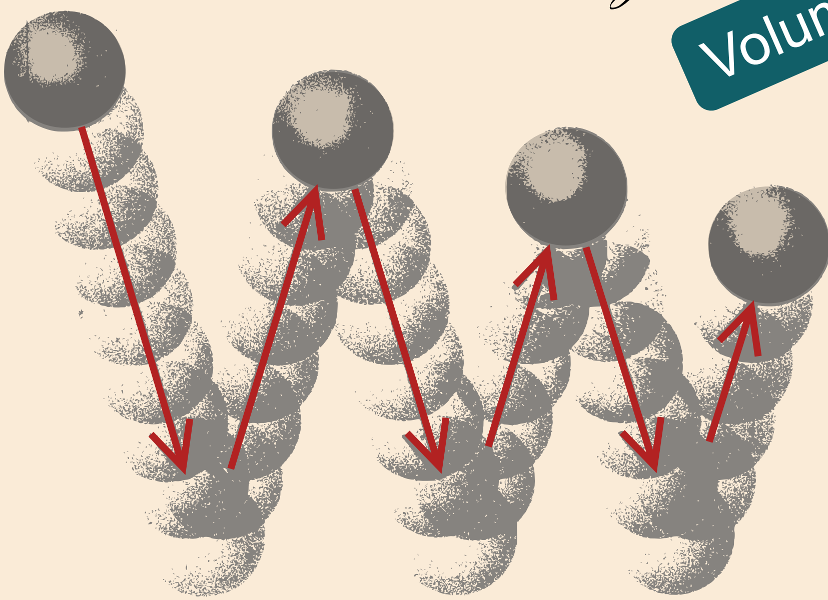
ELEMENTARY TEXTBOOK ON PHYSICS

Edited by G. S. Landsberg

Volume 1



Mir Publishers Moscow

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These three volumes form a course on elementary physics that has become very popular in the Soviet Union. Each sectioh was written by an authority in the appropriate field, while the overall unity and editing was supervised by Academician G.S. Landsberg (1890-1957). This textbook has gone through ten Russian editions and a great deal of effort went into the last edition to introduce SI units and change the terminology and notation for the physical units.

A feature of this course is the relatively small number of formulas and mathematical manipulations. Instead, attention was focussed on explaining physical phenomena in such a way as to combine scientific rigour and a form understandable to school children. Another aspect of the text is the technological application of the physical laws.

These features make the text a world-class textbook.

For students preparing to enter universities and colleges to study physics, and for those it high schools specialising in physics.

ELEMENTARY TEXTBOOK ON PHYSICS

Volume 1

ЭЛЕМЕНТАРНЫЙ УЧЕБНИК ФИЗИКИ

Под редакцией академика

Г. С. ЛАНДСБЕРГА

В 3-х томах

ТОМ 1

МЕХАНИКА. ТЕПЛОТА. МОЛЕКУЛЯРНАЯ ФИЗИКА

Издательство «Наука»
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ELEMENTARY TEXTBOOK ON PHYSICS

Edited by G. S. Landsberg

In three volumes

Volume 1

MECHANICS
HEAT
MOLECULAR PHYSICS

Mir Publishers Moscow

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by Natalia Wadhwa

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From the Preface to the First Russian Edition

The title *Elementary Textbook on Physics* for this book reflects our endeavour to write a book that will acquaint the reader with fundamentals of physics as a science. This should be the goal when teaching physics at high schools, technical schools, or vocational training schools. We hope that this book can be used as the main physics textbook in all such schools since the underlying principles have universal application.

These principles impart certain features to the book which distinguish it from existing high-school textbooks. The features require some explanation, which should mainly be the task of the teachers. Hence we address this preface to those who teach physics.

Lecturers at universities and engineering colleges have a sad impression that the students who have finished high school possess a poor grasp of physics. What is most striking is not their lack of information or theoretical concepts but rather the absence of clear and correct ideas about the relations between these concepts. Students often cannot say what forms the basis of a definition, what is the result of an experiment, and what should be treated as a theoretical generalisation of experimental knowledge. New facts are often taken as self-evident corollaries, and for this reason the importance of the facts remains unrecognised. Sometimes, different formulations of the same statements are mistaken as different laws.

Naturally, the teaching of physics at universities differs considerably from teaching at high schools in the volume of material, depth of presentation, and in the systematic use of mathematical tools. However, at an early stage, physics (or an introduction to it) should be taught as a *science* and not as an aggregate of individual facts. In other words, the facts should be used as a basis for creating in the minds of students a clear idea about the scientific approach typical of physics. It goes without saying that this approach is experimental.

Nobody would venture to deny that physics is an experimental science or that its laws are established with the help of experiments. In some textbooks, however, these statements are presented as declarations to which the first pages are devoted. Subsequently, the experiments are mainly used as illustrations, and the fact that physical concepts are inseparable from experiments slips away from the students. It is necessary, however, to persuade students that logically formulated definitions become meaningful only after having been verified experimentally, i.e. as a *result of measurements*. Every concept introduced in physics acquires a concrete meaning only if a certain method of observation and measurement is associated with it. Without this, a concept cannot find any application in the investigation of actual physical phenomena.

Let us consider, for example, the simple concept of uniform motion. Whether a given motion is uniform depends on the method of observation. The motion of a train, for example, can be rightfully treated as uniform if we use rough methods for measuring each segment of the path and the time intervals. If we use more accurate methods, however, the motion may turn out to be nonuniform. If a motion satisfies the established definition of uniformity for a method of observation, all laws of uniform motion are applicable to it, and all conclusions and calculations are valid for it with an accuracy which is characteristic for the method of measurement.

A clear idea about the *experimental* nature of physical laws is of utmost importance: it makes physics a *natural science* rather than a system of speculative constructions. On the other hand, it sets the limits of applicability of physical laws and theories based on them and shows how science can be developed.

At the first stage of teaching physics, a correct *idealisation* of phenomena and the significance of this idealisation are equally important. Naturally, every teacher or textbook writer appreciates the necessity of idealisation and uses it wherever required. However, the idealisation is sometimes unjustified.

Idealising a phenomenon is to neglect those features which are insignificant for a problem and to retain the required properties. In this respect, the same phenomenon can be idealised to different extents depending on the problem. Moreover, using a correct idealisation, we can sometimes omit certain features of a phenomenon and retain other features that would appear to be inseparable from those we neglected. For example, one of the most widespread and useful idealisations in mechanics is the concept of a perfectly rigid body or incompressible liquid. These idealisations are required for a wide variety of mechanical problems in which deformations do not play a significant role and where the change in the size and shape of a body can be ignored. However, stresses caused by strains in a deformed body are significant in the dynamics of phenomena. For this reason, the idealised concept of a perfectly rigid body as a body *without* deformations makes the simplest problems in mechanics meaningless unless certain reservations are made. It is necessary to establish from the outset that we ignore deformations in a solid or liquid but take into account the stresses which are caused in the idealised body by deformations and which explain all the phenomena under consideration. Without a clear idea about this, it is impossible to understand even the simplest phenomenon and, for example, to say why a load rests on a table in spite of the fact that the force of gravity acts on it, since we cannot see the other force, viz. the elastic tensile force exerted by the table and balancing the force of gravity.

An introduction to the physics as a science and the explanation of such idealised concepts should be made with great care. When used correctly, these concepts can be very helpful and considerably simplify the formulation of laws and calculations. But reticence or an incorrect application of the concepts may lead to the risk inherent in teaching, viz. the formation of erroneous notions which will later handicap further and deeper understanding. By way of example, we can mention the concepts of a magnetic pole or a geometric ray. There is no doubt that these concepts have value, and it would be irrational to do without them. However, the utmost care and explanation are essential to avoid any confusion that the introduction of these concepts might create. Many of us who evaluate inventions or theoretical work are aware, for example, that confidence in the infallibility of geometrical optics based on an erroneous interpretation of the useful concept of geometric ray can lead to serious confusions.

* * *

Like every other form of education, teaching at high school can never be exhaustive. However, it must be planned in such a way that the student could and would *increase his knowledge* but should by no means repeat the *whole thing*. This should be the ultimate aim of everyone venturing to write a textbook. It is imperative that methodological and methodical imperfections like the ones mentioned above be meticulously avoided.

The group of physicists who took upon themselves the task of compiling this *Elementary Textbook on Physics* were guided by these principles. Their decision to do so was not motivated by a desire to alter the conventional contents but by the considerations listed above. For this reason, "simple" questions, which are normally discussed in a few lines, are given extensive coverage in this book. It is because of this approach, and by no means due to an increase in the number of topics, that the volume of this book exceeds that of conventional textbooks.

Moscow, June 1948 *G. Landsberg*

From the Publishers of the Tenth Russian Edition

Elementary Textbook on Physics first appeared in 1948-52 under the editorship of Academician G.S. Landsberg (1890-1957) and immediately became popular with students preparing for entrance examinations in physics. The success of the book was due very much to the fact that each section was written by a specialist. Contributors to the book included the scientists S.E. Khaikin, M.A. Isakovich, M.A. Leontovich, D.I. Sakharov (Vol. 1), S.G. Kalashnikov (Vol. 2), S.M. Rytov, M.M. Sushchinskii (with the participation of I.A. Yakovlev), F.S. Landsberg-Baryshanskaya, and F.L. Shapiro (Vol. 3).

The distinguishing feature of this course is that it contains a comparatively small number of formulas and mathematical calculations. The main attention in this textbook is devoted to the explanation of the essence of physical phenomena. The material is presented on a high scientific level and at the same time in a form comprehensible to school students. Another feature of the book is that it describes a large number of technical applications of physical laws. In this respect, the book has no analogue among textbooks on physics written on this level in the world.

During a quarter of a century, *Elementary Textbook on Physics* has seen nine editions. The previous ninth edition was issued in 1975. Although separate sections of the book have been updated during the preparation of earlier editions, the present (tenth) edition is the result of a considerable revision to incorporate SI units as well as new terminology.

The chapters of Volume 2 on magnetic phenomena have been heavily revised. In the previous editions, these phenomena were presented on the basis of Coulomb's law for magnetic charges. While preparing the manuscript for the tenth edition, these chapters were rewritten on the basis of the concept of the magnetic field of moving charges and currents. To meet the requirements of the SI system of units, the formulas on electromagnetism are presented in a rationalised form. Magnetic induction **B** is used as the main force characteristic of the magnetic field rather than the magnetic field strength **H**, as it was done in the previous editions.

The text has been partially renewed and supplemented by I.Ya. Barit, L.G. Landsberg, F.S. Landsberg-Baryshanskaya, V.I. Lushchikov, S.M. Rytov, I.V. Savel'ev, M.M. Sushchinskii, M.S. Khaikin, S.M. Shapiro, O.A. Shustin, and I.A. Yakovlev. The first two volumes were edited by I.V. Savel'ev and the third volume by the persons mentioned above.

Introduction

The knowledge acquired by a student at high school and the observation of the surroundings (among other things, the information concerning the unimaginable potentialities of the modern technology) inevitably lead to the question: how could man, with his low physical ability and imperfect organs of senses which allow him a very limited scope to directly observe the physical phenomena, create modern technology with its huge possibilities which extend far beyond the fictions by J. Verne? Almost everybody would answer this question without thinking: *this miracle was brought about by the science of nature*. The physical science plays a particularly important role in this triumph of man.

What are the tools at the disposal of the physical science that allow it to reign the world?

First of all, physics clearly deals with phenomena in real world, and hence the first step in gaining knowledge about these phenomena involves *observations*.

However, scientific observation is not a simple problem. Let us watch, for example, falling bodies. It can be easily seen that a body dropped from a small height strikes the ground with a small force, while the impact as a result of a fall from a large height can be much stronger and may even destroy the falling body. The observation of rain drops does not reveal, however, any difference in the impacts of the drops from low and high clouds. Everybody knows that a pilot who falls from an aeroplane is smashed to death, while a pilot who jumps with a parachute even from a larger height lands smoothly. Aircraft bombs, especially heavy ones, hit with a tremendous force that enables them to pierce multistorey buildings. Thus, a comparatively simple phenomenon of falling may proceed in different ways. If we want to control this phenomenon, we must find the relationship between its different aspects, viz. establish how certain characteristics of the motion of a body are influenced by the shape and mass of the body, the height from which it falls, etc. and (this is most important) draw *general conclusions* from these facts, which explain why the body falls in this way and not in another way.

The same problems emerge in studying any other phenomenon. We must establish what affects a phenomenon and learn how to suppress or enhance its individual aspects. For this purpose, we must be able to analyse the phenomenon, single out its individual elements and, if possible, change the conditions in which it occurs. This means a transition from the simple observation to an *experiment*. Here it is very important not to limit ourselves to general qualitative impressions about the phenomenon but to find *quantitative characteristics* of its individual elements in the form of measurable quantities. In other words, we must determine the concepts which may serve as the quantitative characteristics of the phenomenon and find the methods of measurement of relevant quantities. Having determined these quantities, we can establish numerical relations between them, i.e. formulate the *laws* governing the phenomenon in a quantitative (analytical) form. Thus, in the above example of falling bodies, we introduce the concepts of the velocity of the falling body, its acceleration (i.e. the change in velocity), the height of fall, air resistance, the mass of the body, the force of gravity acting on it, and so on. To find the laws of fall means to establish a relation between these quantities.

The most important problem in the experimental investigation of phenomena is to establish quantitative laws indicating how some quantities vary with a change in other quantities. These laws show how the conditions under which these phenomena occur should be changed to attain desired results. Physical laws help us to understand the essence of phenomena and thus pave the way for creating a *theory* of phenomena, i.e. the general ideas which explain why a phenomenon obeys established laws and why it is related with other phenomena which at first glance seem to be very far from it.

For example, while analysing the fall of bodies, we establish the laws of fall by determining the part played by the resistance of air, the dependence of this resistance on the shape of the body and velocity of its motion. In this way, we gradually construct the complete theory of the phenomenon, which shows, among other things, that eddies formed in air at a rapid motion of the body may play a significant role in fall. The importance of the so-called streamlined shape of the body, i.e. the shape for which eddy formation and the drag associated with it are less pronounced, is illucidated. The analysis of these questions makes it possible to solve a number of important problems in aircraft industry, in designing motor cars of a rational shape and high-speed trains, and so on.

It is clear from what has been said above that experiments play a major role in physics. With the help of experiments, the regularities of phenomena are established, and the theory of the phenomena can be constructed. In turn, the theory allows us to predict new, unknown features of the phenomena and outlines the conditions under which these features can be manifested. Such conclusions drawn from the theory are again verified in experiments, which sometimes lead to a refinement or improvement of the theory. Thus, a complex and unclear phenomenon gradually becomes quite comprehensible, and we learn how to control it at our discretion. This ability to control natural phenomena forms the basis of modern technology.

The above arguments about the role of experiments explain why physics is called an *experimental science*. It should not be thought, however, that in order to establish laws and construct theories it is sufficient just to analyse the results of a thoroughly performed experiment. The concentration of all mental and creative abilities of scientists is required to construct the majestic edifice of science.

In the experiment on falling bodies discussed above, the phenomenon under consideration was very simple. Nevertheless, even for this phenomenon it was not easy to distinguish the features which play the major role from those which are less important. It is quite difficult to *idealise* the phenomenon by ignoring the secondary effects and singling out the essential features. In many cases, the problem is complicated by the fact that diverse processes are interlinked in real phenomena. For example, a significant role in a phenomenon can be played by electrical or thermal processes which result in forces that impart an acceleration to bodies, or some optical effects can be exhibited and can even become decisive, and so on.

Let us consider a thunderstorm. Here, thermal phenomena and effects of molecular physics (evaporation and condensation of water vapour), electrical phenomena (the role of charge centres in drop formation, the emergence of electric field between clouds and the resulting electric discharge), optical and acoustic phenomena (lightning and thunder), and various mechanical phenomena (falling drops, wind, motion of clouds and eddy formation) are closely interconnected.

It is clear that in such cases a complex phenomenon should be divided into simpler phenomena, since it is easier to study it by parts. The observation of complex phenomena reveals that with such a division we can single out groups of similar phenomena, like optical, thermal, electrical and other phenomena (as we did in the example with the thunderstorm). Therefore, while studying physics it is expedient to combine the material under analysis in such groups, although it is often impossible to draw a demarcation line between them. Accordingly, the division of the material into parts (and even their order) is optional and can be made arbitrarily.

In this textbook, we start the analysis of phenomena by considering mechanics (including fluid mechanics) since the corresponding phenomena are comparatively simple. Moreover, a knowledge of the laws of mechanics turns out to be helpful in other branches of physics. We shall consider in the course of the three volumes thermal phenomena and molecular physics which is closely related to them. This is followed by a wide range of electrical and electromagnetic phenomena. Oscillations and waves are combined into an individual part including all mechanical, acoustic and electromagnetic oscillations. Then optical phenomena are considered, whose analysis is based to a large extent on wave theory. We conclude with an introduction to atomic physics.

Part One

Mechanics

Chapter 1

Kinematics

1.1. Motion of Bodies

Mechanical motion of a body is the *change in its position with time relative to other bodies*.

We encounter motion of bodies very frequently in our everyday life, science and technology. We observe the motion of human beings and animals, the motion of water in rivers and seas, and the motion of air (wind). Transport facilities, various mechanisms, machines, instruments, projectiles, etc. execute motion. The Earth and other planets move in space as well as comets, meteors (Fig. 1), the Moon, artificial satellites of the Earth and spacecraft launched to other planets of the Solar system. The Sun moves relative to other stars which, in turn, move relative to one another. Atoms, molecules, electrons, protons, alpha-particles (Fig. 2) and other elementary particles (the smallest parts of matter) are in motion. Practically all physical phenomena involve the motion of bodies. Thus, we begin the

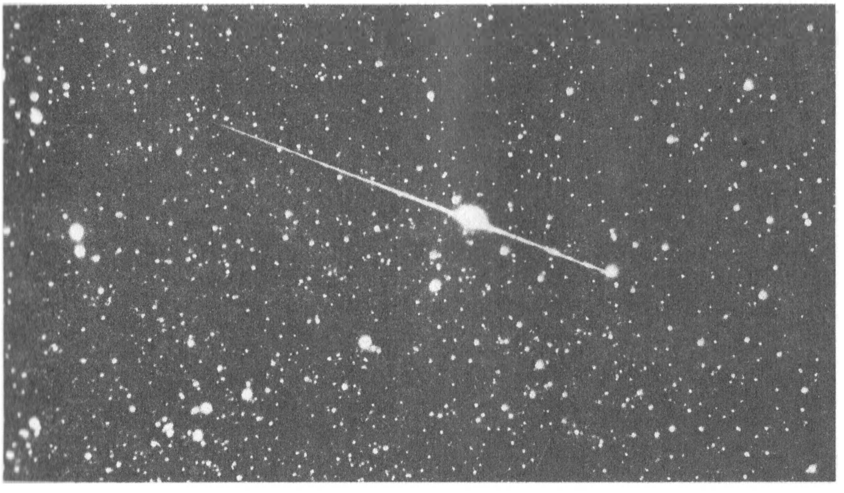

Fig. 1.
A meteor in the night sky.

