be used for the analysis of slopes and deflections of beams due to bending. The following sections deal with various methods of determining beam deflections.

2.7.1 Double Integration Method

Double integration method for evaluating beam deflection is devised by the mathematical definition of curvature [Eq. (2.35)] and flexure formula [Eq. (2.32)]. The coordinates of the elastic curve (x, y) are related to elastic rigidity EI and bending moment $M(x)$ at the section as

$$EI\frac{d^2y}{dx^2} = M(x) \tag{2.36}$$

Integration of this equation w.r.t. x results into the following *slope equation* which represents the slope at any point in the section,

$$EI\frac{dy}{dx} = \int M(x)dx + C_1 \tag{2.37}$$

where C_1 is integration constant. Similarly, expression for deflection at any point is found by integration of the slope equation, as

$$EIy = \int\left(\int M(x)dx\right)dx + C_1x + C_2 \tag{2.38}$$

where C_2 is another integration constant. The two support constraints, serving as boundary conditions, help in getting values of two integration constants in the above two equations. The method involves two times

integration of the curvature equation, therefore, it is called double-integration method.

In this regard, the following points should be noted:

1. The slope and deflection equations are true for continuous functions. Therefore, breakups must be taken for integration between sections of different loadings.

2. For cantilevers, the origin of x is generally selected at fixed support to make $C_1 = C_2 = 0$.

3. A uniformly distributed loading (UDL), in a particular section of the span, must be so rearranged to the whole span of integration that results into actual loading by incorporating negative expressions in $M(x)$.

4. Loads in the form of moments must be included in expressions of $M(x)$ with zero power of its distance from origin.

5. In case of uniformly varying loads (UVL), intensity at the selected section must be considered.

2.7.2 Macaulay's Method

In double integration method, separate expressions are required for each section of the beam, producing a number of differential equations and respective integration constants. In *Macaulay's method*, a single equation is written for the bending moment for all portions of the beam such that the integration constants are applicable to all portions.

2.7.3 Moment-Area Method

Moment-area method is an alternative method of solving the governing differential equations of beam having few constraints. It is one of the easier methods based on two theorems, known as *Mohr's theorems*.

2.7.3.1 Mohr's theorems Mohr's theorems are the finite integrals of the slope equation [Eq. (2.38)] and deflection equation [Eq. (2.37)], showing the relative slope and relative deflection between the two points. The theorems are stated as follows:

1. *Mohr's First Theorem* Angle between the tangents of two points A and B in a elastic curve is given by

$$\theta_{AB} = \int_{x_A}^{x_B} \frac{M(x)}{EI} dx \tag{2.39}$$

2. *Mohr's Second Theorem* Deviation of a point B relative to the tangent drawn to the elastic curve at any other point A in a direction perpendicular to original position of beam is given by

$$t_{B/A} = \int_{x_A}^{x_B} \frac{M(x)}{EI} dx \times x_B \tag{2.40}$$

where x_B is the distance of B from point A.

These theorems can be easily interpreted from previous discussions. The quantities θ_{AB} and $t_{B/A}$ are shown in Fig. 2.30.

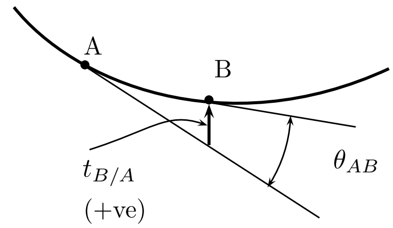


Figure 2.30 Moment-area method.

Important to note that $t_{B/A}$ is the relative deflection, not the absolute one w.r.t. original position. Table 2.5 shows interpretations of θ_{AB} and $t_{B/A}$ from their signs.

Table 2.5 Interpretations of θ_{AB} and $t_{B/A}$

Sign	Positive	Negative
θ_{AB}	Sagging	Hogging
$t_{B/A}$	B is above A	B is below A

Eq. (2.39) and Eq. (2.40) are used to determine values of θ_{AB} and $t_{B/A}$ from the bending moment diagrams drawn by adding elemental areas of moments of different types of loadings one by one in the following useful forms:

$$\theta_{AB} = \frac{1}{EI} \sum (\text{Area})_{AB}$$
$$t_{B/A} = \frac{1}{EI} \sum (\text{Area})_{AB} \times \bar{x}_B$$

where $\sum (\text{Area})_{AB}$ is the resultant area in BM diagram, and $\bar{x}_B$ is the distance of centroids of elemental BM areas from point B of which deflection is to be calculated.

2.7.3.2 Area Moments by Parts The basic input to moment-area method is in determining the area and moment of the moment diagram in parts for individual loads. Let a load over a section the beam results following profile of the bending moment:

$$y = kx^n$$

where x varies from 0 to b and y varies from 0 to h [Fig. 2.31]. The area and centroids of diagram under the curve $y(x)$ are determined as follows:

1. *Moment-Area* Area under this curve is

$$\begin{aligned} A &= \int_0^b kx^n dx \\ &= \frac{k[x^{n+1}]_0^b}{n+1} \\ &= \frac{kb^n b}{n+1} \\ &= \frac{bh}{n+1} \end{aligned}$$

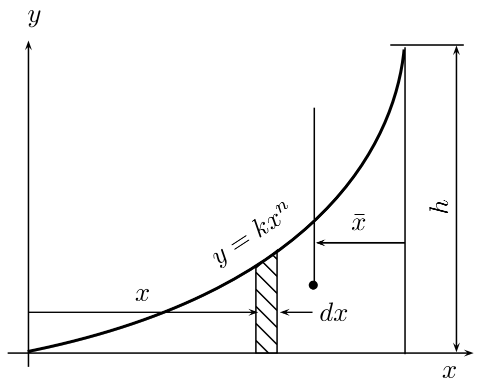


Figure 2.31 | Moment-area by parts.

2. *Centroid of Moment-Area* The distance of its centroid measured from origin ($\bar{x}_o$) is determined by following equation

$$\begin{aligned} \frac{bh}{n+1}\bar{x}_o &= \int_0^b kx^n dxx \\ &= \frac{k[x^{n+2}]_0^b}{n+2} \\ &= \frac{kb^n b^2}{n+1} \\ &= \frac{b^2 h}{n+2} \end{aligned}$$

Therefore,

$$\bar{x}_o = \frac{b\,(n+1)}{n+2}$$

If centroid is measured from the base, then

$$\begin{aligned} \bar{x} &= b - \bar{x}_o \\ &= \frac{b}{n+2} \end{aligned}$$

Expressions for area A and centroidal distance $\bar{x}$ can be used to calculate area and centroid of moment of any type of loadings [Fig. 2.32].

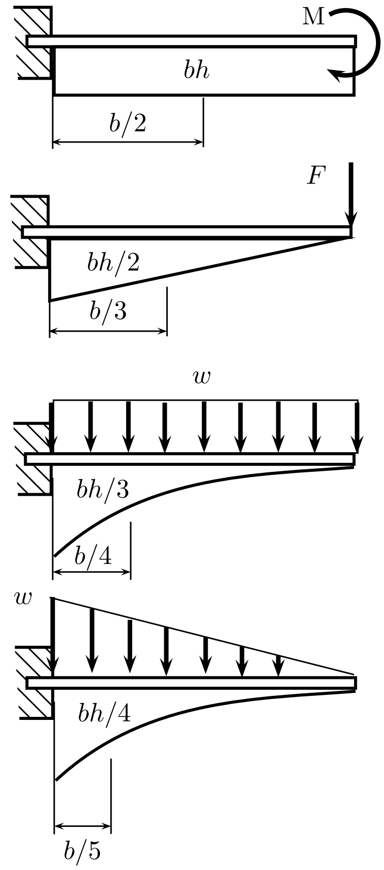


Figure 2.32 | Moment diagram by parts.

Consider a simply supported beam of span l, which is subjected to uniformly distributed load w [Fig. 2.33].

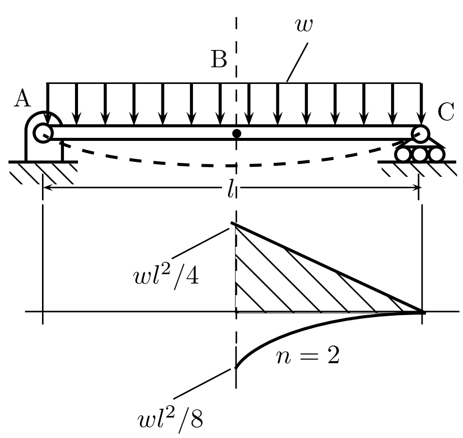


Figure 2.33 | Simply supported beam.

The bending moment diagram of the beam is also shown in Figure 2.33. Maximum bending moment is

$$\begin{aligned} M_{max} &= \frac{wl}{2} \times \frac{l}{2} - w\frac{l}{2} \times \frac{l}{4} \\ &= \frac{wl^2}{4} - \frac{wl^2}{8} \\ &= \frac{wl^2}{8} \end{aligned}$$

Slope of the elastic curve at point A or C will be given by

$$\begin{aligned} \theta_{AB} &= \sum (\text{Area})_{AB} \\ &= \frac{1}{2} \times \frac{wl^2}{4} \times \frac{l}{2} - \frac{1}{2+1} \times \frac{wl^2}{8} \times \frac{l}{2} \\ &= \frac{wl^3}{16} - \frac{wl^3}{48} \\ &= \frac{wl^3}{24} \end{aligned}$$

The area of the moment diagram is symmetric and parabolic ($n = 2$). Therefore, are of bending moment diagram between points A and B can be calculated as

$$\begin{aligned} \sum (\text{Area}_{AB} \times x_B) &= \frac{wl^2}{4} \times \frac{l}{2} \times \frac{l/2}{3} - \frac{wl^2}{8} \times \frac{l}{2} \times \frac{l/2}{2+2} \\ &= \frac{wl^4}{48} - \frac{wl^4}{128} \\ &= -\frac{5wl^4}{384} \end{aligned}$$

Therefore, deflection of point B with respect to point A will be given by

$$\begin{aligned} t_{B/A} &= \frac{\sum (\text{Area}_{AB} \times x_B)}{EI} \\ &= -\frac{5wl^4}{384EI} \end{aligned}$$

Negative sign in the deflection shows that the point B is deflected in the downward direction.

2.7.4 Strain Energy Method

The concepts of strain, strain-displacement relationships are very useful in computing energy-related quantities such as work and *strain energy.*

2.7.4.1 Strain Energy The work done in deformation is transformed into elastic strain energy U that is stored in the body. The area of stress-strain curve represents the strain energy. The expressions for strain energy for axial loads, moments, and twisting moments are derived as follows.

1. *Axial Force* Consider a member of length l and axial rigidity AE subjected to an axial force F applied gradually ($0 \rightarrow F$) due to which it gets extended by δ. The average force on the member is $F/2$, and its displacement δ is given by Hooke's law as
$$\delta = \frac{Fl}{AE}$$
The strain energy U stored in the member will be equal to the work done by the axial force:
$$\begin{aligned} U &= \frac{1}{2}F \times \delta \\ &= \frac{1}{2}F \times \frac{Fl}{AE} \\ &= \frac{1}{2} \times \frac{F^2 l}{AE} \end{aligned}$$
The expression of strain energy for infinitesimal element of length dx is written as
$$U_F = \frac{1}{2}\int_0^l \frac{F^2}{AE} dx$$
2. *Bending Moment* Consider a member of length l and flexural rigidity EI subjected to a bending moment M applied gradually ($0 \rightarrow M$) due to which it gets bent by angle θ. The average bending moment on the member is $M/2$, and its angular deformation θ is given by Hooke's law as
$$\theta = \frac{M}{EI} l$$
The strain energy in this case will be
$$U_M = \frac{1}{2}\int_0^l \frac{M^2}{EI} dx$$
3. *Twisting Moment* Consider a member of length l and *torsional rigidity* GI_p subjected to a twisting moment T applied gradually ($0 \rightarrow T$) due to which it gets twisted by θ. The average twisting moment on the member is $T/2$, and its twist angle θ is given by Hooke's law as
$$\theta = \frac{T}{GI_p} l$$
The strain energy in this case will be
$$U_T = \frac{1}{2}\int_0^l \frac{T^2}{GI} dx$$

2.7.4.2 Castigliano's First Theorem In the special case, when the structure is linear elastic and the deformations are caused by external forces only, (the complementary energy U^* is equal to the strain energy U) the displacement of the structure in the direction of force F is expressed by

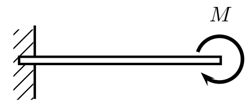
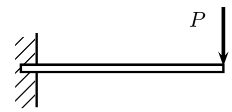
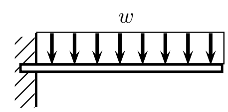
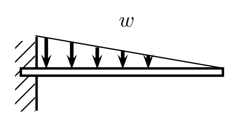
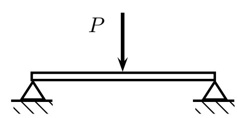
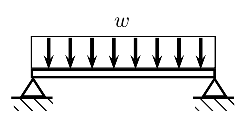
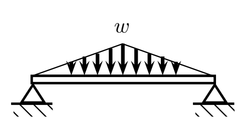
$$\delta = \frac{\partial U}{\partial F}$$

This equation is known as *Castigliano's first theorem.* The use of this theorem is equivalent to the virtual work transformation by the unit-load theorem.

2.7.4.3 Procedure of Strain Energy Method The procedure of strain energy method is based on the calculation of strain energy caused by various loads. Then the Castigliano's theorem is used to determine the deflection on any point by differentiating the total strain energy by the force acting on that point.

Some important findings of simple loads are tabulated in Table 2.6 and Table 2.7.

Table 2.6 Slopes and deflections

Beam	M_{max}	θ_{max}	δ_{max}
	M	$\frac{Ml}{EI}$	$\frac{Ml^2}{2EI}$
	Pl	$\frac{Pl^2}{2EI}$	$\frac{Pl^3}{3EI}$
	$\frac{wl^2}{2}$	$\frac{wl^3}{6EI}$	$\frac{wl^4}{6EI}$
	$\frac{wl^2}{6}$	$\frac{wl^3}{24EI}$	$\frac{wl^4}{30EI}$
	$\frac{Pl}{4}$	$\frac{Pl^2}{16EI}$	$\frac{Pl^3}{48EI}$
	$\frac{wl^2}{8}$	$\frac{wl^3}{24EI}$	$\frac{5wl^4}{384EI}$
	$\frac{wl^2}{12}$	$\frac{5wl^3}{192EI}$	$\frac{wl^4}{120EI}$

2.8 TORSION IN SHAFTS

A shaft is a cylindrical piece of metal used to carry rotating machine parts, such as pulleys and gears, to transmit power or motion.

2.8.1 Shear Stresses

When a shaft is subjected to torsional loads (moments of forces), then the material element is subjected to shear stresses. Consider a material element of thickness dx at radius x at any location along the length in a solid shaft of radius r (diameter d) and length l [Fig. 2.34].

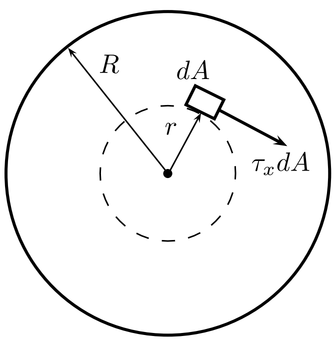


Figure 2.34 Torsion of shaft.

Let shear stress τ_x be induced in material element in tangential direction (perpendicular to x), then accompanying shear stress of the same magnitude (τ_x) will occur longitudinal to the shaft. Let θ be the twist in the shaft. The shear strain γ_x will be $x\theta/l$. Shear stress is given by

$$\tau_x = G\frac{x\theta}{l}$$

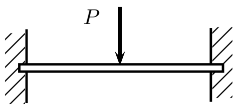
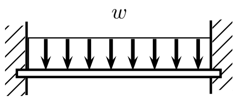
The maximum value of shear stress will be in the outermost lamina ($x = r$):

$$\tau_r = G\frac{r\theta}{l} \tag{2.41}$$

The resulting moment of the forces due to shear stresses in each material element must be equal to the external torque applied:

$$\begin{aligned} T &= \int_0^r \tau_x \left(2\pi x dx\right) x \\ &= \int_0^r G\frac{x\theta}{l} \left(2\pi x dx\right) x \\ &= \frac{G\theta}{l} I_p \end{aligned} \tag{2.42}$$

Table 2.7 Deflections in fixed beams

Beam	δ_{max}
	$\dfrac{Pl^3}{192EI}$
	$\dfrac{wl^4}{384EI}$

where I_p is the polar moment of inertia of the cross-section of the shaft. Combining Eqs. (2.41) and (2.42) results into the following equation for shafts subjected to torsional loads,

$$\frac{\tau_r}{r} = \frac{T}{I_p} = \frac{G\theta}{L} \tag{2.43}$$

This equation is known as the *torsion formula* which is analogous to the flexure formula [Eq. (2.32)] for bending stresses.

The polar moment of inertia of a cross-section can be determined using perpendicular axis theorem. For example, polar moment of inertia of a circular cross-section having diameter d is found as

$$\begin{aligned} I_z &= I_x + I_y \\ &= 2I_x \\ &= 2\frac{\pi d^4}{64} \\ &= \frac{\pi d^3}{32} \end{aligned}$$

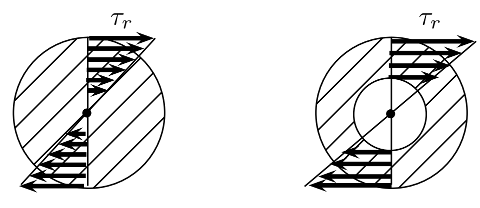
The *torsion formula* is valid for solid and hollow shafts both. For a hollow shaft of outer diameter d_o and inner diameter d_i [Fig. 2.35], the polar modulus is determined as

$$I_p = \pi\frac{\left(d_o^4 - d_i^4\right)}{32}$$

The following points can be deduced from the torsion formula [Eq. (2.43)]:

1. The maximum shear stress generated in a solid shaft[12] is at $x = d/2$, given by
$$\tau_{max} = \frac{T}{\pi d^3/16}$$

2. The shear stress (τ_x) varies linearly with radius x [Fig. 2.35] because
$$\tau_x = \frac{x}{r}\tau_{max}$$



(a) Solid shaft (b) Hollow shaft

Figure 2.35 Shear stress in shafts.

3. *Torsional stiffness* k_T is used to express the resistance of a shaft against torsion:
$$k_T = \frac{T}{\theta}$$

4. If a single uniform shaft is subjected to torques (including support reactions) at different locations along its length, then for equilibrium, the vector sum of torques must be zero:
$$\sum_{i=1}^{n} T_i = 0$$

2.8.2 Torsional Strain Energy

Continuing with the above case, the strain energy stored in a material element of length dl is

$$dE_T = \frac{1}{2} \times \tau_x \frac{\tau_x}{G}\,(2\pi x dx dl)$$

The expressions of *torsional strain energy* are derived separately for solid and hollow shafts.

[12] In rectangular shafts ($a > b$), the maximum shear stress is

$$\tau_{max} = \frac{T}{ab^3}\left[3 + 1.8\frac{b}{a}\right]$$

2.8.2.1 Solid Shaft The strain energy for solid shaft of radius r is determined as

$$
\begin{aligned}
U_T &= \int_0^r \int_0^l dU_T \\
&= \int_0^r \int_0^l \frac{{\tau_x}^2}{2G} 2\pi x dx dl \\
&= \frac{{\tau_{max}}^2}{2Gr^2} \times \int_0^r \int_0^l 2\pi x^3 dx dl \\
&= \frac{{\tau_{max}}^2}{4G} \times \pi r^2 l \\
&= \frac{{\tau_{max}}^2}{4G} \times \text{Volume of the shaft}
\end{aligned}
$$

2.8.2.2 Hollow Shaft The strain energy for a hollow shaft of outer diameter D and inner diameter d is given by

$$I_p = \pi \frac{\left(d_o^4 - d_i^4\right)}{32}$$

Torsional shear stress is maximum at outer lamina:

$$\tau_{max} = \frac{T}{I_p} \times \frac{d_o}{2}$$

Shear stress at any radius x is given by

$$\tau_x = \tau_{max} \frac{x}{d_o/2}$$

The strain energy in any elemental lamina of radial thickness dx and length dl is given by

$$dU_T = \frac{1}{2} \tau_x \frac{\tau_x}{G} (2\pi x dx dl)$$

Therefore, the total strain energy due to torsion in the hollow shaft is given by

$$
\begin{aligned}
U_T &= \int_{d_i/2}^{d_o/2} \int_0^l dE_T \\
&= \int_{d_i/2}^{d_o/2} \int_0^l \frac{{\tau_x}^2}{2G} 2\pi x dx dl \\
&= \frac{{\tau_{max}}^2}{4G} \times \left\{ 1 + \left(\frac{d_i}{d_o}\right)^2 \right\} \\
&\quad \times \text{Volume of the shaft}
\end{aligned}
$$

2.8.3 Composite Shafts

The shafts are combined in series and parallel to meet the design requirements:

1. *Shafts in Parallel* Two or more shafts combined in parallel are called *composite shafts*. In this combination, the angle of twist θ will be the same and total torque will be sum of the torques taken by segmental shafts:

$$\theta_i = \theta$$
$$\sum_{i=1}^{n} T_i = T$$

2. *Shafts in Series* If two or more shafts are combined in series, the torque will be the same in each segment but twist will be different:

$$T_i = T$$
$$\sum_{i=1}^{n} \theta_i = \theta$$

2.8.4 Thin-Walled Tubes

The theory of the torsion of thin-walled closed section beams is known as the *Bredt-Batho theory*. Consider a thin-walled tube of thickness t (assumed very small) is subjected to external torque T [Fig. 2.36].

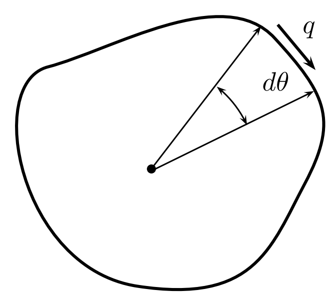


Figure 2.36 | Thin-walled shaft.

The average shearing stress (q) will be given by the following equilibrium equation:

$$
\begin{aligned}
T &= \int_0^{2\pi} \tau (r d\theta t) r \\
&= \tau \left\{ \int_0^{2\pi} (r d\theta) r \right\} t \\
&= \tau \times 2At \\
\tau &= \frac{T}{2At}
\end{aligned}
$$

where A is hollow area of the cross-section. This equation is known as *Bredt-Batho formula*.

2.8.5 Helical Springs

Helical spring is a spiral wound wire having a constant coil diameter and uniform pitch. During expansion or

contraction, the spring wire is predominantly subjected to torsion directly from the forces applied at the ends. Consider a spring having n turns, mean coil diameter D (radius $R = D/2$), wire diameter d. The spring is made of material having modulus of rigidity G [Fig. 2.37]. The length of the spring wire is equal to $2\pi Rn$.

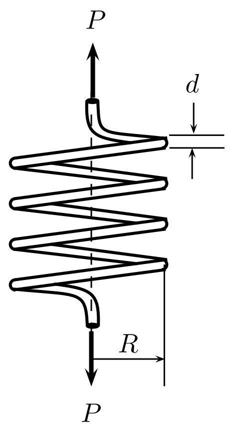


Figure 2.37 | Helical spring.

2.8.5.1 Deflection

When a force P (say for contraction) is applied at the ends of the spring, the spring wire is subjected to torque

$$T = P \times R$$

Therefore, using torsion formula [Eq. (2.32)], the resulting angle of twist (θ) in the wire material is

$$\begin{aligned} \theta &= \frac{PR}{\pi d^4/32} \times \frac{2\pi Rn}{G} \\ &= \frac{64PR^2n}{Gd^4} \end{aligned}$$

Therefore, the deflection is determined as

$$\begin{aligned} \delta &= R\theta \\ &= \frac{64PR^3n}{Gd^4} \\ &= \frac{8PD^3n}{Gd^4} \end{aligned} \tag{2.44}$$

2.8.5.2 Spring Stiffness

The definition of spring stiffness is based on the *Hooke's law for springs*, written as

$$\begin{aligned} P &\propto \delta \\ &= k\delta \end{aligned}$$

where k is the *stiffness of the spring*, also known as *spring constant*. Using Eq. (2.44),

$$k = \frac{Gd^4}{64R^3n}$$

Therefore, stiffness constant of a spring is inversely proportional to the number of turns.

2.8.5.3 Maximum Shear Stress

The spring material is subjected to two sets of shear stresses [Fig. 2.38]:

1. *Torsional Shear Stress* Torque PR generates a shear stress in the spring wire which is maximum at inner point:

$$\tau_1 = \frac{16PR}{\pi d^3}$$

2. *Direct Shear Stress* The spring wire is subjected to direct shear stress which is uniformly distributed throughout the cross-section of the spring wire:

$$\tau_2 = \frac{4P}{\pi d^2}$$

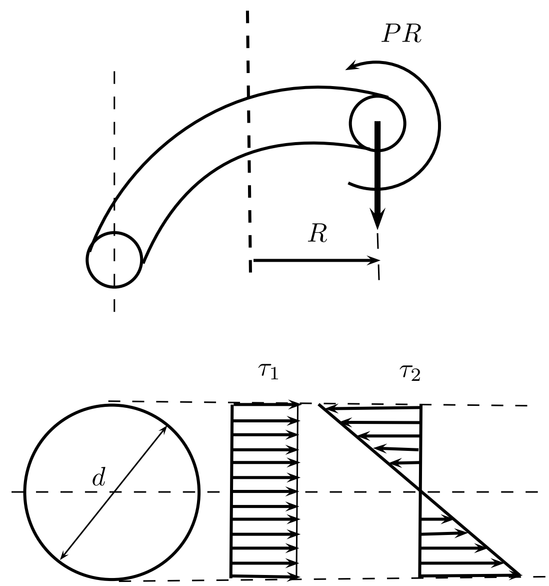


Figure 2.38 | Shear stresses in spring.

The total shear stress at any point will be the sum of the torsional shear stress and the direct shear stress, as explained above. The point of maximum shear stress can be investigated for two cases:

1. *Compressive Force* When the force P is trying to contract or compress the spring, τ_1 is acting downward and τ_2 is acting upward at the outer point and downward at the inner point. So, the maximum value is found at the inner point.

2. *Tensile Force* When force P is trying to expand the spring, τ_1 is acting upward and τ_2 is acting upward at the inner point and downward at the outer point. So, the maximum value of stress is again found at the inner point.

Thus, the shear stress in both cases is maximum at the inner points, given by

$$\begin{aligned}\tau_{max} &= \tau_1+\tau_2\\ &= \frac{4P}{\pi d^2}+\frac{16PR}{\pi d^3}\\ &= \frac{16PR}{\pi d^3}\left[1+\frac{d}{4R}\right] \qquad (2.45)\end{aligned}$$

Since $d/(4R)$ is very small, τ_{max} is primarily caused by the torsion of spring wire.

Cracks in the helical springs used in railway carriage usually start from the inner side of the coil because the springs are subjected to higher cyclic loading and maximum value of shear stress.

2.8.5.4 Wahl's Formula For heavy coil springs, such as those used in railways, $d/(4R)$ is considerable (takes about 14% share). The expression of maximum shear stress, derived earlier [Eq. (2.45)] is based on the assumption that spring wire is straight. To take into account the initial curvature of spring wire, A M Wahl developed the following formula:

$$\tau_{max} = \frac{16PR}{\pi d^3}\left[\frac{4c-1}{4c-4}+\frac{0.615}{c}\right] \qquad (2.46)$$

where $c = D/d$ is *spring index*. This equation is known as *Wahl's formula* in which the left side quantity is known as *Wahl's constant* (K) and is written as

$$K = \frac{4c-1}{4c-4}+\frac{0.615}{c}$$

In practice, the coil element is subjected to combined shear, bending, twisting stresses. When a closed coil spring is subjected to a couple about its axis, then only bending stresses are induced.

2.8.5.5 Combined Springs Springs are often arranged in various combinations to obtain the desired stiffness. In some applications, two concentric springs used to provide greater spring force are wound in opposite directions to prevent buckling. The equivalent stiffness and deflection of combined springs are determined as follows:

1. *Springs in Parallel* To take care of heavy loads, two or more springs can be used in parallel, then equivalent force is sum of the partial loads on each spring:

$$F = \sum_{i=1}^{n} F_i$$

Since deflection in each spring will be equal (say, to δ), therefore,

$$F_i = \delta k_i$$

Hence, the effective stiffness of the parallel springs is given by

$$\begin{aligned}k &= \frac{F}{\delta}\\ &= \frac{\sum_{i=1}^{n} F_i}{\delta}\\ &= \frac{\sum_{i=1}^{n} \delta k_i}{\delta}\\ &= \sum_{i=1}^{n} k_i\end{aligned}$$

2. *Springs in Series* Sometimes two or more springs are made to work in a series for greater deflections. In such cases, each spring is subjected to the same load (say it F), therefore

$$\delta_i = \frac{F}{k_i}$$

In such cases, the deflection of combined springs is equal to the sum of the deflection of all the springs with the same axial load.

$$\begin{aligned}\delta &= \sum_{i=1}^{n} \delta_i\\ \frac{F}{k} &= \sum_{i=1}^{n} \frac{F}{k_i}\\ \frac{1}{k} &= \sum_{i=1}^{n} \frac{1}{k_i}\end{aligned}$$

This represents the effective stiffness of the springs in series.

2.9 COMBINED STRESSES

Let a shaft be subjected to combined stresses due to bending moment M as well as torque T. This is the case of plane stress where Mohr's circle can be used to evaluate maximum value of shear stress and principle stresses [Fig. 2.39].

The maximum values of shear stress (τ_{max}) and normal stress (σ_{max}) are determined as follows:

$$\begin{aligned}\tau_{max} &= \sqrt{\left(\frac{\sigma}{2}\right)^2+\tau}\\ &= \frac{16}{\pi d^3}\sqrt{T^2+M^2}\\ \sigma_{max} &= \frac{\sigma}{2}+\tau_{max}\\ &= \frac{32}{\pi d^3}\times\frac{1}{2}\left(M+\sqrt{T^2+M^2}\right)\end{aligned}$$

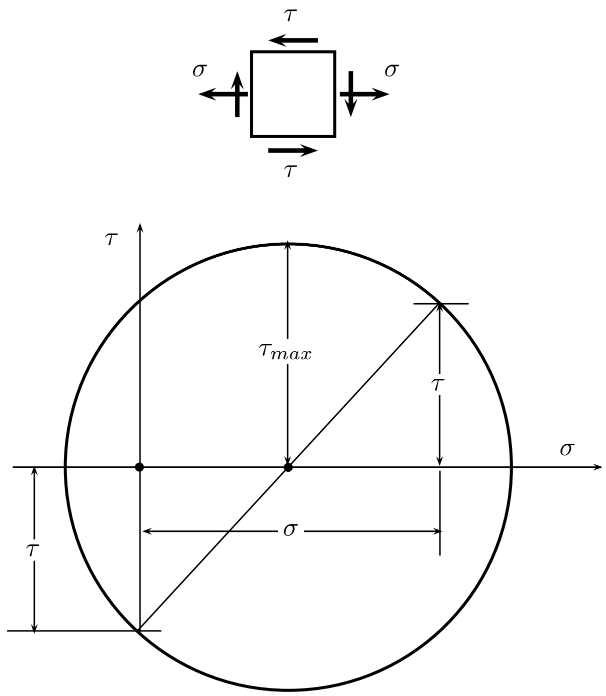


Figure 2.39 Combined stresses.

In view of the above, the equivalent torque for determining torsional shear stress and the equivalent bending moment for determining bending stress can be expressed as

$$T_e = \sqrt{T^2 + M^2}$$
$$M_e = \frac{1}{2}\left(M + \sqrt{T^2 + M^2}\right)$$

These expressions are used in design of shafts against static and dynamics loads after incorporating various factors to account for dynamics and fatigue in practical applications.

2.10 THICK-WALLED SHELLS

A shell is treated as a thick-walled shell when its wall thickness is about at least one-tenth of its radius. Derivations of stresses in thick-walled cylinders and spheres are explained in following subsections.

2.10.1 Thick-Walled Cylinders

The analysis of stresses in thick cylinders is based on the *Lame's theory* with certain assumption for simplification. Consider a material element of thickness dr, angular width $d\theta$ and unit depth at radius r of a thick-walled cylinder subjected to radial stresses σ_{r+dr} at the outer surface and σ_r at the inner surface along with hoop stresses σ_θ along the periphery [Fig. 2.40].

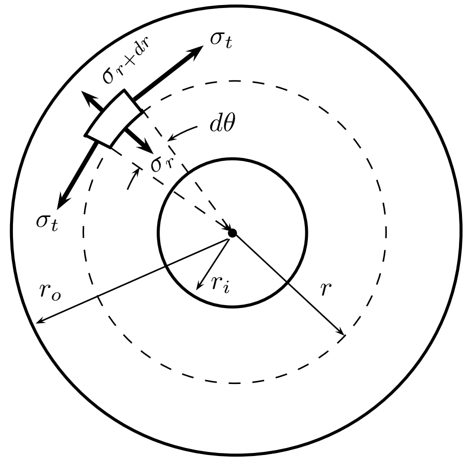


Figure 2.40 Thick-walled cylinder.

The relationships for radial and hoop stresses is based on two equations:

1. For equilibrium in radial direction

$$\sigma_r dr + r d\sigma_r - \sigma_\theta dr = 0$$
$$r\frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0 \tag{2.47}$$

2. Strain in z (depth) direction is (constant with respect to radius)

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu\left(\sigma_r + \sigma_\theta\right)\right]$$

Let $\sigma_r + \sigma_\theta = 2A$ (a constant), therefore, Eq. (2.47) can be written as

$$r\frac{d\sigma_r}{dr} + 2\sigma_r = 2A$$
$$\int \frac{dr}{A - \sigma_r} = 2\int \frac{dr}{r}$$
$$\left(A - \sigma_r\right) r^2 = B$$

where B is the integration constant. Thus,

$$\sigma_r = A - \frac{B}{r^2} \tag{2.48}$$

Using Eq. (2.47),

$$\sigma_\theta = A + \frac{B}{r^2} \tag{2.49}$$

Thus, the stresses are related to radius in a non-linear fashion. Equations (2.48) and (2.49) are known

as *Lame's equations*, which are useful for stresses in thick cylinders. The integration constants A and B are determined by applying boundary conditions, as explained in the following two special cases:

1. *Internal Pressure* When the shell is subjected to only internal pressure p_i (i.e., $p_i \neq 0$, $p_o = 0$), the radial stress at the outer surface will be zero:

$$0 = A - \frac{B}{r_o^2} \tag{2.50}$$

$$-p_i = A - \frac{B}{r_i^2} \tag{2.51}$$

Subtracting Eq. (2.51) from Eq. (2.50),

$$p_i = B\left(\frac{1}{r_i^2} - \frac{1}{r_o^2}\right)$$

$$B = p_i \frac{r_o^2 r_i^2}{r_o^2 - r_i^2}$$

Putting this value of B in Eq. (2.50),

$$\begin{aligned} A &= \frac{B}{r_o^2} \\ &= p_i \frac{r_i^2}{r_o^2 - r_i^2} \end{aligned}$$

Taking $k = r_o/r_i$, the hoop stress and radial stress at any radius r are determined, respectively, as follows:

$$\begin{aligned} \sigma_\theta &= A + \frac{B}{r^2} \\ &= \frac{B}{r_o^2} + \frac{B}{r^2} \\ &= \frac{B}{r_o^2}\left(1 + \frac{r_o^2}{r^2}\right) \\ &= \frac{r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right) p_i \\ &= \frac{1}{k^2 - 1}\left(1 + \frac{r_o^2}{r^2}\right) p_i \end{aligned}$$

$$\begin{aligned} \sigma_r &= A - \frac{B}{r^2} \\ &= \frac{B}{r_o^2} - \frac{B}{r^2} \\ &= \frac{B}{r_o^2}\left(1 - \frac{r_o^2}{r^2}\right) \\ &= \frac{r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right) p_i \\ &= \frac{1}{k^2 - 1}\left(1 - \frac{r_o^2}{r^2}\right) p_i \end{aligned}$$

These expressions can be used to determine the profile of σ_θ and σ_r w.r.t. radius r [Fig. 2.41].

Hoop stresses at the inner and outer surfaces, respectively, will be

$$\begin{aligned} \sigma_{\theta-i} &= \frac{1}{k^2 - 1}\left(1 + \frac{r_o^2}{r_i^2}\right) p_i \\ &= \left(\frac{k^2 + 1}{k^2 - 1}\right) p_i \end{aligned}$$

$$\begin{aligned} \sigma_{\theta-o} &= \frac{1}{k^2 - 1}\left(1 + \frac{r_o^2}{r_o^2}\right) p_i \\ &= \left(\frac{2}{k^2 - 1}\right) p_i \end{aligned}$$

Also, radial stress at $r = r_o$ is zero, but it is equal to p_i (compressive) at $r = r_i$. Thus, using Mohr's circle for the stresses on a material element at inner radius [Fig. 2.41], the maximum shear stress at $r = r_i$ is determined as

$$\begin{aligned} \tau_{max} &= \frac{1}{2}\left(\sigma_{\theta-i} + p_i\right) \\ &= \frac{1}{2}\left\{\left(\frac{2}{k^2 - 1}\right) p_i + p_i\right\} \\ &= \frac{1}{2}\left(\frac{2}{k^2 - 1} + 1\right) p_i \\ &= \frac{1}{2}\left(\frac{k^2 + 1}{k^2 - 1}\right) p_i \end{aligned}$$

2. *External Pressure* When the shell is subjected to only external pressure p_o (i.e., $p_i = 0$, $p_o \neq 0$), the maximum stress is hoop stress at $r = r_i$:

$$\sigma_{max} = -\left(\frac{2{r_o}^2}{{r_o}^2 - {r_i}^2}\right) p_o$$

Using Lame's equations, Eqs. (2.48) and (2.49), for radial stress along with negative sign for compressive stresses,

$$-p_o = A - \frac{B}{r_o^2}$$

$$0 = A - \frac{B}{r_i^2}$$

From the above two equations,

$$p_o = -B\left(\frac{1}{r_i^2} - \frac{1}{r_o^2}\right)$$

$$B = -p_i \frac{r_o^2 r_i^2}{r_o^2 - r_i^2}$$

$$\begin{aligned} A &= \frac{B}{r_i^2} \\ &= -p_o \frac{r_o^2}{r_o^2 - r_i^2} \end{aligned}$$

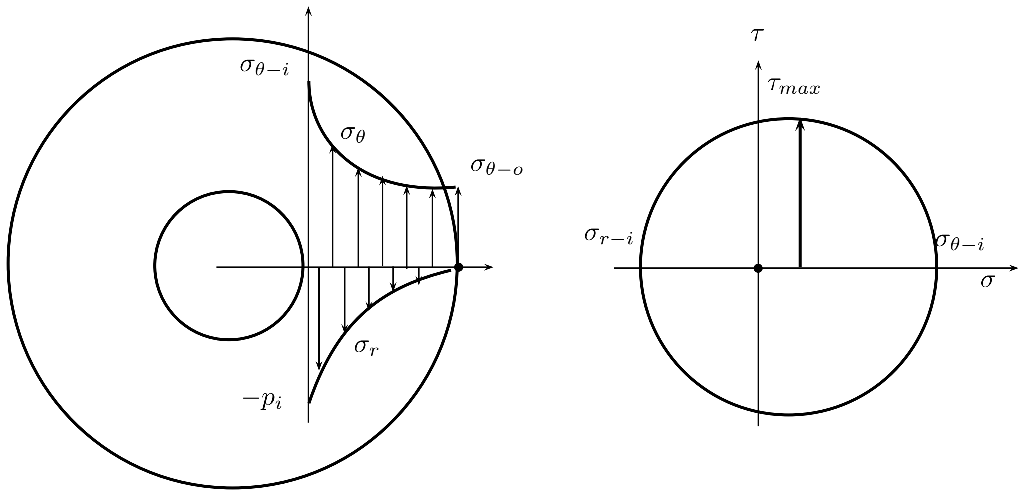


Figure 2.41 Stresses in thick-walled cylinders.

Taking $k = r_o/r_i$, the hoop stress at any radius r shall be given by

$$\begin{aligned}\sigma_\theta &= A+\frac{B}{r^2}\\ &= \frac{B}{r_i^2}+\frac{B}{r^2}\\ &= \frac{B}{r_i^2}\left(1+\frac{r_i^2}{r^2}\right)\\ &= -\frac{r_o^2}{r_o^2-r_i^2}\left(1+\frac{r_i^2}{r^2}\right)p_o\\ &= -\frac{k^2}{k^2-1}\left(1+\frac{r_i^2}{r^2}\right)p_o\end{aligned}$$

Similarly, radial stress at any radius r shall be given by

$$\begin{aligned}\sigma_r &= A-\frac{B}{r^2}\\ &= \frac{B}{r_i^2}-\frac{B}{r^2}\\ &= \frac{B}{r_i^2}\left(1-\frac{r_i^2}{r^2}\right)\\ &= -\frac{r_o^2}{r_o^2-r_i^2}\left(1-\frac{r_i^2}{r^2}\right)p_o\\ &= -\frac{k^2}{k^2-1}\left(1-\frac{r_i^2}{r^2}\right)p_o\end{aligned}$$

Thus, the values of hoop stresses at inner and outer circumferences can be determined as

$$\begin{aligned}\sigma_{\theta-i} &= -\frac{k^2}{k^2-1}\left(1+\frac{r_i^2}{r_i^2}\right)p_o\\ &= -\frac{2k^2}{k^2-1}p_o\end{aligned}$$

$$\begin{aligned}\sigma_{\theta-o} &= -\frac{k^2}{k^2-1}\left(1+\frac{r_i^2}{r_o^2}\right)p_o\\ &= -\frac{k^2}{k^2-1}\left(1+\frac{1}{k^2}\right)p_o\\ &= -\frac{k^2+1}{k^2-1}p_o\end{aligned}$$

For thick cylinders subjected to internal pressure only, the hoop stress is tensile while radial stress is compressive. When the cylinder is subjected to external pressure only, both radial and hoop stresses are compressive in nature. In both situations, the maximum hoop stress occurs at inner circumference and decreases towards the outer circumference. Hence, the maximum pressure inside the shell is limited by the hoop stress at inner circumference.

2.10.2 Compound Cylinders

The hoop stress in thick-walled cylinders is maximum at the inner circumference and decreases towards the outer circumference. This indicates poor utilization of material in the outer layers. *Autofrettage* is a method of pre-stressing of thick cylinders by winding a wire a number of times over the cylinder surface in a tight condition. The stresses can be distributed more uniformly by using a compound cylinder made by heating a larger cylinder and cooling a smaller cylinder, and then inserting a small cylinder into the larger cylinder. This will initially induce compressive hoop stresses in the inner tube and tensile hoop stresses in the outer tube [Fig. 2.42]. The smaller cylinder is subjected to external pressure at the interface (p). If outer radius of smaller cylinder is r, Poisson's ratio is μ_1 and modulus of elasticity is E_1, then the radial and hoop stresses σ_{r1} and $\sigma_{\theta 1}$ at the outer circumference of the smaller

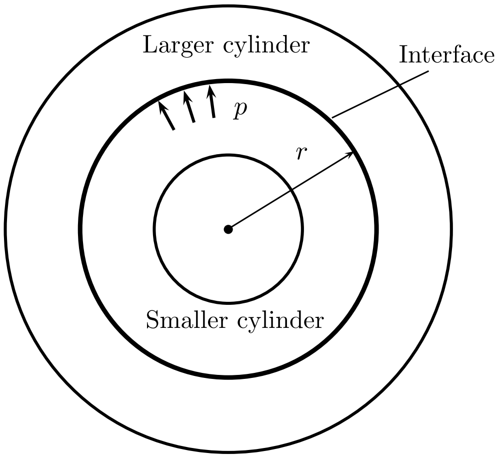


Figure 2.42 | Compound cylinder.

cylinder are determined as

$$\sigma_{r1} = -p$$
$$\sigma_{\theta 1} = -\frac{k_1^2+1}{k_1^2-1}p$$

where k_1 is the ratio of external and internal radii of the smaller cylinder.

The changes in diameter of the cylinder will be caused by strain in the tangential direction (i.e., along the hoop stresses). Thus, decrease in the radius of smaller cylinder would be given by

$$\begin{aligned}\delta_1 &= \frac{r}{E_1}\left(\sigma_{\theta 1} - \mu_1 \sigma_{r1}\right)\\ &= \frac{r}{E_1}\left(-\frac{k_1^2+1}{k_1^2-1}p - \mu_1 \times -p\right)\\ &= \frac{r}{E_1}\left(\mu_1 - \frac{k_1^2+1}{k_1^2-1}\right)p\end{aligned}$$

Similarly, the larger cylinder is subjected to internal pressure at the interface (p). If the inner radius of the larger cylinder is r, Poisson's ratio is μ_2, and modulus of elasticity is E_2, then the radial and hoop stresses at the inner circumference (i.e., σ_{r2} and $\sigma_{\theta 2}$) of the larger cylinder are determined as

$$\sigma_{r2} = -p$$
$$\sigma_{\theta 2} = \left(\frac{k_2^2+1}{k_2^2-1}\right)p$$

where k_2 is the ratio of external and internal radii of the larger cylinder. Therefore, the increase in radii of the larger cylinder would become

$$\begin{aligned}\delta_2 &= \frac{r}{E_2}\left(\sigma_{\theta 2} - \mu_2 \sigma_{r2}\right)\\ &= \frac{r}{E_2}\left(\frac{k_2^2+1}{k_2^2-1} + \mu_2\right)p\end{aligned}$$

Thus, the difference in radii of the larger and smaller cylinders before shrinking would be given by

$$\begin{aligned}\delta &= \delta_2 - \delta_1\\ &= \frac{r}{E_2}\left(\frac{k_2^2+1}{k_2^2-1} + \mu_2\right)p - \frac{r}{E_1}\left(\mu_1 - \frac{k_1^2+1}{k_1^2-1}\right)p\end{aligned}$$

If both the cylinders are made of same material (i.e., $\mu_1 = \mu_2$ and $E_1 = E_2$), then

$$\begin{aligned}\delta &= \frac{r}{E}\left(\frac{k_2^2+1}{k_2^2-1} + \mu\right)p - \frac{r}{E}\left(\mu - \frac{k_1^2+1}{k_1^2-1}\right)p\\ &= \frac{rp}{E}\left(\frac{k_2^2+1}{k_2^2-1} + \frac{k_1^2+1}{k_1^2-1}\right)\end{aligned}$$

2.10.3 Thick-Walled Spheres

Consider a thick-walled sphere with internal and external radii r_i and r_o respectively. For analysis of stresses and strains, consider a concentric spherical element of thickness dr and angular width and angular widths $d\theta$ at radius r. Due to external or internal pressures in the sphere, the radial stresses on this element are σ_{r+dr} at outer surface and σ_r at inner surface and hoop stresses are σ_θ in both the lateral directions [Fig. 2.43].

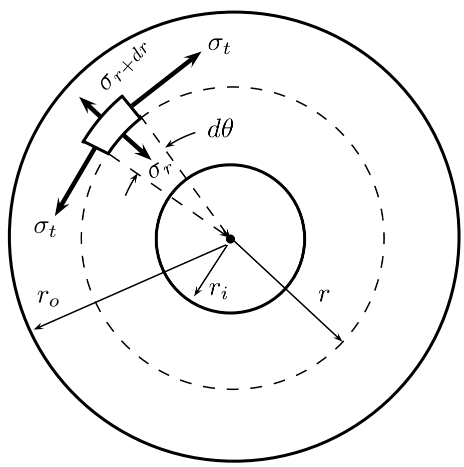


Figure 2.43 | Thick-walled sphere.

This case of thick-walled sphere is special because the element considered in the study is three-dimensional on which equal hoop stresses σ_θ act along the surface. Thus, the following conditions are employed in finding the governing relationships for radial and hoop stresses:

1. *Equilibrium of Hemisphere* The relationship between radial stress σ_r and hoop stress σ_r can be found by using the equilibrium of a hemispherical

portion of this sphere. This is written as

$$\begin{aligned}
2\pi r dr \sigma_\theta &= \pi (r+dr)^2 (\sigma_r + d\sigma_r) - \pi r^2 \sigma_r \\
2\pi r dr \sigma_\theta &= \pi r (r d\sigma_r + 2 dr \sigma_r) \\
\sigma_\theta &= \frac{r}{2}\left(\frac{d\sigma_r}{dr}\right) + \sigma_r \\
\sigma_\theta - \sigma_r &= \frac{r}{2}\frac{d\sigma_r}{dr} \qquad (2.52)
\end{aligned}$$

2. *Definition of Strains* If the stresses in the element cause increase in radius from r to $r+u$, the strains in polar coordinates are found as follows:

(a) The circumferential strain is determined as

$$\begin{aligned}
\varepsilon_\theta &= \frac{(r+u)\,d\theta}{r d\theta} \\
&= \frac{u}{r} \\
u &= r\varepsilon_\theta
\end{aligned}$$

(b) The radial strain is written as

$$\begin{aligned}
\varepsilon_r &= \frac{d(r+u) - dr}{dr} \\
&= \frac{du}{dr}
\end{aligned}$$

Therefore, relationship between ε_r and ε_θ, by their definitions, is

$$\begin{aligned}
\varepsilon_r &= \frac{d}{dr}(r\varepsilon_\theta) \\
&= \varepsilon_\theta + r\frac{d\varepsilon_\theta}{dr} \qquad (2.53)
\end{aligned}$$

3. *Tri-Axial Stresses* Considering the tri-axial stresses acting on the element, the strains can be expressed as

(a) The radial strain in the element is written as

$$\begin{aligned}
\varepsilon_r &= \frac{\sigma_r - \mu(\sigma_\theta + \sigma_\theta)}{E} \\
&= \frac{\sigma_r - 2\mu\sigma_\theta}{E}
\end{aligned}$$

(b) The circumferential strain is determined as

$$\begin{aligned}
\varepsilon_\theta &= \frac{\sigma_\theta - \mu(\sigma_\theta + \mu\sigma_r)}{E} \\
&= \frac{-\mu\sigma_r + (1-\mu)\,\sigma_\theta}{E}
\end{aligned}$$

Using Eq. (2.53)

$$\begin{aligned}
\sigma_r - 2\mu\sigma_\theta &= -\mu\sigma_r + (1-\mu)\,\sigma_\theta - \mu r\frac{d\sigma_r}{dr} \\
&\quad + (1-\mu)\,r\frac{d\sigma_\theta}{dr} \\
(\mu+1)(\sigma_\theta - \sigma_r) &= \mu r\frac{d\sigma_r}{dr} - (1-\mu)\,r\frac{d\sigma_\theta}{dr} \\
(\mu+1)\frac{r}{2}\frac{d\sigma_r}{dr} &= \mu r\frac{d\sigma_r}{dr} - (1-\mu)\,r\frac{d\sigma_\theta}{dr} \\
-(1-\mu)\,r\frac{d\sigma_\theta}{dr} &= (1-\mu)\frac{r}{2}\frac{d\sigma_r}{dr} \\
\frac{d\sigma_\theta}{dr} &= -\frac{1}{2}\frac{d\sigma_r}{dr} \qquad (2.54)
\end{aligned}$$

Differentiating Eq. (2.52),

$$\begin{aligned}
\sigma_\theta - \sigma_r &= \frac{r}{2}\frac{d\sigma_r}{dr} \\
\frac{d\sigma_\theta}{dr} - \frac{d\sigma_r}{dr} &= \frac{r}{2}\frac{d^2\sigma_r}{dr^2} + \frac{1}{2}\frac{d\sigma_r}{dr}
\end{aligned}$$

Using Eq. (2.54),

$$\begin{aligned}
-\frac{1}{2}\frac{d\sigma_r}{dr} - \frac{d\sigma_r}{dr} &= \frac{r}{2}\frac{d^2\sigma_r}{dr^2} + \frac{1}{2}\frac{d\sigma_r}{dr} \\
r\frac{d^2\sigma_r}{dr^2} + 4\frac{d\sigma_r}{dr} &= 0
\end{aligned}$$

Above differential equation can be solved by substituting

$$\psi = \frac{d\sigma_r}{dr}$$

to get

$$\begin{aligned}
r\frac{d\psi}{dr} + 4\psi &= 0 \\
\frac{d\psi}{\psi} &= -4\frac{dr}{r} \\
\ln\psi &= -4\ln r + \ln c_1 \\
\ln\psi &= \ln\left(\frac{c_1}{r^4}\right) \\
\psi &= \frac{c_1}{r^4} \\
\frac{d\sigma_r}{dr} &= \frac{c_1}{r^4} \\
\sigma_r &= -\frac{c_1}{3r^3} + c_2
\end{aligned}$$

Hoop stress σ_θ can be found as

$$\begin{aligned}
\sigma_\theta &= \frac{r}{2}\frac{d\sigma_r}{dr} + \sigma_r \\
&= \frac{r}{2}\frac{d}{dr}\left(-\frac{c_1}{3r^3} + c_2\right) - \frac{c_1}{3r^3} + c_2 \\
&= \frac{c_1}{2r^3} - \frac{c_1}{3r^3} + c_2 \\
&= \frac{c_1}{6r^3} + c_2
\end{aligned}$$

Substituting $c_1 = 6B$ and $c_2 = A$,

$$\sigma_\theta = A + \frac{B}{r^3}$$
$$\sigma_r = A - \frac{2B}{r^3}$$

These two equations represent the initial conditions, and the constants A and B can be determined using the boundary conditions, like in case of thick-walled cylinders.

2.11 THIN-WALLED SHELLS

Shells are said to be thin when the thickness of wall (t) is very small as compared to their diameter (d). Generally, for thin-walled shells,

$$t \leq \frac{d_i}{20}$$

In these shells, the stress is assumed to be uniform throughout the thickness of the shell.

2.11.1 Thin-Walled Cylinder

Consider the half-longitudinal section of a cylinder having length l, mean diameter d, thickness t, and made of material of Young's modulus E subjected to internal pressure p [Fig. 2.44].

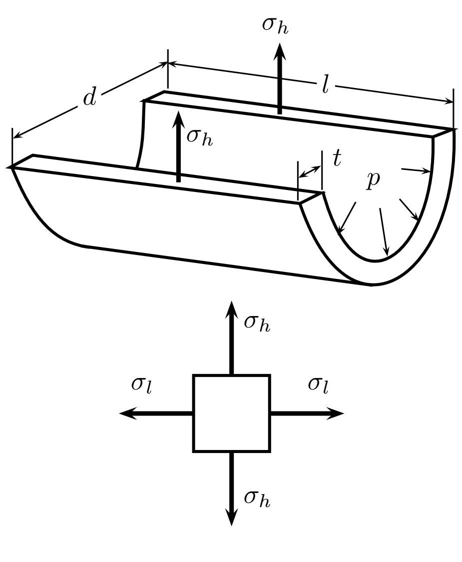


Figure 2.44 Thin-walled cylinder.

The stresses and strains are determined as follows:

1. *Stresses* The following are the stresses involved in a thin-cylinder subjected to internal pressure:

   (a) *Hoop Stress* - Average hoop stress in the cylinder is given by

   $$pdl = \sigma_h lt \times 2$$
   $$\sigma_h = \frac{pd}{2t}$$

   (b) *Longitudinal Stress* - The average longitudinal stress in the cylinder is given by

   $$p \times \frac{\pi d^2}{4} = \sigma_l \times \pi dt$$
   $$\sigma_l = \frac{pd}{4t}$$

   Since $\sigma_h > \sigma_l$, longitudinal joints are critical for design think walled cylinders.

   (c) *Shear Stress* - The maximum shear stress in thin cylinders is found using Mohr's circle:

   $$\begin{aligned}\tau_{max} &= \frac{1}{2}(\sigma_h - \sigma_l) \\ &= \frac{1}{2}\left(\frac{pd}{2t} - \frac{pd}{4t}\right) \\ &= \frac{1}{2}\frac{pd}{2t}\left(1 - \frac{1}{2}\right) \\ &= \frac{pd}{8t}\end{aligned}$$

2. *Strains* The following are the strains in a material element of the cylinder:

   (a) *Circumferential Strain* - The circumferential strain is determined as

   $$\begin{aligned}\varepsilon_c &= \frac{1}{E}(\sigma_h - \mu\sigma_l) \\ &= \frac{pd}{4tE}(2-\mu)\end{aligned}$$

   (b) *Longitudinal Strain* - Longitudinal strain shall be given by

   $$\begin{aligned}\varepsilon_l &= \frac{1}{E}(\sigma_l - \mu\sigma_h) \\ &= \frac{pd}{4tE}(1-2\mu)\end{aligned}$$

   (c) *Volumetric Strain* - Using above expressions, the volumetric strain shall be given by

   $$\begin{aligned}\varepsilon_v &= 2\varepsilon_c + \varepsilon_l \\ &= \frac{pd}{4tE}(5-4\mu) \\ &= \frac{\sigma_h}{2E}(5-4\mu)\end{aligned}$$

2.11.2 Thin-Walled Spheres

The stresses in thin-walled spheres can be determined by taking half of the section of the cylinder with mean diameter d, thickness t, and made of material of Young's modulus E subjected to internal pressure p [Fig. 2.45]. The stresses and strains are determined as follows:

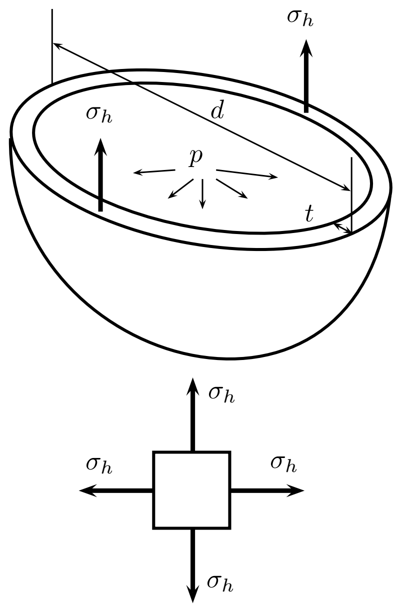


Figure 2.45 | Thin-walled sphere.

1. <ins>*Hoop Stress*</ins> The hoop stress (σ_h) in the thin-walled sphere subjected to internal pressure is given by

$$p \times \frac{\pi d^2}{4} = \sigma_h \times \pi dt$$
$$\sigma_h = \frac{pd}{4t}$$

2. <ins>*Strains*</ins> Following are the strains in a material element of the sphere:

   (a) *Circumferential Strain* - The circumferential strain (ε_c) is given by

$$\varepsilon_c = \frac{\sigma_h}{E}(1-\mu) = \frac{pd}{4tE}(1-\mu)$$

   (b) *Volumetric Strain* - Using above expressions, the volumetric strain (ε_v) in the sphere would be given by

$$\varepsilon_v = 3\varepsilon_c = \frac{3pd}{4tE}(1-\mu)$$

2.12 STRUTS AND COLUMNS

Long or slender structural members loaded axially in compression are called *columns*. Instead of failing by direct compression, such members can bend and deflect laterally. This type of failure of columns is called *buckling*. The phenomenon of buckling is explained by the concept of equilibrium:

1. <ins>*Stable Equilibrium*</ins> When an axial load is applied such that the column remains straight and undergoes only axial compression, the columns is said to be in *stable equilibrium*, which means that it returns to the straight position.

2. <ins>*Neutral Equilibrium*</ins> Upon increasing the axial load gradually, the column can bend, and reach a *neutral equilibrium* where the column can undergo small lateral deflections with no change in the axial force. The corresponding value of the axial load is called the *critical load*.

3. <ins>*Unstable Equilibrium*</ins> At the higher values of axial load, the column is in *unstable equilibrium* where it can collapse by bending.

The ratio of the effective length of a column to the least radius of gyration of its cross-section is called the *slenderness ratio*, l/r. Columns are said to be long if slenderness ratio is less than 10. *Struts* are compression members of trusses.

2.12.1 Axial Loading

2.12.1.1 Euler's Theory Consider a long column of length l, cross-sectional area A, moment of inertia I, made of isotropic material of Young's modulus E, hinged at both ends and subjected to compressive load P. Upon gradual application of the load P, a point will come when the column will start buckling below its yield strength point and beyond this point, the column will become unstable. This is called *crippling point* and the load corresponding to this point is called *crippling load*.

The Euler's theory[13] takes following assumptions

1. Column is initially perfectly straight and axially loaded.

2. Section of the column is uniform throughout.

3. Material is perfectly elastic, homogeneous, and isotropic and obeys Hooke's law.

[13] In 1757, mathematician Leonhard Euler derived a formula that gives the maximum axial load that a long, slender, ideal column can carry without buckling.

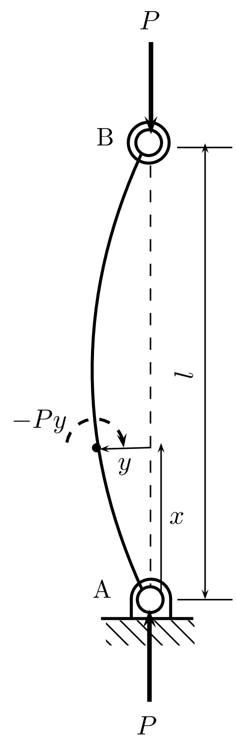


Figure 2.46 Euler's theory of columns.

4. Direct stress is very small compared to the bending stress corresponding to the buckling condition.
5. Length of the column is very large compared to lateral dimension.
6. Self-weight of column is ignorable.
7. The column will fail by buckling alone.

Euler's theory can be used for calculation of crippling loads for four types of end conditions of columns:

1. Hinged–hinged ends
2. Fixed–free ends
3. Fixed–fixed ends
4. Fixed–hinged ends

Fixed end means slope of elastic curve at that point is fixed but the end is free to move longitudinally; whereas for hinged ends, the slope is not fixed but the end is also free to move longitudinally. A free end can move transversely too. While designing a screw jack against the buckling, free–fixed condition is assumed.

Consider the case of a long column having its both ends hinged [Fig. 2.46]. The crippling load can be determined by taking elastic-line equation (flexure formula):

$$EI\frac{d^2y}{dx^2} - Py = 0$$

Solution of this differential equation can be written as

$$y = C_1 \cos\left(\sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(\sqrt{\frac{P}{EI}}\right)$$

The arbirary constants C_1 and C_2 can be determined using boundary conditions:

1. At $x = 0$, $y = 0$, so

$$C_1 = 0$$

2. At $x = l$, $y = 0$, so

$$0 = C_2 \sin\left(\sqrt{\frac{P}{EI}}\right)$$

which gives

$$\sqrt{\frac{P}{EI}} = n\pi, \qquad \text{where } n = 1, 2, 3, ..$$

For $n = 1$, the above equation becomes

$$P = \frac{\pi^2 EI}{l^2} \tag{2.55}$$

This is known as *Euler's formula*. It represents the crippling load for long columns having both ends hinged. The formula can be applied for other three sets of end conditions for long column by taking their equivalent length (l_e) in the Euler's formula:

$$P = \frac{\pi^2 EI}{l_e^2}$$

Figure 2.47 shows the expressions of crippling loads and equivalent lengths for long columns having different end conditions.

The radius of gyration can be considered to be an indication of the stiffness of a section based on the shape of the cross-section when used as a compression member. The smallest value of the radius of gyration is used for structural calculations as this is the plane in which the member is most likely to buckle.

Square or circular shapes are ideal choices for columns as there is no smallest radius of gyration. They have the same value because the radius is constant. If r is the radius of gyration, then $I = Ar^2$, so for a hinged–hinged long column,

$$\frac{P}{A} = \frac{\pi^2 EI}{(l/r)^2}$$

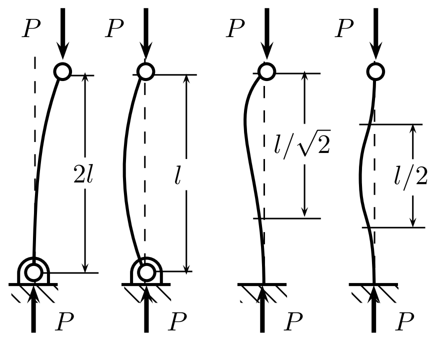


Figure 2.47 Shape and crippling load for long columns

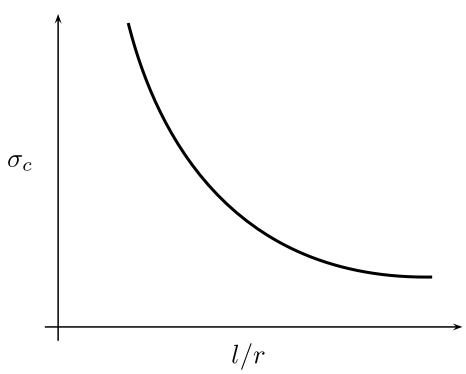


Figure 2.48 Limitation of Euler's theory.

In the above expression, l/r is called *slenderness ratio*. The plot σ_c versus l/k is depicted in Fig. 2.48.

The curve is entirely defined by the magnitude of E. At higher values of l/k (i.e., for long columns), the crippling stress falls rapidly. The Euler's theory is applicable only when the stress accompanying the bending that occurs during buckling does not exceed the proportional limit ($\sigma_c < \sigma_y$). Thus, conventionally, long columns are defined as those columns for which Euler's formula applies.

2.12.1.2 Rankine's Formula

Euler's formula is valid for long columns and does not take into account the direct compressive stress. Therefore, Rankine's formula takes into account the crushing load $P_c = (\sigma_c A)$ and the Euler's crippling force, say P_e, [Eq. (2.55)] to calculate equivalent crippling load P as two springs work in series:

$$\begin{aligned}
\frac{1}{P} &= \frac{1}{P_c} + \frac{1}{P_e} \\
P &= \frac{P_c}{1 + P_c/P_e} \\
&= \frac{\sigma_c A}{1 + \frac{\sigma_c A}{\pi^2 EI/l^2}} \\
&= \frac{\sigma_c A}{1 + \frac{\sigma_c}{\pi^2 E}\left(\frac{l^2}{I/A}\right)} \\
&= \frac{\sigma_c A}{1 + \alpha\,(l/k)^2}
\end{aligned}$$

where

$$\begin{aligned}
\alpha &= \frac{\sigma_c}{\pi^2 E} \\
k &= \sqrt{\frac{I}{A}}
\end{aligned}$$

Here, k is the radius of gyration of the cross-sectional area of the column. To calculate the minimum value of crippling load, k should be based on the axis for which the cross-section has minimum value of moment of inertia. Some values of α are found as

$$\alpha = \begin{cases} 1/7500 & \text{Mild steels} \\ 1/1600 & \text{Cast irons} \\ 1/750 & \text{Timbers} \end{cases}$$

2.12.1.3 Johnson's Parabolic Formula

This is application for columns with both ends pinned.

$$\frac{P}{A} = \sigma_c - g\left(\frac{l}{k}\right)^2$$

where

$$\begin{aligned}
g &= \frac{{\sigma_c}^2}{4\pi^2 E} \\
&= \frac{\sigma_c}{4}\alpha
\end{aligned}$$

2.12.2 Eccentric Loading

The columns are usually designed to support axial loads. Under uncertain conditions, columns can also be subjected to axial loads with a small eccentricity (e) from the axis, as depicted in Fig. 2.49. The eccentric load produces bending of columns even when the load is small; the column deflects from the onset of loading. In such cases, the allowable load for the column is determined by two factors: magnitude of deflection (y) and bending stress (σ), rather than by the critical load as in the case of axially loaded columns.

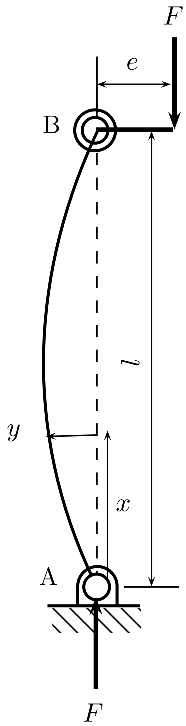


Figure 2.49 Eccentrically loaded column.

2.12.2.1 Rankine Formula The Rankine formula is based on the concept that the maximum stress in the column will be the sum of the additional bending stress due to eccentric loading and the axial stress caused by axial load:

$$\begin{aligned}\sigma_c &= \frac{P}{A}+\frac{P\cdot e}{I}y_c\\ &= \frac{P}{A}\left(1+\frac{ey_c}{k^2}\right)\\ P &= \frac{\sigma_c A}{1+ey_c/k^2}\end{aligned}$$

Thus,

$$P = \frac{\sigma_c A}{\left(1+ey_c/k^2\right)\left\{1+\alpha\left(l/k\right)^2\right\}}$$

2.12.2.2 Secant Formula Equation of elastic curve for the eccentrically loaded column [Fig. 2.49] can be found using the flexure formula:

$$\begin{aligned}EI\frac{d^2y}{dx^2} &= -P\left(y+e\right)\\ EI\frac{d^2y}{dx^2}+Py &= -Pe\end{aligned}$$

Solution of this differential equation is written as

$$y = A\cos\left(x\alpha\right)+B\sin\left(x\alpha\right)-e$$

where

$$\alpha = \sqrt{\frac{P}{EI}}$$

Integration constants A and B are determined using two boundary conditions:

1. At $x=0$, $y=0$, so $A=e$
2. At $x=l$, $y=0$, so $B=e\tan\left(\frac{\alpha l}{2}\right)$

Finally,

$$y = e\cos\left(x\alpha\right)+e\tan\left(\frac{\alpha l}{2}\right)\sin\left(x\alpha\right)-e$$

This equation represents the deflection y as a function of x. Using this equation, the mid-span deflection (at $x=l/2$) is determined as

$$\begin{aligned}y_{max} &= e\cos\left(\frac{l}{2}\alpha\right)+e\tan\left(\frac{\alpha l}{2}\right)\sin\left(\frac{l}{2}\alpha\right)-e\\ &= e\left[\sec\left(\frac{l}{2}\alpha\right)-1\right]\\ &= e\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)-1\right]\end{aligned}$$

Maximum eccentricity of load at mid-span is

$$y_{max}+e = e\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)$$

If y_c is the distance of the outermost lamina, and $k=\sqrt{I/A}$ is the minimum radius of gyration of the section, then the maximum bending stress in the column is determined as

$$\begin{aligned}\sigma_b &= \frac{M_{max}}{I/y_c}\\ &= \frac{P\left(y_{max}+e\right)}{I/y_c}\\ &= \frac{P\left(y_{max}+e\right)y_c}{Ak^2}\end{aligned}$$

The maximum compressive stress in the column is determined as

$$\begin{aligned}\sigma_c &= \frac{P}{A}+\sigma_b\\ &= \frac{P}{A}\left\{1+\frac{\left(y_{max}+e\right)y_c}{k^2}\right\}\end{aligned}$$

Substituting the value of y_{max},

$$\begin{aligned}\sigma_c &= \frac{P}{A}\left\{1+\frac{\left(y_{max}+e\right)y_c}{k^2}\right\}\\ &= \frac{P}{A}\left\{1+\frac{ey_c}{k^2}\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\right\}\end{aligned}$$

This expression is called *secant formula* for eccentrically loaded columns.

IMPORTANT FORMULAS

Stress and Strain

1. Normal stress and strain

$$\sigma = \frac{F}{A}$$

$$\varepsilon = \frac{\delta l}{l}$$

2. True stress and true strain

$$\sigma = \sigma_0 (1+\varepsilon_0)$$
$$\varepsilon = \ln\left(\frac{A_0}{A}\right)$$

3. Hooke's law states that strain within elastic limits developed in a material is proportional to applied stress.

$$E = \frac{\sigma}{\varepsilon}$$
$$G = \frac{\tau}{\gamma}$$
$$K = \frac{\sigma}{\varepsilon_v}$$

4. Poisson's ratio

$$\mu = \frac{\text{Lateral strain}}{\text{Axial strain}}$$

5. Elastic constants

$$G = \frac{E}{2(1+\mu)}$$
$$K = \frac{E}{3(1-2\mu)}$$
$$E = \frac{9KG}{3K+G}$$

$$-1.0 < \mu < 0.5$$

$$G_{max} = \frac{E}{2}$$
$$K_{min} = \frac{E}{3}$$

6. Deformation formulas:

(a) Circular taper

$$\delta = \frac{4Pl}{\pi d_1 d_2 E}$$

(b) Square taper

$$\delta = \frac{Pl}{d_1 d_2 E}$$

(c) Simple taper

$$\delta = \frac{Pl \ln(d_1/d_2)}{Et(d_1-d_2)}$$

(d) Self-weight

$$\delta = \frac{1}{2}\left(\frac{Wl}{AE}\right)$$

7. Tri-axial stress

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y+\sigma_z)]$$
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \mu(\sigma_z+\sigma_x)]$$
$$\varepsilon_z = \frac{1}{E}[\sigma_z - \mu(\sigma_x+\sigma_y)]$$

8. Thermal load

$$\sigma_t = E\alpha\Delta T$$
$$\varepsilon_t = \alpha\Delta T$$

Dynamic Loading

1. Static load

$$\delta = \delta_{st} = \frac{W}{k}$$

2. Impact load

$$\frac{\delta}{\delta_{st}} = 1+\sqrt{1+\frac{2h}{\delta_{st}}}$$

3. Shock load

$$\delta = 2\delta_{st}$$

Mohr's Circle

1. Stresses

$$R_\sigma^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\mathrm{OO'} = \frac{\sigma_x+\sigma_y}{2}$$
$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

2. Strains

$$R_\sigma = R_\varepsilon \times \frac{E}{1+\mu}$$

Beam Loadings

1. SF and BM

$$F(x) = \frac{1}{x_s}\int_0^{x_s} f(x)dx = \frac{dM(x)}{dx}$$
$$M(x) = \int_0^{x_s} f(x)dx = \int F(x)dx$$
$$w(x) = \frac{dF(x)}{dx}$$

2. Flexure formula

$$\frac{\sigma_y}{y} = \frac{M}{I} = \frac{E}{\rho}$$

3. Maximum bending stress

$$\sigma_y^{max} = \frac{M}{I}y_{max} = \frac{M}{Z}$$
$$Z = \frac{I}{y_{max}}$$

4. Bending shear stress

$$\tau_y = \frac{F_s}{Ib}A\bar{y}$$

(a) Rectangular section

$$\tau_{max} = \frac{3}{2}\tau_{avg}$$

(b) Solid circular section

$$\tau_{max} = \frac{4}{3}\tau_{avg}$$

(c) Triangular section

$$\tau_{max} = \frac{3}{2}\tau_{avg}$$

5. Curvature of elastic curve

$$\rho = \frac{1}{d^2y/dx^2}$$

6. Double-integration method

$$EI\frac{d^2y}{dx^2} = M(x)$$

$$EI\frac{dy}{dx} = \int M(x)dx + C_1$$

$$EIy = \int\left(\int M(x)dx\right)dx + C_1x + C_2$$

7. Moment-area method

$$\theta_{AB} = \int_{x_A}^{x_B} \frac{M(x)}{EI}dx = \frac{1}{EI}\sum(\text{Area})_{AB}$$

$$t_{B/A} = \int_{x_A}^{x_B} \frac{M(x)}{EI}dx \cdot x_B = \frac{1}{EI}\sum(\text{Area})_{AB} \cdot \bar{x}_B$$

8. Moment-area by parts

$$A = \frac{bh}{n+1}$$

$$\bar{x} = \frac{b}{n+2}$$

9. Strain energy

$$U_F = \frac{1}{2}\int_0^l \frac{F^2}{AE}dx$$

$$U_M = \frac{1}{2}\int_0^l \frac{M^2}{EI}dx$$

$$U_T = \frac{1}{2}\int_0^l \frac{T^2}{GI}dx$$

Torsion

1. Torsion formula

$$\frac{\tau_r}{r} = \frac{T}{I_p} = \frac{G\theta}{L}$$

2. Polar modulus of hollow shaft

$$I_p = \pi\frac{\left(d_o^4 - d_i^4\right)}{32}$$

3. Torsional shear stress

$$\tau_{max} = \frac{T}{\pi d^3/16}$$

$$E_T = \frac{{\tau_{max}}^2}{4G} \times \text{volume}$$

4. Shafts in parallel

$$\sum_{i=1}^{n} T_i = T$$

$$\theta_i = \theta$$

5. Shafts in series

$$\sum_{i=1}^{n} \theta_i = \theta$$

$$T_i = T$$

6. Thin-walled tubes

$$\tau = \frac{T}{2At}$$

7. Spring

$$k = \frac{Gd^4}{64R^3n}$$

$$\tau_1 = \frac{4P}{\pi d^2}$$

$$\tau_2 = \frac{16PR}{\pi d^3}$$

$$\tau_{max} = \frac{16PR}{\pi d^3}\left[1 + \frac{d}{4R}\right]$$

(a) Wahl's formula

$$\tau_{max} = \frac{16PR}{\pi d^3} \times \left[\frac{4c-1}{4c-4} + \frac{0.615}{c}\right]$$

(b) Springs in parallel

$$k = \sum_{i=1}^{n} k_i$$

(c) Springs in series

$$\frac{1}{k} = \sum_{i=1}^{n} \frac{1}{k_i}$$

Combined Stresses

$$T_e = \sqrt{T^2 + M^2}$$

$$M_e = \frac{1}{2}\left(M + \sqrt{T^2 + M^2}\right)$$

Thick-Walled Shells

1. Thick-walled cylinders

$$\sigma_r = A - \frac{B}{r^2}$$

$$\sigma_\theta = A + \frac{B}{r^2}$$

2. Thick-walled spheres

$$\sigma_\theta = A + \frac{B}{r^3}$$

$$\sigma_r = A - \frac{2B}{r^3}$$

Thin-Walled Shells

1. Thin-walled cylinders

$$\sigma_h = \frac{pd}{2t}$$

$$\sigma_l = \frac{pd}{4t}$$

$$\tau_{max} = \frac{pd}{8t}$$

$$\varepsilon_c = \frac{pd}{4tE}(2-\mu)$$

$$\varepsilon_l = \frac{pd}{4tE}(1-2\mu)$$

$$\varepsilon_v = \frac{\sigma_h}{2E}(5-4\mu)$$

2. Thin-walled spheres:

$$\sigma_h = \frac{pd}{4t}$$

$$\tau_{max} = \sigma_h$$

$$\varepsilon_c = \frac{pd}{4tE}(1-\mu)$$

$$\varepsilon_v = \frac{3pd}{4tE}(1-\mu)$$

Theory of Columns

1. Euler's theory

$$P = \frac{\pi^2 EI}{l^2}$$

2. Rankine's formula

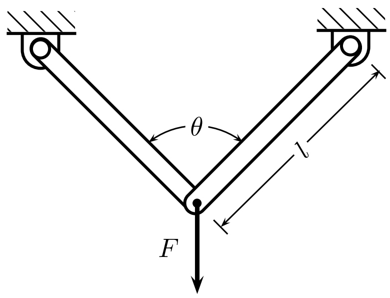
$$P = \frac{\sigma_c A}{1 + \alpha\,(l/k)^2}$$

$$\alpha = \frac{\sigma_c}{\pi^2 E}$$

$$k = \frac{I}{A}$$

SOLVED EXAMPLES

1. Both the bars of the symmetric truss as shown in the figure are made of the same material of modulus of elasticity E, equal cross- sectional area A and length l.



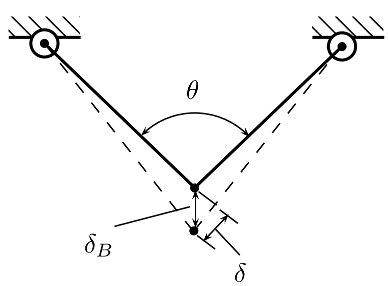
Determine the vertical deflection of point B on application of load F at this point.

Solution. The load F shall be equally divided into the members such that the force on each bar is given by

$$\frac{F}{2} = 2F_1 \cos(\theta/2)$$
$$F_1 = \frac{F}{2\cos(\theta/2)}$$

The deflection in the bars due to these forces is

$$\delta = \frac{Fl}{AE} = \frac{F_1 l}{2AE\cos(\theta/2)}$$



The deflection of point B will be given by

$$\delta_B = \frac{\delta}{\cos(\theta/2)} = \frac{F_1 l}{2\cos^2(\theta/2)\,AE}$$

2. A rod of material with $E = 200 \times 10^3$ MPa and $\alpha = 10^{-3}$ mm/mm-°C is fixed at both the ends. It is uniformly heated such that the increase in temperature is 30°C. What is the stress developed in the rod?

Solution. Thermal stress will be compressive because the rod is fixed at both ends. Stress is calculated as

$$\sigma_t = -E\alpha\Delta T = -6000 \text{ N/mm}^2$$

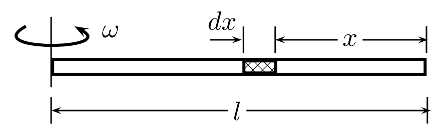
3. When a weight of 100 N falls on a spring of stiffness 1 kN/m from a height of 2 m, what is the deflection caused in the first fall?

Solution. During impact, the potential energy of the weight is converted into that of spring. Hence, deflection (x) will be given by

$$\frac{1}{2}kx^2 = mgh$$
$$x = \sqrt{\frac{2mgh}{k}} = 0.6324 \text{ m}$$

4. A rod of length l, cross-section area A and specific weight w (per unit length) rotates at angular velocity ω about an axis passing through one end of the rod. Determine the extension produced in the rod due to centrifugal forces.

Solution. The condition presented in the problem is depicted in the following figure.



The extension of a small element of length dx at distance x from the free end is given by

$$\delta = \int_0^l \frac{1}{AE}\left(l - \frac{x}{2}\right)\omega^2 \frac{w}{g} x\,dx = \frac{w\omega^2}{gAE}\left[l\frac{x^2}{2} - \frac{x^3}{6}\right]_0^l = \frac{w\omega^2 l^3}{gAE}\left(\frac{1}{2} - \frac{1}{6}\right) = \frac{w\omega^2 l^3}{3gAE}$$

5. The principal strains at a point in a body under bi-axial state of stress are 1000×10^{-6} and -600×10^{-6}. What is the maximum shear strain at that point?

Solution. The radius of Mohr's circle for strains is given by

$$r = \frac{1000 - (-600)}{2} \times 10^{-6} = 800 \times 10^{-6}$$

The maximum shear strain is given by

$$\begin{aligned}\gamma &= 2\times r\\ &= 1600\times 10^{-6}\end{aligned}$$

6. Hoop stress and longitudinal stress in a boiler shell under internal pressure are 100 MN/m^2 and 50 MN/m^2, respectively. Young's modulus of elasticity and Poisson's ratio of the shell material are 200 GN/m^2 and 0.3, respectively. Determine the hoop strain in boiler shell.

Solution. Given that

$$\begin{aligned}\sigma_h &= 100\ \mathrm{MN/m^2}\\ \sigma_l &= 50\ \mathrm{MN/m^2}\\ E &= 200\ \mathrm{GN/m^2}\\ \mu &= 0.3\end{aligned}$$

Therefore,

$$\begin{aligned}\varepsilon_h &= \frac{\sigma_h}{E}(1-2\mu)\\ &= 0.425\times 10^{-3}\end{aligned}$$

7. The normal stresses at a point are $\sigma_x = 80$ MPa and $\sigma_y = 20$ MPa; the shear stress at this point is $\tau_{xy} = 40$ MPa. What is the maximum principal stress at this point?

Solution. The radius of the Mohr's circle is determined as

$$\begin{aligned}R_\sigma &= \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}\\ &= 50\sqrt{2}\ \mathrm{MPa}\end{aligned}$$

The σ-coordinate of center of Mohr's circle is

$$\begin{aligned}c &= \frac{\sigma_x+\sigma_y}{2}\\ &= \frac{10+2}{2}\\ &= 50\ \mathrm{MPa}\end{aligned}$$

Hence, the maximum principal stress is

$$\begin{aligned}\sigma_1 &= R_\sigma + c\\ &= 100\ \mathrm{MPa}\end{aligned}$$

8. A simply supported beam of constant flexural rigidity and length $2L$ carries a concentrated load P at its mid span and the deflection under the load is δ. A cantilever beam of the same flexural rigidity and length L is subjected to load P at its free end. Determine the deflection at the free end of the cantilever.

Solution. The deflection in simply supported beam is

$$\delta = \frac{P(2L)^3}{48EI}$$

The deflection in cantilever beam is

$$\begin{aligned}\delta' &= \frac{P(L)^3}{3EI}\\ &= 2\delta\end{aligned}$$

9. A solid shaft of diameter 150 mm, length 1200 mm can be subjected to maximum shear stress 80 MPa. A hole of 75 mm diameter is now drilled throughout the length of the shaft. Determine the percentage reduction in torque-carrying capacity of the shaft for the same maximum shear stress.

Solution. Internal diameter of the hollow shaft is made half of the external diameter. Maximum shear stress in solid shaft is equal to that in the hollow shaft of half diameter:

$$\frac{T}{\pi d^4/32}\times\frac{d}{2} = \frac{T'}{\pi\left(d^4-(d/2)^4\right)/32}\times\frac{d}{2}$$

$$T' = \frac{15}{16}T$$

Percentage reduction in the torque carrying capacity is

$$\begin{aligned}&= \frac{T-15T/16}{T}\times 100\\ &= 6.25\%\end{aligned}$$

10. A length of 10 mm diameter steel wire is coiled to a close coiled-helical spring having 10 coils of 75 mm mean diameter, and the spring has a stiffness k. Determine the percentage increase in stiffness if the same length of the wire is coiled to 15 coils of 50 mm mean diameter.

Solution. Stiffness k is determined as

$$k = \frac{Gd^4}{64R^3n}$$

Therefore,

$$\begin{aligned}\frac{k'}{k} &= \frac{75^3\times 10}{50^3\times 15}\\ k' &= 2.25k\end{aligned}$$

Percentage increase in stiffness is

$$\begin{aligned}\%\ \text{Increase} &= \frac{2.25-1}{1}\times 100\\ &= 125\%\end{aligned}$$

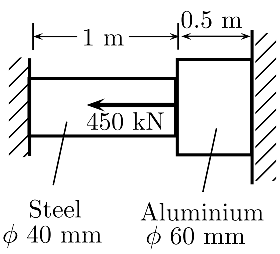
11. Determine the maximum principal strain in a thin cylindrical tank, having a radius of 30 cm and wall thickness of 8 mm when subjected to an internal

pressure of 1.5 MPa. Take Young's modulus $E =$ 200 GPa and Poissons ratio $\mu = 0.2$.

Solution. The maximum principal strain is the circumferential strain, and is given by

$$\begin{aligned}\varepsilon &= \frac{pd}{4tE}(2-\mu)\\ &= \frac{1.5\times10^6\times0.60}{4\times0.006\times200\times10^9}\times(2-0.2)\\ &= 3.375\times10^{-4}\end{aligned}$$

12. A composite bar is made of aluminium and steel portions of lengths 0.5 m and 1 m, respectively and diameters of 60 mm and 40 mm, respectively. A load 450 kN is applied at the junction of the portions as shown in figure.



The modulus of elasticity of steel and aluminium are 210 GPa and 70 GPa, respectively. Determine the loads shared by the aluminium and steel portions. Also determine the stresses developed in both the portions.

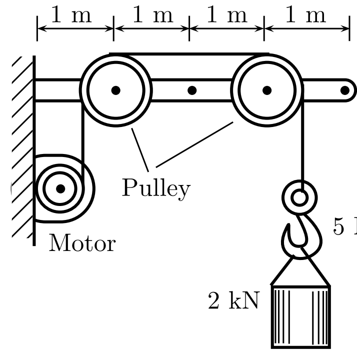
Solution. Let F_{st} kN be the force shared by steel. Aluminium portion shall take $(450-F_{st})$ kN. The extension of Al would be compensated by the compression in steel, so

$$\begin{aligned}\frac{4F_{st}\times10^3\times0.5}{\pi\times0.04^2\times210\times10^9} &= \frac{4\,(450-F_{st})\times10^3\times1}{\pi\times0.06^2\times70\times10^9}\\ F_{st} &= \frac{4}{3}(450-F_{st})\\ F_{st}+\frac{4}{3}F_{st} &= 600\\ F_{st} &= 257.14\text{ kN}(-)\\ F_{al} &= 192.86\text{ kN}\end{aligned}$$

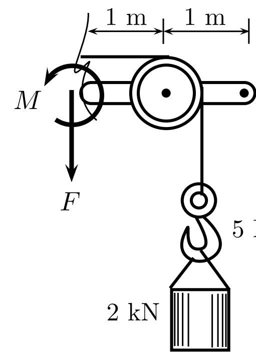
The stresses are determined as

$$\begin{aligned}\sigma_{al} &= \frac{4\times192.86}{\pi\times0.06^2}\\ &= 68.20\text{ MPa}\\ \sigma_{st} &= \frac{4\times257.14}{\pi\times0.04^2}\\ &= 204.62\text{ MPa}\end{aligned}$$

13. Determine the shear force and bending moment at point C in the following configuration:



Solution. Consider the right portion of the section at point C:



Thus, shear force F and bending moment M are determined as

$$\begin{aligned}F &= 2\times10^3+5\\ &= 2005\text{ N}\\ M &= 2005\times0.125-2005\times(1+0.125)\\ &= 2005\text{ Nm}\end{aligned}$$

14. A high-strength steel band saw, 20 mm wide and 0.8 mm thick, runs over a pulley 600 mm in diameter. The modulus of elasticity of the saw material is 200 GPa. Determine the maximum flexural stress developed in the saw due to bending in the pulley. Also calculate the minimum radius of the pulley which can be used without exceeding a flexural stress of 400 MPa.

Solution. The maximum flexural stress is calculated by using flexure formula

$$\frac{\sigma}{y} = \frac{E}{\rho}$$

where

$$\begin{aligned}y &= \frac{0.8}{2}\\ &= 0.4\text{ mm}\\ \rho &= \frac{600}{2}\\ &= 300\text{ mm}\\ E &= 200\text{ GPa}\end{aligned}$$

Therefore, the maximum flexural stress is

$$\begin{aligned}\sigma &= E\frac{y}{\rho}\\ &= 266.67 \text{ MPa}\end{aligned}$$

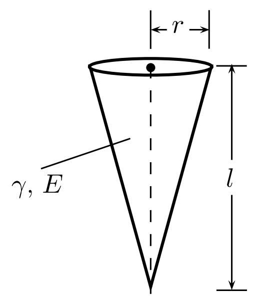
Given that maximum flexural stress

$$\sigma = 400 \text{ MPa}$$

Substituting this value in flexure formula, the minimum radius when σ does not exceed 400 MPa is

$$\begin{aligned}\rho &= \frac{E}{\sigma}y\\ &= 200 \text{ mm}\end{aligned}$$

15. A cone of height L and diameter d is made of a material having weight density $\gamma = \rho g$ and modulus of elasticity E.

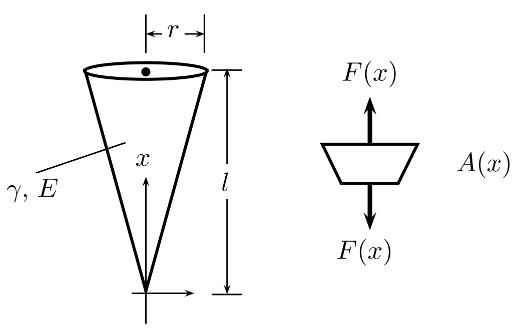


Determine how far its end is displaced due to gravity when it is suspended in the vertical position keeping its base on the upper side.

Solution. Area of cross-section at distance x from the apex of the cone is given by

$$A(x) = \pi\left(\frac{r}{L}x\right)^2$$

The forces acting on any cross-section of the cone are shown in the following diagram:



Gravity force acting on the section of length dx is

$$F = \frac{A(x)x}{3}\gamma$$

Extension of the element is

$$\begin{aligned}\delta &= \int_0^L \frac{F}{A(x)E}dy\\ &= \int_0^L \frac{x\gamma}{3E}dx\\ &= \frac{\gamma l^2}{6E}\end{aligned}$$

16. A cylindrical pressure vessel has an inner radius 1.0 m and a wall thickness of 20 mm. It is made from steel plates welded together along the 45° seam. Determine the normal and shear stress components along this seam if the vessel is designed to an internal pressure of 6 MPa.

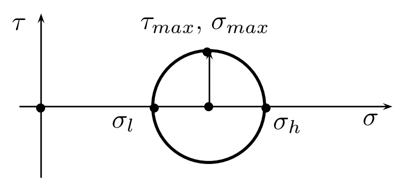
Solution. Given

$$\begin{aligned}d &= 1.5\times 2\\ &= 3.0 \text{ m}\\ t &= 0.02 \text{ m}\end{aligned}$$

The vessel can be treated as thin-walled cylinder for which hoop stress and longitudinal stress are determined as

$$\begin{aligned}\sigma_h &= \frac{pd}{2t}\\ &= 300 \text{ MPa}\\ \sigma_l &= \frac{pd}{4t}\\ &= 150 \text{ MPa}\end{aligned}$$

Mohr's circle is drawn as follows:



The stresses along the 45° seam are found as follows:

$$\begin{aligned}\tau_{max} &= \frac{\sigma_h - \sigma_l}{2}\\ &= 75 \text{ MPa}\\ \sigma_{max} &= \frac{\sigma_h + \sigma_l}{2}\\ &= 225 \text{ MPa}\end{aligned}$$

17. A 5 m long pin-ended column of square cross-section is to be made of steel. Assuming $E = 200$ GPa, $\sigma = 250$ MPa, and using factor of safety $N = 2.5$ in computing Euler's critical load for buckling, determine the size of the cross-section if the column has to support 200 kN load.

Solution. Let a be size of the square cross-section for which moment of inertia is givne by

$$I = \frac{a^4}{12}$$

Euler's critical load is determined as

$$\begin{aligned} N \times P_c &= \frac{\pi^2 EI}{L^2} \\ 2.5 \times 200 \times 10^3 &= \frac{\pi^2 \times 200 \times 10^9 \times a^4}{5^2 \times 12} \\ a &= 93.36 \text{ mm} \\ &\approx 100 \text{ mm} \end{aligned}$$

The column should also bear the normal stress

$$\begin{aligned} \sigma &= \frac{2.5 \times 200 \times 10^3}{0.1^2} \\ &= 50 \text{ MPa} \end{aligned}$$

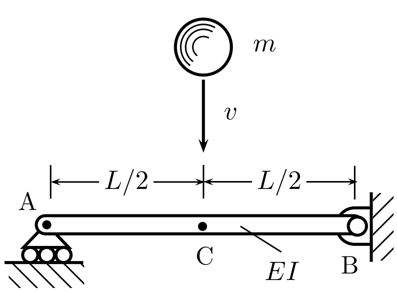
which is within the allowable stress of 250 MPa.

18. During a manufacturing operation, a steel rod of diameter $d = 20$ mm and length $l = 2$ m acquires an elastic strain energy of $u = 50$ Nm per unit volume when it is subjected to yielding on axial load. Modulus of elasticity of the steel is $E = 200$ GPa. Determine the yield strength of the specimen.

Solution. Strain energy (per unit volume) is determined as

$$\begin{aligned} \frac{\sigma_y^2}{2E} &= \frac{u}{\pi d^2 \times l/4} \\ \sigma_y &= \sqrt{\frac{8uE}{\pi d^2 \times l}} \\ &= \sqrt{\frac{8 \times 15 \times 200 \times 10^9}{\pi 0.02^2 \times 2}} \\ &= 97.72 \text{ MPa} \end{aligned}$$

19. A block of m kg moving with a velocity v hits squarely the prismatic beam AB at its mid-point C.



Ignoring the change in potential energy, determine the deflection of point C of the beam.

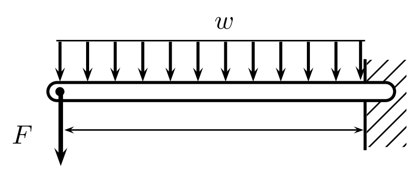
Solution. Let δ be the deflection at point C. For the simply supported beam loaded with an equivalent load P at mid-point,

$$\begin{aligned} \delta &= \frac{PL^3}{48EI} \\ P &= \frac{48\delta EI}{L^3} \end{aligned}$$

Strain energy of the beam shall be equal to the kinetic energy of the moving mass:

$$\begin{aligned} \frac{1}{2} P\delta &= \frac{1}{2} mv^2 \\ \frac{48\delta^2 EI}{L^3} &= mv^2 \\ \delta &= \sqrt{\frac{48mv^2 L^3}{48EI}} \end{aligned}$$

20. Determine the deflection of the free end of a cantilever beam AB which supports a UDL w over the full span L and a concentrated load F at the free end. Modulus of elasticity is E and area-moment of inertia is I.



Solution. Consider a section at distance x from the free end. Taking positive direction downward, the moment at this section is

$$M(x) = Fx + w\frac{x^2}{2}$$

Using energy method,

$$\delta = \int_0^L \frac{M}{EI} \frac{\delta M}{\delta F} dx$$

The deflection of free end is determined as

$$\begin{aligned} \delta &= \int_0^L \frac{Fx + wx^2/2}{EI} x dx \\ &= \frac{1}{EI} \int_0^L \left(Fx^2 + w\frac{x^3}{2} \right) dx \\ &= \frac{1}{EI} \left[F\frac{x^3}{3} + w\frac{x^4}{8} \right]_0^L \\ &= \frac{FL^3}{3EI} + \frac{wL^4}{8EI} \end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. The second moment of a circular area about the diameter is given by (D is the diameter).

(a) $\pi D^4/4$ (b) $\pi D^4/16$
(c) $\pi D^4/32$ (d) $\pi D^4/64$

(GATE 2003)

Solution. Natural axis of the circular area is its diameter (D) about which the second moment is

$$I = \frac{\pi D^4}{64}$$

Ans. (d)

2. A concentrated load of P acts on a simply supported beam of span L at a distance $L/3$ from the left support. The bending moment at the point of application of the load is given by

(a) $PL/3$ (b) $2PL/3$
(c) $PL/9$ (d) $2PL/9$

(GATE 2003)

Solution. Reaction at loaded end is

$$R \times L = P \times \left(L - \frac{L}{3}\right)$$
$$R = \frac{2P}{3}$$

Therefore, bending moment of the reaction at the point of application is

$$M = R \times \frac{L}{3}$$
$$= \frac{2PL}{9}$$

Ans. (d)

3. Two identical circular rods of the same diameter and length are subjected to the same magnitude of axial tensile force. One of the rods is made out of mild steel having the modulus of elasticity of 206 GPa. The other rod is made out of cast iron having the modulus of elasticity of 100 GPa. Assume both the materials to be homogeneous and isotropic and the axial force causes the same amount of uniform stress in both the rods. The stresses developed are within the proportional limit of the respective materials. Which of the following observations is correct?

(a) Both rods elongate by the same amount
(b) Mild steel rod elongates more than the cast iron rod
(c) Cast iron rod elongates more than the mild steel rod
(d) As the stresses are equal, strains are also equal in both the rods

(GATE 2003)

Solution. Given

$$E_{steel} = 206 \text{ GPa}$$
$$E_{Al} = 100 \text{ GPa}$$

Also, σ is same in both rods, and length is also same. Axial strain and elongation are given by

$$\varepsilon = \frac{\sigma}{E}$$
$$\delta = \frac{\sigma}{E} l$$

Known that

$$E_{steel} > E_{c.i.}$$

Therefore, elongation in steel will be less than that in cast iron.

Ans. (c)

4. Two beams, one having square cross-section and another circular cross-section, are subjected to the same amount of bending moment. If the cross-sectional area as well as the material of the both the beams are the same, then

(a) maximum bending stress developed in both the beams is the same
(b) the circular beam experiences more bending stress that the square one
(c) the square beam experiences more bending stress than the circular one
(d) as the material is same, both the beams will experience the same deformation

(GATE 2003)

Solution. If a is the size of the square (s) section and d is the diameter of circular (c) section, then

$$a^2 = \frac{\pi}{4} d^2$$
$$a = \frac{\sqrt{\pi}}{2} d$$

Ratio of section modulus is

$$\frac{Z_s}{Z_c} = \frac{\pi d^3/32}{a^3/6}$$
$$= 1.18164$$

Bending stress is given by

$$\sigma = \frac{M}{Z}$$

Material of beam is the same. The ratio of stresses is

$$\begin{aligned}\frac{\sigma_c}{\sigma_s} &= \frac{Z_c}{Z_s} \\ &= 1.18164\end{aligned}$$

Therefore,

$$\sigma_c > \sigma_s$$

Ans. (b)

5. Maximum shear stress developed on the surface of a solid circular shaft under pure torsion is 240 MPa. If the shaft diameter is doubled, then the maximum shear stress developed corresponding to the same torque will be

(a) 120 MPa (b) 60 MPa
(c) 30 MPa (d) 15 MPa

(GATE 2003)

Solution. Given that

$$\tau_1 = 240 \text{ MPa}$$

Maximum shear stress in a solid shaft is

$$\tau = \frac{16T}{\pi d^3}$$

Therefore, the maximum shear stress developed in the shaft is

$$\begin{aligned}\frac{\tau_2}{\tau_1} &= \left(\frac{d_1}{d_2}\right)^3 \\ \tau_2 &= \frac{\tau_1}{8} \\ &= \frac{240}{8} \\ &= 30 \text{ MPa}\end{aligned}$$

Ans. (c)

6. A simply supported laterally loaded beam was found to deflect more than a specified value. Which of the following measures will reduce the deflection?

(a) Increase the area moment of inertia
(b) Increase the span of the beam
(c) Select a different material having lesser modulus of elasticity
(d) Magnitude of the load to be increased

(GATE 2003)

Solution. Deflection of simply supported is inversely proportional to area moment of inertia.

Ans. (a)

7. A shaft subjected to torsion experiences a pure shear stress τ on the surface. The maximum principal stress on the surface, which is at 45° to the axis, will have a value

(a) $\tau \cos 45°$ (b) $2\tau \cos 45°$
(c) $\tau \cos^2 45°$ (d) $2\tau \sin 45° \cos 45°$

(GATE 2003)

Solution. In pure shear of τ, the principle stress at 45° will be

$$\sigma_1 = 2\tau \sin 45° \cos 45°$$

Ans. (d)

Linked Answer Questions

The state of stress at a point P in a two-dimensional loading is such that the Mohr's circle is a point located at 175 MPa on the positive normal stress axis.

8. Determine the maximum and minimum principal stresses respectively from the Mohr's circle

(a) +175 MPa, −175 MPa
(b) +175 MPa, +175 MPa
(c) 0, −175 MPa
(d) 0, 0

(GATE 2003)

Solution. Given that the Mohr's circle is a point. Therefore,

$$R_\sigma = 0$$

Also, center of the Mohr's circle is located at

$$OO' = 175 \text{ MPa}$$

The radius of Mohr's circle is

$$\begin{aligned}R_\sigma &= 0 \\ \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} &= 0\end{aligned}$$

For $R_\sigma = 0$,

$$\begin{aligned}\tau_{xy} &= 0 \\ \frac{\sigma_x - \sigma_y}{2} &= 0\end{aligned}$$

Therefore,

$$\sigma_x = \sigma_y$$

Also,

$$\begin{aligned} OO' &= \frac{\sigma_x+\sigma_y}{2} \\ &= 175 \\ \sigma_x &= \sigma_y \\ &= +175 \text{ MPa} \end{aligned}$$

Ans. (b)

9. Determine the directions maximum and minimum principal stresses at the point P from the Mohr's circle

(a) 0, 90° (b) 90°, 0
(c) 45°, 135° (d) all directions

(GATE 2003)

Solution. The orientation of principle plane ϕ is where shear stress is zero. For the given case, shear stress is zero in all directions.

Ans. (d)

10. In terms of Poisson's ratio (ν), the ratio of Young's modulus (E) to shear modulus (G) of elastic materials is

(a) $2(1+\nu)$ (b) $2(1-\nu)$
(c) $(1+\nu)/2$ (d) $2(1-\nu)$

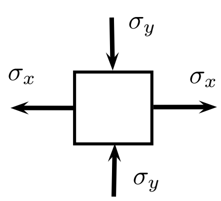
(GATE 2004)

Solution. Using following formula:

$$G = \frac{E}{2(1+\nu)}$$
$$\frac{E}{G} = 2(1+\nu)$$

Ans. (a)

11. The figure shows the state of stress at a certain point in a stressed body. The magnitudes of normal stresses in the x and y directions are 100 MPa and 20 MPa, respectively.



The radius of Mohr's stress circle representing this state of stress is

(a) 120 (b) 80
(c) 60 (d) 40

(GATE 2004)

Solution. Given that

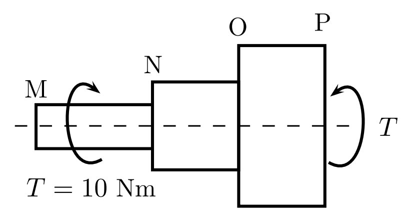
$$\begin{aligned} \sigma_x &= 100 \text{ MPa} \\ \sigma_y &= -20 \text{ MPa} \\ \tau_{xy} &= 0 \text{ MPa} \end{aligned}$$

Radius of Mohr's circle is

$$\begin{aligned} R_\sigma &= \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}{}^2} \\ &= \sqrt{\left(\frac{100-(-20)}{2}\right)^2 + 0^2} \\ &= 60 \text{ MPa} \end{aligned}$$

Ans. (c)

12. A torque of 10 Nm is transmitted through a stepped shaft as shown in the figure. The torsional stiffness of individual sections of lengths MN, NO and OP are 20 Nm/rad, 30 Nm/rad and 60 Nm/rad, respectively.



The angular deflection between the ends M and P of the shaft is

(a) 0.5 rad (b) 1.0 rad
(c) 5.0 rad (d) 10.0 rad

(GATE 2004)

Solution. Torsional stiffness is torque divided by angle of twist. Torsional stiffness in the three segments, S_1, S_2 and S_3, is given as

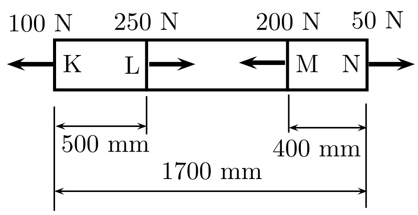
$$\begin{aligned} T_{S1} &= 20 \text{ Nm/rad} \\ T_{S2} &= 30 \text{ Nm/rad} \\ T_{S3} &= 30 \text{ Nm/rad} \\ T &= 10 \text{ Nm} \end{aligned}$$

The net angular deflection (θ) will be sum of the angular deflection in the three segments, which are applied with the same amount of torque, T. Hence,

$$\begin{aligned} \theta &= \frac{T}{T_{S1}} + \frac{T}{T_{S2}} + \frac{T}{T_{S3}} \\ &= \frac{10}{20} + \frac{10}{30} + \frac{10}{60} \\ &= 1 \text{ rad} \end{aligned}$$

Ans. (b)

13. The figure below shows a steel rod of 25 mm^2 cross-sectional area. It is loaded at four points, K, L, M and N. Assume $E_{\text{steel}} = 200$ GPa.



The total change in length of the rod due to loading is

(a) 1 μm (b) −10 μm
(c) 16 μm (d) −20 μm

(GATE 2004)

Solution. Given that

$$A = 25 \times 10^{-6}\ \text{m}^2$$
$$E = 200 \times 10^{9}\ \text{Pa}$$

Using Hooke's law,

$$\begin{aligned}\delta &= \frac{100 \times 0.5}{AE} - \frac{150 \times 0.8}{AE} + \frac{50 \times 0.4}{AE}\\ &= \frac{-50}{AE}\\ &= -0.00001\ \text{m}\\ &= -10\ \mu\text{m}\end{aligned}$$

Ans. (b)

14. A solid circular shaft of 60 mm diameter transmits a torque of 1600 Nm. The value of maximum shear stress develop is

(a) 37.72 MPa (b) 47.72 MPa
(c) 57.72 MPa (d) 67.72 MPa

(GATE 2004)

Solution. Given that

$$T = 1600\ \text{Nm}$$
$$d = 0.06\ \text{m}$$

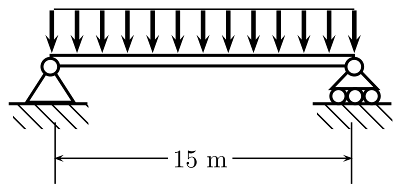
The maximum shear stress is

$$\begin{aligned}\tau_{\max} &= \frac{16T}{\pi d^3}\\ &= 37.725 \times 10^{6}\ \text{Pa}\\ &= 37.72\ \text{MPa}\end{aligned}$$

Ans. (a)

Common Data Questions

A steel beam of breadth 120 mm and height 750 mm is loaded as shown in the figure. Assume $E_{\text{steel}} = 200$ GPa.



15. The beam is subjected to a maximum bending moment of

(a) 3375 kNm (b) 4750 kNm
(c) 6750 kNm (d) 8750 kNm

(GATE 2004)

Solution. Given that

$$w = 120 \times 10^{3}\ \text{N/m}$$
$$l = 15\ \text{m}$$

The maximum bending moment is

$$\begin{aligned}M_{\max} &= \frac{wl^2}{8}\\ &= \frac{120 \times 10^{3} \times 15^2}{8}\\ &= 3375\ \text{kNm}\end{aligned}$$

Ans. (a)

16. The value of maximum deflection of the beam is

(a) 93.75 mm (b) 83.75 mm
(c) 73.75 mm (d) 63.75 mm

(GATE 2004)

Solution. Given that

$$b = 0.120\ \text{m}$$
$$h = 0.750\ \text{m}$$
$$E = 200 \times 10^{9}\ \text{GPa}$$

Moment of inertia is determined as

$$\begin{aligned}I &= \frac{bh^3}{12}\\ &= \frac{0.120 \times 0.750^3}{12}\\ &= 4.218 \times 10^{-3}\end{aligned}$$

The deflection is maximum at mid-span, given by

$$\delta_{\max} = \frac{5}{384}\frac{wl^4}{EI}$$

Putting the given values, one obtains

$$\delta_{\max} = \frac{5}{384}\frac{120\times10^3\times15^4}{200\times10^9\times4.218\times10^{-3}}$$
$$= 0.09375 \text{ m}$$
$$= 93.75 \text{ mm}$$

Ans. (a)

17. A uniform, slender cylindrical rod is made of a homogeneous and isotropic material. The rod rests on a frictionless surface. The rod is heated uniformly. If the radial and longitudinal thermal stresses are represented by σ_r and σ_z respectively, then

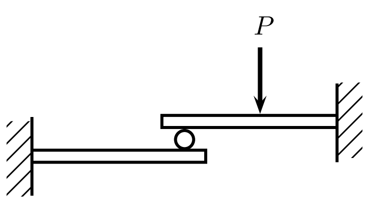
(a) $\sigma_r = 0$, $\sigma_z = 0$ (b) $\sigma_r \neq 0$, $\sigma_z = 0$
(c) $\sigma_r = 0$, $\sigma_z \neq 0$ (d) $\sigma_r \neq 0$, $\sigma_z \neq 0$

(GATE 2005)

Solution. As the surface is frictionless, there will be no longitudinal force, so the longitudinal stress is zero. Also, the rod is free, therefore, there will be no radial stress.

Ans. (a)

18. Two identical cantilever beams are supported, as shown in the figure, with their free ends in contact with a rigid roller.



After a load P is applied, the free ends will have

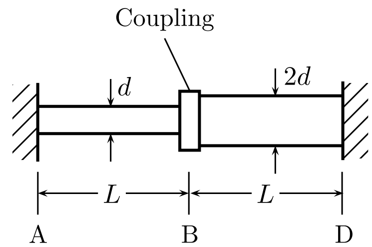
(a) equal deflections but not equal slopes
(b) equal slopes but not equal deflections
(c) equal slopes as well as equal deflections
(d) neither equal slopes nor equal deflections

(GATE 2005)

Solution. There will be equal deflections at the free ends of both the cantilevers because both free ends are loaded against the roller to result vertically downward deflection. But their slopes will be different because the right side cantilever is loaded differently.

Ans. (a)

19. The two shafts AB and BC of equal length and diameters d and $2d$ are made of the same material. They are joined at B through a shaft coupling, while the ends A and C are built-in (cantilevered). A twisting moment T is applied to the coupling.



If T_A and T_C represent the twisting moments at the ends A and C, respectively, then

(a) $T_C = T_A$ (b) $T_C = 8T_A$
(c) $T_C = 16T_A$ (d) $T_A = 16T_C$

(GATE 2005)

Solution. Using flexure formula

$$\frac{T}{I_p} = \frac{G\theta}{l}$$

For circular shafts of same material (G), equal lengths (l), and joined at ends for equal angle of twist (θ),

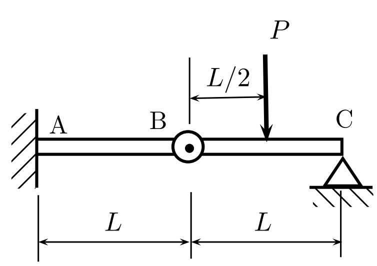
$$T \propto d^4$$

Applying this to present case,

$$T_A = \left(\frac{d}{2d}\right)^4 T_C$$
$$T_C = 16T_A$$

Ans. (c)

20. A beam is made up of two identical bars AB and BC, by hinging them together at B. The end A is built-in (cantilevered) and the end C is simply supported.



With the load P acting as shown, the bending moment at A is

(a) zero (b) $PL/2$
(c) $PL/3$ (d) indeterminate

(GATE 2005)

Solution. The reaction (R) at hinged B is determined by taking BC as free body:

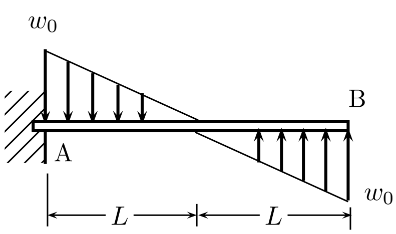
$$R = \frac{P}{2}$$

The moment at A is determined by taking cantilever AB with applied load R vertically upward at B:

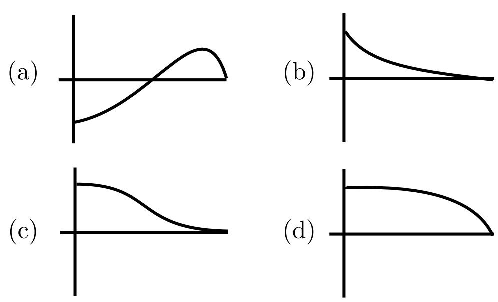
$$\begin{aligned} M &= R \times L \\ &= \frac{P}{2} \times L \end{aligned}$$

Ans. (b)

21. A cantilever beam carries the anti-symmetric load shown, where w_0 is the peak intensity of the distributed load.



Qualitatively, the correct bending moment diagram for this beam is

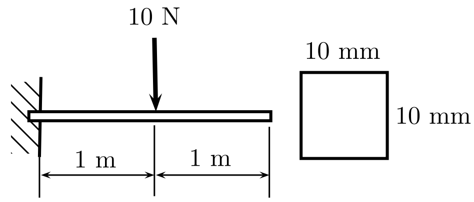


(GATE 2005)

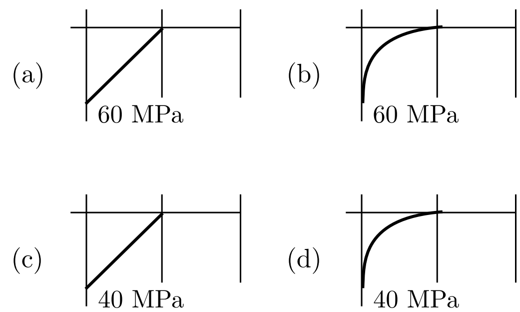
Solution. Upward (sagging) moments are taken positive from both sides of the sections. Starting from the right end, the moment will be zero at the free end. Further, the moment will increase towards the mid-point after which increase will be gradually hampered by the uniformly varying load (UVL) increasing from mid point to the fixed end.

Ans. (c)

22. A cantilever beam has the square cross-section 10 mm × 10 mm. It carries a transverse load of 10 N.



Considering only bottom fibers of the beam, the correct representation of the longitudinal variation of bending stress is



(GATE 2005)

Solution. Moment of inertia is constant throughout the section and is equal to

$$\begin{aligned} I &= \frac{bh^3}{12} \\ &= \frac{0.01 \times 0.01^3}{12} \end{aligned}$$

Bending stress at any point is proportional to bending moment M, which is zero in the half span free end side, and varies linearly in half span fixed end side from zero at mid span to maximum value at the fixed end, given by

$$M = 10 \times 1 \text{ Nm}$$

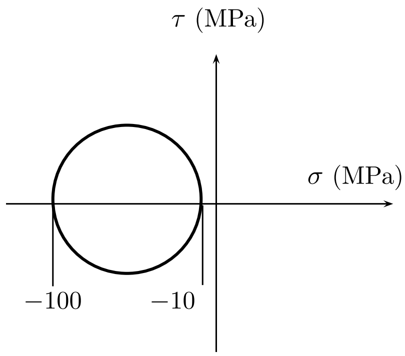
Bending stress at left end is

$$\begin{aligned} \sigma_b &= \frac{M}{I} \times \frac{h}{2} \\ &= \frac{12 \times 10 \times 0.01}{0.01 \times 0.01^3 \times 2} \\ &= 60 \text{ MPa} \end{aligned}$$

Hence, the bending stress will also vary linearly as M varies.

Ans. (a)

23. The Mohr's circle of plane stress for a point in a body is shown.



The design is to be done on the basis of the maximum shear stress theory for yielding. Then, yielding will just begin if the designer chooses a ductile material whose yield strength is

(a) 45 MPa (b) 50 MPa
(c) 90 MPa (d) 100 MPa

(GATE 2005)

Solution. Radius of the Mohr's circle is the maximum shear stress,

$$\tau_{\max} = \frac{100-10}{2} = 45 \text{ MPa}$$

Therefore, the minimum yield strength is two times that of the maximum shear stress:

$$\begin{aligned}\sigma_y &= 2\tau_{\max}\\ &= 2\times 45\\ &= 90 \text{ MPa}\end{aligned}$$

Ans. (c)

24. For a circular shaft of diameter d subjected to torque T, the maximum value of the shear stress is:

(a) $\frac{64T}{\pi d^3}$ (b) $\frac{32T}{\pi d^3}$
(c) $\frac{16T}{\pi d^3}$ (d) $\frac{8T}{\pi d^3}$

(GATE 2006)

Solution. Polar moment of inertia for circular shaft of diameter d is

$$I_p = \frac{\pi d^4}{32}$$

Maximum shear stress generated is given by

$$\begin{aligned}\frac{\tau_{\max}}{d/2} &= \frac{T}{I_p}\\ &= \frac{T}{\pi d^4/32}\\ \tau_{\max} &= \frac{16T}{\pi d^3}\end{aligned}$$

Ans. (c)

25. A pin-ended column of length L, modulus of elasticity E and second moment of the cross-sectional area I is loaded eccentrically by a compressive load P. The critical buckling load (P_{cr}) is given by

(a) $\frac{EI}{\pi^2 L^2}$ (b) $\frac{EI}{3\pi^2 L^2}$
(c) $\frac{\pi EI}{L^2}$ (d) $\frac{\pi^2 EI}{L^2}$

(GATE 2006)

Solution. For pin-ended column,

$$P_{\text{cr}} = \frac{\pi^2 EI}{L^2}$$

Ans. (d)

26. A steel bar of 40 mm × 40 mm square cross-section is subjected to an axial compressive load of 200 kN. If the length of the bar is 2 m and $E = 200$ GPa, the elongation of the bar will be:

(a) 1.25 mm (b) 2.70 mm
(c) 4.05 mm (d) 5.40 mm

(GATE 2006)

Solution. Given that

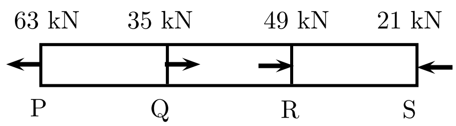
$$\begin{aligned}A &= 0.04\times 0.04 \text{ m}^2\\ F &= 200\times 10^3 \text{ N}\\ L &= 2 \text{ m}\\ E &= 200 \text{ GPa}\end{aligned}$$

Therefore, elongation (δ) is given by

$$\delta = \frac{FL}{AE} = 1.25 \text{ mm}$$

Ans. (a)

27. A bar having a cross-sectional area of 700 mm^2 is subjected to axial loads at the positions indicated.



The value of stress in the segment QR is:

(a) 40 MPa (b) 50 MPa
(c) 70 MPa (d) 120 MPa

(GATE 2006)

Solution. Given that

$$A = 700 \times 10^{-6} \text{ m}^2$$

Net tensile force on the segment QR is

$$\begin{aligned} F &= 63 - 35 \\ &= 28 \text{ kN} \end{aligned}$$

Therefore, stress (σ) is

$$\begin{aligned} \sigma &= \frac{F}{A} \\ &= \frac{28 \times 10^3}{700 \times 10^{-6}} \\ &= 40 \text{ MPa} \end{aligned}$$

Ans. (a)

28. The ultimate tensile strength of a material is 400 MPa and the elongation up to maximum load is 35%. If the material obeys power law of hardening ($\sigma = K\varepsilon^n$), then the true stress-true strain relation (stress in MPa) in the plastic deformation range is:

(a) $s = 540\varepsilon^{0.30}$ (b) $s = 775\varepsilon^{0.30}$
(c) $s = 540\varepsilon^{0.35}$ (d) $s = 775\varepsilon^{0.35}$

(GATE 2006)

Solution. Given that

$$\begin{aligned} \sigma_u &= 400 \text{ MPa} \\ \varepsilon_u &= 0.35 \end{aligned}$$

True stress and strain are calculated as

$$\begin{aligned} \sigma &= \sigma_u (1 + \varepsilon_u) \\ &= 540 \text{ MPa} \\ \varepsilon &= \ln(1 + \varepsilon_u) \\ &= 0.300105 \end{aligned}$$

The power law equation (given in the question) is known as Holloman-Ludwig equation, which describes the material flow behavior:

$$\sigma = K\varepsilon^n$$

where, K is strength coefficient, and n is strain hardening exponent.

Alternate Solution. Instead of trying to solve for K and n, for which two equations are not provided, it is better to solve the problem by elimination. Out of given options, (b) is giving correct answer.

$$\sigma = 775\varepsilon^{0.30} \text{ MPa}$$

Ans. (c)

Linked Answer Questions

A simply supported beam of span length 6 m and 75 mm diameter carries a uniformly distributed load of 1.5 kN/m.

29. What is the maximum value of bending moment?

(a) 9 kNm (b) 13.5 kNm
(c) 81 kNm (d) 125 kNm

(GATE 2006)

Solution. Given that

$$\begin{aligned} l &= 6 \text{ m} \\ d &= 0.075 \text{ m} \\ w &= 1.5 \text{ kN/m} \end{aligned}$$

Bending moment is maximum at mid-span, and is given by

$$\begin{aligned} M_{\max} &= \frac{1}{8} w l^2 \\ &= 6.750 \text{ kNm} \end{aligned}$$

Ans. (a)

30. What is the maximum value of bending stress?

(a) 162.98 MPa (b) 325.95 MPa
(c) 625.95 MPa (d) 651.90 MPa

(GATE 2006)

Solution. Section modulus of circular section of diameter d is given by

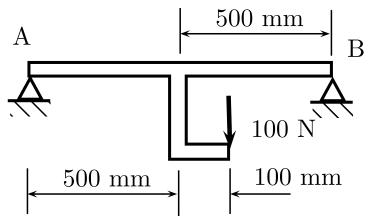
$$\begin{aligned} Z &= \frac{\pi d^4}{64} \times \frac{2}{d} \\ &= \frac{\pi d^3}{32} \end{aligned}$$

Using the value of $M_{\max}$, maximum bending stress is calculated as

$$\begin{aligned} \sigma_{\max} &= \frac{M_{\max}}{Z} \\ &= \frac{6.750 \times 10^3}{\pi \times 0.075^3 / 32} \\ &= 162.975 \text{ MPa} \end{aligned}$$

Ans. (a)

31. In a simply supported beam loaded as shown below, the maximum bending moment in Nm is



(a) 25 (b) 30
(c) 35 (d) 60

(GATE 2007)

Solution. Reaction at B is

$$R_B = \frac{100 \times (0.5+0.10)}{1} = 60 \text{ N}$$

Bending moment would be maximum at point load, and will be given by

$$M = R_B \times 0.5 = 30 \text{ Nm}$$

Ans. (b)

32. A steel rod of length L and diameter D, fixed at both ends, is uniformly heated to a temperature rise of ΔT. The Young's modulus is E and the coefficient of linear expansion is α. The thermal stress in the rod is

(a) 0 (b) $\alpha\Delta T$
(c) $E\alpha\Delta T$ (d) $E\alpha\Delta TL$

(GATE 2007)

Solution. The rod is fixed at both ends. Therefore, apparent longitudinal thermal strain is

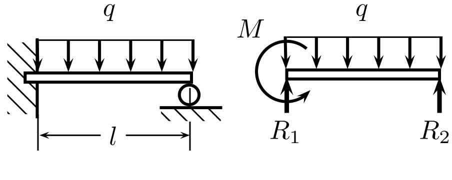
$$\varepsilon = \frac{\alpha\Delta TL}{L} = \alpha\Delta T$$

Thermal stress is

$$\sigma = \varepsilon E = \alpha\Delta TE$$

Ans. (c)

33. A uniformly loaded propped cantilever beam and its free body diagram are shown below.



The reactions are

(a) $R_1 = 5ql/8,\ R_2 = 3ql/8,\ M = ql^2/8$
(b) $R_1 = 3ql/8,\ R_2 = 5ql/8,\ M = ql^2/8$
(c) $R_1 = 5ql/8,\ R_2 = 3ql/8,\ M = 0$
(d) $R_1 = 3ql/8,\ R_2 = 5ql/8,\ M = 0$

(GATE 2007)

Solution. This is a statistically indeterminate beam for which static equilibrium conditions are insufficient to compute all the reactions and draw free body diagrams. For this, additional information regarding deflections (compatibility conditions) is used, as follows.

Assuming flexural rigidity of the beam to be EI. If end (2) is made free, then its deflection will be

$$\delta = \frac{ql^4}{8EI}$$

The deflection caused by the reaction R_2 is

$$\delta = -\frac{R_2 l^3}{3EI}$$

Equating above two equations,

$$R_2 = \frac{3}{8}ql$$

For static equilibrium,

$$R_1 + R_2 = ql$$
$$R_1 = \frac{5}{8}ql$$

Taking moments about (1),

$$M = -R_2 \times l + ql \times \frac{l}{2} = -\frac{3}{8}ql \times l + q\frac{l^2}{2} = \frac{1}{8}ql^2$$

Ans. (a)

34. A 200×100×50 mm steel block is subjected to a hydrostatic pressure of 15 MPa. The Young's modulus and Poisson's ratio of the material are 200 GPa and 0.3 respectively. The change in the volume of the block in mm^3 is

(a) 85 (b) 90
(c) 100 (d) 110

(GATE 2007)

Solution. Given that

$$\sigma = -15 \text{ MPa}$$
$$E = 200 \text{ GPa}$$
$$\mu = 0.3$$

Linear strain in any direction is given by

$$\varepsilon = -\frac{\sigma}{E}(1-2\mu) = 0.00003$$

Volumetric strain is calculated as

$$\varepsilon_v = 3\varepsilon = -0.00009$$

Initial volume of the block is

$$V = 200 \times 100 \times 50 = 1.0 \times 10^6 \text{ mm}^3$$

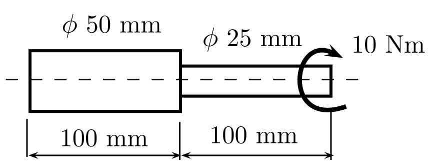
Therefore, change in volume (mm^3) is

$$\Delta V = V \times \varepsilon_v = -90 \text{ mm}^3$$

Therefore, volume decreases by 90 mm^3.

Ans. (b)

35. A stepped steel shaft shown below is subjected to 10 Nm torque.



If the modulus of rigidity is 80 GPa, the strain energy in the shaft in N mm is

(a) 4.12 (b) 3.46
(c) 1.73 (d) 0.86

(GATE 2007)

Solution. Given for the left side step

$$T = 10 \text{ Nm}$$
$$l = 0.10 \text{ m}$$
$$G = 80 \times 10^9 \text{ Pa}$$
$$d_1 = 0.025 \text{ m}$$
$$d_2 = 0.050 \text{ m}$$

The angle of twist is determined as

$$\theta = \frac{Tl}{JG}$$
$$\theta_1 = 0.000325949 \text{ rad}$$

Using the torsional formula, for the same T, l and G, ratio of angles of twist in the two steps is

$$\frac{\theta_1}{\theta_2} = \left(\frac{d_2}{d_1}\right)^4 = 16$$

Therefore,

$$\theta_2 = \frac{\theta_1}{16}$$

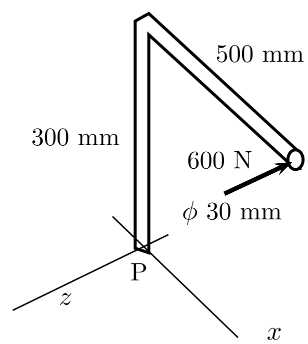
Strain energy for gradually applied torque is

$$E = \frac{1}{2} \times T(\theta_1 + \theta_2) = 0.00173161 \text{ Nm}$$

Ans. (c)

Linked Answer Questions

A machine frame shown in the figure below is subjected to a horizontal force of 600 N parallel to z-direction.



36. The normal and shear stresses in MPa at point P are, respectively,

(a) 67.9 and 56.6 (b) 56.6 and 67.9
(c) 67.9 and 0.0 (d) 0.0 and 56.6

(GATE 2007)

Solution. Given that

$$d = 0.030 \text{ m}$$

Torque and bending moment acting at point P are, respectively,

$$T = 600 \times 0.5 = 300 \text{ Nm}$$
$$M = 600 \times 0.3 = 180 \text{ Nm}$$

Therefore, normal (bending) stress and shear (torsional) stresses are

$$\sigma = \frac{32M}{\pi d^3} = 67.9061 \text{ MPa}$$

$$\tau = \frac{16T}{\pi d^3} = 56.5884 \text{ MPa}$$

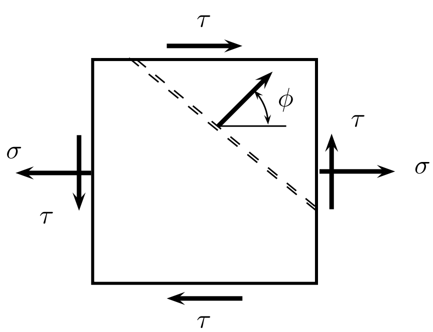
Ans. (a)

37. The maximum principal stress in MPa and the orientation of the corresponding principal plane in degrees are, respectively,

(a) −32.0 and −29.52
(b) 100.0 and 60.48
(c) −32.0 and 60.48
(d) 100.0 and −29.52

(GATE 2007)

Solution. The element is subjected to stresses as shown below



The principle stress is given by

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = 99.9459 \text{ MPa}$$

The expression for orientation of principle plane, where shear stress is zero, is given by

$$\tan\phi = \frac{2\tau}{\sigma}$$
$$\phi = 59.0318°$$

Ans. (b)

38. The transverse shear stress acting in a beam of rectangular cross-section, subjected to a transverse shear load is

(a) variable with maximum at the bottom of the beam
(b) variable with maximum at the top of the beam
(c) uniform
(d) variable with maximum of the neutral axis

(GATE 2008)

Solution. Transverse shear stress in rectangular, circular and I-sections is maximum at neutral axis.

Ans. (d)

39. A rod of length L and diameter D is subjected to a tensile load P. Which of the following is sufficient to calculate the resulting change in diameter?

(a) Young's modulus
(b) Shear modulus
(c) Poisson's ratio
(d) Both Young's modulus and shear modulus

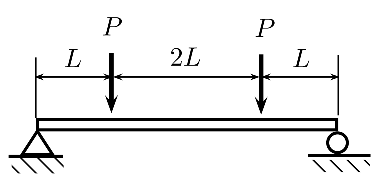
(GATE 2008)

Solution. Young's modulus is required to calculate longitudinal strain. To calculate the lateral strain, Poisson's ratio is also required, which can be calculated as

$$G = \frac{E}{2(1+\mu)}$$

Ans. (d)

40. The strain energy stored in the beam with flexural rigidity EI and loaded as shown in the figure is

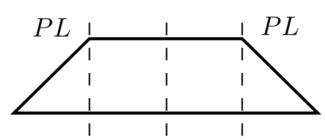


(a) $\frac{P^2L^3}{3EI}$ (b) $\frac{2P^2L^3}{3EI}$

(c) $\frac{4P^2L^3}{3EI}$ (d) $\frac{8P^2L^3}{3EI}$

(GATE 2008)

Solution. For the given configuration, reactions at both ends will be equal to P. Bending moment diagram is shown below.



Strain energy in an beam element subjected to bending moment M is given by

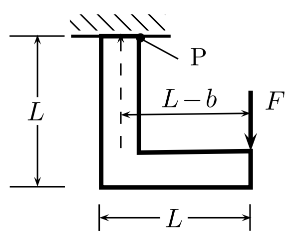
$$U = \frac{1}{2}\frac{M^2}{EI}dx$$

As this is a case of symmetric loading, strain energy in the beam is given by

$$\begin{aligned}U &= 2\times\left\{\frac{1}{2}\int_0^L \frac{(Px)^2}{EI}dx + \frac{1}{2}\int_L^{2L}\frac{(-PL)^2}{EI}dx\right\}\\ &= \int_0^L \frac{(Px)^2}{EI}dx + \int_L^{2L}\frac{(-PL)^2}{EI}dx\\ &= \left[\frac{P^2x^3}{3EI}\right]_0^L + \left[\frac{P^2L^2x}{EI}\right]_L^{2L}\\ &= \frac{P^2L^3}{3EI} + \left[\frac{2P^2L^3}{EI} - \frac{P^2L^3}{EI}\right]\\ &= \frac{P^2L^3}{3EI} + \frac{P^2L^3}{EI}\\ &= \frac{4P^2L^3}{3EI}\end{aligned}$$

Ans. (c)

41. For the component loaded with a force F as shown in the figure, the axial stress at the corner point P is



(a) $\frac{F(3L-b)}{4b^3}$ (b) $\frac{F(3L+b)}{4b^3}$

(c) $\frac{F(3L-4b)}{4b^3}$ (d) $\frac{F(3L-2b)}{4b^3}$

(GATE 2008)

Solution. The corner point P is subjected bending stress due to moment $F(L-b)$ and direct stress due to force F, therefore, total axial stress is

$$\begin{aligned}\sigma_a &= \frac{F(L-b)\times b}{2b\times(2b)^3/12} + \frac{F}{(2b)^2}\\ &= \frac{F(3L-2b)}{4b^3}\end{aligned}$$

Ans. (d)

42. A solid circular shaft of diameter 100 mm is subjected to an axial stress of 50 MPa. It is further subjected to a torque of 10 kNm. The maximum principal stress experienced on the shaft is closest to

(a) 41 MPa (b) 82 MPa

(c) 164 MPa (d) 204 MPa

(GATE 2008)

Solution. Maximum shear stress due to applied torque

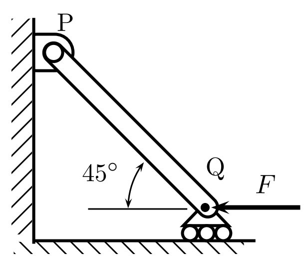
$$\begin{aligned}\tau &= \frac{16T}{\pi d^3}\\ &= \frac{16\times10\times10^3}{\pi\times0.1^3}\\ &= 50.91\text{ MPa}\end{aligned}$$

The axial stress is already of the value $\sigma = 50$ MPa. Therefore, a component at surface is subjected to τ and σ for which the maximum principle stress is given by

$$\begin{aligned}\sigma_1 &= \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}\\ &= \frac{50}{2} + \sqrt{\left(\frac{50}{2}\right)^2 + 50.91^2}\\ &= 81.71\text{ MPa}\end{aligned}$$

Ans. (b)

43. The rod PQ of length L and with flexural rigidity EI is hinged at both ends.



For what minimum force F is it expected to buckle?

(a) $\frac{\pi^2EI}{L^2}$ (b) $\frac{\sqrt{2}\pi^2EI}{L^2}$

(c) $\frac{\pi^2EI}{\sqrt{2}L^2}$ (d) $\frac{\pi^2EI}{2L^2}$

(GATE 2008)

Solution. For both ends hinged condition, buckling load is given by

$$F_c = \frac{\pi^2EI}{L^2}$$

This shall be achieved by F when

$$F = \frac{F_c}{\cos 45°} = \frac{\sqrt{2}\pi^2 EI}{L^2}$$

Ans. (b)

44. A compression spring is made of music wire of 2 mm diameter having a shear strength and shear modulus of 800 MPa and 80 GPa, respectively. The mean coil diameter is 20 mm, free length is 40 mm and the number of active coils is 10. If the mean coil diameter is reduced to 10 mm, the stiffness of the spring is, approximately,

(a) decreased by 8 times
(b) decreased by 2 times
(c) increased by 2 times
(d) increased by 8 times

(GATE 2008)

Solution. Given that

$$D_1 = 20 \text{ mm}$$
$$D_2 = 10 \text{ mm}$$

Keeping other values constants, stiffness (k) of the spring is proportional to cube of mean coil diameter ($k \propto D^3$), hence

$$k_2 = \left(\frac{D_2}{D_1}\right)^3 k_1 = \left(\frac{10}{20}\right)^3 k_1 = \frac{1}{8}k_1$$

Ans. (a)

45. A two-dimensional fluid element rotates like a rigid body. At a point within the element, the pressure is 1 unit. Radius of the Mohr's circle, characterizing the state of stress at that point is

(a) 0.5 unit (b) 0 unit
(c) 1 unit (d) 2 unit

(GATE 2008)

Solution. Pressure is a compressive stress in all directions, therefore, in a two-dimensional plane,:

$$\sigma_x = -1$$
$$\sigma_y = -1$$
$$\tau_{xy} = 0$$

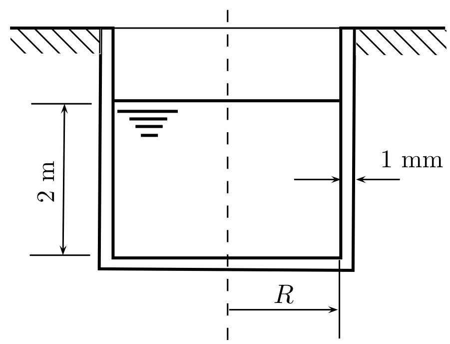
Radius of Mohr's circle is

$$R_\sigma = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}{}^2} = 0 \text{ unit}$$

Ans. (b)

Linked Answer Questions

A cylindrical container of radius $R = 1$ m, wall thickness 1 mm is filled with water upto a depth of 2 m and suspended along its upper rim. The density of water is 1000 kg/m^3 and acceleration due to gravity is 10 m/s^2. The self weight of the cylinder is negligible. The formula for hoop stress in a thin-walled cylinder can be used at all points along the height of the cylindrical container.



46. The axial and circumferential stress (σ_a, σ_c) experienced by the cylinder wall at mid depth (1 m as shown) are

(a) (10, 10) MPa (b) (5, 10) MPa
(c) (10, 5) MPa (d) (5, 5) MPa

(GATE 2008)

Solution. For cylindrical vessels, axial stress σ_a is half of the circumferential stress, therefore, from given options $\sigma_a = 5$ MPa and $\sigma_c = 10$ MPa

Ans. (b)

47. If the Young's modulus and Poisson's ratio of the container material are 100 GPa and 0.3, respectively, the axial strain in the cylinder wall at mid-depth is

(a) 2×10^{-5} (b) 6×10^{-5}
(c) 7×10^{-5} (d) 1.2×10^{-5}

(GATE 2008)

Solution. Given that

$$\sigma_a = 5 \text{ MPa}$$
$$\sigma_c = 10 \text{ MPa}$$
$$E = 100 \text{ GPa}$$
$$\mu = 0.3$$

Axial strain in the cylinder wall at mid-span is

$$\begin{aligned}\varepsilon &= \frac{1}{E}(\sigma_a - \mu\sigma_c)\\ &= \frac{1}{100\times10^9}\left(5\times10^6 - 0.3\times10\times10^6\right)\\ &= 2\times10^{-5}\end{aligned}$$

Ans. (a)

48. If the principal stresses in a plane stress problem are $\sigma_1 = 100$ MPa, $\sigma_2 = 40$ MPa, the magnitude of the maximum shear stress (in MPa) will be

(a) 60 (b) 50
(c) 30 (d) 20

(GATE 2009)

Solution. Given that

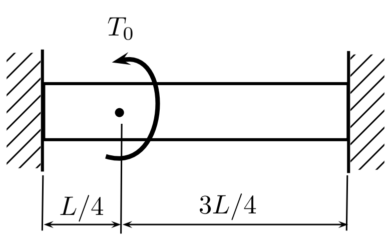
$$\sigma_1 = 100 \text{ MPa}$$
$$\sigma_2 = 40 \text{ MPa}$$

The magnitude of the maximum shear stress is

$$\begin{aligned}\tau_{\max} &= \frac{\sigma_1 - \sigma_2}{2}\\ &= \frac{100-40}{2}\\ &= 30 \text{ MPa}\end{aligned}$$

Ans. (c)

49. A solid shaft of diameter d and length L is fixed at both the ends. A torque T_0 is applied at a distance $L/4$ from the left end as shown in the figure.



The maximum shear stress in the shaft is

(a) $\dfrac{16T_0}{\pi d^3}$ (b) $\dfrac{12T_0}{\pi d^3}$

(c) $\dfrac{8T_0}{\pi d^3}$ (d) $\dfrac{4T_0}{\pi d^3}$

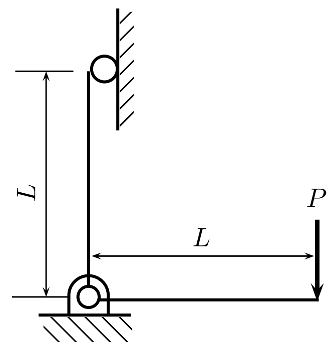
(GATE 2009)

Solution. The torque (T_0) wherever applied will create equal reaction. The maximum shear stress is determined as

$$\begin{aligned}\tau &= \frac{T_0\times d/2}{\pi d^4/32}\\ &= \frac{16T_0}{\pi d^3}\end{aligned}$$

Ans. (a)

50. A frame of two arms of equal length L is shown in the figure. The flexural rigidity of each arm of the frame is EI.



The vertical deflection at the point of application of load P is

(a) $\dfrac{PL^3}{3EI}$ (b) $\dfrac{2PL^3}{3EI}$

(c) $\dfrac{2PL^3}{EI}$ (d) $\dfrac{4PL^3}{3EI}$

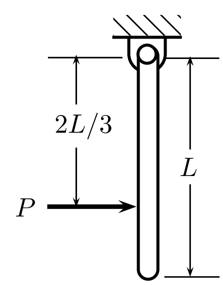
(GATE 2009)

Solution. The horizontal arm will act like cantilever of length L and flexural rigidity EI, subjected to load P at the free end, therefore, the deflection at free end is

$$\delta = \frac{PL^3}{3EI}$$

Ans. (a)

51. A uniform rigid rod of mass M and length L is hinged at one end as shown in the figure. A force P is applied at a distance of $2L/3$ from the hinge so that the rod swings to the right.



The reaction at the hinge is

(a) $-P$ (b) 0
(c) $P/3$ (d) $2P/3$

(GATE 2009)

Solution. Taking moments about the free end,

$$P \times \frac{L}{3} = R \times L$$
$$R = \frac{P}{3}$$

Ans. (c)

52. The state of plane stress at a point is given by $\sigma_x = -200$ MPa, $\sigma_y = 100$ MPa, $\tau_{xy} = 100$ MPa. The maximum shear stress in MPa is

(a) 111.8 (b) 150.1
(c) 180.3 (d) 223.6

(GATE 2010)

Solution. Given that

$$\sigma_x = -200 \text{ MPa}$$
$$\sigma_y = 100 \text{ MPa}$$
$$\tau_{xy} = 100 \text{ MPa}$$

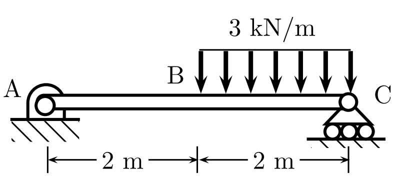
The maximum shear stress is

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \sqrt{\left(\frac{-200-100}{2}\right)^2 + 100^2}$$
$$= 180.278 \text{ MPa}$$

Ans. (c)

Linked Answer Questions

A massless beam has a loading pattern as shown in the figure. The beam is of rectangular cross-section with a width of 30 mm and height of 100 mm.



53. The maximum bending moment occurs at

(a) location B
(b) 2675 mm to the right of A
(c) 2500 mm to the right of A
(d) 3225 mm to the right of A

(GATE 2010)

Solution. If R_C is reaction at C, then taking moments of forces about point A gives

$$R_C \times 2 = 2 \times 3000 \times (2+1)$$
$$R_C = 4500 \text{ N}$$

Sum of the forces in vertical direction should be zero, hence

$$R_A + R_C = 3000 \times 2$$
$$R_A = 1500 \text{ N}$$

Bending moment increases from A to C continuously due reaction R_A but at the same time from point B, bending moment in the opposite direction is seen due to uniformly applied load. Therefore, bending moment at distance x from A beyond B is

$$M = R_A x - 3000 \times \frac{(x-2)^2}{2}$$

For it to be maximum or minimum,

$$\frac{dM}{dx} = 0$$
$$R_A - 3000 \times (x-2) = 0$$
$$x = 2.5 \text{ m}$$

Ans. (c)

54. The maximum magnitude of bending stress (in MPa) is given by

(a) 60.0 (b) 67.5
(c) 200.0 (d) 225.0

(GATE 2010)

Solution. Maximum magnitude of bending moment is at $x = 2.5$ in the following expression

$$M = R_A x - 3000 \times \frac{(x-2)^2}{2}$$

Therefore,

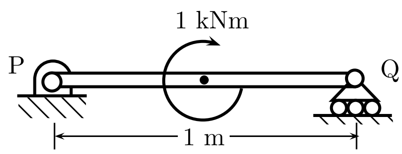
$$M_{\max} = 1500 \times 2.5 - 3000 \times \frac{(2.5-2)^2}{2}$$
$$= 3375 \text{ Nm}$$

Maximum bending stress is

$$\sigma = \frac{6M_{\max}}{bh^2}$$
$$= 67.5 \text{ MPa}$$

Ans. (b)

55. A simply supported beam PQ is loaded by a moment of 1 kNm at the mid-span of the beam as shown in the figure.



The reaction forces R_P and R_Q at supports P and Q, respectively, are

(a) 1 kN downward, 1 kN upward
(b) 0.5 kN upward, 0.5 kN downward
(c) 0.5 kN downward, 0.5 kN upward
(d) 1 kN upward, 1 kN upward

(GATE 2011)

Solution. Taking moments about Q,

$$R_P \times 1 = -1 \times 10^3$$
$$R_P = -1 \times 10^3 \text{ N } (\downarrow)$$

Also,

$$R_P + R_Q = 0$$
$$R_Q = -R_P$$
$$= 1 \times 10^3 \text{ N } (\uparrow)$$

Ans. (a)

56. A column has a rectangular cross-section of 10 mm × 20 mm and a length of 1 m. The slenderness ratio of the column is close to

(a) 200 (b) 346
(c) 477 (d) 1000

(GATE 2011)

Solution. Given that

$$l = 1000 \text{ mm}$$
$$A = 10 \times 20 \text{ mm}^2$$
$$l = 1 \text{ m}$$

Minimum moment of inertia of the column is

$$I_{min} = \frac{20 \times 10^3}{12}$$
$$= 1.667 \times 10^3 \text{ mm}^4$$

Radius of gyration of the column is

$$r = \sqrt{\frac{I_{\min}}{A}}$$
$$= \sqrt{\frac{1.667 \times 10^3}{20 \times 10}}$$
$$= 2.88675$$

Slenderness ratio of the column is

$$\text{SR} = \frac{l}{r}$$
$$= 346.41$$

Ans. (b)

57. A thin cylinder of inner radius 500 mm and thickness 10 mm is subjected to an internal pressure of 5 MPa. The average circumferential (hoop) stress in MPa is

(a) 100 (b) 250
(c) 500 (d) 1000

(GATE 2011)

Solution. Given that

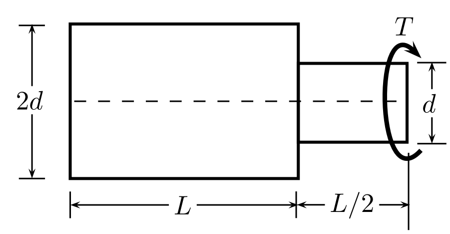
$$d = 2 \times 0.5$$
$$= 1 \text{ m}$$
$$t = 0.010 \text{ m}$$
$$p = 5 \text{ MPa}$$

Therefore, average circumferential stress is

$$\sigma_h = \frac{pd}{2t}$$
$$= \frac{5 \times 1}{2 \times 0.010}$$
$$= 250 \text{ MPa}$$

Ans. (b)

58. A torque T is applied at the free end of a stepped rod of circular cross-sections as shown in the figure. The shear modulus of the material of the rod is G.



The expression for d to produce an angular twist θ at the free end is

(a) $\left(\dfrac{32TL}{\pi\theta G}\right)^{1/4}$ (b) $\left(\dfrac{18TL}{\pi\theta G}\right)^{1/4}$

(c) $\left(\dfrac{16TL}{\pi\theta G}\right)^{1/4}$ (d) $\left(\dfrac{2TL}{\pi\theta G}\right)^{1/4}$

(GATE 2011)

Solution. The twist θ will be sum of the angles of twist in the two segments, therefore

$$\theta = \frac{32Tl}{\pi (2d)^4 G} + \frac{32Tl/2}{\pi (d)^4 G} = \frac{18Tl}{\pi d^4 G}$$

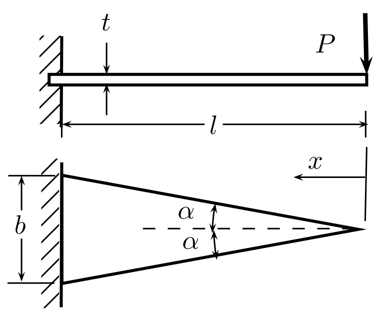
Therefore,

$$d = \left(\frac{18Tl}{\pi G \theta}\right)^{1/4}$$

Ans. (b)

Linked Answer Questions

A triangular shaped cantilever beam of uniform thickness is shown in the figure. The Young's modulus of the material of the beam is E. A concentrated load P is applied at the free end of the beam.



59. The area moment of inertia about the neutral axis of a cross-section at a distance x measured from the free end is

(a) $\dfrac{bxt^3}{6l}$ (b) $\dfrac{bxt^3}{12l}$

(c) $\dfrac{bxt^3}{24l}$ (d) $\dfrac{xt^3}{12}$

(GATE 2011)

Solution. The width at the point x will be bx/l, therefore, moment of inertia is

$$I_x = \frac{1}{12} \times \frac{bx}{l} \times t^3 = \frac{bxt^3}{12l}$$

Ans. (b)

60. The maximum deflection of the beam is

(a) $\dfrac{24Pl^3}{Ebt^3}$ (b) $\dfrac{12Pl^3}{Ebt^3}$

(c) $\dfrac{8Pl^3}{Ebt^3}$ (d) $\dfrac{6Pl^3}{Ebt^3}$

(GATE 2011)

Solution. The deflection will be maximum at free end, and will be given by

$$\delta_{\max} = \frac{Pl^3}{3EI}$$

where moment of inertia is taken averaged at $2l/3$, given by

$$I = \frac{2}{3}\frac{bt^3}{12}$$

$$\delta_{\max} = \frac{6Pl^3}{Ebt^3}$$

Ans. (d)

61. A cantilever beam of length L is subjected to a moment M at the free end. The moment of inertia of the beam cross-section about the neutral axis is I and the Young modulus is E. The magnitude of the maximum deflection is

(a) $\dfrac{ML^2}{2EI}$ (b) $\dfrac{ML^2}{EI}$

(c) $\dfrac{2ML^2}{EI}$ (d) $\dfrac{4ML^2}{EI}$

(GATE 2012)

Solution. Area moment of bending moment with respect to free end will be $ML \times L/2$, therefore, deflection will be

$$\delta = \frac{ML^2}{2EI}$$

Ans. (a)

62. For a long slender column of uniform cross-section, the ratio of critical buckling load for the case with both ends clamped to the case with both ends hinged is

(a) 1 (b) 2

(c) 4 (d) 8

(GATE 2012)

Solution. Critical buckling load for both ends fixed/clamped is

$$P_1 = \frac{4\pi^2 EI}{l^2}$$

while that for both end hinged column is

$$P_2 = \frac{\pi^2 EI}{l^2}$$

Therefore,

$$\frac{P_1}{P_2} = 4$$

Ans. (c)

63. A thin-walled spherical shell is subjected to an internal pressure. If the radius of the shell is increased by 1% and the thickness is reduced by 1%, with the internal pressure remaining the same, the percentage change in the circumferential (hoop) stress is

(a) 0 (b) 1
(c) 1.08 (d) 2.02

(GATE 2012)

Solution. For thin-walled spherical shell of diameter $d = 2r$ and thickness t, hoop stress is given by

$$\sigma_{h1} = \frac{2pr}{4t}$$

With an increase of 1% in radius and 1% decrease in thickness, the hoop stress becomes

$$\begin{aligned}\sigma_{h2} &= \frac{2pr}{4t} \times \frac{1.01}{0.99}\\ &= 1.0202\sigma_{h1}\end{aligned}$$

The percentage change is

$$\begin{aligned}\frac{\sigma_{h2} - \sigma_{h1}}{\sigma_{h1}} \times 100 &= \frac{1.0202\sigma_{h1} - \sigma_{h1}}{\sigma_{h1}} \times 100\\ &= \frac{1.0202 - 1}{1} \times 100\\ &= 2.02\end{aligned}$$

Ans. (d)

64. A solid steel cube constrained on all six faces is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is α, Young's modulus is E and the Poisson's ratio is ν, the thermal stress developed in the cube due to heating is

(a) $-\dfrac{\alpha(\Delta T)E}{1-2\nu}$ (b) $-\dfrac{2\alpha(\Delta T)E}{1-2\nu}$

(c) $-\dfrac{3\alpha(\Delta T)E}{1-2\nu}$ (d) $-\dfrac{\alpha(\Delta T)E}{3(1-2\nu)}$

(GATE 2012)

Solution. Strain developed in each axis of the cube will be

$$\varepsilon = \alpha \Delta T E$$

The stress σ developed in the cube is given by

$$\begin{aligned}\varepsilon &= -\frac{\sigma}{E}(1-2\nu)\\ \sigma &= -\frac{\varepsilon}{\sigma(1-2\nu)}E\\ &= -\frac{\alpha \Delta T E}{(1-2\nu)}\end{aligned}$$

Ans. (a)

65. The state of stress at a point under plane stress condition is

$$\begin{aligned}\sigma_{xx} &= 40 \text{ MPa}\\ \sigma_{yy} &= 100 \text{ MPa}\\ \tau_{xy} &= 40 \text{ MPa}\end{aligned}$$

The radius of the Mohr's circle representing the given state of stress in MPa is

(a) 40 (b) 50
(c) 60 (d) 100

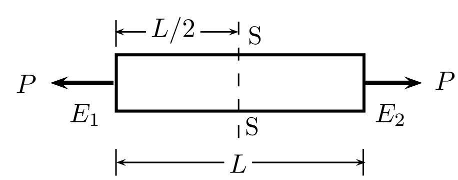
(GATE 2012)

Solution. Radius of Mohr's circle (MPa) is

$$\begin{aligned}R_\sigma &= \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}{}^2}\\ &= 50 \text{ MPa}\end{aligned}$$

Ans. (b)

66. A rod of length L having uniform cross-sectional area A is subjected to a tensile force P as shown in the following figure.



If the Young's modulus of the material varies linearly from E_1 to E_2 along the length of the rod, the normal stress developed at the section SS is

(a) $\dfrac{P}{A}$ (b) $\dfrac{P(E_1 - E_2)}{A(E_1 + E_2)}$

(c) $\dfrac{PE_2}{AE_1}$ (d) $\dfrac{PE_1}{AE_2}$

(GATE 2013)

Solution. The normal stress is the normal force per unit area of the cross-section, thus independent of the modulus of elasticity.

Ans. (a)

67. The threaded bolts A and B of the same material and length are subjected to identical tensile load. If the elastic strain energy stored in bolt A is 4 times that of the bolt B and the mean diameter of bolt A is 12 mm, the mean diameter of bolt B in mm is

(a) 16 (b) 24
(c) 36 (d) 48

(GATE 2013)

Solution. Let d_A and d_B be the diameters of threaded bolts A and B, respectively. If F is applied on a bar of length l and cross-sectional area A, the strain energy stored in the bolt can be expressed as

$$U = \frac{F^2 L}{2AE}$$

where E is the modulus of elasticity. Thus, keeping all other parameters constants, except area A,

$$U \propto \frac{1}{A} \propto \frac{1}{d^2}$$

Thus,

$$\frac{U_A}{U_B} = \frac{d_B^2}{d_A^2}$$
$$\frac{4}{1} = \frac{d_B^2}{12^2}$$
$$d_B = 12 \times \sqrt{4} = 24 \text{ mm}$$

Ans. (b)

68. A long thin-walled cylindrical shell, closed at both ends, is subjected to an internal pressure. The ratio of the hoop stress (circumferential stress) to longitudinal stress developed in the shell is

(a) 0.5 (b) 1.0
(c) 2.0 (d) 4.0

(GATE 2013)

Solution. For thin-walled cylindrical shell of diameter d and thickness t, subjected to internal pressure p_1, the hoop stress and longitudinal stress are given by

$$\sigma_h = \frac{p_i d}{2t}$$
$$\sigma_l = \frac{p_i d}{4t}$$

Thus,

$$\frac{\sigma_h}{\sigma_l} = 2$$

Ans. (c)

69. A simply supported beam of length L is subjected to a varying distributed load

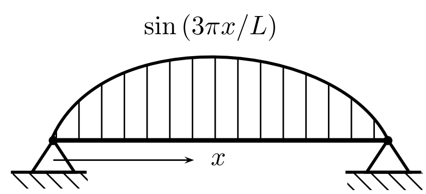
$$w = \sin\left(3\pi \frac{x}{L}\right)$$

where the distance x is measured from the left support. The magnitude of the vertical force in N at the left support is

(a) 0 (b) $\frac{L}{3\pi}$
(c) $\frac{L}{\pi}$ (d) $\frac{2L}{\pi}$

(GATE 2013)

Solution. The sinusoidal force is shown in the figure.



The total load over the span of the beam is determined as

$$W = \int_0^L w\,dx = \int_0^L \sin\left(3\pi \frac{x}{L}\right) dx = -\frac{L}{3\pi}\left\{\cos \frac{3\pi x}{L}\right\}_0^L = -\frac{L}{3\pi}\left\{\cos \frac{3\pi L}{L} - \cos 0\right\} = \frac{2L}{3\pi}$$

This load would be equally distributed into vertical reactions at both ends, thus

$$R = \frac{W}{2} = \frac{L}{3\pi}$$

Ans. (b)

MULTIPLE CHOICE QUESTIONS

1. A bar produces a lateral strain of magnitude -60×10^{-5} m/m, when subjected to tensile stress of magnitude 300 MPa along the axial direction. Find the elastic modulus of the material, if the Poisson's ratio is 0.3.

(a) 100 GPa (b) 150 GPa
(c) 200 GPa (d) 400 GPa

2. What is the relationship between the linear elastic properties: Young's modulus (E), rigidity modulus (G) and bulk modulus (K)?

(a) $\frac{1}{E} = \frac{9}{K} + \frac{3}{G}$ (b) $\frac{3}{E} = \frac{9}{K} + \frac{1}{G}$
(c) $\frac{9}{E} = \frac{3}{K} + \frac{1}{G}$ (d) $\frac{9}{E} = \frac{1}{K} + \frac{3}{G}$

3. A beam is said to be of uniform strength, if

(a) the bending moment is the same throughout the beam
(b) the shear stress is the same throughout the beam
(c) the deflection is the same throughout the beam
(d) the bending stress is the same at every section along its longitudinal axis

4. The deformation of a bar under its own weight as compared to that when subjected to a direct axial load equal to its own weight will be

(a) the same (b) one-fourth
(c) half (d) double

5. The number of independent elastic constants required to express the stress-strain relationship for a linearly elastic isotropic material is

(a) one (b) two
(c) three (d) four

6. A tapered bar (diameter of end sections being d_1 and d_2) and a bar of uniform section with diameter d have the same length and are subjected the same axial pull. Both the bars will have the same extension if d is equal to

(a) $(d_1 + d_2)/2$ (b) $d_1 d_2$
(c) $\sqrt{d_1 d_2/2}$ (d) $\sqrt{(d_1 + d_2)/2}$

7. The relationship between Lame's constant λ, Young's modulus E, and the Poisson's ratio μ is

(a) $\lambda = \frac{E\mu}{(1+\mu)(1-2\mu)}$

(b) $\lambda = \frac{E\mu}{(1-\mu)(1+2\mu)}$

(c) $\lambda = \frac{E\mu}{(1-\mu)(1-2\mu)}$

(d) $\lambda = \frac{E\mu}{(1+\mu)(1+2\mu)}$

8. The stretch in steel rod of circular section, having a length l subjected to a tensile load P and tapering uniformly from a diameter d_1 at one end to a diameter d_2 at the other end, is given by

(a) $\frac{Pl}{4Ed_1d_2}$ (b) $\frac{Pl\pi}{Ed_1d_2}$
(c) $\frac{Pl}{\pi Ed_1d_2}$ (d) $\frac{4Pl}{\pi Ed_1d_2}$

9. If Poisson's ratio of a material is 0.5, then the elastic modulus for the material is

(a) three times its shear modulus
(b) four times its shear modulus
(c) equal to its shear modulus
(d) indeterminate

10. Circumferential and longitudinal strains in a cylindrical boiler under internal steam pressure are ε_1 and ε_2, respectively. The change in volume of the boiler per unit volume will be

(a) $\varepsilon_1 + 2\varepsilon_2$ (b) $\varepsilon_1 {\varepsilon_2}^2$
(c) $2\varepsilon_1 + \varepsilon_2$ (d) ${\varepsilon_1}^2 + \varepsilon_2$

11. If a material had a modulus of elasticity 210 GPa and a modulus of rigidity 80 GPa, then the approximate value of the Poisson's ratio of the material would be

(a) 0.26 (b) 0.31
(c) 0.47 (d) 0.50

12. A steel rod of 1 cm^2 cross-sectional area is 100 cm long and has a Young's modulus of elasticity 210 GPa. It is subjected to an axial pull of 20 kN. The elongation of the rod will be

(a) 0.05 cm (b) 0.1 cm
(c) 0.15 cm (d) 0.20 cm

13. A vertical hanging bar of length L and weighing w N/unit length carries a load W at the bottom. The tensile force in the bar at a distance y from the support will be given by

(a) $W+wL$ (b) $W+w(L-y)$

(c) $(W+w)\dfrac{y}{L}$ (d) $W+\dfrac{W}{w}(L-y)$

14. The rigidity modulus of a material whose $E = 210$ GPa and Poisson's ratio is 0.25, will be

(a) 84×10^9 Pa (b) 50×10^9 Pa
(c) 30×10^9 Pa (d) 20×10^9 Pa

15. In a beam of uniform strength,

(a) moment of resistance is the same throughout the length
(b) flexural rigidity is the same throughout the length
(c) maximum fiber stress is the same throughout the length
(d) bending moment is the same throughout the length

16. A solid cube of steel of sides 1 m is immersed in water at a depth of 1 km. The resulting decrease in volume is 0.073×10^{-3} m^3. The decrease in length of any one of the sides of the cube will be nearly

(a) 0.072×10^{-3} m^3 (b) 0.008×10^{-3} m^3
(c) 0.024×10^{-3} m^3 (d) 0.648×10^{-3} m^3

17. The value of Poisson's ratio for any material cannot exceed

(a) 0.2 (b) 1.414
(c) 1.0 (d) 0.5

18. The unit of elastic modulus is the same as those of

(a) stress, shear modulus, and pressure
(b) strain, shear modulus, and force
(c) shear modulus, stress and force
(d) stress, strain, and pressure

19. If the cross-section of a member is subjected to a uniform shear stress of intensity τ, modulus of rigidity is G, then the strain energy stored per unit volume is equal to

(a) $\dfrac{2\tau^2}{G}$ (b) $2\tau^2 G$

(c) $\dfrac{\tau^2}{2G}$ (d) $\dfrac{G^2}{2\tau}$

20. For a linearly elastic, isotropic and homogeneous material, the number of elastic constants required to relate stress and strain is

(a) 2 (b) 3
(c) 4 (d) 6

21. The elastic constants, modulus of elasticity E and modulus of rigidity K are related through Poisson's ratio μ as

(a) $E = 2K(1-2\mu)$ (b) $E = 3K(1-2\mu)$
(c) $E = 2K(1+2\mu)$ (d) $E = 3K(1+2\mu)$

22. The Young's modulus of elasticity of a material is 2.5 times its modulus of rigidity. The Poisson's ratio for the material will be

(a) 0.25 (b) 0.33
(c) 0.50 (d) 0.75

23. A weight falls on a plunger fitted in a container filled with oil thereby producing a pressure of 1.5 N/mm^2 in the oil. The bulk modulus of oil is 2800 N/mm^2. Given this situation, the volumetric compressive strain produced in the oil will be

(a) 400×10^{-6} (b) 800×10^{-6}
(c) 268×10^{-6} (d) 535×10^{-6}

24. If the principal stresses and maximum shearing stresses are of equal numerical value at a point in a stressed body, the state of stress can be termed as

(a) isotropic
(b) uni-axial
(c) pure shear
(d) generalized plane state of stress

25. Principal stresses at a point in plane stressed element are

$$\sigma_x = \sigma_y = 500 \text{ kg/cm}^2$$

Normal stress on the plane inclined at 45° to x-axis is

(a) 0 (b) 500 kg/cm^2
(c) 707 kg/cm^2 (d) 1000 kg/cm^2

26. Maximum shear stress in a Mohr's circle

(a) is equal to radius of Mohr's circle
(b) is greater than radius of Mohr's circle
(c) is less than radius of Mohr's circle
(d) could be any of the above

27. A point in a two-dimensional state of strain is subjected to pure shearing strain of magnitude γ_{xy} radians. Which one of the following is the maximum principal strain?

(a) γ_{xy} (b) $\gamma_{xy}/\sqrt{2}$
(c) $\gamma_{xy}/2$ (d) $2\gamma_{xy}$

28. In a strained material, one of the principal stresses is twice the other. The maximum shear stress in the same case is $\tau_{\max}$. Then, what is the value of the maximum principal stress?

(a) $\tau_{\max}$ (b) $2\tau_{\max}$
(c) $4\tau_{\max}$ (d) $8\tau_{\max}$

29. Two-dimensional state of stress at a point in a plane stressed element is represented by a Mohr circle of zero radius. Then both principal stresses

(a) are equal to zero
(b) are equal to zero and shear stress is also equal to zero
(c) are of equal magnitude but of opposite sign
(d) are of equal magnitude and of same sign

30. A plane stressed element is subjected to the state of stress given by

$$\sigma_x = \tau_{xy} = 10 \text{ MPa}, \quad \sigma_y = 0$$

The maximum shear stress in the element is equal to

(a) $5\sqrt{3}$ MPa
(b) 10 MPa
(c) $5\sqrt{5}$ MPa
(d) 15 MPa

31. In case of bi-axial state of normal stress, the normal stress on 45° plane is equal to

(a) the sum of the normal stresses
(b) difference of the normal stresses
(c) half the sum of the normal stresses
(d) half the difference of the normal stresses

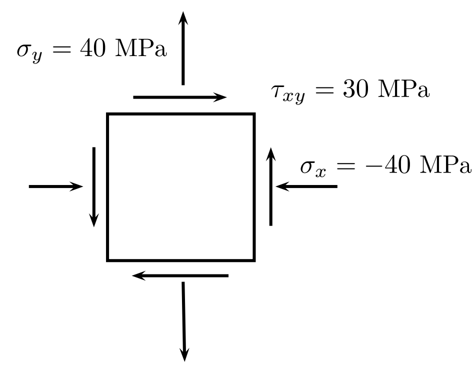
32. If a prismatic bar is subjected to an axial tensile stress, σ, then shear stress induced on a plane inclined at θ with the axis will be

(a) $(\sigma/2)\sin 2\theta$ (b) $(\sigma/2)\cos 2\theta$
(c) $(\sigma/2)\cos^2\theta$ (d) $(\sigma/2)\sin^2\theta$

33. For a plane stress case, $\sigma_1 = 50$ MPa, $\sigma_2 = -100$ MPa, $\tau_{12} = 40$ MPa, the maximum and minimum principal stresses are, respectively,

(a) 60 MPa, −110 MPa
(b) 50 MPa, 110 MPa
(c) 40 MPa, −120 MPa
(d) 70 MPa, 130 MPa

34. The state of stress at a point in a loaded member is shown in the figure.



The magnitude of maximum shear stress is

(a) 10 MPa (b) 30 MPa
(c) 50 MPa (d) 100 MPa

35. A solid circular shaft is subjected to a maximum shearing stress of 140 MPa. The magnitude of the maximum normal stress developed in the shaft is

(a) 140 MPa (b) 80 MPa
(c) 70 MPa (d) 60 MPa

36. The principal stresses at a point in an elastic material are 60 N/mm^2 tensile, 20 N/mm^2 tensile and 50 N/mm^2 tensile. If the material properties are $\mu = 0.35$, $E = 10^5$ N/mm^2, then the volumetric strain of the material is

(a) 9.3×10^{-5} (b) 3.9×10^{-4}
(c) 10.5×10^{-5} (d) 21×10^{-5}

37. If the two principal strains at a point are 1000 μm and 600 μm, then the maximum shear strain is

(a) 800 μm (b) 500 μm
(c) 1600 μm (d) 200 μm

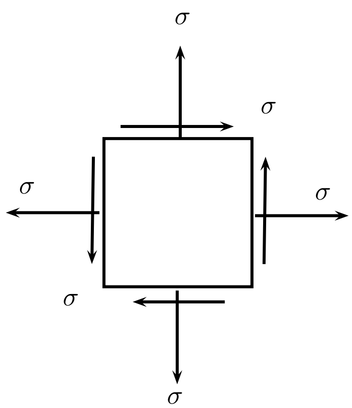
38. The normal stresses at a point are $\sigma_x = 10$ MPa and $\sigma_y = 2$ MPa; the shear stress at this point is $\tau_{xy} = 4$ MPa. The maximum principal stress at this point is:

(a) 16.54 MPa (b) 14.11 MPa
(c) 11.65 MPa (d) 10.00 MPa

39. A 1.5 mm thick sheet is subject to unequal bi-axial stretching and the true strains in the directions of stretching are 0.05 and 0.09. The final thickness of the sheet in mm is

(a) 1.414 (b) 1.304
(c) 1.362 (d) 289

40. The maximum principal stress for the stress state shown in the figure is



(a) σ (b) 2σ
(c) 3σ (d) 1.5σ

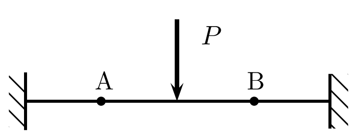
41. The point of contraflexure is a point where

(a) shear force changes sign
(b) bending moment changes sign
(c) shear force is maximum
(d) bending moment is maximum

42. The bending moment (M) is constant over a length segment l of a beam. The shearing force will also be constant over this length and is given by

(a) $\dfrac{M}{l}$ (b) $\dfrac{M}{2l}$
(c) $\dfrac{M}{4l}$ (d) Indeterminate

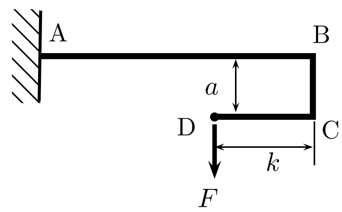
43. A beam AB of length $2l$ having a concentrated load P at its mid-span is hinge supported at its two ends A and B on two identical cantilevers as shown in the figure.



The correct value of bending moment at A is

(a) zero (b) $Pl/2$
(c) Pl (d) $2Pl$

44. A structural member ABCD is loaded as shown in the given figure.



The shearing force at any section on the length BC of the member is

(a) zero (b) F
(c) Fa/k (d) Fk/a

45. A cantilever beam of rectangular cross-section is 1 m deep and 0.6 m thick. If the beam were to be 0.6 m deep and 1 m thick, then the beam would

(a) be weakened by 0.5 times
(b) be weakened by 0.6 times
(c) be strengthened by 0.6 times
(d) have the same strength

46. In case of a beam circular cross-section subjected to transverse loading, the maximum shear stress developed in the beam is greater than the average shear stress by

(a) 50% (b) 33%
(c) 25% (d) 10%

47. What is the nature of distribution of shear stress in a rectangular beam?

(a) linear (b) parabolic
(c) hyperbolic (d) elliptic

48. Two beams of equal cross-sectional area are subjected to equal bending moment. If one beam has square cross-section and the other has circular cross-section, then

(a) both beams will be equally strong
(b) circular cross-section beam will be stronger
(c) square cross-section beam will be stronger
(d) the strength of the beam will depend on the nature of loading

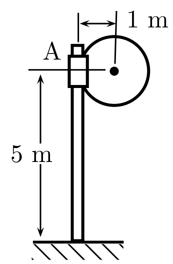
49. A wooden beam of rectangular cross-section 10 cm deep by 5 cm wide carries maximum shear force of 20 kN. Shear stress at natural axis of the beam section is

(a) zero (b) 4 MPa
(c) 6 MPa (d) 80 MPa

50. A rectangular section beam subjected to a bending moment M varying along its length is required to develop same maximum bending stress at any cross section. If the depth of the section is constant then its width will vary as

(a) M (b) $\sqrt{M}$
(c) M^2 (d) $1/M$

51. Consider the sign board mounting shown in the figure.



The wind load acting perpendicular to the plane of the figure is 100 N. We wish to limit the deflection, due to bending, at point A of the hollow cylindrical pole of outer diameter 150 mm, to 5 mm. What is the wall thickness for the pole. Assume $E = 200$ GPa.

(a) 3.36 mm (b) 4.25 mm
(c) 6.72 mm (d) 7.61 mm

52. A horizontal beam with square cross-section is simply supported with sides of the square horizontal and vertical, and carries a distributed loading that produces maximum bending stress σ in the beam. When the beam is placed with one of the diagonals in horizontal direction, the maximum bending stress will be

(a) $\sigma/\sqrt{2}$ (b) σ
(c) $\sqrt{2}\sigma$ (d) 2σ

53. The ratio of average shear stress to the maximum shear stress in a beam with a square cross-section is

(a) 1 (b) 2/3
(c) 3/2 (d) 2

54. The area moment of inertia of a square of size 1 about its diagonal is

(a) 1/3 (b) 1/4
(c) 1/12 (d) 1/6

55. The shape of the bending moment diagram for a uniform cantilever beam carrying a uniformly distributed load over its length is

(a) a straight line (b) a hyperbola
(c) a ellipse (d) a parabola

56. Maximum deflection of a cantilever beam of length l carrying uniformly distributed load w per unit length will be (where E = modulus of elasticity of beam material, I = moment of inertia of beam cross-section.)

(a) $\dfrac{wl^4}{EI}$ (b) $\dfrac{wl^4}{4EI}$

(c) $\dfrac{wl^4}{8EI}$ (d) $\dfrac{wl^4}{384EI}$

57. A bar having length L and uniform cross-section with area A is subjected to both tensile force P and torque T. If G is the shear modulus and E is the Young's modulus, the internal strain energy stored in the bar is

(a) $\dfrac{T^2L}{2GI_p}+\dfrac{P^2L}{AE}$ (b) $\dfrac{T^2L}{GI_p}+\dfrac{P^2L}{2AE}$

(c) $\dfrac{T^2L}{2GI_p}+\dfrac{P^2L}{2AE}$ (d) $\dfrac{T^2L}{GI_p}+\dfrac{P^2L}{AE}$

58. For the state of stress of pure shear τ, the strain energy stored per unit volume in the elastic, homogeneous isotropic material having elastic constants E and μ will be

(a) $\dfrac{\tau^2(1+\mu)}{E}$ (b) $\dfrac{\tau^2(1+\mu)}{2E}$

(c) $\dfrac{2\tau^2(1+\mu)}{E}$ (d) $\dfrac{\tau^2(2+\mu)}{2E}$

59. A simply supported beam with width b and depth d carries a central load W and undergoes deflection δ at the center. If the width and depth are interchanged, the deflection at the center of the beam would attain the value

(a) $(d/b)\,\delta$ (b) $(d/b)^2\,\delta$
(c) $(d/b)^3\,\delta$ (d) $(d/b)^{3/2}\,\delta$

60. A cantilever of span L is subjected to a concentrated load P and a moment M at the free end. Deflection at the free end is given by

(a) $\dfrac{2PL^3}{2EI}+\dfrac{ML^2}{3EI}$ (b) $\dfrac{ML^2}{2EI}+\dfrac{PL^3}{3EI}$

(c) $\dfrac{ML^2}{3EI}+\dfrac{PL^3}{2EI}$ (d) $\dfrac{ML^2}{2EI}+\dfrac{PL^3}{48EI}$

61. A cantilever beam carries a load W uniformly distributed over its entire length. If the same load is placed at the free end of the same cantilever, then the ratio of maximum deflection in the first case to that in the second case will be

(a) 3/8 (b) 8/3
(c) 5/8 (d) 8/5

62. A simply supported beam carrying a concentrated load W at mid-span deflects by δ_1 under the load. If the same beam carries the load W such that it is distributed uniformly over the entire length and undergoes a deflection δ_2 at the mid-span. The ratio $\delta_1 : \delta_2$ is

(a) $2:1$ (b) $\sqrt{2}:1$
(c) $1:1$ (d) $1:2$

63. A beam having uniform cross-section carries a uniformly distributed load of intensity q per unit length over its entire span, and its mid span deflection is δ. The value of mid span deflection of the same beam when the same load is distributed with intensity $2q$ per unit length at one end to zero at the other end is

(a) $\delta/3$ (b) $\delta/2$
(c) $2\delta/3$ (d) δ

64. A simply supported beam of rectangular section 4 cm by 6 cm carries a mid-span concentrated load such that the 6 cm side lies parallel to line of action of loading; deflection under load is δ. If the beam is now supported with the 4 cm side parallel to line of action of loading, the deflection under the load will be

(a) 0.44δ (b) 0.67δ
(c) 1.5δ (d) 2.25δ

65. The elastic strain energy stores in a rectangular cantilever beam of length L, subjected to a bending moment M applied at the end is

(a) $\dfrac{M^2L}{EI}$ (b) $\dfrac{ML^2}{2AE}$

(c) $\dfrac{ML^2}{3EI}$ (d) $\dfrac{ML^2}{16EI}$

66. A point load W acts at the center of a simply supported beam. If the load is changed to a uniformly distributed load, then the ratio of maximum deflections in the two cases will be

(a) 1.2 (b) 1.3
(c) 1/4 (d) 8/5

67. Total strain energy stored in a simply supported beam of span L and flexural rigidity EI subjected to a concentrated load W at the center is equal to

(a) $\dfrac{W^2L^3}{4EI}$ (b) $\dfrac{W^2L^3}{6EI}$

(c) $\dfrac{W^2L^3}{24EI}$ (d) $\dfrac{W^2L^3}{96EI}$

68. In a cantilever beam, if the length is doubled while keeping the cross-section and the concentrated load acting at the free end the same, the deflection at the free end will be increased by

(a) 2.66 times (b) 3 times
(c) 6 times (d) 8 times

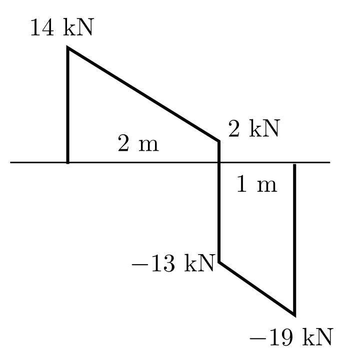
69. A point, along the length of a beam subjected to loads, where bending moment changes its sign, is known as the point of

(a) inflexion (b) maximum stress
(c) zero shear force (d) contraflexure

70. A beam carrying a uniformly distributed load rests on two supports b distance apart with equal overhangs a at each end. The ratio b/a for zero bending moment at mid-span is

(a) 1/2 (b) 1
(c) 2/3 (d) 2

71. The shear force diagram of a loaded beam is shown in the following figure.



The maximum bending moment in the beam is

(a) 16 kNm (b) 11 kNm
(c) 28 kNm (d) 8 kNm

72. The ratio of the area under the bending moment diagram to the flexural rigidity between any two points along a beam gives the change in

(a) deflection (b) slope
(c) shear force (d) bending moment

73. A square bar of side 4 cm and length 100 cm is subjected to an axial load F. The same bar is then used as a cantilever beam and subjected to an end load F. The ratio of the strain energies, stored in the bar is the second case to that stored in the first case, is:

(a) 16 (b) 400
(c) 1000 (d) 2500

74. For the two shafts connected in parallel

(a) torque in each shaft is the same
(b) shear stress in shaft is the same
(c) angle of twist of each shaft is the same
(d) torsional stiffness of each shaft is the same

75. The ratio of torque carrying capacity of a solid shaft to that of a hollow shaft is given by ($k = Di/D_o$, D_i = Inside diameter of hollow shaft, D_o = Outside diameter of hollow shaft, shaft materials are the same.):

(a) $1-k^4$ (b) $\left(1-k^4\right)^{-1}$
(c) k^4 (d) $1/k^4$

76. Under axial load, each section of a closed-coil helical spring is subjected to

(a) tensile stress and shear stress due to load
(b) compressive stress and shear stress due to torque
(c) tensile stress end shear stress due to torque
(d) torsional and direct shear stresses

77. A helical spring has N turns of coil diameter D and a second spring, made of same wire diameter and of same material has $N/2$ turns of coil of diameter $2D$. If the stiffness of the first spring is k, then the stiffness of the second spring will be

(a) $k/4$ (b) $k/2$
(c) $2k$ (d) $4k$

78. Two helical tensile springs of the same material and also having identical mean coil diameter and weight, have wire diameters d and $d/2$. The ratio of their stiffness is

(a) 1 (b) 4
(c) 64 (d) 128

79. In the calculation of induced shear stress in helical springs, the Wahl's correction factor is used to take care of

(a) combined effect of transverse shear stress and bending stress in the wire
(b) combined effect of bending stress and curvature of the wire
(c) combined effect of transverse shear stress and curvature of the wire
(d) combined effect of torsional shear stress and transverse shear stress in the wire

80. A long helical spring, having a spring stiffness of 12 kN/m and number of turns 20, breaks into two parts with number of turns 10 in both the parts. If the two parts are connected in series, then the stiffness of the resultant spring will be

(a) 6 kN/m (b) 12 kN/m
(c) 24 kN/m (d) 30 kN/m

81. Two closed-coil springs are made from the same diameter wire, one wound on 2.5 cm diameter core and the other on 1.25 cm diameter core. If each spring had n coils, then the ratio of their spring constants would be

(a) 1/16 (b) 1/8
(c) 1/4 (d) 1/2

82. A solid shaft of diameter 100 mm, length 1000 mm is subjected to a twisting moment T. The maximum shear stress developed in the shaft is 60 N/mm^2. A hole of 50 mm diameter is now drilled throughout the length of the shaft. To develop a maximum shear stress of 60 N/mm^2 in the hollow shaft, the torque T must be reduced by

(a) $T/4$ (b) $T/8$
(c) $T/12$ (d) $15T/16$

83. Two hollow shafts of the same material have the same length and outside diameter. Shaft 1 has internal diameter equal to one-third of the outer diameter and shaft 2 has internal diameter equal to half of the outer diameter. If both the shafts are subjected to the same torque, the ratio of their twists θ_1/θ_2 will be equal to

(a) 16/81 (b) 8/27
(c) 19/27 (d) 243/256

84. Maximum shear stress in a solid shaft of diameter D and length L twisted through an angle θ is τ. A hollow shaft of same material and length having outside and inside diameters of D and $D/2$, respectively, is also twisted through the same angle of twist θ. The value of maximum shear stress in the hollow shaft will be

(a) $16\tau/15$ (b) $8\tau/7$
(c) $4\tau/3$ (d) τ

85. A closed-coil helical spring is acted upon by an axial force. The maximum shear stress developed in the spring is τ. Half of the length of the spring is cut off and the remaining spring is acted upon by the same axial force. The maximum shear stress in the spring in the new condition will be

(a) $\tau/2$ (b) τ
(c) 2τ (d) 4τ

86. A solid shaft of diameter D carries a twisting moment that develops maximum shear stress τ. If the shaft is replaced by a hollow one of outside diameter D and inside diameter $D/2$, then the maximum shear stress will be

(a) 1.067τ (b) 1.143τ
(c) 1.333τ (d) 2τ

87. A length of 10 mm diameter steel wire is coiled to a closed-coil helical spring having 8 coils of 75 mm mean diameter, and the spring has a stiffness k. If the same length of the wire is coiled to 10 coils of 60 mm mean diameter, then the spring stiffness will be

(a) k (b) $1.25k$
(c) $1.56k$ (d) $1.95k$

88. Two shafts of same length and material are joined in series. If the ratio of their diameters is 2, then the ratio of their angles of twist will be

(a) 2 (b) 4
(c) 8 (d) 16

89. A steel shaft of outside diameter 100 mm is solid over one-half of its length and hollow over the other half. Inside diameter of the hollow portion is 50 mm. The shaft is held rigidly at two ends and a pulley is mounted at its mid-section. It is twisted by applying torque on the pulley. If the torque carried by the solid portion of the shaft is 1600 Nm, then the torque carried by the hollow portion of the shaft will be

(a) 1600 kNm (b) 1500 kNm
(c) 1400 kNm (d) 1200 kNm

90. A closed-coil helical spring is cut in two equal parts along its length. Stiffness of the two springs so obtained will be

(a) double of that of the original spring
(b) same as that of the original spring
(c) half of that of the original spring
(d) one-fourth of that of the original spring

91. Two shafts A and B are made of the same material. The diameter of shaft B is twice that of shaft A. The ratio of power which can be transmitted by shaft A to that of shaft B is:

(a) 2 (b) 4
(c) 8 (d) 16

92. The deflection of a spring with 20 active turns under a load of 1000 N is 10 mm. The spring is made into two pieces each of 10 active coils and placed in parallel under the same load. The deflection of the system is

(a) 20 mm (b) 10 mm
(c) 5 mm (d) 2.5 mm

93. A shaft is subjected to combined twisting moment T and bending moment M. What is the equivalent bending moment?

(a) $\sqrt{M^2+T^2}/2$
(b) $\sqrt{M^2+T^2}$
(c) $\left(M+\sqrt{M^2+T^2}\right)/2$
(d) $\left(M+\sqrt{M^2+T^2}\right)$

94. A shaft was initially subjected to bending moment and then was subjected to torsion. If the magnitude of bending moment is found to be the same as that of the torque, then the ratio of maximum bending stress to shear stress would be

(a) 0.25 (b) 0.5
(c) 2.0 (d) 4.0

95. Bending moment M and torque T are applied on a solid circular shaft in two situations. If the maximum bending stress equals to maximum shear stress developed, then M is equal to

(a) $T/2$ (b) T
(c) $2T$ (d) $4T$

96. When bending moment M and torque T is applied on a shaft, then equivalent torque is

(a) $M+T$ (b) $\sqrt{M^2+T^2}$
(c) $\sqrt{M^2+T^2}/2$ (d) $\left(M+\sqrt{M^2+T^2}\right)/2$

97. A solid shaft can resist a bending moment of 3.0 kNm and a twisting moment of 4.0 kNm together, then the maximum torque that can be applied is:

(a) 7.0 kNm (b) 3.5 kNm
(c) 4.5 kNm (d) 5.0 kNm

98. The safe rim velocity of a flywheel is influenced by the

(a) centrifugal stresses
(b) fluctuation of energy
(c) fluctuation of speed
(d) mass of the flywheel

99. Where does the maximum hoop stress in a thick-walled cylinder under external pressure occur?

(a) At the outer surface
(b) At the inner surface
(c) At the mid-thickness
(d) At the 2/3rd outer radius

100. A thick-walled cylinder is subjected to internal pressure of 100 N/mm^2. If hoop stress developed at the outer radius of the cylinder is 100 N/mm^2, the hoop stress developed at the inner radius is

(a) 100 N/mm^2 (b) 200 N/mm^2
(c) 300 N/mm^2 (d) 400 N/mm^2

101. A thick-walled hollow cylinder having outer and inner radii of 90 mm and 40 mm, respectively, is subjected to an external pressure of 800 MN/m^2. The maximum circumferential stress in the cylinder will occur at a radius of

(a) 40 mm (b) 60 mm
(c) 65 mm (d) 90 mm

102. In a thick-walled cylinder pressurized inside, the hoop stress is maximum at

(a) the center of the wall thickness
(b) the outer radius
(c) the inner radius
(d) both the inner and the outer radii

103. A thick-walled cylinder is subjected to an internal pressure of 60 MPa. If the hoop stress on the outer surface is 150 MPa, then the hoop stress on the internal surface is

(a) 105 MPa (b) 180 MPa
(c) 210 MPa (d) 135 MPa

104. A penstock pipe of 10 m diameter carries water under a pressure head of 100 m. If the water thickness is 9 mm, what is the tensile stress in the pipe wall in MPa?

(a) 2725 (b) 545.0
(c) 272.5 (d) 1090

105. A thin cylinder contains fluid at a pressure of 500 N/mm^2, the internal diameter of the shell is 0.6 m and the tensile stress in the material is to be limited to 9000 N/mm^2. The shell must have a minimum wall thickness of nearly

(a) 9 mm (b) 11 mm
(c) 17 mm (d) 21 mm

106. A thin-walled cylindrical vessel of wall thickness t and diameter D is filled with gas to a gauge pressure of p. The maximum shear stress on the vessel will then be

(a) $\dfrac{pD}{t}$ (b) $\dfrac{pD}{2t}$

(c) $\dfrac{pD}{4t}$ (d) $\dfrac{pD}{8t}$

107. The maximum principal strain in a thin cylindrical tank, having a radius of 25 cm and wall thickness of 5 mm when subjected to an internal pressure of 1 MPa, is (taking Youngs modulus as 200 GPa and Poissons ratio as 0.2)

(a) 2.25×10^{-4} (b) 2.25×10^{-5}
(c) 2.25×10^{-6} (d) 2.25×10^{-7}

108. If diameter of a long column is reduced by 20%, the percentage of reduction in Euler buckling load is

(a) 4 (b) 36
(c) 49 (d) 59

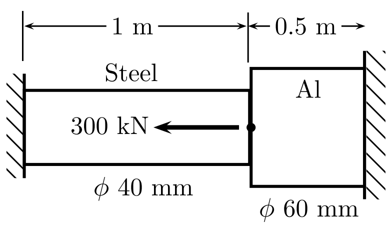
109. While designing a screw in a screw jack against buckling failure, the end conditions for the screw are taken as

(a) both ends fixed
(b) both ends hinged
(c) one end fixed and other the end hinged
(d) one end fixed and the other end free

NUMERICAL ANSWER QUESTIONS

1. In an axial tensile test on 15 mm diameter bar of gauge length 200 mm, the load at proportionality limit is 25 kN and the corresponding changes in length and diameter are 0.25 mm and 0.0065 mm, respectively. Calculate the modulus of elasticity and the Poisson's ratio of the material.

2. A composite bar is made of aluminium and steel portions of lengths 500 mm and 1000 mm, respectively, and diameters of 60 mm and 40 mm, respectively. A load 300 kN is applied at the junction of the portions as shown in figure.



The modulus of elasticity of steel and aluminium are 210 GPa and 70 GPa respectively. Calculate the strains and stresses induced in aluminium and steel portions of the bar.

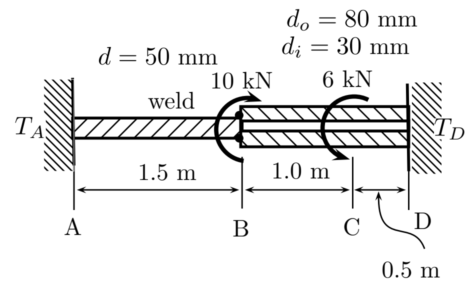
3. A thin rectangular steel plate 100 mm×50 mm undergoes deformations of 0.05 mm and 0.01 mm in longitudinal and lateral directions. The modulus of elasticity of steel is 200 GPa and Poisson's ratio is 0.3. Calculate the stresses in the longitudinal and lateral directions.

4. A steel bar 50 mm×50 mm×300 mm is under a compressive load of 100 kN in longitudinal direction. The lateral strains are prevented by constraints. The apparent modulus of elasticity is 200 GPa and Poisson's ratio is 0.3. Calculate the change in the volume of the bar.

5. A weight of 100 N is dropped onto the flange attached to a 20 mm diameter rod of length 2 m. The tensile strength of the material is 150 MPa and modulus of elasticity is 200 GPa. Calculate the maximum height from where the weight can be dropped. Also calculate the maximum extension in the rod.

6. A cylindrical pressure tank of mean diameter 1.6 m is fabricated by butt welding a 8 mm plate along a helix which forms an angle of 20° with the transverse plane. The end caps are spherical having wall thickness 10 mm. The internal gauge pressure is 0.8 MPa. Calculate the stresses in the direction and perpendicular to the weld.

7. A compound shaft is made by fillet welding at the periphery of 50 mm diameter shaft with a hollow shaft of inside and outside diameter 80 mm and 30 mm, respectively. The shaft is fixed at both ends. The hollow portion is subjected to torsional loads of 6 kN and 10 kN at the edge of joint and at mid span, respectively.



Calculate the shear stresses developed at the weld.

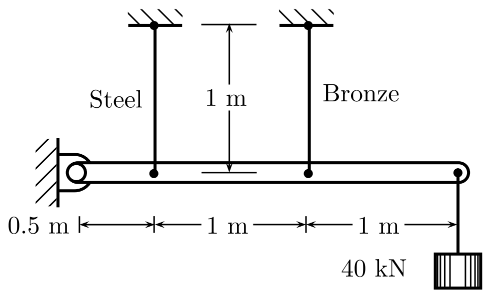
8. A steel column section SC 250 has sectional area 109 cm^2 and second moment of inertia 12500 cm^4 and 3260 cm^4 in lateral directions. Yield stress of the steel is 250 MPa and modulus of elasticity is 200 GPa. Calculate the maximum length of column to avoid buckling if both ends are hinged, and also if both ends are fixed rigidly.

9. A railway wagon weighing 210 kN is stopped by a buffer of 4 steel springs while moving at a speed of 15 km/hr. Mean diameter of the springs is 220 mm with 10 number of turns/coils. The maximum allowable compression of springs is 300 mm. Bulk modulus of rigidity of the material is 85 GPa. Calculate the stiffness and wire diameter of the spring.

10. A 50 N weight is dropped on a close coiled spring from a height of 150 mm. The spring has 10 coils of mean diameter 30 mm and spring wire diameter 5 mm. The modulus of rigidity is 80 GPa. Calculate the deflection and maximum shear stress in the spring.

11. A short concrete post of size 200 mm × 200 mm is reinforced axially with six symmetrically placed steel bars, each 600 mm^2 in area. An axial load of 800 kN is applied on it. The bulk moduli of elasticity for steel and concrete are 180 GPa and 18 GPa, respectively. Calculate the axial stress in the steel bar and the concrete.

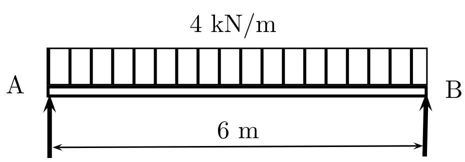
12. A horizontal rigid bar of negligible mass and length 2.5 m is hinged at A and supported by two rods made of bronze and steel, each 1 m long, as shown in figure. A load of 40 kN is also supported at its free end.



The cross-sectional areas of the steel and bronze rods are 500 mm^2 and 200 mm^2, respectively. Moduli of elasticity of their material are 180 GPa and 80 GPa, respectively. Calculate the stresses in the steel and bronze rods.

13. A steel rod 5 m long is secured between two walls. The cross-sectional area of the rod is 600 mm^2. The load is rod is zero at 25° C. The rod is cooled to $-10°$ C. Coefficient of thermal expansion is 12 μm/m°C and modulus of elasticity is 180 GPa. Calculate the stress in the rod if walls are rigid. Calculate the stress in the rod if walls spring together a total distance of 0.2 mm with drop in temperature.

14. A simply supported beam 120 mm wide and 180 mm deep in cross-section and 6 m long carries a uniformly distributed load of 4 kN/m, as shown in figure.



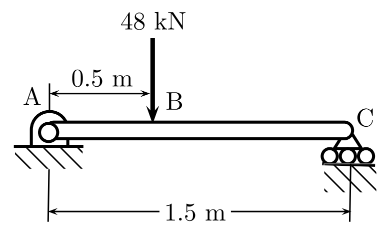
Calculate the shear stress developed at horizontal layer 30 mm from top at a section 1.0 m from the left end. Also calculate the maximum shear stress developed in the beam.

15. A steel tube of inner diameter 100 mm and wall thickness 5 mm is subjected to a torsional moment of 1000 Nm. Determine the principal stresses on the outer surface of the tube. Also calculate the orientation of the principal plane on the outer surface of the tube.

16. A solid phosphor bronze shaft 60 mm in diameter is rotating at 800 rpm and transmitting power under torsion only. An electrical resistance strain gauge mounted on the surface of the shaft gives strain reading as 3.98×10^{-4}. The modulus of elasticity for phosphor bronze is 105 GPa and Poisson ratio is 0.3. Estimate the maximum shear stress in the shaft and the maximum power that can be transmitted by the shaft.

17. A timber beam 150 mm wide and 200 mm deep carries a uniformly distributed load over a span of 4 m and is simply supported. The permissible stresses are 30 N/mm^2 longitudinally. Determine the load carrying capacity of the timber. Calculate the maximum load if permissible shear stress in transverse direction is 3 N/mm^2.

18. A beam of rectangular cross-section 50 mm and 100 mm deep is simply supported over a span of 1.5 m.



It carries a concentrated load of 48 kN at a distance 0.5 m from the left support. Young's modulus of the material is 200 GPa. Calculate the maximum tensile stress in the beam. Also calculate the vertical deflection of the beam at a point 0.5 m from the right support.

19. A composite shaft is to carry a torque of 125 Nm. It consists of a 60 mm diameter steel tube fitted tightly over a hollow brass tube of external and internal diameters of 40 mm and 30 mm, respectively. The bulk modulus of steel and brass are 80 GPa and 40 GPa, respectively. Determine the maximum stresses in the steel and brass portions.

20. A solid shaft is to transmit 300 kW at 120 rpm. The shear stress is not to exceed 100 MPa. It is desired to reduce the weight of the shaft by replacing it by a hollow shaft with diameter ratio of 0.6 keeping other parameters same. Determine the diameter of solid shaft and percentage saving of the material by using hollow shaft.

21. In a torsion test, the specimen is a hollow shaft with 50 mm external and 30 mm internal diameter. An applied torque of 1.6 kNm is found to produce an angular twist of 0.4° measured on a length of 0.2 m of the shaft. The Young's modulus of elasticity obtained from a tensile test has been found to be 200 GPa. Calculate the modulus of elasticity and the Poisson's ratio of the material.

ANSWERS

Multiple Choice Questions

1.	(b)	**2.**	(d)	**3.**	(d)	**4.**	(c)	**5.**	(b)	**6.**	(b)	**7.**	(a)	**8.**	(d)	**9.**	(a)	**10.**	(a)
11.	(b)	**12.**	(b)	**13.**	(b)	**14.**	(a)	**15.**	(c)	**16.**	(c)	**17.**	(d)	**18.**	(a)	**19.**	(c)	**20.**	(a)
21.	(b)	**22.**	(a)	**23.**	(d)	**24.**	(c)	**25.**	(b)	**26.**	(a)	**27.**	(c)	**28.**	(c)	**29.**	(d)	**30.**	(c)
31.	(c)	**32.**	(a)	**33.**	(a)	**34.**	(c)	**35.**	(a)	**36.**	(b)	**37.**	(c)	**38.**	(c)	**39.**	(b)	**40.**	(b)
41.	(b)	**42.**	(d)	**43.**	(a)	**44.**	(a)	**45.**	(b)	**46.**	(b)	**47.**	(b)	**48.**	(c)	**49.**	(c)	**50.**	(a)
51.	(a)	**52.**	(c)	**53.**	(c)	**54.**	(c)	**55.**	(d)	**56.**	(c)	**57.**	(c)	**58.**	(a)	**59.**	(b)	**60.**	(b)
61.	(a)	**62.**	(b)	**63.**	(d)	**64.**	(c)	**65.**	(a)	**66.**	(d)	**67.**	(d)	**68.**	(d)	**69.**	(d)	**70.**	(c)
71.	(a)	**72.**	(b)	**73.**	(d)	**74.**	(c)	**75.**	(b)	**76.**	(d)	**77.**	(a)	**78.**	(c)	**79.**	(b)	**80.**	(b)
81.	(b)	**82.**	(d)	**83.**	(d)	**84.**	(d)	**85.**	(b)	**86.**	(a)	**87.**	(c)	**88.**	(d)	**89.**	(b)	**90.**	(a)
91.	(c)	**92.**	(d)	**93.**	(c)	**94.**	(c)	**95.**	(a)	**96.**	(b)	**97.**	(d)	**98.**	(a)	**99.**	(b)	**100.**	(b)
101.	(a)	**102.**	(c)	**103.**	(c)	**104.**	(b)	**105.**	(c)	**106.**	(d)	**107.**	(a)	**108.**	(d)	**109.**	(d)		

Numerical Answer Questions

1.	113 GPa, 0.3467	**2.**	45.47 kPa, -136.42 kPa	**3.**	123.08 MPa, 76.92 MPa
4.	111.43 mm^3	**5.**	175.96 mm, 2.54 mm	**6.**	75.32 MPa, 44.68 MPa
7.	85.14 MPa	**8.**	4.85 m, 9.719 m	**9.**	1.02 MN/m, 56.5 mm
10.	27.7 mm, 424.53 MPa	**11.**	110.5 MPa, 11.05 MPa	**12.**	205.06 MPa, 153.85 MPa
13.	93.6 MPa, 57.6 MPa	**14.**	308.6 kPa, 833.33 kPa	**15.**	24.81 MPa, 45°
16.	32.15 MPa, 114.22 kW	**17.**	15 kN/m, 30 kN/m	**18.**	192 MPa, 3.2 mm
19.	3.89 MPa, 1.22 MPa	**20.**	0.10 m, 29.79%	**21.**	85.82 GPa, 0.165

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (b) The lateral strain will be given by

$$\begin{aligned}\varepsilon_2 &= \mu\frac{\sigma_1}{E}\\ E &= \frac{\mu\sigma_1}{\varepsilon}\\ &= 150\text{ GPa}\end{aligned}$$

2. (d) The relationship among E, K and G is

$$E = \frac{9KG}{3K+G}$$

which can also be expressed as

$$\frac{9}{E} = \frac{1}{K} + \frac{3}{G}$$

3. (d) A beam of uniform strength is designed such that the bending stress is same at every section in its longitudinal axis.

4. (c) The deformation of a bar due to its own weight is half as if weight is acting on the weightless bar.

5. (b) Two independent elastic constants (out of μ, E, G, K) are required to express the stress-strain relationship for a linearly elastic isotropic material.

6. (b) The equivalent area for extension of tapered bar is geometric mean of areas at end sections.

7. (a) Lame's constant is used in plane strain condition:

$$\lambda = \frac{E\mu}{(1+\mu)(1-2\mu)}$$

8. (d) Extension of circular taper shafts is given by

$$\delta = \frac{4Pl}{\pi E d_1 d_2}$$

9. (a) Given that $\mu = 0.5$. Using

$$G = \frac{E}{2(1+\mu)}$$
$$E = 3G$$

10. (a) Volume of a cylinder is

$$V = l \times \frac{\pi}{4} D^2$$

Hence, volumetric strain (ε_V) is

$$\frac{dV}{V} = \frac{dl}{l} + 2\frac{dD}{D}$$
$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_2$$

11. (b) Given that

$$E = 210 \times 10^9 \text{ GPa}$$
$$G = 80 \times 10^9 \text{ GPa}$$

Using

$$G = \frac{E}{2(1+\mu)}$$
$$\mu = 0.31$$

12. (b) Given,

$$A = 1 \text{ cm}^2$$
$$l = 100 \text{ cm}$$
$$E = 210 \times 10^9 \text{ Pa}$$
$$F = 20 \times 10^3 \text{ N}$$

Elongation of the rod is

$$\delta = \frac{Fl}{AE}$$
$$= 0.1 \text{ cm}$$

13. (b) Tensile force (F) will be equal to carrying load plus the weight of the bar of the length below the point $(L-y)$:

$$F = W + w(L-y)$$

14. (a) Given that

$$E = 210 \text{ GPa}$$
$$\mu = 0.25$$

Using

$$G = \frac{E}{2(1+\mu)}$$
$$= 84 \times 10^9 \text{ Pa}$$

15. (c) In a beam of uniform strength, the maximum bending stress is the same throughout the length.

16. (c) The volumetric strain of the cube is

$$\varepsilon_v = 3\frac{\delta a}{a}$$
$$\frac{0.073 \times 10^{-3}}{1^3} = 3 \times \frac{\delta a}{1}$$
$$\delta a = 0.024 \times 10^{-3}$$

17. (d) The value of Poisson's ratio can vary from -1 to 0.5 only.

18. (a) The unit of elastic modulus is N/m^2 which is the same for stress, shear modulus and pressure.

19. (c) Strain energy in shear is determined as

$$U_\tau = \frac{\tau^2}{2G}$$

20. (a) For linear elastic isotropic and homogenous material, only two elastic constants (out of μ, E, G, and K) are required to relate stress and strain.

21. (b) Bulk modulus of elasticity (K) is related to modulus of elasticity (E) as

$$K = \frac{E}{3(1-2\mu)}$$

22. (a) Given $E = 2.5G$. Therefore, using

$$G = \frac{E}{2(1+\mu)}$$
$$= \frac{2.5G}{2(1+\mu)}$$
$$\mu = 0.25$$

23. (d) The volumetric compressive strain in the oil is

$$-\frac{dv}{v} = \frac{p}{K}$$
$$= \frac{1.5}{2800}$$
$$= 5.357 \times 10^{-4}$$
$$= 535.7 \times 10^{-6}$$

24. (c) The Mohr's circle will be a circle with radius τ and center at (0,0). It is a case of pure shear, like in torsional load in the shafts.

25. (b) For given case,

$$\begin{aligned}\sigma_1 &= \sigma_2\\ &= 500\ \text{kg/cm}^2\\ \sigma_x &= \sigma_1\end{aligned}$$

26. (a) The y-axis of the Mohr's circle represents the shear stress, and its maximum possible value is the radius of the circle.

27. (c) Radius of Mohr's circle for strain will be $\gamma_{xy}/2$, which is also the maximum principal strain.

28. (c) For σ_1 being the maximum principal stress, the maximum shear stress is given by

$$\begin{aligned}\tau_{\max} &= \frac{\sigma_1-(\sigma_1/2)}{2}\\ \sigma_1 &= 4\tau_{\max}\end{aligned}$$

29. (d) The point of Mohr's circle can be anywhere but the principal stresses will be of the same magnitude and sign.

30. (c) Maximum shear stress is radius of Mohr's circle:

$$\begin{aligned}\tau_{\max} &= \sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_{xy}^2}\\ &= \sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_{xy}^2}\\ &= 5\sqrt{5}\ \text{MPa}\end{aligned}$$

31. (c) If normal stress is σ, then the normal stress on a plane at $\theta = 45°$ will be

$$\begin{aligned}\sigma_\theta &= \sigma\cos^2\theta\\ &= \frac{\sigma}{2}\end{aligned}$$

32. (a) In Mohr's circle for this case, the radius will be $\sigma/2$, at angle 2θ. Hence, the shear stress will be

$$\tau_\theta = \frac{\sigma}{2}\sin 2\theta$$

33. (a) The center and radius of Mohr's circle are determined as

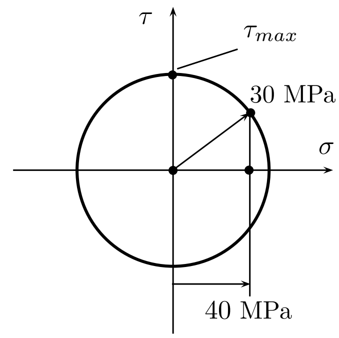
$$\begin{aligned}OO' &= \frac{50-100}{2}\\ &= -25\ \text{MPa}\\ R_\sigma &= \sqrt{\left(\frac{50+100}{2}\right)^2+40^2}\\ &= 85\ \text{MPa}\end{aligned}$$

The maximum principal stresses is given by

$$\begin{aligned}\sigma_1 &= OO'+R_\sigma\\ &= -25+85\\ &= 60\ \text{MPa}\\ \sigma_2 &= OO'-R_\sigma\\ &= -25-85\\ &= -110\ \text{MPa}\end{aligned}$$

Thus, option (a) is correct.

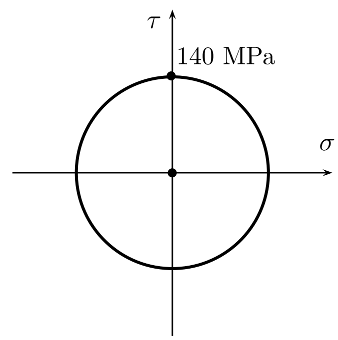
34. (c) The Mohr's circle for the given stresses is shown in the figure below.



Thus, the maximum shear stress shall be given by

$$\begin{aligned}\tau_{max} &= \sqrt{40^2+30^2}\\ &= 50\ \text{MPa}\end{aligned}$$

35. (a) The Mohr's circle for the given case is shown in the figure below



Thus, the maximum normal stress is equal to the radius of Mohr circle, that is, 140 MPa.

36. (b) The strain in three directions is given by

$$\begin{aligned}\varepsilon_x &= \frac{\sigma_x-\mu(\sigma_y+\sigma_z)}{E}\\ \varepsilon_y &= \frac{\sigma_y-\mu(\sigma_z+\sigma_x)}{E}\\ \varepsilon_z &= \frac{\sigma_z-\mu(\sigma_x+\sigma_y)}{E}\end{aligned}$$

The volumetric stain is determined as

$$\begin{aligned}\varepsilon_v &= \varepsilon_x+\varepsilon_y+\varepsilon_z \\ &= \frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-2\mu) \\ &= \frac{60+20+50}{10^5}(1-2\times 0.35) \\ &= 3.9\times 10^{-4}\end{aligned}$$

37. (c) The maximum shear strain is given by

$$\begin{aligned}\gamma &= 2\times\frac{\varepsilon_1-\varepsilon_2}{2} \\ &= 2\times\frac{1000-(-600)}{2} \\ &= 1600\ \mu\text{m}\end{aligned}$$

38. (c) The radius of the Mohr's circle is determined as

$$\begin{aligned}R_\sigma &= \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} \\ &= \sqrt{\left(\frac{10-2}{2}\right)^2+4^2} \\ &= 4\sqrt{2}\ \text{MPa}\end{aligned}$$

The σ-coordinate of center of Mohr's circle is

$$\begin{aligned}c &= \frac{\sigma_x+\sigma_y}{2} \\ &= \frac{10+2}{2} \\ &= 6\ \text{MPa}\end{aligned}$$

Hence, maximum principal stress is

$$\begin{aligned}\sigma_1 &= R_\sigma+c \\ &= 11.65\ \text{MPa}\end{aligned}$$

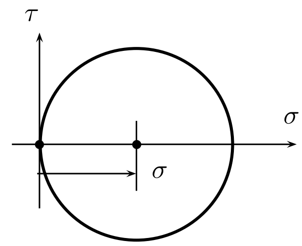
39. (b) The true strain (ε) is related to engineering strain (ε_0) as

$$\begin{aligned}\varepsilon &= \ln(1+\varepsilon_0) \\ e^\varepsilon &= 1+\varepsilon_0 \\ \varepsilon_0+1 &= e^\varepsilon\end{aligned}$$

For volume constancy

$$\begin{aligned}t_2\times e^{0.05}\times e^{0.09} &= 1.5 \\ t_2 &= 1.304\ \text{mm}\end{aligned}$$

40. (b) Mohr's circle for the given stress state is shown below.



Thus, the maximum principal stress is 2σ.

41. (b) Point of the contraflexure is the point where bending moment value is zero or changes its sign.

42. (d) The shear force is related to bending moment as

$$F = \frac{dM}{dx}$$

For given case, M =constant, hence, $F = 0$.

43. (a) Hinged joints are not made to support bending moment.

44. (a) The length BC is subjected to axial load F and constant bending moment Fk, and there is no shear force.

45. (b) The section modulus for rectangular beam is given by

$$Z = \frac{bd^2}{6}$$

Thus, for the two cases

$$\begin{aligned}\frac{Z_2}{Z_1} &= \frac{0.6}{1}\times\left(\frac{1}{0.6}\right)^2 \\ &= \frac{1}{0.6}\end{aligned}$$

46. (b) For circular cross-section,

$$\tau_{\max} = \frac{4}{3}\tau_{\text{avg}}$$

47. (b) For a rectangular section of height h and width b, the shear stress is given by following parabolic equation

$$\tau_y = \frac{F_s}{b\times bh^3/12}\times b\,(h/2-y)\left(\frac{h/2-y}{2}+y\right)$$

Maximum shear stress at $y = 0$ is related to the average value as

$$\tau_{\max} = 1.5\times\tau_{\text{avg}}$$

48. (c) Cross-sectional area of square and circular section is the same, hence,

$$\begin{aligned}\frac{\pi d^2}{4} &= a^2 \\ d &= 1.128a\end{aligned}$$

The beam strength is expressed as section modulus

$$Z_s = \frac{a^2}{12} = 0.083a^2$$

$$Z_c = \frac{\pi d^3}{32} = 0.1409a^3$$

Hence, $Z_c > Z_s$.

49. (c) For solid rectangular sections:

$$\tau_{\max} = \frac{3}{2}\tau_{avg} = \frac{3}{2} \times \frac{20 \times 10^3}{0.1 \times 0.05} = 6 \text{ MPa}$$

50. (a) If bending stress σ is to be developed throughout the length, then at any section

$$\sigma = \frac{6M}{bd^2}$$
$$b \propto M$$

51. (a) Given,

$$F = 100 \text{ N}$$
$$l = 5 \text{ m}$$
$$r = 1 \text{ m}$$
$$d_o = 0.150 \text{ m}$$
$$\delta_A = 5 \text{ mm}$$
$$E = 200 \text{ GPa}$$

The system can be modeled as a cantilever subjected to point load $F = 100$ N and twisting moment $T = 100 \times 1$ m but the deflection shall be caused the point load only:

$$\delta_A = \frac{FL^3}{3EI}$$

$$I = \frac{FL^3}{3E\delta_A} = \frac{100 \times 5^3}{3 \times 200 \times 10^9 \times 0.005} = 4.1667 \times 10^{-6} \text{ m}^4$$

Internal diameter is given by

$$\frac{\pi}{64}\left(0.15^4 - d_i^4\right) = 4.1667 \times 10^{-6}$$
$$d_i = 0.1432 \text{ m}$$

Thickness of the pole is

$$t = \frac{d_o - d_i}{2} = 3.36 \text{ mm}$$

52. (c) Only distance of outer lamina will change from b to $b/\sqrt{2}$, and the moment of inertia will be same in both cases, therefore, maximum bending stress in the second case will be $\sqrt{2}\sigma$.

53. (c) For beams with rectangular cross-section, the maximum shear stress is given by

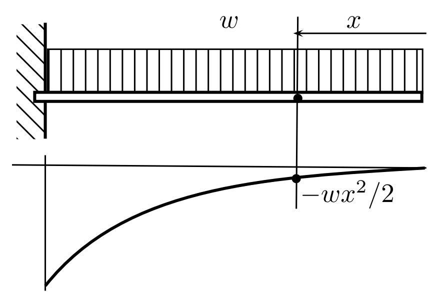
$$\tau_m = \frac{3}{2}\tau_a$$

This is also valid for square cross-sections.

54. (c) The area moment of inertia of a square of size a about its diagonal axis or natural axis parallel to side is given by

$$I = \frac{a^4}{12}$$

55. (d) The cantilever subjected uniformly distributed load, say, w per unit length is shown in the figure.



The bending moment is proportional to x^2, thus, representing a parabolic shape.

56. (c) Maximum deflection (at free end) of a cantilever beam of length l subjected to UDL w per unit length is given by

$$\delta_{\max} = \frac{wl^4}{8EI}$$

57. (c) The strain energy due to axial load P and torsion T are, respectively,

$$U_F = \frac{1}{2}\frac{\sigma^2 L}{AE} = \frac{P^2 L}{2AE}$$

$$U_T = \frac{1}{2}\frac{T^2 L}{I_p G} = \frac{T^2 L}{2I_p G}$$

Hence, for the given case,

$$U = \frac{P^2L}{2AE} + \frac{T^2L}{2I_pG}$$

58. (a) The bulk modulus of rigidity G is related to E as

$$G = \frac{E}{1+\mu}$$

Thus, strain energy for pure shear is given by

$$U_\tau = \frac{\tau^2}{E}(1+\mu)$$

59. (b) The beam deflection is inversely proportional to the moment of inertia. In the first case, the moment of inertia is

$$I = \frac{bd^3}{12}$$

and in second case

$$I' = \frac{db^3}{12}$$

Hence, the deflection in second case will be

$$\begin{aligned} \delta' &= \delta \times \frac{bd^3}{db^3} \\ &= \delta\left(\frac{d}{b}\right)^2 \end{aligned}$$

60. (b) The deflection at free end of the cantilever will be equal to the sum of the two components, given by

$$\delta = \frac{ML^2}{2EI} + \frac{PL^3}{3EI}$$

61. (a) The deflection in the first and second cases are, respectively,

$$\begin{aligned} \delta_1 &= \frac{WL^3}{8EI} \\ \delta_2 &= \frac{WL^3}{3EI} \\ \frac{\delta_1}{\delta_2} &= \frac{3}{8} \end{aligned}$$

62. (b) For the two cases

$$\begin{aligned} \delta_1 &= \frac{WL^3}{48EI} \\ \delta_2 &= \frac{5WL^3}{384EI} \end{aligned}$$

Therefore,

$$\delta_1 : \delta_2 = 1.6 : 1 \approx \sqrt{2} : 1$$

63. (d) Assuming the beam to be simply supported, the mid-span deflection in the first case is

$$\delta = \frac{5ql^4}{384EI}$$

In the second case, with UDL, using superimposing method, the mid span deflection is

$$\delta = \frac{5ql^4}{384EI}$$

Thus, there will be same deflection in both the cases.

64. (c) The deflection is inversely proportional to section modulus of the beam, written as

$$\delta \propto \frac{1}{bd^2}$$

Hence, for the given two cases

$$\begin{aligned} \delta_2 &= \frac{4\times 6^2}{6\times 4^2}\delta \\ &= 1.5\delta \end{aligned}$$

65. (a) The work done in deformation is transformed into elastic strain energy U that is stored in the body. The area of stress–strain curve represents the strain energy. For the given case, the average bending moment acted on the member is M, and its displacement is θ. This displacement is given by Hooke's law as

$$\theta = \frac{M}{EI}l$$

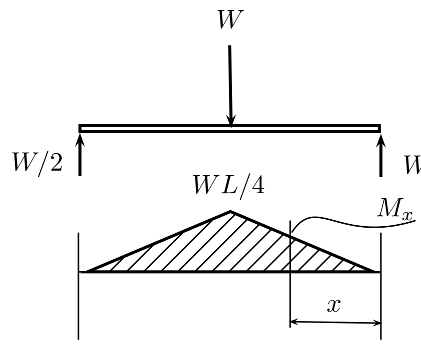
The strain energy in this case will be

$$\begin{aligned} U_M &= \int_0^L \frac{M^2}{EI}dx \\ &= \frac{M^2L}{EI} \end{aligned}$$

66. (d) The deflections in two cases are, respectively, $(wl = W)$

$$\begin{aligned} \delta_1 &= \frac{1}{48}\frac{Wl^3}{EI} \\ \delta_2 &= \frac{5}{384}\frac{Wl^3}{EI} \\ \frac{\delta_1}{\delta_2} &= \frac{384}{5\times 48} \\ &= \frac{8}{5} \end{aligned}$$

67. (d) The beam and bending moment diagram is shown in the figure below.



The strain energy for the given case is calculated as

$$\begin{aligned} U_M &= 2\times\frac{1}{2}\int_0^{L/2}\frac{M^2}{EI}dx \\ &= \frac{1}{EI}\int_0^{L/2}\left(\frac{W}{2}x\right)^2 dx \\ &= \frac{W^2L^3}{3\times4\times8EI} \\ &= \frac{W^2L^3}{96EI} \end{aligned}$$

Alternate Solution. The strain energy can also be determined directly as

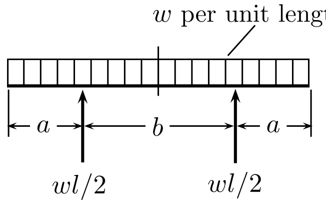
$$\begin{aligned} U_M &= \frac{1}{2}\delta\times W \\ &= \frac{1}{2}\times\frac{PL^3}{48EI}\times W \\ &= \frac{W^2L^3}{96EI} \end{aligned}$$

68. (d) The deflection at the free end of a cantilever subjected to a concentrated load at its free end is given by

$$\begin{aligned} \delta &= \frac{PL^3}{3EI} \\ &\propto L^3 \end{aligned}$$

69. (d) The point of inflexion of a curve is a point at which the concavity or curvature changes its sign, while point of contraflexure is the point where bending moment changes its sign.

70. (c) The given condition of the beam is shown in the figure below.



The bending moment at mid-span shall be zero for

$$\begin{aligned} b\frac{wl}{2}-\frac{wl}{2}\frac{b/2+a}{2} &= 0 \\ b-\frac{b}{4}-\frac{a}{2} &= 0 \\ \frac{b}{a} &= \frac{2}{3} \end{aligned}$$

71. (a) The area of the SF diagram in the two parts is

$$\begin{aligned} M_1 &= \frac{14+2}{2}\times2 \\ &= 16\text{ kNm} \\ M_2 &= \frac{-13-19}{2}\times1 \\ &= -16\text{ kNm} \end{aligned}$$

Thus, maximum bending moment is 16 kNm.

72. (b) As in area moment method, the slope between any two points is determined as

$$\theta_{AB} = \int_{x_A}^{x_B}\frac{M(x)}{EI}dx$$

73. (d) Given that

$$\begin{aligned} a &= 0.04\text{ m} \\ l &= 1\text{ m} \end{aligned}$$

The ratio of strain energy in the two cases is determined as

$$\begin{aligned} R &= \frac{\frac{1}{2}\int_0^l\left\{(Fx)^2/(EI)\right\}dx}{F^2l/(2AE)} \\ &= \frac{F^2l^3/(6EI)}{F^2l/(2AE)} \\ &= \frac{Al^2}{3I} \\ &= \frac{a^2l^2}{3\times\frac{a^4}{12}} \\ &= \frac{4l^2}{a^2} \\ &= 2500 \end{aligned}$$

74. (c) Angle of twist in each shaft is the same for parallel arrangements of the shafts.

75. (b) The ratio of torque carrying capacity is proportional to the polar modulus given by

$$\begin{aligned} I_p &= \frac{\pi D_o^3/32}{\pi\left(D_o^4-D_i^4\right)/32} \\ &= \frac{1}{1-k^4} \end{aligned}$$

76. (d) The spring element is subjected to torsional and direct shear stress.

77. (a) Stiffness (k) is given by

$$k = \frac{Gd^4}{64R^3 n} \propto \frac{1}{R^3 n}$$

Hence,

$$k' = \frac{2}{2^3}k = \frac{k}{4}$$

78. (c) The weight of springs is same but with wire diameter are d and $d/2$. Thus, the ratio of number of turns will be given by

$$n_1 d^2 = n_2 \left(\frac{d}{2}\right)^2$$

$$\frac{n_1}{n_2} = \frac{1}{4}$$

The stiffness of the spring is determined as

$$k = \frac{Gd^4}{64R^3 n}$$

$$k \propto \frac{d^4}{n}$$

Hence, the ratio will be $2^4 \times 4 = 64$.

79. (b) To take into account the initial curvature of spring wire, Wahl's factor is used in design of springs as

$$\tau_{\max} = \frac{16PR}{\pi d^3}\left[\frac{4c-1}{4c-4} + \frac{0.615}{c}\right]$$

where $c = D/d$ is spring index.

80. (b) If original stiffness is k, then after breaking into two equal parts, the stiffness of each part will be $2k$. If these parts are combined in series, then equivalent stiffness is

$$k_e = \frac{1}{2 \times 1/(2k)} = k$$

81. (b) Stiffness (spring constant) (k) is given by

$$k = \frac{Gd^4}{64R^3 n} \propto \frac{1}{R^3}$$

Therefore,

$$\frac{k_1}{k_2} = \frac{R_2^3}{R_1^3} = \frac{(1.25/2)^3}{1.25^3} = \frac{1}{8}$$

82. (d) Maximum shear stress in solid shaft is equal to that in hollow shaft of half diameter:

$$\frac{T}{\frac{\pi}{32}d^4} \times \frac{d}{2} = \frac{T'}{\frac{\pi}{32}\left(d^4 - (d/2)^4\right)} \times \frac{d}{2}$$

$$T' = \frac{15}{16}T$$

83. (d) Angle of twist is inversely proportional to polar modulus:

$$\frac{\theta_1}{\theta_2} = \frac{{d_1}^2 - (d/2)^4}{{d_1}^2 - (d/3)^4} = \frac{243}{256}$$

84. (d) Maximum stress for solid shaft is

$$\tau = \frac{16T}{\pi D^3}$$

For hollow shafts,

$$\tau' = \frac{32T}{\pi\left(D^4 - (D/2)^2\right)} \times \frac{D}{2} = \frac{16}{15}\tau$$

85. (b) The maximum shear stress developed in the spring is related to the load and its length as

$$\tau = \frac{16PR}{\pi d^3}$$

which is independent of the number turns or length.

86. (a) For the solid shaft,

$$\tau \propto \frac{1}{D^3}$$

For hollow shaft

$$\tau_2 \propto \frac{D}{\left(D^4 - (D/2)^4\right)} \propto \frac{16}{15D^3}$$

Therefore,

$$\tau_2 = \frac{16}{15}\tau = 1.06\tau$$

87. (c) Let d, R and n be the wire diameter, mean coil radius, and number of coils of the spring, then stiffness k is determined as

$$k = \frac{Gd^4}{64R^3n}$$

where G is modulus of rigidity of spring material. Therefore, for the given two cases

$$\frac{k_2}{k} = \frac{75^3 \times 8}{60^3 \times 10}$$
$$k_2 = 1.56k$$

88. (d) Given, $d_2 = 2d_1$. The polar modulus of solid shafts is given by

$$I_p = \frac{\pi d^4}{32}$$

The angle of twist is given by

$$\theta = \frac{TL}{GI_p} \propto \frac{1}{d^4}$$

Thus, the ratio of angles of twist would be $2^4 = 16$.

89. (b) Keeping other parameters constant, the torque shall be given by

$$T \propto I_p$$

Torque carried by the hollow portion of the shaft is

$$T = \frac{\pi\left(100^4 - 50^4\right)}{\pi\left(100^4\right)} \times 1600 = \left\{1 - \left(\frac{50}{100}\right)^4\right\} \times 1600 = 1500 \text{ kNm}$$

90. (a) The stiffness of spring is given by

$$k = \frac{Gd^4}{64R^3n} \propto \frac{1}{n}$$

91. (c) The power that can be transmitted by a shaft is

$$P = T\omega = \frac{\pi d^3\tau}{16} \times \frac{2\pi N}{60} \propto d^3$$

Hence,

$$\frac{T_a}{T_b} = \left(\frac{2}{1}\right)^3 = 8$$

92. (d) Given that

$$n = 20$$
$$P = 1000 \text{ N}$$
$$\delta = 10 \text{ mm}$$

The spring stiffness is

$$k = \frac{1000}{10} = 100 \text{ N/mm}$$

The stiffness is inversely proportional to the number of turns, thus, the stiffness of two pieces will be $2 \times 100 = 200$ N/mm. The equivalent stiffness of the parallel combination is $2 \times 200 = 400$ N/mm. Hence, the deflection for the same load is

$$\delta = \frac{1000}{400} = 2.5 \text{ N/mm}$$

93. (c) The equivalent bending moment is given by

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$

94. (c) If M is the bending moment and torque applied on a shaft of diameter, say, d, then, bending stress and shear stresses are

$$\sigma = \frac{32M}{\pi d^3}$$
$$\tau = \frac{16M}{\pi d^3}$$

Therefore,

$$\frac{\sigma}{\tau} = 2$$

95. (a) Maximum bending stress in a solid circular shaft of diameter d due bending moment M is given by

$$\sigma_b = \frac{32M}{\pi d^3}$$

Maximum shear stress due to torque T is given by

$$\tau = \frac{16T}{\pi d^4}$$

For the two situations

$$\sigma_b = \tau$$

Hence,

$$M = \frac{T}{2}$$

96. (b) When bending moment M and torque T is applied on a shaft, the equivalent torque and equivalent bending moment are

$$T_e = \sqrt{T^2 + M^2}$$
$$M_e = \frac{1}{2}\left(M + \sqrt{T^2 + M^2}\right)$$

97. (d) The combined torque is given by

$$\begin{aligned} T_e &= \sqrt{T^2 + M^2} \\ &= \sqrt{3^2 + 4^2} \\ &= 5.0 \text{ kN} \end{aligned}$$

98. (a) The rotation of the flywheel produces centrifugal stress (σ_θ) which in turn affects the safe rim velocity (v) as

$$\sigma_\theta = \rho v^2$$

where ρ is the density of the flywheel.

99. (b) Maximum hoop stress in the thick cylinder occurs at inner radius.

100. (b) When $p_i \neq 0$, $p_o = 0$, the hoop stresses at inner and outer surfaces will be

$$\sigma_{\theta-i} = \left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) p_i$$
$$\sigma_{\theta-o} = \left(\frac{2r_i^2}{r_o^2 - r_i^2}\right) p_i$$

Given that $\sigma_{\theta-o} = 100$, $p_i = 100$, thus

$$\begin{aligned} \frac{2r_i^2}{r_o^2 - r_i^2} &= 1 \\ \frac{r_o^2 - r_i^2}{r_i^2} &= 2 \\ \frac{r_o^2}{r_i^2} &= 3 \end{aligned}$$

Therefore,

$$\begin{aligned} \sigma_{\theta-i} &= \left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) p_i \\ &= \left(\frac{\frac{r_o^2}{r_i^2} + 1}{\frac{r_o^2}{r_i^2} - 1}\right) p_i \\ &= \frac{3+1}{3-1} \times 100 \\ &= 200 \text{ N/mm}^2 \end{aligned}$$

101. (a) When $p_o \neq 0$, $p_i = 0$ - the maximum stress is hoop stress at $r = r_i$

$$\sigma_{\max} = -\left(\frac{2{r_o}^2}{{r_o}^2 - {r_i}^2}\right) p_o$$

102. (c) When $p_i \neq 0$, $p_o = 0$, then the maximum hoop stress and corresponding shear stress at $r = r_i$ are

$$\sigma_{\max} = \left(\frac{{r_o}^2 + {r_i}^2}{{r_o}^2 - {r_i}^2}\right) p_i$$
$$\tau_{\max} = \frac{{r_o}^2}{{r_o}^2 - {r_i}^2} p_i$$

The maximum hoop stress occurs at inner circumference and the hoop stress decreases towards the outer circumference. Hence, the maximum pressure inside the shell is limited corresponding to the condition.

103. (c) For thick-walled cylinders, the relationships between hoop stress and longitudinal stress with radius is

$$\sigma_r = A - \frac{B}{r^2}$$
$$\sigma_\theta = A + \frac{B}{r^2}$$

where A and B are constants. When subjected to internal pressure p_i, the radial stress at outer surface will be zero, thus

$$0 = A - \frac{B}{r_o^2}$$
$$-p_i = A - \frac{B}{r_i^2}$$

From the above two equations,

$$p_i = B\left(\frac{1}{r_i^2} - \frac{1}{r_o^2}\right)$$
$$B = p_i \frac{r_o^2 r_i^2}{r_o^2 - r_i^2}$$
$$A = \frac{B}{r_o^2}$$
$$= p_i \frac{r_i^2}{r_o^2 - r_i^2}$$

Hoop stress at inner surfaces is

$$\sigma_{\theta-i} = A + \frac{B}{r_i^2}$$
$$= p_i \frac{r_i^2}{r_o^2 - r_i^2} + p_i \frac{r_o^2 r_i^2}{r_o^2 - r_i^2}\frac{1}{r_i^2}$$
$$= \left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) p_i$$

Hoop stress at outer surface is

$$\sigma_{\theta-o} = A + \frac{B}{r_o^2}$$
$$= p_i \frac{r_i^2}{r_o^2 - r_i^2} + p_i \frac{r_o^2 r_i^2}{r_o^2 - r_i^2}\frac{1}{r_o^2}$$
$$= \left(\frac{2r_i^2}{r_o^2 - r_i^2}\right) p_i$$
$$= \left(\frac{2}{\frac{r_o^2}{r_i^2} - 1}\right) p_i$$

Thus, ratio of thickness is

$$\frac{r_o^2}{r_i^2} = \frac{2p_i}{\sigma_{\theta-o}} + 1$$

Given

$$p_i = 60 \text{ MPa}$$
$$\sigma_{\theta-o} = 150 \text{ MPa.}$$

Thus

$$\frac{r_o^2}{r_i^2} = \frac{2 \times 60}{150} + 1$$
$$= 1.8$$

Thus, the hoop stress at internal surface is

$$\sigma_{\theta-i} = \frac{r_o^2 + r_i^2}{2r_i^2}\sigma_{\theta-o}$$
$$= \left(\frac{r_o^2}{r_i^2} + 1\right)\frac{\sigma_{\theta-i}}{2}$$
$$= (1.8 + 1)\frac{150}{2}$$
$$= 210 \text{ MPa}$$

104. (b) Given that

$$d = 10 \text{ m}$$
$$h = 100 \text{ m}$$
$$t = 0.009 \text{ m}$$

Hence, the pressure at pipe will be given by

$$p = \rho g h$$

The hoop stress in the pipe is given by

$$\sigma_h = \frac{pd}{2t}$$
$$= 545.0 \times 10^6 \text{ N/m}^2$$
$$= 545.0 \text{ MPa}$$

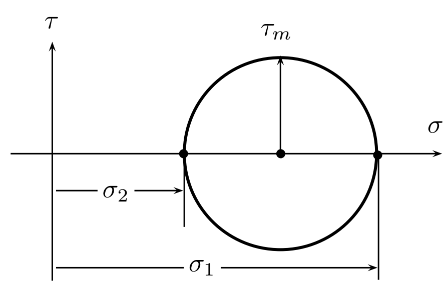
105. (c) Hoop stress in a thin-walled cylinder is given by

$$\sigma_t = \frac{pd}{2t}$$
$$t = \frac{pd}{2\sigma_t}$$
$$= 16.667 \text{ mm}$$

106. (d) An element in thin-walled cylindrical vessel is subjected to the hoop stress and longitudinal stress given by

$$\sigma_\theta = \frac{pd}{2t}$$
$$\sigma_l = \frac{pd}{4t}$$

The maximum shear stress will be equal to the radius of Mohr's circle.



Thus,

$$\begin{aligned}\tau_m &= \frac{\sigma_\theta - \sigma_l}{2}\\ &= \frac{pd}{8t}\end{aligned}$$

107. (a) For the given case of thin-walled cylinder, the maximum principal strain is the circumferential strain given by

$$\begin{aligned}\varepsilon_h &= \frac{pd}{4tE}(2-\mu)\\ &= \frac{1\times 10^6\times 0.50}{4\times 0.005\times 200\times 10^9}\times(2-0.2)\\ &= 0.00025\end{aligned}$$

108. (d) Euler buckling load is given by

$$\begin{aligned}F &= \frac{\pi^2 EI}{l^2}\\ &= \frac{\pi^2 E\times \pi d^4/64}{l^2}\\ &\propto d^4\end{aligned}$$

Reduction of d into $0.8d$ will reduce the F_2 to $0.8^4 F_1$ that is $0.4096F_1$. Hence, percentage reduction is

$$1-0.4096 = 59\%$$

109. (d) For screw jacks, the bottom point is considered as fixed end while another end is free.

Numerical Answer Questions

1. The modulus of elasticity is calculated as

$$\begin{aligned}E &= \frac{25\times 10^3\times 0.2}{(\pi\times 0.015^2/4)\times(0.25/1000)}\\ &= 113 \text{ GPa}\end{aligned}$$

Poisson ratio of the material is calculated as

$$\begin{aligned}\mu &= \frac{0.0065/15}{0.25/200}\\ &= 0.3467\end{aligned}$$

2. For the given configuration of the material elements, the extension of Al and compression of steel should be equal. If the force shared by steel is F_{st} kN, then force shared by aluminium will be $300-F_{st}$ kN. Thus

$$\begin{aligned}\frac{4F_{st}\times 10^3\times 1}{\pi 0.04^2\times 210\times 10^9} &= \frac{4(300-F_{st})\times 10^3\times 1}{\pi 0.06^2\times 70\times 10^9}\\ F_{st} &= \frac{4}{3}(300-F_{st})\end{aligned}$$

Also

$$\begin{aligned}F_{st}+\frac{4}{3}F_{st} &= 400\\ F_{st} &= 171.428 \text{ kN}(-)\\ F_{al} &= 128.57 \text{ kN}\end{aligned}$$

The stresses induced are calculated as

$$\begin{aligned}\sigma_{st} &= -\frac{4\times 171.428}{\pi\times 0.04^2}\\ &= -136.42 \text{ kPa}\\ \sigma_{al} &= \frac{4\times 128.57}{\pi\times 0.06^2}\\ &= 45.47 \text{ kPa}\end{aligned}$$

3. The strains are calculated as

$$\begin{aligned}\varepsilon_x &= \frac{0.05}{100}\\ &= 5\times 10^{-4}\\ \varepsilon_y &= \frac{0.01}{50}\\ &= 2\times 10^{-4}\end{aligned}$$

Therefore, the stresses are calculated as

$$\begin{aligned}\sigma_x &= \frac{E(\varepsilon_x-\mu\varepsilon_y)}{1-\mu^2}\\ &= 123.08 \text{ MPa}\\ \sigma_y &= \frac{E(\varepsilon_y-\mu\varepsilon_x)}{1-\mu^2}\\ &= 76.92 \text{ MPa}\end{aligned}$$

4. Let σ_x be the stress in longitudinal direction and $\sigma_y = \sigma_z$ (due to symmetry) the stresses in lateral directions. The stress in longitudinal direction is

$$\begin{aligned}\sigma_x &= -\frac{100\times 10^3}{50\times 50}\\ &= -40 \text{ MPa}\end{aligned}$$

Lateral strain in y direction will be given by

$$\varepsilon_y = \frac{1}{E}\{\sigma_y - \mu(\sigma_x+\sigma_z)\}$$

The lateral strains in y or z directions are zero, and $\sigma_y = \sigma_z$, therefore

$$\begin{aligned}\sigma_y - \mu(\sigma_x+\sigma_z) &= 0\\ \sigma_y &= \sigma_x\frac{\mu}{1-\mu}\\ &= -17.14 \text{ MPa}\end{aligned}$$

The strain in longitudinal is determined as

$$\begin{aligned}\varepsilon_x &= \frac{1}{E}\{\sigma_x - \mu(\sigma_y + \sigma_y)\}\\ &= 1.4858\times 10^{-4}\end{aligned}$$

Since strain is only in x direction, therefore, the change in volume will be given by

$$\begin{aligned}\delta V &= \varepsilon_x \times V\\ &= 111.435 \text{ mm}^3\end{aligned}$$

5. The potential energy of the weight is transferred into strain energy. The static deflection of the rod will be calculated as

$$\begin{aligned}\delta &= \frac{\sigma}{E} l\\ &= \frac{150\times 10^6}{200\times 10^9}\times 2\\ &= 0.75 \text{ mm}\end{aligned}$$

If h is the maximum height of the weight and δ is the static deflection of the rod, then energy conservation gives

$$\begin{aligned}\frac{\sigma^2}{2E} A\times l &= mg\left(h + \frac{\sigma}{E} l\right)\\ h &= \frac{\sigma^2 A\times l}{2mgE} - \frac{\sigma}{E} l\\ &= \frac{\sigma}{E} l\left(\frac{\sigma\times A}{2mg} - 1\right)\\ &= 175.96 \text{ mm}\end{aligned}$$

The maximum extension is given by

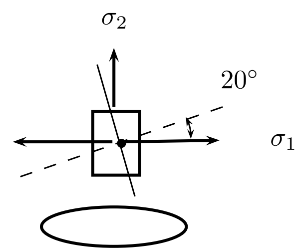
$$\begin{aligned}\delta_m &= \delta\left\{1 + \sqrt{1 + \frac{2h}{\delta}}\right\}\\ &= 7.5\times 10^{-4}\times 3.385\\ &= 2.539 \text{ mm}\end{aligned}$$

6. The tank is diameter d = 1.6 m, weld thickness 8 mm is subjected to 0.8 MPa internal pressure, therefore, hoop stress and longitudinal stress are calculated as

$$\begin{aligned}\sigma_1 &= \frac{pd}{2t}\\ &= \frac{0.8\times 1.6}{2\times 0.008}\\ &= 80 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\sigma_2 &= \frac{pd}{4t}\\ &= \frac{0.8\times 1.6}{4\times 0.008}\\ &= 40 \text{ MPa}\end{aligned}$$

These stresses are shown in figure



The stresses along and perpendicular to the weld (at $\theta = 20°$ plane) are

$$\begin{aligned}\sigma_{1'} &= \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos\theta\\ &= 75.32 \text{ MPa}\\ \sigma_{2'} &= \frac{\sigma_1+\sigma_2}{2} - \frac{\sigma_1-\sigma_2}{2}\cos\theta\\ &= 44.68 \text{ MPa}\end{aligned}$$

7. The angles of twist in the three segments of torques are calculated as

$$\begin{aligned}\theta_{AB} &= \frac{32T_A\times 1.5}{\pi\times 0.05^4 G}\\ \theta_{BC} &= \frac{32(T_A-10)\times 10^3\times 1.0}{\pi\times(0.08^4-0.03^4)G}\\ \theta_{CD} &= \frac{32(T_A-4)\times 10^3\times 0.5}{\pi\times(0.08^4-0.03^4)G}\end{aligned}$$

As both ends are fixed, therefore

$$\begin{aligned}\theta_{AB}+\theta_{BC}+\theta_{CD} &= 0\\ T_A\left(\frac{1.5}{0.05^4}+\frac{1.5}{0.08^4-0.03^4}\right) - \frac{(10+2)\times 10^3}{0.08^4-0.03^4} &= 0\end{aligned}$$

Therefore,

$$\begin{aligned}T_A &= \frac{298.87\times 10^6}{277.35\times 10^3}\\ &= 1.07 \text{ kNm}\\ T_D &= T_A - 4\\ &= -2.92 \text{ kNm}\end{aligned}$$

Shear stress at the weld is calculated as

$$\begin{aligned}\tau &= \frac{16T_A}{\pi d^3}\\ &= \frac{16\times 1.07\times 10^3}{\pi\times 0.04^3}\\ &= 85.14 \text{ MPa}\end{aligned}$$

8. The minimum radius of gyration of the column is calculated as

$$\begin{aligned}k &= \sqrt{\frac{3260}{109}}\\ &= 0.05469 \text{ m}\end{aligned}$$

Maximum length l_c for both ends hinged column is given by

$$\sigma_c = \frac{\pi^2 E}{(l/k)^2}$$
$$l_c = \pi k\sqrt{\frac{E}{\sigma_c}}$$
$$= 4.859 \text{ m}$$

Maximum length l_c for both ends fixed column is given by

$$\sigma_c = \frac{\pi^2 E}{l^2/(2k)^2}$$
$$l_c = 2\pi k\sqrt{\frac{E}{\sigma_c}}$$
$$= 9.719 \text{ m}$$

9. The kinetic energy of wagons weighing 250 kN at speed 15 kmph= 4.16 m/s is converted into potential energy of the four springs:

$$4\times\frac{1}{2}k\times 0.30^2 = \frac{1}{2}\times\frac{210\times 10^3}{9.81}4.16^2$$
$$k = 1.02 \text{ MN/m}$$

Given that

$$G = 85\times 10^9 \text{ Pa}$$
$$R = 0.11 \text{ m}$$
$$n = 10 \text{ m}$$

The diameter of the spring wire is given by

$$k = \frac{Gd^4}{64R^3 n}$$
$$d = \sqrt[4]{\frac{64R^3 nk}{G}}$$
$$= 56.5 \text{ mm}$$

10. Given that

$$G = 80\times 10^9 \text{ Pa}$$
$$d = 0.005 \text{ m}$$
$$R = 0.015 \text{ m}$$
$$n = 10$$

The spring constant (stiffness) is determined as

$$k = \frac{Gd^4}{64R^3 n}$$
$$= 23.148 \text{ kN/m}$$

If δ is the spring deflection, the work done by the weight will be stored into potential energy of the spring, therefore

$$50\,(0.15+\delta) = \frac{1}{2}k\delta^2$$
$$7.5+50\delta = 11.574\times 10^3\delta^2$$

Therefore,

$$\delta = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{50\pm 591.4}{2\times 11.574\times 10^3}$$
$$= 0.0277 \text{ m}$$
$$= 27.7 \text{ mm}$$

The equivalent static load is given by

$$P = k\delta$$
$$= 641.20 \text{ N}$$

Maximum shear stress is determined a

$$\tau = \frac{16PR}{\pi d^3}\left(1+\frac{d}{4R}\right)$$
$$= 424.53 \text{ MPa}$$

11. Given that

$$E_s = 180\times 10^9 \text{ Pa}$$
$$E_c = 18\times 10^9 \text{ Pa}$$

The axial stresses in the steel bar and concrete σ_s and σ_c are related to axial deformation of the post as

$$\frac{\sigma_s\times l}{E_s} = \frac{\sigma_c\times l}{E_c}$$
$$\sigma_s = 10\sigma_c$$

The cross-sectional area of steel and concrete are, respectively,

$$A_s = 6\times 600$$
$$= 3600 \text{ mm}^2$$
$$A_c = 200^2 - 6\times 600$$
$$= 36400 \text{ mm}^2$$

The net load of 1000 kN is to bear by both the portions of the post, therefore,

$$\sigma_s A_s + \frac{\sigma_s}{10}A_c = 12000\times 10^3$$
$$\sigma_s\left(A_s+\frac{A_c}{10}\right) = 12000\times 10^3$$
$$\sigma_s = 110.5 \text{ MPa}$$

Axial stress in the concrete is given by

$$\sigma_c = \frac{\sigma_s}{10}$$
$$= 11.05 \text{ MPa}$$

12. Let F_s and F_b are the loads shared by the steel and bronze rods respectively. As the rod is rigid, therefore, the deformation of steel and bronze rods are related as

$$\begin{aligned}\frac{\delta_s}{0.5} &= \frac{\delta_b}{1.5}\\ \frac{F_s\times 1.0}{500\times 180} &= \frac{0.5}{1.5}\times\frac{F_b\times 1.0}{200\times 80}\\ F_s &= 1.875F_b\end{aligned}$$

For equilibrium, taking moments about hinged point A,

$$\begin{aligned}40\times 2.5 &= F_s\times 0.5+F_b\times 1.5\\ F_b &= \frac{40\times 2.5}{0.5\times 1.875+1.5}\\ &= 41.02\text{ kN}\\ F_s &= 1.875F_b\\ &= 76.923\text{ kN}\end{aligned}$$

The stresses in steel and bronze rods are given by

$$\begin{aligned}\sigma_s &= \frac{76.923\times 10^3}{500\times 10^{-6}}\\ &= 153.85\text{ MPa}\\ \sigma_b &= \frac{41.02\times 10^3}{200\times 10^{-6}}\\ &= 205.06\text{ MPa}\end{aligned}$$

13. Given that

$$\begin{aligned}l &= 5\text{ m}\\ A &= 600\text{ mm}^2\\ \Delta T &= 25-(-10)\\ &= 30^\circ\text{ C}\\ \alpha &= 12\ \mu\text{m/m}^\circ\text{C}\\ E &= 180\times 10^9\text{ Pa}\end{aligned}$$

The thermal stress in the rod is given by

$$\begin{aligned}\sigma_t &= E\alpha\Delta T\\ &= 64.8\text{ MPa}\end{aligned}$$

The reduction of stress due to spring of 0.5 mm is given by

$$\begin{aligned}\sigma &= 180\times 10^9\times\frac{0.2\times 10^{-3}}{5}\\ &= 7.20\text{ MPa}\end{aligned}$$

Thus, balance thermal stress is

$$\begin{aligned}\sigma &= 64.8-7.20\\ &= 57.6\text{ MPa}\end{aligned}$$

14. The moment of inertia of the section is

$$\begin{aligned}I &= \frac{120\times 180^3}{12}\\ &= 58.32\times 10^6\text{ mm}^4\end{aligned}$$

The shear force at the section 1 m from left end is

$$\begin{aligned}V &= \frac{4\times 6}{2}-4\times 1\\ &= 8\text{ kN}\end{aligned}$$

To calculated the shear stress at layer 30 mm from top of the section, area above the layers is $A = 120\times 30$ mm^2 and its centroid is situated at $\bar{y} = 75$ mm from the NA of the beam. Therefore

$$\begin{aligned}A'\bar{y} &= 120\times 30\times 75\\ &= 270\times 10^3\text{ mm}^3\end{aligned}$$

Therefore, the maximum shear stress is

$$\begin{aligned}\tau &= \frac{V}{Ib}A'\bar{y}\\ &= \frac{8}{58.32\times 120}\times 270\\ &= 0.3086\text{ N/mm}^2\\ &= 308.6\text{ kPa}\end{aligned}$$

Maximum value of shear force at the ends, given by

$$\begin{aligned}V &= \frac{4\times 6}{2}\\ &= 12\text{ kN.}\end{aligned}$$

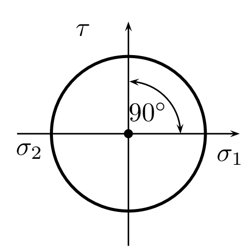
The maximum shear stress occurs at the top lamina of the section, given by

$$\begin{aligned}\tau &= 1.5\frac{V}{bh}\\ &= 833.33\text{ kPa}\end{aligned}$$

15. The inner diameter is 100 mm and outer diameter is 105 mm. The shear stress on the outer plane of the tube is calculated as

$$\begin{aligned}\tau &= \frac{16\times 1000\times 0.105}{\pi\,(0.105^4-0.100^4)}\\ &= 24.81\text{ MPa}\end{aligned}$$

The principal stresses are shown in figure.



The principal stresses are given by

$$\begin{aligned}\sigma_1 &= \tau \\ &= 24.81 \text{ MPa} \\ \sigma_2 &= -\tau \\ &= -24.81 \text{ MPa}\end{aligned}$$

As seen in the figure, the principal planes are oriented at $90/2 = 45°$ from the axis of the tube.

16. The surface of the shaft is under pure shear with shear stress τ and principal stresses equal to τ of the opposite direction. Therefore, the strain will be given by

$$\begin{aligned}\varepsilon &= \frac{\tau}{E} + \mu\frac{\tau}{E} \\ \tau &= \frac{E\varepsilon}{1+\mu} \\ &= \frac{105\times10^9\times3.98\times10^{-4}}{1+0.3} \\ &= 32.146 \text{ MPa}\end{aligned}$$

Torque transmitted by the shaft is

$$\begin{aligned}\tau &= \frac{16T}{\pi d^3} \\ T &= \frac{\pi d^3\tau}{16} \\ &= \frac{\pi\times0.06^3\times32.146\times10^6}{16} \\ &= 1.3633 \text{ kNm}\end{aligned}$$

Power transmitted is

$$\begin{aligned}P &= T\times\frac{2\pi N}{60} \\ &= 1363.3\times\frac{2\pi\times800}{60} \\ &= 114.22 \text{ kW}\end{aligned}$$

17. Let w N/m is the UDL on the beam. Maximum bending moment is

$$M = \frac{wl^2}{8}$$

Given that $\sigma_b = 30$ N/mm^2. Therefore,

$$\begin{aligned}\frac{M}{bd^2/6} &\le \sigma_b \\ w &\le \frac{8\sigma_b bd^3}{6l^2} \\ &= 15 \text{ kN/m}\end{aligned}$$

Let w N/m is the UDL on the beam. Maximum shear force in the beam is

$$F = \frac{wl}{2}$$

Given that $\tau = 3$ N/mm^2. Therefore,

$$\begin{aligned}\frac{F}{(bd^3/12)\times b}\times b\frac{d}{2}\times\frac{d}{4} &\le \tau \\ \frac{12\times(wl/2)}{8bd} &\le \tau \\ w &= \frac{4bd\tau}{3l} \\ &= 30 \text{ kN/m}\end{aligned}$$

18. The reactions at supports are calculated as

$$\begin{aligned}R_C &= \frac{48\times0.5}{1.5} \\ &= 16 \text{ kN} \\ R_A &= 48-16 \\ &= 32 \text{ kN}\end{aligned}$$

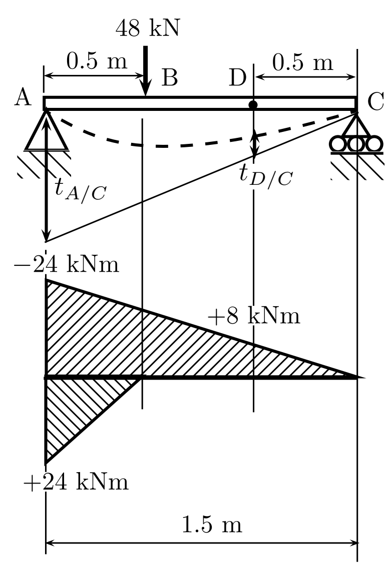
The bending moment will be maximum at the point of application of the concentrated load in the simply supported beam, given by

$$\begin{aligned}M_{max} &= 32\times0.5 \\ &= 16 \text{ kNm}\end{aligned}$$

Maximum tensile stress is calculated as

$$\begin{aligned}\sigma &= \frac{M_{max}}{\frac{50\times100^2}{6}} \\ &= 192 \text{ N/mm}^2\end{aligned}$$

Area of moments and dimensions are shown in the figure



The rigidity of the beam is expressed as

$$\begin{aligned} EI &= 200\times 10^9 \times \frac{0.05\times 0.1^3}{12} \\ &= 833.33\times 10^3 \end{aligned}$$

The deflection at D is calculated as

$$\begin{aligned} EI\delta_D &= \frac{0.5}{1.5} t_{A/C} - t_{D/C} \\ &= \frac{1}{2}\times 24\times 1.5^2 \times \frac{1}{3} - \frac{1}{2}\times 8\times 0.5^2 \times \frac{1}{3} \\ &= 2.667\times 10^3 \text{ Nm}^2 \end{aligned}$$

Hence,

$$\begin{aligned} \delta_D &= \frac{2.667\times 10^3}{EI} \\ &= \frac{2.667\times 10^3}{833.33\times 10^3} \\ &= 3.2 \text{ mm} \end{aligned}$$

19. If T_a Nm is the torque carried by steel portion, then $125 - T_A$ Nm will be that carried by the brass portion. If l is the length of composite shaft, the angles of twist in steel and brass portions are determined as

$$\begin{aligned} \theta_s &= \frac{32T_s\times l}{\pi\,(0.06^4 - 0.04^4)\, 80\times 10^9} \\ \theta_b &= \frac{32\,(100 - T_s)\times l}{\pi\,(0.04^4 - 0.03^4)\, 40\times 10^9} \end{aligned}$$

The angles of twist in the composite shaft (parallel) will be same, therefore

$$\begin{aligned} T_s &= 2\,(125 - T_s)\,\frac{0.06^4 - 0.04^4}{0.04^4 - 0.03^4} \\ T_s &= 11.88\,(125 - T_s) \\ &= 115.35 \text{ Nm} \\ T_b &= 100 - T_s \\ &= 9.64 \text{ Nm} \end{aligned}$$

The shear stresses are calculated as

$$\begin{aligned} \tau_s &= \frac{16\times 115.35\times 0.06}{\pi\,(0.06^4 - 0.04^4)} \\ &= 3.389 \text{ MPa} \\ \tau_b &= \frac{16\times 9.64\times 0.04}{\pi\,(0.04^4 - 0.03^4)} \\ &= 1.22 \text{ MPa} \end{aligned}$$

20. The torque transmitted is determined as

$$\begin{aligned} T\times \frac{2\pi\times 120}{60} &= 300\times 10^3 \\ T &= 23.873 \text{ kNm} \end{aligned}$$

The diameter of the solid shaft is determined as

$$\begin{aligned} \tau &= \frac{16T}{\pi d^3} \\ 100\times 10^6 &= \frac{16\times 23.873\times 10^3}{\pi d^3} \\ d &= 0.1067 \text{ m} \end{aligned}$$

The external diameter (d_o) of hollow shaft with $d_i = 0.6 d_o$ is determined as

$$\begin{aligned} \tau &= \frac{16 T d_o}{\pi\,(d_o^4 - d_i^4)} \\ 100\times 10^6 &= \frac{16\times 23.873\times 10^3}{\pi d_o^3\,(1 - 0.6^4)} \\ d &= 0.11175 \text{ m} \end{aligned}$$

For the same length, the saving in material is

$$\begin{aligned} \%\text{ Saving} &= 1 - \frac{d_o^2 - d_i^2}{d^2} \\ &= 1 - \frac{\left(1 - 0.6^2\right) d_o^2}{d^2} \\ &= 29.79\% \end{aligned}$$

21. The twist and torque in the hollow shaft are related to rigidity as

$$\begin{aligned} \frac{1.6\times 10^3}{\pi\,(0.05^4 - 0.03^4)/32} &= G\frac{0.4\times \pi/180}{0.2} \\ G &= 85.82 \text{ GPa} \end{aligned}$$

The modulus of rigidity (G) is related to modulus of elasticity (E) through the Poisson ratio (μ) as

$$\begin{aligned} G &= \frac{E}{2\,(1+\mu)} \\ 85.82 &= \frac{200}{2\,(1+\mu)} \\ \mu &= 0.165 \end{aligned}$$

CHAPTER 3

THEORY OF MACHINES

Machine is a mechanism that consists of fixed and moving parts and modifies mechanical energy to assist in performing of human tasks. *Theory of machines* is an applied science of the relationships between geometry and relative motion of the parts of machine, and concerns to the forces which act on those parts. It involves analysis of as well as synthesis. Analysis is the study of motions and forces concerning different parts of an existing mechanism, whereas synthesis involves design of different parts. The study of mechanisms, therefore, is divided into kinematics and dynamics. The branch of *kinematics* deals with the relative motions of different parts of a mechanism without taking into account the forces producing the motions, thus it is solely based on geometric point of view. *Dynamics* involves determination of forces impressed upon different parts of a mechanism. It has sub-branches of kinetics and statics. *Kinetics* is study of forces when the body is in motion whereas *statics* deals with forces when the body is stationary.

3.1 MECHANISMS AND MACHINES

Mechanisms and machines are composed of kinematic links and pairs. Mechanisms modify the external force and deliver some advantage in motion and force. For this, a term *mechanical advantage* is defined for a mechanism as the ratio of the output force (or torque) to the input force (or torque). The power input and output remain the same during such modification.

3.1.1 Rigid and Resistant Bodies

A *rigid body* does not suffer any distortion under the action of external forces. *Resistant bodies* are semi-rigid bodies, normally flexible but under certain loading conditions, act as rigid bodies. Resistant bodies constitute parts of machines through which requisite motion and forces are transmitted, such as belts, fluids, springs. For example, a belt is rigid when subjected to tensile force. Similarly, fluid acts as rigid body in hydraulic press during compression. Same is the case with springs.

3.1.2 Kinematic Links

Kinematic link is a resistant body or an assembly of resistant bodies which go on to make a part of a machine and enable modification and transmission of mechanical work through the relative motion between the parts. Each link or element can consist of several parts which are manufactured as separate units. A link need not necessarily be a rigid body, but it must be a resistant body.

Depending upon the number of joints on which turning pairs can be placed, kinematic links are classified

as *binary*, *ternary*, *quaternary* links, which have 2, 3, 4 joints, respectively [Fig. 3.1].

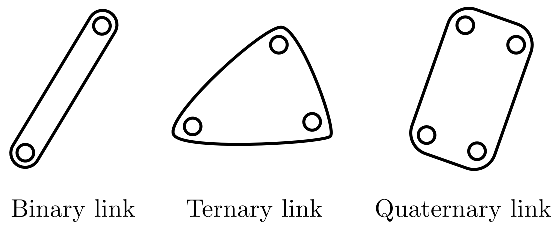


Figure 3.1 Types of links.

A *structure* only modifies and transmits forces without resulting in any work. The simplest structure is one with three bars.

3.1.3 Kinematic Pairs

Two links when connected together in such a way that their relative motion is completely or successfully constrained, constitute a *kinematic pair*.

3.1.3.1 Types of Constrained Motions A kinematic pair can be constituted by various means which decide the type of relative motion between the links. If this relative motion is one and only type, then it is said to be *constrained motion*. This can be of three types:

1. Completely Constrained Motion A *completely constrained motion* takes place in one definite direction. The motion is complete by its own links. For example, a square bar can only slide in a square slot.

2. Incompletely Constrained Motion If motion of a link is possible in more than one direction and governed by the direction of force, the motion is called *incompletely constrained motion*. For example, a circular bar can rotate and reciprocate in a round hole.

3. Successfully Constrained Motion When incompletely constrained motion is made to be only one direction by using some external means, it is called *successfully constrained motion*. For example, the vertical motion of a shaft in footstep bearing is constrained by load upon it while it can undergo rotation only. Similarly, the rotatory motion of a piston inside the cylinder is constrained by a piston pin.

3.1.3.2 Classification of Kinematic Pairs Kinematic pairs are classified according to the nature of relative motion, contact, and constraint. This is explained as follows:

1. Nature of Relative Motion Various types of kinematic pairs are explained as follows [Fig. 3.2]:

   (a) *Sliding Pair* - If two links of a pair are connected in such a way that they can have only sliding motion, they are called *sliding pair*, such as piston-cylinder, ram-guide [Fig. 3.2]. A sliding pair has a completely constrained motion.

   (b) *Turning Pair* - If two links of a pair are connected in such a way that they can have only turning motion, they are called *turning pair*, such as a shaft with collars at both ends fitted into a circular hole, and the crankshaft in a journal bearing. A turning pair has a completely constrained motion.

   (c) *Rolling Pair* - When a link of a pair has a rolling motion relative to other, the pair is called *rolling pair*, or *cylindrical pair*, for example, a rolling wheel on a flat surface, pulley in a belt drive. The ball-bearing shaft constitute a very interesting example in which balls of bearing make rolling pair with both the shaft and the bearing.

   (d) *Screw Pair* - If a link of a pair has a turning as well as sliding motion relative to the other, the pair is called *screw pair*. The lead screw and nut of a lathe machine constitute screw pair.

   (e) *Spherical Pair* - When one link with spherical interface turn inside another link, it forms *spherical pair*, for example, ball and socket joint.

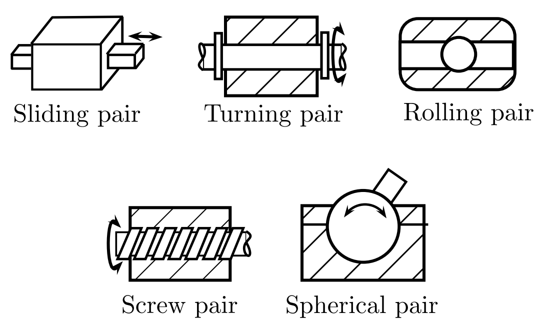


Figure 3.2 Relative motion in kinematic pairs.

2. Nature of Contact By virtue of the nature of contact, kinematic pairs can be two types:

   (a) *Lower Pair* - If the two links in a pair have surface contact while in motion, the pair so formed is called a *lower pair*. The relative motion is purely sliding or turning, and the contact surface is similar in both links, for example, shaft revolving in a bearing, steering gear mechanism, universal coupling.

(b) *Higher Pair* - If the two links in a pair have point or line contact while in motion, the pair so formed is known as a *higher pair*. The contact surface of the two links is dissimilar. For example, cam and follower mechanism, toothed gears, ball and roller bearings.

3. Nature of Constraint The concept of closed and unclosed kinematic pairs is explained as follows [Fig. 3.3]:

(a) *Closed Pair* - If the elements of a pair are held together mechanically, it is known as a *closed pair*. The contact between the two elements can be broken only by destruction of at least one of them. All lower pairs and some of the higher pairs are closed pairs.

(b) *Unclosed Pair* - If the elements of a pair are not held together mechanically, instead, either due to force of gravity or some spring action, it is called *unclosed pair*. For example, cam and follower.

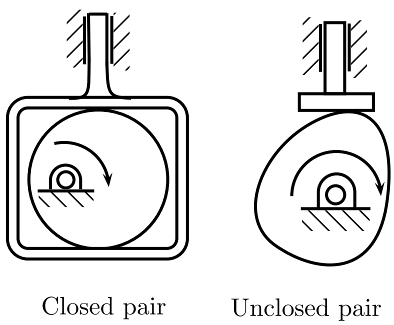


Figure 3.3 Nature of constraints.

3.1.4 Kinematic Chains

A *kinematic chain* is a combination of kinematic pairs in which each link constitutes two pairs and its relative motion is completely constrained. Thus, a kinematic chain has a single degree of freedom [*Section* 3.1.9]. A *redundant chain* does not allow any motion of a link relative to the other.

A simplest kinematic chain is a four bar chain. A chain having more than four links is called *compound kinematic chain.*

3.1.5 Inversions of Kinematic Chain

Primary function of a mechanism is to transmit or to modify motion and it can work as a machine. Different types of motions are possible from a given mechanism by fixing one of its kinematic links. The mechanisms obtained in this way can be very different in appearance and in the purposes for which they are used. Each mechanism is termed as the *inversion* of the original kinematic chain.

As many inversions are possible as the number of links in the mechanism. Inversion has no effect on relative motion between links of the mechanism, but changes the absolute motion.

3.1.6 Four-Bar Chains

A *four-bar mechanism* consists of four bars joined together in closed series with pin joints [Fig. 3.4]. The four links can have different lengths solely depending on the purpose of inversions. Let l_1, l_2, l_3, l_4 be the lengths

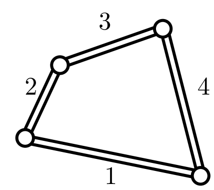


Figure 3.4 Four-bar chain mechanism.

of links of the four-bar mechanism in ascending order ($l_1 < l_2 < l_3 < l_4$). Based on the lengths of the fixed and free links, the inversions of this four-bar mechanism can be divided into two groups:

1. Class I According to *Grashof's law*, if there is to be continuous relative motion or rotation between two links, the sum of the shortest and largest links of a planar four-bar linkage cannot be greater than the sum of remaining two links.

$$l_1 + l_4 \leq l_2 + l_3$$

This type of mechanism is called of class-I, which comprises the following mechanisms:

(a) *Crank Lever Mechanism* - By making the largest link as crank (adjacent link fixed).

(b) *Drag Link Quick Return Mechanism* - By fixing the shortest link.

(c) *Double Crank Mechanism* - By having opposite links of equal length and fixing the shortest link.

2. Class II This inversion is opposite to Class-I in reference to Grashof's law.

$$l_1 + l_4 > l_2 + l_3$$

Therefore, there is no continuous rotation between the two links, and the resulting mechanism is a

rocker-rocker mechanism or *double rocker mechanism*.

Therefore, in a four-bar chain, for the link adjacent to the (fixed) short link to be a crank, the sum of the shortest and the longest links should be less than the sum of the other two links.

Various inversions of a four-bar chain are described as follows:

1. Crank-Lever Mechanism A *crank-lever mechanism*, also known as *crank-rocker mechanism* is obtained by fixing the largest link of the four-bar chain.

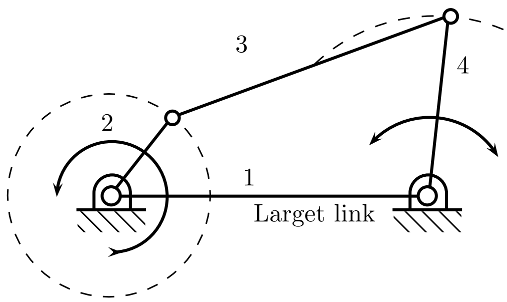


Figure 3.5 Crank-lever mechanism.

The crank rotates full revolution but the follower can only oscillate [Fig. 3.5].

2. Drag-Link Mechanism *Drag link mechanism* is a *quick-return mechanism* having a complete revolution of crank and follower. It is obtained by fixing the shortest link [Fig. 3.6].

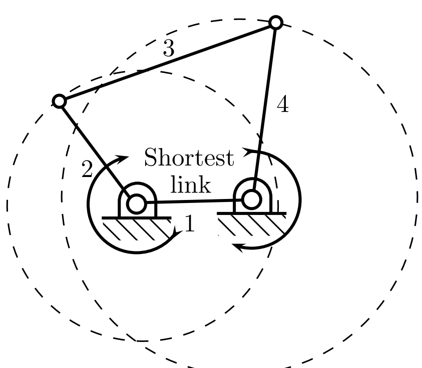


Figure 3.6 Drag-link mechanism.

3. Double Crank Mechanism A *double crank mechanism* consists of two cranks, as seen in following applications:

(a) *Couple Wheel Locomotive* - In coupled wheel locomotive mechanism, the opposite links are of equal lengths:

$$l_1 = l_3$$
$$l_2 = l_4$$

(b) *Pantograph* - A pantograph is used to produce a path described by a point either to an enlarged or reduced scale [Fig. 3.7].

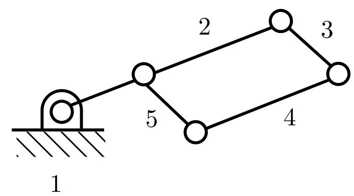


Figure 3.7 Pantograph.

This mechanism has a peculiarity in that instead of one fixed link, only a point is fixed as a pivot while input motion is given by moving a point on some link along some given planar curve. Applications of pantograph include profile grinding, indicator rig, etc.

(c) *Parallelogram* - In parallelograms also, the opposite links are of the same length.

3.1.7 Slider Crank Mechanism

A *slider crank mechanism* is a modification of the four-bar chain in which a turning pair is replaced by sliding pair; the mechanism consists of one sliding pair and three turning pairs [Fig. 3.8]. It is used to convert rotary motion into reciprocating motion and vice versa in reciprocating machines.

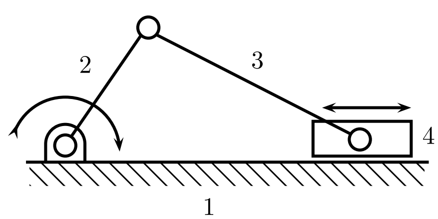


Figure 3.8 Slider-crank mechanism.

The inversions of slider-crank mechanism and their applications are described as follows:

1. Frame-Fixed In this inversion, link 1 (frame) is fixed and an adjacent linkage is made crank. Following are the important applications of this inversion:

(a) Reciprocating engines
(b) Reciprocating compressors

2. <u>*Crank-Fixed*</u> Fixing of link 2 (crank) makes link 3 (connecting rod) to rotate about joint 2-3. The inversion applies in the following applications:

(a) *Whitworth Quick Return Mechanism* - This mechanism is used in metal cutting machines in which forward stroke takes a little longer and cuts the metal, whereas the return stroke is idle and takes a shorter period [Fig. 3.9].

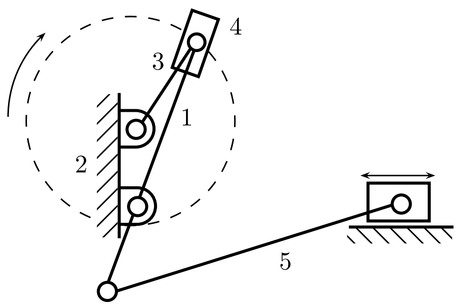


Figure 3.9 | Whitworth quick return motion.

(b) *Rotary Engine* - This mechanism is obtained by replacing the slider by a piston and making link 1 (frame) to act as pivoted cylinder. Instead of one cylinder, seven, or nine cylinders symmetrically placed at regular intervals in the same plane are used. All the cylinders rotate about the same axis and form a balanced system.

3. <u>*Connecting Rod-Fixed*</u> This mechanism is obtained by fixing the link 3 (connecting rod), which makes link 1 to oscillate about the joint of 2-3. The following are the important applications of this inversion:

(a) *Oscillating Cylinder Engine* - In this application, the piston reciprocates inside the cylinder pivoted to the fixed link.
(b) *Slotted Lever-Crank Mechanism* - If the cylinder is made to work as a guide, and the piston in the form of a slider, it results into the slotted lever-crank mechanism [Fig. 3.10].

4. <u>*Slider-Fixed*</u> If link 4 (slider) is fixed, it makes link 3 (connecting rod) to oscillate about fixed pivot 1-2. Inversion by fixing the slider is applied in hand pump.

3.1.8 Double Slider Crank Mechanism

It is possible to replace two turning pairs by two sliding pairs of four-bar mechanism to get a *double slider crank mechanism* [Fig. 3.11].

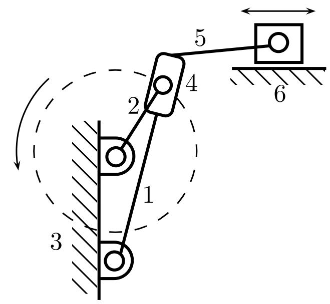


Figure 3.10 | Slotted lever-crank mechanism.

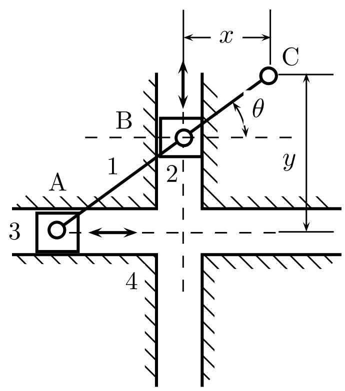


Figure 3.11 | Double slider crank mechanism.

Following are inversions and their applications of this mechanism:

1. <u>*Frame-Fixed*</u> The inversion obtained by fixing the frame is seen in *elliptical trammel* [Fig. 3.11]. With the movement of the sliders, any point on the (connecting) link (3), except mid-point, traces an *ellipse* on a fixed plate. The mid-point of the link 3 traces a circle. To examine, let the link (3) AB makes an angle θ with the x-axis. Considering the displacements of the sliders from the center-line of the trammel,

$$x = \mathrm{BC}\cos\theta$$
$$y = \mathrm{AC}\sin\theta$$

Therefore,

$$\frac{x}{\mathrm{BC}} = \cos\theta$$
$$\frac{y}{\mathrm{AC}} = \sin\theta$$

Squaring and adding

$$\frac{x^2}{\mathrm{BC}^2} + \frac{y^2}{\mathrm{AC}^2} = 1 \tag{3.1}$$

This is an equation of ellipse, indicating that the path traced by point C is an ellipse which has its

semi-major and semi-minor axes equal to AC and BC, respectively.

For the special case, when C is the mid-point of AB, AC = BC, and Eq. (3.1) transforms into the equation of a circle of diameter equal to the length of the link.

2. *Slider-Fixed* The inversion achieved by fixing either of the sliders is applied in *scotch yoke* which is used to convert the rotary motion into a sliding motion [Fig. 3.12].

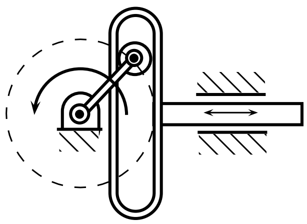


Figure 3.12 Scotch yoke.

This mechanism is most commonly used in control valve actuators in high pressure oil and gas pipelines. As the connecting link AB rotates like a crank, the horizontal portion of the link (frame) reciprocates in the fixed link (any one of the sliders).

3. *Crank-Fixed* The inversion achieved on crank fixed is applied in *Oldham Coupling* which is used to join two rotating parallel shafts at a small distance [Fig. 3.13].

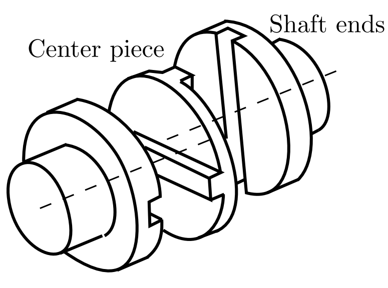


Figure 3.13 Oldham coupling.

Two flanges, each having a rectangular slot, are keyed, one on each shaft. The two flanges are positioned such that the slot in one is at right angle to the slot in the other. All the rotating elements have the same angular velocity at every instant. The path followed by the center of intermediate piece is circle of diameter equal to distance between the two parallel shafts.

Maximum sliding velocity of the intermediate piece is equal to peripheral velocity of the center of the disc along its circular path (i.e., $c \times \omega$ where c is parallel distance and ω is angular velocity of shafts). Angular velocity of the center of cross about the center of its circle is two times that of angular velocity of the cross.

3.1.9 Degrees of Freedom

The number of *degrees of freedom* or *mobility* of a system is the number of independent variables that must be specified to completely define the condition of the system. Same applies to rigid body, kinematic pair, and kinematic chain, discussed as follows:

1. *Rigid Body* Degree of freedom for a rigid body is defined as the number of possible motions in which the body can move in the given space.

   An unconstrained rigid body in space can be described in six independent motions:

   (a) Translation along x, y, z axes

   (b) Rotation about x, y, z axes

   Thus, a rigid body possesses six degrees of freedom. Connection with other bodies through pairs impose certain constraints on the relative motion, hence, the number of degrees of freedom is reduced.

2. *Kinematic Pair* Degree of freedom of a pair is defined as the number of independent relative motions a pair can have in the given space.

3. *Kinematic Chain* To obtain constrained or definite motions of the links of a mechanism, it is necessary to know how many inputs are required to specify the position of the mechanism. In some mechanisms, only one input is necessary that determines the motion of other links, and it is said to have one degree of freedom. The degree of freedom of a structure is zero. A structure with negative degree of freedom is known as a *super-structure*.

   The following are useful equations to calculate degrees of freedom for kinematic chains.

   (a) *Gruebler's Equation* - A rigid link in a plane has three degrees of freedom; an planar assembly having n links shall posses total degrees of freedom $3n$, before the links are joined together. Each revolutionary pair or joint will remove two degrees of freedom (e.g. x_i, y_i). Thus, if n is number of links of a mechanism including fixed links, f_1 is the number of pin joints or revolute pairs or pairs that permit one degree of freedom (i.e. the number of pint joints plus the number of sliding

pairs or total number of lower pairs), then, degrees of freedom, say F, are found as

$$F = 3(n-1) - 2f_1 \tag{3.2}$$

This is called *Gruebler's equation*.

(b) *Kutzbach Equation* - When there are such pairs which remove only one degree of freedom (f_2, number of roll sliding pair or total number of higher pairs), then the Gruebler's equation [Eq. (3.2)] is modified into

$$F = 3(n-1) - 2f_1 - f_2 \tag{3.3}$$

This is called the *Kutzbach equation*. Alternative form of this expression can be obtained in terms of number of binary joints J, number of higher pairs f_2, for n number of links as

$$J + \frac{f_2}{2} = \frac{3}{2}n - 2 \tag{3.4}$$

or

$$n = \frac{2}{3}\left(J + 2 + \frac{f_2}{2}\right) \tag{3.5}$$

In the above equation, one ternary joint is equivalent to two binary joints, and one quaternary joint is equivalent to three binary joints [*Section* 3.1.2].

3.2 UNIVERSAL JOINT

A *universal joint* is used to connect two non-parallel and intersecting shafts and misaligned shafts, for example, to transmit power from the gearbox to rear axle in an automobile. Universal joint is also known as *Hooke's joint*[1].

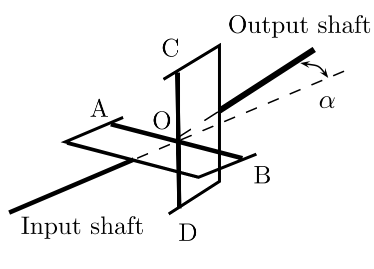


Figure 3.14 | Hooke's joint.

Consider a universal joint, connecting two horizontal shafts 1 and 2 at an angle α [Fig. 3.14]. The shafts are supported on the bearings. Each shaft has a fork at its end. The four ends of the two forks of shafts are connected by a center piece, the right angle arms of which rest in the bearings, provided in the fork ends of both shafts.

[1]During 1667-1675, Robert Hooke analyzed the joint and found that its speed of rotation was non-uniform. The first recorded use of the term universal joint for this device was by Hooke.

3.2.1 Shaft Rotations

Let θ be the (absolute) angle rotated by shaft-1 (driver), ϕ be the (absolute) angle rotated by shaft-2 (driven). Using projections of the fork ends, the following relation can be found between the angle moved by the two shafts:

$$\frac{\tan\phi}{\tan\theta} = \sec\alpha \tag{3.6}$$

This is the basic equation for the rotation of shaft-2 with respect to that of shaft-1. This equation will be utilized in deriving expressions for speed ratio and acceleration of the shaft-2.

3.2.2 Shaft Speeds

3.2.2.1 Speed Ratio Differentiating Eq. (3.6) w.r.t. time t

$$\begin{aligned}
\sec^2\phi\frac{d\phi}{dt}\cos\alpha &= \sec^2\theta\frac{d\theta}{dt} \\
\frac{\omega_2}{\omega_1} &= \frac{d\phi/dt}{d\theta/dt} \\
&= \frac{\sec^2\theta}{\sec^2\phi\cos\alpha} \\
&= \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}
\end{aligned}$$

The following are the three interesting situations where the expression of ω_2/ω_1 can be described with respect to the angular position of shaft-2 (θ):

1. $\omega_2 = \omega_1$:

$$\tan\theta = \pm\sqrt{\cos\alpha} \tag{3.7}$$

2. Minimum ω_2/ω_1:

$$\sin^2\theta = 0$$
$$\theta = 90^\circ, 270^\circ \tag{3.8}$$
$$\left(\frac{\omega_2}{\omega_1}\right)_{\min} = \cos\alpha \tag{3.9}$$

3. Maximum ω_2/ω_1:

$$\cos^2\theta = 1$$
$$\theta = 0^\circ, 180^\circ \tag{3.10}$$
$$\left(\frac{\omega_2}{\omega_1}\right)_{\max} = \frac{1}{\cos\alpha} \tag{3.11}$$

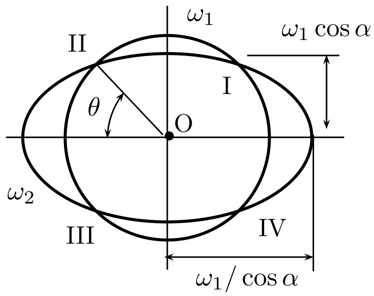


Figure 3.15 Velocities of shafts in Hooke's joint.

3.2.2.2 Speed Variation

The variation in the speed of the driven shaft ω_2 w.r.t. ω_1 can be drawn in the *polar velocity diagram* for a given value of α [Fig. 3.15].

In the diagram, ω_1 is an exact circle but ω_2 is an ellipse. The maximum variation in the velocity of driven shaft w.r.t. its mean velocity ω_1 is

$$\begin{aligned}\frac{\omega_{\max}-\omega_{\min}}{\omega_1} &= \frac{1}{\cos\alpha} - \cos\alpha \\ &= \frac{\sin^2\alpha}{\cos\alpha} \\ &= \tan\alpha\sin\alpha\end{aligned}$$

For small values of α in rad,

$$\tan\alpha \approx \sin\alpha \approx \alpha^2$$

therefore,

$$\frac{\omega_{\max}-\omega_{\min}}{\omega_1} \propto \alpha^2$$

3.2.3 Angular Acceleration

Angular acceleration of the driven shaft is

$$\begin{aligned}\dot{\omega}_2 &= \frac{d\omega}{dt} \\ &= \omega_1 \frac{d}{dt}\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}\right) \\ &= \frac{-\omega_1{}^2\cos\alpha.\sin^2\alpha.\sin 2\theta}{\left(1-\sin^2\alpha.\cos^2\theta\right)^2}\end{aligned}$$

For minimum or maximum $\dot{\omega}_2$,

$$\cos 2\theta \approx \frac{2\sin^2\alpha}{2-\sin^2\alpha} \tag{3.12}$$

As is evident in Fig. 3.15, the acceleration is maximum in II and IV quadrants and minimum in I and III quadrants. The acceleration is zero at four values of θ:

$$\begin{aligned}\sin 2\theta &= 0 \\ \cos 2\theta &= 1 \\ \theta &= 0^\circ, 90^\circ, 180^\circ, 270^\circ\end{aligned}$$

The angle between the two shafts α should be kept as minimum as possible and excessive masses should not be attached to the driven shaft, otherwise, very high alternating stresses due to angular acceleration and retardation will be set up in the parts of the joint, which are undesirable.

3.2.4 Double Hooke's Joints

A *double Hooke's joint* can be obtained by joining two Hooke's joints using an intermediate shaft. If the misalignment between each shaft and the intermediate shaft is equal, the driving and the driven shafts remain in exact angular alignment, though the intermediate shaft rotates with varying speed.

In a double Hooke joint, the angular arrangement of the two Hooke's joints decides the velocity ratio at any instant. It is immaterial whether the driven shaft makes the angle with the axis of driving shaft to its left or right. The velocity ratio depends upon the position of forks, as explained as follows:

1. *Constant Velocity Ratio* The constant velocity ratio is achieved when driving and driven shafts make an equal angle with the intermediate shaft and forks of intermediate shaft lie in the same plane. This is the reason why double hook joints are used, because then dynamic stresses are reduced to zero.
2. *Varying Velocity Ratio* If the above condition is not satisfied, then the speed ratio varies between maximum and minimum values, given by

$$\begin{aligned}\left(\frac{\omega_2}{\omega_1}\right)_{\max} &= \frac{1}{\cos^2\alpha} \\ \left(\frac{\omega_2}{\omega_1}\right)_{\min} &= \cos^2\alpha\end{aligned}$$

Double Hooke's joints are used in connecting two drive-shafts.

3.3 KINEMATIC ANALYSIS

Each *kinematic link* of a machine has a relative motion in a definite path, which can be either straight, circular or curved. Kinematic analysis usually aims at determining motion characteristics of various links in a mechanism. Such information is essential for computing forces and thereby dimensions of the links, enabling design of various links in a mechanism.

Two types of methods are available for kinematic analysis: graphical method and analytical method. Graphical methods are essential in developing a conceptual

understanding about the subject. They provide the fastest method of checking the results, though less accurate.

3.3.1 Velocity of a Link

In kinematic analysis, all the motions are measured relative to some reference axes or planes. Usually, the earth is taken to be a fixed reference plane, and all such motions relative to it are termed as *absolute motion*. When motion of a body is measured with respect to another body, in motion or steady state, it is called *relative motion*. In mechanism, the motion of a link can be measured with respect to fixed points as well as moving points on the links.

3.3.1.1 Linear Velocity Let a rigid link OA of length r rotate about a fixed point O with a uniform velocity ω. In a small interval of time δt, the link turns through a small angle $\delta\theta$ and the point A moves to new location A′ [Fig. 3.16]. Displacement of point A equal to $r\delta\theta$, therefore, *linear velocity* of the point A relative to point O is defined as

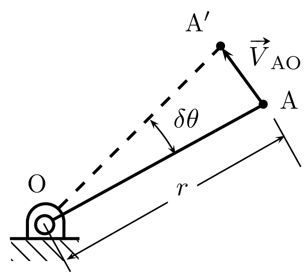


Figure 3.16 Linear velocity of a link.

$$V_{\mathrm{AO}} = \lim_{\delta t\to 0} \frac{r\delta\theta}{\delta t}$$

$$= r \lim_{\delta t\to 0} \frac{\delta\theta}{\delta t}$$

$$= r\omega_{\mathrm{AO}} \tag{3.13}$$

where ω_{AO} represents the angular velocity of the link, given by

$$\omega_{\mathrm{AO}} = \lim_{\delta t\to 0} \frac{\delta\theta}{\delta t} \tag{3.14}$$

The direction of V_{AO} is along the displacement of A. When $\delta t \to 0$, AA′ will be perpendicular to OA. This emerges from the fact that A can neither approach nor recede from O and thus, the only possible motion of A relative to O is in a direction perpendicular to OA.

The magnitude of the instantaneous linear velocity V_{AO} ($=r\omega_{\mathrm{AO}}$) of a point on a rotating body is proportional to its distance from the axis of rotation:

$$V_{\mathrm{AO}} \propto r$$

Therefore, velocity of an intermediate point B on the link OA [Fig. 3.17] can be found as

$$\frac{V_{\mathrm{BO}}}{V_{\mathrm{AO}}} = \frac{\mathrm{BO}}{\mathrm{AO}}$$

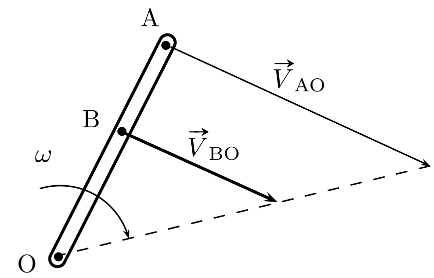


Figure 3.17 Velocity of an intermediate point.

Consider a case of link AB in which absolute velocities of points A and B are $\vec{V}_{\mathrm{A}}$ and $\vec{V}_{\mathrm{B}}$, respectively [Fig. 3.18]. The *relative velocity* of point B with respect to A is given by

$$\vec{V}_{\mathrm{BA}} = \vec{V}_{\mathrm{B}} - \vec{V}_{\mathrm{A}}$$

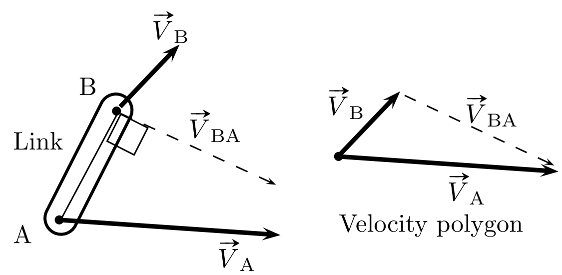


Figure 3.18 Relative velocity in a link.

If $\vec{V}_{\mathrm{A}}$ is completely known, but $\vec{V}_{\mathrm{B}}$ is known in direction (the tangent to the path followed B) only, then magnitude of $\vec{V}_{\mathrm{B}}$ is also determined because direction of $\vec{V}_{\mathrm{BA}}$ will be perpendicular to the link AB. This observation is used in graphical methods.

A *vector polygon* is a graphical depiction of vector equations of velocities of two or more points in a link or mechanism. It contains information about the magnitude and direction of velocities [Fig. 3.18].

3.3.1.2 Angular Velocity *Angular velocity* of a link is defined as the linear velocity divided by its radius. Similar to Eq. (3.14), in reference to the previous case of Fig. 3.18, angular velocity of link AB is given by

$$\omega_{\mathrm{AB}} = \frac{V_{\mathrm{AB}}}{\mathrm{AB}}$$

3.3.1.3 Instantaneous Center For two bodies having relative motion with respect to one another, instantaneous center of rotation is an imaginary point common to the two bodies such that any of the two bodies can be assumed to have motion of rotation with respect to the other about the imaginary point. In general, instantaneous center of rotation is not a stationary point since as the body moves from one position to another, the velocities of their points keep changing.

Let a point A on a rigid body have a linear velocity $\vec{V}_{\mathrm{A}}$ and the body itself has an angular velocity $\vec{\omega}$ [Fig. 3.22]. These two quantities completely define the velocities of all points or particles in the rigid body. To appreciate this sentence, let a perpendicular be erected at point A to $\vec{V}_{\mathrm{A}}$ and the distance $r_{\mathrm{A}}{=}V_{\mathrm{A}}/\omega$, measured along it to locate instantaneous center 'I' of the body [Fig. 3.19].

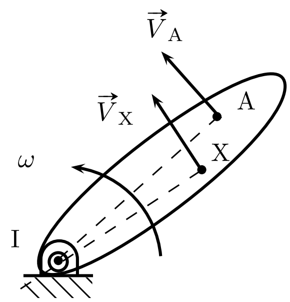


Figure 3.19 | Instantaneous center.

The magnitude of linear velocity of any other point X, at any given instant, will then be given as

$$V_{\mathrm{X}} = \mathrm{IX} \times \omega$$

Based on the above equation, the instantaneous center (I) is the intersection point of the normals of velocities at any two points in the body.

Instantaneous center between the two links (say, 2 and 3) is denoted by I_{23}.

3.3.1.4 Number of Instantaneous Centers In a mechanism, the number of instantaneous centers is the number of possible combinations of two links. A mechanism, having n links, will have the number of instantaneous centers given by

$$\begin{aligned} N &= {}^{n}C_{r} \\ &= \frac{n\,(n-1)}{2} \end{aligned}$$

Arnold Kennedy's theorem states that three bodies, having relative motion with respect to one another, have three instantaneous centers, all of which lie on the same straight line.

When extended to kinematic chains, Arnold Kennedy's theorem implies that with every combination of three links, there are three I's and if two of them are known, the third one will lie on the line joining them. This concept can be used to locate the instantaneous centers of mechanism, for example, four-bar mechanisms [Fig. 3.20] and slider-crank mechanism [Fig. 3.21].

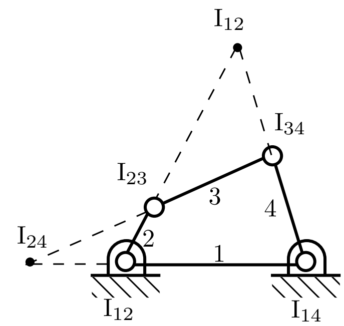


Figure 3.20 | Four-bar mechanisms.

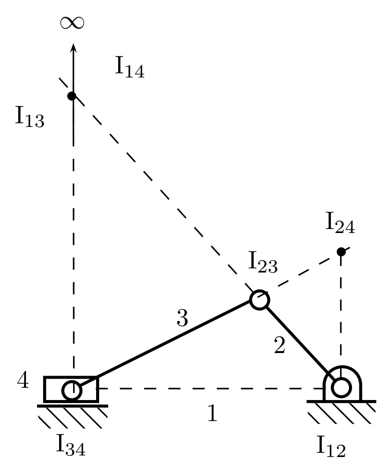


Figure 3.21 | Slider-crank mechanism.

Few simple examples of the location of the instantaneous centers are illustrated in Fig. 3.22.

3.3.2 Acceleration in Mechanism

Acceleration is an important aspects in the design of mechanisms as it directly indicates the inertia forces of the members. *Linear acceleration* is defined as the rate of change of *linear velocity* with respect to time:

$$\vec{a} = \frac{d\vec{V}}{dt}$$

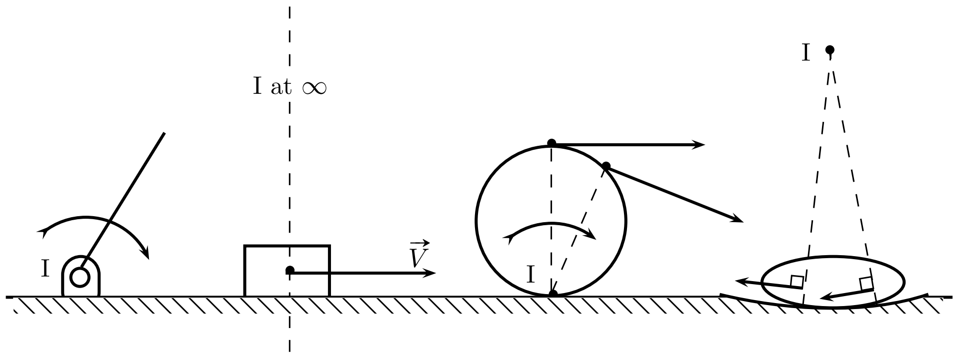


Figure 3.22 Examples of locating I.

Similarly, *angular acceleration* is defined as the rate of change of angular velocity with respect to time,

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

Angular acceleration of the particle, having a circular motion of radius r and angular velocity ω, is given by

$$\alpha = r\omega^2$$

If a link OA moves with angular velocity ω_{AO} [Fig. 3.13] then the angular acceleration of point A on the link w.r.t. O is given by

$$\alpha_{\mathrm{AO}} = \mathrm{AO} \times {\omega_{\mathrm{OA}}}^2$$

When a slider moves over a link which itself is moving, the acceleration of the slider involves an important component known as *Coriolis component*[2]. It is the tangential component of acceleration of a slider with respect to the coincident point on the link. To examine this, let a link OA rotate about a fixed point O. Let ω, α represent the angular velocity, angular acceleration of OA, respectively, v, f represent linear velocity and linear acceleration of a point P on the slider of the link at radial distance r from the center O [Fig. 3.23]. The components of acceleration of a point P on the slider can be determined as follows:

1. Acceleration Along OA The change in velocity of point P along the rotating link OA is given by

$$\delta v_P^{\parallel} = \{(v + f\delta t)\cos\delta\theta - (\omega + \alpha\delta t)r'\sin\theta\} - v$$

With limiting case of $\delta t \to 0$, $\cos\delta\theta \to 1$, $\sin\delta\theta \to 0$. Hence, acceleration of P along OA is

$$a_P^{\parallel} = \lim_{\delta t \to 0} \frac{\delta v_P^{\parallel}}{\delta t} = f - \omega^2 r \qquad (3.15)$$

[2]The mathematical expression for the Coriolis component appeared in an 1835 paper by French scientist Gaspard-Gustave Coriolis.

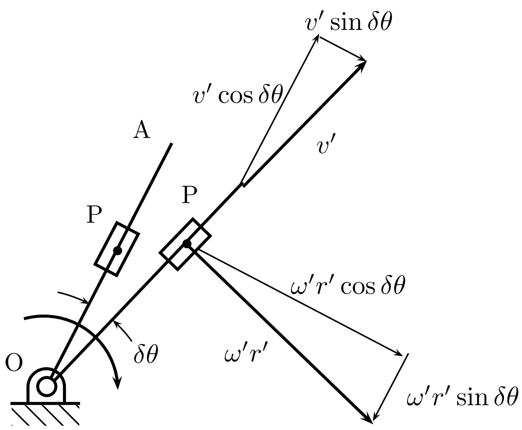


Figure 3.23 Coriolis component.

2. Acceleration Normal to OA The change in velocity of point P perpendicular to the rotating link OA is

$$\delta v_P^{\perp} = (v + f\delta t)\sin\delta\theta + (\omega + \alpha\delta t)(r + \delta r)\cos\delta\theta - \omega r$$

With limiting case of $\delta t \to 0$, $\cos\delta\theta \to 1$, $\sin\delta\theta \to 0$. Hence, acceleration of P perpendicular to OA as

$$a_P^{\perp} = \lim_{\delta t \to 0} \frac{\delta v_P^{\perp}}{\delta t} = 2\omega v + r\alpha \qquad (3.16)$$

In this equation, the component $2\omega v$ is known as *Coriolis component.* The remaining component is the tangential acceleration.

The total acceleration of the point P would be the vector sum of the orthogonal components of acceleration, given

by Eqs. (3.15) and (3.16):

$$a_P^{\parallel} = f - \omega^2 r$$
$$a_P^{\perp} = 2\omega v + r\alpha$$

The *Coriolis component* of acceleration exists in mechanisms having a slider in a rotating link with angular acceleration, for example, quick-return mechanism.

3.4 CAM FOLLOWER MECHANISM

A *cam* mechanism usually consists of two moving elements, *cam* and *follower*, mounted on a fixed frame. A *cam* can be defined as a machine element having a curved outline or a curved groove, which, by its oscillation or rotation motion, gives a predetermined specified motion to another element known as *follower*. In a cam-follower mechanism, the follower usually has a line-contact with the cam, thus, they constitute a higher pair mechanism.

Cam devices are versatile through which almost any arbitrarily-specified motion can be obtained. In some instances, they offer the simplest and most compact way to transform motions. Cams are widely used in automatic machines, internal engines, machine tools, printing control mechanisms, and so on.

3.4.1 Types of Cams

Different types of cams are shown in Fig. 3.24. These are explained as follows:

1. Wedge Cam   A *wedge cam* consists of a wedge which in general has a translation motion. The contact between the cam and follower is maintained by using a spring.
2. Disc Cam   In *disc cams*, the follower moves radially from the center of rotation of the cam; the transmission line passes through center of cam.
3. Spiral Cam   A *spiral cam* contains a spiral groove which mesh with a pin-gear follower. Rotation of the cam is reversed to reset the the follower.
4. Cylindrical Cam   A *cylindrical cam* is a cylinder having a circumferential contour in the surface, which mesh with a pin-gear follower.
5. Conjugate Cam   A *conjugate cam* is made of two disc cams, keyed together in such way that makes a positive constraint in touch with two rollers of a follower. These cams are preferred due to low wear and noise in high speed, and high dynamic load.
6. Globoidal Cams   A *globoidal cam* is similar to cylindrical cam but it can have convex or concave surface with a circumferential contour to impart oscillatory motion to the follower.
7. Spherical Cam   In a *spherical cam*, the follower oscillates about an axis perpendicular to the axis of rotation of cam having a spherical surface.

3.4.2 Types of Followers

Different types of followers are shown in Fig. 3.25. These

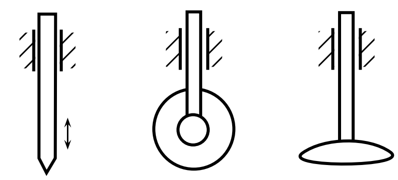


Figure 3.25 | Types of cam followers.

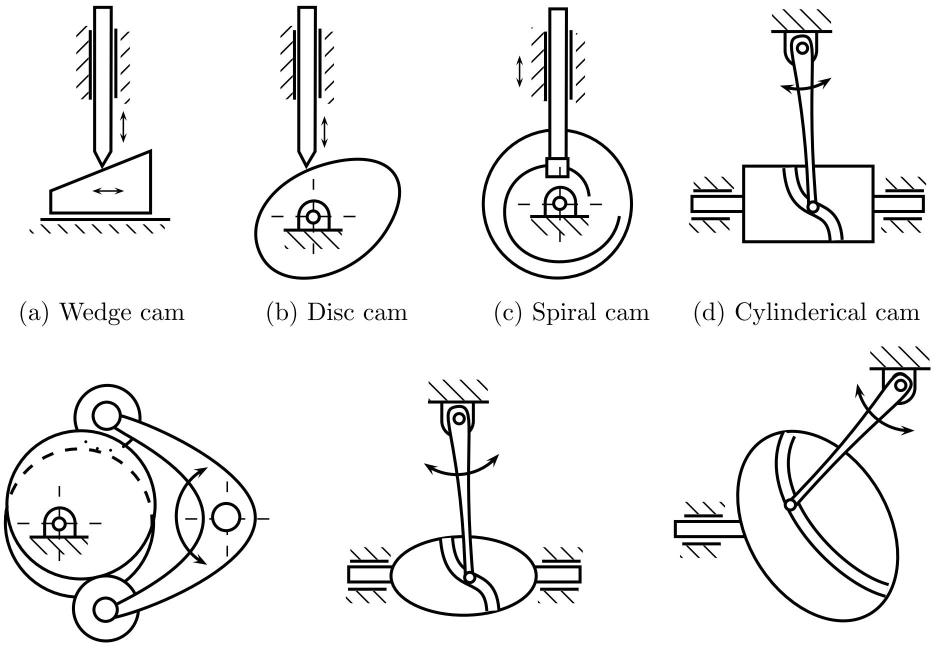
are explained as follows:

1. Knife Edge Follower   *Knife edge followers* are not used mostly due to great wear at knife edge and considerable side thrust.
2. Roller Follower   In the case of steep rise, a *roller follower* jams the cam and hence is not preferred for such design.
3. Mushroom Follower   *Mushroom followers* are of two types: flat faced and spherical faced. There is no side thrust except due to friction at the contact of cam and the follower.

3.4.3 Terminology

The following are the important design dimensions and geometries associated with the cam and follower [Fig. 3.26].

1. Basic Circle   The smallest circle to the cam profile and concentric to the center of rotation of the cam is known as *base circle*.
2. Trace Point   The *trace point* is the point of tracing on the follower. It is actually the edge of knife edge follower or the center of roller follower.
3. Prime Curve   The curve drawn by the trace point by fixing the cam at an angle is called *prime curve*.
4. Prime Circle   The smallest circle to pitch curve is called *prime circle*.



(a) Wedge cam (b) Disc cam (c) Spiral cam (d) Cylinderical cam

(e) Conjugate cam (f) Globoidal cam (g) Spherical cam

Figure 3.24 | Types of cams.

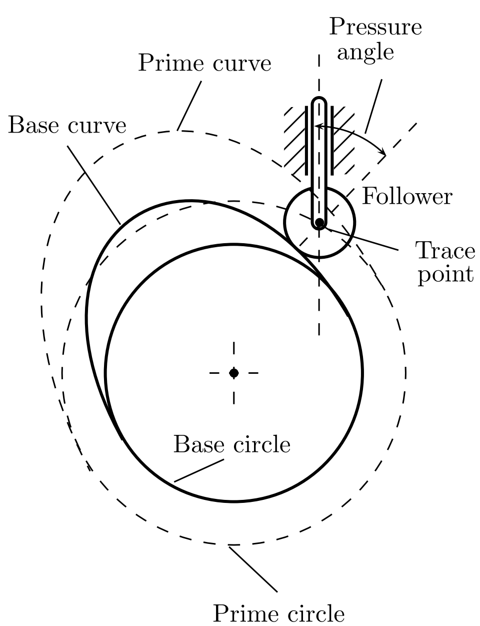


Figure 3.26 | Terminology of cam-follower.

5. *Pressure Angle* *Pressure Angle* is the angle between normal to the pitch curve at a point and direction of motion of follower.

For the same rise or fall as the pressure angle increases size (or base circle) of cam decreases. The size of base circle controls the pressure angle for given rise. The increase in the base circle diameter increases the length of the arc of the circle upon which the wedge (the raised portion) is to be made.

6. *Pitch Point* *Pitch point* is the point on a cam profile for which the *pressure angle* attains maximum value.

7. *Pitch Circle* The circle passing through pitch point and concentric to the center of rotation of the cam is called *pitch circle*.

3.4.4 Motion of the Follower

As a cam rotates about its axis, it imparts a specific motion to the follower which repeats in each revolution of the cam. The position of the follower, say measured from its lowest position, depends upon the angular position of cam:

$$s = f(\theta) \tag{3.17}$$

This relation can be visualized by plotting the angular displacement (θ) of the cam on x-axis and the linear displacement (s) of the follower on y-axis.

The function $s = f(\theta)$ represents the design scheme of the displacement of the follower. In this relation, the

following terms are used to describe various elements of the motion of the cam and the follower:

1. *Elements of Motion of Cam*
   (a) *Angle of Ascent* - *Angle of ascent* (ϕ_a) is the angle through which the cam turns during the time the follower rises.
   (b) *Angle of Dwell* - *Angle of dwell* (δ) is the angle through which the cam turns while the follower remains stationary at the highest or the lowest position.
   (c) *Angle of Descent* - *Angle of descent* (ϕ_d) is the angle through which the cam turns during the time the follower returns to the initial position.
   (d) *Angle of Action* - *Angle of action* is the total angle moved by the cam during the time between the beginning of rise and end of return of the follower.
2. *Elements of Motion of the Follower*
   (a) *Lift* - *Lift* is the maximum displacement of the follower.
   (b) *Ascent* - *Ascent* or *rise* is the movement of the follower away from the center of cam.
   (c) *Dwell* - *Dwell* is the period when there is no movement of the follower. In internal combustion engines, a shorter dwell period means a smaller period of valve opening, resulting in less fuel per cycle and lesser power production. Thus, the minimum value of dwell angle cannot be reduced from a certain value.

Using Eq. (3.17), the profiles of velocity and acceleration of the follower motion can also be determined as

$$v = \frac{ds}{dt}$$
$$f = \frac{dv}{dt}$$

Angular speed of cam is represented by

$$\omega = \frac{d\theta}{dt}$$

The following sections deal with the basic displacement functions of the ascent or descent of the follower. The ascent or descent of the follower takes place when the cam rotates an angle ϕ. The lift is represented by h.

3.4.4.1 Simple Harmonic In simple harmonic profile, the ascent and descent of the follower takes place in a half cycle (π) of the sinusoidal function. Therefore, half cycle of the harmonic functions is equivalent to the rotation cam (2ϕ).

The ascent takes place when cam rotates angle ϕ. Therefore, the simple harmonic displacement (s) can be related to angular displacement ($\theta = 0 \to \phi$) as

$$s = \frac{h}{2}\left\{1 - \cos\left(\frac{\pi\theta}{\phi}\right)\right\}$$

Velocity function is found as

$$\begin{aligned} V &= \frac{ds}{dt} \\ &= \frac{h}{2}\left\{\frac{\pi}{\phi}\omega \sin\left(\frac{\pi\theta}{\phi}\right)\right\} \end{aligned}$$

Acceleration function is found as

$$\begin{aligned} f &= \frac{dv}{dt} \\ &= \frac{h}{2}\left\{\frac{\pi^2}{\phi^2}\omega^2 \cos\left(\frac{\pi\theta}{\phi}\right)\right\} \end{aligned}$$

The functions of s, v and f are drawn in a combined plot [Fig. 3.27]. The velocity increases from zero to maximum at quarter of the simple harmonic motion;

$$\begin{aligned} \frac{\pi\theta}{\phi} &= \frac{\pi}{2} \\ \theta &= \frac{\phi}{2} \end{aligned}$$

Therefore, the maximum value of velocity is given by

$$v_{\max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi}\right)$$

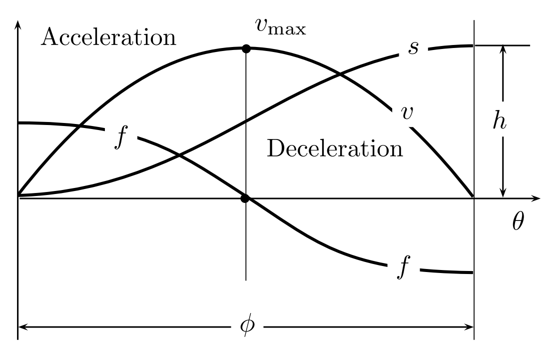


Figure 3.27 Simple harmonics.

At the beginning of rise ($\theta = 0$), acceleration (f) of the follower suddenly increases from zero to maximum value, given by

$$f_{\max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi}\right)^2$$

The maximum acceleration at the beginning itself causes higher inertia loads of the follower, therefore, the simple harmonic motion of the follower is usually applied for moderate speeds only.

3.4.4.2 Constant Acceleration

A *constant acceleration* profile gives a constant acceleration in the first half of the ascent and a constant deceleration in the second of the ascent. Consider a case when the velocity of follower at the beginning of the rise is zero ($v_0 = 0$). At any instant of time t taken from the beginning, the displacement s during ascent can be related to acceleration f using the second equation of linear motion:

$$\begin{aligned} s &= v_0 t + \frac{1}{2} f t^2 \\ &= \frac{1}{2} f t^2 \end{aligned} \tag{3.18}$$

This is an equation of parabola, thus, the profile of constant acceleration is also known as *parabolic motion*. Since there a constant acceleration in the first half of the ascent and a constant deceleration in the second half of the ascent, the ascents in the first and the second halves will be the same.

The arbitrary time t can be related to angular position θ of the cam as

$$t = \frac{\theta}{\omega}$$

The ascent and descent of the follower takes place when the cam rotates an angle 2ϕ; ascent upto lift h takes place when cam rotates an angle ϕ. For the constant angular speed ω, the cam takes time ϕ/ω for lift h of the follower. Taking the lift upto to the mid-point where the acceleration changes its sign and using Eq. (3.18),

$$\begin{aligned} \frac{h}{2} &= \frac{1}{2} f \left(\frac{\phi}{2\omega} \right)^2 \\ f &= 4h \left(\frac{\omega}{\phi} \right)^2 \end{aligned}$$

Putting these values in Eq. (3.18), the displacement function is found as

$$\begin{aligned} s &= \frac{1}{2} \times 4h \left(\frac{\omega}{\phi} \right)^2 \left(\frac{\theta}{\omega} \right)^2 \\ &= \frac{2h}{\phi^2} \theta^2 \end{aligned}$$

Therefore, the function of velocity is found as

$$\begin{aligned} v &= \frac{ds}{dt} \\ &= \frac{4h\omega}{\phi^2} \theta \end{aligned}$$

This equation represents a linear profile of the velocity, having constant slope, equal to acceleration f. The functions s, v, and f are drawn in a combined plot [Fig. 3.28].

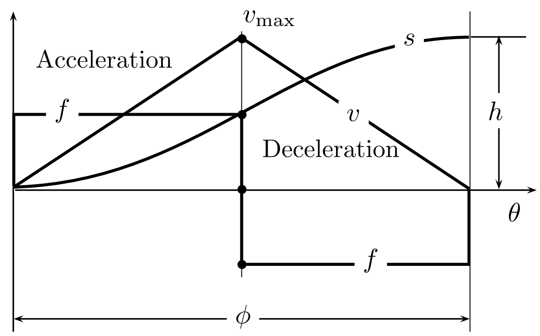


Figure 3.28 Constant acceleration motion.

Velocity is maximum when the acceleration changes its sign ($\theta = \phi/2$):

$$v_{\max} = \frac{2h\omega}{\phi} \tag{3.19}$$

At mid-way, an infinite jerk is produced, hence this profile of the follower motion is adopted only upto moderate speeds.

3.4.4.3 Constant Velocity

Constant velocity of the follower implies that the displacement of the follower is proportional to the cam displacement and the slope of the displacement curve is constant:

$$s \propto h \frac{\theta}{\phi} \tag{3.20}$$

Therefore,

$$\begin{aligned} v &= \frac{ds}{dt} \\ &= \frac{h\omega}{\phi} \end{aligned}$$

Since the velocity function is constant, the acceleration throughout the lift is zero. The functions s, v and f are drawn in a combined plot [Fig. 3.29].

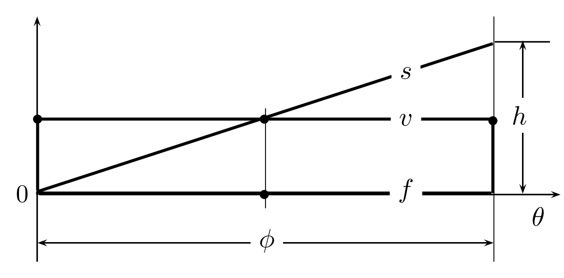


Figure 3.29 Constant velocity profile.

The acceleration of the follower during rise and return period is zero but it is infinite at the beginning and end

of the motion due to abrupt changes in the velocity. On account of this reason, the constant velocity profile is generally not used in practice.

3.4.4.4 Cycloidal Profile A *cycloid* is the locus of a point on a circle rolling on a straight line. Following is the relation of rise (s) with cam rotation (θ):

$$s = \frac{h}{\pi}\left(\frac{\pi\theta}{\phi} - \frac{1}{2}\sin\frac{2\pi\theta}{\phi}\right)$$

Therefore, the functions of velocity and acceleration are found as

$$\begin{aligned} v &= \frac{ds}{dt} \\ &= \frac{h}{\pi}\left(\frac{\pi\omega}{\phi} - \frac{\pi\omega}{\phi}\cos\frac{2\pi\theta}{\phi}\right) \\ &= \frac{h\omega}{\phi}\left(1 - \cos\frac{2\pi\theta}{\phi}\right) \\ f &= \frac{dv}{dt} \\ &= \frac{2h\pi\omega^2}{\phi^2}\sin\frac{2\pi\theta}{\phi} \end{aligned}$$

The functions s, v and f are drawn in a combined plot [Fig. 3.30]. The velocity is maximum at mid-point of the lift ($\theta = \phi/2$)

$$v_{\max} = \frac{2h\omega}{\phi} \tag{3.21}$$

At this point, the acceleration is zero.

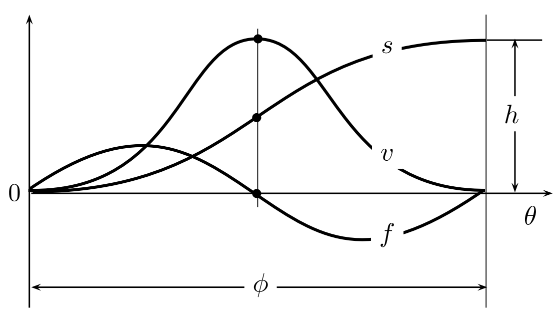


Figure 3.30 Cycloid profile motion.

Maximum value of acceleration occurs at $\theta = \phi/4$:

$$f_{\max} = \frac{2h\pi\omega^2}{\phi^2}$$

There are no abrupt changes in velocity and acceleration, hence, cycloidal program is the most suitable one for high speed follower motion.

3.5 GEARS

If power transmitted between two shafts is small, motion between them can be obtained by using plain cylinders or discs, called *friction wheels*. However, as the power increases, slip occurs between the discs and the motion no longer remains definite. Therefore, *gears* are used to transmit motion from one shaft to another by successive engagements of *teeth* without any intermediate link or connector. In this method, the surfaces of two bodies make a tangential contact and have a rolling motion along the tangent at the point of contact. No motion is possible along the common normal as that will either break the contact or one body will tend to penetrate into the other.

3.5.1 Classification

Gears can be broadly classified according to the relative positions of their shaft axes, discussed under following headings.

3.5.1.1 Gears for Parallel Shafts The motion between two parallel shafts is equivalent to the rolling of two cylinders, assuming no slipping. Gears under this group are the following:

1. *Spur Gears* Straight *spur gears* are the simplest form of gears having straight teeth parallel to the gear axis. The contact of two teeth takes place over the entire width along a line parallel to the axes of rotation. As the gears rotate, the line of contact goes on shifting parallel to the shaft.

   Although there is no axial thrust, but there is a sudden application of load, associated with high impact stresses and excessive noise at high speeds.

2. *Helical Gears* In *helical gears*, the teeth are part of helix instead of straight across the gear parallel to the axis. The mating gears have the same helix angle, but in opposite direction. As the gear rotates, the contact shifts along the line of contact in involute helicoid across the teeth.

   Load application is gradual, because at the beginning of engagement, the contact occurs at the point of leading edge of the curved teeth. Helical gears are, therefore, used at higher velocities and can carry higher loads compared to straight spur gears.

   The inclined direction of forces on the teeth in helical gears results axial thrust.

3. *Double Helical Gears* To avoid the problem of axial thrust in helical gears, *double helical gears* are made of two helical gears with opposite helix angles, which can be up to 45°.

If the left helix and right helix of a double helical gear meet at a common apex and there is no groove in between two pairs of gears, the gear is called *herringbone gear*.

4. Rack and Pinion In this case, the spur rack can be considered to be a spur gear of infinite pitch radius with its axis of rotation placed at infinity parallel to that of pinion. The pinion rotates while the rack translates. This mechanism is used to convert either rotary motion into linear motion or vice versa.

3.5.1.2 Gears for Intersecting Shafts The motion between two intersecting shafts is equivalent to the rolling of two cones, assuming no slipping. Therefore, the gears used for intersecting shafts are called *bevel gears*. The following are their types:

1. Straight Bevel Gears *Straight bevel gears* are provided with straight teeth, radial to the point of intersection of the shaft axes and vary in cross-section through the length inside the generator of the cone. Their main application is in connecting low speed shafts at right angles. In such applications, straight bevel gears of the same size are known as *miter gears*.

   Straight bevel gears can be viewed as a modified version of straight spur gears in which the teeth are made in conical direction instead of parallel to the axis. Also, like in straight spur gears, the line of contacts of straight bevel gears is a straight line. These gears become noisy at higher speed.

2. Spiral Bevel Gears To avail high speed applications, bevel gears are made such that their teeth are inclined at an angle to the face of the bevel, and then they are known as spiral bevel gears or helical bevels.

   Spiral bevel gears are quieter in action but are subjected to axial thrust. These gears find application in the differential drive of automobiles.

3. Zero Bevel Gear *Zero bevel gears* are made with zero spiral angle. Their curved teeth produces quieter motion.

3.5.1.3 Gears for Skew Shafts The following gears are used to join two skew (non-parallel and non-intersecting) shafts:

1. Hypoid Gears The *hypoid gears* are made of the frusta of *hyperboloids of revolution*. Two matching hypoid gears are made by revolving the same line which is in fact their line of contact, therefore, these gears are not interchangeable.

   The relative motion between these gears consists partly of rolling and partly of sliding, along the common line of contact.

2. Worm Gears *Worm gears* are used to connect skewed shafts (i.e. non-parallel and non-intersecting), but not necessarily at right angles. Teeth on worm gear are cut continuously like the threads on a screw. The gear meshing with the worm gear is known as *worm wheel* and the combination is known as worm and worm wheel. At least one teeth of the worm must make a complete turn around the pitch cylinder, and thus forms screw thread.

   Unlike with ordinary gear trains, the direction of transmission in worm drive is not reversible when using large reduction ratios. Due to the greater friction involved between the worm and worm-wheel, usually a single start (one spiral) worm is used.

   If a multi-start worm (multiple spirals) is used then the ratio reduces accordingly and the braking effect of a worm and worm-gear would need to be discounted as the gear will be able to drive the worm.

3.5.2 Gear Terminology

The following are important dimensions and geometries concerned with toothed gears [Fig. 3.31]:

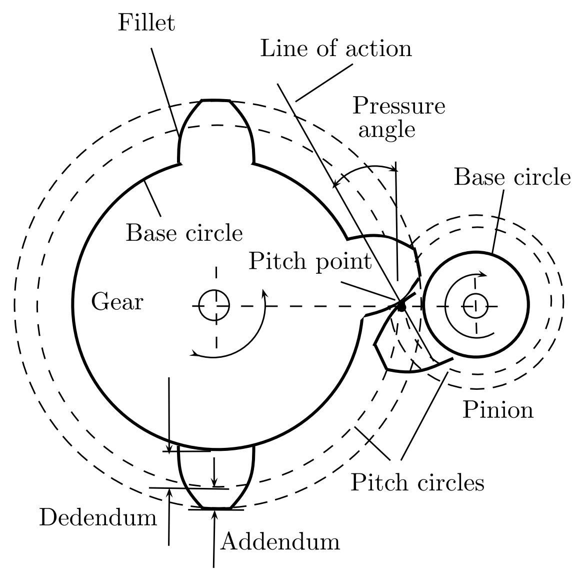


Figure 3.31 Gear terminology.

1. Pitch Circle *Pitch circle* is the apparent circle that two gears can be taken like smooth cylinders rolling without slipping.

2. Addendum Circle *Addendum!circle* is the outermost profile circle of a gear. *Addendum* (a) is the

radial distance between the pitch circle and the addendum circle.

3. Dedendum Circle *Dedendum!circle* is the innermost profile circle. *Dedendum* is the radial distance between the pitch circle and the dedendum circle.

4. Clearance *Clearance* is the radial distance from the top of the tooth to the bottom of the tooth space in the mating gear.

5. Backlash *Backlash* is the tangential space between teeth of mating gears at pitch circles.

6. Full Depth *Full depth* is the sum of the dedendum and the addendum.

7. Face Width *Face width* is the length of tooth parallel to axes.

8. Diametral Pitch *Diametral pitch* (p) is the number of teeth per unit diameter.

   If d is diameter of pitch circle of the gear with T number of teeth, then diametral pitch is calculated as

$$p = \frac{T}{d}$$

9. Module *Module* (m) is the inverse of diametral pitch:

$$m = \frac{1}{p} = \frac{d}{T}$$

10. Circular Pitch *Circular pitch* is the space in pitch circle used by each teeth:

$$\begin{aligned} p_c &= \frac{\pi d}{T} \\ &= m\pi \\ &= \frac{\pi}{p} \end{aligned}$$

11. Gear Ratio *Gear Ratio* (G) is the ratio of numbers of teeth of larger gear to smaller gear.

12. Pressure Line *Pressure line* is the common normal at the point of contact of mating gears along which the driving tooth exerts force on the driven tooth. It is also called *line of action*.

13. Pressure Angle *Pressure angle* (ϕ) is the angle between the pressure line and common tangent to pitch circles. It is also called *angle of obliquity*. High pressure angle requires wider base and stronger teeth.

14. Pitch Angle *Pitch angle* is the angle captured by a tooth. If there are T teeth in a gear, the pitch angle is determined as

$$\text{Pitch angle} = \frac{360^\circ}{T} \tag{3.22}$$

15. Contact Ratio *Contact ratio* is the ratio of angle of action and pitch angle:

$$\text{Contact ratio} = \frac{\text{Angle of action}}{\text{Pitch angle}} \tag{3.23}$$

16. Path of Approach *Path of approach* is the distance along the pressure line traveled by the contact point from the point of engagement to the pitch point.

17. Path of Recess *Path of recess* is the distance along the pressure line traveled by the contact point from the pitch point to the point of disengagement.

18. Path of Contact *Path of contact* is the sum of path of approach and path of recess.

19. Arc of Approach *Arc of approach* is the distance traveled by a point on either pitch circle of the two wheels from the point of engagement to the pitch.

20. Arc of Recess *Arc of recess* is the distance traveled by a point on either pitch circle of the two wheels from the pitch point to the point of disengagement.

21. Arc of Contact *Arc of contact* is the distance traveled by a point on either pitch circle of the two wheels during the period of contact of a pair of teeth.

22. Angle of Action *Angle of action* is the angle turned by a gear during arc of contact.

3.5.3 Law of Gearing

Consider two rigid bodies 1 and 2, representing a portion of the two gears in mesh, rotate about the centers O_1 and O_2, respectively [Fig. 3.32]. The common tangent TT and common normal NN pass through contact point C or C_1 and C_2, the points of contact on respective bodies.

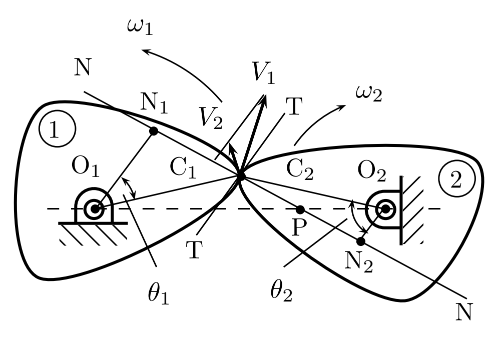


Figure 3.32 Law of gearing.

The components of relative velocity of the two gears can be determined as follows:

1. Along the Common Normal Relative velocity along the common normal N-N is given by

$$V_1 \cos\theta_1 - V_2 \cos\theta_2 = 0$$
$$\omega_1 \mathrm{O_1C_1} \cos\alpha - \omega_2 \mathrm{O_2C_2} \cos\beta = 0$$
$$\omega_1 \mathrm{O_1N_1} - \omega_2 \mathrm{O_2N_2} = 0$$

Therefore,

$$\frac{\omega_1}{\omega_2} = \frac{\mathrm{O_2N_2}}{\mathrm{O_1N_1}} = \frac{\mathrm{N_2P}}{\mathrm{N_1P}} = \frac{\mathrm{O_2P}}{\mathrm{O_1P}} \tag{3.24}$$

Hence, the angular velocities of the two gears shall remain constant if the common normal at the point of contact of the two teeth passes through a fixed point P which divides the line of centers in the inverse ratio of angular velocities of two gears. This is known as *law of gearing* and the point P is called the *pitch point*.

2. Along the Common Tangent The relative velocity of the mating bodies along the common tangent (TT) at the point of contact is called *velocity of sliding*:

$$\begin{aligned} V_s &= V_1 \sin\theta_1 - V_2 \sin\theta_2 \\ &= \omega_1 \mathrm{O_1C_1} \frac{\mathrm{C_1N_1}}{\mathrm{O_1C_1}} - \omega_2 \mathrm{O_2C_2} \frac{\mathrm{C_2N_2}}{\mathrm{O_2C_2}} \\ &= \omega_1 (\mathrm{N_1P} - \mathrm{C_1P}) - \omega_2 (\mathrm{N_2P} - \mathrm{C_2P}) \\ &= (\omega_1 + \omega_2)\,\mathrm{CP} + (\omega_1 \mathrm{N_1P} - \omega_2 \mathrm{N_2P}) \end{aligned}$$

Applying the *law of gearing* [Eq. (3.24)],

$$V_s = \mathrm{CP} \times (\omega_1 + \omega_2) \tag{3.25}$$

At the pitch point, CP = 0, thus, the velocity of sliding is also zero.

3.5.4 Teeth Profiles

The profile of teeth must satisfy the *law of gearing* so that the mating gears can run without any breakage. There are two types of teeth profiles:

1. *Cycloidal profile*
2. *Involute profile*

Involute profile is preferred due to favorable features, and is discussed later. A gear of one type of teeth can mate only with the gear of same type of teeth, otherwise law of gearing will not be satisfied.

3.5.4.1 Cycloidal Teeth Cycloidal profile is composed of two types of profiles [Fig. 3.33]:

1. *Hypocycloid profile*[3] - inside the pitch curve
2. *Epicycloid profile*[4] - outside the curve.

These curves are produced when a small generating circle rolls inside and outside the perimeter of the pitch circle of the two mating gears. Therefore, cycloidal teeth are also called *double curve teeth* [Fig. 3.33].

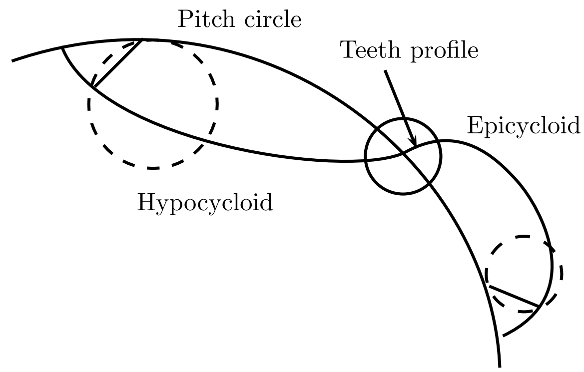


Figure 3.33 Cycloidal profile.

The cycloidal teeth of the two mating gears are produced with the same generating circle. When mating gears of cycloidal teeth run, the common normal passes through a fixed point P (pitch point) in each position during action [Fig. 3.34]. Thus, the *law of gearing* is satisfied.

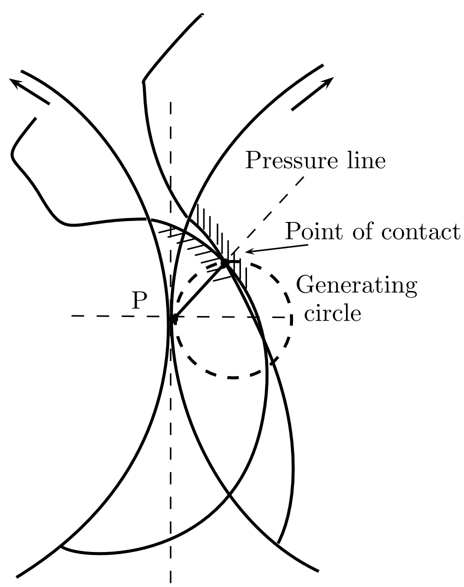


Figure 3.34 Meshing of cycloidal teeth.

[3]Locus of a point on the circle when that circle rolls without slipping inner circumference another circle.

[4]Epicycloid is the locus of a point on the circle when that circle rolls without slipping on outer circumference of another circle.

The pressure angle varies from maximum at the beginning of engagement to zero at pitch point and then maximum but equal in reverse direction at disengagement. As is evident from Fig. 3.34, the contact between the teeth takes place along the common point between the hypocycloid and epicycloid portions of the mating teeth. Therefore, for cycloidal profile, path of approach is equal to arc of approach, and path of recess is equal to the arc of recess.

Advantage of cycloidal profile is that the phenomenon of interface does not exist. Since, these are made up of two curves, their accurate profile is difficult to produce. This has rendered this system obsolete. The details of cycloidal teeth are not discussed in the present context.

3.5.4.2 Involute Teeth

The *involute teeth*[5] consist of a single involute curve which is the locus of point on a straight (generating) line which rolls without slipping on the circumference of a base circle [Fig. 3.35].

During the messing of involute teeth of mating gears, the path of contact or pressure line is the common tangent to the base circle and it passes through the fixed point P which implies that involute teeth follow the *law of gearing* [Fig. 3.35]. When the pitch point is shifted or the center-to-center distance is increased, there is no change in velocity ratio, but there is increase in the pressure angle. Diameter of the base circle of the gear profile is a manufacturing property which remains invariant for the gear.

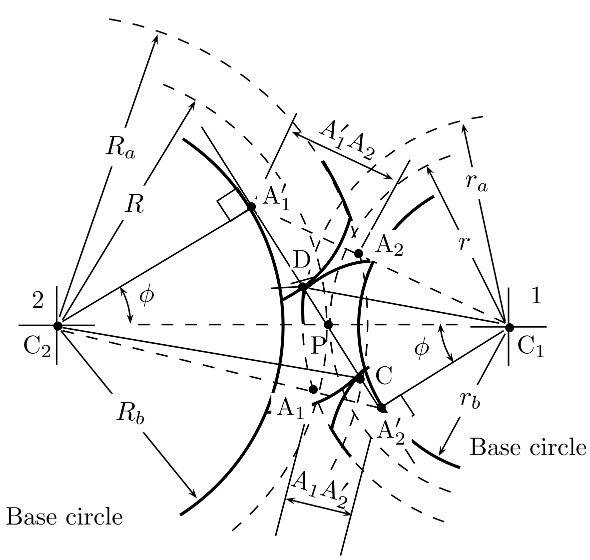


Figure 3.35 | Meshing of involute teeth.

[5]An involute function is defined as

$$\text{Inv}(\phi) = \tan\phi - \phi$$

The base circle radii are related to pitch radii of the mating gears as

$$R_b = R\cos\phi$$
$$r_b = r\cos\phi$$

Initial contact occurs at point C where addendum of driven wheel intersects the line of action and final contact occurs at point D where addendum circle of the driver intersects the line of action. Between these two points, the line of action passes through the pitch point P. Using this information and trigonometry, the following properties can be determined for gears having involute teeth:

1. *Path of Contact* The path of contact consists of two components:

   (a) *Path of Approach* - The distance along the pressure line traveled by the contact point from the point of engagement (C) to the pitch point (P):

   $$\begin{aligned}\text{CP} &= \text{CA}_1' - \text{PA}_1' \\ &= \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi\end{aligned}$$

   The path of approach can attain a maximum value when the point of engagement lies at A_2' in Fig. 3.35. Therefore, the maximum value of path of approach is given by

   $$\text{A}_2'\text{P} = r\sin\phi$$

   (b) *Path of Recess* - The distance along the pressure line traveled by the contact point from the pitch point (P) to the point of disengagement (D):

   $$\begin{aligned}\text{PD} &= \text{DA}_2' - \text{PA}_2' \\ &= \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi\end{aligned}$$

   The path of recess can attain a maximum value when the point of disengagement lies at A_1' in Fig. 3.36. Therefore, the maximum value of path of recess is given by

   $$\text{PA}_1' = R\sin\phi$$

   Now, the path of contact is given by

   $$\begin{aligned}\text{CD} &= \text{CP} + \text{PD} \\ &= \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi \\ &\quad + \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi\end{aligned}$$

   The path of contact can attain a maximum value when the point of contact traces the pressure line from point E to point F. Therefore, the maximum value of path of contact is given by

   $$\text{A}_2'\text{A}_1' = (R + r)\sin\phi$$

2. *Arc of Contact* Using the definition of involute curve, the distance traveled by the contact point is equal to the arc on the base circle. For a gear of pitch circle radius R, the base circle radius is $R\cos\phi$. Therefore, an arc of length x on the base circle will trace same angle on the pitch circle radius, where its length will be $x/\cos\phi$. Using this, the components of arc of approach for involute teeth meshing are determined as follows:

   (a) *Arc of Approach* - Arc of approach is the distance traveled by a point on either pitch circle of the two wheels during the period of contact from engagement to pitch point:

   $$\text{Arc CP} = \frac{\text{CP}}{\cos\phi}$$

   (b) *Arc of Recess* - Arc of recess is the distance traveled by a point on either pitch circle of the two wheels during the period of contact from pitch point to disengagement:

   $$\text{Arc PD} = \frac{\text{PD}}{\cos\phi}$$

   Therefore, arc of contact is given by

   $$\begin{aligned}\text{Arc CD} &= \text{Arc CP} + \text{Arc PD}\\ &= \frac{\text{CD}}{\cos\phi}\end{aligned}$$

3. *Number of Pairs of Teeth in Contact* The pair of teeth lying in between the point of engagement and point of disengagement will be meshing with each other. Therefore, the number of pairs of teeth in contact can be determined as

   $$\begin{aligned}n &= \frac{\text{Arc of contact}}{\text{Circular pitch}}\\ &= \frac{\text{CDP}}{p_c\cos\phi}\end{aligned}$$

   If z_w and z_p are the number of teeth on the mating gears of equal module (m) or circular pitch ($p_c = m\pi$), then

   $$\begin{aligned}R &= \frac{z_w}{2\pi}p_c\\ r &= \frac{z_p}{2\pi}p_c\end{aligned}$$

   Maximum number of teeth pairs can be in mesh when the point of contact traces the pressure line from point E to point F:

   $$\begin{aligned}n_{\max} &= \frac{(R+r)\sin\phi}{p_c\cos\phi}\\ &= (z_w+z_p)\,\frac{\tan\phi}{2\pi}\end{aligned}$$

3.5.5 Interference

Any tooth profile can be used successfully for a spur gear as long as the mating tooth profile is compatible for producing a constant speed ratio. The compatible tooth profile is technically known as *conjugate* with respect to the tooth profile of the first gear. Mating of non-conjugate (non-involute) teeth is known as *interference*, in which the contacting teeth have different velocities which can lock the two mating gears.

A radial profile (a non-involute type) is generally adopted for the part of teeth inside the base circle. Due to inconjugate teeth, the tip of the pinion would try to dig out the flank of the tooth of the wheel. Therefore, interference occurs in the mating of two gears. Similarly, if the addendum radius of the wheel is made greater than $\text{C}_2\text{A}_2'$ [Fig. 3.35], the tip of the wheel tooth will be in contact with a portion of the non-involute profile of the pinion tooth.

To have no interference of the teeth, the addendum should not penetrate into the base circle of the mating gear; the addendum circles of the wheel and pinion must intersect the line of action between A_2' and A_1' [Fig. 3.35]. These points are called *interference points.* To avoid interference, the limiting value of addendum of the wheel is $\text{A}_1\text{A}_2'$ whereas that of pinion is $\text{A}_1'\text{A}_2$ and the latter is clearly greater than the former ($\text{A}_1'\text{A}_2 > \text{A}_1\text{A}_2'$). Therefore, for equal addenda[6] of the wheel and the pinion, the addendum radius of the wheel decides whether the interference will occur or not.

The addendum of wheel can be determined geometrically as follows:

$$\begin{aligned}\text{C}_2\text{A}_2'^{\,2} &= \text{C}_2\text{A}_1'^{\,2} + \text{A}_1'\text{A}_2'^{\,2}\\ &= \text{C}_2\text{A}_1'^{\,2} + \left(\text{A}_1'\text{P} + \text{PA}_2'\right)^2\\ &= (R\cos\phi)^2 + (R\sin\phi + r\sin\phi)^2\\ &= R^2 + \left(r^2 + 2rR\right)\sin^2\phi\\ &= R^2\left[1 + \frac{1}{R^2}\left(r^2+2rR\right)\sin^2\phi\right]\\ &= R^2\left[1 + \left(\frac{r^2}{R^2} + \frac{2r}{R}\right)\sin^2\phi\right]\end{aligned}$$

Therefore,

$$\text{C}_2\text{A}_2' = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi}$$

[6] The radial distance between the pitch circle and the addendum circle. Addenda is plural form of word ‘addendum’.

Maximum value of the addendum of the wheel is

$$\begin{aligned}
A_1A_2' &= C_2A_2' - R \\
&= R\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - R \\
&= R\left[\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - 1\right]
\end{aligned}$$

If the selected value of addendum of larger gear (wheel) is a_w times of module m, then

$$A_1A_2' \geq a_w m$$

$$R\left[\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - 1\right] \geq a_w m$$

Let T be the number of teeth on wheel, then

$$m = \frac{2R}{T}$$

Gear ratio is expressed as

$$\frac{R}{r} = G$$

From the above relations,

$$T \geq \frac{2a_w}{\sqrt{1+(1/G)\,(1/G+2)\sin^2\phi} - 1} \tag{3.26}$$

When addendum is equal to module ($a_w = 1$),

$$T \geq \frac{2}{\sqrt{1+(1/G)\,(1/G+2)\sin^2\phi} - 1} \tag{3.27}$$

This is the expression for number of teeth on a larger gear (wheel) to avoid interference in meshing of gears at pressure angle ϕ, gear ratio G, and addendum equal to module. A gear-meshing having pressure angle $\phi = 20°$ and unit gear ratio ($G = 1$) will be interference-free if $T > 12.32 \approx 13$.

3.5.6 Rack and Pinion

In rack and pinion arrangement, the pinion rotates while the rack translates. This mechanism is used to convert either rotary motion into linear motion or vice versa. In this mechanism, the spur rack can be considered to be a spur gear of infinite pitch radius with its axis of rotation placed at infinity parallel to that of the pinion; instead of base circle, the larger gear is formed over a straight line.

The condition for interference running of a rack-pinion arrangement can also be examined, as depicted in Fig. 3.36. To avoid interference, the addendum of pinion should not penetrate to the pitch line (PA_1) of rack. Let z_p be the number teeth on pinion and ϕ be the pressure angle. The depth A_1A_2' can be determined as follows:

$$\begin{aligned}
A_1A_2' &= (r\sin\phi)\sin\phi \\
&= r\sin^2\phi \\
&= \frac{mz_p}{2}\sin^2\phi
\end{aligned}$$

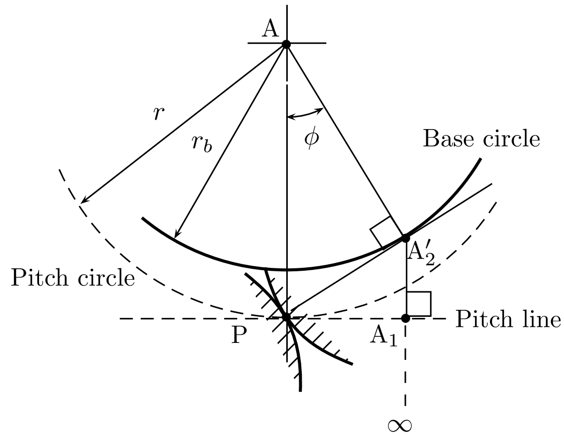


Figure 3.36 | Interference in rack-pinion.

For addendum equal to module, the running will be free of interference if

$$\begin{aligned}
A_1A_2' &\geq m \\
\frac{mz_p}{2}\sin^2\phi &\geq m \\
z_p &\geq \frac{2}{\sin^2\phi}
\end{aligned}$$

For interference-free running of a rack pinion having $\phi = 20°$, $z_p > 18$, and for $\phi = 14.5°$, $z_p > 32$.

3.5.7 Helical Gears

On the basis of the direction of helix (ψ) cut on *helical gears*, these can be either right handed or left handed. Angle between axes of two shafts is given by

$$\theta = \psi_1 - \psi_2 \tag{3.28}$$

The type of contact between teeth of helical gear teeth depends on θ. For parallel shafts ($\theta = 0$), the contact exists along a diagonal line, while for skew shafts ($\theta \neq 0$), there exists a point contact between gear teeth. Therefore, crossed-helical or spiral gears are not used for heavy loads, but parallel helical gears are stronger than spur gears because of diagonal contacts. When helical gears are used in skew shafts, they are called spiral gear or crossed-helical gears.

In the case of helical gears, normal circular pitch of two mating gears must be the same. If the helix angle

of a helical gear is increased, the load carrying capacity of the tooth increases and the form factor increases with increase in helix angle.

3.5.8 Gear Trains

Desired speed ratio in a gear system can be achieved by combination of various gears with different number of teeth and inversions. These combinations are called *gear trains*. Gear trains are necessary when a large or certain velocity ratio or mechanical advantage is required, or shafts are kept at a distance. Some important types of gear trains are discussed in following sections.

3.5.8.1 Simple Gear Train When each gear of the gear train is mounted on a separate shaft and all the gear axes remain in position fixed to frame, it is called *simple gear train*.

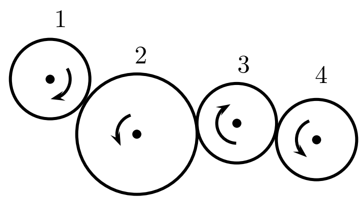


Figure 3.37 Simple gear train.

For the simple gear train shown in Fig. 3.37, the speed ratio, also called *train value*, is given by

$$N_4 = N_1 \times \frac{T_2}{T_1} \times \frac{T_3}{T_2} \times \frac{T_4}{T_3}$$

$$\frac{N_4}{N_1} = \frac{T_4}{T_1}$$

Thus, in simple gear trains, the intermediate gears do not have any influence on the velocity ratio. Intermediate gears work as *idler gears*, serving two purposes: first, they control the direction of rotation of output gear, and second, they bridge the gap between the shafts.

3.5.8.2 Compound Gear Train A gear pair is called compounded if they are mounted on the same shaft and are made into an integral part in some way. A *compound gear train* consists of one or more compound gear pairs. The compounding involves large speed reduction.

Compound gear trains are of two classes: reverted and non-reverted. In *reverted gear trains*, the first and last gears are co-axial [Fig. 3.38], otherwise it is called *non-reverted gear train* [Fig. 3.39]. Reverted gear trains are used in lathe machines where back gear is used to impart slow speed of the chuck.

3.5.8.3 Epicyclic Gear Train A gear train having a relative motion of axes is called *planetary* or *epicyclic gear train*. The axis of at least one of the gear also moves relative to the frame. For example, in Fig. 3.40, if the gear S is made fixed, the axis of gear P will rotate about the axis of S. Such a gear train has two degrees of freedom. Planetary gear trains are quite useful in making the reduction unit more compact than a compound gear train.

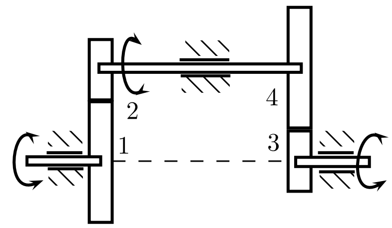


Figure 3.38 Compound gear train (reverted).

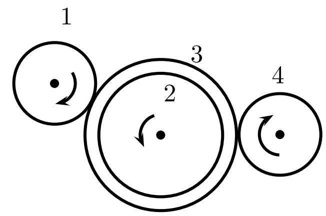


Figure 3.39 Compound gear train (non-reverted)

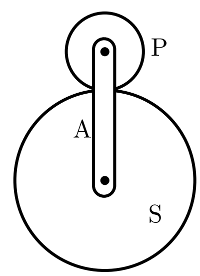


Figure 3.40 Epicyclic gear train.

To simplify the analysis, an epicyclic gear train can be converted into a simple reverted gear train by fixing the arm and releasing the fixed gear. The relative motion between any two links does not change but the absolute motion does change. Based on this understanding, the following procedure is adopted in analyzing the epicyclic gear trains:

1. Lock arm to obtain simple reverted train such that except arm, all gears are free to rotate.
2. Consider any gear and turn it in say clockwise direction through one revolution. Establish corresponding revolutions of all other gears, note it in a tabular form in a first row.

3. Rotate the chosen gear in step 2 x times, by multiplying each entry of the first row by an arbitrary variable x and enter the values of the product in the second row of the table.

4. Revolve arm by y revolutions by adding y to all the entries of second row, and enter the results in third row.

In the above steps, x and y are the two unknown variables in terms of which the third row gives corresponding revolutions of each gear and arm. Values of x and y are determined by the given condition, for example, sun is fixed on frame, so its speed is zero, and so on. The revolutions of all the gears are determined by putting values of x and y.

To better understand the procedure, consider an epicylic gear train shown in Fig. 3.41. The gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star-shaped carrier C. The size of different wheels are such that the planet carrier C rotates at 1/4th of the speed of the sun wheel S. The minimum number of teeth on any wheel is 12. The problem is to determine the number of teeth on sun gear.

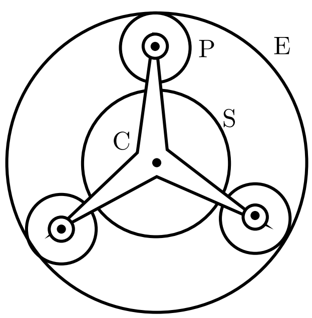


Figure 3.41 Epicyclic gear train.

The tabular method is shown in Table 3.1.

Table 3.1 Tabular method for epicyclic gear train

Condition	C	S	P	E
C fixed, S one revolution	0	1	$-\dfrac{T_S}{T_P}$	$-\dfrac{T_S}{T_E}$
C fixed, S x revolutions	0	x	$-\dfrac{T_S}{T_P}x$	$-\dfrac{T_S}{T_E}x$
Add y to all	y	$y+x$	$y-\dfrac{T_S}{T_P}x$	$y-\dfrac{T_S}{T_E}x$

Given that $N_C = N_S/4$. Therefore, using the speeds of gears shown in the bottom row,

$$N_C = \frac{N_S}{4}$$
$$y = \frac{y+x}{4}$$
$$\frac{x}{y} = 3$$

Also,

$$N_E = 0$$
$$y - \frac{T_S}{T_E}x = 0$$
$$T_E = \frac{x}{y}T_S$$
$$T_E = 3T_S$$

Taking the dimensional constraints,

$$T_S + 2T_P = T_E$$
$$T_P = \frac{T_E - T_S}{2}$$
$$T_P = T_S$$

Observing above equations,

$$T_E > T_S$$

Thus, the minimum number of teeth is $T_S = 12$ (given). Using above relations,

$$T_S = 12$$
$$T_E = 36$$
$$T_P = 12$$

3.6 DYNAMIC ANALYSIS OF SLIDER CRANK MECHANISM

Figure 3.42 shows the slider crank mechanism under consideration where l and r are the lengths of connecting rod and crank, respectively. The crank has angular speed ω and angular acceleration α.

Ratio of lengths l and r is denoted as

$$n = \frac{l}{r}$$

The relation between angular position of connecting rod (θ) and crank (ϕ) is determined as

$$l \sin\phi = r \sin\theta$$
$$\sin\phi = \frac{\sin\theta}{n} \tag{3.29}$$

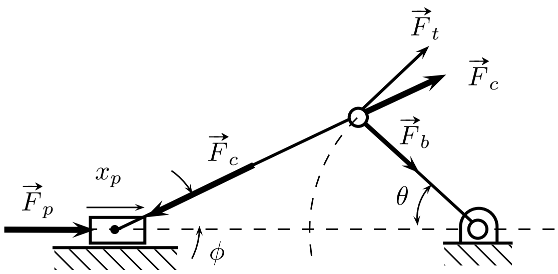


Figure 3.42 Slider-crank mechanism.

3.6.1 Motion in Links

3.6.1.1 Slider The distance of slider at extreme outer position is $(l+r)$, therefore, the position or displacement function of slider, measured from the extreme outer position, at any instant is given by

$$\begin{aligned} x_p &= (l+r) - l\cos\phi - r\cos\theta \\ &= r\left\{(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right\} \end{aligned}$$

Therefore, velocity of the slider, directed towards crank pin, at any instant is determined as

$$\begin{aligned} v_p &= \frac{dx_p}{dt} \\ &= r\omega\left\{\sin\theta + \frac{\sin 2\theta}{2n}\right\} \end{aligned}$$

Similarly, acceleration of the slider is given by

$$\begin{aligned} f_p &= \frac{dv_p}{dt} \\ &= r\omega^2\left\{\cos\theta + \frac{\cos 2\theta}{n}\right\} \end{aligned}$$

3.6.1.2 Connecting Rod The angle ϕ denotes the angular position of connecting rod with respect to frame. Therefore, angular speed of connecting rod is determined using Eq. (3.29):

$$\begin{aligned} \cos\phi\frac{\partial\phi}{\partial t} &= \omega\cos\theta \\ \omega_{\text{CR}} &= \frac{\partial\phi}{\partial t} \\ &= \frac{\omega}{n}\times\frac{\cos\theta}{\cos\phi} \end{aligned}$$

For small values of ϕ $(\cos\phi\approx 1)$:

$$\omega_{\text{CR}} = \omega\frac{\cos\theta}{n}$$

Angular acceleration of connecting rod is derived as

$$\begin{aligned} \alpha_{\text{CR}} &= \frac{d\omega_{\text{CR}}}{dt} \\ &= -\omega^2\frac{\sin\theta}{n} \end{aligned}$$

For special case $\theta = \phi = 0$:

$$\begin{aligned} (\omega_{\text{CR}})_{\max} &= \frac{\omega}{n} \\ \alpha_{\text{CR}} &= 0 \end{aligned}$$

3.6.2 Dynamic Forces

3.6.2.1 Piston Effort The net effective force acting on the piston is known as *piston effort*. It consists of three elements:

1. *Pressure Force*

$$F_p = p_1A_1 - p_2A_2$$

2. *Inertia Force*

$$F_i = m\omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

3. *Gravity Force*

$$F_g = mg$$

The piston effort is given by

$$F_P = F_p - F_i \pm F_g$$

Gravity force is considered only in vertical motion of the piston with suitable sign.

3.6.2.2 Crank Effort The force acting on the crank at the joint of connecting rod as a result of the force on the piston is known as *crank effort*. Resolving the piston force along the connecting rod,

$$F_{\text{CR}} = \frac{F_P}{\cos\phi}$$

The force exerts a thrust on cylinder-wall, given by

$$F_{\text{wall}} = F_{\text{CR}}\sin\phi$$

The force on the crank generates crank effort, given by

$$F_t = F_{\text{CR}}\sin(\phi+\alpha)$$

The force transmitted to the crank bearing is

$$F_B = F_{\text{CR}}\cos(\phi+\alpha)$$

3.6.3 Turning Moment

Turning moment on the crank shaft is derived as

$$\begin{aligned} T &= F_t \times r \\ &= \frac{F_P}{\cos\phi} \times \sin(\theta+\alpha) \times r \\ &= \frac{rF_P}{\cos\phi}\{\sin\theta\cos\phi + \cos\theta\sin\phi\} \\ &= F_P r\left\{\sin\theta + \cos\theta\sin\phi\left(\frac{1}{\cos\phi}\right)\right\} \\ &= F_P r\left\{\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right\} \end{aligned}$$

This is the expression of turning moment in terms of piston effort (F_p), crank radius r for given angular position (θ) of the crank. For given values of F_p and r, the turning moment is can be represented as

$$T = f(\theta)$$

Using this function, $T(\theta)$ can be plotted for different values of θ. This plot is called *turning moment diagram.*

3.7 FLYWHEEL

In every machine, there is at least one point at which energy is supplied and at least one other point at which energy is delivered. Between these two points, there is undesired variation in energy and speed of the machine. A *flywheel* is a balanced spinning wheel with significant moment of inertia. It is used to control variation in speed during each cycle of an engine by making moment of inertia of rotating parts quite large. A flywheel acts as a reservoir of energy which stores and releases the energy as per its requirements.

3.7.1 Mean Speed of Rotation

Let a machine be attached with a flywheel with moment of inertia[7] I. The maximum and minimum seeds of rotation of the flywheel are ω_1 and ω_2, respectively. Therefore, the mean speed of rotation of the flywheel is

$$\omega = \frac{\omega_1+\omega_2}{2}$$

To quantify the fluctuation of speed, a term *coefficient of fluctuation of speed* (k_s) is defined as

$$k_s = \frac{\omega_1-\omega_2}{\omega}$$

[7]Moment of Inertia of a disc is

$$I_{\text{disc}} = \int_0^R \rho\times 2\pi b r dr \times r^2 = 2\frac{\rho b\pi R^2\times R^2}{4} = \frac{mR^2}{2}$$

Using the definition of mean speed ω, and taking $r = \omega_1/\omega_2$:

$$\begin{aligned} \frac{k_s}{2} &= \frac{r-1}{r+1} \\ rk_s + k_s &= 2r-2 \\ r(2-k_s) &= 2+k_s \\ r &= \frac{2+k_s}{2-k_s} \end{aligned}$$

This expression relates speed ratio r to the *coefficient of fluctuation of speed* (k_s).

3.7.2 Energy Fluctuation

Let E be the kinetic energy of the flywheel at mean speed ω:

$$E = \frac{1}{2}I\omega^2$$

The maximum fluctuation of energy ($e_{\max}$) is determined as

$$\begin{aligned} e_{\max} &= \frac{1}{2}I(\omega_1^2-\omega_2)^2 \\ &= I(\omega_1-\omega_2)\frac{\omega_1+\omega_2}{2} \\ &= I\frac{(\omega_1-\omega_2)}{\omega}.\omega^2 \\ &= I\omega^2 k_s \\ &= 2\times\frac{I\omega^2}{2}k_s \\ &= 2k_s E \end{aligned}$$

Thus,

$$k_s = \frac{e_{\max}}{2E}$$

A flywheel stores the extra energy (more than average) supplied by engine during power stroke, and to reduce variations in the power generation curve, it returns the stored energy. Therefore, a flywheel must have sufficient moment of inertia to become capable of absorbing the maximum energy variation.

3.7.3 Turning Moment Diagrams

A *turning moment diagram* indicates the variation in the turning moment or torque due to the pressure variation in the cylinder for one complete revolution of the power cycle. The profile and frequency of *turning moment diagram* depends upon the type of engine or power pack being in use.

Let torque equation (T–θ) on turning diagram be presented as

$$T = f(n\theta)$$

Thus, the cycle repeats in every $2\pi/n$ radian. The mean torque of the cycle will be given by

$$\bar{T} = \frac{n}{2\pi}\int_0^{2\pi/n} T d\theta$$

The excess energy is calculated by first finding θ_A and θ_B between which maximum fluctuation (addition or delivery) of energy occurs above the mean value. For this, the following equation is solved for N different roots of θ:

$$T - \bar{T} = 0$$

The variation in energy is calculated between the successive roots of θ using the following equation (n varying from 1 to N):

$$e = \int_{\theta_n}^{\theta_{n+1}} (T - \bar{T}) d\theta$$

Out of such variations, θ_A and θ_B are chosen between which the maximum variation of energy is given by

$$e_{\max} = \int_{\theta_A}^{\theta_B} (T - \bar{T}) d\theta$$

Maximum and minimum speed occurs at the points of maximum and minimum kinetic energy.

3.8 BELT DRIVE

A belt is a looped strip of flexible material used to mechanically link two or more rotating shafts. A belt drive offers smooth transmission of power between two shafts at considerable distance. The drive is lubrication-free and requires minimum maintenance. The pulleys are usually less expensive than chain drive sprockets and exhibit little wear over long periods of operation.

The *belt drive* is not a positive drive. Let s_1 and s_2 be the slips in driver pulley and driven pulley, respectively. Let N_1 and N_2 denote speeds of driver and driven pulleys having diameter D and d, respectively. For belt thickness t, speed ratio of the drive is given by

$$\frac{N_2}{N_1} = (1 - s_1)(1 - s_2)\frac{D + t}{d + t}$$

Therefore, the total slip is

$$s = (1 - s_1)(1 - s_2)$$

Belt drives have drawback of slippage, wear in belts, and unsuitability in adverse service conditions. Slip in *timing belts* (used in time crank and cam) is zero because they are toothed belts.

3.8.1 Types of Belt Drives

In a two-pulley system, the belt can either drive the pulleys in the same direction (open drive), or the belt can be crossed so that the direction of the shafts is opposite (crossed drive) [Fig. 3.43].

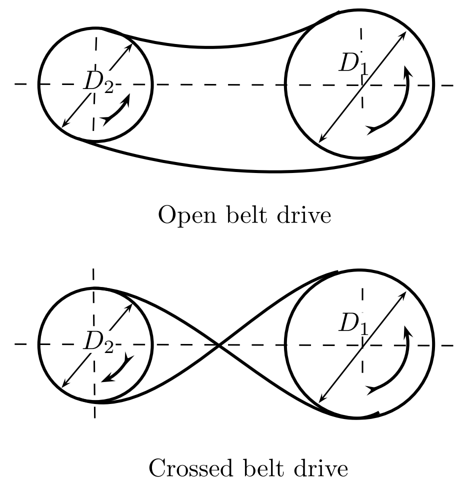


Figure 3.43 Open and crossed belt drives.

The two types of belt drives are explained as follows:

1. Open Belt Drive An *open belt drive* is used to rotate the driven pulley in the direction of driving pulley. Power transmission results makes one side of the belt more tightened (*tight side*) as compared to the other (*slack side*). In horizontal drives, tight side is always kept in the lower side of two pulleys because the sag of the upper side slightly increases the angle of wrap of the belt on the two pulleys.

2. Crossed Belt Drive A *crossed belt drive* is used to rotate driven pulley in the opposite direction of the driving pulley. Higher the value of wrap enables more power can be transmitted than an open belt drive. However, bending and wear of the belt are important concerns.

3.8.2 Length of Belts

The length of the belt depends upon the type of belt drive. The expressions are derived as follows:

1. Open Belt Drive Consider an open belt drive consisting of two pulleys of diameters D and d kept apart by a center distance c [Fig. 3.44].

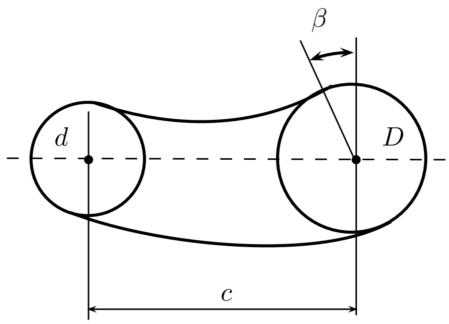


Figure 3.44 Open belt drive.

Angle of lap β is related as

$$\begin{aligned}\sin\beta \approx \beta &= \frac{D-d}{2c}\\ \cos\beta &= \sqrt{1-\sin^2\beta}\\ &= \left\{1-\left(\frac{D-d}{2c}\right)^2\right\}^{1/2}\\ &\approx 1-\frac{1}{2}\frac{(D-d)^2}{4c^2}\end{aligned}$$

These expressions for β and $\cos\beta$ is used as

$$\begin{aligned}L &= 2\left\{\frac{\pi}{2}-\beta\right\}\frac{d}{2}\\ &\quad +2\left\{\frac{\pi}{2}+\beta\right\}\frac{D}{2}\\ &\quad +2c\left\{1-\frac{1}{2}\frac{(D-d)^2}{4c^2}\right\}\\ &= 2\left\{\frac{\pi}{2}-\frac{D-d}{2c}\right\}\frac{d}{2}\\ &\quad +2\left\{\frac{\pi}{2}+\frac{D-d}{2c}\right\}\frac{D}{2}\\ &\quad +2c\left\{1-\frac{1}{2}\frac{(D-d)^2}{4c^2}\right\}\\ &= \pi\frac{(D+d)}{2}+2c+\frac{(D-d)^2}{4c}\end{aligned}$$

2. *Crossed Belt Drive* Consider a crossed belt drive consisting of two pulleys of diameters D and d kept apart by a distance c [Fig. 3.45].

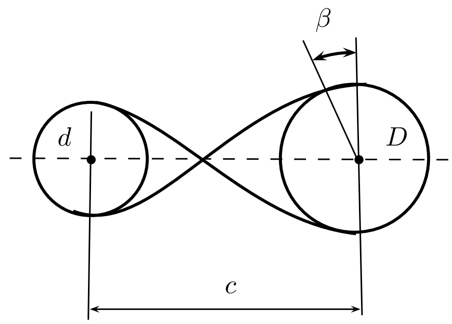


Figure 3.45 Crossed belt drive.

Angle of lap β is related as

$$\begin{aligned}\sin\beta \approx \beta &= \frac{D+d}{2c}\\ \cos\beta &= \sqrt{1-\sin^2\beta}\\ &= \left\{1-\left(\frac{D+d}{2c}\right)^2\right\}^{1/2}\\ &\approx 1-\frac{1}{2}\frac{(D+d)^2}{4c^2}\end{aligned}$$

Length of the belt is expressed as

$$\begin{aligned}L &= 2\left(\frac{\pi}{2}+\beta\right)\frac{d}{2}+2\left(\frac{\pi}{2}-\beta\right)\frac{D}{2}+2c\cos\beta\\ &= 2\left\{\frac{\pi}{2}+\frac{D+d}{2c}\right\}\frac{d}{2}+2\left\{\frac{\pi}{2}-\frac{D+d}{2c}\right\}\frac{D}{2}\\ &\quad +2c\left\{1-\frac{1}{2}\frac{(D+d)^2}{4c^2}\right\}\\ &= \pi\frac{(D+d)}{2}+2c+\frac{(D+d)^2}{4c}\end{aligned}$$

3.8.3 Power Transmission

Let a belt of mass m per unit length rotate with peripheral velocity v over a pulley of radius r [Fig. 3.46]. An elemental length $\delta l = r\delta\theta$ of the belt passing over the pulley is subjected to centrifugal force, which creates centrifugal tension T_c both on tight and slack sides. Let μ be the coefficients of friction and R be the reaction between belt and pulley.

3.8.3.1 Initial Tension The belt is assembled with an initial tension. When power is transmitted, the tension in tight side increases from T_i to T_1 and on the slack side decreases from T_i to T_2. If the belt is assembled to obey Hooke's law and its length remains constant, then

$$\begin{aligned}T_1-T_i &= T_i-T_2\\ T_i &= \frac{T_1+T_2}{2}\end{aligned}$$

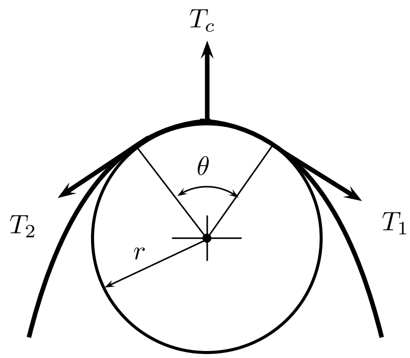


Figure 3.46 | Centrifugal tension in belt drives.

3.8.3.2 Centrifugal Tension The centrifugal tension in the belt is determined as

$$m\,(r d\theta)\,\frac{v^2}{2} = 2T_c\frac{d\theta}{2}$$
$$T_c = mv^2$$

3.8.3.3 Ratio of Driving Tensions Equilibrium of the belt element is examined in two orthogonal directions:

1. *Radial Direction*

$$R + T_c d\theta = (T + dT)\,\frac{d\theta}{2} + T\frac{d\theta}{2}$$
$$R = (T - T_c)\,d\theta \tag{3.30}$$

2. *Tangential Direction*

$$T + dT - T = \mu R$$
$$dT = \mu R \tag{3.31}$$

Using Eqs. (3.30) and (3.31),

$$dT = \mu\,(T - T_c)\,d\theta$$
$$\int_{T_1}^{T_2} \frac{dT}{T - T_c} = \int_0^{\theta} \mu d\theta$$
$$\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta}$$

where T_1 and T_2 are actual tensions in the belt, $T_1 - T_c$ and $T_2 - T_c$ are the driving tension or effective tensions on the pulley. The equation indicates that if the diameter of the driving and driven pulleys are unequal, the belt will slip first on the pulley having smaller angle of lap θ (i.e. smaller pulley in open belt, friction will be less). Therefore, the *angle of lap* (also called *angle of wrap* or *contact*) is taken for the angle subtended by the segment of belt in contact with smaller pulley at the center.

The idler pulleys are used to tighten the belts and increases the angle of lap. Idler pulleys are held against the belt by their own weight in addition to an adjustable weight. Idler pulley are not provided crowning, and are kept on loose sides, nearer to smaller pulley.

3.8.3.4 Power Transmitted The power transmitted through the belt drive is determined as

$$\begin{aligned} P &= (T_1 - T_2)\,v \\ &= \{(T_1 - T_c) - (T_2 - T_c)\}\,v \\ &= (T_1 - T_c)\left\{1 - e^{-\mu\theta}\right\} v \\ &= \left(T_1 - mv^2\right)\left\{1 - e^{-\mu\theta}\right\} v \end{aligned}$$

For maximum power transmission,

$$\frac{dP}{dv} = 0$$
$$\frac{d}{dv}\left(T_1 - mv^2\right) v = 0$$
$$T_1 - 3mv^2 = 0$$
$$T_c = \frac{T_1}{3}$$

Belt-velocity for maximum power transmission is

$$v^* = \sqrt{\frac{T_1}{3m}}$$

At this velocity, the maximum possible power transmission is

$$\begin{aligned} P &= \left(T_1 - \frac{T_1}{3}\right)\left\{1 - e^{-\mu\theta}\right\} v \\ &= \frac{2T_1}{3}\left\{1 - e^{-\mu\theta}\right\} v \end{aligned}$$

3.8.3.5 Effect of Centrifugal Tension If centrifugal tension is neglected, then

1. *Ratio of Driving Tensions*

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

2. *Power Transmitted*

$$P = T_1\left\{1 - e^{-\mu\theta}\right\} v$$

Therefore, the effect of centrifugal is to reduce the power transmitted. The maximum efficiency of belt drive remains unaffected to 66.67%.

3.8.4 Crowning of Pulleys

Pulleys of flat drives are crowned by producing slightly conical or convex surface on the rim. When the belt slips off the pulley, the crown helps it to adhere to the cone surface due to pull on the belt. Crowning is always done on the driving pulley because tension is on entry side.

3.8.5 Law of Belting

According to *law of belting*, the center line of the belt when it approaches a pulley must lie in the mid-plane of the pulley. Hence, in non-parallel shafts, rotation is possible in only one direction.

3.8.6 Elastic Creep

Presence of friction between pulley and belt causes differential tension in the belt. This differential tension causes the belt to elongate or contract, and thus create a relative motion between the belt and the pulley surface. This slip is called *elastic creep*. This reduces the speed of the belt and power transmission.

3.8.7 V-Belts

In V-belt drives, the angle of groove β results in a higher coefficient of friction, known as *effective coefficient of friction*, given by

$$\mu' = \frac{\mu}{\sin(\beta/2)} \tag{3.32}$$

which is always greater than μ. This is the main reason behind use of V-belts. V-belt groove angle is 34—36° while pulley groove angle is 40°. To avoid locking, V-belts are designed not to touch the bottom of the V-groove.

3.9 FRICTION

The sliding of one solid body in contact with a second solid body is always restricted by a force called the force of *friction*. It acts in opposite direction to that of the relative motion and is tangential to the surface of the two bodies at the point of contact.

Friction is an important aspect in every machine because it involves wearing of machine components and consumes energy that dissipates into heat. Sometimes friction is also desirable for functioning of a machine, such as friction clutches, belt drives.

3.9.1 Theory of Friction

Consider a body of weight W resting on a smooth and dry plane surface. The normal reaction at the surface is R_n [Fig. 3.47].

If a small horizontal force F is applied to the body to move it over the surface, until the body is unable to move, the equilibrium equation of the body is given by

$$R_n = W$$
$$F = F'$$

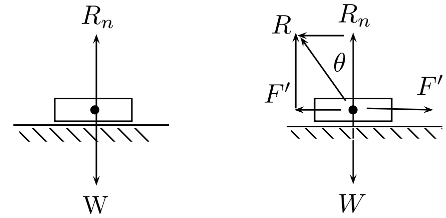


Figure 3.47 Coefficient of friction.

where F' is the horizontal force resting on the motion of the body. Let R be the resultant of R_n and F', expressed as

$$R_n = R\cos\theta$$
$$F' = R\sin\theta$$

If the pull F is increased continuously, the above angle θ would attain a limiting value ϕ [Fig. 3.48] when the body will just move and with μ as the *coefficient of friction*,

$$F' = \mu R_n$$
$$\tan\phi = \frac{F'}{R_n} = \mu$$

Here, ϕ is called *angle of friction*.

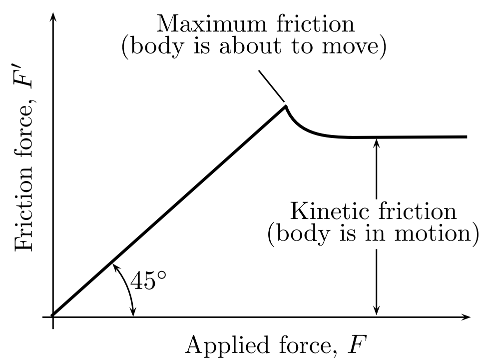


Figure 3.48 Friction force versus applied force.

The cone of friction is the imaginary cone generated in the case of non-coplanar forces by revolving the static resultant reaction R about the normal. Its cone angle will be 2ϕ [Fig. 3.49].

3.9.1.1 Friction Circle Greasy friction occurs in heavy loaded, slow running bearings. When a shaft rests in its bearing, the weight of the shaft W acts through its center of gravity. The reaction of the bearing acts in line with W in the vertically upward direction. The shaft rests on the bearing in metal-to-metal contact.

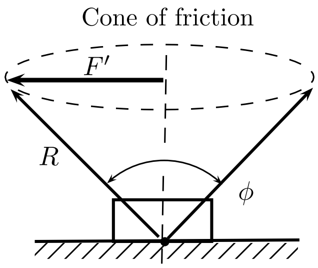


Figure 3.49 | Cone of friction.

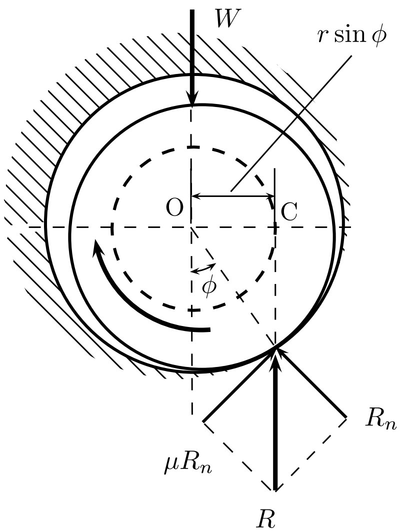


Figure 3.50 | Friction circle.

When a torque is applied to the shaft, it rotates and seat of pressure creeps or climbs up the bearing in a direction opposite to that of rotation. Metal-to-metal contact still exists and greasy friction criterion applies as the oil film will be of molecular thickness. For equilibrium, the resultant reaction R must vertically act upward and must be equal to W, however, these two forces at a distance OC, constitute a couple [Fig. 3.50]. If r is the radius of journal (shaft), the friction torque will be given by

$$T = W \times r \sin\phi$$

A circle drawn with radius $r\sin\phi$ is known as the *friction circle* of the journal.

3.9.2 Inclined Plane

Consider a body of weight W over an inclined plane at an angle α to the horizontal. The limiting angle of friction between the surfaces is ϕ. Let a force F be applied to cause the body slide with uniform velocity parallel to the slope. On the limiting case, it will be equal to the angle of friction ϕ.

Three situations are possible for body over inclined plane: at rest, moving up, and moving down the plane. The expressions for forces and efficiency in such cases are derived as follows.

3.9.2.1 Body at Rest For the equilibrium of the body at rest on the plane [Fig. 3.51], the limiting resultant friction force R is given by

$$\begin{aligned} W\sin\alpha &= \mu W\cos\alpha \\ \tan\alpha &= \mu \\ &= \tan\phi \\ \alpha &= \phi \end{aligned}$$

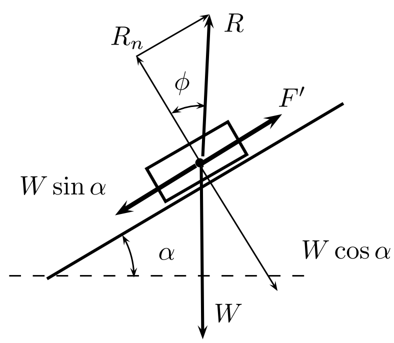


Figure 3.51 | Body at rest on the plane.

Angle of repose is the maximum angle to which an inclined plane can be raised before an object resting on it starts moving under the action of its won weight and friction resistance. Above equation shows that angle of response is equal to the angle of friction.

3.9.2.2 Body Moving Up the Plane For the equilibrium of the body moving up the plane [Fig. 3.52], using *Lami's theorem*:

$$\frac{R}{\sin\theta} = \frac{F}{\sin(\alpha+\phi)} = \frac{W}{\sin\{\theta-(\alpha+\phi)\}}$$

Therefore

$$\frac{F}{W} = \frac{\sin(\phi+\alpha)}{\sin\{\theta-(\alpha+\phi)\}} \tag{3.33}$$

In the above equation, the pulling force will be minimum if the denominator on right hand side is maximum, therefore,

$$\begin{aligned} \sin\{\theta-(\alpha+\phi)\} &= 1 \\ \theta-(\alpha+\phi) &= \frac{\pi}{2} \\ \theta-\left(\frac{\pi}{2}+\alpha\right) &= \phi \end{aligned}$$

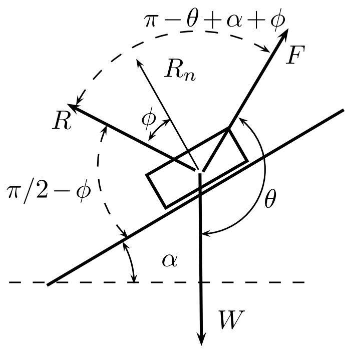


Figure 3.52 | Motion up the plane.

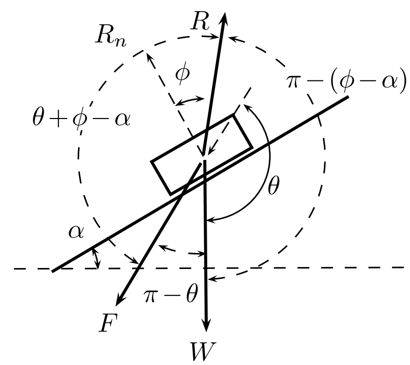


Figure 3.53 | Motion down the plane.

This equation indicates that the pulling force (F) will be minimum if the angle between F and inclined plane is equal to the angle of friction. In that case, the minimum pulling force will be given by

$$F = W\sin(\alpha+\phi)$$

In this case, *efficiency* of inclined plane is the ratio of the pulling force required to move the body without friction ($\mu = 0$) and with friction. Therefore, using Eq. (3.33),

$$\begin{aligned}\eta &= \frac{F_0}{F}\\ &= \frac{\sin\alpha}{\sin(\theta-\alpha)} \times \frac{\sin\{\theta-(\alpha+\phi)\}}{\sin(\alpha+\phi)}\\ &= \frac{\cot(\phi+\alpha)-\cot\theta}{\cot\alpha-\cot\theta}\end{aligned}$$

In the above equation, when pulling force F is applied in horizontal direction ($\theta = \pi/2$),

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

3.9.2.3 Body Moving Down the Plane

For the equilibrium of the body moving down the plane [Fig. 3.53], using *Lami's theorem*:

$$\begin{aligned}\frac{F}{\sin\{\pi-(\phi-\alpha)\}} &= \frac{W}{\sin\{\theta-\alpha+\phi\}}\\ \frac{F}{W} &= \frac{\sin(\phi-\alpha)}{\sin\{\theta+(\phi-\alpha)\}}\end{aligned} \tag{3.34}$$

If friction is neglected in the above case, then

$$F = -\frac{-W\sin\alpha}{\sin(\theta-\alpha)}$$

Negative sign in the above expression indicates that the weight component adds the effort to move the body in the downward direction, therefore, force is required in opposite direction to oppose the downward motion.

In this case, *efficiency of inclined plane* is the ratio of the force required to move the body with friction and without friction ($\mu = 0$). Therefore, using Eq. (3.34),

$$\begin{aligned}\eta &= \frac{F}{F_0}\\ &= \frac{\sin(\phi-\alpha)}{\sin\{\theta+(\phi-\alpha)\}} \times \frac{\sin(\theta-\alpha)}{-\sin\alpha}\\ &= \frac{\cot\alpha-\cot\theta}{\cot(\phi-\alpha)+\cot\theta}\end{aligned}$$

In the above equation, when pulling force F is applied in horizontal direction ($\theta = \pi/2$),

$$\eta = \frac{\tan(\phi-\alpha)}{\tan\alpha}$$

3.9.3 Friction in Screw Threads

Screw and nut combinations are used to convert rotary motion into translational motion and transmit power. Screw threads are mainly of two types namely, square threads and V-threads. The V-threads offer more resistance to the motion than square threads, therefore, the square threads are used in screw jacks, whereas V-threads are used for tightening of two components.

The axial distance traveled by thread in one turn is called lead. Pitch (p) is the distance between two adjacent threads parallel to axis of the screw. For single start threads, lead is equal to pitch. *Helix angle* (α) is the slope or inclination of the threads with horizontal. For single start screws [Fig. 3.54], if d is the mean diameter of the helix, then

$$\tan\alpha = \frac{p}{\pi d}$$

In the V-threads shown in Fig. 3.55, the reaction on thread is inclined by angle β with vertical direction of load W. Therefore, due to wedge effect, the *effective*

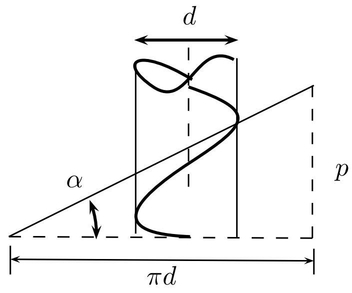


Figure 3.54 Helix angle of screw threads.

coefficient of friction is given by

$$\mu' = \frac{\mu}{\cos(\beta/2)}$$

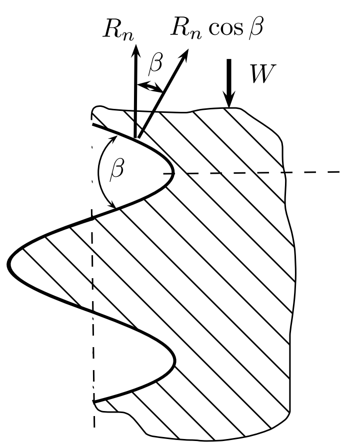


Figure 3.55 Friction in V-threads.

The following derivations are for square threads but V-threads can be analyzed by converting these derivations into equivalent square threads with increase in friction by cosine of half V-angle ($\beta/2$).

3.9.3.1 Lifting Consider a square thread screw used as a jack to lift a load W. Using Eq. (3.33) for $\theta = \pi/2$,

$$F = W\frac{\sin(\phi+\alpha)}{\sin\{\pi/2-(\phi+\alpha)\}}$$
$$= W\tan(\alpha+\phi)$$

If force f is applied at the end of the lever length l, then

$$f \times l = F \times \frac{d}{2}$$
$$f = \frac{Wd}{2l}\tan(\phi+\alpha)$$

Screw efficiency in lifting a load is defined as the ratio of work done in lifting the load per revolution and work done by the applied force per revolution. In one revolution of lifting, the load W travels axial distance equal to lead (l), while the point of applied force F moves the circumferential distance πd, therefore, screw efficiency is given by

$$\eta = \frac{W \times l}{F \times \pi d}$$
$$= \frac{\tan\alpha}{\tan(\alpha+\phi)} \tag{3.35}$$

For maximum screw efficiency,

$$\frac{d\eta}{d\alpha} = 0$$

This results in

$$\alpha = \frac{\pi-\phi}{2}$$

Putting this value in Eq. (3.35), the maximum screw efficiency is found as

$$\eta_{\max} = \frac{1-\sin\phi}{1+\sin\phi}$$

Mechanical advantage is the ratio of weight lifted and applied force:

$$\frac{W}{F} = \frac{2L}{d}\tan(\phi+\alpha)$$

3.9.3.2 Lowering Now, consider a square thread screw used as a jack to lower a load W. Using Eq. (3.34) for $\theta = \pi/2$,

$$F = W\frac{\sin(\phi-\alpha)}{\sin\{\pi/2+(\phi-\alpha)\}}$$
$$= W\tan(\phi-\alpha)$$

If force f is applied at the end of the lever length L, then

$$f \times L = F \times \frac{d}{2}$$
$$f = \frac{Wd}{2l}\tan(\alpha-\phi)$$

This equation indicates that the angle of friction (ϕ) should always be more than the helix angle of the screw (α). Otherwise, the load will slide down of its own weight W. When $\alpha = \phi$, the nut will be on the point of reversing, and using Eq. (3.35) the *screw efficiency in lowering* will be

$$\eta = \frac{\tan\phi}{\tan(2\phi)}$$

For small values of ϕ, $\tan\phi \approx \phi$, hence

$$\eta = \frac{1}{2}$$

Derived with condition $\alpha = \phi$, the above equation indicates that the reversal of nut is avoided if the efficiency of the thread is less than 50% approximately.

3.9.4 Pivots and Collars

Pivots and collars are used to support a rotating shaft subjected to axial loads. *Collars* are provided at any position along the shaft and bears the axial load on a mating surface. *Pivots*, sometimes called footstep bearing, are recesses in which shafts is inserted at one end to bear the axial load [Fig. 3.56]. The surfaces of collars and pivots can be either flat or conical.

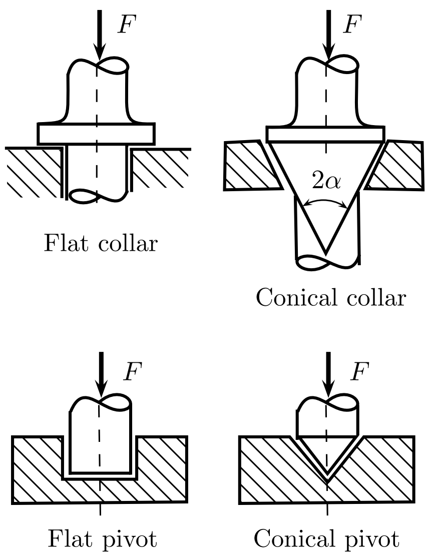


Figure 3.56 | Flat and conical pivots.

The friction torque of a collar or pivot bearing can be determined either by uniform pressure theory or uniform wear theory. Each assumption leads to a different value of friction torque.

3.9.4.1 Uniform Pressure Theory

The *uniform pressure theory* assumes that intensity of pressure on the bearing surface is constant. This can be examined in flat and conical collars/pivots:

1. *Flat Collars and Pivots* Consider a flat collar of radii internal radius R_i, external radius R_o, subjected to axial force F. The uniform pressure intensity at any point in the collar is given by

$$p = \frac{F}{\pi\left({R_o}^2 - {R_i}^2\right)}$$

The friction torque will be given by

$$\begin{aligned} T &= \int_{R_i}^{R_o} r \times p \times 2\pi r \times dr \\ &= \int_{R_i}^{R_o} 2p\pi r^2 dr \\ &= \mu F \times \underbrace{\frac{2}{3} \times \frac{{R_o}^3 - {R_i}^3}{{R_o}^2 - {R_i}^2}}_{\bar{R}} \end{aligned} \qquad (3.36)$$

Friction torque for flat pivot, based on uniform pressure theory, can be derived using the above expression by taking $R_i = 0$, $R_o = R$:

$$T = \mu F \times \frac{2R}{3}$$

This expression is comparable to Eq. (3.36), where average radius is $\bar{R}$ given by

$$\bar{R} = \frac{2}{3} \times \frac{{R_o}^3 - {R_i}^3}{{R_o}^2 - {R_i}^2}$$

2. *Conical Collars and Pivots* Consider a conical collar of radii internal radius R_i, external radius R_o, half cone angle α, subjected to axial force F. The uniform pressure intensity at any point in the collar is given by

$$F = \int_{R_i}^{R_o} p \times 2\pi r \times \frac{dr}{\sin\alpha}$$

$$p = \frac{F \sin\alpha}{\pi\left({R_o}^2 - {R_i}^2\right)}$$

The friction torque is given by

$$T = \frac{\mu}{\sin\alpha} F \times \underbrace{\frac{2}{3} \times \frac{{R_o}^3 - {R_i}^3}{{R_o}^2 - {R_i}^2}}_{\bar{R}} \qquad (3.37)$$

Friction torque for conical pivot, based on uniform pressure theory, can be derived using the above expression by taking $R_i = 0$, $R_o = R$:

$$T = \frac{2}{3} \times \frac{\mu}{\sin\alpha} \times FR$$

This expression is comparable to Eq. (3.37), where the average radius $\bar{R}$ is given by

$$\bar{R} = \frac{2}{3} \times \frac{{R_o}^3 - {R_i}^3}{{R_o}^2 - {R_i}^2}$$

3.9.4.2 Uniform Wear Theory

The *uniform wear theory* assumes the uniform wearing of the bearing surface. For this, the intensity of pressure should be inversely proportional to the elemental areas. In this

case, for the two locations at r_1 and r_2 with the same width b, the pressure intensities are given by

$$p_1 \times 2\pi r_1 b = p_2 \times 2\pi r_2 b$$
$$p_1 r_1 = p_2 r_2$$

Hence

$$pr = \text{constant}$$

This can be examined in flat and conical collars/pivots:

1. *Flat Collars and Pivots* By this theory, for a flat collar of radii internal radius R_i, external radius R_o, subjected to axial force F, the pressure intensity p at any radius r will be given by

$$p = \frac{F}{2\pi r\,(R_o - R_i)}$$

The friction torque will be

$$\begin{aligned} T &= \int_{R_i}^{R_o} 2p\pi r^2 dr \\ &= \int_{R_i}^{R_o} 2\frac{F}{2\pi r\,(R_o - R_i)}\pi r^2 dr \\ &= \mu F \times \underbrace{\frac{R_o + R_i}{2}}_{\bar{R}} \end{aligned} \tag{3.38}$$

Friction torque for flat pivot, based on uniform wear theory, can be derived using the above expression by taking $R_i = 0$, $R_o = R$:

$$T = \frac{1}{2}\mu F R$$

This expression is comparable to Eq. (3.38), where the average radius $\bar{R}$ is given by

$$\bar{R} = \frac{R_o + R_i}{2}$$

2. *Conical Collars and Pivots* Similarly, for a conical collar of radii internal radius R_i, external radius R_o, half cone angle α, subjected to axial force F, the pressure intensity p at any radius r will be given by

$$p = \frac{F}{2\pi r\,(R_o - R_i)}$$

The friction torque will be given by

$$T = \int_{R_i}^{R_o} r \times \left(\mu p \times 2\pi r \times \frac{dr}{\sin\alpha}\right) \tag{3.39}$$

$$= \frac{\mu F}{\sin\alpha} \times \frac{R_o + R_i}{2} \tag{3.40}$$

Friction torque for conical pivot can be derived using above expression by taking $R_i = 0$, $R_o = R$:

$$T = \frac{\mu}{\sin\alpha} \times F \times \frac{R}{2}$$

This expression is comparable to Eq. (3.40), where average radius is $\bar{R}$ given by

$$\bar{R} = \frac{R_o + R_i}{2}$$

In the above cases, the effect of half cone angle α is to increase the effective coefficient of friction as

$$\mu' = \frac{\mu}{\sin\alpha}$$

Also, by keeping $\alpha = \pi/2$ ($\sin\alpha = 1$) on the expressions for conical bearings (both collar and pivot), the expressions for flat bearings are derived.

The expressions obtained by uniform pressure theory and uniform wear theory give different values. In all the above cases, uniform wear theory gives smaller friction torque than that by uniform pressure theory.

3.9.5 Friction Clutches

A *clutch* is a device used to transmit the rotary motion from one shaft to another. In *friction clutches*, the connection of the two shafts is affected by friction between the two mating concentric surfaces when pressed against each other.

In case of multi-plate clutch having n_1 and n_2 plates on driving and driven shafts, the number of friction surfaces shall be given by

$$n = n_1 + n_2 - 1$$

The expressions of friction torque derived for pivots and collars can be used to determine the maximum torque and power transmission. In design of clutches, the objective is to make the clutch capable for maximum torque transmission, therefore, using *uniform wear theory* gives safer values.

3.10 GOVERNOR

Governor is a device used to maintain the speed of an engine within specified limits when the engine works in varying of load. This can be distinguished from that of a flywheel which controls energy fluctuations per cycle (Table 3.2). The functioning of flywheel is independent of speed. The operation of a governor is intermittent while that of a flywheel is continuous.

3.10.1 Types of Governors

Based on the source of controlling force, the governors are of two types:

Table 3.2 Governor versus flywheel

	Governor	Flywheel
1.	Provided on prime-movers	Provided in engine and machines
2.	Regulates supply of fuel	Stores mechanical energy
3.	Takes care of long range variation in load	Take care of variation in the cycle
4.	Works only when load changes	Works in each cycle

1. *Centrifugal Governors* In *centrifugal governors*, two or more masses, known as *governor balls*, are caused to revolve about the axis of a shaft, which is driven through suitable gearing from the engine crankshaft. Each ball is acted upon by a force which acts in the radially inward direction, and is provided by a deadweight, a spring or a combination of two. This force is termed as the *controlling force* and it must increase in magnitude as the distance of the ball from the axis of rotation increases. The inward or outward movement of the balls is transmitted by the governor mechanism to the valve which controls the amount of energy supplied to the engine.

2. *Inertia Governors* In inertia governors, the balls are so arranged that the inertia forces caused by an *angular acceleration* or retardation of the governor shaft tend to alter their position. The obvious advantage of this type of governor lies in its more rapid response to the effect of a change of load. This advantage is offset, however, by the practical difficulty of arranging for the complete balance of the revolving parts of the governor. For this reason, centrifugal governors are much more frequently used than are inertia governors, and thus, only the former type will be dealt with here.

The concept of some important governors is explained as follows

3.10.1.1 Watt Governor *Watt governor*[8], although now obsolete, is interesting as being the forerunner of the later examples of governors. It consists of two balls attached to the spindle through four arms. The two upper arms meet at the pivot on the spindle axis. The lower arms are connected at a sleeve by pin joints. The movement of the sleeve is restricted by stops [Fig. 3.57].

[8]Watt governor is the original form of governor as used by Watt on some of his early steam engines.

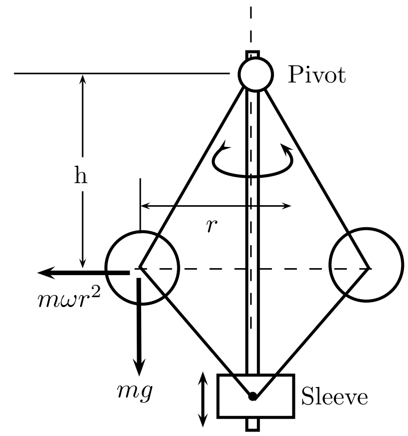


Figure 3.57 Watt governor.

Consider the situation when the rotation speed is N rpm ($=\omega$ rad/s) and the balls are located at radius r and height h measured from pivot. The centrifugal forces acting on all four balls is $m\omega^2 r$. The weight of the balls is mg. Taking moment of these forces acting on the balls about the pivot gives

$$m\omega^2 r \times h - mg \times r = 0$$

$$h = \frac{g}{\omega^2}$$

Using this equation, the height h can be expressed in terms of speed N (rpm) as

$$h = \frac{895}{N^2}$$

Differentiating the above equation w.r.t. N, the change in height is found to be related with change in speed as

$$\delta h \propto -\frac{\delta N}{N^3}$$

Therefore, with increasing speeds, δh becomes insignificant, and governor stops functioning. Therefore, Watt governors are suitable for slow speed engines only.

3.10.1.2 Porter Governor The porter governor is a modified Watt governor in which a heavy central mass M is placed to the sleeve. The action is exactly the same as that of the Watt governor [Fig. 3.58].

With equilibrium equation of balls, the expression for height h can be derived as

$$h = \frac{mg + (Mg \pm F)(1+q)/2}{mg} \times \frac{895}{N^2}$$

where F is the frictional force on movement of central mass, and q is a constant defined as

$$q = \frac{\tan\alpha}{\tan\beta}$$

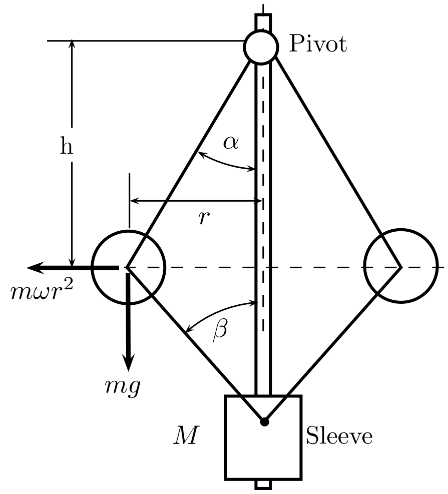


Figure 3.58 Porter governor.

The quantity q will have a different values for each radius of rotation of the governor balls, unless the upper and lower arms are of equal length.

3.10.1.3 Proel Governor A Proel governor is similar to the Porter governor in that it is has a heavily weighted sleeve, but differs from it in the arrangement of balls [Fig. 3.59]. These are carried on the extension of the lower arm instead of being carried out at the junction of the upper and lower arms. The action of this governor is similar to Watt governor.

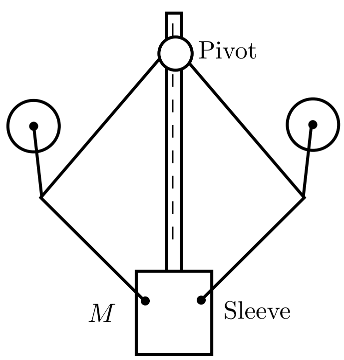


Figure 3.59 Proel governor.

3.10.1.4 Hartnell Governor The Hartnell governor is of the spring loaded type Fig. 3.60. It consists of two bell crank levers pivoted at points offset to central axis. The frame is attached to the governor spindle and rotates with it. Each ball lever carries a ball at the end of vertical arms, and roller at the other end of horizontal arm. A helical spring provides equal downward forces on the two rollers through sleeve.

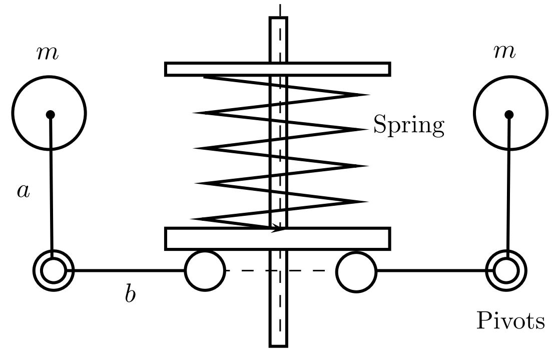


Figure 3.60 Hartnell governor.

Let r_1 and r_2 be the radius of balls at two speeds ω_1 and ω_2. The centrifugal forces acting on the balls in both conditions will be

$$F_1 = m{\omega_1}^2 r_1$$
$$F_2 = m{\omega_2}^2 r_2$$

Taking moments of forces acting on the balls w.r.t. pivots for both situations separately,

$$F_1 a_1 = \frac{Mg+f_s+f}{2} b_1 - mgc_1$$
$$F_2 a_2 = \frac{Mg+f_s+f}{2} b_2 - mgc_2$$

where f_s is spring force, f is friction force, and c_1 and c_2 are the offsets of the balls. Taking,

$$a_1 \approx a_2 \approx a, \quad b_1 \approx b_2 \approx b, \quad c_1 = c_2 = 0$$

Therefore,

$$F_1 a = \frac{Mg+f_{s_1}+f \times b}{2}$$
$$F_2 a = \frac{Mg+f_{s_2}+f \times b}{2}$$

Subtracting the above two equations,

$$(F_2 - F_1)\, a = (f_{s_2} - f_{s_1})\, \frac{b}{2}$$

The difference in spring force is

$$f_{s_2} - f_{s_1} = \frac{2a}{b}(F_2 - F_1)$$

3.10.2 Sensitiveness and Stability

The following are the important terms related to sensitiveness and stability of governors:

1. *Sensitiveness* Sensitiveness of a governor is correctly defined as the ratio of the difference

between the maximum and minimum equilibrium speeds to the mean equilibrium speed:

$$\text{Sensitiveness} = \frac{N_1 - N_2}{N}$$

where N_1 and N_2 shows the speed range between which the governor is insensitive, and N is the mean equilibrium speed. This is also referred as the *coefficient of insensitiveness* as a measure of insensitiveness of a governor.

2. *Isochronism* A governor is said to be *isochronous* when it has the same equilibrium speed for all the positions of sleeve or the balls; any change of speed results in moving the balls into extreme positions. Thus, an isochrounous governor will have infinite sensitiveness.

3. *Stability* A governor is said to be stable when for each speed within the working range, there is only one radius of rotation of the governor balls at which the governor is in equilibrium.

4. *Hunting* *Sensitiveness* of a governor is a desirable quality. However, if a governor is too sensitive, it can fluctuate continuously because when the load on the engine falls, the sleeve rises rapidly to a maximum position. This shuts off the fuel supply. If the frequency of fluctuations in engine speed happens to coincide with the natural frequency of oscillations of the governor, then due to resonance, the amplitude of oscillations becomes very high with the result that the governor tends to intensify the speed variations instead of controlling it. Such a situation is known as *hunting*.

3.10.3 Controlling Force

In a centrifugal governor, the resultant of all external forces which control the movement of the ball can be regarded as a single inward radial force acting at the center of the ball. The variation of this force F with radius of rotation r of the ball, which is necessary to keep the ball in equilibrium at various configurations (i.e. for different values of r). The force F is known as *controlling force* which is function of single variable r:

$$F = f(r)$$

Let the ball rotates at a speed ω then centrifugal force needed for maintaining the radius r is $m\omega^2 r$. So, for given value of ω

$$F \propto r$$

The governor is said to be stable when slope of speed curve is less than that of controlling force curve. For a given governor to be stable at all radii (i.e. throttling can be controlled and will bring speed to the desired value

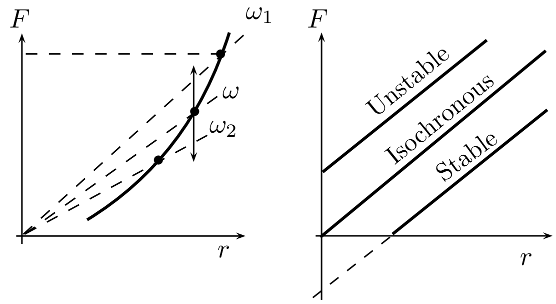


Figure 3.61 Controlling force diagram.

without hunting), the controlling force curve should pass $r = +\text{ve}$ or 0 [Fig. 3.61]. For stability,

$$\frac{F}{r} < \frac{\partial F}{\partial r}$$

The friction force at the sleeve gives rise to the insensitiveness in the governor. At any given radius r, there will be two different speeds one being when sleeve moves up and other being when sleeve moves down. Fig. 3.61 shows that when the speed increases from ω (N) to ω_1 (N_1), r increases to r_1, and F increases to F_1 and sleeve closes the throttling value. Similarly, when $\omega_1 \to \omega_2$ (N_2), and r decreases to r_2.

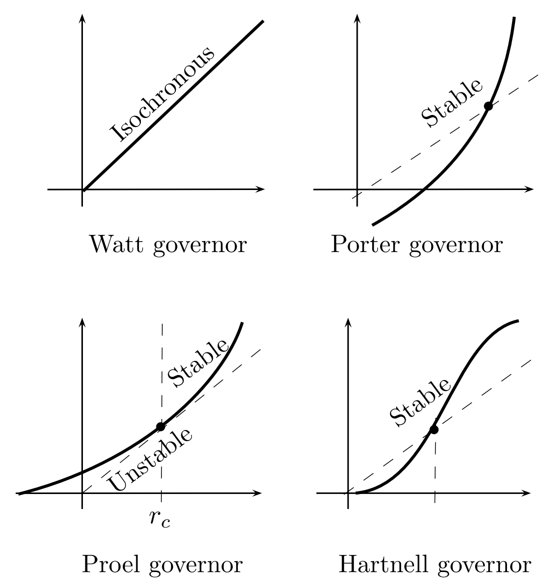


Figure 3.62 Stability and isochronism of gonvernors

Figure 3.62 depicts the plots of controlling force versus radius of rotation of balls for different types of governors. It is evident that Watt governor is an isochronous governor. Porter governor is a stable governor while Proel governor is stable after a certain radius. Hartnell governor is also a stable governor.

IMPORTANT FORMULAS

Fundamentals

1. Degree of freedom
   (a) Gruebler's equation
   $$F = 3(n-1) - 2f_1$$
   (b) Kutzbach equation
   $$F = 3(n-1) - 2f_1 - f_2$$
   (c) Number of links
   $$n = \frac{2}{3}\left(J + 2 + \frac{f_2}{2}\right)$$

Universal Joint

1. Shaft rotation
$$\frac{\tan\phi}{\tan\theta} = \sec\alpha$$
$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\theta}$$
2. For $\omega_1 = \omega_2$
$$\tan\theta = \pm\sqrt{\cos\alpha}$$
3. Minimum and maximum speed ratios
$$\left(\frac{\omega_2}{\omega_1}\right)_{min} = \cos\alpha$$
$$\left(\frac{\omega_2}{\omega_1}\right)_{max} = \frac{1}{\cos\alpha}$$
4. Maximum variation
$$\frac{\omega_{max} - \omega_{min}}{\omega_1} = \tan\alpha\sin\alpha$$
5. For minimum or maximum $\dot{\omega}_2$
$$\cos 2\theta = \frac{2\sin^2\alpha}{2 - \sin^2\alpha}$$
6. Double Hooke's joint
$$\left(\frac{\omega_2}{\omega_1}\right)_{max} = \frac{1}{\cos^2\alpha}$$
$$\left(\frac{\omega_2}{\omega_1}\right)_{min} = \cos^2\alpha$$

Kinematic Analysis

1. Velocity of point A w.r.t. O
$$V_{AO} = r\omega_{AO}$$
2. Velocity of intermediate point
$$\frac{V_{BO}}{V_{AO}} = \frac{\text{BO}}{\text{AO}}$$
3. Velocity of link AB
$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$
4. Angular velocity of link AB
$$\omega_{AB} = \frac{V_{AB}}{\text{AB}}$$
5. I-centers of n links
$$N = {}^nC_r = \frac{n(n-1)}{2}$$
6. Arnold-Kennedy's theorem states that three bodies, having relative motion with respect to one another, have three instantaneous centers, all of which lie on the same straight line.
7. Angular acceleration
$$\alpha_{AO} = \text{AO} \times {\omega_{OA}}^2$$
8. Acceleration of slider
$$a_P^{\parallel} = f - \omega^2 r$$
$$a_P^{\perp} = 2\omega v + r\alpha$$
9. Coriolis component $= 2\omega v$

Cam Profiles

1. Simple harmonics
$$s = \frac{h}{2}\left\{1 - \cos\left(\frac{\pi\theta}{\phi}\right)\right\}$$
$$v = \frac{h}{2}\left\{\frac{\pi}{\phi}\omega\sin\left(\frac{\pi\theta}{\phi}\right)\right\}$$
$$v_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi}\right)$$
$$f = \frac{h}{2}\left\{\frac{\pi^2}{\phi^2}\omega^2\cos\left(\frac{\pi\theta}{\phi}\right)\right\}$$
$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi}\right)^2$$
2. Constant acceleration
$$s = \frac{1}{2}ft^2$$
$$f = 4h\left(\frac{\omega}{\phi}\right)^2$$
$$s = \frac{2h}{\phi^2}\theta^2$$
$$v = \frac{4h\omega}{\phi^2}\theta$$
$$v_{max} = \frac{2h\omega}{\phi}$$
3. Constant velocity
$$v = \frac{h\omega}{\phi}$$
4. Cycloidal profile
$$s = \frac{h}{\pi}\left(\frac{\pi\theta}{\phi} - \frac{1}{2}\sin\frac{2\pi\theta}{\phi}\right)$$
$$v = \frac{h\omega}{\phi}\left(1 - \cos\frac{2\pi\theta}{\phi}\right)$$
$$v_{max} = \frac{2h\omega}{\phi}$$
$$f = \frac{2h\pi\omega^2}{\phi^2}\sin\frac{2\pi\theta}{\phi}$$
$$f_{max} = \frac{2h\pi\omega^2}{\phi^2}$$

Gears

$$p = \frac{T}{d}$$
$$m = \frac{d}{T}$$
$$p_c = \frac{\pi d}{T} = m\pi = \frac{\pi}{p}$$

1. Sliding velocity
$$V_s = \text{CP} \times (\omega_1 + \omega_2)$$
$$\text{Inv}(\phi) = \tan\phi - \phi$$
2. Base circles
$$R_b = R\cos\phi$$
$$r_b = r\cos\phi$$

3. Path of approach

$$\mathrm{CP} = \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi$$
$$\mathrm{CP}_{max} = r\sin\phi$$

4. Path of recess

$$\mathrm{PD} = \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi$$
$$\mathrm{PD}_{max} = R\sin\phi$$

5. Path of contact

$$\mathrm{CD} = \mathrm{CP} + \mathrm{PD}$$

Maximum path of contact

$$\mathrm{A}_2'\mathrm{A}_1' = (R+r)\sin\phi$$

6. Arc of approach

$$\text{Arc CP} = \frac{\mathrm{CP}}{\cos\phi}$$

7. Arc of recess

$$\text{Arc PD} = \frac{\mathrm{PD}}{\cos\phi}$$

8. Arc of contact

$$\text{Arc CD} = \frac{\mathrm{CD}}{\cos\phi}$$

9. Number of pairs of teeth in contact

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{\mathrm{CP}}{p\cos\phi}$$
$$n_{max} = \frac{(R+r)\sin\phi}{p_c\cos\phi} = (z_w + z_p)\frac{\tan\phi}{2\pi}$$

10. To avoid interference,

$$T \geq \frac{2}{\sqrt{1+(1/G)(1/G+2)\sin^2\phi - 1}}$$

Rack and pinion

$$T \geq \frac{2}{\sin^2\phi}$$

11. Angle between helical gears

$$\theta = \psi_1 - \psi_2$$

Slider-Crank Mechanism

$$n = \frac{l}{r}$$
$$\sin\phi = \frac{\sin\theta}{n}$$
$$x_p = r\left\{(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right\}$$
$$v_p = r\omega\left\{\sin\theta + \frac{\sin 2\theta}{2n}\right\}$$
$$a_p = r\omega^2\left\{\cos\theta + \frac{\cos 2\theta}{n}\right\}$$
$$= \frac{\omega}{n} \times \frac{\cos\theta}{\cos\phi} \approx \omega\frac{\cos\theta}{n}$$
$$\alpha_{CR} = -\omega^2\frac{\sin\theta}{n}$$
$$F_i = m\omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$
$$T = F_P r \times \left\{\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right\}$$

Flywheel

$$\omega = \frac{\omega_1 + \omega_2}{2}$$
$$k_s = \frac{\omega_1 - \omega_2}{\omega}$$
$$r = \frac{2+k_s}{2-k_s}$$
$$E = \frac{1}{2}I\omega^2$$
$$e_{max} = \frac{1}{2}I(\omega_1^2 - \omega_2)^2 = 2k_sE$$
$$k_s = \frac{e_{max}}{2E}$$
$$e_{max} = \int_{\theta_A}^{\theta_B}(T - \bar{T})d\theta$$

Belt Drives

$$\frac{N_2}{N_1} = (1-s_1)(1-s_2)\frac{D+t}{d+t}$$
$$s = (1-s_1)(1-s_2)$$
$$L_o = \pi\frac{(D+d)}{2} + 2c + \frac{(D-d)^2}{4c}$$
$$L_c = \pi\frac{(D+d)}{2} + 2c + \frac{(D+d)^2}{4c}$$
$$T_i = \frac{T_1 + T_2}{2}$$
$$T_c = mv^2$$
$$\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta}$$
$$P = \left(T_1 - mv^2\right)\left\{1 - e^{-\mu\theta}\right\}v$$
$$v^* = \sqrt{\frac{T_1}{3m}}$$
$$\mu' = \frac{\mu}{\sin(\beta/2)}$$

Friction

$$R_n = R\cos\theta$$
$$F' = R\sin\theta$$
$$\tan\phi = \frac{F'}{R_n} = \mu$$

1. Body moving up

$$\eta = \frac{\cot(\phi+\alpha) - \cot\theta}{\cot\alpha - \cot\theta}$$

2. Body moving down

$$\eta = \frac{\cot\alpha - \cot\theta}{\cot(\phi-\alpha) + \cot\theta}$$

3. Screw thread friction

$$\mu' = \frac{\mu}{\cos(\beta/2)}$$

(a) Lifting

$$\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}$$

(b) Lowering

$$\eta = \frac{\tan\phi}{\tan(2\phi)}$$

Collar and Pivot

1. Uniform pressure theory

$$T = \frac{2\mu}{3\sin\alpha F} \times \frac{{R_o}^3 - {R_i}^3}{{R_o}^2 - {R_i}^2}$$

2. Uniform wear theory

$$T = \frac{\mu F}{\sin\alpha} \times \frac{R_o + R_i}{2}$$

SOLVED EXAMPLES

1. The distance between two parallel shafts is $d =$ 15 mm and they are connected by an Oldham's coupling. The driving shaft revolves at $N = 120$ rpm. Calculate the maximum speed of sliding of the tongue of the intermediate piece along its groove.

Solution. Given that

$$d = 0.015 \text{ m}$$
$$N = 120 \text{ rpm}$$

Angular speed of the shafts is given by

$$\omega = \frac{2\pi N}{60} = 12.57 \text{ rad/s}$$

The maximum speed of sliding of the tong of the intermediate piece along its groove is given by

$$v = \omega \times d = 0.188 \text{ m/s}$$

2. The speed of driving shaft of a Hooke's joint of angle 20° is 600 r.p.m. Determine the maximum speed of the driven shaft.

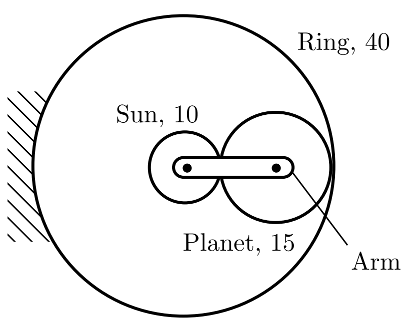
Solution. Given that

$$\alpha = 20^\circ$$
$$\omega_1 = 600 \text{ r.p.m.}$$

Maximum speed of the driven shaft is

$$\omega_2 = \frac{\omega_1}{\cos\alpha} = 638 \text{ r.p.m.}$$

3. The following figure shows an epicyclic gear train. Sun gear is driven clockwise at 120 rpm. The ring gear is held stationary. Determine the rotational speed of the arm for the number of teeth shown on the gears.



Solution. Given that

$$T_S = 10$$
$$T_P = 15$$
$$T_R = 40$$

Initiating the tabular algorithm by fixing the arm:

$$N_A = y$$
$$N_S = x + y$$
$$N_P = -\frac{T_S}{T_P}x + y = -\frac{10}{15}x + y = -\frac{2}{3}x + y$$
$$N_R = -\frac{T_S}{T_P} \times \frac{T_P}{T_R}x + y = -\frac{10}{15} \times \frac{15}{40}x + y = -\frac{1}{4}x + y$$

Given that

$$N_R = 0$$
$$N_S = 120 \text{ rpm (clockwise)}$$

Thus,

$$x + y = 120$$
$$-\frac{1}{4}x + y = 0$$

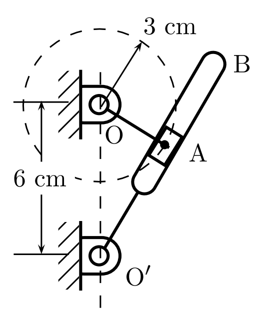
This gives

$$x = 96$$
$$y = 120 - 96 = 24$$

Hence, the rotational speed of arm

$$N_A = y = 24 \text{ rpm}$$

4. Determine the ratio of time taken for forward motion to that for the return motion for the quick return mechanism shown in figure.



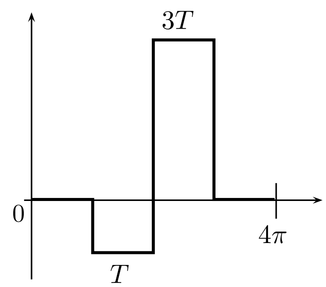
Solution. The angle β of return motion is determined as

$$\beta = 2\cos^{-1}\left(\frac{3}{6}\right) = 120^\circ$$

The ratio is determined as

$$R = \frac{360-\beta}{\beta} = 2$$

5. A four-stroke engine is designed for 20 kW at 300 rpm. The turning moment diagram is shown as follows:



The load on the engine is constant. Determine the moment of inertia of the flywheel to keep the total fluctuation of the crankshaft speed within 1.5%.

Solution. Given that

$$P = 20\times 10^3 \text{ W}$$
$$N = 300 \text{ rpm}$$
$$k_s = 0.03$$

The angular speed is

$$\omega = \frac{2\pi\times 300}{60} = 31.41 \text{ rad/s}$$

Power of the engine can be determined from turning moment diagram:

$$P\times\frac{2\times 60}{N} = 3T\pi - T\pi$$

Therefore, torque is

$$T = \frac{2\times 60\times P}{2\pi\times N} = 1273.23 \text{ Nm}$$

The average torque is

$$\bar{T} = \frac{P}{\omega} = \frac{20\times 10^3}{31.41} = 636.74 \text{ Nm}$$

The excess power is

$$I\omega^2 k_s = \left(3T-\bar{T}\right)\pi$$
$$I = \frac{\left(3T-\bar{T}\right)\pi}{\omega^2 k_s} = \frac{(3\times 1273.23-636.74)\,\pi}{31.41^2\times 0.03} = 338 \text{ kg·m}^2$$

6. The driving shaft of a Hooke's joint rotates at a uniform speed of 360 rpm and the maximum variation in speed of the driven shaft is ±3% of the mean speed. Determine the maximum and minimum speeds of the driven shaft.

Solution. The speed of driving shaft is

$$\omega_1 = 420 \text{ rpm}$$

The permitted angle between the shaft axes is

$$\alpha^2 = 2\times 0.03$$
$$\alpha = 0.245 \text{ rad} = 14.03^\circ$$

Maximum and minimum speeds of the driven shaft will be given by

$$\omega_{2\max} = \frac{\omega_1}{\cos\alpha} = 371.07 \text{ rpm}$$
$$\omega_{2\min} = \omega_1\cos\alpha = 349.26 \text{ rpm}$$

7. A shaft rotating at $N = 420$ rpm is supported by a thrust bearing having external and the internal diameters $d_o = 400$ mm and $d_i = 240$ mm, respectively. Axial load on the bearing is $W = 150$ kN while the intensity of pressure is limited to $p = 325$ kN/m^2. Determine the power lost in overcoming the friction if the coefficient of friction is $\mu = 0.05$. Determine the number of collars required in the bearing to limit the level of pressure intensity.

Solution. Using the data given, external and internal radii of the thrust bearing are

$$r_o = \frac{0.400}{2} = 0.200 \text{ m}$$
$$r_i = \frac{0.240}{2} = 0.120 \text{ m}$$
$$W = 150\times 10^3 \text{ N}$$
$$p = 325\times 10^3 \text{ N/m}^2$$
$$N = 420 \text{ rpm}$$
$$\mu = 0.05$$

Using the uniform pressure theory, the power lost in overcoming the friction is

$$P = \frac{2}{3}\mu W \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \times \frac{2\pi N}{60}$$
$$= 53.87 \text{ kW}$$

The number of collars to bear the load W is given by

$$W = n \times p\pi\left(r_o^2 - r_i^2\right)$$
$$n = 5.73$$
$$\approx 6$$

8. A belt drive is to be designed for transmission of 10.5 kW power from a pulley 800 m in diameter running at 360 rpm. The angle embraced is 165° and the coefficient of friction between the belt and the pulley is taken to be 0.3. The safe working stress for the leather belt is 2.5 MPa and its density is 1000 kg/m^3. Thickness of the belt is 12 mm. Determine the width of the belt taking into account the effect of centrifugal tension in the belt.

Solution. Given that

$$P = 10.5 \times 10^3 \text{ W}$$
$$D = 0.8 \text{ m}$$
$$N = 360 \text{ rpm}$$
$$\theta = 165^\circ = 2.88 \text{ rad}$$
$$\mu = 0.3$$
$$\sigma = 2.5 \times 10^6 \text{ Pa}$$
$$\rho = 1 \times 10^3 \text{ kg/m}^3$$
$$t = 0.012 \text{ m}$$

The linear velocity of the belt is given by

$$v = \frac{\pi D N}{60}$$
$$= 15.07 \text{ m/s}$$

The ratio of the belt tensions is

$$\frac{T_1}{T_2} = e^{\mu\theta}$$
$$= 2.37$$

The power transmitted is

$$P = (T_1 - T_2)\, v$$
$$= \left(T_1 - \frac{T_1}{2.37}\right) v$$

Therefore,

$$T_1 = \frac{10.5 \times 10^3 \times 2.37}{1.37 \times 15.07}$$
$$= 508.57 \text{ N}$$

Let b be the width of the belt. The centrifugal tension is given by

$$T_c = \rho b t \times 1 \times v^2$$
$$= 2725.25 b \text{ N}$$

The maximum tension in the belt is given by

$$T_1 + T_c = \sigma b t$$

Therefore, the width of the belt is

$$b = \frac{T_1 + T_c}{\sigma t}$$
$$= 18.46 \text{ mm}$$

9. A cone clutch of cone angle 20° is used to connect a stationary flywheel of moment of inertia 0.5 kg·m^2 to an electric motor rotating at constant speed of 720 rpm. The mean diameter of the contact surfaces is 100 mm and the coefficient of friction between them is 0.2. Calculate the minimum torque required to produce slipping of the clutch for an axial force of 300 N. Also calculate the time needed to attain the full speed and the energy lost during slipping.

Solution. Given that

$$\alpha = \frac{20^\circ}{2}$$
$$= 10^\circ$$
$$d_m = 0.1 \text{ m}$$
$$r_m = \frac{d_m}{2}$$
$$= \frac{0.10}{2} = 0.050 \text{ m}$$
$$\mu = 0.2$$
$$F = 300 \text{ N}$$
$$I = 0.5 \text{ kgm}^2$$
$$N = 720 \text{ rpm}$$

Therefore, the maximum torque required will be given by

$$T = \mu \frac{F}{\sin\alpha} \frac{r_o + r_i}{2}$$
$$= 17.28 \text{ Nm}$$

Angular speed of the flywheel is

$$\omega = \frac{2\pi N}{60}$$
$$= 75.36 \text{ rad/s}$$

The time (t) required to absorb the energy will be given by

$$\frac{T}{I} = \frac{\omega}{t}$$

Therefore,

$$t = \frac{I\omega}{T}$$
$$= 2.18 \text{ s}$$

GATE PREVIOUS YEARS' QUESTIONS

1. The mechanism used in a shaping machine is

(a) a closed 4-bar chain having 4 revolute pairs
(b) a closed 6-bar chain having 6 revolute pairs
(c) a closed 4-bar chain having 2 revolute and 2 sliding pairs
(d) an inversion of the single slider-crank chain

(GATE 2003)

Solution. If the cylinder in slider crank chain is made to work as a guide, and the piston is in the form of a slider, it results into slotted lever crank mechanism.

Ans. (d)

2. The lengths of the links of a 4-bar linkage with revolute pairs only are p, q, r and s units. Given that $p < q < r < s$. Which of these links should be the fixed one, for obtaining a 'double crank' mechanism?

(a) link of length p (b) link of length q
(c) link of length r (d) link of length s

(GATE 2003)

Solution. Double crank mechanisms are obtained by fixing the shortest link in a four-bar chain.

Ans. (a)

3. For a certain engine having an average speed of 1200 rpm, a flywheel approximated as a solid disc, is required for keeping the fluctuation of speed within 2% about the average speed. The fluctuation of kinetic energy per cycle is found to be 2 kJ. What is the least possible mass of the flywheel if its diameter is not exceed by 1 m?

(a) 40 kg (b) 51 kg
(c) 62 kg (d) 73 kg

(GATE 2003)

Solution. Given that

$$\begin{aligned} \omega &= \frac{1200 \times 2\pi}{60} \\ &= 40\pi \text{ rad/s} \\ k_s &= 0.02 \\ r &= \frac{1}{2} \\ &= 0.5 \text{ m} \\ e &= 2 \times 10^3 \text{ J per cycle} \end{aligned}$$

Moment of inertia of the wheel is

$$I = \frac{1}{2} m r^2$$

Fluctuation of kinetic energy per cycle is

$$e = I k_s \omega^2$$

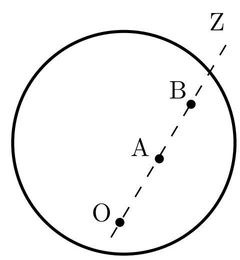
Solving, one obtains

$$m = 50.6606 \text{ kg}$$

Ans. (b)

Common Data Questions

The circular disc, shown in its plan view in the figure, rotates in a plane parallel to the horizontal plane about the point O at a uniform angular velocity ω. Two other points A and B are located on the line OZ at distances r_A, and r_B from O, respectively.



4. The velocity of point B with respect to point A is a vector of magnitude

(a) 0
(b) $\omega(r_B - r_A)$ and direction opposite to the direction of motion of point B
(c) $\omega(r_B - r_A)$ and direction same as the direction of motion of point B
(d) $\omega(r_B - r_A)$ and direction being from O to Z

(GATE 2003)

Solution. Velocity of A and B w.r.t. O is

$$\vec{V}_A = \vec{r}_A \times \vec{\omega}$$
$$\vec{V}_B = \vec{r}_B \times \vec{\omega}$$

Therefore, velocity of B w.r.t. A is

$$\begin{aligned} \vec{V}_{B/A} &= \vec{V}_B - \vec{V}_A \\ &= (\vec{r}_B - \vec{r}_A) \times \vec{\omega} \\ |\vec{V}_{B/A}| &= (r_B - r_A) \times \omega \end{aligned}$$

Ans. (c)

5. The acceleration of point B with respect to point A is a vector of magnitude

(a) 0
(b) $\omega^2(r_B - r_A)$ and direction same as the direction of motion of point B
(c) $\omega^2(r_B - r_A)$ and direction opposite to the direction of motion of point B
(d) $\omega^2(r_B - r_A)$ and direction being from Z to O

(GATE 2003)

Solution. Acceleration of B w.r.t. A is given by

$$\vec{a}_{B/A} = \vec{\omega} \times \vec{V}_{B/A}$$
$$|\vec{a}_{B/A}| = \omega^2(r_B - r_A)$$

which will be along Z to O, as seen from vector product.

Ans. (d)

Linked Answer Questions

The overall gear ratio in a 2-stage speed reduction gear box (with all spur gear) is 12. The input and output shafts of the gear box are collinear. The counter shaft which is parallel to the input and output shaft has a gear (Z_2 teeth) and pinion (Z_3 =15 teeth) to mesh with pinion (Z_1 =16 teeth) on the input shaft and gear (Z_4 = teeth) on the output shaft, respectively. It was decided to use a gear ratio of 4 with 3 module in the first stage and 4 module in the second stage.

6. Z_2 and Z_4 are

(a) 64 and 45 (b) 45 and 64
(c) 48 and 60 (d) 60 and 48

(GATE 2003)

Solution. Given that

$$\frac{N_1}{N_4} = 12$$
$$Z_1 = 16$$
$$Z_3 = 15$$
$$\frac{Z_2}{Z_1} = 4$$
$$m = 4 \text{ mm}$$

Therefore,

$$Z_2 = 4 \times 16 = 64$$

Also,

$$N_4 = N_1 \times \frac{Z_1}{Z_2} \times \frac{Z_3}{Z_4}$$
$$Z_4 = \frac{N_1}{N_4} \times \frac{Z_1}{Z_2} \times Z_3 = 45$$

Ans. (a)

7. The center distance in the second stage is

(a) 90 mm (b) 120 mm
(c) 160 mm (d) 240 mm

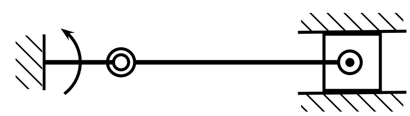
(GATE 2003)

Solution. The central distance in the second stage is

$$c = \frac{m}{2}(Z_3 + Z_4) = \frac{4}{2}(15 + 45) = 120 \text{ mm}$$

Ans. (b)

8. For a mechanism shown below, the mechanical advantage for the given configuration is



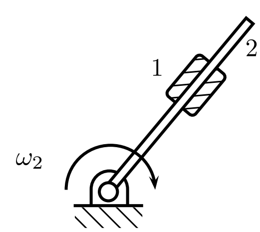
(a) 0 (b) 0.5
(c) 1.0 (d) ∞

(GATE 2004)

Solution. Mechanical advantage is the ratio of output force and input force. In this case, output force is zero.

Ans. (a)

9. In the figure shown, the relative velocity of link 1 with respect to link 2 is 12 m/s. Link 2 rotates at a constant speed of 120 rpm. The magnitude of Coriolis component of the acceleration of link 1 is



(a) 302 m/s^2 (b) 604 m/s^2
(c) 906 m/s^2 (d) 1208 m/s^2

(GATE 2004)

Solution. Given that

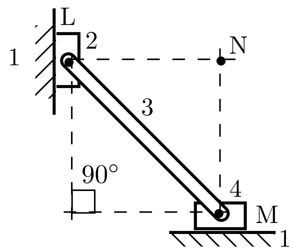
$$\omega_2 = \frac{120 \times 2\pi}{60} = 12.57 \text{ rad/s}$$
$$v = 12 \text{ m/s}$$

The Coriolis component is

$$2\omega_2 v = 2 \times 12.57 \times 12$$
$$= 301.593 \text{ m/s}^2$$

Ans. (a)

10. The following figure shows a planar mechanism with single degree of freedom.



The instantaneous center I_{24} for the given configuration is located at a position

(a) L (b) M
(c) N (d) ∞

(GATE 2004)

Solution. The normals of velocities of 2 and 4 meet at point N, therefore, it will be the location of I_{24}.

Ans. (c)

11. Match the following:

Mechanism	Motion
P. Scott-Russel Mechanism	1. Intermittent motion
Q. Geneva mechanism	2. Quick return motion
R. Off-set slider-crank mechanism	3. Simple harmonic motion
S. Scotch yoke mechanism	4. Straight line motion

(a) P-2 Q-3 R-1 S-4 (b) P-3 Q-2 R-4 S-1
(c) P-4 Q-1 R-2 S-3 (d) P-4 Q-3 R-1 S-2

(GATE 2004)

Solution. Scott-Russel mechanism is a straight line mechanism. The Geneva drive or Maltese cross is a gear mechanism that translates a continuous rotation into an intermittent rotary motion. The rotating drive wheel has a pin that reaches into a slot of the driven wheel advancing it by one step. The drive wheel also has a raised circular blocking disc that locks the driven wheel in position between steps. Off-set Slider-crank mechanism provides a quick return mechanism. Scotch yoke mechanism converts harmonic motion into rotary motion.

Ans. (c)

12. Match the following;

Type of gears	Arrangement of shafts
P. Bevel gears	1. Non-parallel off-set shafts
Q. Worm gears	2. Non-parallel intersecting shafts
R. Herringbone gears	3. Non-parallel, non-intersecting shafts
S. Hypoid gears	4. Parallel shafts

(a) P-4, Q-2, R-1, S-3
(b) P-2, Q-3, R-4, S-1
(c) P-3, Q-2, R-1, S-4
(d) P-1, Q-3, R-4, S-2

(GATE 2004)

Solution. Gears can be classified according to the relative positions of their shaft axes. These are

(a) Gears for parallel shafts: Spur Gears, Helical Gears, Double Helical and Herringbone Gears, Rack and Pinion
(b) Gears for intersecting shafts: Straight Bevel Gears, Spiral Bevel Gears, Zero Bevel Gears.
(c) Gears for skew shafts (non-parallel and non-intersecting): Hypoid Gears, Spiral Gears, Worm Gears.

Ans. (b)

13. Match the following with respect to spatial mechanisms.

Type of Joint	Degree of Constraint
P. Revolute	1. Three
Q. Cylindrical	2. Five
R. Spherical	3. Four
	4. Two
	5. Zero

(a) P-1, Q-3, R-3 (b) P-5, Q-4, R-3
(c) P-2, Q-3, R-1 (d) P-4, Q-5, R-3

(GATE 2004)

Solution. Degree of freedom (DOF) of joints and degree of constraints (DOC) are as follows

Joint	DOF	DOC
Revolute	1	5
Cylindrical	2	4
Spherical	3	3

Ans. (c)

14. The number of degrees of freedom of a planar linkage with 8 links and 9 simple revolute joints is

(a) 1 (b) 2
(c) 3 (d) 4

(GATE 2005)

Solution. Given that

$$n = 8$$
$$f_1 = 9$$

For planar mechanisms with revolute pairs, the degree of freedome is

$$\text{DOF} = 3(n-2) - 2f_1$$
$$= 3$$

Ans. (c)

15. In a cam-follower mechanism, the follower needs to rise through 20 mm during 60° of cam rotation, the first 30° with a constant acceleration and then with a deceleration of the same magnitude. The initial and final speeds of the follower are zero. The cam rotates at a uniform speed of 300 rpm. The maximum speed of the follower is

(a) 0.60 m/s (b) 1.20 m/s
(c) 1.68 m/s (d) 2.40 m/s

(GATE 2005)

Solution. As the magnitude of acceleration (a) in both parts of 30° travel of the follower is the same, the travel distance will also be equal. As $N = 300$, let t (seconds) be the time for 30° travel determined as

$$t = \frac{60}{300 \times 360} \times 30$$
$$= 0.0166667$$

Given that the initial and final speeds are zero, let t be the time for 30° travel, then using

$$s = ut + \frac{1}{2}at^2$$
$$0.02 = \frac{1}{2}at^2$$
$$a = 144 \text{ m/s}^2$$

The maximum velocity will be at the end of first half, calculated as

$$v = at$$
$$= 144\frac{1}{60}$$
$$= 2.4 \text{ m/s}$$

Ans. (d)

16. A rotating disc of 1 m diameter has two eccentric masses of 0.5 kg each at radii of 50 mm and 60 mm at angular positions of 0° and 150°, respectively. A balancing mass of 0.1 kg is to be used to balance the rotor. What is the radial position of the balancing mass?

(a) 50 mm (b) 120 mm
(c) 150 mm (d) 280 mm

(GATE 2005)

Solution. Let the balancing mass be located at (x, y) point with respect to center of the disc. For equilibrium along x-axis and y-axis,

$$0.1\omega^2 x = 0.5 \times -0.06\cos 30^\circ \omega^2 + 0.5 \times 0.05\omega^2$$
$$0.1\omega^2 y = 0.5\,(0.06 \sin 30^\circ)\,\omega^2$$

Solving the above equations,

$$x = -0.00980762$$
$$y = -0.15$$

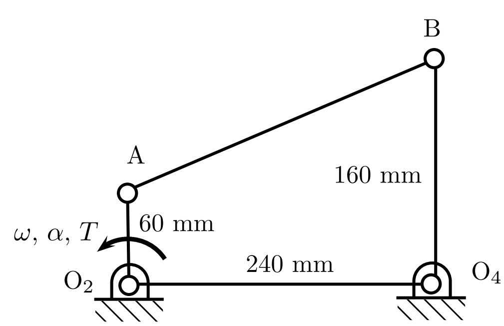
The radial distance of the balancing mass is

$$r = \sqrt{x^2 + y^2}$$
$$= 0.15032 \text{ m}$$
$$= 150.32 \text{ mm}$$

Ans. (c)

Common Data Questions

An instantaneous configuration of a four-bar mechanism, whose plane is horizontal, is shown in the figure below. At this instant, the angular velocity and angular acceleration of link O_2A are $\omega = 8$ rad/s and $\alpha = 0$, respectively, and the driving torque (T) is zero. The link O_2A is balanced so that its center of mass falls at O_2.



17. Which kind of 4-bar mechanism is O_2ABO_4?

(a) Double crank mechanism
(b) Crank-rocker mechanism
(c) Double-rocker mechanism
(d) Parallelogram mechanism

(GATE 2005)

Solution. Given that

$$l_1 = 60 \text{ mm}$$
$$l_2 = 160 \text{ mm}$$
$$l_3 = 240 \text{ mm}$$

The length AB is determined as

$$l_4 = \sqrt{100^2 + 240^2} = 260 \text{ mm}$$

Therefore,

$$l_1 + l_4 \ (= 320) < l_3 + l_4 \ (= 400)$$

Hence, the linkage follows Grashof's law (Class I); there is continuous motion between the two links. For double crank mechanism the shortest link should be fixed, which is not the present case, and so the answer cannot be double-rocker mechanism. It is also not the parallelogram. Therefore, it is the case of crank-rocker mechanism.

Ans. (b)

18. At the instant considered, what is the magnitude of the angular velocity of O_4B?

(a) 1 rad/s	(b) 3 rad/s
(c) 8 rad/s	(d) 64/3 rad/s

(GATE 2005)

Solution. V_A and V_B both are parallel to Q_2Q_4. As AB is rigid link. For given instant, $V_A = V_B$. Angular velocity of BO_4 is

$$\omega_{BO_4} = \frac{V_B}{\text{BO}_4} = \frac{V_A}{BO_4} = \frac{8 \times 60}{160} = 3 \text{ rad/s}$$

Ans. (b)

19. At the same instant, if the component of the force at joint A along AB is 30 N, then the magnitude of the joint reaction at O_2

(a) is zero
(b) is 30 N
(c) is 78 N
(d) cannot be determined from the given data

(GATE 2005)

Solution. Reaction at O_2 will be along the link AO_2 to balance the load of 30 N which is determined by resolving in reverse direction as

$$R = 30 \times \frac{260}{100} = 78 \text{ N}$$

Ans. (c)

20. For a four-bar linkage in toggle position, the value of mechanical advantage is

(a) 0.0	(b) 0.5
(c) 1.0	(d) ∞

(GATE 2006)

Solution. Mechanical advantage is the ratio of output force and input force. In toggle position, the input force is zero, therefore, mechanical advantage is infinite.

Ans. (d)

21. The number of inversions for a slider crank mechanism is

(a) 6	(b) 5
(c) 4	(d) 3

(GATE 2006)

Solution. There are four kinematic links in the slider crank mechanism, therefore, the number of inversions is also four.

Ans. (c)

22. Match the items in columns I and II.

Column I	Column II
P. Addendum	1. Cam
Q. I-center	2. Beam
R. Section Modulus	3. Linkage
S. Prime Circle	4. Gear

(a) P-4, Q-2, R-3, S-1
(b) P-4, Q-3, R-2, S-1
(c) P-3, Q-2, R-1, S-4
(d) P-3, Q-4, R-1, S-2

(GATE 2006)

Solution. Addendum is the top portion of a gear teeth. Instantaneous center (I-center) of rotation is an imaginary point such that one or more

linkages of the two bodies can be assumed to have motion of rotation with respect to the other about the imaginary point. Section modulus is the area moment of the beam cross-section divided by the maximum distance of top lamina from the neutral axis. Prime circle is the smallest circle to pitch curve of a cam.

Ans. (b)

23. A disk clutch is required to transmit 5 kW at 2000 rpm. The disk has a friction lining with coefficient of friction equal to 0.25. Bore radius of friction lining is equal to 25 mm. Assume uniform contact pressure of 1 MPa. The value of outside radius of the friction lining is

(a) 39.4 mm (b) 49.5 mm
(c) 97.9 mm (d) 142.9 mm

(GATE 2006)

Solution. Given that

$$\begin{aligned} P &= 5000 \text{ W} \\ N &= 200 \text{ rpm} \\ \mu &= 0.25 \\ r_i &= 0.025 \text{ m} \\ p &= 1 \times 10^6 \text{ Pa} \end{aligned}$$

Friction torque is calculated as

$$\begin{aligned} P &= T \times \frac{2\pi N}{60} \\ T &= 23.8732 \text{ Nm} \end{aligned}$$

For uniform contact pressure p,

$$\begin{aligned} T &= \frac{2}{3}\mu \times p\pi \left(r_o^2 - r_i^2\right) \times \frac{\left(r_o^3 - r_i^3\right)}{\left(r_o^2 - r_i^2\right)} \\ &= \frac{2}{3}\mu p\pi \left(r_o^3 - r_i^3\right) \end{aligned}$$

Therefore,

$$r_o = 39.4121 \text{ mm}$$

Ans. (a)

24. Twenty degree full depth involute profiled 19-tooth pinion and 37-tooth gear are in mesh. If the module is 5 mm, the center distance between the gear pair will be

(a) 140 mm (b) 150 mm
(c) 280 mm (d) 300 mm

(GATE 2006)

Solution. Given that

$$\begin{aligned} z_p &= 19 \\ z_g &= 37 \\ m &= 5 \text{ mm} \end{aligned}$$

Central distance is given by

$$\begin{aligned} c &= m\frac{(z_g + z_p)}{2} \\ &= 140 \text{ mm} \end{aligned}$$

Ans. (a)

25. If C_f is the coefficient of speed fluctuation of a flywheel then the ratio of $\omega_{\max}/\omega_{\min}$ will be:

(a) $\dfrac{1-2C_f}{1+2C_f}$ (b) $\dfrac{2-C_f}{1+2C_f}$

(c) $\dfrac{1+2C_f}{1-2C_f}$ (d) $\dfrac{2+C_f}{2-C_f}$

(GATE 2006)

Solution. Given that

$$\begin{aligned} C_f &= \frac{\omega_{\max} - \omega_{\min}}{\left(\omega_{\max} + \omega_{\min}\right)/2} \\ &= \frac{2r-2}{r+1} \end{aligned}$$

where

$$r = \frac{\omega_{\max}}{\omega_{\min}}$$

Therefore,

$$\begin{aligned} rC_f + C_f &= 2r - 2 \\ r\left(C_f - 2\right) &= -C_f - 2 \\ r &= \frac{2+C_f}{2-C_f} \end{aligned}$$

Ans. (d)

26. Match the items in columns I and II.

Column I	Column II
P. Higher pair	1. Gruebler's equation
Q. Lower pair	2. Line contact
R. Quick return	3. Euler's equation
S. Mobility	4. Planar
	5. Shaper
	6. Surface contact

(a) P-2, Q-6, R-4, S-3
(b) P-6, Q-2, R-4, S-1
(c) P-6, Q-2, R-5, S-3
(d) P-2, Q-6, R-5, S-1

(GATE 2006)

Solution. If the two links in a pair have point or line contact while in motion, the pair so formed is known as a higher pair. If the two links in a pair have surface contact while in motion, the pair so formed is called a lower pair. Quick return motion is used in shaper machine. Mobility of a linkage is determined by Gruebler's equation.

Ans. (d)

27. In a four-bar linkage, S denotes the shortest link length, L is the longest link length, P and Q are the lengths of other two links. At least one of the three moving links will rotate by 360° if

(a) $S + L \leq P + Q$ (b) $S + L > P + Q$
(c) $S + P \leq L + Q$ (d) $S + P > L + Q$

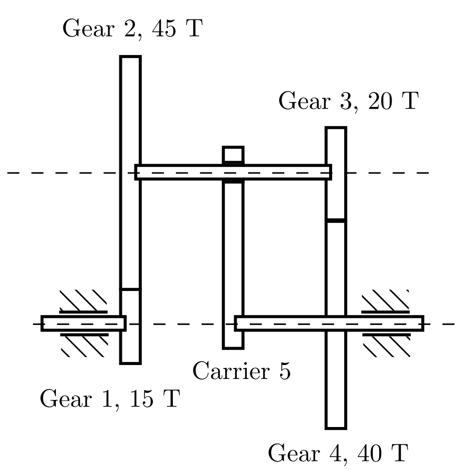
(GATE 2006)

Solution. According to Grashof's law, if there is to be continuous relative motion or rotation between two links, the sum of the shortest and largest links of a planar four-bar linkage cannot be greater than the sum of the remaining two links.

Ans. (a)

Common Data Questions

A planetary gear train has four gears and one carrier. Angular velocities of the gears are ω_1, ω_2, ω_3 and ω_4, respectively. The carrier rotates with angular velocity $5\omega_1$.



28. What is the relation between the angular velocities of Gear 1 and Gear 4?

(a) $(\omega_1 - \omega_5)/(\omega_4 - \omega_5) = 6$
(b) $(\omega_4 - \omega_5)/(\omega_1 - \omega_5) = 6$
(c) $(\omega_1 - \omega_2)/(\omega_4 - \omega_3) = -2/3$
(d) $(\omega_2 - \omega_5)/(\omega_4 - \omega_5) = 8/9$

(GATE 2006)

Solution. Ratio of angular velocities with respect to carrier for left side pair of gears is

$$\frac{\omega_1 - \omega_5}{\omega_2 - \omega_5} = \frac{45}{15} = 3$$

Similarly, for the right side pair gears, the ratio is

$$\frac{\omega_3 - \omega_5}{\omega_4 - \omega_5} = \frac{40}{20} = 2$$

Using $\omega_2 = \omega_3$, and multiplying the above two equations,

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$$

Ans. (a)

29. For $\omega_1 = 60$ rpm clockwise (cw) when looked from the left, what is the angular velocity of the carrier and its direction so that Gear 4 rotates in counterclockwise (ccw) direction at twice the angular velocity of Gear 1 when looked from the left

(a) 130 rpm, cw (b) 223 prm, ccw
(c) 256 rpm, cw (d) 156 rpm, ccw

(GATE 2006)

Solution. Given that

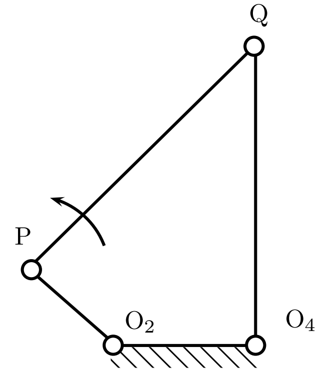
$$\omega_1 = 60 \text{ rpm}$$
$$\omega_4 = -2\omega_1 = -120 \text{ rpm}$$

Therefore, using

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$$
$$\omega_5 = -156 \text{ rpm ccw}$$

Ans. (d)

30. The input link O_2P of a four-bar linkage is rotated at 2 rad/s in counterclockwise direction as shown.



The angular velocity of the coupler PQ in rad/s, at an instant when $\angle O_4O_2P=180°$, is

(a) 4 (b) $2\sqrt{2}$
(c) 1 (d) $1/\sqrt{2}$

(GATE 2007)

Solution. Given that

$$\omega_{O_2P} = 2 \text{ rad/s}$$

When $\angle O_4O_2P=180°$, direction of $V_P \perp PO_4$, and $PQ \perp QO_4$, therefore, O_4 will be the I-center of PQ, of which angular velocity will be

$$\omega_{PQ} = \frac{V_P}{PO_4} = \frac{a\times 2}{2a} = 1 \text{ rad/s}$$

Ans. (c)

31. The speed of an engine varies from 210 rad/s to 190 rad/s. During a cycle, the change in kinetic energy is found to be 400 Nm. The inertia of the flywheel in kg·m^2 is

(a) 0.10 (b) 0.20
(c) 0.30 (d) 0.40

(GATE 2007)

Solution. Given that

$$\omega_1 = 210 \text{ rad/s}$$
$$\omega_2 = 190 \text{ rad/s}$$
$$E = 400 \text{ Nm}$$

Moment of inertia is

$$I = \frac{2E}{\omega_1^2 - \omega_2^2} = 0.1 \text{ kg·m}^2$$

Ans. (a)

Common Data Questions

A gear set has a pinion with 20 teeth and a gear with 40 teeth. The pinion runs at 30 rev/s and transmits a power of 20 kW. The teeth are on the 20° full depth system and have a module of 5 mm. The length of the line of action is 19 mm.

32. The center distance for the above gear set in mm is

(a) 140 (b) 150
(c) 160 (d) 170

(GATE 2007)

Solution. Given that

$$z_p = 20$$
$$z_g = 40$$
$$N = 30 \text{ rev/s}$$
$$P = 20 \text{ kW}$$
$$\phi = 20°$$
$$m = 5$$

For 20° full depth system, addendum is equal to m. Central distance is given by

$$c = \frac{m(z_p + z_g)}{2} = 150 \text{ mm}$$

Ans. (b)

33. The contact ratio of the contacting tooth is

(a) 1.21 (b) 1.25
(c) 1.29 (d) 1.33

(GATE 2007)

Solution. Contact ratio is written as

$$\text{CR} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

Arc of contact is related to the path of approach as

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi}$$
$$\text{CR} = \frac{\text{Path of contact}}{\text{Circular pitch}}$$

The line of action is 19 mm, $m = 5$ mm and $\phi = 20°$. Therefore,

$$\text{CR} = \frac{19}{\pi m\cos\phi} = 1.28721 \text{ mm}$$

Ans. (c)

34. The resultant force on the contacting gear tooth in N is

(a) 77.23 (b) 212.20
(c) 225.80 (d) 289.43

(GATE 2007)

Solution. Tangential force on the contacting tooth is

$$F_t = \frac{T}{r_p} = \frac{P}{r_p\omega} = 212.207 \text{ N}$$

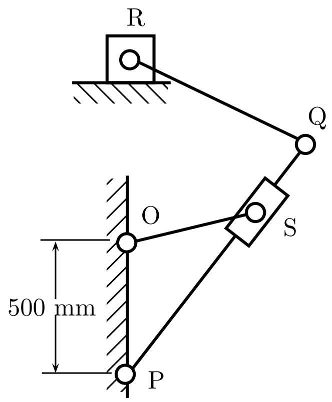
The resultant force along the pressure line is

$$F = \frac{F_t}{\cos\phi} = 225.826 \text{ N}$$

Ans. (c)

Linked Answer Questions

A quick return mechanism is shown below. The crank OS is driven at 2 rev/s in counterclockwise direction.



35. If the quick return ratio is 1:2, then the length of the crank in mm is

(a) 250 (b) $250\sqrt{3}$
(c) 500 (d) $500\sqrt{3}$

(GATE 2007)

Solution. Given that the quick return ratio is

$$R = \frac{1}{2}$$
$$OP = 500 \text{ mm}$$

If α is $\angle$POS in extreme points when OS$\perp$PS, then

$$\frac{2\alpha}{360^\circ - 2\alpha} = \frac{1}{2}$$
$$\alpha = 60^\circ$$

In the same situation, length is given by

$$\text{OS} = \text{OP}\cos\alpha = 250 \text{ mm}$$

Ans. (a)

36. The angular speed of PQ in rev/s when the block R attains maximum speed during forward stroke (stroke with lower speed) is

(a) 1/3 (b) 2/3
(c) 2 (d) 3

(GATE 2007)

Solution. Maximum speed in forward stroke will occur when PO and OS are in the same vertical line along with OS in vertically upward direction. In such a case, the velocity of point S on link OS is

$$V_S = 2\times 250 = 500$$

For PQ, P acts as I-center, therefore, angular speed of PQ or PS will be given by

$$\omega_{PS} = \frac{V_S}{PS} = \frac{2\times 250}{500+250} = \frac{2}{3}$$

Ans. (b)

37. A planar mechanism has 8 links and 10 rotary joints. The number of degrees of freedom of the mechanism using Gruebler's criterion, is

(a) 0 (b) 1
(c) 2 (d) 3

(GATE 2008)

Solution. Given that

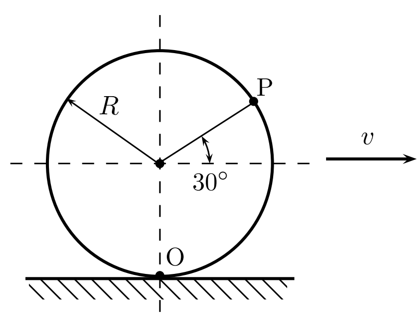
$$L = 8$$
$$j = 10$$

Gruebler's criterion for planar mechanisms is used as

$$F = 3(L-1) - 2j = 3(8-1) - 2\times 10 = 1$$

Ans. (b)

38. A circular disc of radius R rolls without slipping at a velocity v.



The magnitude of the velocity at point P (see figure) is

(a) $\sqrt{3}v$ (b) $\sqrt{3}v/2$

(c) $v/2$ (d) $2v/\sqrt{3}$

(GATE 2008)

Solution. Angular velocity of the rolling (ω) at contact point O is given by

$$\omega = \frac{v}{R}$$

Therefore, the velocity at point P, by definition is

$$v_{OP} = \omega \times OP$$

where

$$\begin{aligned} \text{OP} &= \frac{R + R\sin\theta}{\cos\theta} \\ &= R\left(\frac{1+1/2}{\sqrt{3}/2}\right) \\ &= \sqrt{3}R \end{aligned}$$

Therefore,

$$\begin{aligned} v_{OP} &= \frac{v}{R} \times \sqrt{3}R \\ &= \sqrt{3}v \end{aligned}$$

Ans. (a)

39. In a cam design, the rise motion is given by a simple harmonic motion (SHM)

$$s = \frac{h}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right)$$

where h is the total rise, θ is camshaft angle, β is the total angle of the rise interval. The jerk is given by

(a) $\frac{h}{2}\left(1-\cos\frac{\pi\theta}{\beta}\right)$ (b) $\frac{\pi}{\beta}\frac{h}{2}\sin\left(\frac{\pi\theta}{\beta}\right)$

(c) $\frac{\pi^2}{\beta^2}\frac{h}{2}\cos\left(\frac{\pi\theta}{\beta}\right)$ (d) $-\frac{\pi^3}{\beta^3}\frac{h}{2}\sin\left(\frac{\pi\theta}{\beta}\right)$

(GATE 2008)

Solution. Jerk is defined as the rate of variation in acceleration per unit time, therefore, jerk is given by

$$\frac{d^3s}{dt^3} = -\frac{\pi^3}{\beta^3}\frac{h}{2}\sin\left(\frac{\pi\theta}{\beta}\right)$$

Ans. (d)

40. A journal bearing has a shaft diameter of 40 mm and a length of 40 mm. The shaft is rotating at 20 rad/s and the viscosity of the lubricant is 20 mPa-s. The clearance is 0.020 mm. The loss of torque due to the viscosity of the lubricant is, approximately,

(a) 0.040 Nm (b) 0.252 Nm

(c) 0.400 Nm (d) 0.652 Nm

(GATE 2008)

Solution. Given that

$$\begin{aligned} l &= 0.04 \text{ m} \\ r &= 0.020 \text{ m} \\ \omega &= 20 \text{ rad/s} \\ \mu &= 20 \times 10^{-3} \text{ Pa-s} \\ h &= \frac{0.020}{1000} \text{ m} \end{aligned}$$

The viscous torque will be on cylindrical surface and bottom circular surface both, therefore, the loss of torque is written as

$$T = \underbrace{2\pi\omega\mu\frac{lr^3}{h}}_{\text{Circular}} + \underbrace{\frac{1}{2}\pi\omega\mu\frac{r^4}{h}}_{\text{Cylindrical}}$$

Using the values in above equation,

$$T = 0.0452389 \text{ Nm}$$

Ans. (a)

41. A clutch has outer and inner diameters 100 mm and 40 mm, respectively. Assuming a uniform pressure of 2 MPa and coefficient of friction of liner material 0.4, the torque carrying capacity of the clutch is

(a) 148 Nm (b) 196 Nm

(c) 372 Nm (d) 490 Nm

(GATE 2008)

Solution. Given that

$$\begin{aligned} R_o &= \frac{100}{2} \\ &= 50 \text{ mm} \\ &= 0.05 \text{ m} \\ R_i &= \frac{40}{2} \\ &= 20 \text{ mm} \\ &= 0.02 \text{ m} \\ p &= 2 \text{ MPa} \\ \mu &= 0.4 \end{aligned}$$

Hence, with the assumption of uniform pressure intensity p, the friction torque is given by

$$\begin{aligned} T &= \int_{R_i}^{R_o} r \times \mu p \times 2\pi r dr \\ &= \frac{2}{3}\pi\mu\left(R_o{}^3 - R_i{}^3\right) \\ &= \frac{2}{3}\pi \times 0.4\left(0.05^3 - 0.02^3\right) \\ &= 196.035 \text{ Nm} \end{aligned}$$

Ans. (b)

42. Match the type of gears with their most appropriate description.

Type of Gears	Description
P. Helical	1. Axes are non-parallel and non-intersecting
Q. Spiral bevel	2. Axes are parallel and teeth are inclined to the axis
R. Hypoid	3. Axes are parallel and teeth are parallel to the axis
S. Rack and pinion	4. Axes are perpendicular and intersecting, and teeth are inclined to the axis
	5. Axes are perpendicular and used for large speed reduction
	6. Axes are parallel and one of the gears has infinite radius

(a) P-2, Q-4, R-1, S-6
(b) P-1, Q-4, R-5, S-6
(c) P-2, Q-6, R-4, S-2
(d) P-6, Q-3, R-1, S-5

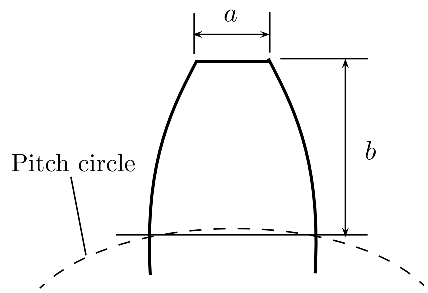
(GATE 2008)

Solution. Gears can be classified according to the relative positions of their shaft axes. These are

(a) Gears for parallel shafts: Spur Gears, Helical Gears, Double Helical and Herringbone Gears, Rack and Pinion.
(b) Gears for intersecting shafts: Straight Bevel Gears, Spiral Bevel Gears, Zero Bevel Gears.
(c) Gears for skew shafts (non-parallel and non-intersecting): Hypoid Gears, Spiral Gears, Worm Gears.

Ans. (a)

43. One tooth of a gear having 4 mm module and 32 teeth is shown in the figure. Assume that the gear tooth and the corresponding tooth space make equal intercepts on the pitch circumference.



The dimensions a and b, respectively, are closest to

(a) 6.08 mm, 4 mm (b) 6.48 mm, 4.2 mm
(c) 6.28 mm, 4.3 mm (d) 6.28 mm, 4.1 mm

(GATE 2008)

Solution. Given that

$$\begin{aligned} m &= 4 \\ T &= 32 \end{aligned}$$

Therefore,

$$\begin{aligned} d &= mT \\ &= 128 \end{aligned}$$

Arc projected by one teeth

$$\begin{aligned} a &= \frac{2\pi}{2T} \times \frac{d}{2} \\ &= 6.28319 \text{ mm} \end{aligned}$$

The angle subtended by half teeth is

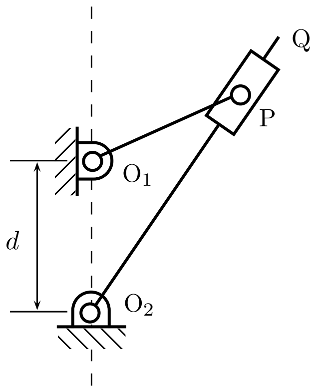
$$\theta = \frac{2\pi}{4T}$$

Therefore,

$$\begin{aligned} b &= m + \frac{d}{2}(1 - \cos\theta) \\ &= 4.07641 \text{ mm} \end{aligned}$$

Ans. (d)

44. A simple quick return mechanism is shown in the figure. The forward-to-return ratio of the quick return mechanism is 2:1.

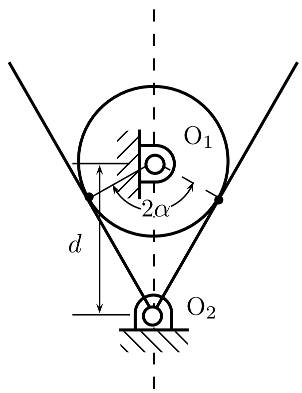


If the radius of the crank O_1P is 125 mm, then the distance d (in mm) between the crank center to lever pivot center point should be

(a) 144.3 (b) 216.5
(c) 240.0 (d) 250.0

(GATE 2009)

Solution. Let α be $\angle O_2O_1P$ in extreme points as shown in the figure.



For the given quick return ratio,

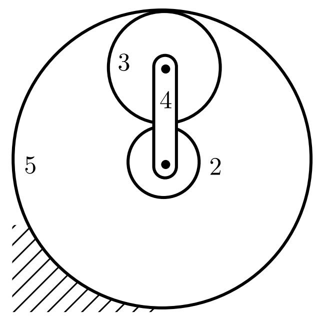
$$\frac{2\alpha}{360^\circ - 2\alpha} = \frac{1}{2}$$
$$\alpha = 60^\circ$$

Therefore, in the same condition

$$d = \frac{125}{\cos\alpha} = 250 \text{ mm}$$

Ans. (d)

45. An epicyclic gear train is shown schematically in the following figure. The sun gear 2 on the input shaft is a 20 teeth external gear. The planet gear 3 is a 40 teeth external gear. The ring gear 5 is a 100 teeth internal gear. The ring gear 5 is fixed and the gear 2 is rotating at 50 rpm ccw (ccw=counter-clockwise and cw=clockwise).



The arm 4 attached to the output shaft will rotate at

(a) 10 rpm ccw (b) 10 rpm cw
(c) 12 rpm cw (d) 12 rpm ccw

(GATE 2009)

Solution. Given

$$T_2 = 20$$
$$T_3 = 40$$
$$T_5 = 100$$

Observations with starting arm fixed are as follows.

Arm	Gear 2	Gear 3	Gear 4
0	1	$-\frac{T_2}{T_3}$	$-\frac{T_2}{T_3} \times \frac{T_3}{T_5}$
0	x	$-\frac{T_2}{T_3}x$	$-\frac{T_2}{T_5}x$
y	$x+y$	$-\frac{T_2}{T_3}x+y$	$-\frac{T_2}{T_5}x+y$

Therefore,

$$-\frac{T_2}{T_3}x + y = 0$$
$$x = 5y$$
$$x + y = 60$$
$$y = 10 \text{ ccw}$$

Ans. (a)

46. Match the approaches given below to perform stated kinematics/dynamics analysis of machine.

Analysis	Approach
P. Continuous relative rotation	1. D'Alembert's principle
Q. Velocity and acceleration	2. Gruebler's criterion
R. Mobility	3. Grashof's law
S. Dynamic-static analysis	4. Kennedy's theorem

(a) P-1, Q-2, R-3, S-4
(b) P-3, Q-4, R-2, S-1
(c) P-2, Q-3, R-4, S-1
(d) P-4, Q-2, R-1, S-3

(GATE 2009)

Solution. According to Grashof's law, if there is to be a continuous relative motion or rotation between the two links, the sum of the shortest and largest links of a planar four-bar linkage cannot be greater than the sum of the remaining two links. Kennedy's theorem states that three bodies, having relative motion with respect to one another, have three instantaneous centers, all of which lie on the same straight line. If n is number of links of a mechanism including fixed links, f_1 is number of pin joints or revolute pairs or pairs that permit one degree of freedom, then $F = 3(n-1)-2f_1$, which is called Gruebler's equation. D' Alembert's principle states that external forces acting on a body and the resultant inertia forces on it are in equilibrium.

Ans. (b)

47. Mobility of a statically indeterminate structure is

(a) ≤ -1 (b) 0
(c) 1 (d) ≥ 2

(GATE 2010)

Solution. For determinate structures, mobility is zero, and for statistically indeterminate structures mobility is less than -1.

Ans. (a)

48. There are two points P and Q on a planar rigid body. The relative velocity between the two points

(a) should always be along PQ
(b) can be oriented along any direction
(c) should always be perpendicular to PQ
(d) should be along QP when the body undergoes pure translation

(GATE 2010)

Solution. If the rigid body rotates with angular velocity ω, then relative velocity will be

$$\vec{V}_{P/Q} = \overrightarrow{PQ} \times \vec{\omega}$$

which is always perpendicular to $\overrightarrow{PQ}$.

Ans. (c)

49. Which of the following statements is incorrect?

(a) Grashof's rule states that for a planar crank-rocker four-bar mechanism, the sum of the shortest and longest link lengths cannot be less than the sum of the remaining two link lengths.
(b) Inversions of a mechanism are created by fixing different links one at a time.
(c) Geneva mechanism is an intermittent motion device
(d) Gruebler's criterion assumes mobility of a planar mechanism to be one.

(GATE 2010)

Solution. According to Grashof's law, if there is to be continuous relative motion or rotation between two links, the sum of the shortest and largest links of a planar four-bar linkage cannot be greater than the sum of the remaining two links.

$$l_1 + l_4 \leq l_2 + l_3$$

Ans. (a)

50. Tooth interference in an external involute spur gear pair can be reduced by

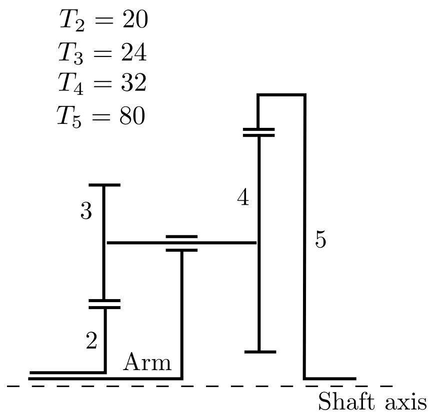
(a) decreasing the center distance between gear pair
(b) decreasing the module
(c) decreasing the pressure angle
(d) increasing the number of gear teeth

(GATE 2010)

Solution. Increasing the number of teeth will make available involute profile teeth in contacts.

Ans. (d)

51. For the epicyclic gear arrangement shown in the figure, $\omega_2 = 100$ rad/s clockwise (cw) and $\omega_{\text{arm}} = 80$ rad/s counter clockwise (ccw).



The angular velocity ω_s (in rad/s) is

(a) 0 (b) 70 cw
(c) 140 ccw (d) 140 cw

(GATE 2010)

Solution. Taking clockwise direction as positive, the tabular method is shown in the table below for the given configuration.

ω_A	ω_2	ω_3	ω_4	ω_5
		Arm fixed, 2 by x		
0	x	$-x\dfrac{20}{24}$	$-x\dfrac{20}{24}$	$-x\dfrac{20}{24}\dfrac{32}{80}$
		Rotate assembly by y		
y	$x+y$	$-x\dfrac{20}{24}+y$	$-x\dfrac{20}{24}+y$	$-x\dfrac{20}{24}\dfrac{32}{80}+y$

Given that

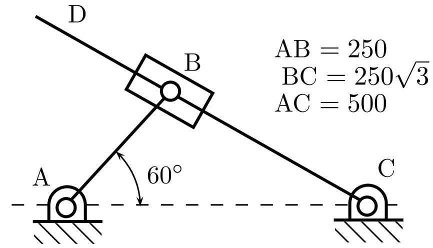
$$x+y = \omega_2 = 100$$
$$y = \omega_{\text{arm}} = -80$$

Therefore,

$$x = 180$$
$$\omega_5 = -x\frac{20}{24}\times\frac{32}{80}+y$$
$$= -140$$

Ans. (c)

52. For the configuration shown in the figure, the angular velocity of link AB is 10 rad/s counterclockwise.



The magnitude of the relative sliding velocity (in m/s) of slider B with respect to rigid link CD is

(a) 0 (b) 0.86
(c) 1.25 (d) 2.5

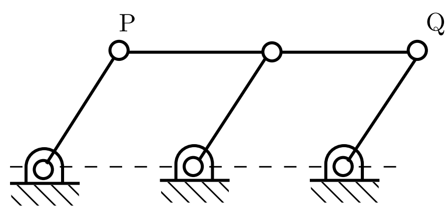
(GATE 2010)

Solution. In the given situation, AB⊥BC. Therefore, relative velocity of the slider is equal to

$$V_{B/A} = 10\times 0.250$$
$$= 2.5 \text{ m/s}$$

Ans. (d)

53. A double parallelogram mechanism is shown in the figure. Note that PQ is a single link.



The mobility of the mechanism is

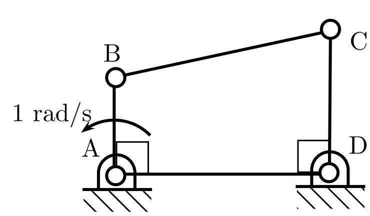
(a) −1 (b) 0
(c) 1 (d) 2

(GATE 2011)

Solution. This is a four-bar chain mechanism with single degree of freedom. Here, the mid-link and its revolute pairs are redundant and have no effect on mobility.

Ans. (c)

54. For the four-bar linkage shown in the figure, the angular velocity of link AB is 1 rad/s. the length of link CD is 1.5 times the length of link AB.



In the configuration shown, the angular velocity of link CD in rad/s is

(a) 3 (b) 3/2
(c) 1 (d) 2/3

(GATE 2011)

Solution. Given that

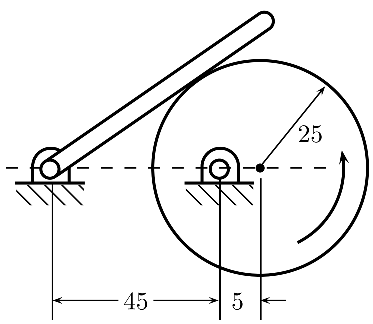
$$\omega_{AB} = 1 \text{ rad/s}$$
$$\frac{\text{CD}}{\text{AB}} = 1.5$$

For given configuration,

$$V_{B/A} = V_{C/D}$$
$$\omega_{AB} \times \text{AB} = \omega_{CD} \times \text{CD}$$
$$\omega_{CD} = \omega_{AB} \times \frac{\text{AB}}{\text{CD}} = \frac{2}{3}$$

Ans. (d)

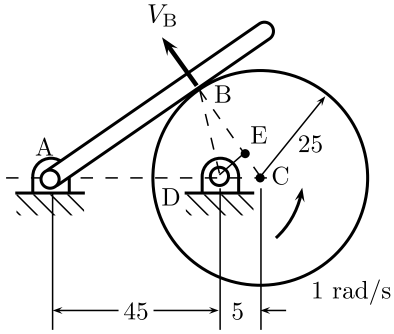
55. In the mechanism given below, if the angular velocity of the eccentric circular disc is 1 rad/s, the angular velocity (rad/s) of the follower link for the instant shown in the figure is (all measurements are in mm)



(a) 0.05 (b) 0.1
(c) 5.0 (d) 10.0

(GATE 2012)

Solution. The dimensions are shown in the figure:



In the triangle ABC,

$$\text{AB} = \sqrt{(45+5)^2 - 25^2} = 43.30 \text{ mm}$$

A line DE⊥BC is drawn. ΔABC and ΔDEC are similar. Therefore,

$$\frac{\text{DE}}{\text{AB}} = \frac{\text{DC}}{\text{AC}}$$
$$\text{DE} = \frac{\text{DC}}{\text{AC}} \times \text{AB} = \frac{5}{45+5} \times 43.30 = 4.33 \text{ mm}$$

Velocity of point E will be

$$V_{\text{E}} = \text{DE} \times \omega = 4.33 \times 1 = 4.33 \text{ mm/s}$$

Tangential velocity of the point B on the link AB will be equal to the velocity of point E w.r.t. D., that is, 4.33 mm/s. Angular velocity of link AB is calculated as

$$\omega_{\text{AB}} = \frac{V_{\text{B}}}{\text{AB}} = \frac{4.33}{43.30} = 0.1 \text{ rad/s}$$

Ans. (b)

56. Circular disc of uniform thickness 20 mm, radius 200 mm and mass 20 kg, is used as flywheel. If it rotates at 600 rpm, the kinetic energy of the flywheel, in Joules, is

(a) 395 (b) 790
(c) 1580 (d) 3160

(GATE 2012)

Solution. Given that

$$R = 0.2 \text{ m}$$
$$m = 20 \text{ kg}$$
$$N = 60 \text{ rpm}$$

Moment of inertia of solid disc is given by

$$I = \frac{mR^2}{2}$$

Angular speed of rotation (rad/s) is

$$\omega = \frac{2\pi N}{60}$$

The kinetic energy of flywheel is

$$\begin{aligned}\text{K.E.} &= \frac{1}{2}I\omega^2\\ &= \frac{1}{2}\frac{mR^2}{2}\left(\frac{2\pi N}{60}\right)^2\\ &= 789.568 \text{ J}\end{aligned}$$

Ans. (b)

57. The following are the data for two crossed helical gears used for speed reduction. Gear I: Pitch circle diameter in the plane of rotation 80 mm and helix angle 30°. Gear II: Pitch circle diameter in the plane rotation 120 mm and helix angle 22.5°. If the input speed is 1440 rpm, the output speed in rpm is

(a) 1200 (b) 900
(c) 875 (d) 720

(GATE 2012)

Solution. Given that

$$\begin{aligned}d_1 &= 80 \text{ mm}\\ \alpha_1 &= 30^\circ\\ d_2 &= 120 \text{ mm}\\ \alpha_2 &= 22.5^\circ\\ N_1 &= 1440 \text{ rpm}\end{aligned}$$

Speed ratio between helical gears is

$$\begin{aligned}\frac{N_2}{N_1} &= \frac{d_1\cos\alpha_1}{d_2\cos\alpha_2}\\ N_2 &= 899.884 \text{ rpm}\end{aligned}$$

Ans. (b)

58. A solid disc of radius r rolls without slipping on the horizontal floor with angular velocity ω and angular acceleration α. The magnitude of acceleration of the point of contact on the disc is

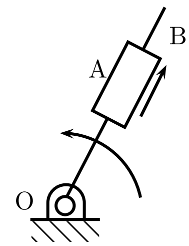
(a) zero (b) $r\alpha$
(c) $\sqrt{(r\alpha)^2 + (r\omega^2)^2}$ (d) $r\omega^2$

(GATE 2012)

Solution. As the disc rolls without slipping, both velocity and acceleration at the point of contact will be zero.

Ans. (a)

59. A link OB is rotating with a constant angular velocity of 2 rad/s in counter clockwise direction and a block is sliding radially outward on it with an uniform velocity of 0.75 m/s with respect to the rod, as shown in the figure.



If OA = 1 m, the magnitude of the absolute acceleration of the block at location A in m/s^2 is

(a) 3 (b) 4
(c) 5 (d) 6

(GATE 2013)

Solution. Given that

$$\begin{aligned}v &= 0.75 \text{ m/s}\\ r &= 1 \text{ m}\\ \omega &= 2 \text{ rad/s}\end{aligned}$$

Acceleration of the block at location A are:

(a) Radial acceleration:

$$\begin{aligned}\alpha_r &= \frac{v^2}{r}\\ &= \frac{0.75^2}{1}\\ &= 4 \text{ m/s}^2\end{aligned}$$

(b) Coriolis component (in tangential direction):

$$\begin{aligned}\alpha_t &= 2\omega v\\ &= 3 \text{ m/s}^2\end{aligned}$$

The net acceleration is

$$\begin{aligned}\alpha &= \sqrt{\alpha_t^2 + \alpha_r^2}\\ &= 5\end{aligned}$$

Ans. (c)

60. A planar closed kinematic chain is formed with rigid links PQ= 2.0 m, QR= 3.0 m, RS= 2.5 m and SP= 2.7 m with all revolute joints. The link to be fixed to obtain a double rocker (rocker–rocker) mechanism is

(a) PQ (b) QR
(c) RS (d) SP

(GATE 2013)

Solution. RS is the smallest link which should be fixed to achieve a double rocker mechanism.

Ans. (c)

61. A flywheel connected to a punching machine has to supply energy of 400 Nm while running at a mean angular speed of 20 rad/s. If the total fluctuation of speed is not to exceed ±2%, the mass moment of inertia of the flywheel in kg·m^2 is

(a) 25 (b) 50
(c) 100 (d) 125

(GATE 2013)

Solution. Given that the coefficient of speed fluctuation (k_s) and energy fluctuation (e), respectively,

$$\begin{aligned} k_s &= [0.02-(-0.02)] \\ &= 0.04 \\ e &= 400 \text{ Nm} \end{aligned}$$

Also,

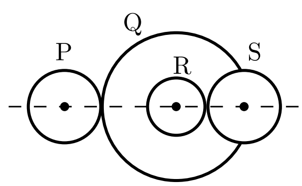
$$\omega_m = 20 \text{ rad/s}$$

Thus, the mass moment of inertia can be found as

$$\begin{aligned} e &= I\omega_m^2 k_s \\ I &= \frac{e}{\omega_m^2 k_s} \\ &= \frac{400}{20^2 \times 0.04} \\ &= 25 \text{ kg·m}^2 \end{aligned}$$

Ans. (a)

62. A compound gear train with gears P, Q, R and S has number of teeth 20, 40, 15 and 20, respectively. Gears Q and R are mounted on the same shaft as shown in the figure below.



The diameter of the gear Q is twice that of the gear R. If the module of the gear R is 2 mm, the center distance in mm between gears P and S is

(a) 40 (b) 80
(c) 120 (d) 160

(GATE 2013)

Solution. Given that

$$\begin{aligned} T_P &= 20 \\ T_Q &= 40 \\ T_R &= 15 \\ T_S &= 20 \\ d_Q &= 2d_R \\ m_R &= 2 \text{ mm} \end{aligned}$$

The diameter of gear R is

$$\begin{aligned} d_R &= m_R T_R \\ &= 2\times 15 \\ &= 30 \text{ mm} \end{aligned}$$

Diameter of gear Q is

$$\begin{aligned} d_Q &= 2d_R \\ &= 2\times 30 \\ &= 60 \text{ mm} \end{aligned}$$

The diameter of gear P is determined as

$$\begin{aligned} \frac{d_P}{d_Q} &= \frac{T_P}{T_Q} \\ d_P &= \frac{T_P}{T_Q}\times d_Q \\ &= \frac{20}{40}\times 60 \\ &= 30 \text{ mm} \end{aligned}$$

Diameter of gear S is determined as

$$\begin{aligned} \frac{d_S}{d_R} &= \frac{T_S}{T_R} \\ d_S &= \frac{T_S}{T_R}\times d_R \\ &= \frac{20}{15}\times 30 \\ &= 40 \text{ mm} \end{aligned}$$

The center distance between gear P and S is

$$\begin{aligned} c_{PS} &= \frac{d_P}{2}+\frac{d_Q}{2}+\frac{d_R}{2}+\frac{d_S}{2} \\ &= \frac{30}{2}+\frac{60}{2}+\frac{30}{2}+\frac{40}{2} \\ &= 15+30+15+20 \\ &= 80 \text{ mm} \end{aligned}$$

Ans. (b)

MULTIPLE CHOICE QUESTIONS

1. A kinematic chain consists of n links. The maximum number of possible inversions for this chain is

(a) n (b) $n!$
(c) $n-1$ (d) $n!-1$

2. The distance between two parallel shafts is 20 mm and they are connected by an Oldham's coupling. The driving shaft revolves at 180 rpm. What will be the maximum speed of sliding of the tongue of the intermediate piece along its groove?

(a) 0.575 m/s (b) 0.454 m/s
(c) 0.376 m/s (d) indeterminate

3. Slotted lever crank mechanism is an inversion of slider crank chain, obtained by fixing

(a) crank (b) connecting rod
(c) slider (d) frame

4. Oldham's coupling is an inversion of

(a) four-bar mechanism
(b) crank and lever mechanism
(c) single slider crank mechanism
(d) double slider crank mechanism

5. Which of the following is the higher pair?

(a) belt and pulley (b) turning pair
(c) screw pair (d) sliding pair

6. The connection between the piston and cylinder in a reciprocating engine corresponds to

(a) completely constrained kinematic pair
(b) incompletely constrained kinematic pair
(c) successfully constrained kinematic pair
(d) single link

7. The Whitworth quick return mechanism is formed in a slider-crank chain when the

(a) coupler link is fixed
(b) longest link is a fixed link
(c) slider is a fixed link
(d) smallest link is a fixed link

8. Scotch yoke mechanism is used to generate

(a) sine function (b) square roots
(c) logarithms (d) inversions

9. Which one of the following is an open pair?

(a) ball and socket joint
(b) journal bearing
(c) lead screw and nut
(d) cam and follower

10. In a single slider four-bar linkage, when the slider is fixed, it forms a mechanism of

(a) hand pump
(b) reciprocating engine
(c) quick return
(d) oscillating cylinder

11. A point on a link connecting a double slider crank chain will trace a

(a) straight line (b) circle
(c) parabola (d) ellipse

12. ABCD is a mechanism with link lengths AB = 200, BC = 300, CD = 400 and DA = 300. Which one of the following links should be fixed for the resulting mechanism to be a double crank mechanism? (All length are in mm)

(a) AB (b) BC
(c) CD (d) DA

13. Which one of the following mechanisms represents an inversion of the single slider crank chain?

(a) Elliptical trammel
(b) Oldham's coupling
(c) Whitworth quick return mechanism
(d) Pantograph mechanism

14. A mechanism has n number of links (including fixed links) and f_1 number of pins or revolute pairs or pairs that permit one degree of freedom. According to Gruebler's equation, the number of degrees of freedom is given by

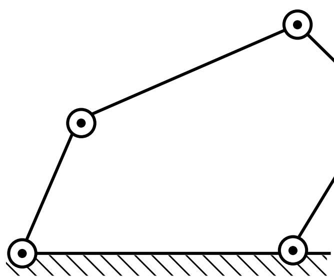
(a) $F = 3(n-1)-2f_1$ (b) $F = 2(n-1)-3f_1$
(c) $F = 3(n-1)+2f_1$ (d) $F = 2(n-1)+3f_1$

15. A mechanism has n number of links (including fixed links) and f_1 number of pin or revolute pairs

or pairs that permit one degree of freedom. The mechanims also has f_2 number of pairs which remove only one degree of freedom (f_2 i.e. number of roll sliding pair or total number of higher pairs). According to Kutzbach equation, the number of degrees of freedom is given by

(a) $F = 3(n-1) - 2f_1 - f_2$
(b) $F = 2(n-1) - 3f_1 - f_2$
(c) $F = 3(n-1) + 2f_1 - f_2$
(d) $F = 2(n-1) + 3f_1 - f_2$

16. A five-link planar mechanism with five revolute pairs is shown in the figure.



The number of degrees of freedom of this mechanism is

(a) 4 (b) 3
(c) 2 (d) 1

17. Universal joint is used to connect two shafts which are

(a) non-parallel and intersecting shafts and misaligned shafts
(b) parallel and intersecting shafts and misaligned shafts
(c) parallel and intersecting shafts and aligned shafts
(d) non-parallel and intersecting shafts and aligned shafts

18. Which one of the following statements is not correct?

(a) Hooke's joint is used to connect two rotating co-planar, non-intersecting shafts
(b) Hooke's joint is used to connect two rotating co-planar, intersecting shafts
(c) Oldham's coupling is used to connect two parallel rotating shafts
(d) Hooke's joint is used in the steering mechanism for automobiles

19. In a Hooke's joint, the driving shaft rotates at constant angular speed but driven shaft rotates at

(a) constant angular speed throughout its revolution
(b) constant angular speed for half revolution while varying speed for another half
(c) varying speed which gets it maximum value one times in each revolution
(d) varying speed which gets it maximum value two times in each revolution

20. Two shafts at an angle α are connected by hook's joint. The angular speeds of driving shaft and driving shaft are constant at ω_1. At any position θ of the driving shaft, the angular speed of the driven shaft is ω_2. The ratio of speeds of driven shaft and driving shaft (ω_2/ω_1) will be given by

(a) $\cos^2\alpha/\left(1-\sin^2\alpha\cos^2\theta\right)$
(b) $\sin^2\alpha/\left(1-\sin^2\alpha\cos^2\theta\right)$
(c) $\cos\alpha/\left(1-\sin^2\alpha\cos^2\theta\right)$
(d) $\sin\alpha/\left(1-\sin^2\alpha\cos^2\theta\right)$

21. Two shafts at an angle α are connected by hook's joint. The maximum variation in the velocity of driven shaft w.r.t. its mean velocity is expressed as

(a) $\tan\alpha\sin\alpha$ (b) $\tan\alpha\cos\alpha$
(c) $\cot\alpha\sin\alpha$ (d) $\cot\alpha\cos\alpha$

22. Two shafts are connected at angle α by the Hooke joint. For maximum acceleration of the driven shaft, at angle θ of the driving shaft is given by

(a) $\cos 2\theta = 2\sin^2\alpha/\left(2-\sin^2\alpha\right)$
(b) $\cos 2\theta = 2\cos^2\alpha/\left(2-\cos^2\alpha\right)$
(c) $\sin 2\theta = 2\sin^2\alpha/\left(2-\cos^2\alpha\right)$
(d) $\sin 2\theta = 2\cos^2\alpha/\left(2-\sin^2\alpha\right)$

23. The constant velocity ratio is achieved in a double Hooke's joint when

(a) driving and driven shafts make equal but opposite angle with the intermediate shaft and forks of intermediate shaft lie in the same plane
(b) driving and driven shafts make equal but opposite angle with the intermediate shaft and forks of intermediate shaft lie in the orthogonal plane
(c) driving and driven shafts make equal angle with the intermediate shaft and forks of intermediate shaft lie in the same plane
(d) driving and driven shafts make equal angle with the intermediate shaft and forks of intermediate shaft lie in the orthogonal plane

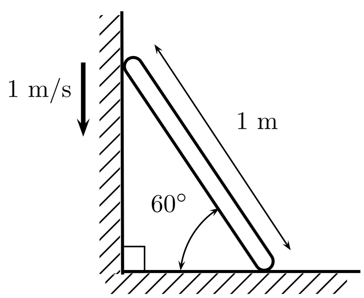
24. If driving and driven shafts make equal angle with the intermediate shaft and forks of intermediate shaft lie in orthogonal planes, then maximum and minimum ratio of speeds of the shafts (ω_2/ω_1) is given by

(a) $\cos\alpha,\ 1/\cos\alpha$
(b) $1/\cos\alpha,\ \cos\alpha$
(c) $\cos^2\alpha,\ 1/\cos^2\alpha$
(d) $1/\cos^2\alpha,\ \cos^2\alpha$

25. The speed of driving shaft of a Hooke's joint of angle $19.5°$ is 500 rpm (given $\sin 19.5° = 0.33$, $\cos 19.5° = 0.94$). The maximum speed of the driven shaft is nearly

(a) 168 rpm (b) 444 rpm
(c) 471 rpm (d) 531 rpm

26. A rod of length 1 m is sliding in a corner as shown in figure.



At an instant when the rod makes an angle of 60 degrees with the horizontal plane, the velocity of point A on the rod is 1 m/s. The angular velocity of the rod at this instant is

(a) 2 rad/s (b) 1.5 rad/s
(c) 0.5 rad/s (d) 0.73 rad/s

27. A body in motion will be subjected to Coriolis acceleration when that body is

(a) in plane rotation with variable velocity
(b) in plane transition with variable velocity
(c) in plane motion which is a resultant of plane translation and rotation
(d) restrained to rotate while sliding over another body

28. The instantaneous center of rotation of a rigid thin disc rolling on a plane rigid surface is located at

(a) the center of the disc
(b) an infinite distance on the plane surface
(c) the point of contact
(d) the point on the circumference situated vertically opposite to the contact point

29. In order to draw the acceleration diagram, it is necessary to determine the Coriolis component of acceleration in the case of

(a) crank and slotted lever quick return mechanism
(b) slider crank mechanism
(c) four bar mechanism
(d) pantograph

30. When a slider moves with a velocity v on a link rotating at an angular speed of ω, the Coriolis component of acceleration is given by

(a) $\sqrt{2}v\omega$ (b) $v\omega$
(c) $v\omega/2$ (d) $2v\omega$

31. The total number of instantaneous centers for a mechanism consisting of 'n' links is

(a) $n/2$ (b) n
(c) $(n-1)/2$ (d) $n(n-1)/2$

32. What is the number of instantaneous centers of rotation for a 6-link mechanism?

(a) 4 (b) 6
(c) 12 (d) 15

33. Instantaneous center of a body rolling with sliding on a stationary curved surface lies

(a) at the point of contact
(b) on the common normal at the point of contact
(c) on the common tangent at the point of contact
(d) at the center of curvature of the stationary surface

34. For a spring-loaded roller-follower driven with a disc cam,

(a) the pressure angle should be larger during rise than that during return for ease of transmitting motion
(b) the pressure angle should be smaller during rise than that during return for ease of transmitting motion
(c) the pressure angle should be large during rise as well as during return for ease of transmitting motion
(d) the pressure angle does not affect the ease of transmitting motion

35. The choice of displacement diagram during rise or return of a follower of a cam-follower mechanism

is based on dynamic considerations. For high speed cam follower mechanism, the most suitable displacement for the follower is

(a) cycloidal motion
(b) simple harmonic motion
(c) parabolic or uniform acceleration motion
(d) uniform motion or constant velocity motion

36. In a plane cam mechanism with reciprocating roller follower, the follower has a constant acceleration in the case of

(a) cycloidal motion
(b) simple harmonic motion
(c) parabolic motion
(d) 3-4-5 polynomial motion

37. The motion transmitted between the teeth of two spur gears in mesh is generally

(a) sliding
(b) rolling
(c) rotary
(d) partly sliding partly rolling

38. For a standard gear tooth profile, one module is equal to

(a) dedendum (b) addendum
(c) diametral pitch (d) circular pitch

39. A 1.5 kW motor is running at 1440 rev/min. It is to be connected to a stirrer running at 36 rev/min. The gearing arrangement suitable for this application is

(a) differential gear (b) helical gear
(c) spur gear (d) worm gear

40. To avoid interference in mating gears at pressure angle ϕ and gear ratio G and module equal to addendum, the minimum number of teeth on the gear shall be given by

(a) $\dfrac{2}{\sqrt{1+(1/G)\,(1/G+2)\sin^2\phi}-1}$

(b) $\dfrac{2}{\sqrt{1+(1/G)\,(1/G-2)\sin^2\phi}-1}$

(c) $\dfrac{2}{\sqrt{1+(1/G)\,(1/G+2)\cos^2\phi}-1}$

(d) $\dfrac{2}{\sqrt{1+(1/G)\,(1/G-2)\cos^2\phi}-1}$

41. Gears used to connect non-parallel and non-intersecting shafts include

(a) hypoid gears (b) spiral gears
(c) worm gears (d) all of the above

42. The pair of gears used to convert either rotary motion into linear motion or vice versa is called

(a) hypoid gear
(b) worm gear
(c) rack and pinion
(d) none of the above

43. Straight bevel gears of the same size and two gears at right angle to each other are known as

(a) miter gears (b) orthogonal gears
(c) rack and pinion (d) helical gears

44. Gears used in drive to the differential of an automobile are

(a) straight bevel gears
(b) spiral bevel gears
(c) zero bevel gears
(d) all of the above

45. If pitch circle diameter of a gear is d and there are total T teeth in the gear, then circular pitch of the gear is defined as

(a) d/T (b) $\pi d/T$
(c) T/d (d) $\pi T/d$

46. If two gears of module m but different number of teeth T and t are mating, then the central distance between these two gears will be given by

(a) $\dfrac{T+t}{2m}$ (b) $\dfrac{T+t}{2\pi}$

(c) $\dfrac{m\,(T+t)}{2}$ (d) $\dfrac{m\,(T+t)}{2\pi}$

47. Pressure angle is defined as the angle between

(a) pressure line and common tangent to pitch circles
(b) pressure line and common tangent to base circles
(c) pressure line and center-line joining the pitch circles
(d) none of the above

48. According to the law of gearing for constant velocity ratio of the two mating bodies

(a) the common normal at the point of contact of two tooth should always pass through a fixed point which divides the line of centers in the ratio of angular velocities of two gears
(b) the common tangent at the point of contact of two tooth should always pass through a fixed point which divides the line of centers in the ratio of angular velocities of two gears
(c) the common tangent at the point of contact of two tooth should always pass through a fixed point which divides the line of centers in the inverse ratio of angular velocities of two gears
(d) the common normal at the point of contact of two tooth should always pass through a fixed point which divides the line of centers in the inverse ratio of angular velocities of two gears

49. If R and r are the radius of pitch circles and ϕ is the pressure angle, then the maximum value of path of approach is

(a) $R\sin\phi$ (b) $r\sin\phi$
(c) $R\cos\phi$ (d) $r\cos\phi$

50. If R and r are the radius of pitch circles and ϕ is the pressure angle, then maximum value of path of recess is

(a) $R\sin\phi$ (b) $r\sin\phi$
(c) $R\cos\phi$ (d) $r\cos\phi$

51. In a gear mesh, R and r are the radius of pitch circles, R_a and r_a are the radius of addendum circles, ϕ is the pressure angle. The path of approach is expressed as

(a) $\sqrt{R^2 - R_a^2\sin^2\phi} - R\cos\phi$
(b) $\sqrt{R^2 - R_a^2\cos^2\phi} - R\sin\phi$
(c) $\sqrt{R_a^2 - R^2\sin^2\phi} - R\cos\phi$
(d) $\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi$

52. In a gear mesh, R and r are the radius of pitch circles, R_a and r_a are the radius of addendum circles, ϕ is the pressure angle. The path of recess is expressed as

(a) $\sqrt{r_a^2 - r^2\sin^2\phi} - r\cos\phi$
(b) $\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$
(c) $\sqrt{r^2 - r_a^2\sin^2\phi} - r\cos\phi$
(d) $\sqrt{r^2 - r_a^2\cos^2\phi} - r\sin\phi$

53. Which type(s) of tooth profiles of gears satisfy the law of gearing?

(a) cycloidal teeth
(b) involute teeth
(c) both (a) and (b)
(d) none of the above

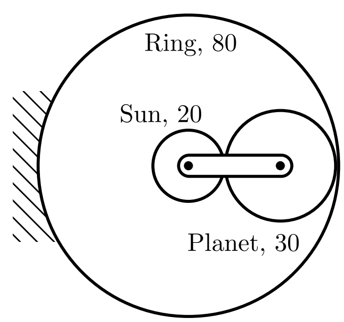
54. The interference will not occur if

(a) dedendum of gear is greater than that of gear
(b) dedendum of pinion is greater than that of pinion
(c) addendum of gear is greater than that of pinion
(d) addendum of pinion is greater than that of gear

55. An involute pinion and gear are in mesh if both have the same size of addendum then there will be inference between

(a) the tip of gear wheel tooth and flank of pinion
(b) the tip of teeth of the gear and pinion both
(c) the flanks of gear and pinion both
(d) all of the above

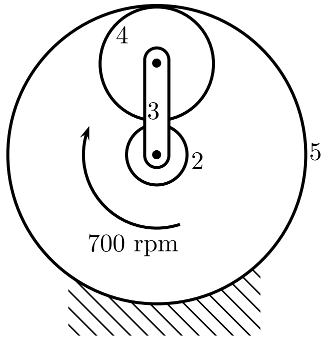
56. The sun gear in the figure is driven clockwise at 100 rpm. The ring gear is held stationary. For the number of teeth shown on the gears, the arm rotates at



(a) 0 rpm (b) 20 rpm
(c) 33.33 rpm (d) 66.67 rpm

57. Figure shows a planetary gear train. Gears 2, 4 and 5 have 24, 40 and 144 teeth, respectively. Gear 5 is fixed. Gear 2 is rotating clockwise at 700 rpm.

$T_2 = 24, T_4 = 40, T_5 = 144$



What will be the rpm of the arm and gear 4?

(a) 100, −260 (b) −260, 100
(c) 260, 100 (d) −100, 260

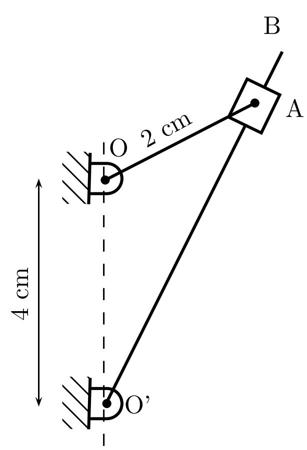
58. To make a worm drive reversible, it is necessary to increase

(a) center distance
(b) worm diameter factor
(c) number of starts
(d) reduction ratio

59. In spur gears, the circle on which the involute is generated is called the

(a) pitch circle (b) clearance circle
(c) base circle (d) addendum circle

60. Figure shows a quick return mechanism. The crank OA rotates clockwise uniformly.



The ratio of time for forward motion to that for return motion is

(a) 0.5 (b) 2.0
(c) $\sqrt{2}$ (d) 1

61. If reduction ratio of about 50 is required in a gear drive, then the most appropriate gearing would be

(a) spur gears
(b) bevel gears
(c) double helical gears
(d) worm and worm wheel

62. A rack is a gear of

(a) infinite diameter
(b) infinite module
(c) zero pressure angle
(d) large pitch

63. For spur with gear ratio greater than one, the interference is most likely to occur near the

(a) pitch point
(b) point of beginning of contact
(c) point of end of contact
(d) root of the tooth

64. There are six gears A, B, C, D, E, F in a compound train. The number of teeth in the gears are 20, 60, 30, 80, 25 and 75, respectively. The ratio of the angular speeds of the driven (F) to the driver (A) of the drive is

(a) 1/24 (b) 1/8
(c) 4/15 (d) 12

65. A fixed gear having 100 teeth meshes with another gear having 25 teeth. The center lines of both the gears being joined by an arm so as to form an epicylic gear train. The number of rotations made by the smaller gear for one rotation of the arm is

(a) 3 (b) 4
(c) 5 (d) 6

66. Balancing of a rigid rotor can be achieved by appropriately balancing masses in

(a) a single plane (b) two planes
(c) three planes (d) four planes

67. For an involute gear with pressure angle ϕ, the ratio of pitch circle radius to base circle radius is

(a) $\sin\phi$ (b) $\cos\phi$
(c) $\sec\phi$ (d) $\csc\phi$

68. In full length 14.5° involute system, the smallest number of teeth in a pinion which meshes with rack without interference is

(a) 12 (b) 16
(c) 25 (d) 32

69. In a flat collar pivot bearing, the moment due to friction is proportional to (r_o and r_i are the outer and inner radii, respectively)

(a) $\frac{r_o^2 - r_i^2}{r_o - r_i}$ (b) $\frac{r_o^2 - r_i^2}{r_o + r_i}$

(c) $\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}$ (d) $\frac{r_o^3 - r_i^3}{r_o - r_i}$

70. In involute gears, the pressure angle is

(a) dependent on the size of teeth
(b) dependent on the size of gears
(c) always constant
(d) always variable

71. Which one of the following is true for involute gears?

(a) interference is inherently absent
(b) variation in center distance of shafts increases radial force
(c) a convex flank is always in contact with concave flank
(d) pressure angle is constant throughout the teeth engagement

72. Which of the following statements are correct?

1. For constant velocity ratio transmission between two gears, the common normal at the point of contact must always pass through a fixed point on the line joining the centers of rotation of the gears.
2. For involute gears the pressure angle changes with change in center distance between gears.
3. The velocity ratio of compound gear train depends upon the number of teeth of the input and ouput gears only.
4. Epicylic gear trains involve rotation of at least one gear axis about some other gear axis.

(a) 1, 2 and 3 (b) 1, 3 and 4
(c) 1, 2 and 4 (d) 2, 3 and 4

73. A fixed gear having 200 teeth is in mesh with another gear having 50 teeth. The two gears are connected by an arm. The number of turns made by the smaller gear for one revolution of arm about the center of the bigger gear is

(a) 2/3 (b) 3
(c) 4 (d) 5

74. An involute pinion and gear are in mesh. If both have the same size of addendum, then there will be an interference between the

(a) tip of the gear and flank of pinion
(b) tip of the pinion and flank of gear
(c) flanks of both gear and pinion
(d) tips of both gear and pinion

75. Match List I with List II and select the correct answer.

List I	List II
A. Helical gears	1. Non-interchangeable
B. Herring bone gears	2. Zero axial thrust
C. Worm gears	3. Quiet motion
D. Hypoid gears	4. Extreme speed reduction

(a) A-1, B-2, C-3, D-4
(b) A-3, B-2, C-1, D-4
(c) A-3, B-1, C-2, D-4
(d) A-3, B-2, C-4, D-1

76. The work surface above the pitch surface of the gear tooth is termed as

(a) addendum (b) dedendum
(c) flank (d) face

77. In a simple gear train, if the number of idler gears is odd, then the direction of motion of driven gear will

(a) be same as that of the driving gear
(b) be opposite to that of the driving gear
(c) depend upon the number of teeth on the driving gear
(d) depend upon the total number of teeth on all gears of the train

78. Consider the following statements in case of reverted gear train

1. The direction of rotation of the first and the last gear is the same.
2. The direction of rotation of the first and the last gear is opposite.
3. The first and the last gears are on the same shaft.
4. The first and the last gears are on separate but co-axial shafts.

Which of these statements is/are correct?

(a) 1 and 3 (b) 2 and 3
(c) 2 and 4 (d) 4 alone

79. When two spur gears having involute profiles on their teeth engagement, the line of action is tangential to the

(a) pitch circles
(b) dedendum circles
(c) addendum circles
(d) base circles

80. Certain minimum number of teeth on the involute pinion is necessary in order to

(a) provide an economical design
(b) avoid interference
(c) reduce noise in operation
(d) overcome fatigue failure of the teeth

81. The velocity ratio between pinion and gear in a gear drive is 2.3, the module of teeth is 2.0 mm and sum of number of teeth on pinion and gear is 99. What is the center distance between pinion and the gear?

(a) 49.5 mm (b) 99 m
(c) 148.5 mm (d) 198 mm

82. External gear with 60 teeth meshes with a pinion of 20 teeth, module being 6 mm. What is the center distance in mm?

(a) 120 (b) 180
(c) 240 (d) 300

83. A gear having 100 teeth is fixed and another gear having 25 teeth revolves around it, center lines of both the gears being joined by an arm. How many revolutions will be made by the gear of 25 teeth for one revolution of arm?

(a) 3 (b) 4
(c) 5 (d) 6

84. The number of degrees of freedom of an epicylcic gear train is

(a) zero (b) one
(c) two (d) three

85. A high pressure angle for spur gears leads to

(a) minimum axial thrust
(b) greater backlash
(c) more interference
(d) wide base and stronger tooth

86. In a reciprocating engine mechanism, the crank and connecting rod of same length r meters are at right angles to each other at a given instant, when the crank makes an angle 45° with inner dead center. If the crank rotates with a uniform velocity of ω rad/s, the angular acceleration of the connecting rod will be?

(a) $2\omega^2$ (b) $\omega^2/\sqrt{2}$
(c) $-\omega^2\sqrt{2}$ (d) zero

87. Static balancing is satisfactory for low speed rotors but with increasing speeds, dynamic balancing becomes necessary. This is because,

(a) the unbalanced couples are cause only at higher speeds
(b) the unbalanced forces are dangerous at higher speeds
(c) the effects of unbalances are proportional to the square of the speed
(d) the effects of unbalances are directly proportional to the speed

88. A system in dynamic balance implies that

(a) the system is critically damped
(b) there is no critical speed in the system
(c) the system is also statically balanced
(d) there will be absolutely no wear of bearings

89. Introduction of the flywheel in a rotating system smoothens

(a) the bending moment on the rotating shaft
(b) the twisting moment on the shaft
(c) the bending stress on the shaft
(d) the axial force along the shaft

90. A four-stroke engine develops 18.5 kW at 250 rpm. The turning moment diagram is rectangular for both expansion and compression strokes. The turning moment is negative during compression stroke and is zero during suction and exhaust strokes. The turning moment for the expansion stroke is 2.8 times that of the compression stroke. Assuming constant load, determine the moment of inertia of the flywheel to keep the total fluctuation of the crankshaft speed within 1% of the average speed of 250 rpm.

(a) 845.74 kg.m^2 (b) 318.79 kg.m^2
(c) 144.56 kg.m^2 (d) 63.00 kg.m^2

91. Which of the following statement is correct?

(a) Flywheel reduces speed fluctuations during a cycle for a constant load, but flywheel does not control the mean speed of the engine if the load changes.
(b) Flywheel does not reduce speed fluctuation during a cycle for a constant load, but flywheel does not control the mean speed of the engine if the load changes.
(c) Governor controls speed fluctuations during a cycle for a constant load, but governor does not control the mean speed of the engine if the load changes.
(d) Governor controls speed fluctuations during a cycle for a constant load, and governor also control the mean speed of the engine if the load changes.

92. A flywheel of moment of inertia 9.8 kg·m^2 fluctuates by 30 rpm for a fluctuation in energy of 1936 Joules. The mean speed of the flywheel is (rpm)

(a) 600 (b) 900
(c) 968 (d) 2940

93. The radius of gyration of a solid disc type flywheel of diameter D is

(a) D (b) $D/\sqrt{2}$
(c) $D/2$ (d) $D/(2\sqrt{2})$

94. Flywheels are fitted for single cylinder and multi-cylinder engine of the same power rating. Which of the following statement is true?

(a) The flywheel will be smaller for single cylinder engine as compared to that for multi-cylinder engine.
(b) The flywheel will be smaller for multi-cylinder engine as compared to that for single cylinder engine.
(c) The flywheels for two engines will be identical.
(d) The size of flywheel for a engine depends on the compression ratio.

95. If the rotating mass of a rim type flywheel is distributed on another rim type flywheel whose mean radius is half mean radius of the former, then energy stored in the latter at the same speed will be

(a) four times the first one
(b) same as the first one
(c) one-fourth of the first one
(d) one and half times the first one

96. A flywheel is fitted to the crankshaft of an engine having E amount of indicated work per revolution and permissible limits of coefficients of fluctuation of energy and speeds as k_e and k_s, respectively. The kinetic energy of the flywheel is then given by

(a) $\dfrac{2k_eE}{k_s}$ (b) $\dfrac{k_eE}{2k_s}$
(c) $\dfrac{k_eE}{k_s}$ (d) $\dfrac{k_sE}{2k_e}$

97. In a 4-stroke IC engine, the turning moment during the compression stroke is

(a) positive throughout
(b) negative throughout
(c) positive during major portion of the stroke
(d) negative during major portion of the stroke

98. In case of a flywheel, the maximum fluctuation of energy is the

(a) sum of maximum and minimum energies
(b) difference between the maximum and minimum energies
(c) ratio of the maximum and minimum energies
(d) ratio of the minimum and maximum energies

99. If the size of a flywheel in a punching machine is increased

(a) then the fluctuation of speed and fluctuation of energy will both decrease
(b) then the fluctuation of speed will decrease and the fluctuation of energy will increase
(c) then the fluctuation of speed will increase and the fluctuation of energy will decrease
(d) then the fluctuation of speed and fluctuation of energy both will increase

100. What is the value of the radius of gyration of disc type flywheel as compared to rim type flywheel for the same diameter?

(a) $\sqrt{2}$ times (b) $1/\sqrt{2}$ times
(c) 2 times (d) 1/2 times

101. In a flywheel, the safe stress is 25.2 MN/m^2 and the density is 7 g/cm^3. Then what is the maximum peripheral velocity?

(a) 30 m/s (b) 45 m/s
(c) 60 m/s (d) 120 m/s

102. The ratio of tension on the tight side to that on the slack side in a flat belt drive is

(a) proportional to the product of coefficient of friction and lap angle
(b) an exponential function of the product of coefficient of friction and lap angle
(c) proportional to the lap angle
(d) proportional to the coefficient of friction

103. Given that T_1 and T_2 are the tensions on the tight and slack sides of the belt respectively, the initial tension of the belt, taking into account centrifugal tension T_c, is equal to

(a) $(T_1+T_2+T_c)/3$
(b) $(T_1+T_2+2T_c)/2$
(c) $(T_1+T_2+3T_c)/3$
(d) $(T_1-T_2+3T_c)/2$

104. The difference between tensions on the tight and slack sides of a belt drive is 3000 N. If the belt speed is 15 m/s, the transmitted power in kW is

(a) 45
(b) 22.5
(c) 90
(d) 100

105. The percentage improvement in power capacity of a flat belt drive, when the wrap angle at the driving pulley is increased from 150° to 210° by an idler arrangement for a friction coefficient of 0.3 is

(a) 25.21
(b) 33.92
(c) 40.17
(d) 67.85

106. A 50 kW motor using six V-belts is used in a pump mill. If one of the belts break after a month of continuous running then

(a) the broken belt is to be replaced by a similar belt
(b) all the belts are to be replaced
(c) the broken belt and two adjacent belts are to be replaced
(d) the broken belt and one adjacent belt are to be replaced

107. In multiple V-belt drives, when a single belt is damaged, it is preferable to change the complete set to

(a) reduce vibration
(b) reduce slip
(c) ensure uniform loading
(d) ensure proper alignment

108. In a flat belt drive, if the slip between the driver and the belt is 1%, that between belt and follower is 3%, and driver and follower pulley diameters are equal, then the velocity ratio of the drive will be

(a) 0.99
(b) 0.98
(c) 0.97
(d) 0.96

109. When a belt drive is transmitting maximum power

(a) effective tension is equal to centrifugal tension
(b) effective tension is half of centrifugal tension
(c) driving tension on slack side is equal to the centrifugal tension
(d) driving tension on tight side is thrice the centrifugal tension

110. If μ is the actual coefficient of friction in a belt moving in grooved pulley, the groove angle being 2α, the virtual coefficient of friction will be

(a) $\mu/\sin\alpha$
(b) $\mu/\cos\alpha$
(c) $\mu\sin\alpha$
(d) $\mu\cos\alpha$

111. Centrifugal tension in belts is

(a) useful because it maintains some tension even when no power is transmitted
(b) not harmful because it does not take part in power transmission
(c) harmful because it increases belt tension and reduces power transmitted
(d) a hypothetical phenomenon and does not actually exist in belts

112. The creep in a belt drive is due to the

(a) material of the pulleys
(b) material of the belt
(c) unequal size of the pulleys
(d) unequal tension on tight and slack sides of the belt

113. The length of the belt in the case of a cross-belt drive is given in terms of center distance between pulleys (C), diameters of the pulleys D and d as

(a) $2C+\dfrac{\pi}{2}(D+d)+\dfrac{(D+d)^2}{4C}$

(b) $2C+\dfrac{\pi}{2}(D-d)+\dfrac{(D+d)^2}{4C}$

(c) $2C+\dfrac{\pi}{2}(D+d)+\dfrac{(D-d)^2}{4C}$

(d) $2C+\dfrac{\pi}{2}(D-d)+\dfrac{(D-d)^2}{4C}$

114. In case of belt drives, the effect of the centrifugal tension is to

(a) cause the belt to leave the pulley and increase the power to be transmitted
(b) cause the belts to stay on the pulley and increase the power to be transmitted
(c) reduce the driving power of the belt
(d) stretch the belt in longitudinal direction

115. Which one of the following statements relating to belt drives is correct?

(a) The rotational speeds of the pulleys are directly proportional to their diameters
(b) The length of the crossed belt increases as the sum of the diameters of the pulleys increases
(c) The crowning of the pulleys is done to make the drive sturdy
(d) The slip increases the velocity ratio

116. In a flat belt drive the belt can be subjected to a maximum tension T and centrifugal tension T_c. What is the condition for transmission of maximum power?

(a) $T = T_c$ (b) $T = \sqrt{3}T_c$
(c) $T = 2T_c$ (d) $T = 3T_c$

117. The controlling force curve of an spring loaded governor is given by

$$F = ar - c$$

where r is the radius of rotation of the governor balls, and a and c are constants. The governor is

(a) stable (b) unstable
(c) isochronous (d) insensitive

118. A Hartnell governor is a governor of

(a) inertia type (b) pendulum type
(c) centrifugal type (d) dead weight type

119. A governor is said to be isochronous when the equilibrium speed for all radii or rotation the balls within the working range

(a) is not constant (b) is constant
(c) varies uniformly (d) has uniform acceleration

120. A Hartnell governor has its controlling force F given by

$$F = p + qr$$

where r is the radius of balls and p and q are constants. The governor becomes isochronous when

(a) $p = 0$ and q is positive
(b) p is positive and $q = 0$
(c) p is negative and q is positive
(d) p is positive and q is also positive

121. A spring loaded governor is found unstable. It can be made stable by

(a) increasing the spring stiffness
(b) decreasing the spring stiffness
(c) increasing the ball weight
(d) decreasing the ball weight

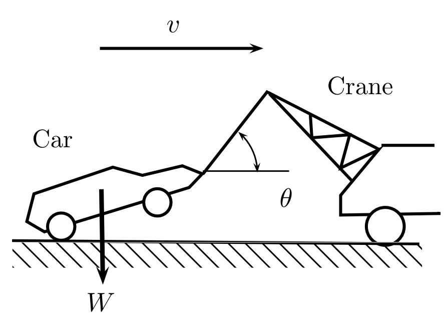
122. For a spring controlled governor to be stable, the controlling force (F) is related to the radius (r) by the equation

(a) $F = ar - b$ (b) $F = ar + b$
(c) $F = ar$ (d) $F = a/r - b$

123. The sensitivity of an isochronous governor is

(a) zero (b) one
(c) two (d) infinity

124. A car weighing W kN is to be lifted at angle θ and moved with velocity v m/s by crane as shown in the figure. The coefficient of friction between tires and road is μ.



What will be the minimum power requirement of the crane for moving the car?

(a) $W \times v$ kW (b) $\mu W v$ kW
(c) $\mu W v \cos\theta$ kW (d) $\mu W v \sin\theta$ kW

125. If ϕ is the friction angle, the efficiency of power screw is maximum when the lead angle is

(a) $\pi/2 - \phi/2$ (b) $\pi/2 - \phi$
(c) $\pi - \phi/2$ (d) $\pi - \phi$

126. A wheel of mass m and radius r is in accelerated rolling motion without slip under a steady axial

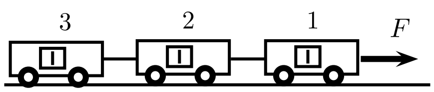
torque T. If the coefficient of kinetic friction is μ the friction force from the ground on the wheel is

(a) μmg
(b) T/r
(c) zero
(d) none of the above

127. A box rests in the rear of a truck moving with a deceleration of 2 m/s^2. To prevent the box from sliding, the approximate value of static coefficient of friction between the box and the bed of the truck should be

(a) 0.1 (b) 0.2
(c) 0.3 (d) 0.4

128. Three freight wagons, each of mass m, are pulled by a tractive effort F by a locomotive as shown in figure.



The rolling friction between the wagon wheels and track is negligible. What will be tractive forces on the third wagon?

(a) $2F$ (b) F
(c) $F/2$ (d) $F/3$

129. A friction circle is drawn when a journal rotates in bearing. Its radius depends on the coefficient of friction and the

(a) magnitude of the forces on the journal
(b) angular velocity of the journal
(c) clearance between the journal and the bearing
(d) radius of the journal

130. Frictional torque for threads at mean radius while raising load is given by

(a) $WR\tan(\phi-\alpha)$ (b) $WR\tan(\phi+\alpha)$
(c) $WR\tan\alpha$ (d) $WR\tan\phi$

131. To ensure self-locking in a screw jack, it is essential that helix angle is

(a) larger than friction angle
(b) smaller than friction angle
(c) equal to friction angle
(d) such as to give maximum efficiency in lifting

132. The maximum efficiency of a self-locking screw is

(a) 50% (b) 70%
(c) 75% (d) 80%

133. In a collar thrust bearing, the number of collars have been doubled, while maintaining to coefficient of friction and axial thrust same. This will result in

(a) same frictional torque and same bearing pressure
(b) double frictional torque and half bearing pressure
(c) double frictional torque and same bearing pressure
(d) same frictional torque and half bearing pressure

134. Which one of the following is the correct expression for the torque transmitted by a conical clutch of outer radius R_o, inner radius R_i and semi-cone angle α, assuming uniform pressure (where W = total axial load and μ = coefficient of friction)?

(a) $\dfrac{\mu W(R_o+R_i)}{2\sin\alpha}$ (b) $\dfrac{\mu W(R_o+R_i)}{3\sin\alpha}$

(c) $\dfrac{2\mu W\left(R_o^3-R_i^3\right)}{3\sin\alpha\,(R_o^2-R_i^2)}$ (d) $\dfrac{3\mu W\left(R_o^3-R_i^3\right)}{4\sin\alpha\,(R_o^2-R_i^2)}$

135. In the multiple disc clutch, if there are 6 discs on the driving shaft and 5 discs on the driven shaft, then the number of pairs of contact surfaces will be equal to

(a) 11 (b) 12
(c) 10 (d) 22

136. The turning moment diagram for a single cylinder double acting steam engine consists of positive and negative loops above and below the average torque line. For the positive loop, the ratio of the speeds of the flywheel at the beginning and the end is which one of the following?

(a) less than unity
(b) equal to unity
(c) greater than unity
(d) zero

137. Flywheel absorbs energy during those periods of crank rotation when

(a) the twisting moment is greater than the resisting moment
(b) the twisting moment is equal to the resisting moment
(c) the twisting moment is less than the resisting moment
(d) the load on the engine falls

NUMERICAL ANSWER QUESTIONS

1. A crank and slotted lever mechanism used in a shaper has a center distance of 300 mm between the center of oscillation of the slotted lever and the center of rotation of the crank. The radius of the crank is 120 mm. Calculate the angle run by the crank during cutting. Also calculate the ratio of the time of cutting to the time of return.

2. The angle between the axes of two shafts connected by Hooke's joint is 20°. Calculate the angle turned by the driving shaft when the velocity ratio is maximum and when the the velocity ratio is unity.

3. The driving shaft of a Hooke's joint rotates at a uniform speed of 420 rpm. The allowed variation in speed of the driven shaft is ± 4% of the mean speed. Determine the greatest permissible angle between the axes of the shafts. Also determine the maximum and minimum speeds of the driven shaft.

4. Two shafts at 20° are joined by Hooke's joint. The driving shaft rotates at a uniform speed of 120 rpm. The driven shaft carries a steady load of 10 kW. Calculate the maximum value of acceleration of the driven shaft. Also, calculate the mass of the flywheel of the driven shaft if its radius of gyration is 200 mm and the output torque of the driven shaft does not vary by more than 10% of the input shaft.

5. Two shafts at some distance are joined by a double Hooke's joint such that the forks of the intermediate shaft lie in planes perpendicular to each other. The driving shaft rotates at constant speed of 480 rpm. The angle of the driving and of the driven shaft with the intermediate shaft is 25°. Calculate the maximum and minimum speed of the driven shaft.

6. Two shafts with an inclination angle 165° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 rpm. The driven shaft carries a flywheel of mass 15 kg and 100 mm radius of gyration. Calculate the maximum angular acceleration and torque on the driven shaft.

7. An electric motor driven power screw moves a nut in a horizontal plane against a force of 80 kN at speed of 400 mm/min. The screw has a single square thread of 8 mm pitch on a major diameter of 50 mm. The coefficient of friction at the screw threads is 0.1. Calculate the required power of the electric motor.

8. A turn buckle with right and left hand single start threads is used to couple two wagons. Its thread pitch is 16 mm and mean diameter 50 mm. The coefficient of friction between the nut and screw is 0.15. Calculate the work done in drawing the wagons together a distance of 320 mm against a steady load of 2000 N. Also calculate the work done if the load increases from 2000 N to 4000 N over the distance of 160 mm.

9. A screw jack is used to lift a 25 kN load that rotates with the screw. The jack is made of threaded screw having diameter 30 mm and pitch of 6 mm. The coefficient of friction between in the thread is = 0.15. Determine the ratio of torques required to raise and lower the load. Also determine the efficiency of the machine.

10. A thrust bearing supports an axial load of 100 kN at 360 rpm. It has diameters of external and internal contracting surfaces area 380 mm and 280 mm, respectively. Coefficient of friction in the lubricated surface is 0.05 and the permitted intensity of pressure is 300 kN/m^2. Calculate the power lost in overcoming the friction. Also calculate the number of of collars required for the bearing.

11. A conical pivot is used to supports a load of 20 kN at 120 rpm. The pivot has cone-angle of 90° and its external radius is three times the internal radius. Coefficient of friction in the lubricated surface is 0.06 and the permitted intensity of pressure is 350 kN/m^2. Calculate the external radius of the collar and the power lost to overcome the friction.

12. A single plate clutch is used to transmit 30 kW at 1000 rpm. The outer diameter of both the effective plates is 400 mm. Coefficient of friction between contacting surfaces is 0.20 and the allowed pressure intensity between the plates is 75 kN/m^2. Calculate the inner diameter of the plate. Also calculate the axial force to engage the clutch plate.

13. A multi-plate disc clutch transmits 150 kW of power at 1500 rpm. Coefficient of friction for the friction surfaces is 0.2. Axial intensity of pressure is not to exceed 150 kN/m^2. The internal radius is 100 mm and is half of the external radius.

Calculate the torque transmitted per friction surface in the clutch. Also calculate the the number of plates needed to transmit the required torque.

14. A leather belt is required to transmit 8.5 kW from a pulley 1.2 m in diameter running at 300 rpm. The angle embraced is 160° and the coefficient of friction between the belt and the pulley is 0.25. The safe working stress for the leather belt is 2.0 MPa, density of the leather is 1 Mg/m^3 and thickness of the belt is 15 mm. Calculate the ratio of belt tension in tight and slack sides. Also calculate the width of the belt considering the centrifugal tension into account.

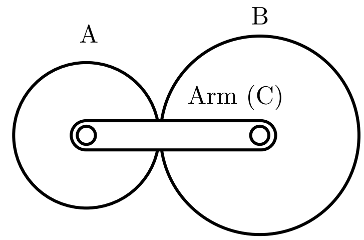
15. A belt drive is used to transmit 7.5 kW of power between parallel shafts apart 2 m. The driving and the driven shafts rotate at 80 rpm and 200 rpm, respectively. The diameter of the smaller pulley is 450 mm. The belt is 120 mm wide and 10 mm thick. The coefficient of friction is 0.2. Calculate the stress in the belt if the two pulleys are connected by open belt drive and crossed belt drive.

16. In a flat belt drive, the initial tension is 1500 N. The coefficient of friction between the belt and the pulley is 0.2 and the angle of lap on the smaller pulley is 160°. The smaller pulley has a radius of 250 mm and rotates at 600 rpm. Calculate the power transmitted through the belt.

17. Meshing of two gears of 10 mm module produces a velocity ratio of 3 and pressure angle of 20°. The larger gear has 60 teeth. The addenda on pinion and gear wheel is equal to the module. Calculate the number of pairs of teeth in contact. Also calculate the angle of action of the pinion and the gear wheel.

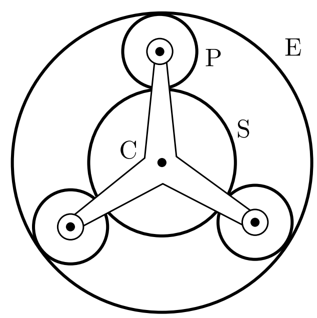
18. Meshing of two gears of 6 mm module produces a velocity ratio of 4 and pressure angle of 20°. The smaller gear rotates at 180 rpm. Addendum of the gears is equal to the module. Determine the minimum number of teeth on each wheel to avoid interference. Calculate the corresponding number of pairs of teeth in contact.

19. In an epicylic gear train, an arm carries two gears A and B having 48 and 60 teeth, respectively, as shown in the figure.



Calculate the speed of gear B if the arm rotates at 200 rpm in the anticlockwise direction about the center of the fixed gear A. Also calculate the speed of gear B if the gear A instead of being fixed, makes 360 rpm in the clockwise direction.

20. An epicylic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star-shaped carrier C. The size of different wheels are such that the planet carrier C rotates at 1/6th of the speed of the sun wheel S. The minimum number of teeth on any wheel is 18. The driving torque on the sun wheel is 150 Nm.



Calculate the number of teeth on sun gear, and the required torque to keep the internal gear E stationary.

21. In a press-working station, a machine is used to punch-out 35 mm diameter holes on a 25 mm thick plate at a rate of 8 holes/min. The punch has a stroke of 100 mm. The mean speed of flywheel is 25 m/s at radius of gyration. Specific energy requirement of the operation is 10 J/mm^2 of the sheared area. Determine the power of the motor. Calculate the mass of the flywheel to limit the total fluctuation of speed to 3% of the mean speed.

22. Two standard full depth gears of pressure angle 14.5° have a module of 5 mm. The gear ratio is 4. The addendum of the gear is equal to the module. Calculate the minimum number of teeth required in gear to avoid interference. What should be pressure angle if number of teeth is to be kept to 60 without interference?

ANSWERS

Multiple Choice Questions

1. (a)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (d)	**10.** (a)
11. (d)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (d)	**17.** (a)	**18.** (a)	**19.** (d)	**20.** (c)
21. (a)	**22.** (a)	**23.** (c)	**24.** (d)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (a)	**30.** (d)
31. (d)	**32.** (d)	**33.** (d)	**34.** (b)	**35.** (a)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (d)	**40.** (a)
41. (d)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (b)	**47.** (a)	**48.** (d)	**49.** (b)	**50.** (a)
51. (d)	**52.** (b)	**53.** (c)	**54.** (d)	**55.** (a)	**56.** (b)	**57.** (a)	**58.** (c)	**59.** (c)	**60.** (b)
61. (d)	**62.** (a)	**63.** (b)	**64.** (c)	**65.** (c)	**66.** (b)	**67.** (c)	**68.** (d)	**69.** (c)	**70.** (c)
71. (d)	**72.** (b)	**73.** (d)	**74.** (a)	**75.** (c)	**76.** (a)	**77.** (b)	**78.** (d)	**79.** (d)	**80.** (b)
81. (b)	**82.** (a)	**83.** (d)	**84.** (a)	**85.** (d)	**86.** (c)	**87.** (c)	**88.** (c)	**89.** (b)	**90.** (a)
91. (a)	**92.** (a)	**93.** (b)	**94.** (b)	**95.** (c)	**96.** (b)	**97.** (d)	**98.** (b)	**99.** (a)	**100.** (b)
101. (c)	**102.** (b)	**103.** (b)	**104.** (a)	**105.** (d)	**106.** (b)	**107.** (c)	**108.** (d)	**109.** (d)	**110.** (a)
111. (c)	**112.** (d)	**113.** (a)	**114.** (c)	**115.** (a)	**116.** (d)	**117.** (a)	**118.** (c)	**119.** (b)	**120.** (a)
121. (b)	**122.** (a)	**123.** (d)	**124.** (b)	**125.** (a)	**126.** (a)	**127.** (b)	**128.** (d)	**129.** (d)	**130.** (b)
131. (b)	**132.** (a)	**133.** (d)	**134.** (c)	**135.** (c)	**136.** (c)	**137.** (a)			

Numerical Answer Questions

1. 132.84°, 1.71	**2.** 0°, 44.11°	**3.** 22.92°, 593 rpm, 403 rpm
4. 19.74 rad/s, 100 kg	**5.** 584.37 rpm, 394.26 rpm	**6.** 759.26 rad/s^2, 113.89 Nm
7. 1.582 kW	**8.** 802.99 W, 802.99 W	**9.** 2.52, 29.49%
10. 39.02 kW, 7	**11.** 14.3 mm, 2.202 kW	**12.** 0.1 mm, 1.0 kN
13. 267 Nm, 5	**14.** 2.0, 25.54 mm	**15.** 2.45 MPa, 2 MPa
16. 12.82 kW	**17.** 1.67, 26.35°, 10.63°	**18.** 64, 16, 1.646
19. 360 rpm, 660 rpm	**20.** 18, 600 Nm	**21.** 3.665 kW, 1282.8 kg
22. 116, 20.31°		

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (a) Inversions of a mechanism are the different mechanisms obtained by fixing different links in a kinematic chain. Thus, the number of inversions is equal to the number of links in the mechanism.

2. (c) Let d be the distance between shafts. Given that

$$N = 180 \text{ rpm}$$
$$d = 0.020 \text{ m}$$

Angular speed of the shafts is given by

$$\omega = \frac{2\pi N}{60} = 18.85 \text{ rad/s}$$

The maximum speed of sliding of the tong of the intermediate piece along its groove is given by

$$v = \omega \times d = 0.376 \text{ m/s}$$

3. (b) If the cylinder is made to work as a guide, and the piston in the form of a slider, it results into slotted lever crank mechanism.

4. (d) Oldham's coupling is an inversion of the double slider crank mechanism.

5. (a) If the two links in a pair have point or line contact while in motion, the pair so formed is known as a higher pair, such as cam-follower mechanism, toothed gears, ball and roller bearings, belt and pulley.

6. (c) When incompletely constrained motion is made to be only one direction by using some external means, it is called a successfully constrained motion.

7. (d) Whitworth quick return mechanism is obtained by fixing the smallest link (crank) of a slider crank mechanism.

8. (a) A scotch yoke is used to convert the rotary motion into a sliding motion. Mathematically, it generates sine function.

9. (d) If the elements of a pair are not held together mechanically, instead, either due to force of gravity or some spring action, it is called unclosed (open) pair. For example, cam and follower.

10. (a) Hand pump mechanism is obtained by fixing the slider in the slider-crank mechanism.

11. (d) The path traced by a point on the link connecting a double slider crank chain is an ellipse.

12. (a) Double crank mechanism is achieved by fixing the shortest link.

13. (c) Whitworth quick return mechanism is obtained from single slider crank chain by fixing its crank.

14. (a) Gruebler's equation is written as

$$F = 3(n-1) - 2f_1$$

15. (a) Kutzbach equation is written as

$$F = 3(n-1) - 2f_1 - f_2$$

16. (d) Given that the number of hinged joints

$$f_1 = 5$$

Number of links

$$n = 5$$

Degree of freedom is determined as

$$\begin{aligned} F &= 3\,(n-1) - 2f_1 \\ &= 3 \times (5-1) - 2 \times 5 \\ &= 2 \end{aligned}$$

As one link is already fixed, balance degree of freedom is $2-1=1$.

17. (a) Universal joint is used to connect two non-parallel and intersecting shafts and misaligned shafts.

18. (a) Universal joint (called Hooke's joint) is used to connect two non-parallel and intersecting shafts and misaligned shafts, e.g. to transmit power from the gearbox to rear axle in an automobile.

19. (d) The driving shaft rotates at constant angular speed but driven shaft rotates at varying speed which gets it maximum value two times in each revolution.

20. (c) At any position θ of the driving shaft, the ratio of speeds of driven shaft and driving shaft (ω_2/ω_1) will be given by

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}$$

21. (a) The maximum variation in the velocity of driven shaft w.r.t. its mean velocity is expressed as

$$\frac{(\omega_2)_{max} - (\omega_2)_{min}}{\omega_1} = \tan\alpha\sin\alpha$$

22. (a) For maximum acceleration of the driven shaft, at angle θ of the driving shaft is given by

$$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}$$

23. (c) The constant velocity ratio is achieved in a double Hooke's joint when driving and driven shafts make equal angle with the intermediate shaft and forks of intermediate shaft lie in the same plane.

24. (d) Maximum and minimum ratio of speeds of the shafts (ω_2/ω_1) is given by

$$\begin{aligned} \left(\frac{\omega_2}{\omega_1}\right)_{max} &= \frac{1}{\cos^2\alpha} \\ \left(\frac{\omega_2}{\omega_1}\right)_{min} &= \cos^2\alpha \end{aligned}$$

25. (d) Given that

$$\begin{aligned} \alpha &= 19.5^\circ \\ \omega_1 &= 500 \text{ rpm} \end{aligned}$$

Maximum speed of the driven shaft is

$$\omega_2 = \frac{\omega_1}{\cos\alpha} = 531.91 \text{ rpm}$$

26. (a) The linear velocity at the top end in y direction is

$$V_{\mathrm{A}} = -1 \text{ m/s}$$

Velocity in x direction of the bottom end is calculated as

$$V_{\mathrm{B}} = -V_{\mathrm{A}} \tan 60^\circ = 1.732 \text{ m/s}$$

The net velocity of the link is determined as

$$V_{AB} = \sqrt{V_A^2 + V_B^2} = 2 \text{ m/s}$$

The angular velocity is given by

$$\omega_{AB} = \frac{V_{AB}}{AB} = 2 \text{ rad/s}$$

27. (d) The Coriolis component of acceleration is equal to $2\omega v$, which is possible when the body is restrained to rotate while sliding over another body.

28. (c) For two bodies having relative motion with respect to one another, instantaneous center (I) of rotation is an imaginary point common to two bodies such that any of the two bodies can be assumed to have motion of rotation with respect to the other about the imaginary point.

29. (a) In crank and slotted level mechanism, the slider moves (v) in accelerating (ω) lever where value of $2\omega v$ exists.

30. (d) The lateral acceleration of slider moving with speed v on a crank at angular speed ω and acceleration α is

$$a^{\perp} = 2\omega v + r\alpha$$

In above equation, component $2\omega v$ is known as Coriolis component.

31. (d) Number of instantaneous centers for a mechanism consisting of 'n' links is

$$N = \frac{n(n-1)}{2}$$

32. (d) In a mechanism, the number of instantaneous centers is the number of possible combinations of two links. Mathematically, it is the number of combinations of n links taken two at a time, given by

$$N = {}^nC_r = \frac{n(n-1)}{2}$$

For $n = 6$,

$$N = 15$$

33. (d) The relative motion takes place along the common tangent between the two links at the point of contact. Consequently, the instantaneous center lies on the common normal at the point of contact. Its position on the common normal depends on the ratio of sliding and angular velocities.

34. (b) Pressure angle is the angle between normal to the pitch curve at a point and direction of motion of follower. Thus, for easy lift pressure angle should be minimum while for return it can be greater.

35. (a) There are no abrupt changes in velocity and acceleration, hence, cycloidal program is the most suitable one for high speed follower motion.

36. (c) In parabolic motion of the follower, expressed as

$$s = ut + \frac{1}{2}at^2$$

the acceleration remains constant.

37. (b) According to profile of gear teeth and law of gearing, the teeth roll over each other.

38. (b) Module of a gear is defined as

$$m = \frac{d}{T}$$

For a standard gear tooth profile, one module is equal to addendum.

39. (d) To achieve such a high speed ratio, worm gear should be used.

40. (a) The minimum number of teeth on the gear shall be given by

$$T \geq \frac{2}{\sqrt{1 + (1/G)(1/G + 2)\sin^2\phi} - 1}$$

41. (d) Hypoid gears, spiral gears and worm gears are used to connect non-parallel and non-intersecting shafts.

42. (c) Rack and pinion mechanism is used to convert either rotary motion into liner motion or vice versa.

43. (a) Straight bevel gears of the same size and two gears at right angle to each other are known as miter gears.

44. (b) Spiral bevel gears find application in drive to the differential of an automobile.

45. (b) Circular pitch in a gear is the space in pitch circle used by each teeth:

$$p_c = \frac{\pi d}{T}$$

46. (b) The diameters of the pitch circles will be mT and mt. Hence, the central distance will be given by

$$\begin{aligned} c &= \frac{mT}{2} + \frac{mt}{2} \\ &= \frac{m(T+t)}{2} \end{aligned}$$

47. (a) Pressure angle is the angle between pressure line and common tangent to pitch circles.

48. (d) According to the law of gearing, the angular velocities of the two gears remain constant, the common normal at the point of contact of two tooth should always pass through a fixed point which divides the line of centers in the inverse ratio of angular velocities of two gears.

49. (b) The maximum possible value of the path of approach is $r \sin\phi$.

50. (a) The maximum possible value of the path of recess is $R \sin\phi$

51. (d) The path of contact is the distance along the pressure line traveled by contact point from the point of engagement to the pitch point. It is expressed as

$$\text{CP} = \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi$$

52. (b) Path of recess is the distance along the pressure line traveled by contact point from the pitch point to the point of disengagement. It is expressed as

$$\text{PD} = \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$$

53. (c) Both cyloidal and involute teeth profiles satisfy the law of gearing.

54. (d) To avoid interference in wheel and pinion, addendum of pinion should be greater than that of gear.

55. (a) An involute pinion and gear are in mesh if both have the same size of addendum then there will be inference between the tip of gear wheel tooth and flank of pinion.

56. (b) Given that

$$\begin{aligned} N_R &= 0 \\ N_S &= 100 \text{ rpm (clockwise)} \end{aligned}$$

Using the tabular algorithm, assuming the arm to be fixed, one finds

$$\begin{aligned} N_A &= y \\ N_S &= x+y \\ N_P &= -\frac{20}{30}x+y \\ &= -\frac{2}{3}x+y \\ N_R &= -\frac{20}{30}\times\frac{30}{80}x+y \\ &= -\frac{1}{4}x+y \end{aligned}$$

Thus,

$$\begin{aligned} x+y &= 100 \\ -\frac{1}{4}x+y &= 0 \end{aligned}$$

Solving above equations, one finds

$$\begin{aligned} x &= \frac{400}{5} \\ &= 80. \\ y &= 100-80 \\ &= 20 \end{aligned}$$

Hence, speed of arm

$$\begin{aligned} N_A &= y \\ &= 20 \text{ rpm} \end{aligned}$$

57. (a) Keeping the arm (3) as fixed and applying the tabular logarithm gives

$$\begin{aligned} N_3 &= y \\ N_2 &= x+y \\ N_4 &= -\frac{24}{40}x+y \\ N_5 &= -\frac{24}{40}\times\frac{40}{144}x+y \end{aligned}$$

Gear 5 is fixed:

$$\begin{aligned} 1\frac{1}{6}x+y &= 0 \\ x &= 6y \end{aligned}$$

Given $N_2 = 700$:

$$x+y = 700$$
$$y = 100$$
$$x = 600$$

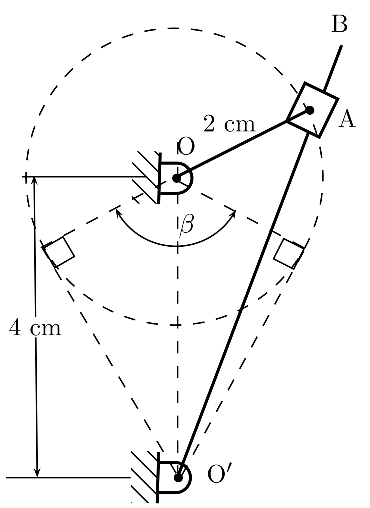
Thus

$$N_3 = y = 100 \text{ rpm}$$
$$N_4 = -\frac{3}{5}x+y = -260 \text{ rpm}$$

58. (c) Unlike with ordinary gear trains, the direction of transmission in worm drive is not reversible when using large reduction ratios, due to the greater friction involved between the worm and worm-wheel, when usually a single start (one spiral) worm is used. If a multi-start worm (multiple spirals) is used then the ratio reduces accordingly and the braking effect of a worm and worm-gear can be discounted as the gear can drive the worm.

59. (c) The involute teeth consist of only single involute curve which is the locus of point on a straight (generating) line which rolls without slipping on the circumference of a base circle.

60. (b) The angular rotation of the crank for forward and return strokes are shown in the figure below.



The value of angle β is determined as

$$\beta = 2\sin^{-1}\left(\frac{2}{4}\right) = 120^\circ$$

The ratio is determined as

$$R = \frac{360-\beta}{\beta} = 2$$

61. (d) Worm and worm wheel gears are used for large gear ratios.

62. (a) A rack is a gear of infinite diameter because instead of base circle, the gears are formed over straight line.

63. (b) For spur with gear ratio greater than one, that is, the addendum of the wheel is more than a limit, the tip of the wheel tooth will be in contact with a portion of the non-involute profile of the pinion tooth. This takes place at the beginning of the contact.

64. (c) The speed ratio is given by

$$\frac{N_F}{N_A} = \frac{20}{60}\times\frac{60}{30}\times\frac{30}{80}\times\frac{80}{25}\times\frac{25}{75} = \frac{20}{75} = \frac{4}{15}$$

65. (c) The larger gear ($T_S = 100$) is fixed, the smaller gear ($T_P = 25$) is joined with an arm (A). The tabular method shall be used as below

Condition	N_A	N_S	N_P
A fixed, S (1)	0	1	$-100/25$
C fixed, S(x)	0	x	$-100x/25$
Add y to all	y	$y+x$	$-100x/25+y$

Given that

$$N_S = 0$$
$$y+x = 0$$
$$x = -y$$
$$N_P = -4x+y$$
$$N_A = y$$
$$\frac{N_P}{N_A} = \frac{-4x+y}{y} = 4+1 = 5$$

66. (b) Along with the unbalanced mass, to balance the moments, the masses should be placed on two planes.

67. (c) For pressure angle ϕ of a gear, the base circle radius (r_b) is related to pitch circle radius (r) as

$$r_b = r\cos\phi$$
$$\frac{r}{r_b} = \sec\phi$$

68. (d) To avoid interference

$$T > \frac{2}{\sin^2 \phi}$$

Regarding interference in rack-pinion, for $\phi = 20°$, $T > 18$, and for $\phi = 14.5°$, $T > 32$.

69. (c) Using uniform pressure theory, for a flat collar of radii internal radius r_i, external radius r_o, the friction torque will be given by

$$T = \frac{2\mu F}{3} \frac{{r_o}^3 - {r_i}^3}{{r_o}^2 - {r_i}^2}$$

70. (c) In involute gears, the pressure angle is always constant, which is the angle between center line and tangent to base circles.

71. (d) Pressure angle is the angle between pressure line and common tangent to pitch circles. In involute teeth profile, it remains constant throughout the teeth contact path.

72. (b) According to law of gearing, if it is desired that angular velocities of the two gears remain constant, the common normal at the point of contact of two tooth should always pass through a fixed point which divides the line of centers in the inverse ratio of angular velocities of two gears. For involute gears, the pressure angle remains constant.

73. (d) Given that

$$T_g = 200$$
$$T_p = 50$$

Using tabular procedure,

$$\frac{N_p}{N_a} = \frac{T_g}{T_p} + 1 = 5$$

74. (a) There will be inference between the tip of gear wheel tooth and flank of pinion.

75. (c) Helical gears are used for quieter motion. Herring bone gears are made of double helix to avoid axial thrust. Worm gears give extreme speed reduction. Hypoid gears are non-interchangeable.

76. (a) Addendum circle is the outermost profile circle.

77. (b) The idler gears are used to reverse the direction of rotation of gears. Odd number of idler gears will give opposite direction of driving gear.

78. (d) In reverted gear trains, the first and last gears are co-axial.

79. (d) For gears with involute profile, line of action is tangential to the base circles of the gears

80. (b) To avoid interference,

$$T \geq \frac{2}{\sqrt{1 + (1/G)\,(1/G + 2)\sin^2 \phi} - 1}$$

81. (b) The center distance is given by

$$\begin{aligned} c &= \frac{m}{2}\,(T_g + T_p) \\ &= \frac{2}{2} \times 99 \\ &= 99 \text{ mm} \end{aligned}$$

82. (a) The center distance will be given by

$$\begin{aligned} c &= \frac{m}{2}\,(T_g - T_p) \\ &= 120 \text{ mm} \end{aligned}$$

83. (d) The number of revolutions of pinion is

$$\begin{aligned} N_p &= \frac{T_g}{T_p} + 1 \\ &= 6 \end{aligned}$$

84. (a) In epcicyclic gear train, the motion of one link decides the motion of others. Hence, the degree of freedom is one.

Alternatively, for an epicyclic gear train

$$n = 3$$
$$f_2 = 1$$
$$f_1 = 2$$

Using Kutzbach equation:

$$\begin{aligned} F &= 3(n-1) - 2f_1 - f_2 \\ &= 3\,(3-1) - 2 \times 2 - 1 \\ &= 1 \end{aligned}$$

85. (d) The size of the base circle controls the pressure angle. The increase in the base circle diameter increases the length of the arc of the circle upon which the wedge (the raised portion) is to be made. Hence, the teeth becomes stronger.

86. (c) Given that

$$\begin{aligned} l &= r \\ n &= l/r \\ &= 1 \\ \theta &= 45° \end{aligned}$$

Angular acceleration of connecting rod is

$$\begin{aligned} \alpha_{CR} &= -\omega^2 \frac{\sin\theta}{n} \\ &= -\frac{\omega^2}{\sqrt{2}} \end{aligned}$$

87. (c) The acceleration is proportional to square of the angular speed, and so is the unbalanced forces. Thus, at higher speeds, dynamic balancing is also necessary.

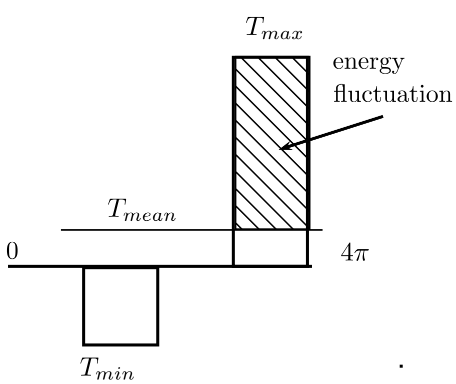
88. (c) A system of rotating masses is said to be in static balance if the combined mass center of the system lies on the axis of rotation. A system of rotating masses is said to be in dynamic balance if there exists neither any resultant centrifugal force nor any resultant couple when the system rotates.

89. (b) The absorption of energy by flywheel smoothens the twisting moment on the rotating shafts.

90. (a) Given that

$$\begin{aligned} P &= 18.5\times 10^3 \text{ W} \\ N &= 250 \text{ rpm} \\ k_s &= 2\times\frac{1}{100} \\ &= 0.02 \end{aligned}$$

Turning moment diagram is shown below:



Also given,

$$T_{max} = 2.8T_{min}$$

The angular speed is

$$\begin{aligned} \omega &= \frac{2\pi N}{60} \\ &= \frac{2\times\pi\times 250}{60} \\ &= 26.18 \text{ rad/s} \end{aligned}$$

The maximum torque is determined as

$$\begin{aligned} T_{max}\left(1-\frac{1}{2.8}\right)\pi &= \frac{18.5\times 10^3}{250/(2\times 60)} \\ T_{max} &= 4396.92 \text{ Nm} \end{aligned}$$

The average torque is

$$\begin{aligned} T_{mean} &= \frac{P}{\omega} \\ &= \frac{18.5\times 10^3}{26.18} \\ &= 706.65 \text{ Nm} \end{aligned}$$

The excess power is

$$\begin{aligned} I\omega^2 k_s &= (T_{max}-T_{mean})\,\pi \\ I &= \frac{(T_{max}-T_{mean})\,\pi}{\omega^2 k_s} \\ &= \frac{(4396.92-706.65)\,\pi}{26.18^2\times 0.02} \\ &= 845.74 \text{ kg.m}^2 \end{aligned}$$

91. (a) Flywheel reduces speed fluctuations during a cycle for a constant load, but the governor controls the mean speed of the engine in case of changes in the load.

92. (a) The fluctuation of energy of the flywheel is determined as

$$\begin{aligned} e &= \frac{1}{2}I(\omega_1^2-\omega_2)^2 \\ &= I(\omega_1-\omega_2)\frac{\omega_1+\omega_2}{2} \\ &= I(\omega_1-\omega_2)\bar{\omega} \end{aligned}$$

Hence, the mean speed is given by

$$\begin{aligned} \bar{\omega} &= \frac{e}{I(\omega_1-\omega_2)} \\ &= \frac{1936}{9.8\times\frac{2\pi\times 30}{60}} \\ &= 62.88 \text{ rad/s} \\ &= 600.48 \text{ rpm} \end{aligned}$$

93. (b) Moment of inertia of a solid disc type flywheel of diameter D is

$$I = \frac{1}{2}mD^2$$

Therefore, radius of gyration is

$$\begin{aligned} k &= \sqrt{\frac{I}{m}} \\ &= \frac{D}{\sqrt{2}} \end{aligned}$$

94. (b) There will be less variation of turning moment in the multi-cylinder engine as compared to that for single cylinder engine.

95. (c) Energy stored is kinetic energy is

$$\begin{aligned} E &= \frac{1}{2}mk^2\times\omega^2 \\ E &\propto k^2 \end{aligned}$$

Therefore, energy stored in the flywheel of half mean radius of the former flywheel will be

$$\frac{E_2}{E_1} = \frac{(1/2)^2}{1^2} = \frac{1}{4}$$

96. (b) Kinetic energy of the flywheel is

$$\text{KE} = \frac{1}{2}I\omega^2 = \frac{\Delta E}{2k_s} = \frac{k_e E}{2k_s}$$

97. (d) During compression stroke, the power is used to compress the charge and valves are closed/opened before or after dead centers. Hence, turning moment is negative during major portion of the stroke.

98. (b) Let E be kinetic energy of the flywheel at ω, e is maximum fluctuation of energy. The fluctuation of energy

$$e = \frac{1}{2}I(\omega_1^2 - \omega_2)^2 = I\omega^2 k_s$$

where k_s is the coefficient of fluctuation of speed.

99. (a) Fluctuation of energy depends upon the fluctuation of speed. Hence, increased size of flywheel will reduce both.

100. (b) Moment of inertia of disc type flywheel is

$$I = \frac{1}{2}mr^2$$

while that of rim type flywheel is

$$I = mr^2$$

101. (c) Given that

$$\rho = 7 \times 10^{-3} \times 10^{-2}\ \text{kg/m}^2$$
$$\sigma = 25.2 \times 10^6\ \text{N/m}^2$$

Using

$$\sigma = \rho v^2$$
$$v = \sqrt{\frac{\sigma}{\rho}} = 60.35\ \text{m/s}$$

102. (b) The ratio is of tensions on tight side (T_1) and slack side (T_2) is expressed as

$$\frac{T_1}{T_2} = \exp(\mu\theta)$$

where μ is the coefficient of friction, and θ is the lap angle (rad).

103. (b) Let the belt is assembled with an initial tension. When power is transmitted, the tension in tight side increases from T_i to T_1 and on slack side decreases from T_i to T_2. If the belt is assembled to obey Hooke's law and its length remains constant, then

$$T_1 - T_i = T_i - T_2$$
$$T_i = \frac{T_1 + T_2}{2}$$

The centrifugal tension T_c is added in tension of the both sides of the belt. Therefore,

$$T_i = \frac{T_1 + T_c + T_2 + T_c}{2} = \frac{T_1 + T_2 + 2T_c}{2}$$

104. (a) The power transmitted through the belt drive is calculated as

$$P = (T_1 - T_2)\,v = 3000 \times 15 = 45\ \text{kW}$$

105. (d) The ratio of tensions on tight and slack sides is related to lap angle θ (rad) and coefficient of friction μ as

$$\frac{T_1}{T_2} = \exp(\mu\theta)$$

Thus, the power transmitted can be expressed as

$$P = (T_1 - T_2)\,v = T_2\left[\exp(\mu\theta) - 1\right]v$$

If the lap angle is changed from θ_1 to θ_2, the change is power is determined as

$$\frac{P_2 - P_1}{P_1} = \frac{e^{\mu\theta_2} - e^{\mu\theta_1}}{e^{\mu\theta_1} - 1}$$

Given that

$$\theta_1 = 150^\circ = 2.617\ \text{rad}$$
$$\theta_2 = 210^\circ = 3.665\ \text{rad}$$
$$\mu = 0.3$$

Hence,

$$\frac{P_2 - P_1}{P_1} = \frac{0.81}{1.1926} = 67.91\%$$

106. (b) All the belts should be changed to avoid uneven tension and over-loading of smaller length belts.

107. (c) To ensure uniform loading of each belt, it is preferable to change the complete set.

108. (d) Given that

$$s_1 = 0.01$$
$$s_2 = 0.03$$
$$D_1 = D_2$$

Velocity ratio is

$$\frac{N_2}{N_1} = (1-s_1)(1-s_2)\frac{D_1}{D_2} = 0.96$$

109. (d) For maximum power transmission in belt drives

$$T_c = \frac{T_1}{3}$$

110. (a) Virtual coefficient of friction for the groove angle 2α is

$$\mu' = \frac{\mu}{\sin(\alpha)}$$

111. (c) For belts with centrifugal tension,

$$\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta}$$

where T_1 and T_2 are actual tensions in the belt, $T_1 - T_c$ and $T_2 - T_c$ are the driving or effective tensions on the pulley.

112. (d) Presence of friction between pulley and belt causes differential tensions in the belt. This results into elongation or contraction, and a relative motion between the belt and the pulley surface. This slip is called elastic creep. The net effect of creep is to reduce speed of the belt, resulting in reduction of power transmission.

113. (a) Length of cross-belt drive is given by

$$L = \pi\frac{(D+d)}{2} + 2C + \frac{(D+d)^2}{4C}$$

114. (c) In belt drives, the power transmitted is reduced if centrifugal effect is considered for given value of tension.

115. (a) The angular speeds of the pulleys are directly proportional to the their diameters.

116. (d) For maximum power transmission in belt drives

$$T_c = \frac{T_i}{3}$$

117. (a) The controlling force curve is passing through r in the positive axis, therefore, the governor is stable.

118. (c) In Hartnell governor, the centrifugal forces acting on the balls is used to control the speed.

119. (b) A governor with infinite sensitiveness is called isochronous governor. This means that for all positions of sleeve or the balls, the governor has the same speed and any change of speed results into moving the balls into extreme positions.

120. (a) For an isochronous governor

$$F \propto r$$

Therefore, $p = 0$, and q should be positive. For an stable governor

$$F = qr - p$$

121. (b) For unstable governors, the relation between controlling force F and radius r varies as

$$F = pr + q$$

while for stable governors

$$F = pr - q$$

In above equation, q is a function of spring stiffness, therefore, it is possible by reducing the spring stiffness, which will reduce the force F.

122. (a) The governor is said to be stable when slope of the speed curve is less than that of controlling force curve. For a given governor to be stable at all radii, the controlling force curve should pass $r = +$ve or 0. In this case

$$\frac{F}{r} < \frac{\partial F}{\partial r}$$

For this, the equation of controlling force is

$$F = ar - b$$

123. (d) A governor with infinite sensitiveness is called isochronous governor. This means that for all positions of sleeve or the balls, the governor has the same speed and any change of speed results in moving the balls into extreme positions.

124. (b) The horizontal force required to move the car is μW kN, therefore, the power requirement is $\mu W v$ kW.

125. (a) Screw efficiency is maximum when

$$\alpha = \frac{\pi - \phi}{2}$$

126. (a) The reaction at contact surface will be mg. Hence, the rolling friction will be μmg.

127. (b) The inertia force on the box shall be ma, to resist it, the friction force will be μmg. Therefore,

$$\begin{aligned} \mu mg &= ma \\ \mu &= \frac{a}{g} \\ &= \frac{2}{9.81} \\ &= 0.203 \end{aligned}$$

128. (d) Considering the three sets of wagons as a free body on which force F acts, the acceleration of each wagon can be calculated as

$$\begin{aligned} F &= 3m \times a \\ a &= \frac{F}{3m} \end{aligned}$$

Hence, the tractive force on the last wagons will be

$$\begin{aligned} F_3 &= m \times a \\ &= m\frac{F}{3m} \\ &= \frac{F}{3} \end{aligned}$$

129. (d) If r is the journal radius, and ϕ is the coefficient of friction, then a circle drawn with radius $r \sin \phi$ is known as the friction circle of the journal.

130. (b) For square thread screw used as a jack to lift a load W, the friction force is

$$F = W \tan(\alpha + \phi)$$

Hence, the friction torque at mean radius R is

$$T = WR \tan(\alpha + \phi)$$

131. (b) The angle of friction (ϕ) should always be more than the helix angle of the screw (α). Otherwise ($f \geq 0$), the load will slide down of its own weight W.

132. (a) The angle of friction should always be more than the helix angle of the screw. Otherwise, the load will slide down of its own weight W. Reversal of nut on the screw is avoided if the efficiency of the thread is less than 50% approximately.

133. (d) The thrust will be distributed into collars. Hence, the frictional torque will be reduced in each collar but overall for all the collars it will be the same.

134. (c) For a conical collar of radii internal radius R_i, external radius R_o, half cone angle α, subjected to axial force F, under the uniform pressure theory, the friction torque is given by

$$T = \frac{2\mu F}{3 \sin \alpha} \times \frac{{R_o}^3 - {R_i}^3}{{R_o}^2 - {R_i}^2}$$

135. (c) Given that

$$\begin{aligned} n_1 &= 6 \\ n_2 &= 5 \end{aligned}$$

Number of surfaces in contact is

$$\begin{aligned} N &= n_1 + n_2 - 1 \\ &= 10 \end{aligned}$$

136. (c) During the positive rule, the energy will be supplied to flywheel for storage. Hence, speed will be less at the beginning and more at end of the positive loop.

137. (a) Flywheel absorbs extra energy available in the system, that is, when the twisting moment is greater than the resisting moment.

Numerical Answer Questions

1. Angle run by the crank in return stroke is

$$\begin{aligned} \alpha &= 2 \times \cos^{-1} \frac{120}{300} \\ &= 132.84^\circ \end{aligned}$$

Therefore, the cutting ratio is

$$\begin{aligned} R &= \frac{360 - \alpha}{\alpha} \\ &= 1.71 \end{aligned}$$

2. Given that

$$\alpha = 20^\circ$$

The velocity ratio is maximum when

$$\begin{aligned} \cos^2 \theta &= 1 \\ \theta &= 0,\ 180^\circ \end{aligned}$$

The velocity ratio is unity when

$$\tan\theta = \pm\sqrt{\cos\alpha}$$
$$\theta = 44.11^\circ$$

3. Maximum variation in speed of the driven shaft is

$$\alpha^2 = 2\times 0.04$$
$$\alpha = 0.283 \text{ rad}$$
$$= 16.20^\circ$$

Speed of the driving shaft is

$$\omega_1 = 420 \text{ rpm}$$

Maximum and minimum speeds of the driven shaft are

$$\omega_{2\max} = \frac{\omega_1}{\cos\alpha}$$
$$= 593.96 \text{ rpm}$$
$$\omega_{2\min} = \omega_1\cos\alpha$$
$$= 403.30 \text{ rpm}$$

4. Given that

$$\alpha = 20^\circ$$
$$N_1 = 120 \text{ rpm}$$
$$P = 10\times 10^3 \text{ W}$$
$$k = 0.200 \text{ m}$$
$$\Delta T = 10\%$$

Maximum torque will be generated when the acceleration is maximum, which is determined by angle θ, given by

$$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}$$
$$\theta = 41.43^\circ$$
$$\omega_1 = \frac{2\pi N_1}{60}$$
$$= 12.57 \text{ rad/s}$$

Maximum acceleration of the driven shaft for $\theta = 41.43^\circ$ is given by

$$\frac{d\omega_2}{dt} = \frac{-\omega_1^2\cos\alpha\sin^2\alpha\sin 2\theta}{\left(1-\sin^2\alpha\cos^2\theta\right)^2}$$
$$= \frac{17.233}{0.872}$$
$$= 19.74 \text{ rad/s}$$

The average torque on driving shaft will be given by

$$T = \frac{P}{\omega_1}$$
$$= 795.57 \text{ Nm}$$

The permissible variation in torque (15%) will be observed by the flywheel of mass m, given by

$$T\times\Delta T = mk^2\frac{d\omega_2}{dt}$$
$$m = 100.75 \text{ kg}$$

5. Given that

$$N_1 = 480 \text{ rpm}$$
$$\alpha = 25^\circ$$

When the forks of the intermediate shaft lie in planes perpendicular to each other, the maximum speed is given by

$$(N_2)_{\max} = \frac{N_1}{\cos^2\alpha}$$
$$= 584.37 \text{ rpm}$$

When the forks of the intermediate shaft lie in planes perpendicular to each other, the minimum velocity is given by

$$(N_2)_{\min} = N_1\cos^2\alpha$$
$$= 394.26 \text{ rpm}$$

6. Given that

$$\alpha = 180-165$$
$$= 15^\circ$$
$$N_1 = 1000 \text{ rpm}$$
$$m = 15 \text{ kg}$$
$$k = 0.1 \text{ m}$$

For maximum acceleration of the driven shaft, the angle run by the drive shaft is given by

$$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}$$
$$\theta = 40.87^\circ$$

The angular speed of the driving shaft is given by

$$\omega_1 = \frac{2\pi N_1}{60}$$
$$= 104.72 \text{ rad/s}$$

Maximum acceleration of the driven shaft is given by at $\theta = 40.87^\circ$

$$\frac{d\omega_2}{dt} = \frac{-\omega_1^2\cos\alpha\sin^2\alpha\sin 2\theta}{\left(1-\sin^2\alpha\cos^2\theta\right)^2}$$
$$= 759.26 \text{ rad/s}^2$$

The maximum torque on the driven shaft will be

$$\begin{aligned} T &= mk^2 \times \frac{d\omega_2}{dt} \\ &= 113.89 \text{ Nm} \end{aligned}$$

7. The speed of the nut is 300 mm/min, hence, the angular speed of the screw will be given by

$$\begin{aligned} N &= \frac{300}{8} \\ &= 50 \text{ rpm} \end{aligned}$$

Given that

$$\begin{aligned} W &= 80 \times 10^3 \text{ N} \\ v &= 400 \text{ mm/min} \\ p &= 8 \text{ mm} \\ d_o &= 50 \text{ mm} \\ \mu &= 0.1 \end{aligned}$$

Mean diameter of the screw will be

$$\begin{aligned} d &= d_o - \frac{p}{2} \\ &= 46 \text{ mm} \end{aligned}$$

The helix angle of the screw is given by

$$\begin{aligned} \alpha &= \tan^{-1} \frac{p}{\pi d} \\ &= 3.168° \end{aligned}$$

The friction angle is given by

$$\begin{aligned} \phi &= \tan^{-1} \mu \\ &= 5.71° \end{aligned}$$

The required power of the motor to move the load will be given by

$$\begin{aligned} P &= W \tan(\phi + \alpha) \times \frac{d}{2} \times \frac{2\pi N}{60} \\ &= 1.582 \text{ kW} \end{aligned}$$

8. Given that

$$\begin{aligned} p &= 16 \text{ mm} \\ d &= 50 \text{ mm} \\ \mu &= 0.2 \end{aligned}$$

Therefore, the helix angle and friction angle of the screw are

$$\begin{aligned} \alpha &= \tan^{-1} \frac{p}{\pi d} \\ &= 5.81° \\ \phi &= \tan^{-1} \mu \\ &= 8.53° \end{aligned}$$

The torque required to overcome the friction between screw and nut is given by

$$\begin{aligned} T &= W \tan(\phi + \alpha) \times \frac{d}{2} \\ &= 12.78 \text{ Nm} \end{aligned}$$

For one revolution of the turn buckle of the opposite starts, the wagons will move a distance equal to $2p = 32$ mm. Therefore, to draw the wagons for total distance 320 mm, total 320/32 turns will be required, during which the work done will be given by

$$\begin{aligned} W &= T\theta \\ &= 12.78 \times 2\pi \times \frac{320}{32} \\ &= 802.99 \text{ W} \end{aligned}$$

The work required to draw the wagons against load of 4000 N over the distance 160 mm will be

$$\begin{aligned} W' &= 802.99 \times \frac{4000}{2000} \times \frac{160}{320} \\ &= 802.99 \text{ W} \end{aligned}$$

9. Given that

$$\begin{aligned} p &= 6 \text{ mm} \\ d &= 30 \text{ mm} \\ \mu &= 0.15 \\ W &= 25 \text{ kN} \end{aligned}$$

The helix angle α is given by

$$\begin{aligned} \tan\alpha &= \frac{p}{\pi d} \\ &= 0.06366 \\ \alpha &= 3.64° \end{aligned}$$

The friction angle ϕ is given by

$$\begin{aligned} \phi &= \tan^{-1} \mu \\ &= 8.53° \end{aligned}$$

The torque to raise and raise the load are given respectively by

$$\begin{aligned} T_r &= W \tan(\phi + \alpha) \times \frac{d}{2} \\ T_l &= W \tan(\phi - \alpha) \times \frac{d}{2} \end{aligned}$$

Therefore, ratio of the torques is

$$\begin{aligned} R &= \frac{\tan(\phi + \alpha)}{\tan(\phi - \alpha)} \\ &= 2.52 \end{aligned}$$

The efficiency of the screw is given by

$$\begin{aligned}\eta &= \frac{\tan\alpha}{\tan(\alpha+\phi)}\\ &= 0.2949\\ &= 29.49\%\end{aligned}$$

10. Given that

$$\begin{aligned}r_o &= \frac{0.380}{2}\\ &= 0.190 \text{ m}\\ r_i &= \frac{0.280}{2}\\ &= 0.140 \text{ m}\\ W &= 100\times10^3 \text{ N}\\ p &= 300\times10^3 \text{ N/m}^2\\ N &= 360 \text{ rpm}\\ \mu &= 0.05\end{aligned}$$

Assuming constant pressure intensity, The power lost in overcoming the friction is given by

$$\begin{aligned}P &= \frac{2}{3}\mu W\frac{r_o^3-r_i^3}{r_o^2-r_i^2}\times\frac{2\pi N}{60}\\ &= 31.34 \text{ kW}\end{aligned}$$

The number of collars to bear the load W is given by

$$\begin{aligned}W &= n\times p\pi\left(r_o^2-r_i^2\right)\\ n &= 6.43\\ &\approx 7\end{aligned}$$

11. Given that

$$\begin{aligned}\alpha &= \frac{90^\circ}{2}\\ &= 45^\circ\\ W &= 20\times10^3 \text{ N}\\ r_o &= 3r_i\\ N &= 120 \text{ rpm}\\ p &= 350\times10^3 \text{ N/m}^2\\ \mu &= 0.06\end{aligned}$$

For the uniform intensity

$$p = \frac{W}{\pi\left(r_o^2-r_i^2\right)}$$

Replacing r_o by $2.5r_i$, one gets

$$\begin{aligned}r_i &= \sqrt{\frac{20}{\pi\times350\left(3^2-1^2\right)}}\\ &= 0.04768 \text{ m}\\ r_o &= 0.1430 \text{ m}\end{aligned}$$

The is given by

$$\begin{aligned}P &= \frac{2}{3\sin\alpha}\mu W\frac{r_o^3-r_i^3}{r_o^2-r_i^2}\times\frac{2\pi N}{60}\\ &= 2.202 \text{ kW}\end{aligned}$$

12. Given that

$$\begin{aligned}P &= 30\times10^3 \text{ W}\\ N &= 1000 \text{ rpm}\\ p_m &= 75\times10^3 \text{ N/m}^2\\ r_o &= \frac{0.400}{2}\\ &= 0.2 \text{ m}\\ \mu &= 0.20\\ n &= 2\end{aligned}$$

The torque transmitted through the clutch is

$$\begin{aligned}T &= \frac{P}{\frac{2\pi N}{60}}\\ &= \frac{30\times10^3}{\frac{2\pi\times1000}{60}}\\ &= 286.47 \text{ Nm}\end{aligned}$$

Considering the uniform wear theory, the axial force F is given by

$$\begin{aligned}p_m &= \frac{F}{2\pi r_i\left(r_o-r_i\right)}\\ F &= p_m\times2\pi r_i\left(r_o-r_i\right)\end{aligned}$$

The frictional torque should be equal to the maximum torque to be transferred, hence

$$\begin{aligned}T &= \mu F\frac{r_o+r_i}{2}\times n\\ &= \mu p_m\pi r_i\left(r_o^2-r_i^2\right)n\\ \frac{T}{\mu p_m\pi n} &= \left(r_o^2-r_i^2\right)r_i\end{aligned}$$

Putting the known values, one finds

$$r_i^3-0.04r_i+0.00304 = 0$$

Using Newton–Raphson method with initial value $r_i = 0.20$ m,

$$\begin{aligned}x_{n+1} &= x_n-\frac{f(x)}{f'(x)}\\ r_i &= 0.105 \text{ m}\end{aligned}$$

Considering the uniform wear theory, the axial force F is given by

$$\begin{aligned} F &= p_m \times 2\pi r_i (r_o - r_i) \\ &= 1.414 \text{ kN} \end{aligned}$$

13. Given that

$$\begin{aligned} P &= 150 \times 10^3 \text{ W} \\ N &= 1500 \text{ rpm} \\ \mu &= 0.2 \\ p_m &= 150 \times 10^3 \text{ N/m}^2 \\ r_i &= 0.10 \text{ m} \\ r_o &= \frac{r_i}{0.5} = 0.2 \text{ m} \end{aligned}$$

Assuming uniform wear theory, the maximum axial force per plate is given by

$$\begin{aligned} p_m &= \frac{F}{2\pi r_i (r_o - r_i)} \\ F &= p_m \times 2\pi r_i (r_o - r_i) \\ &= 9.42 \text{ kN} \end{aligned}$$

The torque that can be transmitted through single surface is given by

$$\begin{aligned} T &= \mu F \frac{r_o + r_i}{2} \\ &= 282.74 \text{ Nm} \end{aligned}$$

Total torque is calculated as

$$\begin{aligned} T_t &= \frac{P}{2\pi N/60} \\ &= 954.92 \text{ Nm} \\ n &= \frac{T_t}{T} \\ &= 3.37 \end{aligned}$$

Hence, at least 4 surfaces will be required, which is possible by 5 plates, 3 on driven and 2 on driving shafts, thus, $3 + 2 - 1 = 4$.

14. Given that

$$\begin{aligned} P &= 8.5 \times 10^3 \text{ W} \\ D &= 1.2 \text{ m} \\ N &= 300 \text{ rpm} \\ \theta &= 160^\circ \\ &= 2.792 \text{ rad} \\ \mu &= 0.25 \\ \sigma &= 2 \times 10^6 \text{ Pa} \\ \rho &= 1 \times 10^3 \text{ kg/m}^3 \\ t &= 0.015 \text{ m.} \end{aligned}$$

The linear velocity of the belt is given by

$$\begin{aligned} v &= \frac{\pi D N}{60} \\ &= 18.84 \text{ m/s} \end{aligned}$$

The ratio of the belt tensions is

$$\begin{aligned} \frac{T_1}{T_2} &= e^{\mu\theta} \\ &= 2.0 \end{aligned}$$

The power transmitted is

$$\begin{aligned} P &= (T_1 - T_2)\, v \\ T_1 &= 902.33 \text{ N} \end{aligned}$$

The centrifugal tension is calculated as

$$\begin{aligned} T_c &= \rho bt \times 1 \times v^2 \\ &= 5324.184b \text{ N} \end{aligned}$$

Maximum tension in the belt is given by

$$\begin{aligned} T_1 + T_c &= \sigma bt \\ b &= \frac{T_1 + T_c}{\sigma t} \\ &= 25.54 \text{ mm} \end{aligned}$$

15. Given that

$$\begin{aligned} b &= 0.12 \text{ m} \\ t &= 0.1 \text{ m} \\ P &= 7.5 \times 10^3 \text{ W} \\ c &= 2.0 \text{ m} \\ d &= 0.45 \text{ m} \\ N_1 &= 80 \text{ rpm} \\ N_2 &= 200 \text{ rpm} \\ \mu &= 0.2 \end{aligned}$$

The diameter of the driving shaft will be given by

$$\begin{aligned} \frac{D}{d} &= \frac{160}{80} \\ D &= 0.9 \text{ m} \end{aligned}$$

The velocity of belt at mean radius of rotation will be given by

$$\begin{aligned} v &= \frac{2\pi N_2}{60}\left(\frac{d}{2} + \frac{t}{2}\right) \\ &= 5.759 \text{ m/s} \end{aligned}$$

If T_1 and T_2 are the tensions on the belt, the power being transmitted is given by

$$\begin{aligned} P &= (T_1 - T_2)\, v \\ T_1 - T_2 &= 1302.18 \text{ N} \end{aligned}$$

For the open belt drives, the angle of lap is given by

$$\theta = \pi - 2\sin^{-1}\left(\frac{D-d}{2c}\right)$$
$$= 2.916 \text{ rad}$$

The ratio of tensions is given by

$$\frac{T_1}{T_2} = e^{\mu\theta}$$
$$= 1.79$$
$$T_2\,(1.79-1) = 1302.18 \text{ N}$$
$$T_2 = 1644.65 \text{ N}$$
$$T_1 = 2943.94 \text{ N}$$

The stress in the belt is given by

$$\sigma = \frac{T_1}{bt}$$
$$= 2.45 \text{ MPa}$$

Lap angle for the cross belt drives is

$$\theta = \pi + 2\sin^{-1}\left(\frac{D+d}{2c}\right)$$
$$= 3.83 \text{ rad}$$

The ratio of tensions is given by

$$\frac{T_1}{T_2} = e^{\mu\theta}$$
$$= 2.15$$
$$T_2\,(2.15-1) = 1302.18 \text{ N}$$
$$T_2 = 1131.15\text{N}$$
$$T_1 = 2431.98 \text{ N}$$

The stress in the belt is given by

$$\sigma = \frac{T_1}{bt}$$
$$= 2.02 \text{ MPa}$$

16. Given that

$$T_i = 1500 \text{ N}$$
$$\mu = 0.2$$
$$\theta = 160^\circ$$
$$= 2.792 \text{ rad}$$
$$r = 0.250 \text{ m}$$
$$N = 600 \text{ rpm}$$

The linear velocity of the belt is

$$v = \frac{2\pi r N}{60}$$
$$= 15.71 \text{ m/s}$$

The ratio of belt tension is given by

$$\frac{T_1}{T_2} = e^{\mu\theta}$$
$$= 1.748$$

The initial tension is related to belt tensions as

$$T_i = \frac{T_1+T_2}{2}$$
$$T_2 = \frac{2T_i}{1.748+1}$$
$$= 1091.70 \text{ N}$$
$$T_1 = 1908.30 \text{ N}$$

The power transmitted through the belt is given by

$$P = (T_1 - T_2)\,v$$
$$= 12.82 \text{ kW}$$

17. Given that

$$m = 10 \text{ mm}$$
$$G = 3$$
$$\phi = 20^\circ$$
$$T = 60$$
$$t = \frac{T}{G}$$
$$= 30$$
$$a = m$$

The pitch radius of the gears will be given by

$$R = \frac{mT}{2}$$
$$= 300 \text{ mm}$$
$$r = \frac{mt}{2}$$
$$= 100 \text{ mm}$$

The addendum radius of the gears are given by

$$R_a = R + a$$
$$= 310 \text{ mm}$$
$$r_a = r + a$$
$$= 110 \text{ mm}$$

The path of contact is

$$= \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi$$
$$+ \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$$
$$= 26.34 + 22.97$$
$$= 49.32 \text{ mm}$$

The number of teeth in contact is given by

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{\text{Path of contact}/\cos\phi}{\pi m} = 1.67$$

Angle of action on the pinion is given by

$$\delta_p = \frac{\text{Arc of contact} \times 360°}{2\pi r} = \frac{49.32 \times 360}{\cos 20° \times 2\pi \times 100} = 30.07°$$

Angle of action on the gear is given by

$$\delta_g = \frac{\text{Arc of contact} \times 360°}{2\pi R} = \frac{49.32 \times 360}{\cos 20° \times 2\pi \times 300} = 10.02°$$

18. Given that

$$\phi = 20°$$
$$G = 4$$
$$m = 6 \text{ mm}$$
$$a_w = 1$$
$$N_p = 180 \text{ rpm}$$

To avoid interference

$$T = \frac{2a_w}{\sqrt{1+(1/G)(1/G+2)\sin^2\phi} - 1} = 61.77$$

Since, gear ratio is 4, therefore, T should be divisible by 4 and more than 61.77, therefore,

$$T = 64$$
$$t = \frac{64}{4} = 16$$

Using

$$T = 64$$
$$t = 16$$
$$a = m = 6 \text{ mm}$$

The pitch radius of the gears will be given by

$$R = \frac{mT}{2} = 192 \text{ mm}$$
$$r = \frac{mt}{2} = 48 \text{ mm}$$

The addendum radius of the gears are given by

$$R_a = R + a = 198 \text{ mm}$$
$$r_a = r + a = 54 \text{ mm}$$

The path of contact is

$$= \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi + \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi = 15.89 + 13.27 = 29.16 \text{ mm}$$

The number of teeth in contact is given by

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{\text{Path of contact}/\cos\phi}{\pi m} = 1.646$$

It means that at least teeth shall be in contact continuously and another teeth will be in contact for 64.6% of the cycle.

19. The tabular method is shown below.

Arm	Gear A	Gear B
Arm fixed		
0	$+1$	$-T_A/T_B$
Arm fixed, gear A rotates x revolutions		
0	$+x$	$-(T_A/T_B)x$
Add y revolutions to all elements		
y	$x+y$	$y-(T_A/T_B)x$

When arm rotates at 200 rpm and gear A is fixed

$$y = 200$$
$$x + y = 0$$
$$x = -200$$

Therefore, speed of the gear B is

$$N_B = y - \frac{T_A}{T_B}x = \left(1 + \frac{48}{60}\right)200 = 360 \text{ rpm}$$

When gear A rotates at 400 rpm (clockwise) and arm rotates at 200 rpm (anti-clockwise), then

$$\begin{aligned} y &= 200 \\ x+y &= -400 \\ x &= -600 \end{aligned}$$

The speed of gear B is given by

$$\begin{aligned} N_B &= y - \frac{T_A}{T_B}x \\ &= 200 + \frac{46}{60}\times 600 \\ &= 660 \text{ rpm} \end{aligned}$$

20. The tabular method is shown below:

C	S	P	E
C fixed, S one revolution:			
0	1	$-T_S/T_P$	$-T_S/T_E$
C fixed, S x revolutions:			
0	x	$-(T_S/T_P)\,x$	$-(T_S/T_E)\,x$
Add y to all elements:			
y	$y+x$	$y-(T_S/T_P)\,x$	$y-(T_S/T_E)\,x$

Given that

$$\begin{aligned} N_C &= \frac{N_S}{6} \\ y &= \frac{y+x}{6} \\ \frac{x}{y} &= 5 \end{aligned}$$

Also,

$$\begin{aligned} N_E &= 0 \\ y - \frac{T_S}{T_E}x &= 0 \\ T_E &= \frac{x}{y}T_S \\ &= 5T_S \end{aligned}$$

Taking the dimensional constraints

$$\begin{aligned} T_S + 2T_P &= T_E \\ T_P &= \frac{T_E - T_S}{2} \end{aligned}$$

Observing above equations, one can see that

$$T_E > T_S$$

Hence, the minimum number of teeth on the sun wheel, thus

$$\begin{aligned} T_S &= 18 \\ T_E &= 90 \\ T_P &= 36 \end{aligned}$$

The driving torque on the sun is 150 Nm, if the angular speed is ω_E, the energy will be conserved, hence, the torque on C will be

$$\begin{aligned} T_C &= 150 \times \frac{\omega_S}{\omega_C} \\ &= 150 \times \frac{N_S}{N_C} \\ &= 750 \text{ Nm} \end{aligned}$$

Taking the whole gear train as a free body, the torque to keep E stationary is

$$\begin{aligned} T &= 750 - 150 \\ &= 600 \text{ Nm} \end{aligned}$$

21. Given that the number of holes per minute $n = 8$, diameter of the hole $d_h = 35$ mm, thickness of the plate $t_h = 25$ mm, energy rate $e_h = 10.0$ Nm/mm^2. Energy required per hole is

$$\begin{aligned} P_1 &= \pi d t e_h \\ &= 27.489 \text{ kJ} \end{aligned}$$

The power required for punching work will be given by

$$\begin{aligned} P &= \pi d t e_h \times \frac{n}{60} \\ &= 3.665 \text{ kW} \end{aligned}$$

Stroke length of the punch $l = 100$ mm, mean speed of the flywheel is 25 m/s. As the piston stroke is 100 mm, hence, in one round, it will travel $2\times 100 = 200$ mm. Average punching velocity is

$$\begin{aligned} v_p &= \frac{200\times n}{60} \\ &= 26.67 \text{ mm/s} \end{aligned}$$

Hence, time spent in actual punching a single hole, that is, in running through plate thickness will be

$$\begin{aligned} t &= \frac{t_h}{v_p} \\ &= \frac{25}{26.67} \\ &= 0.9373 \text{ s} \end{aligned}$$

The energy supplied by the motor during running the plate thickness will be

$$\begin{aligned} E &= P \times t \\ &= 3.665 \times 0.9373 \\ &= 3.435 \text{ kJ} \end{aligned}$$

But a single hole requires 27.489 kJ which shall be compensated by the energy supplied by the flywheel during punching of a single hole, determined as

$$\begin{aligned} e &= 27.489 - 3.435 \\ &= 24.05 \text{ kJ} \end{aligned}$$

The kinetic energy the flywheel ($E = mv^2/2$) is related to coefficient of speed fluctuation k_s ($= 0.03$) and energy fluctuation e as

$$\begin{aligned} \frac{mv^2}{2} &= \frac{e}{2k_s} \\ m &= 1282.8 \text{ kg} \end{aligned}$$

22. Given that

$$\begin{aligned} a_w &= 1 \\ G &= 4 \\ \phi &= 14.5^\circ \end{aligned}$$

Therefore, minimum number of teeth required on the gear to avoid interference is given by

$$\begin{aligned} T &= \frac{2a_w}{\sqrt{1 + (1/G)\,(2 + 1/G)\sin^2\phi} - 1} \\ &= 114.42 \text{ say } 116 \end{aligned}$$

Hence, number of teeth on gear is 116 and on pinion 29. Given that $T = 60$. Therefore, minimum number of teeth required on the gear to avoid interference is given by

$$\begin{aligned} T &= \frac{2a_w}{\sqrt{1 + (1/G)\,(2 + 1/G)\sin^2\phi} - 1} \\ 60 &= \frac{2 \times 1}{\sqrt{1 + (1/4)\,(2 + 1/4)\sin^2\phi} - 1} \\ \phi &= 20.31^\circ \end{aligned}$$

CHAPTER 4

VIBRATIONS

Vibrations are the oscillations of a structural system about the equilibrium position. In general sense, these are *periodic motions*, repeating in a certain interval of time. All the structural system possessing mass and elasticity are capable of vibrations to some extent. If uncontrolled, vibrations can lead catastrophic situations and unusual consequences. Vibrations are induced by unbalanced forces and can also be induced for benefits. Therefore, the design of engineering systems requires consideration of the vibrational factors. Vibration isolators are used to protect structures from excessive forces developed in the operation of rotating machine. The theory of vibrations is concerned with the study of oscillatory motions of bodies and the forces associated with them.

4.1 FUNDAMENTALS

This section describes the basic concepts and analysis of mechanical vibrations.

4.1.1 Basic Phenomenon

All bodies having mass and elasticity are capable of vibrations. Mass is an inherent property of the body. Elasticity of the material permits relative motion among its parts. When body particles are displaced from the equilibrium position by the application of external force, the internal forces of the body, in the form of stresses and inertia, try to bring the body to its original equilibrium position. The swinging of pendulum and the motion of plucked spring are typical examples.

Vibration of a system involves transfer of potential energy to kinetic energy, and kinetic energy to potential energy. In a damped system, some energy is dissipated in each cycle of vibration which must be replaced by an external source, if state of steady vibration is to be maintained.

4.1.2 Harmonic Motion

Harmonic motion is the simplest form of vibrations which is represented in terms of *amplitude* x_0, time t and *frequency* ω, and phase angle ϕ in trigonometric function [Fig. 4.1]:

$$x = x_0 \sin(\omega t + \phi) \tag{4.1}$$

A harmonic motion having amplitude x_0 and rotating at constant angular velocity ω can be represented in exponential form or as a complex quantity

$$\begin{aligned} z &= x_0 e^{j\omega t} \\ &= x_0 \cos\omega t + j x_0 \sin\omega t \end{aligned} \tag{4.2}$$

where z is referred as *complex sinusoid*.

The associated terms of sinusoidal representation are defined as follows:

1. *Amplitude* *Amplitude* (x_0) is the maximum displacement of a particle under the harmonic motion from equilibrium position. This peak value indicates the maximum strain that the vibrating part is undergoing. The average value of displacement can be found as

$$\bar{x} = \lim_{T\to\infty} \frac{1}{T}\int_0^T x\,(t)\,dt$$

The square of the displacement is associated with the energy of the vibration for which the mean square value is a measure:

$$\bar{x}^2 = \lim_{T\to\infty} \frac{1}{T}\int_0^T x^2\,(t)\,dt$$

2. *Period* *Period* is the time required to execute one cycle of the harmonic motion.

3. *Frequency* *Frequency* (f) of a harmonic motion is the number of cycles executed in unit time. It is the inverse of time period (T):

$$f = \frac{1}{T}$$

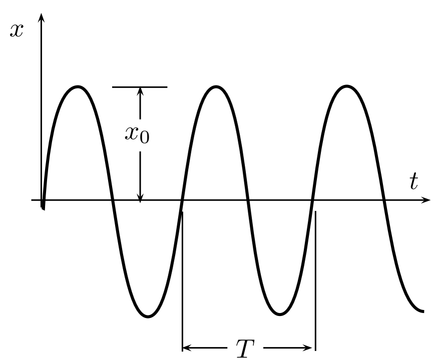


Figure 4.1 Harmonic motion.

The unit of frequency, cycle per second, is designated as one hertz (Hz).

As the system executes one cycle, the argument of the trigonometric function runs through 2π radians. Thus,

$$1 \text{ cycle} = 2\pi \text{ radians}$$

Therefore, the periodicity of motion is also expressed in terms of *circular frequency*, measured in rad/s:

$$\begin{aligned} f &= \frac{\omega}{2\pi} \text{ cycles/s} \\ &= \omega \text{ rad/s} \end{aligned}$$

Frequency is also expressed in terms of revolution per minute (rpm):

$$f = \frac{60\omega}{2\pi} \text{ rpm}$$

4. *Phase Angle* *Phase angle* (ϕ) represents the lead or lag between the response and a purely sinusoidal response. If $\phi > 0$, the response is said to *lag* a pure sinusoid, and if $\phi < 0$, the response is said to *lead* the sinusoid.

4.1.3 Work Done per Cycle

Let a vibrating force $F = F_0 \sin\omega t$ act on a particle and causes displacement $x = x_0 \sin(\omega t - \phi)$. In one cycle of the harmonic motion, the system executes 2π radian. Therefore, *work done per cycle* is determined[1] as

$$\begin{aligned} W &= \int_0^{2\pi/\omega} F\frac{dx}{dt}dt \\ &= \int_0^{2\pi/\omega} F_0 \sin\omega t \times \frac{d}{dt}\{x_o \sin(\omega t - \phi)\}\,dt \\ &= \int_0^{2\pi/\omega} F_0 \sin\omega t \times x_0\omega \cos(\omega t - \phi)\,dt \\ &= F_0 x_0 \omega \int_0^{2\pi/\omega} \sin\omega t \cos(\omega t - \phi)\,dt \\ &= \frac{F_0 x_0 \omega}{2} \int_0^{2\pi/\omega} (\sin 2\omega t + \sin\phi)\,dt \\ &= \frac{F_0 x_0 \omega}{2} \left[-\frac{\cos 2\omega t}{2\omega} + \sin\phi \times t\right]_0^{2\pi/\omega} \\ &= \frac{F_0 x_0 \omega}{2} \left[-(0-0) + \left(\sin\phi \times \frac{2\pi}{\omega} - 0\right)\right] \end{aligned}$$

This finally results

$$W = \pi F_0 x_0 \sin\phi \tag{4.3}$$

This indicates that the work done per cycle in by a harmonic force depends upon the phase difference between the force and displacement. Eq. (4.3) can be examined for two extreme conditions:

[1] Using

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$
$$\int \sin\theta d\theta = -\cos\theta$$

1. When force and displacement functions are in same phase ($\phi = 0$):

$$W = 0$$

2. When force and displacement functions are orthogonal ($\phi = \pi/2$):

$$W = \pi F_o x_o$$

4.1.4 Superposing Waves

Figure 4.2 depicts two *sinusoidal waves* in a polar or vector diagram:

$$x_1 = x_{01} \sin \omega_1 t$$
$$x_2 = x_{02} \sin \omega_2 t$$

The relative phase angle $(\omega_1 - \omega_2)\, t$ is the angle between these vectors.

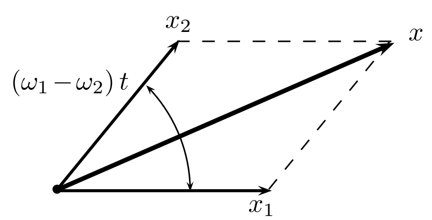


Figure 4.2 Superposing waves.

If these waves coincide on a common medium in the same direction, the resulting wave of superposition is given by their vector sum:

$$\begin{aligned} x &= x_1 + x_2 \\ &= x_{01} \sin \omega_1 t + x_{02} \sin \omega_2 t \\ &= x_0 \sin (\omega_2 - \omega_1)\, t \end{aligned}$$

where

$$x_0 = \sqrt{x_{01}{}^2 + x_{02}{}^2 + 2 x_{01} x_{02} \cos (\omega_2 - \omega_1)\, t}$$

4.1.5 Classification of Vibrations

Some of the important attributes of classifications are as follows.

4.1.5.1 Degrees of Freedom

The number of degrees of freedom of a mechanical system is the number of kinematically independent coordinates necessary to completely describe the motion of each element of the system. Based on this, the vibration systems can be classified into the following:

1. Discrete Systems — A vibration system having a finite number of degrees of freedom is called *discrete system*.
2. Single Degree of Freedom — A system having only one degree of freedom is called a *single degree of freedom* (SDOF) system.
3. Multiple Degree of Freedom — A system with two or more degree of freedom is called a *multiple degree of freedom* (MDOF) system.
4. Continuous System — A system with an infinite number of degrees of freedom is called a *continuous system*.

The number of degrees of freedom indicates the number of differential equations or variables required to define a system. Therefore, complexity in predicting the behavior of a system increases with increase in the number of degrees of freedom.

4.1.5.2 Characteristics

Linearity and non-linearity of a mechanical system directly affect the difficulty in predicting the behavior of system, discussed as follows:

1. Linear System — If all the basic components of a vibratory system: the spring, the mass, and the damper, behave linearly, the resulting vibration is governed by linear differential equations. Therefore, such a vibration is known as *linear vibration*. The principle of superposition is valid for linear systems only.
2. Non-linear System — A system is *non-linear* if its motion is governed by non-linear differential equations. This can be caused by the non-linear behavior of one or more components of the system. The principle of superposition is invalid for non-linear vibrations.

Mathematical techniques and methods are well developed for analysis of linear systems while those for non-linear systems are still under development.

4.1.5.3 External Inputs

The vibrations can be free or forced, described as follows.

1. Free Vibrations — In a *free vibration*, the system oscillates under the action of inherent inertia and elastic forces of the system, initiated by a small disturbance; vibrations occur in the absence of external forces. The oscillation of a simple pendulum is a typical example of free vibrations.

   If a system is left after an initial disturbance to vibrate on its own, the frequency with which it oscillates naturally, without external forces, is known as the *natural frequency* of the system. A

vibratory system having n degrees of freedom will have n distinct natural frequencies of vibration.

2. Forced Vibrations In contrast to free vibrations, *forced vibrations* take place under the excitation of external forces. The oscillations in machines, such as engines, are forced vibrations.

   If the frequency of the external force coincides with the natural frequencies of the system, a condition known as *resonance* occurs; the system undergoes dangerously large oscillations. Failure of structures, such as building, bridges, turbines, and airplane wings, is generally associated with the occurrence of resonance.

   If the external force is periodic, the vibrations are *harmonic*. If the external input is aperiodic, vibrations are said to be *transient*. If the excitation force is known at all times, the excitation is said to be *deterministic*. If the excitation force is *stochastic* (unknown, unpredictable) the excitation is said to be *random* or *non-deterministic*. Examples of random excitations are wind velocity, road roughness, and ground motion during earthquakes. In these cases, a large collection of records of the excitation can exhibit some statistical regularity. It is possible to estimate averages, such as the mean and mean square values of the excitation.

4.1.5.4 Energy Dissipation A vibration system can have elements that dissipate energy. In this respect, the systems are classified as follows:

1. Undamped Vibrations Vibrations without energy dissipation are called *undamped vibrations*.

2. Damped Vibrations If an energy dissipating element is present in the system, the vibrations are called *damped vibrations*.

In many physical systems, the amount of damping is so small that it can be disregarded for most of the engineering purposes. However, consideration of damping becomes extremely importance in analyzing systems near resonance.

4.1.6 Elements of Vibration Systems

A mechanical system consists of three basic elements: inertia, stiffness, and damping. *Inertia* components store kinetic energy, *stiffness* components store potential energy, and *damping* components dissipate the energy of the system. In addition to these, external forces provide energy to the system. These elements are described under the following subsections.

4.1.6.1 Spring Elements Springs act as reservoir of potential energy, the energy by virtue of displacement or deflection, but they don't require motion (velocity) to do so. A helical-coil spring serves as the model for all linear structural components, such as bars undergoing longitudinal motion, shafts under rotational motion, and beams under transverse vibrations; all store potential energy and can be modeled as springs. The characteristics of a stiffness component are described as follows:

1. Stiffness A spring is a flexible mechanical link between two particles in a mechanical system. In reality, a spring itself is a continuous system. However, the inertia of the spring is usually small compared to other elements in the mechanical systems, and is neglected. Under this assumption, the force applied to each end of the spring is the same.

   The length of the spring when it is not subjected to external force is called *unstretched length*. Since the spring is made of a flexible material, the force F that must be applied to the spring to change its length by x should be continuous function of x. A linear spring obeys a force-displacement law in following format:

$$F = kx$$

   where k is called the *spring stiffness* or *spring constant* which has dimensions of force per unit length.

   Using the *constitutive equation*, the stiffness of a structural member can be appropriately defined in the following form:

$$k = \left(\frac{dF}{dx}\right)_{x=0}$$

   For example, consider a cantilever beam of length l with an end mass m. For simplicity, mass of the beam is assumed negligible. From the subject of strength of material, the static deflection of the beam at the free end is given by

$$\delta = \frac{Wl^3}{3EI}$$

   where, $W(= mg)$ is the weight of the mass, E is Young's modulus of beam material, and I is the moment of inertia of the cross-section of the beam. Therefore, the spring constant for the system is

$$\begin{aligned} k &= \frac{W}{\delta} \\ &= \frac{3EI}{l^3} \end{aligned}$$

   The modeling of stiffness components in the form of combination of springs (in series or parallel) is convenient by such components by a single spring of an equivalent stiffness k_e such that the

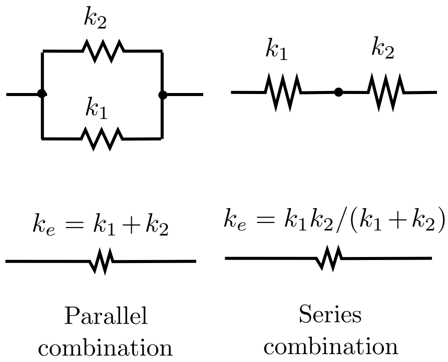


Figure 4.3 | Combination of springs elements.

system undergoes same displacement for a given force [Fig. 4.3].

2. <u>*Potential Energy*</u> The work done by a force is calculated as force multiplied by distance. Figure 4.4 shows the work done by the spring force as its point of application moves from a position x_1 to x_2. This work is stored as potential energy (U) in the spring, given by

$$U_{1\to 2} = \int_{x_1}^{x_2} (-kx)\, dx$$
$$= k\frac{{x_1}^2}{2} - k\frac{{x_2}^2}{2}$$

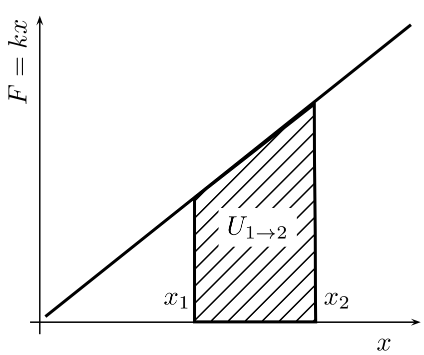


Figure 4.4 | Potential energy in springs.

Since the work depends upon the initial and final positions of the point of application of the spring force and not on the path of the system, the spring force is *conservative* in nature. Therefore, the potential energy function[2] can be defined for spring as

$$U(x) = \frac{1}{2}kx^2$$

where x is the change in the length of the spring from its original length.

4.1.6.2 Inertia Elements The inertia element of a mechanical is assumed to be a rigid body which acts as a reservoir of kinetic energy, the energy by virtue of the velocity of the body. Using the *Newton's second law of motion*, the product of the mass and its acceleration is equal to the force applied to the mass. The work is equal to the force multiplied by the displacement in the direction of the force. The work done on a mass is stored in the form of the kinetic energy (T).

The mass of a body acts as inertia force against the linear motion. For angular motions, distribution of mass in the body affects the inertia against the rotation. For this, *moment of inertia* (I) is used as measure of inertia in angular motions, defined as

$$I = \int_0^r r^2 dm$$

where r is the distance of center of infinitesimal mass dm from the axis of rotation of the body [Fig. 4.5].

According to *d'Alembert's principle*, while dealing with dynamics, inertia force or torques ($m\ddot{x}$ and $I\ddot{\theta}$) should be taken into account.

4.1.6.3 Damping Elements In many practical systems, the vibrational energy is gradually converted to heat or sound, which results into a gradual reduction in the energy and the amplitude of the vibrations. The mechanism of gradual conversion of the vibrational energy is known as *damping*. A damper is assumed to have neither mass nor elasticity. Damping forces exist only if there is a relative velocity between two ends of the damper.

There are mainly four types of damping mechanisms used in mechanical systems: viscous damping, Coulomb damping, structural damping, and slip damping. These are explained as follows:

1. <u>*Viscous Damping*</u> *Viscosity* is the property of a fluid by virtue of which it offers resistance to the motion of one layer over the adjacent one. Some amount of energy is dissipated in overcoming

[2] Similarly, a *torsional spring* is a link in a mechanical system where application of torque leads to an angular displacement between the ends of the torsional spring. A linear torsional spring has a relationship between an applied moment (M) and the angular displacement (θ) as

$$M = k_t\theta$$

where k_t is the torsional stiffness which has dimensions of force times length. Therefore, the potential energy function for a torsional spring is

$$U(\theta) = \frac{1}{2}k_t\theta^2$$

which is similar to that of linear spring.

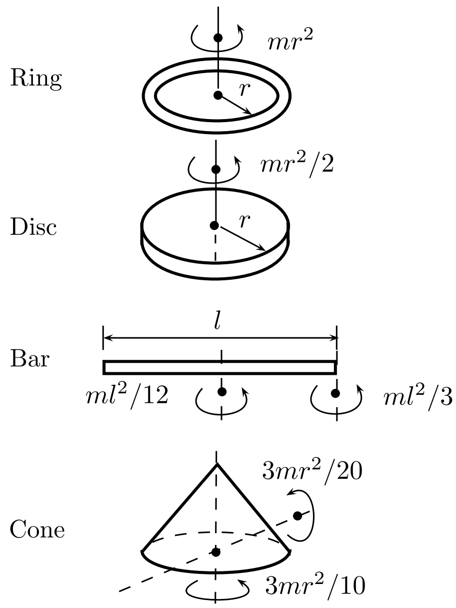


Figure 4.5 | Moment of inertia.

this viscous resistance. Therefore, when a system is allowed to vibrate in a viscous medium, the resulting damping is called as *viscous damping*[3].

Consider two plates of equal area A separated by a fluid film of coefficient of viscosity μ and thickness t. The upper plate is allowed to move parallel to the fixed plate with a velocity $\dot{x}$ [Fig. 4.6].

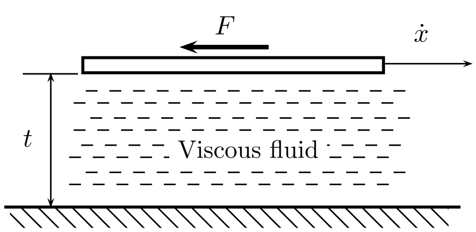


Figure 4.6 | Viscous damping.

Using *Newton's law of viscosity*, the net viscous force F required to maintain this motion is expressed as

$$
\begin{aligned}
F &= \frac{\mu A}{t}\dot{x} \\
&= c\dot{x}
\end{aligned}
$$

where c is called the *viscous damping coefficient*, used as the measure of viscous damping.

The equivalent damping coefficient for a combination of viscous dampers can be determined as in case of springs [Fig. 4.7].

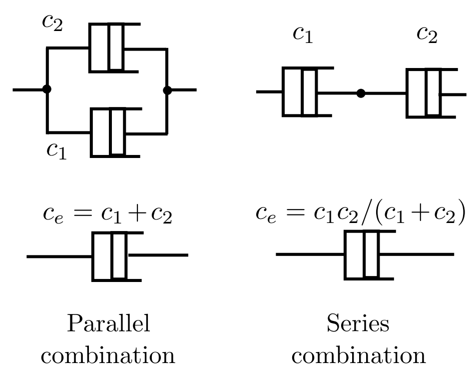


Figure 4.7 | Combination of viscous dampers.

The rate of energy dissipated per cycle is determined as

$$
\begin{aligned}
E &= \oint F \cdot dx \\
&= \oint c\dot{x}dx \\
&= \int_0^{2\pi/\omega} c\dot{x}^2 dt
\end{aligned}
\tag{4.4}
$$

The primary objective of damping in oscillatory systems is to limit the amplitude of the vibration at resonance. For a simple harmonic motion:

$$
\begin{aligned}
x &= x_0 \sin \omega t \\
\dot{x} &= \omega x_0 \cos \omega t
\end{aligned}
$$

Therefore, the amplitude at resonance can be represented as

$$
\begin{aligned}
F &= c\dot{x} \\
F &= cx_0\omega \\
x_0 &= \frac{F}{c\omega}
\end{aligned}
$$

Using Eq. (4.4), the energy dissipated per cycle is

$$
\begin{aligned}
E &= \int_0^{2\pi/\omega} c\omega^2 x_0^2 \left(\frac{1+\cos 2\omega t}{2}\right) dt \\
&= \pi c\omega x_0^2
\end{aligned}
$$

[3]If a non-ferrous conducting object is moved in a direction perpendicular to the lines of magnetic flux which is produced by a permanent magnet, then as the object moves, eddy current, proportional to the velocity, is induced in the object. This eddy current sets up a magnetic field so as to oppose the original magnetic field that has induced it. This provides a resistance to the motion of the object in the magnetic field. For analysis purposes, this is also considered mechanical damping of viscous type.

Thus, energy dissipation per cycle under viscous damping is proportional to the square of the amplitude of motion, therefore, the hysteresis curve is an ellipse [Fig. 4.8]. For non-linear damping, the energy is not a quadratic function of amplitude, therefore, the curve is no longer an ellipse.

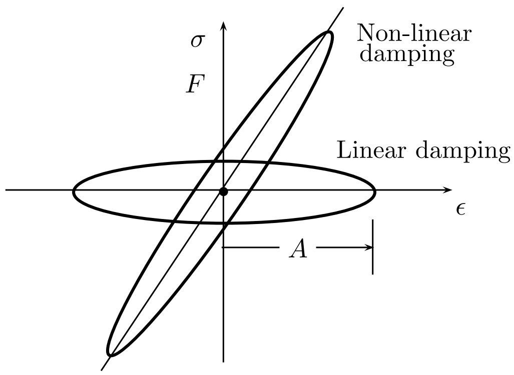


Figure 4.8 Hysteresis in viscous damping.

The damping properties of a material can also be presented in alternatives forms:

(a) *Specific Damping Capacity* - *Specific damping capacity* is defined as the energy loss per cycle divided by the peak potential energy:

$$\beta = \frac{E}{U}$$

(b) *Loss Coefficient* - *Loss coefficient* is defined as the ratio of damping energy loss per radian divided by the peak potential or strain energy:

$$\eta = \frac{E}{2\pi U}$$

For non-viscous damping, no such simple expression exists. However, an equivalent damping coefficient c_e can be determined by equating the energy dissipated by the viscous damping to that of non-viscous damping, assuming harmonic motion.

2. Coulomb Damping When a body is allowed to slide over the other, the surfaces offer frictional resistance to the relative motion. Some amount of energy is always dissipated in overcoming the friction. The damping induced by friction is called *Coulomb damping* or *dry friction damping*.

The general expression for coulomb damping is

$$F = \mu R_n$$

where μ is the coefficient of friction, and R_n is the normal reaction.

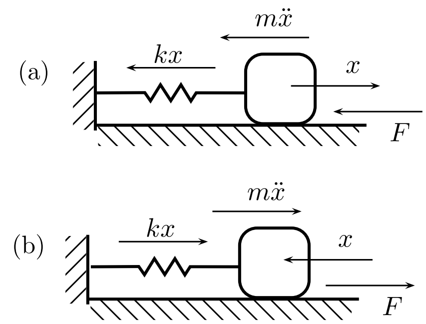


Figure 4.9 Coulomb damping.

Consider a spring-mass system subjected to Coulomb damping [Fig. 4.9].

The reduction in the amplitude of the vibration is examined in two directions separately:

(a) Rightward Movement For the rightward movement of mass m connected with spring of stiffness k, the equilibrium equation is

$$m\ddot{x} + kx + F = 0$$

By applying the boundary conditions: at $t = 0$, $x = x_0$ and $\dot{x} = 0$,

$$x = \left(x_0 - \frac{F}{k}\right)\cos\sqrt{\frac{k}{m}}t + \frac{F}{k}$$

The equation holds good for half cycle. When $t = \pi/\omega$ ($\cos\pi = -1$), the half cycle gets completed during which the displacement is obtained from above equation as

$$x_{1/2} = -\left(x_0 - \frac{2F}{k}\right)$$

Thus, the initial amplitude x_0 is reduced by $2F/k$ in half cycle. The natural frequency of oscillation of the system is $\omega_n = \sqrt{k/m}$.

(b) Leftward Movement The dynamic equation for leftward movement of the mass, for reversed sign convention of x' $(= -x)$, is

$$m\ddot{x} + kx + F = 0$$

Therefore, the amplitude again reduces by $2F/k$ in the half cycle. The natural frequency of oscillation of the system remains constant at $\omega_n = \sqrt{k/m}$.

The decay in amplitude per cycle in Coulomb damping is found as

$$\Delta = \frac{4F}{k}$$

This is a constant quantity for constant friction force F and stiffness k. The motion will cease, however, when the amplitude becomes less than the elongation of spring at which the spring force is insufficient to overcome the static friction force.

3. *Structural Damping* *Structural damping* is offered by the elastic properties of the structure itself. This type of damping is due to the internal friction of the molecules of elastic materials. When a material is subjected to cyclic reversal of loading, a hysteresis loop appears on the stress–strain diagram, indicating that more work is required for straining the material than what is recovered during the return of the cycle [Fig. 4.10]. The

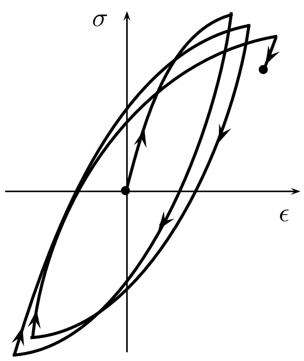


Figure 4.10 Hysteresis loop in cyclic loading.

difference of the work is measured by the area of the hysteresis loop as the energy dissipated per unit volume per cycle. Therefore, structural damping is also called *hysteresis damping*. The magnitude of this type of damping is very small as compared to that of other modes of damping.

Experiments indicate that the energy dissipated per cycle is proportional to the stiffness of the material (k) and square of the amplitude (x_0), but independent of frequency of the vibration:

$$E = \alpha x_0^2$$

where k is a constant (having units of force per unit displacement) representing the influence of shape, size, and stiffness of the structure. The equivalent damping coefficient can be determined as

$$\pi c_e \omega x_0^2 = \alpha x_0^2$$
$$c_e = \frac{\alpha}{\pi\omega}$$

The structural damping *loss coefficient* is

$$\eta = \frac{E}{2\pi U} = \frac{\alpha x_0^2}{2\pi\left(kx_0^2/2\right)} = \frac{\alpha}{\pi k}$$

where k is the stiffness of the structure.

4. *Slip Damping* Damping is also caused by the friction between the internal planes of a structure, that slip or slide as the deformation takes place. Microscopic slip occurs on the interfaces of machine elements which causes dissipation of vibration energy. This results into damping of vibrations which is called *slip damping*.

4.2 UNDAMPED FREE VIBRATION

Resonance is the situation when natural frequency of vibration coincides with that of excitation in a given machine. Therefore, determination of natural frequency and amplitude of vibrations of a machine element is essential in designing process.

The following are the three basic methods employed for vibrational analysis:

1. Equilibrium method
2. Energy method
3. Rayleigh's energy method

These are described in the following sections along with the examples of undamped free vibrations of single degree of freedom.

4.2.1 Equilibrium Method

The equilibrium method considers the equilibrium of external and internal forces in the system. It is also known as *Newton's method* because it employs the *Newton's second law of motion*. The law states that the rate of change of momentum of a mass is equal to the force acting on it.

$$F_i = m\ddot{x}$$

This force F is the the inertia force, as explained by the *d'Alembert's principle*. The principle states that a body, which is not in static equilibrium by virtue of some displacement, can be brought to static equilibrium by introducing on it an inertia force which is equal to the mass times the acceleration of the body and

acts through the center of gravity of the body but in opposite direction to the acceleration. Therefore, in static equilibrium, the vector sum of the resultant external force (F) acting on a body and the inertia force (F_i) is equal to zero:

$$F + F_i = 0$$

Equivalent or extended form of Newton's method is the *principle of virtual work*. For example, when mass of a spring-mass system is given a virtual displacement δx, the total virtual work done by all the forces is set equal to zero to obtain

$$-k\ddot{x}\delta x - kx\delta x = 0$$
$$m\ddot{x} + kx = 0$$

The following two examples explain the general procedure of the equilibrium method:

1. Spring-Mass System Consider a spring mass system constrained to move in a rectilinear manner along the axis of the spring. Spring of constant stiffness k is fixed at one end and carries a mass m at its free end [Fig. 4.11].

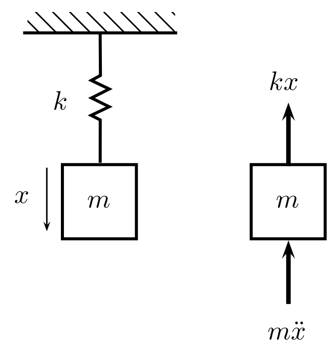


Figure 4.11 Spring-mass system.

The body is displaced x distance from its equilibrium position vertically downwards. This equilibrium position is called *static equilibrium*. The spring force kx and the inertia force $m\ddot{x}$ both act in upward direction. For equilibrium,

$$m\ddot{x} + kx = 0 \tag{4.5}$$

This is the differential equation of motion of the spring-mass system, which can be solved for x. Therefore, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{m}} \tag{4.6}$$

2. Simple Pendulum Consider a pendulum system consisting of a hanging body of mass m attached to a massless string of length l [Fig. 4.12].

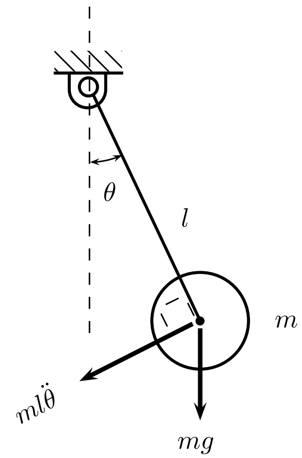


Figure 4.12 Pendulum.

If mass is displaced at an small angle θ, then the equation of equilibrium of forces acting on the mass is found as

$$ml\ddot{\theta} + mg\theta = 0$$

This equation can be solved for θ. The natural frequency of the pendulum is found as

$$\omega = \sqrt{\frac{g}{l}}$$

4.2.2 Energy Method

Kinetic energy and potential energy are the two forms of microscopic energy of a system, which can be related to motion and the influence of some external effects:

1. Kinetic Energy The energy that a system possesses by virtue of its motion relative to some reference frame is called *kinetic energy* (T). When all the parts of a system, having mass m, move with the same velocity $\dot{x}$ with respect to some fixed reference frame, the kinetic energy is expressed as

$$T = m\frac{\dot{x}^2}{2}$$

2. Potential Energy The energy that a system possesses by virtue of its elevation in a gravitational field is called *potential energy* (U). The gravitational field can be gravity, magnetism, electricity, or surface tension. When all parts of a system, having mass m, are at elevation x relative to center of a potential field, say gravity g, the potential energy of the system is expressed as

$$\mathrm{U} = mgx$$

In a vibratory system, the energy is partly potential and partly kinetic. *Energy method* considers the system as conservative; no energy is lost due to friction or energy dissipating non-elastic members. Thus, the sum of the kinetic energy and potential energy is constant:

$$T+U = \text{constant}$$
$$\frac{d}{dt}(T+U) = 0 \tag{4.7}$$

Differentiation of the above equation w.r.t. time (t) gives the differential equation of the equilibrium of the system. Following two examples explain the general procedure of energy method:

1. Spring-Mass System Consider a spring–mass system constrained to move in a rectilinear manner along the axis of the spring. Spring of constant stiffness k is fixed at one end and carries a mass m at its free end [Fig. 4.11]. The potential energy and kinetic energy at any instant of time will be given by

$$U = \frac{1}{2}kx^2$$
$$T = \frac{1}{2}m\dot{x}^2$$

Using Eq. (4.7),

$$\frac{d}{dt}\left\{\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right\} = 0$$
$$m\ddot{x} + kx = 0 \tag{4.8}$$

This equation of motion for the spring-mass system is the same as Eq. (4.5).

2. Cylinder Rolling on Cylindrical Surface Consider a solid cylinder of radius r rolling without slipping on a cylindrical surface of radius R [Fig. 4.13].

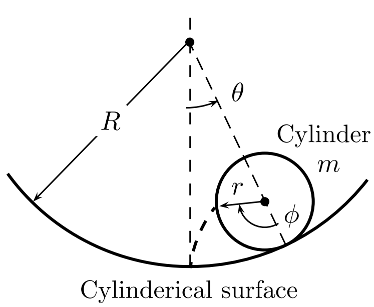


Figure 4.13 Cylinder on cylindrical surface.

Let the cylinder roll (without slipping) by angle ϕ during which it traces an angle θ at the center of cylindrical surface. Therefore,

$$r\phi = R\theta$$
$$r\dot{\phi} = R\dot{\theta}$$

During rolling, both the translation and rotation of the cylinder take place by the following velocities:

(a) Translational velocity

$$\dot{x} = (R-r)\,\dot{\theta}$$

(b) Rotational velocity

$$\omega = \dot{\phi} - \dot{\theta} = \left(\frac{R}{r} - 1\right)\dot{\theta}$$

If m is the mass of cylinder, kinetic energy (T) and potential energy (U) of the system at any angle θ are written as

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\left(mr^2\right)\omega^2 = \frac{3}{4}(R-r)^2\,\dot{\theta}^2$$
$$U = mg\,(R-r)\,(1-\cos\theta)$$

For small values of θ: $\sin\theta \approx \theta$, therefore, using Eq. (4.7),

$$\ddot{\theta} + \frac{2g}{3\,(R-r)}\theta = 0$$

Natural frequency of the oscillations is found as

$$\omega_n = \sqrt{\frac{2g}{3\,(R-r)}}$$

4.2.3 Rayleigh's Energy Method

The principle of conservation of energy for an undamped system [Eq. (4.7)] is restated as

$$T+U = \text{constant}$$

This can have an alternative form that the maximum kinetic energy at the mean position will be equal to the maximum potential energy at the extreme position:

$$T_{max} = U_{max} \tag{4.9}$$

The application of this equation is known as Rayleigh's energy method, which directly gives the natural frequency of the system.

The method has an alternative form. If motion of various masses of a system can be expressed in terms of a single displacement x of some specific point; the system is simply one of a single degree of freedom, the kinetic energy of the system can be written as

$$T = \frac{1}{2}m_e\dot{x}^2$$

where m_e is the *effective* or *equivalent lumped mass* at the specified point. For the equivalent stiffness k_e of the system at the specified point, the natural frequency can be written as

$$\omega_n = \sqrt{\frac{k_e}{m_e}} \tag{4.10}$$

This is evident in the following examples:

1. Spring–Mass System For the spring-mass system [Fig. 4.11], the displacement is represented as

$$x = x_0 \sin \omega_n t$$

Differential of above equation w.r.t. time (t) gives expression of velocity as

$$\dot{x} = \omega_n x_0 \cos \omega_n t$$

Maximum velocity at mean position is $\omega_n a$, therefore, maximum kinetic energy at mean position is $m(\omega_n x_0)^2/2$. Similarly, the maximum potential energy at the extreme position is $kx_0^2/2$. Thus,

$$\frac{1}{2}m(\omega_n x_0)^2 = \frac{1}{2}kx_0^2$$

Therefore, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{m}}$$

This is the same as obtained in Eq. (4.6).

2. Effect of Mass of Spring Consider the above example when mass of the spring (m_s) is not ignorable. Length of spring is l. The velocity of any spring element at distance y from the base can be assumed in a linear fashion as

$$\dot{y} = \frac{y}{l}\dot{x}$$

Therefore, the kinetic energy of the spring–mass system can be written as

$$\begin{aligned} T_s &= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\int_0^l \left(\dot{x}\frac{y}{l}\right)^2 \frac{m_s}{l}dy \\ &= \frac{1}{2}\left(m + \frac{m_s}{3}\right)\dot{x}^2 \end{aligned}$$

The equivalent mass of the system is found as

$$m_e = m + \frac{m_s}{3}$$

Taking $k_e = k$, natural frequency of the vibrations is found using Eq. (4.10),

$$\omega_n = \sqrt{\frac{k}{m + m_s/3}}$$

4.3 FREE DAMPED VIBRATION

In the absence of energy dissipation, a free vibration can persist forever. Evidently, this never occurs in nature. All the free vibrations die down after a time due to damping.

Consider a mass m attached with a spring of stiffness k and a viscous damper of *damping coefficient* (c) [Fig. 4.14].

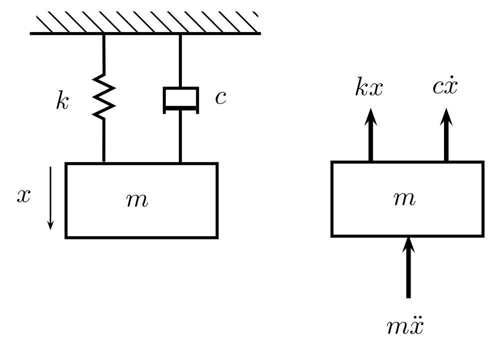


Figure 4.14 Free damped system.

The body is displaced by distance x vertically downward from its equilibrium position. Using *Newton's method*, the equilibrium equation for the system is written as

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{4.11}$$

The solution of this equation can be in the following form:

$$x = e^{ut}$$

where u is a constant. Velocity and acceleration functions are written as

$$\dot{x} = ue^{ut}, \quad \ddot{x} = u^2 e^{ut}$$

Putting these values in Eq. (4.11) gives the following values of the constant:

$$u_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

Therefore, solution of Eq. (4.11) is written as

$$x = A_1 e^{u_1 t} + A_2 e^{u_2 t} \tag{4.12}$$

The *critical damping coefficient* c_c is the value of damping coefficient c for which

$$\left(\frac{c_c}{2m}\right)^2 - \frac{k}{m} = 0$$
$$c_c = 2\sqrt{km}$$

The ratio of c and c_c is termed as *damping ratio*, denoted by ξ:

$$\xi = \frac{c}{c_c}$$

Therefore, Eq. (4.12) can be written in terms of ξ as

$$x = A_1 e^{\left(-\xi+\sqrt{\xi^2-1}\right)\omega_n t} + A_2 e^{\left(-\xi-\sqrt{\xi^2-1}\right)\omega_n t} \tag{4.13}$$

where

$$\omega_n = \sqrt{\frac{k}{m}}$$

is the *natural frequency* of undamped free vibrations in the same system. This can be written in terms of c_c as

$$\omega_n = \frac{c_c}{2m}$$

In Eq. (4.13), the displacement (x) consists of two exponential functions of damping ratio ξ, which can be positive, negative or zero. Depending upon the value of ξ w.r.t. unity, free-damped vibration systems are classified into following:

1. Over-damped system ($\xi > 1$)
2. Critically damped system ($\xi = 1$)
3. Under-damped system ($\xi < 1$)

These are discussed in the following subsections.

4.3.1 Over-Damped System

When $\xi > 1$, the system is called *over-damped*. Using Eq. (4.13), the displacement function is re-written as

$$x = \underbrace{A_1 e^{\left(-\xi+\sqrt{\xi^2-1}\right)\omega_n t}}_{x_1} + \underbrace{A_2 e^{\left(-\xi-\sqrt{\xi^2-1}\right)\omega_n t}}_{x_2}$$

This expression contains two exponential functions, x_1 and x_2, with negative power of e; both the elements decrease exponentially with time. Therefore, the motion is aperiodic or non-oscillatory [Fig. 4.15].

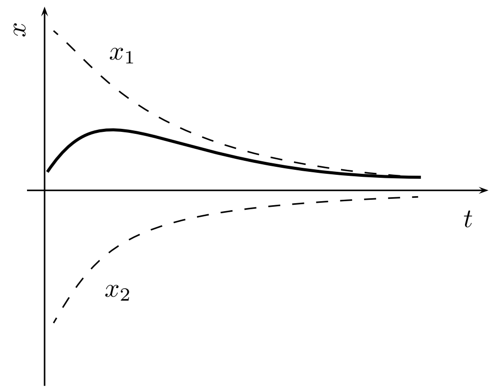


Figure 4.15 Aperiodic motion ($\xi > 1$).

The value of arbitrary constants A_1 and A_2 can be found for initial condition ($t = 0$) when the displacement and velocity are equal to $x(0)$ and $\dot{x}(0)$. Once the system is disturbed, it will take infinite time to come back to equilibrium position.

4.3.2 Critically Damped System

When $\xi = 1$, the system is called to be *critically damped*. Using Eq. (4.13), the displacement function for this case is written as

$$x = (A_1 + A_2 t)\, e^{-\omega_n t}$$

where A_1 and A_2 are the arbitrary constants which can be determined from the initial conditions. This is an exponential function with negative power of e; the displacement decreases exponentially with time. Therefore, the motion is aperiodic or non-oscillatory. Figure 4.16 shows three different patterns of the function which depend upon the direction and value of initial velocity $\dot{x}(0)$, evident through the arbitrary constants A_1 and A_2.

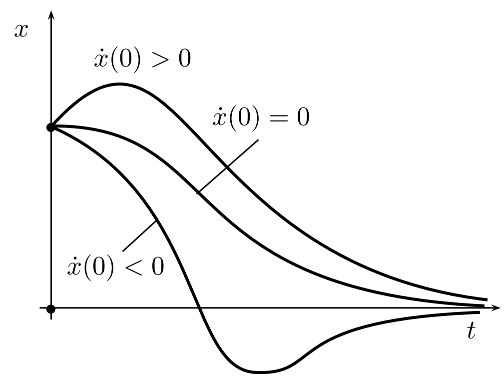


Figure 4.16 Aperiodic motion ($\xi = 1$).

The situation of critical damping measures the relative amount of damping in a particular system. Critical damping means that the amount of damping which will make the system stop vibrating within the least possible time.

4.3.3 Under-Damped System

When $\xi < 1$, the system is called to be under-damped. Using Eq. (4.13), the displacement function for this case is written as

$$x = e^{-\xi\omega_n t}\left[A_1 e^{j\sqrt{1-\xi^2}\omega_n t} + A_2 e^{-j\sqrt{1-\xi^2}\omega_n t}\right]$$

Using $e^{jx} = \cos x + j \sin x$,

$$x = A e^{-\xi\omega_n t} \sin(\omega_d t + \phi) \tag{4.14}$$

where

$$\omega_d = \sqrt{1-\xi^2}\omega_n$$

is the *frequency of damped vibrations*, ϕ is the phase difference, and A is the amplitude.

The displacement function is a multiplication of exponentially decreasing amplitude and a sinusoidal component; the motion is periodic with frequency w_d but the amplitude decreases exponentially in every cycle [Fig. 4.17].

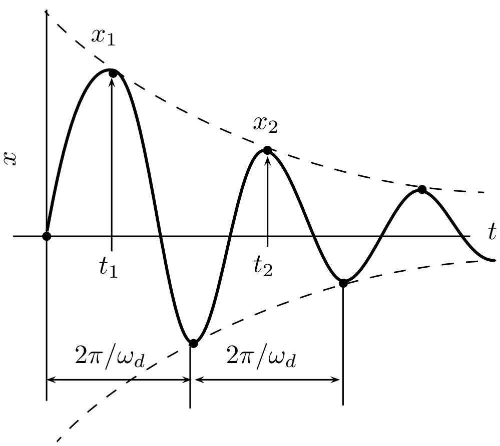


Figure 4.17 Aperiodic motion ($\xi < 1$).

To evaluate the cyclic decrement, let t_1 and t_2 ($= t_1 + t_d$) denote the times corresponding to two successive amplitudes x_1 and x_2, respectively. Here, t_d is the time period given by

$$t_d = \frac{2\pi}{\omega_d}$$

Using Eq. (4.14), the ratio of two successive amplitudes is

$$\begin{aligned}\frac{x_1}{x_2} &= \frac{Ae^{-\xi\omega_n t_1}\sin(\omega_d t_1+\phi)}{Ae^{-\xi\omega_n t_2}\sin(\omega_d t_2+\phi)}\\ &= e^{-\xi\omega_n(t_1-t_2)}\frac{\sin(\omega_d t_1+\phi)}{\sin(\omega_d t_1+2\pi+\phi)}\\ &= e^{\xi\omega_n(t_2-t_1)}\\ &= e^{\xi\omega_n t_d}\\ \frac{x_1}{x_2} &= e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}\end{aligned} \tag{4.15}$$

In this context, a term *logarithmic decrement* (δ) is defined as the natural logarithm of the ratio of any two successive amplitudes on the same side of the mean line:

$$\delta = \ln\left(\frac{x_1}{x_2}\right) \tag{4.16}$$

If the system executes n cycles, the ratio of amplitudes can be expressed as

$$\delta = \frac{1}{n}\ln\left(\frac{x_1}{x_n}\right)$$

Using Eqs. (4.15) and (4.16), the logarithmic decrement δ is related to ξ as

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \tag{4.17}$$

Using Eq. (4.17), the damping factor can be presented in terms of δ as

$$\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} \tag{4.18}$$

The amplitude, frequency (ω_d), and logarithmic decrement (δ) in damped vibrations depend upon the damping factor ξ; The amplitude decreases with increase in the amount of damping or ξ. Figure 4.18 shows the variation of ω_d/ω_n and δ with respect to ξ.

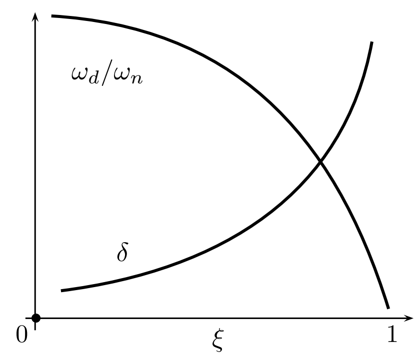


Figure 4.18 ω_d/ω_n and δ with respect to ξ.

4.4 FORCED VIBRATION

Forced vibrations take place under the excitation of external forces. The oscillations in machines, such as engines, rotating unbalance, are typical examples of forced vibrations. Forced vibrations can also be modeled as a spring–mass damper system with an external dynamic force. Based on this, the vibrations due to rotating unbalance and support excitation can also be studied.

4.4.1 Spring–Mass–Damper System

Consider a mass m, attached with spring of stiffness k and a viscous damper of coefficient c, is subjected to a dynamic force F [Fig. 4.19].

The body is displaced x distance from its equilibrium position vertically downwards. For this system, the differential equation of equilibrium is written as

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t \tag{4.19}$$

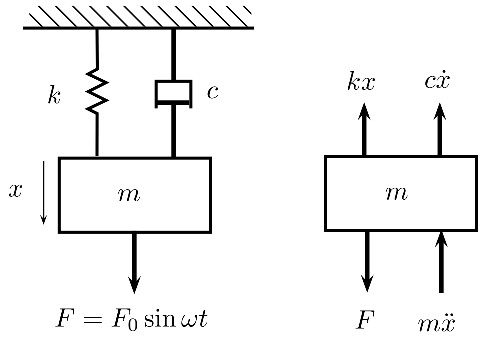


Figure 4.19 Forced vibration.

The solution of above equation has two components, complementary function (x_c) and particular integral (x_p):

$$x = x_c + x_p$$

1. *Complementary Function* This component is the solution of left-hand side of Eq. (4.19) without force function $F_0 \sin \omega t$; a second order differential equation:

$$x_c = Ae^{-\xi\omega_n t} \sin\left(\sqrt{1-\xi^2}\omega_n t + \phi_n\right)$$

2. *Particular Integral* This is the steady-state component of the solution. Because the force function is sinusoidal, the particular integral should also be a sinusoidal function in the following form

$$x_p = x_0 \sin(\omega t - \phi) \tag{4.20}$$

where x_0 is the amplitude, and ϕ is the phase difference by which the displacement lags the vector force.

A damped vibration dies down rapidly with time, instantaneously or slowly, depending upon the amount of damping. Therefore, the solution of Eq. (4.19) consists of only particular integral, given by Eq. (4.20), as the steady-state solution:

$$x = x_0 \sin(\omega t - \phi) \tag{4.21}$$

The velocity and acceleration of this solution are as follows:

1. *Velocity* Differentiating Eq. (4.21) w.r.t. time,

$$\begin{aligned}\dot{x} &= \omega x_0 \cos(\omega t - \phi)\\ &= \omega x_0 \sin\left(\omega t - \phi + \frac{\pi}{2}\right)\end{aligned} \tag{4.22}$$

2. *Acceleration* Differentiating Eq. (4.22) w.r.t. time,

$$\begin{aligned}\ddot{x} &= \omega^2 x_0 \cos\left(\omega t - \phi + \frac{\pi}{2}\right)\\ &= -\omega^2 x_0 \sin(\omega t - \phi + \pi)\end{aligned} \tag{4.23}$$

Thus, the functions $\dot{x}$ and $\ddot{x}$ are ahead of the displacement x by $\pi/2$ and π radians, respectively. Using Eqs. (4.21)–(4.23), the forces acting on the system (i.e., damping force, inertia force, spring force, external force) can be shown on a phase diagram [Fig. 4.20].

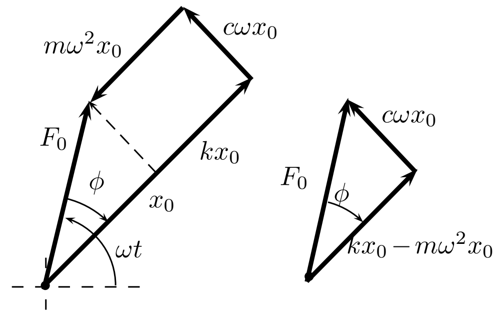


Figure 4.20 Phase diagram in forced vibrations.

The phase diagram provides the expression of steady-state amplitude (x_0) and phase angle (ϕ) in Eq. (4.21):

1. *Amplitude* Using Pythagoras' theorem for the triangle of forces:

$${F_0}^2 = (c\omega x_0)^2 + \left(x_0 k - m\omega^2 x_0\right)^2$$

The amplitude of the forced vibration is found as

$$\begin{aligned}x_0 &= \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}\\ &= \frac{x_{st}}{\sqrt{(1 - \omega^2/{\omega_n}^2)^2 + (2\xi\omega/\omega_n)^2}}\end{aligned} \tag{4.24}$$

where $x_{st} = F_0/k$ is the static displacement caused by a static force F_0 in the absence of damper.

2. *Phase Angle* The phase angle ϕ between force and displacement vectors is found as

$$\begin{aligned}\tan\phi &= \frac{c\omega}{k - m\omega^2}\\ &= \frac{2\xi\omega/\omega_n}{1 - \omega^2/{\omega_n}^2}\end{aligned} \tag{4.25}$$

The ratio x_0/x_{st} is known as *magnitification factor* Λ:

$$\begin{aligned}\Lambda &= \frac{x_0}{x_{st}}\\ &= \frac{1}{\sqrt{(1 - \omega^2/{\omega_n}^2)^2 + (2\xi\omega/\omega_n)^2}}\end{aligned} \tag{4.26}$$

Equations (4.24) and (4.25) can be used to plot x/x_{st} and ϕ w.r.t. ω/ω_n for different values of ξ [Fig. 4.21].

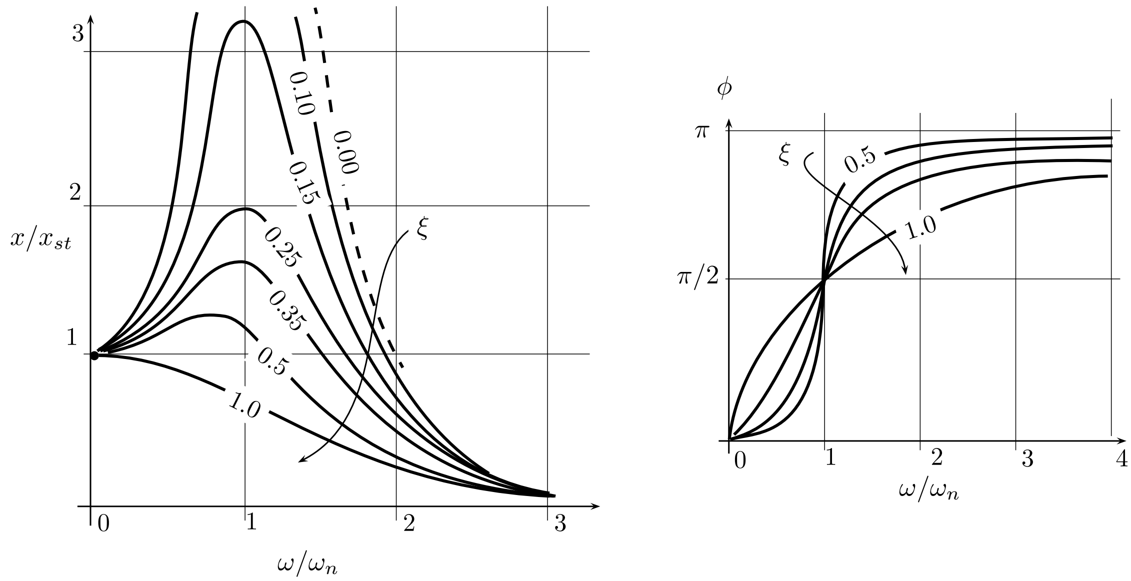


Figure 4.21 x/x_{st} and ϕ w.r.t. ω/ω_n.

The magnification factor (Λ) and phase angle (ϕ) are functions of frequency ratio and damping factor. These can be examined as follows:

1. *Effect of Frequency Ratio* The effect of frequency ratio can be examined in the following three cases:

   (a) *Small Frequencies ($\omega \ll \omega_n$)* At small frequencies ($\omega \ll \omega_n$), both inertia and damping forces are small because velocity ($\dot{x}$) and acceleration ($\ddot{x}$) are very small. The case is equivalent when the system is subjected to a static load F, which is balanced by the spring force. The amplitude and phase angle are as follows:

$$\Lambda \approx 1$$
$$\phi \approx 0$$

   The forces in this case are

$$c\dot{x} = 0$$
$$m\ddot{x} = 0$$
$$kx = F_0$$

   Thus, the magnification factor is unity. It is independent of ξ; damping coefficient has no role to play because there is no motion. In this situation, the external force is balanced by spring force.

   (b) *Resonance ($\omega = \omega_n$)* If the frequency of the external force ω coincides with the natural frequencies (ω_n) of the system, a condition known as *resonance* occurs. The corresponding values are as follows:

$$\Lambda = \frac{1}{2\xi}$$
$$\phi_r = \frac{\pi}{2}$$

   In this case,

$$c\dot{x} = F$$
$$m\ddot{x} = kx$$

   The external force is balanced by the damping force while the inertia force is balanced by the spring force.

   (c) *Large Frequencies ($\omega \gg \omega_n$)* At large frequencies ($\omega \gg \omega_n$),

$$\Lambda \approx 0$$
$$\phi \approx \pi$$

   In this case,

$$c\dot{x} = 0$$
$$m\ddot{x} = F$$
$$kx = 0$$

   The external force is balanced by the inertia force. Damping has no effect on the system.

2. *Effect of Damping Factor* It is interesting to find the frequency ratio $r = \omega/\omega_n$ for the peak value of the magnification factor Λ. For this, the

denominator of Eq. (4.26) should be minimum. This can be obtained by

$$\frac{d}{dr}\left\{\sqrt{\left(1-\frac{\omega^2}{\omega_n{}^2}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}\right\}=0$$

This results in

$$\frac{\omega}{\omega_n}=\sqrt{1-2\xi^2}$$

Thus, the maximum value of Λ occurs for $0<\xi<1/\sqrt{2}$. The value of frequency ratio cannot be more than 1, therefore, the peak values of Λ for different values of ξ occurs for $\omega<\omega_1$. The corresponding value of Λ is found as

$$\begin{aligned}\Lambda_{max} &= \frac{1}{\sqrt{\left(1-\omega^2/\omega_n{}^2\right)^2+\left(2\xi\omega/\omega_n\right)^2}}\\ &= \frac{1}{\sqrt{\left(1-1-2\xi^2\right)^2+4\xi^2\left(1-2\xi^2\right)}}\\ &= \frac{1}{\sqrt{4\xi^4+4\xi^2-8\xi^4}}\\ &= \frac{1}{2\xi\sqrt{1-2\xi^2}}\end{aligned}$$

This is the peak value of Λ which occurs at frequency ratio $\omega/\omega_n=\sqrt{1-2\xi^2}$.

4.4.2 Rotating Unbalance

Rotating machines are not supposed to have any unbalanced mass because it induces vibrations in the system. Consider a *rotating unbalance* mass m_e at eccentric radius e with constant angular speed ω. The machine is supported by a spring–mass–damper system of stiffness k and damping coefficient c [Fig. 4.22]

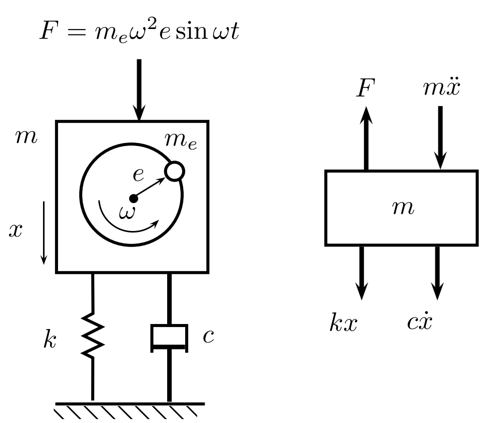


Figure 4.22 Rotating unbalance.

The unbalanced dynamic force acting on the mass m shall be written as

$$F=m_e\omega^2 e\sin\omega t$$

The model of forced vibrations in spring-mass-damper system [*Section* 4.4.1] can be applied to this system by taking the equivalent amplitude of force

$$F_0=m_e\omega^2 e$$

Accordingly, the amplitude and phase angle are found as follows:

1. <u>*Amplitude*</u>

$$\begin{aligned}x_0 &= \frac{m_e e\omega^2}{\sqrt{\left(k-m\omega^2\right)^2+\left(c\omega\right)^2}}\\ \frac{x_0}{x_{st}} &= \frac{\omega^2/\omega_n{}^2}{\sqrt{\left(1-\omega^2/\omega_n{}^2\right)^2+\left(2\xi\omega/\omega_n\right)^2}}\end{aligned} \tag{4.27}$$

where $x_{st}=m_e\cdot e/m$.

2. <u>*Phase Angle*</u> The phase angle ϕ between the force and the displacement vectors is found as

$$\begin{aligned}\tan\phi &= \frac{c\omega}{k-m\omega^2}\\ &= \frac{2\xi\omega/\omega_n}{1-\ \omega^2/\omega_n{}^2}\end{aligned} \tag{4.28}$$

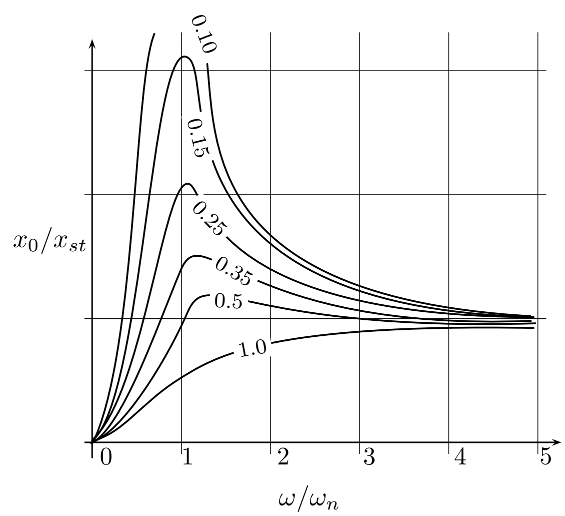


Figure 4.23 x/x_{st} for rotating unbalance.

The effect of ω/ω_n and ξ on x/x_{st} can be observed using Eqs. (4.27) and (4.28) [Fig. 4.23]:

1. Effect of Frequency Ratio The amplitude of all the curves is zero at $\omega = 0$, but markedly high near resonance ($\omega = \omega_n$). At higher frequencies, the amplitude ratio is almost unity where the effect of damping is negligible.

2. Effect of Damping Factor The peak value of amplitude is found when

$$\frac{\omega}{\omega_n} = \frac{1}{\sqrt{1-2\xi^2}}$$

Therefore, the peak value occurs for $0 < \xi < 1/\sqrt{2}$. The denominator of the above frequency ratio cannot be more than unity, therefore, the frequency ratio is always greater than 1, therefore, peak value occurs at spead ratio near and higher than 1. The corresponding peak value of the amplitude is found as

$$(x_0)_{max} = \frac{m_e e/m}{2\xi\sqrt{1-\xi^2}}$$

4.4.3 Support Excitation

Figure 4.24 shows a spring-mass-damper system in which the support of the system itself vibrates with the following displacement equation:

$$y = y_0 \sin(\omega t + \alpha)$$

The relative displacement on the spring and damper is $x - y$. Therefore, the differential equation of equilibrium can be written as

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$
$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

This can be written as

$$m\ddot{x} + c\dot{x} + kx = y_0\sqrt{k^2 + (c\omega)^2}\sin(\omega t + \phi)$$

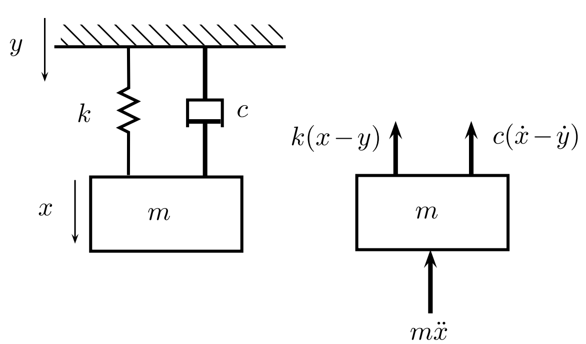


Figure 4.24 Support excitation.

The model of forced vibrations in spring-mass-damper system [*Section* 4.4.1] can be applied to this system by taking the equivalent amplitude of force

$$F_0 = y_0\sqrt{k^2 + (c\omega)^2}$$

The amplitude and phase angle are found as follows:

1. Amplitude

$$x_0 = \frac{y_0\sqrt{k^2 + (c\omega)^2}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$\frac{x_0}{y_0} = \frac{\sqrt{1 + (2\xi\omega/\omega_n)^2}}{\sqrt{(1 - \omega^2/{\omega_n}^2)^2 + (2\xi\omega/\omega_n)^2}} \tag{4.29}$$

2. Phase Angle The phase angle ϕ force and displacement vectors is found as

$$\tan\phi = \frac{c\omega}{k - m\omega^2} = \frac{2\xi\omega/\omega_n}{1 - \omega^2/{\omega_n}^2} \tag{4.30}$$

4.4.4 Transmissibility

Machines are isolated from undesired vibrations by mounting the machines on springs and providing dashpot mechanisms such as shock absorbers in motor cycle and automobiles.

For a system with unbalance force due to rotating mass [Fig. 4.22], the force transmitted (F_{tr}) to the base or foundation is the sum of spring force and the dashpot force:

$$F_{tr} = \sqrt{(kx)^2 + (c\omega x)^2} = x\sqrt{k^2 + c^2\omega^2}$$

The ratio of transmitted force and dynamic force is known as *transmissibility* (Tr). For the present case,

$$\text{Tr} = \frac{F_{tr}}{F_0} = \frac{\sqrt{1 + (2\xi\omega/\omega_n)^2}}{\sqrt{(1 - \omega^2/{\omega_n}^2)^2 + (2\xi\omega/\omega_n)^2}}$$

This expression can be used to investigate the effect of system characteristics on the transmissibility by plotting Tr w.r.t. ω/ω_n for different values of ξ [Fig. 4.25].

The purpose of providing the spring and dampers to the machine is to make the force transmitted less than

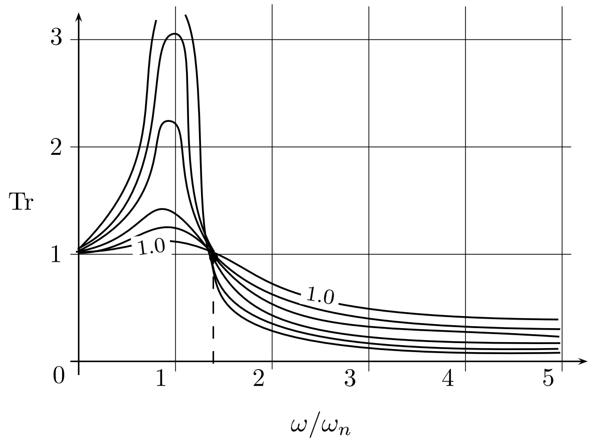


Figure 4.25 Tr versus ω/ω_n.

the impressed or periodic force; the body is isolated from large accelerations of the base when Tr < 1. The value of Tr depends upon the ratio ω/ω_n and ξ. Therefore, the effects of ω/ω_n and ξ on Tr are examined as follows:

1. *Effect of Frequency Ratio* The transmissibility curve for different damping factors have the same value Tr = 1 for $\omega/\omega_n = \sqrt{2}$. The isolation is achieved (Tr < 1.0) when $\omega/\omega_n > \sqrt{2}$. The system is dangerous if $\omega/\omega_n < \sqrt{2}$. It follows that ω/ω_n must be as large as possible for the required stiffness of the spring.

2. *Effect of Damping Factor* For $0 < \xi < 1$ and $\omega < \omega_n$, the maximum value of Tr is obtained at

$$\frac{\omega}{\omega_n} = \frac{1}{2\xi}\left(\sqrt{1+8\xi^2}-1\right)^{1/2}$$

At this frequency, the peak value of Tr is obtained as

$$\mathrm{Tr}_m = 4\xi^2\sqrt{\frac{\sqrt{1+8\xi^2}}{2+16\xi^2+(16\xi^4-8\xi^2-2)\sqrt{1+8\xi^2}}}$$

If $\xi = 0$, then transmissibility is written as

$$\mathrm{Tr} = \frac{1}{1-(\omega/\omega_n)^2}$$

4.4.5 Whirling of Rotating Shafts

When a rotor is mounted on a shaft, its center of mass does not usually coincide with the center line of the shaft. Therefore, when the shaft rotates, it is subjected to a centrifugal force which makes the shaft to bend in the direction of eccentricity of the center of mass. The shaft tends to bow out at certain speed and whirl in a complicated manner. This increases the eccentricity of the mass, and hence the centrifugal force. In this way, the effect is cumulative and ultimately the shaft can even fail.

Critical speed or *whirling speed* is the speed at which the shaft tends to vibrate violently in transverse direction. This is also called *whipping speed*. In general, the critical speeds of any circular shaft coincide with the natural frequencies of vibrations of the non-rotating shaft on its bearings. Below the critical speeds, the shaft offers some elastic resistance to a sidewise force, and this is no longer true at the critical speed. It has been observed that at critical speed, the shaft again becomes almost straight. But at some other speed, the same phenomenon recurs, the only difference being that the shaft now bends in two bows and so on.

Consider a rotor of mass m assembled on a shaft of stiffness k with an eccentricity e. The shaft rotates with angular velocity ω and rotor gets additional deflection due to centrifugal force [Fig. 4.26].

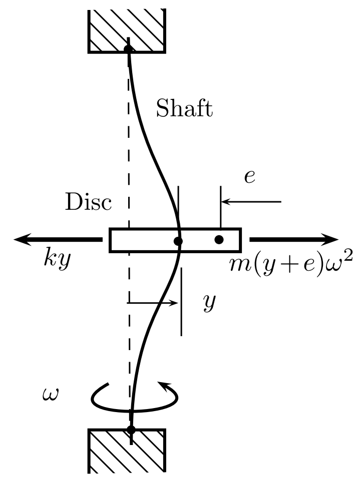


Figure 4.26 Critical speed of shaft.

The unbalanced mass is in equilibrium under the centrifugal force $m\,(y+e)\,\omega^2$ and force resisting the deflection ky:

$$ky = m\,(y+e)\,\omega^2$$

$$y = \frac{e}{(\omega/\omega_n)^2-1} \tag{4.31}$$

where ω_n ($=\sqrt{k/m}$) is the natural frequency. From Eq. (4.31), when $\omega = \omega_n$, the deflection y is infinitely large (resonance occurs) and the speed ω is the critical speed. By increasing the frequency ω beyond ω_n, y approaches the value $-e$ or the center of mass of the rotor approaches the center line of the rotation. This principle is used in running high-speed turbines by speeding up the rotor rapidly beyond the critical speed and the rotor runs steadily.

IMPORTANT FORMULAS

Fundamentals

1. Harmonic motion

$$x = x_0 \sin(\omega t + \phi)$$
$$\bar{x} = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\,dt$$
$$\bar{x}^2 = \lim_{T\to\infty} \frac{1}{T} \int_0^T x^2(t)\,dt$$
$$f = \frac{1}{T} \text{ Hz}$$
$$f = \frac{\omega}{2\pi} \text{ cycles/s}$$
$$= \omega \text{ rad/s}$$
$$= \frac{60\omega}{2\pi} \text{ rpm}$$

2. Work done per cycle

$$W = \pi F_o x_o \sin\phi$$

3. Spring

$$U(x) = \frac{1}{2}kx^2$$

$$k_e = k_1 + k_2 \quad \text{Parallel}$$
$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{Series}$$

4. Damping coefficient

$$F = c\dot{x}$$
$$E = \oint F \cdot dx = \pi c \omega x_0^2$$
$$\beta = \frac{E}{U}, \quad \eta = \frac{E}{2\pi U}$$

5. Coulomb damping

$$\Delta = \frac{4F}{k}$$

6. Structural damping

$$\eta = \frac{E}{2\pi U}$$
$$c_e = \frac{\alpha}{\pi\omega} = \frac{\alpha}{\pi k}$$

Undamped Free Vibrations

$$m\ddot{x} + kx = 0$$
$$\omega_n = \sqrt{\frac{k}{m}}$$

1. Energy method

$$\frac{d}{dt}(T + U) = 0$$

2. Rayleighs energy method

$$T_{max} = U_{max}$$

3. Cylinder on cylindrical face

$$\omega_n = \sqrt{\frac{2g}{3(R-r)}}$$

4. Pendulum

$$\omega = \sqrt{\frac{g}{l}}$$

Free Damped Vibrations

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$c_c = 2\sqrt{km}, \quad \xi = \frac{c}{c_c}, \quad \omega_n = \frac{c_c}{2m}$$
$$\omega_d = \sqrt{1-\xi^2}\,\omega_n$$
$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{1}{n}\ln\left(\frac{x_1}{x_n}\right)$$
$$= \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$
$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

Forced Vibrations

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$$

$$x_0 = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$
$$= \frac{x_{st}}{\sqrt{(1-\omega^2/\omega_n{}^2)^2 + (2\xi\omega/\omega_n)^2}}$$

$$\tan\phi = \frac{c\omega}{k - m\omega^2} = \frac{2\xi\omega/\omega_n}{1-\omega^2/\omega_n{}^2}$$

$$\Lambda_{max} = \frac{1}{2\xi\sqrt{1-2\xi^2}}$$
$$\text{when } \frac{\omega}{\omega_n} = \sqrt{1-2\xi^2}$$

Rotating Unbalance

$$F = m_e\omega^2 e \sin\omega t$$
$$F_0 = m_e\omega^2 e$$
$$\frac{x_0}{x_{st}} = \frac{\omega^2/\omega_n{}^2}{\sqrt{(1-\omega^2/\omega_n{}^2)^2 + (2\xi\omega/\omega_n)^2}}$$

$$\tan\phi = \frac{c\omega}{k - m\omega^2}$$
$$= \frac{2\xi\omega/\omega_n}{1-\ \omega^2/\omega_n{}^2}$$

$$(x_0)_{max} = \frac{m_e e/m}{2\xi\sqrt{1-\xi^2}}$$
$$\text{when } \frac{\omega}{\omega_n} = \frac{1}{\sqrt{1-2\xi^2}}$$

Support Excitation

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$\frac{x_0}{y_0} = \frac{\sqrt{1 + (2\xi\omega/\omega_n)^2}}{\sqrt{(1-\omega^2/\omega_n{}^2)^2 + (2\xi\omega/\omega_n)^2}}$$

$$\tan\phi = \frac{c\omega}{k - m\omega^2}$$
$$= \frac{2\xi\omega/\omega_n}{1-\omega^2/\omega_n{}^2}$$

Transmissibility

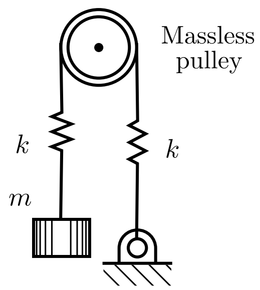
$$\text{Tr} = \frac{\sqrt{1 + (2\xi\omega/\omega_n)^2}}{\sqrt{(1-\omega^2/\omega_n{}^2)^2 + (2\xi\omega/\omega_n)^2}}$$

Critical Speed of Shaft

$$y = \frac{e}{(\omega/\omega_n)^2 - 1}$$
$$\omega_n = \sqrt{\frac{k}{m}}$$

SOLVED EXAMPLES

1. An extensible string of stiffness k in each side of the massless pulley supports a mass m:



Determine the natural frequency of the system.

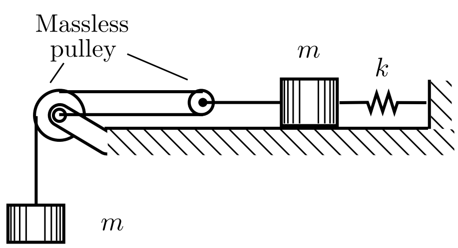
Solution. The system can be modeled as equivalent spring-mass system having stiffness (springs in series):

$$\frac{1}{k_e} = \frac{1}{k} + \frac{1}{k}$$
$$k_e = \frac{k}{2}$$

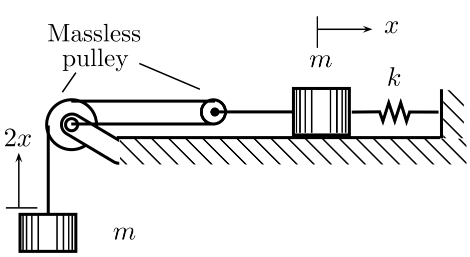
Therefore, the natural frequency of the system is

$$\omega = \sqrt{\frac{k}{2m}}$$

2. Determine the natural frequency of the system.



Solution. Let T be the tension in the string. The system has dependent motion. For movement x of mass on the plane, the mass over the pulley shall move by $2x$:



The spring is under static extension δ_{st} given by

$$\delta_{st} k = T_{st} = 2mg$$

Therefore, the respective equations of motion can be written as

$$m\ddot{x} + k(x + \delta_{st}) - 2T = 0$$
$$m \times 2\ddot{x} + T - mg = 0$$

Eliminating T from above equations, one finds

$$5m\ddot{x} + kx = 0$$

Therefore, comparing with spring-mass model:

$$\omega = \sqrt{\frac{k}{5m}}$$

3. A simple U tube manometer is filled with liquid of specific gravity s. The cross-sectional area of tube is a and length of the liquid column is l. Determine the natural frequency of oscillations of the liquid column. If the value of length of the column is 20 cm, what will be the natural frequency of oscillations?

Solution. For a displacement x, the total energy of the system is given by

$$\frac{1}{2}\rho a l \dot{x}^2 + (\rho a g x)\, x = 0$$

Therefore, differentiating w.r.t. t on both sides,

$$\rho a l \dot{x}\ddot{x} + 2\rho a g x \dot{x} = 0$$
$$\rho a l \ddot{x} + 2\rho a g x = 0$$

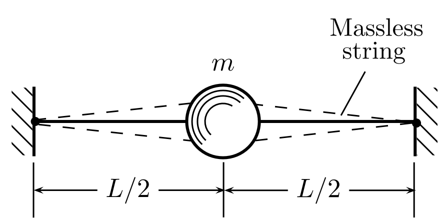
Hence, the natural frequency of oscillations is

$$f = \frac{1}{2\pi}\sqrt{\frac{2g}{l}}$$

For $l = 0.2$ m,

$$\omega = 1.57 \text{ Hz}$$

4. A light ball of mass m is tightly stretched by two strings with initial tension T:



Determine the natural frequency of the ball if it is plucked vertically to a small distance.

Solution. Let the ball is given a slight angular deflection (θ) in vertical position. The ball is vertically displaced by

$$\theta = \frac{x}{L/2}$$

The ball will be under equilibrium of inertia force and resolved tension on both the strings (taking $\sin\theta \approx \theta$):

$$m\ddot{x} + 2T\theta = 0$$
$$m\ddot{x} + 2T \times \frac{x}{L/2} = 0$$
$$m\ddot{x} + \frac{4T}{L} \times x = 0$$

Therefore,

$$\omega = \sqrt{\frac{4T}{mL}}$$

5. A 10 kg mass is supported on a spring of stiffness 4 kN/m and has a dash pot which produces a resistance of 20 N at velocity of 0.25 m/s. Determine the natural frequency and damping ratio of the system.

Solution. Given that

$$m = 10 \text{ kg}$$
$$k = 4 \times 10^3 \text{ N/m}$$

Natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{m}} = 20 \text{ rad/s}$$

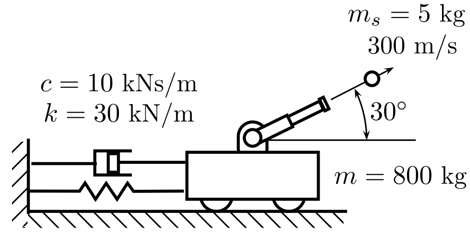
Damping coefficient is

$$c = \frac{20}{0.25} = 80 \text{ Ns/m}$$

The damping ratio is determined as

$$\xi = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} = 0.2$$

6. A gun-carrying vehicle fires a shell of mass 5 kg at speed 300 m/s inclined at 30° from the horizontal. The combined mass of the gun and the vehicle is 800 kg. The recoil mechanism is critically damped and has an equivalent stiffness of 30 kN/m.



Determine the maximum recoil of the gun-vehicle unit.

Solution. Given that

$$m_s = 5 \text{ kg}$$
$$v = 300 \text{ m/s}$$
$$\theta = 20^\circ$$

For gun-vehicle system,

$$m = 800 \text{ kg}$$
$$k = 30 \times 10^3 \text{ N/m}$$
$$c = 9.8 \times 10^3 \text{ Ns/m}$$

Natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{m}} = 6.12 \text{ rad/s}$$

The critically damped displacement can be written as

$$x = (A_1 + A_2 t)\, e^{-\omega_n t}$$

The unknown constant A_1 and A_2 can be determined using initial conditions. Taking $x(0) = 0$,

$$A_1 = 0$$
$$A_2 = \dot{x}(0)$$

Initial recoil velocity of the gun-vehicle can be determined using the principle of conservation of linear momentum:

$$800 \times \dot{x}(0) = 5 \times 300 \times \cos 30^\circ$$
$$\dot{x}(0) = 1.62 \text{ m/s}$$

Thus, the displacement can be written as

$$x = 1.62te^{-6.12t}$$

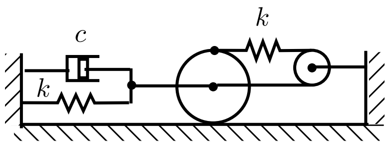
For maximum value of x,

$$t = \frac{1}{6.17} = 0.162 \text{ s}$$

Thus, the maximum displacement will be given by

$$x = 1.62 \times 0.162 \times e^{-1} = 96.5 \text{ mm}$$

7. Center of a sphere of mass m and radius r is attached to a spring-dashpot system of stiffness k and damping constant c on the left side. It is also attached to a spring of stiffness k with a string passing over pulley on the right side.



Determine the natural frequency and damping ratio of the system.

Solution. The sphere will oscillate about the bottom contact point. For slight angular deflection θ of the sphere, the spring and dashpot shall extend by $x = r\theta$. The spring attached to the string over pulley shall extend by $2r\theta$ on left side and $r\theta$ on right side, total extension $3r\theta$.

Moment of inertia of the sphere about the bottom contact point will be

$$\begin{aligned} I &= \frac{mr^2}{2} + mr^2 \\ &= \frac{3}{2}mr^2 \end{aligned}$$

Taking moments of forces about the bottom contact point

$$\begin{aligned} \frac{3mr^2}{2}\frac{\ddot{x}}{r} + r\,(kx + c\dot{x}) + 2r\,(k \times 3x) &= 0 \\ \frac{3m}{2}\ddot{x} + c\dot{x} + 10kx &= 0 \end{aligned}$$

Thus, the natural frequency of the system is

$$\begin{aligned} \omega &= \sqrt{\frac{10k}{3m/2}} \\ &= \sqrt{\frac{20k}{3m}} \end{aligned}$$

Damping ratio is

$$\begin{aligned} \xi &= \frac{c}{2\sqrt{10k \times 3m/2}} \\ &= \frac{c}{2\sqrt{15km}} \end{aligned}$$

8. A damped system has stiffness $k = 450$ kN/m and time period 2.0 s. The ratio of a consecutive amplitudes is 4.0. Determine the amplitude and phase of the steady state motion when a dynamic force $F = 2.5\cos 3t$ N acts on the system.

Solution. Given that

$$\begin{aligned} k &= 650 \text{ N/m} \\ t_d &= 2.0 \text{ s} \\ \frac{x_0}{x_1} &= 4.2 \\ F_0 &= 2.5 \text{ N} \\ \omega &= 3 \text{ rad/s} \end{aligned}$$

Using

$$\begin{aligned} \delta &= \frac{1}{n}\ln\left(\frac{x_1}{x_n}\right) \\ &= \frac{1}{1}\ln(4.2) \\ &= 1.38 \end{aligned}$$

The damping ratio can be found as

$$\begin{aligned} \xi &= \frac{\delta}{\sqrt{\delta^2 + 4\pi^2}} \\ &= 0.21 \end{aligned}$$

Damped frequency of the system is

$$\begin{aligned} \omega_n &= \frac{2\pi}{t_d} \\ &= \frac{2\pi}{t_d\sqrt{1-\zeta^2}} \\ &= 3.21 \text{ rad/s} \end{aligned}$$

Thus, frequency ratio is

$$\begin{aligned} \frac{\omega}{\omega_n} &= \frac{3}{3.2} \\ &= 0.9375 \end{aligned}$$

Static displacement is

$$\begin{aligned} x_{st} &= \frac{F_0}{k} \\ &= \frac{2.5}{650} \\ &= 3.846 \text{ mm} \end{aligned}$$

Amplitude of steady state vibrations is

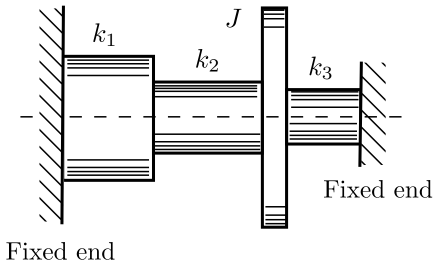
$$\begin{aligned} x_0 &= \frac{x_{st}}{\sqrt{(1-\omega^2/{\omega_n}^2)^2 + (2\xi\omega/\omega_n)^2}} \\ &= \frac{3.846}{\sqrt{0.01466 + 0.155}} \\ &= \frac{3.846}{0.4119} \\ &= 9.33 \text{ mm} \end{aligned}$$

Phase lag is

$$\begin{aligned} \tan\phi &= \frac{2\xi\omega/\omega_n}{1-(\omega^2/{\omega_n}^2)} \\ &= \frac{0.39375}{0.121} \\ &= 3.251 \\ \phi &= 72.9^\circ \end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. Consider the arrangement shown in the figure below where J is the combined polar mass moment of inertia of the disc and the shafts; k_1, k_2, k_3 are the torsional stiffness of the respective shafts.



The natural frequency of torsional oscillation of the disc is given by

(a) $\sqrt{(k_1+k_2+k_3)/J}$
(b) $\sqrt{(k_1k_2+k_2k_3+k_3k_1)/(J(k_1+k_2))}$
(c) $\sqrt{(k_1k_2k_3)/(J(k_1k_2+k_2k_3+k_3k_1))}$
(d) $\sqrt{(k_1k_2+k_2k_3+k_3k_1)/(J(k_2+k_3))}$

(GATE 2003)

Solution. Equivalent setup is k_1 and k_2 in series, which is parallel to k_3, therefore, equivalent stiffness is

$$\begin{aligned} k_e &= \frac{1}{1/k_1+1/k_2}+k_3 \\ &= \frac{k_1k_2+k_2k_3+k_3k_1}{k_1+k_2} \end{aligned}$$

Natural frequency of vibrations is

$$\begin{aligned} \omega_n &= \sqrt{\frac{k_e}{J}} \\ &= \sqrt{\frac{k_1k_2+k_2k_3+k_3k_1}{J(k_1+k_2)}} \end{aligned}$$

Ans. (b)

2. A flexible rotor-shaft system comprises a 10 kg rotor disc placed in the middle of a massless shaft of diameter 30 mm and length 500 mm between bearings (shaft is being taken mass-less as the equivalent mass of the shaft is included in the rotor mass) mounted at the ends. The bearings are assumed to simulate simply supported boundary conditions. The shaft is made of steel for which the value of E is 2.1×10^{11} Pa. What is the critical speed of rotation of the shaft?

(a) 60 Hz (b) 90 Hz
(c) 135 Hz (d) 180 Hz

(GATE 2003)

Solution. Given that

$$\begin{aligned} d &= 0.030 \text{ m} \\ l &= 0.5 \text{ m} \\ m &= 10 \text{ kg} \\ E &= 2.1\times10^{11} \text{ Pa} \end{aligned}$$

Moment of inertia of the shaft is

$$I = \frac{\pi d^4}{64}$$

For simply supported beams, stiffness

$$\begin{aligned} k &= \frac{W}{\delta} \\ &= \frac{48l^3}{EI} \end{aligned}$$

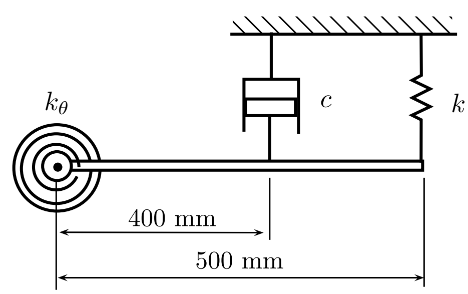
Therefore, critical speed of the shaft is

$$\begin{aligned} f_n &= \frac{\omega_n}{2\pi} \\ &= \frac{1}{2\pi}\sqrt{\frac{k}{m}} \\ &= 90.1203 \text{ Hz} \end{aligned}$$

Ans. (b)

Common Data Questions

A uniform rigid slender bar of mass 10 kg, hinged at the left end is suspended with the help of spring and damper arrangement as shown in the figure where $k = 2$ kN/m, $C = 500$ Ns/m and the stiffness of the torsional spring k_θ is 1 kNm/rad. Ignore the hinge dimensions.



3. The undamped natural frequency of oscillations of the bar about the hinge point is

(a) 42.43 rad/s (b) 30 rad/s
(c) 17.32 rad/s (d) 14.14 rad/s

(GATE 2003)

Solution. Given that

$$\begin{aligned} m &= 10 \text{ kg} \\ k &= 2 \times 10^3 \text{ N/m} \\ k_\theta &= 1 \times 10^3 \text{ N-m/rad} \\ c &= 500 \text{ Ns/m} \end{aligned}$$

For small angular displacement θ, taking torsional moments about hinge,

$$\frac{m \times 0.5^2}{3}\ddot{\theta} + 0.4^2\dot{\theta}c + 0.5^2\theta k + \theta k_\theta = 0$$
$$\frac{m \times 0.5^2}{3}\ddot{\theta} + 0.4^2\dot{\theta}c + \left(0.5^2 k + k_\theta\right)\theta = 0$$

Therefore, the natural frequency of vibrations is

$$\begin{aligned} \omega_n &= \frac{0.5^2 k + k_\theta}{m \times 0.5^2/3} \\ &= 42.4264 \text{ rad/s} \end{aligned}$$

Ans. (a)

4. The damping coefficient in the vibration equation is given by

(a) 500 Nms/rad (b) 500 N/(m/s)
(c) 80 Nms/rad (d) 80 N/(m/s)

(GATE 2003)

Solution. Equivalent damping coefficient is

$$\begin{aligned} c_e &= 0.4^2 c \\ &= 80 \text{ Ns/m} \end{aligned}$$

Ans. (c)

5. A vibrating machine is isolated from the floor using springs. If the ratio of excitation frequency of vibration of machine to the natural frequency of the isolation system is equal to 0.5, the transmissibility of ratio of isolation is

(a) 1/2 (b) 3/4
(c) 4/3 (d) 2

(GATE 2004)

Solution. Given that

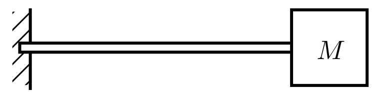
$$\xi = 0$$
$$\frac{\omega}{\omega_n} = 0.5$$

Transmissibility is determined as

$$\begin{aligned} \text{Tr} &= \frac{\sqrt{1 + (2\xi\omega/\omega_n)^2}}{\sqrt{(1 - \omega^2/{\omega_n}^2)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} \\ &= \frac{1}{1 - (\omega/\omega_n)^2} \\ &= \frac{4}{3} \end{aligned}$$

Ans. (c)

6. A mass M of 20 kg is attached to the free end of a steel cantilever beam of length 1000 mm having a cross-section of 25×25 mm. Assume the mass of the cantilever to be negligible and E_{steel} = 200 GPa.



If the lateral vibration of this system is critically damped using a viscous damper, the damping constant of the damper is

(a) 1250 Ns/m (b) 625 Ns/m
(c) 312.50 Ns/m (d) 156.25 Ns/m

(GATE 2004)

Solution. Given that

$$\begin{aligned} M &= 20 \text{ kg} \\ l &= 1 \text{ m} \\ A &= 0.025 \times 0.025 \text{ m}^2 \\ E &= 200 \times 10^9 \text{ Pa} \end{aligned}$$

The moment of inertia and stiffness of cantilever is determined as

$$\begin{aligned} I &= \frac{bd^4}{12} \\ \delta &= \frac{Pl^3}{3EI} \\ k &= \frac{W}{\delta} \\ &= \frac{3EI}{l^3} \end{aligned}$$

Critical damping coefficient is

$$\begin{aligned} c_c &= 2\sqrt{k \times M} \\ &= 1250 \text{ Ns/m} \end{aligned}$$

Ans. (a)

7. A simple pendulum of length 5 m, with a bob of mass 1 kg, is in simple harmonic motion. As it passes through its mean position, the bob has a speed of 5 m/s. The net force on the bob at the mean position is

(a) zero (b) 2.5 N
(c) 5 N (d) 25 N

(GATE 2005)

Solution. At mean position, net force on the bob will be zero because acceleration is zero.

Ans. (a)

8. There are four samples P, Q, R and S, with natural frequencies 64, 96, 128 and 256 Hz, respectively. These are mounted on test setups for conducting vibration experiments. If a loud pure note of frequency 144 Hz is produced by some instrument, which of the samples will show the most perceptible induced vibration?

(a) P (b) Q
(c) R (d) S

(GATE 2005)

Solution. For most perceptible vibrations, the induced frequency should be nearer to the natural frequency.

Ans. (c)

9. In a spring-mass system, the mass is 0.1 kg and the stiffness of the spring is 1 kN/m. By introducing a damper, the frequency of oscillation is found to be 90% of the original value. What is the damping coefficient of the damper?

(a) 1.2 Ns/m (b) 3.4 Ns/m
(c) 8.7 Ns/m (d) 12.0 Ns/m

(GATE 2005)

Solution. Given that

$$\begin{aligned} m &= 0.1 \text{ kg} \\ k &= 1000 \text{ N/m} \\ \omega_d &= 0.9\omega_n \\ \frac{\omega_d}{\omega_n} &= 0.9 \end{aligned}$$

Therefore, critical damping coefficient is

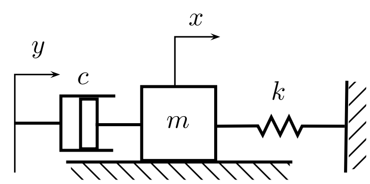
$$\begin{aligned} c_c &= 2\sqrt{km} \\ &= 20 \text{ Ns/m} \end{aligned}$$

Using,

$$\begin{aligned} \omega_d &= \sqrt{1-\xi^2}\omega_n \\ \frac{c}{c_c} &= \xi \\ &= \sqrt{1-\left(\frac{\omega_d}{\omega_n}\right)^2} \\ &= 0.43589 \\ c &= 8.7178 \text{ Ns/m} \end{aligned}$$

Ans. (c)

10. The differential equation governing the vibrating system is:



(a) $m\ddot{x}+c\dot{x}+k(x-y)=0$
(b) $m(\ddot{x}-\ddot{y})+c(\dot{x}-\dot{y})+kx=0$
(c) $m\ddot{x}+c(\dot{x}-\dot{y})+kx=0$
(d) $m(\ddot{x}-\ddot{y})+c(\dot{x}-\dot{y})+k(x-y)=0$

(GATE 2006)

Solution. The relative motion at damper, as compared to simple spring-mass-damper system, is $(\dot{x}-\dot{y})$, therefore the equivalent differential equation for the given system is

$$m\ddot{x}+c(\dot{x}-\dot{y})+kx=0$$

Ans. (c)

11. A machine of 250 kg mass is supported on springs of total stiffness 100 kN/m. Machine has an unbalanced rotating force of 350 N at the speed of 3600 rpm. Assuming a damping factor of 0.15, the value of transmissibility ratio is:

(a) 0.0531 (b) 0.9922
(c) 0.0162 (d) 0.0028

(GATE 2006)

Solution. Given that

$$\begin{aligned} m &= 250 \text{ kg} \\ k &= 100\times 10^3 \text{ N/m} \\ N &= 3600 \text{ rpm} \\ \omega &= \frac{2\pi N}{60} \\ &= 376.991 \text{ rad/s} \\ \xi &= 0.15 \end{aligned}$$

Natural frequency of vibrations in the system is

$$\omega_n = \sqrt{\frac{k}{m}}$$
$$= 20 \text{ rad/s}$$

Transmissibility ratio is defined as

$$Tr = \frac{F_{tr}}{F_0}$$
$$= \frac{\sqrt{1+(2\xi\omega/\omega_n)^2}}{\sqrt{(1-\omega^2/{\omega_n}^2)^2+(2\xi\omega/\omega_n)^2}}$$
$$= 0.016206$$

Ans. (c)

Linked Answer Questions

A vibratory system consists of a mass 12.5 kg, a spring of stiffness 1000 N/m, and a dashpot with damping coefficient of 15 Ns/m.

12. The value of critical damping of the system is

(a) 0.223 Ns/m (b) 17.88 Ns/m
(c) 71.4 Ns/m (d) 223.6 Ns/m

(GATE 2006)

Solution. Given that

$$m = 12.5 \text{ kg}$$
$$k = 1000 \text{ N/m}$$
$$c = 15 \text{ Ns/m}$$

Therefore, critical damping coefficient is

$$c_c = 2\sqrt{km}$$
$$= 2\sqrt{1000 \times 12.5}$$
$$= 223.607 \text{ Ns/m}$$

Ans. (d)

13. The value of logarithmic decrement is

(a) 1.35 (b) 1.32
(c) 0.68 (d) 0.66

(GATE 2006)

Solution. Damping factor is

$$\xi = \frac{c}{c_c}$$
$$= 0.067082$$

Logarithmic increment is

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$
$$= 0.42244$$

Ans. (d)

14. For an under-damped harmonic oscillator, resonance

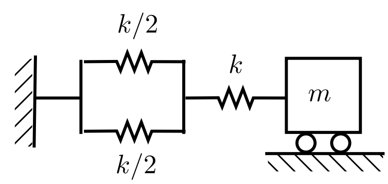
(a) occurs when excitation frequency is greater than undamped natural frequency
(b) occurs when excitation frequency is less than undamped natural frequency
(c) occurs when excitation frequency is equal to undamped natural frequency
(d) never occurs

(GATE 2007)

Solution. In under-damped vibrations, $\xi < 1$, and in such cases, vibrations can not find any probability of resonance.

Ans. (d)

15. The natural frequency of the system shown below is



(a) $\sqrt{k/(2m)}$ (b) $\sqrt{k/m}$
(c) $\sqrt{2k/m}$ (d) $\sqrt{3k/m}$

(GATE 2007)

Solution. The equivalent spring constant of the parallel springs is

$$k_e = 2 \times \frac{k}{2}$$
$$= k$$

Therefore, it constitutes two springs of stiffness k in series, therefore, equivalent spring constant is $k/2$. Hence, the natural frequency is given by

$$\omega_n = \sqrt{\frac{k}{2m}}$$

Ans. (a)

16. The equation of motion of a harmonic oscillator is given by

$$\frac{d^2x}{dt^2} + 2\xi\omega_n\frac{dx}{dt} + {\omega_n}^2 x = 0$$

and the initial condition at $t = 0$ are $x(0) = \chi$, $dx/dt(0) = 0$. The amplitude of $x(t)$ after n complete cycles is

(a) $\chi \exp\left(-2n\pi\xi/\sqrt{1-\xi^2}\right)$

(b) $\chi \exp\left(2n\pi\xi/\sqrt{1-\xi^2}\right)$

(c) $\chi \exp\left(-2n\pi\sqrt{1-\xi^2}/\xi\right)$

(d) χ

(GATE 2007)

Solution. Comparing with the equilibrium equation for general spring–mass–damper system,

$$m\ddot{x}+c\dot{x}+kx=0$$

Thus,

$$\frac{c}{m}=2\xi\omega_n, \quad \frac{k}{m}=\omega_n$$

For the above equation, the amplitude after n cycles is written as

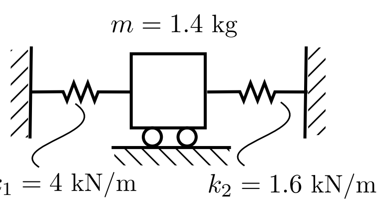
$$\delta=\frac{1}{n}\ln\frac{x_1}{x_n}=\frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

Therefore,

$$\begin{aligned} x_n &= x_1 \exp(n\delta) \\ &= \chi \exp\left(2n\pi\xi/\sqrt{1-\xi^2}\right) \end{aligned}$$

Ans. (b)

17. The natural frequency of the spring–mass system shown in the figure is closest to



(a) 8 Hz (b) 10 Hz
(c) 12 Hz (d) 14 Hz

(GATE 2008)

Solution. The equivalent spring constant of two springs in parallel is

$$\begin{aligned} k_e &= k_1+k_2 \\ &= 4000+1600 \\ &= 5600 \text{ N/m} \end{aligned}$$

Therefore, the natural frequency of vibrations of the system is

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{k}{m}} \\ &= 10.87 \text{ Hz} \end{aligned}$$

Ans. (b)

18. A uniform rigid rod of mass $m = l$ kg and length $L = 1$ m is hinged at its center and laterally supported at one end by a spring of spring constant $k = 300$ N/m. The natural frequency ω_n in rad/s is

(a) 10 (b) 20
(c) 30 (d) 40

(GATE 2008)

Solution. Given

$$\begin{aligned} k &= 300 \text{ N/m} \\ m &= 1 \text{ kg} \end{aligned}$$

Therefore, the natural frequency of vibrations of the system is

$$\begin{aligned} \omega_n &= \sqrt{\frac{k}{m/3}} \\ &= \sqrt{\frac{300}{1/3}} \\ &= 30 \text{ rad/s} \end{aligned}$$

This is because the mass of the spring is not under the uniform force, but it varies linearly from zero at corner to maximum at spring support. In such cases, equivalent mass is found to be one-third of the original mass.

Ans. (c)

19. The rotor shaft of a large electric motor supported between short bearings at both the ends shows a deflection of 1.8 mm in the middle of the rotor. Assuming the rotor to be perfectly balanced and supported at knife edges at both the ends, the likely critical speed (in rpm) of the shaft is

(a) 350 (b) 705
(c) 2810 (d) 4430

(GATE 2009)

Solution. Given that $\delta = 1.8\times10^{-3}$ m.Thus, the critical speed is given by

$$\begin{aligned} \omega_n &= \frac{60}{2\pi}\sqrt{\frac{g}{\delta}} \\ &= 704.968 \text{ rpm} \end{aligned}$$

Ans. (b)

20. An automotive engine weighing 240 kg is supported on four springs with linear characteristics. Each of the front two springs have a stiffness of 16 MN/m, while the stiffness of each rear spring

is 32 MN/m. The engine speed (in rpm), at which resonance is likely to occur, is

(a) 6040 (b) 3020
(c) 1424 (d) 955

(GATE 2009)

Solution. Given that

$$m = 240 \text{ kg}$$
$$k_1 = 16 \times 10^6 \text{ N/m}$$
$$k_2 = 32 \times 10^6 \text{ N/m}$$

Therefore, the equivalent spring constant

$$k_e = 2k_1 + 2k_2 = 96 \times 10^6 \text{ N/m}$$

The natural frequency of vibration

$$w_n = \frac{60}{2\pi}\sqrt{\frac{k_e}{m}} = 6039.51 \text{ rpm}$$

Ans. (a)

21. A vehicle suspension system consists of a spring and a damper. The stiffness of the spring is 3.6 kN/m and the damping constant of the damper is 400 Ns/m. If the mass is 50 kg, then the damping factor (ξ) and damped natural frequency (ω_n), respectively, are

(a) 0.471 and 1.19 Hz
(b) 0.471 and 7.48 Hz
(c) 0.666 and 1.35 Hz
(d) 0.666 and 8.50 Hz

(GATE 2009)

Solution. The critical damping constant (Ns/m)

$$c_c = 2\sqrt{km} = 848.528 \text{ Ns/m}$$

Critical damping factor

$$\xi = \frac{c}{c_c} = 0.471405$$

Natural frequency (rad/sec)

$$\omega_n = \sqrt{\frac{k}{m}} = 8.48528 \text{ rad/s}$$

Damped natural frequency

$$\omega_d = \frac{\sqrt{1-\xi^2}\omega_n}{2\pi} = 1.19101 \text{ Hz}$$

Ans. (a)

22. The natural frequency of a spring–mass system on earth is ω_n. The natural frequency of this system on the moon ($g_{moon} = g_{earth}/6$) is

(a) ω_n (b) $0.408\omega_n$
(c) $0.204\omega_n$ (d) $0.167\omega_n$

(GATE 2010)

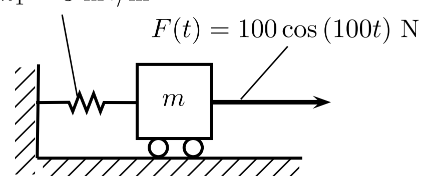
Solution. Natural frequency of spring mass system with stiffness k and mass m is

$$\omega_n = \sqrt{\frac{k}{m}}$$

which is independent of g.

Ans. (a)

23. A mass m attached to a spring is subjected to a harmonic force as shown in the figure.



The amplitude of the forced motion is observed to be 50 mm. The value of m (in kg) is

(a) 0.1 (b) 1.0
(c) 0.3 (d) 0.5

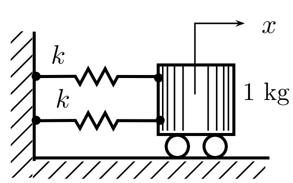
(GATE 2010)

Solution. For equilibrium,

$$F_0 = x_0 k - m\omega^2 x_0$$
$$m = \frac{1}{\omega^2}\left(k - \frac{F_0}{x_0}\right) = 0.1 \text{ kg}$$

Ans. (a)

24. A mass of 1 kg is attached to two identical springs each with stiffness $k = 20$ kN/m as shown in the figure.



Under frictionless condition, the natural frequency of the system in Hz is close to

(a) 32 (b) 23
(c) 16 (d) 11

(GATE 2011)

Solution. Given that

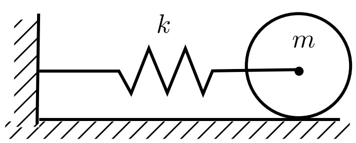
$$k = 20 \times 10^3 \text{ N/m}$$
$$m = 1 \text{ kg}$$

Equivalent stiffness of springs is $2k$, therefore, the natural frequency of vibrations is

$$f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = 31.831 \text{ Hz}$$

Ans. (a)

25. A disc of mass m is attached to a spring of stiffness k as shown in the figure.



The disc rolls without slipping on a horizontal surface. The natural frequency (in rad/s) of vibration of the system is

(a) $\sqrt{k/m}$ (b) $\sqrt{2k/m}$
(c) $\sqrt{2k/(3m)}$ (d) $\sqrt{3k/(2m)}$

(GATE 2011)

Solution. Moment of inertia of the disc w.r.t. the base point at circumference is

$$I = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$$

Equilibrium equation is

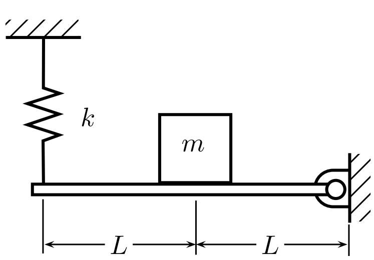
$$\frac{3}{2}mr^2\ddot{\theta} + kr^2\theta = 0$$

Therefore, the natural frequency is

$$\omega_n = \sqrt{\frac{2k}{3m}} \text{ rad/s}$$

Ans. (c)

26. A concentrated mass m is attached at the center of a rod of length $2L$ as shown in the figure.



The rod is kept in a horizontal equilibrium position by a spring of stiffness k. For a very small amplitude of vibration, neglecting the weights of the rod and spring, the undamped natural frequency of the system is

(a) $\sqrt{k/m}$ (b) $\sqrt{2k/m}$
(c) $\sqrt{k/(2m)}$ (d) $\sqrt{4k/m}$

(GATE 2012)

Solution. Equilibrium equation for displacement x of the mass is

$$m\ddot{x} \times L + 2L \times 2x \times k = 0$$
$$m\ddot{x} + 4kx = 0$$

Therefore, the undamped natural frequency of the system is

$$\omega = \sqrt{\frac{4k}{m}}$$

Ans. (d)

27. If two nodes are observed at a frequency of 1800 rpm during whirling of a simply supported long slender rotating shaft, the first critical speed of the shaft in rpm is

(a) 200 (b) 450
(c) 600 (d) 900

(GATE 2013)

Solution. The natural frequency of a simply supported long slender shaft is proportional to the number of modes n. For first critical speed, $n = 1$ for which the frequency is given by

$$f_c = \frac{1}{2} \times 1800 = 900$$

Ans. (d)

28. A single degree of freedom system having mass 1 kg and stiffness 10 kN/mm initially at rest is subjected to an impulse force of magnitude 5 kN for 10^4 s. The amplitude in mm of the resulting free vibration is

(a) 0.5 (b) 1.0
(c) 5.0 (d) 10.0

(GATE 2013)

Solution. The amplitude of vibrations is the initial deflection of the mass, given by

$$x_0 = \frac{F}{k} = 0.5 \text{ mm}$$

Ans. (a)

MULTIPLE CHOICE QUESTIONS

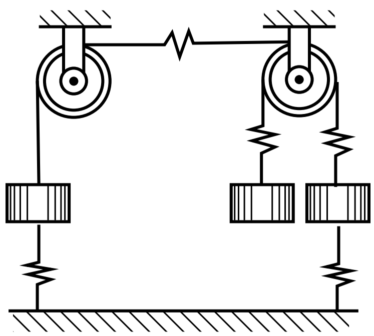
1. A vibrating force $F = F_0 \sin \omega t$ acts on a particle and causes displacement $x = x_0 \sin(\omega t - \phi)$. The work done by force on the particle per cycle is given by

(a) $\pi F_0 x_0 \sin\phi$ (b) $\pi F_0 x_0 \cos\phi$
(c) $F_0 x_0 \sin\phi$ (d) $F_0 x_0 \cos\phi$

2. Vibration systems that have a finite number of degrees of freedom are called

(a) finite system
(b) discrete system
(c) continuous system
(d) homogenous system

3. A spring-mass system is shown in the following figure.



The number of degree of freedom of the system is

(a) 1 (b) 2
(c) 3 (d) 4

4. The time period of simple pendulum depends on its effective length l and the local acceleration due to gravity g. What is the number of dimensionless parameters involved?

(a) two (b) one
(c) three (d) zero

5. Rayleigh's method of computing the fundamental natural frequency is based on

(a) conservation of energy
(b) conservation of momentum
(c) conservation of masses
(d) laws of statics

6. If the vibrations are assumed to have no source of energy dissipation, they are called

(a) damped vibrations
(b) undamped vibrations
(c) forced vibrations
(d) free vibrations

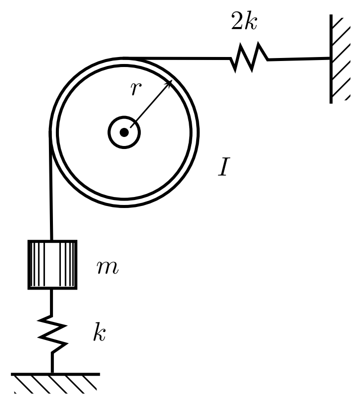
7. The equation of free vibrations of a system is

$$\ddot{x} + 36\pi^2 x = 0$$

Its frequency is

(a) 6 Hz (b) 3π Hz
(c) 3 Hz (d) 6π Hz

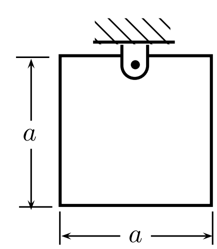
8. A spring–mass pulley system is shown in the figure below.



The radius and moment of inertia of the pulley are r and I, respectively. The natural frequency of the system is

(a) $\sqrt{5k/(m+I/r^2)}$ (b) $\sqrt{5k/(m-I/r^2)}$
(c) $\sqrt{3k/(m+I/r^2)}$ (d) $\sqrt{3k/(m-I/r^2)}$

9. A homogeneous square plate of side a and mass m is suspended from the mid-point of one of the sides, as shown in the following figure.



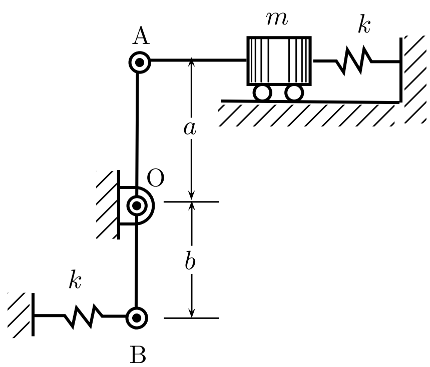
For small angular displacement θ what will be the natural frequency of the system?

(a) $\sqrt{3g/(5a)}$ (b) $\sqrt{5g/(3a)}$
(c) $\sqrt{6g/(5a)}$ (d) $\sqrt{5g/(6a)}$

10. What should be the length of the pendulum for time period of 1 second?

(a) 2.5 cm (b) 9.81 cm
(c) 24.84 cm (d) indeterminate

11. A spring–mass system is shown in the figure below in which the weightless rod AB is pivoted at point O.



The natural frequency of the system is

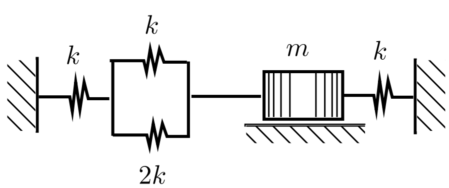
(a) $\sqrt{k(a^2-b^2)/(mb^2)}$

(b) $\sqrt{k(a^2+b^2)/(mb^2)}$

(c) $\sqrt{k(a^2-b^2)/(ma^2)}$

(d) $\sqrt{k(a^2+b^2)/(ma^2)}$

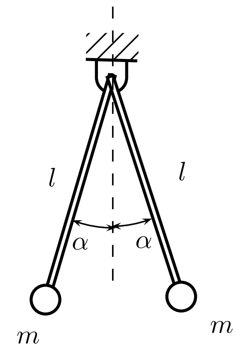
12. A spring–mass system is shown in the figure below.



The natural frequency of the system is

(a) $\sqrt{3k/(5m)}$ (b) $\sqrt{5k/(3m)}$
(c) $\sqrt{8k/(5m)}$ (d) $\sqrt{5k/(8m)}$

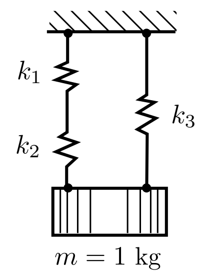
13. The assembly shown in the figure is composed of two massless rods of length l with two particles, each of mass m.



The natural frequency of this assembly for small oscillations is

(a) $\sqrt{g/l}$ (b) $\sqrt{2g/(l\cos\alpha)}$
(c) $\sqrt{g/(l\cos\alpha)}$ (d) $\sqrt{g\cos\alpha/l}$

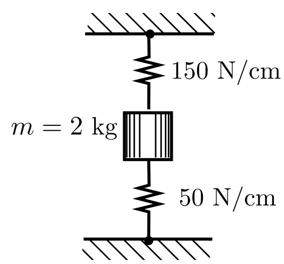
14. A mass of 1 kg is suspended by means of 3 springs as shown in figure.



The spring constants k_1, k_2 and k_3 are 1 kN/m, 3 kN/m and 2 kN/m, respectively. The natural frequency of the system is, approximately,

(a) 46.90 rad/s (b) 52.44 rad/s
(c) 60.55 rad/s (d) 77.46 rad/s

15. A vibratory system is shown in given figure.



The natural frequency of vibration in rad/s is

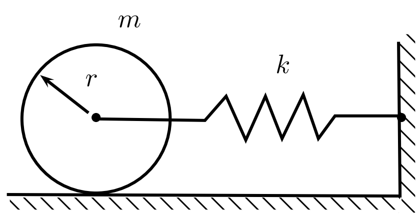
(a) 43.3 (b) 86.6
(c) 100 (d) 200

16. A machine mounted on a single coil spring has a period of free vibration of T. If the spring is cut into four equal parts and placed in parallel and the

machine is mounted on them, then the period of free vibration of the new system will become

(a) $16T$ (b) $4T$
(c) $T/4$ (d) $T/16$

17. A disc of mass m and radius r is attached to a spring of stiffness k. During its motion, the disc rolls on the ground.



When released from some stretched position, the center of the disc will execute harmonic motion with a time period of

(a) $2\pi\sqrt{\dfrac{m}{k}}$ (b) $4\pi\sqrt{\dfrac{m}{k}}$

(c) $2\pi\sqrt{\dfrac{3m}{2k}}$ (d) $2\pi\sqrt{\dfrac{2m}{3k}}$

18. A simple spring–mass system has a natural frequency of ω_n. If the spring stiffness is halved and the mass is doubled, then the natural frequency will become

(a) $\omega_n/2$ (b) $2\omega_n$
(c) $4\omega_n$ (d) $8\omega_n$

19. If air resistance is neglected, while it is executing small oscillations the acceleration of the bob of a simple pendulum at the mid-point of its swing will be

(a) zero
(b) a minimum but not equal to zero
(c) a maximum
(d) not determinable unless the length of the pendulum and the mass of the bob are known

20. If a mass 'm' oscillates on a spring having a mass m_s and stiffness k, then the natural frequency of the system is

(a) $\sqrt{k/(m+m_s/3)}$ (b) $\sqrt{k/(m_s+m/3)}$
(c) $\sqrt{3k/(m+m_s)}$ (d) $\sqrt{k/(m+m_s)}$

21. A mass suspended at the bottom of two springs in series having stiffness 10 N/mm and 5 N/mm. The equivalent spring stiffness of two springs is nearly

(a) 0.3 N/mm (b) 3.3 N/mm
(c) 5 N/mm (d) 15 N/mm

22. A rod of uniform diameter is suspended from one of its ends in vertical plane. The mass of the rod is m and length l, the natural frequency of this rod for small amplitude is

(a) $\sqrt{\dfrac{g}{l}}$ (b) $\sqrt{\dfrac{g}{3l}}$

(c) $\sqrt{\dfrac{2g}{3l}}$ (d) $\sqrt{\dfrac{3g}{2l}}$

23. If the damping factor in a vibrating system is unity, then the system will

(a) have no vibrations
(b) be highly damped
(c) be under damped
(d) be critically damped

24. When the mass of a critically damped single degree of freedom system is deflected from its equilibrium position and released, it will

(a) return to equilibrium position without oscillation
(b) oscillate with increasing time period
(c) oscillate with decreasing amplitude
(d) oscillate with constant amplitude

25. A pendulum system consisting of a hanging body of mass m attached to a massless string of length l. If the mass is displaced at an small angle, the natural frequency of the system will be

(a) $\sqrt{g/(2l)}$ (b) $\sqrt{g/l}$
(c) $\sqrt{2g/l}$ (d) $\sqrt{4g/l}$

26. In coulomb damping by a friction force F on the spring-mas system of stiffness k, the reduction in amplitude per cycle is

(a) F/k (b) $2F/k$
(c) $4F/k$ (d) $8F/k$

27. Critical damping factor for spring mass system of stiffness k and mass m is

(a) $\sqrt{km}$ (b) $2\sqrt{km}$
(c) $3\sqrt{km}$ (d) $4\sqrt{km}$

28. For a critical damped system of natural frequency ω_n, the displacement (x) is related to time (t) as

(a) $x = (A_1 + A_2 t)\, e^{-\omega_n t}$

(b) $x = A_1 e^{-\omega_n t}$
(c) $x = (A_1 + A_2 t)\, e^{-2\omega_n t}$
(d) $x = A_1 e^{-2\omega_n t}$

29. If ω_n is the natural frequency of a free system and damping coefficient is ξ, then the damped frequency of the system will be given by

(a) $\xi\omega_n$ (b) $(1-\xi)\,\omega_n$
(c) $\sqrt{1-\xi}\omega_n$ (d) $\sqrt{1-\xi^2}\omega_n$

30. Logarithmic decrement (δ) is the natural logarithm of the ratio of any two successive amplitudes on the same side of the mean line. It is expressed in terms of damping ratio ξ as

(a) $2\pi\xi/\sqrt{1-\xi^2}$ (b) $2\pi\xi^2/\sqrt{1-\xi^2}$
(c) $\pi\xi/\sqrt{1-\xi}$ (d) $\pi\xi/\sqrt{1-\xi^2}$

31. The damping ratio (ξ) can be expressed in terms of logarithmic decrement δ as

(a) $4\delta/\sqrt{4\pi^2+\delta^2}$ (b) $\delta/\sqrt{4\pi^2+\delta^2}$
(c) $4\delta/\sqrt{4\pi^2-\delta^2}$ (d) $\delta/\sqrt{4\pi^2-\delta^2}$

32. If initial amplitude of a vibrating system x_1 and after executing n cycles, the amplitude is x_n. Then logarithmic decrement is determined as

(a) $\delta^2 = 2\ln(x_1/x_n)/n$
(b) $\delta^2 = 2\ln(x_1/x_n)/n^2$
(c) $\delta^2 = \ln(x_1/x_n)/n$
(d) $\delta^2 = \ln(x_1/x_n)/n^2$

33. A railroad bumper is designed as spring in parallel with a viscous damper. What is the bumper's damping coefficient such that the system has a damping ratio of 1.25 when the bumper is engaged by a 20,000 kg railroad car and has a stiffness 200 kN/m?

(a) 126.49 kNs/m (b) 158.11 kNs/m
(c) 79.06 kNs/m (d) 63.24 kNs/m

34. The natural frequency of an undamped vibrating system is 100 rad/s. A damper with a damping factor of 0.8 is introduced into the system. The frequency of vibration of the damped system, in rad/s, is

(a) 60 (b) 75
(c) 80 (d) 100

35. Logarithmic decrement of a damped single degree of freedom system is δ. If the stiffness of the spring is doubled and the mass is made half, then the logarithmic decrement of the new system will be equal to

(a) $\delta/4$ (b) $\delta/2$
(c) δ (d) 2δ

36. A mass of 1 kg is attached to the end of a spring with a stiffness of 0.7 N/mm. The critical damping coefficient of this system is

(a) 1.40 Ns/m (b) 18.522 Ns/m
(c) 52.92 Ns/m (d) 529.20 Ns/m

37. The equation of motion for a single degree of freedom system with viscous damping is

$$4\ddot{x}+9\dot{x}+16x=0$$

The damping ratio of the system is

(a) 9/128 (b) 9/16
(c) $9\sqrt{2}/16$ (d) 9/8

38. The equation of motion for a damped viscous vibration is

$$3\ddot{x}+9\dot{x}+27x=0$$

The damping factor is

(a) 0.25 (b) 0.50
(c) 0.75 (d) 1.00

39. A mass m, attached with spring of stiffness k and viscous damper of coefficient c, is subjected to dynamic force $F_0 \sin\omega t$. If $x_{st}(=F/k)$ is the static deflection of the spring, the amplitude of the vibrations shall be given by

(a) $F_0/\sqrt{(k-m\omega^2)^2+(c\omega)^2}$
(b) $F_0/\sqrt{(k+m\omega^2)^2+(c\omega)^2}$
(c) $F_0/\sqrt{(k-m\omega^2)^2+(c\omega)^2}$
(d) $F_0/\sqrt{(k+m\omega^2)^2+(c\omega)^2}$

40. In case of resonance in a forced damped vibration having damping ratio ξ, the ratio of the amplitude of vibrations (x_0) and static deflection (x_{st}) is given by

(a) $\dfrac{1}{\xi}$ (b) $\dfrac{1}{\sqrt{\xi}}$
(c) $\dfrac{1}{2\xi}$ (d) $\dfrac{1}{\sqrt{2\xi}}$

41. In a forced vibration with viscous damping, the maximum amplitude occurs when the forced frequency is

(a) equal to natural frequency
(b) slightly less than the natural frequency
(c) slightly greater than the natural frequency
(d) zero

42. The ratio of the maximum dynamic displacement due to a dynamic force to the deflection due to the static force of the same magnitude is called the

(a) displacement ratio
(b) deflection ratio
(c) force factor
(d) magnification factor

43. A 75 kg industrial sewing machine has a rotating unbalance of 0.20 kgm. The machine operates at 150 Hz and is mounted on a foundation of equivalent stiffness 1.75×10^6 N/m and damping ratio 0.15. The steady state amplitude of the vibrations due to unbalance is

(a) 2.73 mm (b) 3.27 mm
(c) 7.23 (d) 2.37 mm

44. A spring damper mechanical system supports a rotor of mass 1000 kg. It is known that rotor has unbalance mass of 1.5 kg located at 5.5 cm radius. It is found that resonance occurs at 1500 rpm. If damping ratio is 0.15, what will be the amplitude of vibrations if the rotor is made to run at 1000 rpm?

(a) 0.14 mm (b) 0.27 mm
(c) 0.33 mm (d) 0.45 mm

45. A machine of 100 kg mass has a 20 kg rotor with 0.5 mm eccentricity. The mounting springs have stiffness 85 kN/m and damping is negligible. If the operating speed is 20π rad/s and the unit is constrained to move vertically, the dynamic amplitude of the machine will be

(a) 0.470×10^{-4} m (b) 1.000×10^{-4} m
(c) 1.270×10^{-5} m (d) 2.540×10^{-4} m

46. If ω is the frequency of the dynamic force, ω_n is the natural frequency of the system, then for damping ratio $\xi = 0$, the transmissibility is determined as

(a) $1/(1-\omega/\omega_n)$ (b) $1/(1-2\omega/\omega_n)$
(c) $1/\left(1-\omega^2/\omega_n^2\right)$ (d) $1/\left(1-2\omega^2/\omega_n^2\right)$

47. A 120 kg turbine, mounted on four identical springs in parallel, having stiffness 3×10^5 N/m each, operates at 2000 rpm. What is the percentage of isolation?

(a) 16.54% (b) 29.69%
(c) 70.30% (d) 83.45%

48. Transmitted force through a spring–mass–damper system will be greater than the transmitted through rigid support for all values of damping factors, if the frequency ratio ω/ω_n is

(a) more than $\sqrt{2}$ (b) less than $\sqrt{2}$
(c) equal to one (d) less than one

49. For effective vibration isolation, if the forcing frequency is ω, the natural frequency ω_n of the system must be

(a) $\omega/4$ (b) ω
(c) 4ω (d) 10ω

50. Match List I with List II and select the correct answer using the codes given below

List I (Transmissibility)	List II (Frequency ratio)
A. 1	1. $\omega/\omega_n > \sqrt{2}$
B. Less than 1	2. $\omega/\omega_n = \sqrt{2}$
C. Greater than 1	3. $\omega/\omega_n >> \sqrt{2}$
D. Tending to 0	4. $\omega/\omega_n << \sqrt{2}$

(a) A-1, B-2, C-3, D-4
(b) A-2, B-1, C-4, D-3
(c) A-2, B-1, C-3, D-4
(d) A-1, B-2, C-4, D-3

51. If $\omega/\omega_n = \sqrt{2}$, where ω is the frequency of excitation and ω_n is the natural frequency of vibrations, then the transmissibility of vibrations will be

(a) 0.5 (b) 1.0
(c) 1.5 (d) 2.0

52. When a shaking force is transmitted through the spring, damping becomes detrimental when the ratio of its frequency to the natural frequency is greater than

(a) 0.25 (b) 0.50
(c) 1.00 (d) $\sqrt{2}$

53. High damping reduces the transmissibility if the frequency ratio ω/ω_n

(a) is less than $\sqrt{2}$
(b) is greater than $\sqrt{2}$
(c) is less than $1/\sqrt{2}$
(d) is greater than $1/\sqrt{2}$

54. In a forced damped vibration system, if ξ is the damping ratio, ω_n is natural frequency, ω is the frequency of dynamic force. The ratio of static deflection (x_{st}) and amplitude of vibrations (x_0) is given by

(a) $\sqrt{\left(1+\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}$

(b) $\sqrt{\left(1+\frac{\omega^2}{\omega_n^2}\right)^2-\left(2\xi\frac{\omega}{\omega_n}\right)^2}$

(c) $\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}$

(d) $\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2-\left(2\xi\frac{\omega}{\omega_n}\right)^2}$

55. Critical speed for shaft of lateral stiffness k and rotor of mass m having eccentricity e is given by

(a) $\sqrt{2k/m}$ (b) $\sqrt{k/m}$
(c) $\sqrt{k/(2m)}$ (d) $\sqrt{ek/(2m)}$

56. The rotor of a turbine is generally rotated at

(a) the critical speed
(b) a speed much below the critical speed
(c) a speed much above the critical speed
(d) the speed has no relation to critical speed

57. A uniform cantilever beam undergoes transverse vibrations. The number of natural frequencies associated with the beam is

(a) 1 (b) 10
(c) 100 (d) infinite

58. Whirling speed of a shaft coincides with the natural frequency of its

(a) longitudinal vibration
(b) transverse vibration
(c) torsional vibration
(d) coupled bending and torsional vibration

59. The critical speed of a shaft is affected by the

1. eccentricity of the shaft
2. span of the shaft
3. diameter of the shaft

Of these statements

(a) 1 and 2 are correct
(b) 1 and 3 are correct
(c) 2 and 3 are correct
(d) all are correct

60. The danger of breakage and vibration is maximum

(a) below critical speed
(b) near critical speed
(c) above critical speed
(d) none of the above

61. A shaft has an attached disc at the center of its length. The disc has its center of gravity located at a distance 2 mm from the axis of the shaft. When the shaft is allowed to vibrate in its natural blow shaped mode, it has a frequency of vibration of 10 rad/s. When the shaft is rotated a 300 revolution per minute, it will whirl with a radius of

(a) 2 mm (b) 2.25 mm
(c) 2.50 mm (d) 3.00 mm

62. A slender shaft supported on two bearings at its ends carries a disc with an eccentricity e from the axis of rotation. The critical speed of the shaft is N. If the disc is replaced by a second one of the same weight but mounted with an eccentricity $2e$, critical speed of the shaft in the second case is

(a) $\frac{N}{2}$ (b) $\frac{N}{\sqrt{2}}$
(c) N (d) $2N$

63. The critical speed of rotating shaft depends upon

(a) mass
(b) stiffness
(c) mass and stiffness
(d) mass, stiffness and eccentricity

64. The critical speed of a shaft is affected by the

(a) diameter and the eccentricity of the shaft
(b) span and the eccentricity of the shaft
(c) diameter and the span of the shaft
(d) span of the shaft

65. The natural frequency of transverse vibration of a mass-less beam of length L having a mass m attached at its mid span is given by (EI is the flexural rigidity of the beam)

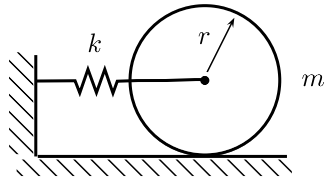
(a) $\sqrt{mL^3/(48EI)}$ rad/s
(b) $\sqrt{48mL^3/(EI)}$ rad/s
(c) $\sqrt{48EI/(mL^3)}$ rad/s
(d) $\sqrt{3EI/(mL^3)}$ rad/s

NUMERICAL ANSWER QUESTIONS

1. Body of mass 5 kg is supported on a spring of stiffness 1960 N/m and has a dash pot connected to it, which produces a resistance of 0.98 N at velocity of 0.5 m/s. Calculate the natural frequency and damping ratio of the system.

2. An unknown mass m is attached to one end of a spring of stiffness k having natural frequency 20 Hz. When 2 kg mass is attached with m, the natural frequency of the system is lowered by 20%. Determine the unknown mass m and spring stiffness k.

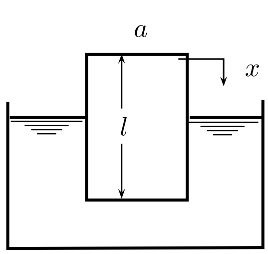
3. A circular cylinder of mass m and radius r is connected by a spring of stiffness k as shown in the figure.



Calculate the natural frequency of the system if $m = 4$ kg, $k = 50$ kN/m.

4. The exhaust from a single cylinder four-stroke diesel engine is connected to a silencer and the pressure therein is to be measured with a simple U-tube manometer. If the engine speed is 1000 rpm, what will be the frequency of pressure fluctuations in the silencer? What will the minimum length of the manometer tube so that the natural frequency of the oscillation of the liquid column will be 5 times slower than the frequency of the pressure fluctuations in the silencer?

5. A homogeneous solid cylinder of length l, cross-sectional area a and specific gravity s (< 1.0) is floating in water with its axis vertical.



Calculate the natural frequency of the system for $s = 0.8$ and $l = 2$ m.

6. A gun barrel of mass 800 kg has a recoil spring of stiffness 300 kN/m. The barrel recoils 1.5 m on firing. Determine the initial recoil velocity of the barrel. Also calculate the critical damping factor for the system.

7. A mass of 5 kg is attached to a spring having a stiffness 20 kN/m. The mass slides on a horizontal surface, the coefficient of friction between mass and surface is 0.2. Calculate the damped frequency of of the system. Also calculate the reduction in the amplitude per cycle.

8. The disc of a torsional pendulum has a moment of inertia of 0.05 kg-m^2 and is immersed in a viscous fluid. The brass shaft attached to it is of 15 cm diameter and 50 cm long. When the pendulum is vibrating, the observed amplitudes on the same side of the rest position for successive cycles are 12 and 8 degrees. Calculate the logarithmic decrement and damping torque of the system.

9. A machine of mass 2 ton is acted upon by an external force of 3000 N at a frequency of 2000 rpm. To reduce the effects of vibration, isolator of rubber having a static deflection of 4 mm under the machine load and an estimated damping factor $\xi = 0.2$ are used. Calculate the force transmitted to the foundation and amplitude of the forced vibrations.

10. A vertical single stage air compressor having a mass of 600 kg is mounted on springs having stiffness of 2×10^5 N/m and damping factor $\xi = 0.3$. The rotating parts are completely balanced and the equivalent reciprocating parts weight 25 kg. The stroke length is 0.3 m. Calculate the amplitude of the vertical vibrations. Also calculate the phase difference between the motion and excitation force when the compressor is operated at 300 rpm.

11. A 120 kg machine is mounted at the mid-span of a 1.5 m long simply supported beam of elastic modulus 200 GPa and cross-sectional moment of inertia 1.53×10^{-6} m^4. An experiment is run on the system during which machine is subjected to harmonic excitation of magnitude 2 kN at a range of excitation frequencies. The largest steady state amplitude recorded during the experiment is 2.5 mm. Calculate the natural frequency and magnification factor of the system.

ANSWERS

Multiple Choice Questions

1.	(a)	**2.**	(b)	**3.**	(d)	**4.**	(d)	**5.**	(a)	**6.**	(b)	**7.**	(c)	**8.**	(c)	**9.**	(c)	**10.**	(c)
11.	(d)	**12.**	(d)	**13.**	(d)	**14.**	(b)	**15.**	(c)	**16.**	(b)	**17.**	(c)	**18.**	(a)	**19.**	(a)	**20.**	(a)
21.	(b)	**22.**	(c)	**23.**	(d)	**24.**	(a)	**25.**	(b)	**26.**	(c)	**27.**	(b)	**28.**	(a)	**29.**	(d)	**30.**	(a)
31.	(b)	**32.**	(c)	**33.**	(b)	**34.**	(a)	**35.**	(c)	**36.**	(c)	**37.**	(b)	**38.**	(b)	**39.**	(a)	**40.**	(c)
41.	(b)	**42.**	(d)	**43.**	(a)	**44.**	(a)	**45.**	(b)	**46.**	(c)	**47.**	(c)	**48.**	(a)	**49.**	(a)	**50.**	(b)
51.	(b)	**52.**	(d)	**53.**	(a)	**54.**	(c)	**55.**	(b)	**56.**	(c)	**57.**	(d)	**58.**	(b)	**59.**	(c)	**60.**	(b)
61.	(b)	**62.**	(c)	**63.**	(c)	**64.**	(c)	**65.**	(c)										

Numerical Answer Questions

1.	19.79 rad/s, 0.0098	**2.**	3.55 kg, 56.06 kN	**3.**	91.28 rad/s
4.	52.35 rad/s, 0.179 m	**5.**	2.48 rad/s	**6.**	29.05 m/s, 30.983 kNs/m
7.	63.24 rad/s, 1.962 mm	**8.**	0.40, 60.79 Nm/rad	**9.**	348 N, 63.6 mm
10.	8.35 mm, −27.78°	**11.**	190.4 rad/s, 18.38		

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (a) The work done by force $F = F_0 \sin \omega t$ for displacement $x = x_0 \sin(\omega t - \phi)$ of the particle per cycle is given by

$$\begin{aligned} W &= \int_0^{2\pi} F \frac{dx}{dt} dt \\ &= \pi F_0 x_0 \sin \phi \end{aligned}$$

2. (b) Vibration systems that have a finite number of degrees of freedom are called discrete system.

3. (d) There are five elements: 3 masses and 2 pulleys, but the movement of left side mass is related to its pulley, hence, the system has 4 degrees of freedom.

4. (d) The present case has two variables l and g ($n = 2$) and basic dimensions are L and T. Hence, the number of dimensionless parameters is $2 - 2 = 0$.

5. (a) Rayleigh's method considers the fact that the maximum kinetic energy at the mean position is equal to the maximum potential energy at the extreme position.

6. (b) Undamped vibrations occur without energy dissipation.

7. (c) Frequency is

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{36\pi^2}{1}} \\ &= 3 \text{ Hz} \end{aligned}$$

8. (c) Using conservation of energy, the potential energy (U) and kinetic energy (T) for small displacement x of the mass (m) are determined, respectively, as

$$\begin{aligned} U &= \frac{1}{2}kx^2 + \frac{1}{2}2kx^2 \\ &= \frac{1}{2}(3k)x^2 \\ T &= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\left(\frac{\dot{x}}{r}\right)^2 \\ &= \frac{1}{2}\left(m + \frac{I}{r^2}\right)\dot{x}^2 \end{aligned}$$

Hence, equivalent stiffness of the system is $3k$ while equivalent mass is $m + I/r^2$, therefore, natural

frequency of the system will be

$$\omega_n = \sqrt{\frac{3k}{m+I/r^2}}$$

9. (c) The moment of inertia of the plate about the pivot will be given by

$$\begin{aligned} I_o &= \frac{ma^2}{3}+\frac{ma^2}{12} \\ &= \frac{5ma^2}{12} \end{aligned}$$

For small angular displacement θ, the center of gravity will move up by $a\theta/2$. Using Newton's method

$$\begin{aligned} \frac{5ma^2}{12}\ddot{\theta}+mg\frac{a}{2}\theta = 0 \\ \frac{5ma^2}{12}\ddot{\theta}+\frac{mga}{2}\theta = 0 \end{aligned}$$

Hence, natural frequency of the system is

$$\begin{aligned} \omega_n &= \sqrt{\frac{mga/2}{5ma^2/12}} \\ &= \sqrt{\frac{12\times mga}{2\times 5ma^2}} \\ &= \sqrt{\frac{6g}{5a}} \end{aligned}$$

10. (c) The natural frequency of the pendulum is related to its length l as

$$\omega_n = \sqrt{\frac{g}{l}}$$

Hence, time period T is determined as

$$\begin{aligned} T &= \frac{2\pi}{\omega_n} \\ &= 2\pi\sqrt{\frac{l}{g}} \\ l &= \frac{T^2 g}{4\pi^2} \\ &= \frac{1^2\times 9.81}{4\pi^2} \\ &= 0.2484 \text{ m} \\ &= 24.84 \text{ cm} \end{aligned}$$

11. (d) Using Newton's method for small angular movement of the rod AB about O, one finds

$$\begin{aligned} ma\ddot{\theta}a+ka\theta a+kb\theta b = 0 \\ ma^2\ddot{\theta}+k\left(a^2+b^2\right)\theta = 0 \end{aligned}$$

Hence, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k\left(a^2+b^2\right)}{ma^2}}$$

12. (d) The equivalent stiffness of the spring combinations is determined in two parts. For the left springs,

$$\begin{aligned} k_e' &= \frac{1}{1/k+1/(2k)}+k \\ &= \frac{5}{3}k \end{aligned}$$

For complete combination of the springs in the system

$$\begin{aligned} \frac{1}{k_e} &= \frac{3}{5k}+\frac{1}{k} \\ k_e &= \frac{5}{8}k \end{aligned}$$

Thus, the natural frequency of the system is

$$\begin{aligned} \omega_n &= \sqrt{\frac{k_e}{m}} \\ &= \sqrt{\frac{5k}{8m}} \end{aligned}$$

13. (d) The moment of inertia of the assembly about pivot is $2\times ml^2$. If the assembly is displaced by angle θ, the equilibrium equation can be written as

$$2\times ml^2\ddot{\theta}+2mgl\cos\alpha\theta = 0$$

Hence, natural frequncy of the system is

$$\omega_n = \sqrt{\frac{g\cos\alpha}{l}}$$

14. (b) Effective spring constant is

$$\begin{aligned} k_e &= \frac{1}{1/1+1/3}+2 \\ &= 2.75 \text{ kN} \end{aligned}$$

The natural frequency of the system is determined as

$$\begin{aligned} \omega &= \sqrt{\frac{k_e}{m}} \\ &= 52.44 \text{ rad/s} \end{aligned}$$

15. (c) The two springs will act in parallel combination. Therefore, the equivalent stiffness of the system is

$$\begin{aligned} k_e &= k_1+k_2 \\ &= 150+50 \\ &= 200 \text{ N/cm} \end{aligned}$$

Hence, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{200 \times 100}{2}} = 100 \text{ rad/s}$$

16. (b) The cutting of spring into four equal parts shall make stiffness of each spring $4k$. If these are combined in parallel, the stiffness will be $4 \times 4k = 16k$. The time period is proportional to $\sqrt{k}$, therefore, the new system will have time period of $\sqrt{16}T = 4T$.

17. (c) The problem can be solved by using energy method. For the small displacement x of the center of the disc, the kinetic and potential energy of the system are, respectively,

$$T = \frac{1}{2}\frac{mr^2}{2}\left(\frac{\dot{x}}{r}\right)^2 + \frac{1}{2}m\dot{x}^2 = \frac{1}{2}\left(\frac{3m}{2}\right)\dot{x}^2$$

$$U = \frac{1}{2}kx^2$$

Thus, equivalent mass of the system is $m_e = 3m/2$. Hence, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{3m/2}} = \sqrt{\frac{2k}{3m}}$$

Time period of the harmonic motion will be

$$T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{3m}{2k}}$$

18. (a) Natural frequency is given by

$$\omega_n \propto \sqrt{\frac{k}{m}}$$

Therefore,

$$\omega_n' = \sqrt{\frac{1/2}{2}}\omega_n = \frac{\omega_n}{2}$$

19. (a) The acceleration of the bob of a simple pendulum is maximum at the end points whereas it is zero at mid-point.

20. (a) The equivalent oscillating mass of vibrations is

$$m_e = m_s + \frac{m}{3}$$

21. (b) Equivalent stiffness of springs in series is

$$k_e = \frac{k_1 k_2}{k_1 + k_2} = 3.33 \text{ N/mm}$$

22. (c) Natural frequency for the pendulum of length l with equivalent mass $m/3$ will be given by

$$\omega_n = \frac{1}{2\pi}\sqrt{\frac{g}{3l}}$$

23. (d) For critically damped system: $\xi = 1$

24. (a) For critically damped system ($\xi = 1$)

$$x = (A_1 + A_2 t)\, e^{-\omega_n t}$$

Thus, the mass will return to equilibrium position without oscillation.

25. (b) The natural frequency of pendulum is

$$\omega_n = \sqrt{\frac{g}{l}}$$

26. (c) In vibrations under frictional damping, the reduction in amplitude per cycle is

$$\Delta = \frac{4F}{k}$$

27. (b) Critical damping factor is

$$c_c = 2\sqrt{km}$$

28. (a) For critically damped systems ($\xi = 1$)

$$x = (A_1 + A_2 t)\, e^{-\omega_n t}$$

29. (d) The damped frequency of the system is

$$\omega_d = \sqrt{1 - \xi^2}\omega_n$$

30. (a) Logarithmic decrement (δ) is

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

31. (b) Logarithmic decrement (δ) is

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

Hence,

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

32. (c) The logarithmic decrement will be given by

$$\delta = \frac{1}{n} \ln \left(\frac{x_1}{x_n} \right)$$

33. (b) Given that

$$\begin{aligned} m &= 20000 \text{ kg} \\ k &= 2 \times 10^5 \text{ N/m} \\ \xi &= 1.25 \end{aligned}$$

Damping coefficient is

$$\begin{aligned} c &= \xi \times c_c \\ &= 2\xi\sqrt{km} \\ &= 158.11 \text{ kNs/m} \end{aligned}$$

34. (a) Given that

$$\begin{aligned} \omega_n &= 100 \text{ rad/s} \\ \xi &= 0.8 \end{aligned}$$

Damping frequency is

$$\begin{aligned} \omega_d &= \omega_n \sqrt{1 - \xi^2} \\ &= 60 \text{ rad/s} \end{aligned}$$

35. (c) Logarithmic decrement depends upon the damping factor (ξ), and not on the mass or stiffness of the system.

36. (c) Given that

$$\begin{aligned} m &= 1 \text{ kg} \\ k &= 0.7 \text{ N/mm} \end{aligned}$$

Critical camping coefficient is

$$\begin{aligned} c_c &= 2\sqrt{km} \\ &= 52.91 \text{ Ns/m} \end{aligned}$$

37. (b) Given that

$$\begin{aligned} m &= 4 \\ c &= 9 \\ k &= 16 \end{aligned}$$

Damping ratio is

$$\begin{aligned} \xi &= \frac{c}{c_c} \\ &= \frac{c}{2\sqrt{km}} \\ &= \frac{9}{2\sqrt{16 \times 4}} \\ &= \frac{9}{16} \end{aligned}$$

38. (b) From the given equation,

$$\begin{aligned} m &= 3 \\ k &= 27 \\ c &= 9 \end{aligned}$$

Hence, the damping factor is

$$\begin{aligned} \xi &= \frac{c}{c_c} \\ &= \frac{c}{2\sqrt{km}} \\ &= 0.5 \end{aligned}$$

39. (a) Amplitude of the displacement is

$$x_0 = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

40. (c) The ratio of amplitude of vibrations at resonance (x_0) and static deflection (x_{st}) is given by

$$\frac{x_0}{x_{st}} = \frac{1}{2\xi}$$

41. (b) The maximum amplitude in forced vibrations occurs when

$$\frac{\omega}{\omega_n} = \sqrt{1 - 2\xi^2}$$

Thus, the forced frequency (ω) should be slightly less than the natural frequency (ω_n).

42. (d) Magnification factor is the ratio of the maximum dynamic displacement due to a dynamic force to the deflection due to the static force of the same magnitude.

43. (a) Given that

$$\begin{aligned} m &= 75 \text{ kg} \\ m_0 e &= 0.20 \text{ kgm} \\ \omega &= 150 \times 2\pi \\ &= 942.47 \text{ rad/s} \\ k &= 1.75 \times 10^6 \text{ N/m} \\ \xi &= 0.15 \end{aligned}$$

Natural frequency of the system is

$$\begin{aligned} \omega_n &= \sqrt{\frac{k}{m}} \\ &= 152.75 \text{ rad/s} \end{aligned}$$

Ratio of forced and natural frequencies is

$$\begin{aligned} r &= \frac{\omega}{\omega_n} \\ &= 6.17 \end{aligned}$$

Maximum steady state amplitude is

$$\begin{aligned} x_0 &= \frac{m_0 e \times \omega^2/k}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\xi\omega/\omega_n\right)^2}} \\ &= \frac{0.1015}{37.11} \\ &= 2.73 \times 10^{-3} \text{ m} \end{aligned}$$

44. (a) Given that

$$\begin{aligned} m &= 1000 \text{ kg} \\ m_0 &= 1.5 \text{ kg} \\ e &= 0.055 \text{ m} \\ \xi &= 0.15 \end{aligned}$$

Resonance frequency is the natural frequency:

$$\begin{aligned} \omega_n &= 1500 \frac{2\pi}{60} \\ &= 157.08 \text{ rad/s} \\ \omega &= 1000 \frac{2\pi}{60} \\ &= 104.72 \text{ rad/s} \\ \frac{\omega}{\omega_n} &= 1.5 \end{aligned}$$

Using,

$$\begin{aligned} \omega_n &= \sqrt{\frac{k}{m}} \\ k &= \omega_n^2 m \\ &= 24.67 \times 10^6 \text{ N/m} \end{aligned}$$

The steady state amplitude is given by

$$\begin{aligned} x_0 &= \frac{m_0 e \times \omega^2/k}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\xi\omega/\omega_n\right)^2}} \\ &= \frac{(m_0 e/m) \times \left(\omega^2/\omega_n^2\right)}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\xi\omega/\omega_n\right)^2}} \\ &= \frac{1.856 \times 10^{-4}}{1.328} \\ &= 1.397 \times 10^{-4} \text{ m} \\ &\approx 0.14 \text{ mm} \end{aligned}$$

45. (b) Given that

$$\begin{aligned} m &= 100 \text{ kg} \\ k &= 85 \text{ kN/m} \\ \omega &= 20\pi \text{ rad/s} \\ F_0 &= 20 \times 0.5 \times 0.0001 \times \omega^2 \text{ N} \\ c &= 0 \end{aligned}$$

Therefore,

$$\begin{aligned} x_0 &= \frac{F_0}{\sqrt{\left(k - m\omega^2\right)^2 + (c\omega)^2}} \\ &= 1.274 \times 10^{-5} \text{ m} \end{aligned}$$

46. (c) Transmissibility for undamped machines is

$$\text{Tr} = \frac{1}{1 - (\omega/\omega_n)^2}$$

47. (c) Equivalent stiffness of the four springs is

$$\begin{aligned} k_e &= 4k \\ &= 12 \times 10^5 \text{ N/m} \end{aligned}$$

Natural frequency of the vibrations is

$$\begin{aligned} \omega_n &= \sqrt{\frac{k}{m}} \\ &= \sqrt{\frac{12 \times 10^5}{120}} \\ &= 100 \text{ rad/s} \end{aligned}$$

The forced frequency is

$$\begin{aligned} \omega &= 2000 \times \frac{2\pi}{60} \\ &= 209.43 \text{ rad/s} \end{aligned}$$

For free systems ($\xi = 0$),

$$\begin{aligned} \text{Tr} &= \frac{1}{(\omega/\omega_n)^2 - 1} \\ &= 0.2969 \end{aligned}$$

Hence, isolation is determined as

$$\begin{aligned} &= 1 - \text{Tr} \\ &= 1 - 0.2969 \\ &= 70.30\% \end{aligned}$$

48. (a) The transmissibility is Tr is less than 1.0 when $\omega/\omega_n > \sqrt{2}$.

49. (a) The condition is $\omega_n < \omega$.

50. (b) The correct combination is tabulated as follows:

(Transmissibility)	(Frequency ratio)
A. 1	1. $\omega/\omega_n = \sqrt{2}$
B. Less than 1	2. $\omega/\omega_n > \sqrt{2}$
C. Greater than 1	3. $\omega/\omega_n >> \sqrt{2}$
D. Tending to 0	4. $\omega/\omega_n << \sqrt{2}$

51. (b) Transmissibility (Tr) is equal to 1 for $\omega/\omega_n = \sqrt{2}$ for all the values of ξ.

52. (d) The system is dangerous if

$$\frac{\omega}{\omega_n} < \sqrt{2}$$

53. (a) Damping is advantageous when

$$\frac{\omega}{\omega_n} > \sqrt{2}$$

54. (c) In forced damped vibrations,

$$x_0 = \frac{x_{st}}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\xi\omega/\omega_n\right)^2}}$$

55. (b) Critical speed for the shaft is

$$\omega_n = \sqrt{\frac{k}{m}}$$

56. (c) High speed elements (e.g. shaft, rotor) are rotated beyond the critical speed.

57. (d) Transverse vibrations can occur in infinite nodes, thus, can have infinite number of natural frequencies.

58. (b) The whirling speed of shaft coincides with the natural frequency of its transverse vibrations.

59. (c) Critical speed for the shaft is

$$\omega = \sqrt{\frac{k}{m}}$$

Thus, span and diameter of the shaft affect the natural critical speed. The eccentricity affects the amplitude of transverse vibrations, not the critical speed.

60. (b) Resonance occurs at critical speed in which amplitudes of vibrations are maximum.

61. (b) Given that

$$\begin{aligned} e &= 2 \text{ mm} \\ \omega_n &= 10 \text{ rad/s} \\ \omega &= 300 \times 2\pi/60 \\ &= 10\pi \text{ rad/s} \end{aligned}$$

The radius is found as

$$\begin{aligned} y &= \frac{e}{\left(\omega_n/\omega\right)^2 - 1} \\ &= 2.25 \text{ mm} \end{aligned}$$

62. (c) Critical speed of a shaft is equal to natural frequency of vibrations,

$$\omega_n = \sqrt{\frac{k}{m}}$$

which is independent of eccentricity.

63. (c) Critical speed for the shaft is

$$\omega_n = \sqrt{\frac{k}{m}}$$

64. (c) Critical speed for the shaft is

$$\omega_n = \sqrt{\frac{k}{m}}$$

The stiffness depends upon the diameter and span of the shaft.

65. (c) The natural frequency of vibrations is

$$\omega_n = \sqrt{\frac{k}{m}}$$

For given condition of beam

$$\begin{aligned} k &= \frac{W}{\delta} \\ &= \frac{48EI}{L^3} \end{aligned}$$

Numerical Answer Questions

1. Given that

$$\begin{aligned} m &= 5 \text{ kg} \\ k &= 1960 \text{ N/m} \end{aligned}$$

Damping coefficient is

$$\begin{aligned} c &= \frac{0.98}{0.5} \\ &= 1.96 \text{ Ns/m} \end{aligned}$$

Natural frequency of the system is

$$\begin{aligned} \omega_n &= \sqrt{\frac{k}{m}} \\ &= \sqrt{\frac{1960}{5}} \\ &= 19.79 \text{ rad/s} \end{aligned}$$

The damping ratio is determined as

$$\begin{aligned}\xi &= \frac{c}{c_c}\\ &= \frac{c}{2\sqrt{km}}\\ &= \frac{1.96}{2\sqrt{1960\times 5}}\\ &= 0.0098\end{aligned}$$

2. The natural frequency of a simple spring–mass system in initial stage is given by

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

After attaching 1 kg mass with the unknown mass m, the natural frequency becomes

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{m+1}}$$

Given

$$\begin{aligned}f_2 &= \left(1-\frac{20}{100}\right) f_1\\ \sqrt{\frac{k}{m+1}} &= 0.8\sqrt{\frac{k}{m}}\\ \frac{k}{m+1} &= 0.8^2\frac{k}{m}\\ \frac{m}{m+1} &= 0.64\\ m &= 3.55 \text{ kg}\end{aligned}$$

Given, $f_1 = 20$ Hz, therefore

$$\begin{aligned}20 &= \frac{1}{2\pi}\sqrt{\frac{k}{m}}\\ k &= 56.059 \text{ kN}\end{aligned}$$

3. For a small displacement x of the cylinder, it shall rotate angle $\theta = x/r$. Hence, the total energy of the system at any moment will be given by

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}kx^2 = c$$

where

$$\begin{aligned}\dot{x} &= r\dot{\theta}\\ I &= \frac{1}{2}mr^2\end{aligned}$$

Hence, differentiating both sides w.r.t. time t

$$\frac{3m}{2}\ddot{\theta} + k\theta = 0$$

Given that

$$\begin{aligned}m &= 4 \text{ kg}\\ k &= 5\times 10^3 \text{ N/m}\end{aligned}$$

Therefore, the natural frequency is given by

$$\begin{aligned}\omega_n &= \sqrt{\frac{2k}{3m}}\\ &= 91.28 \text{ rad/s}\end{aligned}$$

4. For a four-stroke engine, there is one exhaust for two revolutions, hence, the frequency of pressure fluctuation for 1000 rpm will be

$$\begin{aligned}\omega_e &= \frac{1000}{2}\\ &= 500 \text{ rpm}\\ &= 52.35 \text{ rad/s}\end{aligned}$$

Natural frequency of oscillations in the U-tube manometer for given length l is expressed as

$$\omega_n = \sqrt{\frac{2g}{l}}$$

Given that

$$\omega_e = 5\omega_n$$

Therefore,

$$\begin{aligned}52.35 &= 5\times\sqrt{\frac{2g}{l}}\\ l &= 0.179 \text{ m}\end{aligned}$$

5. For a given displacement x of the cylinder, the equilibrium equation will be given by

$$\begin{aligned}s\rho_w al\ddot{x} + \rho_w axg &= 0\\ sl\ddot{x} + xg &= 0\\ \omega_n &= \sqrt{\frac{g}{sl}}\end{aligned}$$

Given that

$$\begin{aligned}s &= 0.8\\ l &= 2 \text{ m}\end{aligned}$$

The natural frequency of oscillations is given by

$$\begin{aligned}\omega_n &= \sqrt{\frac{g}{sl}}\\ &= 2.48 \text{ rad/s}\end{aligned}$$

6. Using Rayleigh method, the maximum values of kinetic energy of the barrel and potential energy of the spring must be equal, hence

$$\frac{1}{2}kx^2 = \frac{1}{2}m\dot{x}^2$$
$$\dot{x} = \sqrt{\frac{kx^2}{m}}$$

Therefore, for initial recoil $x = 1.5$ m. the initial recoil velocity will be

$$\dot{x} = \sqrt{\frac{300\times10^3\times1.5^2}{800}}$$
$$= 29.05 \text{ m/s}$$

The critical damping factor is determined as

$$c_c = 2\sqrt{km}$$
$$= 30.983 \text{ kNs/m}$$

7. Given that

$$m = 5 \text{ kg}$$
$$k = 20\times10^3 \text{ N/m}$$

Therefore, natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{m}}$$
$$= 63.24 \text{ rad/s}$$

Given that $\mu = 0.2$. The friction force is given by

$$F = \mu mg$$
$$= 9.81 \text{ N}$$

The rate of reduction in amplitude per cycle is

$$\delta = \frac{4F}{k}$$
$$= 1.962 \text{ mm}$$

8. The logarithmic decrement, by definition is

$$\delta = \ln\frac{\theta_n}{\theta_{n+1}}$$
$$= \ln\frac{12}{8}$$
$$= 0.405$$

Damping ratio (ξ) is calculated as

$$\xi = \sqrt{\frac{\delta^2}{4\pi^2+\delta^2}}$$
$$= 0.0643$$

Properties of the shaft are

$$d = 0.15 \text{ m}$$
$$l = 0.5 \text{ m}$$
$$G = 4.5\times10^{10} \text{ N/m}^2$$

Torsional stiffness of the shaft is

$$k_t = \frac{T}{\theta}$$
$$= \frac{G}{l}I_p$$
$$= \frac{G}{l}\times\frac{\pi d^4}{32}$$
$$= 4.47\times10^6 \text{ Nm/rad}$$

Given that

$$I = 0.05 \text{ kgm}^2$$

Damping torque is

$$c_c = \xi\times2\sqrt{k_tI}$$
$$= 60.79 \text{ Nm/rad}$$

9. Given that

$$x_{st} = 0.003 \text{ m}$$
$$m = 1000 \text{ kg}$$
$$F_0 = 3000 \text{ N}$$
$$\xi = 0.2$$

The frequency of external force is

$$\omega = \frac{2\pi N}{60}$$
$$= 209.43 \text{ rad/s}$$

Spring stiffness is

$$k = \frac{mg}{x_{st}}$$
$$= 4.9 \text{ MN/m}$$

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{m}}$$
$$= 49.52 \text{ rad/s}$$

The force transmitted to the foundation is

$$\frac{F_{tr}}{F_0} = \frac{\sqrt{1+(2\xi\omega/\omega_n)^2}}{\sqrt{(1-\omega^2/\omega_n{}^2)^2+(2\xi\omega/\omega_n)^2}}$$
$$= 0.116$$
$$F_{tr} = 348 \text{ N}$$

The amplitude of the vibration of the machine is given by

$$\frac{x_0}{x_{st}} = \frac{1}{\sqrt{\left(1-\omega^2/{\omega_n}^2\right)^2+\left(2\xi\omega/\omega_n\right)^2}}$$
$$= 0.059$$
$$x_o = 6.36\times10^{-4}\ \text{m}$$
$$= 63.6\ \text{mm}$$

10. Given that

$$m = 600\ \text{kg}$$
$$k = 2\times10^5\ \text{N/m}$$
$$\xi = 0.3$$
$$m_o = 25\ \text{kg}$$
$$e = \frac{0.3}{2}$$
$$= 0.15\ \text{m}$$
$$N = 300$$

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{m}}$$
$$= 18.25\ \text{rad/s}$$

Frequeny of external force is

$$\omega = \frac{2\pi N}{60}$$
$$= 31.41\ \text{rad/s}$$

Therefore, the amplitude is given by

$$\frac{x_o}{m_o e/m} = \frac{\left(\omega/\omega_n\right)^2}{\sqrt{\left(1-\omega^2/{\omega_n}^2\right)^2+\left(2\xi\omega/\omega_n\right)^2}}$$
$$x_o = 8.35\times10^{-3}\ \text{m}$$
$$= 8.35\ \text{mm}$$

Phase difference is calculated as

$$\phi = \tan^{-1}\left(\frac{2\xi\left(\omega/\omega_n\right)}{1-\left(\omega/\omega_n\right)^2}\right)$$
$$= -27.78^\circ$$

11. Given that

$$E = 200\times10^9\ \text{Pa}$$
$$I = 1.53\times10^{-6}\ \text{m}^4$$
$$L = 1.5\ \text{m}$$
$$m = 120\ \text{kg}$$
$$x_0 = 2.5\ \text{mm}$$

Stiffness of the beam is

$$k = \frac{48EI}{L^3}$$
$$= 4.352\times10^6\ \text{N/m}$$

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{m}}$$
$$= \sqrt{\frac{4.352\times10^6}{120}}$$
$$= 190.43\ \text{rad/s}$$

Static deflection of the mass is

$$x_{st} = \frac{F_0}{k}$$
$$= \frac{2\times10^3}{4.352\times10^6}$$
$$= 45.95\ \text{mm}$$

Magnification factor is calculated as

$$\Lambda = \frac{x_{st}}{x_0}$$
$$= \frac{45.95}{2.5}$$
$$= 18.38$$

CHAPTER 5

DESIGN

A machine consists of various elements which have motion with respect to each other. Elements of a machine need strength, rigidity, wear resistance, minimum dimension and weight, manufacturability, safety, standards, maintainability and reliability. The subject of design employs scientific principles, technical information and imagination in the description of a machine to perform specific functions with maximum economy and efficiency. Theory of machines offers kinematics and dynamics of the linkages or elements of a machine while machine design enables in achieving the design of elements so that they can meet the kinematics and dynamics.

5.1 STATIC LOADING

The mechanism of failure of a material depends on the load characteristics. Static loads are slowly applied and remain constant with time, while dynamic loads are either suddenly applied (impact load) or repeatedly varied with time (fatigue load).

5.1.1 Modes of Failure

A mechanical component is called failed if it is unable to perform its desired functions satisfactorily. In static loading, the failure can be in any one of the following modes:

1. *Elastic Deflection* A member of structure, such as shafts, beam, column, can have elastic deflection beyond the allowable limits. In such cases, the material does not fail but it is unable to function. Elastic deflection vanishes after removal of loads from the member. This mode of failure is concerned with stability of the machine elements, where modulus of elasticity and rigidity of the material play a key role.

2. *Yielding* A component made of ductile material loses its engineering usefulness due to significant amount of plastic deformation after exceeding the *yield stress.* Only ductile materials can yield significantly. Presence of cracks in a ductile material can cause sudden fracture at stress levels below the yield strength, even under static loads.

3. *Fracture* Components made of brittle materials fail due to sudden fracture, without any plastic deformation or significant change in shape. In such cases, *ultimate strength* of the material is of primary importance.

5.1.2 Factor of Safety

Design of a machine component basically involves selection of suitable material having desired properties and dimensions, making them capable to bear loads. For this, strength of the member should be higher than the effective load. To ensure sufficient reserve strength in the components, the design process usually considers a *factor of safety* (N), defined as

$$N = \frac{\text{Failure stress}}{\text{Allowable stress}}$$

In general, failure stress for ductile materials is the tensile stress (σ_y) or shear stress (τ_y) at yield, while it is the ultimate stress (σ_u) for brittle materials.

The factor of safety is virtually determined by past practice, methods of stress analysis, and most importantly, the gravity of the consequences that failure of a part can entail. In aircraft engineering, where severe weight restrictions are imposed on the structures, the factor of safety is kept in the range 1.5–2. In the design of stationary engineering structures intended for prolonged service, the factor of safety is rather large, in the range 2–5.

5.1.3 Static Failure Theories

Theories of failures offer criteria for defining the safe stress to avoid failure of materials. Historically, several theories have been formulated to explain the failures with varying accuracy and applicability. It depends on the ductility or brittleness of the selected materials and type of stresses.

Mohr's circle of plane stresses shows that the maximum shear stress at yielding (τ_y) is equal to one-half of the tensile yield strength (σ_{yt}).

In the following subsections, a factor of safety 'N' is considered with respect to failure stress.

5.1.3.1 Maximum Normal Stress Theory The *maximum normal stress theory*, also known as *Rankine's theory*[1], predicts that the failure will occur whenever one of the two principal stresses become equal to or exceed the yield point stress. For a material having yield strengths, σ_{yt} and σ_{yc}, in tension and compression, respectively, the maximum normal stress theory gives the following values of the failure stresses in tension and compression, respectively:

$$\frac{\sigma_{yt}}{N} \geq \sigma_t$$
$$\frac{\sigma_{yc}}{N} \geq \sigma_c$$

[1]Named after W. J. M. Rankine (1820-1872), an eminent engineering educator of England.

The theory can be depicted as a region of safety for a bi-axial stress system with the assumption $\sigma_{yt} = \sigma_{yc}$ [Fig. 5.1].

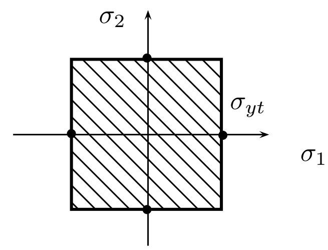


Figure 5.1 | Maximum normal stress theory.

Rankine's theory is suitable for tensile and compressive stresses, and is applicable mainly for brittle materials.

5.1.3.2 Maximum Shear Stress Theory The *maximum shear stress theory*, also known as *Tresca-Guest theory*[2], states that failure of an element occurs when the maximum shear stress exceeds shear stress in a tensile specimen at yield [Fig. 5.2].

Mohr's circle shows that for a material element subjected to tri-axial stress system, maximum shear stress is half of the maximum difference in normal stresses in any two directions. Therefore, maximum shear stress theory offers the following criteria for failure:

$$\frac{\sigma_{yt}}{N} \geq \begin{array}{c} |\sigma_1 - \sigma_2| \\ |\sigma_2 - \sigma_3| \\ |\sigma_3 - \sigma_1| \end{array}$$

This criteria can be depicted as a region of safety for a bi-axial stress system with an assumption $\sigma_{yt} = \sigma_{yc}$ [Fig. 5.2].

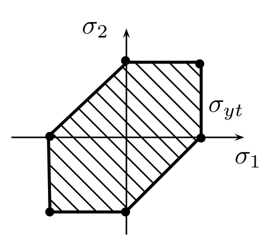


Figure 5.2 | Maximum shear stress theory.

Tresca-Guest theory is applied for shear stresses acting on a ductile material and is most suitable

[2]Maximum shear stress theory was originally stated by Coulombs (1736-1806) in 1773, later proposed by Tresca in 1864 and experimentally supported by J. J. Guest about 1900 in England.

for bending and torsion stresses, and also for fatigue loadings.

5.1.3.3 Maximum Strain Theory The *maximum strain theory*, often called *Saint Venant's theory*[3], predicts the failure of materials based on the strain at yield point. For a bi-axial stress system, the maximum strain theory offers the following criteria for failure:

$$\frac{1}{E} \times \frac{\sigma_y}{N} \geq \left(\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}\right)$$

where E and μ are the modulus of elasticity and Poisson's ratio for the material. Thus,

$$\frac{\sigma_y}{N} \geq (\sigma_1 - \mu\sigma_2)$$

This can be depicted as a region of safety for a bi-axial stress system with the assumption $\sigma_{yt} = \sigma_{yc}$ [Fig. 5.3].

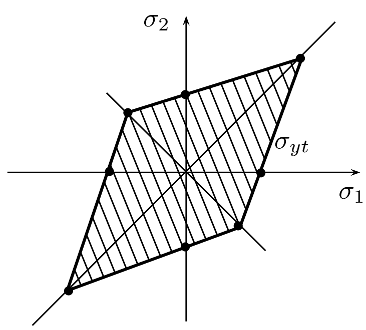


Figure 5.3 Maximum strain theory.

The theory is applicable for both tensile and compressive stresses. Since *Hooke's law* has been utilized, the maximum strain theory is limited to validity to the linear elastic range.

5.1.3.4 Maximum Strain Energy Theory The *maximum strain energy theory*, also known as *Beltrami-Haigh theory*[4], predicts that the failure occurs when the total strain energy per unit volume exceeds the total strain energy per unit volume of tensile specimen at yield.

For a material element subjected to a tri-axial stress system, strain energy per unit volume under the influence of principal stresses σ_i $(i = 1, 2, 3)$ is given by

$$U = \frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3) \tag{5.1}$$

where ε_i $(i = 1, 2, 3)$ are the strains in corresponding directions. Using Hooke's law and incorprating the Poisson's ratio,

$$\varepsilon_1 = \frac{\sigma_1 - \mu(\sigma_2 + \sigma_3)}{E}$$

and similar expressions for ε_2 and ε_3. Using Eq. (5.1), strain energy is written as

$$U = \frac{{\sigma_1}^2 + {\sigma_2}^2 + {\sigma_3}^2}{2E} - \frac{2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}{2E}$$

Strain energy of the material at yield is

$$U_f = \frac{1}{2}\frac{(\sigma_{yt}/N)^2}{E}$$

According to the maximum strain energy theory,

$$U_f \geq U$$
$$\left(\frac{\sigma_{yt}}{N}\right)^2 \geq {\sigma_1}^2 + {\sigma_2}^2 + {\sigma_3}^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

For a bi-axial stress system (i.e. $\sigma_3 = 0$):

$$\left(\frac{\sigma_{yt}}{N}\right)^2 = {\sigma_1}^2 + {\sigma_2}^2 - 2\mu(\sigma_1\sigma_2)$$
$$\frac{1}{N} \geq \left(\frac{\sigma_1}{\sigma_{yt}}\right)^2 + \left(\frac{\sigma_2}{\sigma_{yt}}\right)^2 - 2\mu\frac{\sigma_1}{\sigma_{yt}}\frac{\sigma_2}{\sigma_{yt}}$$

This is an expression of ellipse[5], having semi-major and semi-minor axes, $\sigma_{yt}/\left(N\sqrt{1-\mu}\right)$, $\sigma_{yt}/\left(N\sqrt{1+\mu}\right)$, presenting the safe region for design [Fig. 5.4].

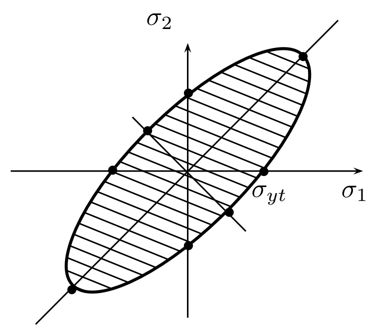


Figure 5.4 Maximum strain energy theory.

The maximum strain energy theory[6] utilizes *Hooke's law*, therefore, it is valid in linear elastic range only.

5.1.3.5 Maximum Distortion Energy Theory The *maximum distortion energy theory*, also known as *Huber-Mises-Hencky theory*[7], predicts that the failure occurs

[3]Work of Barre de Saint Venant (1767-1886), a great French mathematician.
[4]The theory was proposed by Beltrami in 1885.

[5]The ellipse having major and minor axes on x and y axes is given by

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

[6]This theory is in disagreement with experimental observations, so it is not used by designers, but is of historical significance in the development of the widely used distortion energy theory.
[7]The maximum distortion energy theory was proposed first by Huber (Poland, 1904) with later contributions by Von Mises (Germany, 1913) and Hencky.

when distortion energy in a volume reaches the distortion energy in the same volume of a tensile specimen at yield.

For material element subjected to a tri-axial stress system, the total strain energy per unit volume is given by

$$U = \frac{1}{2E}\left[{\sigma_1}^2 + {\sigma_2}^2 + {\sigma_3}^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

where σ_i $(i = 1, 2, 3)$ are the principle stresses, E is the modulus of elasticity, and μ is the Poisson's ratio of the material.

Strain energy in a component consists of two elements: the energy associated solely with change in volume, termed as *dilatation energy*, and the energy associated with the change in shape, termed as *distortion energy*. These are determined as follows:

1. *Dilatation Energy* To calculated the dilation energy, the material is assumed to be subjected to a uniform stress $\bar{\sigma}$, equal to the average of the three principle stresses:

   $$\bar{\sigma} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

   The uniform stress $\bar{\sigma}$ produces only volumetric changes while $\sigma_i - \bar{\sigma}$ are responsible for distortion of the material element.

   Therefore, dilatation energy is the strain energy contributed by the average normal stress:

   $$\begin{aligned}\bar{U} &= \frac{1}{2}\left[3\bar{\sigma}^2 - 2\mu \times 3\bar{\sigma}^2\right] \\ &= \frac{3(1-2\mu)\,\bar{\sigma}^2}{2E}\end{aligned}$$

2. *Distortion Energy* Distortion energy is determined using its definition:

   $$\begin{aligned}U_d &= U - \bar{U} \\ &= \frac{1+\mu}{6E}\left\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right\}\end{aligned} \tag{5.2}$$

   Using this, the distortion energy for the tensile specimen at yield is determined by taking $\sigma_1 = \sigma_{yt}$ and $\sigma_2 = \sigma_3 = 0$ and factor of safety N:

   $$U_d = \frac{1+\mu}{3E}\left(\frac{\sigma_{yt}}{N}\right)^2 \tag{5.3}$$

   The failure, specially in ductile materials, is related only to the distortion energy, with no contribution from the dilatation energy.

   Using Eqs. (5.2) and (5.3), the maximum distortion energy theory is expressed as

   $$\left(2\frac{\sigma_{yt}}{N}\right)^2 \geq (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \tag{5.4}$$

   The value of stress on the right hand of the above equation is called *von Mises effective stress*. The equation represents a sphere. For a bi-axial state of stress, it is a circle [Fig. 5.5].

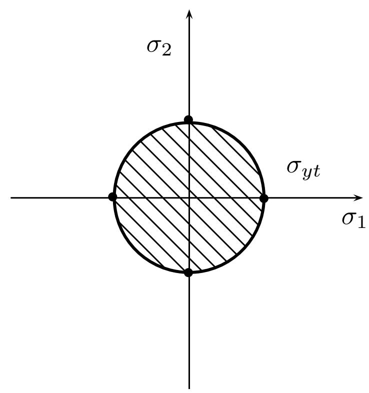


Figure 5.5 Maximum distortion energy theory.

For a general case with shear stresses,

$$\begin{aligned}2{\sigma_{yt}}^2 \geq\; &(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \\ &+ 6\left({\sigma_{12}}^2 + {\sigma_{23}}^2 + {\sigma_{31}}^2\right)\end{aligned}$$

Based on this expression, the following conclusions can be drawn:

1. Principal planes $(\sigma_{12} = \sigma_{23} = \sigma_{31} = 0)$:

   $$2{\sigma_{yt}}^2 \geq (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

2. Plane stresses $(\sigma_{33} = \sigma_{23} = \sigma_{31} = 0)$:

   $$\sigma_{yt} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 + 3{\sigma_{12}}^2}$$

3. Pure shear ($\sigma_{12} \neq 0$, all other stresses zero):

   $$\sigma_{yt} = \sqrt{3}\sigma_{12}$$

Of all the theories dealing with ductile behavior, the distortion energy theory agrees best with experimental data. It is the most suitable for steel elements subjected to tensile and shear stresses. However, it is invalid for uniform stress state $(\sigma_1 = \sigma_2 = \sigma_3)$ because then the material is not subjected to only volumetric changes.

The theories of failure for ductile materials are summarized in Table 5.1 and Fig. 5.6.

5.1.3.6 Coulomb-Mohr Theory

Brittle materials fail by fracture instead of yielding. Since fracture in tension is considered due to normal tensile stress alone, Rankine's theory of maximum normal stress is applicable in this case.

Table 5.1 Theories of failure

Failure criteria	Safe region	Researcher
Normal Stress	Squar	Rankine
Shear Stress	Hexagonal	Tresca–Guest
Strain	Rhomboid	St. Venant
Strain Energy	Ellipse	Beltrami–Haigh
Distortion Energy	Circle	Von Mises

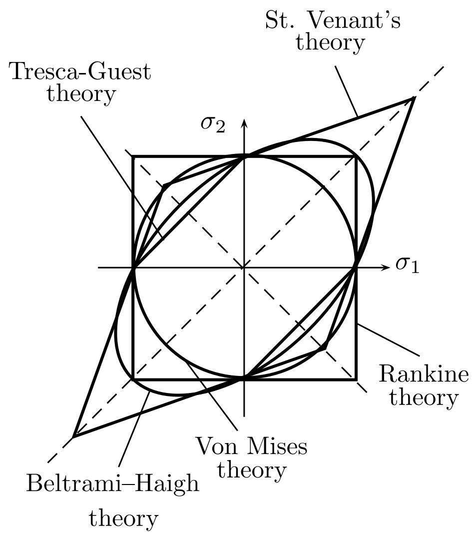


Figure 5.6 Comparison of static failure theories.

Brittle fracture in compression is caused by a combination of normal compressive stress and shear stress, therefore, it requires a special theory of failure. The criteria of failure prescribed by the *Coulomb-Mohr theory* for two-dimensional case is shown in Fig. 5.7. The safe region for a brittle material subjected to compressive stress, as predicted by the Coulomb-Mohr theory, is a line having a slope σ_{ut}/σ_{uc}, where σ_{ut} and σ_{uc} are the ultimate strength in tensile and compression, respectively. Therefore, incorporating a factor of safety N, the safe region for a material, subjected to tensile stress σ_t and compressive stress σ_c in orthogonal directions, can be represented by the following equation:

$$\frac{\sigma_t}{\sigma_{ut}} + \frac{\sigma_c}{\sigma_{uc}} = \frac{1}{N}$$

This is the basic expression of Coulomb-Mohr theory.

5.2 DYNAMIC LOADING

Machine parts subjected to repeated stresses fail below the ultimate or yield stress. Therefore, static failure the-

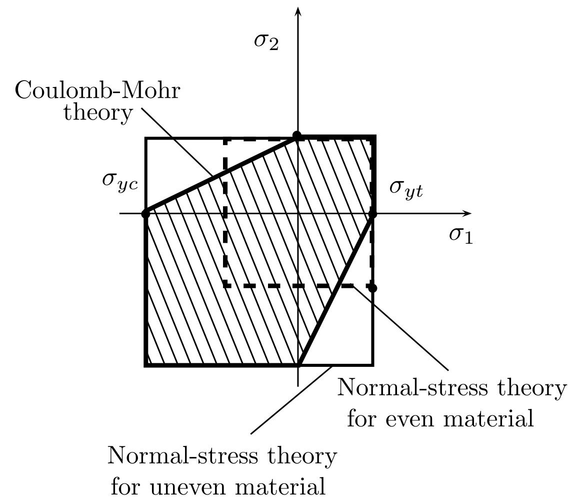


Figure 5.7 Coulomb-Mohr theory.

ories can lead to unsafe design of components subjected to dynamic loadings.

5.2.1 Fatigue Failure

Machine members subjected to repeated stresses are found to have failed even when the actual maximum stresses were below the ultimate strength of the material, and quite frequently at stress values even below the yield strength. The most distinguishing feature is that the failure had occurred only after the stresses have been repeated a very large number of times. Hence, such type of failures is called *fatigue failure*[8].

A fatigue failure begins with a small initial crack which usually develops at a point of local discontinuity in the material, such as a change in cross-section, a key-way, notch, cracks, hole or other area of stress concentration. Once a crack is initiated, the stress concentration comes into effect and the crack progresses more rapidly. As the stressed area decreases in size, the stress increase in magnitude until, finally, the remaining area is unable to sustain the load, and the component fails suddenly.

A fatigue failure, therefore, is characterized by two distinct regions: the first is due to progressive development of the crack, while the second is due to the sudden fracture. The zone of sudden fracture is very similar in appearance to the fracture of a brittle material, such as cast iron failed in tension.

[8]The term 'fatigue' was first applied to such situations by Poncelet in 1839. August Wohler made the first scientific investigation into fatigue failure in 1870 when he identified the endurance limit for steels. R. R. Moore adapted the simply supported rotating beam in fully reversed pure bending.

5.2.2 Stress Cycle

Machine elements, such as rotating shaft, are often subjected to sinusoidal dynamic loads. Figure 5.8 shows a rotating shaft subjected to lateral force which results into variable bending stresses. The resulting stress cycle is also shown where the maximum, minimum, variable and average stresses are indicated.

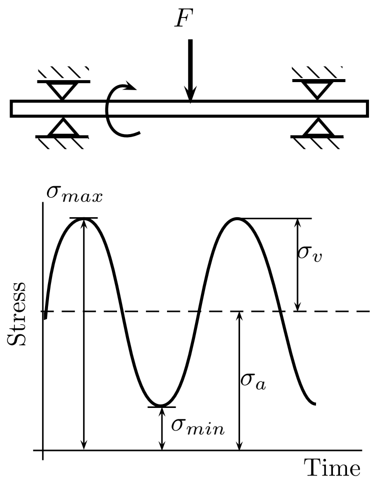


Figure 5.8 Cyclic stresses on a rotating shaft.

Following are the characteristics of a stress cycle:

1. Variable Stress

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

2. Average Stress

$$\sigma_a = \frac{\sigma_{max} + \sigma_{min}}{2}$$

These are used in the criteria of fatigue failures.

5.2.3 Stress Concentration

Stress concentration severely affects the fatigue strength of machine parts. Therefore, it is essentially considered in design of machine parts subjected to fatigue loading by defining *stress concentration factor* (k_f) as the ratio of endurance limit of a notch free specimen to endurance limit of a notched specimen [Fig. 5.9].

Stress concentration cannot be completely eliminated, but can be reduced in intensity by suitably providing fillets, reducing shank area in threaded fasteners, and by other means.

In static loading of ductile materials, the increase in stress at the stress raiser causes the local yielding of components resulting in equal distribution of stresses. The effect of stress concentration is critical in dynamic loading. Hence, the *stress concentration factor* is used for dynamic loading only.

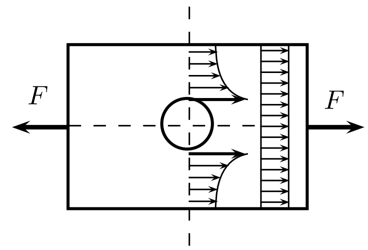


Figure 5.9 Stress concentration.

5.2.4 Notch Sensitivity

Notch sensitivity (q) for fatigue loading is defined in terms of actual stress concentration factor k_f and the theoretical stress concentration factor k_t by the following expression:

$$q = \frac{k_f - 1}{k_t - 1}$$

The value of q is different for different materials and this normally lies between 0 to 0.7. It is small for ductile materials and increases with decrease the ductility.

5.2.5 Hysteresis of Stress

Hysteresis is the incomplete recovery of strain in a material that is subjected to a stress cycle during its unloading due to energy consumption. The unloading path is not followed exactly at the time of reloading and a small loop is left between the unloading and reloading curves. This represents an energy loss, termed as *hysteresis loss* and the loop is known as *hysteresis loop* [Fig. 5.10]. Fatigue failure can also be described in terms of hysteresis loop as the repeatedly applied stress causes hysteresis loss upto a limit to change its basic properties from ductile to brittle.

5.2.6 Endurance Limit

Failure of a material under different loads takes place after some cycles of reversal. Performance of materials in high-cycle fatigue situations is commonly characterized by a curve giving the relationship between reversible stress (σ_v) and number of cycles (N) of reversals for failure. This curve is known as *endurance curve*. The curve is also known as *Woehler curve* or *S–N diagram* [Fig. 5.11].

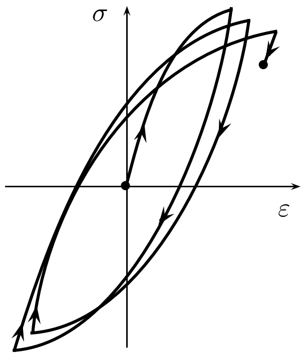


Figure 5.10 | Hysteresis of cyclic stress.

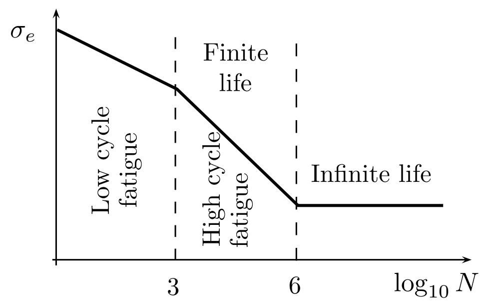


Figure 5.11 | Endurance curve.

The stress at which the material of the shaft completes a million (10^6) number of cycles of reversals before failure is called *endurance limit* of the material. The portion of the curve upto 10^3 cycles is known as *low cycle fatigue* where the curve is approximately a straight line. Beyond this limit is the *high cycle fatigue*.

Larger specimens have a higher probability of containing a more serious stress concentration nuclei because of greater volume. Therefore, larger machine parts exhibit poorer fatigue strength than smaller parts.

5.2.7 Design for Finite Life

A machine element can be designed for a definite life. This involves calculation of endurance strength (σ_e) of the material for desired finite life cycles (N) using the plot $\sigma - \log_{10}N$ with simple linear extension [Fig. 5.11].

5.2.8 Cumulative Damage Criteria

Design of a component for a finite life can have a different situation when a machine part is subjected to different stresses for different proportions of its fatigue life. In such cases, it is essential to know the balance life of the component for the design stress based on the past history of fatigue loading.

According to *Miner's rule*, also known as *cumulative damage criteria*[9], if there are n stresses of lives L_i's with variable loadings (composed of average and variable stresses), and N_i are the actual number of cycles with those stress components, respectively, then

$$\sum_{i=1}^{n} \frac{N_i}{L_i} = C \tag{5.5}$$

where C is experimentally found to be between 0.7 and 2.2. Usually for design purposes, C is assumed to be 1.

Miner's rule can be thought of as assessing what proportion of life is consumed by stress reversal at each magnitude.

5.2.9 Fatigue Failure Criterion

Fluctuating stresses in a component are characterized by the mean stress and the stress amplitude [Fig. 5.8]. Presence of mean stress shifts the $\sigma_e - N$ curve downward [Fig. 5.11]. Thus, a machine part under a higher mean stress will have a shorter fatigue life for desired stress level.

In fatigue diagrams, the mean stress (σ_a) is plotted on the abscissa and stress amplitude (σ_v) is plotted on the ordinate [Fig. 5.12]. When σ_v is zero, the load is purely static and criterion of failure is the ultimate strength (σ_{ut}) for brittle materials or yield strength (σ_{yt}) for ductile materials. Opposite to this, when mean stress is zero, the stress is completely reversing, the criterion of failure is the endurance limit (σ_e).

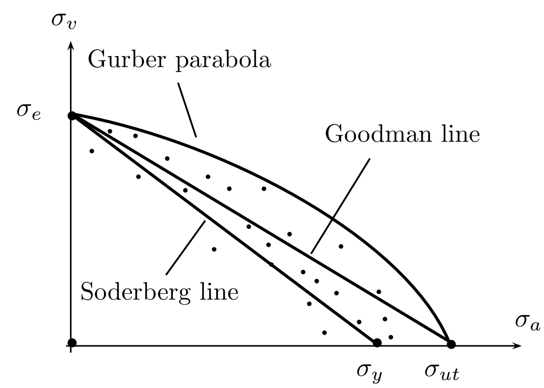


Figure 5.12 | Fatigue failure criteria.

When the component is subjected to both σ_a and σ_v, the actual failure occurs at different scattered points. There exists a border, which divides safe region from unsafe region for various combinations of σ_a and σ_v.

[9]First it was proposed by A. Palmgren in 1924, and popularized in 1945 by M. A. Miner. The rule is generally called Miner's rule or the Palmgren-Miner linear damage hypothesis.

Important criterion proposed to construct the borderline are explained as follows [Fig. 5.12]:

1. *Gerber Parabola* A parabolic curve joining σ_e on the ordinate and σ_{ut} on the abscissa is called the *Gerber parabola* of fatigue failure. This criterion fits the failure points of experimental data in the best manner. The following equation presents this failure criterion:

$$\left(\frac{\sigma_a}{\sigma_{ut}}\right)^2 + \frac{\sigma_v}{\sigma_e} = 1 \tag{5.6}$$

To consider a factor of safety N, the above equation is modified as

$$\left(N\frac{\sigma_a}{\sigma_{ut}}\right)^2 + N\frac{\sigma_v}{\sigma_e} = 1 \tag{5.7}$$

2. *Soderberg Line* A straight line joining σ_e on the ordinate and σ_y on the abscissa is called the *Soderberg line* of fatigue failure. Following is the equation for this failure criterion:

$$\frac{\sigma_a}{\sigma_y} + \frac{\sigma_v}{\sigma_e} = 1 \tag{5.8}$$

To consider a factor of safety N, the equation takes the following form:

$$\frac{\sigma_a}{\sigma_y} + \frac{\sigma_v}{\sigma_e} = \frac{1}{N} \tag{5.9}$$

This indicates that the factor of safety shifts the criterion's line towards origin.

3. *Goodman Line* A straight line joining σ_e on the ordinate and σ_{ut} on the abscissa is called the *Goodman line* of fatigue failure. This is represented by the following equation:

$$\frac{\sigma_a}{\sigma_{ut}} + \frac{\sigma_v}{\sigma_e} = 1 \tag{5.10}$$

If a factor of safety N is used, then

$$\frac{\sigma_a}{\sigma_{ut}} + \frac{\sigma_v}{\sigma_e} = \frac{1}{N} \tag{5.11}$$

Goodman line is a safer option for design consideration because it is completely inside the Gerber parabola. Soderberg line is a more conservative failure criterion.

Figure 5.13 shows the modified criteria of Goodman lines for fluctuating axial or bending stresses (σ) and fluctuating torsional shear stresses (τ).

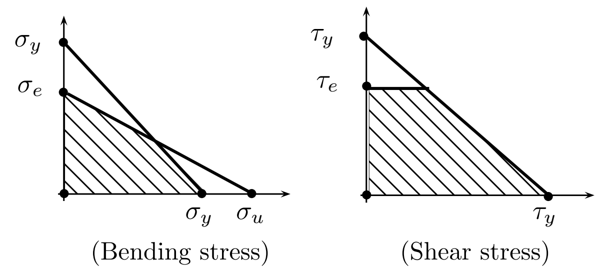


Figure 5.13 | Modified Goodman lines.

5.3 RIVETED JOINTS

5.3.1 Rivets

A *rivet* is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called *shank* and lower portion of shank is called *tail*. Rivets are used to make permanent fastening between the plates in structural work, ship building, bridges, tanks, boiler shells, etc. For making a riveted joint, a hole has to be drilled in the plates to be connected. This reduces the tearing strength of the plates.

The joints produced by riveting are not water tight, their fatigue strength is poor, and protruding rivet heads are undesirable in various applications.

Mild steel and wrought iron are commonly used materials for producing rivet. Copper and aluminium rivets are used where corrosion resistance or light weight is required.

5.3.2 Caulking and Fullering

To achieve complete tightness, the joints are caulked or fullered [Fig. 5.14]. In *caulking*, the edge of one plate is pressed tightly against the other plate by means of a blunt chisel like tool called caulking tool.

Fullering is similar to caulking except that fullering makes use of tool having thickness at the end equal to that of the plate. There is less risk of the plate being damaged in fullering.

5.3.3 Terminology

Following terms are used in design of riveted joints:

1. *Pitch* Pitch is the distance between center of one rivet to that of the next adjacent rivet in the same row.

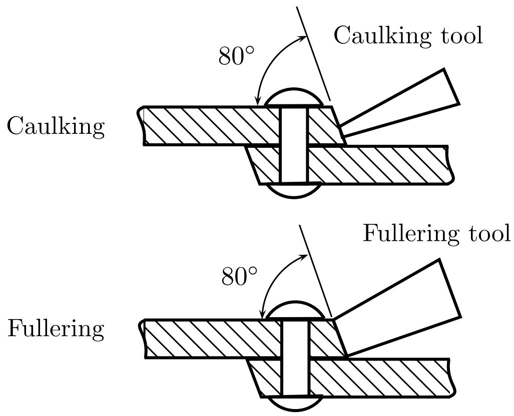


Figure 5.14 | Caulking and fullering.

2. Margin Margin or marginal distance is the distance between the edges of the plate to the center line of the adjacent row of rivets.
3. Diagonal pitch Diagonal pitch is the distance between the centers of rivets to the center of the next rivet in the adjacent row measured diagonally.
4. Row Pitch Row pitch is the distance between two adjacent rows of rivets.

5.3.4 Failure of Rivet Joints

A rivet joint can fail in three ways: tearing of plate, shearing of rivets, and crushing of rivets. Riveted joints are designed against these modes of failure, discussed as follows:

1. *Tearing of Plate* The plates can tear along the line of minimum cross-section, that is, along the line through the center of holes [Fig. 5.15]. Only a pitch length (p) of the joint is considered since every rivet is responsible for that much length of the plate only.

   If σ_t is the permissible tensile stress for the plate material, and t is the thickness of the plate, d is the diameter of the hole in the plate, then tearing resistance or the pull required to tear off the rivet per pitch length is given by

$$F = (p - d)\, t\sigma_t$$

2. *Shearing of Rivets* Rivets can fail in shear at the plane where two plates meet together. There can be single or double shear [Fig. 5.16].

   If τ is the shear strength of rivet material, then shearing strength of a rivet under single shear having shank diameter d is given by

$$F = \frac{\pi}{4} d^2 \tau$$

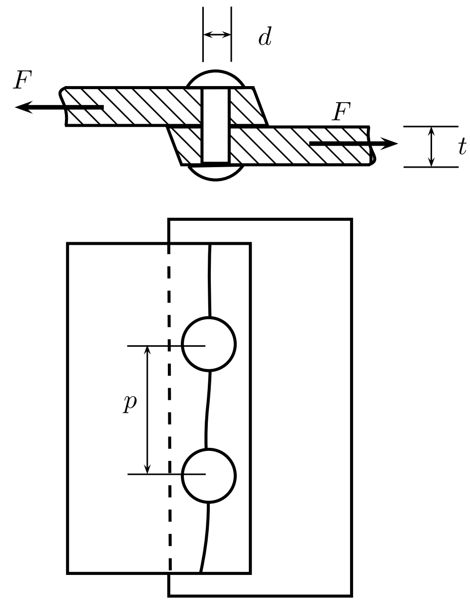


Figure 5.15 | Tearing of plate.

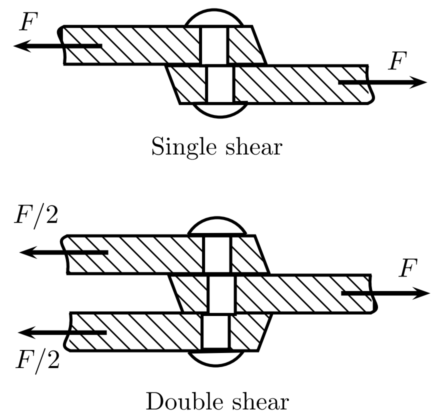


Figure 5.16 | Shearing of rivets.

   For double shear,

$$F = 2\frac{\pi}{4} d^2 \tau$$

3. *Crushing of Rivets* The rivets can fail under crushing in which the rivet hole becomes oval and hence the joint becomes loose. This type of failure is also known as *bearing failure*. It is a special type of compressive stress which occurs at the interface of two surfaces. The distribution of the stress depends on the shape of the surface.

   If σ_c is the crushing stress of rivet material, then the crushing strength of rivet joint having n number rivets in one row is given by

$$F = n \times dt\sigma_c$$

In practice, diameter of rivet hole for plate thickness greater than 8 mm is taken as

$$d = 6.04\sqrt{t}$$

This is known as *Unwin's formula*.

The riveted joints fail wherever the stress exceeds the corresponding strength of any mode of failure. To quantify the strength a rivet joint, *efficiency of a riveted joint* is defined as the ratio of the minimum strength of rivet joint and the strength of solid plate.

5.4 WELDED JOINTS

Welded joints are permanent in nature; the component parts once joined together cannot be dissembled without breaking the welds. These are employed as a substitute for riveted joints, and alternative for casting or forging. Welded joints are produced by a manufacturing process known as *welding*[10] and the machine part assembled by welding are known as *weldments*.

Welded assemblies are tight and leak proof as compared with riveted assemblies. Welded joints avoid stress concentration due to holes in case of riveted joints and eliminate the excess weight if joint is by other fastening means. Welding is often a cost-effective method of fabrication.

5.4.1 Types of Welds

Welded joints are classified according to the relative position of the mating components [Fig. 5.17]:

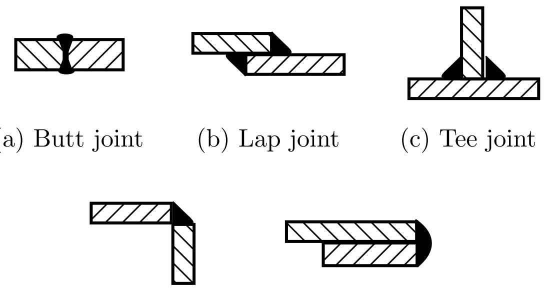
(a) Butt Joint A *butt joint* is formed by lying the two components in the same plane and mating the edges to be welded. For plates thicker than 6 mm, the edges of the plates are to be beveled on single side, while for thickness more than 20 mm, both sides of edges are beveled to form double beveled butt joints.

(b) Lap Joint Two overlapping components form a *lap joint*. A *fillet weld* consists of an approximately triangular cross-section joining two surfaces at right angle to each other.

(c) Tee Joint When end face of one component is welded to a side of the other component, the joint formed is called *tee joint*. The weld can be made on single side or both sides.

(d) Corner Joint A *corner joint* consists of an approximately triangular cross-section, joining two surfaces at right angle to each other.

(e) Edge Joint An *edge joint* is formed by mating the parallel edges of the mating components. Edge joints are not recommended for plates thicker than 6 mm.



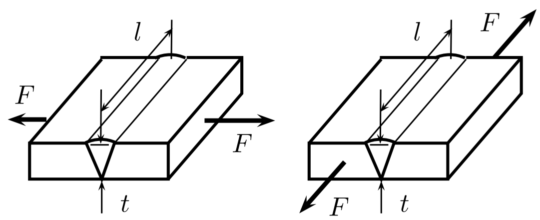
(a) Butt joint (b) Lap joint (c) Tee joint

(d) Corner joint (d) Edge joint

Figure 5.17 Types of welds.

5.4.2 Stress in Butt Welds

Throat is the minimum height of cross-section of the weld which does not include the bulge or reinforcement provided to compensate for flaws in the weld. Consider a simple butt weld joint of *throat* t and length l, subjected to a force F in two situations [Fig. 5.18].



(Tensile force) (Shear force)

Figure 5.18 Loads on a butt weld.

A butt weld can be subjected to two types of stresses:

1. Tensile Stress If the weld is subjected a tensile force F across the weld length, the average tensile stress in the weld is given by

$$\sigma = \frac{F}{tl}$$

2. Shear Stress If the same butt weld is subjected to shearing force F in opposite direction along the weld length [Fig. 5.18], then the average shear stress in the weld is given by

$$\tau = \frac{F}{tl}$$

[10]Various techniques of welding are discussed separately in the Chapter 13.

The design of butt weld should meet both the criteria of failure.

5.4.3 Stress in Fillet Welds

Consider a simple fillet weld joining two plates of thickness t (also called *leg*) and length l, subjected to tensile force F [Fig. 5.19].

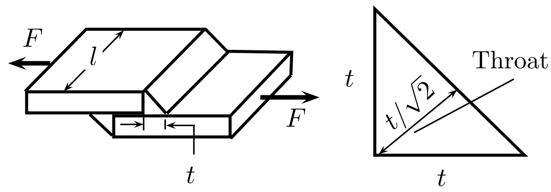


Figure 5.19 | Fillet weld.

The failure of fillet welds is usually due to shear at the throat area of the weld for any direction of applied load. In this case, the throat is equal to $t/\sqrt{2}$. Therefore, the average shear stress in the single fillet weld is given by

$$\tau = \frac{F}{(t/\sqrt{2})\, l} = \frac{\sqrt{2}F}{tl}$$

For double fillet weld joint on both sides of plates, the average shear stress in the single fillet weld is given by

$$\tau = \frac{\sqrt{2}F}{2tl} = \frac{F}{\sqrt{2}tl}$$

5.4.4 Eccentrically Loaded Welds

Consider a bracket attached to the support by means of two welds (leg t, total length l). It is subjected to an eccentric force F at eccentricity e from the center of gravity G of welds [Fig. 5.20].

The eccentric force F can be replaced by an equal and similarly directed force F acting through the center of gravity G along with a couple M $(= F \cdot e)$. This results into two elements of stresses in the welds:

1. *Primary Shear Stress* The force F results direct shear stress (τ_1), known as *primary shear stress*, assumed to be uniformly distributed over the throat area of all the welds:

$$\tau_1 = \frac{F}{(t/\sqrt{2}) \times l}$$

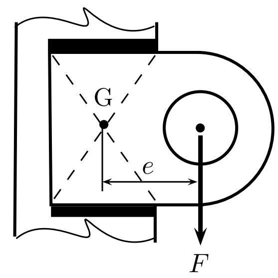


Figure 5.20 | Eccentrically loaded welds.

2. *Secondary Shear Stress* The couple M causes torsional shear stresses, known as *secondary shear stresses*, in the throat area of the welds. For a point at distance r from G, secondary shear stress (τ_2) is given by

$$\tau_2 = \frac{Mr}{J}$$

where J is the polar moment of inertia of all the weld lengths about the point G.

The secondary shear stress at any point is proportional to its distance from the center of gravity(G) and it is maximum at the farthest point from G.

The resulting stress at any point can be obtained by vector sum of the forces caused by the two stresses:

$$\tau = \tau_1 + \tau_2$$

The maximum value of τ is considered in the design.

5.4.5 Bending Moment on Welds

Consider a cantilever beam of rectangular cross-section welded to a support by means of two fillet welds w_1 and w_2 (leg t, total length l) at the top and bottom edges [Fig. 5.21].

The applied eccentric force F can be replaced by an equal and similarly directed force F acting through the plane of welds along with a bending moment of couple M $(= p \cdot e)$. This results into two elements of stresses in the welds:

1. *Shear Stress* The force F results in direct shear stress (τ_1) assumed to be uniformly distributed over the throat area of all the welds:

$$\tau_1 = \frac{F}{(t/\sqrt{2})\, l}$$

2. *Bending Stress* The bending moment M causes bending stress (σ_b) at a point located at distance

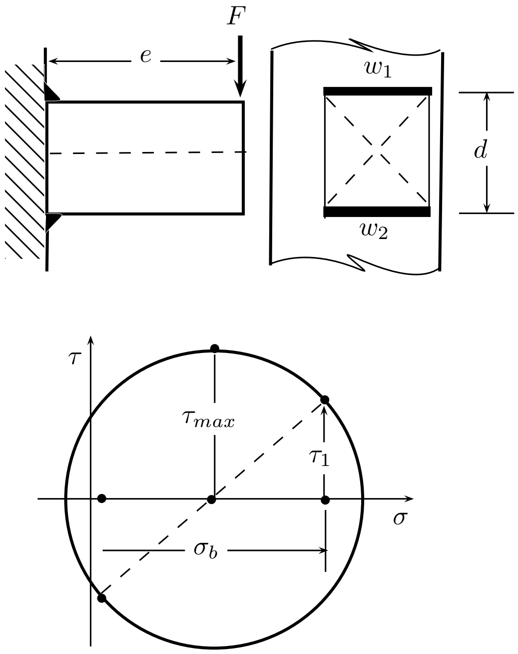


Figure 5.21 Bending moments on welds.

y from neutral axis of the all welds. Using flexure formula,

$$\sigma_b = \frac{My}{I}$$

where I is the moment of inertia of all welds based on the throat area.

Using the Mohr's circle [Fig. 5.21], the resulting maximum shear stress at any point is found as

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + {\tau_1}^2}$$

This shear stress should not exceed the shear strength of the weld.

5.5 THREADED JOINTS

A *threaded joint* is a temporary joint designed to hold two or more parts together by means of threading. The *thread* is produced by machining or forming a helical groove on the surface of a cylindrical body and mating hole.

The largest diameter of the screw thread is called the *major diameter* or *nominal diameter* and it is used to express the size of the thread. The smallest diameter of screw thread is called the *minor* or *root diameter*. The cross-section at the root diameter is the weakest section.

5.5.1 Types of Screw Fastening

Common types of screw fastening are as follows:

1. *Bolt and Nut* A bolted joint is used in assemblies where the components have relatively small thickness and where space is not enough to accommodate the bolt head, the nut and the wrench.

   There are two methods for increasing the shock absorbing capacity of bolts:

   (a) Reduce the shank diameter to core diameter of threads or even less.
   (b) Increase the length of shank portion of the bolt.

   The threaded portion of the bolt is the weakest part and maximum elastic energy is absorbed in this region. In a *bolt of uniform strength*, the entire bolt is stressed to the same limiting value, resulting in maximum energy absorption.

2. *Cap screw* A screw is mated into a threaded hole in one of the components being assembled, not into the nut. The screw joint is used in assemblies where one of the components is thick enough to accommodate the threaded length of the screw.

3. *Studs* *Studs* are cylindrical rods threaded at both ends. One end of the stud is screwed into one of the connecting components, while the nut is tightened at the other end. Studs are particularly suitable for positioning the covers of a cylinder head.

4. *Machine Screws* Machine screws are small cap screws with various types of heads, most of which have slots for a screw driver. Machine screws are used in assembly of small machines.

5. *Set Screws* Set screws are used to prevent relative motion between the two parts. The threaded portion of the screw passes through a tapped hole in one of the parts and the point of the screw presses against the other piece, and thus holding two parts together by friction. The points of set screws are hardened to combat early wear.

5.5.2 ISO Thread Designation

The designation of ISO metric threads offer two types of pitches: coarse and fine. The *coarse pitch* is the default pitch for a given diameter. *Fine pitches* are defined for use in applications where the height of the normal coarse pitch would be unsuitable, such as threads in thin-walled pipes. Here, the terms "coarse" or "fine" are not meant for quality of the thread.

A screw thread with a fine pitch is specified by the letter M followed by the value of the nominal diameter

and the pitch. For example, M20 × 2 indicates a screw thread with a 20 mm nominal diameter and a 2 mm pitch. If the pitch is coarse, the value of pitch is omitted and the screw thread is specified as M20.

5.5.3 Stress Analysis of Bolt

In simple case, the cross-section at the *root diameter* (d) is the weakest section. Tensile strength of the bolt at this cross-section is given by

$$F = \frac{\pi d^2}{4} \times \frac{\sigma_{yt}}{N}$$

where σ_{yt} is the tensile yield point of bolt material and N is the factor of safety.

Threads of the bolt in contact with the nut are sheared at the core diameter. Therefore, the bolt should have a minimum shear strength at root diameter:

$$F = \frac{\pi d^2}{4} \times \frac{\tau_y}{N}$$

5.5.4 Eccentric Load on Bolted Joints

Figure 5.22 shows a bolt subjected to eccentric load F having an eccentricity e w.r.t. the center of gravity (G) of the bolts. The eccentric force F can be replaced by an equal and similarly directed force F acting through the plane of joint along with a bending moment of couple M ($= F.e$). This results in two elements of stresses in the welds:

1. *Primary Shear Stress* The force F results in direct shear stress (τ_1) assumed to be uniformly distributed over the area of all the bolts. For n bolts of root diameter d,

$$F = n\frac{\pi d^2}{4}\tau_1$$

2. *Secondary Shear Stress* The bending moment M causes shear stresses, known as secondary shear stresses, such that their moments about G is equal to M [Fig. 5.23]. For bolt at distance r from G, secondary shear stress (τ_2) is given by

$$\tau_2 = \frac{Mr}{J}$$

where J is the polar moment of inertia of root area of all the bolts about the point G. Therefore, the secondary shear stress at any point is proportional to its distance from the center of gravity. Obviously, it is maximum at the farthest point.

The maximum value of shear stress can be determined by adding the vectors of forces at the most stressed bolt.

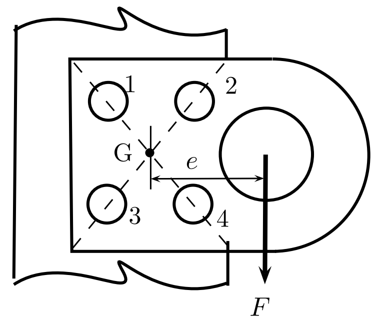


Figure 5.22 | Eccentric load on bolted joint.

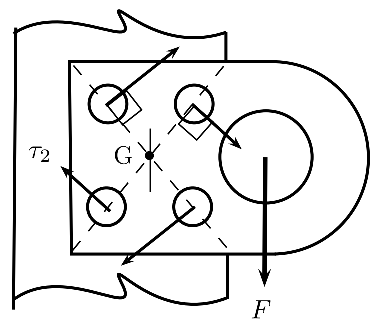


Figure 5.23 | Secondary shear stress.

5.5.5 Elastic Analysis of Bolted Joints

Consider an assembly of two components formed by a single bolt joint [Fig. 5.24]. This study is based on the assumption that the stresses are within the linear elastic limit.

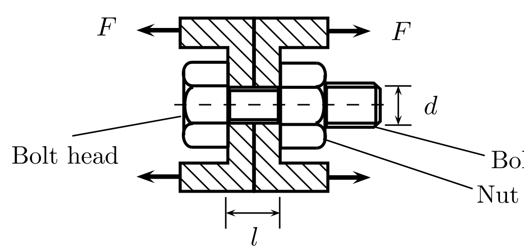


Figure 5.24 | Elastic analysis of bolted joints.

5.5.5.1 Pre-Load

Initially tightening the bolt (by means of spanner) induces a pre-load F_i in the assembly, that has two effects [Fig. 5.25]:

1. *Tension in Bolt* The bolt is subjected to an initial tension or preload, F_i that causes an elongation of

the bolt δ_b given by

$$\delta_b = \frac{F_i}{k_b} \tag{5.12}$$

where k_b is the stiffness of bolt.

2. *Compression in Parts* The components of the assembly are subjected to compressive load F that causes a total deformation δ_c, given by

$$\delta_c = \frac{F_i}{k_c} \tag{5.13}$$

where k_c is the equivalent stiffness of the components (in series).

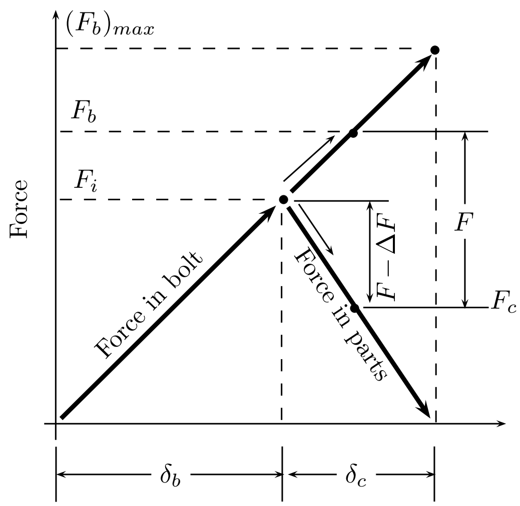


Figure 5.25 Elastic analysis of bolted joint.

5.5.5.2 Service Loads When the parts are clamped and put in service, the external load F has two effects:

1. *Increased Tension in the Bolt* The tension in the bolt increases ΔF which results in additional elongation $\Delta\delta$, given by

$$\Delta\delta = \frac{\Delta F}{k_b} \tag{5.14}$$

2. *Decreased Compression in the Parts* Additional elongation of the bolts relieves the compression on the two parts by corresponding reduction in load $F-\delta F$, given by

$$\Delta\delta = \frac{F-\Delta F}{k_c} \tag{5.15}$$

Eliminating $\Delta\delta$ from the above two equations,

$$\Delta F = F\frac{k_b}{k_b+k_c}$$

The force acting on the bolt is

$$F_b = F_i + \Delta F$$

Figure 5.25 shows that the limiting value $(F_b)_{max}$:

$$\frac{F_i}{\delta_b} = \frac{(F_b)_{max}}{\delta_b+\delta_c}$$

This occurs when the compression of the two parts become zero; the joint will begin to open, since the parts can no longer expand to maintain the tightness. Using Eqs. (5.12) and (5.13), one gets

$$(F_b)_{max} = F_i\frac{k_b+k_c}{k_c}$$

In this equation, effective stiffness of the bolt and parts are constant. Thus, the limiting value of tension in bolt is directly proportional to the initial tension.

5.5.6 Bolt Tightening

In certain applications, such as where gaskets are used, the bolts require a minimum torque to ensure initial tightening to meet two objectives:

1. *Thread friction and Pre-load* The torque required to overcome the thread friction and induce a pre-load F_i can be taken as

$$T_1 = \frac{F_i d_m}{2}\left(\frac{\mu\sec\theta+\tan\alpha}{1-\mu\sec\theta\tan\alpha}\right)$$

where μ is the coefficient of friction, θ is half of the V angle, α is helix angle, and d_m is the mean diameter of the threads.

Equivalent coefficient of friction in the above expression is taken as

$$\begin{aligned}\tan\phi' &= \mu' \\ &= \frac{\mu}{\cos\theta} \\ &= \mu\sec\theta\end{aligned}$$

2. *Collar friction* The collar friction between the nut and the washer can be determined using uniform wear theory:

$$T_2 = \frac{\mu F_i}{2}\left(\frac{D_o+D_i}{2}\right)$$

where D_o is the diameter of an imaginary circle across the flats of the hexagonal nut and D_i is the diameter of the hole in the washer.

Total torque requirement is

$$T = T_1 + T_2$$

This can be now be reduced for ISO metric screw threads of nominal diameter d and the following properties:

$$\theta = 30^\circ$$
$$\alpha \approx 2.5^\circ$$
$$d_m \approx 0.9d$$
$$\frac{D_o + D_i}{2} \approx 1.4d$$

Taking $\mu = 0.15$,

$$T_1 \approx 0.098 F_i d$$
$$T_2 \approx 0.105 F_i d$$

Thus, the total torque requirement is

$$\begin{aligned} T &= T_1 + T_2 \\ &\approx 0.2 F_i d \end{aligned}$$

5.6 DESIGN OF SHAFTS

A *shaft* is a cylindrical component rotating about its axis, supported at bearings at two or more points longitudinally. Shafts are generally in circular section and transmit power from one rotating element to another through rotating elements, such as pulleys, gears, sprockets, and wheels. This is always accompanied by the transverse and axial loads on the shaft.

Length of the shaft should be kept minimum to minimize the stress and deflection. Stress raisers, like key-ways, should be located away from the section of large bending moments. Natural frequency of vibration should be at least more than 1.5 times of the forced frequency of rotating elements. Hollow shafts possess better stiffness mass ratio, and a higher natural frequency, but they are costly.

5.6.1 Design for Static Loads

Shafts are generally manufactured from ductile materials, therefore, principle shear stress theory can be used for shaft design which can sustain various combination of loads. Using Mohr's circle, shear strength of a ductile material is related to yield strength σ_y as

$$\tau_y = \frac{\sigma_y}{2}$$

5.6.1.1 Simple Bending Suppose a hollow shaft is to be designed for a bending moment M. Internal and external diameters of the shaft are d_i and d_o. Using flexure formula, the maximum bending strength of the shaft is determined as

$$\frac{\sigma_y}{N} = \frac{32M}{\pi d_o^3 (1 - k^4)}$$

where $k = d_i/d_o$ and N is the factor of safety. For solid shafts ($k = 0$):

$$\frac{\sigma_y}{N} = \frac{32M}{\pi d_o^3}$$

5.6.1.2 Simple Torsion Suppose a hollow shaft is to be designed for a torsional moment T. Internal and external diameters of the shaft are d_i and d_o. Using *torsion formula*, the maximum shear stress in the shaft is given by

$$\frac{\tau_y}{N} = \frac{16T}{\pi d_o^3 (1 - k^4)}$$

where $k = d_i/d_o$ and N is the factor of safety. For solid shafts ($k = 0$):

$$\frac{\tau_y}{N} = \frac{16T}{\pi d_o^3}$$

5.6.1.3 Combined Bending and Torsion Most transmission shaft supporting gears and pulleys are subjected to a combined load of bending and torsional moments. Suppose a hollow shaft is to be designed for a combined loads of bending moment M and torsional moment T. Internal and external diameters of the shaft are d_i and d_o. Using Mohr's circle for combined stresses, the combined loads can be replaced by an equivalent bending moment (M_e) or equivalent torsional moment (T_e), given by

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$
$$T_e = \sqrt{M^2 + T^2}$$

Using flexure formula and *torsion formula*, the maximum bending and shear stresses in the shaft are determined as

$$\frac{\sigma_y}{N} = \frac{32M}{\pi d_o^3 (1 - k^4)}$$
$$\frac{\tau_y}{N} = \frac{16T_e}{\pi d_o^3 (1 - k^4)}$$

where $k = d_i/d_o$. For solid shafts, $k = 0$.