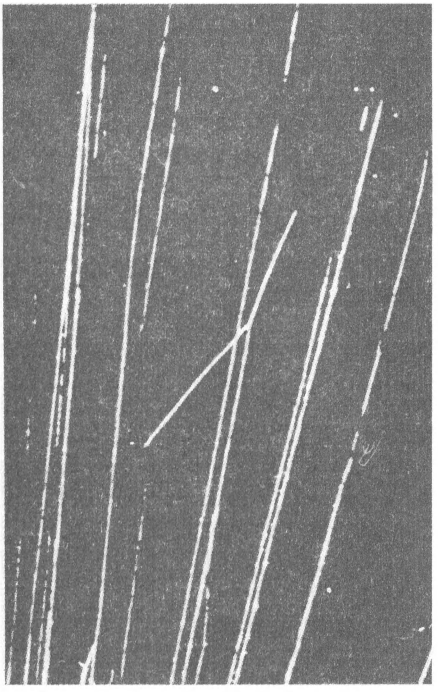

Fig. 2.
Rapidly moving alpha-particles leave behind cloudy tracks of water drops in a Wilson cloud chamber.

study of physics with an analysis of the motion of bodies. This branch of physics is called *mechanics*.

The word "mechanics" takes its origin from the Greek word *mechane* which means machine or tool. Even in ancient times, Egyptians, and later Greeks, Romans and other peoples constructed various machines for transportation, construction and military purposes (Fig. 3). The operation of these machines was based on the movement of levers, wheels, ropes, and other mechanisms, which caused the motion of loads being lifted or displaced. The investigation of the operation of these machines gave birth to the science about the motion of bodies, viz. mechanics.

Mechanics also includes the study of conditions under which bodies remain at rest, viz. equilibrium conditions. Such problems play a decisive role in construction and industrial engineering. When a house built of toy blocks, a large building or a bridge break, this means that equilibrium conditions for these bodies have been violated.

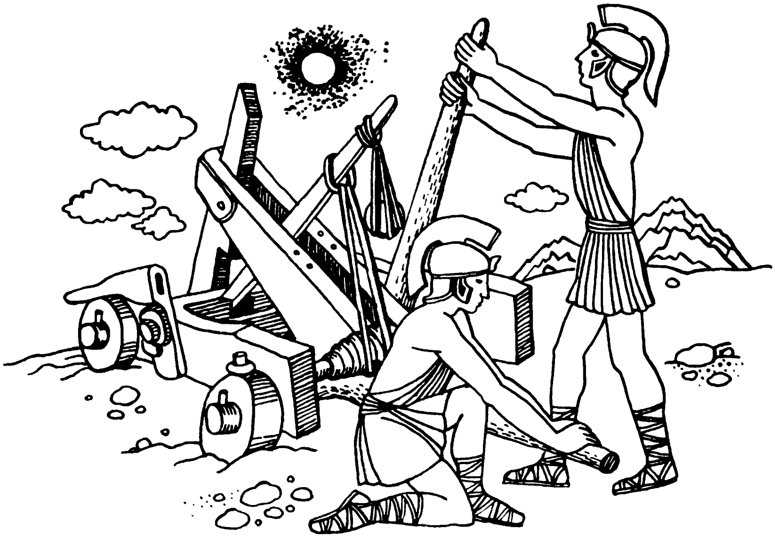

Fig. 3.
Missile weapon of ancient Greeks.

Motion is observed not only for material bodies. We can speak of the motion of a light spot displaced on a wall as a result of the rotation of a mirror or the motion of a shadow behind an illuminated object, as well as the motion of a flying bullet or a stone thrown from a certain height.

Light and radio signals require a short time to travel considerable distances (they can, for example, cover the distance from the Earth to the Moon in just 2.5 s). For this reason, for comparatively short distances it may seem in ordinary conditions on the surface of the Earth that light or a radio signal covers the distance between two points instantaneously. This, however, is not true: like all material bodies, light must spend a certain (although short) time to cover such a distance. But it is very difficult to detect and measure the time taken by light to cover various distances. This was done for the first time only in the 17th century, although the study of motion of material bodies and acoustic signals began in ancient times.

The motion of light and acoustic signals is more complicated than the displacement of material bodies. It will be discussed in Vol. 3.

1.2. Kinematics. Relative Nature of Motion and State of Rest

To study motion, we must first of all know how to *describe* it. For the time being, we are not interested in the cause of motion. The branch of mechanics which studies motion without investigating the reasons behind it is called *kinematics*.

The motion of each body can be analysed relative to any other bodies. A given body moves differently relative to different bodies: a suitcase lying on a shelf in a moving train is at rest relative to the carriage but moves relative to the Earth. A balloon carried by wind moves relative to the Earth but is at rest relative to air. An aeroplane flying in a squadron is at rest relative to the other planes in the squadron but moves relative to the Earth at a high velocity (say, 800 km/h); the velocity of this plane relative to a plane moving in the opposite direction with the same velocity is 1600 km/h.

In movies, the same motion is often shown relative to different bodies. For example, a train is shown moving against the background of a landscape (motion relative to the Earth), and then a compartment of a carriage with trees running past the window (motion relative to the carriage).

Every motion, as well as the state of rest (as a special case of motion), *is relative*. In order to answer the question whether a body is at rest or in motion and what the nature of this motion is, it is necessary to indicate the bodies relative to which the motion of the given body is considered. Otherwise, the statement about the motion of the body is meaningless.

The bodies relative to which a given motion is considered form a *reference system*. The choice of the reference system for analysing a given motion depends on the conditions of the problem. For example, if an enemy plane is to be hit from the ground, the aiming should depend on the velocity of the plane in the reference system fixed to the Earth (in the example considered above, this velocity was 800 km/h), but to hit the same plane from another plane flying towards the first one, one should proceed from the velocity of the target in the reference system fixed to the second plane flying from the opposite direction (1600 km/h). When motion on the surface of the Earth is studied, the reference system is usually fixed to the Earth (although, as was noted above, a train, a plane or any other body can also be taken as a reference system). Studying the motion of the Earth as a whole or the motion of planets, the Sun and stars are taken as the reference system. It will be shown in Chap. 2 that this system is especially convenient for studying the laws of dynamics.

? 1.2.1. Will a small flag flutter when fixed to the basket of a balloon carried by wind?

1.3. Trajectory of Motion

To describe the motion of a body, we must indicate how the positions of its points change with time. During the motion, each point of the body

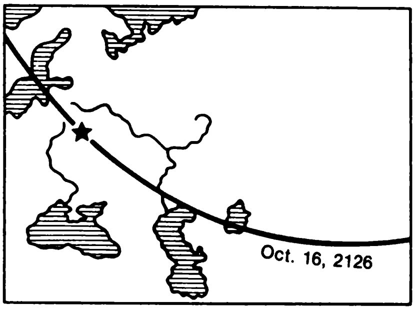

Fig. 4.
The trajectory of the centre of the Moon's shadow during the forthcoming solar eclipse of October 16, 2126.

describes a certain line, viz. the *trajectory of motion*. Moving a piece of chalk over a blackboard, we leave a track on it, viz. the trajectory of motion of the edge of the chalk. A writing is the trajectory of the tip of a pen. The luminous track of a meteor in the night sky (see Fig. 1) and the cloudy tracks of alpha-particles (see Fig. 2) are the trajectories of the meteor and alpha-particles. Astronomers expecting a solar eclipse calculate the trajectory of motion of the Moon's shadow over the surface of the Earth beforehand. Figure 4 shows such a trajectory for the nearest total solar eclipse.

Since the motion is relative, *the trajectory of motion may depend on the choice of the reference system*. For instance, in the absence of wind, to an observer in a carriage at rest the rain drops seem to fall vertically: rain drops leave behind vertical tracks on the window glass. If, however, the train starts off, the rain drops seem to fall obliquely with respect to the moving carriage: rain drops leave behind inclined tracks on the window, their slope being the larger, the higher the velocity of the train. Figure 5 shows the trajectory of point P on the rim of a wheel relative to the surface of the Earth (the wheel is rolling over a straight road). Naturally, the trajectory of point P relative to the cart is the rim circumference itself.

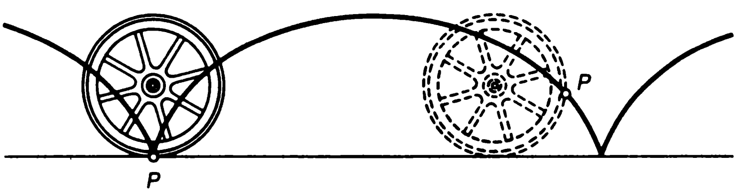

Fig. 5.
Point P on the rim of a rolling wheel describes a cycloid trajectory relative to the surface of the Earth.

1.4. Translatory and Rotary Motion of a Body

The trajectories of different points of a body may be different. This can be visually demonstrated by rapidly moving, for example, a splinter smouldering at two ends in a dark room. The human eye has the property of persistence of vision for about 0.1 s. Therefore, we perceive the trajectories of the smouldering ends of the splinter as luminous lines and can compare the two trajectories (Fig. 6).

The simplest motion of a body is such that all its points move similarly, describing identical trajectories. This type of motion is known as *translatory* motion. We can observe a translatory motion by displacing the splinter so that it always remains parallel to itself. The trajectories in a translatory motion can be either straight lines (Fig. 7*a*) or curves (Fig. 7*b*). It can be proved that *in translatory motion any straight line drawn in a body remains parallel to itself*. This typical feature is convenient to use when one has to determine whether or not a given motion is translatory. In a rolling cylinder, for example, straight lines crossing its axis do not remain parallel to themselves since rolling is not a translatory motion. When a T-square and a triangle move over a drawing board, any straight line drawn in them remains parallel to itself, and, hence, their motion is translatory (Fig. 8). A needle in a sewing machine, a piston in the cylinder of a steam engine or internal combustion engine, the hood of a motor car (but not its wheels!) moving on a straight road are examples of bodies in translatory motion.

Another simple type of motion is *rotary motion*, or *rotation*. In rotary motion, all points of the body move in circles whose centres lie on a straight line. This line is called the *rotation axis* (straight line OO' in

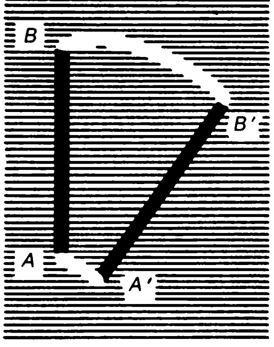

Fig. 6.
The trajectories AA' and BB' of the smouldering ends of a splinter are different.

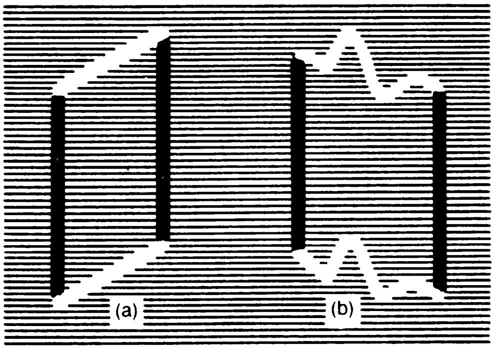

Fig. 7.
Translatory motion of a splinter.

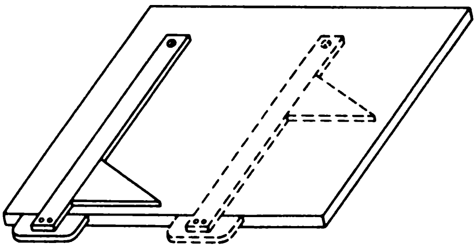

Fig. 8.
A T-square and a triangle are in translatory motion relative to the surface of a drawing board.

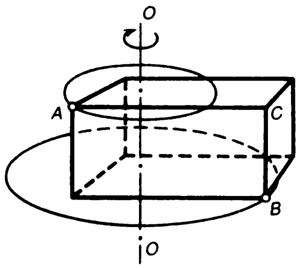

Fig. 9.
Rotation of a bar around the axis OO'. The trajectories of points A and B are shown.

Fig. 9). The circles lie in parallel planes that are perpendicular to the rotation axis. The points lying on the rotation axis remain at rest. Rotation is not a translatory motion since only straight lines that are parallel to the rotation axis remain parallel to themselves in this motion (like the straight line BC in Fig. 9).

The diurnal motion of the Earth is a rotary motion. The oscillations of the pendulum of a clock is also an example of rotation. Rotation is frequently encountered in engineering: wheels, pulleys, shafts and axles of various mechanisms, crankshafts, air screws, pointers of measuring instruments, etc. execute rotary motions.

? **1.4.1.** Is the motion of the pedals (without free running) of a moving bicycle translatory?

1.5. Motion of a Point

To describe the motion of a body, we must know how its various points move. But if a body is in translatory motion, all its points move in the same way. Therefore, to describe a translatory motion it is sufficient to describe the motion of its any points. If different points of a body move differently, it is still possible sometimes to confine the description to the motion of a single point. This refers to the case when we are interested only in the change of the position of the body as a whole, for example, as in the analysis of the flight of a bullet or an aeroplane, the motion of a ship in the sea, the motion of a planet around the Sun, and so on. Thus, while studying the motion of a planet around the Sun, it is sufficient to describe the motion of its centre.

Consequently, in some cases the description of the motion of a body is reduced to the description of the motion of a point.

Various types of motion of a point differ first of all in the shape of the trajectories. If the trajectory is a straight line, the motion of the point is referred to as *rectilinear* (*motion in a straight line*). If the trajectory is a curve, the motion is said to be *curvilinear*. The motion of a body as a whole can be treated as rectilinear or curvilinear only if we can limit ourselves to a description of motion of a single point of the body. Generally speaking, some points of a body can move in straight lines, while its other points are in a curvilinear motion.

The motion of a point in a straight line is the simplest type of motion. The next nineteen sections of this book will be devoted to an analysis of this type of motion.

? 1.5.1. Which points of a cylinder rolling over a plane move in a straight line?

1.6. Description of Motion of a Point

The trajectory of motion indicates all the positions occupied by a point. However, even if we know the trajectory, we cannot say whether the point moved rapidly or slowly on individual segments of the trajectory, whether it stopped or moved continuously, and so on. To obtain a *complete* description of motion, we must also know at what instant of time the point occupied a certain position on the trajectory. For this purpose, it is necessary to mark all points of the trajectory in a certain way and "correlate" each point to the moment of time when the moving point passed through this mark.

On railways and highways, such a marking-out is made by installing mileposts along the track. They help to determine at what distance from the initial point a train or a car is. The number on a post passed by the train directly indicates the distance s from an initial point which is usually a large city situated on this road.

Let us start with an analysis of the motion of a point in a straight line. In this case, the straight line along which the point moves can be taken as the x-axis with the origin O at an arbitrary point (Fig. 10). Then the position of the point on the trajectory will be determined by the segment laid from point O to a given point (segments OA and OB in Fig. 10). To distinguish points lying on different sides of point O, the position of points for which the segment is laid in the direction of the x-axis is determined by the length of the segment with the plus sign (point A in Fig. 10), while the

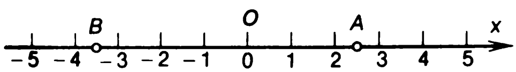

Fig. 10.
Marking of a rectilinear trajectory.

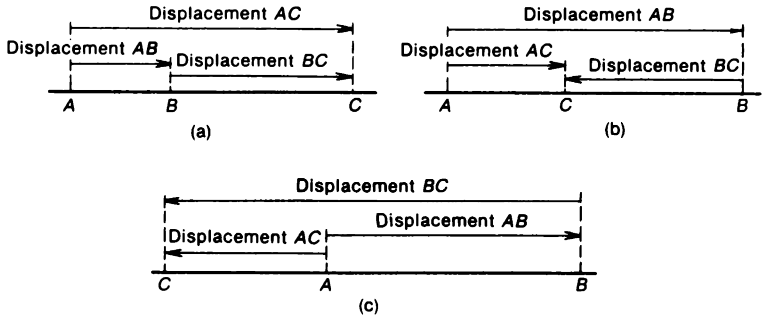

Fig. 11.
Composition of displacements: (*a*) in the same direction and (*b*) and (*c*) in opposite directions.

position of points for which the segment is laid in the opposite direction is determined by the length of the segment with the minus sign (point B in Fig. 10). The length of the segment taken with the appropriate sign is called the *coordinate* x of the point. For example, the coordinate of point A in Fig. 10 is $x_A = 2.5$, while the coordinate of point B is $x_B = -3.5$.

Suppose that a point has moved from A to B (Fig. 11). The segment AB directed from the initial to the final position is called the *displacement of the point*. [1] The length of a segment is always expressed by a positive number. We shall call this number the *magnitude of displacement*.

If the point made two consecutive displacements AB and BC, the resultant displacement is AC. Figure 11 shows that if the displacements being combined have the same direction (Fig. 11*a*), the direction of the resultant displacement coincides with the direction of the components, and the magnitude of the resultant displacement is equal to the sum of the magnitudes of the components. If, however, the displacements being combined have opposite directions (Fig. 11*b* and *c*), the direction of the resultant displacement coincides with the direction of the component whose magnitude is larger. The resultant displacement in this case is equal in magnitude to the difference of the magnitudes of the components:

$$\text{magnitude of } AC = |\text{magnitude of } AB - \text{magnitude of } BC|.$$

The distance covered by the point along its trajectory is known as the *path length*. The path length, usually denoted by s, is expressed by a positive number. If the direction of motion does not change during some period of time, the path length (in the case of rectilinear motion) coincides with the magnitude of the displacement. If the direction of motion changes, the time interval under consideration (for example, the time t_{AC} during which the point is displaced by AC) should be divided into the inter-

[1] The displacement of a point is a vector (see Sec. 1.23). — *Eds.*

vals during which the direction of motion remains unchanged, the distance covered by the point on each interval should be calculated, and all these path lengths should be added. For example, if in the case depicted in Fig. 11*b* the direction of motion has not changed over displacements AB and BC, the distance covered during the time t_{AC} is equal to the sum of the magnitudes of displacements AB and BC.