The standard procedure for design of shafts against combined bending and torsion is specified in ASME code which takes the permissible shear stress τ_y as the minimum of $0.3 \times \sigma_{yt}$ and $0.18 \times \sigma_{ut}$. Key-ways are assumed to reduce these values by 25%. The code is based maximum shear stress theory and takes into account the shock and fatigue in operating conditions. Accordingly, the maximum shear stress in a shaft is given by

$$\frac{\tau_y}{N} = \frac{16}{\pi d^3}\sqrt{(k_b M)^2 + (k_t T)^2}$$

where k_b and k_t are *combined shock and fatigue factors* applied to bending and torque, respectively.

5.6.2 Design for Dynamic Loads

Using Soderberg criteria, the ASME code with the specified permissible shear stress, can also be used for design against fluctuating loads:

$$\frac{\tau}{N} = \frac{16}{\pi d^3}\sqrt{\left(M_a + k_b\frac{\sigma_y}{\sigma_e}M_v\right)^2 + \left(T_a + k_t\frac{\sigma_y}{\sigma_e}T_v\right)^2}$$

where the subscripts a and v indicates the average and variable components of bending moment and torque, and σ_y and σ_e are the yield and endurance strength of the shaft material.

5.7 DESIGN OF KEYS

A key is a machine element fitted in an axial direction into the mating grooves, called *key-ways*, cut in the shaft and the mating member, such as pulleys, gears. Primary function of the key is to prevent the relative rotation between the shaft and mating member. In most of the applications, the secondary function of the keys is to prevent relative movements in axial direction of the shaft.

The cutting of key-ways reduces the strength and rigidity of the shaft. Sometimes, setscrews are used to seat the key firmly in the key-way and to prevent axial movement of the component parts.

5.7.1 Types of Keys

The following are the common types of keys [Fig. 5.26]:

1. Square Key A square key is sunk half in the shaft and half in the hub of a gear, pulley or crank. In general, a square key has its sides equal to one-fourth of the shaft diameter.

2. Rectangular Key Rectangular or flat keys are used where the weakening of the shaft is serious and where added stability of the connection is desired, as in machine tools.

   Flat keys with uniform cross-section are called *feather keys*, otherwise a taper of 1:100 is provided on the thickness while the width is kept uniform. Feather key permits axial movement of the hub on the shaft.

   Tapered keys are often provided with *gib head* which facilitates their removal when the other end is inaccessible but gib-head is dangerous for workmen.

3. Saddle Key Saddle keys fit in the key-way of the hub only; there is no key-way on the shaft. Saddle keys are suitable for light service or in cases where relative motion between shaft and hub is required for adjustment and key way cannot be provided on the shaft. The power is transmitted by friction between that inner hollow cylindrical face of the key and shaft. Sometimes, flat saddle keys are used in which the inner surface of the key is left flat and the shaft is planed off to accommodate the key.

4. Woodruff Key A woodruff key[11] is an adjustable sunk key in the form of an almost semi-circular disk of uniform thickness. The key requires a circular type of key-way in the shaft. It can be used with tapered shaft because it aligns itself in the seat. The larger depth of key-way prevents tendency of the key to slip over the shaft but it weakens the shaft more than that by the use of straight keys.

5. Tangent Key Tangent keys are made in two parts, each tapered to ensure a tight fit. The face of the tangent keys is kept inclined to the tangent on the shaft, therefore, single set of tangent keys is suitable for heavy loads in one direction only. Double tangent keys are employed for reversible drives.

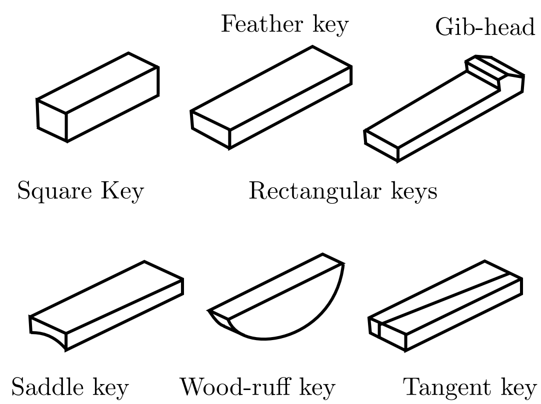


Figure 5.26 Types of keys.

5.7.2 Design of Square and Flat Keys

Consider a flat key of width w, thickness t, and length l. It is fitted on a shaft of diameter d in order to transmit a torque T during which two equal and opposite forces F are exerted by the shaft and the hub [Fig. 5.27].

[11]The key is named after the Woodruff Manufacturing Co. Hartford (USA), which first manufactured it in 1892.

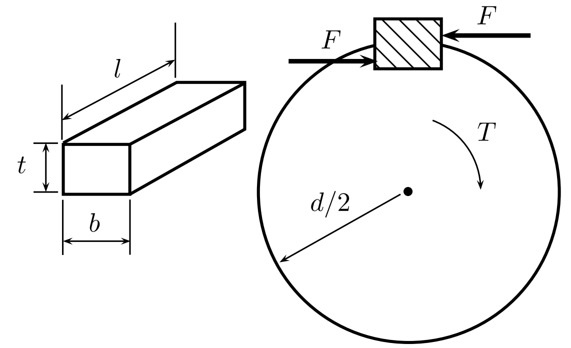


Figure 5.27 Design of flat keys.

The tangential force (F) is related to torque T on the shaft as

$$F = \frac{T}{d/2}$$

The design of key is based on two criteria:

1. *Shear Stress* The tangential force F produces a shear stress (τ) on the tangential area (bl) of the key:

$$\tau = \frac{F}{bl}$$

2. *Crushing Stress* The tangential force F produces a crushing stress (σ_c) on the area of a radial plane of the key:

$$\sigma_c = \frac{F}{l \times t/2}$$

The dimensions of the key should fulfill both the criteria.

5.8 DESIGN OF GEARS

Mechanical gears are used for power transmission at low speed and high torque. A gear is designed for specified power transmission (kW), speed (rpm), and speed ratio.

5.8.1 Standard Systems of Gear Teeth

Due to problems in manufacturing, cyloidal or composition teeth are obsolete and presently involute teeth are preferred under the following two systems:

1. 14.5° Full depth involute system
2. 20° Full depth involute system

The increase of pressure angle from 14.5° to 20° increases the load carrying capacity. It also increases the length of the contact and avoids possibility of interference.

5.8.2 Minimum Number of Teeth

Involute profile poses the difficulty of interference when the number of teeth is reduced below the minimum number of teeth. The interference can be overcome in three ways:

1. Using stub teeth of which height is less than the full depth teeth
2. Using composite profile with cyloidal curve at the root of the tooth
3. Increasing the center distance

To avoid interference in involute teeth, the limiting number of teeth on the pinion is given by

$$z_p = \frac{2a_w}{\sqrt{1+(1/G)\,(2+1/G)\sin^2\phi} - 1}$$

where ϕ is pressure angle, G is gear ratio, a_w is the ratio of addendum and module of the gear, which can be taken equal to 1. For unit gear ratio ($G = 1$) and ϕ =14.5°, 20°, 25°: $z_p \approx 22$, 13, 9, respectively.

Similarly, the limiting number of teeth on pinion to avoid interference in a rack and pinion is given by

$$z_p = \frac{2}{\sin^2\phi}$$

For $\phi = 20°$, $z_p = 14$.

5.8.3 Transmitted Load

The power is transmitted by means of a force exerted by the tooth of the driving gear on the meshing tooth of the driven gear. According to the *law of gearing*, the resultant force always acts along the pressure line. The tangential component of this force (F_t), known as *transmitted force*, is determined based on the torque and power to be transmitted as

$$F_t = \frac{\text{Design power}}{\text{Pitch circle velocity}}$$

The effect of the radial component, which induces compressive stress, is almost neglected. It is also assumed that at any time only one pair of teeth is in contact and takes the total load.

Two factors are considered in determining the transmitted force:

1. *Service Factor* In most of the applications, the torque developed by the source of power varies during the work cycle. Similarly, the torque required by the driven machine also varies. Although these variations are balanced by using flywheel,

but gears are to be designed on maximum torque. This is accounted by using a *service factor* (C_s) in the rated torque:

$$C_s = \frac{\text{Design torque}}{\text{Rated torque}}$$

For 24 hours intermittent operation, $C_s \approx 1.25$.

2. <u>*Service Factor*</u> To overcome the difficulty in calculating the exact magnitude of dynamic load, a *velocity factor* C_v, developed by Barth, is used.

Using the above factors, the effective tangential force (F_T) between two meshing teeth is found as

$$F_T = \frac{C_s F_t}{C_v}$$

Let the dimensional parameters of gear teeth be defined as: m as the module, b as the face width, P_c as the circular pitch ($= \pi m$).

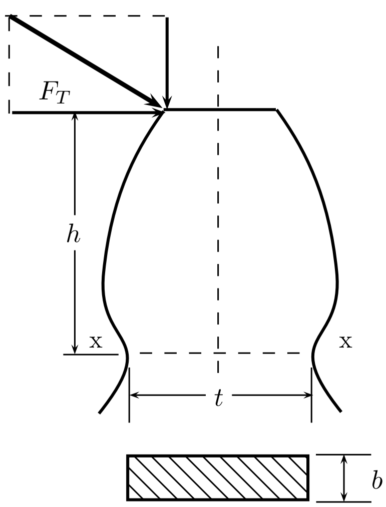


Figure 5.28 | Gear teeth as cantilever.

At the section of minimum cross-sectional area, as the force F_T acts at height h (tip of tooth) [Fig. 5.28]. The bending moment on the teeth is found as

$$M_b = F_T h$$

Section modulus of the teeth is

$$I = \frac{bt^3}{12}$$

The distance of the most stressed fiber from the natural axis of the section is

$$y = \frac{t}{2}$$

The maximum bending stress on the tooth is incorporated as

$$\begin{aligned}
\sigma_b &= \frac{F_T h t/2}{bt^3/12} \\
F_T &= b\sigma_b \frac{t^2}{6h} \\
&= mb\sigma_b \left(\frac{t^2}{6hm}\right) \\
&= mb\sigma_b Y
\end{aligned}$$

where Y is a dimensionless parameter known as the *Lewis form factor*:

$$Y = \frac{t^2}{6hm}$$

The quantity Y/π is called *form factor*.

5.8.4 Beam Strength

Beam strength of a gear tooth is the maximum tangential force that can be transmitted without bending failure. Considering the dynamic factor (C_v) and the stress concentration factor (k_f), the beam strength is determined as

$$F_t = \sigma_b m b Y \left(\frac{C_v}{k_f}\right)$$

This is known as the *Lewis equation*[12]. The face width can be assumed, approximately, $9-12$ mm. Thus, b and m are determined from this equation. The gear with minimum $\sigma_b Y$ is to be selected for design, and for the same materials, pinion is selected.

5.8.5 Dynamic Loading

Due to inaccuracy in cutting process of teeth, the load on teeth in contact suddenly increases. Buckingham equation is used to check the strength of gear for this type of dynamic loadings. The dynamic load (F_d) is assumed to consist of transmitted force (F_T) and increment load (F_i) due to various factors:

$$F_d = F_T + F_i$$

5.8.6 Check for Wear

Wear strength is provided by *surface endurance strength* (σ_{es}) for repeated loads. Its approximate value is taken as 1.75× BHN MPa for steel, and 0.5× BHN MPa

[12] In 1892, Wilfred Lewis published a paper titled, "The investigation of the strength of gear tooth", in which he considered gear tooth as a cantilever beam of uniform strength.

for cast iron. Wear strength of a gear tooth is simply determined by the *Buckingham equation*:

$$F_w = \frac{{\sigma_{es}}^2}{1.4} b D_p \sin\phi \left(\frac{1}{E_p} + \frac{1}{E_g}\right) \frac{2z_p}{z_g - z_p}$$

where D_p is the pitch circle diameter of pinion. Other symbols used in this expression are as follows: pinion (p), gear (g), surface hardness is BHN^{p,g}, allowable bearing stress $\sigma_b^{p,g}$, endurance limit for repeated loads $\sigma_e^{p,g}$, Young's modulus $E^{p,g}$, and ϕ is pressure angle. For safe design,

$$F_w \geq 1.5 F_d$$

where F_d is the dynamic load on a tooth.

5.9 ROLLING CONTACT BEARING

Rolling contact bearings are called *anti-friction bearings* because of very low value of the resultant coefficient of friction ($\mu \approx 0.001$). In roller bearings, the shape of the roller is cylindrical, spherical, and tapered cylinder. Balls are generally made of carbon chrome steel. All parts of the bearing: inner and outer races, and balls should be equally hardened.

5.9.1 Static Load Carrying Capacity

Static load carrying capacity (C_0) of a rolling contact bearing is defined as the maximum static load for which the combined permanent deformation of the rolling element and races at any point does not exceed 0.0001 times that of the rolling element diameter.

Based on Hertz contact stresses, *Stribeck's equation* defines the static load carrying capacity of a bearing with certain assumptions. For number of balls z and diameter d, *Striebeck's equation* is written as

$$C_0 = k\frac{d^2 z}{5}$$

where k is a constant of proportionality.

5.9.2 Dynamic Load Carrying Capacity

Dynamic load carrying capacity (C) of a rolling contact bearing is defined as the radial load in radial bearing (and axial load in axial bearing) which a bearing can endure for a minimum life of one million (10^6) revolutions.

5.9.3 Equivalent Dynamic Load

When a bearing is subjected to both a radial load F_r and axial load F_a, an *equivalent dynamic load* (P) is calculated for selection of bearing or calculation of bearing life. It is determined as

$$P = X(VF_r) + Y(F_a)$$

where V is the race rotation factor, and X and Y are the radial and thrust factors, respectively. Their values depend upon ratios F_a/F_r and F_a/C_0 (specified in manufacturer's catalogs).

5.9.4 Reliability

If a bearing has 90% of reliability, it means that 90 bearings will fail over 100 for given life. Life (L) for reliability (R) is given by

$$\frac{L}{L_{90}} = \left[\frac{\ln(1/R)}{\ln(1/R_{90})}\right]^{1/b}$$

where

$$b = 1.17$$
$$R_{90} = 0.90$$

5.9.5 Load Life Relation

Let N be the speed of rotation of the bearing in revolution per hour, L_h be the life of bearing in hours. Therefore, life of bearing in million ($= 10^6$) revolutions is given by

$$L = \frac{60NL_h}{10^6}$$

For the dynamic load carrying capacity C, the equivalent dynamic load P is related to life of bearing (L) as

$$L = \left(\frac{C}{P}\right)^n$$
$$C = PL^{1/n}$$

where

$$n = \begin{cases} 3 & \text{Ball bearings} \\ 10/3 & \text{Roller bearings} \end{cases}$$

5.9.6 Cyclic Loads

If a rolling contact bearing is subjected loads P_i for N_i revolutions ($i = 1, 2, \cdots, n$), then equivalent load (P_e) for the complete work cycle is determined as

$$P_e = \sqrt[3]{\frac{\sum N_i P_i^3}{\sum N_i}}$$

5.10 SLIDING CONTACT BEARING

Sliding contact bearings are ordinary journal bearings where lubrication is used to reduce friction between mating surfaces. These bearings can have two basic modes of lubrication: thick film lubrication and thin film lubrication, discussed as follows.

5.10.1 Thick Film Lubrication

Thick film lubrication is a state of lubrication in which two surfaces of the bearing are completely separated by a thick film of lubricant. Due to the absence of surface-to-surface contact, the viscous resistance arises from the viscosity of the lubricant. The lubricating oil is fed at the point of minimum stress so that it can penetrate inside the bearing.

Thick film lubrication has two regimes of lubrication:

1. *Hydrodynamic Lubrication* In *hydrodynamic lubrication*, the load supporting film is maintained by the shape and dynamic motion of the surfaces, such as in engines. The rotating journal climbs the bearing surface and forces the lubricant into the wedge-shaped region between the journal and bearing. The pressure is generated gradually within the system. Due to this feature, these bearings are also called *self-acting bearing*.
2. *Hydrostatic Lubrication* Hydrostatic lubrication is achieved by creating the load supporting film by an external source, like a pump. Therefore, these bearings are called *externally pressurized bearings*. Hydrostatic bearings, although costly, offer advantages, such as high load carrying capacity, no starting friction, no rubbing action.

5.10.2 Thin Film Lubrication

Thin film lubrication, also known as *boundary lubrication*, is the condition in which the lubricant film is relatively thin and there is partial metal to metal contact, such as in machine tool slides. The conditions resulting in boundary lubrication are excessive load, insufficient surface area or oil supply, low speed and misalignment.

5.10.3 Design Considerations

Consider a bearing of diameter D inside which a journal of diameter d and length l, which rotates at eccentricity e and speed N in rpm ($N' = N/60$ in revolution per second). The absolute viscosity of the lubricant is Z (Pa·s) and the coefficient of friction is μ. The minimum film thickness is h_0 ($\approx 0.0001D$) [Fig. 5.29].

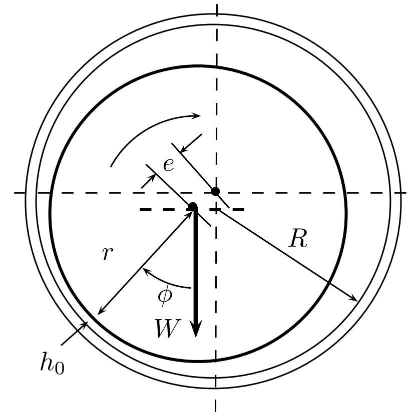


Figure 5.29 Sliding contact bearing.

The diametral and radial clearances in the bearing are, respectively,

$$c_d = D - d$$
$$c_r = \frac{c_d}{2}$$

Eccentricity ratio is defined as

$$\epsilon = \frac{e}{c_r}$$

Film thickness at any angle ϕ is determined as

$$h = \frac{c_d}{2}(1 + \epsilon\cos\phi)$$

The minimum film thickness is related as

$$e = \frac{c}{2} - h_0$$

For a radial load W on the journal, the average pressure (p) on the bearing is given by

$$p = \frac{W}{dl}$$

Coefficient of friction is the ratio of tangential force to the radial load acting on the bearing.

The following are the important characteristics of sliding contact bearings:

1. *Bearing Characteristic Number* *Bearing characteristic number* is a dimensionless number, defined as

$$\text{Bearing characteristic number} = \frac{ZN}{p}$$

At low speeds, the lubricant cannot separate the surfaces of the journal and bearing, resulting in a metal-to-metal contact. With increase in

speed N, more liquid is forced into the wedge-shaped clearance. This results in transition from thin film lubrication, which transforms into thick film lubrication at a certain speed. This transition is depicted in the plot of bearing characteristic number (ZN/p) versus coefficient of friction (μ) [Fig. 5.30].

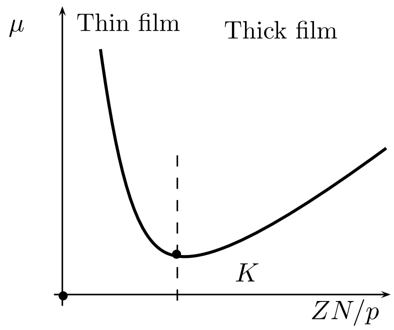


Figure 5.30 Bearing modulus.

The coefficient of friction between the journal and bearing becomes minimum at an optimum speed and then increases with further speed.

2. Bearing Modulus The value of bearing characteristic number corresponding to the minimum coefficient of friction is called *bearing modulus* (K). In order to avoid seizure, the operating value of bearing characteristic number (ZN/p) should be at least 5 to 6 times of K while for fluctuating loads, it should be at least $15K$.

3. Sommerfeld Number The theory of hydrodynamic lubrication is based on the Reynold's equation which was solved by Raimondi and Boyde in a tabular form that gives correlation between many dimensionless numbers. These tables are referred using a dimensionless number known as the *Sommerfeld number*[13] (S) which contains all the normally specified variables:

$$S = \frac{ZN'}{p}\left(\frac{r}{c}\right)^2$$

4. Coefficient of Friction The following are the equations for coefficient of friction:

   (a) *Petroff's Equation* - [14] The following equation is known as the *Petroff's equation* which gives reasonable estimate of co-efficient of friction for lightly loaded bearings:

   $$\mu = 2\pi^2\left(\frac{ZN}{p}\right)\frac{D}{c_d}$$

   In above equation, ZN/p is the bearing characteristic number and D/c_d is the clearance ratio. Both are dimensionless parameters of the bearing. Clearance ratio normally ranges from 500 to 1000 in bearings.

   (b) *Mckee's Equation* - The following equation is *Mckee's empirical relation* for determining coefficient of friction sliding contact bearings:

   $$\mu = 0.002 + 0.326\left(\frac{ZN}{p}\right)\frac{D}{c_d}$$

[13] Arnold Sommerfeld (1868–1951) was a German theoretical physicist. He was nominated a record 81 times for the Nobel Prize. He introduced the 2nd quantum number (azimuthal quantum number) and the 4th quantum number (spin quantum number). He also introduced the fine-structure constant, and pioneered X-ray wave theory.

[14] In 1883, Petroff published his work on bearing friction based on simplified assumptions.

5.11 BRAKES AND CLUTCHES

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member in order to retard the motion. The kinetic energy of the machine element is absorbed by mechanical brakes, which is later dissipated in the form of heat. Figure 5.31 shows a single-shoe brake which is pressed against the rim of a revolving brake wheel drum. The shoe is made of softer material than the rim of the wheel.

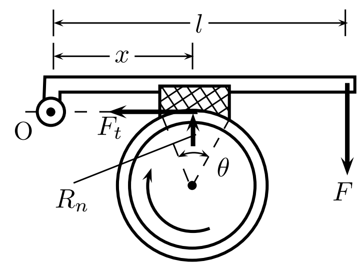


Figure 5.31 Single shoe brake.

When force F is applied on the lever, it produces a normal reaction R_n at the rim. Taking moments about the fulcrum O, one obtains

$$a \times R_n = F \times b$$
$$R_n = \mu\frac{Fb}{a}$$

If μ is the coefficient of friction, the tangential braking force F_t acting on the wheel drum is given by

$$F_t = \mu R_n$$
$$= \mu\frac{\mu Fb}{a}$$

The concepts of V-belts and friction clutches, discussed in Chapter 3, are used for study of clutches.

IMPORTANT FORMULAS

Static Failure Theories

1. Rankine's theory (σ)
$$\frac{\sigma_{yt}}{N} \geq \sigma_t$$
2. Tresca-Guest theory (τ)
$$\frac{\sigma_{yt}}{N} \geq |\sigma_1 - \sigma_2|$$
3. Saint Venant's theory (ε)
$$\frac{\sigma_y}{N} \geq (\sigma_1 - \mu\sigma_2)$$
4. Beltrami-Haigh theory (U)
$$\frac{\sigma_{yt}}{N} = \sqrt{{\sigma_1}^2 + {\sigma_2}^2 - 2\mu(\sigma_1\sigma_2)}$$
5. Von Mises theory (U_d)
$$\left(2\frac{\sigma_{yt}}{N}\right)^2 \geq \sum (\sigma_1 - \sigma_2)^2$$
For pure shear
$$\sigma_{yt} = \sqrt{3}\sigma_{12}$$
6. Coulomb-Mohr theory
$$\frac{\sigma_t}{\sigma_{ut}} + \frac{\sigma_c}{\sigma_{uc}} = \frac{1}{N}$$

Dynamic Loading

1. Stress cycle
$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$
$$\sigma_a = \frac{\sigma_{max} + \sigma_{min}}{2}$$
$$q = \frac{k_f - 1}{k_t - 1}$$
2. Miner's rule
$$\sum_{i=1}^{n} \frac{N_i}{L_i} = C$$
3. Fatigue failure criteria:
   (a) Gerber parabola
$$\left(N\frac{\sigma_a}{\sigma_{ut}}\right)^2 + N\frac{\sigma_v}{\sigma_e} = 1$$
   (b) Soderberg line
$$\frac{\sigma_a}{\sigma_y} + \frac{\sigma_v}{\sigma_e} = \frac{1}{N}$$
   (c) Goodman line
$$\frac{\sigma_a}{\sigma_{ut}} + \frac{\sigma_v}{\sigma_e} = \frac{1}{N}$$

Rivet Joints

1. Tearing strength
$$F = (p - d)\,t$$
2. Shearing strength
$$F = \frac{\pi d^2}{4}\tau$$
3. Crushing strength
$$F = ndt\sigma_e$$

Welded Joints

1. Shear in fillet weld
$$F = \frac{\sqrt{2}F}{tl}$$
2. Eccentrically loaded welds
$$\tau_1 = \frac{\sqrt{2}F}{tl}, \quad \tau_2 = \frac{Mr}{J}$$
3. Bending moment on welds
$$\tau_1 = \frac{\sqrt{2}F}{tl}, \quad \sigma_b = \frac{My}{I}$$

Threaded Joints

1. Stress analysis
$$F = \frac{\pi d^2}{4}\frac{\sigma_{yt}}{N} \text{ or } \frac{\pi d^2}{4}\frac{\tau_y}{N}$$
2. Eccentric loading
$$\tau_1 = \frac{4F}{n \times \pi d^2}, \quad \tau_2 = \frac{Mr}{J}$$

Design of Shafts

1. Bending and torsion
$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$
$$T_e = \sqrt{M^2 + T^2}$$
$$\frac{\tau_y}{N} = \frac{16}{\pi d^3}\sqrt{(k_b M)^2 + (k_t T)^2}$$
2. Dynamic loading
$$\left(\frac{\pi d^3 \tau}{16N}\right)^2 = \left(M_a + k_b\frac{\sigma_y}{\sigma_e}M_v\right)^2 + \left(T_a + k_t\frac{\sigma_y}{\sigma_e}T_v\right)^2$$

Gear Design

1. Tangential load
$$F_T = mb\sigma_b\left(\frac{t^2}{6hm}\right)$$
$$Y = \frac{t^2}{6hm}$$
2. Beam strength
$$F_t = \sigma_b mbY\left(\frac{C_v}{k_f}\right)$$

Rolling Contact Bearings

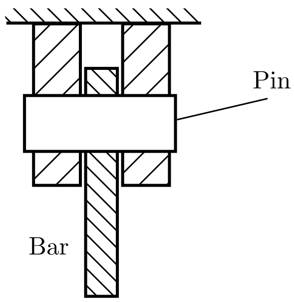
$$P = X\,(VF_r) + Y\,(F_a)$$
$$\frac{L}{L_{90}} = \left[\frac{\ln(1/R)}{\ln(1/R_{90})}\right]^{1/1.17}$$
$$L = \frac{60NL_h}{10^6}$$
$$L = \left(\frac{C}{P}\right)^n$$
$$C = PL^{1/n}$$
where
$$n = \begin{cases} 3 & \text{Ball bearings} \\ 10/3 & \text{Roller bearings} \end{cases}$$
$$P_e = \sqrt[3]{\frac{\sum N_i P_i^3}{\sum N_i}}$$

Sliding Contact Bearings

1. Bearing characteristic number
$$\frac{ZN}{p}$$
2. Sommerfeld number
$$S = \frac{ZN'}{p}\left(\frac{r}{c}\right)^2$$
3. Petroff's equation
$$\mu = 2\pi^2\left(\frac{ZN}{p}\right)\frac{D}{c_d}$$
4. Mckee's equation
$$\mu = 0.002 + 0.326\left(\frac{ZN}{p}\right)\frac{D}{c_d}$$

SOLVED EXAMPLES

1. A steel bar of rectangular cross-section 10 mm×50 mm is attached to a support by means of a round pin of diameter 20 mm as shown in the figure.



The allowable stresses for the bar in tension and the pin in shear are 150 MPa and 80 MPa, respectively. What is the maximum permissible value of tensile load that the bar can carry?

Solution. The width of the minimum cross section of the bar is $50-20=30$ mm. Thus, the allowable tensile load for safe stress of bar is

$$\begin{aligned} F_1 &= 150\times 10\times 30 \\ &= 45 \text{ kN} \end{aligned}$$

Similarly, the allowable load for safe shear stress in the pin is

$$\begin{aligned} F_2 &= 80\times 2\times \frac{\pi\times 20^2}{4} \\ &= 50.26 \text{ kN} \end{aligned}$$

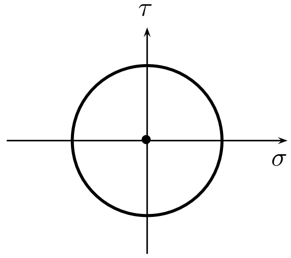
Thus, the safest option is 45 kN.

2. Yield strength of steel is found 210 N/mm^2. Using a factor of safety of 3 and applying maximum principal stress theory of failure, determine the permissible stress in the steel shaft subjected to torsion.

Solution. Given that

$$\sigma_y = 250 \text{ MPa}$$

Mohr's circle is drawn for a shaft subjected to torsion:



The maximum principal stress will be equal to the maximum shear stress:

$$\begin{aligned} \tau_{max} &\leq \frac{\sigma_y}{N} \\ &= \frac{250}{3} \\ &= 70 \text{ MPa} \end{aligned}$$

3. A critical element of a component is under bi-axial stresses being 350 MPa and 150 MPa. Determine the maximum working stress using distortion energy theory.

Solution. Given that

$$\begin{aligned} \sigma_1 &= 350 \text{ MPa} \\ \sigma_2 &= 150 \text{ MPa} \\ \sigma_{12} &= 0 \end{aligned}$$

Using distortion theory for plane stress conditions:

$$\begin{aligned} \sigma_y &= \sqrt{\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2+3\sigma_{12}^2} \\ &= 304.13 \text{ MPa} \end{aligned}$$

4. A large uniform steel plate of 150 mm width containing a rivet-hole of size 7.5 mm each is subjected to uniform uni-axial tension of 75 MPa. Determine the maximum stress in the plate due to stress concentration.

Solution. Due to rivet-hole, the maximum stress due to stress concentration is

$$\begin{aligned} \sigma_{max} &= 3\sigma \\ &= 3\times 75 \\ &= 225 \text{ MPa} \end{aligned}$$

5. A ductile component is subjected to dynamic stress varying between 150 MPa and 250 MPa. The ultimate tensile strength of the material is 450 MPa, yield point in tension is 250 MPa, and endurance limit for reversed bending is 150 MPa. Determine the factor of safety for the design of the component.

Solution. Given that

$$\begin{aligned} \sigma_{max} &= 250 \text{ MPa} \\ \sigma_{min} &= 150 \text{ MPa} \\ \sigma_{ut} &= 450 \text{ MPa} \\ \sigma_{yt} &= 350 \text{ MPa} \\ \sigma_e &= 250 \text{ MPa} \end{aligned}$$

The average and variable stresses are

$$\sigma_a = \frac{250+150}{2} = 200 \text{ MPa}$$
$$\sigma_v = \frac{250-150}{2} = 50 \text{ MPa}$$

Using Soderberg criteria (for ductile materials)

$$\frac{1}{N} = \frac{\sigma_a}{\sigma_{yt}} + \frac{\sigma_v}{\sigma_e} = \frac{200}{350} + \frac{50}{250}$$
$$N = 1.29$$

6. A shaft is subjected to torque 1500 Nm and bending moment 750 Nm. The combined shock and fatigue factor for bending is 1.5 and for torsion is 2. Determine the equivalent twisting moment for the shaft.

Solution. Given that

$$T = 1500 \text{ Nm}$$
$$M = 750 \text{ Nm}$$
$$k_m = 1.5$$
$$k_t = 2$$

Equivalent twisting moment is

$$T_e = \sqrt{(k_m M)^2 + (k_t T)^2} = 3204 \text{ Nm}$$

7. The dynamic load capacity a bearing is 25 kN. Determine the maximum radial load it can sustain to operate at 540 rev/min for 2500 hours.

Solution. Given that

$$C = 25 \text{ kN}$$
$$N = 540 \text{ rpm}$$
$$L_h = 2500 \text{ hours}$$

Life of bearing in million revolutions (L) is

$$L = \frac{60\times 540\times 2500}{10^6} = 81$$

The designation indicates a ball bearing, thus

$$C = PL^{1/3}$$
$$P = \frac{C}{L^{1/3}} = \frac{25}{81^{1/3}} = 5.77 \text{ kN}$$

8. The life of a ball bearing at a load of 15 kN is 10,000 hours. Determine the life (in hours), if the load is increased to 20 kN for the same operating condition.

Solution. Given that

$$P_1 = 15 \text{ kN}$$
$$L_1 = 10000 \text{ hours}$$
$$P_2 = 20 \text{ kN}$$

For ball bearings:

$$C = PL^{1/3}$$
$$L_2 = L_1 \times \left(\frac{P_1}{P_2}\right)^3 = 10000 \times \frac{1}{(20/15)^3} = 4218 \text{ hours}$$

9. A journal bearing supports a shaft that rotates at 1800 rpm. Clearance to radius ratio is 1/120. Viscosity of the lubricant is $\mu = 30\times 10^{-3}$ Pa·s and bearing pressure is 2.4 MPa. Determine Sommerfeld number.

Solution. Given that

$$\mu = 30\times 10^{-3} \text{ Pa·s}$$
$$N = 1800 \text{ rpm}$$
$$p = 2.4 \text{ MPa}$$
$$r/c = 120$$

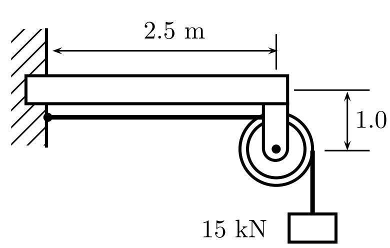
The speed in rps is determined as

$$N' = \frac{N}{60} = 30 \text{ rps}$$

Sommerfeld number is given by

$$S = \frac{\mu N'}{p}\left(\frac{r}{c}\right)^2 = 5.4\times 10^{-3}$$

10. A cantilever beam of rectangular cross-section is used to support a static load of 15 kN attached to a string passing through a pulley of radius 0.5 m. The ratio of depth to width of the cross-section is 3.



The yield strength of the beam material is 250 N/mm^2. Using the maximum normal stress theory of failure, determine the width of the cross-section of the beam for factor of safety 2.5. Weight of the beam material can be ignored.

Solution. Maximum bending moment is at the fixed end:

$$\begin{aligned} M &= 15\times 2.5+15\times 0.5+15\,(1.0-0.5) \\ &= 52.5 \text{ kNm} \end{aligned}$$

The allowable bending stress in the material is

$$\begin{aligned} \sigma_b &= \frac{250}{2.5} \\ &= 100 \text{ N/mm}^2 \end{aligned}$$

The width of cross-section w is given by

$$\begin{aligned} \sigma_b &= \frac{M\times 3w/2}{w\times (3w)^3/12} \\ &= \frac{2M}{3w^3} \\ w &= \sqrt[3]{\frac{2M}{3\sigma_b}} \\ &= \sqrt[3]{\frac{2\times 52.5\times 10^3}{3\times 100\times 10^6}} \\ &= 70 \text{ mm} \end{aligned}$$

11. A 26-teeth 5 mm module 20° full depth pinion receives 5.1 kW at 750 rpm. Determine the tangential force on the gear.

Solution. Given that

$$\begin{aligned} z &= 26 \\ m &= 0.005 \text{ m} \\ \phi &= 20^\circ \\ P &= 3\times 10^3 \text{ W} \\ N &= 750 \text{ rpm} \end{aligned}$$

Pitch circle velocity of the pinion is

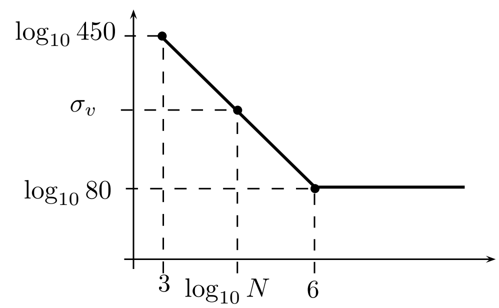
$$\begin{aligned} V &= \frac{\pi d N}{60} \\ &= \frac{\pi m z N}{60} \\ &= 5.1 \text{ m/s} \end{aligned}$$

Tangential tooth load is given by

$$\begin{aligned} F_t\times V &= P \\ F_t &= \frac{P}{V} \\ &= \frac{5.1\times 10^3}{5.1} \\ &= 1 \text{ kN} \end{aligned}$$

12. A shaft is subjected to alternating stress 125 MPa. Fatigue strength to sustain 1000 cycle is 450 MPa. Determine the life of the shaft for corrected endurance strength of 80 MPa.

Solution. Given that $\sigma_e = 450$ MPa for $1000 = 10^3$ cycles, and $\sigma_e = 80$ MPa for 10^6 cycles. Life for alternating stress 125 MPa can be determined using $\log_{10}\sigma$–$\log_{10} N$ curve:



Thus, number of cycles is given by

$$\begin{aligned} \log_{10} N &= 6-\frac{\log_{10} 125-\log_{10} 80}{\log_{10} 450-\log_{10} 80}\times(6-3) \\ &= 6-\frac{0.1938}{0.75}\times(6-3) \\ &= 5.224 \\ N &= 167.85\times 10^3 \text{ cycles} \end{aligned}$$

13. A thin cylindrical pressure vessel of 180 mm diameter and 2 mm thickness is subjected to an internal pressure varying from 8 to 12 MPa. Assume that the yield, ultimate and endurance strength of material are 625, 850 and 410 MPa, respectively. Determine the factor of safety for Goodman's criteria.

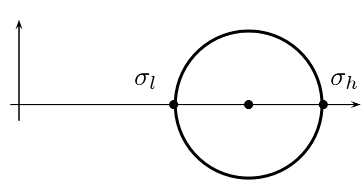
Solution. Given that

$$\begin{aligned} d &= 0.240 \text{ m} \\ t &= 0.002 \text{ m} \end{aligned}$$

Hoop stress and longitudinal stress on thin cylinders are determined as

$$\begin{aligned} \sigma_h &= \frac{pd}{2t} \\ \sigma_l &= \frac{pd}{4t} \end{aligned}$$

These stresses are drawin on Mohr's circle



Thus, maximum stress is the hoop stress. Average and variable hoop stresses are

$$\sigma_a = \frac{12+8}{2} \times \frac{d}{2t}$$
$$= 450 \text{ MPa}$$
$$\sigma_v = \frac{12-8}{2} \times \frac{d}{2t}$$
$$= 90 \text{ MPa}$$

Using Goodman's criteria:

$$\frac{1}{N} = \frac{\sigma_a}{\sigma_{ut}} + \frac{\sigma_v}{\sigma_e}$$
$$= \frac{450}{850} + \frac{90}{410}$$
$$N = 1.33$$

14. A 24-teeth 20° full depth spur gear having module 3 mm, face width of 32 mm is transmitting 3.5 kW at 1200 rpm. Velocity factor and form factor can be taken as 1.5 and 0.3, respectively. Determine the maximum stress in the gear tooth.

Solution. Given that

$$m = 6 \text{ mm}$$
$$T = 24$$
$$b = 32 \text{ mm}$$
$$\phi = 20°$$
$$P = 3.5 \times 10^3 \text{ W}$$
$$N = 1200\text{rev/s}$$
$$k_f = 1.5$$
$$Y = 0.3$$
$$d = mT$$
$$= 144 \text{ mm}$$

Pitch circle velocity

$$v = \frac{\pi \times 0.144 \times 1200}{60}$$
$$= 9.04 \text{ m/s}$$

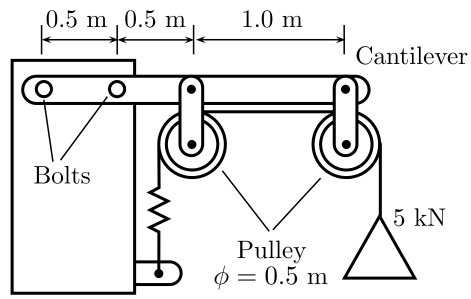
Transmitted load is calculated as

$$F_T = \frac{3.5 \times 10^3}{9.04}$$
$$= 387.17 \text{ N}$$

Using Lewis equation,

$$\sigma = \frac{F_T k_f}{mbY}$$
$$= 10 \text{ MPa}$$

15. A steel cantilever of cross-section 20×60 mm is bolted to a frame with two M16 bolts (B_1 and B_2). It is used to support a load of 5 kN attached to a string passing through two pulleys as shown in the figure.



Determine the maximum resultant shear stress on the bolts:

Solution. The spring will exert equal tension 5 kN. The bolts will be subjected to two types of loads:

(a) Primary shear load (F_1) on bolts is due to direct shear stress resulted by load $5+5 = 10$ kN, which will be equally divided between the two bolts. Therefore, primary shear load on both bolts is

$$F_1 = \frac{5+5}{2}$$
$$= 5 \text{ kN}$$

(b) Secondary shear load (F_2) on P is due to moment resulted by eccentric loading w.r.t. CG of the bolts:

$$M = 5 \times 0.5 + 5 \times 2.0$$
$$= 12.5 \text{ kN}$$

This moment shall be balanced by equal and opposite shear forces on both bolts. Therefore,

$$F_2 \times 0.5 = 12.5 \text{ kN}$$
$$F_2 = 25 \text{ kN}$$

Root diameter of M16 bolt,

$$d = 16 \text{ mm}$$

Resultant shear stress is maximum at inner side bolt:

$$F = F_2 + F_1$$
$$= 30 \text{ kN}$$
$$\tau = \frac{F}{\pi d^2/4}$$
$$= \frac{7.5 \times 10^3}{\pi \times 0.016^2/4}$$
$$= 149.2 \text{ MPa}$$

GATE PREVIOUS YEARS' QUESTIONS

1. A wire rope is designated as 6×19 standard hoisting. The numbers 6×19 represent

(a) diameter in millimeter $\times$ length in meter
(b) diameter in centimeter $\times$ length in meter
(c) number of strands $\times$ number of wires in each strand
(d) number of wires in each strand $\times$ number of strands

(GATE 2003)

Solution. The specification of a wire rope shows "number of strands $\times$ number of wires in each strand".

Ans. (c)

2. Square key of side $d/4$ each and length l is used to transmit torque T from the shaft of diameter d to the hub of a pulley. Assuming the length of the key to be equal to the thickness of the pulley, the average shear stress developed in the key is given by

(a) $\frac{4T}{ld}$ (b) $\frac{16T}{ld^2}$
(c) $\frac{8T}{ld^2}$ (d) $\frac{16T}{\pi d^3}$

(GATE 2003)

Solution. Average shear stress will be

$$\tau = \frac{T}{d/2} \times \frac{1}{d/4 \times l} = \frac{8T}{d^2 l}$$

Ans. (c)

3. In a band brake, the ratio of tight side band tension to the tension on the slack side is 3. If the angle of overlap of band on the drum is 180° the coefficient of friction required between drum and the band is

(a) 0.20 (b) 0.25
(c) 0.30 (d) 0.35

(GATE 2003)

Solution. Given that

$$\frac{T_1}{T_2} = 3$$
$$\theta = 180^\circ = \pi \text{ rad}$$

Using

$$\frac{T_1}{T_2} = \exp(\mu\theta)$$
$$\mu = \frac{\ln 3}{\pi} = 0.349699$$

Ans. (d)

4. Two mating spur gears have 40 and 120 teeth, respectively. The pinion rotates at 1200 rpm and transmits a torque of 20 N.m. The torque transmitted by the gear is

(a) 6.6 Nm (b) 20 Nm
(c) 40 Nm (d) 60 Nm

(GATE 2004)

Solution. The gear will rotate with $40/120 = 1/3$ speed of pinion, therefore, torque will be 3 times, that is, 60 Nm.

Ans. (d)

5. In terms of theoretical stress concentration factor (k_t) and fatigue stress concentration factor (k_f), the notch sensitivity q is expressed as

(a) $\frac{k_f - 1}{k_t - 1}$ (b) $\frac{k_f - 1}{k_t + 1}$
(c) $\frac{k_t - 1}{k_f - 1}$ (d) $\frac{k_f + 1}{k_t - 1}$

(GATE 2004)

Solution. The notch sensitivity q for fatigue loading is defined as

$$q = \frac{k_f - 1}{k_t - 1}$$

Ans. (a)

6. The S–N curve for steel becomes asymptotic nearly at

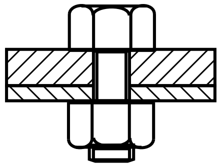
(a) 10^3 cycles (b) 10^4 cycles
(c) 10^6 cycles (d) 10^9 cycles

(GATE 2004)

Solution. The S–N curve for steel becomes asymptotic nearly at 10^6 cycles.

Ans. (c)

7. In a bolted joint, two members are connected with an axial tightening force of 2200 N.



If the bolt used has metric threads of 4 mm pitch, the torque required for achieving the tightening force is

(a) 0.7 Nm (b) 1.0 Nm
(c) 1.4 Nm (d) 2.8 Nm

(GATE 2004)

Solution. Given that

$$F_a = 2200 \text{ N}$$
$$p = 4 \text{ mm}$$

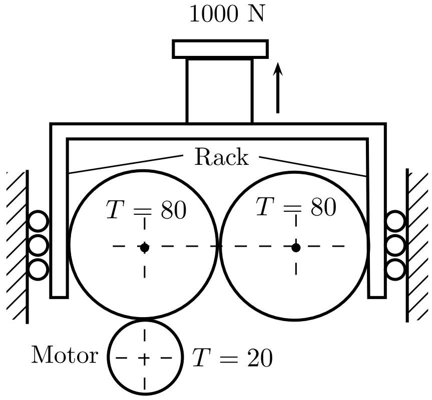
During tightening of one revolution, the axial tightening force moves a distance of pitch, therefore, tightening torque T (Nm) is given by energy balance as

$$\begin{aligned} T \times 2\pi &= F_a \times p \\ T &= \frac{F_a \times p}{2\pi} \\ &= \frac{2200 \times 0.004}{2\pi} \\ &= 1.40056 \text{ Nm} \end{aligned}$$

Ans. (c)

Common Data Questions

A compacting machine shown in the figure below is used to create a desired thrust force by using a rack and pinion arrangement. The input gear is mounted on the motor shaft. The gears have involute teeth of 2 mm module.



8. If the drive efficiency is 80%, then torque required on the input shaft to create 1000 N output thrust is

(a) 20 Nm (b) 25 Nm
(c) 32 Nm (d) 50 Nm

(GATE 2004)

Solution. The output thrust of 1000 N will be divided equally in both arms, therefore, tangential force of 500 N need to be generated on both gears of diameter 0.16 m.

If T is the torque to be produced by motor at speed ω_1, then the energy will be transmitted to the gears to produce the torque $F \times D/2$ with efficiency of 80% and speed of ω_2 $(= \omega_1/4)$. Therefore,

$$\begin{aligned} T\omega_1 \times 0.8 &= 2 \times F \times D/2 \times \omega_2 \\ T &= 25 \text{ Nm} \end{aligned}$$

Ans. (b)

9. If the pressure angle of the rack is 20°, then force acting along the line of action between the rack and the gear is

(a) 250 N (b) 342 N
(c) 532 N (d) 600 N

(GATE 2004)

Solution. Given that

$$\phi = 20^\circ$$

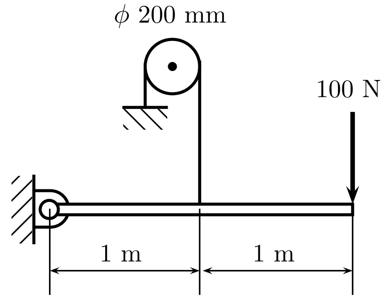
Force along the line of action is

$$\begin{aligned} F &= \frac{500}{\cos\phi} \\ &= 532 \text{ N} \end{aligned}$$

Ans. (c)

Linked Answer Questions

A band brake consists of a lever attached to one end of the band. The other end of the band is fixed to the ground. The wheel has a radius of 200 mm and the wrap angle of the band is 270°. The braking force applied to the lever is limited to 100 N, and the coefficient of friction between the band and the wheel is 0.5. No other information is given.



10. The maximum tension that can be generated in the band during braking is

(a) 1200 N (b) 2110 N
(c) 3224 N (d) 4420 N

(GATE 2005)

Solution. Given that

$$\begin{aligned} \mu &= 0.5 \\ \theta &= 270^\circ \\ &= 3\pi/2 \text{ rad} \\ T_2 &= 100 \times 2 \\ &= 200 \text{ N} \end{aligned}$$

Maximum tension is given by

$$\begin{aligned} \frac{T_1}{T_2} &= e^{\mu\theta} \\ T_1 &= 2110 \text{ N} \end{aligned}$$

Ans. (b)

11. The maximum wheel torque that can be completely braked is

(a) 200 Nm (b) 382 Nm
(c) 604 Nm (d) 844 Nm

(GATE 2005)

Solution. As the maximum tension is $T_1 = 2110$ N and radius is $r = 0.2$ m, therefore, maximum torque is

$$\begin{aligned} T_{\max} &= T_1 \times r \\ &= 382.029 \text{ Nm} \end{aligned}$$

Ans. (b)

12. Which one of the following is a criterion in the design of hydrodynamic journal bearings?

(a) Sommerfeld number
(b) Rating life
(c) Specific dynamic capacity
(d) Rotation factor

(GATE 2005)

Solution. Sommerfeld number is a criterion in the design of hydrodynamic journal bearings.

Ans. (a)

13. A cylindrical shaft is subjected to an alternating stress of 100 MPa. Fatigue strength to sustain 1000 cycles is 490 MPa. If the corrected endurance strength is 70 MPa, estimated shaft life will be

(a) 1071 cycles (b) 15000 cycles
(c) 281914 cycles (d) 928643 cycles

(GATE 2006)

Solution. Given that

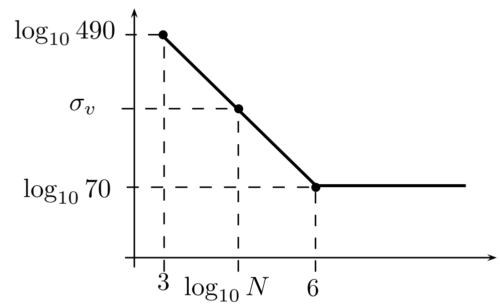
(i) For $1000 = 10^3$ cycles,

$$\sigma_e = 490 \text{ MPa}$$

(ii) For 10^6 cycles,

$$\sigma_e = 70 \text{ MPa}$$

$\log_{10} \sigma_e$–$\log_{10} N$ diagram is shown in the figure:



Therefore, for desired strength of 100 MPa, the $\log_{10} N$ based on the slope of $\log_{10} \sigma_e$–$\log_{10} N$ linear diagram is given by

$$\begin{aligned} \log_{10} N &= 6 - \frac{\log_{10} 100 - \log_{10} 70}{\log_{10} 490 - \log_{10} 70} \times (6-3) \\ N &= 281914 \text{ cycles} \end{aligned}$$

Ans. (c)

14. According to Von Mises distortion energy theory, the distortion energy under three dimensional stress state is represented by

(a) $\dfrac{1}{2E}\left(\sum \sigma_i^2 - 2\mu \sum \sigma_i \sigma_j\right)$

(b) $\dfrac{1-2\mu}{6E}\left(\sum \sigma_i^2 + 2 \sum \sigma_i \sigma_j\right)$

(c) $\dfrac{1+\mu}{3E}\left(\sum \sigma_i^2 - \sum \sigma_i \sigma_j\right)$

(d) $\dfrac{1}{3E}\left(\sum \sigma_i^2 - \mu \sum \sigma_i \sigma_j\right)$

(GATE 2006)

Solution. The strain energy can be divided into two parts: the energy associated solely with change in volume, termed dilatation energy, and the energy associated with the change in shape, termed distortion energy. Distortion energy is calculated as the total strain energy minus contribution due to uniform stresses.

Ans. (c)

15. A 60 mm long and 6 mm thick fillet weld carries a steady load of 15 kN along the weld. The shear strength of the weld material is equal to 200 MPa. The factor of safety is

(a) 2.4 (b) 3.4
(c) 4.8 (d) 6.8

(GATE 2006)

Solution. Given that

$$l = 0.06 \text{ m}$$
$$s = 6 \text{ mm}$$
$$F = 15 \text{ kN}$$
$$\tau_{max} = 200 \text{ MPa}$$

Throat of the weld is

$$t = s\cos 45^\circ$$

Factor of safety (N) is given by

$$F = N \times \tau_{\max} l t$$
$$15 \times 10^3 = N \times 200 \times 10^6 \times 0.06 \times 0.006 \times \cos 45^\circ$$
$$N = 3.39411$$

Ans. (b)

16. A ball bearing operating at a load F has 8000 hours of life. The life of the bearing, in hours, when the load is doubled to $2F$ is

(a) 8000 (b) 6000
(c) 4000 (d) 1000

(GATE 2007)

Solution. Given that $L_1 = 8000$ hours. Therefore,

$$P_1 = F$$
$$P_2 = 2F$$

Using the load-life equation for ball bearings:

$$L \propto \frac{1}{P^3}$$
$$L_2 = \left(\frac{F}{2F}\right)^3 L_1$$
$$= \left(\frac{1}{2}\right)^3 \times 8000$$
$$= 1000 \text{ hours}$$

Ans. (d)

17. A thin spherical pressure vessel of 200 mm diameter and 1 mm thickness is subjected to an internal pressure varying from 4 to 8 MPa. Assume that the yield, ultimate and endurance strength of material are 600, 800 and 400 MPa, respectively. The factor of safety as per Goodman's relation is

(a) 2.0 (b) 1.6
(c) 1.4 (d) 1.2

(GATE 2007)

Solution. Given that

$$d = 0.200 \text{ m}$$
$$t = 0.001 \text{ m}$$

Hoop stress for internal pressure p:

$$\sigma = \frac{pd}{4t}$$

Average and variable hoop stresses are

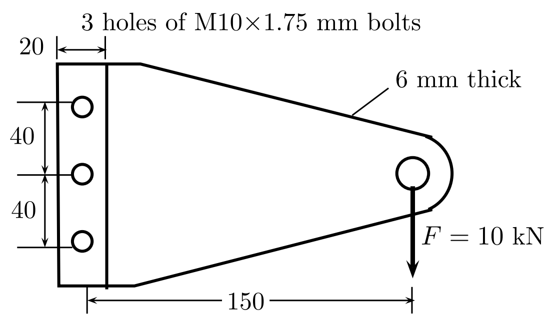
$$\sigma_a = \frac{6+4}{2} \times \frac{d}{4t}$$
$$= 250 \text{ MPa}$$
$$\sigma_v = \frac{6-4}{2} \times \frac{d}{4t}$$
$$= 50 \text{ MPa}$$

Using Goodman's criteria:

$$\frac{1}{N} = \frac{\sigma_a}{\sigma_{ut}} + \frac{\sigma_v}{\sigma_e}$$
$$N = 1.6$$

Ans. (b)

18. A bolted joint is shown below.



(All dimensions in the figure are in mm)

The maximum shear stresses, in MPa, in the bolts at A and B respectively are

(a) 242.6, 42.5 (b) 42.5, 242.6
(c) 42.5, 42.5 (d) 242.6, 242.6

(GATE 2007)

Solution. The eccentric force $F = 10$ kN will transfer a vertical force of 10 kN and a torque $T = 10 \times 0.15 = 1.5$ kNm. CG of the bolt joint is situated at B, therefore, the bolt at point B will not face any torsional load, therefore, will not bear shear stress due to torsional load. The bolt at point A will bear both torsional and normal shear stress. Thus, option (a) is correct. There is no need to calculate the values.

Ans. (a)

19. A natural feed journal bearing of diameter 50 mm and length 50 mm operating at 20 revolution/second carries a load of 2.0 kN. The lubricant used has a viscosity of 20 mPas. The radial clearance is 50 μm. The Sommerfeld number for the bearing is

(a) 0.062 (b) 0.125
(c) 0.250 (d) 0.785

(GATE 2007)

Solution. Given that

$$d = 0.050$$
$$l = 0.050$$
$$N' = 20 \text{ rps}$$
$$\mu = 20 \times 10^{-3} \text{ Pa}\cdot\text{s}$$
$$c = 2 \times 50 = 100\ \mu\text{m}$$

Bearing load per unit projected area is

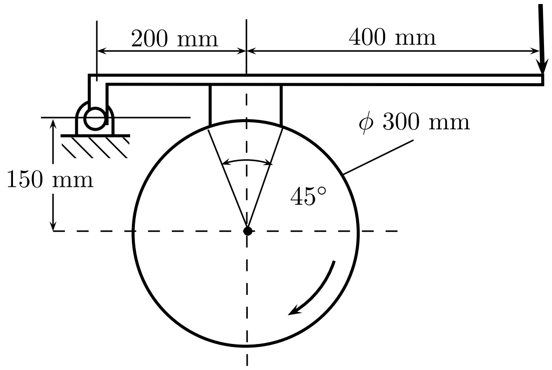
$$p = \frac{2.0 \times 10^3}{d \times l} = 800000 \text{ N/m}^2$$

Sommerfeld number is

$$= \frac{\mu N'}{p}\left(\frac{D}{c}\right)^2 = 0.125$$

Ans. (b)

20. A brake-block shown below has a face width of 300 mm and a mean coefficient of friction of 0.25.



For an activating force of 400 N, the braking torque in Nm is

(a) 30 (b) 40
(c) 45 (d) 60

(GATE 2007)

Solution. Given that

$$F = 400 \text{ N}$$
$$D = 0.300 \text{ m}$$

Normal reaction at the wheel is

$$R = \frac{F \times 0.6}{0.2} = 1200$$

Braking torque,

$$T = \mu R \times \frac{D}{2} = 45 \text{ Nm}$$

Ans. (c)

21. The piston rod of diameter 20 mm and length 700 mm in a hydraulic cylinder is subjected to a compressive force of 10 kN due to the internal pressure. The end conditions for the rod can be assumed as guided at the piston end and hinged at the other end. The Young's modulus is 200 GPa. The factor of safety for the piston rod is

(a) 0.68 (b) 2.75
(c) 5.62 (d) 11.0

(GATE 2007)

Solution. Given that

$$d = 0.02 \text{ m}$$
$$l = 0.7 \text{ m}$$
$$F = 10 \text{ kN}$$

The given condition of rod is equivalent to hinged-fixed condition of the column, for which crippling load is given by

$$F_c = 2\frac{\pi^2 EI}{l^2} = 63278.1 \text{ N}$$

Therefore, factor of safety

$$N = \frac{F_c}{F} = 6.327$$

Ans. (c)

22. An axial residual compressive stress due to a manufacturing process is present on the outer surface of a rotating shaft subjected to bending. Under a given load, the fatigue life of the shaft in the presence of the residual compressive stress is

(a) decreased

(b) increased or decreased, depending on the external bending load

(c) neither decreased nor increased

(d) increased

(GATE 2008)

Solution. The presence of compressive stress suppresses the effect of effective average and variable stresses on the surface, therefore, net effect is increased compressive strength.

Ans. (d)

23. A spur gear has module of 3 mm, number of teeth 16, a face width of 36 mm and a pressure angle of 20°. It is transmitting a power of 3 kW at 20 rev/s. Taking a velocity factor of 1.5, and a form factor of 0.3, the stress in the gear tooth is about

(a) 32 MPa (b) 46 MPa

(c) 58 MPa (d) 70 MPa

(GATE 2008)

Solution. Given that

$$\begin{aligned} m &= 3 \text{ mm} \\ z &= 16 \\ b &= 36 \text{ mm} \\ \phi &= 20^\circ \\ P &= 3\times 10^3 \text{ W} \\ N' &= 20 \text{ rev/s} \\ k_f &= 1.5 \\ Y &= 0.3 \end{aligned}$$

Pitch circle diameter is

$$d = mz = 0.048 \text{ m}$$

Pitch circle velocity

$$\begin{aligned} v &= 20\times 2\pi \times \frac{0.048}{2} \\ &= 3.0159 \text{ m/s} \end{aligned}$$

Therefore, using transmitted load is calculated as

$$\begin{aligned} F_T &= \frac{P}{v} \\ &= 994.718 \text{ N} \end{aligned}$$

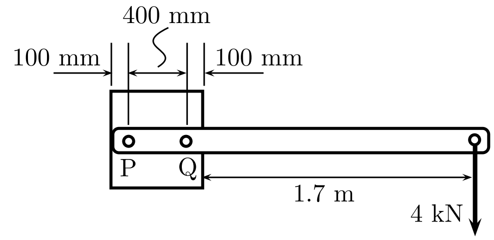
Using Lewis equation, one gets

$$\begin{aligned} \sigma &= \frac{F_T k_f}{mbY} \\ &= 46.0518 \text{ MPa} \end{aligned}$$

Ans. (b)

Linked Answer Questions

A steel bar of 10×50 mm is cantilevered with two M12 bolts (P and Q) to support a static load of 4 kN as shown in the figure.



24. The primary and secondary shear loads on the bolt P, respectively, are

(a) 2 kN, 20 kN (b) 20 kN, 2 kN

(c) 20 kN, 0 kN (d) 0 kN, 20 kN

(GATE 2008)

Solution. Primary shear load (F_1) on bolts is due to direct shear stress resulted by load 4 kN, which will be equally divided between two bolts. Therefore, primary shear load on both bolts is

$$\begin{aligned} F_1 &= \frac{4}{2} \\ &= 2 \text{ kN} \end{aligned}$$

Secondary shear load (F_2) on P is due to moment resulted by eccentric loading of 4 kN with eccentricity

$$\begin{aligned} e &= 1700+100+200 \\ &= 2.0 \text{ m} \end{aligned}$$

The eccentric load will be balanced by equal and opposite shear forces on both bolts. Therefore,

$$\begin{aligned} F_2 \times 0.4 &= 4\times 2.0 \\ &= 20 \text{ kN} \end{aligned}$$

Ans. (a)

25. The resultant shear stress on bolt P is closest to

(a) 132 MPa (b) 159 MPa

(c) 178 MPa (d) 195 MPa

(GATE 2008)

Solution. Given that

$$d = 12 \text{ mm (M12 bolts)}$$

Resultant load on bolt P is

$$\begin{aligned} F &= F_2 - F_1 \\ &= 18 \text{ kN} \end{aligned}$$

Therefore, the resultant shear stress on P is given by

$$\tau = \frac{F}{\pi d^2/4} = \frac{18 \times 10^3}{\pi \times 0.012^2/4} = 159.155 \text{ MPa}$$

Ans. (b)

26. A solid circular shaft of diameter d is subjected to a combined bending moment M and torque, T. The material property to be used for designing the shaft using the relation $16/\left(\pi d^3\right) \times \sqrt{M^2+T^2}$ is

(a) ultimate tensile strength (σ_u)

(b) tensile yield strength (σ_y)

(c) torsional yield strength (σ_{sy})

(d) endurance strength (σ_e)

(GATE 2009)

Solution. The combined effect of T and M is expressed as shear stress

$$\tau_{\max} = \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

Ans. (c)

27. A forged steel link with uniform diameter of 30 mm at the center is subjected to an axial force that varies from 40 kN in compression to 160 kN in tension. The tensile (σ_u), yield (σ_y) and corrected endurance (σ_e) strengths of the steel material are 600 MPa, 420 MPa and 240 MPa, respectively. The factor of safety against fatigue endurance as per Soderberg's criterion is

(a) 1.26 (b) 1.37

(c) 1.45 (d) 2.00

(GATE 2009)

Solution. Given that

$$\begin{aligned} d &= 0.03 \text{ m} \\ F_{max} &= 160 \text{ kN} \\ F_{min} &= 40 \text{ kN} \\ \sigma_y &= 420 \text{ MPa} \\ \sigma_e &= 240 \text{ MPa} \end{aligned}$$

Therefore,

$$\begin{aligned} \sigma_{\max} &= \frac{F_{max}}{\pi d^2/4} \\ &= 56.5884 \text{ MPa} \\ \sigma_{\min} &= \frac{F_{max}}{\pi d^2/4} \\ &= 226.354 \text{ MPa} \\ \sigma_a &= \frac{\sigma_{max}+\sigma_{min}}{2} \\ &= 141.471 \text{ MPa} \\ \sigma_v &= \frac{\sigma_{max}-\sigma_{min}}{2} \\ &= 84.8826 \text{ MPa} \end{aligned}$$

Using Soderberg's criterion,

$$\frac{1}{N} = \frac{\sigma_a}{\sigma_y} + \frac{\sigma_v}{\sigma_e} = \frac{1}{1.4482}$$

Factor of safety is

$$N = 1.4482$$

Ans. (c)

Linked Answer Questions

A 20° full depth involute spur pinion of 4 mm module and 21 teeth is to transmit 15 kW at 960 rpm. Its face width is 25 mm.

28. The tangential force transmitted (in N) is

(a) 3552 (b) 2611

(c) 1776 (d) 1305

(GATE 2009)

Solution. Given that

$$\begin{aligned} \phi &= 20^\circ \\ P &= 15 \times 10^3 \text{ W} \\ m &= 4 \text{ mm} \\ T &= 21 \text{ teeth} \\ N &= 960 \text{ rpm} \end{aligned}$$

Therefore,

$$\begin{aligned} R &= \frac{mT}{2} \\ &= 0.042 \text{ m} \\ F_t &= \frac{P}{R \times 2\pi N/60} \\ &= 3552.57 \text{ N} \end{aligned}$$

Ans. (a)

29. Given that the tooth geometry factor is 0.32 and the combined effect of dynamic load and allied factors intensifying the stress is 1.5; the minimum allowable stress (in MPa) for the gear material is

(a) 242.0 (b) 166.5
(c) 121.0 (d) 74.0

(GATE 2009)

Solution. Given that

$$Y = 0.32$$
$$k_f = 1.5$$

The minimum allowable stress (i.e. beam strength) is

$$\begin{aligned}\sigma &= \frac{F_t \times k_f}{mbY}\\ &= \frac{3552.57 \times 1.5}{0.004 \times 0.025 \times 0.32}\\ &= 166.527 \text{ MPa}\end{aligned}$$

Ans. (b)

30. A band brake having band-width of 80 mm, drum diameter of 250 mm, coefficient of friction of 0.25 and angle of wrap of 270° is required to exert a friction torque of 1000 N-m. The maximum tension (in kN) developed in the band is

(a) 1.88 (b) 3.56
(c) 6.12 (d) 11.56

(GATE 2010)

Solution. Given that

$$\begin{aligned}\mu &= 0.25\\ \theta &= 270^\circ\\ &= \frac{3\pi}{2} \text{ rad}\\ T &= 1000 \text{ Nm}\\ d &= 0.250 \text{ m}\end{aligned}$$

Ratio of tension on both sides of the belt is

$$\begin{aligned}\frac{T_1}{T_2} &= e^{\mu\theta}\\ T_2 &= \frac{T_1}{e^{\mu\theta}}\\ &= \frac{T_1}{\exp(0.25 \times 3\pi/2)}\\ &= \frac{T_1}{3.24}\end{aligned}$$

The torque will be given by

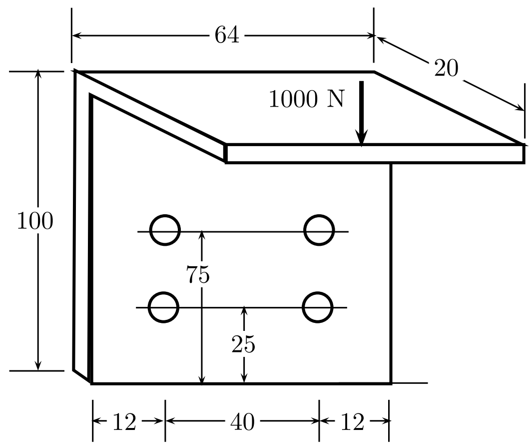
$$(T_1 - T_2)\frac{d}{2} = 1000$$

Solving the above two equation,

$$T_1 = 11.5584 \text{ kN}$$

Ans. (d)

31. A bracket (shown in figure) is rigidly mounted on wall using four rivets. Each rivet is 6 mm in diameter and has an effective length of 12 mm.



Direct shear stress (in MPa) in the most heavily loaded rivet is

(a) 4.4 (b) 8.8
(c) 17.6 (d) 35.2

(GATE 2010)

Solution. Given that

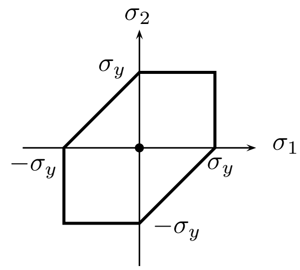
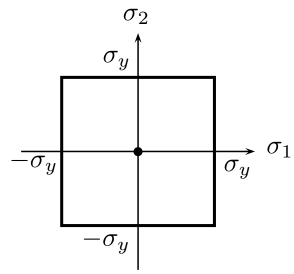
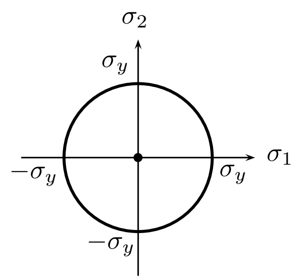
$$P = 1000 \text{ N}$$
$$d = 0.006 \text{ m}$$

The load is equally shared by the four rivets:

$$\begin{aligned}\tau &= \frac{P/4}{\pi d^2/4}\\ &= 8.84194 \text{ MPa}\end{aligned}$$

Ans. (b)

32. Match the following criteria of material failure, under biaxial stresses σ_1 and σ_2, and yield stress σ_y, with their corresponding graphic representations:

P.	Maximum normal stress criterion	L.	
Q.	Maximum distortion energy criterion	M.	
R.	Maximum shear stress criterion	N.	

(a) P-M, Q-L, R-N
(b) P-N, Q-M, R-L
(c) P-M, Q-N, R-L
(d) P-N, Q-L, R-M

(GATE 2011)

Solution. The static failure theories are summarized as follows:

Failure criteria	**Safe region**
Normal stress	Squar
Shear stress	Hexagonal
Strain	Rhomboid
Strain energy	Ellipse
Distortion energy	Circle

Ans. (c)

33. Two identical ball bearings P and Q are operating at loads 30 kN and 45 kN, respectively. The ratio of the life of bearing P to the life of bearing Q is

(a) 81/16 (b) 27/8
(c) 9/4 (d) 3/2

(GATE 2011)

Solution. According to load life equation for ball bearings, life (L) is inversely proportional to the cube of load P. Therefore, the ratio of the life of bearing P to the life of bearing Q is

$$\begin{aligned} \frac{L_P}{L_Q} &= \left(\frac{45}{30}\right)^3 \\ &= \frac{27}{8} \end{aligned}$$

Ans. (b)

34. A solid circular shaft needs to be designed to transmit a torque of 50 Nm. If the allowable shear stress of the material is 140 MPa, assuming a factor of safety of 2, the minimum allowable design diameter in mm is

(a) 8 (b) 16
(c) 24 (d) 32

(GATE 2012)

Solution. Given that

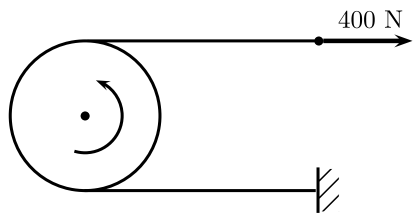
$$\begin{aligned} T &= 50 \text{ Nm} \\ n &= 2 \\ \tau_{max} &= \frac{140}{n} \\ &= 70 \text{ MPa} \end{aligned}$$

Therefore, the diameter of the shaft is found as

$$\begin{aligned} \tau_{\max} &= \frac{16T}{\pi d^3} \\ d &= \sqrt[3]{\frac{16T}{\pi \tau_{max}}} \\ &= \sqrt[3]{\frac{16 \times 50}{\pi \times 70 \times 10^6}} \\ &= \sqrt[3]{3.638} \\ &= 0.0153797 \text{ m} \\ &= 15.3797 \text{ mm} \\ &\approx 16 \text{ mm} \end{aligned}$$

Ans. (b)

35. A force of 400 N is applied to the brake drum of 0.5 m diameter in a band brake system as shown in the figure, where the wrapping angle is 180°.



If the coefficient of friction between the drum and the band is 0.25, the braking torque applied, in Nm is

(a) 100.6 (b) 54.4
(c) 22.1 (d) 15.7

(GATE 2012)

Solution. Given that

$$
\begin{aligned}
T_1 &= 400 \text{ N} \\
r &= \frac{0.5}{2} \text{ m} \\
\theta &= 180^\circ \\
&= \pi \text{ rad} \\
\mu &= 0.25
\end{aligned}
$$

Therefore, using

$$
\begin{aligned}
\frac{T_1}{T_2} &= e^{\mu\theta} \\
T_2 &= \frac{T_1}{e^{\mu\theta}} \\
&= \frac{400}{\exp(0.25\times\pi)} \\
&= 182.375
\end{aligned}
$$

Braking torque is

$$
\begin{aligned}
T &= (T_1 - T_2)\, r \\
&= 54.4062 \text{ Nm}
\end{aligned}
$$

Ans. (b)

36. A fillet welded joint is subjected to transverse loading F as shown in the figure. Both legs of the fillets are of 10 mm size and the weld length is 30 mm.

If the allowable shear stress of the weld is 94 MPa, considering the minimum throat area of the weld, the maximum allowable transverse load in kN is

(a) 14.44 (b) 17.92
(c) 19.93 (d) 22.16

(GATE 2012)

Solution. Given that

$$
\begin{aligned}
s &= 10 \text{ mm} \\
\tau_{max} &= 94 \text{ MPa} \\
l &= 30 \text{ mm}
\end{aligned}
$$

Transverse load is

$$
\begin{aligned}
F &= \frac{s}{\sqrt{2}} l \times \tau_{\max} \\
&= \frac{0.01}{\sqrt{2}\times 0.030\times 94\times 10^6} \\
&= 19.9404 \text{ kN}
\end{aligned}
$$

Ans. (c)

37. The homogeneous state of stress for a metal part undergoing plastic deformation is

$$
T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -10 \end{bmatrix}
$$

where the stress component values are in MPa. Using von Mises yield criterion, the value of estimated shear yield stress, in MPa is

(a) 9.50 (b) 16.07
(c) 28.52 (d) 49.41

(GATE 2012)

Solution. Given that

$$
\begin{aligned}
\sigma_{11} &= 10 \text{ MPa} \\
\sigma_{12} &= 5 \text{ MPa} \\
\sigma_{13} &= 0 \text{ MPa} \\
\sigma_{22} &= 20 \text{ MPa} \\
\sigma_{23} &= 0 \text{ MPa} \\
\sigma_{33} &= -10 \text{ MPa}
\end{aligned}
$$

For a general case of stress tensor along with shear stresses, von Mises yield criterion for yield stress is written as

$$
\begin{aligned}
2{\sigma_t^y}^2 &= (\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 \\
&\quad + 6\left({\sigma_{12}}^2 + {\sigma_{23}}^2 + {\sigma_{31}}^2\right)
\end{aligned}
$$

Therefore,

$$
\sigma_t^y = 27.8388 \text{ MPa}
$$

Shear stress (MPa) is found based on the case of pure shear as

$$
\begin{aligned}
\tau^y &= \frac{\sigma_t^y}{\sqrt{3}} \\
&= 16.0728 \text{ MPa}
\end{aligned}
$$

Ans. (b)

38. A bar is subjected to fluctuating tensile load from 20 kN to 100 kN. The material has yield strength of 240 MPa and endurance limit in reversed bending is 160 MPa. According to the Soderberg principle, the area of cross-section in mm^2 of the bar for a factor of safety of 2 is

(a) 400 (b) 600
(c) 750 (d) 1000

(GATE 2013)

Solution. Given that

$$F_{min} = 20 \text{ kN}$$
$$F_{max} = 100 \text{ kN}$$
$$\sigma_y = 240 \text{ MPa}$$
$$\sigma_e = 160 \text{ MPa}$$

Let A mm^2 be the cross-sectional area of the bar. The average and variable stresses in the bar are

$$\sigma_a = \frac{F_{min} + F_{max}}{2A} = \frac{60 \times 10^3}{A} \text{ MPa}$$

$$\sigma_v = \frac{F_{max} - F_{min}}{2A} = \frac{40 \times 10^3}{A} \text{ MPa}$$

Using Soderberg criteria:

$$\frac{\sigma_a}{\sigma_y} + \frac{\sigma_v}{\sigma_e} = \frac{1}{N}$$

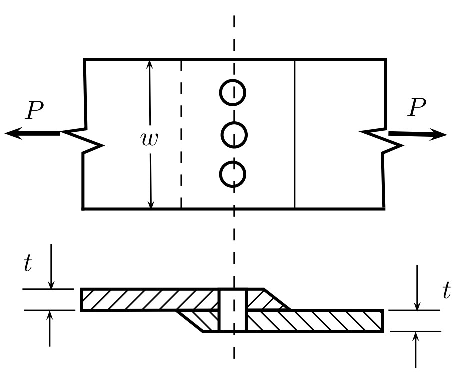
$$\frac{60 \times 10^3}{A \times 240} + \frac{40 \times 10^3}{A \times 160} = \frac{1}{2}$$

$$A = 1000 \text{ mm}^2$$

Ans. (d)

Common Data Questions

A single riveted lap joint of two similar plates, shown in the figure, has the following geometrical and material details:



Width of the plate	w	=	200 mm
Thickness of the plate	t	=	5 mm
Number of rivets	n	=	3
Diameter of the rivet	d_r	=	10 mm
Diameter of the rivet hole	d_h	=	11 mm
Tensile strength of plate	σ_p	=	200 MPa
Shear strength of rivet	σ_s	=	100 MPa
Bearing strength of rivet	σ_c	=	150 MPa

39. If the rivets are to be designed to avoid crushing failure, the maximum permissible load P in kN is

(a) 7.50 (b) 15.00
(c) 22.50 (d) 30.00

(GATE 2013)

Solution. The maximum permissible load P to avoid crushing of the rivets is determined as

$$F = n \times dt\sigma_c = 3 \times 10 \times 5 \times 150 = 22500 \text{ N} = 22.5 \text{ kN}$$

Ans. (c)

40. If the plates are to be designed to avoid tearing failure, the maximum permissible load P in kN is

(a) 83 (b) 125
(c) 167 (d) 501

(GATE 2013)

Solution. The maximum permissible load P to avoid tearing of the plates is determined as

$$F = (w - nd_h)\, t\sigma_t = (200 - 3 \times 11) \times 5 \times 200 = 167000 \text{ N} = 167 \text{ kN}$$

Ans. (c)

MULTIPLE CHOICE QUESTIONS

1. From a tension test, the yield strength of steel is found to be 200 N/mm^2. Using a factor of safety of 2 and applying maximum principal stress theory of failure, the permissible stress in the steel shaft subjected to torque will be

(a) 50 N/mm^2 (b) 57.7 N/mm^2
(c) 86.6 N/mm^2 (d) 100 N/mm^2

2. Permissible bending moment in a circular shaft under the pure bending is M according to maximum principal stress theory of failure. According to maximum shear stress theory of failure, the permissible bending moment in the shaft is

(a) $M/2$ (b) M
(c) $\sqrt{2}M$ (d) $2M$

3. A component made of ductile material loses its engineering usefulness due to large amount of

(a) elastic deformation before yield stress is reached
(b) plastic deformation after yield stress is reached
(c) fracture after elastic and plastic deformation
(d) all of the above

4. The failure of components made of brittle materials seen as

(a) yielding before plastic deformation
(b) elastic deformation and plastic deformation
(c) sudden fracture after plastic deformation
(d) sudden fracture without any plastic deformation

5. Rankine's theory predicts that failure under bi-axial stress will occur whenever

(a) one of the two principal stresses becomes equal to or exceed the yield point stress
(b) both of the two principal stresses become equal to or exceed the yield point stress
(c) the maximum shear stress becomes equal to or exceed the yield point stress
(d) the maximum shear stress becomes equal to or exceed the shear stress at yield point

6. If maximum normal stress at the time of failure in the simple uni-axial test is σ_f, the corresponding failure value of principal shear strain τ_f for the test will be equal to

(a) $2\sigma_f$ (b) σ_f
(c) $\sigma_f/2$ (d) not related to σ_f

7. A material is subjected to two-dimensional stresses σ_1 and σ_2. If μ and E are the Poisson's ratio and modulus of elasticity of the material, then according to the maximum strain energy theory of failure for factor of safety N, the tensile strength of the material shall be given by

(a) $(\sigma_t/N)^2 = \sigma_1^2+\sigma_2^2-\mu\sigma_1\sigma_2$
(b) $(\sigma_t/N)^2 = \sigma_1^2+\sigma_2^2+\mu\sigma_1\sigma_2$
(c) $(\sigma_t/N)^2 = \sigma_1^2+\sigma_2^2-2\mu\sigma_1\sigma_2$
(d) $(\sigma_t/N)^2 = \sigma_1^2+\sigma_2^2+2\mu\sigma_1\sigma_2$

8. If σ_t and μ are the tensile yield strength and Poisson's ratio of the a material subjected bi-axial stresses, the semi-major and semi-minor axes of the ellipse for maximum strain energy theory are given by

(a) $\sigma_{yt}/\sqrt{1-\mu}$, $\sigma_{yt}/\sqrt{1+\mu}$
(b) $\sigma_{yt}/\sqrt{1-\mu}$, $\sigma_{yt}/\sqrt{1-\mu}$
(c) $\sigma_{yt}/\sqrt{1+\mu}$, $\sigma_{yt}/\sqrt{1-\mu}$
(d) $\sigma_{yt}/\sqrt{1+\mu}$, $\sigma_{yt}/\sqrt{1+\mu}$

9. The maximum strain energy theory is limited to validity in

(a) the complete elastic range
(b) the linear elastic range only
(c) the non-linear elastic range only
(d) all of the above

10. Distortion energy is associated with

(a) solely with change in volume
(b) solely with the change in shape
(c) total strain energy
(d) all of the above

11. According to Von Huber Mises Hencky theory, the failure, specially in ductile materials, is related to

(a) the distortion energy only
(b) dialation energy only
(c) both (a) and (b)
(d) none of the above

12. According to maximum distortion energy theory, a material of yield point stress σ_t can be applied pure shear τ equal to

(a) $\sigma_t/2$ (b) $\sigma_t/3$
(c) $\sigma_t/\sqrt{2}$ (d) $\sigma_t/\sqrt{3}$

13. Maximum distortion energy theory is not applicable when

(a) $\sigma_1 = \sigma_2 = \sigma_3 \neq 0$
(b) $\sigma_1 = \sigma_2 \neq \sigma_3 = 0$
(c) $\sigma_1 \neq \sigma_2 = \sigma_3 = 0$
(d) $\sigma_1 \neq \sigma_2 \neq \sigma_3 = 0$

14. Columb-Mohr theory of failure is based suitable for

(a) ductile materials
(b) brittle materials
(c) both (a) and (b)
(d) none of the above

15. Which theory of failure will you use for aluminium components under steady loading:

(a) principal stress theory
(b) principal strain theory
(c) strain energy theory
(d) maximum shear stress theory

16. A small element at the critical section of a component is in a bi-axial state of stress with the two principal stresses being 360 MPa and 140 MPa. The maximum working stress according to distortion energy theory is

(a) 220 MPa (b) 110 MPa
(c) 314 MPa (d) 330 MPa

17. A large uniform plate of 100 mm width containing a rivet-hole of size 5 mm is subjected to uniform uni-axial tension of 120 MPa. The maximum stress in the plate is

(a) 120 MPa (b) 150 MPa
(c) 150 MPa (d) 360 MPa

18. When failure occurs only after the stresses have been repeated a very large number of times, such type of failure is called

(a) ductile failure (b) brittle failure
(c) fatigue failure (d) creep

19. Stress concentration factor is considered in stress analysis for

(a) static loading only
(b) dynamic loading only
(c) both (a) and (b)
(d) none of the above

20. If fatigue stress concentration factor is k_f and theoretical stress concentration factor is k_t, then notch sensitivity q for fatigue loading is expressed as

(a) $(k_f+1)/(k_t+1)$ (b) $(k_f-1)/(k_t+1)$
(c) $(k_f+1)/(k_t-1)$ (d) $(k_f-1)/(k_t-1)$

21. In general, to calculate the endurance strength (σ_e) of a material for a desired finite life cycles, N, which type of curves are used?

(a) σ–$\ln N$ (b) σ–$\log_{10} N$
(c) $\ln\sigma$–$\ln N$ (d) $\log_{10}\sigma$–$\log_{10} N$

22. Select the correct statement about the fatigue failure criterion lines

(a) A parabolic curve joining σ_e on the ordinate and σ_{ut} on the abscissa is called Gerber line.
(b) A straight line joining σ_e on the ordinate and σ_{yt} on the abscissa is called Soderberg line.
(c) A straight line joining σ_e on the ordinate and σ_{ut} on the abscissa is called Goodman line.
(d) All of the above

23. The methods of introducing pre-stressed surface layer in compression include

(a) shot blasting (b) peening
(c) tumbling (d) all of the above

24. In leaf springs, the inner spring of leaf will usually form a crack first because

(a) the load is directly applied on this spring
(b) the radius of curvature of inner leaf is more than that of the outer one
(c) the radius of curvature of inner leaf is less than that of the outer one
(d) all the leaves have different stresses

25. The design considerations for members subjected to fluctuating loads with the same factor of safety yield the most conservative estimates when using

(a) Gerber relation
(b) Soderberg relation
(c) Goodman relation
(d) none of the above

26. Fatigue strength of a rod subjected to cyclic axial force is less than that of a rotating beam of the same dimensions subjected to steady lateral force because

(a) axial stiffness is less than bending stiffness
(b) of absence of centrifugal effects in the rod
(c) the number of discontinuities vulnerable to fatigue are more in the rod
(d) at a particular time the rod has only one type of stress whereas the beam has both the tensile and compressive stresses

27. Carburized machine components have high endurance limit because carburization

(a) raises the yield point of the material
(b) produces a better surface finish
(c) introduces a compressive layer on the surface
(d) suppresses any stress concentration produced in the component

28. If h is the leg of a weld joint, then its throat is equal to

(a) h (b) $h/\sqrt{2}$
(c) $h/2$ (d) $2h$

29. In eccentric loading of welds, the shear stress assumed to be uniformly distributed over the throat area of all the welds is known as

(a) primary shear stress
(b) secondary shear stress
(c) tertiary shear stress
(d) distributed load

30. In eccentric loading of welds, the stress which vary from point to point as proportional to its distance from the center of gravity, is known as

(a) primary shear stress
(b) secondary shear stress
(c) tertiary shear stress
(d) distributed load

31. The permissible stress in a filled weld is 100 N/mm^2. The fillet weld has equal leg length of 15 mm each. The allowable shearing load on weld per cm length of the weld is

(a) 22.5 N (b) 15.0 N
(c) 10.6 N (d) 7.5 kN

32. A circular rod of diameter d is welded to a flat plate along its circumference by fillet weld of thickness t. Assuming τ_w as the allowable shear stress for the weld material, what is the value of the safe torque that can be transmitted?

(a) $\pi d^2 t\tau_w$ (b) $\pi d^2 t\tau_w/2$
(c) $\pi d^2 t\tau_w/\sqrt{8}$ (d) $\pi d^2 t\tau_w/\sqrt{2}$

33. If the ratio of the diameter of rivet hole to the pitch of rivets is 0.25, then the tearing efficiency of the joint is

(a) 0.50 (b) 0.75
(c) 0.25 (d) 0.87

34. The diameter of a rivet connecting plate of thickness t in mm is given by Unwin's formula

(a) $d = 6.04\sqrt{t}$ (b) $d = 4.05\sqrt{t}$
(c) $d = 1.9\sqrt{t}$ (d) $d = 1.5\sqrt{t}$

35. A fine screw thread having major diameter of 8 mm and pitch of 1 mm metric size is designated as

(a) M8$\times$ 1 (b) M8
(c) 1M8 (d) 8M$\times$1

36. Bolts in the flanged end of pressure vessel are usually pre-tensioned. Indicate which of the following statements is not true.

(a) Pre-tensioning helps to seal the pressure vessel
(b) Pre-tensioning increases the fatigue life of the bolts
(c) Pre-tensioning reduces the maximum tensile stress in the bolts
(d) Pre-tensioning helps to reduce the effect of pressure pulsations in the pressure vessel

37. A bolt of uniform strength can be developed by

(a) keeping the core diameter of threads equal to the diameter of unthreaded portion of the bolt
(b) keeping the core diameter smaller than the diameter of the unthreaded portion
(c) keeping the nominal diameter of threads equal tot he diameter of unthreaded portion
(d) one end fixed and other end free

38. The bolts in a rigid flanged coupling connecting two shafts transmitting power are subjected to

(a) shear force and bending moment
(b) axial force
(c) torsion and bending moment
(d) torsion

39. The hemispherical end of a pressure vessel is fastened to the cylindrical portion of the pressure vessel with the help of gasket, bolts and lock nuts. The bolts are subjected to

(a) tensile stress
(b) compressive stress
(c) shear stress
(d) bearing stress

40. How can shock absorbing capacity of a bolt be increased?

(a) by tightening it properly
(b) by increasing the shank
(c) by grinding the shank
(d) by making the shank equal to the core diameter

41. Eight bolts are to be selected for fixing the cover plate of a cylinder subjected to a maximum load of 980 kN. If the design stress for the bolt material is 315 N/mm^2, what is the diameter of each bolt?

(a) 10 mm (b) 22 mm
(c) 30 mm (d) 36 mm

42. The bolts in a rigid flanged coupling connecting two shafts transmitting power are subjected to

(a) shear force and bending moment
(b) axial force
(c) torsion
(d) torsion and bending moment

43. Match List I with List II and select the correct answer.

List I (Type of keys)	List II (Characteristics)
A. Woodruff key	1. Loose fitting, light duty
B. Kennedy key	2. Heavy duty
C. Feather key	3. Self-aligning
D. Flat key	4. Normal industrial use

(a) A-2, B-3, C-1, D-4
(b) A-3, B-2, C-1, D-4
(c) A-2, B-3, C-4, D-1
(d) A-3, B-2, C-4, D-1

44. A key connecting a flange coupling to a shaft is likely to fail in

(a) shear (b) tension
(c) torsion (d) bending

45. Which key is preferred for the condition where a large amount of impact type torque is to be transmitted in both directions of rotation?

(a) Woodruff key (b) Feather key
(c) Gib-head key (d) Tangent key

46. In the assembly design of shaft, pulley, and key, the weakest member is

(a) pulley (b) key
(c) shaft (d) none

47. A square key of side $d/4$ is to be fitted on a shaft of diameter d and in the hub of a pulley, if the material of the key and shaft is same and the two are to be equally strong in shear, what is the length of the key?

(a) $\pi d/2$ (b) $2\pi d/3$
(c) $3\pi d/4$ (d) $4\pi d/5$

48. For the design of a shaft, subjected to bending moment M and torque T, the equivalent bending moment is given by

(a) $\sqrt{M^2+T^2}$
(b) $\left(M+\sqrt{M^2+T^2}\right)/2$
(c) $(M+T)/2$
(d) $\sqrt{M^2+T^2}/4$

49. A shaft subjected to fluctuating loads for which the normal torque (T) and bending moment (M) are 1000 Nm and 500 Nm respectively. If the combined shock and fatigue factor for bending is 1.5 and for torsion is 2, then the equivalent twisting moment for the shaft is

(a) 2000 Nm (b) 2050 Nm
(c) 2100 Nm (d) 2136 Nm

50. A solid shaft transmits a torque T. The allowable shearing stress is τ. What is the diameter of the shaft?

(a) $\sqrt[3]{\dfrac{16T}{\pi\tau}}$ (b) $\sqrt[3]{\dfrac{32T}{\pi\tau}}$

(c) $\sqrt[3]{\dfrac{16T}{\tau}}$ (d) $\sqrt[3]{\dfrac{T}{\tau}}$

51. Which material is used for bushes in the bushed pin type of flexible coupling?

(a) Gun metal (b) plastic
(c) Rubber (d) Aluminium

52. Which factor is used to consider magnitude of dynamic load in gear design?

(a) Lewis form factor
(b) Form factor
(c) Service factor
(d) Velocity factor

53. The failure of fillet welds at the throat area is usually due to

(a) shear (b) tension
(c) bending (d) torsion

54. The limiting wear load of spur gear is proportional to (where E_p =Young's modulus of pinion material; E_g =Young's modulus of gear material)

(a) $\dfrac{1}{E_p+E_g}$ (b) $\dfrac{E_p+E_g}{E_pE_g}$

(c) $1+\dfrac{E_p}{E_g}$ (d) $1+\dfrac{E_g}{E_p}$

55. In the formulation of Lewis equation for toothed gearing, it is assumed that tangential tooth load F_t acts on the

(a) pitch point
(b) tip of the tooth
(c) root of the tooth
(d) whole face of the tooth

56. A system of lubrication in which the load supporting fluid film is created by the shape and relative motion of the sliding surfaces is known as

(a) hydrodynamic lubrication
(b) hydrostatic lubrication
(c) thin film lubrication
(d) none of the above

57. A system of lubrication in which the load supporting fluid film, separating the two surfaces, is created by an external source is known as

(a) hydrodynamic lubrication
(b) hydrostatic lubrication
(c) thin film lubrication
(d) none of the above

58. A system of lubrication in which the lubricant film is relatively thin and there is partial metal-to-metal contact is known as

(a) thin film lubrication
(b) boundary lubrication
(c) both (a) and (b)
(d) none of the above

59. Hydrostatic bearings, although costly, offer the following advantages

(a) High load carrying capacity
(b) No starting friction
(c) No rubbing action
(d) All of the above

60. If c_r is radial clearance and e is eccentricity in a journal bearing, then the film thickness at any angle ϕ from the minimum thickness point shall be equal to

(a) $c_r\left(1+\dfrac{e}{2c_r}\cos\phi\right)$ (b) $c_r\left(1-\dfrac{e}{2c_r}\cos\phi\right)$

(c) $c_r\left(1+\dfrac{e}{c_r}\cos\phi\right)$ (d) $c_r\left(1-\dfrac{e}{c_r}\cos\phi\right)$

61. If μ is the absolute viscosity, N is speed of journal, and p is bearing pressure, then the bearing characteristic number is determined as

(a) $p/(\mu N)$ (b) $\mu p/N$
(c) $\mu N/p$ (d) $N/(\mu p)$

62. If a bearing has 90% of reliability for life L_{90}, it means that 90 bearings will fail over 100 for given life. Then life for reliability R is given by

(a) $\sqrt[3]{\ln\left(\dfrac{1}{R}\right)/\ln\left(\dfrac{1}{R_{90}}\right)}\times L$

(b) $\sqrt[3]{\ln\left(\dfrac{1}{R_{90}}\right)/\ln\left(\dfrac{1}{R}\right)}\times L$

(c) $\sqrt[1.17]{\ln\left(\dfrac{1}{R}\right)/\ln\left(\dfrac{1}{R_{90}}\right)}\times L$

(d) $\sqrt[1.17]{\ln\left(\dfrac{1}{R_{90}}\right)/\ln\left(\dfrac{1}{R}\right)}\times L$

63. If for a bearing, revolution per hour is N, life in hours is L_h, therefore, life in million revolutions is

(a) $60NL_h/10^6$ (b) $60L_h/\left(N \times 10^6\right)$
(c) $NL_h/\left(60 \times 10^6\right)$ (d) $L_h/\left(60 \times N \times 10^6\right)$

64. The life of a ball bearing at a load of 10 kN is 8000 hours. Its life in hours, if the load is increased to 20 kN, keeping all other conditions the same, is

(a) 400 (b) 2000
(c) 1000 (d) 500

65. Match List I with List II and select the correct answer.

List I		List I	
A.	Automobile wheel mounting on axle	1.	Magneto bearing
B.	High speed grinding spindle	2.	Angular contact bearing
C.	IC Engine connecting rod	3.	Taper roller bearing
D.	Leaf spring eye mounting	4.	Hydrodynamic journal bearing
		5.	Sintered metal bearing
		6.	Teflon/Nylon bush

(a) A-3, B-1, C-6, D-5
(b) A-1, B-2, C-3, D-4
(c) A-1, B-2, C-4, D-6
(d) A-3, B-1, C-4, D-6

66. The dynamic load capacity of 6306 bearing is 22 kN. The maximum radial load it can sustain to operate at 600 rev/min, for 2000 hours is:

(a) 4.16 kN (b) 3.60 kN
(c) 6.25 kN (d) 5.29 kN

67. To restore stable operating condition in a hydrodynamic journal bearing, when it encounters higher magnitude loads,

(a) oil viscosity should be decreased
(b) oil viscosity should be increased
(c) oil viscosity index should be increased
(d) oil viscosity index should be decreased

68. In thick film hydrodynamic journal bearings, the coefficient of friction

(a) increases with increase in load
(b) is independent of load
(c) decreases with increase in load
(d) can increase or decrease with increase in load

69. A journal bearing of diameter 25 cm and length 40 cm carries a load of 150 kN. The average bearing pressure is

(a) 0.15 kN/cm^2 (b) 1.5 kN/cm^2
(c) 15 kN/cm^2 (d) 150 kN/cm^2

70. Deep groove ball bearings are used for

(a) heavy thrust load body
(b) small angular displacement of shafts
(c) radial load at high speed
(d) combined thrust and radial loads at high speed

71. The most suitable bearing for carrying very heavy loads with slow speed is

(a) hydrodynamic bearing
(b) ball bearing
(c) roller bearing
(d) hydrostatic bearing

72. In a journal bearing, the radius of the friction circle increases with the increase in

(a) load
(b) radius of the journal
(c) speed of the journal
(d) viscosity of the lubricant

73. A full journal bearing having clearance to radius ratio of 1/100, using a lubricant with $\mu = 28 \times 10^{-3}$ Pa·s supports the shaft journal running at N = 2400 rpm. If bearing pressure is 1.4 MPa, the Sommerfeld number is

(a) 8×10^{-3} (b) 8×10^{-5}
(c) 0.48 (d) 0.48×10^{-2}

74. It is seen from the curve that there is a minimum value of the coefficient of friction (μ) for a particular value of the bearing characteristic number. What is this value called?

(a) McKee number
(b) Reynolds number
(c) Bearing modulus
(d) Sommerfeld number

75. Anti-friction bearings are

(a) sleeve bearings
(b) gas lubricated bearings
(c) ball and roller bearings

(d) journal bearings

76. If the load on a ball bearing is halved, its life

(a) remains unchanged
(b) increases two times
(c) increases four times
(d) increases eight times

77. On what does the basic static capacity of a ball bearing depend?

(a) It is directly proportional to number of balls in a row and diameter of ball
(b) It is directly proportional to square of ball diameter and inverse of number of rows of balls
(c) It is directly proportional to number of balls in a row and square of diameter of ball
(d) It is inversely proportional to square of diameter of ball and directly proportional to number of balls in a row

78. In journal bearing design, the factor ZN/p is called the 'bearing characteristic number, where Z is absolute viscosity of the lubricant, N is the speed of journal in rpm, and p is the bearing pressure on the projected bearing area. Let K be the value of ZN/p corresponding to the minimum amount of friction. For hydrodynamic lubrication of the bearing, ZN/p should be

(a) larger than K (b) smaller than K
(c) equal to K (d) equal to zero

79. Hydrodynamic bearings are preferred over rolling element bearings in some applications in which

(a) power losses are minimum
(b) higher loads are to be carried at relatively high speed
(c) higher loads are to be carried at relatively low speed
(d) they are available 'off the shelf' in the market

80. The life of a ball-bearing (L) is related to dynamic load carrying capacity P as

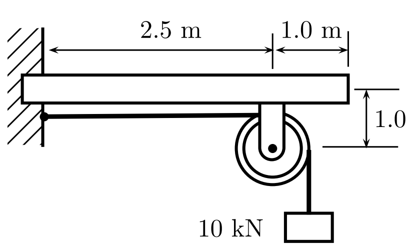
(a) $L \propto P^{-1/3}$ (b) $L \propto P^{-3}$
(c) $L \propto P^{-3.3}$ (d) $L \propto P^{-2}$

81. Starting friction is low in

(a) hydrostatic lubrication
(b) hydrodynamic lubrication
(c) mixed (or semi-fluid) lubrication
(d) boundary lubrication

NUMERICAL ANSWER QUESTIONS

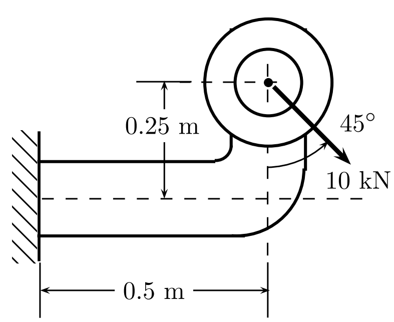
1. A cantilever beam of rectangular cross-section is used to support a pulley as shown in the figure:



The tension in the wire rope is 10 kN. The yield strength of the beam material is 250 N/mm^2. The factor of safety is 1.5. The ratio of depth to width of the cross-section is 3. Calculate the maximum bending moment in the beam and the width of the cross-section of the beam using the maximum normal stress theory of failure.

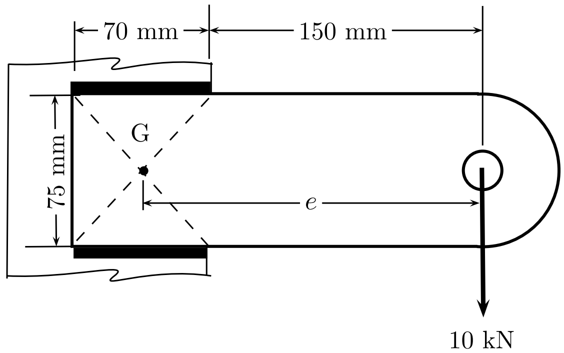
2. Two plates of width 300 mm are fixed together in a butt joint by means of four rivets. The plates are subjected to tensile force 80 kN. The tensile yield strength of both the plates and rivets is 240 N/mm^2. The yield strength in shear is 57.7% of the tensile yield strength, and the factor of safety is 2.5. The stress concentration is negligible. Determine the diameter of the rivets and thickness of the plates.

3. A gray cast iron bracket having a rectangular cross-section is used to support a load 10 kN acting at 45° to the vertical.



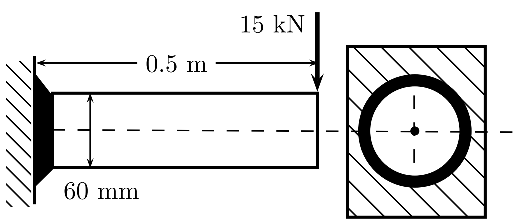
The depth of the cross-section is twice the width. The yield strength of gray cost iron is 160 N/mm^2. The factor of safety is 2.5. Calculate the width of the cross-section of the beam using the maximum normal stress theory of failure.

4. A bracket joined by two beads of weld supports an eccentric load of 10 kN.



The permissible shear stress for the weld is 150 N/mm^2. Determine the size of the welds.

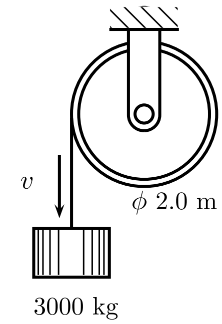
5. A shaft of diameter 60 mm is welded by a ring of fillet weld bead to support an eccentric load of 15 kN.



The permissible shear stress for the weld is 150 N/mm^2. Note that the moment of inertia about diametral axis of a circular ring of thickness t and radius r is πtr^3. Calculate the size of the welds. Also calculate the maximum bending stress in the weld.

6. A shaft of diameter 32 mm transmits 20 kW at 780 rpm through a gear mounted on a key made of steel for which yield strength in tensile is 450 MPa and same in compression. The factor of safety is kept at 2.5 and width or height of the key is kept one-fourth of the diameter of the shaft. Using distortion energy theory, determine the length of the key.

7. A mass of 3000 kg attached to a string is lowered at a velocity of 2.0 m/s from a drum of diameter 2 m and mass 100 kg.



Radius of gyration of the drum is 1.0 m. On applying the brake, the mass is brought to rest in a distance of 1.0 m. Determine the energy absorbed by the brake and its torque capacity.

8. A centrifugal clutch is used to transmit power at running speed of 1440 rpm through four shoes, each having a mass of 2 kg. In the engaged position of the clutch, the radius to the center of gravity of the shoe is 100 mm, while the inner radius of the drum is 200 mm. The coefficient of friction is 0.25. The spring force at the beginning of engagement is 1 kN. Determine the speed at which the engagement begins. Also calculate the the power transmitted by the clutch at 1400 rpm.

9. A taper roller bearing has a dynamic load capacity of 30 kN. The desired life for 90% of the bearings is 9000 hours at 250 rpm. Calculate the life of the bearing in million revolutions. Also calculate the equivalent radial load that the bearing can carry.

10. Work cycle of a ball bearing is divided into three parts of radial loads:

(a) 2000 N at 1200 rpm for one quarter cycle,

(b) 4000 N at 800 rpm for one half cycle, and

(c) 1500 N at 1600 rpm for the remaining cycle.

The expected life of the bearing is 12000 hours. Calculate the life of the bearing in million revolutions. Also calculate the equivalent radial load that the bearing can carry.

11. Desired life of a ball bearing for reliability of 99% and radial load of 10 kN is 10000 hours at 1600 rpm. Calculate the life of bearing in millions for 99% reliability, and the dynamic load carrying capacity life of bearing for reliability of 90%.

ANSWERS

Multiple Choice Questions

1. (d)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (b)
11. (a)	**12.** (d)	**13.** (a)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (d)
21. (b)	**22.** (d)	**23.** (d)	**24.** (b)	**25.** (b)	**26.** (c)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (b)
31. (c)	**32.** (c)	**33.** (b)	**34.** (a)	**35.** (a)	**36.** (a)	**37.** (a)	**38.** (a)	**39.** (a)	**40.** (d)
41. (c)	**42.** (a)	**43.** (b)	**44.** (a)	**45.** (d)	**46.** (b)	**47.** (a)	**48.** (b)	**49.** (d)	**50.** (a)
51. (c)	**52.** (d)	**53.** (a)	**54.** (b)	**55.** (b)	**56.** (a)	**57.** (b)	**58.** (c)	**59.** (d)	**60.** (c)
61. (c)	**62.** (c)	**63.** (a)	**64.** (c)	**65.** (d)	**66.** (d)	**67.** (b)	**68.** (c)	**69.** (a)	**70.** (d)
71. (d)	**72.** (b)	**73.** (a)	**74.** (c)	**75.** (c)	**76.** (d)	**77.** (c)	**78.** (a)	**79.** (c)	**80.** (a)
81. (a)									

Numerical Answer Questions

1. 51.92 mm	**2.** 21.44 mm, 3.89 mm	**3.** 50.26 mm
4. 4.05 mm	**5.** 8.84 mm, 300 MPa	**6.** 22 mm
7. 35.63 kJ, 35.63 kNm	**8.** 675.23 rpm, 106.98 kW	**9.** 135, 6.755 kN
10. 792, 27.65 kN	**11.** 7153, 192.68 kN	

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (d) For a shaft subjected to torsion, the maximum principal stress will be equal to the maximum shear stress. It means, that tensile stress should be below the yield strength, i.e. $200/2 = 100\ \text{N/mm}^2$.

2. (b) Consider a shaft of diameter d. By maximum principal stress theory, bending stress on pure bending M will be

$$\sigma = \frac{32M}{\pi d^3}$$

The maximum shear stress in pure bending will be

$$\tau = \frac{\sigma}{2}$$

This is the same ratio by using maximum shear stress theory. Therefore, value of bending moment will be same.

3. (b) A ductile material fails by yielding.

4. (d) Brittle materials fail by sudden fracture without deformation.

5. (a) The maximum normal stress theory also known as Rankine's theory predicts that failure will occur whenever one of the two principal stresses become equal to or exceed the yield point stress.

6. (c) Using Mohr's circle, the corresponding failure value of principal shear strain τ_f for the test will be equal to $\sigma_f/2$.

7. (c) According to the maximum strain energy theory of failure for factor of safety N, the tensile strength of the material will be given by

$$\left(\frac{\sigma_t}{N}\right)^2 = \sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2$$

8. (a) The semi-major and semi-minor axes of the ellipse for maximum strain energy theory are given by

$$\frac{\sigma_{yt}}{\sqrt{1-\mu}},\ \frac{\sigma_{yt}}{\sqrt{1+\mu}}$$

9. (b) Since Hooke's law has been utilized, the maximum strain energy theory is limited to

validity to the linear elastic range, just as the normal strain theory.

10. (b) It is postulated that the strain energy can be divided into two parts: the energy associated solely with change in volume, termed dilatation energy, and the energy associated with the change in shape, termed distortion energy.

11. (a) The maximum distortion energy theory, also known as Von Huber Mises Hencky theory, predicts that the failure, specially in ductile materials, is related only to the distortion energy, with no contribution from the dilatation energy.

12. (d) According to maximum distortion energy theory, the maximum shear stress for yield point σ_t is given by

$$\tau = \frac{\sigma_t}{\sqrt{3}}$$

13. (a) For $\sigma_1 = \sigma_2 = \sigma_3 \neq 0$, distortion energy becomes zero because then material is subjected to uniform stress state that will cause only volumetric changes. Hence, the distortion energy theory is not applicable for this special case.

14. (b) Columb-Mohr theory of failure is used for brittle materials (e.g. cast iron) subjected to compressive and tensile stresses.

15. (d) Aluminium is a highly ductile material for which maximum shear stress theory is most suitable.

16. (c) Given that

$$\begin{aligned} \sigma_1 &= 360 \text{ MPa} \\ \sigma_2 &= 140 \text{ MPa} \\ \sigma_{12} &= 0 \end{aligned}$$

Distortion energy theory for plane stress,

$$\begin{aligned} \sigma_y &= \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 + 3\sigma_{12}^2} \\ &= 314.32 \text{ MPa} \end{aligned}$$

17. (d) Due to rivet-hole, the maximum stress due to stress concentration is 3σ.

18. (c) The most distinguishing characteristics of fatigue failure is that the failure occurs only after the stresses have been repeated a very large number of times.

19. (b) In static loading of ductile materials, stress concentration is compensated by local yielding. Therefore, stress concentration factor is not considered in static loading.

20. (d) The notch sensitivity q for fatigue loading is expressed as

$$q = \frac{k_f - 1}{k_t - 1}$$

21. (b) To calculate the endurance strength (σ_e) of a material for a desired finite life cycles, N, σ-$\log_{10} N$ curves are used.

22. (d) These three truly defined lines divides safe region from unsafe region for various combinations of σ_a and σ_v.

23. (d) There are several methods of introducing pre-stressed surface layer in compression, for example, shot blasting, peening, tumbling or cold working by rolling.

24. (b) Due to less radius of curvature of inner springs, there are more bending and fatigue stresses. Hence, inner springs crack first.

25. (b) A straight line joining σ_e on the ordinate and σ_y on the abscissa is called Soderberg line, which is the most conservative criterion.

26. (c) The whole volume of the rod is subjected to a cyclic axial stress, where the probability of discontinuities is throughout the volume, while in the case of rotating beam, the maximum bending moment can be at mid-section only.

27. (c) Carburizing [discussed in Chapter 10] is case hardening process which introduces a compressive layer on the surface which enhances the endurance limit of the component.

28. (b) Throat t is the smallest area in the weld, and is determined in terms of leg h as

$$t = \frac{h}{\sqrt{2}}$$

29. (a) The eccentric force result into direct shear stress, known as primary shear stress, assumed to be uniformly distributed over the throat area of all the welds.

30. (b) In eccentric loading of welds, the secondary shear stress at any point is proportional to its distance from the center of gravity.

31. (c) Given that

$$\begin{aligned} \tau_{\max} &= 100 \text{ N/mm}^2 \\ t &= 15 \text{ mm} \\ l &= 10 \text{ mm} \end{aligned}$$

Therefore,

$$\begin{aligned} F &= \tau_{\max} \times l \times \frac{t}{\sqrt{2}} \\ &= 10.6 \text{ kN} \end{aligned}$$

32. (c) The safe torque is given by

$$T = \frac{d}{2}\tau_w \times \pi d \times \frac{t}{\sqrt{2}} = \frac{\pi d^2 \tau_w t}{2\sqrt{2}}$$

33. (b) If p is the pitch and D is the diameter of the hole, the tearing efficiency of a circumferential joint is defined as

$$\eta = \frac{p-D}{p} = 1-\frac{D}{p} = 1-0.25 = 0.75$$

34. (a) In practice, diameter of rivet hole for plate thickness greater than 8 mm is taken as

$$d = 6.04\sqrt{t}$$

This is known as Unwin's formula.

35. (a) A metric ISO screw thread is designated by the letter M followed by the value of the nominal diameter D and the pitch P, both expressed in millimeters and separated by the multiplication sign, $\times$ (e.g., $M8 \times 1$).

36. (a) Pre-tensioning of these bolts helps this when the pressure vessel is loaded and the bolt gets elastic deformation, thus helping to seal the pressure vessel.

37. (a) In a bolt of uniform strength, the entire bolt is stressed to the same limiting value, thus resulting into maximum energy absorption.

38. (a) The bolts of rigid flanged coupling have to bear shear stress and bending moments.

39. (a) The fastening bolts are under tensile stress.

40. (d) The shock absorbing capacity of bolt can be increased if the shank of bolt is turned down to a diameter equal to the root diameter of threads or even less.

41. (c) The diameter will be given by

$$8 \times 315 \times \frac{\pi d^2}{4} = 980 \times 10^3$$
$$d = 22.251 \text{ mm}$$

42. (a) The torque generates tangential force which results into bending and shear on the bolts.

43. (b) Woodruff keys are semicircular shaped, so self aligning. Woodruff key is that it eliminates milling a key-way near shaft shoulders, which already have stress concentrations. Kennedy keys are used in heavy duty applications. A key attached to one member of a pair and which permits relative axial movement of the other is known as feather key. Flat keys are used in normal industrial applications.

44. (a) In general, keys are design against shear and compression, hence, out of given options, only shear is the correct.

45. (d) The tangent keys are fitted in pair at right angles. Each key can withstand torsion in one direction only. These are used in large heavy duty shafts.

46. (b) Key is the weakest member in the assembly.

47. (a) For the given case,

$$\frac{16T}{\pi d^3} = \frac{\frac{T}{d/2}}{l \times d/2}$$
$$l = \frac{\pi d}{2}$$

48. (b) Equivalent bending moment is

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$

49. (d) Given that

$$T = 1000 \text{ Nm}$$
$$M = 500 \text{ Nm}$$
$$k_m = 1.5$$
$$k_t = 2$$

Equivalent twisting moment is

$$T_e = \sqrt{(k_m M)^2 + (k_t T)^2} = 2136 \text{ Nm}$$

50. (a) The shear stress is related to torque as

$$\tau = \frac{16T}{\pi d^3}$$
$$d = \sqrt[3]{\frac{16T}{\pi \tau}}$$

51. (c) Rubber bushes are used in flexible couplings.

52. (d) To overcome the difficulty in calculating the exact magnitude of dynamic load, a velocity factor is used.

53. (a) The failure of fillet welds is usually due to shear at the throat area of the weld.

54. (b) Wear strength is provided by surface endurance strength for repeated loads σ_{es} which is approximately equal to 1.75× BHN MPa for steel and 0.5× BHN MPa for cast iron. Wear strength is given by

$$F_w = \frac{{\sigma_{es}}^2}{1.4} b D_p \sin\phi \times \left(\frac{1}{E_p} + \frac{1}{E_g}\right) \frac{2z_p}{z_g - z_p}$$

55. (b) At section of minimum cross-section area, as the force F_t acts at height h above the pitch (tip of tooth), the acting bending moment is

$$M_b = F_T h$$

56. (a) Hydrodynamic lubrication is defined as a system of lubrication in which the load supporting fluid film is created by the shape and relative motion of the sliding surfaces.

57. (b) Hydrostatic lubrication is defined as a system of lubrication in which the load supporting fluid film, separating the two surfaces, is created by an external source, like a pump, supplying sufficient fluid under pressure.

58. (c) Thin film lubrication, which is also called boundary lubrication is defined as a condition where the lubricant film is relatively thin and there is partial metal-to-metal contact.

59. (d) Hydrostatic bearings offer high load carrying capacity, no starting friction, no rubbing action.

60. (c) The film thickness at any angle ϕ from the minimum thickness point is given by

$$h = c_r\left(1 + \frac{e}{c_r}\cos\phi\right)$$

61. (c) Bearing characteristic number is $\mu N/p$.

62. (c) The life for reliability R is given by

$$L_R = \sqrt[1.17]{\frac{\ln(1/R)}{\ln(1/R_{90})}} \times L$$

63. (a) Life of the bearing in million revolutions is determined as

$$L = \frac{60 N L_h}{10^6}$$

64. (c) Given that

$$\begin{aligned} P_1 &= 10 \text{ kN} \\ L_1 &= 8000 \text{ hours} \\ P_2 &= 20 \text{ kN} \end{aligned}$$

For ball bearings:

$$\begin{aligned} C &= PL^{1/3} \\ L_2 &= L_1 \times \left(\frac{P_1}{P_2}\right)^3 \\ &= 8000 \times \frac{1}{2^3} \\ &= 1000 \text{ hours} \end{aligned}$$

65. (d) The IC engine connecting rod bearings are hydrodynamic journal bearings where pressure is generated due to rotation of the journal. Taper roller bearings are used in automobile axles for wheel mounting to avoid for high capacity. The magneto bearing is a bearing which can be dismantled and which consists of a deep groove ball bearing inner ring and an outer ring with one shoulder only, thus, these are used in grinding spindles for their easy replacement and installations. Taflon/Nylon bushes are used in the eye mounting of leaf springs for smooth operation.

66. (d) Given that

$$\begin{aligned} C &= 22 \text{ kN} \\ N &= 600 \text{ rpm} \\ L_h &= 2000 \text{ hours} \end{aligned}$$

Life of bearing in million revolutions (L) is

$$\begin{aligned} L &= \frac{60 \times 600 \times 2000}{10^6} \\ &= 72 \end{aligned}$$

The designation indicates a ball bearing, thus

$$\begin{aligned} C &= PL^{1/3} \\ P &= \frac{C}{L^{1/3}} \\ &= \frac{22}{72^{1/3}} \\ &= 5.288 \text{ kN} \end{aligned}$$

67. (b) Viscosity index is an arbitrary measure for the change of viscosity with temperature, that it is relevant when temperature is increasing. To restore the load, the viscosity of the oil should be increased.

68. (c) In thick film hydrodynamic journal bearings, coefficient of friction between the journal and bearing first decreases to minimum value at an optimum speed and then increases with further speed.

69. (a) Average bearing pressure is calculated as

$$\begin{aligned} p &= \frac{150}{40 \times 25} \\ &= 0.15 \text{ kN/cm}^2 \end{aligned}$$

70. (d) Deep groove ball bearings are able to bear thrust and radial loads at high speed.

71. (d) Hydrostatic bearings uses external device for making the lubrication film. Thus, suitable for carrying heavy loads with slow speeds.

72. (b) Radius of friction circle is $r\sin\phi$.

73. (a) Given that

$$\begin{aligned}\mu &= 28\times10^{-3}\ \text{Pa·s}\\ N &= 2400\ \text{rpm}\\ p &= 1.4\ \text{MPa}\\ r/c &= 100\end{aligned}$$

The speed in rps is determined as

$$N' = \frac{N}{60}\ \text{rps}$$

Sommerfeld number is given by

$$\begin{aligned}S &= \frac{\mu N'}{p}\left(\frac{r}{c}\right)^2\\ &= 8\times10^{-3}\end{aligned}$$

74. (c) The value of bearing characteristic number corresponding to the minimum coefficient of friction is called bearing modulus, denoted by K.

75. (c) In rolling contact bearings, the rolling contacts results in low coefficient of friction. Therefore, rolling contact bearings are called anti-friction bearings. However, this is misnomer. There is always friction at the contacting surfaces between the rolling element and the inner and outer cages.

76. (d) If dynamic load carrying capacity is C N, equivalent dynamic load is P, then, there is a relation between load and life of bearing

$$L = \left(\frac{C}{P}\right)^n$$

$$C = P\times L^{1/n}$$

where

$$n = \begin{cases}3 & \text{Ball bearings}\\ 10/3 & \text{Roller bearings}\end{cases}$$

77. (c) The static load carrying capacity (C_0) of the bearing is defined as the static load which corresponds to a total permanent deformation and balls and races at the most heavily stressed point of contact equal to 0.0001 times of ball diameter.

If number of balls is z with diameter d then Seebeck's equation gives value this

$$C_0 = k\frac{d^2z}{5}$$

where k is a constant of proportionality.

78. (a) The bearing should not be operated near the critical value K. In order to avoid seizure, the operating value of bearing characteristic number ($\mu N/p$) should be at least 5 to 6 times of K while for fluctuating loads it should be at least $15K$.

79. (c) In hydrodynamic bearings, the load supporting fluid film is created by the shape and relative motion of the sliding surfaces. Thus these bearings are preferred where speed is high.

80. (a) For ball bearings

$$C = P\times L^{1/3}$$

81. (a) Hydrostatic lubrication uses external device to introduce the film of lubrication, thus, starting friction is low.

Numerical Answer Questions

1. Maximum bending moment is at the fixed end:

$$\begin{aligned}M &= 10\times2.5+10\times1.0\\ &= 35\ \text{kNm}\end{aligned}$$

The allowable bending stress in the material is

$$\begin{aligned}\sigma_b &= \frac{250}{1.5}\\ &= 166.7\ \text{N/mm}^2\end{aligned}$$

The width of cross-section w is given by

$$\begin{aligned}\sigma_b &= \frac{M\times3w/2}{w\times(3w)^3/12}\\ &= \frac{2M}{3w^3}\\ w &= \sqrt[3]{\frac{2M}{3\sigma_b}}\\ &= \sqrt[3]{\frac{2\times35\times10^3}{3\times166.7\times10^6}}\\ &= 51.92\ \text{mm}\end{aligned}$$

2. The diameter of the rivets is

$$4 \times \frac{\pi d^2}{4} \times \frac{0.577 \times 240}{2.5} = 80 \times 10^3$$
$$d = 21.44 \text{ mm}$$

The thickness of the plates shall be

$$(300 - 4 \times 21.44)\, t \times \frac{240}{2.5} = 80 \times 10^3$$
$$t = 3.89 \text{ mm}$$

3. Maximum bending moment is at the top edge of the fixed end, given by

$$\begin{aligned} M &= 10 \sin 45^\circ \times 0.25 + 10 \cos 45^\circ \times 0.5 \\ &= 5.30 \text{ kNm} \end{aligned}$$

The allowable bending stress in the material is

$$\begin{aligned} \sigma_b &= \frac{160}{2.5} \\ &= 64 \text{ N/mm}^2 \end{aligned}$$

If the width of cross-section w mm, the bending stress due to M is given by

$$\begin{aligned} \sigma_b &= \frac{M \times 2w/2}{w \times (2w)^3/12} \\ &= \frac{12M}{8w^3} \\ &= \frac{12 \times 5.30 \times 10^6}{8w^3} \\ &= \frac{7.95 \times 10^6}{w^3} \text{ N/mm}^2 \end{aligned}$$

The direct tensile stress due to horizontal component of the force is

$$\begin{aligned} \sigma_t &= \frac{10 \times 10^3 \sin 45^\circ}{w \times 2w} \\ &= \frac{3.535 \times 10^3}{w^2} \text{ N/mm}^2 \end{aligned}$$

Therefore

$$64 = \frac{3.535 \times 10^3}{w^2} + \frac{7.95 \times 10^6}{w^3}$$

Solving by Newton-Raphson Method with initial value $w = 50$ mm:

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$
$$w = 50.26 \text{ mm}$$

4. The welds are positioned symmetrically. Bending moment at center of gravity G is

$$\begin{aligned} M &= 10 \times 10^3 \times \left(150 + \frac{75}{2}\right) \\ &= 1875.0 \times 10^3 \text{ N mm} \end{aligned}$$

Let t be the size (throat) of the weld in mm. Total area of the welds is

$$\begin{aligned} A &= (75 + 75)\, t \\ &= 150t \end{aligned}$$

Primary shear stress at any point is

$$\begin{aligned} \tau_1 &= \frac{10 \times 10^3}{150t} \\ &= \frac{66.67}{t} \text{ N/mm}^2 \end{aligned}$$

The distance r of the farthest point in the weld from G calculated as

$$\begin{aligned} y &= \sqrt{\left(\frac{75}{2}\right)^2 + \left(\frac{75}{2}\right)^2} \\ &= 53.03 \text{ mm} \end{aligned}$$

The polar moment of inertia of the two welds (about the axis normal to plate) is

$$\begin{aligned} I &= 2 \times 75t \left[\frac{75^2}{12} + \left(\frac{75}{2}\right)^2\right] \\ &= 281.125 \times 10^3 \times t \text{ mm}^4 \end{aligned}$$

Secondary shear stress is

$$\begin{aligned} \tau_2 &= \frac{My}{I} \\ &= \frac{353.69}{t} \text{ N/mm}^2 \end{aligned}$$

This secondary shear stress will act perpendicular the line joining the farthest point and center of gravity, thus making 45° from horizontal. Thus, the effective shear stress will be made of the horizontal and vertical components,

$$\begin{aligned} \tau_h &= \tau_2 \cos 45^\circ \\ &= \frac{250.09}{t} \text{ N/mm}^2 \\ \tau_v &= \tau_2 \sin 45^\circ + \tau_1 \\ &= \frac{316.77}{t} \text{ N/mm}^2 \\ \tau &= \sqrt{\tau_h^2 + \tau_v^2} \\ &= \frac{403.59}{t} \text{ N/mm}^2 \end{aligned}$$

The allowable stress is 150 N/mm^2. Therefore

$$\frac{453.94}{t} = 100$$
$$t = 4.05 \text{ mm}$$

5. Given that $d = 60$ mm. Let t be the size (throat) of the weld in mm. Total area of the welds is

$$\begin{aligned} A &= \pi d t \\ &= 188.49t \text{ mm}^2 \end{aligned}$$

Primary shear stress at any point is calculated as

$$\begin{aligned} \tau_1 &= \frac{10 \times 10^3}{188.49t} \\ &= \frac{53.05}{t} \text{ N/mm}^2 \end{aligned}$$

Bending stress at the top layer of the weld is

$$\begin{aligned} \sigma_b &= \frac{M \times r}{\pi t r^3} \\ &= \frac{15 \times 10^3 \times 500 \times 30}{\pi \times t \times 30^3} \\ &= \frac{2652.58}{t} \text{ N/mm}^2 \end{aligned}$$

Maximum shear stress in the weld is

$$\begin{aligned} \tau &= \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_1^2} \\ &= \frac{1327.35}{t} \text{ N/mm}^2 \end{aligned}$$

Permissible shear stress in the weld is 150 N/mm^2. Therefore,

$$\begin{aligned} \frac{1327.35}{t} &= 150 \\ t &= 8.84 \text{ mm} \end{aligned}$$

Primary shear stress at any point is

$$\begin{aligned} \tau_1 &= \frac{53.05}{t} \\ &= 6 \text{ N/mm}^2 \end{aligned}$$

The maximum value of bending stress is

$$\begin{aligned} \sigma_b &= \frac{2652.58}{t} \\ &= 300 \text{ N/mm}^2 \end{aligned}$$

6. Given that diameter of shaft $d = 30$ mm. Width and height of the key are

$$b = h = \frac{d}{4} = 8 \text{ mm}$$

The torque on the gear is

$$\begin{aligned} T &= \frac{20 \times 10^3}{2\pi \times 780/60} \\ &= 244.85 \times 10^6 \text{ N-mm} \end{aligned}$$

The force on the key is

$$\begin{aligned} F &= \frac{T}{d/2} \\ &= \frac{244.85 \times 10^6}{32/2} \\ &= 15.30 \times 10^3 \text{ N} \end{aligned}$$

Maximum allowable stress is

$$\begin{aligned} \sigma &= \frac{450}{2.5} \\ &= 180 \text{ N/mm}^2 \end{aligned}$$

Using distortion energy theory, maximum allowable shear stress is

$$\begin{aligned} \tau &= \frac{\sigma}{\sqrt{3}} \\ &= 103.92 \text{ N/mm}^2 \end{aligned}$$

For shear stress limitation,

$$\begin{aligned} \frac{F}{l \times b} &= \tau \\ l &= \frac{F}{b \times \tau} \\ &= \frac{15.30 \times 10^3}{8 \times 103.92} \\ &= 18.40 \text{ mm} \end{aligned}$$

For compressive stress limitation,

$$\begin{aligned} \frac{F}{l \times h/2} &= \sigma \\ l &= \frac{2F}{h \times \tau} \\ &= \frac{2 \times 15.30 \times 10^3}{8 \times 180} \\ &= 21.25 \text{ mm} \end{aligned}$$

Therefore, $t = 21.25 \approx 22$ mm is safe.

7. The energy absorbed by the brake is

$$\begin{aligned} E &= \frac{3000 \times 2^2}{2} + 3000 \times 9.81 \times 1 + \frac{100}{2} \times 1^2 \left(\frac{2.0}{1.0}\right)^2 \\ &= 35.63 \text{ kJ} \end{aligned}$$

The angle turned by the drum is

$$\begin{aligned} \theta &= \frac{1.0}{1.0} \\ &= 1.0 \text{ rad} \end{aligned}$$

Therefore, the torque capacity is

$$\begin{aligned} T &= \frac{E}{\theta} \\ &= 35.63 \text{ kNm} \end{aligned}$$

8. The spring is preloaded at 1000 N, therefore, the speed at the engagement will be given by

$$\begin{aligned} 1000 &= 2\times\omega_1^2\times 0.100 \\ \omega_1 &= 70.71 \text{ rad/s} \\ N_1 &= 675.23 \text{ rpm} \end{aligned}$$

The initial angular speed is

$$\begin{aligned} \omega_2 &= \frac{2\pi\times 1440}{60} \\ &= 150.79 \text{ rad/s} \end{aligned}$$

Given that

$$\begin{aligned} \mu &= 0.25 \\ r &= 0.10 \text{ m} \\ r_i &= 0.200 \text{ m} \\ m &= 2 \text{ kg} \\ n &= 4 \end{aligned}$$

The friction torque that can be transmitted at 1400 rpm is

$$\begin{aligned} T_f &= \mu n m \left(\omega_2^2-\omega_1^2\right) r\times r_i \\ &= 709.50 \text{ Nm} \end{aligned}$$

The power that can be transmitted as

$$\begin{aligned} P &= T_f\omega_2 \\ &= 106.98 \text{ kW} \end{aligned}$$

9. In one hour, the revolutions are $60\times N$. In 8000 hours, the bearing will rotate the following millions of revolutions:

$$\begin{aligned} L &= \frac{60NL_h}{10^6} \\ &= 135 \end{aligned}$$

Given, $C = 26$ kN. Equivalent radial load is

$$\begin{aligned} P &= \frac{C}{L^{3/10}} \\ &= 6.755 \text{ kN} \end{aligned}$$

10. To simplify the solution, let the bearing undergoes work cycle of pne minute duration. Given that the bearing rotates in three sets of number of revolutions (N_i's):

$$\begin{aligned} N_1 &= \frac{1}{4}\times 1200 \\ &= 300 \\ N_2 &= \frac{1}{2}\times 800 \\ &= 400 \\ N_3 &= \frac{1}{4}\times 1600 \\ &= 400 \end{aligned}$$

The total number of rotation in one cycle of one minute duration (which is the average speed in rpm) is

$$\begin{aligned} N &= \sum N \\ &= 1100 \text{ rpm} \end{aligned}$$

In one hour, the revolutions are $60N$. Therefore, in 12000 hrs the bearing shall rotate following million revolutions:

$$\begin{aligned} L &= \frac{60NL_h}{10^6} \\ &= \frac{60\times 1100\times 12000}{10^6} \\ &= 792 \end{aligned}$$

The equivalent load on the bearing is

$$\begin{aligned} P &= \sqrt[3]{\frac{\sum NP^3}{\sum N}} \\ &= 2988.16 \text{ N} \end{aligned}$$

The dynamic load carrying capacity is

$$\begin{aligned} C &= PL^{1/3} \\ &= 27.65 \text{ kN} \end{aligned}$$

11. The revolutions in million for 99% reliability are

$$\begin{aligned} L_{99} &= \frac{60NL_h}{10^6} \\ &= \frac{60\times 1600\times 10000}{10^6} \\ &= 960 \end{aligned}$$

The life for 90% reliability is given by

$$\begin{aligned} \frac{L_{90}}{L_{99}} &= \left(\frac{\ln(1/0.9)}{\ln 1/0.99}\right)^{1/1.17} \\ L_{90} &= 7153.04 \text{ millions} \end{aligned}$$

Given that radial load $P = 10$ kN. The dynamic load carrying capacity is calculated as

$$\begin{aligned} C &= PL_{90}^{1/3} \\ &= 192.68 \text{ kN} \end{aligned}$$

PART II

FLUID MECHANICS & THERMAL SCIENCES

CHAPTER 6

FLUID MECHANICS

Fluid is a material capable of flowing, which can be in liquid or gas phase. A liquid possesses a definite volume while a gas is compressible but with no definite volume. An *ideal fluid* is incompressible and has neither viscosity nor surface tension. Real fluids are compressible, viscous and possess surface tension. Fluid mechanics is the study of fluids either in motion or at rest; the subject can be broadly divided into two parts: *fluid statics*, the study of fluids at rest, and *fluid dynamics*, the study of fluids in motion.

6.1 FLUID PROPERTIES

Understanding the properties of fluids is essential to analyze their behavior in operating conditions. Some of the fluid properties are discussed below.

6.1.1 Mass Density

Mass density (ρ) of a fluid is the mass per unit volume. As the molecular activity and spacing increase with temperature, fewer molecules exist in a given volume of fluid. Therefore, the mass density of a fluid decreases with increasing temperature. Under the action of pressure, a large number of molecules can be forced into a given volume, therefore, the mass density of a fluid increases with increasing pressure.

6.1.2 Specific Weight

Specific weight (w) of a fluid is the weight per unit volume. It is related to mass density (ρ) as

$$w = \rho g$$

where g is the gravity.

6.1.3 Specific Volume

Specific volume (v) of a fluid is the volume of the fluid per unit mass. It is reciprocal of density (ρ):

$$v = \frac{1}{\rho} \tag{6.1}$$

6.1.4 Specific Gravity

Specific gravity (s) of a given fluid is the ratio of the mass density of the fluid to the mass density of a standard fluid. Water at 4°C and H_2, or N_2 at specific

temperature and pressure are used as standard fluids for liquids and gas, respectively. Specific gravity of water is 1, while that for mercury (Hg) is 13.6.

6.1.5 Viscosity

Viscosity is the property by virtue of which a fluid offers resistance to the movement of one layer of the fluid over an adjacent layer. It is due to cohesion and primarily molecular momentum exchange between fluid layers.

Figure 6.1 shows a flow of fluid in which the upper layer, moving faster with velocity $u+du$, tries to draw the lower slowly moving layer with velocity u, thus exerting a shear force (F) along the direction of flow (x) on the lower layer.

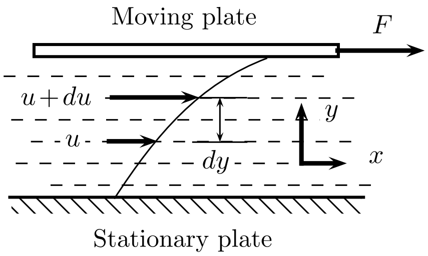


Figure 6.1 Newton's law of viscosity.

The vertical distance (y-axis) between these two layers is dy. The lower layer also tries to retard the upper one, according to Newton's third law of motion, with an equal and opposite force (F). Such a flow is called *shear flow* of the fluid. If the tangential force F acts over an area of contact A, then the shear stress, τ, is defined as

$$\tau = \frac{F}{A}$$

6.1.5.1 Newton's Law of Viscosity According to the *Newton's law of viscosity*, the shear stress between two fluid layers is proportional to the *rate of shear stress.* The rate of shear flow, or rate of change in velocity along y-axis is du/dy [Fig. 6.1]. Therefore, the Newton's law of viscosity is written as

$$\tau \propto \frac{du}{dy}$$
$$\tau = \mu \frac{du}{dy}$$

where μ is called coefficient of *dynamic viscosity*, or simply viscosity. This is used as a measure of viscosity of a fluid[1]. The direction of τ depends upon the direction of du/dy. Physically, the value of viscosity is an indicator of the resistance in the fluid against the relative motion between the adjacent layers.

Units of dynamic viscosity μ are Ns/m^2 or Pa s (Pascal×second) in SI and Poise (= dyne s/cm^2 =0.1 Pa.s) in CGS.

6.1.5.2 Kinematic Viscosity Another important way to express viscosity is by using the term, *kinematic viscosity* (ν), which is related to dynamic viscosity μ and density ρ, as

$$\nu = \frac{\mu}{\rho} \tag{6.2}$$

Units of kinematic viscosity are m^2/s in SI system and Stokes[2]=(10^{-4} m^2/s) in CGS.

Kinematic viscosity is considered as a kinematic quantity because its unit does not contain any unit of mass. Physically, kinematic viscosity represents the relative ability of a fluid to diffuse a disturbance in momentum as compared to its ability of sustaining the original momentum. Therefore, kinematic viscosity can also be quantified as *momentum diffusivity* and the shear stress as *momentum flux.*

6.1.5.3 Mechanism of Viscosity The viscosity in a fluid is caused mainly by two factors:

1. *Intermolecular Force of Cohesion* Due to strong *cohesive forces* between the molecules, any layer in a moving fluid tries to drag the adjacent layer to move with equal speed, and thus produces the effect of viscosity. Since cohesion decreases with temperature, the liquid viscosity does likewise.
2. *Molecular Momentum Exchange* As the random molecular motion of the fluid particles increases with increase in temperature, the viscosity increases accordingly. Therefore, except for very special cases (e.g., at very high pressure), the viscosity of both liquids and gases ceases to be a function of temperature.

6.1.5.4 Variation of Viscosity Viscosity of fluid varies with temperature and pressure both, the effect depends on the state of fluid (gas or liquid) [Fig. 6.2]:

1. *Changes in Viscosity of Liquids* A higher temperature implies a more vigorous random motion, thereby weakening the effective intermolecular attraction. Viscosity of liquids is governed by intermolecular forces of attraction (i.e. cohesive forces), therefore, viscosity decreases with increasing temperature. For many liquids, the temperature dependence of viscosity can be represented

[1] Viscosity is measured by Redwood viscometer.

[2] George Gabriel Stokes, (1819-1903) was a mathematician and physicist. At Cambridge, he made important contributions to fluid dynamics including the Navier-Stokes equations, optics, and mathematical physics including Stokes' theorem.

reasonably well by the *Arrhenius equation*,

$$\mu = Ae^{B/T}$$

where T is the absolute temperature, and A and B are constants.

2. *Changes in Viscosity of Gases* Primarily because of lesser molecular density, intermolecular attraction is not dominant in gases, and the viscosity originates mainly because of the transfer and exchange of molecular momentum. Thus, viscosity of gases commonly increases with increase in temperature.

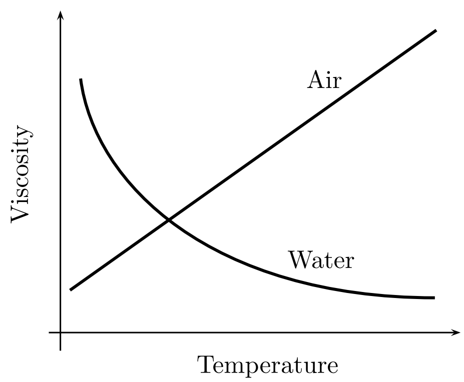


Figure 6.2 Variation of viscosity.

6.1.5.5 Non-Newtonian Fluids Common fluids (e.g. air, water, mercury) follow the *Newton's law of viscosity*, and therefore, are known as *Newtonian fluids*. Other fluids, the constitutive relationship between the shear stress and the shear deformation rate being more complicated than a simple linear one, do not follow the linear relationship, and therefore, are known as *non-Newtonian fluids*.

Although non-Newtonian fluids do not have the property of viscosity (as the viscosity is defined through the Newton's law of viscosity only), their characteristics can be cast in a Newtonian form by introducing an *apparent viscosity*, which is the ratio of the local shear stress to the shear strain rate at that point. The apparent viscosity is not a true property for non-Newtonian fluids since its value depends upon the shear rate.

Few of the non-Newtonian fluids are called *purely viscous fluids* because their shear stress is a function only of the shear rate in the following relationship,

$$\tau = \mu \left(\frac{du}{dy} \right)^n \tag{6.3}$$

where value of index n decides the characteristics of fluid, as depicted in Fig. 6.3.

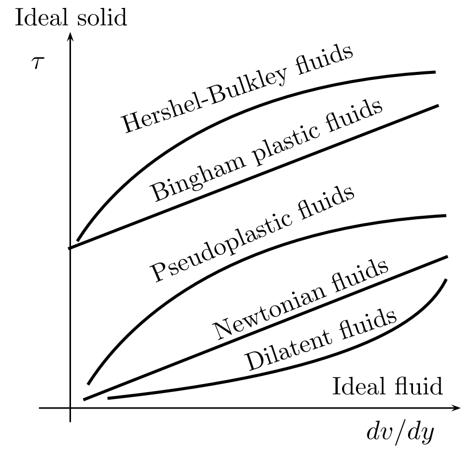


Figure 6.3 Purely viscous non-Newtonian fluids.

Thixotropic and rheopectic fluids are two common classes of *purely time dependent* non-Newtonian fluids which have a hysteresis loop whose shape depends upon the time-dependent rate of shear [Fig. 6.4].

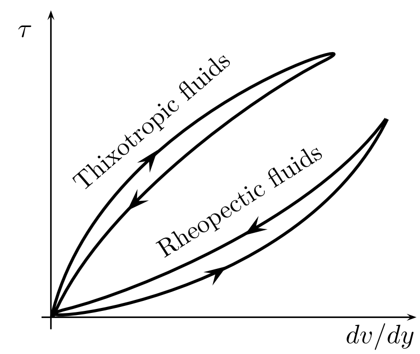


Figure 6.4 Purely time dependent fluids.

Different types of non-Newtonian fluids are discussed in the following headings:

1. *Pseudoplastic Fluids* The fluids having $n < 1$ in Eq. (6.3) are called *pseudoplastic* (shear thinning) where there is decrease in apparent viscosity with increase in shear rate. The reason of a fluid being pseudoplastic can be attributed to the breakdown of loosely bonded aggregates by the shearing effect of flow. An example of pseudoplastic fluids is a non-aqueous suspension of polymers.

2. *Dilatent Fluids* The fluids having $n > 1$ in Eq. (6.3) are called *dilatent* (or shear thickening) where there is increase in apparent viscosity with increase in shear rate. This characteristic of a fluid can be due to the shift of a closely packed particulate system to a more open arrangement

under shear which can entrap some of the liquids. Examples of dilatent fluids are aqueous suspension of magnetite, galena, and ferro-silicon.

3. Bingham Plastic Fluids A *Bingham plastic fluid* has initial yield stress after which it acts a Newtonian fluid. A typical example is toothpaste which does not flow out of a tube until a finite shear stress is applied by squeezing. Other examples of the Bingham plastic fluids are sewage sludge, mud clay.

4. Herschel-Bulkley Fluids Like Bingham plastic fluids, a *Herschel-Bulkley fluid*[3] also has initial yield stress but after this initial stress, the fluid becomes pseudoplastic. An ordinary paint comes under this category.

5. Rheopectic Fluids A dilatent fluid having a hysteresis loop of shear stresses is called *rheopectic fluid*; the apparent viscosity of rheopectic fluids increases with increase in time for a given shear rate. Examples of this category include gypsum pastes and printer inks.

6. Thixotropic Fluid A pseudoplastic fluid with hysteresis loop is known as *thixotropic fluid*; the apparent viscosity of a thixotropic fluid decreases with time under constant shear. The water suspension in bentonitic clay used in petroleum industry as drilling fluid comes under this category.

6.1.5.6 Viscous Torque The viscosity in fluids adds the requirement of additional torque to rotate a body over another separated by a fluid layer. The following are few examples of resistant viscous torques [Table 6.1]:

1. Rotating Cylinder Consider a cylindrical body of radius r and height h, rotating in a fluid medium at angular speed ω and the film thickness is h. The tangential velocity at the periphery of the cylinder is $r\omega$. Using the Newton's law of viscosity, the viscous torque acting on the rotating cylinder will be given by

$$\begin{aligned} T &= \mu \frac{r\omega}{h} \times 2\pi r l \times r \\ &= 2\pi\omega\mu \frac{lr^3}{h} \end{aligned}$$

2. Rotating Disc Consider a disc of radius r, rotating over a fluid film of thickness h, at angular speed ω. The relative tangential velocity at radius x is $x\omega$. Using Newton's law of viscosity, the viscous torque acting on the rotating cylinder will be given by

$$\begin{aligned} T &= \int_0^r x \times \mu \frac{x\omega}{h} 2\pi x dx \\ &= \int_0^r \mu \frac{\omega}{h} 2\pi x^3 dx \\ &= \frac{1}{2}\pi\omega\mu \frac{r^4}{h} \end{aligned}$$

3. Rotating Cone Consider a cone of radius r and half-cone angle θ, rotating over a fluid film of thickness h, at angular speed ω. Consider a cylindrical element of radius x and thickness dx. The area of disc over which the viscous shear stresses act is given by

$$da = 2\pi x \frac{dx}{\sin\theta}$$

Therefore, viscous torque on the rotating cone will be given by

$$\begin{aligned} T &= \int_0^r x \times \mu \frac{x\omega}{h} \times 2\pi x \frac{dx}{\sin\theta} \\ &= \frac{1}{2}\pi\omega\mu \frac{r^4}{h\sin\theta} \end{aligned}$$

4. Rotating Hemisphere Consider a hemisphere of radius r, rotating over a fluid film of thickness h, at angular speed ω. Consider a cylindrical element of radius $r\sin\theta$ and thickness $rd\theta$ [Fig. 6.5].

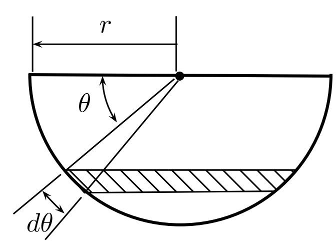


Figure 6.5 | Rotating hemisphere.

Therefore, viscous torque on the rotating cone will be given by

$$\begin{aligned} T &= \int_0^{\pi/2} (r\cos\theta) \times \mu \frac{r\cos\theta\omega}{h} \times 2\pi\,(r\cos\theta)\, rd\theta \\ &= 2\pi\omega\mu \frac{r^3}{h} \int_0^{\pi/2} \cos^3\theta d\theta \end{aligned}$$

[3]This non-Newtonian fluid model was introduced by Herschel and Bulkley in 1926.

Solving with the help of Gamma function, [4]

$$T = \frac{4}{3}\pi\omega\mu\frac{r^4}{h}$$

6.1.6 Vapor Pressure

When a liquid is confined in a closed vessel, the ejected vapor molecules accumulated in the space between free liquid surface and top of the vessel exert a partial pressure on the liquid surface. This pressure is known as *vapor pressure*[5] of the liquid.

The following phenomenon and concepts are related to vapor pressure:

1. Solubility of Gas in Liquids The solubility of a gas in a liquid depends on temperature, the partial pressure of the gas over the liquid, the nature of the solvent, and the nature of the gas. *Henry's law* states that the solubility of gas in liquids is proportional to partial pressure of the gas.

2. Cavitation If the pressure in the liquid reduces to the vapor pressure, the vaporization of liquid starts resulting in the pockets of dissolved gases and vapors. These bubbles of vapor thus formed are collected and carried by flowing liquid into a region of high pressure where they collapse, giving rise to high impact pressure such that material from adjoining boundaries gets eroded and cavities are formed. This phenomenon is called *cavitation*.

3. Use of Mercury in Manometers Mercury (Hg) has low vapor pressure, and hence it is an excellent fluid for use in barometers. On contrary, various volatile fluids (e.g. benzene) have very high vapor pressure, so, they vaporize easily.

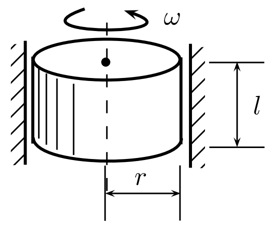
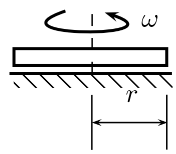
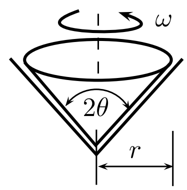
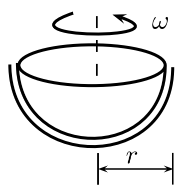
6.1.7 Compressibility

The normal compressive stress of any fluid element at rest is known as *hydrostatic pressure* which arises as a result of innumerable molecular collisions in the entire fluid. The degree of compressibility of a substance is characterized by *bulk modulus of elasticity* (K), defined as

$$K = \frac{\delta p}{-\delta v/v}$$

where p is the absolute pressure. With increase in temperature, K decreases for liquids and increases for gases.

Table 6.1 Viscous torques

	Object	**Viscous Torque**
1.	Cylindrical shaft of length l, radius r rotating over a concentric cylinder 	$2\pi\omega\mu\frac{lr^3}{h}$
2.	Disc of radius r rotating over a plate 	$\frac{1}{2}\pi\omega\mu\frac{r^4}{h}$
3.	Inverted cone of radius r, cone angle θ rotating in concentric cone 	$\frac{1}{2}\pi\omega\mu\frac{r^4}{h\sin\theta}$
4.	Hemisphere of radius r rotating in a concentric hemisphere 	$\frac{4}{3}\pi\omega\mu\frac{r^4}{h}$

[4] Gamma function $\Gamma(x)$ is used to evaluate the integrals as

$$\int_0^{\pi/2} \sin^m\theta\cos^n\theta d\theta = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$$

with $\Gamma(1) = 1$, $\Gamma(n+1) = n\Gamma(n) = n!$, $\Gamma(2) = 1\Gamma(1) = 1$, $\Gamma(3) = 2\Gamma(2) = 2$, $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

[5] Values of vapor pressure of water (H_2O) at 20°C and 100°C are 2.34 kPa and 101.3 kPa, respectively. These are equivalent to water column of height 0.2385 m and 10.32 m, respectively. Therefore, to avoid *cavitation* in pump systems, the length of the suction pipe should not be more than 10.32 m.

Bulk modulus of elasticity (K) is not constant, but it increases with increase in pressure because as the fluid mass is compressed, its resistance to further compression increases. The value of bulk modulus of the gases varies with the condition of the undergoing process. In this case,

$$K = \begin{cases} p & \text{Isothermal condition} \\ \gamma p & \text{Adiabatic condition} \end{cases}$$

The value of K for liquids is very high as compared with those of gases, therefore, liquids are usually considered as *incompressible fluids*. For example, bulk modulus of water and air are 2.06×10^9 Pa and 1.03×10^5, respectively.

The *compressibility* (β) of a fluid is the inverse of the bulk modulus, K:

$$\beta = \frac{1}{K}$$

Acoustic velocity c also depends upon the value of K as

$$c = \begin{cases} \sqrt{K/\rho} & \text{Liquids} \\ \sqrt{\gamma RT} & \text{Gases} \end{cases}$$

where ρ is density of liquid, γ is the ratio of specific heats, R is gas constant. If the flow velocity is small as compared to the local acoustic velocity, the compressibility of gases can be neglected in the study.

6.1.8 Surface Tension

Surface tension (σ) is the property of a fluid surface film that allows it to resist an external force. It is defined as the force exerted by the fluid on one side of a line of unit length drawn over the free fluid surface.

Cohesion is the intermolecular attraction in the same material while *adhesion* is the intermolecular attraction between different materials. Cohesion resists tensile stress, while adhesion enables one material to stick to another body. Surface tension is caused by cohesion (but capillarity is due to both types of attraction) and its magnitude decreases with increase in temperature. It is also dependent on the material in contact with the liquid surface.

Consider an interface between two phases of fluids (liquid and gas) [Fig. 6.6]. The molecules located in the bulk of the liquid have identical interaction with all the neighboring molecules. However, the molecules on the interface have liquid molecules on one side and gas molecules on the other. This implies that the interface has sufficient energy to overcome the net driving force on its molecules and allow them to be located on the interface. This energy is qualitatively known as *interfacial energy* or *surface energy*. To appreciate the effect of surface tension, it is also measured as the *surface energy* per unit surface area of the fluid film.

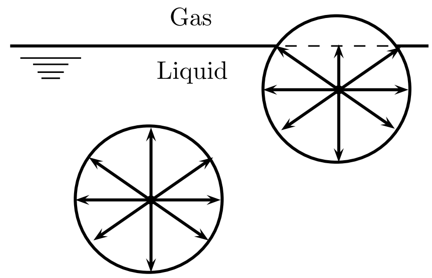


Figure 6.6 | Surface energy.

Although easily deformed, droplets of water tend to be pulled into spherical shape by the cohesive forces of the surface layer. In the absence of other forces, including gravity, drops of virtually all liquids would be perfectly spherical. According to the *Young–Laplace equation*[6], the spherical shape minimizes the necessary "wall tension" of the surface layer. Due to surface tension, falling water drops become spherical, liquid jet breaks, and soap bubbles are formed. Surface tension in fluids results into phenomenon of excess pressure in bubbles, droplets, jets. It also results in angle of contact and capillary action.

In many fluid mechanics problems, the surface tension is not considered since inertia, gravitational and viscous forces are much more important.

6.1.8.1 Excess Pressure

When a droplet is separated initially from the surface of the main body of the liquid, then due to surface tension, there is a net inward force exerted over the entire surface of the droplet which causes contraction and increase in internal pressure. This increase in internal pressure is called *excess pressure*.

The phenomenon of excess pressure happens in bubbles, droplets, and jets. Expressions for excess pressure in these cases are derived as follows:

[6]Young–Laplace equation is a non-linear partial differential equation that describes the capillary pressure difference sustained across the interface between two static fluids, such as water and air, due to the phenomenon of surface tension or wall tension:

$$\Delta p = \frac{2\sigma}{R} \tag{6.4}$$

where δp is the pressure difference across the fluid interface, σ is surface tension, R is mean radius of the curvature. The equation is named after Thomas Young, who developed the qualitative theory of surface tension in 1805, and Pierre-Simon Laplace who completed the mathematical description in the following year. Eq. (6.4) can be used to determine the excess pressure in droplets, jets, and soap bubbles for which the value of mean radii of curvatures (R) comes out to be $r/2$, r and $2r$, respectively. Similarly, in capillary tube of diameter d, the radius of curvatures comes out to be $d/(2\cos\theta)$ where θ is *angle of contact*, and $\Delta p = \rho gh$.

1. *Bubbles* If a bubble of radius r is cut into two parts, the force due excess pressure Δp in area πr^2 shall be equal to the force caused surface tension σ in the periphery $2\pi r$ on both inner and outside edges:

$$\pi r^2 \Delta p = 2 \times 2\pi r\sigma$$
$$\Delta p = \frac{4\sigma}{r}$$

2. *Droplets* If a droplet of radius r is cut into two parts, the force due excess pressure Δp in area πr^2 is found to be equal to the force caused by surface tension σ in the periphery $2\pi r$:

$$\pi r^2 \Delta p = 2\pi r\sigma$$
$$\Delta p = \frac{2\sigma}{r}$$

3. *Jets* If a jet of radius r is cut into two parts longitudinally, the force due excess pressure Δp in rectangular area $2r \times l$ balances the force caused by surface tension σ in the longitudinal edge $2r \times l$:

$$2rl\Delta p = 2l\sigma$$
$$\Delta p = \frac{\sigma}{r}$$

6.1.8.2 Angle of Contact

Due to surface tension, the curved surface of the liquid interface makes an angle with the solid surface. This phenomenon is quantified as *angle of contact* (θ), defined as the angle between tangent at liquid surface at the point of contact and the surface inside the liquid. Value of the angle of contact depends upon the cohesion in the liquid and its adhesion with others, as can be seen in two possible cases:

1. *Adhesion > Cohesion* In this case, the liquid wets the solid surface in contact. It decreases the pressure within the liquid, hence liquid surface rises. This results in a concave liquid surface upward and the angle of contact is less than 90° [Fig. 6.7]. For example, water-glass-air interface has $\theta \approx 0°$.

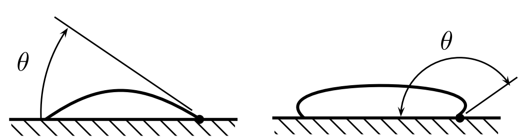


Figure 6.7 | Angle of contact.

2. *Adhesion < Cohesion* In this case, the liquid does not wet the solid surface and liquid surface is concave downward and angle of contact is greater than 90° [Fig. 6.7]. For example, mercury-glass-air interface has $\theta \approx 130°$.

The surface acting agents, known as *surfactants*, are wetting agents that lower the surface tension of a liquid which permits easier spreading and lowering of the interfacial tension between the two liquids.

6.1.8.3 Capillarity

The phenomenon of rise or fall of liquid surface relative to adjacent general level of liquid is called *capillarity.* This happens due to pressure difference across the liquid-gas interface. It is related to the angle of contact and surface tension both. The following cases are related to capillary action:

1. *Simple Capillary Rise* Consider a capillary (a very fine tube of glass) of diameter d submerged partially in liquid (say water) of density ρ. Due to surface tension (σ), liquid inside the capillary raises to height h from the general level [Fig. 6.8].

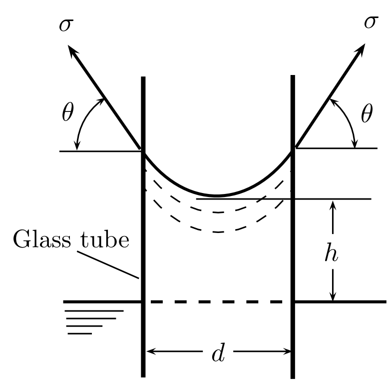


Figure 6.8 | Capillarity.

The capillary rise is given by the equilibrium of the weight of the column and forces due to surface tension acting on the periphery, at the angle of contact:

$$\rho g \frac{\pi d^2}{4} h = \pi d \times \sigma \cos\theta$$
$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

In order to avoid correction for the effect of capillarity in manometers, diameter of the manometer tube is kept more than 6 mm.

2. *Rise in Tube inserted in Two Liquids* If a tube of radius r is inserted in a liquid of specific gravity s_1 above which another liquid of specific gravity s_2 lies, the capillary rise is given by the equilibrium of weight of the liquid at the top and surface tension:

$$\rho g(s_1 - s_2)(\pi r^2)h = \sigma\cos\theta(2\pi r)$$
$$h = \frac{2\sigma\cos\theta}{r\rho g(s_1 - s_2)}$$

3. Rise between Parallel Plates If two vertical parallel plates each of width l apart by distance t are held partially immersed in a liquid of surface tension σ and density ρ, then weight of liquid lifted in between plates due capillary action will be equal to the vertical resolution of surface tension force:

$$\rho \times g \times hlt = 2l\cos\theta\sigma$$
$$h = \frac{2\sigma\cos\theta}{\rho gt}$$

4. Pull required for Ring Consider a ring of mean diameter d kept at the surface of a fluid. If a vertical force F is applied upon the ring, then the surface tension working at inner and outer peripheries in vertically downward direction will resist the pull:

$$F = 2\times\pi D\sigma$$

6.1.8.4 Marangoni Effect Surface tension is a function of temperature. The *Marangoni effect*, also called the *Gibbs-Marangoni effect*[7], is the mass transfer along the interface between two fluids due to surface tension gradient, which can be caused by concentration gradient or by a temperature gradient. The Marangoni effect is used for drying silicon wafers after a wet processing step during the manufacture of integrated circuits.

6.2 FLUID PRESSURE

Pressure (p) at any point is the compressive force (F) exerted by fluid per unit area (A):

$$p = \frac{\delta F}{\delta A}$$

The SI unit for pressure is the pascal[8] (Pa), equal to one newton per square meter ($\mathrm{N/m^2}$).

The pressure at a point, by definition, gives the impression of being a vector, however, it is the same in all directions. Therefore, it has magnitude but not a specific direction, and thus it is a scalar quantity.

6.2.1 General Equation

Consider a static cubic mass of fluid element of dimensions δx, δy and δz subjected to pressure p at centroid [Fig. 6.9].

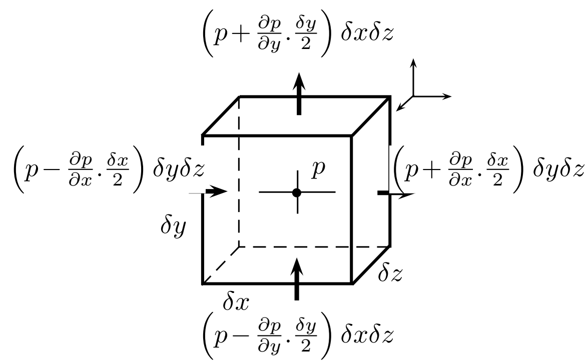


Figure 6.9 Pressure in fluid.

The pressure derivatives in the three directions are found as

$$\frac{\partial p}{\partial x} = 0$$
$$\frac{\partial p}{\partial y} = 0$$
$$\frac{\partial p}{\partial z} = -\rho g$$

These derivatives can be represented as

$$\nabla p = \rho g\hat{k}$$

where divergence is denoted by symbol (∇)

$$\nabla = \frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}$$

Integrating above equation for constant density (ρ):

$$p = -\rho gz + C$$

At any point x, $p = p_a$, $z = z_0 + h$, therefore,

$$p_x = p_a + \rho gh$$

Thus, the hydrostatic pressure (p) (in a static mass of liquid) at any point depends only upon the vertical depth of the point below the free surface and the specific weight of the liquid. It does not depend upon the shape and size of the bounding container. This finding is employed as the *Torricelli's theorem* by using Bernoulli's equation in determining the velocity at depth h as $\sqrt{2gh}$.

The height h in the above equation is known as the *pressure head* (h). It represents the height of column of homogeneous fluid of density ρ that will produce a given intensity of pressure p:

$$h = \frac{p}{\rho g} \tag{6.5}$$

[7] Marangoni effect was first identified by physicist James Thomson (Lord Kelvin's brother) in 1855. The general effect is named after Italian physicist Carlo Marangoni, who published it in 1865. Later, a complete theoretical treatment of the subject was given by J. Willard Gibbs in 1878.

[8] The unit of pressure Pascal is named after Blaise Pascal, the French mathematician, physicist, inventor, writer, and philosopher in 1971. Pascal's law is also in his name.

For a compressible fluid, density being a function of pressure,

$$\rho = f(p)$$

where $f(p)$ depends upon the state of transition. Thus,

$$\int \frac{dp}{\rho g} = -\int dz \tag{6.6}$$

6.2.2 Variation in Atmospheric Pressure

Air is compressible fluid; its density is variant with height. Let ρ_0 and p_0 denote the density and pressure of air at the ground level ($z = 0$). It is interesting to predict the pressure (p) of air at a certain height z. The condition of the atmosphere can be assumed to be isothermal or adiabatic.

6.2.2.1 Isothermal Atmosphere In the isothermal state of atmosphere,

$$\frac{p}{\rho} = \frac{p_0}{\rho_0}$$
$$\rho = \rho_0 \left(\frac{p}{p_0}\right)$$

Putting this value of ρ in Eq. (6.6),

$$\int \frac{p_0 dp}{\rho_0 p} = -g \int dz$$
$$\frac{p_0}{\rho_0} \int \frac{dp}{p} = -g \int dz$$
$$-gz = \frac{p_0}{\rho_0} \ln p + C$$

At $z = 0$, $p = p_0$:

$$z = -\frac{p_0}{\rho_0 g} \ln \left(\frac{p}{p_0}\right)$$

For a perfect gas, $p_0 = \rho_0 R T_0$:

$$z = -\frac{RT_0}{g} \ln \left(\frac{p}{p_0}\right) \tag{6.7}$$
$$\frac{p}{p_0} = -\exp\left(-\frac{gz}{RT_0}\right)$$

This indicates exponential variation of the pressure in isothermal state of the atmosphere.

6.2.2.2 Adiabatic Atmosphere In case of adiabatic state of atmosphere,

$$\frac{p}{\rho^k} = \frac{p_0}{\rho_0^k}$$
$$\rho = \rho_0 \left(\frac{p}{p_0}\right)^{1/k}$$

Putting this value of ρ in Eq. (6.6),

$$\int \frac{{p_0}^k dp}{\rho_0 p^k} = -g \int dz$$
$$\frac{1}{\rho_0} \int \frac{dp}{\left(\frac{p}{p_0}\right)^{1/k}} = -g \int dz$$
$$-gz = \frac{p_0}{\rho_0} \left(\frac{p}{p_0}\right)^{1-1/k} \frac{1}{1-1/k} + C$$

At $z = 0$, $p = p_0$:

$$z = \frac{k}{k-1} \frac{p_0}{g\rho_0} \left[1 - \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}\right]$$

For a perfect gas, $p_0 = \rho_0 R T_0$:

$$z = -\frac{RT_0}{g} \ln \left(\frac{p}{p_0}\right) \tag{6.8}$$
$$\frac{p}{p_0} = \left\{1 - z\frac{g}{RT_0}\left(\frac{k-1}{k}\right)\right\}^{\frac{k}{k-1}}$$

This indicates the variation of the pressure in adiabatic state of the atmosphere.

6.2.3 Pascal's Law

Pascal's law states that when certain pressure is applied at any point in a fluid at rest, the pressure is equally transmitted in all directions and to every other point in the fluid.

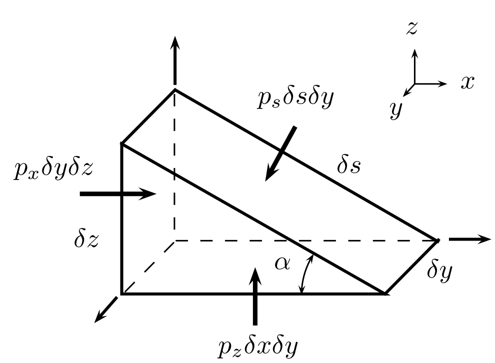


Figure 6.10 Pascal's law.

To analytically prove the Pascal's law, consider a prismatic element of fluid of base δx, height δz and hypotenuse δs at angle α with x-direction and depth δy ($\delta x = \delta s \cos\alpha$, and $\delta z = \delta s \sin\alpha$) [Fig. 6.10].

The equilibrium of the element can be examined in two directions:

1. x-direction:

$$p_x \delta z \delta y = p_s(\delta s \delta y)\sin\alpha$$
$$p_x = p_s$$

2. z-direction[9]:

$$p_z \delta x \delta y = p_s(\delta s \delta y)\cos\alpha + \underbrace{w\left(\frac{\delta x \delta z}{2}\delta y\right)}_{\text{negligible}}$$
$$p_z = p_s$$

Here x-direction is a general case, and so is true for y-direction too, $p_y = p_s$. Thus, there is same pressure p_s in all directions.

The pressure at the bottom of a vessel is independent of size or shape of the vessel and only depends upon the height of fluid over the base. This phenomenon is called *Pascal's paradox*, as it is a consequence of the Pascal's law. Pascal's law is the basic principle of the working of U-tube manometers, hydraulic press, jack, lift, etc.

6.2.4 Pressure Nomenclature

The pressure at a point in a fluid can be measured absolutely or with respect to a standard value, like atmospheric pressure. The standard value of atmospheric pressure can be expressed in the following units:

$$\begin{aligned} p_a &= 0.760 \text{ m of Hg} \\ &= 101.325 \text{ kPa} \\ &= 1.01325 \text{ bar} \\ &= 10.3 \text{ m of water} \end{aligned}$$

In this respect, the pressure of a fluid can be expressed in three forms [Fig. 6.11]:

1. ***Absolute Pressure*** A pressure measured absolutely w.r.t. datum or complete vacuum is called absolute pressure (p).

2. ***Gauge Pressure*** The pressure measured above the *atmospheric pressure* (p_a) is called *gauge pressure* (p_g):

$$p = p_a + p_g$$

3. ***Vacuum Pressure*** The pressure measured below the atmospheric pressure (p_a) is known as *vacuum pressure* (p_v).

$$p = p_a - p_v$$

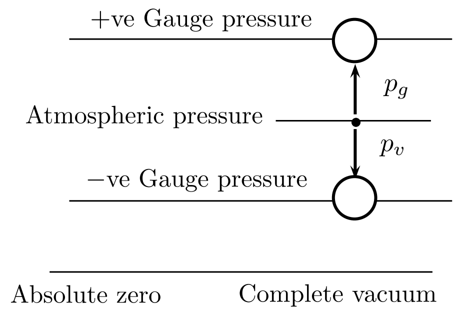


Figure 6.11 Pressure nomenclature.

6.2.5 Pressure Measurements

The following are the common instruments used for pressure measurements.

6.2.5.1 Barometer A *barometer*[10] is an inverted tube, partially submerged into mercury (specific gravity 13.6) or any other liquid. The closed end of tube is kept at vacuum. Due to atmospheric pressure outside the tube, mercury rises upto corresponding height (h) in the tube [Fig. 6.12].

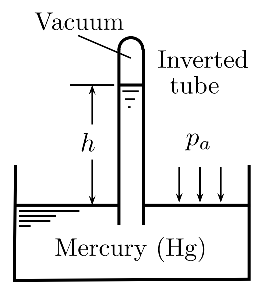


Figure 6.12 Barometer.

For density of mercury ρ_m:

$$p_a = \rho_m g h$$
$$h = \frac{p_a}{\rho_m g}$$

Therefore, h is taken as the direct measure of the atmospheric pressure. Therefore, the atmospheric pressure is known as *barometric pressure*. For example, the standard value of atmospheric pressure (1.01325 bar) is equivalent to 0.760 m of Hg and 10.3 m of water column.

6.2.5.2 Piezometer A *piezometer* is a tube of which one end is attached to a fluid carrying pipe and the other end is open vertically to atmosphere [Fig. 6.13].

[9]The factor due to gravity is negligible only for small mass or low density.

[10]The word "barometer" is derived from the Greek word "baros", meaning weight, and the Greek word "metron", meaning measure.

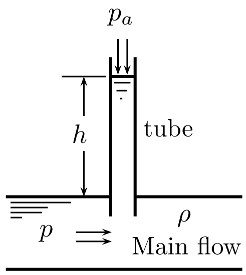


Figure 6.13 Piezometer.

The pressure of the fluid in pipe rises the level (h) of fluid in the tube corresponding to the gauge pressure. For density of fluid ρ,

$$p = p_a + \rho g h$$
$$h = \frac{p - p_a}{\rho g}$$

Thus, the piezometer measures gauge pressure in terms of the column of the fluid in the pipe.

6.2.5.3 Bourdon Gauge *Bourdon gauge*[11] is used to measure gauge pressure, directly from dial gauge. Due to increase in internal pressure, the elliptical cross-section of (steel or bronze) tube tends to become circular, thus causes the tube to straighten and move the gears of the Bourdon gauge.

6.2.5.4 Manometer A common simple manometer consists of a U-shaped tube of glass filled with high density liquid, typically mercury.

Figure 6.14 shows a manometer for measuring the pressure differential between the two mains A and B. In this case, the equilibrium of the manometric liquid can be seen at datum level:

$$\begin{aligned} p_A + \rho_A g z_A &= p_B + \rho_A g z_B + \rho_m g h \\ p_A - p_B &= \rho_m g h + \rho_A g z_B - \rho_A g z_A \\ &= \rho_m g h + g\,(\rho_A z_B - \rho_A z_A) \end{aligned}$$

Tube of a manometer can be made with smaller diameter to make it more sensitive. It is easy to analyze the relation between fluid heights in manometer tubes. Equilibrium of forces acting at a datum level will be equal on both sides.

6.2.6 Hydrostatic Forces

Integration of the pressure of fluid over a given surface area results in the magnitude of the force exerted by the fluid on the surface. This force is called the *hydrostatic force* and point of its application is known as the *center of pressure*.

[11] A mechanical gauge build by E. Bourdon.

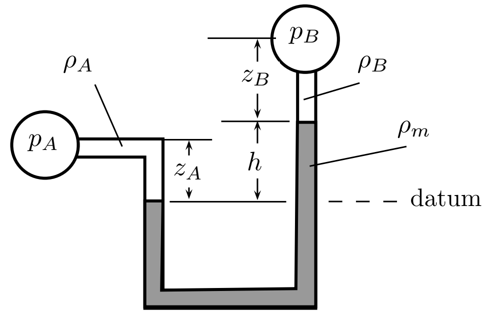


Figure 6.14 Manometer.

Normally, the atmospheric pressure provides uniform forces on both sides of the plane, therefore, it has no effect on either magnitude or position of pressure. Hence, in computation of hydrostatic force (also known as *total pressure*) only gauge pressure has to be considered.

The design of containers and hydro-structures requires computation of the hydrostatic forces on various solid surfaces adjacent to the fluid. The hydraulic forces basically relate to the actual or apparent weight of fluid over the surface. The concept is explained for two types of cases: plane surfaces and curved surfaces, discussed as follows.

6.2.6.1 On Plane Surfaces The hydrostatic force and its centroid for plane surfaces are determined as follows:

1. *Hydrostatic Force* Let a plane surface of random area submerged in a fluid of density ρ, at an angle θ from horizontal [Fig. 6.15].

   Consider an infinitesimal element of this surface of area dA at distance y from point O. The depth of this element is $y \sin\theta$. The hydrostatic force acting on this element is

   $$dF = \rho g (y \sin\theta) dA$$

   Total force on one side of the plane surface is

   $$\begin{aligned} F &= \int dF \\ &= \int \rho g (y \sin\theta) dA \\ &= \rho g \bar{x} \times A \end{aligned} \tag{6.9}$$

   where

   $$\bar{x} = \bar{y} \sin\theta$$

   This is the depth of the centroid (G) of total area A, which can be simply calculated by

   $$\bar{x} = \frac{1}{A} \int x dA$$

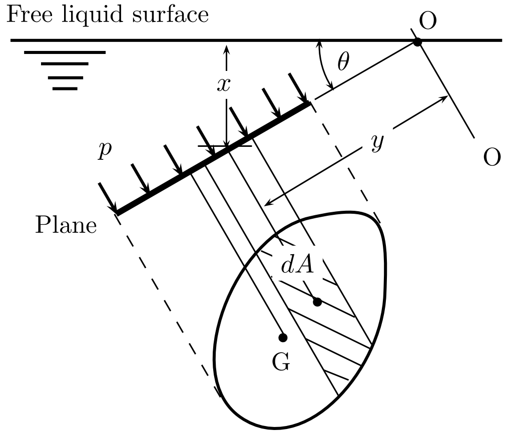


Figure 6.15 Forces on a plane surface.

Hence, the total pressure is equal to the product of the pressure intensity at centroid of area ($\rho g\bar{x}$) and the area of the plane surface (A).

2. *Center of Pressure* Center of pressure is the point at which the hydrostatic force (F) can be assumed to act. For equilibrium, moments of forces on the surface with respect to the axis passing through point O must be equal to that of the hydrostatic force:

$$
\begin{aligned}
F\bar{y}_p &= \int (\rho g y \sin\theta dA)\, y \\
&= \rho g \sin\theta \int y^2 dA \\
&= \rho g \sin\theta I_O
\end{aligned}
$$

where I_O is the area moment of inertia of the plane w.r.t. the axis passing through the point O.

Using the parallel axis theorem, I_O can be related to the area moment of inertia w.r.t. an axis passing through centroid G of area and parallel to O-O (I_G) as

$$I_O = I_G + \bar{y}^2 A$$

Using this relationship, the coordinates of the center of pressure on y-axis is found as

$$\bar{y}_p = \bar{y} + \frac{I_G}{A\bar{y}}$$

By putting $\bar{x} = \bar{y}\sin\theta$ in the above equation, x-coordinate of the center of pressure is found as

$$\bar{x}_p = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}} \tag{6.10}$$

Center of pressure ($\bar{x}_p$) lies on the axis of symmetry at a vertical depth $\bar{x}_p$ below free surface.

Equations (6.9) and (6.10) can be applied to determine the hydrostatic forces and the center of pressure for various situations, depicted in Fig. 6.16. These situations are explained as follows:

1. *Effect of Inclination* Using Eq. (6.10), the effect of variation in θ can be seen on the location of the center of pressure:

   (a) For vertical planes ($\theta = 90°$):

   $$\bar{x}_p = \bar{x} + \frac{I_G}{A\bar{x}}$$

   (b) For horizontal planes ($\theta = 0°$):

   $$\bar{x}_p = \bar{x}$$

2. *Rectangle Surface* Consider a submerged rectangular surface of height d and width b, inclined at angle θ from the horizontal. For this plane surface,

   $$A = bd$$
   $$I_G = \frac{bd^3}{12}$$

   Using Eq. (6.10),

   $$\bar{x}_p = \bar{x} + \frac{(d\sin\theta)^2}{12\bar{x}}$$

3. *Circular Surface* Consider a submerged circular disc of diameter d, inclined at angle θ from horizontal. For this plane surface,

   $$A = \frac{\pi}{4}d^2$$
   $$I_G = \frac{\pi d^4}{64}$$

   Using Eq. (6.10),

   $$\bar{x}_p = \bar{x} + \frac{(d\sin\theta)^2}{16\bar{x}}$$

4. *Triangular Surface* Consider a submerged triangular surface of height d and width b, inclined at angle θ from horizontal. For this plane surface,

   $$A = \frac{bd}{2}$$
   $$I_G = \frac{bd^3}{36}$$

   Using Eq. (6.10),

   $$\bar{x}_p = \bar{x} + \frac{(d\sin\theta)^2}{18\bar{x}}$$

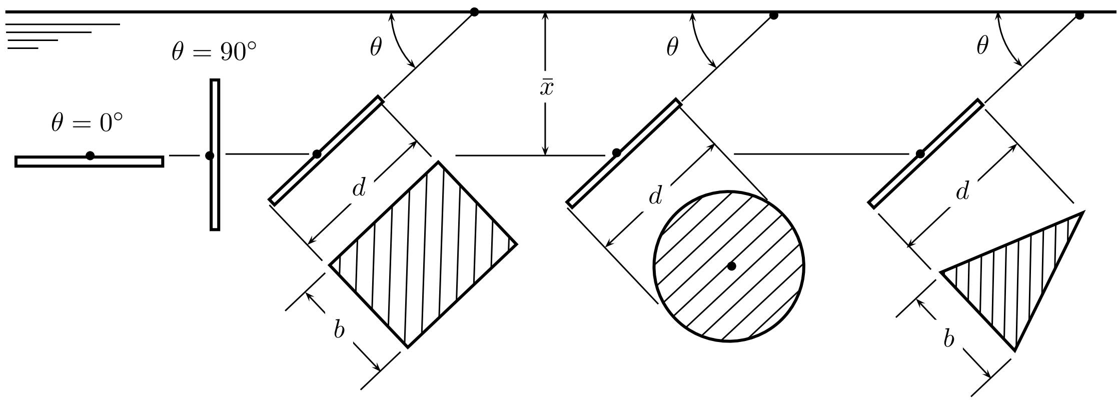


Figure 6.16 Center of pressure.

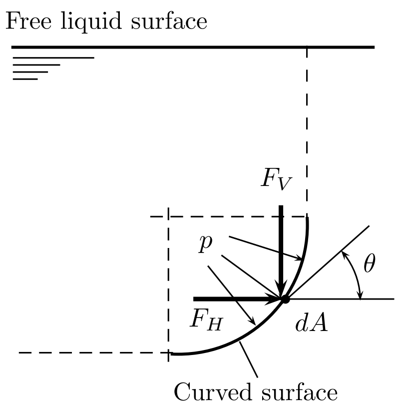


Figure 6.17 Forces on curved surface.

6.2.6.2 On Curved Surfaces The case of hydrostatic force on a curved surface differs from that of plane surfaces, because at every point, the direction of pressure is different [Fig. 6.17].

In this case, the hydrostatic force consists of two components:

1. *Horizontal Component* Horizontal component (F_H) is the total pressure on the projected area of the curved surface on the vertical plane acting on center of gravity of the projection:

$$F_H = \int p \cos\theta dA$$

2. *Vertical Component* Vertical component (F_V) is the total weight of the liquid contained in the portion extending vertically above the curved surface upto free liquid surface:

$$F_V = \int p \sin\theta dA$$

The resultant hydrostatic force is the vector sum of the two components, whose magnitude and inclination from the horizontal are found as

$$F = \sqrt{{F_H}^2 + {F_V}^2}$$
$$\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right)$$

In some cases, if pressure acts underside of a curved surface, the vertical component F_V acts upward and is equal to the weight of an imaginary volume of liquid extending vertically from the curved surface up to free surface. Similarly, in some problems, instead of a free surface, a pressure gauge reading (p_g) can be given. In such cases, an apparent free surface can be assumed to be located at $p_g/\rho g$ height above the level where gauge pressure is p_g.

6.3 BUOYANCY AND FLOATATION

When a body is immersed in a fluid, the fluid offers pressure on the body surface in the form of total pressure. The horizontal components of these hydrostatic forces on the two sides work opposite, so get canceled. But vertical components are not equal, and force at lower surface is more than that on top surface. Due to this difference, a net upward lift acts on the body. This upward lift opposite to the action of gravity results in the tendency of an immersed body to be lifted up in the fluid. This tendency is known as *buoyancy*. The net upward force is called *buoyant force* and its center of application is called *center of buoyancy*.

6.3.1 Archimedes Principle

Archimedes principle[12] states that when a body is immersed in a fluid either partially or totally, it is lifted up by a force equal to the weight of the fluid displaced by the body.

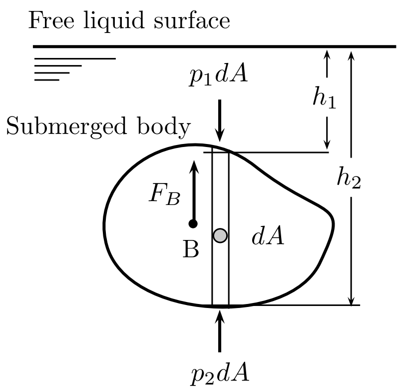


Figure 6.18 | Archimedes principle.

To realize the Archimedes principle, consider a vertical cylindrical element of a body totally immersed in a fluid of density ρ. The cross-sectional area of the element is dA and its height is y [Fig. 6.18]. The buoyant force, by definition, is

$$
\begin{aligned}
F_B &= \int (p_2 - p_1) dA \\
&= \rho g \int y dA \\
&= \rho g \forall
\end{aligned}
$$

where $\forall$ is the volume of the fluid displaced by the immersed body. Since, the weight acts through the center of gravity, the center of buoyancy force coincides with the centroid of volume displaced.

Applications of the Archimedes principle can be seen in the following cases:

1. When a body floats at the interface of two fluids of specific gravities s_1 and s_2 with volume V_1 sunk in the first fluid, and volume V_2 in the second fluid, then force of buoyancy is offered by both the fluids on respective replaced volumes:

$$F_B = \rho_w g (s_1 V_1 + s_2 V_2)$$

2. Let a body of volume V and weight W have two weights W_1 and W_2 when sunk in two liquids of specific gravities s_1 and s_2, respectively. The volume of the body would be determined as

$$
\begin{aligned}
W &= W_1 + \rho_w g s_1 V = W_2 + \rho_w g s_2 V \\
V &= \frac{W_2 - W_1}{\rho_w g (s_1 - s_2)}
\end{aligned}
$$

3. If a block of ice, floating over water in a vessel, slowly melts in it, then water level in the vessel remains same. Analytically, the mass of ice after it melts, and the mass of ice that is submerged inside the water are equal, so:

$$
\begin{aligned}
V_{melt} \times \rho_w &= V_{ice} \times s_{ice} \rho_w \\
V_{melt} &= s_{ice} \times V_{ice}
\end{aligned}
$$

4. Archimedes principle is employed in *hydrometer*, used to measure specific gravity of a fluid. It is a cylindrical body having marks at the middle to show its volume immersed in water.

   When hydrometer is immersed in a liquid of specific gravity ($s < 1$), the bubble of the hydrometer rises above the water mark by height h. Therefore,

$$
\begin{aligned}
W &= \rho_w \forall g \\
&= s \rho_w \left\{ \forall + \frac{\pi d^2}{4} h \right\} g \\
h &= \frac{4W}{\rho_w g \pi d^2} \left\{ \frac{1}{s} - 1 \right\}
\end{aligned}
$$

6.3.2 Principles of Flotation

The Archimedes principle states that the buoyancy force on a body floating in a fluid is equal to the weight of the fluid displaced by the body. If the buoyancy force is somehow greater than the weight of the body, the body will be lifted until it reaches a stable position.

6.3.2.1 Volume Displaced The volume of the fluid displaced ($\forall$) by a floating body is equal to the volume of the body (V) multiplied by the relative specific gravity (s/s_f):

$$
\begin{aligned}
s \rho_w g V &= s_f \rho_w g \forall \\
\forall &= V \times \frac{s}{s_f} \qquad (6.11)
\end{aligned}
$$

where s and s_f are the specific gravities of the body and the fluid, respectively.

6.3.2.2 Metacentric Height Consider a body of mass m, weight W $(= mg)$, floating in a fluid of density ρ [Fig. 6.19]. When it is plucked to a small angular displacement θ about a horizontal axis, the body

[12]The principle is named after its discoverer, Archimedes of Syracuse (287-212 BC), a Greek mathematician, physicist, engineer, inventor, and astronomer.

oscillates about a point known as *metacenter* (M), the point of intersection of a line passing through the CG of the body G and center of buoyancy B, and a vertical line passing through the new (extreme) center of buoyancy. For all values of the small initial tilt (θ), metacenter M remains practically fixed. The distance between metacenter (M) and center of gravity (G) is known as *metacentric height* (GM).

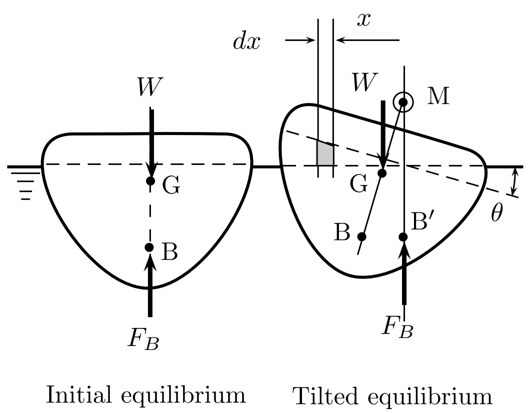


Figure 6.19 Flotation.

For equilibrium of the floating body, the moment of buoyancy force must be equal to moment of lifting force on mass displaced:

$$F_B \times \mathrm{BB}' = 2 \times \int \{x\theta dxl\}\, \rho g x$$

$$\rho g v \times \mathrm{BM}\theta = 2\rho g\theta \times \int x^2 l dx$$

$$\mathrm{BM} = \frac{I_O}{\forall} \tag{6.12}$$

where I_O is the area moment of inertia of the cross-section of body at free surface level about central axis, and $\forall$ is the volume displaced. Using Eq. (6.12), the metacentric height (GM) is found as

$$\mathrm{GM} = \mathrm{BM} - \mathrm{BG}$$
$$= \frac{I_O}{\forall} - \mathrm{BG}$$

6.3.2.3 Floatation Stability

When an immersed body is given small angular displacement, it can be stable, unstable, or in neutral equilibrium, depending upon the direction of the metacentric height, described as follows:

1. *Stable* When B is above G (i.e. GM is positive), then body will return to its original position after some oscillations, hence the body will be stable.
2. *Unstable* When B is below G (i.e. GM is negative), then the body will be unstable. It will get a rotation such that B comes above G and the body becomes stable.
3. *Neutral Equilibrium* When center of buoyancy B coincides with center of gravity of the body G, then GM will be zero, and the body will remain in neutral equilibrium at same position.

6.3.2.4 Float Oscillations

The following nomenclature is given to the oscillations of a floating body [Fig. 6.20]:

1. *Pitching* - the oscillations about transverse axis
2. *Rolling* - the oscillations about longitudinal axis
3. *Yawing* - the oscillations about vertical axis

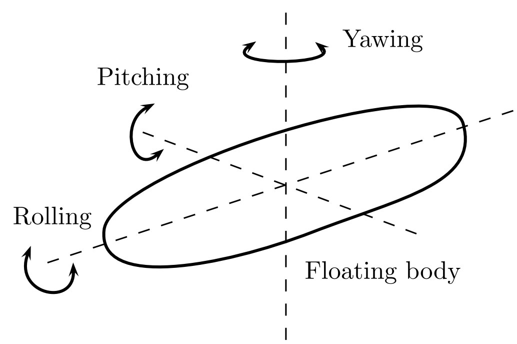


Figure 6.20 Oscillations of a floating body.

Let the floating body have radius of gyration k about an axis passing through G and parallel to free liquid surface:

$$k = \sqrt{\frac{I_G}{A}}$$

Using d'Alembert's principle, the dynamic equilibrium of a floating body can be written as

$$mg\,(\mathrm{GM}\sin\theta) = -mk^2\ddot{\theta}$$
$$k^2\ddot{\theta} + g\mathrm{GM}\theta = 0$$

Solution of this equation can be written as

$$\theta = A\sin\omega t + B\cos\omega t$$

where

$$\omega = \sqrt{\frac{g \times \mathrm{GM}}{k^2}} \tag{6.13}$$

This is the *circular frequency* of oscillations (rad/s). Using this, the time period of the oscillations is found as

$$T = \frac{2\pi}{\omega}$$
$$= \frac{2\pi k}{\sqrt{g \times \mathrm{GM}}} \tag{6.14}$$

Thus, the time period of oscillations depends upon GM and k.

For stability and lower time-period of oscillations, metacentric height should be large, that is more stable and safer. A body has higher radius of gyration in pitching axis than that in rolling:

$$k_{pitching} \geq k_{rolling}$$

Therefore, if a floating body is stable in rolling, then it will also be stable in pitching.

6.4 SOLID BODY MOTIONS

This section presents analysis of the pressure and slope of the free surface of liquids contained in a container subjected to accelerations. Since the liquid layers are not in relative motion, viscous forces do not appear here, instead only container is subjected to acceleration due to which liquid free surface changes its slope and consequently pressure distribution inside the liquid also changes. This type of motion is also called *solid body motion*.

6.4.1 Linear Acceleration

Consider a fluid element of a container moving with acceleration, $\vec{a} = a_x\hat{\imath}+a_z\hat{k}$. The equilibrium in x and z directions is given by the following equations:

$$\frac{\partial p}{\partial x} = -\rho a_x \tag{6.15}$$

$$\frac{\partial p}{\partial z} = -\rho(a_z+g) \tag{6.16}$$

The pressure differential is written as

$$\begin{aligned} dp &= \frac{\partial p}{\partial x}dx+\frac{\partial p}{\partial z}dz \\ &= -\rho a_x dx-\rho(a_z+g)dz \end{aligned}$$

The pressure at any point (x, z) is given by

$$\begin{aligned} p &= \int dp \\ &= -\rho\left\{a_x x+(a_z+g)\,z\right\}+C \end{aligned}$$

At a point A on the free surface at height $z = h$, the pressure is $p = p_a$:

$$p_a = -\rho\left\{a_x x+(a_z+g)\,h\right\}+C$$

Subtracting the above two equations,

$$p = p_a+\rho\,(a_z+g)\,(h-z) \tag{6.17}$$

where $(h-z)$ shows depth of the point from free surface. This shows that the horizontal component of acceleration (a_x) has no effect on the pressure p. The pressure (i.e. gauge pressure) at any point inside the liquid, subjected to acceleration $\vec{a} = (a_x\hat{k}+a_z\hat{k})$, is equal to the height of liquid column above that point multiplied by (a_z+g). Thus, (a_z+g) is the effective acceleration in vertical upward direction.

Using Eqs. (6.15) and (6.16), the slope of the free surface is written as

$$\begin{aligned} \frac{dz}{dx} &= -\frac{\partial p/\partial x}{\partial p/\partial z} \\ &= -\frac{a_x}{a_z+g} \end{aligned}$$

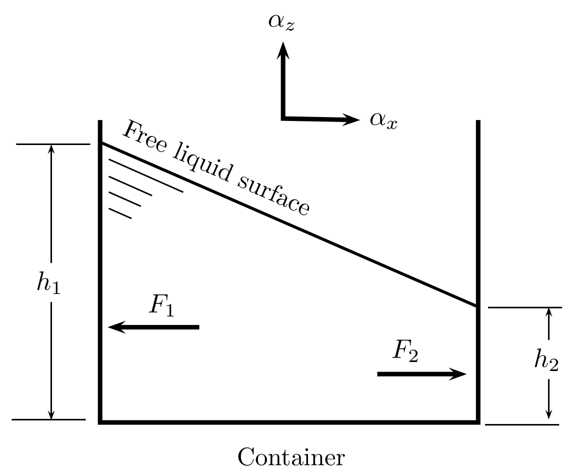


Figure 6.21 | Linear acceleration.

Using the concept of hydrostatic force [*Section* 6.2.6], the forces on walls of the container [Fig. 6.21] can be determined as follows:

1. Force on the left wall:

$$\begin{aligned} F_1 &= \int pbdz \\ &= \int_0^{h_1} \rho\,(a_z+g)\,(z)bdz \\ &= \rho(a_z+g)b\frac{{h_1}^2}{2} \end{aligned}$$

2. Force on the right wall:

$$\begin{aligned} F_2 &= \int_0^{h_2} pbdz \\ &= \int_0^{h_2} \rho\,(a_z+g)\,(z)bdz \\ &= \rho(a_z+g)b\frac{{h_2}^2}{2} \end{aligned}$$

The differences in levels of free liquid surface at left and right ends is determined as

$$-\frac{a_x}{a_z+g} = \frac{h_1-h_2}{l}$$
$$(h_1-h_2) = -\frac{a_x}{a_z+g}l$$

The difference of forces on the left and right walls is

$$\begin{aligned}F_1-F_2 &= \rho(a_z+g)b\frac{{h_1}^2-{h_2}^2}{2}\\ &= \rho(a_z+g)b\times\frac{h_1+h_2}{2}(h_1-h_2)\\ &= -\rho\underbrace{\left(b\frac{h_1+h_2}{2}\times l\right)}_{\text{Volume}}a_x\\ &= -ma_x\end{aligned}$$

This follows the Newton's second law of motion.

6.4.2 Angular Rotation

Consider an elemental liquid mass of width dr, area dA at radius r in a liquid of density ρ, contained in cylindrical vessel of radius R. The cylinder is subjected to angular rotation ω about its central axis. Let the liquid has its initial level z_i w.r.t. bottom [Fig. 6.22].

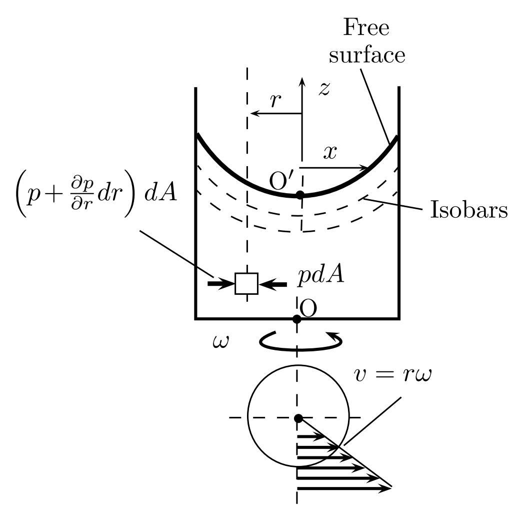


Figure 6.22 Angular rotation.

Due to rotation of the container, the liquid mass also rotates and due centrifugal forces, the pressure increases at the outer circles. The mass rises at the outer area while it falls down at the center. Let the maximum level at outermost periphery be z_{max} and minimum level at the center be z_0.

The equilibrium of the liquid element in radial direction and vertical direction gives the following equations:

$$\frac{\partial p}{\partial r} = \rho\omega^2 r$$
$$\frac{\partial p}{\partial z} = -\rho g$$

Pressure differential is given by

$$dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz$$

Therefore,

$$\begin{aligned}p &= \int dp\\ &= \rho\left\{\frac{\omega^2 r^2}{2} - gz\right\} + C\end{aligned}$$

At the center point O′ on the free surface, $r = 0$, $z = z_0$: $p = p_a$. Therefore, eliminating C from the above equation, one gets

$$p = p_a + \rho\frac{\omega^2 r^2}{2} - \rho g(z-z_0)$$

where $(z-z_0)$ represents height of any point from the minimum level z_0. For any point at the free surface $p = p_a$, so

$$(z-z_0) = \frac{\omega^2 r^2}{2g}$$

This is an equation of parabola with an origin at lowest point O′ on the surface at height z_0 from the point O. Taking new origin at the point O′, the profile of free surface is given by

$$z = \frac{\omega^2 r^2}{2g} \tag{6.18}$$

This is the governing equation of solid angular motion of the liquid. Using this, the following points can be drawn:

1. *Rise and Fall of Free Surface* Volume of a paraboloid revolution is half of the circumscribing cylinder. Therefore, the volume of liquid rise and fall are equal. If z_i is the initial level, then the minimum level (z_0) at the center and the maximum level z_m at the outer periphery are related as

$$\begin{aligned}\pi r^2 z_i &= \pi r^2 z_0 + \frac{\pi r^2(z_m-z_0)}{2}\\ z_i &= z_0 + \frac{z_m-z_0}{2}\\ 2z_i &= 2z_0 + z_m - z_0\\ z_0 &= 2z_i - z_m\end{aligned}$$

2. *Velocity Head* Using Eq. (6.18), the difference of heights between any two points in a vertical line is written as

$$z_2 - z_1 = \underbrace{\frac{{V_2}^2-{V_1}^2}{2g}}_{\Delta\text{Velocity heads}}$$

Thus, the difference of between two points is equal to the difference in the velocity head.

3. *Rotating Closed-Cylindrical Vessel* If a cylindrical vessel completely filled with a liquid and closed at the top and bottom ends is rotated about its central axis, then an imaginary paraboloid revolution of liquid is formed with vertex at the intersection of axis of rotation. The pressure is exerted on top end and beside the weight of liquid mass. The weight of liquid in this paraboloid revolution adds the force on bottom end. Therefore, forces at top and bottom are determined as follows:

$$\begin{aligned} F_T &= \int_0^R \rho \frac{\omega^2 r^2}{2g} 2\pi r dr \\ &= \pi R^2 \times \rho \frac{\omega^2 R}{4} R \\ F_B &= F_T + \rho \pi R^2 H g \end{aligned}$$

4. *Rotating Open-Cylindrical Vessel* Consider a right circular cylinder of radius R and height H. The cylinder is open at the top, filled with liquid, and is rotating about its vertical axis at speed ω. The volume of water spilled is the volume of the paraboloid revolution of the free surface:

$$\begin{aligned} V &= \frac{1}{2}\pi R^2 \left(\frac{\omega^2 R^2}{2g}\right) \\ &= \frac{\pi \omega^2 R^4}{4g} \end{aligned}$$

If the cylinder is rotated at such a speed that half the liquid spills out, then pressure at the center of bottom face of the cylinder is zero [Fig. 6.23]. The

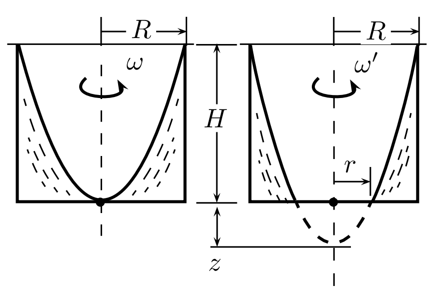


Figure 6.23 Angular rotation.

angular speed (ω) will given by

$$H = \frac{\omega^2 R^2}{2g}$$

If angular speed is increased to ω' ($> \omega$), then the free surface in liquid will maintain the parabolic shape but its origin will be below the bottom face of cylinder. The radius of circular area exposed will be related to depth of apex below the bottom face:

$$\begin{aligned} z &= \frac{\omega'^2 r^2}{2g} \\ H + z &= \frac{\omega'^2 R^2}{2g} \end{aligned}$$

where H is the height of the cylinder and z is the depth.

6.5 KINEMATICS OF FLUID FLOW

Geometry of fluid flow is studied in two basic streams: kinematics and kinetics. *Kinematics!fluid flow* describes the fluid motion and its effects without considering nature of the forces causing the motion. This forms the foundation for studies on dynamic behavior of fluids. *Kinetics of fluid flow* considers the forces causing the motion.

A *flow field* is a region in which the flow velocity is defined at each and every point in space at any instant of time. The fluid motion is described by the following two classical approaches for the analysis of fluid flow.

1. *Langrangian Approach* In this method, the fluid motion is described by tracing the kinematic behavior of individual particle constituting the fluid. This is done by specifying their position as a function of time.

2. *Eulerian Approach* This method, devised by Leonhard Euler, seeks the velocity and its variation with time at each and every location in the flow field. Thus, the hydrodynamic parameters are the function of space and time.

6.5.1 Fluid Velocity

Consider movement of a fluid particle P at point P(x, y, z) with position vector ($\vec{s}$) in the space occupied by a fluid in motion [Fig. 6.24] is

$$\vec{s} = x\hat{i} + y\hat{j} + z\hat{k}$$

After a infinitesimal time δt, the position of the particle changes to P$(x+\delta x,\ y+\delta y,\ z+\delta z)$ with displacement vector

$$\delta\vec{s} = \delta x\hat{i} + \delta y\hat{j} + \delta z\hat{k}$$

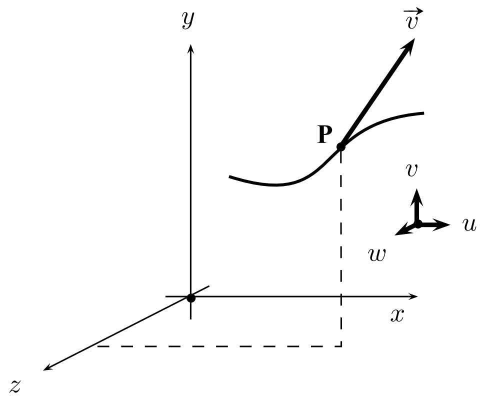


Figure 6.24 | Components of velocity.

The velocity vector ($\vec{v}$) at any point P at instant t is related to displacement vector ($\vec{s}$) as

$$\begin{aligned}\vec{v} &= \lim_{\delta t \to 0} \frac{\delta \vec{s}}{\delta t} \\ &= \frac{d\vec{s}}{dt} \\ &= u\hat{i} + v\hat{j} + w\hat{k}\end{aligned}$$

where u, v and w are the components of velocity vector in x, y and z direction, respectively. These are determined as

$$\begin{aligned}u &= \lim_{\delta t \to 0} \frac{\delta x}{\delta t} \\ &= \frac{dx}{dt} \\ v &= \lim_{\delta t \to 0} \frac{\delta y}{\delta t} \\ &= \frac{dy}{dt} \\ w &= \lim_{\delta t \to 0} \frac{\delta z}{\delta t} \\ &= \frac{dz}{dt}\end{aligned}$$

Velocity is also a function of space and time (x, y, z, t).

6.5.2 Fluid Acceleration

Acceleration[13] at a particle at point P (x, y, z) is represented as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \tag{6.19}$$

where

$$\begin{aligned}a_x &= \frac{du}{dt} \\ &= \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t} + \frac{\partial u}{\partial t}\end{aligned}$$

and similar expressions are for a_y and a_z. Using these details, Eq. (6.19) can be written in vector form:

$$\vec{a} = \underbrace{\frac{\partial \vec{v}}{\partial t}}_{\text{Local}} + \underbrace{\vec{v}.\nabla\vec{v}}_{\text{Convective}}$$

This expression has two components:

1. ***Local Acceleration*** Local component of the acceleration exists by virtue of a change in velocity with respect to coordinates.

   For example, in a *uniform flow*, the velocity at time remains the same at any point in the stream:

   $$\left(\frac{\partial \vec{v}}{\partial s}\right)_t = 0$$

   Therefore,

   $$\begin{aligned}\vec{v}.\nabla\vec{v} &= 0 \\ \vec{a} &= \frac{\partial \vec{v}}{\partial t}\end{aligned}$$

2. ***Convective Acceleration*** Convective component of the acceleration exists by virtue of a change in velocity with respect to time.

   For example, in a *steady flow*, the velocity remains constant at a point in the stream:

   $$\left(\frac{\partial v}{\partial t}\right)_s = 0$$

[13] Time derivative of a velocity function $f(x(t), y(t), z(t), t)$ is written as

$$\begin{aligned}\frac{df}{dt} &= \frac{d}{dt}f(x(t), y(t), z(t), t) \\ &= \frac{df}{dt} + \frac{dx}{dt}\frac{df}{dx} + \frac{dy}{dt}\frac{df}{dy} + \frac{dz}{dt}\frac{df}{dz} \\ &= \frac{df}{dt} + u\frac{df}{dx} + v\frac{df}{dy} + w\frac{df}{dz} \\ &= \frac{df}{dt} + \vec{V}.\nabla f\end{aligned}$$

It shows that even in a time dependent flow field, any given element can suffer changes in f as it moves from place to place. For the given case, it is called convective acceleration.

Therefore,

$$\vec{a} = \vec{v}.\nabla\vec{v}$$

The acceleration vector need not be tangential to stream line. The convective acceleration can be resolved into two components, one along the stream lines (tangential) and another normal to stream lines. As a result, convective acceleration exists if stream lines come closer to each other.

In a flow through a nozzle, convective acceleration at any location is

$$a_{conv} = v\frac{\partial v}{\partial s}$$

where

$$\frac{\partial v}{\partial s} = \frac{v_2 - v_1}{s}$$

6.5.3 Types of Fluid Flow

Fluid flows can be classified based on several characteristics, explained as follows.

6.5.3.1 Steady and Unsteady Types

1. Steady Flow *Steady flow* is a flow in which at any point various fluid characteristics (e.g. velocities, pressure, density, temperature) do not change with time[14] but differ in space of the flow domain:

$$\left(\frac{\partial\vec{v}}{\partial t}\right)_s = \left(\frac{\partial p}{\partial t}\right)_s = \left(\frac{\partial\vec{a}}{\partial t}\right)_s = 0$$

2. Unsteady Flow In *unsteady flows*, the fluid characteristics vary with time.

6.5.3.2 Uniform and Non-uniform Types

1. Uniform Flow The flow is said to be *uniform flow* if the velocity vector of flow does not change with space coordinates within the flow domain at any instant of time. The derivative of velocity w.r.t. space coordinate is always zero:

$$\left(\frac{\partial\vec{v}}{\partial s}\right)_t = 0$$

2. Non-Uniform Flow The flow is said to be *non-uniform* if at any instant of time, the velocity vector of flow varies with space coordinates within the flow domain. So, the derivative of velocity with respect to space coordinate is non-zero:

$$\left(\frac{\partial\vec{v}}{\partial s}\right)_t \neq 0$$

[14]Acceleration is zero at any point.

6.5.3.3 Rotational and Irrotational Flow

1. Rotational Flow *Rotational flow* is a flow in which fluid particles rotate about their own mass center while moving in the direction of flow.

2. Irrotational Flow *Irrotational flow* is a flow in which fluid particles do not rotate about their own mass center.

6.5.3.4 Laminar and Turbulent Types

1. Laminar Flows *Laminar flow* is a flow in which fluid particles move in layers with one layer of the fluid sliding smoothly over another. Here, viscosity of the fluid plays a dominant role. As such, if fluid is very viscous, the flow is treated as a laminar flow.

2. Turbulent Flow *Turbulent flow* is a flow in which fluid particles move in entirely disorderly manner that results in a rapid and continuous mixing of fluid and leads to transfer of momentum. Turbulent flow is considered unsteady flow in the long term.

6.5.3.5 Dimensions of Fluid Flow In general, a fluid flow is three-dimensional, with pressures and velocities and other flow properties varying in all the three directions. Sometimes, the dimensions are reduced for simplification where it is assumed that the greatest changes only occur in two directions or even only in one direction. The flow can be unsteady (the parameters can vary with time). Thus, the dimensionality of a flow is decided based on the following concepts:

1. Three-Dimensional Flow A flow is said to be *three-dimensional*, when flow parameters (such as velocity, pressure) at a given instant of time are functions of all three-dimensional coordinates (e.g. flow in a tapered pipe).

2. Two-Dimensional Flow A flow is *two-dimensional*, if flow parameters are function of two coordinates (e.g. flow in a straight pipe).

3. One-Dimensional flow A flow is *one-dimensional*, if flow parameters depend upon only one direction (e.g. non-viscous flow in a straight pipe[15]).

6.5.3.6 Incompressible Types

1. Compressible Flow When density of fluid changes with space variables, the flow is called *compressible flow*.

[15]Since flow must be zero at the pipe wall, yet non-zero in the center (i.e. there is a difference in parameters across a section). It should be dealt as two-dimensional flow if very high accuracy is required.

2. *Incompressible Flow* When density of fluid remains constant throughout the flow region, the flow is called *incompressible flow*.

6.5.4 Fluid Flow Lines

Three types of lines are associated with a fluid flow: stream lines, path lines, and streak lines. These are described as follows:

1. *Stream Line* *Stream line*[16] is an imaginary curve drawn through a flowing fluid in such a way that the tangent to it at any point gives the direction of instantaneous velocity of flow at that point. There can be no flow across any stream line of a flow system.

   The velocity vector and the infinitesimal line segment along stream line are represented, respectively, as

$$\vec{v} = u\hat{i}+v\hat{j}+w\hat{k}$$
$$\vec{ds} = dx\hat{i}+dy\hat{j}+dz\hat{k}$$

   Therefore,

$$\vec{v}\times\vec{s} = 0$$
$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \tag{6.20}$$

   This is the differential equation of stream lines in a Cartesian frame of reference.

   The distance between stream lines along the flow indicates the acceleration [Fig. 6.25].

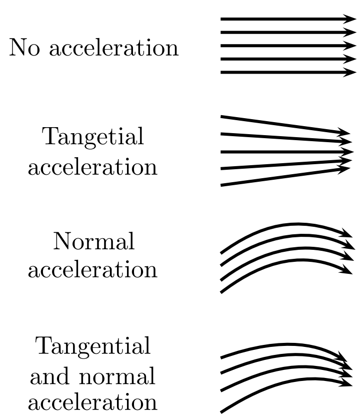


Figure 6.25 Stream lines and acceleration.

   In a steady flow, the pattern of stream line pattern also remains unchanged with time. A related term called *stream tube* is defined as an imaginary tube formed by a group of stream lines. No flow is possible across the stream tube.

2. *Path Line* *Path line* is the curve traced by a single fluid particle as it moves over a period of time. Thus, a family of path lines represents the trajectories of different particles.

3. *Streak Line* *Streak line*[17] is the curve made by fluid particles passing through a fixed point at any given instant. This line is of particular interest in experimental flow visualization.

In a steady flow, a fluid particle always moves along a stream line, so its stream lines, path lines, and streak lines coincide with each other.

6.6 MOTION OF FLUID PARTICLES

Motions of fluid particles are broadly classified as [Fig. 6.26]:

1. Pure translation
2. Linear deformation
3. Angular deformation
4. Pure rotation

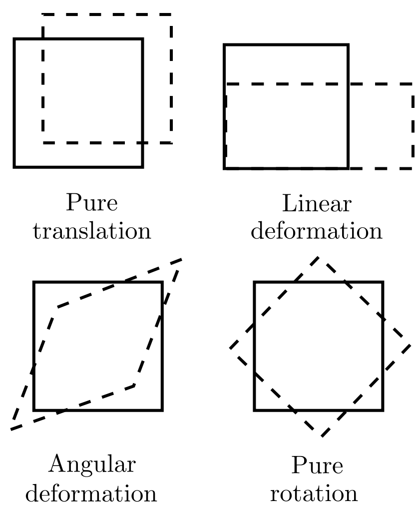


Figure 6.26 Motion of fluid particles.

Consider a rectangular element of size δx and δy in a fluid flow at a point $P(x, y)$ in a two-dimensional

[16]The analytical description of flow velocities by the *Eulerian approach* is geometrically depicted through the concept of stream lines.

[17]The equation of a streak line at time t can be derived by the Lagrangian method.

coordinate system orthogonal to axis of rotation (z-axis). During δt time interval, the particle has gone through translation and rotation.

6.6.1 Rotation

Rotation of a fluid particle is defined as the average angular velocity of any two orthogonal infinitesimal linear elements in the particle perpendicular to the axis of rotation.

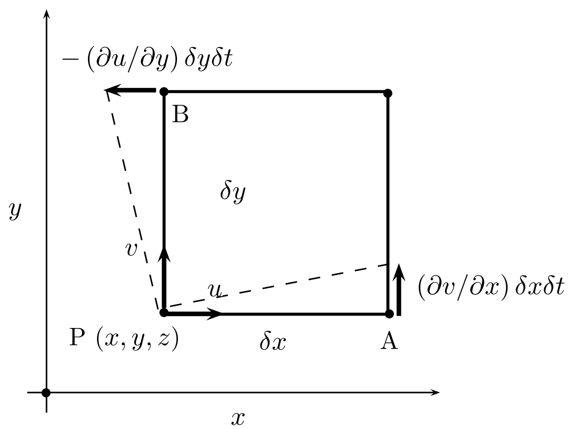


Figure 6.27 Rotation of an element.

For the rotation of the fluid element shown in Fig. 6.27, angular velocity of edge PA about z-axis is (anti-clock-wise taken positive):

$$\omega_{PA} = \frac{1}{\delta t}\frac{\left(\frac{\partial v}{\partial x}\delta x\right)\delta t}{\delta x} = \frac{\partial v}{\partial x}$$

Angular velocity of edge PB about z-axis is

$$\omega_{PB} = -\frac{\partial u}{\partial y}$$

Thus, rotation of this particle about z-axis, as per definition, is

$$\omega_z = \frac{1}{2}(\omega_{PA}+\omega_{PB}) = \frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

Similarly, rotation about x and y axes, respectively, are

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)$$
$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)$$

So, rotation vector of the fluid particle at any point is

$$\vec{\omega} = \omega_x\hat{i}+\omega_y\hat{j}+\omega_z\hat{k} = \frac{1}{2}\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ u & v & w\end{bmatrix} = \frac{1}{2}(\nabla\times\vec{v}) \tag{6.21}$$

Thus, rotation vector of fluid element is half the curl of the velocity vector.

6.6.2 Shear Strains

The rotation of a fluid particle is always associated with shear stresses due to viscosity of the fluid. The following are expressions for shear strains:

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)$$
$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)$$
$$\varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)$$

A highly viscous flow is invariably a rotational one.

6.6.3 Circulation

The circulation Γ around a closed contour is defined as the line integral of the tangential component of the velocity [Fig. 6.28].

The circulation of velocity vector $\vec{v}$ around a contour line (s) is given by

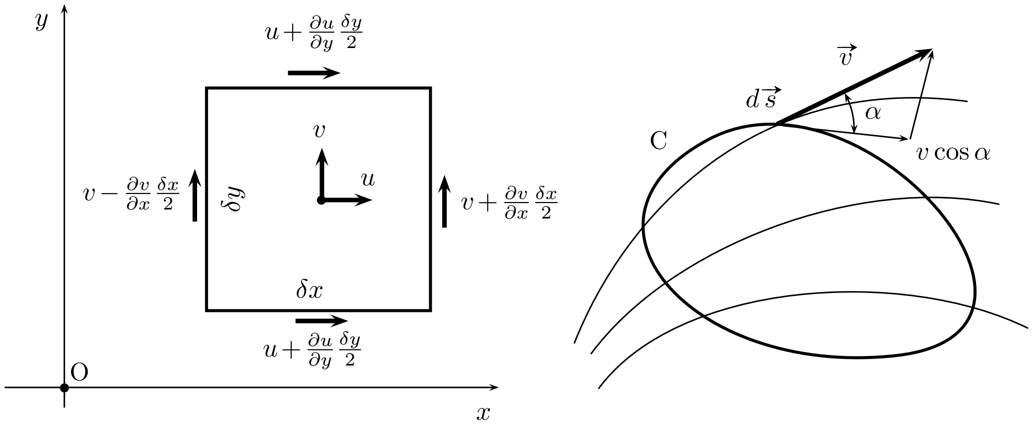
$$\Gamma = \int_C \vec{v}.d\vec{s} = \int_C v\cos\alpha ds = \int_C (udx+vdy+wdz)$$

where $v\cos\alpha$ is component of velocity vector $\vec{v}$ along the infinitesimal part ds of the curve C.

A free vortex flow is represented as (c is a constant)

$$v_\theta r = c$$
$$v_r = 0$$

Therefore, circulation of a free vortex flow along any closed contour, excluding the singular point (the origin) should be zero. This is consistent with the condition to be satisfied by an irrotational flow field.



(a) Circulation around an element (b) Circulation around a curve

Figure 6.28 Circulation.

6.6.4 Vorticity

Vorticity is defined as the circulation of infinitesimal closed curve per unit area of the curve. In vector form,

$$\vec{\Omega} = \nabla \times \vec{v}$$

If an imaginary line is drawn in the fluid so that the tangent to it at each point is in the direction of the vorticity vector $\vec{\Omega}$ at that point, the line is called a *vortex line*. Therefore, a general equation for vortex line can be written as

$$\vec{\Omega} \times d\vec{s} = 0$$

For $x - y$ system,

$$\Omega_z = 2 \times \omega_z$$

Vorticity is equal to twice the rotation component about the axis perpendicular to the plane in which the area is lying and this is a vector perpendicular to the plane of a small curve. For an irrotational flow,

$$\Omega_z = 0$$

6.6.5 Stream Function

Stream function (ψ) is a single higher order function that can be used to replace two variables (u and v) in a two-dimensional and steady-state flow, which satisfies the continuity equation:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0$$
$$\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0$$

Thus, *stream function* is defined as the scalar function of time t and space $s(x, y)$ such that its partial derivative w.r.t. any direction gives the velocity component at right angle in anti-clockwise rotation to the direction:

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x} \tag{6.22}$$

Using the definition of the stream function [Eq. (6.22)], following relevant points can be deduced:

1. ***Existence of Flow*** The definition of the stream function itself is based on the continuity equation. Thus, the existence of ψ shows the existence of a two-dimensional incompressible and steady flow.

2. ***Stream Line*** The derivative of stream function is written as

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy$$
$$= \frac{v}{d}x - udy$$

For $d\psi = 0$,

$$\frac{dx}{u} = \frac{dy}{v}$$

This is the equation of stream line [Eq. (6.20)], which implies that $d\psi = 0$ on a stream line. Thus, the value of stream function remains constant along a stream line. Stream lines can be represented as (where c is a constant),

$$\psi(x, y) = c$$

A stream line is the curve of constant steam function, whose slope is given by

$$\left(\frac{dy}{dx}\right)_\psi = -\frac{\partial\psi/\partial x}{\partial\psi/\partial y} = -\frac{v}{-u} = \frac{v}{u} \tag{6.23}$$

Intersection of two streamlines in a flow shows stagnation point at which velocity vector ceases.

3. *Discharge* Discharge, the rate of volume flow, can be expressed in terms of ψ as

$$q = \vec{v}.\delta x\delta y\hat{k} = w\delta x\delta y v\delta x + u\delta y = \delta\psi$$

Hence, for a two-dimensional case, the discharge per unit height between two stream lines of stream functions ψ_1 and ψ_2 is $\psi_2 - \psi_1$.

4. *Stream Function in Polar Coordinates* The continuity equation in polar coordinates is written as

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta) = 0$$

$$\frac{\partial}{\partial r}\left(-\frac{\partial\psi}{\partial\theta}\right) + \frac{\partial}{\partial\theta}\left(\frac{\partial\psi}{\partial r}\right) = 0$$

Therefore, the potential functions can be defined in polar coordinates as

$$v_r = -\frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

$$v_\theta = \frac{\partial\psi}{\partial r}$$

6.6.6 Velocity Potential

When the components of rotation at all points in a flow field become zero, the flow is said to be *irrotational*. Thus, the necessary and sufficient condition for a flow field to be irrotational is

$$\vec{\nabla}\times\vec{v} = 0 \tag{6.24}$$

This condition of irrotationality of a fluid flow can be related to a scalar function, known as the *velocity potential* (ϕ), defined as

$$\vec{v} = \nabla\phi$$

Thus, *velocity potential* can be defined as a scalar function of time t and space $s(x, y, z)$ such that its negative derivative w.r.t. any direction gives the fluid velocity along that direction:

$$u = \frac{\partial\phi}{\partial x}, \quad v = \frac{\partial\phi}{\partial y}, \quad w = \frac{\partial\phi}{\partial z} \tag{6.25}$$

Unlike the stream function, velocity potential contains coordinates (x, y and z) of three-dimensional space; it is not limited to two coordinates.

Using the definition of the potential function [Eq. (6.25)], the following relevant points can be deduced:

1. *Potential Line* A *potential line* is a curve of constant velocity potential (ϕ) whose slope is given by,

$$\left(\frac{dy}{dx}\right)_\phi = -\frac{\partial\phi/\partial x}{\partial\phi/\partial y} = -\frac{-u}{-v} = -\frac{u}{v}$$

Using Eq. (6.23)

$$\left(\frac{dy}{dx}\right)_\phi \times \left(\frac{dy}{dx}\right)_\psi = -1$$

Therefore, stream lines are orthogonal to the potential lines. *Flow net* is a grid obtained by drawing a series of stream lines and equipotential lines in a two-dimensional flow of an ideal fluid. This shows velocity distribution as no flow occurs along two equipotential lines [Fig. 6.29].

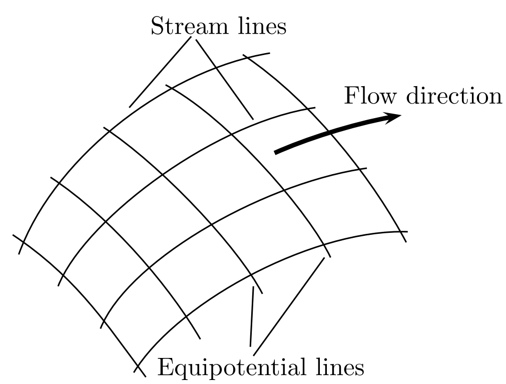


Figure 6.29 | Flow net.

2. *Irrotational flow* The definition of potential function is based on the irrotationality of the flow. Therefore, existence of a potential function indicates an irrotational flow. Using Eq. (6.24), for irrotational flow,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

For a two-dimensional irrotational flow:

$$\omega_z = 0$$

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

3. Continuity Equation For an incompressible steady flow, *equation of continuity* [Eq. (6.30)] reads

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Using Eq. (6.25),

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}=0$$

For a two-dimensional incompressible steady flow,

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=0$$

This is the *Laplace equation* of ϕ in two-dimensional case. Thus, for an incompressible, steady and irrotational flow, the potential function satisfies the Laplace's equation.

4. Cauchy-Riemann Equations *Cauchy-Riemann equations* are the interrelationship between ϕ and ψ for a two-dimensional flow:

$$\frac{\partial \phi}{\partial x}=\frac{\partial \psi}{\partial y}$$
$$\frac{\partial \phi}{\partial y}=-\frac{\partial \psi}{\partial x}$$

These expressions also indicate that in a two-dimensional flow, stream lines and potential lines are orthogonal to each other.

6.7 PRINCIPLES OF FLUID FLOW

The characteristics of fluid flows are based on the conservation principles of mass, energy, and momentum. In fluid mechanics, these conservation laws are represented in terms of field variables, such as velocity (v), pressure (p), density (ρ). Since the field variables depend on space coordinates (x, y, z) and time t, the governing equations are in the form of partial differential equations with respect to these independent variables.

6.7.1 Conservation of Mass

Matter cannot be created or destroyed. When considered in analysis of fluid flows, this is known as the principle of *conservation of mass*. This can be applied to fixed volumes in space, known as *control volume* for which the total mass flow-in will be equal to mass flow-out [Fig. 6.30].

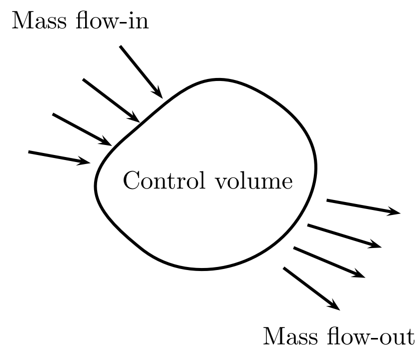


Figure 6.30 | Flow through control volume.

Consider an infinitesimal parallelepiped space element in a fluid flow of Cartesian coordinates with dimensions δx, δy and δz along x, y and z axes, respectively [Fig. 6.31]. The element has u, v and w as the components of velocity v at centroid in x, y and z directions, respectively.

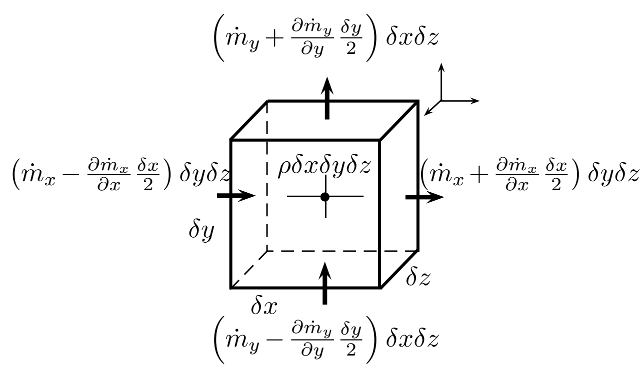


Figure 6.31 | Conservation of mass.

The rates of mass flow in the three orthogonal directions are written as

$$\dot m_x=\rho u\times\delta y\delta z$$
$$\dot m_y=\rho v\times\delta x\delta z$$
$$\dot m_z=\rho w\times\delta x\delta y$$

The net mass of fluid that has remained in the parallelepiped per unit time due to velocity difference along x-direction is

$$\delta\dot m_x=-\frac{\partial}{\partial x}\left\{\rho\left(u\delta y\delta z\right)\right\}$$

Similarly, it can be calculated along y and z directions [Fig. 6.32]. Thus, the total remnant mass is

$$\delta\dot m=\delta\dot m_x+\delta\dot m_y+\delta\dot m_z \tag{6.26}$$

$\delta\dot m$ is also given by

$$\delta\dot m=\frac{\partial}{\partial t}\left\{\rho\left(\delta x\delta y\delta z\right)\right\} \tag{6.27}$$

From the above equation, the law of mass conservation is represented by the following *Euler's equation of continuity* [Eq. (6.33)]:

$$\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}+\frac{\partial \rho w}{\partial z}+\frac{\partial \rho}{\partial t}=0 \tag{6.28}$$

Assuming ρ as an isotropic property, Eq. (6.28) results in

$$\rho\left\{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right\}+\frac{\partial \rho}{\partial t}=0 \tag{6.29}$$

This is the *general equation of continuity* which contains a "time-derivative" of the fluid density, $\partial\rho/\partial t$.

For an incompressible flow, ρ is constant, and steady flow

$$\left(\frac{\partial \rho}{\partial t}\right)_s=0$$

Thus, Eq. (6.29) reduces to the following equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{6.30}$$

The general equation [Eq. (6.29)] can also be written in the following forms:

1. Vector Form

$$\frac{\partial \rho}{\partial t}+\rho\nabla\cdot\vec{v}=0$$

For incompressible flows,

$$\nabla\cdot\vec{v}=0$$

2. Integral Form

$$\frac{\partial}{\partial t}\int_{\forall}\rho dv+\int_{S}\rho v dS=0$$

For incompressible flows,

$$\rho v A=C$$

3. Polar Coordinates

$$\frac{\partial}{\partial r}(rv_r)+\frac{\partial v_\theta}{\partial \theta}=0$$

6.7.2 Conservation of Energy

A more general approach for obtaining the relationship between parameters of a fluid flow[18] is to apply the principle of conservation of energy. Consider a point in a fluid flow at height z from datum, where pressure is p, velocity is v [Fig. 6.33].

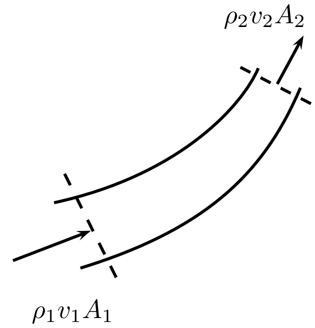


Figure 6.32 | Mass continuity in stream tube.

6.7.2.1 Bernoulli's Equation The relationship among these quantities for an incompressible, inviscous, and steady stream line flow through a pipe of varying cross-section can be written as

$$\frac{\partial v}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial s}+g\frac{\partial z}{\partial s}+v\frac{\partial v}{\partial s}=0 \tag{6.31}$$

This equation is known as the *Euler's equation*. For steady flow, this equation can be written as

$$\frac{dp}{\rho}+vdv+gdz=0$$

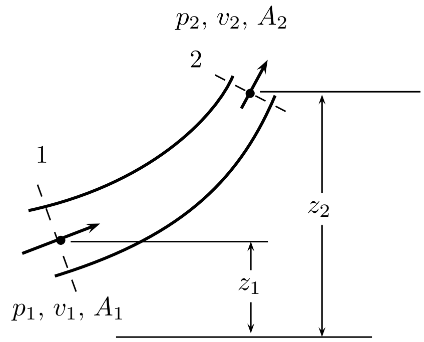


Figure 6.33 | Bernoulli's equation.

This can also be written in an *integrated form*:

$$\frac{p}{\rho g}+\frac{v^2}{2g}+z=C \tag{6.32}$$

This equation is called *Bernoulli's equation*[19] between the points on the same stream line where v is the average velocity of flow. This equation presents the principle of conservation of energy in an steady, incompressible, inviscid and streamlined flow.

[18]An ideal fluid is characterized by the absence of viscosity and thermal diffusivity. Because of this property, the motion of a an ideal fluid particle is adiabatic.

[19]Bernoulli's principle is named after the Swiss scientist Daniel Bernoulli who published it in his book Hydrodynamica in 1738.

6.7.2.2 Energy Heads The Bernoulli's equation along a stream line essentially states that the total of flow energy, kinetic energy and potential energy per unit weight remains conserved as it is transmitted from one point to another in the flow along a stream line under the assumption of steady, incompressible and inviscid (not viscous) flow. The energy associated per unit weight is technically known as *head*. Eq. (6.32) consists of the following energy heads:

1. *Pressure head*, $p/\rho g$ (*flow energy*)[20]
2. *Velocity head*, $v^2/2g$ (kinetic energy)
3. *Gravity head*, z (potential energy)

The sum $(z + p\rho g)$ is known as *piezometric head*, and its variation is represented by a *hydraulic gradient line*.

6.7.2.3 Kinetic Energy Correction Factor In any general flow, velocity (v) is a function of space and time (x, y, z, t). So, an average velocity at any section w.r.t. its flow area A can be calculated by

$$\bar{v} = \frac{1}{A}\int v dA$$

Kinetic energy correction factor (α) is defined to equalize kinetic energy based on average velocity to actual kinetic energy:

$$\alpha = \frac{1}{A}\int \left(\frac{v}{\bar{v}}\right)^3 dA$$

Value of α is always greater than 1.

6.7.3 Conservation of Momentum

The following are the two sets of forces that act on a fluid:

1. *External Body Forces* These are long range external body forces that penetrate matter and act actually on all the material in any element. Gravity force is the only external body force considered in present study.
2. *Molecular Forces* These are the short range molecular forces that are internal to the fluid, as a result of interactions among elements within a thin surface layer. These forces result into stresses, such as pressure, shear stress due to viscosity.

[20]The work done to maintain the flow for small displacement dx in the presence of pressure per unit weight and is given by

$$W = \frac{pAdx}{\rho Adxg} = \frac{p}{\rho g}$$

Consider a volume $\forall$ bounded by a material surface S that moves with the flow, always containing the same fluid elements. Its momentum is

$$\int_{\forall} d\forall \rho \vec{v}$$

Hence, the rate of change of momentum is

$$\frac{d}{dt}\int_{\forall} d\forall \rho \vec{v} = \int_{\forall} d\forall \rho \frac{d\vec{v}}{dt}$$

This must be equal to the net force on the element. This statement represents the principle of *conservation of momentum* in fluid flow.

6.7.3.1 Momentum Flux Momentum of a particle in a fluid flow is represented in terms of *momentum flux*, defined as

$$\phi = (\rho v dA)\, v$$

Newton's second law of motion states that the momentum of a particle remains conserved if no forces act on the system.

A simple study is carried out to explain the concept of momentum of a fluid. Consider an incompressible, inviscid, steady flow through a pipe elbow of angle θ [Fig. 6.34].

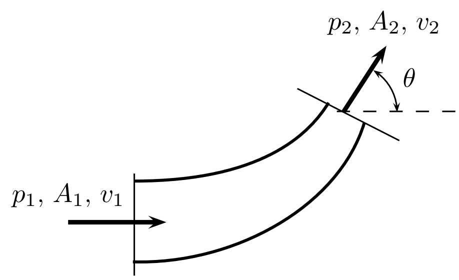


Figure 6.34 Momentum of fluid flow.

Due to velocity change from v_1 to v_2, momentum flux also changes resulting into a net force on elbow. The fluid entering at 1 exerts a force on elbow consisting of pressure component $p_1 A_1$ and momentum component $\rho \dot{Q} v_1$. Similarly, a force consisting of the same components is exerted outward. Horizontal and vertical components of this net force are given by

$$F_x = \left\{p_1 A_1 + \rho \dot{Q} v_1\right\} - \left\{p_2 A_2 + \rho \dot{Q} v_2\right\}\cos\theta$$
$$F_y = -\left\{p_2 A_2 + \rho \dot{Q} v_2\right\}\sin\theta$$

6.7.3.2 Euler's Equation A fluid mass is subjected to different types of forces, such as gravity forces, pressure forces, viscous forces, turbulent forces, surface tension force, and compressible force. The subject of

fluid dynamics is the study of these forces and their effect on fluid motion as the applications of Newton's second law of motion.

Consider a cylindrical element of cross-sectional area dA and length ds in a stream line of steady flow. Let θ be the angle between direction of flow and line of action of gravity force. The angle θ will be given by

$$\cos\theta = \frac{d\partial z}{\partial s}$$

As the flow is steady, local acceleration and convective accelerations can be written as

$$\frac{\partial v}{\partial t} = 0$$
$$v\frac{\partial v}{\partial s} \neq 0$$

Therefore,

$$\begin{aligned} a_s &= \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s} \\ &= v\frac{\partial v}{\partial s} \end{aligned}$$

The resultant pressure on the element is equal to the mass of the fluid multiplied by the acceleration a_s along the stream line, as

$$\begin{aligned} \rho(dAds)\times a_s = pdA \\ -\left(p+\frac{\partial p}{\partial s}ds\right) dA\cos\theta \\ -\rho g\times(dAds)\cos\theta \end{aligned}$$

$$v\frac{\partial v}{\partial s} + g\frac{\partial z}{\partial s} + \frac{\partial p}{\rho\partial s} = 0$$

Multiplying with ∂z gives

$$vdv + gdz + \frac{dp}{\rho} = 0 \qquad (6.33)$$

This expression is called *Euler's equation* of motion. This equation has been used to derive Bernoulli's equation [Eq. (6.32)].

6.7.3.3 Momentum Correction Factor

A *momentum correction factor* (β) is used to equalize momentum based on average velocity to actual momentum. It is given by

$$\beta = \frac{1}{A}\int\left(\frac{v}{\bar{v}}\right)^2 dA$$

Value of β is always greater than 1.

6.7.3.4 Navier-Stokes Equations

Navier-Stokes equations[21] describe the motion of fluid substances. These equations include Newton's second law to fluid motion, together with the assumption that the fluid stress is the sum of a diffusing viscous term (proportional to the gradient of velocity), plus a pressure term. The derivation and application of these equations is a lengthy process, and is out of present context.

[21]Navier-Stokes equations are named after Claude-Louis Navier and George Gabriel Stokes.

6.8 FLOW MEASUREMENTS

Bernoulli's equation [Eq. (6.32)] can be applied in flow measurements, such as in venturimeter, orifice-meter, nozzle meter, rotameter, elbowmeter, pitot tube, discussed as follows.

6.8.1 Venturimeter

A *venturimeter*[22] is a device used to measure discharge (volume flow rate per unit time) through a pipe. This is composed of two nozzles (cones) joined with their smaller ends. The taper angle of the first cone (convergent) at which liquid enters is approximately 20°, while that of divergent cone is made lesser (approximately 5°) to facilitate an easy exit without flow separation and minimum energy losses. In order to avoid *cavitation* at the throat, diameter at the throat is kept between 1/3 to 1/4 of pipe diameter. To measure the pipe pressures, two piezometers are fitted, one at the throat, another at the inlet. Velocity increases at the throat due to shorter area available for the same discharge [Fig. 6.35].

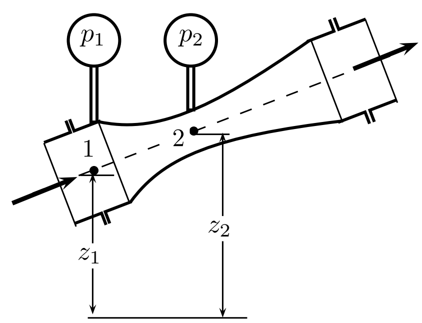


Figure 6.35 Venturimeter.

To make a general case, venturimeter is inclined to horizontal. For horizontal units, $z_2 = z_1$ [Fig. 6.35]. Equation of continuity gives $a_1v_1 = a_2v_2$, so

$$\frac{v_1}{v_2} = \frac{a_2}{a_1}$$

[22]The Venturi effect is the reduction in fluid pressure that results when a fluid flows through a constricted section of pipe. The Venturi effect is named after Giovanni Battista Venturi (1746-1822), an Italian physicist.

Bernoulli's equation for these two sections can be written as

$$\frac{p_2 - p_1}{\rho g} + \frac{{v_2}^2 - {v_1}^2}{2g} + (z_2 - z_1) = 0$$
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{{v_2}^2 - {v_1}^2}{2g}$$
$$h = \frac{{v_2}^2 - {v_1}^2}{2g}$$

where h is difference in piezometric heads. Therefore,

$$h = \frac{{v_2}^2}{2g}\left\{1 - \left(\frac{v_1}{v_2}\right)^2\right\}$$
$$= \frac{{v_2}^2}{2g}\left\{1 - \left(\frac{a_2}{a_1}\right)^2\right\}$$
$$v_2 = a_1 \frac{\sqrt{2gh}}{\sqrt{{a_1}^2 - {a_2}^2}}$$

The discharge is determined as

$$\dot{Q} = a_2 v_2$$
$$= a_1 a_2 \frac{\sqrt{2gh}}{\sqrt{{a_1}^2 - {a_2}^2}}$$

To adjust the energy losses due to swirl or separation, a *coefficient of discharge* (c_d) is used in the expression of discharge through pipe:

$$\dot{Q} = c_d a_1 a_2 \frac{\sqrt{2gh}}{\sqrt{{a_1}^2 - {a_2}^2}}$$

The following relations are used when differential manometer is used to measure h with a different liquid (e.g. mercury) of specific gravity s_m [Fig. 6.36]:

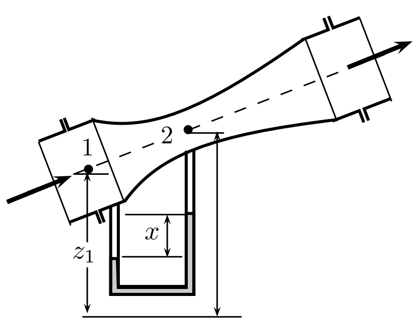


Figure 6.36 Venturimeter.

1. $\underline{s_m > s}$ Equilibrium equation results in

$$s\rho_w g\{l - x + (z_2 - z_1)\} = p_1 + s\rho_w g l - p_2 - s_m \rho_w g x$$

This gives

$$h = \left(\frac{p_1}{s\rho_w g} + z_1\right) - \left(\frac{p_2}{s\rho_w g} + z_2\right)$$
$$= \frac{s_m}{s}x - x$$
$$= x\left(\frac{s_m}{s} - 1\right)$$

2. $\underline{s_m < s}$ Equilibrium equation results in

$$p_2 + s\rho_w g\{l + x + (z_2 - z_1)\} = p_1 + s\rho_w g l + s_m \rho_w g x$$

This gives

$$h = \left(\frac{p_1}{s\rho_w g} + z_1\right) - \left(\frac{p_2}{s\rho_w g} + z_2\right)$$
$$= x - \frac{s_m}{s}x$$
$$= x\left(1 - \frac{s_m}{s}\right)$$

3. $\underline{s_m = s}$ This will result into a situation when $x = 0$ and the flow cannot be measured.

6.8.2 Orifice Meter

An *orifice meter* is a device used for discharge measurement in which an orifice plate is used to suddenly contract the area of flow in a pipe. The calculations are same as that for a venturimeter. Energy losses are associated when the fluid flows through an orifice plate, therefore, *coefficient of discharge* (c_d) is in the range of 0.67. The throat area (a_2) is found little bit advanced of orifice plate.

6.8.3 Nozzle Meter

A *nozzle meter* is similar to an orifice meter but with a smooth convergent cone to make c_d about 0.98.

6.8.4 Rotameter

A *rotameter* is also called *variable area meter*. It consists of a rotor heavier than the fluid and hence it sinks to the bottom when liquid is at rest. As the liquid begins to flow through the meter, it lifts the rotor until it reaches a steady level corresponding to discharge. The rotation of float helps to keep it steady at which hydrostatic and dynamic thrust of the liquid on underside of rotor will be equal to hydrostatic thrust on the upper side plus the apparent weight of the rotor [Fig. 6.37].

6.8.5 Elbowmeter

An *elbowmeter* is an important device for precise discharge measurements. This consists of an elbow attached to a pipe for flow measurement. A differential

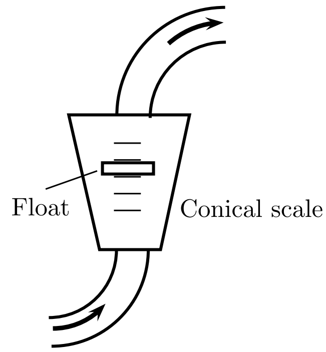


Figure 6.37 Rotameter.

manometer is fitted on this whose one end is fitted at inner radius, another end at out radius [Fig. 6.38].

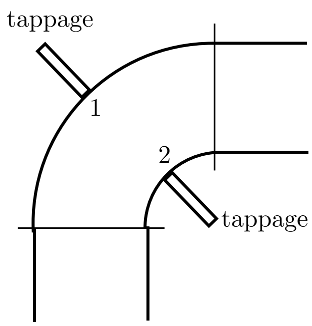


Figure 6.38 Elbowmeter.

When liquid flows through an elbow, pressure increases with radius due to approximately free vortex flow. This pressure differential is used to proportionate the discharge by introducing a discharge correction factor c_d.

6.8.6 Pitot Tube

A *pitot tube*[23] is used to measure flow in a very large pipe or container. This consists of a very fine L-bent pipe [Fig. 6.39].

When fluid strikes at the projected end against the flow, the velocity at point A vanishes into dynamic head h above the free surface. Bernoulli's equation between points 1 and A can be written as

$$\underbrace{h_0 + \frac{v^2}{2g}}_{\text{Point 1}} = \underbrace{(h_0 + h) + 0}_{\text{Point A}}$$

[23]The pitot tube was invented by the French engineer Henri Pitot in the early 18th century, and was modified by French scientist Henry Darcy.

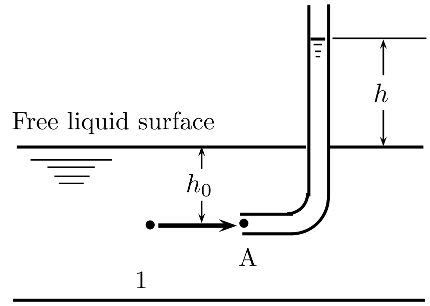


Figure 6.39 Simple pitot tube.

$$v = \sqrt{2gh}$$

This pattern of formula is known as the *Torricelli's formula* which indicates that the velocity of flow is proportional to the square root of the head.

There are energy losses associated with the flow. Therefore, the actual velocity is always less than the theoretical velocity based on the Bernoulli's equation. These losses are quantified by using a *coefficient of velocity* (c_v) in the calculation of actual velocity of flow. Therefore, after introducing *coefficient of velocity* in *Torricelli's formula*,

$$v = c_v\sqrt{2gh}$$

Since, an instantaneous change of direction is not possible, the streamlines continue to converge beyond the orifice until they become parallel at a short distance from the throat. Thus, the minimum area of the stream is lower than the area of the throat or orifice. This section of stream is known as *vena contracta*. To quantify this phenomenon, a *coefficient of contraction* (c_c) is used in the calculation of discharge. The coefficient of discharge is the product of *coefficient of velocity* (c_v) and *coefficient of contraction* (c_c) as

$$c_d = c_v \times c_c$$

6.9 VORTEX MOTION

A liquid is said to be in *vortex motion* when fluid particles rotates about an axis. Vortex motion can be of two types:

1. <u>Free Vortex Motion</u> In a *free vortex motion*, fluid particles rotate without any external force being impressed upon the fluid; it does not involve expenditure of energy from external source, such as in whirlpool in a river, flow around a circular bend, flow after impeller in centrifugal pump, flow before turbine vanes. Angular momentum is

conserved in free vortex motion:

$$\frac{\partial}{\partial t}(mv_\theta r) = 0$$
$$v_\theta r = C$$

A point $r = 0$ where $v_\theta = \infty$ is called *singular point*.

Circulation in free vortex motion is determined as

$$\Gamma = (2\pi r)\, v_\theta = 2\pi C$$

But the flow is irrotational.

2. Forced Vortex Motion In a *forced vortex motion*, whole mass rotates with constant angular velocity which requires expenditure of energy from external sources, such as in a flow inside impeller of centrifugal pump, runner. The flow field is described in polar coordinate system as

$$v_\theta = \omega r$$
$$v_r = 0$$

All the fluid particles rotate with the same angular velocity ω like a solid body. The vorticity for the above flow field can be determined as

$$\Omega = 2\omega$$

Therefore, a forced vortex motion is not irrotational.

6.10 FLOW FROM A LINE SOURCE

A two-dimensional flow emanating from a point in the x–y plane and imagined to flow uniformly in all directions is called a *source*. Since the two-dimensional source is a line in the z-direction, it is known as a *line source* [Fig. 6.40].

Total flow per unit time per unit length of the line source is called *flow strength*, (m) of the source. The tangential velocity (v_θ) at a radial distance r from the source is given by

$$v_\theta = \frac{m}{r}$$

In this case, the stream function, potential function and circulation are given by

$$\psi = m\theta$$
$$\phi = m \ln r$$
$$\Gamma = 0$$

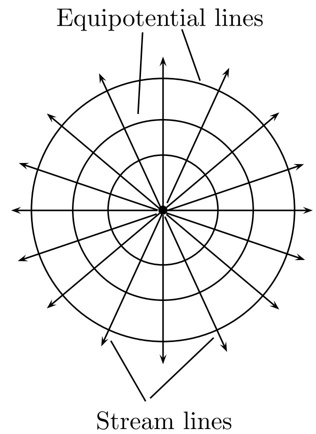


Figure 6.40 Flow from a line source.

6.11 LAMINAR PIPE FLOW

A pipe is a closed conduit in which fluid flows under pressure. Pipes are commonly circular in cross-section. The flowing fluid in pipe is always resisted due to viscosity and boundary wall characteristics of pipe wall. In order to overcome it, certain energy of the flowing fluid is absorbed and converted into heat.

Consider a pipe of radius R, diameter D $(= 2R)$, length l, inclined at angle θ from vertical, through which a liquid of density ρ and coefficient of viscosity μ is flowing in laminar way. The x-coordinate is along the length and z-coordinate is vertical [Fig. 6.41].

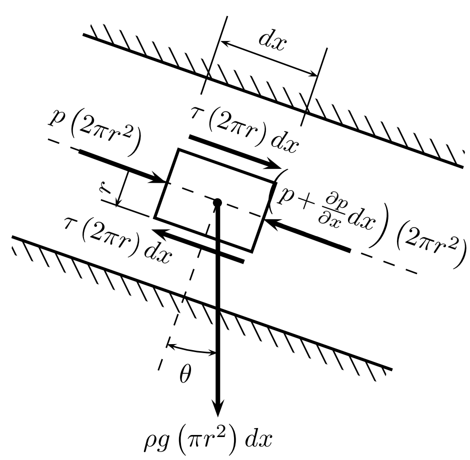


Figure 6.41 Laminar flow through a pipe.

The angle θ can be expressed as

$$\cos\theta = -\frac{dz}{dx} = -\frac{\partial z}{\partial x}$$

6.11.1 Velocity Profile

In laminar flow through a pipe, different layers are generated in the form of concentric tubes, and one tube smoothly slides over another. Thus, the inner laminar tube can be assumed as a cylinder. This occurs at low velocity so that forces due to viscosity predominate over the inertia forces.

Consider equilibrium of an infinitesimal cylindrical fluid element of thickness dr, radius r, length δx. Pressure differential[24] across its length is $\partial p/\partial x$ and shear stress acting on outer surface is τ_r which results in shear force $2\pi r\tau\delta x$ working opposite to the flow direction (x). Therefore, shear stress at that point is determined by writing the equilibrium of forces along x-axis as

$$\frac{\partial p}{\partial x}\delta x\pi r^2 + \tau_r 2\pi r\delta x = \rho\left(\pi r^2\delta x\right) g\cos\theta$$

$$\begin{aligned}\tau_r &= -\frac{\partial p}{\partial x}\frac{r}{2} - \rho g\frac{\partial z}{\partial x}\frac{r}{2}\\ &= -\rho g\frac{\partial}{\partial x}\left(\frac{p}{\rho g} + z\right)\frac{r}{2}\\ &= -\rho g\frac{\partial h}{\partial x}\frac{r}{2}\end{aligned} \tag{6.34}$$

where h is the piezometric head at the element. If the total piezometric head loss is h_f in length l, then gradient is written as

$$-\frac{\partial h}{\partial x} = \frac{h_f}{l} \tag{6.35}$$

Equation (6.34) shows that variation of τ_r w.r.t. r is linear with zero at $r = 0$ and maximum at $r = R$ [Fig. 6.41].

Newton's law of viscosity provides

$$\tau_r = -\mu\frac{\partial v}{\partial r}$$

Therefore, using Eq. (6.34), one gets

$$\begin{aligned}-\rho g\frac{\partial h}{\partial x}\frac{r}{2} &= -\mu\frac{\partial v}{\partial r}\\ \int dv &= \frac{1}{2\mu}\rho g\frac{\partial h}{\partial x}\int r\,dr\\ v &= \frac{1}{4\mu}\rho g\frac{\partial h}{\partial x}r^2 + C\end{aligned}$$

[24]Here, r, x are variables used to show the coordinates of a point. So partial differential is taken in expression for $\cos\theta$ and pressure differential.

At $r = R$, $v = 0$, therefore,

$$v = -\frac{1}{4\mu}\rho g\frac{\partial h}{\partial x}\left(R^2 - r^2\right) \tag{6.36}$$

Eq. (6.36) indicates a parabolic distribution of velocity [Fig. 6.42].

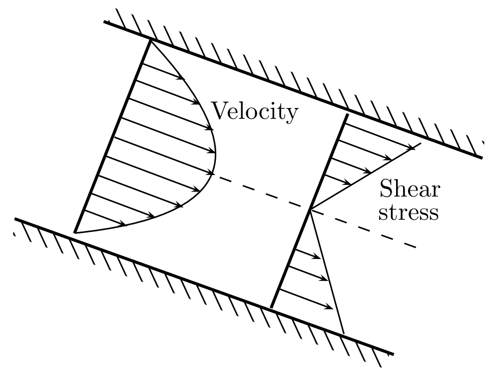


Figure 6.42 Velocity and shear stress profiles.

The following parameters can be determined:

1. <u>*Maximum Velocity*</u> The maximum velocity occurs at $r = 0$:

$$v_{max} = -\frac{1}{4\mu}\rho g\frac{\partial h}{\partial x}R^2$$

$$v = v_{max}\left\{1 - \left(\frac{r}{R}\right)^2\right\}$$

2. <u>*Average Velocity*</u> The average velocity $\bar{v}$ is found as

$$\begin{aligned}\bar{v} &= \frac{1}{\pi R^2}\int_0^R v\,(2\pi r)\,dr\\ &= \frac{2v_{max}}{R^2}\int\left\{1 - \left(\frac{r}{R}\right)^2\right\} r\,dr\\ &= \frac{2v_{max}}{R^2}\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R\\ &= \frac{2v_{max}}{R^2}\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right)\\ &= \frac{v_{max}}{2}\end{aligned}$$

The point at which $\bar{v}$ occurs is given by

$$\bar{v} = \left\{1 - \left(\frac{\bar{r}}{R}\right)^2\right\} v_{max}$$

$$\bar{r} = \frac{R}{\sqrt{2}}$$

6.11.2 Discharge

Discharge through pipe is determined as

$$
\begin{aligned}
Q &= \int_0^R v\,(2\pi r)\,dr \\
&= -\frac{1}{4\mu}\rho g\frac{\partial h}{\partial x}\int_0^R \left(R^2-r^2\right)(2\pi r)\,dr \\
&= -\frac{\pi}{2\mu}\rho g\frac{\partial h}{\partial x}\int_0^R \left(R^2-r^2\right) r dr \\
&= -\frac{\pi}{2\mu}\rho g\frac{\partial h}{\partial x}\left[R^2\frac{r^2}{2}-\frac{r^4}{4}\right]_0^R \\
&= -\frac{\pi}{8\mu}\rho g\frac{\partial h}{\partial x}R^4 \\
&= -\frac{\pi D^4}{128\mu}\frac{\partial p}{\partial x} \qquad (6.37)
\end{aligned}
$$

6.11.3 Piezometric Head Gradient

Using Eq. (6.37), the gradient of piezometric head ($h = p/\rho g$) can be written as

$$
\begin{aligned}
-\frac{\partial h}{\partial x} &= \frac{8\mu Q}{\pi\rho g R^4} \\
&= \frac{8\mu\left(\pi D/2^2\bar{v}\right)}{\pi\rho g D/2^4} \\
&= \frac{32\mu\bar{v}}{\rho g D^2} \qquad (6.38)
\end{aligned}
$$

This equation is known as *Hagen-Poiseuille's equation.* It suggests that the gradient in absolute terms is proportional to $\bar{v}/D^2$:

$$
-\frac{\partial h}{\partial x} \propto \frac{\bar{v}}{D^2}
$$

The following are the relevant terms and expressions related to the piezometric head gradient:

1. Fanning Friction Coefficient Based on Eq. (6.38), the head loss in a pipe flow is generally written in the following form:

$$
h_f = \frac{4fl}{D}\times\frac{\bar{v}^2}{2g} \qquad (6.39)
$$

where f is called *Fanning friction coefficient.*

2. Darcy-Weisbach Friction Factor Another important term used in calculations is the *Darcy-Weisbach friction factor*, which is four times of the Fanning friction factor f, thus equal to $4f$. Using Eqs. (6.38) and (6.39),

$$
4f = 64\frac{\mu}{\rho\bar{v}D}
$$

Eq. (6.39) can also be used for other than circular cross-sections by taking D as $D_c = 4A/P$ where A and P are area and perimeter of the cross-section, respectively.

3. Reynolds Number A dimensionless parameter, *Reynolds number*[25] (Re), is defined as the ratio of inertia force and viscous force. For the present case,

$$
\mathrm{Re} = \frac{\rho v D}{\mu}
$$

So, the relation between $4f$ and Re is

$$
4f = \frac{64}{\mathrm{Re}} \qquad (6.40)
$$

Laminar flow exists in the range of Re < 2000.

Moody's diagram is a f (x-axis) versus Re (y-axis) plot for different types of pipe surfaces. In this plot, curve for laminar flow is linear, which is depiction of the above inverse relationship [Eq. (6.40)].

6.11.4 Shear Stress

Substituting value of $-\partial h/\partial x$ from Eq. (6.38) into Eq. (6.34), one gets

$$
\begin{aligned}
\tau_r &= \rho g\left(\frac{32\mu\bar{v}}{\rho g D^2}\right)\frac{r}{2} \\
&= \frac{32\mu\bar{v}}{D^2}\frac{r}{2}
\end{aligned}
$$

This equation represents the distribution of shear stress in laminar pipe flows [Fig. 6.42]. The equation indicates that the maximum value of τ_r would occur at $r = D/2$, as

$$
\begin{aligned}
\tau_{max} &= \frac{32\mu\bar{v}}{D^2}\frac{D/2}{2} \\
&= \frac{8\mu\bar{v}}{D}
\end{aligned}
$$

Above expression of τ_{max} is in absolute terms. Shear stress at radius r (τ_r) can be written as

$$
\tau_r = \frac{2\tau_{max}}{D^2}r
$$

Alternatively, τ_r can be determined by substituting value of $-\partial h/\partial x$ from Eq. (6.39) into Eq. (6.34) as

$$
\tau_r = \rho g\left(\frac{4fl}{D}\times\frac{\bar{v}^2}{2g}\right)\frac{r}{2}
$$

[25] The concept was introduced by George Gabriel Stokes in 1851, but the Reynolds number is named after Osborne Reynolds (1842-1912), who popularized its use in 1883.

The maximum value of τ_r at $r = D/2$ is

$$\tau_{max} = \rho\left(\frac{4fl}{D}\times\frac{\bar{v}^2}{2}\right)\frac{D/2}{2}$$
$$= f\times\frac{\rho\bar{v}^2}{2}$$

Using this,

$$\bar{v}^* = \sqrt{\frac{\tau_{max}}{\rho}}$$

The average velocity $\bar{v}^*$ shows the velocity of shear, hence, it is called *shear velocity* or *friction velocity.*

6.11.5 Power Transmission

The power required to overcome the head gradient is determined as

$$P = \rho g\dot{Q}h_f$$
$$= \rho g\left(\frac{\pi D^2}{4}\right)\frac{32\mu\bar{v}l}{\rho g D^2}$$
$$= 8\pi\mu\bar{v}^2 l$$

This expression is in absolute terms. Thus,

$$P \propto \mu\bar{v}^2 l$$

Alternative form is

$$P = \tau_{max}\pi D l\bar{v}$$

Suppose H is the total head supplied at the inlet of the pipe, so remnant head at the exit is $(H - h_f)$. Using the value of h_f from Eq. (6.39), the power available at the exit is

$$P_r = \rho g\dot{Q}\left(H - h_f\right)$$
$$= \rho g\left(\frac{\pi D^2}{4}\bar{v}\right)\left(H - \frac{4fl}{D}\times\frac{\bar{v}^2}{2g}\right)$$

For maximum power transmitted

$$\frac{dP_r}{d\bar{v}} = 0$$
$$H = 3h_f$$

Thus, power transmitted P_r is maximum when the loss of head due to friction is one third of the total head supplied. Efficiency at maximum power transmission is

$$\eta = \frac{H - h_f}{h_f}$$
$$= 66.7\%$$

Therefore, the efficiency for maximum power transmission is 66.7%.

6.11.6 Losses in Pipe Flow

Fluid flow is always associated with shear stresses and to overcome this, external power is required. Similarly, when there is contraction or expansion in flow area, there are associated head losses due to formation of swirl. In this reference, losses in a pipe flow are broadly classified into major and minor losses.

6.11.6.1 Major Losses Major losses are associated with the friction or viscosity. Expression for major loss in a pipe flow has been derived in previous analysis in terms of piezometric head [Eq. (6.39)]:

$$h_f = \frac{4fl}{D}\times\frac{\bar{v}^2}{2g}$$

6.11.6.2 Minor Losses Minor losses are due to change in velocity because of change of flow area. The following are the expressions for minor losses

1. At pipe inlet (square edge)

$$h_i = \frac{1}{2}\times\frac{v^2}{2g}$$

2. Due to sudden expansion

$$h_e = \frac{\left(v_1 - v_2\right)^2}{2g}$$

3. Due to bends

$$h_b = k\frac{{v_2}^2}{2g}$$

where $k = f(\theta, R/D)$

4. Due to sudden contraction/nozzle

$$h_c = \left(\frac{1}{{c_c}^2 - 1}\right)\frac{{v_2}^2}{2g}$$

where c_c is the *coefficient of contraction.*

5. At pipe outlet

$$h_o = \frac{v^2}{2g}$$

6. Due to conical expansion (cone angle α)

$$h_{ce} = k\frac{\left(v_1 - v_2\right)^2}{2g}$$

with $k = f(\alpha, D_2/D_1)$.

6.11.7 Equivalent Length

When many pipes of different materials are joined together (in series or parallel fashion) to flow the same discharge, the analysis of flow is made easy by using the concept of *Equivalent length* (l_e). The equivalent length is defined as the length of an imaginary pipe of standard diameter (D) with the same head loss (h_f), discharge ($\dot{Q}$) and friction factor (f) of the referred pipe. Putting expression of $\bar{V} = 4\dot{Q}/(\pi D^2)$ into Eq. (6.39),

$$\begin{aligned} h_f &= \frac{4fl}{D} \frac{\left(4\dot{Q}\right)^2}{2g\left(\pi D^2\right)^2} \\ &= \frac{32\dot{Q}^2}{\pi g} \times \frac{fl}{D^5} \end{aligned}$$

Therefore[26],

$$h_f \propto \frac{fl}{D^5}$$

The following paragraphs explain the calculation of equivalent length for series and parallel combinations of pipes:

6.11.7.1 Pipes in Series In series combination of n pipes, the total head loss is the sum of partial head losses in different pipes:

$$\frac{f_e l_e}{{D_e}^5} = \sum_{i=1}^{n} \frac{f_i l_i}{{D_i}^5} \tag{6.41}$$

Therefore, n number of pipes of diameter d can be replaced by a single pipe of diameter D:

$$\begin{aligned} \frac{fl}{D^5} &= n\frac{fl}{d^5} \\ n &= \left(\frac{D}{d}\right)^{5/2} \end{aligned}$$

6.11.7.2 Pipes in Parallel In parallel combination, there is same head loss in all the pipes:

$$\frac{f_i l_i}{{D_i}^5} = C$$

Total discharge is the sum of the partial discharges in different pipes:

$$\dot{Q} = \sum_{i=1}^{n} \dot{Q}_i$$

[26]This is not in absolute terms as f is taken as a constant.

6.11.8 Water Hammer

When valve at a section in a pipe is closed suddenly, the fluid immediately next to the valve is compressed by rest of liquid flowing in downstream. The pressure at the valve increases because velocity is reduced to zero. At the same time, walls of the pipe will be stretched due to elasticity and also due to compressibility of fluid. All this leads to propagation of *pressure wave* in the upstream direction along the pipe. After traveling through the length upto exit, entire mass comes to rest and under excess pressure. At this unbalanced situation, restarting of pressure waves takes place and it propagates towards the closed valve. Some liquid can flow back to the reservoir. Ideally, there will be pressure waves traveling back and forth. This phenomenon is called *water hammer*.

The pressure waves can cause major problems, from noise and vibration to pipe collapse. Air vessels are pressurized vessels used to protect the system from water hammer. They typically have an air cushion above the fluid level in the vessel, which can be regulated or separated by a bladder.

6.12 STOKES' LAW

Laminar flow can exist around a solid body which moves through a fluid of infinite extent in such a way that viscosity is the only fluid property affecting the pattern of motion and the accelerating force is negligible, such as dust particle falling in air, fine sand falling in water. The drag on moving body depends upon size of the object, velocity and viscosity of the fluid.

Stokes' law[27] is an expression for the drag (F_D) experienced by a sphere of diameter D, moving with constant velocity v in a fluid of viscosity μ [Fig. 6.43], written as

$$F_D = 3\pi\mu v D \qquad \text{valid for Re} < 0.1$$

This equation holds good for values of Reynolds number less than 0.1.

Including the buoyancy force, if the sphere is dropping down through a fluid (say water) under its own weight then equilibrium equation is written as

$$\underbrace{\frac{\pi}{6}D^3\rho_w g}_{\text{Buoyancy force}} + F_D = \frac{\pi}{6}D^3\rho g$$

[27]Neglecting inertia forces which are not important in highly viscous flows, Professor George Gabriel Stokes, an Irish-born mathematician, used Navier-Stokes equations to derive the Stokes' law. It played a critical role in the research leading to at least 3 Nobel Prizes.

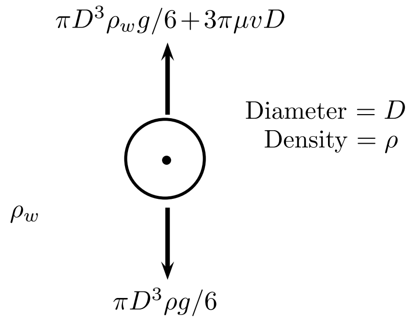


Figure 6.43 Stokes' law.

Therefore, the drag force acting on the body is given by

$$F_D = \frac{\pi}{6}D^3\left(\rho-\rho_w\right)g$$
$$3\pi\mu VD = \frac{\pi}{6}D^3\left(\rho-\rho_w\right)g$$
$$v = \frac{D^2}{18\mu}\left(\rho-\rho_w\right)g$$

The velocity in the above expression is called *terminal fall velocity*. The drag force (F_D) can also be expressed as

$$F_D = C_D A\rho\frac{v^2}{2}$$

where

$$A = \frac{\pi}{4}D^2$$
$$C_D = \frac{24}{\mathrm{Re}}$$
$$\mathrm{Re} = \frac{\rho vD}{\mu}$$

The Stokes' law is applied in determining the terminal velocities of fine sediment particles falling through fluids. It is the basis of the *falling-sphere viscometer*. This can be used to explain why small water droplets or ice crystals can remain suspended in air as clouds until they grow to a critical size and start falling as rain or snow and hail.

6.13 BOUNDARY LAYER THEORY

When a fluid flows over a plate or surface in general, the layer of the fluid in direct contact will have zero velocity while, due to viscosity, layers over this will be getting velocity from zero to main stream velocity. Thus, a small region of vertical *velocity gradient* is produced at the immediate vicinity of the boundary surface. This region is known as *boundary layer* where effect of viscosity is found.

Reynolds number (Re_x) for boundary layer at distance x from edge is defined as

$$\mathrm{Re}_x = \frac{\rho u_\infty x}{\mu}$$

Re_x changes from 3×10^5 to 6×10^5. The thickness of the boundary layer depends on the Reynolds number.

Boundary layers are also formed at the inlet of a pipe flow and after a distance x_e, called *entry length*, the flow becomes fully developed (the whole section comes under the effect of viscosity) [Fig. 6.44].

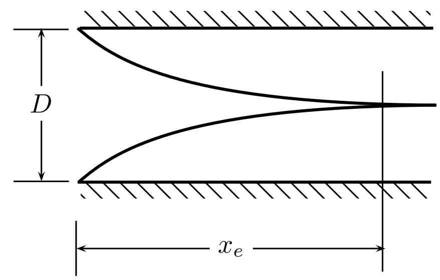


Figure 6.44 Entry length in pipe flows.

The entry length, x_e, is given by

$$x_e \approx \begin{cases} 0.005D & \text{Laminar flow} \\ 10D \text{ to } 60D & \text{Turbulent flow} \end{cases}$$

6.13.1 Boundary Layer Thicknesses

Normal *thickness of boundary layer* is the distance from boundary surface at which the velocity reaches 0.99 of the main stream velocity. At the beginning near the edge, the boundary layer is laminar due to effect of viscosity. Further along the flow direction, there comes a *transition zone* where boundary layer changes from laminar to turbulent.

Let a fluid of density ρ and viscosity μ move with free stream velocity u_∞ and form a boundary layer of normal thickness δ over a surface of length l and width b. Velocity at any arbitrary point at height y and distance x from edge is u. The loss of velocity, called *viscosity defect*, at this point is $u_\infty - u$ [Fig. 6.45].

The difference in stream velocity and velocities inside the boundary layer results in a reduction in mass flow, momentum flux, and kinetic energy of the fluid in the boundary layer as compared to the free stream of the same thickness. These deficits are determined as follows.

1. Actual mass flow rate per unit width at any section in boundary layer of normal thickness δ is

$$\Delta\dot{m} = \int_0^\delta \rho u dy$$

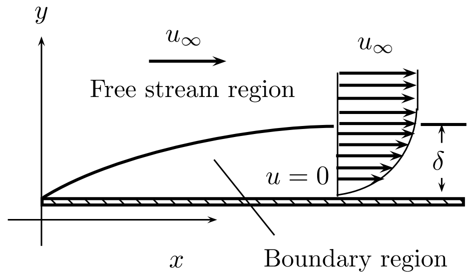


Figure 6.45 Boundary layer.

2. Momentum flux at the section is

$$\Delta m_\theta = \int_0^\delta \rho u^2 dy$$

3. Loss of kinetic energy per unit mass is $\left(u_\infty^2 - u^2\right)/2$. Thus, loss of kinetic energy is

$$\Delta E = \int_0^\delta \rho u \frac{\left(u_\infty^2 - u^2\right)}{2} dy$$

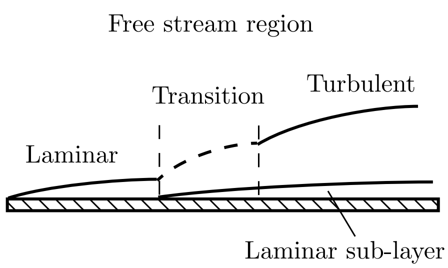


Figure 6.46 Boundary layers.

Therefore, to remove effect of viscosity in calculation of mass flow rate, momentum flux and kinetic energy, the following three imaginary thicknesses are to be added into normal thickness:

1. Displacement Thickness *Displacement thickness*, (δ^*) is the distance by which the boundary surface would have to be displaced outward so that the total actual discharge would be the same as that of ideal frictionless fluid past the displacement thickness:

$$\delta^* u_\infty = \int_0^\delta (u_\infty - u)\, dy$$
$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$$

2. Momentum Thickness *Momentum thickness* (θ) is the distance from the actual boundary surface such that momentum flux corresponding to main stream velocity through this distance is equal to deficiency of momentum due to boundary layer formation:

$$\rho u_\infty^2 \theta = \int_0^\delta \left(u_\infty^2 - u^2\right) dy$$
$$\theta = \int_0^\delta \left\{1 - \left(\frac{u}{u_\infty}\right)^2\right\} dy$$

3. Energy Thickness *Energy thickness* (δ_E) is the distance from the actual boundary surface such that the energy flux corresponding to free stream velocity through this distance is equal to deficiency of energy due to boundary layer formation:

$$\frac{1}{2}\rho {u_\infty}^3 \delta_E = \int_0^\delta \rho u \frac{\left(u_\infty^2 - u^2\right)}{2} dy$$
$$\delta_E = \int_0^\delta \frac{u}{u_\infty} \left\{1 - \left(\frac{u}{u_\infty}\right)^2\right\} dy$$

6.13.2 Laminar Boundary Layer

6.13.2.1 Velocity Distribution For a laminar boundary layer, the following parabolic velocity distribution is near practical results.

$$(u_\infty - u) = (\delta - y)^2 \tag{6.42}$$

6.13.2.2 Boundary Layer Thickness Using Eq. (6.42) along with the principles of fluid flow, the expression[28] for normal thickness can be found as

$$\frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}$$
$$\delta = 5\sqrt{\frac{x\nu}{u_\infty}}$$

where $\nu (= \mu/\rho)$ is the kinematic viscosity. Therefore,

$$\delta \propto \sqrt{x}$$

6.13.2.3 Shear Stress Shear stress at $y = 0$ is given by

$$\frac{\tau_0}{\rho v_\infty^2/2} = \frac{0.644}{\sqrt{\mathrm{Re}_x}} = C_f$$

where C_f is called *local drag coefficient*. The above expression is also known as Blasius law. Using this, one gets,

$$\tau_0 \propto \frac{1}{\sqrt{x}}$$

[28]Derivation of boundary layer thickness is out of context.

6.13.2.4 Drag Force Total drag force on plate is written as

$$F_D = \int_0^l \tau_0 b dx$$
$$= \bar{C}_f bl \frac{\rho v_\infty^2}{2}$$

where $\bar{C}_f$ is the overall (average) drag coefficient, given by

$$\bar{C}_f = \frac{1.328}{\sqrt{\text{Re}_l}}$$

Other expressions are

$$\frac{\delta^*}{x} = \frac{1.729}{\sqrt{\text{Re}_x}}$$
$$\frac{\theta}{x} = \frac{0.644}{\sqrt{\text{Re}_x}}$$

6.13.3 Turbulent Boundary Layer

After the transition zone, in turbulent boundary layer, logarithmic profile as written below gives near practical results.

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/n}$$

where value of index, n depends upon Re_x and it decreases somewhat with increasing Re_x. Here, $n \approx 7$ works fine. It can be seen that

$$u \propto log(y)$$

In this turbulent zone, in immediate vicinity of boundary, there also exists a laminar sub-layer with linear velocity distribution and extreme velocity gradient. The thickness of this laminar sublayer is given by

$$\delta' = 11.6 \frac{\nu}{\sqrt{\frac{\tau_0}{\rho}}}$$
$$= 11.6 \frac{\nu}{u^*}$$

where u^* is *shear velocity* [Fig. 6.46].

In the turbulent zone (Re_x varies from 5×10^5 to 5×10^7), the following logarithmic velocity distribution is found:

$$\frac{\delta}{x} = \frac{0.376}{{\text{Re}_x}^{1/5}}$$
$$\delta \propto x^{4/5}$$
$$\bar{C}_f = \frac{0.074}{{\text{Re}_l}^{1/5}}$$

The thickness of turbulent boundary layer increases at much faster rate than in laminar boundary layer. Average drag coefficient in turbulent layer is higher than that in laminar one.

The average drag coefficient for length l, including turbulent boundary layer, is given by

$$\bar{C}_f = \frac{0.074}{{\text{Re}_l}^{1/5}} - \frac{1700}{\text{Re}_l}$$

6.13.4 Flow Separation

When a boundary layer flow occurs against adverse pressure gradient:

$$\frac{dp}{dx} > 0$$

The momentum of fluid particles gets reduced and energy is dissipated as heat by formation of eddies. Thus, the flow starts separating from the boundary. This phenomenon is called *flow separation*.

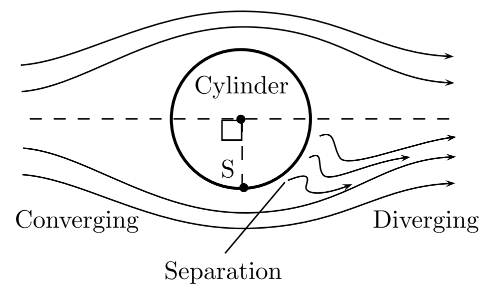


Figure 6.47 Flow separation across a cylinder.

Figure 6.47 shows the case of flow separation in the flow past a circular cylinder in an infinite medium. The flow area is converging upto 90° beyond which it is diverging. In the viscous region (near the solid boundary), the viscous forces opposes the combined effect of pressure and inertia. It is evident that beyond 90°, the inertia force is opposed by both viscous and pressure force. Thus, in this phenomenon, the fluid particles in the boundary layer are separated from the wall and driven in the upstream direction, however, flowing stream pushes back these separated layers together with it and develops a broad pulsating wake.

Since the wake zone pressure is less than that of the forward stagnation point, the cylinder experiences a drag force which is basically attributed to the pressure difference. The drag force brought about by this pressure difference is known as form drag, whereas the shear stress at the wall gives rise to skin friction drag. Generally, these two drag forces together are responsible for the resultant drag on body.

IMPORTANT FORMULAS

Fluid Properties

1. Newton's law of viscosity

$$\tau = \mu \frac{du}{dy}$$
$$\nu = \frac{\mu}{\rho}$$

2. Viscous torque

(a) Rotating cylinder

$$T = 2\pi\omega\mu \frac{lr^3}{h}$$

(b) Rotating disc

$$T = \frac{1}{2}\pi\omega\mu \frac{r^4}{h}$$

(c) Rotating cone

$$T = \frac{1}{2}\pi\omega\mu \frac{r^4}{h\sin\theta}$$

(d) Rotating hemisphere

$$T = \frac{4}{3}\pi\omega\mu \frac{r^4}{h}$$

3. Compressibility

$$K = \frac{\delta p}{-\delta v/v} = \frac{1}{\beta} = \begin{cases} p & \text{Isothermal} \\ \gamma p & \text{Adiabatic} \end{cases}$$

4. Excess pressure

$$\text{Bubbles } \Delta p = \frac{4\sigma}{r}$$
$$\text{Droplets } \Delta p = \frac{2\sigma}{r}$$
$$\text{Jets } \Delta p = \frac{\sigma}{r}$$

5. Capillarity

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

Fluid Pressure

1. Hydrostatic force on planes

$$F = \rho g \bar{x} \times A$$
$$\bar{x} = \bar{y}\sin\theta$$
$$\bar{x} = \frac{1}{A}\int x dA$$
$$\bar{x}_p = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$$

(a) Rectangular plane

$$\bar{x}_p = \bar{x} + \frac{(d\sin\theta)^2}{12\bar{x}}$$

(b) Circular disc

$$\bar{x}_p = \bar{x} + \frac{(d\sin\theta)^2}{16\bar{x}}$$

(c) Triangular plane

$$\bar{x}_p = \bar{x} + \frac{(d\sin\theta)^2}{18\bar{x}}$$

2. Hydrostatic force on surfaces

$$F_H = \int p\cos\theta dA$$
$$F_V = \int p\sin\theta dA$$
$$F = \sqrt{{F_H}^2 + {F_V}^2}$$
$$\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right)$$

Buoyancy and Floatation

$$F_B = \rho g \forall$$
$$\forall = v \times \frac{s}{s_f}$$
$$\text{GM} = \frac{I_O}{\forall} - \text{BG}$$
$$\omega = \sqrt{\frac{g \times \text{GM}}{k^2}}$$
$$T = \frac{2\pi k}{\sqrt{g \times \text{GM}}}$$

$$k_{pitching} \geq k_{rolling}$$

Stability of Blocks

1. Rectangular block

$$\frac{d}{h} > \sqrt{6s(1-s)}$$

2. Cylindrical block

$$\frac{d}{h} > \sqrt{8s(1-s)}$$

3. Inverted cone

$$\frac{d}{h} > \sqrt{4\left(\frac{1}{s^{1/3}} - 1\right)}$$

Solid Body Motion

1. Linear acceleration

$$p = p_a + \rho(a_z + g)(h - z)$$
$$\frac{dz}{dx} = -\frac{a_x}{a_z + g}$$

2. Rotation

$$p = p_a + \rho\frac{\omega^2 r^2}{2} - \rho g(z - z_0)$$
$$z = \frac{\omega^2 r^2}{2g}$$
$$z_0 = 2z_i - z_m$$

Kinematics

1. Acceleration

$$\vec{a} = \underbrace{\frac{\partial \vec{v}}{\partial t}}_{\text{Local}} + \underbrace{\vec{v} \cdot \nabla \vec{v}}_{\text{Convective}}$$

2. Stream line equation

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

3. Rotation

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{bmatrix} = \frac{1}{2}(\nabla \times \vec{v})$$

4. Shear strains

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

5. Circulation

$$\Gamma = \int_C \vec{v}\cdot d\vec{s} = \int_C (udx+vdy+wdz)$$

6. Vorticity

$$\vec{\Omega} = \nabla \times \vec{v}$$
$$\Omega_z = 2\times\omega_z$$

7. Stream function

$$u = -\frac{\partial\psi}{\partial y}, \quad v = \frac{\partial\psi}{\partial x}$$

8. Potential function

$$u = \frac{\partial\phi}{\partial x}, \quad v = \frac{\partial\phi}{\partial y}, \quad w = \frac{\partial\phi}{\partial z}$$

$$\left(\frac{dy}{dx}\right)_\phi \times \left(\frac{dy}{dx}\right)_\psi = -1$$

9. Cauchy-Riemann equations

$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}, \quad \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x}$$

Principles of Fluid Flow

1. Conservation of mass

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0$$
$$\rho vA = C$$
$$\frac{\partial}{\partial r}(rv_r)+\frac{\partial v_\theta}{\partial\theta} = 0$$

2. Conservation of Energy

$$\frac{p}{\rho g}+\frac{v^2}{2g}+z = C$$

$$\bar{v} = \frac{1}{A}\int vdA$$

3. Conservation of momentum

$$\frac{d}{dt}\int_\forall d\forall\rho\vec{v} = \int_\forall d\forall\rho\frac{d\vec{v}}{dt}$$

Momentum flux

$$\phi = (\rho vdA)\,v$$

Momentum correction factor

$$\beta = \frac{1}{A}\int\left(\frac{v}{\bar{v}}\right)^2 dA$$

Pressure Measurements

1. Venturimeter

$$\dot{Q} = a_1 a_2\frac{\sqrt{2gh}}{\sqrt{{a_1}^2-{a_2}^2}}$$

2. Pitot tube

$$v = c_v\sqrt{2gh}$$

Vortex Motion

1. Free vortex motion

$$v_\theta r = C$$
$$\Gamma = 2\pi C$$

2. Forced vortex motion

$$v_\theta = \omega r$$
$$v_r = 0$$
$$\Omega = 2\omega$$

Laminar Pipe Flow

1. Gradient

$$-\frac{\partial h}{\partial x} = \frac{h_f}{l}$$

2. Velocity

$$v = -\frac{1}{4\mu}\rho g\frac{\partial h}{\partial x}\left(R^2-r^2\right) = v_{max}\left\{1-\left(\frac{r}{R}\right)^2\right\}$$
$$\bar{v} = \frac{v_{max}}{2}$$
$$\bar{r} = \frac{R}{\sqrt{2}}$$

3. Discharge

$$Q = -\frac{\pi D^4}{128\mu}\frac{\partial p}{\partial x}$$

4. Head loss

$$-\frac{\partial h}{\partial x} = \frac{32\mu\bar{v}}{\rho g D^2}$$
$$h_f = \frac{4fl}{D}.\frac{\bar{v}^2}{2g}$$
$$\mathrm{Re} = \frac{\rho vD}{\mu}$$
$$4f = \frac{64}{\mathrm{Re}}$$

5. Shear stress

$$\tau_r = \rho g\left(\frac{4fl}{D}\times\frac{\bar{v}^2}{2g}\right)\frac{r}{2}$$
$$\tau_{max} = f\frac{\rho\bar{v}^2}{2}$$

6. Power

$$P = 8\pi\mu\bar{v}^2 l = \tau_{max}\pi Dl\bar{v}$$
$$h_f \propto \frac{fl}{D^5}$$

Stokes' Law

$$F_D = 3\pi\mu vD \qquad \mathrm{Re} < 0.1$$
$$v = \frac{D^2}{18\mu}(\rho-\rho_w)\,g$$

Boundary Layer

$$\mathrm{Re}_x = \frac{\rho u_\infty x}{\mu}$$

$$\delta^* = \int_0^\delta\left(1-\frac{u}{u_\infty}\right)dy$$
$$\theta = \int_0^\delta\left\{1-\left(\frac{u}{u_\infty}\right)^2\right\}dy$$
$$\delta_E = \int_0^\delta\frac{u}{u_\infty}\left\{1-\left(\frac{u}{u_\infty}\right)^2\right\}dy$$

1. Laminar boundary layer

$$(u_\infty-u) = (\delta-y)^2$$
$$\frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}$$
$$\frac{\tau_0}{\frac{\rho{v_\infty}^2}{2}} = \frac{0.644}{\sqrt{\mathrm{Re}_x}} = C_f$$
$$\tau_0 \propto \frac{1}{\sqrt{x}}$$

2. Turbulent boundary layer

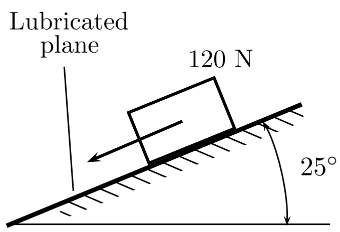
$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/n}$$
$$\delta' = 11.6\frac{\nu}{\sqrt{\frac{\tau_0}{\rho}}} = 11.6\frac{\nu}{u^*}$$
$$\frac{\delta}{x} = \frac{0.376}{{\mathrm{Re}_x}^{1/5}}$$
$$\delta \propto x^{4/5}$$
$$\bar{C}_f = \frac{0.074}{{\mathrm{Re}_l}^{1/5}}$$

Flow Separation

$$\frac{dp}{dx} > 0$$

SOLVED EXAMPLES

1. A rectangular block weighing 120 N slides down a 25° inclined plane which is lubricated by a 2.5 mm thick film of oil of relative density 0.85 and viscosity 9.5 poise.



Determine the terminal velocity of the block if the contact area is 0.25 m^2.

Solution. Given that

$$\begin{aligned} W &= 120 \text{ N} \\ \theta &= 25^\circ \\ h &= 0.0025 \text{ m} \\ \mu &= 9.0/10 \\ &= 0.95 \text{ Pa s} \\ A &= 0.25 \text{ m}^2 \end{aligned}$$

The block is in equilibrium under shear force and weight component:

$$\begin{aligned} W\sin\theta - \tau A &= 0 \\ \tau &= \frac{W\sin\theta}{A} \end{aligned}$$

Let u be the terminal velocity. Using Newton's law of viscosity,

$$\begin{aligned} \mu\frac{u}{h} &= \frac{W\sin\theta}{A} \\ u &= 0.53 \text{ m/s} \end{aligned}$$

2. Velocity of fluid in a viscous flow over a plate is given by the following function:

$$u = 5y - \frac{y^2}{2} \text{ m/s}$$

where y (m) is the distance of the point from bottom surface. If the coefficient of dynamic viscosity is 2.15 Pa s. Determine the shear stress at $y = 3$ m.

Solution. Rate of change of velocity w.r.t. y is

$$\frac{du}{dy} = 5 - y$$

Using Newton's law of viscosity,

$$\begin{aligned} \tau &= \mu\frac{du}{dy} \\ &= \mu(5-y) \\ &= 2.15\times(5-3) \\ &= 4.3 \text{ Pa} \end{aligned}$$

3. Surface tension at air–water interface is 0.073 N/m, Determine the excess pressure in an air bubble of diameter 0.02 mm.

Solution. Given that

$$\begin{aligned} \sigma &= 0.073 \text{ N/m} \\ d &= 0.02 \text{ mm} \end{aligned}$$

Excess presssure in air bubble is given by

$$\begin{aligned} \Delta p &= \frac{4\sigma}{d} \\ &= 14.6 \text{ kPa} \end{aligned}$$

4. A circular annular plate having outer and inner diameter of 2.5 m and 1.25 m, respectively, is immersed in water with its plane making an angle of 45° with the horizontal. The center of the circular annular plate is 2.0 m below the free surface. What is the hydrostatic thrust on one side of the plate?

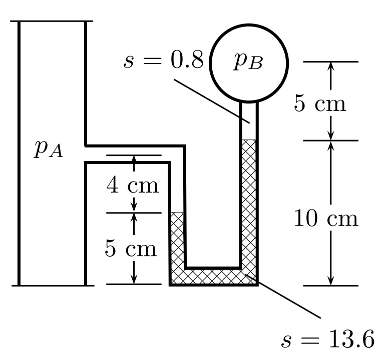
Solution. Given that

$$\begin{aligned} A &= \frac{\pi}{4}\left(2.5^2 - 1.25^2\right) \\ \bar{x} &= 2.0 \\ \rho g &= 9.81\times 1000 \end{aligned}$$

Hydrostatic force is given by

$$\begin{aligned} F &= \rho g\bar{x}A \\ &= 72.27 \text{ kN} \end{aligned}$$

5. A pressure gauge (B) filled with oil is used to measure the pressure at point A in a water main. If the pressure recorded is $p_B = 100$ kPa, determine the pressure at point A.



The pressure at point A is given by

$$\begin{aligned} p_A + 0.04 \times \rho g &= 0.05 \times 13.6 \times \rho g + 0.05 \times 0.8 \times \rho g + p_B \\ p_A &= 6.67 + p_B \\ &= 106.67 \text{ kPa} \end{aligned}$$

6. A rectangular plate 1.75 m × 4.0 m is immersed in oil of specific gravity 0.85 with 1.75 m side horizontal and just at the water surface. The plane of plate makes an angle of 30° with the horizontal. Determine the pressure force on one side of the plate.

Solution. Given that

$$\begin{aligned} \bar{x} &= 2.0 \times \sin 30^\circ \\ &= 1.0 \text{ m} \\ A &= 1.75 \times 4.0 \\ &= 7 \text{ m}^2 \\ \rho &= 0.85 \times 1000 \\ &= 850 \text{ kg/m}^3 \end{aligned}$$

The hydrostatic force on plane surface is

$$\begin{aligned} F &= \rho g(\bar{x} A) \\ &= 58.31 \text{ kN} \end{aligned}$$

7. A steel block having mass of 20 kg is suspended by a wire and lowered until submerged into a tank containing oil of relative density 0.85. Taking the relative density of aluminium as 7.8, calculate the tension in the wire will be ($g = 9.8$ m/s^2).

Solution. Volume of the steel block submerged into the oil is

$$V = \frac{20}{7.8 \times 1000}$$

The oil would exert a buoyancy force that will reduce the tension in the string:

$$\begin{aligned} T &= 20 \times 9.81 - \frac{0.85}{7.8} 20 \times 9.81 \\ &= 174.81 \text{ N} \end{aligned}$$

8. A body weighs 20 N and 25 N when weighed under submerged conditions in liquids of relative densities 1.2 and 0.8, respectively. Determine the volume of the body.

Solution. Given that

$$\begin{aligned} W_1 &= 20 \text{ N} \\ W_2 &= 25 \text{ N} \\ s_1 &= 1.2 \\ s_2 &= 0.8 \end{aligned}$$

Let the volume of the body be V and assume density to be 1. Thus, the weights under submerged condition are given by

$$\begin{aligned} W_1 &= (1 - s_1)\, V g \\ W_2 &= (1 - s_2)\, V g \end{aligned}$$

From the above two equations,

$$\begin{aligned} V &= \frac{W_2 - W_1}{(s_2 - s_1)\, g} \\ &= 12.5 \text{ l} \end{aligned}$$

9. A pitot tube is used to measure the velocity of air having density 1.2 kg/m^3. At a point, the difference in the stagnation and static pressures of a pitot static tube is found to be 450 Pa. Determine the velocity of air at the point in m/s

Solution. The velocity is given by

$$\begin{aligned} v &= \sqrt{\frac{2p}{\rho}} \\ &= 27.38 \text{ m/s} \end{aligned}$$

10. A pipe of 18 cm diameter and 15 km length is used to transport oil of viscosity $\mu = 0.1$ Nm/s^2 from a tanker to the shore with a velocity of 0.5 m/s. The flow is laminar. Determine the power required to maintain the flow.

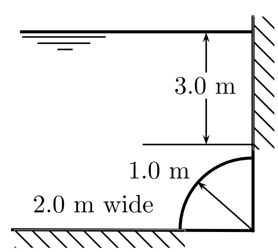
Solution. Given that

$$\begin{aligned} \mu &= 0.1 \text{ Nm/s}^2 \\ \bar{v} &= 0.5 \text{ m/s} \\ l &= 15000 \text{ m} \end{aligned}$$

Power requirement in laminar flow through a pipe is

$$\begin{aligned} P &= 8\pi\mu\bar{v}^2 l \\ &= 9.42 \text{ kW} \end{aligned}$$

11. A door in a tank is in the form of a quadrant of a cylinder of 1.0 m radius and 2.0 m width.



Determine the resultant force acting on the door

Solution. Horizontal component of the force is given by

$$F_H = \rho g \bar{h} A$$

where

$$\begin{aligned}\bar{h} &= 3.0+\frac{1.0}{2}\\ &= 3.5 \text{ m}\\ A &= 2.0\times 1.0\\ &= 2.0 \text{ m}^2\end{aligned}$$

Therefore,

$$F_H = 68.67 \text{ kN}$$

The vertical component of the force is equal to the weight of the water above the door, given by

$$\begin{aligned}F_V &= \rho g \times 2.0 \times \left(2.5\times 1.0 - \frac{\pi\times 1.0^2}{4\times 4}\right)\\ &= 45.19 \text{ kN}\end{aligned}$$

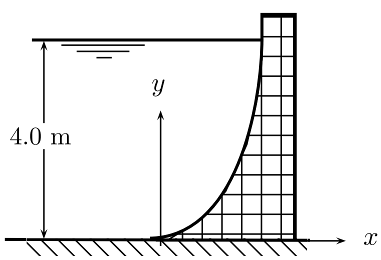
Hence, the net hydrostatic force is

$$\begin{aligned}F &= \sqrt{F_H^2+F_V^2}\\ &= 82.20 \text{ kN}\end{aligned}$$

Inclination of this force is

$$\begin{aligned}\theta &= \tan^{-1}\frac{F_V}{F_H}\\ &= 33.34°\end{aligned}$$

12. A dam has a parabolic shape $y = x^2$ of the inner side wall. Water level in the dam is 5 m from the bottom.



Determine the horizontal and vertical forces on the wall per unit width.

Solution. Horizontal component of the force is

$$F_H = \rho g \bar{h} A$$

where

$$\begin{aligned}\bar{h} &= \frac{4.0}{2} = 2.0 \text{ m}\\ A &= 4.0\times 1.0 = 4.0 \text{ m}^2\\ g &= 9.81 \text{ m/s}^2\end{aligned}$$

Therefore,

$$F_H = 78.48 \text{ kN}$$

Depth of the wall in x direction is $\sqrt{4} = 2$ m. Vertical component of the force is equal to the weight of the water vertically above the wall:

$$\begin{aligned}F_V &= \rho g \times \left(\int_0^2 x^2 dx\right)\times 1.0\\ &= \rho g \times \frac{2^3}{3}\times 1.0\\ &= 26.16 \text{ kN}\end{aligned}$$

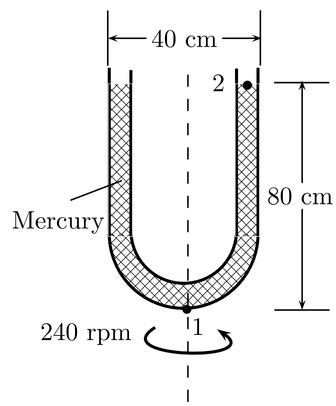
Hence, the net hydrostatic force is

$$\begin{aligned}F &= \sqrt{F_H^2+F_V^2}\\ &= 82.72 \text{ kN}\end{aligned}$$

Inclination of this force is

$$\begin{aligned}\theta &= \tan^{-1}\frac{F_V}{F_H}\\ &= 33.34°\end{aligned}$$

13. A U-tube of diameter 40 cm contains mercury upto a height of 80 cm. It is rotated about its center at 240 rpm in solid body motion.



Determine the pressure at the bottom point (1) of the tube. Take $p_2 = 100$ kPa.

Solution. Angular speed of rotation is

$$\begin{aligned}\omega &= \frac{2\pi N}{60}\\ &= 25.13 \text{ rad/s}\end{aligned}$$

The head developed at outer circumference (r = 0.2 m) due to rotation of the tube is

$$\begin{aligned}z &= \frac{\omega r^2}{2g}\\ &= 1.28 \text{ m}\end{aligned}$$

It means that the center of isobaric surface has moved below the bottom of the tube by

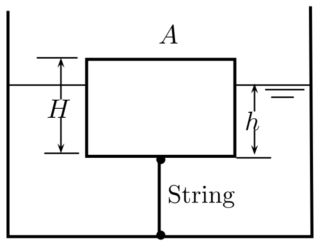
$$\begin{aligned}\Delta z &= 1.28 - 0.80\\ &= 0.48 \text{ m}\end{aligned}$$

Thus, pressure at bottom (point 1) is

$$\begin{aligned}p_1 &= p_2 - \Delta z \times \rho_m \times g\\ &= 100 - 0.48\times 13.6\times 9.81\\ &= 35.96 \text{ kPa}\end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. A cylindrical body of cross-sectional area A, height H and density ρ_s is immersed to a depth h in a liquid of density ρ, and tied to the bottom with a string.



The tension in the string is

(a) ρghA (b) $(\rho_s - \rho)\, ghA$
(c) $(\rho - \rho_s)\, ghA$ (d) $(\rho h - \rho_s H)\, gA$

(GATE 2003)

Solution. Weight of the body is $\rho_s gHA$, and buoyancy force is ρghA. Tension in the string at bottom is

$$\begin{aligned} T &= \rho ghA - \rho_s gHA \\ &= (\rho h - \rho_s H)\, gA \end{aligned}$$

Ans. (d)

2. A water container is kept on a weighing balance. Water from the tap is falling vertically into the container with a volume flow rate of Q; the velocity of the water when it hits the water surface is U. At a particular instant of time the total mass of the container and water is m. The force registered by the weighing balance at this instant of time is

(a) $mg + \rho QU$ (b) $mg + 2\rho QU$
(c) $mg + \rho QU^2/2$ (d) $\rho QU^{/}2$

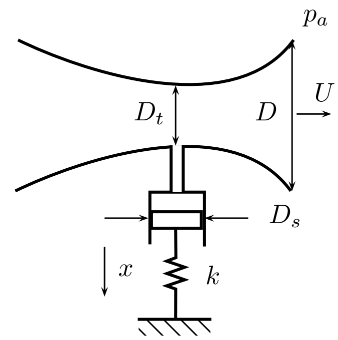
(GATE 2003)

Solution. Weight of the container and water is mg. The momentum exchange at striking is ρQU, therefore, net force is

$$F = mg + \rho QU$$

Ans. (a)

3. Air flows through a venturi and into atmosphere. Air density is ρ; atmospheric pressure is p_a; throat diameter is D_t; exit diameter is D and exit velocity is U. The throat is connected to a cylinder containing a frictionless piston attached to a spring. The spring constant is k. The bottom surface of the piston is exposed to atmosphere. Due to the flow, the piston moves by distance x.



Assuming incompressible frictionless flow, x is

(a) $\dfrac{\rho U^2}{2k} \times \pi D_s^2$

(b) $\dfrac{\rho U^2}{8k}\left(\dfrac{D^2}{D_t^2} - 1\right)\pi D_s^2$

(c) $\dfrac{\rho U^2}{2k}\left(\dfrac{D^2}{D_t^2} - 1\right)\pi D_s^2$

(d) $\dfrac{\rho U^2}{8k}\left(\dfrac{D^4}{D_t^4} - 1\right)\pi D_s^2$

(GATE 2003)

Solution. Using the continuity equation for obtaining the velocity at inlet:

$$v_1 = U\left(\frac{D}{D_t}\right)^2$$

Using Bernoulli's equation

$$\begin{aligned} p_1 - p_a &= \frac{\rho}{2}\left(U^2 - v_1^2\right) \\ &= \frac{\rho}{2}U^2\left\{1 - \left(\frac{D}{D_t}\right)^4\right\} \end{aligned}$$

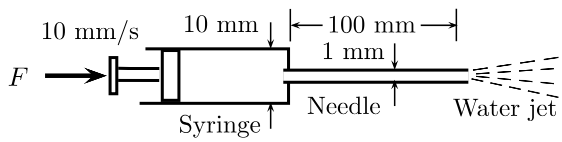
But

$$\begin{aligned} -kx &= \frac{\pi D_s^2}{4}\left(p_1 - p_a\right) \\ x &= \frac{\pi D_s^2}{4}\frac{\rho}{2k}U^2\left\{\left(\frac{D}{D_t}\right)^4 - 1\right\} \\ &= \frac{\rho U^2}{8k}\left\{\left(\frac{D}{D_t}\right)^4 - 1\right\}\pi D_s^2 \end{aligned}$$

Ans. (d)

Common Data Questions

A syringe with a frictionless plunger contains water and has at its end, a 100 mm long needle of 1 mm diameter. The internal diameter of the syringe is 10 mm. Water density is 1000 kg/m^3. The plunger is pushed in at 10 mm/s and the water comes out as a jet.



4. Assuming ideal flow, the force F, in Newtons, required on the plunger to push out the water is

(a) 0 (b) 0.04
(c) 0.13 (d) 1.15

(GATE 2003)

Solution. Using subscripts 1 for syringe and 2 for needle:

$$d_1 = 0.01 \text{ m}$$
$$d_2 = 0.001 \text{ m}$$
$$l_2 = 0.1 \text{ m}$$
$$\rho = 1000 \text{ kg/m}^3$$
$$v_1 = 0.01 \text{ m/s}$$

Therefore,

$$v_2 = v_1\left(\frac{d_1}{d_2}\right)^2 = 1 \text{ m/s}$$

The needle opens in atmosphere, therefore,

$$p_2 = 0$$

Using Bernoulli's equation

$$\frac{p_1 - p_2}{\rho g} = \frac{{v_2}^2 - {v_1}^2}{2g}$$
$$p_1 = 499.95 \text{ Pa}$$

Force required is

$$F = p_1 \times \frac{\pi}{4}{d_1}^2 = 0.039266 \approx 0.04 \text{ N}$$

Ans. (b)

5. Neglect losses in the cylinder and assume fully developed laminar viscous flow throughout the needle; the Darcy friction factor is 64/Re, where Re is the Reynolds number. Given that the viscosity of water is 1.0×10^{-3} kg/sm, the force F in newtons required on the plunger is

(a) 0.13 (b) 0.16
(c) 0.3 (d) 4.4

(GATE 2003)

Solution. Given that

$$f = \frac{64}{\text{Re}}$$
$$\mu = 1.0 \times 10^{-3} \text{ kg/sm}$$

Therefore, Reynolds number is

$$\text{Re} = \frac{\rho V_2 d_2}{\mu} = 1000$$

Friction coefficient is

$$f = \frac{64}{\text{Re}} = 0.064$$

Head loss in the pipe flow is calculated as

$$h_f = \frac{f d_2}{l_2} \times \frac{{V_2}^2}{2g} = 0.326198 \text{ m}$$

Using Bernoulli's equation,

$$\frac{p_1 - p_2}{\rho g} = \frac{{V_2}^2 - {V_1}^2}{2g} + h_f$$
$$p_1 = 3699.95 \text{ Pa}$$

Force required is

$$F = p_1 \times \frac{\pi}{4}{d_1}^2 = 0.290593 \approx 0.3 \text{ N}$$

Ans. (c)

6. An incompressible fluid (kinematic viscosity, 7.4×10^{-7} m^2/s , specific gravity, 0.88) is held between two parallel plates. If the top late is moved with a velocity of 0.5 m/s while the bottom one is held stationary, the fluid attains a linear velocity profile in the gap of 0.5 mm between these plates; the shear stress in Pascals on the surface of top plate is

(a) 0.651×10^{-3} (b) 0.651
(c) 6.51 (d) 0.651×10^3

(GATE 2004)

Solution. Given that

$$\nu = 7.4\times 10^{-7}\ \text{m}^2/\text{s}$$
$$s = 0.88$$
$$dv = 0.5-0 = 0.5\ \text{m/s}$$
$$dy = 0.0005\ \text{m}$$

Therefore, density is

$$\rho = s\times\rho_w = 880\ \text{kg/m}^3$$

Shear stress is given by

$$\tau = \nu\rho\frac{dv}{dy} = 0.6512\ \text{Pa}$$

Ans. (b)

7. A fluid flow is represented by the velocity field $\vec{v} = ax\hat{i}+ay\hat{j}$, where a is a constant. The equation of streamline passing through a point (1,2) is

(a) $x-2y=0$ (b) $2x+y=0$
(c) $2x-y=0$ (d) $x+2y=0$

(GATE 2004)

Solution. Given that

$$u = ax$$
$$v = ay$$

The equation of stream line is given by

$$\frac{dx}{u} = \frac{dy}{v}$$
$$\frac{dx}{ax} = \frac{dy}{ay}$$
$$\frac{dx}{x} = \frac{dy}{y}$$

Integrating both sides,

$$\ln x = \ln y + \ln c$$
$$x = cy$$

For given point $(x=1, y=2)$, $c=1/2$. Therefore, equation of stream line is

$$2x-y=0$$

Ans. (c)

8. For a fluid flow through a divergent pipe of length L having inlet and outlet radii R_1 and R_2, respectively, and a constant flow rate of Q, assuming the velocity to be axial and uniform at any cross-section, the acceleration at the exit is

(a) $\dfrac{2Q(R_1-R_2)}{\pi L {R_2}^3}$ (b) $\dfrac{2Q^2(R_1-R_2)}{\pi^2 L {R_2}^3}$

(c) $\dfrac{2Q^2(R_1-R_2)}{\pi^2 L {R_2}^5}$ (d) $\dfrac{2Q^2(R_2-R_1)}{\pi^2 L {R_2}^5}$

(GATE 2004)

Solution. Let the velocity vector be written as

$$\vec{v} = u\hat{i}+v\hat{j}+w\hat{k}$$

which is a function of (x,y,z,t). The acceleration at a point $P(x,y,z)$ is given by

$$\vec{a} = a_x\hat{i}+a_y\hat{j}+a_z\hat{k}$$

where

$$a_x = \frac{du}{dt}$$

For the given case, $v=w=0$. Axial velocities at inlet and outlet are

$$u_1 = \frac{Q}{\pi {R_1}^2}$$
$$u_2 = \frac{Q}{\pi {R_2}^2}$$

Average of longitudinal velocity gradient is

$$\frac{\partial u}{\partial x} = \frac{u_1-u_2}{L}$$

Acceleration in x-direction is

$$a_x = u\frac{\partial u}{\partial x} = \frac{Q^2\left({R_1}^2-{R_2}^2\right)}{\pi^2 L {R_1}^2 {R_2}^4}$$

Assuming $R_1 \approx R_2$,

$$a_x = \frac{2Q^2(R_1-R_2)}{\pi^2 L {R_2}^5}$$

Ans. (c)

9. A closed cylinder having a radius R and height H is filled with oil density ρ. If the cylinder is rotated about its axis at an angular velocity of ω, the thrust at the bottom of the cylinder is

(a) $\pi R^2\rho g H$
(b) $\pi R^2\rho\omega^2 R^2/4$
(c) $\pi R^2\left(\pi^2 R^2+\rho g H\right)$
(d) $\pi R^2\left(\rho\omega^2 R^2/4+\rho g H\right)$

(GATE 2004)

Solution. The head developed due to rotation with speed ω at radius r is

$$h_r = \frac{\omega^2 r^2}{2g}$$

The force exerted by the fluid at the top cover of the cylinder is

$$\begin{aligned} F_1 &= \int_0^R \rho \frac{\omega^2 r^2}{2g} 2\pi r dr \\ &= \frac{1}{4}\pi\rho\omega^2 R^4 \end{aligned}$$

The same will be vertically downward at the bottom. Also, the weight of the fluid is

$$W = \rho\pi R^2 H g$$

Therefore, the total thrust at the bottom is

$$\begin{aligned} F_1 + W &= \frac{1}{4}\pi\rho\omega^2 R^4 + \rho\pi R^2 H g \\ &= \pi R^2 \left(\frac{1}{4}\rho\omega^2 R^2 + \rho H g\right) \end{aligned}$$

Ans. (d)

10. For air flow over a flat plate, velocity (U) and boundary layer thickness (δ) can be expressed, respectively, as

$$\begin{aligned} \frac{U}{U_\infty} &= \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left\{\frac{y}{\delta}\right\}^3 \\ \delta &= \frac{4.64x}{\sqrt{\mathrm{Re} \times x}} \end{aligned}$$

If the free stream velocity is 2 m/s, and air has kinematic viscosity of 1.5×10^{-5} m^2/s and density of 1.23 kg/m^3, the wall shear stress at $x = 1$ m, is

(a) 2.36×10^2 N/m^2
(b) 43.6×10^{-3} N/m^2
(c) 4.36×10^{-3} N/m^2
(d) 2.18×10^{-3} N/m^2

(GATE 2004)

Solution. Given that

$$\begin{aligned} U_\infty &= 2 \text{ m/s} \\ \nu &= 1.5 \times 10^{-5} \text{ m}^2/\text{s} \\ \rho &= 1.23 \text{ kg/m}^3 \\ x &= 1 \text{ m} \end{aligned}$$

Therefore,

$$\begin{aligned} \mathrm{Re}_x &= \frac{U_\infty x}{\nu} \\ &= 133333 \end{aligned}$$

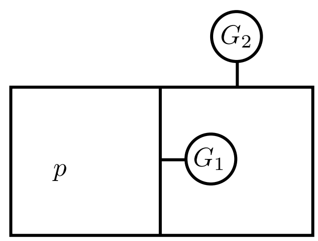
$$\begin{aligned} \delta &= \frac{4.64x}{\sqrt{\mathrm{Re} \times x}} \\ &= 0.0127072 \\ \frac{U}{U_\infty} &= \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left\{\frac{y}{\delta}\right\}^3 \\ \left(\frac{\partial U}{\partial y}\right)_{y=0} &= \frac{3}{2\delta}U_\infty \end{aligned}$$

Therefore, the wall shear stress at $x = 1$ is

$$\begin{aligned} \tau_0 &= \mu\left(\frac{\partial U}{\partial y}\right)_{y=0} \\ &= \rho\nu \times \frac{3}{2\delta}U_\infty \\ &= 4.36 \times 10^{-3} \text{ N/m}^2 \end{aligned}$$

Ans. (c)

11. The pressure gauges G_1 and G_2 installed on the system show pressures of $p_{G1} = 5.00$ bar and p_{G2}=1.00 bar.



The value of unknown pressure p is

(a) 1.01 bar (b) 2.01 bar
(c) 5.00 bar (d) 7.01 bar

(GATE 2004)

Solution. Given that

$$\begin{aligned} p_{G_1} &= 5.00 \text{ bar} \\ p_{G_2} &= 1.00 \text{ bar} \end{aligned}$$

Let $p_a = 1.01$ bar be the atmospheric pressure. From the figure,

$$\begin{aligned} p_{G1} &= p - p_2 \\ p_{G2} &= p_2 - p_a \\ p_2 &= p_{G2} + p_a \end{aligned}$$

Therefore,

$$\begin{aligned} p &= p_{G1} + p_2 \\ &= p_{G1} + p_{G2} + p_a \\ &= 5.00 + 1.00 + 1.01 \\ &= 7.01 \text{ bar} \end{aligned}$$

Ans. (d)

12. A venturimeter of 20 mm throat diameter is used to measure the velocity of water in a horizontal pipe of 40 mm diameter. If the pressure difference between the pipe and throat sections is found to be 30 kPa then, neglecting frictional losses, the flow velocity is

(a) 0.2 m/s (b) 1.0 m/s
(c) 1.4 m/s (d) 2.0 m/s

(GATE 2005)

Solution. Given that

$$\begin{aligned} d_1 &= 0.04 \text{ m} \\ d_2 &= 0.02 \text{ m} \\ p_1 - p_2 &= 30000 \text{ Pa} \end{aligned}$$

Let $\rho = 1000$ kg/m^2. For continuity,

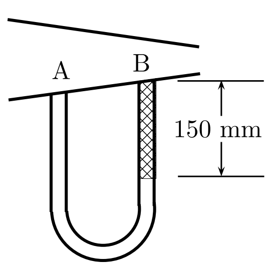
$$\begin{aligned} v_1 \times \frac{\pi}{4} {d_1}^2 &= v_2 \times \frac{\pi}{4} {d_2}^2 \\ v_2 &= \left(\frac{d_1}{d_2}\right)^2 v_1 \\ &= 4v_1 \end{aligned}$$

Using Bernoulli's equation,

$$\begin{aligned} \frac{p_1 - p_2}{\rho g} &= -\frac{v_1^2 - v_2^2}{2g} \\ v_1 &= \sqrt{\frac{p_1 - p_2}{\rho g} \times \frac{2g}{4^2 - 1}} \\ &= 2.0 \text{ m/s} \end{aligned}$$

Ans. (d)

13. A U-tube manometer with a small quantity of mercury is used to measure the static pressure difference between two locations A and B in a conical section through which an incompressible fluid flows. At a particular flow rate, the mercury column appears as shown in the figure.



The density of mercury is 13600 kg/m^3 and g = 9.81 m/s^2. Which of the following is correct?

(a) Flow direction is A to B and $p_A - p_B = 20$ kPa
(b) Flow direction is B to A and $p_A - p_B = 1.4$ kPa
(c) Flow direction is A to B and $p_B - p_A = 20$ kPa
(d) Flow direction is B to A and $p_B - p_A = 1.4$ kPa

(GATE 2005)

Solution. Given that

$$\begin{aligned} h &= 0.150 \text{ m} \\ \rho_m &= 13600 \text{ kg/m}^3 \end{aligned}$$

The pressure difference between A and B is

$$\begin{aligned} p_A - p_B &= (\rho_m - \rho_w) g \Delta h \\ &= 21.4839 \text{ kPa} \end{aligned}$$

As $p_A > p_B$, the flow is from A to B.

Ans. (c)

14. A leaf is caught in a whirlpool. At a given instant, the leaf is at a distance of 120 m from the center of the whirlpool. The whirlpool can be described by the following velocity distribution:

$$\begin{aligned} v_r &= -\frac{60 \times 10^3}{2\pi r} \text{ m/s} \\ v_t &= \frac{300 \times 10^3}{2\pi r} \text{ m/s} \end{aligned}$$

where r (in meters) is the distance from the center of the whirlpool. What will be the distance of the leaf from the center when it has moved through half a revolution?

(a) 48 m (b) 64 m
(c) 120 m (d) 142 m

(GATE 2005)

Solution. To complete half revolution, the leaf has to travel around $\pi \times 120$ m distance with tangential velocity, which is minimum at $r = 120$ m, equal to

$$v_t = 397.887 \text{ m/s}$$

This velocity will continuously increase with time. Therefore, minimum time (seconds) taken for half revolution is

$$\begin{aligned} t_{1/2} &= \frac{\pi \times 120}{397.887} \\ &= 0.947482 \end{aligned}$$

In the same time, the leaf will travel a distance s towards the center with radial velocity which will be minimum at $r = 120$ m given by

$$v_r = 79.5775 \text{ m/s}$$

and s will be given by

$$s = v_r \times t_{1/2} = 75.3982 \text{ m}$$

Therefore, the approximate distance of the leaf from the center is

$$r = 120 - 75.3982 = 44.6018 \text{ m}$$

Thus, the answer is closest to option (a).

Ans. (a)

15. For a Newtonian fluid,

(a) shear stress is proportional to shear strain.
(b) rate of shear stress is proportional to shear strain.
(c) shear stress is proportional to rate of shear strain.
(d) rate of shear stress is proportional to rate of shear strain.

(GATE 2006)

Solution. Newtonian fluids follow Newton's law of viscosity, according to which, rate of shear stress is proportional to the rate of shear strain.

Ans. (d)

16. In a two-dimensional velocity field with velocities u and v along the x and y directions, respectively, the convective acceleration along the x-direction is given by

(a) $u\,(\partial u/\partial x) + v\,(\partial u/\partial y)$
(b) $u\,(\partial u/\partial x) + v\,(\partial v/\partial y)$
(c) $u\,(\partial v/\partial x) + v\,(\partial u/\partial y)$
(d) $v\,(\partial u/\partial x) + u\,(\partial u/\partial y)$

(GATE 2006)

Solution. Vector form of the acceleration is

$$\vec{a} = \underbrace{\frac{\partial \vec{v}}{\partial t}}_{\text{Local}} + \underbrace{\vec{v}\cdot\nabla\vec{v}}_{\text{Convective}}$$

where (for two-dimensional case)

$$V = u\hat{i} + v\hat{j}$$

Therefore, convective acceleration along x direction ($\hat{i}$) is

$$a = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

Ans. (a)

17. A two-dimensional flow field has velocities along the x and y directions given by $u = x^2 t$ and $v = -2xyt$, respectively, where t is the time. The equation of stream lines is:

(a) $x^2 y$ =constant
(b) xy^2 =constant
(c) xy =constant
(d) not possible to determine

(GATE 2006)

Solution. Given that

$$u = x^2 t$$
$$v = -2xyt$$

The stream line is represented as

$$\frac{dx}{u} = \frac{dy}{v}$$

Therefore,

$$\frac{dy}{dx} = \frac{v}{u}$$

Hence, for the given case,

$$\frac{dy}{dx} = \frac{-2xyt}{x^2 t} = \frac{-2y}{x}$$
$$\frac{dy}{y} = \frac{-2dx}{x}$$

Integrating both sides,

$$\ln y = -2\ln x$$
$$x^2 y = \text{constant}$$

Ans. (a)

18. The velocity profile in a fully developed laminar flow in a pipe of diameter D is given by

$$u = u_0\left(1 - \frac{4r^2}{D^2}\right)$$

where r is the radial distance from the center. If the viscosity of the fluid is μ, the pressure drop across the length L of the pipe is

(a) $\mu u_0 L/D^2$ (b) $4\mu u_0 L/D^2$
(c) $8\mu u_0 L/D^2$ (d) $16\mu u_0 L/D^2$

(GATE 2006)

Solution. Given that

$$u = u_0\left(1 - \frac{4r^2}{D^2}\right)$$

Using Newton's law of viscosity and equilibrium, the shear stress at radius r is

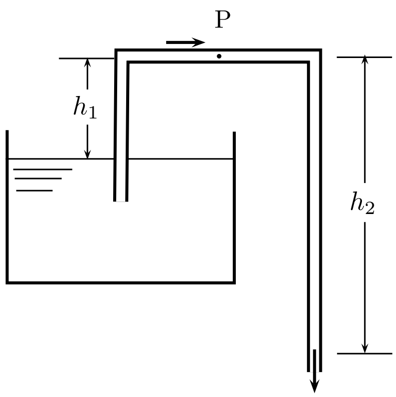
$$\tau_r = -\rho g \frac{\partial h}{\partial x} \times \frac{r}{2} = -\mu \frac{\partial u}{\partial r}$$

Therefore,

$$\frac{\rho g h}{L} = -\frac{16\mu u_0}{D^2}$$
$$\Delta p = \rho g h = -\frac{16\mu u_0 L}{D^2}$$

Ans. (d)

19. A siphon draws water from a reservoir and discharges it out at atmospheric pressure.



Assuming ideal fluid and the reservoir is large, the velocity at point P in the siphon tube is:

(a) $\sqrt{2gh_1}$ (b) $\sqrt{2gh_2}$
(c) $\sqrt{2g(h_2-h_1)}$ (d) $\sqrt{2g(h_2+h_1)}$

(GATE 2006)

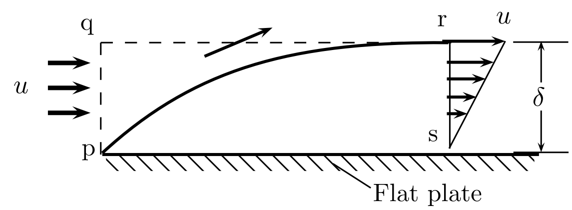
Solution. The velocity at point P will be equal to that at Q, where head is h_2-h_1. Therefore,

$$v = \sqrt{2g\Delta h} = \sqrt{2g(h_2-h_1)}$$

Ans. (c)

Linked Answer Questions

A smooth flat plate with a sharp leading edge is placed along a gas stream flowing at $u_0 = 10$ m/s. The thickness of the boundary layer at section $r-s$ is 10 mm, the breadth of the plate is 1 m (into the paper) and the density of the gas $\rho = 1.0$ kg/m^3. Assume that the boundary layer is thin, two-dimensional, and follows a linear velocity distribution, $u = u_0 y/\delta$, at the section $r-s$, where y is the height from plate.



20. The mass flow rate (in kg/s) across the section q–r is:

(a) zero (b) 0.05
(c) 0.10 (d) 0.15

(GATE 2006)

Solution. Given that

$$\rho = 1 \text{ kg/m}^3$$
$$\delta = 0.01 \text{ m}$$
$$b = 1 \text{ m}$$
$$u_0 = 10 \text{ m/s}$$

Mass flow rate across the section is

$$\dot{m} = \rho b \int_0^{\delta} u_0 \frac{y}{\delta} dy = \frac{1}{2}\rho b u_0 \delta = 0.05 \text{ kg/s}$$

Ans. (b)

21. The integrated drag force (in N) on the plate, between p–s, is:

(a) 0.67 (b) 0.33
(c) 0.17 (d) zero

(GATE 2006)

Solution. Assuming unit drag coefficient, the drag force will be given by

$$F_D = \rho A \frac{u^2}{2} = \frac{1}{2}\rho \left\{ \int_0^{\delta} \left(\frac{u_0 y}{\delta}\right)^2 dy \right\} = \frac{1}{6}\rho b u_0^2 \delta = 0.1667 \text{ N}$$

Ans. (c)

22. Consider an incompressible laminar boundary layer flow over a flat plate of length L, aligned with the direction of an incoming uniform free stream. If F is the ratio of the drag force on the front half of the plate to the drag force on the near half, then

(a) $F < 1/2$ (b) $F = 1/2$
(c) $F = 1$ (d) $F > 1$

(GATE 2007)

Solution. The viscous force will be more on the later half portion because boundary layer thickness is less in this portion, in turn, velocity gradient is more, resulting in more shear stress and finally more drag.

Ans. (d)

23. In a steady flow through a nozzle, the flow velocity on the nozzle axis is given by $v = u_0(1+3x/L)\,i$, where x is the distance along the axis of the nozzle from its inlet plane and L is the length of the nozzle. The time required for a fluid particle on the axis to travel from the inlet to the exit plane of the nozzle is

(a) $\dfrac{L}{u_0}$ (b) $\dfrac{L\ln 4}{3u_0}$

(c) $\dfrac{L}{4u_0}$ (d) $\dfrac{L}{2.5u_0}$

(GATE 2007)

Solution. Given that flow velocity on the nozzle axis,

$$\frac{dx}{dt} = v = u_0\left(1+3\frac{x}{L}\right)$$

Therefore,

$$dt = \frac{dx}{u_0(1+3x/L)}$$

Integrating on both sides of the equation,

$$t = \frac{L}{3u_0}\left[\ln\left(1+3\frac{x}{L}\right)\right]_0^L = \frac{L}{3u_0}\ln 4$$

Ans. (b)

24. Consider steady laminar incompressible axi-symmetric fully developed viscous flow through a straight circular pipe of constant cross sectional area at a Reynolds number of 5. The ratio of inertia force to viscous force on a fluid particle is

(a) 5 (b) 1/5
(c) 0 (d) ∞

(GATE 2007)

Solution. Reynolds number is defined as

$$\text{Re} = \frac{\text{Inertial force}}{\text{Viscous force}} = 5$$

Ans. (a)

25. Which of the following statements about steady incompressible forced vortex flow is correct?

P: Shear stress is zero at all points in the flow.
Q: Vorticity is zero at all points in the flow.
R: Velocity is directly proportional to the radius from the center of the vortex.
S: Total mechanical energy per unit mass is constant in the entire flow field.

(a) P and Q (b) R and S
(c) P and R (d) P and S

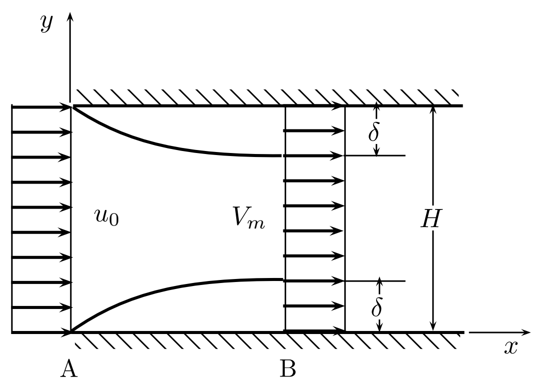
(GATE 2007)

Solution. In steady, incompressible, vortex flow, shear stress is zero at all points because there is no velocity gradient. Vorticity is also zero.

Ans. (a)

Linked Answer Questions

Consider a steady incompressible flow through a channel as shown below.



The velocity profile is uniform with a value of u_0 at the inlet section A. The velocity profile at section B down stream is

$$u = \begin{cases} v_m \frac{y}{\delta}, & 0 \le y \le \delta \\ v_m, & \delta \le y \le H-\delta \\ v_m \frac{H-y}{\delta}, & H-\delta \le y \le H \end{cases}$$

26. The ratio v_m/u_0 is

(a) $(1-2\delta/H)^{-1}$ (b) $(1+2\delta/H)^{-1}$
(c) $(1-\delta/H)^{-1}$ (d) $(1+\delta/H)^{-1}$

(GATE 2007)

Solution. Using the equation of mass conservation assuming unit depth of the channel, one finds

$$u_0 \times H = v_m \int_0^{\delta} \frac{y}{\delta} dy + V_m (H - 2\delta) + v_m \int_{H-\delta}^{H} \frac{H-y}{\delta} dy$$

$$\frac{v_m}{u_0} = \frac{1}{1-\delta/H}$$

Ans. (c)

27. The ratio $\frac{p_A - p_B}{\rho u_0^2/2}$ (where p_A and p_B are the pressures at section A and B, respectively, and ρ is the density of the fluid) is

(a) $(1-\delta/H)^{-2}-1$ (b) $(1-\delta/H)^{-2}$
(c) $(1-2\delta/H)^{-2}-1$ (d) $(1-\delta/H)^{-1}$

(GATE 2007)

Solution. Bernoulli's equation can be applied between sections A and B as

$$\frac{p_A}{\rho g} + \frac{{u_0}^2}{2g} = \frac{p_B}{\rho g} + \frac{1}{2gH}\bar{v}^2$$

The average velocity is calculated as

$$\bar{v}^2 = 2\int_0^{\delta} \left(\frac{v_m y}{\delta}\right)^2 dy + v_m^2 (H - 2\delta)$$

$$\frac{p_A - p_B}{1/2{u_0}^2} = \frac{H(3H-4\delta)}{3(H-\delta)^2} - 1$$

The most nearest answer is (a).

Ans. (a)

28. For the continuity equation given by $\vec{\nabla} \cdot \vec{v} = 0$ to be valid, where $\vec{v}$ is the velocity vector, which one of the following is a necessary condition?

(a) steady flow
(b) irrotational flow
(c) inviscid flow
(d) incompressible flow

(GATE 2008)

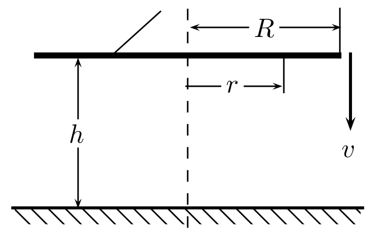
Solution. Continuity equation for incompressible flow is

$$\vec{\nabla} \cdot \vec{v} = 0$$

Ans. (d)

Linked Answer Questions

The gap between a moving circular plate and a stationary surface is being continuously reduced, as the circular plate comes down at a uniform speed v towards the stationary bottom surface, as shown in the figure. In the process, the fluid contained between the two plates flows out radially. The fluid is assumed to be incompressible and inviscid.



29. The radial velocity v_r at any radius r when the gap width is h, is

(a) $v_r = \frac{vr}{2h}$ (b) $v_r = \frac{vr}{h}$

(c) $v_r = \frac{2vr}{h}$ (d) $v_r = \frac{vh}{r}$

(GATE 2008)

Solution. Consider the movement of plate under the radius r. For constant volume,

$$v_r \times 2\pi r \times h = v \times \pi r^2$$

$$v_r = \frac{v \times r}{2h}$$

Ans. (a)

30. The radial component of the fluid acceleration at $r = R$ is

(a) $\frac{3v^2R}{4h^2}$ (b) $\frac{v^2R}{4h^2}$

(c) $\frac{v^2R}{2h^2}$ (d) $\frac{v^2R}{4h^2}$

(GATE 2008)

Solution. If H is initial level (a constant) of the top plate, and it takes time t to reach down at height h, then

$$h = H - vt$$

Radial velocity at any radius r is

$$\begin{aligned} v_r &= \frac{v \times r}{2h} \\ &= \frac{v \times r}{2(H - vt)} \end{aligned}$$

Therefore, radial acceleration is

$$\begin{aligned} a_r &= \frac{dv_r}{dt} \\ &= \frac{v^2 \times r}{2(H - vt)^2} \\ &= \frac{v^2 \times r}{2h^2} \end{aligned}$$

At $r = R$,

$$a_r = \frac{v^2 \times R}{2h^2}$$

Ans. (c)

31. Consider steady, incompressible and irrotational flow through a reducer in a horizontal pipe where the diameter is reduced from 20 cm to 10 cm. The pressure in the 20 cm pipe just upstream of the reducer is 150 kPa. The fluid has a vapor pressure of 50 kPa and a specific weight of 5 kN/m^3. Neglecting frictional effects, the maximum discharge (in m^3/s) that can pass through the reducer without causing cavitation is

(a) 0.05 (b) 0.16
(c) 0.27 (d) 0.38

(GATE 2009)

Solution. Given that

$$\begin{aligned} D_1 &= 0.2 \text{ m} \\ D_2 &= 0.1 \text{ m} \\ p_1 &= 150 \text{ kPa} \\ p_2 &= 50 \text{ kPa} \\ \rho g &= 5 \text{ kN/m}^3 \\ g &= 9.81 \text{ m/s}^2 \end{aligned}$$

Using continuity equation for incompressible fluid:

$$\begin{aligned} \frac{\pi {D_1}^2}{4} v_1 &= \frac{\pi {D_2}^2}{4} v_2 \\ v_1 &= \left(\frac{D_2}{D_1}\right)^2 v_2 \\ &= \frac{v_2}{4} \end{aligned}$$

Using Bernoulli's equation,

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g}$$

Solving the above equation and using the given values, one gets

$$v_2 = 20.458 \text{ m/s}$$

Therefore, discharge is

$$\begin{aligned} Q &= \frac{\pi {D_2}^2}{4} v_2 \\ &= 0.160677 \text{ m}^3\text{/s} \end{aligned}$$

Ans. (b)

32. You are asked to evaluate assorted fluid flows for their suitability in a given laboratory application. The following three flow choices, expressed in terms of the two-dimensional velocity fields in the xy-plane, are made available.

P. $u = 2y, \quad v = -3x$
Q. $u = 3xy, \quad v = 0$
R. $u = -2x, \quad v = 2y$

Which flow(s) should be recommended when the application requires the flow to be incompressible and irrotational?

(a) P and R (b) Q
(c) Q and R (d) R

(GATE 2009)

Solution. The conditions are as follows

(a) For incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(b) For irrotational flow

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

Checking for the given fields,

P. $0 + 0 = 0, \quad 2 \neq -3$
Q. $3y + 0 \neq 0$
R. $2 - 2 = 0, \quad 0 = 0$

Therefore, only R is incompressible and irrotational.

Ans. (d)

33. Water at 25°C is flowing through a 1.0 km long galvanized iron pipe of 200 mm diameter at the rate of 0.07 m^3/s. If the value of Darcy friction factor for this pipe is 0.02 and density of water is 1000 kg/m^3, the pumping power (in kW) required to maintain the flow is

(a) 1.8 (b) 17.4
(c) 20.5 (d) 41.0

(GATE 2009)

Solution. Given that

$$L = 1000 \text{ m}$$
$$D = 0.2 \text{ m}$$
$$Q = 0.07 \text{ m}^3/\text{s}$$
$$4f = 0.02$$
$$\rho = 1000 \text{ kg/m}^3$$

Therefore,

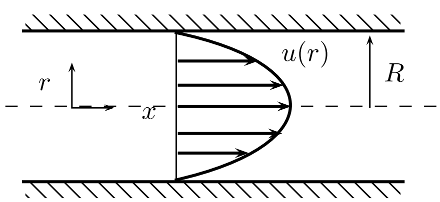
$$v = \frac{4Q}{\pi D^2} = 2.22817 \text{ m/s}$$
$$h_f = \frac{4fL}{D}\frac{v^2}{2g} = 25.3045 \text{ m}$$
$$P = \rho g Q h_f = 17.3766 \text{ kW}$$

Ans. (b)

34. The velocity profile of a fully developed laminar flow in a straight circular pipe, as shown in the figure, is given by the expression

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left(1-\frac{r^2}{R^2}\right)$$

where dp/dx is a constant.



The average velocity of fluid in the pipe is

(a) $-\frac{R^2}{8\mu}\times\frac{dp}{dx}$ (b) $-\frac{R^2}{4\mu}\times\frac{dp}{dx}$

(c) $-\frac{R^2}{2\mu}\times\frac{dp}{dx}$ (d) $-\frac{R^2}{\mu}\times\frac{dp}{dx}$

(GATE 2009)

Solution. With the given function, the average velocity is given by

$$\bar{u} = \frac{1}{\pi R^2}\int_0^R u(r)\times 2\pi r\times dr = -\frac{R^2}{8\mu}\left(\frac{dp}{dx}\right)$$

This is already explained in theory.

Ans. (a)

35. For the stability of a floating body, under the influence of gravity alone, which of the following is TRUE?

(a) Metacentre should be below center of gravity
(b) Metacentre should be above center of gravity
(c) Metacentre and center of gravity must lie on the same horizontal line
(d) Metacentre and center of gravity must lie on the same vertical line

(GATE 2010)

Solution. Floating body will be stable (i.e. it will return to its original position after some oscillations) when center of buoyancy (B) is above center of gravity (G). Thus, metacentric height (GM) should be positive.

Ans. (b)

36. The maximum velocity of a one-dimensional incompressible fully developed viscous flow, between two fixed parallel plates, is 6 m/s. The mean velocity (in m/s) of the flow is

(a) 2 (b) 3
(c) 4 (d) 5

(GATE 2010)

Solution. Given that $v_{max} = 6$ m/s. Maximum and average velocities in laminar flow between the plates are related as

$$\bar{v} = \frac{2}{3}v_{max} = 4 \text{ m/s}$$

Ans. (c)

37. Velocity vector of a flow field is given as

$$\vec{v} = 2xy\hat{i} - x^2z\hat{j}$$

The vorticity vector at (1, 1, 1) is

(a) $4\hat{i}-\hat{j}$ (b) $4\hat{i}-\hat{k}$
(c) $\hat{i}-4\hat{j}$ (d) $\hat{i}-4\hat{k}$

(GATE 2010)

Solution. Given that Vorticity is defined as

$$\vec{\Omega} = \nabla\times\vec{v}$$

where ∇ is divergence is defined as

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Given that

$$\vec{v} = 2xy\hat{i} - x^2 z\hat{j}$$

Therefore,

$$\vec{\Omega} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy & -x^2 z & 0 \end{bmatrix}$$
$$= x^2\hat{i} + 0\times\hat{j} + (-2xz - 2x)\hat{k}$$
$$= x^2\hat{i} + (-2xz - 2x)\hat{k}$$

The vorticity vector at (1, 1, 1) is

$$\vec{\Omega} = 1^2\hat{i} + (-2\times 1\times 1 - 2\times 1)\hat{k}$$
$$= \hat{i} - 4\hat{k}$$

Ans. (d)

38. A lightly loaded full journal bearing has a journal of 50 mm, bush bore of 50.05 mm and bush length of 20 mm. If rotational speed of journal is 1200 rpm and average viscosity of liquid lubricant is 0.03 Pa s, the power loss (in W) will be

(a) 3.7 (b) 74
(c) 118 (d) 237

(GATE 2010)

Solution. Given that

$$d_j = 0.050 \text{ m}$$
$$d_b = 0.0505 \text{ m}$$
$$l = 0.020 \text{ m}$$
$$N = 1200 \text{ rpm}$$
$$\mu = 0.03 \text{ Pa.s}$$

Therefore,

$$h = \frac{d_b - d_j}{2}$$
$$= 0.00025 \text{ m}$$
$$\omega = \frac{2\pi N}{60}$$
$$= 125.664 \text{ rad/s}$$
$$r = \frac{d_j}{2}$$
$$= 0.025 \text{ m}$$

Therefore, power loss will be

$$P = \text{Friction torque} \times \omega$$
$$= \frac{2\pi\omega\mu l r^3}{h} \times \omega$$
$$= 3.72 \text{ W}$$

Ans. (a)

39. A smooth pipe of diameter 200 mm carries water. The pressure in the pipe at section S1 (elevation: 10 m) is 50 kPa. At Section S2 (elevation: 12 m) the pressure is 20 kPa and velocity is 2 m/s. Density of water is 1000 kg/m^3 and acceleration due to gravity is 9.8 m/s^2. Which of the following is TRUE?

(a) flow from S1 to S2 and head loss is 0.53 m
(b) flow from S2 to S1 and head loss is 0.53 m
(c) flow from S1 to S2 and head loss is 1.06 m
(d) flow from S2 to S1 and head loss is 1.06 m

(GATE 2010)

Solution. Given that

$$p_1 = 50\times 10^3 \text{ Pa}$$
$$z_1 = 10 \text{ m}$$
$$p_2 = 20\times 10^3 \text{ Pa}$$
$$z_2 = 12 \text{ m}$$
$$\rho = 1000 \text{ kg/m}^3$$
$$g = 9.8 \text{ m/s}^2$$

As the pipe of of constant diameter, velocity will be remain the same at all sections. Therefore, using Bernoulli's equation, assuming head loss h_f in flow from S1 to S2, one gets

$$\frac{p_1}{\rho g} + z_1 = \frac{p_2}{\rho g} + z_2 + h_f$$
$$h_f = 1.06 \text{ m}$$

Ans. (c)

40. Match the following.

Column-I	Column-II
P. Compressible flow	U. Reynolds number
Q. Free surface flow	V. Nusselt number
R. Boundary layer flow	W. Weber number
S. Pipe flow	X. Froude number
T. Heat convection	Y. Mach number
	Z. Skin friction coefficient

(a) P-U, Q-X, R-V, S-Z, T-W

(b) P-W, Q-X, R-Z, S-U, T-V
(c) P-Y, Q-W, R-Z, S-U, T-X
(d) P-Y, Q-W, R-Z, S-U, T-V

(GATE 2010)

Solution. Reynolds number (Re) is defined as the ratio of inertia force and viscous force, and it is used in the analysis of pipe flow. Mach number is ratio of sonic velocity and velocity of gas, which is used in the analysis of compressible flows. The Weber number (We) is the ratio between the inertial force and the surface tension force. Nusselt number (Nu) is defined as ratio of heat convected through the fluid and heat conducted through the fluid. The Froude number (Fr) is a dimensionless value that describes different flow regimes of open channel flow.

Ans. (d)

41. A stream line and an equipotential line in a flow field

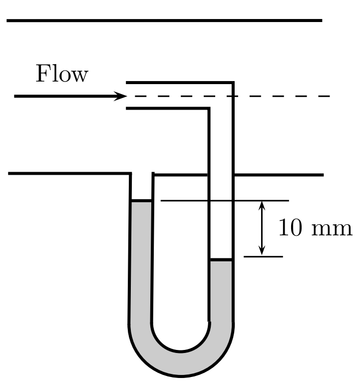
(a) are parallel to each other
(b) are perpendicular to each other
(c) intersect at an acute angle
(d) are identical

(GATE 2011)

Solution. Equipotential lines and stream lines are orthogonal to each other.

Ans. (b)

42. The following figure shows the schematic for the measurement of velocity of air (density = 1.2 kg/m^3) through a constant - area duct using a pitot tube and a water-tube manometer. The differential head of water (density = 1000 kg/m^3) in the two columns of the manometer is 10 mm. Take acceleration due to gravity as 9.8 m/s^2.



The velocity of air in m/s is

(a) 6.4 (b) 9.0
(c) 12.8 (d) 25.6

(GATE 2011)

Solution. Given that

$$\rho_a = 1.2 \text{ kg/m}^3$$
$$\rho_w = 1000 \text{ kg/m}^3$$
$$h = 0.01 \text{ m}$$
$$g = 9.8 \text{ m/s}^2$$

Using Bernoulli's equation for air,

$$\frac{{v_1}^2 - {v_2}^2}{2g} = \frac{p_2 - p_1}{\rho_a g}$$

where

$$v_2 = 0$$
$$p_2 - p_1 = \rho_w g h$$

Therefore

$$\begin{aligned} v_1 &= \sqrt{2 \times \frac{\rho_w g h}{\rho_a}} \\ &= \sqrt{2 \times \frac{1000 \times 9.81 \times 0.01}{1.2}} \\ &= 12.7802 \text{ m/s} \end{aligned}$$

Ans. (c)

43. Oil flows through a 200 mm diameter horizontal cast iron pipe (friction factor $4f = 0.0225$) of length 500 m. The volumetric flow rate is 0.2 m^3/s. The head loss (in m) due to friction is (assume $g = 9.81$ m/s^2)

(a) 116.18 (b) 0.116
(c) 18.22 (d) 232.36

(GATE 2012)

Solution. Given that

$$d = 0.2 \text{ m}$$
$$4f = 0.0225$$
$$l = 500 \text{ m}$$
$$Q = 0.2 \text{ m}^3/\text{s}$$
$$g = 9.81 \text{ m/s}^2$$

Average velocity of flow is

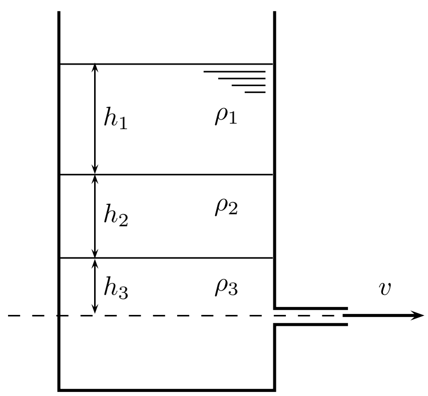
$$\begin{aligned} v &= \frac{4Q}{\pi d^2} \\ &= 6.37 \text{ m/s} \end{aligned}$$

Therefore, head loss in pipe is

$$\begin{aligned} h_f &= 4f\frac{l}{d}\times\frac{v^2}{2g} \\ &= 0.0225\times\frac{500}{0.2}\times\frac{6.37^2}{2\times 9.81} \\ &= 116.194 \text{ m} \end{aligned}$$

Ans. (a)

44. A large tank with a nozzle attached contains three immiscible inviscid fluids as shown.



Assuming that the changes in h_1, h_2 and h_3 are negligible, the instantaneous discharge velocity is

(a) $\sqrt{2gh_3\left(1+\dfrac{\rho_1 h_1}{\rho_3 h_3}+\dfrac{\rho_2 h_2}{\rho_3 h_3}\right)}$

(b) $\sqrt{2g\left(h_1+h_2+h_3\right)}$

(c) $\sqrt{2g\left(\dfrac{\rho_1 h_1+\rho_2 h_2+\rho_3 h_3}{\rho_1+\rho_2+\rho_3}\right)}$

(d) $\sqrt{2g\left(\dfrac{\rho_1 h_2 h_3+\rho_2 h_3 h_1+\rho_3 h_1 h_2}{\rho_1 h_1+\rho_2 h_2+\rho_3 h_3}\right)}$

(GATE 2012)

Solution. Gauge pressure at the nozzle level is

$$p = \left(\rho_1 h_1+\rho_2 h_2+\rho_3 h_3\right) g$$

Using Bernoulli's equation,

$$\frac{v^2}{2g} = \frac{p}{\rho_3 g}$$

one obtains

$$\begin{aligned} v &= \sqrt{\frac{2p}{\rho_3}} \\ &= \sqrt{\frac{2\left(\rho_1 h_1+\rho_2 h_2+\rho_3 h_3\right) g}{\rho_3}} \\ &= \sqrt{2gh_3\left(1+\frac{\rho_1 h_1}{\rho_3 h_3}+\frac{\rho_2 h_2}{\rho_3 h_3}\right)} \end{aligned}$$

Ans. (a)

45. In incompressible fluid flows over a flat plate with zero pressure gradient. The boundary layer thickness is 1 mm at a location where the Reynolds number is 1000. If the velocity of the fluid alone is increased by a factor of 4, then the boundary layer thickness at the same location, in mm, will be

(a) 4 (b) 2
(c) 0.5 (d) 0.25

(GATE 2012)

Solution. Boundary layer thickness is given by

$$\frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}$$

where as Reynolds number (Re_x) is related to velocity as

$$\mathrm{Re}_x = \frac{\rho v d}{\mu}$$

Therefore, keeping constant x, ρ, d, and μ, one finds

$$\delta \propto \frac{1}{\sqrt{v}}$$

Therefore, for the given case ($\delta_1 = 1$ mm)

$$\begin{aligned} \delta_2 &= \frac{1}{\sqrt{4}}\delta_1 \\ &= 0.5 \text{ mm} \end{aligned}$$

Ans. (c)

46. For steady, fully developed flow inside a straight pipe of diameter D, neglecting gravity effects, the pressure drop D_p over a length L and the wall shear stress τ_w are related by

(a) $\tau_w = \Delta pD/(4L)$ (b) $\tau_w = \Delta pD^2/\left(4L^2\right)$
(c) $\tau_w = \Delta pD/(2L)$ (d) $\tau_w = 4\Delta pL/D$

(GATE 2013)

Solution. Neglecting the gravity effects, the equilibrium equation for shear forces and pressure force can be written as

$$\pi DL\tau_w = \Delta p \times \frac{\pi D^2}{4}$$
$$\tau_w = \frac{\Delta pD}{4L}$$

Ans. (a)

47. Water is coming out from a tap and falls vertically downwards. At the tap opening, the stream diameter is 20 mm with uniform velocity of 2 m/s. Acceleration due to gravity is 9.81 m/s^2. Assuming steady, inviscid flow, constant atmospheric pressure everywhere and neglecting curvature and surface tension effects, the diameter is mm of the stream 0.5 m below the tap is approximately

(a) 10 (b) 15
(c) 20 (d) 25

(GATE 2013)

Solution. Using Bernoulli's equation

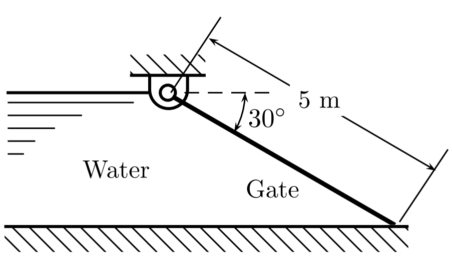
$$\frac{v_2^2}{2g} = \frac{v_1^2}{2g} + h_1 - h_2$$
$$v_2^2 = \sqrt{v_1^2 + 2g(h_1 - h_2)}$$
$$= \sqrt{2^2 + 2\times 9.81 \times 0.5}$$
$$= 3.716 \text{ m/s}$$

For continuity

$$v_2 \times \pi\frac{d_2^2}{4} = v_1 \times \pi\frac{d_1^2}{4}$$
$$d_2 = \sqrt{\frac{v_1}{v_2}}d_1$$
$$= 14.67 \text{ mm}$$
$$\approx 15 \text{ mm}$$

Ans. (b)

48. A hinged gate of length 5 m, inclined at 30° with the horizontal and with water mass on its left, is shown in figure below.



Density of water is 1000 kg/m^3. The minimum mass of the gate in kg per unit width (perpendicular to the plane of paper), required to keep it closed is

(a) 5000 (b) 6600
(c) 7546 (d) 9623

(GATE 2013)

Solution. Given that

$$b = 1 \text{ m}$$
$$d = 5 \text{ m}$$
$$\theta = 30°$$
$$\rho = 1000 \text{ kg/m}^3$$

The moment of inertia of the gate about its NA parallel to hinge is

$$I_{xx} = \frac{bd^3}{12}$$
$$= \frac{1\times 5^3}{12}$$
$$= 10.4167 \text{ m}^4$$

The center of gravity of the gate is

$$\bar{x} = \frac{5}{2}\sin 30°$$
$$= 1.25 \text{ m}$$

The center of pressure is

$$\bar{x}_p = \bar{x} + \frac{I\sin^2\theta}{A\bar{x}}$$
$$= 1.25 + \frac{10.4167\times\sin^2 30°}{5\times 1\times 1.25}$$
$$= 1.667$$

The hydrostatic force will act normal to the plane of the gate. Taking moments of the hydrostatic force and weight of the gate gives

$$mg\times\frac{d}{2}\cos\theta = \rho g\bar{x}A\times\frac{\bar{x}_p}{\sin\theta}$$
$$m = \frac{2\rho g\bar{x}A\times\bar{x}_p}{d\sin\theta\cos\theta}$$
$$= 9624.428 \text{ kg}$$

Ans. (d)

MULTIPLE CHOICE QUESTIONS

1. Newton's law of viscosity relates

(a) velocity gradient and rate of shear strain
(b) rate of shear deformation and shear stress
(c) shear deformation and shear stress
(d) pressure and volumetric strain

2. Which one of the following sets of conditions clearly apply to an ideal fluid?

(a) viscous and compressible
(b) non-viscous and incompressible
(c) non-viscous and compressible
(d) viscous and incompressible

3. Fluids that require a gradually increasing shear stress to maintain a constant strain rate are known as

(a) rhedopectic fluids
(b) thixotropic fluids
(c) pseudoplastic fluids
(d) Newtonian fluids

4. A static fluid can have

(a) non-zero normal and shear stress
(b) negative normal stress and zero shear stress
(c) positive normal stress and zero shear stress
(d) zero normal stress and non-zero shear stress

5. A 90 N rectangular solid block slides down a 30° inclined plane. The plane is lubricated by a 3 mm thick film of oil of relative density 0.90 and viscosity 8.0 poise. If the contact area is 0.3 m^2, the terminal velocity (m/s) of the block will be equal to

(a) 0.0562 (b) 0.5625
(c) 0.1687 (d) 1.687

6. The space between two parallel plates kept 3 mm apart is filled with an oil of dynamic viscosity 0.2 Pa s. What is the shear stress (in N/m^2) on the lower fixed plate if the upper one is moved with a velocity of 1.5 m/s?

(a) 0.1 (b) 1.0
(c) 10 (d) 100

7. The velocity distribution near a solid wall at a section in a laminar flow is given by

$$u = 5\sin(5\pi y) \text{ for } y \le 0.10 \text{ m}$$

The dynamic viscosity of the fluid is 5 poise. What will be the value of shear stress at $y = 0.05$ m in N/m^2?

(a) 27.6 (b) 276.0
(c) 55.52 (d) 555.2

8. A sleeve 10 cm long encases a vertical metal rod 3.0 cm in diameter with a radial clearance of 2 mm. If when immersed in an oil of viscosity 6.0 poise, the effective weight of the sleeve is 7.5 N, at what velocity the sleeve will slide down the rod?

(a) 10.64 m/s (b) 5.32 m/s
(c) 2.66 m/s (d) 1.33 m/s

9. A circular disc of radius R is kept at a small height h above a fixed bed by means of a layer of oil of viscosity μ. If the disc is rotated at an angular velocity ω, the viscous torque on the disc will be given by

(a) $2\pi\mu\omega R^4/h$ (b) $\pi\mu\omega R^4/h$
(c) $\pi\mu\omega R^4/(2h)$ (d) $\pi\mu\omega R^4/(4h)$

10. A solid cone of radius R and half vertex angle α is to rotate at an angular velocity ω over an oil film of thickness h in a conical housing. If the dynamic viscosity of oil is μ, the viscous torque on the cone will be given by

(a) $2\pi\mu\omega R^4/(h\sin\alpha)$
(b) $\pi\mu\omega R^4/(h\sin\alpha)$
(c) $\pi\mu\omega R^4/(2h\sin\alpha)$
(d) $\pi\mu\omega R^4/(4h\sin\alpha)$

11. If the surface tension at air-water interface is 0.073 N/m, the pressure difference between inside and outside of an air bubble of diameter 0.01 mm would be

(a) 58.4 kPa (b) 29.2 kPa
(c) 14.6 kPa (d) 7.3 kPa

12. If the surface tension at the soap-air interface is 0.088 N/m, the internal pressure in a soap bubble of 2 cm diameter would be

(a) 35.2 N/m^2 (b) 17.6 N/m^2
(c) 8.8 N/m^2 (d) 4.4 N/m^2

13. A glass U-tube has two limbs of internal diameter 6 mm and 16 mm, respectively, and contains water. Calculate the difference in water level in the two limbs due to capillary action. Surface tension of water-glass is 0.073 N/m and angle of contact can be assumed to be zero.

(a) 31.0 mm (b) 3.10 mm
(c) 30.4 mm (d) 3.00 mm

14. If surface tension of soap solution is 0.040 N/m, the work done in blowing a soap bubble of diameter 12 cm shall be equal to

(a) 7.238×10^{-3} J (b) 3.619×10^{-3} J
(c) 1.809×10^{-3} J (d) Indeterminate

15. The volume of water is to be reduced by 1.5%. If its bulk modulus of elasticity is 2.2×10^9 Pa, the increase in pressure will be

(a) 1.3×10^4 kPa (b) 2.3×10^4 kPa
(c) 3.3×10^4 kPa (d) 4.3×10^4 kPa

16. The viscosity of

(a) liquids increases with temperature
(b) gases increases with temperature
(c) fluids decreases with temperature
(d) fluids increases with temperature

17. If the relationship between the shear stress τ and the rate of shear strain du/dy is expressed as

$$\tau = k\left(\frac{du}{dy}\right)^n$$

The fluid with exponent $n<1$ is known as

(a) Pseudoplastic fluid
(b) Bingham plastic fluid
(c) Dilatent fluid
(d) Newtonian fluid

18. If the capillary rise of water in a 2 mm diameter tube is 1.5 cm, the height of capillary rise in a 0.5 mm diameter tube, in cm, will be

(a) 10.0 (b) 1.5
(c) 6.0 (d) 24.0

19. A vertical shaft has a hemispherical bottom of radius R which rotates inside a bearing of identical shape at its end. An oil film of thickness h and viscosity μ is maintained in the bearing. The viscous torque in the shaft when it rotates with an angular velocity ω will be given by

(a) $4\pi\omega\mu R^4/(3h)$ (b) $8\pi\omega\mu R^4/(3h)$
(c) $16\pi\omega\mu R^4/(3h)$ (d) $32\pi\omega\mu R^4/(3h)$

20. If μ is dynamic viscosity and ρ is the density of fluid, then kinematic viscosity is determined as

(a) μ (b) μ/ρ
(c) $\rho\mu$ (d) ρ/μ

21. The viscosity in a fluid is caused mainly by

(a) intermolecular force of cohesion
(b) molecular momentum exchange
(c) both (a) and (b)
(d) none of the above

22. Select wrong statement regarding viscosity

(a) In general, the viscosity of both liquids and gases ceases to be a function of pressure
(b) viscosity of the liquids decreases with increase in temperature
(c) viscosity of gases increases with increase in temperature
(d) none of the above

23. With usual symbols, the viscosity of non-newtonian fluids can be represented as

(a) $\tau=\mu du/dy$ (b) $\tau=n\mu du/dy$
(c) $\tau=\mu\sqrt{du/dy}$ (d) $\tau=\mu\,(du/dy)^n$

24. The partial pressure exerted by the molecules of a liquid confined in a closed vessel is known as

(a) gauge pressure
(b) ambient pressure
(c) vapor pressure
(d) absolute pressure

25. The viscous torque on cylindrical shaft of radius r and length l, rotating at speed ω (rad/s) in concentric cylinder with radial clearance h is given by

(a) $\pi\omega\mu lr^3/(3h)$ (b) $\pi\omega\mu lr^3/(2h)$
(c) $\pi\omega\mu lr^3/h$ (d) $2\pi\omega\mu lr^3/h$

26. The viscous torque on inverted hemisphere of radius r, cone angle θ rotating at speed ω (rad/s) in concentric hemisphere with radial clearance h is given by

(a) $2\pi\omega\mu r^4/h$ (b) $4\pi\omega\mu r^4/(3h)$
(c) $3\pi\omega\mu r^4/(2h)$ (d) $\pi\omega\mu r^4/(2h)$

27. An important law which states that gas solubility is proportional to partial pressure, is known as

(a) Newton's law (b) Henry's law
(c) Stokes law (d) Hooke's law

28. Due to which property of liquids, falling water drops become spherical, liquid jet breaks, and soap bubbles are formed?

(a) viscosity (b) surface tension
(c) bulk modulus (d) all of the above

29. Following phenomenon are associated with surface tension in fluids

(a) excess pressure (b) angle of contact
(c) capillary (d) all of the above

30. The phenomenon of rise or fall of liquid surface relative to adjacent general level of liquid is called capillarity . It is associated with

(a) angle of contact
(b) surface tension
(c) both (a) and (b)
(d) none of the above

31. A capillary of diameter d is submerged partially in liquid of density ρ and surface tension σ. If the angle of contact is θ, the capillary rise will be

(a) $\sigma\cos\theta/(\rho g d)$ (b) $2\sigma\cos\theta/(\rho g d)$
(c) $4\sigma\cos\theta/(\rho g d)$ (d) $8\sigma\cos\theta/(\rho g d)$

32. If a tube of radius r is inserted in a liquid of specific gravity s_1 above which another liquid of specific gravity s_2 lies such that the angle of contact at free surface is θ, then capillary rise will be equal to

(a) $\dfrac{\sigma\cos\theta}{r\rho g(s_1-s_2)}$ (b) $\dfrac{2\sigma\cos\theta}{r\rho g(s_1-s_2)}$

(c) $\dfrac{\sigma\sin\theta}{r\rho g(s_1-s_2)}$ (d) $\dfrac{\sigma\sin\theta}{r\rho g(s_1-s_2)}$

33. The ratio of gauge pressure within a spherical droplet and that in a bubble of the same fluid and same size will be

(a) 1/4 (b) 1/2
(c) 1 (d) 2

34. The height to which a liquid will rise in an open capillary tube is inversely proportional to

(a) temperature of liquid
(b) density of liquid
(c) air pressure
(d) surface tension

35. Which one of the following is bulk modulus K of a fluid? (with usual symbols)

(a) $\rho\partial p/\partial\rho$ (b) $(\partial p/\partial\rho)/\rho$
(c) $\rho\partial\rho/\partial p$ (d) $(\partial\rho/\partial p)/\rho$

36. Surface tension is due to

(a) viscous forces
(b) cohesion
(c) adhesion
(d) the difference between adhesive and cohesive forces

37. The shear stress developed in in a lubricating oil of viscosity 9.81 poise, filled between two parallel plates 1 cm part and moving with relative velocity of 2 m/s is

(a) 20 N/m^2 (b) 19.62 N/m^2
(c) 29.62 N/m^2 (d) 40 N/m^2

38. What is the pressure inside a soap bubble, over the atmospheric pressure if its diameter is 2 cm and the surface tension is 0.1 N/m?

(a) 0.4 N/m^2 (b) 4.0 N/m^2
(c) 40.0 N/m^2 (d) 400.0 N/m^2

39. In an experiment to determine the rheological behavior of a material, the observed relation between shear stress τ, and the rate of shear strain du/dy, is

$$\tau = \tau_0 + c\left(\frac{du}{dy}\right)^{0.5}$$

The material is

(a) a Newtonian fluid
(b) a thixotropic fluid
(c) a Bingham plastic fluid
(d) an ideal plastic fluid

40. The reading of the pressure gauge fitted on a vessel is 25 bar. The atmospheric pressure is 1.03 bar and the value of g is 9.81 m/s^2. The absolute pressure in the vessel is

(a) 23.97 bar (b) 25.00 bar
(c) 26.03 bar (d) 34.84 bar

41. The barometric pressure at the base of a mountain is 750 mm Hg and at the top 600 mm Hg. If the average air density is 1 kg/m^3, the height of the mountain is, approximately,

(a) 2000 m (b) 3000 m
(c) 4000 m (d) 5000 m

42. If a hydraulic press has a ram of 12.5 cm diameter and plunger of 1.25 cm diameter, what force would be required on the plunger to raise a mass of 1 ton on the ram?

(a) 981 N (b) 98.1 N
(c) 9.81 N (d) 0.98 N

43. Which law states that when a certain pressure is applied at any point in a fluid at rest the pressure is equally transmitted in all directions and to every other point in the fluid?

(a) Newton's law
(b) Pascal's law
(c) Stokes law
(d) Archimedes' principle

44. If a rectangle of height d is submerged into a liquid at an angle θ with horizontal such that its center of gravity is $\bar{x}$, then center of pressure will be at a depth given by

(a) $\bar{x}+\dfrac{d\sin\theta^2}{6\bar{x}}$ (b) $\bar{x}+\dfrac{d\sin\theta^2}{12\bar{x}}$

(c) $\bar{x}+\dfrac{d\sin\theta^2}{16\bar{x}}$ (d) $\bar{x}+\dfrac{d\sin\theta^2}{18\bar{x}}$

45. If a triangle of height d is submerged into a liquid at an angle θ with horizontal such that its center of gravity is $\bar{x}$, then center of pressure will be at a depth given by

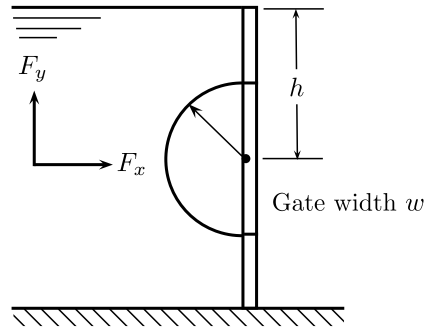
(a) $\bar{x}+\dfrac{d\sin\theta^2}{6\bar{x}}$ (b) $\bar{x}+\dfrac{d\sin\theta^2}{12\bar{x}}$

(c) $\bar{x}+\dfrac{d\sin\theta^2}{16\bar{x}}$ (d) $\bar{x}+\dfrac{d\sin\theta^2}{18\bar{x}}$

46. In hydrostatics, it is observed that total weight of the water in the tank is much less than the total pressure on the bottom of the tank. This is known as

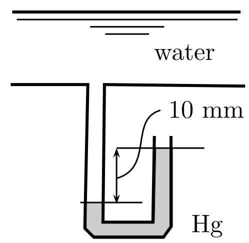
(a) Pascal's paradox
(b) Newton's paradox
(c) Stokes paradox
(d) Archimedes' principle

47. The horizontal and vertical hydrostatic forces F_x and F_y on the semi-circular gate, having a width w into the plane of figure, are



(a) $F_x = \rho ghrw$ and $F_y = 0$
(b) $F_x = 2\rho ghrw$ and $F_y = 0$
(c) $F_x = \rho ghrw$ and $F_y = \rho gwr^2/2$
(d) $F_x = 2\rho ghrw$ and $F_y = \pi\rho gwr^2/2$

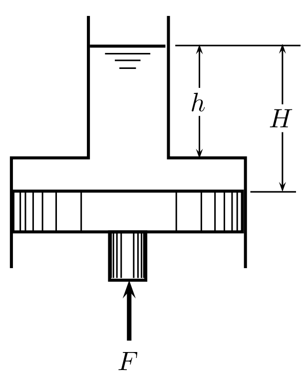
48. A mercury manometer is used to measure the static pressure at a point in a water pipe as shown in the figure.



The level difference of mercury in the two limbs is 10 mm. The gauge pressure at that point is

(a) 1236 Pa (b) 1333 Pa
(c) zero (d) 98 Pa

49. The force F required to support the liquid of density ρ for a height h in the column, as shown in the figure, is given by



(a) ρgHA (b) $\rho g\,(H-h)\,A$
(c) $\rho g\,(H+h)\,A$ (d) ρghA

50. In a inclined manometer, in 30° to horizontal, measures the pressure differential between two locations of a pipe carrying water. If the manometric liquid is mercury (specific gravity 13.6) and the manometer showed a level difference of 20 cm, then the pressure head difference of water between the two tapping will be

(a) 1.26 m (b) 1.36 m
(c) 2.52 m (d) 2.72 m

51. An open tank contains water to depth of 2 m and oil over it to a depth of 1 m. If the specific gravity of oil is 0.8, then the pressure at the interface of the two fluid layers will be

(a) 7848 N/m^2 (b) 8720 N/m^2
(c) 9747 N/m^2 (d) 9750 N/m^2

52. The vertical component of the force on a curved surface submerged in a static liquid is equal to the

(a) mass of the liquid above the curved surface
(b) weight of the liquid above the curved surface
(c) product of pressure at CG multiplied by the area of the curved surface
(d) product of pressure at CG multiplied by the projected area of the curved surface

53. The normal stress is the same in all directions at a point in a fluid only when

(a) the fluid is frictionless
(b) the fluid is frictionless and incompressible
(c) the fluid has zero viscosity and is at rest
(d) one fluid layer has no motion relative to an adjacent layer

54. A circular annular plate having outer and inner diameter of 1.4 m and 0.6 m, respectively, is immersed in water with its plane making an angle of 60° with the horizontal. The center of the circular annular plate is 1.85 m below the free surface. The hydrostatic thrust on one side of the plate is

(a) 1975 N (b) 19.75 kN
(c) 11.4 kN (d) 22.8 kN

55. If a vessel containing liquid moves downward with constant acceleration g, then

(a) the pressure throughout the liquid mass is atmospheric
(b) the pressure in the liquid mass is greater than the hydrostatic pressure
(c) there will be vacuum in the liquid
(d) the pressure throughout the liquid mass is greater than atmospheric

56. The depth of center of pressure for a rectangular lamina immersed vertically in water up to height h is given by

(a) $h/2$ (b) $h/4$
(c) $2h/3$ (d) $3h/2$

57. When can a piezometer be not used for pressure measurement in pipes?

(a) The pressure difference is low
(b) The velocity is high
(c) The fluid in the pipe is a gas
(d) The fluid in the pipe is highly viscous

58. A U-tube open at both ends and made of 8 mm diameter glass tube mercury up to a height of 10 cm in both the limbs. If 19 cm^3 of water is added to one of the limbs, what is the difference in mercury levels in the two limbs at equilibrium?

(a) 4.5 mm (b) 1.0 mm
(c) 2.8 mm (d) 3.2 mm

59. Which property of mercury is the main reason for its use in barometers?

(a) high density
(b) negligible capillarity effect
(c) very low vapor pressure
(d) low compressibility

60. A rectangular plate 0.75 m × 2.4 m is immersed in a liquid of relative density 0.85 with its 0.75 m side horizontal and just at the water surface. If the plane of plate makes an angle of 60° with the horizontal, what is the approximate pressure force on one side of the plate?

(a) 7.80 kN (b) 15.60 kN
(c) 18.00 kN (d) 24.00 kN

61. In order to increase sensitivity of U-tube manometer, one leg is usually inclined by an angle θ. What is the sensitivity of inclined tube compared to sensitivity of U-tube?

(a) $\sin\theta$ (b) $1/\sin\theta$
(c) $1/\cos\theta$ (d) $\tan\theta$

62. A circular area of 1.2 m diameter is immersed vertically in a liquid of unit weight 800 N/m^3 with

its top edge just on the liquid surface. The depth of center of pressure on one side, measured below the liquid surface is

(a) 0.80 m (b) 0.75 m
(c) 0.60 m (d) 0.64 m

63. In an isothermal atmosphere, the pressure

(a) is constant with elevation
(b) decreases linearly with elevation
(c) cannot be related to elevation
(d) decreases near the surface but approaches a constant value

64. Calculation of meta-centric height of a floating body involves second moment of area. The axis about which this moment is to be calculated passes through the

(a) top horizontal surface of the body
(b) bottom horizontal surface of the body
(c) center of gravity of the body
(d) center of buoyancy

65. If a cylindrical wooden pole (gravity 0.6), 20 cm in diameter and 1 m in height is placed in a pool of water in a vertical position, then it will

(a) float in stable equilibrium
(b) float in unstable equilibrium
(c) float in neutral equilibrium
(d) start moving horizontally

66. The fraction of the volume of a solid piece of metal of relative density 8.25 floating above the surface of a container of mercury of relative density 13.6 is

(a) 1.648 (b) 0.607
(c) 0.393 (d) 0.352

67. According to Archimedes' principle, when a body is immersed in a fluid either partially or totally, it is lifted up by a force equal to the

(a) weight of the the body
(b) weight of the fluid volume equal to the body
(c) weight of the fluid displaced by the body
(d) none of the above

68. If a body weighs W in air has two weights W_1 and W_2 in two liquids of specific gravities s_1 and s_2, respectively, then its volume can be calculated as

(a) $\dfrac{W_2-W_1}{\rho_w g\,(s_1+s_2)}$ (b) $\dfrac{W_2+W_1}{\rho_w g\,(s_1-s_2)}$

(c) $\dfrac{W_2+W_1}{\rho_w g\,(s_1+s_2)}$ (d) $\dfrac{W_2-W_1}{\rho_w g\,(s_1-s_2)}$

69. When a body floats at interface of two fluids of specific gravities s_1 and s_2 with volume v_1 on sink in first fluid, and volume v_2 on second fluid, then force of buoyancy is

(a) $\rho_w g\,(s_1 v_1 + s_2 v_2)$
(b) $\rho_w g\,(s_1 v_1 - s_2 v_2)$
(c) $\rho_w g\,(v_1+v_2)\,(s_1+s_2)$
(d) $\rho_w g\,(v_1-v_2)\,(s_1-s_2)$

70. If a block of ice, floating over water in a vessel, slowly melts in it, then water level in the vessel will

(a) decrease (b) increase
(c) remains same (d) all of the above

71. Select the correct nomenclature given to the oscillations of a floating body

(a) pitching - transverse axis
(b) rolling - longitudinal axis
(c) yawing - vertical axis
(d) all of the above

72. The least radius of gyration of a ship is 9 m and the metacentric height is 750 mm. The time period of oscillation of the ship is

(a) 42.41 s (b) 75.4 s
(c) 20.85 s (d) 85 s

73. A rectangular block having minimum width d and height h and specific gravity s would remain stable floating in water if

(a) $d/h > \sqrt{6s\,(1-s)}$ (b) $d/h < \sqrt{6s\,(1-s)}$
(c) $d/h > \sqrt{6s\,(1+s)}$ (d) $d/h < \sqrt{6s\,(1+s)}$

74. A cylindrical block having diameter d and height h and specific gravity s would remain stable floating in water if

(a) $d/h > \sqrt{8s\,(1+s)}$ (b) $d/h < \sqrt{8s\,(1+s)}$
(c) $d/h > \sqrt{8s\,(1-s)}$ (d) $d/h < \sqrt{8s\,(1-s)}$

75. An inverted cone having base diameter d and height h and specific gravity s would remain stable floating in water if

(a) $d/h > \sqrt{4\left(s^{-1/2} - 1\right)}$
(b) $d/h < \sqrt{4\left(s^{-1/2} - 1\right)}$
(c) $d/h > \sqrt{4\left(s^{-1/3} - 1\right)}$
(d) $d/h < \sqrt{4\left(s^{-1/3} - 1\right)}$

76. A floating body is said to be in stable equilibrium when

(a) metacenter lies above center of gravity of the body
(b) metacenter coincides with center of gravity of the body
(c) metacenter lies below the center of gravity of the body
(d) none of the above

77. The following terms are related to floating bodies:

Center of gravity	=	G
Meta center	=	M
Weight of floating body	=	W
Buoyant force	=	B

Match List I with List II and select the correct answer.

List I (Condition)	List II (Result)
A. G is above M	1. Stable equilibrium
B. G and M coincide	2. Unstable equilibrium
C. G is below M	3. Floating body
D. $B \geq W$	4. Neutral equilibrium

(a) A-1, B-3, C-2, D-4
(b) A-3, B-1, C-4, D-2
(c) A-2, B-3, C-4, D-1
(d) A-2, B-4, C-1, D-3

78. A large metacentric height in a vessel

(a) improves stability and makes periodic time to oscillation longer
(b) impairs stability and makes periodic time of oscillation shorter
(c) has no effect on stability or the periodic time of oscillation
(d) improves stability and makes the periodic time of oscillation shorter

79. A block of aluminium having mass of 12 kg is suspended by a wire and lowered until submerged into a tank containing oil of relative density 0.8. Taking the relative density of aluminium as 2.4, the tension in the wire will be ($g = 9.8$ m/s^2)

(a) 12000 N (b) 800 N
(c) 120 N (d) 80 N

80. A wooden rectangular block of length l is made to float in water with its axis vertical. The center of gravity of the floating body is $0.15l$ above the center of buoyancy. What is the specific gravity of the wooden block?

(a) 0.6 (b) 0.65
(c) 0.7 (d) 0.75

81. What is the vertical component of pressure force on submerged curved surface equal to?

(a) Its horizontal component.
(b) The force on a vertical projection of the curved surface.
(c) The product of the pressure at centroid and surface area.
(d) The gravity force of liquid vertically above the curved surface up to the free surface.

82. What is the depth of center of pressure of a vertical immersed surface from free surface of liquid? (Symbols have their usual meaning.)

(a) $\bar{x} + \dfrac{I_G}{A\bar{x}}$ (b) $\bar{x} + \dfrac{I_G A}{\bar{x}}$
(c) $\bar{x} + \dfrac{I_G \bar{x}}{A}$ (d) $\bar{x} + \dfrac{A\bar{x}}{I_G}$

83. The distance from the center of buoyancy to the meta center is given by I/V_d, where V_d is the volume of fluid displaced. What does I represent?

(a) Moment of inertia of a horizontal section of the body taken at the surface of the fluid.
(b) Moment of inertia about its vertical centroidal axis
(c) Polar moment of inertia
(d) Moment of inertia about its horizontal centroidal axis

84. What is the vertical distance of the center of pressure below the centroid of the plane area?

(a) $I_G / \left(A\bar{h}\right)$ (b) $I_G \sin\theta / \left(A\bar{h}\right)$
(c) $I_G \sin^2\theta / \left(A\bar{h}\right)$ (d) $I_G \sin^2\theta / \left(A\bar{h}^2\right)$

85. A body weighs 30 N and 15 N when weighed under submerged conditions in liquids of relative densities 0.8 and 1.2, respectively. What is the volume of the body?

(a) 12.50 l (b) 3.82 l
(c) 18.70 l (d) 75.50 l

86. A tank has in its side a very small horizontal cylinder fitted with a frictionless piston. The head of liquid above the piston is h and the piston area a, the liquid having a specific weight γ. What is the force that must be exerted on the piston to hold it in position against the hydrostatic pressure?

(a) $2\gamma ha$ (b) γha
(c) $2\gamma ha/3$ (d) $\gamma ha/2$

87. What acceleration would cause the free surface of a liquid contained in an open tank moving in a horizontal track to dip by 45°.

(a) $g/2$ (b) $2g$
(c) g (d) $3g/2$

88. A partially filled tank is carried out on a truck which is moving with a constant acceleration. The water surface in the tank will

(a) move up in the front and move down at the rear
(b) move up at the rear and move down in front
(c) move down in front and also at the rear but move up in the center
(d) remain undisturbed

89. A rectangular water tank, full to the brim, has its length breadth and height in the ratio of 2:1:2. The ratio of hydrostatic force at the bottom to that at any larger vertical surface is

(a) 1/2 (b) 1
(c) 2 (d) 4

90. A right circular cylinder, open at the top is filled with liquid of relative density 1.2. It is rotated about its vertical axis at such speed that half of the liquid spills out. The pressure at the center of the bottom will be

(a) zero
(b) one fourth of the value when the cylinder was full
(c) half of the value when the cylinder was full
(d) not determinable from the given data

91. When a vertical cylindrical vessel containing water is rotated about its axis, then the free surface of water becomes

(a) a cycloid of revolution
(b) an ellipsoid of revolution
(c) a hyperboloid of revolution
(d) a parabolic of revolution

92. Which one of the following stream functions ψ is a possible irrotational flow field?

(a) $\psi = y^2 - x^2$ (b) $\psi = Ax^2y^2$
(c) $\psi = A\sin(xy)$ (d) $\psi = Ax + By^2$

93. A two-dimensional flow field is given by $\phi = 3xy$. The stream function is represented by

(a) $\psi = 3(x+y)^2$
(b) $\psi = 3\left(x^2+y^2\right)/2$
(c) $\psi = 3\left(x^2-y^2\right)/2$
(d) None of the above

94. The velocity components for two dimensional incompressible flow of a fluid are

$$u = x - 4y, \qquad v = -y - 4x$$

It can be concluded that

(a) the flow does not satisfy the continuity equation
(b) the flow is rotational
(c) the flow is irrotational
(d) none of the above

95. The stream lines and the lines of constant velocity potential in an inviscid rotational flow field form

(a) parallel grid lines placed in accordance with their magnitude
(b) intersecting grid net with arbitrary orientation
(c) an orthogonal grid system
(d) none of the above

96. In a two-dimensional flow, the velocity components in x and y directions in terms of stream function (ψ) are

(a) $u = \partial\psi/\partial x, \quad v = \partial\psi/\partial y$
(b) $u = \partial\psi/\partial y, \quad v = \partial\psi/\partial x$
(c) $u = -\partial\psi/\partial y, \quad v = \partial\psi/\partial x$
(d) $u = \partial\psi/\partial x, \quad v = -\partial\psi/\partial y$

97. Which one of the following statements is true to two-dimensional flow of ideal fluids?

(a) Potential function exists if stream function exists.
(b) Stream function may or may not exist.
(c) Both potential function and stream function must exist for every flow.
(d) Stream function will exist but potential function may or may not exist.

98. The curl of a given velocity field (i.e. $\nabla \times \vec{v}$) indicates the rate of

(a) change of flow at a point
(b) twisting of the lines of flow
(c) deformation
(d) translation

99. The area of a 2 m long tapered duct increases as

$$A = 0.5 - 0.2x$$

where x is the distance in meters. At a given instant of discharge of 0.5 m^3/s is flowing in the duct and is found to increase at a rate of 0.2 m^3/s. The local acceleration in m^2/s at $x = 0$ will be

(a) 1.4 (b) 1.0
(c) 0.4 (d) 0.667

100. If the stream function is given by $\psi = 3xy$, then the velocity at a point (2,3) will be

(a) 7.21 unit (b) 10.82 unit
(c) 2.236 unit (d) 3.211 unit

101. A stream function is given by $x^2 - y^2$. The potential function of the flow will be

(a) $2xy + f(x)$ (b) $2xy +$ constant
(c) $2\left(x^2 - y^2\right)$ (d) $2xy + f(y)$

102. A stream line is a line

(a) which is along the path of the particle
(b) which is always parallel to the main direction of flow
(c) along which there is no flow
(d) on which tangent drawn at any point gives the direction of velocity

103. For irrotational and incompressible flow, the velocity potential and stream function are given by ϕ and ψ, respectively. Which one of the following sets is correct?

(a) $\nabla^2\phi = 0$, $\nabla^2\psi = 0$
(b) $\nabla^2\phi \neq 0$, $\nabla^2\psi = 0$
(c) $\nabla^2\phi = 0$, $\nabla^2\psi \neq 0$
(d) $\nabla^2\phi \neq 0$, $\nabla^2\psi \neq 0$

104. The stream function in a flow field is given by $\psi = 2xy$. In the same flow field, what is the velocity at a point (2,1)?

(a) 4 unit (b) 5.4 unit
(c) 1.73 unit (d) 4.47 unit

105. Stream lines, path lines and streak lines are virtually identical for

(a) uniform flow
(b) flow of ideal fluids
(c) steady flow
(d) non-uniform flow

106. In a flow field, the stream lines and equipotential lines

(a) are parallel
(b) cut at any angle
(c) are orthogonal everywhere in the flow field
(d) cut orthogonally except at the stagnation points

107. Existence of velocity potential implies that

(a) flow is in continuum
(b) flow is irrotational
(c) fluid is ideal
(d) fluid is compressible

108. The continuity equation for a steady flow states that

(a) velocity field is continuous at all points in flow field
(b) the velocity is tangential to the streamlines
(c) the stream function exists for steady flows
(d) the efflux rate of mass through the control surface is zero

109. The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

is valid only for

(a) ideal fluid flow
(b) incompressible fluid whether the flow is steady or not
(c) steady flow, whether it is compressible or not
(d) steady flow and compressible fluids

110. A fluid jet is discharging from a 100 mm nozzle and the vena contracta formed has a diameter of 90 mm. If the coefficient of velocity is 0.95, then the coefficient of discharge for the nozzle is

(a) 0.855 (b) 0.810
(c) 0.902 (d) 0.769

111. Bernoulli's equation with usual symbols

$$\frac{p}{\rho g}+\frac{v^2}{2g}+h=c$$

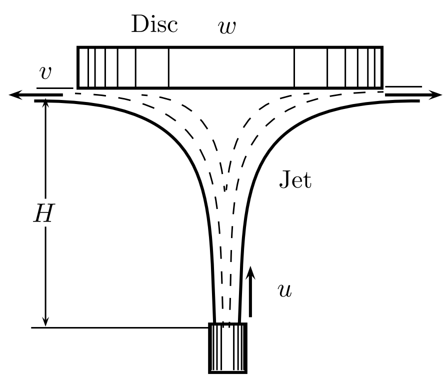
is applicable for

(a) steady, frictionless, and incompressible flow along a stream line
(b) uniform and frictionless flow along a stream line when ρ is a function of p
(c) steady and frictionless flow along a streamline when ρ is a function of p
(d) steady, uniform, and incompressible flow along a streamline

112. Navier–Stokes equation represents the conservation of

(a) energy (b) mass
(c) pressure (d) momentum

113. A vertical jet of water leaving the nozzle with a velocity u strikes a disc weighing w as shown in the figure.



The jet is then deflected horizontally. The disc will be held in equilibrium at at distance y where the fluid velocity is v when y is equal to

(a) $\left(u^2-v^2\right)/(2g)$ (b) $u^2/(2g)$
(c) w/u^2 (d) w/v^2

114. If a fluid jet discharging from a 50 mm diameter orifice has a 40 mm diameter at its vena contracta, then its coefficient of contraction will be

(a) 0.32 (b) 0.64
(c) 0.96 (d) 1.64

115. The locus of elevations that water will rise in a series of pitot tubes is called

(a) the hydraulic grade line
(b) the energy grade line
(c) the velocity head
(d) the pressure head

116. A nozzle has velocity head of 10 m at outlet. If it is kept vertical, the height reached by the stream is

(a) 100 m (b) 10 m
(c) $\sqrt{10}$ m (d) $1/\sqrt{10}$ m

117. The Euler's equation of motion for the flow of an ideal fluid is derived considering the principle of conservation of

(a) mass and the fluid as incompressible and inviscid
(b) momentum and the fluid as incompressible and viscous
(c) momentum and the fluid as incompressible and inviscid
(d) energy and the fluid as incompressible and inviscid

118. Water hammer in pipes takes place when

(a) fluid is flowing with high velocity
(b) fluid is flowing with high pressure
(c) flowing fluid is suddenly brought to rest by closing a valve
(d) flowing fluid is brought to rest by gradually closing a valve

119. A liquid flows downward through a tapered vertical portion of a pipe. At the entrance and exit of the pipe, the static pressures are equal. If for a vertical height h, the velocity becomes four times, then the ratio of h to the velocity head at entrance will be

(a) 3 (b) 8
(c) 15 (d) 24

120. When is Bernoulli's equation applicable between any two points in a flow field?

(a) The flow is steady, incompressible and rotational
(b) The flow is steady, compressible and irrotational

(c) The flow is unsteady, incompressible and irrotational

(d) The flow is steady, incompressible and irrotational

121. Two venturimeters of different area ratios are connected at different locations of a pipeline to measure discharge. Similar manometers are used across the two venturimeters to register the head differences. The first venturimeter of area ratio 2 registers a head difference h, while the second venturimeter registers $5h$. The area ratio for the second venturimeter is?

(a) 3 (b) 4
(c) 5 (d) 6

122. Match List I with List II and select the correct answer using the codes given below

List I	List II
A. Rotameter	1. Vena contracta
B. Venturimeter	2. End contraction
C. Orifice meter	3. Tapering tube
D. Flow nozzle	4. Convergent-divergent
	5. Bell mouth entry

(a) A-1, B-2, C-3, D-4
(b) A-3, B-4, C-1, D-2
(c) A-5, B-4, C-2, D-1
(d) A-3, B-5, C-1, D-2

123. While measuring the velocity of air ($\rho = 1.2$ kg/m^3), the difference in the stagnation and static pressures of a pitot static tube was found to be 380 Pa. The velocity at that location in m/s is

(a) 24.03 (b) 4.02
(c) 17.8 (d) 25.17

124. Which one of the statements is correct for a forced vortex?

(a) turns in an opposite direction to a free vortex
(b) always occurs in conjunction with a free vortex
(c) has the linear velocity directly proportional to the radius
(d) has the linear velocity inversely proportional to the radius

125. A 12 cm diameter straight pipe is laid at a uniform downgrade and flow rate is maintained such that velocity head in the pipe is 0.5 m. If the pressure in the pipe is observed to be uniform along the length when the down slope of the pipe is 1 in 10, what is the friction factor for the pipe?

(a) 0.012 (b) 0.024
(c) 0.042 (d) 0.050

126. The loss of head in a pipe of certain length carrying a rate of flow of Q is found to be H. If a pipe of twice the diameter but of the same length is to carry a flow rate of 2Q, then the head loss will be

(a) H (b) $H/2$
(c) $H/4$ (d) $H/8$

127. Air enters a rectangular duct measuring 30×40 cm^2 with a velocity of 8.5 m/s and a temperature of 40°C. Kinematic viscosity of the air is 16.95×10^{-6} m^2/s. What will be the Reynolds number?

(a) 1.72×10^5 (b) 2.58×10^5
(c) 0.86×10^5 (d) 0.72×10^5

128. The minimum value of friction factor f that can occur in laminar flow through a circular pipe is

(a) 0.064 (b) 0.032
(c) 0.016 (d) 0.008

129. Two identical pipes of length l, same diameter, friction factor are connected in parallel between two points. For the same total volume flow rate with pipe of same diameter and friction factor, the single length of the pipe will be

(a) $l/2$ (b) $l/\sqrt{2}$
(c) $\sqrt{2}l$ (d) $l/4$

130. In flow through a pipe, the transition from laminar to turbulent flow does not depend on

(a) velocity of the fluid
(b) density of the fluid
(c) diameter of the pipe
(d) length of the pipe

131. The Reynolds number for flow of a certain fluid in a circular tube is specified as 2500. What will be the Reynolds number when the tube diameter is increased by 20% and the fluid velocity is decreased by 40% keeping fluid the same?

(a) 1200 (b) 1800
(c) 3600 (d) 200

132. For laminar flow through a long pipe, the pressure drop per unit length increases

(a) in linear proportion to the cross-sectional area
(b) in proportion to the diameter of the pipe

(c) in inverse proportion to the cross-sectional area

(d) in inverse proportion to the square of cross-sectional area

133. Laminar flow takes place in a circular tube. At what distance from the boundary layer does the local velocity reaches the average velocity?

(a) $0.707R$ (b) $0.293R$
(c) $0.5R$ (d) R

134. Two pipelines of equal lengths are connected in series. The diameter of the second pipe is two times that of the first pipe. The ratio of frictional head losses between the first pipe and the second pipe is

(a) 1:32 (b) 1:16
(c) 1:8 (d) 1:4

135. Minor losses in a piping system are

(a) less than the friction factor losses
(b) due to the viscous stresses
(c) assumed to vary linearly with the velocity
(d) found by using loss coefficients

136. A fully developed laminar viscous flow through a circular tube has the ratio of maximum velocity to average velocity as

(a) 3.0 (b) 2.5
(c) 2.0 (d) 1.5

137. In a laminar flow through a pipe of diameter D, the total discharge Q is expressed as (μ is the dynamic viscosity of the fluid, and $-\partial p/\partial x$ is the pressure gradient).

(a) $-\dfrac{\pi D^4}{128\mu} \times \dfrac{\partial p}{\partial x}$ (b) $-\dfrac{\pi D^4}{64\mu} \times \dfrac{\partial p}{\partial x}$

(c) $-\dfrac{\pi D^4}{32\mu} \times \dfrac{\partial p}{\partial x}$ (d) $-\dfrac{\pi D^4}{16\mu} \times \dfrac{\partial p}{\partial x}$

138. A pipe is connected in series to another pipe whose diameter is twice and length is 32 times that of the first pipe. The ratio of frictional head losses for the first pipe to those for the second pipe is (both pipes have the same frictional constant)

(a) 8 (b) 4
(c) 2 (d) 1

139. A pipeline connecting two reservoirs has its diameter reduced by 20% due to deposition of chemicals. For a given head difference in the reservoirs with unaltered friction factor, this would cause a reduction in discharge of

(a) 42.8% (b) 20%
(c) 17.8% (d) 10.6%

140. A pipe of 20 cm diameter and 30 km length transports oil from a tanker to the shore with a velocity of 0.318 m/s. The flow is laminar. If $\mu = 0.1$ Nm/s^2, the power required for the flow would be

(a) 9.25 kW (b) 8.36 kW
(c) 7.63 kW (d) 10.13 kW

141. For maximum transmission of power through a pipe line with total head H, the head lost due to friction is given by

(a) $0.1H$ (b) $H/3$
(c) $H/2$ (d) $2H/3$

142. The pressure drop in a 100 mm diameter horizontal pipe is 50 kPa over a length of 10 m. The shear stress at the pipe wall is

(a) 0.25 kPa (b) 0.125 kPa
(c) 0.50 kPa (d) 25.0 kPa

143. A compound pipeline consists of two pieces of identical pipes. The equivalent length of same diameter and same friction factor, for the compound pipeline is L_1 when pipes are connected in series, and is L_2 when connected in parallel. What is the ratio of equivalent lengths L_1/L_2?

(a) $32:1$ (b) $8:1$
(c) $2:1$ (d) $\sqrt{2}:1$

144. The power consumed per unit length in laminar flow for the same discharge, varies directly as D^n where D is the diameter of the pipe. What is the value of n?

(a) $1/2$ (b) $-1/2$
(c) -2 (d) -4

145. If a fluid flows through a capillary tube of length L and diameter D, and the mass flow rate and pressure drops are measured, the viscosity of the fluid can be estimated from the

(a) Euler's equation
(b) Bernoulli's equation
(c) Hagen-Poiseuille equation
(d) None of the above

146. The terminal velocity of a small sphere falling in a viscous fluid is

(a) proportional to the diameter of the sphere
(b) inversely proportional to the viscosity of the fluid
(c) inversely proportional to the diameter of the sphere
(d) proportional to the density of the fluid

147. Air (density 1.2 kg/m^3 and kinematic viscosity 15 centistokes) flows over a flat plate at zero angle of incidence, at a velocity of 20 m/s. If Reynolds number at transition is taken as 2.5×10^5, maximum distance, from leading edge up to which the boundary layer remains laminar, is

(a) 375 mm (b) 93.75 mm
(c) 187.5 mm (d) 250 mm

148. In the boundary layer, the flow is

(a) viscous and rotational
(b) inviscid and irrotational
(c) inviscid and rotational
(d) viscous and irrotational

149. In case of laminar boundary layer on a flat plate, the local skin friction coefficient (C_f) is given by (symbols have the usual meaning)

(a) $4.91/\sqrt{\mathrm{Re}_x}$
(b) $0.664/\sqrt{\mathrm{Re}_x}$
(c) $1.328/\sqrt{\mathrm{Re}_x}$
(d) $C_f = 0.322\sqrt{\mathrm{Re}_x}$

150. For laminar flow over a flat plate, the thickness of the boundary layer at a distance from the leading edge is found to be 5 mm. Thickness of the boundary layer at a downstream section which is at twice the distance of the previous section from the leading edge will be

(a) 10 mm (b) $2\sqrt{5}$ mm
(c) $5\sqrt{2}$ mm (d) 2.5 mm

151. If the velocity distribution in a turbulent boundary layer is given by

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/9}$$

the ratio of displacement thickness to nominal layer thickness will be

(a) 1.0 (b) 0.6
(c) 0.3 (d) 0.1

152. The velocity distribution in a turbulent flow in a pipe is often assumed to

(a) be parabolic
(b) be zero at the wall and increase linearly to the center
(c) vary according to the 1/7th power law
(d) be unpredictable and is thus not used

153. Laminar sublayer can develop during flow over a flat plate. It exists in

(a) laminar zone
(b) transition zone
(c) turbulent zone
(d) laminar and transition zones

154. The laminar boundary layer thickness in zero pressure gradient flow over a flat plate along the x-direction varies as

(a) $x^{-1/2}$ (b) $x^{1/7}$
(c) $x^{1/2}$ (d) x

155. The transition Reynolds number for flow over a flat plate is 5×10^5. What is the distance from the leading edge at which transition will occur for flow of water with a uniform velocity of 1 m/s? (for water, the kinematic viscosity, $\nu = 0.858 \times 10^{-6}$ m^2/s)

(a) 1 m (b) 0.43 m
(c) 43 m (d) 103 m

156. Given that

$$\begin{aligned} \delta &= \text{boundary layer thickness} \\ \delta^* &= \text{displacement thickness} \\ \delta_E &= \text{energy thickness} \\ \theta &= \text{momentum thickness} \end{aligned}$$

Shape factor H of a boundary layer is equal to

(a) δ_E/δ (b) δ^*/θ
(c) δ/δ (d) δ/δ^*

157. According to Blasius law, the local skin friction coefficient in the boundary layer over a flat plate is given by

(a) $0.332/\sqrt{\mathrm{Re}}$ (b) $0.664/\sqrt{\mathrm{Re}}$
(c) $0.647/\sqrt{\mathrm{Re}}$ (d) $1.328/\sqrt{\mathrm{Re}}$

158. In which of the following cases must separation of boundary layer occur?

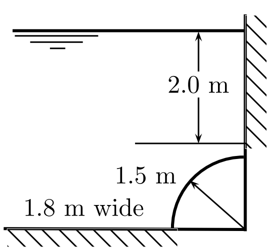
(a) $dp/dx < 0$
(b) $dp/dx = 0$
(c) $dp/dx > 0$
(d) $dp/dx > 0$ and the velocity profile has a point of inflection.

NUMERICAL ANSWER QUESTIONS

1. A rectangular plate 0.6 m wide and 1.2 m deep lies within a water body such that its plane is inclined at 45° to the horizontal and the top edge is 0.70 m below the water surface. Determine the total pressure on one side of the plate and location of the center of pressure below the free surface of water.

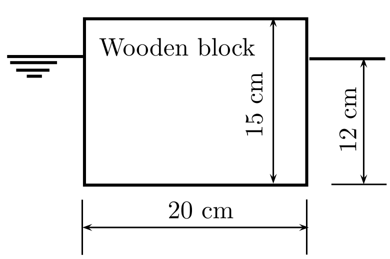
2. A circular plate of diameter 0.75 m is immersed in a liquid of relative density 0.80 with its plane making angle of 30° with the horizontal. The center of the plate is at a depth of 1.50 m below the free surface. Determine the total pressure on one side of the plate and location of the center of pressure below the free surface of water.

3. A door in a tank is in the form of a quadrant of a cylinder of 1.5 m radius and 1.5 m wide, as shown in the figure.



Calculate the the resultant force acting on the door and its inclination from horizontal.

4. A wooden block in the form of a rectangular prism floats with its shortest axis vertical. The block is 40 cm long, 20 cm wide and 15 cm deep with a depth of immersion of 12 cm. Radius of gyration about longitudinal axis of the body is 8 cm. Calculate the metacentric height and time period of oscillations of the body.



5. An open tank 5 m long and 2 m deep and 3 m wide contains oil of relative density 0.9 to a depth of 0.9 m. The tank is accelerated along its length on a horizontal track at a constant value of 3.0 m/s^2. Determine the inclination of the free surface and the pressure at the bottom of the tank at the front and rear edges.

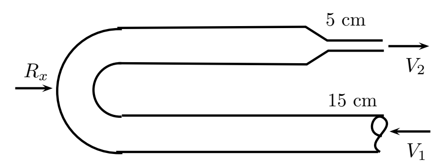
6. The velocity along the center line of a nozzle of length L is given by

$$v = 2t\left(1 - \frac{x}{2L}\right)^2$$

where L is the velocity in m/s, t is the time in seconds from commencement of flow, x is the distance from inlet to nozzle. Calculate the local acceleration when $x = 0.3$ m, $L = 0.8$ m. Also calculate the convective acceleration when $t = 3$ s, $x = 0.3$ m, $L = 0.8$ m.

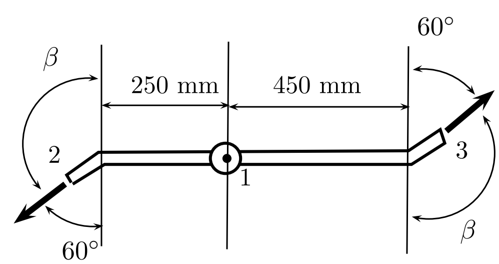
7. A pipeline is 15 cm in diameter and is at an elevation of 100 m at section A. At section B it is at an elevation of 107 m and has a diameter of 30 cm. When the discharge of 50 l/s of water is passed through this pipe, the pressure at the section A is observed to be 30 kPa. The energy loss in the pipe is 2 m. Determine the pressure at section B if flow direction is from A to B. Also determine the pressure at point B if flow is reversed.

8. A discharge of 0.06 m^3/s flows through a horizontal bend as shown in figure:



Calculate the gauge pressure and force on the bolt at section 1.

9. The jets of oil having specific gravity 0.85 is issuing out of a sprinkler of unequal arms of 12 mm in diameter and total discharge of 1.5 l/s.



Assuming zero friction, determine the rotational speed of the sprinkler. Also determine the torque required to hold the sprinkler stationary.