In order to "correlate" the marked points of the trajectory to the moments of time at which the moving point passes through the marks, a certain instant of time is chosen as the initial one, and each position of the moving point on the trajectory is put into correspondence with the time interval that has passed from the initial moment of time. We shall denote time intervals by t.

On a railroad, this correlation can be made by a passenger on a train, who notes with the help of his watch the instants at which the train passes by mileposts. Such a correlation can also be made by observers on the railroad station who mark with the help of a clock the time when the train passes by a station. Referees of sport events, who record with a stopwatch the instant of time when a skier crosses the finish line or an aeroplane flies over a control point, correlate the position of a moving body on its trajectory to some instant of time. In this case, the initial moment is the moment of start.

At a school laboratory, such a correlation can be made with the help of a dropping bottle (Fig. 12) mounted on a moving body, for example, a cart or a spring toy car. The drops of ink falling in equal time intervals mark the position of the body on its trajectory. The moment corresponding to the falling of a certain drop is taken as the initial instant of time.

The stroboscopic method of observation is sometimes used to study the motion. A stroboscope is a device producing intermittent light flashes of short duration at equal intervals of time. A device in which short current pulses cause bright flashes in a special lamp can be used for this purpose. An opaque disc with a slit, rotating in front of an ordinary lamp, also produces stroboscopic light.

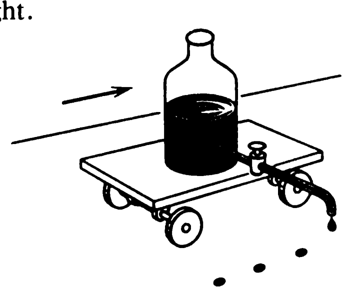

Fig. 12.
Dropping bottle.

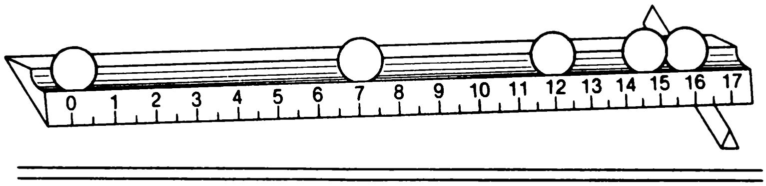

Fig. 13.
A ball rolling down a trough as seen in stroboscopic light (from the photograph).

Suppose, for example, that the motion of a ball rolling down an inclined trough is studied. If the experiment is carried out in a dark room, and the ball is illuminated by stroboscopic light, the ball will be seen only in the positions where it is illuminated by a flash. If we place a ruler with divisions along the trough, it will also be illuminated, and the positions of the ball relative to the ruler at the instants of flashes can be registered (Fig. 13). In order to record all the positions of the ball, the pattern can be photographed by a camera with an open shutter throughout the motion.

By using a stroboscope, we can see simultaneously a number of different positions of an object without resorting to a photograph. If several consecutive flashes of a stroboscope occur in 0.1 s, we shall see several consecutive positions of the ball due to the persistence of vision. A similar pattern can be observed if we rapidly move a bright rod illuminated by a daylight lamp or some other gas-discharge tube. Such lamps fed by an alternating current produce one hundred flashes per second, which allows us to see simultaneously a number of consecutive positions of the rod. One can also see several positions of one's waving hand in a dark projection room during a film show (24 flashes per second).

By "correlating" in some way separate positions of a moving point with the instants of time, we obtain a complete description of the motion of the point. This means that we know all the positions of the point and are able to determine for each of these positions the distance along the trajectory from the initial point and the time interval that has elapsed from the initial moment.

Thus, any description of motion of a point is based on the measurement of distances and intervals of time. It should be noted that the initial point on the trajectory and the initial moment of time can be chosen arbitrarily, from the considerations of convenience of description of a given motion. A moving point must not necessarily be in the position $s = 0$ at the instant of time $t = 0$.

1.7. Measurement of Length

The base unit of length is a *metre* (m). Initially, the standard metre was taken as the distance between two marks on a specially made platinum-

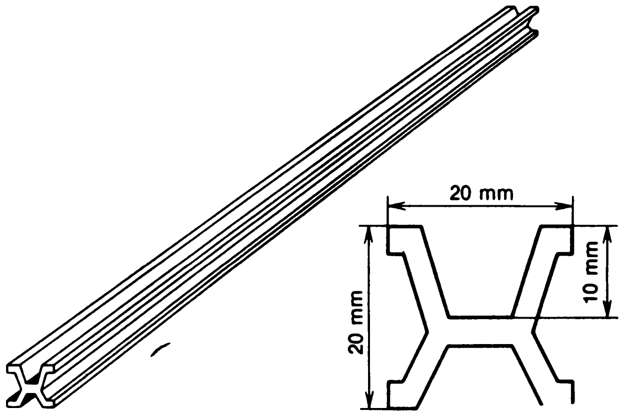

Fig. 14.
The former standard metre (general view and cross section).

iridium rod of length 102 cm, which is stored at the International Bureau of Weights and Measures near Paris (Fig. 14). The material and shape of the rod, as well as the conditions in which it was stored, were chosen so as to ensure the invariability of the sample in the best possible way. In particular, measures were taken to maintain a constant temperature of the rod. Meticulously prepared secondary standards, viz. copies of this sample, are stored at institutes of weights and measures in many countries.

It was planned to choose 1/40 000 000 of the length of the Earth's meridian as the standard metre. However, when it turned out that the precision of measurements on the surface of the Earth was insufficient, it was decided not to replace the existing standard or introduce corrections on the basis of more accurate measurements, but preserve the sample as the unit of length. This sample is about 0.2 mm shorter than 1/40 000 000 of the Earth's meridian.

Besides this base unit, other units are also used in science and engineering, which are larger or smaller than metre by factors of ten: [2]

kilometre (1 km = 1000 m);
centimetre (1 cm = 0.01 m);
millimetre (1 mm = 0.001 m);
micrometre (1 μm = 0.001 mm = 0.000 001 m);
nanometre (1 nm = 0.000 000 001 m).

[2] Decimal multiples and fractional units, as well as their notations, are formed with the help of factors and prefixes, e.g., 10^9 — giga (G), 10^6 — mega (M), 10^3 — kilo (k), 10^2 — hecto (h), 10^{-1} — deci (d), 10^{-2} — centi (c), 10^{-3} — milli (m), 10^{-6} — micro (μ), 10^{-9} — nano (n). — *Eds.*

In Great Britain, the USA and some other countries, the so-called British units of length are still widely used:

1 inch = 25.4 mm;
1 foot = 12 inches = 304.8 mm;
1 statute mile = 1609 m;
1 nautical mile = 1852 m (the length of the one-minute arc of the Earth's meridian).

The fact that there exists a great number of units of length (as well as units for other physical quantities) presents difficulties in practical calculations. For this reason, the international standard definitions of units for all physical quantities have been worked out. The set of these definitions is called the *SI system of units* (for the French *Système International*). This system is used in science and technology in many countries throughout the world.

In this system of units, a metre is defined as the length equal to 1 650 763.73 wavelengths of the orange line emitted by a special lamp in which the krypton-86 gas glows under the action of electric discharge. [3] The number of wavelengths is chosen so that this unit of length coincides to the greatest possible extent with the standard metre stored in Paris. For this reason, the unit of length was not chosen in such a way as to contain a round integral number of wavelengths (say, a million). This new unit can be reproduced (optically) to a higher degree of accuracy than the previous standard metre. It is very convenient that there is no need to resort to a unique sample to reproduce the unit of length. It is sufficient to make a special krypton lamp and observe the light emitted by it.

To measure length in practice, including the distance between two positions of a point on its trajectory, copies of secondary standards are used, viz. rods, rulers or tape measures that are equal to the length of the standard or of its parts (centimetres, millimetres). The zero of the ruler is made to coincide with one end of the segment being measured and the division standing against its other end is marked. If the second end does not coincide with any division on the scale, then one should approximately determine the fraction of a division corresponding to this end.

Various auxiliary attachments are used to reduce inevitable errors in measurements. Figure 15 shows one of such attachments, viz. a *vernier caliper*. The vernier is an additional scale sliding along the main scale. The divisions of the vernier are smaller than the divisions of the main scale by 0.1 of their length. If, for example, the division of the main scale is 1 mm, the division of the vernier has the length of 0.9 mm. It can be seen from the figure that the diameter of the ball being measured is larger than 11 mm but smaller than 12 mm. In order to find the number of decimal fractions of a millimetre in the remaining part of the division, we must find out the vernier division which coincides with one of the divisions of the main scale. In the figure, it is the ninth division of the vernier. This means that the eighth, seventh, etc.

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[3] Since 1983, the metre is defined as the distance covered in vacuum by a plane electromagnetic wave in 1/299 792 458 s. — *Eds*.

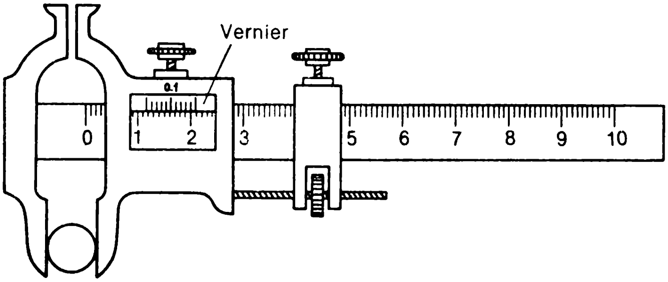

Fig. 15.
Vernier caliper.

divisions of the vernier are ahead of the preceding divisions of the main scale by 0.1 mm, 0.2 mm, etc. while the initial division of the vernier is 0.9 mm ahead of the preceding division of the main scale closest to it. Hence it follows that the ball diameter contains an integral number of millimetres between the zero of the main scale and the beginning of the vernier scale (11 mm), and decimal fractions of a millimetre equal to the number of vernier divisions between the beginning of its scale to the coinciding divisions (0.9 mm). Thus, the ball diameter being measured is 11.9 mm.

Hence, the vernier makes it possible to measure distances to within 1/10 of a scale division.

1.8. Measurement of Time Intervals

The choice of the unit of time can be made on the basis of the duration of some recurring process. From ancient times, the duration of one complete rotation of the Earth about its axis relative to the Sun (a day) was taken as a unit of time. Since the duration of this rotation slightly differs during a year (almost by 1 min), we take for the unit of time the average value of this quantity over a year. Days are divided into hours, minutes and seconds.

The *second* (s) is one of the base units in SI. It is defined as the time interval equal to the sum of 9 192 631 770 periods of radiation corresponding to the transition between two definite energy levels in the cesium-133 atom. The second is approximately equal to 1/86 400 of the mean solar day.

Various recurring processes can be used for constructing timepieces, viz. the instruments for measuring time intervals.

In old times, clepsydras (water clocks) were used, in which time was determined from the amount of water flowing from one vessel to another (Fig. 16). The duration of the same time interval was reproduced with the help of a sand glass in which a definite amount of sand runs through a narrow tube (Fig. 17). The accuracy of these instruments is not high.

Various oscillatory processes, like oscillations of a pendulum, viz. a load suspended by a thread or rod (wall clock pendulum), recur with a much higher accuracy. If the amplitude of a swinging pendulum is not very

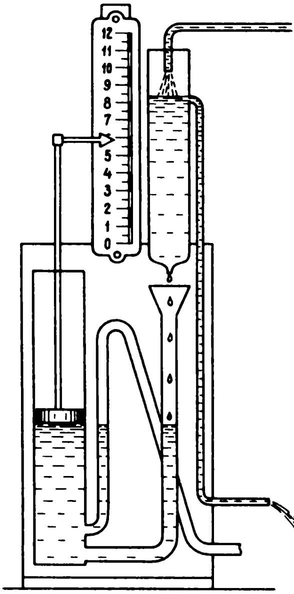

Fig. 16.
Clepsydra.

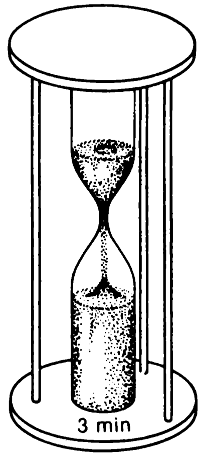

Fig. 17.
Sand glass.

large, the period of oscillations (the time of a swing from an extreme position to and fro) practically does not depend on the amplitude and is determined only by the pendulum length. The fact that the period of oscillations of a pendulum is independent of the amplitude was established by the Italian physicist and astronomer Galileo Galilei (1564-1642) and was later used by the Dutch physicist and mathematician Ch. Huygens (1629-1695) who created the first pendulum clock in 1657. In this clock, oscillations are counted with the help of a system of wheels. After each oscillation, the hands of the clock turn through a certain angle so that the time interval can be determined by the change in the positions of the hands.

Later, a pocket watch was invented. In this instrument, the oscillating pendulum is replaced by a wheel which is attached to a spiral spring (so-called balance-wheel) and oscillates relative to the axle about the equilibrium position at a constant period determined by the properties of the balance-wheel, or the spiral spring. A stopwatch, i.e. the watch started and stopped by pressing a button, is especially convenient in practice. It has a long hand making a revolution per minute, which enables one to measure decimal fractions of a second on the dial plate.

After clocks with a pendulum (and later with a balance-wheel) had been invented, all other types of mechanical clocks became obsolete as less accurate. However, the sand glass is still in use, for example, in medical practice for such treatments (baths and others), where only one definite time interval has to be counted. The dropping bottle and stroboscope described in Sec. 1.6 are also a kind of clock.

In modern technology, time intervals are measured with increasingly high accuracy with the help of oscillating quartz crystals (quartz clock) or vibrations of molecules (molecular clock). Quartz and molecular clocks make it possible to measure time intervals to within 10^{-6}, 10^{-9} and even 10^{-12} fractions of a second.

1.9. Uniform Rectilinear Motion and Its Velocity

A motion in which a body covers equal distances in any equal intervals of time is known as a *uniform* motion. For example, a train moves uniformly over a long even route; its wheels experience impacts at rail joints in equal intervals of time, and mileposts (or telegraph posts erected at approximately equal distances from one another) pass by the window in equal time intervals. A motor car moves uniformly along a straight segment of the road if its engine works smoothly. A skater or a runner is also in a uniform motion at the middle of the distance. Other examples of uniform motion are falling of rain drops, rising of gas bubbles in a glass of soda water and descending of a parachutist with an open parachute.

In different types of uniform motion, the displacement of bodies in equal time intervals may be different, and hence the same displacements will occur in different time. To cover the distance between two telegraph posts, a motor car, for instance, spends less time than a cyclist. A pedestrian may cover 100 m per minute, while an artificial satellite of the Earth may traverse during this time 500 km, and a radio or light signal will travel in the same time interval over a distance of 18 000 000 km. We say that the motor car runs faster than the pedestrian, and the radio signal travels faster than the satellite. In order to characterise this difference in uniform motions quantitatively, a special physical quantity, viz. the velocity of motion, is introduced. [4]

The velocity of a uniform motion is the ratio of the distance covered by a body to the time interval during which this distance is traversed:

$$\text{velocity} = \frac{\text{distance}}{\text{time interval}}\,.$$

To determine the velocity of a body, we must measure the length of the path traversed by the body and the time interval during which it is done,

[4] Velocity is a vector quantity which is characterised by the magnitude, called speed, and direction. Henceforth, we shall use the term "velocity" both for scalar and vector quantities. — *Eds*.

and then divide the result of the first measurement by that of the second one.

Thus, in accordance with the definition of uniform motion, a body will cover twice, thrice and longer distances during time intervals that are twice, thrice and longer, while during half the initial time interval the body will cover half the distance, and so on.

Consequently, the value of the velocity is the same irrespective of the segment of the path and of the time interval over which it is determined. This means that in a uniform motion, velocity is a constant quantity characterising the given motion on any segment of the path and over any time interval. We shall denote velocity by v.

If we denote the time interval by t and the distance covered by s, the velocity of a uniform motion will be expressed by the formula

$$v = s/t. \tag{1.9.1}$$

If we know the velocity v of a uniform motion, we can find the distance covered over any time interval t from the formula

$$s = vt. \tag{1.9.2}$$

This formula shows that in a uniform motion the distance covered by a body increases in proportion to time. It also shows that *the velocity in uniform motion is numerically equal to the distance covered per unit time.* If we know the distance s covered by a body in uniform motion and the velocity v of the motion, we can determine the time interval t required to cover this distance as follows:

$$t = s/v. \tag{1.9.3}$$

These formulas provide the answers to all questions concerning a uniform motion.

Any measurement, including the measurement of the path lengths and time intervals required for determining the velocity of motion, is always made with a certain error. For this reason, even when the measurements give the same value of the velocity of motion over different segments of the trajectory, we can state that the motion is uniform only with the accuracy with which measurements were made. For example, if the time interval over which a train has covered the distance between two posts is determined with the help of the minute hand of watch, it often seems that this time is the same over a segment of the path many kilometres long. To this degree of accuracy, the motion of the train is uniform. If, however, we use a stop-watch and measure time intervals to within fractions of a second, it will be seen that these time intervals slightly differ, and hence the motion of the train is not uniform to such a degree of accuracy.

? **1.9.1.** In blasting, a special slowly burning safety fuse (bickford fuse) is used for blastholes (viz. the holes with explosive materials). What must be the length of the fuse to enable one to run away 150 m at a velocity of 5 m/s from it after lighting, while the fuse burns at a rate of 1 m in 2 min?

1.9.2. A boy whose height is 1.5 m runs at a velocity of 3 m/s in a straight line passing under a lamp hanging at a height of 3 m. Show that the shadow of the boy's head moves uniformly and determine the velocity of its motion.

1.10. The Sign of Velocity in Rectilinear Motion

Let us suppose that at an instant of time t_1 (counted off from the initial moment) a body is at a point with coordinate x_1 (see Sec. 1.6), while at a later instant of time t_2 it is at a point with coordinate x_2. The difference $t_2 - t_1$ gives the time interval t during which the body moves, while the absolute value of the difference $x_2 - x_1$ is equal to the distance s covered by the body. Therefore, formula (1.9.1) can be written in the form

$$v = \frac{|x_2 - x_1|}{t_2 - t_1}. \tag{1.10.1}$$

If we take just the difference $x_2 - x_1$ in the numerator, we obtain the following formula:

$$v = \frac{x_2 - x_1}{t_2 - t_1}. \tag{1.10.2}$$

The quantity v defined by this formula turns out to be an algebraic quantity. Indeed, the difference $t_2 - t_1$ is always positive since t_2 (the later instant) is expressed by a larger number than t_1 (the earlier instant). On the other hand, the difference $x_2 - x_1$ can be either positive (if $x_2 > x_1$) or negative (if $x_2 < x_1$). Its sign depends on the direction in which the body moves. If it moves in the direction of the x-axis, then $x_2 > x_1$, and the quantity v defined by formula (1.10.2) is positive. If the body moves in the opposite direction, then $x_2 < x_1$, and v is negative.