ANSWERS

Multiple Choice Questions

1. (b)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (c)
11. (b)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (b)
21. (c)	**22.** (d)	**23.** (d)	**24.** (c)	**25.** (d)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (d)	**30.** (c)
31. (c)	**32.** (b)	**33.** (d)	**34.** (b)	**35.** (a)	**36.** (c)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (c)
41. (a)	**42.** (b)	**43.** (b)	**44.** (b)	**45.** (d)	**46.** (a)	**47.** (c)	**48.** (a)	**49.** (a)	**50.** (b)
51. (a)	**52.** (b)	**53.** (b)	**54.** (d)	**55.** (a)	**56.** (c)	**57.** (d)	**58.** (c)	**59.** (c)	**60.** (b)
61. (b)	**62.** (b)	**63.** (d)	**64.** (a)	**65.** (b)	**66.** (c)	**67.** (c)	**68.** (d)	**69.** (a)	**70.** (c)
71. (d)	**72.** (c)	**73.** (a)	**74.** (c)	**75.** (c)	**76.** (a)	**77.** (d)	**78.** (d)	**79.** (d)	**80.** (d)
81. (d)	**82.** (c)	**83.** (a)	**84.** (c)	**85.** (b)	**86.** (b)	**87.** (c)	**88.** (b)	**89.** (b)	**90.** (a)
91. (d)	**92.** (a)	**93.** (d)	**94.** (c)	**95.** (c)	**96.** (c)	**97.** (d)	**98.** (a)	**99.** (b)	**100.** (b)
101. (b)	**102.** (d)	**103.** (a)	**104.** (d)	**105.** (c)	**106.** (c)	**107.** (b)	**108.** (d)	**109.** (a)	**110.** (d)
111. (a)	**112.** (d)	**113.** (a)	**114.** (b)	**115.** (c)	**116.** (b)	**117.** (c)	**118.** (c)	**119.** (c)	**120.** (d)
121. (c)	**122.** (b)	**123.** (d)	**124.** (c)	**125.** (b)	**126.** (d)	**127.** (a)	**128.** (d)	**129.** (d)	**130.** (d)
131. (b)	**132.** (c)	**133.** (b)	**134.** (a)	**135.** (d)	**136.** (c)	**137.** (a)	**138.** (d)	**139.** (a)	**140.** (c)
141. (b)	**142.** (b)	**143.** (c)	**144.** (c)	**145.** (c)	**146.** (b)	**147.** (c)	**148.** (a)	**149.** (b)	**150.** (b)
151. (d)	**152.** (c)	**153.** (c)	**154.** (c)	**155.** (b)	**156.** (b)	**157.** (b)	**158.** (c)		

Numerical Answer Questions

1.	7.941 kN, 1.178 m	**2.**	5.850 kN, 1.693	**3.**	111.86 kN, 49.37°
4.	1.27 cm, 1.424 s	**5.**	17°, 1.2 kPa, 14.64 kPa	**6.**	0.945 m/s^2, −14.623 m/s^2
7.	−54.54 kPa, −15.3 kPa	**8.**	461.19 kPa, 10.19 kN	**9.**	70.85 rpm, 1.47 kN

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (b) According to the Newton's law of viscosity, the shear stress between two fluid layers is proportional to the rate of shear stress.

2. (b) An inviscid and incompressible fluid is considered as ideal fluid.

3. (b) Viscosity of thixotropic fluids decreases with duration of stress, thus to maintain constant rate of shear strain, the fluid requires increasing shear stress.

4. (b) A static negative normal stress (i.e. pressure) and zero shear stress (as the fluid is static).

5. (a) Given that weight of the the block, $W = 90$ N, inclination of plane $\theta = 30°$, thickness of oil film $h = 0.003$ m, dynamic viscosity $\mu = 8.0/10 = 0.8$ Pa·s, contact area $A = 0.3$ m^2. Using Newton's law of viscosity, if u is the velocity of the block along with plane (or terminal velocity) the shear stress is given by

$$\tau = \mu \frac{u}{h}$$

At the terminal velocity, the block should be in equilibrium, therefore, net forces along the plane should be zero. Mathematically, if τ is shear stress,

then along the plane

$$W\sin\theta - \tau A = 0$$

Substituting the given values in the above equation, one obtains

$$u = 0.5625 \text{ m/s}$$

6. (d) Given that

$$\begin{aligned} dy &= 3\times 10^{-3} \text{ m} \\ \mu &= 0.2 \text{ Pa s} \\ du &= 1.5 \text{ m/s} \end{aligned}$$

Using Newton's law of viscosity,

$$\begin{aligned} \tau &= \mu\frac{du}{dy} \\ &= 100 \text{ N/m}^2 \end{aligned}$$

7. (a) Given that

$$\begin{aligned} u &= 5\sin(5\pi y) \\ \mu &= 5/10 \\ &= 0.5 \text{ Pa s} \end{aligned}$$

Therefore,

$$\frac{du}{dy} = 5\cos(5\pi y)\times 5\pi$$

Using Newton's law of viscosity for $y = 0.05$ m,

$$\begin{aligned} \tau &= \mu\frac{du}{dy} \\ &= 27.76 \text{ N/m}^2 \end{aligned}$$

8. (c) Let u be the sliding velocity. Given that

$$\begin{aligned} \mu &= 6.0/10 \\ &= 0.6 \text{ Pa s} \\ h &= 2\times 10^{-3} \text{ m} \\ l &= 0.1 \text{ m} \\ r &= \frac{0.03}{2} \\ &= 0.015 \text{ m} \\ W &= 7.5 \text{ N} \end{aligned}$$

For equilibrium,

$$\begin{aligned} \tau\times 2\pi r\times l &= W \\ \tau &= \frac{W}{2\pi r\times l} \\ &= 795.77 \text{ Pa} \end{aligned}$$

Using Newton's law of viscosity,

$$\begin{aligned} \tau &= \mu\frac{u}{h} \\ u &= \frac{\tau\times h}{\mu} \\ &= 2.65 \text{ m/s} \end{aligned}$$

9. (c) The torque on elemental disc element of width dr at radius r will be given by

$$dT = r\times\mu\frac{\omega r}{h}\times 2\pi r dr$$

Total viscous torque over radius R on the disc is

$$\begin{aligned} T &= \int_0^R dT \\ &= \int_0^R r\times\mu\frac{\omega r}{h}\times 2\pi r dr \\ &= \left[\frac{\mu\omega}{h}2\pi\frac{r^4}{4}\right]_0^R \\ &= \frac{\pi\mu\omega R^4}{2h} \end{aligned}$$

10. (c) Consider a conical element of radii r and $r+dr$. The conical surface area in touch with the oil film is equal to

$$da = 2\pi r\frac{dr}{\sin\alpha}$$

Using Newton's law of viscosity,

$$\tau = \mu\frac{\omega r}{h}$$

Viscous torque on this element is

$$dT = \mu\frac{\omega r}{h}\times 2\pi r\frac{dr}{\sin\alpha}\times r$$

Total viscous torque on the element is

$$\begin{aligned} T &= \int_0^R dT \\ &= \int_0^R \frac{2\pi\mu\omega}{h\sin\alpha}\times r^3 dr \\ &= \frac{\pi\mu\omega R^4}{2h\sin\alpha} \end{aligned}$$

11. (b) Given that

$$\begin{aligned} \sigma &= 0.073 \text{ N/m} \\ d &= 0.01 \text{ mm} \end{aligned}$$

Excess pressure due to surface tension in bubble is

$$\begin{aligned} \Delta p &= \frac{4\sigma}{d} \\ &= 29.2 \text{ kPa} \end{aligned}$$

12. (b) Given that

$$\sigma = 0.088 \text{ N/m}$$
$$d = 0.02 \text{ m}$$

A soap bubble has two surfaces. Excess pressure is given by

$$\Delta p = \frac{4\sigma}{d} = 17.6 \text{ N/m}^2$$

13. (b) Given that

$$d_1 = 0.006 \text{ m}$$
$$d_2 = 0.016 \text{ m}$$
$$\sigma = 0.073 \text{ N/m}$$
$$\theta = 0^\circ$$

Assuming

$$\rho = 1000 \text{ kg/m}^3$$
$$g = 9.81 \text{ m/s}^2$$

Capillary rise in a diameter d is given by

$$h = \frac{4\sigma\cos\theta}{\rho g h}$$

The difference in capillary rise in the two limbs is given by

$$\Delta h = \frac{4\sigma\cos\theta}{\rho g}\left(\frac{1}{d_1} - \frac{1}{d_2}\right) = 3.1 \text{ mm}$$

14. (b) Given that

$$\sigma = 0.040 \text{ N/m}$$
$$d = 0.12 \text{ m}$$
$$r = \frac{d}{2} = 0.06 \text{ m}$$

The soap bubble has two spherical surfaces where surface tension can act, hence, the work done will be equal to

$$W = \sigma \times 4\pi r^2 \times 2 = 3.619 \times 10^{-3} \text{ J}$$

15. (b) Given that the percentage decrease in volume of the water and bulk modulus, respectively

$$\frac{\Delta V}{V} = 1.5\%$$
$$K = 2.2 \times 10^9 \text{ Pa}$$

Therefore

$$\Delta p = -\frac{\Delta V}{V}K = 3.3 \times 10^4 \text{ kPa}$$

16. (b) The viscosity of gases increases with the increase in temperature, whereas that of liquids decreases.

17. (a) For pseudoplastic fluids, $n < 1$.

18. (c) The capillary rise is given by

$$\Delta h = \frac{4\sigma\cos\theta}{\rho g d}$$
$$\Delta h \propto \frac{1}{d}$$

Hence,

$$\Delta h_2 = \frac{2}{0.5} \times 1.5 = 6.0 \text{ cm}$$

19. (a) Viscous torque on the hemispherical body is given by

$$T = \frac{4\pi\omega\mu R^4}{3h}$$

20. (b) Kinematic viscosity is the determined as

$$\nu = \frac{\mu}{\rho}$$

21. (c) The viscosity in a fluid is caused mainly by intermolecular force of cohesion and molecular momentum exchange.

22. (d) Since cohesion decreases with increase in temperature, the liquid viscosity does likewise. In liquids, viscosity is governed by cohesive forces, therefore, viscosity decreases with increase in temperature. In gases, the molecular activity plays a dominant role, therefore, viscosity increases with increase in temperature.

23. (d) The non-Newtonian fluids, for example, polymer solution, paints, blood, do not follow the linear relationship, and therefore, their viscosity is represented as

$$\tau = \mu\left(\frac{du}{dy}\right)^n$$

24. (c) When a liquid is confined in a closed vessel the ejected vapor molecules accumulated in the space between free liquid surface and top of the vessel exerts a partial pressure on the liquid surface which is known vapor pressure of the liquid.

25. (d) The viscous torque on cylindrical shaft of radius r and length l, rotating at speed ω (rad/s) in concentric cylinder with radial clearance h is determined as

$$T = 2\pi\omega\mu\frac{lr^3}{h}$$

26. (b) The viscous torque on inverted hemisphere of radius r, cone angle θ rotating at speed ω (rad/s) in concentric hemisphere with radial clearance h is determined as

$$T = \frac{4}{3}\pi\omega\mu\frac{r^4}{h}$$

27. (b) Henry's law is states that solubility of gas is proportional to its partial pressure.

28. (b) Due to surface tension in liquids, falling water drops of become spherical, liquid jet breaks, and soap bubbles are formed.

29. (d) Surface tension in fluids results in excess pressure, angle of contact and capillary action.

30. (c) The phenomenon of capillarity is related to angle of contact and surface tension both.

31. (c) The equilibrium equation is written as

$$\begin{aligned}\rho g\frac{\pi d^2}{4}h &= \sigma\cos\theta\pi d\\ h &= \frac{4\sigma\cos\theta}{\rho g d}\end{aligned}$$

32. (b) Considering the condition of equilibrium,

$$\begin{aligned}\sigma\cos\theta(2\pi r) &= \rho g(s_1 - s_2)(\pi r^2)h\\ h &= \frac{2\sigma\cos\theta}{r\rho g(s_1 - s_2)}\end{aligned}$$

33. (d) The ratio is given by

$$\begin{aligned}\frac{\Delta p_{\text{droplet}}}{\Delta p_{\text{bubble}}} &= \frac{4\sigma/R}{2\sigma/R}\\ &= 2\end{aligned}$$

34. (b) For a capillary of diameter d submerged partially in liquid of density ρ, surface tension σ, the capillary rise is given by

$$\begin{aligned}h &= \frac{4\sigma\cos\theta}{\rho g d}\\ h &\propto \frac{1}{\rho}\end{aligned}$$

35. (a) According the definition of bulk modulus

$$\begin{aligned}K &= \frac{\partial p}{-\partial v/v}\\ &= \rho\frac{\partial p}{\partial \rho}\end{aligned}$$

36. (c) Surface tension is caused by cohesion.

37. (b) Given that

$$\begin{aligned}\mu &= \frac{9.81}{10}\\ &= 0.981\ \text{Ns/m}\\ t &= 0.1\ \text{m}\\ v &= 2\ \text{m/s}\end{aligned}$$

Using Newton's law of viscosity,

$$\begin{aligned}\tau &= \mu\frac{v}{t}\\ &= 19.62\ \text{N/m}^2\end{aligned}$$

38. (c) Pressure differential in bubble is

$$\begin{aligned}\Delta p &= \frac{8\sigma}{d}\\ &= 40\ \text{N/m}^2\end{aligned}$$

39. (b) The equation represents a thixotropic fluid. Bingham plastic is a viscoplastic material that behaves as a rigid body at low stress but flows as a viscous fluid at high stress. It is named after Eugen C Bingham, who proposed its mathematical model.

40. (c) The pressure gauge gives reading of gauge pressure. Thus, the absolute pressure shall be

$$\begin{aligned}p &= 25 + 1.03\\ &= 26.03\ \text{bar}\end{aligned}$$

41. (a) The height of the mountain is given by

$$\begin{aligned}p &= \rho g h\\ h &= \frac{13.6\times 10^3}{1}\times(0.750 - 0.600)\\ &= 2040\ \text{m}\end{aligned}$$

42. (b) The force required on the plunger is

$$\begin{aligned}F &= \frac{1.25^2}{12.5^2}\times 1000\times 9.81\\ &= 98.1\ \text{N}\end{aligned}$$

43. (b) Pascal's law states that when a certain pressure is applied at any point in a fluid at rest the pressure is equally transmitted in all directions and to every other point in the fluid.

44. (b) Center of pressure for rectangle is

$$\bar{x}_p = \bar{x} + \frac{(d\sin\theta)^2}{12\bar{x}}$$

45. (d) Center of pressure for triangle is

$$\bar{x}_p = \bar{x} + \frac{(d\sin\theta)^2}{18\bar{x}}$$

46. (a) The phenomenon is known as Pascal's paradox because it is a consequence of the Pascal's law.

47. (c) Horizontal force F_x is the total pressure on projected area of curved surface on the vertical plane acting on center of gravity of the projection, thus

$$\begin{aligned} F_x &= \rho gh \times (r \times w) \\ &= \rho ghrw \end{aligned}$$

Vertical force F_y is the buoyancy force on the gate

$$\begin{aligned} F_y &= \rho g \times \frac{\pi r^2}{2} \times w \\ &= \frac{1}{2}\pi\rho g r^2 w \end{aligned}$$

48. (a) Taking

$$\begin{aligned} \rho_m &= 13.6 \text{ kg/m}^3 \\ h &= 0.010 \text{ mm} \\ g &= 9.81 \text{ m/s}^2 \end{aligned}$$

The balance of pressure on the both limbs of the manometer gives

$$\begin{aligned} \rho_m gh &= p + \rho_w gh \\ p &= (\rho_m - \rho_w)\, gh \\ &= 1236 \text{ Pa} \end{aligned}$$

49. (a) The net head available on the piston is H, thus the pressure will be ρgH, and net force will be ρgHA

50. (b) The pressure difference will be given by

$$\begin{aligned} \delta p &= 13.6 \times 10^3 \times 9.81 \times 0.2 \sin 30^\circ \\ &= 13.341 \text{ m of water} \end{aligned}$$

51. (a) The pressure at the interface will be the pressure exerted by oil:

$$\begin{aligned} p &= 0.8 \times 9810 \times 1 \\ &= 7848 \text{ N/m}^2 \end{aligned}$$

52. (b) The vertical component of the force on the curved surface is total weight of the liquid contained in the portion extending vertically above the curved surface upto free liquid surface.

53. (b) If the fluid is frictionless (inviscous) and incompressible, the normal stress in all directions is same.

54. (d) Given that

$$\begin{aligned} A &= \frac{\pi}{4}\left(1.4^2 - 0.6^2\right) \\ \bar{x} &= 1.85 \text{ m} \\ \rho g &= 9.81 \times 1000 \text{ N/m}^3 \end{aligned}$$

Hydrostatic force is

$$\begin{aligned} F &= \rho g \bar{x} A \\ &= 22806.07 \text{ N} \end{aligned}$$

55. (a) The effective acceleration due to gravity will be $g - g = 0$, hence, the pressure throughout the liquid mass will be atmospheric.

56. (c) Center of pressure is given by

$$\bar{x}_p = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$$

For the given case, $\theta = 90^\circ$, hence

$$\begin{aligned} \bar{x}_p &= \frac{h}{2} + \frac{bh^3/12 \times 1}{bh \times h/2} \\ &= \frac{2h}{3} \end{aligned}$$

57. (d) The high viscosity of fluid will generate head loss in the piezometer column, hence, readings will not be correct.

58. (c) The height of water column will be given by

$$\begin{aligned} 19 &= \frac{\pi 0.8^2}{4} l \\ l &= 37.79 \text{ cm} \end{aligned}$$

The difference in the mercury levels is given by

$$\begin{aligned} 37 \times 1 &= x \times 13.6 \\ x &= 2.78 \text{ cm} \end{aligned}$$

59. (c) Mercury has low vapor pressure, therefore, an excellent fluid to be used in barometers.

60. (b) Given that

$$\begin{aligned} \bar{x} &= 1.2 \times \sin 60^\circ \\ &= 1.039 \text{ m} \\ A &= 0.75 \times 2.4 \text{ m}^2 \\ \rho &= 0.85 \times 1000 \\ &= 850 \text{ kg/m}^3 \end{aligned}$$

The hydrostatic force on plane surface is given by

$$\begin{aligned} F &= \rho g(\bar{x}A) \\ &= 15.594 \text{ kN} \end{aligned}$$

61. (b) The vertical height of the column is

$$h = l \sin\theta$$

Hence, the comparative sensitivity is $1/\sin\theta$.

62. (b) Given,

$$\begin{aligned} d &= 1.2 \text{ m} \\ \theta &= 90^\circ \end{aligned}$$

Therefore,

$$\begin{aligned} \bar{x} &= \frac{d}{2} \\ &= 0.6 \text{ m} \\ I_G &= \frac{\pi d^4}{64} \\ A &= \frac{\pi d^2}{4} \end{aligned}$$

The center of pressure is given by

$$\begin{aligned} \bar{x}_p &= \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}} \\ &= \bar{x} + \frac{d^2}{16\bar{x}} \\ &= 0.75 \text{ m} \end{aligned}$$

63. (d) In case of isothermal state of atmosphere, the pressure varies with height exponentially, in following manner

$$\frac{p}{p_0} = -\exp\left(-\frac{gz}{RT_0}\right)$$

while for adiabatic state of atmosphere

$$\frac{p}{p_0} = \left\{1 - z\frac{g}{RT_0}\left(\frac{k-1}{k}\right)\right\}^{k/(k-1)}$$

64. (a) In calculation of GM, I_o is area moment of inertia of the cross-section of body at free surface level about central axis

65. (b) Given that

$$\begin{aligned} d &= 0.2 \text{ m} \\ H &= 1 \text{ m} \\ s &= 0.6 \end{aligned}$$

For stability of the cylinder,

$$\begin{aligned} \frac{D}{H} &\geq \sqrt{8s(1-s)} \\ \frac{0.2}{1} &\geq \sqrt{8\times 0.6\times 0.4} \\ 0.2 &\geq 1.38 \end{aligned}$$

which is not true, therefore, the cylinder is unstable.

66. (c) If V is the volume of the solid, the volume of the solid above the surface will be given by

$$\begin{aligned} V - V_d &= \left(1 - \frac{8.25}{13.6}\right)V \\ \frac{V - V_d}{V} &= 39.33\% \end{aligned}$$

67. (c) Archimedes principle states that when a body is immersed in a fluid either partially or totally, it is lifted up by a force equal to the weight of the fluid displaced by the body.

68. (d) Weight of the body remains the same for two liquids, hence

$$\begin{aligned} W_1 + \rho_w g s_1 v &= W_2 + \rho_w g s_2 v \\ v &= \frac{W_2 - W_1}{\rho_w g (s_1 - s_2)} \end{aligned}$$

69. (a) The force of buoyancy will be exerted by both the liquids. Total force is written as

$$F_B = \rho_w g (s_1 v_1 + s_2 v_2)$$

70. (c) If a block of ice, floating over water in a vessel, slowly melts in it, then water level in the vessel remains the same.

71. (d) Pitching, rolling, and yawing are the nomenclature given to the oscillations of a floating body about transverse, longitudinal and vertical axes, respectively.

72. (c) Given that

$$\begin{aligned} k &= 9 \text{ m} \\ \text{GM} &= 0.75 \text{ m} \end{aligned}$$

Time period of oscillations is

$$\begin{aligned} T &= 2\pi\frac{k}{\sqrt{g\times \text{GM}}} \\ &= 20.85 \text{ s} \end{aligned}$$

73. (a) A rectangular block having minimum width d and height h and specific gravity (s) will remain stable floating in water if

$$\frac{d}{h} > \sqrt{6s(1-s)}$$

74. (c) A rectangular block having minimum width d and height h and specific gravity s will remain stable floating in water if

$$\frac{d}{h} > \sqrt{8s(1-s)}$$

75. (c) An inverted cone having base diameter d and height h and specific gravity s shall remain stable floating in water if

$$\frac{d}{h} > \sqrt{4\left(\frac{1}{s^{1/3}}-1\right)}$$

76. (a) A floating body is said to be in stable equilibrium when metacenter lies above center of gravity of the body.

77. (d) The correct match is as follows:

List I (Condition)		List II (Result)	
A.	G is above M	1.	Unstable equilibrium
B.	G and M coincide	2.	Neutral equilibrium
C.	G is below M	3.	Stable equilibrium
D.	$B \geq W$	4.	Floating body

78. (d) Large metacentric height of floating body ensures stability and shortens the time of oscillation.

79. (d) Using Archimedes' principle,

$$\begin{aligned} T &= 12 \times 9.8\left(1-\frac{0.8}{2.4}\right) \\ &= 78.4 \text{ N} \end{aligned}$$

80. (d) If s is the specific gravity, the volume displaced is given by

$$\begin{aligned} V_d \rho_w g &= V \rho_w g s \\ V_d &= sV \end{aligned}$$

For the given configuration with unit cross-sectional area, the submerged volume is $2 \times (0.5-0.15) = 0.75$ liters, hence, $s = 0.75$.

81. (d) Vertical component is the total weight of the liquid contained in the portion extending vertically above the curved surface upto free liquid surface.

82. (c) Coordinate of center of pressure is determined as

$$\bar{x}_p = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$$

83. (a) In the expression, I is the area moment of inertia of the cross-section of body at free surface level about central axis.

84. (c) The x-coordinate of the center of pressure is given by

$$\bar{x}_p = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$$

The vertical distance of the center of pressure below the centroid of the plane area is

$$\bar{x}_p - \bar{x} = \frac{I_G \sin^2\theta}{A\bar{x}}$$

85. (b) Let V be the volume of the body and assume density to be 1. Thus, the weights under submerged condition will be given by

$$\begin{aligned} W_1 &= (1-s_1)\,Vg \\ W_2 &= (1-s_2)\,Vg \end{aligned}$$

From the above two equations, one gets

$$\begin{aligned} W_1 - W_2 &= (s_2 - s_1)\,Vg \\ V &= \frac{W_1 - W_2}{(s_2 - s_1)\,g} \\ &= 3.822 \text{ l} \end{aligned}$$

86. (b) The hydrostatic force is given by

$$F = \gamma \times h \times a$$

87. (c) The slope of free surface is

$$\begin{aligned} \tan\theta &= -\frac{a_x}{g} \\ a_x &= g \times \tan 45^\circ \\ &= g \end{aligned}$$

88. (b) The water surface in the tank will move up at the rear and move down in the front.

89. (b) Hydrostatic force at the bottom is

$$F_1 = \rho g h\,(2 \times 1)$$

Hydrostatic force on the larger vertical surface is

$$F_2 = \frac{1}{2}\rho g h\,(2 \times 2)$$

Therefore,

$$F_1 = F_2$$

90. (a) If a right circular cylinder open at the top is filled with liquid and rotated about its vertical axis at such speed that half the liquid spills out, the imaginary paraboloid revolution will be formed with apex at center, then pressure at the center of bottom face of the cylinder is zero.

91. (d) At any point at the free surface of rotating fluid ($p = p_a$)

$$(z - z_0) = \frac{\omega^2 r^2}{2g}$$

This is an equation of parabola.

92. (a) Using the stream function,

$$\frac{\partial \psi}{\partial x} = v$$
$$\frac{\partial \psi}{\partial y} = -u$$

For $\psi = y^2 - x^2$,

$$v = \frac{\partial \psi}{\partial x} = -2x$$
$$u = -\frac{\partial \psi}{\partial y} = -2y$$

and

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} = -2$$

Thus, the flow is irrotational.

93. (d) Given that

$$\phi = 3xy$$

Using Cauchy-Riemann equations:

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} = -\frac{\partial \phi}{\partial y} = -3x$$
$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial x} = 3y$$

Therefore,

$$\psi = -\frac{3}{2}\left(x^2 - y^2\right)$$

94. (c) The flow is irrotational because

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$
$$-4 = -4$$

95. (c) Stream lines and potential lines are orthogonal to each other.

96. (c) Stream function is defined as

$$\frac{\partial \psi}{\partial x} = v$$
$$\frac{\partial \psi}{\partial y} = -u$$

97. (d) Existence of stream function shows the possibility of flow and its continuity while potential function indicates irrotational flow only.

98. (a) Rotation vector at any point is

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k} = \frac{1}{2}\left(\nabla \times \vec{v}\right)$$

99. (b) Given that

$$A = 0.5 - 0.2x$$
$$\frac{dQ}{dt} = 0.2$$

Local acceleration is given by

$$Q = vA$$
$$\frac{dQ}{dt} = \frac{dv}{dt}A$$
$$\frac{dv}{dt} = \frac{1}{A}\frac{dQ}{dt} = 1.0$$

100. (b) Given that

$$\psi = 3xy$$

Using the definition of stream function,

$$u = -\frac{\partial \psi}{\partial y}$$
$$3x = 6$$
$$v = \frac{\partial \psi}{\partial x}$$
$$3y = 9$$

Velocity at a point (2,3)

$$v_{(2,3)} = \sqrt{u^2 + v^2} = 10.8166 \text{ unit}$$

101. (b) Given that stream function is

$$\psi = x^2 - y^2$$

Using the definition of potential function,

$$\begin{aligned} v &= \frac{\partial \psi}{\partial x} \\ &= 2x \\ u &= -\frac{\partial \psi}{\partial y} \\ &= 2y \end{aligned}$$

The correct option is

$$\phi = 2xy + \text{constant}$$

102. (d) Stream line is an imaginary curve drawn through a flowing fluid in such a way that the tangent to it at any point gives the direction of velocity of flow at that point.

103. (a) For irrotational flow:

$$\nabla^2 \phi = 0$$

For incompressible flow:

$$\nabla^2 \psi = 0$$

104. (d) Using the definition of stream function

$$\begin{aligned} u &= -\frac{\partial \psi}{\partial y} \\ &= -2x \\ v &= \frac{\partial \psi}{\partial x} \\ &= 2y \end{aligned}$$

Therefore, velocity at (2,1) is

$$\begin{aligned} v_{(2,1)} &= \sqrt{(-4)^2 + 2^2} \\ &= 4.47 \text{ unit} \end{aligned}$$

105. (c) For a steady flow, stream lines, path lines, and streak lines coincide each other.

106. (c) In a flow field, stream lines and equipotential lines are orthogonal to each other.

107. (b) The existence of stream function shows existence of a two-dimensional incompressible steady flow while the existence of potential function implies that the flow is irrotational.

108. (d) Continuity equation (conservation of mass) indicates that the efflux rate of mass through the control surface is zero.

109. (a) The equation is valid only for incompressible and steady flow in which density does not vary with time and space.

110. (d) The coefficient of vena contracta is

$$\begin{aligned} c_c &= \frac{90^2}{100^2} \\ &= 0.81 \end{aligned}$$

The coefficient of discharge (c_d) is related to coefficient of velocity (c_v) and coefficient of vena contracta (c_c) as

$$\begin{aligned} c_d &= c_v \times c_c \\ &= 0.81 \times 0.95 \\ &= 0.769 \end{aligned}$$

111. (a) Bernoulli's equation is valid for steady, inviscid, incompressible flow along a stream line.

112. (d) Navier-Stokes equations include Newton's second law to fluid motion, together with the assumption that the fluid stress is the sum of a diffusing viscous term (proportional to the gradient of velocity), plus a pressure term.

113. (a) Using Bernoulli's equation, one finds

$$\begin{aligned} \frac{u^2}{2g} &= h + \frac{v^2}{2g} \\ h &= \frac{u^2 - v^2}{2g} \end{aligned}$$

114. (b) Coefficient of vena contracta (c_c) is

$$\begin{aligned} c_c &= \left(\frac{40}{50}\right)^2 \\ &= 0.64 \end{aligned}$$

115. (c) The rise of elevation in pitot tube is equal to the velocity head.

116. (b) The height reached by stream would be equal to the velocity head.

117. (c) In fluid dynamics, the equation of motion (conservation of momentum) of an inviscid flow is known as Euler's equation.

118. (c) Water hammer in pipes takes place when flowing fluid is suddenly brought to rest by closing a valve.

119. (c) Bernoulli's equation between entrance and exit is

$$\begin{aligned} \frac{p}{\rho g} + \frac{v^2}{2g} + h_1 &= \frac{p}{\rho g} + \frac{(4v)^2}{2g} \\ h &= 15\frac{v^2}{2g} \end{aligned}$$

120. (d) Bernoulli's equation is applicable for an incompressible, inviscous, and steady stream line flow through a pipe of varying cross-section

121. (c) If a_1 is the area of pipe (constant) and a_2 is the throat area of the venturimeter, the discharge from venturi is related to area ratio as

$$Q = c_d a_1 a_2 \frac{\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$
$$h \propto \frac{1}{a_1^2}\left(\frac{a_1^2}{a_2^2} - 1\right)$$
$$\propto \left(r^2 - 1\right)$$

Thus, for the given data,

$$\frac{h}{5h} = \frac{2^2 - 1}{r^2 - 1}$$
$$r = 4$$

122. (b) Rotameter consists of a conical scale made of tapering tube. Venturimeter uses convergent-divergent tube. An orifice meter consists of an orifice plate to suddenly contract the area of flow in a pipe where vena contracta is formed.

123. (d) The velocity is given by

$$v = \sqrt{\frac{2p}{\rho}}$$
$$= 25.1667 \text{ m/s}$$

124. (c) In forced vortex motion, whole mass rotates with constant angular velocity and expenditure of energy from external source occurs. Thus, linear velocity is directly proportional to the radius.

125. (b) Fanning friction coefficient $(4f)$ is used to express h_f by its definition as

$$h_f = \frac{4f \times l}{D} \times \frac{\bar{v}^2}{2g}$$
$$4f = 0.024$$

126. (d) The head loss is related to discharge and diameter as

$$h_f \propto \frac{\dot{Q}^2}{D^5}$$
$$h_2 = \frac{2^2}{2^5} H$$
$$= \frac{H}{8}$$

127. (a) Reynolds number for rectangular ducts is given by

$$\text{Re} = \frac{\rho v D}{\mu} = \frac{vD}{\nu}$$

where

$$D = \frac{4A}{p}$$
$$= \frac{4 \times 0.3 \times 0.4}{2\,(0.3 + 0.4)}$$
$$= 0.3428 \text{ m}$$
$$\text{Re} = \frac{8.5 \times 0.3428}{16.95 \times 10^{-6}}$$
$$= 1.72 \times 10^5$$

128. (d) For laminar flow in a circular pipe (Re = 2000),

$$\text{Re} = \frac{16}{f}$$
$$f = 0.008$$

129. (d) The head loss is

$$h_f = \frac{32\dot{Q}^2 fl}{\pi g D^5}$$

For parallel combination of two pipes with discharge $\dot{Q}$ in each pipe, the head loss shall be given by

$$h_f = \frac{32\dot{Q}^2 fl}{\pi g D^5}$$

Instead of two pipes, if a single pipe of same diameter is used, then for the same discharge, the head loss is given by

$$h_f = \frac{32\left(2 \times \dot{Q}\right)^2 fl_e}{\pi g D^5}$$

Therefore,

$$\frac{32\left(2 \times \dot{Q}\right)^2 fl_e}{\pi g D^5} = \frac{32\dot{Q}^2 fl}{\pi g D^5}$$
$$l_e = \frac{l}{4}$$

130. (d) In pipe flow, the transition from laminar to turbulent flow is decided based on the value of Reynolds number, which is expressed as

$$\text{Re} = \frac{\rho v D}{\mu}$$

Re is independent of the length of the pipe.

131. (b) The Reynolds number is defined as

$$\text{Re} = \frac{\rho v D}{\mu}$$

Reynolds number for the second case will be

$$\text{Re}_2 = \frac{v_2}{v_1} \times \frac{D_2}{D_1} \times \text{Re}_1$$
$$= 0.6 \times 1.2 \times 2500$$
$$= 1800$$

132. (c) Using Hagen-Poiseuille equation,

$$
\begin{aligned}
-\frac{\partial h}{\partial x} &= \frac{32\mu V}{\rho g D^2} \\
&\propto \frac{1}{D^2} \\
&\propto \frac{1}{A}
\end{aligned}
$$

133. (b) The average velocity occurs at radius

$$
\begin{aligned}
R_m &= \frac{R}{\sqrt{2}} \\
&= 0.707R
\end{aligned}
$$

Therefore, distance from the boundary is

$$
\begin{aligned}
x &= R - R_m \\
&= 0.293R
\end{aligned}
$$

134. (a) In series connection, the discharge will be same in both the pipes. The head loss is related to diameter as

$$
\begin{aligned}
h_f &\propto \frac{1}{D^5} \\
\frac{h_2}{h_1} &= \frac{1}{2^5} \\
&= \frac{1}{32}
\end{aligned}
$$

135. (d) Minor losses in a piping system are directly calculated by using loss coefficients.

136. (c) For laminar flow through pipes:

$$v_{\max} = 2\bar{v}$$

137. (a) Discharge in laminar pipe flow is expressed as

$$
\begin{aligned}
Q &= -\frac{\pi R^4}{8\mu}\rho g \frac{\partial h}{\partial x} \\
&= -\frac{\pi D^4}{128\mu}\frac{\partial p}{\partial x}
\end{aligned}
$$

138. (d) Head loss due to friction is given by

$$
\begin{aligned}
h_f &= \frac{4fl}{D}\frac{\left(4\dot{Q}\right)^2}{2g\left(\pi D^2\right)^2} \\
&= \frac{32\dot{Q}^2}{\pi g} \times \frac{fl}{D^5}
\end{aligned}
$$

For constant Q (series connection) and f

$$h_f \propto \frac{l}{D^5}$$

For the given case,

$$
\begin{aligned}
\frac{h_{f1}}{h_{f2}} &= \left(\frac{2}{1}\right)^5 \times \frac{1}{32} \\
&= 1
\end{aligned}
$$

139. (a) For constant head loss due to friction and other parameters, the discharge is given by

$$
\begin{aligned}
h_f &= \frac{32\dot{Q}^2}{\pi g} \times \frac{fl}{D^5} \\
Q &\propto D^{5/2}
\end{aligned}
$$

For the given case

$$
\begin{aligned}
\frac{Q_2}{Q_1} &= 0.8^{5/2} \\
&= 0.572
\end{aligned}
$$

Hence, the percentage reduction is

$$
\begin{aligned}
&= 1 - 0.572 \\
&= 42.756\%
\end{aligned}
$$

140. (c) Given that

$$
\begin{aligned}
\mu &= 0.1 \text{ Nm/s}^2 \\
V &= 0.318 \text{ m/s} \\
l &= 30000 \text{ m} \\
d &= 0.2 \text{ m}
\end{aligned}
$$

Hence, the power required to overcome the head gradient is

$$
\begin{aligned}
P &= 8\pi\mu\bar{V}^2 l \\
&= 7.624 \text{ kW}
\end{aligned}
$$

141. (b) Power transmitted is maximum when the loss of head due to friction is one-third of the total head supplied.

142. (b) Shear stress at radius r is

$$
\begin{aligned}
\tau_r &= -\frac{\partial p}{\partial x}\frac{r}{2} \\
&= \frac{50 \times 10^3}{10} \times \frac{0.1}{4} \\
&= 0.125 \text{ kPa}
\end{aligned}
$$

143. (c) The equivalent length is defined as the length of an imaginary pipe of standard diameter D with same head loss h_f, same discharge $\dot{Q}$ and same friction factor f of the referred pipe. In the given case, let the length of each pipe is L. For the series combination, equivalent is $L_1 = 2L$ and for parallel combination it is $L_2 = L$. Hence, $L_1 : L_2 = 2 : 1$.

144. (c) According to Hagen-Poiseuille equation

$$h_f \propto \frac{1}{D^2}$$

The power consumed to meet this head loss at constant discharge is

$$\begin{aligned} P &= \rho g Q h_f \\ &\propto \frac{1}{D^2} \end{aligned}$$

145. (c) Following is the Hagen-Poiseuille equation:

$$-\frac{\partial h}{\partial x} = \frac{32\mu v}{\rho g D^2}$$

The above equation can be used to determine μ in terms of flow rate (Q) and pressure drop dh.

146. (b) The terminal velocity (v), using Stokes law, is expressed as

$$v = \frac{D^2}{18\mu}(\rho - \rho_w)\, g$$

Thus, inversely proportional to viscosity (μ).

147. (c) Given that

$$\begin{aligned} v &= 20 \text{ m/s} \\ \text{Re} &= 2.5 \times 10^5 \\ \nu &= 15 \times 10^{-2} \times 10^{-4} \text{ m}^2/\text{s} \end{aligned}$$

Reynolds number at distance x is given by

$$\begin{aligned} \text{Re} &= \frac{vx}{\nu} \\ x &= \frac{\text{Re} \times \nu}{v} \\ &= 0.1875 \text{ m} \end{aligned}$$

148. (a) Boundary layer is generated only due to viscosity and it results in rotational flow.

149. (b) In laminar boundary layer,

$$\begin{aligned} \frac{\tau_0}{\rho {v_\infty}^2/2} &= \frac{0.644}{\sqrt{\text{Re}_x}} \\ &= C_f \end{aligned}$$

where C_f is called local drag coefficient.

150. (b) The laminar boundary layer follows parabolic profile, therefore,

$$\delta \propto \sqrt{x}$$

Thickness of boundary layer at twice the original distance distance will be given by

$$\begin{aligned} \delta_2 &= 5\sqrt{\frac{2x}{x}} \\ &= 5\sqrt{2} \text{ mm} \end{aligned}$$

151. (d) Displacement thickness is determined as

$$\begin{aligned} \delta^* &= \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy \\ &= \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{1/9}\right) dy \\ &= \left[y - \left(\frac{9y^{10/9}}{10\delta^{1/9}}\right)\right]_0^\delta \\ &= \delta - \frac{9}{10}\delta \\ &= 0.1 \times \delta \end{aligned}$$

152. (c) For a laminar boundary layer,

$$(u_\infty - u) = (\delta - y)^2$$

In turbulent boundary layer,

$$\begin{aligned} \frac{u}{u_\infty} &= \left(\frac{y}{\delta}\right)^{1/n} \\ u &\propto \log(y) \end{aligned}$$

153. (c) In turbulent zone, in immediate vicinity of boundary, there also exists a laminar sub-layer with linear velocity distribution and extreme velocity gradient.

154. (c) If $\nu(= \mu/\rho)$ is kinematic viscosity, the expression for normal boundary layer thickness is

$$\begin{aligned} \frac{\delta}{x} &= \frac{5}{\sqrt{\text{Re}_x}} \\ &= 5\sqrt{\frac{x\nu}{u_\infty}} \\ \delta &\propto \sqrt{x} \end{aligned}$$

155. (b) Given

$$\begin{aligned} \text{Re}_x &= 5 \times 10^5 \\ U_\infty &= 1 \text{ m/s} \\ \nu &= 0.858 \times 10^{-6} \text{ m}^2/\text{s} \end{aligned}$$

Using

$$\begin{aligned} \text{Re}_x &= \frac{xU_\infty}{\nu} \\ x &= 0.43 \text{ m} \end{aligned}$$

156. (b) A shape factor is used in boundary layer flow to determine the nature of the flow

$$H = \frac{\delta^*}{\theta}$$

where H is the shape factor, δ is the displacement thickness and θ is the momentum thickness.

157. (b) In laminar boundary layer, shear stress at $y = 0$ is given by the Blasius law, written as

$$\frac{\tau_0}{\rho v_\infty{}^2/2} = \frac{0.644}{\sqrt{\mathrm{Re}_x}} = C_f$$

158. (c) When a boundary layer flow occurs against adverse pressure gradient

$$\frac{dp}{dx} > 0$$

the momentum of fluid particles gets reduced and energy is dissipated as heat by formation of eddies. Thus, the flow starts separating from the boundary. This phenomenon is called flow separation.

Numerical Answer Questions

1. Depth of center of gravity of the plate from water datum is

$$\bar{x} = 0.7 + 0.6 \times \sin 45^\circ = 1.1243 \text{ m}$$

Area of the rectangular plate is

$$A = 0.6 \times 1.2 = 0.72 \text{ m}^2$$

Total pressure on the plate is

$$F = \rho g \bar{h} \times A = 7.941 \text{ kN}$$

The moment of inertia of the rectangular plate about the axis passing through center of gravity is

$$I_G = \frac{0.6 \times 1.2^3}{12} = 0.0864 \text{ m}^4$$

The depth of the center of pressure will be given by

$$\bar{x}_p = \bar{x} + \frac{I_G \sin^2 \theta}{A \bar{x}} = 1.178 \text{ m}$$

2. Depth of center of gravity of the plate from water datum is

$$\bar{x} = 1.5 + \frac{0.75}{2} \times \sin 30^\circ = 1.6875 \text{ m}$$

Area of the rectangular plate is

$$A = \frac{\pi d^2}{4} = 0.4418 \text{ m}^2$$

Total pressure on the plate is

$$F = s \rho g \bar{h} \times A = 5.850 \text{ kN}$$

The moment of inertia of the circular plate about the axis passing through center of gravity is

$$I_G = \frac{\pi d^4}{64} = 0.01553 \text{ m}^4$$

The depth of the center of pressure will be given by

$$\bar{x}_p = \bar{x} + \frac{I_G \sin^2 \theta}{A \bar{x}} = 1.693 \text{ m}$$

3. Horizontal component of the force is given by

$$F_H = \rho g \bar{h} A$$

where

$$\bar{h} = 2.0 + \frac{1.5}{2} = 2.75 \text{ m}$$
$$A = 1.8 \times 1.5 = 2.7 \text{ m}^2$$

Therefore

$$F_H = 72.839 \text{ kN}$$

The vertical component of the force is equal to the weight of the water above the door, given by

$$F_V = \rho g \times 1.8 \times \left(3.5 \times 1.5 - \frac{\pi \times 1.5^2}{4 \times 4}\right) = 84.90 \text{ kN}$$

Hence, net hydrostatic force is

$$F = \sqrt{F_H^2 + F_V^2} = 111.86 \text{ kN}$$

The inclination of this force is

$$\theta = \tan^{-1} \frac{F_V}{F_H} = 49.37^\circ$$

4. Given that $k = 0.08$ m. BG is the distance between center of gravity (G) of the body and center of buoyancy force (B). Metacentric height GM is given by

$$\mathrm{GM} = \frac{I_O}{\forall} - \mathrm{BG}$$

where

$$\begin{aligned} I_o &= \frac{0.4 \times 0.2^3}{12} \\ \forall &= 0.2 \times 0.4 \times 0.12 \\ \mathrm{BG} &= \frac{0.15 - 0.12}{2} \end{aligned}$$

Hence

$$\mathrm{GM} = 1.27 \text{ cm}$$

Therefore, time period of oscillations is given by

$$\begin{aligned} T &= 2\pi \frac{k}{\sqrt{g \times \mathrm{GM}}} \\ &= 1.424 \text{ s} \end{aligned}$$

5. Given that $a_x = 3.0$ m/s^2. The inclination of the free surface will be given by

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{a_x}{g}\right) \\ &= 17^\circ \end{aligned}$$

Given that $s = 0.9$, the depth of the oil at the front and rear edges will be

$$\begin{aligned} h_f &= 0.9 - \frac{5}{2}\tan\theta \\ &= 0.136 \text{ m} \\ h_r &= 0.9 + \frac{5}{2}\tan\theta \\ &= 1.664 \text{ m} \end{aligned}$$

Therefore, the pressure at the front and read edges will be

$$\begin{aligned} p_f &= s\rho g h_f = 1.2 \text{ kPa} \\ p_r &= s\rho g h_r = 14.64 \text{ kPa} \end{aligned}$$

6. The local acceleration is given by

$$\frac{dv}{dt} = 2\left(1 - \frac{x}{2L}\right)^2$$

Hence, putting, $x = 0.3$ m, $L = 0.8$ m

$$\frac{dv}{dt} = 0.9453 \text{ m/s}^2$$

The convective acceleration is given by

$$v\frac{dv}{dx} = -\frac{4t^2}{L}\left(1 - \frac{x}{2L}\right)^3$$

Hence, at putting, $t = 3$ s, $x = 0.3$ m, $L = 0.8$ m

$$v\frac{dv}{dx} = -14.623 \text{ m/s}^2$$

7. The flow velocities at A and B are determined as

$$\begin{aligned} v_A &= \frac{4 \times 50 \times 10^{-3}}{\pi \times 0.15^2} \\ &= 2.829 \text{ m/s} \\ v_B &= \frac{4 \times 50 \times 10^{-3}}{\pi \times 0.30^2} \\ &= 0.7074 \text{ m/s} \end{aligned}$$

When the flow is from A to B, the pressure at B is given by Bernoulli's equation as

$$\begin{aligned} \frac{p_B}{\rho g} &= \frac{p_A}{\rho g} + \frac{v_A^2 - v_B^2}{2g} + (z_A - z_B) - h_f \\ p_B &= -54.54 \text{ kPa (gauge)} \end{aligned}$$

When the flow is from B to A, the pressure at B is given by Bernoulli's equation as

$$\begin{aligned} \frac{p_B}{\rho g} &= \frac{p_A}{\rho g} + \frac{v_A^2 - v_B^2}{2g} + (z_A - z_B) + h_f \\ p_B &= -15.3 \text{ kPa (gauge)} \end{aligned}$$

8. Given that discharge is $Q = 0.06$ m^3/s. At section 1, the diameter of pipe is $d_1 = 0.15$ m, therefore, velocity of flow is

$$\begin{aligned} v_1 &= \frac{4Q}{\pi d_1^2} \\ &= 3.395 \text{ m/s} \end{aligned}$$

At section 2, the diameter of pipe is $d_2 = 0.05$ m, therefore, velocity of flow is

$$\begin{aligned} v_2 &= \frac{4Q}{\pi d_2^2} \\ &= 30.56 \text{ m/s} \end{aligned}$$

The gauge pressure at 2 is $p_2 = 0$. Using Bernoulli's equation,

$$\begin{aligned} \frac{v_2^2 - v_1^2}{2g} &= \frac{p_1}{\rho g} \\ p_1 &= 461.19 \text{ kPa} \end{aligned}$$

Considering the bend as free body, the momentum equation in x-direction is

$$\begin{aligned} -p_1 A_1 + R_x &= \rho Q\left[v_2 - (-v_1)\right] \\ R_x &= 10.19 \text{ kN} \end{aligned}$$

9. The cross-sectional area of the jet is

$$a = \frac{\pi \times 0.012^2}{4} = 1.13 \times 10^{-4}\ \mathrm{m}^2$$

The discharge from each nozzle is

$$q = \frac{Q}{2} = 7.5 \times 10^{-4}\ \mathrm{m}^3/s$$

Jet velocities are

$$v_1 = v_2 = \frac{q}{a} = 6.63\ \mathrm{m/s}$$

The radius of arms are

$$r = 0.25\ \mathrm{m}$$
$$R = 0.45\ \mathrm{m}$$

The inclination angle is

$$\beta = 180 - 60 = 120°$$

The moment of momentum is

$$T = -\rho q r\,(u_1 + v_1 \cos\beta) + \rho q R\,(u_2 + v_2 \cos\beta) = -\rho q r\,(r\omega + v_1 \cos\beta) + \rho q R\,(R\omega + v_2 \cos\beta)$$

When friction is zero, $T = 0$, hence

$$\rho q r\,(r\omega + v_1 \cos\beta) = \rho q R\,(R\omega + v_2 \cos\beta)$$

Therefore,

$$\omega = \frac{(v_2 R - v_1 r)\cos\beta}{-(R^2 - r^2)} = -\frac{v_2\,(R - r)\cos\beta}{(R^2 - r^2)} = -\frac{v_2 \cos\beta}{(R + r)}$$

Substituting the values,

$$\omega = 7.435\ \mathrm{rad/s} = 70.85\ \mathrm{rpm}$$

Torque required to hold the sprinkler stationary ($\omega = 0$) is given by

$$T = -\rho q v_2 \cos\beta\,(R + r) = 1.47\ \mathrm{kN}$$

CHAPTER 7

HEAT TRANSFER

Heat is the form of energy that can be transferred from one system to another by virtue of temperature difference. Thermodynamics is concerned with the amount, not the rate, of heat transfer on a system, and it does not tell about how long the process can take or what is the mode of heat transfer. *Heat transfer* is a separate science that deals with the rates of such energy transfers. There is no area of engineering which does not involve heat transfer phenomenon, except adiabatic processes which are hypothetical!

7.1 MODES OF HEAT TRANSFER

Conduction, convection, and radiation are the three modes of heat transfer. The mechanisms of heat transfer in these modes are governed by certain physical rules, discussed as follows:

1. *Conduction* In *conduction*, heat transfer takes place due to temperature difference within a body or between bodies in thermal contact, without flow or mixing of mass. The rate of heat transfer through conduction is governed by the *Fourier's law* of heat conduction.
2. *Convection* In *convection*, heat is transferred to a flowing fluid at the surface over which it flows by combined molecular diffusion and bulk flow. Thus, convection involves conduction and fluid motion. The rate of convective heat transfer is governed by the *Newton's law of cooling*.
3. *Radiation* Thermal *radiation* is in the limited wavelength range of 0.1–10 μm of electromagnetic spectrum (*photons*). It is emitted by all surfaces as a result of the changes in the electronic configurations of the atoms or molecules, irrespective of the temperature. No medium is required for radiative heat transfer but the source should be in visual contact for direct radiation. The maximum rate of heat radiation that can be emitted by a surface at a thermodynamic temperature is based on the *Stefan-Boltzmann law*.

7.2 CONDUCTION

7.2.1 Fourier's Law

According to the *Fourier's law*[1], the rate of heat conducted ($\dot{Q}$) through a body is proportional to the cross-sectional area (A) and *temperature gradient*

[1] Joseph Fourier (1768-1830) was a French mathematician and physicist. Fourier series, Fourier transform, and Fourier's law were named in his honor.

$(\partial T/\partial x)$ in the body:

$$\dot{Q}_x \propto -A\frac{\partial T}{\partial x}$$
$$\dot{Q}_x = -\kappa A\frac{\partial T}{\partial x} \tag{7.1}$$

where the constant of proportionality κ is called *thermal conductivity* of the body material.

7.2.2 Fourier's Equation

The *Fourier's law of conduction* [Eq. (7.1)] can be used to derive general equations of heat conduction in three coordinate systems: rectilinear coordinates, cylindrical coordinates, and spherical coordinates.

7.2.2.1 Rectilinear Coordinates Consider a material element of thickness δx, width δy, height δz made of a material having specific heat c_p, density ρ, conductivities κ_x, κ_y, and κ_z in x, y and z directions, respectively. The rate of heat generation per unit volume is $\dot{q}$ [Fig. 7.1].

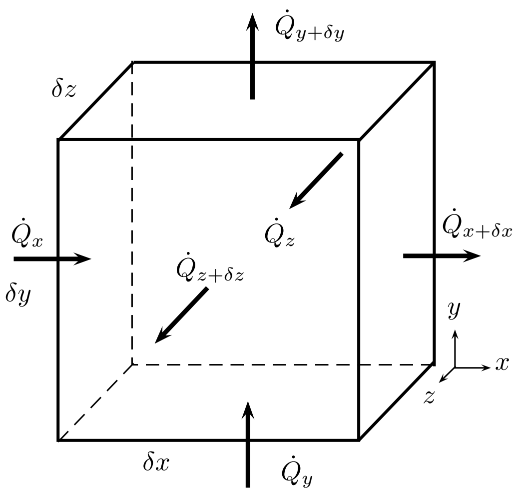


Figure 7.1 Rectilinear element for conduction.

Using the Fourier's law of conduction [Eq. (7.1)], the rate of increase in heat transfer along x-direction can be written as

$$\delta\dot{Q}_x = \frac{\partial\dot{Q}}{\partial x}\delta x$$

where

$$\dot{Q}_x = -\kappa_x \delta y \delta z\frac{\partial T}{\partial x}$$

Similarly, expressions for $\delta\dot{Q}_y$ and $\delta\dot{Q}_z$ can be found. The rate of heat generation and rate of heat stored in the element must be equal to the rate of increase in its heat capacity:

$$c_p\rho\left(\delta x\delta y\delta z\right)\frac{\partial T}{\partial\tau} = \delta\dot{Q}_x + \delta\dot{Q}_y + \delta\dot{Q}_z + \dot{q}\left(\delta x\delta y\delta z\right)$$

$$\kappa_x\frac{\partial^2 T}{\partial x^2} + \kappa_y\frac{\partial^2 T}{\partial y^2} + \kappa_z\frac{\partial^2 T}{\partial z^2} + \dot{q} = c_p\rho\frac{\partial T}{\partial\tau} \tag{7.2}$$

This is the *general equation of heat conduction*, also known as *Fourier's equation* in rectilinear coordinates. For isotropic thermal conductivity ($\kappa_x = \kappa_y = \kappa_z = \kappa$), this equation takes following form:

$$\sum\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\kappa} = \frac{1}{\alpha}\frac{\partial T}{\partial\tau} \tag{7.3}$$
$$\nabla^2 T + \frac{\dot{q}}{\kappa} = \frac{1}{\alpha}\frac{\partial T}{\partial\tau} \tag{7.4}$$

where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$
$$\alpha = \frac{\kappa}{\rho c_p} \tag{7.5}$$

Here, α is called *thermal diffusivity*, a material property relevant in a transient heat transfer only where it represents how fast heat diffuses through a material. It can also be viewed as the ratio of heat conducted through the material to the heat stored per unit volume. Thus, the substances with high thermal diffusivity rapidly adjust their temperature to that of their surroundings because they conduct heat quickly in comparison to their volumetric heat capacity.

7.2.2.2 Cylindrical Coordinates Consider an element of a cylindrical body, having angular width $\delta\theta$ at radius r, thickness δr and depth δz [Fig. 7.2].

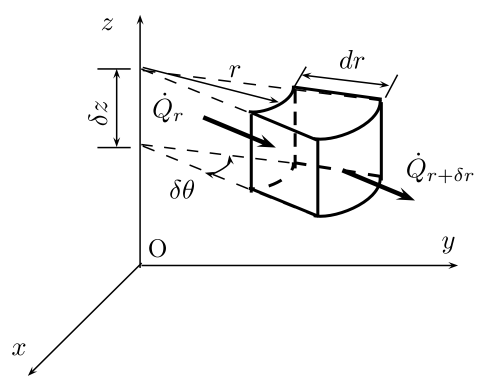


Figure 7.2 Cylindrical element for conduction.

Using Fourier's law [Eq. (7.1)] for isotropic thermal conductivity κ, the rate of of increase in heat conduction in radial direction is written as

$$\delta\dot{Q}_r = \frac{\partial\dot{Q}}{\partial r}\delta r$$

where

$$\dot{Q}_r = -\kappa r\delta\theta\delta z\frac{\partial T}{\partial r}$$

For heat balance,

$$\delta\dot{Q}_r + \dot{q}\,(r\delta\theta\delta r\delta z) = c_p\rho\,(r\delta\theta\delta r\delta z)\,\frac{\partial T}{\partial\tau}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{\kappa} = \frac{1}{\alpha}\frac{\partial T}{\partial\tau} \tag{7.6}$$

This is the *general equation of heat conduction in cylindrical coordinates.*

7.2.2.3 Spherical Coordinates Consider an element of a spherical body, having angular width $\delta\theta$ at radius r and thickness δr. The rate of increase in heat conduction in r (radial) direction is

$$\delta\dot{Q}_r = \frac{\partial\dot{Q}}{\partial r}\delta r$$

where

$$\dot{Q}_r = -\kappa\,(r\delta\theta)^2\,\frac{\partial T}{\partial r}$$

For heat balance,

$$\delta\dot{Q}_r + \dot{q}\left\{(r\delta\theta)^2\,\delta r\right\} = c_p\rho\left\{(r\delta\theta)^2\,\delta r\right\}\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{\kappa} = \frac{1}{\alpha}\frac{\partial T}{\partial\tau} \tag{7.7}$$

This is *general equation of heat conduction in spherical coordinates.*

7.2.3 Thermal Resistance

A heat transfer problem can be analogized with electrical circuit problem, in which temperature difference (ΔT) presents the potential difference and rate of heat conducted ($\dot{Q}$) is analogous to current. *Thermal resistance* (R_{th}) to heat flow is analogous to electrical resistance to current. Thus,

$$\dot{Q} = \frac{\Delta T}{R_{th}}$$

Therefore,

$$R_{th} = \frac{\Delta T}{\dot{Q}}$$

Using this definition, the expressions for thermal resistance can be derived for different shapes of bodies.

7.2.3.1 Rectangular Wall Consider a plane wall of the material of thermal conductivity κ, thickness t, cross-sectional area A, without any heat generation ($\dot{q} = 0$) [Fig. 7.3]. Applying the Fourier's law [Eq. (7.1)] for this case,

$$\begin{aligned}\dot{Q} &= \kappa A\frac{\Delta T}{t}\\ &= \frac{\Delta T}{t/\,(\kappa A)}\end{aligned}$$

Thus, term $t/\,(\kappa A)$ represents thermal resistance R_{th} to the heat conduction in the plane wall.

$$R_{th} = \frac{t}{\kappa A} \tag{7.8}$$

This is the expression of thermal resistance offered by a rectangular wall.

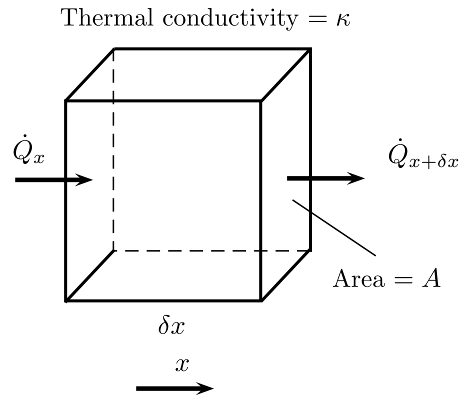


Figure 7.3 Thermal resistance.

7.2.3.2 Cylindrical Element Consider radial steady conduction in a hollow cylindrical body with inner radius r_i, outer radius r_o, length l, made of a material with thermal conductivity κ, with no heat generation. Temperatures at radii r_i and r_o are T_i and T_o, respectively. Using Fourier's equation in cylindrical coordinates [Eq. (7.6)],

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

Integration of both sides of the above equation gives

$$\begin{aligned}r\frac{\partial T}{\partial r} &= C_1\\ \frac{\partial T}{\partial r} &= \frac{C_1}{r}\end{aligned}$$

Again, integration of both sides of the above equation gives

$$T = C_1\ln(r) + C_2$$

Therefore, radial temperature distribution is logarithmic. Applying for $r = r_i$ and $r = r_o$,

$$\begin{aligned}T_i &= C_1\ln(r_i) + C_2\\ T_o &= C_1\ln(r_o) + C_2\end{aligned}$$

Therefore,

$$\begin{aligned}r\frac{\partial T}{\partial r} &= C_1\\ &= \frac{(T_i - T_o)}{\ln(r_o/r_i)}\end{aligned}$$

The rate of heat conduction is

$$\begin{aligned}\dot{Q}_r &= -2\pi r l \kappa \frac{\partial T}{\partial r} \\ &= -2\pi l \kappa \left(r \frac{\partial T}{\partial r} \right) \\ &= -2\pi l \kappa \frac{(T_i - T_o)}{\ln(r_o/r_i)}\end{aligned}$$

Therefore,

$$R_{th} = \frac{\ln(r_o/r_i)}{2\pi\kappa l} \tag{7.9}$$

$$= \frac{\ln(A_o/A_i)}{2\pi\kappa l} \tag{7.10}$$

This is the expression of thermal resistance offered by a cylindrical body.

7.2.3.3 Spherical Element Consider radial steady conduction in a hollow spherical body with inner radius r_i, outer radius r_o, length l, made of a material with thermal conductivity κ, with no heat generation ($\dot{q} = 0$). Temperatures at r_i and r_o are T_i and T_o. Using Fourier's equation in spherical coordinates [Eq. (7.7)],

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = 0$$

Integration of both sides of the above equation gives

$$\begin{aligned}r^2 \frac{\partial T}{\partial r} &= C_1 \\ \frac{\partial T}{\partial r} &= \frac{C_1}{r^2}\end{aligned}$$

Again, integration of both sides of the above equation gives

$$T = -\frac{C_1}{r} + C_2$$

Applying for $r = r_i$ and $r = r_o$

$$\begin{aligned}T_i &= -\frac{C_1}{r_i} + C_2 \\ T_o &= -\frac{C_1}{r_o} + C_2\end{aligned}$$

Therefore,

$$\begin{aligned}r^2 \frac{\partial T}{\partial r} &= C_1 \\ &= \frac{(T_i - T_o)\, r_o r_i}{r_o - r_i}\end{aligned}$$

The rate of heat conduction is

$$\begin{aligned}\dot{Q}_r &= -4\pi r^2 \kappa \frac{\partial T}{\partial r} \\ &= -4\pi\kappa \left(r^2 \frac{\partial T}{\partial r} \right) \\ &= 4\pi\kappa \frac{(T_i - T_o)\, r_o r_i}{r_o - r_i}\end{aligned}$$

Hence, thermal resistance is given by

$$\begin{aligned}R_{th} &= \frac{(r_o - r_i)}{(4\pi r_o r_i)\,\kappa} \\ &= \frac{(A_o - A_i)}{\kappa\sqrt{A_o A_i}}\end{aligned}$$

This is the expression of thermal resistance offered by a spherical body.

7.2.4 Variable Thermal Conductivity

Thermal conductivity (κ) of a material depends upon its temperature (T). The following linear relation is generally considered to represent variable thermal conductivity,

$$\kappa = \kappa_o (1 + \beta T)$$

where κ_o is the thermal conductivity at zero temperature and β is a constant of linearity. To examine the effect of β on heat conduction, consider heat transfer through a wall of cross-sectional area A and thickness t [Fig. 7.4]. The heat conduction through an element of thickness dx at distance x from the inner side is

$$\begin{aligned}\dot{Q} &= -\kappa_o (1 + \beta T) A \frac{\partial T}{\partial x} \\ -\frac{\dot{Q}}{\kappa_o A} \int \partial x &= \int (1 + \beta T)\, \partial T \\ -\frac{\dot{Q}}{\kappa_o A} x &= T + \frac{\beta}{2} T^2 + C\end{aligned}$$

This equation indicates a parabolic temperature distribution in which the temperature gradient at any point depends upon β [Fig. 7.4]. For $\beta = 0$, the temperature profile is linear.

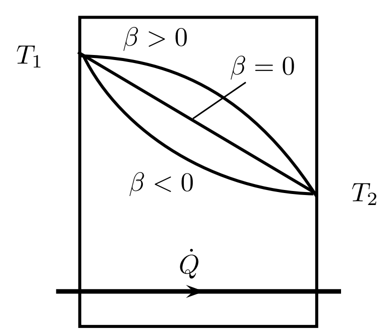


Figure 7.4 Temperature gradient.

Let T_1 and T_2 show the temperatures at x_1 and x_2 locations, then

$$-\frac{\dot{Q}}{\kappa_o A}(x_1 - x_2) = (T_1 - T_2) + \frac{\beta}{2}\left(T_1{}^2 - T_2{}^2\right)$$

$$\dot{Q} = -\kappa_o\left\{1 + \beta\frac{(T_1 + T_2)}{2}\right\} \times A\frac{(T_1 - T_2)}{(x_1 - x_2)}$$

$$= -\bar{\kappa}A\frac{(T_1 - T_2)}{(x_1 - x_2)} \tag{7.11}$$

where $\bar{\kappa}$ is arithmetic average of thermal conductivities between T_1 and T_2:

$$\bar{\kappa} = \kappa_o\left\{1 + \beta\frac{(T_1 + T_2)}{2}\right\}$$

Pure metals have higher thermal conductivity, while the thermal conductivity of an alloy of two metals is usually much lower than that of either metals.

7.2.5 Critical Radius of Insulation

Since the purpose of insulation is to reduce the heat loss, it is always better to provide insulating material of lower conductivity on lower radius, because then heat will find lower multiple of area for heat conduction and thermal conductivity. There is also a limitation on the thickness of insulation beyond which there will be less effective insulation. In such cases, the material of insulation adds conductance for heat transfer.

7.2.5.1 Cylindrical Element Consider a cylindrical element of radius r_i, length l over which an insulation of thermal conductivity κ is laid upto outer radius r_o. Temperature of the element is T_i, the outside temperature is T_∞ and h_o is convection coefficient on the outer surface [Fig. 7.5].

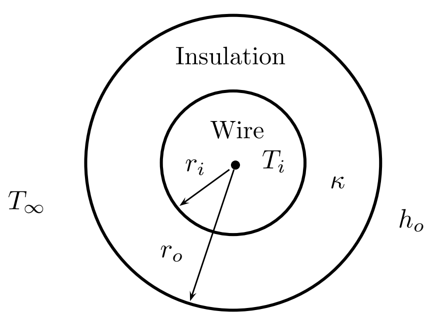


Figure 7.5 Critical thickness.

The heat transfer involves two elements of resistance in series:

1. The convective heat transfer resistance at outer layer:
$$R_1 = \frac{1}{(2\pi r_o l)\,h_o}$$
2. The conductive resistance [Eq. (7.10)] in the insulation:
$$R_2 = \frac{\ln(r_o/r_i)}{2\pi\kappa l}$$

The effective thermal resistance is given by

$$R_e = R_1 + R_2 = \frac{1}{(2\pi r_o l)\,h_o} + \frac{\ln(r_o/r_i)}{2\pi\kappa l}$$

This resistance R_e acts over the temperature potential of $(T_i - T_\infty)$. Therefore, the rate of heat transfer is written as the rate of heat transfer through conduction and convection:

$$\dot{Q} = (T_i - T_\infty)\Big/\left(\frac{1}{2\pi r_o l h_o} + \frac{\ln(r_o/r_i)}{2\pi\kappa l}\right)$$

The variation profile of $\dot{Q}$ with r_o is seen in Fig. 7.6.

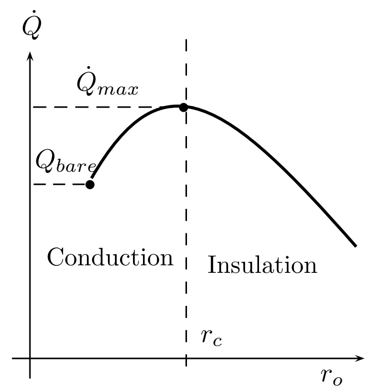


Figure 7.6 Critical radius of insulation.

Rate of heat transfer reaches its maximum value where

$$\frac{d\dot{Q}}{dr_o} = 0$$

$$r_o = \frac{\kappa}{h_o} \tag{7.12}$$

This value of outer radius r_o is known as the *critical radius* of insulation for cylinders. It depends on the thermal conductivity of insulation (κ) and the external convection heat transfer coefficient (h_o). For perfect conduction, radius of the insulation layer should be within r_c; while for perfect insulation, the layer should be more than r_c.

The radius of electrical wires is generally smaller than the critical radius. This permits use of plastic electrical insulation to actually enhance the heat transfer from electrical wire.

7.2.5.2 Spherical Element Consider a sphere of radius r_i, length l, on which an insulation of thermal conductivity κ is laid. Temperature of sphere is T_i, outer temperature is T_∞ and h_o is convection coefficient. In this case, the heat transfer involves two elements of resistance in series:

1. The convective heat transfer resistance at outer layer:
$$R_1 = \frac{1}{4\pi {r_o}^2 h_o}$$
2. The conductive resistance in the insulation:
$$R_2 = \frac{(r_o - r_i)}{(4\pi r_o r_i)\,\kappa}$$

The effective thermal resistance is given by

$$\begin{aligned} R_e &= R_1 + R_2 \\ &= \frac{1}{4\pi {r_o}^2 h_o} + \frac{r_o - r_i}{4\pi r_o r_i \kappa} \end{aligned}$$

This resistance R_e acts over the temperature potential of $(T_i - T_\infty)$. Therefore, the rate of heat transfer through conduction and convection is

$$\dot{Q} = (T_i - T_\infty) \Big/ \left(\frac{1}{4\pi {r_o}^2 h_o} + \frac{r_o - r_i}{4\pi r_o r_i \kappa} \right)$$

Similar profile of $\dot{Q}$ [Fig. 7.6] is seen for spherical elements also. For maxima or minima of $\dot{Q}$

$$\begin{aligned} \frac{d\dot{Q}}{dr_o} &= 0 \\ r_o &= \frac{2\kappa}{h_o} \end{aligned} \tag{7.13}$$

This value of outer radius r_o is called the *critical radius* of insulation for spheres.

Equations (7.12) and (7.13) indicate that for given values of κ and h_o, the critical radius of insulation for spherical body is two times that for a cylindrical body.

7.2.6 Heat Generation

Various electrical systems involve heat generation due to conversion from electrical to thermal energy that affects the temperature distribution, such as in current-carrying conductors. The rate of heat generation in passing a current I through a medium of electrical resistance R is given by

$$\dot{Q}_g = I^2 R$$

If the heat is generated uniformly in the volume (V) of the medium, then the rate of heat generation per unit volume is written as

$$\dot{q} = \frac{\dot{Q}_g}{V}$$

The effect of heat generation on conductive heat transfer has already been incorporated in the general equations in three types of coordinate systems and these can be used to derive expressions of heat transfer in rectilinear, cylindrical and spherical elements, as follows.

7.2.6.1 Rectilinear Element Consider a steady state heat conduction in a plane wall of thickness t, thermal conductivity κ and heat generation rate $\dot{q}$ [Fig. 7.7].

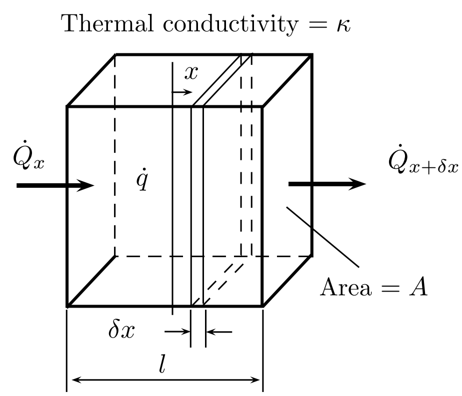


Figure 7.7 Conduction with heat generation.

Using the equation of heat conduction in rectilinear coordinates [Eq. (7.3)],

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{\kappa} = 0 \tag{7.14}$$

This equation is subjected to following constraints:

$$\begin{aligned} \frac{dT}{dx} &= 0, \text{ at } x = 0 \\ T &= T_w, \text{ at } x = \frac{t}{2} \end{aligned}$$

where T_w is the temperature at the wall. Integration of Eq. (7.14) gives

$$\frac{dT}{dx} = -\frac{\dot{q}}{\kappa} x + C_1 \tag{7.15}$$

Using the first constraint,

$$C_1 = 0$$

Integration of Eq. (7.15) gives

$$T = -\frac{\dot{q}}{\kappa}\frac{x^2}{2} + C_2$$

Using the second constraint,

$$T = T_w + \frac{\left\{ (l/2)^2 - x^2 \right\} \dot{q}}{2\kappa}$$

This is the expression of temperature profile in the wall with heat generation. Using this, the following temperatures can be determined:

1. *Temperature at Wall* The heat generated is uniformly conducted towards both sides of the wall. Therefore, for heat balance,

$$h_o(T_w - T_\infty) = \frac{\dot{q}}{2}$$
$$T_w = T_\infty + \frac{\dot{q}}{2h_o}$$

2. *Maximum Temperature* Temperature gradient is zero at the center. Therefore, temperature is maximum at $x = 0$:

$$T_{max} = T_\infty + \frac{\dot{q}}{2h_o} + \frac{\dot{q}}{\kappa}\frac{l^2}{8}$$

7.2.6.2 Cylindrical Elements Consider a steady heat conduction in a current carrying conductor of radius r_i, thermal conductivity κ, and heat generation rate $\dot{q}$ [Fig. 7.8].

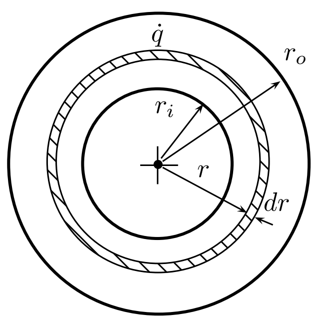


Figure 7.8 Heat conduction in a cylinder.

Using equation of heat conduction in cylindrical coordinates [Eq. (7.6)]:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}}{\kappa} = 0$$

This can be rearranged in the following form:

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{q}}{\kappa}r \tag{7.16}$$

This equation is subjected to two constraints:

$$\frac{dT}{dr} = 0, \text{ at } r = 0 \tag{7.17}$$
$$T = T_w, \text{ at } r = r_i \tag{7.18}$$

Integration of Eq. (7.16) gives

$$r\frac{dT}{dr} = -\frac{\dot{q}}{\kappa}\frac{r^2}{2} + C_1$$
$$\frac{dT}{dr} = -\frac{\dot{q}}{\kappa}\frac{r}{2} + \frac{C_1}{r} \tag{7.19}$$

Using the first constraint [Eq. (7.17)]:

$$C_1 = 0$$

Integration of Eq. (7.19) gives

$$T = -\frac{\dot{q}}{\kappa}\frac{r^2}{4} + C_2$$

Using the second constraint [Eq. (7.18)]:

$$T = T_w + \frac{\left({r_i}^2 - r^2\right)\dot{q}}{4\kappa}$$

This is the expression of temperature profile in the wall with heat generation. Using this, following temperatures can be determined:

1. *Temperature at Wall* The heat generated is uniformly conducted towards outer surface. Therefore, for heat balance,

$$h_o \times 2\pi r_i l\,(T_w - T_\infty) = \pi {r_i}^2 l\dot{q}$$
$$T_w = T_\infty + \frac{\dot{q}r_i}{2h_o}$$

2. *Maximum Temperature* Temperature gradient is zero at center. Therefore, temperature is maximum at $r = 0$:

$$T_{max} = T_\infty + \frac{\dot{q}r_i}{2h_o} + \frac{{r_i}^2\dot{q}}{4\kappa}$$

7.2.6.3 Spherical Elements Consider a steady state heat conduction in a spherical conductor of radius r_s, thermal conductivity κ, and heat generation rate $\dot{q}$. Using equation of heat conduction in spherical coordinates [Eq. (7.7)]:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{q}}{\kappa} = 0 \tag{7.20}$$

This is subjected to two constraints:

$$\frac{dT}{dr} = 0, \quad \text{at } r = 0 \tag{7.21}$$
$$T = T_w, \quad \text{at } r = r_s \tag{7.22}$$

Integration of Eq. (7.20) gives

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot{q}}{\kappa}r^2$$
$$r^2\frac{dT}{dr} = -\frac{\dot{q}}{\kappa}\frac{r^3}{3} + C_1$$
$$\frac{dT}{dr} = -\frac{\dot{q}}{\kappa}\frac{r}{3} + \frac{C_1}{r^2} \tag{7.23}$$

Using the first constraint [Eq. (7.21)]:

$$C_1 = 0$$

Integration of Eq. (7.23) gives

$$T = -\frac{\dot{q}}{\kappa}\frac{r^2}{6} + C_2$$

Using the second constraint [Eq. (7.22)]:

$$T = T_w + \frac{\left(r_s{}^2 - r^2\right)\dot{q}}{6\kappa}$$

This is the expression of temperature profile in the wall with heat generation. Using this, the following temperatures can be determined:

1. Temperature at Wall The heat generated is uniformly conducted towards the outer surface. Therefore, for heat balance,

$$h_o \times 4\pi r_s{}^2 (T_w - T_\infty) = \frac{4\pi}{3} r_s{}^3 \dot{q}$$
$$T_w = T_\infty + \frac{\dot{q} r_s}{3h_o}$$

2. Maximum Temperature Temperature gradient is zero at the center. Therefore, temperature is maximum at $r = 0$:

$$T_{max} = T_\infty + \frac{\dot{q} r_s}{3h_o} + \frac{r_s{}^2 \dot{q}}{6\kappa}$$

7.2.7 Rectangular Fins

Fins are the *extended surfaces* used to enhance the rate of heat transfer between a solid and an adjoining fluid through conduction-convection effects. The set of fins used on outer area of an engine cylinder is a typical example. The enhanced rate of heat transfer is obtained by increasing the surface area across which the convection occurs. Thermal conductivity of the fin material should be high to minimize the temperature variations from its base to free end.

Design of fins involves use of equations of heat conduction and convection. Consider a rectangular fin[2] of uniform cross-sectional area A throughout its length l. The fin is attached to a hot body at temperature T_0 from where heat is to be ejected to the atmosphere at temperature T_∞. Overall convective *heat transfer coefficient* at fin surface is h. Let a small element of fin surface of width δx be at a distance x from the base and uniform perimeter P. Temperature at this section is T [Fig. 7.9].

Heat that enters at x is $\dot{Q}_x$ and heat that exits at $x + \delta x$ is $\dot{Q}_x + \partial \dot{Q}_x/\partial x$, therefore, the equilibrium equation

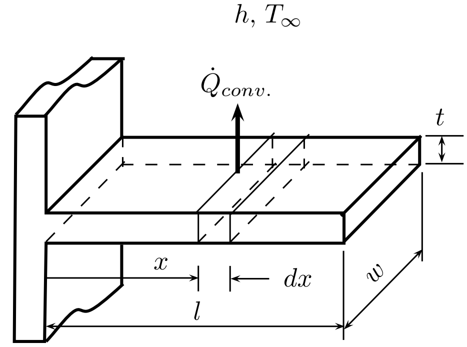


Figure 7.9 Heat transfer in a rectangular fin.

for heat balance is

$$\frac{\partial}{\partial x}\left(-\kappa A \frac{\partial T}{\partial x}\right)\delta x = hP\delta x \times (T - T_\infty)$$
$$\frac{\partial^2 T}{\partial x^2} + \frac{hP}{\kappa A}(T - T_\infty) = 0$$
$$\frac{\partial^2 \theta}{\partial x^2} + m^2\theta = 0 \tag{7.24}$$

where

$$m = \sqrt{\frac{hP}{\kappa A}}$$
$$\theta = T - T_\infty$$

The solution of Eq. (7.24) can be written as

$$\theta = C_1 e^{-mx} + C_2 e^{mx} \tag{7.25}$$

where two arbitrary constants, C_1 and C_2, need two boundary conditions for complete solution of the equation. The first boundary condition is at the base of the fin ($x = 0$) where

$$\theta = T_0 - T_\infty$$
$$= \theta_0$$

Using Eq. (7.25),

$$\theta_0 = C_1 \frac{1}{e^{m\times 0}} + C_2 e^{m\times 0}$$
$$= C_1 + C_2 \tag{7.26}$$

For complete solution of Eq. (7.25), the second boundary condition for Eq. (7.24) can be obtained by any one of the following assumptions:

1. Infinitely long fin
2. Insulated end fin
3. Convective end fin

These three assumptions are examined as follows.

[2]A fin with parabolic profile is very effective in a sense maximum heat transfer at lower material cost.

7.2.7.1 Infinitely Long Fins For a very long fin ($x \to \infty$), the temperature at free end will be equal to the ambient temperature ($\theta \to 0$). Using this feature as the second constraint, Eq. (7.25) is written as

$$\begin{aligned} 0 &= \frac{C_1}{e^{m\times\infty}} + C_2 e^{m\times\infty} \\ C_2 &= 0 \end{aligned}$$

Using Eq. (7.26),

$$C_1 = \theta_0$$

Equation (7.25) takes the following form:

$$\theta = \theta_0 e^{mx}$$

This is the expression of temperature profile. Using this, the rate of heat transfer for long fin can be determined as

$$\begin{aligned} \dot{Q} &= -\kappa A \left(\frac{\partial T}{\partial x}\right)_{x=0} \\ &= -\kappa A \times \theta_0 m e^{m\times 0} \\ &= -\kappa A m \left(T_0 - T_\infty\right) \\ &= \sqrt{hP\kappa A}\left(T_0 - T_\infty\right) \end{aligned} \tag{7.27}$$

This is the equation of heat transfer for long fins.

7.2.7.2 Insulated end fins The insulated free end of the fin will have zero temperature gradient:

$$\left(\frac{\partial T}{\partial x}\right)_l = 0, \quad x = l \tag{7.28}$$

Differentiating Eq. (7.25):

$$\begin{aligned} \frac{\partial T}{\partial x} &= \frac{d}{dx}\left(\frac{C_1}{e^{mx}} + C_2 e^{mx}\right) \\ &= m\left(\frac{-C_1}{e^{mx}} + C_2 e^{mx}\right) \end{aligned}$$

Using the second constraint [Eq. (7.28)],

$$\begin{aligned} m\left(\frac{-C_1}{e^{ml}} + C_2 e^{ml}\right) &= 0 \\ C_1 &= C_2 e^{2ml} \end{aligned}$$

Using Eq. (7.26)

$$C_2 = \frac{\theta_0}{1 + e^{2ml}}$$

Therefore, Eq. (7.25) takes the following form:

$$\begin{aligned} \theta &= \theta_0 \left[\frac{e^{2ml} e^{-mx}}{1 + e^{2ml}} + \frac{e^{mx}}{1 + e^{2ml}}\right] \\ &= \theta_0 \left[\frac{e^{m(l-x)} + e^{-m(l-x)}}{e^{ml} + e^{-ml}}\right] \\ &= \theta_0 \frac{\cosh\{m(l-x)\}}{\cosh(ml)} \end{aligned} \tag{7.29}$$

This is the expression of temperature profile. Using this, temperature gradient at any point is determined as

$$\begin{aligned} \frac{dT}{dx} &= \frac{d\theta}{dx} \\ &= \theta_0 \left[m \frac{\sinh\{m(l-x)\}}{\cosh(ml)}\right] \end{aligned}$$

Therefore, the rate of heat transfer in a fin having insulated free end is given by

$$\begin{aligned} \dot{Q} &= -\kappa A \left(\frac{\partial T}{\partial x}\right)_{x=0} \\ &= -\kappa A \times \theta_0 m \tanh\{m(l-x)\} \end{aligned} \tag{7.30}$$

This is the equation of heat transfer for insulated end fins.

7.2.7.3 Convective End Fins Let the fin have its free end (at $x = l$) where coefficient of convection is h, therefore,

$$\begin{aligned} \frac{\partial T}{\partial x} &= \frac{d}{dx}\left(\frac{C_1}{e^{mx}} + C_2 e^{mx}\right) \\ &= m\left(\frac{-C_1}{e^{mx}} + C_2 e^{mx}\right) \\ \left(\frac{\partial T}{\partial x}\right)_{x=l} &= m\left(\frac{-C_1}{e^{ml}} + C_2 e^{ml}\right) \end{aligned}$$

For heat balance,

$$hA\theta_l = -\kappa A \left(\frac{d\theta}{dx}\right)_{x=l} \tag{7.31}$$

This is the second constraint for Eq. (7.25). Using this along with Eq. (7.26), one gets

$$C_1 = \left\{\frac{(h/(m\kappa) + 1)}{(-h/(m\kappa) + 1)}\right\} e^{2ml} C_2$$

Therefore, rate of heat transfer is found using Eq. (7.25) as

$$\dot{Q} = \sqrt{hP\kappa A}\left[\frac{\tanh\{m(l-x)\} + h/(m\kappa)}{1 + h/(m\kappa)\tanh(ml)}\right]\left(T_0 - T_\infty\right)$$

This is the equation of heat transfer for convective end fins.

An important application of this derivation is found in the estimation of error in temperature measurement. Temperature of hot fluid flowing through a pipe is measured by inserting a thermometer in a glass in touch with the fluid. This portion acts like a fin due to which there comes an error in measurement [Fig. 7.10].

This case can be taken as insulated fin [Eq. (7.29)]:

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh(ml)}$$

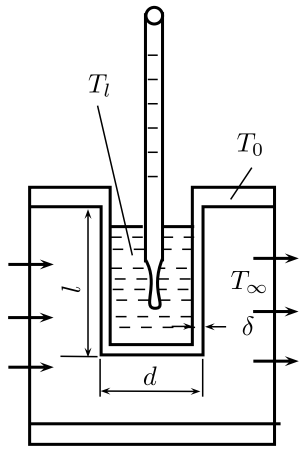


Figure 7.10 Error in temperature measurement.

To reduce temperature error, ml should be large.

The design of fin can be assessed in the following terms:

1. Fin Efficiency *Fin efficiency* is defined as the ratio of the rate of heat transfer of the fin with maximum possible heat transfer if the whole surface is made base temperature uniformly:

$$\eta_{fin} = \frac{\dot{Q}}{\dot{Q}_{max}}$$

This can be investigated for two cases:

(a) *Infinitely Long Fins* - Using Eq. (7.27),

$$\begin{aligned}\eta_{fin} &= \frac{\kappa A m \theta_0}{P l h \theta_0}\\ &= \frac{1}{ml}\end{aligned}$$

(b) *Insulated-End Fins* - Using Eq. (7.30),

$$\begin{aligned}\eta_{fin} &= \frac{\kappa A m \tanh(ml)\,\theta_0}{P l h \theta_0}\\ &= \frac{\tanh(ml)}{ml}\end{aligned}$$

2. Fin Effectiveness *Fin effectiveness* is defined as the ratio of the rate of heat transfer from the fin and that without the fin. For a fin having A_f and A_b as the fin-surface area and base area, respectively, the effectiveness is given by

$$\begin{aligned}\varepsilon_{fin} &= \frac{\dot{Q}_{with\ fin}}{\dot{Q}_{without\ fin}}\\ &= \frac{\eta_{fin} A_f h \theta_0}{A_b h \theta_0}\\ &= \eta_{fin}\frac{A_f}{A_b}\end{aligned} \tag{7.32}$$

This can be investigated for two cases:

(a) *Infinitely Long and Insulated-End Fins* - For these fins,

$$\begin{aligned} l &\to \infty\\ \tanh(ml) &\approx 1\end{aligned}$$

Using Eq. (7.33),

$$\varepsilon_{fin} = \sqrt{\frac{\kappa P}{hA}}$$

(b) *Insulated-End Fins* - Using Eq. (7.30),

$$\begin{aligned}\varepsilon_{fin} &= \frac{\sqrt{hP\kappa A}\tanh(ml)\,\theta_0}{hA\theta_0}\\ &= \frac{\tanh(ml)}{\sqrt{\frac{hA}{\kappa P}}}\end{aligned} \tag{7.33}$$

Using the above expressions, the following conclusions can be drawn:

(a) Thermal conductivity κ of the fin material should be very high. For this, highly conductive materials, such as aluminium, copper are used for fins.

(b) The ratio perimeter to cross-sectional area of the fin should be as high as possible. For this, large numbers of thin plates or slender pins are preferred.

(c) Fin effectiveness is high in applications involving a low convection coefficient h. Therefore, fins are generally placed on fluids having lower convection coefficient, such as fins in liquid-to-gas heat exchangers are placed on the gas side.

7.2.8 Transient Heat Conduction

When the rate of heat transfer vary with time, the process is called *unsteady or transient heat transfer*. This condition mainly occurs in starting stages of most of the engineering devices, such as boilers, heat exchangers, regenerator, fins, etc.

7.2.8.1 Lumped Heat Analysis *Lumped heat analysis* is the simplest and most convenient method that can be used to solve transient conduction problems. The cooling or heating of a system is called *Newtonian* if the system possesses infinite thermal conductivity; and for such a system, temperature gradient at any point will be zero.

Consider a body having volume V, surface area A, and initial temperature T_0. The body is immersed in an infinite volume of a fluid of density ρ, specific heat c, at temperature T_∞. The coefficient of convection in the

fluid is h. As at any moment t, there is temperature difference $T-T_\infty$. The temperature differential results in heat transfer between the body and fluid, resulting in a change in the heat capacity of the body [Fig. 7.11].

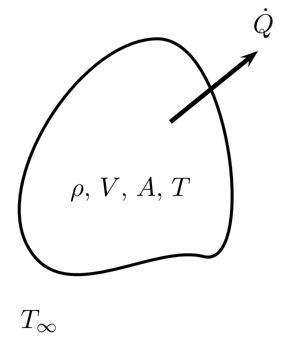


Figure 7.11 | Lumped heat analysis.

The equilibrium equation for heat transfer can be written as

$$h\left(T-T_\infty\right)A = -\rho Vc\frac{\partial T}{\partial t}$$
$$\int\frac{\partial T}{\left(T-T_\infty\right)} = -\frac{hA}{\rho Vc}\int\partial t$$
$$\ln\left(T-T_\infty\right) = -\frac{hA}{\rho Vc}t+C$$

This equation is subjected to a boundary condition:

$$T = T_0, \quad \text{at } t = 0$$

Therefore,

$$\ln\left(\frac{T-T_\infty}{T_0-T_\infty}\right) = -\frac{hA}{\rho Vc}t$$
$$\frac{T-T_\infty}{T_0-T_\infty} = \exp\left(-\frac{hA}{\rho Vc}t\right) \tag{7.34}$$

This represents an exponential profile of temperature w.r.t. to time. Using this, the following quantities can be determined

1. ***Total Heat Transfer*** Total heat given off upto time t is determined as

$$\begin{aligned}U &= \int_0^t h\left(T-T_\infty\right)Adt\\ &= hA\left(T_0-T_\infty\right)\int_0^t \exp\left(-\frac{hA}{\rho Vc}t\right)dt\\ &= hA\left(T_0-T_\infty\right)\times\frac{-\rho Vc}{hA}\left[\exp\left(-\frac{hA}{\rho Vc}t\right)\right]_0^t\\ &= \rho Vc\left(T_0-T_\infty\right)\left[\exp\left(-\frac{hA}{\rho Vc}t\right)-1\right]\end{aligned}$$

2. ***Time Constant*** The rate of cooling at any time t is determined as

$$\frac{dT}{dt} = -\left(T_0-T_\infty\right)\frac{hA}{\rho Vc}\exp\left(-\frac{hA}{\rho Vc}t\right)$$

At the starting point, $t = 0$

$$\frac{dT}{dt} = -\left(T_0-T_\infty\right)\frac{hA}{\rho Vc}$$

Therefore, for given values of h and c, a body having higher value of $A/\rho V$ shall face higher cooling rate. Thus, a hollow sphere will cool at a faster rate than a solid sphere of the same outer diameter and same initial temperature.

In above equation, the quantity $\rho Vc/\left(hA\right)$ has a unit of time, therefore, let it be expressed with symbol t^*. Using Eq. (7.34), the temperature at this time interval (t^*) is

$$\begin{aligned}\frac{T_*-T_\infty}{T_0-T_\infty} &= \exp\left(-\frac{hA}{\rho Vc}\times\frac{\rho Vc}{hA}\right)\\ &= \exp\left(-1\right)\\ &= 0.368\end{aligned}$$

Therefore, time interval t^* is the time required by a body of infinite conductivity to reach 36.8% of value of initial temperature difference. This is known as *time constant*. This is an important parameter for thermocouples. The time required by a thermocouple to attain 63.2% of value of the initial temperature difference is called *sensitivity of thermocouple*.

The temperature profile w.r.t. time [Eq. (7.34)] can also be written in terms of two dimensionless numbers, Biot number (Bi) and Fourier number (Fo):

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-\mathrm{Bi}\cdot\mathrm{Fo}}$$

These dimensionless numbers are defined as follows:

1. ***Biot Number*** *Biot number*[3] is defined as

$$\mathrm{Bi} = \frac{hl_c}{\kappa} \tag{7.35}$$

where $l_c(= V/A)$ is called *critical length* of the body. This lumped heat analysis (also known as *Lumped heat capacity approach*) is valid only for $\mathrm{Bi} < 0.1$.

2. ***Fourier Number*** *Fourier number* is defined as

$$\mathrm{Fo} = \frac{\alpha t}{{l_c}^2} \tag{7.36}$$

Fourier number is a dimensionless measure of time used in transient conduction problems.

[3]Biot (1774-1862) was a French physicist, astronomer, and mathematician who established the reality of meteorites, made an early balloon flight, and studied the polarization of light.

7.2.8.2 Semi-Infinite Body

A *semi-infinite body* is one in which at any instant of time, there is always a point where the effect of heating (or cooling) at one of its boundary is not felt at all. At this point, the temperature remains unchanged. The transient temperature change in a plane infinitely thick wall is similar to that of a semi-infinite body until enough time has passed for the surface temperature effect to penetrate through it.

Consider a body extending to infinity in x-direction. The entire body is initially at a temperature T_0. The surface temperature at $x = 0$ is suddenly raised to T_s [Fig. 7.12].

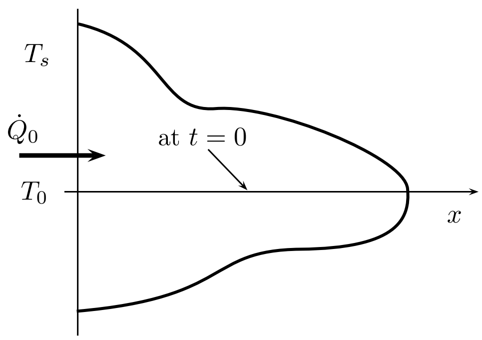


Figure 7.12 Semi-infinite body analysis.

The governing equation is

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2}$$

where

$$\theta = T - T_s$$

with the initial and boundary conditions:

1. $\theta = T_0 - T_s = \theta_0$ at $t = 0$ for all x
2. $\theta = T_s - T_s = 0$ at $x = 0$ for all $t > 0$
3. $\theta \to \theta_0$ as $x \to \infty$

Further discussion on this topic is out of the context of this book.

7.3 CONVECTION

Convective heat transfer occurs along with the movement of fluid particles by which heat is carried from one point to another. Thus, velocity of fluid plays a dominant role in convective heat transfer. The velocity can be achieved due to density difference, then, the convection is called *natural convection*. Another way is by forcing the fluid through compressor or pump or any other means, then convection is called *forced convection*. These two types of convection modes are studied separately.

Convective mode of heat transfer is governed by the Newton's law of cooling, and the conservation principles of mass, momentum, and energy, applicable in fluid flow. The derivation of expressions for convective heat transfer is complex and out of the context of this book.

7.3.1 Newton's Law of Cooling

Consider a body having surface area A, surface temperature T_s. The outside temperature is T_∞ [Fig. 7.13].

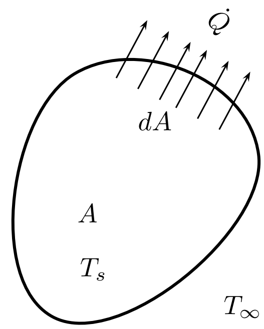


Figure 7.13 Convective heat transfer.

According to *Newton's law of cooling*, the rate of heat transfer ($\dot{Q}$) in convective heat transfer is proportional to the temperature difference ($T_s - T_\infty$) and area of heat transfer (dA):

$$\begin{aligned} d\dot{Q} &\propto dA\,(T_s - T_\infty) \\ &= h dA\,(T_s - T_\infty) \end{aligned}$$

where h is called *coefficient of convection*. Total heat transfer through the given area is

$$\begin{aligned} \dot{Q} &= \int d\dot{Q} \\ &= (T_s - T_\infty) \int h dA \end{aligned}$$

Like boundary layer in fluid flow, there exists thermal boundary layer in convective heat transfer within which temperature gradient exists and accordingly h varies over the surface [Fig. 7.14].

The velocity of the fluid relative to an immersed body is called *free stream velocity*. It is usually taken to be equal to the upstream velocity, also called *approach velocity*, which is the velocity of approaching fluid far ahead of the body.

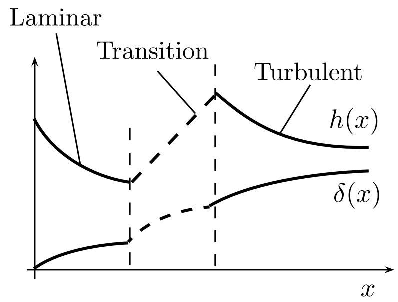


Figure 7.14 Variation of δ and h.

7.3.2 Dimensionless Numbers

Due to complexity, the study of convective heat transfer is based on dimensional analysis, empirical relations and practical observations. The analysis involves use of *dimensionless numbers* or parameters having physical interpretation that relates the condition of flow and heat transfer, discussed as follows.

7.3.2.1 Reynolds Number *Reynolds number* (Re_L) is the ratio of inertia to viscous forces in a region of characteristic length L:

$$\begin{aligned}\mathrm{Re}_L &= \frac{\rho V^2/L}{\mu V/L^2}\\ &= \frac{\rho V L}{\mu}\end{aligned}$$

Inertia forces represent the increase in momentum of the fluid. Thus, inertia forces dominate for higher values of Re and viscous forces dominate for small values of Re. In fluid flow, the Reynolds number determines the existence of laminar or turbulent flow, and also affects the boundary layer thickness.

7.3.2.2 Prandtl Number *Prandtl number*[4] (Pr) is the property of a fluid defined as the ratio of kinematic viscosity and thermal diffusivity:

$$\begin{aligned}\mathrm{Pr} &= \frac{\nu}{\alpha}\\ &= \frac{\mu c}{\kappa}\end{aligned}$$

For liquid metals $\mathrm{Pr} = 0.003 - 0.01$, air $\mathrm{Pr} \approx 0.7$ and for water $\mathrm{Pr} \approx 7$.

The *Reynolds number* and the *Grashof number* [*Section* 7.3.2.6] are subscripted with a length scale variable, but Prandtl number contains no such length scale in its definition and is dependent only on the fluid and the fluid state. As such, Prandtl number is often found in property tables alongside viscosity and thermal conductivity.

The Prandtl number controls the relative thickness of the momentum and thermal boundary layers. When Pr is small, it means that the heat diffuses very quickly compared to the velocity (momentum). This means that for liquid metals the thickness of the thermal boundary layer is much bigger than the velocity boundary layer.

For mercury, heat conduction is very effective compared to convection; thermal diffusivity is dominant. For engine oil, convection is very effective in transferring energy from an area, compared to pure conduction: momentum diffusivity is dominant.

7.3.2.3 Nusselt Number *Nusselt number*[5] (Nu) is defined as ratio of heat convected through the fluid and heat conducted through the fluid:

$$\begin{aligned}\mathrm{Nu} &= \frac{hA\Delta T}{\kappa A \Delta T/x}\\ &= \frac{hx}{\kappa}\end{aligned}$$

The Nusselt number is a dimensionless version of the temperature gradient at the surface between the fluid and the solid, and it thus provides a measure of the convection occurring from the surface. Therefore, in most of the empirical relations, the Nusselt number forms the basis of analyzing the convective heat transfer.

Nusselt number always increases with increase in Reynolds number in case of forced convection and with an increase in the Grashof number in case of free convection.

7.3.2.4 Stanton Number *Stanton number*[6] (St) is defined as the ratio of convective heat flow from a wall and mass heat flow rate:

$$\begin{aligned}\mathrm{St} &= \frac{hl^2\Delta T}{\rho l^3 c \Delta T}\\ &= \frac{h}{\rho V c}\end{aligned}$$

The Stanton number is relevant in analyzing the geometric similarity of the momentum boundary layer and the thermal boundary layer; it represents a relationship

[4] Ludwig Prandtl (1875-1953) was a German scientist, a pioneer in the development of rigorous systematic mathematical analyses to form the basis of the applied science of aeronautical engineering. His studies identified the boundary layer, thin-airfoils, and lifting-line theories.

[5] Wilhelm Nusselt (1882-1957) was a German engineer. He studied and taught mechanical engineering at the Munich Technical University. During this teaching tenure, he developed the dimensional analysis of heat transfer, without any knowledge of the Buckingham π theorem or any other developments of Lord Rayleigh.

[6] The Stanton number is named after Thomas Edward Stanton (1865-1931).

between the shear force at the wall (due to viscous drag) and the total heat transfer at the wall (due to thermal diffusivity) [*Section* 7.3.5].

7.3.2.5 Peclet Number *Peclet number*[7] (Pc) is defined as

$$\mathrm{Pc} = \frac{\text{Advection of heat}}{\text{Conduction of heat}} = \frac{\rho v c \Delta T}{\kappa \Delta T / x} = \frac{v x}{\alpha}$$

where v is velocity. Therefore,

$$\mathrm{Pc} = \mathrm{Re} \times \mathrm{Pr}$$

Advection is the movement of material by flowing fluid.

7.3.2.6 Grashof Number *Grashof number*[8] (Gr) is defined as

$$\mathrm{Gr} = \frac{\text{Inertia force} \times \text{Buoyancy}}{\text{Viscous force}^2} = \frac{\beta g \Delta T {l_c}^3}{\nu^2}$$

where β is fluid coefficient of thermal expansion, g is gravity, l_c is critical length.

Grashof number frequently arises in the study of situations involving natural convection. At higher Grashof numbers, the boundary layer is turbulent; at lower Grashof numbers, the boundary layer is laminar.

7.3.2.7 Rayleigh Number The Grashof number describes the relationship between buoyancy and viscosity, while the Prandtl number describes the relationship between momentum diffusivity and thermal diffusivity. *Rayleigh number*[9] (Ra) is defined as the product of Grashof number and the Prandtl number:

$$\mathrm{Ra} = \mathrm{Gr} \times \mathrm{Pr} = \frac{g \beta \Delta T l_c^3}{\nu^2} \mathrm{Pr} = \frac{g \beta \Delta T l_c^3}{\nu \times \alpha}$$

The Rayleigh number can be viewed as the ratio of buoyancy force and the product of thermal and momentum diffusivities. When the Rayleigh number is below the critical value for a given fluid, heat transfer is primarily in the form of conduction; when it exceeds the critical value, heat transfer is primarily in the form of convection.

7.3.2.8 Schmidt Number *Schmidt number*[10] (Sc) is defined as the ratio of the momentum diffusivity (viscosity) and the mass diffusivity:

$$\mathrm{Sc} = \frac{\text{Viscous diffusion rate}}{\text{Mass diffusion rate}} = \frac{\mu}{\rho \alpha_m}$$

where α_m is mass diffusivity. The Schmidt number is used to characterize fluid flows having simultaneous momentum and mass diffusion convection processes.

Mass diffusivity or diffusion coefficient is a proportionality constant between the molar flux due to molecular diffusion and the gradient in the concentration of the species. This coefficient has SI unit of m^2/s.

Lewis number (Le) relates Pr and Sc as

$$\mathrm{Le} = \frac{\mathrm{Sc}}{\mathrm{Pr}}$$

Lewis number is a measure of the relative thermal and concentration boundary layer thicknesses.

7.3.3 Forced Convection

7.3.3.1 Over a Horizontal Plate Consider a forced convection in a fluid of density ρ, thermal conductivity κ, kinematic viscosity ν, stream temperature T_∞, over a horizontal plate of length l at surface temperature T_s [Fig. 7.15].

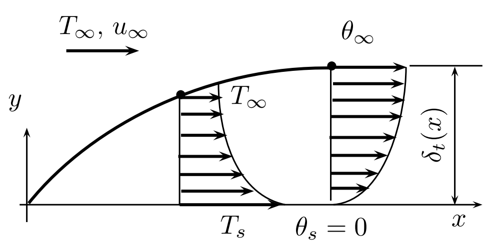


Figure 7.15 Thermal boundary layer.

The following equation derived by *Polhausen* presents relation between boundary layer thickness δ and thermal boundary layer thickness δ_t as

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{1/3}$$

[7]Peclet number is named after the French physicist Jean Claude Eugene Peclet (1793-1857).

[8]Grashof number is named after the German engineer Franz Grashof.

[9]Lord Rayleigh, (1842-1919) was an English physicist who, with William Ramsay, discovered the element argon, for which he earned the Nobel Prize for Physics in 1904. He also discovered Rayleigh scattering, explaining why the sky is blue, and predicted the existence of the surface waves now known as Rayleigh waves.

[10]Schmidt number was named after the German engineer Ernst Heinrich Wilhelm Schmidt (1892-1975)

The fluid properties are to be taken at average fluid or film temperature:

$$T_f = \frac{T_s + T_\infty}{2}$$

Nusselt number at any point at distance x from edge (on single side) is given by:

1. *Laminar Boundary Layer* Nusselt number for laminar boundary layer ($\mathrm{Pr} \geq 0.6$) is given by

$$\begin{aligned} \mathrm{Nu}_x &= \frac{hx}{\kappa} \\ &= 0.332 \times \mathrm{Re}_x{}^{1/2}\mathrm{Pr}^{1/3} \quad (7.37) \\ \overline{\mathrm{Nu}_l} &= \frac{\bar{h}l}{\kappa} = 0.644 \times \mathrm{Re}_l{}^{1/2}\mathrm{Pr}^{1/3} \end{aligned}$$

Thus,

$$\bar{h} = 2 \times h_{x=l}$$

For liquid metals ($\mathrm{Pr} \ll 0.005$)

$$\mathrm{Nu}_x = 0.565 \times \mathrm{Pc}_x{}^{1/2}$$

2. *Turbulent Boundary Layer* For the boundary layer fully turbulent from the leading edge of the plate, the local Nusselt number is given by

$$\begin{aligned} \mathrm{Nu}_x &= \frac{hx}{\kappa} = 0.0296 \times \mathrm{Re}_x{}^{4/5}\mathrm{Pr}^{1/3} \\ \overline{\mathrm{Nu}_l} &= \frac{\bar{h}l}{\kappa} = \frac{4}{3}\mathrm{Nu}_l \end{aligned}$$

3. *Laminar and Turbulent Boundary Layer* For boundary layer consisting of laminar and turbulent sections, the average Nusselt number (for $0.6 < \mathrm{Pr} < 60$) is given by

$$\begin{aligned} \overline{\mathrm{Nu}_l} &= \frac{\bar{h}l}{\kappa} \\ &= \left(0.037 \times \mathrm{Re}_l{}^{4/5} - 870\right)\mathrm{Pr}^{1/3} \end{aligned}$$

7.3.3.2 In a Fully Developed Pipe Flow Consider convective heat transfer in a fully developed flow of a fluid of density ρ, thermal conductivity κ, kinematic viscosity ν, of uniform temperature T_∞, in a pipe of length l, diameter D, kept at uniform surface temperature T_s. If fluid enters pipe at temperature T_i and exits at T_o, then fluid properties are taken at mean temperature $(T_i + T_o)/2$.

1. *Laminar Flow* ($\mathrm{Re} < 2000$)

   (a) Constant flux assumption

$$\overline{\mathrm{Nu}_D} = \frac{\bar{h}D}{\kappa} = 4.364$$

   (b) Constant wall temperature assumption

$$\overline{\mathrm{Nu}_D} = \frac{\bar{h}D}{\kappa} = 3.66$$

2. *Turbulent Flow* ($\mathrm{Re} > 2000$)

$$\mathrm{Nu}_D = 0.027 \times \mathrm{Re}_D{}^{4/5}\mathrm{Pr}^{1/3}\left(\frac{\mu_f}{\mu_s}\right)^{0.14}$$

where μ_f and μ_s are dynamic viscosities at mean film temperature $T_f = (T_s + T_\infty)/2$, and surface temperature T_s, respectively. This equation is known as *Dittus-Boelter equation* which includes laminar portion also and is valid if

$$l/D \gg 60$$

7.3.4 Natural Convection

In natural convection, the density difference between the inside and the outside of the boundary layer gives rise to buoyancy force and sustained flow. This is known as *Boussinesq approximation.*

Consider a case of natural convection over a horizontal plate. A fluid of density ρ, thermal conductivity κ, kinematic viscosity ν, of uniform temperature T_∞, flows over a horizontal plate of length l kept at uniform surface temperature T_s. The local Grashof number at location x in this case is

$$\mathrm{Gr}_x = \frac{\beta g x^3 (T - T_\infty)}{\nu^2}$$

Local thickness is given by empirical relation

$$\frac{\delta}{x} = 3.93 \times \mathrm{Pr}^{-1/2}(0.952 + \mathrm{Pr})^{1/4}\mathrm{Gr}_x{}^{-1/4}$$

and local Nusselt number

$$\begin{aligned} \mathrm{Nu}_x &= 0.508\mathrm{Pr}^{1/2} \times (0.952 + \mathrm{Pr})^{-1/4} \times \mathrm{Gr}_x{}^{1/4} \\ &= \frac{2x}{\delta} \end{aligned}$$

Thus,

$$\mathrm{Nu}_x \propto (T - T_\infty)^{1/4}$$

Average Nusselt number is

$$\overline{\mathrm{Nu}_l} = \frac{4}{3}\mathrm{Nu}_l$$

All fluid properties are taken at mean film temperature:

$$T_f = \frac{T_s + T_\infty}{2}$$

7.3.5 Boundary Layer Analogies

Primary objective of a forced convection analysis is to determine the drag coefficient (C_f) and the heat transfer rates (Nu). Therefore, it is desirable to have a relationship between C_f and Nu. This is done based on the similarity between momentum and heat transfer in boundary layers. Reynolds analogy and Chilton-Colburn analogy are the two well-known analogies, explained as follows.

7.3.5.1 Reynolds Analogy Near the surface, the heat transfer is always due to conduction because the fluid near the surface is stationary. The ratio of heat flux to shear stress in laminar flow is given by

$$\begin{aligned}\frac{q_s}{\tau_s} &= -\frac{\kappa \times dT/dy}{\mu \times du/dy}\\ &= -\frac{\kappa}{\mu} \times \frac{dT}{du}\\ &= -\frac{c}{\mathrm{Pr}} \times \frac{dT}{du}\end{aligned}$$

where

$$\mathrm{Pr} = \frac{\mu c}{\kappa}$$

For identical velocity and thermal boundary layers (Pr = 1):

$$\begin{aligned}\frac{q_s}{\tau_s} &= -c\frac{dT}{du}\\ -\frac{q_s}{c\tau_s}\int_0^{u_\infty} du &= \int_{T_s}^{T_\infty} dT\\ -\frac{q_s}{c\tau_s}u_\infty &= T_\infty - T_s\\ \frac{q}{T_s - T_\infty} \times \frac{1}{\rho c u_\infty} &= \frac{\tau_s}{\rho u_\infty^2}\\ \frac{h_x}{\rho c u_\infty} &= \frac{C_{fx}}{2}\end{aligned}$$

The dimensionless group of terms on the left hand side of the above equation is called the *Stanton number* (St) [*Section* 7.3.2.4] which is the Nusselt number divided by the product of Reynolds and Prandtl numbers:

$$\begin{aligned}\mathrm{St}_x &= \frac{\mathrm{Nu}_x}{\mathrm{Re}_x\mathrm{Pr}}\\ &= \frac{C_{fx}}{2}\end{aligned}$$

This relation, expressing the relationship between fluid friction and heat transfer for laminar flow on a flat plate, is called the *Reynolds analogy* (valid only at Pr = 1). This analogy is useful in determining the heat transfer coefficient for fluids with Pr ≈ 1 from the knowledge of friction coefficient which is easier to measure.

7.3.5.2 Chilton-Colburn Analogy Reynolds analogy is of limited use because of the restrictions (Pr = 1) on it, but it is desirable to have an analogy that is applicable over a wide range of Pr. This is done by adding a Prandtl number correction. The *skin drag coefficient* and Nusselt number for a laminar flow over horizontal flat plate [Eq. (7.37)] are given by

$$\begin{aligned}C_{fx} &= \frac{0.644}{\sqrt{Re_x}}\\ \frac{C_{fx}}{2} &= \frac{0.332}{\sqrt{Re_x}}\end{aligned}$$

Also

$$\mathrm{Nu}_x = 0.322\sqrt{\mathrm{Re}_x}.\mathrm{Pr}^{1/3}$$

Therefore

$$\begin{aligned}\mathrm{St}_x &= \frac{\mathrm{Nu}_x}{\mathrm{Re}_x.\mathrm{Pr}}\\ &= \mathrm{Pr}^{-2/3}\frac{C_{fx}}{2}\\ \mathrm{St}_x\mathrm{Pr}^{2/3} &= \frac{C_{fx}}{2}\\ &= j_H\end{aligned}$$

This relationship is known as the *modified Reynolds analogy* or the *Chilton-Colburn analogy* which is valid in the range of $0.5 < \mathrm{Pr} < 60$. Here, the dimensionless empirical parameter j_H is called the *Coluburn j-factor*

7.4 RADIATION

7.4.1 Basic Definitions

When radiation is incident upon a surface, some part of it is reflected back from the surface, some part is absorbed by the surface, and some part is transmitted through the surface [Fig. 7.16].

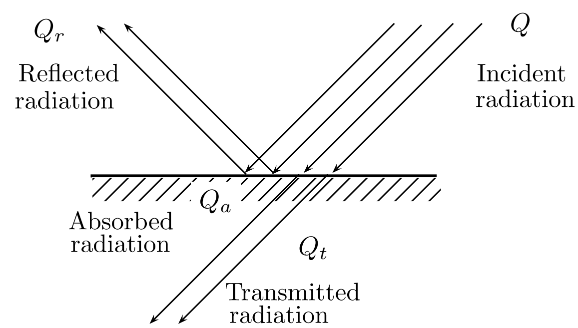


Figure 7.16 | Incidence radiation on a surface.

This behavior of surface with respect of radiation is expressed by using three surface properties, *transmissivity* (τ), *absorptivity* (α), *reflectivity* (ρ), defined as the

respective part divided by total incident radiation [Table 7.1]:

$$\tau = \frac{Q_t}{Q}$$
$$\alpha = \frac{Q_a}{Q}$$
$$\rho = \frac{Q_r}{Q}$$

Therefore,

$$\alpha+\rho+\tau = 1$$

Table 7.1 Surface properties related to radiation

Surface Type	Properties
Oblique surfaces	$\tau = 0$
White surfaces	$\tau = \alpha = 0,\ \rho = 1$
Transparent surfaces	$\tau = 1$
Black bodies	$\alpha = 1$

These surface properties are also function of the wavelength of radiation λ. If the whole spectrum is incident upon a surface, then

$$\rho = \frac{1}{Q}\int_0^\infty \rho_\lambda Q_\lambda d\lambda$$
$$\tau = \frac{1}{Q}\int_0^\infty \tau_\lambda Q_\lambda d\lambda$$
$$\alpha = \frac{1}{Q}\int_0^\infty \alpha_\lambda Q_\lambda d\lambda$$

A *black body* is an idealized physical body that absorbs the incident radiation completely. Because of the perfect absorptivity at all wavelengths, a black body[11] is also the best possible emitter of thermal radiation.

7.4.2 Intensity of Radiation

Intensity of radiation (I) is defined as the heat flux (energy per unit time) incident per unit projected area of the surface with unit solid angle:

$$I = \frac{\text{Heat flux}}{\text{Projected area} \times \text{Solid angle}}$$

Let a radiation heat flux dQ_ϕ be incident upon a surface of infinitesimal area dA at angle ϕ from the normal N of the plane [Fig. 7.17].

[11]The nose of an aeroplane is painted black because black bodies absorb maximum heat.

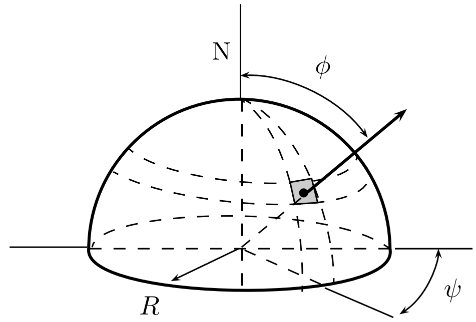


Figure 7.17 Defining the intensity of radiation.

The intensity of the incident radiation at angle ϕ is written as

$$I_\phi = \frac{dQ_\phi}{dA\cos\phi \times d\omega}$$

where $d\omega$ is solid angle[12]. Thus,

$$I_\phi = \frac{dQ_\phi/dA}{\cos\phi \times d\omega} = \frac{dE_\phi}{\cos\phi \times d\omega} \tag{7.38}$$

where dE_ϕ is *emissive power* at angle ϕ, defined as heat flux incident per unit area.

Total radiation intensity is related to monochromatic radiation intensity I_λ as

$$I = \int_0^\infty I_\lambda d\lambda$$

7.4.3 Lambert's Cosine law

According to *Lambert's cosine law*[13], a diffused surface radiates energy in such a manner that the rate of radiated energy in any particular direction is proportional to the cosine of the angle between the direction and normal to the surface:

$$dQ_\phi = dQ_n \cos\phi$$

So,

$$I_\phi = \frac{dQ_n\cos\phi}{dA\cos\phi \times d\omega} = \frac{dQ_n}{dA \times d\omega} = I_n$$

Thus, radiation intensity is independent of direction. The surfaces which follow this law are called *Lambertian* surfaces.

[12]Solid angle is defined as dA/r^2

[13]Lambert's cosine law is named after Johann Heinrich Lambert, from his Photometria, published in 1760.

7.4.4 Hemispherical Emissive Power

An expression for *hemispherical emissive power* (for *Lambertian* surfaces) of a particular wavelength can be found by replacing value of $d\omega$ as [Fig. 7.17]

$$\begin{aligned} d\omega &= \frac{dA}{r^2} \\ &= \frac{(r\sin\psi d\phi)\,(rd\psi)}{r^2} \\ &= \sin\psi d\psi d\phi \end{aligned}$$

Using Eq. (7.38), the hemispherical emissive power for wavelength λ can be written as

$$\begin{aligned} E_\lambda &= \int dE_\lambda \\ &= \int_0^{2\pi} dE_{\phi\lambda} d\phi \\ &= \int I_{n\lambda}\cos\phi d\omega d\phi \\ &= I_{n}\lambda \int_{\psi=0}^{\pi/2}\int_{\phi=0}^{2\pi}\cos\phi\sin\psi d\psi d\phi \\ &= \pi I_{n_\lambda} \end{aligned}$$

Thus, hemispherical emissive power is π times the intensity of radiation.

7.4.5 Planck's Distribution Law

According to *Planck's distribution law*[14], energy of radiation (E) is distributed over the range of wavelengths (λ), as depicted in [Fig. 7.18]. Thus, a particular value of wavelength possesses a specific energy, E_λ called *monochromatic emissive power*, given by

$$E_\lambda = \pi I_\lambda = \frac{C_1}{\lambda^5\left[\exp\left(C_2/\left(\lambda T\right)\right) - 1\right]} \tag{7.39}$$

where

$$\begin{aligned} C_1 &= 0.374\times10^{-5}\ \mathrm{m^3W} \\ C_2 &= 1.439\times10^{-2}\ \mathrm{mK} \end{aligned}$$

Figure 7.18 shows that the emitted radiation is a continuous function of wavelength. At any specified temperature, the energy of radiation increases with wavelength and decreases after reaching a peak value. The amount of energy increases with increase in temperature. The curve shifts leftward for shorter wavelengths; larger fraction of energy is emitted at shorter wavelengths.

[14]Max Planck originally produced this law in 1900 in an attempt to improve upon the Wien approximation, published in 1896 by Wilhelm Wien, which fit the experimental data at short wavelengths but deviated from it at long wavelengths.

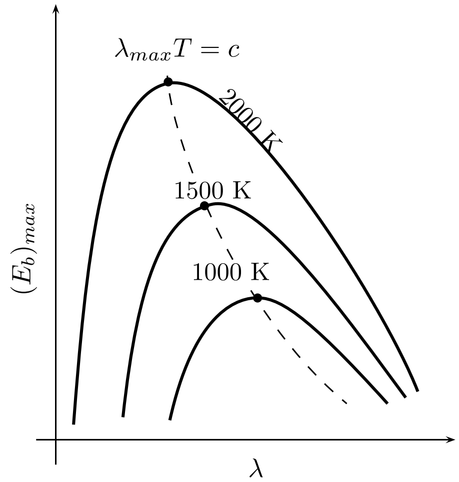


Figure 7.18 | Planck's distribution law.

7.4.6 Wein's Displacement Law

Planck's distribution law [Eq. (7.39)] shows that as the temperature increases, the peak value of the radiation shifts towards shorter wavelengths. The wavelength (λ_m) at which the peak occurs for a specified temperature is given by

$$\frac{dE_\lambda}{d\lambda} = 0$$

Using Eq. (7.39),

$$\lambda_m T = 0.2898\times10^{-2}\mathrm{mK} \tag{7.40}$$

This relation is called *Wein's displacement law*[15]. The peak value of energy of radiation (at $\lambda = \lambda_m$) is given by

$$\frac{E_{\lambda_m}}{T^5} = 1.307\times10^{-5}\mathrm{W/m^2K^5} \tag{7.41}$$

Using Eqs. (7.40) and (7.41),

$$\begin{aligned} \lambda_m &\propto \frac{1}{T} \\ E_{\lambda_m} &\propto T^5 \end{aligned}$$

It is established that Wein's displacement law fails at higher wavelengths and lower temperatures.

7.4.7 Stefan-Boltzmann Law

The total emissive power of a surface can be obtained by integration of the spectral black body emissive power E_λ over the entire wavelength spectrum:

$$E_b = \int_0^\infty E_\lambda d\lambda$$

[15]Wilhelm Wien (1864-1928) was a German physicist who, in 1893, used classical theories about heat and electromagnetism to deduce Wien's displacement law.

Using the Plank's law [Eq. (7.39)] for E_λ,

$$E_b = \sigma T^4 \tag{7.42}$$

where

$$\begin{aligned}\sigma &= \frac{C_1}{15}\left(\frac{\pi}{C_2}\right)^4 \\ &= 5.67\times 10^{-8}\mathrm{W/m^2K^4}\end{aligned}$$

Eq. (7.42) is known as the *Stefan-Boltzmann law* or *Stefan's law*, and the constant σ is called *Stefan-Boltzmann constant*[16]. The total emissive power of a black body corresponds to the area under the entire curve for a specific temperature in Fig. 7.18.

For a specified temperature T, the radiation emitted by a black body over a wavelength from 0 to λ can be obtained as

$$E_{0\to\lambda(T)} = \int_0^\lambda E_\lambda d\lambda$$

A dimensionless quantity *black body radiation function* (f_λ) is defined as

$$f_{\lambda(T)} = \frac{E_{0\to\lambda(T)}}{E_{(T)}}$$

7.4.8 Emissivity

Emissivity (ϵ) of a surface is defined as ratio of emissive power of the surface at given temperature and radiation from a black body at the same given temperature:

$$\epsilon = \frac{E}{E_b}$$

Using Eq. (7.42), radiation from a surface (can be non-black) can be written in terms of radiation from a black body as

$$\begin{aligned}E &= \epsilon E_b \\ &= \epsilon\sigma T^4\end{aligned}$$

The emissivity depends on the wavelength λ, therefore, average value of emissivity of a non-black surface can be written as

$$\epsilon = \frac{\int_0^\infty \epsilon_\lambda E_\lambda d\lambda}{E_b}$$

7.4.9 Kirchhoff's Law

Absorptivity of a surface (α) is not a surface property, but depends upon the intensity of radiation of the incident radiation. Hence, the absorptivity of a surface at a given temperature is measured when the surface is in thermal equilibrium with the black body at the same temperature.

Kirchhoff's law[17] establishes a relationship between the emissive power of a surface to its absorptivity. It states that at thermal equilibrium, the emissivity of a body (or surface) equals its absorptivity. If a body at temperature T_b is placed in a thermal atmosphere at temperature T_s, then rate of absorption is $\alpha_s\sigma {T_s}^4$ and rate of emission is $\alpha_b\sigma {T_b}^4$, because $\alpha = \epsilon$.

7.4.10 Net Radiation

Radiative heat transfer between surfaces depends on the orientation of the surfaces relative to each other as well as their radiation properties and absolute temperatures. Thus, the net radiation between two surfaces is examined under the following headings.

7.4.10.1 Black Surfaces Consider infinitesimal elements of area dA_1 and dA_2 at distance r on two black surfaces S_1 and S_2 of the total surface areas A_1 and A_2 at temperatures T_1 and T_2, making angles ϕ_1 and ϕ_2 with the lines joining them, respectively [Fig. 7.19].

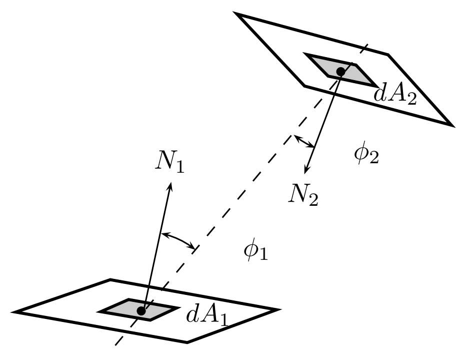


Figure 7.19 Net radiation.

Effective areas of these elemental surfaces normal to common line are $\cos\phi_1 dA_1$ and $\cos\phi_2 dA_2$, respectively. Solid angle made by area $\cos\phi_2 dA_2$ on area $\cos\phi_2 dA_1$ is given by

$$d\omega = \frac{\cos\phi_2 dA_2}{r^2}$$

[16]Stefan-Boltzmann law was deduced by Jozef Stefan (1835-1893) in 1879 on the basis of experimental measurements made by John Tyndall and was derived from theoretical considerations, using thermodynamics, by Ludwig Boltzmann (1844-1906) in 1884.

[17]Gustav Robert Kirchhoff (1824-1887) was a German physicist who contributed to the fundamental understanding of electrical circuits, spectroscopy, and the emission of black-body radiation by heated objects. He coined the term "black body" radiation in 1862.

If intensity of radiation from surface dA_1 is I_1, then radiative heat transfer from surface dA_1 to dA_2 is

$$
\begin{aligned}
d\dot{Q}_{1\to 2} &= I_1 \cos\phi_1 dA_1 \cdot d\omega \\
&= \frac{E_1}{\pi} \cos\phi_1 dA_1 \cdot d\omega \\
\dot{Q}_{1\to 2} &= \int d\dot{Q}_{1\to 2} \\
&= \sigma {T_1}^4 \int\int \frac{\cos\phi_1 dA_1 \cos\phi_2 dA_2}{\pi r^2} \\
\dot{Q}_{2\to 1} &= \int d\dot{Q}_{2\to 1} \\
&= \sigma {T_2}^4 \int\int \frac{\cos\phi_1 dA_1 \cos\phi_2 dA_2}{\pi r^2}
\end{aligned}
$$

Net radiative heat exchange is

$$
\begin{aligned}
\dot{Q}_{net\ 1\to 2} &= \dot{Q}_{1\to 2} - \dot{Q}_{2\to 1} \\
&= \sigma \left({T_1}^4 - {T_2}^4\right) \int\int \frac{\cos\phi_1 dA_1 \cos\phi_2 dA_2}{\pi r^2} && (7.43)
\end{aligned}
$$

$$(7.44)$$

The integral part on the right hand side of the above equation can be expressed as

$$\int\int \frac{\cos\phi_1 dA_1 \cos\phi_2 dA_2}{\pi r^2} = F_{12}A_1 = F_{21}A_2$$

where F_{12} and F_{21} are called *shape factors*. Therefore,

$$
\begin{aligned}
\dot{Q}_{net\ 1\to 2} &= \sigma \left({T_1}^4 - {T_2}^4\right) F_{12}A_1 \\
&= \sigma \left({T_1}^4 - {T_2}^4\right) F_{21}A_2 1
\end{aligned}
\tag{7.45}
$$

Thus, factor F_{12} represents fraction of area A_2 that is seen by area A_1, and the same is true for F_{21}.

7.4.10.2 Gray Surfaces Eq. (7.44) can be used for gray surfaces by introducing emissivity ϵ_1 and ϵ_2 of the respective surfaces. The radiative heat transfer from surface dA_1 to dA_2 is

$$
\begin{aligned}
d\dot{Q}_{1\to 2} &= \epsilon_2 I_1 \cos\phi_1 dA_1 . d\omega \\
&= \epsilon_2 \frac{\epsilon_1 E_1}{\pi} \cos\phi_1 dA_1 . d\omega \\
\dot{Q}_{1\to 2} &= \int d\dot{Q}_{1\to 2} \\
&= \epsilon_1\epsilon_2\sigma {T_1}^4 \int\int \frac{\cos\phi_1 dA_1 \cos\phi_2 dA_2}{\pi r^2}
\end{aligned}
$$

Similarly, radiative heat transfer from surface dA_2 to dA_1 is

$$
\begin{aligned}
\dot{Q}_{2\to 1} &= \int d\dot{Q}_{2\to 1} \\
&= \epsilon_2\epsilon_1\sigma {T_2}^4 \int\int \frac{\cos\phi_1 dA_1 \cos\phi_2 dA_2}{\pi r^2}
\end{aligned}
$$

The net heat transfer is

$$
\begin{aligned}
\dot{Q}_{net} &= \dot{Q}_{1\to 2} - \dot{Q}_{2\to 1} \\
&= \epsilon_1\epsilon_2\sigma \left({T_1}^4 - {T_2}^4\right) \int\int \frac{\cos\phi_1 dA_1 \cos\phi_2 dA_2}{\pi r^2} \\
&= \epsilon_1\epsilon_2\sigma \left({T_1}^4 - {T_2}^4\right) F_{12}A_1 \\
&= \epsilon_1\epsilon_2\sigma \left({T_1}^4 - {T_2}^4\right) F_{21}A_2
\end{aligned}
$$

7.4.11 Shape Factor

Shape factor takes into the account the effects of orientation on radiation heat transfer between two surfaces. It is purely a geometric quantity and is independent of the surface properties and temperature. It is also called the *view factor*, *configuration factor*, and *angle factor*.

Shape factor F_{ij} indicates the fraction of the radiation leaving the surface i that directly incidents on the surface j. Thus, the following arithmetic rules are valid in radiative heat exchanges:

1. Shape factors between two surfaces
$$F_{12}A_1 = F_{21}A_2$$
2. Sum of all the shape factors of a surface is equal to 1:
$$\sum_{j=1}^{n} F_{ij} = 1$$
$$
\begin{aligned}
F_{11} + F_{12} + F_{13} .... + F_{1n} &= 1 \\
F_{21} + F_{22} + F_{23} .... + F_{2n} &= 1 \\
\cdots &= 1
\end{aligned}
$$

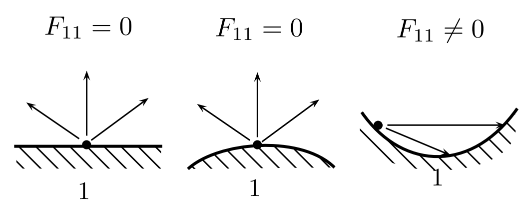


Figure 7.20 | Shape factors.

3. A convex or plane surface cannot see itself [Fig. 7.20], therefore,
$$F_{11} = 0$$
4. A concave surface can see itself [Fig. 7.20], therefore,
$$F_{11} \neq 0$$
5. For two discs of same radius on common normal,
$$F_{12} = F_{21} = 0.17$$

7.4.12 Electrical Network Analogy

In radiative heat exchange between two gray surfaces, emissivity and shape factor must be included into *Stefan-Boltzmann law* to achieve effective heat exchange. This radiative heat exchange can easily be analogized with electrical circuit in which $E_b = \sigma T^4$ presents potentials on surfaces [Fig. 7.19]. The electric circuit analogy[18] considers the resistances to the radiation in following headings:

1. Surface resistance
2. Shape factor resistance

These are explained as follows.

7.4.12.1 Surface Resistance The concept of surface resistance is based on the definitions of two quantities:

1. *Irradiance* (H), defined as flux of energy that irradiates the surface.
2. *Radiosity* (B), defined as the total flux of radiative energy away from the surface.

   The radiosity of a black body is equal to its emissive power because a black body does not reflect any radiation ($\rho = 0$, $\epsilon = 1$).

The radiosity can be expressed as the sum of the irradiated energy that is reflected by the surface (ρH) and the radiation emitted by it (ϵE_b). Thus,

$$B = \underbrace{\rho H}_{\text{reflected}} + \underbrace{\epsilon E_b}_{\text{emitted}}$$

Therefore,

$$H = \frac{B - \epsilon E_b}{\rho}$$

The net heat flux leaving the surface is

$$\begin{aligned} q_{net} &= B - H \\ &= B - \left(\frac{B - \epsilon E_b}{\rho}\right) \\ &= B - \frac{B}{\rho} + \frac{\epsilon E_b}{\rho} \\ &= \frac{\epsilon}{\rho} E_b - \frac{1-\rho}{\rho} B \end{aligned}$$

For a surface that is gray $(\alpha = \epsilon)$ and opaque $(\tau = 0)$

$$\begin{aligned} \rho + \alpha &= 1 \\ \rho + \epsilon &= 1 \end{aligned}$$

[18] An electric circuit analogy for heat exchange among diffuse gray bodies was developed by *Oppenheim* in 1956.

Therefore,

$$\begin{aligned} Q_{net} &= q_{net} A \\ &= A\left(\frac{\epsilon}{1-\epsilon} E_b - \frac{\epsilon}{1-\epsilon} B\right) \\ &= \frac{E_b - B}{(1-\epsilon)/\epsilon A} \end{aligned}$$

The above equation is a form of *Ohm's law*, which tells us that $(E_b - B)$ can be viewed as a driving potential for transferring heat away from a surface through an effective surface resistance:

$$R_s = \frac{(1-\epsilon)}{\epsilon A} \tag{7.46}$$

With Eq. (7.46), the *surface resistances* on surfaces 1 and 2 [Fig. 7.19] are, respectively,

$$\begin{aligned} R_{s1} &= \frac{1-\epsilon_1}{A_1 \epsilon_1} \\ R_{s2} &= \frac{1-\epsilon_2}{A_2 \epsilon_2} \end{aligned}$$

The surface resistance of a black body is zero.

7.4.12.2 Shape Factor Resistance *Shape factor resistance* between the two surfaces can be written as

$$R_{sf} = \frac{1}{A_1 F_{12}} = \frac{1}{A_2 F_{21}}$$

Thus, surface resistance and shape factor resistances between two surfaces act in series across the potential $\sigma(T_1^4 - T_2^4)$, shown in Fig. 7.21.

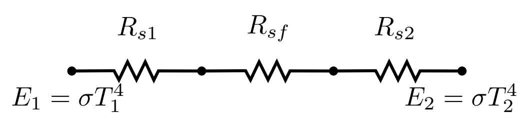


Figure 7.21 | Electrical analogy.

Thus, the heat exchange between two surfaces can be written as

$$\begin{aligned} \dot{Q}_{net\ 1\to 2} &= \frac{\sigma\left({T_1}^4 - {T_2}^4\right)}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}} \\ &= \frac{A_2\left(\epsilon_2 F_{12} + \epsilon_1 F_{21}\right)}{\epsilon_1 \epsilon_2 A_1 A_2 F_{12}} \end{aligned}$$

Electrical network analogy can be demonstrated in the following cases of radiative heat transfer:

1. Net heat transfer between two large gray parallel planes of area A_1 and A_2, emissivities ϵ_1 and ϵ_2 is

given by

$$\begin{aligned}\dot{Q}_{net} &= \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1-\epsilon_1}{A\epsilon_1} + \frac{1}{A} + \frac{1-\epsilon_2}{A\epsilon_2}} \\ &= \frac{A\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1-\epsilon_1}{\epsilon_1} + 1 + \frac{1-\epsilon_2}{\epsilon_2}} \\ &= \frac{A\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1}{\epsilon_1} - 1 + 1 + \frac{1}{\epsilon_2} - 1} \\ &= \frac{A\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \\ &= \frac{A\sigma\left(T_1{}^4 - T_2{}^4\right) \times \epsilon_1\epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1\epsilon_2}\end{aligned}$$

2. Net heat transfer between two large gray concentric cylinders of areas A_1 and A_2, emissivities ϵ_1 and ϵ_2 is given by

$$\begin{aligned}\dot{Q}_{net} &= \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1} + \frac{1-\epsilon_2}{A_2\epsilon_2}} \\ &= \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}\end{aligned}$$

7.4.13 Heat Shields

A heat shield is designed to shield a substance from absorbing excessive heat from an outside source by either dissipating, reflecting or simply absorbing the heat. Suppose n number of parallel shields of emissivity ϵ are kept between the two parallel surfaces S_1 and S_2 of the same emissivities of the same area A. Resistance to radiation between surface S_1 and first shield is

$$\begin{aligned}R_1 &= \frac{1-\epsilon}{\epsilon} + 1 + \frac{1-\epsilon}{\epsilon} \\ &= 2\frac{1}{\epsilon} - 1\end{aligned}$$

The $(n-1)$ resistances applied to radiation between (any) two consecutive shields (in series) is

$$R_s = R_1$$

Resistance to radiation between nth shield and surface S_2 is

$$R_2 = R_1$$

Thus, the total resistance to radiation is

$$\begin{aligned}R &= R_1 + (n-1)\,R_1 + R_1 \\ &= (n+1)\,R_1\end{aligned}$$

The resistance to heat transfer without shield is R_1, therefore,

$$\frac{\dot{Q} \text{ with } n \text{ shields}}{\dot{Q} \text{ without shields}} = \frac{1}{n+1}$$

7.5 HEAT EXCHANGER

A *heat exchanger* is a device that enables the transfer of heat from one fluid (liquid or gas) to another without direct contact between the fluids. For heat transfer, the fluids must be at different temperatures and they must come into thermal contact. The heat exchange involves convection in each fluid and conduction through the separating wall. Heat can flow only from the hotter to the cooler fluids, as per the second law of thermodynamics.

7.5.1 Types of Heat Exchangers

Heat exchangers come in different shapes and sizes. They can be classified according to construction specifications or direction of flow of the two fluids, discussed as follows.

7.5.1.1 Constructional Features The construction of most heat exchangers falls into one of two categories:

1. Tube and Shell Type *Tube and shell type heat exchanger* consists of a set of tubes in a container called a shell. At the ends of the tubes, the tube side fluid is separated from the shell side fluid by the tube sheet. The tubes are rolled and press-fitted or welded into the tube sheet to provide a leak tight seal.

   In systems where the two fluids are at vastly different pressures, the higher pressure fluid is typically directed through the tubes and the lower pressure fluid is circulated on the shell side. This is due to economy, because the heat exchanger tubes can be made to withstand higher pressures than the shell of the heat exchanger for a much lower cost.

2. Plate Type A *plate type heat exchanger* consists of plates instead of tubes to separate the hot and cold fluids. The hot and cold fluids flow in alternate between each of the plates. Plate type heat exchangers are not widely used because of the inability to reliably seal the large gaskets between each of the plates. Because of this problem, plate type heat exchangers have only been used in small, low pressure applications such as oil coolers for engines.

   Because each of the plates has a very large surface area, the plates provide each of the fluids with an extremely large heat transfer area. Therefore, a plate type heat exchanger, as compared to a similarly sized tube and shell heat exchanger, is capable of transferring much more heat.

7.5.1.2 Flow Arrangement Heat exchangers can also be characterized on the basis of the direction of flow the

two fluids have relative to each other. The three common types of heat exchangers are as follows:

1. *Parallel Flow* In *parallel flow heat exchangers*, both the tube side fluid and the shell side fluid flow in the same direction. In this case, the two fluids enter the heat exchanger from the same end with a large temperature difference.

2. *Counter Flow* In *counter flow heat exchangers*, the two fluids flow in opposite directions. Each of the fluids enter the heat exchanger from opposite ends. Because the cooler fluid exits the counter flow heat exchanger at the end where the hot fluid enters the heat exchanger, the cooler fluid will approach the inlet temperature of the hot fluid.

3. *Cross Flow* In *cross flow heat exchangers*, one fluid flows through tubes and the second fluid passes around the tubes at 90° angle. Cross flow heat exchangers are usually found in applications where one of the fluids change their states (2-phase flow), such as condenser of steam turbine.

Practically, most large heat exchangers are not purely parallel flow, counter flow, or cross flow; they are usually a combination of the two or all three types of heat exchangers.

A *regenerative heat exchanger* is one in which the same fluid is both the cooling fluid and the cooled fluid. The hot fluid leaving a system gives up its heat to regenerate or heat up the fluid returning to the system.

7.5.2 Heat Transfer Coefficients

Due to continuous flow of fluids and chemical reactions, tubes are subjected to fouling that in turn act as thermal resistance and affects *over all heat transfer coefficients*. Total heat exchange in a heat exchanger tube can be written as

$$\dot{Q} = U_o A_o \Delta T_m = U_i A_i \Delta T_m$$

where U_i and U_o refer to overall heat transfer coefficients with respect to inner and outer surface areas A_o and A_i, respectively, and ΔT_m is the mean effective temperature difference between the fluids.

If h_i and h_o represent respective convective *heat transfer coefficients*, and r_i and r_o represent respective radii and also R_{fi} and R_{fo} represent respective fouling factors, then

$$\begin{aligned} \frac{1}{U_o A_o} &= \frac{1}{U_i A_i} \\ &= \left\{ \frac{1}{h_o A_o} + \frac{R_{fi}}{A_i} \right\} + \frac{\ln (r_o / r_i)}{2\pi \kappa l} + \left\{ \frac{R_{fo}}{A_o} + \frac{1}{h_i A_i} \right\} \end{aligned}$$

7.5.3 Heat Exchanger Analysis

The flow in heat exchanger is considered under steady-state and fully insulated at outer surface. Consider an infinitesimal part of the heat exchanging tube of area dA, where temperatures of hot and cold fluids are T_h and T_c, respectively. Figure 7.22 shows the temperature profile along the direction of flow in parallel flow and counter flow heat exchangers.

The profile of temperature variation depends upon the heat capacity of the fluid. The profiles are different for both types of heat exchangers, therefore, expressions of LMTD are also different, derived as follows. Heat exchange from infinitesimal area dA at any point of the heat exchanger [Fig. 7.22] is given by

$$d\dot{Q} = U dA \,(T_h - T_c) \tag{7.47}$$

where U is the overall heat transfer coefficient and A is the heat transfer area of the heat exchanger. This heat is taken from hot fluid and given to cold fluid. The temperature of hot fluid reduces, while that of cold fluid increases.

There are two types of problems associated with designing of heat exchangers:

1. *Designing Heat Exchanger* When desired temperatures at inlets and outlets of both the fluids is known and number of tubes required in heat exchanger is to be calculated, then logarithmic mean temperature difference (LMTD) method is used.

2. *Determining the Capacity* When heat exchanger is given and one wants to know the heat exchange capacity, so this is a reverse problem. Then, effectiveness-number of transfer units (NTU) method is used.

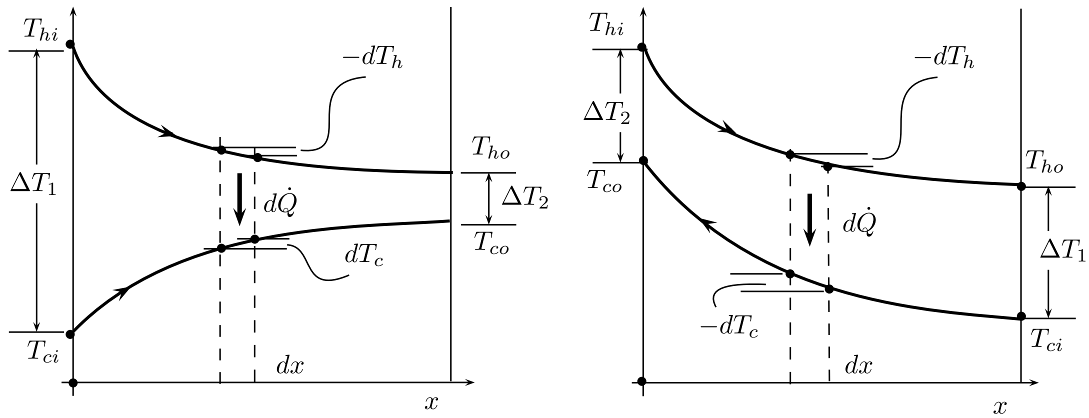
The two methods are explained in the following headings.

7.5.4 LMTD Method

7.5.4.1 Parallel Flow In parallel flow heat exchanger, the temperature of hot fluid reduces towards direction of flow, while that of cold fluid increases. Therefore,

1. *Hot Fluid*

$$\begin{aligned} -C_h dT_h &= U dA \,(T_h - T_c) \\ dT_h &= -\frac{U dA \Delta T}{C_h} \\ \dot{Q} &= C_h \,(T_{hi} - T_{ho}) \end{aligned} \tag{7.48}$$



(a) Parallel flow (b) Counter flow

Figure 7.22 | Temperature profile heat exchangers.

2. *Cold Fluid*

$$d\dot{Q} = C_c dT_c$$
$$dT_c = \frac{UdA\Delta T}{C_c} \tag{7.49}$$
$$\dot{Q} = C_c\left(T_{co} - T_{ci}\right)$$

The temperature difference along the fluid flow is

$$d\left(\Delta T\right) = dT_h - dT_c$$

Using Eqs. (7.48) and (7.49),

$$d\left(\Delta T\right) = -UdA\Delta T\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Integrating both sides of above equation,

$$\int_1^2 \frac{d\left(\Delta T\right)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_1^2 dA$$
$$\ln\left(\Delta T_2/\Delta T_1\right) = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)A$$

where ΔT_1 and ΔT_2 are the temperature differences at inlet and outlet, respectively.

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left\{\frac{\left(T_{hi} - T_{ho}\right)}{\dot{Q}} + \frac{-\left(T_{ci} - T_{co}\right)}{\dot{Q}}\right\}$$
$$= -\frac{UA}{\dot{Q}}\left\{\left(T_{hi} - T_{ci}\right) - \left(T_{ho} - T_{co}\right)\right\}$$
$$= -\frac{UA}{\dot{Q}}\left(\Delta T_1 - \Delta T_2\right)$$
$$\dot{Q} = UA \times \frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2/\Delta T_1\right)}$$
$$= UA \times \text{LMTD}$$

where

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2/\Delta T_1\right)} \tag{7.50}$$

This is the expression of LMTD for parallel flow heat exchangers.

7.5.4.2 Counter Flow In counter flow heat exchangers, heat $d\dot{Q}$ given by Eq. (7.47) is taken from hot fluid and given to cold fluid but directions of fluid flow are opposite. The heat exchanges in hot and cold fluids are determined as follows:

1. *Hot Fluid*

$$d\dot{Q} = -C_h dT_h$$
$$dT_h = -\frac{UdA\Delta T}{C_h} \tag{7.51}$$
$$\dot{Q} = C_h\left(T_{hi} - T_{ho}\right) \tag{7.52}$$

2. *Cold Fluid*

$$d\dot{Q} = -C_c dT_c$$
$$dT_c = -\frac{UdA\Delta T}{C_c} \tag{7.53}$$
$$\dot{Q} = C_c\left(T_{co} - T_{ci}\right) \tag{7.54}$$

Thus, the temperature difference along the flow direction of the hot fluid is

$$d\left(\Delta T\right) = dT_h - dT_c$$

Using Eqs. (7.51) and (7.53),

$$d\left(\Delta T\right) = -UdA\Delta T\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

Integrating both sides of the above equation,

$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} - \frac{1}{C_c}\right)\int_1^2 dA$$
$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -U\left(\frac{1}{C_h} - \frac{1}{C_c}\right) A$$

where ΔT_1 and ΔT_2 are temperature differences at inlet and outlet. Using Eqs. (7.52) and (7.54),

$$\begin{aligned}\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) &= -U\left\{\frac{(T_{hi} - T_{ho})}{\dot{Q}} - \frac{(T_{co} - T_{ci})}{\dot{Q}}\right\} \\ &= \frac{UA}{\dot{Q}}\{-T_{hi} + T_{ho} + T_{co} - T_{ci}\} \\ &= \frac{UA}{\dot{Q}}\{(T_{ho} - T_{ci}) - (T_{hi} - T_{co})\} \\ &= \frac{UA}{\dot{Q}}(\Delta T_2 - \Delta T_1)\end{aligned}$$

Therefore, the rate of heat exchange is

$$\begin{aligned}\dot{Q} &= UA \times \frac{\Delta T_2 - \Delta T_2}{\ln(\Delta T_2/\Delta T_1)} \\ &= UA \times \text{LMTD}\end{aligned}$$

where

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} \tag{7.55}$$

This is the expression of LMTD for counter flow heat exchangers.

The expressions of LMTD for parallel flow [Eq. (7.50)] and counter flow [Eq. (7.55)] heat exchangers are same. The temperature differences in these expressions, ΔT_2 and ΔT_1, are the maximum and minimum temperature differences existing in the system.

7.5.5 NTU Method

Effectiveness-NTU method is based on the definition of two dimensionless quantities:

1. *Effectiveness* *Effectiveness* (ϵ) of a heat exchanger is a dimensionless parameter, defined as the ratio of actual heat transfer and maximum heat transfer possible:

   $$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}}$$

   The fluid with small heat capacity experiences a large change in temperature, thus it will first reach the maximum temperature where the exchange of heat will stop. Therefore, for both parallel and counter flow heat exchangers, the maximum heat transfer would be given by

   $$\dot{Q}_{max} = C_{min}(T_{hi} - T_{ci})$$

   where C_{min} is the minimum heat capacity of fluids in the system. The effectiveness of a heat exchanger is defined for the maximum possible heat transfer $\dot{Q}_{max}$ which can be hypothetically achieved in a heat exchanger of infinite length.

2. *Number of Transfer Units* To quantify the number heat units being transferred between the fluids in a heat exchanger, the *number of transfer units* (NTU) is defined as

   $$\text{NTU} = \frac{UA}{C_{min}}$$

   This dimensionless group can be viewed as a comparison of the heat capacity of the heat exchanger, expressed in W/K, with the heat capacity of the flow.

For convenience sake in the heat exchanger analysis, heat capacity ratio can be defined as

$$r = \frac{C_{min}}{C_{max}}$$

Effectiveness is a function of NTU and r:

$$\varepsilon = f(\text{NTU,r})$$

The value of heat capacity ratio r can vary from 0 to 1. For the given value of NTU, the effectiveness is maximum when $r = 0$ (phase-change process) and minimum when $r = 1$.

The expressions for ε are derived for the two types of heat exchangers in the following sections.

7.5.5.1 Parallel Flow

In parallel flow heat exchangers, there is only one case when maximum heat transfer is possible (when $T_{ho} = T_{co}$) [Fig. 7.23] and the maximum possible heat exchange is again

$$\begin{aligned}\dot{Q}_{max} &= C_c(T_{co} - T_{ci}) \\ &= C_h(T_{hi} - T_{ho})\end{aligned} \tag{7.56}$$

Rate of heat transfer at any point through a small area dA is

$$d\dot{Q} = UdA(T_h - T_c)$$

From Eq. (7.56), it can also be written as

$$d\dot{Q} = -C_h dT_h = C_c dT_c$$

where dT_h and dT_c show the temperature difference across the small area dA in hot and cold lines, respectively. The above equation can be written as

$$dT_h = -\frac{d\dot{Q}}{C_h}$$
$$dT_c = \frac{d\dot{Q}}{C_c}$$

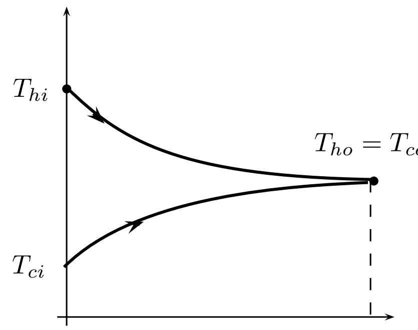


Figure 7.23 NTU method - parallel flow.

Subtraction of the above two equations gives

$$d\left(T_h - T_c\right) = -d\dot{Q}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$
$$= -UdA\left(T_h - T_c\right)\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Integration of the above equation gives

$$\ln\left(\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}}\right) = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)A \tag{7.57}$$

Using the definition of effectiveness, for hot and cold fluids, one gets

1. *Hot Fluid*

$$\varepsilon = \frac{C_h\left(T_{hi} - T_{ho}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)}$$
$$T_{ho} = T_{hi} - \frac{C_{min}}{C_h}\left(T_{hi} - T_{ci}\right)\varepsilon \tag{7.58}$$

2. *Cold Fluid*

$$\varepsilon = \frac{C_c\left(T_{co} - T_{ci}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)}$$
$$T_{co} = T_{ci} + \frac{C_{min}}{C_c}\left(T_{hi} - T_{ci}\right)\varepsilon \tag{7.59}$$

Subtraction of Eq. (7.59) from Eq. (7.58) gives

$$\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}} = 1 - C_{min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\varepsilon$$

Along with Eq. (7.57), one obtains effectiveness as

$$\varepsilon = \frac{1 - \exp\left\{-UA\left(1/C_h + 1/C_c\right)\right\}}{C_{min}\left(1/C_h + 1/C_c\right)}$$

This equation can be examined for the following two situations:

1. *When $C_c < C_h$* When heat capacity of cold fluid is minimum:

$$C_{min} = C_c$$
$$\varepsilon = \frac{1 - \exp\left\{-\frac{UA}{C_{min}}\left(1 + C_c/C_h\right)\right\}}{\left(1 + C_c/C_h\right)}$$

2. *When $C_c > C_h$* When heat capacity of hot fluid is minimum:

$$C_{min} = C_h$$
$$\varepsilon = \frac{1 - \exp\left\{-\frac{UA}{C_{min}}\left(1 + C_h/C_c\right)\right\}}{\left(1 + C_h/C_c\right)}$$

Therefore, from the above two cases, a common expression for effectiveness of a parallel flow heat exchanger is

$$\varepsilon = \frac{1 - \exp\left\{-\mathrm{NTU}\left(1 + r\right)\right\}}{1 + r} \tag{7.60}$$

In a parallel flow heat exchanger, two limiting cases of heat capacity ratio r can be examined as follows:

1. *Phase Change Process ($r = 0$)* For a parallel flow heat exchanger involving the phase change of a fluid during heat exchange ($C_{max} = \infty$), $r = 0$. Therefore, the effectiveness is given by

$$\varepsilon = 1 - \exp\left\{-\mathrm{NTU}\right\} \tag{7.61}$$

This is the same as Eq. (7.64) for counter flow heat exchangers.

2. *Constant Heat Capacity ($r = 1$)* When $C_{min} = C_{max} = C$ or $r = 1$, using Eq. (7.60),

$$\varepsilon = \frac{1 - \exp\left(-2 \times \mathrm{NTU}\right)}{2}$$

7.5.5.2 Counter Flow

The slope of temperature distribution curve depends upon the heat capacities C_h and C_c of hot and cold fluids, respectively, as

$$\frac{dT_h}{dx} \propto \frac{1}{C_h}$$
$$\frac{dT_c}{dx} \propto \frac{1}{C_c}$$

Therefore, more the heat capacity, lesser is the slope, and vice versa. Based on this concept, two extreme situations for a counter flow heat exchanger are shown in Fig. 7.24.

The following are two possible situations when rate of heat transfer is maximum:

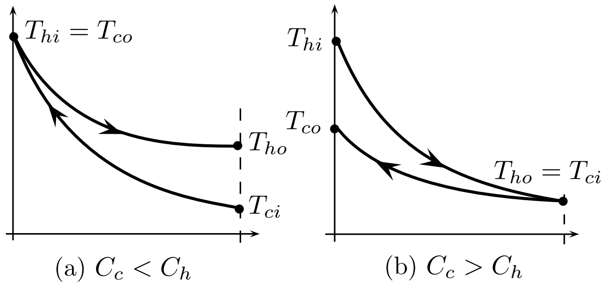


Figure 7.24 Counter-flow heat exchanger.

1. Outlet temperature of cold fluid is equal to the inlet temperature of hot fluid:

$$T_{co} = T_{hi}$$

This is possible if $C_h > C_c$. In this case, maximum rate of heat transfer is given by

$$\begin{aligned}\dot{Q}_{max} &= C_c\,(T_{co} - T_{ci})\\ &= C_{min}\,(T_{hi} - T_{ci})\end{aligned}$$

2. Outlet temperature of hot fluid is equal to the inlet temperature of cold fluid:

$$T_{ho} = T_{ci}$$

This is possible if $C_c > C_h$. In this case, maximum rate of heat transfer is given by

$$\begin{aligned}\dot{Q}_{max} &= C_h\,(T_{hi} - T_{ho})\\ &= C_{min}\,(T_{hi} - T_{ci})\end{aligned}$$

Conclusively, in both the situations, maximum rate of heat transfer is $C_{min}\,(T_{hi} - T_{ci})$. Therefore, the definition of effectiveness is written as

$$\begin{aligned}\varepsilon &= \frac{\dot{Q}}{C_{min}\,(T_{hi} - T_{ci})}\\ &= \frac{C_c\,(T_{co} - T_{ci})}{C_{min}\,(T_{hi} - T_{ci})}\end{aligned}$$

Here, $(T_{hi} - T_{ci})$ is the maximum temperature difference in the system. The rate of heat transfer $\dot{Q}$ in the heat exchanger of area A with temperature difference ΔT can be written as

$$\dot{Q} = UA\Delta T$$

The rate of heat transfer through a small area dA at any point is

$$d\dot{Q} = U dA\,(T_h - T_c)$$

The rate of change in temperature is related to heat capacities as

$$d\dot{Q} = -C_h dT_h = -C_c dT_c$$

This equation gives,

$$dT_h = -\frac{d\dot{Q}}{C_h}$$

$$dT_c = -\frac{d\dot{Q}}{C_c}$$

Subtracting the above equations, one obtains

$$\begin{aligned}d\,(T_h - T_c) &= d\dot{Q}\left(\frac{1}{C_c} - \frac{1}{C_h}\right)\\ &= U dA\,(T_h - T_c)\left(\frac{1}{C_c} - \frac{1}{C_h}\right)\end{aligned}$$

Integrating both sides of the above equation,

$$\begin{aligned}\int_1^2 \frac{d\,(T_h - T_c)}{(T_h - T_c)} &= \int_1^2 U\left(\frac{1}{C_c} - \frac{1}{C_h}\right) dA\\ \ln\frac{T_{ho} - T_{ci}}{T_{hi} - T_{co}} &= U\left(\frac{1}{C_c} - \frac{1}{C_h}\right) A\end{aligned} \tag{7.62}$$

By definition of effectiveness,

$$\begin{aligned}\varepsilon &= \frac{C_h\,(T_{hi} - T_{ho})}{C_{min}\,(T_{hi} - T_{ci})}\\ T_{ho} &= T_{hi} - \frac{C_{min}}{C_h}\,(T_{hi} - T_{ci})\,\varepsilon\end{aligned}$$

Similarly,

$$\begin{aligned}\varepsilon &= \frac{C_c\,(T_{co} - T_{ci})}{C_{min}\,(T_{hi} - T_{ci})}\\ T_{co} &= T_{ci} + \frac{C_{min}}{C_c}\,(T_{hi} - T_{ci})\,\varepsilon\end{aligned}$$

Putting these expressions in Eq. (7.62),

$$\frac{T_{hi} - \frac{C_{min}}{C_h}\,(T_{hi} - T_{ci})\,\varepsilon - T_{ci}}{T_{hi} - T_{ci} - \frac{C_{min}}{C_c}\,(T_{hi} - T_{ci})\,\varepsilon} = e^{U\left(\frac{1}{C_c} - \frac{1}{C_h}\right)A}$$

$$\frac{(T_{hi} - T_{ci}) - \frac{C_{min}}{C_h}\,(T_{hi} - T_{ci})\,\varepsilon}{(T_{hi} - T_{ci}) - \frac{C_{min}}{C_c}\,(T_{hi} - T_{ci})\,\varepsilon} = e^{U\left(\frac{1}{C_c} - \frac{1}{C_h}\right)A}$$

$$\begin{aligned}\frac{1 - \frac{C_{min}}{C_h}\varepsilon}{1 - \frac{C_{min}}{C_c}\varepsilon} &= \exp\left\{U\left(\frac{1}{C_c} - \frac{1}{C_h}\right) A\right\}\\ 1 - \frac{C_{min}}{C_h}\varepsilon &= \exp\left\{U\left(\frac{1}{C_c} - \frac{1}{C_h}\right) A\right\}\\ &\quad - \frac{C_{min}}{C_c}\exp\left\{U\left(\frac{1}{C_c} - \frac{1}{C_h}\right) A\right\}\end{aligned}$$

$$\varepsilon = \frac{1 - \exp\{-UA\,(1/C_h - 1/C_c)\}}{C_{min}/C_h - (C_{min}/C_c)\exp\{-UA\,(1/C_h - 1/C_c)\}}$$

This expression of effectiveness can be again examined for two situations:

1. If $C_{min} = C_h$, then

$$\varepsilon = \frac{1-\exp\left\{-\frac{UA}{C_{min}}\left(1-C_{min}/C_c\right)\right\}}{1-\left(C_{min}/C_c\right)\exp\left\{-\frac{UA}{C_{min}}\left(1-C_{min}/C_c\right)\right\}}$$
$$= \frac{1-\exp\{-\mathrm{NTU}\,(1-r)\}}{1-r\exp\{-\mathrm{NTU}\,(1-r)\}}$$

2. If $C_{min} = C_c$, then

$$\varepsilon = \frac{1-\exp\{-UA\,(1/C_h-1/C_{min})\}}{C_{min}/C_h-\frac{C_{min}}{C_{min}}\exp\{-UA\,(1/C_h-1/C_{min})\}}$$
$$= \frac{1-\exp\{-NTU\,(r-1)\}}{r-\exp\{-NTU\,(r-1)\}}$$
$$= \frac{1-1/\exp\{-NTU\,(1-r)\}}{r-1/\exp\{-NTU\,(1-r)\}}$$
$$= \frac{\exp\{-NTU\,(1-r)\}-1}{r\exp\{-NTU\,(1-r)\}-1}$$
$$= \frac{1-\exp\{-\mathrm{NTU}\,(1-r)\}}{1-r\exp\{-\mathrm{NTU}\,(1-r)\}}$$

Thus, in both the situations, the effectiveness of a counter flow heat exchanger (for $0 \geq r < 1$) is given by

$$\varepsilon = \frac{1-\exp\{-\mathrm{NTU}\,(1-r)\}}{1-r\exp\{-\mathrm{NTU}\,(1-r)\}} \tag{7.63}$$

In a counter flow heat exchangers, two limiting cases of heat capacity ratio r can be examined as follows:

1. *Phase Change Process ($r = 0$)* For an heat exchanger involving the phase change of a fluid during heat exchange ($C_{max} = \infty$), $r = 0$. Therefore, the effectiveness is given by

$$\varepsilon_{r\to 0} = 1-\exp\{-\mathrm{NTU}\} \tag{7.64}$$

This is same as Eq. (7.61) for parallel flow heat exchangers. It is interesting to see that this is valid for all types of heat exchangers. For this special case, the behavior of the heat exchanger is independent of the flow arrangement.

2. *Constant Heat Capacity ($r = 1$)* When $C_{min} = C_{max} = C$ or $r = 1$, Eq. (7.63) becomes indeterminate. Therefore, using the l'Hopital's rule[19],

$$\varepsilon_{r\to 1} = \lim_{r\to 1}\frac{(\partial/\partial r)\left(1-\exp\{-\mathrm{NTU}\,(1-r)\}\right)}{(\partial/\partial r)\left(1-r\exp\{-\mathrm{NTU}\,(1-r)\}\right)}$$

Taking

$$x = \exp\{-\mathrm{NTU}\,(1-r)\}$$

one obtains

$$\varepsilon_{r\to 1} = \lim_{r\to 1}\left\{\frac{-\mathrm{NTU}\times x}{-x-r\mathrm{NTU}\times x}\right\}$$
$$= \frac{-\mathrm{NTU}}{-1-\mathrm{NTU}}$$
$$= \frac{\mathrm{NTU}}{1+\mathrm{NTU}}$$

Alternatively, the above expression can be derived by considering the fact that when $r = 1$, the temperature differential between the two fluids remains constant throughout the heat exchanger:

$$T_{hi}-T_{co} = T_{ho}-T_{ci} \tag{7.65}$$

The rate of heat exchange can be written in two forms:

$$\dot{Q} = UA\,(T_{ho}-T_{ci})$$
$$\dot{Q} = C\,(T_{co}-T_{ci})$$

Using these expressions, NTU can be expressed in terms of temperatures as

$$\mathrm{NTU} = \frac{UA}{C} = \frac{T_{co}-T_{ci}}{T_{ho}-T_{ci}}$$

Using the value T_{hi} from Eq. (7.65),

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}}$$
$$= \frac{UA\,(T_{hi}-T_{co})}{C\,(T_{hi}-T_{ci})}$$
$$= \mathrm{NTU}\times\frac{(T_{hi}-T_{co})}{(T_{hi}-T_{ci})}$$
$$= \mathrm{NTU}\times\frac{(T_{ho}-T_{ci})}{(T_{ho}-T_{ci}+T_{co}-T_{ci})}$$
$$= \frac{\mathrm{NTU}}{1+(T_{co}-T_{ci})\,/\,(T_{ho}-T_{ci})}$$
$$= \frac{\mathrm{NTU}}{1+\mathrm{NTU}}$$

[19] L'Hopital's rule states that for functions $f(x)$ and $g(x)$ which are differentiable on the given domain, and

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$$

where a can be any real number, infinity or negative infinity, then one can write

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

The rule is named after French mathematician Guillaume de l'Hopital (1661—1704).

IMPORTANT FORMULAS

Conduction

1. Fourier's law

$$\dot{Q}_x = -\kappa A \frac{\partial T}{\partial x}$$

Thermal diffusivity

$$\alpha = \frac{\kappa}{\rho c_p}$$

Heat conduction equations

(a) Rectilinear coordinates

$$\nabla^2 T + \frac{\dot{q}}{\kappa} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

(b) Cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{\kappa} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

(c) Spherical coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{\kappa} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

2. Thermal resistance

(a) Rectangular wall

$$R_{th} = \frac{t}{\kappa A}$$

(b) Cylindrical body

$$R_{th} = \frac{\ln(r_o/r_i)}{2\pi\kappa l}$$

(c) Spherical body

$$R_{th} = \frac{(r_o - r_i)}{(4\pi r_o r_i)\,\kappa}$$

3. Variable thermal conductivity

$$\kappa = \kappa_o(1+\beta T)$$
$$\dot{Q} = -\bar{\kappa}A\frac{(T_1-T_2)}{(x_1-x_2)}$$
$$\bar{\kappa} = \kappa_o\left\{1+\beta\frac{(T_1+T_2)}{2}\right\}$$

4. Critical radius of insulation

(a) Cylindrical element

$$r_o = \frac{\kappa}{h_o}$$

(b) Spherical element

$$r_o = \frac{2\kappa}{h_o}$$

5. Heat generation

(a) Rectangular element

$$T = T_w + \frac{\left\{l^2/4 - x^2\right\}\dot{q}}{2\kappa}$$
$$T_w = T_\infty + \frac{\dot{q}}{2h_o}$$
$$T_{max} = T_\infty + \frac{\dot{q}}{2h_o} + \frac{\dot{q}}{\kappa}\frac{l^2}{8}$$

(b) Cylindrical element

$$T = T_w + \frac{\left({r_i}^2 - r^2\right)\dot{q}}{4\kappa}$$
$$T_w = T_\infty + \frac{\dot{q}r_i}{2h_o}$$
$$T_{max} = T_\infty + \frac{\dot{q}r_i}{2h_o} + \frac{{r_i}^2\dot{q}}{4\kappa}$$

(c) Spherical element

$$T = T_w + \frac{\left({r_s}^2 - r^2\right)\dot{q}}{6\kappa}$$
$$T_w = T_\infty + \frac{\dot{q}r_s}{3h_o}$$
$$T_{max} = T_\infty + \frac{\dot{q}r_s}{3h_o} + \frac{{r_s}^2\dot{q}}{6\kappa}$$

6. Rectangular fins

(a) Long fins

$$\dot{Q} = \sqrt{hP\kappa A}\,(T_0 - T_\infty)$$

(b) Insulated fins

$$\dot{Q} = -\kappa A\theta_0 m \times \tanh\{m(l-x)\}$$

7. Fin efficiency

$$\eta_{fin} = \frac{\dot{Q}}{\dot{Q}_{max}}$$

(a) Infinitely long fins

$$\eta_{fin} = \frac{1}{ml}$$

(b) Insulated-end fins

$$\eta_{fin} = \frac{\tanh(ml)}{ml}$$

8. Fin effectiveness

$$\varepsilon_{fin} = \frac{\dot{Q}_{with\ fin}}{\dot{Q}_{without\ fin}} = \eta_{fin}\frac{A_f}{A_b}$$

(a) Infinitely long fins

$$\varepsilon_{fin} = \sqrt{\frac{\kappa P}{hA}}$$

(b) Insulated-end fins

$$\varepsilon_{fin} = \frac{\tanh(ml)}{\sqrt{\frac{hA}{\kappa P}}}$$

9. Transient heat conduction

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-\frac{hA}{\rho V c}t}$$
$$t^* = \frac{\rho V c}{hA}$$
$$\frac{T_* - T_\infty}{T_0 - T_\infty} = 0.368$$
$$\mathrm{Bi} = \frac{hl_c}{\kappa}$$
$$\mathrm{Fo} = \frac{\alpha t}{{l_c}^2}$$
$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-\mathrm{Bi}\cdot\mathrm{Fo}}$$

Convection

1. Newton's law of cooling

$$d\dot{Q} = h dA\,(T_s - T_\infty)$$
$$\dot{Q} = (T_s - T_\infty)\int h dA$$

2. Non-dimensional numbers

$$\mathrm{Re}_L = \frac{\rho V L}{\mu}$$
$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu c}{\kappa}$$
$$\mathrm{Nu} = \frac{hx}{\kappa}$$
$$\mathrm{St} = \frac{h}{\rho V c}$$
$$\mathrm{Pc} = \frac{Vx}{\alpha} = \mathrm{Re}\times\mathrm{Pr}$$
$$\mathrm{Gr} = \frac{\beta g \Delta T {l_c}^3}{\nu^2}$$
$$\mathrm{Ra} = \mathrm{Gr}\times\mathrm{Pr} = \frac{g\beta\Delta T l_c^3}{\nu\times\alpha}$$
$$\mathrm{Sc} = \frac{\mu}{\rho\alpha_m}$$
$$\mathrm{Le} = \frac{\mathrm{Sc}}{\mathrm{Pr}}$$

3. Forced convection over horizontal plate

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{1/3}$$
$$T_f = \frac{T_s + T_\infty}{2}$$

(a) Laminar boundary layer

$$\mathrm{Nu}_x = 0.332\times{\mathrm{Re}_x}^{1/2}\mathrm{Pr}^{1/3}$$
$$\overline{\mathrm{Nu}_l} = 0.644\times{\mathrm{Re}_l}^{1/2}\mathrm{Pr}^{1/3}$$

(b) Turbulent boundary layer

$$\mathrm{Nu}_x = 0.0296\times{\mathrm{Re}_x}^{4/5}\mathrm{Pr}^{1/3}$$
$$\overline{\mathrm{Nu}_l} = \frac{4}{3}\mathrm{Nu}_l$$

Fully-developed pipe flow

(a) Laminar flow
At constant flux

$$\overline{\mathrm{Nu}_D} = \frac{\overline{h}D}{\kappa} = 4.364$$

At constant wall temperature

$$\overline{\mathrm{Nu}_D} = \frac{\overline{h}D}{\kappa} = 3.66$$

(b) Turbulent flow

$$\mathrm{Nu}_D = 0.027\times{\mathrm{Re}_D}^{4/5}\mathrm{Pr}^{1/3}\times\left(\frac{\mu_f}{\mu_s}\right)^{0.14}$$

4. Boundary layer analogies

(a) Reynolds analogy

$$\mathrm{St}_x = \frac{\mathrm{Nu}_x}{\mathrm{Re}_x\mathrm{Pr}} = \frac{C_{fx}}{2}$$

(b) Chilton-Colburn analogy

$$\mathrm{St}_x\mathrm{Pr}^{2/3} = \frac{C_{fx}}{2} = j_H$$

Radiation

1. Surface properties

$$\tau = \frac{Q_t}{Q},\quad \alpha = \frac{Q_a}{Q},\quad \rho = \frac{Q_r}{Q}$$
$$\alpha+\rho+\tau = 1$$
$$I_\phi = \frac{dE_\phi}{\cos\phi\times d\omega}$$
$$I = \int_0^\infty I_\lambda d\lambda$$

2. Lambert's cosine law

$$dQ_\phi = dQ_n\cos\phi$$
$$I_\phi = I_n$$

3. Hemispherical emissivity

$$E_\lambda = \pi I_{n_\lambda}$$

4. Planck's distribution law

$$E_\lambda = \pi I_\lambda = \frac{C_1}{\lambda^5\left\{\exp\left(\frac{C_2}{\lambda T}\right)-1\right\}}$$
$$C_1 = 0.374\times10^{-5}\ \mathrm{m^3W}$$
$$C_2 = 1.439\times10^{-2}\ \mathrm{mK}$$

5. Wein's displacement law

$$\lambda_m T = 0.2898\times10^{-2}\mathrm{mK}$$
$$\frac{E_{\lambda_m}}{T^5} = 1.307\times10^{-5}\mathrm{W/m^2K^5}$$

6. Stefan-Boltzmann law

$$E_b = \int_0^\infty E_\lambda d\lambda = \sigma T^4$$
$$\sigma = \frac{C_1}{15}\left(\frac{\pi}{C_2}\right)^4 = 5.67\times10^{-8}\mathrm{W/m^2K^4}$$

7. Emissivity

$$E = \epsilon E_b = \epsilon\sigma T^4$$

8. Net radiation

$$\dot{Q}_{net} = \epsilon_1\epsilon_2\sigma\left({T_1}^4-{T_2}^4\right)F_{12}A_1 = \epsilon_1\epsilon_2\sigma\left({T_1}^4-{T_2}^4\right)F_{21}A_2$$

9. Shape factor rules

(a) Two surfaces

$$F_{12}A_1 = F_{21}A_2$$

(b) Shape factors of a surface

$$F_{11}+F_{12}+\ldots.+F_{1n} = 1$$

(c) Convex or plane surface

$$F_{11} = 0$$

(d) Concave surface

$$F_{11} \neq 0$$

10. Electrical analogy

(a) Surface resistance

$$R_s = \frac{(1-\epsilon)}{\epsilon A}$$

(b) Shape factor resistance

$$R_{sf} = \frac{1}{A_1F_{12}} = \frac{1}{A_2F_{21}}$$

11. Heat shields

$$\frac{\dot{Q}\text{ with }n\text{ shields}}{\dot{Q}\text{ without shields}} = \frac{1}{n+1}$$

Heat Exchangers

1. LMTD method

$$\mathrm{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

2. NTU method

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}}$$
$$\dot{Q}_{max} = C_{min}\,(T_{hi}-T_{ci})$$
$$\mathrm{NTU} = \frac{UA}{C_{min}}$$
$$r = \frac{C_{min}}{C_{max}}$$

Parallel flow HE

$$\varepsilon = \frac{1-\exp\{-\mathrm{NTU}\,(1+r)\}}{1+r}$$

Counter flow HE

$$\varepsilon = \frac{1-\exp\{-\mathrm{NTU}\,(1-r)\}}{1-r\exp\{-\mathrm{NTU}\,(1-r)\}}$$

SOLVED EXAMPLES

1. A furnace is made of a red brick wall of thickness 250 cm and conductivity 1.2 W/mK. Determine the thickness of a layer of diatomic earth of conductivity 0.15 W/mK that can be used to replace the brick wall for the same heat loss and temperature drop.

Solution. Given that

$$\begin{aligned}\kappa_1 &= 1.2 \text{ W/mK}\\ \kappa_2 &= 0.15 \text{ W/mK}\\ x_1 &= 0.25 \text{ m}\end{aligned}$$

Using Fourier's equation:

$$Q = \frac{\kappa A \Delta t}{x}$$

For constant $\dot{Q}$ and κ:

$$x_2 = \frac{\kappa_2}{\kappa_1} x_1$$

Therefore,

$$x_1 = 3.12 \text{ cm}$$

2. A 25 cm thick plate of insulation has its two surfaces kept at 350°C and 225°C. Thermal conductivity of the wall varies linearly with temperature and its values at 350°C and 180°C are 25 W/mK and 15 W/mK, respectively. Determine the steady heat flux through the wall.

Solution. Overall thermal conductivity of the wall is

$$\begin{aligned}\bar{\kappa} &= \frac{\kappa_1 + \kappa_2}{2}\\ &= 20 \text{ W/mK}\end{aligned}$$

$$\begin{aligned}\frac{\dot{Q}}{A} &= \bar{\kappa}\frac{\Delta T}{l}\\ &= 10 \text{ kW}\end{aligned}$$

3. A steam-carrying steel-pipe has inner and outer diameter of 85 mm and 110 mm, respectively. It is covered with an insulation having a thermal conductivity of 1.8 W/mK. The convective heat transfer coefficient between the surface of insulation and the surrounding air is 14.5 W/m^{2}K. Determine the critical radius of insulation.

Solution. The critical radius of insulation for cylindrical elements is

$$\begin{aligned}r_c &= \frac{\kappa}{h}\\ &= \frac{1.8}{14.5}\\ &= 12.41 \text{ cm}\end{aligned}$$

4. A current wire of 18 mm diameter is exposed to air where convective heat transfer coefficient is $h = 25$ W/m^{2}K. The wire is to be laid with an insulating layer of thermal conductivity 0.6 W/mK. Determine the thickness of the insulation for maximum heat dissipation.

Solution. Maximum heat dissipation occurs when insulation is laid upto critical radius. For cylindrical (wire) elements:

$$\begin{aligned}r_c &= \frac{\kappa}{h_o}\\ &= \frac{0.6}{25}\\ &= 24 \text{ mm}\end{aligned}$$

Thus, insulation thickness should be

$$\begin{aligned}t &= 24 - \frac{18}{2}\\ &= 15 \text{ mm}\end{aligned}$$

5. The temperature distribution in a stainless fin (thermal conductivity 0.2 W/cm-°C) of constant cross-sectional area of 4 cm^2 and length of 2 cm, exposed to an ambient temperature of 25°C (with a surface heat transfer coefficient of 0.004 W/cm-°C) is given by

$$T - T_\infty = 6x^2 - 8x + 5$$

where T is in °C and x in in cm. Base temperature of the fin is 120°C. Estimate the rate of heat dissipation through the fin surface.

Solution. Given that

$$\begin{aligned}\kappa &= 0.2 \text{ W/cm}^\circ\text{C}\\ A &= 4 \text{ cm}^2\\ h &= 0.004 \text{ W/cm}^\circ \text{ C}\end{aligned}$$

Using

$$\begin{aligned}\dot{Q} &= -\kappa A \left(\frac{\partial T}{\partial x}\right)_{x=0}\\ &= -0.2 \times 4\,(-8)\\ &= 6.4 \text{ W}\end{aligned}$$

6. A boundary layer over a flat plate has hydrodynamic boundary layer thickness of 0.75 mm. The fluid has dynamic viscosity 30×10^{-6} Pa·s, specific heat 1.8 kJ/kgK and thermal conductivity 0.05 W/mK. Determine the thermal boundary layer thickness.

Solution. Given that

$$\delta = 0.75 \text{ mm}$$
$$\mu = 30 \times 10^{-6} \text{ Pa.s}$$
$$c = 1.8 \times 10^{3} \text{ J/kgK}$$
$$\kappa = 0.05 \text{ W/mK}$$

Prandtl number is calculated by

$$\begin{aligned}\text{Pr} &= \frac{\mu c}{\kappa} \\ &= 1.08\end{aligned}$$

Using

$$\begin{aligned}\frac{\delta}{\delta_t} &= \text{Pr}^{1/3} \\ \delta_t &= \frac{\delta}{\text{Pr}^{1/3}} \\ &= 0.73 \text{ mm}\end{aligned}$$

7. Determine the heat transfer coefficient from a 120 mm diameter steel pipe placed horizontally in ambient at 25°C. Nusselt number is 30 and thermal conductivity of air is 0.05 W/mK.

Solution. Given that

$$D = 0.12 \text{ m}$$
$$\text{Nu} = 30$$
$$\kappa = 0.05 \text{ W/mK}$$

Heat transfer coefficient (W/m^{2}K) is given by

$$\begin{aligned}\text{Nu} &= \frac{hD}{\kappa} \\ h &= \frac{\text{Nu} \times \kappa}{D} \\ &= 12.5 \text{ W/m}^2\text{K}\end{aligned}$$

8. Determine the the net radiation per square meter between two very large plates at temperatures 327°C and 127°C. The emissivity of the hot and cold plates are 0.75 and 0.65, respectively. Stefan Boltzmann constant is 5.67×10^{-8} W/m^{2}K.

Solution. Given that

$$\begin{aligned}A &= 1 \text{ m}^2 \\ T_1 &= 327 + 273 \\ &= 600 \text{ K} \\ T_2 &= 127 + 273 \\ &= 400 \text{ K} \\ \epsilon_1 &= 0.75 \\ \epsilon_2 &= 0.65 \\ \sigma &= 5.67 \times 10^{-8} \text{ W/m}^2\text{K}\end{aligned}$$

Therefore, using

$$\begin{aligned}Q_{1-2} &= \frac{A\sigma\left({T_1}^4 - {T_2}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \\ &= 3.15 \text{ kW/m}^2\end{aligned}$$

9. Two long parallel surfaces have emissivity of 0.85 each. Determine the number of thin parallel shields of equal emissivity to reduce the exchange by 80%.

Solution. The resistance to heat transfer without shield is R_1, therefore,

$$\begin{aligned}\frac{\dot{Q} \text{ with } n \text{ shields}}{\dot{Q} \text{ without shields}} &= \frac{1}{n+1} \\ \frac{1 - 0.80}{1} &= \frac{1}{n+1} \\ n + 1 &= \frac{1}{0.2} \\ n &= 5 - 1 \\ &= 4\end{aligned}$$

10. A counter flow shell and tube heat exchanger is used to heat water with hot exhaust gases. The water (c_p = 4180 J/kgC) flows at a rate of 2 kg/s while the exhaust gas (c_p = 1030 J/kgC) flows at the rate of 5.25 kg/s. If the heat transfer surface area is 32.5 m^2 and the overall heat transfer coefficient is 200 W/m^{2}C, what is the NTU for the heat exchanger?

Solution. Given that

$$\begin{aligned}U_o &= 200 \text{ W/m}^2\text{C} \\ A &= 32.5 \text{ m}^2 \\ C_c &= 2 \times 4180 \\ \dot{m}_h &= 5.2 \text{ kg/s} \\ C_h &= 1030 \times 5.25 \\ &= C_{min}\end{aligned}$$

Therefore,

$$\begin{aligned}\text{NTU} &= \frac{U_o A}{C_{min}} \\ &= \frac{200 \times 32.5}{1030 \times 5.25} \\ &= 1.2\end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. A plate having 10 cm^2 area each side is hanging in the middle of a room of 100 m^2 total surface area. The plate temperature and emissivity are, respectively, 800 K and 0.6. The temperature and emissivity values for the surfaces of the room are 300 K and 0.3, respectively. Boltzmann's constant $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4. The total heat loss from the two surfaces of the plate is

(a) 13.66 W (b) 27.32 W
(c) 27.87 W (d) 13.66 W

(GATE 2003)

Solution. Given that

$$\begin{aligned} A_1 &= 10 \times 10^{-4} \text{ m}^2 \\ A_2 &= 100 \text{ m}^2 \\ \epsilon_1 &= 0.6 \\ T_1 &= 800 \text{ K} \\ \epsilon_2 &= 0.3 \\ T_2 &= 300 \text{ K} \\ \sigma &= 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \end{aligned}$$

Heat exchange between two surfaces is

$$\begin{aligned} \dot{Q}_{net\ 1\to 2} &= \frac{\sigma\left({T_1}^4 - {T_2}^4\right)}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}} \\ &= 27.3173 \text{ W} \end{aligned}$$

Ans. (b)

2. In a counter flow heat exchanger, for the hot fluid, the heat capacity = 2 kJ/kg K, mass flow rate = 5 kg/s, inlet temperature = 150°C, outlet temperature = 100°C. For the cold fluid, heat capacity = 4 kJ/kg K, mass flow rate = 10 kg/s, inlet temperature = 20°C. Neglecting heat transfer to the surroundings, the outlet temperature of the cold fluid in °C is

(a) 7.5 (b) 32.5
(c) 45.5 (d) 70.0

(GATE 2003)

Solution. Given that

$$\begin{aligned} c_h &= 2 \times 10^3 \text{ J/kg.K} \\ m_h &= 5 \text{ kg/s} \\ T_{hi} &= 150^\circ \\ T_{ho} &= 100^\circ \\ c_c &= 4 \times 10^3 \text{ J/kg.K} \\ m_c &= 10 \text{ kg/s} \\ T_{ci} &= 20^\circ\text{C} \end{aligned}$$

For heat balance,

$$\begin{aligned} m_c c_c \left(T_{co} - T_{ci}\right) &= m_h c_h \times \left(T_{hi} - T_{ho}\right) \\ T_{co} &= \frac{m_h c_h \times \left(T_{hi} - T_{ho}\right)}{m_c c_c} + T_{ci} \\ &= 32.5^\circ\text{C} \end{aligned}$$

Ans. (b)

3. Consider a laminar boundary layer over a heated flat plate. The free stream velocity is U_∞. At some distance x from the leading edge the velocity boundary layer thickness is δ_v and the thermal boundary layer thickness is δ_T. If the Prandtl number is greater than 1, then

(a) $\delta_v > \delta_T$
(b) $\delta_T > \delta_v$
(c) $\delta_T \approx \delta_T \sim (U_\infty x)^{-1/2}$
(d) $\delta_T \approx \delta_T \sim x^{-1/2}$

(GATE 2003)

Solution. Polhausen equation relates the boundary layer thickness δ and thermal boundary layer thickness δ_t:

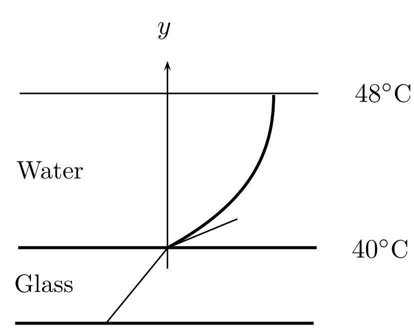
$$\frac{\delta_v}{\delta_T} = Pr^{1/3}$$

As Pr$> 1 \Rightarrow \delta_v > \delta_T$.

Ans. (a)

Common Data Questions

Heat is being transferred by convection from water at 48°C to a glass plate whose surface that is exposed to the water is at 40°C. The thermal conductivity of water is 0.6 W/mK and the thermal conductivity of glass is 1.2 W/mK. The spatial gradient of temperature in the water at the water-glass interface is $dT/dy = 1 \times 10^4$ K/m.



4. The value of the temperature gradient in the glass at the water-glass interface in K/m is

(a) -2×10^4 (b) 0.0
(c) 0.5×10^4 (d) 2×10^4

(GATE 2003)

Solution. Given that

$$\kappa_w = 0.6 \text{ W/mK}$$
$$\kappa_g = 1.2 \text{ W/mK}$$
$$\left(\frac{dT}{dy}\right)_w = 1\times 10^4 \text{ K/m}$$

For the same heat transfer rate (say per unit area)

$$\dot{Q} = \kappa_w \left(\frac{\partial T}{\partial y}\right)_w = \kappa_g \left(\frac{\partial T}{\partial y}\right)_g$$

Therefore,

$$\left(\frac{\partial T}{\partial y}\right)_g = \frac{\kappa_w}{\kappa_g}\left(\frac{\partial T}{\partial y}\right)_w = 5000 \text{ K/m}$$

Ans. (c)

5. The heat transfer coefficient h in W/m^2 K is

(a) 0.0 (b) 4.8
(c) 6 (d) 750

(GATE 2003)

Solution. Given that

$$T_w = 48^\circ$$
$$T_g = 40^\circ$$

Heat transfer coefficient will be given by

$$h\,(T_w - T_g) = \kappa_w \left(\frac{\partial T}{\partial y}\right)_w$$
$$h = 750 \text{ W/m}^2$$

Ans. (d)

6. One-dimensional unsteady state heat transfer equation for a sphere with heat generation at the rate of $\dot{q}$ can be written as

(a) $\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right) + \dfrac{\dot{q}}{\kappa} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial r}$

(b) $\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial T}{\partial r}\right) + \dfrac{\dot{q}}{\kappa} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial r}$

(c) $\dfrac{\partial^2 T}{\partial r^2} + \dfrac{\dot{q}}{\kappa} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial r}$

(d) $\dfrac{\partial^2}{\partial r^2}(rT) + \dfrac{\dot{q}}{\kappa} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial r}$

(GATE 2004)

Solution. Can be derived using basic principles as discussed in the theory.

Ans. (b)

7. A stainless steel tube ($\kappa_s = 19$ W/mK) of 2 cm ID and 5 cm OD is insulated with 3 cm thick asbestos ($\kappa_a = 0.2$ W/mK). If the temperature difference between the innermost and outermost surfaces is 600°C, the heat transfer rate per unit length is

(a) 0.94 W/m (b) 9.44 W/m
(c) 944.72 W/m (d) 9447.21 W/m

(GATE 2004)

Solution. Given that

$$\kappa_s = 19 \text{ W/mK}$$
$$\kappa_a = 0.2 \text{ W/mK}$$
$$r_1 = \frac{0.02}{2} = 0.01 \text{ cm}$$
$$r_2 = \frac{0.05}{2} = 0.025 \text{ cm}$$
$$t_a = 0.03 \text{ cm}$$
$$r_3 = r_2 + t_a$$
$$\Delta T = 600^\circ\text{C}$$
$$l = 1 \text{ m}$$

Thermal resistances of steel and asbestos are

$$R_s = \frac{\ln r_2/r_1}{2\pi\kappa_s l} = 0.00767538 \text{ K/W}$$
$$R_a = \frac{\ln r_3/r_2}{2\pi\kappa_a l} = 0.627434 \text{ K/W}$$

Equivalent resistance of R_s and R_a in series is

$$R = R_s + R_a = 0.63511 \text{ K/W}$$

Therefore, rate of heat transfer is

$$\dot{Q} = \frac{\Delta T}{R} = 944.719 \text{ W/m}$$

Ans. (c)

8. A spherical thermocouple junction of diameter 0.706 mm is to be used for the measurement of temperature of a gas stream. The convective heat transfer coefficient on the bead surface is 400 $\text{W/m}^2\text{K}$. Thermophysical properties of thermocouple material are $\kappa = 20$ W/mK, $c = 400$

J/kg·K and $\rho = 8500$ kg/m^3. If the thermocouple initially at 30°C is placed in a hot stream of 300°C, the time taken by the bead to reach 298°C, is

(a) 2.35 s (b) 4.9 s
(c) 14.7 s (d) 29.4 s

(GATE 2004)

Solution. Given that

$$\begin{aligned} d &= 0.706 \times 10^{-3} \text{ m} \\ r &= \frac{d}{2} \\ h &= 400 \text{ W/m}^2\text{K} \\ \kappa &= 20 \text{ W/mK} \\ c &= 400 \text{ J/kg K} \\ \rho &= 8500 \text{ kg/m}^3 \\ T_1 &= 30°\text{C} \\ T_2 &= 298°\text{C} \\ T_\infty &= 300°\text{C} \end{aligned}$$

Surface area and volume of the material are

$$\begin{aligned} A &= 4\pi r^2 \\ V &= \frac{4}{3}\pi r^3 \end{aligned}$$

Using

$$\frac{T_2 - T_\infty}{T_1 - T_\infty} = \exp\left(-\frac{hA}{\rho V c}t\right)$$

Time required for temperature reaching from T_1 to T_2 is

$$\begin{aligned} t &= \frac{\rho V c}{hA} \ln\left(\frac{T_1 - T_\infty}{T_2 - T_\infty}\right) \\ &= 4.90609 \text{ s} \end{aligned}$$

Ans. (b)

9. In a condenser, water enters a 30°C and flows at the rate 1500 kg/h. The condensing steam is at a temperature of 120°C and cooling water leaves the condenser at 80°C. Specific heat of water is 4.187 kJ/kg K. If the overall heat transfer coefficient is 2000 W/m^2 K, the heat transfer area is

(a) 0.707 m^2 (b) 7.07 m^2
(c) 70.7 m^2 (d) 141.4 m^2

(GATE 2004)

Solution. Given that

$$\begin{aligned} \dot{m}_w &= \frac{1500}{3600} \text{ kg/s} \\ T_{ci} &= 30°\text{C} \\ T_{co} &= 80°\text{C} \\ T_{hi} &= T_{ho} = 120° \\ c_w &= 4.187 \text{ kJ/kgK} \\ U_o &= 2000 \text{ W/m}^2\text{K} \end{aligned}$$

Therefore,

$$\begin{aligned} \text{LMTD} &= \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} \\ &= \frac{(120-30)-(120-80)}{\ln\{(120-30)/(120-80)\}} \end{aligned}$$

Temperature difference in water is

$$\begin{aligned} \Delta T_w &= 80 - 30 \\ &= 50°\text{C} \end{aligned}$$

Let A be the heat transfer area. For heat balance,

$$\begin{aligned} A \times U_o \times \text{LMTD} &= \dot{m}_w c_w \Delta T_w \\ A &= \frac{\dot{m}_w c_w \Delta T_w}{U_o \times \text{LMTD}} \end{aligned}$$

Putting values in the above equation, one gets

$$A = 0.707368 \text{ m}^2$$

Ans. (a)

10. In the case of one-dimensional heat conduction in a medium with constant properties, T is the temperature at position x, at time t. then $\partial T/\partial t$ is proportional to

(a) T/x (b) $\partial T/\partial x$
(c) $\partial^2 T/(\partial x \partial t)$ (d) $\partial^2 T/\partial x^2$

(GATE 2005)

Solution. Using one-dimensional heat conduction equation,

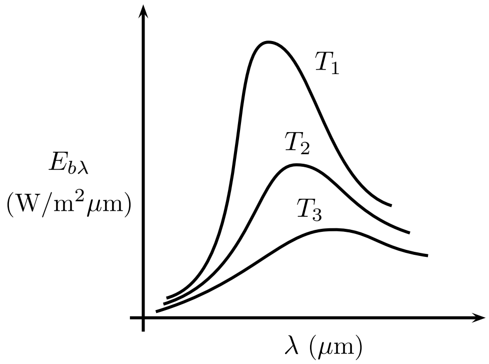
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

one finds that

$$\frac{\partial T}{\partial t} \propto \frac{\partial^2 T}{\partial x^2}$$

Ans. (d)

11. The following figure was generated from experimental data relating spectral black body emissive power to wavelength at three temperatures $T_1 > T_2 > T_3$.



The conclusion is that the measurements are

(a) correct because the maxima in $E_{b\lambda}$ shows the correct trend
(b) correct because Planck's law is satisfied
(c) wrong because the Stefan Boltzmann law is not satisfied
(d) wrong because Wien's displacement law is not satisfied

(GATE 2005)

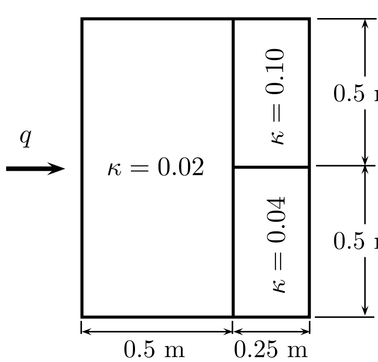
Solution. According to Wien's displacement law

$$\lambda_m T = 0.2898 \times 10^{-2} \text{ mK}$$

For which when λ_m increases, T decreases. It is true when $T_1 < T_2 < T_3$.

Ans. (d)

12. Heat flows through a composite slab, as shown below. The depth of the slab is 1 m. The κ values are in W/m K.



The overall thermal resistance in K/W is

(a) 17.2 (b) 21.9
(c) 28.6 (d) 39.2

(GATE 2005)

Solution. Thermal resistance of rectangular wall of thickness width l, area A, and thermal conductivity κ is given by

$$R = \frac{l}{\kappa A}$$

For given case, thermal resistance R_i for the three segments are

$$R_1 = \frac{0.5}{0.02 \times (1 \times 1)} = 25 \text{ K/W}$$
$$R_2 = \frac{0.25}{0.10 \times (0.5 \times 1)} = 5 \text{ K/W}$$
$$R_3 = \frac{0.25}{0.04 \times (0.5 \times 1)} = 12.5 \text{ K/W}$$

The equivalent thermal resistance is

$$R = R_1 + \frac{1}{1/R_2 + 1/R_3}$$
$$= 28.5714 \text{ K/W}$$

Ans. (c)

13. A small copper ball of 5 mm diameter at 500 K is dropped into an oil bath whose temperature is 300 K. The thermal conductivity of copper is 400 W/mK, its density 9000 kg/m^3 and its specific heat 385 J/kg K. If the heat transfer coefficient is 250 W/m^{2}K and lumped analysis is assumed to be valid, the rate of fall of the temperature of the ball at the beginning of cooling will be in K/s

(a) 8.7 (b) 13.9
(c) 17.3 (d) 27.7

(GATE 2005)

Solution. Given that

$$d = 0.005 \text{ m}$$
$$r = \frac{d}{2} = 0.0025 \text{ m}$$
$$T_0 = 500 \text{ K}$$
$$T_\infty = 300 \text{ K}$$
$$\kappa = 400 \text{ W/mk}$$
$$\rho = 9000 \text{ kg/m}^3$$
$$c = 385 \text{ J/kgK}$$
$$h = 250 \text{ W/m}^2\text{K}$$

Therefore,

$$A = 4\pi r^2$$
$$V = \frac{4\pi}{3} r^3$$

For lumped heat analysis,

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left(-\frac{hA}{\rho V c} t\right)$$

Using the above equation, one gets

$$\frac{dT}{dt} = -(T_0 - T_\infty)\frac{hA}{\rho V c}$$
$$= -17.316 \text{ K/s}$$

Ans. (c)

14. A solid cylinder (surface 2) is located at the center of a hollow sphere (surface 1). The diameter of the sphere is 1 m, while the cylinder has a diameter and length of 0.5 m each. The radiation configuration factor F_{11} is

(a) 0.375 (b) 0.625
(c) 0.75 (d) 1

(GATE 2005)

Solution. Given that

$$d_1 = 1 \text{ m}$$
$$d_2 = 0.5 \text{ m}$$
$$l_2 = 0.5 \text{ m}$$

Surface areas of the given surfaces are

$$A_1 = 4\pi \times {d_1}^2$$
$$= 3.14159 \text{ m}^2$$
$$A_2 = \pi \times d_2 \times l_2 + 2 \times \frac{\pi \times {d_2}^2}{4}$$
$$= \pi \times 0.5 \times 0.5 + 2 \times \frac{\pi \times 0.5^2}{4}$$
$$= 1.1781 \text{ m}^2$$

Also,

$$F_{22} = 0$$

Using

$$F_{21} + F_{22} = 1$$
$$F_{21} = 1$$

Also,

$$F_{12} = \frac{A_2}{A_1}F_{21}$$
$$= 0.375$$
$$F_{11} = 1 - F_{12}$$
$$= 0.625$$

Ans. (b)

15. Hot oil is cooled from 80°C to 50°C in an oil cooler which uses air as the coolant. The air temperature rises from 30°C to 40°C. The designer uses a LMTD value of 26°C. The type of heat exchanger is

(a) parallel flow (b) double pipe
(c) counter flow (d) cross flow

(GATE 2005)

Solution. LMTD for parallel and counter flow heat exchanger is determined as

$$\text{LTDM}_p = \frac{50-10}{\ln(50/10)}$$
$$= 24.8534°\text{C}$$
$$\text{LTDM}_c = \frac{40-20}{\ln(40/20)}$$
$$= 28.8539°\text{C}$$

The LMTD value used by the design is 26°C which is in between the above two, therefore, this is the case of counter flow heat exchanger.

Ans. (d)

Linked Answer Questions

An un-insulated air conditioning duct of rectangular cross-section 1 m × 0.5 m, carrying air at 20°C with a velocity of 10 m/s is exposed to an ambient of 30°C. Neglect the effect of duct construction material. For air in the range of 20–30°C, data are as follows: thermal conductivity = 0.025 W/mK, viscosity= 18 μPa s, Prandtl number = 0.73, density = 1.2 kg/m^3. The laminar flow Nusselt number is 3.4 for constant wall temperature conditions, and for turbulent flow, Nu = 0.023 Re$^{0.8}$ Pr$^{0.33}$.

16. The Reynolds number for the flow is

(a) 444 (b) 890
(c) 4.44×10^5 (d) 5.22×10^5

(GATE 2005)

Solution. Given that

$$\text{Size of duct} = 1 \times 0.5 \text{ m}^2$$
$$T_a = 20°\text{C}$$
$$V = 10 \text{ m/s}$$
$$T_\infty = 30°\text{C}$$
$$\kappa = 0.025 \text{ W/mK}$$
$$\mu = 18 \times 10^{-6} \text{ Pa s}$$
$$\text{Pr} = 0.73$$
$$\rho = 1.2 \text{ kg/m}^3$$

Hydraulic radius of rectangular section is calculated as

$$L_c = \frac{4A}{P}$$
$$= \frac{4(1 \times 0.5)}{2(1+0.5)}$$
$$= 0.667$$

Reynolds number is calculated as

$$\text{Re} = \frac{\rho V L_c}{\mu} = 4.44 \times 10^5$$

Ans. (c)

17. The heat transfer per meter length of the duct, in watts, is

(a) 3.8 (b) 5.3
(c) 89 (d) 769

(GATE 2005)

Solution. For turbulent flows Re > 2000. Therefore,

$$\text{Nu} = \frac{\bar{h}L_c}{\kappa} = 0.023\text{Re}^{0.8}\text{Pr}^{0.33}$$

Using the given and calculated values,

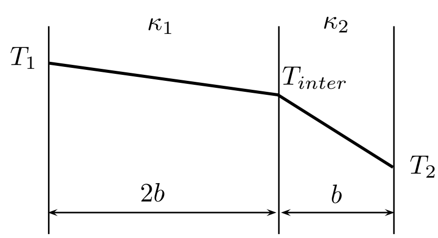
$$\bar{h} = \frac{\text{Nu} \times \kappa}{L_c} = 25.6395 \text{ W/m}^2\text{K}$$

Heat transfer per meter length of the duct is

$$Q = \bar{h} \times 2(1+0.5)(30-20) = 769.185 \text{ W}$$

Ans. (c)

18. In a composite slab, the temperature at the interface (T_{inter}) between two materials is equal to the average of the temperatures at the two ends.



Assuming steady state one dimensional heat conduction, which of the following statements is true about the respective thermal conductivities?

(a) $2\kappa_1 = \kappa_2$ (b) $\kappa_1 = \kappa_2$
(c) $2\kappa_1 = 3\kappa_2$ (d) $\kappa_1 = 2\kappa_2$

(GATE 2006)

Solution. Let T_1 and T_2 be the temperatures at two ends of the composite slab. Given that

$$T_{\text{inter}} = \frac{T_1 + T_2}{2}$$

For unit area, the rate of heat transfer is

$$\dot{Q} = \frac{\kappa_1}{2b}(T_{\text{inter}} - T_1) = \frac{\kappa_2}{b}(T_2 - T_{\text{inter}})$$

Solving the above two equations gives

$$\kappa_1 = 2\kappa_2$$

Ans. (d)

19. With an increase in thickness of insulation around a circular pipe, heat loss to surroundings due to

(a) convection increases, while that due to conduction decreases
(b) convection decreases, while that due to conduction increases
(c) convection and conduction decreases
(d) convection and conduction increases

(GATE 2006)

Solution. With the increase in thickness of insulation, the area for convective heat transfer increases, therefore, convection increases. Opposite to this, increase in thickness increases the thermal resistance for conductive heat transfer.

Ans. (a)

20. The temperature distribution within the thermal boundary layer over a heated isothermal flat plate is given by

$$\frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

where T_w and T_∞ are the temperatures of plate and free stream, respectively, and y is the normal distance measured from the plate. The local Nusselt number based on the thermal boundary layer thickness δ_t is given by

(a) 1.33 (b) 1.50
(c) 2.0 (d) 4.64

(GATE 2007)

Solution. Nusselt number is equal to the dimensionless temperature gradient at the surface. In this case,

$$\left(\frac{dT}{dy}\right)_{y=0} = \frac{3}{2} = 1.5$$

Ans. (b)

21. In a counter flow heat exchanger, hot fluid enters at 60°C and cold fluid enters at 30°C. Mass flow rate of the hot fluid is 1 kg/s and that of the cold fluid is 2 kg/s. Specific heat of the hot fluid is 10

kJ/kg K and that of the cold fluid is 5 kJ/kgK. The log mean temperature difference (LMTD) for the heat exchanger in °C is

(a) 15 (b) 30
(c) 25 (d) 45

(GATE 2007)

Solution. Since heat capacities of the fluids are the same, therefore, temperature difference will be the same at all locations, and it will be taken as LMTD. Therefore,

$$\begin{aligned}\text{LMTD} &= 60-30\\ &= 30^\circ\text{C}\end{aligned}$$

In such cases, the following expression is invalid:

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_2}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

Ans. (b)

22. The average heat transfer coefficient on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temperature with time as it cools. Assume the plate temperature to be uniform at any instant of time and radiation heat exchange with the surroundings negligible. The ambient temperature is 25°C, the plate has a total surface area of 0.1 m^2 and a mass of 4 kg. The specific heat of the plate material is 2.5 kJ/kgK. The convective heat transfer coefficient in W/m^2K at the instant when the plate temperature is 225°C and the change in plate temperature with time $dT/dt = -0.02$ K/s, is

(a) 200 (b) 20
(c) 15 (d) 10

(GATE 2007)

Solution. Given that

$$\begin{aligned}T_\infty &= 25^\circ\text{C}\\ A &= 0.1\ \text{m}^2\\ m &= 4\ \text{kg}\\ c &= 2.5\ \text{kJ/kgK}\\ T_s &= 225^\circ\text{C}\end{aligned}$$

Therefore

$$\begin{aligned}-hA\,(T_s - T_\infty) &= mc\frac{dT}{dt}\\ h &= 9.6\ \text{W/m}^2\text{K}\end{aligned}$$

Ans. (d)

23. A building has to be maintained at 21°C (dry bulb) and 14.5°C (wet bulb). The dew point temperature under these conditions is 10.17°C. The outside temperature is −23°C (dry bulb) and the internal and external surface heat transfer coefficients are 8 W/m^2K and 23 W/m^2K, respectively. If the building wall has a thermal conductivity of 1.2 W/mK, the minimum thickness (in m) of the wall required to prevent condensation is

(a) 0.471 (b) 0.407
(c) 0.321 (d) 0.125

(GATE 2007)

Solution. Given that temperature of air (DBT) inside the room is 21°C. Outside temperature $T_o = -23^\circ$C. For condensation over inside room wall, the temperature will becomes DPT = 10.17°C. Also, $h_i = 8$ W/m^2K, $h_o = 23$ W/m^2K, $\kappa = 1.2$ W/mK. If thickness of wall is t, then heat transfer rate to avoid temperature of DPT over inside wall will be equal to the net heat transfer rate throughout the wall. Therefore, using electrical analogy of thermal resistances, one finds

$$\begin{aligned}\frac{\text{DBT} - T_o}{1/h_i + t/\kappa + 1/h_o} &= \frac{\text{DBT} - \text{DPT}}{1/h_i}\\ \frac{21-(-23)}{1/8 + t/1.2 + 1/23} &= \frac{21 - 10.17}{1/8}\\ t &= 0.407244\ \text{m}\end{aligned}$$

Ans. (b)

Linked Answer Questions

Consider steady one-dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of 80 MW/m^3. The left and right faces are kept at constant temperature of 160°C and 120°C, respectively. The plate has a constant thermal conductivity of 200 W/mK.

24. The location of maximum temperature within the plate from its left face is

(a) 15 mm (b) 10 mm
(c) 5 mm (d) 0 mm

(GATE 2007)

Solution. Given that

$$\begin{aligned}t &= 20\ \text{mm}\\ \dot{q} &= 80\times 10^6\ \text{W/m}^3\\ T_1 &= 160^\circ\text{C}\\ &= 433\ \text{K}\\ T_2 &= 120^\circ\text{C}\\ &= 393\ \text{K}\\ \kappa &= 200\ \text{W/mK}\end{aligned}$$

The steady state equation in rectangular coordinates is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\kappa} = 0$$

$$\frac{\partial T}{\partial x} = -\frac{\dot{q}}{\kappa}x + C_1 = 0$$

$$T = -\frac{\dot{q}}{2\kappa}x^2 + C_1 x + C_2 = 0$$

Applying boundary conditions in the above equation,

(a) At $x = 0$, $T = 433$ K

$$C_2 = 433$$

(b) At $x = 0.020$, $T = 393$ K

$$C_1 = 2000$$

For maximum temperature,

$$\frac{\partial T}{\partial x} = 0$$

$$-\frac{\dot{q}}{\kappa}x + C_1 = 0$$

Therefore,

$$x = \frac{C_1 \kappa}{\dot{q}} = \frac{2000 \times 200}{80 \times 10^6} = 0.005 \text{ m} = 5 \text{ mm}$$

Ans. (c)

25. The maximum temperature within the plate in °C is

(a) 160 (b) 165
(c) 200 (d) 250

(GATE 2007)

Solution. Putting $x = 0.005$ in

$$T = -\frac{\dot{q}}{2\kappa}x^2 + C_1 x + C_2$$

one gets

$$T = 438 \text{ K} = 165°\text{C}$$

Ans. (b)

26. For flow of fluid over a heated plate, the following fluid properties are known:

Viscosity	= 0.001 Pa s
Specific heat (c_p)	=1 kJ/kg K
Thermal conductivity	= 1 W/m K

The hydrodynamic boundary layer thickness at a specified location on the plate is 1 mm. The thermal boundary layer thickness at the same location is

(a) 0.001 mm (b) 0.01 mm
(c) 1 mm (d) 1000 mm

(GATE 2008)

Solution. Polhausen equation relates the boundary layer thickness δ and thermal boundary layer thickness δ_t:

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{1/3}$$

where Pr is Prandtl number, a property of the fluid defined as the ratio of kinematic viscosity and thermal diffusivity:

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu c}{\kappa}$$

Given that

$$c = 1 \text{ kJ/kg K}$$
$$\kappa = 1 \text{ W/m K}$$
$$\mu = 0.001 \text{ Pa.s}$$

Therefore,

$$\mathrm{Pr} = \frac{0.001 \times 1000}{1} = 1$$

Also, $\delta = 1$ mm, therefore,

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} = \frac{1}{1^{1/3}} = 1 \text{ mm}$$

Ans. (c)

27. The logarithmic mean temperature difference (LMTD) of a counter flow heat exchanger is 20°C. The cold fluid enters at 20°C and the hot fluid enters at 100°C. Mass flow rate of the cold fluid is twice that of the hot fluid. Specific heat at constant pressure of the hot fluid is twice that of

the cold fluid. The exit temperature of the cold fluid is

(a) 40°C
(b) 60°C
(c) 80°C
(d) Cannot be determined

(GATE 2008)

Solution. Given that

$$\begin{aligned} \text{LMTD} &= 20°\text{C} \\ T_{hi} &= 100°\text{C} \\ T_{ci} &= 20°\text{C} \\ \dot{m}_c &= 2\dot{m}_h \\ c_h &= 2c_c \end{aligned}$$

For counter flow heat exchangers,

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

where

$$\begin{aligned} \Delta T_2 &= T_{hi} - T_{co} \\ \Delta T_1 &= T_{ho} - T_{ci} \\ \Delta T_2 &> \Delta T_1 \end{aligned}$$

With LMTD definition, the value of ΔT_1, ΔT_2 can be determined. Using

$$\dot{m}_c c_c (T_{co} - T_{ci}) = \dot{m}_h c_h (T_{hi} - T_{ho})$$

$$2\dot{m}_h c_c (T_{co} - T_{ci}) = \dot{m}_h 2c_c (T_{hi} - T_{ho})$$

one finds

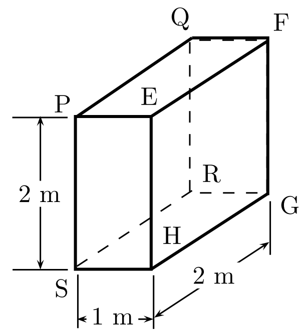
$$\begin{aligned} T_{co} - T_{ci} &= T_{hi} - T_{ho} \\ T_{co} + T_{ho} &= T_{hi} + T_{ci} \\ &= 120 \\ T_{ho} &= 120 - T_{co} \end{aligned}$$

Putting this value into LMTD equation gives

$$T_{co} = 68.36°\text{C}$$

Ans. (b)

28. For the three-dimensional object shown in the figure below, five faces are insulated. This sixth face (PQRS), which is not insulated, interacts thermally with the ambient, with a convective heat transfer coefficient of 10 W/m^{2}K. The ambient temperature is 30°C. Heat is uniformly generated inside the object at the rate of 100 W/m^3.



Assuming the face PQRS to be at uniform temperature, its steady state temperature is

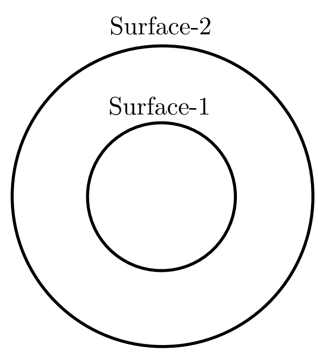
(a) 10°C (b) 20°C
(c) 30°C (d) 40°C

(GATE 2008)

Solution. Since heat generated will be transferred through PQRS, its temperature should be greater than the ambient temperature 30°C. In the given options, it is 40°C

Ans. (d)

29. A hollow enclosure is formed between two infinitely long concentric cylinders of radii 1 m and 2 m, respectively. Radiative heat exchange takes place between the inner surface of the larger cylinder (surface-2) and the outer surface of the smaller cylinder (surface-1). The radiating surfaces are diffused and the medium in the enclosure is non-participating.



The fraction of the thermal radiation leaving the larger surface and striking itself is

(a) 0.25 (b) 0.50
(c) 0.75 (d) 1.00

(GATE 2008)

Solution. For concave surfaces,

$$F_{11} = 0$$

For surface-1,

$$\begin{aligned} F_{11} + F_{12} &= 1 \\ F_{12} &= 1 \end{aligned}$$

Also,

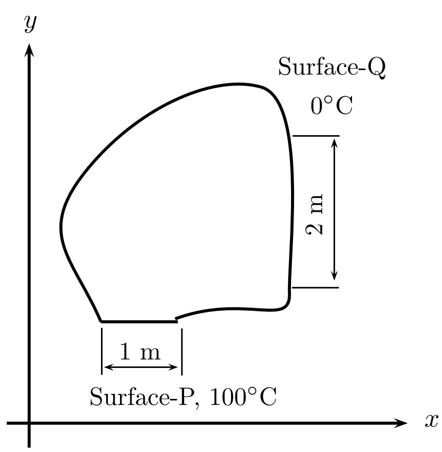
$$\begin{aligned}F_{21} &= F_{12}\frac{A_1}{A_2}\\ &= 1\times\frac{1}{2}\\ &= 0.5\end{aligned}$$

For surface-2,

$$\begin{aligned}F_{22}+F_{21} &= 1\\ F_{22} &= 1-0.5\\ &= 0.5\end{aligned}$$

Ans. (b)

30. Steady two-dimensional heat conduction takes place in the body shown in the figure below. The normal temperature gradient over surfaces P and Q can be considered to be uniform. The temperature gradient $\partial T/\partial x$ at surface Q is equal to 10 K/m. Surfaces P and Q are maintained at constant temperatures as shown in the figure, while the remaining part of the boundary is insulated. The body has a constant thermal conductivity of 0.1 W/mK.



The values of $\partial T/\partial x$ and $\partial T/\partial y$ at surface P are

(a) $\partial T/\partial x = 20$ K/m, $\partial T/\partial y = 0$ K/m
(b) $\partial T/\partial x = 0$ K/m, $\partial T/\partial y = 10$ K/m
(c) $\partial T/\partial x = 10$ K/m, $\partial T/\partial y = 10$ K/m
(d) $\partial T/\partial x = 0$ K/m, $\partial T/\partial y = 20$ K/m

(GATE 2008)

Solution. The variation of temperature at Q in x-axis is zero:

$$\frac{\partial T}{\partial x} = 0 \text{ K/m}$$

As heat transfer takes place at P and Q only, therefore, for heat balance, the area multiplied by temperature gradient at points P and Q should be equal to each other. Hence, at P

$$\begin{aligned}\frac{\partial T}{\partial y} &= 10\times 2\\ &= 20 \text{ K/m}\end{aligned}$$

Ans. (d)

31. A coolant fluid at 30°C flows over a heated flat plate maintained at a constant temperature of 100°C. The boundary layer temperature distribution at a given location on the plate can may be approximated as

$$T = 30+70\exp(-y)$$

where y (in m) is the distance normal to the plate and T is in °C. If thermal conductivity of the fluid is 1.0 W/mK, the local convective heat transfer coefficient (in W/m^{2}K) at that location will be

(a) 0.2 (b) 1
(c) 5 (d) 10

(GATE 2009)

Solution. Given that

$$\begin{aligned}T_\infty &= 30°\text{C}\\ T_s &= 100°\text{C}\\ T &= 30+70\exp(-y)\\ \kappa &= 1.0 \text{ W/mK}\end{aligned}$$

For heat balance over a unit area, at $y = 0$,

$$\begin{aligned}-\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0} &= h\,(T_s-T_\infty)\\ -1\times -70 &= h\,(100-30)\\ h &= 1.0 \text{ W/m}^2\text{K}\end{aligned}$$

Ans. (b)

32. In a parallel flow heat exchanger operating under steady state, the heat capacity rates (product of specific heat at constant pressure and mass flow rate) of the hot and cold fluids are equal. The hot fluid, flowing at 1 kg/s with C_p = 4 kJ/kgK, enters the heat exchanger at 102°C while the cold fluid has an inlet temperature of 15°C. The overall heat transfer coefficient for the heat exchanger is estimated to be 1 kW/m^{2}K and the corresponding heat transfer surface area is 5 m^2. Neglect heat transfer between the heat exchanger and the ambient. The heat exchanger is characterized by the following relation:

$$2\varepsilon = 1-\exp(-2\text{NTU})$$

The exit temperature (in °C) for the cold fluid is

(a) 45 (b) 55
(c) 65 (d) 75

(GATE 2009)

Solution. Given that

$$\begin{aligned}\dot{m}_h &= 1 \text{ kg/s}\\ c_h &= 4\times10^3 \text{ kJ/kgK}\\ T_{hi} &= 102°\text{C}\\ T_{ci} &= 15°\text{C}\\ U &= 1\times10^3 \text{ kW/m}^2\text{K}\\ A &= 5 \text{ m}^2\\ C_h &= C_c\\ &= C_{min}\\ &= \dot{m}_h c_h\end{aligned}$$

The NTU is calculated as

$$\begin{aligned}\text{NTU} &= \frac{UA}{C_{\min}}\\ &= 1.25\end{aligned}$$

But given that

$$\begin{aligned}\varepsilon &= \frac{1-\exp(-2\text{NTU})}{2}\\ &= 0.458958\end{aligned}$$

By the definition of effectiveness,

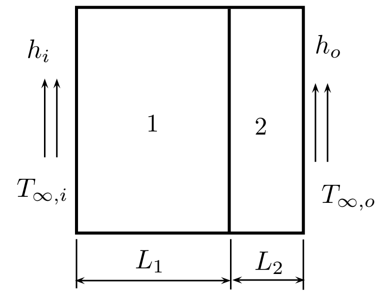
$$\begin{aligned}\varepsilon &= \frac{\dot{Q}}{Q_{\max}}\\ &= \frac{T_{co}-T_{ci}}{T_{hi}-T_{ci}}\end{aligned}$$

Putting the given values in the above equation, one obtains

$$\begin{aligned}0.458958 &= \frac{T_{co}-15}{102-15}\\ T_{co} &= 54.9293°\text{C}\end{aligned}$$

Ans. (b)

33. Consider steady-state heat conduction across the thickness in a plane composite wall (as shown in the figure) exposed to convection conditions on both sides. Given that $h_1 = 20$ W/m^{2}K; $h_o = 50$ W/m^{2}K; $T_{\infty,i} = 20$°C, $T_{\infty,o} = -2$°C, $\kappa_1 = 20$ W/mK, $\kappa_2 = 50$ W/mK, $L_1 = 0.30$ m, $L_2 = 0.15$ m.



Assuming negligible contact resistance between the wall surfaces, the interface temperature, T (in °C), of the two walls will be

(a) −0.50 (b) 2.75
(c) 3.75 (d) 4.50

(GATE 2009)

Solution. From the given details, equivalent thermal resistance for unit cross-sectional area ($A = 1$ m^2) is

$$\begin{aligned}R_{th} &= \frac{1}{h_1A}+\frac{L_1}{\kappa_1A}+\frac{L_2}{\kappa_2A}+\frac{1}{h_0A}\\ &= 0.088\end{aligned}$$

Heat transfer through wall and through a component of the wall should be the same, therefore,

$$\begin{aligned}\frac{T_{\infty,i}-T_{\infty,0}}{R_{\text{th}}} &= \frac{T_{\infty,i}-T_m}{1/(h_1A)+L_1/(\kappa_1A)}\\ T_m &= 3.75°\text{C}\end{aligned}$$

Ans. (c)

Common Data Questions

Radiative heat transfer is intended between the inner surfaces of two very large isothermal parallel metal plates. While the upper plate (designated as plate 1) is a black surface and is the warmer one being maintained at 727°C, the lower plate (plate 2) is a diffused and gray surface with an emissivity of 0.7 and is kept at 227°C. Assume that the surfaces are sufficiently large to form a two-surface enclosure and steady state conditions to exist. Stefan Boltzmann constant is given as 5.67×10^{-8} W/m^{2}K^4.

34. The irradiation (in kW/m^2) for the upper plate (plate 1) is

(a) 2.5 (b) 3.6
(c) 17.0 (d) 19.5

(GATE 2009)

Solution. Given that

$$\begin{aligned}\epsilon_2 &= 0.7\\ T_1 &= 727+273\text{ K}\\ T_2 &= 227+273\text{ K}\\ \sigma &= 5.67\times10^{-8}\text{ W/m}^2\text{K}^4\end{aligned}$$

Therefore,

$$\begin{aligned}Q_2 &= \epsilon_2\sigma {T_2}^4\\ &= 2.480\text{ kW/m}^2\end{aligned}$$

Ans. (a)

35. If plate 1 is also a diffused and gray surface with an emissivity value of 0.8, the net radiation heat exchange (in kW/m^2) between plate 1 and plate 2 is

(a) 17.0 (b) 19.0
(c) 23.0 (d) 31.7

(GATE 2009)

Solution. Now,

$$\begin{aligned}\epsilon_1 &= 0.8\\ \epsilon_2 &= 0.7\end{aligned}$$

Therefore,

$$\begin{aligned}F_{12} &= \frac{1}{1/\epsilon_1+1/\epsilon_2-1}\\ &= \frac{1}{1/0.8+1/0.7-1}\\ &= 0.595745\\ Q_{12} &= F_{12}\sigma\left({T_1}^4-{T_2}^4\right)\\ &= 31.667\text{ kW/m}^2\end{aligned}$$

Ans. (d)

36. A phenomenon is modeled using n-dimensional variables with k primary dimensions. The number of non-dimensional variables is

(a) k (b) n
(c) $n-k$ (d) $n+k$

(GATE 2010)

Solution. Buckingham's π theorem states that the number of independent dimensionless groups that can be formed from a set of n variables having k basic dimensions is $(n-k)$.

Ans. (c)

37. A fin has 5 mm diameter and 100 mm length. The thermal conductivity of fin material is 400 W/mK. One end of the fin is maintained at 130°C and its remaining surface is exposed to ambient air at 30°C. If the convective heat transfer coefficient is 40 W/m^{2}K, the heat loss (in W) from the fin is

(a) 0.08 (b) 5.0
(c) 7.0 (d) 7.8

(GATE 2010)

Solution. Given that

$$\begin{aligned}d &= 0.005\text{ m}\\ l &= 0.1\text{ m}\\ \kappa &= 400\text{ W/mK}\\ T_0 &= 130^\circ\text{C}\\ T_\infty &= 30^\circ\text{C}\\ h &= 40\text{ W/m}^2\text{K}\end{aligned}$$

Therefore, cross-sectional area and perimeter are

$$\begin{aligned}A &= \frac{\pi d^2}{4}\\ p &= \pi d\end{aligned}$$

Assuming long fin, heat transfer rate is

$$\begin{aligned}\dot{Q} &= \sqrt{hp\kappa A}\,(T_0-T_\infty)\\ &= \sqrt{40\times\pi\times0.005\times\pi\times0.05^2/4}\times(130-30)\\ &= 7.02481\text{ W}\end{aligned}$$

Ans. (c)

38. In a condenser of a power plant, the steam condenses at a temperature of 60°C. The cooling water enters at 30°C and leaves at 45°C. The logarithmic mean temperature difference (LMTD) of the condenser is

(a) 16.2°C (b) 21.6°C
(c) 30°C (d) 37.5°C

(GATE 2011)

Solution. Given that

$$\begin{aligned}T_{hi} &= T_{ho}\\ &= 60^\circ\text{C}\\ T_{ci} &= 30^\circ\text{C}\\ T_{co} &= 45^\circ\text{C}\end{aligned}$$

The temperature differences at ends are

$$\begin{aligned}\Delta T_1 &= T_{ho}-T_{co}\\ &= 60-45\\ &= 15^\circ\text{C}\\ \Delta T_2 &= T_{hi}-T_{ci}\\ &= 60-30\\ &= 30^\circ\text{C}\end{aligned}$$

Therefore, LMTD is given by

$$\begin{aligned}\text{LMTD} &= \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} \\ &= \frac{30-15}{\ln(30/15)} \\ &= 21.6404°\text{C}\end{aligned}$$

Ans. (b)

39. A pipe of 25 mm outer diameter carries steam. The heat transfer coefficient between the cylinder and surroundings is 5 W/m^2 K. It is proposed to reduce the heat loss from the pipe by adding insulation having a thermal conductivity of 0.05 W/mK. Which one of the following statements is TRUE?

(a) The outer radius of the pipe is equal to the critical radius
(b) The outer radius of the pipe is less than the critical radius
(c) Adding the insulation will reduce the heat loss
(d) Adding the insulation will increase the heat loss

(GATE 2011)

Solution. Given that

$$\begin{aligned}r &= \frac{25}{2} = 12.5 \text{ mm} \\ h &= 5 \text{ W/m}^2\text{K} \\ \kappa &= 0.05 \text{ W/mK}\end{aligned}$$

Critical radius of insulation for cylindrical insulation is

$$\begin{aligned}r_c &= \frac{\kappa}{h} \\ &= 0.010 \text{ m} \\ &= 10 \text{ mm}\end{aligned}$$

Critical radius is a limitation on the thickness of insulation beyond which insulation will be less effective. For the given case,

$$r_c < r$$

Hence, adding the insulation will reduce heat loss.

Ans. (c)

40. The ratios of the laminar hydrodynamic boundary layer thickness to thermal boundary layer thickness of flows of two fluids P and Q on a flat plate are 1/2 and 2, respectively. The Reynolds number based on the plate length for both the flows is 10^4. The Prandtl and Nusselt numbers for P are 1/8 and 35, respectively. The Prandtl and Nusselt numbers for Q are, respectively,

(a) 8 and 140 (b) 8 and 70
(c) 4 and 70 (d) 4 and 35

(GATE 2011)

Solution. Ratio of laminar hydrodynamic boundary layer thickness (δ) and thermal boundary layer thickness (δ_t) is given by

$$\begin{aligned}\frac{\delta}{\delta_t} &= \text{Pr}^{\frac{1}{3}} \\ \text{Pr} &= \left(\frac{\delta}{\delta_t}\right)^3\end{aligned}$$

Therefore,

$$\begin{aligned}\text{Pr}_Q &= 2^3 \\ &= 8\end{aligned}$$

Nusselt number will be given by

$$\text{Nu} \propto \text{Re}^{1/3}\text{Pr}^{1/3}$$

Therefore, for constant Re,

$$\begin{aligned}\frac{\text{Nu}_Q}{\text{Nu}_P} &= \left(\frac{8}{1/8}\right)^{1/3} \\ \text{Nu}_Q &= 4 \times 35 \\ &= 140\end{aligned}$$

Ans. (a)

41. A spherical steel ball of 12 mm diameter is initially at 1000 K. It is slowly cooled in a surrounding of 300 K. The heat transfer coefficient between the steel ball and the surrounding is 5 W/m^2K. The thermal conductivity of steel is 20 W/mK. The temperature difference between the center and the surface of the steel ball is

(a) Large because conduction resistance is far higher than the convective resistance
(b) Large because conduction resistance is far less than the convective resistance
(c) Small because conduction resistance is far higher than the convective resistance
(d) Small because conduction resistance is far less than the convective resistance

(GATE 2011)

Solution. Given that

$$\begin{aligned}r &= \frac{d}{2} \\ &= 0.006 \text{ m}\end{aligned}$$

Critical length of the spherical body is

$$l_c = \frac{4\pi r^3/3}{4\pi r^2}$$

Biot number is

$$\begin{aligned}\mathrm{Bi} &= \frac{hl_c}{\kappa}\\ &= 0.005\end{aligned}$$

which is very small, which shows conductive resistance is far less than the convective resistance. The temperature at any instant is given by

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-\mathrm{Bi}\cdot\mathrm{Fo}}$$

As Biot number is very small, the difference of T and T_0 will be very small.

Ans. (d)

42. Which one of the following configurations has the highest fin effectiveness?

(a) Thin, closely spaced fins
(b) Thin, widely spaced fins
(c) Thick, widely spaced fins
(d) Thick, closed spaced fins

(GATE 2012)

Solution. Fin effectiveness is defined as the ratio of heat transfer from the fin and that without the fin. In the given options, thin and closely spaced fins will have maximum fin effectiveness because they will provide more heat transfer rate through fins than those without fins.

Ans. (a)

43. For an opaque surface, the absorptivity (α), transitivity (τ), and reflectivity (ρ) are related by the equation

(a) $\alpha+\rho=\tau$ (b) $\alpha+\rho+\tau=0$
(c) $\alpha+\rho=1$ (d) $\alpha+\rho=0$

(GATE 2012)

Solution. For opaque surfaces, $\tau = 0$. Therefore,

$$\begin{aligned}\alpha+\rho+\tau &= 1\\ \alpha+\rho &= 1\end{aligned}$$

Ans. (c)

44. Water ($c_p = 4.18$ kJ/kgK) at 80°C enters a counter flow heat exchanger with a mass flow rate of 0.5 kg/s. Air ($c_p = 1$ kJ/kgK) enters at 30°C with a mass flow rate of 2.09 kg/s. If the effectiveness of the heat exchanger is 0.8, the LMTD (in °C) is

(a) 40 (b) 20
(c) 10 (d) 5

(GATE 2012)

Solution. Given that

$$\begin{aligned}c_h &= 4.18 \text{ kJ/kg K}\\ c_c &= 1.00 \text{ kJ/kg K}\\ t_{hi} &= 80^\circ\text{C}\\ t_{ci} &= 30^\circ\text{C}\\ \dot m_h &= 0.5 \text{ kg/s}\\ \dot m_c &= 2.09 \text{ kg/s}\\ \varepsilon &= 0.8\end{aligned}$$

Using the definition of effectiveness:

$$\begin{aligned}0.8 &= \frac{4.18\times 0.5\,(80 - t_{ho})}{1\times 2.09\,(80-30)}\\ t_{ho} &= 40^\circ\text{C}\end{aligned}$$

Similarly, for heat balance

$$\begin{aligned}(t_{co} - 30) &= \frac{4.18}{2.09}\times 0.5\times 40\\ t_{co} &= 70^\circ\text{C}\end{aligned}$$

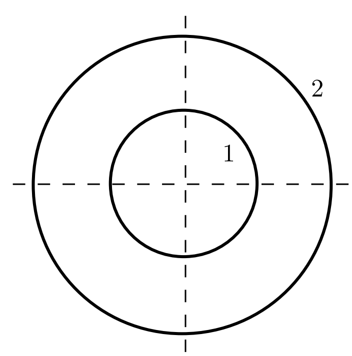
Therefore, temperature differences at inlet and outlet points are

$$\begin{aligned}\Delta T_1 &= 80-70 = 10^\circ\text{C}\\ \Delta T_2 &= 40-30 = 10^\circ\text{C}\end{aligned}$$

Hence, LMTD is also 10°C.

Ans. (c)

45. Consider two infinitely long thin concentric tubes of circular cross-section as shown in the figure.



If D_1 and D_2 are the diameters of the inner and outer tubes, respectively, then the view factor F_{22} is given by

(a) $D_2/D_1 - 1$ (b) Zero
(c) D_1/D_2 (d) $1 - D_1/D_2$

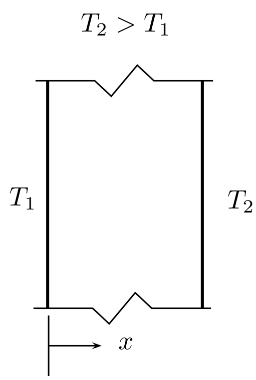
(GATE 2012)

Solution. View factor F_{22} is given by

$$\begin{aligned} F_{22} &= 1 - F_{21} \\ &= 1 - F_{12} \times \frac{A_1}{A_2} \\ &= 1 - 1 \times \frac{\pi D_1 \times l}{\pi D_2 \times l} \\ &= 1 - \frac{D_1}{D_2} \end{aligned}$$

Ans. (d)

46. Consider one-dimensional steady state heat conduction, without heat generation, in a plane wall; with boundary conditions as shown in the figure below.



The conductivity of the wall is given by

$$\kappa = \kappa_0 + bT$$

where κ_0 and b are positive constants and T is the temperature. As x increases, the temperature gradient dT/dx will

(a) remain constant
(b) be zero
(c) increase
(d) decrease

(GATE 2013)

Solution. The rate of heat transfer at any section at x is expressed using Fourier's equation as

$$\dot{Q} = -(\kappa_0 + bT) A \frac{dT}{dx}$$

$$\frac{dT}{dx} \propto \frac{1}{(\kappa_0 + bT)}$$

As seen from the given figure, the temperature increases with increase in x, thus using the above equation, dT/dx will decrease.

Ans. (d)

47. Consider one-dimensional steady state heat conduction along x-axis ($0 \leq x \leq L$) through a plane wall with the boundary surfaces ($x = 0$ and $x = L$) maintained at temperatures 0°C and 100°C. Heat is generated uniformly throughout the wall. Choose the CORRECT statement.

(a) The direction of heat transfer will be from the surface at 100°C to surface at 0°C.
(b) The maximum temperature inside the wall must be greater than 100°C
(c) The temperature distribution is linear within the wall
(d) The temperature distribution is symmetric about the mid-plane of the wall

(GATE 2013)

Solution. This is one-dimensional steady state head conduction and the heat generation. For steady state heat conduction through wall at $x = L$, the temperature inside the wall should be higher than the temperature at $x = L$, that is, 100°C.

Ans. (b)

48. A steel ball of diameter 60 mm is initially in thermal equilibrium at 1030°C in a furnace. It is suddenly removed from the furnace and cooled in ambient air at 30°C, with convective heat transfer coefficient $h = 20$ W /m^{2}K. The thermophysical properties of steel are: density $\rho = 7800$ kg/m^3, conductivity $\kappa = 40$ W/mK, and specific heat $c = 600$ J/kgK. The time required in seconds to cool the steel ball in air from 1030°C to 430°C is

(a) 519 (b) 931
(c) 1195 (d) 2144

(GATE 2013)

Solution. Given that

$$\begin{aligned} R &= \frac{0.06}{2} \\ &= 0.03 \text{ m} \\ T_0 &= 1030^\circ\text{C} \\ T_\infty &= 30^\circ\text{C} \\ h &= 20 \text{ W/m}^2\text{K} \\ \kappa &= 40 \text{ W/mK} \\ \rho &= 7800 \text{ kg/m}^3 \\ c &= 600 \text{ J/kgK} \\ T &= 430^\circ\text{C} \end{aligned}$$

For spherical objects of radius R

$$\begin{aligned} \frac{V}{A} &= \frac{4\pi R^3/3}{4\pi R^2} \\ &= \frac{R}{3} \end{aligned}$$

The time required for the body to be cooled from T_0 to T temperature in transient state is given by

$$\frac{T-T_\infty}{T_0-T_\infty} = \exp\left(-\frac{3ht}{\rho Rc}\right)$$
$$\ln\left(\frac{T-T_\infty}{T_0-T_\infty}\right) = -\frac{3ht}{\rho Rc}$$

Therefore,

$$\begin{aligned} t &= -\ln\left(\frac{T-T_\infty}{T_0-T_\infty}\right)\times\frac{\rho Rc}{3h} \\ &= -\ln\left(\frac{430-30}{1030-30}\right)\times\frac{7800\times0.03\times600}{3\times20} \\ &= 2144.12 \end{aligned}$$

Ans. (d)

49. Two large diffused gray parallel plates, separated by a small distance, have surface temperatures of 400 K and 300 K. If the emissivities of the surfaces are 0.8 and the Stefan-Boltzmann constant is 5.67×10^{-8} W/m^{2}K^4, the net radiation heat exchange rate in kW/m^2 between the two plates is

(a) 0.66 (b) 0.79
(c) 0.99 (d) 3.96

(GATE 2013)

Solution. Given that

$$\begin{aligned} T_1 &= 400 \text{ K} \\ T_2 &= 300 \text{ K} \\ \epsilon_1 &= \epsilon_2 \\ &= 0.8 \\ \sigma &= 5.67\times10^{-8} \text{ W/m}^2\text{K}^4 \end{aligned}$$

The radiative heat exchange between two large parallel gray plates kept at T_1 and T_2 temperatures is given by

$$\begin{aligned} Q_{12} &= \frac{\sigma\left(T_1^4-T_2^4\right)}{1/\epsilon_1+1/\epsilon_2-1} \\ &= \frac{5.67\times10^{-8}\left(400^4-300^4\right)}{1/0.8+1/0.8-1} \\ &= \frac{992.25}{1.5} \\ &= 661.5 \text{ W/m}^2 \\ &= 0.6615 \text{ W/m}^2 \end{aligned}$$

Ans. (a)

Common Data Questions

Water (specific heat, $c_p = 4.18$ kJ/kgK) enters a pipe at a rate of 0.01 kg/s and a temperature of 20°C. The pipe, of diameter 50 mm and length 3 m, is subjected to a wall heat flux $\dot{q}_w$ W/m^2.

50. If $\dot{q}_w = 2500x$, where x is in meters and in the direction of flow ($x = 0$ at the inlet), the bulk mean temperature of the water leaving the pipe in °C is

(a) 42 (b) 62
(c) 74 (d) 104

(GATE 2013)

Solution. If T_o is the mean bulk temperature of the water leaving the pipe, then equilibrium of heat transfer for small element of the pipe of length dx would be written as

$$\begin{aligned} \dot{m}cdT &= 2500xdx\times\pi d \\ \dot{m}c\int_{T_i}^{T_o} dT &= 2500\pi d\int_0^L xdx \\ \dot{m}c\int_{T_i}^{T_o} dT &= 2500\pi d\times\frac{L^2}{2} \\ T_o &= \frac{2.5\pi\times0.05\times3^2/2}{0.01\times4.18}+20 \\ &= 42.27+20 \\ &= 62.27^\circ\text{C} \end{aligned}$$

Ans. (b)

51. If $\dot{q}_w = 5000$, and the convection heat transfer coefficient at the pipe outlet is 1000 W/m^{2}K, the temperature in °C at the inner surface of the pipe at the outlet is

(a) 71 (b) 76
(c) 79 (d) 81

(GATE 2013)

Solution. Given that

$$\begin{aligned} \dot{q}_w &= 5.0 \text{ kW/m}^2 \\ h &= 1.0 \text{ kW/m}^2\text{K} \end{aligned}$$

If T_o is the mean bulk temperature of the water leaving the pipe, then equilibrium of heat transfer for small element of the pipe of length dx would be written as

$$\begin{aligned} T_o &= \frac{\dot{q}_w\times\pi dL}{\dot{m}c}+T_i \\ &= 76.37^\circ\text{C} \end{aligned}$$

Let T_p be the temperature of inner surface of the pipe. The heat flux in terms of heat transfer coefficient ($h = 1.0$ kW/m^{2}K) is expressed as

$$\begin{aligned} h\left(T_p-T_o\right) &= \dot{q}_w \\ T_p &= \frac{\dot{q}_w}{h}+T_o \\ &= 81.37^\circ\text{C} \end{aligned}$$

Ans. (d)

MULTIPLE CHOICE QUESTIONS

1. Thermal diffusivity of a substance is

(a) inversely proportional to thermal conductivity
(b) directly proportional to thermal conductivity
(c) directly proportional to the square of thermal conductivity
(d) inversely proportional to the square of thermal conductivity

2. A composite slab has two layers of different materials with thermal conductivity κ_1 and κ_2. If each layer has the same thickness, the equivalent thermal conductivity of the slab would be

(a) $\sqrt{\kappa_1\kappa_2}$ (b) $\kappa_1+\kappa_2$
(c) $(\kappa_1+\kappa_2)/(\kappa_1\kappa_2)$ (d) $(\kappa_1\kappa_2)/(\kappa_1+\kappa_2)$

3. For a given heat flow and for the same thickness, the temperature drop across the material will be maximum for

(a) copper (b) steel
(c) glass wool (d) refractory brick

4. Two insulating materials of thermal conductivities κ and 2κ are available for lagging a pipe carrying a hot fluid. If the radial thickness of each material is the same, then

(a) material with higher thermal conductivity should be used for the inner layer and one with lower thermal conductivity for the outer layer
(b) material with lower thermal conductivity should be used for the inner layer and one with higher thermal conductivity for the outer layer
(c) it is immaterial in which sequence the insulating materials are used
(d) it is not possible to judge unless numerical values of the dimensions are given

5. If a material possess thermal conductivity κ, density ρ and specific heat c_p, its thermal diffusivity α is equal to

(a) $\kappa/(\rho c_p)$ (b) $\rho c_p/\kappa$
(c) $\kappa c_p/\rho$ (d) $\rho/(\kappa c_p)$

6. The equation of heat conduction for isotropic thermal conductivity κ and thermal diffusivity α is written in rectilinear coordinates as

(a) $\sum \dfrac{\partial^2 T}{\partial x^2}+\dfrac{\dot{q}}{\kappa}=\dfrac{1}{\alpha}\dfrac{\partial T}{\partial \tau}$

(b) $\sum \dfrac{\partial^2 T}{\partial x^2}-\dfrac{\dot{q}}{\kappa}=\dfrac{1}{\alpha}\dfrac{\partial T}{\partial \tau}$

(c) $\sum \dfrac{\partial^2 T}{\partial x^2}+\dfrac{\dot{q}}{\kappa}=-\dfrac{1}{\alpha}\dfrac{\partial T}{\partial \tau}$

(d) $-\sum \dfrac{\partial^2 T}{\partial x^2}+\dfrac{\dot{q}}{\kappa}=\dfrac{1}{\alpha}\dfrac{\partial T}{\partial \tau}$

7. Heat conduction equation in cylindrical coordinates can be written as

(a) $\frac{\partial}{\partial r}\left(r\partial T/\partial r\right)/r+\dot{q}/\kappa=(\partial T/\partial \tau)/\alpha$
(b) $\frac{\partial}{\partial r}\left(r^2\partial T/\partial r\right)/r^2+\dot{q}/\kappa=(\partial T/\partial \tau)/\alpha$
(c) both (a) and (b)
(d) none of the above

8. The temperatures across a wall is measured as T_1 and T_2 ($< T_1$). If the thermal conductivity varies linearly with temperature as per following relation,

$$\kappa=\kappa_0\left(1+\beta T\right)$$

the average thermal conductivity of the wall would be

(a) $\kappa_0\left(1+\beta\left(T_1+T_2\right)\right)$
(b) $\kappa_0\left(1+\beta\left(T_1-T_2\right)\right)$
(c) $\kappa_0\left(1+\beta\left(T_1+T_2\right)/2\right)$
(d) $\kappa_0\left(1+\beta\left(T_1-T_2\right)/2\right)$

9. Lumped system analysis is the simplest and most convenient method that can be used to solve transient conduction problems. This analysis can be used only when Biot number (Bi) is

(a) less than 0.1 (b) more than 0.1
(c) less than 1.0 (d) more than 1.0

10. A body of volume V, surface area A, at its initial temperature T_0, is immersed in an infinite volume of a fluid of density ρ, specific heat c, at temperature T_∞. If the coefficient of convection is h, then using lumped heat analysis, the temperature difference $T-T_\infty$ at any moment t, would be given by

(a) $(T_0 - T_\infty)\exp\left(\dfrac{hA}{\rho Vc}\right)$

(b) $(T_0 - T_\infty)\exp\left(\dfrac{\rho Vc}{hA}\right)$

(c) $(T_0 - T_\infty)\exp\left(-\dfrac{hA}{\rho Vc}\right)$

(d) $(T_0 - T_\infty)\exp\left(-\dfrac{\rho Vc}{hA}\right)$

11. A body, in which at any instant of time, there is always a point where the effect of heating (or cooling) at one of its boundary is not felt at all, is known as

(a) finite body
(b) semi-infinite body
(c) infinite body
(d) unaffected by

12. In descending order of magnitude, the thermal conductivity of (a) pure iron, (b) liquid water, (c) saturated water vapor and (d) aluminium can be arranged as

(a) a, b, c, d (b) b, c, a, d
(c) d, a, b, c (d) d, c, b, a

13. A steel steam pipe 10 cm inner diameter and 11 cm outer diameter is covered with an insulation having a thermal conductivity of 1 W/mK. If the convective heat transfer coefficient between the surface of insulation and the surrounding air is 8 $\text{W/m}^2\text{K}$, the critical radius of insulation is

(a) 10 cm (b) 11 cm
(c) 12.5 cm (d) 15 cm

14. A solid sphere and a hollow sphere of the same material and size are heated to the same temperature and allowed to cool in the same surroundings. If the temperature difference between the body and that of the surroundings is T, then

(a) both spheres will cool at the same rate for small values of T
(b) both spheres will cool at the same rate only for large values of T
(c) hollow sphere will cool at a faster rate for all the values of T
(d) solid sphere will cool at a faster rate for all the values of T

15. A furnace is made of a red brick wall of thickness 0.5 m and conductivity 0.7 W/mK. For the same heat loss and temperature drop, this can be replaced by a layer of diatomic earth of conductivity 0.14 W/mK and thickness?

(a) 0.05 m (b) 0.1 m
(c) 0.2 m (d) 0.5 m

16. The temperature distribution, at a certain instant of time in a concrete slab during curing is given by

$$T = 3x^2 + 3x + 16$$

where x is in cm and T is in K. The rate of temperature change with time is given by (assuming diffusivity to be 0.0003 cm^2/s)

(a) +0.0009 K/s (b) +0.0048 K/s
(c) −0.0012 K/s (d) −0.0018 K/s

17. A 0.5 m thick plane wall has its two surfaces kept at 300°C and 200°C. Thermal conductivity of the wall varies linearly with temperature and its values at 300°C and 200°C are 25 W/mK and 15 W/mK, respectively. Then the steady heat flux through the wall is

(a) 8 kW/m^2 (b) 5 kW/m^2
(c) 4 kW/m^2 (d) 3 kW/m^2

18. What will be the geometric radius of heat transfer for a hollow sphere of inner and outer radii r_1 and r_2?

(a) $\sqrt{r_1 r_2}$ (b) $r_1 r_2$
(c) r_2/r_2 (d) $r_2 - r_1$

19. A plane wall is 25 cm thick with an area of 1 m^2, and has a thermal conductivity of 0.5 W/mK. If a temperature difference of 60°C is imposed across it, what is the heat flow?

(a) 120 W (b) 140 W
(c) 160 W (d) 180 W

20. A composite hollow sphere with steady internal heating is made of two layers of materials of equal thickness with thermal conductivities in the ratio of 1:2 for inner to outer layers. Ratio of inside to outside diameter is 0.8. What is the ratio of temperature drop across the inner and outer layers?

(a) 0.4 (b) 1.6
(c) 0.8 (d) 2.5

21. A composite wall having three layers of thickness 0.3 m, 0.2 m and 0.1 m and of thermal conductivities 0.6, 0.4 and 0.1 W/mK, respectively, is

having surface area 1 m^2. If the inner and outer temperatures of the composite wall are 1840 K and 340 K, respectively, what is the rate of heat transfer?

(a) 150 W (b) 1500 W
(c) 75 W (d) 750 W

22. For conduction through a spherical wall with constant thermal conductivity and with inner side temperature greater than the outer wall temperature, (one-dimensional heat transfer), what is the type of temperature distribution?

(a) Linear
(b) Parabolic
(c) Hyperbolic
(d) None of the above

23. A wall of thickness 0.6 m width has a normal area 1.5 m^2 and is made up of material of thermal conductivity 0.4 W/mK. The temperatures on the two sides are 8000°C and 1000°C. What is the thermal resistance of the wall?

(a) 1 W/K (b) 1.8 W/K
(c) 1 K/W (d) 1.8 K/W

24. A composite wall of a furnace has 2 layers of equal thickness having thermal conductivities in the ratio of 3:2. What is the ratio of the temperature drop across the two layers?

(a) $2:3$ (b) $3:2$
(c) $1:2$ (d) $\ln 2 : \ln 3$

25. In which one of the following materials is the heat energy propagation minimum due to conduction of heat transfer?

(a) lead (b) copper
(c) water (d) air

26. A composite wall of a furnace has 3 layers of equal thickness having thermal conductivities in the ratio of 1:2:4. What will be the temperature drop ratio across the three respective layers?

(a) $1:2:4$
(b) $4:2:1$
(c) $1:1:1$
(d) $\log 4 : \log 2 : \log 1$

27. A large concrete slab 1 m thick has one-dimensional temperature distribution:

$$T = 4 - 10x + 20x^2 + 10x^3$$

where t is temperature and x is distance from one face towards other face of wall. If the slab material has thermal diffusivity of 2×10^{-3} m^2/h, what is the rate of change of temperature at the other face of the wall?

(a) 0.1°C/h (b) 0.2°C/h
(c) 0.3°C/h (d) 0.4°C/h

28. Upto critical radius of insulation,

(a) added insulation will increase the rate of heat loss
(b) added insulation will decrease the rate of heat loss
(c) convection heat loss will be less than conduction heat loss
(d) none of the above

29. If for a cylindrical element, thermal conductivity of insulation is κ and coefficient of convective heat transfer of outside is h, the critical radius of insulation is

(a) h/κ (b) $h/(2\kappa)$
(c) κ/h (d) $2\kappa/h$

30. If for a spherical element, thermal conductivity of insulation is κ and coefficient of convective heat transfer of outside is h, the critical radius of insulation is

(a) h/κ (b) $h/(2\kappa)$
(c) κ/h (d) $2\kappa/h$

31. For a current wire of 20 mm diameter exposed to air ($h = 20$ W/m^2K), maximum heat dissipation occurs when thickness of insulation ($\kappa = 0.5$ W/mK) is

(a) 20 mm (b) 25 mm
(c) 15 mm (d) 10 mm

32. It is proposed to coat a 1 mm diameter wire with enamel paint ($\kappa = 0.1$ W/mK) to increase its heat transfer with air. If the air side heat transfer coefficient is 100 W/m^2K, the optimum thickness of enamel paint should be

(a) 0.25 mm (b) 0.5 mm
(c) 1 mm (d) 2 mm

33. A steel ball of mass 1 kg and specific heat 0.4 kJ/kg is at a temperature of 60°C. It is dropped into 1 kg water at 20°C. The final steady state temperature of water is

(a) 23.5°C (b) 30°C
(c) 35°C (d) 40°C

34. It is desired to increase the heat dissipation rate over the surface of an electronic device of spherical shape of 5 mm radius exposed to convection with $h = 10$ W/m^{2}K by encasing it in a spherical sheath of conductivity 0.04 W/mK. For maximum heat flow, the diameter of the sheath should be

(a) 18 mm (b) 16 mm
(c) 12 mm (d) 8 mm

35. A copper wire of radius 0.5 mm is insulated with a sheathing of thickness 1 mm having a thermal conductivity of 0.5 W/mK. The outside convective heat transfer coefficient is 10 W/m^{2}K. If the thickness of insulation sheathing is raised by 10 mm, then the electrical current carrying capacity of the wire will

(a) increase
(b) decrease
(c) remain the same
(d) vary depending upon the electrical conductivity of the wire

36. A cylinder made of metal of conductivity 40 W/mK is to be insulated with a material of conductivity 0.1 W/mK. If the convective heat transfer coefficient with the ambient atmosphere is 5 W/m^{2}K, the critical radius of insulation is

(a) 2 cm (b) 4 cm
(c) 8 cm (d) 50 cm

37. The time constant of thermocouple is

(a) the time taken to attain 100% of initial temperature difference
(b) the time taken to attain 63.2% of initial temperature difference
(c) the time taken to attain 50% of initial temperature difference
(d) the minimum time taken to record a temperature reading

38. The curve for unsteady state cooling or heating of bodies is a

(a) parabolic curve asymptotic to time axis
(b) exponential curve asymptotic to time axis
(c) exponential curve asymptotic both to time and temperature axes
(d) hyperbolic curve asymptotic both to time and temperature axes

39. The efficiency of a pin fin with insulated tip is

(a) $\tanh mL/\sqrt{hA/\kappa P}$
(b) $\tanh mL/(mL)$
(c) $mL/\tanh mL$
(d) $\sqrt{hA/\kappa P}/\tanh mL$

40. Addition of fin to the surface increases the heat transfer if $\sqrt{hA/\kappa P}$ is

(a) equal to one
(b) greater than one
(c) less than one
(d) greater than one but less than two

41. The temperature distribution in a stainless fin (thermal conductivity 0.17 W/cm-°C) of constant cross-sectional area of 2 cm^2, and length of 1 cm, exposed to an ambient of 40°C (with a surface heat transfer coefficient of 0.0025 W/cm°C) is given by

$$T - T_\infty = 3x^2 - 5x + 6$$

where T is in °C and x in in cm. If the base temperature is 100°C, then the heat dissipated by the fin surface will be

(a) 6.8 W (b) 3.4 W
(c) 1.7 W (d) 0.017 W

42. With usual symbols, the rate of heat transfer ($\dot{Q}$) for long fins is expressed as

(a) $\sqrt{\dfrac{hA}{p\kappa}}(T_0 - T_\infty)$ (b) $\sqrt{\dfrac{hP}{\kappa A}}(T_0 - T_\infty)$

(c) $\sqrt{\dfrac{h\kappa}{pA}}(T_0 - T_\infty)$ (d) $\sqrt{\dfrac{\kappa A}{hP}}(T_0 - T_\infty)$

43. A plane wall of thickness $2L$ has a uniform volumetric heat source q (W/m^3). It is exposed to local ambient temperature T_∞ at both the ends ($x = \pm L$). The surface temperature T_s of the wall under steady state condition (where h and κ have their usual meaning) is given by

(a) $T_\infty + qL/h$ (b) $T_\infty + qL^2/(2\kappa)$
(c) $T_\infty + qL^2/h$ (d) $T_\infty + qL^3/(2\kappa)$

44. Which one of the following is correct? Fins are used to increase the heat transfer from a surface by

(a) increasing the temperature difference
(b) increasing the effective surface area
(c) increasing the convective heat transfer coefficient
(d) none of the above

45. In free convection heat transfer, transition from laminar to turbulent flow is governed by the critical value of the

(a) Reynolds number
(b) Grashof number
(c) Reynolds number, Grashof number
(d) Prandtl number, Grashof number

46. According to Newton's law of cooling, the rate of heat transfer in convection is

(a) proportional to the temperature difference
(b) proportional to the area of heat transfer
(c) both (a) and (b)
(d) none of the above

47. Nusselt number always increases with increase in

(a) Reynolds number in case of forced convection
(b) Grashof number in case of free convection
(c) both (a) and (b)
(d) none of the above

48. Prandtl number is, a property of a fluid, defined as the ratio of

(a) dynamic viscosity and thermal diffusivity
(b) thermal diffusivity and dynamic viscosity
(c) kinematic viscosity and thermal diffusivity
(d) thermal diffusivity and kinematic viscosity

49. Grashof number frequently arises in the study of situations involving natural convection. One can say that

(a) at higher Grashof numbers, the boundary layer is turbulent
(b) at lower Grashof numbers, the boundary layer is laminar
(c) both (a) and (b)
(d) none of the above

50. When the Rayleigh number is below the critical value for a given fluid, the heat transfer is primarily in the form of

(a) conduction
(b) convection
(c) both (a) and (b)
(d) none of the above

51. For a laminar boundary layer over a horizontal plate, the average Nusselt number for the length of the plate with usual symbols is given by

(a) $\overline{\mathrm{Nu}}_l = 0.322 \times \mathrm{Re}_l^{1/2}\mathrm{Pr}^{1/3}$
(b) $\overline{\mathrm{Nu}}_l = 0.322 \times \mathrm{Re}_l^{1/3}\mathrm{Pr}^{1/2}$
(c) $\overline{\mathrm{Nu}}_l = 0.644 \times \mathrm{Re}_l^{1/2}\mathrm{Pr}^{1/3}$
(d) $\overline{\mathrm{Nu}}_l = 0.644 \times \mathrm{Re}_l^{1/3}\mathrm{Pr}^{1/2}$

52. In a turbulent boundary layer over a horizontal plate, the local Nusselt number with usual symbols is given by

(a) $\overline{\mathrm{Nu}}_x = 0.322 \times \mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}$
(b) $\overline{\mathrm{Nu}}_x = 0.322 \times \mathrm{Re}_x^{4/5}\mathrm{Pr}^{1/3}$
(c) $\overline{\mathrm{Nu}}_x = 0.0296 \times \mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}$
(d) $\overline{\mathrm{Nu}}_x = 0.0296 \times \mathrm{Re}_x^{4/5}\mathrm{Pr}^{1/3}$

53. The ratio of average Nusselt number and local Nusselt number at trailing edge in a turbulent boundary layer over a horizontal plate is

(a) 2 (b) 3/2
(c) 4/3 (d) 5/3

54. Nusselt number for a fully developed flow in a pipe of diameter D having constant flux is equal to

(a) 1.25 (b) 3.22
(c) 3.66 (d) 4.364

55. According to Colburn analogy, the skin drag coefficient C_f at any point x is related to Stanton number (St) and Prandtl number (Pr) as

(a) $\mathrm{St}_x\mathrm{Pr}^{2/3} = C_{fx}$ (b) $\mathrm{St}_x^{2/3}\mathrm{Pr} = C_{fx}$
(c) $\mathrm{St}_x^{2/3}\mathrm{Pr} = C_{fx}/2$ (d) $\mathrm{St}_x\mathrm{Pr}^{2/3} = C_{fx}/2$

56. Water (Prandtl number $\approx$ 6) flows over a flat plate which is heated over the entire length. Which one of the following relationship between the hydrodynamic boundary layer thickness (δ) and the thermal boundary layer thickness (δ_t) is true?

(a) $\delta_t > \delta$
(b) $\delta_t < \delta$
(c) $\delta_t = \delta$
(d) cannot be predicted

57. If velocity of water inside a smooth tube is doubled, the turbulent flow heat transfer coefficient between the water and the tube will

(a) remain unchanged
(b) increase to double its value
(c) increase but will not reach double its value

(d) increase to more than double its value

58. Which one of the following non-dimensional numbers is used for transition from laminar to turbulent flow in free convection?

(a) Reynolds number
(b) Grashof number
(c) Peclet number
(d) Rayleigh number

59. For steady, uniform flow through pipes with constant heat flux supplied to the wall, what is the value of Nusselt number?

(a) 48/11 (b) 11/48
(c) 24/11 (d) 11/24

60. The constant term $\beta g \theta L^3/\nu^2$ is for

(a) Nusselt number
(b) Prandtl number
(c) Grashof number
(d) Mach number

61. The ratio of energy transferred by convection to that by conduction in called

(a) Stanton number (b) Nusselt number
(c) Biot number (d) Peclet number

62. Hydrodynamic and thermal boundary layer thicknesses are equal for Prandtl number

(a) equal to zero (b) less than 1
(c) equal to 1 (d) more than 1

63. The ratio of surface convection resistance to internal conduction resistance is known as

(a) Grashof number (b) Biot number
(c) Stanton number (d) Prandtl number

64. Match List I with List II and select the correct answer.

	List I		List II
A.	Stanton number	1.	Natural convection of ideal gases
B.	Grashof number	2.	Mass transfer
C.	Peclet number	3.	Forced convection
D.	Schmidt number	4.	Forced convection for small Prandtl number

(a) A-2, B-4, C-3, D-1
(b) A-3, B-1, C-4, D-2
(c) A-3, B-4, C-1, D-2
(d) A-2, B-1, C-3, D-4

65. The ratio of thickness of thermal boundary layer to the thickness of hydrodynamic boundary layer is equal to Pr^n, where n is

(a) $-1/3$ (b) $-2/3$
(c) 1 (d) -1

66. Match List I with List II and select the correct answer.

	List I		List II
A.	Reynolds number	1.	Film coefficient, pipe diameter, thermal conductivity
B.	Prandtl number	2.	Flow velocity, acoustic velocity
C.	Nusselt number	3.	Heat capacity, dynamic viscosity, thermal conductivity
D.	Mach number	4.	Flow velocity, pipe diameter, kinematic viscosity

(a) A-4, B-1, C-3, D-2
(b) A-4, B-3, C-1, D-2
(c) A-2, B-3, C-1, D-4
(d) A-2, B-1, C-3, D-4

67. Consider the development of laminar boundary layer for a moving non-reacting fluid in contact with a flat plate of length l along the flow direction. The average value of heat transfer coefficient can be obtained by multiplying the local heat transfer coefficient at the trailing edge by the factor

(a) 0.75 (b) 1.0
(c) 1.5 (d) 2.0

68. A cube at high temperature is immersed in a constant temperature bath. It loses heat from its top, bottom, and side surfaces with heat transfer coefficient of h_1, h_2 and h_3, respectively. The average heat transfer coefficient for the tube is

(a) $(h_1+h_2+h_3)/3$
(b) $(h_1h_2h_3)^{1/3}$
(c) $1/h_1+1/h_2+1/h_3$
(d) none of the above

69. Given that

Nu = Nusselt number
Re = Reynolds number
Pr = Prandtl number
Sh = Sherwood number
Sc = Schmidt number
Gr = Grashof number

The functional relationship for free convective mass transfer is given as

(a) Nu=f(Gr, Pr) (b) Sh=f(Sc, Gr)
(c) Nu=f(Re, Pr) (d) Sh=f(Re, Sc)

70. Heat lost from a 100 mm diameter steam pipe placed horizontally in ambient at 30°C. If the Nusselt number is 25 and thermal conductivity of air is 0.03 W/mK, then the heat transfer coefficient will be

(a) 7.5 W/m^{2}K (b) 16.2 W/m^{2}K
(c) 25.2 W/m^{2}K (d) 30 W/m^{2}K

71. For flow over a flat plate, the hydrodynamic boundary layer thickness is 0.5 mm. The dynamic viscosity is 25×10^{-6} Pa s, specific heat is 2.0 kJ/kgK and thermal conductivity is 0.05 W/mK. The thermal boundary layer thickness would be

(a) 0.1 mm (b) 0.5 mm
(c) 1 mm (d) 2 mm

72. For fully developed turbulent flow in a pipe with heating, the Nusselt number Nu, varies with Reynolds number Re and Prandtl number Pr as

(a) $\mathrm{Re}^{0.5}\mathrm{Pr}^{1/3}$ (b) $\mathrm{Re}^{0.8}\mathrm{Pr}^{0.2}$
(c) $\mathrm{Re}^{0.8}\mathrm{Pr}^{0.4}$ (d) $\mathrm{Re}^{0.8}\mathrm{Pr}^{0.3}$

73. Which one of the following numbers represents the ratio of kinematics viscosity to the thermal diffusivity?

(a) Grashof number (b) Prandtl number
(c) Mach number (d) Nusselt number

74. A thin flat plate 2 m × 2 m is hanging freely in air. The temperature of the surroundings is 25°C. Solar radiation is falling on one side of the plate at the rate of 500 W/m^2. What should be the convective heat transfer coefficient in W/m^2°C, if the temperature of the plate is to remain constant at 30°C?

(a) 25 (b) 50
(c) 100 (d) 200

75. Schmidt number is ratio of which of the following?

(a) Product of mass transfer coefficient and thermal diffusivity of fluid
(b) Kinematic viscosity to thermal diffusivity of fluid
(c) Kinematic viscosity to thermal coefficient of fluid
(d) Thermal diffusivity to diffusion coefficient of fluid

76. The value of thermal conductivity of thermal insulation applied to a hollow spherical vessel containing very hot material is 0.5 W/mK. The convective heat transfer coefficient at the outer surface of insulation is 10 W/m^{2}K. What is the critical radius of the sphere?

(a) 0.1 m (b) 0.2 m
(c) 1.0 m (d) 2.0 m

77. Air at 20°C blows a hot plate of 50×60 cm^2 made of carbon steel maintained at 220°C. The convective heat transfer coefficient is 25 W/m^{2}K. What will be the heat loss from the plate?

(a) 1500 W (b) 2500 W
(c) 3000 W (d) 4000 W

78. For a fluid flowing over a flat plate, the Nusselt number at a point 0.5 m from the leading edge is 100. If the thermal conductivity of the fluid is 0.025 W/mK, the coefficient of convective heat transfer is

(a) 2000 W/m^{2}K
(b) 5 W/m^{2}K
(c) 5×10^{-4} W/m^{2}K
(d) 1.25×10^{-4} W/m^{2}K

79. Which one of the following statements is correct? For a hemisphere, the solid angle is measured

(a) in radians and its maximum value is π
(b) in degree and its maximum value is 180°
(c) in steradians and its maximum value is 2π
(d) in steradians and its maximum value is π

80. What is the radiation intensity in a particular direction?

(a) Radiant energy per unit time per unit area of the radiating surface
(b) Radiant energy per unit time per unit solid angle per unit area of the radiating surface
(c) Radiant energy per unit time per unit solid angle per unit projected area of the radiating surface in the given direction

(d) Radiant energy per unit time per unit projected area of the radiating surface in the given direction

81. What is the basic equation of thermal radiation from which all other equations of radiation can be derived?

(a) Stefan–Boltzmann equation
(b) Planck's equation
(c) Wien's equation
(d) Rayleigh–Jeans criterion

82. The wavelength of the radiation emitted by a body depends upon

(a) the nature of its surface
(b) the area of its surface
(c) the temperature of its surface
(d) all the above factors

83. In a radiative heat transfer, a gray surface is the one

(a) which appears gray to the eye
(b) whose emissivity is independent of wavelength
(c) which has reflectivity equal to zero
(d) which appears equally bright from all directions

84. If T is the temperature corresponding to optimum wavelength, then according to Wein's displacement law, the maximum monochromatic emissive power is proportional to

(a) T (b) T^2
(c) T^4 (d) T^5

85. Wein's displacement law fails at

(a) higher wavelengths and lower temperatures
(b) higher wavelengths and temperatures
(c) lower wavelengths and higher temperatures
(d) lower wavelengths and temperatures

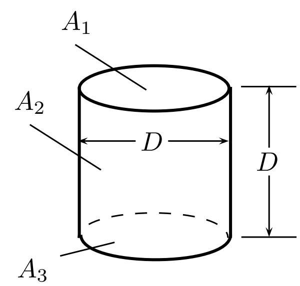
86. Emissivity (ϵ) of a surface is defined as

(a) the emissive power of the surface at a given temperature
(b) the radiation from a black body at a given temperature
(c) the ratio of emissive power of the surface and radiation from a black body at a given temperature
(d) the ratio of radiation from a black body and emissive power of the surface at a given temperature

87. Using electrical analogy, the surface resistance of a surface of area A_1 and emissivity ε is expressed as

(a) $(1-\varepsilon)/(A\varepsilon)$ (b) $(1+\varepsilon)/(A\varepsilon)$
(c) $1/(A\varepsilon)$ (d) $1/\varepsilon$

88. For the circular tube of equal length and diameter shown in the figure.



The view factor F_{13} is 0.17. The view factor F_{12} in this case will be

(a) 0.17 (b) 0.21
(c) 0.79 (d) 0.83

89. Two long parallel surfaces each of emissivity 0.7 are maintained at different temperatures and accordingly have radiation heat exchange between them. It is desired to reduce 75% of this radiant heat transfer by inserting thin parallel shields of emissivity on other ends. The number of shields should be

(a) 1 (b) 2
(c) 3 (d) 4

90. For infinite parallel plates with emissivities ϵ_1 and ϵ_2, the interchange factor for radiation from surface 1 to surface 2 is given by

(a) $(\epsilon_1\epsilon_2)/(\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2)$
(b) $1/\epsilon_1+1/\epsilon_2$
(c) $\epsilon_1+\epsilon_2$
(d) $\epsilon_1\epsilon_2$

91. What is the net radiation interchange per square meter for two very large plates at temperatures 800 K and 500 K? (The emissivity of the hot and cold plates are 0.8 and 0.6, respectively, and Stefan Boltzmann constant is 5.67×10^{-8} W/m^{2}K.)

(a) 1.026 kW/m^2 (b) 10.26 kW/m^2
(c) 102.6 kW/m^2 (d) 1026.0 kW/m^2

92. Sun's surface at 5800 K emits radiation at a wavelength of 0.5μm. A furnace at 300°C will emit through a small opening, a radiation at a wavelength of nearly

(a) 10 μm (b) 5 μm
(c) 0.25 μm (d) 0.025 μm

93. Consider two infinitely long black body concentric cylinders with a diameter ratio of 3. The shape factor for the outer cylinder with itself will be

(a) 0 (b) 1/3
(c) 2/3 (d) 1

94. If the temperature of a solid surface changes from 27°C to 627°C, then its emissive power will increase in the ratio of

(a) 3 (b) 9
(c) 27 (d) 81

95. Match List I with List II and select the correct answer.

	List I		List II
A.	Stefan Boltzmann law	1.	$q = hA\,(T_1 - T_2)$
B.	Newton's law of cooling	2.	$E = \alpha E_b$
C.	Fourier's law	3.	$q = \dfrac{\kappa A}{L}\,(T_1 - T_2)$
D.	Kirchhoff's aw	4.	$q = \sigma A\left({T_1}^4 - {T_2}^4\right)$
		5.	$q = \kappa A\,(T_1 - T_2)$

(a) A-4, B-1, C-3, D-2
(b) A-4, B-1, C-3, D-2
(c) A-2, B-1, C-3, D-4
(d) A-3, B-5, C-1, D-4

96. Solar radiation of 1200 W/m^2 falls perpendicularly on a gray opaque surface of emissivity 0.5. If the surface temperature is 50°C and surface emissive power is 600 W/m^2, the radiosity of that surface will be

(a) 600 W/m^2 (b) 1000 W/m^2
(c) 1200 W/m^2 (d) 1800 W/m^2

97. The shape factor of a hemispherical body placed on a flat surface with respect to itself is

(a) zero (b) 0.25
(c) 0.5 (d) 1.0

98. Two large parallel gray plates with a small gap, have exchange radiation at the rate of 1000 W/m^2 when their emissivities are 0.5 each. By coating one plate, its emissivity is reduced to 0.25. Temperatures remain unchanged. The new rate of heat exchange will become

(a) 500 W/m^2 (b) 600 W/m^2
(c) 700W/m^2 (d) 800W/m^2

99. Fraction of radiative energy leaving one surface that strikes the other surface is called

(a) radiative flux
(b) emissive power of the first surface
(c) shape factor
(d) re-radiation flux

100. Two spheres A and B of the same material have radii 1 m and 4 m and temperature 4000 K and 2000 K, respectively. The energy radiated by sphere A is

(a) greater than that of sphere B
(b) less than that of sphere B
(c) equal to that of sphere B
(d) equal to double that of sphere B

101. Two long parallel surfaces, each of emissivity 0.7 are maintained at different temperatures and accordingly have radiation exchange between them. It is desired to reduce 75% of this radiant heat transfer by inserting thin parallel shields of equal emissivity (0.7) on both sides. What would be the number of shields?

(a) 1 (b) 2
(c) 3 (d) 4

102. Two radiating surfaces $A_1 = 6$ m^2 and $A_2 = 4$ m^2 have the shape factor $F_{12} = 0.1$; the shape factor F_{21} will be

(a) 0.18 (b) 0.15
(c) 0.12 (d) 0.10

103. The radiative heat transfer rate per unit area (W/m^2) between two plane parallel gray surfaces (emissivity = 0.9) maintained at 400 K and 300 K is

(a) 992 (b) 812
(c) 464 (d) 567

104. For evaporators and condensers in given conditions, the logarithmic temperature difference (LMTD) for parallel flow is

(a) equal to that for counter flow
(b) greater than that for counter flow
(c) smaller than that for counter flow
(d) very much smaller than that for counter flow

105. In a certain heat exchanger, both the fluids have identical mass flow rate-specific heat product. The hot fluid enters at 76°C and leaves at 47°C and the cold fluid entering at 26°C leaves at 55°C. The effectiveness of the heat exchanger is

(a) 0.16 (b) 0.58
(c) 0.72 (d) 1.0

106. Logarithmic mean temperature difference (LMTD) for a heat exchanger is expressed as

(a) $\dfrac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$ (b) $\dfrac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_1/\Delta T_2)}$

(c) $2\dfrac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$ (d) $2\dfrac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_1/\Delta T_2)}$

107. When T_{c1} and T_{c2} are the temperatures of cold fluid at entry and exit, respectively, and T_{h1} and T_{h_2} are the temperatures of hot fluid at entry and exit points. When cold fluid has lower heat capacity rate as compared to hot fluid, then effectiveness of the heat exchanger is given by

(a) $\dfrac{T_{c1} - T_{c2}}{T_{h_1} - T_{c1}}$ (b) $\dfrac{T_{h2} - T_{h1}}{T_{c2} - T_{h1}}$

(c) $\dfrac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}}$ (d) $\dfrac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}}$

108. In a heat exchanger with one fluid evaporating or condensing, the minimum surface area will be required for

(a) parallel flow
(b) counter flow
(c) cross flow
(d) same in all above

109. A designer chooses the values of fluid flow ranges and specific heats in such a manner that the heat capacities of the two fluids are equal. A hot fluid enters the counter flow heat exchanger at 100°C and leaves at 60°C. The cold fluid enters the heat exchanger at 40°C. The mean temperature difference between the two fluids (in °C) is

(a) $(100+60+40)/3$ (b) 60
(c) 40 (d) 20

110. A counter flow shell and tube heat exchanger is used to heat water with hot exhaust gases. The water (c_p = 4180 J/kgC) flows at a rate of 2 kg/s while the exhaust gas (c_p = 1030 J/kgC) flows at the rate of 5.25 kg/s. If the heat transfer surface area is 32.5 m^2 and the overall heat transfer coefficient is 200 W/m^{2}C, what is the NTU for the heat exchanger?

(a) 1.2 (b) 2.4
(c) 4.5 (d) 8.6

111. A counter flow heat exchanger is used to heat water from 20°C to 80°C by using hot exhaust gas entering at 140°C and leaving at 80°C. The logarithmic mean temperature difference for the heat exchanger is

(a) 80°C
(b) 60°C
(c) 110°C
(d) indeterminate as 0/0 is involved

112. In a counter flow heat exchanger, the product of specific heat and mass flow rate is same for the hot and cold fluids. If NTU is equal to 0.5, then the effectiveness of the heat exchanger is

(a) 1.0 (b) 0.5
(c) 0.33 (d) 0.2

113. In a balanced counter flow heat exchange with

$$\dot{m}_h c_h = \dot{m}_c c_c$$

the NTU is equal to 1.0. What is the effectiveness of the heat exchanger?

(a) 0.5 (b) 1.5
(c) 0.33 (d) 0.2

114. If n variables in a physical phenomenon contain m fundamental dimensions, then the variables can be arranged into how many dimensionless terms?

(a) n (b) m
(c) $n-m$ (d) $n+m$

115. Dimensionless time is represented by

(a) Biot number
(b) Fourier number
(c) Euler number
(d) Reynolds number

NUMERICAL ANSWER QUESTIONS

1. A hollow sphere of 20 cm internal diameter and 32 cm external diameter of a material having thermal conductivity 55 W/mK is used as a container for a liquid chemical mixture. Its inner and outer surface temperatures are 325°C and 95°C, respectively. Calculate the thermal resistance and the rate of heat flow through of the sphere.

2. The temperatures across a 50 cm thick furnace wall are 350°C and 55°C. The thermal conductivity of the furnace material varies with temperature as per the following profile

$$\kappa = 0.005T - 5\times 10^{-6}T^2$$

where temperature T is in °C. What is the rate of heat loss from the furnace wall?

3. The door of a cold storage plant is made from two 5 mm thick glass sheets separated by a uniform air gap of 3 mm. The temperature of the air inside the room is −25°C and the ambient temperature is 30°C. The heat transfer coefficient between the glass and air is 20 W/m^{2}K on the both sides of the door. There is no convective heat transfer in the air as air is still. Thermal conductivities of the glass and air are 0.75 W/mK and 0.02 W/mK, respectively. What is the rate of leaking of heat into the room. Calculate the percentage increase in the rate of heat loss if the air gap is reduced to 1 mm.

4. A steam boiler furnace is made of a layer of fireclay 15 cm thick and a layer of red brick 40 cm thick. The wall temperature inside the boiler furnace is 1200°C and that on the outside is 50°C. Thermal conductivities of fire clay and red brick are 0.533 W/mK and 0.7 W/mK, respectively. Calculate the rate of heat loss per square meter of the furnace wall. Also calculate the thickness of the fire-clay layer to reduce the rate of heat loss by 50%.

5. A sphere made of aluminium weighing 6 kg, initially at a temperature of 300°C, is suddenly immersed in a fluid at 20°C. The convective heat transfer coefficient is 60 W/m^{2}K. The material properties of aluminium are $\rho = 2700$ kg/m^3, $c = 900$ J/kgK, $\kappa = 205$ W/mK. Lumped capacity method is applied for the analysis. What is the time required by the sphere to reach 36.8% of the initial temperature difference? How much time it would take in cooling upto a temperature of 100°C?

6. Air at 27°C is flowing along a heated flat plate at 227°C at a velocity of 5 m/s. The plate is 5 m long and 1.75 m wide. The kinematic viscosity of air at 27°C (300 K) is 15.89×10^{-6} m^2/s. The properties of air at 127°C are $\rho = 0.8711$ kg/m^3, $c_p = 1.014$ kJ/kg K, $\nu = 26.41\times 10^{-6}$ m^2/s, $\kappa = 33.8\times 10^{-3}$ W/mK, Pr = 0.690. Calculate the thickness of hydrodynamic boundary layer at a point 50 cm from the leading edge of the plate. Also calculate the rate of heat transfer from the first 50 cm of the plate.

7. A study considers the sun a spherical black body. It is established that the intensity of the radiation emitted by the sun is maximum at a wavelength of 0.5 μm. Given that Wien's displacement coefficient $C_2 = 2890$ μmK, Stefan–Boltzmann's coefficient is $\sigma = 5.67\times 10^{-8}$ W/K^4. Calculate the surface temperature using Wien's displacement law and the emissive power using Stefan–Boltzmann law.

8. A gray surface is maintained at a temperature of 927°C. The maximum spectral emissive power at that temperature is 2.74×10^{10} W/m^2. Calculate the emissivity of the gray surface and the wavelength corresponding to the maximum spectral intensity of radiation.

9. A pipe carrying steam having an outside diameter of 25 cm runs in a large room and is exposed to air at a temperature of 27°C. The pipe surface temperature is 337°C. The emissivity of the pipe surface is 0.85. Calculate the rate of heat loss to surroundings per meter length of the pipe. Also calculate the percentage reduction in the rate of heat loss by radiation if the pipe is now enclosed in a 50 cm diameter brick conduit of emissivity 0.90.

10. In a counter flow heat exchanger, 2.78 kg/s of an oil having a specific heat of 2.095 kJ/kg K is cooled from 95°C to 40°C by 2.1 kg/s of water (c_p = 4.18 kJ/kgK) entering at 20°C. The overall heat transfer coefficient of the exchanger is 500 W/m^2 K. Calculate the outlet temperature of water and net heat exchange area of the unit.

11. Water ($c_p = 4.186$ kJ/kg K) enters a counter flow heat exchanger at 20°C flowing at a rate 0.5 kg/s. It is heated by an oil ($c_p = 2.095$ kJ/kg K) entering at 105°C at a rate of 0.2 kg/s. The overall heat transfer coefficient is 1050 W/m^{2}K. Calculate the rate of heat exchange per unit area. Also calculate the outlet temperature of the water and oil.

ANSWERS

Multiple Choice Questions

1. (b)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (c)
11. (b)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (d)	**17.** (c)	**18.** (a)	**19.** (a)	**20.** (d)
21. (d)	**22.** (b)	**23.** (c)	**24.** (a)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (d)
31. (a)	**32.** (b)	**33.** (a)	**34.** (a)	**35.** (b)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (c)
41. (c)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (d)	**46.** (c)	**47.** (c)	**48.** (c)	**49.** (c)	**50.** (a)
51. (c)	**52.** (d)	**53.** (c)	**54.** (d)	**55.** (d)	**56.** (b)	**57.** (c)	**58.** (a)	**59.** (a)	**60.** (c)
61. (b)	**62.** (c)	**63.** (b)	**64.** (b)	**65.** (a)	**66.** (b)	**67.** (d)	**68.** (a)	**69.** (a)	**70.** (a)
71. (b)	**72.** (d)	**73.** (b)	**74.** (a)	**75.** (b)	**76.** (c)	**77.** (a)	**78.** (b)	**79.** (c)	**80.** (c)
81. (b)	**82.** (c)	**83.** (b)	**84.** (d)	**85.** (a)	**86.** (d)	**87.** (a)	**88.** (d)	**89.** (c)	**90.** (a)
91. (b)	**92.** (b)	**93.** (c)	**94.** (d)	**95.** (b)	**96.** (c)	**97.** (a)	**98.** (b)	**99.** (b)	**100.** (c)
101. (c)	**102.** (b)	**103.** (b)	**104.** (a)	**105.** (b)	**106.** (a)	**107.** (d)	**108.** (d)	**109.** (d)	**110.** (a)
111. (d)	**112.** (c)	**113.** (a)	**114.** (c)	**115.** (b)					

Numerical Answer Questions

1. 5.42 °C/kW, 42.39 kW	**2.** 455.01 W/m^2	**3.** 208.7 W/m^2, 61.22%
4. 1348 W/m^2, 60 cm	**5.** 1092.53 s, 1376.7 s	**6.** 6.3 mm, 4270 kW
7. 5780 K, 63.28 MW/m^2	**8.** 0.84, 2.408 μm	**9.** 4.93 kW/m^2, 4.5%
10. 56.49°C, 22.68 m^2	**11.** 31.67 kW, 5.72°C	

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (b) Thermal diffusivity is defined as

$$\alpha = \frac{\kappa}{\rho c_p}$$

2. (d) In the given case, the two resistance, will act in series, therefore, effective thernal conductivity shall be given by

$$\kappa_e = \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}$$

3. (d) The rate of heat flow is expressed as

$$\dot{Q} = -\kappa A \frac{dT}{dx}$$

Hence, the temperature gradient is inversely proportional to the thermal conductivity. Out of the given options, refractory materials offer least thermal conductivity.

4. (b) Material with lower thermal conductivity should be used for the inner layer (to get minimum κA so that it offers higher thermal resistance in the initial level itself which will increase the temperature gradient, therefore, one with higher thermal conductivity should be placed on the outer surface.

5. (a) Thermal diffusivity of a material is related to its thermal conductivity κ, density ρ and specific heat c_p as

$$\alpha = \frac{\kappa}{\rho c_p}$$

6. (a) The equation of heat conduction for isotropic materials is written as

$$\sum \frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\kappa} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$

7. (a) Heat conduction equation in cylindrical coordinates can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{q}}{\kappa}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

The same in spherical coordinates is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{\dot{q}}{\kappa}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

8. (c) Average thermal conductivity of the wall is

$$\bar{\kappa}=\kappa_0\left\{1+\beta\frac{T_1+T_2}{2}\right\}$$

9. (a) Lumped system analysis, also known as lumped heat capacity approach, can be used only when Bi < 0.1.

10. (c) According the lumped heat analysis, the temperature difference is given by

$$\frac{T-T_\infty}{T_0-T_\infty}=\exp\left(-\frac{hA}{\rho Vc}\right)$$

11. (b) A semi-infinite body is the one in which at any instant of time, there is always a point where the effect of heating (or cooling) at one of its boundary is not felt at all.

12. (c) The conductivity of solids is greater than liquids, and that of liquid is greater than vapor. Thermal conductivity of aluminium is greater than that of the pure iron.

13. (c) Critical radius of insulation is

$$\begin{aligned} r_c &= \frac{\kappa}{h} \\ &= \frac{1}{8} \\ &= 12.5 \text{ cm} \end{aligned}$$

14. (c) A hollow sphere will cool at a faster rate than a solid sphere of the same outer diameter and at the same initial temperature.

15. (b) Given that

$$\begin{aligned} \kappa_1 &= 0.7 \text{ W/mK} \\ \kappa_2 &= 0.14 \text{ W/mK} \\ x_1 &= 0.5 \text{ m} \end{aligned}$$

Using Fourier's equation:

$$Q=\frac{\kappa A\Delta t}{x}$$

For constant $\dot{Q}$ and κ:

$$\begin{aligned} x_2 &= \frac{\kappa_2}{\kappa_1}x_1 \\ x_1 &= 0.1 \text{ m} \end{aligned}$$

16. (d) Given that

$$\alpha = 0.0003 \text{ cm}^2/\text{s}$$

Using general heat conduction equation,

$$\frac{\partial T}{\partial \tau}=\alpha\frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial T}{\partial \tau}=-0.0018 \text{ K/s}$$

17. (c) Overall thermal conductivity is

$$\begin{aligned} \kappa &= \frac{\kappa_1+\kappa_2}{2} \\ &= 20 \text{ W/mK} \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{\dot{Q}}{A} &= \kappa\frac{\Delta T}{l} \\ &= 4 \text{ kW} \end{aligned}$$

18. (a) The rate of heat conduction in spherical elements is given by

$$\dot{Q}_r=\kappa\times 4\pi r_o r_i\frac{(T_i-T_o)}{r_o-r_i}$$

This can be compared with the following equation:

$$\dot{Q}_r=\kappa\times 4\pi r^2\times\frac{(T_i-T_o)}{r_o-r_i}$$

19. (a) The heat flow rate is given by

$$\begin{aligned} \dot{Q} &= \kappa A\frac{\Delta T}{t} \\ &= 120 \text{ W} \end{aligned}$$

20. (d) Given that

$$\frac{\kappa_1}{\kappa_2}=\frac{1}{2}$$

$$\frac{d_1}{d_2}=0.8$$

Being equal thickness of each layer, the diameter will be given by

$$\begin{aligned} d_m &= \frac{d_1+d_2}{2} \\ &= 0.9d_2 \end{aligned}$$

For the constant heat transfer rate, the ratio of temperature drop is given by

$$\begin{aligned} Q &= -2\pi l\kappa \frac{\Delta T}{\ln(r_o/r_i)} \\ \Delta T &= \frac{\ln(r_o/r_i)}{\kappa} \\ \frac{\Delta T_i}{\Delta T_o} &= \frac{\ln(0.8/0.9)}{\ln(0.9)} \times \frac{2}{1} \\ &= 2.426 \end{aligned}$$

21. (d) Thermal resistance of a rectangular body is given by

$$R = \frac{t}{\kappa A}$$

For the given case, thermal resistances are in series, hence, rate of heat transfer is

$$\begin{aligned} \dot{Q} &= \frac{(1840-340)\times 1}{0.3/0.6+0.2/0.4+0.1/0.1} \\ &= 750 \text{ kW} \end{aligned}$$

22. (b) The temperature distribution is given by the following parabolic equation

$$T = T_w + \frac{\left({r_s}^2 - r^2\right)\dot{q}}{6\kappa}$$

23. (c) Thermal resistance is defined as

$$R = \frac{\Delta T}{\dot{Q}}$$

For rectangular walls,

$$\begin{aligned} R &= \frac{t}{\kappa A} \\ &= \frac{0.6}{1.5\times 4} \\ &= 1 \text{ K/W} \end{aligned}$$

24. (a) For constant heat transfer rate, the temperature difference is inversely proportional to the thermal conductivity.

25. (d) Thermal conductivity of air is least out of the given options.

26. (a) Temperature difference is proportional to thermal conductivity of the wall.

27. (b) For isotropic thermal conductivity in one-dimensional heat conduction,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\kappa} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

where, α is called thermal diffusivity, defined as

$$\alpha = \frac{\kappa}{\rho c_p}$$

Given that

$$\begin{aligned} t &= 1 \text{ m} \\ \alpha &= 2\times 10^{-3} \text{ m}^2/\text{hr} \\ \frac{\partial^2 T}{\partial x} &= 40 + 60x \end{aligned}$$

Therefore

$$\begin{aligned} \frac{\partial T}{\partial \tau} &= \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)_{x=t} \\ &= 0.2^\circ\text{C/h} \end{aligned}$$

28. (b) Critical radius of insulation is the limit on the thickness of insulation beyond which there will be less effective insulation. In such cases, the material of insulation adds conductance for heat transfer.

29. (c) Critical radius of insulation for cylindrical elements is

$$r_c = \frac{\kappa}{h}$$

30. (d) Critical radius of insulation for spherical elements is

$$r_c = \frac{2\kappa}{h}$$

31. (a) Critical radius of insulation for cylindrical elements is determined as

$$\begin{aligned} r_c &= \frac{\kappa}{h} \\ &= \frac{0.5}{20} \\ &= 25 \text{ mm} \end{aligned}$$

Thickness of the insulation shall be

$$\begin{aligned} t &= 25 - \frac{10}{2} \\ &= 20 \text{ mm} \end{aligned}$$

32. (b) The critical radius of insulation is given by

$$\begin{aligned} r_c &= \frac{\kappa}{h} \\ &= \frac{0.1}{100} \\ &= 1 \text{ mm} \end{aligned}$$

The diameter of the wire is 1 mm, thus, the thickness of enamel paint is

$$\begin{aligned} t &= r_c - \frac{d}{2} \\ &= 1 - \frac{1}{2} \\ &= 0.5 \text{ mm} \end{aligned}$$

33. (a) If T_f is the final temperature, then for heat balance

$$\begin{aligned}1\times 0.4\,(60-T_f) &= 1\times 4.18\,(T_f-20)\\ T_f &= 23.49^\circ\text{C}\end{aligned}$$

34. (a) Given that

$$\begin{aligned}h &= 10\ \text{W/m}^2\text{K}\\ \kappa &= 0.04\ \text{W/mK}\end{aligned}$$

The critical radius of insulation is given by

$$\begin{aligned}r_c &= \frac{\kappa}{h}\\ &= 0.004\ \text{m}\end{aligned}$$

Radius of electronic device is 5 mm, therefore, diameter of the sheath is

$$\begin{aligned}&= 2\times(4+5)\\ &= 18\ \text{mm}\end{aligned}$$

35. (b) Given that

$$\begin{aligned}r_i &= 0.5\ \text{mm}\\ r_o &= 0.5+1+10\\ &= 11.5\ \text{mm}\\ \kappa &= 0.5\ \text{W/mK}\\ h &= 10\ \text{W/m}^2\text{K}\end{aligned}$$

Critical radius of insulation for wire is

$$\begin{aligned}r_c &= \frac{\kappa}{h_o}\\ &= \frac{0.5}{10}\\ &= 0.05\ \text{m}\\ &= 5\ \text{mm}\end{aligned}$$

Since $r_c < r_o$, there will be decrease in rate of heat transfer.

36. (a) Given that

$$\begin{aligned}\kappa &= 0.1\ \text{W/mK (of insulation)}\\ h &= 5\ \text{W/m}^2\text{K}\end{aligned}$$

Critical radius of insulation is

$$\begin{aligned}r_c &= \frac{\kappa}{h}\\ &= 0.02\ \text{m}\\ &= 2\ \text{cm}\end{aligned}$$

37. (b) Time constant is the time required by a body of infinite conductivity to reach 36.8% of value of initial temperature difference, that is, the temperature difference would be reduced by $100-36.8 = 63.2\%$.

38. (c) Cooling rate is an exponential function of time:

$$\frac{T-T_\infty}{T_0-T_\infty} = \exp\left(-\frac{hA}{\rho V c}t\right)$$

So, it is exponential to both time and temperature axes.

39. (b) Efficiency of a fin is defined as the ratio of the rate of heat transfer of the fin with maximum possible heat transfer if the whole surface is made base temperature uniformly:

$$\eta_{fin} = \frac{\dot{Q}}{\dot{Q}_{max}}$$

For insulated fins,

$$\eta_{fin} = \frac{\tanh(ml)}{ml}$$

40. (c) Fin effectiveness (ε) is defined as the ratio of heat transfer from the fin and that without the fin. For insulated fins

$$\varepsilon = \sqrt{\frac{\kappa P}{hA}}$$

For $\varepsilon \gg 1$

$$\begin{aligned}\sqrt{\frac{\kappa P}{hA}} &\gg 1\\ hA &\ll \kappa P\end{aligned}$$

In this case, fins are more effective on hot side of fluid for dissipating heat.

41. (c) Given that

$$\begin{aligned}\kappa &= 0.17\ \text{W/cm}^\circ\text{C}\\ A &= 2\ \text{cm}^2\\ h &= 0.0025\ \text{W/cm}^\circ\text{C}\end{aligned}$$

Using

$$\begin{aligned}\dot{Q} &= -\kappa A\left(\frac{\partial T}{\partial x}\right)_{x=0}\\ &= 1.7\ \text{W}\end{aligned}$$

42. (c) For long fins,

$$\dot{Q} = \sqrt{hP\kappa A}\,(T_0-T_\infty)$$

43. (a) For heat balance,

$$\begin{aligned}h_o A\,(T_w-T_\infty) &= \dot{q}AL\\ T_w &= T_\infty+\frac{\dot{q}L}{h_o}\end{aligned}$$

44. (b) The fin surface increases the effective surface area for heat transfer.

45. (d) By dimensional analysis,

$$\mathrm{Nu} = \phi(\mathrm{Pr}, \mathrm{Gr})$$

46. (c) According to Newton's law of cooling, the rate of heat transfer in convective heat transfer is proportional to the temperature difference and area of heat transfer.

47. (c) Nusselt number always increases with increase in Reynolds number in case of forced convection and with an increase in the Grashof number in case of free convection.

48. (c) Prandtl number (Pr) is a property of a fluid, defined as the ratio of kinematic viscosity to thermal diffusivity.

49. (c) Grashof number (Gr) is defined as

$$\mathrm{Gr} = \frac{\text{Inertia Force} \times \text{Buoyancy Force}}{\text{Viscous Force}^2}$$

Thus, at higher Grashof numbers, the boundary layer is turbulent; at lower Grashof numbers, the boundary layer is laminar.

50. (a) When the Rayleigh number is below the critical value for a given fluid, heat transfer is primarily in the form of conduction; when it exceeds the critical value, heat transfer is primarily in the form of convection.

51. (c) Average value of Nusselt number over a length l in a laminar boundary layer is given by

$$\overline{\mathrm{Nu}}_l = 0.644 \times \mathrm{Re}_l^{1/2} \mathrm{Pr}^{1/3}$$

52. (d) The local Nusselt number in a turbulent boundary layer is given by

$$\overline{\mathrm{Nu}}_x = 0.0296 \times \mathrm{Re}_x^{4/5} \mathrm{Pr}^{1/3}$$

53. (c) The average Nusselt number ($\overline{\mathrm{Nu}}_l$) in a turbulent boundary layer over a horizontal plate is related to local Nusselt number (Nu_l) at length l as

$$\overline{\mathrm{Nu}}_l = \frac{4}{3} \mathrm{Nu}_l$$

54. (d) Nusselt number for a fully developed flow in a pipe of diameter D having constant flux assumption is given by

$$\overline{\mathrm{Nu}}_D = \frac{\overline{h}D}{\kappa} = 4.364$$

55. (d) The following equation is called Colburn analogy which is valid for $\mathrm{Pr} = 0.5 - 50$,

$$\mathrm{St}_x \mathrm{Pr}^{2/3} = \frac{C_{fx}}{2}$$

56. (b) Hydrodynamic boundary layer thickness (δ) and the thermal boundary layer thickness (δ_t) are related to the Prandtl number of the fluid Pr (=6) as

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{1/3}$$
$$\delta = 1.82\delta_t$$

57. (c) In turbulent flow through a fully developed flow, Nusselt number depends upon Reynolds number as

$$\mathrm{NU}_D \propto \mathrm{Re}_D^{4/5}$$
$$\frac{\bar{h}D}{\kappa} \propto \left(\frac{\rho V D}{\mu}\right)^{4/5}$$
$$\bar{h} \propto V^{4/5}$$

Thus, the heat transfer coefficient $\bar{h}$ after doubling the velocity will be $2^{4/5} = 1.74$ times.

58. (a) Reynolds number determines whether the flow is laminar or turbulent. When the Rayleigh number is below the critical value for a given fluid, heat transfer is primarily in the form of conduction; when it exceeds the critical value, heat transfer is primarily in the form of convection.

59. (a) Nusselt number for constant heat flux is

$$\mathrm{Nu} = 4.364 = \frac{48}{11}$$

60. (c) Grashof number is defined as

$$\mathrm{Gr} = \frac{\beta g L^3 \Delta T}{\nu^2}$$

61. (b) Nusselt number (Nu) is defined as the ratio of heat convected through the fluid and heat conducted through the fluid.

62. (c) The following equation, derived by Pohlhausen, presents relation between boundary layer thickness δ and thermal boundary layer thickness δ_t as

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{1/3}$$

Thus, $\delta = \delta_t$ when $\mathrm{Pr} = 1$.

63. (b) Biot number (Bi) is defined as

$$\mathrm{Bi} = \frac{h l_c}{\kappa}$$

where l_c $(= V/A)$ is called critical length of the body.

64. (b) Stanton number (St) is defined as the ratio of convective heat flow from a wall and mass heat flow rate. Grashof number frequently arises in the study of situations involving natural convection. Peclet number(Pc) is defined as $\text{Pc} = \text{Re} \times \text{Pr}$. Schmidt number (Sc) is defined as the ratio of the momentum diffusivity (viscosity) and the mass diffusivity.

65. (a) The following equation derived by Pohlhausen presents relation between boundary layer thickness δ and thermal boundary layer thickness δ_t as

$$\frac{\delta}{\delta_t} = \text{Pr}^{1/3}$$

66. (b) Reynolds number (Re) is defined as the ratio of inertia force and viscous force. For pipe flows,

$$\text{Re} = \frac{\rho v D}{\mu}$$

Prandtl number (Pr) is the property of a fluid defined as the ratio of kinematic viscosity and thermal diffusivity:

$$\text{Pr} = \frac{\nu}{\alpha} = \frac{\mu c}{\kappa}$$

Nusselt number (Nu) is defined as the ratio of heat convected through the fluid and heat conducted through the fluid:

$$\text{Nu} = \frac{hA\Delta T}{\kappa A \Delta T/x} = \frac{hx}{\kappa}$$

Mach number is related to sonic velocity (a) and velocity of gas (v) as

$$M = \frac{v}{a}$$

67. (d) For laminar boundary layer, Nusselt number at any point at distance x from edge (on single side):

$$\text{Nu}_x = \frac{hx}{\kappa} = 0.332 \times \text{Re}_x{}^{1/2}\text{Pr}^{1/3}$$

$$\overline{\text{Nu}}_l = \frac{\overline{h}l}{\kappa} = 0.644 \times \text{Re}_l{}^{1/2}\text{Pr}^{1/3}$$

Thus,

$$\overline{h} = 2 \times h_{x=l}$$

68. (a) If a is the cross-sectional area of one face of the tube, and ΔT is the temperature gradient, then average heat transfer coefficient is given by

$$\bar{h} \times 6a \times \Delta T = 2 \times a\,(h_1 + h_2 + h_3)\,\Delta T$$

$$\bar{h} = \frac{h_1 + h_2 + h_3}{3}$$

69. (a) For free convective heat transfer,

$$\text{Nu} = f(\text{Gr}, \text{Pr})$$

70. (a) Given that

$$\begin{aligned} D &= 0.1 \text{ m} \\ \text{Nu} &= 25 \\ \kappa &= 0.03 \text{ W/mK} \end{aligned}$$

Heat transfer coefficient (W/m^{2}K) is given by

$$\begin{aligned} \text{Nu} &= \frac{hD}{\kappa} \\ h &= \frac{\text{Nu}\kappa}{D} \\ &= 7.5 \text{ W/m}^2\text{K} \end{aligned}$$

71. (b) The following equation derived by Pohlhausen presents relation between boundary layer thickness δ and thermal boundary layer thickness δ_t as

$$\frac{\delta}{\delta_t} = \text{Pr}^{1/3}$$

where Pr is Prandtl Number, a the property of a fluid, defined as the ratio of kinematic viscosity to thermal diffusivity:

$$\begin{aligned} \text{Pr} &= \frac{\nu}{\alpha} \\ &= \frac{\mu c}{\kappa} \end{aligned}$$

For the given case, $\delta = 0.5$ mm, and

$$\begin{aligned} \text{Pr} &= \frac{25 \times 10^{-6} \times 2.0 \times 10^3}{0.05} \\ &= 1 \end{aligned}$$

Hence,

$$\begin{aligned} \delta_t &= \delta \\ &= 0.5 \text{ mm} \end{aligned}$$

72. (d) In a fully developed turbulent flow in a pipe with heating, the Nusselt number is given by

$$\text{Nu}_D = 0.027\text{Re}_D{}^{4/5}\text{Pr}^{1/3}\left(\frac{\mu_f}{\mu_s}\right)^{0.14}$$

73. (b) Prandtl number (Pr) is the property of a fluid defined as the ratio of kinematic viscosity and thermal diffusivity.

74. (a) Equilibrium equation for heat transfer is

$$\begin{aligned} hA\,(T-T_\infty) &= \dot{q}\times A \\ h\times(2\times2)\,(30-25) &= 500\times(2\times2) \\ h &= 25\ \mathrm{W/m^2\ ^\circ C} \end{aligned}$$

75. (b) Schmidt number (Sc) is defined as the ratio of the momentum diffusivity (viscosity) and the mass diffusivity.

76. (c) Critical radius of insulation for spheres is

$$\begin{aligned} r_c &= \frac{2\kappa}{h} \\ &= \frac{2\times0.5}{10} \\ &= 0.1\ \mathrm{m} \end{aligned}$$

77. (a) Given that

$$\begin{aligned} T_1 &= 20^\circ\mathrm{C} \\ T_2 &= 200^\circ\mathrm{C} \\ A &= 0.5\times0.6\ \mathrm{m^2} \\ h &= 25\ \mathrm{W/m^2K} \end{aligned}$$

Therefore, the rate of heat loss is given by

$$\begin{aligned} \dot{Q} &= hA\,(T_1-T_2) \\ &= 1500\ \mathrm{W} \end{aligned}$$

78. (b) Given that

$$\begin{aligned} \mathrm{Nu} &= 100 \\ L &= 0.5\ \mathrm{m} \\ \kappa &= 0.025\ \mathrm{W/mK} \end{aligned}$$

Nusselt number is given by

$$\begin{aligned} \mathrm{Nu} &= \frac{hL}{\kappa} \\ h &= \frac{\mathrm{Nu}\kappa}{L} \\ &= 5\ \mathrm{W/m^2K} \end{aligned}$$

79. (c) In geometry, a solid angle (Ω) is the two-dimensional angle in three-dimensional space that an object subtends at a point. It is a measure of how large the object appears to an observer looking from that point. In the SI system, a solid angle is a dimensionless unit of measurement called a steradians (sr). For a unit sphere, the solid angle of the spherical cap is given as

$$\Omega = 2\pi\,(1-\cos\theta)$$

When $\theta = \pi/2$, the spherical cap becomes a hemisphere having a solid angle 2π.

80. (c) Intensity of radiation (I) is defined as heat flux (energy per unit time) incident per unit projected area of the surface with unit solid angle. Mathematically,

$$I = \frac{\text{Heat flux}}{\text{Projected area}\times\text{Solid angle}}$$

81. (b) Planck's law becomes the Rayleigh-Jeans criterion, while in the limit of high frequencies (i.e. small wavelengths) it tends to the Wien's approximation, and in turn, Stefan Boltzmann equation is derived.

82. (c) A particular value of wavelength possesses a specific energy, E_λ called monochromatic emissive power, given by

$$E_\lambda = \pi I_\lambda = \frac{C_1}{\lambda^5\left\{\exp\left(\frac{C_2}{\lambda T}\right)-1\right\}}$$

where,

$$\begin{aligned} C_1 &= 0.374\times10^{-5}\ \mathrm{m^3W} \\ C_2 &= 1.439\times10^{-2}\ \mathrm{mK} \end{aligned}$$

Hence, it depends upon the temperature of the surface.

83. (b) In radiative heat transfer, a typical physical assumption is that a surface's spectral emissivity and absorptivity do not depend on wavelength, so that the emissivity is a constant. This is known as the gray body assumption.

84. (d) According to Wein's displacement law, the maximum monochromatic emissive power is proportional to T^5.

85. (a) Wein's displacement law fails at higher wavelengths and lower temperatures.

86. (d) Emissivity of a surface is defined as the ratio of emissive power of the surface at a given temperature and radiation from a black body at the same given temperature.

87. (a) The surface resistance of a surface is expressed as

$$R_s = \frac{1-\varepsilon}{A\varepsilon}$$

88. (d) Given that

$$\begin{aligned} F_{13} &= 0.17 \\ F_{11} &= 0 \end{aligned}$$

Thus, using

$$\begin{aligned} F_{11}+F_{12}+F_{13} &= 1 \\ F_{12} &= 1-F_{13} \\ &= 0.83 \end{aligned}$$

89. (c) The initial resistance R_1 is to be reduced by n shields to R_n given by

$$R_n = (n+1)\, R_1$$

Now,

$$\frac{R_n}{R_1} = 0.75$$

So,

$$n = 3$$

90. (a) Net heat transfer between two large gray parallel planes of area A_1 and A_2, emissivities ϵ_1 and ϵ_2, is given by

$$\dot{Q}_{net} = \frac{A\sigma\left({T_1}^4 - {T_2}^4\right) \times \varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

91. (b) Given that

$$\begin{aligned} A &= 1\ \text{m}^2 \\ T_1 &= 800\ \text{K} \\ T_2 &= 500\ \text{K} \\ \varepsilon_1 &= 0.8 \\ \varepsilon_2 &= 0.6 \\ \sigma &= 5.67 \times 10^{-8}\ \text{W/m}^2\text{K} \end{aligned}$$

Therefore, using

$$\begin{aligned} Q_{1-2} &= \frac{A\sigma\left({T_1}^4 - {T_2}^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \\ &= 10.26\ \text{kW/m}^2 \end{aligned}$$

92. (b) Using Plank's distribution law:

$$\begin{aligned} \lambda_m T &= 0.2898 \times 10^{-2}\ \text{mK} \\ \lambda &= \frac{0.5 \times 5800}{300 + 273} \\ &= 5.06\ \mu\text{m} \end{aligned}$$

93. (c) Area of the cylinder is $A = \pi d L \propto d$. Taking 1 for outer cylinder and 2 for inner cylinder, one can see that

$$\begin{aligned} F_{11} &= 0 \\ F_{12} &= 1 - F_{11} \\ &= 1 \\ F_{11} &= 1 - F_{12} \\ &= 1 - \frac{F_{21} \times A_2}{A_1} \\ &= 1 - \frac{1 \times 1}{3} \\ &= \frac{2}{3} \end{aligned}$$

94. (d) Emissive power is proportional to T^4.

95. (b) Stefan Boltzmann law determines the maximum rate of radiation from a surface. Newton's law of cooling is the basic principle of convective heat transfer. Fourier's law is the basis of heat conduction. Kirchhoff's law is used to relate absorptivity with emissivity.

96. (c) Radiosity (B) is defined as total flux of radiative energy away from the surface. It is the sum of the irradiated energy that is reflected by the surface (ρH) and the radiation emitted by it (ϵE_b). Thus, for given case, $\rho = 0.5$:

$$\begin{aligned} B &= \underbrace{\rho H}_{\text{reflected}} + \underbrace{\epsilon E_b}_{\text{emitted}} \\ &= 0.5 \times 1200 + 600 \\ &= 1200\ \text{W/m}^2 \end{aligned}$$

97. (a) Since, a convex surface cannot see itself, hence, $F_{11} = 0$.

98. (b) For two large gray parallel planes of area A each and emissivities ϵ_1 and ϵ_2,

$$\dot{Q}_{net} = \frac{A\sigma\left({T_1}^4 - {T_2}^4\right)}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

Hence,

$$\dot{Q}_{net} \propto \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

For the given case,

$$\begin{aligned} \dot{Q}_{net} &= \frac{1/0.5 + 1/0.25 - 1}{0.5 + 1/0.5 - 1} \times 1000 \\ &= 600\ \text{W/m}^2 \end{aligned}$$

99. (b) Emissive power is defined as the radiative energy leaving the surface. It is determined by Stefan–Boltzmann Law as

$$E = \epsilon \sigma T^4$$

100. (c) The emissive power is given by Stefan–Boltzmann law as

$$E = \sigma T^4 \times A$$

Hence,

$$\begin{aligned} \frac{E_A}{E_B} &= \left(\frac{4000}{2000}\right)^4 \left(\frac{1}{4}\right)^2 \\ &= 1 \end{aligned}$$

101. (c) The resistance to heat transfer without shield is R_1, therefore,

$$\frac{\dot{Q} \text{ with } n \text{ shields}}{\dot{Q} \text{ without shields}} = \frac{1}{n+1}$$
$$\frac{0.25}{1} = \frac{1}{n+1}$$
$$n = 3$$

102. (b) The following is the relationship for shape factors

$$F_{12}A_1 = F_{21}A_2$$
$$F_{21} = F_{12}\frac{A_1}{A_2}$$
$$= 0.15$$

103. (b) Given that

$$\epsilon_1 = \epsilon_2$$
$$= 0.9$$
$$T_1 = 400 \text{ K}$$
$$T_2 = 300 \text{ K}$$

The radiative heat exchange between to large parallel planes is given by

$$Q = \frac{\sigma\left(T_1^4 - T_2^4\right)}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$
$$= \frac{5.67\times 10^{-8}\left(400^4 - 300^4\right)}{1/0.9 + 1/0.9 - 1}$$
$$= 811.84 \text{ W/m}^2$$

104. (a) For evaporator and condensers, the temperature of one of the fluids remains constant, therefore, LMTD is equal for parallel and counter flow heat exchangers.

105. (b) As $C_{min} = C_{max}$, the effectiveness by its definition is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}}$$
$$= \frac{C_{min}\left(T_{hi} - T_{ho}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)}$$
$$= \frac{T_{hi} - T_{ho}}{T_{hi} - T_{ci}}$$
$$= \frac{76 - 47}{76 - 26}$$
$$= 0.58$$

106. (a) The LMTD for heat exchangers is expressed as

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2/\Delta T_1\right)}$$

107. (d) Effectiveness (ε) is defined as the ratio of actual heat transfer and maximum heat transfer possible. Mathematically

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}}$$

It will be proved that for both parallel and counter flow heat exchangers, the maximum heat transfer for inclusion in definition of effectiveness is

$$\dot{Q}_{max} = C_{min}\left(T_{hi} - T_{ci}\right)$$

where C_{min} is the minimum heat capacity of fluids in the system.

108. (d) LMTD or NTU for all the given cases will be the same.

109. (d) If T_{c2} is the temperature of cold fluid at exit, and given that heat capacities of both fluids are the same, therefore,

$$T_{c2} - 40 = 100 - 60$$
$$= 80°\text{C}$$

Hence, the mean temperature difference is

$$\Delta T_m = \frac{100 + 60}{2} - \frac{80 + 40}{2}$$
$$= 20°\text{C}$$

110. (a) Given that

$$U_o = 200 \text{ W/m}^2\text{C}$$
$$A = 32.5 \text{ m}^2$$
$$C_c = 2 \times 4180$$
$$C_h = 1030 \times 5.25$$
$$= C_{min}$$

Therefore,

$$\text{NTU} = \frac{U_o A}{C_{min}}$$
$$= 1.2$$

111. (d) LMTD for counter or parallel flow heat exchange is given by

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2/\Delta T_1\right)}$$
$$= \frac{(140 - 80) - (80 - 60)}{\ln\left[(140 - 80)\,/\,(80 - 60)\right]}$$
$$= \frac{0}{0}$$

But the average temperature difference will be equal to 60°C throughout the exchanger.

112. (c) Given that

$$C_{min} = C_{max}$$
$$\text{NTU} = 0.5$$

Hence,

$$\varepsilon = \frac{\text{NTU}}{1+\text{NTU}} = 0.33$$

113. (a) The effectiveness of heat exchangers with equal heat capacities of flowing fluids ($C_{min} = C_{max}$) is related to NTU by the following equation,

$$\varepsilon = \frac{\text{NTU}}{1+\text{NTU}}$$

Given that NTU = 1.0, therefore,

$$\varepsilon = \frac{1.0}{1+1.0} = 0.5$$

114. (c) According to the theorem, the number of independent dimensionless groups that can be formed from a set of n variables having m basic dimensions is $(n-m)$.

115. (b) Fourier number is a dimensionless measure of time used in transient conduction problems. The Graetz number is a dimensionless number that characterizes laminar flow in a conduit. For a conduct of diameter D, length L, it is expressed as

$$\text{Gz} = \frac{D}{L}\text{Re.Pr}$$

Numerical Answer Questions

1. Given that

$$r_i = 0.1 \text{ m}$$
$$r_o = 0.16 \text{ m}$$
$$\kappa = 55 \text{ W/mK}$$
$$T_i = 325°\text{C}$$
$$T_o = 95°\text{C}$$

Thermal resistance of the sphere is

$$R_{th} = \frac{r_o - r_i}{4\pi r_o r_i \times \kappa} = 5.42°\text{C/kW}$$

The rate of heat transfer is given by

$$\dot{Q} = \frac{T_i - T_o}{R_{th}} = 42.39 \text{ kW}$$

2. Given that

$$t = 0.5 \text{ m}$$
$$T_1 = 350°\text{C}$$
$$T_2 = 55°\text{C}$$

Using Fourier's law

$$\dot{Q} = -\int \kappa A \frac{dT}{dt}$$
$$\int \frac{\dot{Q}}{A} dt = -\int \kappa dT$$
$$\frac{\dot{Q}}{A} t = -\int_{T_1}^{T_2} \kappa dT$$
$$= \left[0.005 \times \frac{T^2}{2} - 5\times 10^{-6} \times \frac{T^3}{3}\right]_{55}^{350}$$
$$= 227.5$$
$$\frac{\dot{Q}}{A} = 455.01 \text{ W/m}^2$$

3. The rate of heat transfer through the wall for the unit area would be given by

$$\frac{\dot{Q}}{A} = \frac{30+25}{2\times 1/20 + 2\times 0.005/0.75 + 0.003/0.02} = \frac{55}{0.263} = 208.86 \text{ W/m}^2$$

The rate of heat transfer through the wall for the unit area shall be given by

$$\frac{\dot{Q}}{A} = \frac{30+25}{2\times 1/20 + 2\times 0.005/0.75 + 0.001/0.02} = 336.73 \text{ W/m}^2$$

Percentage increase in the rate of heat loss is

$$\frac{336.73 - 208.86}{208.86} \times 100 = 61.22\%$$

4. The rate of heat transfer through the wall for the unit area would be given by

$$\begin{aligned}\frac{\dot{Q}}{A} &= \frac{1200-50}{0.15/0.533+0.4/0.7}\\ &= 1348.41\ \mathrm{W/m^2}\end{aligned}$$

The rate of heat transfer through the wall for the unit area is given by

$$\begin{aligned}\frac{1348.41}{2} &= \frac{1200-50}{t/0.533+0.4/0.7}\\ t &= 0.60\ \mathrm{m}\\ &= 60\ \mathrm{cm}\end{aligned}$$

5. Given that

$$\begin{aligned}T &= 100^\circ\mathrm{C}\\ T_0 &= 300^\circ\mathrm{C}\\ T_\infty &= 20^\circ\mathrm{C}\\ \rho &= 2700\ \mathrm{kg/m^3}\\ c &= 900\ \mathrm{J/kgK}\\ \kappa &= 205\ \mathrm{W/mK}\end{aligned}$$

The surface area A and volume V of the sphere are calculated as

$$\begin{aligned}V &= \frac{6}{2700}\\ &= 2.22\times10^{-3}\ \mathrm{m^3}\\ \frac{4}{3}\pi r^3 &= 2.22\times10^{-3}\\ r &= 0.0809\ \mathrm{m}\\ A &= 4\pi r^2\\ &= 0.0823\end{aligned}$$

In lumped heat analysis, the time constant is defined as the time required by the sphere to reach 36.8% of the initial temperature difference, expressed as

$$\begin{aligned}t^* &= \frac{\rho V c}{hA}\\ &= 1092.53\ \mathrm{s}\end{aligned}$$

In lumped heat analysis, the temperature and time are related as follows

$$\begin{aligned}\frac{T-T_\infty}{T_0-T_\infty} &= \exp\left(-\frac{hA}{\rho V c}t\right)\\ 0.2857 &= \exp\left(-9.153\times10^{-4}\times t\right)\\ t &= 1376.7\ \mathrm{s}\end{aligned}$$

6. Given that

$$\begin{aligned}u_\infty &= 5\ \mathrm{m/s}\\ \nu &= 15.89\times10^{-6}\ \mathrm{m^2/s}\end{aligned}$$

At $x = 0.5$ m, Reynolds number is calculated as

$$\begin{aligned}\mathrm{Re}_x &= \frac{u_\infty x}{\nu}\\ &= 1.57\times10^5\end{aligned}$$

Since, $\mathrm{Re} < 5\times10^5$, the boundary layer is laminar. Thickness of hydraulic boundary layer is given by

$$\begin{aligned}\delta &= \frac{5x}{\sqrt{\mathrm{Re}_x}}\\ &= 6.302\ \mathrm{mm}\end{aligned}$$

The average film temperature is

$$\begin{aligned}T_f &= \frac{27+227}{2}\\ &= 227^\circ\mathrm{C}\end{aligned}$$

At 227°C, the properties of air are given as

$$\begin{aligned}\rho &= 0.8711\ \mathrm{kg/m^3}\\ c_p &= 1.014\ \mathrm{kJ/kg\ K}\\ \nu &= 26.41\times10^{-6}\ \mathrm{m^2/s}\\ \kappa &= 33.8\times10^{-3}\ \mathrm{W/mK}\\ \mathrm{Pr} &= 0.690\end{aligned}$$

Reynolds number at $x = 0.5$ m is calculated as

$$\begin{aligned}\mathrm{Re}_x &= \frac{u_\infty x}{\nu}\\ &= 94.66\times10^3\end{aligned}$$

The Nusselt number at $x = 0.5$ m is calculated as

$$\begin{aligned}\mathrm{Nu}_x &= 0.332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}\\ \frac{h_x x}{\kappa} &= 90.26\\ h_x &= 6.10\ \mathrm{W/m^2K}\end{aligned}$$

Therefore, the average heat transfer coefficient for the first $x = 0.5$ m distance from leading edge on single side is given by

$$\begin{aligned}\bar{h}_x &= 2\times h_x\\ &= 12.2\ \mathrm{W/m^2K}\end{aligned}$$

Hence, the net heat transfer on both sides is

$$\begin{aligned}\dot{Q} &= 2\times\bar{h}_x A\left(T_s-T_\infty\right)\\ &= 4270\ \mathrm{kW}\end{aligned}$$

7. Given that

$$\lambda_m = 0.5\ \mu\mathrm{m}$$

Using Wien's displacement law,

$$\begin{aligned}\lambda_m T &= C_2\\ T &= \frac{2890}{0.5}\\ &= 5780 \text{ K}\end{aligned}$$

Considering the sun as a black body at temperature $T = 5780$ K, the emissive power is given by

$$\begin{aligned}E_b &= \sigma T^4\\ &= 63.284 \text{ MW/m}^2\end{aligned}$$

8. Given that temperature of the body

$$\begin{aligned}T &= 927+273\\ &= 1200 \text{ K}\\ E\lambda_m &= 2.74\times 10^{10} \text{ W/m}^2\end{aligned}$$

The relation between maximum spectral emissive power (E_b) and corresponding temperature is

$$\begin{aligned}\frac{E_b\lambda_m}{T^5} &= 1.307\times 10^{-5} \text{ W/m}^3\text{.K}^5\\ E_b\lambda &= 3.252\times 10^{10} \text{ W/m}^2\end{aligned}$$

Therefore, the emissivity can be calculated as

$$\begin{aligned}\epsilon &= \frac{E}{E_b}\\ &= \frac{E\lambda_m}{E_b\lambda_m}\\ &= \frac{1.37\times 10^{10}}{3.252\times 10^{10}}\\ &= 0.842\end{aligned}$$

Using Wien's displacement law,

$$\begin{aligned}\lambda_m T &= C_2\\ \lambda &= \frac{2890}{1200}\\ &= 2.408\ \mu\text{m}\end{aligned}$$

9. Given that

$$\begin{aligned}\epsilon_s &= 0.85\\ A_s &= 1\times\pi\times 0.25\\ &= 0.7853 \text{ m}^2\\ T_s &= 300 \text{ K}\\ T_\infty &= 610 \text{ K}\\ A_b &= 1\times\pi\times 0.5\\ &= 1.57 \text{ m}^2\\ \epsilon_b &= 0.9\end{aligned}$$

The heat will be lost in thermal radiation as

$$\begin{aligned}Q_{12} &= \epsilon_s A_s\sigma\left(T_s^4 - T_\infty^4\right)\\ &= 4.933 \text{ kW/m}^2\end{aligned}$$

The net radiation is

$$\begin{aligned}Q_{12} &= \frac{\sigma\left(T_s^4 - T_\infty^4\right)}{(1-\epsilon_s)/(A_s\epsilon_s)+1/A_s+(1-\epsilon_b)/(A_b\epsilon_b)}\\ &= \frac{7.391\times 10^3}{1.569}\\ &= 4.711 \text{ kW/m}^2\end{aligned}$$

Percentage reduction in the rate of heat loss is

$$\begin{aligned}&= \frac{4.933-4.711}{4.933}\times 100\\ &= 4.5\%\end{aligned}$$

10. The equilibrium equation for heat exchange is

$$\begin{aligned}2.1\times 4.18\times (T_{co}-20) &= 2.78\times 2.095\times (95-40)\\ T_{co} &= \frac{320.325}{8.778}+20\\ &= 56.49°\text{C}\end{aligned}$$

The LMTD of the exchanger is calculated as

$$\begin{aligned}\Delta T_1 &= T_{ho}-T_{ci}\\ &= 40-20\\ &= 20°\text{C}\\ \Delta T_2 &= T_{hi}-T_{co}\\ &= 95-56.49\\ &= 38.51°\text{C}\end{aligned}$$

Hence,

$$\begin{aligned}Q &= UA\times\text{LMTD}\\ &= UA\times\frac{\Delta T_2-\Delta T_1}{\ln(\Delta T_2/\Delta T_1)}\\ A &= \frac{Q}{U}\times\frac{\ln(\Delta T_2/\Delta T_1)}{\Delta T_2-\Delta T_1}\\ &= 22.68 \text{ m}^2\end{aligned}$$

11. Given that

$$\begin{aligned}U &= 1050 \text{ W/m}^2 K\\ A &= 1 \text{ m}^2\end{aligned}$$

Heat capacities of both the fluids are

$$\begin{aligned}C_c &= 0.5\times 4.186\\ &= 2.093 \text{ kW/K}\\ C_h &= 0.2\times 2.095\\ &= 0.419 \text{ kW/K}\\ &= C_{min}\end{aligned}$$

The ratio of heat capacities is

$$r = \frac{C_{min}}{C_{max}} = 0.2$$

The number of transfer units (NTU) are

$$\text{NTU} = \frac{UA}{C_{min}} = 2.506$$

Effectiveness of the counter flow heat exchanger is

$$\begin{aligned}\varepsilon &= \frac{1 - \exp\{-\text{NTU}(1-r)\}}{1 - r\exp\{-\text{NTU}(1-r)\}} \\ &= \frac{0.8653}{0.9731} \\ &= 0.8892\end{aligned}$$

Rate of heat transfer through heat exchanger is

$$\begin{aligned}\dot{Q} &= \varepsilon \dot{Q}_{max} \\ &= \varepsilon C_{min}(T_{hi} - T_{ci}) \\ &= 31.67 \text{ kW}\end{aligned}$$

Using the heat balance:

$$\begin{aligned}2.093 \times (T_{co} - 20) &= 31.67 \\ T_{co} &= 35.13^\circ\text{C} \\ 0.419 \times (105 - T_{ho}) &= 31.67 \\ T_{ho} &= 29.41^\circ\text{C}\end{aligned}$$

Difference in the temperature of the fluids is

$$\begin{aligned}&= 35.13 - 29.41 \\ &= 5.72^\circ\text{C}\end{aligned}$$

CHAPTER 8

THERMODYNAMICS

Thermodynamics is the science that deals with interaction of energy and matter. The subject is based on the primitive concepts that have been formulated into the fundamental laws and govern the principles of energy conversion and feasibility of the processes. Since all natural processes involve interaction between energy and matter, thermodynamics encompasses a very large area of application. The emphasis of the present context is on understanding the fundamental concepts and on systematic formulation and solution of problems from the first principles of thermodynamics.

8.1 BASIC CONCEPTS

8.1.1 Thermodynamic Approaches

Behavior of a matter can be studied at two levels of approach:

1. *Microscopic Approach* Microscopic approach is concerned with the behavior of each molecule that cannot be perceived by human senses. Behavior of the concerned matter is described by summing up the behavior of its molecules, such as in kinetic theory of gases. This approach is also called *statistical thermodynamics.*

2. *Macroscopic Approach* Size of the engineering systems is generally much larger than the mean free path of the molecules, therefore, molecular level analysis is not appropriate due to time consuming demerits. In such situations, the only interest of study is to know the overall effects of action of the molecules which can be simply perceived by human senses, such as pressure, temperature, volume. Such an approach is called *macroscopic approach.*

For example, pressure, a macroscopic quantity, is the average rate of change of momentum due to all the molecular collisions made on a unit area. The effects of pressure can be easily felt.

Macroscopic approach conveniently disregards the atomic nature of a substance to view it as a continuous and homogeneous matter. This is called the concept of *continuum*[1]; the substance is treated free from any kind of discontinuity. This idealization permits properties to be treated as point functions, varying continually in space.

The concept of continuum can be explained for density as a property. Consider a mass δm in a small volume δv surrounding a point in a system.

[1]The concept of continuum is also used in fluid mechanics, mechanics of materials and also in heat transfer studies.

The variation of average density $\delta m/\delta v$ can be plotted against δv. The average density tends to approach an asymptote as δv increases [Fig. 8.1].

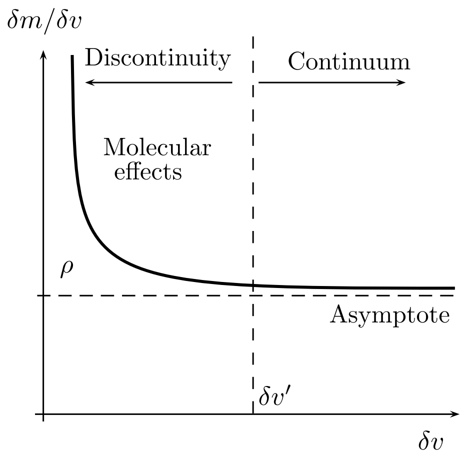


Figure 8.1 | Concept of continuum.

When δv reaches $\delta v'$, so small as to contain relatively few molecules, the average density fluctuates substantially due to random motion of the molecules. In such a situation, the value of quantity $\delta m/\delta v$ cannot be predicted. This threshold volume $\delta v'$ can be regarded as a continuum beyond which there is no effect of molecular motion on the properties of the system. Thus, macroscopic *density* ρ of the system is defined as

$$\rho = \lim_{\delta v \to \delta v'} \frac{\delta m}{\delta v}$$

The concept of continuum can be similarly applied to other properties of the matter.

8.1.2 Thermodynamic Systems

A system in thermodynamics is the collection of matter or region in space chosen for study. Thermodynamic analysis can be simplified by defining an appropriate system which in turn leads a systematic study.

8.1.2.1 System, Surrounding and Universe *Thermodynamic system* is a three-dimensional region of space bounded by one or more surfaces. The boundary can be real or imaginary and can change its size, shape, and location. The region of physical space that lies outside the defined boundaries of a system is called *surrounding*. Whenever a thermodynamic system is defined, the complementary region (surrounding) gets automatically defined. A system and its surroundings together comprise a *universe* [Fig. 8.2].

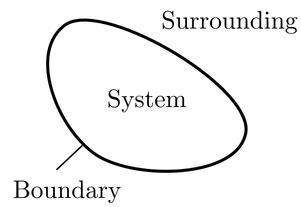


Figure 8.2 | System, surrounding, and universe.

8.1.2.2 System Exchanges Between a given system and its surroundings, the following two types of exchanges can occur:

1. Energy exchange
2. Mass exchange

Here, energy means both heat and work transfers. Heat transfer can takes place through a *diathermal boundary* only. An *adiabatic boundary* does not allow heat exchange to take place.

8.1.2.3 Types of Systems Classification of thermodynamic systems is based on the types of exchanges and depends on selection of a fixed mass or a fixed volume in the space for study. A thermodynamic system can be closed, open, or isolated, explained as follows:

1. *Closed System* A closed system consists of a fixed mass (thus, also known as *control mass*) on which only energy transfer can occur [Fig. 8.3].

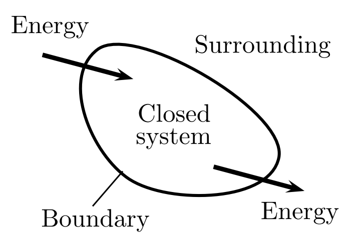


Figure 8.3 | Closed system.

2. *Open System* Both matter and energy cross the boundary of an open system [Fig. 8.4]. Most of the engineering devices involving mass flow, such as a compressor, turbine, nozzle, are examples of open system.

   Based on steadiness[2] of exchange rates, the open systems can be of two types:

[2]The term 'steady' implies no change with time. The opposite of steady is 'unsteady' or 'transient'. The term uniform, however, implies no change with location over a specified region.

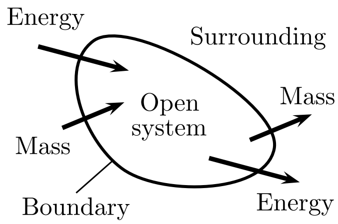


Figure 8.4 | Open system.

(a) Steady Flow System When flow rates of mass and energy remain constant, the system is called *steady flow system*. Most of the engineering devices (e.g. turbine, pumps, heat exchangers, refrigerators) come under this category.

Properties of the working fluid can change from point to point within the control volume, but at any fixed point they remain the same during the entire process.

(b) Unsteady Flow System When flow rates of mass and energy vary with time, the system is called *unsteady flow system*. This condition mainly occurs in starting stages of steady flow systems.

Energy flow associated with a fluid stream is often expressed in rate form by incorporating the mass flow rate ($\dot{m}$), the amount of mass flowing through a cross-section per unit time.

Thermodynamic analysis of open systems involves study of a certain fixed volume in space surrounding the system, known as the *control volume*. Thus, there is no difference between open system and control volume. *Control surface* is the imaginary or real boundary of the control volume, which can be fixed or moveable. The contact surface is shared by both the system and the surrounding.

3. Isolated System An *isolated system* does not have any interaction of mass or energy with its surroundings. Therefore, mass and energy inside an isolated system remain constant [Fig. 8.5].

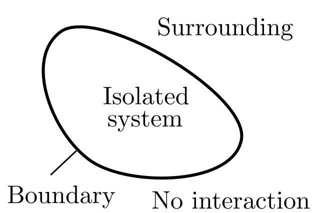


Figure 8.5 | Isolated system.

A quantity of matter homogeneous in chemical composition and physical structure is called a *phase*. A substance can exist in any one of the three phases, namely, solid, liquid, and gas. A *homogeneous system* consists of single phase. A system consisting of more than one phase is called *heterogeneous system*.

8.1.3 State Properties

Physical condition (*state*) of a system is described by certain characteristics, such as mass, volume, temperature, pressure. These characteristics are called *properties of the system*. Thermodynamics is concerned with the properties that are macroscopic in nature and approach [*Section* 8.1.1].

Based on the dependency on the mass or extensiveness, thermodynamic properties can be of two types:

1. Intensive Properties *Intensive properties* are independent of the mass in the system, such as density, pressure, temperature. These properties are generally used to compare systems in an absolute manner, irrespective of the mass of the systems.

   Intensive properties are generally denoted by lowercase letters (temperature T is the obvious exception).

2. Extensive Properties *Extensive properties* depend on the extent of the system, such as volume, energy, momentum. Their magnitude increases with increase in mass of the system. Extensive properties are used to observe the scale of the systems.

   Extensive properties per unit mass are called *specific properties*, which are in fact intensive properties, such as specific volume, specific energy, density.

   Extensive properties are generally denoted by uppercase letters (mass m is the obvious exception), whereas specific extensive properties (i.e. intensive properties) are denoted by lowercase letters.

Extensiveness of a property can be determined by dividing the system into two equal parts with an imaginary partition. Each part will have the same value of intensive properties as the original system but half the value of the extensive properties.

8.1.4 Thermodynamic Equilibrium

A system is said to exist in a state of thermodynamic equilibrium if its isolation from the surrounding does not cause a change in any of the macroscopic properties of the system. This is possible if there exist no unbalance

force, no chemical reaction, and no change in energy. Hence, thermodynamic equilibrium is meant for mechanical, chemical, and thermal equilibrium of a system.

The concept of *thermodynamic equilibrium* is related to the concept of *quasi-static process*, the basis of all theoretical thermodynamic cycles. An *isolated system* always reaches, in the course of time, a state of *thermodynamic equilibrium* and can never depart from it spontaneously.

8.1.5 Two Property Rule

8.1.5.1 Constitutive Relation Definite values of all the properties of a system indicate a definite *state* of the system. However, all the properties of a system cannot be varied independently since they are interrelated through *Constitutive relation* of the following type:

$$f(p, v, T) = 0$$

where p, v, T are some interrelated properties of a system. Interestingly, a system can be perfectly defined by knowing how many variables can be varied independently. Two properties are independent if one property can be varied while the other one is held constant.

Experiments have shown that once a sufficient number of properties are determined, the rest of the properties assume definite values automatically using the constitutive relations.

8.1.5.2 State Postulate The number of properties required to fix the state of a system is given by the *state postulate* or *two property rule*. According to this rule, the state of a simple compressible system is completely specified by two independent intensive properties.

8.1.5.3 Compressible System In *state postulate*, a system is called *simple compressible system* in absence of electrical, magnetic, gravitational, motion, and surface tension effects. These effects are caused by external forces, and are negligible in most engineering problems. Otherwise, an additional property needs to be specified for each effect that is significant. A simple system of compressible substance (gas) can be described by, (p, v), or (p, T), or (T, v), or (p, u), or (u, v), but cannot by (T, u) because u is not independent of T.

8.1.6 Processes and Cycle

A change in one or more properties of a system is called a *change in state*. The succession of states passed through during a change of state is called the *path of the change of state*. When the path is completely specified, the change of state is called a *process*[3]. When a system in a given initial state goes through a number of different changes in state (i.e. through various processes) and finally returns to its initial state, the system undergoes a *cyclic process*, or simply a *cycle*. Therefore, at the conclusion of a cycle, all the properties have the same value as at the beginning. For a cyclic process, the final state is identical with the initial state; cyclic integral of a property is always zero [Fig. 8.6].

Using the *state postulate*, the properties of a system can be taken as the state coordinates to describe the state of the system as a point on a two-dimensional thermodynamic property diagram. Therefore, processes and cycle of a system can be conveniently represented on two-dimensional property diagrams [Fig. 8.6].

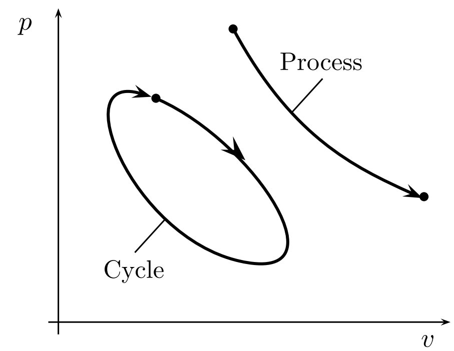


Figure 8.6 Processes and cycle.

For a given state, there is a definite value for each property. The change in a property of a system is independent of the path the system follows during the change of state. Therefore, properties are *point functions*[4].

8.1.7 Modes of Energy

Energy[5] exists in numerous forms, such as thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, nuclear, and all these forms constitute the total energy of a system. All these forms are called different modes of energy. The mode of energy significantly affects the efficiency and type of energy exchange of a thermodynamic system. For thermodynamic studies, the *modes of energy* are grouped into macroscopic and microscopic modes, discussed as follows:

1. Macroscopic Modes of Energy The *macroscopic energy* of a system is related to motion and the influence of some external potentials, such

---

[3]The prefix iso- is often used to designate a process for which a particular property remains constant [*Section* 8.8.7]. For example, *isothermal process*, *isobaric process*, *isochoric (or isometric) process*.

[4]Differentials of point functions are exact or *perfect differentials*.

[5]The term "energy" was coined in 1807 by Thomas Young, and its use in thermodynamics was proposed in 1852 by Lord Kelvin.

as gravity, magnetism, electricity, and surface tension. A system possesses such forms of energy always with respect to some external reference frame. Thus, kinetic energy and potential energy come under macroscopic mode of energy, explained as follows:

(a) Kinetic Energy The energy that a system possesses as a result of its motion relative to some reference frame is called *kinetic energy*. For example, when all the parts of a system, having mass m, move with the same velocity V with respect to some fixed reference frame, the kinetic energy is expressed as

$$T = m\frac{V^2}{2}$$

(b) Potential Energy The energy that a system possesses as a result of its elevation in a gravitational field is called *potential energy*. For example, when all parts of a system, having mass m, are at elevation z relative to the center of a potential field, say gravity g, the potential energy of the system is equal to

$$U_g = mgz$$

Both of these forms of energy are the *organized form of energy*, as these can be readily converted into work.

2. Microscopic Modes of Energy The molecules are always in random motion and possess energy in several forms, such as translational energy, rotational energy, vibrational energy, electronic energy, chemical energy, nuclear energy. These are the *microscopic forms of energy* which are related to the molecular structure of a system and the degree of the molecular activity. These forms of energy are independent of outside reference frame. These are the *disorganized forms of energy*[6] that cannot be readily converted into work.

   The sum of all microscopic forms of energy is called the *internal energy*[7] of the system and is denoted by U. Since internal energy of a system is independent of outside reference frame, therefore, it is a property of the system.

In most of the thermodynamic systems, the effects of magnetic, electrical and surface tension fields are negligible. Thus, the total energy of a system consists of kinetic energy, potential energy, and internal energies, and is expressed as

$$E = U + m\frac{V^2}{2} + mgz$$

Closed system generally remains stationary during a process and thus experience no change in their macroscopic energy. For such systems, referred to as *stationary systems*, the change in total energy is identical to the change in internal energy:

$$\Delta E = \Delta U$$

In absence of motion and gravity,

$$E = U$$

Thermodynamics aims for devising the means for converting disorganized internal energy into useful or organized work, or sometimes interchange between the above two modes of energy. Thermodynamics does not inquire about the absolute value of the total energy but deals only with the change in the total energy.

8.1.8 Equilibrium in Processes

Thermodynamic processes are categorized on the basis of maintaining thermodynamic equilibrium at each state point in the process. As such, a process can be quasi-static or irreversible, described as follows:

1. Quasi-Static Processes Quasi-static means "like-static". Hence, infinite slowness is the characteristic feature of a *quasi-static process*. A quasi-static process is, thus, a succession of infinite equilibrium states. Such processes are also called *reversible processes* because once having taken place, can be reversed, and in so doing leave no change in either the system or surroundings.

   One way to make real processes approximate reversible process is to carry out the process in a series of small or infinitesimal steps. For example, heat transfer can be considered reversible if it occurs by virtue of very small temperature difference between the system and its surrounding.

2. Irreversible Processes An *irreversible process* is a process, if reversed, cannot return both the system and the surroundings to their original states. All of the natural processes are irreversible processes.

Practically, there exist no truly reversible processes in this world; however, the term "reversible" is used to make the analysis simpler, and to determine maximum theoretical efficiencies.

[6]The kinetic energy of an object is an organized form of energy associated with the orderly motion of all molecules in one direction in a straight path or around an axis. In contrast, the kinetic energies of the molecules are completely random and highly disorganized.

[7]The term internal energy and its symbol U first appeared in the works of Rudolph Clausius and William Rankine in the second half of the nineteenth century.

8.2 ZEROTH LAW OF THERMODYNAMICS

Several properties of materials depend on temperature in definite way. This fact forms the basis for accurate temperature measurement. For example, the commonly used mercury-in-glass thermometer is based on the expansion of mercury with temperature.

The *zeroth law of thermodynamics*[8] states that if two bodies are in thermal equilibrium with a third body, they are also in thermal equilibrium with each other. This law serves as the basis for the validity of temperature measurement.

In temperature measurements, the third body is replaced with a thermometer and zeroth law is restated as “two bodies are in thermal equilibrium if both have the same temperature reading even if they are not in contact”.

8.3 ENERGY TRANSFER

The forms of energy not stored in a system can be viewed as the dynamic forms of energy or as energy interactions which are recognized only at the system boundary. Energy can cross the boundary of a closed system in two distinct forms, *heat* and *work*. Therefore, the term *energy* for closed systems is meant for ‘work’ and ‘heat’ both. These are the energies in transit and are identified at the boundary only. An energy interaction is heat transfer if its driving force is temperature difference, otherwise it is work.

A quantity transferred to or from a system during an interaction is not a property since the amount of such quantity depends on more than just the state of the system. In other words, the systems possess energy, but not heat or work. Both forms of energy interactions are associated with process, not with a state. Therefore, heat and work are *path functions*; their magnitudes depend on the path followed during a process as well as the end states.

8.3.1 Heat Transfer

Heat transfer is defined as the energy interaction across a boundary of a system by virtue of a temperature difference. Thus, the temperature difference between two systems represent the potential of heat transfer between the systems.

8.3.1.1 Sign Convention Heat transfer is a directional quantity, and thus, the complete description of heat interaction requires the specification of both the magnitude and direction. Heat flow into a system is taken as positive, and heat flow out of a system is taken as negative [Fig. 8.7].

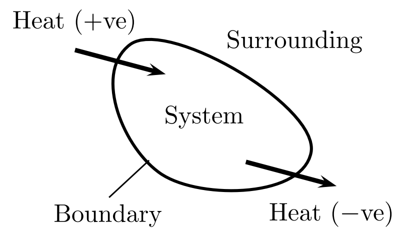


Figure 8.7 | Heat transfer (sign convention).

8.3.1.2 Heat Transfer in a Process The amount of heat transfer during the process between two states (say 1 and 2) is denoted by Q_{12} or just Q. Sometimes, knowledge of heat transfer rate is desired instead of total heat transferred over some time interval. The heat transfer rate is denoted by $\dot{Q}$. When $\dot{Q}$ varies with time (t), the amount of heat transfer during a process is determined by integrating $\dot{Q}$ over the time interval of the process, as

$$Q = \int_1^2 \dot{Q} dt$$

When $\dot{Q}$ remains constant during a process, the above relation reduces to

$$Q = \dot{Q}\,(t_2 - t_1)$$

Being a path function, heat transfer in a process from state 1 to state 2 can be represented as

$$Q_{1-2} = \int_1^2 T dX$$

where T is the temperature at the point in the path and X is another property[9] of the system.

A process in which heat cannot cross the boundary of the system is called *adiabatic process*[10]. Thus, an adiabatic process involves only work interaction. It should not be confused with an *isothermal process*. Even though there is no heat transfer during an adiabatic process, the energy content and thus the temperature of a system can still be changed by other means, such as work.

[8]The zeroth law of thermodynamics was first formulated and labeled by R. H. Fowler in 1931. This was recognized as a fundamental principle more than half a century after the formulation of the first and the second laws of thermodynamics. The law is named zeroth law since it should have preceded the first and second laws of thermodynamics.

[9]The quantity X is actually the entropy of the system [*Section* 8.5.14].

[10]The word adiabatic comes from Greek word *adiabatos*, which means not to be passed.

8.3.2 Work Transfer

If the energy crossing the boundary of a closed system is not heat, it must be work. *Work* is done by a force if it causes a body to move in the direction of the force. A rising piston, rotating shaft, and an electric wire crossing the system boundaries, all are associated with work interactions.

The work done during a process between two states 1 and 2 is denoted by W_{12} or just W. Work done per unit time is called *power*, denoted by P. The unit of power is kJ/s or kW.

In thermodynamics, the work is said to be done by a system if the sole effect on things external to the system can be reduced to the raising of a weight (cannot actually but imaginary).

8.3.2.1 Sign Convention Work transfer is also a directional quantity. Work is arbitrarily taken to be positive when the system does work, and negative when the work is done on the system [Fig. 8.8].

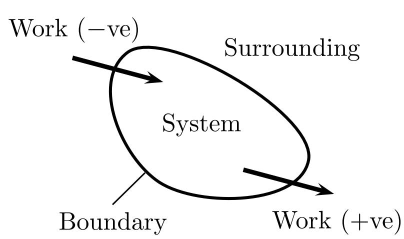


Figure 8.8 | Work transfer (sign convention).

8.3.2.2 Work Done in a Process When volume (v) of a gas changes with pressure (p) in a quasi-static process, the work done by the system (gas) is given by

$$dW = pdv$$
$$W_{12} = \int_{v_1}^{v_2} pdv \tag{8.1}$$

Therefore, work is a *path function*; dW is an inexact or *imperfect differential*.

For irreversible processes, the path cannot be certain, therefore,

$$W \begin{cases} = \int pdv & \text{Reversible processes} \\ \neq \int pdv & \text{Irreversible processes} \end{cases}$$

If the work done on a gas is equal to the change in potential energy (of mass), it results in a situation where $dv = 0$ and yet dW is not equal to zero.

8.3.2.3 Flow Work The *flow work*, significant only in flow process or an open system, represents the energy transferred across the system boundary as a result of the energy imparted to the fluid by a pump, blower, or compressor to make the fluid flow across the control volume. It is analogous to displacement work.

The flow work per unit mass is equal to pv, equivalent to the work required to push the volume of mass from zero to v under constant pressure p.

8.3.2.4 Work in Free Expansion Free expansion of a gas against vacuum is not a quasi-static process. Since vacuum does not offer any resistance to the expansion of the gas, therefore, there is no work transfer involved in the free expansion.

8.3.2.5 Electrical Work In an electrical field, electrons in a wire move under the effect of electromagnetic forces. Thus, electrons crossing the system boundary do electrical work on the system.

In general, potential difference V and the current I can vary with time, therefore, the electrical work done during a time interval from t_1 to t_2 is expressed as

$$W = \int_{t_1}^{t_2} VIdt$$

8.4 FIRST LAW OF THERMODYNAMICS

Experiments have shown that by means of proper apparatus, any form of energy can be converted into other forms, and that during this process absolutely no part of energy is lost. Heat energy and mechanical energy are thus found inter-convertible. Since nothing is lost in such conversions, a unit of one form of the energy must always give certain number of units of another form. The *first law of thermodynamics* is the application of the conservation of energy principle in thermodynamic processes.

8.4.1 Expressions of First Law

The expressions of the first law of thermodynamics are different for closed and open systems (steady and unsteady), discussed as follows:

1. *Closed System* If a closed system [Fig. 8.9] within given time period takes heat dQ, works dW, and change in its internal energy is dU, then, the first law of thermodynamics is represented as

$$dQ = dW + dU \tag{8.2}$$

Therefore, for a closed system, heat transfer is the

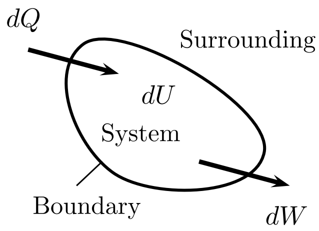


Figure 8.9 First law of thermodynamics.

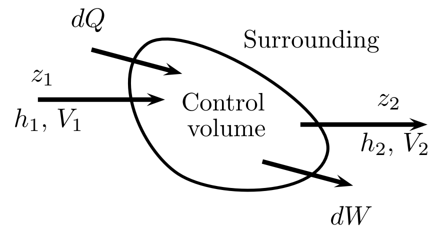


Figure 8.10 Steady flow system.

sum of work transfer and change in internal energy of the system.

2. *Cyclic Processes* If a closed system in a part of its cycle takes heat dQ and works dW, the cyclic integral of the change in internal energy (dU as a property) would be zero. Therefore, using Eq. (8.2), one gets

$$\oint dQ = \oint dW + \oint dU$$

$$\oint dQ = \oint dW \tag{8.3}$$

This is the first law of thermodynamics for cyclic processes on a closed system where $\oint$ represents the cyclic integration. For discrete energy exchanges, Eq. (8.3) takes following form:

$$\left(\sum Q\right)_{cycle} = \left(\sum W\right)_{cycle} \tag{8.4}$$

Therefore, for a cyclic process, the total heat transfer is equal to total work transfer.

3. *Steady Flow Systems* In a *steady flow system* with unit mass flow rate, the first law of thermodynamics is written as

$$\begin{aligned} Q - W = {} & \underbrace{(h_2 - h_1)}_{\text{Change in enthalpy}} \\ & + \underbrace{\frac{{V_2}^2 - {V_1}^2}{2}}_{\text{Change in K.E.}} \\ & + \underbrace{g\,(z_2 - z_1)}_{\text{Change in P.E.}} \end{aligned}$$

where Q and W represent heat and work transfer per unit mass, respectively, h, V, z represent specific enthalpies, velocities and elevation above datum levels with subscripts 1 and 2 for inlet and outlet points, respectively [Fig. 8.10].

This equation is called *steady flow energy equation* [*Section* 8.4.5].

4. *Unsteady Flow Systems* In the present context, following is the common expression for two unsteady flow processes: filling and emptying,

$$\begin{aligned} Q - W = {} & \underbrace{m_2 u_2 - m_1 u_1}_{\text{change in internal energy}} \\ & - \underbrace{(m_2 - m_1)\, e_f}_{\text{change in flow energy}} \end{aligned}$$

where

$$\begin{aligned} u_1 &= c_v T_1 \\ u_2 &= c_v T_2 \\ p_1 v_1 &= m_1 R T_1 \\ p_2 v_2 &= m_2 R T_2 \end{aligned}$$

In the above equation, Q and W represent heat and work transfer, m_1 and m_2 represent the mass in the reservoir before and after the process, respectively, u denotes specific internal energy, and the value of e_f is defined as

$$e_f = \begin{cases} c_p T_r & \text{Filling processes} \\ c_p T_2 & \text{Emptying processes} \end{cases}$$

where T_r is the temperature of reservoir used in filling process.

8.4.2 Energy - A Property

The interactions of heat and work cause a change in the stored energy (E) of the system. During energy transfer, the system undergoes a change from one state to another. When a closed system undergoes a cyclic process, the total heat transfer is equal to the total work transfer. In other words, the cyclic integral of the change in energy of the system is zero; energy is a point function, therefore, a property of the system.

Consider a system undergoing cycles between state 1 to state 2 in two alternative paths A and B and returning by a common path C. So, the system undergoes a cycle A-B [Fig. 8.11].

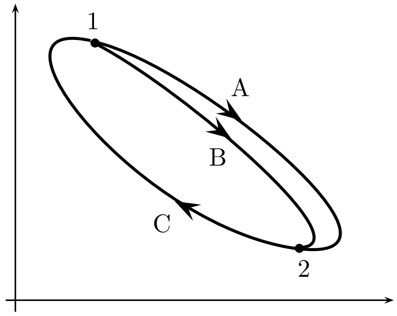


Figure 8.11 Energy - a property.

Using the first law for two processes A and B [Eq. (8.2)]:

$$dQ_C = dE_C + dW_C$$
$$dQ_A = dE_A + dW_A$$

For a cycle consisting of processes C and A [Eq. (8.3)]:

$$\oint dQ = \oint dW$$
$$dQ_C + dQ_A = dW_C + dW_A$$
$$dQ_C - dW_C = -(dQ_A - dW_A)$$
$$dE_C = -dE_A$$

Similarly, for a cycle consisting of processes C and B:

$$dE_C = -dE_B$$

This indicates that the change in energy (dE) between two states of a system is the same, irrespective of the path the system follows between the two states. Therefore, energy of a system is a *point function* and a *property* of the system.

8.4.3 Enthalpy

Enthalpy of a system is the energy in terms of sum of internal energy, and flow energy. For a system of unit mass, the specific enthalpy (h) is given by

$$h = u + pv$$

where u is the specific internal energy and pv is the specific flow energy. Enthalpy is an important form of energy, having special relevance in thermodynamic analysis of open systems.

8.4.4 Specific Heats

Specific heat is the amount of heat required by unit mass of the system for unit rise of its temperature. In this process, pressure and volume play dominant roles. Therefore, specific heat can be measured in two ways, by keeping constant volume or constant pressure. The specific heat measured by keeping volume constant is called *specific heat at constant volume* (c_v). Similarly, the specific heat measured by keeping pressure constant is called *specific heat at constant pressure* (c_p). Their relationship with internal energy and enthalpy can be established as follows:

1. *Constant Volume Process* For a *constant volume process*, $dv = 0$.

$$(\Delta u)_v = \int_{T_1}^{T_2} c_v dT$$

and for a closed system of unit mass

$$\begin{aligned} dQ_v &= du + pdv \\ &= du \\ &= \int_{T_1}^{T_2} c_v dT \end{aligned}$$

Thus, heat transfer at constant volume changes the internal energy of the system in equal amount:

$$c_v = \left(\frac{\partial Q}{\partial T}\right)_v$$

Therefore, specific heat of a substance at constant volume c_v is the rate of change of internal energy with respect to temperature:

$$c_v = \left(\frac{\partial u}{\partial T}\right)$$

2. *Constant Pressure Process* In an *isobaric process* ($dp = 0$),

$$\begin{aligned} d(pv) &= pdv + vdp \\ &= pdv \end{aligned}$$

Therefore,

$$\begin{aligned} dQ_p &= du + pdv \\ &= du + d(pv) \\ &= d(u + pv) \\ &= dh \end{aligned}$$

Hence, heat transfer at constant pressure changes the enthalpy of the system with equal amount. Therefore, specific heat at constant pressure c_p is the rate of change of enthalpy with respect to temperature[11]:

$$c_p = \left(\frac{\partial h}{\partial T}\right)$$

[11]For an ideal gas,

$$pv = nRT$$

A relevant term, *heat capacity* (C), is defined as the amount of heat required by full mass of the system for unit rise of its temperature at constant volume or constant pressure. For a system having mass m,

$$C_v = mc_v$$
$$C_p = mc_p$$

where C_v and C_p are the heat capacities at constant volume and constant pressure, respectively.

8.4.5 Steady Flow Systems

Most of the engineering devices work at constant rate of flow of mass and energy through the control surface and the control volume in course of time attains an invariant state with time. Such a state is called *steady flow state*. At the steady state of a system, any thermodynamic property will have a fixed value at a particular location, and will not alter with time (t) but can vary with space.

Consider a steady flow system in which one stream of fluid enters at point 1 and leaves the control volume at point 2. There is no accumulation of mass or energy within the control volume [Fig. 8.12].

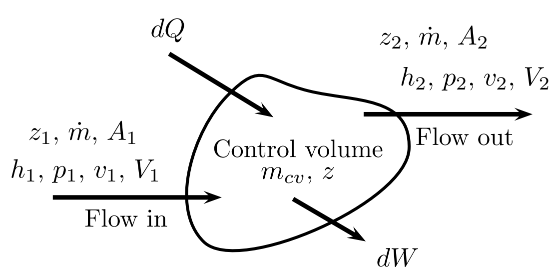


Figure 8.12 Steady flow system.

The symbols, A, $\dot{m}$, p, v, u, V and z are used to represent cross-sectional area, mass flow rate, pressure (absolute), specific volume, specific internal energy, velocity, and elevation above an arbitrary datum, respectively, and m_{cv} represents the mass of control volume. The net rates of heat transfer and work transfer through the control surface are dQ/dt, dW/dt, respectively.

The specific energy e and specific enthalpy h are written, respectively, as

$$e = \frac{V^2}{2} + zg + u$$
$$h = u + pv$$

The equation of continuity is used to relate flow area, specific volume and velocity at inlet and outlet points:

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

The shaft work W_x is the only external work done by the system. Steady flow systems involve flow energy (pv) at inlet and outlet points. Therefore, assuming no accumulation of energy within the system,

$$\dot{m}(e_1 + p_1 v_1) + \frac{dQ}{dt} = \dot{m}(e_2 + p_2 v_2) + \frac{dW_x}{dt}$$
$$e_1 + p_1 v_1 + \frac{dQ}{dm} = e_2 + p_2 v_2 + \frac{dW_x}{dm}$$
$$\left(h_1 + \frac{V_1{}^2}{2} + z_1 g\right) + \frac{dQ}{dm} = \left(h_2 + \frac{V_2{}^2}{2} + z_2 g\right) + \frac{dW_x}{dm}$$

where dQ/dm and dW_x/dm represent the rate of heat and work transfer per unit mass, respectively. This equation can also be written as

$$Q - W_x = (h_2 - h_1) + \frac{V_2{}^2 - V_1{}^2}{2} + g(z_2 - z_1) \tag{8.5}$$

where Q and W_x refer to energy transfer per unit mass. This equation is known as *steady flow energy equation* (SFEE). The differential form of the above equation is

$$dQ - dW_x = dh + VdV + gdz \tag{8.6}$$

The application of SFEE is explained in the following devices:

1. ***Nozzle and Diffuser*** *Nozzles* are used in turbo-machines to convert pressure energy into the kinetic energy of a fluid. A *diffuser* increases the pressure of a fluid at the expense of kinetic energy [Fig. 8.13].

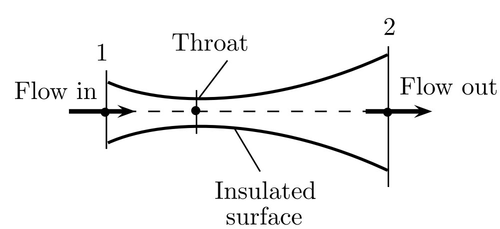


Figure 8.13 Nozzle and diffuser.

---

and internal energy is a function of temperature only. Therefore, enthalpy can be written as

$$h = u(T) + nRT$$
$$= f(T)$$

It means that for an ideal gas, enthalpy is also the function of temperature only.

For a nozzle or diffuser, the flow is assumed to be adiabatic ($dQ = 0$) and there is no work transfer ($dW_x = 0$). Using Eq. (8.5),

$$h_1 + \frac{{V_1}^2}{2} = h_2 + \frac{{V_2}^2}{2}$$
$$V_2 = \sqrt{2\,(h_1 - h_2) + {V_1}^2}$$

This expression[12] is used in determining the exit velocity.

Mach number, an important quantity in study of compressible flow (such as in nozzle or diffuser), is defined as the ratio of the velocity of gas (V) to velocity of sound in the gas ($a = \sqrt{\gamma RT}$)

$$M = \frac{V}{a}$$

2. *Throttling Device* Throttling[13] is the process of passing a fluid through a constricted passage, resulting in an appreciable pressure drop. Throttling

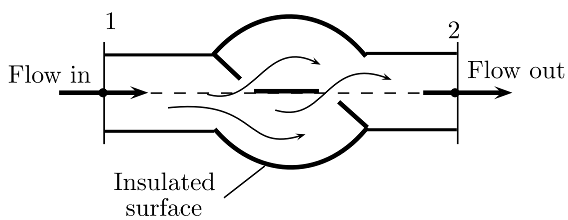


Figure 8.14 Throttling device.

valves are usually small devices [Fig. 8.14], and the flow through them can be assumed to be adiabatic since there is neither sufficient time nor large enough area for any effective heat transfer to take place. Also, there is no work done and change in potential energy:

$$dQ = dW_x = dz = 0.$$

The variation in velocities (V_1, V_2) at inlet and outlet can also be assumed to be negligible. Therefore, using Eq. (8.5),

$$h_1 + \frac{{V_1}^2}{2} = h_2 + \frac{{V_2}^2}{2}$$
$$h_1 = h_2$$

This indicates that the enthalpy of the fluid before throttling is equal to the enthalpy of the fluid after throttling[14]. The drop in pressure is compensated by corresponding change in density or internal energy of the flowing fluid.

3. *Turbine and Compressor* Turbine and engines are used to extract power from the working fluid, whereas compressor and pumps are used to energize the fluid [Fig. 8.15].

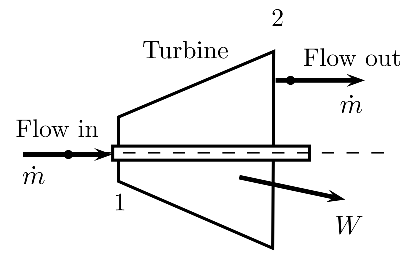


Figure 8.15 Turbine and compressor.

For a well-insulated turbine system ($Q = 0$) and ignoring the changes in kinetic and potential energies,

$$h_1 = h_2 + \frac{W}{m}$$
$$\frac{W}{m} = h_1 - h_2$$

The change in enthalpy of the fluid is equal to the amount of work transfer.

4. *Heat Exchanger* A *heat exchanger* is used to transfer heat from one fluid to another [Fig. 8.16].

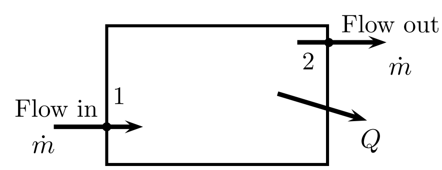


Figure 8.16 Heat exchanger.

When the law of conservation of energy is applied, the rate of change of enthalpy of one fluid is equal to that of the other fluid[15].

[12]In using above equation, the units of h and V should be observed without mistake, because, generally h is in kJ/kg and $V^2/2$ is in J. Therefore, $(h_1 - h_2)$ must be divided by 1000.

[13]The magnitude of the temperature drop or rise during a throttling process is governed by a property called the *Joule-Thomson coefficient*.

[14]This observation is used in measurement of dryness fraction of steam.

[15]Heat exchangers are specifically studied in the subject of heat transfer.

8.4.6 Unsteady Flow Systems

In general, unsteady flow processes are difficult to analyze because the properties of the mass at the inlets and outlets can change during a process.

Consider a device through which a fluid is flowing under unsteady state [Fig. 8.17].

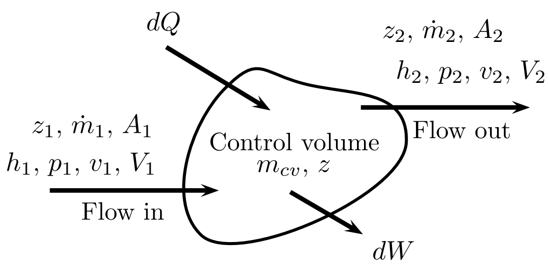


Figure 8.17 Unsteady flow system.

It requires idealization that the fluid flow at any inlet or outlet is uniform and steady, and thus, the fluid properties do not change with time or position over the cross of an inlet or outlet. If they do, they are averaged and treated as constant for the entire process.

As in the steady flow system, the equations of conservation of mass and energy are applied here:

1. *Conservation of Mass* The rate at which the mass of fluid within the control volume (m_{cv}) is accumulated is equal to the net rate of mass flow across the control surface, as given below

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2$$

The change in mass inside the control volume over any finite period of time:

$$\Delta m_{cv} = \Delta m_1 - \Delta m_2$$

2. *Conservation of Energy* Energy of the system within the control volume is written as

$$E_{cv} = \left(U + \frac{m_{cv}V^2}{2} + m_{cv}gz\right)_{cv}$$

Open flow systems involve flow energy (pv) at inlet and outlet points. The rate of increase in this energy is equal to the net rate of energy inflow:

$$\begin{aligned}\frac{dE_{cv}}{dt} &= \left(h_1 + \frac{{V_1}^2}{2} + z_1 g\right)\dot{m}_1 \\ &\quad - \left(h_2 + \frac{{V_2}^2}{2} + z_2 g\right)\dot{m}_2 \\ &\quad + \frac{dQ}{dt} - \frac{dW_x}{dt}\end{aligned}$$

For a finite time interval, the above equation becomes

$$\begin{aligned}\Delta E_{cv} &= Q - W_x \\ &\quad + \int \left(h_1 + \frac{{V_1}^2}{2} + z_1 g\right) dm_1 \\ &\quad - \int \left(h_2 + \frac{{V_2}^2}{2} + z_2 g\right) dm_2\end{aligned}$$

This equation is known as *unsteady flow energy equation* (USFEE). The application of USFEE can be seen in the following processes:

1. *Charging Process* Consider a process in which gas bottle is filled from a pipeline. In the beginning the bottle contains gas of mass m_1, at state (p_1, T_1, v_1, h_1, u_1). The valve is opened and gas flows into the bottle till the mass of gas in the bottle is m_2 at state (p_2, T_2, v_2, h_2, u_2). The supply to the pipeline is very large so that the state of gas in the pipeline (indicated by subscript p) is constant at (p_p, T_p, v_p, h_p, u_p) and velocity of flow is V_p.

The change in internal energy of the control volume is

$$\Delta E_{cv} = m_2 u_2 - m_1 u_1 \tag{8.7}$$

Since addition of energy is from single side (1) only, therefore,

$$\Delta E_{cv} = Q - W + \int \left(h_p + \frac{{V_p}^2}{2}\right) dm_1$$

By putting value of E_{cv} from Eq. (8.7),

$$\begin{aligned}Q - W &= m_2 u_2 - m_1 u_1 \\ &\quad - \left(h_p + \frac{{V_p}^2}{2}\right)(m_2 - m_1)\end{aligned}$$

This is the equation of first law for unsteady charging process.

By ignoring V_p, the equation can be further reduced to

$$Q - W = m_2 u_2 - m_1 u_1 - h_p (m_2 - m_1)$$

If the tank is empty before the start of charging process ($m_1 = 0$), and there is no energy transfer, then

$$\begin{aligned}h_p m_2 &= m_2 u_2 \\ c_p T_p &= c_v T_2 \\ T_2 &= \gamma T_p\end{aligned}$$

Thus, the temperature of gas after charging (T_2) will be equal to γ times the temperature of gas in the pipe (T_p).

2. *Discharging Process* Consider a case of discharging of a bottle in which the extraction is from single side (2) only. Thus,

$$\Delta E_{cv} = Q - W - \int \left(h_2 + \frac{{V_2}^2}{2} \right) dm_2$$

Therefore,

$$Q - W = m_2 u_2 - m_1 u_1 - \left(h_2 + \frac{{V_2}^2}{2} \right) (m_2 - m_1)$$

This is the equation of first law for unsteady discharging process. By ignoring V_2, the equation can be further reduced to

$$Q - W = m_2 u_2 - m_1 u_1 - h_2 (m_2 - m_1)$$

The above analysis leads to the following common equation:

$$Q - W = m_2 u_2 - m_1 u_1 - (m_2 - m_1) e_f$$

where

$$e_f = \begin{cases} c_p T_p & \text{Charging process} \\ c_p T_2 & \text{Discharging process} \end{cases}$$

which represents the specific enthalpy of the working fluid in motion, as in the case of

1. *Charging Process* $c_p T_p$ is the enthalpy of charging fluid (at pipe), and
2. *Discharging Process* $c_p T_2$ is the enthalpy of discharging fluid (at 2).

8.5 SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics does not impose any restriction on the direction of a process; satisfying the first law does not ensure that the process can actually occur. However, natural processes occur only in one direction, for example, heat flows from higher to lower temperatures, water flows downward, time flows in the forward direction. The reverse of these phenomena never happens spontaneously. The spontaneity of the process is driven by a finite potential, such as gradients of temperature, concentration, electric potential. So important is this observation, that it is called the *second law of thermodynamics*, which remedies the inadequacy of the first law in identifying the feasibility of a process.

Mechanical energy can be simply converted into heat energy. For example, heat is produced by friction of moving bodies and in other similar phenomena. The first law can be used to state that, if a process occurs, the net change in energy will be zero:

$$W = Q$$

However, the change in the opposite direction is by far the most difficult task. The apparatus necessary to convert heat into mechanical forms of energy is complicated and does not even theoretically convert all of the supplied heat energy:

$$Q \overset{\rightarrow}{>} W$$

Therefore, work is considered as *high grade energy* while heat as *low grade energy*. The conversion of low grade energy (heat, Q) into high grade energy (work, W) is possible through a cyclic heat engine, but it is incomplete[16].

8.5.1 Energy Reservoirs

In the development of the second law of thermodynamics, hypothetical bodies known as *energy reservoirs*, facilitate understanding of thermodynamic cycles or processes. Thermodynamic analysis involves two types of energy reservoirs:

1. *Thermal Energy Reservoir* A *thermal energy reservoir* (TER) is defined as a large hypothetical body of infinite heat capacity which is capable of absorbing or rejecting an unlimited quantity of heat without suffering appreciable changes in its thermodynamic coordinates. Atmosphere, large rivers, and a two-phase system can be conveniently modeled as thermal energy reservoirs.
2. *Mechanical Energy Reservoir* A *mechanical energy reservoir* (MER) is a large body enclosed by an adiabatic impermeable wall capable of storing work as potential energy (e.g. raised weight or wound spring) or kinetic energy (e.g. flywheel).

8.5.2 Cyclic Heat Engine

A *heat engine cycle* involves net heat transfer and work transfer to the system. Heat engine can be a closed system (e.g. gas confined in a cylinder and piston) or an open system (e.g. steam or gas power plant).

Let heat Q_1 be transferred to the system and work W be the net work done by the system. Heat Q_2 is rejected from the system and the system is brought back to its initial condition [Fig. 8.18].

[16]Sadi Carnot, a French military engineer, first studied this aspect of energy transformation.

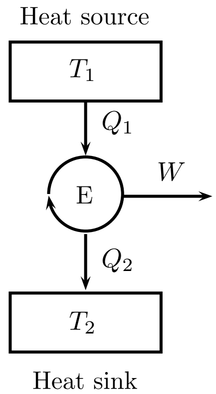


Figure 8.18 | Cyclic heat engine.

The direction of arrow of heat engine cycles is clockwise. Net heat transfer and work transfer in the cycle are

$$Q = Q_1 - Q_2$$

Using the first law of thermodynamics for cyclic systems:

$$Q = W$$

The *efficiency of a heat engine* (η_e) is defined as the ratio of net work output and total heat input to the cycle:

$$\begin{aligned}\eta_e &= \frac{W}{Q_1}\\ &= \frac{Q}{Q_1}\\ &= \frac{Q_1 - Q_2}{Q_1}\\ &= 1 - \frac{Q_2}{Q_1}\end{aligned}$$

The experience shows that $W < Q_1$, therefore, $\eta_e < 1$; all the heat input to the heat engine cannot be converted into work[17].

8.5.3 Refrigerator

A *refrigerator* operates in a cycle to maintain a body at a temperature lower than that of its surrounding. Thus, the direction of a refrigeration cycle is opposite to that of a refrigeration cycle [Fig. 8.19].

To measure the performance of a refrigerator, a *coefficient of performance* (COP_r) is defined as the ratio

[17]Present day engines, such as petrol engine, diesel engine, steam engines, are efficient upto a range of only 30-45%, and engineers are working rigorously to improve the efficiency.

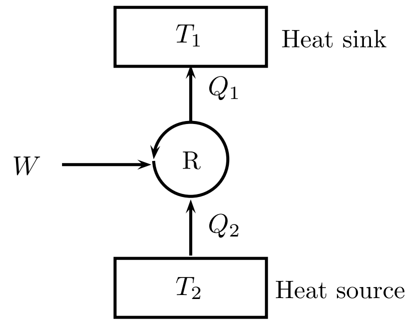


Figure 8.19 | Cyclic refrigerator.

of desired thermal effect and work input to the system:

$$\begin{aligned}\text{COP}_r &= \frac{Q_2}{W}\\ &= \frac{Q_2}{Q_1 - Q_2}\end{aligned} \tag{8.8}$$

8.5.4 Heat Pump

A *heat pump* (HP) operates in a cycle to maintain a body at a temperature higher than that of the surrounding. Consider a body losing heat Q_1 to the surroundings. The cyclic effects in a heat pump are similar to that of a refrigerator [Fig. 8.20].

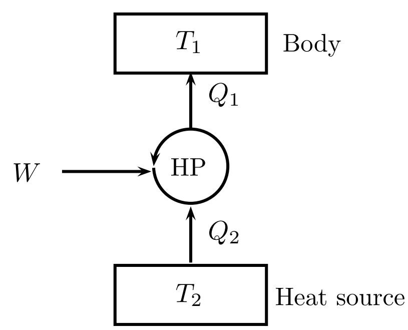


Figure 8.20 | Cyclic heat pump.

Coefficient of performance of a heat pump is defined as

$$\text{COP}_p = \frac{Q_1}{W} \tag{8.9}$$

Using Eqs. (8.8) and (8.9), for heat engine, heat pump, and refrigerator working between the same temperature limits:

$$\text{COP}_p = \text{COP}_r + 1 = \frac{1}{\eta_e}$$

8.5.5 Statements of Second Law

The second law of thermodynamics has two but equivalent forms of statements:

1. *Kelvin Planck Statement* *Kelvin and Planck* stated that it is impossible for a heat engine to produce work in a complete cycle if it exchanges heat only with bodies at a single fixed temperature. Thus, for a cyclic heat engine,

$$Q_2 \neq 0$$

A heat engine has to therefore exchange heat with two thermal energy reservoirs at two different temperatures to produce work in a complete cycle.

2. *Clausius Statement* *Clausius*[18] stated that it is impossible to construct a device which, operating in a cycle, will produce no effect other than the transfer of heat from a cooler to a hotter body. Thus, for a heat pump or refrigerator,

$$W \neq 0$$

Heat, therefore, cannot flow on itself from a body at a lower temperature to a body at a higher temperature. Some work must be expended to achieve this.

Both statements of the second law of thermodynamics are equivalent to each other. This can be shown by proving that violation of one statement implies the violation of second, and vice versa.

8.5.6 Reasons of Irreversibility

Any natural process carried out with a finite gradient (of temperature, pressure, voltage, etc.) is an *irreversible process*. All spontaneous processes are irreversible.

Irreversibility of a process can be due to numerous factors, such as friction, unrestrained expansion, mixing of two fluids, heat transfer across a finite temperature difference, electrical resistance, inelastic deformation of solids, and chemical reactions. These factors can be grouped into two categories:

1. *Lack of Equilibrium* This includes heat transfer through a finite temperature difference, lack of pressure equilibrium, free expansion, throttling, etc.
2. *Dissipative Work* This includes friction, paddle wheel work transfer, transfer of electricity through a resistor.

[18]Rudolf Julius Emanuel Clausius (1822-1888), was a German physicist and mathematician, one of the central founders of the science of thermodynamics. In 1850, he first stated the basic ideas of the second law of thermodynamics. In 1865, he introduced the concept of entropy.

If there is equilibrium and no dissipative effects, all the work done by the system during a process in one direction can be returned to the system during the reverse process. Such processes are *reversible* in nature.

8.5.7 Carnot Cycle

The most interesting cycle, both historically and thermodynamically, is the *Carnot cycle*. It could be carried out with any material as working substance, but can be simply investigated for the case of a perfect gas [Fig. 8.21].

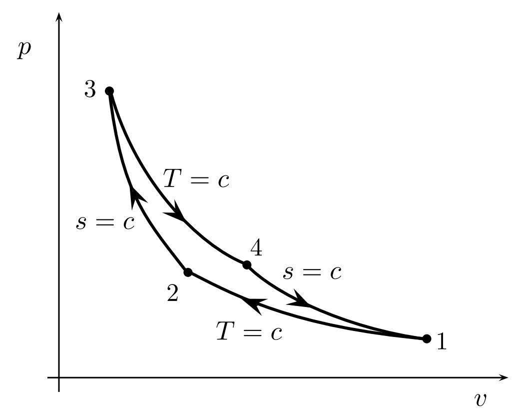


Figure 8.21 | Carnot cycle.

The Carnot cycle comprises the following four reversible processes,

1. Isothermal process ($1 \to 2$): Heat Q_1 is added to the system.
2. Adiabatic process ($2 \to 3$): Work W_e is done by the system.
3. Isothermal process ($3 \to 4$): Heat Q_2 is rejected by the system.
4. Adiabatic process ($4 \to 1$): Work W_c is done on the the system.

The subscripts e and c in W_e and W_c, respectively, are used to signify the expansion and the compression of the system during the respective processes. The net work done by the system and net heat transfer into the system in a cycle is

$$W = W_e - W_c$$
$$Q = Q_1 - Q_2$$

Efficiency of the Carnot cycle is

$$\begin{aligned}\eta_{Carnot} &= \frac{W}{Q} \\ &= \frac{Q_1 - Q_2}{Q_1} \\ &= 1 - \frac{Q_2}{Q_1}\end{aligned}$$

8.5.8 Carnot Principles

Based on the Kelvin-Planck and Clausius statements of the second law of thermodynamics, the two *Carnot principles* are stated as follows:

1. All heat engines operating between a given constant temperature source and a given constant temperature sink, none of them having a higher efficiency than the reversible engine:

$$\eta_{rev} > \eta_{irr}$$

2. The efficiencies of all the reversible engines operating between the same two reservoirs are the same:

$$\eta_{rev,1} = \eta_{rev,2}$$

The first statement of the Carnot principles can be proved analytically by considering two heat engines E_A (irreversible) and E_B (reversible) operating between source temperature T_1, and sink temperature T_2 [Fig. 8.22].

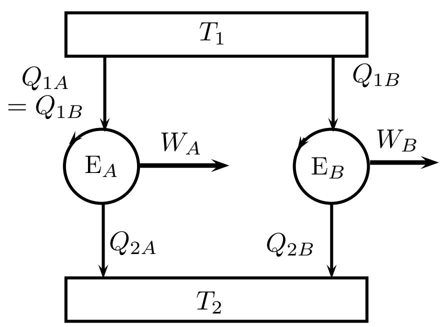


Figure 8.22 Both E_A and E_B as heat engines.

If $\eta_A > \eta_B$, then for the same amount of heat input ($Q_{1A} = Q_{1B}$),

$$\begin{gathered}\frac{W_A}{Q_{1A}} > \frac{W_B}{Q_{1B}} \\ W_A > W_B\end{gathered}$$

When E_B is reversed to act as heat pump $\exists_B$ then the heat Q_{1B} discharged by $\exists_B$ is the heat input to E_A [Fig. 8.23].

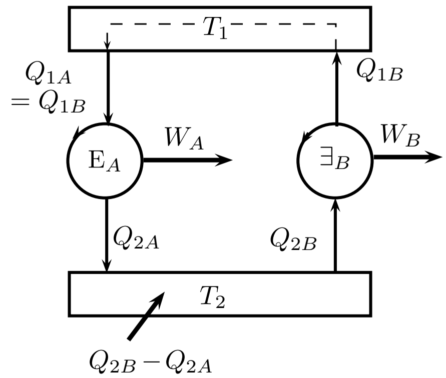


Figure 8.23 Engine E_A and heat pump $\exists_B$.

The net effect is a cyclic heat engine producing net work $W_A - W_B$, while exchanging heat from a single reservoir T_2. This violates the Kelvin Planck statement. Hence, the assumption $\eta_A > \eta_B$ is incorrect, so

$$\eta_B \geq \eta_A$$

The second Carnot principle can be proved by replacing the irreversible engine by another reversible engine having higher efficiency than the first one. After reversing the first engine, the net effect will violate the Clausius statement by producing work while exchanging heat with single reservoir. This proves that both the reversible engines would have equal efficiencies.

8.5.9 Celsius Scale

Temperature measuring instruments should have thermometric properties, for example, the length of a mercury column in a capillary tube, the electrical resistance of a wire, the pressure of a gas in a closed vessel, the e.m.f. generated at the junction of two dissimilar metal wires. To assign numerical values to the thermal state of a given system, it is necessary to establish a temperature scale on which temperature of a system can be read. Therefore, the temperature scale is read by assigning numerical values to certain easily reproducible states. For this purpose, Celsius scale[19] uses the following two points:

1. *Ice Point* The equilibrium temperature of ice with air saturated water at standard atmospheric pressure which is assigned a value of 0°C.
2. *Steam Point* The equilibrium temperature of pure water with its own vapor at standard atmospheric pressure is assigned a value of 100°C.

[19]This scale is called the *Celsius Scale* named after *Anders Celsius*. In 1742 he proposed the Celsius temperature scale. The scale was later reversed in 1745 by Carl Linnaeus, one year after Celsius' death.

8.5.10 Perfect Gas Scale

An ideal gas with unit mole obeys the following constitutive relation[20]:

$$pv = \Re T$$

where $\Re$ (= 8.314 J/mol K) is the *universal gas constant*. The perfect gas temperature scale is based on the observation that the temperature of a gas at constant volume increases with increase in pressure.

The temperature at the triple point (T_{tp}) of water has been assigned a value of 273.16 K. Therefore, temperature (T) of an ideal gas varies proportionally w.r.t. pressure (p):

$$\frac{T}{T_{tp}} = \frac{p}{p_{tp}}$$

$$T = 273.16 \times \frac{p}{p_{tp}} \tag{8.10}$$

Let a series of measurements with different amounts of gas in a bulb be made. The measured pressures at the triple point as well as at the system temperature change depending on the amount of gas in the bulb. A plot of the temperature T, calculated from Eq. (8.10) is shown in Fig. 8.24.

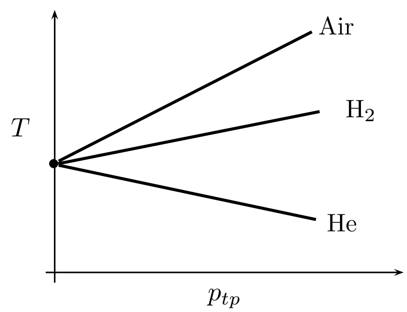


Figure 8.24 T versus p_{tp}.

When these curves are extrapolated to zero pressure, all of them yield the same intercept. This behavior can be expected since all gases behave like ideal gas when their pressure approaches zero.

The correct temperature of the system can be obtained only when the gas behaves like an ideal gas, and hence, the value is to be calculated in limit $p_{tp} \to 0$. Therefore,

$$T_{p_{tp}\to 0} = 273.16 \times \frac{p}{p_{tp}}$$

A constant pressure thermometer can also be used to measure the temperature:

$$T_{v_{tp}\to 0} = 273.16 \times \frac{v}{v_{tp}}$$

Here, v_{tp} is the volume of the gas at the triple point of water and v is the volume of the gas at the system temperature.

8.5.11 Absolute Temperature Scale

The *Carnot principles* state that the efficiency of all engines working between the same temperature levels is the same, and independent of working substance. Therefore, for a reversible cycle (say, Carnot cycle) receiving heat Q_1 and rejecting heat Q_2, the efficiency will solely depend upon the temperatures t_1 and t_2 at which heat is transferred:

$$\begin{aligned}\eta_e &= 1 - \frac{Q_2}{Q_1} \\ &= f(t_1, t_2) \\ \frac{Q_1}{Q_2} &= F(t_1, t_2) \text{ (say)}\end{aligned} \tag{8.11}$$

Let a reversible engine E_1 receive heat from source at t_1 and reject heat at t_2 to another reversible engine E_2 which, in turn, rejects heat to the sink at t_3:

$$\frac{Q_1}{Q_2} = F(t_1, t_2)$$

$$\frac{Q_2}{Q_3} = F(t_2, t_3)$$

Another heat engine E_3 can operate between t_1 and t_3:

$$\frac{Q_1}{Q_3} = F(t_1, t_3)$$

$$\frac{Q_1}{Q_2} = \frac{Q_1/Q_3}{Q_2/Q_3}$$

Therefore,

$$F(t_1, t_2) = \frac{F(t_1, t_3)}{F(t_2, t_3)}$$

The temperatures t_1, t_2, t_3 can assume arbitrary value. Since the ratio Q_1/Q_2 depends only on t_1 and t_2 and is independent of t_3, t_3 is eliminated and the above equation takes the following form:

$$\frac{Q_1}{Q_2} = \frac{\phi(t_1)}{\phi(t_2)}$$

Kelvin proposed the simplest form of function $\phi(t) = T$, therefore,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

This scale is the *absolute temperature scale*, and is better known as *Kelvin scale*[21].

[20]This equation is only an approximation to the actual behavior of the gases. The behavior of all gases approaches the ideal gas limit at sufficiently low pressure.

[21]The efficiency of Carnot cycle can be formulated by analysis of its cycle composed of reversible processes as

$$\eta_{Carnot} = 1 - \frac{T_2}{T_1}$$

The SI system uses the Kelvin scale for measuring temperature which is based on the concept of *absolute zero*, the theoretical temperature at which molecules would have zero kinetic energy. Absolute zero ($-273.16°C$) is set at zero on the Kelvin scale. This means that there is no temperature lower than zero Kelvin, so there are no negative numbers on the Kelvin scale.

8.5.12 Reversible Adiabatic Paths

Consider a reversible cycle consisting of two reversible processes (constant entropy, $s = c$) and an isothermal process (constant temperature, $T = c$). Heat transfer can take place in the isothermal process but not in the reversible processes [Fig. 8.25].

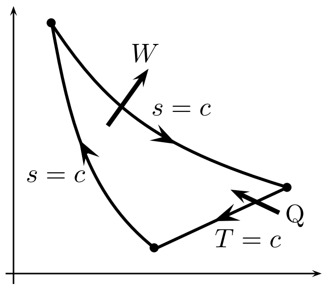


Figure 8.25 Impossible cycle.

The net effect of the cycle will be production of work without discharging heat, violating the second law of thermodynamics. Thus, such a cycle is impossible. Alternatively, two reversible adiabatic processes passing through the same end points must coincide with each other.

8.5.13 Clausius Theorem

Consider a system changing state from an initial equilibrium state 1 to final equilibrium state 2. Let two reversible adiabatic paths $1-1'$ and $2'-2$ be drawn [Fig. 8.26].

A reversible isotherm $1'-2'$ is drawn in such a way that area under $1-1'-2'-2$ is equal to that under $1-2$:

$$W_{1-1'-2'-2} = W_{1-2}$$

Using the first law of thermodynamics for closed system in two alternative paths,

$$Q_{1-2} = U_2 - U_1 + W_{1-2}$$
$$Q_{1-1'-2'-2} = U_2 - U_1 + W_{1-1'-2'-2}$$

This finding proves that the ideal gas temperature and Kelvin temperature are equivalent.

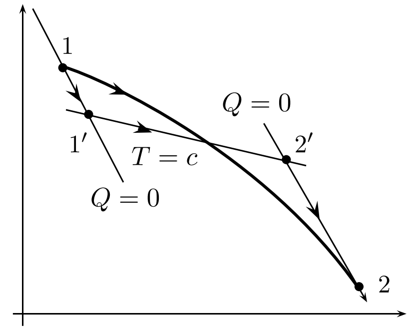


Figure 8.26 Reversible paths.

Hence,

$$Q_{1-2} = Q_{1-1'-2'-2}$$

Thus, a reversible path can be replaced by a reversible adiabatic path, followed by a reversible isotherm, and then by another reversible adiabatic path, such that the heat transfer during the isothermal process is the same as that transferred during the original process.

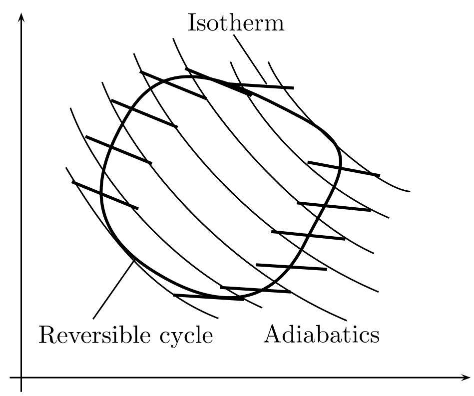


Figure 8.27 Clausius theorem.

Dividing any reversible cyclic process into such transformations results in large number of Carnot cycles [Fig. 8.27] as

$$\oint_R \frac{dQ}{T} = 0 \tag{8.12}$$

Therefore, for a reversible cyclic process, cyclic integral of dQ/T is zero. This is known as *Clausius theorem*.

The Clausius theorem [Eq. (8.12)] is very important for thermodynamic analysis of power cycles. For example, in Carnot cycle,

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$$

where Q_2 is taken negative because it is the heat rejected.

8.5.14 Entropy - A Property

In thermodynamic analyses, the quantity dQ/T quantity is of very frequent use. As established by the Clausius theorem, the cyclic integral dQ/T is zero. This quantity is a point function (i.e. a property of the system). This quantity is the change in *entropy* of the system[22]. The entropy is represented by S. It is an extensive property of system and sometimes referred to as total entropy. Entropy per unit mass is designated as s, an intensive property, and has the unit of kJ/kgK. Therefore,

$$dQ = TdS$$

The entropy change of a system during a process 1-2 can be determined as

$$\int_1^2 \frac{dQ_{rev}}{T} = (S_2 - S_1)$$

which is independent of the path.

Based on the definition of entropy, following equations can be derived for a system of unit mass:

1. First Tds Equation Using the first law of thermodynamics for a closed system [Eq. (8.2)]:

$$Tds = du + pdv \tag{8.13}$$

This is the first Tds equation, also known as *Gibbs equation*.

2. Second Tds Equation By definition of enthalpy

$$\begin{aligned} h &= u + pv \\ dh &= du + d(pv) \\ &= du + pdv + vdp \\ &= Tds + pdv \\ Tds &= dh - vdp \end{aligned}$$

This is the second Tds equation which is obtained by eliminating du from the first Tds equation.

8.5.15 Clausius Inequality

The *Clausius theorem* [Eq. (8.12)] for reversible processes cycle is written as

$$\oint_R \frac{dQ}{T} = 0 \tag{8.14}$$

However, entropy change in an irreversible process is always higher than entropy change in a reversible process between the same end points:

$$\int_1^2 \frac{dQ}{T} \leq (S_2 - S_1)_{irr} \tag{8.15}$$

The above two equations are combined together as

$$\oint \frac{dQ}{T} \leq 0 \tag{8.16}$$

This equation is known as the *Clausius inequality*[23] or the *entropy principle* which is valid for all thermodynamic cycles, reversible or irreversible, including refrigeration cycles. The equality in this equation holds for totally reversible cycles and the inequality for the irreversible ones.

The *Clausius inequality* is used as an alternative form of the second law of thermodynamics, which helps in examining the feasibility of a process (i.e. whether a process is possible or impossible). The inequality can also be used in finding the condition for maximum work. This can be demonstrated through processes discussed as follows.

8.5.15.1 Heat Transfer Consider a case when transfer of heat Q between two bodies takes place at a finite temperature difference $T_1 - T_2$ [Fig. 8.28].

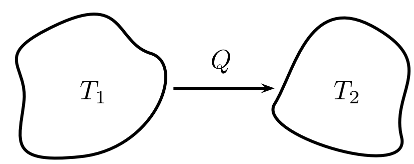


Figure 8.28 Heat transfer.

The total entropy change is the sum of entropy changes in both bodies:

$$\begin{aligned} dS &= -\frac{Q}{T_1} + \frac{Q}{T_2} \\ &= \frac{Q\,(T_1 - T_2)}{T_1 T_2} \end{aligned}$$

For a naturally possible process ($dS > 0$), $T_1 > T_2$, otherwise the process is impossible. This means that heat transfer from a lower body to higher body is impossible (without add of work), thus returning to the second law of thermodynamics.

8.5.15.2 Mixing of Fluids Consider mixing of two fluids of equal heat capacity C at temperatures T_1, T_2. The final temperature of mixture is T_f [Fig. 8.29].

[22]Rudolph Clausius (1822-1888) realized in 1865 that he had discovered a new thermodynamic property, and he chose to name this property entropy, originally entropie (on analogy of Energie) from Greek *entropia* "a turning toward," from *en* "in" + *trope* "a turning".

[23]Clausius inequality was first stated by the German physicis R. J. E. Clausius (1822-1888), one of the founders of thermodynamics.

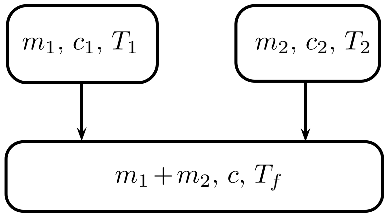


Figure 8.29 | Mixing of fluids.

The total entropy change in mixing is

$$\begin{aligned} dS &= C\ln\left(\frac{T_f}{T_1}\right)+C\ln\left(\frac{T_f}{T_2}\right) \\ &= C\left\{\ln\left(\frac{T_f}{T_1}\right)+\ln\left(\frac{T_f}{T_2}\right)\right\} \\ &= C\ln\left(\frac{T_f^2}{T_1T_2}\right) \end{aligned}$$

Since ${T_f}^2 > T_1T_2$, $dS > 0$, so, it is a possible case.

8.5.15.3 Work from Finite Bodies Maximum work obtainable from two finite bodies of heat capacity C_p at temperatures T_1 and T_2 can be investigated. Let the final temperature reached by both the bodies after extraction of maximum obtainable work is T_f [Fig. 8.30].

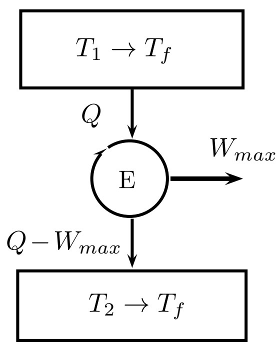


Figure 8.30 | Work from finite bodies.

The net change in entropy is

$$\begin{aligned} C_p\ln\frac{T_f}{T_1}+C_p\ln\frac{T_f}{T_2} &\geq 0 \\ C_p\left(\ln\frac{T_f}{T_1}+\ln\frac{T_f}{T_2}\right) &\geq 0 \\ C_p\ln\frac{T_f^2}{T_1T_2} &\geq 0 \\ T_f &\geq \sqrt{T_1T_2} \end{aligned}$$

The work will be maximum when entropy change is zero:

$$T_f = \sqrt{T_1T_2}$$

Therefore, the maximum possible work is

$$\begin{aligned} W_{max} &= C_p\left[(T_1-T_f)-(T_2-T_f)\right] \\ &= C_p\left(T_1-T_2-2\sqrt{T_1T_2}\right) \end{aligned}$$

8.5.15.4 Work from a Finite Body Consider a finite body of heat capacity C_p at temperature T. To extract work from this body a TER at T_0 can be used as sink [Fig. 8.31].

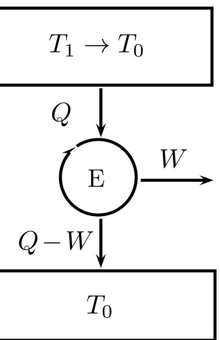


Figure 8.31 | Work from a finite body.

The temperature of TER does not change during the heat transfer. If heat Q is taken out from the finite body and heat engine extracts work W, then the change in entropy of TER is

$$\Delta S_r = \frac{Q-W}{T_0}$$

The heat can be extracted until the temperature of body reaches to that of TER. Therefore, the condition for possibility of this process is

$$C_p\ln\frac{T_0}{T}+\frac{Q-W}{T_0} \geq 0$$

Therefore,

$$W \leq C_p\left[(T-T_0)-T_0\ln\frac{T}{T_0}\right]$$

8.5.16 The Increase of Entropy Principle

Consider a cycle made of arbitrary (reversible or irreversible) processes 1-2, and an internally reversible 2-1.

Using the Clausius inequality for this cycle:

$$\oint \frac{dQ}{T} \leq 0$$

$$\int_1^2 \frac{dQ}{T} + \int_2^1 \frac{dQ}{T} \leq 0$$

$$\int_1^2 \frac{dQ}{T} + S_1 - S_2 \leq 0$$

$$S_2 - S_1 \geq \int_1^2 \frac{dQ}{T}$$

where $S_2 - S_1$ is the change of entropy when the system undergoes any reversible process from state 1 to 2. In the above expression, the equality holds for an internally reversible process and the inequality for an irreversible process. The expression can also be presented in differential form:

$$dS \geq \frac{dQ}{T}$$

Therefore, entropy change of a closed system during an irreversible process is greater than the integral of dQ/T (entropy transfer) evaluated for that process. This can be taken as entropy generated during an irreversible process, due to entirely the presence of irreversibilities. Entropy generation is denoted by S_{gen}, therefore,

$$S_2 - S_1 = \int_1^2 \frac{dQ}{T} + S_{gen}$$

Entropy generation S_{gen} is not a property of the system because it depends on the process. It is either a positive quantity or zero (for reversible processes).

Heat transfer is accompanied with entropy transfer, while work transfer does not involve entropy transfer.

For an isolated system undergoing an irreversible path, the entropy transfer is zero; the entropy change of a system is equal to the entropy generation. Therefore,

$$\Delta S_{isolated} \geq 0$$

Therefore, entropy of an isolated system during a process always increases or in the limiting case of reversible process remains constant. This is known as *increase in entropy principle*.

8.5.17 Isentropic Processes

A process during which entropy remains constant is called an *isentropic process*[24]. This is possible for a closed system if it undergoes an internally reversible and adiabatic process.

An isentropic process can serve as an appropriate model for actual processes, such as in pumps, turbine, nozzles, diffusers. Therefore, in many applications, isentropic processes are used to define isentropic efficiencies for the processes to compare the actual performance of these devices.

[24]An isentropic process is not necessarily a reversible adiabatic process because entropy increase of a substance during a process as a result of irreversibilities can be offset by a decrease in entropy due to heat losses. However, the term isentropic process is customarily used in thermodynamics to imply an internally reversible adiabatic processes.

8.5.18 Perpetual Motion Machines

Any device that violates the first or second law of thermodynamics is called a *perpetual motion machine* (PPM), which can be of two types:

1. PPM-I *Perpetual motion machines of first class* does work without input energy, thus violate the *first law of thermodynamics*.
2. PPM-II *Perpetual motion machines of second class* is an engine without any heat rejection or a refrigerator without work input, thus violate the *second law of thermodynamics*.

Despite numerous attempts, no perpetual motion machine is known to have worked.

8.6 THIRD LAW OF THERMODYNAMICS

Consider a hypothetical situation when enough engines are placed in series such that the heat rejected from the last engine is zero; absolute temperature of the last sink is zero. However, the second law of thermodynamics proves its impossibility. Thus, it appears that a definite zero exists on the absolute temperature scale, which cannot be reached without violation of the second law. In other words, attainable values of absolute temperature are always greater than zero. This is also known as the *third law of thermodynamics*. In terms of the *Fowler-Guggenheim statement*, it is impossible by any procedure, no matter how idealized, to reduce any system to the absolute zero of temperature in a finite number of operations.

8.7 EXERGY AND IRREVERSIBILITY

The *second law of thermodynamics* prohibits complete conversion of a *low grade energy* (heat) into *high grade energy* (shaft work). That part of low grade energy which is available for conversion is referred as *available energy* (AE), while the part, which, according to second

law, must be rejected, is known as *unavailable energy* (UAE). In this context, a new term, *availability*[25], is introduced in the study.

8.7.1 Dead State

A system is said to be in the *dead state* when it is in equilibrium with its surrounding. Thus, a system at the dead state is in chemical, thermal and mechanical equilibrium; the system is at the temperature and pressure of its environment, and has no kinetic or potential energy relative to the environment. The properties of a system at dead state are denoted by subscript zero, for example, p_0, T_0, h_0, u_0, and s_0 for a system of unit mass.

8.7.2 Useful Work

The work produced by a device is not always entirely in a usable form. For example, when a gas of unit mass in a piston cylinder device expands from v_1 to v_2, part of the work done by the gas is used to push the atmospheric air at constant pressure p_0. The difference between the actual work W and the surrounding work is called *useful work*, written as

$$W_u = W - p_0 (v_2 - v_1)$$

The volume of a steady flow system does not change, hence, maximum useful work would remain same.

8.7.3 Exergy

A system can deliver the maximum possible work when it undergoes from the specified initial state to the state of its environment (*dead state*) through a reversible process. This represents the *useful work* potential of the system at the specified initial state, and is called *exergy*. It is also termed as *availability*[26].

Availability or exergy is only the potential of work, not the actual work, that a system can deliver without violating any of the thermodynamic laws. It depends upon the state of both the system and its surrounding. The concept can be examined in the following systems.

8.7.3.1 Carnot Cycle Consider a closed system undergoing a Carnot cycle [Fig. 8.21] in which heat rejected is given by

$$Q_2 = T_2 \frac{Q_1}{T_1}$$

Heat rejection can be minimized by reducing T_2 up to T_0. Hence, the availability of the system is

$$\begin{aligned} W &= Q_1 \left(1 - \frac{T_0}{T_1}\right) \\ &= Q_1 - T_0 \frac{Q_1}{T_1} \end{aligned} \tag{8.17}$$

8.7.3.2 Closed System Consider a closed system which is given input heat Q during which it passes through a path 1-2 [Fig. 8.32].

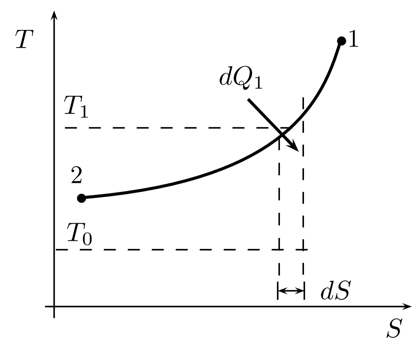


Figure 8.32 Work on T-S diagram.

Using Eq. (8.17), the available work for a small heat transfer dQ is

$$dW = dQ - \frac{dQ}{T_1} T_0$$

The maximum available work (availability) is

$$\begin{aligned} W &= \int_1^2 dW \\ &= \int_1^2 dQ - T_0 \int_1^2 \frac{dQ}{T_1} \\ &= Q - T_0 (S_2 - S_1) \end{aligned}$$

Unavailable energy is

$$\begin{aligned} \text{UAE} &= Q - W \\ &= T_0 (S_2 - S_1) \end{aligned}$$

8.7.3.3 Finite Heat Source Consider a finite source of heat capacity C, undergoing a temperature change from T_1 to T_0. The available energy (availability) of the source is

$$\begin{aligned} W &= \int_{T_0}^{T_1} C dT - T_0 \int_{T_0}^{T_1} C \frac{dT}{T} \\ &= C \left[(T_1 - T_0) - T_0 \ln \left(\frac{T_1}{T_0} \right) \right] \end{aligned}$$

[25] *Josiah Willard Gibbs* is accredited with being the originator of this concept of availability.
[26] The term availability was introduced in US in the 1940s. Its synonym exergy was introduced in Europe in the 1950s, which has found global acceptance partly because it is shorter, it rhymes with energy and entropy.

8.7.3.4 Effect of Temperature Consider heat loss from a hot gas flowing through a conductive pipe [Fig. 8.33]. The process is associated with entropy increase given by

$$dS = \frac{dQ}{T} = mc\frac{dT}{T}$$

$$\frac{dT}{dS} \propto \frac{T}{mc}$$

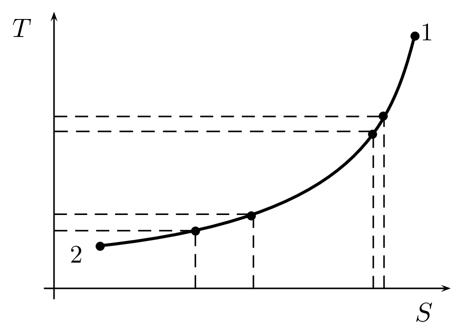


Figure 8.33 Work on T–S diagram.

Therefore, as temperature increases, slope of T–S diagram increases; loss of available energy is more when heat loss occurs at a higher temperature than when the same heat loss occurs at a lower temperature. Eq. (8.16) shows that maximum work is possible only in reversible process because S_2-S_1 is higher than dQ/T in an irreversible process.

8.7.3.5 Steady Flow System The availability of a steady flow system [*Section* 8.4.5] is

$$dW_{max} = \left\{h_1 - T_0 s_1 + \frac{{V_1}^2}{2} + gz_1\right\} - \left\{h_2 - T_0 s_2 + \frac{{V_2}^2}{2} + gz_2\right\}$$

8.7.4 Availability Function

When a system changes its state tending towards that of its surrounding, the work potential diminishes and finally ceases to exist at dead state. Thus, an important quantity, *availability function*, is used to represent the available energy or potential of a system with respect to the surrounding, determined as follows:

1. Closed System For a closed system of unit mass, availability function ϕ is defined as

$$\phi = u - T_0 s + p_0 v$$

By definition, if the system changes its state from 1 to 2, and respective availability functions are ϕ_1, ϕ_2, then, reversible work (availability) is simply derived as

$$W_{max} = \phi_1 - \phi_2$$

2. Steady Flow System Flow systems involve flow work pv and macroscopic energy, therefore, availability function of a steady flow system having unit rate of mass flow [*Section* 8.4.5] is defined as

$$\psi = h - T_0 s + \frac{V^2}{2} + gz$$

By definition, if working fluid changes its state from 1 to 2 through the system, and respective availability functions are ψ_1, ψ_2, then, the reversible work (availability) is simply derived as

$$W_{max} = \psi_1 - \psi_2$$

Another term for an steady flow process, B is defined as

$$B = h - T_0 S$$

which is equal to the availability function when there is no variation in kinetic energy and potential energy of the system.

8.7.5 Irreversibility

Irreversibility, denoted by I, is the difference between actual work done by the system and ideal maximum work possible:

$$I = W - W_{max}$$

It can be related to entropy change of universe (Δs_{univ}), and temperature of surrounding as

$$I = T_0 \Delta S_{univ}$$

This is applicable to both closed and open systems. The change in entropy is calculated as

$$\Delta S_{univ} = \Delta S_{sys} + \Delta S_{sur}$$

Thus, irreversibility represents the amount of heat required to add entropy into the universe at its absolute temperature.

8.8 PROPERTIES OF GASES

Gas and vapor are often used as synonymous words. The vapor phase of a substance is customarily called a gas when it is above the *critical temperature*. Vapor usually implies a gas that is not far from the state of condensation. This section deals with state or constitutive relations between properties of gases.

8.8.1 Pure Substance

A *pure substance* is defined as the one that is homogeneous and invariable in chemical composition throughout its mass, even when processed. This includes atmospheric air, steam-water mixture, combustion products of a fuel. The mixture of air and liquid air is not a pure substance since the relative proportions of oxygen and nitrogen differ in the gas and liquid phases in equilibrium. Water, nitrogen, helium, and carbon dioxide, for example, are all pure substances.

A pure substance does not have to be of a single chemical element or compound. A mixture of various chemical elements or compounds, also qualifies as pure substance as long as the mixture is homogeneous. For example, a mixture of oil and water is not a pure substance because oil is not soluble in water, it will collect on top of the water, forming two chemically dissimilar regions.

8.8.2 Ideal Gas Equation of State

Any equation that relates the macroscopic properties of a system, such as pressure, temperature, and specific volume of a substance, is called an *equation of state*. The equation of state represents the behavior of a pure substance, thus, it is also called *constitutive relation*.

The simplest form of equation of state is the *ideal gas equation*[27] of state for unit mole, written as

$$pv = \Re T \tag{8.18}$$

where $\Re$ is the *universal gas constant* which is equal to 8.314 kJ/kmol-K. It is related to *gas constant* R as

$$R = \frac{\Re}{M}$$

where M is the molar mass of the gas, defined as the mass of one mole of a substance in grams, or the mass of one kmol in kilograms.

Molar mass of a substance has the same numerical value in both unit systems because of the way it is defined. For example, molar mass of oxygen (O_2) is 32 which means the mass of 1 kmol of oxygen is 32 kg.

[27] In 1662, Robert Boyle, an Englishman, observed during his experiments within a vacuum chamber that the pressure of gases is inversely proportional to their volume. In 1802, J. Charles and J. Gay Lusaac, Frenchmen, experimentally determined that at low pressures, the volume of gas is proportional to its temperature. Ideal gas equation was first stated by Clapeyron in 1834 as a combination of Boyle's law and Charles's law. The equation was also derived from kinetic theory by August Kronig in 1856 and Rudolf Clausius in 1857. Universal gas constant was discovered and first introduced into the ideal gas law instead of a large number of specific gas constants by Dmitri Mendeleev in 1874.

Using ideal gas equation for two different states of an ideal gas,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

Ideal gas is a gas model that obeys the *ideal gas equation*. In the range of practical interest, many familiar gases such as air, argon, helium, hydrogen, krypton, neon, nitrogen, oxygen, and even heavier gases, such as carbon dioxide, can be conveniently treated as ideal gases. Other gases follow the ideal gas equation only in the range of high temperatures and low pressures when their density is low.

8.8.3 Compressibility Factor

The deviation from ideal gas behavior at a given temperature and pressure can accurately be accounted for by the introduction of a correction factor called *compressibility factor* (z), which is defined as [Fig. 8.34]:

$$z = \frac{pv}{\Re T}$$

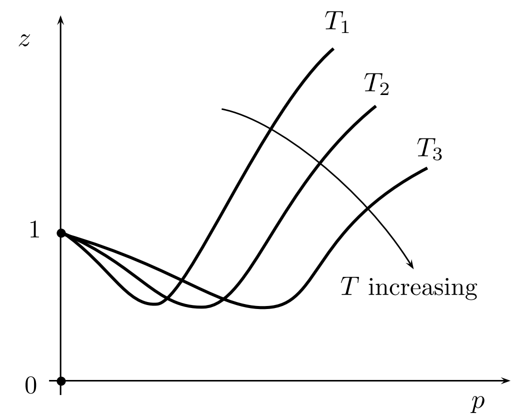


Figure 8.34 Compressibility factor chart.

The *compressibility factor* can be related to the following points:

1. For an ideal gas, by their definition, $z = 1$ but for real gases, z can be greater than or less than unity. The farther away z is from unity, the more the gas deviates from ideal gas behavior.

2. For given absolute temperature T and pressure p, the actual volume of a real gas v_{actual} and volume of an ideal gas v_{ideal} can be related to the compressibility factor z as

$$pv_{actual} = z\Re T$$
$$pv_{ideal} = \Re T$$

Therefore,

$$z = \frac{v_{actual}}{v_{ideal}}$$

8.8.4 Principle of Corresponding States

The *ideal gas equation* is closely valid for real gases at low pressure and high temperature, which can keep the gases far away from saturation. Therefore, the pressure or temperature of a substance is high or low relative to its critical temperature or critical pressure. Experiments have shown that gases behave differently at a given temperature and pressure, but they behave very much the same at temperatures and pressure normalized with respect to their critical temperatures and pressures. This normalization of properties is done by defining the *reduced properties*, as

$$p_r = \frac{p}{p_c}$$
$$T_r = \frac{T}{T_c}$$

The compressibility factor is approximately the same for all gases at the same reduced pressure and temperature. This is called the *principle of corresponding states* which can also be stated as at the same *reduced pressure* and *reduced temperature*, the *reduced volume* of different gases is approximately the same:

$$\begin{aligned} v_r &= \frac{v}{v_c} \\ &= \frac{z\Re T/p}{z_c\Re T_c/p_c} \\ &= \frac{z}{z_c}\frac{T_r}{p_r} \\ &= f\left(p_r, T_r, z\right) \end{aligned}$$

where z_c is critical compressibility factor ($\approx 0.2-0.3$), which can be taken as constant.

Thus, T_r is plotted as a function of reduced pressure p_r and z, generalized compressibility chart is found satisfactory for great variety of substances [Fig. 8.35].

By definition, two different substances are considered to be in corresponding states, if their pressure, volume and temperature are of the same fractions of the critical pressure, critical volume, critical temperature, respectively.

8.8.5 Van der Waals Equation

The ideal gas equation is based on the postulates of the kinetic theory of gases proposed by Clerk Maxwell. Van der Waals[28] intended to improve this equation by two corrections:

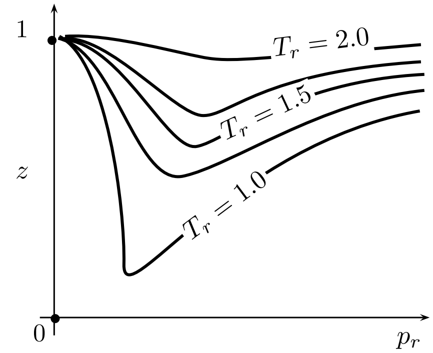


Figure 8.35 Reduced compressibility factor.

1. Intermolecular attraction forces, by incorporating a/v^2
2. Volume occupied by the molecules themselves, by incorporating b

Thus, the *Van der Waals equation* of state has two constants that are determined from the behavior of a substance at the *critical point*. The equation is written as

$$\left(p + \frac{a}{v^2}\right)(v - b) = \Re T \tag{8.19}$$

This law is followed by real gases particularly at high pressure and low temperature. Rearranging this equation,

$$pv^3 - (pb + \Re T)\, v^2 + av - ab = 0$$

This equation has three roots of v with the following characteristics [Fig. 8.36]:

1. Out of three, only one root needs to be real for low temperature ($T < T_c$) (i.e. liquid phase).
2. Three positive real roots exist for certain range of pressure (i.e. liquid plus gas phase).
3. As temperature increases, at critical point all three roots become equal to each other. Above critical temperature ($T > T_c$) only one real root exists for all values of p (i.e. gas phase).

The determination of the constants a and b is based on the observation that the critical isotherm on a p-v diagram [Fig. 8.36] has a horizontal inflection point

[28]Van der Waals (1837-1923) was a Dutch theoretical physicist and thermodynamicist famous for his work on an equation of state for gases (proposed in 1873) and liquids. His name is also associated with van der Waals forces, van der Waals molecules, and van der Waals radii. He won the 1910 Nobel Prize in physics.

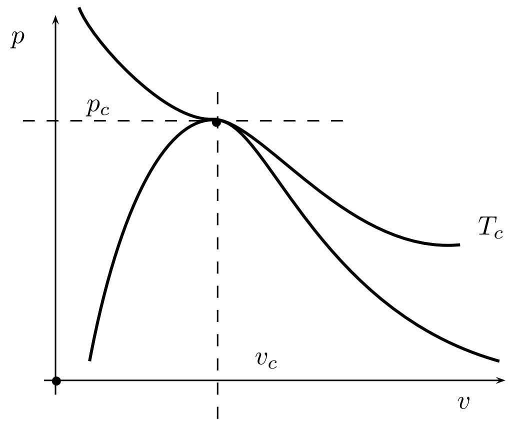


Figure 8.36 Critical properties.

at the critical point. Therefore, the first and second derivatives of p with respect to v at the critical point must be zero:

1. First derivative

$$\left(\frac{\partial p}{\partial v}\right)_{T_c} = 0$$

$$\frac{-\Re T_c}{(v_c-b)^2} + \frac{2a}{{v_c}^3} = 0$$

2. Second derivative

$$\left(\frac{\partial^2 p}{\partial v^2}\right)_{T_c} = 0$$

$$\frac{2\Re T_c}{(v_c-b)^3} - \frac{6a}{{v_c}^4} = 0$$

These two conditions are solved for

$$b = \frac{v_c}{3}$$
$$a = 3p_c{v_c}^2$$

Therefore, the *universal gas constant* is derived as

$$\Re = \frac{8}{3}\frac{p_c v_c}{T_c}$$

Eliminating the individual coefficients a, b, and $\Re$ from Eq. (8.19)

$$\left(p_r + \frac{3}{{v_r}^2}\right)(3v_r - 2) = 8T_r \tag{8.20}$$

This equation is called *reduced equation of state.*

8.8.6 Entropy Change of Ideal Gases

Using the Tds equations, entropy change in ideal gases is derived as follows:

1. Using the first Tds equation:

$$\begin{aligned} ds &= \frac{du}{T} + \frac{pdv}{T} \\ &= c_v\frac{dT}{T} + R\frac{dv}{v} \end{aligned} \tag{8.21}$$

The entropy change between two states 1 and 2 is written as

$$\begin{aligned} s_2 - s_1 &= \int_1^2 ds \\ &= c_v\int_1^2 \frac{dT}{T} + R\int_1^2 \frac{dv}{v} \\ &= c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \end{aligned} \tag{8.22}$$

2. Using the second Tds equation for ideal gases:

$$\begin{aligned} ds &= \frac{dh}{T} - \frac{vdp}{T} \\ &= c_p\frac{dT}{T} - R\frac{dp}{p} \end{aligned} \tag{8.23}$$

The entropy change between two states 1 and 2 is written as

$$\begin{aligned} s_2 - s_1 &= \int_1^2 ds \\ &= c_p\int_1^2 \frac{dT}{T} - R\int_1^2 \frac{dp}{p} \\ &= c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \end{aligned} \tag{8.24}$$

8.8.7 Reversible Processes

Following is the constitutive relation for one unit mass of an ideal gas:

$$pv = RT \tag{8.25}$$

By the definition of specific heats [*Section* 8.4.4], the changes in internal energy and enthalpy of an ideal gas are as follows:

$$u_2 - u_1 = c_v(T_2 - T_1)$$
$$h_2 - h_1 = c_p(T_2 - T_1)$$

The basic formulas for energy interaction (work and heat transfer) are as follows:

$$W_{1\to 2} = \int_{v_1}^{v_2} pdv$$
$$Q_{1\to 2} = \int_{T_1}^{T_2} Tds$$

The entropy change can be calculated as

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{dQ}{T}$$

During a process, a pure substance follows specific rule or law, that can be called *process relation*. Such a law is conveniently expressed in terms of pressure p, and specific volume v.

Following is the relationship between the area on the $p-v$ diagram which represents the work done [Fig. 8.37]:

$$\int_{v_1}^{v_2} pdv + p_1v_1 = -\int_{p_1}^{p_2} vdp + p_2v_2$$

$$p_1v_1 - p_2v_2 = -\int_{p_1}^{p_2} vdp - \int_{v_1}^{v_2} pdv \quad (8.26)$$

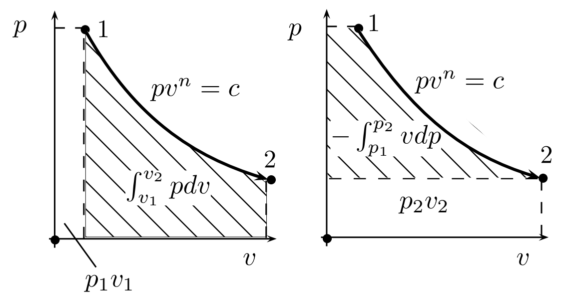


Figure 8.37 $\int_{v_1}^{v_2} pdv$ and $-\int_{p_1}^{p_2} vdp$.

Equation (8.26) is very helpful in thermodynamics of compressors as open system, where $-\int_{p_1}^{p_2} vdp$ is found to be a very interesting quantity.

The relationship between R, c_p and c_v of an ideal gas can be found as

$$\begin{aligned}\frac{R}{c_v} &= \frac{c_p - c_v}{c_v} \\ &= \gamma - 1 \\ \frac{R}{c_p} &= \frac{c_p - c_v}{c_p} \\ &= \frac{\gamma - 1}{\gamma}\end{aligned}$$

The heat capacity ratio (γ) for an ideal gas can be related to the degrees of freedom (n) of a molecule by:

$$\gamma = 1 + \frac{2}{n}$$

This can be examined for monoatomic gases ($n = 3$) and diatomic gases ($n = 5$), as follows:

$$\gamma = \begin{cases} 5/3 & \text{For monoatomic gases} \\ 7/5 & \text{For diatomic gases} \end{cases}$$

In the following subsections, important expressions for energy transfer, such as internal energy, heat transfer, work transfer, are derived for different types of reversible processes on ideal gases.

8.8.7.1 Polytropic Process Polytropic process is the most general process by use of which expressions for all other processes can be easily derived.

1. ***Process Relation*** The process relation for polytropic process is written as

$$pv^n = c \quad (8.27)$$

where n is the index. Along with Eq. (8.25),

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

By differentiating both sides of Eq. (8.27), the slope of $p-v$ curve [Fig. 8.38] is found as

$$\frac{dp}{dv} = -n\frac{p}{v}$$

2. ***Work Done*** Work done can be calculated as [Fig. 8.38]

$$W_{1\to 2} = \int_{v_1}^{v_2} pdv$$

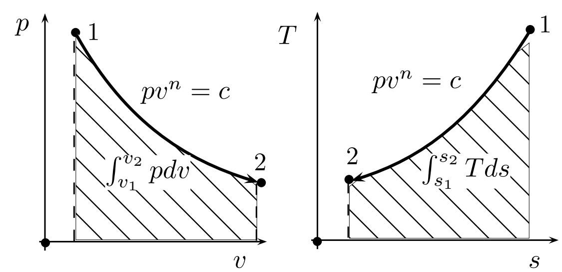


Figure 8.38 Polytropic process.

Therefore,

$$\begin{aligned}W_{1\to 2} &= \int_{v_1}^{v_2} \frac{c}{v^n} dv \\ &= \int_{v_1}^{v_2} cv^{-n} dv \\ &= \left[\frac{c}{-n+1} v^{-n+1} dv\right]_{v_1}^{v_2} \\ &= \frac{1}{(-n+1)} \left[\frac{c}{v^n} vdv\right]_{v_1}^{v_2} \\ &= \frac{p_1v_1 - p_2v_2}{n-1} \\ &= -\frac{R(T_2 - T_1)}{n-1} \quad (8.28)\end{aligned}$$

3. *Heat Transfer* Heat transfer in the process is [Fig. 8.38]

$$\begin{aligned} Q &= W + (u_2 - u_1) \\ &= -\frac{R\,(T_2 - T_1)}{n-1} + c_v\,(T_2 - T_1) \\ &= \left\{-\frac{c_p - c_v}{n-1} + c_v\right\}(T_2 - T_1) \\ &= \underbrace{\left(\frac{n-\gamma}{n-1}\right)c_v}_{c_n}(T_2 - T_1) \end{aligned} \tag{8.29}$$

Therefore, equivalent *specific heat* for a polytropic process can be written as

$$c_n = \left(\frac{n-\gamma}{n-1}\right)c_v \tag{8.30}$$

This simplifies the calculations of heat transfer and entropy change.

Heat transfer Q can be represented in terms of W by using Eq. (8.28) and Eq. (8.29), as

$$\begin{aligned} Q &= -\left(\frac{n-\gamma}{R}c_v\right)W \\ &= -\frac{n-\gamma}{(\gamma-1)\,c_v}c_v W \\ &= \frac{\gamma-n}{\gamma-1}W \end{aligned} \tag{8.31}$$

Work transfer W can related to heat transfer Q as

$$W = \frac{\gamma-1}{\gamma-n}Q \tag{8.32}$$

Equations (8.30) and (8.32) can be used to derive expressions of specific heats and work done for other processes as described in Table 8.1.

Table 8.1 Reversible processes

Process	n	c	W	Q
Polytropic	n	c_n	-	-
Isobaric	0	c_p	$(\gamma-1)\,Q/\gamma$	$c_p\Delta T$
Isochoric	∞	c_v	0	$c_v\Delta T$
Isothermal	1	∞	Q	W
Adiabatic	γ	0	Δu	0

4. *Change in Entropy* Change in entropy is

$$s_2 - s_1 = c_n \ln\left(\frac{T_2}{T_1}\right) \tag{8.33}$$

This equation can be used to determine the sign of entropy change (ds) by knowing the value of n w.r.t. γ:

$$n\begin{cases} > \gamma,\ ds > 0 \\ = \gamma,\ ds = 0 \\ < \gamma,\ ds < 0 \end{cases}$$

Decrease in entropy of the system is seen in working of centrifugal and axial compressors.

5. *Change in Internal Energy* The change in internal energy can be found by

$$u_2 - u_1 = c_v\,(T_2 - T_1)$$

8.8.7.2 Isobaric Process Constant pressure processes are called *isobaric processes.*

1. *Process Relation* The process relation for isobaric process ($n = 0$) is written as

$$p = c$$

2. *Work Done* Work done in isobaric processes [Fig. 8.39] is expressed as

$$W_{1\to 2} = p\,(v_2 - v_1)$$

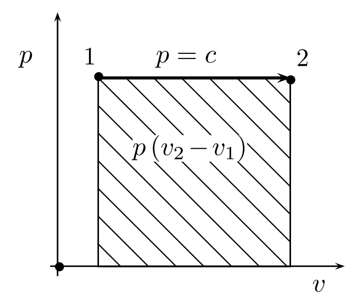


Figure 8.39 Isobaric process.

3. *Heat Transfer* The process occurs at constant pressure, therefore, specific heat at constant pressure is involved in the heat transfer:

$$Q = c_p\,(T_2 - T_1)$$

4. *Change in Entropy* The change in entropy is

$$\begin{aligned} s_2 - s_1 &= \int_1^2 \frac{c_p dT}{T} \\ &= c_p \ln\left(\frac{T_2}{T_1}\right) \end{aligned}$$

Using the second Tds equation for isobaric processes ($dp = 0$):

$$\begin{aligned} Tds &= c_p dT + v dp \\ \left(\frac{dT}{ds}\right)_p &= \frac{T}{c_p} \end{aligned}$$

This is the slope of isobaric path on T–s diagram.

5. *Change in Internal Energy* The change in internal energy

$$u_2 - u_1 = c_v\,(T_2 - T_1)$$

8.8.7.3 Isochoric Process

Constant volume processes are also known as *isochoric process*.

1. *Process Relation* The process relation for isochoric processes ($n = \infty$) is written as

$$v = c$$

2. *Work Done* The volume remains constant during isochoric process [Fig. 8.40], hence the work transfer is zero.

$$W_{1\to 2} = 0$$

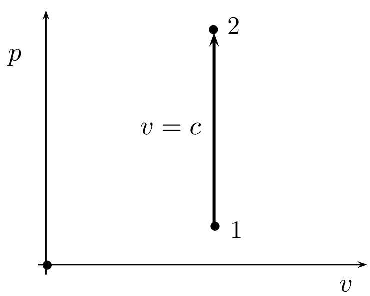


Figure 8.40 Isochoric process.

3. *Heat Transfer* Iscochoric processes occur at constant volume, therefore, heat transfer involves specific heat at constant volume c_v:

$$Q = c_v\,(T_2 - T_1)$$

4. *Change in Entropy* The change in entropy is

$$\begin{aligned} s_2 - s_1 &= \int_1^2 \frac{c_v dT}{T} \\ &= c_v \ln\left(\frac{T_2}{T_1}\right) \end{aligned}$$

Using the first Tds equation for isochoric processes ($dv = 0$):

$$\begin{aligned} Tds &= c_v dT + pdv \\ \left(\frac{dT}{ds}\right)_v &= \frac{T}{c_v} \end{aligned}$$

This is the slope of isochoric path on T–s diagram.

5. *Change in Internal Energy* The change in internal energy:

$$u_2 - u_1 = c_v\,(T_2 - T_1)$$

The work transfer is zero in the process, the heat transfer is equal to change in internal energy:

$$Q = u_2 - u_1$$

8.8.7.4 Isothermal Process

In *isothermal processes*, the temperature of the system remains constant.

1. *Process Relation* The process relation for isothermal processes ($n = 1$) is written as

$$pv = c \tag{8.34}$$

Therefore,

$$\begin{aligned} pdv + vdp &= 0 \\ \int pdv &= -\int vdp \end{aligned}$$

2. *Work Done* Work done in isothermal processes [Fig. 8.41] can be expressed as

$$\begin{aligned} W_{1\to 2} &= \int_{v_1}^{v_2} pdv \\ &= \int_{v_1}^{v_2} \frac{C}{v} dv \\ &= [C \ln v]_{v_1}^{v_2} \\ &= p_1 v_1 \ln\left(\frac{v_2}{v_1}\right) \\ &= p_2 v_2 \ln\left(\frac{v_2}{v_1}\right) \end{aligned}$$

The same expression can also be derived by taking $n \to 1$ in polytropic pdv-work.

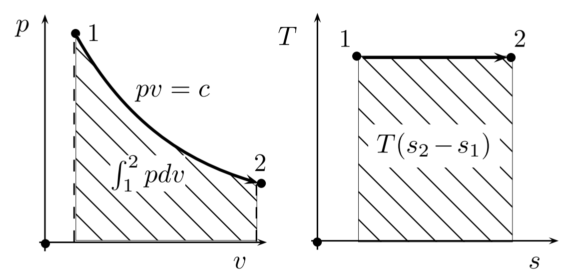


Figure 8.41 Isothermal process.

3. *Change in Internal Energy* Isothermal processes occur at constant temperature:

$$\begin{aligned} u_2 - u_1 &= \int_1^2 c_v dT \\ &= 0 \end{aligned}$$

There is no change in internal energy in isothermal processes.

4. *Heat Transfer* Using the first law, heat transfer is equal to work done:

$$\begin{aligned} Q &= W_{1\to 2} + (u_2 - u_1) \\ &= W_{1\to 2} \end{aligned}$$

5. *Change in Entropy* Change in entropy is determined as

$$s_2 - s_1 = \frac{Q}{T} = \frac{W_{1\to 2}}{T}$$

8.8.7.5 Adiabatic Process *Adiabatic process* does not permit heat transfer. Therefore, such processes are isentropic processes.

1. *Process Relation* Useful expressions can be derived for isentropic processes ($ds = 0$):

(a) Using Eq. (8.22)

$$c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} = 0$$

$$\ln \frac{T_2}{T_1} = \ln \left(\frac{v_1}{v_2}\right)^{R/c_v}$$

$$\left(\frac{T_2}{T_1}\right)_s = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

(b) Using Eq. (8.24):

$$c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = 0$$

$$\ln \frac{T_2}{T_1} = \ln \left(\frac{p_2}{p_1}\right)^{R/c_p}$$

$$\left(\frac{T_2}{T_1}\right)_s = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$

Combining the above two equations, one obtains

$$\left(\frac{T_2}{T_1}\right)_s = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} \tag{8.35}$$

This is very useful equation for dealing with isentropic processes of ideal gases. Using this, the process relation for adiabatic processes can be written as

$$pv^\gamma = c$$

Thus, for adiabatic processes the polytropic index n is equal to γ.

2. *Work Done* Using Eq. (8.28), work done in an adiabatic process [Fig. 8.42] is obtained as

$$W_{1\to 2} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

3. *Heat Transfer* There is no heat transfer involved in adiabatic process.

4. *Change in Entropy* As there is no heat transfer, there is no change in entropy.

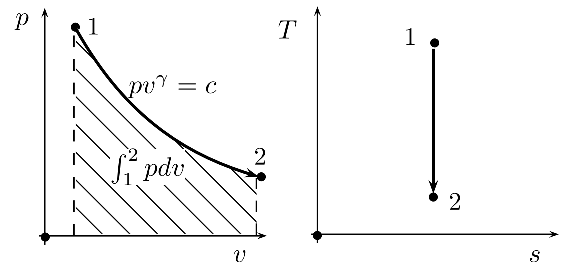


Figure 8.42 Adiabatic process.

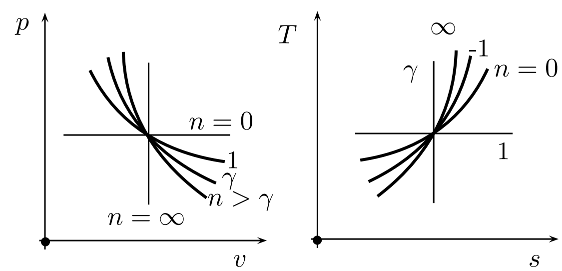


Figure 8.43 Reversible processes.

Figure 8.43 enables comparison of slopes of the reversible processes on p-v and T-s diagrams. It is depicted that heat transfer between two points is maximum for $n = 1$ (isothermal process).

Table 8.2 readily summarizes the formulations of work and heat transfers for all the reversible processes discussed above.

8.8.8 Properties of Gas Mixtures

The properties of gas mixtures are affected by the properties and fraction of the non-reactive constituent gases. To quantify this, *mole fraction* (x_i) is defined as the fraction of moles of given constituent to the total moles of the mixture:

$$x_i = \frac{n_i}{n}$$

where n_i represents the number of moles of ith constituent, and n is the total number of moles in the gas mixture. The summation of all the mole fractions would be equal to unity:

$$\sum_{i=1}^{n} x_i = 1$$

8.8.8.1 Dalton's Law According to the *Dalton's law*, pressure of a mixture of ideal gases is equal to the sum of the partial pressures of constituents of the mixture.

Table 8.2 Reversible processes on ideal gas

Process	c	W	Q
Polytropic			
$pv^n = c$	$\left(\dfrac{n-\gamma}{n-1}\right) c_v$	$-\dfrac{R\,(T_2 - T_1)}{n-1}$	$c_n\,(T_2 - T_1)$
		$= \dfrac{\gamma-1}{\gamma-n} Q$	$= \dfrac{\gamma-n}{\gamma-1} W$
Isobaric			
$p = c$	c_p	$p\,(v_2 - v_1)$	$c_p\,(T_2 - T_1)$
Isochoric			
$v = c$	c_v	0	$c_v\,(T_2 - T_1)$
Isothermal			
$pv = c$	∞	$p_2 v_2 \ln\left(\dfrac{v_2}{v_1}\right)$	W
Adiabatic			
$pv^\gamma = c$	0	$\dfrac{p_1 v_1 - p_2 v_2}{\gamma - 1}$	0

Ideal gas equation for n moles of an ideal gas having volume V at temperature T and pressure p is written as

$$pV = n\Re T$$

Consider a mixture of ideal gases having total volume V at temperature T and pressure p. If a particular constituent i has n_i moles in the mixture, then partial pressure of this constituent is given by

$$p_i = \frac{n_i \Re T}{V} = x_i p$$

Dalton's law can be represented as

$$p = \sum_{i=1}^{n} p_i$$

8.8.8.2 Amagat's Law According to *Amagat's law*, volume of a mixture of ideal gases is equal to the sum of the partial volumes of constituents of the mixture. *Partial volume* is defined as

$$V_i = \frac{n_i \Re T}{p} = x_i V$$

Amagat's law can be represented as

$$V = \sum_{i=1}^{n} V_i$$

Total volume of the mixture is given by

$$V = n\frac{\Re T}{p}$$

8.8.8.3 Gibbs-Dalton Law Under the ideal gas approximation, the properties of a gas are not influenced by the presence of other gases, and each gas component in the mixture behaves as if it exists alone at the mixture temperature and mixture volume. This principle is known as *Gibbs-Dalton law*, which is an extension of the Dalton's law. In view of this, the formula for the properties of gas mixture can be summarized as below

$$\text{Gas constant, } R = \frac{\sum_{i=1}^{n} m_i R_i}{\sum_{i=1}^{n} m_i}$$

$$\text{Molecular weight, } \mu = \sum_{i=1}^{n} x_i \mu_i$$

$$\text{Density, } \rho = \sum_{i=1}^{n} \rho_i$$

$$\text{Internal energy, } U = \sum_{i=1}^{n} u_i m_i$$

$$\text{Enthalpy, } H = \sum_{i=1}^{n} h_i m_i$$

$$\text{Entropy, } S = \sum_{i=1}^{n} s_i m_i$$

8.9 GAS COMPRESSION

Compressor is a type of machine that elevates the pressure of a compressible gas. *Reciprocating compressors* are used to produce compressed gas used for industrial applications for like gas transmission pipelines, petrochemical plants, refineries, cleaning, pneumatic control devices.

8.9.1 Shaft Work

Let the air get compressed in a polytropic process $pv^n = c$ from p_1, v_1 state (1) to p_2, v_2 state (2) [Fig. 8.44].

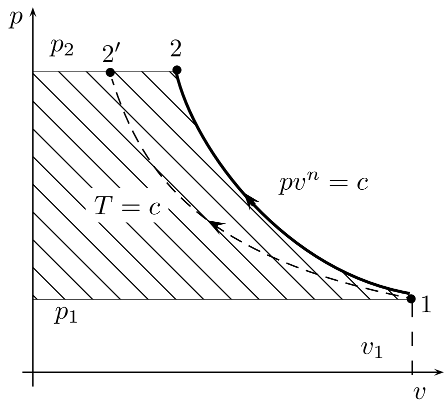


Figure 8.44 Compression on p-v plane.

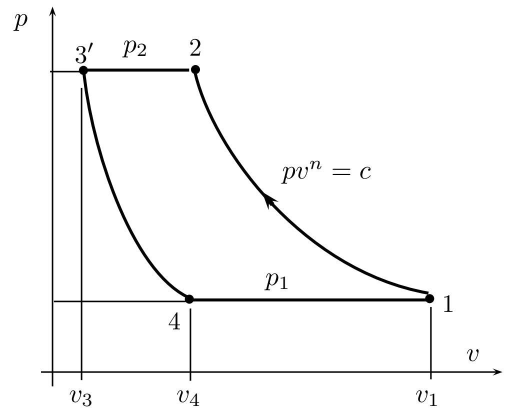


Figure 8.45 Volumetric efficiency.

Total shaft work

$$
\begin{aligned}
W_c &= p_1v_1 + \frac{p_1v_1 - p_2v_2}{n-1} - p_2v_2 \\
&= -\frac{n}{n-1}(p_2v_2 - p_1v_1) \\
&= -\int v dp \\
&= -\frac{n}{n-1} p_1v_1 \left\{ \left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1 \right\}
\end{aligned}
$$

where n is the index of compression.

8.9.2 Volumetric Efficiency

When clearance is provided in the cylinder, this affects the mass flow rate. So, *volumetric efficiency* for compressor is defined as the ratio of actual volume intake to swept volume of the cylinder [Fig. 8.45]:

$$\eta_v = \frac{v_1 - v_4}{v_1 - v_3}$$

Thus,

$$
\begin{aligned}
v_1 &= v_s + v_c \\
v_3 &= v_c
\end{aligned}
$$

where v_c is clearance volume and v_s is swept volume. Also,

$$v_4 = v_3 \left(\frac{p_2}{p_1}\right)^{1/n}$$

where n is the index of expansion. Therefore,

$$\eta_v = 1 - \frac{v_c}{v_s} \left\{ \left(\frac{p_2}{p_1}\right)^{1/n} - 1 \right\}$$

This expression is used to consider the effect of v_c in power calculation[29].

$$
\begin{aligned}
W_c &= -\frac{n}{n-1} mRT_1 \left\{ \left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1 \right\} \\
&= -\frac{n}{n-1} \eta_v p_1 v_1 \left\{ \left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1 \right\}
\end{aligned}
$$

8.9.3 Staged Compression

Staged compression is used to reduce the work requirement. This is accomplished by introducing an intercooler between two stages of compression. The intercooler cools the compressed gas to initial temperature. Thus, staged compression tends to bring the compression process towards an isothermal process [Fig. 8.46].

If index of expansion is same for all stages and air is cooled to initial temperature after each stage, then the maximum work required to compress the air from p_1 to p_2 in N stages is given by

$$W_c = N \times \frac{n}{n-1} mRT_1 \left\{ \left(\frac{p_2}{p_1}\right)^{(n-1)/(nN)} - 1 \right\}$$

and if $N \to \infty$ (isothermal process)

$$W_c \to p_1 v_1 \ln\left(\frac{v_1}{v_2}\right)$$

The objective is to increase the pressure (and not to increase internal energy), so when temperature is maintained to T_1 (i.e. isothermal process), the work is not increased but remains the same for the whole

[29]In this way, the volume flow rate is not used, but mass flow rate is used.

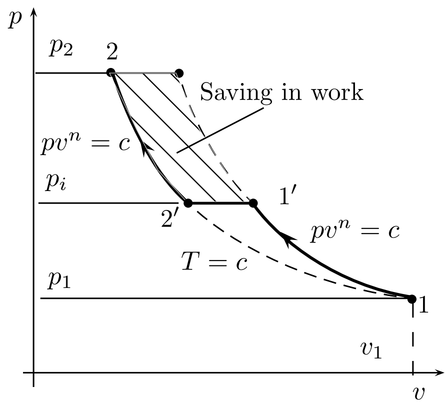


Figure 8.46 | Staged compression.

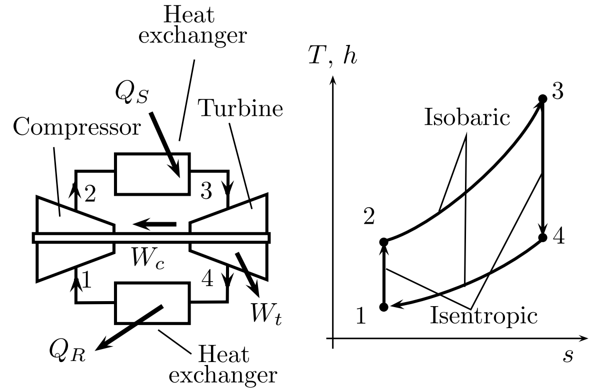


Figure 8.47 | Brayton cycle.

movement of piston. Pressure increases in an adiabatic process, so the work requirement also increases.

For two-staged compression, intermediate pressure p_i is geometrical mean of suction and delivery pressures, irrespective of perfect cooling provided with same indices of compression in both stages and minimum total work of compression,

$$p_i = \sqrt{p_1 p_2}$$

But with imperfect cooling, the work done in higher pressure stage is more than that for lower stage. If m and n are indices of compression in the first and second stages, respectively, then for perfect cooling, the ratio of compression works is determined as

$$\begin{aligned}\left(\frac{p_i}{p_1}\right)^{(m-1)/m} &= \left(\frac{p_2}{p_i}\right)^{(n-1)/n} \\ \frac{W_1}{W_2} &= \frac{m/(m-1)}{n/(n-1)} \\ &= \frac{m(n-1)}{n(m-1)}\end{aligned}$$

8.10 BRAYTON CYCLE

Brayton cycle[30], also known as *Joule cycle*, is the theoretical cycle for gas turbines. It is a modified version of Carnot cycle in which isothermal processes are replaced by isobaric processes. Thus, the cycle consists of two isentropic processes and two constant pressure processes [Fig. 8.47].

[30]The Brayton cycle was first proposed by George Brayton for use in the reciprocating oil burning engine that he developed around 1870.

For the Brayton cycle shown in Fig. 8.47,

$$\begin{aligned}p_2 &= p_3 \\ p_1 &= p_4\end{aligned}$$

Two important ratios, compression ratio (r) and pressure ratio (r_p), are defined, respectively, as

$$\begin{aligned}r &= \frac{V_1}{V_2} \\ r_p &= \frac{p_2}{p_1} \\ &= \frac{p_3}{p_4}\end{aligned}$$

In the process $1 \rightarrow 2$,

$$\begin{aligned}\frac{p_2}{p_1} &= \left(\frac{V_1}{V_2}\right)^{\gamma} \\ r_p &= r^{\gamma} \\ T_1 &= \frac{T_2}{r^{\gamma-1}}\end{aligned}$$

In the process $3 \rightarrow 4$,

$$\begin{aligned}T_4 &= \left(\frac{p_4}{p_3}\right)^{(\gamma-1)/\gamma} T_3 \\ &= \frac{T_3}{r_p{}^{(\gamma-1)/\gamma}} \\ &= \frac{T_3}{r^{\gamma-1}}\end{aligned}$$

8.10.1 Thermal Efficiency

Heat interactions of the cycle are as follows:

$$\begin{aligned}Q_S &= mc_p\,(T_3 - T_2) \\ Q_R &= mc_p\,(T_4 - T_1)\end{aligned}$$

Efficiency of the Brayton cycle is determined as

$$\begin{aligned}\eta_{Brayton} &= 1-\frac{Q_R}{Q_S}\\ &= 1-\frac{T_4-T_1}{T_3-T_2}\\ &= 1-\frac{T_3/r^{\gamma-1}-T_2/r^{\gamma-1}}{T_3-T_2}\\ &= 1-\frac{1}{r^{(\gamma-1)}}\\ &= 1-\frac{1}{r_p{}^{(\gamma-1)/\gamma}}\end{aligned}$$

Using the above expression, the following points can be deduced:

1. The efficiency of a Brayton cycle depends upon the compression ratio (r) and γ:
$$\eta_{Brayton} = f(r,\gamma)$$
2. For the same compression ratio (r_p) with same working fluid, efficiency of Brayton cycle is equal to that of Otto cycle[31]:
$$\eta_{Brayton} = \eta_{Otto}$$
3. The lower limit of temperature T_1 is limited by the atmospheric temperature (T_{min} say). The highest temperature T_3 is limited by the characteristics of material available for burner and turbine construction (say upto T_{max}). This is evident in the $T-s$ plot for the Brayton cycle with different values of compression ratios [Fig. 8.48].

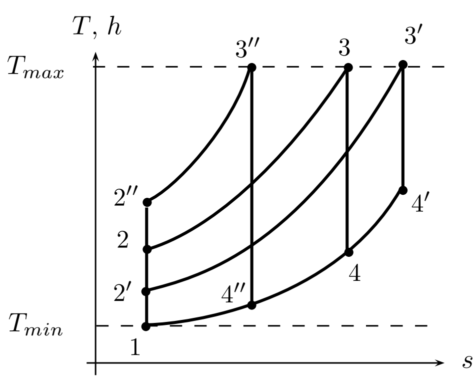


Figure 8.48 Effect of r_p.

Figure 8.49 shows the effect of r_p on $\eta_{Brayton}$. As the pressure ratio is increased, the efficiency steadily increases. In the limit when the compression process ends at T_{max}, the Carnot efficiency is reached, r_p has the maximum value r_{p-max}. The efficiency $\eta_{Brayton}$ is found to be maximum, and equal to

$$(\eta_{Brayton})_{max} = 1-\frac{T_1}{T_3}$$

when

$$r_{p_{max}} = \left(\frac{T_1}{T_3}\right)^{\gamma/(\gamma-1)}$$

8.10.2 Maximum Work Output

Figure 8.49 shows the effect of r_p on $\eta_{Brayton}$ and W_{net}. In the limiting case when r_p reaches to the maximum value r_{p-max}, the net work output becomes zero.

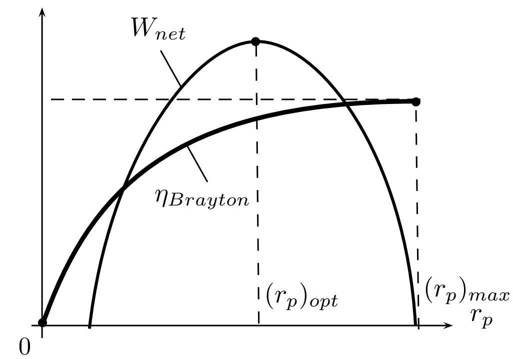


Figure 8.49 W_{net} versus r_p.

The work output of the Brayton cycle is

$$\begin{aligned}W_{net} &= c_p\{(T_3-T_4)-(T_2-T_1)\}\\ &= c_p\left\{\left(T_3-\frac{T_3}{r^{\gamma-1}}\right)-\left(r^{\gamma-1}T_1-T_1\right)\right\}\\ &= c_p\left\{T_3\left(1-\frac{1}{r^{\gamma-1}}\right)-T_1\left(r^{\gamma-1}-1\right)\right\}\\ &= c_p\left\{T_3\left(1-r^{-\gamma+1}\right)-T_1\left(r^{\gamma-1}-1\right)\right\}\end{aligned}$$

For maxima of W_{net} w.r.t. r,

$$\begin{aligned}\frac{dW_{net}}{dr} &= 0\\ T_3r^{-\gamma}-T_1r^{\gamma-2} &= 0\\ \frac{r^{\gamma-2}}{r^{-\gamma}} &= \frac{T_3}{T_1}\\ r^{2(\gamma-1)} &= \frac{T_3}{T_1}\\ \bar{r} &= \left(\frac{T_3}{T_1}\right)^{1/(2(\gamma-1))}\\ \bar{r}_p &= \left(\frac{T_3}{T_1}\right)^{\gamma/(2(\gamma-1))}\\ &= \sqrt{r_{p_{max}}}\end{aligned}$$

[31] Discussed in Chapter 9.

The maximum work output at this pressure ratio is written as

$$\max(W_{net}) = c_p\left\{\sqrt{T_3}-\sqrt{T_1}\right\}^2$$

Mass flow rate is

$$\dot{m} = \frac{3600}{W_{net}}\ \text{kg/kWh}$$

where W_{net} is in kW.

In gas turbine power plants, the ratio of the compressor work to the turbine work, called *back pressure ratio*, is very high. Usually one-half of the turbine work output is used to drive the compressor. Therefore, a power plant with high back work ratio requires a larger turbine to provide the additional requirement of the compressor. Hence, the turbines used in gas turbine power plants are larger than those used in steam turbine power plants of the same net power output.

8.10.3 Isentropic Efficiencies

Figure 8.50 shows the effect of machine efficiencies on the cycle.

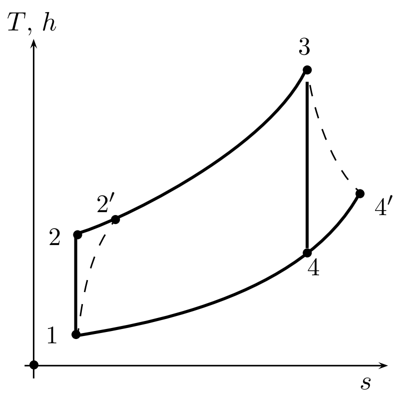


Figure 8.50 | Effect of η_t and η_c.

When turbine efficiency (η_t) and compressor efficiencies (η_c) are involved, then the efficiency of Brayton cycle is written as

$$\eta_{Brayton} = \frac{w_t\eta_t - w_c/\eta_c}{Q}$$

where

$$\eta_t = \frac{dT_{actual}}{dT_{isentropic}} = \frac{(T_3-T_{4'})}{(T_3-T_4)}$$

$$\eta_c = \frac{dT_{isentropic}}{dT_{actual}} = \frac{(T_2-T_1)}{(T_{2'}-T_1)}$$

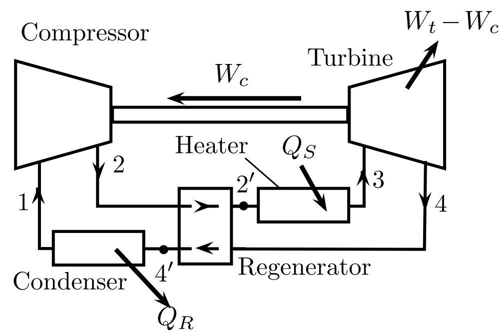
Polytropic efficiencies of compressor and turbine are defined, respectively, as

$$\frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/(\gamma\eta_{pc})}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)\eta_{pt}/\gamma}$$

8.10.4 Regeneration

Regeneration in Brayton cycle is the heat addition at higher temperature, resulting in increase in the mean temperature of heat addition and decrease in the mean temperature of heat rejection. Thus, efficiency of the Brayton cycle is increased but work output remains unchanged.



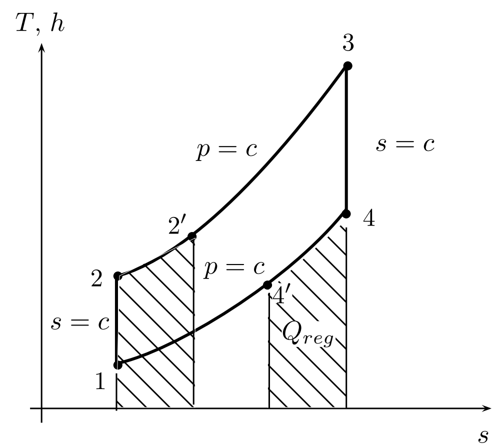


Figure 8.51 | Regeneration on T-s plane.

The ratio of the actual temperature rise of air to the maximum possible temperature rise [Fig. 8.51] is called the *effectiveness of the regenerator* (ϵ):

$$\epsilon = \frac{T_{2'}-T_2}{T_4-T_2}$$

In an ideal regenerative cycle, the compressed air is heated to the turbine exhaust temperature in the

regenerator so that

$$T_2' = T_4$$
$$T_2 = T_4'$$

For the isentropic process $1 \rightarrow 2$,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$
$$= {r_p}^{(\gamma-1)/\gamma}$$

For the isentropic process $3 \rightarrow 4$,

$$\frac{T_4}{T_3} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$
$$= {r_p}^{(\gamma-1)/\gamma}$$

Therefore,

$$\frac{T_2}{T_1} = \frac{T_4}{T_3} = {r_p}^{(\gamma-1)/\gamma}$$

In such a case, the efficiency of the cycle can be found as

$$\eta_{Brayton-reg} = 1 - \frac{mc_p\,(T_{4'} - T_1)}{mc_p\,(T_3 - T_{2'})}$$
$$= 1 - \frac{(T_{4'} - T_1)}{(T_3 - T_{2'})}$$
$$= 1 - \frac{T_1}{T_3}\frac{(T_{4'}/T_1 - 1)}{(1 - T_{2'}/T_3)}$$

Replacing values of $T_{4'}$ and $T_{2'}$,

$$\eta_{Brayton-reg} = 1 - \frac{T_1}{T_3}\frac{(T_2/T_1 - 1)}{(1 - T_4/T_3)}$$
$$= 1 - \frac{T_1}{T_3} \times \frac{T_2}{T_1} \times \frac{(1 - T_1/T_2)}{(1 - T_4/T_3)}$$
$$= 1 - \frac{T_1}{T_3} \times {r_p}^{(\gamma-1)/\gamma}$$

For a fixed ratio T_1/T_3, the cycle efficiency drops with increasing pressure ratio.

In practice, a regenerator is costly, heavy and bulky and causes pressure losses which bring about a decrease in cycle efficiency. Above certain pressure ratio (p_2/p_1), the addition of a regenerator causes a loss of cycle efficiency, when compared to original cycle. In this situation, the compressor discharge temperature T_2 is higher than the turbine exhaust gas temperature T_5. The compressed air will thus be cooled in the regenerator and the exhaust gas will be heated.

8.10.5 Intercooling and Reheat

Efficiency of a Brayton cycle can be increased by the use of staged compression with intercooling or and by using staged heat input called *reheat* at turbine [Fig. 8.52]. It is was found that the efficiency of cycle actually reduces by staging the compression and intercooling because heat to be added is increased but there can be a net gain in efficiency when intercooling is adopted in conjunction with a regenerator. Same is true with reheat. When the number of stages is large, then Brayton cycle tends towards *Ericsson cycle*[32].

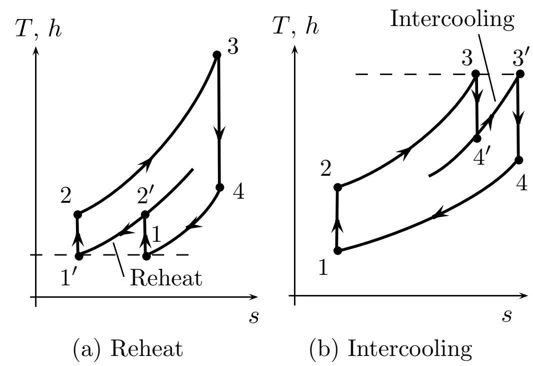


Figure 8.52 Reheat and intercooling.

8.10.6 Comparison with Otto Cycle

The Brayton cycle can be compared with the *Otto cycle* to deduce the following points:

1. For the same compression ratio and work capacity, Brayton cycle handles a larger range of volume and a smaller range of pressure and temperature.

2. A reciprocating engine cannot efficiently handle a large volume flow of low pressure gas, for which the engine size becomes larger, and the friction losses also becomes more. So, the Otto cycle is more suitable in the reciprocating field.

3. An internal combustion engine is exposed to the highest temperature (after combustion of fuel) only for a short while, and it gets time to become cool in the other processes of the cycle. On the other hand, gas turbine plant, a steady flow device, is always exposed to the highest temperature used. So to protect material, the maximum temperature of gas that can be used in a gas turbine plant cannot be as high as in an internal combustion engine. In the steady flow machinery, it is more difficult to carry out heat transfer at constant volume (Otto cycle) than at constant pressure (Brayton cycle).

[32]Discussed in Chapter 9.

IMPORTANT FORMULAS

Basic Concepts

$$E = U + m\frac{V^2}{2} + mgz$$

$$Q = \int_1^2 \dot{Q}dt = \dot{Q}(t_2 - t_1)$$

$$dW = pdv$$

$$W_{12} = \int_{v_1}^{v_2} pdv$$

$$W \begin{cases} = \int pdv & \text{Reversible} \\ \neq \int pdv & \text{Irreversible} \end{cases}$$

First Law of Thermodynamics

1. First law equations

$$dQ = dW + dU$$

$$\oint dQ = \oint dW$$

$$Q - W = (h_2 - h_1) + \frac{{V_2}^2 - {V_1}^2}{2} + g(z_2 - z_1)$$

2. Enthalpy

$$h = u + pv$$

3. Specific heats

$$c_v = \left(\frac{\partial Q}{\partial T}\right)_v = \left(\frac{\partial u}{\partial T}\right)$$

$$c_p = \left(\frac{\partial Q}{\partial T}\right)_p = \left(\frac{\partial h}{\partial T}\right)$$

4. Steady flow systems

   (a) Nozzle and diffuser

$$V_2 = \sqrt{2(h_1 - h_2) + {V_1}^2}$$

   (b) Throttling

$$h_1 = h_2$$

   (c) Turbine and compressor

$$\frac{W}{m} = h_1 - h_2$$

Second Law of Thermodynamics

1. Cyclic heat engine

$$W = Q_1 - Q_2$$

$$\eta_e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

2. Refrigerator

$$\text{COP}_r = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

3. Heat pump

$$\text{COP}_p = \frac{Q_1}{W}$$

$$\text{COP}_p = \text{COP}_r + 1 = \frac{1}{\eta_e}$$

4. Carnot principles

$$\eta_{rev} > \eta_{irr}, \quad \eta_{rev-1} = \eta_{rev-2}$$

5. Clausius theorem

$$\oint_R \frac{dQ}{T} = 0$$

   For Carnot cycle:

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$$

6. Entropy

$$dQ = TdS$$

$$\int_1^2 \frac{dQ_{rev}}{T} = (S_2 - S_1)$$

7. Tds equations

$$Tds = du + pdv$$

$$Tds = dh - vdp$$

8. Clausius inequality

$$\oint \frac{dQ}{T} \leq 0$$

9. Increase of entropy principle

$$S_2 - S_1 \geq \int_1^2 \frac{dQ}{T} = \int_1^2 \frac{dQ}{T} + S_{gen}$$

$$\Delta S_{isolated} \geq 0$$

Exergy and Availability

1. Availability function

$$W_{max} = \phi_1 - \phi_2$$

   (a) Closed system

$$\phi = u - T_0 s + p_0 v$$

   (b) Steady flow system

$$\psi = h - T_0 s + m\frac{V^2}{2} + mgz$$

2. Irreversibility

$$I = W - W_{max}$$

$$I = T_0 \Delta S_{univ}$$

$$\Delta S_{univ} = \Delta S_{sys} + \Delta S_{sur}$$

Properties of Gas

1. Ideal gas equation

$$pv = \Re T$$

$$R = \frac{\Re}{M}$$

2. Compressibility factor

$$z = \frac{pv}{\Re T} = \frac{v_{actual}}{v_{ideal}}$$

3. Corresponding states

$$p_r = \frac{p}{p_c}, \quad T_r = \frac{T}{T_c}$$

4. Van der Waals equation

$$\left(p + \frac{a}{v^2}\right)(v - b) = \Re T$$

$$\left(p_r + \frac{3}{{v_r}^2}\right)(3v_r - 2) = 8T_r$$

5. Entropy change of ideal gases

$$s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

6. Reversible process

$$u_2 - u_1 = c_v\,(T_2 - T_1)$$
$$h_2 - h_1 = c_p\,(T_2 - T_1)$$
$$W_{1\to 2} = \int_{v_1}^{v_2} p dv$$
$$Q_{1\to 2} = \int_{T_1}^{T_2} T ds$$
$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{dQ}{T}$$
$$p_1v_1 - p_2v_2 = -\int_1^2 v dp - \int_1^2 p dv$$
$$c_v = \frac{R}{\gamma - 1}, \quad c_p = \frac{\gamma R}{\gamma - 1}$$

(a) Polytropic process

$$pv^n = c$$
$$c_n = \left(\frac{n-\gamma}{n-1}\right) c_v$$
$$W = -\frac{R\,(T_2 - T_1)}{n-1} = \frac{\gamma - 1}{\gamma - n} Q$$
$$Q = c_n\,(T_2 - T_1) = \frac{\gamma - n}{\gamma - 1} W$$

(b) Isobaric process

$$W = p\,(v_2 - v_1)$$
$$Q = c_p\,(T_2 - T_1)$$

(c) Isochoric process

$$W = 0$$
$$Q = c_c\,(T_2 - T - 1)$$

(d) Isothermal process

$$pv = c$$
$$W = p_2 v_2 \ln\left(\frac{v_2}{v_1}\right) = Q$$

(e) Adiabatic process

$$pv^\gamma = c$$
$$W = \frac{p_1v_1 - p_2v_2}{\gamma - 1}$$
$$Q = 0$$

7. Gas mixtures
Mole fraction

$$x_i = \frac{n_i}{n}$$
$$\sum_{i=1}^{n} x_i = 1$$

Dalton's law

$$p = \sum_{i=1}^{n} p_i$$

Amagat's law

$$V = \sum_{i=1}^{n} V_i$$

Other properties

$$R = \frac{\sum_{i=1}^{n} m_i R_i}{\sum_{i=1}^{n} m_i}$$
$$\mu = \sum_{i=1}^{n} x_i \mu_i$$
$$\rho = \sum_{i=1}^{n} \rho_i$$
$$U = \sum_{i=1}^{n} u_i m_i$$
$$H = \sum_{i=1}^{n} h_i m_i$$
$$S = \sum_{i=1}^{n} s_i m_i$$

8. Gas compression

$$W_c = -\int v dp = -\frac{np_1v_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}$$
$$\eta_v = \frac{v_1 - v_4}{v_1 - v_3} = 1 - \frac{v_c}{v_s}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right\}$$
$$W_c = -\frac{nmRT_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} = -\frac{n\eta_v p_1 v_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}$$

9. Staged compression

$$\frac{W_c}{N} = \frac{nmRT_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{nN}} - 1\right\}$$
$$p_i = \sqrt{p_1 p_2}$$
$$\left(\frac{p_i}{p_1}\right)^{\frac{m-1}{m}} = \left(\frac{p_2}{p_i}\right)^{\frac{n-1}{n}}$$
$$\frac{W_1}{W_2} = \frac{m\,(n-1)}{n\,(m-1)}$$

10. Brayton cycle

$$\eta_{Brayton} = 1 - \frac{1}{r_p^{\left(\frac{\gamma-1}{\gamma}\right)}}$$
$$(\eta_{Brayton})_{max} = 1 - \frac{T_1}{T_3}$$

when

$$r_{p_{max}} = \left(\frac{T_1}{T_3}\right)^{\frac{\gamma}{\gamma-1}}$$

For maximum work

$$\bar{r}_p = \left(\frac{T_3}{T_1}\right)^{\gamma/2(\gamma-1)} = \sqrt{r_{p_{max}}}$$
$$\max(W_{net}) = c_p\left\{\sqrt{T_3} - \sqrt{T_1}\right\}^2$$

Isentropic efficiencies

$$\eta_t = \frac{dT_{actual}}{dT_{isentropic}} = \frac{(T_3 - T_{4'})}{(T_3 - T_4)}$$
$$\eta_c = \frac{dT_{isentropic}}{dT_{actual}} = \frac{(T_2 - T_1)}{(T_{2'} - T_1)}$$

Polytropic efficiencies

$$\frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}\frac{1}{\eta_{pc}}}$$
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}\eta_{pt}}$$

Regenerator effectiveness

$$\epsilon = \frac{T_{2'} - T_2}{T_4 - T_2}$$

SOLVED EXAMPLES

1. Suppose 85 kJ of heat is supplied to a closed system at constant volume. During the next process, the system rejects 90 kJ of heat at constant pressure while 20 kJ of work is done on it. The system is brought to the original state by an adiabatic process. The initial internal energy is 100 kJ. Determine the quantity of work transfer during the process.

Solution. For cyclic processes, the heat interaction is equal to work transfer. Hence, for the given case

$$85-90+0 = 0-20+W$$
$$W = 15 \text{ kJ}$$

2. A steam turbine receives steam steadily at 10 bars with an enthalpy of 3000 kJ/kg and discharges at 1 bar with an enthalpy of 2700 kJ/kg. The power output is 3.5 kW and mass flow rate is 12 kg/s. Ignoring the changes in kinetic and potential energies, determine the rate of heat transfer from the turbine casing to the surroundings.

Solution. Using steady state energy equation

$$12\times 3000 = 12\times 2700+3.5\times 10^3+Q$$
$$Q = 100 \text{ W}$$

3. An electric motor of 5 kW is subjected to a braking test for 1 hour. The heat generated by the frictional forces in the process is transferred to the surroundings at 20°C. Calculate the entropy change in the process.

Solution. The resulting entropy change is given by

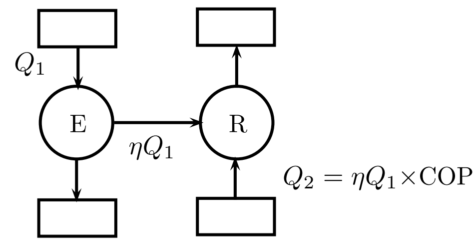
$$\Delta S = \frac{5\times 3600}{20+273} = 61.43 \text{ kJ/K}$$

4. If an engine of 40 % thermal efficiency drives a refrigerator having a coefficient of performance of 5. Determine the the heat input to the engine per kJ of heat removed from the cold body of the refrigerator.

Solution. Given that

$$\eta = 0.4$$
$$\text{COP} = 5$$
$$Q_2 = 1 \text{ kJ}$$

The heat engine and refrigerator are shown in the figure.



Heat input to the engine is given by

$$Q_1 = \frac{Q_2}{\text{COP}\times\eta} = \frac{1}{5\times 0.4} = 0.5 \text{ kJ}$$

5. A heat pump is used to heat a house in the winter and then reversed to cool the house in the summer. The inside temperature of the house is to be maintained at 20°C. The heat transfer through the house walls is 7.9 kJ/s and the outside temperature in winter is 5°C. Calculate the minimum power (approximate) required to drive the heat pump.

Solution. Given that

$$Q_1 = 7.9 \text{ kW}$$
$$T_1 = 20+273 \text{ K}$$
$$T_2 = 5+273 \text{ K}$$

The minimum power requirement is given by

$$W = \frac{T_1-T_2}{T_2}\times Q_1 = 404.43 \text{ W}$$

6. The refrigerating efficiency, that is, the ratio of actual COP to reversible COP of a refrigeration cycle is 0.8, the condenser and evaporator temperature are 51°C and −30°C, respectively. Calculate the work requirement for cooling capacity 2.4 kW of the plant.

Solution. The COP of the reversible refrigeration cycle working between temperatures $T_1 = 51°\text{C}$ (324 K) and $T_2 = -30°\text{C}$ (243 K) is given by

$$\text{COP}_r = \frac{T_2}{T_1-T_2} = \frac{243}{324-243} = 3$$

Hence, the actual COP of the cycle is given by

$$\eta_r = \frac{\text{COP}}{\text{COP}_r}$$
$$\text{COP} = 0.8\times 3.0 = 2.4$$

Required cooling capacity is

$$Q_2 = 2.4 \text{ kW}$$

The work requirement for Q_2 is given by

$$\begin{aligned} W &= \frac{Q_2}{\text{COP}} \\ &= 1.0 \text{ kW} \end{aligned}$$

7. Experiments indicate that about 0.522 kW/m^2 of energy can be collected on a plate operating at 85°C. This energy is to be transferred to a heat engine to produce 1 kW of useful shaft power. The heat engine rejects heat to atmosphere at 25°C. Determine the efficiency of the heat engine and minimum collector area of the plate.

Solution. The heat engine has to work between 358 K and 298 K, therefore, its efficiency is

$$\begin{aligned} \eta &= 1 - \frac{298}{358} \\ &= 16.76\% \end{aligned}$$

The heat input to the engine for 1 kW work output is

$$\begin{aligned} Q_1 &= \frac{W}{\eta} \\ &= 5.967 \text{ kW} \end{aligned}$$

The minimum area of the collector plate is given by

$$\begin{aligned} A &= \frac{5.967}{0.522} \\ &= 11.43 \text{ m}^2 \end{aligned}$$

8. Two moles of an ideal gas is isothermally and irreversibly expanded from 10 to 1 atm. The external pressure is constant at 1 atm and the temperature is 300 K. Calculate the work done ($\Re = 8.3$ J/mol K).

Solution. Given that

$$\begin{aligned} n &= 2 \\ p_1 &= 10 \text{ atm} \\ p_2 &= 1 \text{ atm} \\ p_0 &= p_2 \\ &= 1 \text{ atm} \\ T_1 &= 300 \text{ K} \\ \Re &= 8.3 \text{ J/mol K} \end{aligned}$$

Work done in isothermal reversible process ($pv = c$) is given by

$$\begin{aligned} W_{1\to 2} &= p_1 v_1 \ln\left(\frac{p_1}{p_2}\right) \\ &= n\Re T_1 \ln\left(\frac{p_1}{p_2}\right) \\ &= 4980 \text{ J} \end{aligned}$$

The work done against the atmosphere is given by

$$\begin{aligned} W_0 &= p_0 (v_2 - v_1) \\ &= p_2 v_2 \left(1 - \frac{p_2}{p_1}\right) \\ &= n\Re T_1 \left(1 - \frac{p_2}{p_1}\right) \\ &= 4980 \times 0.9 \\ &= 4482 \text{ J} \end{aligned}$$

9. A gas turbine develops 120 kJ of work while the compressor absorbed 60 kJ of work and the heat supplied is 200 kJ. If a regenerator which would recover 40% of the heat in the exhaust were used, determine the increase in the overall thermal efficiency.

Solution. Given that

$$\begin{aligned} W_t &= 120 \text{ kJ} \\ W_c &= 60 \text{ kJ} \\ Q_1 &= 200 \text{ kJ} \end{aligned}$$

Therefore, initial overall thermal efficiency is

$$\begin{aligned} \eta_1 &= \frac{W_t - W_c}{Q_1} \\ &= 30\% \end{aligned}$$

As the regenerator would recover 40% of energy from the exhaust, therefore, the overall thermal efficiency would be

$$\begin{aligned} \eta_2 &= \frac{W_t - W_c}{Q_1 - 0.4\,(Q_1 - W_t)} \\ &= 35.7\% \end{aligned}$$

Hence, increase in overall efficiency is 5.7%.

10. Air expands from pressure p_1 to pressure p_2 ($p_2 = p_1/10$). If the process of expansion is isothermal, the volume at the end of expansion is 0.55 m^3. If the process of expansion is adiabatic, determine the volume at the end of expansion.

Solution. Given that

$$\begin{aligned} p_2 &= \frac{p_1}{10} \\ v_2 &= 0.55 \text{ m}^3 \end{aligned}$$

In isothermal expansion

$$\begin{aligned} p_1 v_1 &= p_2 v_2 \\ v_1 &= \frac{p_2}{p_1} v_2 \\ &= 0.055 \text{ m}^3 \end{aligned}$$

In adiabatic expansion ($\gamma = 1.4$)

$$\begin{aligned} p_1 v_1^{\gamma} &= p_2 v_2^{\gamma} \\ v_2 &= v_1 \left(\frac{p_1}{p_2}\right)^{1/\gamma} \\ &= 0.285 \text{ m}^3 \end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. A 2 kW, 40 liter water heater is switched on for 20 min. The heat capacity c_p for water is 4.2 kJ/kgK. Assuming all the electrical energy has gone into heating the water, increase of the water temperature in degree centigrade is

(a) 2.7 (b) 4.0
(c) 14.3 (d) 25.25

(GATE 2003)

Solution. Given that

$$\begin{aligned} Q &= 2\times 10^3 \text{ W} \\ t &= 20\times 60 \text{ s} \\ m &= 40 \text{ liter} \\ &= 40 \text{ kg} \\ c &= 4.2 \text{ kJ/kgK} \end{aligned}$$

If increase in water temperature is ΔT°C, for heat balance,

$$\begin{aligned} Q &= mc\Delta T \\ \Delta T &= \frac{Q}{mc} \\ &= \frac{2\times 10^3\times 20\times 60}{40\times 4.2\times 10^3} \\ &= 14.2857^\circ\text{C} \end{aligned}$$

Ans. (c)

2. An industrial heat pump operates between the temperatures of 27°C and 13°C. The rates of heat addition and heat rejection are 750 W and 1000 W, respectively. The COP for the heat pump is

(a) 7.5 (b) 6.5
(c) 4.0 (d) 3.0

(GATE 2003)

Solution. Given that

$$\begin{aligned} Q_1 &= 1000 \text{ W} \\ Q_2 &= 750 \text{ W} \end{aligned}$$

COP of heat pump is given by

$$\begin{aligned} \text{COP}_p &= \frac{Q_1}{Q_1 - Q_2} \\ &= 4.00 \end{aligned}$$

Ans. (c)

3. Considering the relationship $TdS = dU + pdV$ between the entropy (S), internal energy (U), pressure (p), temperature (T) and volume (V), which of the following statements is correct?

(a) It is applicable only for a reversible process
(b) For an irreversible process, $TdS > dU + pdV$
(c) It is valid only for an ideal gas
(d) It is equivalent to the first law for a reversible process

(GATE 2003)

Solution. The derivation of equation

$$TdS = dU + pdV$$

is based on the first law of thermodynamics given by

$$Q = dU + W$$

where

$$\begin{aligned} Q &= TdS \\ W &= pdV \end{aligned}$$

Ans. (d)

4. In a gas turbine, hot combustion products with the specific heats $c_p = 0.98$ kJ/kgK, $c_v = 0.7538$ kJ/kgK enter the turbine at 20 bar, 1500 K and exit at 1 bar. The isentropic efficiency of the turbine is 0.94. The work developed by the turbine per kg of gas flow is

(a) 689.64 kJ/kg (b) 794.66 kJ/kg
(c) 1009.72 kJ/kg (d) 1312.00 kJ/kg

(GATE 2003)

Solution. Given that

$$\begin{aligned} c_p &= 0.98 \text{ kJ/kgK} \\ c_v &= 0.7538 \text{ kJ/kgK} \\ p_3 &= 20\times 10^5 \text{ N/m}^2 \\ T_3 &= 1500 \text{ K} \\ p_4 &= 1\times 10^5 \text{ N/m}^2 \\ \eta_s &= 0.94 \end{aligned}$$

Ratio of specific heats is calculated as

$$\begin{aligned} \gamma &= \frac{c_p}{c_v} \\ &= 1.30008 \end{aligned}$$

The temperature after isentropic expansion in turbine T_4 is given by

$$\begin{aligned} \frac{T_3}{T_4} &= \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} \\ T_4 &= 751.264 \text{ K} \end{aligned}$$

The temperature after actual expansion in turbine $T_{4'}$ is given by the definition of isentropic efficiency as

$$\eta_s = \frac{T_3 - T_{4'}}{T_3 - T_4}$$
$$T_4 = 796.188 \text{ K}$$

Therefore, turbine work is

$$\begin{aligned} W_T &= h_3 - h_4 \\ &= c_p (T_3 - T_4) \\ &= 689.736 \text{ kJ/kg} \end{aligned}$$

Ans. (a)

Common Data Questions

Nitrogen gas (molecular weight 28) is enclosed in a cylinder by a piston, at the initial condition of 2 bar, 298 K and 1 m^3. In a particular process, the gas slowly expands under isothermal condition, until the volume becomes 2 m^3. Heat exchange occurs with the atmosphere at 298 K during this process.

5. The work interaction for the nitrogen gas is

(a) 200 kJ (b) 138.6 kJ
(c) 2 kJ (d) −200 kJ

(GATE 2003)

Solution. Given that

$$\begin{aligned} p_1 &= 2 \times 10^5 \text{ N/m}^2 \\ T_1 &= 298 \text{ K} \\ v_1 &= 1 \text{ m}^3 \\ v_2 &= 2 \text{ m}^3 \\ T_0 &= 298 \text{ K} \end{aligned}$$

Work done on the gas in the isothermal process is

$$\begin{aligned} W &= p_1 v_1 \ln\left(\frac{v_2}{v_1}\right) \\ &= 2 \times 10^5 \times 1 \times \ln\left(\frac{2}{1}\right) \\ &= 138.629 \text{ kW} \end{aligned}$$

Ans. (b)

6. The entropy change for the universe during the process in kJ/K is

(a) 0.4652 (b) 0.0067
(c) 0 (d) −0.6711

(GATE 2003)

Solution. For isothermal process, there is no change in internal energy, therefore,

$$Q = W$$

The change in entropy of the universe is

$$\begin{aligned} \Delta S &= \frac{W}{T_0} \\ &= \frac{138.629}{298} \\ &= 0.465199 \text{ kJ/K} \end{aligned}$$

Ans. (a)

7. A gas contained in a cylinder is compressed, the work required for compression being 5000 kJ. During the process, heat interaction of 2000 kJ causes the surroundings to be heated. The change in internal energy of the gas during the process is

(a) −7000 kJ (b) −3000 kJ
(c) +3000 kJ (d) +7000 kJ

(GATE 2004)

Solution. Given that

$$\begin{aligned} W &= -5000 \text{ kJ} \\ Q &= -2000 \text{ kJ} \end{aligned}$$

Using the first law of thermodynamics,

$$\begin{aligned} Q &= W + \Delta U \\ \Delta U &= Q - W \\ &= 3000 \text{ kJ} \end{aligned}$$

Ans. (c)

8. The compression ratio of a gas power plant cycle corresponding to maximum work output for the given temperature limits of T_{max} and T_{min} will be

(a) $\left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$ (b) $\left(\frac{T_{min}}{T_{max}}\right)^{\frac{\gamma}{2(\gamma+1)}}$

(c) $\left(\frac{T_{min}}{T_{max}}\right)^{2\gamma/(\gamma-1)}$ (d) $\left(\frac{T_{max}}{T_{min}}\right)^{2\gamma/(\gamma+1)}$

(GATE 2004)

Solution. For maximum work output from gas power plant (Brayton cycle), compression ratio is given by

$$r_p = \left(\frac{T_{max}}{T_{min}}\right)^{\gamma/2(\gamma-1)}$$

Ans. (a)

9. A heat engine having an efficiency of 70% is used to drive a refrigerator having a co-efficient of performance of 5. The energy absorbed from

low temperature reservoir by the refrigerator for each kJ of energy absorbed from high temperature source by the engine is

(a) 0.14 kJ (b) 0.71 kJ
(c) 3.5 kJ (d) 7.1 kJ

(GATE 2004)

Solution. Given that

$$\eta = 0.7$$
$$\text{COP}_r = 5$$

Let heat Q_1 taken from high temperature source by the engine to develop work W be

$$W = \eta Q_1$$

The same work will be used to draw heat Q_2 from low temperature by the refrigerator, therefore,

$$\begin{aligned} Q_2 &= \text{COP}_r \times W \\ &= \text{COP}_r \times \eta Q_1 \\ \frac{Q_2}{Q_1} &= \text{COP}_r \times \eta \\ &= 5 \times 0.7 \\ &= 3.5 \text{ kJ} \end{aligned}$$

Ans. (c)

10. A solar collector receiving solar radiation at the rate of 0.6 kW/m^2 transforms it to the internal energy of a fluid at an overall efficiency of 50%. The fluid heated to 350 K is used to run a heat engine which rejects heat at 313 K. If the heat engine is to deliver 2.5 kW power, the minimum area of the solar collector required would be

(a) 8.33 m^2 (b) 16.66 m^2
(c) 39.68 m^2 (d) 79.36 m^2

(GATE 2004)

Solution. Given that

$$\begin{aligned} \dot{q} &= 0.6 \times 10^3 \text{ W/m}^2 \\ \eta_o &= 0.5 \\ T_1 &= 350 \text{ K} \\ T_2 &= 313 \text{ K} \\ W &= 2.5 \times 10^3 \text{ W} \end{aligned}$$

If A is the area of the solar collector, the heat generated by solar collector is

$$Q_1 = \eta \dot{q} A$$

This heat is used as input to the heat source in the engine, therefore,

$$\begin{aligned} Q_1 &= W \frac{T_1}{T_1 - T_2} \\ \eta \dot{q} A &= W \frac{T_1}{T_1 - T_2} \\ A &= W \frac{T_1}{\eta \dot{q}(T_1 - T_2)} \\ &= 78.8288 \text{ m}^2 \end{aligned}$$

Ans. (d)

11. A steel billet of 2000 kg mass is to be cooled from 1250 K to 450 K. The heat released during this process is to be used as a source of energy. The ambient temperature is 303 K and specific heat of steel is 0.5 kJ/kg K. The available energy of this billet is

(a) 490.44 MJ (b) 30.95 MJ
(c) 10.35 MJ (d) 0.10 MJ

(GATE 2004)

Solution. Given that

$$\begin{aligned} m &= 2000 \text{ kg} \\ T_1 &= 1250 \text{ K} \\ T_2 &= 450 \text{ K} \\ T_0 &= 303 \text{ K} \\ c &= 0.5 \text{ kJ/kgK} \end{aligned}$$

The changes in entropy and internal energy of the system (mass) are, respectively,

$$\begin{aligned} \Delta S &= mc \ln\left(\frac{T_1}{T_2}\right) \\ \Delta U &= mc(T_1 - T_2) \end{aligned}$$

Using the above expressions, the availability or available energy (AE) is

$$\begin{aligned} \text{AE} &= \Delta U - T_0 \Delta S \\ &= 490.44 \times 10^6 \text{ J} \end{aligned}$$

Ans. (a)

12. The following four figures have been drawn to represent a fictitious thermodynamics cycle, on the $p - v$ and $T - s$ planes.

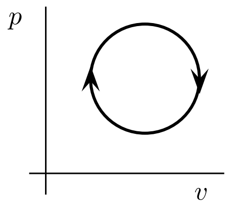


Fig. 1

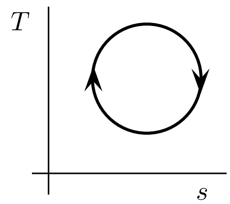


Fig. 2

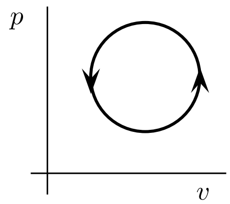


Fig. 3

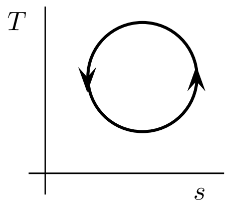


Fig. 4

According to the first law of thermodynamics, equal areas are enclosed by

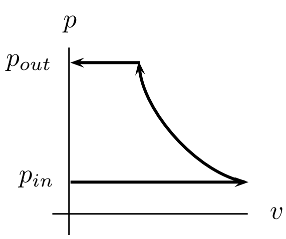
(a) Figures 1 and 2 (b) Figures 1 and 3
(c) Figures 1 and 4 (d) Figures 2 and 3

(GATE 2005)

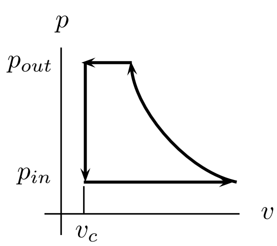
Solution. The direction of cycles in the Figures 1 and 2 are same, and co-ordinates are p–v and T–s.

Ans. (a)

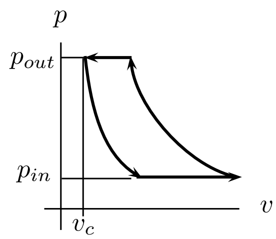
13. A p–v diagram has been obtained from a test on a reciprocating compressor. Which of the following represents that diagram?



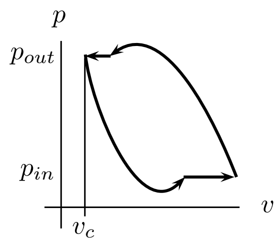
(a)



(b)



(c)



(d)

(GATE 2005)

Solution. Option (d) is most suitable for practical p–v diagram for a compressor. All other options represent cases of ideal cycles.

Ans. (d)

14. A reversible thermodynamic cycle containing only three processes and producing work is to be constructed. The constraints are:

(i) there must be one isothermal process,
(ii) there must be one isentropic process,
(iii) the maximum and minimum cycle pressures and the clearance volume are fixed, and
(iv) polytropic processes are not allowed.

Then, the numbers of possible cycles are

(a) 1 (b) 2
(c) 3 (d) 4

(GATE 2005)

Solution. The cycle has one isentropic process (3-1), which can be used for producing work output, and hence (3-1) is fixed. The other process has to be isothermal process which can only be either used for heat rejection (1-2) because slope its slop is less than isentropic process, and cycle has to complete with constant pressure process in which heat will be added. Hence, all the three processes are fixed, and only one cycle is possible.

Ans. (a)

15. Nitrogen at an initial state of 10 bar, 1 m^3 and 300 K is expanded isothermally to a final volume of 2 m^3. The $p-v-T$ relation is

$$\left(p+\frac{a}{v^2}\right)v = RT$$

where $a > 0$. The final pressure

(a) will be slightly less than 5 bar
(b) will be slightly more than 5 bar
(c) will be exactly 5 bar
(d) cannot be ascertained in the absence of the value of a

(GATE 2005)

Solution. Given that

$$p_1 = 10 \text{ bar}$$
$$v_1 = 1 \text{ m}^3$$
$$T_1 = 300 \text{ K}$$
$$v_2 = 1 \text{ m}^3$$

For the given isothermal process,

$$\left(p_1+\frac{a}{{v_1}^2}\right)v_1 = \left(p_2+\frac{a}{{v_2}^2}\right)v_2$$

For which

$$p_2 = 5 + \frac{a}{2} \text{ bar}$$

As $a > 0$, $p_2 > 5$.

Ans. (b)

16. A 100 W electric bulb was switched on in a 2.5 m × 3 m × 3 m size thermally insulated room having a temperature of 20°C. The room temperature at the end of 24 hours will be

(a) 321°C (b) 341°C
(c) 450°C (d) 470°C

(GATE 2006)

Solution. Taking atmospheric pressure $p = 1$ bar, density (ρ) of air at $T = 20°\text{C}$ (= 293 K) is calculated as

$$\rho = \frac{m}{V} = \frac{p}{RT}$$
$$= \frac{1 \times 10^5}{0.287 \times 293}$$
$$= 1.18919$$

Heat generated in one hour is

$$Q = 100 \times 24 \times 60 \times 60$$

If the final temperature is T_2, heat balance equation is

$$Q = \rho \times 2.5 \times 3 \times 3 \times c_v \, (T_2 - 20)$$
$$T_2 = 469.735°\text{C}$$

Ans. (d)

17. Match items from groups I, II, III, IV and V:

Group I		E Heat	F Work
Group II	When added to the system is	G Positive	H Negative
Group III	Differential	I Exact	J Inexact
Group IV	Function	K Path	L Point
Group V	Phenomenon	M Transient	N Boundary

(a) F-G-J-K-M, E-G-I-K-N
(b) E-G-I-K-M, F-H-I-K-N
(c) F-H-J-L-N, E-H-I-L-N
(d) E-G-J-K-N, F-H-J-K-M

(GATE 2006)

Solution. Heat (E) is considered positive (G) when added to the system. It is an inexact (I) differential because it depends upon the path (K). Heat transfer occurs at boundary (M). Work is considered negative (H) when added to the system. It is an inexact differential because it depends on the path. Therefore, option (d) is correct.

Ans. (d)

Linked Answer Questions

A football was inflated to a gauge pressure of 1 bar when the ambient temperature was 15°C. When the game started the next day, the air temperature at the stadium was 5°C. Assume that the volume of the football remains constant at 2500 cm^3.

18. The amount of heat lost by the air in the football and the gauge pressure of air in the football at the stadium respectively equal

(a) 30.6 J, 1.94 bar (b) 21.8 J, 0.93 bar
(c) 61.1 J, 1.94 bar (d) 43.7 J, 0.93 bar

(GATE 2006)

Solution. Given that

$$V_1 = V_2$$
$$= 2500 \text{ cm}^3$$
$$T_1 = 15°\text{C}$$

Absolute pressure of air is $p_1 = 2$ bar. Mass of the air in the system is

$$m = \frac{p_1 V_1}{RT}$$
$$= \frac{2 \times 10^5 \times 2800 \times 10^{-6}}{287 \times (15 + 273)}$$
$$= 0.00604917 \text{ kg}$$

The heat transfer during the constant volume process is

$$Q = mc_v \Delta T$$
$$= 0.00604917 \times 0.718 \times (10)$$
$$= 0.043433 \text{ kJ}$$

If p_a is the atmospheric pressure, then the gauge pressure after constant volume heat extraction is given by

$$p_2 = p_1 \frac{T_2}{T_1} - p_a$$
$$= 2 \times 10^5 \times \frac{5 + 273}{15 + 273} - 1 \times 10^5$$
$$= 0.930556 \text{ bar}$$

Ans. (d)

19. Gauge pressure of air to which the ball must have been originally inflated so that it would equal 1 bar gauge at the stadium is:

(a) 2.23 bar (b) 1.94 bar
(c) 1.07 bar (d) 1.00 bar

(GATE 2006)

Solution. Initial gauge pressure to get 1 bar gauge pressure (=2 bar absolute) at the stadium is given by

$$p_1 = 2\times\frac{15+273}{5+273}-1$$
$$= 1.07 \text{ bar}$$

Ans. (c)

20. Which of the following relationships is valid only for reversible processes undergone by a closed system of simple compressible substance (neglect changes in kinetic and potential energy)?

(a) $\delta Q = dU+\delta W$ (b) $TdS = dU+\delta W$
(c) $TdS = dU+pdV$ (d) $\delta Q = dU+pdV$

(GATE 2007)

Solution. For a closed system of simple compressible substance,

$$TdS = dU+pdV$$

Ans. (c)

21. A heat transformer is a device that transfers a part of the heat, supplied to it at an intermediate temperature, to high temperature reservoir while rejecting the remaining part to a low temperature heat sink. In such a heat transfer, 100 kJ of heat is supplied at 350 K. The maximum amount of heat in kJ that can be transferred to 400 K, when the rest is rejected to a heat sink at 300 K, is

(a) 12.50 (b) 14.29
(c) 33.33 (d) 57.14

(GATE 2007)

Solution. If maximum heat Q is supplied at 400 K, then net entropy change must be zero. Hence,

$$\frac{100}{350} = \frac{Q}{400}+\frac{100-Q}{300}$$
$$Q = 57.1429 \text{ kJ}$$

Ans. (d)

22. Which of the following statements is correct?

P: A gas cools upon expansion only when its Joule Thomson coefficient is positive in the temperature range of expansion.

Q: For a system undergoing a process, its entropy remains constant only when the process is reversible.

R: The work done by a closed system in an adiabatic process is a point function.

S: A liquid expands upon freezing when the slope of its fusion curve on pressure-temperature diagram is negative.

(a) R and S (b) P and Q
(c) Q, R and S (d) P, Q and R

(GATE 2007)

Solution. In an adiabatic process on a closed system,

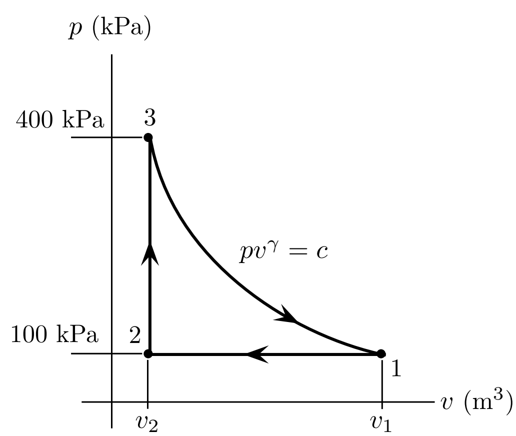
$$\Delta Q = 0$$
$$\Delta W = -\Delta U$$

where U is a property of the system, that is, a point function; therefore, work W is also a point function. Also, on p–T diagram, if fusion curve is negative, it shows expansion of the liquid.

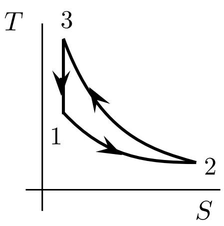
Ans. (a)

Common Data Questions

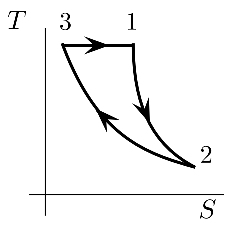
A thermodynamic cycle with an ideal gas as working fluid is shown below:



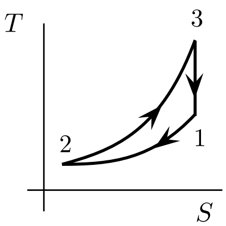
23. The above cycle is represented on T-s plane by



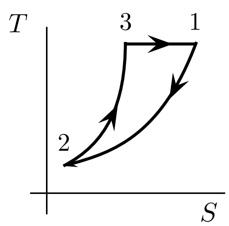
(a)



(b)



(c)



(d)

(GATE 2007)

Solution. On T–s plane, slope of constant volume process is more than that for constant pressure process.

Ans. (c)

24. If the specific heats of the working fluid are constant and the value of specific heat ratio γ is 1.4, the thermal efficiency (%) of the cycle is

(a) 21 (b) 40.9
(c) 42.6 (d) 59.7

(GATE 2007)

Solution. On T–s plane, slope of constant pressure process is more than that for constant volume process.

Ans. (c)

25. Two moles of oxygen are mixed adiabatically with another two moles of oxygen in mixing chamber, so that the final total pressure and temperature of the mixture become same as those of the individual constituents at their initial states. The universal gas constant is given as R. The change in entropy due to mixing, per mole of oxygen, is given by

(a) $-R\ln 2$ (b) 0
(c) $R\ln 2$ (d) $R\ln 4$

(GATE 2008)

Solution. As the process is adiabatic, there is no change in entropy of the system.

Ans. (b)

26. A gas expands in a frictionless piston-cylinder arrangement. The expansion process is very slow, and is resisted by an ambient pressure of 100 kPa. During the expansion process, the pressure of the system (gas) remains constant at 300 kPa. The change in volume of the gas is 0.01 m^3. The maximum amount of work that could be utilized from the above process is

(a) 0 kJ (b) 1 kJ
(c) 2 kJ (d) 3 kJ

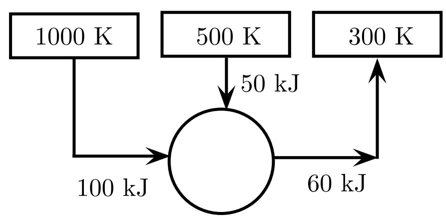
(GATE 2008)

Solution. The gas worked against 100 kPa and expanded by 0.01 m^3, therefore, the maximum amount of work is

$$\begin{aligned} W &= 100 \times 0.01 \\ &= 1 \text{ kJ} \end{aligned}$$

Ans. (b)

27. A cyclic device operates between three thermal reservoirs, as shown in the figure. Heat is transferred to/from the cyclic device. It is assumed that heat transfer between each thermal reservoir and the cyclic device takes place across negligible temperature difference. Interactions between the cyclic device and the respective thermal reservoirs that are shown in the figure are all in the form of heat transfer.



Cyclic device

The cycle can be

(a) a reversible heat engine
(b) a reversible heat pump or a reversible refrigerator
(c) an irreversible heat engine
(d) an irreversible heat pump or an irreversible refrigerator

(GATE 2008)

Solution. The given system constitutes a reversible heat engine because heat is released from high temperature to lower temperature.

Ans. (a)

28. A balloon containing an ideal gas is initially kept in an evacuated and insulated room. The balloon ruptures and the gas fills up the entire room. Which one of the following statements is TRUE at the end of the above processes?

(a) The internal energy of the gas decreases from its initial value, but the enthalpy remains constant

(b) The internal energy of the gas increases from its initial value, but the enthalpy remains constant

(c) Both internal energy and enthalpy of the gas remain constant

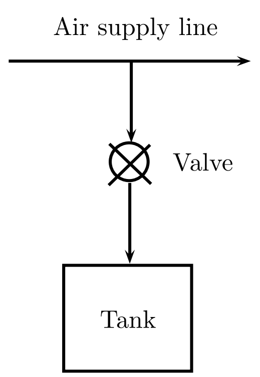
(d) Both internal energy and enthalpy of the gas increase

(GATE 2008)

Solution. The room is evacuated and insulated, therefore, there is no heat exchange, and temperature will also remain constant. Hence, internal energy and enthalpy will remain constant.

Ans. (c)

29. A rigid, insulated tank is initially evacuated. The tank is connected with a supply line through which air (assumed to be ideal gas with constant specific heats) passes at 1 MPa, 350°C. A valve connected with the supply line is opened and the tank is charged with air until the final pressure inside the tank reaches 1 MPa.



The final temperature inside the tank

(a) is greater than 350°C

(b) is less than 350°C

(c) is equal to 350°C

(d) may be greater than, less than, or equal to 350°C, depending on the volume of the tank

(GATE 2008)

Solution. In charging process, the temperature of tank will be less than the supply temperature because the supplied gas has to work to expand in the evacuated tank.

Ans. (b)

30. In a steady state steady flow process taking place in a device with a single inlet and a single outlet, the work done per unit mass flow rate is given by

$$w = -\int_{inlet}^{outlet} v dp$$

where v is the specific volume and p is the pressure. The expression for w given above

(a) is valid only if the process is both reversible and adiabatic

(b) is valid only if the process is both reversible and isothermal

(c) is valid for any reversible process

(d) is incorrect, it must be

$$w = \int_{inlet}^{outlet} p dv$$

(GATE 2008)

Solution. The expression is true for all the steady flow reversible processes.

Ans. (c)

31. If a closed system is undergoing an irreversible process, the entropy of the system

(a) must increase

(b) always remains constant

(c) must decrease

(d) can increase, decrease or remain constant

(GATE 2009)

Solution. Entropy of the system in irreversible processes always increases.

Ans. (a)

32. A frictionless piston-cylinder device contains a gas initially at 0.8 MPa and 0.015 m^3. It expands quasi-statically at constant temperature to a final volume of 0.030 m^3. The work output (in kJ) during this process will be

(a) 8.32 (b) 12.00

(c) 554.67 (d) 8320.00

(GATE 2009)

Solution. For quasi-static isothermal processes

$$\begin{aligned} W &= p_1 v_1 \ln \frac{v_2}{v_1} \\ &= 0.8 \times 10^6 \times 0.015 \ln \left(\frac{0.030}{0.015}\right) \\ &= 8.317 \text{ kW} \end{aligned}$$

Ans. (a)

33. A compressor undergoes a reversible, steady flow process. The gas at inlet and outlet of the compressor is designated as state 1 and state 2, respectively. Potential and kinetic energy changes are to be ignored. The following notations are used: v = specific volume and p =pressure of the gas. The specific work required to be supplied to the compressor for this gas compression process is

(a) $\int_1^2 pdv$ (b) $\int_1^2 vdp$
(c) $v_1(p_2 - p_1)$ (d) $-p_2(v_1 - v_2)$

(GATE 2009)

Solution. The work required for compression in a reversible steady flow process (1→2) is given by

$$W = \int_1^2 vdp$$

Ans. (b)

34. An irreversible heat engine extracts heat from a high temperature source at a rate of 100 kW and rejects heat to a sink at a rate of 50 kW. The entire work output of the heat engine is used to drive a reversible heat pump operating between a set of independent isothermal heat reservoirs at 17°C and 75°C. The rate (in kW) at which the heat pump delivers heat to its high temperature sink is

(a) 50 (b) 250
(c) 300 (d) 360

(GATE 2009)

Solution. Given that for irreversible heat engine,

$$Q_1 = 100 \text{ kW}$$
$$Q_2 = 50 \text{ kW}$$

Work output is

$$W = Q_1 - Q_2 = 50 \text{ kW}$$

For heat pump,

$$T_1 = 75 + 273 = 348 \text{ K}$$
$$T_2 = 17 + 273 = 290 \text{ K}$$

Heat delivered to high temperature sink

$$Q_1 = T_1 \times \frac{W}{T_1 - T_2} = 300 \text{ kW}$$

Ans. (c)

35. One kilogram of water at room temperature is brought into contact with a high temperature thermal reservoir. The entropy change of the universe is

(a) equal to entropy change of the reservoir
(b) equal to entropy change of water
(c) equal to zero
(d) always positive

(GATE 2010)

Solution. If heat Q is transferred from temperature T_1 to T_2 then entropy change of the universe is

$$\Delta S = -\frac{Q}{T_1} + \frac{Q}{T_2} = \frac{Q(T_1 - T_2)}{T_1 T_2}$$

As $T_1 > T_2$, therefore, $\Delta S > 0$.

Ans. (d)

36. A monoatomic ideal gas (γ = 1.67, molecular weight = 40) is compressed adiabatically from 0.1 MPa, 300 K to 0.2 MPa. The universal gas constant is 8.314 kJ/kmol-K. The work of compression of the gas (in kJ/kg) is

(a) 29.7 (b) 19.9
(c) 13.3 (d) 0

(GATE 2010)

Solution. Given that

$$\gamma = 1.67$$
$$M = 40$$
$$\Re = 8.314 \times 10^3 \text{ J/kmol-K}$$
$$p_1 = 0.1 \times 10^6 \text{ Pa}$$
$$p_2 = 0.2 \times 10^6 \text{ Pa}$$
$$T_1 = 300 \text{ K}$$

Gas constant for the ideal gas is

$$R = \frac{\Re}{M} = 207.85 \text{ J/kgK}$$

Temperature after adiabatic compression is

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 396.181 \text{ K}$$

Work done in adiabatic process per kg is

$$\begin{aligned} W &= \frac{R(T_2 - T_1)}{\gamma - 1} \\ &= \frac{207.85 \times (396.181 - 300)}{1.67 - 1} \\ &= 29.8377 \text{ kW} \end{aligned}$$

Ans. (a)

37. Consider the following two processes:

I. A heat source at 1200 K loses 2500 kJ of heat to sink at 800 K.

II. A heat source at 800 K loses 2000 kJ of heat to sink at 500 K.

Which of the following statements is TRUE?

(a) Process I is more irreversible than Process II
(b) Process II is more irreversible than Process I
(c) Irreversibility associated in both the processes is equal
(d) Both the processes are reversible

(GATE 2010)

Solution. If heat Q is transferred from temperature T_1 to T_2 then entropy change of universe is

$$\Delta S = -\frac{Q}{T_1} + \frac{Q}{T_2}$$

For the given two processes,

$$\begin{aligned} \Delta S_1 &= 2500\left(-\frac{1}{1200} + \frac{1}{800}\right) \\ &= 10.4167 \text{ kJ/K} \\ \Delta S_2 &= 2000\left(-\frac{1}{800} + \frac{1}{500}\right) \\ &= 15.00 \text{ kJ/K} \end{aligned}$$

Ans. (b)

38. Heat and work are

(a) intensive properties
(b) extensive properties
(c) point functions
(d) path functions

(GATE 2011)

Solution. Heat and work are path functions because their differentials are inexact or imperfect differentials.

Ans. (d)

39. The contents of a well-insulated tank are heated by a resistor of 23 Ω in which 10 A current is flowing. Consider the tank along with its contents as a thermodynamic system. The work done by the system and the heat transfer to the system are positive. The rates of heat (Q), work (W) and change in internal energy (ΔU) during the process in kW are

(a) $Q = 0$, $W = -2.3$, $\Delta U = +2.3$
(b) $Q = +2.3$, $W = 0$, $\Delta U = +2.3$
(c) $Q = -2.3$, $W = 0$, $\Delta U = -2.3$
(d) $Q = 0$, $W = +2.3$, $\Delta U = -2.3$

(GATE 2011)

Solution. Work done (W) by the system is zero. Heat given to the system is $10^2 \times 23 = 2.3$ kW, and by the first law of thermodynamics

$$\begin{aligned} Q &= W + \Delta U \\ \Delta U &= Q \\ &= 2.3 \text{ kW} \end{aligned}$$

Ans. (b)

40. The values of enthalpy of steam at the inlet and outlet of a steam turbine in a Rankine cycle[33] are 2800 kJ/kg and 1800 kJ/kg, respectively. Neglecting pump work, the specific steam consumption in kg/kW-hour is

(a) 3.60 (b) 0.36
(c) 0.06 (d) 0.01

(GATE 2011)

Solution. Given

$$\begin{aligned} h_1 &= 2800 \text{ kJ/kg} \\ h_2 &= 1800 \text{ kJ/kg} \end{aligned}$$

Power developed by 1 kg/s steam flow is

$$\begin{aligned} W &= (h_1 - h_2) \\ &= 1000 \text{ kW/kg} \end{aligned}$$

Specific steam consumption is

$$\begin{aligned} &= \frac{3600}{W} \\ &= 3.6 \text{ kg/kW-hour} \end{aligned}$$

Ans. (a)

[33]Discussed in Chapter 9.

41. A pump handling a liquid raises its pressure from 1 bar to 30 bar. Take the density of the liquid as 990 kg /m^3. The isentropic specific work done by the pump in kJ/kg is

(a) 0.10 (b) 0.30
(c) 2.50 (d) 2.93

(GATE 2011)

Solution. Given that

$$\begin{aligned} p_1 &= 1 \text{ bar} \\ p_2 &= 30 \text{ bar} \\ \rho &= 990 \text{ kg/m}^3 \end{aligned}$$

Isentropic work done by the pump is

$$\begin{aligned} w_p &= v\,(p_2 - p_1) \\ &= \frac{1}{\rho}\,(p_2 - p_1) \\ &= 2.92929 \text{ kJ/kg} \end{aligned}$$

Ans. (d)

42. An ideal Brayton cycle, operating between the pressure limits of 1 bar and 6 bar, has minimum and maximum temperatures of 300 K and 1500 K. The ratio of specific heats of the working fluid is 1.4. The approximate final temperatures in Kelvin at the end of the compression and expansion processes are, respectively,

(a) 500 and 900 (b) 900 and 500
(c) 500 and 500 (d) 900 and 900

(GATE 2011)

Solution. Given that

$$\begin{aligned} r &= \frac{6}{1} \\ &= 6 \\ T_1 &= 300 \text{ K} \\ T_3 &= 1500 \text{ K} \\ \gamma &= 1.4 \end{aligned}$$

Therefore,

$$\begin{aligned} T_2 &= T_1 r^{(\gamma-1)/\gamma} \\ &= 500.553 \text{ K} \\ T_4 &= T_3 / r^{(\gamma-1)/\gamma} \\ &= 899.005 \text{ K} \end{aligned}$$

Ans. (a)

Common Data Questions

In an experimental set-up, air flows between two stations P and Q adiabatically. The direction of flow depends on the pressure and temperature conditions maintained at P and Q. The conditions at station P are 150 kPa and 350 K. The temperature at station Q is 300 K. The following are the properties and relations pertaining to air: Specific heat at constant pressure, $c_p = 1.005$ kJ/kgK, Specific heat at constant volume, $c_v = 0.718$ kJ/kgK, Characteristic gas constant, $R = 0.287$ kJ/kgK, Enthalpy, $h = c_p T$, Internal energy, $u = c_v T$.

43. If the air has to flow from station P to station Q, the maximum possible value of pressure in kPa at station Q is close to

(a) 50 (b) 87
(c) 128 (d) 150

(GATE 2011)

Solution. Given that

$$\begin{aligned} T_1 &= 350 \text{ K} \\ T_2 &= 300 \text{ K} \\ p_1 &= 150 \text{ kPa} \\ c_p &= 1.005 \text{ kJ/kgK} \end{aligned}$$

Using ideal gas equation for $m = 1$ kg,

$$\begin{aligned} pv &= RT \\ T ds &= dh - v dp \end{aligned}$$

For isentropic process (1 to 2), one finds,

$$c_p \ln \frac{T_2}{T_2} - R \ln \frac{p_2}{p_1} = 0$$

Using the given data, the above equation provides

$$p_2 = 87.4301 \text{ kPa}$$

Ans. (b)

44. If the pressure at station Q is 50 kPa, the change in entropy $(s_p - s_q)$ in kJ/kgK is

(a) −0.155 (b) 0
(c) 0.160 (d) 0.355

(GATE 2011)

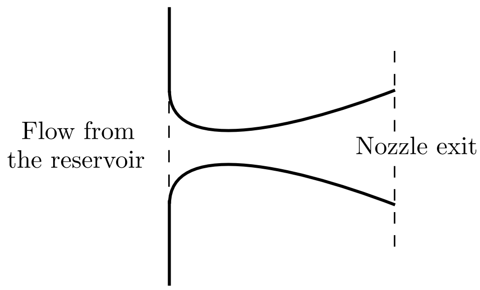
Solution. Given that $p_2 = 50$ kPa, along with previous data, the change in entropy is determined as

$$\begin{aligned} s_p - s_q &= c_p \ln \frac{T_2}{T_2} - R \ln \frac{p_2}{p_1} \\ &= 0.16038 \text{ kJ/kgK} \end{aligned}$$

Ans. (c)

Linked Answer Questions

The temperature and pressure of air in a large reservoir are 400 K and 3 bar, respectively. A converging diverging nozzle of exit area 0.005 m^2 is fitted to the wall of the reservoir as shown in the figure. The static pressure of air at the exit section for isentropic flow through the nozzle is 50 kPa. The characteristic gas constant and the ratio of specific heats of air are 0.287 kJ/kgK and 1.4, respectively.



45. The density of air in kg/m^3 at the nozzle exit is

(a) 0.560 (b) 0.600
(c) 0.727 (d) 0.800

(GATE 2011)

Solution. Given that

$$T_1 = 400 \text{ K}$$
$$p_1 = 300 \text{ kPa}$$
$$\gamma = 1.4$$
$$R = 0.287 \text{ kJ/kgK}$$

Temperature at the nozzle exit is

$$\begin{aligned} T_2 &= T_1\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} \\ &= 239.735 \text{ K} \end{aligned}$$

The density of air at the nozzle exit is

$$\begin{aligned} \rho_2 &= \frac{p_2}{RT_2} \\ &= 0.726703 \text{ kg/m}^3 \end{aligned}$$

Ans. (c)

46. The mass flow rate of air through the nozzle in kg/s is

(a) 1.30 (b) 1.77
(c) 1.85 (d) 2.06

(GATE 2011)

Solution. Given that

$$\begin{aligned} A_2 &= 0.005 \text{ m}^2 \\ c_p &= R\frac{\gamma}{\gamma-1} \\ &= 1.0045 \text{ kJ/kgK} \\ &= 1.0045\times10^3 \text{ J/kgK} \end{aligned}$$

Velocity at nozzle exit is determined by first law of thermodynamics for steady flow processes as

$$\begin{aligned} V_2 &= \sqrt{2c_p\,(T_1-T_2)} \\ &= 567.426 \text{ m/s} \\ \dot{m} &= \rho_2 V_2 A_2 \\ &= 2.06175 \text{ kg/s} \end{aligned}$$

Ans. (d)

47. An ideal gas of mass m and temperature T_1 undergoes a reversible isothermal process from an initial pressure p_1 to a final pressure p_2. The heat loss during the process is Q. The entropy change Δs of the gas is

(a) $mR\ln(p_2/p_1)$
(b) $mR\ln(p_2/p_1)$
(c) $mR\ln(p_2/p_1)-Q/T$
(d) Zero

(GATE 2012)

Solution. For isothermal processes on ideal gas

$$du = 0$$
$$p_1v_1 = p_2v_2$$

The change in entropy is found as

$$\begin{aligned} du &= Tds - pdv \\ ds &= \frac{p}{T}dv \\ &= \frac{mR}{v}dv \\ &= mR\ln\left(\frac{p_1}{p_2}\right) \end{aligned}$$

Ans. (b)

Linked Answer Questions

Air enters an adiabatic nozzle at 300 kPa, 500 K, with a velocity of 10 m/s. It leaves the nozzle at 100 kPa with a velocity of 180 m/s. The inlet area is 80 cm^2. The specific heat of air c_p is 1008 J/kg.K.

48. The exit temperature of the air is

(a) 561 K (b) 532 K
(c) 484 K (d) 468 K

(GATE 2012)

Solution. Given that

$$V_1 = 10 \text{ m/s}$$
$$V_2 = 180 \text{ m/s}$$
$$T_1 = 500 \text{ K}$$
$$c_p = 1008 \text{ J/kgK}$$

The heat balance equation is

$$\frac{V_2^2}{2} - \frac{V_1^2}{2} = c_p\,(T_1 - T_2)$$
$$\frac{180^2}{2} - \frac{10^2}{2} = 1008\,(500 - T_2)$$
$$T_2 = 483.978 \text{ K}$$

Ans. (c)

49. The exit area of the nozzle in cm^2 is

(a) 90.1 (b) 56.3
(c) 4.4 (d) 12.9

(GATE 2012)

Solution. Given that

$$p_1 = 300 \text{ kPa}$$
$$p_2 = 100 \text{ kPa}$$
$$A_1 = 80 \text{ cm}^2$$

Using ideal gas equation, density (ρ) of air is given by

$$\rho = \frac{m}{v} = \frac{p}{RT}$$

Using the above equation, one can write equation for conservation of mass as

$$\frac{p_1}{RT_1} A_1 V_1 = \frac{p_2}{RT_2} A_2 V_2$$
$$A_2 = 12.9 \text{ cm}^2$$

Ans. (d)

50. A cylinder contains 3 m^3 of ideal gas at a pressure of 1 bar. This gas is compressed in a reversible isothermal process till its pressure increases to 5 bar. The work in kJ required for this process is

(a) 804.7 (b) 953.2
(c) 981.7 (d) 1012.2

(GATE 2013)

Solution. Given that

$$v_1 = 3 \text{ m}^3$$
$$p_1 = 100 \text{ kPa}$$
$$p_2 = 500 \text{ kPa}$$

The work done in reversible isothermal process is determined as

$$w = p_1 v_1 \ln\left(\frac{p_2}{p_1}\right)$$
$$= 100 \times 3 \ln\left(\frac{5}{1}\right)$$
$$= 804.71 \text{ kJ}$$

Ans. (a)

51. Specific enthalpy and velocity of steam at inlet and exit of a steam turbine, running under steady state, are as given below:

Point	Specific Enthalpy	Velocity
Inlet	3250 kJ/kg	180 m/s
Exit	2360 kJ/kg	5 m/s

The rate of heat loss from the turbine per kg of steam flow rate is 5 kW. Neglecting changes in potential energy of steam, the power developed in kW by the steam turbine per kg of steam flow rate, is

(a) 901.2 (b) 911.2
(c) 17072.5 (d) 17082.5

(GATE 2013)

Solution. Using steady state energy equation, one finds

$$w = (h_1 - h_2) + q + \frac{V_1^2 - V_2^2}{2}$$
$$= (3250 - 2360) + 5 + \frac{180^2 - 5^2}{2}$$
$$= 890 + 5 + 16.1875$$
$$= 911.1875 \text{ kW}$$

Ans. (b)

52. The pressure, temperature and velocity of air flowing in a pipe are 5 bar, 500 K and 50 m/s, respectively. The specific heats of air at constant pressure and at constant volume are 1.005 kJ/kgK and 0.718 kJ/kgK, respectively. Neglect potential energy. If the pressure and temperature of the surroundings are 1 bar and 300 K, respectively, the available energy in kJ/kg of the air stream is

(a) 170 (b) 187
(c) 191 (d) 213

(GATE 2013)

Solution. The change in specific entropy if the air system reaches to dead state is

$$\begin{aligned} s_1 - s_0 &= c_p \ln\left(\frac{T_1}{T_0}\right) - R\ln\left(\frac{p_1}{p_0}\right) \\ &= 1.005\ln\left(\frac{500}{300}\right) - 0.287\ln\left(\frac{5}{1}\right) \\ &= 0.05147 \text{ kJ/kgK} \end{aligned}$$

The change in availability of the system is

$$\begin{aligned} \psi_1 - \psi_0 &= (h_1 - h_0) + T_0(s_1 - s_0) \\ &= 1.005 \times (500 - 300) - 300 \times 0.05147 \\ &= 185.559 \text{ kJ/kg} \end{aligned}$$

Ans. (b)

Linked Answer Questions

In a simple Brayton cycle, the pressure ratio is 8 and temperatures at the entrance of compressor and turbine are 300 K and 1400 K, respectively. Both compressor and gas turbine have isentropic efficiencies equal to 0.8. For the gas, assume a constant value of c_p (specific heat at constant pressure) equal to 1 kJ/kgK and ratio of specific heats as 1.4. Neglect changes in kinetic and potential energies.

53. The power required by the compressor in kW/kg of gas flow rate is

(a) 194.7 (b) 243.4
(c) 304.3 (d) 378.5

(GATE 2013)

Solution. Given that

$$\begin{aligned} r_p &= p_2/p_1 = 8 \\ T_1 &= 300 \text{ K} \\ T_3 &= 1400 \text{ K} \\ \eta_c &= 0.8 \\ \eta_t &= 0.8 \\ c_p &= 1 \text{ kJ/kgK} \\ \gamma &= 1.4 \end{aligned}$$

The temperature after isentropic compression is

$$\begin{aligned} T_2 &= T_1\left(\frac{p_2}{p_1}^{\frac{\gamma-1}{\gamma}}\right) \\ &= 300 \times 8^{0.4/1.4} \\ &= 300 \times 1.811 \\ &= 543.434 \text{ K} \end{aligned}$$

The temperature after actual expansion is

$$\frac{T_2 - T_1}{T_2' - T_1} = \eta_c$$

$$\begin{aligned} T_2' &= T_1 + \frac{T_2 - T_1}{\eta_c} \\ &= 604.29 \text{ K} \end{aligned}$$

The power required by compressor per kg gas-flow rate is

$$\begin{aligned} w_c &= c_p(T_2' - T_1) \\ &= 1 \times (604.29 - 300) \\ &= 304.29 \text{ kW/kg} \end{aligned}$$

Ans. (b)

54. The thermal efficiency of the cycle in percentage (%) is

(a) 24.8 (b) 38.6
(c) 44.8 (d) 53.1

(GATE 2013)

Solution. The temperature of gas after isentropic expansion is

$$\begin{aligned} T_4 &= T_3\left(\frac{p_1}{p_2}^{\frac{\gamma-1}{\gamma}}\right) \\ &= 1400 \times \left(\frac{1}{8}\right)^{0.4/1.4} \\ &= \frac{1400}{1.811} \\ &= 773.053 \text{ K} \end{aligned}$$

The work output from the turbine per kg of flow rate is

$$\begin{aligned} w_t &= c_p(T_3 - T_4)\eta_t \\ &= 1 \times (1400 - 773.053)\,0.8 \\ &= 501.55 \text{ kW/kg} \end{aligned}$$

Heat input to the cycle per kg of flow rate is

$$\begin{aligned} q_1 &= c_p(T_3 - T_2') \\ &= 1 \times (1400 - 604.29) \\ &= 795.707 \text{ kW/kg} \end{aligned}$$

The efficiency of the cycle is

$$\begin{aligned} \eta &= \frac{w_t - w_c}{q_1} \\ &= \frac{501.55 - 304.29}{795.707} \\ &= 24.79\% \end{aligned}$$

Ans. (a)

MULTIPLE CHOICE QUESTIONS

1. Which one of the following statements is not correct? In a transient flow process

(a) the rates of inflow and outflow of mass are different
(b) the state of matter inside the control volume varies with time
(c) there can be heat and work interactions across the control volume
(d) there is no accumulation of energy inside the control volume

2. Thermodynamic work is the product of

(a) two intensive properties
(b) two extensive properties
(c) an intensive property and change in an extensive property
(d) an extensive property and change in an intensive property

3. Which of the following is an intensive property of a system?

(a) pressure
(b) mass
(c) enthalpy
(d) volume

4. Consider the following properties:

1. Temperature
2. Viscosity
3. Specific entropy
4. Thermal conductivity

Which of the above properties of a system is/are intensive?

(a) 1 only (b) 2 and 3 only
(c) 2, 3 and 4 only (d) 1, 2, 3 and 4

5. Pressure reaches a value of absolute zero

(a) at a temperature of 273 K
(b) under vacuum conditions
(c) at the earth's center
(d) when molecular momentum of system becomes zero

6. Ice kept in a well insulated thermos flask is an example of which system?

(a) Closed system
(b) Isolated system
(c) Open system
(d) Non-flow adiabatic system

7. Which one of the following parameters is significant to ascertain chemical equilibrium of a system?

(a) Clapeyron relation
(b) Maxwell relation
(c) Gibbs function
(d) Helmholtz function

8. A thermodynamic system is considered to be an isolated one if

(a) mass transfer and entropy change are zero
(b) entropy change and energy transfer are zero
(c) energy transfer and mass transfer are zero
(d) mass transfer and volume change are zero

9. In case of power failure, a battery is used to light a bulb, run a fan and heat an electric iron each of 100 W rating for 10 min. In this process, the work done and heat supplied by the battery are given by?

(a) $W = 0$, $Q = 0$
(b) $W = 120$ kJ, $Q = 0$
(c) $W = 60$ kJ, $Q = 120$ kJ
(d) None of the above

10. Work transfer between the system and surroundings

(a) is a point function
(b) is always given by $\int pdv$
(c) is a function of pressure only
(d) depends on the path followed by the system

11. A new temperature scale in degrees N is to be defined. The boiling and freezing points of water on this scale are 400° N and 100° N, respectively. What will be the reading on new scale corresponding to 60°C?

(a) 120° N (b) 180° N
(c) 220° N (d) 280° N

12. Measurement of temperature is based on which law of thermodynamics?

(a) Zeroth law of thermodynamics
(b) First law of thermodynamics
(c) Second law of thermodynamics
(d) Third law of thermodynamics

13. A 4 kW, 20 liter water heater is switched on for 10 min. The heat capacity c_p for water is 4 kJ/kg-K. Assuming all the electrical energy has gone into heating the water, what is the increase of the water temperature?

(a) 15°C (b) 20°C
(c) 26°C (d) 30°C

14. A small steam whistle (perfectly insulated and doing no shaft work) causes a drop of 0.8 kJ/kg in the enthalpy of steam from entry to exit. If the kinetic energy of the steam at entry is negligible, the velocity of the steam at exit is

(a) 4 m/s (b) 40 m/s
(c) 80 m/s (d) 120 m/s

15. A steam turbine receives steam steadily at 10 bar with an enthalpy of 3000 kJ/kg and discharges at 1 bar with an enthalpy of 2700 kJ/kg. The work output is 250 kJ/kg. The changes in kinetic and potential energies are negligible. The heat transfer from the turbine casing to the surroundings is equal to

(a) 0 kJ (b) 50 kJ
(c) 150 kJ (d) 250 kJ

16. A system undergoes a change of state during which 80 kJ of heat is transferred to it and it does 60 kJ of work. The system is brought back to its original state through a process during which 100 kJ of heat is transferred to it. The work done by the system is

(a) 40 kJ (b) 60 kJ
(c) 120 kJ (d) 180 kJ

17. When air is compressed, the enthalpy is increased from 100 to 200 kJ/kg. Heat lost during this compression is 50 kJ/kg. Neglecting the change in kinetic and potential energies, the power required for a mass flow of 2 kg/s of air through the compressor will be

(a) 300 kW (b) 200 kW
(c) 100 kW (d) 50 kW

18. A perpetual motion machine capable of generating increasing amounts of energy without interacting with its surroundings cannot exist. This is best explained by the

(a) First law of thermodynamics
(b) Third law of thermodynamics
(c) Energy conservation principle
(d) Gibbs-Helmholtz

19. How much heat is absorbed by 100 g of water when its temperature decreases from 25°C to 5°C? (Take the specific heat of water to be 4.2 J/g K).

(a) 8400 J (b) −8400 J
(c) 2000/4.2 J (d) −2000/4.2 J

20. For a closed system, the difference between the heat added to the system and the work done by the system is equal to

(a) enthalpy (b) entropy
(c) temperature (d) internal energy

21. The internal energy of a certain system is a function of temperature alone and is given by the formula

$$E = 25 + 0.25t \text{ kJ}$$

If this system executes a process for which the work done by it per degree temperature increase is 0.75 kNm, the heat interaction per degree temperature increase in kJ is

(a) −1.00 (b) −0.50
(c) 0.50 (d) 1.00

22. The air with enthalpy of 100 kJ/kg is compressed by an air compressor to a pressure and temperature at which its enthalpy becomes 200 kJ/kg. The loss of heat is 40 kJ/kg from the compressor as the air passes through it. Neglecting the change in kinetic and potential energies, the power required for an air mass flow of 0.5 kg/s is

(a) 30 kW (b) 50 kW
(c) 70 kW (d) 90 kW

23. For a simple closed system of constant composition, the difference between the net heat and work interactions is identifiable as the change in

(a) enthalpy (b) entropy
(c) flow energy (d) internal energy

24. The cyclic integral of $(\delta Q - \delta W)$ for a process is

(a) positive (b) negative
(c) zero (d) unpredictable

25. Which one of the following expresses reversible work done by the system (steady flow) between states 1 and 2?

(a) $\int_1^2 pdv$ (b) $-\int_1^2 vdp$
(c) $-\int_1^2 pdv$ (d) $\int_1^2 vdp$

26. Thermal electric power plant produces 1000 MW of power. If the coal releases 900×10^7 kJ/h of energy, then what is the rate at which heat is rejected from the power plant?

(a) 500 MW (b) 1000 MW
(c) 1500 MW (d) 2000 MW

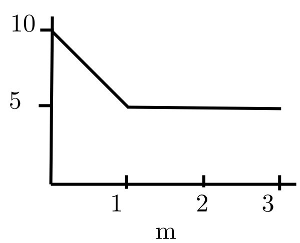
27. Suppose 0.70 kg/s of air enters the compressor with a specific enthalpy of 290 kJ/kg and leaves it with specific enthalpy 450 kJ/kg. Velocities at inlet and exit are 6 m/s and 2 m/s, respectively. Assuming adiabatic process, what is power input to the compressor?

(a) 100 kW (b) 118 kW
(c) 115 kW (d) 112 kW

28. In a throttling process, which one of the following parameters remain constant?

(a) Temperature (b) Pressure
(c) Enthalpy (d) Entropy

29. The given figure shows the variation of force (Newtons) in an elementary system which undergoes a process during which the plunger position changes from 0 to 3 m.



If the internal energy of the system at the end of the process is 2.5 J higher, then the heat absorbed during the process is

(a) 15 J (b) 20 J
(c) 25 J (d) 30 J

30. In a cyclic heat engine operating between a source temperature of 600°C and a sink temperature of 20°C, the least rate of heat rejection per kW net output of the engine is

(a) 0.460 kW (b) 0.505 kW
(c) 0.588 kW (d) 0.650 kW

31. A reversible heat engine operating between hot and cold reservoirs delivers a work output of 54 kJ while it rejects a heat of 66 kJ. The efficiency of this engine is

(a) 0.45 (b) 0.66
(c) 0.75 (d) 0.82

32. A cyclic heat engine does 50 kJ of work per cycle. If the efficiency of the heat engine is 75%, the heat rejected per cycle is

(a) 16.67 kJ (b) 33.30 kJ
(c) 37.50 kJ (d) 66.67 kJ

33. A system undergoes a state change from 1 to 2. According the second law of thermodynamics for the process to be feasible, the entropy change, $s_2 - s_1$ of the system

(a) is positive or zero
(b) is negative or zero
(c) is zero
(d) can be positive, negative or zero

34. For two cycles coupled in series, the topping (upper) cycle has an efficiency of 30% and the bottoming cycle has an efficiency of 20%. The overall combined cycle efficiency is

(a) 50% (b) 44%
(c) 38% (d) 55%

35. A solar energy based heat engine which receives 80 kJ of heat at 100°C and rejects 70 kJ of heat to the ambient at 30°C is to be designed. The thermal efficiency of the heat engine is

(a) 70% (b) 1.88%
(c) 12.5% (d) indeterminate

36. One kilomole of an ideal gas is throttled from an initial pressure of 0.5 MPa to 0.1 MPa. The initial temperature is 300 K. The entropy change of the universe is:

(a) 13.38 kJ/K (b) 401.3 kJ/K
(c) 0.0446 kJ/K (d) −0.0446 kJ/K

37. It is proposed to build a refrigeration plant for cold storage to be maintained at −3°C. The ambient temperature is 27°C. If 5×10^6 kJ/h of energy is to

be continuously removed from the cold storage, the minimum power required to run the refrigerator will be

(a) 14.3 kW (b) 75.3 kW
(c) 154.3 kW (d) 245.3 kW

38. A reversible engine has ideal thermal efficiency of 30%. When it is used as a refrigeration machine with all other conditions unchanged, the coefficient of performance will be

(a) 3.33 (b) 3.00
(c) 2.33 (d) 1.33

39. A heat engine gives an output of 3 kW when the input is 10000 J/s, then the thermal efficiency of the engine will be

(a) 20% (b) 30%
(c) 70% (d) 76.7%

40. A heat engine is supplied with 250 kJ/s of heat at a constant fixed temperature of 227°C. The heat is rejected at 27°C. The cycle is reversible, if the amount of heat rejected is

(a) 273 kJ/s (b) 200 kJ/s
(c) 180 kJ/s (d) 150 kJ/s

41. Which one of the following pairs best expresses a relationship similar to that expressed in the pair "pressure-volume" for a thermodynamic system undergoing a process?

(a) enthalpy-entropy
(b) pressure-enthalpy
(c) pressure-temperature
(d) temperature-entropy

42. For a reversible cycle

(a) $\oint dQ/T > 0$ (b) $\oint dQ/T < 0$
(c) $\oint dQ/T = 0$ (d) $\oint dQ/T = \infty$

43. For a Carnot cycle heat rejected is 1000 kJ. The maximum and minimum temperatures are 750 K and 400 K, respectively. The heat absorbed is, approximately,

(a) 946 kJ (b) 750 kJ
(c) 1000 kJ (d) 1875 kJ

44. A perfect crystal at 0 K has

(a) zero enthalpy
(b) zero entropy
(c) maximum entropy
(d) reached a state of equal entropy and enthalpy, pressure, however, must be constant

45. A mass of a fluid at temperature T_1 is mixed with an equal mass of the same fluid at temperature T_2. The resultant change in entropy of the universe is

(a) zero (b) negligible
(c) always negative (d) always positive

46. A heat pump working on a reversed Carnot cycle has a COP of 5. If it works as a refrigerator taking 1 kW of work input, the refrigeration effect will be

(a) 1 kW (b) 2 kW
(c) 3 kW (d) 4 kW

47. One reversible heat engine operates between 1600 K and T_2 K, and another reversible engine operates between T_2 K and 400 K. If both engines have the same heat input and output, then the temperature T_2 must be equal to

(a) 1000 K (b) 1200 K
(c) 1400 K (d) 800 K

48. A heat engine using lake water at 12°C as source and the surrounding atmosphere at 2°C as sink executes 1080 cycles per minute. If the amount of heat drawn per cycle is 57 J, then the output of the engine will be

(a) 66 W (b) 56 W
(c) 46 W (d) 36 W

49. The operating temperature of a cold storage is −2°C. Heat leakage from the surrounding is 30 kW for the ambient temperature of 40°C. The actual COP of the refrigeration plant used is one-fourth that of an ideal plant working between the same temperatures. The power required to drive the plant is

(a) 1.86 kW (b) 3.72 kW
(c) 7.44 kW (d) 18.60 kW

50. A system of 100 kg mass undergoes a process in which its specific energy increases from 0.3 kJ/kgK to 0.4 kJ/kgK. At the same time, the entropy of the surroundings decreases from 80 kJ/K to 75 kJ/K. The process is

(a) reversible and isothermal
(b) irreversible
(c) reversible

(d) impossible

51. A refrigerating machine working on reversed Carnot cycle takes out 2 kW per minute of heat from the system while between temperature limits of 300 K and 200 K. COP and power consumption of the cycle will be respectively

(a) 1 and 1 kW (b) 1 and 2 kW
(c) 2 and 1 kW (d) 2 and 2 kW

52. For a thermodynamic cycle to be irreversible, it is necessary that

(a) $\oint \frac{\delta Q}{T} = 0$ (b) $\oint \frac{\delta Q}{T} < 0$

(c) $\oint \frac{\delta Q}{T} > 0$ (d) $\oint \frac{\delta Q}{T} \geq 0$

53. A reversible engine operates between temperatures T_1 and T_2. The energy rejected by this engine is received by a second reversible engine at temperature T_2 and rejected to a reservoir at temperature T_3. If the efficiencies of the engines are the same then the relationship between T_1, T_2 and T_3 is given by

(a) $T_2 = (T_1 + T_3)/2$ (b) $T_2 = \sqrt{{T_1}^2 + {T_3}^2}$
(c) $T_2 = \sqrt{T_1 T_3}$ (d) $T_2 = (T_1 + 2T_3)/3$

54. A refrigerating machine having coefficient of performance equal to 2 is used to remove heat at the rate of 1200 kJ/min. What is the power required for this machine?

(a) 80 kW (b) 60 kW
(c) 20 kW (d) 10 kW

55. A Carnot engine operates between 327°C and 27°C. If the engine produces 300 kJ of work, what is the entropy change during head addition?

(a) 0.5 kJ/K (b) 1.0 kJ/K
(c) 1.5 kJ/K (d) 2.0 kJ/K

56. The relation

$$ds = \frac{dQ}{T}$$

where s represents entropy, Q represents heat and T represents temperature (absolute), holds good in which one of the following processes?

(a) Reversible processes only
(b) Irreversible processes only
(c) Both reversible and irreversible
(d) All real processes

57. An inventor says that his new concept of an engine, while working between temperature limits of 27°C and 327°C rejects 45% of heat absorbed from the source. His engine is then equivalent to which one of the following engines?

(a) Carnot engine
(b) Diesel engine
(c) An impossible engine
(d) Ericsson engine

58. Three engines A, B and C operating on Carnot cycle use working substances as argon, oxygen and air, respectively. Which will have higher efficiency?

(a) Engine A
(b) Engine B
(c) Engine C
(d) All engines have same efficiency

59. A series of combination of two Carnot engines operate between the temperatures of 180°C and 20°C. If the engine produces equal amount of work, then what is the intermediate temperature?

(a) 80°C (b) 90°C
(c) 100°C (d) 110°C

60. An engine working on Carnot cycle rejects 40% of absorbed heat from the source, while the sink temperature is maintained at 27°C, then what is the source temperature?

(a) 750°C (b) 477°C
(c) 203°C (d) 67.5°C

61. A reversible heat engine rejects 50% of the heat supplied during a cycle of operation. If this engine is reversed and operates as a heat pump, then what is its coefficient of performance?

(a) 1.0 (b) 1.5
(c) 2.0 (d) 2.5

62. The thermal efficiency of a Carnot heat engine is 30%. If the engine is reversed in operation to work as a heat pump with operating condition unchanged, then what will be COP for heat pump?

(a) 0.30
(b) 2.33
(c) 3.33
(d) Cannot be calculated

63. Increase in entropy of a system represents

(a) increase in availability of energy
(b) increase in temperature
(c) decrease in pressure
(d) degradation of energy

64. A Carnot engine receiving heat at 400 K has an efficiency of 24%. The COP of a Carnot refrigerator working between the same temperature limit is

(a) 1 (b) 2
(c) 3 (d) 4

65. When the lower temperature is fixed, COP of a refrigerating machine can be improved by

(a) operating the machine at higher speeds
(b) operating the machine at lower speeds
(c) raising the higher temperature
(d) lowering the higher temperature

66. In an ideal refrigeration (reversed Carnot) cycle, the condenser and evaporator temperatures are 27°C and −13°C, respectively. The COP of this cycle would be

(a) 6.5 (b) 7.5
(c) 10.5 (d) 15.0

67. The heat absorbed or rejected during a polytropic process is equal to

(a) $\sqrt{(\gamma-n)/(\gamma-1)}\times$ work done
(b) $(\gamma-n)/(\gamma-1)\times$ work done
(c) $(\gamma-1)/(\gamma-n)\times$ work done
(d) $(\gamma-n)^2/(\gamma-1)^2\times$ work done

68. For reversible adiabatic compression in a steady flow process, the work transfer per unit mass is

(a) $\int pdv$ (b) $\int vdp$
(c) $\int Tds$ (d) $\int sdT$

69. If $\int pdv$ and $-\int vdp$ for a thermodynamic system of an ideal gas on evaluation gives the same quantity during a process, then the process undergone by the system is

(a) isenthalpic
(b) isentropic
(c) isobaric
(d) isothermal

70. For an ideal gas, the expression

$$T\left(\frac{\partial s}{\partial T}\right)_p - T\left(\frac{\partial s}{\partial T}\right)_v$$

is always equal to

(a) zero (b) γ
(c) R (d) RT

71. In a process, the temperature and volume are increased to three times the original values. The entropy/kg change is given by

(a) $c_v \ln 3$ (b) $c_p \ln 3$
(c) $c_v \ln 3/3$ (d) $c_p \ln 3/3$

72. The heat flow of a system under isochoric conditions is a direct measurement of

(a) work (b) ΔU
(c) ΔH (d) ΔG

73. For the isothermal expansion of an ideal gas which of the following is true?

1. Energy change $\Delta U = 0$
2. Heat = Work
3. Enthalpy change $\Delta H = 0$

(a) 1 only (b) 2 and 3
(c) 1 and 3 (d) 1 and 2

74. The change in enthalpy of a system is equal to the heat flow between the system and its surroundings under which condition?

(a) constant volume
(b) constant pressure
(c) constant temperature
(d) adiabatic conditions

75. When a system undergoes a process such that

$$\int \frac{dQ}{T} = 0, \quad \text{and } \Delta s > 0$$

the process is

(a) irreversible adiabatic
(b) reversible adiabatic
(c) isothermal
(d) isobaric

76. When a perfect gas enclosed in a cylinder piston device executes a reversible adiabatic expansion process,

1. its entropy will increase
2. its entropy change will be zero
3. the entropy change of the surroundings will be zero

Of these statements

(a) 1 and 3 are correct
(b) 2 alone is correct
(c) 2 and 3 are correct
(d) 1 alone is correct

77. Change in internal energy in a reversible process occurring in a closed system is equal to the heat transferred if the process occurs at constant

(a) pressure (b) volume
(c) temperature (d) enthalpy

78. Change in enthalpy in a closed system is equal to the heat transferred, if the reversible process takes place at constant

(a) temperature (b) internal energy
(c) pressure (d) entropy

79. Which one of the following thermodynamic processes approximates the steaming of food in a pressure cooker?

(a) isenthalpic (b) isobaric
(c) isochoric (d) isothermal

80. In which one of the following processes in a closed system, the thermal energy transferred to a gas is completely converted to internal energy resulting in an increase in gas temperature?

(a) Isochoric process
(b) Adiabatic process
(c) Isothermal process
(d) Free expansion

81. An ideal gas at 27°C is heated at constant pressure till its volume becomes three times. What would be then temperature of gas?

(a) 81°C (b) 627°C
(c) 543°C (d) 327°C

82. In a reversible isothermal expansion process, the fluid expands from 10 bar and 2 m^3 to 2 bar and 10 m^3. During the process the heat supplied is 100 kW. What is the work done during the process?

(a) 33.3 kW (b) 100 kW
(c) 80 kW (d) 20 kW

83. Air is being forced by the bicycle pump into a tyre against a pressure of 4.5 bars. A slow downward movement of the piston can be approximated as

(a) isobaric process
(b) adiabatic process
(c) throttling process
(d) isothermal process

84. Availability of a system at any given state is

(a) a property of the system
(b) the maximum work obtainable as the system goes to dead state
(c) the total energy of the system
(d) the maximum useful work obtainable as the system goes to dead state

85. A heat reservoir at 900 K is brought into contact with the ambient at 300 K for a short time. During this period, 9000 kJ of heat is lost by the heat reservoir. The total loss in availability due to this process is

(a) 18000 kJ
(b) 9000 kJ
(c) 6000 kJ
(d) None of the above

86. For a reversible power cycle, the operating temperature limits are 800 K and 300 K. It takes 400 kJ of heat. The unavailable work will be

(a) 250 kJ (b) 150 kJ
(c) 120 kJ (d) 100 kJ

87. The loss of available energy associated with the transfer of 1000 kJ of heat from a constant temperature system at 600 K to another 400 K when the environmental temperature is 300 K is

(a) 166.67 kJ (b) 250 kJ
(c) 500 kJ (d) 750 kJ

88. In a reversible cycle, the source temperature is 227°C and the sink temperature is 27°C. The maximum available work for a heat input of 100 kJ will be

(a) 100 kJ (b) 60 kJ
(c) 40 kJ (d) 88 kJ

89. What will be the loss of available energy associated with the transfer of 1000 kJ of heat from constant temperature system at 600 K to another at 400 K when the environment temperature is 300 K?

(a) 150 kJ (b) 250 kJ
(c) 500 kJ (d) 700 kJ

90. Which thermodynamic property is evaluated with the help of Maxwell equations from the data of other measurable properties of a system?

(a) Enthalpy (b) Entropy
(c) Latent heat (d) Specific heat

91. A gas having a negative Joule-Thompson coefficient ($\mu < 0$), when throttled, will

(a) become cooler
(b) become warmer
(c) remain at the same temperature
(d) either be cooler or warmer depending on the type of gas

92. Consider the following:

1. Air
2. Gaseous combustion products
3. Steam

Which of these are pure substances, assuming there is no phase change?

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

93. Which one of the following statements is correct?

(a) Compressibility factor is unity for ideal gases
(b) Compressibility factor is zero for ideal gases
(c) Compressibility factor is lesser than unity for ideal gases
(d) Compressibility factor is more than unity for ideal gases

94. Three moles of an ideal gas are compressed to half the initial volume at a constant temperature of 300 K. The work done in the process is

(a) 5188 J (b) 2500 J
(c) -2500 J (d) -5188 J

95. The ratio c_p/c_v for a gas with n degrees of freedom is equal to

(a) $n+1$ (b) $n-1$
(c) $2/n-1$ (d) $1+2/n$

96. When a gas is heated at constant pressure, the percentage of heat supplied which goes the internal energy of the gas is

(a) more for a diatomic gas than for monoatomic gas
(b) same for monoatomic, diatomic, and triatomic gases but less than 100%
(c) 100% for all gases
(d) less for triatomic gas than for a diatomic gas

97. Which of the following represents the real gas law?

(a) $nRT = \left(p+\frac{a}{v^2}\right)(v-b)$

(b) $nRT = \left(p-\frac{a}{v^2}\right)(v+nb)$

(c) $nRT = \left(p-\frac{a}{v^2}\right)(v-b)$

(d) $nRT = \left(p+\frac{a}{v^2}\right)(v-nb)$

98. The specific heats of an ideal gas depend on its

(a) temperature
(b) pressure
(c) volume
(d) molecular weight and structure

99. The entropy of a mixture of pure gases is the sum of the entropy of constituents evaluated at

(a) temperature and pressure of the mixture
(b) temperature of the mixture and the partial pressure of the constituents
(c) temperature and volume of mixture
(d) pressure and volume of the mixture

100. The van der Waals equation of state for non-ideal gases differs from the ideal gas in that it accounts for?

1. The mass of each molecule of the gas
2. The volume of each molecule of the gas
3. The attractive forces between molecules of the gas.

(a) 1 and 3 only (b) 2 and 3 only
(c) 1 and 2 only (d) 1, 2, and 3

101. Which of the following statements about van der Waals equation is valid?

(a) It is valid for all pressures and temperatures.
(b) It represents a straight line on pv versus v plot.
(c) It has three roots of identical values at the critical point.

(d) The equation is valid for diatomic gases only

102. If a real gas obeys the Clausius equation of state $p(v-b) = RT$, then

(a) $(\partial u/\partial v)_T \neq 0$ (b) $(\partial u/\partial v)_T = 0$
(c) $(\partial u/\partial v)_T = 1$ (d) $(\partial u/\partial v)_T = 1/p$

103. Molar specific heat of an ideal gas depends on

(a) its pressure
(b) its temperature
(c) both its pressure and temperature
(d) the number of atoms in a molecule

104. The Mach number of nitrogen flowing at 195 m/s when the pressure and temperature in the undisturbed flow are 690 kN/m^2 absolute and 93°C, respectively, will be

(a) 0.25 (b) 0.50
(c) 0.66 (d) 0.75

105. In a reciprocating compressor, the compression work per kg of air

(a) increases as clearance volume increases
(b) decreases clearance volume increases
(c) is independent of clearance volume
(d) increases with clearance volume only for multistage compressors

106. A single-acting two-stage compressor with complete intercooling delivers air at 16 bar. Assuming an intake state of 1 bar at 15°C, the pressure ratio per stage is

(a) 16 (b) 8
(c) 4 (d) 2

107. Which of the following statements does not apply to the volumetric efficiency of a reciprocating air compressor?

(a) It decreases with increase in inlet temperature
(b) It increases with decrease in pressure ratio
(c) It increases with decrease in clearance ratio
(d) It decreases with increase in clearance to stroke ratio

108. The clearance volume of a reciprocating compressor reflects

(a) piston speed
(b) noise level
(c) volumetric efficiency
(d) temperature of air after compression

109. For two-stage compressor in which index of compression for low pressure stage is m and for high pressure stage is n, the load sharing with perfect intercooling is expressed as

(a) $\dfrac{W_1}{W_2} = \dfrac{m(n-1)}{n(m-1)}$ (b) $\dfrac{W_1}{W_2} = \dfrac{n(n-1)}{m(m-1)}$

(c) $\dfrac{W_1}{W_2} = \dfrac{m}{n}$ (d) $\dfrac{W_1}{W_2} = \dfrac{n-1}{m-1}$

110. In a two-stage reciprocating compressor, the compression from p_1 to p_2 is with perfect intercooling and no pressure loss, and also follows the same polytropic process. If atmospheric pressure is p_a, then the intermediate pressure p_2 is given by

(a) $p_2 = (p_1 + p_3)/2$ (b) $p_2 = \sqrt{p_1 p_2}$
(c) $p_2 = p_a p_3/p_1$ (d) $p_2 = p_a\sqrt{p_3/p_1}$

111. A four-stage compressor with perfect intercooling between stages, compresses air from 1 bar to 16 bar. The optimum pressure in the last intercooler will be

(a) 6 bar (b) 8 bar
(c) 10 bar (d) 12 bar

112. If n is the polytropic index of compression and p_2/p_1 is the pressure ratio for a three-stage compressor with ideal intercooling, the expression for total work of three stages is

(a) $\dfrac{3n}{n-1} p_1 v_1 \left\{ \left(\dfrac{p_2}{p_1}\right)^{(n-1)/n} - 1 \right\}$

(b) $\dfrac{n}{n-1} p_1 v_1 \left\{ \left(\dfrac{p_2}{p_1}\right)^{(n-1)/(3n)} - 1 \right\}$

(c) $\dfrac{n}{n-1} p_1 v_1 \left\{ \left(\dfrac{p_2}{p_1}\right)^{(n-1)/n} - 1 \right\}$

(d) $\dfrac{3n}{n-1} p_1 v_1 \left\{ \left(\dfrac{p_2}{p_1}\right)^{(n-1)/(3n)} - 1 \right\}$

113. The volumetric efficiency of a compressor depends upon

1. clearance volume
2. pressure ratio

3. index of expansion

Of these statements

(a) 1 and 2 are correct
(b) 1 and 3 are correct
(c) 2 and 3 are correct
(d) 1, 2 and 3 are correct

114. Which of the following statements are correct for multi-staging in a reciprocating air compressor?

1. It decreases the volumetric efficiency.
2. The work done can be reduced.
3. A small high-pressure cylinder is required.
4. The size of flywheel is reduced.

Select the correct answer using the codes given below :

(a) 1, 2 and 3 (b) 2, 3 and 4
(c) 1, 3 and 4 (d) 1, 2 and 4

115. In a single stage reciprocating air compressor, the work done on air to compress it from suction pressure to delivery pressure will be minimum when the compression is

(a) isothermal process
(b) adiabatic process
(c) polytropic process
(d) constant pressure process

116. Phenomenon of choking in compressors means

(a) no flow of air
(b) fixed mass flow rate regardless of pressure ratio
(c) reducing mass flow rate with increase in pressure ratio
(d) increased inclination of chord with air stream

117. Consider the following statements:

1. Zeroth law of thermodynamics is related to temperature.
2. Entropy is related to first law of thermodynamics.
3. Internal energy of an ideal gas is a function of temperature and pressure.
4. Van der Waals equation is related to an ideal gas

Which of the above statements is/are correct?

(a) 1 only (b) 2, 3 and 4
(c) 1 and 3 (d) 2 and 4

118. Match the following and chose the correct option from the codes below.

	List I		List II
A.	Irreversibility	1.	Mechanical equivalent
B.	Joule–Thomson experiment	2.	Thermodynamic temperature scale
C.	Joule's experiment	3.	Throttling process
D.	Reversible engine	4.	Loss of availability

(a) A-1, B-2, C-3, D-4
(b) A-1, B-2, C-4, D-3
(c) A-4, B-3, C-2, D-1
(d) A-4, B-3, C-1, D-2

119. In a gas turbine cycle, the turbine output is 600 kJ/kg, the compressor work is 400 kJ/kg and the heat supplied is 1000 kJ/kg. The thermal efficiency of this cycle is

(a) 80% (b) 60%
(c) 40% (d) 20%

120. The use of regenerator in a gas turbine cycle

(a) increases efficiency but has no effect on output
(b) increases output but has no effect on efficiency
(c) increases both efficiency and output
(d) increases efficiency but decreases output

121. Intercooling in gas turbine

(a) decreases net output but increases thermal efficiency
(b) increases net output but decreases thermal efficiency
(c) decreases net output and thermal efficiency
(d) increases net output and thermal efficiency

122. In a gas turbine, efficiency depends on

(a) pressure ratio only
(b) temperature of air only
(c) calorific value of fuel
(d) all of the above

123. If the maximum temperature is T_3 and minimum temperature is T_1 then the optimum pressure ratio in a gas turbine is equal to

(a) $(T_3/T_1)^{\frac{\gamma}{\gamma-1}}$ (b) $(T_3/T_1)^{\frac{\gamma-1}{2\gamma}}$
(c) $(T_3/T_1)^{\frac{\gamma}{2(\gamma-1)}}$ (d) $(T_3/T_1)^{\frac{2(\gamma-1)}{\gamma}}$

124. A gas turbine cycle with infinitely large number of stages during compression and expansion leads to

(a) Stirling cycle (b) Atkinson cycle
(c) Ericsson cycle (d) Brayton cycle

125. In an axial flow gas turbine, the hot gases approach the rotor inlet with an absolute velocity of 600 m/s in a direction 30° from the wheel tangent. The gases leave the rotor axially. If the blade speed is 300 m/s, then the theoretical power output per kg/s of gas flow will be approximately

(a) 132 kW (b) 156 kW
(c) 172 kW (d) 205 kW

126. A gas turbine power plant has a specific output of 350 kJ/kg and an efficiency of 34%. A regenerator is installed and the efficiency increases to 51%. The specific output will be closest to

(a) 350 kJ/kg (b) 468 kJ/kg
(c) 525 kJ/kg (d) 700 kJ/kg

127. A gas turbine works on which one of the following cycles?

(a) Brayton (b) Rankine
(c) Stirling (d) Otto

128. The efficiency of a simple gas turbine can be improved by using a regenerator because

(a) work of compression is reduced
(b) heat required to be supplied is reduced
(c) work output of the turbine in increased
(d) heat rejected is increased

129. Thermal efficiency of gas turbine cycle with regeneration in terms of T_3 (maximum temperature), T_1 (minimum temperature), r_p (pressure ratio) and γ (ratio of specific heats) is given by

(a) $1-(T_1/T_3)\,r_p^{\gamma/(\gamma-1)}$
(b) $1-(T_3/T_1)\,r_p^{\gamma/(\gamma-1)}$
(c) $1-(T_3/T_1)\,r_p^{(\gamma-1)/\gamma}$
(d) $1-(T_1/T_3)\,r_p^{(\gamma-1)/\gamma}$

130. When the Brayton cycle working in the pressure limits of p_1 and p_2 is reversed and operated as a refrigerator, what is the ideal value of COP for such a cycle?

(a) $(p_2/p_1)^{\gamma-1}$
(b) $(p_2/p_1)^{\gamma-1}-1$
(c) $(p_2/p_1)^{(\gamma-1)/\gamma}-1$
(d) none of the above

131. Increasing the number of reheating stages in a gas turbine to infinity makes the expansion tending:

(a) reversible adiabatic
(b) isothermal
(c) isobaric
(d) adiabatic

NUMERICAL ANSWER QUESTIONS

1. The emf in a thermocouple with the test junction at T°C on gas thermometer scale and reference junction at ice point is given by

$$\varepsilon = 0.2T - 5\times10^{-4}T^2 \text{ mV}$$

The mV is calibrated at ice and steam points. Calculate the emf reading at 50°C and corresponding reading of this thermometer where the gas thermometer reads 50°C.

2. In a manufacturing plant, it is required to melt iron at the rate of 1.4 kg/s, from initial temperature 25°C to molten metal at 1700° C. For this, an electric furnace of efficiency 80% is to be used. The melting point of iron is 1535°C, latent heat is 270 kJ/kg and the specific heats in solid and liquid states are 0.502 kJ/kg K in liquid state 0.518 kJ/kgK, respectively. The density of molten metal is 6900 kg/m^3. The furnace volume should be three times the flow rate of the metal per hour. Calculate the kW rating of the electric furnace.

3. A system composed of 5 kg of a substance expands from an initial pressure 500 kPa and a volume 0.3 m^3 to a final pressure 101 kPa for which $pv^{1.2}$ =constant. The internal energy (kJ) of the system is found to be a function of pressure (kPa) and volume (m^3) of the substance as

$$u = 3.6pv + 90$$

The expansion is quasi-static process. Determine the net change in internal energy of the system and net heat transfer to the system.

4. A reversible heat engine operates between two reservoirs at temperatures of 627°C and 47°C. The engine is used to drive a reversible refrigerator which operates between reservoirs at temperatures 47°C and −23°C. The heat transfer to the engine is 2400 kJ and the net work output of the combined plant is 450 kJ. Estimate the cooling effect of the refrigerant and the net heat transfer to the reservoir at 47°C.

5. A heat pump is to be used to heat a house in winter and then reversed to cool the house in summer. The interior temperature is to be maintained at 25°C. Heat transfer through the walls and roof is estimated to be 0.525 kW per degree temperature difference between the inside and outside. If the outside temperature in winter is 8°C, what is the minimum power required to drive the heat pump? If the power output is the maintained at this level, what is the maximum outer temperature for which the inside temperature can be maintained in summer?

6. One kg of water is to be heated from initial temperature 25°C to 100°C by using thermal reservoirs. The specific heat of water is 4.186 kJ/kg. Calculate the entropy change of the universe if heating is done in single stage from 25°C to 100°C by a reservoir at temperature 100°C. What is the entropy change of the universe if heating is done in first by contact with thermal reservoir at 62°C and then by another reservoir at temperature 100°C?

7. A copper rod has length 2 m and diameter 0.02 m. One end of the rod is at 150°C and the other end is at 10°C. The rod is perfectly insulated along its length and the thermal conductivity of copper is 380 W/mK. Calculate the rate of heat transfer along the rod. Also calculate the rate of entropy production due to irreversibility of the heat transfer.

8. A thermally insulated 100-ohm resistor carries a current of 2 A for 1 s. The initial temperature of the resistor is 10°C, its mass is 10 g and its specific heat is 0.85 J/gK. Calculate the changes in entropy of the resistor and the universe, separately.

9. 15 kg of water at 85°C is mixed with 25 kg of water at 35°C, the pressure being taken as constant and the temperature of the surroundings being maintained at 15°C. Specific heat of water is 4.186 kJ/kgK. Determine the temperature of water after mixing, and the decrease in available energy.

10. In an air preheater, the combustion products are to be cooled from from 300°C to 200°C at the rate of flow of 12.5 kg/s. The air enters at 40°C at a flow rate of 11.5 kg/s. The atmospheric temperature is 27°C. For the air, $c_p = 1.005$ kJ/kg and for combustion products, $c_p = 1.09$ kJ/kg K. Calculate the change in the availability of the products and the irreversibility of the process.

11. One kg of an ideal gas undergoes an reversible adiabatic process from initial pressure of 300 kPa, temperature 80°C and volume 0.28 m^3 to a final pressure of 350 kPa and final volume 0.4 m^3. The work done on the gas is 100 kJ. Calculate the increase in the entropy of the gas.

12. A gas turbine plant is based on Brayton cycle in which the air enters to the compressor at 0.1 MPa and 25°C. The pressure ratio is 6 and maximum cycle temperature is 875°C. The isentropic efficiency of both the turbine and compressor is 80%. For air, $\gamma = 1.4$. Determine the cycle efficiency of the plant. Also calculate the percentage improvement in the cycle efficiency if a generator of 75% effectiveness is employed.

13. A heat source at 627°C transfers heat at the rate of 50 kW to a system maintained at 287°C. Atmospheric temperature is 27°C. These temperatures to remain constant during the process. Calculate the rate of entropy production and the rate of irreversibility in the process.

14. The air is to be compressed from 1 bar and 25°C to 75 bar at the rate of 5 m^3/min through a two-stage air compressor working under perfect intercooling. The speed of compressor is 300 rpm. The polytropic index of expansion in both the stages is 1.25. If mechanical efficiency is 75%, what is the power requirement of the compressor? If average piston speed is 3 m/s, and volumetric efficiency is 80% for both stages, what will be the diameter of low pressure cylinder?

15. A two-stage air compressor with perfect cooling takes in air at 1 bar and 27°C to 9 bar with index of compression 1.3 in both the stages. The mass flow rate of air is 0.5 kg/s. For air, $R = 0.287$ J/kgK, $c_p = 1.005$ kJ/kgK. Calculate the power requirement of the compressor and the rate of heat rejection in intercooler.

ANSWERS

Multiple Choice Questions

1. (b)	**2.** (c)	**3.** (a)	**4.** (d)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (d)
11. (d)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (d)
21. (d)	**22.** (c)	**23.** (d)	**24.** (c)	**25.** (b)	**26.** (c)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (b)
31. (a)	**32.** (a)	**33.** (a)	**34.** (d)	**35.** (c)	**36.** (a)	**37.** (c)	**38.** (c)	**39.** (b)	**40.** (d)
41. (d)	**42.** (c)	**43.** (d)	**44.** (b)	**45.** (d)	**46.** (d)	**47.** (d)	**48.** (d)	**49.** (d)	**50.** (d)
51. (c)	**52.** (b)	**53.** (c)	**54.** (d)	**55.** (b)	**56.** (a)	**57.** (c)	**58.** (d)	**59.** (c)	**60.** (b)
61. (c)	**62.** (c)	**63.** (d)	**64.** (c)	**65.** (d)	**66.** (b)	**67.** (b)	**68.** (b)	**69.** (d)	**70.** (c)
71. (b)	**72.** (b)	**73.** (d)	**74.** (b)	**75.** (d)	**76.** (c)	**77.** (b)	**78.** (c)	**79.** (c)	**80.** (a)
81. (b)	**82.** (a)	**83.** (a)	**84.** (d)	**85.** (d)	**86.** (b)	**87.** (b)	**88.** (c)	**89.** (b)	**90.** (b)
91. (b)	**92.** (c)	**93.** (a)	**94.** (d)	**95.** (d)	**96.** (a)	**97.** (d)	**98.** (d)	**99.** (b)	**100.** (d)
101. (c)	**102.** (b)	**103.** (d)	**104.** (b)	**105.** (c)	**106.** (c)	**107.** (a)	**108.** (c)	**109.** (a)	**110.** (b)
111. (b)	**112.** (d)	**113.** (d)	**114.** (b)	**115.** (a)	**116.** (b)	**117.** (a)	**118.** (d)	**119.** (d)	**120.** (a)
121. (b)	**122.** (a)	**123.** (c)	**124.** (c)	**125.** (b)	**126.** (c)	**127.** (a)	**128.** (b)	**129.** (d)	**130.** (d)
131. (b)									

Numerical Answer Questions

1. 24 mV, 52.17°C	**2.** 1.948 MW	**3.** -122.91 kJ, 47.79 kJ
4. 3916.67 kJ, 4754.28 kJ	**5.** 539.9 W, 42.5°C	**6.** 0.098 kJ/K, 0.051 kJ/K
7. 8.36 W, 9.7×10^{-3} W/K	**8.** 1.307 J/K, 1.307 J/K	**9.** 53.75°C, 130.85 kJ
10. −46.29 kJ/kg, 324.4 kW	**11.** 0.260 kJ/K	**12.** 25.56%, 9.19%
13. 33.73 W/K, 10.12 kW	**14.** 60 kW, 300 mm	**15.** 107.7 kW, 43.5 kW

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (b) In a transient flow process, the state of matter inside the control volume varies with time.

2. (c) Pressure is an intensive property while volume is an extensive property. Thus, thermodynamic work is the product of an intensive property and change in an extensive property.

3. (a) Intensive properties are independent of the mass in the system, for example, density, pressure, temperature. Extensive properties are related to mass, for example, volume, energy.

4. (d) Intensive properties are independent of the mass in the system, for example, density, pressure, temperature. Extensive properties are related to mass, for example, volume, energy.

5. (d) Temperature and pressure are the measures of molecular moment of the system. The molecular moment of the system is zero, the absolute temperature and absolute pressure becomes zero.

6. (b) Heat and work cannot be transferred into the ice kept in a well insulated thermos flask, hence, it is an isolated system.

7. (c) Chemical equilibrium is attained when the Gibbs energy of the system is at its minimum value.

8. (c) An isolated system has no interaction with its surroundings.

9. (b) The battery is not meant to heat the system, thus, heat transfer is zero while the work transfer is equal to

$$\begin{aligned} W &= 2\times 100\times 10\times 60 \\ &= 120 \text{ kJ} \end{aligned}$$

10. (d) The change in a property of a system is independent of the path the system follows during the change of state. Therefore, properties are point functions. Work transfer is not a property of the system.

11. (d) The reading on new scale shall be given by

$$\begin{aligned} &= 100+\frac{400-100}{100-0}\times 60 \\ &= 280^\circ \text{ N} \end{aligned}$$

12. (a) Zeroth law of thermodynamics states that if a body A is in thermal equilibrium with body B and body C, then the body B and body C are also in thermal equilibrium with each other. This law is specifically applied in temperature measurements.

13. (d) The increase in temperature is given by

$$\begin{aligned} 4\times 10\times 60 &= 20\times 4\times \Delta T \\ \Delta T &= 30^\circ\text{C} \end{aligned}$$

14. (b) Using steady state energy equation for the steam, one finds

$$\begin{aligned} V_2 &= \sqrt{2\Delta h} \\ &= \sqrt{2\times 0.8\times 1000} \\ &= 40 \text{ m/s} \end{aligned}$$

15. (b) Given that

$$\begin{aligned} h_1 &= 3000 \text{ kJ/kg} \\ h_2 &= 2700 \text{ kJ/kg} \\ W &= 350 \text{ kJ/kg} \end{aligned}$$

Using steady state energy equation,

$$\begin{aligned} h_1 &= h_2+W+Q \\ 3000 &= 2700+250+Q \\ Q &= 50 \text{ kJ} \end{aligned}$$

16. (c) For the first process,

$$\begin{aligned} Q_{12} &= 80 \text{ kJ} \\ W_{12} &= 60kJ \\ \Delta E_{12} &= 80-60 \\ &= 20 \text{ kJ} \end{aligned}$$

For the returning process (sign of internal energy will be opposite.)

$$\begin{aligned} Q_{21} &= 100 \text{ kJ} \\ W_{21} &= Q_{21}-\Delta E_{21} \\ &= 100-(-20) \\ &= 120 \text{ kJ} \end{aligned}$$

17. (a) Given that

$$\begin{aligned} h_1 &= 100 \text{ kJ/kg} \\ h_2 &= 200 \text{ kJ/kg} \\ Q &= 50 \text{ kJ/kg} \\ \dot{m} &= 2 \text{ kg/s} \end{aligned}$$

The steady flow energy equation for this process can be written as

$$\begin{aligned} h_1 &= h_2+\frac{W}{\dot{m}}+Q \\ \frac{W}{2} &= 200-100+50 \\ W &= 300 \text{ kW} \end{aligned}$$

18. (a) The first law of thermodynamics is the application of the conservation of energy principle to heat and thermodynamic processes.

19. (b) Heat absorbed (negative of heat released) is given by

$$\begin{aligned} Q &= -ms\Delta T \\ &= -8400 \text{ J} \end{aligned}$$

20. (d) This is according to the first law of thermodynamics for a closed system.

21. (d) Given that

$$\begin{aligned} \frac{dW}{dt} &= 0.75 \\ E &= 25+0.25t \end{aligned}$$

Using first law of thermodynamics,

$$\begin{aligned} \frac{dQ}{dt} &= \frac{dE}{dt}+\frac{dW}{dt} \\ &= 1.00 \end{aligned}$$

22. (c) For given conditions of air compressor, steady flow energy equation is

$$h_1+W = h_2+Q$$

Per kg work required is

$$\begin{aligned} W &= (h_2-h_1)+Q \\ &= 140 \text{ kJ/kg} \end{aligned}$$

Hence, for mass flow rate of 0.5 kg/s, the work required is 70 kW.

23. (d) If a closed system within given time period takes dQ heat, does dW amount of work, and change in its internal energy is dU, then, the first law is written as

$$dQ = dW + dU$$
$$dQ - dW = dU$$

24. (c) According to the first law of thermodynamics

$$\delta Q - \delta W = \delta u$$

where internal energy u is a property of the system.

25. (b) The work done by an steady flow system is

$$W = -\int_1^2 vdp$$

26. (c) The heat rejected will be given by

$$\begin{aligned} Q_2 &= Q_1 - W \\ &= \frac{900\times 10^7}{3600\times 10^3} - 1000 \\ &= 1500 \text{ MW} \end{aligned}$$

27. (a) Given that

$$\begin{aligned} \dot{m} &= 0.70 \text{ kg/s} \\ h_1 &= 290 \text{ kJ/kg} \\ h_2 &= 450 \text{ kJ/kg} \\ V_1 &= 6 \text{ m/s} \\ V_2 &= 2 \text{ m/s} \end{aligned}$$

The work input to the compressor is given by steady flow energy equation as

$$\begin{aligned} h_1 + \frac{V_1^2}{2} + \frac{W}{\dot{m}} &= h_2 + \frac{V_2^2}{2} \\ 290 + \frac{6^2}{2} + \frac{W}{0.7} &= 450 + \frac{2^2}{2} \\ W &= 100.8 \text{ kW} \end{aligned}$$

28. (c) The enthalpy of the fluid before throttling is equal to the enthalpy of the fluid after throttling. The drop in pressure is compensated by corresponding change in density or internal energy of the flowing fluid.

29. (b) The work done by the system during the process is

$$\begin{aligned} W &= \frac{10+5}{2}\times 1 + (3-1)\times 5 \\ &= 7.5 + 10 \\ &= 17.5 \text{ J} \end{aligned}$$

Therefore, heat absorbed is

$$\begin{aligned} Q &= W + du \\ &= 17.5 + 2.5 \\ &= 20 \text{ J} \end{aligned}$$

30. (b) Given that

$$\begin{aligned} T_1 &= 600 + 273 \\ &= 873 \text{ K} \\ T_2 &= 20 + 273 \\ &= 293 \text{ K} \end{aligned}$$

Least rate of heat rejection per kW net output of the engine is

$$\begin{aligned} \frac{Q_2}{W} &= \frac{T_2}{T_1 - T_2} \\ &= 0.505 \text{ kW} \end{aligned}$$

31. (a) The efficiency of heat engine is given by

$$\begin{aligned} \eta &= \frac{54}{66+54} \\ &= 0.45 \end{aligned}$$

32. (a) Given that

$$\begin{aligned} W &= 50 \text{ kJ} \\ \eta &= 0.75 \end{aligned}$$

Heat rejected per cycle is determined as

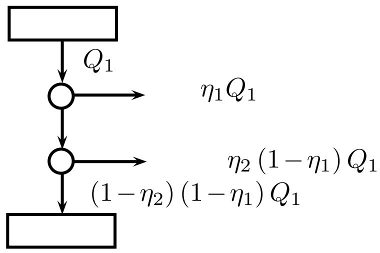
$$\begin{aligned} Q_2 &= Q_1 - W \\ &= \frac{W}{\eta} - W \\ &= \left(\frac{1}{\eta} - 1\right) W \\ &= \left(\frac{1}{0.75} - 1\right)\times 50 \\ &= 16.67 \text{ kJ} \end{aligned}$$

33. (a) According to Clausius inequality or entropy principle

$$\int \frac{dQ}{T} \geq 0$$

This principal can be used to find whether a process is feasible or not.

34. (d) The coupled cycle is shown below in the figure.



The heat input to the cycle is Q_1 and heat rejected to from the cycle is

$$Q_2 = (1-\eta_1)(1-\eta_2)Q_1$$

Therefore, the overall efficiency of the coupled cycles is given by

$$\begin{aligned}\eta_o &= 1-\frac{Q_2}{Q_1}\\ &= 1-(1-\eta_1)(1-\eta_2)\\ &= 1-(1-0.3)(1-0.2)\\ &= 1-0.7\times 0.8\\ &= 44\%\end{aligned}$$

35. (c) The thermal efficiency of the engine is calculated as

$$\begin{aligned}\eta &= 1-\frac{Q_2}{Q_1}\\ &= 1-\frac{70}{80}\\ &= 12.5\%\end{aligned}$$

36. (a) For ideal gas, the throttling is an isothermal process because enthalpy (c_pT) remains constant. The universal gas constant is $\Re = 8.314$ kJ/kmole-K. There is no heat transfer to the surrounding, hence, the entropy change of the universe is

$$\begin{aligned}Tds &= c_pdT - T\left(\frac{\partial v}{\partial T}\right)_p dp\\ \delta s &= -\Re \ln\frac{p_2}{p_1}\\ &= -8.314\ln\frac{0.1}{0.5}\\ &= 13.38\text{kJ/K}\end{aligned}$$

37. (c) Given that

$$\begin{aligned}T_2 &= -3+273\\ &= 270\text{ K}\\ T_1 &= 27+273\\ &= 300\text{ K}\\ Q_2 &= \frac{5\times 10^6}{3600}\\ &= 1.3889\text{ MW}\end{aligned}$$

Therefore, minimum power requirement is given by

$$\begin{aligned}W &= \frac{Q_2}{T_2}(T_1-T_2)\\ &= 154.3\text{ kW}\end{aligned}$$

38. (c) Given that $\eta_e = 0.3$. The COP of the refrigerator is

$$\begin{aligned}\text{COP}_r &= \frac{1}{\eta_e - 1}\\ &= \frac{1}{0.3}-1\\ &= 2.33\end{aligned}$$

39. (b) Thermal efficiency is given by

$$\begin{aligned}\eta &= \frac{3\times 10^3}{1\times 10^4}\\ &= 0.3\\ &= 30\%\end{aligned}$$

40. (d) Minimum heat rejected for reversible cycle is given by

$$\begin{aligned}Q_2 &= \frac{Q_1\times T_2}{T_1}\\ &= \frac{250\times(27+273)}{227+273}\\ &= 150\text{ kJ/s}\end{aligned}$$

41. (d) The pressure–volume relationship represents work transfer while temperature entropy relationship represents heat transfer. Both heat and work transfers are forms of energy transfer.

42. (c) According the Clausius theorem, for a cycle composed of reversible processes:

$$\oint_R \frac{dQ}{T} = 0$$

where Q and T are the heat exchange and temperature, respectively, at any point in the cycle

43. (d) Given that

$$\begin{aligned}Q_2 &= 1000\text{ kJ}\\ T_1 &= 750\text{ K}\\ T_2 &= 400\text{ K}\end{aligned}$$

Hence,

$$\begin{aligned}Q_1 &= Q_2\frac{T_1}{T_2}\\ &= 1875\text{ kJ}\end{aligned}$$

44. (b) At absolute zero temperature (0 K), entropy is zero.

45. (d) It is proved in thermodynamics that entropy always increases in mixing of two fluids.

46. (d) COP for heat pump is

$$\begin{aligned}\text{COP}_p &= \frac{T_1}{T_1 - T_2}\\ 5 &= \frac{1}{1 - T_2/T_1}\\ 1 - \frac{T_2}{T_1} &= 1/5\\ \frac{T_2}{T_1} &= 1 - 1/5\\ &= 4/5\end{aligned}$$

COP for refrigerator is

$$\begin{aligned}\text{COP}_r &= \frac{T_2}{T_1 - T_2}\\ \frac{Q_2}{W} &= \frac{1}{T_1/T_2 - 1}\\ &= \frac{1}{5/4 - 1}\\ &= 4\end{aligned}$$

Therefore, for work input of 1 kW, refrigeration effect will be 4 kW.

47. (d) The given condition means equal efficiency of both engines, therefore,

$$\begin{aligned}\frac{1600 - T_2}{1600} &= \frac{T_2 - 400}{T_2}\\ T_2 &= 800 \text{ K}\end{aligned}$$

48. (d) Given that

$$\begin{aligned}T_1 &= 12 + 273\\ &= 285 \text{ K}\\ T_2 &= 2 + 273\\ &= 275 \text{ K}\\ Q_1 &= 57 \text{ J/cycle}\end{aligned}$$

Hence, the cycle efficiency is calculated as

$$\begin{aligned}\eta &= \frac{T_1 - T_2}{T_1}\\ \frac{W}{57} &= \frac{10}{285}\\ W &= 2 \text{ J}\end{aligned}$$

Per second rate is $2 \times 1080/60 = 36$ W.

49. (d) Given that

$$\begin{aligned}T_1 &= 40°\text{C}\\ &= 271 \text{ K}\\ T_2 &= -2°\text{C}\\ &= 271 \text{ K}\\ Q_2 &= 30 \text{ kW}\end{aligned}$$

The COP of working plant is given by

$$\begin{aligned}\text{COP} &= \frac{Q_2}{W}\\ &= \frac{1}{4}\left(\frac{T_2}{T_1 - T_2}\right)\\ W &= 18.6 \text{ kW}\end{aligned}$$

50. (d) For an irreversible process, entropy change is always higher than entropy change in a reversible process between same end points,

$$\int_i^f \frac{dQ_{irr}}{T} \leq (S_f - S_i)$$

For given case

$$\begin{aligned}\int_i^f \frac{dQ_{irr}}{T} &= 100\,(0.4 - 0.3)\\ &= 10 \text{ kJ/K}\\ (S_f - S_i) &= 75 - 80\\ &= 5 \text{ kJ/K}\end{aligned}$$

Hence

$$\int_i^f \frac{dQ_{irr}}{T} \geq (S_f - S_i)$$

Therefore, the process itself is impossible.

51. (c) Given that

$$\begin{aligned}Q_2 &= 2 \text{ kW}\\ T_1 &= 300 \text{ K}\\ T_2 &= 200 \text{ K}\end{aligned}$$

The COP is given by

$$\begin{aligned}\text{COP} &= \frac{T_2}{T_1 - T_2}\\ &= 2\end{aligned}$$

Power consumption of the cycle is

$$\begin{aligned}W &= \frac{Q_2}{T_2} \times (T_1 - T_2)\\ &= 1 \text{ kW}\end{aligned}$$

52. (b) The Clausius theorem states for a reversible process

$$\oint_R \frac{dQ}{T} = 0$$

but for an irreversible process, entropy change is always higher than the entropy change in a reversible process between same end points. So,

$$\int_i^f \frac{dQ_{irr}}{T} \leq (S_f - S_i)$$

Therefore, by merging above two equations, one can write

$$\oint \frac{dQ}{T} \leq 0$$

Above equation is known as Clausius inequality or entropy principle.

53. (c) For equal efficiencies,

$$1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$$
$$T_2 = \sqrt{T_1 T_3}$$

54. (d) Given that

$$\text{COP} = 2$$
$$Q_2 = 1200 \text{ kJ/min}$$

Hence,

$$W = \frac{Q_2}{\text{COP}} = 600 \text{ kJ/min} = 10 \text{ kW}$$

55. (b) Given that

$$T_1 = 327 + 273 = 600 \text{ K}$$
$$T_2 = 27 + 273 = 300 \text{ K}$$
$$W = 300 \text{ kJ}$$

Heat addition is given by

$$Q_1 = \frac{T_1}{T_1 - T_2} \times W = 600 \text{ kJ}$$

Entropy change during heat addition is

$$\Delta S_1 = \frac{Q_1}{T_1} = 1 \text{ kJ/K}$$

56. (a) The expression of entropy change is true only for reversible processes.

57. (c) The efficiency of any reversible engine cannot be more than that of Carnot cycle. For the given case, the Carnot cycle efficiency is

$$\eta_c = 1 - \frac{27 + 273}{327 + 273} = 0.5$$

while efficiency of the given engine is

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{1 - 0.45}{1} = 0.55$$

Since $\eta > \eta_c$, it is an impossible engine.

58. (d) The efficiency of reversible engine is independent of the working fluid.

59. (c) For equal amount of work, the intermediate temperature will be given by

$$T_m = \frac{T_1 + T_2}{2} = 100^\circ\text{C}$$

60. (b) The source temperature will be given by

$$\frac{T_1}{27 + 273} = \frac{Q_1}{0.4 Q_1}$$
$$T_1 = 477^\circ\text{C}$$

61. (c) The efficiency of the given reversible heat engine is $\eta = 0.5$. The COP of the heat pump from this reversible cycle will be

$$\text{COP} = \frac{1}{\eta} = 2$$

62. (c) Given that $\eta_{HE} = 0.3$. The COP of the heat pump made of a reversed heat engine is given by

$$\text{COP} = \frac{1}{\eta_{HE}} = 3.33$$

63. (d) Increase in entropy of a system represents degradation of energy.

64. (c) The COP of refrigerator is given by

$$\text{COP} = \frac{1}{\eta} - 1 = \frac{1}{0.24} - 1 = 3.166$$

65. (d) The performance parameter for a refrigerator cycle, called the coefficient of performance (COP),

is defined as the ratio of desired thermal effect and work input to the system. It is expressed in terms of lower and higher temperatures as

$$\text{COP} = \frac{T_2}{T_1 - T_2}$$

Hence, COP can be increased by reducing the higher temperature (T_1).

66. (b) Given that

$$\begin{aligned} T_1 &= 27+273 \\ &= 300 \text{ K} \\ T_2 &= -13+273 \\ &= 260 \text{ K} \end{aligned}$$

Hence,

$$\begin{aligned} \text{COP} &= \frac{T_2}{T_1 - T_2} \\ &= 7.5 \end{aligned}$$

67. (b) Heat transfer Q in a polytropic process can be represented in terms of work transfer W as

$$Q = \frac{\gamma - n}{\gamma - 1} W$$

68. (b) For steady flow process (e.g. in reciprocating air compressors), the work transfer is $\int vdp$.

69. (d) For reversible isothermal processes,

$$\int pdv = -\int vdp$$

70. (c) By definition of specific heats, one gets

$$\begin{aligned} c_p &= T\left(\frac{\partial s}{\partial T}\right)_p \\ c_v &= T\left(\frac{\partial s}{\partial T}\right)_v \end{aligned}$$

Hence,

$$\begin{aligned} T\left(\frac{\partial s}{\partial T}\right)_p - T\left(\frac{\partial s}{\partial T}\right)_v &= c_p - c_v \\ &= R \end{aligned}$$

71. (b) Given that

$$\begin{aligned} T_2 &= 3T_1 \\ v_2 &= 3v_1 \end{aligned}$$

As $T \propto v$, it is a constant pressure process for which the entropy change is

$$\begin{aligned} \Delta s &= c_p \ln \frac{T_2}{T_1} \\ &= c_p \ln 3 \end{aligned}$$

72. (b) Constant volume processes are known as isochoric processes, therefore, the work done during isochoric process is zero. Hence, in such processes, heat flow is equal to the change in internal energy.

73. (d) For isothermal expansion, change in internal energy is zero, and hence, heat input is equal to work output.

74. (b) In an isobaric process ($dp = 0$),

$$\begin{aligned} dpv &= pdv + vdp \\ &= pdv \end{aligned}$$

Therefore, one can write

$$\begin{aligned} dQ_p &= du + pdv \\ &= du + d(pv) \\ &= d(u + pv) \\ &= dh \end{aligned}$$

Hence, heat transfer at constant pressure increases the enthalpy of the a system with equal amount.

75. (d) The first equation indicates irreversible process, but as $\Delta s > 0$, therefore, it is neither adiabatic nor isothermal, hence, it is isobaric.

76. (c) Reversible adiabatic processes are isentropic, that is, the entropy change of the system and the universe is zero.

77. (b) In constant volume process, the work done by the system is zero. Therefore, using the first law of thermodynamics, change in internal energy is equal to the heat transfer in the process.

78. (c) The enthalpy is defined as

$$dh = c_p dt$$

Thus, change in enthalpy is equal to the heat transfer in constant pressure process.

79. (c) The volume of the pressure cooker remains constant. Hence, the steaming of food is a constant volume process known as isochoric process.

80. (a) In constant volume process, the work done is zero, hence, heat transfer is equal to the change in internal energy of the system.

81. (b) For an ideal gas in constant pressure process

$$\begin{aligned} T &\propto v \\ T_2 &= 3 \times (27 + 273) \\ &= 900 \text{ K} \\ &= 627^\circ\text{C} \end{aligned}$$

82. (a) The work done in isothermal process is given by

$$W = p_1 v_1 \ln\left(\frac{v_2}{v_1}\right)$$
$$= 32.188 \text{ kW}$$

83. (a) The work is done at a constant pressure 4.5 bar.

84. (d) Availability of a given system is defined as the maximum useful work that is obtainable in a process in which the system comes to the equilibrium with its surroundings (i.e. dead state).

85. (d) As there is no change in temperature of the heat reservoir, the availability remains constant.

86. (b) The unavailable work is given by

$$Q_2 = 400 \times \frac{300}{800}$$
$$= 150 \text{ kJ}$$

87. (b) The loss of available energy in the process is

$$\Delta s = 1000 \times \left(\frac{1}{400} - \frac{1}{600}\right) \times 300$$
$$= 250 \text{ kJ}$$

88. (c) Maximum available work is given by

$$W = Q_1 \times \frac{T_2 - T_1}{T_1}$$
$$= 100 \times \frac{227 - 27}{227 + 273}$$
$$= 40 \text{ kJ}$$

89. (b) Loss of available energy will be given by

$$W_1 = Q_1\left(1 - \frac{T_0}{T_1}\right)$$
$$W_2 = Q_1\left(1 - \frac{T_0}{T_2}\right)$$

Hence,

$$W_1 - W_2 = Q_1 T_0 \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$
$$= 250 \text{ kJ}$$

90. (b) Maxwell equations are the relations of entropy with measurable properties pressure, temperature and volume.

91. (b) The region inside the inversion curve where μ_J is positive is called the cooling region and outside region where μ_J is negative is called heating region.

92. (c) A pure substance is defined as one that is homogeneous and invariable in chemical composition throughout its mass, even when processed, for example, atmospheric air, steam-water mixture, combustion products of a fuel.

93. (a) For ideal gases, the compressibility factor $(pv/n\Re T)$ is unity.

94. (d) Given that

$$T_1 = T_2$$
$$= 300 \text{ K}$$
$$n = 3$$

The work done is

$$W = nRT_1 \ln\frac{V_2}{V_1}$$
$$= 3 \times 8.314 \times 300 \ln\frac{1}{2}$$
$$= -5186 \text{ J}$$

95. (d) The heat capacity ratio (γ) for an ideal gas can be related to the degrees of freedom (n) of a molecule by:

$$\gamma = 1 + \frac{2}{n}$$

96. (a) Heat at constant pressure involves specific heat at constant pressure (c_p):

$$Q = c_p dT$$

The percentage of heat supplied that goes to internal energy is

$$r = \frac{c_v dT}{c_p dT}$$
$$= \frac{1}{\gamma}$$

For a monoatomic gas (degrees of freedom $n = 3$), $\gamma = 1.67$, while for a diatomic gas (degrees of freedom $n = 5$) $\gamma = 1.4$; Ratio of specific heat γ is less for diatomic gases, therefore, the percentage of heat transfer at constant pressure that goes as internal energy will be more for diatomic gaes than than for monoatomic gases.

97. (d) This equation is known as van der Waals equation of state.

98. (d) The specific heat of an ideal gas depends upon its molecular weight and structure.

99. (b) The entropy of a mixture of pure gases is the sum of the entropy of constituents evaluated at temperature of the mixture and the partial pressure of the constituents.

100. (d) The van der Waals equation counts for mass, volume and attractive forces of each molecule of the gas.

101. (c) Van der Waals equation has three roots of identical values at the critical point.

102. (b) For given real gas,

$$p\,(v-b) = RT$$
$$p = \frac{RT}{v-b}$$
$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

Using the first energy equation,

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p$$
$$= T\frac{R}{v-b} - \frac{RT}{v-b}$$
$$= 0$$

103. (d) The molar specific heat is the heat capacity per mole rather than per unit mass. Diatomic gases have larger molar specific heats than monoatomic gases.

104. (b) Given that

$$\mu = 28 \text{ (molecular weight)}$$
$$R = \frac{8314}{28}$$
$$= 296.95 \text{ J/kmol K}$$
$$\gamma = 1.4 \text{ (diatomic gas)}$$
$$V = 195 \text{ m/s}$$
$$p = 690 \text{ kN/m}^2$$
$$T = 93°\text{C}$$
$$= 366 \text{ K}$$

Therefore,

$$M = \frac{V}{\sqrt{\gamma RT}}$$
$$= \frac{195}{390}$$
$$= 0.5$$

105. (c) The shaft work per kg for reciprocating compressor to increase pressure from p_1 to p_2 is given by

$$\frac{W_c}{m} = -\frac{n}{n-1}RT_1\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right\}$$

which is independent of clearance volume.

106. (c) The pressure in the intermediate stage is given by

$$p_m = \sqrt{16\times 1}$$
$$= 4 \text{ bar}$$

Hence, pressure ratio per stage is 4.

107. (a) The volumetric efficiency of a reciprocating compressor is expressed as

$$\eta_v = 1 - \frac{v_c}{v_s}\left\{\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right\}$$

which is independent of the inlet temperature.

108. (c) The clearance volume of a reciprocating compressor reflects volumetric efficiency.

109. (a) For perfect cooling with different indices in two stage compressor, the intermediate pressure and ratio of work inputs are

$$\left(\frac{p_i}{p_1}\right)^{(m-1)/m} = \left(\frac{p_2}{p_i}\right)^{(n-1)/n}$$
$$\frac{W_I}{W_{II}} = \frac{m/\,(m-1)}{n/\,(n-1)}$$

where m, n are indices of compression in first and second stages respectively.

110. (b) For two-staged compression, intermediate pressure p_i is geometrical mean of suction and delivery pressures, irrespective of perfect cooling provided with the same indices of compression in both stages and minimum total work of compression,

$$p_i = \sqrt{p_1 p_2}$$

111. (b) Let pressures increase in the series, 1, p, p^2, p^3, p^4 . Therefore,

$$p^4 = 16$$
$$p = 2$$

Hence, the optimum pressure in last intercooler is

$$p^3 = 8$$

112. (d) The work required for single stage compressor is

$$W_c = -\frac{n}{n-1}p_1 v_1\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right\}$$

Hence, assuming equal work for each stage with geometric mean pressure in intermediate stages, total work for three-stage compressor is

$$W_c = -\frac{3n}{n-1}p_1 v_1\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/(3n)} - 1\right\}$$

113. (d) Volumetric efficiency of reciprocating compressors is determined as

$$\eta_v = 1 - \frac{v_c}{v_s}\left\{\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right\}$$

114. (b) Multi-staging has no effect on volumetric efficiency of reciprocating air compressors.

115. (a) Minimum work is required for isothermal compression.

116. (b) Choking or stonewalling of centrifugal compressor occurs when the compressor is operating at low discharge pressure and very high flow rates. These high flow rates at compressor choke point are actually the maximum that the compressor can push through. Any further decrease in the outlet resistance will not lead to increase in compressor output.

117. (a) Entropy is related to the second law of thermodynamics. Internal energy of an ideal gas is a function of temperature only. Van der Waals equation is related to real gas.

118. (d) Irreversibility is a measure of loss of availability. Joule–Thomson experiment is a throttling process. Joule's experiment is done to determine mechanical equivalent (work) of heat. The concept of reversible engine is used to define thermodynamic temperature scale

119. (d) Given that

$$\begin{aligned} W_t &= 600 \text{ kJ/kg} \\ W_c &= 400 \text{ kJ/kg} \\ Q_1 &= 1000 \text{ kJ/kg} \end{aligned}$$

Therefore, thermal efficiency is

$$\begin{aligned} \eta_{th} &= \frac{W_t - W_c}{Q_1} \\ &= 20\% \end{aligned}$$

120. (a) Regeneration increases cycle efficiency by using rejected heat but has no effect on output.

121. (b) Intercooling reduces the compression work, thus increasing the net output, but more heat is required at combustion chamber, thus thermal efficiency decreases.

122. (a) The efficiency of Brayton cycle ($\eta_{Brayton}$) depends upon the compression ratio (r) and γ as

$$\eta_{Brayton} = 1 - \frac{1}{{r_p}^{(\gamma-1)/\gamma}}$$

123. (c) The optimum pressure ratio for maximum value of W_{net} is given by

$$\bar{r}_p = \left(\frac{T_3}{T_1}\right)^{\gamma/(2(\gamma-1))}$$

124. (c) When the number of stages is large, then Brayton cycle tends towards Ericsson cycle.

125. (b) Theoretical power output per kg/s shall be given by

$$\begin{aligned} P &= \dot{m}\,(V_{w1})\,u \\ &= \dot{m}V_1 \cos\alpha_1 u \\ &= 1 \times 600 \times \cos 30^\circ \times 300 \\ &= 155.88 \text{ kW} \end{aligned}$$

126. (c) Given that for turbine without generator

$$\begin{aligned} w_n &= 350 \text{ kJ/kg} \\ \eta &= 34\% \end{aligned}$$

Thus, the energy input q_1 is

$$\begin{aligned} q_1 &= \frac{w_n}{\eta} \\ &= \frac{350}{0.34} \\ &= 1029.41 \text{ kJ/kg} \end{aligned}$$

For the turbine with regenerator $\eta' = 51\%$, thus, the specific output is

$$\begin{aligned} w_n' &= q_1 \times \eta' \\ &= 1029.41 \times 0.51 \\ &= 524.99 \text{ kJ/kg} \end{aligned}$$

127. (a) Brayton cycle, also known as Joule cycle, is the theoretical cycle for gas turbines.

128. (b) Regeneration in Brayton cycle is the heat addition at higher temperature. Because of regeneration, the mean temperature of heat addition increases and mean temperature of heat rejection decreases. Thus, efficiency of Brayton cycle is increased but work output remains unchanged.

129. (d) Efficiency of gas turbine cycle with regeneration is given by

$$\eta_{Brayton-reg} = 1 - \frac{T_1}{T_3}{r_p}^{(\gamma-1)/\gamma}$$

For a fixed ratio T_1/T_3, the cycle efficiency drops with increasing pressure ratio.

130. (d) The coefficient of performance of the reversed Brayton cycle is determined as

$$\text{COP} = \frac{1}{{r_p}^{(\gamma-1)/\gamma}}$$

where

$$r_p = \frac{p_2}{p_1}$$

One finds that COP decreases with increase in r_p.

131. (b) When the number of inter-cooling and reheat stages is large, then Brayton cycle tends towards Ericsson cycle, which consists of two isobaric and two isothermal processes.

Numerical Answer Questions

1. The emf reading 50°C point is

$$\begin{aligned}\varepsilon &= 0.5T - 4\times 10^{-4}T^2\\ &= 24\text{ mV}\end{aligned}$$

Given that the emf readings at ice point and steam point are

$$\begin{aligned}\varepsilon_{0^\circ} &= 0.0\text{ mV}\\ \varepsilon_{100^\circ} &= 46\text{ mV}\end{aligned}$$

Therefore, it will read

$$\begin{aligned}T &= \frac{100}{46}\times 24\\ &= 52.17^\circ\text{C}\end{aligned}$$

2. The heat required per kg of iron is

$$\begin{aligned}q &= 0.502\,(1535-25)+270+0.518\,(1700-1535)\\ &= 1113.49\text{ kW/kg}\end{aligned}$$

The rating of furnace should be

$$\begin{aligned}Q &= \frac{\dot{m}q}{\eta}\\ &= \frac{1.4\times 1113.49}{0.80}\\ &= 1.948\times 10^3\text{ kW}\\ &= 1.948\text{ MW}\end{aligned}$$

3. The final volume of the system is given by

$$\begin{aligned}v_2 &= \frac{p_1}{p_2}^{1/1.2}v_1\\ &= 1.147\text{ m}^3\end{aligned}$$

Change in internal energy is

$$\begin{aligned}\delta u &= 3.6\times\Delta\,(pv)\\ &= 3.6\times(p_2v_2-p_1v_1)\\ &= -122.91\text{ kJ}\end{aligned}$$

The work done by the system is

$$\begin{aligned}\delta w &= \frac{p_2v_2-p_1v_1}{n-1}\\ &= 170.71\text{ kJ}\end{aligned}$$

Using the first law of thermodynamics,

$$\begin{aligned}\delta q &= \delta w+\delta u\\ &= 47.79\text{ kJ}\end{aligned}$$

4. The engine is working between 900 K and 320 K. The work output from the engine will be

$$\begin{aligned}W_e &= 2400\times\frac{900-320}{900}\\ &= 1546.67\text{ kJ}\end{aligned}$$

The net work output of the plant is 450 kJ, therefore, work transfer to the refrigerator is

$$\begin{aligned}W_r &= W_e - W_{net}\\ &= 1546.67-450\\ &= 1096.67\text{ kJ}\end{aligned}$$

The refrigerator works between 320 K and 250 K. Hence, heat extracted from 250 K reservoir is

$$\begin{aligned}Q_{2'} &= W_r\times\frac{250}{320-250}\\ &= 3916.67\text{ kJ}\end{aligned}$$

Heat transfer by the engine to the 320 K reservoir is

$$\begin{aligned}Q_2 &= 2500\times\frac{320}{900}\\ &= 853.33\text{ kJ}\end{aligned}$$

The heat transfer by the refrigerator to 320 K reservoir is

$$\begin{aligned}Q_{1'} &= W_r\times\frac{320}{320-250}\\ &= 3900.95\text{ kJ}\end{aligned}$$

Hence, total heat transfer to the 320 K reservoir is

$$\begin{aligned}Q_2 &= Q_{1'}+Q_2\\ &= 4754.28\text{ kJ}\end{aligned}$$

5. The heat pump has to transfer heat Q_1 from 298 K to 281 K, given by

$$\begin{aligned}Q_1 &= 0.525(298-281)\\ &= 8.925\text{ kW}\end{aligned}$$

Therefore, the power requirement is

$$\begin{aligned}W &= Q_1\frac{298-281}{281}\\ &= 539.9\text{ W}\end{aligned}$$

In summer, the refrigerator has to work with 539.9 W extracting heat Q_2 from 298 K to atmospheric temperature T, given by

$$Q_2 = 0.525(T-298)\times 1000\text{ W}$$

Therefore,

$$\begin{aligned}W &= Q_2 \frac{T_1 - T_2}{T_2}\\ 539.9 &= Q_2 \times \frac{T - 298}{298}\\ T &= 315.54 \text{ K}\\ &= 42.5°\text{C}\end{aligned}$$

6. One kg of water is to be heated from 298 K to 373 K by thermal reservoir at 373 K. The entropy change of the universe is given by

$$\begin{aligned}\Delta S_{\text{univ}} &= \Delta S_{\text{reservoir}} + \Delta S_{\text{water}}\\ &= -\frac{c_p (T_2 - T_1)}{T_2} + c \ln \frac{T_2}{T_1}\\ &= -\frac{4.186\,(373-298)}{373} + 4.186 \ln\left(\frac{373}{298}\right)\\ &= -0.8417 + 0.9397\\ &= 0.0979 \text{ kJ/K}\end{aligned}$$

In first step, water is to be heated from 298 K to 335 K by thermal reservoir at 335 K. The entropy change of the universe is given by

$$\begin{aligned}\Delta s_{\text{univ},1} &= -\frac{4.186\,(335-298)}{335} + 4.186 \ln\left(\frac{335}{298}\right)\\ &= -0.4623 + 0.4899\\ &= 0.0276 \text{ kJ/K}\end{aligned}$$

In second step, water is to be heated from 335 K to 373 K by thermal reservoir at 373 K. The entropy change of the universe is given by

$$\begin{aligned}\Delta s_{\text{univ},2} &= -\frac{4.186\,(373-335)}{373} + 4.186 \ln\left(\frac{373}{335}\right)\\ &= -0.4264 + 0.4497\\ &= 0.0233 \text{ kJ/K}\end{aligned}$$

The net change in entropy of the universe is

$$\begin{aligned}\Delta s_{\text{univ}} &= \Delta s_{\text{univ},1} + \Delta s_{\text{univ},2}\\ &= 0.0509 \text{ kJ/K}\end{aligned}$$

7. Given that

$$\begin{aligned}l &= 2 \text{ m}\\ d &= 0.02 \text{ m}\\ T_1 &= 150°\text{C}\\ T_2 &= 10°\text{C}\\ k &= 380 \text{ W/mK}\end{aligned}$$

The rate of heat transfer is given by

$$\begin{aligned}\dot{Q} &= k \times \frac{\pi d^2}{4} \times \frac{T_1 - T_2}{l}\\ &= 8.357 \text{ W}\end{aligned}$$

There is no change in entropy of the rod. The heat at the rate of 8.357 W is extracted at 10°C from the surrounding and then the same heat is returned to it at 150°C. Therefore, rate of entropy production is

$$\begin{aligned}\Delta s &= \frac{8.357}{283} - \frac{8.357}{423}\\ &= 9.69 \times 10^{-3} \text{ W/K}\end{aligned}$$

8. Given that

$$\begin{aligned}m &= 10 \text{ g}\\ c &= 0.85 \text{ J/gK}\end{aligned}$$

The resistor is thermally insulated, therefore, the electrical energy (I^2R) will increase its temperature from 10°C to T_2, given by

$$\begin{aligned}mc \times (T - 10) &= 2^2 \times 100 \times 1\\ T &= 57.05°\text{C}\end{aligned}$$

Therefore, the change in entropy of the resistor is

$$\begin{aligned}\Delta s &= 0.85 \times 10 \times \ln\left(\frac{330.05}{283}\right)\\ &= 1.307 \text{ J/K}\end{aligned}$$

The resistor is thermally insulated and there is no change in entropy when electrical energy is supplied. Therefore, there is no change in entropy of the surrounding, in turn, the entropy change of the universe is equal to that for the resistor.

9. The temperature after mixing is calculated as

$$\begin{aligned}T_f &= \frac{15 \times 85 + 25 \times 35}{15 + 35}\\ &= 53.75°\text{C}\end{aligned}$$

The total increase in entropy of the universe is

$$\begin{aligned}\Delta s &= 15 \times 4.186 \ln\left(\frac{326.75}{358}\right)\\ &\quad + 25 \times 4.186 \ln\left(\frac{326.75}{308}\right)\\ &= -5.73 + 6.184\\ &= 0.454 \text{ kJ/K}\end{aligned}$$

The atmospheric temperature is 288 K, therefore, decrease in available energy is

$$\begin{aligned}\Delta \text{AE} &= T_0 \Delta s\\ &= 130.85 \text{ kJ}\end{aligned}$$

10. The availability of flow system is given as

$$\psi = (h - h_0) - T_0\,(s - s_0)$$

Therefore, the initial and final availabilities of the combustion products are

$$\begin{aligned}\psi_1 &= c_p\,(T_1 - h_0) - T_0\,(s_1 - s_0)\\ &= 1.09\,(573 - 300) - 300 \times 1.09 \ln\left(\frac{573}{300}\right)\\ &= 85.97 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\psi_2 &= c_p\,(T_2 - h_0) - T_0\,(s_2 - s_0)\\ &= 1.09\,(473 - 300) - 300 \times 1.09 \ln\left(\frac{473}{300}\right)\\ &= 39.68 \text{ kJ/kg}\end{aligned}$$

The change in the availability of the products is

$$\begin{aligned}\Delta\psi &= \psi_2 - \psi_1\\ &= -46.29 \text{ kJ/kg}\end{aligned}$$

Using the first law of thermodynamics (conservation of energy), the final temperature of air is calculated as

$$\begin{aligned}\dot{m}_a c_a\,(T_{a2} - 40) &= \dot{m}_p c_p\,(300 - 200)\\ T_{a2} &= \frac{\dot{m}_p c_p\,(300 - 200)}{\dot{m}_a c_a} + 40\\ &= 157.89^\circ\text{C}\\ &= 430.89 \text{ K}\end{aligned}$$

The rate of change in entropy of the universe is

$$\begin{aligned}\Delta\dot{S} &= 12.5 \times 1.09 \ln\left(\frac{473}{573}\right)\\ &\quad + 11.5 \times 1.005 \ln\left(\frac{430.89}{313}\right)\\ &= -2.613 + 3.694\\ &= 1.081 \text{ kW/K}\end{aligned}$$

Therefore, the rate of irreversibility of the process is

$$\begin{aligned}\dot{I} &= T_0 \Delta\dot{s}\\ &= 324.40 \text{ kW}\end{aligned}$$

11. The initial state of the gas is

$$\begin{aligned}p_1 &= 300 \text{ kPa}\\ T_1 &= 408 \text{ K}\\ V_1 &= 0.28 \text{ m}^3\end{aligned}$$

Therefore, for $m = 1$ kg, gas constant is given by

$$\begin{aligned}p_1 v_1 &= RT_1\\ R &= 0.205 \text{ kJ/kgK}\end{aligned}$$

The final state of the gas is

$$\begin{aligned}p_2 &= 350 \text{ kPa}\\ V_2 &= 0.4 \text{ m}^3\\ T_2 &= \frac{p_2 V_2}{R}\\ &= 682.9 \text{ K}\end{aligned}$$

Given that $W_{1-2} = 100$ kJ. The process is isobaric and adiabatic, therefore, net heat transfer is zero,

$$\begin{aligned}0 &= -100 + c_v\,(682.92 - 408)\\ c_v &= 0.364 \text{ kJ/kgK}\end{aligned}$$

Specific heat at constant pressure is given by

$$\begin{aligned}c_p &= R + c_v\\ &= 0.569 \text{ kJ/kgK}\end{aligned}$$

The increase in entropy is given by

$$\begin{aligned}\Delta s_{1-2} &= mc_v \ln\left(\frac{p_2}{p_1}\right) + mc_p \ln\left(\frac{V_2}{V_1}\right)\\ &= 0.259 \text{ kJ/K}\end{aligned}$$

12. Given that

$$\begin{aligned}p_1 &= 0.1 \text{ MPa}\\ T_1 &= 298 \text{ K}\\ T_3 &= 1148 \text{ K}\\ r_p &= 6\\ \eta_t &= \eta_c\\ &= 0.8\end{aligned}$$

The temperature after isentropic compression (1–2) is given by

$$\begin{aligned}T_2 &= \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} T_1\\ &= 497.22 \text{ K}\end{aligned}$$

The temperature after isentropic expansion (3–4) is given by

$$\begin{aligned}T_4 &= \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma} T_3\\ &= 668.03 \text{ K}\end{aligned}$$

Since, $\eta_c = 0.8$, therefore, actual temperature of gas after its compression is

$$\begin{aligned}T_{2'} - T_1 &= \frac{(T_2 - T_1)}{\eta_c}\\ T_{2'} &= 547.02 \text{ K}\end{aligned}$$

Since, $\eta_t = 0.8$, therefore, actual temperature of gas after its expansion is

$$\begin{aligned}T_3 - T_{4'} &= \eta_t\,(T_3 - T_4)\\ T_{4'} &= 780.03 \text{ K}\end{aligned}$$

The efficiency of the cycle is

$$\begin{aligned}\eta &= \frac{W_t - W_c}{Q_1}\\ &= \frac{(T_3 - T_{4'}) - (T_{2'} - T_1)}{(T_3 - T_{2'})}\\ &= 19.79\%\end{aligned}$$

By the definition of effectiveness of regenerator, the temperature of gas entering into the heater is given by

$$\begin{aligned}\frac{T_6 - T_{2'}}{T_{4'} - T_{2'}} &= 0.75\\ T_6 &= 721.78 \text{ K}\end{aligned}$$

Therefore, the cycle efficiency with regenerator is

$$\begin{aligned}\eta &= \frac{W_t - W_c}{Q_1}\\ &= \frac{(T_3 - T_{4'}) - (T_{2'} - T_1)}{(T_3 - T_6)}\\ &= 27.91\%\end{aligned}$$

Percentage improvement in the cycle efficiency is

$$\begin{aligned}&= \frac{27.91 - 25.56}{25.56} \times 100\\ &= 9.19\%\end{aligned}$$

13. The increase in entropy is determined as

$$\begin{aligned}\Delta\dot{s} &= \left(\frac{1}{T_2} - \frac{1}{T_1}\right) Q\\ &= \left(\frac{1}{560} - \frac{1}{900}\right) \times 50 \times 10^3\\ &= 33.73 \text{ W/K}\end{aligned}$$

The rate of irreversibility is determined as

$$\begin{aligned}\dot{I} &= T_0 \Delta\dot{s}\\ &= 300 \times 33.73\\ &= 10.12 \text{ kW}\end{aligned}$$

14. Given that

$$\begin{aligned}T_1 &= 298 \text{ K}\\ p_1 &= 1 \text{ bar}\\ p_2 &= 75 \text{ bar}\\ \dot{V}_1 &= \frac{5}{60} \text{ m}^3\text{/s}\\ n &= 1.25\\ \eta_m &= 0.75\end{aligned}$$

The work required in both the stages is same, thus, total power requirement is

$$\begin{aligned}P &= \frac{2}{\eta_m} \times \frac{np_1\dot{V}_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/(2n)} - 1\right]\\ &= 59.99 \text{ kW}\end{aligned}$$

If L is the stroke length of both the cylinders, the average speed of the piston is given by

$$\begin{aligned}2L \times \frac{N}{60} &= 3\\ L &= 0.3 \text{ m}\end{aligned}$$

The diameter of the low pressure cylinder is given by

$$\begin{aligned}\frac{\pi d_{LP}^2}{4} \times L \times \eta_v &= \frac{5}{300}\\ d_{LP} &= 0.297 \text{ m}\\ &\approx 300 \text{ mm}\end{aligned}$$

15. Given that

$$\begin{aligned}T_1 &= 300 \text{ K}\\ p_1 &= 1 \text{ bar}\\ p_2 &= 9 \text{ bar}\\ \dot{m} &= 0.5 \text{ kg/s}\\ n &= 1.3 \text{ bar}\end{aligned}$$

The work required in both the stages is same, thus, total power requirement is

$$\begin{aligned}P &= 2 \times \frac{n\dot{m}RT_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/(2n)} - 1\right]\\ &= 107.66 \text{ kW}\end{aligned}$$

The intermediate pressure is

$$\begin{aligned}p_m &= \sqrt{p_1 p_2}\\ &= 3 \text{ bar}\end{aligned}$$

The temperature of air at intercooler is given by

$$\begin{aligned}T_m &= \left(\frac{p_m}{p_1}\right)^{\frac{n-1}{n}} T_1\\ &= 386.57 \text{ K}\end{aligned}$$

The rate of heat rejection is calculated as

$$\begin{aligned}Q &= \dot{m}c_p (T_m - T_1)\\ &= 43.5 \text{ kW}\end{aligned}$$

CHAPTER 9

APPLICATIONS

Applications of thermal and fluid sciences can be seen in power engineering, internal combustion engines, refrigeration and turbomachinery. Power engineering is oriented towards the study of the properties of steam, steam flow through nozzle, Rankine cycle, and steam turbines. The subject of internal combustion engines includes the fundamental concepts, thermodynamics of air standard cycles, working principles of spark ignition engines and compression ignitions engines, including their performance analysis. The subject of refrigeration and air conditioning is used to study the refrigeration cycles and psychrometry. Different types of hydraulic turbines and their characteristics are considered in the subject of turbomachinery.

9.1 POWER ENGINEERING

9.1.1 Properties of Steam

The state of a substance between liquid and gas is called *vapor*. Steam is the vapor form of water (H_2O), which is used in steam engines, steam turbines and other miscellaneous processes, such as refrigeration, cleaning, washing.

Sensible heat produces temperature difference in substance while *latent heat* produces phase transformation. For every pressure, there is a definite point of temperature above which the substance cannot exist in the liquid form; a substance changes its phase from liquid to vapor. This temperature is called the *saturation temperature of vaporization* (T_{sat}) corresponding to the given pressure (p). Thus, temperature and pressure can be related as

$$T_{sat} = T_{sat}(p_{sat})$$
$$p_{sat} = p_{sat}(T_{sat})$$

The temperature of vaporization rises with the pressure because vaporization involves separation of the molecules. Saturated liquid is the liquid with no vapor, and saturated vapor is the vapor with no liquid. If the temperature (T) of the a given liquid at pressure p is less than the saturation temperature (T_{sat}) corresponding to the pressure p, then the liquid is called *sub-cooled liquid*. Similarly, if the temperature (T) of the given vapor at pressure p is more than the saturation temperature (T_{sat}) corresponding to the pressure p, then the vapor is called *superheated vapor*.

Thermodynamic property diagrams involve two invariant points:

1. ***Critical Point*** Critical point describes the highest temperature and pressure at which a substance

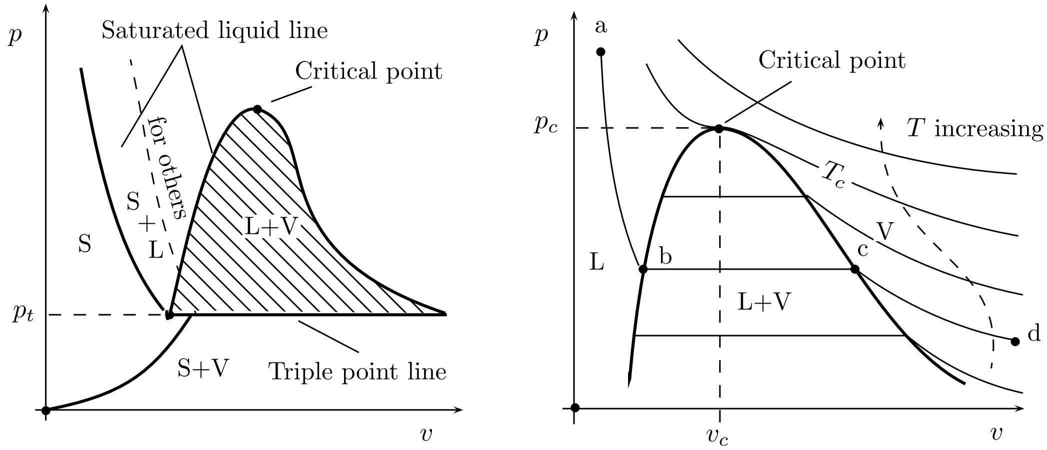


Figure 9.1 p-v diagram.

can coexist in both the liquid and vapor phases in equilibrium; latent heat of vaporization is zero.

For example, *critical point* of water is 374.15°C, 221.2 bar, where specific volume is 0.00317 m^3/kg.

2. *Triple Point* All the three curves of phase transition (sublimation, vaporization, and fusion) meet at triple point. This point is seen as a line in p–v–T surface on the p–v plane view, having a constant pressure but variable volume. Thus, all the three phases (solid, liquid and vapor), coexist in equilibrium on triple point line. At the pressure below the triple point line, the substance cannot exist in liquid phase, and on heating, the substance sublimates from solid to vapor.

   Triple point line of water is 273.16 K and 0.006112 bar at which water exists in all the three phases. It is used as the reference for establishing the *Kelvin temperature scale*.

Specific volume of water decreases when it is heated from 0°C and then increases after 4°C. This is called *peculiarity* (but in the case of other substances, such as CO_2, the volume continuously increases upon heating).

9.1.1.1 Phase-diagrams The equilibrium states of a compressible substance can be specified in terms of directly measurable properties, such as pressure (p), specific volume (v) and temperature (T). In addition to these, specific entropy (s) and specific enthalpy (h), generally determined using thermodynamic relations, are also used to describe the state of a substance. For simplicity, solid, liquid, and vapor phases are denoted by S, L and V, respectively. The following are the commonly used phase diagrams of steam:

1. *p–v Diagram* The p–v diagram is shown in Fig. 9.1. The area under the dome is the two-phase region where both liquid and vapor can exist in equilibrium. On the left side is the liquid region and on the right is the vapor region.

   An isotherm (abcd) appears in three segments:

   (a) Segment a-b is almost vertical, because the change in the volume of liquid is very small for a large change in pressure.

   (b) Segment b-c is horizontal, because the phase transition from liquid to vapor occurs at constant pressure and temperature. This segment represents all possible mixtures of saturated liquid and saturated vapor.

   (c) Segment c-d is less steep because vapor is compressible.

2. *p–T Diagram* The three saturation curves of sublimation, vaporization, and fusion meet at triple point on p–T diagram, where all the three phases, solid, liquid and vapor, coexist in equilibrium [Fig. 9.2].

   Fusion curve can be extended indefinitely, but the vaporization curve terminates at critical point. The p–T diagram helps to conclude about degree of freedom (f) of the system through following points:

   (a) At triple point, no thermodynamic property of the system can be varied independently and the system is said to be invariant ($f = 0$).

   (b) Along any saturation curve, the system is univariant; only one thermodynamic property of the system can be varied independently ($f = 1$).

   (c) The system is bivariant in the single phases region (i.e. area between any two curves) ($f = 2$).

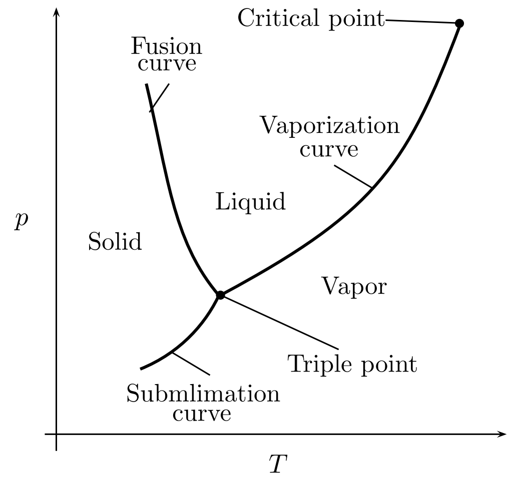


Figure 9.2 | p–T diagram.

3. *T–s Diagram* Energy interaction through a substance can be easily visualized through T–s diagrams. By the definition of entropy, the heat transferred to or from a system equals the area under the T–s curve of the process. Figure 9.3 exhibits the same features as p–v diagram [Fig. 9.1] of steam.

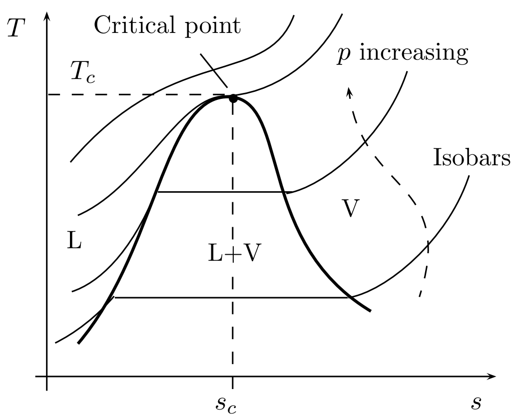


Figure 9.3 | T–s diagram.

4. *p–v–T Surface* Using state postulate, if any two of three state variables (p, v, T) are specified, the third is determined. This implies that the states of the substance can be represented as a surface in a three-dimensional p–v–T space.

Most substances contracts upon freezing and have the p–v–T surface [Fig. 9.4]. The notable exception is water, which expands upon freezing.

There are definite regions of single phase and two phases on the p–v–T surface. A point lying on a line between a single-phase and a two-phase region represents a *saturation state.* The line between the liquid and the liquid-vapor regions is called the *liquid-saturation line*, and any point on that line represents a *saturated-liquid state.* A point on the boundary between the vapor and the liquid-vapor regions is called a *saturated-vapor state.*

The *critical state* is noticeable where the saturated-liquid and saturated-vapor lines meet. The state variables of this unique point are denoted by p_c, v_c and T_c. If a substance is above the critical temperature T_c, it cannot condense into a liquid, no matter how high is the pressure. This merging of the liquid and vapor states above the critical temperature is a characteristic of all known substances. While a pure vapor state can exist at a pressure lower than p_c, at pressures above p_c it is constrained to be a vapor. States with pressures above p_c are described as *supercritical states.*

5. *h–s Diagram* The h–s diagram [Fig. 9.5] has an entirely different shape from the other diagrams of steam. It is better known as the *Mollier diagram*[1] [Fig. 9.5].

As the pressure increases, the saturation temperature increases, and so the slope of the isobar in h–s plane also increases. The isobar lines diverge from one another. The critical isobar is a tangent at the critical point. In the vapor region, the states of equal slopes at various pressures are joined by isothermal lines.

9.1.1.2 Dryness Fraction

If in 1 kg of liquid vapor mixture, x kg is the mass of vapor and $(1-x)$ kg is the mass of liquid, then x is known as the *dryness fraction* or *quality* of steam.

Dryness fraction of a wet steam having m_g and m_f as the masses of vapor and liquid, respectively, is given by

$$x = \frac{m_g}{m_g + m_f}$$

The value of x varies from 0 (saturated water) to 1 (dry saturate steam) [Fig. 9.6].

[1] *Richard Mollier* (1863-1935) was a professor at Dresden University who pioneered the graphical display of the relationship of temperature, pressure, enthalpy, entropy and volume of steam and *moist air* that has since aided the teaching of thermodynamics to many generations of engineers. His enthalpy-entropy diagram for steam was first published in 1904.

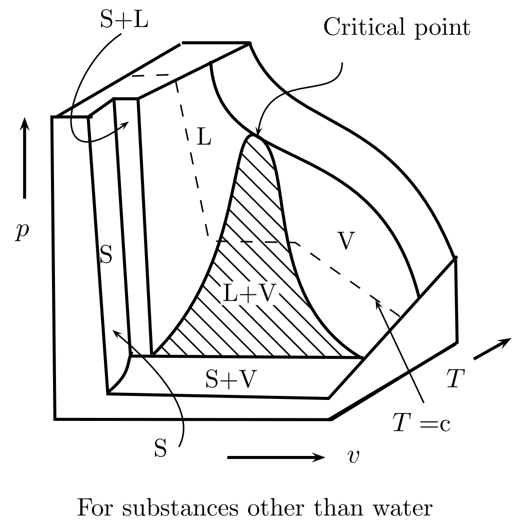


Figure 9.4 $p-v-T$ surfaces.

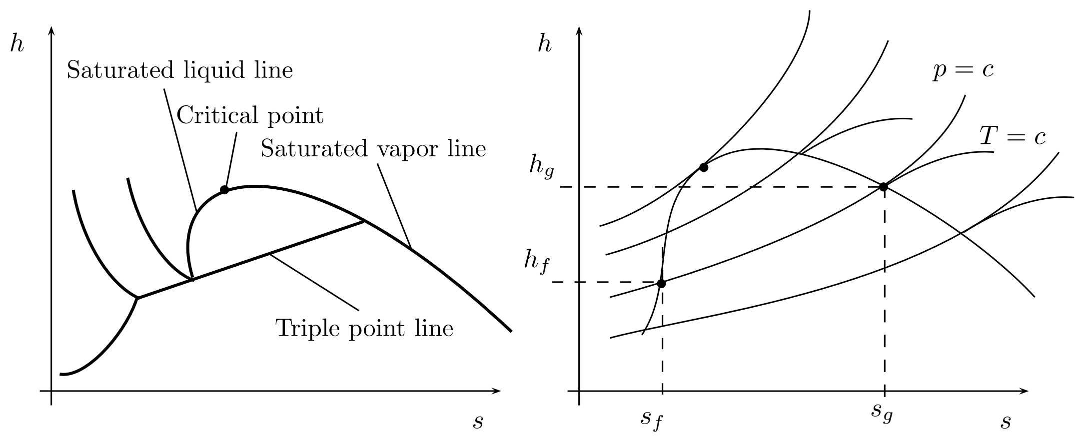


Figure 9.5 h–s diagram.

The specific properties of wet steam are related through dryness fraction x by the following equations:

$$v = v_f + x v_{fg}$$
$$h = h_f + x h_{fg}$$
$$s = s_f + x s_{fg}$$
$$u = u_f + x u_{fg}$$

If wet steam is heated at constant pressure, its temperature shall remain constant. The steam quality increases toward 100% dry saturated steam. Continued heat input will then generate *superheated steam* and its enthalpy is related to the *degree of superheat* ($T_{sup} - T_{sat}$) as

$$h_{sup} = h_{sat} + c(T_{sup} - T_{sat})$$

where c is the specific heat of saturated steam.

The ratio of masses of liquid and vapor phases can be found in terms of their specific volumes:

$$\begin{aligned} m &= m_f + m_g \\ mv &= m_f v_f + m_g v_g \\ m_f v + m_g v &= m_f v_f + m_g v_g \\ m_f (v - v_f) &= m_g (v_g - v) \\ \frac{m_f}{m_g} &= \frac{(v_g - v)}{(v - v_f)} \end{aligned}$$

This is the expression of lever rule for m_f or m_g.

9.1.1.3 Measurement of Dryness The quality of steam can be measured[2] by bringing it from the two

[2]As a convention, the devices for measurement of dryness fraction are called calorimeter.

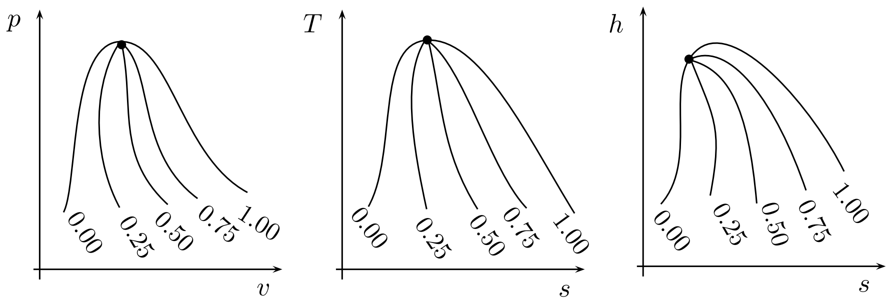


Figure 9.6 Quality lines.

phase region to the single phase of superheated region where the state can be fixed by measuring pressure and temperature, independently. This can be done either by adiabatic throttling or electrical heating. The two methods for measuring the dryness of steam are described as follows:

1. *Throttling* In this method, a *throttling* device is used to throttle the steam into superheated region. A sample of wet steam of mass m and at pressure p_1 is taken from the steam main through a perforated sampling tube. It is throttled by a partially opened valve (or orifice) to a pressure p_2 and temperature T_2 so that after throttling the steam is in the superheated region [Fig. 9.7].

   The enthalpy of steam remains constant in throttling process ($1 \rightarrow 2$):

$$h_1 = h_2$$
$$h_{f1} + x_1 h_{fg1} = h_2$$
$$x_1 = \frac{h_2 - h_{f1}}{h_{fg1}}$$

   A superheat of minimum 5°C is desired to ensure that there is single phase of steam after throttling.

2. *Separating and Throttling* When the steam is very wet, and pressure after throttling is not low enough to make the steam to the superheated region, then a combined separating and throttling calorimeter is used for the measurement of quality [Fig. 9.8]. Steam from the main is first passed through a separator, where some part of the moisture is separated out ($1 \rightarrow 2$) due to the sudden change in direction and falls by gravity, and the partially dry vapor is then throttled ($2 \rightarrow 3$) and taken to the superheated region. Heat equilibrium results

$$h_3 = h_2$$
$$h_3 = h_{f1} + x_2 h_{fg1}$$
$$x_2 = \frac{h_3 - h_{f1}}{h_{fg1}}$$

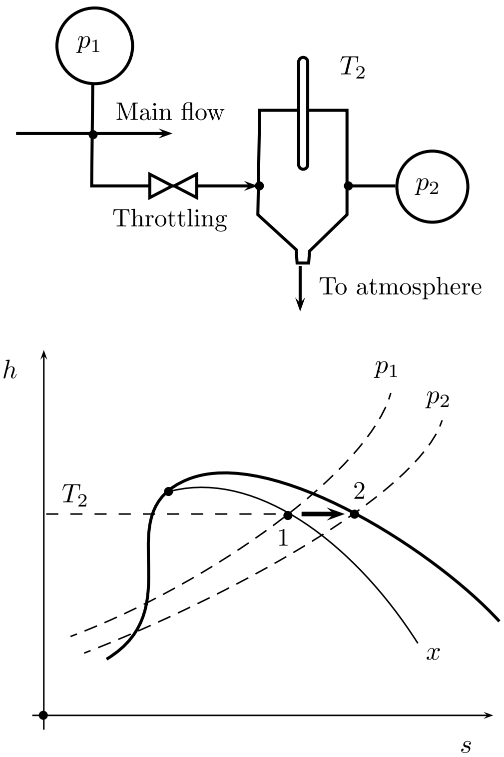


Figure 9.7 Throttling calorimeter.

If m kg steam of quality x_1 is taken in sampling tube, m_1 kg of it is separated and m_2 kg is throttled and then condensed to water and collected, then $m = m_1 + m_2$. At state 2, mass of dry vapor will be $x_2 m_2$ (total wet mass × dryness fraction). Therefore,

$$x_1 = \frac{x_2 m_2}{m_1 + m_2}$$

This is the dryness fraction of the steam.

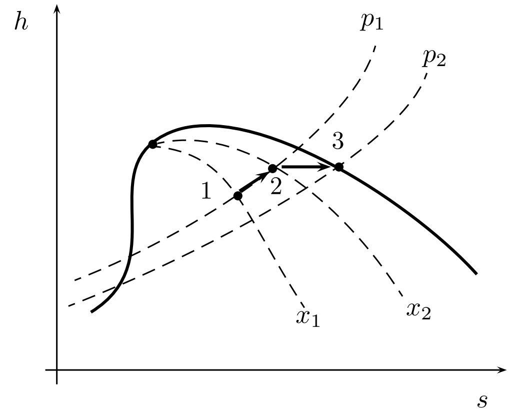


Figure 9.8 Separating and throttling.

9.1.2 Steam Flow through Nozzle

The flow of steam or gas through a *nozzle* is treated as *compressible flow*[3]. The flow is characterized by a definite relationship between the velocity and cross-sectional area of the flow, and the limit of maximum discharge.

Isentropic stagnation enthalpy (h_0) is defined as the maximum enthalpy of the fluid when the fluid is brought to rest isentropically. Isentropic enthalpy of steam having enthalpy h_1 and velocity V_1 is given by

$$h_0 = h_1 + \frac{{V_1}^2}{2}$$

For an ideal gas,

$$c_pT_0 = c_pT_1 + \frac{{V_1}^2}{2}$$
$$\frac{T_0}{T_1} = \left(\frac{\rho_0}{\rho}\right)^{\frac{\gamma-1}{\gamma}}$$

where $\gamma = c_p/c_v$.

[3]In compressible flow,

1. *Sonic velocity* is related to fluid properties as
$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$$
$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = -\left(\frac{v^2 \partial p}{\partial v}\right)_s$$
2. *Mach number* is related to sonic velocity (a) and velocity of gas (V) as
$$M = \frac{V}{a}$$
3. In sonic flow, semi cone angle is called the *Mach angle*, given by
$$\sin\alpha = \frac{1}{M}$$

When moving steam is brought to rest, there is always a loss in pressure rise. This loss is measured by a parameter called *isentropic efficiency* which is defined as

$$\eta_s = \frac{\text{Actual pressure rise}}{\text{Isentropic pressure rise}}$$

9.1.2.1 Steam Tables

Following are the thermodynamic properties of steam:

1. Absolute pressure, p bar or kPa
2. Saturation temperature, T_{sat} °C
3. Specific enthalpy of saturated liquid, h_f kJ/kg
4. Specific enthalpy of vaporization, h_{fg} kJ/kg
5. Specific entropy of saturated liquid, s_f kJ/kg K
6. Specific entropy of saturated vapor, s_g kJ/kg K
7. Specific entropy of vaporization, s_{fg} kJ/kg K
8. Specific volume of saturated liquid, v_f m^3/kg
9. Specific volume of saturated vapor, v_g m^3/kg
10. Change in specific volume during vaporization, v_{fg} m^3/kg

Knowledge of these thermodynamic properties is essential in the analysis and design of a power system cycles with steam as working fluid (e.g. steam turbine). These properties can be determined either theoretically or experimentally. For example, following are the simple relationships:

$$h_{fg} = h_g - h_f$$
$$s_{fg} = s_g - s_f$$
$$v_{fg} = v_g - v_f$$

For convenience to the power engineers, these properties are tabulated in the form of tables, which are known as *steam tables*. The properties of saturated steam and superheated steam are located on the basis of temperature and pressure.

9.1.2.2 Isentropic Flow Analysis

Consider an isentropic flow of steam through a horizontal converging-diverging nozzle[4] [Fig. 9.9].

1. *First Law of Thermodynamics* Steady flow energy equation for unit rate of mass flow is
$$Q - W = dh + VdV + gdz$$

[4]The first use of such a nozzle occurred in 1893 in a steam turbine designed by a Swedish engineer, Carl G. B. de Laval (1845-1913), and therefore, converging-diverging nozzles are often called *Laval nozzles*.

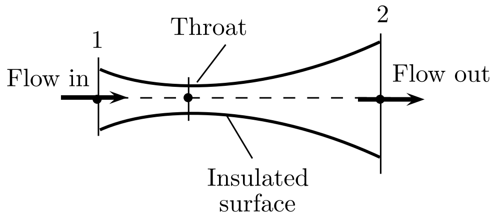


Figure 9.9 Flow through nozzle.

For the present case $Q = W = dz = 0$, therefore,

$$dh = -VdV \tag{9.1}$$

2. *Second Law of Thermodynamics* Using the second T–ds equation ($ds = 0$),

$$Tds = dh - vdp$$
$$dh = vdp \tag{9.2}$$

Eliminating dh from Eqs. (9.1) and (9.2),

$$-VdV = vdp \tag{9.3}$$
$$-\frac{dV}{V} = \frac{vdp}{V^2} \tag{9.4}$$

3. *Conservation of Mass* For mass conservation, specific volume (v), cross-sectional area (A) and velocity (V) are related as

$$\frac{AV}{v} = c$$
$$\frac{dA}{A} + \frac{dV}{V} - \frac{dv}{v} = 0$$
$$\frac{dA}{A} = \frac{dv}{v} - \frac{dV}{V} \tag{9.5}$$

Therefore

$$\begin{aligned}\frac{dA}{A} &= \frac{vdp}{V^2}\left(1 + \frac{V^2 dv}{v^2 dp}\right)\\ &= \frac{vdp}{V^2}\left(1 - \frac{V^2}{c^2}\right)\\ &= \left(1 - M^2\right)\frac{vdp}{V^2}\end{aligned} \tag{9.6}$$

where c is the sonic velocity (c), given by

$$c^2 = -\left(v\frac{\partial p}{\partial v}\right)_s$$

Eq. (9.6) is very important equation[5] which represents the relation between the variation of cross-sectional flow area (dA/A) and the type of flow described by *Mach number* (M). Pressure gradient across a nozzle dp is negative, therefore, the value of M decides the sign of dA/A, as summarized in Table 9.1 and Fig. 9.10.

[5]This equation is valid for both steam and gas with assumption that flow is isentropic and without any friction losses.

Table 9.1 Isentropic flow through nozzle

Flow	M	dA/A	**Section**
Subsonic	< 1	Negative	Converging
Sonic	$= 1$	Zero	Throat
Supersonic	> 1	Positive	Diverging

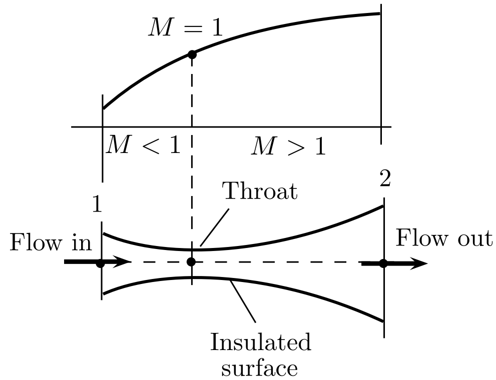


Figure 9.10 Mach number.

9.1.2.3 Critical Pressure Ratio

Isentropic expansion can be represented in polytropic process

$$pv^n = c$$

where n is the expansion index of steam,given by

$$n = \begin{cases} 1.035 + \frac{x}{10} & \text{Wet steam}\\ 1.135 & \text{Saturated steam}\\ 1.3 & \text{Superheated steam}\end{cases}$$

Therefore,

$$v = \left(\frac{C}{p}\right)^{1/n}$$

Using energy equation [Eq. (9.3)],

$$\begin{aligned} -VdV &= vdp\\ -\int VdV &= \int vdp\\ -\int_1^2 VdV &= \int_1^2 \left(\frac{C}{p}\right)^{1/n} dp\\ \frac{{V_2}^2 - {V_1}^2}{2} &= -\frac{n}{n-1}\left(p_2v_2 - p_1v_1\right)\\ V_2 &= \sqrt{\frac{2n}{n-1}\left(p_1v_1 - p_2v_2\right) - {V_1}^2}\end{aligned}$$

The inlet velocity can be assumed to be negligible ($V_1 \approx 0$),

$$V_2 = \sqrt{\frac{2n}{n-1}p_1v_1\left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\}}$$

Specific volume at exit (2) is

$$v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

Denoting pressure ratio p_2/p_1 by r, the mass flow rate is given by

$$\dot{m} = \frac{A_2}{v_2} V_2 \tag{9.7}$$

$$= A_2 \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1} \left\{ r^{\frac{2}{n}} - r^{\frac{n+1}{n}} \right\}} \tag{9.8}$$

For maximum discharge

$$\frac{d\dot{m}}{dr} = 0$$

Solving the above equation for r,

$$r_c = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

This is called *critical pressure ratio* at which the throat velocity is given by

$$V_2 = \sqrt{n p_2 v_2}$$

which is equal to the sonic velocity at throat (2). Above expression can also be written as

$$V_2 = a_1 \sqrt{\frac{2}{n+1}}$$

where a_1 is the sonic velocity at inlet (1).

At critical pressure ratio ($r = r_c$), Eq. (9.8) gives

$$\dot{m}_{max} = A_2 \sqrt{\frac{p_1}{v_1}} \times \begin{cases} 0.636 \text{ wet} \\ 0.677 \text{ superheated} \end{cases}$$

Back pressure of a convergent divergent nozzle is always less than the critical pressure.

9.1.2.4 Choking of Nozzle The flow rate through a nozzle can be increased by reducing the back pressure, if the flow is entirely subsonic. A situation reaches when the flow velocity at the minimum cross-sectional area (throat) eventually reaches the speed of sound; *Mach number* becomes equal to 1. Any further lowering of the back pressure cannot accelerate the flow through the nozzle any more, because that would entail moving the point where $M = 1$ away from the throat, and so the flow gets stuck [Fig. 9.11].

The flow pattern downstream of the nozzle can still change, but the mass flow rate is fixed to maximum. Such a condition is known as *choked flow*.

Figure 9.12 shows the variation of pressure in the span of nozzle for different values of the back pressure.

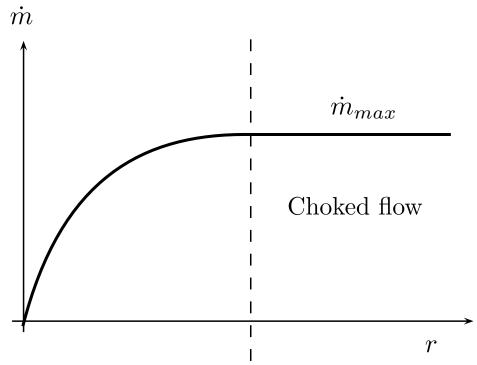


Figure 9.11 $\dot{m}$ versus r in steam flow.

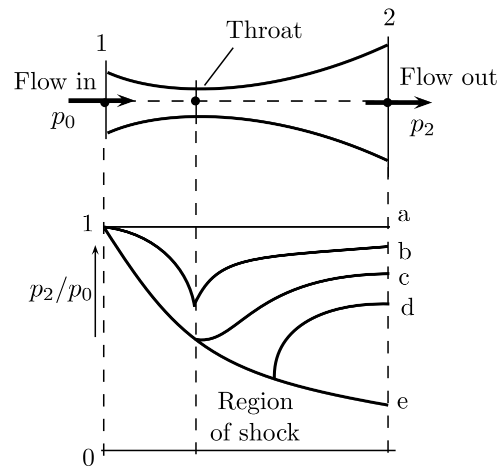


Figure 9.12 Effect of back pressure.

Shock waves occur due to strike of supersonic velocity fluid with the higher density fluid at throat or near the exit. Hence, velocity abruptly changes after shock to supersonic values. After shock, divergent portion acts as diffuser because it increases the pressure and reduces velocity. Shock occurs only after throat where supersonic velocities exist.

9.1.2.5 Effect of Friction Figure 9.13 shows a flow of steam through a nozzle which involves friction ($1 \to 2'$), otherwise the process would be isentropic ($1 \to 2$).

To quantify the effect of friction, *nozzle efficiency* (η_n), is used in the analysis which is defined as the ratio actual enthalpy drop to isentropic enthalpy drop in the nozzle:

$$\eta_n = \frac{h_1 - h_2'}{h_1 - h_2}$$

$$= \frac{{V_2'}^2 - {V_1}^2}{{V_2}^2 - {V_1}^2}$$

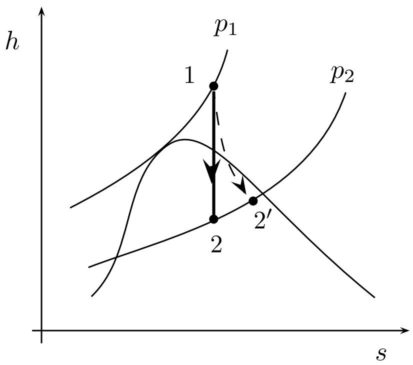


Figure 9.13 | Effect of friction.

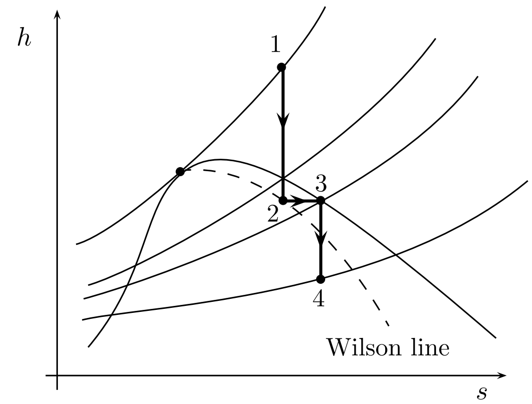


Figure 9.14 | Saturated flow.

For small entry velocities, $V_1 \approx 0$,

$$\eta_n = \frac{{V_2'}^2}{{V_2}^2} = {k_n}^2,$$

where k_n is known as *velocity coefficient* for nozzles.

The net effect of friction is to increase the entropy (s) and final dryness fraction (x) or superheating the steam, and finally reducing the exit velocity, thereby reducing the mass flow rate.

9.1.2.6 Supersaturated Flow

The increase in measured discharge to theoretical discharge is due to time lag in condensation of the steam, which remains dry instead of wet. This is called *supersaturation* of steam. This state is also called *metastable state*. The following are the two basic reasons behind this phenomenon:

1. Absence of Tiny Particles This results in delayed condensation.

2. High Velocity of Steam Molecules This is difficult to be overcome by attractive forces.

The limit to supersaturation is known as *Wilson line* where the steam saturates suddenly ($2 \to 3$) at constant enthalpy (h) [Fig. 9.14].

The net effect of supersaturation is to reduce heat drop slightly during the expansion and a corresponding reduction in exit velocity from the nozzle. This results in increase in values of x, s, $\dot{m}$ while velocity of flow is reduced.

9.1.3 Rankine Cycle

Rankine cycle is the ideal cycle for steam power plants.

9.1.3.1 Simple Rankine Cycle

Simple Rankine cycle consists of the following reversible processes [Fig. 9.15]:

1. Isentropic expansion ($1 \to 2$) of steam in turbine

2. Isobaric cooling ($2 \to 3$) of steam in condenser

3. Isentropic pumping ($3 \to 4$) of saturated liquid (water) to boiler,

4. Isobaric heating ($4 \to 1$) in boiler.

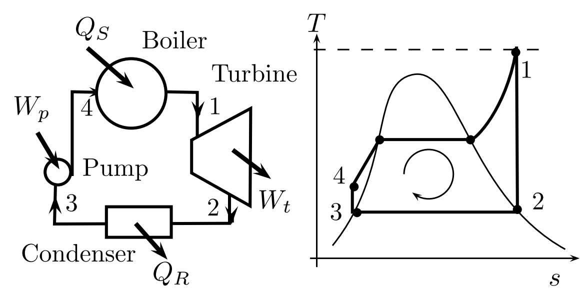


Figure 9.15 | Rankine cycle.

1. Cycle Efficiency Applying the *steady flow energy equation* to each of the processes on the basis of unit mass of working substance and neglecting changes in kinetic energy and potential energy, the heat supplied and the heat rejected are found as

$$Q_S = h_1 - h_4$$
$$Q_R = h_2 - h_3$$

Work done by the steam on the turbine, and the work done by pump on the steam, respectively, are

$$W_t = h_2 - h_1$$
$$W_p = h_4 - h_3$$

Therefore, the efficiency of Rankine cycle is

$$\begin{aligned}\eta_{Rankine} &= \frac{W_{net}}{Q_S} \\ &= \frac{W_t - W_p}{Q_S} \\ &= \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)} \\ &= 1 - \frac{h_2 - h_3}{h_1 - h_4}\end{aligned}$$

2. *Pump Work* Considering the water as incompressible fluid, the work done by the pump on water during isentropic process ($3 \rightarrow 4$, $ds = 0$) can be found using

$$\begin{aligned}T ds &= dh - v dp \\ dh &= v dp\end{aligned}$$

Since the change in specific volume is negligible,

$$\Delta h = v \Delta p.$$

Hence,

$$W_p = h_4 - h_3 = v_3 (p_1 - p_2).$$

Thus, W_p can be assumed to be negligible. The efficiency of Rankine cycle can be written as

$$\eta_{Rankine} \approx \frac{(h_1 - h_2)}{(h_1 - h_4)}$$

3. *Steam Rate* The capacity of a steam power plant is generally expressed in terms of *steam rate* (kg/h-kW), which is defined as the rate of steam flow required to produce unit shaft output power (1 kW):

$$\text{Steam rate} = \frac{3600}{W_t - W_p} \text{ kg/h}$$

where turbine and pump works, W_t and W_p, are in kW.

4. *Mean Temperature of Heat Addition* Rankine cycle can be compared with Carnot cycle in terms of mean temperature of heat addition T_{m1}, defined such that area under 4–1 is equal to area under 4′–1′ [Fig. 9.16]:

$$\begin{aligned}Q_S &= h_1 - h_4 \\ &= T_{m1}(s_1 - s_4) \\ T_{m1} &= \frac{h_1 - h_4}{s_1 - s_4}\end{aligned}$$

Minimum heat rejection is possible at minimum temperature (T_{min}):

$$Q_R = T_{min}(s_1 - s_4)$$

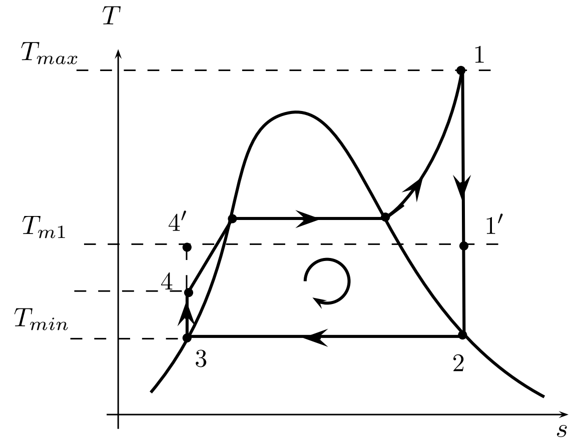


Figure 9.16 T_{m1} in Rankine cycle.

Therefore, cycle efficiency is written as

$$\begin{aligned}\eta_{Rankine} &= 1 - \frac{Q_R}{Q_S} \\ &= 1 - \frac{T_{min}}{T_{m1}}\end{aligned}$$

Hence, the efficiency of Rankine cycle can be increased by two options:

(a) *Decreasing T_{min}* There is a lower limit for T_{min} to the surrounding temperature (T_0) below which temperature cannot be reduced (without additional work).

(b) *Increasing T_{m1}* At fixed surrounding temperature T_0,

$$\eta_{Rankine} = f(T_{m1}) \qquad \text{as } T_{min} \rightarrow T_0.$$

The mean temperature of heat addition can be increased by superheating the steam before it enters into turbine [Fig. 9.17]. The steam is superheated from 1 to 1′ and expansion in turbine takes place from 1′ to 2′. Therefore, there is increase in T_{m1}.

However, the maximum value of T_{m1} is limited by two factors:

(i) *Metallurgy* Metallurgical structure of power plant components cannot permit high temperature.

(ii) *Moisture Content* The dryness fraction of steam should not go below 85% because the entrained water particles in the vapor coming out from the nozzles with high velocity strike the blades and erode their surfaces affecting the longevity of the turbine blades.

Therefore, it is desirable that the expansion in turbine takes place in the vapor region.

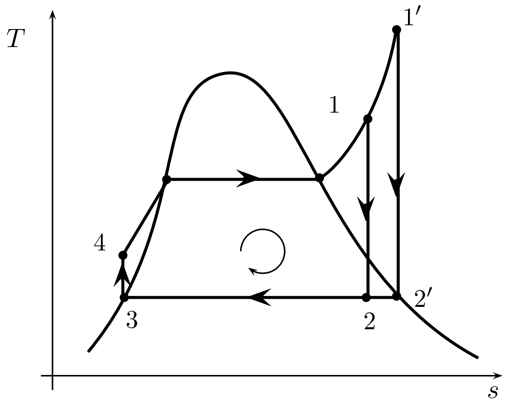


Figure 9.17 | Effect of superheat on T_{m1}.

9.1.3.2 Rankine Cycle with Reheat To get more work output, the simple Rankine cycle can be modified by incorporating reheat of the expanding steam. In this, after partial expansion 1–2′ in a high pressure turbine, the steam is reheated in reheater and expanded in a low pressure turbine 2′–1′ [Fig. 9.18].

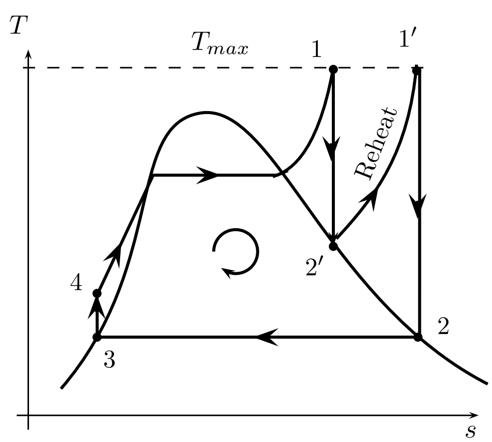


Figure 9.18 | Reheat in Rankine cycle.

To protect the blades and reheater tubes, steam cannot be allowed to expand deep into the two-phase region before it is taken for reheating. In reheater tubes, the moist steam can leave behind solid deposits in the form of scale which is difficult to remove and can hamper the reheating capacity and cause undesired pressure drops. The optimum pressure ($p_{2'}$) for starting reheat is generally kept about 0.2 to 0.25 of the initial steam pressure (p_1).

Reheat cycle acts on high pressures, which requires appreciable pump work.

9.1.3.3 Regenerative Rankine Cycle In a *regenerative Rankine cycle*, feed water is heated by a part of steam extracted from the intermediate stages of the turbine [Fig. 9.19].

Let unit kg mass of steam enters into the turbine at 1, and m kg steam is extracted at 2′, which is used to heat up the $(1-m)$ kg of feed water by mixing in a heater. The remaining $(1-m)$ kg steam expands in further stages upto 2, gets condensed into water in condenser and then pumped to heater while mixing with m kg of steam. The amount m of steam extracted from the turbine is such that at the exit from the heater, the state is saturated liquid at the respective pressures. Then total 1 kg of water is then pumped to the boiler where heat from an external source is supplied.

For adequately insulated heater,

$$mh_{2'} + (1-m)\,h_3 = h_{3'}$$

$$m = \frac{h_{3'} - h_3}{h_{2'} - h_3}$$

Feed water heaters used in regenerative Rankine cycle can be of two types:

1. *Open Heaters* In an *open heater* or contact-type, the extracted or bled steam is allowed to mix with feed water and both leave the heater at a common temperature as seen earlier.

2. *Closed Heaters* In a *closed heater*, the fluid are kept separate, and are not allowed to mix together. The heat transfer takes place through walls and tubes.

9.1.3.4 Binary Vapor Cycle The following are the difficulties at high temperature associated with steam as working fluid:

1. Steam cannot be used above critical state (374°C, 221.2 bar).
2. Latent heat of vaporization (h_{fg}) decreases with increase in pressure.
3. Expansion of steam from high pressure results in high degree of moisture content.
4. Due to low saturation pressure (0.0318 bar) at normal temperature (25°C), the condenser needs to work at vacuum.

An ideal working fluid for Rankine cycle should meet the following thermal requirements:

1. Reasonable saturation pressure at maximum temperature
2. Steep saturated vapor line to minimize moisture problem
3. Saturation pressure at minimum temperature higher than atmospheric temperature

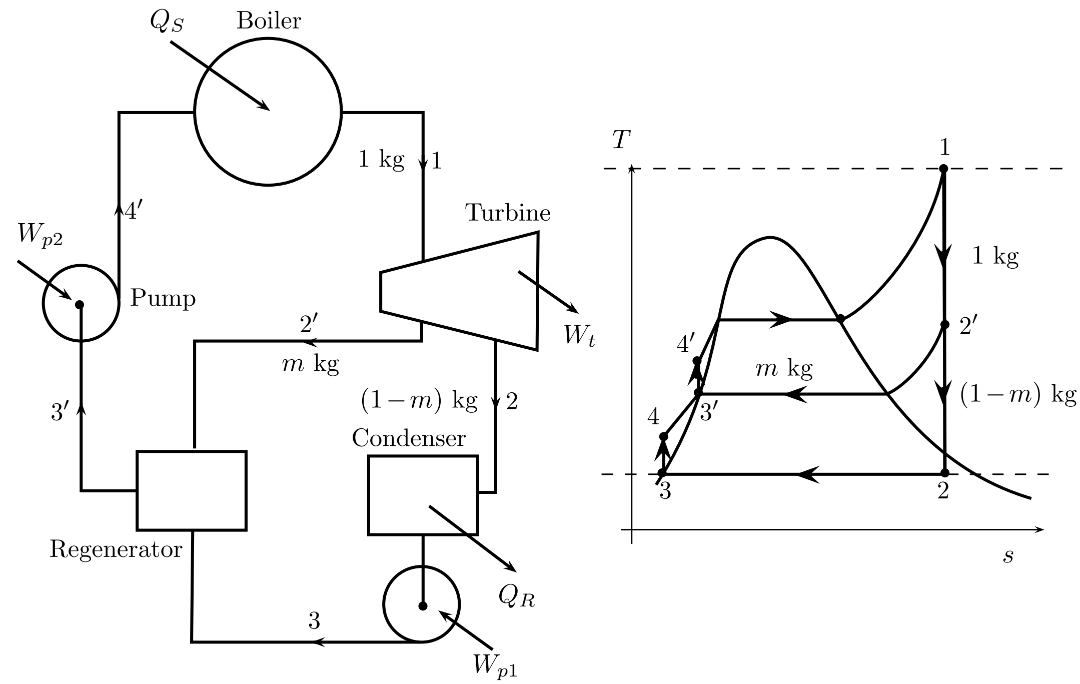


Figure 9.19 Regenerative Rankine cycle.

4. Low value of liquid specific heat to increase mean temperature of heat addition.

These requirements are not met by any single working fluid. For example, critical point of mercury is at 20.6 bar and 540°C for which mean temperature of heat addition will be high. However, mean temperature of heat rejection will also be high. To avoid this, mercury cycle is used along with steam cycle in the form of *binary vapor cycle* [Fig. 9.20].

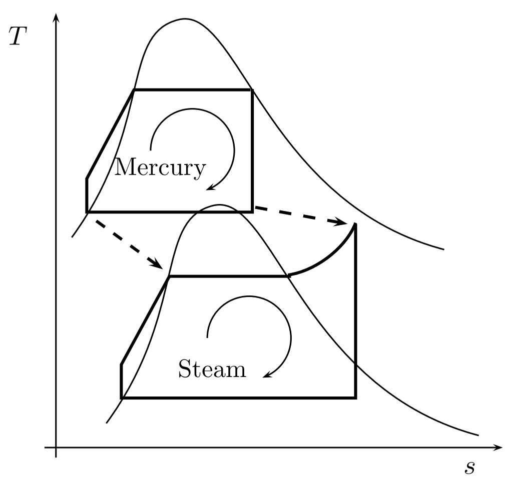


Figure 9.20 Binary Rankine cycle.

Energy balance of the mercury condenser and steam boiler is written as

$$m_w\,(h_1 - h_4) = m_m\,(h_2 - h_3)$$

$$\frac{m_m}{m_w} = \frac{(h_1 - h_4)}{(h_2 - h_3)}$$

A binary vapor cycle is simply a combination of two Rankine cycles, one with mercury as working fluid at higher temperature and the second with steam as working fluid at lower temperature. Mean temperature of heat rejection is the lowest temperature of steam cycle.

9.1.4 Steam Turbines

The motive power in a steam turbine[6] is obtained by the rate of change in momentum of a high velocity jet of steam, impinging on curved blades free to rotate. For this, steam from the boiler is expanded in a nozzle, resulting in the emission of a high velocity jet. This jet of steam impinges on the moving vanes or blades, mounted on a shaft [Fig. 9.21]. Here, it undergoes a change of direction of motion which gives rise to a change in momentum and, therefore, a force. *Carry over losses*

[6]Mechanically, the turbine is superior to the reciprocating engine, as it has only one major moving part, the rotor, and this can (in theory) be balanced perfectly. This is one of the reasons that mechanical efficiency of turbines is better than reciprocating engines.

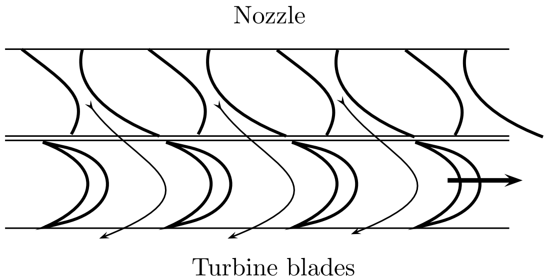


Figure 9.21 Operation of turbines.

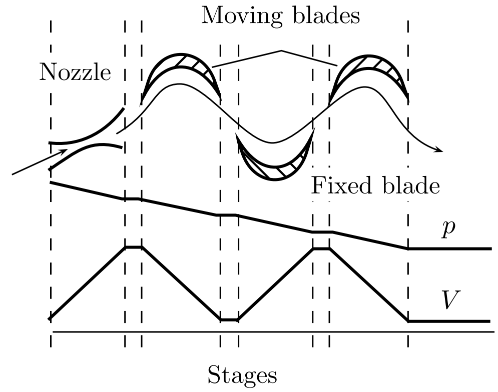


Figure 9.22 Reaction stages.

are associated in generation power due to formation of eddies between nozzle or fixed blades and moving blades.

9.1.4.1 The Classification The study of steam turbines is simplified by classifying them on the basis of their operational principle as impulse turbines and reaction turbines, described as follows:

1. *Impulse Turbines* In *impulse turbines*, the drop in pressure of steam takes place only in nozzles, and not in moving blades. This is obtained by making the blade passage of constant cross-sectional area.

   The single-stage impulse turbine is known as the *de Laval turbine* after its inventor. Pressure or velocity compounding of stages of impulse turbine results in the *Rateau turbines* or the *Curtis turbines*, respectively.

   While an impulse turbine has no change in pressure across the moving blades (pressure drops only occur in fixed blades or nozzles), the pressure is higher on the upstream side of a row of reaction blades than on the downstream side. Therefore, steam tends to leak around the blade tips. This is a particular problem in high pressure stages where the pressure differential is the greatest. This loss is called *tip leakage*.

2. *Reaction Turbines* In reaction turbines, the drop in pressure takes place in *fixed nozzles* as well as in *moving blades* [Fig. 9.22].

   The pressure drop suffered by steam while passing through the moving blades causes additional conversion of pressure energy into kinetic energy within these blades, thus giving rise to reaction and adding to the propelling force. The blade passage cross-sectional area is varied (converging type). Reaction turbines are costlier because of special design of moving blades.

   Parson's turbine is an example of reaction turbine in which equal enthalpy drops occur in the fixed and moving blades.

   In reaction turbines, with reduction of inlet pressure, specific volume increases, thus also increasing the volume flow rate, thereby necessitating increased flow area. This requires increased blade height and mean wheel diameter.

In the low pressure region (in both types of turbines), the blade heights are quite large. The velocity at the blade root will be much smaller than that at the midpoint, and still smaller than at the tip. Hence, for higher efficiencies, the blade angles should vary with the diameter. For this reason, twisted (or warped) blades are used in the later stages of the turbine.

9.1.4.2 Degree of Reaction To quantify the share of power produced in moving blades, *degree of reaction* (R) is defined as the ratio of enthalpy drop in moving blades and total enthalpy drop:

$$R = \frac{\Delta h_s \text{ in moving blade}}{\Delta h_s \text{ in all stages}}$$

Impulse turbines do not involve reaction, therefore, $R = 0$. Parson's turbine is designed for 50% degree of reaction.

9.1.4.3 Velocity Diagram Velocity diagrams simplify the velocity analysis by graphically representing the velocities involved in the turbine rotation, and finally enable in calculation of the power and the efficiency of the turbine stage. Suppose the steam comes out from the nozzle at velocity V_1 and impinges at an angle α_1 with the turbine blade moving at velocity u, thus, relative velocity $\vec{V}_{r1}$ of blade is given by

$$\vec{V}_{r1} = \vec{V}_1 - \vec{u}$$

Relative and absolute velocities of steam at blade exit are V_{r2} and V_2, respectively. Blade angles at inlet and outlet are β_1 and β_2 [Fig. 9.23].

Velocities of the incoming and outgoing steam jets along the axis of rotation are $V_1 \sin\alpha_1$ and $V_2 \sin\alpha_2$

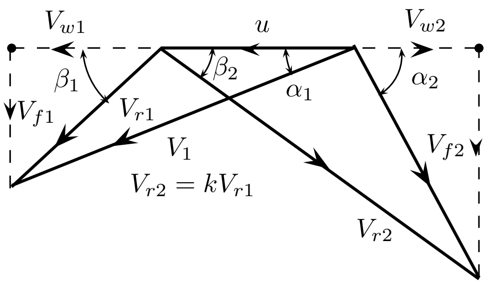


Figure 9.23 Velocity triangle.

respectively. *Blade friction factor* (k) is defined as

$$V_{r2} = k \times V_{r1}$$

Using these elements, the following quantities can be determined for a turbine:

1. *Axial Thrust* Axial thrust shall be given by the change in momentum along the axial direction multiplied by mass flow rate:

$$F_a = \dot{m}\,(V_1 \sin\alpha - V_2 \sin\alpha_2)$$

2. *Tangential Thrust* Tangential thrust is equal to change in momentum along the tangential direction:

$$\begin{aligned} F_t &= \dot{m}\,(V_1 \cos\alpha_1 - V_2 \cos\alpha_2) \\ &= \dot{m}\,(V_{w1} - V_{w2}) \end{aligned}$$

3. *Power* Power (P) is developed by the tangential thrust, therefore

$$\begin{aligned} P &= F_t \times u \\ &= \dot{m}\,(V_{w1} - V_{w2})\,u \end{aligned}$$

4. *Blade Efficiency* *Blade efficiency* is defined as the ratio of power developed in the blades and energy input to the blade. Mathematically,

$$\eta_b = \frac{\dot{m}\,(V_{w1} - V_{w2})\,u}{\dot{m}{V_1}^2/2} \tag{9.9}$$

5. *Gross Stage Efficiency* *Gross stage efficiency* takes into account the effect of friction in nozzle in the blade efficiency:

$$\eta_s = \eta_b \times \eta_n$$

Energy converted to heat due to friction is

$$E = \frac{\dot{m}\left({V_{r1}}^2 - {V_{r2}}^2\right)}{2}$$

These general formulations can now applied in examining efficiency of both types of turbines, described in the following subsections.

9.1.4.4 Efficiency of Impulse Turbines

Referring to Fig. 9.23, the velocity change in tangential direction is written as

$$\begin{aligned} V_{w_1} + V_{w2} &= V_{r1}\cos\beta_1 + V_{r2}\cos\beta_2 \\ &= V_{r1}\cos\beta_1\left(1 + k\frac{\cos\beta_2}{\cos\beta_1}\right) \\ &= V_{r1}\cos\beta_1\,(1 + kB) \\ &= (V_1\cos\alpha_1 - u)\,(1 + kB) \end{aligned}$$

Power developed per kg is

$$\begin{aligned} P &= (V_{w_1} + V_{w2}) \times u \\ &= (V_1\cos\alpha_1 - u)\,(1 + kB)\,u. \end{aligned}$$

Blade efficiency Eq. (9.9) is

$$\begin{aligned} \eta_b &= \frac{(V_1\cos\alpha_1 - u)\,(1 + kB) \times u}{{V_1}^2/2} \\ &= 2\,(1 + kB)\,(\cos\alpha_1 - \rho)\,\rho \end{aligned}$$

where $\rho = u/V_1$ is the velocity ratio. For maximum or minimum efficiency,

$$\frac{d\eta_b}{d\rho} = 0$$

which results in

$$\rho = \frac{\cos\alpha}{2}$$

At this speed ratio, the maximum blade efficiency is found as

$$(\eta_b)_{max} = (1 + kB)\,\frac{\cos^2\alpha}{2} \tag{9.10}$$

9.1.4.5 Efficiency of Parson's Turbines

Parson's reaction turbine is designed to achieve 50% degree of reaction for which enthalpy drops are equal in moving and fixed blades:

$$\begin{aligned} \Delta h_{s moving} &= \Delta h_{s\text{fixed}} \\ \frac{{V_{r2}}^2 - {V_{r1}}^2}{2} &= \frac{{V_1}^2 - {V_2}^2}{2} \end{aligned}$$

Therefore,

$$\begin{aligned} V_1 &= V_{r2} \\ V_2 &= V_{r1} \end{aligned}$$

Such a situation is shown in Fig. 9.24.

Work done per kg of steam is

$$\begin{aligned} W &= u\,(V_1\cos\alpha_1 - u + V_1\cos\alpha_1) \\ &= {V_1}^2\left(2\rho\cos\alpha_1 - \rho^2\right) \end{aligned}$$

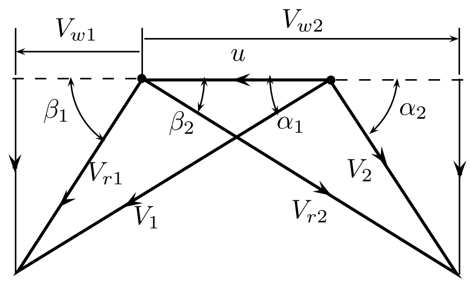


Figure 9.24 | Velocity triangle (50% reaction).

Total energy input per kg of steam is

$$
\begin{aligned}
h_i &= \frac{V_1{}^2}{2} + \underbrace{\frac{V_{r2}{}^2 - V_{r1}{}^2}{2}}_{\text{Reaction at moving blade}} \\
&= \frac{V_1{}^2}{2} + \frac{V_1{}^2 - V_{r1}{}^2}{2} \\
&= V_1{}^2 - \frac{V_{r1}{}^2}{2} \\
&= V_1{}^2 - \frac{\left(V_1{}^2 + u^2 - 2V_1 u \cos\alpha_1\right)}{2} \\
&= \frac{V_1{}^2}{2}\left\{1 - \rho^2 + 2\rho\cos\alpha_1\right\}
\end{aligned}
$$

Therefore, blade efficiency is

$$
\begin{aligned}
\eta_b &= \frac{W}{h_i} \\
&= \frac{2\left(2\rho\cos\alpha_1 - \rho^2\right)}{1 - \rho^2 + 2\rho\cos\alpha_1}
\end{aligned}
$$

For maximum efficiency,

$$
\begin{aligned}
\frac{d\eta_b}{d\rho} &= 0 \\
\rho &= \frac{u}{V_1} \\
&= \cos\alpha_1
\end{aligned}
$$

At this speed ratio, absolute velocity at the outlet V_{r1} is axial, and this maximum efficiency is

$$
(\eta_b)_{max} = \frac{2\cos^2\alpha}{1 + \cos^2\alpha} \tag{9.11}
$$

Such a situation is shown in Fig. 9.25.

For a given value of α_1,

$$
(\eta_b)_{reaction} > (\eta_b)_{impulse}
$$

Therefore, energy input per stage is less, and hence, for the same power generation there are more number of stages required in reaction turbines than impulse turbines.

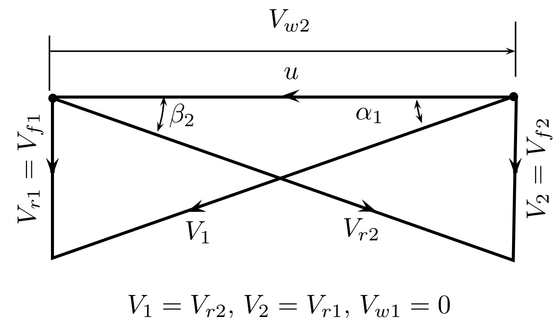


Figure 9.25 | Maximum efficiency.

9.1.4.6 Compounding In steam turbine, a nozzle row followed by one row of blades is called an *stage* of turbine. If steam is allowed to expand in a single row of nozzle, the velocity at exit from the nozzles is very large. Subsequently, the rotational speed of the turbine can be high, in the range of 30,000 rpm, as in *de Laval turbines*. Such high rotational speeds cannot be properly utilized due to friction losses, centrifugal stresses, and energy losses at exit. Therefore, steam turbines are compounded by expanding the steam in a number of stages. This can be achieved in two ways:

1. *Pressure Compounding* *Pressure compounding* is obtained by putting a number of simple impulse stages in series, such that the total enthalpy drop is divided equally among the stages and the pressure drops occur only in the nozzles, not in the moving blades [Fig. 9.26].

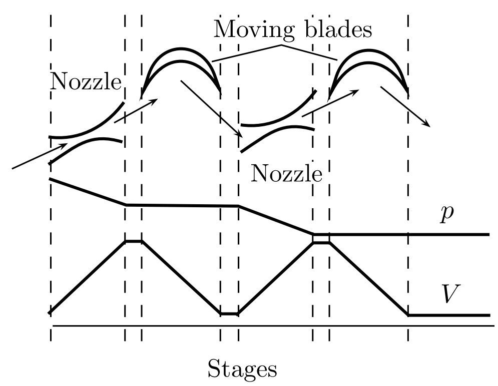


Figure 9.26 | Pressure compounded stages.

The kinetic energy of steam increases in the nozzles at the expense of the pressure drop. The energy is absorbed (partially) by blades in each stage for producing the torque.

2. *Velocity Compounding* In *velocity compounding*, the pressure drop of steam takes place in a single

row of nozzles. The resultant kinetic energy of steam is absorbed by the wheel in a number of rows of moving blades with guide blades in between two such rows [Fig. 9.27].

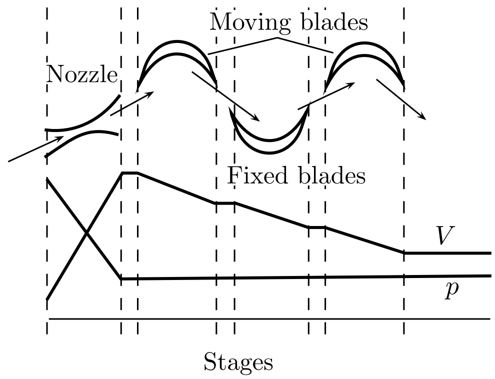


Figure 9.27 Velocity compounded stages.

The Curtis stage turbine[7] is composed of one stage of nozzles as the single-stage turbine, followed by two rows of moving blades instead of one. These two rows are separated by one row of fixed blades attached to the turbine stator, which has the function of redirecting the steam leaving the first row of moving blades to the second row of moving blades. For maximum efficiency of Curtis turbines, the ratio of work in different stages is as follows.

(a) For two stages,

$$W_1 : W_2 = 3 : 1$$

(b) For three stages,

$$W_1 : W_2 : W_3 = 5 : 3 : 1$$

The descending order of efficiency of turbines is

$$\eta_{reaction} > \eta_{impulse} > \eta_{2-row-Curtis}$$

9.1.4.7 Reheat Factor The flow of steam through turbine stages is associated with energy losses. Thus, the portion of the available energy not converted to work. The energy unrecovered from the the steam is termed as *reheat*. A single stage expansion is shown in Fig. 9.28. Reheat is the difference of actual and isentropic enthalpy drops:

$$\begin{aligned}\text{Reheat} &= \Delta h_s - \Delta h_{stage} \\ &= (h_1 - h_{2'}) - (h_1 - h_2) \\ &= h_2 - h_{2'}\end{aligned}$$

[7]The velocity-compounded impulse turbine was first proposed by C.G. Curtis.

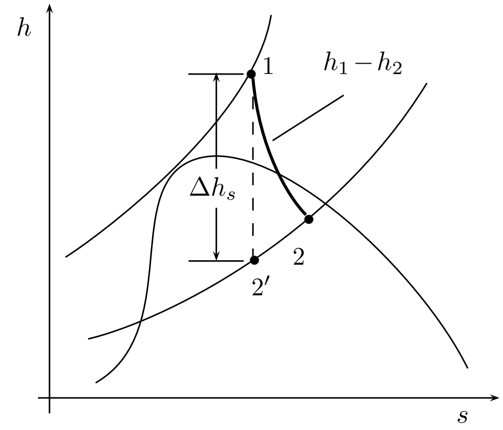


Figure 9.28 Reheat in single stage expansion.

Figure 9.29 shows expansion of steam in a three-stage turbine. The constant pressure lines diverge from one another, thereby, increasing the enthalpy drop for the same pressure drop, but entropy change also increases. Reheat in a single stage is available to do work in the succeeding stage, except the last stage where the reheat is a loss. It is obvious that the sum of the available

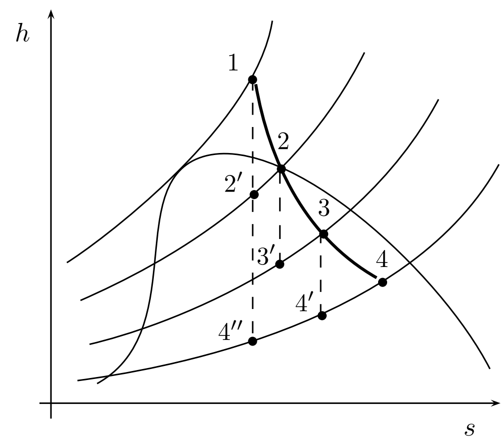


Figure 9.29 Reheat in three-stage expansion.

energies (isentropic enthalpy drops) for each stage is greater than the available energy for the whole turbine or stages. To quantify this aspect, *reheat factor* (RF) is introduced, which is defined as the ratio of cumulative isentropic enthalpy drops and total isentropic enthalpy drop. Mathematically,

$$\text{RF} = \frac{\sum_{n=1}^{n} \Delta h_{s-n}}{\Delta h_s}$$

For expansion shown in Fig. 9.29,

$$\text{RF} = \frac{(h_1 - h_{2'}) + (h_2 - h_{3'}) + (h_3 - h_{4'})}{(h_1 - h_{4''})}$$

The value of RF lies between 1.04 and 1.08, while it is 1 for a single stage turbine.

9.1.4.8 Stage Efficiency *Stage efficiency* (η_{stage}) of a turbine stage is defined as the ratio of the actual enthalpy drop and the cumulative isentropic enthalpy drop. Similarly, the *overall stage efficiency* (η_o) is defined as the ratio of actual enthalpy drop and isentropic enthalpy drop. Therefore,

$$\eta_o = \eta_{stage} \times \mathrm{RF}$$

Since RF ≥ 1,

$$\eta_o \geq \eta_{stage}$$

For a single stage expansion (RF = 1)

$$\eta_o = \eta_{stage}$$

9.1.4.9 Governing The function of a governor is to maintain the shaft speed constant as the load varies. Following are the three ways for the governing of steam turbines:

1. Throttle Governing *Throttle governing* employs a stop valve located in the steam supply line. Partially opened stop valve admits less steam to the turbine, thus producing less power according to the demand. The relationship between consumption and load is given by the *Willan's line*, expressed as

$$M = ck + M_0$$

where M is steam consumption, M_0 is steam consumption at no load, and k is the load with c a constant [Fig. 9.30].

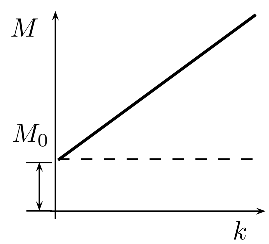


Figure 9.30 Willan's line.

The stop valve is a hydraulically operated, quick opening and shutting valve designed to be either fully open or shut. The throttle valve is used to regulate steam flow during starting or stopping.

2. Nozzle Governing If throttle governing is done at low loads, the turbine efficiency is considerably reduced. The nozzle control can then be a better method of *governing*. The nozzles are made up in sets, which are controlled by separate valves. With decrease of load, the required number of nozzles can be shut off.

3. By-pass Governing To produce more power, additional steam can be admitted through a by-pass valve to the later stages of the turbine. *By-pass governing* operates in a turbine that is throttle governed.

9.2 INTERNAL COMBUSTION ENGINES

Engine is a machine that produces mechanical force and motion from different forms of energy (e.g. a fuel source, compressed air, or electricity). *Combustion* is the act of burning. In *external combustion engines*, such as steam engines, the burning of fuel takes place outside the engine while in *internal combustion engines*, the burning of fuel takes place inside the engine, that produces energy to turn the crankshaft. Internal combustion engines are the most commonly used for mobile propulsion in vehicles and portable machinery. In mobile equipment, internal combustion is advantageous since it can provide high power-to-weight ratios together with excellent fuel energy density.

9.2.1 Fundamentals

Before going through the working principle of IC engines, a brief description of the important engine components related to nomenclature will be more appropriate at this stage.

Cylinder is a cylindrical vessel inside which the piston makes a reciprocating motion. The cylinder is supported in the *cylinder block*. The diameter of cylinder is called the *cylinder bore* (d) and area of the circle of bore diameter is called *piston area* (A_p), written as

$$A_p = \frac{\pi}{4} d^2$$

The varying volume created in the cylinder during the operation is filled with the working fluid, which is subjected to different thermodynamic processes.

Piston is a cylindrical component fitted into the cylinder forming moving boundary of the combustion engine. *Piston rings* are fitted into the slots around the piston, provide a tight seal between the piston and the cylinder wall, thus preventing leakage of combustion gases. The space enclosed in the upper part of the cylinder, by cylinder head and the piston top during combustion process, is called the *combustion chamber*. *Spark plug* is a component to initiate the combustion process in *spark ignition* (SI) engines (not in *compression ignition* (CI) engines) and is usually located on the

cylinder head[8]. The nominal distance through which a piston moves between two successive reversals of its direction of motion is called the *stroke length* (L), or simply the *stroke*. The moment when the direction of the piston motion is reversed at either end of the stroke is called the *dead center* (as there is no motion at that point). *Top dead center* (TDC) is the position when the piston is nearest to the cylinder head (or farthest from crankshaft), while *bottom dead center* (BDC) is the position when the piston is farthest from the cylinder head (or nearest to crankshaft).

The nominal swept volume between the two dead centers is called *displacement* or *swept volume* (v_s), calculated as

$$v_s = \frac{\pi}{4} d^2 L$$

The nominal volume of the combustion chamber above the piston when it is at the TDC is the clearance volume (v_c). The ratio of the total cylinder volume when the piston is at the BDC to the clearance volume is called *compression ratio* (R), written as

$$\begin{aligned} R &= \frac{v_c + v_s}{v_c} \\ &= 1 + \frac{v_s}{v_c} \end{aligned}$$

Here, $v_c + v_s$ is the cylinder volume. The compression ratio of an SI engine varies from 6 to 10, whereas for a CI engine it ranges from 16 to 20.

The passage which connects the intake system to the inlet valve of the engine and through which air or air-fuel mixture is drawn into the cylinder is called the *inlet manifold*. The passage which connects exhaust system to the exhaust valve of the engine and through which the products of combustion escape into the atmosphere is called the *exhaust manifold*.

Mushroom-shaped poppet type valves are provided either on the cylinder head or on the side of the cylinder for regulating the charge coming into the cylinder (*inlet valve*), and for discharging the products of combustion (*exhaust valve*) out of the cylinder.

The *cam shaft* fitted with cam lobes, working in association with *push rods*, *rocker arms*, valve springs and tappets, controls the opening and closing of the two valves. The camshaft is driven by the crankshaft through *timing gears*, also called *cam gears*.

Connecting rod connects the piston (through *gudgeon pin*) and the crankshaft and transmits the gas forces from the piston to the crankshaft. In turn, *crankshaft* converts reciprocating motion of the piston into useful rotary motion of the output shaft. *Balance weights* are provided for static and dynamic balancing of the rotating system. The crankshaft is enclosed in the *crank case*. The net torque imparted to the crankshaft during one complete cycle of operation of the engine fluctuates resulting into a change in the angular velocity of the shaft. In order to achieve a uniform torque, an inertia mass in the form of a *flywheel* is attached to the output shaft. The power impulses of an engine result in torsional vibration in the crankshaft. A *vibration damper* mounted on the front of the crankshaft controls this vibration.

9.2.2 Air Standard Cycles

The working fluid of heat engines is subjected to a series of changes, known as *thermodynamic air standard cycles*, through which the energy absorbed as heat is converted into mechanical work. Internal combustion engine does not operate on a thermodynamic cycle as it involves an open system. However, it is often possible to analyze the open cycle by imagining one or more processes that would bring the working fluid at the exit conditions back to the condition of the starting point. The actual gas power cycles are rather complex.

To reduce the analysis to a manageable level, the following approximations, commonly known as *air standard assumptions*, are made:

1. The working medium is a perfect gas.
2. There is no change in the mass of the working medium.
3. All the processes that constitute the cycle are reversible.
4. Heat is supplied from a constant high temperature source.
5. Some heat is assumed to be rejected to a constant low temperature sink during the cycle.
6. There are no undesired heat losses from the system to the surroundings.
7. The working medium has constant specific heats throughout the cycle.
8. The physical properties (c_p, c_v, μ, γ) of the working fluid are constant.

Another assumption that is often utilized to simplify the analysis even more is that air has constant specific heats whose values are determined at room temperature (25°C). When this assumption is utilized, the air standard assumptions are called *cold air standard assumptions*. A cycle for which air standard assumptions are applicable is frequently referred to as an *air standard cycle*.

[8]The credit of inventing the spark-ignition engine goes to *Nicolaus A. Otto* (1876), whereas, compression-ignition engine was invented by *Rudolf Diesel* (1892). Therefore, they are often referred to as Otto engine and Diesel engine, respectively.

Throughout the study, the heat supplied and heat rejected are denoted by Q_S and Q_R, respectively. The net work output is expressed as W_{net}. The mean effective pressures is referred as p_m. Mass flow rate of working fluid is m, and p, v, T are the pressure, specific volume, and absolute temperature of the working medium with subscripts of the points in the cycle. Compression ratio (r) is the ratio of volumes during the compression process, and pressure ratio is the ratio of pressures.

Mean effective pressure (p_m) is defined as that hypothetical constant pressure acting on piston during its expansion stroke producing the same work output as that from the actual cycle:

$$\begin{aligned} p_m &= \frac{\text{Work output}}{\text{Swept volume}} \\ &= \frac{\eta_{Cycle} \times \text{Heat supplied}}{\text{Swept volume}} \end{aligned}$$

This can be determined using *indicator diagram* as

$$p_m = \frac{\text{Area of diagram}}{\text{Length of diagram}} \times \text{Conversion factor}$$

Important air standard cycles are described under following sections.

9.2.2.1 Carnot Cycle The *Carnot cycle*[9] is represented as a standard of perfection and engines can be compared with it to judge the degree of perfection. This cycle gives the concept of maximizing obtainable work between two different temperature limits. The cycle consists of the following four processes [Fig. 9.31]:

1. Adiabatic compression ($1 \rightarrow 2$)
2. Isothermal heat addition ($2 \rightarrow 3$)
3. Adiabatic expansion ($3 \rightarrow 4$)
4. Isothermal heat rejection ($4 \rightarrow 1$)

Considering the isothermal processes, $1 \rightarrow 2$ and $3 \rightarrow 4$,

$$\begin{aligned} T_1 &= T_2 \\ T_4 &= T_3 \\ Q_S &= RT_3 \ln\left(\frac{v_1}{v_2}\right) \\ Q_R &= RT_1 \ln\left(\frac{v_4}{v_3}\right) \end{aligned}$$

[9]Sadi Carnot, a French engineer, proposed this cycle in 1894.

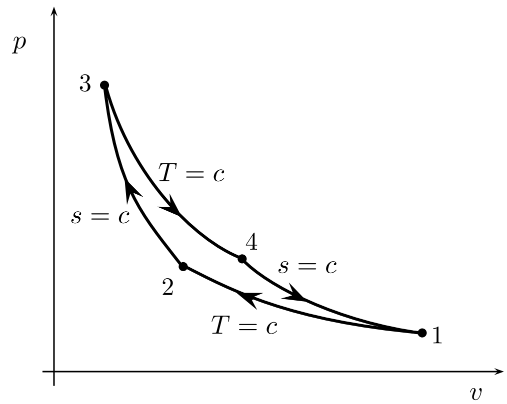


Figure 9.31 Carnot cycle.

Considering adiabatic processes, $2 \rightarrow 3$ and $4 \rightarrow 1$,

$$\begin{aligned} \frac{v_3}{v_2} &= \left(\frac{T_2}{T_3}\right)^{1/(\gamma-1)} \\ \frac{v_4}{v_1} &= \left(\frac{T_1}{T_4}\right)^{1/(\gamma-1)} \\ &= \left(\frac{T_2}{T_3}\right)^{1/(\gamma-1)} \\ &= \frac{v_3}{v_2} \\ &= r \end{aligned}$$

where r is the compression or expansion ratio. Efficiency of the Carnot cycle is given by

$$\begin{aligned} \eta_{Carnot} &= 1 - \frac{Q_R}{Q_S} \\ &= 1 - \frac{RT_1 \ln r}{RT_3 \ln r} \\ &= 1 - \frac{T_1}{T_3} \end{aligned}$$

The following are the key features of the Carnot cycle:

1. Working between two fixed temperatures, Carnot cycle (and other reversible cycles) has maximum possible efficiency compared to all other air-standard cycles.
2. Lower temperature (T_1) is generally the atmospheric temperature or cooling water temperature, and hence fixed quantity. Therefore, cycle efficiency can be increased only by increasing the source temperature.

3. Since $T_1 = T_2$, η_{Carnot} can be expressed in terms of r as

$$\begin{aligned}\eta_{Carnot} &= 1 - \frac{T_1}{T_3} \\ &= 1 - \frac{T_2}{T_3} \\ &= 1 - \frac{1}{r^{\gamma-1}}\end{aligned}$$

Therefore, η_{Carnot} increases as r increases at the expense of large piston displacement.

4. Similarly, η_{Carnot} can be expressed in terms of pressure ratio (r_p) for compression or expansion processes as

$$\begin{aligned}\eta_{Carnot} &= 1 - \frac{T_1}{T_3} \\ &= 1 - \frac{T_2}{T_3} \\ &= 1 - \left(\frac{p_2}{p_3}\right)^{\gamma/(\gamma-1)} \\ &= 1 - \frac{1}{{r_p}^{\gamma/(\gamma-1)}}\end{aligned}$$

Hence, η_{Carnot} increases as r_p increases. This means that Carnot cycle should be operated at high peak pressure to obtain maximum efficiency.

5. The Carnot cycle has a low mean effective pressure because of its very low work output. This problem is solved in Stirling and Ericsson cycles. Carnot cycle does not provide a suitable basis for the operation of an engine.

9.2.2.2 Stirling Cycle The *Stirling cycle* is one of the modified forms of the Carnot cycle to produce higher mean effective pressure. This is done by replacing two isentropic processes in Carnot cycle by constant volume processes [Fig. 9.32].

In Stirling cycle,

$$\begin{aligned}v_3 &= v_2 \\ v_4 &= v_1\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{v_1}{v_2} &= \frac{v_4}{v_3} \\ &= r\end{aligned}$$

Using the p–v diagram of the cycle, the following quantities can be determined:

1. *Heat Supplied* Heat is supplied to the cycle during two processes, $2 \to 3$ and $3 \to 4$:

$$Q_S = c_v\,(T_3 - T_2) + RT_3 \ln\left(\frac{v_4}{v_3}\right)$$

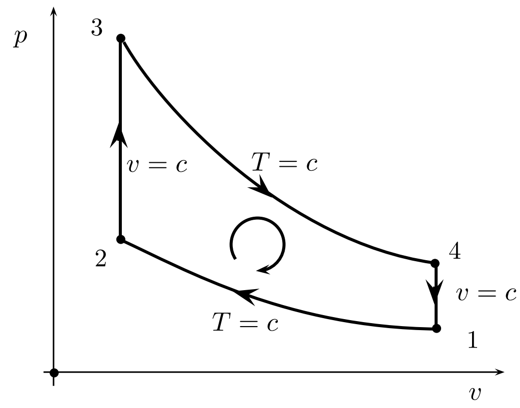


Figure 9.32 Stirling cycle.

2. *Heat Rejected* Heat is rejected by the cycle during two processes, $4 \to 1$ and $1 \to 2$:

$$Q_R = c_v\,(T_4 - T_1) + RT_1 \ln\left(\frac{v_2}{v_1}\right)$$

3. *Net Work Output* Work output of the cycle is the area of p–v diagram, determined as

$$\begin{aligned}W &= \oint p dv \\ &= RT_3 \ln\left(\frac{v_4}{v_3}\right) - RT_1 \ln\left(\frac{v_1}{v_2}\right) \\ &= R\,(T_3 - T_1)\ln r\end{aligned}$$

With equal change in specific volume of the gas, the work output is largest for Stirling cycle and smallest for Carnot cycle.

4. *Mean Effective Pressure* Mean effective pressure of Stirling cycle is

$$\begin{aligned}p_m &= \frac{W}{v_1 - v_2} \\ &= \frac{R\,(T_3 - T_1)\ln r}{v_1 - v_2}\end{aligned}$$

5. *Cycle Efficiency* Efficiency of Sterling cycle is

$$\begin{aligned}\eta_s &= \frac{W}{Q_S} \\ &= \frac{R\,(T_3 - T_1)\ln r}{c_v\,(T_3 - T_2) + RT_3 \ln r}\end{aligned}$$

With perfect heat regeneration ($Q_{2\to3} = Q_{4\to1}$), the only heat added from an external source during the cycle is that which is transferred during the isothermal process 3-4 and the only heat rejected

to an external sink during 1-2. Thus, the efficiency of Stirling cycle reaches to that of Carnot cycle

$$\begin{aligned}\eta_s &= 1 - \frac{Q_R}{Q_S}\\ &= 1 - \frac{RT_1 \ln r}{RT_3 \ln r}\\ &= 1 - \frac{T_1}{T_3}\end{aligned}$$

This cycle was used earlier for hot air engines and became obsolete as Otto and Diesel cycles came into use.

9.2.2.3 Ericsson Cycle

The *Ericsson cycle* is another modified form of Carnot cycle for higher mean effective pressure. This is achieved by replacing both the isentropic processes in Carnot cycle by two isobaric processes [Fig. 9.33].

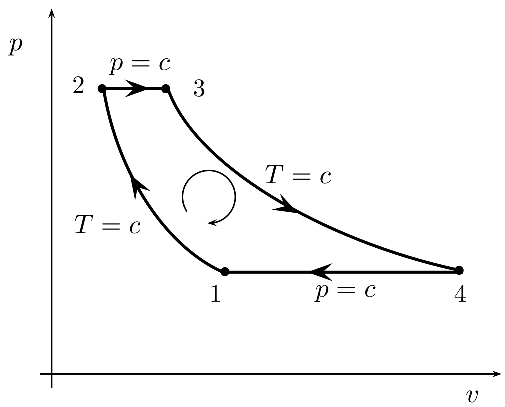


Figure 9.33 Ericsson cycle.

Using the p–v diagram of the cycle, the following quantities can be determined:

1. *Heat Supplied* Heat is supplied to the cycle during two processes, $1 \to 2$ and $2 \to 3$:

$$\begin{aligned}Q_S &= Q_{1\to2} + Q_{2\to3}\\ &= W_{1\to2} + Q_{2\to3}\\ &= RT_1 \ln\left(\frac{p_2}{p_1}\right) + c_p\,(T_3 - T_2)\end{aligned}$$

2. *Heat Rejected* Heat is rejected by the cycle during two processes, $3 \to 4$ and $4 \to 1$:

$$\begin{aligned}Q_R &= Q_{3\to4} + Q_{4\to1}\\ &= W_{3\to4} + Q_{4\to1}\\ &= RT_3 \ln\left(\frac{p_2}{p_1}\right) + c_p\,(T_4 - T_1)\end{aligned}$$

For Ericsson cycle with ideal heat regeneration ($Q_{4\to1} = Q_{2\to3}$), the only heat added from an external source during the cycle is that which is transferred during the isothermal process 3-4 and the only heat rejected to an external sink during 1-2. With this assumption,

$$\begin{aligned}Q_S &= 3 \to 4\\ &= RT_3 \ln\left(\frac{p_2}{p_1}\right)\\ Q_R &= Q_{1\to2}\\ &= RT_1 \ln\left(\frac{p_2}{p_1}\right)\end{aligned}$$

3. *Net Work Output* Work output of the cycle is the area of p–v diagram, determined as

$$\begin{aligned}W &= RT_3 \ln\left(\frac{p_2}{p_1}\right) - RT_1 \ln\left(\frac{p_2}{p_1}\right)\\ &= R\,(T_1 - T_3) \ln r_p\end{aligned}$$

4. *Mean Effective Pressure* The maximum swept volume in the cycle is $v_4 - v_2$, therefore,

$$\begin{aligned}p_m &= \frac{W}{v_4 - v_2}\\ &= \frac{R\,(T_1 - T_3) \ln r_p}{v_4 - v_2}\end{aligned}$$

5. *Cycle Efficiency* The efficiency of Ericsson cycle is determined as

$$\begin{aligned}\eta_{Ericsson} &= 1 - \frac{Q_R}{Q_S}\\ &= 1 - \frac{mRT_1 \ln(p_2/p_1)}{mRT_3 \ln(p_2/p_1)}\\ &= 1 - \frac{T_1}{T_3}\end{aligned}$$

Thus, the efficiency becomes equal to the Carnot cycle efficiency:

Both the Stirling and Ericsson cycles, with perfect regeneration, are totally reversible, as is the Carnot cycle. According to the second statement of the Carnot principle[10], all the three cycles must have the same thermal efficiency when operating between the same temperature limits:

$$\eta_{Stirling} = \eta_{Ericsson} = \eta_{Carnot} = 1 - \frac{T_1}{T_3}$$

where T_3 and T_1 are the maximum temperature and the minimum temperature in the cycle. Thus, the Stirling and Ericsson cycles are also of only theoretical interest.

[10] Refer the Chapter 8.

9.2.2.4 Otto Cycle Having constant volume heat addition, the *Otto cycle*[11] forms the basis for the working of today's spark engines. The Otto cycle is formed by replacing both isothermal processes of Carnot cycle by two isochoric processes [Fig. 9.34]. Thus, the cycle consists of following four processes:

1. Adiabatic compression ($1 \to 2$)
2. Isochoric heat addition ($2 \to 3$)
3. Adiabatic expansion ($3 \to 4$)
4. Isochoric heat rejection ($4 \to 1$)

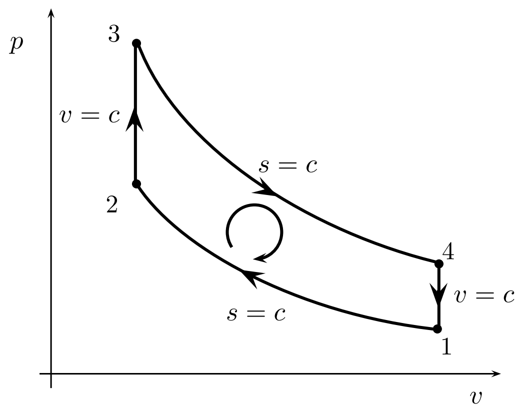


Figure 9.34 Otto cycle.

In this cycle, for isentropic processes $1 \to 2$ and $3 \to 4$,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{(\gamma-1)}$$
$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{(\gamma-1)}$$

But $v_2 = v_3$ and $v_4 = v_1$, therefore

$$\frac{v_1}{v_2} = \frac{v_4}{v_3} = r \text{ (say)}$$

where r is called the *compression ratio* (in the first isentropic process). Hence,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = r^{(\gamma-1)} = x \text{ (say)}$$

[11]The Otto cycle is named after Nikolaus A. Otto who built a successful four stroke engine in 1876 in Germany using the cycle proposed by Frenchman Beau de Rochas in 1862.

Similarly, in processes $1 \to 2$ and $3 \to 4$,

$$\frac{p_2}{p_1} = \frac{p_3}{p_4} = r^\gamma$$

and in $2 \to 3$ and $4 \to 1$,

$$\frac{p_3}{p_2} = \frac{p_4}{p_1} = r_p$$

where r_p is called pressure ratio. Therefore,

$$p_2 = r^\gamma p_1$$
$$p_3 = r_p p_2 = r_p r^\gamma p_1$$
$$p_4 = r_p p_1$$

Using the p–v diagram of the cycle, the following quantities can be determined:

1. ***Heat Supplied*** Heat is supplied to the cycle during isochoric process $2 \to 3$:

$$Q_S = mc_v\,(T_3 - T_2)$$

2. ***Heat Rejected*** Heat is rejected by the cycle during isochoric process $4 \to 1$:

$$Q_R = mc_v\,(T_4 - T_1)$$

3. ***Cycle Efficiency*** Efficiency of the Otto cycle is determined as

$$\begin{aligned}\eta_{Otto} &= 1 - \frac{Q_R}{Q_S} \\ &= 1 - \frac{T_4 - T_1}{T_3 - T_2} \\ &= 1 - \frac{T_4 - T_1}{xT_4 - xT_1} \\ &= 1 - \frac{1}{x}\end{aligned}$$

$$\eta_{Otto} = 1 - \frac{1}{r^{(\gamma-1)}} \tag{9.12}$$

This equation can be used to plot η_{Otto} with respect to r and γ, as depicted in Fig. 9.35.

Efficiency of the Otto cycle η_{Otto} is a function of compression ratio and γ, and is independent of the heat supplied and pressure ratio. It increases with increase with both the compression ratio and specific heat ratio. It is higher with the use of monoatomic gases, such as He ($\gamma = 1.66$), Ar ($\gamma = 1.97$), instead of using bi-atomic ($\gamma = 1.4$ for cold air and $\gamma = 1.3$ for hot air). Beyond certain values of compression ratios, the increase in η_{Otto} is very small, because the curve tends to be asymptotic. However, practically the compression ratio of petrol engines is restricted to maximum of

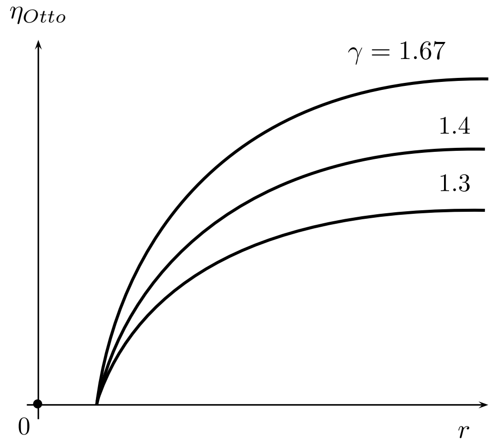


Figure 9.35 η_{Otto} versus r.

9 or 10 due to the phenomenon of autoignition and knocking at high compression ratios.

4. *Net Work Output* The net work output of the cycle is the area under p–v diagram:

$$W = \underbrace{\frac{p_1v_1 - p_2v_2}{\gamma - 1}}_{1\to2} + \underbrace{\frac{p_3v_3 - p_4v_4}{\gamma - 1}}_{3\to4}$$
$$= \frac{p_1v_1}{\gamma-1}\left(\frac{p_3v_3}{p_1v_1} - \frac{p_4v_4}{p_1v_1} - \frac{p_2v_2}{p_1v_1} + 1\right)$$
$$= \frac{p_1v_1}{\gamma-1}\left(\frac{p_3v_2}{p_1v_1} - \frac{p_4v_1}{p_1v_1} - \frac{p_2v_2}{p_1v_1} + 1\right)$$
$$= \frac{p_1v_1}{\gamma-1}\left(\frac{r_pp_2}{p_1r} - r_p - \frac{r^\gamma}{r} + 1\right)$$
$$= \frac{p_1v_1}{\gamma-1}\left(\frac{r_pr^\gamma}{r} - r_p - \frac{r^\gamma}{r} + 1\right)$$
$$= \frac{p_1v_1}{\gamma-1}\left(r_pr^{(\gamma-1)} - r_p - r^{(\gamma-1)} + 1\right)$$
$$= \frac{p_1v_1}{\gamma-1}\left(r_p - 1\right)\left(r^{(\gamma-1)} - 1\right)$$

5. *Mean Effective Pressure* The swept volume of the cycle is $v_2 - v_1$, therefore,

$$p_m = \frac{W}{v_2 - v_1}$$
$$= \frac{W}{v_2\left(r-1\right)}$$
$$= \frac{p_1v_1\left(r_p-1\right)\left(r^{(\gamma-1)}-1\right)}{v_2\left(\gamma-1\right)\left(r-1\right)}$$
$$= \frac{p_1r\left(r_p-1\right)\left(r^{(\gamma-1)}-1\right)}{\left(\gamma-1\right)\left(r-1\right)}$$

Mean effective pressure (p_m) increases with increase in pressure ratio (r_p) at fixed value of compression ratio (r) and ratio of specific heats γ.

9.2.2.5 Diesel Cycle In actual spark ignition engines, the upper limit of compression ratio is limited by the self-ignition temperature of the fuel. This limitation on the compression ratio can be circumvented if air and fuel are compressed separately and brought together at the time of combustion. Such engines work on heavy liquid fuels. These engines are called *compression ignition engines* and they work on a ideal cycle known as *Diesel cycle*[12]. Therefore, diesel engines can be designed to operate at much higher compression ratio, typically between 12 and 24.

Diesel cycle is formed by replacing the isochoric combustion in Otto cycle by isobaric combustion as heat addition process [Fig. 9.36].

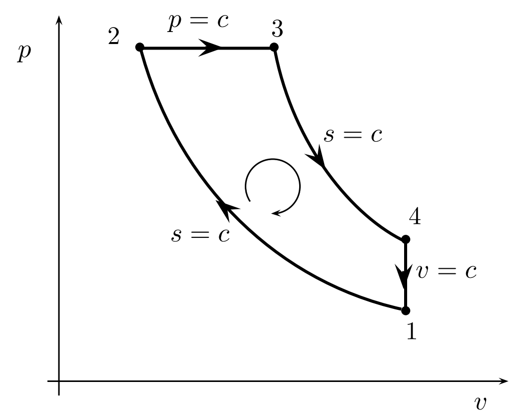


Figure 9.36 Diesel cycle.

There are three volume ratios defined for Diesel cycle:

1. Compression ratio:

$$r = \frac{v_1}{v_2}$$

2. Cut-off ratio:

$$k = \frac{v_3}{v_2}$$

3. Expansion ratio:

$$r_e = \frac{v_4}{v_3}$$

[12] Diesel cycle was invented by Rudolph Diesel in 1897.

Also,

$$\begin{aligned}\frac{v_4}{v_3} &= \frac{v_4 v_2}{v_3 v_2}\\ &= \frac{v_1}{v_2} \times \frac{v_2}{v_3}\\ &= \frac{r}{k}\\ &= r_e\end{aligned}$$

If cut-off takes place at k_f factor of expansion:

$$\begin{aligned}(v_3 - v_2) &= k_f\,(v_1 - v_2)\\ \left(\frac{v_3}{v_2} - 1\right) &= k_f\left(\frac{v_1}{v_2} - 1\right)\\ k - 1 &= k_f\,(r-1)\\ k &= 1 + k_f\,(r-1)\end{aligned}$$

Using these ratios, the temperatures can be found in terms of initial temperature T_1:

1. In the process $1 \rightarrow 2$,

$$T_2 = r^{(\gamma-1)} T_1$$

2. In the process $2 \rightarrow 3$

$$\begin{aligned}T_3 &= kT_2\\ &= kr^{(\gamma-1)} T_1\end{aligned}$$

3. In process $3 \rightarrow 4$,

$$\begin{aligned}T_4 &= T_3 \frac{1}{r_e}^{(\gamma-1)}\\ &= T_3\left(\frac{k}{r}\right)^{(\gamma-1)}\\ &= kr^{(\gamma-1)}\left(\frac{k}{r}\right)^{(\gamma-1)} T_1\\ &= k^{\gamma} T_1\end{aligned}$$

Using the p–v diagram of the cycle, following quantities can be determined:

1. *Heat Supplied* Heat is supplied to the cycle during isobaric process:

$$Q_s = mc_p\,(T_3 - T_2)$$

2. *Heat Rejected* Heat is rejected by the cycle during isochoric process:

$$Q_R = mc_v\,(T_4 - T_1)$$

3. *Cycle Efficiency* Efficiency of the Diesel cycle (η_{Diesel}) is written as

$$\begin{aligned}\eta_{Diesel} &= 1 - \frac{Q_R}{Q_S}\\ &= 1 - \frac{1}{\gamma}\frac{T_4 - T_1}{T_3 - T_2}\\ &= 1 - \frac{1}{\gamma}\frac{T_1\,(k^{\gamma} - 1)}{T_1\left(r^{\gamma-1}k - r^{(\gamma-1)}\right)}\\ &= 1 - \frac{1}{r^{(\gamma-1)}} \times \left\{\frac{(k^{\gamma}-1)}{\gamma\,(k-1)}\right\}\end{aligned}$$

Therefore, η_{Diesel} increases with decrease in k and it approaches η_{Otto} when k is reduced to zero. For same value of r, η_{Diesel} will be equal to η_{Otto} when

$$\begin{aligned}\frac{k^{\gamma}-1}{\gamma\,(k-1)} &= 1\\ k^{\gamma} - \gamma\,(k-1) - 1 &= 0\end{aligned}$$

As $k > 1$, for a given compression ratio (r), Otto cycle is more efficient than the Diesel cycle. But this fact is of no practical importance because, in practice, the operating compression ratio of Diesel engines are much higher (16–20) compared to spark ignition engines working on Otto cycle (6–10).

4. *Net Work Output* The net work output[13] for the Diesel cycle is determined as

$$\begin{aligned}W = &\underbrace{\frac{p_1 v_1 - p_2 v_2}{\gamma - 1}}_{1\rightarrow 2} + \underbrace{p_2\,(v_3 - v_2)}_{2\rightarrow 3}\\ &+ \underbrace{\frac{p_3 v_3 - p_4 v_4}{\gamma - 1}}_{3\rightarrow 4}\end{aligned}$$

5. *Mean Effective Pressure* Swept volume in the process is $v_1 - v_2$, therefore

$$p_m = \frac{W}{v_2\,(r-1)}$$

9.2.2.6 Dual Cycle The chemical reactions of combustion process require some time, thus, the combustion cannot take place at constant volume in the Otto cycle. Similarly, due to uncontrolled combustion in diesel engines, combustion does not occur at constant pressure in the Diesel cycle. Thus, actual cycles are approximated to a *dual cycle*, also called mixed or *limited pressure cycle*, which is a compromise between Otto and Diesel cycles [Fig. 9.37].

[13]Expressions of work output and mean effective pressures can also be derived but there is no need to memorize these because of lengthiness and complications. It is easy to solve numerical questions with fundamental concepts.

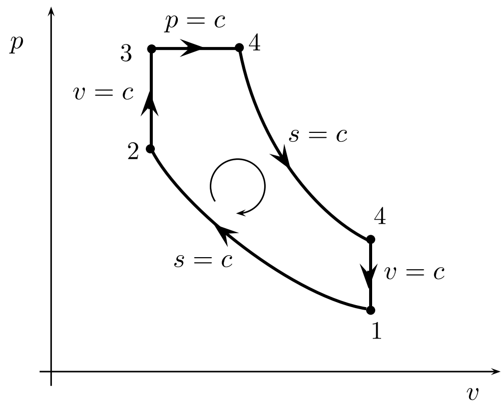


Figure 9.37 Dual cycle.

In a dual cycle, a part of heat is first supplied to the system at constant volume and then the remaining part at constant pressure. The following ratios are important in the analysis of dual cycles:

1. Compression ratio:

$$r = \frac{v_1}{v_2}$$

2. Cut-off ratio:

$$k = \frac{v_4}{v_3}$$

3. Expansion ratio:

$$r_e = \frac{v_5}{v_4}$$

4. Isochoric pressure ratio:

$$r_p = \frac{p_3}{p_2}$$

Heat supplied and rejected are

$$\begin{aligned} Q_S &= c_v\,(T_3 - T_2) + c_p\,(T_4 - T_3) \\ Q_R &= c_v\,(T_5 - T_1) \end{aligned}$$

Using similar process, the efficiency of a dual cycle can be determined as[14]

$$\begin{aligned} \eta_{Dual} &= 1 - \frac{Q_R}{Q_S} \\ &= 1 - \frac{1}{r^{(\gamma-1)}} \cdot \frac{(r_p k^\gamma - 1)}{(r_p - 1) + \gamma r_p\,(k-1)} \end{aligned}$$

[14] Expressions for efficiency, work output, and mean effective pressures for a Dual cycle can also be derived but these are lengthy and complicated. It is easy to solve numerical questions with fundamental concepts.

In the above equation,

1. When $k \to 1$, $\eta_{Dual} \to \eta_{Otto}$
2. When $r_p \to 1$, $\eta_{Dual} \to \eta_{Diesel}$

Figure 9.38 shows a comparison of Otto cycle and Diesel cycle obtained for four situations. The performance of the dual cycle is always between these two cycles. For fixed r (keeping process $1 \to 2$ fixed) and Q_R (keeping process $4 \to 1$ fixed),

$$\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$$

In the other three situations,

$$\eta_{Otto} < \eta_{Dual} < \eta_{Diesel}$$

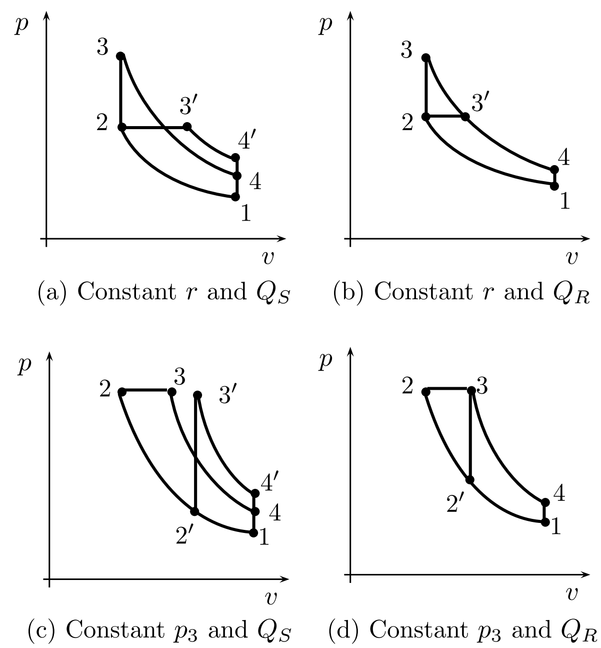


Figure 9.38 Comparison of cycles.

The ideal Otto and Diesel cycles are composed of internally reversible processes, and thus are internally reversible cycles. These cycles are not totally reversible, however, since they involve heat transfer through a finite temperature difference during non-isothermal heat addition and heat rejection processes, which are irreversible. Therefore, the thermal efficiency of an Otto or Diesel cycle is always less than that of Carnot cycle operating between the same temperature limits.

9.2.2.7 Lenoir Cycle Lenoir cycle is used for pulse jet engines. The cycle consists of three processes [Fig. 9.39]:

1. Isochoric heat addition ($1 \to 2$)

2. Isentropic expansion ($2 \rightarrow 3$)
3. Isobaric heat rejection ($3 \rightarrow 1$)

In the process $1 \rightarrow 2$,

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} = r_p$$

where r_p is the pressure ratio. Therefore,

$$T_2 = r_p T_1$$

Similarly, in the process $2 \rightarrow 3$,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{(\gamma-1)/\gamma} = \frac{1}{r_p{}^{(\gamma-1)/\gamma}}$$

$$T_3 = \frac{r_p}{r_p{}^{(\gamma-1)/\gamma}} T_1 = r_p{}^{1/\gamma} T_1$$

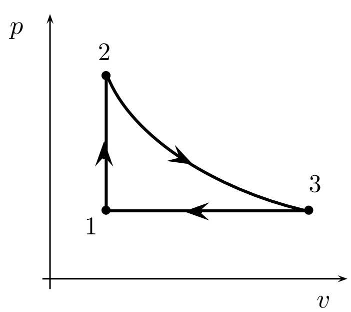


Figure 9.39 | Lenoir cycle.

Heat supplied and rejected are

$$Q_S = mc_v\,(T_2 - T_1)$$
$$Q_R = mc_p\,(T_3 - T_1)$$

Therefore, efficiency of Lenoir cycle is

$$\eta_{Lenoir} = 1 - \frac{Q_R}{Q_S} = 1 - \gamma\frac{T_3 - T_1}{T_2 - T_1} = 1 - \gamma\frac{r_p{}^{1/\gamma}T_1 - T_1}{r_p T_1 - T_1} = 1 - \gamma\left\{\frac{r_p{}^{1/\gamma} - 1}{r_p - 1}\right\}$$

Therefore, efficiency of Lenoir cycle increases with increase in pressure ratio.

9.2.2.8 Atkinson Cycle *Atkinson cycle* is the ideal cycle for Otto engines exhausting to a gas turbine. This is an extended Otto cycle in which the expansion is further allowed to proceed to the lowest pressure [Fig. 9.40].

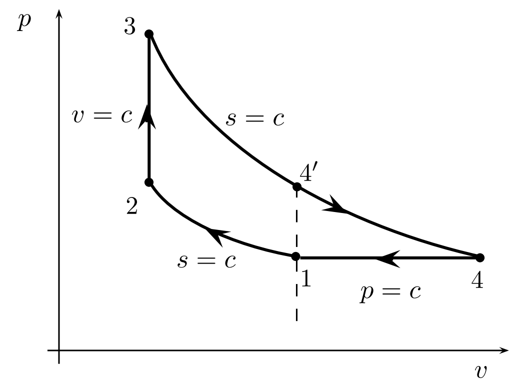


Figure 9.40 | Atkinson cycle.

9.2.3 Working Principles

Based on the number of strokes in cycles, SI and CI engines can be used in either four-stroke or two-stroke engines. In the CI engine, air, instead of a fuel-air mixture is inducted during suction stroke. Due to high compression ratio employed, the temperature at the end of the compression stroke is sufficiently high to self ignite the fuel which is injected into the combustion chamber with the help of a high pressure fuel pump and an injector. The carburettor and ignition system necessary in the SI engine are not required in the CI engine. Major points of differences between SI and CI engines are summarized in Table 9.2.

9.2.3.1 Four-Stroke Engines In a four-stroke engine, the cycle of operations completes in two revolutions (720°) of the crankshaft or four strokes of the piston:

1. *Suction Stroke* Suction starts when the piston is at the TDC and about to move downwards. The inlet valve is open at this time and the exhaust valve is closed. Due to the suction created by the motion of piston towards BDC, the charge consisting of fuel-air mixture (for SI engines), or only air (for CI engines) is sucked into the cylinder. When the piston reaches BDC, the suction stroke ends and inlet valve also closes.
2. *Compression Stroke* The charge taken into the cylinder is compressed by the return stroke into the clearance volume. The mixture is ignited at the end by spark plug (for SI engines), or injection of fuel and auto-ignition (for CI engines).

Table 9.2 SI versus CI engines

Engine	SI Engines	CI Engines
Basic cycle	Otto cycle	Diesel cycle
Compression ratio	6-10	16-20
Thermal efficiency	Lower (due to lower CR)	Higher (due to high CR)
Weight	Light (lower peak pressure)	Heavy (higher peak pressure)
Speed	High	Low
Fuel	Gasoline	Diesel oil
Fuel entry	Air-fuel through carburettor	Fuel through injector pump
Control	Throttle	Fuel pump
Ignition	Spark plug	Auto-ignition

3. *Expansion Stroke* High pressure of the burnt gases forces the piston towards the BDC, and the power is obtained during this stroke.

4. *Exhaust Stroke* The exhaust valve opens and the inlet valve remains closed. The pressure falls into atmospheric level as a part of the burnt gases escape during movement of the piston from BDC to TDC. The exhaust valve closes at the end of the exhaust stroke and some residual gases trapped in the clearance volume remain in the cylinder.

9.2.3.2 Two-Stroke Engines Theoretically, if the two unproductive strokes, namely suction and exhaust, could be served by an alternative arrangement, especially without movement of the piston, then there will be power stroke for each revolution of the crankshaft. In such an arrangement, the power output of the engine should be doubled for the same speed[15]. In actual practice, power output is not exactly double but increased by only about 30% because of associated losses.

A two-stroke cycle completes in one revolution of the crankshaft. The filling process is accomplished by the charge compressed in crankcase or by a blower. During expansion stroke, the charge in the crankcase is compressed. Near the end of expansion stroke, the piston uncovers the exhaust port and the cylinder pressure drops to atmospheric pressure. Further movement of the piston uncovers the transfer port, permitting slightly compressed charge in the crankcase to enter the engine cylinder.

The top of the piston has usually a projection to deflect the fresh charge towards the top of the cylinder before flowing into the exhaust port. This serves double purpose of scavenging the upper part of the cylinder of the combustion products and preventing fresh charge from flowing directly to the exhaust ports.

A comparison of two-stroke and four-stroke engines is given in Table 9.3.

Table 9.3 Two-stroke versus four-stroke engines

Cycle	Two Strokes	Four strokes
Power stroke	One per revolution	One per two revolutions
Flywheel	Lighter	Heavier
Weight	Lighter engine	Heavier engine
Valve	Ports	Actuating mechanism
Initial cost	Less	More
Volumetric efficiency	Less	More
Thermal efficiency	Less	More
Usage objective	Cost, compactness	Efficiency

Two-stroke engines provide a more uniform torque on crankshaft and comparatively less exhaust gas dilution. Therefore, four-stroke engines need heavier flywheel in comparison to two-stroke engines. For the same power, a four-stroke engine is heavier, requires less cooling and lubrication but gives more volumetric and thermal efficiency. In case of SI engines with two-stroke cycle, there is a possibility that some of the fresh charge containing fuel escapes with the exhaust. This results in high fuel consumption and lower thermal efficiency. Two stroke engines lack in flexibility to operate with the same efficiency at all speeds. At part throttle, the fresh mixture is not enough to clean all the exhaust gases and a part of it remains in the cylinder to contaminate the charge. This results in irregular operation of the the engine.

9.2.3.3 Valve Timings *Valve timing* is the scheduled cycle of opening and closing of the inlet and exhaust valves. The following factors are considered in design of valve timing:

1. *Noise and Wear* To avoid noise and wear of valve surfaces, the clearance between cam, tappet, and valve must be slowly lifted out.

[15] Based on this concept, Dugald Clark (1878) invented the two-stroke engine.

2. Ram Effect When the piston reaches BDC and starts to move in compression stroke, the inertia of the entering fresh charge tends to cause it to continue to move into cylinder. Therefore, the intake valve is closed after TDC so that maximum air is taken in. This is called *ram effect*.

3. Energy Saving Opening of exhaust valve earlier to BDC reduces the pressure near the end of power stroke and loss of power occurs, but work required to expel the exhaust is reduced.

Valve timing is often represented in polar diagram, known as *valve timing diagram*, in which rotation of crankshaft is the angular coordinate with assumption of constant angular speed.

9.2.4 Fuel Air Cycles

Fuel air cycle analysis enables to study the effects of fuel-air ratio on the cycles. The analysis is based on the actual properties of the working medium with following approximations:

1. There is no chemical change in either fuel or air prior to combustion.
2. Subsequent to combustion, the charge is always in chemical equilibrium.
3. There is no heat exchange between the gases and the cylinder walls in any process.
4. Fluid motion inside the cylinder is ignorable.

With respect to SI engines, the following additional assumptions are made:

1. The fuel is completely vaporized and perfectly mixed with the air.
2. The burning takes place instantaneously at TDC.

9.2.4.1 Variation from Ideal Cycles The operation of internal combustion engines is based on the theoretical air-standard cycles. Due to air-standard assumptions made for the cycles, the character of fuel-air cycle is different. In actual engine, the following factors are evident:

1. Composition of Gases The working fluid of engines is a mixture of air, fuel vapor and residual gases. The composition of gases (air-fuel mixture) changes due to combustion which directly affects the constitutional properties of the mixture.

2. Variable Specific Heats The difference between c_p and c_v of a gas mixture is equal to the gas constant of the mixture, but there is increase in specific heat of all gases, except monoatomic gases, due to which specific heat ratio decreases, in turn, cycle efficiency reduces.

3. Dissociation Fuel-air cycles involve *dissociation* of mainly CO_2 into CO and O_2 during which heat is absorbed by the gas mixture. The presence of CO and O_2 tends to reduce the dissociation of CO_2, as experienced in rich mixtures. Dissociation does not occur in burnt gases of a lean fuel–air mixture due to low temperature. Maximum extent of dissociation occurs in the burnt gases of chemically correct fuel–air mixture when temperature is expected to be high. The disintegrated constituents recombine at lower temperature during the expansion stroke, thus the heat absorbed during dissociation is thus again released but is carried out by exhaust gases. Figure 9.41 shows the effect of dissociation on break power and specific fuel consumption. The break power is maximum when the mixture ratio is stoichiometric.

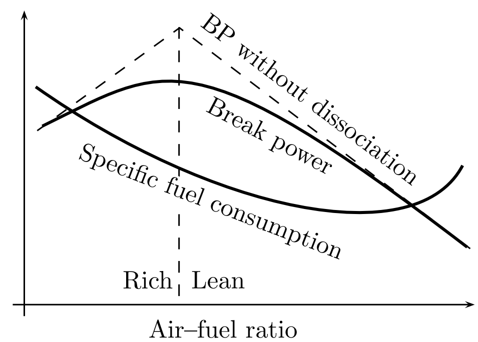


Figure 9.41 | Effect of dissociation.

The effect of dissociation is to lower the maximum temperature and the maximum pressure at the beginning of expansion stroke resulting in loss of power and efficiency. Due to the presence of heterogeneous mixture and excess air, these effects are not so produced in a CI engine as in SI engines.

4. Number of Molecules Variation in the number of molecules in the cylinder directly affects the amount of work the gases can impart on the piston, thus affecting the power output from the engine.

9.2.4.2 Operating Variables Apart from design variables (e.g. design of cylinder, piston, valves), compression ratio and fuel-air ratio are the two main operating variables that affect the cycle performance:

1. Compression Ratio Like air-standard cycle efficiency, the fuel-air cycle efficiency increases with increase in the compression ratio.

2. *Fuel-Air Ratio* Due to reduced energy input per unit mass, the combustion of leaner mixtures results in lower temperature rise. This results in lower specific heat and reduced energy losses due to dissociation. As depicted in Table 9.4, at given compression ratio, the temperature after combustion reaches maximum when the mixture is slightly rich around 6% (F/A = 0.072 or A/F = 14:1).

Table 9.4 Effect of fuel richness

Maximum Quantity	Richness
Peak temperature	6%
Peak pressure	20%
Exhaust temperature	0%
Mean effective pressure	6%

Maximum pressure is achieved in still richer mixture, around 20% (F/A = 0.083 or A/F = 12:1).

9.2.5 Carburation

A *carburettor* is designed to form and provide a combustible mixture of fuel and air in required quantity, and the quality for efficient operation of the engine under all conditions.

9.2.5.1 Mixture Requirements The carburettor must be designed to meet the mixture requirements [Table 9.5], grouped into three ranges of engine operation:

1. *Idling Range* During *idling*, the engine operates at no load. However, the engine requires very less amount of fresh charge (throttle is nearly closed up to 20% load) to work against friction and operate the accessories. The less amount of fresh charge has two effects:

   (a) Mixing of larger portion of exhaust gases with the fresh charge.

   (b) Backward flow of exhaust gases into intake manifold at the start of suction stroke.

   Thus, idling range requires richer mixture to avoid possibility of poor combustion.

2. *Cruising Range* In *cruising range*, the main interest of design is to achieve maximum fuel economy. This is possible at the optimum fuel–air ratio for any given engine speed which can develop the required torque with smooth and reliable operation.

3. *Power Range* Maximum power range requires richer mixture to

   (a) Produce maximum power from the fuel intake

   (b) Prevent over heating of exhaust valve by reducing the flame temperature

   (c) Reduce detonation by reducing the flame temperature

Table 9.5 Air–fuel ratio (octane fuel)

Requirement	A:F
Cold starting/warm up	3:1
Idling	12.5:1
Crushing	16.7:1
Maximum power	14:1

9.2.5.2 Working Principle To understand the working principle of carburettor, consider a simple carburettor consisting of the following elements [Fig. 9.42]:

1. *Venturi* To reduce the pressure to cause the mixing of fuel with air stream, the air is passed through an air passage containing a venturi-shaped restriction of fixed geometry.

2. *Float Chamber* A float chamber acts as a reservoir of fuel in the carburettor to supply fuel at constant level.

3. *Fuel Nozzle* A *fuel nozzle* is located in the *venturi throat*. It is supplied with fuel from a constant level float chamber.

4. *Choke* A *choke* is required to produce the rich mixture by reducing air quantity going to the engine. This is to start the engine with a combustible mixture rich enough to ignite near the spark plug. At low ambient temperature, only a small amount of fuel fed into combustion chamber evaporates so that fuel–air vapor mixture is very lean.

5. *Throttle Valve* Power of an engine is varied by *throttle valve* which is located in down stream of venturi. The mixture gets progressively rich with increased throttle opening because velocity increases in the same manner for both fuel and air, but density of air decreases.

Size of carburettor is specified by diameter of throat in mm and jet size in multiples of 100 mm.

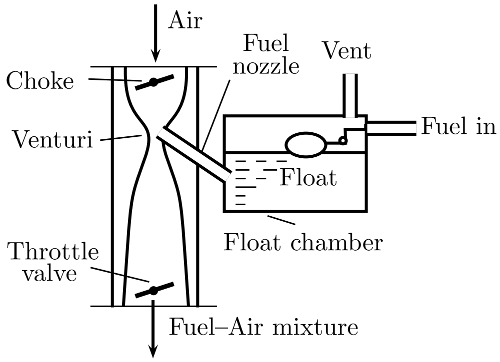


Figure 9.42 Principle of carburettor.

9.2.5.3 Carburettor Systems To meet the requirement of fuel-air mixture supply, a carburettor is provided with the following essential systems:

1. Metering System The *metering system* controls the enriching of mixture due to increased throttle opening operated by *clutch*.

2. Idling System With partially closed throttle, the limited air flow causes insufficient depression to draw fuel through the main fuel nozzle. Thus, an idling tube is provided to directly connect the downstream of venturi to the float chamber through which the fuel gets discharged directly into the engine intake due to low pressure on the downstream. The fuel in the idling tube is drawn by the air flow taken from upstream. This constitutes the *idling system*, which comes into operation during starting, idling range, and low speed operation.

3. Accelerating System Sudden opening of the throttle causes a temporary lean mixture that often misfires and reduces the power output (due to inertia, the flow rate of fuel does not increase). Thus, an accelerating system is used to discharge the additional quantity of fuel into the carburettor air stream to maintain the fuel-air ratio.

4. Economizer The *economizer* provides and regulates enriched mixture required at full throttle operation for speeds above the cruising range.

9.2.6 Combustion in SI Engines

Combustion in SI engines occurs in the following three stages [Fig. 9.43]:

1. *Ignition lag* (low rate and increasing front speed)

2. Propagation of flame (from the first rise in pressure)

3. After burning (low rate and decreasing front speed)

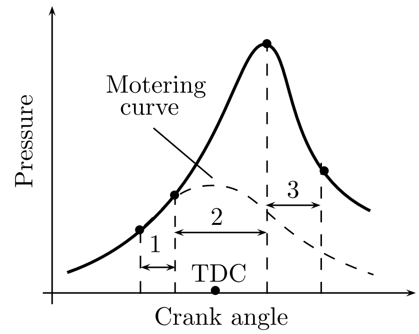


Figure 9.43 Combustion in SI engines.

Spontaneous ignition or autoignition occurs when the temperature of the unburnt mixture at any point exceeds the *self ignition temperature* of the fuel during the period of pre-flame reactions (*ignition delay*). The uneven pressure rise causes noise and disturbs the operation of engine by reducing the mean effective pressure and efficiency. This phenomenon is called *knocking* or *detonation*. This can be avoided by selecting fuel of high autoignition temperature and with long ignition delay period. Any factor which reduces the temperature of unburnt charge should reduce the possibility of detonation. This is possible by reducing compression ratio, mass of inducted charge, inlet temperature and pressure, temperature of combustion chamber, and by retarding the spark timing (having spark closer to TDC). Any factor which increases flame speed will also reduce knock. Flame speed can be increased by increasing turbulence, fuel-air ratio, temperature and pressure at inlet, compression ratio, engine output and speed.

9.2.7 Combustion in CI Engines

Combustion chamber of CI engines is initially charged with air, which is compressed during compression stroke. Towards the end of compression stroke, fuel is injected by the fuel injection system into the cylinder. The fuel atomizes into small droplets which later vaporize and mix with the charge of air at high temperature. The combustion in CI engines takes place in four stages [Fig. 9.44]:

1. Ignition delay (physical and chemical)

2. Rapid combustion (maximum heat release rate)

3. Controlled combustion (by regulating quantity of fuel injected)

4. After burning

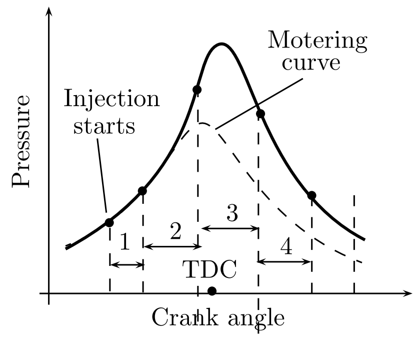


Figure 9.44 Combustion in CI engines.

For longer ignition delay, the burning of the first droplets is delayed. This results into accumulation of fuel droplets in the chamber which can cause too rapid rate of pressure rise. This is known as knocking in CI engines. In SI engines, knocking occurs near or before the end of combustion whereas in CI engines, knocking occurs near the beginning of combustion. This can be avoided by decreasing the ignition delay period, and thus, decreasing the amount of fuel entering when actual burning starts. Knocking in CI engines is reduced by using higher cetane rating (low ignition temperature) of fuel, increasing the injection pressure, injection timing advance, compression ratio, intake temperature and pressure, jacket temperature of water, and fuel temperature. Also, low speed, load and reduced ignition delay reduce the chances of knocking. Knocking in CI engines is also reduced by introducing *pre-ignition chambers*. In open combustion chambers, a masked inlet valve is used to induce the swirl by providing obstruction.

9.2.8 Knock Rating of Fuels

Knock rating determines whether or not a fuel will knock in a given engine under the given operating conditions. Hydrocarbons of extended chain-like structure oxidize easily and knock readily and hard, whereas those of clustered structure iodize less easily and do not knock so readily. Therefore, SI fuels are closed cycle and CI fuels are chain-like or open. Octane rating and cetane rating are the two rating systems for fuels used in SI and CI engines, respectively.

9.2.8.1 Octane Rating Iso-octane (C_8H_{18}) has low boiling point and possesses a very good antiknock quality. On the other hand, n-heptane (n-C_7H_{16}) has very poor antiknock qualities. *Octane rating* is used for fuels of SI engines. This is equivalent percentage of iso-octane with n-heptane. This rating is also represented as *octane number* (ON), which is scaled from 100 to 0.

Since the *motor method* of determining the octane number uses more severe operating conditions than those in research method, the *motor octane number* (MON) is found lower than the *research octane number* (RON). The difference MON and RON is called *fuel sensitivity*. In this reference, a term, *antiknock index* is defined as the average of RON and MON.

9.2.8.2 Cetane Rating *Cetane rating*, used for fuels of CI engines, is the percentage of cetane ($C_{16}H_{34}$, hexadecane) with isocetane (hepta-methylnonane) to make equivalent fuel with respect to knocking. Lower the cetane number, higher is hydrocarbon emission and noise and leads to increased amount of white smoke. Normal paraffins C_nH_{2n+2}, which are straight-chain compounds, have the highest cetane number and lowest specific gravity.

An unnecessary higher octane number in SI engines is a waste, but not harmful in operation; whereas an unnecessary cetane number in CI engines can induce pre-ignition.

9.2.9 Performance Analysis

This section describes the calculation of performance parameters.

9.2.9.1 Physical Dimensions Important physical dimensions of IC engines are

1. Stroke length (L)
2. Engine rpm (N)
3. Number of cylinders (n)
4. Bore diameter (d)

The following are the relevant expressions to above dimensions:

1. Average Piston Speed In one cycle, the distance traveled by piston is $2L$. Time for one cycle is $60/N$ s. So, average piston speed is
$$V_p = 2L\frac{N}{60}$$
2. Swept Volume
$$v_s = n \times \frac{\pi}{4} d^2 L$$
3. Cylinder Volume If v_c is the clearance volume, then cylinder volume is given as
$$v = v_c + v_s$$

9.2.9.2 Performance Parameters The performance of an engine is assessed in the following terms:

1. Engine Power The engine power is measured in three ways:

   (a) *Indicated power* (P_i) is based on forces measured at the combustion chamber. This can be calculated by multiplying mass flow rate of air ($\dot{m}_a$) to work done per cycle (W_{cycle}) by the engine.
   $$P_i = \dot{m}_a \times W_{cycle} \times n$$

   (b) *Brake power* (P_b) is based on the torque developed at crankshaft. This is also called shaft power.

   (c) *Friction power* (P_f) is the power required for motoring the engine (without fuel supply), which shows the power required to start the engine:
   $$P_f = P_i - P_b$$

   Dynamometer power is
   $$W = \frac{\text{Load in kg} \times \text{Speed}}{\text{Dynamometer constant}}$$

2. Mean Effective Pressure Mean effective pressure of cycles is measured in two forms:

   (a) *Indicated Mean Effective Pressure* (p_{im}) is the mean pressure inside the cylinder throughout the cycle based on work done. This is calculated from indicator diagram, and is equal to the area under p–v plot. Therefore,
   $$p_{im} = \frac{\text{Area of indicator diagram}}{\text{Length of diagram}}$$

   An indicator diagram is drawn by a mechanical or electrical instrument attached to the cylinder. A spring constant is used to scale this diagram:
   $$p_{im} = \frac{\text{Area} \times \text{Spring constant}}{\text{Length of diagram}}$$

   Indicated work output (W_i) per cycle is
   $$W_i = p_{im} \times v_s$$

   Indicated power (P_i) is related to W_i as
   $$\begin{aligned} P_i &= W_i \times \text{Cycles per unit time} \\ &= p_{im} \times v_s \times \frac{N}{60} \times n \\ &= \frac{p_{im} \times v_s}{1000} \times \frac{N}{60} \times n \text{ kW} \end{aligned} \tag{9.13}$$

   (b) *Brake Mean Effective Pressure* (p_{mb}) is related to the brake power as
   $$P_b = \frac{p_{bm} \times v_s}{1000} \times \frac{N}{60} \times n \text{ kW} \tag{9.14}$$

   The torque developed at crankshaft is proportional to p_{bm} because
   $$P_b = \frac{2\pi N}{60} T \tag{9.15}$$

3. Engine Efficiencies Engine efficiency is expressed in the following terms:

   (a) *Air standard efficiency* (η_{Cycle}) is the cycle efficiency of the thermodynamic reversible cycle on which the engine cycle is based.

   (b) *Indicated thermal efficiency* (η_i) is based on the indicator diagram.

   (c) *Mechanical efficiency* η_m of an engine is defined as
   $$\eta_m = \frac{P_b}{P_i}$$

   Using Eqs. (9.13) and (9.14), one gets
   $$\eta_m = \frac{p_{bm}}{p_{im}}$$

4. Specific Fuel Consumption *Specific fuel consumption* can be calculated in two forms:

   (a) *Indicated specific fuel consumption* (ISFC)

   (b) *Brake specific fuel consumption* (BSFC)

   These are related as
   $$\text{ISFC} = \text{BSFC} \times \frac{\text{Brake power}}{\text{Indicated power}}$$

9.2.9.3 Friction Power Measurements The internal losses in an engine are essentially of two kinds, viz., pumping losses and friction losses. The friction loss is made up of the friction between the piston and cylinder walls, piston rings, crank shaft and bearings. The effect of such losses is measured as the difference between the indicated power and the brake power of an engine, known as *friction power*. Measurement of friction power is essential for calculation of mechanical efficiency of the engine. The following are the two well-known methods for measurement of friction power:

1. Willan's Method *Willan's method* is based on extrapolation of the graph between fuel consumption and brake power at constant speed on the negative axis of brake power where fuel consumption becomes zero [Fig. 9.45]. The negative brake power represents the friction losses of the engine at the given speed. Thus, the method is also known as *fuel rate extrapolation method*.

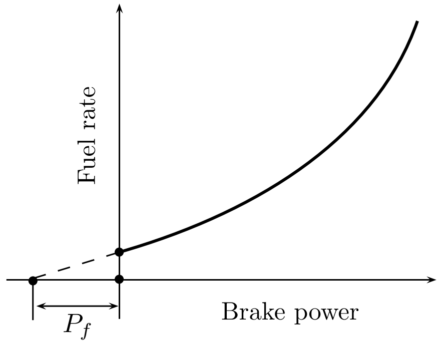


Figure 9.45 Willan's method.

The changing slope of the curve indicates the effect of part load efficiency.

2. _Morse Test_ The *Morse test*[16] is applicable for multi-cylinder engines. In this method, the cylinders of the engine are made inoperative and the reduction in brake power is noted. It is assumed that pumping and friction losses are the same as when the cylinder is inoperative as well as during firing.

9.2.10 Combustion Reactions

Combustion characteristics of fuel-air mixture affect the performance of engine.

9.2.10.1 Combustion The combustion of fuel in internal combustion engines is a fast exothermic chemical reaction in the gaseous phase where oxygen obtained from air is usually one of the reactants.

9.2.10.2 Composition of Air Combustion takes places with the help of air which is not a pure oxygen but a mixture of 21 % O_2, 78% N_2, and 1% argon by volume.

To simplify the study, 100 mol of air are assumed to contain of 21 mol of O_2 and 79 mol of N_2 (air contains 23% of O_2 and 77% N_2 by mass). The count for argon is merged in counts of N_2. Therefore, for each mole of O_2 in air, there are 3.76 mol of N_2.

9.2.10.3 Combustion with Air When a hydrocarbon burns in oxygen, the reaction will only yield carbon dioxide and water. When other elements of fuel, such as carbon, nitrogen, and sulphur, are burned, they yield common oxides.

The general formula for the molecule of fuel used in IC engines can be taken as $C_mH_nO_p$. The basic stoichiometric (chemically correct) equation for fuel air reaction is

$$\begin{aligned}C_mH_nO_p + \zeta \times O_2 + 3.76\zeta \times N_2 &\longrightarrow \\ mCO_2 + \frac{n}{2} \times H_2O + 3.76\zeta \times N_2\end{aligned}$$

where ζ is the chemically correct moles of O_2 per mole of fuel. N_2 does not take part in the reaction.

The number of moles of any element must be equal in both sides of stoichiometric equation. For oxygen,

$$\frac{p}{2} + \zeta = m + \frac{n}{4}$$
$$\zeta = m + \frac{n}{4} - \frac{p}{2}$$

9.2.10.4 Equivalence Ratio *Equivalence ratio* (ϕ) is defined as the ratio of actual fuel–air ratio to the stoichiometric fuel–air ratio. Quality of fuel–air mixture is assessed in terms of ϕ:

$$\phi \begin{cases} > 1 \text{ Rich mixture} \\ = 1 \text{ Stoichiometric mixture} \\ < 1 \text{ Lean mixture} \end{cases}$$

The inverse of equivalence ratio is called *relative fuel–air ratio*, denoted by λ.

9.2.10.5 Oxygen Requirement Theoretical amount in kg of oxygen required per kg of fuel to burn can be determined as

$$m_{O_2} = \frac{32}{12}C + \frac{32}{4}H + \frac{32}{32}S - O$$

where C, H, S, O are moles of carbon, hydrogen sulphur and oxygen, respectively, in the fuel. From this, the total mass of air in kg is

$$m_{air} = \frac{m_{O_2}}{0.23}$$

Fuel is of finite size, and each droplet must be surrounded by more than the necessary number of oxygen molecules to assure oxidation of all the fuel molecules. Thus, required excess air is approximately 20% more than theoretical value.

9.2.10.6 Orsat Apparatus The combustion products can be analyzed in two ways:

1. _Dry Analysis_ The sample of gaseous combustion product is usually cooled down to a temperature, which is below the saturation temperature of the steam present. Therefore, the steam content is not included in the analysis. Such analysis is quoted as *dry analysis* of the products.

2. _Wet Analysis_ Since the products are gaseous, it is usual to quote the analysis by volume. When

[16] The method can be understood better by referring the numerical examples.

steam is included in the exhaust, the analysis is called *wet analysis*.

The most common means of analysis of combustion products is the *Orsat's apparatus*. The apparatus is used to measure volumes of carbon dioxide, oxygen and carbon monoxide within a fixed volume of a sample gas. In this method, a fixed volume of the sampled gas is passed through a specific solution, which absorbs only the required gas. The remaining volume of the gas can then be remeasured and compared with the original volume to find the proportion of a specific gas within the sample.

9.3 REFRIGERATION AND AIR CONDITIONING

Refrigeration is the process of building and maintaining temperature of a system below the temperature of the surroundings. *Air conditioning* is the treatment of air to simultaneously control its temperature, moisture content, cleanliness, odor and circulation, as required by the occupants, a process, or products in the space.

One ton refrigeration is defined as the rate of heat transfer needed to produce one ton ice at 0°C from water at 0°C in one day:

$$1 \text{ ton} = 3.5 \text{ kW} = 210 \text{ kJ/min} = 50 \text{ kcal/min}$$

9.3.1 Refrigerants

A *refrigerant* can be defined as any body or substance that can extract heat from another body or substance. Thus, ice, cold water, cold air, etc. can be treated as refrigerants. Air is a very safe refrigerant which is available free of cost, but *coefficient of performance* (COP) of air-based refrigeration systems is low. It is used in aircraft refrigeration systems.

Most of the problems associated with early refrigerants such as toxicity, flammability, and material incompatibility were completely eliminated by chlorofluorocarbon (CFC) compounds, known as freons. Rowland and Molina[17] argued that the highly stable CFC's cause the depletion of stratospheric ozone layer. Later, CFC-based compounds were also found contributing significantly to the *global warming*. *Halide torch* or leak detector is used to detect leakages of fluorocarbon refrigerants. When a fluorocarbon-based refrigerant is passed over a surface at high temperature (around 500°C), its vapor forms phosgene ($COCl_2$) which in turn is passed over a glowing copper where it forms copper chloride, which changes the color of the flame from pale blue to bright green.

[17]In 1974, Mario Molina and Sherwood Rowland published their widely noted Nature article on the threat to the ozone layer from CFC gases - "freons". They were co-recipients (along with Paul J. Crutzen) of the 1995 Nobel Prize in Chemistry for their role in elucidating the threat to the ozone layer.

Water (H_2O, denoted by R-718) can be used upto 5°C. *Water* refrigeration is used in many applications, such as pre-cooling of vegetables, fruits, chilling of water for drinking.

Ammonia (NH_3, denoted by R-717) has a high value of latent heat of evaporation. It is not miscible in mineral oils, but dissolves in water and forms explosive mixture with air. It reacts with copper and its alloys in the presence of water. Due to toxicity and irritability, ammonia is used in industrial applications and not in air conditioning plants. Oil separator is necessarily provided in ammonia refrigeration systems because oil is partially miscible and causes choking of expansion device. Burning of ammonia with sulphur candle gives white smoke of NH_4SO_4, while that with HCl gives white flame of NH_4Cl.

Carbon dioxide (CO_2) is heavier than air, non-toxic, non-flammable. Its main use is in the form of dry ice that sublimes at −78.3°C.

Sulphur dioxide (SO_2) needs small capacity compressor. It gives gives sneezing smell. Copper is not suitable with SO_2 and CO_2.

9.3.1.1 Desired Properties

The following are the desired properties of refrigerants:

1. ***Vapor Density*** To enable use of smaller compressors and other equipments, the refrigerant should have smaller vapor density. For example, reciprocating compressors are unsuitable for refrigeration systems due to enormously large specific volume of water vapor at low pressure.

2. ***Enthalpy of Vaporization*** To ensure maximum heat absorption during refrigeration, a refrigerant should have high enthalpy of vaporization.

3. ***Thermal Conductivity*** Thermal conductivity of the refrigerant should be high for faster heat transfer during condensation and evaporation.

4. ***Dielectric Strength*** In hermetic arrangements, the motor windings are cooled by refrigerants' vapor on its way to the suction valves of the compressor. Therefore, dielectric strength of a refrigerant is important in hermetically sealed compressor units.

5. ***Critical Temperature*** In order to have large range of isothermal energy transfer, the refrigerant should have critical temperature above the condensing temperature.

6. ***Specific Heat*** Specific heat of a substance can be written as

$$c_{pf} = T\left(\frac{\partial s}{\partial T}\right)_p$$

To have minimum change in entropy during the throttling process, the specific heat should be minimum. For this, liquid saturation line should be almost vertical.

9.3.1.2 Designation of Refrigerants The saturated hydrocarbons are represented as C_nH_{2n+2} and unsaturated carbons by C_nH_{2n}. Migley and Kettering[18] introduced a classification to express all the refrigerants by a very convenient notation called R_{abc}, which has been adopted by ASHRAE. According to this, *methane* based refrigerant compounds are designated in the following format

$$R-[C-1]\,[H+1]\,[F]$$

Chlorine atoms have to satisfy the remaining valency of the carbon atoms, therefore,

$$Cl = 2\,[C+1]-H-F$$

An additional subscript "1" is added to *abc* to designate unsaturated hydrocarbon refrigerants, such as R1150 (Ethylene, C_2H_4), R1130 (Dielene, $C_2H_2Cl_2$).

Designations of some important refrigerants is shown in Table 9.6. R-1000 series is used for unsaturated compounds. R-700 series of designation is used for miscellaneous group of refrigerants with molecular weight in the last digits. R-600 series represents compounds of oxygen and nitrogen. R-500 series is for azeotropes, a mixture of two or more liquids in such a way that its components cannot be altered by simple distillation. For example, *Cerrene* is azeotropic mixture of genetron-100 and Freon-12.

Table 9.6 Designation of refrigerants

R-20	$CHCl_3$	Chloroform
R-12	CCl_2F_2	Dichloro difluoromethane
R-13	$CClF_3$	Monochloro trifluoromethane
R-10	CCl_4	Carbon tetrachloride
R-50	CH_4	Methane
R-170	C_2H_6	Ethane
R-717	NH_3	Ammonia
R-718	H_2O	Water
R-720	Ne	Neon
R-728	N_2	Nitrogen
R-732	O_2	Oxygen

[18] Thomas Migley, Jr. (1889-1944) and Charles Kettering (1876-1958) carried the pioneering work of synthesizing series of chlorofluorocarbons. Kettering was head of research for General Motors from 1920 to 1947. He was also responsible for the invention of Freon refrigerant for refrigeration and air conditioning systems.

9.3.2 Air Refrigeration Cycles

Air refrigeration systems are used in cooling of aircraft cabin and also in the liquefaction of some gases. These systems use air as the working fluid which behaves as a pure substance (does not undergo any phase change during the cycle).

Thermodynamic analysis of air refrigeration cycles is based on the assumption that the working fluid (air) is an ideal gas having a constant mass and specific heat. The cycle is assumed to be a reversible closed-loop cycle. An analysis with the these assumptions is called as *cold air standard cycle analysis*. The analysis is invalid if the cycle includes a throttling process.

9.3.2.1 Reversed Carnot Cycle *Reversed Carnot cycle* is a reversible refrigeration cycle consisting of two isentropic processes and two isothermal processes [Fig. 9.46].

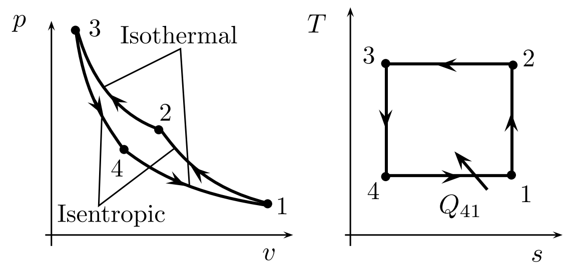


Figure 9.46 Reversed Carnot cycle.

The refrigeration effect and work transfer in the cycle are determined as follows:

1. *Refrigeration Effect* The refrigeration occurs during 4-1 when the cycle extract heat Q_2 from external source:

$$Q_2 = T_1\,(s_1 - s_4)$$

2. *External Work* The work transferred to the working fluid during the cycle is equal to the area on p–v diagram. For the present case, the work transfer to the cycle is also equal to the area of T–s diagram:

$$W = \oint \delta Q$$
$$= -\,(T_2 - T_1)\,(s_1 - s_4)$$

Coefficient of performance (COP) of reversed Carnot cycle can also be determined as

$$\text{COP} = \frac{Q_2}{|W|}$$
$$= \frac{T_1}{T_2 - T_1}$$

Reversed Carnot cycle is not feasible because it suffers from two practical limitations:

1. Isothermal changes are possible with extremely slow motion of working fluid, while the isentropic processes require extremely faster rate of flow.
2. The cycle is impossible without phase changes, which can lead to severe mechanical difficulties. For example, compressor cannot work with liquid phase because of incompressibility.

The idealization of reversed Carnot cycle is used to compare the performance of a refrigeration cycle.

9.3.2.2 Reversed Brayton Cycle *Reversed Brayton cycle*, also known as *Bell-Colemen cycle*, is frequently employed in gas cycle refrigeration systems [Fig. 9.47].

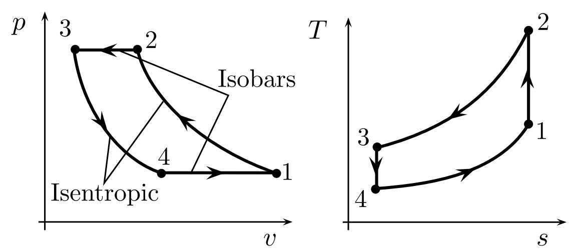


Figure 9.47 | Reverse Brayton cycle.

The working fluid of the cycle undergoes isentropic compression (1→2), isobaric heat rejection (2→3), isentropic expansion (3→4) and isobaric heating (4→1). The refrigeration effect and work transfer in the cycle are determined as follows:

1. *Refrigeration Effect* The refrigeration occurs during 4→1 when the cycle extract heat Q_2 from external source:
$$Q_2 = h_1 - h_4$$
2. *External Work* The work transferred to the working fluid during the cycle is equal to the area on p–v diagram. For the present case, the work transfer to the cycle is determined as
$$\begin{aligned} W &= \oint \delta Q \\ &= Q_{2\to3} - Q_{4\to1} \\ &= (h_3 - h_2) + (h_1 - h_4) \end{aligned}$$

Coefficient of performance (COP) of reversed Brayton cycle can also be determined as

$$\begin{aligned} \text{COP} &= \frac{Q_2}{|W|} \\ &= \frac{1}{r_p{}^{(\gamma-1)/\gamma}} \end{aligned}$$

where r_p is the pressure ratio:

$$r_p = \frac{p_2}{p_1}$$

Thus, COP of the reversed Brayton cycle decreases with increase in pressure ratio (r_p).

9.3.3 Vapor Compression System

Vapor compression systems involve phase transformation (vaporization) of the working fluid during which a large amount of (latent) heat can be transferred to the refrigerant at a near constant temperature.

Expansion valves, orifices or capillary tubes (common in domestic refrigerators and window air conditioners) are used to control the flow of refrigerant from the high pressure (condensing side) of the system into the low pressure (evaporator). Interestingly, flow rate is a function of pressure difference and the degree of liquid sub-cooling on entry; therefore, the capillary tube is self-regulating within certain parameters. The tube bores of 0.8–2 mm with length of 1–4 mm are common.

A *thermostat* is fitted in air conditioning system to stop the equipment or reduce its capacity when the required condition is reached.

9.3.3.1 Simple Cycle In simple vapor compression cycle, dry saturated vapor at state 1 is compressed isentropically to state 2 from where it transforms to condensate state 3 by isobaric heat transfer. The liquid state at 3 is then subjected to throttling (irreversible isenthalpic process) to vapor state at 4 [Fig. 9.48].

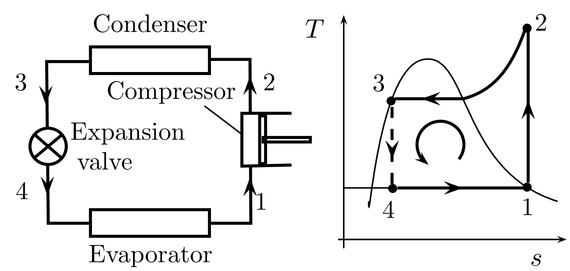


Figure 9.48 | Vapor compression cycle.

The refrigeration effect and work transfer in the cycle are determined as follows:

1. *Refrigeration Effect* Enthalpy remains constant during isentropic expansion ($h_3 = h_4$). Refrigeration occurs during 4→1 when the cycle extract heat Q_2 from external source:
$$\begin{aligned} Q_2 &= h_1 - h_4 \\ &= h_1 - h_3 \end{aligned}$$

2. External Work The work transferred to the working fluid during the isentropic compression ($1 \to 2$):

$$W = -(h_2 - h_1)$$

Coefficient of performance of the cycle can be determined as

$$\text{COP} = \frac{Q_2}{|W|}$$

The COP of the simple vapor compression cycle can be modified by the following options:

1. *Changing p_1 and p_2* - The effect of changes in pressure limits can be visualized on T–s diagram in following manner:
   (a) *Reducing p_1* - COP decreases because compression work has increased.
   (b) *Increasing p_2* - COP decreases because compression work has increased.
   (c) *Both (a) and (b)* - COP decreases because net compression work has increased.

   In using the above options, the effect of these changes occurs in the following sequence:

   $$\text{COP} > \text{COP}_1 > \text{COP}_2 > \text{COP}_3$$

   where the subscripts 1, 2, and 3 indicate the respective options of changing COP.

2. Sub-cooling The net effect of sub-cooling of condensate before throttling is seen as significant reduction in the power consumption and enhanced COP. Refrigeration effect is also increased. Sub-cooling beyond optimum limits offsets the performance.

3. Super-heating Super-heating of vapor before compression offers the following advantages:
   (a) Avoiding possibility of wet compression.
   (b) Increasing the refrigeration effect.
   (c) Reduction in mass flow rate due to increase in specific volume.

   However, super-heating requires more work due to diverging nature of isentropic lines.

9.3.3.2 Cascade Refrigeration System Many industrial applications involve very low temperature and pressure, such as in liquefaction of atmospheric gases and petroleum vapors, manufacturing of dry ice, precipitation hardening of special alloy steels. Refrigeration to such a low temperature is difficult in single-stage systems. Also, sealing against leakage into the low pressure system and large displacement volume forbid use of refrigerants, like freons. Therefore, multi-stage refrigeration, known as *cascade refrigeration system*, is employed in such applications which is a combination of two independent simple vapor compression systems in such a way that the evaporator of the high temperature system becomes the condenser of the low temperature system. The working media of the two cycles are separated from each other.

Consider a cascade refrigeration system working between upper and lower temperatures T_1 and T_2, respectively. The system consists of two refrigeration cycles having coefficient performance COP_1 and COP_2 [Fig. 9.49].

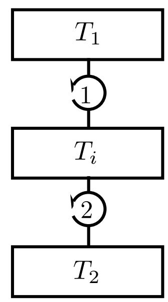


Figure 9.49 Cascade refrigeration system.

The following cases can be analyzed using basic thermodynamics:

1. The *effective COP* of the cascade system is expressed in terms of the COP's of individual cycles as

   $$\text{COP} = \frac{\text{COP}_1 \times \text{COP}_2}{1 + \text{COP}_1 + \text{COP}_2}$$

2. For equal COP of the individual cycles, intermediate temperature of the system is

   $$T_i = \sqrt{T_1 T_2}$$

9.3.4 Psychrometry

Psychrometrics is the study of air-water vapor mixture, better known as *moist air*. In the moist air, the composition of dry air is constant (assumed per kg), but the amount of water vapor present in the air can vary from zero to a maximum depending upon the temperature and pressure of the mixture. The molecular weight and gas constant of air and water vapor are tabulated in Table 9.7.

9.3.4.1 Properties of Moist Air The following are the relevant properties of moist air:

Table 9.7 Molecular weight and gas constant

Gas	M	R (kJ/kgK)
Pure air	28.97	0.287
Water vapor	18.02	0.461

1. *Specific Humidity* *Specific humidity* (w), also known as *humidity ratio*, is the ratio of mass of vapor (m_v) and mass of air (m_a) in a given volume of moist air (of total mass m) at a given temperature (T) and pressure (p):

$$w = \frac{m_v}{m_a}$$

If p_v is partial pressure of water vapor in the moist air, then using the rule of gas mixtures:

$$m_v = \frac{p_v v}{0.461 \times T}$$
$$m_a = \frac{(p - p_a)\,v}{0.287 \times T}$$

Therefore,

$$w = 0.622 \frac{p_v}{p - p_v}$$

For a constant pressure p (say atmospheric pressure),

$$w = f(p_v)$$

Mass of moist air is generally represented in per kg of dry air:

$$\frac{m}{m_a} = \frac{m_a + m_v}{m_a} = (1 + w)\ \text{kg}$$

The moist air is called *saturated* one if its partial pressure (p_v) is equal to the saturation pressure (p_s) corresponding to the given temperature (T) of the moist air. Thus, specific humidity of saturated moist air is written as

$$w_s = 0.622 \frac{p_s}{p - p_s}$$

2. *Relative Humidity* *Relative humidity* (ϕ) is the ratio of mass of water vapor (m_v) in a certain volume (v) of the moist air at a given temperature (T) to the mass of water vapor (m_s) in the same volume (v) of saturated moist air at the same temperature (T):

$$\phi = \frac{m_v}{m_s} = \frac{(p_v v)\,/\,(R_v T)}{(p_s v)\,/\,(R_v T)} = \frac{p_v}{p_s}$$

3. *Degree of Saturation* *Degree of saturation* (μ) is defined as the ratio of specific humidity of a given moist air (w) and specific humidity of the saturated moist air (w_s) at the same temperature (T) and pressure (p):

$$\mu = \frac{w}{w_s}$$

Using the definition of ϕ

$$\mu = \frac{w}{w_s} = \frac{p_v}{p_s} \times \frac{p - p_s}{p - p_v} = \phi \left(\frac{p - p_s}{p - p_v} \right)$$

4. *Enthalpy* Enthalpy of a moist air (in kJ/kg of dry air) at atmospheric pressure and temperature t°C is the sum of the enthalpies of dry air and water vapor:

$$h = 1.005t + w\,(h_{dp} + c_p t)$$

where h_{dp} and c_p are enthalpy and specific heat of steam at dew point temperature, respectively. Their approximate values at $p = 1$ bar and $T = 237.15$ K are as follows:

$$h_{dp} \approx 2500\ \text{kJ/kg}$$
$$c_p \approx 1.88\ \text{kJ/kgK}$$

5. *Dry Bulb Temperature* *Dry bulb temperature* (DBT) is the temperature of air measured by a thermometer freely exposed to the moist air but shielded from radiation.

6. *Dew Point Temperature* *Dew point temperature* (DPT) is the saturation temperature corresponding to partial pressure (p_v) of vapor defined as the temperature when a moist air is cooled such that its vapor partial pressure remains constant when the first drop of liquid is found [Fig. 9.50].

   If moist air is cooled at constant pressure (p), partial pressure of each constituent (p_v and p_a) remains constant (as mole fraction remains constant) until the water vapor reaches the saturation state, after which further cooling causes condensation. This temperature at which condensation starts is called the *dew point temperature* of the moist air which is equal to the saturation temperature corresponding to the partial pressure (p_v) of the vapor in the moist air.

7. *Wet Bulb Temperature* The temperature recorded by a thermometer when the bulb is enveloped by a cotton wick saturated with water is called *Wet bulb temperature* (WBT) [Fig. 9.51]. The temperature of the water in the soaked wick reduces because

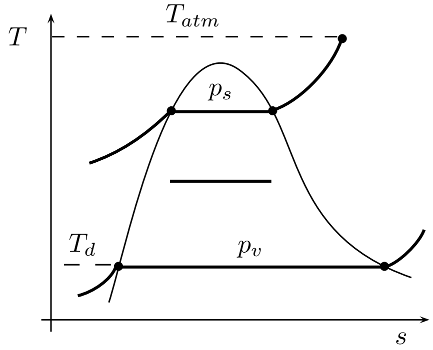


Figure 9.50 | Dew point temperature.

the latent heat of water in the soaked wick is transferred to air by evaporation. WBT is measured at equilibrium when energy removed from the water film by evaporation is equal to the energy supplied to the wick by heat transfer.

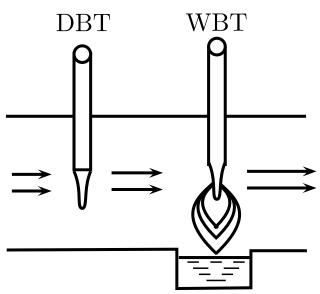


Figure 9.51 | Recoding of WBT and DBT.

When unsaturated moist air flows over along sheet of water in an insulated chamber, the water evaporates and the specific humidity of the air increases, but the air and water are cooled (1→3) as evaporation takes place [Fig. 9.52]. When a thermal equilibrium exists with respect to the water, air, and water vapor, the air gets saturated. The equilibrium temperature is called *adiabatic saturation temperature* or thermodynamic wet bulb temperature. The constant WBT line represents the adiabatic saturation process. It also coincides with the constant enthalpy line.

8. <u>*Wet Bulb Depression*</u> Wet bulb temperature of an unsaturated moist air is always lower than the dry bulb temperature. *Wet bulb depression* of a moist air is defined as the difference between its dry and wet bulb temperatures. It is zero for saturated air.

9.3.4.2 State of Moist Air Moist air is composed of two components ($c = 2$), air and water, in a single phase ($\phi = 1$). Number of *degrees of freedom* of moist air can

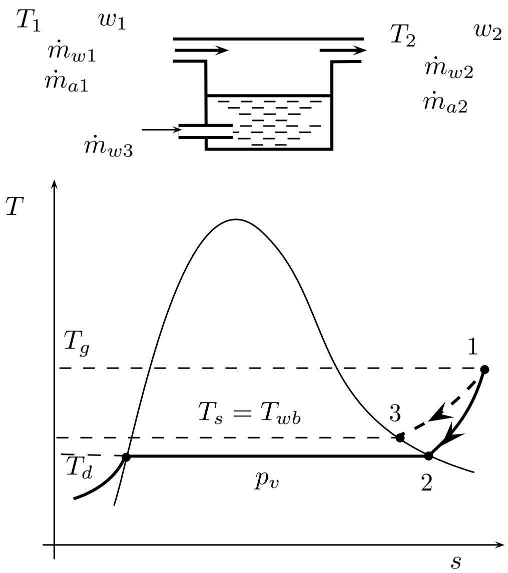


Figure 9.52 | Adiabatic saturation.

be obtained by using *Gibbs phase rule*:

$$\begin{aligned} f &= c - \phi + 2 \\ &= 3 \end{aligned}$$

Thus, the state of a moist air can be defined by three independent variables. Empirical relations are available for determining the properties of moist air, such as modified Apjohn equation, modified Ferrel equation, and Carrier equation.

Carrier equation[19] enables calculation of partial pressure of vapor p_v in a given moist air, if dry bulb temperature T (°C) and wet bulb temperature T' (°C) are known:

$$p_v = p'_v - \frac{(p - p'_v)\,(T - T') \times 1.8}{2800 - 1.3\,(1.8t + 32)}$$

The saturation pressure p'_v corresponding to T' is found from steam tables or charts.

The *psychrometric chart* is a plot between specific humidity w and temperature T (DBT, WBT, and DPT) at atmospheric pressure ($p = p_0$) [Fig. 9.53]. Thus, total two variables are also used here to fix the state of a given moist air because one variable is fixed by taking $p = p_0$.

A point on the psychrometric chart determines the state of the given moist air in terms of DBT, WBT, DPT, enthalpy h of the moist air. It is observed that

[19] Willis H Carrier (1876-1950) is considered the father of air conditioning who placed the science of air conditioning on a firm scientific basis in the year 1911, in his paper Rational Psychrometric Formulae widely known also as the 'Magna Carta of Psychrometrics'. The ASHRAE psychrometric chart was pioneered by him in 1904.

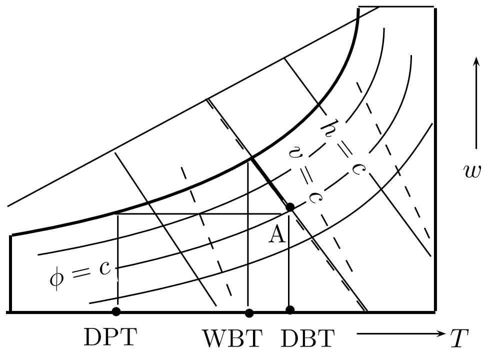


Figure 9.53 Psychrometric chart.

for saturated moist air, DBT, WBT, DPT are the same. These charts are very useful in solving the numerical problems of air conditioning systems.

9.3.4.3 Psychrometric Processes

The moist air can be subjected to many types of processes to meet the requirement of air conditioning systems. Such processes involve heat transfer. If temperature changes during heat addition or rejection, it is called *sensible heating*, otherwise, it is called *latent heating* [Fig. 9.54]. Heat transfer per kg of dry air in process 1→2 is equal to the difference of enthalpies at terminal states:

$$Q = h_2 - h_1$$

Heat transfer to the moist air is characterized by two factors:

1. Sensible Heat Factor *Sensible heat factor* (SHF) is defined as the ratio of sensible heat (SH) to the total heat, that is, latent heat (LH) plus sensible heat, supplied to the moist air [Fig. 9.54]:

$$\text{SHF} = \frac{\text{SH}}{\text{SH+LH}}$$

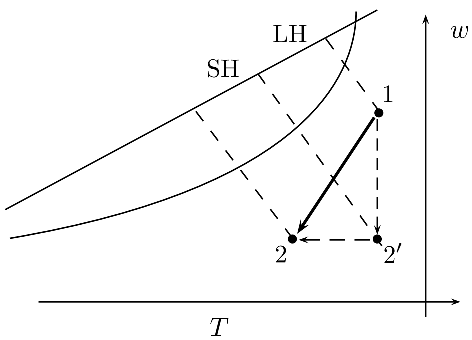


Figure 9.54 Sensible heat factor.

2. By-pass Factor Sensible cooling or heating of moist air is done by passing it through a coil (say at temperature T_s). However, the temperature of the moist air at the outlet of the cooling coil T_2 is always lesser (heating) or more (cooling) than T_s [Fig. 9.55].

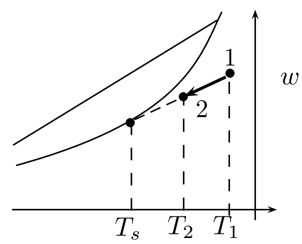


Figure 9.55 By-pass factor.

To quantify this effect, *by-pass factor* (B) of a coil is defined as the ratio of final and initial differences between the temperatures of coil and moist air:

$$\begin{aligned} B &= \frac{T_s - T_2}{T_s - T_1} \quad \text{(heating)} \\ &= \frac{T_2 - T_s}{T_1 - T_s} \quad \text{(cooling)} \end{aligned}$$

Temperature T_s of the cooling coil is called *apparatus dew point temperature*.

By-pass factor B can be thought as from 1 kg of moist air reaching the coil, B kg is by passed without any effect. Mathematically

$$T_1 B + T_s (1 - B) = T_2$$

Coil efficiency is expressed as

$$\eta_{coil} = 1 - B$$

By-pass factor increases with increasing velocity of air because air does not get much time for heat transfer. Figure 9.56 shows various types of psychrometric processes on the psychrometric charts.

Psychrometric processes demonstrated in Fig. 9.56 are discussed as follows:

1. Humidification Increasing humidity by the washer of moisture is called *humidification*. Cooling and humidification are done by spray washer. The adiabatic saturation occurs along the constant WBT line.

2. Chemical Dehumidification Few substances, like silica gel or activated alumina, absorbs water molecules due to affinity during which latent heat

of condensation is released. This is known as *chemical dehumidification* that occurs along a constant enthalpy line towards a lower humidity ratio. Therefore, DBT increases but DPT decreases.

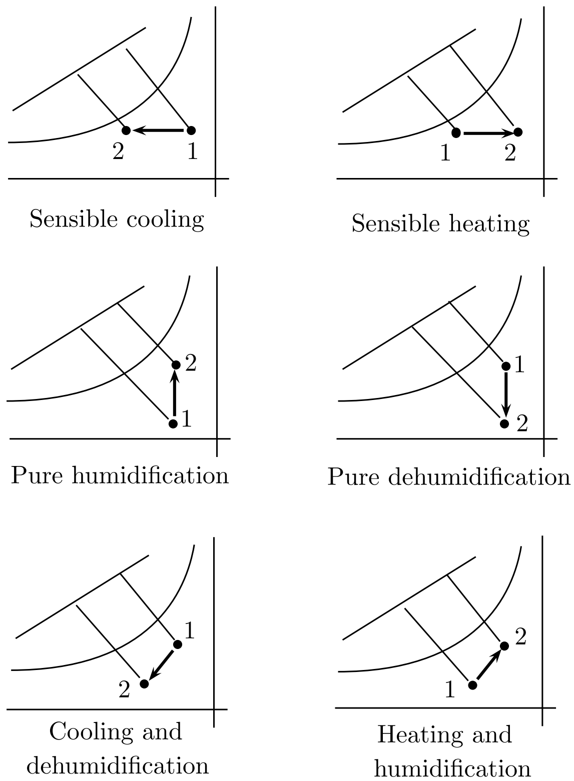


Figure 9.56 Psychrometric processes.

3. Sensible Cooling During *sensible cooling*, the moisture content of air remains constant but temperature decreases. For this, the surface of the cooling coil should be dry and its temperature should be greater than the dew point temperature of air.

4. Sensible Heating *Sensible heating* is obtained by passing the moist air through a heating coil. Moisture content of air remains constant. The heat transfer rate per kg of dry air during this process is given by

$$\mathrm{SH} = h_1 - h_2$$

5. Cooling and Dehumidification When moist air is cooled below its dew-point by bringing it in contact with a cold surface, some of the water vapor in the air condenses and leaves the air stream as liquid. As a result, both the temperature and humidity ratio of air decreases. This process is used in summer air conditioning systems.

6. Heating and Humidification During winter, it is essential to heat and humidify the room air for comfort. This is normally done by first sensibly heating the air and adding water vapor to the air stream through steam nozzles.

7. Evaporative Cooling Evaporative cooling is based on the fact that latent heat of vaporization of water is absorbed from the water body and the surrounding air. As a result, both the water and the air are cooled during the process.

   Evaporative cooling is essentially identical to the adiabatic saturation process that occurs at constant wet bulb temperature.

8. Mixing of Air Streams *Mixing of air streams* at two different states is a common psychrometric process in air conditioning. Depending upon the state of the individual streams, the mixing process can take place with or without condensation of moisture, described as follows:

   (a) Mixing Without Condensation Figure 9.57 shows an adiabatic mixing of two moist air streams during which no condensation of moisture takes place. When two air streams at state points 1 and 2 are mixed with each other, the resulting mixture condition 3 can be obtained from mass and energy balance:

$$m_{a,1}w_1 + m_{a,2}w_2 = (m_{a,1} + m_{a,2})\, w_3$$
$$m_{a,1}h_1 + m_{a,2}h_2 = (m_{a,1} + m_{a,2})\, h_3$$

   Hence, the final enthalpy and humidity ratio of mixture are weighted averages. A generally valid approximation is that the final temperature of the mixture is the weighted average of the inlet temperatures.

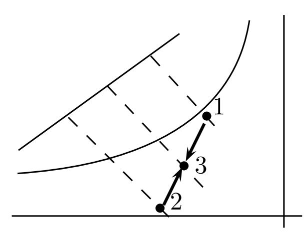


Figure 9.57 Mixing of air streams.

   Therefore, the point on the psychrometric chart representing the mixture lies on a straight line connecting the two inlet states at the inverse ratio of masses:

$$\frac{(1 \to 3)}{(2 \to 3)} = \frac{m_{a2}}{m_{a1}}$$

   (b) Mixing With Condensation When very cold and dry air is mixed with a warm air at high relative humidity, the resulting mixture

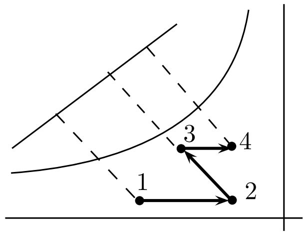


Figure 9.58 Winter air conditioning.

condition can lie in the two-phase region. There will be condensation of water vapor; some amount of water will leave the system as liquid water. Due to release of latent heat, the temperature of air increases. This is seen as the formation of fog or frost in winter when the cold air near the earth mixes with the humid and warm air, which develops towards the evening or after rains.

9.3.5 Air Conditioning

Condition (state) of air is defined by its temperature and moisture content. Air conditioning involves a change in the condition of air. Inevitably "comfort" is a very subjective matter. Around 90% people are comfortable when the air temperature is between 18–22°C and the percentage saturation is between 40–65%. This zone, shown on the psychrometric chart, is known as the *comfort zone*.

Factors affecting comfort air conditioning are temperature, humidity, purity and motion of air. Dry bulb temperature affects heat transfer by convection and evaporation, relative humidity affects heat loss by evaporation, air velocity influences both convective and evaporative heat transfer, and the surrounding surface temperature affects the radiative heat transfer.

9.3.5.1 Summer and Winter Air Conditioning The requirements of air conditioning depends upon the condition of atmospheric air. Therefore, different processes or systems are used for air conditioning in winter and summer:

1. Winter Air Conditioning In winter, air needs to be heated and dehumidified. For this, air is first passed through the air preheater to prevent possible freezing of water and to control the evaporation of water into humidifier. After that air is passed through preheater to bring the air to the original dry bulb temperature [Fig. 9.58].

2. Summer Air Conditioning In summer, air needs to be cooled and dehumidified. Heating is done to bring the air to the designed DBT and RH [Fig. 9.59].

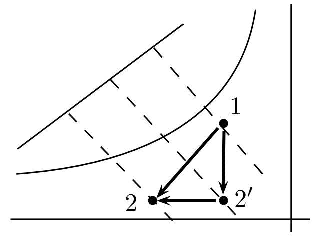


Figure 9.59 Summer air conditioning.

9.3.5.2 Load Calculations Design and analysis of air conditioning systems basically involve determination of the design capacity of cooling or heating equipment, selection of suitable cooling/heating system, selecting supply conditions, design of air transmission and distribution systems. Under a typical summer condition, the building gains sensible and latent heats from the surroundings and internal heat sources. The supply air to the building extracts heat gains from the conditioned space. These heat gains along with other heat gains have to be extracted from the air stream by the cooling coil, so that air at required cold and dry conditions can again be supplied to the building to complete the cycle [Fig. 9.60].

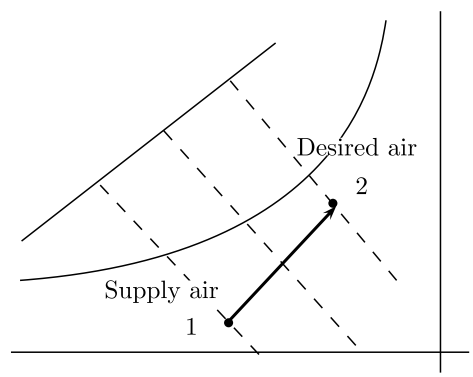


Figure 9.60 Load calculations.

In general, the sensible and latent heat transfer rates on the cooling coil are larger than the building heat gains due to the need for ventilation and return duct losses. The required cooling capacity of an air conditioning system is characterized by following estimates:

1. Room Sensible Heat Factor The effect of room heat on moist air is divided into two parts:

   (a) *Room sensible heat* (RSH) includes solar transmission, sensible heat of infiltrated of air,

internal heat gain from people, power, lights, etc.

(b) *Room latent heat* (RLH) includes latent heat of infiltrated air, heat gain from people, steam, etc.

Room sensible heat factor (RSHF) is defined as

$$\text{RSHF} = \frac{\text{RSH}}{\text{RSH+RLH}}$$

2. Outside Air Sensible and Latent Heats — The air conditioned space involves entry of outside air through ventilation which is associated with *outside air sensible heat* (OASH) and *outside air latent heat* (OALH) [Fig. 9.61]:

$$\text{OASH} = (h_g - h_i)\, m_{OA}$$
$$\text{OALH} = (h_o - h_g)\, m_{OA}$$

where m_{OA} is the mass of outside air, related to volume v and specific volume v_o of the outside air as

$$m_{OA} = \frac{v}{v_o}$$

Here, subscripts i and o are used to denote the conditions of inside and outside air, respectively.

3. Grand Total Heats — Room heats and outside heats form *Grand total heats*:

$$\text{GTSH} = \text{RSH} + \text{OASH}$$
$$\text{GTLH} = \text{RLH} + \text{OALH}$$

4. Effective Room Heats — The sensible and latent components of the effective room heats (ERH) are given by

$$\text{ERSH} = \text{RSH} + B \times \text{OASH}$$
$$\text{ERLH} = \text{RLH} + B \times \text{OALH}$$

where B is the by-pass factor of the cooling coil.

Heat interactions can be drawn on psychrometric chart as lines that denote the changes in the properties of moist air during air conditioning [Fig. 9.61]:

1. Room sensible heat line (RSHL)
2. Gross sensible heat line (GSHL)
3. Effective sensible heat line (ESHL)

The outside air enters at state O, gets modified in the coil through line GSHL upto point P. From this point P, the conditioned air enters the room and gets modified to room conditions upto point I along the line RSHL. GSHL line crosses the saturation line at apparatus dew point (ADP), but due to by-pass factor B, the air is modified only upto P. ESHL is drawn between ADP and I. The net changes in state of outside air is between points O and I.

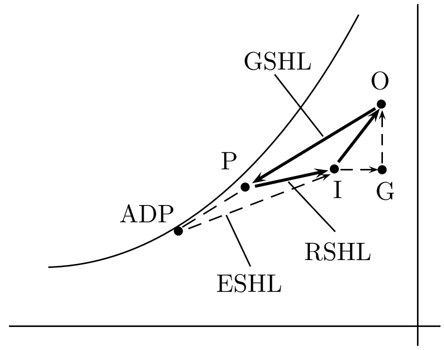


Figure 9.61 | Heat lines on psychrometric chart.

9.4 TURBOMACHINERY

Turbomachinery is an application of the basic principles of fluid mechanics. There are two broad categories of *turbomachinery*, *pumps* (energy absorbing devices) and *turbines* (energy producing devices). Pumps do not necessarily increase the speed of the fluid passing through it; rather they increase the pressure of the fluid. Of course, if the pump were turned off, there might be no flow of the fluid. A *compressor* is a gas pump designed to deliver a very high pressure rise, typically at low to moderate flow rates. Hydraulic turbines are power generating machines, which convert hydraulic energy into mechanical energy. The present context of study concerns hydraulic turbines (Pelton wheel, Francis turbine and Kaplan turbine). Working of centrifugal pumps is also discussed in the last section.

9.4.1 Fundamentals

9.4.1.1 General Layout A *dam* is constructed to store water at requisite head from where water is sent to hydraulic turbine through a large pipe known as *penstock*. A *forebay* is essentially a smaller reservoir at the head of the penstock. Water first enters into forebay, which in turn, distributes it to the penstock through which water is supplied to different turbines in a plant.

The turbine is attached to a generator whose speed of rotation (N) is related to frequency of power f and number of poles per generator (p):

$$N = \frac{120f}{p}$$

9.4.1.2 Hydraulic Heads The available hydraulic energy is represented in terms of the following heads:

1. Gross Head — The difference of head race level and tail race level is called *gross head*. It is the maximum head that can be availed at turbine.

2. *Net Effective Head* The actual head available at the inlet turbine is the gross head minus the head losses due to friction in the flow. It is known as *net effective head*.

9.4.1.3 Turbine Efficiencies

Power generation in hydraulic turbines is characterized by the following efficiencies:

1. *Hydarulic Efficiency* *Hydraulic efficiency* (η_h) is defined as

$$\eta_h = \frac{\text{Runner power}}{\text{Power supplied to runner}}$$

2. *Mechanical Efficiency* *Mechanical efficiency* (η_m) is defined as

$$\eta_m = \frac{\text{Shaft power}}{\text{Runner power}}$$

3. *Overall efficiency* *Overall efficiency* (η_o) is defined as

$$\begin{aligned}\eta_o &= \frac{\text{shaft power}}{\text{power supplied to runner}}\\ &= \eta_h \times \eta_m\end{aligned}$$

9.4.1.4 Types of Turbines

Based on the process of energy conversion, hydraulic turbines are categorized into two types:

1. *Impulse Turbines* In an *impulse turbine*, the available hydraulic energy is converted into kinetic energy at one time only. Pelton wheel is an impulse turbine.

2. *Reaction Turbines* In *reaction turbines*, the available hydraulic energy is converted into kinetic energy gradually by passing high head water into nozzle type blades in an air-tight casing. Francis, Kaplan, and propeller turbines are the reaction turbines.

Table 9.8 shows the classification of hydraulic turbines based on the direction of flow.

Table 9.8 Types of turbines based on direction of flow

Turbine Type	Example
Tangential flow	Pelton wheel
Radial flow	Francis turbine
Axial flow	Kaplan turbine, Propeller turbines
Mixed flow	Modern Francis turbine

Hydraulic turbines are designed for available head and discharge. Obviously, for the same power, high head turbine will require lesser discharge than that for low head turbines. Table 9.9 shows the classification of hydraulic turbines based on the working heads.

Table 9.9 Types of turbines based on working heads

Head	Range	Example
High	$H > 250$ m	Pelton wheel
Medium	60—250 m	Francis turbine
Low	$H < 60$ m	Kaplan, Propeller turbines

Deriaz turbine is used in pump storage plants that can be run by motor and can run a generator. It is a cross of Kaplan and Francis turbines. *Bulb turbine* is a small fixed axial flow propeller turbine operating at low speed. Turbo-generator is housed in an enclosed bulb shaped casing which is installed right in the middle of the flow passage. It is suitable for tidal power plants.

9.4.1.5 Bernoulli's Equation

Let water enter turbine system with pressure p_1, velocity V_1 and exit with pressure p_2, velocity V_2. Applying *Bernoulli's equation*, the energy transported to turbine (between inlet and outlet points) is given by

$$\begin{aligned}H &= \left(\frac{p_1}{\rho g} + \frac{{V_1}^2}{2g}\right) - \left(\frac{p_2}{\rho g} + \frac{{V_2}^2}{2g}\right)\\ &= \underbrace{\left(\frac{p_1 - p_2}{\rho g}\right)}_{\text{Pressure energy}} + \underbrace{\left(\frac{{V_1}^2 - {V_2}^2}{2g}\right)}_{\text{Kinetic energy}}\end{aligned}$$

Based on this equation, the expressions of energy transported through two types turbines can be written as follows:

1. *Impulse Turbines* Impulse turbines act at atmospheric pressure at inlet and outlet of the turbine ($p_1 = p_2$):

$$E = \frac{{V_1}^2 - {V_2}^2}{2g}$$

2. *Reaction Turbines* Reaction turbines gradually convert pressure energy into kinetic energy of the rotor. The velocity at inlet and outlet can be assumed to be same ($V_1 = V_2$):

$$E = \frac{p_1 - p_2}{\rho g}$$

Degree of reaction (R) is defined as the share of pressure energy in the total energy transfer:

$$\begin{aligned}R &= \frac{p_1 - p_2}{\rho g} \times \frac{1}{E}\\ &= 1 - \frac{{V_1}^2 - {V_2}^2}{2gE}\end{aligned}$$

Degree of reaction of an impulse turbine is zero.

9.4.1.6 Euler's Equation

Let water enter into turbine blade rotating at linear speed u_1 and exit at point where blade velocity is u_2. When water enters, the velocities w.r.t. to blades are V_{r1} and V_{r2}. Tangential components of these relative velocities are V_{w1} and V_{w2}. The power transfer to turbine blades is found in terms of energy head as

$$P = \frac{V_{w1}u_1 \pm V_{w2}u_2}{g} \tag{9.16}$$

This equation is known as the *Euler's equation for hydraulic turbines*.

9.4.2 Pelton Wheel

Pelton wheel[20] is a rotor which consists of radially arranged buckets (seldom less than 15) of semi-elliptical cup divided by a sharp-edged ridge known as *splitter* which divides the jet into two equal portions to neutralize the axial thrust. The camber angle of the bucket is limited to $\theta = 165°$ to avoid the hitting of water leaving the bucket on the back of the following bucket. A *notch* is also cut at lower tip of each bucket which prevents the jet striking the preceding bucket being interrupted by the next bucket very soon. This notch also avoids the deflection of water towards the center of the wheel as bucket first meets the jet. Buckets are made of chrome-plated steel or stainless steel.

Hydraulic jet is produced by passing the water through nozzles. The discharge is controlled by a *spear* or needle provided on the nozzles. For horizontal Pelton wheel axis, maximum two jets are used, and for vertical axis, upto six jets are used. *Brake nozzles* are used to direct a jet of water on the back of buckets, thereby bringing the wheel quickly to rest after it is shut down. Pelton wheel casing does not perform any hydraulic function.

9.4.2.1 Velocity Diagram

Figure 9.62 shows the velocity diagram for Pelton wheel. For the net effective head H available at the nozzle, the jet will hit the buckets tangentially at speed V_1:

$$V_1 = C_V\sqrt{2gH}$$

where C_V is velocity coefficient ($\approx$ 0.98). Therefore, hydraulic power supplied to wheel is

$$P_h = \frac{1}{2}\rho Q {V_1}^2$$

For diameter of wheel [21] D (measured from the center of bucket to center of wheel) and angular speed ω (rad/s), tangential speed at mean radius is written as

$$u = \frac{\omega D}{2}$$

Sometimes, speed ratio ψ is used to express u as

$$u = \psi\sqrt{2gH}$$

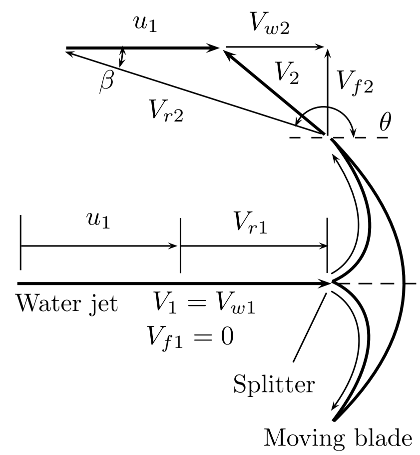


Figure 9.62 | Velocity triangle (Pelton wheel).

The absolute velocity of jet can be resolved into two orthogonal components:

1. *Whirl Velocity* Whirl velocities, denoted by V_w, are the tangential component of absolute velocities that are responsible for bucket motion.
2. *Flow Velocity* Radial components of velocities are called flow velocities that are responsible for axial thrust in the buckets.

These components are determined at both inlet and outlet of the blade as follows:

1. *Velocities at Inlet Point* Tangential speed u and jet velocity V_1 are unidirectional and tangential to wheel:

$$V_{w1} = V_1$$
$$V_{r1} = V_1 - u$$
$$V_{f1} = 0$$

Sometimes, flow ratio ϕ is used to calculate flow velocity as

$$V_{f1} = \phi\sqrt{2gH}$$

2. *Velocities at Outlet Point* The jet slides over bucket surface. *Blade velocity coefficient* k is used to quantify the effect of bucket friction on the relative velocity of jet:

$$V_{r2} = kV_{r1}$$

---

[20]Named after American engineer Lester A. Pelton (1829-1908) who invented it in 1878. According to legend, Pelton modeled the splitter ridge shape after the nostril of a cow's nose.

[21]Jet ratio m is the ratio of wheel diameter D, and nozzle diameter d. Value of m is kept between 11 and 14. Number of buckets, z, are calculated by $z = 0.5m + 15$.

For the bucket tip angle[22] at outlet is β ($= 180 - \theta$):

$$\begin{aligned} V_{f2} &= V_{r2}\sin\beta \\ V_{w2} &= V_{r2}\cos\beta - u \\ &= kV_{r1}\cos\beta - u \end{aligned}$$

9.4.2.2 Runner Power Runner power can be determined using Euler's equation [Eq. (9.16)]

$$\begin{aligned} P &= \rho Q\left(V_{w1}u_1 + V_{w2}u_2\right) \\ &= \rho Q\left[V_1 u + \left(kV_{r1}\cos\beta - u\right)u\right] \\ &= \rho Q\left(V_1 - u\right)\left(1 + k\cos\beta\right)u \end{aligned}$$

9.4.2.3 Hydraulic Efficiency Hydraulic efficiency is given by

$$\begin{aligned} \eta_h &= \frac{P}{P_h} \\ &= \frac{2\left(V_1 - u\right)\left(1 + k\cos\beta\right)u}{{V_1}^2} \\ &= \left(1 - \frac{u}{V_1}\right)\left(1 + k\cos\beta\right)\frac{u}{V_1} \end{aligned}$$

For maximum η_h,

$$\begin{aligned} \frac{d\eta_h}{dV_1} &= 0 \\ \frac{u}{V_1} &= 0.5 \end{aligned}$$

The maximum efficiency at this velocity is

$$(\eta_h)_{max} = \frac{1 + k\cos\beta}{2}$$

9.4.2.4 Governing Requirement of maximum efficiency of Pelton wheel is that the ratio of bucket is initial jet velocity u/V_1 has to be kept at its optimum value of about 0.46. Hence, when u is fixed, V_1 has to be fixed. Therefore, the control must be made by a ratio of the cross-sectional area of the jet so that the flow rate changes in proportion to the change in the flow area keeping the jet velocity V_1 same. Thus, the power of the Pelton wheel is controlled by varying the nozzle discharge by means of an automatically adjusted needle, known as spear valve, provided on the nozzles.

9.4.3 Francis Turbine

Francis turbine[23] is an inward radial flow reaction turbine in which water from penstock enters into an air-tight scroll casing that completely surrounds the runner and provides a uniform distribution of water around the circumference of turbine runner. The scroll casing maintains a constant velocity of water by gradually decreasing the cross-sectional area of flow. Water enters into *speed ring* where *stay vanes* direct it into airfoil-shaped *guide vanes*, known as wicket gates, that regulate the quantity of water and facilitate entry at suitable tip angle on turbine blades. The direction of flow of water changes from radial to axial and produces circumferential forces on the runner. The movable wicket gates mounted on the scroll or stationary part are controlled from the governor servomotor. At exit, a pipe of gradually increasing cross-sectional area, known as *draft tube*, is used to permit negative suction head by having the capability of converting kinetic energy of water into pressure head, thus increasing the power capacity of turbine system. In part loading, the entry is not shock-less, so efficiency of Francis turbine goes down.

9.4.3.1 Velocity Diagram If H is head available at turbine inlet and water exits at velocity V_2, *Euler's equation* [Eq. (9.16)] can be written for Francis turbine as

$$\begin{aligned} P &= H - \frac{{V_2}^2}{2g} \\ &= \frac{\left(V_{w1}u_1 + V_{w2}u_2\right)}{g} \end{aligned}$$

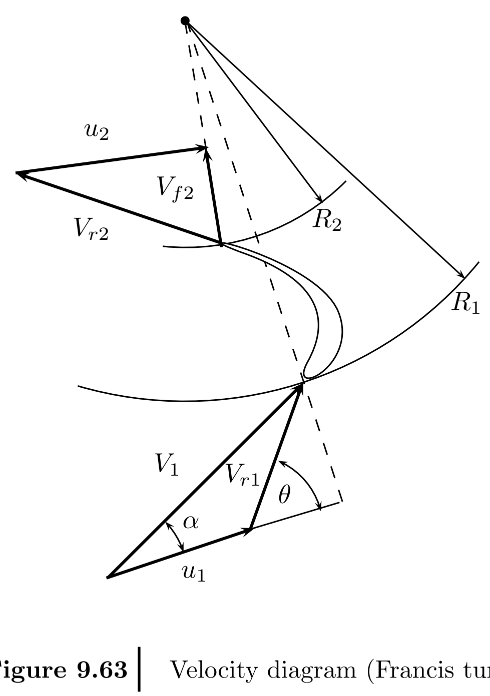


Figure 9.63 Velocity diagram (Francis turbine).

Let the turbine runner rotate at angular speed ω and diameters at inlet and outlet of the runner be D_1 and

[22]Bucket tip angle cannot be made zero, otherwise the outgoing jet from one bucket will hit the successive bucket from the backside, thus will retard the rotation. This angle is kept around 10° to 20°.
[23]Francis turbine is named after its inventor James B. Francis (1815—1892), an American engineer.

D_2. Blade speeds at inlet and outlet of the runner can be determined as

$$u_1 = \frac{\omega D_1}{2}$$
$$u_2 = \frac{\omega D_2}{2}$$

Components of velocities at inlet and outlet are determined as follows:

1. *Velocities at Inlet* Let water enter into a blade of Francis turbine with velocity V_1. For the blade angles shown in Fig. 9.63,

$$V_{w1} = V_1 \sin\alpha$$
$$V_{w1} = V_1 \cos\beta$$

2. *Velocities at Outlet* For radial exit from the blade,

$$V_{w2} = 0$$
$$V_{f2} = V_{r2} \cos\beta$$

If runner heights at inlet and outlet are h_1 and h_2, then discharge is expressed in terms of flow velocities as

$$Q = \pi D_1 h_1 V_{f1}$$
$$= \pi D_2 h_2 V_{f2}$$

Thus, Euler's equation takes the following form for a Francis turbine:

$$H - \frac{{V_2}^2}{2g} = \frac{V_{w1} u_1}{g}$$

9.4.3.2 Hydraulic Efficiency Hydraulic efficiency of Francis turbine is given by

$$\eta_h = \frac{V_{w1} u_1}{gH}$$

9.4.3.3 Draft Tube The *draft tube* is a conduit which connects the runner exit to the tail race, where the water is finally discharged from the turbine. The primary function of the draft tube is to reduce the velocity of the discharged water to minimize the loss of kinetic energy at the outlet. This also permits the turbine to be set above the tail water without any appreciable drop of available head.

Let h_d be the height of draft tube above the tail race and y be the depth of draft tube inside the water downstream or tail race [Fig. 9.64].

The pressure at point 2 is found as

$$\frac{p_2}{\rho g} = \frac{p_a}{\rho g} + y$$

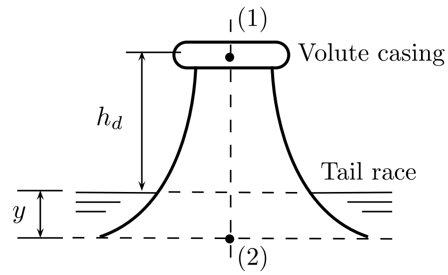


Figure 9.64 Draft tube in Francis turbine.

Let h_{fd} is loss of head due to friction in draft tube. Thus, using the Bernoulli's equation between points (1) and (2):

$$\frac{p_1}{\rho g} + \frac{{V_1}^2}{2g} + (y + h_d) = \frac{p_2}{\rho g} + \frac{{V_2}^2}{2g} + h_{fd}$$

Therefore,

$$\begin{aligned}
\frac{p_1}{\rho g} &= \frac{p_2}{\rho g} + \frac{{V_2}^2}{2g} + h_{fd} - \frac{{V_1}^2}{2g} - (y + h_d) \\
&= \frac{p_a}{\rho g} + y + \frac{{V_2}^2}{2g} + h_{fd} - \frac{{V_1}^2}{2g} - (y + h_d) \\
&= \frac{p_a}{\rho g} + \frac{{V_2}^2}{2g} + h_{fd} - \frac{{V_1}^2}{2g} - h_d \\
&= \frac{p_a}{\rho g} + h_{fd} - h_d - \left(\frac{{V_1}^2 - {V_2}^2}{2g} \right)
\end{aligned}$$

In this equation, $V_1 > V_2$, and h_{fd} is small in comparison to other heads. Therefore, p_1 is less than the atmospheric pressure p_a; the gauge pressure at exit of the volute casing is negative.