Thus, the sign of the quantity defined by (1.10.2) allows us to determine the direction (along x or against x) in which the body moves. This is very convenient. Consequently, for motion in a straight line, we shall conditionally speak of positive and negative velocities. [5]

1.11. Units of Velocity

Formula (1.9.1) shows that if a body covers a unit distance in a unit of time, its velocity v is also equal to unity. Hence, *for the unit of velocity we*

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[5] The quantity determined by formula (1.10.2) is the projection of the velocity vector on the x-axis (Sec. 1.24). — *Eds.*

take the velocity of uniform motion in which a body covers a unit distance in unit time. Thus, the SI unit of velocity is the velocity of motion in which a path of one metre is traversed in a second. This unit is *metre per second* (m/s). We can find the velocity of any motion in metres per second by dividing the path length in metres by the corresponding time interval in seconds.

For any other choice of units of time and/or distance, the unit of velocity will be different. If we take a centimetre and a second as units of length and time, the unit of velocity will be *centimetre per second* (cm/s). It is the velocity of motion in which a distance of 1 cm is traversed in 1 s. If we take a kilometre and an hour as units, the unit of velocity will be *kilometre per hour* (km/h). This is the velocity of motion in which 1 km is covered in 1 h. In a similar way we can compile units of velocity for any other choice of units of length and time.

Clearly, in different units, the velocity of the same motion will have different numerical values. Suppose that we know the numerical value of the velocity of some motion in certain units, say, in metres per second. This value is obtained by dividing the number expressing the path length in metres by the corresponding time interval in seconds. Let us express the velocity of this motion in some other units, for instance, kilometres per hour. Shall we measure again the distance covered (now in kilometres) and the time interval (now in hours)? There is no need to repeat the measurements. The new value of the velocity V [km/h] of the given motion can be obtained from the old value v [m/s] by calculations.

Indeed, let us denote the measured distance by s [m] and time interval by t [s]. The numerical value of velocity is

$$\frac{s\ [\mathrm{m}]}{t\ [\mathrm{s}]} = v\ [\mathrm{m/s}].$$

If we measured the same distance in kilometres and time in hours, the quantities appearing in this formula would change: the distance would be expressed by the quantity S [km] $= s$ [m] $\times$ 1/1000, and the time by the quantity T [h] $= t$ [s] $\times$ 1/3600. In the new units, the velocity is given by

$$V\ [\mathrm{km/h}] = \frac{S\ [\mathrm{km}]}{T\ [\mathrm{h}]} = \frac{s\ [\mathrm{m}] \times 1/1000}{t\ [\mathrm{s}] \times 1/3600} = 3.6v\ [\mathrm{m/s}].$$

This formula makes it possible to go over from the velocity v in metres per second to the velocity V in kilometres per hour. From this formula we can easily obtain the expression for the inverse transition, viz. from kilometres per hour to metres per second:

$$v\ [\mathrm{m/s}] = \frac{1}{3.6}\, V\ [\mathrm{km/h}].$$

For example, for $v = 100$ m/s, the velocity $V = 3.6 \times 100 = 360$ km/h, while for $V = 72$ km/h we obtain $v = (1/3.6) \times 72 = 20$ m/s.

We can also easily obtain a relation between units of velocity themselves. For this purpose, in the formulas obtained above we must take the initial velocity equal to unity. This gives

$$1 \text{ km/h} = \frac{1}{3.6} \text{ m/s}, \qquad 1 \text{ m/s} = 3.6 \text{ km/h}.$$

Using formulas (1.9.1)-(1.9.4), as well as other formulas including length, time and velocity, for calculations, we must express all quantities in matching units. If, for example, velocity is expressed in metres per second, the distances and time intervals should be expressed in metres and seconds. If the distance is given in kilometres and time in hours, the velocity should be expressed in kilometres per hour. If given quantities are expressed in units which do not match one another, a change of units should be made. If, for instance, length is given in kilometres and time in hours, the value of velocity should be expressed in kilometres per hour, and just this value should be used in formulas.

There exists a "natural standard" of velocity. It is the velocity of light in vacuum (for example, in outer space), which is equal approximately to 300 000 km/s.[6] Any radio signal propagates in vacuum at the same velocity. The velocity of light plays a very important role in all branches of physics. It has been established that bodies cannot move at a velocity higher than the velocity of light in vacuum: the velocity of light in vacuum is the limiting velocity of bodies. The velocities of all terrestrial and celestial bodies are always very small in comparison with the velocity of light. The velocity of the Earth in its motion around the Sun, for example, is 30 km/s, i.e. only 0.0001 of the velocity of light. We encounter velocities approaching the velocity of light only in the world of smallest particles constituting matter, viz. electrons, protons and other elementary particles. At such velocities, important features are observed in the motion of bodies. These questions will be discussed in Vol. 3.

In nautical practice, a special unit of velocity called a *knot* is used. A knot is the velocity of motion in which a body covers a nautical mile per hour. One knot is equal to 0.514 m/s. Modern ships which develop a velocity of about 40 knots, i.e. 20 m/s, move at the speed of a hurricane.

It is interesting to note that sometimes a unit of length is used which is based on the velocity of light. It is a *light year*, i.e. the distance covered by light in a year. A light year is approximately equal to 9.4605×10^{15} m. This unit of length is used in astronomy where the

[6] The velocity of light in transparent media is less than in vacuum. For example, the velocity of light in water is 225 000 km/s.

distances of thousands, millions, and even billions of light years are encountered. The star closest to the Earth is at a distance of 3.2 light years, while the most distant galaxies (stellar systems) are at distances of the order of 3×10^9 light years.

1.12. Path vs. Time Graph

If the trajectory of motion of a point is known, the dependence of the distance s covered by the point on the elapsed time interval t gives a complete description of this motion. It was shown above that for a uniform motion such a dependence can be given by formula (1.9.2). The relation between s and t at separate instants of time can also be given in a tabular form. The table contains the values of time intervals and corresponding path lengths. Let us suppose that the velocity of a uniform motion is 2 m/s. Formula (1.9.2) in this case has the form $s = 2t$. We compile the table of path and time for such a motion:

t, s	1	2	3	4	5	6	...
s, m	2	4	6	8	10	12	...

It is often convenient to plot the dependence of one quantity on another instead of expressing it through formulas or tables since graphs provide a more visual pattern of variation of quantities and may simplify calculations. Let us plot the dependence of the path length on time for the motion under consideration. For this we take two mutually perpendicular straight lines (coordinate axes) and call one of these (abscissa axis) the axis of time and the other (ordinate axis) the axis of path. Then we choose the scales for time intervals and path length and take the origin of coordinates as the initial instant and the initial point on the trajectory. On the axes, we mark the values of time and path length for the motion under consideration (Fig. 18). In order to "correlate" the values of path length with the instants of time, we erect perpendiculars to the axes at the points marked on them (e.g. at the points corresponding to 3 s and 6 m). The point of intersection of perpendiculars corresponds to two quantities simultaneously, viz. the path length s and time t. In this way, "correlation" is realised. We can proceed in a similar way for any other instant of time and the corresponding path, and obtain a point on a graph for each such time-path length pair. Figure 18 shows a construction in which both rows in the table are replaced by a number of points. If we made such a construction for all instants of time, we would obtain a solid straight line instead of separate points (this line is also shown in the figure). We shall call this line the *path vs. time graph*, or simply the *path graph*.

In the case under consideration, the path graph turned out to be a straight line. It can be shown that the path graph of a uniform motion is

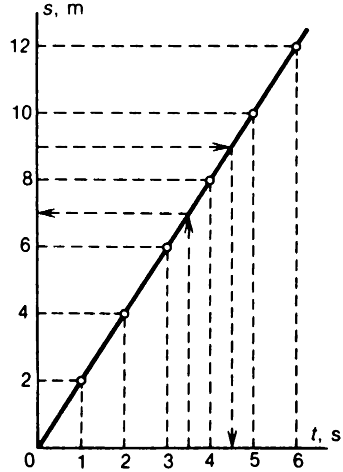

Fig. 18.
The path vs. time graph for a motion at a velocity of 2 m/s.

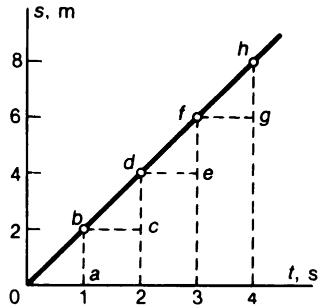

Fig. 19.
To Exercise 1.12.1.

always a straight line. Conversely, if the path vs. time graph of a motion is a straight line, the motion is uniform.

? **1.12.1.** Prove the above statement with the help of Fig. 19.

Repeating the same construction for another velocity of motion, we see that the points of the graph corresponding to the higher velocity lie above the points of the graph for the lower velocity (Fig. 20). Thus, *the higher the velocity of a uniform motion, the steeper is the straight line representing the path graph*, i.e. the larger the angle formed by this straight line with the time axis.

The slope of a graph naturally is determined not only by the numerical value of the velocity but also by the choice of the time and path scales. For instance, the graph depicted in Fig. 21 represents the time dependence of the path for the same motion as that shown by the graph in Fig. 18, although it has a different slope. Hence it is clear that we compare motions on the basis of the slopes of the graphs only if they are plotted on the same scale.

Using path graphs, we can solve various problems on motion. By way of example, the dashed lines in Fig. 18 show constructions required for solving the following problems concerning the given motion: (a) find the distance covered in 3.5 s; (b) determine the time during which a path of

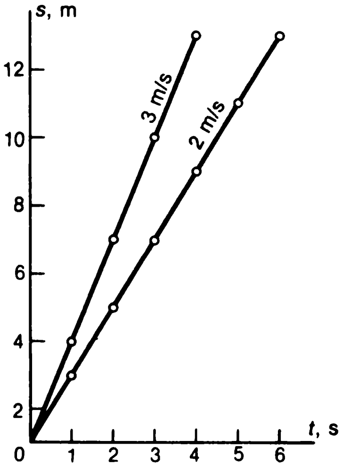

Fig. 20.
Path vs. time graphs for uniform motions at velocities of 2 and 3 m/s.

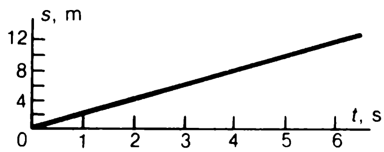

Fig. 21.
A path vs. time graph for the same motion as in Fig. 18, but plotted on a different scale.

9 m has been traversed. The following answers are obtained graphically (dashed lines in the figure): (a) 7 m; (b) 4.5 s.

? **1.12.2.** Using the graph in Fig. 18, determine the distance from the initial point to the moving point in 2 s after it has covered a distance of 6 m.

On the graph describing a uniform motion in a straight line, instead of the path length s we can plot the coordinate x of a moving point along the ordinate axis. Such a description is more informative. In particular, it allows us to determine the direction of motion relative to the x-axis. Besides, taking the origin for the zero coordinate, we can describe the motion of the point at earlier instants of time which are considered to be negative.

For example, the straight line I in Fig. 22 is the graph of motion with a positive velocity of 4 m/s (i.e. in the direction of the x-axis); at the initial instant of time, the moving point was at a point with a coordinate x_0 = 3 m. For the sake of comparison, the same figure shows a graph of motion at the same velocity, but now the moving point is at the initial instant of time at a point with coordinate $x_0 = 0$ (straight line II). The straight line III corresponds to the case when at the moment $t = 0$ the moving point is at a point with the coordinate $x_0 = -7$ m. Finally, the straight line IV describes the motion when the moving point has the coordinate $x = 0$ at an instant $t = -2$ s.

We see that all four graphs have the same slope: the slope depends only

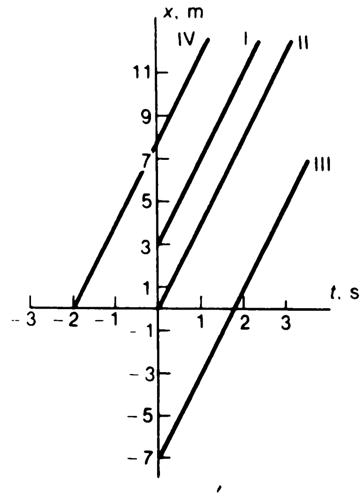

Fig. 22.
Path vs. time graphs for motions with the same velocity but for different initial positions of a moving point.

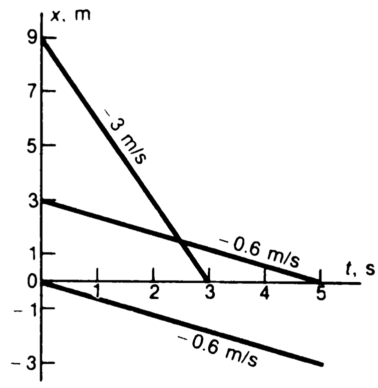

Fig. 23.
Path vs. time graphs for motions with negative velocities.

on the velocity of the moving point and does not depend on its initial position. When the initial position changes, the graph is simply translated along the x-axis upwards or downwards through a corresponding distance.

The graphs of motions with negative velocities (i.e. in the direction opposite to that of the x-axis) are shown in Fig. 23. These are straight lines with negative slopes. The x-coordinate of a point in such a motion decreases with time.

? **1.12.3.** The path vs. time graph for a point moving at a velocity v intercepts a segment s_0 on the ordinate axis. What is the time dependence of the distance s from the initial point? Write the formula expressing this dependence.

1.12.4. A point moving at a velocity v is at a distance s_0 from the initial point at an instant t_0. What is the time dependence of the distance s?

1.12.5. A point moving uniformly along the x-axis has coordinates $x_1 = -3.5$ m and $x_2 = 2.5$ m at instants $t_1 = -2$ s and $t_2 = 6$ s respectively. Determine graphically the moment of time at which the point was at the origin and the x-coordinate corresponding to the initial instant of time. Find the projection of the velocity on the x-axis.

1.12.6. A car starts off from point A 20 min after another car. Using the path graph, find the instant of time and a distance from point A to the point where the second car catches up with the first car if their velocities are 40 km/h and 60 km/h respectively.

1.12.7. Find graphically when two cars starting off simultaneously towards each other from points A and B separated by a distance of 100 km at velocities of 40 and 60 km/h respectively will meet.

Path graphs can be also plotted for the cases when a body moves uniformly for a certain time, then move uniformly but at another velocity during the next time interval, then again changes its velocity, and so on. Figure 26 shows by way of example a path graph for a body which moves during the first hour at a velocity of 20 km/h, during the next hour at 40 km/h, and during the third hour at a velocity of 15 km/h.

? 1.12.8. Plot the path vs. time graph for a motion in which a body has velocities of 10, − 5, 0, 2, and − 7 km/h over consecutive time intervals of 1 hour each. What is the total displacement of the body?

1.13. Velocity vs. Time Graph

The graph of velocity as a function of time can be plotted in the same way as the path vs. time graph. Now we plot the values of velocity on some chosen scale on the ordinate axis which serves as the velocity axis. The abscissa axis is, as before, the time axis. Since the velocity of uniform motion is a constant quantity, the graph is a straight line parallel to the time axis. The higher the velocity, the higher lies the straight line above the time axis (Fig. 24). A negative velocity is represented by a straight line below the abscissa axis. The zero velocity (state of rest) is depicted by a segment on the time axis.

Let us consider a motion at a velocity represented by segment AB. The area of the hatched rectangle in the figure is equal to the product of the segment representing velocity v and the segment corresponding to the time interval t, i.e. is equal to vt. But we know that the path traversed in a uniform motion is also equal to vt (see formula (1.9.2)). Consequently, the path is represented by the hatched area in Fig. 24. Thus, the distance covered in a uniform motion during a certain interval of time is numerically equal to the area bounded by the time axis, velocity graph and two vertical segments corresponding to the beginning and end of the time interval under consideration.

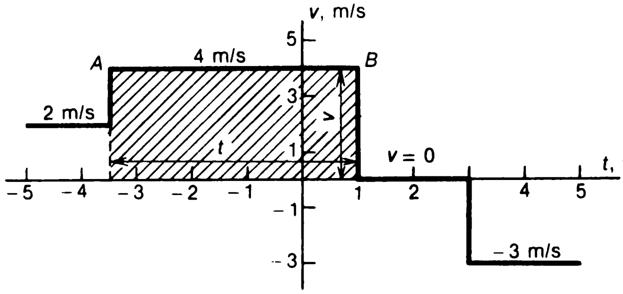

Fig. 24.
Motion of a body with different velocities over different intervals of time. The area of the hatched rectangle is (4 m/s) × 4.5 s = 18 m (traversed path).

1.14. Nonuniform Rectilinear Motion. Average Velocity

It was mentioned in Sec. 1.9 that the statement that a given motion is uniform is valid only within the accuracy of corresponding measurements. For example, using a stopwatch we can discover that the motion of a train which was regarded as uniform in rough measurements turns out to be nonuniform in a more precise measurement.

However, when a train approaches a station, the nonuniformity of its motion can be detected even without a stopwatch. Even rough measurements will show that the time intervals over which the train covers the distance between telegraph posts become longer and longer. To a low degree of accuracy corresponding to the measurements with the help of a simple watch, the motion of the train is uniform at the middle of the span and nonuniform when it approaches a station. Let us put a dropping bottle on a wound-up toy car and set it in motion. In the middle of motion, the distances between the drops are equal (the motion is uniform), but when the spring is almost completely unwound, the drops will be seen to come closer and closer to one another (the motion is nonuniform) (Fig. 25).

For nonuniform motion, we cannot speak about any definite velocity since the ratio of the path length to the corresponding time interval is different for different segments unlike in the case of uniform motion. If, however, we are interested in the motion on some definite segment of the path, this motion as a whole can be characterised by introducing a concept of the average velocity of motion: *the average velocity of nonuniform motion on a given segment of the path is the ratio of the length of this path to the time interval during which this path is traversed*:

$$v_{av} = s/t. \tag{1.14.1}$$

This formula shows that the average velocity is equal to the velocity of uniform motion in which the body would cover the same distance over the same interval of time as in actual motion.

As in the case of uniform motion, the formula $s = v_{av}t$ can be used for

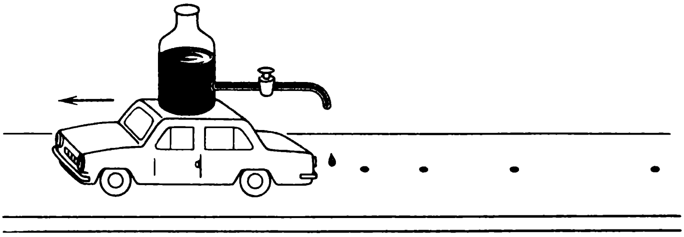

Fig. 25.
Traces of drops uniformly falling from a dropping bottle placed on a moving toy car just before its spring has become completely unwound.

determining the distance covered by the body during a given time interval at this average velocity. Similarly, the formula $t = s/v_{av}$ can be used for determining the time during which a given distance is traversed with a given average velocity. However, these formulas can be used only for the segment of the path and for the time interval for which this average velocity has been calculated. For example, if we know the average velocity for a segment AB and the length of this segment, we can determine the time during which this segment is passed, but we cannot find the time during which half this segment is covered since the average velocity over half the segment in a nonuniform motion is, generally, not equal to the average velocity for the entire segment.

If the average velocity is the same for all segments, it indicates that the motion is uniform, and the average velocity is equal to the velocity of this uniform motion.

If the average velocity is known for consecutive intervals of time of a motion, we can find the average velocity for the total time of motion. Suppose, for example, that a train moves for two hours so that its average velocity is 18 km/h for the first 10 min, 50 km/h for the next hour and a half, and 30 km/h for the remaining time. Let us calculate the distances covered in these time intervals: $s_1 = 18 \times (1/6) = 3$ km, $s_2 = 50 \times 1.5 = 75$ km, and $s_3 = 30 \times (1/3) = 10$ km. Hence the total distance covered by the train is $s = 3 + 75 + 10 = 88$ km. Since this distance has been covered in 2-h interval, the average velocity $v_{av} = 88/2 = 44$ km/h.

This example shows how to calculate the average velocity in the general case if we know the average velocities $v_1, v_2, v_3, \ldots$, with which a body moves over consecutive time intervals $t_1, t_2, t_3, \ldots$. The average velocity of the entire motion is expressed by the formula

$$v_{av} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}.$$

It is important to note that in the general case the average velocity is not equal to the mean value of the average velocities for individual segments of the path.

? **1.14.1.** Show that the average velocity over the entire path is higher than the minimum average velocity and lower than the maximum average velocity for individual segments.
1.14.2. A train covers the first 10 km of the path with an average velocity of 30 km/h, the next 10 km with an average velocity of 40 km/h and the last 10 km with an average velocity of 60 km/h. What is the average velocity of the train over the 30-km path?

1.15. Instantaneous Velocity

In order to describe a given nonuniform motion, we can determine the average velocity for several segments of its path. This, however, gives only a rough idea about the nature of motion.

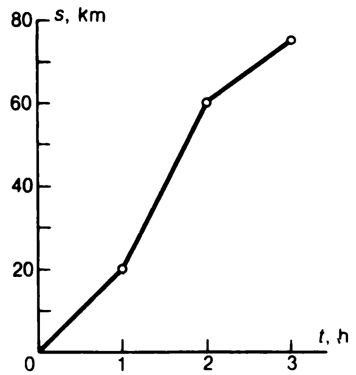

Fig. 26.
The graph gives a rough description of the motion of a car.

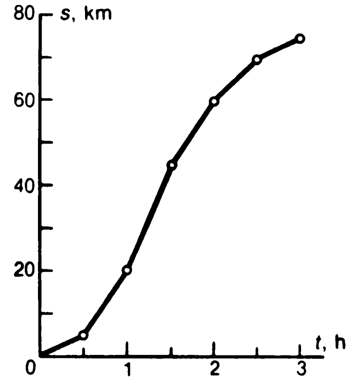

Fig. 27.
A more accurate description than in Fig. 26 of the motion of the car.

As a matter of fact, while determining the average velocity, we as if replace the motion over each time interval by a uniform motion, assuming that the velocity changes abruptly from one time interval to another. The path vs. time graph for a motion in which a point moves at constant but different velocities during different time intervals is depicted by a broken line whose segments have different slopes. Figure 26 shows, for example, the graph of motion of a motor car which moves with an average velocity of 20 km/h during the first hour, with an average velocity of 40 km/h during the second hour, and with an average velocity of 15 km/h during the last hour. For a more accurate description of motion, we have to determine average velocities for shorter time intervals. The path graphs will be broken lines with increasingly large number of segments, which describe the given motion more and more precisely (Figs. 27 and 28).

As we decrease intervals of time, the actual motion within each individual time interval will differ less and less from uniform motion, and finally the difference will become so small that it cannot be detected by the instruments used for determining the average velocity. This is a natural limit for refining the description of motion with a given degree of accuracy in measuring length and time. Within time intervals which are so small that the motion can be regarded as uniform, we can refer the result of measurement to the beginning, end, or in general to any instant of time within the limits of the time interval under consideration.

We shall call the average velocity, measured over such a short time interval that the motion in this interval appears to be uniform for the available instruments, the *instantaneous velocity*, or simply *velocity*.

If a motion is uniform, its instantaneous velocity at any instant of time

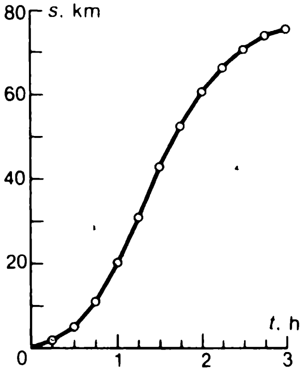

Fig. 28.
A still more accurate description of the motion of the car.

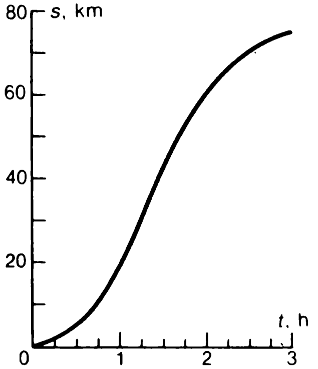

Fig. 29.
A path vs. time graph for the motion of the car is represented by a smooth curve.

is equal to the velocity of this uniform motion: the instantaneous velocity of a uniform motion is constant. On the other hand, the instantaneous velocity of a nonuniform motion is a varying quantity which assumes different values at different instants of time. It is clear from what has been said above that we can assume the instantaneous velocity to vary continuously over the time of motion so that the path vs. time graph can be represented by a smooth curve (Fig. 29). The instantaneous velocity at each instant of time is determined by the slope of the tangent to the curve at a corresponding point.

? **1.15.1.** Show that the average velocity of a nonuniform motion over any segment of the path is higher than the minimum and lower than the maximum value of the instantaneous velocity on this segment.

1.16. Acceleration in Rectilinear Motion

If the instantaneous velocity of a moving body increases, the motion is called *accelerated*, and if the instantaneous velocity decreases, the motion is known as *decelerated*.

In different nonuniform motions, velocity changes differently. For example, a goods train starting from a station is in an accelerated motion, on the span it moves with an acceleration, or uniformly, or with a deceleration, and is in a decelerated motion when approaching a station. A passenger train also moves nonuniformly but its velocity changes at a higher rate than that of a goods train. The velocity of a bullet in the barrel of a rifle increases from zero to hundreds of metres per second over a few thousandths of a second. When the bullet hits the target, its velocity drops to zero also very rapidly. When a rocket is launched, its velocity increases first slowly and then at a higher and higher rate.

Among various accelerated motions, we encounter motions in which instantaneous velocity increases by the same value over equal intervals of time. Such motions are known as *uniformly accelerated*. A ball beginning to roll down an inclined plane or to fall freely to the ground is in a uniformly accelerated motion. It should be noted that the uniformly accelerated nature of this motion is upset by friction and air resistance, which we shall not take into account for the present.

The larger the slope of the plane, the higher the rate of increase in the velocity of the ball rolling down it. The velocity of a freely falling ball increases still more rapidly (by about 10 m/s every second). The change in the velocity of uniformly accelerated motion with time can be characterised quantitatively by introducing a new physical quantity, viz. acceleration.

For a uniformly accelerated motion, the acceleration is the ratio of the increment [7] *of the velocity to the time interval over which this increment was gained*:

$$\text{acceleration} = \frac{\text{velocity increment}}{\text{time interval}}.$$

We shall denote acceleration by a. Comparing this expression with the corresponding expression for velocity in Sec. 1.9, we can say that acceleration is the rate of change in velocity.

Suppose that at an instant t_1 the velocity is v_1, while at another instant t_2 it becomes v_2, so that the velocity increment over the time $t = t_2 - t_1$ is $v_2 - v_1$. Hence, the acceleration is [8]

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v_2 - v_1}{t}. \qquad (1.16.1)$$

It follows from the definition of uniformly accelerated motion that this formula gives the same value of the acceleration irrespective of the time interval t chosen by us. This formula also shows that *the acceleration of*

[7] The increment of a quantity is the difference in its initial and final values. Obviously, the increment can be either positive or negative depending on the nature of the change in the quantity. — *Eds*.

[8] Acceleration is a vector quantity (Sec. 1.23). If we assume that v_1 and v_2 are the velocity projections onto the x-axis (see Footnote 5), the quantity defined by (1.16.1) is the projection of the acceleration vector onto the x-axis. The quantity

$$a = \frac{|v_2 - v_1|}{t}$$

determines the magnitude (i.e. the numerical value) of the acceleration vector (see Footnote 4 in this chapter). — *Eds*.

uniformly accelerated motion is numerically equal to the velocity increment per unit time. The SI unit of acceleration is *metre per second per second* (m/s^2).

If distance and time are measured in other units, the corresponding units will be taken for acceleration. Whatever units are used for distance and time, in the unit of acceleration we have a unit of length in the numerator and the squared unit of time in the denominator. The rule of change-over to other units of length and time is the same as the similar rule for velocities (Sec. 1.11). For example,

$$1 \text{ cm/s}^2 = 36 \text{ m/min}^2.$$

If a motion is not uniformly accelerated, we can introduce the concept of *average acceleration* with the help of the same formula (1.16.1). It characterises the change in velocity over a certain interval of time along the path traversed during this interval. The average acceleration may have different values on different segments of this path (cf. arguments about average velocity in Sec. 1.14).

If we choose such short time intervals that within their limits the average acceleration practically does not change, it will characterise the change in the velocity at any point of this interval. The acceleration obtained in this way is known as *instantaneous acceleration* (normally the word "instantaneous" is omitted, cf. Sec. 1.15). For a uniformly accelerated motion, the instantaneous acceleration is constant and equal to the average acceleration over any interval of time.

1.17. Velocity of Uniformly Accelerated Motion in a Straight Line

Since the acceleration is constant for a uniformly accelerated motion, it is equal to the ratio of the velocity increment over an arbitrary time interval to the duration of this interval. Suppose, for example, that the velocity of a uniformly accelerated motion at the initial instant ("initial velocity") is v_0, and after a time interval t has elapsed, it becomes v. Then the acceleration a can be found from the formula

$$a = \frac{v - v_0}{t}. \qquad (1.17.1)$$

Hence we obtain the formula for velocity:

$$v = v_0 + at. \qquad (1.17.2)$$

If the initial velocity is equal to zero, we have

$$v = at. \qquad (1.17.3)$$

Consequently, if the initial velocity of a uniformly accelerated motion is equal to zero, its velocity is proportional to the time elapsed from the initial moment. The velocity of a ball starting to roll down an inclined plane varies according to this law. The same law (naturally, with a different acceleration) governs the variation of velocity of a freely falling body if its velocity at the initial instant is zero (Sec. 2.26).

The formulas obtained above can be used for calculating the velocity of a body in a uniformly accelerated motion at any instant of time if its initial velocity and acceleration are known. We can also calculate the acceleration from the known initial velocity, the time interval t and the velocity at instant t, and solve some other similar problems.

1.18. The Sign of Acceleration in Rectilinear Motion

In Sec. 1.16, we considered a uniformly accelerated motion (in which velocity increases) and obtained formula (1.16.1) for acceleration. Since $v_2 > v_1$ for an accelerated motion, the acceleration a calculated by this formula was *positive*.

If the velocity decreases with time, the motion is called *decelerated*. In particular, a *uniformly decelerated motion* is one in which velocity decreases by the same value over equal time intervals. A body thrown upwards or a ball pushed up an inclined plane are in a uniformly decelerated motion. The acceleration of such a motion is determined, as in the case of a uniformly accelerated motion, by the ratio of the velocity increment to the time interval over which this increment was gained. Consequently, the acceleration of such a motion is also determined by formula (1.16.1).

For a uniformly decelerated motion, the acceleration calculated by formula (1.16.1) turns out to be *negative* (since $v_2 < v_1$). Consequently, from the sign of the acceleration we can judge whether a motion is uniformly accelerated ($a > 0$) or uniformly decelerated ($a < 0$). [9] The velocity of a uniformly decelerated motion can be calculated by the same formula as for a uniformly accelerated motion:

$$v = v_0 + at, \tag{1.18.1}$$

but acceleration a is negative in this case.

If the initial velocity of a uniformly decelerated motion is positive, it will decrease with time, will become zero, and then will assume negative values. This means that the moving point will come to a halt and then will start to move in the opposite direction. For example, a body thrown upwards stops at a certain moment (the upper point of the ascent) and then

[9] For an accelerated motion in a straight line, the directions of the velocity vector and the acceleration vector coincide. For a decelerated motion, the velocity and acceleration vectors have opposite directions. — *Eds*.

starts to fall down. The moment at which this happens can be found if we know the initial velocity and acceleration by putting v in formula (1.18.1) equal to zero. Suppose, for example, that a body is thrown upwards with a velocity of 5 m/s. We assume that the upward direction is positive. It will be shown later in this book that the acceleration of the body is $a \simeq 10\ \mathrm{m/s^2}$. Consequently, the moment when the body comes to a halt at the upper point of its trajectory is determined from the formula $5 - 10t = 0$, whence $t = 0.5$ s.

Uniformly accelerated and decelerated motions are called *uniformly variable motions*. Sometimes both types of motion are called uniformly accelerated motions bearing in mind that acceleration can be either positive or negative.

1.19. Velocity Graphs for Uniformly Accelerated Motion in a Straight Line

Let us plot the velocity vs. time graphs for a uniformly accelerated motion with the help of formulas of Sec. 1.17. Suppose, for example, that the acceleration is equal to 2 $\mathrm{m/s^2}$ and the initial velocity is zero. Having carried out the construction, we see that the velocity graph is a straight line I passing through the origin (Fig. 30). We can prove that the velocity graph for any uniformly accelerated motion is always a straight line and, conversely, if the velocity graph for some motion is a straight line, the motion is uniformly accelerated (cf. Sec. 1.12). For a higher acceleration, the velocity graph is represented by a straight line II with a larger slope to the time axis.

If the initial velocity differs from zero and is equal to v_0, the velocity graph, as before, is a straight line which passes not through the origin but

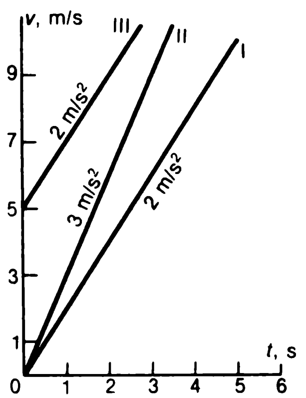

Fig. 30.
Velocity graphs for various uniformly accelerated motions.

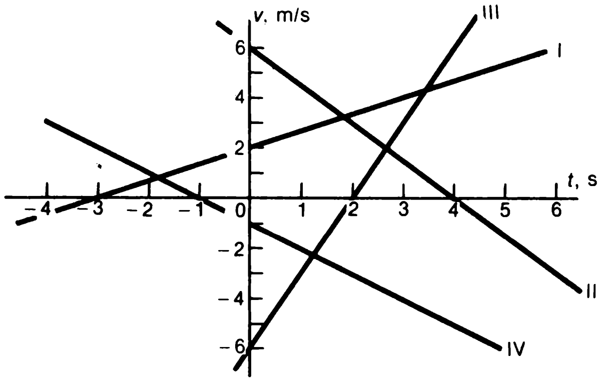

Fig. 31.
Velocity graphs for uniformly accelerated (I, III) and uniformly decelerated (II, IV) motions.

intersects the velocity axis at the point v_0. Figure 30 shows, for example, the velocity graph for a uniformly accelerated motion with the same acceleration of 2 m/s^2 but having an initial velocity of 5 m/s (straight line III). The line has the same slope as the straight line I since the acceleration is the same for the two motions. The slope of the velocity graph also depends on the choice of the time and velocity scales. For this reason, to be able to compare different motions from the form of their velocity graphs, all the graphs must be plotted on the same scale (cf. Sec. 1.12).

For a negative acceleration (uniformly decelerated motion), the velocity graph is also represented by a straight line which now has a negative slope.

Using a velocity graph, we can illustrate all changes in velocity with time for an arbitrary sign of the initial velocity and for an arbitrary sign of the acceleration. For example, straight line I in Fig. 31 corresponds to a positive initial velocity and a positive acceleration, line II to a positive initial velocity and a negative acceleration, line III to a negative initial velocity and a positive acceleration, and line IV to a negative initial velocity and a negative acceleration. The points of intersection of these lines with the time axis correspond to the moments when the velocity changes its sign, i.e. when the direction of motion is reversed. If we are interested only in the numerical value of the velocity and not in its direction, we can say that at these points decelerated motion becomes accelerated. For instance, the numerical value of the velocity of a stone thrown upwards first decreases, and after the upper point has been reached, it starts to increase.

? **1.19.1.** Write the formulas for the x-projections of the velocities of motions represented in Fig. 31.

1.20. Velocity Graph for an Arbitrary Nonuniform Motion

It was shown in Sec. 1.15 that it is possible to plot approximate path graphs for a nonuniform motion by representing this motion as a series of consecutive uniform motions with different velocities. Let us now use this

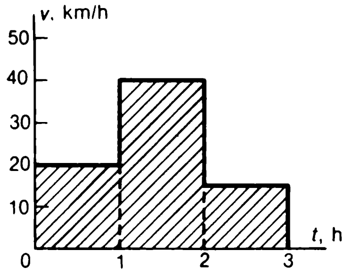

Fig. 32.
Velocity graph of the motion described by the path vs. time graph in Fig. 26.