9.4.4 Kaplan Turbine

Kaplan turbine[24] is a highly efficient and mixed flow reaction turbine with adjustable blades and adjustable wicket gates. The control system is designed so that the variation in blade angle is coupled with the wicket gate setting in a manner which achieves best overall efficiency over a wide range of flow rates. The turbine is suitable for low head and high discharge applications [Fig. 9.65].

Consider a Kaplan turbine having outer diameter D and hub diameter d. Ratio of diameters, d/D is kept between 0.35 and 0.6. Discharge for average flow velocity V_f determined as

$$Q = \frac{\pi}{4} \left(D^2 - d^2 \right) V_f$$

The runner speed at inlet and outlet is the same $u = \omega D$.

[24] Kaplan turbine is named in honor of its inventor Viktor Kaplan (1876-1934).

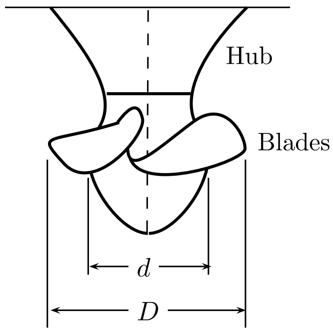


Figure 9.65 Kaplan turbine.

9.4.5 Turbine Characteristics

9.4.5.1 Operating Parameters The operation of hydraulic turbines is characterized by the following six parameters: power (P), efficiency (η), speed (N), head (H), discharge (Q), gate opening (%). Head H is fixed by the location of dam w.r.t. turbine level, N is fixed by the design of generator, and Q can be varied by operating head and gate opening. Therefore, N, H, and Q are considered as the controlling parameters. The proportionality relation of these parameters in terms of D and N can be derived as follows:

1. *Head* The velocity of jet or water at inlet is given by

$$\begin{aligned} V &= \sqrt{2gH} \\ &\propto \sqrt{H} \end{aligned}$$

The tangential velocity of turbine blade is given by

$$\begin{aligned} u &= \frac{\pi D N}{60} \\ u &\propto DN \end{aligned}$$

Also,

$$\begin{aligned} u &\propto \sqrt{H} \\ DN &\propto \sqrt{H} \\ H &\propto D^2N^2 \end{aligned} \tag{9.17}$$

This equation shows the relationship of H with D and N.

2. *Discharge* Discharge Q is found in terms of D and N as

$$\begin{aligned} Q &= \frac{\pi D^2}{4}V \\ &\propto D^2\sqrt{H} \\ &\propto D^3N \end{aligned} \tag{9.18}$$

3. *Power* Power P is derived in terms of D and N as

$$\begin{aligned} P &= \rho g Q H \\ &\propto D^2\sqrt{H}\times H \\ &\propto D^5N^3 \end{aligned} \tag{9.19}$$

Equations (9.17)–(9.19) reflect the characteristic curves of hydraulic turbines w.r.t. changes in size (D) and operating head (H).

9.4.5.2 Specific Speed Design of hydraulic turbines for a particular application is based on head (H), speed (N), and power (P). These factors are combined in a single parameter, known as *specific speed*, denoted by N_s. It is defined as the speed of operation of a geometrically similar model of the turbine, which is so proportioned that it produces unit power (1 kW) when operating at unit head (1 m).

Expression of specific speed can be derived by replacing D in Eq. (9.17) by using Eq. (9.19):

$$\begin{aligned} P &\propto \left(\frac{\sqrt{H}}{N}\right)^5 N^3 \\ &\propto \frac{H^{5/2}}{N^2} \\ N &\propto \frac{H^{5/4}}{\sqrt{P}} \\ N &= k\times\frac{H^{5/4}}{\sqrt{P}} \end{aligned}$$

where k is a constant. However, specific speed is defined when $P = 1$ and $H = 1$, therefore,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

Specific speed of Pelton wheel depends upon the number of jets (n) as

$$N_{s-n} = \sqrt{n}N_s$$

where N_s is specific speed with single jet. Specific speed of given wheel can be increased by increasing number of jets. Number of jets is not more than two for horizontal shaft turbines and is limited to six for vertical shaft turbines. The number of buckets required to maintain optimum efficiency is usually fixed by the empirical relation:

$$\text{Number of buckets} = 15+\frac{53}{N_s}$$

9.4.5.3 Scale Models Scale models (prototype) are used to analyze and simulate the characteristics of

hydraulic turbines under similar operating conditions. Dimensions of a turbine and its prototype are in a definite geometric proportion that depends upon the respective heads and the rotating speeds. This can be examined by using Eq. (9.17):

$$D \propto \frac{\sqrt{H}}{N}$$

For model and prototype turbines

$$\frac{D_m}{D_p} \propto \sqrt{\frac{H_m}{H_p}} \times \frac{N_p}{N_m}$$

where subscripts m and p denote model turbine and prototype. The quantity D_m/D_p is called *scale ratio*, which represents the ratio of diameters of the model turbine and the prototype turbine for achieving similar operating characteristics.

9.4.5.4 Unit Quantities

Unit quantities of a turbine are defined as their value for geometrically similar turbine working under unit head (1 m). These quantities are used in characteristic curves instead of direct quantities to realize the behavior of turbine in all scales. The following three unit quantities are important:

1. *Unit Speed* Since $N \propto \sqrt{H}$, therefore, at $H = 1$, $N = N_u$. Unit speed (N_u) of the turbine is written as
$$N_u = \frac{N}{\sqrt{H}}$$
2. *Unit Discharge* Since $Q \propto \sqrt{H}$, therefore, at $H = 1$, $Q = Q_u$. Unit discharge (Q_u) of the turbine is written as
$$Q_u = \frac{Q}{\sqrt{H}}$$
3. *Unit Power* Since $P \propto H^{3/2}$, therefore, at $H = 1$, $P = P_u$. Unit power (P_u) of the turbine is written as
$$P_u = \frac{P}{H^{3/2}}$$

9.4.6 Centrifugal Pump

Centrifugal pumps are rotodynamic devices in which dynamic pressure is developed by whirling motion of fluid by means of backward curved blades mounted on impeller. This pressure enables lifting of liquid from a lower level to higher level. This is achieved by creating a region of low pressure near the inlet of the pump to suck the liquid and a higher pressure at the outlet of the pump to force the liquid outside it upto desired level.

Pressure is generated in the impeller of a centrifugal pump is directly proportional to the density of the liquid that is in contact with it; negligible pressure will be produced in the presence of air. Therefore, to start a centrifugal pump, it is first subjected to *priming* during which the suction pipe, casing, and portion up to delivery pipe are completely filled with the same liquid to be lifted such that no air pocket is left and then the delivery valve is closed.

A centrifugal pump require low initial cost and offer high efficiency, uniform discharge, easy installation and maintenance. Their high rotational speed requirement facilitates direct coupling with electric motor. Main advantage of a centrifugal pumps lies in high discharging capacity. These pumps can be used with highly viscous liquids but a reciprocating pump can handle only water or less viscous liquids.

9.4.6.1 Fundamental Equations

Operation of a centrifugal pump is characterized by the following parameters:

1. *Static Head* *Static head* (H_s) is the vertical distance between the levels of liquid in sump and tank to which the liquid is delivered. Thus, static head is sum of *suction head* (h_s) and *delivery head* (h_d).
2. *Manometric Head* Manometric head (H_m) is the total head that must be produced by the pump to satisfy the external requirements and energy losses, such as in pump, suction pipe, delivery pipe. Therefore,
$$H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{{V_d}^2}{2g}$$
where h_{fs} and h_{fd} are the head losses in suction and delivery pipes, respectively.

Velocity diagram for centrifugal pump is shown in Fig. 9.66. Work done by centrifugal pump impeller on a unit weight of liquid is given by

$$W = \frac{V_{w1} u_1}{g}$$

However, the pump should compensate the energy losses in pump assembly too:

$$W = H_m + \text{loss of head in pump}$$
$$H_m = W - \text{loss of head in pump}$$

Efficiency of a centrifugal pump can be defined in three ways:

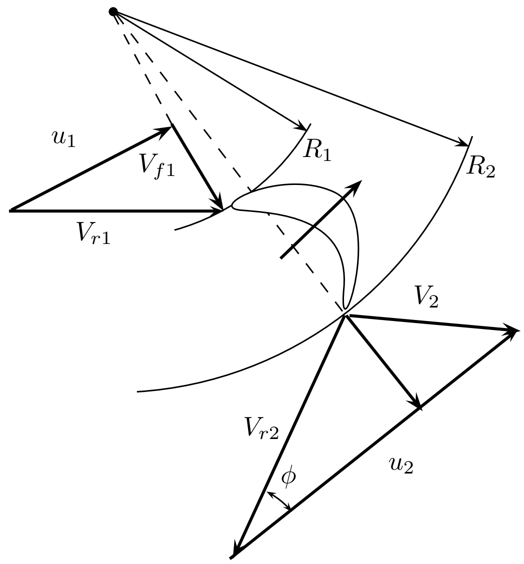


Figure 9.66 Velocity diagram.

1. Manometric Efficiency *Manometric efficiency*[25] of a centrifugal pump is defined as

$$\eta_{mano} = \frac{H_m}{W} = \frac{gH_m}{V_{w1}u_1}$$

2. Mechanical Efficiency *Mechanical efficiency* is given by

$$\eta_{mech} = \frac{W}{\text{Shaft power}}$$

3. Overall Efficiency Overall efficiency is defined as

$$\eta_o = \eta_{mano} \times \eta_{mech}$$

Velocity of flow in delivery pipe V_d will be higher than that in suction pipe V_s, thus diameter of suction pipe is more than that of delivery pipe. The flow will commence through impeller only if

$$\frac{{u_2}^2 - {u_1}^2}{2g} \geq H_m$$

9.4.6.2 Scale Models The model of pump and its prototype are in definite geometric ratio depending on their respective heads and the rotating speeds. It has been established that

$$Q = \pi D B V_f \propto D B V_f$$

Width B is generally represented in a factor of diameter D, therefore, $B \propto D$. The tangential blade velocity

$$u = \frac{\pi DN}{60} = \propto DN$$

Since V_f is proportional to u,

$$\begin{aligned} Q &\propto D^2 \times DN \\ &\propto D^3 \times N \\ P &= \rho g Q H \\ &\propto QH \end{aligned}$$

Since $DN \propto \sqrt{H}$, therefore, $H \propto D^2N^2$. Hence,

$$P \propto D^5 N^3$$

9.4.6.3 Specific Speed Specific speed N_s of a centrifugal pump is defined as the speed of a geometrically similar pump that delivers unit discharge under unit head. If impeller diameter is D and thickness B along with flow velocity V_f, then discharge is given by

$$Q = \pi D B V_f \propto D B V_f$$

Here, B is generally represented in a factor of D, therefore,

$$B \propto D$$

Also, tangential blade velocity

$$u = \frac{\pi DN}{60} \propto DN$$

Velocities u and V_f are both proportional to $\sqrt{H}$, therefore,

$$D \propto \frac{\sqrt{H}}{N}$$

Hence,

$$Q \propto \frac{H}{N^2}\sqrt{H} \propto \frac{H^{3/2}}{N^2}$$

Specific speed is defined when $Q = 1$, $H = 1$, therefore,

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

9.4.6.4 Effect of Blade Angle Velocity of flow at outlet can be determined as

$$V_{f2} = \frac{Q}{\pi D_2 b_2}$$

[25] Power imparted by impeller is $\rho g Q W$ and power delivered by pump is $\rho g Q H_m$

Here, Q is discharge, D_2 is runner diameter, b_2 is runner width at outlet. From the velocity triangle,

$$V_{w2} = u_2 - V_{f2}\cos\beta$$

Therefore, the head developed by a centrifugal pump is

$$\begin{aligned} H &= \frac{V_{w2}u_2}{g} \\ &= \frac{u_2}{g}\left[u_2 - \frac{Q}{\pi D_2 b_2}\cos\beta\right] \end{aligned}$$

This equation can be used to see the effect of blade angle β on H by keeping other parameters constant, as depicted in Fig. 9.67.

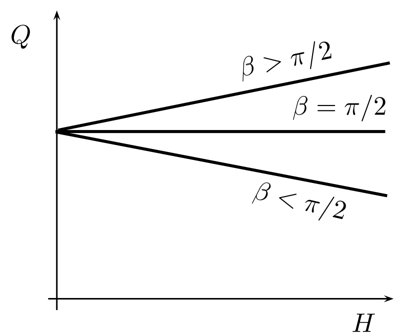


Figure 9.67 Effect of blade angle.

9.4.6.5 Cavitation When the liquid being pumped enters the eye of a centrifugal pump, the decrease in flow area results in an increase in flow velocity accompanied by a decrease in pressure. If the pressure drop is sufficient to cause the formation of vapor bubbles which are swept along the impeller vanes by the flow of the fluid. The vapor bubbles abruptly collapse when they reach in higher pressure region. This causes noise, vibration, reduction in efficiency, and most importantly, damage to the impeller blades. This process of the formation and subsequent collapse of vapor bubbles in a pump is called *cavitation*.

To avoid cavitation, local pressure everywhere inside the pump stays above the vapor pressure. Since pressure is most easily measured at the inlet of the pump, cavitation criteria are typically specified at the pump inlet. Absolute value of suction head is

$$\frac{p_a}{\rho g} - \left\{\frac{{V_s}^2}{2g} + h_s + h_{fs}\right\}$$

To avoid cavitation, the absolute pressure at the pump inlet should not be lower than the vapor pressure p_v of the liquid:

$$\begin{aligned} p_v &\leq \frac{p_a}{\rho g} - \left\{\frac{{V_s}^2}{2g} + h_s + h_{fs}\right\} \\ h_s &\leq \frac{p_a - p_v}{\rho g} - \frac{{V_s}^2}{2g} - h_{fs} \end{aligned}$$

Two parameters are used to design against cavitation:

1. Net Positive Suction Head *Net positive suction head* (NPSH) is defined as the difference between the absolute value of stagnation pressure at pump's inlet and the vapor pressure head,

$$\text{NPSH} = \frac{p_a + p_s}{\rho g} + \frac{{V_s}^2}{2g} - \frac{p_v}{\rho g}$$

NPSH is the head required to make the liquid to flow through the suction pipe to the impeller.

2. Thoma's Cavitation Factor *Thoma's cavitation factor* is defined as

$$\sigma = \frac{\text{NPSH}}{H_m}$$

To avoid caviation,

$$\sigma < 0.103\left(\frac{N_s^p}{1000}\right)^{4/3}$$

9.4.6.6 Pumps in Series and Parallel There are many applications where two or more similar pumps are operated in series or in parallel. This is required to meet the demand of increased flow rate or discharge pressure. Series or parallel arrangement is acceptable for some applications, however, arranging dissimilar pump in series or parallel can lead to problems. The two cases of arrangement are discussed as follows:

1. Series Combination In series combination, the volume flow rate through each pump must be the same but the overall pressure rise is equal to the sum of the pressure rise of each pump.

$$\begin{aligned} Q &= Q_i \\ H &= \sum_{i=1}^{n} H_i \end{aligned}$$

Thus, several pumps are arranged in series to deliver higher pressure at discharge.

2. Parallel Combination When two or more identical pumps are operated in parallel, their individual volume flow rates are summed up.

$$\begin{aligned} H &= H_i \\ Q &= \sum_{i=1}^{n} Q_i \end{aligned}$$

Thus, several pumps are arranged in parallel to deliver higher rate of volume flow.

IMPORTANT FORMULAS

Power Engineering

1. Properties of steam

$$x = \frac{m_g}{m_g + m_f}$$

$$v = v_f + x v_{fg}$$
$$h = h_f + x h_{fg}$$
$$s = s_f + x s_{fg}$$
$$u = u_f + x u_{fg}$$
$$\frac{m_f}{m_g} = \frac{(v_g - v)}{(v - v_f)}$$

$$h_{sup} = h_{sat} + c(T_{sup} - T_{sat})$$

Throttling calorimeter

$$x_1 = \frac{h_2 - h_{f1}}{h_{fg1}}$$

Separating and throttling

$$x_2 = \frac{h_3 - h_{f1}}{h_{fg1}}$$
$$x_1 = \frac{x_2 m_2}{m_1 + m_2}$$

2. Flow through nozzle

$$h_0 = h_1 + \frac{{V_1}^2}{2}$$
$$\frac{dA}{A} = \left(1 - M^2\right) \frac{vdp}{V^2}$$

$$pv^n = c$$
$$n = \begin{cases} 1.035 + \frac{x}{10} & \text{Wet steam} \\ 1.135 & \text{Saturated} \\ 1.3 & \text{Superheated} \end{cases}$$

$$V_2 = \sqrt{\frac{2n}{n-1}(p_1 v_1 - p_2 v_2) - {V_1}^2}$$
$$\frac{\dot{m}}{A_2} = \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1} \left\{ r^{\frac{2}{n}} - r^{\frac{n+1}{n}} \right\}}$$
$$r_c = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$
$$V_2 = \sqrt{n p_2 v_2}$$

$$\dot{m}_{max} = A_2 \sqrt{\frac{p_1}{v_1}} \times \begin{cases} 0.636 \text{ wet} \\ 0.677 \text{ super.} \end{cases}$$

$$\eta_n = \frac{h_1 - h_2'}{h_1 - h_2} = \frac{{V_2'}^2 - {V_1}^2}{{V_2}^2 - {V_1}^2}$$

3. Rankine cycle

$$\eta_{Rankine} = 1 - \frac{h_2 - h_3}{h_1 - h_4}$$
$$\text{Steam rate} = \frac{3600}{W_t - W_p} \text{ kg/h}$$

$$\eta_{Rankine} = 1 - \frac{T_{min}}{T_{m1}}$$

4. Maximum efficiency

(a) Impulse turbine

$$\rho = \frac{\cos\alpha}{2}$$
$$(\eta_b)_{max} = (1 + kB) \frac{\cos^2\alpha}{2}$$

(b) Reaction turbine

$$\rho = \cos\alpha_1$$
$$(\eta_b)_{max} = \frac{2\cos^2\alpha}{1 + \cos^2\alpha}$$

For a given value of α_1:

$$(\eta_b)_{reaction} > (\eta_b)_{impulse}$$

5. Reheat factor

$$\text{RF} = \frac{\sum_{n=1}^{n} \Delta h_{s-n}}{\Delta h_s}$$

Internal Combustion Engines

1. Fundamentals

$$A_p = \frac{\pi}{4} d^2$$
$$v_s = \frac{\pi}{4} d^2 L$$
$$= 1 + \frac{v_s}{v_c}$$
$$p_m = \frac{\text{Work output}}{\text{Swept volume}}$$
$$= \frac{\eta_{Cycle} \times \text{Heat supplied}}{\text{Swept volume}}$$
$$= \frac{\text{Area}}{\text{length}} \times \text{Factor}$$

2. Air standard cycles

(a) Carnot cycle [T-s-T-s]

$$\eta_{Carnot} = 1 - \frac{T_1}{T_3}$$
$$= 1 - \frac{1}{r^{\gamma-1}}$$
$$= 1 - \frac{1}{{r_p}^{\gamma/(\gamma-1)}}$$

(b) Stirling cycle [T-v-T-v]

$$\eta_s = 1 - \frac{T_1}{T_3}$$

(c) Ericsson cycle [T-p-T-p]

$$\eta_{Ericsson} = 1 - \frac{T_3}{T_1}$$

(d) Otto cycle [s-v-s-v]

$$\eta_{Otto} = 1 - \frac{1}{r^{(\gamma-1)}}$$

(e) Diesel cycle [s-p-s-v]

$$\eta_{Diesel} = 1 - \frac{1}{r^{(\gamma-1)}} \times \left\{ \frac{(k^\gamma - 1)}{\gamma(k-1)} \right\}$$

$$k^\gamma - \gamma(k-1) - 1 = 0$$

(f) Lenoir cycle

$$\eta_{Lenoir} = 1 - \gamma \left\{ \frac{{r_p}^{1/\gamma} - 1}{r_p - 1} \right\}$$

3. Performance analysis

$$V_p = 2L\frac{N}{60}$$
$$v_s = n \times \frac{\pi}{4} d^2 L$$
$$v = v_c + v_s$$
$$P_i = \dot{m}_a \times W_{cycle} \times n$$
$$P_f = P_i - P_b$$
$$p_{im} = \frac{\text{area} \times \text{spring constant}}{\text{length of diagram}}$$
$$W_i = p_{im} \times v_s$$
$$P_i = \frac{p_{im} \times v_s}{1000} \times \frac{N}{60} \times n \text{ kW}$$
$$P_b = \frac{p_{bm} \times v_s}{1000} \times \frac{N}{60} \times n \text{ kW}$$
$$= \frac{2\pi N}{60} T$$
$$\eta_m = \frac{P_b}{P_i} = \frac{p_{bm}}{p_{im}}$$

$$\text{ISFC} = \text{BSFC} \times \frac{\text{Brake power}}{\text{Indicated power}}$$

4. Combustion

$$m_{O_2} = \frac{32}{12}\text{C} + \frac{32}{4}\text{H} + \frac{32}{32}\text{S} - \text{O}$$

$$m_{air} = \frac{m_{O_2}}{0.23}$$

Refrigeration

1 ton = 3.5 kW = 50 kcal/min

1. Refrigerant designation

$$\text{R} - [\text{C}-1]\,[\text{H}+1]\,[\text{F}]$$

$$\text{Cl} = 2\,[\text{C}+1] - \text{H} - \text{F}$$

2. Refrigeration cycles

(a) Reversed Carnot cycle

$$Q_2 = T_1\,(s_1 - s_2)$$

$$W = -\,(T_2 - T_1)\,(s_1 - s_4)$$

$$\text{COP} = \frac{T_1}{T_2 - T_1}$$

(b) Reversed Brayton cycle

$$Q_2 = h_1 - h_4$$

$$W = (h_3 - h_2) + (h_1 - h_4)$$

$$\text{COP} = \frac{1}{r_p{}^{(\gamma-1)/\gamma}}$$

$$r_p = \frac{p_2}{p_1}$$

(c) Vapor compression system

$$Q_2 = h_1 - h_4 = h_1 - h_3$$

$$W = -\,(h_2 - h_1)$$

$$\text{COP} = \frac{Q_2}{|W|}$$

3. Psychrometry

$$w = \frac{m_v}{m_a} = 0.622\frac{p_v}{p - p_v}$$

$$\frac{m}{m_a} = (1 + w)\ \text{kg}$$

$$w_s = 0.622\frac{p_s}{p - p_s}$$

$$\phi = \frac{m_v}{m_s} = \frac{p_v}{p_s}$$

$$\mu = \frac{w}{w_s} = \phi\left(\frac{p - p_s}{p - p_v}\right)$$

$$h = 1.005t + w\,(h_{dp} + c_p t)$$

$$h_{dp} \approx 2500\ \text{kJ/kg}$$

$$c_p \approx 1.88\ \text{kJ/kgK}$$

$$p_v = p_v' - \frac{(p - p_v')\,(t - t') \times 1.8}{2800 - 1.3\,(1.8t + 32)}$$

Turbomachinary

$$N = \frac{120f}{p}$$

$$H = \frac{p_1 - p_2}{\rho g} + \frac{{V_1}^2 - {V_2}^2}{2g}$$

$$E = \frac{p_1 - p_2}{\rho g}$$

$$R = \frac{p_1 - p_2}{\rho g} \times \frac{1}{E}$$

$$= 1 - \frac{{V_1}^2 - {V_2}^2}{2gE}$$

$$P = \frac{V_{w1}u_1 \pm V_{w2}u_2}{g}$$

1. Pelton wheel

$$V_1 = C_V\sqrt{2gH}$$

$$P_h = \frac{1}{2}\rho Q {V_1}^2$$

$$u = \frac{\omega D}{2}$$

$$= \psi\sqrt{2gH}$$

$$P = \rho Q\,(V_1 - u)\,(1 + k\cos\beta)\,u$$

$$\eta_h = \left(1 - \frac{u}{V_1}\right)(1 + k\cos\beta)\,\frac{u}{V_1}$$

For $u/V_1 = 0.5$:

$$(\eta_h)_{max} = \frac{1 + k\cos\beta}{2}$$

2. Francis turbine

$$P = \frac{(V_{w1}u_1 + V_{w2}u_2)}{g}$$

$$u_1 = \frac{\omega D_1}{2}$$

$$u_2 = \frac{\omega D_2}{2}$$

$$H - \frac{{V_2}^2}{2g} = \frac{V_{w1}u_1}{g}$$

$$\eta_h = \frac{V_{w1}u_1}{gH}$$

3. Kaplan turbine

$$Q = \frac{\pi}{4}\left(D^2 - d^2\right)V_f$$

4. Turbine characteristics

$$H \propto D^2N^2$$

$$Q \propto D^3N$$

$$P \propto D^5N^3$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$N_{s-n} = \sqrt{n}N_s$$

$$\frac{D_m}{D_p} \propto \sqrt{\frac{H_m}{H_p}} \times \frac{N_p}{N_m}$$

$$N_u = \frac{N}{\sqrt{H}}$$

$$Q_u = \frac{Q}{\sqrt{H}}$$

$$P_u = \frac{P}{H^{3/2}}$$

5. Centrifugal pumps

$$H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{{V_d}^2}{2g}$$

$$W = \frac{V_{w1}u_1}{g}$$

$$\eta_{mano} = \frac{H_m}{W}$$

$$= \frac{gH_m}{V_{w1}u_1}$$

$$P \propto D^5N^3$$

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

SOLVED EXAMPLES

1. In a boiler, feed water supplied per hour is 215 kg, while coal fired per hour is 25 kg. Net enthalpy rise per kg of water is 150 kJ for conversion into steam. If the calorific value of coal is 2000 kJ/kg, determine the boiler efficiency.

Solution. The boiler efficiency is given by

$$\begin{aligned}\eta &= \frac{215 \times 150}{25 \times 2000} \\ &= 64.5\%\end{aligned}$$

2. A single stage impulse turbine with a diameter of 120 cm runs at 3000 rpm. If the blade speed ratio is 0.42, determine the inlet velocity of steam.

Solution. Given that

$$\begin{aligned}D &= 1.2 \text{ m} \\ \frac{u}{V_1} &= 0.42\end{aligned}$$

Therefore,

$$\begin{aligned}u &= \frac{\pi D N}{60} \\ V_1 &= \frac{u}{0.42} \\ &= 450 \text{ m/s}\end{aligned}$$

3. For a single stage impulse turbine with a rotor diameter of 2.5 m and a speed of 2500 rpm when the nozzle angle is 22.5°, the optimum velocity of steam is m/s is?

Solution. Given that

$$\begin{aligned}d &= 2.5 \text{ m} \\ N &= 2500 \text{ rpm} \\ \alpha &= 22.5^\circ\end{aligned}$$

The optimum velocity of steam V_1 (at nozzle exit) for maximum efficiency is given by

$$\begin{aligned}\frac{u}{V_1} &= \frac{\cos\alpha}{2} \\ V_1 &= \frac{2u}{\cos\alpha} \\ &= \frac{2\pi d N}{60 \times \cos\alpha} \\ &= 708.42 \text{ m/s}\end{aligned}$$

4. A spark ignition engine operates at compression ratio 7.5 to produce $24 \times 10^5 \times v_c$ J work per cycle, where v_c is the clearance volume in m^3. Determine the indicated mean effective pressure in the cylinder.

Solution. Given, compression ratio $r = 7.5$, hence

$$\begin{aligned}\frac{v_s + v_c}{v_c} &= 7.5 \\ \frac{v_s}{v_c} &= 6.5\end{aligned}$$

The mean effective pressure is determined as

$$\begin{aligned}p_m &= \frac{24 \times 10^5 \times v_c}{v_s} \\ &= \frac{24 \times 10^5}{6.5} \\ &= 3.69 \text{ bar}\end{aligned}$$

5. A reversed Brayton cycle is used to maintain a body at −25°C. Temperature at the end of isobaric cooling is 27°C and rise in the temperature of air in the refrigerator is 48°C. Determine the net work of compression. Take $c_p = 1$ kJ/kg C.

Solution. Given that

$$\begin{aligned}T_1 &= -25 + 273 \\ &= 248 \text{ K} \\ T_3 &= 27 + 273 \\ &= 300 \text{ K} \\ T_4 &= 248 - 48 \\ &= 200 \text{ K}\end{aligned}$$

For the compression (1→2) and expansion (3→4) processes,

$$\begin{aligned}\frac{T_2}{T_1} &= \frac{T_3}{T_4} \\ T_2 &= 372 \text{ K}\end{aligned}$$

Net compression is

$$\begin{aligned}W_{1-2} &= c_p \{(T_2 - T_1) - (T_3 - T_4)\} \\ &= 24 \text{ kJ/kg}\end{aligned}$$

6. In a vapor compression refrigeration plant, the refrigerant leaves the evaporator at 240 kJ/kg and the condenser at 60 kJ/kg. Determine the rate of cooling per kg of the refrigerant.

Solution. Given that

$$\begin{aligned}h_1 &= 240 \text{ kJ/kg} \\ h_3 &= 60 \text{ kJ/kg}\end{aligned}$$

For throttling,

$$\begin{aligned}h_4 &= h_3 \\ &= 60 \text{ kJ/kg}\end{aligned}$$

Therefore,

$$\begin{aligned} Q_2 &= h_1 - h_4 \\ &= 180 \text{ kJ/kg} \end{aligned}$$

The solution does not require calculation of h_2 (i.e. enthalpy after compression work).

7. The enthalpies at the beginning of compression, at the end of compression, and at the end of condensation are 180 kJ/kg, 220 kJ/kg and 80 kJ/kg, respectively. Determine the COP of the vapor compression refrigeration system.

Solution. The work input at compressor is

$$\begin{aligned} W &= 220 - 180 \\ &= 40 \text{ kJ/kg} \end{aligned}$$

Heat extracted at condenser is

$$\begin{aligned} Q_2 &= 220 - 80 \\ &= 140 \text{ kJ/kg} \end{aligned}$$

Hence, the COP is given by

$$\begin{aligned} \text{COP} &= \frac{140}{40} \\ &= 3.5 \end{aligned}$$

8. Atmospheric air at dry bulb temperature of 17°C enters a heating coil whose surface temperature is maintained at 42°C. The air leaves the heating coil at 27°C. Determine the by-pass factor of the heating coil.

Solution. Given that

$$\begin{aligned} T_s &= 42°\text{C} \\ T_2 &= 27°\text{C} \\ T_1 &= 17°\text{C} \end{aligned}$$

By-pass factor (B) is determined as

$$\begin{aligned} B &= \frac{T_s - T_2}{T_s - T_1} \\ &= \frac{42 - 27}{42 - 17} \\ &= 0.6 \end{aligned}$$

9. A sample of moist air at 101.325 kPa and 26°C has vapor at partial pressure of 1.344 kPa. Saturation pressure of water vapor is 3.36 kPa at 26°C. Determine the humidity ratio and relative humidity of moist air sample.

Solution. Given that

$$\begin{aligned} p &= 101.325 \text{ kPa} \\ \text{DBT} &= 26°\text{C} \\ p_v &= 1.344 \text{ kPa} \\ p_s &= 3.36 \text{ kPa} \end{aligned}$$

Therefore,

$$\begin{aligned} \omega &= 0.622 \frac{p_v}{p - p_v} \\ &= 0.008361 \\ \phi &= \frac{p_v}{p_s} \\ &= 0.4 \\ &= 40\% \end{aligned}$$

10. In an Otto cycle, air is compressed adiabatically from 27°C and 1 bar to 16 bar. Heat is supplied at constant volume until the pressure rises to 40 bar. For the air, $\gamma = 1.4$, $c_v = 0.718$ kJ/kg and $R = 0.2872$ kJ/kg K. Determine the air standard efficiency and mean effective pressure of the cycle.

Solution. Given that

$$\begin{aligned} p_1 &= 1 \text{ bar} \\ p_2 &= 16 \text{ bar} \\ p_3 &= 40 \text{ bar} \end{aligned}$$

Compression ratio is

$$\begin{aligned} r &= \frac{v_1}{v_2} \\ &= \left(\frac{p_2}{p_1}\right)^{1/\gamma} \\ &= 7.24 \end{aligned}$$

Air standard efficiency of the cycle is

$$\begin{aligned} \eta &= 1 - \frac{1}{r^{\gamma - 1}} \\ &= 54.71\% \end{aligned}$$

Initial temperature is

$$\begin{aligned} T_1 &= 27 + 273 \\ &= 300 \text{ K} \end{aligned}$$

Temperature after compression is

$$\begin{aligned} T_2 &= \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} T_1 \\ &= 662.45 \text{ K} \end{aligned}$$

The temperature after heat addition is given by

$$\begin{aligned} T_3 &= \frac{p_3}{p_2} T_2 \\ &= 1656.13 \text{ K} \end{aligned}$$

The heat addition in the cycle is

$$Q_s = c_v (T_3 - T_2) \\ = 713.46 \text{ kJ/kg}$$

The work done by the cycle is

$$W = \eta \times Q_s \\ = 390.33 \text{ kJ/kg}$$

Initial volume of the air per kg is

$$v_1 = \frac{RT_1}{p_1} \\ = 0.861 \text{ m}^3/\text{kg}$$

The volume after compression is

$$v_2 = \frac{v_1}{r} \\ = 0.119 \text{ m}^3/\text{kg}$$

The mean effective pressure, by its definition is

$$p_m = \frac{W}{v_1 - v_2} \\ = 5.26 \text{ bar}$$

11. A single-cylinder compression-ignition engine has 35% brake thermal efficiency and 75% mechanical efficiency. It is supplied with high speed diesel oil of 40 MJ/kg calorific value. Determine the brake specific fuel consumption (BSFC) and indicated specific fuel consumption (ISFC).

Solution. Given that

$$\eta_{bth} = 0.35 \\ \text{CV} = 40 \times 10^3 \text{ kJ/kg} \\ \eta_m = 0.75$$

Brake specific fuel consumption is

$$\text{BSFC} = \frac{3600}{\eta_{bh} \times \text{CV}} \\ = 0.257 \text{ kg/h-kW}$$

Indicated specific fuel consumption is

$$\text{ISFC} = \frac{\eta_m \times 3600}{\eta_{bth} \times \text{CV}} \\ = 0.193 \text{ kg/h-kW}$$

12. A centrifugal pump, driven by a directly coupled 4.5 kW motor of 1500 rpm speed, is proposed to be connected to another motor of 3000 rpm speed. Determine the power of the second motor.

Solution. For centrifugal pumps,

$$P \propto N^3$$

Therefore,

$$P_2 = \frac{3000}{1500}^3 \times 4.5 \\ = 36 \text{ kW}$$

13. An aeroplane is cruising at a speed of 750 kmph at an altitude, where the air temperature is 5°C. Calculate the flight Mach number at this speed.

Solution. The sonic speed of air at 5°C (= 278 K) is determined as

$$c = \sqrt{\gamma RT} \\ = \sqrt{1.4 \times 287 \times 278} \\ = 334.21 \text{ m/s}$$

Speed of the aeroplane is

$$V = 750 \text{ kmph} \\ = \frac{7500 \times 1000}{3600} \\ = 208.33$$

Thus, the Mach number is determined as

$$M = \frac{V}{c} \\ = \frac{208.33}{334.21} \\ = 0.62$$

14. An impulse turbine has nozzle efficiency 0.85, blade velocity ratio 0.6 and mean blade velocity 225 m/s in kJ/kg. Calculate the isentropic heat drop in the nozzle.

Solution. Given that

$$\eta_n = 0.85 \\ u = 225 \text{ m/s} \\ u/V_1 = 0.6$$

Velocity at nozzle exit is

$$V_1 = \frac{u}{0.6} \\ = 375 \text{ m/s}$$

The isentropic heat drop in the nozzle per kg of steam flow rate is given by

$$\Delta h_s = \frac{V_1^2/2}{\eta_n} \\ = \frac{375^2/2}{0.85} \\ = 82.72 \text{ kJ/kg}$$

GATE PREVIOUS YEARS' QUESTIONS

1. In a Rankine cycle, regeneration results in higher efficiency because

(a) pressure inside the boiler increases

(b) heat is added before steam enters the low pressure turbine

(c) average temperature of heat addition in the boiler increases

(d) total work delivered by the turbine increases

(GATE 2003)

Solution. Regeneration in Rankine cycle is the heat addition at higher temperature that increases the mean temperature of heat addition and decreases mean temperature of heat rejection. Thus, efficiency of Rankine cycle is increased but work output remains unchanged.

Ans. (c)

2. Considering the variation of static pressure and absolute velocity in an impulse steam turbine, across one row of moving blades

(a) both pressure and velocity decrease

(b) pressure decreases but velocity increases

(c) pressure remains constant, while velocity increases

(d) pressure remains constant, while velocity decreases

(GATE 2003)

Solution. In impulse turbines, the drop in pressure of steam takes place only in nozzles, not in moving blades. This is obtained by making the blade passage of constant cross-sectional area. Kinetic energy is transferred to blade rotation.

Ans. (d)

3. Match the following:

Column I	Column II
P. Curtis	1. Reaction steam turbine
Q. Rateau	2. Gas turbine
R. Kaplan	3. Velocity compounding
S. Francis	4. Pressure compounding
	5. Impulse water turbine
	6. Axial turbine
	7. Mixed flow turbine
	8. Centrifugal pump

(a) P-2, Q-1, R-7, S-6
(b) P-3, Q-1, R-5, S-7
(c) P-1, Q-3, R-1, S-5
(d) P-3, Q-4, R-7, S-6

(GATE 2003)

Solution. Curtis turbine is a velocity compounded steam turbine. Rateau turbine is a pressure compounded steam turbine. Kaplan turbine is a mixed flow hydraulic turbine with adjustable blades. Francis turbine is an axial flow hydraulic turbine.

Ans. (d)

4. Match the following:

Work Material	Type of Joining
P. Aluminium	1. Submerged arc welding
Q. Die steel	2. Soldering
R. Copper wire	3. Thermit welding
S. Titanium sheet	4. Atomic hydrogen welding
	5. Gas tungsten arc welding
	6. Laser beam welding

(a) P-2, Q-5, R-1, S-3
(b) P-6, Q-3, R-4, S-4
(c) P-4, Q-1, R-6, S-2
(d) P-5, Q-4, R-2, S-6

(GATE 2003)

Solution. Gas tungsten arc welding (GTAW) systems have been designed specifically for aluminum welding with pure tungsten electrodes. Thermit welding is used for die steel. Copper wires are joined by soldering. Titanium sheets are joined by laser beam welding.

Ans. (d)

5. For a spark ignition engine, the equivalence ratio (ϕ) of mixture entering the combustion chamber has values.

(a) $\phi < 1$ for idling and $\phi > 1$ for peak power conditions.
(b) $\phi > 1$ for both idling and peak power conditions.
(c) $\phi > 1$ for idling and $\phi < 1$ for peak power conditions.
(d) $\phi < 1$ for both idling and peak power conditions.

(GATE 2003)

Solution. Equivalence ratio (ϕ) is defined as the ratio of actual fuel–air ratio to the stoichiometric fuel–air ratio. If ϕ is greater than unity, the mixture is said to be rich and if ϕ is less than unity, the mixture is said to be weak. For idling and peak power conditions, rich mixture is required.

Ans. (b)

6. A diesel engine is usually more efficient than a spark ignition engine because

(a) diesel being a heavier hydrocarbon, releases more heat per kg than gasoline
(b) the air standard efficiency of diesel cycle is higher than the Otto cycle, at a fixed compression ratio
(c) the compression ratio of a diesel engine is higher than that of an SI engine
(d) self-ignition temperature of diesel is higher than that of gasoline

(GATE 2003)

Solution. For given compression ratio (r), efficiencies of Otto cycle and Diesel cycles, respectively, are

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}}$$
$$\eta_{Diesel} = 1 - \frac{1}{r^{\gamma-1}}\left\{\frac{k^{\gamma}-1}{\gamma(k-1)}\right\}$$

As the cut-off ratio $k > 1$, the expression in curly bracket is always greater than unity. Therefore, for a given compression ratio (r), Otto cycle is more efficient. But this fact is of no practical importance because, in practice, the operating compression ratio of Diesel engines are much higher (16–20) compared to spark ignition engines working on Otto cycle (6–10).

Ans. (c)

7. An automobile engine operates at a fuel–air ratio of 0.05, volumetric efficiency of 90% and indicated thermal efficiency of 30%. Given that the calorific value of the fuel is 45 MJ/kg and the density of air at intake is 1 kg/m^3, the indicated mean effective pressure for the engine is

(a) 6.075 bar (b) 6.75 bar
(c) 67.5 bar (d) 243 bar

(GATE 2003)

Solution. Given that

$$\frac{F}{A} = 0.05$$
$$\eta_v = 0.9$$
$$\eta_{ith} = 0.3$$
$$C = 45 \times 10^6 \text{ J/kg}$$
$$\rho_a = 1 \text{ kg/m}^3$$

If swept volume per cycle is v_s, then mass of fuel sucked is

$$\begin{aligned} m_f &= \frac{F}{A} m_a \\ &= \frac{F}{A}\rho_a \times \eta_v \times v_s \\ &= 0.045 \times v_s \end{aligned}$$

Indicated thermal efficiency is related to mean effective pressure p_m as

$$\eta_{ith} = \frac{p_m \times v_s}{m_f \times C}$$
$$p_m = 6.07500 \text{ bar}$$

Ans. (a)

8. For an engine operating on air standard Otto cycle, the clearance volume is 10% of the swept volume. The specific heat ratio of air is 1.4. The air standard cycle efficiency is

(a) 38.3% (b) 39.8%
(c) 60.2% (d) 61.7%

(GATE 2003)

Solution. Given that

$$v_c = 0.1 v_s$$
$$\gamma = 1.4$$

The compression ratio is

$$\begin{aligned} r &= \frac{v_c + v_s}{v_c} \\ &= 11 \end{aligned}$$

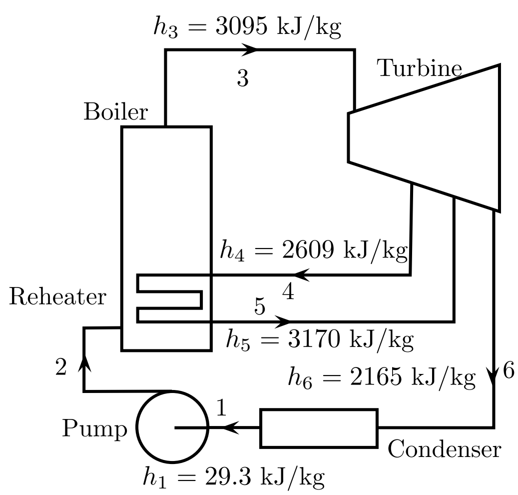
Efficiency of Otto cycle:

$$\begin{aligned} \eta_{\text{Otto}} &= 1 - \frac{1}{r^{\gamma-1}} \\ &= 61.6785\% \end{aligned}$$

Ans. (d)

Common Data Questions

Consider a steam power plant using a reheat cycle as shown. Steam leaves the boiler and enters the turbine at 4 MPa, 350°C, (h_3 = 3095 kJ/kg). After expansion in the turbine to 400 kPa (h_4 = 2609 kJ/kg), the steam is reheated to 350°C (h_5 = 3170 kJ/kg), and then expanded in a low pressure turbine to 10 kPa (h_6 = 2165 kJ/kg).



9. For air with a relative humidity of 80%,

(a) the dry bulb temperature is less than the wet bulb temperature
(b) the dew point temperature is less than wet bulb temperature
(c) the dew point and wet bulb temperatures are equal
(d) the dry bulb and dew point temperatures are equal

(GATE 2003)

Solution. The location of DPT, WBT, and DBT can be seen in the psychrometric chart:

$$\text{DPT} < \text{WBT} < \text{DBT}$$

Ans. (b)

Common Data Questions

A refrigerator based on ideal vapor compression cycle operates between the temperature limits of −20°C and 40°C. The refrigerant enters the condenser as saturated vapor and leaves as saturated liquid. The enthalpy and entropy values for saturated liquid and vapor at these temperatures are given in the table below.

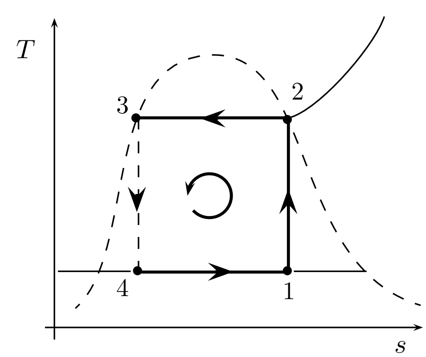
T (°C)	−20	40
h_f (kJ/kg)	20	80
h_g (kJ/kg)	180	200
s_f (kJ/kgK)	0.07	0.3
s_g (kJ/kgK)	0.7366	0.67

10. If refrigerant circulation rate is 0.025 kg/s, the refrigeration effect is equal to

(a) 2.1 kW (b) 2.5 kW
(c) 3.0 kW (d) 4.0 kW

(GATE 2003)

Solution. Given that $\dot{m}$ = 0.025 kg/s and data in the table. The enthalpy h_1 entering into compression is calculated as compression is isentropic process. The given cycle is shown in the figure.



If x_1 is the moisture fraction at 1, then

$$\begin{aligned} s_2 &= s_{f1} + (s_{g1} - s_{f1})\, x_1 \\ 0.67 &= 0.07 + (0.7366 - 0.07)\, x_1 \\ x_1 &= 0.9 \end{aligned}$$

For isenthalpic process (3→4):

$$\begin{aligned} h_1 &= 20 + (180 - 20) \times 0.9 \\ &= 164 \\ h_2 &= 200 \\ h_3 &= 80 \\ h_4 &= h_3 \end{aligned}$$

Refrigeration effect is

$$\begin{aligned} Q_2 &= \dot{m}\,(h_1 - h_4) \\ &= 0.025 \times (164 - 80) \\ &= 2.1 \text{ kW} \end{aligned}$$

Ans. (a)

11. The COP of the refrigerator is

(a) 2.0 (b) 2.33
(c) 5.0 (d) 6.0

(GATE 2003)

Solution. Work input to the compressor is

$$\begin{aligned} W &= \dot{m}\,(h_2 - h_1) \\ &= 0.9 \text{ kW} \end{aligned}$$

Therefore, COP is

$$\begin{aligned} \text{COP} &= \frac{Q_2}{W} \\ &= 2.33 \end{aligned}$$

Ans. (b)

12. A centrifugal pump running at 500 rpm and at its maximum efficiency is delivering a head of 30 m at a flow rate of 60 liters per minute. If the rpm is changed to 1000, then the head H in meters and flow rate Q in liters per minute at maximum efficiency are estimated to be

(a) $H = 60$, $Q = 120$
(b) $H = 120$, $Q = 120$
(c) $H = 60$, $Q = 480$
(d) $H = 120$, $Q = 30$

(GATE 2003)

Solution. Given that

$$\begin{aligned} N_1 &= 500 \text{ rpm} \\ H_1 &= 30 \text{ m} \\ Q_1 &= 60 \text{ l/min} \\ N_2 &= 1000 \text{ rpm} \end{aligned}$$

The relation between head and speed is

$$H \propto N^2$$

therefore,

$$\begin{aligned} H_2 &= H_1 \left(\frac{N_2}{N_1}\right)^2 \\ &= 60 \text{ m} \end{aligned}$$

Using

$$\begin{aligned} Q &\propto N \\ Q_2 &= Q_1 \frac{N_2}{N_1} \\ &= 120 \text{ l/min} \end{aligned}$$

Ans. (a)

13. The thermal efficiency of the plant neglecting pump work is

(a) 15.8% (b) 41.1%
(c) 48.5% (d) 58.6%

(GATE 2004)

Solution. Thermal efficiency is

$$\begin{aligned} \eta &= \frac{\text{Turbine work}}{\text{Heat input}} \\ &= \frac{(h_3 - h_4) - (h_5 - h_6)}{(h_3 - h_1) - (h_5 - h_4)} \\ &= 41.11\% \end{aligned}$$

Ans. (b)

14. The enthalpy at the pump discharge (h_2) is

(a) 0.33 kJ/kg (b) 3.33 kJ/kg
(c) 4.0 kJ/kg (d) 33.3 kJ/kg

(GATE 2004)

Solution. Enthalpy at pump discharge h_2 must be greater than h_1, which is 33.3 kJ/kg out of given options.

Ans. (d)

15. At the time of starting, idling and low speed operation, the carburetor supplies a mixture which can be termed as

(a) lean
(b) slightly leaner than stoichiometric
(c) stoichiometric
(d) rich

(GATE 2004)

Solution. The amount of fresh charge brought in during idling is much less than that during full throttle operation. This results in much larger portion of exhaust gases being mixed with the fresh charge under idling condition. When the intake valve is opened, the pressure difference between combustion chamber and intake manifold results in initial backward flow of exhaust gases into intake manifold. This results in poor

combustion. Thus, it is necessary to provide richer mixture during idling.

Ans. (d)

16. During a Morse test on a 4-cylinder engine, the following measurements of brake power were taken at constant speed.

All cylinders firing	3037 kW
1st cylinder not firing	2102 kW
2nd cylinder not firing	2102 kW
3rd cylinder not firing	2100 kW
4th cylinder not firing	2098 kW

The mechanical efficiency of the engine is

(a) 91.53% (b) 85.07%
(c) 81.07% (d) 61.22%

(GATE 2004)

Solution. When a cylinder does not fire, the difference of power measured with the all cylinder power (P_m) is indicated power (P_i) of that particular cylinder. Therefore, indicated powers of cylinders are 935, 935, 937, 939 kW, respectively. Therefore, total indicated power P_i is 3746 kW. Therefore, mechanical efficiency is

$$\begin{aligned}\eta_m &= \frac{P_m}{P_i}\\ &= \frac{3037}{3746}\\ &= 0.810731\\ &= 81.07\%\end{aligned}$$

Ans. (c)

17. An engine working on air standard Otto cycle has a cylinder diameter of 10 cm and stroke length of 15 cm. The ratio of specific heats for the air is 1.4. If the clearance volume is 196.3 cm^3 and the heat supplied per kg of air per cycle is 1800 kJ/kg, the work output per cycle per kg of air is

(a) 879.1 kJ (b) 890.2 kJ
(c) 895.3 kJ (d) 973.5 kJ

(GATE 2004)

Solution. Given that

$$\begin{aligned}d &= 0.10 \text{ m}\\ L &= 0.15 \text{ m}\\ \gamma &= 1.4\\ v_c &= 196.3\times 10^{-6} \text{ m}^3\\ Q_1 &= 1800\times 10^3 \text{ J/kg/cycle}\end{aligned}$$

Swept volume:

$$\begin{aligned}v_s &= \frac{1}{4}\pi d^2 L\\ &= 0.0011781 \text{ m}^3\end{aligned}$$

Compression ratio:

$$\begin{aligned}r &= \frac{v_s+v_c}{v_c}\\ &= 7.00151\end{aligned}$$

Air standard efficiency of Otto cycle:

$$\begin{aligned}\eta &= 1-\frac{1}{r^{\gamma-1}}\\ &= 0.540883\end{aligned}$$

Work output per cycle per kg of air:

$$\begin{aligned}W &= \eta Q\\ &= 973590 \text{ J}\\ &= 973.590 \text{ kJ}\end{aligned}$$

Ans. (d)

18. In the window air conditioner, the expansion device used is

(a) capillary tube
(b) thermostatic expansion valve
(c) automatic expansion valve
(d) float valve

(GATE 2004)

Solution. A capillary tube is a long, narrow tube of constant diameter. It is used up for refrigerating capacities of approximately 10 kW or less and is common in domestic refrigerators and window air conditioners.

Ans. (a)

19. During chemical dehumidification process of air

(a) dry bulb temperature and specific humidity decrease
(b) dry bulb temperature increases and specific humidity decreases
(c) dry bulb temperature decreases and specific humidity increases
(d) dry bulb temperature and specific humidity increase

(GATE 2004)

Solution. During chemical dehumidification, substances like silica gel, or activated alumina absorbs water molecules due to affinity during which latent heat of condensation is released. The process occurs along a constant enthalpy line

towards a lower humidity ratio. Therefore, DBT increases but WBT and DPT decrease.

Ans. (b)

20. Environment-friendly refrigerant R134$_a$ is used in the new generation domestic refrigerators. Its chemical formula is

(a) $CHClF_2$ (b) $C_2Cl_3F_3$
(c) $C_2Cl_2F_4$ (d) $C_2H_2F_4$

(GATE 2004)

Solution. Methane-based refrigerant compounds are designated by three and two digits respectively.

$$R-[C-1]\,[H+1]\,[F]$$

and, as chlorine atoms satisfy the remaining valency of the carbon atoms,

$$Cl = 2\,[C+1]-H-F$$

For the given refrigerant

$$\begin{aligned} C-1 &= 1 \\ H+1 &= 3 \\ F &= 4 \end{aligned}$$

Hence, the refrigerant is $C_2H_2F_4$.

Ans. (d)

21. Dew point temperature of air at one atmospheric pressure (1.013 bar) is 18°C. The air dry bulb temperature is 30°C. The saturation pressure of water at 18°C and 30°C are 0.02062 bar and 0.04241 bar, respectively. The specific heat of air and water vapor, respectively are 1.005 and 1.88 kJ/kg K and the latent heat of vaporization of water at 0°C is 2500 kJ/kg. The specific humidity (kg/kg of dry air) and enthalpy (kJ/kg of dry air) of this moist air, respectively, are

(a) 0.01051, 52.64 (b) 0.01291, 63.15
(c) 0.01481, 78.60 (d) 0.01532, 81.40

(GATE 2004)

Solution. Given that

$$\begin{aligned} \text{DPT} &= 18^\circ\text{C} \\ \text{DBT} &= 30^\circ\text{C} \\ p_v &= 0.02062 \text{ bar} \\ c_a &= 1.005\times 10^3 \text{ J/kg.K} \\ c_v &= 1.88\times 10^3 \text{ J/kg.K} \\ h_{fg} &= 2500\times 10^3 \text{ J/kgK} \end{aligned}$$

Assuming $p_a = 1.013$ bar,

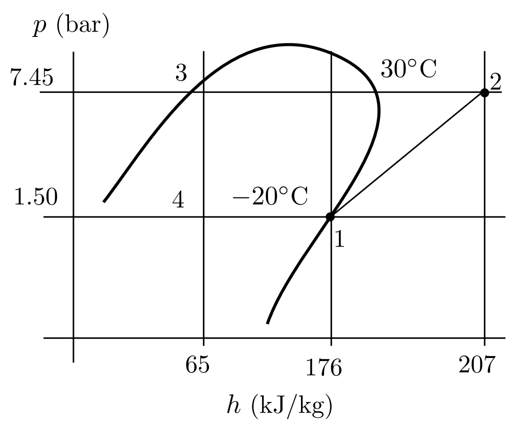
$$\begin{aligned} \omega &= 0.622\frac{p_v}{p_a-p_v} \\ &= 0.0129241 \end{aligned}$$

Specific enthalpy of moist air

$$\begin{aligned} h &= c_a\times \text{DBT}+\omega h_{fg}+\omega c_v\,(\text{DBT}-\text{DPT}) \\ &= 62.7519 \text{ kJ/kg of dry air} \end{aligned}$$

Ans. (b)

22. A R-12 refrigerant reciprocating compressor operates between the condensing temperature of 30°C and evaporator temperature of −20°C. The clearance volume ratio of the compressor is 0.03. Specific heat ratio of the vapor is 1.15 and the specific volume at the suction is 0.1089 m^3/kg. Other properties at various states are given in the figure.



To realize 2 tons of refrigeration, the actual volume displacement rate considering the effect of clearance is

(a) 6.35×10^{-3} m^3/s (b) 63.5×10^{-3} m^3/s
(c) 635×10^{-3} m^3/s (d) 4.88×10^{-3} m^3/s

(GATE 2004)

Solution. Given that

$$\begin{aligned} \frac{v_c}{v_s} &= 0.03 \\ v_1 &= 0.1089 \text{ m}^3\text{/kg} \\ Q_2 &= 2\times 3.516 = 7.032 \text{ kJ} \\ n &= 1.15 \end{aligned}$$

Taking values from figure, one gets

$$\begin{aligned} p_1 &= 1.50 \text{ bar} \\ p_2 &= 7.45 \text{ bar} \\ h_1 &= 176 \text{ kJ/kg} \\ h_2 &= 65 \text{ kJ/kg} \end{aligned}$$

Net refrigeration for mass flow rate $\dot{m}$ is

$$\begin{aligned} Q_2 &= \dot{m}\,(h_1-h_4) \\ \dot{m} &= 0.0633514 \text{ kg/s} \end{aligned}$$

Volumetric efficiency of compressor is

$$\eta_v = 1 + \frac{v_c}{v_s} - \frac{v_c}{v_s}\left(\frac{p_2}{p_1}\right)^{1/n} = 0.909109$$

Therefore, swept volume rate is

$$\dot{v}_s = \eta_v v_1 \times \dot{m} = 6.27191 \times 10^{-3}\ \text{m}^3/\text{s}$$

Ans. (a)

23. A centrifugal pump is required to pump water to an open water tank situated 4 km away from the location of the pump through a pipe of diameter 0.2 m having Darcy's friction factor of 0.01. The average speed of water in the pipe is 2 m/s. If it maintains a constant head of 5 m in the tank, neglecting other minor losses, the absolute discharge pressure at the pump exit is

(a) 0.449 bar (b) 5.503 bar
(c) 44.911 bar (d) 55.203 bar

(GATE 2004)

Solution. Given that

$$L = 4000\ \text{m}$$
$$d = 0.2\ \text{m}$$
$$4f = 0.01$$
$$V = 2\ \text{m/s}$$
$$H_t = 5\ \text{m (head at tank)}$$

Also, assuming

$$\rho_w = 1000\ \text{kg/m}^3$$
$$g = 9.81\ \text{m/s}^2$$

frictional head loss in pipe is

$$h_f = \frac{4f \times L}{D} \times \frac{V^2}{2g} = 40.7747\ \text{m}$$

Total pressure desired at pump outlet including to maintain the atmospheric pressure (10.3 m of water) at tank:

$$p = \rho_w g\,(H_t + h_f + 10.3) = 5.501\ \text{bar}$$

Ans. (b)

24. At a hydro-electric power plant site, available head and flow rate are 24.5 m and 10.1 m^3/s, respectively. If the turbine to be installed is required to run at 4.0 revolution per second (rps) with an overall efficiency of 90%, the suitable type of turbine for this site is

(a) Francis (b) Kaplan
(c) Pelton (d) Propeller

(GATE 2004)

Solution. For the given case,

$$N = 4 \times 60 = 240\ \text{rpm}$$
$$P = 0.9 \times 9810 \times 10.1 \times 24.5 = 2184.74\ \text{kW}$$
$$H = 24.5\ \text{m}$$

Therefore,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = 205.804$$

Specific speed (N_s) is the criteria for selection of turbines as

Turbine	N_s
Pelton	$8.5 - 30$
Francis	$50 - 340$
Kaplan	$255 - 860$

which is the range of Francis turbine.

Ans. (a)

25. Match the columns and select the correct option.

Column I	Column II
P. Reciprocating pump	1. Plant with power output below 100 kW
Q. Axial flow pump	2. Plant with power output between 100 kW to MW
R. Microhydel plant	3. Positive displacement
S. Backward curved vanes	4. Draft tube
	5. High flow rate, low pressure ratio
	6. Centrifugal pump impeller

(a) P-3, Q-5, R-6, S-2
(b) P-3, Q-5, R-2, S-6
(c) P-3, Q-5, R-1, S-6
(d) P-4, Q-5, R-1, S-6

(GATE 2004)

Solution. Reciprocating pumps are positive displacement type. Axial flow pump give high flow rate and low pressure ratio. The capacity of microhydel plant is less than 100 kW. Backward curved vanes are used in centrifugal pump impellers.

Ans. (c)

26. The velocity components in the x and y directions of a two-dimensional potential flow are u and v, respectively, then $\partial u/\partial x$ is equal to

(a) $\partial v/\partial x$ (b) $-\partial v/\partial x$
(c) $\partial v/\partial y$ (d) $-\partial v/\partial y$

(GATE 2005)

Solution. Using continuity equation,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$
$$\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$$

Ans. (d)

Linked Answer Questions

The following table of properties was printed out for saturated liquid and saturated vapor of ammonia. The titles for only the first two rows are available, that is, T (°C) and p (kPa). All that we know is that the other rows (3 to 8) contain data on specific properties, namely, internal energy (kJ/kg), enthalpy (kJ/kg), and entropy (kJ/kg.K).

T (°C)	-20	0	20	40
p (kPa)	190.2	429.6	857.5	1554.9
	88.76	179.69	272.89	368.74
	0.3657	0.7114	1.0408	1.3574
	89.05	180.36	274.30	371.43
	5.6155	5.3309	5.0860	4.8662
	1299.5	1318.0	1332.2	1341.0
	1418.0	1442.2	1460.2	1470.2

27. The specific enthalpy data are in rows

(a) 3 and 7 (b) 3 and 8
(c) 5 and 7 (d) 5 and 8

(GATE 2005)

Solution. One can guess and find columns from 1 to 8 denoting t, p, h_f, s_f, u_f, s_g, u_g, h_g. Therefore, specific enthalpies (h_f and h_g) are represented by 3 and 8 rows.

Ans. (b)

28. When saturated liquid at 40°C is throttled to -20°C, the quality at exit will be

(a) 0.189 (b) 0.212
(c) 0.231 (d) 0.788

(GATE 2005)

Solution. Enthalpy remains constant during throttling. Enthalpy of saturated liquid at 40°C is 368.74 kJ/kg. The quality x after throttling (at exit) will be given by

$$368.74 = 88.76+x\,(1418.0-88.76)$$
$$= 0.210632$$

Ans. (b)

Common Data Questions

In two air standard cycles - one operating on the Otto and the other on the Brayton cycle - air is isentropically compressed from 300 to 450 K. Heat is added to raise the temperature to 600 K in the Otto cycle and to 550 K in the Brayton cycle.

29. If η_O and η_B are the efficiencies of the Otto and Brayton cycles, then

(a) $\eta_O = 0.25$, $\eta_B = 0.18$
(b) $\eta_O = \eta_B = 0.33$
(c) $\eta_O = 0.5$, $\eta_B = 0.45$
(d) it is not possible to calculate the efficiencies unless the temperature after the expansion is given

(GATE 2005)

Solution. Given that

$$T_2 = 450\text{K}$$
$$T_1 = 300\text{K}$$

Using

$$\frac{p_2}{p_1}=\left(\frac{v_1}{v_2}\right)^{\gamma}$$
$$=\left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

Compression ratio (r) for Otto cycle:

$$r=\frac{v_1}{v_2}$$
$$= 2.75568$$

Pressure ratio (r_p) for Brayton cycle:

$$r_p=\frac{p_2}{p_1}$$
$$= 4.13351$$

Therefore,

$$\begin{aligned}\eta_O &= 1-\frac{1}{r^{\gamma-1}}\\ &= 0.33\\ \eta_B &= 1-\frac{1}{r_p^{(\gamma-1)/\gamma}}\\ &= 0.33\end{aligned}$$

Ans. (b)

30. If W_O and W_B are the work outputs per unit mass, then

(a) $W_O > W_B$
(b) $W_O < W_B$
(c) $W_O = W_B$
(d) it is not possible to calculate the work outputs unless the temperature after expansion is given.

(GATE 2005)

Solution. Heat input in Otto cycle is at constant volume and that in Brayton cycle is at constant pressure. Taking

$$\begin{aligned}c_v &= 0.718 \text{ kJ/kg.K}\\ c_p &= 1.005 \text{ kJ/kJ.K}\end{aligned}$$

Work output of Otto cycle:

$$\begin{aligned}W_O &= \eta_O \times c_v\,(600-450)\\ &= 35.9\end{aligned}$$

Work output of Brayton cycle:

$$\begin{aligned}W_B &= \eta_B \times c_p\,(550-450)\\ &= 33.5\end{aligned}$$

Therefore,

$$W_O > W_B$$

Ans. (a)

31. For a typical sample of ambient air (at 35°C, 75% relative humidity and standard atmospheric pressure), the amount of moisture in kg per kg of dry air will be, approximately,

(a) 0.002 (b) 0.027
(c) 0.25 (d) 0.75

(GATE 2005)

Solution. Given that

$$\begin{aligned}\text{DBT} &= 35^\circ\text{C}\\ \phi &= 75\%\\ p_s &= 0.05628 \text{ bar (at } 35^\circ\text{C)}\end{aligned}$$

Using

$$\begin{aligned}\phi &= \frac{p_v}{p_s}\\ p_v &= \phi p_s\\ &= 4221 \text{ Pa}\\ \omega &= 0.622\frac{p_v}{p-p_v}\\ &= 0.0365489\end{aligned}$$

Ans. (b)

32. Water at 42°C is sprayed into a stream of air at atmospheric pressure, dry bulb temperature of 40°C and a wet bulb temperature of 20°C. The air leaving the spray humidifier is not saturated. Which of the following statements is true?

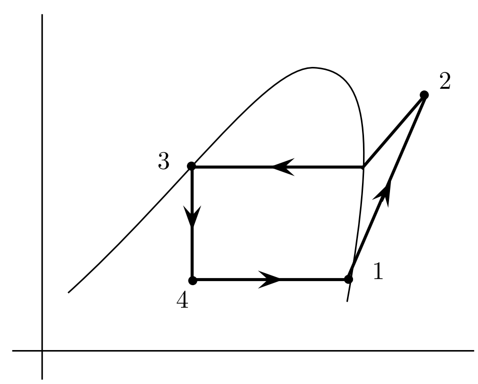
(a) Air gets cooled and humidified
(b) Air gets heated and humidified
(c) Air gets heated and dehumidified
(d) Air gets cooled and dehumidified

(GATE 2005)

Solution. Temperature of water (42°C) is more than DBT (40°C) of air. Therefore, the air will get heated and humidified.

Ans. (b)

33. The vapor compression refrigeration cycle is represented as shown in the figure, with state 1 being the exit of the evaporator.



The coordinate system used in this figure is

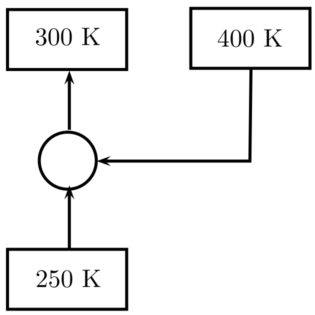
(a) $p-h$ (b) $T-s$
(c) $p-s$ (d) $T-h$

(GATE 2005)

Solution. This is simple vapor compression cycle on p-h plane.

Ans. (a)

34. A vapor absorption refrigeration system is a heat pump with three thermal reservoirs as shown in the figure. A refrigeration effect of 100 W is required at 250 K when the heat source available is at 400 K. Heat rejection occurs at 300 K.



The minimum value of heat required (in W) is

(a) 167 (b) 100
(c) 80 (d) 20

(GATE 2005)

Solution. The system consisting of two cycles will work. First cycle is for refrigeration effect of 100 W between 250 K and 300 K. For this, the work required is calculated as

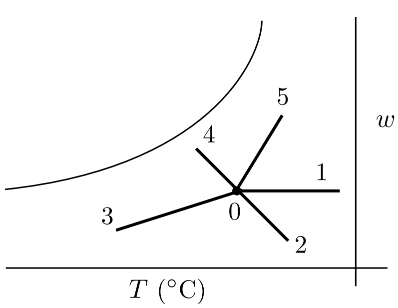
$$W = 100 \times \frac{300-250}{250}$$
$$= 20 \text{ W}$$

Second cycle is to supply this work while operating between 400 K and 300 K, so that it will consume heat Q_1 given by

$$Q_1 = W \times \frac{400}{400-300}$$
$$= 80 \text{ W}$$

Ans. (c)

35. Various psychrometric processes are shown in the figure.



Code	Process
P. 0 - 1	1. Chemical dehumidification
Q. 0 - 2	2. Sensible heating
R. 0 - 3	3. Cooling and dehumidification
S. 0 - 4	4. Humidification with stream injection
T. 0 - 5	5. Humidification with water injection

The matching pairs are

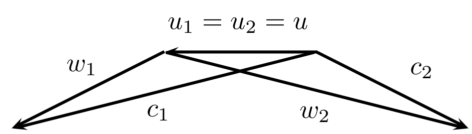
(a) P-1, Q-2, R-3, S-4, T-5
(b) P-2, Q-1, R-3, S-5, T-4
(c) P-2, Q-1, R-3, S-4, T-5
(d) P-3, Q-4, R-5, S-1, T-2

(GATE 2005)

Solution. Vertical axis shows specific humidity which changes for latent heat/humidification. Sensible heating/cooling is seen in horizontal axis only.

Ans. (b)

36. In the velocity diagram shown below, u = blade velocity, c = absolute fluid velocity and w = relative velocity of fluid and the subscripts 1 and 2 refer to inlet and outlet.



This diagram is for

(a) an impulse turbine
(b) a reaction turbine
(c) a centrifugal compressor
(d) an axial flow compressor

(GATE 2005)

Solution. As the velocity increases after impact in the blade ($\omega_2 > \omega_1$), there is a increase in kinetic energy in lapse of pressure. Therefore, it is a case of reaction turbine.

Ans. (b)

37. Determine the correctness or otherwise of the following Assertion [a] and the Reason [r].

Assertion [a]: In a power plant working on a Rankine cycle, the regenerative feed water heating improves the efficiency of the steam turbine.

Reason [r]: The regenerative feed water heating raises the average temperature of heat addition in the Rankine cycle.

(a) Both [a] and [r] are true and [r] is the correct reason for [a].
(b) Both [a] and [r] are true but [r] is NOT the correct reason for [a].
(c) Both [a] and [r] are false
(d) [a] is false and [r] is true.

(GATE 2006)

Solution. The regenerative feed water heating raises the average temperature of heat addition in the Rankine cycle, therefore, efficiency of the cycle increases.

Ans. (a)

38. Determine the correctness or otherwise of the following Assertion [a] and the Reason [r].

Assertion [a]: Condenser is an essential equipment in a steam power plant.

Reason [r]: For the same mass flow rate and the same pressure rise, a water pump requires substantially less power than a steam compressor.

(a) Both [a] and [r] are true and [r] is the correct reason for [a].
(b) Both [a] and [r] are true but [r] is NOT the correct reason for [a].
(c) [a] is true but [r] is false
(d) [a] is false and [r] is true.

(GATE 2006)

Solution. The condenser in steam power plant condenses the steam from the exhaust of the turbine into liquid to allow it to be pumped. For the same mass flow rate and same pressure rise, a water pump will requires substantially less power than a steam compressor because density of water is more than steam and it adds inertia or centrifugal force required in pumping.

Ans. (b)

39. Group I shows different heat addition processes in power cycles. Likewise, Group II shows different heat removal processes. Group III lists power cycles. Match items from Group I, II and III.

P. Pressure constant	S. Pressure constant	1. Rankine cycle
Q. Volume constant	T. Volume constant	2. Otto cycle
R. Temperature constant	U. Temperature constant	3. Carnot cycle
		4. Diesel cycle
		5. Brayton cycle

(a) P-S-5, R-U-3, P-S-1, Q-T-2
(b) P-S-1, R-U-3, P-S-4, P-T-2
(c) R-T-3, P-S-1, P-T-4, Q-S-5
(d) P-T-4, R-S-3, P-S-1, P-S-5

(GATE 2006)

Solution. Brayton cycle involves isobaric heat addition and heat rejection. Carnot cycle involves isothermal heat addition and heat rejection. Rankine cycle also involves isobaric heat addition and heat rejection. Otto cycle involves isochoric heat addition and heat rejection.

Ans. (a)

40. Dew point temperature is the temperature at which condensation begins when the air is cooled at constant.

(a) volume (b) entropy
(c) pressure (d) enthalpy

(GATE 2006)

Solution. To get the dew point temperature, the temperature of moisture is reduced at constant pressure.

Ans. (c)

41. The statements concerning psychrometric chart are given:.

(a) Constant relative humidity lines are uphill straight lines to the right.
(b) Constant wet bulb temperature lines are downhill straight lines to the right.
(c) Constant specific volume lines are downhill straight lines to the right.
(d) Constant enthalpy lines are coincident with constant wet bulb temperature lines.

Which of the statements are correct?

(a) 2 and 3 (b) 1 and 2
(c) 1 and 3 (d) 2 and 4

(GATE 2006)

Solution. In psychrometric chart, both constant WBT and specific volume lines are downhill straight lines to the right.

Ans. (a)

42. A thin layer of water in a field is formed after a farmer has watered it. The ambient air conditions are: temperature 20°C and relative humidity 5%. An extract of steam tables is given below:

Temperature (°C)	Saturation Pressure (kPa)
-15	0.10
-10	0.26
-5	0.40
0.01	0.61
5	0.87
10	1.23
15	1.71
20	2.34

Neglecting the heat transfer between the water and the ground, the water temperature in the field after phase equilibrium is reached equals

(a) 10.3°C (b) −10.3°C
(c) −14.5°C (d) 14.5°C

(GATE 2006)

Solution. In phase equilibrium, the water temperature in the field should be equal to the saturation temperature corresponding to the vapor pressure. Using the extract of steam table, the saturation pressure (p_s) of water corresponding the ambient temperature 20°C is 2.34 kPa. Relative humidity (ϕ) of humid air is related to vapour pressure (p_v) and saturation pressure (p_s) as

$$\phi = \frac{p_v}{p_s}$$

Given that $\phi = 5\%$. Therefore,

$$\begin{aligned} p_v &= \phi \times p_s \\ &= 0.05 \times 2.34 \\ &= 0.117 \text{ kPa} \end{aligned}$$

In the given table, $p_v = 0.117$ is in the range of 0.10–0.26 kPa, and corresponding temperature exists between −15°C and −10°C. Thus, option (c) is correct.

Ans. (c)

43. Given below is an extract from steam tables:

T	p_s	v		h	
		v_f	v_g	h_f	v_g
(°C)	(bar)	(m^3/kg)	(m^3/kg)	(kJ/kg)	(kJ/kg)
45	0.09593	0.001010	15.26	188.45	2394.8
342.24	150	0.001658	0.010337	1610.5	2610.5

Specific enthalpy of water in kJ/kg at 150 bar and 45°C is

(a) 203.60 (b) 200.53
(c) 196.38 (d) 188.45

(GATE 2006)

Solution. Saturation temperature of water at 150 bar (15 MPa) is 342.24°C. It means that water will be in liquid stage below this temperature. Water will also be in liquid state (incompressible) when temperature is 45°C and pressure is more than 0.09593 bar. Therefore, it will have specific enthalpy corresponding to the h_f of the 45°C, that is, 188.45 kJ/kg. Therefore, option (d) is correct.

Ans. (d)

44. In a Pelton wheel, the bucket peripheral speed is 10 m/s, the water jet velocity is 25 m/s and volumetric flow rate of the jet is 0.1 m^3/s. If the jet deflection angle is 120° and the flow is ideal, the power developed is

(a) 7.5 kW (b) 15.0 kW
(c) 22.5 kW (d) 37.5 kW

(GATE 2006)

Solution. Given that

$$\begin{aligned} u &= 10 \text{ m/s} \\ V_1 &= 25 \text{ m/s} \\ Q &= 0.1 \text{ m}^3\text{/s} \\ \beta &= \frac{120°}{2} = 60° \\ k &= 1 \text{ (assuming)} \end{aligned}$$

Power developed in Pelton turbine is given by

$$\begin{aligned} P &= \rho Q\,(V_1 - u)\,\{1 + k\cos\beta\} \\ &= 22.50 \text{ kW} \end{aligned}$$

Ans. (a)

45. A large hydraulic turbine is to generate 300 kW at 1000 rpm under a head of 40 m. For initial testing, a 1:4 scale model of the turbine operates under a head of 10 m. The power generated by the model (in kW) will be

(a) 2.34 (b) 4.68
(c) 9.38 (d) 18.75

(GATE 2006)

Solution. Given that

$$\begin{aligned} P_1 &= 300 \text{ kW} \\ N_1 &= 1000 \text{ rpm} \\ H_1 &= 40 \text{ m} \\ \frac{D_2}{D_1} &= \frac{1}{4} \\ H_2 &= 10 \text{ m} \end{aligned}$$

For hydraulic turbines

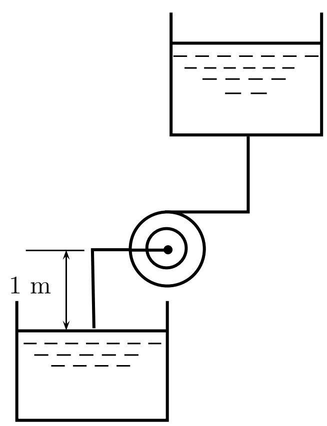
$$\begin{aligned} Q &\propto D^2 V \\ &\propto D^2\sqrt{H} \end{aligned}$$

Therefore,

$$\begin{aligned} P &= \rho g Q H \\ &\propto D^2 H^{3/2} \\ P_2 &= 300 \times \left(\frac{1}{4}\right)^2 \left(\frac{1}{4}\right)^{3/2} \\ &= 2.34375 \text{ kW} \end{aligned}$$

Ans. (a)

46. A horizontal shaft centrifugal pump lifts water at 65°C. The suction nozzle is one meter below pump center line. The pressure at this point equals 200 kPa gauge and velocity is 3 m/s. Steam tables show saturation pressure at 65°C is 25 kPa, and specific volume of the saturated liquid is 0.001020 m^3/ kg.



The pump net positive suction head (NPSH) in meters is:

(a) 24 (b) 26
(c) 28 (d) 30

(GATE 2006)

Solution. Given that

$$\begin{aligned} p_a &= 100 \text{ kPa} \\ p_s &= 200 \text{ kPa} \\ p_v &= 25 \text{ kPa} \\ \rho &= 1000 \text{ kg/m}^3 \\ g &= 9.81 \text{ m/s}^2 \\ V_s &= 3 \text{ m/s} \end{aligned}$$

Using these values,

$$\begin{aligned} \text{NPSH} &= \frac{p_a + p_s}{\rho g} - \frac{p_v}{\rho g} + \frac{V_s{}^2}{2g} \\ &= 30.7849 \text{ m} \end{aligned}$$

Ans. (d)

47. Water has a critical specific volume of 0.003155 m^3/kg. A closed and rigid steel tank of volume 0.025 m^3 contains a mixture of water and steam at 0.1 MPa. The mass of the mixture is 10 kg. The tank is now slowly heated. The liquid level inside the tank

(a) will rise
(b) will fall
(c) will remain constant
(d) may rise or fall depending on the amount of heat transferred

(GATE 2007)

Solution. The tank contains 10 kg of water plus steam in 0.025 m^3 space. If it is heated, the specific volume will be $0.025/10 = 0.0025$ m^3/kg, which is less than the critical specific volume. Also, heating will convert water into steam, therefore, the liquid level will fall.

Ans. (b)

48. Which combination of the following statements is correct? The incorporation of reheater in a steam power plant:

P: always increases the thermal efficiency of the plant
Q: always increases the dryness fraction of steam at condenser inlet
R: always increases the mean temperature of heat addition
S: always increases the specific work output

(a) P and S (b) Q and S
(c) P, R and S (d) P, Q, R and S

(GATE 2007)

Solution. Incorporation of reheater in steam power plan shifts the condenser input towards right, therefore, will increase the dryness fraction at condenser inlet. Also, as the steam is expanded many times before entering into the condenser and rejecting heat there, the work output per unit mass flow rate increases.

Ans. (b)

49. The stroke and bore of a four-stroke spark ignition engine are 250 mm and 200 mm, respectively. The clearance volume is 0.001 m^3. If the specific heat

ratio is $\gamma = 1.4$, the air-standard cycle efficiency of the engine is

(a) 46.40% (b) 56.10%
(c) 58.20% (d) 62.80%

(GATE 2007)

Solution. Given that

$$
\begin{aligned}
L &= 0.250 \text{ m}\\
D &= 0.2 \text{ m}\\
v_c &= 0.001 \text{ m}^3\\
\gamma &= 1.4
\end{aligned}
$$

Swept volume:

$$
\begin{aligned}
v_s &= \frac{\pi D^2}{4} \times L\\
&= 0.00785398 \text{ m}^3
\end{aligned}
$$

Compression ratio:

$$
\begin{aligned}
R &= 1 + \frac{v_s}{v_c}\\
&= 8.85398
\end{aligned}
$$

Air standard efficiency of spark ignition engine (Otto cycle) is

$$
\begin{aligned}
\eta_{\text{Otto}} &= 1 - \frac{1}{R^{\gamma-1}}\\
&= 58.2031\%
\end{aligned}
$$

Ans. (c)

50. Atmospheric air at a flow rate of 3 kg/s (on dry basis) enters a cooling and dehumidifying coil with an enthalpy of 85 kJ/kg of dry air and a humidity ratio of 19 grams/kg of dry air. The air leaves the coil with an enthalpy of 43 kJ/kg of dry air and a humidity ratio of 8 grams/kg of dry air. If the condensate water leaves the coil with an enthalpy of 67 kJ/kg, the required cooling capacity of the coil in kW is

(a) 75.0 (b) 123.8
(c) 128.2 (d) 159.0

(GATE 2007)

Solution. Given that

$$
\begin{aligned}
\dot{m} &= 3 \text{ kg/s}\\
h_1 &= 85 \text{ kJ/kg}\\
\omega_1 &= 0.019 \text{ g/kg of dry air}\\
h_2 &= 43 \text{ kJ/kg}\\
\omega_2 &= 0.008 \text{ g/kg of dry air}\\
h_c &= 67 \text{ kg/s}
\end{aligned}
$$

For heat balance,

$$
\begin{aligned}
Q &= \dot{m}h_2(1+\omega_2) + \dot{m}h_c - \dot{m}h_1(1+\omega_1)\\
&= 75.0 \text{ kW}
\end{aligned}
$$

Ans. (a)

51. The inlet angle of runner blades of a Francis turbine is 90°. The blades are so shaped that the tangential component of velocity at blade outlet is zero. The flow velocity remains constant throughout the blade passage and is equal to half of the blade velocity at runner inlet. The blade efficiency of the runner is

(a) 24% (b) 50%
(c) 80% (d) 89%

(GATE 2007)

Solution. Given, for constant flow velocities equal to half of the blade velocity at runner inlet,

$$
\begin{aligned}
V_{f1} &= V_{f2}\\
&= \frac{u_1}{2}
\end{aligned}
$$

Inlet angle of runner blades is $\theta = 90°$, therefore,

$$
\begin{aligned}
V_1 &= \sqrt{{V_{f1}}^2 + u_1^2}\\
&= \sqrt{\left(\frac{u_1}{2}\right)^2 + u_1^2}\\
&= \frac{u_1\sqrt{5}}{2}
\end{aligned}
$$

Also, for radial exit $V_{w2} = 0$ (i.e. tangential velocity is zero.). Therefore,

$$
\begin{aligned}
V_2 &= V_{f2}\\
&= \frac{u_1}{2}
\end{aligned}
$$

Kinetic energy input to turbine is ${V_1}^2/2$ and outlet energy is ${V_2}^2/2$, therefore, blade efficiency of the runner is

$$
\begin{aligned}
\eta_b &= 1 - \frac{{V_2}^2}{{V_1}^2}\\
&= 1 - \frac{(u_1/2)^2}{\left(u_1\sqrt{5}/2\right)^2}\\
&= 80\%
\end{aligned}
$$

Ans. (c)

52. A model of a hydraulic turbine is tested at a head of 1/4th of that under which the full scale turbine works. The diameter of the model is half of that of the full scale turbine. If N is the rpm of the full scale turbine, then the rpm of the model will be

(a) $N/4$ (b) $N/2$
(c) N (d) $2N$

(GATE 2007)

Solution. Given that

$$\frac{H_2}{H_1} = \frac{1}{4}$$
$$\frac{D_2}{D_1} = \frac{1}{2}$$
$$N_1 = N$$

For hydraulic turbines,

$$H \propto D^2 N^2$$
$$N \propto \frac{H}{D^2}$$

Using this formula, one gets

$$N_2 = \frac{1/4}{1/2^2} N$$
$$= N$$

Ans. (c)

53. Match the items in Columns I and II and select the correct option.

Column I	Column II
P. Centrifugal compressor	1. Axial flow
Q. Centrifugal pump	2. Surging
R. Pelton wheel	3. Priming
S. Kaplan turbine	4. Pure impulse

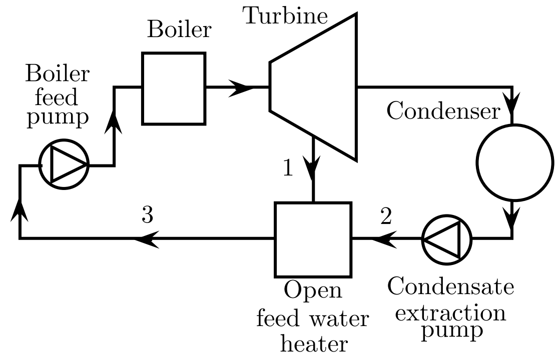
(a) P-2, Q-3, R-4, S-1
(b) P-2, Q-3, R-1, S-4
(c) P-3, Q-4, R-1, S-2
(d) P-1, Q-2, R-3, S-4

(GATE 2007)

Solution. In centrifugal compressors, the oscillation of flow and the resulting rapid variation in pressure difference gives rise to the phenomenon called "surging". Priming is done in centrifugal pump operation in which suction pipe, casing, and portion up to delivery pipe are completely filled with the same liquid to be lifted such that no air pocket is left, and then the delivery valve is closed. Pelton wheel is pure impeller in which there is no reaction. Kaplan turbine is an axial flow turbine.

Ans. (a)

54. A thermal power plant operates on a regenerative cycle with a single open feed water heater, as shown in the figure. For the state points shown, the specific enthalpies are $h_1 = 2800$ kJ/kg, $h_2 = 200$ kJ/kg. The bleed to the feed water heater is 20% of the boiler stream generation rate.



The specific enthalpy at state 3 is

(a) 720 kJ/kg (b) 2280 kJ/kg
(c) 1500 kJ/kg (d) 3000 kJ/kg

(GATE 2008)

Solution. Given that

$$h_1 = 2800 \text{ kJ/kg}$$
$$h_2 = 200 \text{ kJ/kg}$$

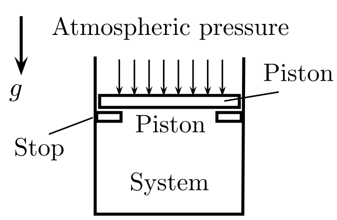
For heat balance,

$$h_3 = 0.2 \times h_1 + 0.8 \times h_2$$
$$= 720 \text{ kJ/kg}$$

Ans. (a)

Common Data Questions

In the figure shown, the system is pure substance kept in a piston-cylinder arrangement. The system is initially a two-phase mixture containing 1 kg of liquid and 0.03 kg of vapor at a pressure 100 kPa. Initially, the piston rests on set of stops, as shown in the figure. A pressure of 200 kPa is required to exactly balance the weight of the piston and the outside atmospheric pressure. Heat transfer takes place in the system until its volume increases by 50%. Heat transfer to the system occurs in such a manner that the piston, when allowed to move, does so in a very slow (quasi-static/quasi-equilibrium) process. The thermal reservoir from which heat is transferred to the system has a temperature of 400°C. Average temperature of the system boundary can be taken as 175°C. The heat transfer to the system is 1 kJ, during which its entropy increases by 10 kJ/K.



Specific volume of liquid (v_f) and vapor (v_g) phases, as well as values of saturation temperatures T_{sat} for corresponding pressure p, are given in the table below.

p (kPa)	T_{sat} (°C)	v_f (m^3/kg)	v_g (m^3/kg)
100	100	0.001	0.1
200	200	0.0015	0.002

55. At the end of the process, which one of the following situations will be true?

(a) Superheated vapor will be left in the system
(b) No vapor will be left in the system
(c) A liquid plus vapor mixture will be left in the system
(d) The mixture will exist at a dry saturate vapor state

(GATE 2008)

Solution. Given that the system is initially a two-phase mixture containing 1 kg of liquid and 0.03 kg of vapor at a pressure 100 kPa. Based on the given specific volumes at 100 kPa and the initial composition of mixture, the initial volume of mixture is

$$\begin{aligned} V_1 &= 1\times 0.001 + 0.03\times 0.1 \\ &= 0.004\ \text{m}^3 \end{aligned}$$

Volume after the expansion of 50% to 200 kPa:

$$\begin{aligned} V_2 &= 1.5\times V_1 \\ &= 0.006\ \text{m}^3 \end{aligned}$$

If vapor phase is saturated at 200 kPa, then its volume should be

$$\begin{aligned} V_{2'} &= 1\times 0.0015 + 0.03\times 0.002 \\ &= 0.00156\ \text{m}^3 \end{aligned}$$

As $V_{2'} < V_2$, therefore, the mixture gets more volume to expand beyond the saturation phase, therefore, liquid will vaporize. If mixture becomes dry saturated, then its volume will be

$$\begin{aligned} V_{2''} &= (1+0.03)\times 0.002 \\ &= 0.00206\ \text{m}^3 \end{aligned}$$

which is again less than V_2, and now, the vapor gets more space for expansion beyond saturation phase (i.e. superheated).

Ans. (a)

56. The work done by the system during the process is

(a) 0.1 kJ (b) 0.2 kJ
(c) 0.3 kJ (d) 0.4 kJ

(GATE 2008)

Solution. To lift the piston, atmospheric pressure $p = 200$ kPa is required in the system, which will be constant throughout the expansion process. Hence, work done is

$$\begin{aligned} W &= p\,(V_2 - V_1) \\ &= 200\times 10^3\,(0.006-0.004) \\ &= 0.4\ \text{kJ} \end{aligned}$$

Ans. (d)

57. The net entropy generation (considering the system and the thermal reservoir together) during the process is closest to

(a) 7.5 J/K (b) 7.7 J/K
(c) 8.5 J/K (d) 10 J/K

(GATE 2008)

Solution. Entropy generated in the system is 10 J/K. Heat $Q = 1$ kJ is extracted from heat source kept at $T_{hs} = 400+273 = 673$ K. Therefore, net entropy generated is

$$\begin{aligned} \Delta s &= 10 - + \frac{Q}{T_{hs}} \\ &= 10 - \frac{1000}{673} \\ &= 8.51412\ \text{J/K} \end{aligned}$$

Ans. (d)

58. Which one of the following in NOT a necessary assumption for the air-standard Otto cycle?

(a) All processes are both internally as well as externally reversible.
(b) Intake and exhaust processes are constant volume heat rejection processes.
(c) The combustion process is a constant volume heat addition process.
(d) The working fluid is an ideal gas with constant specific heats.

(GATE 2008)

Solution. Intake process is a constant volume heat addition process, therefore, option (b) is not true as it includes both exhaust and intake processes.

Ans. (b)

59. Moist air at a pressure of 100 kPa is compressed to 500 kPa and then cooled to 35°C in an after cooler. The air at the entry to the after cooler

is unsaturated and becomes just saturated at the exit of the after cooler. The saturation pressure of water at 35°C is 5.628 kPa. The partial pressure of water vapor (in kPa) in the moist air entering the compressor is closest to

(a) 0.57 (b) 1.13
(c) 2.26 (d) 4.52

(GATE 2008)

Solution. The moist air is compressed from $p_1 =$ 100 kPa to $p_2 = 500$ kPa. It is cooled in after cooler in the constant pressure process to $T_3 =$ 35°C, and saturation pressure $p_3 = 5.628$ kPa. The moist air at the exit of the after cooler (say point 3) is saturated, therefore, p_3 is the partial pressure of water vapor, which should be same as at p_2. Therefore, partial pressure of water vapor at p_2 is 5.628 kPa. Again as total pressure at initial stage (point 1) is one fifth of that after the compression (point 2), therefore, partial pressure will have the same ratio. Hence, partial pressure at 1 is 5.628/5=1.13 kPa.

Ans. (b)

60. Air (at atmospheric pressure) at a dry bulb temperature of 40°C and wet bulb temperature of 20°C is humidified in air washer operating with continuous water recirculation. The wet bulb depression (i.e. the difference between the dry and wet bulb temperatures) at the exit is 25% of that at the inlet. The dry bulb temperature at the exit of the air washer is closest to

(a) 10°C (b) 20°C
(c) 25°C (d) 30°C

(GATE 2008)

Solution. Given that

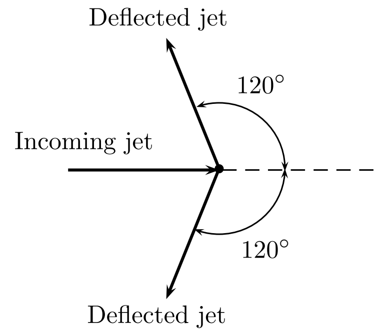
$$\mathrm{DBT}_1 = 40°\mathrm{C}$$
$$\mathrm{WBT} = 20°\mathrm{C}$$

WBT remains constant during humidification. Therefore,

$$\begin{aligned}\mathrm{DBT}_2 &= 0.25\,(\mathrm{DBT}_1 - \mathrm{WBT}) + \mathrm{WBT}\\ &= 0.25\,(40-120)+40\\ &= 25°\mathrm{C}\end{aligned}$$

Ans. (c)

61. Water, having a density of 1000 kg/m^3, issues from a nozzle with a velocity of 10 m/s and the jet strikes a bucket mounted on a Pelton wheel. The wheel rotates at 10 rad/s. The mean diameter of the wheel is 1 m. The jet is split into two equal streams by the bucket, such that each stream is deflected by 120°, as shown in the figure. The friction in the bucket may be neglected.



Magnitude of the torque exerted by the water on the wheel, per unit mass flow rate of the incoming jet, is

(a) 0 (Nm)/(kg/s)
(b) 1.25 (Nm)/(kg/s)
(c) 2.5 (Nm)/(kg/s)
(d) 3.75 (Nm)/(kg/s)

(GATE 2008)

Solution. Given that

$$\begin{aligned}\rho &= 1000\ \mathrm{kg/m^3}\\ V &= 10\ \mathrm{m/s}\\ \omega &= 10\ \mathrm{rad/s}\\ D &= 1\ \mathrm{m}\\ \phi &= \frac{120}{2}\\ &= 60°\\ k &= 1\\ \dot{m} &= 1\ \mathrm{kg/s}\end{aligned}$$

Wheel speed is calculated as

$$\begin{aligned}u &= \omega \times \frac{D}{2}\\ &= 10 \times \frac{1}{2}\\ &= 5\end{aligned}$$

Power generated for unit mass flow rate is

$$T\omega = \dot{m}\,(V-u)\,(1+k\cos\phi)\,u$$

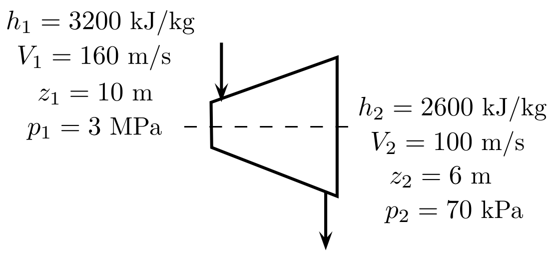
Therefore,

$$\begin{aligned}T \times 10 &= 1 \times (10-5)\,(1+1\cos 60°) \times 5\\ T &= 3.75\ \mathrm{Nm/(kg/s)}\end{aligned}$$

Ans. (d)

Common Data Questions

The inlet and the outlet conditions of stream for an adiabatic steam turbine are as indicated in the notations are as usually followed.



62. If mass flow rate of steam through the turbine is 20 kg/s, the power output of the turbine (in MW) is

(a) 12.157 (b) 12.941
(c) 168.001 (d) 168.785

(GATE 2009)

Solution. Using the values shown in the figure, the work output in steady flow process is given by first law of thermodynamics:

$$\begin{aligned} W &= \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + (z_1 - z_2)\, g\right] \\ &= 20 \times [3200 - 2600 \\ &\quad + \frac{160^2 - 100^2}{2 \times 1000} + \frac{(10-6) \times 9.81}{1000}] \\ &= 12.1568 \text{ MW} \end{aligned}$$

Ans. (a)

63. Assume the above turbine to be part of a simple Rankine cycle. The density of water at the inlet to the pump is 1000 kg/m^3. Ignoring kinetic and potential energy effects, the specific work (in kJ/kg) supplied to the pump is

(a) 0.293 (b) 0.351
(c) 2.930 (d) 3.510

(GATE 2009)

Solution. Specific pump work is given by

$$\begin{aligned} W_p &= v\Delta p \\ &= \frac{1}{\rho}(p_1 - p_2) \\ &= \frac{1}{1000}\left(3 \times 10^6 - 70 \times 10^3\right) \\ &= 2.930 \text{ kJ/kg} \end{aligned}$$

Ans. (c)

64. In air-standard Otto cycle, the compression ratio is 10. The condition at the beginning of the compression process is 100 kPa and 27°C. Heat added at constant volume is 1500 kJ/kg, while 700 kJ/kg of heat is rejected during the other constant volume process in the cycle. Specific gas constant for air = 0.287 kJ/kgK. The mean effective pressure (in kPa) of the cycle is

(a) 103 (b) 310
(c) 515 (d) 1032

(GATE 2009)

Solution. Given that

$$\begin{aligned} r &= 10 \\ p_1 &= 100 \text{ kPa} \\ T_1 &= 27 + 273 \text{ K} \\ Q_1 &= 1500 \text{ kJ/kg} \\ Q_2 &= 700 \text{ kJ/kg} \\ R &= 0.287 \text{ kJ/kg} \end{aligned}$$

Therefore,

$$\begin{aligned} v_1 &= \frac{RT_1}{p_1} \\ &= 0.861 \text{ m}^3 \\ v_2 &= \frac{v_1}{r} \\ &= 0.0861 \text{ m}^3 \\ p_m &= \frac{W}{dv} \\ &= \frac{Q_1 - Q_2}{v_1 - v_2} \\ &= 1.03239 \text{ MPa} \\ &= 1032.39 \text{ kPa} \end{aligned}$$

Ans. (d)

65. In an ideal vapor compression refrigeration cycle, the specific enthalpy of refrigerant (in kJ/kg) at the following states is given as:

Inlet of condenser	:	283
Exit of condenser	:	116
Exit of evaporator	:	232

The COP of this cycle is

(a) 2.27 (b) 2.75
(c) 3.27 (d) 3.75

(GATE 2009)

Solution. Given that

$$\begin{aligned} h_2 &= 283 \text{ kJ/kg} \\ h_3 &= 116 \text{ kJ/kg} \\ h_1 &= 232 \text{ kJ/kg} \end{aligned}$$

The COP is given by

$$\text{COP} = \frac{h_1 - h_3}{h_2 - h_1} = \frac{232 - 116}{283 - 232} = 2.274$$

Ans. (a)

Common Data Questions

In a steam power plant operating on the Rankine cycle, steam enters the turbine at 4 MPa, 350°C and exits at a pressure of 15 kPa. Then it enters the condenser and exits as saturated water. Next, a pump feeds back the water to the boiler. The adiabatic efficiency of the turbine is 90%. The thermodynamic states of water and steam are given in the table.

Steam: 4 MPa, 350°C

h (kJ/kg)	s (kJ/kgK)	v (m^3/kg)
3092.5	6.5821	0.06645

Water: 15 kPa

h_f	h_g	s_f	s_g	v_f	v_g
225.94	2599.1	0.7549	8.0085	0.001014	10.02

h is specific enthalpy (kJ/kg), s is specific entropy (kJ/°C kg) and v the specific volume (m^3/kg); subscripts f and g denote saturated liquid state and saturated vapor state.

66. The net work output (kJ/kg) of the cycle is

(a) 498 (b) 775
(c) 860 (d) 957

(GATE 2010)

Solution. Given that

$$h_1 = 3092.5 \text{ kJ/kg}$$
$$h_g = 2599.1 \text{ kJ/kg}$$
$$h_f = 225.94 \text{ kJ/kg}$$
$$\eta = 0.9$$
$$p_1 = 4 \times 10^6 \text{ Pa}$$
$$p_2 = 15 \times 10^3 \text{ Pa}$$
$$v_p = 0.001014 \text{ m}^3\text{/kg}$$

Quality after isentropic expansion is given by,

$$6.5821 = 0.7549 + 7.2536x$$
$$x = 0.8$$

Therefore,

$$h_2 = h_f + (h_g - h_f)\,x = 225.94 + (2599.1 - 225.94) \times 0.8 = 2132.42 \text{ kJ/kg}$$

Pump work is

$$W_p = v_p\,(p_1 - p_2) = 0.001014 \times \left(4 \times 10^6 - 15 \times 10^3\right) = 4.04079 \text{ kJ/kg}$$

Turbine work is

$$W_t = \eta\,(h_1 - h_2) = 0.9 \times (3092.5 - 2132.42) = 864.068 \text{ kJ/kg}$$

Hence, net work output is

$$W_{\text{net}} = W_t - W_p = 864.068 - 4.04079 = 860.027 \text{ kJ/kg}$$

Ans. (c)

67. Heat supplied (kJ/kg) to the cycle is

(a) 2372 (b) 2576
(c) 2863 (d) 3092

(GATE 2010)

Solution. Heat supplied to the boiler is

$$Q_s = h_1 - (h_3 + W_p) = 2862.52 \text{ kJ/kg}$$

Ans. (c)

68. A turbo-charged four-stroke direct injection diesel engine has a displacement volume of 0.0259 m^3 (25.9 l). The engine has an output of 950 kW at 2200 rpm. The mean effective pressure in MPa is closest to

(a) 2 (b) 1
(c) 0.2 (d) 0.1

(GATE 2010)

Solution. Given that

$$v_s = 0.0259 \text{ m}^3$$
$$P = 950 \text{ kW}$$
$$N = 2200 \text{ rpm}$$

If p_m is mean effective pressure then for four-stroke engines, then

$$P = \frac{p_m v_s N}{2 \times 60}$$
$$p_m = 2.0007 \text{ MPa}$$

Ans. (a)

69. A moist air sample has dry bulb temperature of 30°C and specific humidity of 11.5 g water vapor per kg dry air. Assume molecular weight of air as 28.93. If the saturation vapor pressure of water at 30°C is 4.24 kPa and the total pressure is 90 kPa, then the relative humidity (in %) of air sample is

(a) 50.5 (b) 38.5
(c) 56.5 (d) 68.5

(GATE 2010)

Solution. Given that

$$\begin{aligned}\omega &= \frac{11.5}{1000}\\ &= 0.0115 \text{ g/kg of dry air}\\ p &= 90\times 10^3\text{Pa}\\ p_s &= 4.24\times 10^3\text{Pa}\end{aligned}$$

Using

$$\begin{aligned}\omega &= 0.622\frac{p_v}{p-p_v}\\ 0.0115 &= 0.622\times\frac{p_v}{90\times 10^3 - p_v}\\ p_v &= 1633.78 \text{ Pa}\end{aligned}$$

Relative humidity is obtained as

$$\begin{aligned}\phi &= \frac{p_v}{p_s}\times 100\\ &= \frac{1633.78}{4.24\times 10^3}\times 100\\ &= 38.5326\%\end{aligned}$$

Ans. (b)

70. A hydraulic turbine develops 1000 kW power for a head of 40 m. If the head is reduced to 20 m, the power developed (in kW) is

(a) 177 (b) 354
(c) 500 (d) 707

(GATE 2010)

Solution. Given that

$$\begin{aligned}P_1 &= 1000 \text{ kW}\\ H_1 &= 40 \text{ m}\\ H_2 &= 20 \text{ m}\end{aligned}$$

Power developed by a hydraulic turbine:

$$\begin{aligned}P &= \rho g Q H\\ &\propto D^2\sqrt{H}\times H\\ &\propto H^{3/2}\end{aligned}$$

Hence

$$\begin{aligned}P_2 &= P_1\times\left(\frac{H_2}{H_1}\right)^{3/2}\\ &= 1000\times\left(\frac{20}{40}\right)^{3/2}\\ &= 353.553 \text{ kW}\end{aligned}$$

Ans. (b)

71. The crank radius of a single cylinder internal combustion engine is 60 mm and the diameter of the cylinder is 80 mm. The swept volume of the cylinder in cm^3 is

(a) 48 (b) 96
(c) 302 (d) 603

(GATE 2011)

Solution. Given that

$$\begin{aligned}r &= 6 \text{ cm}\\ d &= 8 \text{ cm}\\ L &= 2r\end{aligned}$$

Swept volume is

$$\begin{aligned}v_s &= \frac{\pi d^2}{4}\times 2r\\ &= \frac{\pi 8^2}{4}\times 2\times 6\\ &= 603.186 \text{ cm}^3\end{aligned}$$

Ans. (d)

72. If a mass of moist air in an airtight vessel is heated to a higher temperature, then

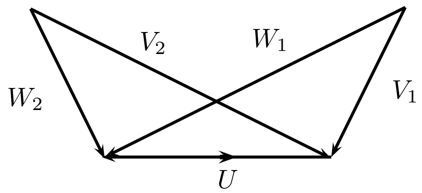
(a) Specific humidity of the air increases
(b) Specific humidity of the air decreases
(c) Relative humidity of the air increases
(d) Relative humidity of the air decreases

(GATE 2011)

Solution. Heating will increase dry bulb temperature (DBT) with constant specific humidity (ω), therefore, relative humidity (ϕ) will decrease.

Ans. (d)

73. Velocity triangles at the inlet and exit of the rotor of a turbo-machine are shown. V denotes the absolute velocity of the fluid, W denotes the relative velocity of the fluid, and U denotes the blade velocity. Subscripts 1 and 2 refers to inlet and outlet respectively.



If $V_2 = W_1$ and $V_1 = W_2$, then the degree of reaction is

(a) 0 (b) 1
(c) 0.5 (d) 0.25

(GATE 2012)

Solution. Degree of reaction is the ratio of energy change in moving blades and that in all stages. The given case is only for 50% reaction turbines.

Ans. (c)

74. Steam enters an adiabatic turbine operating at steady state with an enthalpy of 3251.0 kJ/kg and leaves as a saturated mixture at 15 kPa with quality (dryness fraction) 0.9. The enthalpies of the saturated liquid and vapor at 15 kPa are $h_f =$ 225.94 kJ/kg, and $h_g =$ 2598.3 kJ/kg, respectively. The mass flow rate of steam is 10 kg/s. Kinetic and potential energy changes are negligible. The power output of the turbine in MW is

(a) 6.5 (b) 8.9
(c) 9.1 (d) 27.0

(GATE 2012)

Solution. Given that

$$\dot{m} = 10 \text{ kg/s}$$
$$h_1 = 3251.0 \text{ kJ/kg}$$

Enthalpy at outlet is

$$\begin{aligned} h_2 &= 225.94 + 0.9\,(2598.3 - 225.94) \\ &= 2361.06 \text{ kJ/kg} \end{aligned}$$

Power output is

$$\begin{aligned} P &= \frac{\dot{m}\,(h_2 - h_1)}{1000} \\ &= \frac{10\,(2361.06 - 3251.0)}{1000} \\ &= 8.89936 \text{ MW} \end{aligned}$$

Ans. (b)

75. A room contains 35 kg of dry air and 0.5 kg of water vapor. The total pressure and temperature of air in the room are 100 kPa and 25°C, respectively. Given that the saturation pressure for water at 25°C is 3.17 kPa, the relative humidity of the air in the room is

(a) 67% (b) 55%
(c) 83% (d) 71%

(GATE 2012)

Solution. Given that

$$\begin{aligned} m_a &= 35 \text{ kg} \\ m_v &= 0.5 \text{ kg} \\ p &= 100 \text{ kPa} \\ p_s &= 3.17 \text{ kPa} \end{aligned}$$

The specific humidity in the room is

$$\begin{aligned} \omega &= \frac{m_v}{m_a} \\ &= \frac{0.5}{35} \\ &= 0.01428 \end{aligned}$$

Specific humidity of saturated air is

$$\begin{aligned} \omega_s &= 0.622\frac{p_s}{p - p_s} \\ &= 0.622\frac{3.17}{100 - 3.17} \\ &= 0.0203 \end{aligned}$$

Relative humidity in the room is

$$\begin{aligned} \phi &= \frac{\omega}{\omega_s} \\ &= \frac{0.01428}{0.02036} \\ &= 70.127\% \end{aligned}$$

Ans. (d)

Common Data Questions

A refrigerator operates between 120 kPa and 800 kPa in an ideal vapor compression cycle with R-134a as the refrigerant. The refrigerant enters the compressor as saturated vapor and leaves the condenser as saturated liquid. The mass flow rate of the refrigerant is 0.2 kg/s. Properties for R-134a are as follows.

Saturated R-134a

p kPa	T °C	h_f kJ/kg	h_g kJ/kg	s_f kJ/kg	s_g kJ/kg
120	−22.32	22.5	237	0.093	0.950
800	31.31	95.5	267.3	0.354	0.918

Saturated R-134a

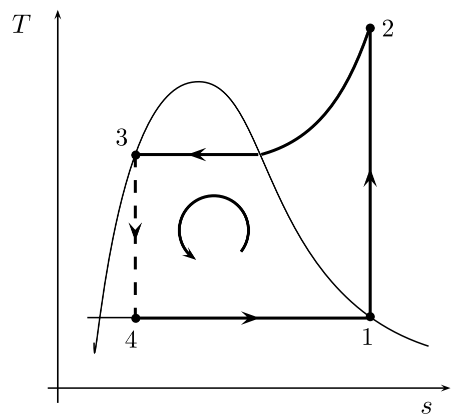
p kPa	T °C	h_f kJ/kg	h_g kJ/kg
800	40	276.45	0.95

76. The power required for the compressor in kW is

(a) 5.94 (b) 1.83
(c) 7.9 (d) 39.5

(GATE 2012)

Solution. The cycle is shown in the figure.



From the given data,

$$h_1 = 237 \text{ kJ/kg}$$
$$h_2 = 276.45 \text{ kJ/kg}$$
$$h_3 = 95.5 \text{ kJ/kg}$$
$$h_4 = 95.5 \text{ kJ/kg}$$
$$\dot{m} = 0.2 \text{ kg/s}$$

Compressor work is given by

$$\begin{aligned} W &= \dot{m}\,(h_2 - h_1) \\ &= 0.2\,(276.45 - 237) \\ &= 7.89 \text{ kW} \end{aligned}$$

Ans. (c)

77. The rate at which heat is extracted in kJ/s from the refrigerated space is

(a) 28.3 (b) 42.9
(c) 34.4 (d) 14.6

(GATE 2012)

Solution. Heat extraction is given by

$$\begin{aligned} Q_2 &= \dot{m}\,(h_1 - h_4) \\ &= 0.2\,(237 - 95.5) \\ &= 28.3 \text{ kW} \end{aligned}$$

Ans. (a)

78. The pressure, dry bulb temperature, and relative humidity of air in a room are 1 bar, 30°C and 70%, respectively. If the saturated steam pressure at 30°C is 4.25 kPa, the specific humidity of the room air in kg water vapor/kg dry air is

(a) 0.0083 (b) 0.0101
(c) 0.0191 (d) 0.0232

(GATE 2013)

Solution. Given that

$$\begin{aligned} p &= 100 \text{ kPa} \\ \phi &= 0.7 \\ p_s &= 4.25 \text{ kPa} \end{aligned}$$

Relative humidity is defined as

$$\phi = \frac{\omega}{\omega_s}$$

Therefore, the specific humidity of the moist air is determined as

$$\begin{aligned} \omega &= \phi \times \omega_s \\ &= 0.7 \times \frac{0.622 \times p_s}{p - p_s} \\ &= 0.7 \times \frac{0.622 \times 4.25}{100 - 4.25} \\ &= \frac{1.85045}{95.75} \\ &= 0.01932 \end{aligned}$$

Ans. (c)

79. In order to have maximum power from a Pelton turbine, the bucket speed must be

(a) equal to the jet speed
(b) equal to half the jet speed
(c) equal to twice the jet speed
(d) independent of the jet speed

(GATE 2013)

Solution. For maximum efficiency of Pelton wheel,

$$\frac{u}{V_1} = 0.5$$
$$u = \frac{V_1}{2}$$

Thus, in order to have maximum power from a Pelton turbine, the bucket speed (u) must be equal to half the jet speed (V_1).

Ans. (b)

MULTIPLE CHOICE QUESTIONS

1. Which one of the following is correct?

(a) pressure and temperature are independent during phase change
(b) an isothermal line is also a constant pressure line in the wet vapor region
(c) entropy decreases during expansion
(d) the term dryness fraction is used to specify the fraction of mass of liquid in a mixture of liquid and vapor

2. With increase in pressure, the latent heat of the steam

(a) remains the same
(b) increases
(c) decreases
(d) behaves unpredictably

3. When wet steam flows through a throttle valve and remains wet at exit

(a) its temperature and quality increases
(b) its temperature decreases but quality increases
(c) its temperature increases but quality decreases
(d) its temperature and quality decreases

4. Constant pressure lines in the superheated region of the Mollier diagram will have

(a) a positive slope
(b) a negative slope
(c) zero slope
(d) both positive and negative slope

5. Which one of the following statements is correct when saturation pressure of water vapor increases?

(a) saturation temperature decreases
(b) enthalpy of evaporation decreases
(c) enthalpy of evaporation increases
(d) specific volume change of phase increases

6. For a pure substance, what are the numbers of the thermodynamic degree of freedom for saturated vapor and superheated vapor, respectively?

(a) 1 and 1 (b) 1 and 2
(c) 2 and 1 (d) 2 and 2

7. Which one of the following is correct? At critical point the enthalpy of vaporization is

(a) dependent on temperature only
(b) maximum
(c) minimum
(d) zero

8. Water ($c_p = 4$ kJ/kgK) is fed to a boiler at 30°C, the enthalpy of vaporization at atmospheric pressure in the boiler is 2400 kJ/kg; the steam coming from the boiler is 0.9% dry. What is the net heat supplied in the boiler?

(a) 2160 kJ/kg (b) 2400 kJ/kg
(c) 2440 kJ/kg (d) 2280 kJ/kg

9. In steam power cycle, reheat factor is usually in the range

(a) 1.02 to 1.05 (b) 1.12 to 1.15
(c) 1.5 to 1.8 (d) 1.9 to 2.1

10. In a steam power plant, feed water heater is a heat exchanger to preheat feed water by

(a) live steam from steam generator
(b) hot air from air preheater
(c) hot flue gases coming out of the boiler
(d) extracting steam from turbine

11. A condenser of a refrigeration system rejects heat at a rate of 120 kW, while its compressor consumes a power of 30 kW. The coefficient of performance of the system would be

(a) 1/4 (b) 1/3
(c) 3 (d) 4

12. In a Rankine cycle, with the maximum steam temperature being fixed from metallurgical considerations, as the boiler pressure increases

(a) the condenser load will increase
(b) the quality of turbine exhaust will decrease
(c) the quality of turbine exhaust will increase
(d) the quality of turbine exhaust will remain unchanged

13. For a given set of operating pressure limits of a Rankine cycle, the highest efficiency occurs for

(a) Saturated cycle
(b) Superheated cycle
(c) Reheat cycle

(d) Regenerative cycle

14. The main advantage of a reheat Rankine cycle is

(a) reduced moisture content in low pressure side of turbine
(b) increase efficiency
(c) reduced load on condenser
(d) reduced load on pump

15. Which one of the following is correct? In ideal regenerative cycle, the temperature of steam entering the turbine is same as that of

(a) water entering the turbine
(b) water leaving the turbine
(c) steam leaving the turbine
(d) water at any section of the turbine

16. In a convergent-divergent nozzle, normal shock can generally occur

(a) along the divergent portion and throat
(b) along the convergent portion
(c) anywhere along the length
(d) near the inlet

17. In flow through steam nozzles, the actual discharge will be greater than the theoretical value when

(a) steam at inlet is superheated
(b) steam at inlet is saturated
(c) steam gets supersaturated
(d) steam at inlet is wet

18. In flow through a convergent nozzle, the ratio of back pressure to the inlet pressure is given by the ratio

$$\frac{p_B}{p_1} = \left(\frac{2}{\gamma+1}\right)^{2\gamma/(\gamma-1)}$$

If the back pressure is lower than p_B given by the above equation, then

(a) the flow in the nozzle is supersonic
(b) a shock wave exists inside the nozzle
(c) the gases expand outside the nozzle and a shock wave appears outside the nozzle
(d) a shock wave appears at the nozzle exit

19. For adiabatic expansion with friction through a nozzle, the following remains constant:

(a) entropy
(b) static enthalpy
(c) stagnation enthalpy
(d) stagnation pressure

20. The effect of friction on flow of steam through nozzle is to

(a) decrease the mass flow rate and to increase the wetness at the exit
(b) increase the mass flow rate and to increase the exit temperature
(c) decrease the mass flow rate and to decrease the wetness at the exit
(d) increase the exit temperature without any effect on the mass flow rate

21. If the velocity of propagation of small disturbance in air at 27°C is 330 m/s, then at a temperature of 54°C, its speed would be

(a) 660 m/s
(b) $330\sqrt{2}$ m/s
(c) $330/\sqrt{2}$ m/s
(d) $330\sqrt{327/300}$ m/s

22. For one-dimensional isentropic flow in a diverging passage, if the initial static pressure is p_1, and the initial Mach number is M_1, then for the downstream flow

(a) $M_2 < M_1$, $p_1 < p_2$
(b) $M_2 < M_1$, $p_1 > p_2$
(c) $M_2 > M_1$, $p_1 > p_2$
(d) $M_2 > M_1$, $p_1 < p_2$

23. The stagnation temperature is of an isentropic flow of air ($\gamma = 1.4$) is 400 K. If the temperature is 200 K at a section, then the Mach number of the flow will be

(a) 1.046 (b) 1.264
(c) 2.236 (d) 3.211

24. Isentropic flow is

(a) irreversible adiabatic flow
(b) reversible adiabatic flow
(c) ideal fluid flow
(d) frictionless reversible flow

25. An aeroplane is cruising at a speed of 800 kmph at an altitude, where the air temperature is 0°C. The flight Mach number at this speed is nearly

(a) 1.5 (b) 0.254
(c) 0.67 (d) 2.04

26. The isentropic heat drop in the nozzle of an impulse turbine with nozzle efficiency 0.9, blade velocity ratio 0.5 and mean blade velocity 150 m/s in kJ/kg is

(a) 50 (b) 40
(c) 60 (d) 75

27. The critical pressure ratio for maximum discharge through a nozzle is

(a) $\left(\dfrac{2}{n+1}\right)^{(n-1)/n}$ (b) $\left(\dfrac{2}{n+1}\right)^{n/(n-1)}$

(c) $\left(\dfrac{2}{n-1}\right)^{(n-1)/n}$ (d) $\left(\dfrac{2}{n-1}\right)^{n/(n-1)}$

28. Under ideal conditions, the velocity of steam at the outlet of a nozzle for a heat drop of 400 kJ/kg will be, approximately,

(a) 1200 m/s
(b) 900 m/s
(c) 600 m/s
(d) the same as the sonic velocity

29. In a steam nozzle, inlet pressure of superheated steam is 10 bar. The exit pressure is decreased from 3 bar to 1 bar. The discharge rate will

(a) remain constant
(b) decrease
(c) increase slightly
(d) increase or decrease depending on whether the nozzle is convergent or convergent-divergent

30. In a steam nozzle, to increase the velocity of steam above sonic velocity by expanding steam below critical pressure

(a) a vacuum pump is added
(b) ring diffusers are used
(c) divergent portion of the nozzle is necessary
(d) abrupt change in cross-section is needed

31. Steam pressure values at the inlet and exit of a nozzle are 16 bar and 5.2 bar, respectively, and discharge is 0.28 m^3/s. Critical pressure ratio is 0.5475. If the exit pressure is reduced to 3.2 bar, then what will be the flow rate in m^3/s?

(a) 0.280 (b) 0.328
(c) 0.356 (d) 0.455

32. In a steam condenser, the partial pressure of steam and air are 0.06 bar and 0.007 bar, respectively. The condenser pressure is

(a) 0.067 bar (b) 0.06 bar
(c) 0.053 bar (d) 0.007 bar

33. The main aim of compounding in steam turbines is

(a) improve efficiency
(b) reduce steam consumption
(c) reduce motor speed
(d) reduce turbine size

34. In a de Laval nozzle expanding superheated steam from 10 to 0.1 bar, the pressure at the minimum cross-section will be

(a) 3.3 bar (b) 5.46 bar
(c) 8.2 bar (d) 9.9 bar

35. For a Parson's reaction turbine, if α_1 and α_2 are fixed blade angles at inlet and exit, respectively, and β_1 and β_2 are the moving blade angles at entrance and exit, respectively, then

(a) $\alpha_1 = \alpha_2, \beta_1 = \beta_2$ (b) $\alpha_1 = \beta_1, \alpha_2 = \beta_2$
(c) $\alpha_1 < \beta_1, \alpha_2 > \beta_2$ (d) $\alpha_1 = \beta_2, \alpha_2 = \beta_1$

36. Match List I with List II and select the correct answer using the codes given below

List-I (Turbine)	List-II (Classification)
A. Parson's	1. Pressure compounded
B. De Laval	2. Reaction
C. Rateau	3. Simple impulse
D. Curtis	4. Velocity compounded

(a) A-3, B-2, C-1, D-4
(b) A-2, B-3, C-4, D-1
(c) A-2, B-3, C-1, D-4
(d) A-3, B-2, C-4, D-1

37. In Parson's reaction turbine stage, blade velocity is 320 m/s at the mean radius and the rotor lade exit angle is 30°. For minimum kinetic energy of the steam leaving the stage, the steam velocity at the exit of the stator will be

(a) $640/\sqrt{3}$ m/s (b) 640 m/s
(c) $320/\sqrt{3}$ m/s (d) $140/\sqrt{3}$ m/s

38. A reaction turbine stage has angles α, β, γ as nozzle angle, inlet blade angle, and outlet blade angle, respectively. The expression for maximum efficiency of the turbine is given by

(a) $2\cos^2\beta/\left(1+\cos^2\beta\right)$
(b) $2\cos^2\gamma/\left(1+\cos^2\gamma\right)$
(c) $2\cos^2\alpha/\left(1+\cos^2\alpha\right)$
(d) $\cos(\alpha+\beta)/\cos^2\gamma$

39. The degree of reaction of a turbine is defined as the ratio of

(a) static pressure drop to total energy transfer
(b) total energy transfer to static pressure drop
(c) change of velocity energy across the turbine to the total energy transfer
(d) velocity energy to pressure energy

40. Given that

$$\begin{aligned} V_b &= \text{Blade speed} \\ V &= \text{Velocity of steam entering the blade} \\ \alpha &= \text{Nozzle angle} \end{aligned}$$

The efficiency of an impulse turbine is maximum when

(a) $V_b = 0.5V\cos\alpha$ (b) $V_b = V\cos\alpha$
(c) $V_b = 0.5V^2\cos\alpha$ (d) $V_b = V^2\cos\alpha$

41. The Rateau turbine belongs to the category of

(a) pressure compounded turbine
(b) reaction turbine
(c) velocity compounded turbine
(d) radial flow turbine

42. A Curtis stage, Rateau stage and a 50% reaction stage in a steam turbine are examples of

(a) different types of impulse stages
(b) different types of reaction stages
(c) a simple impulse stage, a velocity compounded impulse stage and reaction stage
(d) a velocity compounded impulse stage, a pressure compounded stage and a reaction stage

43. A steam plant has the boiler efficiency of 92%, turbine efficiency (mechanical) of 94%, generator efficiency of 95% and cycle efficiency of 44%. If 6% of the generated power is used to run the auxiliaries, the overall plant efficiency is

(a) 34% (b) 39%
(c) 45% (d) 30%

44. For a single stage impulse turbine with a rotor diameter of 2 m and a speed of 3000 rpm when the nozzle angle is 20°, the optimum velocity of steam is m/s is?

(a) 334 (b) 356
(c) 668 (d) 711

45. If in a steam turbine stage, heat drop in moving blade ring is 40 kJ/kg and heat in the fixed blade ring is 60 kJ/kg, then the degree of reaction is

(a) 20% (b) 40%
(c) 60% (d) 70%

46. If in an impulse turbine designed for free vortex flow, tangential velocity of steam at the root radius of 250 mm is 430 m/s and the blade height is 100 mm, then the tangential velocity of steam at the tip will be

(a) 602 m/s (b) 504 m/s
(c) 409 m/s (d) 307 m/s

47. The maximum blade efficiency of single stage impulse turbine in terms of nozzle angle α under ideal conditions is proportional to

(a) $\cos\alpha$ (b) $\sin\alpha$
(c) $\sin^2\alpha$ (d) $\cos^2\alpha$

48. In a reaction turbine, the enthalpy drop in the fixed blade ring is 50 kJ per kg and the enthalpy drop in the moving blade ring is 25 kJ per kg. The degree of reaction of the turbine is

(a) 66.7% (b) 50.0%
(c) 33.3% (d) 6.0%

49. Symmetrical blading is used in a turbine when its degree of reaction is

(a) 25% (b) 50%
(c) 75% (d) 100%

50. The degree of reaction of a turbine is the ratio of enthalpy drop in

(a) moving blades to enthalpy drop in the stage
(b) fixed blades to enthalpy drop in the stage
(c) moving blades to enthalpy drop in fixed blades
(d) fixed blades to enthalpy drop in moving blades

51. The impulse turbine rotor efficiency will have a maximum value $\cos^2\alpha$ where α is the nozzle exit flow angle, if the

(a) blades are equiangular
(b) blade velocity coefficient is unity
(c) blades are equiangular and frictionless

(d) blade solidity is 0.65

52. Energy conversion takes place only in one row of rotor of nozzle blades and later the steam glides over the rotor and guide rows in the case of

(a) De Laval turbine
(b) Rateau turbine
(c) Parson's turbine
(d) Curtis turbine

53. In a 50% reaction turbine stage, the tangential component of absolute velocity at rotor inlet is 537 m/s and blade velocity is 454 m/s. The power output in kW of steam will be

(a) 302 (b) 282
(c) 260 (d) 284

54. Employing superheated steam in turbines leads to

(a) increase in erosion of blading
(b) decrease in erosion of blading
(c) no erosion in blading
(d) no change in erosion of blading

55. In which one of the following steam turbines, steam is taken from various points along the turbine, solely for feed water heating?

(a) extraction turbine
(b) bleeder turbine
(c) regenerative turbine
(d) reheat turbine

56. The degree of reaction of an impulse turbine:

(a) is less than zero
(b) is greater than zero
(c) is equal to zero
(d) increases with steam velocity at the inlet

57. A 4-row velocity compounded steam turbine develops a total power of 6400 kW. What is the power developed by the last row?

(a) 200 kW (b) 400 kW
(c) 800 kW (d) 1600 kW

58. Blade erosion in steam turbines takes place

(a) due to high temperature steam
(b) due to droplets in steam
(c) due to high rotational speed
(d) due to high flow rate

59. In a simple impulse turbine the nozzle angle at the entrance is 30°. For maximum diagram efficiency, what is the blade-speed ratio? (Note: $\sin 30° = 0.50$, $\cos 30° = 0.866$, $\sin 15° = 0.259$, $\cos 15° = 0.966$)

(a) 0.259 (b) 0.750
(c) 0.500 (d) 0.433

60. If the enthalpy drop in the moving blades and fixed blades of a steam turbine is 10 kJ/kg and 15 kJ/kg, respectively, then what is the degree of reaction?

(a) 67% (b) 60%
(c) 40% (d) 33%

61. In steam and other vapor cycles, the process of removing non-condensables is called

(a) scavenging process
(b) deaeration process
(c) exhaust process
(d) condensation process

62. Given that

$$\eta_s = \text{Stage efficiency}$$
$$\eta_n = \text{Nozzle efficiency}$$
$$\eta_b = \text{Blade efficiency}$$

Which one of the following is correct?

(a) $\eta_n = \eta_b \eta_s$ (b) $\eta_b = \eta_n \eta_s$
(c) $\eta_n \times \eta_b \times \eta_s = 1$ (d) $\eta_s = \eta_b \eta_n$

63. In a spark ignition engine working on the ideal Otto cycle, the compression ratio is 5.5. The work output per cycle (i.e. area of the p-v diagram) is equal to $23.625 \times 10^5 \times v_c$ J where v_c is the clearance volume in m^3. The indicated mean effective pressure is

(a) 4.295 bar (b) 5.250 bar
(c) 86.870 bar (d) 106.30 bar

64. The correct sequence of the decreasing order of brake thermal efficiency of the three given basic type of IC engines is

(a) 4-stroke CI engine, 4-stroke SI engine, 2-stroke SI engine
(b) 4-stroke SI engine, 4-stroke CI engine, 2-stroke SI engine
(c) 4-stroke CI engine, 2-stroke SI engine, 4-stroke SI engine
(d) 2-stroke SI engine, 4-stroke SI engine, 4-stroke CI engine

65. An IC engine has a bore and stroke of 2 units each. The area to calculate heat loss can be taken as

(a) 4π (b) 5π
(c) 6π (d) 8π

66. With increasing temperature of intake air, IC engine efficiency

(a) decreases
(b) increases
(c) remains the same
(d) depends on other factors

67. For the same maximum pressure and heat input

(a) the exhaust temperature of petrol engine is more than that of the diesel engine
(b) the exhaust temperature of diesel engine is more than that of the petrol engine
(c) the exhaust temperature of dual cycle engine is less than that of the diesel engine
(d) the exhaust temperature of dual cycle engine is more than that of the petrol engine

68. Which cycle consists of two reversible isotherms and two reversible isobars?

(a) Carnot cycle
(b) Stirling cycle
(c) Ericsson cycle
(d) Brayton cycle

69. In the case of a Diesel cycle, increasing the cut-off ratio will increase

(a) efficiency
(b) mean effective pressure
(c) maximum weight
(d) the engine weight

70. For the same compression ratio and the same heat input, the correct sequence of the increasing order of the thermal efficiencies of the given cycles is

(a) Otto, Diesel, Dual
(b) Diesel, Dual, Otto
(c) Dual, Diesel, Otto
(d) Dual, Otto, Diesel

71. In an air standard Otto cycle, r is the volume compression ratio and γ is an adiabatic index (c_p/c_v), the air standard efficiency is given by

(a) $\eta = 1 - 1/(r^{\gamma} - 1)$ (b) $\eta = 1 - 1/r^{\gamma}$
(c) $\eta = 1 - 1/r^{\gamma-1}$ (d) $\eta = 1 - 1/r^{2\gamma-1}$

72. The bore and stroke of the cylinder of a 6-cylinder engine working on an Otto-cycle are 17 cm and 30 cm, respectively, total clearance volume is 9225 cm^3, then what is the compression ratio?

(a) 7.8 (b) 6.2
(c) 15.8 (d) 5.4

73. The air–fuel ratio for idling speed of an automobile petrol engine is closer to

(a) 10:1 (b) 15:1
(c) 17:1 (d) 21:1

74. In combustion process, the effect of dissociation is to

(a) to reduce the flame temperature
(b) separate the products of combustion
(c) reduce the proportion of carbon monoxide in gases
(d) reduce the use of excess air

75. As compared to air standard cycles, in actual working, the effect of variation in specific heats is to

(a) increase maximum pressure and maximum temperature
(b) decrease maximum pressure and maximum temperature
(c) increase maximum pressure and decrease temperature
(d) decrease maximum pressure and increase maximum temperature

76. The essential function of carburettor in a spark engine is to:

(a) meter the fuel into the air stream and amount dictated by the load and speed.
(b) bring about mixing of air and fuel to get a homogeneous mixture
(c) vaporize the fuel
(d) distribute fuel uniformly to all cylinders in a multi-cylinder engine and also vaporize it

77. Match list I with List II related to operation SI engines and select the correct option.

List-I	List-II
Operating mode	Air fuel ratio
A. Idling	1. 12.5
B. Cold starting	2. 9.0
C. Crusing	3. 16.0
D. Maximum power	4. 22.0
	5. 3.0

(a) A-2, B-4, C-5, D-1
(b) A-1, B-3, C-4, D-2
(c) A-5, B-2, C-1, D-3
(d) A-2, B-5, C-3, D-1

78. In some carburettors, meter rod and economizer device is used for

(a) cold starting
(b) idling
(c) power enrichment
(d) acceleration

79. Which of the following factors increase detonation in the SI engines?

1. Increased spark advance
2. Increased speed
3. Increased air-fuel ratio beyond stoichiometric strength
4. Increased compression ratio

Select the correct answer using the codes given below

(a) 1 and 3 (b) 2 and 4
(c) 1, 2 and 4 (d) 1 and 4

80. Reference fuels for knock rating of SI engine fuels would include

(a) iso-octane and α-methyl naphthalene
(b) normal octane and aniline
(c) iso-octane and n-hexane
(d) n-heptane and iso-octane

81. By higher octane number of SI fuel, it means that the fuel has

(a) higher heating value
(b) higher flash point
(c) lower volatility
(d) longer ignition delay

82. Consider the following measures:

1. Increasing the compression pressure,
2. Increasing the intake temperature,
3. Increasing the length to diameter ratio of the cylinder,
4. Increasing the engine speed.

The measures necessary to reduce the tendency of knock in CI engines would include

(a) 1, 2 and 3 (b) 1, 2 and 4
(c) 1, 3 and 4 (d) 2, 3 and 4

83. In the operation of four-stroke diesel engines, the term ‘squish’ refers to the

(a) injection of fuel in pre-combustion chamber
(b) discharge of gases from the pre-combustion chamber
(c) entry of air into the combustion chamber
(d) stripping of fuel from the core

84. Which one of the following quantities is assumed constant for an internal combustion engine while estimating its friction power by extrapolation through Willan’s line?

(a) brake thermal efficiency
(b) indicated thermal efficiency
(c) mechanical efficiency
(d) volumetric efficiency

85. In a Morse test for a 2-cylinder, 2-stroke, spark ignition engine, the brake power was 9 kW whereas the brake powers of individual cylinders with spark cut-off were 4.25 kW and 3.75 kW, respectively. The mechanical efficiency of the engine is

(a) 90% (b) 80%
(c) 45.5% (d) 52.5%

86. An engine produces 10 kW break power while working with a brake thermal efficiency of 30%. If the calorific value of the fuel used is 40,000 kJ/kg, then what is the fuel consumption?

(a) 0.5 kg/h (b) 3.0 kg/h
(c) 0.3. kg/h (d) 1.0 kg/h

87. A 40 kW engine has a mechanical efficiency of 80%. If the frictional power is assumed to be constant with load, what is the approximate value of the mechanical efficiency at 50% of the rated load?

(a) 45% (b) 55%
(c) 65% (d) 75%

88. The main object of Morse test is to find out

(a) performance of a petrol engine
(b) performance of a diesel engine
(c) frictional power of a petrol engine
(d) indicated power of a single cylinder diesel engine

89. The amount of CO_2 produced by 1 kg of carbon on complete combustion in kg is

(a) 3/11 (b) 3/8
(c) 8/3 (d) 11/3

90. The presence of nitrogen in the products of combustion ensures that:

(a) complete combustion of fuel takes place
(b) incomplete combustion of fuel occurs
(c) dry products of combustion are analyzed
(d) air is used for the combustion

91. The combustion analysis carried out by the Orsat Apparatus renders which one of the following?

(a) the percentage composition by weight on the dry basis
(b) the percentage composition by volume on the dry basis
(c) the percentage composition by weight on the wet basis
(d) the percentage composition by volume on the wet basis

92. Air cooling is used for freon compressors, whereas water jacketing is adopted for cooling ammonia compressors. This is because

(a) latent heat of ammonia is higher than that of freon
(b) thermal conductivity of water is higher than that of air
(c) specific heat of water is higher than that of air
(d) of the larger superheat horn of ammonia compression cycle

93. Global warming is caused by

(a) ozone
(b) carbon dioxide
(c) nitrogen
(d) carbon monoxide

94. The pipes and fitting in an ammonia refrigeration system should be made of

(a) cast steel or wrought iron
(b) aluminium
(c) naval brass
(d) copper

95. The color of the flame of halide torch, in case of leakage of Freon refrigerant, will change to

(a) bright green (b) yellow
(c) red (d) orange

96. The significant advantage of using ammonia as a refrigerant is its

(a) characteristic odor
(b) high latent heat
(c) solubility
(d) inflammability

97. A one ton capacity water cooler cools water steadily from 35°C to 20°C. The specific heat of water is 4.18 kJ/kgK. The water flow rate will be, nearly

(a) 13.33 l/h (b) 33.3 l/h
(c) 200 l/h (d) 250 l/h

98. When a refrigerator system is started from ambient conditions, the evaporator temperature decreases from ambient temperature to design value. This period is known as a pull-down period. The power requirement of compressor during pull down

(a) decreases continuously
(b) increases continuously
(c) remains constant
(d) increases and then decreases

99. A refrigeration system operating on reversed Brayton cycle is used for maintaining 250 K. If the temperature at the end of constant pressure cooling is 300 K and rise in the temperature of air in the refrigerator is 50 K, then the net work of compression will be (assume $c_p = 1$ kJ/kg C)

(a) 250 kJ/kg (b) 200 kJ/kg
(c) 50 kJ/kg (d) 25 kJ/kg

100. In a vapor compression refrigeration plant, the refrigerant leaves the evaporator at 195 kJ/kg and the condenser at 65 kJ/kg. For every kg of refrigerant, the plant can supply per second a cooling load of

(a) 70 kW (b) 100 kW
(c) 130 kW (d) 160 kW

101. In a vapor compression refrigeration system, liquid to suction heat exchanger is used to

(a) keep the COP constant
(b) prevent the liquid refrigerant from entering the compressor
(c) sub-cool the liquid refrigerant leaving the condenser
(d) sub-cool the vapor refrigerant from the evaporator

102. Excessive pressure drop in the liquid line in a refrigerating system causes

(a) high condenser pressure
(b) flashing of the liquid refrigerant
(c) higher evaporator pressure
(d) under cooling of the liquid refrigerant

103. The enthalpies at the beginning of compression at the end of compression and at the end of condensation are 185 kJ/kg, 210 kJ/kg and 85 kJ/kg, respectively. The COP of the vapor compression refrigeration system is

(a) 0.25 (b) 5.0
(c) 4.0 (d) 1.35

104. The effect of super-heating of vapor in the evaporator and sub-cooling of condensate in the condenser, for the same compressor work is

(a) increase the COP
(b) decrease the COP
(c) super-heating increases COP, but sub-cooling decreases COP
(d) super-heating decreases COP, but sub-cooling increases COP

105. For the same condenser and evaporator temperatures, the COP of absorption refrigeration system is less than that of mechanical vapor compression refrigeration system, since in the absorption refrigeration system

(a) a liquid pump is used for compression
(b) a refrigerant as well as a solvent is used
(c) absorber requires heat rejection
(d) low grade energy is used to run the system

106. In the absorption refrigeration cycle, the compressor of the vapor compression refrigeration cycle is replaced by

(a) liquid pump
(b) generator
(c) absorber and generator
(d) absorber, liquid pump and generator

107. In a vapor absorption refrigerator, heat is rejected in:

(a) condenser only
(b) generator only
(c) absorber only
(d) condenser and absorber

108. The atmosphere air at dry bulb temperature of 15°C enters a heating coil maintained at 40°C. The air leaves the heating coil at 25°C. The by-pass factor of the heating coil is

(a) 0.376 (b) 0.4
(c) 0.6 (d) 0.67

109. By-pass factor for a cooling coil

(a) increases with increase in velocity of air passing through it
(b) decreases with increase in velocity of air passing through it
(c) remains unchanged with increase in velocity of air passing through it
(d) may increases or decrease with increase in velocity of air passing through it depending upon the condition of air entering

110. In case of sensible cooling of air, the coil efficiency is given by (BPF=by-pass factor)

(a) BPF−1 (b) 1−BPF
(c) 1/BPF (d) 1+BPF

111. In a saturated air-water mixture, the

(a) dry bulb temperature is higher than the wet bulb temperature
(b) dew point temperature is lower than the wet bulb temperature
(c) dry bulb, wet bulb, and dew point temperatures are the same
(d) dry bulb temperature is higher than the dew point temperature

112. Which property of moist air remains constant during adiabatic cooling

(a) dry bulb temperature
(b) specific humidity
(c) relative humidity
(d) wet bulb temperature

113. When a stream of moist air is passed over a cold and dry cooling coil such that no condensation takes place, then the air stream will get cooled along the line of constant

(a) wet bulb temperature
(b) dew point temperature
(c) relative humidity
(d) enthalpy

114. For air at a given temperature, as the relative humidity is increased isothermally,

(a) the wet bulb temperature and specific enthalpy increase
(b) the wet bulb temperature and specific enthalpy decrease
(c) the wet bulb temperature increases and specific enthalpy decreases
(d) the wet bulb temperature decreases and specific enthalpy increases

115. Ambient air dry-bulb temperature is 45°C and wet bulb temperature is 27°C. Select the lowest possible condensing temperature from the following for an evaporatively cooled condenser.

(a) 25°C (b) 30°C
(c) 42°C (d) 48°C

116. If a sample of moist air of 50% relative humidity at atmospheric pressure is isothermally compressed to a pressure of two atmospheres, then

(a) its relative humidity will reduce to 25%
(b) its relative humidity will remain unchanged
(c) the sample of air will become saturated
(d) saturation pressure will increase to twice the value

117. Evaporative regulation of body temperature fails when the body temperature is

(a) more than wet bulb temperature but less than dry bulb temperature
(b) more than dew point temperature but less than wet bulb temperature
(c) more than dew point temperature but less than dry bulb temperature
(d) less than dew point temperature

118. The state of air supplied by a cooling coil with a bypass factor B lies on the psychrometric chart at the

(a) intersection of RSHF line with saturation curve
(b) intersection of GSHF line with saturation curve
(c) point which divides RSHF line in proportion to B and $(1-B)$
(d) point which divides ESHF line in proportion to B and $(1-B)$

119. The process in a hot water spray washer maintained at a temperature of 40°C through which unsaturated air at 10°C dry bulb temperature and 50% relative humidity passes, is

(a) sensible heating
(b) humidification
(c) heating and humidification
(d) heating and dehumidification

120. It is desired to condition the outside air from 70% RH and 45°C dry bulb to 50% RH and 25°C dry bulb condition. The practical arrangement would be

(a) cooling and dehumidification
(b) dehumidification and pure sensible cooling
(c) cooling and humidification
(d) dehumidification

121. The expression

$$\frac{0.622p_v}{p-p_v}$$

is used to determine

(a) relative humidity
(b) specific humidity
(c) degree of saturation
(d) partial pressure

122. The effective temperature is a measure of the combined effect of

(a) dry bulb temperature and relative humidity
(b) dry bulb temperature and air motion
(c) wet bulb temperature and air motion
(d) dry bulb temperature, relative humidity and air motion

123. If p_v is the partial pressure of vapor, p_s is the partial pressure of vapor for saturated air and p is the barometric pressure, the relationship between relative humidity ϕ and degree of saturation μ is given by

(a) $\mu = \phi\,(p-p_s)\,/\,(p-p_v)$
(b) $\mu = \phi\,(p-p_v)\,/\,(p-p_s)$

(c) $\mu = \phi p_v/p$
(d) $\mu = \phi p_v/p_s$

124. The by-pass factor of single cooling coil in an air conditioner is 0.7. The by-pass factor, if three such cooling coils with the same apparatus dew point are kept one behind the other, will be

(a) 0.210 (b) 0.292
(c) 0.343 (d) 0.412

125. If the volume of moist air with 50% relative humidity is isothermally reduced to half its original volume, then relative humidity of moist air becomes

(a) 25% (b) 60%
(c) 75% (d) 100%

126. Air at 35°C DBT and 25°C dew point temperature passes through the water shower, hose temperature is maintained at 20°C. What is the process involved?

(a) cooling and humidification
(b) sensible cooling
(c) cooling and dehumidification
(d) heating and humidification

127. The wet bulb depression is zero, when relative humidity is equal to:

(a) 100% (b) 60%
(c) 40% (d) zero

128. Air washer can work as

1. Humidifier only
2. Dehumidifier only
3. Filter only

Which of the statements given above is/are correct?

(a) Only I (b) Only 2 and 3
(c) Only 1 and 3 (d) 1, 2 and 3

129. On a psychrometric chart, what does a vertical downward line represent?

(a) Adiabatic saturation
(b) Sensible cooling
(c) Dehumidification
(d) Humidification

130. Which one of the following is correct? During sensible cooling of moist air, its relative humidity

(a) increases
(b) does not change
(c) decreases
(d) affects specific humidity

131. Which one of the following is correct? On psychrometric chart, the constant wet bulb temperature lines coincide with

(a) constant relative humidity lines
(b) constant enthalpy lines
(c) constant dew point temperature lines
(d) constant volume lines

132. In a sample of moist air at standard atmospheric pressure of 101.325 kPa and 26°C, the partial pressure of water vapor is 1.344 kPa. If the saturation pressure of water vapor is 3.36 kPa at 26°C, then what are the humidity ratio and relative humidity of moist air sample?

(a) 0.00826 and 1.31%
(b) 0.00836 and 40%
(c) 0.01344 and 1.32%
(d) 0.01344 and 40%

133. What is the saturation temperature at the partial pressure of water vapor in the air water vapor mixture called?

(a) dry bulb temperature
(b) wet bulb temperature
(c) dew point temperature
(d) saturation temperature

134. The humidity ratio of atmospheric air at 28°C and 760 mm of Hg is 0.016 kN/m^2. What is the partial pressure of the water vapor?

(a) 2.242 kN/m^2 (b) 2.535 kN/m^2
(c) 3.535 kN/m^2 (d) 4.242 kN/m^2

135. When moist air passes through a bed of silica gel, the

(a) dry bulb temperature of air decreases
(b) dry bulb temperature of air increases
(c) specific humidity of air increases
(d) it undergoes adiabatic saturation

136. If the specific humidity of moist air remains the same but its dry bulb temperature increases, its dew point temperature

(a) remains the same

(b) increases
(c) decreases
(d) may increase or decrease depending on its relative humidity

137. In an adiabatic saturation process of air

(a) the enthalpy remains constant
(b) the temperature remains constant
(c) the absolute humidity remains constant
(d) the relative humidity remains constant

138. If air flows over a cooling coil, dehumidification of air will take place if the coil surface temperature is below the following temperature of the entering air:

(a) wet bulb temperature
(b) dry bulb temperature
(c) dew point temperature
(d) adiabatic saturation temperature

139. In an air conditioning process, 5 kJ/min heat is extracted from a room. If the sensible heat factor is 0.8, then the latent heat extracted will be

(a) 4 kJ/min (b) 2 kJ/min
(c) 1 kJ/min (d) 0.25 kJ/min

140. In summer air conditioning, the condition of air passing through the space undergoes a process of

(a) sensible cooling
(b) sensible heating
(c) cooling and dehumidification
(d) heating and humidification

141. In winter air conditioning, the process is

(a) heating, humidification and cooling
(b) heating, humidification and heating
(c) heating, dehumidification and heating
(d) cooling, dehumidification and heating

142. In summer, air may be cooled and dehumidified by spraying chilled water to air in the form of mist. The minimum temperature to which air may be cooled is the

(a) wet bulb temperature
(b) adiabatic saturation temperature
(c) apparatus dew point
(d) dry bulb temperature

143. Match List I with List II and select the correct option.

List-I	List-II
A. Propeller	1. Inward flow reaction
B. Francis	2. Tangential flow impulse
C. Kaplan	3. Axial flow reaction with fixed vanes
D. Pelton	4. Axial flow reaction with adjustable vanes

(a) A-2, B-4, C-1, D-3
(b) A-3, B-4, C-1, D-2
(c) A-2, B-1, C-4, D-3
(d) A-3, B-1, C-4, D-2

144. Match List I with List II and select the correct option.

List-I	List-II
A. Pelton wheel (single jet)	1. Medium discharge, low head
B. Francis turbine	2. High discharge, low head
C. Kaplan turbine	3. Medium discharge, medium head
	4. Low discharge, high head

(a) A-1, B-2, C-3 (b) A-1, B-3, C-4
(c) A-4, B-1, C-3 (d) A-4, B-3, C-2

145. A reaction turbine discharges 30 m^3/s of water under a head of 10 m with an overall efficiency of 92%. The power developed is

(a) 295.2 kW (b) 287.0 kW
(c) 276.0 kW (d) 265.2 kW

146. Based on the direction of flow, which of the following turbine is different from the other three?

(a) Pelton wheel
(b) Kaplan turbine
(c) Modern gas turbine
(d) Parson's turbine

147. The gross head available to a hydraulic power plant is 100 m. The utilized head in the runner of the hydraulic turbine is 72 m. If the hydraulic efficiency of the turbine is 90%, the pipe friction head is estimated to be

(a) 20 m (b) 18 m
(c) 16.2 m (d) 1.8 m

148. The gross head on a turbine is 300 m. The length of penstock supplying water from reservoir to the turbine is 400 m. The diameter of the penstock is 1 m and velocity of water through penstock is 5 m/s. If coefficient of friction is 0.0098, the net head on the turbine would be, nearly

(a) 310 m (b) 295 m
(c) 200 m (d) 150 m

149. In fluid machinery the relationship between saturation temperature and pressure decides the process of

(a) flow separation (b) turbulent mixing
(c) cavitation (d) water hammer

150. In an axial flow impulse turbine, energy transfer takes place due to

(a) change in relative kinetic energy
(b) change in absolute kinetic energy
(c) change in pressure energy
(d) change in energy because of centrifugal force

151. Which of the following water turbines does not require a draft tube?

1. Propeller turbine
2. Pelton turbine
3. Kaplan turbine
4. Francis turbine

Select the correct answer using the codes given below

(a) 1 and 4 (b) Only 2
(c) 1, 2 and 3 (d) 2, 3 and 4

152. The overall efficiency of a Pelton turbine is 70%. If the mechanical efficiency is 85%, what is its hydraulic efficiency?

(a) 82.4% (b) 59.5%
(c) 72.3% (d) 81.5%

153. In a fluid machine, the relevant parameters are volume flow rate, density, viscosity, bulk modulus, pressure difference, power consumption, rotational speed and characteristic dimension. Using the Buckingham π theorem, what would be the number of independent non-dimensional groups?

(a) 3
(b) 4
(c) 5
(d) none of the above

154. What does Euler's equation of turbomachines relate to?

(a) Discharge and power
(b) Discharge and velocity
(c) Head and power
(d) Head and velocity

155. Why are surge tanks used in a pipeline?

(a) to reduce frictional loss in pipe
(b) to ensure uniform flow in pipe
(c) to relieve the pressure due to water hammer
(d) to reduce cavitation

156. If α is the blade angle at the outlet, then the maximum hydraulic efficiency of an ideal impulse turbine is

(a) $(1+\cos\alpha)/2$ (b) $(1-\cos\alpha)/2$
(c) $(1-\sin\alpha)/2$ (d) $(1+\sin\alpha)/2$

157. A Pelton wheel with single jet rotates at 600 rpm. The velocity of the jet from the nozzle is 100 m/s. if the ratio of the vane velocity to jet velocity is 0.44, what is the diameter of the Pelton wheel?

(a) 0.7 m (b) 1.4 m
(c) 2.1 m (d) 2.8 m

158. In case of Pelton turbine installed in a hydraulic power plant, the gross head available is the vertical distance between

(a) forebay and tail race
(b) reservoir level and turbine inlet
(c) forebay and turbine inlet
(d) reservoir level and tail race

159. A jet of water issues from a nozzle with a velocity of 20 m/s and it impinges normally on a flat plate moving away from it at 10 m/s. If the cross-sectional area of the jet is 0.02 m^2 and the density of water is taken as 1000 kg/m^3, then the force developed on the plate will be

(a) 10 N (b) 100 N
(c) 1000 N (d) 2000 N

160. Efficiency of Pelton wheel will be maximum if the ratio of jet velocity to tangential velocity of the wheel is

(a) 0.5 (b) 1
(c) 2 (d) 4

161. Which one of the following is correct? If the number of jets in a Pelton turbine is n, then the specific speed is

(a) $\propto n^2$ (b) $\propto n$
(c) $\propto \sqrt{n}$ (d) Independent of n

162. A Francis turbine is coupled to an alternator to generate electricity with a frequency of 50 Hz. If the alternator has 12 poles, then the turbine should be regulated to run at which one of the following constant speeds?

(a) 250 rpm (b) 500 rpm
(c) 600 rpm (d) 1000 rpm

163. The use of a draft tube in a reaction type water turbine helps to

(a) prevent air from entering
(b) increase the flow rate
(c) convert the kinetic energy to pressure energy
(d) eliminate eddies in the downstream

164. A Francis turbine working at 400 rpm has a unit speed of 50 rpm and develops 500 kW of power. What is the effective head under which this turbine operates?

(a) 62.5 m (b) 64.0 m
(c) 40.0 m (d) 100 m

165. Which one of the following is not correct regarding both Kaplan and propeller turbines?

(a) the runner is axial
(b) the blades are wing type
(c) there are four to eight blades
(d) the blades can be adjusted

166. Kaplan turbine is

(a) a high head mixed flow turbine
(b) a low axial flow turbine
(c) an outward flow reaction turbine
(d) an impulse inward flow turbine

167. When a hydraulic turbine is operated, it is found that it has a high design efficiency and this efficiency remains constant over a wide range of regulation from the design condition. What is the type of this turbine?

(a) Pelton (b) Francis
(c) Kaplan (d) Propeller

168. If the full scale turbine is required to work under a head of 30 m and to run at 428 rpm, then a quarter scale turbine model tested under a head of 10 m must run at

(a) 143 rpm (b) 341 rpm
(c) 428 rpm (d) 988 rpm

169. The specific speed of a turbine is defined as the speed of a member of the same homogenous series of such a size that it

(a) delivers unit discharge at unit head
(b) delivers unit discharge at unit power
(c) delivers unit power at unit discharge
(d) produces unit power under unit head

170. Specific speed of a hydraulic turbine is given by

(a) $N\sqrt{P}/H^{3/4}$ (b) $N\sqrt{Q}/H^{3/4}$
(c) $N\sqrt{P}/H^{5/4}$ (d) $N\sqrt{Q}/H^{5/4}$

171. The specific speeds of a turbine and a pump are (usual symbols), respectively,

(a) $N\sqrt{Q}/H^{3/4}, \quad N\sqrt{P}/H^{5/4}$
(b) $N\sqrt{Q}/H^{3/4}, \quad N\sqrt{P}/H^{3/4}$
(c) $N\sqrt{Q}/H^{5/4}, \quad N\sqrt{P}/H^{5/4}$
(d) $N\sqrt{Q}/H^{5/4}, \quad N\sqrt{P}/H^{3/4}$

172. For fluid machinery, unit speed is proportional to

(a) $\sqrt{H}$ (b) $H^{3/2}$
(c) $1/\sqrt{H}$ (d) $1/H^{3/2}$

173. If H is the head available for a hydraulic turbine the power, speed and discharge are proportional, respectively, to

(a) $H^{1/2}$, $H^{1/2}$, $H^{3/2}$
(b) $H^{3/2}$, $H^{1/2}$, $H^{1/2}$
(c) $H^{1/2}$, $H^{3/2}$, $H^{1/2}$
(d) $H^{1/2}$, $H^{1/2}$, H

174. A hydraulic reaction turbine working under a head of 16 m develops 640 kW of power. What is the unit power of the turbine?

(a) 10 kW (b) 40 kW
(c) 60 kW (d) 160 kW

175. Which one of the following statements is not correct in respect of hydraulic turbines?

(a) speed is inversely proportional to diameter
(b) power is proportional to speed^3
(c) power is proportional to head$^{3/2}$
(d) speed is proportional to head$^{1/2}$

176. The model of propeller 3 m in diameter, cruising at 10 m/s in air, is tested in a wind tunnel on a 1:10 scale model. If a thrust of 50 N is measured on the model at 5 m/s wind speed, then the thrust on the prototype will be

(a) 20000 N (b) 2000 N
(c) 500 N (d) 200 N

177. For a given centrifugal pump,

(a) the discharge varies directly as the speed
(b) the head varies inversely as the speed
(c) the power varies as the square of the speed
(d) the discharge varies as the square of the speed

178. Priming is necessary in

(a) centrifugal pumps to lift water from a greater depth
(b) centrifugal pumps to remove air in the suction pipe and casing
(c) hydraulic turbine to remove air in the turbine casing
(d) hydraulic turbine to increase the speed of turbine and to generate more power

179. In case of centrifugal pump, cavitation will occur if

(a) it operates above the minimum net positive suction head
(b) it operates below the minimum net positive suction head
(c) the pressure at the inlet of the pump is above the atmospheric pressure
(d) the pressure at the inlet of the pump is equal to the atmospheric pressure

180. A centrifugal pump driven by a directly coupled 3 kW motor of 1450 rpm speed, is proposed to be connected to another motor of 2900 rpm speed. The power of the motor should be

(a) 6 kW (b) 12 kW
(c) 18 kW (d) 24 kW

181. Manometric efficiency of a centrifugal pump is defined as the ratio of

(a) suction head to the head imparted by the impeller to water
(b) head imparted by the impeller to water to the suction head
(c) manometric head to the head imparted by the impeller to water
(d) head imparted by the impeller to water to the manometric head

182. A centrifugal pump gives maximum efficiency when its blades are

(a) bent forward (b) end backward
(c) straight (d) wave shaped

183. A centrifugal pump having an impeller of 10 cm diameter discharges 40 liter/s when running at 1000 rpm. The corresponding speed of a geometrically similar pump having an impeller of 40 cm diameter and 0.8 m^3/s discharge will be

(a) 276.4 rpm (b) 298.3 rpm
(c) 312.5 rpm (d) 358.2 rpm

184. For discharge Q, the specific speed of a pump is N_s. For half discharge with the same head, the specific speed will be

(a) N_s (b) $N_s/\sqrt{2}$
(c) $\sqrt{2}N_s$ (d) $2N_s$

185. A pump running at 1000 rpm consumers 1 kW and generates head of 10 m of water. When it is operated at 2000 rpm, its power consumption and head generated would be

(a) 4 kW, 50 m of water
(b) 6 kW, 20 m of water
(c) 3 kW, 30 m of water
(d) 8 kW, 40 m of water

186. Why is a minimum of Net Positive Suction Head (NPSH) required for a hydraulic pump?

(a) to prevent cavitation
(b) to increase discharge
(c) to increase suction head
(d) to increase efficiency

187. Cavitation in a centrifugal pump is likely to occur at the

(a) impeller exit (b) impeller inlet
(c) diffuser exit (d) involute casing

NUMERICAL ANSWER QUESTIONS

1. The steam flows steadily through a 0.25 m diameter pipeline from the boiler to the turbine. At the boiler end, the steam conditions are: $p = 4$ MPa, $t = 400°C$, $h = 3213.6$ kJ/kg, and $v = 0.073$ m^3/kg. At the turbine end, the conditions are: $p = 3.5$ MPa, $t = 392°$ C, $h = 3202.6$ kJ/kg, and $v = 0.084$ m^3/kg. There is a heat loss of 9.5 kJ/kg from the pipeline. Calculate the steam flow rate through the boiler.

2. Steam initially at 1.4 MPa, 300°C expands reversibly and adiabatically in a steam turbine to 50°C. The flow rate of steam is 0.5 kg/s. Steam at 1.4 MPa 300°C has the following properties:

$$s = 6.9534 \text{ kJ/kgK}$$
$$h = 3040.0 \text{ kJ/kg}$$

Steam properties at 50°C has following properties:

$$p_s = 12.349 \text{ kPa}$$
$$s_f = 0.7038 \text{ kJ/kgK}$$
$$s_g = 8.0763 \text{ kJ/kgK}$$
$$h_f = 209.33 \text{ kJ/kg}$$
$$h_{fg} = 2382.7 \text{ kJ/kg}$$

Determine the moisture of the steam after expansion, and the ideal power output of the process.

3. An engine is using 5.4 kg of air per minute while operating at 1200 rpm. The engine requires 0.2256 kg of fuel per hour to produce an indicated power of 1 kW. The air-fuel ratio is 15:1. Indicated thermal efficiency is 38% and mechanical efficiency is 80%. Calculate the brake power of the engine, and the calorific value of the fuel.

4. In an Otto cycle air at 17°C and 1 bar is compressed adiabatically until the pressure is 15 bar. Heat is added at constant volume until the pressure rises to 40 bar. For the air, $\gamma = 1.4$, $c_v = 0.718$ kJ/kg and $R = 0.2872$ kJ/kg K. Determine the air standard efficiency and the mean effective pressure of the cycle.

5. A four-cylinder, two-stroke gasoline engine has cylinder diameter 120 mm, stroke length 160 mm, speed 1650 rpm, area of positive loop of the indicator diagram = 6.00 cm^2, area of the negative loop of the indicator diagram = 0.3 cm^2, length of indicator diagram = 65 mm, spring constant = 3.0 bar/cm. Estimate the mean effective pressure and the indicated power of the engine.

6. A four-cylinder engine running at 1200 rpm delivers 20 kW. The average torque when one cylinder was cut is 110 Nm. The engine uses 360 grams of gasoline per kWh and the calorific value of the fuel is 43 MJ/kg. Calculate the indicated specific fuel consumption rate and the indicated thermal efficiency of the engine.

7. A vapor compression refrigeration system operates between the evaporating and condensing temperatures of 258 K and 313 K, respectively. The power input to the compressor is 10 kW. The refrigerant is R-22 (CHF_2Cl) for which enthalpy at the end of isentropic compression is $h_2 = 287.07$ kJ/kg. Take, at 258 K ($-15°C$): $h_g = 245.36$ kJ/kg, and at 313 K (40°C): $h_f = 95.4$ kJ/kg. Calculate the refrigeration capacity and the COP of the system.

8. An R-134a refrigeration system works between pressure limits 1.65 bar and 10.17 bar, respectively. The heat transfer from the condenser is found to be 80 kJ/min. The refrigerant vapor leaves the evaporator in the saturated state. The condensate leaves the condenser in just saturated state. The refrigerant flow rate through the system is 0.5 kg/min. (For R134a at 1.65 bar $h_g = 389.20$ kJ/kg, and at 10.17 bar $h_f = 256.54$ kJ/kg). Calculate the refrigeration capacity in tonnage and the COP of the system.

9. A moist air is maintained at 1 bar such that its dry bulb temperature is 20°C and dew point temperature is 5°C. The values of saturation pressures of the steam at 20°C and 5°C are 0.02339 bar and 0.00872 bar, respectively. Calculate the relative humidity and the degree of saturation of the moist air.

10. Air at DBT 30°C, ϕ 40% undergoes a constant humidity process until the final state DBT is 20°C. The atmospheric pressure is 1.0132 bar. For the steam at 30°C: $p_s = 0.04246$ bar. Flow rate of the moist air is 2 m^3/s. Calculate the initial and tje final enthalpy of moist air (in kg/kg of dry air), and the rate of cooling produced by the coil.

11. A reaction turbine works at 450 rpm under a head of 115 m. The diameter of the inlet is 1.20 m and the flow area is 0.4 m^2. At the inlet the absolute and relative velocities make angles of 20° and 60°, respectively, with the tangential velocity. The whirl velocity at outlet is zero. Calculate the power and hydraulic efficiency of the turbine.

ANSWERS

Multiple Choice Questions

1. (b)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (d)
11. (c)	**12.** (b)	**13.** (d)	**14.** (a)	**15.** (c)	**16.** (a)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (c)
21. (d)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (c)
31. (a)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (d)	**36.** (c)	**37.** (a)	**38.** (a)	**39.** (c)	**40.** (a)
41. (a)	**42.** (d)	**43.** (a)	**44.** (c)	**45.** (b)	**46.** (d)	**47.** (d)	**48.** (c)	**49.** (b)	**50.** (a)
51. (c)	**52.** (a)	**53.** (b)	**54.** (b)	**55.** (c)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (d)	**60.** (c)
61. (b)	**62.** (d)	**63.** (b)	**64.** (a)	**65.** (c)	**66.** (c)	**67.** (a)	**68.** (c)	**69.** (b)	**70.** (b)
71. (c)	**72.** (d)	**73.** (a)	**74.** (a)	**75.** (b)	**76.** (a)	**77.** (d)	**78.** (d)	**79.** (d)	**80.** (d)
81. (d)	**82.** (a)	**83.** (c)	**84.** (b)	**85.** (a)	**86.** (b)	**87.** (c)	**88.** (c)	**89.** (d)	**90.** (d)
91. (b)	**92.** (a)	**93.** (b)	**94.** (a)	**95.** (a)	**96.** (b)	**97.** (c)	**98.** (d)	**99.** (d)	**100.** (c)
101. (c)	**102.** (b)	**103.** (b)	**104.** (d)	**105.** (d)	**106.** (d)	**107.** (d)	**108.** (c)	**109.** (a)	**110.** (b)
111. (c)	**112.** (d)	**113.** (b)	**114.** (d)	**115.** (a)	**116.** (c)	**117.** (d)	**118.** (d)	**119.** (c)	**120.** (a)
121. (b)	**122.** (d)	**123.** (a)	**124.** (c)	**125.** (d)	**126.** (a)	**127.** (a)	**128.** (d)	**129.** (c)	**130.** (c)
131. (b)	**132.** (b)	**133.** (c)	**134.** (b)	**135.** (b)	**136.** (a)	**137.** (a)	**138.** (c)	**139.** (c)	**140.** (c)
141. (b)	**142.** (c)	**143.** (d)	**144.** (b)	**145.** (c)	**146.** (a)	**147.** (a)	**148.** (b)	**149.** (c)	**150.** (c)
151. (b)	**152.** (a)	**153.** (c)	**154.** (c)	**155.** (c)	**156.** (a)	**157.** (b)	**158.** (d)	**159.** (d)	**160.** (c)
161. (c)	**162.** (b)	**163.** (c)	**164.** (b)	**165.** (d)	**166.** (a)	**167.** (c)	**168.** (d)	**169.** (d)	**170.** (c)
171. (a)	**172.** (c)	**173.** (a)	**174.** (a)	**175.** (c)	**176.** (a)	**177.** (a)	**178.** (b)	**179.** (b)	**180.** (d)
181. (c)	**182.** (c)	**183.** (c)	**184.** (b)	**185.** (d)	**186.** (a)	**187.** (b)			

Numerical Answer Questions

1. 83.72 kg/s	**2.** 0.8476, 405.55 kW	**3.** 67.6 kW, 37.0 MJ/kg
4. 53.87%, 5.21 bar	**5.** 2.6 bar, 52.35 kW	**6.** 0.291 kg/h-kW, 28.74%
7. 10.272, 3.595	**8.** 0.3158, 4.85	**9.** 37.28%, 36.7%
10. 57.23, 46.99, 23.45	**11.** 5263 kW, 89.7%	

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (b) In the wet region, both temperature and pressure remain constant throughout the phase change.

2. (b) For steam dp/dT increases with increase in pressure. Using Clausius–Clapeyron equation:

$$\frac{dp}{dT} = \frac{h_{i-f}}{T\,(v_f - v_i)}$$

The latent heat of steam (h_{i-f}) increases with increase in pressure.

3. (d) As can be observed in the Mollier diagram, both the temperature and quality (dryness fraction) of the steam will decrease.

4. (a) Isobars on Mollier chart has positive slope throughout the wet and super heated regions.

5. (c) Enthalpy of evaporation increases with increasing pressure and temperature.

6. (c) The degrees of freedom are the number of independent variables that can be specified to completely define an equilibrium state. The rest of the variables then become fixed. For example, degree of freedom of saturated water is 1. The number of degrees of freedom are obtained by Gibbs phase rule as

$$f = c - \phi + 2$$

For both saturated vapors, only single variable required because these two phases exist with single component (H_2O). For superheated steams, minimum two variables are required because this is single phase ($p = 1$) and single component $c = 1$. Hence, $f = 2$.

7. (d) Critical point represents the highest temperature and pressure at which both the liquid and vapor phases coexist in equilibrium. The vaporization curve terminates at critical point.

8. (c) The net heat supplied is given by

$$\begin{aligned} Q &= 4\,(100 - 30) + 2400 \times 0.9 \\ &= 2440 \text{ kJ/kg} \end{aligned}$$

9. (a) There are various losses in the stage and the portion of the available energy not converted to work and remaining in the fluid is termed as reheat. Reheat is the difference of actual and isentropic enthalpy drops. The value of reheat factor (RH) lies between 1.04 and 1.08. It can be seen that reheat factor for any single stage expansion is equal to 1.

10. (d) Feed water heaters used in regenerative Rankine cycle and can be of two types; open heaters, and closed heaters. In an open heater or contact-type, the extracted or bled steam is allowed to mix with feed water and both leave the heater at a common temperature. In a closed heater, the fluids are kept separate, and are not allowed to mix together.

11. (c) Given that

$$\begin{aligned} Q_1 &= 120 \text{ kW} \\ W &= 30 \text{ kW} \end{aligned}$$

Therefore,

$$\begin{aligned} \text{COP} &= \frac{Q_1 - W}{W} \\ &= 3 \end{aligned}$$

12. (b) In Rankine cycle, the maximum value of T_{m1} is limited by basically two factors: metallurgy of components may not bear the temperature and moisture content which is limited to not below 85%, as the entrained water particles along with vapor coming out from the nozzles with high velocity strike the blades and erode their surfaces affecting the longevity of the turbine blades.

13. (d) In a regenerative Rankine cycle, the mean temperature of heat addition is increased, thus, increases the cycle efficiency.

14. (a) In Rankine cycle with reheat, as in case of superheat, to protect the blades and reheater tubes, steam is not allowed to expand deep into the two-phase region before it is taken for reheating. In reheater tubes, the moist steam may leave behind solid deposits in the form of scale which is difficult to remove and may hamper the reheating capacity and cause undesired pressure drop.

15. (c) In regenerative Rankine cycle, feed water is heated by a part of steam extracted from the intermediate stages of the turbine.

16. (a) In a convergent-divergent nozzle, normal shock can generally occur along the divergent portion and throat.

17. (c) In flow through steam nozzles, the actual discharge will be greater than the theoretical value when steam gets supersaturated.

18. (c) Shock waves occur due to strike of supersonic velocity fluid with the higher density fluid at throat or near the exit of the nozzle. Hence, velocity abruptly changes after shock to supersonic values. After shock, divergent portion acts as diffuser because it increases the pressure and reduces velocity. Shock occurs only after throat when supersonic velocities exist.

19. (c) The net effect of friction in adiabatic flow or expansion through nozzle is to increase the entropy and final dryness fraction or super heating the steam and finally reducing the exit velocity. This effect reduces the mass flow rate through the nozzle.

20. (c) The net effect of friction is to increase the entropy and final dryness fraction or super heating the steam and finally reducing the exit velocity, and in turn, reducing the mass flow rate.

21. (d) Velocity of propagation of sound (a) in a gas is

$$a = \sqrt{\gamma R T}$$

Using this,

$$\frac{a_2}{a_1} = \sqrt{\frac{T_2}{T_1}}$$
$$a_2 = \sqrt{\frac{54+273}{27+273}} \times 330$$
$$= \sqrt{\frac{327}{300}} \times 330$$

22. (b) In diverging section, dA is negative, therefore, if $p_2 < p_1$, then $M_2 < M_1$.

23. (c) Given that

$$T_s = 400 \text{ K}$$
$$T_1 = 200 \text{ K}$$
$$\gamma = 1.4$$

Using

$$\frac{T_s}{T_1} = 1 + \frac{\gamma - 1}{2} M^2$$

One obtains

$$M = 2.236$$

24. (b) Isentropic flow is reversible adiabatic flow (a reversible process without heat transfer).

25. (c) The sonic speed of air at 0°C (= 273.16 K) is determined as

$$c = \sqrt{\gamma RT}$$
$$= \sqrt{1.4 \times 287 \times 273.16}$$
$$= 331.29 \text{ m/s}$$

Thus, the Mach number for speed $V = 800$ km/h= 222.22 m/s is determined as

$$M = \frac{V}{C}$$
$$= \frac{222.22}{331.29}$$
$$= 0.67$$

26. (a) Given that

$$\eta_n = 0.9$$
$$u = 150 \text{ m/s}$$
$$\rho = u/V_1 = 0.5$$

Velocity at nozzle exit is

$$V_1 = \frac{u}{\rho}$$
$$= \frac{150}{0.5}$$
$$= 300 \text{ m/s}$$

The isentropic heat drop in the nozzle per kg of steam flow rate is given by

$$\Delta h_s = \frac{\frac{1}{2} V_1^2}{\eta_n}$$
$$= \frac{\frac{1}{2} \times 300^2}{0.9}$$
$$= 50 \text{ kJ/kg}$$

27. (b) The critical pressure ratio for maximum discharge through a nozzle is given by

$$r_c = \left(\frac{2}{n+1} \right)^{n/(n-1)}$$

28. (b) Given that

$$\Delta h = 400 \text{ kJ/kg}$$

Velocity will be given by

$$V = \sqrt{2 \Delta h}$$
$$= \sqrt{2 \times 400 \times 10^3}$$
$$= 894.4 \text{ m/s}$$

29. (a) For superheated steam ($n = 1.4$),

$$r_c = \left(\frac{2}{n+1} \right)^{n/(n-1)}$$
$$= 0.5459$$

Given that $p_1 = 10$ bar. Hence, $p_c = 5.459$ bar. The back pressure is already less than this critical pressure, hence, the mass flow rate will remain constant.

30. (c) For steam nozzles, the supersonic flow occurs in divergent portion only.

31. (a) Given that $p_1 = 16$ bar, $p_2 = 5.2$ bar, $r_c = 0.5475$, hence, critical pressure at exit is $p_1 \times r = 8.76$ bar. As p_2 is already less than this, the nozzle is found to be choked. Hence, any further decrease in pressure will not increase the discharge beyond 0.28 m^3/s.

32. (a) The condenser pressure is the sum of partial pressure of the elements. Therefore

$$p = 0.06 + 0.007$$
$$= 0.067 \text{ bar}$$

33. (c) The steam turbines are compounded or stages, where steam, instead of expanding in a single stage, is made to expand in a number of stages, whereby, the turbine speed is reduced while securing the same enthalpy drop of the steam.

34. (b) Given that $p_1 = 10$ bar. For superheated steam, $n = 1.3$. Therefore, pressure at throat is

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{n/(n-1)}$$
$$p_2 = 5.46 \text{ bar}$$

35. (d) To achieve 50% reaction (in Parson's reaction turbine),

$$\Delta h_{s\text{moving blade}} = \Delta h_{s\text{fixed blade}}$$

For which

$$V_1 = V_{r2}$$
$$V_2 = V_{r1}$$

In this situation,

$$\alpha_1 = \beta_2$$
$$\alpha_2 = \beta_1$$

36. (c) Parson's turbine is a reaction turbine with 50% reaction. De Laval is a simple impulse turbine. Rateau and Curtis turbines are pressure compounded and velocity compounded turbines, respectively.

37. (a) Given that

$$u_1 = 320 \text{ m/s}$$
$$\alpha_1 = 30^\circ$$

For maximum stage efficiency,

$$\frac{u_1}{V_1} = \cos\alpha_1$$
$$V_1 = \frac{640}{\sqrt{3}} \text{ m/s}$$

38. (a) For inlet blade angle β in a reaction turbine,

$$\rho = \frac{u}{V_1}$$
$$= \cos\beta$$

The maximum efficiency is

$$(\eta_b)_{\max} = \frac{2\cos^2\beta}{1+\cos^2\beta}$$

39. (c) Degree of reaction (R) is defined as

$$R = \frac{\Delta h_s \text{ in moving blade}}{\Delta h_s \text{ in all stages}}$$

40. (a) For maximum efficiency of impulse turbine

$$\frac{V_b}{V} = \frac{\cos\alpha}{2}$$

41. (a) The Rateau turbine is pressure compounded turbine.

42. (d) The single-stage impulse turbine is also called the de Laval turbine after its inventor. Rateau turbine is a pressure compounded one while Curtis turbine is a velocity compounded turbine.

43. (a) Let Q_1 be the heat given to the boiler. The work output of the plant is determined as

$$W = Q_1 \times 0.92 \times 0.44 \times 0.94 \times 0.95\,(1-0.06)$$
$$\eta_o = \frac{W}{Q_1}$$
$$= 0.339$$
$$\approx 34\%.$$

44. (c) Given that

$$d = 2 \text{ m}$$
$$N = 3000 \text{ rpm}$$
$$\alpha = 20^\circ$$

The optimum velocity of steam V_1 (at nozzle exit) for maximum efficiency is given by

$$\frac{u}{V_1} = \frac{\cos\alpha}{2}$$
$$V_1 = \frac{2u}{\cos\alpha}$$
$$= \frac{2\pi dN}{60 \times \cos\alpha}$$
$$= 668.64 \text{ m/s}$$

45. (b) Degree of reaction is determined as

$$R = \frac{40}{40+60}$$
$$= 40\%$$

46. (d) In a free vortex flow, $vr = c$. Therefore, velocity at the tip will be given by

$$V_\theta \times (250+100) = 250 \times 430$$
$$V_\theta = 307.14 \text{ m/s}$$

47. (d) Maximum blade efficiency of a single stage impulse turbine is given by

$$\eta_{max} = (1+kB)\,\frac{\cos^2\alpha}{2}$$

48. (c) Degree of reaction is determined as

$$R = \frac{25}{25+50}$$
$$= 33.3\%$$

49. (b) Symmetrical blading is used in Parson's reaction turbine where degree of reaction is 50%.

50. (a) The degree of reaction of a turbine is the ratio of enthalpy drop in moving blades to enthalpy drop in the stage.

51. (c) Maximum blade efficiency of a single stage impulse turbine is given by

$$\eta_{max} = (1+kB)\,\frac{\cos^2\alpha}{2}$$

When blades are equiangular ($B = 1$) and frictionless ($k = 1$), the maximum efficiency is given by

$$\eta_{max} = \cos^2\alpha$$

52. (a) The single stage impulse turbine is also called the de Laval turbine where the drop in pressure of steam takes place only in nozzles, and not in the moving blades.

53. (b) The power output per kg of steam shall be given by

$$\begin{aligned} P &= 1\times(2\times 537-454)\times 454 \\ &= 281.48 \text{ kW} \end{aligned}$$

54. (b) Superheated steam reduces erosion of blades due to possible moist air drops.

55. (c) In regenerative Rankine cycle, feed water is heated by a part of steam extracted from the intermediate stages of the turbine.

56. (c) Degree of reaction of an impulse turbine is zero because all the energy is converted in fixed blades only.

57. (b) The ratio of work produced in a 2-stage turbine is 3:1 as one move from higher to lower pressure. This ratio is 5:3:1 in three-stage turbine and changes to 7:5:3:1 in a four-stage turbine. Hence,

$$\begin{aligned} (7+5+3+1)\,W_1 &= 6400 \\ W_1 &= 400 \text{ kW} \end{aligned}$$

58. (b) Droplets due to high moisture content in the steam causes blade erosion in steam turbines.

59. (d) For maximum diagram efficiency of an impulse turbine, the blade-speed ratio is given by

$$\frac{u}{V_1} = \frac{\cos\alpha}{2} = 0.433$$

60. (c) Degree of reaction (R) is defined as

$$\begin{aligned} R &= \frac{\Delta h_s \text{ in moving blade}}{\Delta h_s \text{ in all stages}} \\ &= \frac{10}{10+15} \\ &= 0.4 \\ &= 40\% \end{aligned}$$

61. (b) In steam and other vapor cycles, it is important to remove non-condensable gases that will otherwise accumulate in the system. These are mostly gases that leak from the atmosphere into those portions of the cycle that operate below atmospheric pressure, such as the condenser. Presence of these gases raises the total pressure of the system, thus, decreasing the plant efficiency. The process of removing non-condensables is called deaeration.

62. (d) Blade efficiency is defined as the ratio of power developed by the blades and energy input to the blade. Therefore, stage efficiency is

$$\eta_s = \eta_b \times \eta_n$$

63. (b) Given that compression ratio $r = 5.5$, hence

$$\begin{aligned} \frac{v_s+v_c}{v_c} &= 5.5 \\ \frac{v_s}{v_c} &= 4.5 \end{aligned}$$

The mean effective pressure is determined as

$$\begin{aligned} p_m &= \frac{23.625\times 10^5\times v_c}{v_s} \\ &= \frac{23.625\times 10^5}{4.5} \\ &= 5.25 \text{ bar} \end{aligned}$$

64. (a) CI engine is based on diesel cycle and its efficiency is higher due to high compression ratio. Similarly, a four-stroke engine possesses higher thermal efficiency than two-stroke due to various factors associated with the processes.

65. (c) The heat loss is possible through cylindrical surface and top and bottom circular surfaces, whose area is determined as

$$\begin{aligned} A &= \pi\times 2\times 2+2\times\pi\frac{2^2}{4} \\ &= 6\pi \end{aligned}$$

66. (c) The efficiency of IC engine does not depend upon the intake temperature of the air.

67. (a) For cycles with constant p_3 and Q_S, exhaust temperature will be more for Otto cycle, which is also evident as

$$\eta_{\text{Otto}} < \eta_{\text{Dual}} < \eta_{\text{Diesel}}$$

68. (c) The Ericsson cycle is a modified form of Carnot cycle for higher mean effective pressure. This is achieved by replacing both isentropic processes in Carnot cycle by two isobaric processes.

69. (b) The efficiency of a Diesel cycle is given by

$$\eta_{Diesel} = 1 - \frac{1}{r^{\gamma-1}} \times \frac{k^{\gamma}-1}{\gamma(k-1)}$$

Therefore, η_{Diesel} increases with decrease in cut-off ratio k. The mean effective pressure increases with increase in cut-off ratio because it increases the work output with the same compression ratio.

70. (b) For same compression ratio and heat input, the order of cycles with increasing efficiency is Diesel cycle, dual cycle, and Otto cycle.

71. (c) For Otto cycle, air standard efficiency is expressed as

$$\eta = 1 - \frac{1}{r^{(\gamma-1)}}$$

72. (d) Given that

$$\begin{aligned} n &= 6 \\ d &= 17 \text{ cm} \\ l &= 30 \text{ cm} \\ v_c &= 9225 \text{ cm}^3 \end{aligned}$$

Swept volume (cm^3) is given by

$$\begin{aligned} v_s &= n\frac{\pi d^2}{4} l \\ &= 40856 \end{aligned}$$

The compression ratio is defined as

$$\begin{aligned} r &= \frac{v_c + v_s}{v_c} \\ &= 5.428 \end{aligned}$$

73. (a) For idling, the air–fuel ratio is kept closer to 10:1

74. (a) The effect of dissociation is to lower the maximum temperature and pressure at the beginning of expansion stroke resulting into a loss of power and efficiency.

75. (b) All gases, except monoatomic gases, show an increase in specific heats with the increase in temperature, but difference between c_p and c_v remains unchanged, and specific heat ratio decreases, in turn, cycle efficiency decreases (due to decreased maximum pressure and temperature.)

76. (a) A carburettor is designed to provide a combustible mixture of fuel and air as per the requirements for efficient operation of the engine under all conditions of load and speed.

77. (d) The following are the requirements of air–fuel ratio:

Requirement	A:F
Cold starting/warm up	3:1
Idling	12.5:1
Crushing	16.7:1
Maximum power	14:1

78. (d) The metering system controls the enriching of mixture due to increased throttle opening operated by clutch. The economizer provides and regulates enriched mixture required at full throttle operation.

79. (d) Any factor which reduces the temperature of unburnt charge should reduce the possibility of detonation. Any factor which increases flame speed will reduce knock. Alternatively, knock can be reduced by reducing the distance to be traveled by the flame. This is possible by the factors 1 and 4.

80. (d) Iso-octane (C_8H_{18}) has low boiling point and possesses a very good antiknock quality. On the other hand, n-heptane (n-C_7H_{16}) has very poor antiknock qualities. Octane rating is used for fuels of SI engines. This is equivalent percentage of iso-octane with n-heptane.

81. (d) Iso-octane (C_8H_{18}) has low boiling point and possesses a very good antiknock quality. Octane rating is used for fuels of SI engines.

82. (a) Knock can be reduced by reducing the distance to be traveled by the flame. This is possible by measures 1, 2 and 3.

83. (c) This radial inward motion of the gas mixture of gas mixture is called squish. This helps in quickly spreading of the flame front.

84. (b) In the Willan's method, a graph between fuel consumption and the brake power at constant speed is drawn and it is extrapolated on the negative axis of the brake power where fuel consumption becomes zero, and negative brake power represents the friction losses of the engine at the given speed. The indicated thermal efficiency is assumed to be constant.

85. (a) Given that brake power of engine is 9 kW. Indicated power of individual cylinders is

$$\begin{aligned} P_{i1} &= 9 - 4.25 = 4.75 \text{ kW} \\ P_{i2} &= 9 - 3.75 = 5.25 \text{ kW} \end{aligned}$$

Hence, mechanical efficiency is

$$\eta_m = \frac{9}{4.75+5.25}$$
$$= 90\%$$

86. (b) Fuel consumption rate will be given by

$$\dot{m} = \frac{10\times 60\times 60}{0.3\times 40000}$$
$$= 3.0 \text{ kg/h}$$

87. (c) The friction power is given by

$$P_f = \frac{40}{0.8} - 40$$
$$= 10 \text{ kW}$$

Mechanical efficiency for 50% rated load is

$$\eta_m = \frac{20}{10+20}$$
$$= 66.67\%$$

88. (c) Morse test is used for measuring friction power of an engine.

89. (d) Chemical reaction of burning of C is

$$C + O_2 \rightarrow CO_2$$

Hence, for 12 kg of C, 44 kg of CO_2 is produced. Hence, for 1 kg 44/12=11/3 kg of CO_2 will be produced.

90. (d) Nitrogen forms around 78% volume of the air which did not burn during combustion.

91. (b) Since the products being analyzed in Orsat's apparatus are gaseous, it is usual to quote the analysis by volume.

92. (a) Latent heat of ammonia is 1369 kJ/kg while that of R-11 is 180 kJ/kg.

93. (b) Carbon dioxide (CO_2) is the primary greenhouse gas.

94. (a) The pipes and fitting in an ammonia refrigeration system should be made of cast steel or wrought iron because it reacts with copper and its alloys in the presence of water.

95. (a) Halide torch or leak detector is used to detect leakages of fluorocarbon refrigerants. When a fluorocarbon based refrigerant is passed over a surface at high temperature ($\approx$500°C), its vapor forms phosgene ($COCl_2$) which in turn is passed over a glowing copper where it forms copper chloride, which changes the color of the flame from pale blue to bright green.

96. (b) Ammonia is a simple molecule like water, hence, its latent heat of evaporation is very high and gives high COP.

97. (c) One ton capacity of refrigeration is equal to 3.5 kW. Hence, water flow rate is given by

$$\dot{m}\times 4.18\times (35-20) = 3.5$$
$$\dot{m} = 0.005 \text{ kg/s}$$
$$= 200.95 \text{ l/h}$$

98. (d) To meet the requirement of pull down period, the power requirement will increase and then after it will decrease.

99. (d) Given that

$$T_1 = 250 \text{ K}$$
$$T_3 = 300 \text{ K}$$
$$T_4 = 200 \text{ K}$$

For the compression (1→2) and expansion (3→4) processes,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$
$$T_2 = 375 \text{ K}$$

Net work of compression is

$$W_{1-2} = c_p\{(T_2 - T_1) - (T_3 - T_4)\}$$
$$= 25 \text{ kJ/kg}$$

100. (c) Given that

$$h_1 = 195 \text{ kJ/kg}$$
$$h_2 = 65 \text{ kJ/kg}$$

For throttling, $h_4 = h_3$. Therefore,

$$Q_2 = h_1 - h_4$$
$$= 130 \text{ kJ/kg}$$

For every kg of refrigerant, the plant can supply per second a cooling load of 130 kW.

101. (c) Liquid-suction heat exchangers are commonly installed in refrigeration systems with the intent of ensuring proper system operation and increasing system performance by sub-cooling the refrigerant leaving the condenser.

102. (b) Excessive pressure drop in the liquid line in a refrigerating system causes flashing of the liquid refrigerant. The presence of flash-gas in the liquid lines reduces the efficiency of the refrigeration cycle.

103. (b) The work input at compressor is $210-185=25$ kJ/kg and heat extracted at condenser is $210-85=125$ kJ/kg. Hence, the COP is given by

$$\begin{aligned}\text{COP} &= \frac{125}{25}\\ &= 5.0\end{aligned}$$

104. (d) Super-heating of vapor before compression increases refrigeration rate but there is more power requirement for compression due to diverging nature of isentropic lines. Sub-cooling of condensate (before throttling) significantly reduces the power consumption and enhances COP.

105. (d) Low grade energy (heat) is used to run the an absorption refrigeration system.

106. (d) In the absorption refrigeration cycle, the compressor of the vapor compression refrigeration cycle is replaced by absorber, liquid pump and generator.

107. (d) In a vapor absorption refrigeration system, the heat is rejected in condenser and then in absorber also.

108. (c) Given that

$$\begin{aligned}t_s &= 40^\circ\text{C}\\ t_1 &= 15^\circ\text{C}\\ t_2 &= 25^\circ\text{C}\end{aligned}$$

The by-pass factor is given by

$$\begin{aligned}B &= \frac{t_s-t_2}{t_s-t_1}\\ &= 0.6\end{aligned}$$

109. (a) By-pass factor increases with increasing velocity of air because air does not get much time for heat transfer.

110. (b) The coil efficiency is expressed as

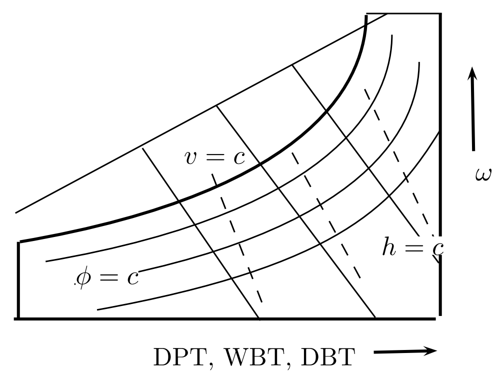
$$\eta_{coil} = 1-\text{BPF}$$

111. (c) The point of saturated air-water mixture lies on the saturation line in psychrometric chart, for which DBT, WBT and DPT are the same.

112. (d) The constant WBT line represents the adiabatic saturation process. It also coincides with the constant enthalpy line.

113. (b) During such a cooling, the partial pressure of the moist air will remain constant, thus, dew point temperature (DPT) will also remain constant. DPT is defined as the temperature when a moist air is cooled such that its vapor partial pressure remains constant when the first drop of liquid is found.

114. (d) This can be observed in the psychrometric chart as shown in the figure below.



Mathematically, the relative humidity (ϕ) are determined as

$$\phi = \frac{p_v}{p_s}$$

For given temperature of air, p_s will remain constant, hence, increased relative humidity will increase the partial pressure p_v. Hence, specific humidity (ω) will increase because

$$\omega = 0.622\frac{p_v}{p-p_v}$$

Increase in ω will result in increase in specific enthalpy (kJ/kg of dry air) because

$$h = 1.005t+\omega\left(h_{dp}+c_p t\right)$$

The constant WBT line represents the adiabatic saturation process. It also coincides with the constant enthalpy line. Thus, the wet bult temperature will also increase.

115. (a) The condensing temperature should be below the wet bulb temperature for evaporative cooling.

116. (c) Relative humidity is determined as

$$\phi = \frac{p_v}{p_s}$$

Thus, when a sample of moist air is isothermally compressed (corresponding saturation pressure p_s remains constant) but partial pressure p_v will be doubled, therefore, relative humidity will be doubled to 100%, that is, saturated.

117. (d) Dew point temperature of the moist air is equal to the saturation temperature corresponding to the partial pressure (p_v) of the vapor in the moist air. Therefore, evaporation will not take

place when body temperature is below the dew point.

118. (d) By-pass factor (say B) is defined as the ratio of final and initial differences between the temperatures of coil and moist air. Thus, the state of air lies at the point which divides ESHF line (along which cooling takes place) in proportion to B and $(1-B)$.

119. (c) The temperature of water spray is more than the DBT of moist air, therefore, it will be a heating process. Also, the moist air is unsaturated therefore, humidification will also take place.

120. (a) Both RH and DBT are to be reduce, therefore it will be a process of cooling and dehumidification.

121. (b) Specific humidity is the ratio of mass of vapor (m_v) and mass of air (m_a) in a given volume of moist air (of total mass m) at a given temperature (T) and pressure (p). Specific humidity is also called humidity ratio. If p_v is partial pressure of water vapor in the moist air, then

$$\begin{aligned}\omega &= \frac{m_v}{m_a} \\ &= \frac{(p_v v)\,/\,(0.461T)}{(p-p_a)\,v/\,(0.287T)} \\ &= 0.622\frac{p_v}{p-p_v}\end{aligned}$$

122. (d) Effective temperature is a measure of combined effect of DBT, relative humidity and air motion.

123. (a) Degree of saturation is given by

$$\mu = \phi\left(\frac{p-p_s}{p-p_v}\right)$$

124. (c) By-pass factor (B) is defined as the ratio of final and initial differences between the temperatures of coil and moist air. If three coils of same by-pass factor (B) are arranged in series, then, equivalent by pass factor will be B^3. For given case, $B^3 = 0.343$.

125. (d) Relative humidity (ϕ) is the ratio of mass of water vapor (m_v) in a certain volume (v) of the moist air at a given temperature (T) to the mass of water vapor (m_s) in the same volume (v) of saturated moist air at the same temperature (T). Mathematically,

$$\begin{aligned}\phi &= \frac{m_v}{m_s} \\ &= \frac{p_v}{p_s}\end{aligned}$$

Therefore, for the given temperature (i.e. p_s) in isothermal conditions

$$\phi \propto p_v$$

reduction of volume will increase the partial pressure in the same proportion, hence, the relative humidity will double, that is, become 100% as the initial relative humidity is 50%..

126. (a) Cooling will reduce the temperature and humidification will be down by water shower.

127. (a) The wet bulb depression (i.e. the difference between the dry and wet bulb temperatures) is zero, hence, the air is fully saturated.

128. (d) The functions of air washer are to cool, humidify, dehumidify and clean the air.

129. (c) Vertical downward process on psychrometric chart shows dehumidification.

130. (c) During sensible cooling of moist air, the relative humidity decreases.

131. (b) On psychrometric charts, the constant enthalpy line and constant wet bulb temperature lines are the same.

132. (b) Given that

$$\begin{aligned}p &= 101.325 \text{ kPa} \\ \text{DBT} &= 26^\circ\text{C} \\ p_v &= 1.344 \text{ kPa} \\ p_s &= 3.36 \text{ kPa}\end{aligned}$$

Therefore,

$$\begin{aligned}\omega &= 0.622\frac{p_v}{p-p_v} \\ &= 0.008361 \\ \mu &= \frac{p_v}{p_s} \\ &= 0.4 \\ &= 40\%\end{aligned}$$

133. (c) Dew point temperature is the saturation temperature corresponding to partial pressure of vapor, and is defined as the temperature when moist air is cooled such that its vapor partial pressure remains constant when the first drop of liquid is formed.

134. (b) Given that

$$\begin{aligned}\omega &= 0.016 \\ p &= 760 \text{ mm}\end{aligned}$$

Using

$$\omega = 0.622\frac{p_v}{p-p_v}$$

one obtains

$$\begin{aligned} p_v &= 19.05 \text{ mm} \\ &= 2.542 \text{ kN/m}^2 \end{aligned}$$

135. (b) Some substances like silica gel and activated alumina have great affinity with water vapor. They are called absorbents. When moist air passes through a bed of silica gel, water vapor molecules get absorbed on its surface. Latent heat of condensation is released, so the DBT of air increases.

136. (a) Dew point temperature (DPT) is the saturation temperature corresponding to partial pressure p_v of vapor. The specific humidity is related to vapor pressure as

$$\begin{aligned} \omega &= 0.622 \frac{p_v}{p - p_v} \\ &= f(p_v) \end{aligned}$$

Thus, when w is constant, p_v remains constant, therefore, corresponding DPT remains constant.

137. (a) The constant WBT line represents the adiabatic saturation process. It also coincides with the constant enthalpy line.

138. (c) When air is cooled below the dew point temperature, condensation occurs and moisture is removed from the air stream.

139. (c) Sensible heat is

$$\begin{aligned} \text{SH} &= 0.8 \times 5 \\ &= 4 \text{ kJ/min} \end{aligned}$$

Therefore, latent heat is $5 - 4 = 1$ kJ/min.

140. (c) In summer air conditioning, the air has to pass through cooling and dehumidification.

141. (b) In winter, air is to be heated and dehumidified. For this, air is first passed through the air preheater to prevent possible freezing of water and to control the evaporation of water into humidifier. After that air is passed through preheater to bring the air to the original DBT

142. (c) The moist air cannot be cooled down below the apparatus dew point temperature.

143. (d) Propeller is an axial flow reaction turbine with fixed vanes. Francis turbine is an inward flow reaction turbine. Kaplan is an axial flow reaction turbine with adjustable vanes (highest hydraulic efficiency). Pelton wheel is a tangential flow impulse turbine.

144. (b) The correct combination is depicted in the following table.

Head	Example
High (> 250 m)	Pelton wheel
Medium (60 – 250 m)	Francis turbine
Low (< 60)	Kaplan, Propeller

145. (c) Given that

$$\begin{aligned} Q &= 30 \text{ m}^3/\text{s} \\ H &= 10 \text{ m} \\ \eta_o &= 92\% \\ \rho &= 1000 \text{ kg/m}^3 \\ g &= 9.8 \text{ m/s}^2 \end{aligned}$$

Therefore, power developed is

$$\begin{aligned} P &= \eta_o \rho g Q H \\ &= 276 \text{ kW} \end{aligned}$$

146. (a) Pelton wheel is a tangential flow turbine while others are axial flow turbines.

147. (a) As the runner power is only 72 m, and hydraulic efficiency is 90%. Hence, the head available at runner entrance is $72/0.9 = 80$ m. In turn, the friction loss is $100 - 80 = 20$ m.

148. (b) Given that

$$\begin{aligned} H_g &= 300 \text{ m} \\ l &= 400 \text{ m} \\ D &= 1 \text{ m} \\ V &= 5 \text{ m/s} \\ 4f &= 0.0098 \end{aligned}$$

Hence, net head will be

$$\begin{aligned} H_n &= H_g - \frac{4fl}{D} \times \frac{V^2}{2g} \\ &= 295 \text{ m} \end{aligned}$$

149. (c) If in any flow system, the pressure at any point in the liquid approaches the vapor pressure, the vaporization of liquid starts resulting in the pockets of dissolved gases and vapors. The bubbles of vapor thus formed are collected and carried by flowing liquid into a region of high pressure where they collapse giving rise to high impact pressure such that material from adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is called cavitation.

150. (c) In an impulse turbine, the available energy is converted into kinetic energy at one time.

151. (b) The draft tube is an integral part of a reaction turbine. Pelton wheel is an impulse turbine, hence, draft tube cannot be used.

152. (a) Given that

$$\eta_o = 0.7$$
$$\eta_m = 0.85$$

Therefore, hydraulic efficiency is determined using the following relationship:

$$\eta_o = \eta_h \times \eta_m$$
$$\eta_h = \frac{\eta_o}{\eta_m} = 0.8235$$

153. (c) According to the theorem, the number of independent dimensionless groups that can be formed from a set of n variables having r basic dimensions is $(n-r)$. For the given case, $n = 8$ and $r = 3$ (M, L, T). Number of independent non-dimensional groups is $8-3 = 5$.

154. (c) Following is the Euler's equation for turbo-machines.

$$H = \left(\frac{p_1}{\rho_w g} + \frac{{V_1}^2}{2g}\right) - \left(\frac{p_2}{\rho_w g} + \frac{{V_2}^2}{2g}\right)$$

155. (c) Surge tanks are used in a pipe line to relieve the pressure due to water hammer.

156. (a) Maximum efficiency of impulse turbine (Pelton wheel) is found to be

$$\eta_{h\ max} = \frac{1 + k\cos\beta}{2}$$

157. (b) Given that

$$N = 600 \text{ rpm}$$
$$V_1 = 100 \text{ m/s}$$
$$\frac{u}{V_1} = 0.44$$

Hence,

$$u = 0.44 \times V_1 = 44 \text{ m/s}$$

But

$$u = \omega \frac{D}{2}$$
$$\omega = \frac{2\pi N}{60}$$
$$D = \frac{2u}{\omega} = 1.4 \text{ m}$$

158. (d) Gross head is level difference of head race level and tail race level. It is the maximum head that can be availed at turbine.

159. (d) Given that

$$V = 20 - 10 = 10 \text{ m/s}$$
$$\rho = 1000 \text{ kg/m}^3$$
$$a = 0.02 \text{ m}^2$$

The force developed is determined as

$$F = (\rho a V)\, V = 2000 \text{ N}$$

160. (c) For maximum efficiency of Pelton wheel,

$$V_1 = 2u$$

161. (c) Specific speed (N_s) of Pelton wheel depends upon the number of jets (n) as

$$N_{s-n} = \sqrt{n} N_s$$

where N_s is specific speed with single jet.

162. (b) Speed of the generator is

$$N = \frac{120 f}{p}$$

where f is frequency of power generation, and p is number of poles per generator. For given case, $f = 50$ Hz, $p = 12$. Hence,

$$N = \frac{120 \times 50}{12} = 500$$

163. (c) At the exit of reaction turbines, a pipe of gradually increasing cross-sectional area, known as draft tube, is used to permit negative suction head by having the capability of converting kinetic energy of water into pressure head, thus increasing the power capacity of turbine system.

164. (b) Given that

$$N_u = 50 \text{ rpm}$$
$$N = 400 \text{ rpm}$$
$$P = 500 \text{ kW}$$

Effective head of the turbine is determined using

$$N_u = \frac{N}{\sqrt{H}}$$
$$H = \left(\frac{N}{N_u}\right)^2 = 64.0 \text{ m}$$

165. (d) Blades can be adjusted only in Kaplan turbine.

166. (a) Kaplan turbine is a highly efficient mixed flow reaction turbine with adjustable blades and adjustable wicket gates. It is used with low head high discharge specifications.

167. (c) Kaplan turbine is a highly efficient mixed flow reaction turbine due to its adjustable blades coupled with adjustable wicket gates.

168. (d) If m demotes model turbine, and p denotes the prototype, then one gets

$$\frac{D_m}{D_p} \propto \sqrt{\frac{H_m}{H_p}} \times \frac{N_p}{N_m}$$

For the given case,

$$\frac{1}{4} = \sqrt{\frac{10}{30}} \times \frac{428}{N_m}$$
$$N_m = 988.42 \text{ rpm}$$

169. (d) Specific speed is defined as the speed of operation of a geometrically similar model of the turbine which is so proportioned that it produces unit power (1 kW) when operating at unit head (1 m).

170. (c) Specific speed for turbines is given by

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

171. (a) Specific speed of a turbine is the speed of a geometrically proportioned model that produces unit power (1 kW) when operating at unit head (1 m). It is determined as

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

Specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump that delivers unit discharge under unit head. It is determined as

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

172. (c) Unit quantities of a turbine are defined as their value for geometrically similar turbine working under unit head (1 m). These quantities are used in characteristic curves instead of direct quantities to realize the behavior of turbine in all scales. Unit speed N_u (as $N \propto \sqrt{H}$), therefore, at $H = 1$, $N = N_u$

$$N_u = \frac{N}{\sqrt{H}}$$

173. (a) It is established that

$$H \propto D^2N^2$$
$$Q \propto D^3N$$
$$P \propto D^5N^3$$

Hence,

$$P \propto H^{3/2}$$
$$N \propto H^{1/2}$$
$$Q \propto H^{1/2}$$

174. (a) Unit quantities of a turbine are defined as their value for geometrically similar turbine working under unit head (1 m). For the given case, $P = 640$ kW, $H = 16$ m. Hence, unit power for a given turbine is determined as

$$P_u = \frac{P}{H^{3/2}}$$
$$= 10 \text{ kW}$$

175. (c) For hydraulic turbines

$$H \propto D^2N^2$$
$$Q \propto D^3N$$
$$P \propto D^5N^3$$

176. (a) In terms of size (diameter D), density ρ, and velocity V, the thrust can be related as

$$F \propto \rho D^2V^2$$

Therefore, thrust on prototype is

$$F = 50 \times 1 \times \left(\frac{10}{1}\right)^2 \left(\frac{10}{5}\right)$$
$$= 20000 \text{ N}$$

177. (a) For centrifugal pump,

$$Q \propto D^3 \times N$$
$$H \propto D^2N^2$$
$$P \propto D^5N^3$$

178. (b) Priming is done in centrifugal pump operation in which suction pipe, casing, and portion up to delivery pipe are completely filled with the same liquid to be lifted such that no air pocket is left and then the delivery valve is closed.

179. (b) Cavitation occurs in centrifugal pumps if it operates below the minimum net positive suction head.

180. (d) For centrifugal pumps

$$P \propto N^3$$

Using the above expression, one finds

$$P_2 = \frac{2900}{1450}^3 \times 3 = 24 \text{ kW}$$

181. (c) Manometric efficiency of a centrifugal pump is defined as the ratio of manometric head (H_m) to the head imparted by the impeller to water (W),

$$\eta = \frac{H_m}{W} = \frac{gH_m}{V_{w1}u_1}$$

182. (c) If Q is discharge, D_2 is runner diameter, b_2 is runner width at outlet, one gets

$$H = \frac{u_2}{g}\left[u_2 - \frac{Q}{\pi D_2 b_2}\cos\beta\right]$$

Keeping other parameters constant, the effect of curvature β on H can be seen as

$$H = A_1 - A_2\cos\beta$$

Maximum efficiency can be obtained when blades are straight ($\beta = 180°$).

183. (c) Given that

$$\frac{D_1}{D_2} = \frac{10}{40}$$
$$\frac{Q_1}{Q_2} = \frac{40}{800}$$

For centrifugal pumps, $Q \propto D^3 N$, therefore

$$\frac{N_2}{N_1} = \left(\frac{D_1}{D_2}\right)^3 \times \frac{Q_2}{Q_1}$$
$$\frac{N_2}{1000} = 0.3125$$
$$N_2 = 312.5 \text{ rpm}$$

184. (b) Specific speed of centrifugal pump is given by

$$N_s = \frac{N\sqrt{Q}}{H^{5/4}} \propto \sqrt{Q}$$

185. (d) For centrifugal pumps

$$P \propto N^3$$
$$H \propto N^2$$

Using above expressions, one finds

$$P = 2^3 \times 1 = 8 \text{ kW}$$
$$H = 2^2 \times 10 = 40 \text{ m}$$

186. (a) Minimum NPSH is required in centrifugal pumps to prevent cavitation.

187. (b) Cavitation in a centrifugal pump is likely to occur at the impeller inlet.

Numerical Answer Questions

1. For constant diameter pipeline, the velocities at inlet and exit are related as

$$\frac{AV_2}{v_2} = \frac{AV_1}{v_1}$$
$$V_2 = \frac{0.084}{0.073}V_1$$
$$V_2 = 1.15V_1$$

The work transfer in the pipeline is zero. Using the steady flow energy equation

$$\frac{V_2^2 - V_1^2}{2} + (h_2 - h_1) + Q = 0$$

$$V_1 = 124.51 \text{ m/s}$$

Given that

$$d = 0.25 \text{ m}$$
$$v_1 = 0.073 \text{ m}^3/\text{kg}$$

So, the mass flow rate is

$$\dot{m} = \frac{\pi d^2/4 \times V_1}{v_1} = 83.72 \text{ kg/s}$$

2. The initial state of the steam is as follows

$$p_1 = 1.4 \text{ MPa}$$
$$T_1 = 300°\text{C}$$
$$s_1 = 6.9534 \text{ kJ/kgK}$$
$$h_1 = 3040.0 \text{ kJ/kg}$$

The final state of the steam at 50°C is

$$s_f = 0.7038 \text{ kJ/kgK}$$
$$s_g = 8.0763 \text{ kJ/kgK}$$

The steam is processed isentropically, therefore, moisture is given by

$$s_f + x(s_g - s_f) = s_1$$
$$x = 0.8476$$

The enthalpy of steam in its final state is

$$h_2 = h_f + x h_{fg}$$
$$= 2228.90 \text{ kJ/kg}$$

Given, $\dot{m} = 0.5$ kg/s. Therefore, net power output of the process is

$$W = \dot{m}(h_1 - h_2)$$
$$= 405.55 \text{ kW}$$

3. Given that

$$N = 1200 \text{ rpm}$$
$$\eta_{ith} = 0.38$$
$$\eta_m = 0.8$$

Therefore, brake thermal efficiency and mass flow rates are

$$\dot{m}_a = \frac{5.4}{60} = 0.09 \text{ kg/s}$$
$$\dot{m}_f = \frac{\dot{m}_a}{15} = 6 \times 10^{-3} \text{ kg/s}$$

Indicated specific fuel consumption is 0.2256 kg per hour for 1 kW.

$$\text{BSFC} = \frac{\text{ISFC}}{\eta_m}$$
$$= \frac{0.2256}{0.8}$$
$$= 0.3195 \text{ kg/h-kW}$$

Brake power of the engine is given by

$$P_b = \frac{\dot{m}_f \times 3600}{\text{BSFC}}$$
$$= 67.6 \text{ kW}$$

Brake thermal efficiency is

$$\eta_{bth} = \eta_{ith} \times \eta_m$$
$$= 0.304$$

The calorific value (CV) is given by

$$\text{BSFC} = \frac{3600}{\eta_{bth} \times \text{CV}}$$
$$\text{CV} = \frac{3600}{\eta_{bth} \times \text{BSFC}}$$
$$= 37064.5 \text{ J/kg}$$

4. Given that

$$p_1 = 1 \text{ bar}$$
$$p_2 = 15 \text{ bar}$$
$$p_3 = 40 \text{ bar}$$

Compression ratio is

$$r = \frac{v_1}{v_2}$$
$$= \left(\frac{p_2}{p_1}\right)^{1/\gamma}$$
$$= 6.92$$

Air standard efficiency of the cycle is

$$\eta = 1 - \frac{1}{r^{\gamma-1}}$$
$$= 53.87\%$$

The initial temperature is

$$T_1 = 17 + 273$$
$$= 290 \text{ K}$$

Temperature after compression is

$$T_2 = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} T_1$$
$$= 628.67 \text{ K}$$

The temperature after heat addition is given by

$$T_3 = \frac{p_3}{p_2} T_2$$
$$= 1676.45 \text{ K}$$

The heat addition in the cycle is

$$Q_s = c_v (T_3 - T_2)$$
$$= 752.31 \text{ kJ/kg}$$

The work done by the cycle is

$$W = \eta \times Q_s$$
$$= 405.26 \text{ kJ/kg}$$

Initial volume of the air per kg is

$$v_1 = \frac{RT_1}{p_1}$$
$$= 0.832 \text{ m}^3/\text{kg}$$

The volume after compression is

$$v_2 = \frac{v_1}{r}$$
$$= 0.055 \text{ m}^3/\text{kg}$$

The mean effective pressure, by its definition is

$$\begin{aligned} p_m &= \frac{W}{v_1 - v_2} \\ &= 5.21 \text{ bar} \end{aligned}$$

5. Mean effective pressure is

$$\begin{aligned} p_m &= \frac{6.0 - 0.3}{6.5} \times 3.0 \\ &= 2.63 \text{ bar} \end{aligned}$$

Given that

$$\begin{aligned} d &= 0.12 \text{ m} \\ l &= 0.16 \text{ cm} \\ N &= 1650 \text{ rpm} \\ n &= 4 \end{aligned}$$

Indicated power of a two-stroke engine is

$$\begin{aligned} P_i &= p_m \times \frac{\pi}{4} d^2 l x \frac{N}{60} \times n \\ &= 52.35 \text{ kW} \end{aligned}$$

6. Given that

$$\begin{aligned} N &= 1200 \text{ rpm} \\ P_b &= 20 \text{ kW} \\ \text{BSFC} &= 0.360 \text{ kg/h kW} \end{aligned}$$

Average torque in three cylinders is $T = 110$ Nm, therefore, average brake power (P_b) for three cylinders is

$$\begin{aligned} P_b &= \frac{2\pi N}{60} \times T \\ &= 13.82 \text{ kW} \end{aligned}$$

Hence, average indicated power of single cylinder is

$$\begin{aligned} P_i &= 20 - 13.82 \\ &= 6.18 \text{ kW} \end{aligned}$$

Total indicated power is $4 \times 6.18 = 24.72$ kW, therefore, indicated specific fuel consumption:

$$\begin{aligned} \text{ISFC} &= \text{BSFC} \times \eta_m \\ &= \text{BSFC} \times \frac{P_b}{P_i} \\ &= 0.29126 \text{ kg/h-kW} \end{aligned}$$

Fuel consumption rate is calculated as

$$\begin{aligned} \dot{m}_f &= \frac{\text{ISFC}}{P_i} \\ &= 2 \times 10^{-3} \text{ kg/s} \end{aligned}$$

Given that CV $= 43 \times 10^6$ J/kg. The indicated thermal efficiency is calculated as

$$\begin{aligned} \eta_{ith} &= \frac{P_i}{\dot{m}_f \times \text{CV}} \\ &= 28.74\% \end{aligned}$$

7. Given that the work input to the compressor starts from $h_1 = 245.36$ kJ/kg (at 258 K) and enthalpy after compressor is $h_2 = 287.07$ kJ/kg. Also given $w_c = 10$ kW, therefore, mass flow rate is given by

$$\begin{aligned} \dot{m}(h_2 - h_1) &= w_c \\ &= \frac{w_c}{h_2 - h_1} \\ &= 0.2397 \text{ kg/s} \end{aligned}$$

One ton is equal to 3.5 kW. The enthalpy after expansion is (at 313 K) is $h_4 = 95.4$ kJ/kg. Therefore, the tonnage is calculated as

$$\begin{aligned} Q_2 &= \frac{\dot{m}(h_1 - h_4)}{3.5} \\ &= 10.272 \end{aligned}$$

The COP of the cycle is given by

$$\begin{aligned} \text{COP} &= \frac{\dot{m}(h_1 - h_4)}{w_c} \\ &= 3.595 \end{aligned}$$

8. Given that

$$\begin{aligned} \dot{m} &= 0.5/60 \text{ kg/s} \\ h_1 &= 389.20 \text{ kJ/kg} \\ h_4 &= 256.54 \text{ kJ/kg} \end{aligned}$$

The refrigeration capacity is calculated as

$$\begin{aligned} Q_2 &= \frac{\dot{m}(h_1 - h_4)}{3.5} \\ &= 0.3158 \text{ tons} \end{aligned}$$

Using the first law of thermodynamics, the compressor work is calculated as

$$\begin{aligned} w_c &= \dot{m}(h_1 - h_4) - \frac{80}{60} \\ &= -0.2278 \text{ kW} \end{aligned}$$

The COP of the cycle is given by

$$\begin{aligned} \text{COP} &= \frac{\dot{m}(h_1 - h_4)}{w_c} \\ &= 4.85 \end{aligned}$$

9. The moist air is maintained at $p = 1$ bar. From the given data, the saturation pressure and partial vapor pressure of the moist air are

$$p_s = 0.02339 \text{ bar}$$
$$p_v = 0.00872 \text{ bar}$$

Relative humidity is calculated as

$$\phi = \frac{p_v}{p_s}$$
$$= = 37.28\%$$

Degree of saturation is calculated as

$$\mu = \left(\frac{p_v}{p - p_v}\right) / \left(\frac{p_s}{p - p_s}\right)$$
$$= \frac{0.00879}{0.02395}$$
$$= 36.7\%$$

10. Given that for initial state,

$$\phi = 40\% = 0.4$$
$$p_s = 0.04246 \text{ bar}$$

Partial pressure of the vapor is

$$\phi = \frac{p_v}{p_s}$$
$$p_v = 0.01698 \text{ bar}$$

Specific humidity of the moist air is

$$w = 0.622 \frac{p_v}{p - p_v}$$
$$= 0.01060$$

For air, $c_p = 1.004$ kJ/kg. Using

$$h = c_p T_{db} + w\,(2501.4 + 1.88 T_{db})$$

Initial (T_{db} =30°C) and final (T_{db} =20°C) enthalpies (in kg/kg of dry air) are

$$h_1 = 57.233$$
$$h_2 = 46.99$$

For air $R = 287.2$ J/kg K. Given $T_1 = 30 + 273 = 303$ K. Initial specific volume of the dry air is

$$v_{a1} = \frac{R_a T_1}{p - p_v}$$
$$= 0.8735 \text{ m}^3/\text{kg}$$

Cooling effect for flow rate of 2 m^3/s is

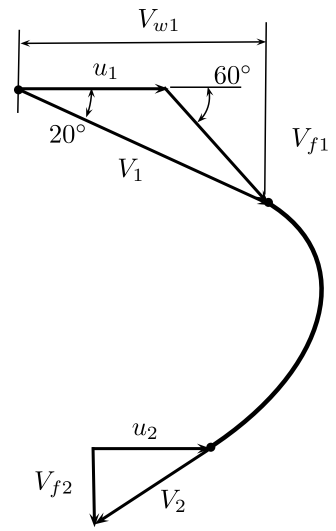
$$Q_2 = \frac{2}{0.8735} \times (57.233 - 46.99)$$
$$= 23.45 \text{ kW}$$

11. Given that

$$\alpha_1 = 20°$$
$$\beta_1 = 60°$$

Tangential velocity at inlet is

$$u_1 = \frac{\pi D_1 N}{60}$$
$$= 28.27 \text{ m/s}$$



Nozzle velocity is given by

$$\frac{V_1}{\sin 120°} = \frac{u_1}{\sin 40°}$$
$$V_1 = 38.09 \text{ m/s}$$

Velocity of flow at inlet is

$$V_{f1} = V_1 \sin 20°$$
$$= 13.03 \text{ m/s}$$

Velocity of whirl at inlet is

$$V_{w1} = V_1 \cos 20°$$
$$= 35.79 \text{ m/s}$$

Since whirl velocity (V_{w2}) at outlet is zero, the power developed in the turbine runner is

$$P = \rho Q\,(u_1 V_{w1} - u_2 V_{w2})$$
$$= 5263 \text{ kW}$$

The hydraulic efficiency of turbine runner is

$$\eta_h = \frac{u_1 V_{w1}}{gH}$$
$$= 89.7\%$$

PART III

MANUFACTURING & INDUSTRIAL ENGINEERING

CHAPTER 10

ENGINEERING MATERIALS

Materials are essential for the development and growth of human civilization. The designation of successive historical epochs, as the Stone, Copper, Bronze, and Iron Ages, reflect the importance of materials. Engineering materials are the foundation of technology. Material science is concerned to the behavior of materials and variation of their properties.

10.1 ATOMIC STRUCTURE

The smallest unit of a material, *atom*, consists of three particles: electron, proton, and neutron. Electrons are negative charged particles of magnitude 1.6×10^{-19} C (Coulomb), whereas protons are positive charged particles of equal magnitude. Neutrons are neutral particles, having no charge. Proton and neutron are the nucleons bounded by nuclear forces into *atomic nuclei*. A neutral atom has the same number of electrons and protons. Neutrons and protons have very similar masses, roughly equal to 1 a.m.u. each. Mass of an electron is negligible with respect to that of proton or neutron.

The following are the basic terminology related to characteristics of an atom:

1. Atomic Number   A chemical element is characterized by the number of protons in its nucleus. *Atomic number* (Z) is the number of protons found in the nucleus of an atom, identical to the number electrons for an electrically neutral atom. The atomic number ranges in integral units from 1 for hydrogen to 92 for uranium, the highest of the naturally occurring elements.

2. Atomic Weight   *Atomic weight*[1] of an element is the average relative weight[2] of its single atom as compared to the weight of one atom of carbon-12 which taken to be 12. All atoms having different atomic weights but belonging to the same element are called *isotopes*.

3. Atomic Mass Unit   *Atomic!mass unit* (a.m.u.) is a unit of mass, equal to 1.6605×10^{-27} kg. One *mole* of an atom is the amount of mass in grams equal

[1]Specific heat of metals in liquid state depends upon the atomic weight as

$$C = \frac{29.93}{\text{atomic weight}}\,\text{kJ/kg-K}$$

For example, specific heat of iron (Fe, atomic weight 56) is equal to 0.5344 kJ/kg-K.

[2]The original standard of atomic weight, established in the 19th century, was hydrogen, with a value of 1. From about 1900 until 1961, oxygen was used as the reference standard, with an assigned value of 16. As a result of discoveries of isotopes of oxygen, a new scale based on carbon-12 was established in 1961.

to the atomic mass in a.m.u. of the atoms. Thus, one mole of carbon has a mass of 12 g.

4. *Avogadro's Number* Avogadro's number is formally defined as the number of carbon-12 atoms in 12 grams of unbound carbon-12 in its rest-energy electronic state. Thus, it is the reciprocal of atomic mass unit:

$$N_A = \frac{1}{1.6605 \times 10^{-27}}$$
$$= 6.0223 \times 10^{26}/\text{kmole}$$
$$= 6.0223 \times 10^{23}/\text{mole}$$

The number of atoms in one mole is called the *Avogadro's number* (N_A).

10.2 CRYSTAL STRUCTURE

Solid materials are broadly classified according to the regularity with which the atoms or ions are arranged with respect to one another:

1. *Amorphous Material* A solid with non-crystalline structure is called *amorphous*, such as silicate glasses, metallic glasses, amorphous carbon, amorphous silicon and many polymers. Many amorphous materials can be crystallized in a controlled fashion. This is the basis for formation of glass–ceramics and strengthening of polyethylene terephthalate (PET) plastics used for manufacturing of containers of beverages and foods.

2. *Crystalline Material* A *crystalline material* possesses a long-range order of atomic arrangement through repeated periodicity at regular intervals in three dimensions of space. All the metals, many ceramic materials and certain polymers form crystalline structure under normal solidification condition. A crystalline structure always has more free energy than a non-crystalline structure.

Most of the properties of crystalline solids depend on their crystal structure, therefore, it is essential to study their crystalline structures. For this reason, atoms are considered as hard spheres having well-defined diameters.

10.2.1 Lattice and Unit Cells

Crystal lattice is a three-dimensional periodic array of points coinciding with atomic positions. The smallest repeatable entity that can be used to completely represent a crystal structure is called *unit cell*. It is the building block which defines the crystal structure by virtue of its geometry and the atom positions inside it [Fig. 10.1].

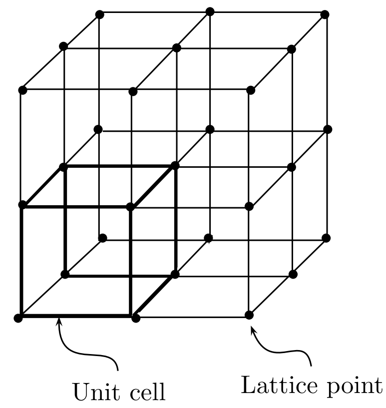


Figure 10.1 Lattice and unit cell.

The unit cell of a crystal lattice has three unique *crystallographic axes* and, in general, three edge lengths.

10.2.2 Crystal Systems

By pure symmetry considerations, there are only fourteen independent ways of arranging points in three-dimensional space, conforming to the definition of space lattice. These 14 space lattices are called *Bravais lattices* after Auguste Bravais. These are grouped into seven *crystal systems* according to the configuration of unit cells: cubic (3), hexagonal (1), tetragonal (2), rhombohedral (1), orthrhombic (4), monoclinic (2), and triclinic (1).

A *crystallographic direction* is defined as a line between two points. The crystallographic directions in cubic crystal are expressed in terms of clear fractions of the resolutions in square brackets. For example, if a, b, and c are the integers corresponding to the reduced projections along the x, y and z axes, respectively, the direction is represented as $[abc]$. Negative indices are represented by a bar over the appropriate index, for example $[1\bar{1}1]$. Common directions [100], [110] and [111] are shown in Fig. 10.2.

A family of directions is represented by <...>. For example, in cubic crystals, all the directions represented by the following indices are equivalent: [100], $[\bar{1}00]$, [010], $[0\bar{1}0]$, [001] and $[00\bar{1}]$.

Crystallographic planes in cubic crystals are expressed in terms of the resolutions of the normal to the plane in a simple bracket (...), as depcited in Fig. 10.2. These fractions are called *Miller indices* of the plane. A group of planes is denoted by {...}.

If a plane is parallel to an axis, its intercept is at infinity, and its Miller index along this axis will be zero. Negative intercepts on an axis are denoted by a bar above the numbers.

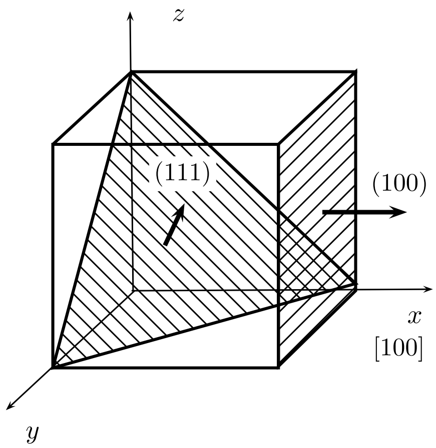


Figure 10.2 Miller indices in a cubic crystal.

The crystal directions of a family are not necessarily parallel to each other. Similarly, not all planes of a family are parallel to each other.

10.2.3 Anisotropy and Isotropy

The properties of materials depend on their crystal structure, and many of these properties are directional in nature. Different directions in a given crystal can have different packing. This aspect causes properties to be different in different directions. This directionality of properties is termed as *anisotropy*.

For example, elastic modulus of body centered cubic (BCC) iron is more along the body diagonal than it is to the cube edge. Similarly, atoms along the edge of face centered cubic (FCC) crystals are more separated than along its face diagonal.

The substances in which measured properties are independent of direction in which they are measured are called *isotropic*. Polycrystalline materials show isotropy because the crystallographic orientations of individual grains are random.

10.2.4 Characteristics of Cells

The following are important characteristics of unit cells which affect the properties of crystalline materials:

1. *Atoms in a Cell* The number of atoms in a unit cell is determined as

$$N = N_{interior} + \frac{N_{face}}{2} + \frac{N_{corner}}{8}$$

where N with subscript indicates number of atoms in respective locations within the unit cell.

2. *Coordination Number* *Coordination number* (CN) is the number of equidistant nearest neighboring atoms of an atom in a unit cell.

3. *Atomic Packing Factor* The atomic packing factor (APF), also called *packing efficiency*, of a crystal is defined as the sum of the sphere volumes of all atoms within a unit cell divided by the unit cell volume.

4. *Density* If n number of atoms having atomic weight Z are associated in each unit cell having cell volume V_c, then the theoretical density (ρ) of the solid is determined as

$$\rho = \frac{nZ}{V_c N_A}$$

where N_A is the Avogadro's number ($= 6.023 \times 10^{23}$ atoms/mol).

10.2.5 Crystal Structure of Metals

Most metals crystallize upon solidification in dense-packed structures because energy is released as the atoms come closer together and bond more tightly with each other. Densely packed structures are lower and more stable energy arrangements. The order of magnitude of the distance between the atoms in these crystal structures is 0.1 nm. About 90% of the metals crystallize in three relatively simple crystal structures: body centered cubic, face centered cubic, and hexagonal close packed, described as follows.

10.2.5.1 Body Centered Cubic Cell A *body centered cubic* (BCC) crystal contains atoms at all eight corners and a single atom is at the center of the cube [Fig. 10.3]. FCC structure is found in Na, K, V, Mo, Ta, W, Cr, α-iron (900–1400°C), etc.

In this structure, the central and corner atoms touch one another along cube diagonals, therefore, unit cell length a and atomic radius are related as

$$a = \frac{4R}{\sqrt{3}}$$

Number of atoms in BCC are 2. The coordination number for the BCC crystal is 8; each central atom as 8 corner atoms as its nearest neighbors. Since coordination number is less for BCC than FCC, so also is the atomic packing factor for BCC (0.68) is less than that of FCC (0.74).

10.2.5.2 Face Centered Cubic Cell A *face centered cubic* (FCC) crystal consists of an atom at each cube corner and an atom in the center of each cube face [Fig. 10.3]. Metals such as Ca, Ni, Cu, Ag, Pt, Au, Pb, Al, γ-iron (except 900–1400°C), show BCC structure.

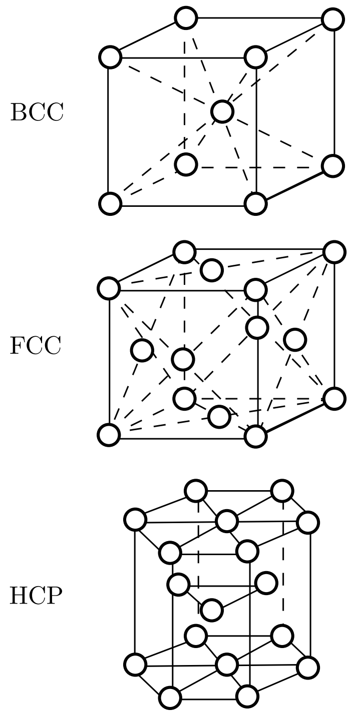


Figure 10.3 Crystal cells.

In this structure, the atom or ion spheres touch one another across a face diagonal, therefore, the cube edge length a and the atomic radius R are related as

$$a = \frac{4R}{\sqrt{2}}$$

For FCC, the number of atoms in unit cell is 4, the coordination number is 12 and packing factor is 0.74 (a value slightly higher than the 0.68 found for BCC metals). This structure is more ductile than BCC.

Interesting is to note that 0.74 is the highest value of APF which is possible for filling space by stacking equal sized hard spheres. For this reason, the FCC structure is sometimes referred to as *cubic close packed* (CCP).

10.2.5.3 Hexagonal Closed Packed Cell A *hexagonal closed packed* (HCP) crystal consists of close packed planes of six atoms at both the bottom and top planes (known as *basal planes*) of the unit cell, separated by 3 atoms at the cell center. This type of crystal is found in Be, Mg, Zn, Cd, Te, Zr, Ti, etc.

There are 6 number of atoms in an HCP crystal. The atoms in the mid-plane have 6 nearest neighbors atoms in both the adjacent planes, thus, coordination number of an HCP crystal is 12.

Important characteristics of various unit cells are summarized in Table 10.1.

Table 10.1 Characteristics of unit cells

Unit cell	N	CN	a/R	**Cell Volume**	**APF**
Simple	1	6	$4/\sqrt{4}$	$64R^3/\left(4\sqrt{4}\right)$	0.52
BCC	2	8	$4/\sqrt{3}$	$64R^3/\left(3\sqrt{3}\right)$	0.68
FCC	4	12	$4/\sqrt{2}$	$64R^3/\left(2\sqrt{2}\right)$	0.74
HCP	6	12	–	–	0.74

10.3 CRYSTAL IMPERFECTIONS

Behavior of materials and their properties are governed by the nature of the atomic bond, crystal structure, lattice defects, microstructure and macrostructure. Some properties of materials, for example, density, elastic moduli, specific heat, thermal expansion coefficient, etc., can be understood on the basis of the arrangement of atoms.

Real crystalline materials are rarely perfect in atomic arrangement. Under mechanical or thermal loads, all solids tend to reshape themselves, resulting in imperfections in their crystals. Lattice defects are disrupted regions in a volume of the lattice, and are characterized by geometry. Thus, a study of the nature of lattice defects in various crystalline solids is essential towards understanding the material properties.

On the basis of their geometry, lattice imperfections are grouped into four categories: (1) point defects, (2) line defects, (3) planar defects, (4) bulk defects, described as follows.

10.3.1 Point Defects

Point defects are imperfect point-like regions in the crystal within about 1 to 2 times of atomic diameter. The characteristics of point defects are different in elemental crystals and compound crystals, therefore, are studied separately.

10.3.1.1 Elemental Crystals The following are the two types of point defects in elemental crystals:

1. *Vacancies* A missing atom from a normal lattice site in a crystal is called a *vacancy* [Fig. 10.4]. Vacancies are usually created during solidification and can also be generated by heating, or irradiating a material.

   The fraction of vacant lattice sites in equilibrium at absolute temperature (T) is given by the *Boltzmann distribution*:

$$\frac{n}{N} = e^{-E/(kT)}$$

where n is the number of vacant sites in N lattice positions, k is the *Boltzmann's constant* ($= 1.38 \times 10^{-23}$ J/atom-K or 8.62×10^{-5} eV/atom-K), and E is the energy required to move an atom from the interior of a crystal to its surface. This shows an exponential increase in the number of vacancies with temperature.

Vacancies can play an important role in diffusion, and influence material properties, such as electrical conductivity of materials.

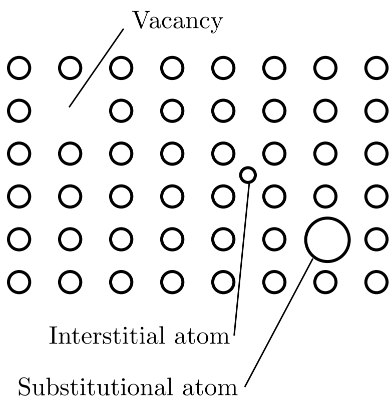


Figure 10.4 | Point defects in elemental crystals.

2. *Interstitialcies* A matrix atom can occupy an interstitial site. Such a defect is known as *self-interstitial* or an *interstitialcy* [Fig. 10.4]. Interstitialcy occurs mainly in crystals of low atomic packing factor.

   An impurity atom, whose size is similar to the matrix atom, can occupy a *substitutional* site. On the other hand, impurity atoms of much smaller size can be accommodated at interstitial sites between the matrix atoms. For example, if nickel is added to copper, the nickel atoms will occupy some of the copper sites. Similarly, carbon atoms occupy interstitial positions in steel. Zinc is a substitutional atom in the copper lattice of brass.

Presence of vacancies and impurity atoms affects the coordination number of atoms around the defect. This results into unbalanced forces and lattice distortion around the defect.

10.3.1.2 Compound Crystals Nature of point defects is more complex in compounds (having ionic, covalent, or intermetallic bonds) than in elemental crystals. Charge neutrality in ionic solids is maintained either by displacing a cation to an interstitial site (called *Frenkel defect*) or by simultaneously removing a cation and an anion (called *Schottky defect*), explained as follows:

1. *Frenkel Defects* When an ion is displaced from a regular position to an interstitial position, the pair of vacancy–interstitial so created is called *Frenkel defect*[3].

   Cations (positively charged ions), usually smaller in size, are displaced easily than *anions* (negatively charged ions). Close-packed structures have fewer interstitial and displaced ions than vacancies because additional energy is required to force the atoms into the interstitial positions.

2. *Schottky Defects* A pair of one cation and one anion can be missing from an ionic crystal, without violating the condition of *charge neutrality*, provided the valency of these opposite charged ions is equal. The pair of vacant sites, thus formed, is called *Schottky defect*[4].

   Schottky defects occur mainly in alkali halides (family of inorganic compounds containing alkali metal and a halogen). These ion-pair vacancies, like single vacancies, facilitate atomic diffusion.

Generally, Schottky defects are more common than Frenkel defects because the interstitial sites in many structures are not large enough to accommodate the displaced ions [Fig. 10.5].

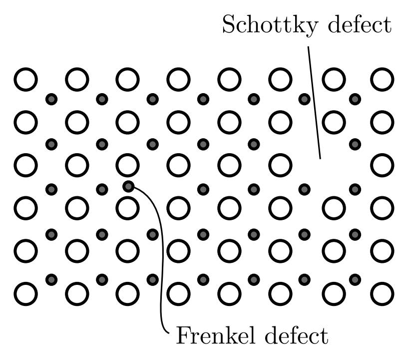


Figure 10.5 | Frenkel and Schottky defects.

10.3.2 Line Defects

If the atomic arrangement differs from that of an ideal crystal along a set of lattice points on a line, the defect is known as a *line defect* or *dislocation*. Dislocation line is the line of demarcation between slipped and unslipped regions, which can be straight, curved, or a closed loop.

[3]The phenomenon is named after the Soviet physicist Yakov Frenkel, who discovered this defect in 1926.

[4]Walter Hermann Schottky (1886-1976) was a German physicist who made many significant contributions in the areas of semiconductor devices, technical physics and technology.

Dislocations occur in high densities when an extra incomplete plane is inserted into the lattice and dislocation line is seen at the end of the plane. Therefore, dislocation in a crystal can be identified by considering an atom-to-atom path around a dislocation line in an orthogonal plane. This path is known as *Burgers circuit*[5].

A Burgers circuit closes for perfect crystals but fails to close and finish in an imperfect crystal. The vector that is required to close the circuit (from starting point to final point, as a sign convention) around a dislocation line is known as *Burgers vector*, $\overrightarrow{b}$. In this context, $\hat{t}$ is used to represent an unit vector representing the direction of the dislocation line. Burgers vector describes both the magnitude and direction of the dislocation.

A dislocation in a crystal is a combination of two limiting cases, known as edge dislocation and screw dislocation, described in the following subsections.

10.3.2.1 Edge Dislocation An *edge dislocation* is formed by adding an extra partial plane of atoms to the crystal. It is also called *Taylor-Orowan dislocation*[6]. The dislocation line appears as the edge of the extra half plane of the atoms in the crystal, and hence the name of edge dislocation.

Burgers vector in this case is perpendicular to the dislocation line. A pure edge dislocation can slip along the Burgers vector in the slip plane (made of $\overrightarrow{b}$ and $\hat{t}$ vectors), while conserving the number of atoms in the incomplete plane [Fig. 10.6].

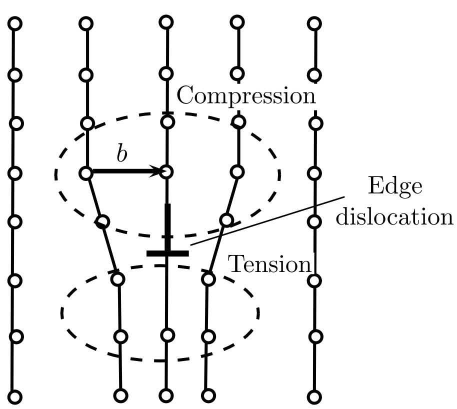


Figure 10.6 Edge dislocation.

[5]This concept was introduced by a Dutch physicist Johannes Martinus Burgers (1895-1981).

[6]The concept of a lattice dislocation as the crystallographic unit responsible for the deformation of crystalline solids is commonly attributed to the papers published by Orowan, Taylor and Polanyi in 1934.

The atoms above the dislocation line are squeezed together and are in a state of compression, whereas the atoms of opposite side are pulled apart and experience tensile stresses. Therefore, the dislocation line is a region of higher energy than the rest of the crystal.

The edge dislocation is considered positive when compressive stresses are present above the dislocation line, and is represented by $\perp$. If the stress state is opposite (compressive stresses exist below the dislocation line), it is considered as negative edge dislocation, and is represented by $\top$.

10.3.2.2 Screw Dislocation A *screw dislocation* is created when the successive planes of atoms normal to the dislocation line are converted by the presence of the dislocation into a spiral ramp or screw surface [Fig. 10.7]. It is also called *Burger's dislocation.* A screw dislocation has Burger vector parallel to the linear defect but there is a distortion of the plane.

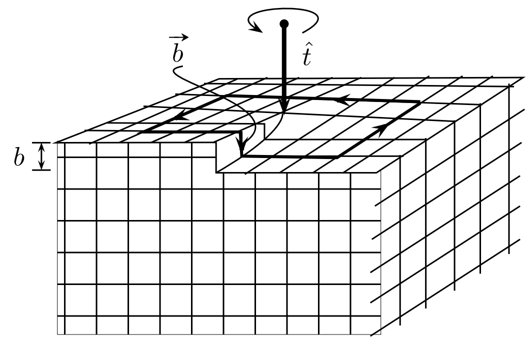


Figure 10.7 Screw dislocation.

Unlike edge dislocation, a screw dislocation does not have a preferred slip plane and can cross-slip onto another plane under favorable stress conditions.

The atoms are displaced in two orthogonal planes and the distortion follows a helical path. Screw dislocation is considered positive if Burgers vector and $\hat{t}$ are in the same direction (as in Fig. 10.7), and negative if these vectors are in the opposite direction.

Dislocations commonly originate during plastic deformation, solidification, and as a consequence of thermal stresses due to rapid cooling. Unlike point defects, these are not thermodynamically stable and can be removed by heating to high temperatures where they cancel each other or move out through the crystal to its surface.

10.3.3 Planar Defects

Planar defects or *interfacial defects* refer to the regions of distortions that lie about a surface having thickness of a few atomic diameters. Since inter-atomic bonds are

distorted or broken through modification of the nearest neighbor atoms across an interface, work is required in creating the interface. Hence, all planar defects have a certain energy per unit area associated with them. Some common planar defects are described as follows:

1. External Surface The environment of an atom at a surface differs from that of an atom in the bulk, especially the number of neighbors (coordination) at the surface is less. Thus, the unsaturated bonds of surface atoms give rise to a surface energy and cause relaxation (decrease in lattice spacing) or reconstruction (changes in the crystal structure).

2. Grain Boundaries Solidification of metals begins with the formation of tiny crystals, called *grains*. A *polycrystalline* material consists of several grains joined together. The interfaces between the grains of different orientations in crystalline solids are called *grain boundaries* which can be several atoms wide and generally have disordered orientation of grains on either side of the boundary. In spite this, polycrystalline solids are still very strong as cohesive forces present within and across the boundary.

   Since the number of bonds per atom at a grain boundary is less than that for an interior atom, interfacial energy is associated with boundaries. Therefore, atoms at the grain boundaries dissolve more readily on etching with a chemical reagent.

3. Tilt Boundary High or low angle tilt or twist boundaries can involve changes in the crystallographic orientations of adjacent grains. Tilt boundary (low angle) is associated with relatively little energy and is composed of edge dislocations lying one above the other.

4. Twin Boundary Twin boundaries occur in pairs such that the change in orientation introduced by one boundary is restored by the other; there is specific mirror lattice symmetry. Twinning occurs on a definite crystallographic plane and in a specific direction.

These imperfections are not thermodynamically stable, rather they are metastable imperfections.

10.3.4 Bulk Defects

Bulk defects include pores, cracks, foreign inclusions and phases, normally introduced during processing and fabrication. These defects are capable of acting as stress raisers, and thus, deleterious to the parent metal's mechanical behavior. However, foreign particles can be added purposefully to strengthen the parent material where foreign particles act as obstacles to movement of dislocations. This is called *dispersion hardening*.

10.4 EFFECTS OF IMPERFECTIONS

Mechanical properties (such as the strength, ductility, toughness, and ductile–brittle transition) depend upon the crystal structure and imperfections. Point defects influence electrical conductivity, mechanical strength and diffusivity. Lattice as well as bulk defects are relevant in material processing involving solidification, deformation, and powder metallurgy.

For example, rimmed steel sheets are particularly susceptible to strain (within 30 days). This is due to the presence of dissolved carbon and nitrogen in steel. This is eliminated by vacuum de-gasing and in aluminium killed steels[7]. Similarly, strength of copper is increased by addition of nickel owing to the formation of a solid solution.

The additional imperfection tend to hinder dislocation motion and consequently the strength increases. Also, hard ceramic particles of alumina (Al_2O_3) are dispersed in a soft ductile matrix of aluminium. These particles resist the movement of dislocations in the aluminium matrix, and hence increase the strength of aluminium metal. This is called *dispersion hardening*. sintered aluminium powder (SAP) is based on this advantage.

Hall-Petch equation[8] is based on the dislocation theory that relates yield point stress σ_{yp} to the grain size d as

$$\sigma_{yp} = \sigma_o + \frac{k}{\sqrt{d}} \tag{10.1}$$

where σ_o is the strength of one crystal and k is a constant. Thus, a solid with smaller grains is stronger due to the presence of more grain boundaries. Therefore, the Hall-Petch equation [Eq. (10.1)] is also known as an expression of *grain-boundary strengthening*.

Large grains have lower free energy than small grains. Lowest energy state occurs in a single crystal (without grain boundaries). *Whiskers* are single crystals, which in some cases are made directly from vapor.

Parallel planes of high atomic density and corresponding large inter-planar spacing exist in the crystal structure. Any movement in the crystal takes place either along these planes or parallel to them. Therefore, slip occurs in most closely packed directions since they require least amount of energy. In FCC (e.g. aluminium,

[7]During steel making process, the dissolved oxygen in the liquid metal can combine with carbon to form carbon monoxide bubbles. This can be eliminated through the addition of deoxidizing agents, such as aluminium, ferrosilicon and manganese. In the case of aluminium, the dissolved oxygen reacts with it to form aluminium oxide (alumina, Al_2O_3). Completely deoxidized steels are known as "killed steels".

[8]The pioneering work of E.O Hall and N.J. Petch in the late 1940s, "The Cleavage Strength of Crystals" N.J. Petch, J. Iron & Steel Inst., 174, 25-28.

gold, copper), there are 12 possible slip directions, so easily deformed. In HCP, twining occurs and in FCC, so slip occurs easily.

10.5 GRAIN FORMATION

In the process of solidification, if all the nucleation sites except one are suppressed, the liquid solidifies into a single grain. The orientation of the grain can be engineered by placing a seed crystal of known orientation. SiC and Si_3N_4 filamentary single crystals (known as *whiskers*) are used as reinforcing materials in composites because they exhibit superior strength and elastic modulus. Single crystal also exhibit excellent high temperature properties because of the absence of grain boundaries. Single crystal turbine blades used in present jet engines exhibit superior high temperature properties. Silicon single crystals are the backbone of the electronic industry because of their excellent electronic properties.

When a melt is cooled slowly under equilibrium conditions in the presence of abundant nucleation sites, the liquid forms a large number of small grains. Although solidified grains physically look alike, the crystallographic orientations of the grains are different and orientation changes randomly from one grain to another. Such a solid is known as a *polycrystalline* material.

The number of grains, and hence, material properties can easily be varied by controlling nucleation process. This can be achieved by adding *inoculants* or *seed crystals*. For example, a fine grain steel is produced by adding refractory elements (e.g. W, V, Cr, Mo). These elements combine with carbon and produce carbides that provide effective seeding sites for nucleation of grains.

Materials in high temperature service usually fail by crack formation and growth along grain boundaries. Thus, high temperature capabilities can be improved by eliminating grain boundaries and using single crystal materials. Single crystal blades of nickel base superalloys are currently used as gas turbine blades to increase the high temperature capability of the turbines.

10.6 GRAIN SIZE MEASUREMENT

A common method of measuring the grain size is by comparing the grains at a fixed magnification with standard grain size charts. Charts are coded with ASTM grain size number (G) which is related with the number of grains per mm^2 at 1× magnification (n_a) as

$$G = -2.9542 + 1.4427 \ln n_a$$

In terms of G, the number of grains (n) per square inch (645 mm^2) at a magnification of 100× is found as

$$n = 2^{G-1}$$

Higher the ASTM grain number, smaller is the grain diameter. The grain diameter (D) in mm and ASTM number (G) are related as

$$D = \frac{1}{100}\sqrt{\frac{645}{2^{G-1}}}$$

In some materials, the grain size could be as small as few nanometers size (1 nm = 10 Å). Such materials are known as *nanocrystalline materials*.

10.7 GRAIN STRUCTURE

Due to cold work, the distortion of the lattice structure hinders the passage of electrons and decreases the electrical conductivity. Increase in internal energy, particularly at the grain boundaries, makes the material more susceptible to inter-granular corrosion, thereby, reducing its corrosion resistance.

Recovery, recrystallization and grain growth are the three processes which influence the grain boundary of a microstructure. These are explained in the following subsections [Fig. 10.8].

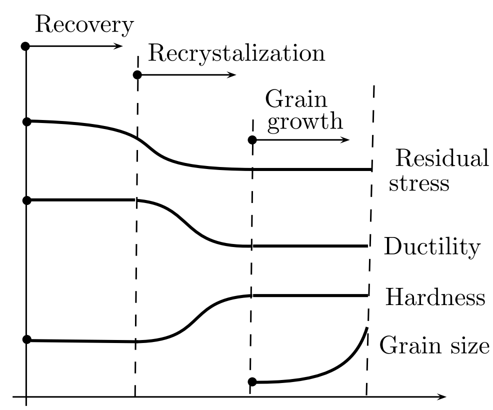


Figure 10.8 Grain boundary modification.

10.7.1 Recovery

Every cold worked microstructure contains deformed grains and tangled dislocations. *Recovery* is a low temperature heating of the metals to relieve internal residual stresses caused by cold working. The thermal energy permits dislocations to move and form grain boundary,

thus, dislocation density in the microstructure virtually remains the same. This results in relatively no change in mechanical properties of the microstructure, but electrical conductivity increases appreciably during the recovery stage because electrons get a smooth path to travel.

10.7.2 Recrystallization

Heating of cold-worked microstructure at higher temperature results in rapid recovery which relieves residual stresses. At high temperatures, new grains nucleate at the grain boundaries, eliminating most of the dislocations. The process of formation of new grains is known as *recrystallization*. Consequently, dislocation density reduces in this process. Certain minimum amount of cold work, known as *critical deformation*, is necessary for recrystallization. The process occurs at a definite temperature, known as *recrystallization temperature*, which depends upon various factors. Generally, it is one-third to half of the melting point of the metal/alloy.

Recrystallization rate, R_c increases exponentially with temperature:

$$R_c \propto e^T$$

Recrystallization temperature T_c is low with higher degree of deformation, finer grain size, low temperature of cold work, high rate of plastic deformation but it is high with presence of a second phase in the metal.

10.7.3 Grain Growth

At more higher temperatures, both recovery and recrystallization occur rapidly, thus producing fine recrystallized grains. At high enough temperatures, favored grains consume the smaller grains. This process is known as *grain growth*, which reduces grain boundary area.

High temperature in the tungsten filaments of incandescent light bulbs results in grain growth and subsequent reduction in strength. This is one of the factors for the failure of the filaments. Grain growth is applied in alumina ceramics for making optical materials used in lighting.

10.8 DEFORMATION

Deformation of solid materials has two elements: change in shape (*distortion*) and change in volume (*dilatation*). However, dilation is considered negligible in solid materials, thus, deformation is used as synonymous to distortion.

10.8.1 Types of Deformation

Deformation can be permanent or temporary. *Permanent deformation* is irreversible; the deformation stays even after removal of the applied forces, while the *temporary deformation* is recoverable as it disappears after the removal of the applied forces. Temporary deformation is also called *elastic deformation*, while the permanent deformation is called *plastic deformation*.

Time dependent recoverable deformation under load is called *anelastic deformation*, while the characteristic recovery of temporary deformation after the removal of load as a function of time is called *elastic aftereffect*.

Progressive permanent deformation under constant load is called *creep*. For visco-elastic materials, both recoverable and permanent deformations occur together which are time dependent.

When a material is subjected to applied forces, the material experiences elastic deformation followed by plastic deformation. Transition from elastic state to plastic state is characterized by the *yield strength* of the material. Microscopically, plastic deformation involves breaking of original atomic bonds, movement of atoms and the restoration of bonds. In other words, plastic deformation is based on irreversible displacements of atoms through substantial distances from their equilibrium positions.

The mechanism of plastic deformation is different for crystalline and amorphous materials. For crystalline materials, deformation is accomplished by means of a process called *slip* that involves motion of dislocations. In amorphous materials, plastic deformation takes place by viscous flow mechanism in which atoms or ions slide past one another under applied stress without any directionality.

It is very difficult to describe the behavior of metals under practical conditions. Therefore, theory of plasticity neglects the following aspects:

1. Anelastic Strain *Anelastic strain* is time-dependent recoverable strain.
2. Hysteresis Behavior *Hysteresis behavior* results from loading and un-loading of material.
3. Bauschinger Effect *Bauschinger effect* shows the dependence of yield stress on loading path and direction.

Plastic deformation is indeed uniform but only up to some extent of strain, whereafter plastic deformation is concentrated to the phenomenon called *necking*. The changeover from uniform plastic deformation to non-uniform plastic deformation is characterized by *ultimate tensile strength* (σ_u).

10.8.2 Stress–Strain Relationship

Hooke's law is the proportionality relation between elastic strain (ε) and stress (σ):

$$\sigma = E\varepsilon$$

where E is the Young's modulus of elasticity of the material.

The following power-law equation, known as *Holloman-Ludwig equation*, describes the material behavior in plastic range:

$$\sigma = K\varepsilon^n \tag{10.2}$$

where K is the *strength coefficient*, and n is the *strain hardening exponent* which can have values from $n = 0$ (perfectly plastic solid) to $n = 1$ (elastic solid). For most metals, n has values between 0.10 and 0.50.

A variation from the above power-law equation [Eq. (10.2)], is also known as *Ludwig equation*:

$$\sigma = \sigma_y + K\varepsilon^n$$

where σ_y is the yield strength of the material. The true strain ε used in the above equations should actually be the plastic strain value, given by the following equation,

$$\varepsilon_p = \varepsilon - \varepsilon_e \approx \varepsilon$$

where elastic strains (ε_e) can be safely neglected because plastic strains are very large when compared with elastic strain values.

10.8.3 Strain Hardening

Ductile materials show increase in strength and *hardness* when plastically deformed at temperatures lower than the crystallization temperature. This is called *work hardening* or *strain hardening*.

The response of a metal to cold working can be quantified by the strain-hardening exponent (n). Higher value of n indicates large degree of strengthening. Perfectly elastic materials ($n = 1$) have linear relationship between stress and strain (i.e. Hooke's law). Rigid materials ($n = \infty$) do not experience any strain, regardless of the applied stress. Perfectly plastic materials ($n = 0$) are non-strain hardening material. A rigid plastic material experiences strain hardening ($n \neq 1$) [Fig. 10.9]. For metals, strain hardening is the result of dislocation interaction and multiplication. The strain-hardening exponent is relatively low for HCP metals but is higher for BCC and particularly, for FCC metals. Metals with a low strain-hardening exponent respond poorly to cold working.

Strain hardening is significant in many metal-forming and fabrication operations. As a result of work hardening, electrical conductivity can be decreased. Work hardening can cause development of internal stresses, increase in corrosion and crack formation and elastic distortion.

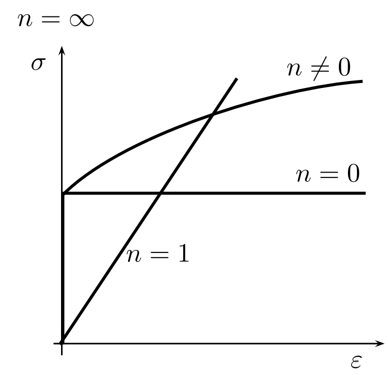


Figure 10.9 Strain hardening.

10.8.4 Deformation Mechanisms

Microscopically, plastic deformation involves motion of dislocations. The following are the two prominent mechanisms of plastic deformation:

1. Slip *Slip* is the most prominent mechanism of plastic deformation in metals, which involves sliding of blocks of crystal over one other along definite crystallographic planes, called *slip planes* but the orientation of the crystals remains the same. Slip occurs when shear stress applied exceeds a critical value in *slip directions*.

   A slip plane is generally the plane of the greatest atomic density, and the slip direction is the close-packed direction within the slip plane. Feasible combination of a slip plane together with a slip direction is considered as a *slip system*.

   A BCC crystal has 48 possible slip systems, therefore, metals with BCC structure have good strength and moderate ductility. FCC crystals have 12 slip systems, thus, the metals with FCC structures have moderate strength and good ductility. Similarly, an HCP crystal has three slip systems and, thus, has a low probability of slip; however, more systems become active at elevated temperatures.

2. Twinning *Twinning* occurs when a portion of crystal takes up an orientation that is related to the orientation of the rest of the untwinned lattice in a definite, symmetrical way. Twinning results in very small gross plastic deformation, but it causes the changes in plane orientation that facilitate slip. Twinning generally occurs when slip is restricted, because the stress necessary for twinning is usually higher than that for slip.

10.9 MULTIPHASE STRUCTURE

10.9.1 Phase

A *phase* can be defined as a homogeneous portion of a system that has uniform physical and chemical characteristics. It is a physically distinct, chemically homogeneous and mechanically separable portion of a system.

A *pure substance*, under equilibrium conditions, can exist as either of a phase namely vapor, liquid or solid, depending upon the conditions of temperature and pressure but the chemical composition remains invariant. Allotropic forms of a substance are considered as separate phases.

A metal can form different phases at different temperatures. This is due to the differences in the amount of energy required at each temperature. This is known as *allotropism* or *polymorphism*. Crystal structures can also be modified by the addition of atoms of other metals, called *alloying*. For instance,

1. Iron exhibits BCC structure up to 910°C and above this temperature, the structure changes to FCC. Upon further heating upto 1394°C, FCC lattice transforms back to BCC before it becomes completely liquid.

2. Monoclinic zirconia under high pressure (more than 37 kbar) transforms to tetragonal zirconia and with release of pressure, the structure reverts back to monoclinic.

3. Hexagonal form of boron nitride (BN) transforms to the hard cubic form upon application of very high pressure.

A single phase system is called *homogeneous*, and systems composed of more than one phase are termed as mixtures or *heterogeneous*. Most of the alloy systems and composites are heterogeneous in nature.

Number of phases that can be in equilibrium depends on many factors, such as temperature, pressure, and composition. In a given microstructure, the region of different structures and composition are referred to as phases and their boundaries are called *phase boundaries*.

10.9.2 Solid Solution

A *solid solution* consists of atoms of at least two different types. The solute atoms occupy either substitutional or interstitial positions in solvent lattice, and crystal structure of the solvent is maintained. Solid solutions are generally denoted by Greek letters (α, β, γ, μ, η, and δ).

10.9.2.1 Types of Solid Solution Depending on the site of occupancy of solute atoms or relative size of the atoms, the solid solutions can be of two types:

1. *Substitutional Solid Solutions* *Substitutional solid solutions* are formed when the solute atom is large enough to replace solvent atoms in their lattice positions. According to the *Hume-Rothery rules*, solvent and solute atoms must differ in atomic size by less than 15% in order to form this type of solution (e.g. Cu-Au, Cu-Zn, Cu-Sn).

2. *Interstitial Solid Solutions* *Interstitial solid solutions* are formed when smaller solute atoms fit in interstitial space between solvent atoms. Elements commonly used to form interstitial solid solutions include H, Li, Na, N, C, and O. Fe-C is an example of interstitial solid solution.

Both substitutional and interstitial atoms change the properties of solvent metal. Most often, these effects are used in design of various alloys.

10.9.2.2 Hume-Rothery Conditions The formation of the substitutional solid solution depends mainly on the following four factors:

1. The crystal structure of the two elements should be the same for complete solid solubility.

2. The size difference between the two atoms should be less than 15%.

3. The chemical affinity between the two atoms should be less.

4. The difference in the valency of solute and solvent atoms should be less.

These rules are popularly known as the the *Hume-Rothery conditions*[9].

10.9.2.3 State of Equilibrium *Equilibrium* of multiphase structures is best described in terms of a thermodynamic quantity, known as *free energy* which is a function of internal energy of the system, and also the randomness or disorder of the atoms (entropy). A system is at equilibrium if its free energy is minimum under the specified combination of temperature, pressure and composition. A state of equilibrium in solid systems is never achieved completely because the rate of approach to equilibrium is extremely low. Such a system is said to be in a non-equilibrium or *metastable state*. A metastable micro-structure can persist definitely experiencing only extreme light and almost imperceptible changes with time.

[9]Named after William Hume-Rothery (1899–1968), a metallurgist who studied the alloying of metals. His research was conducted at Oxford University where in 1958, he was appointed to the first chair in metallurgy.

10.9.2.4 Solidification Temperature

Solidification of pure metals occurs at constant temperature with gradual release of the latent heat of fusion. However, solidification of an alloy (solid solution) takes place over a temperature range between the *solidus* and *liquidus* of the solid solution.

10.9.3 Gibbs Free Energy

Gibbs free energy[10] (G) of a system is defined as

$$G = H - TS \tag{10.3}$$

where T is the absolute temperature at which the process occurs, H and S are the enthalpy and entropy of a system, respectively. Free energy curves for liquid and solid alloys over the entire composition range can be drawn at different temperatures. These diagrams can predict stability of phases at different points. If $\Delta G < 0$, the solution remains stable and solidifies as a single phase, otherwise the solution dissociates into two phases.

10.9.4 Phase Diagrams

Phase diagrams provide a convenient way of representing which state of aggregation is stable for a particular set of temperature, pressure and composition. Phase diagrams also provide information about melting, casting, crystallization, and other phenomena.

Two important concepts related to phase diagrams are described below:

1. *Gibbs Phase Rule* The number of *degrees of freedom* of a system is the number of variables, such as temperature, pressure, or concentration which can be varied independently without changing the number of phases in a defined field. This is determined by the *Gibbs phase rule*[11], a simple relation between the number of equilibrium phases (ϕ), components (c) and independent variables or degrees of freedom (f):

$$f = c - \phi + 2 \tag{10.4}$$

Most of the materials are used at a constant atmospheric pressure, hence Eq. (10.4) deduces to

$$f = c - \phi + 1 \tag{10.5}$$

This is known as the *condensed phase rule.*

2. *Lever Rule* Composition of elements is generally expressed in weight percentage. The relative amounts of phases present in equilibrium at a given temperature are calculated using the *lever rule.* In Fig. 10.10, for calculation of the proportion

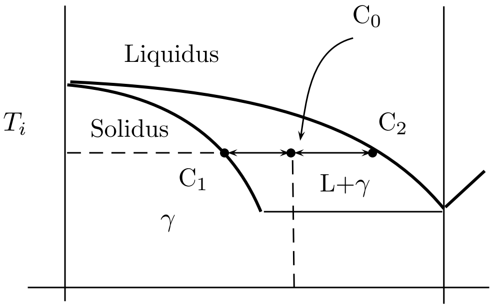


Figure 10.10 Lever rule.

of each phase present at composition C_0 and temperature T_1, a horizontal line is drawn at T_1 such that the line intersects two phase boundaries at composition C_1 and C_2, respectively. Then

$$\%\ \text{Liquid} = \frac{C_1C_0}{C_1C_2} \times 100$$

Phase diagrams can be broadly classified as unary or binary phase diagrams, described as follows.

10.9.4.1 Unary Phase Diagrams

All *unary systems* are presented in pressure versus temperature diagram. The diagram consists of individual phase fields and their boundaries, such as in water. All three phases (solid, liquid, and vapor) come to equilibrium at the invariant point. Two simple unary phase diagrams are described as follows:

1. *Zirconia* Zirconia (ZrO_2) exists in three different forms: monoclinic, tetragonal and cubic phases. Monoclinic phase is stable at atmospheric pressure and temperature. Above 1100°C, this phase transforms into tetragonal phase, which further changes to cubic phase beyond 2370°C, and finally to liquid phase at 2680°C.

2. *Pure Iron* Pure iron (Fe) also exists in three distinct solid phases: α, β, and δ. Fe appears as α phase (BCC) upto 910°C. It transforms into γ phase (FCC) and finally at 1394°C, γ changes to δ before becoming completely liquid at 1538°C.

10.9.4.2 Binary Phase Diagrams

Binary phase diagrams represent the phase fields of a two-component system. Phase diagrams for most of the binary systems are determined at the atmospheric pressure. Therefore, all the binary diagrams are drawn with the composition in x-axis and temperature in the y-axis. In binary

[10] Josiah Willard Gibbs (1839-1903) was an American theoretical physicist, chemist, and mathematician. Yale University awarded Gibbs the first American Ph.D. in engineering in 1863.
[11] Gibbs phase rule was proposed by the physicist J. Willard Gibbs in 1874.

diagrams, single-phase regions are always separated from each other by a two-phase region.

Two components, satisfying the *Hume-Rothery rules* [*Section* 10.9.2.2], become soluble in each other and form substitutional solid solutions. They could be completely soluble in the liquid state as well as in the solid state in all proportions. Such a system is called an *isomorphous system*.

Phase diagram of an isomorphous system can be divided into three distinct phase fields: liquid, solid, and liquid + solid regions. A number of metallic (Cu-Ni, Ge-Si, Ag-Cu) and non-metallic systems (CuO-NiO or cupronickel, Al_2O_3-Cr_2O_3, FeO-MgO, MgO-NiO) exhibit isomorphous behavior. In isomorphous and eutectic phase diagrams, the two solid phases α and β are sometimes called *terminal solid solutions* because they exist over composition ranges near the concentration extremities of the diagrams.

Three common binary diagrams are discussed as follows:

1. Copper-Nickel System The simplest binary phase diagram is of the copper and nickel (Cu-Ni) system, shown in Fig. 10.11, where the α phase is a substitutional solid solution consisting of both Cu and Ni in FCC crystal.

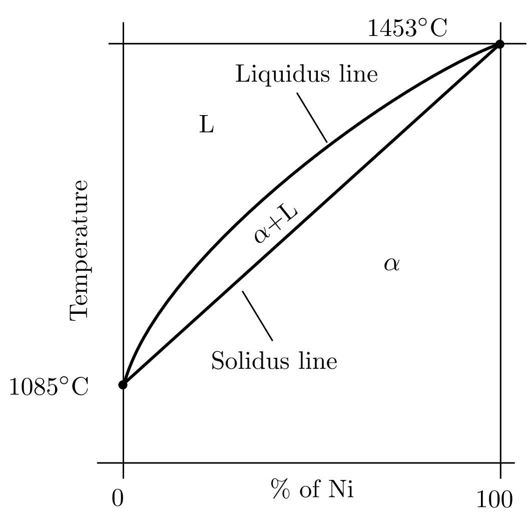


Figure 10.11 | Copper-nickel system.

Melting points of Cu and Ni are 1085°C and 1453°C, respectively. At temperature below about 1080°C, Cu and Ni are mutually soluble in each other in solid state for all compositions. This complete solubility is explained by the fact that both Cu and Ni have the same crystal structure (FCC), nearly identical radii, and electronegatives and similar valences. The Cu-Ni system[12] is called isomorphous because of the complete solubility in liquid and solid phases.

2. Copper-Silver System Copper-silver system is known as *eutectic binary phase diagram*. The diagram contains three single-phase regions of α, β and liquid phases [Fig. 10.12]. The α phase (FCC) is rich in copper (silver as the solute) while the β phase (FCC) is rich in silver (copper as the solute).

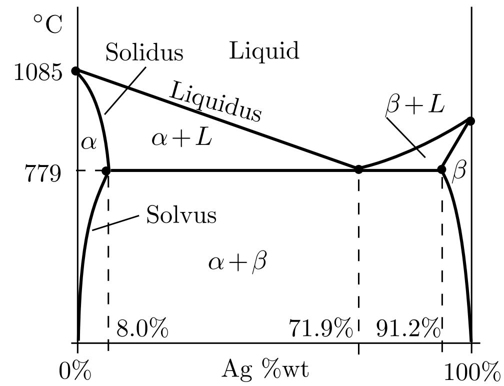


Figure 10.12 | Copper-silver system.

There are also three two-phase regions: $\alpha+L$, $\beta+L$ and $\alpha+\beta$. The solid solubility line separating the α (or β) and $\alpha+\beta$ phase regions is termed as *solvus line*. The boundary between the α (or β) and $\alpha+L$ phases is called *solidus line*. The horizontal line of 799°C can also be considered as solidus line representing the lowest temperature at which a liquid phase can exist for any copper-silver alloy, that is, at equilibrium. The point on this line at 71.9% of Ag is called an invariant point where following eutectic reaction occurs

$$\text{Liquid} \underset{\text{Cooling}}{\overset{\text{Heating}}{\rightleftharpoons}} \alpha+\beta$$

For a eutectic system, three phases (α, β, L) can be in equilibrium but only at points along the eutectic isotherm (799°C).

3. Lead-Tin System The eutectic system of lead (Pb) and tin (Sn) has general shape similar to that for copper-silver system [Fig. 10.13].

The eutectic invariant point is located at 61.9 wt% Sn and 183°C. On several applications, the low melting temperature alloys are prepared having near eutectic compositions, such as solder

[12]Cupronickel alloys exhibit good corrosion resistance and hence, find place in marine applications.

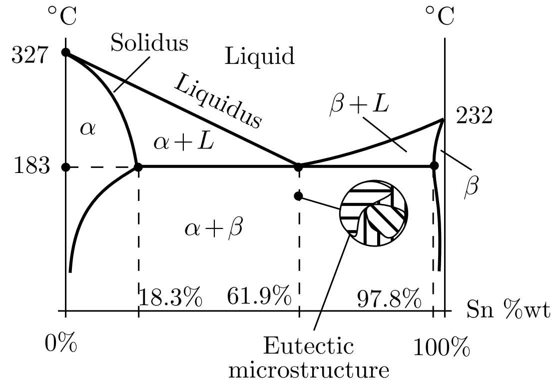


Figure 10.13 | Lead-tin system.

is 60% tin (Sn) and 40% lead[13] (Pb) which completely melts at 185°C.

The Fe-C system is considered as the most important binary alloy systems, therefore, it is discussed separately in *Section* 10.10.

10.9.5 Eutectic Microstructure

Different types of micro-structures are possible with different rates of cooling of alloys belonging to binary eutectic systems. If an alloy is cooled from a temperature within the liquid-phase region down the vertical line of eutectic composition, no changes occur until the temperature reaches the eutectic temperature. Upon crossing the eutectic isotherm, the liquid transforms to the two α and β phases. For example, in case of Pb-Sn system [Fig. 10.13],

$$\text{Liquid (61.9\% wt Sn)} \underset{\text{Cooling}}{\overset{\text{Heating}}{\rightleftharpoons}} \alpha\ \text{(18.3\% wt Sn)} + \beta\ \text{(97.8\% wt Sn)}$$

This transformation redistributes the elements by atomic diffusion in the α and β phases in alternating fashion to replace the liquid [Fig. 10.13].

10.9.6 Invariant Reactions

Invariant reactions in binary systems occur at fixed temperature and pressure between three phases at fixed (invariant) composition. Important invariant reactions are described as follows:

1. *Eutectic Reaction* In a *eutectic reaction*, a liquid upon cooling dissociates into two solids of different compositions:

$$\text{Liquid} \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \text{Solid}_1 + \text{Solid}_2$$

Melting point of an alloy is minimum at eutectic composition[14], as seen in the following systems:

(a) *Lead-Tin System* Soldering alloy (62% tin and 28% lead) has minimum melting point 180°C while melting points of the pure tin and lead are 232°C and 327°C, respectively.

(b) *Copper-Silver System* In Cu-Ag system, the eutectic point occurs for 71.9% of Ag at 799°C:

$$\text{Liquid} \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \alpha + \beta$$

In this system, 799°C is the lowest temperature at which a liquid phase can exist for any copper-silver alloy that is at equilibrium.

(c) *Iron-Carbon System* In Fe-C system, minimum melting point occurs at 4.34 weight %C at 1130°C.

2. *Eutectoid Reaction* In an *eutectoid reaction* (or eutectic-like), a solid upon cooling dissociates into two new solids of different compositions:

$$\text{Solid}_1 \underset{\text{Cooling}}{\overset{\text{Heating}}{\rightleftharpoons}} \text{Solid}_2 + \text{Solid}_3$$

The eutectoid reaction occurs in many alloy systems, such as Al-Cu, Cu-Zn, Fe-C systems. The feature distinguishing "eutectoid" from "eutectic" is that one solid phase instead of a liquid transforms into two other solid phases at a single temperature.

The eutectoid structure, frequently lamellar, is produced by precipitation from the solid solution. The crystal structure of the new phase is known as the *Widmanstätten structure*. The alternate plates of Solid_2 and Solid_3 are formed by a diffusion of controlled reaction. The inter-lamellar spacing depends on the cooling rate.

3. *Peritectic Reaction* In a *peritectic reaction*, a liquid and a solid upon cooling form a new solid at a constant temperature:

$$\text{Liquid} + \text{Solid}_1 \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \text{Solid}_2$$

Peritectic reaction occurs in copper-zinc system at 598°C, 78.6 wt% Zn and 21.4 wt% Cu.

[13]Lead is a mildly toxic metal and there is serious concern about the environmental impact of discarded lead-containing products that can leach into ground water from landfills or pollute the air if incinerated. Hence, some lead free solders have been developed which includes ternary alloys (i.e. composed of three metals of tin-sliver-copper and tin-silver-bismuth solders.

[14]The word 'eutectic' means easily melted.

4. ***Peritectoid Reaction*** Reverse of eutectoid reaction is known as peritectoid reaction in which two solids upon cooling form a new solid:

$$\text{Solid}_1 + \text{Solid}_2 \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} + \text{Solid}_3$$

Peritectic and peritectoid reactions do not give rise to micro-constituents as the eutectic and eutectoid reactions do.

5. ***Monotectic Reaction*** In *monotectic reaction*, a liquid transforms into another liquid of different composition and a solid phase precipitates out:

$$\text{Liquid}_1 \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \text{Liquid}_2 + \text{Solid}$$

The eutectic and eutectoid reactions are similar in that they both involve the decomposition of a single phase into two solid phases. The –oid suffix indicates that a solid, rather than liquid, phase is decomposing. Eutectoid and peritectoid are the two reactions on solid phases. Solid-state reactions occur much more slowly, are subject to greater undercooling, and rarely attain true equilibrium conditions [Table 10.2].

Table 10.2 | Invariant reactions

Reaction	Change upon Cooling
Eutectic	$\text{Liquid}_1 \rightarrow \text{Solid}_1 + \text{Solid}_2$
Eutectoid	$\text{Solid}_1 \rightarrow \text{Solid}_2 + \text{Solid}_3$
Peritectic	$\text{Liquid}_1 + \text{Solid}_1 \rightarrow \text{Solid}_2$
Peritectoid	$\text{Solid}_1 + \text{Solid}_2 \rightarrow \text{Solid}_3$
Monotectoid	$\text{Liquid}_1 \rightarrow \text{Liquid}_2 + \text{Solid}_1$

Eutectic and eutectoid points in binary systems are used as reference points and the compositions below and above the eutectoid composition are referred to as *hypo-eutectoid* (less than eutectoid) and *hyper-eutectoid* (more than euctectoid), respectively. For example, in the Fe-Fe_3C system, at 727°C, γ-phase dissociates into an eutectoid mixture of ferrite (α) and cementite (Fe_3C) which is known as *pearlite*.

10.10 IRON-CARBON PHASE

The phase diagram of Fe-Fe_3C system [Fig. 10.14] is not a true equilibrium because the iron carbide (Fe_3C) is an unstable phase that after prolonged heat treatment decomposes to iron and carbon (the graphite form). The study is simplified with an assumption that sufficient time has been allowed at each new temperature for any necessary adjustment in phase compositions and relative amounts.

There are six important temperatures 727°C, 912°C, 1130°C, 1394°C, 1493°C, and 1538°C at which some phase transformation occurs. Temperature line 727°C is called *lower critical temperature* while *upper critical temperature* line varies from 914°C to 727°C and again from 727°C to 1130°C. Maximum and minimum melting points are 1538°C and 1130°C.

The composition between 0.022% and 0.76% of carbon in iron is called *hypo-eutectoid* (less than eutectoid) composition alloy, and the composition between 0.76% and 2.14% of carbon in iron is called *hyper-euctoid* (more than eutectoid) composition alloy. Cementite (6.67% C) has the maximum possible concentration of carbon in Fe-C system.

10.10.1 Phase Components

Carbon is an interstitial impurity in iron and forms a solid solution in α and δ phases. Table 10.3 shows the points of maximum concentration of carbon and corresponding temperature in Fe-C phases.

Table 10.3 | Maximum carbon content in Fe-C phases

Phase	Structure	Carbon	Temperature
α-Iron	BCC	0.022%	727°C
γ-Iron	FCC	2.140%	1130°C
δ-Iron	BCC	0.100%	1493°C

10.10.1.1 Ferrite Pure iron contains less than 0.008% weight of carbon. Upon heating, pure iron experiences changes in crystal structure before it melts. The stable form at room temperature, called *ferrite*[15] or *α-iron*, has a BCC crystal structure. Only small concentrations of carbon are soluble in α-iron because the shape and size of BCC interstitial space positions which make it difficult to accommodate the carbon atoms. This phase is relatively soft, can be made magnetic at temperature below 768°C and has the maximum possible density of 7880 kg/m^3.

10.10.1.2 Austenite Ferrite (α-iron) experiences a polymorphic transformation to FCC *austenite*[16] (*γ-iron*) at 912°C. The austenite persists upto 1394°C at which it again becomes BCC phase, known as *δ-iron*. Upon further heating, it melts at 1538°C.

Austenite or γ-iron, when alloyed with just carbon, is not stable below 727°C. Maximum solubility of carbon in γ-iron is 2.14%, approximately 100 times the maximum solubility in BCC ferrite phase because FCC interstitial positions are larger. Austenite is a non-magnetic phase.

[15] The word is derived from latine word *ferrum* which means iron.
[16] Named after pioneer metallographer W. C. R. Austen.

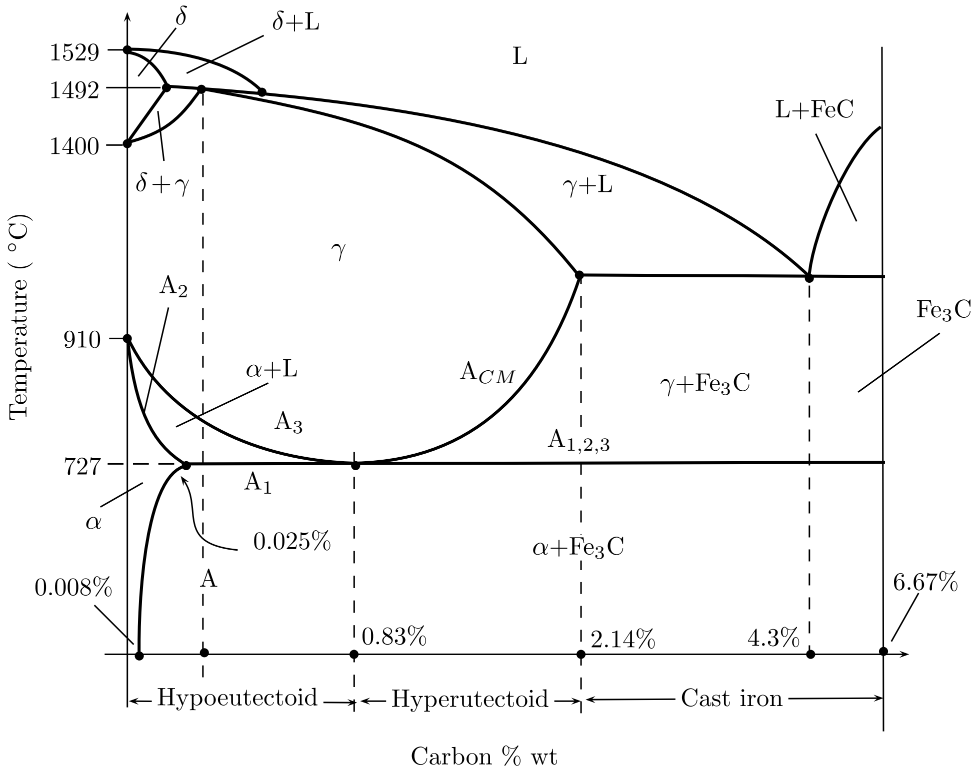


Figure 10.14 Fe-Fe_3C system.

10.10.1.3 Cementite A very hard and brittle phase, *cementite* (Fe_3C), is formed when carbon exceeds 6.67%. Cementite phase is associated with two invariant reactions:

1. ***Eutectic Reaction*** A eutectic invariant reaction occurs at 4.34% C, 1130°C:

$$L \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \gamma + Fe_3C$$

2. ***Eutectoid Reaction*** A *eutectoid* invariant reaction occurs at 0.76% C and 727°C:

$$\gamma \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \underbrace{\alpha}_{75\%} + \underbrace{Fe_3C}_{25\%}$$

Cementite is the only metastable phase; it will remain as compound indefinitely at room temperature. However, if heated to between 650°C and 700°C for several years, it will gradually change into α-iron and carbon in the form of graphite which will remain the same upon subsequent cooling to room temperature. Addition of silicon (Si) to cast iron greatly accelerates this cementite decomposition reaction to form graphite.

10.10.1.4 Pearlite Microstructure for the eutectoid steel that is slowly cooled through the eutectoid temperature consist of alternating lamellae of the two phases α and Fe_3C. Carbon atoms diffuse away from the 0.22% ferrite, extending from the grain boundaries into unreacted austenite grain. The resulting structure is called *pearlite* because it has the appearance of mother of pearl when viewed under microscope at low magnifications. It has properties intermediate between the soft-ductile ferrite and hard-brittle cementite.

10.10.2 Invariant Reactions

The invariant reactions in Fe-C system are as follows:

1. Eutectoid reaction (0.8% C, 727°C)

$$\gamma \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \alpha + Fe_3C$$

2. Peritectic reaction (0.16% C, 1490°C)

$$L + \delta \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \gamma$$

3. Monotectic reaction (0.51% C, 1495°C)

$$L \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} L + \gamma$$

4. Eutectic reaction (4.3% C, 1130°C)

$$\mathrm{L} \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \gamma + \mathrm{Fe_3C}$$

10.10.3 TTT Diagram

Rate of phase transformation also depends upon the temperature. *Temperature-time transformation diagram* (TTT), also known as *isothermal transformation curves*, are useful in planning heat treatments.

Figure 10.15 depicts TTT diagram of steel at eutectoid composition. In the diagram, M_s and M_f stand for martensite start temperature and martensite finish temperature, respectively. The transformation rate at some particular temperature is inversely proportional to the time required for reaction to proceed to 50% completion. At temperature just below the eutectoid, very long time is required for 50% transformation, and therefore, the reaction is very slow. The transformation rate increases with decreasing temperature.

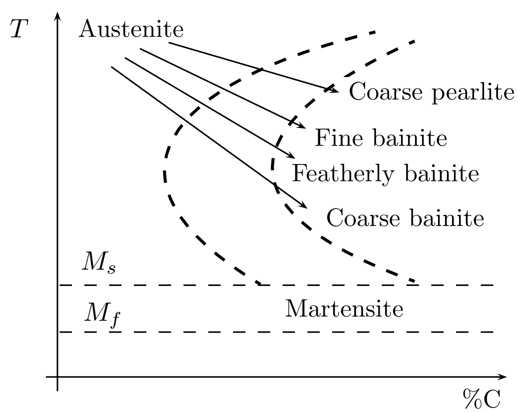


Figure 10.15 TTT diagram (0.8% C steel).

The resulting micro-structures depend upon the rate of phase transformation or cooling. These are described under following sections.

10.10.3.1 Course Pearlite Thickness ratio of ferrite and cementite layers in pearlite is 8:1. At temperature just below the eutectoid (727°C), relatively thick layers of both α ferrite and Fe_3C phases are produced. This micro-structure is called *coarse pearlite* (diffusion rate is relatively high).

10.10.3.2 Fine Pearlite Rate of carbon diffusion decreases at lower temperatures and the layers become progressively thinner. Thin layered structure produced in the vicinity of 540°C is termed as *fine pearlite* or *sorbite*. If the steel is cooled at a faster rate, an even finer structure of austenite decomposition is obtained, which is called *troostite*. The reinforcement by cementite phase and restriction of dislocation motion by phase boundaries in sorbite and troosite account for hardness and strength greater than coarse pearlite.

10.10.3.3 Bainite *Bainite* is obtained at higher cooling rate when alternating ferrite and cementite lamellae become thinner than that in fine pearlite. For temperature approximately between 300°C and 400°C, bainite forms a series of parallel needles of ferrite that are separated by elongated particles of the cementite phase. Such bainite is called *upper bainite*. *Lower bainite* is produced between about 200°C and 300°C, in which ferrite phase exists as thin plates and narrow cementite particles form within the ferrite plates. Due to their fine structures, bainite steels are generally stronger and harder than pearlitic steels and exhibit a desirable combination of strength and ductility.

10.10.3.4 Spherodite If pearlitic or bainitic steel is heated to and kept at a temperature below the eutectoid (727°C) for a sufficiently long period of time (18–24 hours), a new micro-structure is formed in which Fe_3C phase appears as sphere-like particles embedded in a continuous α-phase matrix. This micro-structure is known as *spherodite*. Due to lesser boundary area per unit volume, spheroidite offers lesser restriction to plastic deformation. Thus, spheroidized steels are extremely ductile, much more than either course or fine pearlite. Spheroidized steels are notably tough because crack can encounter only a very small fraction of brittle cementite particles as it propagates through the ductile ferrite matrix.

10.10.3.5 Martensite In Fe-C system, martensite is the hardest, strongest, and the most brittle phase. It is a metastable phase that results from a diffusion free transformation of austenite when austenized Fe-C alloys are rapidly cooled or quenched. Martensite grains nucleate and grow at a very rapid rate equal to the velocity of sound within austenite matrix.

Fast cooling (≈ 500°C/s) restricts the diffusion of carbon to remain as interstitial impurity in martensite, such that they constitute a super-saturated solid solution. To make room for the carbon atoms, FCC structure of austenite is distorted into to a *body centered tetragonal* (BCT) martensite without loss of its contained carbon atoms. This is a simple ferrite (α-iron) BCC structure having iron atoms at corners but it is stretched along one of its dimensions due to placement of carbon atoms in the edges. Therefore, martensite is generally designated as α'. FCC austenite is denser than BCT martensite, therefore, during quenching to martensite, the micro-structure expands.

Due to high lattice distortion, martensite has high residual stresses, and high hardness and strength. However, martensite is too brittle. Therefore, tempering of

martensite is accomplished by heating the martensite steel to a temperature below the eutectoid (727°C) for a specified period. Internal stresses, however, can be relieved at temperature 200°C.

10.11 HEAT TREATMENT OF STEEL

Heat treatment is the process of heating and cooling of a material to change its physical and mechanical properties without changing the original shape and size. Heat treatment of steel is often associated with increasing its strength, but can also be used to improve machinability, formability, restoring ductility, etc. Basic heat treatment processes for steels are described in the following subsections.

10.11.1 Hardening

Hardening involves bringing the steel into austenite range by proper soaking at a temperature greater than 727°C and then rapidly cooling it using a quenching media. *Critical cooling rate* (CCR) required for quenching depends upon carbon percentage. Higher carbon percentage requires lower CCR. Quenching media with decreasing capacity are brine, water, oil, forced air, still air, and furnace itself.

Hardenability is defined as the distance below the surface where the amount of martensite had been reduced to 50%. This is measured by *Jominy end quench test*. Lower grain size promotes the nucleation of pearlite, therefore, hardenability of steel can be increased by increasing the grain size of austenite.

In *dispersion hardening*, a dispersion of hard particles in the matrix of a soft phase provides pinning sites for the movement of dislocations. *Precipitation hardening* is suitable for few alloy systems which show significant decrease in solid solubility of one phase into the matrix phase when temperature falls. The classic example is the Duralumins (96% Al–4% Cu). For this, precipitation hardening involves heating of the aluminium alloy to around 520°C at which the Cu atoms fully dissolve in the Al to give the random substitutional solid solution α. The material is then quenched to retain the new formation and is then found to be harder and tougher than before. The new structure is unstable, and over the period of time, tiny particles of $CuAl_2$ start to precipitate out of the solid solution. The precipitates, although small, are closely spaced and evenly scattered throughout the structure. This process is also called *age-hardening*.

10.11.2 Tempering

Martensite formed during the quenching process is extremely hard and brittle and lacks toughness, therefore, hardening is always followed by tempering. *Tempering* is the process of keeping the material for a long time at higher temperature [Fig. 10.16].

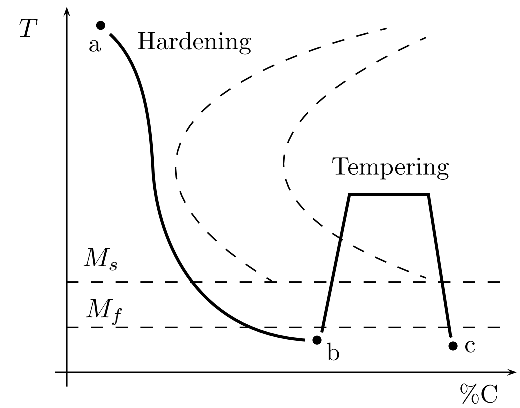


Figure 10.16 Hardening and tempering.

To improve ductility and to remove internal stresses in hardened martensite, tempering is done by heating the quenched steel to a temperature between 200°C and 400°C and then cooling.

10.11.3 Austempering

The martensite obtained after quenching is crack-prone. Therefore, *austempering* (tempering of austenite) is carried out as a variation in the hardening process wherein material is quickly quenched a little above the M_s temperature for prolonged period [Fig. 10.17]. Under isothermal condition, the austenite transforms into bainite, which is stress free and less-likely to cause crack formation in the material. The principle purpose of austempering is to obtain high impact strength and increased notch toughness at a given high hardness level.

10.11.4 Martempering

Martempering is the process of tempering of martensite [Fig. 10.17]. The steel is heated to the austenite range, followed by rapid quenching in water bath to a temperature above M_s. Thereafter, the material is maintained at constant temperature in an oil bath so that entire section is brought to uniform temperature, but not long enough to cause austenite decomposition. Then, it is taken out of bath and cooled sufficiently fast in air to transform austenite into martensite.

Martempering produces martensite and retains austenite in the hardened steel. Over conventional harden-

ing, the process has advantages of reduced possibility of warping, distortion, quenching cracks and volume change.

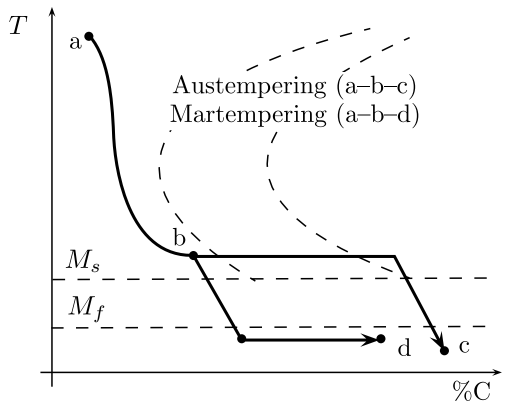


Figure 10.17 Austempering and martempering.

10.11.5 Annealing

Annealing (or full annealing) is the process of heating to 60°C above A_3 line to fully convert the structure into austenite, and subsequently cooling it very slowly in the heating furnace itself. This results in a grain structure of fine pearlite with excess of ferrite or cementite. The process eliminates internal stresses, reduces hardness, increases ductility, enhances machinability, and refines the grain structure.

Different degrees of softening can be achieved by slight variation in the annealing temperature, cooling rate, and atmosphere. For example, oxidation of the alloy can be minimized by annealing in a sealed container under controlled conditions. This process is called *box annealing*. Annealing in a protective atmosphere of inert gas (e.g. argon, nitrogen) to prevent surface discoloration is known as *bright annealing*. In *process or intermediate annealing*, the workpiece (e.g. sheet, wire) is annealed in order to restore the ductility, which would have been exhausted by work hardening during cold working. This enables further cold working of the workpiece.

10.11.6 Stress Relieving

Stress relieving is done by heating the cold worked steels to a relatively low temperature of about 75°C below the transformation temperature line A_1 (727°C) in the Fe-Fe_3C phase diagram [Fig. 10.14]. After removing from the furnace, the parts are cooled in still air. The process is a also known as *finish annealing*. It results in lowering the residual stresses, thereby lessening the risk of distortion in machining.

10.11.7 Spheroidizing

When an alloy having pearlite structure is held for a long period at a temperature close to but less than the eutectoid temperature and cooled to room temperature, the alloy exhibits a microstructrue consisting of grains of ferrite and uniform distribution of nearly spherical particles of cementite. The process is called *spheroidizing*, which is a variation of annealing process.

Ductility of spherodized steel is very high. Uniform distribution of hard carbide particles in the soft ferrite matrix gives high wear resistance. Spherodized steels are used for the manufacture of wire ropes employed extensively in the transport industry. Spherodizing is also used specially to improve machinability of high carbon steels.

10.11.8 Normalizing

Normalizing is the process of heating of the material upto austenising temperature range and then cooling at a faster rate in still air. Pearlite structure obtained would be of uniformly fine grain size. This would result in higher tensile strength (yield point) and hardness than what is possible by annealing.

10.11.9 Case Hardening

Case hardening is meant for improving hardness of outer layer, while keeping internal material stronger and ductile. The following are the important processes for case hardening.

10.11.9.1 Case Carburizing The process of *case carburizing* is applied to low carbon steels. The specimen along with carbonaceous material[17], such as charcoal, is packed in a sealed container and is kept in furnace. The pack is heated to 800-950°C and held upto 20 hours. On heating, the oxygen present in a small amount in the sealed container reacts with the charcoal and forms carbon monooxide (CO) which when in contact with the specimen, releases oxygen while carbon diffuses into outer skin of the specimen as cementite. The released oxygen again forms CO with charcoal and the whole process gets repeated. Hardness is obtained up to a depth of 1–2 mm. This process is also called *pack carburizing*.

10.11.9.2 Cyaniding *Cyaniding* or *liquid carburizing* involves addition of carbon and nitrogen to the case of carbon steels and alloy steels by heating the steel in contact with a molten bath of cyanide (CN) in steel pot. The cyanide bath can consist of 45% sodium

[17]Another way is *gas carburizing* where carbon in gaseous form (e.g. natural gas, propane, methane) is used instead of charcoal.

cyanide (NaCN) and potassium cyanide (KCN) with inert salts, such as 15% sodium chloride (NaCl), 40% sodium carbonate (Na_2CO_3), which provide the necessary fluidity to the cyanide bath. The bath is maintained at a temperature of 750°C to 880°C and contact time is between 15 and 60 min. This produces around 0.15 mm depth of hardened case. The heated parts are removed from the bath and quenched in water. The salt is poisonous and the bath gives off unpleasant fumes which must be extracted through a hood.

10.11.9.3 Nitriding Alloying elements in steels, such as aluminium, chromium, vanadium, and molybdenum, form very hard nitrides when they come in contact with nitrogen. In *nitriding*, also called *carbo-nitriding* or *gas cyaniding*, the steel is put in a sealed container with ammonia gas and then heated to nitriding temperature between 500°C to 575°C for a duration 8–40 hours (the slowest case hardening process). This significantly increases wear resistance and fatigue life of the component. The portion of workpiece which is not to be case hardened is covered with tin.

10.11.9.4 Induction Hardening *Induction hardening* involves heating of component surface by electromagnetic induction. The workpiece, such as crank shaft, is enclosed in the magnetic field of an alternating high frequency (10 kHz to 2 MHz) current conductor to obtain case depth of the order of 0.25 to 1.5 mm. This causes induction heating of the workpiece which is later quenched with water spray. Depth of case hardening is controlled by varying the supply voltage.

10.11.9.5 Flame Hardening *Flame hardening* involves heating of the workpiece by means of a oxyacetylene flame through a torch, followed by a water spray on the heated parts. The heat penetrates only to small depth (3 mm) on the surface and consequently the steel in the outer layers is quenched to martensite and bainite. The process is used for crank shafts, large gears, cams, etc. Bearing surfaces of guide-ways in machine tools are either flame or induction hardened.

10.12 ALLOYING OF STEELS

Steels are alloyed with other elements to improve the properties as compared to the plain carbon steel. For example, grain size of steel can be controlled by the addition of alloying elements, such as aluminium, boron, vanadium, titanium, during manufacturing process.

Role of important alloying elements in steels are discussed as follows:

1. *Carbon* Small amount of carbon (C) in the range of 0.80% gives hardness and tensile strength to the steel and makes it responsive to heat treatment [Fig. 10.18].

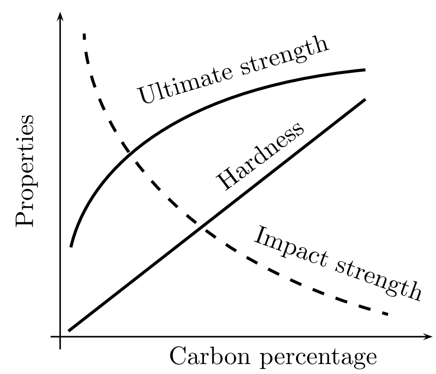


Figure 10.18 Effect of carbon on steels.

Carbon content beyond certain limits in plain carbon steel makes the steel to assume the characteristics of cast iron, which cannot be worked either hot or cold.

2. *Manganese* Manganese (Mn) is normally present in all steel. It functions as a deoxidizer, and imparts strength and responsiveness to heat treatment. It is usually present in quantities from 1% to 2%, but certain special steels can have 10% to 15% of Mn. Mn in iron combines with sulphur, forming MnS which later combines with calcium oxide and is removed.

   Manganese and nickel (Ni) are the two elements that lower the temperature of austenite formation while others raise it. All alloying elements, except cobalt (Co), shift the TTT diagram [Fig. 10.15] to right hand side, resulting decreased critical cooling rate for martensite formation.

3. *Aluminium* Aluminium (Al) is considered as the most active deoxidizer used in producing steels. It is used in controlling the grain size.

4. *Boron* Boron (B) is added to steel in amounts of 0.0005 to 0.003% to improve hardenability; acts as a hardenability intensifier. Boron is also used as inculcator of fine grain size.

5. *Phosphorus* Some amount of phosphorus (P) is present in all steels. In addition to increasing yield strength and reducing ductility at low temperatures, phosphorus is believed to increase resistance to atmospheric corrosion. Phosphorus increases fluidity of molten iron. It is added upto 0.12% for free cutting steel.

   Phosphorus combines with iron making FeP, which embrittles cast iron. High levels of phosphorus can lead to '*cold shortness*' in steel where the

steel tends to become very brittle under extreme cold conditions and thus is vulnerable to cracking.

6. *Silicon* Silicon (Si) is one of the common deoxidizers used during the manufacturing process. It increases toughness of spring steels, chisels, punches used in sheet metal operations. It is also used in special steels in the range of 1.5–2.5% to improve the hardenability. Silicon is added upto to 5% to produce certain electrical characteristics in the so-called silicon electrical steels.

7. *Sulphur* Sulphur (S) is added up to 0.33% in free cutting steels. High level of sulphur can lead to '*hot shortness*' in steel, a condition in which the melting point of steel gets lowered, thereby reducing its strength dramatically under high temperature conditions.

8. *Chromium* Chromium (Cr) increases the depth of hardness penetration. Most chromium-bearing alloys contain 0.50–1.50% chromium. Stainless steels contain chromium in large quantities (12–25%), frequently in combination with nickel, and possess increased resistance to oxidation and corrosion.

9. *Titanium* Titanium (Ti) is the strongest element for carbide formation, therefore, it is added to 18-8 stainless steels (18% chromium and 8% nickel) to make them immune to harmful carbide precipitation.

10. *Columbium* Columbium[18] (Cb) in 18-8 stainless steel has a similar effect as titanium in making the steel immune to harmful carbide precipitation and resulting in inter-granular corrosion. Columbium-bearing welding electrodes are used in welding both titanium and columbium bearing stainless steels, since titanium would be lost in the weld arc, whereas columbium will be carried over into the weld deposit.

11. *Molybdenum* Molybdenum (Mo) in comparatively small quantities ranging from 0.10 to 0.40% is added greatly to the penetration of hardness and increases toughness and creep strength. It tends to help steel resist softening at high temperatures.

12. *Tungsten* Tungsten (W) tends to produce a fine, dense grain and keen cutting edge when used in relatively small quantities. When used in larger quantities of 17–20% in combination with other elements, it produces high speed steels which retains hardness and strength at high temperatures.

13. *Vanadium* Vanadium (V), usually in quantities from 0.15–0.20%, retards grain growth, even after hardening from high temperatures or after periods of extended heating. Tool steels[19] containing vanadium resist shock better than other steels.

14. *Nickel* Nickel (Ni) increases strength and toughness of steel but is one of the least effective elements for increasing hardenability. Steels containing nickel usually have more impact resistance, especially at low temperatures. Blades of gas turbines are made of high nickel alloy.

15. *Copper* Copper (Cu) is normally added in small amounts (0.15–0.25%) to improve resistance to atmospheric corrosion and to increase tensile and yield strengths with only a slight loss in ductility. Higher strength properties can be obtained by precipitation hardening in copper-bearing steels.

16. *Hydrogen* Hydrogen content beyond solubility limits leads to pinhole formation and porosity in the steel. This is called *hydrogen embrittlement*.

These important alloying elements and their effects are summarized in Table 10.4.

Table 10.4 | Alloying elements in steels

Element	Effects
C	Increases hardness
Mn	Increases ductility, strength
Ni, Mn, Si	Increases impact strength
Al	Fine grains, deoxidizer
B	Hardenability
Si	Graphitizer
P, Cu	Raises yield point
P, S, Pb	Increases machinability
Pb	Free cutting steels
H	Causes embrittlement
Cr	Corrosion resistance, hardness
Mo	Creep and hot strength
W	Fine grains, hardness, strength
V	Fatigue strength

10.13 MECHANICAL PROPERTIES

Mechanical properties of engineering materials describe the behavior of the material under action of external

[18] In 1864 and 1865, a series of scientific findings clarified that niobium (Nb) and columbium (Cb) were the same element.

[19] The term "tool steel" is a generic description of steel which has been developed specifically for tooling applications, such as drawing, blanking, mold inserts, stamping, metal slitting, forming and embossing, although they are not limited to just those areas.

forces. These are discussed under the following subsections.

10.13.1 Tensile Strength

The engineering tension test[20] is commonly used to provide basic design information on the strength characteristics of a material. In the test, the specimen is subjected to a gradually increasing uni-axial tensile force and the elongation is measured simultaneously.

The stress-strain diagram for mild steels [Fig. 10.19] can be explained under the following points:

1. *Linearity* The strain vary linearly with stress upto point P just before the elastic limit beyond which deformation is non-linear.

2. *Yielding* Near the elastic limit, there occurs an easiness in the rate of resulting strain for the same applied stress and the material is said to be *yielding*. This point is called *yield point*.

   There is no definite point where exactly material starts to deform plastically. Thus, it is common to assume that stress value at 0.2% offset strain as yield strength (σ_0), denoted by point Y. This *offset yield strength* is also called *proof stress* which is used in design as it avoids the difficulties in measuring proportional or elastic limit. For some materials where there is essentially no initial linear portion, offset strain of 0.5% is frequently used.

   Resilience is defined as the ability of a material to absorb energy when deformed elastically and to return it when unloaded. This is approximately equal to the area under elastic part of the stress-strain curve. This is measured in terms of *modulus of resilience* (U_r), defined as strain energy per unit volume required to stress the material from zero stress to yield stress:

$$\begin{aligned} U_r &= \frac{1}{2}\sigma_y \varepsilon_y \\ &= \frac{{\varepsilon_y}^2}{2E} \end{aligned}$$

   where ε_y is the elastic strain limit and E is the modulus of elasticity.

3. *Ultimate Strength* The stress reaches a maximum at point U uptil which plastic deformation is uniform along the length of the specimen.

   Toughness (U_t) of a material is defined as its ability to absorb energy in the plastic range. It is measured as the work done per unit volume without causing it to rupture. This can be obtained as the area under the stress-strain curve, approximately of rectangular shape:

$$U_t = \begin{cases} (1/2)\,(\sigma_{yp} + \sigma_{ut})\,\varepsilon_f & \text{Ductile materials} \\ (2/3)\,\sigma_{ut}\varepsilon_f & \text{Brittle materials} \end{cases}$$

   Thus, toughness gives an idea about both strength (σ) and ductility (ε) of the material.

4. *Failure* The stress relaxes after exceeding the ultimate stress because of the onset of necking that results in non-uniform plastic deformation before the specimen fractures at point F.

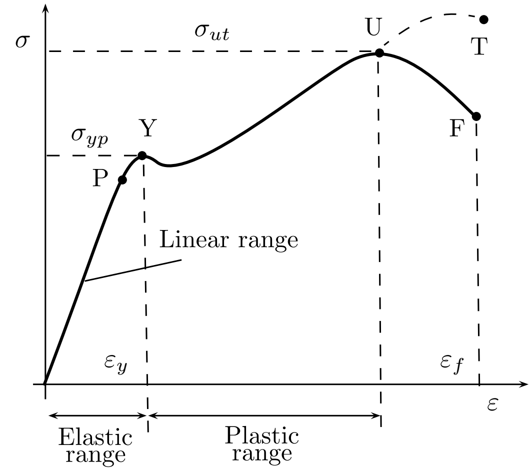


Figure 10.19 | Stress-strain curve for mild steel.

The diagram is drawn based on initial cross-sectional area, but as strain increases, cross-sectional area reduces, and points of stresses based on actual cross-sectional area divert beyond the ultimate strength point, as shown by the dotted line.

Figure 10.20 shows the (scaled) shapes of stress-strain diagrams for cast iron, hard steel, brass, rubber and plastics. Stress-strain curve of brittle materials, such as cast iron, does not follow Hooke's law. The ultimate stress in compression is much larger than that in tension. Therefore, one should try to place the brittle material under compression when developing a design.

True stress–true strain curve in plastic range is typically approximated as

$$\sigma = K\varepsilon^n$$

where K is known as the *strength coefficient* and n is known as the *strain-hardening exponent*.

[20]Standardized test procedure is explained by ASTM standard E0008-04.

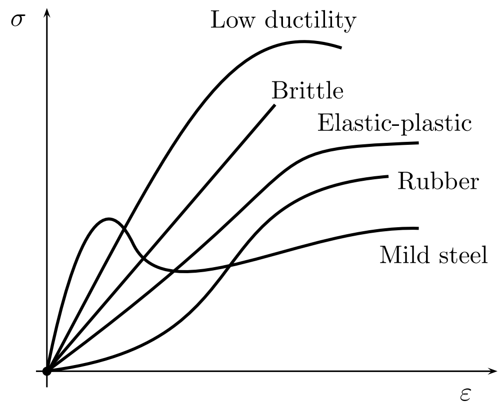


Figure 10.20 $\sigma-\varepsilon$ curves.

10.13.2 Impact Strength

The resistance of a material to fracture under dynamic load is characterized by *impact strength.* It is defined as the specific work required to fracture a test specimen with a stress concentrator (notch) in the middle when broken by a single blow of striker in pendulum type impact testing. This can be measured by the following standardized tests:

1. *Charpy Test* In this test, the specimen is in the shape of a bar of square cross-section (10 mm×10 mm) into which a V-notch (2 mm depth) is machined at midspan. The load is applied as an impact blow from a weighted pendulum hammer that is released from a height h. After the impact, the pendulum continues to swing rising upto maximum height h'. The energy absorbed, computed from the difference between h and h', is a measure of impact energy. Charpy V-notch technique is most commonly used.

2. *Izod Test* Izod test is performed by the same procedure except in the manner of specimen support. In Izod, it is kept vertical; whereas in Charpy, it is kept horizontal.

The impact energy so measured is called *notch toughness*. Impact strength is affected by the rate of loading, temperature, and the presence of stress raisers in the material. It also depends upon the dimension of specimen and sharpness of notch.

10.13.3 Fatigue Strength

Materials subjected to a repetitive or fluctuating stress fail at a stress much lower than that required to cause fracture under steady loads. This is called fatigue failure. Fatigue results in brittle fracture with no gross deformation. The $\sigma-N$ curves, known as *Woehler curves*, are used to determine fatigue strength of a material for definite life. The *fatigue ratio* is defined as

$$\text{Fatigue ratio} = \frac{\text{Fatigue strength}}{\text{Yield point}} \approx 0.35$$

Fatigue properties of a material are affected by corrosion, surface finish, temperature, microstructure, residual stresses, heat treatment, and stress concentration. For steels having the same strength, fatigue life with coarse pearlite is lower than that with spheroidal microstructure because of the shape of carbide particles. Rounded carbides have lower stress concentrating effects which result in longer life.

Shot peening is a cold working process to improve the fatigue resistance of metal by surface compression. Small steel shot is blasted against the surface at high velocity; the result being small indications which create a slight plastic flow of the surface metal. This method produces compressive stresses, which offset the effect of tensile stresses within the metal.

When cyclic stresses are applied on a material, the proportional limit varies according to the direction of applied stress. During plastic deformation, the yield strength of material increases in the direction of plastic flow, but if the stress is applied in the opposite direction, plastic deformation begins at a lower yield stress than that of the initial yield point. This phenomenon is called *Bauschinger effect*[21]. Because of the lower yield stress in the reverse direction of loading, the phenomenon is also known as *Work softening* or *strain softening.*

10.13.4 Creep

Creep is defined as the time-dependent permanent deformation that occurs under stress at elevated temperature. Creep characteristics of a material is determined by applying a constant stress to a heated specimen of the material. The specimen stretches elastically by small amount. The strain is measured as a function of time that gives the creep curve.

The combined influence of applied stress and temperature on the creep rate and rupture time follows the *Arrhenius law*[22]:

$$\text{Creep rate} = A \exp\left(-\frac{B}{T}\right)$$

where T is temperature and A, B are constants.

[21]Named after the German engineer Johann Bauschinger.

[22]The equation was first proposed by the Dutch chemist J. H. van't Hoff in 1884. The Swedish chemist Svante Arrhenius provided a physical justification in 1889.

10.13.5 Hardness

Hardness of engineering materials can be measured by various of scales and tests. Hardness number has no intrinsic significance; it cannot be used as in calculations. The tests are discussed in the following subsections.

10.13.5.1 Moh's scale *Moh's scale* can be used to estimate or compare the hardness of materials. It scales the material hardness from 1 (talc) to 10 (diamond).

10.13.5.2 Brinell Test In Brinell test[23], a standard hardened ball of diameter D is pressed into the surface of the specimen by gradually applying load P for a definite period [Fig. 10.21]. The impression of the ball so obtained is measured by a microscope and the *Brinell Hardness Number* (BHN) is found by the following relationship:

$$\mathrm{BHN} = \frac{P}{\pi D \times \text{Depth of indentation}}$$

If d is the diameter of indentation, the depth of indentation is calculated as

$$\begin{aligned} x &= \frac{D}{2} - \sqrt{\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2} \\ &= \frac{D - \sqrt{D^2 - d^2}}{2} \end{aligned}$$

Therefore,

$$\mathrm{BHN} = \frac{2P}{\pi D \left(D - \sqrt{D^2 - d^2}\right)}$$

The ball is made of steel 10 mm for upto 500 BHN, and tungsten carbide ball for upto 650 BHN. Three types of loads (500 kg, 1500 kg, 3000 kg) are used.

Brinell[24] test leaves a relatively large impression, and it is limited to heavy sections. Thickness of test piece should not be less than 8 times the depth of indentation. Therefore, lesser ball diameter and loads should be used for thin specimens.

The Brinell test is generally suitable for materials of low to medium hardness. Ultimate tensile strength and surface endurance strength of steel are related to its BHN in MPa as

$$\begin{aligned} \sigma_{ut} &= 3.5 \times \mathrm{BHN} \\ \sigma_{es} &= 1.75 \times \mathrm{BHN} \end{aligned}$$

10.13.5.3 Vickers Test *Vickers hardness* is carried out with 136° diamond pyramid of four sides (angle between opposite faces) and loads ranging from 1–120 kg [Fig. 10.22].

[23]Brinell test was conceptualized by Swedish engineer Johan August Brinell in 1900.
[24]Brinelling is a term used to describe permanent indentation on a surface between contacting bodies, such as ball bearing indenting a flat surface under dynamic loads.

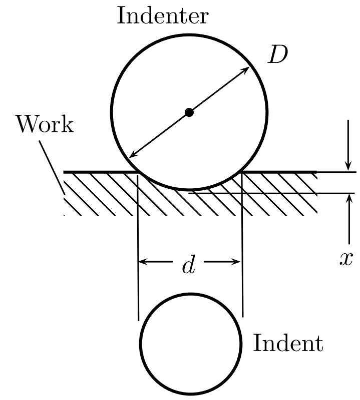


Figure 10.21 | Indentation in Brinell test.

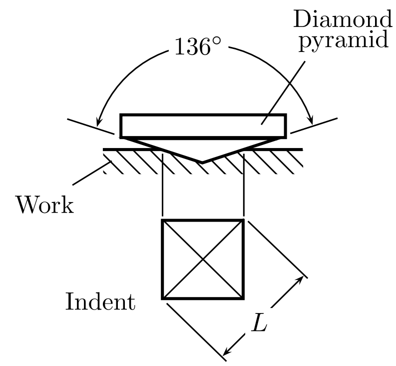


Figure 10.22 | Vickers test.

The diagonal of this indented square (L) is measured and area of the pyramidal surface is calculated as

$$\begin{aligned} A &= \frac{L/\sqrt{2}}{\cos\left((180-136)/2\right)^\circ} \times \frac{L}{\sqrt{2}} \\ &= \frac{L/\sqrt{2}}{\cos 22^\circ} \times \frac{L}{\sqrt{2}} \\ &= \frac{L^2}{1.8543} \end{aligned}$$

If load P is applied which makes the indentation with diagonal length L, then, *Vickers hardness number* (VHN) is determined as

$$\begin{aligned} \mathrm{VHN} &= \frac{P}{A} \\ &= \frac{1.854P}{L^2} \end{aligned}$$

This is the most accurate method of hardness testing.

10.13.5.4 Rockwell Test In *Rockwell test*, the indenter is pressed on the surface with a minor load and then a major load. The difference in the depth of penetration is

a measure of the hardness. This requires much smaller penetrator (diamond or steel cone of 120°). There are two common scales on a Rockwell testing machine: B-scale (soft) and C-scale (hard).

Rockwell hardness number, read directly from a dial on the testing machine, is expressed along with hardness scale. For instance, 60 HRC indicates hardness number 60 on the C scale.

Rockwell test is suitable for thinner sections and the hardest as well as softest material can be tested. This is the most widely used method because of its speed and is free of personal errors.

10.13.5.5 Shore Scleroscope In scleroscope, the hardness measurements are made by dropping a hard object, a small diamond pointed hammer which is closed in a glass tube, on the surface and observing the height of rebound. As it falls, its potential energy is transformed into kinetic energy which is passed on the surface when it strikes. Due to the elastic strain energy of the surface and damping capacity of stiffness of the material, the diamond pointed hammer rebounds, which is a measure of hardness. Scleroscope gives small impression, rapid test, with portable instrumentation, thus, is suitable for hardness testing of large-sized components.

10.13.5.6 Knoop Test The Knoop test[25] uses a diamond ground to a pyramid form that produces a diamond-shaped indentation with long and short diagonals, using loads ranging from 25 g to 5 kg. Due to the light loads, it is called a micro-hardness test [Fig. 10.23].

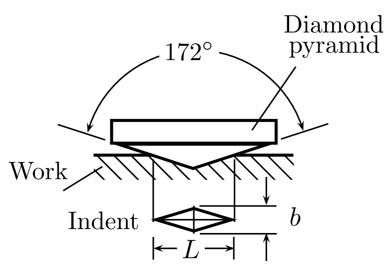


Figure 10.23 Knoop hardness test.

Let L and b $(= L/7.11)$ be the long and short diagonals of the indent. The uncovered projected area of indentation is determined as

$$A = Lb - \times \frac{1}{2} Lb$$
$$= \frac{L^2}{14.2}$$

[25]The test was developed by Frederick Knoop and colleagues at the National Bureau of Standards of the USA in 1939.

Knoop hardness number is calculated by dividing the load with uncovered projected area:

$$\text{HK} = \frac{14.2P}{L^2}$$

Indentation of Knoop hardness test is very small, in the range of 0.01–0.10 mm, therefore, surface preparation is very important. It is mostly used for testing micro-hardness of small parts, thin sections, or case depth work, and brittle materials.

The above mentioned tests and their criteria are summarized in Table 10.5.

Table 10.5 Hardness test and criteria

Test	Criteria
Moh's scale	Comparative hardness
Brinell test	Curved surface area
Vickers test	Area of pyramidal surface
Rockwell test	Depth of impression
Shore scleroscope	Rebound
Knoop test	Uncovered projected area

10.14 ENGINEERING METALS

By virtue of the iron content, engineering metals are broadly grouped into two kinds:

1. *Ferrous Materials* Ferrous materials are those in which iron (Fe) is the principle constituent.
2. *Non-Ferrous Materials* Materials other than ferrous materials are known as non-ferrous materials.

If materials are hard to form, components with these materials are fabricated by casting, thus, they are called *cast alloys*. If material can be deformed, they are known as *wrought alloys*.

10.14.1 Ferrous Materials

Ferrous materials are produced in larger quantities than any other metallic material. One main drawback of ferrous alloys is their corrosion. Other disadvantages include relatively high density (≈7800 kg/m^3), and comparatively low electrical and thermal conductivities.

The main alloying element in ferrous metals is carbon (C) which predominantly affects properties of the material. Ferrous alloys with less than 2.14% C are termed as *steels*, while those with higher carbon

content are termed as *cast irons*. Based on the amount of alloying additions, steels are of two kinds: (plain) carbon steels and alloy-steels, described as follows.

10.14.1.1 Plain Carbon Steels Mechanical properties of plain carbon steels are very sensitive to carbon content. Therefore, steels are classified based on the carbon percentage:

1. *Low Carbon Steels* *Low-carbon steels* (%C < 0.3) are soft steels, also called *mild steels*. Carbon present in these alloys is limited, and is not enough to strengthen these materials by heat treatment; hence these alloys are strengthened by cold working process. Their microstructure consists of ferrite and pearlite, and these alloys are thus, ductile combined with high toughness. These metals are easily machinable and weldable.

   Typical applications of mild steels include bolts, nuts, sheets, plates, tubes, wire fences, automobiles, body sheets, forged parts, tinned plates, fan blades, etc.

2. *Medium Carbon Steels* *Medium carbon steels* ($0.3 <$ %C < 0.6) are stronger than low carbon steels. These alloys can be heat treated to improve their strength. Also, these alloys possess better machinability.

   Medium carbon steels are used for rail road equipments, agriculture equipments, axles, chisels, punches, files, cutting tools, shafts, connecting rods, spindles, gears, turbine buckets, steerings arms, clutch discs, wrenches, etc.

3. *High Carbon Steels* *High carbon steels* (%C > 0.6) are the strongest and hardest of the plain carbon steels, and of course, their ductility is very limited. These are heat treatable, and mostly used in hardened and tempered conditions. They possess very high wear resistance, and are capable of holding sharp edges.

   High carbon steels are used for hand tools, chisels, punches, files, cutting tools, shafts, connecting rods, spindles, gears, turbine buckets, steering arms, clutch discs, wrenches, leaf springs, etc.

10.14.1.2 Alloy Steels Steel is truly an alloy of Fe and C. However, all steels are not called alloy steels. The term refers to steels with other alloying elements in addition to the carbon. Properties of some important alloy steels are discussed as follows:

1. *Free Cutting Steels* Free cutting steels are characterized by good machinability rendered by the sulphur (0.08–0.35%) and lead content. Sulphur present in the form of manganese sulphide (MnS) inclusions improve machinability by causing the formation of a broken chip and by providing a built-in lubricant. Mechanical properties increase with increasing carbon concentration (0.15–0.3%).

2. *Low Alloy Steels* These steels are characterized by a micro-structure consisting of fine grain ferrite as one phase and a hard second phase of martensite and austenite. Alloying elements in these steels are limited to 8%, in which the carbon can be added upto 0.3%. These are used for automobile bodies, bridges, building construction, etc.

3. *Stainless Steels* These steels are called *stainless* because in the presence of oxygen these steels develop a thin hard adherent film of chromium oxide that protects the metal from corrosion. This protective film builds up again if surface is scratched. For this, minimum chromium required is 10–20%. Higher the carbon content, lower is the resistance to corrosion, because carbon combines with chromium in the steel and forms chromium carbide, and reduced availability of chromium lowers the passivity of the steel. However, chromium carbide promotes galvanic corrosion.

4. *High Speed Steels* High speed steels (HSS) are used for high speed cutting tools. These are of two types: molybdenum type (M-series, 10% Mo) and tungsten type (T-series, 12-18% W). M-series steels generally have higher abrasive resistance than T-series steels. HSS can be coated with titanium nitride and titanium carbide for more wear resistance. These steels contain tungsten ($\approx$18%) chromium ($\approx$4%) and vanadium ($\approx$1%) in descending order of percentage composition.

5. *Invar* The nickel-iron alloy Invar[26] contains 36% nickel, and possesses the lowest thermal expansion among all metals and alloys in the range from room temperature upto approximately 230°C. The alloy is ductile and easily weldable, and machinability is similar to austenitic stainless steel. It does not suffer from stress corrosion cracking. Most bimetallic strips have *invar* as one metal because of its low coefficient of thermal expansion and yellow brass as other metal for low temperature or a nickel alloy for higher temperature.

10.14.1.3 Cast Irons Cast irons contain 2–4.5% carbon and 3.5% silicon. Alloys with this carbon content melt at lower temperatures than steel, therefore, casting is the most used fabrication technique for these alloys. Weldability of cast iron is quite low hence brazing, arc-welding or gas welding with special electrodes are used for fabrication.

[26]Invented in 1896 by Swiss scientist Charles Edouard Guillaume, for which he won Nobel Prize in Physics in 1920.

The metastable phase of cementite can readily decompose to form soft ferrite and graphite. The formation of graphite (*graphitization*) can be controlled by modifying composition by addition of the silicon and rate of cooling. Thus, cast irons are classified according to their solidification morphology from the eutectic temperature:

1. Gray Cast Iron When molten cast iron is slowly cooled, its cementite decomposes. Alloying with Si (1–3%) is responsible for decomposition of cementite, which also increases fluidity. The resulting structure contains graphite largely in the form of flakes. This is called *gray cast iron* because its fracture path is along the graphite flakes of gray appearance.

   Graphite flakes act as stress raisers, resulting in negligible ductility, weak in tension, though strong in compression. However, the presence of graphite flakes render vibration damping property by means of internal friction.

2. Wrought Iron Shape of graphite flakes of gray cast iron is changed into nodular form by small addition of magnesium (Mg) or cerium (Ce) to the molten metal prior to pouring. This permits the material to be somewhat ductile and shock resistant. The resulting structure is called *wrought iron* because during the process of its manufacturing it is forged or wrought with large hammers to combine the slag with iron.

   Melting point of wrought iron is very high due to very low percentage of carbon. Hence, wrought iron cannot be cast and heat treated. Due to inherent properties, wrought iron is also called ductile or *nodular cast iron*.

   Due to better corrosion resistance and fatigue resistance, wrought irons are used in pumps, compressors, steel mill rolls, connecting rods, crank shafts, crane hooks, engine bolts, gears, sheet metals dies, etc. Wrought iron is the only ferrous metal that contains siliceous slag (1–3%). During rolling, slag particles get elongated along the direction of rolling, increasing the strength, fatigue strength and corrosion resistance.

3. White Cast Iron *White cast iron* is obtained either by cooling gray cast iron rapidly or by adjusting the composition by keeping the carbon and silicon contents within 2.5% and 1.5%, respectively. These conditions do not permit decomposition of cementite. This is called *white cast iron* because of its white crystalline appearance of the fracture surface.

   Due to the presence of large amounts of iron carbide instead of graphite, the metal is very hard, wear resistant and brittle. Rail wheels are made by chilled castings in which white cast iron is obtained at rim.

4. Malleable Cast Iron *Malleable cast iron* is obtained by annealing white cast iron in an atmosphere of carbon monoxide and carbon dioxide between 800 to 900°C for several hours, during which cementite decomposes into iron and graphite. The graphite exists as cluster or rosettes in a ferrite or pearlite matrix. Consequently, malleable iron has a structure similar to that of nodular iron which promotes ductility, strength and shock resistance, and hence is called *malleable*.

   Malleable cast iron is used for automobile crank shafts, pipe fittings, sprockets, rail roads, conveyor chain links, gear cases, universal joint yokes, etc.

10.14.2 Non-Ferrous Materials

Non-ferrous materials have specific advantages over ferrous materials. They can be fabricated with ease and offer relatively low density, and high electrical and thermal conductivities. This section introduces some typical non-ferrous metals and their alloys of commercial importance.

10.14.2.1 Aluminium Alloys Aluminium alloys are characterized by low density (2700 kg/m^3), high thermal and electrical conductivities, and good corrosion resistant. Aluminium has FCC crystal structure, therefore, its alloys are ductile even at low temperatures. However, their low melting point (660°C) restricts their use at elevated temperatures.

Duralumin is an alloy of 94% Al and 4% Cu (and 0.5% Mn + 0.5% Mg) is used in screw machining, rivets, and aerospace industry. Important application of aluminium alloys include heat exchangers, light reflectors, cooking utensils, aircrafts, fuel and air lines, fuel tanks, rivets, wires, aircraft wheels, aircraft pump parts, water cooled engine cylinder blocks, etc. Aluminium with 11% silicon is used for making engine pistons by die casting technique in which silicon improves fluidity.

10.14.2.2 Copper Alloys Unalloyed Cu is soft, ductile (difficult to machine) and has virtually unlimited capacity for cold work. Copper has two main alloys; brass (Cu + Zn) and bronzes (Cu + Sn). Brass has higher strength than copper and is less expensive. Strength increases upto 40% zinc, but ductility, corrosion resistance decreases. Hence, upto 30% zinc is used in brass. A small amount of lead (3%) is added to increase machinability. Addition of sulphur gives a greatly increased cutting ability. Tellurium can be used as an alternative to sulphur to obtain the same result.

Brass and bronze are used for making gaskets, springs, firing pins, bushings, diaphragms, automobile radiators, lamp fixtures, clutch disks, fuse clips, etc. Applications of some important copper alloys are enlisted as follows:

1. *Gun metal* (90% Cu + 10% Sn) is used in boiler valves and mountings.

2. *Phosphor bronze* (70% Cu + 20% Sn + 1% P) is used in worm gears, springs, wire ropes.

3. *Gliding metal* (95% Cu + 5% Zn) is used in coins, medals.

4. *Muntz metal*[27] (60% Cu + 40% Zn) is corrosion-resistant alloy used for condenser tubes.

5. *Tin bronze* or *phosphor bronze* (phosphorus upto 0.3%) is used as deoxidizer.

6. *Cupronickel* (70% Cu + 30% Ni) is used in coins, utensils, heat exchanger tubes.

7. *German silver* (65% Cu + 23% Zn + 12% Ni) has a color resembling silver.

8. *Constanton* (58.5% Cu + 40% Ni +1.5% Mn) is used in rheostats, thermocouple, heating devices, etc.

10.14.2.3 Magnesium Alloys The most sticking property of magnesium (Mg) is its lowest density (1745 kg/m^3) among all structural metals. Mg has HCP structure, thus, its alloys are difficult to form at room temperatures. Hence, Mg alloys are usually fabricated by casting or hot working. Magnesium alloys find applications in hand-held devices, like saws, tools, automotive parts like steering wheels, seat frames, electronics like casing for laptops, cell phones.

10.14.2.4 Titanium Alloys Titanium (Ti) and its alloys are of relatively low density, high strength and have very high melting point. At the same time, they are easy to machine and forge. However, major limitation is its chemical reactivity at high temperatures, which necessitates special techniques to extract it. Thus, these alloys are expensive. Titanium alloys possess excellent corrosion resistance in diverse atmospheres, and wear properties. These are used in space vehicles, airplane structures, surgical implants, and petroleum and chemical industries.

10.14.2.5 Refractory Metals Refractory metals (e.g. Nb, Mo, W and Ta) possess very high melting points. They also possess high strength and high elastic modulus. Refractory metals are essentially used in space vehicles, x-ray tubes, welding electrodes, and where there is a need for corrosion resistance.

10.14.2.6 Noble Metals These are eight noble metals: Ag, Au, Pt, Pa, Rh, Ru, Ir and Os. All these possess some common properties such as expensive, soft and ductile, oxidation resistant. Ag, Au and Pt are used extensively in jewelry; alloys of Ag and Au are employed as dental restoration materials; Pt is used in chemical reactions as a catalyst and in thermocouples.

10.14.2.7 Other Non-Ferrous Alloys *Monel*[28] is primarily composed of nickel (upto 67%) and copper, with some iron and other trace elements. It is very difficult to machine as it work-hardens very quickly. It is resistant to corrosion and acids, and some alloys can withstand fire in the presence of pure oxygen. Monel's corrosion resistance makes it ideal for marine applications such as piping systems, pump shafts, seawater valves, trolling wire, and strainer baskets. It is also used for handling H_2SO_4, HCl.

Nichrome is an alloy of Ni and Cr, primarily used as an electric resistance heating element in wire and strip forms. Chromium is quite soluble in nickel. The 90/10 nickel chromium alloy is commonly used in thermocouples, in conjunction with a 95/5 Ni/Al alloy. This combination is called *chromel-alumel*.

Babbitt[29], also called *bearing metal*, is any of several alloys used to provide the bearing surface in a plain bearing. A wide variety of Babbitt alloys exist. Some common compositions are: (90% Sn, 10% Cu), (89% Sn, 7% Sb, 4% Cu), (80% Pb, 15% Sb, 5% Sn), where Sb is the symbol of antimony.

10.15 CERAMICS

Ceramics are inorganic and non-metallic materials, which are hard, abrasion resistant, brittle, chemically inert, and poor conductors of heat.

10.15.1 Types of Ceramics

Important types of ceramics are discussed as follows:

1. Glasses Glasses are non-crystalline silicates containing oxides, usually CaO, Na_2O, K_2O and Al_2O_3. Typical property of glasses that is important in engineering applications is their optical transparency and ease in fabrication. Glasses are mainly used in containers, windows, mirrors, etc.

2. Clay Products Clay is an inexpensive ingredient, found naturally in great abundance. Clay products are mainly of two kinds: structural products

[27] Named after George Fredrick Muntz, a metal-roller of Birmingham, who commercialized the alloy, following his patent of 1832.

[28] Monel is a trademark of Special Metals Corporation for a series of nickel alloys.

[29] The original Babbitt metal was invented in 1839 by Isaac Babbitt.

(bricks, tiles, sewer pipes) and white-wares (porcelain, chinaware, pottery, etc.).

3. *Refractories* Refractories are described by their capacity to withstand high temperatures without melting or decomposing; and their inertness in severe environments. Thermal insulation is also an important functionality of refractories.

4. *Abrasive Ceramics* Abrasive ceramics are used to grind, wear, or cut away other softer material. Diamond, silicon carbide, tungsten carbide, silica sand and corundum (crystalline form of aluminium oxide) are some typical examples of abrasive ceramic materials.

5. *Cements* Cement, plaster of Paris and lime come under this cement group of ceramics. Upon mixing with water, these materials form slurry, which sets subsequently and hardens. The cementituous bond develops at room temperature.

6. *Advanced Ceramics* These are newly developed and manufactured in limited range for specific applications. Usually their electrical, magnetic and optical properties and combination of properties are exploited. Their typical applications include heat engines, ceramic armors, electronic packaging, optical fiber communication, etc.

10.15.2 Applications

Some typical ceramics and respective applications are discussed as follows:

1. *Alumina* *Alumina* (Al_2O_3) is used in many applications, such as to contain molten metal, where material is operated at very high temperatures under heavy loads, as insulators in spark plugs, and in dental and medical use.

2. *Aluminium Nitride* Because of electrical insulation but high thermal conductivity, aluminium nitride is used in electronic applications, such as electrical circuits operating at a high frequency.

3. *Silica* Silica (SiO_2) is an essential ingredient in many engineering ceramics, thus, is the most widely used ceramic material. Silica-based materials are used in thermal insulation, abrasives, laboratory glassware, etc. It also finds application in communications media as integral part of optical fibers.

4. *Silicon Carbide* Silicon carbide (SiC) is known as one of the best ceramic material for very high resistance to oxidation at high temperatures (SiC forms a protective skin of SiO_2). It finds many applications, such as coatings on other material for protection from extreme temperatures, abrasive material, as reinforcement in many metallic and ceramic based composites.

10.16 POLYMERS

Industrially, polymers are classified into two main classes: plastics and elastomers, explained as follows:

1. *Plastics* Plastics are either natural or synthetic organic resins, and are processed by forming or molding into shapes. They have a wide range of desired properties, such as light weight, wide range of colors, low thermal and electrical conductivity, less brittleness, good toughness, good resistance to acids, bases and moisture, high dielectric strength (use in electrical insulation), etc.

2. *Elastomers* *Elastomers*, also known as rubbers, are polymers which can undergo large elongations under load, at room temperature, and return to their original shape when the load is released. There are a number of man-made elastomers in addition to natural rubber. These consist of coil-like polymer chains that can reversibly stretch by applying a force.

10.17 COMPOSITES

Composites are made by combining two distinct engineering materials; one is called *matrix* that is continuous and surrounds the other phase, that is, dispersed phase. The properties of composites are a function of the properties of the constituent phases, their relative amounts, and size-and-shape of dispersed phase. The matrix material acts as a medium to transfer and distribute the load to the fibers, which carry most of the applied load. The matrix also provides protection to fibers from external loads and atmosphere.

For uni-axially aligned continuous fibers, the effective value of tensile strength and elasticity of the material can be determined using the rule of mixtures' formula:

$$\sigma = \sigma_f x_f + \sigma_m (1 - x_f)$$
$$E = E_f x_f + E_m (1 - x_f)$$

where x_f is the volume fraction of the fibres, f and m denote properties of fibers and matrix, respectively.

IMPORTANT FORMULAS

Atomic Structure

$$e = 1.6\times10^{-19}\ \text{C}$$
$$\text{a.m.u.} = 1.6605\times10^{-27}\ \text{kg}$$
$$N_A = 6.0223\times10^{23}/\text{mol}$$

Crystal Structure

1. Atoms in a cell
$$N = N_{interior} + \frac{N_{face}}{2} + \frac{N_{corner}}{8}$$
2. Atomic packing factor
$$\text{APF} = \frac{\text{Volumes of all atoms}}{\text{Unit cell volume}}$$
3. Density
$$\rho = \frac{nZ}{V_cA}$$
4. Boltzmann distribution:
$$\frac{n}{N} = e^{\frac{-Q}{kT}}$$

Characteristics of unit cells

Cell	N	CN	a/R	APF
Simple	1	6	$\frac{4}{\sqrt{4}}$	0.52
BCC	2	8	$\frac{4}{\sqrt{3}}$	0.68
FCC	4	12	$\frac{4}{\sqrt{2}}$	0.74
HCP	6	12	–	0.74

Grain Size Measurement

$$G = -2.9542 + 1.4427\ln n_a$$
$$n = 2^{G-1}$$
$$D = \frac{1}{100}\sqrt{\frac{645}{2^{G-1}}}$$
$$R_c \propto e^T$$

Stress–Strain Relationship

1. Hooke's law
$$\sigma = E\varepsilon$$
2. Holloman-Ludwig equation
$$\sigma = K\varepsilon^n$$
3. Ludwig equation
$$\sigma = \sigma_y + K\varepsilon^n$$

Multiphase Structure

1. Gibbs free energy
$$G = H - TS$$
2. Gibbs phase rule
$$f = c - \phi + 2$$
3. Lever rule
$$\%\text{Liquid} = \frac{\text{C}_1\text{C}_0}{\text{C}_1\text{C}_2}\times100$$
4. Invariant reactions

Eutectic	$L_1 \rightarrow S_1+S_2$
Eutectoid	$S_1 \rightarrow S_2+S_3$
Peritectic	$L_1+S_1 \rightarrow S_2$
Peritectoid	$S_1+S_2 \rightarrow S_3$
Monotectoid	$L_1 \rightarrow L_2+S_1$

Iron–Carbon Phase

1. Maximum carbon content

Phase	C%	T°C
α (BCC)	0.022	727
γ (FCC)	2.140	1130
δ (BCC)	0.100	1493

2. Invariant reactions
   (a) Eutectoid reaction
   $$\gamma \rightleftharpoons \alpha + \text{Fe}_3\text{C}$$
   (b) Peritectic reaction
   $$\text{L} + \delta \rightleftharpoons \gamma$$
   (c) Monotectic reaction
   $$\text{L} \rightleftharpoons \text{L} + \gamma$$
   (d) Eutectic reaction
   $$\text{L} \rightleftharpoons \gamma + \text{Fe}_3\text{C}$$

Properties of Materials

1. Stress–strain curve
$$\sigma = K\epsilon^n$$
2. Modulus of resilience
$$U_r = \frac{1}{2}\sigma_y\varepsilon_y = \frac{{\varepsilon_y}^2}{2E}$$
3. Toughness
$$U_t = \begin{cases} \frac{\sigma_{yp}+\sigma_{ut}}{2}\varepsilon_f & \text{Ductile} \\ \frac{2}{3}\sigma_{ut}\varepsilon_f & \text{Brittle} \end{cases}$$
4. Fatigue
$$\text{Fatigue ratio} = \frac{\sigma_e}{\sigma_y} \approx 0.35$$
5. Creep
$$\text{Creep rate} = A\exp\left(-\frac{B}{T}\right)$$
6. Hardness tests
$$\text{BHN} = \frac{2P}{\pi D\left(D-\sqrt{D^2-d^2}\right)}$$
$$\sigma_{ut} = 3.5\times\text{BHN}$$
$$\sigma_{es} = 1.75\times\text{BHN}$$
$$\text{VHN} = \frac{1.854P}{L^2}$$
$$\text{HK} = \frac{14.2P}{L^2}$$

Test	**Criteria**
Moh's scale	Comparative hardness
Brinell test	Curved surface area
Vickers test	Pyramidal surface
Rockwell test	Depth of impression
Sceleroscope	Rebound
Knoop test	Uncovered projected area

Composites

$$\sigma = \sigma_f x_f + \sigma_m(1-x_f)$$
$$E = E_f x_f + E_m(1-x_f)$$

SOLVED EXAMPLES

1. Determine the volume and theoretical density of FCC unit cell of an atom having atomic radius 0.1 nm and atomic weight 60.0 g/mol. Take $N_A = 6.023 \times 10^{23}$ atoms/mol.

Solution. Given that $R = 0.1 \times 10^{-9}$ m. The volume of FCC unit cell is given by

$$\begin{aligned} V_c &= 16\sqrt{2}R^3 \\ &= 0.0226 \text{ nm}^3 \end{aligned}$$

Number of atoms per unit cell for FCC structure are $n = 4$; Molar weight $M = 60$ g/mol. Therefore, density of unit cell is given by

$$\begin{aligned} \rho &= \frac{n \times (M/N_A)}{V_c} \\ &= 17.63 \text{ g/cm}^3 \end{aligned}$$

2. Calculate the atomic packing factor for an FCC unit cell.

Solution. A face centered cubic (FCC) crystal consists of an atom at each cube corner and an atom in the center of each cube face. In this structure, the atom or ion spheres touch one another across a face diagonal, therefore, the cube edge length a and the atomic radius R are related as

$$a = \frac{4R}{\sqrt{2}}$$

The number of atoms in FCC unit cell is 4. Therefore, atomic packing factor is found as

$$\begin{aligned} \text{APF} &= \frac{4 \times \left(4 \times \pi R^3/3\right)}{a^3} \\ &= \frac{4 \times 4 \times \pi R^3/3}{\left(4R/\sqrt{2}\right)^3} \\ &= 0.740 \end{aligned}$$

3. Calculate the atomic packing factor for an BCC unit cell.

Solution. A body centered cubic (BCC) crystal contains atoms at all eight corners and a single atom is at the center of the cube. In this structure, the central atoms and corner atoms touch one another along cube diagonals, therefore, unit cell length a and atomic radius are related as

$$a = \frac{4R}{\sqrt{3}}$$

The number of atoms in BCC is 2. Therefore, atomic packing factor is found as

$$\begin{aligned} \text{APF} &= \frac{2 \times \left(4 \times \pi R^3/3\right)}{a^3} \\ &= \frac{4 \times 4 \times \pi R^3/3}{\left(4R/\sqrt{3}\right)^3} \\ &= 0.68 \end{aligned}$$

4. In an measurement with $100\times$ magnification factor on metal specimen, 65 grains per square inch are observed. Determine the ASTM grain size number of the specimen.

Solution. Given that the number of grains per square inch $N = 65$, therefore, the ASTM grain size number n is given by

$$\begin{aligned} N &= 2^{n-1} \\ \ln N &= (n-1)\ln 2 \\ n &= \frac{\ln N}{\ln 2} + 1 \\ &= 7.02 \end{aligned}$$

5. Lead (Pb) has FCC structure with inter-atomic distance of 3.499 Å. Calculate the number of atoms per square millimeter on plane (100) of the structure.

Solution. Given that

$$2R = 3.499\text{Å}$$

Size of the FCC unit cell is

$$\begin{aligned} a &= \frac{4R}{\sqrt{2}} \\ &= \frac{2 \times 3.499}{\sqrt{2}} \\ &= 4.948\text{Å} \\ &= 4.948 \times 10^{-10} \text{ m} \\ &= 4.948 \times 10^{-7} \text{ mm} \end{aligned}$$

The plane (100) is the face of the FCC which has $1 + 4 \times 1/4 = 2$ atoms. Therefore, planar density is

$$\begin{aligned} \rho &= \frac{2}{a^2} \\ &= \frac{2}{(4.948 \times 10^{-7})^2} \\ &= 8.169 \times 10^{12} \text{ atoms/mm}^2 \end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. During heat treatment of steel, the hardness of various structures in increasing order is

(a) martensite, fine pearlite, coarse pearlite, spheroidite
(b) fine pearlite, coarse pearlite, spheroidite, martensite
(c) martensite, coarse pearlite, fine pearlite, spheroidite
(d) spheroidite, coarse pearlite, fine pearlite, martensite

(GATE 2003)

Solution. Martensite is the hardest phase. Alloys containing pearlitic micro-structure have greater strength and hardness than those with spherodite because there is less area of boundary per unit volume in spherodite.

Ans. (d)

2. Hardness of steel greatly improves with

(a) annealing (b) cyaniding
(c) normalizing (d) tempering

(GATE 2003)

Solution. Only cyaniding is used for increasing hardness of steel, and other processes are used to relieve internal stresses, increase ductility and reduce hardness.

Ans. (b)

3. From the lists given below, choose the most appropriate set of heat treatment process and the corresponding process characteristics:

Process	Characteristics
P. Tempering	1. Austenite is converted into bainite
Q. Austempering	2. Austenite is converted into martensite
R. Martempering	3. Cementite is converted into globular structure
	4. Both hardness and brittleness are reduced
	5. Carbon is absorbed into the metal

(a) P-3, Q-1, R-5 (b) P-4, Q-3, R-2
(c) P-4, Q-1, R-2 (d) P-1, Q-5, R-4

(GATE 2004)

Solution. Tempering reduces both hardness and brittleness. Austempering is used to convert austenite into bainite. Martempering is used to convert austenite into martensite.

Ans. (c)

4. The percentage of carbon in gray cast iron is in the range of

(a) 0.25 to 0.75% (b) 1.25 to 1.75%
(c) 3 to 4% (d) 8 to 10%

(GATE 2004)

Solution. Generally, cast iron contains 2–4.5% C, 3.5% Si. From the given options, the first two and the last option are impossible, thus (c) is the closest to answer.

Ans. (c)

5. The main purpose of spheroidizing treatment is to improve

(a) hardenability of low carbon steels
(b) machinability of low carbon steels
(c) hardenability of high carbon steels
(d) machinability of high carbon steels

(GATE 2006)

Solution. Spherodizing is used especially for improving machinability of high carbon steels, for making ball bearings, and ball races.

Ans. (b)

6. Match the items in Column I and Column II, and select the correct option.

Column I	Column II
P. Charpy test	1. Fluidity
Q. Knoop test	2. Microhardness
R. Spiral test	3. Formability
S. Cupping test	4. Toughness
	5. Permeability

(a) P-4, Q-5, R-3, S-2
(b) P-3, Q-5, R-1, S-4
(c) P-2, Q-4, R-3, S-5
(d) P-4, Q-2, R-1, S-3

(GATE 2006)

Solution. Charpy test is used to measure toughness. Knoop test is used to measure the

hardness. Spiral test is used to measure fluidity. Cupping test is used measure formability.

Ans. (d)

7. If a particular Fe-C alloy contains less than 0.83% carbon, it is called

(a) high speed steel
(b) hypoeutectoid steel
(c) hypereutectoid steel
(d) cast iron

(GATE 2007)

Solution. Fe-C steel alloys with less than 0.83% carbon are called hypoeutectoid steels, whereas those with more than 0.83% are called hypereutectoid steels.

Ans. (b)

8. The effective number of lattice points in the unit cell of simple cubic, body centered cubic, and face centered cubic space lattices, respectively, are

(a) 1, 2, 2 (b) 1, 2, 4
(c) 2, 3, 4 (d) 2, 4, 4

(GATE 2009)

Solution. The total number of atoms per unit cell is given by

$$N = N_{interior} + \frac{N_{face}}{2} + \frac{N_{corner}}{8}$$

For simple cubic, BCC and FCC, the number of atoms per unit cell are 1, 2 and 4, respectively.

Ans. (b)

9. The material property that depends only on the basic crystal structure is

(a) fatigue strength (b) work hardening
(c) fracture strength (d) elastic constant

(GATE 2010)

Solution. Fatigue strength depends upon many factors, such as surface area, size. Work hardening depends upon the work done or strain in the material. Similarly, work hardening and fracture strength also depends on the basic crystal structure of the material. Elastic constants are determined by the basic crystal structure.

Ans. (d)

10. The crystal structure of austenite is

(a) body centered cubic
(b) face centered cubic
(c) hexagonal closed packed
(d) body centered tetragonal

(GATE 2011)

Solution. Ferrite (α-iron) experiences a polymorphic transformation to FCC austenite or γ-iron at 912°C. This austenite persists upto 1394°C.

Ans. (b)

11. During normalizing process of steel, the specimen is heated

(a) between the upper and lower critical temperature and cooled in still air
(b) above the upper critical temperature and cooled in furnace
(c) above the upper critical temperature and cooled in still air
(d) between the upper and lower critical temperature and cooled in furnace

(GATE 2012)

Solution. Normalizing involves heating of the material up to austenizing temperature range (A_3) and at somewhat faster cooling rate in still air.

Ans. (c)

12. For a ductile material, toughness is the measure of

(a) resistance to scratching
(b) ability to absorb energy up to fracture
(c) ability to absorb energy till elastic limit
(d) resistance to indentation

(GATE 2013)

Solution. Toughness (U_t) of a material is defined as its ability to absorb energy in the plastic range. It is measured as the work done per unit volume without causing it to rupture. This can be obtained as the area under the stress-strain curve, approximately of rectangular shape:

$$U_t = \begin{cases} (1/2)\,(\sigma_{yp} + \sigma_{ut})\,\varepsilon_f & \text{Ductile materials} \\ (2/3)\,\sigma_{ut}\varepsilon_f & \text{Brittle materials} \end{cases}$$

Ans. (b)

MULTIPLE CHOICE QUESTIONS

1. Decreasing grain size in a polycrystalline material

(a) increases yield strength and corrosion resistance
(b) decreases yield strength and corrosion resistance
(c) decreases yield strength but increases corrosion resistance
(d) increases yield strength but decreases corrosion resistance

2. In the grain-size determination using standard charts, the relation between the given size number n and the average number of grains N per square inch at a magnification of 100× is

(a) $N = 2^n$ (b) $N = 2^{n-1}$
(c) $N = 2^{n+1}$ (d) $N = 2^n + 1$

3. Chemicals attack atoms within grain boundaries preferentially because they have

(a) lower energy than those in the grains
(b) higher energy than those in the grains
(c) higher number of atoms than in the grains
(d) lower number of atoms than in the grains

4. The complete phase recrystallization and fine grain structure is obtained in casting, forging and rolled parts by

(a) recrystallization annealing
(b) normalizing
(c) spheroidizing
(d) austenizing

5. What is the movement of block of atoms along certain crystallographic planes and directions, termed as?

(a) Glide (b) Twinning
(c) Slip (d) Jog

6. Which one of the following statements is correct in the case of screw dislocations? ($\vec{b}$ = Burgers vector; $\hat{t}$ = Imaginary vector)

(a) $\vec{b}$ is perpendicular to $\hat{t}$
(b) $\vec{b}$ is parallel to $\hat{t}$
(c) $\vec{b}$ is inclined to $\hat{t}$
(d) $\vec{b}$ and $\hat{t}$ are non-coplanar and non-intersecting

7. Which one of the following is correct for 'Burgers vector' in screw dislocation?

(a) It is perpendicular to the dislocation line
(b) It is inclined to the dislocation line
(c) It is parallel to the dislocation line
(d) It is opposite to the dislocation line

8. Which one of the following defects is 'Schottky defect'?

(a) Vacancy defect
(b) Compositional defect
(c) Interstitial defect
(d) Surface defect

9. Surface imperfections which separate two orientations that are mirror images of one another is called

(a) stacking fault (b) grain boundary
(c) tilt boundary (d) twinned boundary

10. Materials which show direction-dependent properties are called

(a) homogeneous (b) viscoelastic
(c) isotropic (d) anisotropic

11. According to Gibbs phase rule, the number of degrees of freedom of an eutectic point in a binary system is

(a) 0 (b) 1
(c) 2 (d) 3

12. Which one of the following factors is more relevant to represent complete solubility of two metals in each other?

(a) Chemical affinity
(b) Valency factor
(c) Crystal structure factor
(d) Relative size factor

13. A reaction in which liquid and solid on cooling get converted into another solid is known as

(a) eutectoid reaction
(b) eutectic reaction
(c) peritectic reaction
(d) peritectoid reaction

14. The iron-carbon diagram and the TTT curves are determined under

(a) equilibrium and non-equilibrium conditions, respectively
(b) non-equilibrium and equilibrium conditions, respectively
(c) equilibrium conditions for both
(d) non-equilibrium conditions for both

15. During monotectic solidification, one liquid

(a) combines with one solid to form a second new solid
(b) solidified into two different solids
(c) forms one solid
(d) forms one solid and another liquid of different composition

16. Martensite is a super-saturated solution of carbon in

(a) α-iron (b) β-iron
(c) γ-iron (d) δ-iron

17. TTT diagram indicates time and temperature transformation of

(a) cementite (b) pearlite
(c) ferrite (d) austenite

18. Globular form of cementite in the structure of steel is obtained through

(a) normalizing (b) malleabilizing
(c) spheroidising (d) carbonizing

19. Pearlite phase in an iron-carbide phase diagram is

(a) eutectic phase
(b) hypoeutectic mixture
(c) eutectoidal mixture
(d) hypereutectic phase

20. Cast steel crankshaft surface is hardened by

(a) nitriding
(b) normalizing
(c) carburizing
(d) induction hardening

21. In case carburizing, carbon is introduced to form high carbon layer at the surface. The carbon is introduced in the form of

(a) graphite flakes (b) pearlite
(c) cementite (d) free carbon

22. Match List I with List II and select the correct option.

List I (Heat treatment)	List II (Effects)
A. Annealing	1. Refines grain structure
B. Nitriding	2. Improves the hardness of the whole mass
C. Martempering	3. Increases surface hardness
D. Normalizing	4. Improves ductility

(a) A-4, B-3, C-2, D-1
(b) A-1, B-3, C-4, D-1
(c) A-4, B-2, C-1, D-3
(d) A-2, B-1, C-3, D-4

23. 'Tempering' of quenched martensite steel is necessary to improve the

(a) hardness of the metal
(b) surface texture of the metal
(c) corrosion resistance of the metal
(d) ductility of the metal

24. Which of the following materials generally exhibits a yield point?

(a) Cast iron
(b) Annealed and hot-rolled mild steel
(c) Soft brass
(d) Cold-rolled steel

25. On completion of heat treatment, the resulting structure will have retained austenite if the

(a) rate of cooling is greater than the critical cooling rate
(b) rate of cooling is less than the critical cooling rate
(c) martensite formation starting temperature is above the room temperature
(d) martensite formation finishing temperature is below the room temperature

26. The alloying element mainly used to improve the endurance strength of steel materials is

(a) nickel (b) vanadium
(c) molybdenum (d) tungsten

27. Small percentage of boron is added to steel to

(a) increase hardenability

(b) reduce machinabililty
(c) increase wear resistance
(d) increase endurance strength

28. For the pipe fitting, such as elbow, tee, union, etc., which of the following is preferred?

(a) Pig iron
(b) Malleable iron
(c) Spheroidal graphite cast iron
(d) High carbon steel

29. Presence of hydrogen in steel causes

(a) reduced neutron absorption cross section
(b) improved weldability
(c) embrittlement
(d) corrosion resistance

30. Main alloying elements in high speed steel in order of increasing proportions are

(a) vanadium, chromium, tungsten
(b) tungsten, titanium, vanadium
(c) chromium, titanium, vanadium
(d) tungsten, chromium, titanium

31. Small amounts of which one of the following elements/pairs of elements is added to steel to increase its machinability?

(a) Nickel
(b) Sulphur and phosphorus
(c) Silicon
(d) Manganese and copper

32. Addition of vanadium in steel results in improvement of

(a) heat treatability by quenching
(b) hardenability
(c) fatigue strength
(d) resistance to oxidation at elevated temperature

33. For improving the strength of steel at elevated temperatures, which one of the following alloying element is used?

(a) Copper (b) Tungsten
(c) Aluminium (d) Zinc

34. Austempering is employed to obtain

(a) 100% martensitic structure
(b) 100% bainitic structure
(c) 50% martensitic and 50% bainitic structure
(d) 100% pearlitic structure

35. Which one of the following is the process to refine the grains of metal after it has been distorted by hammering or cold working?

(a) Annealing
(b) Softening
(c) Recrystallizing
(d) Normalizing

36. The pattern known as Widmanstätten structure is encountered in

(a) tempering (b) normalizing
(c) spheroidizing (d) annealing

37. Which one of the following elements is an austenite stabilizer?

(a) Chromium (b) Tungsten
(c) Nickel (d) Molybdenum

38. Which one of the following elements is a ferrite stabilizer?

(a) Nickel (b) Manganese
(c) Copper (d) Chromium

39. Some high speed steels have cobalt (Co) added to them in amounts ranging from 2 to 15%, since this element improves the

(a) cutting efficiency, especially at high temperature
(b) depth hardening ability of the HSS tool
(c) red hardness of the HSS tool
(d) grain structure of the HSS tool

40. For most brittle materials, the ultimate strength in compression is much larger than the ultimate strength in tension. This is mainly due to

(a) presence of flaws and microscopic cracks and cavities
(b) necking in tension
(c) severity of tensile forces as compared to compressive
(d) non-linearity of stress-strain diagram

41. All of the following statements about yield point phenomenon are true except

(a) dislocation motion is responsible for yield point

(b) presence of carbon in iron is responsible for yield point in steels
(c) Upper and lower yield points are found in many aluminium alloys
(d) Yield point indicates heterogeneous deformation

42. Plastic deformation is caused by

(a) stored elastic energy
(b) dislocation motion
(c) low value of modulus
(d) complex elastic stress

43. Upper and lower yield points are observed in

(a) all pure metals
(b) carbon steels
(c) brittle metals
(d) an α–β brass

44. Creep failure occurs due to

(a) gradual application of load at ordinary temperature
(b) a very rapid rate of loading at subzero temperatures
(c) formation of voids under a steady load at elevated temperature
(d) excessive work hardening

45. The Bauschinger effect is observed when

(a) tension is followed by compression
(b) compression is followed by tension
(c) material is subjected to reversing torsion
(d) all of the above

46. Match List I with List II and select the correct option.

List I (Material)	List II (Characteristics)
A. Perfectly elastic	1. Deformation at same stress level
B. Rigid, perfectly plastic	2. Linear behavior
C. Rigid, linear strain hardening	3. Approximation of for most of the materials
D. Elastic, linearly strain hardening	4. No recovery upon unloading

(a) A-1, B-2, C-3, D-4
(b) A-1, B-2, C-4, D-3
(c) A-2, B-1, C-3, D-4
(d) A-2, B-1, C-4, D-3

47. The volume of a meal specimen remains constant in the plastic deformation from initial diameter d_i to final diameter d_f. The true strain (ε) within the uniform elongation range can be expressed as

(a) $\ln(d_i/d_f)$
(b) $\ln(d_f/d_i)$
(c) $2\ln(d_i/d_f)$
(d) $2\ln(d_f/d_i)$

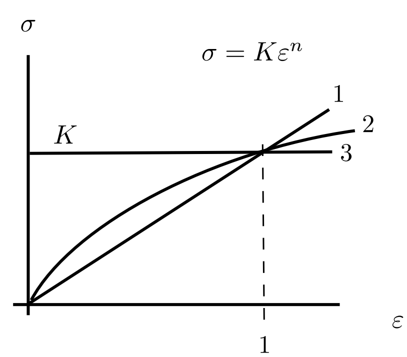
48. A typical true stress (σ) true strain (ε) curve for a metal specimen is found to be

$$\sigma = K\varepsilon^n$$

If during a tensile test, instability sets when the specimen begins to neck and cannot support the load because the cross-sectional area of the necked region is reducing, then the true strain shall be equal to

(a) 0
(b) n
(c) n^2
(d) 1

49. Three true stress (σ) -true strain (ε) curves are marked as 1, 2 and 3, as shown in the figure below



The correct sequence of the type of material that these curve indicate is

(a) perfect elastic, elastic, perfect plastic
(b) perfect plastic, elastic, perfect elastic
(c) elastic, perfect elastic, perfect plastic
(d) elastic, perfect plastic, perfect elastic

50. A test specimen is stressed slightly beyond the yield point and then unloaded. Its yield strength will

(a) decrease
(b) increase
(c) remains same

(d) becomes equal to ultimate tensile strength

51. A measure of Rockwell hardness is the

(a) depth of penetration of indenter
(b) surface area of indentation
(c) projected area of indentation
(d) height of rebound

52. A carbon steel having BHN 100 should have ultimate strength closer to

(a) 100 N/mm^2 (b) 200 N/mm^2
(c) 350 N/mm^2 (d) 1000 N/mm^2

53. When σ and E remain constant, the energy absorbing capacity of part subject to dynamic force is a function of

(a) length
(b) cross-section
(c) volume
(d) none of the above

54. Which of the following would you prefer for checking of the hardness of thin sections?

(a) Herbert cloud burst test
(b) Shore scleroscope
(c) Knoop hardness test
(d) Vickers hardness test

55. Match List I with List II and select the correct option.

List I	List II
A. Toughness	1. Moment area method
B. Endurance strength	2. Hardness
C. Resistance to abrasion	3. Energy absorbed before fracture in a tension test
D. Deflection in a beam	4. Fatigue loading

(a) A-4, B-3, C-1, D-2
(b) A-4, B-3, C-2, D-1
(c) A-3, B-4, C-2, D-1
(d) A-3, B-4, C-1, D-2

56. The property by which an amount of energy is absorbed by a material without plastic deformation is called

(a) toughness (b) impact strength
(c) ductility (d) resilience

57. The correct sequence of creep deformation in a creep curve in order of their elongation is

(a) steady state, transient, accelerated
(b) transient, steady state, accelerated
(c) transient, accelerated, steady state
(d) accelerated, steady state, transient

58. Resilience of a material becomes important when it is subjected to

(a) fatigue
(b) thermal stresses
(c) shock loading
(d) pure static loading

59. Nodular gray cast iron is obtained from the gray cast iron by adding a small amount of

(a) manganese (b) phosphorus
(c) magnesium (d) chromium

60. Addition of which of the following improves machining of copper?

(a) Sulphur (b) Vanadium
(c) Tin (d) Zinc

61. The percentage of phosphorus in phosphor bronze is

(a) 0.1 (b) 1
(c) 11.1 (d) 98

62. Invar is used for measuring tapes primarily due to its

(a) non-metallic properties
(b) high nickel content
(c) low coefficient of thermal expansion
(d) hardenability

63. Babbitt lining is used on brass/bronze to

(a) increase bearing resistance
(b) increase compressive strength
(c) provide anti-friction properties
(d) increase water resistance

NUMERICAL ANSWER QUESTIONS

1. An alloy consists of 90% of aluminium and 10% of copper by weight. The atomic weight of copper is 63.55 g/mol, and that of aluminium is 26.98 g/mol. Calculate the percentage of atoms of copper in the alloy.

2. In an measurement with 100× magnification factor on metal specimen, 50 grains per square inch are observed. What is the ASTM grain size number of the specimen? For the same specimen, how many grains per square inch will there be at a magnification factor of 80×?

3. A cylindrical specimen of steel having an original diameter of 10 mm is tensile tested to fracture and found to have an engineering fracture strength of 425 MPa. Its diameter at fracture is found to be 8.0 mm. Calculate the percentage reduction of area and the true stress in at the time of fracture.

4. A continuous and longitudinally aligned glass fiber reinforced composite consists of 40% volume of glass fibers having a modulus of elasticity 69 GPa and balance of a polymer resin which displays a modulus of 2.4 GPa. The cross-section area of 250 mm^2 is subjected to a stress of 50 MPa. Calculate the longitudinal strain of the composite for a longitudinal stress of 100 MPa.

ANSWERS

Multiple Choice Questions

1. (d)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (d)	**10.** (d)
11. (a)	**12.** (d)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (c)	**20.** (d)
21. (c)	**22.** (a)	**23.** (d)	**24.** (a)	**25.** (d)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (a)
31. (b)	**32.** (c)	**33.** (b)	**34.** (b)	**35.** (a)	**36.** (d)	**37.** (c)	**38.** (d)	**39.** (c)	**40.** (a)
41. (c)	**42.** (b)	**43.** (b)	**44.** (c)	**45.** (d)	**46.** (d)	**47.** (c)	**48.** (a)	**49.** (a)	**50.** (c)
51. (a)	**52.** (c)	**53.** (c)	**54.** (c)	**55.** (c)	**56.** (d)	**57.** (c)	**58.** (c)	**59.** (c)	**60.** (a)
61. (b)	**62.** (c)	**63.** (c)							

Numerical Answer Questions

1. 4.50%
2. 6.644, 78.124 grains/in^2
3. 36%, 664 MPa
4. 1.722×10^{-3}

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (d) Hall-Petch equation gives yield strength of a polycrystalline material:

$$\sigma_y = \sigma_0 + \frac{k}{\sqrt{d}}$$

where σ_o is the strength of single crystal, d is the grain size and k is a constant. Thus, the yield strength increases with decrease in grain size. But corrosion resistance decreases with decreased grain size as it offers more grain boundary of high energy available for chemical reaction.

2. (b) A common method of measuring the grain size is by comparing the grains at a fixed magnification with standard grain size charts. Charts are coded with ASTM grain size number, G, and are related with n_a, number of grains per mm^2 at 1× magnification as

$$G = -2.9542 + 1.4427 \ln n_a$$

In terms of G, number of grains per square inch (645 mm^2) at a magnification of 100× is equal to 2^{G-1}.

3. (b) Since, the number of bonds per atom at a grain boundary is less than that for an interior atom, interfacial energy is associated with boundaries. Atoms at the grain boundaries dissolve more readily on etching with a chemical reagent.

4. (a) The annealed structure would eliminate all internal stresses, reduces hardness, increases ductility, enhances machinability, refines grain structure.

5. (b) Twinning occurs on a definite crystallographic plane and in a specific direction, both of which depend on the crystal structure.

6. (b) Screw dislocation is considered positive if Burgers vector and $\vec{t}$ are parallel, and vice versa.

7. (c) If Burgers vector is parallel to the dislocation line, such a dislocation is called a screw dislocation.

8. (a) A pair of one cation and one anion can be missing from an ionic crystal, without violating the condition of charge neutrality when the valency of these opposite charged ions is equal.

9. (d) Twinned boundary is a special type of grain boundary across which there is specific mirror lattice symmetry.

10. (d) For anisotropic materials, each material property is dependent upon the direction.

11. (a) An invariant point (where reactions intersect), such as eutectic point, has zero degree of freedom. The reaction occurs at a particular composition and temperature.

12. (d) The size difference between the two atoms should be less than 15%. The chemical affinity between the two atoms should be less; a strong chemical affinity promotes the formation of compounds.

13. (c) In peritectic reaction, a liquid and a solid react and form a new solid at a constant temperature.

14. (a) Phase diagrams such as iron-carbon, are under equilibrium, whereas the diagrams of phase transformation, such as TTT curves, are determined under non-equilibrium conditions.

15. (d) In monotectic reaction, a liquid transforms into another liquid of different composition and a solid phase precipitates out.

16. (c) Austenite (FCC), i.e. γ-iron, experiences transformation to a body centered tetragonal (BCT) martensite.

17. (d) Temperature-time diagram indicate the time and temperature transformation of austenite into various phases.

18. (c) The microstructrue consists of grains of ferrite and uniform distribution of nearly spherical particles of cementite.

19. (c) In the Fe-Fe$_3$C system, at 727°C, γ-phase dissociates into an eutectoid mixture of ferrite (α) and cementite (Fe$_3$C), which is known as pearlite.

20. (d) Case carburizing is basically applied to low carbon steels. Induction hardening is used to harden the surface of crankshafts.

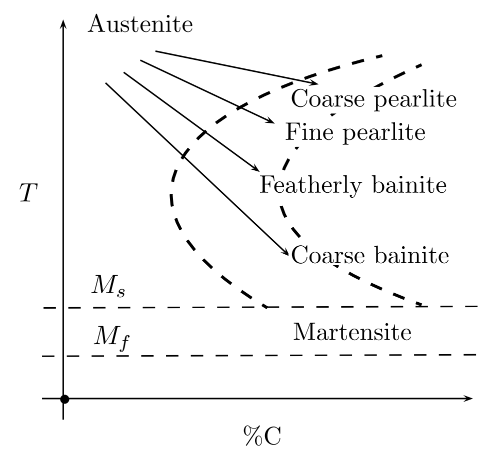
21. (c) In case carburizing, carbon diffuses into the outer skin of the specimen as cementite.

22. (a) Annealing eliminates all internal stresses, reduces hardness, increases ductility, enhances machinability, refines grain structure. Nitriding is a method of surface hardening. Martempering is tempering of martensite which is done on the whole mass of the object. Normalizing is used to refine the grain structure.

23. (d) Tempering is done to improve ductility and remove internal stresses in hardened martensite.

24. (a) Annealed and hot-rolled mild steel generally exhibit a yield point.

25. (d) The phase transformation diagram (TTT) is shown as follows:



One can see that if martensite finishing temperature is below the room temperature, the martensite formation will not complete, and the structure will retain austenite.

26. (b) Vanadium is alloyed in steels to enhance endurance strength.

27. (a) Boron (B) is added to steel in amounts of 0.0005–0.003% to improve hardenability. In combination with other alloying elements, boron acts as an intensifier, thus increasing the depth of hardening during quenching.

28. (b) Malleable iron has a structure similar to that of nodular iron which promotes ductility, strength and shock resistance, and hence is called malleable. These properties are suitable for making of the pipe fittings.

29. (c) If the hydrogen content of the molten steel exceeds the solubility limit of hydrogen in solid iron, the hydrogen will be rejected during solidification, and this leads to pinhole formation and porosity in steel. It is called hydrogen embrittlement.

30. (a) For high speed steels, the generally proportions of main alloying elements is W (18%) > Cr (4%) > V (1%).

31. (b) Sulphur is added up to 0.33% in free cutting steels to increase machinability. Similarly, phosphorus is added up to 0.12% for free cutting steel.

32. (c) Tool steel containing vanadium seem to resist shock better than those which do not contain this element.

33. (b) Tungsten is also used in certain heat resistant steel where the retention of strength at high temperatures is important.

34. (b) In austempering, the austenite would be transformed into bainite under isothermal condition.

35. (a) Annealing eliminates all internal stresses, reduces hardness, increases ductility, enhances machinability, refines grain structure.

36. (d) In cast steels, the austenite grains formed during solidification are relatively coarse, which lead to a large ferrite grain size on further cooling to room temperature. In addition, some ferrite often forms as Widmanstätten structure, primarily due to large prior austenite grain size, which adversely affects impact properties of the steel.

37. (c) Elements which tend to stabilize austenite are called austenite stabilizers, such as Mn, Ni, Co and Cu.

38. (d) Elements which tend to stabilize ferrite are called ferrite stabilizers, for example, Cr, W, Mo, V, Si.

39. (c) Cobalt improves strength at high temperatures and magnetic permeability.

40. (a) The flaws and microscopic cracks and cavities, such as graphite flakes in cast iron, act as stress raisers, resulting negligible ductility, weak in tension, although strong in compression.

41. (c) Upper and lower yield points are found in mild steel, not in aluminium alloys.

42. (b) Plastic deformation is caused by dislocation motion.

43. (b) Upper and lower yield points are observed in carbon steels.

44. (c) At elevated temperatures, failure occurs in form of time-dependent yielding known as creep.

45. (d) When a metal is subjected to tension in the plastic range, and the load is then reversed an applied in compression, the yield stress in compression is found to be lower than that in tension. This phenomenon is known as the Bauschinger effect, which is also observed when compression is followed by tension. This phenomenon is also called strain softening or work softening. This is also seen in torsional loads also.

46. (d) Perfectly elastic materials ($n = 1$) have linear relationship between stress and strain (i.e. Hooke's law). Rigid materials ($n = \infty$) do not experience any strain, regardless of the applied stress. A rigid plastic material experiences strain hardening ($n \neq 1$). Perfectly plastic materials ($n = 0$) are non-strain hardening material, thus, show deformation at same stress level. Elastic linearly strain hardening behavior is the assumption for most of the engineering materials.

47. (c) The true strain by its definition for volume constancy can be derived as

$$\begin{aligned}\varepsilon &= \int_o^l \frac{dl}{l} \\ &= \ln \frac{l_f}{l_i} \\ &= \ln \frac{A_i}{A_f} \\ &= \ln \left(\frac{d_i}{d_f}\right)^2 \\ &= 2 \ln \frac{d_i}{d_f}\end{aligned}$$

48. (a) If A_i is the initial cross-sectional, then that when true strain is ε is given by

$$\varepsilon = \ln \frac{A_i}{A_f}$$
$$A_f = A_i e^{-\varepsilon}$$

The load which the material can bear is given by

$$F = \sigma A_f = \sigma A_i e^{-\varepsilon}$$

When necking begins,

$$\frac{dF}{d\varepsilon} = 0$$
$$A_i \left(\frac{d\sigma}{d\varepsilon} e^{-\varepsilon} - \sigma e^{-\varepsilon} \right) = 0$$
$$\frac{d\sigma}{d\varepsilon} = \sigma$$
$$nK\varepsilon^{n-1} = K\varepsilon^n$$
$$\varepsilon = n$$

49. (a) For a perfect plastic $n = 0$, for perfect elastic $n = 1$.

50. (c) The specimen will pass through a hysteresis cycle in which peak point will be the same as yield point.

51. (a) The depth of penetration is a measure of the Rockwell hardness.

52. (c) The relationship between hardness in BHN and ultimate strength for carbon steel is considered as

$$\sigma_u = 3.5 \times \text{BHN}$$

53. (c) The energy absorbing capacity is directly proportional to the volume of the material.

54. (c) Since the test indentation is very small in a Knoop hardness test, it is mostly used for small parts, thin sections, or case depth work. In Herbert test, steels balls are dropped from a predetermined height over the surface.

55. (c) Toughness is measured as energy absorbed before fracture in the tension test. Endurance strength is the strength against fatigue loading. Resistance to abrasion is called hardness. Deflection in beams is determined using moment area method.

56. (d) Resilience is defined as the ability of a material to absorb energy when deformed elastically and return to original form when unloaded.

57. (c) The correct sequence in the creep curve is instantaneous (accelerated), steady, transient.

58. (c) Resilience is defined as the ability of a material to absorb energy when deformed elastically. This property is important when the material is subjected to shock loading.

59. (c) Shape of graphite flakes of gray cast iron is changed into nodular by small addition of magnesium (Mg) or cerium (Ce) to the molten metal prior to pouring.

60. (a) The softness of pure copper makes it a difficult metal to machine. To retain the higher conductivity of copper or its aesthetic appearance, the addition of sulphur gives a greatly increased cutting ability. Tellurium can be used an alternative to sulphur to obtain the same result.

61. (b) Phosphor bronze contains 70% Cu + 20% Sn and a significant phosphorus content (0.02–0.4%) of up to 1%. It is used in worm gears, springs, wire ropes.

62. (c) The nickel-iron alloy Invar contains 36% nickel, and possesses the lowest thermal expansion among all metals and alloys in the range from room temperature up to approximately 230°C.

63. (c) Babbitt, also called Babbitt metal or bearing metal, is any of several alloys used to provide the bearing surface in a plain bearing. Some common compositions are: (90% Sn, 10% Cu), (89% Sn, 7% Sb, 4% Cu), (80% Pb, 15% Sb, 5% Sn).

Numerical Answer Questions

1. From the given details, 100 g of alloy would contain 90 g of aluminium and 10 g of copper. The number of moles of aluminium atoms is

$$\frac{90g}{26.98} = 3.335$$

Number of moles of the copper atoms is

$$\frac{10}{63.55} = 0.1573$$

Therefore, atomic percentage of copper is

$$\frac{0.1573}{0.1573 + 3.335} \times 100 = 4.50\%$$

2. Given that the number of grains per square inch $N = 50$, therefore, the ASTM grain size number n is given by

$$\begin{aligned} N &= 2^{n-1} \\ \ln N &= (n-1)\ln 2 \\ n &= \frac{\ln N}{\ln 2} + 1 \\ &= 6.64 \end{aligned}$$

For magnification M, other than 100×, the number of grains per square inch N_M and the ASTM grain size number n is related as

$$N_M \left(\frac{M}{100}\right)^2 = 2^{n-1}$$

Therefore, for 80% magnification factor on the same specimen (n remains same), the number of grains per square inch is given by

$$\begin{aligned} N_M &= N\left(\frac{100}{M}\right)^2 \\ &= 78.124 \text{ grains/in}^2 \end{aligned}$$

3. Given that original diameter 10.0 mm and diameter at fracture 8.0 mm. Therefore, percentage reduction in area is

$$\begin{aligned} \%\text{RA} &= \left(1 - \frac{8^2}{10^2}\right) \times 100 \\ &= 36\% \end{aligned}$$

The fracture strength 425 MPa is measured as per the original diameter 10 mm. Therefore, the true stress at the time of fracture at diameter 8 mm is

$$\begin{aligned} \sigma_T &= \left(\frac{10}{8}\right)^2 \times 425 \\ &= 664 \text{ MPa} \end{aligned}$$

4. Given that

$$\begin{aligned} x_f &= 0.4 \\ E_f &= 69 \text{ GPa} \\ E_m &= 2.4 \text{ GPa} \\ A &= 250 \times 10^{-6} \text{ m}^2 \\ \sigma &= 50 \text{ MPa} \end{aligned}$$

The effective modulus of elasticity of the composite is given by

$$\begin{aligned} E &= E_f x_f + E_m (1 - x_f) \\ &= 69 \times 0.4 + 2.4(1 - 0.4) \\ &= 29.04 \text{ GPa} \end{aligned}$$

The longitudinal strain is calculated as

$$\begin{aligned} \varepsilon_l &= \frac{\sigma_l}{E} \\ &= 1.722 \times 10^{-3} \end{aligned}$$

CHAPTER 11

METAL CASTING

Casting, also known as *foundering*, is the oldest manufacturing process where liquid metal is poured into a preformed cavity made of refractory material. The solidified metal thus obtained is the product of *casting*. Casting is an inexpensive process but extensively used in manufacturing because of its simple tools and methods. The molten metal can flow into very small sections so that intricate shapes can be made with large saving of material. Casting is used to produce components, such as cylinder blocks, liners, machine tool beds, pistons, piston rings, mill rolls, wheels, water supply pipes.

11.1 ELEMENTS OF CASTING

Basic elements of casting are enlisted as follows:

1. Molding Box *Molding box* is a box in which sand is filled for making the pattern cavity. A molding box consists of three portions: *cope* (upper flask), *cheek* (middle flask), and *drag* (lower flask).
2. Pattern *Pattern* is a replica of the object to be made by the casting process with some modifications for *pattern allowances*, provision of core prints, elimination of fine details.
3. Cores *Cores* are provided to form additional part of pattern for hollow castings, for example, in pipe fittings. *Core prints* are provided on the patterns for proper seating of cores.
4. Chills *Chills* are the objects made of the same casting metal which are placed in the mold to obtain *directional solidification*.
5. Chaplets *Chaplets* are used to support cores inside the mold cavity against the weight and hydraulic forces.
6. Sprue *Sprue* is the passage through which molten metal from the pouring basin reaches the mold cavity in a controlled way.
7. Runner *Runner* is the passage in parting plane in the molding box through which molten metal flow is regulated before it reaches the mold cavity.
8. Riser *Risers* are provided in gating systems to compensate liquid shrinkage of poured metal.

11.2 SAND CASTING

The following are the basic steps for producing sand castings:

1. Pattern Making The mold is made by packing some readily formed aggregate material, such

as molding sand around the pattern. The cavity created by removal of pattern is ultimately filled with molten metal which later solidifies to casting.

2. Core Making Cores are usually made of sand and placed into the mold cavity to form the interior surfaces of castings.

3. Molding Molding consists of all operations necessary to prepare a mold for receiving molten metal. It involves placing a molding aggregate around a pattern held with a supporting frame, withdrawing the pattern to leave the mold cavity, setting the cores in the mold cavity and finishing and closing the mold. A sand *muller* is a mixing machine that squeezes all the sand thoroughly so that all grains of sand are evenly coated with the binder.

4. Melting and Pouring The preparation of molten metal for casting is referred to as melting. It is usually done in a specifically designated area of the foundry, and the molten metal is transferred to the pouring area where the molds are filled.

5. Cleaning After casting is cooled and removed from the mold box, burned-on sand and scale are removed from the casting to improve the surface appearance of the casting. The complete process of removing gates, runners, risers, flash, and cleaning the casting is called *fettling*. *Tumbling* is the process of removing sand and unwanted fins by keeping the casting in a barrel which is completely closed and then slowly rotated on a horizontal axis at around 30–40 rpm. *Sand-blasting* is the process of cleaning sand and irregularities from foundry casting.

11.3 PATTERN

11.3.1 Pattern Allowances

Certain allowances are given on the pattern over the specified dimensions of finished product so that a casting with the particular specification can be produced. The following are the allowances usually considered on patterns and core boxes:

1. Shrinkage Allowance Shrinkage of metals during cooling consists of two stages:

   (a) *Liquid Shrinkage* - *Liquid shrinkage* occurs during cooling in liquid state. This is compensated in castings by providing *risers*. The extra molten metal is kept in risers that can flow into mold cavity during liquid shrinkage.

   (b) *Solid Shrinkage* - *Solid shrinkage* occurs during cooling in solid state, and thus, depends upon thermal expansion coefficient. *Shrinkage allowances* in patterns are provided against this solid shrinkage.

   Shrinkage allowance depends upon the composition of molten metal cast, mold material, mold design, complexity of pattern and component size. The following are the general considerations for shrinkage allowance:

   (a) Higher shrinkage allowances are required for outer dimensions.

   (b) Negative shrinkage allowances are desired for internal surfaces.

   (c) *Double shrinkage allowance* is used for producing master pattern to take care of shrinkage of actual metal cast and the *master pattern* metal.

   (d) Solidification of gray cast iron occurs in two stages. The shrinkage associated with the first stage is compensated by expansion during the second stage. This is due to *graphitization*.

   (e) Descending order of solid shrinkage in few metals is found as

   $$\text{Steel} > \text{Brass} > \text{Al, Mg} > \text{Cast iron}$$

2. Draft *Draft* is the slope on vertical faces provided to facilitate easy withdrawal of the pattern from the mold, which otherwise can damage the mold cavity. It is applied in the form of taper from the parting line as extra metal over and above the desired casting dimensions. Inner details require higher draft than that in outer ones. As the height of the vertical face increases, draft angle value decreases.

3. Machining Allowance *Machining allowance* is the excess in dimensions of castings to take care of machining. It depends upon the cast material, type of molding used, class of surface finish, complexity of details and, obviously, the overall dimensions of the job. Molding sand has greater strength in compression than in tension, therefore, heavier and intricate sections should be included into drag flask to reduce machining allowances.

4. Shake Allowance During withdrawal of the pattern, the mold cavity is enlarged due to rapping of pattern all around the vertical faces. To compensate this, *shake allowance* is provided as a negative allowance on faces the parallel to the parting line.

5. Distortion Allowances Solidification of casting with complex details is not uniform, but it is accompanied by warping distortions. *Distortion allowances*, also called *camber allowance*, takes into account the warping.

11.3.2 Pattern Materials

Some materials used for making patterns are: wood, metals and alloys, plastics, plaster of Paris, rubbers, wax, and resins. Their selection is based on the following characteristics:

1. *Wood* It is easily available, has low weight, is easily shaped, cheap, and used large castings, but absorbs moisture resulting into warpage.
2. *Metals* They are durable, have smooth surface finish, and are used for large-scale production. For example, cast iron, brass, aluminium, white metals[1].
3. *Plastics* These are light weight, easily formable, smooth surface finish, are durable, and stable. For example, cold setting epoxy resin[2].

Disposable patterns are made of *polyurethane foam* for conventional casting, where the pattern is burned inside the mold cavity during pouring of metal.

11.3.3 Pattern Types

The following are the important types of patterns.

1. *Single Piece Pattern* It is made with single piece in the drag flask.
2. *Split Pattern* It is made for intricate castings, split at parting line. Dowel pins are fitted to the cope half for proper alignment and holes are made in the drag half. A split pattern along with gate and riser system is called *cope and drag pattern* and is used for continuous production.
3. *Gated Pattern* Sometimes gate and runner system are integrated with pattern itself, then the pattern is called *gated pattern.*
4. *Match Plate Pattern* A match plate pattern consists of a match plate (made of wooden or metal) on which many small patterns and gating systems can be screwed. This type of pattern is used for large-scale continuous production in machine moldings.
5. *Loose Piece Pattern* Loose piece patterns are used for making intricate shapes where removal of the pattern is not possible. These patterns are placed inside the mold cavity with the help of a wire and these loose pieces are removed after removal of the main pattern.
6. *Follow Board Pattern* They are made of wooden board to closely fit the contour of the weak or thin patterns, and thus, support them during the ramming.
7. *Sweep Pattern* They are used to produce large and axis symmetrical castings (e.g. bells, large gears, wheels).
8. *Skeleton Pattern* Sometimes, for large castings, patterns are made in parts and these are joined together to make the mold cavity. This type of pattern is called *skeleton pattern.*

[1]White metals are any of several light colored alloys as well as any of the several lead-based or tin-based alloys. White metal patterns are not provided with double shrinkage allowance.

[2]Plastic patterns are made from a mold made of plaster of Paris.

11.4 MOLDING SAND

Molding sand is the base on which casting processes are carried out.

11.4.1 Properties of Molding Sand

The following are the requisite properties of molding sand:

1. *Refractoriness* *Refractoriness* is the capability to withstand higher temperature of the molten metal. Graphite can withstand 4200°C (melting point) while silica, upto 1700°C. Higher grain size of sand gives higher refractoriness.
2. *Green Strength* Molding sand containing moisture is called *green sand. Green strength* is the capability of green sand to withstand its own weight.
3. *Dry Strength* Drying of green sand is done at temperature around 240°C. Dry strength is the ability of dry sand to withstand against self-weight and pressure of molten metal. Fine grains provide larger surface area (per unit mass) for the binder to act upon, thus provide better strength.
4. *Collapsibility* *Collapsibility* is the property of sand to withstand expansion or contraction of the molten mental, thus reducing expansion defects in the castings.
5. *Permeability* *Permeability* is the capability of permitting the gas evolved during the molding process. Bulk density of a sand mix depends upon the shape and size of the grains. It will be very low if grains are of equal size with smooth and round shape. Such grains will result in an increased void and higher permeability. Fine grains would have lower permeability but better surface finish for casting to be produced. Permeability decreases with increase in the amount of ramming.

To access permeability, a *permeability number* (PN) is defined as the rate of flow of air through a standard specimen under standard pressure. Mathematically,

$$\mathrm{PN} = \frac{V \times H}{p \times A \times t} = \frac{501.28}{p \times t}$$

where V is the volume of air (2000 m^3), t is the time (minutes), H is the height of sand specimen (5.08 cm), p is the air pressure (g/cm^2), and A is cross-sectional area of the sand specimen (20.268 cm^2).

11.4.2 Molding Sand Additives

A common molding sand consists of around 10% clay, 3–6% water, and 1–6% additives. Clay together with water acts as a bonding agent and imparts tensile strength and shear strength to the molding sand. The following are the additives added in molding sand to improve some of properties as discussed:

1. *Molasses* They are used to enhance the resistance to drying out.
2. *Cereal* It increases strength and collapsibility.
3. *Iron Oxide* It improves surface finish by decreasing metal penetration into molding sand, increases cooling rate, but decreases green strength and permeability.
4. *Silica Flour* It enhances hot strength.
5. *Coal Dust* It provides better surface finish by providing an gaseous envelope to molten metal thus reducing fusion into mold.
6. *Saw Dust and Wood Flour* They enhance the collapsibility, thus, reduce expansion defects and increase water capacity; but if added in high amounts, they make the mold brittle.

11.5 GATING SYSTEM

A good *gating system* is designed for shortest filling time with proper thermal gradient, without any turbulence and mold erosion.

11.5.1 Elements of Gating System

A gating system is provided with the following elements [Fig. 11.1]:

1. *Pouring Basin* Molten metal is poured into pouring basin which acts as a reservoir of molten metal that feeds it smoothly into the sprue.
2. *Skim Core* *Pouring basin* is provided a skimmer or skim core to prevent the entry slag and dirt float on the top of the molten metal.
3. *Strainer* Strainer made of ceramic materials can be placed in the passage to remove dross and eroded sand particles.
4. *Splash Cores* *Splash cores* are placed where the flow can splash the mold, and thus, reduce the eroding forces.
5. *Skim Bob* To trap the impurities in the molten metal, a gating system is provided with *skim bob* in the form of a enlarged section at some point along the runner. Lighter impurities get trapped at the upper portion, while heavier impurities are dropped into lower portion.
6. *Sprue* *Sprue* is the vertical channel through which molten metal is brought from pouring basin to the parting plane where runners take the metal to the mold cavity.
7. *Runner* *Runners* are generally horizontal channels of trapezoidal section at parting plane that carry molten metal from sprue well to gate of the mold cavity.
8. *Gate* *In-gates* or *gates* are the opening through which molten metal enters the mold cavity. Cross-sectional area of gate determines the flow rate of the molten metal into the mold cavity.
9. *Delay Screen* A *delay screen* is a small piece of perforated thin sheet placed in the pouring basin at the top of the down sprue. This screen actually melts into the sprue because of heat from metal and the process delays the entrance of the metal into the sprue, thus, filling the pouring basin fully and avoiding vortex flow generation.

All the elements of a gating system should normally be circular in cross-section because they have lower surface-to-volume ratio which reduce the heat loss and have less friction.

Directional solidification of molten metal is that which starts from the thinnest section of the casting that solidifies first and continues progressively towards the riser, which should be the last to solidify. The design of gating system predominantly affects the *directional solidification*. It can also be ensured by using chills in the molds and exothermic materials in the risers or in facing sand and also by increasing thickness of certain sections by using exothermic padding.

In *vertical gating*, liquid metal is poured vertically to fill the mold cavity. It involves turbulence, and thus,

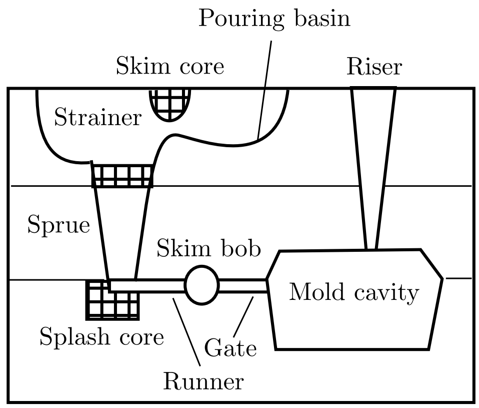


Figure 11.1 Gating system.

is not suggested for non-ferrous metals. To avoid this, *bottom gating* is used in which liquid metal is filled in mold from bottom to top, thus avoiding splashing and also oxidation associated with vertical gating.

11.5.2 Pouring Time

If liquid metal is poured very slowly it can result in solidification before filling the mold cavity. If liquid metal is poured very fast it can result in erosion of the mold surface. Thus, a compromise is adopted for optimum speed. The shortest pouring path should be selected to avoid high pouring temperatures.

Generally, metal starts entering into the mold cavity after the gating systems are completely filled. Molten metal behaves like fluid. Therefore, pouring time can be calculated based on the principles of fluid mechanics.

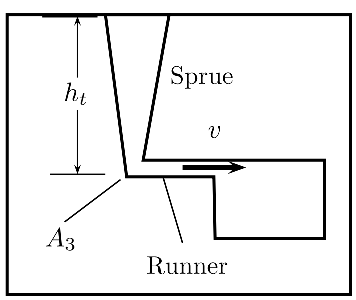


Figure 11.2 Pouring time calculation.

Consider a vertical gating system shown in Fig. 11.2 where the molten metal is dropped into cavity from a height of h_t. Velocity at entrance to mold cavity will be given by

$$v = \sqrt{2gh_t}$$

Therefore, pouring time is

$$t = \frac{\text{Volume of mold cavity}}{A_3\sqrt{2gh_t}}$$

Gray cast iron lose heat very fast, thus pouring time should be very small but for non-ferrous metals a longer pouring time is allowed.

11.5.3 Aspiration Effect

Pressure should not fall below the atmospheric pressure anywhere in the molten metal stream. Otherwise, the gases originating from the baking of organic compounds in the mold will enter the molten metal stream producing porous casting. This phenomenon is known as *aspiration effect*. *Vena contracta* is generated in the molten stream that creates vacuum around this point. To avoid this, a smooth curve at entry is provided.

Consider a gating system in which the areas at two points 2 (upper) and 3 (lower) are A_2 and A_3, where the heads at these points are h_2 and h_3, respectively [Fig. 11.3].

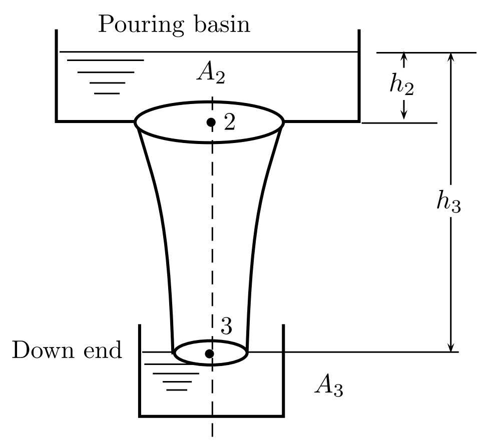


Figure 11.3 Aspiration effect.

Principles of fluid mechanics can be applied as follows:

1. *Bernoulli's equation* Velocities at the two points are found as

$$v_2 = \sqrt{2gh_2}$$
$$v_3 = \sqrt{2gh_3}$$

2. *Principle of Mass Conservation*

$$A_2v_2 = A_3v_3$$
$$\frac{A_3}{A_2} = \frac{v_2}{v_3} = \sqrt{\frac{h_2}{h_3}}$$

To avoid aspiration effect, area for stream should vary in parabolic way with respect to head.

11.5.4 Gating Ratio

Gating ratio refers to the ratio of areas of sprue base, total areas of runner, and total area of gates. In this respect, gating systems can be of two types:

1. Pressurized Gating System If pressure is applied on metal stream, the system is called *pressurized gating system*. Gating ratio is so arranged that a back pressure is maintained on the system by a fluid film restriction at the sprue base. The system is kept full of metal and is usually smaller in volume for a given metal flow rate. Hence, less metal is left in the gating system. Turbulence in metal stream is the concern with this system.

2. Non-Pressurized Gating System When the metal stream flows due to natural gravity force, the system is called *non-pressurized gating system*, sometimes also called as *reverse choke system*. For this, the areas of runner and gates are enlarged after the sprue. The stream velocities are reduced, thus the turbulence and aspiration are minimized.

11.5.5 Cooling and Solidification

Pure metals and alloys behave differently on cooling and solidification. Pure metals have clearly defined melting points, and solidification takes place at a constant temperature. Alloys solidify over a range of temperature when the temperature of molten metal drops below the liquids and is completed when the temperature reaches solidus. Therefore, in pure metal's solidification, if thermal conductivity of the mold is very high, it acts as the center of nucleation for solidification and crystal growth commences from the mold and extends towards the center. Alloys do not have a sharply defined freezing point temperature (like pure metals). The crystal growth depends upon composition gradient, variation of solidus temperature with composition, and thermal gradient within the mold itself. Solidification of alloys over a short range of temperatures results in a wholly columnar structure. However, solidification over a wide range of temperatures results in a dendritic structure.

11.5.5.1 Cooling Time The casting solidfies and cools in the mold itself. The sand mold should be broken only after sufficient cooling of the casting. The cooling time depends upon various factors, such as thickness, mass, and properties of metal and mold.

Consider the case when it is assumed that almost the entire thermal resistance to heat transfer is offered by the mold. The solidification length x is found to be related with cooling time τ as

$$x \propto \sqrt{\tau}$$

11.5.5.2 Solidification Time The *Chvorinov's rule*[3] concludes that *solidification time* (t_s) of a casting is proportional to the square of the ratio of volume (V) to surface area (A) of the casting:

$$\tau_s = K_m \left(\frac{V}{A}\right)^2 \tag{11.1}$$

where K_m is called *mold constant* or *solidification factor* which depends on the properties of the metal, such as density, heat capacity, heat of fusion and superheat, and the properties of mold, such as initial temperature, density, thermal conductivity, heat capacity and wall thickness. The Chvorinov's rule provides powerful general method for tackling the feeding of castings to ensure their soundness.

11.5.6 Design of Riser

In gating system, the design of a riser is also an important aspect. A riser, also called *feeder*, gives molten metal during liquid solidification, otherwise it results into void points (e.g. cavity, porosity or hot tear) in the casting volume. It allows a visual check for ensuring filling up of the mold cavity completely. Also, in the initial stage of pouring, a riser permits air, steam or gases to escape out of the mold.

The following are the general considerations in the design of risers:

1. Risers should be designed to ensure *directional solidification* of the casting, maintaining proper temperature gradient within the solidifying casting.

2. Risers should be close to the heaviest section of the casting, preferably on the top or at the side and connected to the casting by a neck of metal called 'gate' for easy removal of riser from casting during *fettling*.

3. Each of thick sections of a casting should have its own riser.

11.5.6.1 Freezing Ratio The size of riser should be designed for minimum possible volume, but should maintain a solidification time longer than that of casting.

[3] Chvorinov himself showed in his paper "Theory of the Solidification of Castings", Giesserei, 1940, Vol. 27, pp 177-186 that it applied to steel castings from 12 to 6000 kg made in green sand molds.

A special measure for riser design, *freezing ratio*, R, is defined as

$$R = \frac{(A/V)_{\text{casting}}}{(A/V)_{\text{riser}}}$$

where A and V represent the surface area and volume of components under subscripts casting and riser. For proper functioning of the riser, its metal should solidify after solidification of metal in the mold cavity. Therefore, the freezing ratio must be greater than 1, or

$$(A/V)_{\text{casting}} \geq (A/V)_{\text{riser}}$$

To have sufficient head to feed the liquid metal into the mold cavity, riser should have sufficient volume in its space. It is found that minimum volume of riser should be approximately three times that dictated by the liquid shrinkage consideration.

11.5.6.2 Caine's Equation Based on the Chvorinov's rule [Eq. (11.1)], *Caine* developed the following empirical relationship

$$x = \frac{a}{y-b} - c$$

where, x is freezing ratio, y is ratio of volumes of riser and casting, and a, b, c are constants. The equation can be used to draw a curve on x-y plane [Fig. 11.4].

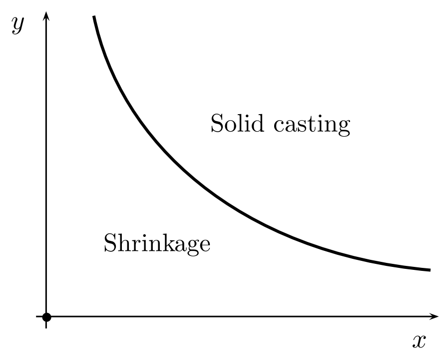


Figure 11.4 | Caine's equation.

It is found that those casting belonging to the upper side of the curve are sound casting. From functional point of view, the best shape for a riser is spherical because it has lowest surface area-to-volume ratio, however, cylindrical risers are used to ease in feeding the metal.

11.5.6.3 Modulus Method The *modulus method*[4] is based on the concept that the freezing time of a casting or a casting section can be approximated by using Chvorinov's rule as

$$\begin{aligned} t &= k^2 \left(\frac{V}{A}\right)^2 \\ &= k^2 m_c^2 \end{aligned}$$

where t is the freezing time of the casting, V is the volume of the casting, A is the surface area of the casting, and k is the constant depending upon the metal and mold properties. The volume-to-surface area ratio is termed as casting modulus (m_c).

11.6 CASTING METHODS

Important types of casting processes are explained in following subsections.

11.6.1 Shell Molding

Shell molding, also known as *croning process*[5], is a process in which the pattern is made in the form of shells. For this, a master pattern made of cast iron is heated to 175–370°C, coated with silicon (as parting agent) and is clamped into a box which contains fine sand mixed with 2.5–4% thermosetting resin binder, such as phenol formaldehyde that coats the sand particles. The box is then rotated upside down, allowing the sand to coat the pattern. The pattern assembly is then placed in an oven for a short period of time to complete the curing process of the resin. The hardened shell is then removed from around the pattern using built-in injector pins.

Two halve shells made are bonded or clamped together in preparation of pouring. The thickness of shell (≈ 5–10 mm) depend upon the curing time. The fine sand used in shell molds has much lower permeability and decomposition of shell sand binder generates high volume of gases.

Shell molding is used for casting of small mechanical parts requiring high precision, gear housing, cylinder heads, connecting rods, turbine blades, etc.

11.6.2 Expendable Pattern Casting

Expendable pattern casting is known as *lost foam casting*, or *lost pattern casting*. This method uses *polystyrene pattern* which evaporates on contact with molten metal to form a cavity for the casting.

In this method, raw expendable polystyrene beads are placed in preheated aluminium die. The beads expand and takes the shape of the die cavity. Additional heat

[4]The concept of modulus method was developed by Wlodawer for practical riser calculations by eliminating the need to calculate actual solidification time in favor of simply determining the relative solidification times of casting sections and risers.

[5]Croning process is named after its inventor J. Croning.

is applied to fuse and bond the beads together. The die is then cooled and opened to remove polystyrene pattern, which in turn is then coated with water-based refractory slurry, dried and placed in a flask. The sand is periodically compacted. Then, the molten metal is poured into the mold. This action immediately vaporizes the pattern (*oblation*) and fills the mold cavity completely replacing the space previously occupied by polystyrene pattern. After metal is solidified, the mold is broken up and the casting is removed.

The metal flow velocity in the mold depends on the rate of degradation of the polymer. The velocity of molten metal at the metal polymer pattern is estimated to be in the range of 0.1–1 m/s. Large thermal gradient are present at metal polymer interface, which causes *directional solidification*.

11.6.3 Investment Casting

Investment casting, also known as *lost wax process*, uses patterns made by injecting semisolid or liquid wax or plastic into a metal die. The pattern is covered with a slurry of refractory material, such as fine silica and finders, ethyl silicate, and acids. Additional layers of slurry are repeatedly coated on the pattern to make it thicker. This one-piece mold is then dried in air and heated to 90–170°C for about 4 hours to melt out the wax. The mold is then poured with molten metal, which later solidifies and the mold is broken up to remove the casting.

11.6.4 Permanent Mold Casting

Permanent mold casting involves permanent molds made of metals, like cast iron, steel, bronze, or graphite. The mold cavity and gating system are machined into the mold itself. In order to increase life, the mold cavity is coated with a refractory slurry or sprayed with graphite on every new casting. The molds are then preheated to about 150–200°C to facilitate metal flow and reduce thermal damage. The process is used for making pistons, cylinder heads, connecting rods, gear blanks, etc.

11.6.5 Die Casting

In die casting, the molten metal is forced into the die cavity at pressure ranging from 0.7 MPa to 700 MPa. There are two versions of this method:

1. Hot Chamber Die Casting *Hot chamber die casting* uses piston which traps a certain amount of molten metal and forces it into the die cavity through a goose neck and nozzle at high pressure ($\approx$ 35 MPa). The metal is held under pressure until it solidifies in the die.

   To improve life and to impart rapid metal cooling, the dies are usually cooled by circulating water or oil through various passageways in the die block.

   Low melting point alloys, such as Zn, Sn, Pb, are usually suitable for this process.

2. Cold Chamber Die Casting In *cold chamber die casting*, the molten metal is prepared outside the main machine from where it is brought and poured into injection cylinder by hand ladle. The metal is forced into the die cavity at pressure usually ranging from 20–70 MPa. The process is suitable for high melting point alloys, such as Al, Mg, Cu.

11.6.6 Centrifugal Casting

Centrifugal casting employs inertia forces caused by rotation of mold at high speeds to uniformly distribute the molten metal on the wall of the mold cavity. The process does not require core. Light impurities are collected at center. There are three methods of centrifugal casting:

1. True Centrifugal Casting This method is used to produce centrally hollow products under the effect of centrifugal force by rotating the hollow mold on the horizontal axis.

2. Semi-centrifugal Casting This method is used to produce products of rotational symmetry, such as pulleys, flywheels, discs, by rotating the mold on the vertical axis.

3. Centrifuging or Pressure Casting In this method, a number of molds are placed on the table and connected to a vertical sprue. The table is rotated about the axis of the vertical sprue. This method is used to produce non-symmetrical objects, such as brackets, bearing caps.

11.6.7 Slush Casting

Slush casting is used with low melting point alloys of Sn, Pb, Zn. The molten metal is poured into the mold, and when the skin has frozen, the mold is turned down or sung to remove the metal still in liquid state. Thus, a thin-walled casting results without any core. This is used for making toys, ornaments, and lighting fixtures like products.

11.6.8 Continuous Casting

The process of *continuous casting*,[6] also known as *strand casting*, involves continuous pouring of molten metal into horizontal or vertical mold or a die opened at both ends, cooling it there rapidly and removing the solidified products in continuous form.

This method is used to produce blooms, billets, slabs, and tubes. *Electromagnetic casting* is a variant of continuous casting, wherein the solidification takes place in electromagnetic field by direct water impingement.

11.7 CASTING DEFECTS

Castings often contain various imperfections which contribute to normal quality variations. These defects not only give a bad appearance to the castings, but also decrease their strength and practical utility. Therefore, it is essential to study the defects, and inculcate precautions to avoid these defects in the final products. The casting defects are grouped into seven categories:

1. *Metallic Projections* These are small metallic projections on casting surfaces. Loose clamping or misalignment of molding flasks results in *mismatch* of the two parts of the castings. Misplacement of molding flasks or pattern results in lifts or shifts. Fins and flashes are thin projections at the parting line that appear due to improper matching of surfaces of the molding flasks. *Swell* is a localized deformation of the vertical mold faces due to pressure of molten metal.

2. *Cavities* *Blows* are trappings of air and appear as internal voids because of poor ventilation, while *gas holes* occur due to poor ventilation of dissolved gases in the melt, particularly hydrogen. *Pin holes* are tiny blow holes inside and below the casting surface. *Porosity* is due to decrease in gas solubility in the metal during solidification. *Shrinkage cavity* occurs because of improper riser design and placement.

3. *Discontinuities* Discontinuities in castings are caused by any hindrance in the contraction of the casting. *Hot tears* are the cracks developed in the metal due to residual stresses in solidified metal caused by solid shrinkage of the metal. *Hot spots* are the portions of casting, which solidify in the end and are often the cause of hot tears.

4. *Defective Surface* Shallow blows on the surfaces are called *scars*, while scars covered with a thin layer of metal are called *blisters*. The dots found on the surface after removing dirt are called *dirt*. *Rat tail* is a minor buckle that occurs due to compression of mold face on the bottom. *Penetration* is due to soft and porous mold face. *Crushers* are the defects on the surfaces occurring due to damages during molding or placing of the cores. Rapid cooling of certain points results in *hard spots* on the casting surface. *Wash* is the extra metal due to erosion of molds caused by metal flow on down surfaces.

5. *Incomplete Casting* *Misrun* is the empty space in the mold cavity that occurs due to prior solidification of the moving face of melt. The metal fails to reach all sections of casting. *Cold shuts* are caused by mating of two cold streams of metals moving in opposite directions. Incomplete filling of the mold cavity is known as *pour short*, while drainage of molten metal from mold cavity is called *run out*.

6. *Incorrect Dimension and Shape* *Buckle* in the casting is due to deformed mold surface caused by thermal expansion of mold. *Drop* is the extra metal caused by erosion of mold surface due to poor green strength.

7. *Inclusions* On solidification of the molten metal, the suspended impurities appear as inclusion, which weakens the casting. Lighter impurities of oxides found in castings are called *dross*.

11.8 INSPECTION OF CASTING

The following are the important non-destructive techniques which can be used for testing the various casting defects:

1. *Visual Inspection* Visual inspection can be used for detecting common defects, such as rough surfaces, shifts, omission of cores.

2. *Ultrasonic Testing* Ultrasonic testing (UST) is used to detect discontinuities in the casting.

3. *Magnetic Particle Inspection* Magnetic particle inspection can be conducted to detect very small voids just below the surface of the casting of a ferromagnetic material.

4. *Radiographic Inspection* Radiographic inspection is performed by using X-rays.

5. *Dye-Penetrant Testing* Dye-penetrant testing (DPT) is used to detect invisible surface defects in non-magnetic castings.

[6] Originally developed in 1860s, to cast non-ferrous metal strip but now used for steel production.

IMPORTANT FORMULAS

Pouring Time

$$v = \sqrt{2gh_t}$$

$$t = \frac{\text{Volume of mold cavity}}{A_3\sqrt{2gh_t}}$$

Aspiration Effect

$$\frac{A_3}{A_2} = \sqrt{\frac{h_2}{h_3}}$$

Cooling Time

$$x \propto \sqrt{\tau}$$

Chvorinov's Rule

$$\tau_s = K_m\left(\frac{V}{A}\right)^2$$

Riser Design

$$\left(\frac{A}{V}\right)_{\text{casting}} \geq \left(\frac{A}{V}\right)_{\text{riser}}$$

Caine's Equation

$$x = \frac{a}{y-b} - c$$

Modulus Method

$$t = k^2\left(\frac{V}{A}\right)^2 = k^2 m_c^2$$

SOLVED EXAMPLES

1. Under uniform cooling, a spherical casting of 25 mm diameter undergoes volumetric solidification shrinkage and volumetric solid contraction of 2.5% and 6.2%, respectively. Determine the diameter of the casting after solidification.

Solution. The size of the cube after solidification and contraction will be

$$\begin{aligned} d &= \sqrt[3]{\left(1-\frac{2.5}{100}\right)\left(1-\frac{6.2}{100}\right)} \times 25 \\ &= \sqrt[3]{0.975 \times 0.938} \times 15 \\ &= 22.86 \text{ mm} \end{aligned}$$

2. A sphere-shaped casting solidifies in 10 min. What will be the solidification time in minutes for another sphere of the same material, which is 8 times heavier than the original casting?

Solution. For the two spheres, the ratio of their diameter (d) and surface area (A) is examined as

$$\begin{aligned} \frac{v_2}{v_1} &= 8 \\ \frac{d_2^3}{d_1^3} &= 8 \\ \frac{d_2}{d_1} &= 2 \\ \frac{A_2}{A_1} &= \frac{d_2^2}{d_1^2} \\ \frac{A_2}{A_1} &= 2^2 \\ &= 4 \end{aligned}$$

The cooling time is found to be proportional to the square of the volume-to-surface area ratio, thus

$$\begin{aligned} t_1 \times \left(\frac{A_1}{V_1}\right)^2 &= t_2 \times \left(\frac{A_2}{V_2}\right)^2 \\ t_2 &= t_1 \times \left(\frac{A_1}{A_2} \times \frac{V_2}{V_1}\right)^2 \\ &= 10 \times \left(\frac{1}{4} \times \frac{8}{1}\right)^2 \\ &= 10 \times 4 \\ &= 40 \text{ min} \end{aligned}$$

3. A spherical drop of molten metal of radius 5 mm was found to solidify in 12 s. In how much time will a similar drop of radius 10 mm would solidify?

Solution. The solidification time is given by

$$t_s \propto \left(\frac{V}{A}\right)^2$$

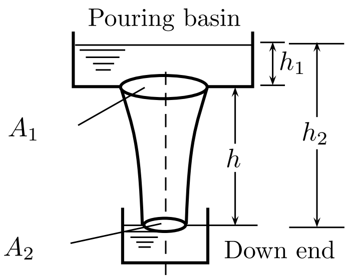
For a sphere of diameter d,

$$t_s \propto d^2$$

Therefore, for the given two cases,

$$\begin{aligned} t_{s2} &= \left(\frac{10}{5}\right)^2 \times 12 \\ &= 2^2 \times 12 \\ &= 48 \end{aligned}$$

4. A down sprue having height $h = 150$ mm and cross-sectional area $A_1 = 450\ \text{mm}^2$ at inlet is fed at constant head from the pouring basin to maintain the flow rate of molten metal $Q = 4.5 \times 10^5$ mm^3/s. Its lower end is open to the atmosphere.



Determine the area of the down-end sprue in mm^2 at its end to avoiding aspiration effect. Take $g = 9.8$ m/s^2.

Solution. Given that

$$\begin{aligned} h &= 150 \text{ mm} \\ A_1 &= 450 \text{ mm}^2 \\ Q &= 4.5 \times 10^5 \text{ mm}^3/\text{s} \\ g &= 9.8 \times 10^3 \text{ mm/s}^2 \end{aligned}$$

Therefore, at entry of the pouring basin,

$$\begin{aligned} A_1 v_1 &= 4.5 \times 10^5 \\ v_1 &= 1000 \text{ mm/s} \end{aligned}$$

But

$$\begin{aligned} v_1 &= \sqrt{2gh_1} \\ h_1 &= \frac{v_1^2}{2g} \\ &= \frac{1000^2}{2 \times 9.81} \\ &= 50.96 \text{ mm} \end{aligned}$$

At exit of the sprue,

$$\begin{aligned} v_2 &= \sqrt{2g \times (h + h_1)} \\ &= \sqrt{2 \times 9.81 \times (150 + 50.96)} \\ &= 1985.69 \text{ mm/s} \end{aligned}$$

For constant Q,

$$\begin{aligned} A_2 v_2 &= 4.5 \times 10^5 \\ A_2 &= 226.62 \text{ mm}^2 \end{aligned}$$

5. A cubical casting of molten metal of size 8 mm was found to solidify in 18 s. Calculate the solidification time for a similar cube of radius 10 mm.

Solution. The solidification time is given by

$$t_s \propto \left(\frac{V}{A}\right)^2$$

For cube of size d,

$$\begin{aligned} t_s &\propto \left(\frac{d^3}{6d^2}\right)^2 \\ &\propto d^2 \end{aligned}$$

Therefore, for the given two cases,

$$\begin{aligned} t_{s2} &= \left(\frac{10}{8}\right)^2 \times 18 \\ &= 18.125 \text{ s} \end{aligned}$$

6. A cylindrical riser (height equal to diameter) is designed to feed a steel slab casting of $300 \times 300 \times 100$ mm^3. Determine the diameter of the riser using Caine's equation:

$$x = \frac{a}{y - b} - c$$

where x is the freezing ratio, y is the riser volume /casting volume. For steels $a = 0.1$, $b = 0.03$, $c = 1.0$.

Solution. Let d (m) be the diameter of the cylindrical riser ($h = d$). Surface area and volume of the steel casting are

$$\begin{aligned} A &= 2 \times 0.300 \times 0.300 + 4 \times 0.300 \times 0.010 \\ &= 0.192 \text{ m}^2 \\ V &= 0.3 \times 0.3 \times 0.01 \\ &= 9 \times 10^{-4} \text{ m}^3 \end{aligned}$$

In the Caine's equation, x and y are calculated as

$$\begin{aligned} x &= \frac{0.192/9 \times 10^{-4}}{(\pi d^2 + \pi d^2/4) \,/\, (\pi d^3/4)} \\ &= \frac{213.33d}{5} \\ &= 42.66d \\ y &= \frac{\pi d^3/4}{9 \times 10^{-4}} \\ &= 872.66d^3 \end{aligned}$$

Therefore,

$$\begin{aligned} 42.66d &= \frac{0.1}{872.66d^3 - 0.03} - 1 \\ 32.22 \times 10^3 d^4 - 1.28d &= 0.1 - 872.66d^3 + 0.03 \end{aligned}$$

Therefore,

$$32.22 \times 10^3 d^4 + 872.66d^3 - 1.28d - 0.13 = 0$$

Solving using Newton–Raphson method,

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

$$\begin{aligned} d &= 0.0433 \text{ m} \\ &\approx 5 \text{ cm} \end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. Hardness of green sand mold increases with

(a) increase in moisture content beyond 6%
(b) increase in permeability
(c) decrease in permeability
(d) increase in both moisture content and permeability

(GATE 2003)

Solution. Decrease in permeability of the green sand will make the mold denser and harder.

Ans. (c)

2. With a solidification factor of 0.97×10^6 s/m^2, the solidification time (in seconds) for a spherical casting of 200 mm diameter is

(a) 539 (b) 1078
(c) 4311 (d) 3233

(GATE 2003)

Solution. Given that

$$K_m = 0.97 \times 10^6 \text{ s/m}^2$$
$$r = \frac{0.2}{2} = 0.1 \text{ m}$$

Solidification time is

$$t_s = K_m \left(\frac{V}{A}\right)^2 = K_m \left(\frac{4\pi r^3/3}{4\pi r^2}\right)^2 = 1077.78 \text{ s}$$

Ans. (b)

3. Misrun is a casting defect which occurs due to

(a) very high pouring temperature of the metal
(b) insufficient fluidity of the molten metal
(c) absorption of gases by the liquid metal
(d) improper alignment of the mold flasks

(GATE 2004)

Solution. Misrun is empty space in the mold cavity that occurs due to prior solidification of the moving face of melt. In the given options, only (b) is the suitable answer.

Ans. (b)

4. Gray cast iron blocks $200 \times 100 \times 10$ mm are to be cast in sand molds. Shrinkage allowance for pattern making is 1%. The ratio of the volume of pattern to that of the casting will be

(a) 0.97 (b) 0.99
(c) 1.01 (d) 1.03

(GATE 2004)

Solution. Shrinkage allowance (α%) is given as addition to the each dimension of the casting. Therefore, if $a \times b \times c$ is the casting, it will require pattern of volume, given by

$$V_{\text{Pattern}} = \left(1 + \frac{\alpha}{100}\right)^3 \times (a \times b \times c)$$
$$\frac{V_{\text{Pattern}}}{V_{\text{Casting}}} = \left(1 + \frac{\alpha}{100}\right)^3$$

For given case $\alpha = 1$, therefore, ratio of pattern and casting volumes is

$$\frac{V_{\text{Pattern}}}{V_{\text{Casting}}} = \left(1 + \frac{1}{100}\right)^3 = 1.01^3 = 1.0303$$

Ans. (d)

5. Match the items of List I with the items of List II and select the correct option.

List I (Equipment)	List II (Processes)
P. Hot chamber machine	1. Cleaning
Q. Muller	2. Core making
R. Dielectric baker	3. Die casting
S. Sand blaster	4. Annealing
	5. Sand mixing

(a) P-2, Q-1, R-4, S-5
(b) P-4, Q-2, R-3, S-5
(c) P-4, Q-5, R-1, S-2
(d) P-3, Q-5, R-2, S-1

(GATE 2005)

Solution. Hot chamber machine is a used in die casting. Muller is used in sand mixing. Dielectric baker is used for core making. Sand blasters are used in cleaning of the casting.

Ans. (d)

6. When the temperature of a solid metal increases,

(a) strength of the metal decreases but ductility increases
(b) both strength and ductility of the metal decrease
(c) both strength and ductility of the metal increase
(d) strength of the metal increases but ductility decreases

(GATE 2005)

Solution. Increase in temperature increases the ductility, but makes the metal weak.

Ans. (a)

7. A mold has a down sprue whose length is 20 cm and the cross-sectional area at the base of the down sprue is 1 cm^2. The down sprue feeds a horizontal runner leading to the mold cavity of volume 1000 cm^3. The time required to fill the mold cavity will be

(a) 4.05 s (b) 5.05 s
(c) 6.05 s (d) 7.25 s

(GATE 2005)

Solution. Head $h = 0.2$ m is available at the base of the down sprue where area is 1 cm^2, therefore, the velocity of the stream entering the cavity is

$$\begin{aligned} v &= \sqrt{2 \times 9.81 \times 0.20} \\ &= 1.98 \text{ m/s} \end{aligned}$$

Time required to fill the volume $V = 1000$ cm^3 is

$$\begin{aligned} t &= \frac{1000 \times 0.01^3}{1 \times 0.01^2 \times 1.98} \\ &= 5.04819 \text{ s} \end{aligned}$$

Ans. (b)

8. An expendable pattern is used in

(a) slush casting
(b) squeeze casting
(c) centrifugal casting
(d) investment casting

(GATE 2006)

Solution. Investment casting method involves use of polystyrene pattern which evaporates upon contact with molten metal to form a cavity for the casting. The process is also known as lost foam casting or expendable patter casting.

Ans. (d)

9. In a sand casting operation, the total liquid head is maintained constant, such that it is equal to the mold height. The time taken to fill the mold with a top gate is t_A. If the same mold is filled with a bottom gate, then the time taken is t_B. Ignore the time required to fill the runner and frictional effects. Assume atmospheric pressure at the top molten metal surfaces. The relation between t_A and t_B is:

(a) $t_B = \sqrt{2}t_A$ (b) $t_B = 2t_A$
(c) $t_B = t_A/\sqrt{2}$ (d) $t_B = 2\sqrt{2}t_A$

(GATE 2006)

Solution. Time required to fill the mold cavity is inversely proportional to the head at its gate because

$$t \propto \frac{v}{\sqrt{2gh}}$$

The volume of mold cavity in both the given cases is the same, but the head available at bottom gating is two times that of top gating, therefore,

$$\begin{aligned} \frac{t_B}{t_A} &= \sqrt{\frac{1}{2}} \\ t_B &= \frac{t_A}{\sqrt{2}} \end{aligned}$$

Ans. (c)

10. Which of the following engineering materials is the most suitable candidate for hot chamber die casting?

(a) Low carbon steel (b) Titanium
(c) Copper (d) Tin

(GATE 2007)

Solution. Titanum articles are manufactured through hot chamber die casting.

Ans. (b)

11. Volume of a cube of side l and volume of a sphere of radius r are equal. Both the cube and the sphere are solid, and composed of the same material. They are being cast. The ratio of the solidification time of the cube to that of the sphere is

(a) $(4\pi/6)^3 (r/l)^6$ (b) $(4\pi/6) (r/l)^2$
(c) $(4\pi/6)^2 (r/l)^3$ (d) $(4\pi/6)^2 (r/l)^4$

(GATE 2007)

Solution. If V and A represent the volume and surface area of the casting, the solidification time is given by

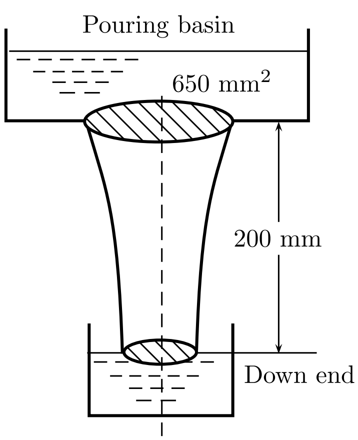
$$t \propto \left(\frac{V}{A}\right)^2$$

Given that $V_{cube} = V_{sphere}$. Therefore,

$$\begin{aligned}\frac{t_{cube}}{t_{sphere}} &= \left(\frac{V_{cube}}{A_{cube}}\right)^2 \left(\frac{A_{sphere}}{V_{sphere}}\right)^2 \\ &= \left(\frac{A_{sphere}}{A_{cube}}\right)^2 \\ &= \left(\frac{4\pi r^2}{6l^2}\right)^2 \\ &= \left(\frac{4\pi}{6}\right)^2 \left(\frac{r}{l}\right)^4\end{aligned}$$

Ans. (d)

12. A 200 mm long down sprue has an area of cross-section of 650 mm^2 where the pouring basin meets the down sprue (i.e. at the beginning of the down sprue). A constant head of molten metal is maintained by the pouring basin. The molten metal flow rate is 6.5×10^5 mm^3/s.



Considering the down-end sprue to be open to atmosphere and an acceleration due to gravity of 10^4 mm/s^2, the area of the down-end sprue in mm^2 at its end (avoiding aspiration effect) should be

(a) 650.0 (b) 350.0
(c) 290.7 (d) 190.0

(GATE 2007)

Solution. Given that

$$\begin{aligned}h &= 200 \text{ mm} \\ A_1 &= 650 \text{ mm}^2 \\ Q &= 6.5\times10^5 \text{ mm}^3/\text{s} \\ g &= 10^4 \text{ mm/s}^2\end{aligned}$$

Therefore, at the entry of the pouring basin,

$$\begin{aligned}A_1 v_1 &= 6.5\times10^5 \\ v_1 &= 1000 \text{ mm/s}\end{aligned}$$

But

$$\begin{aligned}v_1 &= \sqrt{2gh_1} \\ h_1 &= 50 \text{ mm}\end{aligned}$$

At exit of the sprue,

$$\begin{aligned}v_2 &= \sqrt{2g\times(h+h_1)} \\ &= 2236.06 \text{ mm/s}\end{aligned}$$

For constant Q,

$$\begin{aligned}A_2 v_2 &= 6.5\times10^5 \\ A_2 &= 290.68 \text{ mm}^2\end{aligned}$$

Ans. (c)

13. On cooling, a cubical casting of side 40 mm undergoes 3%, 4% and 5% volume shrinkage during the liquid state, phase transition and solid state, respectively. The volume of metal compensated from the riser is

(a) 2% (b) 7%
(c) 8% (d) 9%

(GATE 2008)

Solution. Risers are used to compensate shrinkage before solidification, therefore, volume of the metal compensated from the riser will be $3+4 = 7\%$.

Ans. (b)

14. Two streams of liquid metal, which are not hot enough to fuse properly, result in a casting defect known as

(a) cold shut (b) swell
(c) sand wash (d) scab

(GATE 2009)

Solution. Cold mating of two streams of metal results in cold shuts. Swell is casting defect found in vertical faces due to deformation of mold face due to pressure. Sand wash is the extra metal which results from erosion of mold due to metal flow on down-end surfaces.

Ans. (a)

15. Match the items in Column I and Column II and select the correct option.

Column I	Column II
P. Metallic chills	1. Support for the core
Q. Metallic chaplets	2. Reservoir of the molten metal
R. Riser	3. Control cooling of critical sections
S. Exothermic padding	4. Progressive solidification

(a) P-1,Q-3,R-2,S-4
(b) P-1,Q-4,R-2,S-3
(c) P-3,Q-4,R-2,S-1
(d) P-4,Q-1,R-2,S-3

(GATE 2009)

Solution. Chills are metallic objects made of the same casting metal placed in the mold to increase and provide uniform cooling rate of the casting, and most importantly, to obtain directional solidification to avoid shrinkage cavities at joints. Chaplets are used to support cores inside the mold cavity to take care of its own weight, and overcome the metallic forces. Risers are provided in the gating systems to compensate liquid shrinkage of the poured metal. Exothermic padding is used to control cooling of critical sections.

Ans. (d)

16. In a gating system, the ratio of 1:2:4 represents

(a) sprue base area: runner area: ingate area
(b) pouring basin area: ingate area: runner area
(c) sprue base area: ingate area: casting area
(d) runner area: ingate area: casting area

(GATE 2010)

Solution. The area ratio in the sequence of "sprue: runner: ingate" is very important in gating system. If pressure is applied on metal stream, the gating system is called pressurized one and for this ratio is 1:2:1. However, when stream flow is due to natural gravity force, the ratio is kept at 1:4:4.

Ans. (a)

17. Green sand mold indicates that

(a) Polymeric mold has been cured
(b) Mold has been totally dried
(c) Mold is green in color
(d) Mold contains moisture

(GATE 2011)

Solution. Green sand is a mixture of sand used in metal casting. It is not necessarily green in color, but is called green because it is used while wet, like the "green wood" sometimes used by wood turners.

Ans. (d)

18. A cubic casting of 50 mm side undergoes volumetric solidification shrinkage and volumetric solid contraction of 4% and 6%, respectively. No riser is used. Assume uniform cooling in all directions. The side of the cube after solidification and contraction is

(a) 48.32 mm (b) 49.90 mm
(c) 49.94 mm (d) 49.96 mm

(GATE 2011)

Solution. The size of cube after solidification and contraction will be

$$a = \sqrt[3]{0.96 \times 0.94} \times 50$$
$$= 48.3173 \text{ mm}$$

Ans. (a)

19. A cube-shaped casting solidifies in 5 min. The solidification time in minutes for a cube of the same material, which is 8 times heavier than the original casting will be

(a) 10 (b) 20
(c) 24 (d) 40

(GATE 2013)

Solution. For the two cubes, the ratio of their size (a) and surface area (A) is examined as

$$\frac{V_2}{V_1} = 8$$
$$\frac{a_2^3}{a_1^3} = 8$$
$$\frac{a_2}{a_1} = 2$$
$$\frac{A_2}{A_1} = \frac{a_2^2}{a_1^2} = 2^2 = 4$$

The cooling time is found to be proportional to the square of the volume-to-surface area ratio, thus

$$t_1 \times \left(\frac{A_1}{V_1}\right)^2 = t_2 \times \left(\frac{A_2}{V_2}\right)^2$$
$$t_2 = t_1 \times \left(\frac{A_1}{A_2}.\frac{V_2}{V_1}\right)^2$$
$$= 5 \times \left(\frac{1}{4} \times \frac{8}{1}\right)^2$$
$$= 5 \times 4$$
$$= 20 \text{ min}$$

Ans. (b)

MULTIPLE CHOICE QUESTIONS

1. Let A and V represent the surface area and volume of components under subscripts. A special measure for riser design, freezing ratio, R, is defined as

(a) $(A/V)_{\text{riser}} / (A/V)_{\text{casting}}$
(b) $(A/V)_{\text{casting}} / (A/V)_{\text{riser}}$
(c) $(A/V)_{\text{casting}} \times (A/V)_{\text{riser}}$
(d) none of the above

2. For proper functioning of the riser, freezing ratio must be

(a) equal to zero
(b) less than unity
(c) greater than unity
(d) none of the above

3. The shrinkage of metals during cooling in molds includes

(a) liquid shrinkage
(b) solid shrinkage
(c) both (a) and (b)
(d) none of the above

4. From design point of view, the ratio of the minimum volume of riser to that dictated by the liquid shrinkage consideration is kept at

(a) 1 (b) 2
(c) 3 (d) 4

5. Generally, risers are not required molds for casting of gray cast iron because

(a) liquid shrinkage is negligible
(b) solid shrinkage is negligible
(c) the liquid shrinkage is compensated by solid shrinkage
(d) none of the above

6. Based on the Chvorinov's rule, Caine developed which of the following empirical relationship (x is freezing ratio, y is the ratio of volumes of riser and casting, and a, b, c are constants)

(a) $x = a/(y-b)+c$
(b) $x = a/(y-b)-c$
(c) $x = a/(y+b)+c$
(d) $x = a/(y+b)-c$

7. A spherical casting of 20 mm diameter undergoes volumetric solidification shrinkage and volumetric solid contraction of 2% and 4%, respectively under uniform cooling. The diameter of the casting after solidification and contraction is

(a) 20.00 mm (b) 19.59 mm
(c) 19.39 mm (d) 19.2 mm

8. The decision on the volume of the design riser is based on

(a) Bernoulli's equation
(b) Continuity equation
(c) Newton's law of viscosity
(d) Chvorinov's rule

9. With a solidification factor of 1×10^6 s/m^2, the solidification time (in seconds) for a spherical casting of 100 mm diameter is

(a) 178 (b) 277.78
(c) 300.01 (d) 311

10. While cooling, a cubical casting of side 50 mm undergoes 1%, 2% and 3% volume shrinkage during the liquid state, phase transition and solid state, respectively. The volume of metal compensated from the riser is

(a) 2% (b) 3%
(c) 5% (d) 6%

11. The purpose of chaplets in molding is

(a) to support chills
(b) to support the pattern
(c) to support cores
(d) to achieve directional solidification

12. The velocity of the molten metal stream at the base of a sprue of height h is equal to

(a) $\sqrt{gh}$ (b) $\sqrt{2gh}$
(c) $\sqrt{gh/2}$ (d) $2\sqrt{gh}$

13. In shell molding, the shell is made of

(a) fine sand mixed with resin binder
(b) wax
(c) polystyrene
(d) light metals

14. Shift is a casting defect caused by shifting of

(a) mold box parts (b) pattern
(c) cores (d) all of the above

15. Directional solidification can be ensured by

(a) using chills in the molds
(b) using exothermic materials in the riser or facing sand
(c) increasing thickness of certain sections
(d) all of the above

16. A casting defect formed when two metal streams meet without complete fusion is known as

(a) misrun (b) cold shut
(c) shift (d) hot cracks

17. A casting defect caused by low green strength of the sand when a portion of the sand breaking away from the mold is known as

(a) drop (b) mismatch
(c) misrun (d) flash

18. Select the wrong statements about casting defects

(a) a scar is a shallow blow which occurs on a flat surface of the castings
(b) a blister is a shallow blow like scar with a thin layer of metal covering it
(c) both (a) and (b)
(d) none of the above

19. Core prints are prepared in molds to support

(a) pattern (b) cores
(c) chaplets (d) chills

20. Solidification of casting with complex details is not uniform but it is accompanied by warping distortions. This factor is compensated in pattern as

(a) solidification allowance
(b) camber allowance
(c) draft
(d) shake allowance

21. Excess turbulence in the stream in molding process results in

(a) inclusion of dross or slag
(b) air aspiration into the mold
(c) erosion of the mold walls
(d) all of the above

22. The casting process which uses rotating mold is

(a) slush casting
(b) centrifugal casting
(c) die casting
(d) continuous casting

23. Water-based refractory slurry is used

(a) centrifugal casting
(b) shell molding
(c) die casting
(d) investment casting

24. Approximate value of the thickness of the shell in shell molding, although depend upon curing time, is

(a) 5 mm (b) 10 mm
(c) 15 mm (d) 20 mm

25. The cross-section of all the elements of a gating system should normally be

(a) circular (b) rectangular
(c) oval (d) any of the above

26. Pressure, anywhere in the molten metal stream, should not fall below the atmospheric pressure. This is essential to avoid

(a) atmospheric air effect
(b) respiration effect
(c) aspiration effect
(d) all of the above

27. To avoid aspiration affect, the flow area for stream of the molten metal in vertical sprue should vary with respect to head in

(a) linear
(b) parabolic
(c) exponential
(d) none of the above

28. The stream velocities are reduced, thus the turbulence and aspiration are minimized in

(a) pressurized gating system
(b) non-pressurized gating system
(c) both (a) and (b)
(d) none of the above

29. In a gating system, the ratio of sprue base area: runner area: ingate area is 1:2:4. Thus, it should be a

(a) pressurized gating system
(b) non-pressurized gating system
(c) both (a) and (b)
(d) none of the above

30. During cooling and solidification, the pure metals and alloys have

(a) similar characteristics
(b) different characteristics
(c) unknown characteristics
(d) none of the above

31. When alloys solidify over a short range of temperatures, they form

(a) a wholly columnar structure
(b) a wholly dendritic structure
(c) partially columnar and partially dendritic structure
(d) none of the above

32. According to Chvorinov's rule, solidification time of a casting is

(a) proportional to the square of volume
(b) proportional to the surface area
(c) both (a) and (b)
(d) none of the above

33. Risers should be designed to

(a) ensure directional solidification
(b) maintaining proper temperature gradient
(c) maintain a solidification time longer than that of casting
(d) all of the above

34. In cold chamber die casting process,

(a) molten metal is poured into injection cylinder from where metal is forced into the die cavity
(b) dies are usually cooled by circulating water or oil through various passageways in the die block.
(c) low melting point alloys like Zn, Sn, Pb are cast.
(d) light impurities are collected on the surface layer.

35. Permeability is

(a) the capability of permitting the gas evolved during the molding process
(b) the property to withstand expansion or contraction of the molten mental
(c) the capability of molding sand to withstand its own weight.
(d) the capability to withstand against self-weight and pressure of molten metal.

36. A loose piece pattern is used for

(a) making intricate shapes where removal of all portions of the pattern is not possible
(b) large and axis symmetrical castings
(c) large scale continuous production in machine moldings
(d) intricate castings split at parting line.

37. Cores are used to

(a) produce intricate castings by placing these items inside the mold cavity on core prints.
(b) strengthen and support the pattern
(c) increase the surface finish of castings
(d) enhance the permeability of sand

38. Solid shrinkage of castings is compensated by providing

(a) shrinkage allowance in patterns
(b) bottom risers
(c) shorter length of runner
(d) longer height of sprue

39. Double shrinkage allowance is used for

(a) single piece pattern
(b) split pattern
(c) gated pattern
(d) master pattern

40. Ingates or gates in molding are

(a) a small piece of perforated thin sheet placed in the pouring basin at the top of the down sprue.
(b) the opening though which molten metal enters into mold cavity
(c) the channel through which molten metal is brought into pouring basin plane
(d) the shortest pouring path to avoid high pouring temperatures.

41. Light impurities in centrifugal castings are

(a) collected at outer surface
(b) collected at inner surface
(c) mixed uniformly throughout the casting
(d) thrown away as slug

42. Porosity results in castings due to

(a) decreased solubility of gases in metal during solidification
(b) presence of non-metallic particles in the metal
(c) thermal expansion of mold
(d) prior solidification of moving face of melt

43. Hot chamber die casting is suitable for

(a) high melting point alloys
(b) low melting point alloys
(c) both low and high melting alloys
(d) heavy ferrous metals only

44. Swell is caused by

(a) deformation of vertical mold face
(b) compression of mold face on bottom
(c) thermal expansion of mold
(d) erosion of mold due to metal flow

45. Select the correct statement for rise design

(a) The size of riser should be designed for maximum possible volume but should maintain a solidification time less than that of casting
(b) The size of riser should be designed for minimum possible volume but should maintain a solidification time longer than that of casting
(c) The size of riser should be designed for maximum possible volume but should maintain a solidification time longer than that of casting
(d) The size of riser should be designed for minimum possible volume but should maintain a solidification time less than that of casting

46. Sprue is

(a) a small piece of perforated thin sheet placed in the pouring basin at the top of the down sprue.
(b) the opening though which molten metal enters into mold cavity
(c) the channel through which molten metal is brought into pouring basin plane
(d) the shortest pouring path to avoid high pouring temperatures.

47. Using disposable pattern in investment casting, the molten metal should be poured

(a) slowly
(b) rapidly
(c) normal rate
(d) depends upon the metal

48. If volume of casting is V and surface area is A, the solidification time will be proportional to

(a) V/A (b) A/V
(c) $(V/A)^2$ (d) $(A/V)^2$

49. Hot tears are

(a) the cracks due to residual stresses
(b) the shrinkage cavities due to improper riser
(c) the extra metals due to erosion of mold
(d) the dots found after removing dirt from the surface

50. The investment casting process uses patterns made of

(a) thermosetting resin
(b) synthetic sand
(c) special plastics
(d) wax with polystyrene filler

51. The height of the down-sprue is 175 mm and its cross-sectional area at the base is 200 mm^2. The cross-sectional area of the horizontal runner is also 200 mm^2. Assuming no losses, indicate the correct choice for the time (in seconds) required to fill a mold cavity of volume 10^6 mm^3. (Use $g = 10$ m/s^2).

(a) 2.69 (b) 8.45
(c) 26.72 (d) 84.50

52. A mold has down sprue of height 185 mm and cross sectional area at base 100 mm^2. This sprue feeds a horizontal runner of cross sectional area at ingate 50 mm^2 to fill the mold cavity of 10^6 mm^3. The time required to fill the mold cavity will be

(a) 7.41 s (b) 5.24 s
(c) 3.7 s (d) 2.62 s

53. Bottom gating system is preferred over vertical gating because

(a) it avoids light impurities entering into mold cavity

(b) it avoids splashing and turbulence in the flow
(c) it allows rapid filling of mold cavity
(d) it provides passages to gases

54. Disposable patterns are made of

(a) wood (b) rubber
(c) metal (d) polystyrene

55. The solidification time t_s of a cubical casting is related to its side, a, as

(a) $t_s \propto a^4$ (b) $t_s \propto a^3$
(c) $t_s \propto a^2$ (d) $t_s \propto a$

56. Which of the following materials requires the largest shrinkage allowance, while making a pattern for casting?

(a) Aluminium (b) Brass
(c) Cast iron (d) Carbon steel

57. Light impurities in the molten metal are prevented from reaching the mold cavity by providing a

(a) strainer (b) bottom well
(c) skim bob (d) all of the above

58. Directional solidification in castings can be improved by using

(a) chills and chaplets
(b) chills and padding
(c) chaplets and padding
(d) chills, chaplets and padding

59. Which one of the following methods are used for obtaining directional solidification for riser design?

1. Suitable placement of chills
2. Suitable placement of chaplets
3. Employing padding

Select the correct answer

(a) 1 and 2 (b) 1 and 3
(c) 2 and 3 (d) 1, 2 and 3

60. Gas holes are the casting defects which occurs due to

(a) very high pouring temperature of the metal
(b) insufficient fluidity of the molten metal
(c) absorption of gases by the liquid metal
(d) improper alignment of the mold flasks

61. A spherical drop of molten metal of radius 2 mm was found to solidify in 10 s. A similar drop of radius 4 mm would solidify in

(a) 14.14 s (b) 20 s
(c) 38.20 s (d) 40 s

62. In solidification of metal during casting, compensation for solid concentration is

(a) provided by the oversize pattern
(b) achieved by properly placing risers
(c) obtained by promoting directional solidification
(d) made by providing chills

63. Which one of the following processes produces a casting when pressure forces the molten metal into the mold cavity?

(a) shell molding
(b) investment casting
(c) die casting
(d) continuous casting

64. Small amount of carbonaceous material sprinkled on the inner surface of mold cavity is called

(a) backing sand (b) facing sand
(c) green sand (d) dry sand

65. The relationship between total freezing time t, volume of casting V and its surface area A, according to Chvorinov's rule is

(a) $t = K_m\,(V/A)$ (b) $t = K_m\,(A/V)$
(c) $t = K_m\,(A/V)^2$ (d) $t = K_m\,(V/A)^2$

66. In a green sand molding process, uniform ramming leads to

(a) less chance of gas porosity
(b) uniform flow of molten metal into the mold cavity
(c) greater dimensional stability of the casting
(d) less sand expansion type of casting defect

67. Centrifugally cast products have

(a) large grain structure with high porosity
(b) fine grain structure with high density
(c) fine grain structure with low density
(d) segregation of slug towards the outer skin of the casting

NUMERICAL ANSWER QUESTIONS

1. A molding of an aluminium-alloy casting requires molten-metal flow rate of 180 cm^3/s through a sprue of height 300 mm and diameter of 25 mm at the top. The molten alloy has density of 2650 kg/m^3 and viscosity of 0.005 Pa s. Calculate the sprue diameter at bottom in order to prevent aspiration.

2. A cylindrical casting, having length equal to diameter, solidifies in 15 min. Determine the solidification time for a 27-times heavier cylinder of the same aspect ratio and the same material.

3. A cylindrical riser (height equal to diameter) is designed to feed a cylindrical casting of steel having diameter and height equal to 10 cm both. Determine the diameter of the riser using Caine's equation:

$$x = \frac{a}{y-b} - c$$

where x is the freezing ratio, y is the riser volume /casting volume. For steel, $a = 0.1$, $b = 0.03$, $c = 1.0$.

ANSWERS

Multiple Choice Questions

1.	(b)	**2.**	(c)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(b)	**7.**	(b)	**8.**	(d)	**9.**	(b)	**10.**	(b)
11.	(c)	**12.**	(b)	**13.**	(a)	**14.**	(d)	**15.**	(d)	**16.**	(b)	**17.**	(a)	**18.**	(d)	**19.**	(b)	**20.**	(b)
21.	(d)	**22.**	(b)	**23.**	(d)	**24.**	(a)	**25.**	(a)	**26.**	(c)	**27.**	(b)	**28.**	(b)	**29.**	(b)	**30.**	(b)
31.	(a)	**32.**	(a)	**33.**	(d)	**34.**	(a)	**35.**	(a)	**36.**	(a)	**37.**	(a)	**38.**	(a)	**39.**	(d)	**40.**	(b)
41.	(b)	**42.**	(a)	**43.**	(b)	**44.**	(a)	**45.**	(b)	**46.**	(c)	**47.**	(b)	**48.**	(c)	**49.**	(a)	**50.**	(d)
51.	(a)	**52.**	(b)	**53.**	(b)	**54.**	(d)	**55.**	(c)	**56.**	(d)	**57.**	(c)	**58.**	(b)	**59.**	(b)	**60.**	(c)
61.	(d)	**62.**	(a)	**63.**	(c)	**64.**	(b)	**65.**	(d)	**66.**	(b)	**67.**	(b)						

Numerical Answer Questions

1. 9.66 mm **2.** 135 min **3.** 5 cm

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (b) In riser design, the freezing ratio is defined as

$$\frac{(A/V)_{\text{casting}}}{(A/V)_{\text{riser}}}$$

2. (c) For proper functioning of riser, its metal should solidify after solidification of metal in the mold cavity, therefore, freezing ratio must be greater than unity.

3. (c) The shrinkage of metals during cooling in molds occurs in two stages: liquid shrinkage and solid shrinkage.

4. (c) Minimum volume of riser should be approximately three times that dictated by the liquid shrinkage consideration.

5. (c) Solidification of gray cast iron occurs in two stages. The shrinkage associated with first stage

may well be compensated by expansion during second stage. This is due to graphitization.

6. (b) Based on the Chvorinov's rule, Caine developed following empirical relationship

$$x = \frac{a}{y-b} - c$$

where, x is freezing ratio, y is ratio of volumes of riser and casting, and a, b, c are constants.

7. (b) The diameter of the casting after solidification and contraction will be

$$\begin{aligned} d &= \sqrt[3]{0.98 \times 0.96} \times 20 \\ &= 19.59 \text{ mm} \end{aligned}$$

8. (d) Risers should be designed to maintain a solidification time longer than that of casting, which is determined using Chvorinov's rule.

9. (b) Given that

$$\begin{aligned} K_m &= 1 \times 10^6 \text{ s/m}^2 \\ r &= 0.1/2 = 0.05 \text{ m} \end{aligned}$$

Solidification time is

$$\begin{aligned} t_s &= K_m \left(\frac{V}{A}\right)^2 \\ &= K_m \left(\frac{4\pi r^3/3}{4\pi r^2}\right)^2 \\ &= K_m \left(\frac{r}{3}\right)^2 \\ &= 277.78 \text{ s} \end{aligned}$$

10. (b) Risers are used to compensate shrinkage before solidification, therefore, the volume of the metal compensated from the riser is $1+2 = 3\%$.

11. (c) Chaplets are used to support cores inside the mold cavity to take care of its own weight and overcome the metallic forces.

12. (b) Using Bernoulli's equation, one finds

$$\frac{v^2}{2g} = h$$

13. (a) In shell molding, the shell is made of fine sand mixed with thermosetting resin binder, such as phenol formaldehyde that coats the sand particles.

14. (d) Shift results from shifting of mold box parts, pattern or cores.

15. (d) Directional solidification can be achieved by using chills, exothermic materials or increasing thickness of certain sections.

16. (b) A cold shut is an interface within a casting that is formed when two metals streams meet without complete fusion.

17. (a) A drop occurs on account of a portion of the sand breaking away from the mold, which in turn is seen in the casting.

18. (d) A scar is a shallow blow which occurs on the flat surface of the castings. If it is covered by a thin layer of metal, it is called a blister.

19. (b) For supporting the cores in the mold cavity, an impression, known as core print, in the form of a recess is made in the mold with the help of a projection on pattern.

20. (b) Distortion allowances, also called camber allowance, takes into account this warping or distortion.

21. (d) Excess turbulence is harmful in molding as it results in inclusion of dross, air aspiration and erosion of the mold.

22. (b) Centrifugal casting method uses inertia forces caused by rotation to distribute the molten metal into the mold cavity.

23. (d) In investment casting, removed pattern made of polystyrene pattern from the die is then coated with water-based refractory slurry.

24. (a) The thickness of shell is approximately 5 mm which depends upon curing time.

25. (a) Circular section have lower surface area-to-volume ratio which reduces the heat loss and results in less friction.

26. (c) The gases originating from baking of organic compounds in the mold may enter the molten metal stream producing porous casting. This phenomenon is known as aspiration effect.

27. (b) Flow area for stream should vary in parabolic way with respect to head.

28. (b) In non-pressurized gating system, the areas of runner and gates are enlarged after the sprue base. The stream velocities are reduced, thus the turbulence and aspiration are minimized.

29. (b) In non-pressurized gating system, the areas of runner and gates are enlarged after the sprue base.

30. (b) Pure metals and alloys behave differently on cooling and solidification. In pure metals solidification, crystal growth commences from the mold and extends towards the center. Alloys do not have a sharply defined freezing point temperature.

31. (a) When alloys solidifies over a short range of temperature, it results into a wholly columnar structure. But over a wide range of temperature, then a dendritic structure results.

32. (a) According to Chvorinov's rule, solidification time of a casting is proportional to the square of volume-to-surface area ratio of the casting.

33. (d) Risers should be designed to ensure directional solidification, maintaining proper temperature gradient and its solidification time should be longer than that of the casting.

34. (a) In cold chamber die casting process, molten metal is poured into injection cylinder from where metal is forced into the die cavity.

35. (a) Permeability is the capability of permitting the gas evolved during the molding process.

36. (a) A loose piece pattern is used for making intricate shapes where removal of all portions of the pattern is not possible.

37. (a) Cores are used to produce intricate castings by placing these items inside the mold cavity on core prints.

38. (a) The shrinkage allowance is provided in patterns to compensate the solid shrinkage of castings.

39. (d) Double shrinkage allowance is used for master pattern to take care of shrinkage of actual metal cast as well as the shrinkage of the pattern metal, which is called master pattern metal.

40. (b) In-gates or gates are the opening though which molten metal enters into mold cavity

41. (b) Centrifugal casting method uses inertia forces caused by rotation to distribute the molten metal into the mold cavity. Light impurities due to their light mass are collected at inner surface.

42. (a) Porosity results due to decrease in gas solubility in metal during solidification

43. (b) Hot chamber die casting is suitable for low melting point alloys, such as Zn, Sn, Pb.

44. (a) Swell is found in vertical faces due to deformation of mold face due to pressure.

45. (b) The size of riser should be designed for minimum possible volume, but should maintain a solidification time longer than that of casting.

46. (c) Sprue is the channel through which molten metal is brought into pouring basin plane.

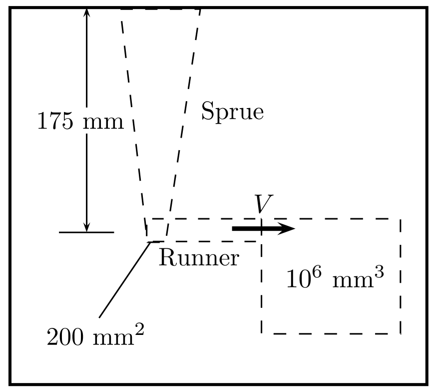
47. (b) Using disposable pattern, the molten metal should be poured rapidly. The velocity of molten metal at the metal polymer pattern from is estimated to be in the range of 0.1-1 m/s.

48. (c) The solidification time is proportional to the $(V/A)^2$

49. (a) Hot tears are cracks caused by residual stresses in the casting caused by solid shrinkage of the metal.

50. (d) Polystyrene is a filler in wax used in investment casting.

51. (a) The given dimensions of mold box are shown in the figure.



The discharge velocity at the runner exit is determined as

$$\begin{aligned} v &= \sqrt{2\times 9.81\times 0.175} \\ &= 1.8529 \text{ m/s.} \\ &= 1852.9 \text{ mm/s} \end{aligned}$$

The filling time is given by

$$\begin{aligned} t &= \frac{10^6}{200\times 1852.9} \\ &= 2.698 \text{ s.} \end{aligned}$$

52. (b) The discharge velocity at the sprue exit is determined as

$$\begin{aligned} v_1 &= \sqrt{2\times 9.81\times 0.185} \\ &= 1.90518 \text{ m/s} \\ &= 1095.18 \text{ mm/s} \end{aligned}$$

The discharge velocity at the runner exit is determined as

$$\begin{aligned} v_2 &= v_1 \times \frac{100}{50} \\ &= 3810.35 \text{ mm/s} \end{aligned}$$

The filling time is given by

$$\begin{aligned} t &= \frac{10^6}{50\times 3810.35} \\ &= 5.24 \text{ s} \end{aligned}$$

53. (b) In bottom gating, liquid metal is filled in mold from bottom to top, thus avoiding splashing and also oxidation (due to aspiration effect) associated with vertical gating.

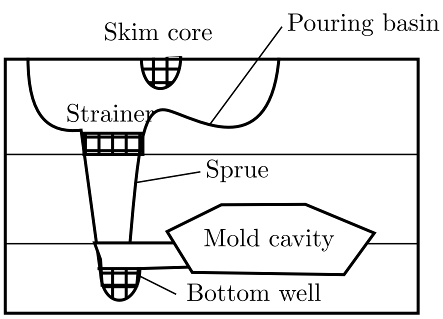
54. (d) There are disposable pattern materials, for example, polyurethane foam for conventional casting where the pattern is burned,

55. (c) Solidification time (t_s) is proportional to $(V/A)^2$. For cubical casting of size a,

$$t_s \propto \left(\frac{a^3}{a^2}\right)^2 = a^2$$

56. (d) Descending order of solid shrinkage in few metals is found as: carbon steel > brass > Al, Mg > cast iron.

57. (c) To avoid the erosion, the molten metal is poured into a pouring basin which acts as reservoir from which it moves smoothly into the sprue. The pouring basin is able to stop the slag entering the mold cavity by means of a skimmer or skim core as shown in the figure.



The skim core holds back the slag and dirt which floats on the top and only allows the clean metal underneath it into the sprue. The strainer does not differentiate between light and heavy impurities. Bottom well is used to trap the heavy impurities.

58. (b) Chills are placed in the mold to increase and uniform cooling rate of the casting by steeper temperature gradient, and most importantly, to obtain directional solidification to avoid shrinkage cavities at joints. Chaplets are used to support cores inside the mold cavity to take care of its own weight and overcome the metallic forces.

59. (b) Chaplets are used to support cores inside the mold cavity to take care of its own weight and overcome the metallic forces. Directional solidification can be achieved by using chills in the molds and exothermic materials in the risers or in facing sand and also by increasing thickness of certain sections by using exothermic padding.

60. (c) Gas holes occur due to poor ventilation of dissolved gases in the melt, particularly hydrogen.

61. (d) The solidification time is given by

$$t_s \propto \left(\frac{V}{A}\right)^2$$

For an sphere of diameter d

$$t_s \propto d^2$$

Therefore, for given two cases

$$t_{s2} = \left(\frac{4}{2}\right)^2 \times 10 = 40 \text{ s}$$

62. (a) Risers are provided in gating systems to compensate liquid shrinkage of poured metal. Shrinkage allowance (i.e. oversize of the pattern) is provided in patterns against the solid shrinkage.

63. (c) In die casting, the molten metal is forced into die cavity (mold made of metal) at high pressure.

64. (b) Sand used next to pattern for better surface finish is composed of high amount of coal dust. This is called facing sand.

65. (d) Chvorinov's rule states that solidification time of a casting t_s is proportional to the square of ratio of volume (V) to surface area (A) of the casting:

$$\tau_s = K_m \left(\frac{V}{A}\right)^2$$

66. (b) Uniform ramming provides greater dimensional stability of the mold.

67. (b) In centrifugal casting, light impurities are collected at center and high density casting are produced.

Numerical Answer Questions

1. Given that $Q = 180$ cm^3/s, $h = 0.30$ m, $d = 0.025$ m. The mass flow rate at inlet of the sprue is calculated as

$$v_1 = \frac{Q}{\pi d^2/4} = \frac{4 \times 180 \times 10^{-6}}{\pi \times 0.025^2} = 0.367 \text{ m/s}$$

The velocity at exist of the sprue will be calculated by using Bernoulli's equation as

$$0.3+\frac{0.367^2}{2g} = \frac{v_2^2}{2g}$$
$$v_2 = 2.453 \text{ m/s}$$

Thus, to avoid aspiration effect, the diameter of the sprue at exist will be determined as

$$\frac{\pi}{4}d_2^2 v_2 = 180\times 10^{-6}$$
$$d_2 = 0.0102 \text{ m}$$
$$= 9.66 \text{ mm}$$

2. Given that $t_1 = 15$ min. Let d be the diameter and length of the original cylinder. The ratio of volume and surface area is

$$\frac{V}{A} = \frac{d\times \pi d^2/4}{2\times \pi d^2/4+\pi d\times d}$$
$$= \frac{\pi d^3}{4\left(\pi d^2\right)\times 3/2}$$
$$= \frac{d}{6}$$

Diameter of a 27-times heavier cylinder shall be

$$d_2 = \sqrt[3]{27}d$$
$$= 3d$$

The cooling time is proportional to the square of the volume to surface area ratio:

$$t_2 = \frac{\left(V_1/A_1\right)^2}{\left(V_2/A_2\right)^2}\times t_1$$
$$= \frac{\left(3d/6\right)^2}{\left(d/6\right)^2}\times 15$$
$$= 3^2\times 15$$
$$= 9\times 15$$
$$= 135 \text{ min}$$

3. Let d (m) be the diameter of the cylindrical riser ($h = d$). Surface area and volume of the steel casting are

$$A = 2\times \frac{\pi}{4}\times 0.1^2+\pi\times 0.1\times 0.1$$
$$= 0.0471 \text{ m}^2$$
$$V = \frac{\pi}{4}0.1^2\times 0.1$$
$$= 7.85\times 10^{-4} \text{ m}^3$$

In the Caine's equation, x and y are calculated as

$$x = \frac{0.0471/7.85\times 10^{-4}}{\left(\pi d^2+\pi d^2/4\right)/\left(\pi d^3/4\right)}$$
$$= \frac{60d}{5}$$
$$= 12d$$
$$y = \frac{\pi d^3/4}{7.85\times 10^{-4}}$$
$$= 1000d^3$$

Therefore,

$$12d = \frac{0.1}{1000d^3-0.03}-1$$
$$12\times 10^3 d^4-0.36d = 0.1-1000d^3+0.03$$

Therefore,

$$12\times 10^3 d^4+1000d^3-0.36d-0.13 = 0$$

Solving, using Newton–Raphson method,

$$x_{n+1} = x_n-\frac{f\left(x\right)}{f'\left(x\right)}$$

$$d = 0.045 \text{ m}$$
$$= 4.5 \text{ cm}$$
$$\approx 5 \text{ cm}$$

CHAPTER 12

FORMING

The processes used for modifying workpiece geometry are called *shaping processes*, and those used for modifying the properties of materials are called *non-shaping processes*, such as heat treatment, surface finishing. The shaping processes can be categorized as mass conserving processes (forming), mass reducing processes (cutting) and mass joining processes (fabrication). Forming is the solid state manufacturing process which is forced into a set of tools called dies where the desired size and shape are obtained through the plastic deformation of the material. Forming processes can be broadly classified as:

1. *Cold Working* Forming performed at a temperature below the recrystallization temperature of work metal is called *cold working*. In cold working, strength and hardness increases due strain hardening, but ductility decreases. Good surface finish and high dimensional accuracy are achieved. If cold working exceeds certain limits, the metal will fracture before reaching the desired size and shape. Therefore, cold working operations are usually carried out in several steps, with intermediate *annealing* operations.

2. *Hot Working* Forming performed at a temperature higher the recrystallization temperature of work metal is called *cold working*. In hot working, refinement of grain size occurs, thus, improving mechanical properties. Even a brittle material can be hot worked. This requires much less force for deformation, but the resulting surface finish and dimensional accuracy are not good. There is no work hardening.

The *coefficient of friction* plays an important role in forming processes and is very important in calculating the forces involved. This is around 0.1 in *cold forming* and 0.6 in *hot forming*. The stresses induced in forming processes are greater than yield stress but lesser than fracture strength. The grains of metal get elongated in the direction of metal flow and form fiber-flow lines. These elongated grains offer more resistance to deformation, thus, better mechanical strength in the direction of flow of metal. This is called *directional effect* of forming processes.

12.1 ROLLING

Rolling is the process of reducing or changing the cross-sectional area of a workpiece by the use of compressive forces exerted by rotating rolls taking advantage of friction between rolls and metal surfaces [Fig. 12.1].

Rolling can be carried out either in warm or cold but it is generally a hot working process (if not mentioned specifically). Hot rolling is often characterized by a con-

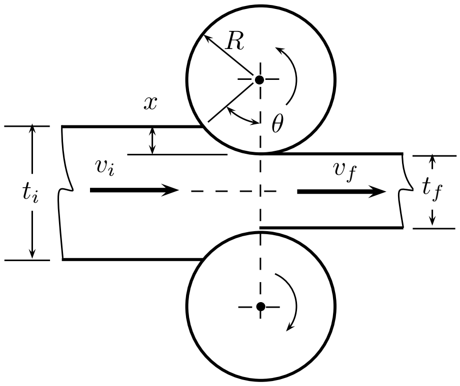


Figure 12.1 | Rolling.

dition called *sticking* when hot workpiece surface adheres to the rolls. Cold rolling is preferred for achieving high dimensional accuracy.

Rolling is characterized by the following parameters:

1. *Friction Force* The rolls pull the work material into the gap through a net frictional force which results in two components of the reaction on the rolls:
   (a) *Roll separating force* along the common normal.
   (b) *Tangential force* in the form of torque required to rotate the rolls.

   For a given reduction in area, the roll separating force increases linearly with roll radius.

2. *Draft* The maximum possible *draft* (Δ) is defined as the difference of initial and final size of the ingot [Fig. 12.1]:
$$\Delta = t_i - t_f$$

3. *Absolute Elongation* *Absolute elongation* (δl) is defined as the difference of the final length (l_f) and the initial length (l_i) of the ingot:
$$\delta l = l_f - l_i$$

4. *Lateral Spread* The reduction in thickness is accomplished with an elongation in both longitudinal and transverse directions. The transverse elongation is called *lateral spread* Δw:
$$\Delta w = w_f - w_i$$
   where w_i and w_f are the initial and final widths, respectively.

5. *Forward and Backward Slips* Since cross-sectional area of the metal decreases after the pass, the velocity of the metal leaving the rolls is always higher than the entering velocity into the rolls. There exists a *neutral point* at which linear velocity of rolls (V_r) at circumference is equal to the velocity of metal flow (V_f) (therefore, there will be no effect of friction).

   Forward slip (s_f) and backward slip (s_b) between the rolls and the workpiece are defined, respectively, as
$$s_f = \frac{v_f - v_r}{v_r}$$
$$s_b = \frac{v_f - v_i}{v_r}$$

6. *Roll Angle* Using principles of geometry, roll angle θ can be expressed as [Fig. 12.1]
$$R\sin\theta \times R\sin\theta = x\,(2R - x)$$
$$x = \frac{R\theta^2}{2}$$
   where x is the depth of the workpiece on the single roll. Therefore, roll angle is determined as
$$\frac{t_i}{2} = \frac{t_f}{2} + \frac{R\theta^2}{2}$$
$$\theta = \sqrt{\frac{t_i - t_f}{R}}$$

7. *Length of Contact* Length of contact can be determined as
$$L = R\theta_i$$
$$= \sqrt{R\,(t_i - t_f)}$$

8. *Roll Pressure* Roll pressure varies along the contact length. It is maximum at the neutral point and trails off on either side of the entry and exit points [Fig. 12.2].

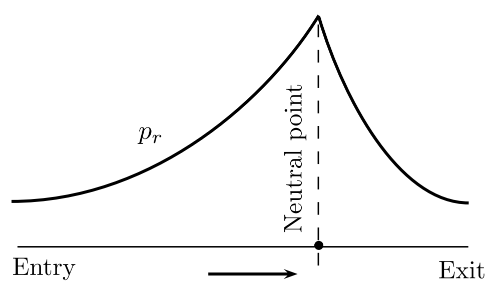


Figure 12.2 | Roll pressure.

Average roll pressure in the roll gap can be calculated as
$$p_r = \frac{F}{A}$$

where F is the roll separating force F and A is the projected area A of a roll on workpiece.

9. Bite Angle *Bite angle* is defined as the roll angle covered by the metal that can be rolled into the rolls without using any pushing or pulling force (i.e. only with use of friction).

   If the metal input has to enter the rolls unaided, the horizontal component of the friction force for unit width must at least be equal to the horizontal component of the roll separating force:

$$\mu \times (p\Delta l \cos\theta_i) \geq p\Delta l \sin\theta_i$$
$$\mu \geq \tan\theta_i$$
$$\theta_i \approx \mu$$

   Thus, bite angle depends on the value of coefficient of friction between the rolls and metal ingot.

10. Minimum Friction Value of minimum coefficient of friction for specified value of $(t_i - t_f)$ is given by

$$\mu \geq \sqrt{\frac{(t_i - t_f)}{R}}$$

   Although friction is necessary for rolling, energy is dissipated in overcoming the friction. Increase in friction increases the rolling force and the power requirements.

Both softening and hardening phenomena occurs in rolling. In the time between passes, static and meta-dynamic recrystallization and grain growth can happen. In order to achieve good mechanical properties in a rolled product, grain growth, and phase transformation can be controlled by controlled rolling and cooling.

Sometimes, *backing rolls* are used to reduce roll deflection. Since the roll separating force depends upon the radius of the drive rolls, these are always kept small in size, whereas backing rolls are provided with large radius to increase rigidity.

12.2 FORGING

Forging involves forming of material by the application of localized compressive forces exerted manually or with power hammers, presses or special forging machines. The process can be carried out on materials in either hot or cold state, but it is generally a hot working operation. Forged parts have good strength and toughness.

Low carbon steels can be forged at temperature higher than those of high carbon steels. Forging of plain carbon steel is done at 1300°C. Cast irons cannot be forged due to their brittleness. Typical forged parts include rivets, bolts, crane hooks, crank shafts, connecting rods, gears, turbine shafts, hand tools, railroads, and a variety of structural components used to manufacture machinery.

When forging is done manually on open dies, it is called *smith forging*. Forging with closed impression dies by a drop hammer is called *drop forging*. If, instead of drop hammer, squeezing is applied in one stroke with the use of press, it is called *press forging*. If machines are used for upsetting, it is called *machine forging*.

Important methods of forging processes are described as follows:

1. Open Die Forging *Open die forging* is the simplest forging process in which the material is shaped by manipulating the work material between blows of specially shaped tools or hammer in open die. The process is flexible but unsuitable for large scale production due to slowness and dependency on the skill of the operator. Therefore, it is most often used in preparing work material for subsequent forging operations. *Fullering* is an open forging operation used to produce a shape with length much greater than its cross-section by simultaneously elongating the workpiece.

2. Closed Die Forging *Closed die forging*, sometimes called *impression die forging*, uses closed shaped dies to control the flow of metal. The heated metal is positioned in the lower cavity and on it one or more blows are struck by the upper die. This makes the metal to flow and completely fill the die cavity. Excess metal is squeezed out around the periphery of the cavity in the form of flash which is trimmed off later.

3. Precision Forging *Precision forging* is used for economy and greater precision in which the metal is deformed in cavity so that no flash is formed and the final dimensions are very close to the desired component dimensions. Aluminium and magnesium alloys are more suitable although steel can also be precision forged. Typical precision forged components are gears, turbine blades, fuel injection nozzles, and bearing casings.

4. Press Forging *Press forging* is performed by using hydraulic press to obtain slow and squeezing action instead of a series of blows as in drop forging. This enables uniform deformation throughout the entire depth of the workpiece. Press forgings generally need smaller draft than drop forgings and have greater dimensional accuracy. Dies are generally heated during press forging to reduce heat loss, promote more uniform metal flow and production of finer details.

5. Upset Forging *Upset forging* involves increasing the cross-section of a material at the expense of its corresponding length. The process uses split dies

having one or several cavities. Upon separation of split die, the heated bar is moved from one cavity to the next. The split dies are then forced together to grip the bar and a heading tool (or ram) advances axially against the bar, upsetting it to completely fill the die cavity. Upon completion of the process, the heading tool comes back and the movable split die releases the stock. Parts produced by upset forging include fasteners, valves, nails, and couplings.

6. Roll Forging *Roll forging* is used to reduce the thickness of round or flat bar with the corresponding increase in length. In this process, with the rotation of rolls through half a revolution, the bar is progressively squeezed and shaped. The bar is then inserted between the next set of smaller grooves and the process is repeated till the desired shape and size are achieved [Fig. 12.3].

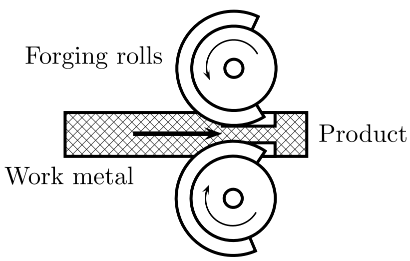


Figure 12.3 Roll forging.

Examples of products by this process include leaf springs, axles, and levers.

7. Swaging In *swaging*, the diameter of a rod or a tube is reduced by forcing it into a confining die. A set of reciprocating dies provides radial blows to cause the metal to flow inward and acquire the form of the die cavity. Screwdriver blades and soldering iron tips are typical examples of swaged products.

Orbital cold forging is a forming process in which the workpiece is subjected to a combined rolling and pressing action between a lower die having a cavity into which the workpiece is compressed, and a swiveling upper die with a conical working face. Typical parts forged by this process are cylindrical shaped and conical parts, such as bevel gears, gear blanks.

12.3 EXTRUSION

Extrusion is used to reduce the cross-section of a block by forcing it to flow through a die orifice under high pressure. The process is based on the plastic deformation of a material due to compressive and shear forces only; no tension stresses are applied. Thus, brittle materials can also be extruded. The process differs from wire drawing in that the metal is pushed in a compression manner rather than pulled under tension.

The following are the two basic types of extrusion processes:

1. Forward Extrusion In *forward extrusion*, the confined metal is forced to flow in the direction of punch travel. Maximum plunger force is required at the start because frictional forces between billet and chamber wall is at the maximum. Force reduces gradually with the plunger movement. Therefore, lubrication is achieved by using oil or graphite or molten glass for extruding steels [Fig. 12.4].

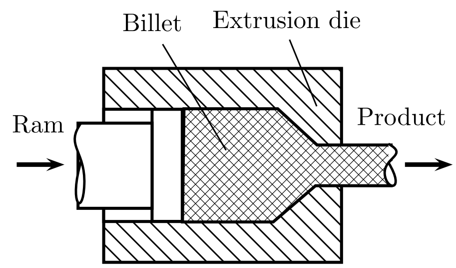


Figure 12.4 Forward extrusion.

Hydrostatic extrusion is a forward hot extrusion in which the billet in the sealed container is surrounded with a fluid or hydrostatic medium that exerts uniform distribution of pressure. Because of the pressurized fluid, lubrication (molten glass) is very effective, and the extruded product has a good surface and dimensional accuracy. Hydrostatic extrusion is used for making reactor fuel rods, cladding of metals, wires, etc. The process is also used for lead coating of copper cables.

2. Backward Extrusion In *backward extrusion*, the confined metal is forced to flow in a direction opposite to that of the punch travel. The descending punch enters the slug and the pressure displaces the metal forward through the opening. Backward extrusion involves less frictional force because of lesser contact of moving metal but this results in more surface defects [Fig. 12.5].

Impact extrusion is a backward extrusion in which slag is placed in a die cavity and struck by a punch, forcing the metal to flow back around the punch through an opening. The process is usually performed on a high speed mechanical press. It is limited to softer metals, such as lead, tin, aluminium, and copper.

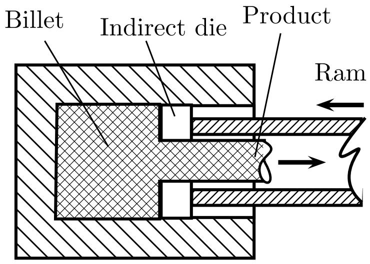


Figure 12.5 Backward extrusion.

Extrusion is used for making small products, such as gears for clocks, watches, typewriters. The advantage of extrusion include the ability to produce a variety of shapes of high strength, good accuracy, and surface finish at high production speeds with a relatively low die cost.

12.4 WIRE DRAWING

Wire drawing is a cold working operation for fabrication of long lengths of small diameter wire (upto 0.01 mm) with good dimensional accuracy. This is done by preparing pointed tip by using swaging process. The ductile material is drawn through the tungsten carbide die by pulling it out. Indirect compression during the drawing causes plastic deformation of the work metal [Fig. 12.6].

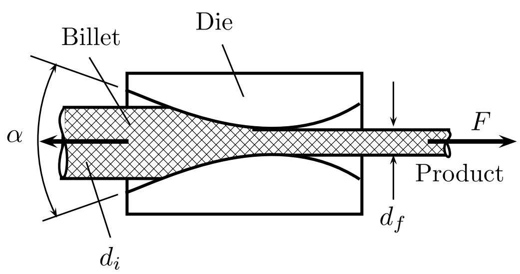


Figure 12.6 Drawing.

To avoid the deformation after the wire has emerged from die, maximum drawing stress is limited to 60% of the yield stress of the emerging product, which limits the reductions to about 35%.

A lubricant is used to reduce friction on the die surface. This can also be achieved by applying coating of ferrous hydroxide (by *sulling process*) or phosphate of manganese iron or zinc (by *phosphating process*). Intermediate annealing is required to restore the ductility.

Let a rod of diameter d_i is reduced to diameter d_f by wire drawing operation [Fig. 12.6]. Yield strength of the work material is σ and coefficient of friction between work material and die is μ. The flow stress for the drawing operation is calculated as

$$\sigma_f = \sigma\frac{1+\phi}{\phi}\left\{1-\left(\frac{d_o}{d_i}\right)^{2\phi}\right\}$$

where

$$\phi = \mu \tan\alpha$$

Power requirement is calculated as

$$P = \sigma_f \times \frac{\pi d_f^2}{4} \times v$$

For maximum reduction, $\sigma_f = \sigma$:

$$\sigma_f = \sigma\frac{1+\phi}{\phi}\left\{1-\left(\frac{d_f}{d_i}\right)^{2\phi}\right\}$$

$$\frac{d_f}{d_i} = \left\{1-\frac{\phi}{1+\phi}\right\}^{\frac{1}{2\phi}}$$

$$= \left(\frac{1}{1+\phi}\right)^{\frac{1}{2\phi}}$$

The percentage reduction in diameter is

$$\left(1-\frac{d_f}{d_i}\right)\times 100$$

Small diameter wire is generally drawn on tandom machines which consists of a series of dies, each held in a water cooled die block. Each die reduces the cross section by a small amount so as to avoid excessive strain in the wire. Intermediate annealing of material between different states of wire can also be done, if required.

12.5 SHEET METAL FORMING

When thickness of the rolled metal is less than 6 mm, it is called sheet metal and above that, it is called plate. Thus, sheet metals possess high ratio of surface area to thickness. Sheet metal forming involves cutting the blanks from sheet metal and then bending or forming the blanks or drawing them into desired shapes. This includes shearing, blanking, punching, bending, drawing, embossing, spinning, bending, etc., which are described in the following subsections.

12.5.1 Shearing

Shearing operations are carried out by applying forces on the sheet metal through a pair of die and punch,

and the material gets sheared from the two edges of die and punch. These sheared surfaces should match to produce clear cuts. To achieve this, *clearance* is provided on punch or die or on both, depending upon the requirement of the process. *Penetration* is the movement of punch end beyond the thickness of the stock sheet to help easy shearing [Fig. 12.7].

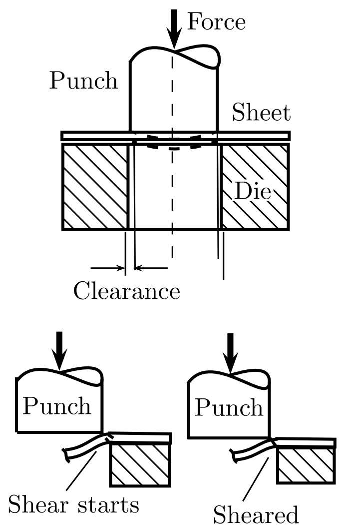


Figure 12.7 Shearing.

12.5.1.1 Punching Force Consider a shearing process in which a sheet metal of thickness t is sheared to form a blank of diameter d. Shearing force is the punching force required to punch to hole:

$$F_p = (\pi d \times t)\,\tau$$

where τ is the shear strength of the sheet material. This expression is valid for $d > t$. When $d < t$, then shearing becomes complicated and the following expression is used:

$$F_p = \frac{dt\sigma_t}{\sqrt[3]{d/t}}$$

where σ_t is the tensile strength of the stock material.

The peak value of punching force can be reduced by distributing the cutting action over a period of time, but the total energy requirement remains the same. This is achieved by grinding a *shear* on the face of the die or punch. Let the punch penetrate by p fraction of sheet metal thickness t. Shear of amount t_s is provided on the punch. Equating the energy requirements in punching operations with shear and without shear:

$$F_s t_s = F_p pt$$
$$F_s = \frac{pt}{t_s} \times F_p$$

Thus, punching force is reduced by fraction pt/t_s.

12.5.1.2 Spring Back When the applied load on deformed metal is removed, the plastic component of the deformation remains permanently, but the elastic component springs back to its original shape. This phenomenon is termed as *spring back.* Materials with higher yield points have more spring back. Spring back depends upon material, shape and thickness. Thicker materials and small holes require more stripping force than thinner materials. Due to spring back, stock material get stuck to punch surface. To avoid it, angular clearance can be provided on the die. Also, a *stripper* can be used that separates the punch from the stock, which tends to grip the punch due to spring back.

12.5.1.3 Shearing Operations There are various operations involving shearing of the sheet metal. Out of these, blanking and punching are considered two broad classifications. *Blanking* is a sheet metal operation in which blanks are sheared from a metal sheet to be used for next processing, while *punching* is producing holes in a blank sheet by removing scrap from holes. Thus, in punching, the sheared slug is discarded as scrap while in blanking, it is a useful part for further processing. These operations are followed by *shaving* for removing of burrs (sharp edges). *Notching* is the process in which cuts are made at the edge of the stock. *Nibbing* is the continuous notching process for cutting an edge in the stock sheet. All these operations are not forming operations because a finite volume of sheet metal is removed. *Silting* is shearing by circular blades or shears which draw the stock material through the shears as they rotate.

12.5.2 Drawing

Drawing is used for making cups, shells, from metal blanks by slowly descending the punch over stock sheet [Fig. 12.8]. Drawing is generally a *deep drawing* in which height of cup drawn is more than the radius. It is called *shallow drawing* if cup height is less than radius of the cup. Corners in die and blank are provided with rounding radius to allow smooth flow of metal.

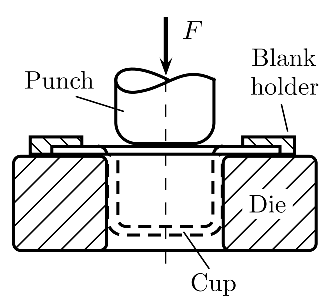


Figure 12.8 Drawing.

The process is characterized by the following parameters:

1. Drawing Ratio *Drawing ratio* (DR) in drawing operation is defined as

$$\text{DR} = \frac{d_b}{d_p}$$

where d_b is the blank diameter and d_p is the punch diameter. Drawing ratio must be less than 2.0 for a feasible operation. If it is more than 2.0, the progressive deep drawing is applied. Also, if it is more than 1.2, blank holder is used.

2. Blank Size Outer diameter of the cup is $d_o = d_i + 2t$ while outer diameter of the flange is $d_f = d_o + 2w$. Then, blank diameter (d_b) will be given by

$$\frac{1}{4}\pi {d_b}^2 t = \left\{\frac{1}{4}\pi {d_f}^2 + h\pi d_o\right\} t$$

$$d_b = \sqrt{{d_f}^2 + 4d_o h}$$

3. Drawing Force The work metal must possess a combination of ductility and strength so that it does not rupture in the critical area (where the metal bends from the punch face to the vertical portion of the punch). Drawing force is given by

$$F_d = \pi d_o t \sigma_t \times \left(\frac{d_b}{d_o} - c\right)$$

where $c = 0.6$ to include friction and bending effects.

The energy required for drawing operation can be determined as

$$W = F_d \times h$$

4. Blank Holding Force During this operation, outer circumference of the blank reduces causing a compressive hoop stress which when exceeds limit can result in a plastic wrinkling of flange of the cup drawn. Blank holders are provided to avoid this. Blank holding force is considered to be one-third of the drawing force. Improper application of the holding force can cause defects, such as wrinkling, tearing.

5. Drawing Speed Aluminium stock can be drawn upto maximum 0.80 m/s and steel stock up to 0.28 m/s. With slower movements, material gets time to flow instead of sudden shearing.

Ironing is a form of drawing operation used for thinning the side walls of drawn cups uniformly.

The earlier tests developed to examine the formability where *cupping tests*, such as the Erichsen and Olsen tests (stretching) and the Swift and Fukui tests (drawing). Cupping tests measure the capability of the material to be stretched before fracturing.

12.5.3 Spinning

Spinning is performed on a lathe machine for making symmetrical vessel-like objects from sheet metal stock, such as cylinder heads, ornaments. A complementary wooden block (mandrel) is set on head stock end on which sheet metal is supported from the back by use of tail stock. Whole assembly is rotated and sheet metal is forced against the mandrel [Fig. 12.9].

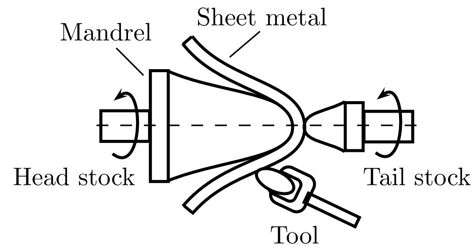


Figure 12.9 | Spinning.

Shear spinning is a spinning operation in which the thicknesss of sheet metal is also reduced during the shaping.

12.5.4 Bending

Bending is a sheet metal forming operation used to introduce finite radius on flat sheets (infinite radius) not only to form shapes but also to provide stiffness to the component by increasing its moment of inertia. Plastic deformation in this process occurs only in the bend region and the material away from the bend is not deformed [Fig. 12.10].

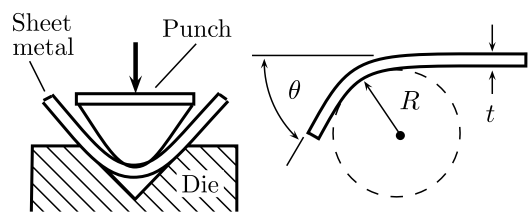


Figure 12.10 | Bending.

During bending, outer fibers of the sheet metal are forced to stretch and come under tension, while the inner fibers come under compression. Therefore, there is a *neutral axis* or plane that separates the tension and compression zones. The position of neutral axis depends on the radius and angle of bend. When material reaches the plastic stage, the neutral axis moves downward since the material opposes compression much better than tension.

Additional bending is required to compensate *spring back*. Bending should be along the grains' flow, otherwise cracks will be generated at outer periphery. Because of the Poisson's ratio, the width of the part in the outer region is smaller and larger in the inner region.

Bend allowance (BA) is the length of the arc of the neutral line axis between the tangent points of a bend in any material, calculated as

$$\text{BA} = \theta\,(R+Kt)$$

where θ is the bend angle in radians, R is inside bend radius, K is a constant known as *stretch factor* or *K-factor*, t is material thickness [Fig. 12.10]. Bend allowance is the addition in the length of the each flange taken between the center of the radius to give the flat pattern length.

Sometimes, *lancing* is done to make bending easy. This is achieved by cutting a small edge at the bending point. During bending operation, contact is made only at upper die and two parallel edges of the V-shaped depression.

12.5.5 Stretch Forming

Stretch forming is another method of producing bent sheets without any local buckling and wrinkling. This is achieved by firmly holding the metal strip under tension during the operation. A die is forced against the metal [Fig. 12.11].

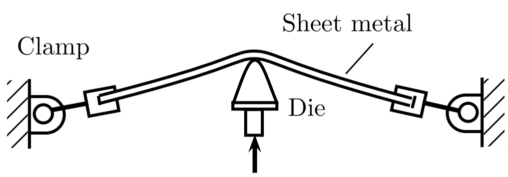


Figure 12.11 Stretch forming.

The deformation in this process is in plastic stage, therefore, spring back is prohibited. Materials having high strain-hardening exponent n have the best stretch formability.

12.5.6 Embossing

Embossing is a cold working operation used for producing raised figures on a sheet stock by means of punch and die of matching cavity contours and leaving very little effect on thickness of work metal [Fig. 12.12].

The process is used to provide dimples on the sheets for rigidity and decoration.

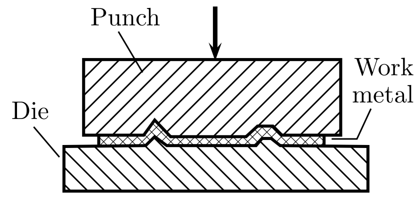


Figure 12.12 Embossing.

12.5.7 Coining

Coining is a cold working and closed-die forging operation used mainly for minting coins and making jewelry from sheet metals. Flow of metal occurs only at the top layer and not in the entire volume. Fine details are produced by applying pressure as high as five to six times the strength of the material. Lubricants are not employed in this process because they can get entrapped in the die cavities and, being incompressible, prevent the full reproduction of fine details of the die.

12.6 HIGH-ENERGY RATE FORMING

High energy rate forming (HERF) involves very high order of the rate of energy flow for a very short interval of time. HREF is based on the principle that the kinetic energy of a moving body is proportional to the square of its velocity, and therefore, a large amount of energy can be supplied by a relatively smaller body moving at a high speed. The following are three common HERF processes:

1. Explosive Forming In *explosive forming*, the metal sheet is clamped over an evacuated die and the whole assembly is kept in a tank confined within a fluid medium (e.g. water). A shock wave in the fluid medium is generated by detonating an explosive charge placed at certain distance from the workpiece in the confined space. Typical explosives include trinitrotoluene (TNT) and dynamite for higher energy, and gun powder for lower energy. The pressure of the shock is sufficiently high to form the metal into die cavity. There is a greater hazard of die failure in the confined operation due to the inevitable lack of control in explosive forming.

2. Electro-Hydraulic Forming Electric discharge in the form of sparks, instead of explosives, can also be used to generate a shock wave in a fluid. An operation using this principle of generating a shock wave is called *electro-hydraulic forming*. The process is also called *under-water spark* or *electric-discharge forming*. In this process, a capacitor

bank is charged through the charging circuit; subsequently, the switch is closed, resulting in a spark within the electrode gap to discharge the capacitors.

3. Electro-Magnetic Forming In *electro-magnetic forming*, electrical energy is first stored in a capacitor bank. It is then discharged through a magnetic coil by closing the switch. The coil produces a magnetic field whose intensity depends on the value of the current. Since the metallic workpiece is in this dynamic magnetic field, a current is induced in the job which sets up its own magnetic field through eddy currents. The forces produced by the two magnetic fields result in a net repelling force between the coil and sheet, which forms the workpiece into the die. The workpiece in process has to be electrically conductive but need not be magnetic.

12.7 POWDER METALLURGY

Powder metallurgy is a metal working process for forming precision metal components from metal powders by compacting in a die.

12.7.1 Process Details

The following are the basic steps of powder metallurgy:

1. Powder Production Metallic powders can be produced by numerous processes: grinding, electrode position, comminution, atomization, chemical reduction, etc. In *atomization*, a liquid metal stream produced by injecting molten metal through a small orifice and the stream is broken up by jets of inert gas, air or water. Powder is also produced by *reduction* of metal oxides using hydrogen or carbon monoxide, as reducing agents. Mechanical *comminution* involves crushing, milling in a ball mill or grinding brittle or less ductile metals into small particles.

2. Powder Mixing The process of mixing includes mixing of various metal powders with lubricants as a result of which the powders are thoroughly intermingled. This is carried out in batch mixers. The temperature during mixing affects the friction between powder particles. With increasing temperature, the friction coefficient between most materials increases and the flow of powders is impaired.

3. Compacting A controlled amount of the mixed powder is introduced into a precision die and then it is pressed or compacted at room temperature and a pressure in the range 100 MPa to 1000 MPa. In doing so, the loose powder is consolidated and densified into a shaped model. The model is generally called *green compact*. As it comes out of the die, the compact has the size and shape of the finished product. Strength of the compact is just sufficient for in-process handling and transportation to the sintering furnace.

4. Sintering Sintering involves heating of the green compact in a protective atmosphere furnace to a suitable temperature below the melting point of the metal. Typical sintering atmospheres are endothermic gas, exothermic gas, dissociated ammonia, hydrogen, and nitrogen. Sintering is responsible for producing physical and mechanical properties by developing metallurgical bond among the powder particles. It also serves to remove the lubricant from the powder, prevents oxidation, and controls carbon content in the part.

Porosity is a unique and inherent characteristic of powder metallurgy which can be exploited to create special products by filling the available pore space with oils, polymers, or metals. This is categorized as: impregnation and infiltration. *Impregnation* is the process in which oil or other fluid is permeated into the pores of a sintered part, for example in oil-impregnated bearings, gears, and similar components. *Infiltration* is an operation in which the pores of the powder metallurgy part are filled with a molten metal using capillary action.

12.7.2 Applications

Some prominent PM products are as follows:

1. Filters PM filters have greater strength and shock resistance than ceramic filters. Fiber metal filters, having porosity upto 95% and more, are used for filtering air and fluids.

2. Cutting Tools and Dies Cemented carbide cutting tool inserts are produced from tungsten carbide powder mixed with cobalt binder.

3. Machinery Parts Gears, bushes and bearings, sprockets, rotors are made from metal powders mixed with sufficient graphite to give the product desired carbon content.

4. Bearing and Bushes Bearing and bushes to be used with rotating parts are made from copper powder mixed with graphite.

5. Magnets Small magnets produced from different compositions of powders of iron, aluminium, nickel and cobalt have shown excellent performance, far superior to those cast.

IMPORTANT FORMULAS

Rolling

$$\theta = \sqrt{\frac{t_i - t_f}{R}}$$

$$L = \sqrt{R\,(t_i - t_f)}$$

$$\theta_i \approx \mu$$

$$\mu \geq \sqrt{\frac{(t_i - t_f)}{R}}$$

Punching Force

$$F_p = (\pi d \times t \quad d > t)\,\tau$$

$$= \frac{dt\sigma_t}{\sqrt[3]{d/t}} \quad d < t$$

$$F_s = \frac{pt}{t_s} \times F_p$$

Drawing

$$d_b = \sqrt{{d_f}^2 + 4d_o h}$$

$$F_d = \pi d_o t \sigma_t \times \left(\frac{d_b}{d_o} - c\right)$$

Bending

$$\mathrm{BA} = \theta\,(R + Kt)$$

SOLVED EXAMPLES

1. A strip with a cross-section 160 mm × 8 mm is being rolled with 25% reduction of area using 500 mm diameter rolls. Determine the angle subtended by the deformation zone at the roll center.

Solution. Given that

$$t_i = 8 \text{ mm}$$
$$t_f = t_i\,(1 - 0.25) = 6 \text{ mm}$$
$$R = 250 \text{ mm}$$

Angle (rad) subtended by the deformation zone on the roll center is

$$\theta = \sqrt{\frac{t_i - t_f}{R}} = 0.089 \text{ rad}$$

2. A strip is to be rolled from a thickness of 40 mm to 25 mm using a two-high mill having rolls of diameter 450 mm. What would be the approximate coefficient of friction for unaided bite?

Solution. The value of minimum coefficient of friction for specified value of $(t_i - t_f)$ is given by

$$\mu_{min} = \sqrt{\frac{(t_i - t_f)}{R}} = \sqrt{\frac{(40 - 25)}{225}} = 0.26$$

3. Determine the diameter of blank to produce a shell of 240 mm diameter and 120 mm height by deep drawing of a 6 mm sheet metal.

Solution. The blank diameter is determined as

$$d_b = \sqrt{d^2 + 4dh} = \sqrt{240^2 + 4 \times 240 \times 120} = 415.69 \text{ mm}$$

4. In blanking of a sheet metal of thickness 6 mm, the maximum punch load 450 kN is used with 25% penetration. Determine the work done during shearing.

Solution. The work done is determined as

$$W = F \times tp = 4.5 \times 10^3 \times \frac{6}{1000} \times \frac{25}{100} = 675 \text{ J}$$

5. In a blanking operation to produce steel washer, the maximum punch load used is 5×10^5 N. The plate thickness is 8 mm and percentage penetration is 30%. Determine the work done during this shearing operation.

Solution. The work done during shearing is

$$W = 5 \times 10^5 \times \frac{8}{1000} \times \frac{30}{100} = 1200 \text{ J}$$

6. Determine the percentage increase in punching force required in a blanking operation of mild steel sheet if diameter of the blank is increased by 25% and thickness is reduced by 4%.

Solution. The punching force is given by

$$F = \pi d t \tau$$

Therefore,

$$F \propto dt$$
$$F_2 = 1.25 \times 0.96 F_1 = 1.2 F_1$$

Thus, punching force is increased by 20%.

7. Determine the diameter of the punch for punching of a metal disk of 76 mm diameter from a 5 mm-thick sheet of steel having shear strength 250 MPa.

Solution. In punching operation of the disc, the clearance is provided on the punch. Diametral clearance is determined as

$$\begin{aligned} c_d &= 0.0032t\tau \\ &= 0.0032 \times 5 \times 250 \\ &= 4 \text{ mm} \end{aligned}$$

Thus, the punch diameter should be

$$\begin{aligned} d_p &= d + c_d \\ &= 76 + 4 \\ &= 80 \text{ mm} \end{aligned}$$

8. Determine the diameter of the smallest hole that can be punched in a 20 mm thick plate. Ultimate shear strength of material is $\tau = 150$ MPa and allowable crushing stress in the punch is $\sigma_c = 450$ MPa.

Solution. Minimum diameter (d) is given by

$$\begin{aligned} \frac{\pi d^2}{4} \times \sigma_c &= \pi d t \tau \\ d &= 4t\frac{\tau}{\sigma_c} \\ &= 30 \text{ mm} \end{aligned}$$

9. A 120 mm diameter billet is to be extruded to 75 mm diameter. The working temperature of 700°C and the extrusion constant is 300 MPa. Calculate the force required for extrusion.

Solution. Given that

$$\begin{aligned} d_i &= 0.12 \text{ mm} \\ d_f &= 0.075 \text{ mm} \\ k &= 300 \times 10^6 \text{ Pa} \end{aligned}$$

Cross-section areas at initial and final stages are

$$\begin{aligned} A_i &= \frac{\pi}{4} {d_i}^2 \\ A_f &= \frac{\pi}{4} {d_f}^2 \end{aligned}$$

Force required for extrusion is given by

$$\begin{aligned} F &= kA_i \ln\left(\frac{A_i}{A_f}\right) \\ &= 3.12 \times 10^6 \text{ N} \end{aligned}$$

10. Calculate the blanking force during punching of 20 mm diameter holes on a 5 mm thick steel sheet. The punch provided a shear of 2 mm and penetration is 30%. Shear strength of the material is 150 N/mm^2.

Solution. Given that

$$\begin{aligned} d &= 2 \text{ mm} \\ t &= 5 \text{ mm} \\ \tau &= 150 \text{ N/mm}^2 \\ t_p &= 0.4t \\ t_s &= 2 \text{ mm} \end{aligned}$$

Therefore, blanking force is

$$\begin{aligned} F &= \pi d t \tau \left(\frac{t_p}{t_s}\right) \\ &= 4.712 \text{ kN} \end{aligned}$$

11. A sheet of 30 mm thickness is rolled to 20 mm thickness by 650 mm diameter rolls rotating at 120 rpm. Calculate the roll strip contact length.

Solution. Given that

$$\begin{aligned} t_i &= 30 \text{ mm} \\ t_f &= 20 \text{ mm} \\ R &= \frac{650}{2} = 325 \text{ mm} \end{aligned}$$

Roll strip contact length is

$$\begin{aligned} L &= \sqrt{R\,(t_i - t_f)} \\ &= \sqrt{325\,(30-20)} \\ &= 57 \text{ mm} \end{aligned}$$

12. A 5 mm thick metal sheet is to be bent at an angle of one radian with a bend radius of 120 mm. Determine the bend allowance for stretch factor 0.6.

Solution. Given that

$$\begin{aligned} t &= 5 \text{ mm} \\ \theta &= 1 \text{ rad} \\ R &= 120 \text{ mm} \\ K &= 0.6 \end{aligned}$$

Therefore, bend allowance (BA) is calculated as

$$\begin{aligned} \text{BA} &= \theta\,(R + Kt) \\ &= 1\,(120 + 0.6 \times 5) \\ &= 123 \text{ mm} \end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. Cold working of steel is defined as working

(a) at its recrystallization temperature
(b) above its recrystallization temperature
(c) below its recrystallization temperature
(d) at two-thirds of the melting temperature of the metal

(GATE 2003)

Solution. A forming process is called cold working if it is done at a temperature below than the recrystallization temperature, otherwise it is called hot working.

Ans. (c)

2. A shell of 100 mm diameter and 100 mm height with the corner radius of 0.4 mm is to be produced by cup drawing. The required blank diameter is

(a) 118 mm (b) 161 mm
(c) 224 mm (d) 312 mm

(GATE 2003)

Solution. Given that

$$d = 100 \text{ mm}$$
$$h = 100 \text{ mm}$$
$$r = 0.4 \text{ mm}$$
$$d \geq 20r$$

Assuming no change in thickness, the blank diameter is given by

$$\pi \frac{{d_b}^2}{4} = \pi \frac{d^2}{4} + \pi d \times h$$
$$d_b = \sqrt{d^2 + 4dh}$$
$$= 223.607 \text{ mm}$$

Ans. (c)

3. A brass billet is to be extruded from its initial diameter of 100 mm to a final diameter of 50 mm. The working temperature of 700°C and the extrusion constant is 250 MPa. The force required for extrusion is

(a) 5.44 MN (b) 2.72 MN
(c) 1.36 MN (d) 0.36 MN

(GATE 2003)

Solution. Given that

$$d_i = 0.1 \text{ mm}$$
$$d_f = 0.05 \text{ mm}$$
$$k = 250 \times 10^6 \text{ Pa}$$

Cross-section areas at initial and final stages are

$$A_i = \frac{\pi}{4} {d_i}^2$$
$$A_f = \frac{\pi}{4} {d_f}^2$$

Force required for extrusion is given by

$$F = k A_i \ln\left(\frac{A_i}{A_f}\right)$$
$$= 2.72198 \times 10^6 \text{ N}$$

Ans. (b)

4. A metal disc of 20 mm diameter is to be punched from a sheet of 2 mm thickness. The punch and the die clearance is 3%. The required punch diameter is

(a) 19.88 mm (b) 19.94 mm
(c) 20.06 mm (d) 20.12 mm

(GATE 2003)

Solution. Given that

$$d = 20 \text{ mm}$$
$$t = 2 \text{ mm}$$

Clearance 3% is based on the thickness, therefore, diameter of the punch is

$$d_p = d - 2 \times \frac{3}{100} \times t$$
$$= 19.88 \text{ mm}$$

Ans. (a)

5. 10 mm diameter holes are to be punched in a steel sheet of 3 mm thickness. Shear strength of the material is 400 N/mm^2 and penetration is 40%. Shear provided on the punch is 2 mm. The blanking force during the operation will be

(a) 22.6 kN (b) 37.7 kN
(c) 61.6 kN (d) 94.3 kN

(GATE 2004)

Solution. Given that

$$d = 0.01 \text{ m}$$
$$t = 3 \text{ mm}$$
$$\tau = 400 \times 10^6 \text{ N/m}^2$$
$$t_p = 0.4t$$
$$t_s = 0.002 \text{ m}$$

Therefore, blanking force is

$$F = \pi d t \tau \left(\frac{t_p}{t_s}\right)$$
$$= 22619.5 \text{ N}$$

Ans. (a)

6. In a rolling process, sheet of 25 mm thickness is rolled to 20 mm thickness. Roll is of diameter 600 mm and it rotates at 100 rpm. The roll strip contact length will be

(a) 5 mm (b) 39 mm
(c) 78 mm (d) 120 mm

(GATE 2004)

Solution. Given that

$$t_i = 0.025 \text{ mm}$$
$$t_f = 0.020 \text{ mm}$$
$$R = \frac{0.600}{2} = 0.300 \text{ mm}$$

Roll strip contact length is

$$L = \sqrt{R\,(t_i - t_f)}$$
$$= 38.7298 \text{ mm}$$

Ans. (b)

7. Match the following

Product		Process	
P.	Molded luggage	1.	Injection molding
Q.	Packaging containers for liquid	2.	Hot rolling
R.	Long structural shapes	3.	Impact extrusion
S.	Collapsible tubes	4.	Transfer molding
		5.	Blow molding
		6.	Coining

(a) P-1, Q-4, R-6, S-3
(b) P-4, Q-5, R-2, S-3
(c) P-1, Q-5, R-3, S-2
(d) P-5, Q-1, R-2, S-2

(GATE 2004)

Solution. Hot rolling is used for making long structural shapes in metals. Impact extrusion is used for making of collapsible tubes.

Ans. (b)

8. A 2 mm thick metal sheet is to be bent at an angle of one radian with a bend radius of 100 mm. If the stretch factor is 0.5, the bend allowance is

(a) 99 mm (b) 100 mm
(c) 101 mm (d) 102 mm

(GATE 2005)

Solution. Given that

$$t = 2 \text{ mm}$$
$$\theta = 1 \text{ rad}$$
$$R = 100 \text{ mm}$$
$$K = 0.5.$$

Therefore, bend allowance (BA) is calculated as

$$\text{BA} = \theta\,(R + Kt)$$
$$= 101 \text{ mm}$$

Ans. (c)

9. A 4 mm thick sheet is rolled with 300 mm diameter rolls to reduce thickness without any change in its width. The friction coefficient at the work-roll interface is 0.1. The minimum possible thickness of the sheet that can be produced in a single pass is

(a) 1.0 mm (b) 1.5 mm
(c) 2.5 mm (d) 3.7 mm

(GATE 2006)

Solution. Given that

$$t_i = 4 \text{ mm}$$
$$R = \frac{300}{2} = 150 \text{ mm}$$
$$\mu = 0.1$$

Maximum thickness of rolled sheet is possible when

$$\mu = \sqrt{\frac{t_i - t_f}{R}}$$
$$t_f = 2.5 \text{ mm}$$

Ans. (c)

10. In a wire drawing operation, diameter of a steel wire is reduced from 10 mm to 8 mm. the mean flow stress of the material is 400 MPa. The ideal force required for drawing (ignoring friction and redundant work) is

(a) 4.48 kN (b) 8.97 kN
(c) 20.11 kN (d) 31.41 kN

(GATE 2006)

Solution. Given that

$$d_f = 8 \text{ mm}$$
$$\sigma_f = 400 \text{ MPa}$$

Drawing force will be

$$F = \frac{\pi d_f^2}{4}\sigma_f = 20.1062 \text{ kN}$$

Ans. (c)

11. Match the items in Columns I and II.

	Column I		Column II
P.	Wrinkling	1.	Yield point elongation
Q.	Orange peel	2.	Anisotropy
R.	Stretcher strains	3.	Large grain size
S.	Earring	4.	Insufficient blank holding force
		5.	Fine grain size
		6.	Excessive blank holding force

(a) P-6, Q-3, R-1, S-2
(b) P-4, Q-5, R-6, S-1
(c) P-2, Q-5, R-3, S-4
(d) P-4, Q-3, R-1, S-2

(GATE 2006)

Solution. Excessive blank holding force in deep drawing causes wrinkling. Orange peel represents large grain size as when metal with a coarse grain size is drawing, the surface roughens and develops an appearance resembling orange peel. Stretcher strain is related to yield point elongation. Earring (wavy surface) is due to anisotropy of properties of materials.

Ans. (d)

12. In open-die forging, a disc of diameter 200 mm and height 60 mm is compressed without any barreling effect. The final diameter of the disc is 400 mm. The true strain is

(a) 1.986 (b) 1.686
(c) 1.368 (d) 0.602

(GATE 2007)

Solution. Given that

$$d_i = 0.2 \text{ m}$$
$$d_f = 0.4 \text{ m}$$

True strain is

$$\varepsilon = \ln\left(\frac{A_f}{A_i}\right) = 2\ln\left(\frac{d_f}{d_i}\right) = 1.386$$

Ans. (c)

13. The thickness of a metallic sheet is reduced from an initial value of 16 mm to a final value of 10 mm in one single pass rolling with a pair of cylindrical rollers each of diameter of 400 mm. The bite angle in degree will be

(a) 5.936 (b) 7.936
(c) 8.936 (d) 9.936

(GATE 2007)

Solution. Given that

$$t_i = 0.016 \text{ m}$$
$$t_f = 0.010 \text{ m}$$
$$R = 0.2\text{m}$$

Bite angle is determined as

$$\tan\theta = \sqrt{\frac{t_i - t_f}{R}}$$
$$\theta = 9.836^\circ$$

Ans. (c)

14. Match the correct combination for following metal working processes.

	Process		Stress
P.	Blanking	1.	Tension
Q.	Stretch forming	2.	Compression
R.	Coning	3.	Shear
S.	Deep drawing	4.	Tension and compression
		5.	Tension and shear

(a) P-2, Q-1, R-3, S-4
(b) P-3, Q-4, R-1, S-5
(c) P-5, Q-4, R-3, S-1
(d) P-3, Q-1, R-2, S-4

(GATE 2007)

Solution. Blanking is a sheet metal operation in which blanks are sheared (shear) from a metal sheet to be used for further processing. Stretch forming is another method of producing bent

(bending) sheets without any local buckling and wrinkling. Coining is a cold working closed die forging operation (by compressive stress) on a sheet used for making coins, medals, etc. Deep drawing is used for making deep cups, shells, from metal blanks by slowly descending the punch over stock sheet in which both tension and compression occur.

Ans. (d)

15. The force requirement in a blanking operation of low carbon steel sheet is 5.0 kN. The thickness of the sheet is t and diameter of the blanked part is d. For the same work material, if the diameter of the blanked part is increased to $1.5d$ and thickness is reduced to 0.4 t, the new blanking force in kN is

(a) 3.0 (b) 4.5
(c) 5.0 (d) 8.0

(GATE 2007)

Solution. Given that $F_1 = 5.0$ kN. Blanking force is

$$\begin{aligned} F_b &\propto dt \\ F_2 &= 1.5 \times 0.4 F_1 \\ &= 3.0 \text{ kN} \end{aligned}$$

Ans. (a)

16. Match the most suitable manufacturing processes for the following parts

Parts		Processes	
P:	Computer chip	1:	Electro-mechanical machining
Q:	Metal forming dies and molds	2:	Ultrasonic machining
R:	Turbine blade	3:	Electro-discharge machining
S:	Glass	4:	Photo-chemical machining

(a) P-4, Q-3, R-1, S-2
(b) P-4, Q-3, R-2, S-1
(c) P-3, Q-1, R-4, S-2
(d) P-1, Q-2, R-4, S-3

(GATE 2007)

Solution. Computer chips are manufactured by photochemical machining. EDM is used for forming dies and molds. Turbine blades are produced by ECM. USM is used for brittle material, such as glass.

Ans. (a)

17. In a single pass rolling operation, a 20 mm thick plate with plate width of 100 mm, is reduced to 18 mm. The roller radius is 250 mm and rotational speed is 10 rpm. The average flow stress for the plate material is 300 MPa. The power required for the rolling operation in kW is closest to

(a) 15.2 (b) 18.2
(c) 30.4 (d) 45.6

(GATE 2008)

Solution. Given that

$$\begin{aligned} t_i &= 20 \text{ mm} \\ t_f &= 18 \text{ mm} \\ b &= 100 \text{ mm} \\ R &= 250 \text{ mm} \\ \omega &= 20\pi \\ \sigma_f &= 300 \text{ MPa} \end{aligned}$$

Length of contact is

$$\begin{aligned} L &= R\theta_i \\ &= \sqrt{R(t_i - t_f)} \\ &= 0.0223 \text{ mm.} \end{aligned}$$

Minimum value of coefficient of friction:

$$\begin{aligned} \mu &= \sqrt{\frac{t_i - t_f}{R}} \\ &= 0.08944 \end{aligned}$$

Frictional torque on one roll

$$\begin{aligned} T &= \mu \times \sigma_f b L \times R \\ &= 1500 \text{ Nm} \end{aligned}$$

Power required to run both rolls

$$\begin{aligned} P &= 2 \times \frac{T\omega}{1000} \\ &= 31.41 \text{ kW} \end{aligned}$$

Ans. (b)

18. In the deep drawing of cups, blanks show a tendency to wrinkle up around the periphery (flange). The most likely cause and remedy of the phenomenon are, respectively,

(a) Buckling due to circumferential compression; Increases blank holder pressure
(b) High blank holder pressure and high friction; Reduces blank holder pressure and apply lubricant
(c) High temperature causing increase in circumferential length; apply coolant to blank

(d) Buckling due to circumferential compression; decrease blank holder pressure

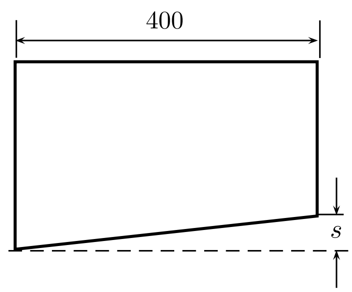
(GATE 2008)

Solution. During deep drawing operation, outer circumference of the blank reduces, thus causing a compressive hoop stress which when exceeds limit may result in a plastic wrinkling of flange of the cup drawn. To avoid this, blank holders are provided over the blank.

Ans. (b)

Linked Answer Questions

In a shear cutting operation, a sheet of 5 mm thickness is cut along a length of 200 mm. The cutting blade is 400 mm long and zero-shear ($s = 0$) is provided on the edge. The ultimate shear strength of the sheet is 100 MPa and penetration to thickness ratio is 0.2. Neglect friction.



19. Assuming force versus displacement curve to be rectangular, the work done (in J) is

(a) 100 (b) 200
(c) 250 (d) 300

(GATE 2010)

Solution. Given that

$$\tau = 100 \times 10^6$$
$$l = 0.2$$
$$t = 0.005$$
$$k = 0.2$$

Therefore, required shear force is

$$\begin{aligned} F &= \tau \times t \times l \\ &= 100.0 \text{ kN} \end{aligned}$$

The punch needs to be displaced by kt distance, therefore, work done is

$$\begin{aligned} W &= F \times kt \\ &= 100 \text{ J} \end{aligned}$$

Ans. (a)

20. A shear of 20 mm ($s = 20$ mm) is now provided on the blade. Assuming force versus displacement curve to be trapezoidal, the maximum force (in kN) exerted is

(a) 5 (b) 10
(c) 20 (d) 40

(GATE 2010)

Solution. When shear of 20 mm is provided in the blade, it will be effective on the length of the sheet by $s = 0.010$ m, and the punch need to be displaced total by $(kt + s)$. As the force-displacement curve will be trapezoidal, the maximum force F in kN will be given by

$$\begin{aligned} F\,(kt + s) &= W \\ F &= \frac{100}{0.2 \times 0.005 + 0.01} \\ &= 9.090 \\ &\approx 10 \text{ kN} \end{aligned}$$

Ans. (b)

21. The maximum possible draft in cold rolling of sheet increases with the

(a) increase in coefficient of friction
(b) decrease in coefficient of friction
(c) decrease in roll radius
(d) increase in roll velocity

(GATE 2011)

Solution. Higher friction increases the maximum draft possible, because

$$(t_i - t_f) \leq \mu^2 R$$

Ans. (a)

22. The operation in which oil is permeated into the pores of a powder metallurgy product is known as

(a) Mixing (b) Sintering
(c) Impregnation (d) Infiltration

(GATE 2011)

Solution. Impregnation is when oil or other fluid is permeated into the pores of a sintered part. For example, oil-impregnated bearings, gears, and similar components.

Ans. (c)

23. The shear strength of a sheet metal is 300 MPa. The blanking force required to produce a blank of 100 mm diameter from a 1.5 mm thick sheet is close to

(a) 45 kN (b) 70 kN
(c) 141 kN (d) 3500 kN

(GATE 2011)

Solution. Given that

$$d = 0.1 \text{ m}$$
$$t = 1.5 \text{ mm}$$
$$\tau = 300 \times 10^6 \text{ N/m}^2$$

Therefore, blanking force is

$$F_b = \pi d t \tau$$
$$= 141.372 \text{ kN}$$

Ans. (c)

24. A solid cylinder of diameter 100 mm and height 50 mm is forged between two frictionless flat dies to a height of 25 mm. The percentage change in diameter is

(a) 0 (b) 2.07
(c) 20.7 (d) 41.4

(GATE 2012)

Solution. During forging, the volume remains constant, therefore, diameter after forging is given by

$$d^2 \times 25 = 100^2 \times 50$$
$$d = 141.421$$

The original diameter is 100 mm, therefore, change is 41.42%.

Ans. (d)

25. Match the following metal forming processes with their associated stresses in the workpiece.

	Process		Stress
P.	Coining	1.	Tensile
Q.	Wire drawing	2.	Shear
R.	Blanking	3.	Tensile and compressive
S.	Deep drawing	4.	Compressive

(a) P-4, Q-1, R-2, S-3
(b) P-4, Q-1, R-3, S-2
(c) P-1, Q-2, R-4, S-3
(d) P-1, Q-3, R-2, S-4

(GATE 2012)

Solution. Coining is a closed die forging process used mainly for minting coins and making jewelry. Wire drawing is primarily the same as bar drawing except that it involves smaller diameter material that can be coiled. Blanking is a sheet metal operation in which blanks are sheared from a metal sheet to be used for next processing. Deep drawing is used for making cups in which height of the cup drawn is more than the radius.

Ans. (a)

26. Calculate the punch size in mm, for a circular blanking operation for which details are given below:

Size of the blank	=	25 mm
Thickness of the sheet	=	2 mm
Radial clearance between punch and die	=	0.06 mm
Die allowance	=	0.05 mm

(a) 24.83 (b) 24.89
(c) 25.01 (d) 25.17

(GATE 2012)

Solution. Size of the punch will be

$$d_p = 25 - 2 \times 0.06 - 0.05$$
$$= 24.83 \text{ mm}$$

Ans. (a)

27. In a single pass rolling process using 410 mm diameter steel rollers, a strip of width 140 mm and thickness 8 mm undergoes 10% reduction of thickness. The angle of bite in radians is

(a) 0.006 (b) 0.031
(c) 0.062 (d) 0.600

(GATE 2012)

Solution. Given that

$$R = \frac{410}{2} \text{ mm}$$
$$t_i = 8 \text{ mm}$$
$$t_f = 0.9 \times 8 \text{ mm}$$

Therefore, bite angle is determined as

$$\tan\theta = \sqrt{\frac{t_i - t_f}{R}}$$
$$\theta = 0.0624695 \text{ rad}$$

Ans. (c)

28. In a rolling process, the state of stress of the material undergoing deformation is

(a) pure compression
(b) pure shear
(c) compression and shear
(d) tension and shear

(GATE 2013)

Solution. The rolls exerts compressive stresses on the metal under going rolling process, while the material deforms by shear due to tangential forces along the roll surface.

Ans. (c)

MULTIPLE CHOICE QUESTIONS

1. Forming is a process in which the desired size and shape are obtained through

(a) plastic deformation
(b) elastic deformation
(c) visco-elastic deformation
(d) all of the above

2. Plastic deformation is always followed by elastic recovery upon removal of the load. In bending, this recovery is known as

(a) wrinkling
(b) springback
(c) lancing
(d) all of the above

3. Negative spring-back is the phenomenon generally associated with

(a) bending
(b) stretch forming
(c) bending with V-die
(d) all of the above

4. Hemming is a process in which the edge of the sheer is folded over itself. It is done to

(a) increase the stiffness of the part
(b) improve appearance
(c) eliminate sharp edges
(d) all of the above

5. Most important characteristic of forming processes is

(a) directional effect
(b) material saving
(c) energy saving
(d) time saving

6. The recrystallization rate (R_c) varies with temperature (T) in the form as

(a) $R_c \propto e^{-T}$ (b) $R_c \propto 1-e^{-T}$
(c) $R_c \propto e^{T}$ (d) $R_c \propto 1-e^{T}$

7. In general, cold working operations are usually carried out in several steps, with intermediate annealing operations to

(a) bring back the metal to recrystallization temperature
(b) strengthen the cold worked metal to avoid fracture
(c) soften the cold worked metal to restore the ductility.
(d) increase the grain size of metal

8. Coefficient of friction plays an important role in forming processes and is very important in calculating the forces involved. It is in the range of

(a) 0.1 in cold forming and 0.6 in hot forming
(b) 0.6 in cold forming and 0.1 in hot forming
(c) both (a) and (b)
(d) none of the above

9. With increasing angle of shear, the force on punch

(a) increases linearly
(b) decreases linearly
(c) decreases as square of angle of shear
(d) increases as square of angle of shear

10. The required diameter of the blank for deep drawing of a cup of diameter d and height h is given by

(a) $\sqrt{d^2-2dh}$ (b) $\sqrt{d^2+2dh}$
(c) $\sqrt{d^2-4dh}$ (d) $\sqrt{d^2+4dh}$

11. The improper application of the holding force can cause

(a) flange wrinkling
(b) wall wrinkling if it is too small
(c) tearing if it is overestimated
(d) all of the above

12. Bending is preferred along the direction of

(a) flow of grains
(b) normal to the flow of grains
(c) both (a) and (b)
(d) none of the above

13. If θ is the bend angle in radians, r is inside bend radius, k is stretch factor, t is material thickness, then bending allowance BA is equal to

(a) $\theta/(r+kt)$ (b) $\theta/(r-kt)$
(c) $\theta(r+kt)$ (d) $\theta(r-kt)$

14. Stretch forming is used for producing bent sheets without any local buckling and wrinkling. This is achieved by keeping the metal strip during the operation under

(a) tension
(b) compression
(c) high temperature
(d) low temperature

15. One of the major advantage of High Energy Rate Forming (HERF) is

(a) hyper-plasticity
(b) energy saving
(c) high rate of production
(d) all of the above

16. In explosive forming process, the material deformation takes place by

(a) direct impact of explosive particles on the material
(b) a shock wave in the liquid medium generated by detonating an explosive
(c) both (a) and (b)
(d) none of the above

17. In cold working,

(a) strength, hardness and ductility increase
(b) strength and hardness increase, but ductility decreases
(c) strength, hardness and ductility decrease
(d) strength and hardness decrease, but ductility increases

18. In coining, which is a cold working process, lubricants are not used because

(a) lubricant may get entrapped in the die cavities and prevent the full reproduction of fine details of the die.
(b) lubricant may get explode in the die cavities due to high pressure in the process
(c) the pressure very low in the process
(d) lubricant are reactive to the work metal under cold working

19. Mechanical properties of hot worked materials are improved because

(a) recrystallization
(b) recovery of grains
(c) grain growth
(d) refinement of grain size

20. Seamless tubes are made by

(a) piercing
(b) forward extrusion
(c) hot rolling
(d) drawing

21. The operation of removing the flash from the forged products is known as

(a) flashing (b) lancing
(c) trimming (d) burring

22. The process carried out on lathe machines for making symmetrical vessel-like objects from sheet metal stock is known as

(a) rolling (b) turning
(c) spinning (d) drawing

23. Upset forging involves

(a) increasing the cross section of a material at the expense of its length
(b) decreasing the cross section of a material at the extension of its length
(c) reducing diameter of a rod by forcing it into a confining die.
(d) none of the above

24. Large size fasteners are produced by

(a) swaging (b) spining
(c) upset forging (d) roll forging

25. Notching is the process in which

(a) blanks are sheared from a metal sheet
(b) burrs are removed from sharp edges.
(c) cuts are made at the edge of the stock
(d) cuts are made at the center point of the stock

26. The collapsible tubes for pastes are produced by

(a) forward extrusion
(b) impact extrusion
(c) hydrostatic extrusion
(d) deep drawing

27. Sometimes in bending operation, lancing is done to

(a) make bending easy
(b) make the bending difficult
(c) increase the bend allowance
(d) decrease the bend allowance

28. There are several methods of introducing pre-stressed surface layer in compression. The following does not include

(a) shot blasting
(b) lancing
(c) tumbling
(d) cold working by rolling

29. If t_i and t_f are the initial and final thicknesses of the roll material, the angle (θ) subtended by work material on rolls of diameter D is determined as

(a) $\sqrt{(t_i - t_f)/(2D)}$ (b) $\sqrt{(t_i - t_f)/D}$
(c) $\sqrt{2(t_i - t_f)/D}$ (d) $2\sqrt{(t_i - t_f)/D}$

30. If μ is the coefficient of friction and R is the radius of the rolls, the maximum possible reduction in thickness of the work material using external force is

(a) μR (b) $\mu^2 R$
(c) $\mu^3 R$ (d) $\mu^4 R$

31. The roll separating force in rolling process can be reduced by

(a) increasing roll diameter
(b) reducing roll diameter
(c) increasing the coefficient of friction
(d) deploying backing rolls

32. The essential property of a material for drawing operation is

(a) elasticity (b) ductility
(c) plasticity (d) endurance

33. A shell of 150 mm diameter and 100 mm height is to be produced by deep drawing of a 5 mm sheet metal. The required blank diameter is

(a) 300 mm (b) 290 mm
(c) 280 mm (d) 270 mm

34. In blanking operation of a sheet metal of thickness 5 mm, the maximum punch load used is 200 kN and percentage penetration is 30%. The work done during the shearing is

(a) 200 J (b) 300 J
(c) 500 J (d) 900 J

35. The punching force required in a blanking operation of mild steel sheet is 500 kN. The diameter of the blank is increased by 20% and thickness is reduced by 4%, then the punching force will be

(a) 434 kN (b) 576 kN
(c) 634 kN (d) 676 kN

36. A metal disc of 50 mm diameter is to be punched out from a sheet of 5 mm thickness. The shear strength of the material is 200 MPa. The required punch diameter is

(a) 50 mm (b) 53.2 mm
(c) 55 mm (d) 58.2 mm

37. For rigid perfectly plastic work material, negligible interface friction and no redundant work, the theoretically maximum possible reduction in the wire drawing operation is

(a) 0.36 (b) 0.63
(c) 1.00 (d) 2.72

38. The force in punching and blanking operations mainly depends on

(a) the modulus of elasticity of metal
(b) the shear strength of metal
(c) the bulk modulus of metal
(d) the yield strength of metal

39. A wire of 0.1 mm diameter is drawn from a rod of 15 mm diameter. Dies giving reductions of 20%, 40% and 80% are available. For minimum error in the final size, the number of stages and reduction at each stage, respectively, would be

(a) 3 stages and 80% reduction for all the three stages
(b) 4 stages and 80% reduction for first three stages followed by a finishing stage of 20% reduction
(c) 5 stages and reduction of 80%, 80%, 40%, 40%, 20% in a sequence
(d) None of the above

40. A 50 mm diameter disc is to be punched out from a carbon steel sheet 1.0 mm thick. The diameter of the punch should be

(a) 49.925 mm
(b) 50.00 mm
(c) 51.50 mm
(d) None of the above

41. Which one of the following process is performed in powder metallurgy to promote self-lubricating properties in sintered parts?

(a) Infiltration (b) Impregnation
(c) Plating (d) Graphitization

42. In blanking operation, the clearance provided is

(a) 50% on punch and 50% on die
(b) on die
(c) on punch
(d) on die or punch depending upon designer's choice

43. In a blanking operation to produce steel washer, the maximum punch load used is 2×10^5 N. The plate thickness is 4 mm and percentage penetration is 25%. The work done during this shearing operation is

(a) 200 J (b) 400 J
(c) 600 J (d) 800 J

44. Thread rolling is restricted to

(a) ferrous materials
(b) ductile materials
(c) hard materials
(d) none of the above

45. In sheet metal work, the cutting force on the tool can be reduced by

(a) grinding the cutting edges sharp
(b) increasing the hardness of tool
(c) providing shear angle on tool
(d) increasing the hardness of die

46. Which one of the following manufacturing processes requires the provision of 'gutters'?

(a) Closed die forging
(b) Centrifugal casting
(c) Investment casting
(d) Impact extrusion

47. Match List I with List II and select the correct answer using the codes given below

List I	List II
A. Malleabililty	1. Wire drawing
B. Hardness	2. Impact loads
C. Resilience	3. Cold rolling
D. Isotropy	4. Indentation
	5. Direction

(a) A-4, B-2, C-1, D-3
(b) A-3, B-4, C-2, D-5
(c) A-5, B-4, C-2, D-3
(d) A-3, B-2, C-1, D-5

48. In sheet metal blanking, shear is provided on punches and dies so that

(a) press load is reduced
(b) good cut edge is obtained
(c) wrapping of sheet is minimized
(d) cut blanks are straight

49. For obtaining a cup of diameter 25 mm and height 15 mm by drawing, the size of round blank should be, approximately,

(a) 42 mm (b) 44 mm
(c) 46 mm (d) 48 mm

50. The mode of deformation of the metal during spinning is

(a) bending
(b) stretching
(c) rolling and stretching
(d) bending and stretching

51. Which one of the following is an advantage of forging?

(a) Good surface finish
(b) Low tooling cost
(c) Close tolerance
(d) Improved physical property

52. In metals subjected cold working, strain hardening effect is due to

(a) slip mechanism
(b) twining mechanism
(c) dislocation mechanism
(d) fracture mechanism

53. Which of the following components can be manufactured by powder metallurgy methods?

1. Carbide tool tips

2. Bearings
3. Filters
4. Brake linings

Select the correct answer using the codes given below:

(a) 1, 3 and 4 (b) 2 and 3
(c) 1, 2 and 4 (d) 1, 2, 3 and 4

54. In powder metallurgy, the operation carried out to improve the bearing property of a bush is called

(a) infiltration (b) impregnation
(c) plating (d) heat treatment

55. A hole is to be punched in a 15 mm thick plate having an ultimate shear strength of 3 N/mm^2. If the allowable crushing stress in the punch is 6 N/mm^2, the diameter of the smallest hole which can be punched is equal to

(a) 15 mm (b) 30 mm
(c) 60 mm (d) 120 mm

56. In the rolling process, roll separating force can be decreased by

(a) reducing the roll diameter
(b) increasing the roll diameter
(c) providing back up rolls
(d) increasing the friction between the rolls and the metal

57. In the forging operation, fullering is done to

(a) draw out the material
(b) bend the material
(c) upset the material
(d) extrude the material

58. In rolling a strip between two rolls, the position of the neutral point in the arc of contact does not depend on

(a) amount of reduction
(b) diameter of the rolls
(c) coefficient of friction
(d) material of the rolls

59. A forging method for reducing the diameter of a bar and in the process making it longer is termed as

(a) fullering (b) punching
(c) upsetting (d) extruding

60. The process of removing the burrs or flash from a forged component in drop forging is called

(a) swaging (b) perforating
(c) trimming (d) fettling

61. Magnetic forming is an example of

(a) cold forming
(b) hot forming
(c) high energy rate forming
(d) roll forming

62. Which one of the following methods is used for the manufacture of collapsible toothpaste tubes?

(a) Impact extrusion (b) Direct extrusion
(c) Deep drawing (d) Piercing

63. Which one of the following is a high energy rate forming process?

(a) Roll forming
(b) Electro-hydraulic forming
(c) Rotary forging
(d) Forward extrusion

64. Metallic powders can be produced by

(a) atomization
(b) pulverization
(c) electro-deposition process
(d) all of the above

65. The true strain for a low carbon steel bar which is doubled in length by forging is

(a) 0.307 (b) 0.5
(c) 0.693 (d) 1.0

66. What is the force required in punching a 40 mm diameter hole through a 2 mm thick annealed aluminium alloy sheet at room temperature? The shear strength of the alloy is 700 MPa.

(a) 92.56 kN (b) 112.00 kN
(c) 175.92 kN (d) 351.84 kN

NUMERICAL ANSWER QUESTIONS

1. A number of cold rolling passes are required in a two-high rolling mill to reduce the thickness of a plate from 50 mm to 25 mm. The roll diameter is 700 mm and the coefficient of friction at the roll work interface is 0.1. It is required that the draft in each pass must be the same. Assuming no front and back tensions, determine the number of passes required in the rolling.

2. A 210 mm wide copper alloy strip is rolled from a thickness of 24 mm to 14 mm through a set of rolls having a radius of 320 mm and speed of 120 rpm. The coefficient of friction is 0.1. The alloy has strength coefficient $K = 910$ MPa and strain hardening exponent $n = 0.5$. Calculate the power required for rolling.

3. A sheet metal operation is performed on 2 mm thick steel sheet to produce circular discs of 24 mm diameter. The shear strength of the work material is 312 MPa. Determine the size of punch for the blanking operation, and the size of die for the piercing operation.

4. A sheet metal operation is performed to produce holes of 120 mm diameter in a 8 mm thick steel sheet. With normal clearance, the operation involves 35% penetration of the punch. The work material has shear strength of 450 MPa. Calculate the work done in the punching operation. Also calculate the shear angle on the punch in order to bring the work within the capacity of a 300 kN press.

5. A cup of circular cross-section with diameter 42 mm, height 54 mm and corner radius of 2 mm is to be drawn from a steel sheet of 0.8 mm thickness. Maximum allowed percentage reduction in first, second and third draws are 38%, 30%, and 25%, respectively. Determine the required size of blank, and the number of draws required in the process.

6. A metallic strip having 210 mm × 8 mm cross-section area is rolled for 25% reduction in cross-sectional area using rolls of radius 180 mm. Shear yield stresses of the material before and after rolling are 0.4 kN/mm^2 and 0.42 kN/mm^2, respectively. Determine the angle subtended by the deformation zone at the roll center, and the minimum coefficient of friction if the metal input has to enter the rolls unaided.

7. A rectangular section, 100 mm wide and 60 mm thick, is to be hot rolled through a single pass with rolls of 400 mm diameter. Calculate the maximum possible reduction (of thickness) in the pass if the coefficient of friction is 0.3. Also determine the rolling load if the mean rolling pressure is 500 MPa.

8. A 12.5 mm diameter rod is to be reduced to 10 mm diameter by drawing in a single pass at a speed of 100 m/min. The die angle is 5° and coefficient of friction between the die and steel rod is 0.15. The strength of the work material is 400 MPa. Calculate the power requirement for the drawing process. Also calculate the maximum possible reduction in diameter of the rod.

9. Calculate the minimum reduction possible in a single pass rolling of 8 mm thick sheet with 400 mm diameter rolls. The friction coefficient at the work-roll interface is 0.1.

10. A steel wire is reduced from 6 mm to 4 mm by wire drawing operation. The mean flow stress of the material is 450 MPa. Calculate the force required for wire drawing by ignoring friction and redundant work.

11. A disc of diameter 250 mm and height 80 mm is compressed to 450 mm diameter disc by open-die forging without any barreling effect. Calculate the true strain in the material during the operation.

12. Determine the force required in punching a 30 mm diameter hole through a 5 mm thick annealed aluminium alloy sheet at room temperature. The shear strength of the alloy is 600 MPa.

13. A cylindrical specimen of alloy steel has a diameter of 250 mm and height of 150 mm. It is upset at room temperature by open die forging with flat dies to a height of 75 mm. The coefficient of friction is 0.15. The material has strength coefficient $K = 800$ MPa, and strain hardening exponent $n = 0.19$. Average forging pressure of a solid cylindrical workpiece is estimated as

$$\bar{p} = \sigma_f \left(1 + \frac{\mu d_2}{3h_2}\right)$$

where d_2 and h_2 are the final height and diameter of the workpiece. Calculate the average upsetting force for the process.

ANSWERS

Multiple Choice Questions

1. (a)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (d)
11. (d)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (b)
21. (c)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (c)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (b)
31. (b)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (b)	**36.** (b)	**37.** (b)	**38.** (b)	**39.** (d)	**40.** (c)
41. (b)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (c)	**46.** (a)	**47.** (b)	**48.** (a)	**49.** (c)	**50.** (d)
51. (d)	**52.** (a)	**53.** (d)	**54.** (a)	**55.** (b)	**56.** (a)	**57.** (a)	**58.** (c)	**59.** (a)	**60.** (c)
61. (c)	**62.** (a)	**63.** (b)	**64.** (c)	**65.** (c)	**66.** (c)				

Numerical Answer Questions

1. 8	**2.** 4309.6 kW	**3.** 23.8 mm, 24.2 mm
4. 3.8 kJ, 9.34°	**5.** 104.09 mm, 3	**6.** 7.396°, 0.129
7. 18 mm, 3000 kN	**8.** 53 kW, 19.45%	**9.** 6 mm
10. 5.65 kN	**11.** 1.75	**12.** 141.37 kN
13. 83.32 MN		

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (a) Forming is defined as a process in which the desired size and shape are obtained through the plastic deformation of the material.

2. (b) Plastic deformation is always followed by elastic recovery upon removal of the load. In bending, this recovery is known as springback.

3. (c) The phenomenon of negative spring-back is generally associated with V-die bending, whereby the bend angle becomes larger after the bend has been created and the load is removed.

4. (d) Hemming is a process in which the edge of the sheer is folded over itself to increase the stiffness of the part, improve appearance and eliminate the sharp edges.

5. (a) The elongated grains offer more resistance to deformation, thus, better mechanical strength in the direction of flow of metal. This is called directional effect of forming processes.

6. (c) Recrystallization rate (R_c) increases exponentially with temperature (T):

$$R_c \propto e^T$$

7. (c) If the cold working exceeds certain limits, the metal will fracture before reaching the desired size and shape. Intermediate annealing restores the ductility, thus making the material suitable for further cold working.

8. (a) Coefficient of friction is around 0.1 in cold forming and 0.6 in hot forming.

9. (b) The shearing force decreases linearly with the angle of shear.

10. (d) If t is the thickness of the sheet which remains constant, for the volume constancy in the deep drawing,

$$\frac{\pi d_b^2}{4}t = \left(\frac{\pi d^2}{4} + \pi dh\right)t$$

$$d_b = \sqrt{d^2 + 4dh}$$

11. (d) The improper application of the holding force can cause wrinkling of flange, wall and tearing.

12. (a) Bending should be along the grains flow otherwise cracks will be generated at outer periphery.

13. (c) If θ is the bend angle in radians, r is inside bend radius, k is stretch factor, t is material thickness, then bending allowance BA is expressed as

$$\text{BA} = \theta\,(r + kt)$$

14. (a) Stretch forming is another method of producing bent sheets without any local buckling and wrinkling. This is achieved by keeping the metal strip under tension during the operation.

15. (a) One major advantage of HERF is that because the shaping of the material occurs so quickly, the material can alter in its metallurgical characteristics, becoming much more ductile. This change in characteristics is known as hyper-plasticity.

16. (b) In explosive forming process, a shock wave in the fluid medium (normally water) is generated by detonating an explosive charge in a confined space.

17. (b) In cold working, strength and hardness increases due strain hardening, but ductility decreases.

18. (a) Lubricants are not employed in this process because they can get entrapped in the die cavities and, being incompressible, prevent the full reproduction of fine details of the die.

19. (d) In hot working, refinement of grain size occurs, thus, improving mechanical properties.

20. (b) Forward hot extrusion is used for mass production of seamless tubes with the use of spider die held in position by legs.

21. (c) Removal of flash from the forged parts is termed as trimming.

22. (c) Spinning is done on lathe machines for making symmetrical vessel like objects from sheet metal stock, for example, cylinder heads, ornaments.

23. (a) Upset forging involves increasing the cross-section of a material at the expense of its corresponding length. Parts produced by upset forged parts include fasteners, valves, nails, and couplings.

24. (c) Parts produced by upset forged parts include fasteners, valves, nails, and couplings.

25. (c) Notching is the process in which cuts are made at the edge of the stock.

26. (b) Backward cold extrusion is also called impact extrusion is used for making collapsible tubes for pastes.

27. (a) Sometimes, lancing is done to make bending easier. This is achieved by cutting a small edge at the bending point.

28. (b) Lancing is done to make bending easy. This is achieved by cutting a small edge at the bending point.

29. (c) The angle θ is expressed as

$$\theta = \sqrt{\frac{2\,(t_i - t_f)}{D}}$$

30. (b) The maximum reduction is written as

$$\mu = \sqrt{\frac{2\,(t_i - t_f)}{D}}$$
$$t_i - t_f = \mu^2 R$$

31. (b) For a given reduction in area, the roll separating force (which tends to bend the rolls) increases linearly with roll radius. Sometimes, backing rolls are used to reduce roll deflection.

32. (b) Ductility is the most important property of material in drawing processes.

33. (b) The blank diameter is determined as

$$\begin{aligned} d_b &= \sqrt{d^2 + 4dh} \\ &= \sqrt{150^2 + 4 \times 150 \times 100} \\ &= 287.22 \end{aligned}$$

34. (b) The work done is determined as

$$\begin{aligned} W &= F \times tp \\ &= 2 \times 10^3 \times \frac{5}{1000} \times \frac{30}{100} \\ &= 300\ \text{J} \end{aligned}$$

35. (b) The punching force is determined as

$$\begin{aligned} F &= \pi d t \tau \\ &= 1.2 \times 0.96 \times 500 \\ &= 576\ \text{kN} \end{aligned}$$

36. (b) In punching operation of the disc, the clearance is provided on the punch. The clearance is determined as

$$\begin{aligned} c &= 0.0032 t\tau \\ &= 0.0032 \times 5 \times 200 \\ &= 3.2\ \text{mm} \end{aligned}$$

Thus, the punch diameter should be 53.2 mm.

37. (b) The drawing stress, σ_d for the simplest case of ideal deformation (i.e. no friction or redundant work) can be obtained by

$$\sigma_d = \sigma_y \ln\left(\frac{A_i}{A_f}\right)$$

The limiting situation can be developed based on the fact, in the ideal case of a perfectly plastic material with yield stress $\sigma_y = \sigma_d$ when the material will yield and fail. Thus,

$$\ln\left(\frac{A_i}{A_f}\right) = 1$$
$$\frac{A_i}{A_f} = 2.71$$

Thus, limiting reduction is

$$\frac{A_i - A_f}{A_i} = 1 - \frac{A_f}{A_i} = 1 - \frac{1}{2.71} = 0.63$$

38. (b) The punching force is determined as

$$F = \pi d t \tau$$

Thus, it is proportional to the shear strength of the sheet material.

39. (d) The reduction can be checked for all the three options:

$$15 \times 0.8^3 = 7.68$$
$$15 \times 0.8^3 \times 0.2 = 1.536$$
$$15 \times 0.8^2 \times 0.4^2 \times 0.2 = 0.3072$$

Hence, none of the above options is correct.

40. (c) There is an optimum range for the clearance, which is 2 to 10% of the sheet thickness, for the best results. In blanking operation of the disc, the clearance is provided on the punch. Thus, out of the given options 51.50 mm is correct which is greater than diameter of the disc 50 mm.

41. (b) Impregnation is the process in which oil or other fluid is permeated into the pores of a sintered powdered metallurgy part.

42. (c) For blanking operation, die size is equal to the blank size, therefore clearance is provided on the punch.

43. (a) The work done during shearing is

$$W = 2 \times 10^5 \times \frac{4}{1000} \times \frac{25}{100} = 200\ \text{J}$$

44. (b) In thread rolling, a hardened tool (die) with the thread profile is pressed onto a rotating workpiece made of ductile material. This process produces screws with greater strength than machined threads due to the cold working, as well as better material yield.

45. (c) To reduce the required punching force on the punch, angular face, called shear, is ground on the face of the punch or die.

46. (a) Gutters are the spaces provided for excess material and ensure complete and defect-free forged parts.

47. (b) Malleability is the most important property of material in forming processes of cold rolling. Isotropy is the phenomena of material properties being independent of the direction. Resilience is defined as the ability of a material to absorb energy when deformed elastically and to return it when unloaded.

48. (a) To reduce the required punching force on the punch, angular face, called shear, is ground on the face of the punch or die.

49. (c) Given that

$$d = 25\ \text{mm}$$
$$h = 15\ \text{mm}$$

Blank diameter is

$$D = \sqrt{d^2 + 4dh} = 46\ \text{mm}$$

50. (d) Spinning is done on lathe machines for making symmetrical vessel like objects from sheet metal stock.

51. (d) The forged parts have good strength and toughness; they can be used reliably for highly stressed and critical applications.

52. (a) The phenomenon where ductile metals become stronger and harder when they are deformed plastically is called strain hardening or work hardening. Slip is the prominent mechanism of plastic deformation in metals.

53. (d) Cemented carbide cutting tool inserts are produced by powder metallurgy from tungsten carbide powder mixed with cobalt binder. Gears,

bushes and bearings, sprockets, rotors are made from metal powders mixed with sufficient graphite to give to product the desired carbon content. Permanent metal powder filters have greater strength and shock resistance than ceramic filters.

54. (a) Infiltration is an operation in which the pores of the powder metallurgy part are filled with a molten metal using capillary action.

55. (b) The diameter of hole is given by

$$\frac{\pi D^2}{4} \times \sigma = \pi \times 15 \times 3 \times D$$
$$D = 30 \text{ mm}$$

56. (a) The roll separating force (which tends to bend the rolls) increases linearly with roll radius.

57. (a) Fullering is an open forging operation that is used to produce a shape with length much greater than its cross-section by redistributing the material and simultaneously elongating the workpiece, using a die with convex surface. It is used in preparing work material for subsequent forging operations.

58. (c) Neutral point is the point where linear velocity of rolls at circumference and velocity of metal flow will be equal (therefore, there will be no effect of friction). This point is called neutral point.

59. (a) Fullering is used to produce a shape with length much greater than its cross-section by redistributing the material and simultaneously elongating the workpiece. Upset forging involves increasing the cross-section of a material at the expense of its corresponding length.

60. (c) Removal of flash from the forged parts is termed as trimming.

61. (c) Electro-magnetic forming is an example of high energy rate forming.

62. (a) Backward cold extrusion is also called impact extrusion is used for making collapsible tubes for pastes. This is limited for soft and ductile materials with thicker die walls.

63. (b) Electric discharge in the form of sparks, instead of explosives, can also be used to generate a shock wave in a fluid. An operation using this principle of generating a shock wave is called electro-hydraulic forming.

64. (c) There are numerous choices for the processes to produce metallic powders, for example, grinding, electro-deposition, communition, atomization, chemical reduction.

65. (c) Given that $l = 2l_0$. True strain is determined as

$$\begin{aligned}\varepsilon &= \int_0^l \frac{dl}{l} \\ &= \ln\left(\frac{l}{l_0}\right) \\ &= \ln 2 \\ &= 0.693\end{aligned}$$

66. (c) Given that

$$\begin{aligned}d &= 0.03 \text{ m} \\ t &= 0.003 \text{ m} \\ \tau &= 700 \text{ MPa}\end{aligned}$$

Punching force is calculated as

$$\begin{aligned}F &= \pi d t \tau \\ &= \pi \times 0.03 \times 0.003 \times 700 \\ &= 175.92 \text{ kN}\end{aligned}$$

Numerical Answer Questions

1. Given that

$$\begin{aligned}t_i &= 50 \text{ mm} \\ t_f &= 25 \text{ mm} \\ R &= 700/2 \\ &= 350 \text{ mm} \\ \mu &= 0.1\end{aligned}$$

For zero tension in front and back ends, the maximum reduction (draft per pass) is

$$\begin{aligned}\mu &= \sqrt{\frac{\Delta t}{R}} \\ \Delta t &= \mu^2 R \\ &= 0.1^2 \times 350 \\ &= 3.5 \text{ mm}\end{aligned}$$

Thus, the total number of passes required is determined as

$$n = \frac{t_i - t_f}{\Delta t} = \frac{53-25}{3.5} = 8$$

2. The length of contact surface along the roll radius is

$$L = \sqrt{R(t_i - t_o)} = \sqrt{0.32(0.024-0.014)} = 0.0565 \text{ m}$$

The true strain in the specimen is calculated as

$$\varepsilon = \ln\left(\frac{t_i}{t_o}\right) = \ln\left(\frac{0.024}{0.014}\right) = 0.539$$

The average flow stress is given by

$$\bar{\sigma_f} = \frac{K\varepsilon^n}{n+1} = \frac{910\times 0.539^{0.5}}{1+0.5} = 445.39 \text{ MPa}$$

The average thickness of the strip is

$$\bar{t} = \frac{t_i + t_o}{2} = \frac{0.024+0.014}{2} = 0.019 \text{ m}$$

The roll force is calculated as

$$F = Lw\bar{\sigma_f}\left(1+\frac{\mu L}{2\bar{t}}\right) = 0.0565\times 0.210\times 445.39\times\left(1+\frac{0.1\times 0.0565}{2\times 0.019}\right) = 6.07\times 10^6 \text{ N}$$

The total power required for rolling is calculated as

$$P = FL\times\frac{2\pi N}{60} = 6.07\times 10^6\times 0.0565\times\frac{2\pi\times 120}{60} = 4309.6 \text{ kW}$$

3. Given that

$$\tau = 312 \text{ MPa}$$
$$t = 2 \text{ mm}$$

The required value of clearance per side is calculated as

$$c = 0.0032t\sqrt{\tau} = 0.0032\times 2\times\sqrt{312} = 0.113 \approx 0.1 \text{ mm}$$

For blanking operation, die size is equal to the blank size, that is, 24 mm while punch size is calculated as

$$d_p = d_b - 2c = 24 - 0.1\times 2 = 23.8 \text{ mm}$$

For piercing operation, the punch size is equal to the blank size while die size is calculated as

$$d_d = d_b + 2c = 24 + 0.1\times 2 = 24.2 \text{ mm}$$

4. The clearance per side is

$$c = 0.0032t\sqrt{\tau} = 0.0032\times 8\times\sqrt{450} = 0.543 \text{ mm}$$

Maximum punching load (without shear) is

$$F = \pi D t\tau = \pi\times 120\times 8\times 450 = 1.357 \text{ MN}$$

For 35% penetration, work done in punching is

$$W = 1.036\times 10^3\times\left(8\times\frac{35}{100}\right) = 3.8 \text{ kJ}$$

Let p is the additional penetration required with 300 kN press, therefore,

$$300\times(8\times 0.35+p) = 3.8\times 10^3$$
$$p = 9.87 \text{ mm}$$

The shear angle is calculated as

$$\tan\phi = \frac{p}{D/2}$$
$$\phi = \tan^{-1}\left(\frac{9.87}{120/2}\right) = 9.34^\circ$$

5. Given that

$$d = 42 \text{ mm}$$
$$h = 54 \text{ mm}$$

For volume constancy, the size of the blank is calculated as

$$d_b = \sqrt{d^2 + 4dh} = 104.09 \text{ mm}$$

The percentage reduction in the drawing is

$$\left(1 - \frac{d}{d_b}\right) \times 100 = \left(1 - \frac{42}{104.09}\right) \times 100 = 59.65\%$$

The reduction is more than 38%. Therefore, more than one pass will be required. The cup diameter after the first draw is given by

$$38 = \left(1 - \frac{d_1}{d_b}\right) \times 100$$
$$38 = \left(1 - \frac{d_1}{104.09}\right) \times 100$$
$$d_1 = 64.53 \text{ mm}$$

The cup diameter after the second draw is given by

$$30 = \left(1 - \frac{d_2}{d_1}\right) \times 100$$
$$30 = \left(1 - \frac{d_2}{64.53}\right) \times 100$$
$$d_2 = 45.17 \text{ mm}$$

The cup diameter after the third draw is given by

$$25 = \left(1 - \frac{d_3}{d_2}\right) \times 100$$
$$25 = \left(1 - \frac{d_2}{45.17}\right) \times 100$$
$$d_3 = 33.87 \text{ mm}$$

Thus, at least three passes of draw will be required.

6. The initial thickness of the strip is $t_i = 8$ mm and final thickness is $t_f = 0.75 \times t_i = 6$ mm. The radius of rolls is 180 mm. Hence,

$$\theta = \sqrt{\frac{t_i - t_f}{R}} = \sqrt{\frac{8-6}{120}} = 0.129 \text{ rad} = 7.396^\circ$$

Minimum value of coefficient of friction is

$$\mu = \sqrt{\frac{t_i - t_f}{R}} = \sqrt{\frac{8-6}{180}} = 0.129$$

7. Given that

$$R = 400/2 = 200 \text{ mm}$$
$$\mu = 0.3$$
$$\bar{p} = 500 \text{ MPa}$$
$$w = 100 \text{ mm}$$

Using

$$\mu = \sqrt{\frac{t_1 - t_2}{R}}$$
$$t_1 - t_2 = 18 \text{ mm}$$

Length of contact is

$$L = \sqrt{R(t_1 - t_2)} = \sqrt{200 \times 18} = 60 \text{ mm}$$

Rolling load is

$$F = \bar{p} \times L \times w = 500 \times 60 \times 100 = 3000 \text{ kN}$$

8. Given that

$$\sigma = 400 \text{ MPa}$$
$$\mu = 0.15$$
$$\alpha = 5/2 = 2.5^\circ$$
$$d_o = 10 \text{ mm}$$
$$d_i = 12.5 \text{ mm}$$

The flow stress for the drawing operation is calculated as

$$\phi = \mu \tan\alpha = 0.15 \cot\frac{5^\circ}{2} = 3.436$$

$$\sigma_f = \sigma \frac{1+\phi}{\phi}\left\{1 - \left(\frac{d_o}{d_i}\right)^{2\phi}\right\} = 404.98 \text{ MPa}$$

Power requirement is calculated as

$$\begin{aligned} P &= \sigma_f \times \frac{\pi \times d_o^2}{4} \times v \\ &= 404.98 \times \frac{\pi \times 10^2}{4} \times \frac{100}{60} \\ &= 53.01 \text{ kW} \end{aligned}$$

For maximum reduction, $\sigma_f = \sigma$:

$$\begin{aligned} \sigma_f &= \sigma \frac{1+\phi}{\phi} \left\{ 1 - \left(\frac{d_o}{d_i} \right)^{2\phi} \right\} \\ \frac{d_o}{d_i} &= \left\{ 1 - \frac{\phi}{1+\phi} \right\}^{\frac{1}{2\phi}} \\ &= \left(\frac{1}{1+\phi} \right)^{\frac{1}{2\phi}} \\ &= 0.805 \end{aligned}$$

The percentage reduction in diameter is

$$\left(1 - \frac{d_o}{d_i} \right) \times 100 = 19.45\%$$

9. Given that

$$\begin{aligned} t_i &= 8 \text{ mm} \\ R &= \frac{400}{2} \\ &= 200 \text{ mm} \\ \mu &= 0.15 \end{aligned}$$

Maximum thickness of rolled sheet is possible when

$$\begin{aligned} \mu &= \sqrt{\frac{t_i - t_f}{R}} \\ t_f &= 6 \text{ mm} \end{aligned}$$

10. Given that

$$\begin{aligned} d_f &= 4 \text{ mm} \\ \sigma_f &= 450 \text{ MPa} \end{aligned}$$

Drawing force will be

$$\begin{aligned} F &= \frac{\pi d_f^2}{4} \sigma_f \\ &= 5.65 \text{ kN} \end{aligned}$$

11. Given that

$$\begin{aligned} d_i &= 0.25 \text{ m} \\ d_f &= 0.45 \text{ m} \end{aligned}$$

True strain is given by

$$\begin{aligned} \varepsilon &= \ln \left(\frac{A_f}{A_i} \right) \\ &= 2 \ln \left(\frac{d_f}{d_i} \right) \\ &= 1.75 \end{aligned}$$

12. Given that

$$\begin{aligned} d &= 0.015 \text{ m} \\ t &= 0.005 \text{ m} \\ \tau &= 600 \text{ MPa} \end{aligned}$$

Punching force is calculated as

$$\begin{aligned} F &= \pi d t \tau \\ &= \pi \times 0.015 \times 0.005 \times 600 \\ &= 141.37 \text{ kN} \end{aligned}$$

13. The true strain in the specimen is calculated as

$$\begin{aligned} \varepsilon &= \ln \left(\frac{h_1}{h_2} \right) \\ &= \ln \left(\frac{150}{75} \right) \\ &= 0.693 \end{aligned}$$

The corresponding true stress is given by

$$\begin{aligned} \sigma_f &= K \varepsilon^n \\ &= 800 \times 0.693^{0.19} \\ &= 746.18 \text{ MPa} \end{aligned}$$

For volume constancy, the diameter of the finished product is given by

$$\begin{aligned} \frac{\pi}{4} d_2^2 \times 0.075 &= \frac{\pi}{4} 0.25^2 \times 0.15 \\ d_2 &= 0.353 \text{ m} \end{aligned}$$

Using average pressure formula,

$$\begin{aligned} \bar{p} &= \sigma_f \left(1 + \frac{\mu d_2}{3 h_2} \right) \\ &= 746.18 \left(1 + \frac{0.15 \times 0.353}{30.075} \right) \\ &= 921.78 \text{ MPa} \end{aligned}$$

The upsetting force is calculated as

$$\begin{aligned} F &= \bar{p} \times \frac{\pi}{4} d_2^2 \\ &= 921.78 \times \frac{\pi}{4} \times 0.353^2 \\ &= 90.21 \text{ MN} \end{aligned}$$

CHAPTER 13

JOINING

Joining or *fabrication* is the process of joining two similar or dissimilar metallic components. A wide range of joining techniques are used in various manufacturing operations, such as mechanical fasteners, adhesives, welding, brazing, and soldering. Most joining operations are more akin to assembly than to metal processing as in welding, brazing and soldering. This chapter is concerned with the joining processes of metal processing nature: welding, brazing, and soldering.

13.1 WELDING

Welding processes are broadly divided into two classes:

1. *Liquid State Welding* In *liquid state welding*, the joint is formed due to fusion of metals. The surfaces of two parts are brought to a temperature above the melting point then allowed to solidify producing weldments. Heat can be obtained by chemical reaction, electric arc, electrical resistance, frictional heat, sound or light energy.
2. *Solid State Welding* In *solid state welding*, the bulk material does not melt but the adhesion is produced by a metallic bonding of energized crystals. The category includes cold welding, ultrasonic welding, explosive welding, diffusion welding, forge welding, friction welding, etc.

If no filter metal is used during welding then it is termed as *auto-genous welding* process. Except in resistance welding, all processes use filler material to fill the gap between the parts to be joined.

13.1.1 Oxyacetylene Gas Welding

Oxyacetylene gas welding involves a regulated supply of acetylene and oxygen into a welding torch which is ignited to generate heat. Gas welding, primarily meaning oxyacetylene gas welding, is the most suitable option for welding of aluminium and its alloys.

13.1.1.1 Basic Principle *Oxyacetylene gas welding* is based upon the following chemical reaction between acetylene (C_2H_2) and oxygen (O_2) mixed in equal volumes:

$$C_2H_2 + O_2 \rightarrow 2CO + H_2 + 488 \text{ kJ/mol}$$

Products CO and H_2 further combine with atmospheric oxygen and generate more heat:

$$4CO + H_2 + 3O_2 \rightarrow 4O_2 + 2H_2O + 812 \text{ kJ/mol}$$

The second stage of burning is distributed in more area and flame temperature is around 1200°C to 2000°C. This energy can be used for preheating the workpieces (specifically of steels and cast irons) to prevent cold cracks.

13.1.1.2 Types of Flames Arc is generated in the form of flames whose structure depends upon the ratio of acetylene and oxygen. The oxyacetylene *flame* can be of three types:

1. Natural Flame *Neutral flame* is produced by stoichiometric ratio of C_2H_2 and O_2 resulting in complete combustion. This is the most preferred flame in gas welding. The flame suitable for mild steel, gray cast iron, alloy steel, lead, and copper bronze.

2. Carburized Flame *Carburized flame* is developed when less oxygen is provided to the gas. This flame is composed of three cones: white inner cone (2900°C), reddish feather middle cone, blue outer cone. Excess carbon can diffuse into base metals and can deleteriously affect the weld properties. Gas welding of steels with carburized flame can make them hard and brittle. Free carbons can cause poor corrosion resistance in stainless steels. This flame is used for oxygen-free copper alloys, high carbon steels, cast irons, high speed steels, cemented carbides, aluminium alloys, nickel alloys, etc.

3. Oxydized Flame *Oxydized flame* uses excess oxygen and produces a loud noise. It is composed of only two cones: inner (3300°C, maximum overall) and outer. This flame is less luminous as compared to carburized flame. Since oxygen is a rapid supporter of combustion, therefore, oxidizing flame is never used for general purpose welding, specially of ferrous alloys. However, oxidized flame is some times used for welding of copper and zinc base alloys where oxide film is necessary to check vaporization of zinc and also to reduce further oxidation.

Stelliting is deposition of stellite (an alloy of cobalt, tungsten, and chromium) during the process of oxy-acetylene welding or electric arc welding when it is done on the bodies requiring hard and wear resistance surface, such as cutting tools, lathe centers, rock drills, press tools, punches, dies.

13.1.1.3 Storage Systems Oxygen is generally kept in a green color cylinder and acetylene in a red color cylinder. According to the pressure used on acetylene, its storage is of two types:

1. High Pressure System In *high pressure system*, oxygen is stored in a seamless steel cylinder at 13.8 to 18.2 MPa, while acetylene is stored at 1.75 MPa in liquid state. Acetylene is highly explosive above 200 kPa, so it is soaked in a porous material of 80% calcium silicate ($CaSiO_3$).

2. Low Pressure System In *low pressure system*, acetylene is generated in situ from calcium carbide by the following reaction:

$$CaC_2 + 2H_2O \rightarrow C_2H_2 + Ca\,(OH)_2$$

The gases are brought together through pipes and meet at the tip of welding torch (made of high thermal conductive materials such as copper to cool the tip). A small spark is needed to start the arc. This is achieved by using friction lighter for safety.

Two types of regulators are used for controlling the flow of gases: nozzle type and stem type. Disadvantage of nozzle type regulator is that its outlet gas pressure gradually reduces with the cylinder pressure. Opposite to this occurs in stem type regulator. Therefore, these two types are used in series.

Piping of copper is never used with acetylene because it forms copper acetylide, which is an unstable compound and dissociates violently with shocks. Therefore, only brass tubing is used with acetylene.

13.1.1.4 Techniques of Gas Welding Moving of welding torch in direction of tip is called *forehand welding* or *right hand welding* which results in preheating of workpieces. If welding torch moves opposite to tip direction, the method is called *backhand welding* or *left hand welding*. This results in annealing of the welded area and is used for thicker sections [Fig. 13.1].

In both techniques, white cone should be at a distance 1.5–3 mm from the workpiece and torch is held at angle around 30–°C. When welding rod is provided with filler material, it is necessary to hold filler rod at a distance of 10 mm approximate from the flame and 1.5–3 mm from the weld metal pool.

13.1.2 Electric Arc Welding

13.1.2.1 Fundamental Concepts *Electric arc welding* is based on the principle that when liberated electrons strike the anode at high velocity, about 60-75% of the total heat is generated at the anode at temperature 6000°C. If workpiece is made anode, it is called *straight polarity* otherwise *reversed polarity*. More heat is generated on workpiece in straight polarity.

Arc length is kept approximately equal to diameter of electrodes. To finish arc welding, arc is extinguished slowly by decreasing current supply so that arc crater is completely filled.

The following are important characteristics of electric arc welding:

1. Arc Length Arc length between the electrode tip and the workpiece determines the resistance and consequently the potential drop across the arc. Longer the arc, higher is the arc voltage. This determines the flow of current across the arc.

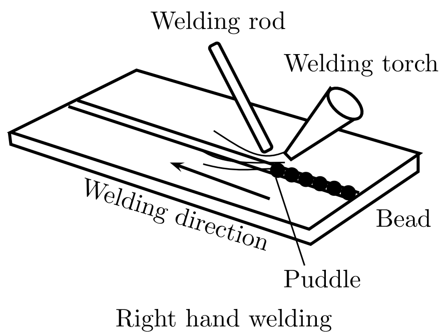


Figure 13.1 Gas welding techniques.

Figure 13.2 shows VI characteristics of arc in arc welding. The curve is drawn at constant arc length. The voltage reduces (against Ohm's law) upto certain limits (around 50 A), and increases thereafter. This is explained with the fact that at high voltage, the arc is almost cylindrical, thus a thick high current arc loses less heat and essentially burns hotter. This results in high thermal conductivity of the spacing between workpiece and electrode tip. However, beyond certain limits, the current path becomes more than the arc gap, thus increasing the resistance.

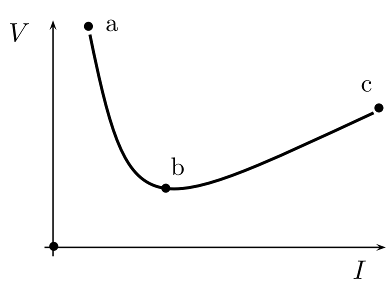


Figure 13.2 VI characteristics of arc.

A change in arc length affects both the voltage and welding current. A longer arc requires higher voltage and lower welding current, and vice versa. A linear relation can be assumed between welding voltage V and arc length L:

$$V = a + bL \tag{13.1}$$

where a and b are constants.

2. *Power Source Characteristics* Figure 13.3 shows two types of VI characteristics of power source along with VI characteristics of arc for arc lengths l and $l+\delta l$. Suitability of power source characteristics depends upon the response of welding current for a change in arc length. This can be examined w.r.t. manual arc welding and semiautomatic arc welding techniques:

(a) *Drooping (Constant Current)* Arc length in manual arc welding is subjected to unintentional variations. Therefore, to keep the welding current at constant level, drooping characteristics is desirable for manual arc welding.

(b) *Flat (Constant Voltage)* Arc in semiautomatic welding plants is maintained between the workpiece and wire, which is fed uniformly as it melts away from the tip. An increase in arc length results in increased voltage and fall in welding current. This reduces the melting rate of wire and the arc length returns to its original value. Thus, to enable variation in welding current w.r.t. arc length, flat characteristics is desirable for semiautomatic welding plants.

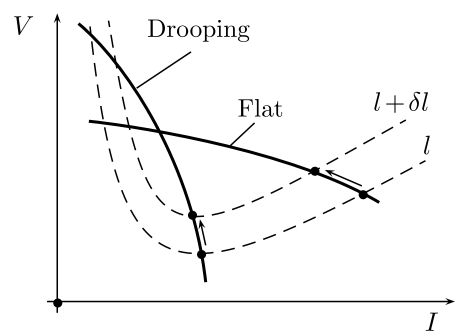


Figure 13.3 VI characteristics of power source.

The highest voltage is the open circuit voltage (V_o) of the power source. VI characteristics of power source can be expressed in linear equation:

$$V = V_o \left(1 - \frac{I}{I_s}\right) \tag{13.2}$$

where I_s is short circuit current.

3. *Arc Power* The arc power can be determined as

$$P = VI$$

In this equation, V and I can be replaced by using Eqs. (13.1) and (13.2) to find an expression of arc power as a function of arc length l. Thus, optimum arc length for maximum arc power can be determined using

$$\frac{dP}{dl} = 0$$

Approximate value of open circuit voltage (V_o) is 80 V, arc starting voltage is 40–50 V, while continuous welding voltage (V) is 20–30 V.

Direct current (DC) supply is preferred for cast iron and non-ferrous metals. Alternating current (AC) produces good penetration and is suitable for aluminium but starting of AC arc is more difficult because of alternating behavior. Welding transformers do not have provision for changing the polarity.

4. *Welding Speed* Consider an arc welding of weld length l, cross-sectional area A. If u is the specific energy required for melting, the heat input to the weld is given by

$$H = u\,(Al)$$

If welding is carried out at speed v, voltage V and current I and welding efficiency η, the heat input can be written as

$$u\,(Al) = \eta\,(VI) \times \frac{l}{v}$$
$$v = \eta \frac{VI}{uA}$$

Size of electrode is based on the thickness of cross-section of the workpiece and weld position. Welding current is based on the thickness.

5. *Types of Electrodes* There are two types of electrodes:

   (a) *Consumable electrode* acts as electrode and filler both, for example steel, cast iron, copper, brass, Al, bronze. Metal arc welding uses consumable electrodes.

   (b) *Non-consumable electrode* acts as electrode only, such as carbon graphite for direct current and tungsten for AC. Tungsten arc welding uses non-consumable electrodes.

6. *Electrode Coating* Using bare wire as electrodes results in many apparent defects in arc welding. The arc is difficult to manipulate, resulting in lack of good fusion, welds become porous, containing slag, and it can also absorb oxygen and nitrogen from atmospheric air. Therefore, electrodes are coated with suitable chemicals or fluxes which render a steadier arc and improve the quality of weld.

   Important coating materials are cellulose (for deeper penetration), rutile (natural titanium dioxide TiO_2) or titania (for stable arc), iron oxide, reducing agents like ferrosilicon, ferromanganese and aluminium, calcium carbonate or limestone (to prevent embrittlement of the welds by reducing hydrogen content in the deposited metal), asbestos winding (to strengthen the coating), calcium fluoride or fluor spar (to control the slag fluidity).

7. *Arc Blow* *Arc blow* is the deflection of arc caused by magnetic field due to flow of the welding current. This results in excessive spatter, incomplete fusion and lower welding speed. This happens more with strongly magnetic materials like nickel alloys. The remedy of this problem is: using alternating current instead of direct current if possible, reducing the amount of current, using shorter arc length, placing more than one ground leads to distribute the current uniformly, etc.

8. *Duty Cycle* AWS defines *duty cycle* as the percentage of time in 10 minute period that a welding machine can be used at its rated output without over loading.

13.1.2.2 Carbon Arc Welding In *carbon arc welding*, the coalescence of the base metal is achieved through the arc between carbon or graphite electrodes and the workpiece. DC is used with single electrode, while AC is used with twin electrodes. Tip of these light electrodes is made conical. A very high welding speed is possible with the process. Generally, no shielding atmosphere is used but if required inert gases, such as helium, nitrogen, argon, etc., are used. The process is most suitable for thinner sheets (upto 2 mm) as the workpiece distortions are negligible.

13.1.2.3 Inert-Gas-Shielded-Arc Welding In *inert gas shielded arc welding*, coalescence is produced from the arc between a metal electrode and the work shielded. Inert gases, such as argon, helium, CO, can be used as shielding environment. The electrodes can be consumable or non-consumable. Thus, two methods can be employed under this technique:

1. *Gas Tungsten Arc Welding* Gas tungsten arc welding (GTAW), formerly known as *tungsten inert gas* (TIG) welding, uses non-consumable thoriated tungsten electrodes with constant current source along with argon as shielding gas. Thus, stable arc gap is maintained at a constant current level.

   Direct current with straight polarity is used for most of the materials. To prevent contamination

of electrodes through oxidation, shielding gas is maintained for some more time after extinguishing the arc. Nitrogen (N_2) is used as shielding gas welding of copper and its alloys. Typical applications of GTAW include the welding of aircraft, missile, nuclear and electronic components, automotive parts, etc.

2. Gas Metal Arc Welding *Gas metal arc welding* (GMAW), also known as *metal inert gas* (MIG) welding, is used for thicker metals and fillet welds. Consumable electrode in the form of a bare wire reel is fed at a constant rate through the feed rollers. Generally, a constant voltage, direct current source is used in this process

13.1.2.4 Submerged Arc Welding In *submerged arc welding* (SAW), the welding zone remains submerged under a cover of granulated *flux* consisting of lime, silica, manganese oxide, calcium fluoride. The flux is fed into the weld zone by gravity flow through a nozzle. The arc is not visible from outside. This prevents spatter and sparks, and suppresses the intense ultraviolet radiation and fumes. The flux also acts as a thermal insulator and promotes deep penetration of heat into the workpiece. A small amount of flux melts and floats over the molten metal pool and helps in removing impurities from the weld joint.

The process uses bare metallic wire electrode which is fed continuously at uniform feed rate to maintain proper arc length. The size of electrodes vary from 1.2 to 10 mm in diameter.

SAW works with AC and DC both without spatter. High current values (1200 to 3500 A) and current density (20 to 70 kA/in^2) permit a high rate of metal transfer and welding speeds. Therefore, the process is mostly used for very high welding speed in single pass for sections having thickness in the range of 1-75 mm.

The process can be performed only in horizontal position as the flux and slag are held in position by gravity. To avoid possibility of entrapped slag or voids, generally one welding pass is preferred.

13.1.3 Resistance Welding

In *resistance welding*, a low voltage (V) and high current (I) power supply is passed through the resistance between two surfaces of the metals to cause local (I^2R) heating at the joint accompanied with pressure to complete the weld [Fig. 13.4].

If welding current I is supplied for interval t through the resistance R between workpieces, the heat generated (Q) is given by

$$Q = \eta I^2 Rt$$

where η is a coefficient to compensate energy losses associated with conduction and radiation. A transformer is used to reduce the voltage from 120–240 V to around 4–12 V and raises the amperage sufficiently to produce a good heating current. Critical variable in the process is contact resistance (around 100 $\mu\Omega$). The current density is around 4.5 to 6.2 kA/cm^2. This current is regulated by primary winding of the transformer.

Copper alloys are generally used for making electrodes to take advantage of minimum electrical resistance offered by them. Heat balance in the metals is achieved by making the electrodes of proportionate size with respect to thickness on which they act.

Due to inherent advantage of the process for high speed and easy set-up, resistance welding is suitable for mass production. Resistance welding is suitable for lap joints. However, process cannot be used with high conductive metals or thicker sections. Equipment of resistance welding is costlier than that for the manual arc welding.

Various versions of resistance welding are described in the following sections.

13.1.3.1 Resistance Spot Welding In *resistance spot welding*, the workpieces are clamped together, overlapping between two electrodes through which current is passed to make the weld nugget [Fig. 13.4].

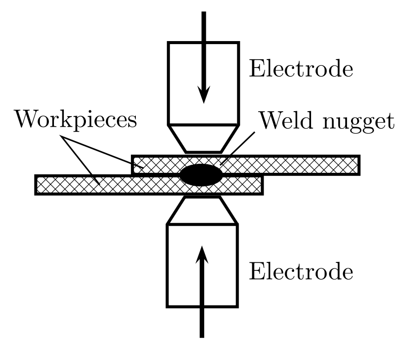


Figure 13.4 Resistance spot welding.

The resistance at the spot is inversely proportional to the pressure. The pressure is released and after a short time the current is turned off. This results in a good bond in the weld nugget having a discolored indentation.

The method is suitable for welding of metallic plates having thickness upto 12 mm where mechanical strength rather than water or air tightness is required. This method is extensively used in automobile and aircraft industry. Various stainless steel utensils and cutlery are spot welded.

13.1.3.2 Resistance Seam Welding *Resistance seam welding* (RSEW) is in effect a continuous-spot welding operation wherein, instead of using pointed electrodes, the two overlapping pieces of sheet metal are passed

between two rotating cylindrical (wheel) electrodes which are held together under pressure and are supplied intermittent current through high power switches, such as ignitrons, thyratrons [Fig. 13.5].

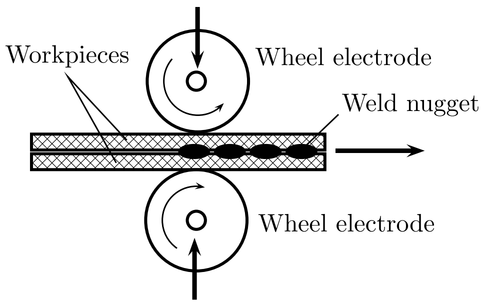


Figure 13.5 | Resistance seam welding.

The fusion zone can be continuous zone of overlapping nuggets or the nuggets can be located at some interval. The first case is called *stitch seam welding* while the later is called *roll spot welding.*

The electrodes are made of copper alloys and heat is dissipated by cooling the electrode and weld area with water. This process can produce highly efficient water-tight and gas-tight welds for industrial and household products. However, the process is limited to sheet metals upto 4.00 mm thick.

13.1.3.3 Resistance Projection Welding

Resistance projection welding (RPW) is in effect a modification of spot welding wherein projection welds are produced at localized points in workpieces held under pressure between suitable electrodes [Fig. 13.6].

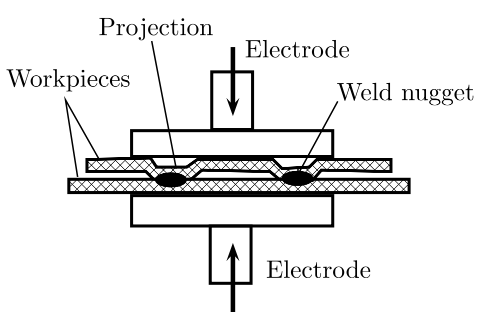


Figure 13.6 | Resistance projection welding.

Before starting the welding, small projections are embossed on the sheet metal through a press. These projections help in local heating during the welding process, which usually collapse during the process. Although embossing of workpieces adds to expenses, the process produces a large number of welds simultaneously in one stroke. Therefore, this process is extensively used in automobile and refrigerator industry for joining of fasteners, nuts, studs and such small parts on the larger parts or sheets.

13.1.3.4 Resistance Butt Welding

In *resistance butt welding* (RPW), the workpieces having the same cross-sections are held together under pressure, while heat is generated in the contact surface by electrical resistance. The joint is upset somewhat by this process but this defect is eliminated by subsequent rolling or grinding. Thus, the process is also known as *upset welding* (UW). This process is extensively used for making electric resistance welded (ERP) pipes with butt joints.

13.1.3.5 Flash Welding

In *flash welding*[1], the workpieces are brought together in very light contact and they act like electrodes. A high voltage supply starts a flash between the two surfaces and continues as the parts advance slowly. Thus, heat is produced from arcing (flashing) between surfaces. The process is completed by applying sufficient forging pressure to effect a weld (such as in RPW), therefore, this is also called *upset welding* (UW) [Fig. 13.7].

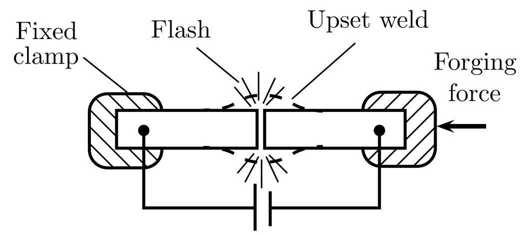


Figure 13.7 | Flash welding.

This method is used for joining high speed steel (HSS) tool tip to cheaper carbon steel shank in manufacturing of large twist drills. Although many non-ferrous alloys are flash welded satisfactorily, alloys containing high percentage of lead, zinc, tin and copper are not recommended for this process.

13.1.3.6 Percussion Welding

In *percussion welding* (PEW), a variation of flash welding, the arc is produced by the rapid discharge of stored energy across the air gap between the ends of workpieces. This is achieved by holding the two pieces are apart, one in a stationary clamp and the other on a moveable clamp backed up against heavy spring pressure. When moveable clamp is released, it moves rapidly, carrying with it the workpiece to be welded. Intense arcing occurs over the surfaces of the workpieces when they are about 1.5 mm apart. The arc is extinguished by the percussion blow of the two

[1]Flash welding is not considered as resistance welding by some experts because it relies on arc effects for heating rather than on the resistance in the metal.

pieces coming together to effect the weld. Therefore, the process is called percussion welding. The welding is very fast, usually occurs in less than 0.1 s, thus developing high localized heat. However, the process is limited to butt welds only.

13.1.3.7 High Frequency Resistance Welding *High frequency resistance welding* (HFRW) is a variation of the *resistance seam welding* (RSEW), wherein a high frequency (upto 450 kHz) current is employed.

13.1.4 Solid State Welding Processes

In *solid state welding*, the bulk material does not melt but the adhesion is produced by a metallic bonding of energized crystals. The category of *solid state welding* includes processes in which joint is produced by a metallic bonding of energized crystals. No heat is supplied from an external source. These are described in the following headings.

13.1.4.1 Cold Welding In *cold welding*, the joint is produced between overlaping plates of ductile materials by introducing high frequency vibratory energy into the weld area through a transducer. The oscillatory shear stresses at the weld interface break and expel the surface oxides. This produces metal-to-metal contact, forming a sound weld nugget.

13.1.4.2 Ultrasonic Welding In *ultrasonic welding*, the joint is produced between ductile materials by their shearing deformation through either impact in die or slow squeezing in rolls at room temperature. Before cold welding, the mating surfaces must be wire-brushed thoroughly to remove oxide films. The process is used for lap welding of sheets, foils and thin wires in automotive and electronic industries.

13.1.4.3 Explosive Welding The joint *explosive welding* (EXW) is formed by detonating an explosive over the flayer plate which impacts with target plate with huge velocity. This impact mechanically interlocks the two mating surfaces. The pressure developed is extremely high. This technique is used for cladding of metals to prevent corrosion, and also for joining dissimilar metals.

13.1.4.4 Diffusion Welding In *diffusion welding*, the joint is produced between clean flat parts with fine surface finish, primarily by diffusion and, to a lesser extent, from plastic deformation of the mating surfaces. In order to have sufficiently high diffusion rate, the process temperature is about $0.7T_m$ where T_m is the lowest melting temperatures of the base metals. The pressure is just under the yield stress at the operating temperature.

13.1.4.5 Forge Welding *Forge welding* welding was the first method welding. It involves heating the metals in a forge to a plastic condition and then uniting them by pressure.

13.1.4.6 Friction Welding In *friction welding* (FW), the heat required for welding is generated through the friction at the interface of the workpieces by inducing relative motion between them. This technique is used to join cylindrical dissimilar materials. The process has three versions: inertia friction welding, linear friction welding, and friction stir welding.

13.1.5 Advance Welding Techniques

13.1.5.1 Thermit Welding *Thermit welding*[2] is the only fusion welding process employing an exothermic chemical reaction for producing heat. The process uses a thermit mixture, mostly of iron oxide (Fe_3O_4) and aluminium (Al) in the mass ratio of 3 : 1, and a reducing agent. The process is extensively used for joining very thick parts, such as rail ends, ship hulls, broken large castings.

In this process, a wax pattern of the weld is build up by pouring wax into the joint and carefully ramming the refractory sand over it to form like a mold. The wax is burnt out by a preheating flame. Thermit reaction is started in a separate crucible by heating the mixture to a temperature above 1206°C. For this, special ignition powder is ignited by burning magnesium ribbon dipped in mixture or with a match.

The following thermit reaction takes place during the process:

$$8\mathrm{Al} + 3\mathrm{Fe_3O_4} \rightarrow \mathrm{Fe} + 4\mathrm{Al_2O_3} + \mathrm{Heat}$$

The reaction requires about 30 s and attains a temperature around 2500°C. During the reaction, aluminium oxide (Al_2O_3) is formed, which floats as slag on the top of molten steel setting below.

Mild steel pieces are added to thermit powder to reduce pouring temperature. For cast iron welds, ferosilicon can be added into the mixture. When reaction is complete, the metal is allowed to flow into the mold. This results in a weld which solidifies from the inside toward the outside.

13.1.5.2 Electroslag Welding In *electroslag welding* (ESW), heat is produced from the resistance of current in an electrically conductive molten flux. Consumable electrodes are fed continuously into the pool of molten slag which maintains a temperature in the range of 2000°C.

The process is used for welding very large vertical plates without any edge preparation in single pass,

[2]Thermit welding is named after the company that invented it.

such as in producing high pressure vessels, frames of heavy presses, rolling mill frames, ship hulls, locomotive frames. This method is followed by heat treatment of the weld.

13.1.5.3 Electron Beam Welding In *electron-beam welding* (EBW), kinetic energy of high velocity electrons is used as the heat source for welding. The process is carried out in 10^{-5} torr[3] vacuum. The heat energy imparted during the process is at a much higher rate than it can be conducted away from the weld zone. This results in a high depth-to-width ratio. The technique can be used to produce high-quality, deep and narrow welds.

13.1.5.4 Laser Beam Welding *Laser* is defined as a concentrated beam of coherent monochromatic radiations, a high energy source of heat to melt (even evaporate) the joint for fusion welding. The laser beam can be highly focused to as small a diameter as 10 μm. It has high energy density, therefore, deep penetrating capability. Therefore, *laser beam welding* (LBW) is used for deeper penetration welds even with dissimilar metals and multilayer materials. The process can be carried out in open atmosphere. The technique is mostly used in electronic industry.

13.1.5.5 Atomic Hydrogen Arc Welding In *atomic hydrogen arc welding* (AHAW), a single phase alternating current is maintained between two non-consumable tungsten electrodes. Diatomic hydrogen (H_2), reactive in nature, is then introduced into the electric arc, where it dissociates into two H^+ ions and simultaneously absorbing a large amount of heat from the arc. When hydrogen strikes a relatively cold surface in weld zone, it recombines into its molecular form and releases the stored heat very rapidly. The temperature of electric arc reaches over 6000°C [Fig. 13.8].

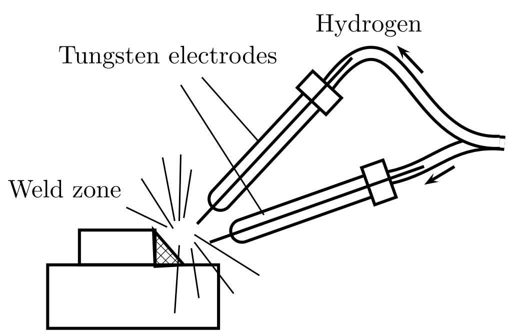


Figure 13.8 Atomic hydrogen arc welding.

[3]The torr is a non-SI unit of pressure with the ratio of 1 to 760 bar, chosen to be roughly equal to the fluid pressure exerted by 1 mm of Hg.

In this process, hydrogen itself act as shielding gas. Weld metal can be added to the joint by a welding rod. AC is preferred to achieve uniform wear in both the electrodes in the process.

13.1.5.6 Plasma Arc Welding A plasma is ionized hot gas composed of nearly equal numbers of electrons and ions. In *plasma arc welding* (PAW), a highly concentrated plasma is initiated between the tungsten electrode and the orifice, using a low-current pilot arc. Filler metal can be fed into plasma arc. Inert gases are used to form a shielding ring around the arc and weld-zone.

13.2 BRAZING

Brazing is the metal joining processes in which a filler metal, having a melting temperature of more than 450°C but lower than the melting temperature of parent metal, is used to fill the joint gap with capillary action. The filler metal has different composition than that of the base metals. Alloys of copper, sliver and aluminium are the most common brazing filler metals.

The gap between the mating surfaces directly affects the strength of the brazed joint. There is an optimum gap to achieve maximum tensile strength of the joint.

Use of flux is inevitable in brazing to protect the surface from oxidation during joining operation and remove oxide films from the joint. Wetting agents are also added in the fluxes to improve the fluidity for desired capillary action.

13.3 SOLDERING

Soldering is the metal joining processes in which a filler metal, having a melting temperature of less than 450°C and also lower than the melting temperature of parent metal, is used to fill the joint gap with capillary action. Strength of joint largely depends upon the adhesive quality of the solder which never reaches the strength of parent metals.

The soldering filler metal is called *solder*. The most commonly used solder is lead and tin alloy containing tin ranging from 5 to 70% and lead 30 to 95%. Higher the content of tin, lower the melting point of alloy. These are available in the form of bars, solid and flux-cored wires, preforms, sheet, foil, ribbon and paste or cream. Soldering is extensively used in electronic industry in producing circuits.

IMPORTANT FORMULAS

Electric Arc Welding

1. Arc length (L)

$$V = a + bL$$

2. Power characteristics

$$V = V_o\left(1 - \frac{I}{I_s}\right)$$
$$P = VI$$

3. Heat generated

$$Q = u(Al)$$

4. Welding speed

$$v = \eta\frac{VI}{uA}$$

SOLVED EXAMPLES

1. A continuous supply of 30 V and 200 A is used in arc-welding of carbon steel to produce weld bead of 40 mm^2 cross-sectional area. Carbon steel requires specific energy 12 J/mm^3 for melting. Electrode material requires the same specific energy. Calculate the average welding speed for 82% efficiency in energy utilization.

Solution. Given that

$$\begin{aligned} V &= 30 \text{ V} \\ I &= 200 \text{ A} \\ A &= 40 \text{ mm}^2 \\ u &= 12 \text{ J/mm}^3 \\ \eta &= 0.82 \end{aligned}$$

Let time t is required for producing a weld bead of length l. Average welding speed will be l/t. For energy balance,

$$\begin{aligned} u \times (lA) &= \eta VIt \\ \frac{l}{t} &= \eta\frac{VI}{uA} \\ &= 0.82 \times \frac{30 \times 200}{12 \times 40} \\ &= 10.25 \text{ mm/s} \end{aligned}$$

2. Two 2 mm thick steel sheets are being spot welded using a current of 5500 A for 0.12 s on each spot. The diameter of electrodes is 4 mm, while the effective resistance is 275 $\mu\Omega$. Density of metal is 7800 kg/m^3, while specific energy of melting is 1.5 MJ/kg. Calculate the amount of heat generated and percentage of heat dissipated to the surrounding.

Solution. Heat supplied to the the weld:

$$\begin{aligned} Q &= I^2Rt \\ &= 5500^2 \times 275 \times 10^{-6} \times 0.12 \\ &= 998.25 \text{ J} \end{aligned}$$

Heat consumed in melting of the nugget is

$$\begin{aligned} Q_w &= \rho \times \frac{\pi d^2}{4} \times 2t \times u \\ &= 918.91 \text{ J} \end{aligned}$$

Percentage heat dissipated is

$$\begin{aligned} 1 - \frac{Q_w}{Q} &= 1 - \frac{918.91}{998.25} \\ &= 7.94\% \end{aligned}$$

3. A projection of welding of 3 spots is performed using a current of 32000 A for a period of 0.01 s. The interface resistance between the plates is 250 $\mu\Omega$. The nugget dimensions are 6.5 mm in diameter and 2 mm in thickness. Heat required for the melting of steel is 12 J/mm^3. Calculate the heat required for welding of the spots, and the percentage of heat dissipated.

Solution. The welding involves melting of three spots of 6 mm diameter and 2.5 mm thickness. Heat required is calculated as

$$\begin{aligned} Q_w &= 3 \times \frac{\pi \times 6.5^2}{4} \times 2.0 \times 12 \\ &= 2.38 \text{ kJ} \end{aligned}$$

Energy supplied to the spots is

$$\begin{aligned} Q &= I^2Rt \\ &= 32000^2 \times 250 \times 10^{-6} \times 0.01 \\ &= 2.56 \text{ kJ} \end{aligned}$$

Heat dissipated to the surrounding is

$$\begin{aligned} &= \frac{Q - Q_w}{H} \\ &= \frac{1.470 - 1.2723}{1.470} \\ &= 7.03\% \end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. In oxyacetylene gas welding, temperature at the inner cone of the flame is around

(a) 3500°C (b) 3200°C
(c) 2900°C (d) 2550°C

(GATE 2003)

Solution. Temperature in inner white cone is around 3100°C, and the maximum temperature occurs at the next of inner cone. Therefore, the nearest answer is 3200°C.

Ans. (b)

2. Two 1 mm thick steel sheets are to be spot welded at a current of 5000 A. Assuming effective resistance to be 200 $\mu\Omega$ and current flow time of 0.2 s, heat generated during the process will be

(a) 0.2 J (b) 1 J
(c) 5 J (d) 1000 J

(GATE 2004)

Solution. Given that

$$I = 5000 \text{ A}$$
$$R = 200\ \mu\Omega$$
$$t = 0.2 \text{ s}$$

Heat generated is given by

$$\begin{aligned} Q &= I^2 Rt \\ &= 5000^2 \times 200 \times 0.2 \\ &= 1000 \text{ J} \end{aligned}$$

Ans. (d)

3. The strength of a brazed joint

(a) decreases with increase in gap between the two joining surfaces
(b) increases with increase in gap between the two joining surfaces
(c) decreases up to a certain gap between the two joining surfaces, beyond which it increases
(d) increases up to a certain gap between the two joining surfaces, beyond which it decreases

(GATE 2005)

Solution. The essential feature of brazing and one of its chief advantages is that the joining operation is always performed at temperatures below the melting point of the materials of the brazed article. The bond is formed by the molten brazing alloy which fills the space between the mating surfaces of the two components (the so-called joint gap) by the mechanism of capillary flow. Therefore, the strength of the brazed joint increases upto a certain gap between the two joining surfaces beyond which it decreases.

Ans. (d)

4. Spot welding of two 1 mm thick sheets of steel (density = 8000 kg/m^3) is carried out successfully by passing a certain amount of current for 0.1 s through the electrodes. The resultant weld nugget formed is 5 mm in diameter and 1.5 mm thick. If the latent heat of fusion of steel is 1400 kJ/kg and the effective resistance in the welding operation is 200 $\mu\Omega$, the current passing through the electrodes is, approximately,

(a) 1480 A (b) 3300 A
(c) 4060 A (d) 9400 A

(GATE 2005)

Solution. Given that

$$R = 200\ \mu\Omega$$
$$t = 0.1 \text{ s}$$
$$\rho = 8000 \text{ kg/m}^3$$
$$L = 1400 \text{ kJ/kg}$$

Volume (V) of nugget

$$\begin{aligned} V &= \frac{\pi}{4}(0.005)^2 \times 0.0015 \\ &= 2.94 \times 10^{-8} \text{ m}^3 \end{aligned}$$

For heat balance,

$$\begin{aligned} I^2 Rt &= \rho V L \\ I &= \sqrt{\frac{\rho V L}{Rt}} \\ &= \sqrt{\frac{8000 \times 2.94 \times 10^{-8} \times 1400}{200 \times 0.1}} \\ &= 4060 \text{ A} \end{aligned}$$

Ans. (c)

5. In an arc welding process, the voltage and current are 25 V and 300 A, respectively. The arc heat transfer efficiency is 0.85 and welding speed is 8 mm/s. The net heat input (in J/mm) is

(a) 64 (b) 797
(c) 1103 (d) 79700

(GATE 2006)

Solution. Given that

$$\begin{aligned} V &= 25 \text{ V} \\ I &= 300 \text{ A} \\ v &= 8 \text{ mm/s} \\ \eta &= 0.85 \end{aligned}$$

Therefore,

$$\begin{aligned} Q &= \eta \frac{VI}{v} \\ &= 0.85 \times \frac{25 \times 300}{8} \\ &= 796.875 \text{ J/mm} \end{aligned}$$

Ans. (b)

6. Which one of the following is a solid state joining process?

(a) Gas tungsten arc welding
(b) Resistance spot welding
(c) Friction welding
(d) Submerged arc welding

(GATE 2007)

Solution. In solid state welding, the bulk material does not melt but the adhesion is produced by a metallic bonding of energized crystals. The category includes cold welding, ultrasonic welding, explosive welding, diffusion welding, forge welding, friction welding, etc. In friction welding (FW), the heat required for welding is generated through the friction at interface of the workpieces by inducing relative motion between them. The technique is used to join cylindrical dissimilar materials. The process has three versions: inertia friction welding, linear friction welding, and friction stir welding.

Ans. (c)

7. A direct current welding machine with linear power source characteristics provides open circuit voltage of 80 V and short circuit current of 800 A. During welding with the machine, the measured arc current is 500 A corresponding to an arc length of 5.0 mm and the measured current is 460 A corresponding to an arc length of 7.0 mm. The linear voltage (V) - arc length (L) characteristic of the welding arc can be given as (where V is in volt and L is in mm)

(a) $V = 20 + 2L$ (b) $V = 20 + 8L$
(c) $V = 80 + 2L$ (d) $V = 80 + 8L$

(GATE 2007)

Solution. Given that

$$\begin{aligned} V_o &= 80 \\ I_s &= 800 \end{aligned}$$

Assuming for the given condition, the linear relation between I and arc length L is

$$I = a + bL$$

Given that

$$\begin{aligned} 500 &= a + b5 \\ 460 &= a + b7 \end{aligned}$$

Therefore,

$$\begin{aligned} 2b &= -40 \\ b &= -20 \\ a &= 500 - 5 \times -20 \\ &= 600 \end{aligned}$$

Therefore

$$I = 600 - 20L$$

Using the voltage relationship,

$$\begin{aligned} V &= V_o \left(1 - \frac{I}{I_s}\right) \\ &= 80 - \frac{I}{10} \\ &= 80 - 60 + 2L \\ &= 20 + 2L \end{aligned}$$

Ans. (a)

8. Two metallic sheets, each of 2.0 mm thickness, are welded in a lap joint configuration by resistance spot welding at a welding current of 10 kA and welding time of 10 milliseconds. A spherical fusion zone extending up to the full thickness of each sheet is formed. The properties of the metallic sheets are given as

Ambient temperature	=	293 K
Melting temperature	=	1793 K
Density	=	7000 kg/m^3
Latent heat of fusion	=	300 kJ/kg
Specific heat	=	800 J/kg K

Assume:

(i) contact resistance along sheet–sheet interface is 500 $\mu\Omega$ and along electrode-sheet interface is zero.
(ii) non-conductive heat loss through the bulk sheet materials; and
(iii) the complete weld fusion zone at the melting temperature.

The melting efficiency (in %) of the process is

(a) 50.37 (b) 60.37
(c) 70.37 (d) 80.37

(GATE 2007)

Solution. Given that

$$I = 10 \times 10^3 \text{ A}$$
$$R = 500 \times 10^{-6}\ \Omega$$
$$t = 0.010 \text{ s}$$

Heat generated due to supplied current is

$$\begin{aligned} Q &= I^2 Rt \\ &= \left(10 \times 10^3\right)^2 \times 500 \times 10^{-6} \times 0.010 \\ &= 500 \text{ J} \end{aligned}$$

The volume of heat zone is half-spherical expanded upto thickness, therefore, radius of heat zone will be equal to the thickness of sheet, that is, 0.002 m. Therefore, melting volume is calculated as

$$\begin{aligned} V &= \frac{4\pi}{3} r^3 \\ &= \frac{4\pi}{3}\left(\frac{0.002}{2}\right)^3 \\ &= 4.18 \times 10^{-9} \text{ m}^3 \end{aligned}$$

Now, heat actually utilized by the process is

$$\begin{aligned} Q_w &= 7000 \times V\{300 \times 10^3 + 800\,(1793 - 293)\} \\ &= 351.858 \text{ J} \end{aligned}$$

Therefore, melting efficiency is

$$\begin{aligned} \eta &= \frac{Q_w}{Q} \\ &= \frac{351.858}{500} \\ &= 70.37\% \end{aligned}$$

Ans. (c)

9. In arc welding of a butt joint, the welding speed is to be selected such that the highest cooling rate is achieved. Melting efficiency and heat transfer efficiency are 0.5 and 0.7, respectively. The area of the weld cross-section is 5 mm^2 and the unit energy required to melt the metal is 10 J/mm^2. If the welding power is 2 kW, the welding speed in mm/s is closest to

(a) 4 (b) 14
(c) 24 (d) 34

(GATE 2008)

Solution. The cross-sectional area of the weld is $a = 5$ mm^2, and energy required to melt the metal is $u = 10$ J/mm^2. The welding power is $P = 2$ kW with melting and heat transfer efficiencies $\eta_m = 0.5$ and $\eta_h = 0.7$. Therefore, heat balance equation for welding speed v can be written as,

$$\begin{aligned} u \times v \times a &= P \times \eta_m \times \eta_h \\ 10 \times 5 \times v &= 2 \times 10^3 \times 0.5 \times 0.7 \\ v &= \frac{2 \times 10^3 \times 0.5 \times 0.7}{0 \times 5} \\ &= 14 \text{ mm/s} \end{aligned}$$

Ans. (b)

10. Two pipes of inner diameter 100 mm and outer diameter 110 mm each are joined by flash butt welding using 30 V power supply. At the interface, 1 mm of material melts from each pipe which has a resistance of 42.4 Ω. If the unit melt energy is 64.4 MJ/m^3, then the time required for welding in seconds is

(a) 1 (b) 5
(c) 10 (d) 20

(GATE 2010)

Solution. Given that

$$d_i = 0.10 \text{ m}$$
$$d_o = 0.11 \text{ m}$$
$$l = 0.001$$
$$V = 30\text{V}$$
$$R = 42.4\ \Omega$$
$$q = 64.4 \times 10^6 \text{ J/m}^3$$

If power is supplied for time t, then for heat balance

$$\begin{aligned} \frac{V^2}{R} t &= q \times \frac{\pi}{4}\left({d_o}^2 - {d_i}^2\right) l \\ \frac{30^2}{42.4} \times t &= 64.4 \times 10^6 \times \frac{\pi}{4}\left(0.11^2 - 0.10^2\right) 0.001 \\ t &= 5.00401 \text{ s} \end{aligned}$$

Ans. (b)

11. Which one among the following welding processes uses non-consumable electrode?

(a) Gas metal arc welding
(b) Submerged arc welding
(c) Gas tungsten arc welding
(d) Flux coated arc welding

(GATE 2011)

Solution. Gas tungsten arc welding (GTAW), formerly known as tungsten inert gas (TIG)

welding, uses non-consumable thoriated tungsten electrodes with constant current source along with argon as shielding gas. Thus, stable arc gap is maintained at a constant current level.

Ans. (c)

12. In a DC arc welding operation, the voltage-arc length characteristic was obtained as $V_{\text{arc}} = 20+5l$ where the arc length l was varied between 5 mm and 7 mm. Here V_{arc} denotes the arc voltage in Volts. The arc current was varied from 400 to 500 A. Assuming linear power source characteristic, the open circuit voltage and short circuit current for the welding operation are:

(a) 45 V, 450 A (b) 75 V, 550 A
(c) 95 V, 950 V (d) 150 V, 1500 A

(GATE 2012)

Solution. For given measurements related to arc lengths, the following two equations are found:

(i) For $l = 5$ mm,

$$\begin{aligned} V_{\text{arc}} &= 20+5\times 5 \\ &= 55 \text{ V} \\ I &= 400 \text{ A} \end{aligned}$$

(ii) For $l = 7$ mm,

$$\begin{aligned} V_{\text{arc}} &= 20+5\times 7 \\ &= 45 \text{ V} \\ I &= 500 \text{ A} \end{aligned}$$

Arc voltage and current are related to open circuit voltage V_o and short circuit current I_s as

$$V_{\text{arc}} = V_o\left(1-\frac{I}{I_s}\right)$$

Therefore, putting the known values of two different cases into the above equation, one finds two equations as

$$55 = V_o\left(1-\frac{400}{I_s}\right)$$
$$45 = V_o\left(1-\frac{500}{I_s}\right)$$

These equations can be written as

$$55I_s = V_o\,(I_s-400)$$
$$45I_s = V_o\,(I_s-500)$$

Dividing the second equation by the first equation, one obtains

$$\begin{aligned} \frac{55}{45} &= \frac{I_s-400}{I_s-500} \\ I_s-400 &= 1.222I_s-611.11 \\ I_s &= \frac{611.11-400}{1.222-1} \\ &= 950 \text{ A} \end{aligned}$$

Keeping this value in any of the equation, say

$$\begin{aligned} 45 &= V_o\left(1-\frac{500}{950}\right) \\ V_o &= \frac{45\times 950}{950-500} \\ &= 95trmV \end{aligned}$$

The the solution is

$$\begin{aligned} V_o &= 95 \text{ V} \\ I_s &= 950 \text{ A} \end{aligned}$$

Ans. (c)

13. Match the correct pairs.

	Process		Characteristics
P.	Friction welding	1.	Non-consumable electrode
Q.	Gas metal arc welding	2.	Joining of thick plates
R.	Tungsten inert gas welding	3.	Consumable electrode wire
S.	Electroslag welding	4.	Joining of cylindrically dissimilar materials

(a) P-4, Q-3, R-1, S-2

(b) P-4, Q-2, R-3, S-1

(c) P-2, Q-3, R-4, S-1

(d) P-2, Q-4, R-1, S-3

(GATE 2013)

Solution. Friction welding (FW) is a solid-state welding processes which uses heat generated by friction between workpieces in relative motion. It is used to join cylindrical dissimilar materials. Tungsten inert gas welding uses non-consumable electrodes made of tungsten. Gas metal arc welding uses consumable electrodes made of the work material itself. Electroslag welding is a welding process, in which the heat is generated by an electric current passing between the consumable electrode (filler metal) and the workpiece through a molten slag covering the weld surface. This technique is used for joining thick plates.

Ans. (a)

MULTIPLE CHOICE QUESTIONS

1. Thick steel plate cut with oxygen normally shows signs of cracking. This tendency of cracking can be minimized by

(a) slow speed cutting
(b) cutting in two or more stages
(c) preheating the plate
(d) using oxyacetylene flame

2. Arc length in arc welding is generally kept approximately equal to

(a) half the diameter of electrode
(b) diameter of electrodes
(c) two times the diameter of electrode
(d) equal to minimum plate thickness

3. In GTAW, a constant and stable arc gap is maintained at a constant current level. It is due to the fact that

(a) tungsten electrodes are not consumed
(b) constant current source is used
(c) argon is used shielding gas
(d) all of the above

4. In GTAW, the shielding gas flow should be maintained for some more time after extinguishing the arc. This is done mainly so that

(a) the joint may get sufficiently cooled
(b) to maintain the protective atmosphere for the weld bead
(c) the electrode gets sufficiently cooled under protective atmosphere
(d) all of the above

5. Power supply for gas metal arc welding is always at

(a) constant voltage
(b) constant current
(c) both (a) and (b)
(d) none of the above

6. The welding process capable for welding of thick sections upto 75 mm in a single pass is

(a) tungsten inert gas welding
(b) gas metal arc welding
(c) submerged arc welding
(d) all of the above

7. Power source for plasma arc welding is

(a) constant voltage type DC with electrode negative
(b) constant voltage type DC with electrode positive
(c) constant current type DC with electrode negative
(d) constant current type DC with electrode positive

8. In resistance welding, the source of supply is

(a) a low voltage and high current
(b) a low voltage and low current
(c) a high voltage and high current
(d) a high voltage and low current

9. The electrodes in resistance welding are generally made of

(a) mild steel (b) stainless steel
(c) aluminium alloys (d) copper alloys

10. The welding technique most suitable for mass production is

(a) electric arc welding
(b) gas welding
(c) resistance welding
(d) none of the above

11. The welding process which is used for welding very large plates without any edge preparation in single pass using consumable electrodes, is known as

(a) submerged arc welding
(b) inert gas shielded arc welding
(c) oxyacetylene welding
(d) electro-slag welding

12. Electron beam welding uses kinetic energy of high velocity electrons as the heat source. This process is carried out in

(a) atmospheric pressure
(b) room temperature
(c) vacuum
(d) water

13. Weldability of cast iron is quite low. Welding techniques suitable for cast iron is

(a) brazing
(b) gas welding
(c) both (a) and (b)
(d) none of the above

14. If no filler metal is used during welding then the process is termed as

(a) autogenous welding
(b) no filler welding
(c) filler free welding
(d) all of the above

15. In liquid state welding along with filler, the joint is formed due to fusion of

(a) base metals only
(b) filler metal only
(c) both filler and base metals
(d) none of the above

16. In welding operation, penetration is defined as

(a) the depth up to which the weld metal combines with the base metal
(b) depression in the weld metal pool at the point where arc strikes the metal plate
(c) small particles or globules of metal scattered around the vicinity of the weld
(d) none of the above

17. In electric arc welding, the continuous welding voltage is expressed in terms of open circuit voltage (V_o), the short circuit current (I_s) and continuous welding current (I) as

(a) $V_o(1-I_s/I)$ (b) $V_o(1+I_s/I)$
(c) $V_o(1-I/I_s)$ (d) $V_o(1+I/I_s)$

18. In thermit welding, iron oxide and aluminium oxide are mixed in the mass ratio

(a) 1 : 1 (b) 2 : 1
(c) 3 : 1 (d) 4 : 1

19. Which type of fusion welding is used for joining very thick parts like rail ends with pool molten metal?

(a) SAW (b) MIG
(c) GTAW (d) Thermit

20. The selection of diameter of electrodes in arc welding is based on

(a) workpiece material
(b) thickness of workpieces
(c) voltage
(d) current

21. Which current is preferred for welding of cast iron and non-ferrous metals?

(a) low frequency AC current
(b) high frequency AC current
(c) DC current
(d) all of the above

22. The welding process that cannot be used for highly electrical conductive materials is

(a) arc welding
(b) gas welding
(c) resistance welding
(d) laser welding

23. In electric arc welding, arc blow is observed when using

(a) alternating current
(b) direct current
(c) weak magnetic materials
(d) all of the above

24. Arc blow can be reduced by

(a) using alternating current instead of direct current
(b) reducing amount of current and arc length
(c) placing more than one ground
(d) all of the above

25. Which of the welding processes uses non-consumable electrodes?

(a) SAW (b) MIG
(c) GTAW (d) Thermit

26. To finish arc welding, arc is generally extinguished slowly by decreasing current supply. This is done to

(a) completely fill the arc crater
(b) reduce the arc blow
(c) increase penetration of welds
(d) none of the above

27. Great amount of heat is generated on workpiece if the arc welding adopts

(a) reversed polarity
(b) straight polarity
(c) both (a) and (b)
(d) none of the above

28. In thermite welding, the thermit reaction is started by

(a) friction lighter for safety
(b) electric lighter for safety
(c) burning of magnesium ribbon dipped in mixture
(d) all of the above

29. The most important feature of laser beam technique that it can be used for

(a) deeper penetration welds only with similar metals and multilayer materials
(b) low penetration welds only with similar metals and multilayer materials
(c) low penetration welds only with similar metals and single layer materials
(d) deeper penetration welds even with dissimilar metals and multilayer materials

30. The shielding gas in gas tungsten arc welding (GTAW) is

(a) hydrogen (b) argon and neon
(c) helium (d) carbon dioxide

31. Select the wrong statement about brazing.

(a) The molten metal fills the closely fitting space by capillary action
(b) The filler metals for brazing melt below the melting point of the metal to be joined
(c) Smaller the gap in the joint, higher is the shear strength of the joint
(d) There is no optimum gap to achieve maximum tensile strength of the joint

32. Use of flux is essential in brazing in order to

(a) fill the gap between workpieces completely
(b) enhance the capillary action to build the joint
(c) prevent oxidation and to remove oxide films from workpiece surfaces
(d) increase penetration of the filler material

33. Select wrong statement about soldering:

(a) The filler metal fills the joint by capillary action (as in brazing)
(b) Temperature in soldering are relatively low (unlike brazing)
(c) Solders are not used for load bearing structural members
(d) After soldering, the flux residue should not be removed to avoid corrosion

34. Reduction is a process of producing metal powders by

(a) injecting molten metal through a small orifice in a reduced pressure chamber
(b) reducing the size of metals by milling in a ball mill
(c) removal of oxygen from metal oxides using reducing agents, such as hydrogen, carbon monoxide
(d) all of the above

35. Blending of metal powder is carried out for

(a) blending of powders of different metals, shapes and sizes to get desired properties
(b) mixing of lubricants with the powders to improve their flow characteristics during processing
(c) mixing of various additives, such as binders, to develop sufficient strength and facilitate sintering
(d) all of the above

36. Resistance spot welding is performed on two plates of 1.5 mm thickness with 6 mm diameter electrode, using 15000 A current for a time duration of 0.25 s. Assuming the interface resistance to be 0.0001 Ω, the heat generated to form the weld is

(a) 5625 J (b) 8437 J
(c) 22500 J (d) 33750 J

37. Spot welding of conductive metals like copper is

(a) easier than ferrous metals
(b) difficult than ferrous metals
(c) similar to other metals
(d) conductivity is not relevant in spot welding

38. Submerged arc welding (SAW) uses

(a) bare rods
(b) coated electrodes
(c) core wires
(d) tungsten electrodes

39. In an arc-welding operation of carbon steel, the continuous voltage and currents are 25 V and 195 A, respectively. The cross-sectional area of the weld bead is 35 mm^2. The specific energy to melt a metal for carbon steel is 12.3 J/mm^3. Assuming the same material for the electrode and efficiency of 80%, what will be the average speed of welding?

(a) 7 mm/s (b) 8 mm/s
(c) 9 mm/s (d) 10 mm/s

40. A 20 V supply is used in a shielded arc welding operation on a carbon steel with electrodes of the same material. The cross-sectional area of the weld is triangular in shape with 12 mm leg. The specific energy to melt a metal for carbon steel is 12.3 J/mm^3. The welding speed is 10 mm/s and efficiency of heat input is 75%. What will be the required current?

(a) 590 A (b) 650 A
(c) 705 A (d) 750 A

41. The temperature of the weld zone in the atomic hydrogen welding is around

(a) 2000 K (b) 3000 K
(c) 4000 K (d) 6000 K

42. Two plates of the same metal having equal thickness are to be butt welded with electric arc. When the plate thickness changes, welding is achieved by

(a) adjusting the current
(b) adjusting the duration of current
(c) changing the electrode size
(d) changing the electrode coating

43. The gas tungsten arc welding (GTAW) is particularly suitable for

(a) aluminium, magnesium
(b) titanium
(c) refractory metals
(d) all of the above

44. Laser beam welding is particularly suitable for welding deep and narrow joints because laser beam can be focused on as small a diameter as

(a) 1 mm (b) 100 μm
(c) 10 μm (d) 5 μm

45. The main characteristics of laser beam welding include

(a) In this process, vacuum is not required, unlike in electron beam welding
(b) the process can be easily automated
(c) the process does not generate X-rays
(d) all of the above

46. Penetration in the welds can be improved by

(a) increasing the rate of heat input
(b) lowering the welding speed
(c) ensuring proper fitment and alignment of weld surfaces
(d) all of the above

47. The welding process used extensively for lap welding of sheet, foil, and thin wires is

(a) laser beam welding
(b) ultrasonic welding
(c) electron beam welding
(d) all of the above

48. The temperature generated in the weld zone during ultrasonic welding is in the range of

(a) room temperature
(b) melting point
(c) recrystallization temperature
(d) one-third to one-half of the melting point

49. For butt welding 40 mm thick steel plates, when the expected quantity of such jobs is 5000 per month over a period of 10 years, choose the best suitable welding process out of the following available alternatives.

(a) Submerged arc welding
(b) Oxyacetylene gas welding
(c) Electron beam welding
(d) MIG welding

50. The welding process which does not uses mechanical energy is

(a) ultrasonic welding
(b) friction welding
(c) thermit welding
(d) gas welding

51. In atomic hydrogen arc welding (AHAW), the arc is generated between

(a) joining parts
(b) consumable electrode and workpiece
(c) consumable electrodes

(d) non-consumable electrodes

52. Gas metal arc welding (GMW) is used for

(a) thicker metals with consumable wire electrode
(b) thinner metals with consumable wire electrode
(c) thicker metals with non-consumable wire electrode
(d) thinner metals with non-consumable wire electrode

53. In gas welding, the combustion takes place by mixing oxygen with

(a) H_2 (b) CO_2
(c) CO (d) C_2H_2

54. The ratio of of oxygen and acetylene in gas welding with carburizing flame is

(a) 1:1 (b) 1.2:1
(c) 1:1.2 (d) indeterminate

55. In oxyacetylene welding, the tubes for acetylene are made of

(a) copper (b) brass
(c) steel (d) aluminium

56. Piping made of copper is never used with acetylene because

(a) copper is insufficient to provide strength
(b) there are chances of leakages of acetylene
(c) acetylene reacts with copper forming unstable compound
(d) all of the above

57. Carburizing flame is used to weld metals such as

(a) steels and cast iron
(b) copper alloys
(c) aluminium alloys
(d) all of the above

58. In forehand welding technique,

(a) movement of welding torch is in the direction of tip which results in preheating of workpieces
(b) movement of welding torch is in opposite direction of tip which results in preheating of workpieces
(c) movement of welding torch is in direction of tip which does not result in preheating of workpieces
(d) movement of welding torch is in opposite direction of tip which does not result in preheating of workpieces

59. In high pressure system of storage of gases, the acetylene is stored in

(a) gaseous form
(b) liquid form
(c) sold form
(d) may be liquid or gas

60. To start the arc in gas welding,

(a) friction lighter is used for safety of operator
(b) electric lighter is used for safety of operator
(c) the chemical reaction does not require a lighter
(d) the requirement of type of lighter depends upon the ambient conditions

61. Preheating before welding is done to

(a) make the steel softer
(b) burn away oil, grease, etc., from the plate surface
(c) prevent cold cracks
(d) prevent plate distortion

62. In a gas welding of mild steel using an oxy-acetylene flame, the total amount of acetylene consumed was 10 liters. The oxygen consumption from the cylinder is

(a) 5 liters (b) 10 liters
(c) 15 liters (d) 20 liters

63. Soldering wire is essentially a

(a) lead tin (b) tin silver
(c) bismuth lead (d) nickel tin

64. In brazing, the flux does not

(a) protect the surface from oxidation during joining operation
(b) dissolve oxides from the surfaces to be joined
(c) reduce surface tension of molten filler metal
(d) reduce the fluidity of the filler metal

65. Select the correct statement

(a) The strength of brazed joint is lower than soldered joint and welded joint.
(b) The strength of brazed joint is higher than soldered joint and welded joint.
(c) The strength of brazed joint is lower than soldered joint but higher than welded joint.
(d) The strength of brazed joint is higher than soldered joint but lower than welded joint.

NUMERICAL ANSWER QUESTIONS

1. An electric-arc welding has the following VI characteristics:

$$V = 18+42l \text{ V}$$

where l is the length of the arc in cm. The power source characteristic is approximated by a straight line with an open circuit voltage 72 V and a short circuit current 1200 A. Determine the optimum arc length and the corresponding arc power.

2. A projection of welding of three spots is obtained with a current of 35000 A for a period of 0.01 s. The effective resistance of the joint is 120 $\mu\Omega$. The nugget dimensions are 6 mm in diameter and 1.5 mm in thickness. Heat required for the melting of steel is 10 J/mm^3. Calculate the heat required for the welding of the spots. Also calculate the percentage of heat dissipated to the surrounding.

3. Two 2 mm thick steel sheets are being spot welded at a current of 5000 A and current flow time is 0.15 s. The diameter of electrodes is 5 mm while the effective resistance is 250 $\mu\Omega$. The density of metal is 7800 kg/m^3, while specific energy for melting is 1.4 MJ/kg. Calculate the amount of heat generated and the percentage of heat dissipated.

ANSWERS

Multiple Choice Questions

1.	(c)	**2.**	(b)	**3.**	(d)	**4.**	(c)	**5.**	(a)	**6.**	(c)	**7.**	(c)	**8.**	(a)	**9.**	(d)	**10.**	(c)
11.	(d)	**12.**	(c)	**13.**	(c)	**14.**	(a)	**15.**	(c)	**16.**	(a)	**17.**	(c)	**18.**	(c)	**19.**	(d)	**20.**	(b)
21.	(c)	**22.**	(c)	**23.**	(b)	**24.**	(d)	**25.**	(c)	**26.**	(a)	**27.**	(b)	**28.**	(c)	**29.**	(d)	**30.**	(b)
31.	(d)	**32.**	(c)	**33.**	(d)	**34.**	(c)	**35.**	(d)	**36.**	(a)	**37.**	(b)	**38.**	(a)	**39.**	(c)	**40.**	(c)
41.	(d)	**42.**	(c)	**43.**	(d)	**44.**	(c)	**45.**	(d)	**46.**	(d)	**47.**	(b)	**48.**	(d)	**49.**	(a)	**50.**	(b)
51.	(d)	**52.**	(a)	**53.**	(d)	**54.**	(c)	**55.**	(b)	**56.**	(c)	**57.**	(d)	**58.**	(a)	**59.**	(a)	**60.**	(a)
61.	(c)	**62.**	(b)	**63.**	(a)	**64.**	(d)	**65.**	(d)										

Numerical Answer Questions

1. 4.2 mm, 19.8 kW **2.** 1.27 kJ, 13.44% **3.** 937.5 J, 8.52%

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (c) Preheating of the workpieces (specifically of steels and cast irons) is essential to prevent cold cracks.

2. (b) Arc length is kept approximately equal to diameter of electrodes.

3. (d) The tungsten electrodes, constant current source, and argon as shielding gas provide GTAW a constant and stable arc gap at a constant current level.

4. (c) The electrodes should be sufficiently cooled in a protective atmosphere rather than oxidizing in normal atmosphere because tungsten oxide has low melting point, and thus, electrode may be contaminated and consumed.

5. (a) Power supply for gas metal arc welding is always a constant voltage type.

6. (c) Submerged arc welding (SAW) is mostly used for faster welding requirement. Therefore, very high welding speed with single pass is suitable with 1–75 mm thick sections.

7. (c) Power source for plasma arc welding is constant current type DC with electrode negative producing electrode wear.

8. (a) In resistance welding, a low voltage and high current power supply is used to produce heat in the resistance between two surfaces of the components.

9. (d) Copper in alloyed form is generally used for making electrodes to take advantage of minimum electrical resistance offered.

10. (c) Due it inherent advantage for high speed and easy set-up, resistance welding is the most suitable for mass production.

11. (d) Electro-slag welding is used for welding very large plates without any edge preparation in single pass using consumable electrodes.

12. (c) Electron beam welding is carried out in 10^5 torr vacuum.

13. (c) Weldability of cast iron is quite low. Therefore, brazing is used and arc and gas welding with special electrodes are used.

14. (a) If no filler metal is used during welding then it is termed as autogenous welding process.

15. (c) In liquid state welding, the joint is formed due to fusion of base metals and filler both.

16. (a) Penetration is the depth up to which the weld metal combines with the base metal as measured from the top surface of the joint.

17. (c) The continuous welding voltage is expressed in terms of open circuit voltage (V_o), the short circuit current (I_s) and continuous welding current (I) as

$$V = V_o\left(1 - \frac{I}{I_s}\right)$$

18. (c) Iron oxide and aluminium oxide are mixed in 3 : 1 mass ratio, as required for their chemical reaction.

19. (d) Thermit welding is a type of fusion welding used for joining very thick parts like rail ends, ship hulls, broken large castings, an application of thermit reaction between iron oxide and aluminium oxide.

20. (b) Size of electrode is based on the thickness of the cross-section to be welded and weld position.

21. (c) Direct current (DC) supply is preferred for cast iron and non-ferrous metals.

22. (c) Metals with high electrical conductivity cannot be weld by resistance welding.

23. (b) Arc blow is the deflection of arc caused by magnetic field due to flow of welding current. This can be reduced by using alternating current instead of direct current.

24. (d) Arc blow can be reduced by: using alternating current instead of direct current if possible, reducing amount of current, using shorter arc length, placing more than one ground leads to distribute the current uniformly.

25. (c) Gas tungsten arc welding (GTAW), formerly known as tungsten inert gas (TIG) welding, uses thoriated tungsten electrodes which are not consumed in GTAW, therefore, a constant and stable arc gap is maintained at a constant current level.

26. (a) To finish arc welding, arc is extinguished slowly by decreasing current supply so that arc crater is completely filled.

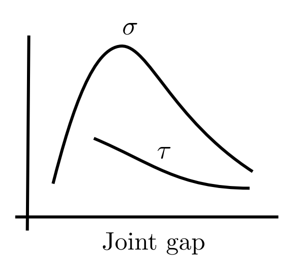
27. (b) When workpiece is made anode, great amount heat is generated on workpiece, and this is called straight polarity; when acting as cathode, it called reversed polarity.

28. (c) Thermite reaction is started by burning of magnesium ribbon dipped in mixture.

29. (d) Laser beam technique is used for deeper penetration welds even with dissimilar metals and multilayer materials.

30. (b) Gas tungsten arc welding uses thoriated tungsten electrodes with constant current source along with argon as shielding gas.

31. (d) In brazing, the clearance between mating surfaces is an important parameter, as it directly affects the strength of the brazed joint. The variation of tensile strength (σ) and shear strength (τ) of the brazed joint with joint clearance is shown in the figure below.



Thus, there is an optimum gap to achieve maximum tensile strength of the joint.

32. (c) The use of flux is essential in brazing in order to prevent oxidation and remove oxide films from workpiece surfaces.

33. (d) Two types of fluxes can be used in soldering. Inorganic acids or salts (e.g. zinc ammonium chloride solutions,) and non-corrosive resin based fluxes (in electrical applications). After soldering, the inorganic flux residues should be removed by washing thoroughly with water to avoid subsequent corrosion.

34. (c) Reduction is a process of producing metal powders by reduction of metal oxides (removal of oxygen) using reducing agents, such as hydrogen, carbon monoxide. Reducing the size of metals by milling in a ball mill is called comminution. Atomization produces metal powder by injecting molten metal through a small orifice.

35. (d) Blending of metal powder is carried out to blend the metal powders of different materials, shapes, sizes. Further they are mixed with lubricant and additives for further processing.

36. (a) Heat generated to form the weld is determined as

$$\begin{aligned} Q &= I^2Rt \\ &= 15000^2 \times 0.0001 \times 0.25 \\ &= 5625 \text{ J} \end{aligned}$$

37. (b) Spot welding uses the resistance of spots on the workpiece. If the metal is electrical conductor, that is, offering less resistance, less heat will be generated in spots, thus making the process difficult.

38. (a) SAW uses bare rods as electrodes which is submerged in a slurry of flux, thus coating of electrodes is not required.

39. (c) Let the weld length be l mm, the time required is t. The welding speed will be l/t. For equilibrium, the heat input to the melt will be equal to the electrical energy, thus

$$\begin{aligned} u\,(lA) &= \eta VIt \\ \frac{l}{t} &= \eta\frac{VI}{uA} \\ &= 0.80 \times \frac{25 \times 195}{12.3 \times 35} \\ &= 9.06 \text{ mm/s} \end{aligned}$$

40. (c) The cross-sectional area of the weld is 0.5×10^2 mm^2. For equilibrium, the heat input to the melt is equal to the electrical energy, thus

$$\begin{aligned} uvA &= \eta VI \\ I &= \frac{uvA}{\eta V} \\ &= \frac{12.3 \times 10 \times 0.5 \times 12^2}{0.75 \times 20} \\ &= 590.4 \text{ A} \end{aligned}$$

41. (d) The atomic reaction takes place in the atomic hydrogen welding process in which the temperature may reach above 6000 K.

42. (c) Size of the electrode is based on the thickness of cross-section to be welded and weld position, and also welding current is based on the thickness as it is approximately proportional to square of thickness.

43. (d) The gas tungsten arc welding (GTAW) is particularly suitable for aluminium, magnesium, titanium, refractory metals and also for thin sections.

44. (c) Laser beam can be focused on as small a diameter as 10 μm, it has high energy density and there is deep penetration capability.

45. (d) Unlike in electron beam welding, laser beam process neither requires vacuum nor generates X-rays. Since laser beams can easily be focused optically, the process can be easily automated.

46. (d) Penetration in the welds can be improved by increasing the rate of heat input, lowering the welding speed, and ensuring proper fitting and alignment of weld surfaces. In addition to these, design of the joint can also be modified.

47. (b) The ultrasonic welding process is used extensively for lap welding of sheet, foil, and thin wires.

48. (d) The temperature generated in the weld zone during ultrasonic welding are in the range of one-third to one-half the melting point of the metals, therefore, no melting and fusion takes place.

49. (a) Submerged arc welding is considered as an excellent and efficient process to use on nearly all ferrous metal welds of exceptionally good quality. Carbon, alloy and stainless steels upto 12 mm thick can be safely welded in single pass, while thicker cross-section requires multi-pass welding. Gas welding is more suitable for thin plates and sheets as its flame is not as piercing as that of arc welding. Welding time is also comparatively longer in gas welding and heat affected zone (HAZ) and distortion are larger than in arc welding.

50. (b) In friction welding, the heat required for welding is, as the name implies, generated through

friction at the interface of the two members being joined.

51. (d) In AHAW, two tungsten electrodes are used with hydrogen as shielding gas which is reactive in nature. When H_2 comes in contact with electric arc, it breaks into two H^+ ions which react with oxygen forming water vapor along with intense heat.

52. (a) Gas metal arc welding (GMAW) is used for thicker metals and fillet welds. Consumable electrode in the form of a wire reel is fed at a constant rate through the feed rollers.

53. (d) The oxyacetylene gas welding is based upon the chemical reaction between C_2H_2 and O_2.

54. (c) The chemical reaction between oxygen and acetylene is written as

$$C_2H_2 + O_2 \rightarrow 2CO + H_2 + 488 \text{ kJ/mol}$$

Thus, for neutral flame, the ratio is 1:1, and for carburizing flame, the volume of C_2H_2 should be more.

55. (b) Only brass tubing is used with acetylene.

56. (c) Piping made of copper is never used with acetylene because it forms copper acetylide which is an unstable compound and dissociates violently with shocks. Therefore, only brass tubing is used with acetylene.

57. (d) Carburizing flame is used for oxygen-free copper alloys, high carbon steels, cast irons, high speed steels, cemented carbides, aluminium alloys, nickel alloys, etc.

58. (a) Moving of welding torch in the direction of tip results into preheating of workpieces, and this is called forehand welding.

59. (a) In high pressure system, the oxygen is stored in a seamless steel cylinder at 13.8 to 18.2 MPa, while acetylene is stored at 1.75 MPa in liquid state. Acetylene is highly explosive above 200 kPa, so it is soaked in 80% calcium silicate ($CaSiO_3$), a porous material.

60. (a) In gas welding, the two gases are brought together through pipes and meet at the tip of welding torch. A small spark is needed to start the arc. This is achieved by using friction lighter for safety.

61. (c) Preheating of the workpieces (specifically of steels and cast irons) is essential to prevent cold cracks.

62. (b) The chemical reaction between oxygen and acetylene is written as

$$C_2H_2 + O_2 \rightarrow 2CO + H_2 + 488 \text{ kJ/mol}$$

Thus, for neutral flame, the ratio is 1:1.

63. (a) The most commonly used solder is an alloy containing tin ranging from 5 to 70% and lead from 30 to 95%. Higher the content of tin, lower is the melting point of alloy.

64. (d) In many brazing applications, fluxes need to be employed whose function is to promote wetting and flowing of the brazing alloy by dissolving non-metallic (mainly oxide) films, present on the surface of both the parent metal and brazing alloy.

65. (d) The strength of brazed joint is higher than soldered joint but lower than welded joint.

Numerical Answer Questions

1. For the given straight line characteristic,

$$\begin{aligned} V_o &= 72 \text{ V} \\ I_s &= 1200 \text{ A} \end{aligned}$$

Power characteristic is

$$\begin{aligned} V &= V_o\left(1 - \frac{I}{I_s}\right) \\ &= 72\left(1 - \frac{I}{1200}\right) \end{aligned}$$

Arc characteristics:

$$V = 18 + 42l$$

Using the above two equations,

$$\begin{aligned} 72 - \frac{72I}{1200} &= 18 + 42l \\ I &= 900 - 700l \end{aligned}$$

Power requirement is calculated as

$$\begin{aligned} P &= VI \\ &= (900 - 700l)(18 + 42l) \end{aligned}$$

For maximum power requirement,

$$\begin{aligned} \frac{dP}{dl} &= 0 \\ 25200 - 58800l &= 0 \\ l &= 4.28 \text{ mm} \end{aligned}$$

The power requirement for $l = 0.428$ cm is

$$\begin{aligned} P &= (900 - 700l)\,(18 + 42l) \\ &= 21.59 \text{ kW} \end{aligned}$$

2. The welding involves melting of three spots of 6 mm diameter and 2.5 mm thickness. Heat required is calculated as

$$\begin{aligned} Q_w &= 3 \times \frac{\pi \times 6^2}{4} \times 1.5 \times 10 \\ &= 1.2723 \text{ kJ} \end{aligned}$$

The electrical energy supplied to the spots is

$$\begin{aligned} Q &= I^2Rt \\ &= 35000^2 \times 120 \times 10^{-6} \times 0.01 \\ &= 1.470 \text{ kJ} \end{aligned}$$

Heat dissipated to the surrounding is

$$\begin{aligned} \frac{Q - Q_w}{Q} &= \frac{1.470 - 1.2723}{1.470} \\ &= 13.44\% \end{aligned}$$

3. Heat generated in the weld:

$$\begin{aligned} Q &= I^2Rt \\ &= 5000^2 \times 250 \times 10^{-6} \times 0.15 \\ &= 937.5 \text{ J} \end{aligned}$$

Given that

$$\begin{aligned} \rho &= 7800 \text{ kg/m}^3 \\ u &= 1.4 \times 10^6 \text{ J/kg} \end{aligned}$$

Volume of the weld nugget is

$$\begin{aligned} V &= \frac{\pi \times 0.005^2}{4} \times 0.002 \times 2 \\ &= 7.854 \times 10^{-8} \text{ m}^3 \end{aligned}$$

Heat consumed in melting of the nugget is

$$\begin{aligned} Q_w &= \rho V u \\ &= 857.65 \text{ J} \end{aligned}$$

Percentage heat dissipated is

$$\begin{aligned} 1 - \frac{Q_w}{Q} &= 1 - \frac{857.65}{937.5} \\ &= 8.52\% \end{aligned}$$

CHAPTER 14

MACHINING

Machining is the process of removing materials from a workpiece in form of chips. If the workpiece is a metal, the process is often termed as *metal cutting*. The body having cutting edge, which remove the excess material is called *cutting tool* and the machine which provides the necessary relative motions between the work and tool is known as *machine tool*. *Feed* is the rate at which tool is fed into the workpiece in each cutting step. In comparison to shaping processes, material removal processes are much more versatile and offer better control on dimensions and surface finish.

Machining is called *orthogonal machining*, when there is no component of velocity in the direction perpendicular to the plane of cutting, or in other words, cutting edge is straight and relative velocity is perpendicular to cutting edge. Otherwise, it is called *oblique machining*.

Chip removal can be done in three ways: *single-point cutting* (e.g. turning, shaping), *multi-point cutting* (e.g. milling), *abrasive machining* (e.g. grinding). Single point cutting tools have only one cutting edge, such as cutting tools used in lathe and shaper. A multi-point cutting tool has more than one cutting edge, such as milling cutters, twist drills. In abrasive machining, the abrasive particles act as cutting edges.

14.1 MECHANISM OF METAL CUTTING

14.1.1 Chip Formation

The form of the chips indicates behavior of the work material under machining condition, specific energy requirement, interaction at the chip–tool interface, etc. It depend mainly upon the materials of workpiece and tool, geometry of the cutting tool, cutting velocity, feed and depth of cut, and machining environment. Thus, mechanisms of chip formation can be studied separately for ductile and brittle materials:

1. *Chip Formation in Ductile Materials* During continuous machining of ductile materials, the uncut layer of the work material just ahead of the cutting tool (edge) is subjected to almost all sided compression [Fig. 14.1].

   The force exerted by the tool on the chip arises out of the normal force (N) and frictional force (F). The compression results into slip (shear deformation) at the plane of maximum shear stress. The shear region gradually moves along the tool rake surface and then goes beyond the point of chip-tool engagement. The succeeding portion of the chip starts undergoing compression followed by shear. This phenomenon repeats rapidly and

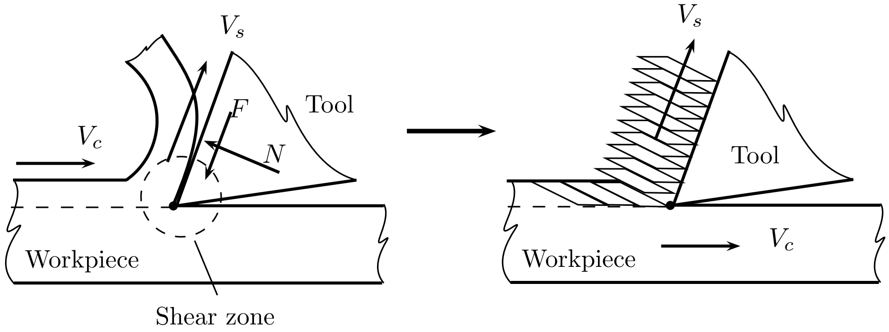


Figure 14.1 Chip formation in ductile material.

results in the formation and removal of chips, layer by layer.

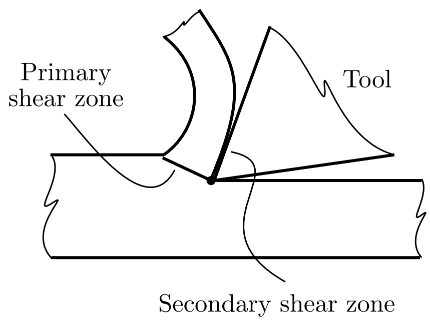


Figure 14.2 Deformation zones.

Serrations are found at the upper surface of chips while the lower surface becomes smooth due to further plastic deformation caused by intensive rubbing with the tool at high pressure and temperature.

Normal stress is maximum at the cutting edge and reduces linearly, while shearing stress is more or less uniform over rake surface and then reduces. This divides the rake surface into two zones: *sticking zone* and *sliding zone* [Fig. 14.2]. Under normal machining at moderate speed, the thickness of shear zone is very small and it can be idealized as the shear plane. The inclination of this shear plane with machined surface is called *shear angle* ϕ.

2. Chip Formation in Brittle Materials Chip formation in brittle materials starts with the development of a small crack at the tool tip due to wedging action of the cutting edge. Stress concentration takes place at the sharp crack-tip which quickly propagates under stress and total separation takes place from the parent workpiece through the minimum resistance path [Fig. 14.3].

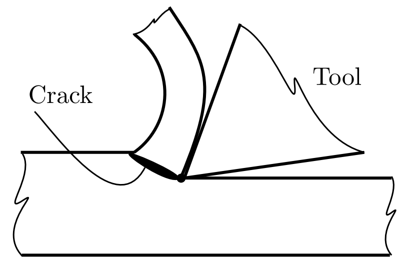


Figure 14.3 Chip formation in brittle materials.

Machining of brittle material produces discontinuous chips and mostly of irregular size and shape.

14.1.2 Built Up Edge

In high-speed cuttings, the temperature increases and lump, known as *built-up edge* (BUE), is formed at the cutting edge that grows continuously and breaks due to increased force caused by flow of material. Fragments of BUE tend to adhere on the machined surface. Thus, formation of BUE is prohibited. Steels containing free ferrite are soft and their machining is with no tool wear. However, they are likely to form BUE and consequently have poor surface finish. The presence of pearlite improves *machinability*, giving good surface finish with no BUE, but decreases tool life. In the presence of cementite, because of its high hardness, tool life decreases.

Continuous chip without BUE is formed with ductile work material, small uncut thickness, high cutting speed, larger rake angle, and suitable cutting fluid.

Temperature rise at shear plane weakens the material and causes further strain. If the metal is a poor conductor, this process when repeated several times and results in large strain at the point of initial strain. Then, a new shear plane will develop some distance from the

first, and deformation shifts to this point. In this way, *inhomogeneous chips* are formed.

14.1.3 Chip Thickness Ratio

Figure 14.4 shows important parameters associated with metal cutting. These are *uncut thickness* (t_1), *chip thickness* (t_2), *rake angle* (α), *clearance angle* (α_c), *cutting velocity* (V_c), *chip velocity* (V_{chip}), and *shear velocity* (V_{shear}).

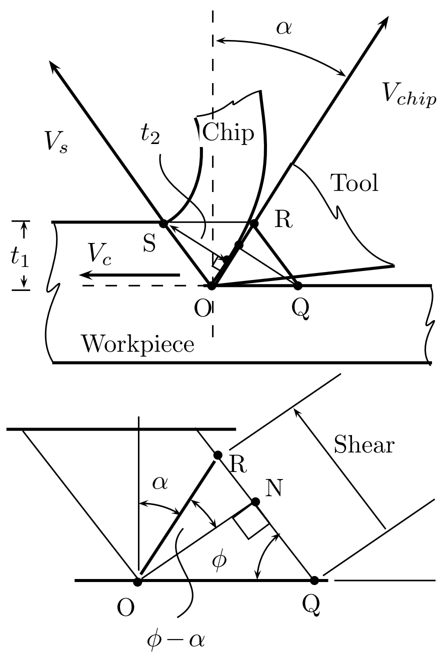


Figure 14.4 Chip formation and shear strain.

Cutting ratio or *chip thickness ratio* (r) in orthogonal cutting is defined as the ratio of uncut thickness (t_1) to chip thickness (t_2):

$$r = \frac{t_1}{t_2}$$

Width of the shear plan can be determined as

$$\mathrm{SO} = \frac{t_2}{\cos(\phi-\alpha)} = \frac{t_1}{\sin\phi}$$

Therefore, the cutting ratio (r) is determined as

$$\begin{aligned} r &= \frac{t_1}{t_2} \\ &= \frac{\sin\phi}{\cos(\phi-\alpha)} \end{aligned} \tag{14.1}$$

14.1.4 Shear Angle

The inclination of this shear plane with machined surface is called *shear angle* ϕ [Fig. 14.4]. This can be determined using Eq. (14.1):

$$\tan\phi = \frac{r\cos\alpha}{1-r\sin\alpha}$$

14.1.5 Shear Strain

Figure 14.4 also demonstrates the geometry of shearing. Inclination of the shear plane normal (ON) w.r.t. to the machine surface and rake of the tool (OR) are written as

$$\begin{aligned} \angle \mathrm{OQN} &= \phi \\ \angle \mathrm{RON} &= \phi-\alpha \end{aligned}$$

The magnitude of shear strain is given by

$$\begin{aligned} \gamma &= \frac{\mathrm{QR}}{\mathrm{ON}} \\ &= \frac{\mathrm{QN}}{\mathrm{ON}} + \frac{\mathrm{NR}}{\mathrm{ON}} \\ &= \cot\phi + \tan(\phi-\alpha) \end{aligned}$$

14.1.6 Chip Velocity

Using the principle of *conservation of mass* in the cutting process:

$$\begin{aligned} V_{chip}\times t_2 &= V_c \times t_1 \\ V_{chip} &= \frac{t_1}{t_2}\times V_c \\ &= rV_c \end{aligned}$$

Thickness of chip is greater than the depth of cut ($t_2 > t_1$), therefore, the velocity of the chip V_{chip} has to be lower than the cutting speed V_c. Using Eq. (14.1),

$$V_{chip} = \frac{\sin\phi}{\cos(\phi-\alpha)} V_c \tag{14.2}$$

14.1.7 Shear Velocity

Inter-relationship among V_{chip}, V_c, and the velocity of shear V_{shear} can be obtained by using the constancy of the volume displaced by tool, volume cut from the workpiece, and volume taken out by the chip. This can be expressed as a velocity triangle [Fig. 14.5].

Using Lami's theorem:

$$\frac{V_c}{\sin\left(\pi-\left(\phi+\frac{\pi}{2}-\alpha\right)\right)} = \frac{V_{shear}}{\sin\left(\frac{\pi}{2}-\alpha\right)} = \frac{V_{chip}}{\sin\phi}$$

$$\frac{V_c}{\cos(\phi-\alpha)} = \frac{V_{shear}}{\cos\alpha} = \frac{V_{chip}}{\sin\phi}$$

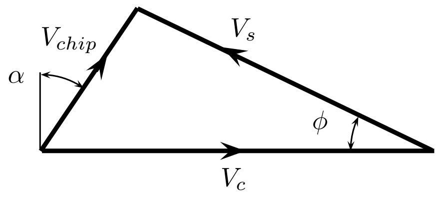


Figure 14.5 Orthogonal cutting.

14.1.8 Shear Strain Rate

If t_m is the mean thickness of primary shear zone, then shear strain rate is given by

$$\dot{\gamma} = \frac{V_{shear}}{t_m} = \frac{V_c \cos\alpha}{\cos(\phi - \alpha)\, t_m}$$

14.1.9 Ernst–Merchant Analysis

The scientific analysis of *cutting forces* was carried out by H. Ernst and M. E. Merchant in 1941. Figure 14.6 shows free body diagram of the chip, considered as a rigid body. The chip is in equilibrium under the action of the following forces:

1. *Shear Force* The resistance to shear in chip formation, acting along the shear plane (F_s).
2. *Normal Reaction* The reaction offered by workpiece, acting normal to shear plane (F_n).
3. *Friction Force* The frictional resistance offered by tool along the tool rake (F).
4. *Normal Reaction* The reaction offered by tool normal to the rake (N).

These forces can be drawn within a circle pivoted at the cutting edge [Fig. 14.6].

This circle is known as *Merchant's circle diagram* or *composite cutting force circle*, first suggested by Merchant. This circle can be used for the analysis of forces acting on the chip. Such an analysis is called *Merchant's analysis*[1].

If coefficient of friction between chip and tool rake is μ (friction angle λ), then forces acting at the rake are related as

$$\frac{F}{N} = \mu = \tan\lambda$$

[1]In 1945, M.E. Merchant developed the earliest steady state orthogonal cutting model using a minimum energy hypothesis and found the relations between the shear angle and the rake angle. The study was published in his papers (a) "Mechanics of metal cutting process: Part I orthogonal cutting and a type-2 of chip", J. Appl. Phys., 16, 267-275 (1945) and (b) "Part II plasticity consideration on orthogonal cutting", ibid., 16, 318-324 (1945).

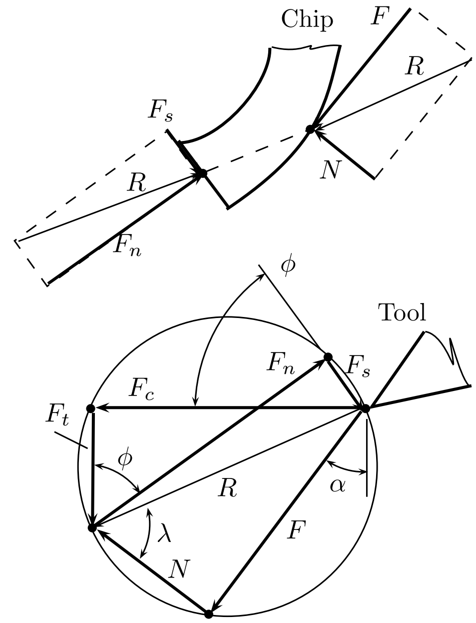


Figure 14.6 Merchant's circle.

Merchant's circle reflects the effect of friction on shear force (F_s) and shear angle (ϕ). Large friction force results in thick chip having low shear angle. Power consumption is low when friction force is minimized. Therefore, Merchant's analysis has two basic objectives:

1. *Cutting Forces* The shear force acting on the chip at shear plane is determined as

$$F_s = b\frac{t_1}{\sin\phi}\tau_s$$

where t is the width of the workpiece and τ_s is the shear strength of the work material. The resultant of F_s and F_n is also equal to R. Therefore, using Merchant's circle [Fig. 14.6], one obtains

$$R = \frac{F_s}{\cos(\phi + \lambda - \alpha)}$$

The resultant force (R) acting on tool rake can be resolved into horizontal and vertical components to find the cutting force (F_c) and thrust force (F_t), respectively. Therefore,

$$F_c = R\cos(\lambda - \alpha)$$
$$F_t = R\sin(\lambda - \alpha)$$

Friction and normal forces at tool rake can be determined as

$$F = F_c \sin\alpha + F_t \cos\alpha$$
$$N = F_c \cos\alpha - F_t \sin\alpha$$

Coefficient of friction can be expressed as

$$\mu = \frac{F}{N}$$
$$= \frac{F_c \sin\alpha + F_t \cos\alpha}{F_c \cos\alpha - F_t \sin\alpha}$$
$$= \frac{F_t + F_c \tan\alpha}{F_c - F_t \tan\alpha}$$

2. *Power Consumption* Power consumption in metal cutting can be determined as

$$P = F_c \times V_c$$

For minimum power consumption,

$$\frac{dP}{d\phi} = 0$$

which is possible when

$$2\phi + \lambda - \alpha = \frac{\pi}{2}$$
$$\phi = \frac{\pi}{4} + \frac{\alpha - \lambda}{2} \qquad (14.3)$$

This equation is known as Ernst–Merchant formula which indicates that the shear angle decreases with decrease in rake angle and increase in friction at the tool-chip interface. This results in increased chip thickness that causes increase in power consumption and heat dissipation.

Apart from Ernst–Merchant formula [Eq. (14.3)], the following are important relationships:

(a) *Stabler theory*

$$\phi = \frac{\pi}{4} - \lambda + \frac{\alpha}{2}$$

(b) *Lee–Shaffer theory*

$$\phi = \frac{\pi}{4} - \lambda + \alpha$$

14.2 CUTTING HEAT

During machining, heat is generated at three regions: The first point (1) is the primary shear zone where major part of the cutting energy is converted into heat. The second point (2) is the secondary deformation zone at the chip-tool interface where heat is further generated due to rubbing and shear of the chip on the tool rake. Rubbing action between the tool and the work generates heat at the third point (3) [Fig. 14.7].

The apportionment of the heat shared by the chip, cutting tool and the blank depends upon the configuration, size and thermal conductivity of the tool work material and the cutting condition. With the increase in cutting velocity, the chip shares heat increasingly. At extremely slow cutting, the heat is carried out in the chip and through the workpiece, cutting tool and atmosphere. However, when cutting speed is increased, there is not enough time for heat dissipation. Normally, the ratio of heat shared among chip, tool and work is 70:20:10.

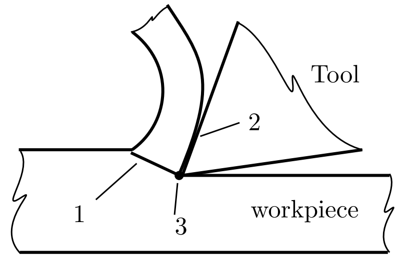


Figure 14.7 Heat generation in metal cutting.

14.3 TOOL MATERIALS

An ideal cutting tool material should be harder than the work and should resist wear, impact, thermal shock. The material should also be stable at high temperature, and should be chemically inert to the work material and cutting fluid. No single cutting tool material incorporates all these qualities. Instead, trade-offs occur among the various tool materials.

There has been a progression in cutting tool materials from high-speed steels to carbide, and on to ceramics and other super-hard materials, discussed as follows:

1. *High Speed Steels* High-speed steels offer four times higher cutting speeds in comparison to carbon steels. There are over 30 grades of high-speed steel in three main categories: tungsten, molybdenum, and molybdenum-cobalt based grades. Use of coatings, particularly titanium nitride, allows high-speed steel tools to cut faster and last longer. This provides a high surface hardness, resists corrosion, and minimizes friction in cutting.

2. *Cemented Carbides* Cemented carbide is a powder metallurgy product consisting of fine carbide particles cemented together with a binder of cobalt. Carbide tools and coated carbide tools cut about 3 to 5 times faster than high-speed steels. The major categories of hard carbides include tungsten carbide, titanium carbide, tantalum carbide and niobium carbide. These are used in solid round tools or in the form of replaceable inserts. Brazed tip tools have cutting insert held in the tool shank by silver brazing. Tungsten carbide is exclusively used as the tool material for brazed tip tools.

3. *Ceramics* Ceramic cutting tools are harder and more heat-resistant than carbides, but are more brittle. They are well suited for machining cast iron, hard steels, and the super-alloys. Ceramic cutting tools can be alumina based and silicon nitride based. The alumina-based ceramics are used for high speed semi- and final-finishing of ferrous and some non-ferrous materials.

4. *Super-Hard Tool Materials* Super-hard tool materials are divided into two categories: cubic boron nitride (CBN) and polycrystalline diamond (PCD). Their cost can be 30 times that of a carbide insert, so their use is limited to well-chosen and cost-effective applications. Cubic boron nitride is used for machining very hard ferrous materials, such as steel dies, alloy steels and hard-facing materials. Polycrystalline diamond is used for non-ferrous machining and for machining abrasive materials, such as glass and some plastics.

Stellites are cobalt-chromium-tungsten cast alloys (45% Co + 18% W + 34% Cr+ 2% C) are wear and corrosion resistant. These are produced by powder metallurgy. Properties of stellite metal cutting tools are in between those of high speed steels and cemented carbides. However, stellites as cutting tool material have became obsolete due to poor grindability.

UCON (50% Columbium (Cb) + 30% Ti + 20% W) is a cutting material, developed by Union Carbide, USA. This is produced by rolling of the powder metallurgy ingot.

14.4 CUTTING FLUIDS

Cutting fluids are the materials applied to the tool-workpiece-chip zone to facilitate the cutting operation primarily by cooling and lubricating the cutting zone. Secondary functions of cutting fluids are to improve the tool life, reduce thermal deformation of the workpiece, improving surface finish and flush away chips from the cutting zone. Thus, cutting fluids are broadly classified into coolants or lubricants.

A good cutting fluid should possess high thermal conductivity, high heat capacity, low viscosity (but good lubrication properties), chemical inertness with tools and workpiece. It should be transparent so that cutting zone can be easily visible. Cost and availability are also important factors.

Cutting fluids are classified into various groups, such as aqueous solution, mineral oils, soluble oils or emulsions, soaps, greases, waxes. Oils have high film strength. An emulsion is a mixture of two immiscible liquids, usually mixtures of oil and water in various proportions. Soaps are generally reaction products of sodium or potassium salts with fatty acids. Greases are highly viscous semi-solid lubricants. Water-based fluids are very effective coolants, but as lubricants they are not as effective as oils. Some solid materials are also used as lubricants in machining operations, such as graphite, molybdenum disulphide, glass. Metal working fluids are usually blended with additives, including oxidation inhibitors, rust preventatives, odor control agents, antiseptics, and foam inhibitors. Important additives in oils are sulphur, chlorine, and phosphorus.

The effectiveness of cutting fluids depends on numerous factors, such as the method of application of the fluid, cutting temperature and speed, and type of machining. Heavy machining processes (e.g. broaching or screwing with tap) generally require middle or heavy cutting oils. Cutting speed in drilling operation is generally low due to two cutting edges of drill tool. Moreover, geometry of formed chip is different. Therefore, cooling effect of cutting fluid is more important in drilling process. For this reason, emulsion oils and sulphur or chlorine additive mineral oils are generally used in drilling. Water-based cutting fluids are more suitable in turning, milling and grinding processes.

Gray cast iron could be machined dry because graphite flakes act as solid lubricants. Cast iron is brittle during machining as they break into small size chips, therefore, friction between cutting tool and chip is less. Copper alloys have better *machinability* and could be machined dry, otherwise water-based oils are used. Some copper alloys produce tough and stingy chips, thus mineral lard oils are used for their machining. Chips of aluminium and aluminium alloys have the tendency to weld onto the tool. Therefore, kerosene or light mineral oils mixed with kerosene are the suitable cutting fluids for machining of aluminium workpieces. Free cutting steels are machined dry. Sulphurized mineral oils are preferred for machining of nickel alloys. There is a risk of the chip catching fire in machining of magnesium alloys, therefore, these alloys are machined at low speeds and instead of water-based fluids, mineral oils of low viscosity are used as cutting fluids.

Cutting fluids present biological and environmental hazards that require proper recycling and disposal. This adds to the cost of machining. Therefore, dry cutting or dry machining has become an increasingly important approach. More specifically, dry cutting has been associated with high-speed machining, because higher cutting speeds transfer a greater amount of heat from cutting zone to the chip.

14.5 CUTTING TOOL GEOMETRY

The geometry of a single point cutting tool is described in terms of following six angles measured outside the volume of the tool:

1. *Back rake angle* (α_b)
2. *Side rake angle* (α_s)
3. *Back clearance angle* (c_b)
4. *Side clearance angle* (c_s)
5. *End cutting angle* (ϕ)
6. *Side cutting angle* (ψ)

Referring Fig. 14.8, these angles are seen on the respective views: ψ, ϕ and r from top view, α_b, c_b and lip angle from side view, and α_s and c_s are seen in front or end view.

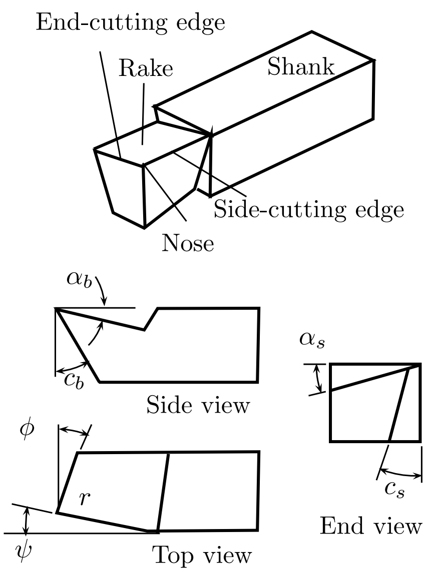


Figure 14.8 | Cutting tool geometry.

Cutting edge angles control chip formation, cutting forces and strengthen the cutting tool. *Rake angle* tends to determine the direction of chip flow across the face of the tool. Cutting efficiency is best with positive rake angle since the tool penetrates the workpiece easier and tends to shear the metal off rather than compress and push it off. Negative rake angle tools are stronger and are used at higher speeds with carbide cutting tools.

The functional angle of greatest important is the true rake angle. It is generally defined as the slope of the tool face in the direction of chip flow w.r.t. radial reference plane passing through the point of tool and axis of the rotating workpiece. For example, in turning operation, tool has zero back rake angle, but effective is positive rake angle.

Nose radius (r) strengthens the tool point by thinning the chip where it approaches the point of the tool and by spreading the chip over a larger area of the point. It also produces better surface finish. A larger nose radius is desirable for heavy depth of cut, heavy feeds and interrupted cutting.

Tool designation or *tool signature* is a sequence of numbers listing various angles in degrees and the size of nose radius in mm. For this, American Standards Association (ASA) has standardized the tool identification, known as *coordinate system*, which uses seven elements comprising the signature of a single point cutting tool, written in the following sequence:

$$\alpha_b - \alpha_s - c_b - c_s - \phi - \psi - r$$

For example,

$$6^\circ - 10^\circ - 7^\circ - 7^\circ - 10^\circ - 30^\circ - 9.5 \text{ mm}$$

Other known systems for tool designation are British system, continental system, international or *orthogonal rake system.*

14.6 MACHINABILITY

Machinability is a measure of two primary factors:

1. *Tool Life* This influences productivity and economy in machining.
2. *Surface Finish* This affects performance and service life of the machined product.

Magnitude of cutting forces, cutting temperature, and chip forms are also considered for assessing machinability because all these directly affect the power requirements and dimensional accuracy of the product. These characteristics are influenced by material properties of the work and tool, cutting tool geometry, process parameters, machining environments, rigidity and stability of the machine tool, machining operation, etc. For example, low carbon steels with less than 0.15% carbon machine poorly, because they are soft and gummy and adhere to the cutting tool. An increase in carbon content upto 0.2% increases machinability. Pearlite grains become so numerous that they interfere with the penetration of cutting tool. A pearlite structure is very low machinability rating. Machinability of a cold drawn steel is higher than an equivalent hot rolled steel. Spherodizing of pearlite structures increases machinability. Sulphur and lead are additives in free cutting steels. Sulphur combines with manganese and forms sulphide inclusions in the grain structure while lead inclusions themselves cause discontinuity.

14.6.1 Tool Life

All cutting tools are perishable; they have a finite working life. When cutting tool is unable to cut, consumes

reasonable energy, and cannot produce acceptable finish, it is considered to have failed. Worn and dull tools cannot be used until they break because this can be a safety hazard. This also creates more scrap, impacts on tool and part costs, thus reducing the productivity. But considerable time is lost whenever a tool is replaced and reset. Therefore, tool life is an important aspect in metal cutting operations.

14.6.1.1 Tool Wear Aside from plastic deformation and mechanical breakage, cutting tools wear in different ways. There are five basic types of wear that directly affect tool life:

1. Diffusion Wear *Diffusion wear* occurs when work material or chip slides with tool face and the temperature at their interface can be sufficient to cause the alloying atoms from harder metal diffuse into the softer matrix of work material.
2. Abrasion Wear *Abrasion wear* occurs when hard particles existing in softer work material act as cutting edges for hard tool face over which the work material slides. These particles plough craters into the surface of the hard tool metal.
3. Adhesion Wear *Adhesion wear* occurs when high temperature involved in cutting operations causes a welding action on the surfaces of the tool and workpiece.
4. Chemical Wear *Chemical wear* is caused by chemical reactions between the tool and workpiece in presence of cutting fluids.
5. Oxidation Wear *Oxidation wear* is the result of oxidation of the carbide tools at high temperatures which decreases the strength and causes wear at the edges.

The wear is visible in two locations on a tool: flank and rake [Fig. 14.9].

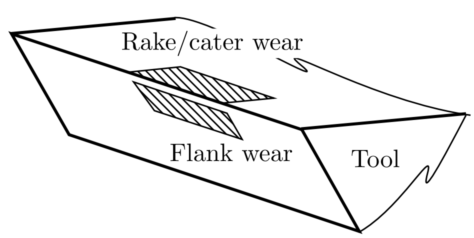


Figure 14.9 Flank and crater wears.

Flank wear is normal and slow type of tool wear which accelerates if the work material is highly abrasive. Wear on rake takes place as *cratering* or depression behind the cutting edge on the face of the tool. If the crater grows large enough and contacts the cutting edge, the tool fails immediately.

The tool material must be at least 35 to 50% harder than the work material. Unfortunately, expected fall in hardness of work due to rise in temperature at high speed is neutralized by higher rate of deformation.

14.6.1.2 Taylor's Tool Life Equation Experimental investigations have shown the following form of direct relationship between tool life T and cutting speed V_c:

$$V_c T^n = C$$

This is known as *Taylor's tool life equation*, where constant C depends upon tool and work material, tool geometry, and cutting condition (but does not depend upon V_c) [Fig. 14.10].

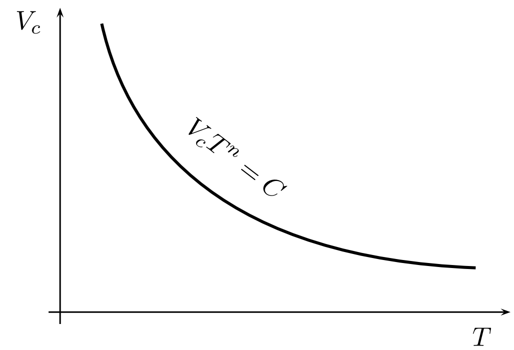


Figure 14.10 Tool life verses cutting velocity.

Taylor's tool life equation predicts that higher cutting speed leads to lower tool life. Taking logarithm of both sides,

$$\ln V_c + n \ln T = \ln C$$

$$\frac{dV_c}{V_c} + n\frac{dT}{T} = 0$$

$$n = -\frac{dV_c/V_c}{dT/T}$$

Slope of the logarithmic plot (between $\ln V_c$ and $\ln T$) is $-n$ [Fig. 14.10].

Generalized form of *Taylor's equation* is

$$V_c = \frac{C'}{T^n {t_1}^p b^q}$$

where C', p, q ($q < p$). Therefore, tool life is more sensitive to uncut thickness than the width. Cutting speed has greatest effect on tool life and tool temperature while depth of cut has greatest influence over forces.

Machinability is a term that indicates the response of work material to the cutting process. This assessment is

quoted through a term "*machinability index*" (k), which is defined as

$$k = \frac{V_{60}}{V_{60R}}$$

where V_{60} is the cutting speed for the material that ensures a tool life of 60 min, V_{60R} is the same for the reference material. SAE 1112 steel (a free cutting steel: 0.13% C, 0.06—1.10% Mn, 0.08—0.03% S) has been considered to have its machinability index 100%.

14.6.2 Surface Finish

The action of cutting tool in machining never results in absolutely smooth and even surfaces of parts. This appears in the form of lack of flatness and out of roundness. The surface invariably bears some traces of unevenness, roughness, notches, etc., both in direction of cutting and direction of feed. Built-up edge has the greatest influence on surface roughness of all the factors.

Every surface, more or less, is composed of minute hills and valleys and when the condition of service requires that two such surfaces operate one against the other, early stages of action will result in the leveling down of hills on each member. This initial wear will result in increased clearance.

Surface finish plays an important role in the performance of machine elements. Friction and wear increases with surface roughness, thus adversely affecting the performance of bearings. Rough surfaces have reduced contact area in interference fit which reduces the holding capacity of the joints. Endurance strength of the component is greatly reduced due to poor surface finish.

Some sort of roughness is always required to absorb the lubricating oil film which cannot be retained in perfectly smooth surfaces. Surface generated by super finishing is the smoothest one while that by planing is roughest.

14.6.2.1 Terminology The following is the basic terminology of the surface finish [Fig. 14.11]:

1. Lay *Lay* represents the direction of predominant surface pattern produced and it reflects the machining operation used to produce it.
2. Waviness *Waviness* refers to the irregularities which are outside the roughness width cut-off values. It is the widely spaced component of the surface texture. Waviness height is the peak-to-valley distance of the surface profile.
3. Surface roughness is specified using two methods:

   (a) *Center line average*

$$\text{CLA} = \frac{1}{l}\int_0^l y(x)dx$$

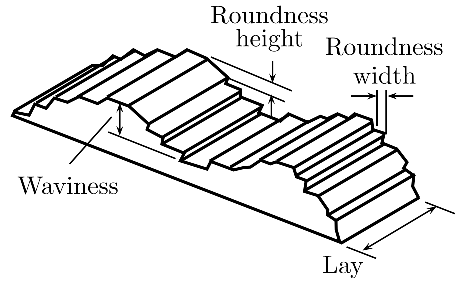


Figure 14.11 Terminology of surface finish.

   (b) *Root mean square*

$$\text{RMS} = \left\{\frac{1}{l}\int_0^l y^2 dx\right\}^{1/2}$$

In the above equations, $y(x)$ represents the profile.

14.6.2.2 Drawing Symbols Surface finishes are usually specified with a "check mark" on the blueprint in which the following details are noted:

1. Surface finishes are specified in micro-inches and are located on the left side of the symbol above the check mark 'V'. An example is shown in Fig. 14.12.

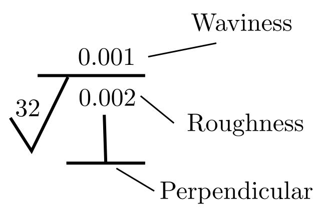


Figure 14.12 Symbols of surface finish.

2. The direction of lay is indicated by a symbol as enlisted in Table 14.1.

   The symbol indicates the direction of trace left by a cutting instrument with respect to projection plane in the drawing. For example, the symbol '=' indicates that the trace left by a cutting instrument is parallel to the projection plane in the drawing.
3. The waviness requirement (if specified) is usually given in thousands of an inch and is located on the top right of the symbol.
4. The roughness width requirement (if specified) is usually given in thousands of an inch and is located on the bottom right of the symbol.

Table 14.1 Symbols for direction of lay

Symbol	Meaning
$=$	Parallel
$\perp$	Perpendicular
$\times$	Diagonally crossed
M	No grain direction
C	Concentric circles
R	Radial around center

14.6.2.3 Marks of Feed Motion A tool leaves sign when it moves across the workpiece during feed motion. These feed marks are prominent with higher the feed rate (f) and smaller the nose-radius (r). Surface finish in machining can be determined in terms of peak-to-valley distance (h_{max}) using the following expressions which are purely based on geometry.

1. Tool without nose radius

$$h_{max} = \frac{f}{\tan\psi + \cot\phi}$$

where ψ and ϕ are side- and end-cutting edge angles, respectively.

2. Tool with nose radius r

$$h_{max} = \frac{f^2}{8r}$$

3. Slab milling

$$h_{max} = \left(\frac{f}{2NZ}\right)^2 \frac{1}{D}$$

where Z is number of teeth in the cutting, D is the average diameter of the cutter, N is number of revolutions per minute and f is feed per minute.

14.7 ECONOMICS OF MACHINING

The concept of *economics of machining* was initiated by W. W. Gilbreth on realizing that cutting speed has predominating effect upon the tool life. Slow cutting speed results in high completion time of operations, in turn, the cost associated with labor, machining, and overheads increases. Too high speeds of cutting, however, accelerate the wear of the tool, which also increases the cost of tool and tool changing. Many factors, such as desired surface finish and tolerances, horsepower of the machine tool, speeds and feed rates, influence the choice of speed and feed rates.

The problem of selecting the cutting speed can be looked in two ways: maximum production rate and minimum production cost. The cost of production consists of the following elements:

1. *Non-Production Cost* This is the cost of preparing the machine tool for machining the workpiece. This includes setting the machine tool, loading, unloading the stock, and other delays.

2. *Machining Cost* This is the cost associated with actual time taken in machining the workpiece. For this, machining time is multiplied by the sum of costs associated with direct labor and overheads. Thus, machining cost depends on the rate of machining.

3. *Tooling Cost* This consists of the cost of tool, tool changing and tool regrinding.

Let z_1 be the direct labor and overhead rate (Rs./min), z_2 be the tool cost per grind including depreciation (Rs.) and t be the tool changing time. Taylor's tool life equation relates cutting speed (V_c) and tool life (T):

$$V_c T^n = C$$

This can be used to derive the variation of production cost and production rate w.r.t. cutting speed [Fig. 14.13]. The production cost is minimum at certain point

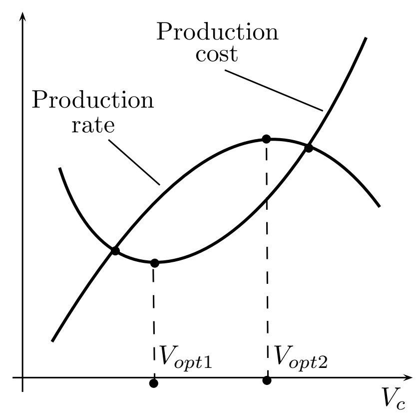


Figure 14.13 Economics of machining.

of cutting velocity (V_{opt1}), while the production rate is highest at another point of velocity (V_{opt2}), by assuming $z_2 \approx 0$. The area between this range is called optimum area of machining. The expressions of these velocities can be found as

$$V_{opt1} = \frac{C}{\{(1/n-1)\,(t+z_2/z_1)\}^n}$$

$$V_{opt2} = \frac{C}{\{(1/n-1)\,t\}^n}$$

Respective expressions for tool life are

$$T_{opt1} = \left(\frac{1}{n} - 1\right)\left(t + \frac{z_2}{z_1}\right)$$
$$T_{opt2} = \left(\frac{1}{n} - 1\right) t$$

14.8 MACHINING PROCESSES

The machined surfaces are produced by combination of two types of motions: *cutting motion* (for the cutting action), and *feed motion* (for gradual feeding). *Generatrix* is the line generated by cutting motion and *directrix* is the line generated by feed motion. Final surface geometry is envelope of generatrix and the process is called *generation*. Plan surfaces (as in shaping) are produced with straightline generatrix and directrix both. Cylinderical surfaces are machined with generatrix as a circle and directrix as a straight line.

14.8.1 Shaping

Shaping is a machining operation carried on shaper (machines). In this process, a single-point cutting tool is attached to the tool head which is mounted on a reciprocating ram. The work is kept stationary. Generally, the cutting is done during the forward stroke of the ram. The work is fed into the tool after each stroke using a separate mechanism.

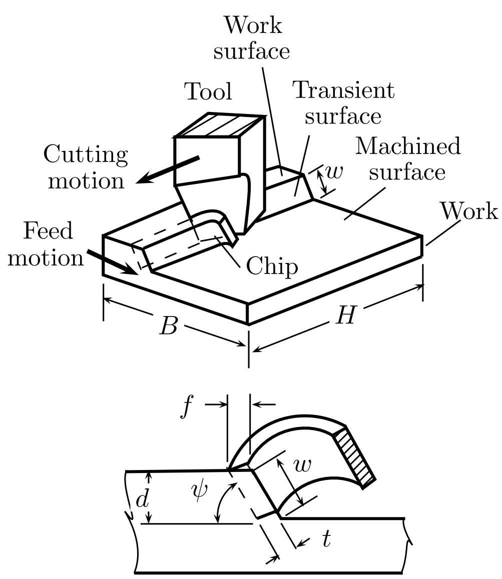


Figure 14.14 Shaping.

Chip formation in shaping operations is shown in Fig. 14.14. If tool is fed at feed rate f (mm per stroke) in the shaping process, thickness and width of the chip are determined as

$$t = f \cos\psi$$
$$w = \frac{d}{\cos\psi}$$

where ψ is the side-cutting edge angle of the cutting tool and d is the depth of cut.

Conventional shapers are driven by quick return mechanism. Let N be the speed of bull gear (double strokes per unit time) in revolution per unit time. Therefore, time taken in complete cycle of the bull gear (comprising cutting and idle strokes) is $t = 1/N$. Let R be the return ratio (idle time/cutting time), and L be stroke length of the ram of the shaper mechanism. Let V_c be the average cutting speed during the cutting stroke. Therefore, time taken in complete cycle of the bull gear (comprising cutting and idle strokes) can also be determined as

$$\frac{1}{N} = \frac{L}{V_c} + R\frac{L}{V_c}$$
$$V_c = (1+R)\,NL$$

Average velocity of ram is

$$V = \frac{2L}{N}$$

Number of double strokes per minute is

$$N = \frac{V_c}{(1+R)\,L}$$

Number of strokes required for shaping of a job of width B and height h can be determined as

$$n = \frac{B}{f} \times \frac{H}{d}$$

Time required for these number of strokes at depth of cut d is

$$T = \frac{B}{f} \times \frac{H}{d} \times \frac{1}{N}$$
$$= \frac{B}{f} \times \frac{H}{d} \times \frac{(1+R)\,L}{V_c}$$

Material removal rate (MRR) can be determined as

$$\text{MRR} = f d V_c$$

14.8.2 Turning

Turning is performed on lathe (machines). In this process, a single-point cutting tool is attached to the tool post mounted on cross slide. Cross slide is mounted on carriage which can move over the guide ways made

on the bed of the machine. The work is attached to a rotating chuck mounted on head stock. The work can also be supported at tail stock through a dead center. Feed is provided by the longitudinal motion of the carriage, while depth of cut is provided by the cross-slide.

For turning operations, *feed* is defined as the lateral movement of the cutting tool per revolution of the work piece (mm/revolution) and *depth of cut* is the distance the tool penetrates below the original surface of the work (mm). Chip formation in turning operations is shown in Fig. 14.15.

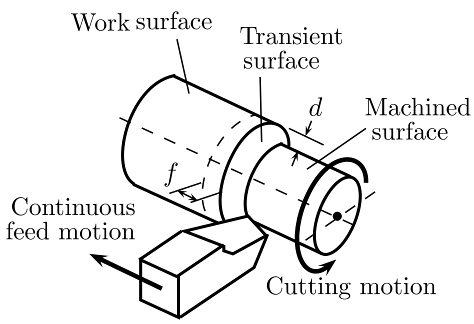


Figure 14.15 Turning.

If turning is done at feed rate f, then dimensions of uncut thickness (t) and width of the chip (w) are found, respectively, as

$$t = f\cos\psi$$
$$w = \frac{d}{\cos\psi}$$

where ψ is the side cutting edge angle of the cutting tool. Average cutting speed is determined as

$$V_c = \frac{\pi DN}{60}$$

where D is the mean diameter D and N is the speed of rotation of the work piece in rpm. Material removal rate is given by

$$\begin{aligned} \text{MRR} &= \pi D f d\frac{N}{60} \\ &= V_c f d \end{aligned}$$

Turning time for a workpiece of length l can be determined as

$$T = \frac{l}{fN}$$

14.8.3 Drilling

Drilling is a common hole-making operation performed with the help of standard-point twist drills attached to a rotating spindle. The tool in this process has two principle cutting edges. The work is kept stationary while the rotating drill is advanced along the hole to be drilled. Chip formation in drilling operations is shown in Fig. 14.16

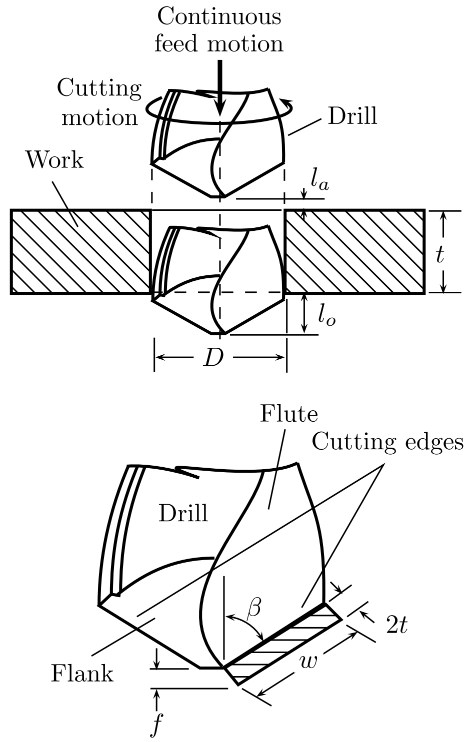


Figure 14.16 Drilling.

If the feed rate per revolution of the drill is f, then uncut thickness (t_1) and width of the chip (w) on each cutting edge would be based on half of the feed rate, $f/2$. Thus,

$$t_1 = \frac{f}{2}\sin\beta$$
$$w = \frac{D/2}{\sin\beta}$$

where β is the half-point angle of the drill.

$$\text{MRR} = \frac{\pi D^2}{4} fN$$

where N is the rpm of the drill.

True rake angle of the cutting face at any radius r is related to the helix angle ψ, half point angle β and diameter of the drill D as

$$\tan\alpha \approx \frac{2r\tan\psi}{D\sin\beta}$$

14.8.4 Milling

Milling is an interrupted machining process performed by a rotating multi-point cutting tool to generate the surface by progressive chip removal. The cutting tool used in milling is known as milling cutter. Equally spaced peripheral teeth will intermittently engage and machine the work piece. Figure 14.17 shows two broad types of milling processes.

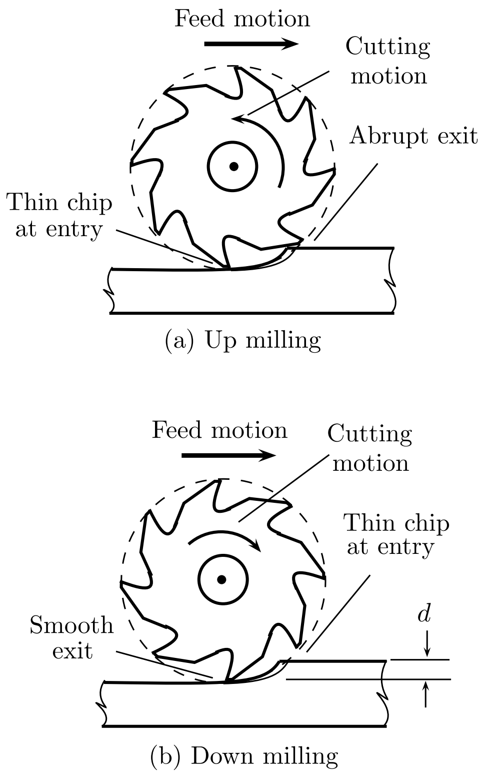


Figure 14.17 Up and down milling processes.

The cutting speed (V_c) is the velocity of the milling cutter tool relative to the work. This is usually expressed as a spindle rotation speed (revolution per minute), N, by the following relation

$$N = \frac{V}{\pi D}$$

where D is the cutter diameter.

In milling, the feed usually means the size of the chip formed by each tooth in the milling cutter, expressed as f (mm per tooth). Feed rate (travel rate of the table) f_r in mm/min is determined as

$$f_r = Nzf$$

where z is the number of teeth on the milling cutter, and N is the spindle rotation speed.

14.8.5 Grinding

Grinding is an abrasive machining process is performed by means of a rotating abrasive wheel, vaguely similar to a milling cutter. Grinding wheels are composed of many small grains of abrasive partiles bonded together, each acting as miniature cutting point. Cutting tools constituted by projected abrasive particles. Four motions are involved in grinding: traversing of wheel, feed of wheel rotation, and rotation of work. *Precision grinding* is concerned with producing good surface finish and high degree of accuracy. Surface finish upto 0.2 μm can be obtained by grinding.

Figure 14.18 shows the structure of a grinding wheel, and the formation of grinding chip being produced by a single abrasive grain, having a large negative rake angle. The chip form has low shear angle.

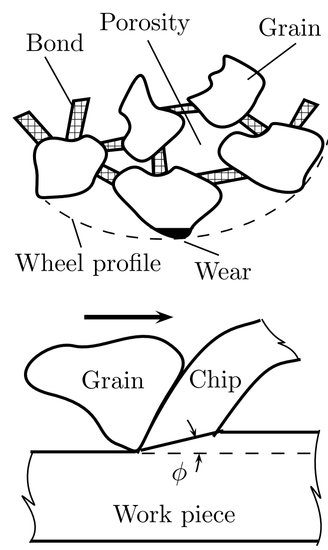


Figure 14.18 Structure of grinding wheel.

The material removal in grinding is shown in Fig. 14.19. Using simple trigonometry, length of average chip is found as

$$L = \sqrt{Dd}$$

Material removal rate in grinding is given by

$$\text{MRR} = VdB$$

where V is grinding speed, d is the depth of cut, and B is the width of grinding wheel. Grinding wheels are specified through the following alphanumeric system:

Grain-Grit-Grade-Structure-Bond

These are explained as follows:

1. *Abrasive Grains* Common abrasives for grinding are aluminium oxide (Al_2O_3), silicon carbide

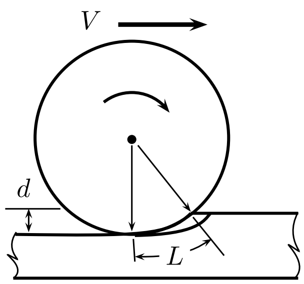


Figure 14.19 | Grinding operation.

(SiC), cubic boron nitride (CBN) and diamond. An abrasive grain is selected based on its sharpness, hardness, and friability. Friability is the tendency for an abrasive grain to fracture under pressure (opposite to toughness) [Table 14.2].

Table 14.2 | Applications of abrasive grains

Abrasive	Work Material
Al_2O_3	Steels, high speed steel, bronze, aluminium
SiC	Cast iron, stainless steel, brass, bronze, copper, aluminium
CBN	Tool steels, stainless steel, cobalt, nickel, super alloys
Diamond	Glass, tungsten carbide, ceramics

2. *Grit Number* *Grit number* indicates the size of an abrasive grain, as a function of sieve size. The larger the grit number, the smaller is the size of grain. The larger the grain size, the more will be the material removal capacity of the grinding wheel.

3. *Grade* Strength of the bond is represented by grade. A hard wheel has a stronger bond, and abrasive grains can bear large forces without getting dislodged from the wheel. Hard wheels are not suitable for hard work materials because in such cases, the grains wear out easily. This is known as *glazing* of the grinding wheel.

4. *Structure* Structure is the measure of porosity of the bonded abrasive. For ductile work material, coarse grit and hard and open wheel is recommended.

5. *Bond* Common bonding materials to hold the abrasive grains are vitrified clay, resinoid materials, slicates, rubber, shellac, metal matrix, oxychloride.

14.9 FINISHING OPERATIONS

14.9.1 Honing

Honing is an abrasive machining process that produces a precision surface on a metal workpiece by scrubbing an abrasive stone against it along a controlled path [Fig. 14.20].

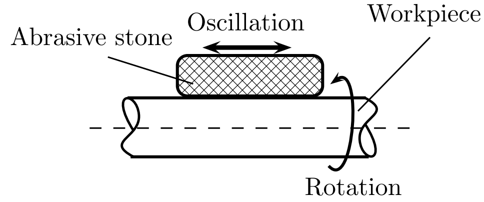


Figure 14.20 | Honing.

Typical applications are the finishing of cylinders for internal combustion engines, air bearing spindles and gears.

14.9.2 Lapping

Lapping is a finishing operation on flat or cylindrical surfaces, performed by charging a lap made of soft material (e.g. cast iron, leather or cloth) with abrasive particles and rubbing it over the work surface with slight pressure [Fig. 14.21].

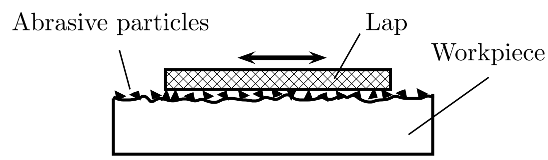


Figure 14.21 | Lapping.

14.9.3 Reaming

A reamer is a multiple cutting-edge tool having straight or helical fluted edges. *Reaming* is a finishing operation carried out with a reamer for making drilled holes dimensionally more accurate. Reamers produce better results at low cutting speeds and cause less chatter. Reaming is not as accurate as single point boring because it tends to follow the path of drilled hole. Reaming feeds are much higher than drilling feeds because there are more teeth in reamers and very less material is removed in every feed [Fig. 14.22].

Reamers are always made with even number of teeth to generate even or symmetrical surfaces. In *machine*

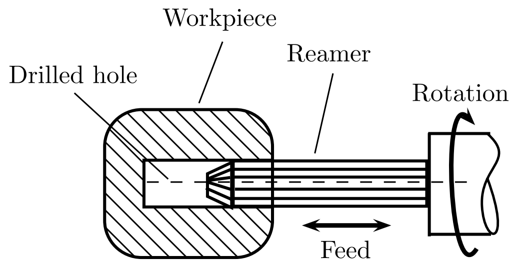


Figure 14.22 | Reaming.

reaming, all cutting is done by chamfer part which forms truncated cone on the end of the reamer. In contrast to this, cutting by a hand reamer is done along the longitudinal cutting edge.

14.9.4 Broaching

A *broach* is a long multi-point cutting tool having a series of transverse cutting edges arranged lengthwise, progressively increasing in size to make successively deeper cuts. *Broaching* is used to finish internal or external surfaces, such as circular, square, irregular holes, internal spline, key ways, teeth of internal gears, flat surface or other types on contour in a workpiece in a single pass. Broach is self-fed due to the gradual increment in the successive teeth during the rectilinear working movement between the tool and workpiece.

14.9.5 Tapping

Tapping is a machining process that uses a multi-point cutting tool (tap) to produce uniform, internal, helical threads. A *tap* is simply a hardened tool steel screw with length wise groves called fleets, milled or ground across the threads. The leading end is tapered to facilitate entry of the tool into the work material. Once started, the tap is automatically drawn into the hole by threads. Hence, it is not be forced in but to be rotated only. The feed per revolution of the tap is equal to the lead of the thread. Taps with long chamfer can be run at higher speeds because long chamfer reduces chip load per tooth.

14.10 MODERN MACHINING PROCESSES

The difficulty in adopting the traditional manufacturing processes can be attributed mainly to the following factors:

1. High hardness and low machinability of new materials.
2. Surface finish, dimensional and geometric complexities.
3. A higher production rate and economy.

Modern manufacturing processes harness energy sources that are considered unconventional by the yesterday's standards. Material removal can now be accomplished with electrochemical reaction, high temperature plasmas, high-velocity jets of liquids and abrasives, magnetic fields, explosives, and the shock waves from powerful electric sparks. Important unconventional processes are described under the following subsections.

14.10.1 Abrasive-Jet Machining

14.10.1.1 Basic Principle In *abrasive jet machining* (AJM), the material is removed by the impact of a high velocity stream of gas and abrasive mixture focused on to the workpiece. High velocity impact of an abrasive particle causes a tiny brittle fracture on the work surface and the flowing gas carries away the dislodged small workpiece particle [Fig. 14.23].

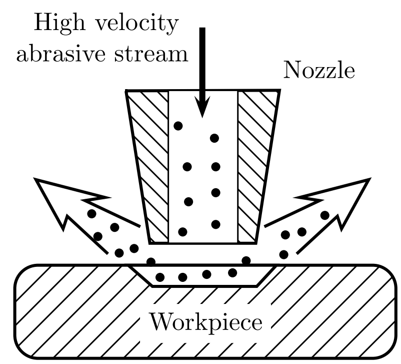


Figure 14.23 | Abrasive jet machining.

The process is similar to conventional sand blasting, but differs in the way that the abrasive is much finer and the process parameters are effectively controlled.

14.10.1.2 Basic Elements The following are essential components of the process:

1. <u>*Abrasive*</u> Mainly two types of abrasives are used: aluminium oxide (Al_2O_3) and silicon carbide (SiC). Aluminium oxide abrasives are preferred in most applications. These are available is powder form with a nominal grain diameter of 10-50 μm. A reuse of the abrasive powder is not recommended as the cutting capacity decreases after the

first application and contamination clogs the small orifices in the nozzle.

When the mass fraction of the abrasives in the jet increases, the MRR initially increases, but with a further increase in the mixing ratio, it reaches a maximum and then drops. Also, as the mass flow rate of the abrasive increases, the MRR also increases.

2. *Gas* AJM units normally operate at a pressure of 0.2–1 N/mm^2. The composition of gas (mostly air or CO_2) affects the MRR indirectly as the velocity-pressure relation depends on this composition. High velocity causes a high MRR even if the mass flow rate of the abrasive is kept constant.

3. *Nozzle* The nozzle material must be very hard to avoid any significant wear due to its continuous contact with abrasive grains. Normally, tungsten carbide (WC) is used for this purpose. For a normal operation, the cross-sectional area of the orifice is 0.05–0.2 mm^2 with rectangular or circular shape. Typical range of velocity of jet is 300 m/s.

   The distance between the nozzle tip and the work surface is called *nozzle tip distance* (NTD). When the NTD increases, the velocity of the abrasive particles impinging on the work surface increases due to their acceleration after they leave the nozzle. This in turn, increases the MRR. With further increase in the NTD, the velocity reduces due to the drag of the atmosphere which initially checks the increase in the MRR and finally reduces it.

14.10.1.3 Merits AJM offers the following advantages:

1. Suitable for brittle and fragile materials, such as germanium, silicon, glass, ceramics, and mica.
2. Preferred specially for drilling, cutting, deburring, etching, and cleaning.
3. Low capital cost.

14.10.1.4 Demerits Major limitations of AJM are as follows:

1. Slow MRR ($\approx$ 40 mg/min, 15 mm^3/min).
2. Embedding of abrasive in workpiece.
3. Tapering of the drilled holes.
4. Rounding-off corners due to free flow of abrasives.
5. Possibility of stray abrasive action.
6. Unfit for ductile materials (because abrasive particles tend to become embeded).

14.10.1.5 Applications AJM is fit for cutting fragile materials without damage. The process is used for frosting glass, removing oxides from metal surfaces, deburring, etching patterns, drilling and cutting thin sections of metal and shaping crystalline materials.

14.10.2 Ultrasonic Machining

The term *ultrasonic* refers to the frequency above the audible range of human ear (more than 16 kHz).

14.10.2.1 Basic Principle The basic *ultrasonic machining* (USM) involves a tool (made of ductile and tough material) vibrating with a very high frequency and a continuous flow of abrasive grains carried in a liquid between the small gap between the tool and the work surface. The tool is gradually fed with a uniform speed. The slurry of abrasive grains suspended in a liquid is fed into the cutting zone under pressure. The abrasive particles are driven into the work surface by the oscillating tool. This action gradually chips away minute particles of material in a pattern controlled by the tool shape and contour [Fig. 14.24].

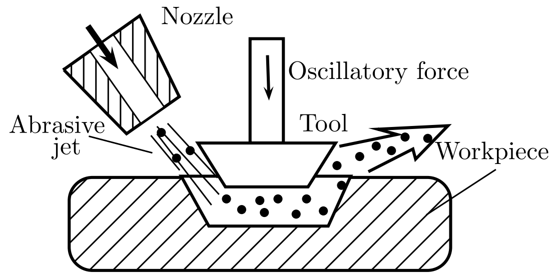


Figure 14.24 | Ultrasonic machining.

Small wear particle are carried away by the abrasive slurry. The tool material, being tough and ductile, wears out a much slower rate.

14.10.2.2 Basic Elements USM consists of the following elements:

1. *Transducer* The transducer is utilized to convert the electrical energy into high frequency vibratory motion of the tool by using piezoelectric materials, such as quartz, lead zirconate titanate (PZT). The tool vibrations are in the order of 15–30 kHz with amplitudes of the order of 10–100 μm.

2. *Tool Cone* *Tool cone* amplifies the mechanical energy produced by the transducer to give the required force-amplitude ratio. Titanium, Monel[2],

[2]Monel is a trademark of Special Metals Corporation for a series of nickel alloys, primarily composed of nickel (up to 67%) and copper.

and stainless steels are generally used as tool cone materials.

3. *Sonotrode* The tip of the tool, called *sonotrode*, is attached to the cone by means of silver brazing or by screws. Tools are made of tough and ductile materials, such as brass, low carbon steel, stainless steel. Length of the tool should be short since massive tools absorb the vibration energy reducing the efficiency of machining. Long tools also cause over stressing to the tool at brazed point.

4. *Abrasives* Typical abrasives used are alumina (Al_2O_3), silicon carbide (SiC), and boron carbide (B_4C). Alumina abrasives wear fast but are good for glass and ceramics. Boron carbide is harder than SiC but is more expensive. It shows faster MRR and can be used with higher frequencies. It is best for machining of tungsten carbide (WC), tool steel and precision stones. Diamond dust is sometimes used for good accuracy, surface finish and cutting rate in diamonds and rubies.

5. *Liquid for Abrasive* Abrasive grains are suspended in liquid with 20–60% concentration by volume. The liquid serves following purposes:

   (a) An acoustic bond between the tool and the workpiece.
   (b) A coolant on the tool face.
   (c) Carrier for abrasive and debris both.

   Water is the most commonly used liquid, although benzene and glycerol are also used.

14.10.2.3 Merits Ultrasonic machining is best suited for hard and brittle materials. It is the only way to produce economically complex cavities without breaking the workpiece. Tooling cost of the the process is low. It does not involve thermal stresses.

14.10.2.4 Demerits Major limitations of this process are

1. Low MRR.
2. Limited depth of hole produced.
3. High tool wear.
4. Unable to produce sharp corners.

14.10.2.5 Applications USM is suitable for machining shallow die cavities and forms in hard and brittle materials, such as hardened steel and sintered carbides. The process is also used for thread cutting in ceramics by rotating the workpiece in a controlled manner.

14.10.3 Electrochemical Machining

14.10.3.1 Basic Principle Michael Faraday[3] discovered that if two electrodes are placed in a bath containing a conducting liquid and when a direct potential is applied across them, the metal can be depleted from the anode and plated on the cathode. This process is universally used in electroplating by making the workpiece the cathode.

Electrochemical machining (ECM) is performed by reversing the process of electroplating (deplating), thus utilizing the principle of electrolysis for metal removal. High rate of electrolyte movement in the tool-workpiece gap washes metal ions away by anodic dissolution from the electrically conductive workpiece (anode: positive pole) before they have a chance to plate onto the tool (cathode: negative pole). The cavity produced on workpiece is the female mating image of the tool [Fig. 14.25].

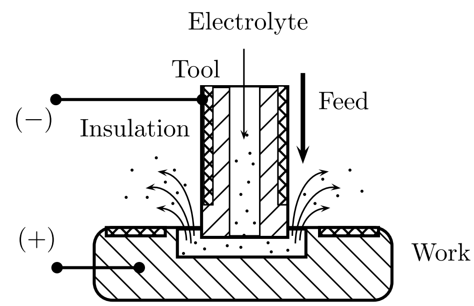


Figure 14.25 Electrochemical machining.

The tool-work gap needs to be maintained at a very small value of the order of 0.25 mm for satisfactory metal removal rates. The electrolyte needs to be pumped through this gap at high pressures (0.7–3.0 MPa). For the recirculation of electrolyte, it is necessary to clean the electrolyte of the debris formed due to metal removal.

14.10.3.2 Basic Elements ECM consists of the following elements:

1. *Electrolyte* An electrolyte acts as current carrier. The electrolyte in ECM should have high electrical conductivity, low viscosity, high specific heat, chemical stability, resistance to form passivating film on the workpiece surface, non-corrosiveness and non-toxicity. The conductivity of electrolyte depends on salt concentration, dissolved gases, machined debris and temperature. Inorganic salt solutions satisfy these requirements,

[3] Michael Faraday, (1791-1867) was an English chemist and physicist who contributed to the fields of electromagnetism and electrochemistry.

such as sodium chloride or potassium chloride mixed in water or sodium nitrate.

The electrolyte enters the gap between the electrode and the work at pressure ranging from 1.4 to 2.4 MPa. The flow of electrolyte serves the function of removing heat and hydrogen bubbles created in the chemical reactions of the process. The flow should be without cavitation, stagnation and vortex formation. This can be achieved by avoiding sharp corners in the flow path.

2. *Tool Material* Tool materials for ECM should be electrically and thermally conductive, and highly resistant to corrosion. The surface finish of the electrode tool is reproduced on the surface of the workpiece. Thus, the formed tool is generally made of aluminium, copper, brass, bronze, titanium, cupronickel[4] or stainless steel.

   The electrode is always made slightly smaller than the cavity desired because the erosion action progressing outward from the electrode always produces a cavity slightly larger than the electrode.

3. *Power Supply* A DC power supply in the range of 5–25 V is used to maintain current densities in range of 1.5–8.0 A/mm^2. The voltage is kept relatively low to minimize arcing across the gap.

14.10.3.3 Merits Electrochemical machining offers the following advantages:

1. High feed rate in ECM results in high MRR comparable with the conventional methods.
2. Metal removal rate in this process exceeds other non-traditional machining processes. It depends on ion exchange rate, therefore, the process is used for machining complex cavities in high-strength materials, for example, hardened tool steel or even carbide.
3. Low voltage decreases the equilibrium machining gap and results in a surface finish of the order of 0.4 μm and tolerance control of the order of ± 0.02 mm or less.
4. Tool wear is almost non-existent and the process is suitable for machining hard materials. All types of tools, dies and molds can be made by ECM.
5. The process involves only chemical forces, therefore, the workpiece does not need heavy mechanical clamping. There remains now residual stress in the workpiece.

[4]Cupronickel is a copper-nickel alloy which is highly resistant to corrosion in seawater, because its electrode potential is adjusted to be neutral with regard to seawater. It is used for piping, heat exchangers and condensers in seawater systems.

14.10.3.4 Demerits The following are the limitations of the ECM process:

1. Sharp interior edges and corners are difficult to produce.
2. Every new job needs a new design for tool (cathode).
3. The design of the process variables and selection electrolyte is a highly skilled job, and more often trial and error method is applied.
4. Use of corrosive media as electrolytes makes ECM difficult to handle. Chemical degradation of electrolyte are potential hazards for environment.
5. The process is suitable for only electrically conductive materials.
6. Large amount of electrical power is required to perform the process, which makes it expensive.

14.10.3.5 Applications The process is extensively used for mass production of turbine blades, engine parts and nozzles. ECM permits the machining after hardening, thus eliminates the risk of distortion or any other change. Fragile parts that are otherwise difficultly machinable can be shaped by ECM. Negligible tool wear gives high degree of accuracy, therefore, ECM is well-suited for mass production.

14.10.4 Electric-Discharge Machining

An arc is produced when two current-carrying wires are short-circuited. During this, a small portion of metals is also eroded away, leaving a small crater. This phenomenon is used in *electric-discharge machining* (EDM). The process is also known as *spark erosion machining*.

14.10.4.1 Basic Principle EDM involves a controlled erosion of electrically conductive materials by the initiation of rapid and repetitive spark discharges between the tool and workpiece separated by a small gap. [Fig. 14.26]. Each electrical spark produces sufficient heat to melt a portion of the workpiece and, usually, some of the tooling material also. Due to rapid heating, the dielectric fluid evaporates in the arc gap which increases the resistance until the arc is interrupted. The associated shock wave and flowing dielectric fluid remove the gas bubbles which later collapse. This also flushes the debris from the workpiece surface. The controlled pulsing of the direct current between the tool and the work produces the intense spark discharge. The arc is always struck at a point closest to the tool. Thus, a complimentary tool surface will be produced in the workpiece.

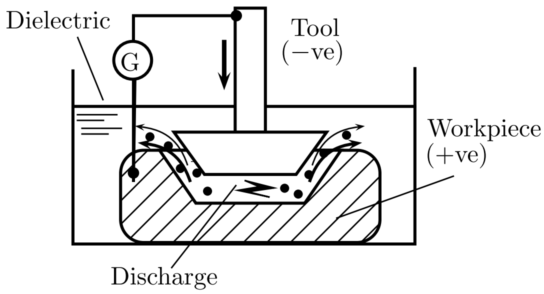


Figure 14.26 | Electric-discharge machining.

Electrochemical dissolution is based on the *Faraday's laws*, stated as follows:

1. The amount of electrochemical dissolution or deposition (m) is proportional to amount of charge (Q) passed.
2. The amount of material deposited or dissolved further depends on electrochemical equivalence (ECE) of the material, that is, the ratio of the atomic weight (A) and valency (v).

Therefore,

$$m \propto Q\frac{A}{v}$$
$$m = Q\frac{A}{Fv}$$

where constant F (= 96500 coulombs) is the *Faraday's constant*. Let current I flow for time t. The charge (Q) is equal to the current (I) multiplied by time (t). Therefore,

$$m = \frac{ItA}{Fv}$$

Material removal rate (MRR) in EDM is given by

$$\text{MRR} = \frac{m}{t\rho} = \frac{IA}{Fv\rho}$$

where ρ is the density of the element. For alloys made with n elements, if V_a is the total volume dissolved, ρ_a is density of the alloy, then mass of material dissolved for any i^{th} element with weight percentage α_i is

$$m_i = V_a\rho_a\alpha_i$$

Each element i will take certain amount of charge Q_i to get dissolved by mass m_i given by

$$m_i = \frac{Q_iA_i}{Fv_i}$$
$$Q_i = \frac{Fm_iv_i}{A_i} = \frac{FV_a\rho\alpha_iv_i}{A_i}$$

Total charge passed is

$$Q_T = \sum Q_i$$

14.10.4.2 Basic Elements Basic EDM system consists of a shaped tool (electrode) and the workpiece connected to a DC power supply and placed in a dielectric (electrically non-conducting) fluid. These are discussed as follows:

1. *Work Material* The EDM process can be used on any material that is an electrical conductor. Latent heat of melting are important physical properties of the material which affect the volume removed per discharge.
2. *Tool Material* Normally, brass, copper-tungsten, and graphite are the tool (electrode) materials. However, graphite is unsuitable for machining of carbides. Tool wear is an important factor because it affects dimensional accuracy and the shape produced. It is related to the melting points of the materials involved. The lower the melting point, the higher is the wear rate. Consequently, graphite electrodes have the highest rate of tool wear.
3. *Dielectric Fluid* The gap between tool and workpiece is maintained about 0.01–0.5 mm by a servo-mechanism. The dielectric fluid is pumped through the gap at a pressure of 200 kPa or less. It acts as an insulator, a flushing medium, and a cooling medium. The common dielectric fluids are mineral oils, although kerosene, liquid paraffin, silicon oils are also used. Distilled and deionized water is also used in specialized applications.
4. *Power Supply* The capacitor discharge is repeated at rates of 50–500 kHz (pulse duration 2–2000 μs) with voltage usually ranging 50–380 V and currents of 0.1–500 A. The MRR and surface roughness increase with increasing current density and decreasing frequency of sparks.

14.10.4.3 Merits EDM offers the following advantages:

1. Suitable for machining complex cavities in high-strength materials.
2. High orders of surface finish (3.2 μm) and tolerance ($\pm$0.05 mm).
3. Easy for automation and mass production.

14.10.4.4 Demerits EDM has the following limitations:

1. Suitable for only electrically conductive materials.

2. Inevitable taper, over-cut and corner radii.
3. Every new job needs a new design for tool.
4. Tool wear affecting dimensional accuracy.
5. Surface finish reducing on higher MRR.
6. High specific power consumption.

14.10.4.5 Applications EDM is extensively used for producing three-dimensional complex cavities to a high degree of accuracy which are later used as tools or die sets for production of castings, forgings, stampings, and extrusion. These are also used in electronic industries.

A variation of EDM is *wire EDM* in which a slow-moving wire travels along a prescribed path, cutting the workpiece, with the discharge sparks acting like cutting teeth (similar to contour cutting with a band saw). This process is used to cut thick plates and for making punches, tools, and dies from hard metals. The wire is made of electrically conductive materials, such as brass, copper, tungsten.

14.10.5 Electron-Beam Machining

14.10.5.1 Basic Principle *Electron-beam machining* (EBM) is a thermoelectric process in which a stream of high-speed electrons impinges on the work surface whereby the kinetic energy converts into thermal energy of extremely high density that melts or vaporizes the work material in a very localized area.

14.10.5.2 Basic Elements To avoid a collision of the accelerating electrons with the air molecules, the process has to be conducted in vacuum (about 10^{-5} mm Hg). Electron beam gun uses voltages in the range of 50–200 kV to generate a continuous stream of electrons and accelerated to 50–80% of the speed of light. Effective control of the electromagnetic lens on the workpiece enables drilling fine holes and cutting narrow slots.

14.10.5.3 Merits EBM offers the following advantages:

1. Specially adopted for micro-machining.
2. Suitable for high-strength materials.
3. Offers high degree of automation and mass production.
4. Extremely close tolerances.

14.10.5.4 Demerits EBM has the following limitations:

1. Vacuum chamber restricting the size of workpiece.
2. Time is required for evacuating the chamber.
3. Emission of X-ray due to interaction of electron beam with workpiece.
4. High specific energy consumption.
5. High equipment cost and need for skilled operators.

14.10.5.5 Applications EBM finds applications in drilling fine holes of the order of 25–125 μm and silting narrow cuts upto 25 μm on 0.25–6.3 mm thick plates. EBM is used for perforating the filters and screens in textile and chemical industries.

14.10.6 Laser-Beam Machining

The term '*laser*' is the acronym for light amplification by stimulated emission of radiation. It is a highly coherent (in space and time) beam of electromagnetic radiation with wavelength varying from 0.1–70 μm.

14.10.6.1 Basic Principle *Laser beam machining*, a thermoelectric process, is accomplished largely by material evaporation in a controlled manner using highly focused and high energy density of laser. Most of the energy is absorbed in a very thin layer at the surface (typical thickness 0.01 μm).

14.10.6.2 Basic Elements The work material should be less reflective to laser in order to enable efficient absorption of the incident energy. It should also have low specific heat and low latent heats of fusion and vaporization. Typical lasers used in manufacturing are CO_2, Nd:YAG (neodymium:yttrium-aluminium-garnet), Nd:glass, ruby, and excimer. For developing a high power, normally a pulsed ruby laser is used. The power requirements for a machining operation restricts the effectively usable wavelength range to 0.4–0.6 μm. The typical output energy of a laser is 20 J. With a pulse duration of 1 millisecond, the peak power reaches a value 20 kW.

Laser beams can be used in combination with gas stream. For example, high pressure inert-gas-assisted laser cutting is used for stainless steel and aluminium as it leaves an oxide-free edge.

14.10.6.3 Merits LBM offers the following advantages:

1. Suitable for machining micro-holes (up to 250 μm) and narrow slots.
2. Suitable for all materials except few.
3. Does not require vacuum.
4. Offers inherent flexibility with fiber-optic beam delivery.

14.10.6.4 Demerits LBM has the following limitations:

1. Produces usually rough surfaces and heat affected zones.
2. High specific energy consumption.
3. Low efficiency (≈ 0.3-0.5 %).
4. Unsuitable for highly conductive and reflective materials.
5. Hazards of lasers to the retina of eye.

14.10.6.5 Applications LBM is extensively used for drilling metals, non-metals and composites. It is used to machine micro-holes in filter screens, carburetors, fuel injection nozzles, wire-drawing die (50 μm), etc. It is used for marking and engraving of parts with letters, numbers and codes.

14.10.7 Plasma-Arc Machining

A *plasma* is defined as a superheated and electrically ionized gas.

14.10.7.1 Basic Principle In *plasma-arc machining* (PAM) operates by directing a high velocity plasma stream at the work, thus melting it and blowing the molten metal through the kerf. The temperatures generated in the process are very high (≈10000°C).

14.10.7.2 Basic Elements A plasma is generated by subjecting a flowing gas to the electron bombardment of an arc. For this, the arc is set up between the electrode and the anodic nozzle. A gas (e.g. nitrogen, argon, hydrogen or their mixture) is forced to flow through this arc. The plasma flows through a water-cooled nozzle to direct the stream to the desired location. Gases or water are often directed to surround the plasma jet to help confine the arc and clean the kerf of molten metal as it forms. Underwater plasma cutting also reduces noise and helps in getting rid of plasma fumes and glare.

14.10.7.3 Merits PAM offers the following advantages:

1. MRR higher than EDM and LBM.
2. Suitable also for electricity non-conducive materials.
3. Suitable for the plates of thickness upto 150 mm.

14.10.7.4 Demerits PAM has the following limitations:

1. Emission of ultraviolet and infrared radiations.
2. Metallurgical transformations in work surface.
3. Expensive equipments.
4. Requires skilled operators.

14.10.7.5 Applications The process is extensively used for profile cutting of metals such as stainless steel, aluminium, Monel, and super alloy plates, which are difficult to cut by oxyfuel techniques.

14.11 JIGS AND FIXTURES

Jigs and *fixtures* are essential work-holding and tool guiding devices that greatly influence the economics of mass production, primarily by reducing the production time. These devices facilitate clamping of workpiece on the machine tool in correct relationship with the cutting tool.

Jigs and fixtures are often used interchangeably and sometimes in pairs, but there is a difference between these two terms, explained as follows:

1. *Jig* Jigs have various reference surfaces and points for accurate alignment of parts and tools. The term jig is confined to the devices employed for holding the workpiece and for guiding the tool in performing the options of drilling, reaming and tapping. For this reason, the jig carries hardened steel bushes or tool guides. This eliminates the need of centering and marking operations.
2. *Fixture* A fixture is used to fix, that is, constrain all degrees of freedom by holding and clamping the workpiece relative to cutting tool on the machine table at the desired position but it does not guide the tool. Thus, a fixture serves three functions: location, support and clamping. Fixtures are generally designed for specific purposes of machining operations, such as milling, turning, planing, shaping or grinding. Fixtures are also used in welding of intricate shapes.

IMPORTANT FORMULAS

Orthogonal Cutting

$$r = \frac{t_1}{t_2} = \frac{\sin\phi}{\cos(\phi-\alpha)}$$
$$\tan\phi = \frac{r\cos\alpha}{1-r\sin\alpha}$$
$$\gamma = \cot\phi + \tan(\phi-\alpha)$$
$$V_{chip} = rV_c$$
$$\frac{V_c}{\cos(\phi-\alpha)} = \frac{V_{shear}}{\cos\alpha} = \frac{V_{chip}}{\sin\phi}$$
$$\dot{\gamma} = \frac{V_{shear}}{t_m} = \frac{V_c\cos\alpha}{\cos(\phi-\alpha)\,t_m}$$

Merchant's Analysis

$$F_s = b\frac{t_1}{\sin\phi}\tau_s$$
$$R = \frac{F_s}{\cos(\phi+\lambda-\alpha)}$$
$$F_c = R\cos(\lambda-\alpha)$$
$$F_t = R\sin(\lambda-\alpha)$$
$$F = F_c\sin\alpha + F_t\cos\alpha$$
$$N = F_c\cos\alpha - F_t\sin\alpha$$
$$\mu = \frac{F}{N} = \frac{F_t + F_c\tan\alpha}{F_c - F_t\tan\alpha}$$
$$P = F_c \times V_c$$
$$\phi = \frac{\pi}{4} + \alpha - \lambda$$

Cutting Tool Designation

$$\alpha_b - \alpha_s - c_b - c_s - \phi - \psi - r$$

Tool Life

$$V_cT^n = C$$
$$n = -\frac{dV_c/V_c}{dT/T}$$
$$V_c = \frac{C'}{T^n {t_1}^p b^q}$$

Surface Finish

1. Terminology

$$\text{CLA} = \frac{1}{l}\int_0^l y(x)dx$$
$$\text{RMS} = \left\{\frac{1}{l}\int_0^l y^2 dx\right\}^{1/2}$$

2. Feed marks

(a) Tool without nose radius

$$h_{max} = \frac{f}{\tan\psi + \cot\phi}$$

(b) Tool with nose radius r

$$h_{max} = \frac{f^2}{8r}$$

(c) Slab milling

$$h_{max} = \left(\frac{f}{2NZ}\right)^2 \frac{1}{D}$$

Economics of Machining

1. Minimum production cost

$$V_{opt1} = \frac{C}{\{(1/n-1)(t+z_2/z_1)\}^n}$$
$$T_{opt1} = \left(\frac{1}{n}-1\right)\left(t+\frac{z_2}{z_1}\right)$$

2. Maximum production rate

$$V_{opt2} = \frac{C}{\{(1/n-1)\,t\}^n}$$
$$T_{opt2} = \left(\frac{1}{n}-1\right)t$$

Machining Processes

1. Shaping

$$t = f\cos\psi$$
$$w = \frac{d}{\cos\psi}$$
$$N = \frac{V_c}{(1+R)\,L}$$
$$n = \frac{B}{f} \times \frac{H}{d}$$
$$T = \frac{B}{f} \times \frac{H}{d} \times \frac{1}{N} = \frac{B}{f} \times \frac{H}{d} \times \frac{(1+R)\,L}{V_c}$$
$$\text{MRR} = fdV_c$$

2. Turning

$$t = f\cos\psi$$
$$w = \frac{d}{\cos\psi}$$
$$V_c = \frac{\pi DN}{60}$$
$$\text{MRR} = \pi Dfd\frac{N}{60} = V_cfd$$
$$T = \frac{l}{fN}$$

3. Drilling

$$t_1 = \frac{f}{2}\sin\beta$$
$$w = \frac{D/2}{\sin\beta}$$
$$\text{MRR} = \frac{\pi D^2}{4}fN$$
$$\tan\alpha \approx \frac{2r\tan\psi}{D\sin\beta}$$

4. Milling

$$N = \frac{V}{\pi D}$$
$$f_r = Nzf$$

5. Grinding

$$L = \sqrt{Dd}$$
$$\text{MRR} = VdB$$

6. Electric discharge machining

$$m = Q\frac{A}{Fv} = \frac{ItA}{Fv}$$
$$F = 96500 \text{ C}$$
$$\text{MRR} = \frac{m}{t\rho} = \frac{IA}{Fv\rho}$$

SOLVED EXAMPLES

1. A tool has the following characteristics of tool life and speed in dry operation:

$$VT^{0.14} = 120 \text{ m/min}$$

Use of cutting fluid shows an increase of 15% in the value of C. Calculate the corresponding percent increase in tool life for cutting speed of 100 m/min.

Solution. Given that

$$n = 0.14$$
$$C = 120 \text{ m/min}$$

Percentage increase in tool life for cutting speed $V = 90$ m/min is determined as

$$\begin{aligned}\% \text{ increase} &= \frac{T_2}{T_1} - 1\\ &= \left(\frac{C_2/V}{C_1/V}\right)^{1/n} - 1\\ &= \left(\frac{1.15}{1}\right)^{1/0.14} - 1\\ &= 1.713\\ &= 171.3\%\end{aligned}$$

2. A typical metal cutting operation is performed using a cutting tool of positive rake $\alpha = 12°$. Shear angle is $\phi = 24°$. Calculate the value of friction angle using Merchant's formula.

Solution. Using Merchant's formula

$$\begin{aligned}2\phi + \lambda - \alpha &= 90°\\ \lambda &= 90° - 2\phi + \alpha\\ &= 54°\end{aligned}$$

3. A single point cutting tool can be used upto 15 hours at 65 m/min. If Taylor's constant $C = 300$, calculate the percentage reduction in tool life on doubling the cutting velocity.

Solution. Given that

$$T_1 = 15 \times 60 = 900 \text{ min}$$
$$V_1 = 65 \text{ m/min}$$
$$C = 300$$
$$V_2 = 2V_1$$

Using Taylor's tool life equation:

$$\begin{aligned}VT^n &= C\\ \ln V_1 + n \ln T_1 &= \ln C\\ n &= \frac{\ln (C/V_1)}{\ln T_1}\\ &= 0.2248\\ 2 \times 65 \times T_2^{0.2248} &= 300\\ T_2 &= 58.91 \text{ min}\end{aligned}$$

4. In an orthogonal cutting, rake angle of the tool is 25° and friction angle is 27°. Using Merchant's shear angle relationship, calculate the value of shear angle.

Solution. Given that

$$\alpha = 25°$$
$$\lambda = 27°$$

Shear angle is given by

$$\begin{aligned}2\phi + \lambda - \alpha &= 90°\\ \phi &= 44°\end{aligned}$$

5. Determine the time required in drilling a 8 mm diameter hole through a 24 mm thick mild steel plate with a drill bit running at 360 rpm and a feed of 0.5 mm per revolution.

Solution. The time required to travel a distance of 24 mm is given by

$$\begin{aligned}t &= \frac{24 \times 60}{360 \times 0.5}\\ &= 8 \text{ s}\end{aligned}$$

6. Through holes of 12 mm diameter are to be drilled in a steel plate of 25 mm thickness. Drill spindle speed is 240 rpm, feed 0.25 mm/rev and drill point angle is 120°. Assuming drill over travel of 3 mm, calculate the time for producing the hole.

Solution. Given that

$$d = 12 \text{ mm}$$
$$t = 25 \text{ mm}$$
$$N = 240 \text{ rpm}$$
$$f = 0.25 \text{ mm/rev}$$
$$\alpha = 120°$$
$$t_o = 3 \text{ mm}$$

The tool has to travel through tool point height, plate thickness and over travel, therefore, time is

$$\begin{aligned}T &= \frac{d/\left(2 \tan(\alpha/2)\right) + t + t_o}{f \times N/60}\\ &= 31.46 \text{ s}\end{aligned}$$

7. In a single point turning operation of steel with a cemented carbide tool, Taylor's tool life exponent is 0.2. Determine the increase in the tool life if the cutting speed is halved.

Solution. Given that

$$n = 0.2$$
$$V_2 = \frac{V_1}{2}$$

Using Taylor's tool life equation:

$$V_c T^n = C$$
$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{1/n}$$
$$= 32T_1$$

8. A machining operation producing chip thickness ratio is 0.35 when back rake angle is 15°. Determine the shear strain in work material.

Solution. Given that

$$r = 0.35$$
$$\alpha = 15^\circ$$

Shear angle is given by

$$\tan\phi = \frac{r\cos\alpha}{1-r\sin\alpha}$$
$$= 0.3717$$
$$\phi = 20.39^\circ$$

Magnitude of shear strain is given by

$$\gamma = \cot\phi + \tan(\phi-\alpha)$$
$$= 2.78$$

9. The rake angle of a cutting tool is 10°, shear angle 35° and cutting velocity 25 m/min. Calculate the velocity of chip along the tool face.

Solution. Given,

$$\alpha = 10^\circ$$
$$\phi = 35^\circ$$
$$V_c = 25 \text{ m/min}$$

Chip velocity is determined as

$$V_{chip} = \frac{V_c \sin\phi}{\cos(\phi-\alpha)}$$
$$= 15.82 \text{ m/min}$$

10. In a turning operation with orthogonal machining, the spindle rotates at 240 rpm and tool is fed at rate of 0.2 mm per revolution. The depth of the cut is 0.5 mm. The rake angle is 10° and shear angle is 35°. Using Merchant's theory, determine the chip-thickness and coefficient of friction.

Solution. Given that

$$t_1 = 0.5 \text{ mm}$$
$$\alpha = 10^\circ$$
$$\phi = 35^\circ$$

Therefore,

$$\frac{t_1}{t_2} = \frac{\sin\phi}{\cos(\phi-\alpha)}$$
$$t_2 = 0.79 \text{ mm}$$

Friction angle λ is

$$2\phi + \lambda - \alpha = 90^\circ$$
$$\lambda = 30^\circ$$

Coefficient of friction is

$$\mu = \tan\lambda$$
$$= 0.58$$

11. An orthogonal cutting of mild steel has the following data:

Cutting speed	:	35 m/min
Depth of cut	:	0.25 mm
Tool rake angle	:	10°
Chip thickness	:	0.8 mm
Cutting force	:	925 N
Thrust force	:	475 N

Using Merchant's analysis, the friction angle during the machining will be

Solution. Given that

$$F_c = 925 \text{ N}$$
$$F_t = 475 \text{ N}$$
$$\alpha = 10^\circ$$

The friction angle is

$$\tan\lambda = \frac{F_c\sin\alpha + F_t\cos\alpha}{F_c\cos\alpha - F_t\sin\alpha}$$
$$= \frac{925\sin 10^\circ + 475\cos 10^\circ}{925\cos 10^\circ - 475\sin 10^\circ}$$
$$= \frac{628.40}{828.46}$$
$$= 0.758$$
$$\lambda = 37.18^\circ$$

GATE PREVIOUS YEARS' QUESTIONS

1. Quality screw threads are produced by

(a) thread milling
(b) thread chasing
(c) thread cutting with single point tool
(d) thread casting

(GATE 2003)

Solution. Thread chasing is the process of cutting a thread on a lathe with a chasing tool that comprises several single point tools banked together in a single tool called chaser. Chasing is used for production of large threads for a die head.

Another important process is thread tapping, a machining process that is used for cutting internal threads using a tap having threads of desired form on its periphery. A tap has cutting teeth and flutes parallel to its axis that act as channels to carry away the chips formed by the cutting action. The tap cuts threads through its combined rotary and axial motion.

Ans. (b)

2. As tool and work are not in contact in EDM process

(a) no relative motion occurs between them
(b) no water of tool occurs
(c) no power is consumed during metal cutting
(d) no force between tool and work occurs

(GATE 2003)

Solution. Electric discharge machining (EDM) involves a controlled erosion of electrically conductive materials by the initiation of rapid and repetitive spark discharges between the tool and workpiece separated by a small gap.

Ans. (d)

3. A batch of 10 cutting tools could produce 500 components while working at 50 rpm with a tool feed of 0.25 mm/rev and depth of cut of 1 mm. A similar batch of 10 tools of the same specification could produce 122 components while working at 80 rpm with a feed of 0.25 mm/rev and 1 mm depth of cut. How many components can be produced with one cutting tool at 60 rpm?

(a) 29 (b) 31
(c) 37 (d) 42

(GATE 2003)

Solution. Tool of the same specification is used in all the three cases, therefore, index n in Taylor's equation will be the same. If k is the tool constant, then

$$\begin{aligned} V_1 &= 50 \times 0.25 \\ T_1 &= \frac{500}{10} \times k \\ V_2 &= 80 \times 0.25 \\ T_2 &= \frac{122}{10} \times k \end{aligned}$$

Using

$$V_1 {T_1}^n = V_2 {T_2}^n$$
$$50 \times 0.25 \left(\frac{500}{10} \times k\right)^n = 80 \times 0.25 \left(\frac{122}{10} \times k\right)^n$$
$$n = 0.3332$$

If z is the number of components produced by one tool then

$$\begin{aligned} V_3 &= 60 \times 0.25 \\ &= 15 \\ T_3 &= \frac{z}{1} \times k \\ &= zk \end{aligned}$$

Using Taylor's tool life equation,

$$\begin{aligned} V_1 {T_1}^n &= V_3 {T_3}^n \\ z &= 28.9289 \end{aligned}$$

Ans. (a)

Common Data Questions

A cylinder is turned on a lathe with orthogonal machining principle. Spindle rotates at 200 rpm. The axial feed rate is 0.25 mm per revolution. Depth of cut is 0.4 mm. The rake angle is 10°. In the analysis, it is found that the shear angle is 27.75°.

4. The thickness of the produced chip is

(a) 0.511 mm (b) 0.528 mm
(c) 0.818 mm (d) 0.846 mm

(GATE 2003)

Solution. Given that

$$\begin{aligned} t_1 &= 0.4 \text{ mm} \\ \alpha &= 10^\circ \\ \phi &= 27.75^\circ \end{aligned}$$

Therefore,

$$\frac{t_1}{t_2} = \frac{\sin\phi}{\cos(\phi-\alpha)}$$
$$t_2 = 0.818184 \text{ mm}$$

Ans. (c)

5. In the above problem, the coefficient of friction at the chip tool interface obtained using Ernst and Merchant theory is

(a) 0.18 (b) 0.36
(c) 0.71 (d) 0.908

(GATE 2003)

Solution. Friction angle λ is

$$2\phi+\lambda-\alpha = 90^\circ$$
$$\lambda = 44.5^\circ$$

Coefficient of friction is

$$\mu = \tan\lambda$$
$$= 0.982697$$

Ans. (d)

6. The mechanism of material removal in EDM process is

(a) melting and evaporation
(b) melting and corrosion
(c) erosion and cavitation
(d) cavitation and evaporation

(GATE 2004)

Solution. Electric discharge machining (EDM) involves a controlled erosion of electrically conductive materials by the initiation of rapid and repetitive spark discharges between the tool and workpiece separated by a small gap.

Ans. (a)

7. Through holes of 10 mm diameter are to be drilled in a steel plate of 20 mm thickness. Drill spindle speed is 300 rpm, feed is 0.2 mm/rev and drill point angle is 120°. Assuming drill over travel of 2 mm, the time for producing a hole will be

(a) 4 s (b) 25 s
(c) 100 s (d) 110 s

(GATE 2004)

Solution. Given that

$$d = 10 \text{ mm}$$
$$t = 20 \text{ mm}$$
$$N = 300 \text{ rpm}$$
$$f = 0.2 \text{ mm/rev}$$
$$\alpha = 120^\circ$$
$$t_o = 0.002$$

The tool has to travel through tool point height, plate thickness and over travel, therefore, time is

$$T = \frac{\frac{d/2}{\tan(\alpha/2)}+t+t_o}{f\times N/60}$$
$$= 24.8868 \text{ mm}$$

Ans. (b)

8. In an orthogonal cutting test on mild steel, the following data were obtained:

Cutting speed	:	40 m/min
Depth of cut	:	0.3 mm
Tool rake angle	:	+5°
Chip thickness	:	1.5 mm
Cutting force	:	900 N
Thrust force	:	450 N

Using Merchant's analysis, the friction angle during the machining will be

(a) 26.6° (b) 31.5°
(c) 45° (d) 63.4°

(GATE 2004)

Solution. Given that

$$F_c = 900 \text{ N}$$
$$F_t = 450 \text{ N}$$
$$\alpha = 5^\circ$$

The friction angle is

$$\tan\lambda = \frac{F_c\sin\alpha+F_t\cos\alpha}{F_c\cos\alpha-F_t\sin\alpha}$$
$$\lambda = 31.5651^\circ$$

Ans. (b)

9. In a machining operation, doubling the cutting speed reduces the tool life to 1/8th of the original value. The exponent n in Taylor's tool life equation $VT^n = C$ is

(a) 1/8 (b) 1/4
(c) 1/3 (d) 1/2

(GATE 2004)

Solution. The speed goes from V to $2V$ and tool life decreases from T to $T/8$, therefore, using Taylor's equation

$$VT^n = C$$
$$VT^n = 2V\left(\frac{T}{8}\right)^n$$
$$\ln V + n\ln T = \ln 2 + \ln V + n\ln\frac{T}{8}$$
$$n = \frac{\ln 2}{3\ln 2} = \frac{1}{3}$$

Ans. (c)

10. Typical machining operations are to be performed on hand-to-machine materials by using the processes listed below. Choose the best set of operation-process combinations

Operation	Process
P. Deburring (internal surface)	1. Plasma arc machining
Q. Die sinking	2. Abrasive flow machining
R. Fine hole drilling in thin sheets	3. Electric discharge machining
S. Tool sharpening	4. Ultrasonic machining
	5. Laser beam machining
	6. Electrochemical grinding

(a) P-1, Q-5, R-3, S-4
(b) P-1, Q-4, R-1, S-2
(c) P-5, Q-1, R-2, S-6
(d) P-2, Q-3, R-5, S-6

(GATE 2004)

Solution. Laser beam machining is used for fine hole drilling in thin sheets. Electrochemical grinding is used for tool sharpening.

Ans. (d)

11. A zig-zag cavity in a block of high strength alloy is to be finish machined. This can be carried out by using

(a) electric discharge machining
(b) electrochemical machining
(c) laser beam machining
(d) abrasive flow machining

(GATE 2005)

Solution. Electrochemical machining is used for high strength alloys as it does not need to exert cutting force and desired shapes can be formed.

Ans. (b)

12. A 600 mm×300 mm flat surface of a plate is to be finish machined on a shaper. The plate has been fixed with the 600 mm side along the tool travel direction. If the tool over-travel at each end of the plate is 20 mm, average cutting speed is 8 m/min, feed rate is 0.3 mm/stroke and the ratio of return time to cutting time of the tool is 1:2, the time required for machining will be

(a) 8 min (b) 12 min
(c) 16 min (d) 20 min

(GATE 2005)

Solution. The tool need to travel total distance of $600+20\times 2$ mm in each stroke. For each stroke of machining, time required is

$$t = \frac{640}{8000} = 0.08 \text{ min}$$

Number of strokes required is

$$n = \frac{300}{0.3} = 1000 \text{ min}$$

Total cutting time is $n\times t$. Including return strokes, the total time required is

$$t_t = (1+R)n\times t = \frac{3}{2}\times 80 = 120 \text{ min}$$

Ans. (b)

13. Two tools P and Q have signatures 5°-5°-6°-8°-30°-0 and 5°-5°-7°-7°-8°-15°-0 (both ASA), respectively. They are used to turn components under the same machining conditions. If h_P and h_Q denote the peak-to-valley heights of surfaces produced by the tools P and Q, the ratio h_P/h_Q will be

(a) $(\tan 8° + \cot 8°)\,/\,(\tan 8° + \cot 30°)$
(b) $(\tan 15° + \cot 8°)\,/\,(\tan 30° + \cot 8°)$
(c) $(\tan 15° + \cot 7°)\,/\,(\tan 30° + \cot 7°)$
(d) $(\tan 7° + \cot 15°)\,/\,(\tan 7° + \cot 30°)$

(GATE 2005)

Solution. Tool geometry is denoted as

$$\alpha_b - \alpha_s - c_b - c_s - \phi - \psi - r$$

Therefore, for the tool P:

$$\psi = 30^\circ$$
$$\phi = 8^\circ$$

For tool Q:

$$\psi = 15^\circ$$
$$\phi = 8^\circ$$

Surface finish (peak-to-valley heights) in machining with feed f is calculated for tools without nose as

$$h = \frac{f}{\tan\psi + \cot\phi}$$

Therefore,

$$\frac{h_p}{h_q} = \frac{\tan 15^\circ + \cot 8^\circ}{\tan 30^\circ + \cot 8^\circ}$$

Ans. (b)

14. If each abrasive grain is viewed as a cutting tool, then which of the following represents the cutting parameters in common grinding operations?

(a) Large negative rake angle, low shear angle and high cutting speed

(b) Large positive rake angle, low shear angle and high cutting speed

(c) Large negative rake angle, high shear angle and low cutting speed

(d) Zero rake angle, high shear angle and high cutting speed

(GATE 2006)

Solution. The abrasive grain will be equivalent with a tool with large negative rake angle, low shear angle, and can be used with high cutting speed.

Ans. (a)

15. Arrange the processes in the increasing order of their maximum material removal rate:
Electrochemical machining (ECM)
Ultrasonic machining (USM)
Electron beam machining (EBM)
Laser beam machining (LBM)
Electric discharge machining (EDM)

(a) USM, LBM, EBM, EDM, ECM

(b) EBM, LBM, USM, ECM, EDM

(c) LBM, EBM, USM, ECM, EDM

(d) LBM, EBM, USM, EDM, ECM

(GATE 2006)

Solution. Processes with increasing MRR are in the order LBM, EBM, USM, ECM, EDM.

Ans. (c)

Common Data Questions

In an orthogonal machining operation:

Uncut thickness	=	0.5 mm
Cutting speed	=	20 m/min
Width of cut	=	5 mm
Chip thickness	=	0.7 mm
Thrust force	=	200 N
Cutting force	=	1200 N
Rake angle	=	15°.

Assume Merchant's theory.

16. The values of shear angle and shear strain, respectively, are

(a) 30.3° and 1.98 (b) 30.3° and 4.23
(c) 40.2° and 2.97 (d) 40.2° and 1.65

(GATE 2006)

Solution. Given that

$$t_1 = 0.5 \text{ mm}$$
$$t_2 = 0.7 \text{ mm}$$
$$\alpha = 15^\circ$$

Therefore,

$$r = \frac{t_1}{t_2}$$
$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$$
$$\phi = 40.24^\circ$$

Shear strain is given by

$$\gamma = \cot\phi + \tan(\phi - \alpha)$$
$$= 1.65$$

Ans. (d)

17. The coefficient of friction at the tool-chip interface is:

(a) 0.23 (b) 0.46
(c) 0.85 (d) 0.95

(GATE 2006)

Solution. Using Merchant's theory,

$$2\phi + \lambda - \alpha = \frac{\pi}{2}$$
$$\lambda = 24.6^\circ$$

Coefficient of friction is

$$\mu = \tan\lambda$$
$$= 0.457$$

Ans. (b)

18. The percentage of total energy dissipated due to friction at the tool-chip interface is:

(a) 30% (b) 42%
(c) 58% (d) 70%

(GATE 2006)

Solution. Given that

$$V = 20 \text{ m/min}$$
$$F_c = 1200 \text{ N}$$
$$F_t = 200N$$

Friction force

$$F = F_c \sin\alpha + F_t \cos\alpha$$
$$= 503.768 \text{ N}$$

Friction heat transfer at tool tip is

$$Q_2 = r \times F \times V$$
$$= 7196.69 \text{ J/min}$$

Heat generated is

$$Q_1 = F_c \times V$$
$$= 24000 \text{ J/min}$$

Therefore, percentage of energy dissipated is

$$\frac{Q_2}{Q_1} \times 100 = 29.9862\%$$

Ans. (a)

19. In orthogonal turning of a low carbon steel bar of diameter 150 mm with uncoated carbide tool, the cutting velocity is 90 m/min. The feed is 0.24 mm/rev and the depth of cut is 2 mm. The chip thickness obtained is 0.48 mm. If the orthogonal rake angle is zero, and the principal cutting edge angle is 90°, the shear angle in degrees is

(a) 20.56 (b) 26.56
(c) 30.56 (d) 36.56

(GATE 2007)

Solution. Given that

$$f = 0.24 \text{ mm/rev}$$
$$t_2 = 0.48 \text{ mm}$$

Therefore, the ratio is

$$r = \frac{f}{t_2}$$
$$= 0.5$$

Shear angle is given by

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$$
$$\phi = 26.56^\circ$$

Ans. (b)

20. In electric discharge machining (EDM), if the thermal conductivity of tool is high and the specific heat of workpiece is low, then the tool wear rate and the material removal rate are expected to be, respectively,

(a) high and high (b) low and low
(c) high and low (d) low and high

(GATE 2007)

Solution. High conductivity of tool and low specific heat of workpiece will result in high tool wear and high MMR.

Ans. (a)

21. In orthogonal turning of medium carbon steel, the specific machining energy is 2.0 J/mm^3. The cutting velocity, feed and depth of cut are 120 m/min, 0.2 mm/rev and 2 mm, respectively. The main cutting force in N is

(a) 40 (b) 80
(c) 400 (d) 800

(GATE 2007)

Solution. Given that

$$U = 2.0 \text{ J/mm}^3$$
$$V_c = 120 \text{ m/min}$$
$$= 2000 \text{ mm/s}$$
$$f = 0.2 \text{ mm/rev}$$
$$d = 2 \text{ mm}$$

Rate of material removal is

$$\text{MRR} = V_c \times f \times d$$

There seems a mistake in the given data. The specific energy U should be 2 kJ/mm^3 to get $F_c = 800$ N. The cutting force is written as

$$F_c = \frac{U \times \text{MRR}}{V_c}$$
$$= U \times f \times d$$
$$= 0.80 \text{ N}$$

Ans. (a)

22. In orthogonal turning of low carbon steel pipe with principal cutting edge angle of 90°, the main cutting force is 1000 N and the feed force is 800 N. The shear angle is 20° and orthogonal rake angle is zero. Employing Merchant's theory, the ratio of friction force to normal force acting on the cutting tool is

(a) 1.56 (b) 1.25
(c) 0.80 (d) 0.64

(GATE 2007)

Solution. Given that $\phi = 20°$ and $\alpha = 0°$. Using

$$2\phi + \lambda - \alpha = \frac{\pi}{2}$$
$$\lambda = 50°$$

The ratio of forces is obtained as

$$\frac{F}{N} = \tan\lambda$$
$$= 1.19175$$

Ans. (b)

Linked Answer Questions

A low carbon steel bar of 147 mm diameter with a length of 630 mm is being turned with uncoated carbide insert. The observed tool lives are 24 min and 12 min for cutting velocities of 90 m/min and 120 m/min, respectively. The feed and depth of cut are 0.2 mm/rev and 2 mm, respectively. Use the unmachined diameter to calculate the cutting velocity.

23. When tool life is 20 min, the cutting velocity in m/min is

(a) 87 (b) 97
(c) 107 (d) 114

(GATE 2007)

Solution. Using Taylor's tool life equation:

$$VT^n = C$$
$$\left(\frac{24}{12}\right)^n = \frac{120}{90}$$
$$n = 0.415037$$

For $T = 20$,

$$V = 90 \times \left(\frac{24}{20}\right)^{1/0.415}$$
$$= 97.0746 \text{ m/min}$$

Ans. (b)

24. Neglect over travel or approach of the tool. When tool life is 20 min, the machining time in minutes for a single pass is

(a) 5 (b) 10
(c) 15 (d) 20

(GATE 2007)

Solution. Given that

$$D = 147 \text{ mm}$$
$$L = 630 \text{ mm}$$

Corresponding rpm for velocity $V = 97.07$:

$$N = \frac{V}{\pi D}$$
$$= 210.203 \text{ rpm}$$

Machining time to cover length L with feed f. In one revolution, tool travels distance f (=0.2 mm/rev), therefore, machining time to cover length L with speed N rpm is

$$t_m = \frac{L}{fN}$$
$$= 14.9855 \text{ s}$$

Ans. (c)

25. Internal gear cutting operation can be performed by

(a) milling
(b) shaping with rack cutter
(c) shaping with pinion cutter
(d) hobbing

(GATE 2008)

Solution. Out of given options, only shaping with pinion cutter is correct and possible.

Ans. (c)

26. In a single point turning tool, the side rake angle and orthogonal rake angle are equal. ϕ is the principal cutting edge angle and its range is $0° \leq \phi \leq 90°$. The chip flows in the orthogonal plane. The value of ϕ is closest to

(a) 0° (b) 45°
(c) 60° (d) 90°

(GATE 2008)

Solution. As the side rake angle is equal to orthogonal rake angle, therefore, principal cutting edge is perpendicular to cutting speed, and the cutting is pure orthogonal. In this case, the principal cutting edge angle also becomes zero.

Ans. (a)

27. A researcher conducts electrochemical machining (ECM) on a binary alloy (density 6000 kg/m^3) of

iron (atomic weight 56, valency 2) and metal P (atomic weight 24, valency 4). Faraday's constant 96500 C/mol. Volumetric material removal rate of the alloy is 50 mm^3/s at a current of 2000 A. The percentage of the metal P in the alloy is closest to

(a) 40 (b) 25
(c) 15 (d) 79

(GATE 2008)

Solution. Electrochemical dissolution is based on Faraday's laws. Each element i will take certain amount of charge Q_i to get dissolved by m_i, which is given by

$$m_i = \frac{Q_i A_i}{F v_i}$$
$$Q_i = \frac{F m_i v_i}{A_i} = \frac{F V_a \rho \alpha_i v_i}{A_i}$$

Total charged pass is

$$Q_T = \sum Q_i$$

Given that

$$\rho_a = 6000 \text{ kg/m}^3$$
$$A_1 = 65$$
$$v_1 = 2$$
$$A_2 = 24$$
$$v_2 = 4$$
$$F = 96500 \text{ C/mol}$$
$$V_a = 50 \times 10^{-6} \text{ m}^3$$
$$I = 2000 \text{ A}$$

For two elements, assuming for $t = 1$ of current I, if α is fraction of P metal (2) in the alloy, then

$$I \times t = F V_a \rho_a \times \frac{(1-\alpha)\, v_1}{A_1} + F V_a \rho_a \times \frac{\alpha v_2}{A_2}$$

Putting the known values in the above equation, one obtains

$$\alpha = 25.4828\%$$

Ans. (b)

Linked Answer Questions

Orthogonal turning is performed on a cylindrical workpiece with shear strength of 250 MPa. The following conditions are used; cutting velocity is 180 m/min, feed is 0.20 mm/rev, depth of cut is 3 mm, chip thickness ratio = 0.5. The orthogonal rake angle is 7°. Apply Merchant's theory for analysis.

28. The shear plane angle (in degrees) and the shear force, respectively, are

(a) 52; 320 N (b) 52; 400 N
(c) 28; 400 N (d) 28; 320 N

(GATE 2008)

Solution. Given that

$$\tau = 250 \text{ MPa}$$
$$V_c = 180 \text{ m/min}$$
$$f = 0.2 \text{ mm/rev}$$
$$d = 3 \text{ mm}$$
$$r = 0.5$$
$$\alpha = 7^\circ$$

The shear plane angle is given by

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} = 0.503$$
$$\phi = 27.85^\circ$$

Shear force is calculated as

$$F_s = \tau \times \frac{f}{\sin\phi} = 321 \text{ N}$$

Ans. (d)

29. Cutting and friction forces, respectively, are

(a) 568 N; 387 N (b) 565 N; 381 N
(c) 440 N; 342 N (d) 480 N; 356 N

(GATE 2008)

Solution. Friction angle γ is calculated as

$$2\phi + \lambda - \alpha = 90$$
$$\lambda = 90 + \alpha - 2\phi = 41^\circ$$

Radius of Merchant's circle is

$$R = \frac{F_s}{\cos(\phi + \lambda - \alpha)} = 687.083 \text{ N}$$

Cutting force is related to R as

$$F_c = R\cos(\lambda - \alpha) = 567.671 \text{ N}$$

Similarly, friction force is related to R as

$$F = R\sin\lambda = 453.379 \text{ N}$$

Hence, none of the options is correct, but based on F_c, option (a) is suitable.

Ans. (a)

30. Friction at the tool-chip interface can be reduced by

(a) decreasing the rake angle
(b) increasing the depth of cut
(c) decreasing the cutting speed
(d) increasing the cutting speed

(GATE 2009)

Solution. Increased cutting speed will reduce the friction.

Ans. (d)

31. Minimum shear strain in orthogonal turning with a cutting tool of zero rake angle is

(a) 0.0 (b) 0.5
(c) 1.0 (d) 2.0

(GATE 2009)

Solution. Shear strain in orthogonal cutting for $\alpha = 0$ is

$$\begin{aligned}\gamma &= \cot\phi + \tan(\phi - \alpha)\\ &= \cot\phi + \tan\phi\\ &= \frac{2}{\sin 2\phi}\end{aligned}$$

It is maximum when $2\phi = \pi/2$, therefore,

$$\gamma_{\max} = 2$$

Ans. (d)

32. Electrochemical machining is performed to remove material from an iron surface of 20 mm × 20 mm under the following conditions

Inter electrode gap	=	0.2 mm
Supply voltage DC	=	12 V
Specific resistance of electrolyte	=	2 Ω-cm
Atomic weight of iron	=	55.85
Valency of iron	=	2
Faraday's constant	=	96500 C/mol

The material removal rate (in g/s) is

(a) 0.3471 (b) 3.471
(c) 34.71 (d) 347.1

(GATE 2009)

Solution. Given that inter-electrode gap $\delta = 0.0002$ m, specific resistance $\rho = 0.02$ Ωm, cross-sectional area $A = 0.02 \times 0.02$ m^2, supply voltage $V = 12$ V (dc), molecular weight $M = 55.85$, valency, $v = 2$, Faraday's constant $F = 96500$ C/mol. Therefore, the electrical resistance offered by the gap is

$$\begin{aligned}R &= \frac{\rho l}{A}\\ &= 0.01\ \Omega\end{aligned}$$

Current is given by

$$\begin{aligned}I &= \frac{V}{R}\\ &= 1200\ \text{A}\end{aligned}$$

MRR is calculated as

$$\begin{aligned}\text{MRR} &= \frac{I}{F} \times \frac{M}{v}\\ &= 0.34711\ \text{g/s}\end{aligned}$$

Ans. (a)

Linked Answer Questions

In a machining experiment, tool life was found to vary with the cutting speed in the following manner:

Cutting speed (m/min)	Tool life (minutes)
60	81
90	36

33. The exponent (n) and constant (C) of the Taylor's tool life equation are

(a) $n = 0.5$ and $C = 540$
(b) $n = 1$ and $C = 4860$
(c) $n = -1$ and $C = 0.74$
(d) $n = -0.5$ and $C = 1.155$

(GATE 2009)

Solution. Using Taylor's equation $VT^n = C$:

$$\begin{aligned}\frac{60}{90} \times \left(\frac{81}{36}\right)^n &= 1\\ n &= 0.5\end{aligned}$$

Therefore,

$$\begin{aligned}C &= 60 \times 81^{0.5}\\ &= 540\end{aligned}$$

Ans. (a)

34. What is the percentage increase in tool life when the cutting speed is halved?

(a) 50% (b) 200%
(c) 300% (d) 400%

(GATE 2009)

Solution. The tool life when cutting speed is half is given by

$$\begin{aligned} T_2 &= (2)^{1/0.5} T_1 \\ &= 4T_1 \end{aligned}$$

Therefore, the percentage increase is

$$\begin{aligned} &= \frac{4T_1 - T_1}{T_1} \times 100 \\ &= 300\% \end{aligned}$$

Ans. (c)

35. For tool A, Taylor's tool life exponent (n) is 0.45 and constant (C) is 90. Similarly for tool B, n=0.3 and C=60. The cutting speed (in m/min) above which tool A will have a higher tool life than tool B is

(a) 26.7 (b) 42.5
(c) 80.7 (d) 142.9

(GATE 2010)

Solution. Given that for tool A, $n_A = 0.45$, $C_A = 90$, and for tool B, $n_B = 0.3$, $C_B = 60$. For cutting speed V (m/min), tool life of tool A (t_A) is given by

$$\begin{aligned} V{T_A}^{n_A} &= C_A \\ T_A &= \left(\frac{C_A}{V}\right)^{1/n_A} \end{aligned}$$

Similarly for tool B

$$T_B = \left(\frac{C_B}{V}\right)^{1/n_B}$$

Therefore, for tool A to have higher tool life than that for B,

$$\begin{aligned} T_A &> T_B \\ \left(\frac{C_A}{V}\right)^{1/n_A} &> \left(\frac{C_B}{V}\right)^{1/n_B} \\ \left(\frac{90}{V}\right)^{1/0.45} &> \left(\frac{60}{V}\right)^{1/0.3} \\ V &> 26.6667 \text{ m/s} \end{aligned}$$

Ans. (a)

36. A single point cutting tool with 12° rake angle is used to machine a steel work piece. The depth of cut, that is, uncut thickness is 0.81 mm. The chip thickness under orthogonal machining condition is 1.8 mm. The shear angle is, approximately,

(a) 22° (b) 36°
(c) 56° (d) 76°

(GATE 2011)

Solution. Given that

$$\begin{aligned} \alpha &= 12^\circ \\ t_1 &= 0.81 \text{ mm} \\ t_2 &= 1.8 \text{ mm} \end{aligned}$$

Shear angle (ϕ) in orthogonal machining is related to thickness ratio ($r = t_1/t_2$) as

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$$

Therefore,

$$\phi = 38.176^\circ$$

Ans. (b)

37. Match the following non-traditional machining processes with the corresponding material removal mechanisms.

List-I (Process)		List-II (Mechanism)	
P.	Chemical machining	1.	Erosion
Q.	Electro-chemical machining	2.	Corrosive reaction
R.	Electro discharge machining	3.	Ion displacement
S.	Ultrasonic machining	4.	Fusion and vaporization

(a) P-2, Q-3, R-4, S-1
(b) P-2, Q-4, R-3, S-1
(c) P-3, Q-2, R-4, S-1
(d) P-2, Q-3, R-1, S-4

(GATE 2011)

Solution. Electrochemical machining (ECM) process uses electrical current to remove the metal, but unlike electric discharge machining, it relies on the principle of electrolysis for metal removal. The basic ultrasonic machining (USM) process involves a tool (made of ductile and tough material) vibrating with a very high frequency and a continuous flow of abrasive slurry in the small gap between the tool and the work surface.

Ans. (a)

38. In abrasive jet machining, as the distance between the nozzle tip and the work surface increases, the material removal rate

(a) increases continuously

(b) decrease continuously

(c) decreases, becomes stable and then increases

(d) increases, becomes stable and then decreases

(GATE 2012)

Solution. The distance between the nozzle tip and the work surface is called nozzle tip distance (NTD). When the NTD increases, the velocity of the abrasive particles impinging on the work surface increases due to their acceleration after they leave the nozzle. This, in turn increases the MRR. With further increase in the NTD, the velocity reduces due to the drag of the atmosphere which initially checks the increase in the MRR and finally reduces it.

Ans. (d)

39. Details pertaining to an orthogonal metal cutting process are given below

Chip thickness ratio	=	0.4
Undeformed thickness	=	0.6 mm
Rake angle	=	$+10°$
Cutting speed	=	2.5 m/s
Mean thickness of primary shear zone	=	25 μm

The shear strain rate in s^{-1} during the process is

(a) 0.1781×10^5 (b) 0.7754×10^5

(c) 1.0104×10^5 (d) 4.397×10^5

(GATE 2012)

Solution. Given that

$$
\begin{aligned}
r &= 0.4\\
\alpha &= 10°\\
t_m &= 25\times10^{-6}\text{ m}\\
V_c &= 2.5\text{ m/s}
\end{aligned}
$$

where t_m is the mean thickness of primary shear zone. Shear angle (ϕ) is given by

$$
\begin{aligned}
\tan\phi &= \frac{r\cos\alpha}{1-r\sin\alpha}\\
\phi &= \tan^{-1}\left(\frac{r\cos\alpha}{1-r\sin\alpha}\right)\\
&= 22.9441°
\end{aligned}
$$

Shear strain rate is given by

$$
\begin{aligned}
\dot{\gamma} &= \frac{V_c\cos\alpha}{\cos(\phi-\alpha)\,t_m}\\
&= 1.01049\times10^5\text{ m/s}
\end{aligned}
$$

Ans. (c)

40. In a single pass drilling operation, a through hole of 15 mm diameter is to be drilled in a steel plate of 50 mm thickness. Drill spindle speed is 500 rpm, feed is 0.2 mm/rev and drill point angle is 118°. Assuming 2 mm clearance at approach and exit, the total drill time in seconds is

(a) 35.1 (b) 32.4

(c) 31.2 (d) 30.1

(GATE 2012)

Solution. Drilling speed is

$$
\begin{aligned}
V &= \frac{500}{60}\times0.2\\
&= 1.667\text{ mm/s}
\end{aligned}
$$

Drill has to travel distance (mm)

$$
\begin{aligned}
s &= 50+2\times\frac{15/2}{\tan59°}+2\times2\\
&= 63.0129\text{ mm}
\end{aligned}
$$

Drilling time (seconds) is

$$
\begin{aligned}
t &= \frac{s}{V}\\
&= 37.8077\text{ s}
\end{aligned}
$$

Ans. (a)

41. A steel bar 200 mm in diameter is turned at a feed of 0.25 mm/rev with a depth of cut of 4 mm. The rotational speed of the workpiece is 160 rpm. The material removal rate in mm^3/s is

(a) 160 (b) 167.6

(c) 1600 (d) 1675.5

(GATE 2013)

Solution. Given that

$$
\begin{aligned}
d &= 200\text{ mm}\\
f &= 0.25\text{ mm/rev}\\
t_1 &= 4\text{ mm}\\
N &= 160\text{ rpm}
\end{aligned}
$$

Material removal rate in turning operation is determined as

$$
\begin{aligned}
\text{MRR} &= f\times t_1\frac{\pi dN}{60}\\
&= 1675.5\text{ mm}^3\text{/s}
\end{aligned}
$$

Ans. (d)

42. Two cutting tools are being compared for a machining operation. The tool life equations are:

$$\text{Carbide tool}: \quad VT^{1.6} = 3000$$
$$\text{HSS tool}: \quad VT^{0.6} = 200$$

where V is the cutting speed in m/min and T is the tool life in min. The carbide tool will provide higher tool life if the cutting speed in m/min exceeds

(a) 15.0 (b) 39.4
(c) 49.3 (d) 60.0

(GATE 2013)

Solution. For the same cutting speeds say V, the tool life of carbide tool (say T_1) will be equal to that of HSS tool (say T_2), if

$$\frac{T_1^{1.6}}{T_1^{0.6}} = \frac{3000}{200}$$
$$T_1 = 15 \text{ min}$$

Cutting speed of carbide tool is

$$V \times 15^{1.6} = 3000$$
$$V = 39.39 \text{ m/min}$$

Ans. (b)

43. During the electrochemical machining (ECM) of iron (atomic weight= 56, valency= 2) at current of 1000 A with 90% current efficiency, the material removal rate was observed to be 0.26 g/s. If titanium (atomic weight= 48, valency = 3) is machined by the ECM process at the current of 2000 A with 90% current efficiency, the expected material removal rate in gm/s will be

(a) 0.11 (b) 0.23
(c) 0.30 (d) 0.52

(GATE 2013)

Solution. The material removal rate (MRR) in ECM process is given by

$$\text{MRR} = \frac{\eta I A}{F v \rho}$$

where A is the atomic weight, v is the valency, F is Faraday's constant equal to 96500 C/mol, ρ is the density of the alloy. Thus, assuming the same density of both the materials,

$$\text{MRR} \propto \frac{\eta I A}{v}$$
$$\text{MRR}_2 = \frac{0.9}{0.9} \times \frac{2000}{1000} \times \frac{48}{56} \times \frac{2}{3} \times 0.26$$
$$= 0.297 \text{ g/s}$$

Ans. (c)

Linked Answer Questions

In orthogonal turning of a bar of 100 mm diameter with a feed of 0.25 mm/rev, depth of cut of 4 mm and cutting velocity of 90 m/min, it is observed that the main (tangential) cutting force is perpendicular to the friction force acting at the chip-tool interface. The main (tangential) cutting force is 1500 N.

44. The orthogonal rake angle of the cutting tool in degrees is

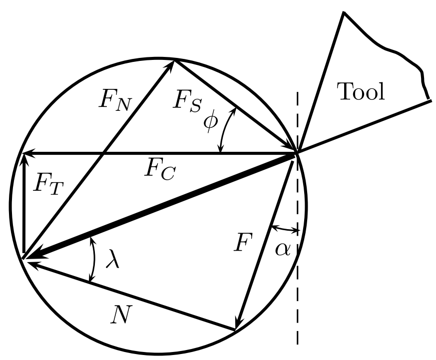
(a) zero (b) 3.58
(c) 5 (d) 7.16

(GATE 2013)

Solution. Given that

$$d = 100 \text{ mm}$$
$$f = 0.25 \text{ mm/rev}$$
$$t_1 = 4 \text{ mm}$$
$$V_c = 90 \text{ m/min}$$
$$F_c = 1500 \text{ N}$$

The rake angle is shown in the Merchant's diagram as below:



Since F_c is normal to friction force F, hence, the rake angle α should be zero.

Ans. (a)

45. The normal force acting at the chip-tool interface in Newton is

(a) 1000 (b) 1500
(c) 2000 (d) 2500

(GATE 2013)

Solution. The normal force at the chip-tool interface N is

$$N = F_c \cos\alpha - F_t \sin\alpha$$
$$= 1500 \times 1 - F_t \times 0$$
$$= 1500 \text{ N}$$

Ans. (b)

MULTIPLE CHOICE QUESTIONS

1. In an orthogonal cutting, ϕ is shear angle, α is rake angle. The trigonometric relationship between cutting velocity V_c, shear velocity V_s and chip velocity V_{chip} can be written as

(a) $V_c/\cos(\phi-\alpha) = V_s/\cos\alpha = V_{chip}/\sin\alpha$
(b) $V_c/\cos(\phi+\alpha) = V_s/\cos\alpha = V_{chip}/\sin\alpha$
(c) $V_c/\cos(\phi-\alpha) = V_s/\sin\alpha = V_{chip}/\sin\alpha$
(d) $V_c/\cos(\phi-\alpha) = V_s/\cos\alpha = V_{chip}/\cos\alpha$

2. Normally, cutting tools are provided with nose radius for

(a) machining of ductile materials
(b) machining of brittle materials
(c) enhancing tool life and surface finish
(d) preventing formation of built-up edge

3. As the rake angle decreases or the friction at the tool-chip interface increases,

(a) the shear angle increases and the chip becomes thinner
(b) the shear angle decreases and the chip becomes thicker
(c) the shear angle decreases and the chip becomes thinner
(d) the shear angle increases and the chip becomes thicker

4. Gray cast iron could be machined dry because

(a) its ductility is high
(b) its shearing strength is very low
(c) it reacts with cutting fluids
(d) graphite flakes act as solid lubricants

5. In orthogonal cutting, the chip thickness in orthogonal cutting depends upon

(a) shear angle (b) rake angle
(c) uncut thickness (d) all of the above

6. In which of the following operations on lathe, the spindle speed will be minimum

(a) turning (b) thread cutting
(c) spinning (d) knurling

7. In lathe machine, the tool carriage and tail stock are guided on

(a) same guide-way on the same bed
(b) different guide-ways on the same bed
(c) not guided on guide-ways
(d) none of the above

8. Taylor's tool life equation is expressed as

(a) $VT^n = C$ (b) $V^nT = C$
(c) $V/T^n = C$ (d) $V^n/T = C$

9. When rake angle increases,

(a) cutting forces reduces and tool becomes thinner
(b) cutting forces increases and tool becomes thicker
(c) cutting forces reduces and tool becomes thicker
(d) cutting forces increases and tool becomes thinner

10. Highest cutting temperature is possible with

(a) stellite (b) tungsten carbide
(c) ceramics (d) diamond

11. In shaper and planar machines, the feed of workpiece is

(a) its advance along the tool movement
(b) its advance normal to the tool movement
(c) its advance per teeth of bull gear
(d) its advance during forward stroke

12. The desired qualities of cutting fluids are

(a) high thermal conductivity, low viscosity
(b) high thermal conductivity, high viscosity
(c) low thermal conductivity, high viscosity
(d) low thermal conductivity, low viscosity

13. The frequency of the tool in case of ultrasonic machining (USM) is in the range of

(a) 10–15 kHz (b) 15–25 kHz
(c) 25–35 kH (d) 35–50 kHz

14. The electron beam machining (EBM) is possible only in a small chamber with

(a) vacuum (b) air
(c) water (d) argon

15. Which is the correct relationship among shear angle ϕ, rake angle α and cutting ratio r in orthogonal cutting?

(a) $\tan\phi = r\cos\alpha/(1-r\sin\alpha)$
(b) $\tan\phi = r\sin\alpha/(1-r\cos\alpha)$
(c) $\tan\phi = r\cos\alpha/(1+r\sin\alpha)$
(d) $\tan\phi = r\sin\alpha/(1+r\cos\alpha)$

16. If Taylor's equation is $VT^n = C$, t is the tool changing time, z_1 is the direct labour and over head rate, z_2 is the tool cost per grind, then the optimum tool life for minimum cost is expressed as

(a) $\left(\frac{1}{n}-1\right)\left(t+\frac{z_2}{z_1}\right)$ (b) $\left(\frac{1}{n}+1\right)\left(t+\frac{z_2}{z_1}\right)$
(c) $\left(\frac{1}{n}-1\right)\left(t-\frac{z_2}{z_1}\right)$ (d) $\left(\frac{1}{n}+1\right)\left(t-\frac{z_2}{z_1}\right)$

17. Tool life testing on a lathe under dry cutting conditions gave n and C of Taylor tool life equation as 0.12 and 130 m/min, respectively. When a coolant was used, C increased by 10%. Find the percent increase in tool life with the use of coolant at a cutting speed of 90 m/min.

(a) 221.27% (b) 121.27%
(c) 21.27% (d) 127.27%

18. Continuous chips without BUE are formed with

(a) ductile work material, small uncut thickness, high cutting speed, larger rake angle
(b) ductile work material, small uncut thickness, low cutting speed, larger rake angle
(c) ductile work material, small uncut thickness, low cutting speed, smaller rake angle
(d) brittle work material, small uncut thickness, high cutting speed, larger rake angle

19. Factors that contribute the formation of built-up-edge (BUE) are

(a) adhesion of workpiece material to the rake face
(b) growth of successive layers of adhered metal
(c) tendency of the workpiece material for strain hardening
(d) all of the above

20. In a cutting operation, the friction angle at tool chip interface is more than the rake angle, then the thrust force will be

(a) downward
(b) upward
(c) zero
(d) cannot be determined

21. In a cutting operation, the coefficient of friction at tool chip interface is zero, then the resultant force on the tool shall be

(a) postive
(b) negative
(c) zero
(d) equal to normal force

22. The presence of aluminium and silicon in steels is always harmful because these elements

(a) chemically react with cutting fluid
(b) make the steel very hard to machine
(c) combine with oxygen and form hard and abrasive elements
(d) none of the above

23. Built-up edge is generally seen in cutting of

(a) low carbon steels
(b) aluminium
(c) martensite stainless steels
(d) all of the above

24. During orthogonal cutting of mild steel with a 10° rake angle tool the chip thickness ratio was obtained as 0.4. The shear angle (in degrees) evaluated from this data is

(a) 6.53 (b) 20.22
(c) 22.94 (d) 50.00

25. In ECM, the material removal is due to

(a) corrosion (b) erosion
(c) fusion (d) ion displacement

26. If you have to machine at a cutting speed of 1 m/s, which one of these tools will you choose in order to have less frequent tool changes?

(a) A (b) B
(c) A and B both (d) indeterminate

27. What is the approximate percentage change is the life of a tool with zero rake angle used in orthogonal cutting when its clearance angle α_c is changed from 10° to 7°? (Hint: Flank wear rate is proportional to $\cot\alpha_c$)

(a) 30% increase (b) 30% decrease
(c) 70% increase (d) 70% decrease

28. In an orthogonal machining operation, the chip thickness and the uncut thickness are equal to 0.45 mm. If the tool rake angle is 0°, the shear plane angle is

(a) 60° (b) 45°
(c) 30° (d) 18°

29. In machining using abrasive material, increasing abrasive grain size

(a) increases the material removal rate
(b) decreases the material removal rate
(c) first decreases and then increases the material removal rate
(d) first increases and then decreases the material removal rate

30. In a typical metal cutting operation, using a cutting tool of positive rake $\alpha = 10°$, it was observed that the shear angle was 20°. The friction angle is

(a) 45° (b) 30°
(c) 60° (d) 40°

31. Selection electrolyte for ECM is as follows:

(a) non-passivating electrolyte for stock removal and passivating electrolyte for finish control
(b) passivating electrolyte for stock removal and non-passivating electrolyte for finish control
(c) selection of electrolyte is dependent on current density
(d) electrolyte selection is based on tool-work electrodes

32. Helix angle of fast helix drill is normally

(a) 5° (b) 35°
(c) 60° (d) 90°

33. Tool life of 10 hours is obtained when cutting with single point tool at 63 m/min. If Taylor's constant $C = 257.35$, tool life on doubling the velocity will be

(a) 5 hours (b) 25.7 min
(c) 38.3 min (d) unchanged

34. The rake angle in a drill

(a) increases from center to periphery
(b) decreases from center to periphery
(c) remains constant
(d) is irrelevant to the drilling operation

35. Cutting power consumption in turning can be significantly reduced by

(a) increasing rake angle of the tool
(b) increasing the cutting angles of the tool
(c) widening the nose radius of the tool
(d) increasing the clearance angle

36. Among the conventional machining processes, maximum specific energy is consumed in

(a) turning (b) drilling
(c) planing (d) grinding

37. In an orthogonal cutting, rake angle of the tool is 20° and friction angle is 25.5°. Using Merchant's shear angle relationship, the value of shear angle will be

(a) 39.5° (b) 42.25°
(c) 47.75° (d) 50.25°

38. To drill a 10 mm diameter hole through a 20 mm thick mild steel plate with a drill bit running at 300 rpm and a feed of 0.25 mm per revolution, time taken in seconds will be

(a) 8 (b) 16
(c) 24 (d) 32

39. Throwaway tungsten carbide tool tips are manufactured by

(a) forging
(b) brazing
(c) powder metallurgy
(d) extrusion

40. In the Taylor's tool life equation

$$VT^n = C$$

The value of n is 0.5. The tool has a life of 180 minutes at a cutting speed of 18 m/min. If the tool life is reduced to 45 minutes, then the cutting speed will be

(a) 9 m/min (b) 18 m/min
(c) 36 m/min (d) 72 m/min

41. In a single point turning operation with a cemented carbide and steel combination having a Taylor exponent of 0.25, if the cutting speed is halved, then the tool life will become

(a) half (b) two times
(c) eight times (d) sixteen times

42. Low helix angle drill are preferred for drilling holes in

(a) plastics (b) copper
(c) cast steel (d) carbon steel

43. Feed gear box for a screw cutting lathe is designed on the basis of

(a) geometric progression
(b) arithmetic progression
(c) harmonic progression
(d) none

44. Machining of titanium is difficult due to

(a) high thermal conductivity of titanium
(b) chemical reaction between tool and work
(c) low tool chip contact area
(d) none of the above

45. In metal cutting, use of low feeds and high cutting speeds is desired when the objective is

(a) high metal removal rate
(b) dry machining
(c) use of soft cutting tool
(d) surface finish

46. A drill for drilling deep holes in aluminium should have

(a) high helix angle
(b) taper shank
(c) small point angle
(d) no lip

47. If the chip-tool contact length is reduced slightly by grinding the tool face, then

(a) both cutting force and interface temperature would decrease
(b) both cutting force and interface temperature would increase
(c) cutting force would decrease but interface temperature would increase
(d) cutting force would increase but interface temperature would decrease

48. It is required to cut screw threads of 2 mm pitch on a lathe. The lead screw has a pitch of 6 mm. If the spindle speed is 60 rpm, then the speed of the lead screw will be

(a) 10 rpm (b) 20 rpm
(c) 120 rpm (d) 180 rpm

49. Electron beam machining can be carried out in

(a) open air
(b) a shielding gas environment
(c) a pressurized inert gas chamber
(d) vacuum

50. What is the correct sequence of the following parameters in order of their maximum to minimum influence on tool life?

1. Feed rate
2. Depth of cut
3. Cutting speed

Select the correct answer using the codes given below:

(a) 1, 2, 3 (b) 3, 2, 1
(c) 2, 3, 1 (d) 3, 1, 2

51. Tool geometry of a single point cutting tool is specified by the following elements:

1. Back rake angle (α_b)
2. Side rake angle (α_s)
3. End cutting edge angle (ϕ)
4. Side cutting edge angle (ψ)
5. Side relief angle (c_s)
6. End relief angle (c_b)
7. Nose radius (r)

The correct sequence of these tool elements used for correctly specifying tool geometry is

(a) 1, 2, 3, 6, 5, 4, 7
(b) 1, 2, 6, 5, 3, 4, 7
(c) 1, 2, 5, 6, 3, 4, 7
(d) 1, 2, 6, 3, 5, 4, 7

52. Given that

Average cutting speed	=	9 m/min
Return ratio	=	1:2
Feed rate per stroke	=	0.3 mm
clearance at each end of cut	=	25 mm

and that the plate is fixed with 700 mm side along the direction of tool travel, the time required for finishing one flat surface of a plate of size 700 mm × 30 mm in a shaper, will be

(a) 10 min (b) 12.5 min
(c) 15 min (d) 20 min

53. Select the wrong statement about the machining processes:

(a) Honing is the finishing operation producing a precision surface.
(b) Tapping is a faster way of producing internal holes, threads, etc.
(c) Grinding is performed by means of a rotating abrasive wheel.
(d) Reamers are always made with odd number of teeth.

54. Consider the following statements about nose radius:

1. It improves tool life.
2. It reduces the cutting force.
3. It improves the surface finish.

Of these statements

(a) 1 and 2 are correct
(b) 2 and 3 are correct
(c) 1 and 3 are correct
(d) 1, 2 and 3 are correct

55. The straight grades of cemented carbide cutting tool materials contain

(a) tungsten carbide only
(b) tungsten carbide and titanium carbide
(c) tugsten carbide and cobalt
(d) tungsten carbide and cobalt carbide

56. Carbon boron nitride

(a) has a very high hardness which is comparable to that of diamond
(b) has a hardness which is slightly more than that of HSS
(c) is used for making cylinder blocks of aircraft engines
(d) is used for making optical glasses

57. The limits to the maximum hardness of a work material, which can be machined with HSS tools even at low speeds is set by which one of the following tool failure mechanism?

(a) attrition
(b) abrasion
(c) diffusion
(d) plastic deformation under compression

58. In orthogonal cutting, the depth of cut is 0.5 mm at a cutting speed of 2 m/s. If the chip thickness is 0.75 mm, the chip velocity is

(a) 1.33 m/s (b) 2 m/s
(c) 2.5 m/s (d) 3 m/s

59. The rake angle in a twist drill

(a) varies from minimum near the dead center to maximum value at the periphery
(b) is maximum at the dead center and zero at the periphery
(c) is constant at every point of the cutting edge
(d) is a function of the size of the chisel edge

60. Which of the following processes does not cause tool wear?

(a) ultrasonic machining
(b) electrochemical machining
(c) electric discharge machining
(d) anode mechanical machining

61. In metal cutting operation, the approximate ratio of heat distributed among chip, tool and work, in that order is

(a) 80:10:10 (b) 33:33:33
(c) 20:60:10 (d) 10:10:80

62. In a single point turning operation of steel with a cemented carbide tool, Taylor's tool life exponent is 0.25. If the cutting speed is halved, the tool life will increase

(a) two times (b) four times
(c) eight times (d) sixteen times

63. Which of the following processes results in the best accuracy of the hole made?

(a) Drilling (b) Reaming
(c) Broaching (d) Boring

64. A straight milling cutter of 100 mm diameter and 10 teeth rotating at 200 rpm is used to remove a layer of 3 mm thickness from a steel bar. If the table feed is 400 mm/min, the feed per tooth in this operation will be

(a) 0.2 mm (b) 0.4 mm
(c) 0.5 mm (d) 0.6 mm

65. In an orthogonal cutting test, the cutting force and thrust force were observed to be 1000 N and 500 N, respectively. If the rake angle of tool is zero, the coefficient of friction chip-tool interface will be

(a) 1/2 (b) 2
(c) $1/\sqrt{2}$ (d) $\sqrt{2}$

66. In a tool life test, doubling the cutting speed reduces the tool life to 1/8th of the original. The Taylor's tool life index is

(a) 1/2 (b) 1/3
(c) 1/4 (d) 1/8

67. Crater wear starts at some distance from the tool tip because

(a) cutting fluid cannot penetrate that region
(b) stress on rake face is maximum at that region
(c) tool strength is minimum at that region
(d) tool temperature is maximum at that region

68. In a shaper machine, the mechanism for tool feed is

(a) Geneva mechanism
(b) Whitworth mechanism
(c) Ratchet and Pawl mechanism
(d) Ward-Leonard system

69. Specific cutting energy is more in grinding process compared to turning because

(a) grinding (cutting) speed is higher
(b) the wheel has multiple cutting edges (grains)
(c) ploughing force is significant due to small chip size
(d) grinding wheel undergoes continuous wear

70. Which one of the following is the hardest cutting tool material next only to diamond?

(a) cemented carbides
(b) ceramics
(c) silicon
(d) cubic boron nitride

71. In a machine tool gear box, the smallest and largest spindles are 100 rpm and 1120 rpm respectively. If there, are 8 speed in all, the fourth speed will be

(a) 400 rpm (b) 280 rpm
(c) 800 rpm (d) 535 rpm

72. Which of the following materials are used in grinding wheels?

1. Aluminium oxide
2. Cubic boron nitride
3. Silicon carbide

Select the correct answer using the codes given below:

(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

73. In a machining operation chip thickness ratio is 0.3 and the back rake angle of the tool is 10°. What is the value of the shear strain?

(a) 0.31 (b) 0.13
(c) 3.00 (d) 3.49

74. The rake angle of a cutting tool is 15°, shear angle 45° and cutting velocity 35 m/min. What is the velocity of chip along the tool face?

(a) 28.5 m/min (b) 27.9 m/min
(c) 25.3 m/min (d) 23.5 m/min

75. Flank wear occurs mainly on which of the following?

(a) nose part and top face
(b) cutting edge only
(c) nose part, front relief face, and side relief face of the cutting tool
(d) face of the cutting tool at a short distance from the cutting edge

76. In turning a solid round bar, if the travel of the cutting tool in the direction of feed motion is 1000 mm, rotational speed of the workpiece is 500 rpm, and the rate of feed is 0.2 mm/revolution, then the machining time will be

(a) 10 s (b) 100 s
(c) 5 min (d) 10 min

77. Tool life is affected mainly with

(a) feed (b) depth of cut
(c) coolant (d) cutting speed

78. With increasing mean grain diameter of the abrasive material in ultrasonic machining (USM), the material removal rate would

(a) increase
(b) decrease
(c) increase and then decrease
(d) decrease and then increase

79. In ultrasonic machining process, the material removal rate will be higher for materials with

(a) higher toughness
(b) higher ductility
(c) lower toughness
(d) higher fracture strain

NUMERICAL ANSWER QUESTIONS

1. An orthogonal cutting is performed with depth of cut of 0.25 mm and cutting speed of 2.5 m/s. Thickness of the chip is found 1 mm. Calculate the chip velocity.

2. In an orthogonal cutting experiment with a tool of rake angle $\alpha = 7°$, the chip thickness was found to be 2.5 mm when the uncut chip thickness was set to 1 mm. Calculate the shear angle and the friction angle using Merchant's formula.

3. In a certain machining operation, the lives of two tools, A and B, are governed by following equations, respectively:

$$VT^{0.125} = 2.5$$
$$VT^{0.250} = 7$$

where V is the cutting speed in m/s and T is the tool life in seconds. Calculate the cutting speed for which both the tools will have the same life.

4. An orthogonal cutting operation is being carried out in which $t_1 = 0.010$ mm, $V_c = 125$ m/min, $\alpha = 10°$ and width of cut $w = 6$ mm. It is observed that $t_2 = 0.014$ mm, cutting force $F_c = 550$ N, tangential force $F_t = 250$ N. Calculate the friction force and the percentage of total energy dissipated in friction at tool-chip contact.

5. For a cutting tool, the Taylor's equation is written as

$$V\sqrt{T} = 600$$

where V is in m/min, T is in minutes. If cutting speed is reduced by 50%, by what percentage tool life will enhance?

6. Machining of steel is performed at cutting speed of 250 m/min, tool rake angle of 12°, width of cut 3 mm, and uncut thickness 0.4 mm. The coefficient of friction between the tool and thick chip is 0.45. The work material has shear strength of 360 MPa. Calculate the shear angle, and the cutting and thrust components of the machining force using Merchant's formula.

7. A 250 mm diameter and 20 mm thick grinding wheel is used to produce a flat surface on steel of size 100×250 mm at table speed of 10 m/min. The grinding wheel rotates for tangential speed of 20 m/s and it is fed at an average rate of 5 mm per pass. Determine the number of passes required for the grinding of 100 mm depth of the workpiece.

8. In a drilling operation under a given condition, the tool life was found to decrease from 20 min to 5 min due to increase in the drill speed from 200 rpm to 400 rpm. Calculate the tool life of that drill under the same condition if the dril speed is 300 rpm.

9. During turning a carbon steel rod of 160 mm diameter by a carbide tool with rake angle 0° and side cutting edge angle 15° at speed of 400 rpm, feed of 0.32 mm/rev and depth of cut 4.0 mm, it is found that the axial components of the cutting force is 800 N. The chip thickness is found to be 0.8 mm. Calculate the friction angle and the power consumption in the process.

10. An HSS tool is used for turning operation. The tool life is 60 min when turning is carried at 30 m/min speed. The tool life reduces to 2.0 min if the cutting speed is doubled. Determine the suitable speed in rpm for turning 300 mm diameter so that tool life is 30 min.

11. For an operation on a lathe machine, the tool changing time is 3 min, tool regrind time is 3 min. Machine running cost per min is Rs. 0.50 and depreciation of the regrind is Rs. 5.0. The constant and exponent of Taylor's tool life equation are 60 and 0.2, respectively. Determine the optimum cutting speed for minimum cost and for maximum production.

12. Tool life testing on a lathe under dry cutting conditions gave n and C of Taylor tool life equation as 0.15 and 150 m/min, respectively. When a coolant was used, C increased by 20%. Find the percent increase in tool life with the use of coolant at a cutting speed of 120 m/min.

13. A machining operation is performed using a cutting tool of positive rake $\alpha = 12°$. The shear angle was found to be 25°. Calculate the friction angle in the operation.

14. Tool life of 20 hours is obtained when cutting with single point tool at 40 m/min. If Taylor's constant $C = 350$, what would be the tool life on doubling the velocity?

ANSWERS

Multiple Choice Questions

1.	(a)	**2.**	(c)	**3.**	(b)	**4.**	(d)	**5.**	(d)	**6.**	(b)	**7.**	(b)	**8.**	(a)	**9.**	(a)	**10.**	(d)
11.	(b)	**12.**	(a)	**13.**	(b)	**14.**	(a)	**15.**	(a)	**16.**	(a)	**17.**	(b)	**18.**	(a)	**19.**	(d)	**20.**	(a)
21.	(a)	**22.**	(c)	**23.**	(d)	**24.**	(c)	**25.**	(d)	**26.**	(b)	**27.**	(b)	**28.**	(b)	**29.**	(a)	**30.**	(c)
31.	(b)	**32.**	(a)	**33.**	(b)	**34.**	(a)	**35.**	(a)	**36.**	(d)	**37.**	(b)	**38.**	(b)	**39.**	(c)	**40.**	(c)
41.	(d)	**42.**	(a)	**43.**	(a)	**44.**	(c)	**45.**	(d)	**46.**	(a)	**47.**	(c)	**48.**	(b)	**49.**	(d)	**50.**	(d)
51.	(b)	**52.**	(b)	**53.**	(d)	**54.**	(c)	**55.**	(c)	**56.**	(a)	**57.**	(a)	**58.**	(a)	**59.**	(c)	**60.**	(b)
61.	(a)	**62.**	(d)	**63.**	(b)	**64.**	(a)	**65.**	(a)	**66.**	(b)	**67.**	(d)	**68.**	(c)	**69.**	(b)	**70.**	(d)
71.	(b)	**72.**	(d)	**73.**	(d)	**74.**	(b)	**75.**	(d)	**76.**	(d)	**77.**	(d)	**78.**	(a)	**79.**	(c)		

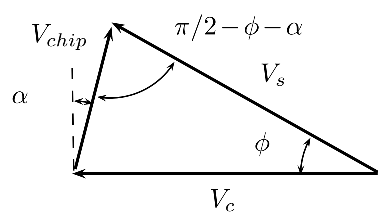
Numerical Answer Questions

1.	0.625 m/s	**2.**	22.65°, 51.7°	**3.**	0.8928 m/s
4.	341.70 N, 48.81%	**5.**	300%	**6.**	38.89°, 1095 N, 232 N
7.	20	**8.**	8.89 min	**9.**	15°, 10.36 kW
10.	0.203	**11.**	31.43 m/s, 36.50 m/s	**12.**	237.27%
13.	52°	**14.**	36.95 min		

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (a) The velocity diagram of orthogonal cutting is shown in the figure below.



The correct relationship between the velocities is expressed as

$$\frac{V_c}{\cos(\phi-\alpha)} = \frac{V_s}{\cos\alpha} = \frac{V_{chip}}{\sin\alpha}$$

2. (c) The nose radius in cutting tool strengthens the tool and increase the surface finish.

3. (b) Using Merchant's theory,

$$2\phi+\lambda-\alpha = \frac{\pi}{2}$$

When the rake angle decreases and or the friction at the tool-chip interface increases, the shear angle decreases and the chip becomes thicker.

4. (d) Gray cast iron could be machined dry because graphite flakes act as solid lubricants. Also, copper alloys have better machinability and could be machined dry, otherwise water-based oils are used.

5. (d) The cutting ratio is related to shear angle ϕ and rake angle α as

$$\frac{t_1}{t_2} = \frac{\sin\phi}{\cos(\phi-\alpha)}$$

Thus, chip thickness t_2 depends upon t_1, α and ϕ.

6. (b) The minimum spindle speed is 80 rpm when the machine is set up for thread cutting.

7. (b) In lathe machine, the tool carriage and tail stock are guided on different guideways on the same bed.

8. (a) Taylor's tool life equation is expressed as

$$VT^n = C$$

where V is cutting velocity (m/min) and T is the tool life (min), n is the index and C is a constant, which depends upon the tool and work materials.

9. (a) When rake angle increases, the edge of the tool becomes sharper, thus reducing the cutting forces and tool becomes thinner.

10. (d) Highest cutting temperature of 1600°C is possible with diamond as tool material.

11. (b) Feed of workpiece is always normal to the cutting direction.

12. (a) The cutting fluid is expected to cool the cutting zone with spray, which is possible with high thermal conductivity and low viscosity to maintain the lubrication.

13. (b) The amplitude and frequency of the tool in USM are in the range of 10–15 μm and 19–25 kHz.

14. (a) To avoid a collision of the accelerating electrons with the air molecules, the process has to be conducted in vacuum (about 105 mm Hg); this makes the process unsuitable for very large workpieces.

15. (a) In orthogonal cutting, the shear angle ϕ, rake angle α and cutting ratio r are related as

$$\tan\phi = \frac{r\cos\alpha}{1-r\sin\alpha}$$

16. (a) The optimum tool life for minimum cost is expressed as

$$T_{opt} = \left(\frac{1}{n}-1\right)\left(t+\frac{z_2}{z_1}\right)$$

where t is the tool changing time, z_1 is the direct labour and over head rate, z_2 is the tool cost per grind, and n is the index of Taylor's tool life equation.

17. (b) Given that

$$n = 0.12$$
$$C = 130 \text{ m/min}$$

The percentage increase in tool life at cutting speed $V = 90$ m/min is determined as

$$\begin{aligned}\frac{T_2}{T_1}-1 &= \left(\frac{C_2/V}{C_1/V}\right)^{1/n}-1\\ &= \left(\frac{1.1}{1}\right)^{1/0.12}-1\\ &= 1.2127\\ &= 121.27\%\end{aligned}$$

18. (a) Continuous chips without BUE are formed with ductile work material, small uncut thickness, high cutting speed, larger rake angle, and suitable cutting fluid.

19. (d) The formation of built-up edge is contributed by adhesion, growth of layers, and straining hardening.

20. (a) The thrust force is related to net reaction R as

$$F_t = R\sin(\lambda-\alpha)$$

The net reaction is always positive, hence, F_t will be positive (downward) when $\lambda > \alpha$.

21. (a) The thrust force is related to net reaction R as

$$\begin{aligned}F_t &= R\sin(\lambda-\alpha)\\ &= -R\sin\alpha\\ &= -N \text{ (upward)}\end{aligned}$$

22. (c) The presence of aluminium and silicon in steel is always harmful because these elements combine with oxygen and form aluminium oxide and silicate. Being hard and abrasive, these compounds increase tool wear and reduce machinability.

23. (d) Built-up edge is formed in machining of low carbon steels and aluminium due to their soft grades, while martensite stainless steels are abrasive, hence, form built-up edge.

24. (c) Given that

$$\alpha = 10^\circ$$
$$r = 0.4$$

Shear angle is determined as

$$\begin{aligned}\phi &= \tan^{-1}\left(\frac{r\cos\alpha}{1-r\sin\alpha}\right)\\ &= 22.94^\circ\end{aligned}$$

25. (d) Electrochemical machining (ECM) process uses electrical current to remove the metal, but unlike electric discharge machining, it is relies on the principle of electrolysis for metal removal.

26. (b) At $V = 1$ m/s, the tool life of the tools are, respectively,

$$\begin{aligned}T_A &= \left(\frac{2.5}{1}\right)^{1/0.125}\\ &= 1525.87 \text{ s}\\ T_B &= \left(\frac{7}{1}\right)^{1/0.250}\\ &= 2401 \text{ s}\end{aligned}$$

As $T_B > T_A$, the second tool B will be preferred.

27. (b) Tool life can be measure of inverse of the flank wear (which is proportional to $\cot\alpha_c$). Thus, percentage change in tool life is

$$\frac{\tan 7^\circ}{\tan 10^\circ} - 1 = -30.36\%$$

28. (b) Given that

$$t_2 = t_1 = 0.45 \text{ mm}$$
$$\alpha = 0^\circ$$

Thickness ratio and shear plane angle are determined as

$$\begin{aligned} r &= \frac{t_1}{t_2} \\ &= 1 \\ \phi &= \tan^{-1}\left(\frac{r\cos\alpha}{1 - r\sin\alpha}\right) \\ &= \tan^{-1} 1 \\ &= 45^\circ \end{aligned}$$

29. (a) The grain size of the abrasive material affects the surface finish and material removal rate (MRR). Small grit size produces better surface finish, while larger grit size permits larger MRR.

30. (c) Given that

$$\alpha = 10^\circ$$
$$\phi = 20^\circ$$

Friction angle (λ) is given by Merchant's circle diagram as

$$\begin{aligned} 2\phi + \lambda - \alpha &= 90^\circ \\ \lambda &= 90^\circ - 2\phi + \alpha \\ &= 60^\circ \end{aligned}$$

31. (b) For stock removal, the electrolyte should possess resistance to form passivating film on the workpiece surface. For finish control, the electrolyte should form a passivating film on the finished workpiece.

32. (a) A high helix drill is also known as a fast helix drill. Its wide flutes and high-helix help to quickly clear the chips. Out of the given options, 5° is correct.

33. (b) Given that

$$\begin{aligned} T_1 &= 10 \times 60 \\ &= 600 \text{ min} \\ V_1 &= 63 \text{ m/min} \\ C &= 257.35 \\ V_2 &= 2V_1 \end{aligned}$$

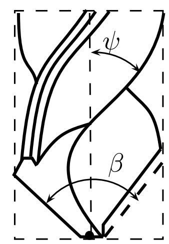
Using Taylor's tool life equation:

$$\begin{aligned} VT^n &= C \\ \ln V_1 + n \ln T_1 &= \ln C \\ n &= \frac{\ln(C/V_1)}{\ln T_1} \\ &= 0.220 \\ 2 \times 63 \times T_2^{0.2199} &= 257.35 \\ T_2 &= 25.73 \text{ min} \end{aligned}$$

34. (a) The normal rake angle α at radius r is approximated in twist drill of diameter D as

$$\alpha \approx \tan^{-1}\left\{\frac{(2r/D)\tan\psi}{\sin\beta}\right\}$$

where ψ is the helix angle, and β is the half point angle, as demonstrated in the figure of twist drill geometry.



Thus, rake angle increases from the center to periphery.

35. (a) Increasing the rake angle reduces the cutting force and so the power consumption.

36. (d) The chip thickness in grinding is very small, hence, for given thickness of a workpiece, the grinding has to be carried out layer by layer, thus, its specific energy is high.

37. (b) Given that

$$\alpha = 20^\circ$$
$$\lambda = 25.5^\circ$$

The shear angle is given by

$$\begin{aligned} 2\phi + \lambda - \alpha &= 90^\circ \\ \phi &= 42.25^\circ \end{aligned}$$

38. (b) In one revolution, the drill travels a distance equal to feed of 0.25 mm. The drill rotates at speed of 300 rpm. Therefore, the time required to travel a distance of 20 mm is given by

$$\begin{aligned} t &= \frac{20}{0.25 \times 300/60} \\ &= \frac{20 \times 60}{300 \times 0.25} \\ &= 16 \text{ s} \end{aligned}$$

39. (c) Throwaway tungsten carbide tool tips are manufactured by powder metallurgy.

40. (c) Tool speed for tool life of 45 min will be given by

$$V \times 45^{0.5} = \left(\frac{180}{45}\right)^{0.5} \times 18$$
$$= 36 \text{ m/min}$$

41. (d) According to Taylor's tool life equation

$$T \propto \frac{1}{V^{1/n}}$$

Therefore,

$$\frac{T_2}{T_1} = \frac{1}{0.5^{1/0.25}}$$
$$= 16$$

42. (a) Low helix angle in drill is preferred for low strength materials such as plastics.

43. (a) Feed gear box of lathe machine is designed on the basis of geometric progression of speed ratios.

44. (c) Most titanium alloys are poor thermal conductors. Heat generated during cutting does not dissipate through the part and machine structure, but concentrates in the cutting area.

45. (d) Surface finish in machining depends on feed (f) and tool geometry parameters as

(a) Tool without nose radius

$$h_{max} = \frac{f}{\tan\psi + \cot\phi}$$

(b) Tool with nose radius r

$$h_{max} = \frac{f^2}{8r}$$

(c) Slab milling

$$h_{max} = \left(\frac{f}{2NZ}\right)^2 \frac{1}{D}$$

Hence, low feeds, and high cutting speeds will result in increased surface finish.

46. (a) High helix angle is desired for drilling deep holes for proper chip flow out of the hole.

47. (c) The reduction in contact length will increase stress concentration and greater heat generation, and also, lesser contact area will decrease the cutting force.

48. (b) Speed of lead screw is $2 \times 60/6 = 20$ rpm.

49. (d) To avoid a collision of the accelerating electrons with the air molecules, electron beam machining has to be conducted in vacuum (about 10^{-5} mm Hg). This makes the process unsuitable for very large workpieces.

50. (d) A generalized form of Taylor's equation is

$$V_c = \frac{C'}{T^n t_1^p b^q}$$

where C', p, q are other constants. It has been observed that $p > q$, so tool life is more sensitive to uncut thickness (feed rate) than the width. Cutting speed has greatest effect on tool life and tool temperature while depth of cut has greatest influence over forces.

51. (b) The tool designation, also called cutting tool signature, is a sequence of numbers listing various angles in degrees and the size of nose radius in mm. The Americal Standards Association (ASA) has standardized the tool identification, known as coordinate system, which uses seven elements comprising the signature of a single point cutting tool, always written in the following sequences

$$\alpha_b - \alpha_s - c_b - c_s - \phi - \psi - r$$

52. (b) Given that

$$V_c = 9 \text{ m/min}$$
$$R = 1/2$$
$$f = 0.3 \text{ mm/stroke}$$
$$c = 25 \text{ mm/end of cut}$$
$$l = 700 \text{ mm}$$
$$b = 30 \text{ mm}$$

The time taken (t) is equal to number of strokes required multiplied by the time required for one stroke, therefore,

$$t = \frac{b}{f} \times \frac{l + 2c}{V_c} \times (1 + R)$$
$$= 12.5 \text{ min}$$

53. (d) Reamers are always made with even number of teeth to generate even or symmetrical surfaces.

54. (c) Nose radius strengthens the tool, thus increases the tool life and also increase the surface finish. It has no effect in cutting forces.

55. (c) Tungsten carbide tool are binded in cobalt binder.

56. (a) Super-hard tool materials are divided into two categories: cubic boron nitride (CBN) and polycrystalline diamond (PCD). Cubic boron

nitride is used for machining very hard ferrous materials such as steel dies, alloy steels and hard-facing materials.

57. (a) Abrasive wear in cutting tool depends upon the hardness of the workpiece.

58. (a) Given that

$$t_1 = 0.5 \text{ mm}$$
$$V_c = 2 \text{ m/s}$$
$$t_2 = 0.75 \text{ mm}$$

Chip velocity is given by

$$V_{chip} \times t_2 = t_1 \times V_c$$
$$V_{chip} = \frac{t_1}{t_2} \times V_c$$
$$= 1.33 \text{ m/s}$$

59. (c) In a twist drill, the rake angle remains constant throughout its cutting edge.

60. (b) Electrochemical machining (ECM) process uses electrical current to remove the metal, but unlike electric discharge machining, it is relies on the principle of electrolysis for metal removal.

61. (a) About 90% power is converted into heat. At extremely slow cutting, the heat is carried out in the chip and through the workpiece, cutting tool and atmosphere. However, when cutting speed is increased, there is not enough time for heat dissipation. Normally, the ratio is chip (70%), tool (20%) and work (10%). Option 80:10:10 is most suited one.

62. (d) Given that

$$n = 0.25$$
$$V_2 = \frac{V_1}{2}$$

Using Taylor's tool life equation:

$$V_c T^n = C$$
$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{1/n}$$
$$= 8T_1$$

63. (b) Reaming is a finishing operation.

64. (a) The cutter has 10 teeth and rotates at 200 rpm. Table speed is 400 mm/min. Therefore, feed per tooth in straight milling is

$$f \times 10 \times 200 = 400$$
$$= 0.2 \text{ mm}$$

65. (a) Using Merchant's circle,

$$F_c = R\cos(\lambda - \alpha)$$
$$F_t = R\sin(\lambda - \alpha)$$

For the given case $\alpha = 0$, hence

$$\tan\gamma = \mu$$
$$= \frac{F_t}{F_c}$$
$$= \frac{1}{2}$$

66. (b) The following is the Taylor's equation

$$V_c T^n = C$$

For given case, n should be 1/3.

67. (d) Cratering occurs behind the cutting edge on the rake face due to tool-chip contact and the temperature depended diffusion.

68. (c) Whitworth quick return mechanism is used in shaper machine for tool feed.

69. (b) The grinding wheel has multiple cutting edges of grains spread through its grind area, hence, specific cutting energy is more in grinding process compared to turning.

70. (d) Super-hard tool materials are divided into two categories: cubic boron nitride (CBN) and polycrystalline diamond (PCD). CBN is the hardest tool material next only to diamond.

71. (b) Machine tool gear boxes are designed for spindle speeds in geometric progression. For given details,

$$x = 100$$
$$xr^7 = 120$$

Hence,

$$r = 1.412$$

The fourth speed will be

$$xr^3 = 281 \text{ rpm}$$

72. (d) Aluminium oxide (Al_2O_3) and silicon carbide (SiC) are used as grains for grinding wheel.

73. (d) Given that

$$r = 0.3$$
$$\alpha = 10^\circ$$

Shear angle is given by

$$\tan\phi = \frac{r\cos\alpha}{1-r\sin\alpha} = 0.3116$$
$$\phi = 17.3^\circ$$

The magnitude of shear strain is given by

$$\gamma = \cot\phi + \tan(\phi-\alpha) = 3.49$$

74. (b) Given that

$$\alpha = 15^\circ$$
$$\phi = 45^\circ$$
$$V_c = 35 \text{ m/min}$$

Chip velocity is determined as

$$V_{chip} = \frac{V_c\sin\phi}{\cos(\phi-\alpha)} = 27.93 \text{ m/min}$$

75. (d) Flank wear occurs on the face of cutting tool at a short distance from the cutting edge.

76. (d) The time required will be given by

$$t = \frac{1000}{0.2\times 500} = 10 \text{ min}$$

77. (d) A generalized form of Taylor's equation is

$$V_c = \frac{C'}{T^n {t_1}^p b^q}$$

where C', p, q are other constants. It has been observed that $q < p$, so tool life is more sensitive to uncut thickness than the width. Cutting speed has greatest effect on tool life and tool temperature while depth of cut has greatest influence over forces.

78. (a) Material removal rate of USM is proportional to mean grain diameter, thus increases with the size of abrasive grains.

79. (c) Material removal rate will be higher for materials with lower toughness.

Numerical Answer Questions

1. Given that

$$t_1 = 0.25 \text{ mm}$$
$$V_c = 2.5 \text{ m/s}$$
$$t_2 = 1.0 \text{ mm}$$

Chip velocity is given by

$$V_{chip}\times t_2 = t_1\times V_c$$
$$V_{chip} = 0.625 \text{ m/s}$$

2. Given that

$$\alpha = 7^\circ$$
$$t_2 = 2.5 \text{ mm}$$
$$t_1 = 1 \text{ mm}$$

Hence,

$$r = \frac{t_1}{t_2} = 0.4$$

Shear angle is

$$\phi = \tan^{-1}\left(\frac{r\cos\alpha}{1-r\sin\alpha}\right) = 22.65^\circ$$

Using Merchant's formula:

$$2\phi+\lambda-\alpha = 90^\circ$$
$$\lambda = 51.7^\circ$$

3. For the same tool life T at cutting speed V^*

$$\left(\frac{2.5}{V^*}\right)^{1/0.125} = \left(\frac{7}{V^*}\right)^{1/0.250}$$
$$\left(\frac{2.5}{V^*}\right) = \left(\frac{7}{V^*}\right)^{0.125/0.250}$$
$$\frac{2.5}{V^*} = \frac{\sqrt{7}}{\sqrt{V^*}}$$
$$\sqrt{V^*} = \frac{2.5}{\sqrt{7}}$$
$$V^* = 0.8928 \text{ m/s}$$

The corresponding tool life for both tools is

$$t^* = \left(\frac{2.5}{0.8928}\right)^{1/0.125} = 3779.95 \text{ s}$$

4. The resultant force is given by

$$R = \sqrt{F_c^2+F_t^2} = \sqrt{550^2+250^2} = 604.15 \text{ N}$$

The angle of friction is given by

$$\begin{aligned} F_c &= R\cos(\gamma-\alpha) \\ 550 &= 604.15\cos(\gamma-10) \\ \gamma &= 34.44^\circ \end{aligned}$$

The friction force is calculated as

$$\begin{aligned} F &= R\sin\gamma \\ &= 604.15\sin 34.44^\circ \\ &= 341.70 \text{ N} \end{aligned}$$

The percentage of energy is given by

$$\begin{aligned} \frac{FV_{chip}}{F_cV_c} &= \frac{F}{F_c}\frac{t_1}{t_2} \\ &= \frac{341.70}{550}\times\frac{0.010}{0.014} \\ &= 48.81\% \end{aligned}$$

5. The tool life T_2 will be given by

$$\begin{aligned} V_1\sqrt{T_1} &= V_2\sqrt{T_2} \\ T_2 &= \frac{V_1}{V_2}^2 T_1 \\ &= 4T_1 \end{aligned}$$

Percentage increase in tool life is

$$\begin{aligned} &= \frac{4T_1-T_1}{T_1}\times 100 \\ &= 300\% \end{aligned}$$

6. Given that $\alpha = 12^\circ$. Therefore,

$$\begin{aligned} \gamma &= \tan^{-1}\mu \\ &= \tan^{-1} 0.45 \\ &= 24.22^\circ \end{aligned}$$

Using Merchant's formula:

$$\begin{aligned} 2\phi+\lambda-\alpha &= 90^\circ \\ \phi &= \frac{90+\alpha-\lambda}{2} \\ &= 38.89^\circ \end{aligned}$$

The shear force of the cutting process is

$$\begin{aligned} F_s &= \frac{wt_1\tau_s}{\sin\phi} \\ &= \frac{3\times 0.4\times 360}{\sin 38.89^\circ} \\ &= 688.08 \text{ N} \end{aligned}$$

Resultant machining force is calculated as

$$\begin{aligned} R &= \frac{F_s}{\cos(\phi+\lambda-\alpha)} \\ &= \frac{688.08}{\cos(38.89+24.22-12)} \\ &= 1095.97 \text{ N} \end{aligned}$$

Cutting and thrust forces are calculated as

$$\begin{aligned} F_c &= R\cos(\gamma-\alpha) \\ &= 1095.97\cos(24.22-12) \\ &= 1070.19 \text{ N} \\ F_t &= R\sin(\gamma-\alpha) \\ &= 1095.97\sin(24.22-12) \\ &= 231.77 \text{ N} \end{aligned}$$

7. As the size of wheel D is in the range of W to $2W$, the approach distance is $250/2 = 125$ mm. Thus, the total distance to travel in one pass is $250+250 = 500$ mm. Table speed is 10×10^3 mm/min. Hence, time required for one pass is

$$\begin{aligned} t_1 &= \frac{500}{10\times 10^3} \\ &= 0.05 \text{ min} \end{aligned}$$

The depth is 100 mm and feed rate is 5 mm per pass, therefore, total number of passes required is

$$\begin{aligned} n &= \frac{100}{5} \\ &= 20 \end{aligned}$$

8. For the given data, the exponent n of the Taylor's tool life equation is given by

$$\begin{aligned} n &= \frac{\ln(V_1/V_2)}{\ln(T_2/T_1)} \\ &= \frac{\ln(200/400)}{\ln(5/20)} \\ &= 0.5 \end{aligned}$$

Tool life for 300 rpm and $n = 0.5$ is

$$\begin{aligned} 300\times T^{0.5} &= 200\times 20^{0.5} \\ T &= 8.89 \text{ min} \end{aligned}$$

9. From the given tool geometry:

$$\begin{aligned} \alpha &= 0^\circ \\ \psi &= 15^\circ \end{aligned}$$

Using Merchant's analysis,

$$\begin{aligned} F_f &= F_T\cos\psi \\ F_r &= F_T\sin\psi \end{aligned}$$

Net tangential component of the cutting force is

$$F_T = \frac{F_f}{\cos\psi} = 828.22 \text{ N}$$

The cutting force is calculated as

$$F_C = F_T \frac{\cos(\psi-\alpha)}{\sin(\psi-\alpha)} = 3090.96 \text{ N}$$

The friction force and normal force are

$$F = F_C \sin\alpha + F_T \cos\alpha = 828.22 \text{ N}$$
$$N = F_C \cos\alpha - F_T \sin\alpha = 3090.96 \text{ N}$$

The angle of friction is calculated as

$$\lambda = \tan^{-1}\frac{F}{N} = 15^\circ$$

The power consumption is determined as

$$P = F_c \times V_c = F_c \times \frac{\pi DN}{60} = 3090.96 \times \frac{\pi \times 0.16 \times 400}{60} = 10.357 \text{ kW}$$

10. Exponent n of the Taylor's tool life equation is

$$n = \frac{\ln(V_1/V_2)}{\ln(T_2/T_1)} = \frac{\ln(2\times 30/30)}{\ln(60/2)} = 0.203$$

Cutting speed for 30 min tool-life is

$$V \times 30^{0.203} = 30 \times 60^{0.205}$$
$$V = 34.58 \text{ m/min}$$

Cutting speed in rpm is given by

$$V = \pi DN$$
$$N = \frac{V}{\pi D} = 36.69 \text{ rpm}$$

11. Given that

$$n = 0.2$$
$$C = 60$$

Tool changing time is $t = 3$ min. Direct running cost z_1 and depreciation cost z_2 are

$$z_1 = 3 \times 0.5 = 1.5 \text{ Rs.}$$
$$z_2 = 5.0 \text{ Rs.}$$

Cutting speed for minimum cost is

$$V_{opt1} = \frac{C}{[(1/n-1)(t+z_2/z_1)]^n} = 31.43 \text{ m/s}$$

Cutting speed for maximum production is

$$V_{opt2} = \frac{C}{[(1/n-1)\,t]^n} = 36.50 \text{ m/s}$$

12. Given that

$$n = 0.15$$
$$C = 150 \text{ m/min}$$

The percentage increase in tool life at cutting speed $V = 120$ m/min is determined as

$$\frac{T_2}{T_1} - 1 = \left(\frac{C_2/V}{C_1/V}\right)^{1/n} - 1 = \left(\frac{1.2}{1}\right)^{1/0.15} - 1 = 2.37 = 237.27\%$$

13. Given that

$$\alpha = 12^\circ$$
$$\phi = 25^\circ$$

Using Merchant's theory,

$$2\phi + \lambda - \alpha = 90^\circ$$
$$\lambda = 90^\circ - 2\phi + \alpha = 52^\circ$$

14. Given that

$$T_1 = 20 \times 60 = 1200 \text{ min}$$
$$V_1 = 40 \text{ m/min}$$
$$C = 350$$
$$V_2 = 2V_1$$

Using Taylor's tool life equation:

$$VT^n = c$$
$$\ln V_1 + n \ln T_1 = \ln C$$
$$n = \frac{\ln(c/V_1)}{\ln T_1} = 0.3$$
$$2 \times 40 \times T_2^{0.3} = 350$$
$$T_2 = 136.95 \text{ min}$$

CHAPTER 15

METROLOGY AND INSPECTION

Metrology is the science of measurement and industrial inspection. Instrument and gauges are the measuring tools that provide comparative and quantitative measurements of a product or component's dimensional, form and orientation attributes. Metrologists are intimately concerned with the design, manufacturing and testing of measuring tools.