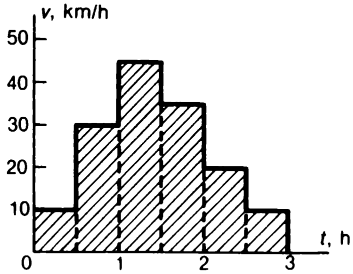

Fig. 33.
Velocity graph for the motion described by the path vs. time graph in Fig. 27.

method for constructing approximate velocity graphs. They will depict average velocities for the time intervals into which we divide a given motion.

For example, from the path graph shown in Fig. 26, we see that the average velocities of the point for the first, second and third hour are equal to 20, 40 and 15 km/h respectively. Assuming that the motion is uniform during each hour (as it was done for plotting the graph), we obtain the velocity graph shown in Fig. 32. Within each hour, the velocity graph is represented by a segment parallel to the time axis (Sec. 1.13). By choosing smaller time intervals, we obtain a new, more accurate velocity graph (Fig. 33) corresponding to a more accurate path graph (see Fig. 27). Here we assume that the motion is uniform during every half an hour. A more accurate velocity graph (Fig. 34) corresponds to a still more accurate path graph (Fig. 28), and so on.

We see that as time intervals decrease, the abrupt changes in the average velocity at the points of transition from one time interval to another become smaller and smaller, and the adjacent steps differ less and less in

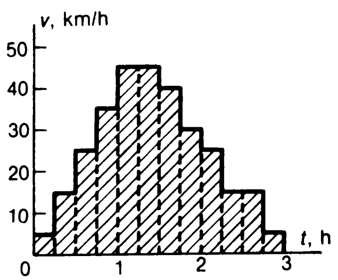

Fig. 34.
The velocity graph corresponding to the motion described by the path vs. time graph in Fig. 28.

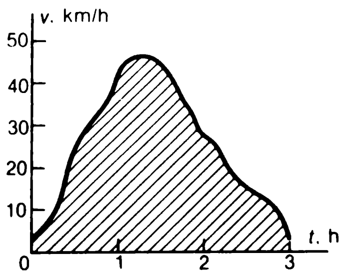

Fig. 35.
The velocity graph for the motion described by the path vs. time graph in Fig. 29.

height. Ultimately, for sufficiently small time intervals, measuring instruments become unable to detect these changes. Then the velocity graph can be represented by a smooth curve instead of a broken line (Fig. 35 corresponding to Fig. 29). This curve gives the values of instantaneous velocity at each instant of time.

1.21. Calculation of the Path Traversed in Nonuniform Motion with the Help of Velocity Graph

It was shown in Sec. 1.13 how the distance covered in a uniform motion can be found with the help of the velocity graph. But how can we determine the distance covered in a nonuniform motion?

Suppose first that the motion is depicted approximately, for instance, as in Fig. 32. Then the areas of rectangles hatched in the figure will represent the paths traversed during the first, second, and third hour respectively. The total area occupied by these rectangles is therefore equal to the total path length. For a more accurate diagram of motion the total path will be determined in the same way, i.e. as the area bounded by the velocity graph (hatched areas in Figs. 33 and 34). Hence we conclude that the area bounded by the graph gives the total distance covered also in the case when the given nonuniform motion is represented exactly, i.e. by a smooth curve (Fig. 35).

The distance covered over any time interval is numerically equal to the area bounded by the time axis, the velocity graph and the vertical segments corresponding to the beginning and end of the given time interval. Thus, the conclusion drawn at the end of Sec. 1.13 for the special case of uniform motion turns out to be valid for the general case of an arbitrary nonuniform motion as well.

1.22. Distance Covered in a Uniformly Variable Motion

We shall use the graphical method for determining the path length for a uniformly accelerated motion. Let the velocity graph for such a motion be represented by straight line BC (Fig. 36). The distance covered during the time $t = OA$ is numerically equal to the area of the trapezium $OBCA$:

$$s = \text{area of } OBCA = \frac{OB + AC}{2} OA.$$

But $OB = v_0$ (initial velocity), $AC = v_0 + at$ (velocity at the moment t) and a is the acceleration. Consequently,

$$s = \frac{v_0 + (v_0 + at)}{2} t = v_0 t + \frac{at^2}{2}. \qquad (1.22.1)$$

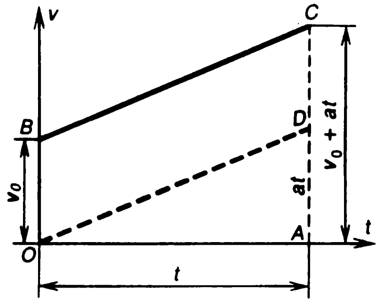

Fig. 36.
Graphical derivation of the formula for a distance covered in a uniformly accelerated motion.

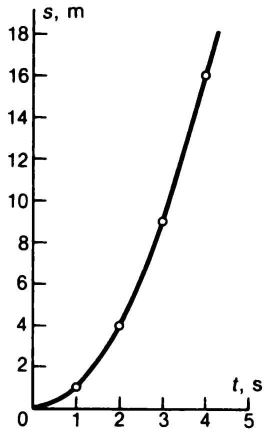

Fig. 37.
A path vs. time graph for a uniformly accelerated motion.

This formula is valid both for uniformly accelerated and decelerated motions. In the former case, v_0 and a have the same sign, while in the latter case their signs are opposite. [10]

If the initial velocity of motion is zero, the graph is not a trapezium but the right triangle ODA with the legs $OA = t$ and $AD = v = at$. Thus, the formula expressing the path length now has the form

$$s = \frac{at}{2}t = \frac{at^2}{2}. \tag{1.22.2}$$

This formula could be directly obtained from (1.22.1) by putting $v_0 = 0$.

Figure 37 shows the path graph for a uniformly accelerated motion with zero initial velocity. It corresponds to formula (1.22.2) for $a = 2\ \text{m/s}^2$. It is a curve which becomes steeper and steeper with time. The distance from the points of the graph to the time axis is proportional to the squares of the distances to the path axis. Such a curve is known as *parabola*.

Formula (1.22.2) shows that when the initial velocity is zero, the distance covered by a body in the first second of a uniformly accelerated motion ($t = 1$ s) is numerically equal to half acceleration. If we know the path traversed over time t with zero initial velocity, the acceleration can be

[10] Strictly speaking, formulas (1.22.1) and (1.22.2) determine the x-coordinate of a moving point at time t rather than the distance s covered by it. If $v_0 > 0$ and $a > 0$, the values of s and the x-coordinate coincide. If $v_0 > 0$ and $a < 0$, formula (1.22.1) gives the path length traversed to the moment at which the sign of the velocity is reversed (i.e. the velocity reverses its direction). — *Eds.*

determined from the formula

$$a = 2s/t^2. \qquad (1.22.3)$$

If the initial velocity v_0 is zero, the distance s covered by the moment t can be expressed in terms of velocity v at this moment, or vice versa, the velocity can be expressed in terms of the distance. Indeed, in this case, $v = at$ and $s = at^2/2$. Eliminating t from these expressions, we obtain

$$s = v^2/2a, \qquad (1.22.4)$$

$$v = \sqrt{2as}. \qquad (1.22.5)$$

Finally, if we know the distance traversed and acceleration, we can determine the time of motion by using formula (1.22.2):

$$t = \sqrt{2s/a}. \qquad (1.22.6)$$

The laws governing a uniformly accelerated motion were established for the first time by Galileo who studied (in 1638) the motion of a ball along an inclined trough. At that time, there were no sufficiently accurate watches, and Galileo measured the time of motion with the help of a sort of clepsydras by weighing water flowing out of a vessel through a small hole. Galileo let the ball roll down the inclined trough (with zero initial velocity) and measured the path lengths over time intervals corresponding to a certain amount of water flowing out of the vessel. In spite of the imperfection of the measuring technique, Galileo managed to establish that the distance covered by the ball is proportional to the square of the time during which this path was traversed.

? **1.22.1.** Write formulas similar to (1.22.4) and (1.22.5) for the case when the initial velocity v_0 differs from zero.

1.22.2. Using formula (1.22.1), show that the distances covered by a point in a uniformly accelerated motion during any equal successive time intervals acquire the same increment.

1.22.3. Using formula (1.22.2), show that for a uniformly accelerated motion with zero initial velocity the increment of the distance covered in two successive equal time intervals is equal to twice the distance covered during the first of these time intervals.

1.22.4. An electric locomotive moves along a horizontal road and approaches a slope at a velocity of 8 m/s. Then it moves down the slope with an acceleration of 0.2 m/s^2. Determine the length of the slope if the locomotive has traversed it in 30 s.

1.22.5. An electric locomotive starts off with a uniform acceleration as a boy running uniformly at a velocity of 2 m/s passes by it. Determine the velocity of the locomotive at the moment it catches up with the boy.

1.22.6. A motor car moving with a constant acceleration has reached a velocity of 20 m/s over a certain distance from the starting point. What was its velocity at the middle of this distance?

1.22.7. Find the path traversed by a body over the time during which its velocity has increased from 4 to 12 m/s at an acceleration of 2 m/s^2.

1.23. Vectors

So far, we have considered only the motion of a point along a given straight line. For determining the displacement of the point in this case, it is sufficient to know its initial position, direction of motion and the path length. Similarly, knowing the initial position of the point, the numerical value of its velocity and its sign, we can specify the position of the point in one, two, etc. seconds.

If, however, the point does not move in a straight line, these data are insufficient. Let us watch the motion of an aeroplane (flying at the same altitude) on a map. Suppose, for instance, that the plane has moved from point A to point B (Fig. 38). The segment AB is the displacement of the plane. Knowing the previous position of the body and its displacement, we can determine its new position. However, unlike the motion in a straight line, we must now determine not only the length of segment AB, but the direction in which this displacement took place in space. For displacement in another direction, the plane would be at some other point even if the displacement is the same (for instance, at point M separated from A by the same distance as point B). This means that ***displacement is characterised not only by its numerical value but also by its direction in space.***

Similarly, the velocities and accelerations of bodies should be characterised by numerical values as well as by directions in space. In physics, one often deals with quantities which are characterised not only by numerical values but also by their direction in space (like velocity and acceleration). It will be shown that forces of interaction between bodies and the electric field strength are quantities of this type.

The quantities which are characterised by the numerical value and

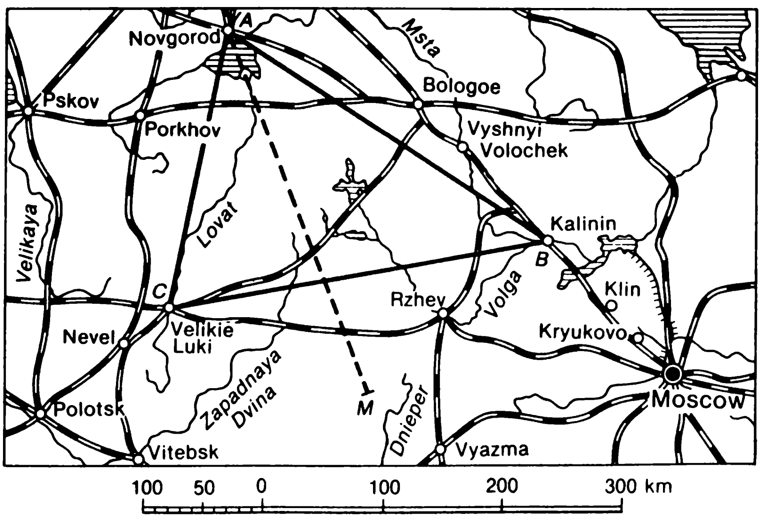

Fig. 38.
Displacements which do not lie on the same straight line. Composition of displacements.

direction in space are called vectors. Thus, displacement, velocity and acceleration are vectors.

The numerical value of a vector is called its *magnitude* (*absolute value* or *modulus*). The magnitude of a vector is always *positive*. In figures, vectors are represented as segments of a straight line with an arrow at the end. The length of the segment determines the magnitude of the vector on a given scale, while the arrow indicates its direction. Vectors are denoted either by boldface letters (**a**, **A**) or by an ordinary letter with an arrow over it ($\vec{a}, \vec{A}$), or, finally, by two letters with an arrow ($\overrightarrow{AB}, \overrightarrow{BC}$), where the first letter denotes the beginning and the second letter the end of the segment representing a vector. The magnitudes of vectors are denoted by the same letters as the vectors but with normal type face and without arrows (a, A, AB, BC), or with the help of the symbol of vector enclosed between vertical lines ($|\mathbf{a}|$, $|\mathbf{A}|$).

In contrast to vectors, the quantities which are characterised by numerical values and which cannot be ascribed any direction in space are known as *scalar quantities* or *scalars*. Time, density of a substance, volume of a body, temperature, and distance (but not displacement!) are scalars. Scalar quantities are equal if their numerical values coincide. Vectors are equal if they coincide in magnitude and direction.

Suppose that a body performed two displacements in succession. For example, an aeroplane first traversed a path represented by vector $\overrightarrow{AB}$ and then the path shown by $\overrightarrow{BC}$ (Fig. 38). The resultant displacement will be depicted by vector $\overrightarrow{AC}$. It is called the sum of the two given displacements. It can be seen that the sum of two displacements is obtained as the side of the triangle whose two other sides are the component displacements. Such a summation rule is called the *vector sum rule*, or the *triangle rule for vector addition* (Fig. 39*a*). Hence it follows that the magnitude of the sum of two vectors is not equal to the sum of the magnitudes of the component vectors in the general case: the magnitude of the sum lies between the sum and the difference of the magnitudes of the components. Only if the vectors being added lie on the same straight line, the magnitude of their sum is equal to the sum of the magnitudes of these vectors (if they have the same direction) or to the absolute value of their difference (if they are directed against each other).

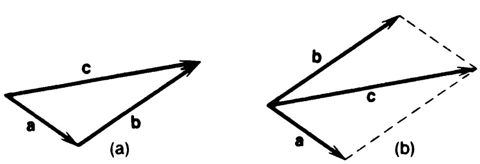

Fig. 39.
Addition of two vectors: (*a*) according to the triangle rule and (*b*) according to the parallelogram rule.

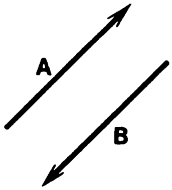

Fig. 40.
Two vectors differing only in sign: $\mathbf{A} = -\mathbf{B}$.

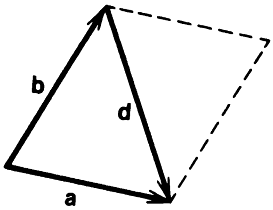

Fig. 41.
Vector subtraction: $\mathbf{d} = \mathbf{a} - \mathbf{b}$.

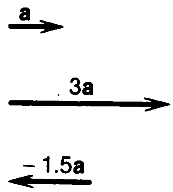

Fig. 42.
Multiplication of a vector by a number.

Vectors can also be added according to the *parallelogram rule* which is equivalent to the triangle rule. While constructing the parallelogram, the two vector components are laid off from a single point and form the sides of the parallelogram. Then the diagonal of the parallelogram drawn from the same point gives the resultant vector.

Vectors having opposite directions are ascribed opposite signs. Figure 40 depicts vectors that are equal in magnitude and differ in direction, i.e. differ in sign: $\mathbf{A} = -\mathbf{B}$.

Subtraction of vectors can be defined in a similar way: to subtract a vector means to add the vector of the same magnitude and opposite direction. One of the diagonals of the vector parallelogram is the sum of the vectors represented by its sides, while the other diagonal represents their difference (Fig. 41).

If more than two vectors are added (for instance, if a body performs more than two successive displacements), the sum of the vectors (the resultant displacement) is obtained by successive addition of the second vector to the first, the third vector to their sum, and so on. If a given displacement is repeated twice, thrice, etc., the resultant displacement has the same direction as the vector of a single displacement, but its magnitude is twice, thrice, etc. as large as the magnitude of a single displacement. Thus, we can introduce the *multiplication of a vector by a number* (*scalar*): a vector multiplied by a number (scalar) is a vector having the same direction if the number (scalar) is positive and the opposite direction if the number (scalar) is negative. The magnitude of the resultant vector is equal to the magnitude of the initial vector multiplied by the absolute value of the number (scalar). Figure 42 represents vectors $\mathbf{a}$, $3\mathbf{a}$ and $-1.5\mathbf{a}$.

? **1.23.1.** Prove that the following laws can be applied to displacements: commutative law ($\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$), associative law ($\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$), and distributive law for the multiplication by a scalar ($m(\mathbf{a} + \mathbf{b}) = m\mathbf{a} + m\mathbf{b}$).

1.24. Decomposition of a Vector into Components

Any vector can be represented as the sum of several vectors. For example, a displacement of a body can be represented as several consecutive displacements as a result of which the body is transferred from the initial to the final state. The replacement of a vector by the vector sum of several other vectors is called the *decomposition of the vector into components*. Naturally, the components of a vector are also vectors. A vector can be decomposed into components in an infinite number of ways. For instance, a vector can be decomposed along two given directions. In this case, the vector being decomposed serves as the diagonal of the parallelogram whose sides coincide with the given directions of the components (Fig. 43).

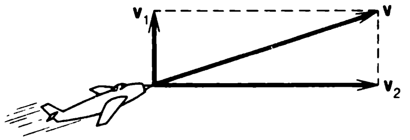

Fig. 43.
Decomposition of the velocity of an aeroplane gaining altitude into the vertical and horizontal components.

If the direction of only one component is specified, the problem on the decompositon of the vector has no definite answer. Figure 44 shows that an infinitely large number of parallelograms with a given diagonal (the vector being decomposed) and a given direction of a side (the direction of one of the components) can be constructed.

? **1.24.1.** An aeroplane is to land at a point A 300 km south-west of the airport of departure, but before that it must drop a message container at an airport B 400 km south-east of the airport of departure. Determine the magnitude of displacement $\overrightarrow{AB}$.

Vectors are frequently decomposed along the directions of axes in a rectangular system of coordinates (Fig. 45*a*). Figure 45*b* represents a vector **a** (or $\overrightarrow{AB}$). We drop perpendiculars to the x- and y-axes from points A and B. The point of intersection of a perpendicular with an axis is called the *projection* of the corresponding point (A or B) on the given axis (Ox or

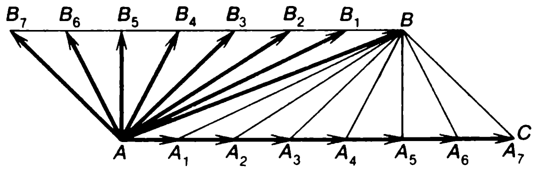

Fig. 44.
The decomposition of vector $\overrightarrow{AB}$ for which only the direction $\overrightarrow{AC}$ of one component is specified. Vector $\overrightarrow{AB}$ can be represented as the sum of vectors $\overrightarrow{AA_1}$ and $\overrightarrow{AB_1}$, $\overrightarrow{AA_2}$ and $\overrightarrow{AB_2}$, $\overrightarrow{AA_3}$ and $\overrightarrow{AB_3}$, and so on.

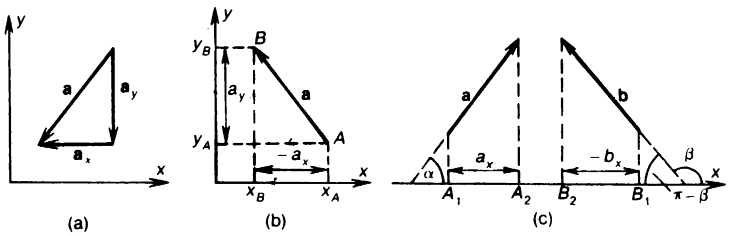

Fig. 45.
(*a*) An example of decomposition of a vector into components parallel to the coordinate axes; (*b*) and (*c*) projections of a vector on coordinate axes.

Oy). The coordinates of these projections are shown in the figure. The difference $x_B - x_A$ is denoted by a_x and is known as the *projection of vector* **a** on the x-axis. Similarly, the difference $y_B - y_A$ is denoted by a_y and is called the *projection of vector* **a** on the y-axis. The projections are called the *components* of the vector along the coordinate axes (a_x is the component of vector **a** along the x-axis, and so on). The projections (components) are scalars.

For the vector shown in Fig. 45*b*, $x_B < x_A$. As a result, the x-projection of this vector is negative ($a_x < 0$). Since $y_B > y_A$, the y-projection of the vector is positive ($a_y > 0$). Figure 45*b* shows the lengths of the segments enclosed between the projections of the beginning and end of the vector on the axes. These lengths must be expressed by positive numbers. For this reason, the value of the length of the segment contained between the projections of points A and B on the x-axis has the form $-a_x$ (since $a_x < 0$, $-a_x > 0$). Note that the x-projection of vector **a** shown in Fig. 45*c* is positive, while the projection of vector **b** is negative.

Let us give another definition of the projection of a vector. Figure 45*c* shows vectors **a** and **b** and their projections on an arbitrary x-axis. The projection of vector **a**, a_x, is equal to the length of the segment A_1A_2 taken with the plus sign (since $a_x > 0$), while the projection of vector **b**, b_x, is equal to the length of the segment B_2B_1 taken with the minus sign (since $b_x < 0$). It should be recalled that the length of the segment B_2B_1, which is expressed by a positive number equal to $-b_x$, is indicated in the figure.

Figure 45*c* shows that the length of the segment A_1A_2, a_x, is equal to the length of the segment representing vector **a** (i.e. the magnitude of vector **a**) multiplied by the cosine of the angle α between the direction of the x-axis and the direction of the vector. Consequently, $a_x = a \cos \alpha$. The length of the segment B_2B_1 is equal to the length of the segment representing vector **b** (i.e. the magnitude of vector **b**) multiplied by the cosine of the angle $\pi - \beta$. The projection of vector **b** is equal to this length taken with the minus sign. Consequently, $b_x = -b \cos (\pi - \beta) = b \cos \beta$.

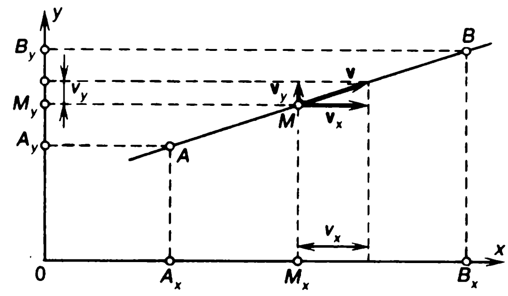

Fig. 46.
Projecting the motion of point M on coordinate axes.

Thus, irrespective of the angle formed by the direction of the vector with the direction of the x-axis, the projection of the vector on this axis is defined by the formula

$$a_x = a \cos \alpha. \tag{1.24.1}$$

If $\alpha < \pi/2$, then $a_x > 0$, and if $\alpha > \pi/2$, then $a_x < 0$. When $\alpha = \pi/2$, the projection of the vector on the axis is zero.

Obviously, the magnitude and direction of a vector (and hence the vector itself) are completely determined by the projections of the vector on the coordinate axes.[11] In particular, for vectors lying in the (x, y)-plane, the magnitude is defined by the formula $a = \sqrt{a_x^2 + a_y^2}$. The "lengths" and signs of the projections determine the direction of the vector.

Let a point move in a straight line. We choose a coordinate system xOy and project the moving point on the coordinate axes (Fig. 46). Figure shows the projections M_x and M_y of the point which occupies a position M at a given instant of time. As the point moves, its projections move as well. If the point M performs a displacement AB, its projections perform displacements A_xB_x and A_yB_y over the same time along the corresponding axes. The figure shows that the *projections of the displacement* of the moving point M *are equal to the displacements of its projections* M_x and M_y along the coordinate axes. If the point moves uniformly, the projections are also in the uniform motion. Dividing the displacements of the point and of its projections by the time t of motion, we obtain the velocities v, v_x and v_y of the point M and its projections M_x and M_y.

It can be shown that the *projection of the velocity of a point is equal to the velocity of motion of its projection*. Similarly, we can show that if a point moves in a straight line nonuniformly, the projections of its instantaneous velocity and acceleration are equal to the instantaneous velocities and accelerations of its projections. Conversely, if we know the displacements, velocities or accelerations of the projections of a moving point on the coordinate axes, we can find the displacement, velocity or acceleration by adding up the obtained components of the required vector with the help of the parallelogram rule.

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[11] We consider here *free* vectors, i.e. the vectors which can move arbitrarily, remaining parallel to themselves. — *Eds.*

Thus, instead of analysing the motion of a point in an arbitrary direction, we can always consider the motion only along definite straight lines, viz. coordinate axes. In some cases, the choice of axes is dictated by the conditions of the problem. For instance, while studying the motion of a thrown body, it is convenient to choose the vertical and horizontal coordinate axes.

1.25. Curvilinear Motion

If a point moves along a curvilinear trajectory, we shall, as before, call the segment connecting its initial and final positions the displacement of the point. The displacement in this case does not lie on the path as in the case of a rectilinear motion (Fig. 47). Nevertheless, for a curvilinear motion too the trajectory can be marked out and individual positions of the moving point can be correlated with the corresponding instants of time. The path length should only be measured along the curvilinear trajectory (as it is shown in the figure) and not along a straight line.

The magnitude of velocity in curvilinear motion is defined in the same way as in motion along the straight line, viz. as the ratio of the path traversed by the point along its trajectory over a sufficiently short time interval to the duration of this interval. As long as we are speaking about the *magnitude* of the velocity and the path length, we can introduce the same concepts of uniform and nonuniform (in particular, uniformly variable) motion along a curvilinear trajectory as for motion in a straight line. We can use the same formulas for calculating the path length and the magnitude of velocity as for rectilinear motion. The difference appears only when we take into account the direction of motion.

1.26. Velocity of Curvilinear Motion

Which direction should be ascribed to the velocity of a curvilinear motion? In this case, there is no definite direction of motion. This question can be

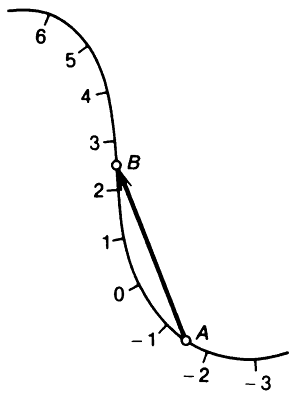

Fig. 47.
Marking of a curvilinear trajectory. The displacement $\vec{AB}$ of the point between its positions A and B does not lie on the trajectory.

answered if we introduce the concept of *instantaneous direction of velocity* in the same way as we introduce the concept of instantaneous velocity of rectilinear motion in Sec. 1.15.

For this purpose, we shall analyse a curvilinear motion over short time intervals. The shorter the intervals we choose, the smaller is the difference between the corresponding small segment of the path and a rectilinear segment, say, its chord. If the time interval is sufficiently small, the given motion will be indistinguishable from rectilinear motion. Besides, for a sufficiently small path length, the chord will be practically indistinguishable from the tangent drawn at any point of this segment of the trajectory. For this reason, for the instantaneous direction of the velocity we take the direction of the tangent at the point of the trajectory where the moving body is at a given instant of time. Normally, the word "instantaneous" is omitted, when the direction of velocity is considered.

The particles of a rotating grindstone move in circles. If we touch the rotating grindstone with the end of a steel rod (Fig. 48), we shall see sparks, viz. tiny incandescent particles separated from the grindstone and flying at a velocity they had at the last moment of their motion together with the grindstone. Moving the steel rod over the circumference of the grindstone, we see that the directions in which sparks fly are different at different points, but they always coincide with tangents to the circle at the points where the rod touches the stone.

? **1.26.1.** To prevent the splashes of bicycle tyres from getting on the cyclist, flaps in the form of arcs of a circle with the centres at the wheel axle are mounted above the wheels. Draw schematically a bicycle with a cyclist and mark on your drawing the smallest size of the flaps for which the cyclist will be protected from splashes.

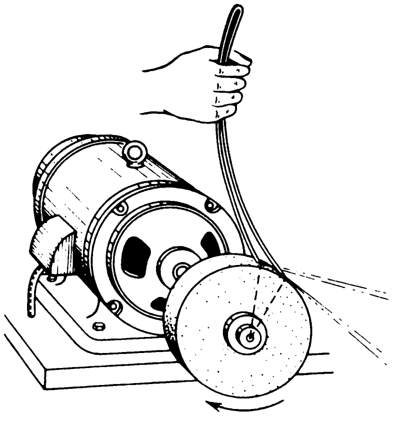

Fig. 48.
The sparks from an object ground on a grindstone fly along tangents to the circle.

1.27. Acceleration in Curvilinear Motion

Analysing the curvilinear motion of a body, we see that its velocity is different at different instants of time. Even if the magnitude of the velocity remains unchanged, its direction does change. In the general case, both magnitude and direction of velocity vary.

Thus, in a curvilinear motion the velocity continuously changes, which means that the body moves with an *acceleration*. To determine this acceleration (its magnitude and direction), we have to find the change of velocity *as a vector*, i.e. to determine the increment of the magnitude of velocity and the change in its direction.

Suppose that a point in a curvilinear motion (Fig. 49) has a velocity $\mathbf{v}_1$ at a certain instant of time and velocity $\mathbf{v}_2$ after a short time interval. The velocity increment is the difference between the vectors $\mathbf{v}_2$ and $\mathbf{v}_1$. Since these vectors have different directions, we must find their vector difference. The increment of the velocity will be expressed by the vector $\Delta\mathbf{v}$,[12] which is represented by the side of a parallelogram whose diagonal is $\mathbf{v}_2$ and the other side is $\mathbf{v}_1$. The acceleration $\mathbf{a}$ is the ratio of the velocity increment to the interval of time t over which this increment was gained:

$$\mathbf{a} = \frac{\Delta\mathbf{v}}{t}.$$

The direction of $\mathbf{a}$ coincides with that of vector $\Delta\mathbf{v}$.

By choosing a sufficiently small t, we arrive at the concept of *instantaneous acceleration* (cf. Sec. 1.16). For an arbitrary t, vector $\mathbf{a}$ is the average acceleration over the time interval t.

The direction of acceleration in curvilinear motion does not coincide with the direction of velocity as in the case of rectilinear motion where these directions either coincide or are opposite. In order to find the direction of acceleration in a curvilinear motion, it is sufficient to compare the directions of the velocities at two close points on the trajectory. Since the velocities are directed along the tangents to the trajectories, the direction of

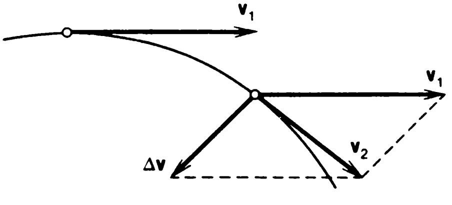

Fig. 49.
The change in velocity of a curvilinear motion.

[12] The Greek capital delta Δ denotes the increment of a scalar or vector quantity. For example, $\Delta A = A_2 - A_1$ is the increment of the magnitude of vector $\mathbf{A}$, $\Delta\mathbf{A} = \mathbf{A}_2 - \mathbf{A}_1$ is the increment of vector $\mathbf{A}$. — *Eds*.

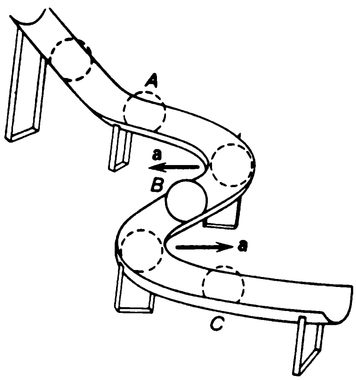

Fig. 50.
Acceleration in a curvilinear motion is always directed away from the concave segment of the trajectory.

acceleration can be determined from the shape of the trajectory. Indeed, since the difference $\mathbf{v}_2 - \mathbf{v}_1$ of the velocities at two close points is always directed towards the bending of the trajectory, it means that acceleration is always directed away from the concavity of the trajectory. For example, when a ball rolls along a bent trough (Fig. 50), its acceleration on segments *AB* and *BC* is directed as shown by the arrows irrespective of whether the ball moves from *A* to *C* or in the opposite direction.

Let us consider the uniform motion of a point along a curvilinear trajectory. As was mentioned above, it is an accelerated motion. Let us determine the acceleration. For this purpose, it is sufficient to consider the acceleration for the special case of uniform motion in a circle. We take two close positions *A* and *B* of the moving point, separated by a short time interval t (Fig. 51*a*). The velocities of the moving point at *A* and *B* are equal in magnitude but have different directions. We shall determine the difference in these velocities with the help of the triangle rule (Fig. 51*b*). Triangles *OAB* and *O′A′B′* are similar as isosceles triangles with equal angles at vertices. The length of the side *A′B′* representing the increment of velocity over the time interval t can be put equal to at, where a is the

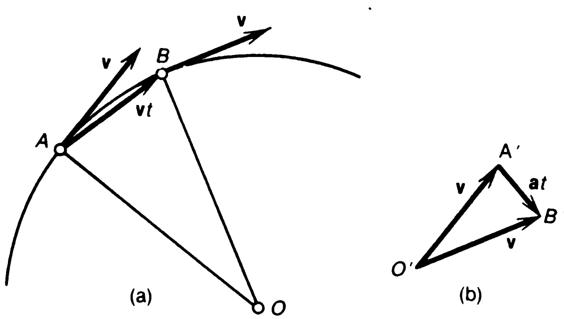

Fig. 51.
To the derivation of the formula for centripetal acceleration.

magnitude of the required acceleration. The side AB analogous to it is the chord of the arc AB. Since the arc is small, the length of its chord is approximately equal to the arc length, viz. vt. Further, $O'A' = O'B' = v$, $OA = OB = R$, where R is the radius of the trajectory. It follows from the similarity of the triangles that the ratios of their analogous sides are equal:

$$\frac{at}{vt} = \frac{v}{R} .$$

Hence we can find the magnitude of the required acceleration:

$$a = \frac{v^2}{R} . \qquad (1.27.1)$$

The direction of acceleration is normal to the chord AB. For sufficiently short time intervals, we can assume that the tangent to the arc virtually coincides with its chord. This means that acceleration can be assumed to be perpendicular (normal) to the tangent to the trajectory, i.e. along the radius to the centre of the circle. For this reason, such an acceleration is called *normal*, or *centripetal acceleration*.

If the trajectory differs from a circle and is an arbitrary curve, we must take in formula (1.27.1) the radius of the circle which is the closest approximation of the curve at a given point. The direction of the normal acceleration in this case is perpendicular to the tangent to the trajectory at the given point. If the acceleration of a curvilinear motion is constant in magnitude and direction, it can be defined as the ratio of the velocity increment to the time interval during which this increment was gained irrespective of the duration of this interval. Hence, in this case the acceleration can be found from the formula

$$\mathbf{a} = \frac{\mathbf{v} - \mathbf{v}_0}{t} , \qquad (1.27.2)$$

which is similar to (1.17.1) for rectilinear motion at a constant acceleration. Here $\mathbf{v}_0$ is the velocity of a body at the initial instant and $\mathbf{v}$ is the velocity at moment t.

1.28. Motion in Different Reference Systems

In Sec. 1.2 it was explained that the same motion of a body can be different depending on the system to which this motion is referred. Let us consider the case when one of the reference systems is in translatory motion relative to another system. Clearly, the second reference system is also in translatory motion relative to the first one.

By way of example, we take for such systems the Earth and a railway flat-car moving along a straight segment of the path. Let a man walk along

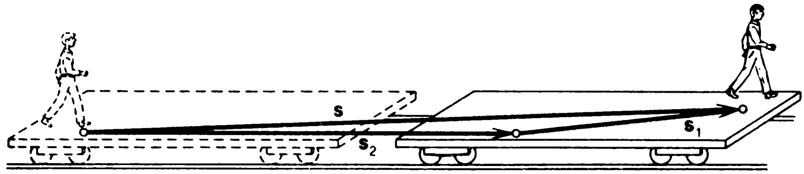

Fig. 52.
Composition of displacements in motions relative to different reference systems.

the flat-car. How can we determine the motion of the man relative to the Earth if we know his motion relative to the flat-car and the motion of the flat-car relative to the Earth?

If we represent the displacement of the man relative to the flat-car by vector $\mathbf{s}_1$ and the displacement of the flat-car relative to the Earth by vector $\mathbf{s}_2$, it can be seen from Fig. 52 that the displacement of the man relative to the Earth will be represented by vector $\mathbf{s}$ which is the diagonal of the parallelogram formed by vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ as its sides. This means that the following vector equality holds:

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2. \tag{1.28.1}$$

Similarly, we can determine the displacement of a body in other cases. It can be shown that as we go over from one reference system to another, *the displacement of the body and the displacement of the reference system are added as vectors*.

If the motion of a man relative to the flat-car and the motion of the flat-car relative to the Earth are uniform and rectilinear, the motion of the man relative to the Earth will also be a uniform motion in a straight line. In this case, dividing both sides of (1.28.1) by the time interval t during which the displacements occurred, we obtain

$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2, \tag{1.28.2}$$

where $\mathbf{v}_1$ is the velocity of the man relative to the flat-car, $\mathbf{v}_2$ is the velocity of the flat-car relative to the Earth, and $\mathbf{v}$ is the velocity of the man relative to the Earth. This means that *the velocity of the body and the velocity of the reference system are also added as vectors* in this case.