15.1 LIMITS, TOLERANCES, AND FITS

Large number of parts of exactly same dimension cannot be produced commercially; actual dimension are always little larger or smaller than the desired ones. The desired dimension of a component is called the *nominal dimension*. The amount of deviation of the actual dimension from the nominal dimension is called the *tolerance*. The extreme permissible dimensions of a part are called *limits*.

Manufacturing tolerance is essential to facilitate *interchangeability* of parts that enable proper functioning of the assembled parts. Interchangeability refers to assembling a member of unit components taken at random from stock so as to build up a complete assembly without fitting or adjustment.

In graphical representations, *basic size* is the exact theoretical size from which limits of size are derived by the application of allowances and tolerances. It is represented by *zero line*. *Allowance* is the minimum clearance space between mating parts intentionally provided to secure the desired fit. *Interference* is the amount by which dimensions of the mating parts overlap. This is also called *negative allowance*.

15.1.1 Limit Systems

A limit system consists of a series of tolerances arranged to suit the specific range of sizes in order to enable specific classes of fit between mating components. The following are the two bases of limit systems [Fig. 15.1]:

1. Basic Hole System *Basic hole system* is a system of fits in which design size of the hole is the basic size from which allowance is subtracted to obtain the diameter of the shaft. The system is preferred because standard tooling (e.g. drills, reamers, broaches, plug gauges) whose size is not adjustable, and shaft can then easily be machined to fit.

2. Basic Shaft System *Basic shaft system* is a system of fits in which the design size of the shaft is the basic size to which allowance is added to obtain the diameter of the hole.

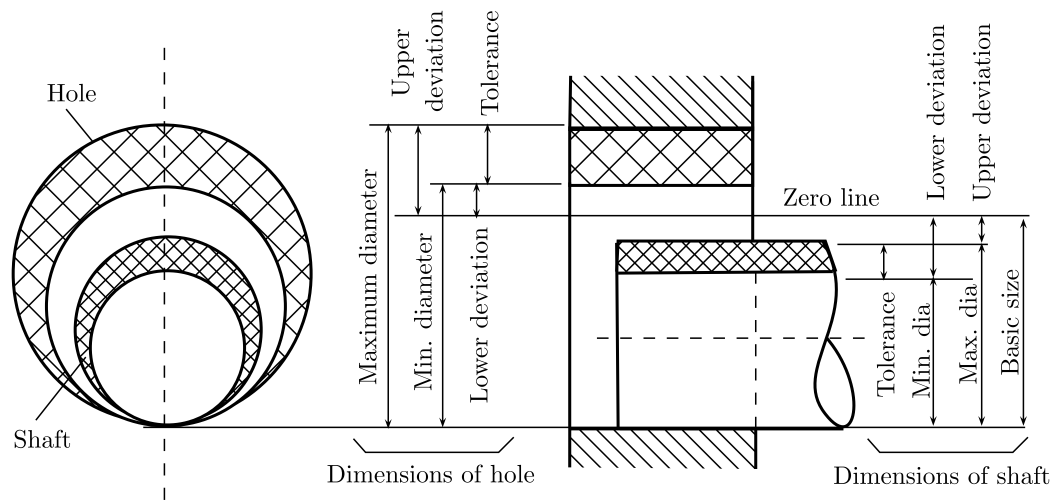


Figure 15.1 Basic size and tolerances.

Deviation is the algebraic difference between the actual size and basic size. *Upper deviation* is the difference between the maximum limit size and the basic size. It is indicated as 'ES' for holes, and 'es' for shafts. *Lower deviation* is the algebraic difference between the minimum limit of a size and the basic size. It is indicated by 'EI' for holes and 'ei' for shafts.

The upper and lower deviations can be related to tolerance (IT) as by following formulas:

1. For shafts:

$$\text{es} = \text{ei} + \text{IT}$$

2. For holes:

$$\text{ES} = \text{EI} + \text{IT}$$

Fundamental deviation (δ) is either the upper or the lower deviation which is nearest to the zero line. This can be either for the shaft or for the hole.

15.1.2 Tolerance Systems

The tolerances on the dimensions can be specified in three ways:

1. Unilateral Tolerance System In *unilateral tolerance system*, the tolerance is provided on only one direction from the basic size, e.g. 3765^{+5}_{+2}, 3767^{+13}_{+0}.

   Unilateral tolerance system permits the variation in tolerance on a hole or shaft without seriously affecting the fit, therefore, it is the most commonly used in interchangeable manufacturing, especially where precision fits are required.

2. Bilateral Tolerance System In *bilateral tolerance system*, the tolerance is split into two parts and shown on both sides of the nominal size. For example, 3774^{+6}_{-7}, 3770^{+10}_{-11}.

   Bilateral system clearly indicates the theoretically desired size and deviations permitted on either side of the basic size. Thus, when tolerance is varied on a part, the basic dimensions of one or both the mating parts need changes to retain the same fit.

3. Limit Dimensioning System In *limit dimensioning*, the size and deviation of part are specified by only maximum and minimum dimensions. For example, $3735 - 3760$.

15.1.3 Fits

The degree of tightness or looseness between mating members is known as the *fit* of the members. It depends on the actual value of the individual tolerances of the mating components. The fits can be broadly classified into three categories [Fig. 15.2].

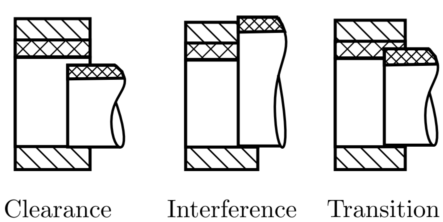


Figure 15.2 Types of fits.

These are explained as follows:

1. <u>*Clearance Fit*</u> *Clearance fit* is one having limits of size so prescribed that a clearance always results when mating parts are assembled. A clearance fit has positive allowance; there is minimum positive clearance between high limit of the shaft and low limit of the hole. A clearance fit can be loose (no precision) or running fit (maintaining film lubrication).

2. <u>*Interference Fit*</u> *Interference fit* is one having limits of size so prescribed that an interference always results when mating parts are assembled. It is achieved by keeping the lower limit of the shaft greater than the upper limit of the hole.

3. <u>*Transition Fit*</u> *Transition fit* is one having limits of size so prescribed that the assembly of mating parts can result in either a clearance fit or an interference fit. This is achieved by keeping the upper limit on the shaft larger than the lower limit on the hole, and lower limit on the shaft smaller than the upper limit on the hole.

15.1.4 IS:919-1963

The system of limits and fits recommended in IS:919-1963 suggests 18 grades (IT01, IT0, IT1 to IT16) of fundamental tolerances and 25 types of fundamental deviations denoted by alphabets both for holes (A to ZC) and shafts (a to zc) in diametral steps up to 500 mm [Fig. 15.3].

Each grade has an associated value or formula for calculation of standard tolerance in terms of mean diameter D given by

$$D = \sqrt{D_1 D_2} \quad \text{mm}$$

and a parameter i, defined as

$$i = 0.45\sqrt[3]{D} + 0.001D \ \mu\text{m}$$

where D is in mm. The tolerances are determined based on the tolerance grade. Such as for IT7, the value of tolerance is $16i$. Other important values of tolerances according to tolerance grades are shown in Table 15.1

Table 15.1 | Tolerance grades

Grade	Value
IT5	$7i$
IT6	$10i$
IT7	$16i$
IT8	$26i$
IT9	$40i$
IT10	$64i$
IT11	$100i$
IT12	$160i$
IT13	$250i$
IT14	$400i$
IT15	$640i$
IT16	$1000i$

The shaft h for which the upper deviation is zero is called the *basic shaft* and the hole H for which the lower deviation is zero is called the *basic hole* [Fig. 15.3]. The values of the fundamental deviation (δ) for the hole from A to H are positive, whereas that for the shaft from a to h are negative. The values of the fundamental deviation for the hole from J to K are either positive or negative, whereas that for shaft form j to k are either positive or negative.

A hole or shaft is completely described if the basis size, followed by the appropriate letter and by the number of the tolerance grade is given. For example, a 25 mm H-hole with tolerance IT8 is indicated as 25 $H8$, a 25 mm f-shaft with tolerance grade IT7 is indicated as 25 $f7$.

A fit is indicated by combining the designations for both the hold and shaft with the hole designation written first, regardless of the system. For example, 25 $H8f7$. Figure 15.3 shows that a fit comprising hole having tolerance between A and H and a shaft between a and h will certainly give a clearance fit.

To demonstrate the procedure, consider a shaft specified by $28h8$. Basic size 28 mm comes in the range 18–30 mm. Therefore, mean diameter is

$$\begin{aligned} D &= \sqrt{D_1 \times D_2} \\ &= 23.2379 \text{ mm} \end{aligned}$$

Tolerance,

$$\begin{aligned} i &= 0.45\sqrt[3]{D} + 0.001D \ \mu\text{m} \\ &= 1.3074 \ \mu\text{m} \\ &= 0.0013074 \text{ mm} \end{aligned}$$

For basic shaft h upper deviation is zero. For IT8, the tolerance is $26i = 0.0339$ mm. Therefore, dimension of the shaft will vary between 28.000 mm and $28.000 - 0.0339$ mm = 27.966 mm.

15.2 LINEAR MEASUREMENT

Linear metrology is the science of linear measurement for determination of the distance between two points in a straight line. The principle of linear measurement is to compare the dimensions to be measured and aligned with standard dimensions marked on the measuring instrument. Some important instruments for linear measurements are described as follows:

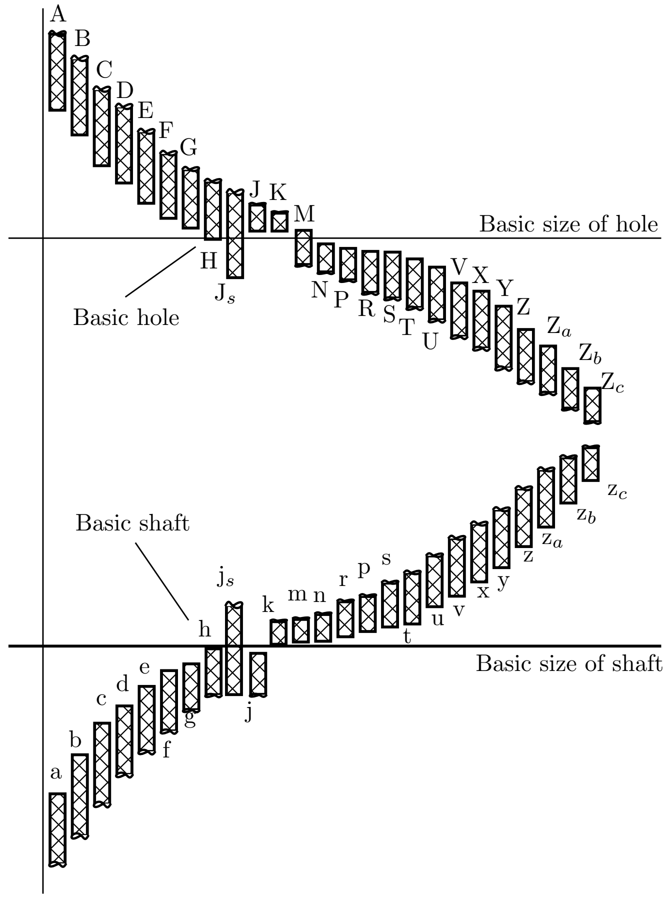


Figure 15.3 | Fundamental tolerances in IS:919-1963.

1. Steel Rule *Steel rule* or *scale* is the simplest and the most commonly used linear measuring instrument. It measures an unknown length by comparing with the one previously calibrated.

2. Calipers A *caliper* is an end-standard measuring instrument to measure the distance between two points. Calipers typically use a precise slide movement for inside, outside, depth or step measurements. A caliper consists of two legs hinged at the top, with the ends of the legs spanning the part to be measured. The legs are made of alloy steel and are identical in shape, with the contact points equidistant from the fulcrum.

3. Vernier Caliper *Vernier caliper* is a combination of inside and outside calipers and has two sets of jaws in which one jaw (with a depth gauge) slides along a rule. The device is based on the observation of Pierre Vernier (1631) that human eye cannot discern the exact distance between two lines, but can tell when two lines coincide so as to form one straight line. Least count of a vernier caliper is determined by dividing the smallest division on the main scale by the total number of divisions on the vernier scale.

4. Micrometers *Micrometers* have greater accuracy than vernier calipers and are used in most of the engineering precision work involving interchangeability of component parts. The function of a micrometer is based on the principle of screw and nut. When a screw is turned through one revolution, the nut advances by one pitch distance. If circumference of the screw is divided into n equal parts, then rotation of one division will cause the nut to advance through pitch/n length (least count). Micrometers having accuracy of 0.01 mm are generally available which can increase upto 0.001 mm.

5. Slip Gauges *Slip gauges* are used in the manufacturing shops as length standards. They are not used for regular and continuous measurement. These are rectangular blocks having cross-section usually 32 mm $\times$ 9 mm but of thickness in standard series. Measuring surface of the gauge

blocks is finished to a very high degree of flatness and accuracy.

The gauges are wrung together by bringing them into contact with each other at right angles and then pressing them with a twisting motion and simultaneously turning them parallel. If gauges are in a good condition, wringing will take place easily.

6. Angle Plates Angle plates are used with surface plates for measurements. The two surfaces of face plates are perpendicular to each other. Angle plates are made from cast iron (minimum hardness 180 HB) in various sizes. These are provided with T-slots and long holes to facilitate their clamping and holding.

7. V-Blocks V-blocks are used for checking the roundness of cylindrical workpieces. The V-angle of the block is 90°. Primary use of V-blocks is to hold or move cylindrical workpieces along a precisely fixed axis.

8. Surface Gauges Surface gauge is a versatile instrument used with surface plates for layout work. The gauge consists of a heavy rigid base and spindle that carries a scriber. The rigid base has a perfectly flat bottom surface.

9. Feeler Gauges *Feeler gauges* are mostly used in engineering to measure the clearance between two parallel flat faces, such as piston and cylinder. These are called feeler gauges because these are neither forced to enter to gap nor to slide freely, but the use should feel the correctness personally. Feeler gauges consist of a set of gauging blades of different grades and thickness, which are assembled in a protective sheath.

10. Comparators *Comparators* are used for quick checking of large number of identical dimensions. These instruments cannot be used as an absolute measuring device, but only for comparing two dimensions.

11. Dial Indicators A dial indicator consists of a spring loaded plunger whose tip is used for measuring or gauging a surface. Movement of the plunger is magnified through the intermediate gearing to show with the pointer.

15.3 ANGULAR MEASUREMENT

Common angular measuring instruments read degrees directly from a circular scale scribed on the dial or circumference, such as *protractors* and *angle gauges*. Most common angle is right or perpendicular angle, therefore, squares are the most common devices for drawing them. Some other common instruments for angular measurements are described as follows:

1. Angle Gauges Angles gauges[1] are a series of fixed angles used for comparative assessment of the angles between two surfaces. Angles can be build up by proper combination of gauges.

2. Sine Bar *Sine bar* is a high precision angle measuring instrument. It is used in conjunction with a set of slip gauges. It is kept on two hardened rollers of accurately equal diameters spaced at a known dimension at each end. Thus, top and bottom surfaces of the bar are absolutely parallel to the center line of the rollers.

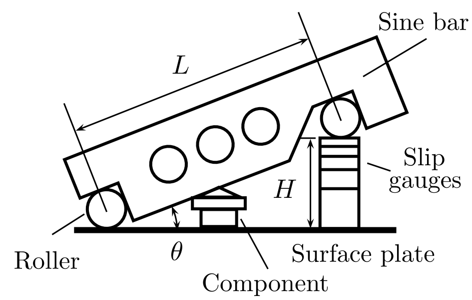


Figure 15.4 Sine bar.

As demonstrated in Fig. 15.4, the angle of the component surface is measured as

$$\theta = \sin^{-1} \frac{H}{L}$$

The bar is made of high-carbon, high chromium corrosion resistant steel and it is essentially hardened, ground, lapped and stabilized. *Relief holes* are drilled in its body to make it lighter and facilitate handling.

3. Clinometers *Clinometers* are the optical devices for measuring elevation angles above horizontal using spirit level mounted on a rotary member, which is carried in a housing. One face of the housing forms the base of the instrument. Angle of inclination of the rotary member carrying the level relative to its base is measured by a circular scale on the housing. [Fig. 15.5].

Clinometers are mainly used to determine the included angle of two adjacent faces of workpiece, large cutting tools, and milling cutter inserts.

4. Autocollimator Small angular tilts of a reflecting surface can be easily measured by an optical

[1]Tomlinson developed angle gauges in 1941.

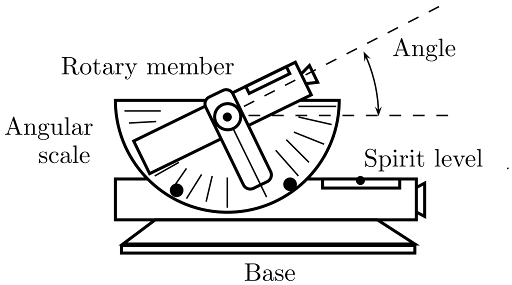


Figure 15.5 Clinometer.

instrument called *autocollimator*. The instrument consists of an infinite telescope and a *collimator* combined into one unit. This provides very sensitive and accurate readings.

When a parallel (collimated[2]) beam of light is projected through the lens, the beam is reflected back from the plane mirror along its own path to focus exactly at the position of the light source [Fig. 15.6].

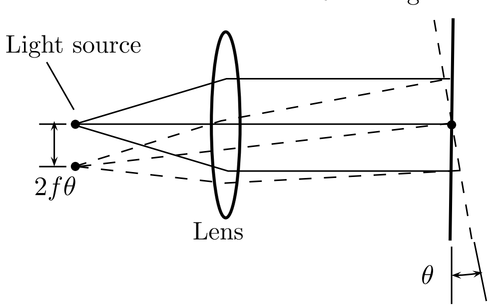


Figure 15.6 Principle of autocollimator.

If the reflector mirror is tilted through a small angle θ, the parallel beam deflects through twice the angle 2θ, thus shifting the focus of the image at a distance $2f\theta$ where f is the focal length of the lens.

5. *Taper Measurements* External tapers can be measured by using two rollers of the same diameter d [Fig. 15.7].

[2]Collimated light has parallel rays, therefore spreads minimally as it propagates. The word is related to "collinear" and implies light that does not disperse with distance (ideally), or that will disperse minimally (in reality).

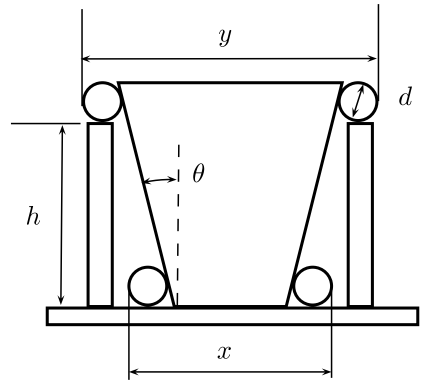


Figure 15.7 Measurements of external tapers.

The taper angle can be determined as

$$y = x + 2\frac{d}{2} + h\cot\theta$$
$$\theta = \cot^{-1}\left(\frac{y - x - d}{h}\right)$$

Internal tapers can be measured with the help of two balls of different sizes [Fig. 15.8].

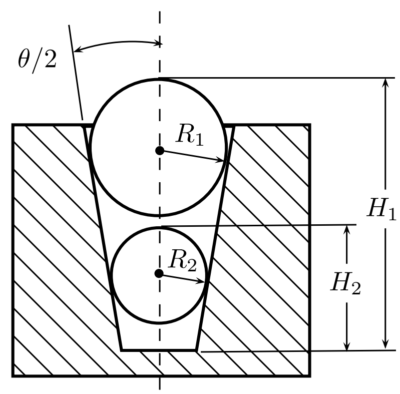


Figure 15.8 Measurements of internal tapers.

The taper angle can be determined as

$$\sin\left(\frac{\theta}{2}\right) = \frac{R_1 - R_2}{(H_1 - R_1) - (H_2 - R_2)} = \frac{R_1 - R_2}{(H_1 - H_2) - (R_1 - R_2)}$$

15.4 GAUGE DESIGN

Gauges or *limit gauges* are high precision instruments used for inspecting a given dimension whether it is

within the specified tolerance limits or not rather than measuring. For this, a gauge consists of two elements [Fig. 15.9]:

1. A "GO" section which presents the maximum limit on the possible dimension of the shaft.
2. A "NO GO" section that presents the minimum limit on the dimension of the shaft.

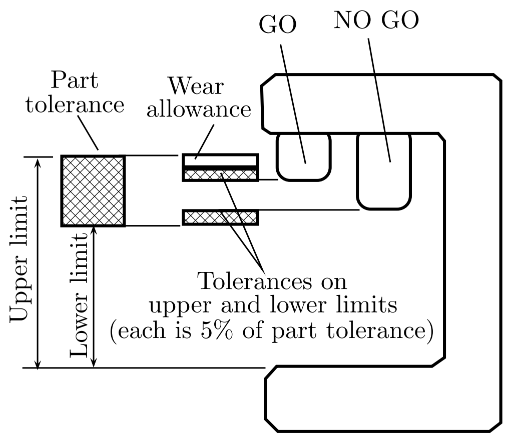


Figure 15.9 Tolerances on gauges.

Tolerance is also essential in manufacturing of the GO and NO GO sections of the gauges. To prohibit a part with higher tolerance, tolerance in gauges is generally applied inside the part tolerance [Fig. 15.9]. Value of gauge tolerance is kept approximately 10% of the workpiece tolerance, which is equally distributed between the GO and NO GO sections. About 5% of the work tolerance is normally allowed as wear allowance to take care of abrasion of GO element, which experiences the relative movement with the part.

15.5 INTERFEROMETRY

Huygens' theory proposes that light is considered as wave motion propagated either as an electromagnetic wave of sinusoidal form. Wave characteristics of light are not apparent under ordinary conditions but when the two waves interact with each other, the wave effect is visible and it can be made useful for measuring applications. This is possible by interferometry techniques which enable the determination of size of end standards (slip gauges and end bars) directly in terms of wavelength of light source. White light is the combination of all the colors of the visible spectrum. This combination of all wavelengths of a visible spectrum and its form is not suitable for interferometry. Monochromatic light such as mercury, mercury 198, cadmium, krypton, krypton 86, thallium, sodium, and laser beams are used.

15.6 SURFACE MEASUREMENT

Surface finish plays an important role in the performance of machine elements. Friction and wear increases with surface roughness, thus adversely affecting the performance of bearings. Rough surfaces have reduced contact area in interference fit which reduces the holding capacity of the joints. Endurance strength of the component is greatly reduced due to poor surface finish.

There are variety of methods available for measurement of the roughness and determination of the statistical properties of surfaces. *Surface finish*[3] measurements can be grouped into two sets:

1. *Comparison-Based Methods* In this category, the surface texture is assessed by observation of the surface with the surface produced by same techniques. These methods include touch inspection, visual inspection, scratch inspection, microscope inspection, etc. These methods involve subjective judgment.

2. *Direct Measurement Methods* It is possible to get the numerical value of surface finish by using stylus probe type of instruments.

   One of the widely used techniques is the *mechanical profilometer.* This instrument directly traces the surface using a narrow diamond *stylus*, which produces a time-varying voltage output proportional to the height of the surface profile. Tomlison surface meter and Tayler–Habson's "Talysurf" are the two profilometers.

   An *optical profilometer* is a non-contact method for providing much of the same information as a stylus based mechanical profilometer. Non-contact profilometer does not touch the surface and the scan speeds are dictated by the light reflected from the surface and the speed of the acquisition electronics. Optical profilometers are more reliable because these are not damaged by surface wear. There are many different techniques which are currently being employed, such as laser triangulation (triangulation sensor), confocal microscopy (used for profiling of very small objects), low coherence interferometry and digital holography.

   Fiber-based optical profilometers scan surfaces with optical probes which send light interference signals back to the profilometer detector via an optical fiber.

[3]Refer the Chapter 14: Machining, where the fundamental concepts and definitions related to surface finish has been discussed in detail.

IMPORTANT FORMULAS

IS:919-1963

$$D = \sqrt{D_1 D_2} \text{ mm}$$
$$i = 0.45\sqrt[3]{D} + 0.001D$$

Sine Bar

$$\theta = \sin^{-1}\frac{H}{L}$$

Gauge Design

1. Value of gauge tolerance is kept approximately 10% of the workpiece tolerance which is equally distributed between the GO and NO GO sections.
2. About 5% of the work tolerance is normally allowed as wear allowance to take care of abrasion of GO element, which experiences the relative movement with the part.

SOLVED EXAMPLES

1. An interchangeable assembly is specified as follows:

(a) Holes of size $28.000^{+0.070}_{0.000}$ mm
(b) Shafts of size $28.000^{+0.030}_{0.010}$ mm

Determine the maximum possible clearance in the assembly.

Solution. Maximum dimension of hole is 28.070 mm, while minimum dimension of shaft is 27.990 mm. Therefore, the maximum clearance will be

$$\begin{aligned} c &= 28.070 - 27.990 \\ &= 0.08 \text{ mm} \end{aligned}$$

2. A shaft and hole pair is designated as $110h_{10}F_8$. The diameter of 110 mm lies in the range of 80–120 mm. Fundamental deviation for hole F is $-5.5D^{0.41}$. The values of standard tolerances for the grades of IT8 and IT10 are $25i$ and $64i$, respectively. Determine the specified size of hole and shaft.

Solution. Given that $D_1 = 80$ mm, $D_2 = 120$ mm, hence the mean diameter of the pair is calculated as

$$\begin{aligned} D &= \sqrt{D_1 D_2} \\ &= 97.979 \text{ mm} \end{aligned}$$

The tolerance limit i is determined as

$$\begin{aligned} i &= 0.45\sqrt[3]{D} + 0.001D \\ &= 2.1725\ \mu\text{m} \\ &= 0.00217 \text{ mm} \end{aligned}$$

Fundamental deviation for hole F is

$$\begin{aligned} \delta_F &= -5.5D^{0.41}\ \mu\text{m} \\ &= 0.036 \text{ mm} \end{aligned}$$

Tolerance for IT8 is

$$\begin{aligned} \text{IT8} &= 25i \\ &= 0.05425 \text{ mm} \end{aligned}$$

Therefore, the size of hole is

$$110^{+0.036+0.0543}_{+0.036} = 110^{+0.0903}_{+0.036} \text{ mm}$$

The shaft h is the basic shaft for which the upper deviation is zero. The tolerance of IT10 is

$$\begin{aligned} \text{IT10} &= 64i \\ &= 0.139 \text{ mm} \end{aligned}$$

Therefore, the size of hole is

$$110^{+0.000}_{-0.139} \text{ mm}$$

3. GO and NO GO snap gauges are to be designed for a shaft $24.000^{+0.050}_{+0.020}$ mm. Gauge tolerances can be taken as 10% of the hole tolerance. Following the ISO system of gauge design, determine the sizes of GO and NO-GO gauges.

Solution. Tolerance is

$$0.050 - 0.020 = 0.030 \text{ mm}$$

10% of the tolerance is 0.003 mm. Therefore, the size of GO gauge is

$$24.000 + 0.020 + 0.003 = 24.023 \text{ mm}$$

Similarly, the size of NO GO gauge is

$$24.000 + 0.060 - 0.003 = 24.057 \text{ mm}$$

4. A cutting tool has a radius of 2.0 mm. Determine the feed rate for achieving a theoretical surface roughness of $R_a = 6\ \mu$m.

Solution. The surface finish expressed as arithmetic mean roughness $R_a = 6$ mm can be taken to the half of the peak to valley depth h, which is related to feed rate (f) and nose radius $r = 2$ mm in turning operation as

$$\begin{aligned} h &= \frac{f^2}{8r} \\ f &= \sqrt{8hr} \\ &= \sqrt{4R_a r} \\ &= 6.92 \text{ mm} \end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. When a cylinder is located in a V-block, the number of degrees of freedom which area arrested is

(a) 2 (b) 4
(c) 7 (d) 8

(GATE 2003)

Solution. The cylinder placed on V-block can move longitudinally and rotate, therefore, out of six degrees of freedom, two are not arrested. The total number of arrested degrees of freedom is $6-2=4$.

Ans. (c)

2. A threaded nut of M16, ISO metric type, having 2 mm pitch with a pitch diameter of 14.701 mm is to be checked for its pitch diameter using two or three numbers of balls or rollers of the following sizes:

(a) Rollers of 2 mm ϕ
(b) Rollers of 1.155 mm ϕ
(c) Balls of 2 mm ϕ
(d) Balls of 1.155 mm ϕ

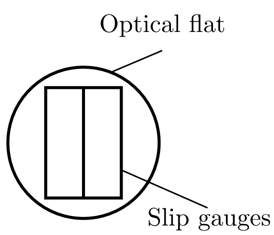
(GATE 2003)

Solution. Given that pitch of the nut, $p = 2$ mm. The pitch diameter can be measured by using three wire method, for which the approximate value of diameter of the wire (i.e. equivalent to roller) in case of ISO metric thread is given by

$$\begin{aligned} d &= 0.57735p \\ &= 1.1547 \text{ mm} \end{aligned}$$

Ans. (d)

3. Two slip gauges of 10 mm width measuring 1.000 mm and 1.002 mm are kept side by side in contact with each other lengthwise. An optical flat is kept resting on the slip gauges as shown in the figure.



Monochromatic light of wavelength 0.0058928 mm is used in the inspection. The total number of straight fringes that can be observed on both slip gauges is

(a) 2 (b) 6
(c) 8 (d) 13

(GATE 2003)

Solution. An optical flat is an optical-grade piece of glass lapped and polished to be extremely flat on one or both sides. They are used with a monochromatic light to determine the flatness of other optical surfaces by interference. A surface polished to a flatness of $d = \lambda/4$ will show straight fringes when tested with a $\lambda/4$ flat. Given that

$$\begin{aligned} \lambda &= 0.0058928 \text{ mm} \\ \Delta\lambda &= 1.002 - 1.000 = 0.002 \text{ mm} \end{aligned}$$

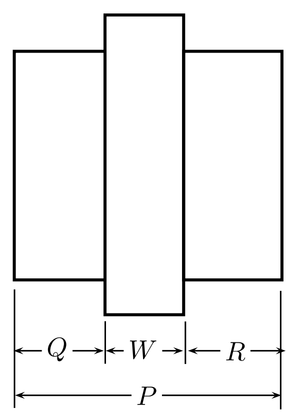
The number of fringes is given by

$$\begin{aligned} n &= 2\frac{\lambda}{\Delta\lambda} \\ &= 5.89 \approx 6 \end{aligned}$$

Ans. (b)

4. A part shown in the figure is machined to the sizes given below

$$\begin{aligned} P &= 35.00 \pm 0.08 \text{ mm} \\ Q &= 12.00 \pm 0.02 \text{ mm} \\ R &= 13.00^{+0.04}_{-0.02} \text{ mm} \end{aligned}$$



With 100% confidence, the resultant dimension W will have the specification

(a) 9.99 ± 0.03 mm (b) 9.99 ± 0.13 mm
(c) 10.00 ± 0.03 mm (d) 10.00 ± 0.13 mm

(GATE 2003)

Solution. It is observed that

$$W = P - Q - R$$

The above equation is applied to all dimensions, or one can simply use the maximum and minimum dimensions of P, Q, R to get the maximum and minimum dimensions of W in the following equations

$$W_{max} = P_{max} - Q_{min} - R_{min}$$
$$W_{min} = P_{min} - Q_{max} - R_{max}$$

Ans. (a)

5. The dimensional limits on a shaft of 25 $h7$ are

(a) 25.000, 25.021 mm
(b) 25.000, 24.979 mm
(c) 25.000, 25.007 mm
(d) 25.000, 24.993 mm

(GATE 2003)

Solution. Size 25 mm comes between 18 mm and 30 mm. Therefore, the mean diameter is

$$\begin{aligned} D &= \sqrt{D_1 \times D_2} \\ &= \sqrt{18 \times 30} \\ &= 23.2379 \text{ mm} \end{aligned}$$

Tolerance limit is

$$\begin{aligned} i &= 0.45\sqrt[3]{D} + 0.001D \ \mu\text{m} \\ &= 1.3074 \ \mu\text{m} \\ &= 0.0013074 \text{ mm} \end{aligned}$$

Tolerance for IT7 is $16i = 0.0209$. For basic shaft h, the upper deviation is zero. Therefore, dimension of the shaft will vary between 25.000 mm and $25.000 - 0.0246$ mm $= 24.979$ mm.

Ans. (b)

6. In an interchangeable assembly, shafts of size $25.000^{+0.040}_{-0.010}$ mm mate with the holes of size $25.000^{+0.020}_{-0.000}$ mm. The maximum possible clearance in the assembly will be

(a) 10 microns (b) 20 microns
(c) 30 microns (d) 60 microns

(GATE 2004)

Solution. Maximum clearance will be

$$\begin{aligned} \text{Maximum clearance} &= +0.020 + 0.010 \\ &= 0.030 \text{ mm} \\ &= 30 \text{ microns} \end{aligned}$$

Ans. (c)

7. GO and NO GO plug gauges are to be designed for a hole $20.000^{+0.050}_{+0.010}$ mm. Gauge tolerances can be taken as 10% of the hole tolerance. Following ISO system of gauge design, sizes of GO and NO GO gauge will be, respectively,

(a) 20.010 mm and 20.050 mm
(b) 20.014 mm and 20.046 mm
(c) 20.006 mm and 20.054 mm
(d) 20.014 mm and 20.054 mm

(GATE 2004)

Solution. Tolerance is $+0.050 - 0.010 = 0.040$ mm. 10% of it is 0.004 mm. Therefore, Size of GO gauge is $20.000 + 0.010 + 0.004 = 20.014$ mm. Similarly, size of NO GO gauge is $20.000 + 0.050 - 0.004 = 20.046$ mm.

Ans. (b)

8. Match the following columns and select the correct option:

Column-I (Feature)		Column-II (Instrument)	
P.	Pitch and angle errors of screw thread	1.	Autocollimator
Q.	Flatness error of a surface plate	2.	Optical interferometer
R.	Alignment error of a machine slideway	3.	Dividing head and dial gauge
S.	Profile of a cam	4.	Spirit level
		5.	Sine bar
		6.	Tool maker's microscope

(a) P-6, Q-2, R-4, S-6
(b) P-5, Q-2, R-1, S-6
(c) P-6, Q-4, R-1, S-3
(d) P-1, Q-4, R-4, S-2

(GATE 2004)

Solution. Sine bar is used for checking angles, typically used with gauge blocks. It is a precision bar that has been hardened and then ground and lapped to very precise dimensions. Optical interferometer is an instrument for making precise measurements for beams of light of factors, such as length, surface irregularities, and index of refraction. An autocollimator is an optical instrument for non-contact measurement of angles. Tool maker's microscope is an ideal measuring instrument for simplifying inspection and precision measurement of diameter, forming tools, gauges as well as template checking of thread and angles.

Ans. (b)

9. In order to have an interference fit, it is essential that the lower limit of the shaft should be

(a) greater than the upper limit of the hole
(b) lesser than the upper limit of the hole
(c) greater than the lower limit of the hole
(d) lesser than the lower limit of the hole

(GATE 2005)

Solution. Interference fit is the one having limits of size so prescribed that either a clearance or an interference may result when mating parts are assembled. For interference fit, it is essential that the lower limit of the shaft be greater than the upper limit of the hole.

Ans. (a)

10. A ring gauge is used to measure

(a) outside diameter but not roundness
(b) roundness but not outside diameter
(c) both outside diameter and roundness
(d) only external threads

(GATE 2006)

Solution. A ring gauge is a cylindrical ring of steel whose inside diameter is finished to gauge tolerance and is used for checking the external diameter of a cylindrical object. Roundness can be checked only.

Ans. (a)

11. A hole is specified as $40^{0.050}_{0.000}$ mm. The mating shaft has a clearance fit with minimum clearance of 0.01 mm. The tolerance on the shaft is 0.04 mm. The maximum clearance in mm between the hole and the shaft is

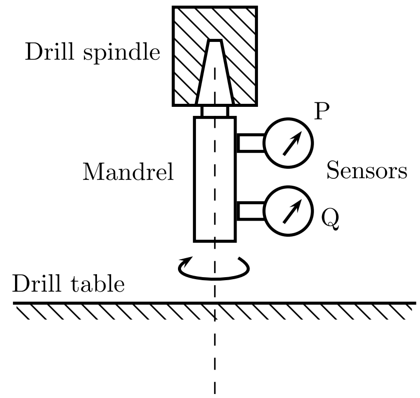
(a) 0.04 (b) 0.05
(c) 0.10 (d) 0.11

(GATE 2007)

Solution. The hole size is 40.000 to 40.050 mm. For minimum clearance of 0.01 mm, maximum shaft size is $40 - 0.01 = 39.99$ mm (on minimum hole size). For tolerance of 0.04 mm on the shaft, the minimum size of the shaft is $39.99 - 0.04 = 39.95$ mm. Therefore, maximum clearance on this system is $40.050 - 39.95 = 0.10$ mm.

Ans. (c)

12. A displacement sensor (a dial indicator) measures the lateral displacement of a mandrel mounted on the taper hole inside a drill spindle. The mandrel axis is an extension of the drill spindle taper hole axis and the protruding portion of the mandrel surface is perfectly cylindrical. Measurements are taken with the sensor placed at two positions P and Q, as shown in the figure.



The readings are recorded as R_x = maximum deflection minus minimum deflection, corresponding to sensor position at X, over one rotation. If $R_p = R_Q > 0$, which one of the following would be consistent with the observation?

(a) The drill spindle rotational axis is coincident with the drill spindle taper hole axis
(b) The drill spindle rotational axis intersects the drill spindle taper hole axis at point P
(c) The drill spindle rotational axis is parallel to the drill spindle taper hole axis
(d) The drill spindle rotational axis intersects the drill spindle taper hole at point Q

(GATE 2008)

Solution. The same readings of the two sensors indicates that the the drill spindle rotational axis is parallel to the drill spindle taper hole axis. The axes are not coincident because of the definition of the reading of the sensors.

Ans. (c)

13. What are the upper and lower limits of the shaft represented by 60f_8. Using the following data: Diameter 60 lies in the diameter step of 50–80 mm. Fundamental tolerance unit i in μm$= 0.45\sqrt[3]{D} + 0.001D$, where D is the representative size in mm. Tolerance value for IT8$= 25i$. Fundamental deviation for f shaft$= -5.5D^{0.41}$.

(a) Lower limit = 59.924 mm, upper limit = 59.970 mm
(b) Lower limit = 59.954 mm, upper limit = 60.000 mm
(c) Lower limit = 59.970 mm, upper limit = 60.016 mm
(d) Lower limit = 60.000 mm, upper limit = 60.046 mm

(GATE 2009)

Solution. From the given data, mean diameter is calculated as

$$D = \sqrt{50\times 80} \\ = 63.245 \text{ mm}$$

The tolerance unit is calculated as

$$i = 0.45\sqrt[3]{D} + 0.001D \\ = 1.856\ \mu\text{m}$$

Tolerance value for IT8 is $25i = 0.046403$ mm. Fundamental deviation is

$$\delta = -5.5\times D^{0.41} \\ = 030.115 \text{ mm (absolute)}$$

Thus

$$\begin{aligned}\text{Upper limit} &= 60 - 030.115 \\ &= 59.969 \text{ mm} \\ \text{Lower limit} &= 59.969 - 0.046403 \\ &= 59.924\end{aligned}$$

Ans. (a)

14. A shaft has a dimension, $\phi 35^{0.009}_{-0.025}$. The respective values of fundamental deviation and tolerance are

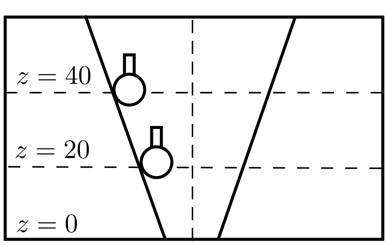
(a) 0.025, ±0.008 (b) 0.025, 0.016
(c) 0.009, ±0.008 (d) 0.009, 0.016

(GATE 2010)

Solution. The fundamental deviation is either the upper or the lower deviation, which is nearest to the zero line. Tolerance is the maximum possible deviation in the dimension. Therefore, for the given case, fundamental deviation is 0.009 and tolerance is $0.025 - 0.009 = 0.016$.

Ans. (d)

15. A taper hole is inspected using a coordinate measuring machine (CMM) with a probe of 2 mm diameter. At a height, $z = 10$ mm from the bottom, 5 points are touched and a diameter of circle (not compensated for probe size) is obtained as 20 mm. similarly, a 40 mm diameter is obtained at a height $z = 40$ mm.



The smaller diameter (in mm) of hole at $z = 0$ is

(a) 13.334 (b) 15.334
(c) 15.442 (d) 15.542

(GATE 2010)

Solution. Let the bottom is at height z_0 from the apex point downside. Therefore, for given measurements

$$\frac{40/2}{40+z_0} = \frac{20/2}{10+z_0} \\ z_0 = 20$$

Similarly, if d_0 is diameter at bottom then

$$\frac{d_0/2}{z_0} = \frac{20/2}{10+z_0} \\ d_0 = 13.3333$$

Ans. (a)

16. A hole has dimension $\phi 9^{+0.015}_{0.0}$ mm. The corresponding shaft is of dimension $\phi 9^{+0.010}_{+0.001}$ mm. The resulting assembly has

(a) loose running fit (b) close running fit
(c) transition fit (d) interference fit

(GATE 2011)

Solution. The shaft size is $9.001 - 9.010$, and hole size is $9.000 - 9.015$. Therefore, it is neither tolerance fit nor interference fit, so it is transition fit.

Ans. (c)

17. In an interchangeable assembly, shafts of size $25.000^{+0.040}_{-0.010}$ mate with holes of size $25.000^{+0.030}_{+0.020}$ mm. The maximum interference (in microns) in the assembly is

(a) 40 (b) 30
(c) 20 (d) 10

(GATE 2012)

Solution. Maximum interference is difference between maximum size of shaft and minimum size of hole, therefore, equal to

$$25.040 - 25.020 = 0.020 \text{ mm} \\ = 20.0\ \mu\text{m}$$

Ans. (c)

18. Cylindrical pins of $25^{+0.020}_{+0.010}$ mm diameter are electroplated in a shop. Thickness of the plating is $30^{+2.0}$ micron. Neglecting gauge tolerances, the size of the GO gauge in mm to inspect the plated components is

(a) 25.042 (b) 25.052
(c) 25.074 (d) 25.084

(GATE 2013)

Solution. The size of the pin after plating can be

$$25^{+0.020}_{+0.010} + 0.030^{+0.002} = 25.03^{+0.022}_{+0.012}$$

The maximum size of the pin for GO gauge is

$$25.03 + 0.022 = 25.052 \text{ mm}$$

Ans. (b)

19. A metric thread of pitch 2 mm and thread angle 60° is inspected for its pitch diameter using 3-wire method. The diameter of the best size wire in mm is

(a) 0.866 (b) 1.000
(c) 1.154 (d) 2.000

(GATE 2013)

Solution. Given that pitch of the thread

$$p = 2 \text{ mm}$$

For 60° thread angle, the best wire size is calculated as

$$\begin{aligned} d &= 0.57135 \times p \\ &= 0.57135 \times 2 \\ &= 1.154 \end{aligned}$$

Ans. (c)

MULTIPLE CHOICE QUESTIONS

1. In an interchangeable assembly, the holes of size $25.000^{+0.040}_{0.000}$ mm mate with shafts of size $25.000^{+0.020}_{0.000}$ mm. The maximum possible clearance in the assembly will be

(a) 0.010 mm (b) 0.020 mm
(c) 0.030 mm (d) 0.040 mm

2. In order to have a clearance fit, it is essential that the upper limit of the shaft should be

(a) greater than the upper limit of the hole
(b) lesser than the upper limit of the hole
(c) greater than the lower limit of the hole
(d) lesser than the lower limit of the hole

3. A shaft is specified as $50^{0.050}_{0.000}$ mm. The mating hole has a clearance fit with minimum clearance of 0.01 mm. The tolerance on the hole is 0.04 mm. The maximum clearance in mm between the hole and the shaft is

(a) 0.05 mm (b) 0.01 mm
(c) 0.50 mm (d) 0.10 mm

4. A shaft has dimension $\phi 10^{+0.010}_{0.02}$ mm. The corresponding hole is of dimension $\phi 10^{+0.050}_{+0.020}$ mm. The resulting assembly has

(a) loose running fit
(b) close running fit
(c) transition fit
(d) interference fit

5. Which of the following instruments can be used for most precision measurement of the thickness of a thin sheet?

(a) Linear scale (b) Vernier caliper
(c) Micrometer (d) Combination set

6. A metric vernier caliper is having 25 divisions on vernier scale which matches with 24 divisions of main scale. If one main scale division is 0.5 mm, the least count of the vernier caliper is

(a) 0.02 mm (b) 0.01 mm
(c) 0.005 mm (d) 0.001 mm

7. V-block is used for a job in workshops to check its

(a) straightness (b) taper
(c) height (d) roundness

8. A ratchet screw in micrometer is provided to

(a) lock the reading measured
(b) maintain constant pressure on the job
(c) prevent wearing of screw threads
(d) allow zero adjustment

9. Which of the following measuring instruments is most accurate?

(a) Vernier caliper (b) Slip gauges
(c) Sine bar (d) Autocollimator

10. Which of the following is not provided on combination set?

(a) Square head (b) Center head
(c) Bevel protractor (d) Vernier caliper

11. A universal surface gauge is commonly employed for

(a) scribing parallel lines at desired heights from the plane surface
(b) comparing the correctness of two similar heights
(c) locating centers of round rod held in V-block
(d) all of the above

12. The standard length of a sine bar is measured between

(a) outer edges of the bar
(b) inner edges of the bar
(c) outer circumferences of the rollers
(d) centers of the rollers

13. The surface plates are usually made of gray cast iron due to its

(a) high degree of flatness and rust free surface
(b) least tendency to wrap
(c) free from residual stresses
(d) lubrication by graphite flakes

14. The main advantage of vernier caliper over the micrometer is that

(a) it can be easily used and provides for quick measurements
(b) it is more accurate
(c) it is more rigid
(d) it can be used for both external and internal measurements

15. A ring gauge is used to measure

(a) outside diameter but not roundness
(b) roundness but not outside diameter
(c) both outside diameter and roundness
(d) only external threads

16. Plug gauges are used to check

(a) accuracy of holes
(b) diameter of solid shafts
(c) length of holes
(d) all of the above

17. Snap gauges are used for checking

(a) internal diameter of hollow shafts
(b) diameters of holes
(c) external diameter of shafts
(d) all of the above

18. Which one of the following processing sequences will give the best accuracy as well as surface finish?

(a) Drilling, reaming, grinding
(b) Drilling, boring, grinding
(c) Drilling, reaming, lapping
(d) Drilling, reaming, electroplating

19. Drilled holes and hone holes, could be designated in the following grades:

(a) $H8$, $H12$ (b) $H6$, $H11$
(c) $H6$, $H8$ (d) $T8$, $H6$

20. A universal precision gauge, also called planar gauge, is used for

(a) setting planar and shaper cutting tools to establish the correct depth of the cut
(b) measurement of parallel faces in a slot
(c) height gauge using a dial test indicator
(d) all of the above

21. The symmetrical spacing of the airy points of a bar of length L is

(a) $L/\sqrt{2}$ (b) $L/\sqrt{3}$
(c) $L\sqrt{3}$ (d) $L\sqrt{2}$

22. Angular measurements can be done using

(a) protractors (b) sine bar
(c) combination set (d) all of the above

23. Which of the following cannot be used for angular measurements?

(a) Angle plate (b) Sine bar
(c) Bevel protractor (d) Combination set

24. The process of joining two slip gauges for precision measurement is known as

(a) wringing (b) sliding
(c) slipping (d) adhesion

25. Feeler gauges are mostly used in engineering to measure

(a) the clearance between two parts
(b) radius of circular plates

(c) surface roughness
(d) thread dimensions

26. Which device is mainly used for locating the center of round bars held in V-block by drawing straight lines and by tilting the job through different angle?

(a) Surface gauge
(b) Surface plate
(c) Angle plate
(d) Center plate

27. Ideal surface roughness, as measured by the maximum height of unevenness, is best achieved when the material is removed by

(a) an end mill
(b) a grinding wheel
(c) a tool with zero nose radius
(d) a ball mill

28. A cutting tool has a radius of 1.8 mm. The feed rate for a theoretical surface roughness of $R_a = 5$ μm is

(a) 0.36 mm/rev (b) 0.189 mm/rev
(c) 0.036 mm/rev (d) 0.0189 mm/rev

29. In turning operation, the feed could be doubled to increase the metal removal rate. To keep the same level of surface finish, the nose radius of the tool should be

(a) halved (b) kept unchanged
(c) doubled (d) made four times

30. The value of surface roughness h obtained during the turning at a feed f with a round nose tool having radius r is given as

(a) $f/(8r)$ (b) $f^2/(8r)$
(c) $f^3/(8r)$ (d) $f^3/\left(8r^2\right)$

31. Fixed angle plate is used for

(a) measurement of angles
(b) holding and supporting the job at right angle
(c) locating the center of round bars
(d) none of the above

32. Tolerance in machined components is essential to facilitate

(a) interchangeability
(b) manufacturing and inspection
(c) functional requirement
(d) all of the above

33. In a system of limits and fit specification, the fundamental tolerances indicate the

(a) fundamental deviation of the part
(b) minimum size permitted for the part
(c) maximum size permitted for part
(d) degree of accuracy of manufacturing

34. Allowance in limits and fits refers to

(a) maximum clearance between shaft and hole
(b) minimum clearance between shaft and hole
(c) difference between maximum and minimum size of hole
(d) difference between maximum and minimum size of shaft

35. For the basic shaft h, the

(a) upper deviation is zero
(b) lower deviation is zero
(c) both upper and lower deviations are zero
(d) cannot be determined

36. The fit indicates

(a) degree of tightness or looseness between mating members
(b) minimum clearance space between mating parts
(c) maximum clearance space between mating parts
(d) tolerances in the mating parts

37. Tolerances are specified in manufacturing parts

(a) to obtain desired fit
(b) to allow inexactness of dimensions in manufacturing
(c) to achieve proper allowances
(d) none of the above

38. Expressing a dimension as $20.5^{+0.00}_{-0.01}$ mm is the case of

(a) unilateral tolerance
(b) bilateral tolerance
(c) limiting dimensions
(d) all of the above

39. The system of limits and fits IS:919-1963 specifies the following number of fundamental tolerances and fundamental deviations, respectively,

(a) 18, 22 (b) 22, 18
(c) 25, 18 (d) 18, 25

40. The standard tolerance unit i is related to mean diameter of the fit D (mm) as

(a) $0.45\sqrt{D}+0.001D$ mm
(b) $0.45\sqrt[3]{D}+0.001D$ mm
(c) $0.45\sqrt{D}+0.01D$ mm
(d) $0.45\sqrt[3]{D}+0.01D$ mm

41. For tolerance grade IT7, the value of tolerance is related to standard tolerance unit i as

(a) $7i$ (b) $16i$
(c) $21i$ (d) $40i$

42. For tolerance grade IT9, the value of tolerance is related to standard tolerance unit i as

(a) $7i$ (b) $16i$
(c) $21i$ (d) $40i$

43. Optical micrometer is used to measure

(a) small angular displacements
(b) small linear displacements
(c) surface roughness
(d) surface profiles

44. In a sine bar, the standard length is measured

(a) from edge to edge
(b) between inner circumference of two rollers
(c) between outer circumference between two rollers
(d) between centers of the two rollers

45. A bevel protractor is used for

(a) angular displacements
(b) linear displacements
(c) flatness measurements
(d) surface roughness

46. A collimator is a

(a) source of a bundle of parallel light rays
(b) source of point light
(c) sort of alignment telescope
(d) device used in interferometric measurements

47. The specification for size 25 mm for hole H with tolerance grade IT8 and shaft f with tolerance grade IT7 is written as

(a) $H8f7\times25$ (b) $25f7H8\times25$
(c) $25H8f7$ (d) $25f7H8$

48. A fit is specified as 25 $H8e8$. The tolerance value for a nominal diameter of 25 mm in IT8 is 33 microns and fundamental deviation for the shaft is 40 microns. The maximum clearance of the fit in microns is

(a) -7 (b) 7
(c) 73 (d) 106

49. In the specification of dimensions and fits,

(a) allowance is equal to bilateral tolerance
(b) allowance is equal to unilateral tolerance
(c) allowance is independent of tolerance
(d) allowance is equal to the difference between maximum and minimum dimension specified by the tolerance.

50. Autocollimator is used to check

(a) roughness
(b) flatness
(c) angle
(d) automobile balance

51. Balls of diameter 30 mm and 15 mm were use to measure the taper of a ring gauge. During inspection, the ball of 30 mm diameter was protruding by 2.5 mm above the top surface of the ring. This surface was located at a height of 50 mm from the top of the 15 mm diameter ball. What is the taper angle?

(a) 19.2° (b) 16.42°
(c) 8.21° (d) 9.6°

52. The fit on a hole-shaft system is specified as $H7s6$. The type of fit is

(a) clearance fit
(b) running fit (sliding fit)
(c) push fit (transition fit)
(d) force fit (interference fit)

53. In the tolerance specification 25 $D6$, the letter D represents

(a) grade of tolerance
(b) upper deviation

(c) lower deviation
(d) type of fit

54. A shaft (diameter $20^{+0.05}_{-0.15}$ mm) and a hole (diameter $20^{+0.20}_{+0.1}$), when assembled would yield

(a) transition fit
(b) interference fit
(c) clearance fit
(d) none of these

55. Vernier caliper is a tool for measuring

(a) small angular tilts
(b) linear dimensions
(c) both (a) and (b)
(d) none of the above

56. Select the wrong statement about tools of linear measurements

(a) The least count of vernier caliper is determined by dividing the smallest division on the main scale by the total number of divisions on the vernier scale.
(b) The least count of micrometer is determined by dividing the pitch of the screw by total number of divisions on the thimble.
(c) both (a) and (b)
(d) none of the above

57. Thread micrometer is used to measure

(a) pitch
(b) root
(c) outside diameter
(d) all of the above

58. Select the correct statement about comparators.

(a) Comparators cannot be used as an absolute measuring device.
(b) It is possible to use dial indicator as a comparator.
(c) Both (a) and (b)
(d) None of the above

NUMERICAL ANSWER QUESTIONS

1. Determine dimensional limits on a hole of $28H7$.

2. A hole is specified as $\phi\ 45^{0.040}_{0.000}$ mm. The mating shaft has a clearance fit with minimum clearance of 0.02 mm. The tolerance on the shaft is 0.05 mm. Determine the maximum clearance in mm between the hole and the shaft.

3. A shaft and hole pair is designated as $95h7F9$. The diameter of 95 mm lies in the range of 80–120 mm. Fundamental deviation for hole F is $-5.5D^{0.41}$. The values of standard tolerances for the grades of IT7 and IT9 are $16i$ and $40i$, respectively. Determine the specified size of hole and shaft.

4. GO and NO GO plug gauges are to be designed for a hole $25.000^{+0.040}_{-0.010}$ mm. Gauge tolerances can be taken as 10% of the hole tolerance. Following ISO system of gauge design, determine the sizes of GO and NO GO gauges.

ANSWERS

Multiple Choice Questions

1. (d)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (d)	**10.** (d)
11. (d)	**12.** (d)	**13.** (d)	**14.** (d)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (c)	**19.** (d)	**20.** (d)
21. (b)	**22.** (d)	**23.** (a)	**24.** (a)	**25.** (a)	**26.** (a)	**27.** (c)	**28.** (b)	**29.** (d)	**30.** (b)
31. (b)	**32.** (d)	**33.** (d)	**34.** (b)	**35.** (a)	**36.** (a)	**37.** (b)	**38.** (a)	**39.** (d)	**40.** (b)
41. (d)	**42.** (d)	**43.** (c)	**44.** (d)	**45.** (a)	**46.** (d)	**47.** (c)	**48.** (d)	**49.** (d)	**50.** (c)
51. (a)	**52.** (d)	**53.** (a)	**54.** (c)	**55.** (b)	**56.** (c)	**57.** (d)	**58.** (c)		

Numerical Answer Questions

1. 28.000, 25.0209 mm **2.** 0.10 **3.** $95^{+0.0707}_{+0.036}$, $95^{+0.000}_{-0.0868}$
4. 24.995 mm, 25.040 mm

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (d) Maximum dimension of hole is 25.040 mm, whereas minimum dimension of the shaft is 25.000 mm, therefore, the maximum clearance will be 0.040 mm.

2. (b) Clearance fit is the one having limits of size so prescribed that a clearance always results when mating parts are assembled. A clearance fit has positive allowance, that is, there is minimum positive clearance between high limit of the shaft and low limit of the hole.

3. (a) The shaft size is 50.000 to 50.050 mm. For minimum clearance of 0.01 mm, minimum hole size is $50.000 + 0.01 = 50.01$ mm (on minimum shaft size). For tolerance of 0.04 mm on the hole, the maximum hole size is $50.01 + 0.04 = 50.05$ mm. Therefore, the maximum clearance on this system is $50.05 - 50.000 = 0.05$ mm.

4. (a) The upper limit of the shaft 10.010 mm is less than the lower limit of the shaft 10.020 mm, hence, there is a clearance of 0.010 mm, which would result into loose running fit.

5. (c) Out of the given options, the least count of micrometer is the least.

6. (a) The least count of vernier caliper shall be $0.5/25 = 0.02$ mm

7. (d) V-block is used to check roundness of a job.

8. (b) A ratchet screw in micrometer is provided to maintain constant pressure on the job.

9. (d) Small angular tilts of a reflecting surface can be easily measured by an optical instrument called autocollimator. Out of the given options, autocollimator is the most accurate.

10. (d) A combination set is a combination of a square head, center head, bevel protractor, spirit level and graduated rule.

11. (d) A universal surface gauge is an improved version of the surface gauge or scribing block. It is commonly employed for scribing parallel lines at desired heights from the plane surface, comparing the correctness of two similar heights, locating centers of round rod held in V-block, etc.

12. (d) Center-to-center distance between the rollers is used for calculation of angles.

13. (d) The surface plates are usually made of gray cast iron due to its lubrication due by graphite flakes

14. (d) Vernier calipers can be used for both external and internal measurements.

15. (a) A ring gauge is a cylindrical ring of steel whose inside diameter is finished to gauge tolerance and is used for checking the external diameter of a cylindrical object. Only roundness can be checked.

16. (a) Plug gauges are used to check the accuracy of holes.

17. (c) Snap gauges are used for checking external diameter of shafts.

18. (c) The processes carried out on a job in sequence of drilling, reaming, lapping will give the best accuracy and surface finish.

19. (d) The precision of drilled holes is very low as compared to hone holes. Hence, their lower deviation cannot be zero. Further, the fundamental deviation of drilled holes should also be more than that of hone holes.

20. (d) Originally, planar gauges were used for setting planar and shaper cutting tools to establish the correct depth of cut. In addition to this, a universal precision gauge can also be used for measurement of parallel faces in a slot as an adjustable parallel, for setting up and leveling of workpieces, and as height gauge using a dial test indicator.

21. (b) Airy points are used for precision measurement (metrology) to support a length standard in such a

way as to minimize bending or droop. The points are symmetrically arranged around the center of the length standard and are separated by a distance equal to $L/\sqrt{3}$.

22. (d) Angular measurements can be done using protractors, sine bar, or combination set.

23. (a) Fixed angle plate is used for holding and supporting the job at right angle, while an adjustable angle plate is used for clamping and holding the job at an adjustable angle. Thus, angle plate cannot be used for angular measurement.

24. (a) The gauges are wrung together by bringing them into contact with each other at right angles and then pressing them with a twisting motion, and simultaneously turning them parallel. If gauges are in a good condition, wringing will take place easily.

25. (a) Feeler gauges are mostly used in engineering to measure the clearance between two parts.

26. (a) Surface gauge is mainly used for locating the center of round bars held in V-block by drawing straight lines and by tilting the job through different angle.

27. (c) A tool with zero nose radius gives surface roughness

$$h_{max} = \frac{f}{\tan\psi + \cot\phi}$$

while with nose radius

$$h_{max} = \frac{f^2}{8r}$$

28. (b) Given that

$$\begin{aligned} R_a &= 0.005 \\ r &= 1.8 \end{aligned}$$

The surface finish expressed as arithmetic mean roughness $R_a = 0.005$ mm can be assumed to the half of the peak-to-valley depth h which is related to feed rate (f) and nose radius $r = 1.8$ mm in turning operation as

$$\begin{aligned} h &= \frac{f^2}{8r} \\ f &= \sqrt{8hr} \\ &= \sqrt{4R_a r} \\ &= 0.189 \text{ mm/rev} \end{aligned}$$

29. (d) Surface finish in turning operation is related to feed f and nose radius r as

$$h_f = \frac{f^2}{8r}$$

To keep the same level of surface finish,

$$r \propto f^2$$

Therefore, the nose radius should be made four times.

30. (b) Surface roughness in turning with feed f and tool nose radius r is given by

$$h = \frac{f^2}{8r}$$

31. (b) Fixed angle plate is used for holding and supporting the job at right angle, while an adjustable angle plate is used for clamping and holding the job at an adjustable angle.

32. (d) Manufacturing tolerance is essential to facilitate interchangeability of parts and assembly and will not interfere with the proper functioning of the assembled parts.

33. (d) The systems of limits and fits is recommended in IS:919-1963 of fundamental tolerances, which indicate the degree of accuracy of the manufacturing.

34. (b) The allowance is the minimum clearance space between mating parts intentionally provided to secure the desired fit.

35. (a) The shaft h for which the upper deviation is zero is called the basic shaft.

36. (a) The degree of tightness or looseness between mating members is known as the fit of the members.

37. (b) It is impossible to manufacture a product to a specified dimension exactly, hence, manufacturing parts are specified with tolerances on the basic dimensions.

38. (a) As upper deviation in the dimension is zero, hence, it is the case of unilateral tolerance.

39. (d) The systems of limits and fits recommended in IS:919-1963 suggests 18 grades (IT01, IT0, IT1 to IT16) of fundamental tolerances and 25 types of fundamental deviations denoted by alphabets both for holes (A to ZC) and shafts (a to zc) in diametral steps up to 500 mm.

40. (b) Each grade has an associated value or formula for calculation of standard tolerance in terms of mean diameter D given by

$$D = \sqrt{D_1 D_2}$$

and i defined as

$$i = 0.45\sqrt[3]{D} + 0.001D \text{ mm}$$

41. (d) For tolerance grade IT7, the value of tolerance is $16i$, where i is the standard tolerance unit.

42. (d) For tolerance grade IT9, the value of tolerance is $40i$, where i is the standard tolerance unit.

43. (c) Optical micrometer is used to measure surface roughness.

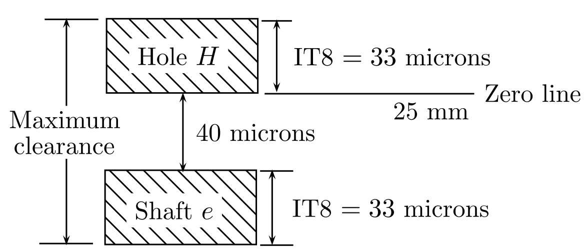
44. (d) In a sine bar, the standard length is measured between centers of the two rollers

45. (a) A bevel protractor is used for angular displacements. It is provided with one or two swinging arms, which can be used to help measure the angle.

46. (d) A collimator is simply a device used in interferometric measurements.

47. (c) A fit is indicated by combining the designations for both the hole and shaft with the hole designation written first, regardless of the system, for example, $25H8f7$.

48. (d) The specification is described in the figure below



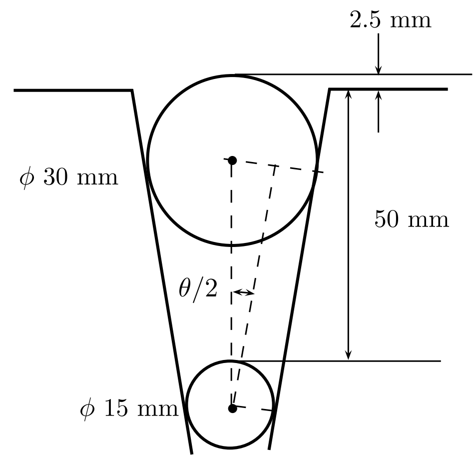
Therefore,

$$\text{Maximum clearance} = 33 + 40 + 33$$
$$= 106 \text{ microns}$$

49. (d) Tolerance is the amount of deviation of the actual dimension from the nominal dimension is called the tolerance. The allowance is the minimum clearance space between mating parts intentionally provided to secure the desired fit.

50. (c) An autocollimator is used to measure small angular tilts of a reflecting surface placed in front of the objective lens of the autocollimator.

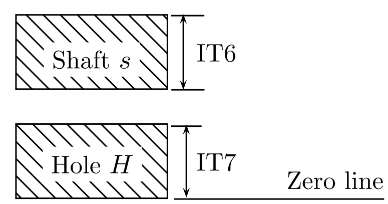
51. (a) The measurements are shown in the figure.



The taper angle is determined as

$$\theta = 2\sin^{-1}\left(\frac{(30-15)/2}{(2.5+50)-(30-15)/2}\right)$$
$$= 19.188^\circ$$

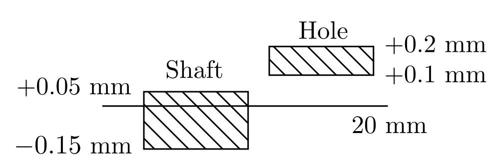
52. (d) In the specification $H7s6$, H and s indicate the fundamental deviation of hole and shaft, respectively, and grades of tolerances IT7 and IT6, respectively. The hole H is the basic hole for which lower deviation is zero. As per the standards, h is the basic shaft for which upper deviation is zero, and shafts alphabets above h are above zero line, as demonstrated in the figure.



Thus, the fit is interference fit.

53. (a) A 25 mm H-hole with tolerance IT8 is indicated as $25H8$, a 25 mm f-shaft with tolerance grade IT7 indicated as $25f7$. In the given case, D represents the grade of tolerance.

54. (c) The tolerances are shown in the figure.



Thus, this is clearance fit.

55. (b) Vernier caliper is a measuring tool used for linear measurements.

56. (c) Both the statements about calculation of least counts of vernier caliper and micrometer are correct.

57. (d) Thread micrometer is used to measure pitch, root and outside diameter.

58. (c) Comparators cannot be used as an absolute measuring device but can only compare two dimensions. It is possible to use dial indicator as a comparator by mounting the dial indicator on the stand at any suitable length.

Numerical Answer Questions

1. Size 28 mm comes between 18 mm and 30 mm. Therefore, mean diameter is

$$\begin{aligned} D &= \sqrt{D_1 \times D_2} \\ &= 23.2379 \text{ mm} \end{aligned}$$

Tolerance unit is

$$\begin{aligned} i &= 0.45\sqrt[3]{D} + 0.001D \ \mu\text{m} \\ &= 1.3073 \ \mu\text{m} \\ &= 0.0013073 \text{ mm} \end{aligned}$$

For basic hole H, the lower deviation is zero. Thus, the upper deviation IT7 $= 16i = 0.0209$ mm. Therefore dimension will vary between 28.000 mm and $25.000 + 0.0209 = 25.0209$ mm.

2. The hole size is 45.000—45.040 mm. For minimum clearance of 0.02 mm, maximum shaft size is $45 - 0.02 = 44.98$ mm (on minimum hole size). For tolerance of 0.05 mm on the shaft, the minimum size of the shaft is $44.98 - 0.05 = 44.93$ mm. Therefore, maximum clearance on this system is $45.040 - 44.93 = 0.11$ mm.

3. Given, $D_1 = 80$ mm, $D_2 = 120$ mm, hence the mean diameter of the pair is calculated as

$$\begin{aligned} D &= \sqrt{D_1 D_2} \\ &= 97.979 \end{aligned}$$

The tolerance limit i is determined as

$$\begin{aligned} i &= 0.45\sqrt[3]{D} + 0.001D \\ &= 2.1725 \ \mu\text{m} \\ &= 0.00217 \text{ mm} \end{aligned}$$

Fundamental deviation for hole F is

$$\begin{aligned} \delta_F &= -5.5D^{0.41} \ \mu\text{m} \\ &= 0.036 \text{ mm} \end{aligned}$$

Tolerance for IT7 is

$$\begin{aligned} \text{IT8} &= 16i \\ &= 0.03472 \end{aligned}$$

Therefore, the size of hole is

$$\begin{aligned} &= 95^{+0.036+0.03472}_{+0.036} \\ &= 95^{+0.0707}_{+0.036} \end{aligned}$$

The shaft h is the basic shaft for which the upper deviation is zero. The tolerance of IT9 is

$$\begin{aligned} &= 40i \\ &= 0.0868 \end{aligned}$$

Therefore, the size of hole is

$$= 95^{+0.000}_{-0.0868}$$

4. Tolerance is $+0.040 + 0.010 = 0.050$ mm. 10% of it is 0.005 mm. Therefore, Size of GO-gauge is $25.000 - 0.010 + 0.005 = 24.995$ mm. Similarly, size of NO GO gauge is $25.000 + 0.050 - 0.005 = 25.045$ mm.

CHAPTER 16

COMPUTER INTEGRATED MANUFACTURING

Computer integrated manufacturing (CIM) describes the integration of *computer aided design* (CAD) and *computer aided manufacturing* (CAM). In the early 1960s, Ivan Sutherland developed the SKETCHPAD system, a milestone of research achievement in computer graphics. The evolution of computer graphics has since resulted in the development of CAD. On the other hand, CAM was inspired by *numerical control* (NC) machines which were first introduced in the early 1950's. The communication between CAD and CAM systems became possible by reuse of the product model designed in CAD systems in CAM systems.

16.1 COMPUTER INTEGRATED MANUFACTURING

Computer integrated manufacturing (CIM) is a general term used to describe the computerized integration of the conventionally isolated functions of manufacturing, such as product design, planning, production, distribution, and management. It essentially needs large scale integrated communication system and extensive database. For this, functions of various elements of a manufacturing system are treated by subsystems such that the output of a subsystem serves as the input to another subsystem. Organizationally, the subsystems can be broadly grouped into two sets of functions:

1. *Business Planning* Forecasting, scheduling, material requirement planning, invoicing and accounting.

2. *Business Execution* Production and process control, material handling, testing and inspection.

Improved product quality and increased flexibility in use of capital are the two broad benefits of using CIM. Additionally, CIM offers the following benefits:

1. Responsiveness to short product life cycles and dynamics of global competition.

2. Process control resulting in consistent product quality and uniformity.

3. Control on production, scheduling, and management of the total manufacturing operations.

4. Improved productivity by optimum utilization of resources.

16.2 COMPUTER AIDED DESIGN

Computer aided design (CAD) systems describe software systems capable of creating, modifying, and analyzing

an engineering design. This involves computers to aid in the process of product design and development.

CAD originated from early *computer graphic systems*, and evolved with the development of interactive computer graphics and geometric modeling. The development of SKETCHPAD system at MIT in 1963 by Dr. Ivan Sutherland was the turning point in the development of CAD systems. SKETCHPAD was the first system that allowed a designer to interact with a computer graphically by drawing on a cathode ray tube (CRT) monitor with a light pen.

In the early 1970s CAD systems were little more than drafting software used to create 2D drawings, limited to simple geometry, such as lines, circular arcs and ellipse arcs. Therefore, they were often referred to as *computer aided drafting*. Advances in programing and computer hardware, notably, solid modeling in the 1970s, made the CAD applications more versatile. CAD further evolved with the development of *geometric modeling* based on the mathematical description of geometry, from simple two-dimensional (2D) drafting to three-dimensional (3D) wire frame, to 3D surfaces, and now 3D solid modeling. *Geometric modeling* enables creation of new geometric models from the inbuilt blocks available in the system, moving the images around on the screen, zooming in on a certain feature, and so on. CAD systems are now being extensively used throughout the engineering processes from conceptual design and detailed engineering, to strength, dynamic analysis of components and assembly planning.

Computer aided design is a faster and more accurate method of engineering design than the conventional methods. This has become possible through the following benefits offered by CAD systems:

1. Increased design productivity.
2. Increased available geometric forms.
3. Improved quality of the design.
4. Improved communication documentation.
5. Creation of manufacturing data base.
6. Design standardization.

16.3 COMPUTER AIDED MANUFACTURING

Computer aided manufacturing (CAM) describes use of the computers and computer technology to assist in all phases of manufacturing, including process and production planning scheduling, manufacture, quality control, and management. Historically, CAM technology was sparked by the invention of NC machine tools that were developed to manufacture complex shapes in an accurate and repeatable manner. NC machines are directed by part programs following industrial data standard, RS274D, known as ISO 6983, internationally. The standard defines a set of M and G codes which specify a sequence of cutting tool movements as well as the direction of rotation, speed of travel and various auxiliary functions, such as coolant flow.

The first generation of CAM emerged when Automatically Programed Tool (APT) was developed to help control NC machines at the Massachusetts Institute of Technology (MIT) in the 1950s. APT is a universal programing language for NC machines and has been widely adopted in industry. APT provides a convenient way to define geometry elements and generate cutter locations for NC programs by computers. APT was created before graphical interfaces were available, so it relies on text to specify the geometry and tool paths needed to machine a part. This poses a significant potential for errors in defining comprehensive geometries and tool positioning commands. This problem was overcome by introduction of graphics-based CAM in 1980s, allowing part geometry to be described in the form of points, lines, arcs, and so on, rather than requiring a translation to a text-oriented notation.

16.4 INTEGRATION OF CAD AND CAM SYSTEMS

In initial stages, the development of CAD systems had little effect on CAM development due to the different capabilities and file formats used by drawings and NC programs. The result was that a lot of CAM programing time was spent in redefining the part geometry, which had already been defined in CAD. The realization of this fact led to the appearance of the first integrated CAD/CAM systems by enabling CAM systems to work with the geometric model created by the CAD system itself. This was evident with decreased time to market dynamics, lower development and design cost and the ability to rapidly translate ideas into models.

Present day CAD/CAM systems, such as Unigraphics, Pro/E, IDEAS, CATIA, have many modules packed together and are running on their own proprietary databases. These systems have both CAD and CAM capabilities and the geometric data from CAD can be used in the CAM module without conversion on the same interface. This allows information transfer from the design stage to the planning stage without re-entering the data manually on part geometry. The CAD database is directly processed by CAM for operating and controlling the production machinery and material handling equipment as well as for performing automated testing and inspection for product quality.

16.5 NUMERICAL CONTROL

Numerical control (NC) of machine tools is a method of automation in which functions of machine tools are controlled by programs using letters, numbers and symbols. These programs contain precise instructions about the manufacturing procedure as well as the movements. NC machines are suitable for jobs with complex part geometry or mathematically defined contours and requiring high accuracy and repeatability, such as airfoils, turbine blades. NC is most appropriate for small or medium lot sizes, ranging from one unit upto several hundred units, and also where there are frequent changes in the product design. An important application of NC is seen in the form of inspection machines, known as *coordinate measuring machines* (CMM), and in controlling robots.

16.5.1 NC Machine Tools

The most prominent feature of NC machine tool is that a variety of machining operations are performed on the same machining center, thus eliminating the non-productive waiting time when such operations are performed on different machines. In addition to this, the provision of automatic tool changing, indexing of tables, and several pallets add to the productivity of the machining centers.

16.5.2 Principle of Operation

An NC system consists of three basic components [Fig. 16.1]:

1. Part Program *Part program* is the set of detailed step-by-step commands that direct the action of the processing equipment. It requires basic information in three categories: part geometry, process information (e.g. cutting process parameters, spindle speed, feed rate) and technology details (e.g. cutting tool, cutting tool selection, etc.). Part programs are written keeping in view of the codes and symbols understood by the machine control unit.

2. Machine Control Unit *Machine control unit*[1], abbreviated as MCU, performs various controlling functions under the instructions contained in part program. In modern NC technology, MCU is a microcomputer and related to control hardware that stores the part program of instructions and executes it by converting each command into mechanical actions of the processing equipment, one command at a time. The MCU hardware also includes components to interface with the processing equipment and feed-back control elements.

3. Processing Unit The processing unit performs the actual productive work in defined processing steps according to the directions given by the MCU, which is driven by the instructions contained in the part program.

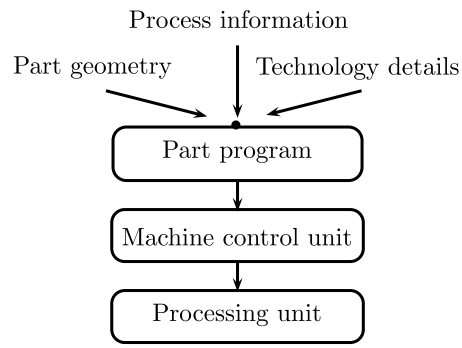


Figure 16.1 Elements of NC systems.

NC machines are designed to be highly automatic and capable of combining several operations in one setup that formerly required several machines. They are designed to reduce the time consumed by the non-cutting elements in the operation cycle, such as changing tools, loading and unloading the work part. A *machining center* is a machine tool capable of performing multiple machining operations on a single workpiece in one setup.

16.5.3 Coordinate Systems

Programs of NC processing equipment need a definite axis system to specify the position of the work head w.r.t. the work part. NC systems employ two types of axis systems, one for flat and prismatic work parts and the other for rotational parts. Both systems are based on Cartesian coordinate system.

16.5.4 Motion Control Systems

Different types of movement of the work head are accomplished by the motion control system. Once the motion is completed, some processing action is accomplished by the work head at the location. The motion control systems for NC can be broadly divided into two types:

1. Positioning Systems *Positioning system*, also called point-to-point mode, moves the work table to a programed location irrespective of the path

[1]Because the MCU is a computer, the term computer numerical control (CNC) is used to distinguish this type of NC from its technological predecessors that were based entirely on hard-wired electronics.

following between two points. This system offers capability of motion in all three axes, one at a time, that is, machining can be performed at a time along single axis only. This mode is useful for drilling and punching machines [Fig. 16.2].

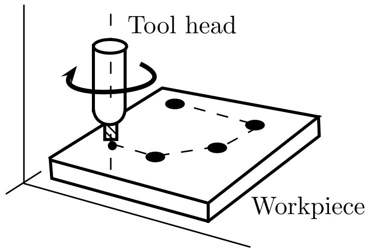


Figure 16.2 | Positioning system.

Reference point for motion control can be defined in two ways:

(a) *Absolute Positioning* In *absolute positioning*, the work head positions are defined relative to the origin of the coordinate system.

(b) *Incremental Positioning* In *incremental positioning*, the next work head position is defined relative to the present location.

2. *Continuous Path Systems* Continuous path systems are capable of continuous simultaneous control of the motion in two or more axes. This enables control of the tool trajectory relative to the work part for the purpose of generating two-dimensional curves, surfaces, or three-dimensional contours in the work part. Continuous path systems can be grouped into three sets:

(a) *Straight Line Mode* In *straight line mode*, the machine performs continuous motion in each axis direction. This mode is suitable for straight line milling operations [Fig. 16.3].

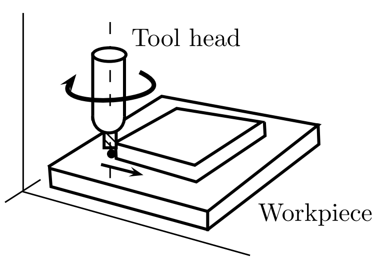


Figure 16.3 | Straight line mode.

(b) *Two-Axis Contouring* Two-axis *contouring* offers simultaneous motion capability in any of the two axis. Any 3D profile to be machined can be completed using the concept of 2.5D mode [Fig. 16.4].

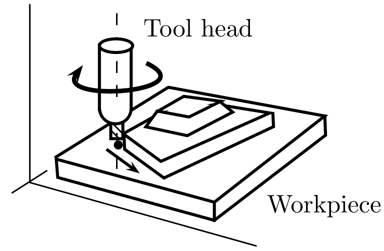


Figure 16.4 | Two-axis contouring.

(c) *Three-Axis Contouring* Three-axis contouring is the highest form of control which gives capability of simultaneous three or more axes motion. This is useful for machining of complex 3D profiles encountered in industrial practice [Fig. 16.5].

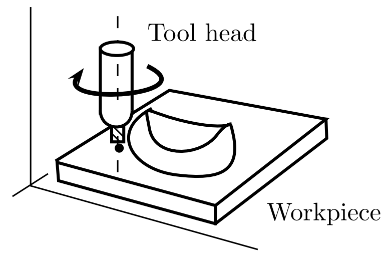


Figure 16.5 | Three-axis contouring.

16.5.5 NC Positioning Systems

An NC positioning system converts the coordinate axis values in the NC part program into relative positions of the tool and work-part during processing. This controls the position of work table by means of a rotating lead-screw driven by a stepper motor or servomotor. For each revolution of the lead screw, the work table moves a distance equal to pitch of the lead screw. In turn, the velocity of the work table (feed rate) is determined by the pitch and rotational speed of the lead screw [Fig. 16.6].

The following are the two types of positioning systems:

1. *Open Loop Positioning System* An *open loop positioning systems* operates without verifying that the actual position achieved in the move is the same as the desired position [Fig. 16.6]. A stepper motor is driven by a series of electrical pulses, which are generated by the MCU in an NC system. Each pulse causes the motor to rotate a fraction of one revolution called *step angle*, denoted by α.

Consider an open loop positioning system in which the stepper motor has n_s number of step

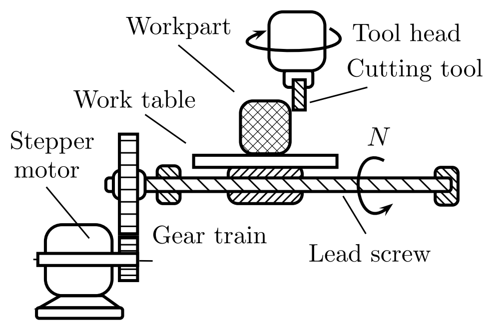


Figure 16.6 NC Positioning system.

angles of equal value α ($= 360/n_s$ degrees) and lead screw has a pitch (p). Transfer of angular rotation of the stepper motor to lead screw is reduced through a gear train having a gear ratio ($r_g > 1$) (defined as the number of turns of the motor for each single turn of the lead screw). The angle turned by the motor shaft (θ_m) for number of pulses n_p would be given by

$$\theta_m = n_p \alpha$$

The angle turned by lead screw (θ_l) for n_p pulses is given by

$$\theta_l = \frac{\theta_m}{r_g} = \frac{n_p \alpha}{r_g}$$

Linear movement of the work table for n_p pulses is determined as

$$x = p \times \frac{\theta_l}{360} = \frac{p n_p \alpha}{360 r_g}$$

Thus, the number of pulses required for movement x are given by

$$n_p = \frac{360 x r_g}{p\alpha}$$

Angular speed N (rpm) of the lead screw for pulse train frequency f_p (Hz) is given by

$$\frac{N}{60} r_g = \frac{f_p}{n_s}$$
$$N = \frac{60 f_p}{n_s r_g}$$

The table speed (v) can be determined as

$$v = Np = \frac{60 f_p p}{n_s r_g}$$

2. *Closed Loop Positioning System* A *closed loop system* functions same as the open loop positioning system but with an additional feature of feedback measurements to ensure that the work table is moved to the desired position. For this, an *optical encoder* is used as feed sensor. The additional feature of optical encoder is treated in reverse manner to that for pulses in the stepper motor.

Closed loop systems are desired for machines that perform continuous path operations, such as milling, turning, in which there are significant forces resisting the forward motion of the cutting tool.

16.5.6 Manual NC Part Programming

NC part programing is used for preparing the sequence of instructions for part processing. Part programs are fed to the NC machine control unit (MCU) using an input medium. The instructions on part programs can be grouped into four sets: geometric instructions (movement between tool head and part head), processing instructions (cutting speed, feed, tools, cutting fluids, etc.), travel instructions (positioning and movement interpolation), and switching instructions (tool changes, coolant supplies, etc.). Part programing can be done manually, computer assisted or using CAD/CAM, and by manual data input.

In manual NC part programing, NC codes are written using a low-level machine language. The instructions include a combination of binary and decimal number systems, called the *binary coded decimal* (BCD) system, alphabetical characters, and other symbols.

A word is formed out of a sequence of those characters which specifies a detail about the operation, such as coordinate position, feed rate, spindle speed. A *block* is a collection of words which specifies a complete NC instruction. The organization of words within a block is known as a *block format*, which is usually in the sequence shown in Fig. 16.7.

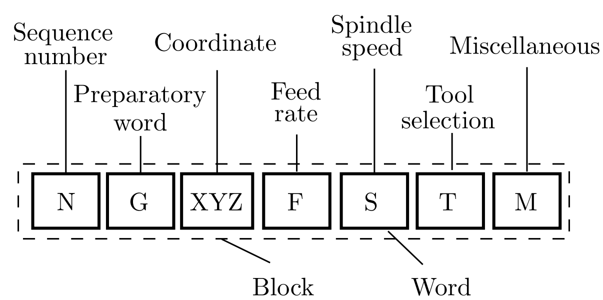


Figure 16.7 Block format of part programs.

Block of a part program starts with a sequence number N, having minimum three digits (e.g. N001,

N023). Preparatory word G is followed by two numerical digit information, G00 to G99. It prepares the MCU for the instructions and data contained in the block. For example, block format N010 G00 X7.0 Y2.0 I5.0 J2.0 can be decoded as G00 prepares the controller for a point-to-point rapid transverse move between the previous point (5, 4) and the end point defined in the current command (7, 2).

16.5.7 Merits of NC

The numerical control offers the following advantages over conventional manufacturing:

1. *Non-Productive Time* NC cannot optimize the metal cutting process itself but can reduce the proportion of time when the machine is not cutting the metal. Thus, NC minimizes the non-productive time, and parts are produced in less time, therefore it is likely to be less expensive. This indirectly saves labor cost also.
2. *Accuracy and Repeatability* Automation and absence of interrelated human factors make the NC products more accurate, even for small batches. Thus, NC maintains consistent quality of the products for the entire batch.
3. *Quality* Consistent quality in the entire batch also reduces the inspection time. Even with use of inspection probes in advanced CNC machines, the measurement function also becomes a part of the program.
4. *Jigs and Fixtures* NC eliminates the need of expensive jigs and fixtures, depending upon the part geometry. The number of setups can usually be reduced significantly using NC.
5. *Part Geometry* Complex profiles can be programed in NC, hence eliminating the need for special form tools.
6. *Machining Center* A single CNC machine center can perform a variety of machining operations, thus eliminating the need for different types of machines.
7. *Machining Time and Cost* Machining time and costs are more accurately calculated and analyzed based on the part program itself.
8. *Human Factors* NC machines can be utilized continuously, ad hence eliminates the effect of operator fatigue in machining time. A single operator can look after two or three NC automatic machines simultaneously. No operator is required in NC except in setting up of the tools and the work, therefore, results in less scrap due to operator errors.
9. *Drafting* Automated drafting machines serve as one of the output devices for a CAD/CAM system.

16.5.8 Demerits of NC

The following are the basic demerits of NC:

1. *High Cost of NC Machines* The cost of an NC machine and cutting tools is five to ten times higher than conventional machines. Thus, NC machines require very high initial investment.
2. *Sophisticated Technology* NC machines use complex and sophisticated technology, thus require skilled staff with specialized training for both software and hardware. Part programers need to be trained to write instructions in desired languages for machines on the shop floor and also be acquainted with the manufacturing process.
3. *Preventive Maintenance* Breakdowns of NC systems are costly and can be time consuming due to their complexity. Their preventive maintenance is essential.
4. *Need of Programing* NC systems are based on programing which requires time and fluency.
5. *Design Requirements* The production time spent by the NC machine cutting metal is significantly greater than that with manually operated machines. This causes certain components, such as the spindle speed, drive gears and feed screws to wear more rapidly.

16.6 GROUP TECHNOLOGY

Group technology (GT) is a manufacturing philosophy in which similar parts are identified and grouped together into *part families* to take advantage of their similarity in manufacturing and design. The similar characteristics of parts in each family facilitates use of similar processing and results in manufacturing efficiencies, that is, reduction in setup time, lower in-process inventories, better scheduling, improved tool control, and use of standardized process plans.

Part family is a collection of parts which are similar either because of geometric shape and size or because similar processing steps are required in their manufacture. The concept of part family is central to design retrieval systems and most current computer-aided process planning schemes. This helps in organizing the layout to specialize in the manufacture of a particular part family. However, the biggest difficulty in changing over to group technology from a traditional production layout is the problem of grouping parts into part families.

IMPORTANT FORMULAS

Benefits of CAD Systems

1. Design productivity
2. Available geometric forms
3. Quality of the design
4. Manufacturing data base
5. Design standardization

Components of NC Systems

1. Part program
2. Machine control unit
3. Processing unit

NC Positioning System

1. Motor shaft angle

$$\theta_m = n_p\alpha$$

2. Lead screw angle

$$\theta_l = \frac{n_p\alpha}{r_g}$$

3. Table movement

$$x = \frac{pn_p\alpha}{360r_g}$$

4. Pulse requirement

$$n_p = \frac{360xr_g}{p\alpha}$$

5. Lead screw rpm

$$N = \frac{60f_p}{n_s r_g}$$

6. Table speed

$$v = \frac{60f_p p}{n_s r_g}$$

SOLVED EXAMPLES

1. Stepper motor of a point-to-point controlled NC machine has specification sensitivity of 3°/pulse. Pitch of the lead screw is 2.4 mm. Determine the expected positioning accuracy.

Solution. For one revolution of the lead screw, total $360/3 = 120$ pulses will be generated and the table would move by 2.4 mm. Thus, for one pulse, the accuracy is

$$\frac{2.4}{120} = 0.02 \text{ mm}$$
$$= 20\ \mu\text{m}$$

2. A multispindle automat performs four operations with times 100, 120, 135, and 155 seconds at each of its work centers. Determine the cycle time (time required to manufacture one work piece) of the work center.

Solution. The cycle time to manufacture one work piece would be the maximum time taken in any of the operations. For the present case, it is 155 s.

3. The work table of a positioning system is driven by a lead screw having pitch 6 mm. The lead screw is connected to the output shaft of a stepper motor through a gear box whose ratio is 8:1. The stepper motor has 72 step angles. The table must move a distance of 240 mm from its present position at a linear velocity of 540 mm/min. Determine the number of pulses required to move the table and required motor speed and the pulse rate to achieve the desirable table velocity.

Solution. Given

$$p = 6 \text{ mm}$$
$$x = 240 \text{ mm}$$

The angle θ_l turned by lead screw of pitch $p = 6$ mm for table distance of $x = 240$ mm is given by

$$\theta_l = \frac{360}{6}\times 240$$
$$= 14400^\circ$$

The value of step angle of the stepper motor is

$$\delta = \frac{360}{72}$$
$$= 5^\circ$$

Number of pulse required by the stepper motor is

$$n_p = \frac{14400\times 8}{5}$$
$$= 23040$$

The speed of rotation of the lead screw is

$$N_l = \frac{540}{6}$$
$$= 90 \text{ rpm}$$

Thus, the speed of stepper motor is

$$N_s = N_l\times 8$$
$$= 440 \text{ rpm}$$

GATE PREVIOUS YEARS' QUESTIONS

1. During the execution of a CNC part program block

N020 G02 X45.0 Y25.0 R5.0

the type of tool motion will be

(a) circular interpolation - clockwise
(b) circular interpolation - counterclockwise
(c) linear interpolation
(d) rapid feed

(GATE 2004)

Solution. The preparatory word G02 is used for circular interpolation in clockwise direction. Thus, option (a) is correct.

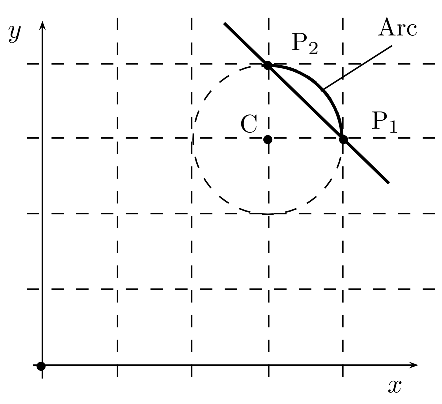
Ans. (a)

2. In a 2-D CAD package, clockwise circular arc of radius 5, specified from P_1 (15, 10) to P_2 (10, 15) will have its center at

(a) (10, 10)
(b) (15, 15)
(c) (15, 10)
(d) (10, 15)

(GATE 2004)

Solution. The drawing is shown in the figure:



The center C of the arc between P_1 and P_2 is at (10, 10).

Ans. (a)

3. Which among the NC operations given below are continuous path operations?

Arc welding (AW)
Milling (M)
Drilling (D)
Punching of sheet metal (P)
Laser cutting of sheet metal (LC)
Spot welding (SW)

(a) AW, LC and M
(b) AW, D, LC and M
(c) D, LC, P and SW
(d) D, LC and SW

(GATE 2005)

Solution. Drilling, laser cutting, and spot welding are the examples of continuous path operations.

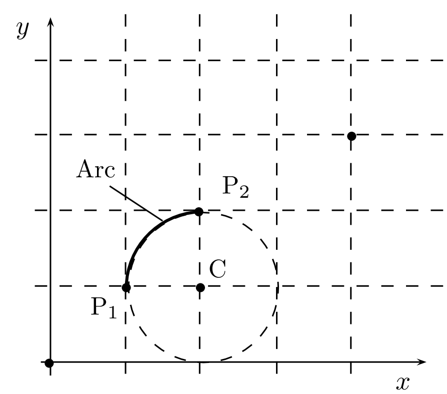
Ans. (b)

4. A tool of an NC machine has to move along a circular arc from (5, 5) to (10, 10) while performing an operation. The center of the arc is at (10, 5). Which one of the following NC tool path commands perform the above mentioned operation?

(a) N010 G02 X10 Y10 X5 Y5 R5
(b) N010 G03 X10 Y10 X5 Y5 R5
(c) N010 G01 X5 Y5 X10 Y10 R5
(d) N010 G02 X5 Y5 X10 Y10 R5

(GATE 2005)

Solution. The desired movement of the tool is shown in the figure:



The point has to move from P_1 (5, 5) to P_2 (10, 10) and radius (R) of the arc is $10 - 5 = 5$. The movement is in clockwise direction. The preparatory word G01 is used for linear interpolation, G02 is used for circular interpolation in clockwise direction, and G03 is used for circular

interpolation in counterclockwise direction. Thus, option (d) is correct.

Ans. (d)

5. NC contouring is an example of

(a) continuous path positioning
(b) point-to-point positioning
(c) absolute positioning
(d) incremental positioning

(GATE 2006)

Solution. NC contouring is an example of continuous path positioning.

Ans. (a)

6. Which type of motor is not used in axis or spindle drives of CNC machine tools?

(a) induction motor
(b) stepper motor
(c) dc servo motor
(d) linear servo motor

(GATE 2007)

Solution. The position of work table of CNC machines is controlled by means of a rotating lead-screw driven by a stepper motor or servomotor. Induction motor is used only in conventional machine tools.

Ans. (a)

7. For generating a Coon's surface, we require

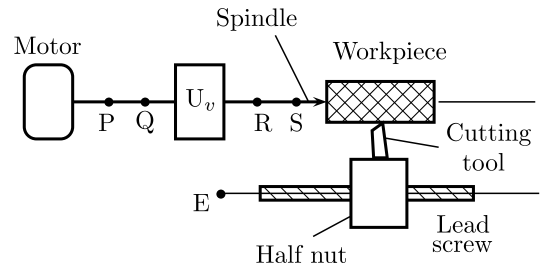
(a) a set of grid points on the surface
(b) a set of grid control points
(c) four bounding curves defining the surface
(d) two bounding curves and a set of grid control points

(GATE 2008)

Solution. Coon's patch defines a surface by four curve segments that are connected in end-to-end to form curve chain.

Ans. (c)

8. The figure shows an incomplete schematic of a conventional lathe to be used for cutting threads with different pitches. The speed gear box U_r is shown and the feed box U_s is to be placed. P, Q, R and S denote locations and have no other significance. Changes in U_r should not affect the pitch of the thread being cut and changes in U_s should not affect the cutting speed.



The correct connections and the correct placement of U_s are given by

(a) Q and E are connected, U_s is placed between P and Q
(b) S and E are connected, U_s is placed between R and S
(c) Q and E are connected, U_s is placed between Q and E
(d) S and E are connected, U_s is placed between S and E

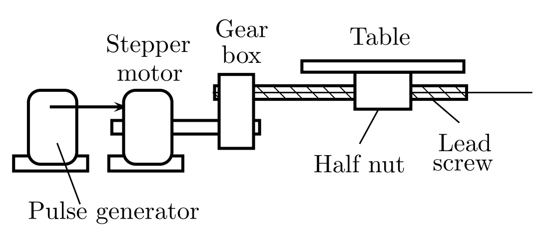
(GATE 2008)

Solution. Comparing the diagram with the layout of a conventional lathe, U_v indicates back gear arrangement. Given that changes in speed gear box (U_r) should not affect the pitch of the thread being cut, and the changes in feed box (U_s) should not affect the cutting speed. This is possible by connecting the final speed of the spindle with the speed of lead screw through feed gear box. Thus, option (d) is correct.

Ans. (d)

Linked Answer Questions

In the feed drive of a point-to-point open loop CNC drive, a stepper motor rotating at 200 steps/rev drives a table through a gear box and lead screw–nut mechanism (pitch = 4 mm, number of starts = 1). The gear ratio (= output rotation speed/input rotational speed) is given by $U = 1/4$. The stepper motor (driven by voltage pulse from pulse generator) executes 1 step/pulse of the pulse generator. The frequency of the pulse train from generator is $f = 10,000$ pulses per min.



9. The basic length unit (BLU), that is, the table movement corresponding to 1 pulse of the pulse generator is

(a) 0.5 microns (b) 5 microns
(c) 50 microns (d) 500 microns

(GATE 2008)

Solution. Given that the number of steps (n_s) in the stepper motor are 200 in 360° (one cycle) of rotation. Gear ratio (U or r_g) is 1/4. Lead screw is single start and its pitch (p) is 4 mm. The frequency of the pulse train from generator is $f_p = 10,000$ pulses per min. For each pulse, the stepper motor rotates by 1 step, that is, angle of $\alpha = 360/200 = 1.8°$. The gear box reduces this angle to $1.8/4 = 0.45°$. For this, the work table shall move by

$$\begin{aligned}\text{BLU} &= 4 \times \frac{0.45}{360} \\ &= 5 \text{ microns}\end{aligned}$$

Ans. (b)

10. A customer insists on a modification to change the BLU of the CNC drive to 10 microns without changing the table speed. The modification can be accomplished by

(a) changing U to 1/2 and reducing f to $f/2$
(b) changing U to 1/8 and increasing f to $2f$
(c) changing U to 1/2 and keeping f unchanged
(d) keeping U unchanged and increasing f to $2f$

(GATE 2008)

Solution. BLU is given by

$$\text{BLU} = \frac{x}{n_p} = \frac{p\alpha}{360 \times r_g}$$

To double the BLC, r_g should be halved. However, the table speed (v) is related as

$$v = \frac{60 f_p p}{n_s r_g}$$

For constant v, p and n_s,

$$f_p \propto r_g$$

Thus, both f_p and r_g should be halved simultaneously.

Ans. (a)

11. Which of the following is the correct data structure for solid models?

(a) solid part → faces → edges → vertices
(b) solid part → edges → faces → vertices
(c) vertices → faces → edges → solid parts
(d) vertices → edges → faces → solid parts

(GATE 2009)

Solution. The correct data structure for solid models is solid part → faces → edges → vertices.

Ans. (a)

12. Match the following:

P. M05	1. Absolute coordinate system
Q. G01	2. Dwell
R. G04	3. Spindle stop
S. G90	4. Linear interpolation

(a) P-2, Q-3, R-4, S-1
(b) P-3, Q-4, R-1, S-2
(c) P-3, Q-4, R-2, S-1
(d) P-4, Q-3, R-2, S-1

(GATE 2009)

Solution. M05 instructs for spindle stop, laser, flame, power off. G01 is for feed rate, G04 is for dwell time, and G90 for absolute coordination system.

Ans. (c)

13. In a CNC program block, N002 G02 G91 X40 Z40 ... G02 and G91 refer to

(a) circular interpolation in counterclockwise direction and incremental dimension
(b) circular interpolation in counterclockwise direction and absolute dimension
(c) circular interpolation in clockwise direction and incremental dimension
(d) circular interpolation in clockwise direction and absolute dimension

(GATE 2010)

Solution. In CNC part program, G02 is used for circular interpolation in clockwise direction (whereas G03 is used for circular interpolation in counterclockwise direction). G91 is used for incremental positioning system (whereas G90 is used for absolute positioning system). Thus, option (c) is correct.

Ans. (c)

14. A CNC vertical milling machine has to cut a straight lot of 10 mm width and 2 mm depth by a cutter of 10 mm diameter between points (0,0) and (100,100) on the XY plane (dimensions in mm). The feed rate used for milling is 50 mm/min. Milling time for the slot (in seconds) is

(a) 120 (b) 170
(c) 180 (d) 240

(GATE 2012)

Solution. The cutter has to travel the diagonal length plus distance equal to its diameter (over-travel), therefore, the time required is

$$t = \frac{100\sqrt{2}+10}{50} \times 60$$
$$= 181.706 \text{ s}$$

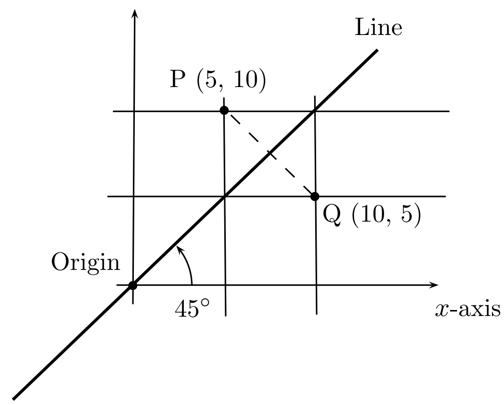
Ans. (c)

15. In a CAD package, mirror image of a 2D point P (5, 10) is to be obtained about a line which passes through the origin and makes an angle of 45° counterclockwise with the x-axis. The coordinates of the transformed point will be

(a) (7.5, 5) (b) (10, 5)
(c) (7.5, −5) (d) (10, −5)

(GATE 2013)

Solution. The point and its mirror image is shown in the figure below.



The mirror image of point P (5, 10) about the given line is at Q (10, 5).

Ans. (b)

MULTIPLE CHOICE QUESTIONS

1. The subsystems in computer integrated manufacturing include

(a) business planning and support
(b) manufacturing process planning
(c) process automation and control
(d) all of the above

2. The benefits of computer integrated manufacturing include

(a) responsiveness to the dynamics of global competition
(b) consistent product quality and uniformity
(c) improved productivity by optimum utilization of resources
(d) all of the above

3. SKETCHPAD system, the milestone of research in computer graphics was developed by

(a) Ivan Sutherland
(b) Requicha
(c) Bedworth
(d) Jung Hyun Han

4. The standards that define NC programing is/are

(a) ISO 6983:1982
(b) ANSI/EIA 274D-1998
(c) STEP-NC
(d) all of the above

5. In current NC programing G-code, the cutter motion is mainly specified in terms of

(a) position
(b) feed rate of axes
(c) both (a) and (b)
(d) none of the above

6. ‘G-code’ is a

(a) low level language
(b) middle level language
(c) high level language
(d) none of the above

7. Computer aided manufacturing (CAM) was inspired by

(a) computer aided design (CAD) systems
(b) computer graphics
(c) numerical controlled (NC) machines
(d) none of the above

8. The most easier step to break the isolation of CAD and CAM systems is to

(a) reuse the product model designed in CAD systems in CAM systems
(b) re-enter the CAD model into CAM systems manually
(c) both (a) and (b)
(d) none of the above

9. The basic benefits of CAD systems include

(a) increased design productivity
(b) creation of manufacturing database
(c) improved communication documentation
(d) all of the above

10. The important feature of CAD/CAM in machining is its capability to

(a) describe the cutting tool path for various operations
(b) save the machining time
(c) increase material removal rate
(d) enhance the tool life

11. The standard ISO 6983:1982 defines a set of M and G codes which specify

(a) a sequence of cutting tool movements
(b) the direction of rotation
(c) speed of travel and various auxiliary functions
(d) all of the above

12. Automatically programed tool (APT) is

(a) a universal programing language for NC machines
(b) a convenient way to define geometry elements and generate cutter locations for NC programs by computers.
(c) a tool to create a part drawing and convert the drawing into NC programs
(d) all of the above

13. Current major commercial CAD/CAM systems include

(a) Unigraphics (b) Pro/E
(c) IDEAS (d) all of the above

14. An NC system consists of

(a) part program of instructions
(b) machine control unit
(c) processing equipment
(d) all of the above

15. Demerits of NC machines include

(a) high initial cost
(b) sophisticated technology
(c) requirement of highly skilled staff
(d) all of the above

16. In an NC machining operation, the tool has to be moved from point (5, 4) to point (7, 2) along a circular path with center at (5, 2). Before starting the operation, the tool is at (5, 4). The correct G and M code for this motion is

(a) N010 G03 X7.0 Y2.0 I5.0 J2.0
(b) N010 G02 X7.0 Y2.0 I5.0 J2.0
(c) N010 G01 X7.0 Y2.0 I5.0 J2.0
(d) N010 G00 X7.0 Y2.0 I5.0 J2.0

17. In computer aided drafting practice, an arc is defined by

(a) two end points only
(b) center and radius
(c) radius and one end point
(d) two end points and center

18. In a point-to-point control NC machine, the slides are positioned by an integrally mounted stepper motor drive. If the specification of the motor is 1°/pulse, and the pitch of the lead screw is 3.6 mm, what is the expected positioning accuracy?

(a) 1 μm (b) 10 μm
(c) 50 μm (d) 100 μm

19. A multispindle automat performs four operations with times 50, 60, 65, and 75 seconds at each of its work centers. The cycle time (time required to manufacture one workpiece) in seconds will be

(a) 250 (b) 62.5
(c) 18.75 (d) 75

20. In a CNC machine tool, encoder is used to sense and control

(a) table position
(b) table velocity
(c) spindle speed
(d) coolant flow

21. In a point-to-point type of NC system,

(a) control of position and velocity of the tool is essential
(b) control of only position of the tool is sufficient
(c) control of only velocity of the tool is sufficient
(d) neither position nor velocity need to be controlled

22. With reference to NC machines, which of the following statements is wrong.

(a) Both closed-loop and open-loop control systems are used
(b) Paper tapes, floppy tapes and cassettes are used for data storage
(c) Digitizers may be used as interactive input devices
(d) Post processor is an item of hardware

NUMERICAL ANSWER QUESTIONS

1. The work table of a positioning system is driven by a lead screw having pitch 8 mm. The lead screw is connected to the output shaft of a stepper motor through a gear box whose ratio is 6:1. The stepper motor has 48 step angles. Determine the number of pulses and the speed of stepper motor required to move the table a distance of 220 mm from its present position at a linear velocity of 440 mm/min.

2. An NC work table operates by closed-loop positioning. The system consists of a servomotor, lead screw and optical encoder. The lead screw has a pitch of 8 mm and is coupled to the motor shaft with a gear ratio of 6:1. The optical encoder generates 48 pules per revolution of its output shaft. The table has been programmed to move a distance 220 mm at a feed rate of 440 mm/min. Determine the number of pulses should be received by the control system to verify that the table has moved exactly 220 mm.

ANSWERS

Multiple Choice Questions

1.	(d)	**2.**	(d)	**3.**	(a)	**4.**	(d)	**5.**	(c)	**6.**	(a)	**7.**	(c)	**8.**	(a)	**9.**	(d)	**10.**	(a)
11.	(d)	**12.**	(d)	**13.**	(d)	**14.**	(d)	**15.**	(d)	**16.**	(d)	**17.**	(d)	**18.**	(b)	**19.**	(d)	**20.**	(a)
21.	(b)	**22.**	(a)																

Numerical Answer Questions

1.	7920, 330 rpm	**2.**	1320

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (d) The subsystems of CIM include business planning and support, product design, manufacturing process planning, process automation and control, and factory floor monitoring systems.

2. (d) Computer integrated manufacturing offers benefits, such as responsiveness, consistency and improved productivity.

3. (a) In the early 1960s, Ivan Sutherland developed the SKETCHPAD system, a milestone of research achievement in computer graphics.

4. (d) The standards that define NC programing code are ANSI/EIA 274D-1998 and from the international community, ISO 6983:1982. This may eventually be replaced by a new ISO standard called STEP-NC that will allow direct exchange of information between CAD/CAM model and the CNC machine tool.

5. (c) Current NC programing is based on 'G-code', where the cutter motion is mainly specified in terms of position and the feed rate of axes.

6. (a) 'G-code' is a low level language, which delivers only limited information to CNC.

7. (c) Computer Aided Manufacturing (CAM) was inspired by Numerical Controlled (NC) Machines.

8. (a) The most easier step to break the isolation of CAD and CAM systems is to reuse the product model designed in CAD systems in CAM systems.

9. (d) A CAD system offers numerous benefits, such as increased design productivity, availability of geometric forms, quality of the design, communication documentation, creation of manufacturing database, design standardization, etc.

10. (a) The important feature of CAD/CAM in machining is its capability to describe the cutting tool path for various operations.

11. (d) The standard defines a set of M and G codes which specify a sequence of cutting tool movements as well as the direction of rotation, speed of travel and various auxiliary functions such as coolant flow.

12. (d) Automatically programed tool (APT) is a universal programing language for NC machines. It is a convenient way to define geometry elements and generate cutter locations for NC programs by computers. It can be used to create a part drawing and convert it into NC programs.

13. (d) Current major commercial CAD/CAM systems are Unigraphics, Pro/E, IDEAS, CATIA, etc.

14. (d) An NC system consists of three basic components: part program of instructions, machine control unit, processing equipment.

15. (d) The cost of an NC machine and cutting tools is five to ten times higher than that of conventional machines. NC machines use complex and sophisticated technology, thus require highly skilled staff.

16. (d) G00 prepares the controller for a point-to-point rapid transverse move between the previous point (i.e. [5, 4]) and the end point defined in the current command (i.e. [7, 2]).

17. (d) The arc can be defined at least three points.

18. (b) For one revolution of the lead screw, total 360 pulses will be generated, by which feed of 3.6 mm will be provided by it. Thus, for 1° or single pulse, the accuracy is calculated as

$$\begin{aligned}\frac{3.6}{360} &= 0.01 \text{ mm} \\ &= 10\ \mu\text{m}\end{aligned}$$

19. (d) The cycle time to manufacture one work piece would be the maximum time taken in any of the operations. For the present case, it is 75 s.

20. (a) Encoder is a device used to convert linear or rotational position information into an electrical output signal.

21. (b) In point-to-point system of control positioning, the locations are specified for the work head, irrespective of the path and velocity.

22. (a) Two types of positioning control system are used in NC systems: open-loop systems, and closed-loop systems:

(i) An open loop system operates without verifying that the actual position achieved in the move is the same as the desired position.

(ii) A closed loop system uses feedback measurements to confirm the final position of the work table.

Numerical Answer Questions

1. The angle θ turned by lead screw of pitch $p = 8$ mm for table distance of $x = 220$ mm is given by

$$\begin{aligned}\theta_l &= \frac{360}{8} \times 220 \\ &= 9900°\end{aligned}$$

The value of step angle of the stepper motor is

$$\begin{aligned}\delta &= \frac{360}{48} \\ &= 7.5°\end{aligned}$$

Number of pulses required by the stepper motor is

$$\begin{aligned} n_l &= \frac{15000 \times 6}{7.5} \\ &= 7920 \end{aligned}$$

The speed of rotation of the lead screw is

$$\begin{aligned} N_l &= \frac{440}{8} \\ &= 55 \text{ rpm} \end{aligned}$$

Thus, the speed of stepper motor is

$$\begin{aligned} N_s &= N_l \times 6 \\ &= 330 \text{ rpm} \end{aligned}$$

2. In closed-loop system, the pulse is generated by the optical encoder as feedback. For one revolution, the output shaft, that is, lead screw, the table will travel a distance equal to pitch of the lead screw. Thus, the number of pulses required for movement of 250 mm is given by

$$\begin{aligned} n_p &= \frac{220}{8} \times 48 \\ &= 1320 \text{ pulses} \end{aligned}$$

CHAPTER 17

PRODUCTION PLANNING & CONTROL

Production planning and control (PPC) are the two important components which entail the acquisition and allocation of limited resources to production activities so as to satisfy customer demand over a specified time period. *Production* is a process of converting raw material into semi-finished products, and thereby adding the value of utility to the products, encompassing the activities of procurement, allocation, and utilization of resources. *Planning* is the systematic preparation for future activities based on assumptions and projections about how the object (being planned) and its environment will develop in the future. The *control* function involves supervising the operations with the aid of control mechanism and feedback information about the progress of work.

17.1 FUNCTIONS OF PPC

Production planning and control (PPC) are inter-related in a complex manner such that they look like single function of management of an enterprise. Their ultimate objective is to contribute the profits of the enterprise.

Production planning is a partial planning approach for a particular function of an organization. It encompasses coordination of parallel activities related to the production processes in order to find suitable measures to eliminate non-allowable deviations from planned production. Planning undertakes many assumptions and uncertainties in outlining the future production processes. However, an uncertain information becomes certain during production, and then it is taken as an input for control decisions to adjust the process performance. Thus, the distinction between planning and control can be justified by the uncertainty about resource availability, process performance and process results.

It will be more appropriate to describe the functions of production planning and control under respective headings [Fig. 17.1].

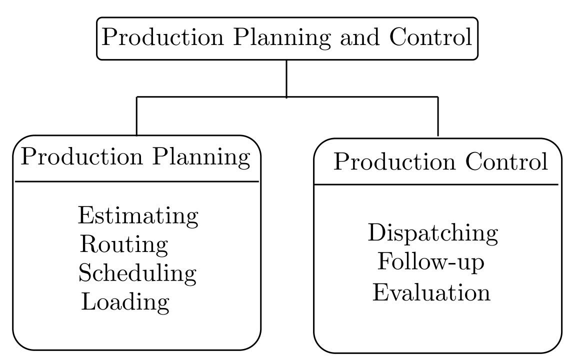


Figure 17.1 Functions of PPC.

17.1.1 Production Planning

Production planning precedes control function. It is the formulation of a new plan in-line with the objective of production. It begins with the analysis of given data on the basis of which a scheme of utilization of firm's resources can be outlined to result in desirable outputs in an efficient manner. It determines the optimal schedule and sequence of operations, economic batch size for production, machine assignments, and dispatching priorities for sequencing.

Production planning is in fact synonymous to process analysis that leads to elimination of undesirable process elements as well as improvement of certain processes. It involves the following functions:

1. *Estimating* *Estimating* function involves deciding the quantity of production and associated cost on the basis of sale forecast. This function uses forecasting techniques in assessing the production quantity.

2. *Routing* *Routing* is to determine and ensure the best and cheapest route or sequence of operations to be followed by the raw material in acquiring the shape of the finished product. Thus, routing involves two elements: operations, and their sequence.

   Routing is performed through *route sheets* for different manufacturing orders. Route sheet of a product defines each step of the production operation and lays down the precise route of operations. It includes important details, such as product identification number, symbol for identification of parts, number of pieces in each lot, operation data, production rate. An assembly of a number of component parts, like a printer or a laptop, requires separate route sheets for each of its parts, sub-assemblies and final assembly.

   In continuous manufacturing systems, such as cement or food industries, routing function is built-in with the original design of the plant and sequencing of machines. However, it is a major planning activity in the case of intermittent production, particularly in customer-oriented products.

   Routing can be generalized or detailed. Generalized routes are established by work-stations or workshops, whereas detailed routes are incorporated within a work-station for different work centers or machines.

3. *Loading* Once the route has been established, specific jobs are then assigned to work centers in view of relative priorities and optimum utilization of capacity of each work center. This is called *loading*. Total time required to perform operations at each work center can be computed by using standard process sheets. These details are then added to the work already planned for each work station. This forms a chart, known as *machine loading charts* or *Gantt charts* that show utilization of men and machines as per priority established in routing and scheduling.

   Loading can be done at various levels of an enterprise. Loading at the level of work station or machine is called *detailed loading*. Loading can also be *generalized* by assigning specific jobs to a group of machines or department as a single unit.

4. *Scheduling* *Scheduling* is deciding the priorities for each job and planning the time-table of production through considerate allocation of start and finishing time for each operation and entire series as routed. Objective of scheduling is to prevent unbalanced use of time among work centers and to utilize resources within established cycle time. Scheduling is closely related to routing because routing cannot be done without scheduling. Therefore, routing and scheduling are generally done by the same team or person of an enterprise. Routing and scheduling need immediate change in the event of contingencies, such as machine breakdowns, delays in supply of raw materials.

   Master schedule is the key plan of production, prepared on the basis of production program. It contains details of the products to be produced, their quantities, and time of delivery. Two major types of master schedules are *rolling master schedules* and *jumping master schedules*. In rolling master schedule, only a short period of production is added, whereas in jumping master schedule a long period can be covered. In the mass production system, involving repetitive or continuous type of manufacturing processes, the master schedule is prepared on the basis of anticipated demand depending on market survey, competitor position, and consumers' preferences regarding quality, price, and design. In intermittent production system, master schedule is prepared based on the orders from customers. Manufacturing schedule are based on the master schedule.

17.1.2 Production Control

Production control is a device to attain the highest efficiency in production by producing the required quantity of production of the required quality at the required time by the best and the cheapest method. This involves control on production quantity, materials and tools, spares and maintenance, labor efficiency, delivery schedule, etc. A production activity of an enterprise is said to be in control when the actual performance

is within the objective of planned performance. The following are the functions of production control:

1. *Dispatching* *Dispatching* is putting the plan into effect by authorized release of resources to plant locations along with the necessary instructions to commence the production in accordance with the requirements of route sheets and schedule charts. This task is performed by a person called the *dispatcher*. Dispatching can be centralized (by central office) or decentralized (i.e. dispatching as per plans, only timing of start is then advised.).

   Quantity of production is controlled during dispatching the manufacturing order. The enterprise is required to exercise effective control over its inventory (both material and tools) to prevent over-stock and out-of-stock situations. Immediate replacement of obsolete and breakdown parts is essential to continue production. However, machine efficiency is significantly affected by the system of periodic maintenance of plant and machinery.

2. *Follow-up* Every production program involves determination of the progress of work, removing bottlenecks in the flow of work, and ensuring that the production operations are taking place in accordance with the plan. This process is called *follow-up*. Follow-up is comparing the actual performance to the planned performance in order to identify the discrepancies in the production for appropriate corrective actions.

3. *Evaluation* Deviation of actual performance from planned schedule is an usual phenomenon. This can result from breakdown of machines, unavailability of raw materials, poor performance of workmen, etc. A break at any point in the supply chain hampers the complete series of operations. Therefore, periodic *evaluation* is essential to formulate corrective action in order to bring the operations back on the schedule.

A detailed discussion on the techniques and procedures of production planning and control is beyond the scope of present context. However, important procedures are described in the following sections of this chapter and in the next two chapters.

17.2 FORECASTING

Estimating stage of production planning seeks for appropriate estimates of production quantity. This is considerably affected by growing competition, frequent changes in customer's demand, and the trend towards automation. Thus, decisions in production should not be based purely on guesses, rather on a careful analysis of data concerning the future course of events. When estimates of future conditions are made on a systematic basis of historical and current data, the process is called *forecasting* and the statement thus obtained is defined as *forecast*.

17.2.1 Forecasting Methods

The ultimate objective of forecasting is to reduce the uncertainty in management decision-making with respect to costs, profit, sales, production, pricing, capital investment, and so forth. Forecasting methods are broadly categorized into two sets: qualitative forecasting methods and quantitative forecasting methods [Fig. 17.2].

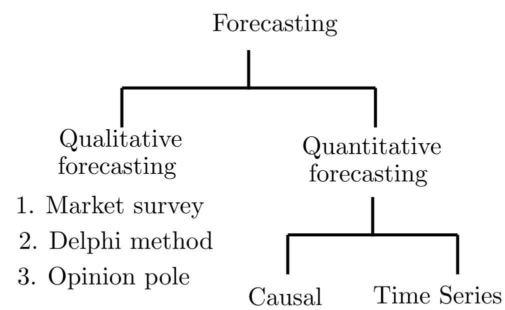


Figure 17.2 Forecasting methods.

Forecasting methods are also classified into intrinsic forecasting and extrinsic forecasting. Intrinsic forecasting uses historical data often availability within the organization itself. Extrinsic forecasting involves external factors.

17.2.1.1 Qualitative Forecasting *Qualitative forecasting* is based on intuitive information that does not have a well-defined analytic structure. This option is essentially used in absence of historical data, such as for a new product. Qualitative forecasting tends to be subjective and is often biased, depending upon the potentially optimistic or pessimistic position of the forecasting team.

Some common methods of qualitative forecasting are described as follows:

1. *Market Survey* Market survey involves structured questionnaire submitted to potential customers in the market. It is fairly expensive and time consuming task.

2. *Opinion Polls* Opinion polls are conducted to assess the opinion of the knowledge personnel and experts in the field whose views carry lot of weight.

3. *Delphi Technique* The Delphi technique[1] is a group process used to survey and collect the opinions of experts on a particular subject. Delphi technique in forecasting is based on a panel of experts in a way that eliminates the potential dominance of the most prestigious, the most verbal and best salespeople. The expert opinion is consensus instead of a compromise; the experts review each others' ideas.

4. *Life Cycle Analogy Forecasting* This technique is applicable to new product or service. It is based on the assumption that most products have a fairly well-defined life cycle consisting of the stages of gradual growth in early stage, maturity, and eventual decline in the demand.

17.2.1.2 Quantitative Forecasting-Causal

Quantitative forecasting methods are based on the apparent causal relationship between two variables such that change in a measurable variable causes a predictable change in each other. The measurable variable effecting the change is called a *leading indicator*. Sufficient number of leading indicators enable bringing excellent forecasting results. For some objects, market survey adds the forecasting. Quantitative forecasting methods based on causality are often time-consuming and expensive primarily because of developing the relationship and obtaining the causal data.

The best example of causal forecasting is the *regression analysis*. It is a statistical technique in which past demand data is used to establish a functional relationship between two variables. One variable is known or assumed to be known; and used to forecast the value of the other unknown variable. Trend line analysis is another statistical method of causal forecasting. In this analysis, the trend line (line of best fit) is drawn on a scatter diagram, which represents the trend in the data. The trend line tells whether a particular data set has increased or decreased over a period of time.

17.2.1.3 Quantitative Forecasting-Time Series

Most common quantitative forecasting methods are based on the assumption that past demand follows some pattern, which can be used in developing projections for future demand. The first step in making a forecast is gathering information from the past in the form of statistical data recorded at successive intervals of time. Such a data is usually referred to as *time series*. Demand data is plotted on a time scale to study and look for consistent shapes and patterns.

Such methods are commonly known as time-series forecasting because time is the only real independent variable in statistical data. Since these methods are based on internal data (sale), they are some times called *intrinsic forecasts*. Understanding the reason behind the trend of demand in time series helps in forecasting. The pattern pattern can be random (unpredictable), trending (apparently definite) or seasonal (periodic).

Some common time-series forecasting models are described as follows:

1. *Simple Average Method* A *simple average method* forecasts the demand of the next time period as the average of demands occurring in all previous time periods.

2. *Simple Moving Average Method* In *simple moving average method*, the average of the demands from several of the most recent periods is taken as the demand forecast for the next time period. The number of past periods to be used in calculations is selected in the beginning and is kept constant.

3. *Weighted Moving Average Method* In *weighted moving average method*, unequal weights are assigned to the past demand data while calculating simple moving average as the demand forecast for the next time period. Usually, the most recent data is assigned the highest weight factor.

4. *Exponential Smoothing* In *exponential smoothing method*[2], weights are assigned in exponential order. The forecast for time period t is related to the demand for the previous period D_{t-1} and forecast of previous period F_{t-1} as[3]

$$F_t = F_{t-1} + \mu\left(D_{t-1} - F_{t-1}\right)$$

where μ is called *smoothing constant*. This method takes care of error $\left(D_{t-1} - F_{t-1}\right)$ in old forecasting. The weights decrease exponentially from most recent demand data to older demand data.

By direct substitution of the defining equation for simple exponential smoothing back into itself,

$$\begin{aligned} F_t = {} & \mu F_{t-1} + \mu\left(1-\mu\right) F_{t-2} \\ & + \mu\left(1-\mu\right)^2 F_{t-3} + ... \\ & + \mu\left(1-\mu\right)^{t-1} F_0 \end{aligned}$$

Thus, F_t is found to be weighted average with general proportion of μ, $\mu\left(1-\mu\right)$, $\mu\left(1-\mu\right)^2$.., which is geometric progression and discrete form

[1] The Delphi technique was named after the Ancient Greek oracle at Delphi, who could predict the future. An oracle refers to a statement from someone of unquestioned wisdom and knowledge or of infallible authority. The technique was developed by Olaf Helmer and his associates at the Rand Corporation in the early 1950s.

[2] Exponential smoothing was first suggested by Charles C. Holt in 1957.

[3] The formulation is attributed to Brown and is known as Brown's simple exponential smoothing.

of exponential function as

$$e^{\mu} = \sum_{n=0}^{\infty} \frac{\mu^n}{n!}$$

Therefore, this method of forecasting is called exponential smoothing.

17.2.2 Forecasting Errors

The numeric difference in the forecasted demand and actual demand is known as *forecasting error* (e). The cost of a forecasting error can be substantial. Forecasting can be improved by examining some objective evaluations of alternative forecasting techniques. Various statistical measures can be used to measure forecasting errors of various models.

Regardless of the object of forecasting, the following are some important fundamental characteristics of forecasting error:

1. When forecasts are almost wrong, the potential error in the forecast can be accommodated through use of buffer capacity.
2. Forecasting is easier for a product line because forecasting errors for an individual product tend to cancel each other.
3. Forecasts for short time periods are more accurate due to possible disruptions in the product demand.
4. Every forecast is incomplete without mentioning estimate of forecast error.
5. Forecasting cannot be substituted by calculation of demand based on actual data for a given time period.

The commonly used measures for summarizing historical errors include following:

1. Mean Absolute Deviation *Mean absolute deviation* (MAD) is determined as

$$\text{MAD} = \frac{\sum |e|}{n}$$

2. Bias *Bias* is defined as

$$\text{Bias} = \frac{\sum e}{n}$$

3. Mean Square Error *Mean square error* (MSE) is determined as

$$\text{MSE} = \frac{\sum e^2}{n-1}$$

where e is the forecasting error for a period in the data, and n is the total number of periods of forecasting.

Forecasting can be monitored by a parameter known as *tracking signal.* It is based on the ratio of cummulative forecast error to the corresponding value of MAD:

$$\text{Tracking signal} = \frac{\sum e}{\text{MAD}}$$

17.3 AGGREGATE PLANNING

Aggregate planning is aimed at pre-estimating the procurement quantity and scheduling the output over an intermediate range by determining optimum levels of the production rate, employment, inventory and other controllable variables. It involves aggregate decisions rather than stock-keeping unit (SKU) level decisions. The resources cannot be changed as fast a rate as the demand. Therefore, aggregate planning offers strategies to the production system so that it can absorb the fluctuations in demand by trade-offs among controllable factors, such as capacity, time, inventory. This can also be done by accepting back orders and subcontracting.

Several methods have been suggested by researchers to absorb fluctuating demand. Notably among those are *linear decision rule*, *graphical charting*, and mathematical programing. Linear decision rule (LDR) was developed by Holt, Modagiliani, Muth and Simon, therefore, it is known as *HMMS rule.* The model initiated with two sets of decision variables, namely, work force and production rate. Graphical techniques are popular because they are easy to understand and use. Linear programming (e.g. transportation model) is a mathematical approach that produces optimal plan for minimizing the costs.

17.4 DISAGGREGATION

The aggregate plan works on the level of workforce and production for medium and long-term periods. For practical implementations, the aggregate plan for each product family is subjected to disaggregation into operational production plans over a short scheduling period. In simple words, the data of aggregate plan are further broken down to detailed levels.

Disaggregation is not a complex process because it involves several steps and a variety of trade-offs that must be made before a final item-by-item production schedule is obtained. The most successful schemes use a hierarchical planning model that explicitly ties together the decisions at each level of planning in order to be internally consistent. The disaggregated plan is assigned the limited resources to meet the cumulative production

data for each product family over a given periodicity of production.

17.5 MATERIAL REQUIREMENT PLANNING

For dependent demand situations, normal reactive inventory control systems, such as economic order quantity models, are not suitable because they result in high inventory costs and unreliable delivery schedules. Unavailability of even one component can cause discontinuity in the production. *Material requirement planning*[4] (MRP) is a special technique to plan the requirement of materials for production. It deals with the materials which directly depend upon the requirement of production. This technique employs production plan or schedule to arrange for the raw materials, rather than depending upon EOQ models. MRP is a simple system of calculating arithmetically the requirement of input materials at different points of time based on the actual production plan. It can be simply defined as a planning and scheduling system to meet time-phased material requirements for production operations, without any probability.

Bill of materials (BOM) is a detailed list of materials required to produce a product. It is constructed in a way that reflects the manufacturing process so that it can be used in material requirement planning.

Fig. 17.3 shows a product structure which consists of three assemblies: A, B and C. There are five materials (1 to 5) required to produce this product.

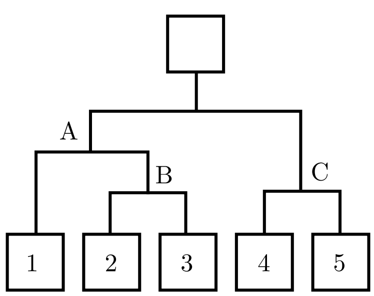


Figure 17.3 Product structure.

Using product structure, the bill of materials recognizes the dependence of certain components on subassemblies, which in turn depend on the final product.

[4]The concept of material requirement planning was introduced by Joseph Orlicky in the early 1960s.

17.6 BREAK-EVEN POINT ANALYSIS

The *break-even point analysis* is a valuable planning and control technique that uses the relationship between production rate and costs of profits. The analysis is performed on a chart that shows a *break-even point* where profit is zero [Fig. 17.4].

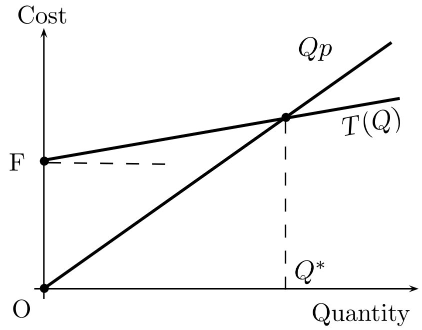


Figure 17.4 Break-even analysis.

Production costs consist of two elements:

1. ***Fixed Cost*** Costs that remain relatively constant regardless of the level of activity are known as *fixed costs* or *indirect costs*. It is considered as preparation expenses to produce a product or service.

2. ***Variable Cost*** Costs that are generally proportional to output are called *variable costs*, or *direct costs*. Such costs are relatively easy to determine.

Revenues results from sales of output. *Profit* represents the difference between revenue and total cost.

Suppose a factory is planning to produce a quantity Q units and sale it at the price rate Rs. p/unit. Fixed cost for the production is Rs. F and variable cost is Rs. V/unit. The revenue on sales of Q units will be Qp. Break-even quantity is determined as

$$Q^* p - F + V Q^* = 0$$
$$Q^* = \frac{F}{p - V}$$

Total cost $T(Q^*)$ and profit $P(Q^*)$ at break-even point are found as

$$T(Q^*) = F + V Q^*$$
$$P(Q^*) = (p + V)\, Q^* - F$$

The business should be immediately closed when variable cost line becomes vertical.

17.7 LOT SIZING RULES

Production planning essentially require the lot size, which is most economical for the manufacture of goods. For this purpose, the cost of production of each lot size is to be worked out and the lot size giving the least cost for unit is selected for manufacturing purpose. *lot-sizing rules* tend to offer lumpy demand conditions and result in inventory for the lower-level components (raw materials).

Some of the commonly used *lot-sizing rules* are summarized as follows:

1. Fixed Order Quantity In this policy, the order quantity is fixed on the basis of intuitive method. The method is suitable for selected items having ordering costs sufficiently high to rule out in net requirement quantity per period.

2. Economic Order Quantity The economic order quantity (EOQ) method is preferable when relatively constant independent demand exists. It is a statistical technique using averages, whereas the MRP procedure assumes known demand reflected in the master production schedule.

3. Lot for Lot Lot-for-lot technique is based on the principle that an MRP system should produce only as needed, with no safety stock and no anticipation of further orders. The technique is efficient when frequent orders are economical and just-in-time inventory technique is implemented.

4. Fixed Period Requirement In this policy, the lot quantity per order is specified to cover certain number of periods.

5. Periodic Order Quantity In *periodic order quantity* (POQ) lot sizing, ordering intervals related to the quantity is computed, and hence, carrying cost tends to be lower. Therefore, it is more effective than EOQ.

6. Part-Period Balancing Pure EOQ approach does not take into account the variations in requirements over time. A simple heuristic method that does this is the *part period balancing* (PPB) method developed by DeMatteis. A part period is defined as a unit of measure that is equivalent to carrying one unit of an item (a part) in inventory for one period. The PPB heuristic is based on the observation that the optimal order quantity in the basic EOQ model occurs when the total ordering or setup cost equals the total holding cost.

17.8 ASSEMBLY LINE BALANCING

The work in an *assembly line* passes through a series of workstations in a uniform time interval, known as workstation cycle time (t). Assembly work performed at each workstation takes its own time that depends upon the work elements. Work time at different workstations can be different. Objective of *assembly line balancing* is allocating operations to each work station of the assembly line without violating the precedence and without exceeding the cycle time. Balance in an assembly line is examined through the following features:

1. Cycle Time Theoretically, workstation cycle time for an assembly is defined as

$$t_c = \frac{\text{production time per day}}{\text{required output per day}}$$

2. Number of Workstations Theoretical minimum number of work stations (n_t) required to satisfy the workstation cycle time constraint is determined as

$$n_t = \frac{\sum t}{t_c}$$

3. Cycle Efficiency *Cycle efficiency* or line efficiency (η) of the assembly line is determined as

$$\begin{aligned}\eta &= \frac{n_t}{n_a}\\ &= \frac{\sum t}{n_a t_c}\end{aligned}$$

where n_a is the actual number of workstations.

The objective of the assembly line balancing can be viewed as to minimize the number of workstations (n_t), which is equivalent to maximizing the *cycle efficiency* (η).

4. Balance Delay *Balance delay* is the amount of idle time on production assembly lines caused by the uneven division of work among operators or stations. It is related to line efficiency as

$$\text{Balance delay} = 1 - \eta$$

IMPORTANT FORMULAS

Functions of PPC

1. Production planning
   (a) Estimating
   (b) Routing
   (c) Scheduling
   (d) Loading
2. Production control
   (a) Dispatching
   (b) Follow-up
   (c) Evaluation

Forecasting

1. Forecasting methods
   (a) Qualitative methods
      i. Market survey
      ii. Delphi method
      iii. Opinion pole
   (b) Quantitative methods
      i. Causal
      ii. Time-series

   Exponential smoothing

$$F_t = F_{t-1} + \mu\,(D_{t-1} - F_{t-1})$$

2. Forecasting errors

$$\text{MAD} = \frac{\sum |e|}{n}$$
$$\text{Bias} = \frac{\sum e}{n}$$
$$\text{MSE} = \frac{\sum e^2}{n-1}$$

Break-Even Point Analysis

$$Q^* = \frac{F}{p-V}$$
$$T(Q^*) = F + VQ^*$$
$$P(Q^*) = (p+V)\,Q^* - F$$

SOLVED EXAMPLES

1. Manufacturing of a product requires fixed investments of Rs. 1,20,000 in a particular year. The estimated sales for this period is 4,00,000. The variable cost per unit for this product is Rs. 6. Determine the break-even point of production if the unit price of the product is Rs. 30.

Solution. Break-even point is calculated as

$$\begin{aligned} Q^* &= \frac{F}{p-V} \\ &= \frac{120000}{28-6} \\ &= 5000 \end{aligned}$$

2. Exponential smoothing factor 0.4 is used to forecast the demand for second period. Actual demand for the first period was 82 units, while forecast was 86 units. Determine the forecast for the next period using exponential smoothing.

Solution. In exponential smoothing method, the forecast for time period t is related to the demand for the previous period D_{t-1} and forecast of previous period F_{t-1}, using smoothing constant μ as

$$\begin{aligned} F_t &= F_{t-1} + \mu\,(D_{t-1} - F_{t-1}) \\ &= 86 + 0.4\,(82-86) \\ &= 84.4 \end{aligned}$$

3. Time study of a production operation revealed a mean cycle time of 15 min. The worker was evaluated to be performing at 85% efficiency. Assuming the allowances to be 12% of the normal time, calculate the standard time for the job.

Solution. The actual average time is 15 min. Adding the rating factor of 0.85, the normal time is $15 \times 0.85 = 12.75$ min. Adding 12% allowances, one finds standard time $12.75 \times 1.12 = 14.28$ min.

4. In a time study exercise, the time observed for an activity was 86 s. The operator had a performance rating of 120. A personal time allowance of 10% is given. Determine the standard time for the activity in seconds.

Solution. Given that the actual time is 86 s with 1.2 rating factor. Therefore, normal time is $86 \times 1.2 = 103.2$ s. Adding 10% allowance, the standard time is $103.2 \times 1.1 = 113.52$ s.

5. In an assembly line for assembling toys, six workers are assigned tasks, which take times of 20, 25, 26, 29, 24 and 26 min, respectively. Calculate the balance delay for the line.

Solution. Balance delay is a measure of the line inefficiency which results from idle time due to imperfect allocation of work among station. In the given case, the maximum time is 29 min. Average time for each activity is

$$\begin{aligned} t_a &= \frac{20+25+26+29+22+26}{6} \\ &= \frac{150}{6} \\ &= 25 \text{ min} \end{aligned}$$

The balance delay is calculated as

$$\begin{aligned} \text{Balance delay} &= 1 - \frac{25}{29} \\ &= 0.137 \\ &= 13.7\% \end{aligned}$$

GATE PREVIOUS YEARS' QUESTIONS

1. A residential school stipulates the study hours as 8.00 pm to 10.30 pm. Warden makes random checks on a certain student 11 occasions a day during the study hours over a period of 10 days and observes that he is studying on 71 occasions. Using 95% confidence interval, the estimated minimum hours of his study during that 10-day period is

(a) 8.5 hours (b) 13.9 hours
(c) 16.1 hours (d) 18.4 hours

(GATE 2003)

Solution. Probability of observing studying is given by

$$P = \frac{71}{11 \times 10}$$

Total studying hours in 10 days are $10 \times 2.5 = 25$ hours. Therefore, minimum number of hours of study are $25 \times P = 16.1364$.

Ans. (c)

2. The sale of cycles in a shop in four consecutive months are given as 70, 68, 82, 95. Exponentially smoothing average method with a smoothing factor of 0.4 is used in forecasting. The expected number of sales in the next month is

(a) 59 (b) 72
(c) 86 (d) 136

(GATE 2003)

Solution. Given that

$$\begin{aligned} \mu &= 0.4 \\ D_1 &= 95 \\ D_2 &= 82 \\ D_3 &= 68 \\ D_4 &= 70 \end{aligned}$$

Forecast for the fifth month is given by

$$\begin{aligned} F_5 &= \mu D_1 + \mu(1-\mu) D_2 + \mu(1-\mu)^2 D_3 \\ &\quad + \mu(1-\mu)^3 D_4 \\ &= 73.52 \end{aligned}$$

Ans. (b)

3. For a product, the forecast and the actual sales for December 2002 were 25 and 20, respectively. If the exponential smoothing constant (α) is taken as 0.2, the forecast sales for January 2003 would be

(a) 21 (b) 23
(c) 24 (d) 27

(GATE 2004)

Solution. Therefore, forecast for next period by exponential smoothing is

$$\begin{aligned} F_t &= F_{t-1} + \alpha(D_{t-1} - F_{t-1}) \\ &= 24 \end{aligned}$$

Ans. (c)

4. A standard machine tool and an automatic machine tool are being compared for the production of a component. The following data refers to the two machines. Setup time, machining time and machine rate for standard machine tool are: 30 min, 22 min, and Rs. 200 per hour, and those for automatic machine tool are 2 hours, 5 min and Rs. 800 per hour, respectively. The break-even production batch size above which the automatic machine tool will be economical to use, will be

(a) 4 (b) 5
(c) 24 (d) 225

(GATE 2004)

Solution. If n is the production batch size, the total costs of production in the two machines are calculated as

$$T(n)_1 = \left(\frac{30}{60} + \frac{22}{60} n\right) \times 200$$
$$T(n)_2 = \left(2 + \frac{5}{60} n\right) \times 800$$

For the break-even production, the total cost of production in two machines will be equal, thus

$$\begin{aligned} T(n)_1 &= T(n)_2 \\ n &= 225 \end{aligned}$$

Ans. (d)

5. A soldering operation was work-sampled over two days (16 hours) during which an employee soldered 108 joints. Actual working time was 90% of the total time and the performance rating was estimated to be 120%. If the contract provides allowance of 20% of the total time available, the standard time for the operation would be

(a) 8 min (b) 8.9 min
(c) 10 min (d) 12 min

(GATE 2004)

Solution. Standard time is equal to actual time multiplied by rating factor plus allowances

$$\text{ST} = \frac{16 \times 60 \times 0.9}{108} \times \frac{120}{100} \times \left(1 + \frac{20}{100}\right)$$
$$= 11.52 \text{ min}$$

Ans. (d)

6. An electronic equipment manufacturer has decided to add a component sub-assembly operation that can produce 80 units during a regular 8-hour shift. This operation consists of three activities as below (ST = Standard Time)

	Activity	ST (min)
M.	Mechanical assembly	12
E.	Electric wiring	16
T.	Test	3

For line balancing the number of workstations required for the activities M, E and T would, respectively, be

(a) 2, 3, 1 (b) 3, 2, 1
(c) 2, 4, 2 (d) 2, 1, 3

(GATE 2004)

Solution. For line balancing, the minimum number of components must be produced by maximum time taking activities, that is, M. For this activity, minimum component produced is $3 \times 30 = 90$ to meet the demand of 80. Others are similarly balanced.

Activity	M	E	T
ST (min)	12	16	3
8-hour output	40	30	160
Workstations	2	3	1

Ans. (a)

7. When 3-2-1 principle in used to support and locate a three-dimensional workpiece during machining, the number of degrees of freedom that are restricted is

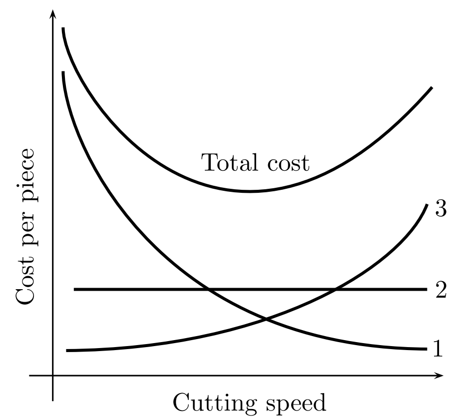
(a) 7 (b) 8
(c) 9 (d) 10

(GATE 2005)

Solution. Total 9 degrees of freedom are restricted in 3-2-1 principle of location.

Ans. (c)

8. The figure below shows a graph, which qualitatively relates cutting speed and cost per piece produced. The three curves, 1, 2 and 3, respectively, represent



(a) machining cost, non-productive cost, tool changing cost
(b) non-productive cost, machining cost, tool changing cost
(c) tool changing cost, machining cost, non-productive cost
(d) tool changing cost, non-productive cost, machining cost

(GATE 2005)

Solution. In the graph shown, 1-machining cost, 2-non-productive cost, 3-tool changing cost.

Ans. (a)

9. An assembly activity is represented on an operation process chart by the symbol

(a) AA (b) A
(c) D (d) O

(GATE 2005)

Solution. Operation process chart is a compact overall view of the whole system of operations involved in the manufacture of a product. Two symbols ◯ and □ are used to represent the operation and flow of the material, respectively.

Ans. (d)

10. The sales of a product during the last four years were 860, 880, 870 and 890 units. The forecast for the fourth year was 876 units. If the forecast for the fifth year, using simple exponential smoothing, is equal to the forecast using a three period moving average, the value of the exponential smoothing constant μ is

(a) 1/7 (b) 1/5
(c) 2/7 (d) 2/5

(GATE 2005)

Solution. By moving average, forecast for fifth year is

$$F_5 = \frac{880+870+890}{3}$$

By exponential smoothing with constant μ, forecast for fifth year is

$$F_5 = 876+\mu(890-876)$$
$$\mu = \frac{2}{7}$$

Ans. (c)

11. A component can be produced by any of the four processes I, II, III and IV. Process I has a fixed cost of Rs. 20 and variable cost of Rs. 3 per piece. Process II has a fixed cost of Rs. 50 and variable cost of Re. 1 per piece. Process III has a fixed cost of Rs. 40 and variable cost of Rs. 2 per piece. Process IV has a fixed cost of Rs. 10 and variable cost of Rs. 4 per piece. If the company wishes to produce 100 pieces of the component, from economic point of view it should choose

(a) Process I (b) Process II
(c) Process III (d) Process IV

(GATE 2005)

Solution. The total cost for the four processes is calculated for 100 pieces as

Process	I	II	III	IV
FC	20	50	40	10
VC	3	1	2	4
TC	320	150	240	410

Therefore, the minimum cost is Rs. 150.

Ans. (b)

12. A welding operation is time-studied during which an operator was pace-rated as 120%. The operator took, on an average, 8 minutes for producing the weld-joint. If a total of 10% allowances are allowed for this operation, the expected standard production rate of the weld-joint (in 8-hour day) is

(a) 45 (b) 50
(c) 55 (d) 60

(GATE 2005)

Solution. Given that

$$R = 1.2$$
$$t_R = 8 \text{ min}$$
$$\text{Allowances} = 10\%$$
$$\text{Time available} = 8\times 60 = 480 \text{ min}$$

The normal time is

$$t_n = t_R \times R = 9.6 \text{ min}$$

Standard time is

$$t_s = t_n \times \left(1+\frac{10}{100}\right) = 10.56 \text{ min}$$

Number of jobs in given time is

$$n = \frac{480}{10.56} = 45.4545$$

Ans. (a)

13. In an MRP system, component demand is

(a) forecasted
(b) established by the master production schedule
(c) calculated by the MRP system from the master production schedule
(d) ignored

(GATE 2006)

Solution. Material Requirement Planning (MRP) is a simple system of calculating arithmetically the requirements of the input materials at different points of time based on actual production plan.

Ans. (c)

14. The following table gives details of an assembly line.

Workstation	Total task time at the workstation (in minutes)
I	7
II	9
III	7
IV	10
V	9
VI	6

What is the line efficiency of the assembly line?

(a) 70% (b) 75%
(c) 80% (d) 85%

(GATE 2006)

Solution. Cycle efficiency of an assembly line of n stations is given by

$$\begin{aligned}\eta &= \frac{\sum t_i}{n \times t_{\max}} \\ &= \frac{48}{6 \times 10} \times 100 \\ &= 80\%\end{aligned}$$

Ans. (c)

15. A moving average system is used for forecasting weekly demand. $F_1(t)$ and $F_2(t)$ are sequences of forecasts with parameters m_1 and m_2, respectively, where m_1 and m_2 $(m_1 > m_2)$ denote the numbers of weeks over which the moving averages are taken. The actual demand shows a steep increase from d_1 to d_2 at a certain time. Subsequently,

(a) neither $F_1(t)$ nor $F_2(t)$ will catch up with the value d_2
(b) both $F_1(t)$ and $F_2(t)$ will reach d_2 in the same period
(c) $F_1(t)$ will attain the value d_2 before $F_2(t)$
(d) $F_2(t)$ will attain the value d_2 before $F_1(t)$

(GATE 2008)

Solution. Forecasting $F(t)$ by moving average system takes average of actual demands of some given period (m), therefore, neither $F_1(t)$ nor $F_2(t)$ will catch up with the value d_2.

Ans. (a)

16. Which of the following forecasting methods takes a fraction of forecast error into account for the next period forecast?

(a) simple average method
(b) moving average method
(c) weighted moving average method
(d) exponential smoothing method

(GATE 2009)

Solution. In exponential smoothing method, the forecast for time period t is related to the demand for the previous period D_{t-1} and forecast of the previous period F_{t-1} as

$$F_t = F_{t-1} + \mu(D_{t-1} - F_{t-1})$$

where μ is called smoothing constant. The method takes care of error $(D_{t-1} - F_{t-1})$ in old forecasting.

Ans. (d)

17. The demand and forecast for February are 12000 and 10275, respectively. Using single exponential smoothing method (smoothing coefficient = 0.25), forecast for the month of March is

(a) 431 (b) 9587
(c) 10706 (d) 11000

(GATE 2010)

Solution. Forecast by exponential smoothing is

$$\begin{aligned}F_t &= F_{t-1} + \mu(D_{t-1} - F_{t-1}) \\ &= 10275 + 0.25(12000 - 10275) \\ &= 10706.3\end{aligned}$$

Ans. (c)

18. Vehicle manufacturing assembly line is an example of

(a) product layout (b) process layout
(c) manual layout (d) fixed layout

(GATE 2010)

Solution. Product layout is generally used in systems where a product has to be manufactured or assembled in large quantities. Machining equipments, and work centers are arranged in the order in which they have to be used.

Ans. (a)

19. Which one of the following is not a decision taken during the aggregate production planning stages?

(a) Scheduling of machines
(b) Amount of labor to be committed
(c) Rate at which production should happen
(d) Inventory to be carried forward

(GATE 2012)

Solution. Aggregate production planning is concerned with the determination of production, inventory, and work force levels to meet fluctuating demand requirements over a planning horizon that ranges from six months to one year.

Ans. (a)

20. In simple exponential smoothing forecasting, to give higher weightage to recent demand information, the smoothing constant must be close to

(a) -1 (b) zero
(c) 0.5 (d) 1

(GATE 2013)

Solution. In exponential smoothing forecasting, the forecast for t^{th} period is determined as

$$F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1})$$

Thus, to give higher weightage to recent demand, that is, $F_t \to D_{t-1}$, $\alpha = 1$.

Ans. (d)

MULTIPLE CHOICE QUESTIONS

1. The quantitative approaches for forecasting does not include

(a) moving averages
(b) exponential smoothing
(c) Delphi
(d) none of the above

2. When using a simple moving average to forecast demand, one would

(a) give equal weight to all demand data
(b) assign more weight to the recent demand data
(c) include new demand data in the average without discarding the earlier data
(d) include new demand data in the average after discarding some of the earlier demand data

3. Which one of the following forecasting techniques is not suited for making forecasts for planning production schedules in the short range?

(a) Moving average
(b) Exponential moving average
(c) Regression analysis
(d) Delphi

4. In a time series forecasting model, the demand for five time periods was 10, 13, 15, 18 and 22. A linear regression fit resulted in an equation

$$F = 6.9 + 2.9t$$

where F is the forecast for period t. The sum of absolute deviations for the five data is:

(a) 2.2 (b) 0.2
(c) -1.2 (d) 24.3

5. In a forecasting model, at the end of period 13, the forecast for period 14 is 75. Actual value in the periods 14 to 16 are constant at 100. If the assumed simple exponential smoothing parameter is 0.5, then the MSE at the end of period 16 is

(a) 820.31 (b) 273.44
(c) 43.75 (d) 14.58

6. A machine is purchased for Rs. 32000, and its assumed life is 20 years. The scrap value at the end of its life is Rs. 8000. If the depreciation is charged by the diminishing balance method, then the percentage reduction in its value, at the end of the first year is

(a) 6.7% (b) 7.1%
(c) 7.2% (d) 7.6%

7. A 750 hours life test is performed on ten components. If one component fails after 350 hours of operation and all others survive the test, then the failure per hour is

(a) 0.000141 (b) 0.000133
(c) 0.00141 (d) 0.00133

8. In an assembly line for assembling toys, five workers are assigned tasks which take times of 10, 8, 6, 9 and 10 minutes respectively. The balance delay for the line is

(a) 43.5% (b) 14.8%
(c) 14.0% (d) 16.3%

9. Preliminary work sampling studies show that machine was idle 25% of the time based on a sample of 100 observations. The number of observations needed for a confidence level of 95% and an accuracy of $\pm 5\%$ is

(a) 400 (b) 1200
(c) 3600 (d) 4800

10. Standardization of products is done to

(a) eliminate unnecessary varieties in design
(b) simplify manufacturing process
(c) make interchangeable manufacture possible
(d) reduce material cost

11. The dispatching function refers to

(a) dispatch of finished goods on order
(b) movement of in process material from shop to shop
(c) dispatch of bills and invoice to the customer
(d) authorizing a production work order to be launched

12. For sales forecasting, pooling of expert opinions is made use of in

(a) statistical correlation
(b) Delphi technique
(c) moving average method
(d) exponential smoothing

13. Fixed investments for manufacturing a product in a particular year is Rs. 80,000. The estimated sales for this period is 2,00,000. The variable cost per unit for this product is Rs. 4. If each unit sold is at Rs. 20, then the break-even point would be

(a) 4000 (b) 5000
(c) 10000 (d) 20000

14. Production scheduling is simpler, and high volume of output and high labor efficiency are achieved in the case of

(a) fixed position layout
(b) process layout
(c) product layout
(d) a combination of line and process layout

15. For a small scale industry, the fixed cost per month is Rs. 5000/-. The variable cost per product is Rs. 20/- and sale price is Rs. 30/- per price. The break-even production per month will be

(a) 300 (b) 460
(c) 500 (d) 10000

16. In manufacturing management, the term 'dispatching' is used to describe

(a) dispatch of sales order
(b) dispatch of factor mail
(c) dispatch of finished product to the user
(d) dispatch of work orders through shop floor

17. Process I requires 20 units of fixed cost and 3 units of variable cost per piece, while process II required 50 units of fixed cost and 1 unit of variable cost per piece. For a company producing 10 piece per day

(a) process I should be chosen
(b) process II should be chosen
(c) either of the two processes could be chosen
(d) a combination of process I and process II should be chosen

18. A production line is said to be balanced when

(a) there are equal number of machines at each workstation
(b) there are equal number of operators at each workstation
(c) the waiting time for service at each station is the same
(d) the operation time at each station is the same

19. Match List I with List II and select the correct answer.

List I (Methods)	List II (Problems)
A. Moving average	1. Assembly
B. Line balancing	2. Purchase
C. Economic batch size	3. Forecasting
D. Johnson algorithm	4. Sequencing

(a) A-1, B-3, C-2, D-4
(b) A-1, B-3, C-4, D-2
(c) A-3, B-1, C-4, D-2
(d) A-3, B-1, C-2, D-4

20. A company intends to use exponential smoothing technique for making a forecast for one of its products. The previous year's forecast has been 78 units and the actual demand for the corresponding period turned out to be 74 units. If the value of the smoothing constant α is 0.2, the forecast for the next period will be

(a) 73 units (b) 75 units
(c) 77 units (d) 78 units

21. Routing in production planning and control refers to the

(a) balancing of load on machines
(b) authorization of work to be performed
(c) progress of work performed
(d) sequence of operations to be performed

22. It is given that the actual demand is 59 units; a previous forecast 64 units and smoothing factor 0.3. What will be the forecast for next period, using exponential smoothing?

(a) 36.9 units (b) 57.5 units
(c) 60.5 units (d) 62.5 units

23. Which one of the following is a qualitative technique of demand forecasting?

(a) correlation and regression analysis
(b) moving average method
(c) Delphi technique
(d) exponential smoothing

24. Which one of the following statements is not correct for the exponential smoothing method of demand forecasting?

(a) Demand for the most recent data is given more weightage

(b) This method requires only the current demand and forecast demand
(c) This method assigns weight to all the previous data
(d) This method gives equal weightage to all the periods

25. In an assembly line, when the workstation times are unequal, the overall production rate of an assembly line is determined by the

(a) fastest station time
(b) slowest station time
(c) average of all station times
(d) average of slowest and fastest station times

26. In an assembly line, what is the balance delay?

(a) Line efficiency$\times$ 100
(b) 100-line efficiency (in %)
(c) Line efficiency/100
(d) None of the above

27. If the fixed cost of the assets for a given period doubles, then how much will the break-even quantity become?

(a) Half the original value
(b) Same as the original value
(c) Twice the original value
(d) Four times the original value

28. In the production of a product, the fixed costs are Rs. 6,000/- and the variable cost is Rs. 10/- per product. If the sale price of the product is Rs. 12/-, the break-even volume of the product to be made will be

(a) 2000 (b) 3000
(c) 4000 (d) 6000

29. Which of the following is the measure of forecast error?

(a) Mean absolute deviation
(b) Trend value
(c) Moving average
(d) Price fluctuation

30. In inventory planning, extra inventory is unnecessarily carried to the end of the planning period when using one of the following lot size decision policies. Identify.

(a) Lot-for-lot production
(b) Economic order quantity (EOQ) lot size
(c) Period order quantity (POQ) lot size
(d) Part period total cost balancing

31. One of the following statements about periodic reordering system (PRS) is not true. Identify,

(a) PRS requires continuous monitoring of inventory levels
(b) PRS is useful in control of perishable items
(c) PRS provides basis for adjustments to account for variations in demand
(d) In PRS, inventory holding costs are higher than in fixed recorder quantity system

32. Fifty observations of a production operation revealed a mean cycle time of 10 min. the worker was evaluated to be performing at 90% efficiency. Assuming the allowances to be 10% of the normal time and the standard time (in seconds) for the job are, respectively,

(a) 10 min, 11.11 min
(b) 9.0 min, 9.9 min
(c) 11.11 min, 10 min
(d) 9.9 min, 9.0 min

33. In a time study exercise, the time observed for an activity was 54 s. The operator had a performance rating of 120. A personal time allowance of 10% is given. The standard time for the activity, in seconds, is

(a) 54 (b) 60.8
(c) 72 (d) 58.32

34. The individual human variability in time studies to determine the production standards is taken care of by

(a) personal allowances
(b) work allowances
(c) rating factor
(d) None of the above

35. A time standard for a data entry clerk is to be set. A job is rated at 120 percent, it takes 30 seconds to enter each record and the allowances are 15%. What is the normal time?

(a) 25 s
(b) 30 s
(c) 36 s
(d) 40 s

36. A stopwatch time study on an operator with a performance rating of 120 yielded a time of 2 min. If allowances of 10% of the total available time are to be given, the standard time of the operation is

(a) 2 min (b) 2.4 min
(c) 2.64 min (d) 2.67 min

NUMERICAL ANSWER QUESTIONS

1. Actual demand and forecasts with smoothing constant 0.2 in first order smoothing are tabulated as below:

Month	Forecast	Demand
Apr	100	200
May	–	50
Jun	–	150

Estimate the forecast for the months of May and June.

2. For a product, selling price is Rs 100 per unit, variable cost of production is Rs 60.00 per unit and fixed cost is Rs 10,00,000.00. Determine the break-even quantity. Due to inflation the variable costs have increased by 10% while fixed costs have increased by 5%. Determine the required percentage increase in the sale price for same value of break-even quantity.

3. The management is interested to know the percentage of idle time of an equipment. The trial study showed that percentage of idle time would be 25%. Determine the number of random observations necessary for 95% level of confidence and $\pm 5\%$ accuracy.

4. There is 8 hours duty and a job should take 40 minutes to complete it. But after 8 hours, an operator is able to complete only 10 such jobs. Determine the operator's performance.

5. Manufacturing of a product requires fixed investments of Rs. 4,50,000 in a particular year. The estimated sales for this period is 8,00,000. The variable cost per unit for this product is Rs. 10. Determine the break-even point of production if unit price of the product is Rs. 50.

ANSWERS

Multiple Choice Questions

1.	(c)	**2.**	(d)	**3.**	(d)	**4.**	(a)	**5.**	(b)	**6.**	(a)	**7.**	(c)	**8.**	(c)	**9.**	(d)	**10.**	(c)
11.	(b)	**12.**	(b)	**13.**	(b)	**14.**	(c)	**15.**	(c)	**16.**	(d)	**17.**	(a)	**18.**	(d)	**19.**	(a)	**20.**	(c)
21.	(d)	**22.**	(d)	**23.**	(c)	**24.**	(d)	**25.**	(b)	**26.**	(b)	**27.**	(c)	**28.**	(b)	**29.**	(a)	**30.**	(d)
31.	(A)	**32.**	(b)	**33.**	(c)	**34.**	(c)	**35.**	(c)	**36.**	(c)								

Numerical Answer Questions

1.	120, 106	**2.**	25000, 8%	**3.**	120
4.	83.33%	**5.**	11250		

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (c) Delphi method is a survey of expert opinion in which the experts review each others' ideas. Thus, it is not a quantitative but an intuitive method of forecasting.

2. (d) In simple moving average method, the average of the demands from several of the most recent periods is taken as the demand forecast for the next time period. The number of past periods to

be used in calculations is selected in the beginning and is kept constant.

3. (d) An intuitive method, Delphi is a survey of expert opinion in which the experts review one anothers idea, which is impossible for production schedules in the short range.

4. (a) The absolute deviation ($|D-F|$) is tabulated as

D	F	$\|D-F\|$
10	9.8	0.2
13	12.7	0.3
15	15.6	0.6
18	18.5	0.5
22	21.4	0.6
	Total	2.2

5. (b) The given data can be tabulated as below

Period	Forecast	Actual Demand
14	75	100
15	–	100
16	–	100

Given that $\mu = 0.5$, thus the forecasts for periods 15 and 16 are determined as

$$\begin{aligned} F_{15} &= F_{14} + \mu(D_{14} - F_{14}) \\ &= 75 + 0.5(100 - 75) \\ &= 87.5 \\ F_{16} &= F_{15} + \mu(D_{15} - F_{15}) \\ &= 87.5 + 0.5(100 - 87.5) \\ &= 93.75 \end{aligned}$$

Thus, the table is populated along with square of the error (E) as

Period	F	D	E	E^2
14	75	100	25	625
15	87.5	100	12.5	156.25
16	93.75	100	6.25	39.06

Hence, mean square error (MSE) is determined as

$$\begin{aligned} \text{MSE} &= \frac{625 + 156.25 + 39.06}{3} \\ &= \frac{820.31}{3} \\ &= 273.43 \end{aligned}$$

6. (a) According to diminishing balance method, the rate of depreciation (%) is determined as

$$\begin{aligned} r &= \left(1 - \sqrt[n]{\frac{S}{C}}\right) \times 100 \\ &= \left(1 - \sqrt[20]{\frac{8000}{32000}}\right) \\ &= (1 - 0.933) \times 100 \\ &= 6.69\% \end{aligned}$$

7. (c) Failure per hour is determined as

$$\frac{9/750 + 1/350}{10} = 0.001486$$

8. (c) Balance delay is a measure of the line inefficiency which results from idle time due to imperfect allocation of work among workstation. In the given case, the maximum time is 10 minutes. Average time for each activity is

$$\begin{aligned} t_a &= \frac{10 + 8 + 6 + 9 + 10}{5} \\ &= \frac{43}{5} \\ &= 8.6 \text{ min} \end{aligned}$$

The balance delay is calculated as

$$\begin{aligned} 1 - \frac{8.6}{10} &= 0.14 \\ &= 14.0\% \end{aligned}$$

9. (d) Given that $p = 0.25$. For 95% confidence and $\pm 5\%$ accuracy, the number of observations are determined as

$$\begin{aligned} n &= 40^2\left(\frac{1}{p} - 1\right) \\ &= 40^2 \times \left(\frac{1}{0.25} - 1\right) \\ &= 40^2 \times 3 \\ &= 4800 \end{aligned}$$

10. (c) Standardization of products is done to make interchangeable manufacture possible.

11. (b) Dispatching deals with the release of resources along with the necessary instruction for the production in accordance with planned items and sequences.

12. (b) Delphi method is a survey of expert opinion in which the experts review each others' ideas.

13. (b) Break-even point is calculated as

$$Q^* = \frac{F}{p-V} = \frac{80000}{20-4} = 5000$$

14. (c) Product layout are designed to result into high volume of output of the specified product.

15. (c) Break-even quantity is given by

$$Q^* = \frac{F}{p-V} = \frac{5000}{30-10} = 500$$

16. (d) Dispatching deals with the release of resources along with the necessary instruction for the production in accordance with planned items and sequences.

17. (a) For the process I, the total cost is $20+10\times3 = 50$, while that for process II is $50+1\times10 = 60$. Hence, process I should be chosen.

18. (d) For line balancing, the operation time at each station is the same. In turn, the minimum number of component must be produced by maximum time taking activities.

19. (a) Moving average is used in forecasting. Line balancing is used in assembly. Economic batch size is used in purchase. Johnson's algorithm is used in sequencing.

20. (c) In exponential smoothing method, weights are assigned in exponential order. The forecast for time period t is related to the demand for the previous period D_{t-1} and forecast of previous period F_{t-1} as

$$F_t = F_{t-1} + \mu\,(D_{t-1} - F_{t-1})$$

where μ is called smoothing constant.

21. (d) The main aim of routing is to determine and ensure the best and cheapest sequence of operations.

22. (d) In exponential smoothing method, the forecast for time period t is related to the demand for the previous period D_{t-1} and forecast of previous period F_{t-1}, using smoothing constant μ as

$$\begin{aligned} F_t &= F_{t-1} + \mu\,(D_{t-1} - F_{t-1}) \\ &= 64 + 0.3\,(59-64) \\ &= 62.5 \end{aligned}$$

23. (c) Delphi method is a survey of expert opinion in which the experts review each others' idea.

24. (d) In exponential smoothing method, weights are assigned in exponential order. The weights decrease exponentially from most recent demand data to older demand data.

25. (b) The overall production rate of an assembly line is determined by the workstation with the slowest time.

26. (b) Balance delay time is the amount of idle time on production assembly lines caused by the uneven division of work among operators or stations. It is related to line efficiency as

$$\text{Balance delay} = 1 - \text{Line efficiency}$$

27. (c) The break-even quantity is directly proportional to the fixed cost.

28. (b) Break-even quantity of the product is given by

$$\begin{aligned} 6000 + 10\times Q &= 12\times Q \\ Q &= 3000 \end{aligned}$$

29. (a) The numerical difference in the forecasted demand and actual demand is known as forecasting error. It is measured in terms of mean absolute deviation (MAD) and Bias.

30. (d) In periodic order quantity (POQ) lot sizing, ordering intervals related to the quantity is computed and hence, carrying cost tends to be lower. Therefore, it is more effective than EOQ. In part period total cost balancing, holding costs and setup costs are balanced as closely as possible for each lot size decision. Thus, while making the balance, any extra inventory is unnecessarily carried to the end of the planning period.

31. (A) The periodic reordering system does not require continuous monitoring but requires periodic monitoring of inventory levels. In part period quantity balancing, the order sizes are determined.

32. (b) The actual average time is 10 min. Adding the rating factor of 0.9, the normal time is $10\times0.9 = 9$ min. Adding 10% allowances, one finds standard time $9\times1.1 = 9.9$ min.

33. (c) Given that the actual time is 54 s with 1.2 rating factor. Therefore, normal time is $54\times1.2 = 64.8$ s. Adding 10% allowance, the standard time is $64.8\times1.1 = 71.28$ s.

34. (c) Personal allowance is allowed to compensate for the time spent by worker in meeting the physical needs, for instance a periodic break in the production routine. In time study for determination of the standard time, rating factor

is considered to take care of individual human variability. Rating is that process during which the time study analyst compares the performance (speed or tempo) of the operator under observation with the observer's own concept of normal performance.

35. (c) Normal time is actual time multiplied by rating factor. For given case, normal time is $30 \times 1.2 = 36$ s.

36. (c) Given that the rating $R = 1.2$, actual time = 2 minutes, allowance=10%. The normal time will be $2 \times 1.2 = 2.4$ min. Including the allowances of 10% in this normal time, the standard time comes to $2.4 \times 1.1 = 2.64$ min.

Numerical Answer Questions

1. Given that $\alpha = 0.2$. Forecast for the month of May is

$$\begin{aligned} F_t &= F_{t-1} + \alpha (D_{t-1} - F_{t-1}) \\ &= 100 + 0.2 (200 - 100) \\ &= 120 \end{aligned}$$

Forecast for the month of June is calculated as

$$\begin{aligned} F_t &= F_{t-1} + \alpha (D_{t-1} - F_{t-1}) \\ &= 120 + 0.2 (50 - 120) \\ &= 106 \end{aligned}$$

2. Given that

$$\begin{aligned} F &= \text{Rs. } 1000000 \\ V &= \text{Rs. } 60/\text{unit} \\ p &= 100 \text{ per unit} \end{aligned}$$

Break-even quantity is determined as

$$\begin{aligned} Q^* &= \frac{F}{p - V} \\ &= \frac{1000000}{100 - 60} \\ &= 25000 \text{ units} \end{aligned}$$

The new sale price shall be given by

$$\begin{aligned} \frac{F}{p - V} &= \frac{1.05F}{p' - 1.1V} \\ p' - 1.1V &= 1.05 (p - V) \\ p' &= 1.05p + 0.05V \\ &= 1.05 \times 100 + 0.05 \times 60 \\ &= \text{Rs. } 108 \end{aligned}$$

The percentage increase in price rate is

$$\begin{aligned} \% \text{ increase} &= \frac{p' - p}{p} \times 100 \\ &= 8\% \end{aligned}$$

3. The number of samples (n) required for desired accuracy s and confidence level expressed in terms of k is given by

$$n = \left(\frac{k}{s}\right)^2 \left(\frac{1}{p} - 1\right)$$

For desired accuracy of 5% and 95% confidence level,

$$\begin{aligned} \left(\frac{k}{s}\right)^2 &= 40 \\ p &= 0.25 \end{aligned}$$

Therefore,

$$\begin{aligned} n &= \left(\frac{k}{s}\right)^2 \left(\frac{1}{p} - 1\right) \\ &= 40 \times \left(\frac{1}{0.25} - 1\right) \\ &= 120 \end{aligned}$$

4. In 8 hours duty the operator is expected to complete $8 \times 60/40 = 12$ jobs but he is found able to complete only 16 jobs, Thus, operator's performance is

$$\begin{aligned} R &= \frac{10}{12} \\ &= 83.33\% \end{aligned}$$

5. Break-even point is calculated as

$$\begin{aligned} Q^* &= \frac{F}{p - V} \\ &= \frac{450000}{50 - 10} \\ &= 11250 \end{aligned}$$

CHAPTER 18

INVENTORY CONTROL

Inventory refers to any kind of resource that has economic value and is maintained to fulfill the present and future needs of an organization. In a production system, *inventory* accounts for a large percentage of working material. It can be of raw material, goods in process, finished goods, and goods assisting in production. Creation and maintenance of inventory is very essential for continuity of production by providing decoupling function between stages of production, ensuring full utilization of sources, and successfully meeting the variations in customer demands. It also takes advantage of lowering the total material cost by availing the quantity discounts. Inventories involve capital tied up, storage and handling cost, deterioration, pilferage, and obsolescence. Inventory control is a procedure of production control by means of which inventory of appropriate quantity is availed for manufacturing operations without entailing the cost involved in overstocking or under-stocking at any instant.

18.1 BASIC CONCEPTS

Informal procedure of inventory management by intuitive determinations can work well with a small company as the number of items are few. However, for an enterprise requiring a wide variety of inventory items having different usage rates, informal systems tend to create problems that result in higher costs and interruptions in production. In such cases, a formal system of inventory management can produce substantial savings. This section describes basic principles that are important for designing a formal system of inventory management.

18.1.1 Types of Inventory

Inventory can be following types:

1. *Lot Size Inventory* Lot size inventory is the periodic inventory to meet the average replenishment in production. Decision on lot size and timings are very important for economical use of personnel and equipment. Continuous production is suitable for high-volume items. However, low-volume items should be preferably produced periodically and in economic lots.

2. *Transit Inventory* Movement of inventory cannot be instantaneous. To prevent delay in supply to work centers, some inventory is essentially in transit between work centers. This is called *transit or process inventory.*

3. *Buffer Inventory* Additional inventory is required to accommodate the uncertainties of the demand and the *lead time*. This stock is called *buffer* or *safety inventory.* However, there is a minor difference between these two terms; buffer inventory

refers inventory reserved to meet customer demand when customer ordering patterns vary, whereas *safety inventory* refers the inventory reserved to meet the customer demand when internal constraints or inefficiencies disrupt in the process flow.

4. Decoupling Inventory Stock points are created between adjacent stages of production in order to achieve decoupling of the stages. This is essential to prevent disruptive effect of breakdown on the entire system of production. Stock points are supplied with *decoupling inventory.*

By convention, manufacturing inventory generally refers to items that contribute or become part of a product. Manufacturing inventory is typically classified into raw materials, finished products, component parts, supplies, and work-in-process inventory.

18.1.2 Costs of Inventory

Apart from purchase cost, two types of costs are associated with inventory:

1. Carrying Cost The cost associated with carrying the inventory in the stores is called *carrying cost* or *holding cost* ($H(Q)$). It includes rent, interest of the money locked up, insurance premium, salaries of store keepers, deterioration, etc.

   If the average inventory held is $\bar{Q}$ units and average holding cost is h (Rs. per unit per annum), then carrying cost is written as

$$H(Q) = \bar{Q}h$$

2. Ordering Cost The cost associated with the procedure for the placement of purchase orders of the inventory is called *ordering cost* or *setup cost* ($O(Q)$). It includes the cost of stationary, postage, telephone, traveling expenses, material handling, etc. The ordering cost is independent of the batch size of the order.

   If A is the ordering cost per order and annual demand is D for ordered quantity Q, then total D/Q orders will be placed per annum. Therefore, ordering cost is

$$O(Q) = \frac{D}{Q}A$$

The total *annual inventory cost* $T(Q)$ is the sum of total annual ordering cost $O(Q)$ and total annual holding or carrying cost $H(Q)$, mathematically

$$T(Q) = O(Q) + H(Q)$$

The costs associated with non-delivery of a demanded item is called *stock-out cost* or *shortage cost.* The backlogging of orders often involves extra cost for administration, price discounts for late deliveries, material handling, and transportation. There can be situation of loss in sale due to selection of alternative suppliers. *Service level* is a measure of the degree of stock-out protection provided by a given amount of safety inventory.

18.1.3 Inventory Demand

Size of the demand is the number of units required in each period. The demand pattern of an item can be of two types:

1. Deterministic Demand The inventory model using the assumption of constant and known *demand* for the item and *lead time* are called *deterministic models.* Here, the stock is replenished as soon as the stock reaches the point of exhaustion. In such a situation, there is no need to maintain any extra stock.

2. Probabilistic Demand When the demand over a period is uncertain, but can be predicted by a probability distribution, the demand is called *probabilistic demand.*

18.1.4 Inventory Replenishment

The size of replenishment orders affects inventory level to be maintained at various stock points. Inventory policies are aimed at determining when to replenish the inventories and how much to order at one time. This is compounded by price discounts and by the need to prevent disruption in operations due to delays in supply time and temporary increase in requirements.

The period of time between two consecutive placements of orders is called *ordering cycle. Lead time* is the time between placing an order and its delivery. *Reorder point* (ROP) is defined as the amount of stock equal to the demand during the *lead time*, which will be consumed by the time the fresh delivery is due to arrive.

Overstocking, the hard core of inventory mismanagement, results in the wastage of scarce resources of a business enterprise. Inventories are financed through borrowings, thus involve interest charges payable to the creditors, irrespective of the profits. Understocking is as disastrous or even more fatal to the manufacturing enterprise. It involves the risk of short supply and as a result the capacity cannot be utilized optimally.

18.1.5 Inventory Control Systems

An inventory system provides the organizational structure and the operating policies for maintaining and

controlling the inventory. Inventory systems can be broadly classified into two categories:

1. *Static Inventory System* These are single purchase decision systems for a single period without replenishment. The inventory cost is optimized between the cost of having and the cost of not having an item in stock.

2. *Dynamic Inventory System* These are multi-purchase decision systems, concerned with consumable spares, that make replenishment decisions on an ongoing basis over time. Majority of inventory problems belong to this type of situation.

18.2 EOQ MODELS

Objective of the EOQ models is to determine the optimal order quantity that minimizes the total incremental cost of holding an inventory and processing order when demand occurs at a constant rate. The optimum level of quantity is called *economic order quantity* (EOQ). There are various EOQ models for different situations, but the following two models are relevant in the present context of study.

18.2.1 Simple EOQ Model

Simple EOQ model[1] is based on the assumptions evident from the inventory replenishment cycle depicted in Fig. 18.1. The demand is deterministic and constant. The depletion rate of the inventory is also constant. The inventory is replenished immediately when stock level reaches zero level.

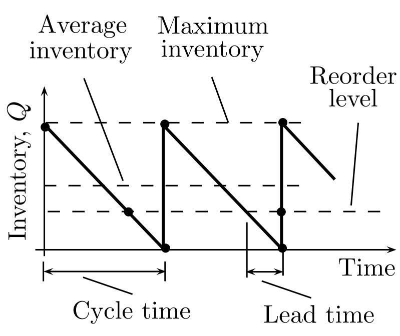


Figure 18.1 Simple EOQ model.

Let Q units be the order quantity to cover annual demand D (units per annum). The components of inventory costs are determined as follows:

[1]Simple EOQ model was developed in 1913 by F. Wilson Harris but R. H. Wilson did its in-depth analysis.

1. *Ordering Cost* There will be D/Q number of orders per year. Therefore, annual ordering cost of the inventory is determined as

$$O(Q) = \frac{D}{Q}A$$

where A is the cost of one order.

2. *Holding Cost* The inventory is maximum (Q) at the replenishment and gradually decreases to zero. Annual average holding cost of the inventory is determined as

$$H(Q) = \frac{Q}{2}h$$

where h is the annual holding cost per unit of the inventory.

Total inventory cost is determined as

$$T(Q) = \frac{D}{Q}A + \frac{Q}{2}h \tag{18.1}$$

Figure 18.2 shows the variation of $O(Q)$, $H(Q)$, $T(Q)$ with respect to order quantity Q.

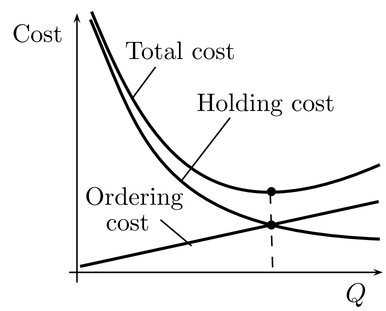


Figure 18.2 Total cost in simple EOQ model.

The order quantity associated with the minimum value of $T(Q)$ can be determined as

$$\frac{dT(Q)}{dQ} = 0$$

$$-\frac{DA}{Q^2} + \frac{h}{2} = 0$$

$$Q = \sqrt{\frac{2AD}{h}}$$

This value of order quantity is called the *economic order quantity* and it is denoted by Q^*. Therefore,

$$Q^* = \sqrt{\frac{2AD}{h}} \tag{18.2}$$

This equation is called *Wilson Harris formula.*

Total annual inventory cost at economic order quantity can be determined by using Eq. (18.1) as

$$\begin{aligned} T(Q^*) &= \frac{D}{Q^*}A + \frac{Q^*}{2}h \\ &= \sqrt{2ADh} \end{aligned}$$

The optimal number of production runs (n^*) is

$$n^* = \frac{D}{Q^*}$$

The optimum purchase cycle time (t^* in days) is

$$\begin{aligned} t^* &= \frac{Q^*}{\text{Demand per day}} \\ &= \frac{Q^*}{D/365} \\ &= \frac{365}{n^*} \end{aligned}$$

Using Eq. (18.2), the economic order quantity is found to be inversely proportional to the square root of the holding cost:

$$Q^* \propto \frac{1}{\sqrt{h}}$$

Let s be the stock-out cost per incidence. Thus, the system has to attain maximum inventory level Q^* at holding cost of $h+s$. The carrying cost associated with safety stock will be s. Therefore, the safety stock can be determined as

$$Q_s = Q^* \times \sqrt{\frac{s}{s+h}}$$

18.2.2 Build-Up EOQ Model

Build-up EOQ model is applied when inventory is not replenished in one shot but rather continuously over a time period. Let the total order quantity Q be the build-up at a constant *production rate* p over a period t_p ($= Q/p$) [Fig. 18.3].

The average inventory level would be determined not only by the lot size Q, but will also be affected by the production rate (p) and the *depletion rate* (d):

$$\begin{aligned} \bar{Q} &= \frac{p-d}{2}t_p \\ &= \frac{Q}{2}\left(1-\frac{d}{p}\right) \end{aligned}$$

Total inventory cost is

$$T(Q) = \frac{D}{Q}A + \frac{Q}{2}\left(1-\frac{d}{p}\right)h$$

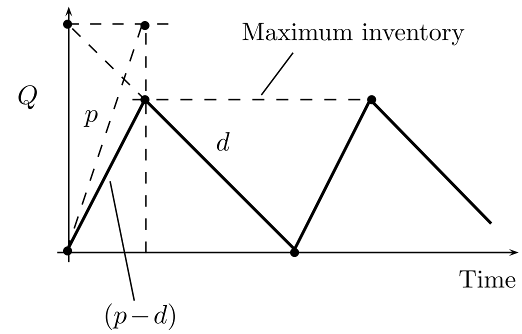


Figure 18.3 Build-up EOQ model.

Minimization of $T(Q)$ with respect to Q gives the expression for build-up EOQ as

$$Q^* = \sqrt{\frac{2AD}{h\,(1-d/p)}}$$

The effective holding cost in build-up model against simple EOQ model is given by

$$h' = h\left(1-\frac{d}{p}\right)$$

18.3 PROBABILISTIC INVENTORY MODELS

Simple inventory models are based on constant demand and supply lead time. However, in real applications, the demand can be uncertain and lead time often varies significantly. In such situations, the risks of stock-out can be reduced by carrying safety or buffer stock, which requires additional funds. Therefore, probabilistic inventory models are developed to balance the risks and minimize the incremental costs. A detailed discussion on these models is out of context of the book.

Single period model is used in uncertain demand situations. The key trade-off in this model is to balance the cost of overstocking the inventory if there are leftovers at the end of the cycle with the cost of understocking in terms of the profit foregone if the demand turns out to be higher than the stock at hand.

Order quantity - reorder point (Q-ROP) model is developed for situations when item is continuously demanded at constant rate for a long time horizon but with wide variations in lead time. This model takes into consideration both the expected demand during lead time and safety stock.

Least unit cost (LUC) technique chooses a lot size that equals the demand of some k (> 0) periods in future. The average holding and ordering cost per unit

is computed for each $k = 1, 2, 3$, etc. The computation starts from $k = 1$ and increasing k by 1 until the average cost per unit starts increasing. The best k is the last one up to which the average cost per unit decreases.

18.4 SELECTIVE APPROACHES

In a large production system, all items of the inventory cannot be controlled uniformly. Therefore, selective approaches are used for inventory control on priority basis. These techniques are based on nature and usage rate of inventories. Some of them are described as follows:

1. ABC Analysis *ABC analysis* is based on the concept "thick on the best, thin on the rest." All inventory items are classified in three distinct categories: (A) High consumption - strict control, (B) Moderate consumption - fair control, and (C) Low consumption - open storage. The analysis is also used as an acronym of Always Better Control.

   The analysis is accomplished by plotting the usage value of the items to obtain the ABC distribution curve which is also called the *Pareto curve*[2] [Fig. 18.4].

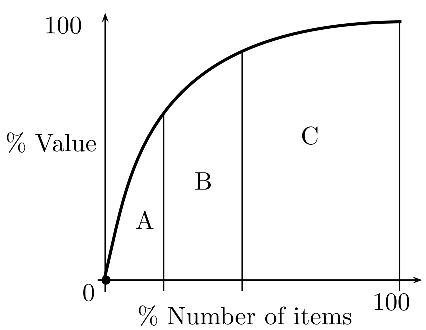


Figure 18.4 ABC Classification.

2. VED Analysis *VED analysis* is applied for spare parts. The items are classified as vital, essential and desirable based on their importance to the production process. Vital items are the items without which the production process would come to a standstill. Essential items would adversely affect the efficiency of the production system although the system would not altogether stop for want of these items. Desirable items do not cause an immediate loss of production.

[2]The Pareto curve is named after the Italian economist Vilfredo Pareto (1848-1923). In 1906, he made the famous observation that twenty percent of the population owned eighty percent of the property in Italy, later generalised by Joseph M. Juran into the Pareto principle (also termed the 80-20 rule).

3. HML Analysis In *HML analysis*, the items are classified based on their unit value (not annual usages) as high value, medium value, and low value items. This analysis helps in administrative decisions in an organization, such as procurement authority which is generally based on the hierarchy and price of material.

4. SDE Analysis *SDE analysis* is done based on purchasing problems, such as long lead time, scarcity and low availability, geographically scattered sources, uncertainty in supply. The items are classified as Scarce, Difficult, and Easy items.

5. FSN Analysis In *FSN analysis*, the items are classified into Fast, Slow-moving, and Non-moving items, based on their issue rates or consumption pattern. This analysis enables in controlling the obsolescence. High consumption items desired attention for their uninterrupted procurement. Non-moving items (almost nil consumption) indicate obsolete inventories due to changes in their specifications.

6. GOLF Analysis *GOLF analysis* is carried out mainly based on the source of material. GOLF stands for Government, Ordinary, Local and Foreign. This classification helps in describing the special procedure to be followed for procurement of materials from specialized sources.

7. XYZ Analysis *XYZ analysis* is based on the value of the inventory undertaken during the closing of annual accounts. X items are those having high value, whereas Z items are of low value. This analysis is used to identify items causing locking up money in the stock.

18.5 JUST-IN-TIME PRODUCTION

In a *just-in-time* (JIT) production system[3], materials are produced only at the time when they are needed in required quantity. For this, the companies make agreements with their vendors. This is also called *kanban system*. Literal meaning of *kanban* is visual record, hence it is also known as *card system*.

Jidoka, along with just-in-time, is one of the two main pillars of the Toyota production system. Japanese word "ji-do-ka" means automation with a human mind, and implies the ability of production lines to be stopped in the event of such problems as equipment malfunctions, quality problems or work being late. This is desired to prevent passing on defects.

[3]This system was found by Taiichi Ohono, vice president of Toyota Motor Company of Japan.

IMPORTANT FORMULAS

Basic Concepts

1. Types of inventory
   (a) Lot size inventory
   (b) Transit inventory
   (c) Buffer inventory
   (d) Decoupling inventory
2. Inventory costs

$$O(Q) = \frac{D}{\bar{Q}}A$$
$$H(Q) = \bar{Q}h$$
$$T(Q) = O(Q) + H(Q)$$

Simple EOQ Model

$$O(Q) = \frac{D}{Q}A$$
$$H(Q) = \frac{Q}{2}h$$
$$T(Q) = \frac{D}{Q}A + \frac{Q}{2}h$$
$$Q^* = \sqrt{\frac{2AD}{h}}$$
$$T(Q^*) = \sqrt{2ADh}$$
$$n^* = \frac{D}{Q^*}$$
$$Q_s = Q^* \times \sqrt{\frac{s}{s+h}}$$

Build-Up EOQ Model

$$\bar{Q} = \frac{Q}{2}\left(1 - \frac{d}{p}\right)$$
$$T(Q) = \frac{D}{Q}A + \frac{Q}{2}\left(1 - \frac{d}{p}\right)h$$
$$Q^* = \sqrt{\frac{2AD}{h\left(1 - \frac{d}{p}\right)}}$$
$$h' = h\left(1 - \frac{d}{p}\right)$$

SOLVED EXAMPLES

1. An aircraft manufacturing company has an annual demand of one million rivets. Company personnel estimate that the cost of placing one order and annual carrying cost per piece are both Rs.1200. If the stock-out costs are estimated to be nearly Rs. 650 each time the company runs out-of-stock, determine the safety stock justified by the carrying cost.

Solution. Using

$$Q_s = Q^* \times \sqrt{\frac{s}{s+h}}$$
$$= \sqrt{\frac{2AD}{h}} \times \sqrt{\frac{s}{s+h}}$$
$$= 838.27$$

2. In computing Wilson's economic lot size for an item, by mistake the demand rate estimate used was 25% higher than the true demand rate. Determine the error in computation of the total cost of setups plus inventory holding per unit time.

Solution. For Wilson's economic lot size, the total cost is given by

$$T(Q^*) = \sqrt{2ADh}$$

Therefore, the percentage increase in $T(Q^*)$ due to 25% higher demand is

$$\% \text{ increase} = \sqrt{1.25} - 1$$
$$= 11.8\%$$

3. For inventory control, following data is given for a factory.

Annual demand	=	26,400 units
Order cost per order	=	Rs. 75
Carrying cost	=	Rs. 15/unit/year
Production rate	=	120 units/day
Working days	=	240/year

Determine the number of production runs and total incremental cost (rupees).

Solution. Economic lot size (Build-up) is determined as

$$Q^* = \sqrt{\frac{2AD}{h(1-d/p)}}$$
$$= 1180 \text{ units}$$

Thus, the number of production runs is

$$R = \frac{D}{Q^*}$$
$$= 22.37$$

The incremental cost, that is, total inventory cost is determined as

$$T(Q^*) = \sqrt{2ADh\left(1 - \frac{d}{p}\right)}$$
$$= \text{Rs. } 1284.5$$

GATE PREVIOUS YEARS' QUESTIONS

1. Market demand for springs is 8,00,000 per annum. A company purchases these springs in lots and sells them. The cost of making a purchase order is Rs. 1,200. The cost of storage of springs is Rs. 120 per stored piece per annum. The economic order quantity is

(a) 400 (b) 2,828
(c) 4,000 (d) 8,000

(GATE 2003)

Solution. Given that

$$D = 800000 \text{ per annum}$$
$$A = \text{Rs. } 1200/\text{order}$$
$$h = \text{Rs. } 120/\text{piece-annum}$$

Therefore, the economic order quantity is

$$Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \times 1200 \times 800000}{120}} = 4000$$

Ans. (c)

2. There are two products P and Q with the following characteristics:

Product	Demand (Units)	Order Cost (Rs./order)	Holding Cost (Rs./unit /year)
P	100	50	4
Q	400	50	1

The economic order quantity (EOQ) of products P and Q will be in the ratio

(a) 1:1 (b) 1:4
(c) 1:2 (d) 1:8

(GATE 2004)

Solution. Given that for product P

$$D = 100 \text{ units}$$
$$A = \text{Rs. } 50/\text{order}$$
$$h = \text{Rs. } 4/\text{order}$$

and for product Q

$$D = 400 \text{ units}$$
$$A = \text{Rs. } 50/\text{order}$$
$$h = \text{Rs. } 1/\text{order}$$

The economic order quantity is given by

$$Q^* = \sqrt{\frac{2AD}{h}}$$

Therefore, the ratio of EOQ for products P and Q will be

$$\frac{Q_P^*}{Q_Q^*} = \sqrt{\frac{50}{50} \times \frac{100}{400} \times \frac{1}{4}} = \frac{1}{1}$$

Ans. (a)

3. A company has an annual demand of 1000 units, ordering cost of Rs. 100/order and carrying cost of Rs. 100/unit year. If the stock-out costs are estimated to be nearly Rs. 400 each time the company runs out-of-stock, the safety stock justified by the carrying cost will be

(a) 4 (b) 20
(c) 40 (d) 100

(GATE 2004)

Solution. If s is cost of stock-out cost, then the safety level of stock is determined as

$$Q_s = Q^* \times \sqrt{\frac{s}{s+h}}$$

Given that

$$D = 1000 \text{ units}$$
$$A = \text{Rs. } 100/\text{order}$$
$$h = \text{Rs. } 100/\text{unit-year}$$
$$s = \text{Rs. } 400$$

Therefore,

$$Q_s = \sqrt{\frac{2AD}{h}} \times \sqrt{\frac{s}{s+h}} = \sqrt{\frac{2 \times 100 \times 1000}{100}} \times \sqrt{\frac{400}{400+100}} = 40$$

Ans. (c)

4. The distribution of lead time demand for an item is as follows:

Lead Time Demand	Probability
80	0.20
100	0.25
120	0.30
140	0.25

The reorder level is 1.25 times the expected value of the lead time demand. The service level is

(a) 25% (b) 50%
(c) 75% (d) 100%

(GATE 2005)

Solution. Service level is a measure of the degree of stock-out protection provided by a given amount of safety inventory. Given that the expected value of the lead time demand is

$$= \frac{80\times 0.2+100\times 0.25+120\times 0.30+140\times 0.25}{0.20+0.25+0.30+0.25}$$
$$= 112$$

The reorder level is

$$= 112\times 1.25$$
$$= 140$$

Thus, the reorder level (140) is equal to the maximum lead time demand (140). therefore, the service level is 140/140 = 100%.

Ans. (d)

5. Consider the following data for an item.

Annual demand	:	2500 units per year
Ordering cost	:	Rs. 100 per order
Inventory holding rate	:	25% of unit price

Unit rates for < 500 is Rs. 10 and for ≥ 500 is Rs. 9. The optimum order quantity (in units) is

(a) 447 (b) 471
(c) 500 (d) $\geq$600

(GATE 2006)

Solution. Given that

$$D = 2500 \text{ units per year}$$
$$A = \text{Rs. } 100/\text{order}$$
$$h = 10\% \text{ of price rate}$$

Assuming EOQ (Q^*) less than 500,

$$h = 0.25\times \text{Rs. } 10 \text{ per unit}$$
$$= \text{Rs. } 2.5 \text{ per unit}$$

For this,

$$Q^* = \sqrt{\frac{2AD}{h}}$$
$$= \sqrt{\frac{2\times 100\times 2500}{2.5}}$$
$$= 447.214$$

For $Q^* > 500$,

$$h = 0.25\times \text{Rs. } 9 \text{ per unit}$$
$$= \times\text{Rs. } 2.25 \text{ per unit}$$

Therefore,

$$Q^* = \sqrt{\frac{2\times 100\times 2500}{2.25}}$$
$$= 471.405$$

which is not found true. Since Q^* for both cases is less than 500, therefore, answer is the first one, that is,

$$Q^* = 447.214$$

Ans. (a)

6. A firm is required to procure three items (P, Q and R). The prices quoted for these items (in rupees) by suppliers S1, S2 and S3 are given in table.

Item	Suppliers (Rs.)		
	S1	S2	S3
P	110	120	130
Q	115	140	140
R	125	145	165

The management policy requires that each item has to be supplied by only one supplier and one supplier supply only one item. The minimum total cost (in rupees) of procurement to the firm is

(a) 350 (b) 360
(c) 385 (d) 395

(GATE 2006)

Solution. The minimum cost is possible when rates for each item are minimum. Out of given table, the minimum value is selected and minimum cost is found to be

$$C = 110+140+145$$
$$= \text{Rs. } 395$$

Ans. (d)

7. A stockist wishes to optimize the number of perishable items he needs to stock in any month

in his store. The demand distribution for this perishable item is

Demand (units)	2	3	4	5
Probability	0.10	0.35	0.35	0.20

The stockist pays Rs. 70 for each item and he sells each at Rs. 90. If the stock is left unsold in any month, he can sell the item at Rs. 50 each. There is no penalty for unfulfilled demand. To maximize the expected profit, the optimal stock level is

(a) 5 units
(b) 4 units
(c) 3 units
(d) 2 units

(GATE 2006)

Solution. Average demand per month is calculated as

$$A = \frac{2\times 0.1+3\times 0.35+4\times 0.35+5\times 0.2}{0.10+0.35+0.35+0.20}$$
$$= 3.65 \text{ units}$$

Therefore, the optimal stock level should meet the average demand of 3.65 units, that is 4 units. Break-even value of stock level (say x) for the maximum demand 5 is

$$\begin{aligned}(90-70)\,x &= (5-x)\,(90-50)\\ 20x &= 40\,(5-x)\\ x &= 10-2x\\ x &= 3.33\\ &\approx 3 \text{ units}\end{aligned}$$

which also corresponds to the average demand.

Ans. (c)

8. Capacities of production of an item over 3 consecutive months in regular time are 100, 100 and 80, and in overtime are 20, 20, and 40. The demands over those 3 months are 90, 130 and 110. The cost of production in regular time and overtime are, respectively, Rs. 20 per item and Rs. 24 per item. Inventory carrying cost is Rs. 2 per item per month. The levels of starting and final inventory are nil. Back order is not permitted. For minimum cost of plan, the level of planned production in overtime in the third month is

(a) 40 (b) 30
(c) 20 (d) 0

(GATE 2007)

Solution. As the demand for the first month is 90 while regular production capacity is 100, therefore, 10 pieces can be made in the first month which will cost only carrying cost of Rs. 2 per month. Again for the second month, demand is 130 but capacity is 100, therefore, 20 pieces should be made in this period by using overtime. Again demand for the third month is 110 but capacity is 80, therefore, 30 pieces must be produced in overtime.

Ans. (b)

9. The maximum level of inventory of an item is 100 and it is achieved with infinite replenishment rate. The inventory becomes zero over one and half month due to consumption at a uniform rate. This cycle continues throughout the year. Ordering cost is Rs. 100 per order and inventory carrying cost is Rs. 10 per item per month. Annual cost (in rupees) of the plan, neglecting material cost, is

(a) 800 (b) 2800
(c) 4800 (d) 6800

(GATE 2007)

Solution. Number of replenishment cycles per year $= 12/1.5 = 8$. Per year ordering cost is $8\times$Rs. 100 = Rs. 800. Annual carrying cost is $100/2\times 8\times$Rs. 10 = Rs. 4000. Total cost is Rs. $800+4000$ = Rs. 4800.

Ans. (c)

10. In a machine shop, pins of 15 mm diameter are produced at a rate of 1000 per month and the same is consumed at a rate of 500 per month. The production and consumption continues simultaneously till the maximum inventory is reached. Then inventory is allowed to reduced to zero due to consumption. The lot size of production is 1000. If backlog is not allowed, the maximum inventory level is

(a) 400 (b) 500
(c) 600 (d) 700

(GATE 2007)

Solution. In one month, one lot of 1000 units is produced and 500 is consumed. Therefore, per month addition in inventory is 500 units, which is consumed in next month to allow minimum inventory to zero. Therefore, maximum inventory level is 500.

Ans. (b)

11. The net requirements of an item over 5 consecutive weeks are 50-0-15-20-20. The inventory carrying cost and ordering cost are Re. 1 per item per week and Rs. 100 per order, respectively. Starting inventory is zero. Use "Least Unit Cost Technique" for developing the plan. The cost of the plan (in rupees) is

(a) 200 (b) 250
(c) 255 (d) 260

(GATE 2007)

Solution. Least unit cost (LUC) technique chooses a lot size that equals the demand of some K periods in future, where $K > 0$. The average holding and ordering cost per unit is computed for each $K = 1, 2, 3$, etc. starting from $K = 1$ and increasing K by 1 until the average cost per unit starts increasing. The best K is the last one up to which the average cost per unit decreases. For the given case, on following this procedure the cost comes out to be 250.

Ans. (b)

12. A company uses 2555 units of an item annually. Delivery lead time is 8 days. The reorder point (in number of units) to achieve optimum inventory is

(a) 7 (b) 8
(c) 56 (d) 60

(GATE 2009)

Solution. As 2555 units are required in 365 days, therefore, 8 days quantity is

$$\begin{aligned}\text{Reorder level} &= \frac{2555}{365} \times 8 \\ &= 56\end{aligned}$$

Ans. (c)

13. Annual demand for window frames is 10000. Each frame costs Rs. 200 and ordering cost is Rs. 300 per order. Inventory holding cost is Rs. 40 per frame per year. The supplier is willing to offer 2% discount if the order quantity is 1000 or more, and 4% if the order quantity is 2000 or more. If the total cost is to be minimized, the retailer should

(a) order 200 frames every time
(b) accept 2% discount
(c) accept 4% discount
(d) order economic order quantity

(GATE 2010)

Solution. Given that

$$\begin{aligned}D &= 10000 \text{ units} \\ p &= \text{Rs. } 200 \\ A &= \text{Rs. } 300/\text{order} \\ h &= \text{Rs. } 40/\text{frame-year}\end{aligned}$$

Therefore, the economic order quantity is

$$\begin{aligned}Q^* &= \sqrt{\frac{2AD}{h}} \\ &= 387.298\end{aligned}$$

Then total cost of purchase and inventory is

$$\begin{aligned}T(Q^*) &= \frac{D}{Q^*}A + \frac{Q^*}{2}h + p \times D \\ &= 2.015 \times 10^6\end{aligned}$$

The two offers can also be examined separately. For the first offer,

$$\begin{aligned}p &= 200 \times 0.98 \\ Q &= 10000 \\ T(Q) &= \frac{D}{Q}A + \frac{Q}{2}h + p \times D \\ &= 1.983 \times 10^6\end{aligned}$$

For the second offer,

$$\begin{aligned}p &= 200 \times 0.96 \\ Q &= 20000 \\ T(Q) &= \frac{D}{Q}A + \frac{Q}{2}h + p \times D \\ &= 1.9615 \times 10^6\end{aligned}$$

In view of above, the last offer costs least.

Ans. (c)

14. The word "kanban" is most appropriately associated with

(a) economic order quantity
(b) just-in-time production
(c) capacity planning
(d) product design

(GATE 2011)

Solution. In a just-in-time (JIT) production system, materials are produced only at the time when they are needed and in required quantity. For this, the companies make agreements with their vendors. This is also called *kanban system*. Literal meaning of *kanban* is visual record, hence it is also known as card system.

Ans. (b)

MULTIPLE CHOICE QUESTIONS

1. For a inventory, the ordering cost per order and holding cost per unit are equal. If annual demand becomes four times, then the size of economic order quantity will become

(a) four times (b) two times
(c) remain the same (d) indeterminate

2. For annual demand D, ordering cost per order A, and holding cost per unit h, the number of orders for minimum total inventory cost is equal to

(a) $\sqrt{hD/A}$ (b) $\sqrt{hD/(2A)}$
(c) $\sqrt{A/(hD)}$ (d) $\sqrt{2A/(hD)}$

3. For annual demand D, ordering cost per order A, and holding cost per unit h, the total cost of inventory for economic order quantity is equal to

(a) $\sqrt{hD/A}$ (b) $\sqrt{2AD/h}$
(c) $\sqrt{2ADh}$ (d) $\sqrt{ADh}$

4. Keeping other parameters constant, a 10% variation in annual demand will cause

(a) 5% variation in economic order quantity
(b) 5% variation in total cost for economic order quantity
(c) both (a) and (b)
(d) none of the above

5. Just-In-Time production is also known as

(a) lean production
(b) group technology
(c) cellular production
(d) all of the above

6. Jidoka is the principle of stopping work immediately whenever

(a) any severe problem or abnormality occurs
(b) any disastrous event occurs
(c) any problem or abnormality occurs
(d) none of the above

7. A company has an annual demand of one million units, ordering cost per order and annual carrying cost per piece, both are Rs. 1000. If the stock-out costs are estimated to be nearly Rs. 500 each time the company runs out-of-stock, the safety stock justified by the carrying cost will be

(a) 400 (b) 416
(c) 800 (d) 816

8. A manufacturing company consumes 3650 units of an item annually. Delivery lead time is 10 days. The reorder point (in number of units) to achieve optimum inventory is

(a) 40 (b) 60
(c) 80 (d) 100

9. If order quantity is equal to economic order quantity, then ordering cost is equal to

(a) zero (b) holding cost
(c) total cost (d) indeterminate

10. In ABC analysis, all inventory items are grouped as:

(a) high consumption – strict control
(b) moderate consumption – fair control
(c) low consumption – open storage
(d) all of the above

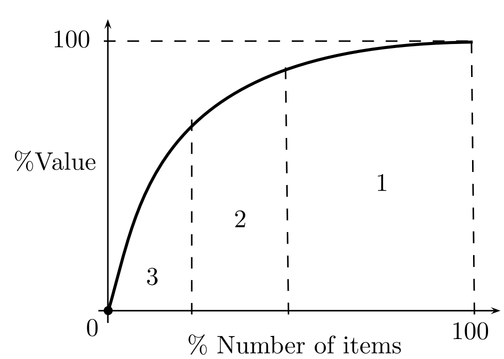
11. The ABC analysis for inventory control does not require the following information

(a) complete list of items
(b) unit cost of each item
(c) periodic consumption
(d) available stock for each item

12. The Pareto principle was generalized for ABC analysis by

(a) Vilfredo Pareto (b) Joseph M. Juran
(c) F. Wilson Harris (d) R. H. Wilson

13. A Pareto curve for ABC classification is shown in the figure that follows.



The inventory under the curve marked as 1, 2, and 3, respectively, indicate items under class of

(a) A, B, C (b) A, C, B
(c) B, A, C (d) C, B, A

14. The ABC analysis can be used for

(a) review of stocking levels
(b) periodicity of physical verification
(c) identifying items for alternate stocking
(d) all of the above

15. A company has an annual demand D of certain equipments. If its holding cost per annum per piece is equal to the ordering cost per order, then economic order quantity will be equal to

(a) $\sqrt{D}$ (b) $\sqrt{2D}$
(c) $\sqrt{D/2}$ (d) indeterminate

16. In ABC analysis, the items are rank in descending order on the basis of their

(a) usage values (b) unit cost
(c) holding cost (d) ordering cost

17. The stock maintained to meet the uncertainties of demand and supply is known as

(a) safety stock (b) buffer stock
(c) additional stock (d) ready stock

18. The inventory of few items for which lump sum demand is made only at some specific period of time, not throughout the year, is known as

(a) safety inventory
(b) buffer inventory
(c) anticipated inventory
(d) all of the above

19. The stock kept between two machines with different product rates is called

(a) decoupling inventory
(b) anticipated inventory
(c) buffer inventory
(d) machine inventory

20. Ordering cycle is the time span between placement of two successive orders. This ordering cycle can be under

(a) continuous review, in which a close watch is kept on the inventory, and the moment stock level touches a certain lower limit, the order is placed
(b) periodic review, in which the orders are placed at equally spaced intervals.
(c) both (a) and (b)
(d) none of the above

21. In optimal-replenishment system, also known as S-s policy, the control decides

(a) re-order cycle
(b) re-order quantity
(c) both (a) and (b)
(d) none of the above

22. Which of the following is not an objective of inventory control?

(a) Ensure economy of costs
(b) Avoid accumulation of materials
(c) Maximize the investment in inventory
(d) None of the above

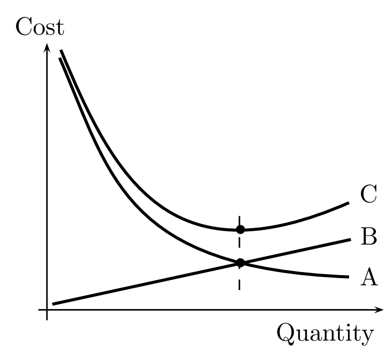
23. In computing Wilson's economic lot size for an item, by mistake the demand rate estimate used was 40% higher than the true demand rate. Due to this error in the lot size computation, the total cost of setups plus inventory holding per unit time, would rise above the true optimum by, approximately,

(a) 1.4% (b) 6.3%
(c) 18.3% (d) 8.7%

24. The lead time is defined as

(a) the time between placing an order and its delivery
(b) the time between two consecutive placements of orders
(c) the time between replenishment and zero stock
(d) none of the above

25. The quantity versus costs plot is shown in the figure below.



The cost marked as A, B and C, respectively, are

(a) ordering cost, holding cost, total cost
(b) holding cost, ordering cost, total cost
(c) total cost, ordering cost, holding cost
(d) total cost, holding cost, ordering cost

26. In VED analysis of spare parts are classified as on the basis of their

(a) usage value
(b) unit cost
(c) consumption quantity
(d) importance to the production

27. The classes are defined in VED analysis are

(a) vital items, without which the production process would come to a standstill
(b) essential items, which would adversely affect the efficiency of the production system although the system would not altogether stop for want of these items.
(c) desirable items, required but may not immediately cause a loss of production.
(d) all of the above

28. The only selective approach suitable for spare parts inventory is

(a) ABC analysis (b) VED analysis
(c) FSN analysis (d) SOS analysis

29. In HML analysis, the items are classified based on their

(a) unit value (b) annual usages
(c) usage value (d) all of the above

30. The SDE analysis is done based on

(a) purchase value
(b) purchase quantity
(c) purchasing problems
(d) purchase rate

31. In FSN analysis, the items are classified based on their

(a) issue rates
(b) purchase difficulty
(c) unit value
(d) usage value

32. The GOLF analysis is carried out mainly based on the

(a) procedure of purchase
(b) value of the order
(c) source of material
(d) usages value

33. The SOS analysis is made based on the nature of

(a) suppliers (b) supplies
(c) purchasers (d) manufacturers

34. Match the basis attribute of the selective approaches for inventory control:

List-I (Approach)		List-II (Basis)
A. ABC	1.	Importance to production
B. VED	2.	Usage value
C. HML	3.	Purchase problem
D. SDE	4.	Unit cost

(a) A-1, B-2, C-4, D-3
(b) A-2, B-1, C-4, D-3
(c) A-1, B-2, C-3, D-4
(d) A-2, B-1, C-3, D-4

35. If the order quantity is Q, the production and depletion rates are p and q, respectively, then the average inventory level will be equal to

(a) $(1-d/p)\,Q$ (b) $(1-p/d)\,Q$
(c) $(1-d/p)\,Q/2$ (d) $(1-p/d)\,Q/2$

36. If for an inventory, the production and depletion rates are the same, then the average level of inventory will be

(a) equal to the order quantity
(b) half of the order quantity
(c) economic order quantity
(d) zero

37. In company, the maximum inventory level of certain material is 500 units, where replenishment is in single lot only. The inventory is consumed at uniform rate and remains nil for one month in every cycle of purchase throughout the year. Ordering cost is Rs. 200 per order and inventory carrying cost is Rs. 20 per item per month. Annual cost (in rupees) of the inventory ordering and holding is

(a) 1200 (b) 30000
(c) 31200 (d) 61200

38. If the demand for an item is doubled and the ordering cost halved, the economic order quantity

(a) remains unchanged
(b) increases by a factor of 2
(c) is doubled
(d) is halved

39. The items which required least attention through ABC analysis are kept in the category of

(a) A
(b) B
(c) C
(d) none of the above

40. If orders are placed once a month to meet an annual demand of 6000 units, then the average inventory would be

(a) 200 (b) 250
(c) 300 (d) 500

41. Annual demand for a product costing Rs 100 per piece is 900. Ordering cost pre order is Rs 100 and inventory holding cost is Rs 2 per unit per year. The economic lot size is

(a) 200 (b) 300
(c) 400 (d) 500

42. ABC analysis in materials management is a method of classifying the inventories based on

(a) the value of annual usage of the items
(b) economic order quantity
(c) volume of material consumption
(d) quantity of materials used

43. In the EOQ model, if the unit ordering cost is doubled, the EOQ

(a) is halved
(b) is doubled
(c) increases 1.414 times
(d) decreases 1.414 times

44. In the inventory control, if the yearly demand for certain material is fixed, the economic order quantity gives minimum

(a) inventory carrying cost per year
(b) acquisition cost per year
(c) total cost per year
(d) number of orders per year

45. EOQ is taken at the point where the cost of carrying equals the cost of

(a) ordering the materials
(b) the material
(c) the safety stock
(d) both the material and the safety stock

46. In a just-in-time (JIT) production system, materials are produced

(a) only at the time when they are needed
(b) only for required quantity
(c) both (a) and (b)
(d) none of the above

47. The economic order quantity does not depend upon

(a) ordering cost per order
(b) holding cost per unit
(c) unit price
(d) all of the above

48. The material that under indirect inventory include

(a) lubricants (b) fuels
(c) cutting fluids (d) all of the above

49. The simple EOQ model does not assume that

(a) demand is certain
(b) inventory is replenished when it is equal to economic lot size
(c) ordering cost is constant for every order
(d) quantity discounts are not allowed

50. The ordering cost of an inventory is associated with the cost of

(a) placing orders for procurement of the inventory
(b) stationery associated with the order
(c) transportation from supplier to depots
(d) labours associated in the manufacturing of the inventory

NUMERICAL ANSWER QUESTIONS

1. A company has to trade one million units of certain washers annually by purchasing in lots. The cost of making a purchase order is one thousand rupees. The cost of storage of springs is hundred rupees per stored unit per annum. Determine the economic order quantity, and the annual cost of ordering and holding the inventory in thousands of rupee.

2. For inventory control, the following data is given for a factory:

Annual requirement	=	15,000 units
Preparation cost	=	Rs. 25/order
Inventory holding cost	=	Rs. 5/unit/year
Production rate	=	100 units/day
Working days	=	250/year

Calculate the number of production runs and the total incremental cost.

3. For a railway locomotive company, the annual demand for driver's cabin windows glass is 10000. Each frame costs Rs. 500 and ordering cost is Rs. 250 per order. The annual inventory holding cost is Rs. 50 per unit. Calculate the economic order quantity. Let the supplier offers 5% discount if the order quantity is 1000 or more, and 10% if order quantity is 2000 or more. Determine the lot size to minimize the total cost.

4. A product has annual demand of 4200 units with an average consumption rate of 8 units/day. The price of the product is Rs. 5 per unit. The inventory system has ordering cost of Rs. 24 per order, annual carrying cost 20% of the value of the inventory, and lead time of 12 days. Determine the economic order quantity and the purchase cycle time.

ANSWERS

Multiple Choice Questions

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(c)	**5.**	(a)	**6.**	(c)	**7.**	(d)	**8.**	(c)	**9.**	(a)	**10.**	(d)
11.	(d)	**12.**	(b)	**13.**	(d)	**14.**	(d)	**15.**	(b)	**16.**	(a)	**17.**	(b)	**18.**	(c)	**19.**	(a)	**20.**	(c)
21.	(c)	**22.**	(c)	**23.**	(c)	**24.**	(a)	**25.**	(a)	**26.**	(d)	**27.**	(d)	**28.**	(b)	**29.**	(a)	**30.**	(c)
31.	(a)	**32.**	(c)	**33.**	(a)	**34.**	(b)	**35.**	(c)	**36.**	(d)	**37.**	(c)	**38.**	(a)	**39.**	(c)	**40.**	(b)
41.	(b)	**42.**	(a)	**43.**	(c)	**44.**	(c)	**45.**	(a)	**46.**	(c)	**47.**	(c)	**48.**	(d)	**49.**	(b)	**50.**	(a)

Numerical Answer Questions

1. 4470, 245 **2.** 24.49 units, Rs. 1224.74 **3.** 316 units, 2000 units
4. 502 units, 62.75 days

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (b) Economic order quantity is given by

$$Q^* = \sqrt{\frac{2AD}{h}}$$

For $A = h$,

$$\begin{aligned} Q^* &= \sqrt{2D} \\ &\propto \sqrt{D} \end{aligned}$$

2. (b) The economic order quantity is given by

$$Q^* = \sqrt{\frac{2AD}{h}}$$

Thus, the number of orders (n) is

$$n = \frac{D}{Q^*} = \sqrt{\frac{hD}{2A}}$$

3. (c) Total inventory cost for economic order quantity is

$$T(Q) = \sqrt{2ADh}$$

4. (c) The economic order quantity is given by

$$Q^* = \sqrt{\frac{2AD}{h}}$$
$$\frac{dQ^*}{Q^*} = \frac{1}{2}\left(\frac{dD}{D}\right)$$

Similarly, the variation in total cost on economic order quantity will be

$$T(Q^*) = \sqrt{2ADh}$$
$$\frac{dT(Q^*)}{T(Q^*)} = \frac{1}{2}\left(\frac{dD}{D}\right)$$

5. (a) Just-in-time (JIT) production means producing the right item at the right time in the right quantity

6. (c) Japanese word “ji-do-ka” means automation with a human mind, and implies intelligent workers and machines identifying errors and taking quick countermeasures.

7. (d)

$$Q_s = Q^* \times \sqrt{\frac{s}{s+h}}$$

Given that

$$D = 10^6 \text{ units}$$
$$A = \text{Rs. } 1000/\text{order}$$
$$h = \text{Rs. } 1000/\text{unit-year}$$
$$s = \text{Rs. } 500$$

Using

$$Q_s = \sqrt{\frac{2AD}{h}} \times \sqrt{\frac{s}{s+h}}$$
$$= 816.49$$

8. (c) As 3000 units are required in 365 days, therefore, 8 days quantity is

$$\text{Reorder level} = \frac{3650}{365} \times 10$$
$$= 100 \text{ units}$$

9. (a) For economic order quantity, the ordering cost is equal to holding cost.

10. (d) ABC (i.e. Always Better Control) is based on the concept “thick on the best, thin on the rest.”

11. (d) The ABC analysis requires a complete list of items, their unit cost and periodic consumption. The available stock has no relevance here.

12. (b) The Pareto curve is named after the Italian economist Vilfredo Pareto, but it was later generalized by Joseph M. Juran into the Pareto principle.

13. (d) The inventory under the class A are costly but their numbers is less, and so is for B and C classes.

14. (d) The ABC analysis is used to review the stocking levels, their physical verification and think for alternatives.

15. (b) When $A = h$, the economic order quantity is given by

$$Q^* = \sqrt{\frac{2AD}{h}}$$
$$= \sqrt{2D}$$

16. (a) In ABC analysis, the items are rank in descending order of their usage values.

17. (b) Buffer stock is is kept to meet uncertainties in demand and supply. Buffer inventory refers inventory reserved to meet customer demand when customer ordering patterns vary, whereas safety inventory refers the inventory reserved to meet the customer demand when internal constraints or inefficiencies disrupt in the process flow.

18. (c) Items, such as fans, air coolers, require in specific season and reason, not throughout the year, hence, these are called anticipated inventory.

19. (a) For continuous production, some stock is kept between the machines with different rate of production. This is called decoupling inventory.

20. (c) Ordering cycle is the time span between placement of two successive orders. This ordering cycle can be under continuous review or periodic review, as described in the options.

21. (c) The optimal replenishment system is a control system that combines regular review cycles and

order points are also used. Stock levels are reviewed periodically but orders are placed when inventories has fallen to a predetermined re-order level s, and the order replenishes the inventory to a maximum level S.

22. (c) The objective inventory control is the minimize the investment in inventory.

23. (c) For Wilson's economic lot size, the total cost is given by

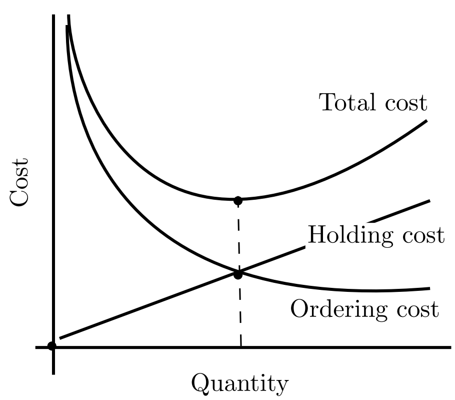
$$T(Q^*) = \sqrt{2ADh}$$

Therefore, the percentage increase in $T(Q^*)$ due to 40% higher demand is

$$\begin{aligned}\% \text{ increase} &= \sqrt{1.4} - 1\\ &= 18.32\%\end{aligned}$$

24. (a) The lead time is the time between placing an order and its delivery.

25. (a) The correct marking of costs is shown in the figure below.



26. (d) In VED analysis of items are classified as vital, essential and desirable based on their importance to the production process.

27. (d) In VED analysis, the items are classified as vital, essential and desirable as described in the options.

28. (b) The VED analysis can be done only on the spare parts, not on the raw materials.

29. (a) In HML analysis, the items are classified based on their unit value (not annual usages).

30. (c) The SDE analysis is done based on purchasing problems, such as long lead time, scarcity and hardly availability, geographically scattered sources, uncertainty in supply.

31. (a) In FSN analysis, the items are classified based on their issue rates as fast, slow moving, and non-moving items.

32. (c) The GOLF analysis is carried out mainly based on the source of material. GOLF stands for Government, Ordinary, Local and Foreign.

33. (a) The SOS analysis is made based on the nature of supplies (seasonal, non-seasonal, etc.)

34. (b) The correct match is shown below.

Approach		Basis	
A.	ABC	1.	Usage value
B.	VED	2.	Importance to production
C.	HML	3.	Unit cost
D.	SDE	4.	Purchase problem

35. (c) The production time can be determined as

$$t_p = \frac{Q}{p}$$

The average inventory is

$$\begin{aligned}\left(\frac{p-d}{2}\right) t_p &= \frac{Q\,(p-d)}{2p}\\ &= \frac{Q}{2}\left(1-\frac{d}{p}\right)\end{aligned}$$

36. (d) For $p = d$, the average inventory is

$$\frac{Q}{2}\left(1-\frac{d}{p}\right) = 0$$

37. (c) Number of replenishment cycles per year $= 12/2 = 6$. Per year ordering cost is Rs. $200 \times 6 = 1200$. Annual carrying cost is Rs. $500/2 \times 6 \times 20 = 30000$. Total cost is Rs. $1200 + 30000 = 31200$.

38. (a) The economic order quantity is expressed as

$$Q^* = \sqrt{\frac{2AD}{h}}$$

where A is ordering cost, A is annual demand and h is holding cost.

39. (c) ABC (i.e. Always Better Control) is based on the concept "thick on the best, thin on the rest." This is a selective control procedure for some items having higher importance strictly so that adequate control is achieved at a lower cost.

40. (b) The monthly order quantity will be $6000/12 = 500$, thus average inventory will be $500/2 = 250$.

41. (b) Given,

$$\begin{aligned}A &= 100\\ D &= 900\\ h &= 2\end{aligned}$$

Economic lot size is given by

$$Q^* = \sqrt{\frac{2AD}{h}} = 300$$

42. (a) ABC (i.e. Always Better Control) is based on the concept "thick on the best, thin on the rest."

43. (c) EOQ is determined as

$$Q^* = \sqrt{\frac{2AD}{h}}$$

where A is unit ordering cost and h is unit holding cost.

44. (c) The objective is to determine the EOQ which minimizes the total cost of an inventory system when demand occurs at a constant rate.

45. (a) For EOQ, the carrying cost is equal to ordering cost.

46. (c) In a just-in-time (JIT) production system, materials are produced only at the time when they are needed and in required quantity.

47. (c) The economic order quantity depends upon annual demand, holding cost, ordering cost.

48. (d) Indirect stock include those items which do not form the part of finished product but aids in manufacturing process.

49. (b) The simple EOQ model assumes that the inventory is replenished when it is zero or equal to safety stock.

50. (a) The ordering cost is associated with the cost of placing orders for procurement of the inventory. It includes the cost of stationary, postage, telephone, traveling expenses, material handling etc.

Numerical Answer Questions

1. Given that

$$\begin{aligned} D &= 10^6 \text{ per annum} \\ A &= \text{Rs. } 1000/\text{order} \\ h &= \text{Rs. } 100/\text{piece/annum} \end{aligned}$$

Therefore,

$$\begin{aligned} Q^* &= \sqrt{\frac{2AD}{h}} \\ &= 4472.1 \end{aligned}$$

The total annual cost of inventory shall be

$$\begin{aligned} T(Q) &= \frac{D}{Q^*}A + \frac{h}{2}Q^* \\ &= \frac{10^6}{4472.1} \times 1000 + \frac{100}{2} \times 4472.1 \\ &= \text{Rs. } 245.96 \times 10^3 \end{aligned}$$

2. Given (build-up EOQ model)

$$\begin{aligned} D &= 15000 \text{ units/year} \\ A &= \text{Rs. } 25/\text{order} \\ h &= \text{Rs. } 5/\text{unit/year} \\ p &= 100 \text{ units per day} \\ d &= 15000/250 = 60 \text{ units per day} \end{aligned}$$

Economic lot size is determined as

$$\begin{aligned} Q^* &= \sqrt{\frac{2AD}{h\,(1-d/p)}} \\ &= \sqrt{\frac{2 \times 25 \times 15000}{5\,(1-60/100)}} \\ &= 612.37 \text{ units} \end{aligned}$$

Thus, the number of production runs is

$$\begin{aligned} n^* &= \frac{D}{Q^*} \\ &= \frac{15000}{612.37} \\ &= 24.49. \end{aligned}$$

The incremental cost, that is, the total inventory cost is determined as

$$\begin{aligned} T(Q^*) &= \sqrt{2ADh\left(1-\frac{d}{p}\right)} \\ &= \text{Rs. } 1224.74 \end{aligned}$$

3. Given that

$$\begin{aligned} D &= 10000 \text{ units} \\ p &= \text{Rs. } 500 \\ A &= \text{Rs. } 250/\text{order} \\ h &= \text{Rs. } 50/\text{frame/year} \end{aligned}$$

Economic order quantity is

$$\begin{aligned} Q^* &= \sqrt{\frac{2AD}{h}} \\ &= 316.22 \text{ units} \end{aligned}$$

Total cost of purchase and inventory is

$$\begin{aligned} T(Q^*) &= \frac{D}{Q^*}A + \frac{Q^*}{2}h + p \times D \\ &= \text{Rs. } 5.016 \times 10^6 \end{aligned}$$

The price of annual demand can bee seen for the two offers.

(1) For the first offer,

$$\begin{aligned} p &= 500 \times 0.90 \\ &= \text{Rs. } 450 \\ Q &= 1000 \end{aligned}$$

Therefore,

$$\begin{aligned} T(Q) &= \frac{D}{Q}A + \frac{Q}{2}h + p \times D \\ &= \text{Rs. } 4.52 \times 10^6 \end{aligned}$$

(2) For the second offer,

$$\begin{aligned} p &= 500 \times 0.80 \\ &= \text{Rs. } 400 \\ Q &= 1000 \end{aligned}$$

$$\begin{aligned} T(Q) &= \frac{D}{Q}A + \frac{Q}{2}h + p \times D \\ &= \text{Rs. } 4.05 \times 10^6 \end{aligned}$$

In view of above, the last offer costs least, hence the lot size should be 2000.

4. The EOQ is determined as

$$\begin{aligned} Q^* &= \sqrt{\frac{2AD}{h}} \\ &= \sqrt{\frac{2 \times 24 \times 4200}{0.80}} \\ &= 501.99 \\ &\approx 502 \text{ units} \end{aligned}$$

Purchase cycle time is

$$\begin{aligned} t^* &= \frac{Q^*}{\text{Demand per day}} \\ &= \frac{502}{8} \\ &= 62.75 \text{ days} \end{aligned}$$

CHAPTER 19

OPERATIONS RESEARCH

Operations research (OR) is the application of scientific methods primarily in addressing the problems of decision making by providing systematic and rational approaches. The term 'operations research' (coined by McClosky and Trefthen in 1940 in the UK) came from military operations during World War-II when scientists dealt with strategic and tactical problems. These techniques were later applied in business, industry and research. Operations research provides a quantitative basis for decisions regarding the operations kept under control. Operations research is based on the scientific approach through theoretical models to understand and explain the phenomenon of operating systems.

19.1 SIMPLEX METHOD

Dantzig's *simplex method* is a general procedure of iterative nature for obtaining systematically the optimal solution to a linear programming problem (LPP). The method is based on the property that if objective function does not take the maximum value in a vertex, then there is an edge starting at that vertex along which the value of the function grows.

The procedure is comparable with the *graphical method* wherein *feasible solution* is located at corner points of the feasible region determined by the constraints of the system. The simplex method is just the same but it is done by table. The algorithm is very simple and efficient, as it considers only those feasible solutions which are provided by the corner points, and that too not all of them. Thus, the technique considers minimum number of feasible solutions to obtain an optimum one.

The procedure begins by assigning values to an appropriately selected set of variables introduced into the problem, and primary (decision) variables of the problem are all set equal to zero. This assumption is analogous to starting the evaluation in graphical approach at the point of origin. The algorithm then replaces one of the initial variables by one of decision variable which contributes most to the desired optimal value. This is repeated until the algorithm terminates indicating the optimal solution of the problem.

19.1.1 Problem Definition

In vector form, the problem is to maximize or minimize the objective function:

$$Z = c_j x_j \tag{19.1}$$

This is subjected to constraints:

$$a_{ij} x_j \leq b_i \tag{19.2}$$

where all x_j and b_i are non-negative (≥ 0).

The following notations are employed:

$$\begin{aligned}
Z &= \text{Objective function} \\
x_i &= \text{Variables of the problem} \\
c_j &= \text{Coefficients of } x_j \text{ in the objective function} \\
a_{ij} &= \text{Coefficient of } x_j \text{ in the } i\text{th constraint} \\
b_i &= \text{Right hand side value of the } i\text{th constraint}
\end{aligned}$$

The coefficients c_j represent the contribution of variables per unit in the objective function.

Implied summation is used in writing the above expressions. However, the problem can be written without implied summation as follows:

1. Objective function [Eq. (19.1)]

$$Z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots$$

2. Constraints [Eq. (19.2)]

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots &\leq b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots &\leq b_2 \\
\ldots.. &\leq .. \\
a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + \cdots &\leq b_i
\end{aligned}$$

19.1.2 Conditions for Applicability

Simplex method is applicable to an LPP which satisfies the following two conditions:

1. Right hand side of each constraint (b_i's) is non-negative (≥ 0).

   To ensure this, the constraints having negative b_i's should be multiplied by '-1' in order to reverse the direction of inequality. For example, the equivalent of constraint $x_1 - 3x_2 \geq -6$ is $-8x_1 + 3x_2 \leq 6$.

2. Each of the decision variables (x_i's) of the problem is non-negative (≥ 0).

   To ensure this, the decision variables without any restriction (if any) on their sign are dealt by treating such variables at the difference of two non-negative variables. For example, an unrestricted variable x_1 can be assumed to be $x_2 - x_3$ where x_2 and x_3 are non-negative variables (≥ 0). This shall also need modifications in all the constraints and objective function (Z).

19.1.3 Simplex Algorithm

The simplex method proceeds by preparing a series of tables called *simplex tableaus*. Steps of the method are described under the following headings.

19.1.3.1 Standardization of the Problem Unless otherwise specified, the following are the modifications required to have constraints in the desired format irrespective of minimization or maximization types of problems:

1. *Modification of Constraints* Irrespective of the problem type, the constraints of LPP require modifications using the following variables:

   (a) *Slack Variables* - Inequalities ($\leq$) of constraints ($a_{ij}x_j \leq b_i$) need conversion into equations by introducing positive non-negative *slack variable*, denoted by s_i (≥ 0), on the left hand side to get the following form of constraints:

   $$a_{ij}x_j + s_i = b_i \tag{19.3}$$

   These are called *slack variables* because they take up any slack between the left and right hand side of the inequalities upon conversion into equation. Slack variables can take values $0 \leq s_i \leq b_i$.

   (b) *Artificial Variables* - *Artificial variables* are used to deal with the inequality of the constraints $a_{ij}x_j \geq b_i$ type for obtaining the initial solution. Such constraints take the following form:

   $$a_{ij}x_j + A_1 = b_i \tag{19.4}$$

   Use of artificial variables in turn helps in preventing use of negative slack variables.

2. *Modification of Objective Function* The objective function of the problem should contain every variable in the system, including slack variables (s_i's) and artificial variables (A_i's), if any. These are incorporated as follows:

   (a) *Slack Variables* - The coefficients of variables in objective function represent the cost involved in using the raw materials or labor hours. However, unused resources represented by slack variables have no cost and effect on the profits. Thus, the coefficients of slack variables in the objective function are assigned zeros, as follows:

   $$Z = c_ix_i + 0 \times s_i \tag{19.5}$$

   (b) *Artificial Variables* - Artificial variables do not represent any quantity relating to the decision problem, they must be driven out of the system and must not reflect in the final solution. This can be ensured by assigning high cost to them. Therefore, a value M, higher than finite number, is assigned to each artificial variable in the objective function:

   $$Z = c_ix_i + 0 \times s_i \pm MA_i \tag{19.6}$$

To effect the proper direction of iterations, negative sign of M is used in maximization type problems and positive sign is used in minimization type problems. For this reason, the technique of using artificial variables is known as *big-M method*.

Conclusively, the *objective function* (Z) is modified by incorporating all the slack variables (s_i) with their zero coefficients, and all the artificial variables (A_i) with the same coefficient M having $-$ve sign for maximization and +ve sign for minimization type problems.

19.1.3.2 Obtaining the Simplex Tableau Initial simplex tableau is written in following format:

	c_j	c_1	c_2	c_3	$\cdots$	
e_i	CSV	x_1	x_2	s_1	$\cdots$	b_i
e_1	s_1	a_{11}	a_{12}	a_{13}	$\cdots$	b_1
e_2	s_2	a_{21}	a_{22}	a_{23}	$\cdots$	b_2

This tableau reflects the following additional elements:

1. *Identity Matrix* After writing the initial simplex tableau in the above format, the next step is to locate the identity (matrix) and variables involved in the simplex tableau. The identity contains all zero except diagonal column of positive 1's. The identity must have this square form with all zeros and a diagonal of (+ ve) ones.

   An *identity matrix square* is obtained when slack variable are introduced with coefficient of unity. It appears automatically where coefficients of other variables in the identity are equal to zero.

2. *Basic Variables* The variables in the identity matrix are called *basic variables* and the remaining are called *non-basic variables*.

   In general, if the number of variables (x_i 's and s_i) is n, and number of constraints (excluding $x_1, x_2 \geq 0$) is m, then, the number of basic variables is m, and the number of non-basic variables is $n-m$. In turn, size of square of identity is determined by the number of constraints.

3. *Current Solution Variables* The basic variables form the basis of solution, and are known as the *current solution variables* (CSV) in the simplex tableau of the latest feasible solution. In every iteration, one of the CSV's is replaced by a new basic variable and corresponding value (equal to b_i). Additional column is added on the left side to repeat the respective value of coefficients of the basic variables in the current solution (e_i) to ease the calculations.

19.1.3.3 Obtaining Feasible Solution A feasible solution is obtained by assigning zero to all variables except basic variables, and then assign the values of the constraints (b_i's) of the variable in the identity.

	c_j	c_1	c_2	c_3	$\cdots$	
e_i	CSV	x_1	x_2	s_1	$\cdots$	b_i
e_1	x_1	a_{11}	a_{12}	a_{13}	$\cdots$	b_1
e_2	x_2	a_{21}	a_{22}	a_{23}	$\cdots$	b_2

For the above table, the initial solution will be $x_1 = b_1$, $x_2 = b_2$, $s_1 = 0$, and so on.

Solution from the initial simplex tableau is not always the optimal solution but indicates the value of optimal function at the origin of x_1 and x_2 coordinates.

19.1.3.4 Testing the Optimality The feasible solution reflected in CSV of the simplex tableau is tested for optimality under the following steps:

1. *Obtaining z_j* Coefficients z_j are defined with implied summation as

$$z_j = a_{ij} e_i \tag{19.7}$$

   Their value is obtained under each (jth) variable column head. For this, each element of the jth column (a_{ij}) is multiplied by the corresponding coefficient of the solution variables appearing in the basis (e_i), and then the products are added up to get z_j. Here, z_j represents the reduction in profit if one unit of any of the variables is added to the matrix.

2. *Obtaining Δ_j* A new set of coefficients Δ_j is defined as

$$\Delta_j = c_j - z_j \tag{19.8}$$

   The row constituted by Δ_j is called *net after opportunity cost row.* The calculations are done in the simplex tableau as shown below:

	c_j	c_1	c_2	c_3	$\cdots$	
e_i	CSV	x_1	x_2	s_1	$\cdots$	b_i
e_1	x_1	a_{11}	a_{12}	a_{13}	$\cdots$	b_1
e_2	x_2	a_{21}	a_{22}	a_{23}	$\cdots$	b_2
	z_j	z_1	z_2	z_3	$\cdots$	
	Δ_j	Δ_1	Δ_2	Δ_3	$\cdots$	

3. *Optimality Test* Except when an artificial variable is included in the basis, a simplex tableau depicts an optimal solution if all entries in the net after opportunity cost row (Δ_j) are non-positive for maximization type problems or non-negative for minimization type problems:

$$\Delta_j = \begin{cases} \leq 0 & \text{maximization type} \\ \geq 0 & \text{minimization type} \end{cases} \tag{19.9}$$

19.1.3.5 Improving the Solution If the solution is not found optimal using Eq. (19.9), the next step is to improve the solution by the following steps:

1. *Finding Key Element* Presence of positive Δ_j (for maximization type problems) or negative Δ_j (for minimization type problems) indicates that the solution can be improved. This is done by replacing the least potent basic varible in CSV by the most potent non-basic variable, described as follows:

   (a) The variable with largest positive Δ_j value (for maximization type) or negative Δ_j (for minimization type) is selected as the incoming variable to CSV. The corresponding column is called *key column* or *pivot column*, of the row $j = k$.

   (b) The values of b_i's are divided by the corresponding values in the key column to get b_i/a_{ik}, which is called *replacement ratios*. The row with the least non-negative quotient is then selected as the *key row* and corresponding variable to this represents the *outgoing variable* from CSV.

   The key column and key row meet at *key element*, can be marked by asterisk $(*)$ in the superscript. The outgoing variable is replaced by incoming variable along with corresponding e_i.

2. *Optimizing the Solution* To find the next optimized solution of the problem, another simplex tableau is derived by obtaining identity through the following steps:

   (a) Divide each element of the key row (including b_i) by the key element to get the corresponding values in the tableau. The row so derived is called the *replacement row*, and the values of this row are called *replacement ratios*, in which the value at the key element is 1.

   (b) Get the other elements of key column zero by subtracting each row (other than key row) by replacement row multiplied by the corresponding row element in the key column.

   This process is also known as *pivoting operation* that determines a revised solution.

3. *Iteration* This new simplex tableau represents the new solution which needs to be again subjected to the optimality test in previous manner. If the solution is not optimal, it must be improved in the similar manner again and again until it satisfies the condition for optimality [Eq. (19.9)].

The simplex algorithm is also explained in the section of solved examples of this chapter. Besides the simplex method, several other efficient methods for solving large linear-programming problems have been developed. Few of them are the revised simplex method, the duplex method, the revised duplex method, Dantzig–Wolfe decomposition method. However, discussion on these methods (suitable for large linear-programming problems) is not useful in the present context of the book.

19.1.4 Exceptional Cases

An LPP can have the following exceptional situations:

1. *Degeneracy and Cycling* For n variables and m constraint problems, there would be m basic and $n-m$ non-basic variables. The basic variable would assume positive values. However, during any stage of iteration, one or more basic-variables can become zero. When this happens, it is said that degeneracy had set in. At this point, there is no assurance that the value of object function would improve in the next iteration, and the new solution can remain degenerate or it can become a loop. This is called *cycling*.

2. *Infeasibility* Infeasibility is said to exist when a given problem has no feasible solution. It is evident graphically when no common point is found in the feasible region (two-dimensional) of all the constraints of a problem. In simplex algorithm, the presence of artificial variable at a positive value in the final solution indicates state of infeasibility. Such a problem arises when the constraints are conflicting each other. The infeasibility solely depends upon the constraints and has nothing to do with the objective function [Fig. 19.1].

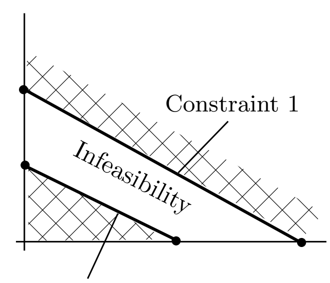


Figure 19.1 | Infeasibility in LPP.

3. *Unboundness* *Unboundness* of the objective function occurs when one or more of the decision variables is permitted to increase infinitely without violating any of the constraints. Thus, value of objective function can be increased indefinitely [Fig. 19.2].

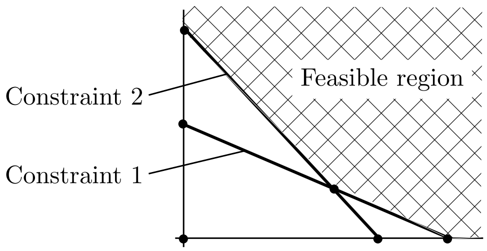


Figure 19.2 | Unboundness in LPP.

In simplex method, the row with the smallest non-negative replacement ratio is selected for the outgoing variable. Absence of non-negative replacement ratio, (all ratios are negative) indicates unbounded solution.

4. *No Feasible Solution* If there is no feasible region, the LPP is said to have no feasible solution [Fig. 19.3].

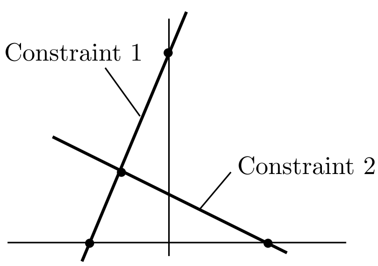


Figure 19.3 | No feasible solution in LPP.

19.1.5 Duality

The optimal solution of maximization type problem can yield complete information about optimal solution of minimization type problem, and vice versa. Then, one is called *primal*, and other is called *dual.* Such an existence of two interdependent problems is called *duality.* When the primal problem is of maximization type, the dual would be of the minimization type.

Every problem, in general form with implied summation, to maximize $c_j x_j$, subjected to $a_{ij} x_j \le b_i$, and $x_j \ge 0$, can be converted so as to minimize $b_i x_i$, subjected to $y_i a_{ij} \le c_j$, and $y_i \ge 0$, using the following conversions for duality:

$$b_i \to c_j, \quad c_j \to b_i, \quad a_{ij} \to a_{ji}, \quad \ge \to \le$$

Let a maximization type problem for an objective function

$$Z = 20x_1 + 30x_2$$

be subjected to constraints:

$$\begin{aligned} 2x_1 + 3x_2 &\le 66 \\ 4x_1 + 3x_2 &\le 84 \\ x_1, x_2 &\ge 0 \end{aligned}$$

Its dual is minimization of the objective function

$$Z = 66y_1 + 84y_2$$

and subjected to constraints:

$$\begin{aligned} 2y_1 + 4y_2 &\ge 20 \\ 3y_1 + 3y_2 &\ge 30 \\ y_1, y_2 &\ge 0 \end{aligned}$$

19.1.6 Limitations of Simplex Method

Following are the limitations of simplex method.

1. *Linearity* The object function and every constrain must be linear, but in practice, the variations are non-linear.
2. *Additivity* The activities must be additive w.r.t. the measurement of effectiveness and usage of each resources
3. *Divisibility* The solutions are integral.
4. *Deterministic* The coefficients can change with time, and at the time of implementation the solution can become wrong.

19.2 TRANSPORTATION PROBLEM

Transportation problems involve a mathematical approach that produces optimal plan for minimizing the transportation costs of goods and services from several supply centers to several demand centers. Transportation problems can be solved using simplex algorithm, but involve a large number of variables and constraints. Therefore, separate algorithms, such as *stepping-stone method*, *modified distribution method*, have been developed to solve transportation problems.

19.2.1 Problem Definition

Consider a system of n number of sources S_i $(i = 1, 2, 3 \cdots, m)$ and m number of destinations D_j $(j = 1, 2, 3 \cdots, n)$. Let a_i be the capacity of source S_i and let b_j be the requirement at destination D_j. Let c_{ij} represent the cost of transportation of a unit from source S_i to destination D_j. Let x_{ij} (≥ 0) be the number of units transported from source S_i to destination D_j. The

problem is to determine x_{ij} that minimizes the total transportation cost. With these details, a transportation problem can be presented in the following format:

1. Objective function (aggregate transportation cost):

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{19.10}$$

2. Constraints (rim requirements):

$$\sum_{j=1}^{n} x_{ij} = a_i \quad \text{for } i = 1, 2, 3 \cdots m$$
$$\sum_{i=1}^{m} x_{ij} = b_j \quad \text{for } j = 1, 2, 3 \cdots n$$
$$x_{ij} \geq 0 \quad \text{for all } i, j$$

This can also be portrayed in a matrix of $m \times n$ size by means of a transportation tableau:

Sources	Destinations (D_j)				Supply
(S_i)	1	2	$\cdots$	n	(a_i)
1	c_{11}	c_{22}	$\cdots$	c_{1n}	a_1
2	c_{21}	c_{22}	$\cdots$	c_{2n}	a_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
m	c_{m1}	c_{m2}	$\cdots$	c_{mn}	a_m
Demand (b_j)	b_1	b_2	$\cdots$	b_n	

The allotment of the transport quantity x_{ij} is written in a similar tableau

Origin	Destinations				Supply
	1	2		n	
1	x_{11}	x_{22}	$\cdots$	x_{1n}	a_1
2	x_{21}	x_{22}	$\cdots$	x_{2n}	a_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
m	x_{m1}	x_{m2}	$\cdots$	x_{mn}	a_m
Demand b_j	b_1	b_2	$\cdots$	b_n	

Before proceeding for the solution, a transportation problem should be looked for the following aspects:

1. *Balanced Transportation Problem* Transportation problem is said to be balanced when the total demand is equal to the total supply:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{19.11}$$

2. *Basic Feasible Solution* The number of constraints in the above transportation problems (x_{mn}) is $m+n$ (sum of the numbers of a_i and b_j). The number of variables required for forming a basis is one less, that is, $m+n-1$. This means, with known values of $m+n-1$ variables, the other will be automatically determined by the compatibility derived using Eq. (19.11). Therefore, the number of basic variables is $m+n-1$, and others are non-basic. Thus, a feasible solution of transportation problem of size $m \times n$ is called *basic feasible solution* if it contains no more than $m+n-1$ non-negative allocations.

19.2.2 Solution Procedure

Solution of a transportation problem consists of the following steps.

19.2.2.1 First Infeasible Solution Feasible solution of a transportation problem is defined as a set of non-negative allocations that satisfies the supply and requirement constraints (called *rim requirements*). This can be obtained by north-west corner rule, least cost method, or *Vogel's approximation method* (VAM).

A feasible solution achieved by the Vogel's approximation method[1] yields a very good initial solution which some items can be an optimal solution. This method consists of the following four steps:

1. Enter the cost difference between the two least cost cells, (i.e. smallest and second smallest element in each column) below the corresponding column and do the same in each row. To right of the row, put these differences in bracket.
2. Select the row or column with the greatest difference and allocate as much as possible to the lowest cost (so that the maximum units are transported with minimum cost, at the first sight) while satisfying the supply and requirement constraints. To do this, first allocate the minimum supply and requirement for the lowest cost cell.
3. Reduce supply and demand units by the amount assigned to the cell and cross the completely satisfied column and row. Repeat the above 1 to 3 steps until all allocations have been made.
4. Write down the reduced transportation table after omitting rows or columns.

The Vogel's approximation method is also called the *penalty method* because the cost differences that it uses are the penalties of not using the least cost routes. Since the objective function is the minimization of the transportation cost, in each iteration that route is selected which involves the maximum penalty of not being used.

[1]was developed by William R. Vogel.

19.2.2.2 Testing the Optimality Optimality test can be performed only on a feasible solution, that satisfies following two conditions:

1. Number of allocations is $m+n-1$.
2. All allocations are in independent[2] positions.

If above conditions are not satisfied, it becomes the case of degeneracy. The optimality test can be performed on a feasible solution by either stepping-stone method or modified distribution method. The stepping stone method is applied by calculating the opportunity cost of each empty cell.

The modified distribution (MODI) method, also called *u*-*v* method, is based on the concept of the dual variables that are used to evaluate the empty cells. The test is performed in the following steps:

1. Find u_i and v_j for each row and column such that
$$u_i+v_j = c_{ij}$$
This can be started by assigning 0 to the first row ($u_1 = 0$).
2. Fill the vacant cell by its u_i+v_j value.
3. Subtract this table/matrix from the original model to find *cost evaluation matrix* (CEM).

The CEM represents the maximum error in allocations in comparison to the optimal solution; the solution could be optimum if this cell should have been chosen for allocation amount equal to the negative value of the cell. A negative cell element of the CEM indicates that the basic solution is not optimal and it can be improved. A feasible solution is optimum if all cells of the CEM are positive.

19.2.2.3 Improving the Solution If the solution is not optimal, rearrangement is made by transferring units from an occupied cell to an empty cell that has the largest opportunity cost but satisfying the rim requirements. The iteration toward an optimal solution is done in the following steps:

1. Identify the CEM cell having most negative entry.
2. Check mark the empty cell on the most negative entry. This cell is called *identified cell*.
3. Assign +ve sign to the identified cell and draw alternating horizontal and vertical lines such that corner is at allocated cells.
4. Assign alternate +ve and −ve signs to corners.
5. Add the absolute of largest negative value of the CEM to all to find next feasible solution.

Again check for optimality test and iterate towards optimal solution (i.e. until all elements of the CEM become non-negative) in the way mentioned above.

The procedure is also explained in the section of solved examples of this chapter.

19.2.3 Exceptional Cases

A transportation problem can have the following exceptional situations:

1. *Maximization Problem* A transportation tableau can represent unit profits, instead of unit costs, and the objective is the maximization of profit. Such cases are solved by converting the profit matrix into cost matrix, thus, objective function needs to be minimized. This is obtained by subtracting the profit matrix from highest profit value in the matrix. The problem is then solved in usual way, and the objective function is determined with reference to the original profit matrix.
2. *Unbalanced Problem* Transportation problems are solved assuming a balanced problem in which aggregate supply is equal to the aggregate demand. If they are unequal, the problem is known as an *unbalanced transportation problems*.

   Unbalanced problems are solved by converting them into a balance problem through the following changes:

   (a) *Supply* > *Demand* - Such problems are solved by creating a dummy destination having zero cost of transportation.

   (b) *Supply* < *Demand* - Such problems are solved by creating a dummy source having zero unit cost of transportation.
3. *Prohibited Routes* Some routes in a transportation system can be strictly prohibited due to a variety of unfavorable reasons. In such cases, the problem of optimization is solved by assigning a very large cost, represented by M, to each of the prohibited routes, thus prevent the allocations in those routes. The problem is then solved in the usual way.
4. *Degeneracy* In transportation problem, if the number of allocations is less than $(m+n-1)$ then this is the case of degeneracy. In such cases, the solution cannot be improved upon because the algorithm cannot be applied.

[2]Independent allocations means it is not possible to increase or decrease any allocation without either changing the position of the allocation or violating the row and column constraint.

19.3 ASSIGNMENT PROBLEM

Resources possess varying abilities for performing different jobs, therefore, the costs of performing those jobs are different. Optimal assignment of resources for production is an essential aspect of production planning and control. An *assignment problem* can be viewed as a reduced or degenerate form of transportation problem obtained by incorporating the following changes:

1. Sources are the assignees and destinations are tasks.
2. Taking demand (b_i) and supply (a_i) as 1, (there will be only one assignment); the units available at each origin and units demanded at each destination are all equal to one.
3. The numbers of origins and destinations should be made exactly equal. This makes the problem a square matrix.

The objective is to minimize the cost of production by optimal assignments of the job to most suitable worker or machine.

19.3.1 Problem Definition

An assignment problem is a $n \times n$ square problem of cost matrix c_{ij} to assign jobs J_i to workers W_j such that working time or cost is minimum. Formulation of this problem requires that the decision variables take only one of the two values 1 or 0, accordingly as an assignment (of a worker to a job) is made or not. The decision variable x_{ij} answers whether ith worker is assigned jth job:

$$x_{ij} = \begin{cases} 1 & \text{true} \\ 0 & \text{false} \end{cases} \tag{19.12}$$

Assignment problem is to minimize the objective function:

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{19.13}$$

This is subjected to the following constraints:

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for } i = 1, 2, 3 \cdots n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for } j = 1, 2, 3 \cdots n$$

$$x_{ij} = 0 \text{ or } 1 \quad \text{for all } i, j$$

The problem can also be portrayed in a $n \times n$ matrix of a transportation tableau:

Workers	Jobs (J_i)				x_{ij}
(W_j)	1	2		n	
1	c_{11}	c_{22}	$\cdots$	c_{1n}	1
2	c_{21}	c_{22}	$\cdots$	c_{2n}	1
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
n	c_{n1}	c_{n2}	$\cdots$	c_{nn}	1
x_{ij}	1	1	$\cdots$	1	

19.3.2 Solution of Problem

For the $n \times n$ assignment problem, the maximum number of possible allocations is $n!$, but only few of them can represent optimal allocations. Because of degeneracy, an assignment problem cannot be easily solved by either simplex algorithm or transportation algorithm. Solutions with simplex methods require $n \times n$ decision variables and $2n$ inequalities as constraints. Solution with transportation problem require $n-1$ dummy allocations. Thus, different algorithms have been developed to solve assignment problems efficiently. Out of these, *Hungarian assignment method*[3] (HAM) is most simple and frequently used. The method is performed through the following steps:

1. *Opportunity Cost Table* Obtain a reduced cost table, known as *opportunity cost table*, through the following steps:
   (a) Find minimum of c_{ij} in each row. Subtract this value in the respective row elements. Thus, each row has at least one zero.
   (b) Find minimum of c_{ij} in each column. Subtract this value in the respective column elements. Thus, each column has at least one zero.
2. *Feasible Solution and Optimality Test* Draw minimum number of horizontal and vertical lines to cover all the zeros. For this, first cover the row or column having maximum number of zeros, and then in descending order. If the number of these lines is equal to n then optimal solution is obtained, and the allocations are made after scanning the rows and columns for single zero. Work is assigned first to the worker having minimum number of zeros in his row, and similarly for all. If solution is not optimal, follow the next step.
3. *Improving the Solution* Find the smallest uncovered elements and subtract it from all uncovered elements and add it to all elements at the intersection of lines. Other elements will not change. Repeat step (2) until an optimal solution is obtained.

[3]Hungarian assignment method was developed by Harold Kuhn in 1955. The algorithm is largely based on the earlier works of two Hungarian mathematicians, so the name comes.

The procedure is also explained in the section of solved examples of this chapter.

19.3.3 Exceptional Cases

An assignment problem can have the following exceptional situations:

1. *Maximization Assignments* Assignments model can be used to optimize the profit or revenue. Such problems can be solved by converting them into minimization type assignment problems. For this, each cell element of the cost matrix (c_{ij}) is subtracted from the largest value of c_{ij} in the matrix. The problem is then solved in usual manner.
2. *Unbalanced Assignments* A assignment problem can be unbalanced (non-square matrix); number of jobs is not equal to the number of workers. Such problems are solved by introducing a dummy worker or job with zero cost (in both cases). The problem is then solved in usual manner.
3. *Multiple Optimal Solutions* Assignment algorithm can face tie situation when it is possible to have two or more ways to cover zeros and assign the job to workers. This indicates existence of multiple optimal solutions but with the same optimal value of the objective function. Selection of optimal solution then depends upon the discretion of the decision maker.
4. *Restricted Assignments* Some cells (workers and jobs) in an assignment problem can be strictly restricted due to a variety of unfavorable reasons. In such cases, the problem of optimization is solved by assigning a very large cost, represented by M, to each of the restricted cell, thus preventing allocations in those cells. The problem is then solved in usual way.

19.4 SEQUENCING

Routing is an important function of production planning. Most of the effectiveness measures, such as time, cost, distance, depend on the order of performing a series of jobs. The jobs can be scheduled by using priority sequencing rules whenever the workstation becomes available for further processing. Some of the priority sequencing rules are enlisted as follows:

1. *First-Come, First-Served Rule* The first-come, first-served (FCFS) rule gives the job arriving at the workstation first the highest priority.
2. *Earliest Due Date Rule* Earliest Due Date (EDD) rule orders the jobs in the order of earliest due date.
3. *Shortest Processing Time Rule* The shortest processing time (SPT) rule orders the jobs in the order of increasing processing times.

However, the priority sequencing rules cannot be applied in large production systems. Theoretically, n number of jobs can be performed on single machine in $n!$ number of ways. Similarly, n number of jobs can be performed through m number of machines or workers in $(n!)^m$ possible sequences. Out of the theoretically possible sequences, only a few or one can be the optimal sequence. The selection of optimal sequence of jobs through a system is called *sequencing*. *Sequencing problem* arises when the system has more than one option of the order in performing a series of jobs.

19.4.1 Problem Definition

Suppose n number of jobs J_i $(i = 1, 2, \cdots, n)$ are to be performed on m number of machines M_j $(j = 1, 2, \cdots, m)$ in order from M_1 to M_m. A machine can process one job at time. Let c_{ij} be the cost (or time) of completion of ith job on jth machine, which is independent of the order of performing jobs. This description can be represented in a tabular format:

	Machines			
Jobs	M_1	M_2	$\cdots$	M_m
J_1	c_{11}	c_{22}	$\cdots$	c_{1m}
J_2	c_{21}	c_{22}	$\cdots$	c_{2m}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
J_n	c_{n1}	c_{n2}	$\cdots$	c_{nm}

The objective of the model is to determine the sequence of jobs (J_i) which will allow execution of all jobs so that the cost or time, from the beginning of the first job till the completion of the last job is minimum.

19.4.2 Solution of Problem

There is no general method of solving $n \times m$ sequencing problem. Therefore, these problems are solved by converting them into $n \times 2$ sequencing problems. Therefore, sequencing of n jobs through 2 machines is discussed first.

19.4.2.1 Processing n Jobs through Two Machines

Sequencing of n jobs through two machines can be solved by *Johnson's algorithm*. Tabular form of the $n \times 2$ problem is shown as follows:

	Machines	
Jobs	M_1	M_2
J_1	c_{11}	c_{12}
J_2	c_{21}	c_{22}
$\cdots$	$\cdots$	$\cdots$
J_n	c_{n1}	c_{n2}

Johnson's algorithm consists of the following steps:

1. Make a horizontal box with n number of cells for sequencing of n jobs in a manner that reduces the cost of performing all the jobs.

Sequence	1	2	3	$\cdots$	$\cdots$	n
Jobs						

   Since a job has to be performed first by machine M_1 then by machine M_2, left side of the box will be filled by for minimum c_{i1}'s and right side will be filled for minimum c_{i2}'s.

2. Find the minimum of c_{i1}'s and c_{i2}'s. If c_{i1}'s is minimum out of these, put corresponding J_i in the first empty column from left in the box. If c_{i2}'s is minimum out of these, put corresponding J_i in the first empty column from right in the box.

   If more c_{i1}'s or c_{i2}'s are minimum and equal, then several alternatives of optimal solution are possible by putting corresponding J_i in the box. Value of objective function remains the same.

   A job can be allocated in the box only once. Therefore, an allocated job should be crossed from the table.

3. Repeat the above process until all jobs are fitted in the box. The sequence of J_i's from left to right in the box is optimal sequence.

4. Determine the cost of performing all the jobs in optimal sequence by adding the time taken in the two machines for each job.

The procedure is also explained in the section of solved examples of this chapter.

19.4.2.2 Processing n Jobs through Three Machines

Problems of sequencing n jobs through three machines can be written in the following format:

	Machines		
Jobs	M_1	M_2	M_3
J_1	c_{11}	c_{12}	c_{13}
J_2	c_{21}	c_{22}	c_{23}
$\cdots$	$\cdots$	$\cdots$	$\cdots$
J_n	c_{n1}	c_{n2}	c_{n3}

These types of problems can be solved by converting them into processing of n jobs through two machines. This can be done if any one of the following two conditions is valid:

1. Minimum processing time on machine M_1 is at least greater than the maximum processing time on M_2:

$$\min(c_{i1}) \geq \max(c_{i2}) \quad \text{for } i = 1, 2, \cdots, n$$

2. Minimum processing time on machine M_3 is at least great as the maximum processing time on machine M_2:

$$\min(c_{i3}) \geq \max(c_{i2}) \quad \text{for } i = 1, 2, \cdots, n$$

If any of the above conditions is valid, the three machine problem is converted into two machines problem by taking combined time in M_1 and M_2 as an apparent single machine, and similarly that on M_2 and M_3 to time on another apparent machine.

	Apparent machines	
Jobs	M_{a1}	M_{a2}
J_1	$c_{11}+c_{12}$	$c_{12}+c_{13}$
J_2	$c_{21}+c_{22}$	$c_{22}+c_{23}$
$\cdots$	$\cdots+\cdots$	$\cdots+\cdots$
J_n	$c_{n1}+c_{n2}$	$c_{n2}+c_{n3}$

The problem is then solved like processing of n jobs through two machines.

19.5 QUEUING THEORY

A queue is a waiting line. It is formed when arrival rate of customers is greater than the serving rate during a period of time. Any service system involving queuing situation has to achieve an economic balance between the percentage utilization of server and cost of waiting line. *Queuing theory*[4] utilizes mathematical models and performance measures to assess and improve the flow of customers through a queuing system.

19.5.1 Elements of Queuing Models

A queuing model consists of the following elements [Fig. 19.4]:

[4] Queuing theory was born in the early 1900s with the work of A. K. Erlang, a Danish mathematician, statistician and engineer of the Copenhagen Telephone Company.

1. Input Process The queuing model where in customers' arrival times are known with certainty are called *deterministic*. The essence of queuing theory is that it takes into account the randomness of the arrival process and the randomness of the service process. Hence, the distribution of inter-arrival time is seen as per a prescribed probability law, mainly *Poisson distribution*.

2. Queue Discipline This is the law according to which the customers form a queue and the manner in which they are served. Following are few examples of queue disciplines:

   (a) First-come first-served (FCFS)

   (b) Last-come first-served (LCFS)

   (c) Service in random order (SIRO)

   (d) Priority service.

   These queue disciplines can be seen in reservation counter (FCFS), government office files (LCFS), treatment of VIP (priority service), etc.

3. Service Mechanism Service mechanism is the facility available in the system to serve the customers. *Service time* is the amount of time needed to serve a customer, and *service rate* is number of customer served per unit time. Service times are assumed to be exponentially distributed about some average service time.

4. Capacity Capacity is the number of customers who can be in the queue. Some of the queuing systems has limited capacity beyond which customers are not permitted to enter into the system until the space is availed by serving the customers already in the queue.

5. Notation Using *Kendall's notation*, the queuing model is denoted in terms of arrival rate distribution (A), service rate distribution (B), number of servers (C), capacity of the system (D) and service discipline (E) as

   (A/B/C) : (D/E)

   In this, M denotes Poisson distribution of arrival rate and exponential distributed service rate. For example, a queuing model "(M/M/1) : (∞/FCFS)" means arrival in Poisson distribution, service in Poisson distribution, infinite (∞) customers, and first come first served (FCFS) basis of queue discipline. Also, this is an example of single server, infinite population queuing model.

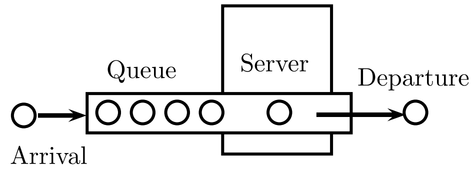


Figure 19.4 Queuing system.

19.5.2 Model (M/M/1):(∞/FCFS)

Consider a (M/M/1) : (∞/FCFS) system in which customer arrive at average rate λ (customers per unit time) and they are served at average rate of μ (customers per unit time). Average utilization, termed as *traffic intensity* (ρ), is expressed as

$$\rho = \frac{\lambda}{\mu}$$

Using the Poisson distribution in arrival of the customers, the probability that n customers will arrive in system during a period of interval t is given by

$$p\left(n, t\right) = e^{-m}\frac{m^n}{n!}$$

where $m = \lambda t$ is number of customers arrived in time interval t.

Using exponential distribution in serving the customers, the probability that no more than t time period is needed to serve a customer is given by

$$p\left(\bar{t}\right) = 1 - e^{-\mu t}$$

where μt is average number of customer served in time period t.

The following expressions can be derived using the concept of probability:

1. The probability of having exactly n customers in the system:
$$p_n = \rho^n\left(1-\rho\right)$$
   Using this, following expressions can be derived:

   (a) The probability of having exactly one customer in the system:
$$p_1 = \rho\left(1-\rho\right)$$

   (b) The probability that there are no customers in the queue:
$$p_0 = 1-\rho$$

(c) Expected number of customers in the system:

$$\begin{aligned} n_s &= \sum_{n=0}^{\infty} p_n \\ &= \frac{\rho}{1-\rho} \\ &= \frac{\lambda/\mu}{1-\lambda/\mu} \\ &= \frac{\lambda}{\mu-\lambda} \end{aligned}$$

(d) Expected number of customers in the queue:

$$\begin{aligned} n_q &= n_s - \rho \\ &= \frac{\rho^2}{1-\rho} \end{aligned}$$

2. Mean waiting time in queue:

$$t_q = \frac{\rho}{\mu-\lambda}$$

3. Mean time in the system:

$$\begin{aligned} \bar{t}_s &= \frac{n_s}{\lambda} \\ &= \frac{1}{\mu-\lambda} \end{aligned}$$

Little's law tells us that the average number of customers in the store n_q is the effective arrival rate λ times the average waiting time in the queue t_q.

$$n_q = \lambda t_q$$

This can be proved as

$$\begin{aligned} n_q &= \frac{\rho^2}{1-\rho} \\ &= \frac{\lambda}{\mu} \times \frac{\rho}{1-\lambda/\mu} \\ &= \lambda \times \frac{\rho}{\mu-\lambda} \\ &= \lambda t_q \end{aligned}$$

19.6 PERT AND CPM

Project evaluation and review technique (PERT) and critical path method (CPM) are the *network based techniques* of project scheduling. A *project* is a well-defined task having definable beginning and end points. It requires resources for the completion of the inter-related constituent activities. Project scheduling aims to develop an optimal sequence of activities of the project so that the project completion time and cost are properly balanced and kept at the optimum level. Project scheduling techniques are described as follows:

1. *Critical Path Method* This technique is based on only single estimate of completion time for each activity of the project of repetitive nature, therefore, CPM is an activity-oriented technique.

2. *Project Evaluation and Review Technique* This technique is applicable for the projects having non-repetitive and stochastic in nature. The technique emphasizes on the completion of task rather than the individual activities of project, therefore, it is an event-oriented technique.

   Each activity has three values of time estimates from β-distribution [Fig. 19.5]:

   (a) *Optimistic Time* - The shortest possible time required for the completion of activity is called *optimistic time* (t_o).
   (b) *Most-Likely Time* - The time required for the completion of activity under normal circumstances is called *most-likely time* (t_m).
   (c) *Pessimistic Time* - The longest possible time required for the completion of activity is called *pessimistic time* (t_p).

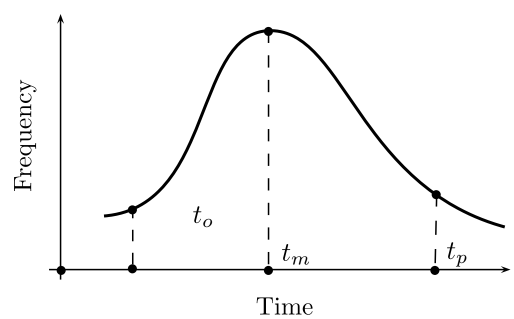


Figure 19.5 β-distribution of time.

PERT utilizes weighted average of the probability distribution of expected completion time of an activity, given by

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Variance and standard deviation of the time estimates are determined as

$$\begin{aligned} v &= \left(\frac{t_p - t_o}{6}\right)^2 \\ \sigma &= \sqrt{v} \\ &= \frac{t_p - t_o}{6} \end{aligned}$$

19.6.1 Project Network Components

Project network is the graphical representation of the project activities and events arranged in a logical sequence and depicting the inter-relationships among them: Thus, a network consists of two major components [Fig. 19.6]:

1. Activity An *activity* is a physically identifiable part of a project which consumes both time and resources. It is represented by an arrow in a network diagram. Tail of an arrow represents the start of activity while head of the arrow represents its completion of the activity. This representation also includes description and estimated completion time over the arrow.

   An activity has two terminal events which can be starting or completion points of other activities. Thus, an activity can have two types of associated activities:

   (a) *Predecessor Activity* - All those activities which must be completed before the start of activity under consideration, are called its *predecessor activities*.

   (b) *Successor Activity* - All those activities which have to follow the activity under consideration are called its *successor activities*.

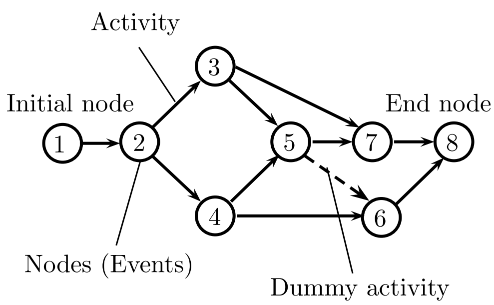


Figure 19.6 Elements of project network.

   A *dummy activity* is used to maintain the predefined precedence relationship only during the construction of the project network. It does not consume any time and resource, therefore, it is represented by a dotted arrow. For example, the dummy activity shown in Fig. 19.6 indicates that the activity 4-5 is the predecessor of the activity 6-8.

   Dummy activities can be used to maintain precedence relationships only when actually required. Their use should be minimized in the network diagram.

2. Event Beginning and ending of an activity are represented as *events*. Each event is shown as a node represented by a circle. An unbroken chain of activities between any two events is called a *path*.

Numbers should be so assigned to the events that they reflect the logical sequence of events in the network.

19.6.2 Critical Path

A project network can have numerous paths between the initial event and the last event of the project. Duration of a particular path would be the sum of durations of all the activities lying on that path. The path that has the longest duration is called the *critical path* and the activities that lie on the critical path are called *critical activities*. A delay in any of the critical activities can affect all other succeeding activities. A network can have more than one critical path of the same duration.

Critical path is identified by a systematic procedure comprising two series of computations:

1. Forward Pass Computation This is the method of computation of *earliest start time* (ES) of the events. The computation begins from the initial event ($\mathrm{ES}_1 = 0$) and moves towards the final event to arrive at the *earliest start time* of all the events. When two or more arrows terminate at an event, then ES is taken as the maximum value of ES's of all such activities. Earliest finishing time (EF) can be calculated by using

$$\mathrm{EF}_{ij} = \mathrm{ES}_{ij} + t_{ij}$$

2. Backward Pass Computation This is the method of computation of *latest start time* (LS) of the events. The computation begins from the final event (by assigning LS = EF) and moves towards the initial event to arrive at the *latest start time* of all the events. When two or more arrows terminate at an event, then LS is equal to the minimum of LS's of all such activities. Latest finishing time (LF) is calculated by using

$$\mathrm{LF}_{ij} = \mathrm{LS}_{ij} + t_{ij}$$

Slack time of an event is the difference between LS and ES. For a critical activity, the earliest start time (ES) and the latest start time (LS) are the same. Therefore, the path connecting events with zero slack is the critical path. Since ES = LS, a succeeding activity in a critical path shall commence immediately after its proceeding activity is completed. If there is any delay in either starting or completion of a critical activity, the project implementation period will get extended.

The procedure is explained in the section of *solved examples* of this chapter.

19.6.3 Activity Float Analysis

The free time available for an activity is called *float*. All paths in a network other than the critical path are called non-critical paths. If the activities in non-critical path are so delayed that they exceed the duration of the critical path, the overall project completion time will get delayed. Hence, a detailed study of non-critical activities with regard to the float is worth doing since it helps in better control of the project implementation and allocation of resources.

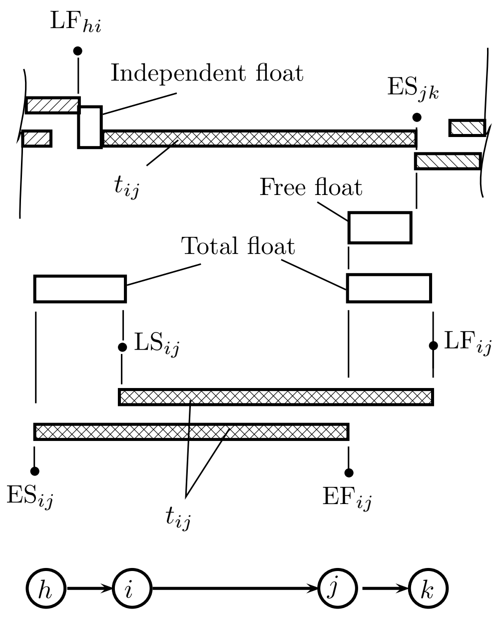


Figure 19.7 | Activity float analysis.

An event has only one estimate of slack but an activity has three types of float estimates [Fig. 19.7]:

1. *Total Float* *Total float* of an activity is the time by which an activity can be delayed without affecting the project completion time:

$$\begin{aligned}\text{Total float}_{ij} &= \text{LS}_{ij} - \text{ES}_{ij}\\ &= \text{LF}_{ij} - \text{EF}_{ij}\end{aligned}$$

2. *Free Float* *Free float* of an activity is the delay that can be permitted in an activity so that succeeding activities in the path are not affected. For this, the earliest start time of the head event of the activity shall not exceed. Therefore, free float of an activity ij is

$$\text{Free float} = \text{ES}_{jk} - \text{EF}_{ij}$$

3. *Independent Float* *Independent float* of an activity is the spare time available for the activity if its preceding activities are completed at their latest, and its succeeding activities start at their earliest. Therefore, independent float of an activity ij is

$$\text{Independent float} = \text{ES}_{jk} - \text{LF}_{hi} - t_{ij}$$

19.6.4 Time-Cost Trade-Off Analysis

Project cost is linked with the requirements of time and resources in completing a project. Project costs are generally a function of time. The shorter the period, lesser the overhead charges, but it would increase the direct cost.

The *critical path method* in network analysis can evaluate the alternate ways to expedite some of the activities and then analyze their effect in the cost of the project. This analysis is referred as the *time-cost trade-off analysis*.

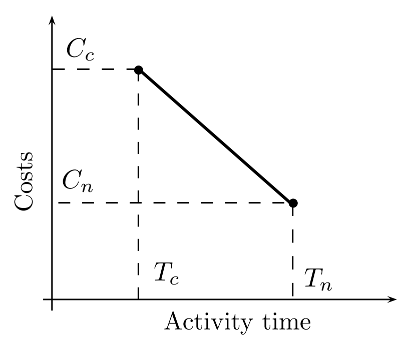


Figure 19.8 | Incremental cost.

Crashing is a process for reducing the duration of an activity by allocating more resources. This can affect the critical path. The minimum possible duration of an activity is called *crash duration* (T_c), and corresponding cost of the activity is called *crash cost* (C_c).The crash cost of an activity is always greater than the normal cost C_n. Assuming linear approximation [Fig. 19.8], the *incremental cost* is the slope of crashing an activity:

$$\text{Incremental cost} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

An activity with the least incremental cost should be crashed on priority. Crashing is preferred only for critical activities. Crashing of non-critical activities would not reduce the project duration.

IMPORTANT FORMULAS

Simplex Method

1. Problem definition
   Maximize objective function:
   $$Z = c_j x_j$$
   Constraints:
   $$a_{ij} x_j \leq b_i$$
   $$x_j, b_i \geq 0$$
2. Simplex algorithm
   (a) Standardization
   $$a_{ij} x_j + s_i = b_i$$
   $$a_{ij} x_j + A_1 = b_i$$
   $$Z = c_i x_i + 0 \times s_i \pm M A_i$$
   (b) Obtaining simplex tableau: Number of basic variables is m is number of constraints
   (c) Obtaining feasible solution
   (d) Testing the optimality
   $$\Delta_j = \begin{cases} \leq 0 & \text{Maximization} \\ \geq 0 & \text{Minimization} \end{cases}$$
   (e) Improving the solution

Transportation Method

1. Problem definition:
   Minimize objective function
   $$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$
   Constraints
   $$\sum_{j=1}^{n} x_{ij} = a_i \quad \text{for } i = 1, 2 \cdots m$$
   $$\sum_{i=1}^{m} x_{ij} = b_j \quad \text{for } j = 1, 2 \cdots n$$
   $$x_{ij} \geq 0 \quad \text{for all } i, j$$
2. Solution procedure:
   (a) First feasible solution by VAM
   (b) Testing the optimality by MODI method: A feasible solution has $m+n-1$ allocations.
   (c) Improving the solution

Assignment Problem

1. Problem definition
   Minimize objective function:
   $$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$
   subjected constraints:
   $$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for } i = 1, 2, \cdots n$$
   $$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for } j = 1, 2, \cdots n$$
   $$x_{ij} = 0 \text{ or } 1 \quad \text{for all } i, j$$
2. Solution procedure (HAM)
   (a) Opportunity cost table
   (b) Feasible solution and optimality test: If the number of the lines is equal to n then optimal solution is obtained.
   (c) Improving the solution

Sequencing

1. Problem definition: To determine the sequence of jobs (J_i) for minimum cost or time.
2. Solution: Johnson's algorithm

Model (M/M/1) : (∞/FCFS) Taking Poisson distribution in arrival and exponential distribution in serving:

1. Exactly n customers in the system:
   $$p_n = \rho^n (1 - \rho)$$
   (a) Exactly one customer in the system:
   $$p_1 = \rho(1 - \rho)$$
   (b) No customers in the queue:
   $$p_0 = 1 - \rho$$
   (c) Expected customers in the system:
   $$n_s = \sum_{n=0}^{\infty} p_n = \frac{\rho}{1 - \rho}$$
   (d) Expected customers in the queue:
   $$n_q = n_s - \rho = \frac{\rho^2}{1 - \rho} = \frac{\lambda}{\mu - \lambda}$$
2. Mean waiting time in the queue:
   $$\bar{t}_q = \frac{\rho}{\mu - \lambda}$$
3. Mean time in the system:
   $$\bar{t}_s = \frac{n_s}{\lambda} = \frac{1}{\mu - \lambda}$$
4. Little's law
   $$n_q = \lambda t_q$$

PERT and CPM

1. Expected completion time
   $$t_e = \frac{t_o + 4t_m + t_p}{6}$$
   $$v = \left(\frac{t_p - t_o}{6}\right)^2$$
   $$\sigma = \sqrt{v} = \frac{t_p - t_o}{6}$$
2. Critical path
   (a) Forward pass
   $$\mathrm{EF}_{ij} = \mathrm{ES}_{ij} + t_{ij}$$
   (b) Backward pass
   $$\mathrm{LF}_{ij} = \mathrm{LS}_{ij} + t_{ij}$$
   For a critical path
   $$\mathrm{EF}_{ij} = \mathrm{LF}_{ij}$$
3. Activity float analysis
   (a) Total float
   $$= \mathrm{LS}_{ij} - \mathrm{ES}_{ij} = \mathrm{LF}_{ij} - \mathrm{EF}_{ij}$$
   (b) Free float
   $$= \mathrm{ES}_{jk} - \mathrm{EF}_{ij}$$
   (c) Independent float
   $$= \mathrm{ES}_{jk} - \mathrm{LF}_{hi} - t_{ij}$$

SOLVED EXAMPLES

1. Determine the maximum value of the following objective function:

$$Z = 60x_1 + 50x_2$$

This is subjected to the following constraints:

$$x_1 + 2x_2 \le 40$$
$$3x_1 + 2x_2 \le 60$$
$$x_1, x_2 \ge 0$$

Solution. The first step of the simplex method is to standardize the problem by converting inequalities of the constraints into equations by introducing slack variables (s_1, s_2) as follows.

$$x_1 + 2x_2 + s_1 = 40$$
$$3x_1 + 2x_2 + s_2 = 60$$

Here, $0 \le s_1 \le 60$ and $0 \le s_2 \le 60$, both depending on the value of x_1 and x_2. Introduction of slack variables also modifies the objective function. The problem now is to maximize

$$Z = 60x_1 + 50x_2 + 0 \times s_1 + 0 \times s_2$$

subjected to constraints

$$x_1 + 2x_2 + s_1 = 40$$
$$3x_1 + 2x_2 + s_2 = 60$$
$$x_1, x_2, s_1, s_2 \ge 0$$

The initial simplex tableau is obtained as

	c_j	60	50	0	0	
e_i	CSV	x_1	x_2	s_1	s_2	b_i
0	s_1	1	2	1	0	40
0	s_2	3	2	0	1	60

From the above table, the first feasible solution is $x_1 = x_2 = 0$, $s_1 = 60$, $s_2 = 96$. Substituting these values in the objective function one gets $Z = 0$. This tableau is subjected to optimality test by calculating z_j as follows:

$$z_1 = 0 \times 1 + 0 \times 3 = 0$$
$$z_2 = 0 \times 2 + 0 \times 3 = 0$$
$$z_3 = 0 \times 1 + 0 \times 0 = 0$$
$$z_4 = 0 \times 0 + 0 \times 1 = 0$$

Subsequently, $\Delta_j = c_j - z_j$ are calculated from the following table:

	c_j	60	50	0	0		
e_i	CSV	x_1	x_2	s_1	s_2	b_i	b_i/a_{ik}
0	s_1	1	2	1	0	40	40/1
0	s_2	$\boxed{3}$	2	0	1	60	60/3
	z_j	0	0	0	0		
	Δ_j	60*	50	0	0		

Since the problem is of maximization type and all $\Delta_j \ge 0$, therefore, the initial solution is not found optimal. Hence, the solution needs an improvement.

In the present case, x_1 is the incoming variable (Δ_1 is most positive) replacing outgoing variable s_2 (least non-negative quotient $60/3 < 40/1$). To improve the solution, the key element is made 1, as shown below.

c_j	60	50	0	0	
CSV	x_1	x_2	s_1	s_2	b_i
s_1	1	2	1	0	40
x_1	3/3	2/3	0/3	1/3	60/3

Other elements of the key column are made zero and the revised simplex tableau is found as

	c_j	60	50	0	0	
e_i	CSV	x_1	x_2	s_1	s_2	b_i
0	s_1	0	4/3	1	-1/3	20
60	x_1	1	2/3	0	1/3	20
	z_j	60	40	0	20	
	Δ_j	0	10	0	-20	-

Positive value $\Delta_2 = 10$ indicates improvement in the solution. Therefore, it needs another iteration.

	c_j	60	50	0	0		
e_i	csv	x_1	x_2	s_1	s_2	b_i	$\frac{b_i}{a_{ik}}$
0	s_1	0	$\boxed{4/3}$	1	-1/3	20	15
60	x_1	1	2/3	0	1/3	20	30

Therefore, s_1 in CSV would be replaced by x_2. The solution is improved as follows:

	c_j	60	50	0	0	
e_i	CSV	x_1	x_2	s_1	s_2	b_i
50	x_2	0	1	3/4	-1/4	15
60	x_1	1	0	$-1/2$	1/2	10
	z_j	60	50	15/2	35/2	
	Δ_j	0	0	-15/2	-35/2	-

Negative values of all Δ_j in this tableau confirm that the solution is optimal. The solution is given by the two basic variables $x_1 = 10$, $x_2 = 15$. Substituting these values in the objective function, one gets $Z = 1350$.

2. Solve the following balanced transportation problem:

	D_1	D_2	D_3	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Solution. The following are the steps involved in solving the transportation problem:

(a) Vogel's approximation is applied as follows.

	D_1	D_2	D_3	Supply	
A	6	4	1	50	$4-1$
		[15]	[35]	[35/0]	$=3$
B	3	8	7	40	$7-3$
	[20]	[20]		[20/0]	$=4$
C	4	4	2	60	$4-2$
		[60]	2	[0]	$=2$
	20	95	35	150	
	[20]	[35]	[0]		
	[0]	[20]	[0]		
		[0]	[0]		
	$4-3$	$4-4$	$2-1$		
	$=1$	$=0$	$=1$		
		0	1		

The quantity in the square brackets shows the steps of allocations. Number of allocations is 5 which is equal to $3+3-1$. Therefore, it is a feasible solution.

(b) Using MODI method for optimality test, first u_i and v_i are determined for the allocated cells:

	D_1	D_2	D_3	u_i
A		4	1	1
B	3	8		5
C		4		1
v_i	-2	3	0	

(c) Vacant cells are now filled with their $u_i + v_i$ values:

	D_1	D_2	D_3	u_i
A	-1	4	1	1
B	3	8	5	5
C	-1	4	0	1
v_i	-2	3	0	

(d) CEM is determined by subtracting the above matrix from the original cost matrix:

	D_1	D_2	D_3
A	7	0	0
B	0	0	2
C	5	0	2

Since all the cells in this CEM are positive, the initial feasible solution is found to be optimal.

3. Solve the following 5×5 assignment problem for minimum cost of transportation:

	Job				
Workers	J_1	J_2	J_3	J_4	J_5
W_1	20	15	23	25	26
W_2	23	29	18	24	30
W_3	11	17	12	18	20
W_4	17	21	29	27	22
W_5	27	29	20	14	24

Solution. To obtain the opportunity cost table, first subtract minimum c_{ij} of each row from respective row elements:

	Job				
Workers	J_1	J_2	J_3	J_4	J_5
W_1	5	0	8	10	11
W_2	5	11	0	6	12
W_3	0	6	1	7	9
W_4	0	4	12	10	5
W_5	13	15	6	0	10

Similarly, subtract minimum c_{ij} of each column from respective column elements:

	Job				
Workers	J_1	J_2	J_3	J_4	J_5
W_1	5	0	8	10	6
W_2	5	11	0	6	7
W_3	0	6	1	7	4
W_4	0	4	12	10	0
W_5	13	15	6	0	5

Now, minimum number of lines are drawn such that they cover all zeros. It starts from the row or column with the maximum number of zeros:

Workers	Job				
	J_1	J_2	J_3	J_4	J_5
W_1	5	0	8	10	6
W_2	5	11	0	6	7
W_3	0	6	1	7	4
W_4	0	4	12	10	0
W_5	13	15	6	0	5

Since the number of lines drawn are equal to 5 $(= n)$, the optimal solution is obtained. The assignments are shown with squares:

Workers	Job				
	J_1	J_2	J_3	J_4	J_5
W_1	5	$\boxed{0}$	8	10	6
W_2	5	11	$\boxed{0}$	6	7
W_3	$\boxed{0}$	6	1	7	4
W_4	0	4	12	10	$\boxed{0}$
W_5	13	15	6	$\boxed{0}$	5

The assignment is done in following sequence:

(1) $W_1 - J_2$

(2) $W_2 - J_3$

(3) $W_3 - J_1$

(4) $W_5 - J_4$

(5) $W_4 - J_5$

The minimum value of objective function is obtained as

$$\begin{aligned} Z &= c_{12}+c_{23}+c_{31}+c_{45}+c_{54} \\ &= 15+18+11+22+14 \\ &= 80 \end{aligned}$$

4. Determine the optimal sequence of the following jobs through two machines:

Jobs	Machines	
	M_1	M_2
J_1	5	6
J_2	11	12
J_3	7	8
J_4	9	7
J_5	6	10
J_6	14	11

Solution. The solution is shown below:

	Sequence					
	1	2	3	4	5	6
Jobs	J_1	J_5	J_3	J_2	J_6	J_4
Minimum c_{i1}'s	5	6	7	11		
Minimum c_{i1}'s					11	7

The optimum time of the sequence is found using the following table:

Jobs	M_1		M_1	
	In	Out	In	Out
J_1	0	5	5	11
J_5	5	11	11	21
J_3	11	18	21	29
J_2	18	29	29	41
J_6	29	43	43	54
J_4	43	52	54	61

Thus, the minimum elapsed time is 61. Machine M_1 is idle only in the last job for $61-52 = 9$. Machine M_2 is idle after ending J_2 for $43-41 = 2$.

5. In a single server infinite population queuing model, arrivals follow a Poisson distribution with mean $\lambda = 6$ per hour. The service times are exponential with mean service time equal to 6 minutes. Determine the expected length of the queue.

Solution. Given that

$$\begin{aligned} \lambda &= 6 \text{ per hour} \\ \mu &= 60/12 \\ &= 10 \text{ per hour} \end{aligned}$$

Therefore,

$$\begin{aligned} \rho &= \lambda/\mu \\ &= 0.6 \end{aligned}$$

Expected length of queue is

$$\begin{aligned} l &= \frac{\rho^2}{1-\rho} \\ &= 0.9 \end{aligned}$$

6. At an assembly machine, parts arrive according to a Poisson process at the rate of 0.5 parts per minute. Processing time has exponential distribution with mean of 1.5 min. Determine the probability that a random part arrival finds that there are already 6 parts in the system (in machine + in queue)?

Solution. Given that

$$\lambda = 0.5 \text{ per minute}$$
$$\mu = \frac{1.5}{2}$$
$$= 0.75 \text{ per minute}$$

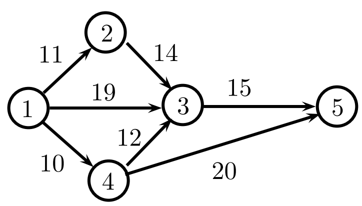
Hence,

$$\rho = \lambda/\mu$$
$$= 0.714$$

The probability that a random part arrival finds that there are already 6 parts in the system is

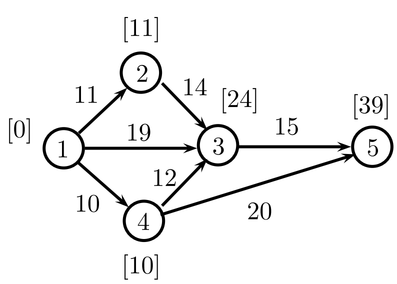
$$P = \rho^n (1 - \rho)$$
$$= 0.714^{10} (1 - 0.714)$$
$$= 0.0378$$

7. A project has the following network diagram where completion time of each activity are indicated on the arrow. Determine the maximum completion time of the project.

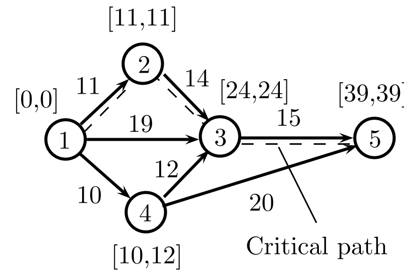


Solution. Solution procedure of the problem has two sets of calculations:

(a) Forward computation is used to determine the earliest finishing time (EF) of each event, as shown in the square brackets over the corresponding events:

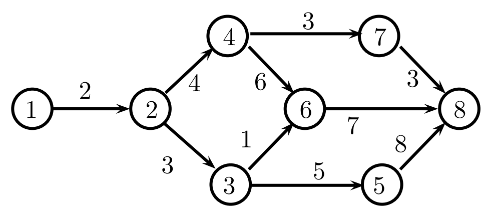


(b) Backward computation is done to compute the latest finishing time (LF), shown after comma in square brackets over the corresponding events:

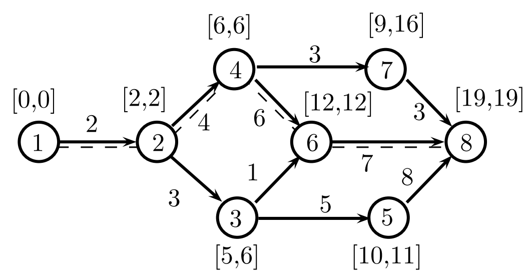


The events for which EF is equal to LF constitute the critical path (1-2-3-5), shown with dotted lines. Maximum completion time of the project is 39 units of time.

8. Find the critical paths of the following project network. Completion time of each activity is shown around the corresponding arrow. Also calculate the maximum completion time of the project.



Solution. The backward and forward computations are done on the network. The calculated earliest finishing time (EF's) and latest finishing time (LF's) of the events are shown within the square brackets [EF,LF] in the diagram below.



The patch constituted by the events of equal EF and LF is the critical path. For present case, critical path is given by

$$1-2-4-6-7-8$$

Maximum completion time of the project is equal to the EF or LF of the last event. Therefore, the project completion time is 19 units of time.

GATE PREVIOUS YEARS' QUESTIONS

1. The symbol used for transport in work-study is

(a) $\Rightarrow$ (b) T
(c) $\square$ (d) ∇

(GATE 2003)

Solution. In work study, $\Rightarrow$ is used for transport. Square box $\square$ is used for Inspection, and ∇ is used for storage [The question is out of syllabus].

Ans. (a)

2. Two machines of the same production rate are available for use. On machine 1, the fixed cost is Rs. 100 and the variable cost is Rs. 2 per piece produced. The corresponding numbers for the machine 2 are Rs. 200 and Re. 1, respectively. The sale price of the first 800 units is Rs. 3.50 per unit and subsequently it is only Rs. 3.00. The breakeven production rate for each machine is

(a) 75 (b) 100
(c) 150 (d) 600

(GATE 2003)

Solution. Both the machines have the same production rate and may be assumed to be working simultaneously to calculate the break-even production rate Q. Taking $Q < 800$,

$$3.50 \times 2Q = (100 + 2Q) + (200 + Q)$$
$$Q = 75$$

(which was assumed true.)

Ans. (a)

3. A manufacturer produces two types of products, 1 and 2, at production levels of x_1 and x_2, respectively. The profit is given as $2x_1 + 5x_2$. The production constraints are

$$x_1 + 3x_2 \leq 40$$
$$3x_1 + x_2 \leq 24$$
$$x_1 + x_2 \leq 10$$
$$x_1, x_2 > 0$$

The maximum profit which can meet the constraints is

(a) 29 (b) 38
(c) 44 (d) 75

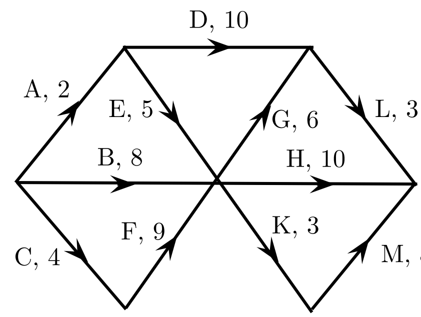
(GATE 2003)

Solution. Using graphical method, profit is maximum at $x_1 = 7$, $x_2 = 3$, therefore, maximum profit is

$$P = 2x_1 + 5x_2$$
$$= 29$$

Ans. (a)

4. A project consists of activities A to M shown in the net in the following figure with the duration of the activities marked in days.



The project can be completed

(a) between 18 and 19 days
(b) between 20 and 22 days
(c) between 24 and 26 days
(d) between 60 and 70 days

(GATE 2003)

Solution. Taking arbitrarily, the project path through C-F-K-M, project timing is $4 + 9 + 3 + 8 = 24$ days, which is the true in the range option (c) only.

Ans. (c)

5. In PERT analysis, a critical activity has

(a) maximum float (b) zero float
(c) maximum cost (d) minimum cost

(GATE 2004)

Solution. The free time available for an activity is called float. The term 'slack' is used with events while 'float' is used for activities. The path connecting events with zero slack is the critical path.

Ans. (b)

6. A maintenance service facility has Poisson arrival rates, negative exponential service time and operates on a 'first-come first-served' queue discipline. Breakdowns occur on an average of 3 per day with a range of zero to eight. The maintenance crew can service an average of 6 machines per day with a range of zero to seven.

The mean waiting time for an item to be serviced would be

(a) 1/6 day (b) 1/3 day
(c) 1 day (d) 3 days

(GATE 2004)

Solution. Given that arrival rate $\lambda = 3$ per day, service rate $\mu = 6$ per day, therefore, mean waiting time in the queue is

$$\bar{t}_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{1}{6}$$

Ans. (a)

7. A company produces two types of toys: P and Q. Production time of Q is twice that of P and the company has a maximum of 2000 time units per day. The supply of raw material is just sufficient to produce 1500 toys (of any type) per day. Toy type Q requires an electric switch which is available at the rate of 600 pieces per day only. The company makes a profit of Rs. 3 and Rs. 5 on type P and Q, respectively. For maximization of profits, the daily production quantities of P and Q toys should, respectively, be

(a) 100, 500 (b) 500, 100
(c) 800, 600 (d) 1000, 1000

(GATE 2004)

Solution. Let x_1 and x_2 is the daily production of P and Q respectively, then,

$$x_1 + 2x_2 \leq 2000$$
$$x_1 \leq 1500$$
$$x_2 \leq 600$$

Maximize the profit

$$P = 3x_1 + 5x_2$$

Solving by graphical methods gives solution matching with options

$$x_1 = 800$$
$$x_2 = 600$$

Ans. (c)

8. Consider a single server queuing model with Poisson arrivals (λ = 4/hour) and exponential service (μ = 4/hour). The number in the system is restricted to a maximum of 10. The probability that a person who comes and leaves without joining the queue is

(a) 1/11 (b) 1/10
(c) 1/9 (d) 1/2

(GATE 2005)

Solution. Given that $n = 10$. When $\rho = 1$ and the number in the system is restricted to a maximum of n, the probability that a customer leaves without joining the queue is

$$P'_n = \frac{1}{n+1} = \frac{1}{11}$$

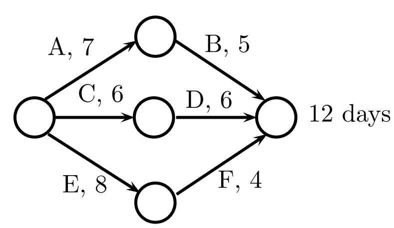
Ans. (a)

9. A project has six activities (A to F) with respective activity durations 7, 5, 6, 6, 8, 4 days. The network has three paths A-B, C-D, and E-F. All the activities can be crashed with the same crash cost per day. The number of activities that need to be crashed to reduce the project duration by one day is

(a) 1
(b) 2
(c) 3
(d) 6

(GATE 2005)

Solution. Th project network is shown in the figure:



The project duration through all the three paths is the same, that is, 12 days. To reduce the project duration by one day, at least one activity should be reduced by one day from all the three paths. Therefore, at least three activities will need to be crashed.

Ans. (c)

10. Company has two factories S1, S2 and two warehouses D1, D2. The supplies from S1 and S2 are 50 and 40, respectively. Warehouse D1 requires a minimum of 20 units and a maximum of 40 units. Warehouse D2 requires a minimum of 20 units and, over and above, it can take as much as can be supplied. A balanced transportation problem is to be formulated for the above situation. The number of supply points, the number of demand points, and the total supply (or total demand) in

the balanced transportation problem respectively are

(a) 2, 4, 90 (b) 2, 4, 110
(c) 3, 4, 90 (d) 3, 4, 110

(GATE 2005)

Solution. The balanced transportation problem for the given case is shown below:

Sources (S_i)	Destinations (D_j) D1′	D1″	D2	D2′	Supply (a_i)
S1	c_{11}	c_{22}	$\cdots$	c_{1n}	50
S2	c_{21}	c_{22}	$\cdots$	c_{2n}	40
S12′	c_{21}	c_{22}	$\cdots$	c_{2n}	∞
Demand (b_j)	20	40	20	∞	90

The total supply from the sources is 90. The minimum and maximum demands of destination D1 are distributed in two real sources D1′ and D1″. A dummy source S12′ is required to meet the infinite demand of destination D2′.

Ans. (c)

Linked Answer Questions

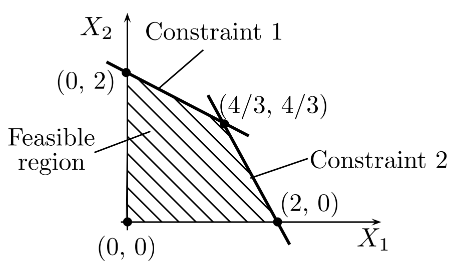
Consider a linear programming problem with two variables and two constraints. The objective function is: Maximize $X_1 + X_2$. The corner points of the desirable feasible region are (0, 0), (0, 2), (2, 0) and (4/3, 4/3).

11. If an additional constraint $X_1 + X_2 \leq 5$ is added, the optimum solution is

(a) (5/3, 5/3) (b) (4/3, 4/3)
(c) (5/2, 5/2) (d) (5, 0)

(GATE 2005)

Solution. The feasible region is shown in the figure:



The solution of the problem exists at the corner points only, that is, (4/3, 4/3) given in the option (c), which also satisfies the additional constraint $X_1 + X_2 \leq 5$.

Ans. (b)

12. Let Y_1 and Y_2 be the decision variables of the dual and v_1 and v_2 be the slack variables of the dual of the given linear programming problem. The optimum dual variables are

(a) Y_1 and Y_2 (b) Y_1 and v_1
(c) Y_1 and v_2 (d) v_1 and v_1

(GATE 2005)

Solution. Slack variable take up any slack between the left and right hand side of the inequalities upon conversion into equation. The optimal solution of the given LPP (i.e. primal) is found at the corner point, where the lines of both the constraints intersect each other. Therefore, the optimum solution of the dual will also be at intersection point, and will not have slack variables in optimal solution. Therefore, option (a) is correct.

Ans. (a)

13. The number of customers arriving at a railway reservation counter is Poisson distributed with an arrival rate of eight customers per hour. The reservation clerk at this counter takes six minutes per customer on an average with an exponentially distributed service time. The average number of the customers in the queue will be

(a) 3 (b) 3.2
(c) 4 (d) 4.2

(GATE 2006)

Solution. Let λ be the average arrival rate, μ be the average service rate. Given that $\lambda = 8$ customers per hour, $\mu = 10$ customers per hour. Here, average utilization termed as traffic density (ρ) is

$$\rho = \frac{\lambda}{\mu} = 0.8$$

For Poisson distributed arrival and exponentially distributed service, the average number of customers in the queue will be

$$n_q = \frac{\rho^2}{1-\rho^2} = 3.2$$

Ans. (b)

14. A manufacturing shop processes sheet metal jobs, wherein each job must pass through two machines (M_1 and M_2, in that order). The processing time (in hours) for these jobs is

Machine	Jobs P	Q	R	S	T	U
M_1	15	32	8	27	11	16
M_2	6	19	13	20	14	7

The optimal make-span (in hours) of the shop is

(a) 120 (b) 115
(c) 109 (d) 79

(GATE 2006)

Solution. Using Johnson's algorithm, the sequence of jobs is R-T-S-Q-U-P. With this, the cycle times for M1 is 109 and for M2 is 115. Therefore, processing time is 115 hours.

Ans. (b)

Linked Answer Questions

Consider a PERT network for a project involving six tasks (a to f).

Task	Predecessor	Expected Task Time (in days)	Variance of the Task Time (in days^2)
a	–	30	25
b	a	40	64
c	a	60	81
d	b	25	9
e	b, c	45	36
f	d, e	20	9

15. The expected completion time of the project is

(a) 238 days (b) 224 days
(c) 171 days (d) 155 days

(GATE 2006)

Solution. The critical path is a-c-e-f, for which the time is

$$t = 30+60+45+20 = 155 \text{ days}$$

Ans. (d)

16. The standard deviation of the critical path of the project is

(a) $\sqrt{151}$ days (b) $\sqrt{155}$ days
(c) $\sqrt{200}$ days (d) $\sqrt{238}$ days

(GATE 2006)

Solution. Standard deviation (σ) and variance (v) are related as

$$\sigma = \sqrt{v}$$

For critical path in the given case, standard deviation (σ) is

$$\sigma = \sqrt{25+81+36+9}$$
$$= \sqrt{151} \text{ days}$$

Ans. (a)

17. In an M/M/1 queuing system, the number of arrivals in an interval of length t is a Poisson random variable (i.e. the probability of there being n arrivals in an interval of length t is $e^{-\lambda t}(\lambda t)^n/n!$). The probability density function $f(t)$ of the inter-arrival time is given by

(a) $\lambda^2\left(e^{-\lambda^2 t}\right)$
(b) $e^{-\lambda^2 t}/\lambda^2$
(c) $\lambda e^{-\lambda t}$
(d) $e^{-\lambda t}/\lambda$

(GATE 2008)

Solution. In Poisson distribution, the probability density function $f(t)$ is $\lambda e^{-\lambda t}$.

Ans. (c)

18. A set of 5 jobs is to be processed on a single machine. The processing time (in days) is given in the table below. The holding cost for each job is Rs. K per day.

Job	Processing Time
P	5
Q	2
R	3
S	2
T	1

A schedule that minimizes the total inventory cost is

(a) T-S-Q-R-P (b) P-R-S-Q-T
(c) T-R-S-Q-P (d) P-Q-R-S-T

(GATE 2008)

Solution. The sequence of jobs can be simply assessed by doing on priority the job with least processing time, therefore, the schedule can be T-S-Q-R-P.

Ans. (a)

19. For the standard transportation, linear program with m sources and n destinations and total supply equaling total demand, an optimal solution (lowest cost) with the smallest number of non-zero x_{ij} values (amounts from source i to destination j) is desired. The best upper bound for this number is

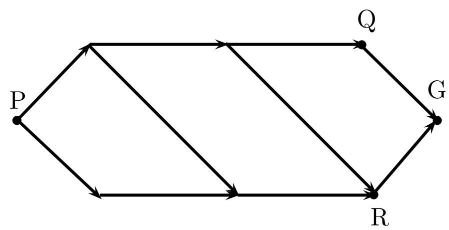
(a) mn (b) $2(m+n)$
(c) $m+n$ (d) $m+n-1$

(GATE 2008)

Solution. In transportation problems, a feasible solution is called basic feasible solution if it contains no more than $m+n-1$ non-negative allocations.

Ans. (d)

20. For the network below, the objective is to find the length of the shortest path from node P to node G. Let d_{ij} be the length of the directed arc from node i to node j. Let s_j be the length of the shortest path from P to node j.



Which of the following equations can be used to find s_G?

(a) $s_G = \text{Min}\{s_Q, s_R\}$

(b) $s_G = \text{Min}\{s_Q - d_{QG}, s_R - d_{RG}\}$

(c) $s_G = \text{Min}\{s_Q + d_{QG}, s_R + d_{RG}\}$

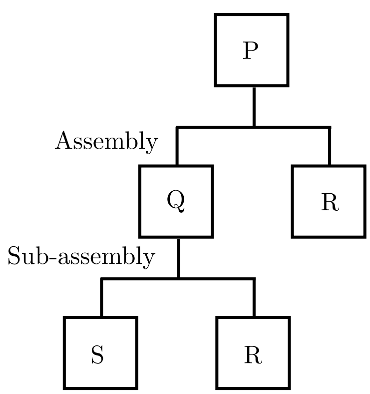
(d) $s_G = \text{Min}\{d_{QG}, d_{RG}\}$

(GATE 2008)

Solution. For the shortest path, all possible distances must be minimum. In the given case, Q and R represent two ways to reach G from P. As s_Q and s_R are known, therefore, the total distance must be calculated including d_{QG} and d_{RG} both.

Ans. (c)

21. The product structure of an assembly P is shown in the figure.



Estimated demand for end product P is as follows.

Weak	Demand
1	1000
2	1000
3	1000
4	1000
5	1200
6	1200

Ignore lead times for assembly and sub-assembly. Product capacity (per week) for component R is the bottleneck operation. Starting with zero inventory, the smallest capacity that will ensure a feasible production plan up to week 6 is

(a) 1000 (b) 1200
(c) 2200 (d) 2400

(GATE 2008)

Solution. From the figure, it is found that two times of R is required for given output of the product. If the initial capacity is C, then at the end of every week, the demand quality will be deducted from the inventory. Therefore, at the end of the 6th week, one will get

$$\begin{aligned} 6C - 2\times\sum_{n=1}^{6} D_n &\geq 0 \\ 6C - 2\times 6400 &\geq 0 \\ C &\geq 2133 \end{aligned}$$

Therefore, option (c) is nearest to the answer.

Ans. (c)

Common Data Questions

Consider the linear program (LP): Maximize $4x+6y$, subjected to

$$\begin{aligned} 3x+2y &\leq 6 \\ 2x+3y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

22. After introducing slack variables s and t, the initial basic feasible solution is represented by the table below (basic variables are $s = 6$ and $t = 6$, and the objective function value is 0)

	-4	-6	0	0	0
s	3	2	1	0	6
t	2	3	0	1	6
	x	y	s	t	RHS

After some simplex iterations, the following table is obtained

	0	0	0	2	12
s	5/3	0	1	-1/3	2
y	2/3	1	0	1/3	2
	x	y	s	t	RHS

From this, one can conclude that

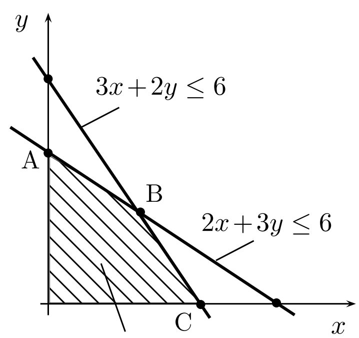
(a) the LP has a unique optimal solution
(b) the LP has an optimal solution that is not unique
(c) the LP is infeasible
(d) the LP is unbounded

(GATE 2008)

Solution. The net after opportunity cost row $\Delta_j = c_j - z_j$ is obtained as follows:

	c_j	4	6	0	0	
e_i		x	y	s	t	RHS
0	s	5/3	0	1	-1/3	2
6	y	2/3	1	0	1/3	2
	z_j	4	6	0	2	
	Δ_j	0	0	0	-2	

A simplex tableau depicts an optimal solution if all entries in the net after opportunity cost row (Δ_j) are non-positive for maximization type problems. Thus, the solution ($x = 0$, $y = 2$, $s = 2$, $t = 0$) is optimal, an value of objective function is $4x+6y = 12$. The graphical solution is shown in the figure:



Feasible region

The solution can exist at corner points (A, B, C). Apart from point A, the two corner points can also be examined. For point C ($x = 2$, $y = 0$), the value of objective function is 8, there not not optimal. Third corner points C (6/5, 6/5) is also an optimal solution. Thus, the option (b) is correct.

Ans. (b)

23. The dual of the LP is

(a) Minmize $6u+6v$ subjected to

$$3u+2v \geq 4$$
$$2u+3v \geq 6$$
$$u, v \geq 0$$

(b) Minmize $6u+6v$ subjected to

$$3u+2v \leq 4$$
$$2u+3v \leq 6$$
$$u, v \geq 0$$

(c) Minmize $4u+6v$ subjected to

$$3u+2v \geq 6$$
$$2u+3v \geq 6$$
$$u, v \geq 0$$

(d) Minmize $4u+6v$ subjected to

$$3u+2v \leq 6$$
$$2u+3v \leq 6$$
$$u, v \geq 0$$

(GATE 2008)

Solution. Every problem, in general form with implied summation, to maximize $c_j x_j$, subjected to $a_{ij} x_j \leq b_i$, and $x_j \geq 0$, can be converted so as to minimize $b_i x_i$, subjected to $y_i a_{ij} \leq c_j$, and $y_i \geq 0$, using the following conversions for duality:

$$b_i \rightarrow c_j$$
$$c_j \rightarrow b_i$$
$$a_{ij} \rightarrow a_{ji}$$
$$\geq \rightarrow \leq$$

Therefore, dual of the given LPP is to minimize $6u+6v$ subjected to

$$3u+2v \geq 4$$
$$2u+3v \geq 6$$
$$u, v \geq 0$$

Ans. (a)

24. The expected time (t_e) of a PERT activity in terms of optimistic time (t_o), pessimistic time (t_p) and most likely time (t_l) is given by

(a) $t_e = (t_o + 4t_l + t_p)/6$
(b) $t_e = (t_o + 4t_p + t_l)/6$
(c) $t_e = (t_o + 4t_l + t_p)/3$
(d) $t_e = (t_o + 4t_p + t_l)/6$

(GATE 2009)

Solution. The PERT is used when activity time estimates are stochastic in nature. For each activity, the three values of time are estimated and

represented in β-distribution. The expected time of an activity is

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

In the question, most likely time t_m is denoted by t_l.

Ans. (a)

25. Consider the following linear programming problem (LPP):

$$\begin{aligned} \text{Maximize } z &= 3x_1 + 2x_2 \\ x_1 &\leq 4 \\ x_2 &\leq 6 \\ 3x_1 + 2x_2 &\leq 18 \\ x_1 \geq 0, x_2 &\geq 0 \end{aligned}$$

(a) The LPP has a unique optimal solution
(b) The LPP is infeasible
(c) The LPP is unbounded
(d) The LPP has multiple optimal solutions

(GATE 2009)

Solution. Graphical method represents two solutions at (2, 6) and (4, 3) of equal $z = 18$, therefore, LPP has multiple optimal solutions.

Ans. (d)

26. Six jobs arrived in a sequence as given below

Job	Processing Time (days)
I	4
II	9
III	5
IV	10
V	6
VI	8

Average flow time (in days) for the above jobs using shortest processing time (SPT) rule is

(a) 20.83
(b) 23.16
(c) 125.00
(d) 139.00

(GATE 2009)

Solution. Shortest processing time (SPT) rule is based on the principle that the shortest job is handled first and completed. It orders the jobs in the order of increasing processing times. Whenever a machine is freed, the shortest job ready at the time will begin processing. This algorithm is optimal for finding the minimum total completion time and weighted completion time. The flow time is the cumulative sum of processing time each job by each job. In the given case, first the jobs are reordered in the ascending order of processing time as follows.

Job	Processing Time (days)	Cumulative Time (days)
I	4	4
III	5	9
V	6	15
VI	8	23
II	9	32
IV	10	42
	Sum	125

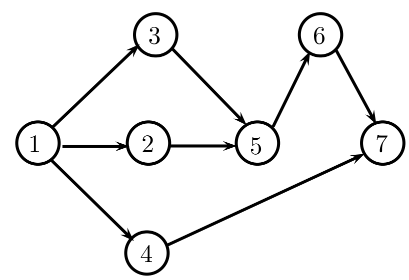
The average flow time is given by

$$\begin{aligned} \text{AFT} &= \frac{125}{6} \\ &= 20.8333 \end{aligned}$$

Ans. (a)

Common Data Questions

Consider the following PERT network:



The optimistic time, most likely time and pessimistic time of all the activities are given in the table below

Activity	Optimistic Time (Days)	Most-Likely Time (Days)	Pessimistic Time (Days)
1-2	1	2	3
1-3	5	6	7
1-4	3	5	7
2-5	5	7	9
3-5	2	4	6
5-6	4	5	6
4-7	4	6	8
6-7	2	3	4

27. The critical path duration of the network (in days) is

(a) 11 (b) 14
(c) 17 (d) 18

(GATE 2009)

Solution. Based on the following equation,

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

the expected time of all the activities t_e is calculated as follows:

Activity	t_o	t_m	t_p	t
1-2	1	2	3	2
1-3	5	6	7	6
1-4	3	5	7	5
2-5	5	7	9	7
3-5	2	4	6	4
5-6	4	5	6	5
4-7	4	6	8	6
6-7	2	3	4	3

Only three paths are possible, and their duration is found as

1. Path 1-3-5-6-7
$$T_e = 6+4+5+3 = 18$$
2. Path 1-2-5-6-7
$$T_e = 2+7+5+3 = 17$$
3. Path 1-4-7
$$T_e = 5+6 = 11$$

The longest duration path (d) is the critical path whose duration is 18 days.

Ans. (d)

28. The standard deviation of the critical path is

(a) 0.33 (b) 0.55
(c) 0.77 (d) 1.66

(GATE 2009)

Solution. The standard deviation of critical path is the sum of the standard deviation of its activity components, calculated as

$$\sigma = \sqrt{v} = \left(\frac{t_p - t_o}{6}\right)$$

Therefore, for critical path (a) 1-3-5-6-7,

Activity	t_o	t_p	v
1-3	5	7	$(2/6)^2$
3-5	2	6	$(4/6)^2$
5-6	4	6	$(2/6)^2$
6-7	2	4	$(2/6)^2$
Total			7/9

Standard deviation

$$\begin{aligned}\sigma &= \sqrt{v}\\ &= 0.881917\end{aligned}$$

Ans. (c)

29. Little's law is relationship between

(a) stock level and lead time in an inventory system
(b) waiting time and length of the queue in a queuing system
(c) number of machines and job due dates in a scheduling problem
(d) uncertainty in the activity time and project completion time

(GATE 2010)

Solution. Little's law tells us that the average number of customers in the store n_q is the effective arrival rate λ times the average waiting time in the queue t_q.

$$n_q = \lambda t_q$$

Ans. (b)

30. Simplex method of solving linear programming problem uses

(a) all the points in the feasible region
(b) only the corner points of the feasible region
(c) intermediate points within the infeasible region
(d) only the interior points in the feasible region.

(GATE 2010)

Solution. Simplex algorithm seeks for feasible solution located at a corner point of the feasible region. The method is based on the property that if objective function does not take the maximum value in a vertex, then there is an edge starting at that vertex along which the value of the function grows.

Ans. (b)

31. The project activities, precedence relationships and durations (in days) are described in the table. The critical path of the project is

Activity	Precedence	Duration
P	-	3
Q	-	4
R	P	5
S	Q	5
T	R,S	7
U	R,S	5
V	T	2
W	U	10

(a) P-R-T-V (b) Q-S-T-V
(c) P-R-U-W (d) Q-S-U-W

(GATE 2010)

Solution. Critical path is the path that takes maximum time. The given options take time 17, 19, 23, and 24 respectively, in which 24 is maximum for Q-S-U-W.

Ans. (d)

Common Data Questions

Four jobs are to be processed on a machine as per data listed in the table.

Job	Processing Time (days)	Due Date
1	4	6
2	7	9
3	2	19
4	8	17

32. If the Earliest Due Date (EDD) rule is used to sequence the jobs, the number of jobs delayed is

(a) 1 (b) 2
(c) 3 (d) 4

(GATE 2010)

Solution. Jobs with earliest due date will be first processes, therefore, following sequence results

Job	Processing Time	Flow Time	Due Date	Tardiness
1	4	4	6	-2
2	7	11	9	2
4	8	19	17	2
3	2	21	19	2

Therefore, total three jobs (2,3,4) will be delayed.

Ans. (c)

33. Using the Shortest Processing Time (SPT) rule, the total tardiness is

(a) 0 (b) 2
(c) 6 (d) 8

(GATE 2010)

Solution. The shortest processing time rule orders the jobs in the order of increasing processing times. Therefore, the following sequence of jobs would be followed:

Job	Processing Time	Flow Time	Due Date	Tardiness
3	2	2	19	-17
1	4	6	6	0
2	7	13	9	4
4	8	21	17	4

Total tardiness $= -17+4+4 = -9$, and only two jobs (2, 4) will be delayed.

Ans. (d)

34. Cars arrive at a service station according to Poisson's distribution with a mean rate of 5 per hour. The service time per car is exponential with a mean of 10 min. At steady state, the average waiting time in the queue is

(a) 10 min (b) 20 min
(c) 25 min (d) 50 min

(GATE 2011)

Solution. Given that

$$\lambda = 5 \text{ per hour}$$
$$\mu = 6 \text{ per hour}$$

Therefore,

$$\rho = \frac{\lambda}{\mu}$$

Average waiting time in the queue is

$$\begin{aligned} t_q &= \frac{\rho}{\mu - \lambda} \\ &= \frac{5}{6} \text{ hours} \\ &= 50 \text{ min} \end{aligned}$$

Ans. (d)

35. Consider the following system of equations

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 0 \\ x_2 - x_3 &= 0 \\ x_1 + x_2 &= 0 \end{aligned}$$

This system has

(a) a unique solution
(b) no solution
(c) infinite number of solutions
(d) five solutions

(GATE 2011)

Solution. Solving the first two equations provides the last equation, therefore, there will be infinite number of solutions.

Ans. (c)

Common Data Questions

One unit of product P_1 requires 3 kg of resource R_1 and 1 kg of resource R_2. One unit of product P_2 requires 2 kg of resource R_1 and 2 kg of resource R_2. The profit, per unit selling product, P_1 and P_2 are Rs. 2000 and Rs. 3000, respectively. The manufacturer has 90 kg of resource R_1 and 100 kg of resource R_2.

36. The manufacturer can make a maximum profit of rupees

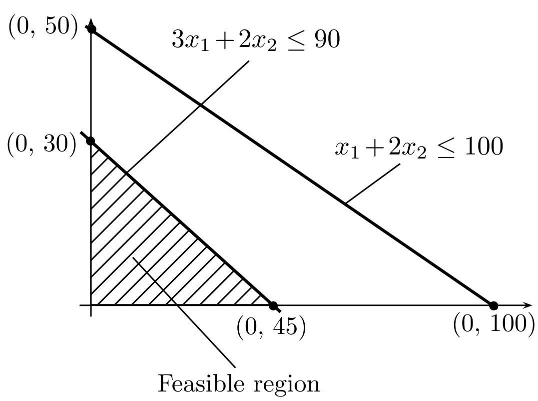
(a) 60,000
(b) 1,35,000
(c) 1,50,000
(d) 2,00,000

(GATE 2011)

Solution. Let x_1 and x_2 be the number of products P_1 and P_2, respectively. The problem can formulated as LPP to maximize $2000x_1 + 3000x_2$, subjected to constrains

$$3x_1 + 2x_2 \leq 90$$
$$x_1 + 2x_2 \leq 100$$
$$x_1, x_2 \geq 0$$

The constraints are shown on the graph:



The solution can exist at corner points of the feasible. The optimality of the solution can be examined separately:

(1) At (0, 45):

$$\text{Profit} = 2000 \times 0 + 3000 \times 45$$
$$= \text{Rs. } 1,35,000$$

(2) At (30, 0):

$$\text{Profit} = 2000 \times 30 + 3000 \times 0$$
$$= \text{Rs. } 60,000$$

Therefore, maximum profit is Rs. 1,35,000 at (0, 45)

Ans. (c)

37. The unit worth of resource R_2, that is, dual price of resource R_2 in Rs. per kg is

(a) 0
(b) 1350
(c) 1500
(d) 2000

(GATE 2011)

Solution. For maximum profit, the manufacturer has to produce $x_2 = 45$ number of product P_2. Given that 2 kg of resource R_2 is utilized for each product P_2. Therefore, unit worth (as defined in question) of resource R_2 is

$$\text{Unit worth} = \frac{135000}{45x2}$$
$$= \text{Rs. } 1500/\text{kg}$$

Ans. (c)

Linked Answer Questions

For a particular project, eight activities are to be carried out. Their relationships with other activities and expected duration (days) are mentioned in the table below.

Activity	Predecessors	Duration
A	–	3
B	A	4
C	A	5
D	A	4
E	B	2
F	D	9
G	C, E	6
H	F, G	2

38. The critical path for the project is

(a) A-B-E-G-H (b) A-C-G-H
(c) A-D-F-H (d) A-B-C-F-H

(GATE 2012)

Solution. All four options can be checked for critical path which requires maximum period in days

(1) A-B-E-G-H:

$$\begin{aligned}\text{Duration} &= 3+4+2+6+2\\ &= 17\end{aligned}$$

(2) A-C-G-H:

$$\begin{aligned}\text{Duration} &= 3+5+6+2\\ &= 16\end{aligned}$$

(3) A-D-F-H:

$$\begin{aligned}\text{Duration} &= 3+4+9+2\\ &= 18\end{aligned}$$

(4) A-B-C-F-H: Unavailable path

Therefore, option (d) is the critical path.

Ans. (c)

39. If the duration of activity F alone is changed from 9 to 10 days, then the

(a) critical path remains the same and the total duration to complete the project changes to 19 days
(b) critical path and the total duration to complete the project remains the same
(c) critical path changes but the total duration to complete the project remains the same
(d) critical path changes and the total duration to complete the project changes to 17 days

(GATE 2012)

Solution. Critical path is the same as A-D-F-H, and total duration to complete the project is increased to $17+(10-9) = 18$ days.

Ans. (a)

40. A linear programming problem is shown below: Maximize $3x+7y$, subject to

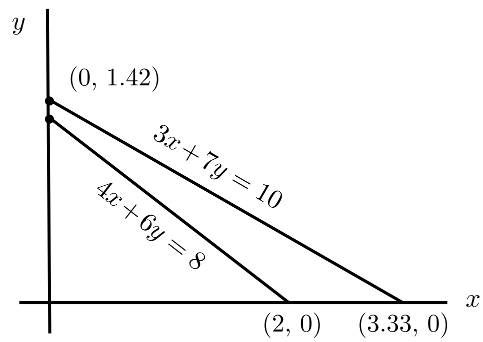
$$\begin{aligned}3x+7y &\le 10\\ 4x+6y &\le 8\\ x,y &\ge 0\end{aligned}$$

It has

(a) an unbounded objective function
(b) exactly one optimal solution
(c) exactly two optimal solutions
(d) infinitely many optimal solutions

(GATE 2013)

Solution. The objective function is given in a constraint, hence there are exactly two optimal solution for the problem at the ends (0, 1.42) and (3.33, 0) as shown in the graph:



Ans. (c)

41. Customers arrive at a ticket counter at the rate of 50 per hour and tickets are issued in the order of their arrival. The average time taken for issuing a ticket is 1 min. Assuming that customer arrivals form a Poisson process and service times are exponentially distributed, the average waiting time in queue in minutes is

(a) 3 (b) 4
(c) 5 (d) 6

(GATE 2013)

Solution. Given that

$$\begin{aligned}\mu &= 60 \text{ per hour}\\ \lambda &= 50 \text{ per hour}\end{aligned}$$

The waiting time is calculated as

$$\begin{aligned}t_q &= \frac{\lambda}{\mu(\mu-\lambda)}\\ &= \frac{50}{60-50}\\ &= 0.083 \text{ hours}\\ &= 5 \text{ min}\end{aligned}$$

Ans. (c)

MULTIPLE CHOICE QUESTIONS

1. In single server queuing model if the arrival rate is λ and service rate is μ, then what is the probability of the system being idle?

(a) λ/μ (b) μ/λ
(c) $1-\lambda/\mu$ (d) $(1-\lambda)/\mu$

2. The constraints in a linear programming model is applied on

(a) value of objective function
(b) value of decision variables
(c) use of available resources
(d) all of the above

3. The most distinguishing feature of a linear programming problem is

(a) the linear equations among all the variables
(b) simplicity of objective function and constraints
(c) decision variables being non-negative
(d) all of the above

4. Non-negativity of decision variables is essential aspect of the linear programming problems because

(a) it gives a controllable value of decision variables
(b) it assures a real-world problem where resources cannot be negative
(c) variables are related in uniformity
(d) none of the above

5. A feasible solution of a linear programming problem

(a) must satisfy all the constraints simultaneously
(b) must satisfy some of the constraints at a time
(c) can only be corner points of the feasible region
(d) must optimize the value of objective function

6. If two constraints do not intersect in the positive quadrant in a linear problem model, then

(a) the solution is unbounded
(b) the problem is infeasible
(c) one of the constraint is redundant
(d) none of the above

7. The feasible region of an LP model is said to be unbounded when

(a) finite number of solutions exists
(b) infinite number of solutions exists
(c) objective function can be decreased to any value
(d) none of the constraint is satisfied by any solution

8. Slack variables are used in the simplex method to

(a) convert inequalities of constraints ($\leq$) into equations
(b) convert inequalities of constraints ($\geq$) into equations
(c) get the initial feasible solution as the optimal one
(d) get the initial solution without negative value of decision variables

9. Artificial variables are used in the simplex method to

(a) to deal the constraints of $\leq$, thus provide initial solution without negative value of slack variables
(b) to deal the constraints of $\geq$, thus provide initial solution without negative value of slack variables
(c) to deal the constraints of $\leq$, thus provide optimal solution without negative value of slack variables
(d) to deal the constraints of $\geq$, thus provide optimal solution without negative value of slack variables

10. The coefficient of artificial variable in the objective function is

(a) zero because artificial variables do not represent any quantity relating to the decision problem
(b) unity to represent artificial variables in the optimal solution
(c) negative sign of M is used in maximization type problems and positive sign is used in minimization type problems.
(d) positive sign of M is used in maximization type problems and negative sign is used in minimization type problems.

11. If a negative value appears in the solution values of decision variables, then the solution is called to be

(a) infeasible (b) optimal
(c) unbounded (d) degenerate

12. Except when an artificial variable is included in the basis, a simplex tableau depicts an optimal solution if all entries in the after opportunity cost row ($\Delta_j = c_j - z_j$) for all values of j are

(a) ≤ 0 for maximization problem and ≥ 0 for minimization problem
(b) ≥ 0 for maximization problem and ≤ 0 for minimization problem
(c) ≥ 0 for both maximization and minimization problems
(d) ≤ 0 for both maximization and minimization problems

13. An optimal solution is achieved for a maximization problem by the simplex method when

(a) $c_j - z_j \geq 0$ (b) $c_j - z_j \leq 0$
(c) $c_j - z_j = 0$ (d) $z_j \leq 0$

14. A variable which does not appear in the basic variable column of a simplex table for a solution is assigned

(a) zero value (b) a negative value
(c) a positive value (d) infinite value

15. An optimal solution of an LP model is degenerate when

(a) multiple optimal solutions exist
(b) solution is infeasible
(c) the solution is unbounded
(d) the values of any one or more basic variable become equal to zero.

16. If any value of basic variable of the final simplex table is negative, then the solution is

(a) infeasible (b) unbounded
(c) optimal (d) degenerate

17. If all a_{ij} in the incoming variable column of the simplex tableau are negative, then

(a) the solution unbounded
(b) there exist multiple solutions
(c) there exist no solution
(d) the iteration is degenerate

18. The dual of the primal maximization of linear programming problem having m constraints and n non-negative variables should

(a) have n constraints
(b) m non-negative variables
(c) be a minimization problem
(d) all of the above

19. For any primal problem and its dual

(a) optimal value of the objective function is same
(b) the value of basic variables is same in the optimal solution
(c) primal will have an optimal solution if an only if dual does too
(d) all of the above

20. The right hand side constant of a constraint in a primal problem appears in the corresponding dual as

(a) a right hand side constant of a constraint
(b) a coefficient of decision variables
(c) a coefficient in the objective function
(d) none of the above

21. If dual has an unbounded solution, the primal has

(a) feasible solution
(b) infeasible solution
(c) unbounded solution
(d) none of the above

22. An initial solution of a transportation problem can be obtained by the following method:

(a) North-West Corner Method
(b) Least Cost Method
(c) Vogel's Approximation Method
(d) All of the above

23. The initial solution can be obtained by the method(s) should satisfy the following conditions

(a) the solution must be feasible, that is, it must satisfy all the supply and demand constraints (known as rim constraints)
(b) the number of positive allocations must be equal to $m+n-1$ where m is the number of rows and n is the number of columns.
(c) Both (a) and (b)
(d) None of the above

24. A dummy source or destination is added in the transportation model to

(a) balance the problem where demand is not equal to supply
(b) prevent solution from become infeasible
(c) prevent solution from become degenerate
(d) none of the above

25. The degeneracy in the transportation problem may occur when

(a) obtaining an initial basic solution having less than $m+n-1$ allocations.
(b) two or more occupied cells with same minimum allocation are simultaneously unoccupied
(c) Both (a) and (b)
(d) None of the above

26. In a transportation problem, the occurrence of degeneracy means

(a) the problem is unbalanced
(b) the problem is balanced
(c) the solution is infeasible
(d) all of the above

27. Applying the modified distribution (MODI) method, the basic feasible solution of the transportation problem is optimal when all the cells of cost evaluation matrix (CEM) are

(a) zero (b) positive
(c) negative (d) all of the above

28. The basic demerit of North-West Corner method to find the initial solution to the transportation problem is that

(a) it leads to a degenerate initial solution
(b) it does not take into account cost of the transportation
(c) it is most complicated and time consuming method
(d) all of the above

29. The solution to a transportation problem with m rows and n columns is said to be feasible when

(a) it satisfies all the rim constraints
(b) the number of positive allocations is equal to $m+n-1$
(c) both (a) and (b)
(d) none of the above

30. The largest opportunity cost value in an unused cell in a transportation problem is selected to improve the current solution because

(a) it represents reduction per unit cost
(b) it improves the total cost
(c) it ensures compliance of rim constraints
(d) all of the above

31. The smallest negative value is selected at the corners of the closed path to assign it to the unused cell because

(a) it improves the total cost
(b) it represents reduction per unit cost
(c) it ensures that the solution is feasible
(d) all of the above

32. The opportunity cost in the MODI method of testing the optimality in a transportation problem is analogous to simplex method has values of

(a) x_i's (b) c_j's
(c) z_j's (d) Δ_j's

33. The unoccupied cells in the transportation problem is analogous to

(a) current solution variables
(b) non-basic variables
(c) optimal values of the objective function
(d) net after opportunity cost cells.

34. In transportation problem, an opportunity cost value is used for unused cell to test optimality, it should be

(a) equal to zero
(b) most positive number
(c) most negative number
(d) post positive or negative

35. While improving the solution of a transportation problem, degeneracy may occur when

(a) two or more occupied cells on the closed path with negative sign are tied for lowest circled value
(b) the closed path indicates a diagonal value
(c) two or more occupied cells are on the closed path but neither of them represent a corner of the path
(d) none of the above

36. When the total supply is not equal to the total demand, the transportation problem is said to be

(a) balanced
(b) unbalanced
(c) degenerate
(d) none of the above

37. The assignment problem can be expressed as a transportation problem by the following changes:

(a) Taking demand and supply as unity.
(b) The numbers of origins and destinations should be made exactly equal.
(c) Both (a) and (b)
(d) None of the above

38. In an assignment problem, a dummy worker or job is used when the problem is

(a) balanced
(b) unbalanced
(c) degenerate
(d) all of the above

39. Hungarian method is the most simple and frequently used for solving assignment problems (say of order n). In this method, the solution is optimal when minimum number of horizontal and vertical lines covering all the zeros of opportunity cost table is equal to

(a) n (b) $n-1$
(c) $2n-1$ (d) n^2

40. Assignment problems of maximization type are converted into minimization type by

(a) adding each of a_{ij} from minimum of a_{ij}
(b) subtracting each of a_{ij} from largest of a_{ij}
(c) subtracting each of a_{ij} of column j from largest of a_{ij} in j
(d) subtracting each of a_{ij} of row i from largest of a_{ij} in i

41. For an assignment problem of order n, the maximum number of possible allocations is

(a) n solutions (b) $n!$ solutions
(c) $n+1$ solutions (d) $n!+1$ solutions

42. Every basic solution of a general assignment problem order n should have allocations equal to

(a) n (b) $2n$
(c) $2n+1$ (d) $2n-1$

43. If the assignment problem is to be solved by using modified distribution method, the number of dummy allocations that need to be added is

(a) n (b) $2n$
(c) $n-1$ (d) $2n-1$

44. If there are n jobs for two machines, the maximum number of possible sequences are

(a) n (b) $n!$
(c) n^2 (d) $n+1$

45. In a single server infinite population queuing model, arrivals follow a Poisson distribution with mean $\lambda = 4$ per hour. The service times are exponential with mean service time equal to 12 min. The expected length of the queue will be

(a) 4 (b) 3.2
(c) 1.25 (d) 5

46. Sequencing problem can be solved using the

(a) Gantt chart
(b) Johnson's algorithm
(c) both (a) and (b)
(d) none of the above

47. If in an simplex tableau when all $c_j - z_j$ values imply optimal solution but at least one of artificial variables appear in the basis with a positive value, then the solution is called

(a) infeasible (b) unbounded
(c) degenerate (d) multiple

48. The objective of network analysis is to minimize

(a) project duration
(b) project cost
(c) both (a) and (b)
(d) none of the above

49. Slack of an event (i), also called float, is the difference between

(a) latest occurrence time (L_i) and its earliest occurrence time (E_i)
(b) latest occurrence time (L_j) and earliest occurrence time (E_j) of its succeeding event
(c) its latest occurrence time (L_i) and latest occurrence time (E_i) of its succeeding event
(d) its latest occurrence time (L_i) and earliest occurrence time (E_j) of its succeeding event

50. Slack time of an activity is the difference between

(a) latest finishing time (LF) and earliest start time (ES)
(b) earliest finishing time (EF) and earliest start time (ES)
(c) latest start time (LS) and earliest start time (ES)
(d) none of the above

51. In general, the PERT deals with the project of

(a) repetitive nature
(b) non-repetitive nature
(c) deterministic nature
(d) probabilistic nature

52. In PERT, the span of time between the optimistic and pessimistic time estimates of an activity is

(a) σ (b) 3σ
(c) 6σ (d) 12σ

53. An activity, which is used to maintain the pre-defined precedence relationship only during the construction of the project network, is called a

(a) dummy activity
(b) critical activity
(c) normal activity
(d) uncritical activity

54. If E_i represents the earliest occurrence time of an event i and L_i represents the latest allowable time of an event i, then for the activities ij in the critical path(s) of a project network,

(a) $E_i = L_i$ and $E_j = L_j$
(b) $L_i - E_i = L_j - E_j$
(c) $L_i - E_i = L_j - E_j = d$ (constant)
(d) none of the above

55. The activity that can be delayed without affecting the execution of the immediate succeeding activity is determined by

(a) total float
(b) free float
(c) independent float
(d) all of the above

56. The key operating characteristic(s) for a queuing system include

(a) utilization factor
(b) percent idle time
(c) average waiting time
(d) all of the above

57. If there are n jobs to be performed, one at a time, on each of m machines, the possible sequences would be

(a) $(n!)^m$ (b) $(m!)^n$
(c) n^m (d) m^n

58. Sequencing problem is the problem of

(a) finding an optimal sequence of completing certain number of jobs so as to minimize the total elapsed time between completion of the first and the last job
(b) finding an optimal sequence of completing certain number of jobs so as to maximize the total elapsed time between completion of the first and last job
(c) finding an optimal sequence of completing certain number of jobs so as to minimize the elapsed time of the first job
(d) finding an optimal sequence of completing certain number of jobs so as to minimize the elapsed time of the last job

59. At a production machine, parts arrive according to a Poisson process at the rate of 0.35 parts per minute. Processing time for parts have exponential distribution with mean of 2 min. What is the probability that a random part arrival finds that there are already 8 parts in the system (in machine + in queue)?

(a) 0.0247 (b) 0.0576
(c) 0.0173 (d) 0.082

60. The first algorithm for linear programming was given by

(a) Bellman (b) Dantzig
(c) Kulm (d) von Neumann

61. A dummy activity is used in PERT network to describe

(a) precedence relationship
(b) necessary time delay
(c) resource restriction
(d) resource idleness

62. The cost of providing service in a queuing system increases with

(a) increased mean time in the queue

(b) increased arrival rate
(c) decreased mean time in the queue
(d) decreased arrival rate

63. If at the optimum in a linear programming problem, a dual variable corresponding to a particular primal constraint is zero, then it means that

(a) right hand side of the primal constraint can be altered without affecting the optimum solution
(b) changing the right hand side of the primal constraint will disturb the optimum solution
(c) the objective function is unbounded
(d) the problem is degenerate

64. In PERT, the distribution of activity times is assumed to be

(a) Normal (b) Gamma
(c) Beta (d) Exponential

65. In M/M/I system, service is at the rate of 6 per hour, and arrival rate is 4 per hour. The fraction for which the system would remain idle is

(a) 2/3 (b) 1/3
(c) 1/2 (d) 1/6

66. Service time in queuing theory is usually assumed to follow

(a) normal distribution
(b) Poisson's distribution
(c) Erlang distribution
(d) exponential distribution

67. In a 6×6 transportation problem, degeneracy would arise if the number of filled slots were

(a) equal to 36 (b) more than 12
(c) equal to 12 (d) less than 11

68. If the arrival takes place every 10 min with a service time of 4 min per unit, then the mean arrival rate, mean service rate, and the probability that one would have to wait will be, respectively,

(a) 10, 4, 0.25 (b) 0.1, 0.25, 0.4
(c) 10, 0.4, 0.25 (d) 0.1, 0.25, 0.1

69. In a M/M/1 queuing system, expected waiting time of a unit that actually waits is given by

(a) $\lambda/(\mu(\mu-\lambda))$ (b) $\lambda/(\lambda-\mu)$
(c) $1/(\mu-\lambda)$ (d) $\lambda/\left(\mu^2(\mu-\lambda)\right)$

70. The solution in a transportation model of dimension $m\times n$ is said to be degenerate if it has

(a) exactly $(m+n-1)$ allocations
(b) fewer than $(m+n-1)$ allocations
(c) more than $(m+n-1)$ allocations
(d) $m\times n$ allocations

71. For event 1, the earliest occurrence time is 8 weeks and the latest occurrence time is 26 weeks. For event 2, the earliest occurrence time is 32 weeks and the latest occurrence time is 37 weeks. If the activity time is 11 weeks, the total float will be

(a) 11 (b) 13
(c) 18 (d) 24

72. A variable which has no physical meaning, but is used to obtain an initial basic feasible solution to the linear programming problem is referred to as

(a) basic variable
(b) non-basic variable
(c) artificial variable
(d) basis

73. For a M/M/1:∞/FCFS queue, the mean arrival rate is equal to 10 per hour and the mean service rate is 15 per hour. The expected queue length is

(a) 1.33 (b) 1.53
(c) 2.75 (d) 3.20

74. In the solution of a linear programming problem by simplex method, if during an iteration, all ratios of right hand side b_i to the coefficients of entering variable a are found to be negative, it implies that the problem has

(a) infinite number of solutions
(b) infeasible solution
(c) degeneracy
(d) unbound solution

75. The variance of the completion time for a project is the sum of variances of

(a) all activity times
(b) non-critical activity times
(c) critical activity times
(d) activity times of first and last activities of the project

76. The critical path of a network is the path that

(a) takes the shortest time
(b) takes the longest time
(c) has the minimum variance
(d) has the maximum variance

NUMERICAL ANSWER QUESTIONS

1. In a machine shop, certain type of machines breakdown at an average rate of 6 per hour in accordance with Poisson process. The estimated cost of idle machine is Rs. 15 per hour. The company finds a repairman A who takes 6 min on an average to repair a machine and his wages are Rs. 8 per hour. Calculate the total cost per day if repairman A is hired for repair. The company finds another repairman B who takes 5 min to repair and the wages are Rs. 10 per hour. Determine the saving (in rupees) that the company can make.

2. In a queuing system, customers arrive during the rush hours at the rate of 24 customers per hour. The average number of customers that can be processed by the server is 30 per hour. The conditions for use of single-channel queuing model applies. Determine the probability that the cashier is idle. Also determine the average number of customers in the queuing system.

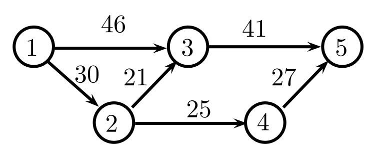
3. The time duration (in minutes) for four jobs A, B, C, and D in three machines 1, 2, and 3 is tabulated as follows:

Job	**M/c 1**	**M/c 2**	**M/c 3**
A	13	5	9
B	5	3	7
C	6	4	5
D	7	2	6

Using Johnson's algorithm, determine the optimum sequence of operation and the minimum elapsed time in carrying out all the four jobs through all the three machines.

4. A train reservation facility has 5 counters each capable of handing 20 requests per hour. The customers arrive with single reservation request at a mean rate of 90/hour. Determine the average length of this facility and the mean time a customer spends at the facility.

5. Determine the maximum completion time for the project shown in the following network diagram:



ANSWERS

Multiple Choice Questions

1. (c) **2.** (d) **3.** (a) **4.** (b) **5.** (a) **6.** (b) **7.** (b) **8.** (a) **9.** (b) **10.** (c)
11. (a) **12.** (a) **13.** (a) **14.** (a) **15.** (d) **16.** (a) **17.** (a) **18.** (d) **19.** (c) **20.** (c)
21. (b) **22.** (d) **23.** (c) **24.** (a) **25.** (c) **26.** (c) **27.** (b) **28.** (b) **29.** (a) **30.** (a)
31. (c) **32.** (d) **33.** (d) **34.** (c) **35.** (a) **36.** (b) **37.** (b) **38.** (b) **39.** (a) **40.** (b)
41. (b) **42.** (d) **43.** (c) **44.** (b) **45.** (a) **46.** (c) **47.** (a) **48.** (a) **49.** (a) **50.** (a)
51. (d) **52.** (c) **53.** (a) **54.** (a) **55.** (b) **56.** (d) **57.** (a) **58.** (a) **59.** (c) **60.** (b)
61. (a) **62.** (c) **63.** (c) **64.** (c) **65.** (b) **66.** (d) **67.** (a) **68.** (b) **69.** (a) **70.** (b)
71. (c) **72.** (c) **73.** (a) **74.** (b) **75.** (c) **76.** (b)

Numerical Answer Questions

1. 2.0 hour, Rs. 44 **2.** 1/5, 4 **3.** BACD, 43 minutes
4. 9, 6 **5.** 87 units

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (c) The probability that given system is idle

$$P_0 = 1 - \rho \quad \text{when } \lambda < \mu$$

2. (d) The constraints are applied on the decision variables and use of available resources, which in turn applies to the objective function also.

3. (a) The problem is said to be linear when the relationship among the variables is in the form of linear equations.

4. (b) The resources available in the work can be utilized when the value is non-negative, hence, decision variables are always non-negative.

5. (a) A solution is said to be feasible solution when it satisfies all the constraints simultaneously. The solution is optimal when it optimizes the value of the objective function.

6. (b) Infeasibility of LP happens when the constraints are conflicting each other, and hence, do not give a common feasible region.

7. (b) An LP problem is said to have unbounded solution when infinite number of the solutions exist for which the value of objective function increases away from the origin because the variables can be arbitrarily increased to large values.

8. (a) Inequalities of constraints ($\leq$) are converted into equations by introducing non-negative slack variables.

9. (b) Artificial variables are used in the simplex method to deal the constraints of $\geq$ by converting them into equality, thus providing initial solution without negative value of slack variables

10. (c) Artificial variables do not represent any quantity relating to the decision problem, hence, the technique of using artificial variables, known as big M method, is used. Negative sign of M is used in maximization type problems and positive sign is used in minimization type problems.

11. (a) The decision variables cannot be negative, hence, the solution is infeasible.

12. (a) Except when an artificial variable is included in the basis, a simplex tableau depicts an optimal solution if all entries in the after opportunity cost row ($\Delta_j = c_j - z_j$) for all values of j are ≤ 0 for maximization problem and ≥ 0 for minimization problem.

13. (a) Except when an artificial variable is included in the basis, a simplex tableau depicts an optimal solution if all entries in the after opportunity cost row ($\Delta_j = c_j - z_j$) for all values of j are ≤ 0 for maximization problem and ≥ 0 for minimization problem.

14. (a) If a variable does not appear in the basic variable column, that is, it is not included in the solution, hence, it is assigned zero value.

15. (d) Degeneracy is the one which indicates the condition that during any stage of iteration, the values of any one or more basic variables become equal to zero.

16. (a) The solution will be infeasible, because basic variables cannot be negative.

17. (a) This indicates that the entering variable could be increased infinitely with any of the current basic variables being removed from this basis.

18. (d) The optimal solution of maximization type problem may yield complete information about optimal solution of minimization type problem, and vice versa. Then, one is called primal, and the other is called dual.

19. (c) By definition of dual of a primal LP, primal will have an optimal solution if an only if dual does too.

20. (c) The right hand side constant of a constraint in a primal problem appears in the corresponding dual as a coefficient in the objective function

21. (b) If dual has an unbounded solution, the primal has infeasible solution.

22. (d) An initial solution of a transportation problem can be obtained by any of these three methods.

23. (c) The initial solution obtained by any of the three methods should satisfy the rim constraints and the number of positive allocations must be equal to $m+n-1$.

24. (a) If supply is more than demand, a dummy destination is made with equal to additional requirement with zero cost of transportation. If demand is more than the supply, a dummy source is added with zero unit cost of transportation

25. (c) The degeneracy occurs when allocations are less than $m+n-1$. The options (a) is applicable only for the initial basic solution while option (b) is applicable for any stage. Hence, both are correct.

26. (c) In a transportation problem, the occurrence of degeneracy means the solution is infeasible.

27. (b) If all cells are positive in CEM, then the solution is optimal.

28. (b) The basic demerit of North-West Corner method to find the initial solution to the transportation problem is that it does not take into account cost of the transportation.

29. (a) The solution is said to be feasible when it satisfies all the supply and demand constraints (known as rim constraints). The degeneracy will occur when allocations are less than $m+n-1$.

30. (a) The largest opportunity cost value in an unused cell in a transportation problem represents reduction per unit cost, hence, it is selected to improve the current solution.

31. (c) The smallest negative value is selected at the corners of the closed path to assign it to the unused cell because it ensures that the solution is feasible.

32. (d) The opportunity cost in MODI method of the testing the optimality of a solution of the transportation problem is equivalent to the values of $\Delta_j = c_j - z_i$ in the simplex method.

33. (d) The unoccupied cells in the transportation problem represent the values of non-basic variables in the current solution.

34. (c) The first step in improving the solution is to identify the cell from cost evaluation matrix with most negative entry.

35. (a) When two or more occupied cells on the closed path with negative sign are tied for lowest circled value, it is not certain whether the solution will improve or not, hence, it represents a case of degeneracy.

36. (b) When total supply is not equal to to total demand, the transportation problem is called unbalanced, and solved by using dummy destination or dummy source.

37. (b) The assignment problem can be expressed as a transportation problem by two steps:

(i) Taking demand and supply as 1, that is, there will be only one assignment in a given row or column.

(ii) The numbers of origins and destinations should be made exactly equal. This makes the problem a square matrix.

38. (b) If assignment problem is unbalanced, that is number of jobs is not equal to number of workers, then according to the need, a dummy worker or job is generated with zero cost

39. (a) If number of these lines is equal to n then optimal solution is obtained.

40. (b) If the assignment problem is of maximization type, it is converted into minimization type. The largest value of a_{ij} is located and then each of a_{ij} is subtracted from it. The problem is solved as minimization problem.

41. (b) For an assignment problem of order n, the number of maximum allocations possible is $n!$.

42. (d) The assignment problem is analogous to the transportation problem with size $m = n$ for which the solution is feasible when allocations are $m+n-1$. Thus, for assignment problem where $m = n$, allocations should be $2n-1$.

43. (c) Using modified distribution method for transportation problem of order n, the allocations are $2n-1$, but an assignment problem can have only n allocations in the optimal solution. Hence, the number of additional allocations is $2n-1-n = n-1$.

44. (b) If there are n jobs in two machines then there are $n!$ sequences possible.

45. (a) Given that

$$\begin{aligned}\lambda &= 4 \text{ per hour}\\ \mu &= \frac{60}{12}\\ &= 5 \text{ per hour}\end{aligned}$$

Therefore,

$$\begin{aligned}\rho &= \frac{\lambda}{\mu}\\ &= 0.8\end{aligned}$$

Expected length of queue is

$$\begin{aligned}n_q &= \frac{\rho^2}{1-\rho}\\ &= 3.2\end{aligned}$$

46. (c) Gantt Chart and Johnson's Algorithm are the methods of solving sequencing problems.

47. (a) The solution is infeasible because artificial variables cannot be in the optimal solution.

48. (a) The objective of network analysis is to minimize the production duration.

49. (a) Slack of an event (i), also called float, is the difference between latest occurrence time (L_i) and its earliest occurrence time (E_i). It has no relation with the succeeding event.

50. (a) Slack time of an activity is the difference between latest finish time and earliest start time.

51. (d) The PERT is used when activity time estimates are stochastic in nature. For each activity, three values of time are estimated and represented in β-distribution. Thus, the technique is useful for all jobs or project that have an element of uncertainty in the estimation of duration, as is the case with new types of projects.

52. (c) In PERT, for each activity, time estimated are based on β-distribution by which the variance (v), standard deviation σ, optimistic time t_o and pessimistic time (t_p) are related as

$$\sigma = \sqrt{v} = \frac{t_p - t_o}{6}$$

53. (a) An activity, which is used to maintain the pre-defined precedence relationship only during the construction of the project network, is called a dummy activity.

54. (a) The path connecting events with zero slack is the critical path. In turn, a succeeding activity in a critical path would commence immediately after its proceeding activity is completed.

55. (b) Free float of an activity is the delay that can be permitted in an activity so that succeeding activities in the path are not affected.

56. (d) All the given options are the operating characteristics of a queuing system.

57. (a) The possible sequences would be $(n!)^m$

58. (a) Sequencing problem is the problem of finding an optimal sequence of completing certain number of jobs so as to minimize the total elapsed time between completion of the first and last job.

59. (c) Given that

$$\lambda = 0.35 \text{ per minute}$$
$$\mu = \frac{1}{2} = 0.5 \text{ per minute}$$

Hence,

$$\rho = \frac{\lambda}{\mu} = 0.7$$

The probability that a random part arrival finds that there are already 8 parts in the system is

$$P_n = \rho^n (1-\rho) = 0.7^8 (1-0.7) = 0.01729$$

60. (b) Dantzig (1914–2005), an American mathematical scientist, is known for his development of the simplex algorithm, an algorithm for solving linear programming problems.

61. (a) An activity, which is used to maintain the pre-defined precedence relationship only during the construction of the project network is called a dummy activity.

62. (c) The deployment of additional sources by increasing the cost of service will reduce the mean time in the queue.

63. (c) Unboundness of the objective function occurs when there is no constraint on the solution so that one or more of the decision variables can be increased indefinitely without violating any of the restrictions. In simplex method, the row with smallest non-negative replacement ratio is selected for the outgoing variable. If there is no non-negative replacement ratio, (i.e. all ratios are negative) then this indicates an unbounded solution.

64. (c) In PERT analysis, activity times are assumed to be under β-distribution.

65. (b) Given that arrival rate $\lambda = 4$, service rate $\mu = 6$. The probability that the system is idle is

$$P_0 = 1-\rho = 1-\frac{\lambda}{\mu} = 1-\frac{4}{6} = \frac{1}{3}$$

66. (d) Service time is amount of time needed to serve a customer, and service rate is the number of customers served per unit time. In queuing theory, it is assumed that service times are exponentially distributed about some average service time.

67. (a) In a $n \times n$ transportation problem, degeneracy will arise if the number of filled slots were equal to $n \times n$, and then it cannot be tested for optimality.

68. (b) Given that arrival rate

$$\lambda = \frac{1}{10} = 0.1 \text{ units per minute}$$

and service rate

$$\mu = \frac{1}{4} = 0.25 \text{ units per minute}$$

Therefore, the probability that one would have to wait is

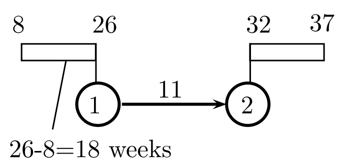
$$\rho = \frac{\lambda}{\mu} = 4$$

69. (a) Mean waiting time in queue is given by ($\rho = \lambda/\mu$)

$$t_q = \frac{\rho}{\mu-\lambda} = \frac{\lambda}{\mu(\mu-\lambda)}$$

70. (b) In transportation problem, if the number of allocations is less than $(m+n-1)$ then this is the case of degeneracy.

71. (c) Total float represents the maximum time by which the completion of activity can be delayed without affecting the project completion time. By this definition, for the calculation of total float of a particular activity, it is assumed that all its proceeding activities start at their respective earliest start times. For the given case, timings are shown in the figure:



To determine the total float of the activity 1-2, it is assumed that the predecessor activity has to start at its earliest start time. From the given data, the event 1 can start on 8th week, and can be delayed upto 16th week. Therefore, the activity 1-2 can be delayed by a total of $26-8 = 18$ weeks, that is, total float.

72. (c) To deal the constraints of $\geq$ to provide initial solution without negative value of slack variables, artificial variables are used.

73. (a) Given that $\lambda = 10$, $\mu = 15$. Using

$$\rho = \frac{\lambda}{\mu} = 0.667$$

the expected length of queue is

$$n_q = \frac{\rho^2}{1-\rho} = \frac{0.667^2}{1-0.667^2} = 1.33$$

74. (b) Infeasibility is said to exist when a given problem has no feasible solution, and is evident graphically when no common point is found in the feasible region (two dimensional) of all the constraints of a problem. In terms of simplex algorithm, when in the final solution, an artificial variable is in the basis at a positive value, then there is no feasible solution to the problem.

75. (c) Variation in critical activities can affect the completion time of the project. Therefore, variation of the completion time for a project is taken as the sum of the variances of the critical activity times.

76. (b) The path that has the longest duration is called the critical path.

Numerical Answer Questions

1. Given that for machine, the breakdown (arrival) rate is $\lambda = 6$ per hour. For the repairman A, service rate is $\mu_A = 60/6 = 10$ break downs per hour. If A is hired, the average number units in the system

$$n_{sA} = \frac{\lambda}{\mu_A - \lambda} = \frac{6}{10-6} = 1.5 \text{ hours}$$

Hence, the total machine hours lost in 8-hour ship is $8\times 1.5 = 12$ hours, whose cost is Rs. $12\times 15 = 180$, while the wages are Rs. $8\times 8 = 64$. Thus, the total cost is Rs. $180+64 = 244$ per day.

For the repairman B, service rate is $\mu_B = 60/5 = 12$ breakdowns per hour.

$$n_{sB} = \frac{\lambda}{\mu_B - \lambda} = \frac{6}{12-6} = 1.0 \text{ hours}$$

Hence, the total machine hours lost in 8-hour shift is $8\times 1 = 8$ hours, whose cost is Rs. $8\times 15 = 120$, while the wages are Rs. $8\times 10 = 80$. Thus, total cost is Rs. $120+80 = 200$ per day. Hence, the total saving for the company is Rs. $244-200 = 44$ per day.

2. Given that $\lambda = 24$ customers per hour and mean service rate $\mu = 30$ customer per hour. The

utilization parameter is

$$\rho = \frac{\lambda}{\mu} = \frac{24}{30} = \frac{4}{5}$$

Probability that the cashier is idle is

$$P_0 = 1-\rho = \frac{1}{5}$$

Expected number of customers in the system is

$$n_s = \frac{\rho}{1-\rho} = 4$$

3. This is a problem of 3 machines, which will be converted into two machines as below

Job	**M/c 1+2**	**M/c 2+3**
A	18	14
B	8	10
C	10	9
D	9	8

Using Johnson's algorithm, steps are as follows

(a) Smallest value is 8 for B (1st column) and D (2nd column) therefore

1	**2**	**3**	**4**
B			D

(b) Second smallest value is 9 for C (2nd column) therefore

1	**2**	**3**	**4**
B		C	D

(c) The balance space is for A

1	**2**	**3**	**4**
B	A	C	D

The minimum cycle time is calculated as

Job	**M/c 1**	**M/c 2**	**M/c 3**
B	5	5+3=8	8+7=15
A	5+13=18	18+5=23	23+9=32
C	18+6=24	24+4=28	32+5=37
D	24+7=31	31+2=33	6+37=**43**

Thus, minimum elapsed time is 43 minutes.

4. Given that

$$\mu = 5\times 20 = 100 \text{ per hour}$$
$$\lambda = 90 \text{ per hour}$$

Utilization parameter is

$$\rho = \frac{\lambda}{\mu} = \frac{90}{100} = \frac{9}{10}$$

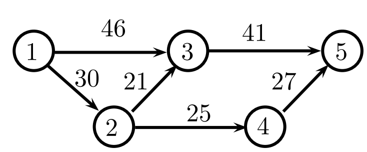
The average length of this facility, that is, expected number of customers in the system are

$$n_s = \frac{\rho}{1-\rho} = \frac{9/10}{1/10} = 9$$

Mean time a customer spends at the facility is

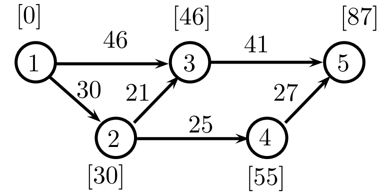
$$\bar{t}_s = \frac{1}{\mu-\lambda} = \frac{1}{100-90} = 0.1 \text{ hr} = 6 \text{ min}$$

5. The project network is shown below:



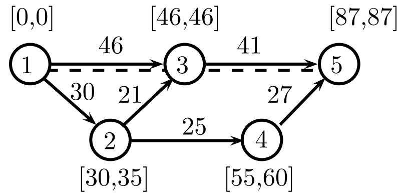
Solution procedure of the problem has two sets of calculations:

(a) Forward computation is used to determine the earliest finishing time (EF) of each event, as shown in the square brackets over the corresponding events:



(b) Backward computation is done to compute the latest finishing time (LF), shown after comma

in square bracket over the corresponding events:



The events for which EF is equal to LF constitute the critical path (1-3-5), shown with dotted lines. Maximum completion time of the project is 87 units of time.

QUESTION BAND FOR PRACTICE

(c) A-2, B-4, C-3, D-1
(d) A-2, B-2, C-1, D-3

7. The mechanism obtained by fixing the slider in the slider crank mechanism is

(a) reciprocating compressors
(b) hand pump
(c) slotted lever crank mechanism
(d) Whitworth quick return mechanism

8. The maximum efficiency of a screw jack having square threads with friction angle ϕ is

(a) $\dfrac{1-\tan(\phi/2)}{1+\tan(\phi/2)}$ (b) $\dfrac{1-\tan\phi}{1+\tan\phi}$

(c) $\dfrac{1-\sin\phi}{1+\sin\phi}$ (d) $\dfrac{1-\sin(\phi/2)}{1+\sin(\phi/2)}$

9. Match List I with List II and select the correct answer using the codes given below:

List I	List II
A. Node point	1. Balancing of reciprocating masses
B. Critical damping	2. Torsional vibration of shafts
C. Magnification factor	3. Forced vibration of spring-mass system
D. Hammer blow	4. Damped vibration
	5. Inclined water tube

(a) A-1, B-4, C-3, D-2
(b) A-2, B-4, C-3, D-1
(c) A-1, B-3, C-4, D-2
(d) A-2, B-3, C-4, D-1

10. Match List I with List II and select the correct answer from the codes given below:

List I (Terminology)	List II (Relevant Term)
A. Interference	1. Arc of approach, arc of recess, circular pitch
B. Dynamic load on tooth	2. Lewis equation
C. Static load	3. Minimum number of teeth on pinion
D. Contact ratio	4. Inaccuracies in tooth profile

(a) A-3, B-4, C-1, D-2
(b) A-1, B-2, C-3, D-4
(c) A-4, B-3, C-2, D-1
(d) A-3, B-4, C-2, D-1

11. In an oil lubricated journal bearing, coefficient of friction between the journal and the bearing

(a) remains constant at all speeds
(b) is minimum at zero speed and increases monotonically with increase in speed
(c) is maximum at zero speed and decreases monotonically with increase in speed
(d) becomes minimum at an optimum speed and then increases with further increase in speed

12. A shaft is subjected to a maximum bending stress of 80 N/mm^2, and maximum shearing stress equal to 30 N/mm^2 at a particular section. If the yield point in tension of the material is 280 N/mm^2, and maximum shear stress theory of failure is used, then the factor of safety obtained will be

(a) 2.5 (b) 2.8
(c) 3.0 (d) 3.5

13. An ideal fluid is

(a) compressible and viscous
(b) is one which obeys perfect law of gases
(c) compressible and gaseous
(d) incompressible and inviscous

14. If the surface tension of water–air interface is 0.073 N/m, the gauge pressure inside a rain drop of 1 mm diameter will be

(a) 0.146 N/m^2 (b) 73 N/m^2
(c) 146 N/m^2 (d) 292 N/m^2

15. In the case of laminar flow over a flat of the length L, held parallel to the relative motion of a fluid, the mean drag coefficient is given by

(a) $0.664/\sqrt{\mathrm{Re}_x}$ (b) $1.6328/\sqrt{\mathrm{Re}_x}$
(c) $0.455/\sqrt{\mathrm{Re}_x}$ (d) $0.38/\sqrt{\mathrm{Re}_x}$

16. According to Newton's law of viscosity, the shear stress between two fluid layers is proportional to

(a) the shear strain
(b) the rate of shear strain
(c) pressure at that point
(d) pressure gradient between layers

17. The velocity distribution in the boundary layer over a flat plate, set parallel to the direction of an

incompressible free stream, is given by one-sixth power law of the form

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/6}$$

What is the ratio of displacement thickness to boundary layer thickness?

(a) 1 : 8 (b) 1 : 7
(c) 7 : 72 (d) 3 : 8

18. Lumped heat transfer analysis of a solid object suddenly exposed to a fluid medium at a different temperature is valid when

(a) Biot number < 0.1
(b) Biot number > 0.1
(c) Fourier number > 0.1
(d) Fourier number < 0.1

19. A heat exchanger with heat transfer surface area A and overall heat transfer coefficient U handles two fluids of heat capacities C_{max} and C_{min}. The parameters NTU (number of transfer units) used in the analysis of heat exchanger is specified as

(a) $\dfrac{AC_{min}}{U}$ (b) $\dfrac{U}{AC_{min}}$
(c) UAC_{min} (d) $\dfrac{UA}{C_{min}}$

20. The radiative heat transfer rate per unit area (W/m^2) between two plane parallel gray surfaces whose emissivity is 0.9 and maintained at 400 K and 300 K is

(a) 992 (b) 812
(c) 567 (d) 464

21. Identify the process for which the two integrals '$\int pdv$' and '$-\int vdp$' evaluated between any two given states give the same value

(a) isenthalpic (b) isothermal
(c) isentropic (d) polytropic

22. Number of components (c), phase (ϕ) and degrees of freedom (f) are related by Gibbs phase rule as

(a) $c-\phi-f=2$ (b) $f-c-\phi=2$
(c) $c+f-\phi=2$ (d) $\phi+f-c=2$

23. A pump is installed at a height of 5 m above the water level in the sump. Frictional loss on the suction side is 0.6 m. If the atmospheric pressure is 10.3 m of water and vapor pressure head is 0.4 m (abs), the NPSH (Net Positive Suction Head) will be

(a) 3.7 m (b) 4 m
(c) 4.3 m (d) 4.6 m

24. At which location of a converging–diverging nozzle, does the shock boundary layer interaction takes place?

(a) Converging portion
(b) Throat
(c) Inlet
(d) Diverging portion

25. At a particular section of a reaction turbine, the diameter of the blade is 1.8 m, the velocity of flow of steam is 49 m/s and the quantity of steam flow is 5.4 m^3/s. The blade height at this section will be, approximately,

(a) 4 cm (b) 2 cm
(c) 1 cm (d) 0.5 cm

26. Eutectoid reaction in Fe-C system occurs at

(a) 600° C (b) 723° C
(c) 1147° C (d) 1493° C

27. A typical true stress (σ) true strain (ε) curve is approximated by equation

(a) $\sigma = K\varepsilon^n$ (b) $\sigma^n = K\varepsilon$
(c) $\sigma^n = \varepsilon$ (d) $\sigma = \varepsilon^n$

28. A strip is to be rolled from a thickness of 30 mm to 15 mm using a two-high mill having rolls of diameter 300 mm. The coefficient of friction for unaided bite should nearly be

(a) 0.32 (b) 0.5
(c) 0.25 (d) 0.07

29. The voltage current characteristics of a DC generator for arc welding is a straight line between an open circuit voltage of 80 V and short circuit current of 300 A. The generator settings for maximum arc power will be

(a) 80 V and 150 A (b) 40 V and 300 A
(c) 40 V and 150 A (d) 80 V and 300 A

30. In which of the following welding processes, flux is used in the form of granules?

(a) AC arc welding
(b) Submerged arc welding
(c) Argon arc welding
(d) DC arc welding

31. According to Merchant's diagram, the correct relationship among shear angle ϕ, friction angle λ and rake angle α for minimum power consumption is

(a) $2\phi+\lambda-\alpha=\pi/2$ (b) $2\phi+\lambda-\alpha=\pi/4$
(c) $\phi+\lambda-\alpha=\pi/2$ (d) $\phi+\lambda-\alpha=\pi/4$

32. In a cutting operation, the friction angle at tool–chip interface is less than the rake angle, then the thrust force will be

(a) downward (b) upward
(c) zero (d) indetermiante

33. Coefficient of expansion is practically nil in a particular alloy. What is this alloy?

(a) Hadfield manganese steel
(b) Invar
(c) Vitallium
(d) Stellite

34. Apart from Chvorinov's rule, the solidification time also depends upon

(a) the properties of the metal
(b) the properties of the mold
(c) initial temperature
(d) all of the above

35. The following data pertain to a single stage impulse steam turbine:

Nozzle angle	=	20°
Blade velocity	=	200 m/s
Relative steam velocity at entry	=	350 m/s
Blade inlet	=	30°
Blade exit angle	=	25°

If blade friction is neglected, the work done per kg steam is:

(a) 124 kJ (b) 164 kJ
(c) 169 kJ (d) 174 kJ

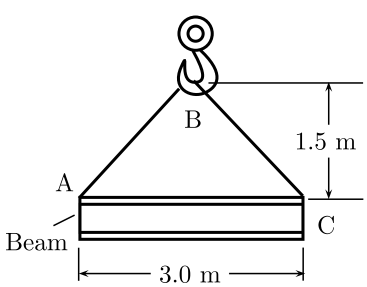
NUMERICAL ANSWER QUESTIONS

1. A system has a temperature-dependent heat capacity at constant volume given by

$$c_v = 0.042T^2$$

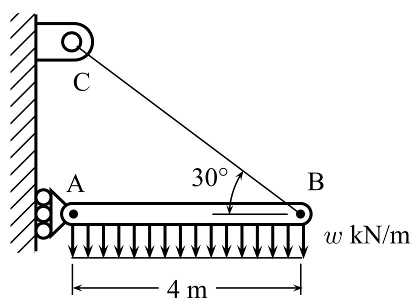
where T is in Kelvin. The system is originally at 350 K. It is desired to cool this system by using a heat engine working with a thermal reservoir at 200 K. Estimate the maximum amount of work that can be recovered during the cooling. Also estimate the entropy change of the thermal reservoir.

2. A beam weighing 50 kN is being lifted through two cables.



Determine the average tension in each of the two cables if the beam is given an upward speed of 5 m/s in 2.0 s.

3. A fluid carrying pipe AB is supported by a hinge at A and a steel-wire BC of diameter 6 mm.



Calculate the UDL w (kN/m) exerted by the flowing fluid inside the pipe so that maximum extension in the wire is limited to 15 mm. Take $E = 200$ GP for steel wire.

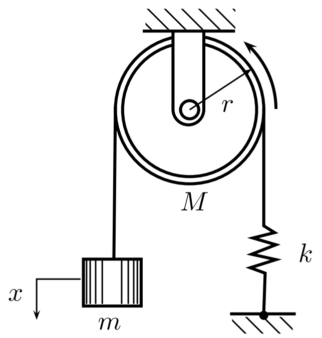
4. Pinion and gear of module 3 mm are in mesh to achieve gear ratio 2:3. The number of teeth on gear is 102. What is the center distance between pinion and the gear?

5. A gun barrel of mass 750 kg has a recoil spring of stiffness 350 kN/m. The barrel recoils 2.0 m on firing. Determine the initial recoil velocity of the

barrel. Determine the critical damping factor in the system.

6. A spring-mass system has stiffness k and inertia m. Determine the percentage reduction in the natural frequency of the system if an additional spring of stiffness $k/3$ is added in series with the original spring.

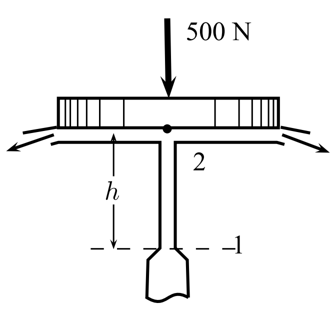
7. A mass is attached through a massless rope passing over a pulley of mass M and is attached with spring of stiffness k at the other end, as shown in the figure.



Calculate the natural frequency of the given system if $m = 2$ kg, $M = 10$ kg, $k = 50$ kN/m.

8. The peak bending stress at critical section of a component varies between 120 MPa and 360 MPa. The ultimate tensile strength of the material is 630 MPa, yield point in tension is 450 MPa, and endurance limit for reversed bending is 360 MPa. Calculate the factor of safety.

9. A vertical upward nozzle of diameter 10 cm exerts a jet at velocity of 10 m/s of oil of specific gravity 0.80.



The jet is used to support a flat horizontal disc weighting 500 N. Calculate the equilibrium height of the plate above the nozzle exit.

10. A stream of air at 10°C is flowing along a heated plate at 144°C at a speed of 75 m/s. The plate is 80 cm long and 60 cm wide. The transition of boundary layer takes place at Re $= 5\times10^5$. At 350 K, the properties of air are $\rho = 0.9950$ kg/m^3, $c_p = 1.009$ kJ/kg K, $\nu = 20.92\times10^{-6}$ m^2/s, $\kappa = 30.0\times10^{-3}$ W/mK, Pr $= 0.700$. Calculate the rate of energy dissipation from the plate.

11. Air expands from pressure p_1 to pressure p_2 ($p_2 = p_1/10$). If the process of expansion is isothermal, the volume at the end of expansion is 0.55 m^3. If the process of expansion is adiabatic, determine the volume at the end of expansion.

12. A gas turbine plant is based on the Brayton cycle working between minimum temperature 25°C and maximum temperature 825°C. The mass flow rate of the gas is 0.5 kg/s. For the gas $c_p = 1.005$ kJ/kg. Calculate the maximum power output and the corresponding efficiency of the cycle.

13. GO and NO-GO snap gauges are to be designed for a shaft $25.000^{+0.060}_{+0.010}$ mm. Gauge tolerances can be taken as 10% of the hole tolerance. Following ISO system of gauge design, determine the sizes of GO and NO-GO gauges.

14. A product has the following data of forecast and actual demand for 10th and 12th period of production.

Period	Forecast	Actual Demand
10	96	110
11	–	114
12	–	120

Forecasting of 11th and 12th period can be done by simple exponential smoothing parameter 0.5. Determine the MSE at the end of 12th period.

15. A manufacturing company consumes 4000 units of an item annually. Delivery lead time is 14 days. Determine the reorder point (in number of units) to achieve optimum inventory.

ANSWERS

Multiple Choice Questions

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(c)	**7.**	(b)	**8.**	(c)	**9.**	(b)	**10.**	(d)
11.	(d)	**12.**	(b)	**13.**	(d)	**14.**	(d)	**15.**	(a)	**16.**	(b)	**17.**	(b)	**18.**	(a)	**19.**	(d)	**20.**	(b)
21.	(b)	**22.**	(d)	**23.**	(c)	**24.**	(d)	**25.**	(b)	**26.**	(b)	**27.**	(a)	**28.**	(a)	**29.**	(c)	**30.**	(b)
31.	(a)	**32.**	(a)	**33.**	(b)	**34.**	(d)	**35.**	(a)										

Numerical Answer Questions

1.	141.75 kJ, 1732.5 kJ/K	**2.**	9.81 kN, 2 kN	**3.**	4.58 kN/m
4.	255 mm	**5.**	32.4 kNs/m	**6.**	50%
7.	84.51 Hz	**8.**	1.15	**9.**	1.86 m
10.	19.45 kW	**11.**	0.285 m^3	**12.**	126.61 kW, 47.90%
13.	25.057 mm	**14.**	149.75	**15.**	153

EXPLANATIONS AND HINTS

Multiple Choice Questions

1. (b) Using Rankine's theory, the buckling load for a column fixed at both ends is

$$F_c = \frac{4\pi^2 EI}{l^2}$$

2. (b) Given $K = E$. Using

$$\begin{aligned} K &= \frac{E}{3(1-2\mu)} \\ 1-2\mu &= \frac{1}{3} \\ \mu &= \frac{1}{3} \end{aligned}$$

3. (a) The deflection δ is $R\theta$, thus

$$\begin{aligned} \delta &= \frac{64PR^3n}{Gd^4} \\ &= \frac{8PD^3n}{Gd^4} \\ &\propto \frac{D^3}{d^4} \end{aligned}$$

4. (c) Auto-frettage is a method of pre-stressing of thick cylinders by winding a number of wire over the cylinder surface in a tight condition.

5. (c) Number of inversions are equal to the number of links in the mechanism.

6. (c) The purpose of flywheel is to control the turning moment by absorbing the kinetic energy. Governor is used to control the speed on par with load. Dunkerley's method of finding natural frequency of multi-degree of freedom system, such as critical speed of rotating shaft. D'Alembert's principle states that external forces acting on a body and the resultant inertia forces on it are in equilibrium.

7. (b) If slider is fixed, it makes connecting rod to oscillate about fixed pivot. Inversion by fixing the slider is applied in hand pump.

8. (c) Maximum screw efficiency is written as

$$\eta_{\max} = \frac{1-\sin\phi}{1+\sin\phi}$$

9. (b) Node point is related to torsional vibration of the shafts. Critical damping is related to damped vibrations. Magnification factor is related to forced vibration of spring-mass system. Hammer blow is related to balancing of reciprocating masses.

10. (d) To avoid interference, some minimum number of teeth are required. Dynamic load on tooth comes due inaccuracies in tooth profile. Static load on the teeth is determined by using Lewis equation. Contact ratio depends upon the arc of approach, arc of recess and circular pitch.

11. (d) In oil lubricated journal bearings, coefficient of friction between the journal and the bearing becomes minimum at an optimum speed and then increases with further speed.

12. (b) Given that

$$\sigma = 80 \text{ N/mm2}$$
$$\tau = 30 \text{ N/mm}^2$$

Therefore, maximum shear stress in the element is

$$\begin{aligned}\tau_{\max} &= \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}\\ &= 50 \text{ N/mm}^2\end{aligned}$$

Therefore, factor of safety is

$$\begin{aligned}\text{FOS} &= \frac{280}{2 \times 50}\\ &= 2.8\end{aligned}$$

13. (d) The ideal fluid is incompressible and inviscous.

14. (d) Given that

$$\sigma = 0.073 \text{ N/m}$$
$$d = 0.001 \text{ m}$$

Pressure drop in a drop of diameter d due to surface tension σ is given by

$$\begin{aligned}\Delta p &= \frac{4\sigma}{d}\\ &= 292 \text{ N/m}^2\end{aligned}$$

15. (a) Shear stress at $y = 0$ is given by

$$\begin{aligned}\frac{\tau_0}{\rho {v_\infty}^2/2} &= \frac{0.644}{\sqrt{\text{Re}_x}}\\ &= C_f\end{aligned}$$

where C_f is called local drag coefficient.

16. (b) According to Newton's law of viscosity, the shear stress between two fluid layers is proportional to the rate of shear stress.

17. (b) Displacement thickness, (δ^*) is the distance by which the boundary surface would have to be displaced outward so that the total actual discharge would be same as that of ideal frictionless fluid past the displacement thickness. Mathematically,

$$\begin{aligned}\delta^* &= \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy\\ &= \frac{\delta}{7}\end{aligned}$$

18. (a) Lumped heat capacity approach is valid only when Bi < 0.1.

19. (d) To realize the number heat units being transferred between the fluids in a heat exchanger, Number of Transfer Units (NTU) defined as

$$\text{NTU} = \frac{UA}{C_{min}}$$

20. (b) Given that

$$\begin{aligned}T_1 &= 400 \text{ K}\\ T_2 &= 300 \text{ K}\\ \epsilon_1 &= \epsilon_2\\ &= 0.9\end{aligned}$$

The net radiative heat transfer between two large parallel gray surface is given by

$$\begin{aligned}\dot{Q} &= \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}\\ &= 812.7 \text{ W}\end{aligned}$$

21. (b) For reversible isothermal processes

$$\int p dv = -\int v dp$$

22. (d) In a phase diagram, the number of phases that are in equilibrium is given by the Gibb's phase rule. It is a simple relation between the number of equilibrium phases (ϕ), components (c) and independent variables or degrees of freedom (f), expressed as

$$f = c - \phi + 2$$

23. (c) NPSH is the head required to make the liquid to flow through the suction pipe to the impeller. For given case

$$\begin{aligned}\text{NPSH} &= 10.3 - 0.4 - 5 - 0.6\\ &= 4.3 \text{ m}\end{aligned}$$

24. (d) Shock waves occur due to strike of supersonic velocity fluid with the higher density fluid at throat or near the exit. Shock occurs only after throat when supersonic velocities exist.

25. (b) Given that

$$D = 1.8 \text{ m}$$
$$V_f = 49 \text{ m/s}$$
$$Q = 5.4 \text{ m}^3/\text{s}$$

Blade height is given by

$$Q = \pi D H V_f$$
$$H = 0.0194 \text{ m}$$
$$\approx 2 \text{ cm}$$

26. (b) Following are the invariant reactions in Fe-C system.

1. Eutectoid (0.8% C)

$$\gamma \overset{723^\circ C}{\rightleftharpoons} \underbrace{\alpha}_{88.5\%} + \underbrace{\text{Fe}_3\text{C}}_{11.5\%}$$

2. Peritectic (0.16% C)

$$\text{L} + \delta \overset{1493^\circ C}{\rightleftharpoons} \gamma$$

3. Monotectic (0.51% C)

$$\text{L} \overset{1495^\circ C}{\rightleftharpoons} \text{L} + \gamma$$

4. Eutectic (4.3% C)

$$\text{L} \overset{1130^\circ C}{\rightleftharpoons} \gamma + \text{Fe}_3\text{C}$$

27. (a) A typical true stress (σ) true strain (ε) curve is approximated by equation

$$\sigma = K\varepsilon^n$$

where K is the strength coefficient and n is called strain hardening exponent.

28. (a) The value of minimum coefficient of friction for specified value of $(t_i - t_f)$ is given by

$$\mu_{min} = \sqrt{\frac{(t_i - t_f)}{R}}$$
$$= \sqrt{\frac{(30-15)}{150}}$$
$$= 0.316$$

29. (c) Given that

$$V_o = 80 \text{ V}$$
$$I_s = 300 \text{ A}$$

Using

$$V = V_o\left(1 - \frac{I}{I_s}\right)$$

The generator power is given by

$$P = VI$$

For maximum arc power,

$$\frac{\partial P}{\partial I} = 0$$
$$I = 150 \text{ A}$$
$$V = 40 \text{ V}$$

30. (b) In submerged arc welding, the welding zone is completely covered by means of a large amount of granulated flux.

31. (a) For minimum power consumption, Merchant diagram of forces involved in cutting provides

$$2\phi + \lambda - \alpha = \frac{\pi}{2}$$

32. (a) The trust force is related to net reaction R as

$$F_t = R\sin(\lambda - \alpha)$$

The net reaction is always positive, hence, F_t will be negative (downward) when $\lambda = \alpha$.

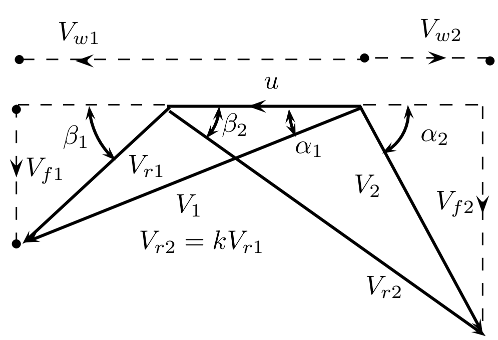
33. (b) The nickel-iron alloy, Invar contains 36% nickel, and possesses the lowest thermal expansion among all metals and alloys. Vitallium is a trademark of an alloy of 60% cobalt, 20% chromium and 5% molybdenum and other substances.

34. (d) The solidification time depends on the properties of the metal, such as density, heat capacity, heat of fusion and superheat, and the mold, such as initial temperature, density, thermal conductivity, heat capacity and wall thickness.

35. (a) Given that

$$\alpha_1 7 = 20^\circ$$
$$u = 200 \text{ m/s}$$
$$V_{r1} = 350 \text{ m/s}$$
$$\beta_1 = 30^\circ$$
$$\beta_2 = 25^\circ$$
$$k = 0$$

Velocity triangle is shown below.



From the diagram,

$$\begin{aligned} V_{w1} &= V_{r1}\cos\beta_1 \\ &= 350\cos 30^\circ \\ &= 303.10 \text{ m/s} \\ V_{r2} &= V_{r1} \\ &= 350 \text{ m/s} \\ V_{w2} &= V_{r2}\cos\beta_2 \\ &= 350\cos 25^\circ - 200 \\ &= 317.20 \text{ m/s} \end{aligned}$$

Thus, the work done per kg of steam is

$$\begin{aligned} W &= (V_{w1}+V_{w2})\,u \\ &= (303.10+317.20)\times 200 \\ &= 124.06 \text{ kW} \end{aligned}$$

Numerical Answer Questions

1. The heat extracted from the system during its cooling from 350 K to 200 K is calculated as

$$\begin{aligned} Q_1 &= \int_{350}^{200} 0.042T^2 dT \\ &= \frac{0.042}{3}\left(200^3 - 350^3\right) \\ &= -488.25\times 10^3 \text{ J} \end{aligned}$$

The entropy change of the system during above process is

$$\begin{aligned} \Delta S &= \int_{350}^{200} \frac{0.042T^2 dT}{T} \\ &= \frac{0.042}{2}\left(200^2 - 350^2\right) \\ &= -1732.5 \text{ J/K} \end{aligned}$$

If W is the maximum work recoverable during the cooling, net heat $Q_1 - W$ will be transferred to the thermal reservoir at 200 K. Therefore, the entropy change of the reservoir will be

$$\Delta S_r = \frac{Q_1 - W}{200}$$

For maximum work, the entropy change of the universe should be zero. Hence

$$\begin{aligned} 1732.5 &= \frac{488.25\times 10^3 - W}{200} \\ W &= 141.75 \text{ kJ} \end{aligned}$$

The entropy change of the thermal reservoir is

$$\begin{aligned} \Delta S_r &= \frac{488.25 - 141.75}{200} \\ &= 1732.5 \text{ kJ/K} \end{aligned}$$

which is equal to that of the system with opposite sign.

2. The effective weight of the beam will be

$$\begin{aligned} W &= \left(9.81 + \frac{5}{2.0}\right)\frac{50\times 10^3}{9.81} \\ &= 62.74 \text{ kN} \end{aligned}$$

Therefore, tension in the cables will be

$$\begin{aligned} 2T\cos 45^\circ &= 62.74 \\ T &= 44.36 \text{ kN} \end{aligned}$$

3. Length of the cable is

$$l_{BC} = \frac{4}{\cos 30^\circ} 4.62 \text{ m}$$

For extension of 15 mm, the cable exerts force given by

$$\begin{aligned} \frac{\delta}{l_{BC}} &= \frac{4T_{BC}}{\pi d^2 E} \\ T_{BC} &= \frac{0.015\times\pi\times 0.006^2\times 200\times 10^9}{4.62\times 4} \\ &= 18.36 \text{ kN} \end{aligned}$$

The load taken by the cable BC can be determined by taking moments about point A:

$$\begin{aligned} w\times 4\times\frac{4}{2} &= T_{BC}\sin 30\times 4 \\ w &= 4.58 \text{ kN/m} \end{aligned}$$

4. Given that

$$\begin{aligned} T_g &= 102 \\ m &= 3 \text{ mm} \end{aligned}$$

Number of teeth on pinion is

$$\begin{aligned} T_p &= 102\times\frac{2}{3} \\ &= 68 \end{aligned}$$

Center distance is given by

$$\begin{aligned} c &= \frac{m}{2}\left(T_g + T_p\right) \\ &= \frac{3}{2}\times 170 \\ &= 255 \text{ mm} \end{aligned}$$

5. Using Rayleigh method,

$$\frac{1}{2}kx^2 = \frac{1}{2}m\dot{x}^2$$
$$\dot{x} = \sqrt{\frac{kx^2}{m}}$$

Therefore, for initial recoil $x = 2.0$ m. the initial recoil velocity will be

$$\dot{x} = \sqrt{\frac{350\times10^3\times2.0^2}{750}}$$
$$= 43.20 \text{ m/s}$$

The critical damping factor is determined as

$$c_c = 2\sqrt{km}$$
$$= 32.4 \text{ kNs/m}$$

6. Natural frequency of the original system is

$$\omega = \sqrt{\frac{k}{m}}$$

Equivalent spring stiffness in series is

$$\frac{1}{k_e} = \frac{1}{k} + \frac{1}{k/3}$$
$$k_e = \frac{k}{4}$$

Natural frequency of the new system is

$$\omega' = \sqrt{\frac{k_e}{m}}$$
$$= \sqrt{\frac{k}{4m}}$$
$$= \frac{\omega}{2}$$

Thus, ω reduces by 50%.

7. For a small displacement x of the mass, the pulley shall rotate angle $\theta = x/r$. Hence, the total energy of the system at any moment will be given by

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}kx^2 = c$$

where

$$\dot{x} = r\dot{\theta}$$
$$I = \frac{1}{2}Mr^2$$

Hence, differentiating both sides w.r.t. time t

$$\left(m + \frac{M}{2}\right)\ddot{\theta} + k\theta = 0$$

Therefore, natural frequency of vibrations is

$$\omega = \sqrt{\frac{k}{m + \frac{M}{2}}}$$

Given

$$m = 2 \text{ kg}$$
$$M = 10 \text{ kg}$$
$$k = 50\times10^3 \text{ N/m}$$

The natural frequency is given by

$$\omega = 84.51 \text{ rad/s}$$

8. Given that

$$\sigma_{max} = 360 \text{ MPa}$$
$$\sigma_{min} = 120 \text{ MPa}$$
$$\sigma_{ut} = 650 \text{ MPa}$$
$$\sigma_{yt} = 450 \text{ MPa}$$
$$\sigma_e = 360 \text{ MPa}$$

The average and variable stresses are

$$\sigma_a = \frac{360+120}{2}$$
$$= 240 \text{ MPa}$$
$$\sigma_v = \frac{360-120}{2}$$
$$= 120 \text{ MPa}$$

Factor of safety n is determined Using Soderberg equation as

$$\frac{1}{n} = \frac{\sigma_a}{\sigma_{yt} + \frac{\sigma_v}{\sigma_e}}$$
$$= \frac{240}{450} + \frac{120}{360}$$
$$n = 1.15$$

9. Given that

$$d = 0.10 \text{ m}$$
$$V_1 = 10 \text{ m/s}$$
$$W = 500 \text{ N}$$
$$s = 0.80$$
$$\rho = 800 \text{ kg/m}^3$$
$$\rho_w g = 9810 \text{N/m}^3$$

Discharge of the nozzle is

$$Q = \frac{\pi d^4}{4} \times V_1$$
$$= 0.0785 \text{ m}^3$$

Using Bernoulli's equation, velocity of impact is

$$0+\frac{V_1^2}{2g}+0=0+\frac{V_2^2}{2g}+h$$
$$V_2=\sqrt{V_1^2-2gh}$$

The equilibrium equation for the impact is

$$\begin{aligned}W &= \rho Q V_2\\ &= \rho Q\sqrt{V_1^2-2gh}\\ h &= \frac{1}{2g}\left(V_1^2-\left(\frac{W}{\rho Q}\right)^2\right)\\ &= 1.86 \text{ m}\end{aligned}$$

10. Given that $u_\infty = 75$ m/s. The average film temperature is

$$\begin{aligned}T_f &= \frac{10+144}{2}\\ &= 77^\circ\text{C}\end{aligned}$$

At 77°C (350 K), the properties of air are given as

$$\begin{aligned}\rho &= 0.9950 \text{ kg/m}^3\\ c_p &= 1.009 \text{ kJ/kg K}\\ \nu &= 20.92\times10^{-6} \text{ m}^2\text{/s}\\ \kappa &= 30.0\times10^{-3} \text{ W/mK}\\ \text{Pr} &= 0.700.\end{aligned}$$

The value of x_L upto which laminar flow exists, as decided by $\text{Re}=5\times10^5$, is calculated as

$$\begin{aligned}\text{Re}_x &= \frac{u_\infty x_L}{\nu}\\ x_L &= \frac{5\times10^5\times20.92\times10^{-6}}{75}\\ &= 0.1395 \text{ m}\end{aligned}$$

The value of Reynolds number at the trailing edge of the plate is

$$\begin{aligned}\text{Re}_L &= \frac{u_\infty L}{\nu}\\ &= 2.86\times10^6\end{aligned}$$

The average Nusselt number for the plate is calculated as

$$\begin{aligned}\bar{\text{N}}\text{u}_x &= \left(0.037\text{Re}_L^{4/5}-870\right)\text{Pr}^{1/3}\\ \frac{\bar{h}_L L}{\kappa} &= 4032.16\\ \bar{h}_L &= 151.2 \text{ W/m}^2\text{K}\end{aligned}$$

The net heat transfer on both sides of the plate is

$$\begin{aligned}\dot{Q} &= 2\times\bar{h}_L A\left(T_s-T_\infty\right)\\ &= 19.45 \text{ kW}\end{aligned}$$

11. Given that

$$\begin{aligned}p_2 &= \frac{p_1}{10}\\ v_2 &= 0.55 \text{ m}^3\end{aligned}$$

In isothermal expansion

$$\begin{aligned}p_1v_1 &= p_2v_2\\ v_1 &= \frac{p_2}{p_1}v_2\\ &= 0.055 \text{ m}^3\end{aligned}$$

In adiabatic expansion ($\gamma=1.4$)

$$\begin{aligned}p_1v_1^\gamma &= p_2v_2^\gamma\\ v_2 &= v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}\\ &= 0.055\left(\frac{10}{1}\right)^{1/1.4}\\ &= 0.285 \text{ m}^3\end{aligned}$$

12. Given that

$$\begin{aligned}T_1 &= 298 \text{ K}\\ T_3 &= 1098 \text{ K}\\ c_p &= 1.005 \text{ kJ/kg}\\ \dot{m} &= 0.5 \text{ kg/s}\end{aligned}$$

The maximum power output is calculated as

$$\begin{aligned}W_{\max} &= \dot{m}c_p\left(\sqrt{T_3}-\sqrt{T_1}\right)^2\\ &= 126.61 \text{ kW}\end{aligned}$$

The cycle efficiency for maximum power output is

$$\begin{aligned}\eta &= 1-\sqrt{\frac{T_1}{T_3}}\\ &= 47.90\%\end{aligned}$$

13. Tolerance is $+0.060-0.010=0.030$ mm. 10% of it is 0.003 mm. Therefore, size of GO gauge is

$$\begin{aligned}&= 25.000+0.010+0.003\\ &= 25.013 \text{ mm}\end{aligned}$$

Similarly, size of NO-GO gauge is

$$\begin{aligned}&= 25.000+0.060-0.003\\ &= 25.057 \text{ mm}\end{aligned}$$

14. Given that $\mu = 0.5$, thus the forecasts for period 15 and 16 are determined as

$$
\begin{aligned}
F_{11} &= F_{10}+\mu\,(D_{10}-F_{10})\\
&= 96+0.5\,(110-96)\\
&= 103\\
F_{12} &= F_{11}+\mu\,(D_{11}-F_{11})\\
&= 103+0.5\,(114-103)\\
&= 108.5
\end{aligned}
$$

Thus, the table is populated along with square of the error (E) as

Period	F	D	E	E^2
10	96	110	14	196
11	103	114	11	121
12	108.5	120	11.5	132.25

Thus, mean square error (MSE) is determined as

$$
\begin{aligned}
\text{MSE} &= \frac{196+121+132.5}{3}\\
&= \frac{449.25}{3}\\
&= 149.75
\end{aligned}
$$

15. As 4000 units are required in 365 days, therefore, 14 days quantity is

$$
\begin{aligned}
\text{Reorder level} &= \frac{4000}{365}\times 14\\
&= 153 \text{ units}
\end{aligned}
$$

Index

The Graduate Aptitude Test in Engineering (GATE) essentially examines the aptitude of an engineering graduate. It differs from other examinations as it calls for an aptitude based learning and approach on engineering topics. To qualify this exam, an aspirant should have perfect understanding on the fundamental concepts, ability to interpret the fundamental quantities and their relationships, and apply these to problem solving.

Latest changes in GATE

- Introduction of new paper "Ecology & Evolution"
- Online mode of examination for all 22 papers
- Inclusion of Numerical Answer Questions (NAQs), in which the candidate has to enter the numerical value of the answer (instead of selecting one option from MCQs)
- Exclusion of linked answer questions (LAQs) and common data questions (CDQs)

This book is aimed to present the subjects of mechanical engineering in a systematic, integrated and precise manner, primarily to meet the requirements of the aspirants of GATE (ME) and help them qualify the examination without unreasonable strain. The book is a self-explanatory and a fully-competent text for similar competitive examinations and undergraduate courses in mechanical engineering.

It helps to solve problems faced by aspirants in terms of extensive syllabus coverage, existing incongruity with syllabus, unavailability of a standard book, etc. Throughout the content, an intuitive and systematic approach is used to reinforce the fundamental concepts. The emphasis on fundamental concepts unveils inbuilt opportunities for developing the aptitude, which is the vital element for success in GATE.

Key Features of the Book

- Two introductory articles, "Strategy for success in GATE" and "Methodological Concepts in Engineering Studies" guide students through the preparation phase of the examination
- Theoretical concepts explained with case studies in simple steps to develop fundamental understanding of core subjects
- Almost every mathematical aspect of the subject covered in form of equations, illustrations, graphs, etc. to enhance problem solving skills
- Important formulas placed at the end of the chapter for a quick review of the useful derivations
- Rightly answered and adequately explained case studies and solved examples
- Solutions of previous 11 years' GATE questions at the end of relevant chapters
- Sufficient questions cover all topics starting from simplest case and adding complexities gradually
  - Multiple Choice Questions (MCQs)
  - Numerical Answer Questions (NAQs)
- Question Bank for practice covering all chapters and question types

About the Authors

Ajay Kumar Tamrakar, Senior Divisional Mechanical Engineer, West Central Railway, under cadre of IRSME (2005 batch), obtained All India 2nd rank with 99.98 percentile in GATE 2001and did his M.Tech (IISc, Bangalore).

Dineshkumar Harursampath, Founder-Head, NMCAD Lab Department of Aerospace Engineering (Mechanical Division), IISc, Bangalore, obtained his B.Tech (IITM), M.E. (IISc Bangalore) and Ph.D. (Georgia Tech, Atlanta).

Wiley India Pvt. Ltd.
4435-36/7, Ansari Road, Daryaganj
New Delhi-110 002
Customer Care +91 11 43630000
Fax +91 11 23275895
csupport@wiley.com
www.wileyindia.com
www.wiley.com

WILEY

ISBN 978-81-265-4542-1
9788126545421