It can be proved that formula (1.28.2) is valid for nonuniform motions as well if we assume that $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}$ are the instantaneous velocities of the body and of the reference system.

If the flat-car moves uniformly in a straight line, then irrespective of the motion of the man along it, his velocity relative to the Earth differs from his velocity relative to the flat-car only by a constant addend ($\mathbf{v}_2$). This means that all *the changes in the velocity* of the man will be the same in the two systems, and hence the accelerations of the man in both systems will be identical.

Thus, *if two reference systems are in translatory, uniform and rectilinear motion relative to each other, the accelerations of bodies relative to both reference systems are the same. Naturally, the velocities of motion of the bodies relative to these systems will be different.*

?

1.28.1. Prove that, if a man is in a rectilinear nonuniform motion relative to a flat-car and the flat-car moves uniformly in a straight line relative to the Earth, the motion of the man relative to the Earth can be curvilinear.

1.28.2. A swimmer covers 3 km in three hours in still water, and a log covers 1 km downstream during the same time. What distance will be covered by the swimmer upstream in the same time?

1.28.3. A steamer covers the distance between locations A and B in 2 h downstream and in 3 h upstream. What time will it take for a log to float from A to B?

1.28.4. A boat covers a certain distance downstream in one third of the time taken by it to cover the same distance upstream. What is the ratio of the velocities of the boat and of the river flow?

1.28.5. A train passes by a telegraph post in 15 s, and through a 450-m long tunnel in 45 s. When it meets another train which is 300 m long, they pass each other in 21 s. Find the velocity of the second train.

1.28.6. A crawler tractor moves at a velocity of 5 m/s. What is the velocity of the (a) upper and (b) lower parts of the crawler relative to the Earth? What are the velocities of these parts relative to the tractor?

1.28.7. A motorboat develops a velocity of 10 km/h in still water. The velocity of the river flow is 5 km/h. What time does it take for a boat to traverse a path of 10 km upstream and to return to the initial position downstream?

1.29. Kinematics of Motion in Outer Space

It was mentioned above that to describe the motion of a point we must measure the path length traversed by the point along its trajectory and "correlate" each position of the point with the corresponding instant of time. Naturally, while studying the motion of a spacecraft or celestial bodies like planets, the Moon and stars, a direct marking out of trajectories is ruled out. The only method of measuring distance to or from a spacecraft (and in general of determining its position) is the transmission of signals that can propagate in space, viz. light and radio signals. For instance, a spacecraft or a planet can be observed through a telescope, with the help of a radar or with the help of signals received from the spacecraft.

In principle, there is no difference in these observations in comparison with those carried out on Earth, where we also make use of light signals (for observing moving objects with naked eye or for photographing them) and radio signals (for radar observations). However, there is an important quantitative difference between observations within the limits of terrestrial distances and observations over huge distances in space. Indeed, since it takes a certain time for every signal to propagate from a moving body to an observer, the moving body turns out to be in a different place at the instant of observation: *the moment of observation of an event lags behind the mo-*

ment when this event took place by the time required for the signal to propagate from the moving body to the observer.

True, the velocity of light and radio signals is so high that the displacement of the body during the time of propagation of the signal is small in comparison with the distance to the body. For example, if we could see a bullet flying at a velocity of 800 m/s over a distance of 1 km without taking into account the lag in the light from the bullet, we would make an error of 3 mm in determining the position of the bullet. Celestial bodies, however, can move for very large distances from an observer, which involves a large increase in the error. For example, the error introduced by the disregard of the time of propagation of light or a radio signal for a spacecraft moving away from the Earth with the same velocity of 800 m/s and reaching the Jupiter orbit (at the minimum separation between the Earth and Jupiter) would amount to 1700 km!

Thus, for large distances we cannot ignore the time of signal propagation. For instance, if we have to send a command (say, to switch on engines) to a spacecraft at the moment when it occupies a certain position relative to celestial bodies, the command should be sent with a lead equal to the time lag of the signal. Besides, the same time lag should be naturally taken into account while determining the position of the spacecraft. For the case of the spacecraft reaching the Jupiter orbit considered above, the time lag of the signal and the required lead must be 2100 s. Clearly, the time lag will be the larger, the longer the distance between the spacecraft and the Earth. For example, for a spacecraft reaching the Pluto orbit the required lead would amount to 20 000 s, while the error in determining the position, introduced by the disregard of the time lag of the signal, would now reach 16 000 km.

Under terrestrial conditions, the time lag of radio signals is used in radar observations. A radar sends a high-power radio signal in the direction in which the appearance of a target is being expected. The target can be an aeroplane, a rocket, a rain cloud or a meteor in the atmosphere — in general, any object capable of reflecting a radio signal. The signal reflected from the object is received by the radar. A special instrument measures the time elapsed between the moment of transmission of the signal and its reception. Since the signal had to traverse the distance between the radar and the target twice, the distance to the target is obviously equal to half the time interval between the signal transmission and reception multiplied by the velocity of the radio signal. The moment of reflection of the signal from the target is equal to the half-sum of the moments of transmission and reception of the signal.

By the time the signal is received by the radar, the target has moved (relative to the position in which the signal reached the target) by the distance equal to the distance to the target multiplied by the ratio of the velocities of the target and the signal. For instance, if an aeroplane flying at a velocity of 2000 km/h is detected at a distance of 1000 km, it will be displaced by 2 m while the signal returns from it to the radar.

For the first time, the velocity of light in space was measured with the help of the time lag of a light signal, coming from a long distance, relative to the moment when the signal was sent. At the end of the 17th century, the Danish scientist O. Roemer, who observed the eclipses of a Jupiter satellite, noted that when the Earth approaches the Jupiter in its yearly motion about the Sun the time intervals between eclipses decrease in comparison with those corresponding to the motion of the Earth away from the Jupiter. He explained this difference by the fact that the time lag of the events occurring near the Jupiter (eclipses of its satellite) decreases when the Earth approaches the Jupiter and increases when the Earth moves away from it. The total difference in the time lag must be equal to the time required for light to cross the diameter of the Earth's orbit. The velocity of light is thus equal to the diameter of the Earth's orbit divided by the maximum difference in the time lags of the observed eclipses. Roemer's method is described in greater detail in Vol. 3 of this book.

It follows from what has been said above that while "correlating" the observed position of a spacecraft (or a celestial body) with the corresponding instant of time, one should refer the position (say, observed through a telescope) not to the moment of observation but to an earlier instant, taking into account the time lag of the signal. This explains the importance of the knowledge of the velocity of light or radiowaves for analysing the motions of objects in space (spacecraft, planets, stars, etc.). The farther the object, the more important it is to take into account the time of light propagation. We see distant stars not in the position they occupy today but at the sites they occupied years or thousand years back. On the other hand, the time lag is small for "terristrial" motions: light would take only 0.13 s to travel even along the equator of the globe.

However, there are motions on the Earth for which we must also take into account the time of propagation of light to "correlate" the positions of a body with the moments of time. We are speaking of motions with velocities comparable with the velocity of light. Elementary particles may have velocities quite close to the velocity of light. Obviously, we must take into account the time of propagation of a light signal for determining the positions of such particles, since their displacements are very large even for a very short time. Ordinary bodies like aeroplanes, rockets or projectiles (if we speak of the fastest moving large objects) move so slowly in comparison with a light signal that the correction to their motion remains small as long as the distances are short enough.

Chapter 2
Dynamics

2.1. Problems of Dynamics

In the previous chapter we did not consider the causes of motions of bodies. We shall now analyse these causes. The branch of mechanics dealing with these problems is known as *dynamics*.

Every motion is of relative nature (see Secs. 1.2 and 1.28). The same motion is completely different in different reference systems and so are the reasons behind it. In some reference systems, the causes of motion are especially simple. Such systems include, for example, the Earth. Therefore, we shall begin the study of dynamics by taking the Earth as the system of reference.

2.2. Law of Inertia

Observations and experience show that bodies acquire acceleration relative to the Earth, i.e. change their velocity relative to the Earth, only when acted upon by other bodies. When a body acquires an acceleration relative to the Earth, we can always specify the body causing this acceleration. For example, if we throw a ball, it is set in motion, i.e. it acquires an acceleration under the action of muscles of the hand. While catching the ball, we decelerate and stop it also acting on it with our hands. The cork of a pneumatic "pistol" (Fig. 53) is set in motion by the air compressed by a moving piston. A bullet escaping from a gun at a high velocity under the action of powder gas gradually loses its velocity due to the action of air. The velocity of a stone thrown upwards decreases under the action of the force of attraction of the Earth; the stone comes to a halt and starts moving downwards with ever increasing velocity (which is also due to the attraction to the Earth).

In all these and other similar cases, the change in velocity, i.e. the emergence of acceleration, is the result of action of other bodies on a given object. In some cases, this action is observed in direct contact (hand, compressed air), while in other cases the action at a distance takes place (the action of the Earth on the stone).

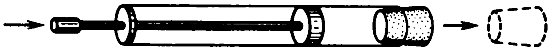

Fig. 53.
Pneumatic "pistol".

What happens when no other bodies act on a given object? In this case, the object either remains at rest relative to the Earth or moves uniformly in a straight line relative to it, i.e. moves without an acceleration. It is practically impossible to verify in simple experiments that in the absence of action from other bodies a given object moves relative to the Earth without acceleration since it is impossible to eliminate the actions of all surrounding bodies *completely*. But the more thoroughly these actions are eliminated, the closer the motion of the given object to a uniform motion in a straight line.

It is most difficult to eliminate the action of friction emerging between a moving body and the support over which it rolls or slides, or the medium (air, water) in which it moves. For example, a steel ball rolling over a horizontal surface covered with sand comes to a halt very soon. If, however, the ball is well polished and is rolling over a smooth surface (say, glass), it keeps its velocity almost unchanged for quite a long time. [1]

In some physical instruments the motion of elementary particles is realised and every particle experiences virtually no action from any other material particles (for this purpose, the instrument must be thoroughly evacuated). Under these conditions, the motion of particles is very close to uniform motion in a straight line (owing to the high velocity and small mass of the particles, the attraction of the Earth is not practically manifested in such experiments).

The first accurate experiments associated with the study of motion of bodies were carried out by Galileo at the end of the 16th and beginning of the 17th centuries. These experiments allowed Galileo to establish the following basic law of dynamics.

A body remains in the state of rest or in a uniform motion in a straight line relative to the Earth if no other bodies act on it. [2]

There is no acceleration in the state of rest and in a uniform motion in a straight line. Consequently, the law established by Galileo indicates that for a body to move with an acceleration relative to the Earth, it must be

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[1] In this case, the action of the Earth is naturally not eliminated but is balanced by the elastic action exerted on the ball by the glass surface. — *Eds.*

[2] This statement is of approximate nature. A more rigorous formulation is as follows: *a body preserves the state of rest or uniform rectilinear motion relative to a heliocentric reference system*, i.e. a system whose centre coincides with the Sun while the coordinate axes are directed to stationary stars (see Sec. 2.3). — *Eds.*

acted upon by other bodies. *The body is accelerated due to the action of other bodies.*

The property of bodies to retain their velocity in the absence of action of other bodies on them is called *inertia* (from Latin *inertia* that means idleness, laziness). For this reason, the above law is known as the *law of inertia,* and the motion in the absence of action of other bodies is called inertial motion.

The law of inertia was the first step towards establishing the basic laws of mechanics, which were not clear at all in Galileo's time. Later (at the end of the 17th century) the great English mathematician and physicist Sir Isaac Newton (1642-1727) formulated the general laws of motion of bodies, the law of inertia being included as the first law of motion. For this reason, the law of inertia is often called *Newton's first law.*

Thus, bodies acquire acceleration under the action of other bodies. If different parts of a body experience different actions, they acquire different accelerations, and will have different velocities after some time. As a result, the nature of motion of the body as a whole may change. For example, if the velocity of a carriage has changed abruptly, friction against the floor will entrain the feet of a passenger, but the floor will exert no action on the body and head of the passenger, and these parts will continue to move by inertia. Therefore, when the carriage, say, slows down, the velocity of the feet decreases, and the body and head whose velocities remain unchanged overtake the feet. As a result, the passenger's body will be inclined in the direction of motion. On the contrary, upon an abrupt increase in the velocity of the train, the body and head, which retain the previous velocity by inertia, will lag behind the feet which are entrained by the carriage, and the passenger's body will lean backwards.

Such manifestations of inertia of bodies are widely used in engineering and in everyday life. For example, shaking dust particles out of a duster, shaking an extra drop of ink off a fountain pen, shaking down the mercury column in a clinical thermometer, all use the property of inertia of bodies (dust particles, ink drops and mercury in the thermometer).

Inertia is used in detonating fuses of gun shells. When a shell abruptly stops upon hitting a target, the detonating cap contained in the shell (but not connected rigidly with it) continues to move and strikes the needle of the fuse connected to the shell.

2.3. Inertial Reference Systems

Reference systems in which the law of inertia is observed are called *inertial reference systems.* Galileo's experiments showed that the Earth is an inertial reference system. But the Earth is not the only system of this type.

There exists an infinite number of such systems. For instance, a train moving at a constant velocity over a straight segment of its path is also an inertial reference system. A body acquires an acceleration relative to the train only under the action of other bodies.

In general, any reference system which is in a uniform, rectilinear translatory motion relative to any other inertial system (say, the Earth) is also an inertial system. Indeed, it has been shown in Sec. 1.28 that accelerations of bodies in such systems are indentical. This means that a body which does not experience the action of other bodies will move without acceleration relative to these systems just as it moves relative to the Earth.

If a reference system is in a translatory motion relative to an inertial system but its motion is not uniform and rectilinear, and it has an acceleration or rotates, such a system cannot be inertial. Indeed, a body may have an acceleration relative to this system even when other bodies do not act on it. For example, a body which is at rest relative to the Earth will have an acceleration relative to a decelerating train or a train moving over a rounded segment of its path although there are no bodies causing this acceleration.

It is important to note that Galileo's experiments, like any other experiments, were carried out only to a certain degree of accuracy. More precise investigations made it possible to establish later that the Earth can be considered an inertial reference system only approximately: the law of inertia is violated in motions relative to it. The reference system fixed to the Sun and other stars can be considered inertial to a higher degree of accuracy. The Earth moves with an acceleration relative to the Sun and stars and rotates about its axis. However, the violations of the law of inertia for the Earth as a reference system are very small. They will be discussed in Chap. 6 of this book, but for the time being we shall regard the Earth as an inertial system of reference.

With the exception of Chap. 6, we shall be dealing everywhere with inertial reference systems. In most problems concerning the motion along the surface of the Earth, the latter will be taken as a reference system. While studying the motion of planets, we shall fix the reference system to the Sun and stars.

2.4. Galileo's Relativity Principle

We shall consider various mechanical experiments carried out in a carriage of a train moving uniformly over a rectilinear segment of the path, and then repeat the same experiments in the train in a state of rest or just on the surface of the Earth. We shall assume that the train moves smoothly and curtains are drawn over its windows so that it is impossible to say whether the train is moving or is at rest. Suppose now that a passenger kicks a ball lying on the floor of the carriage and measures the velocity acquired by the ball relative to the carriage, and a man standing on the surface of the Earth kicks another similar ball lying on the ground with the same force and measures the velocity acquired by the second ball relative to the Earth. It turns out that the balls have acquired identical velocities in their "own" reference systems. Similarly, an apple will fall from a shelf of the carriage according to the same law which governs its falling from a tree to the ground. Carrying out various mechanical experiments in the carriage, we cannot find out whether the carriage moves relative to the surface of the Earth or is at rest.

Similar experiments and observations indicate that bodies acquire identical accelerations as a result of identical actions on them by other bodies in all inertial reference systems: *all inertial systems are absolutely equivalent relative to the causes of accelerations*. This statement was formulated for the first time by Galileo and is called after him *Galileo's relativity principle.*

Thus, when we speak of the velocity of a body, we must always indicate the inertial reference system relative to which this velocity is measured since this velocity will be different in different inertial reference systems even if no other bodies act on the body. On the other hand, the acceleration of the body will be the same relative to all inertial systems. For example, a given body may have zero velocity relative to the carriage but at the same time it moves relative to the Earth at a velocity of 100 km/h, and at a velocity of 30 km/s relative to the reference system fixed to the Sun and stars (this is the velocity of the Earth's motion around the Sun). If, however, the passenger kicks the ball, its acceleration will be the same (say, 25 m/s^2) relative to the train as well as relative to the Earth and relative to the Sun and stars. For this reason, *the acceleration is said to be absolute* relative to different reference systems while *the velocity is relative.*

2.5. Forces

Actions of bodies on others resulting in accelerations are called *forces*. All forces can be divided into two main categories: forces acting *in direct contact* and forces which emerge irrespective of whether bodies are in contact or not, viz. the forces that can act *at a distance.*

For a body to act on another body in direct contact, it must be in a special state. For example, for a hand to act on a ball, its muscles must be contracted, and the spring of a toy pistol must be compressed so that it can act on the cork. Compression, extension, bending, etc. are changes in the shape or volume of bodies in comparison with their initial state. Such changes are known as *deformations,* and as a result of such changes a body is said to be deformed. Muscles, springs, etc. must be in a deformed state to exert a certain force on the bodies in contact with them. These forces mostly act only as long as the bodies are deformed and disappear together with deformations. Such forces are called *elastic forces*. Besides elastic forces, friction can also appear in direct contact between bodies. For instance, friction between the wheel of a railway carriage and the brake block pressed against it or friction acting on a body moving in a viscous liquid (resistance of the medium).

For forces acting at a distance, the pattern of interaction of bodies is much more complicated than for elastic forces. The most important example of forces acting at a distance is the force of universal gravitation or the force of gravity as its special case (the force of attraction by the Earth). Falling of a body, i.e. the appearance of a downward acceleration when a body is lifted above the surface of the Earth and released, indicates that the Earth exerts a force on it, although there is no direct contact between the Earth and the falling body.

Forces of universal gravitation acting between the objects encountered by us in everyday life are negligibly small in comparison with other forces acting among them. For example, the elastic force, exerted when a rubber cord 1 m long and having a thickness of 1 mm is stretched just by 1 mm, is million times stronger than the force of universal gravitation acting between two 1-kg loads separated by 1-m distance. But if one (or both) of the attracting bodies is a huge celestial body, the force of universal gravitation becomes very strong. For example, the Earth attracts a 1-kg load with a force which is 10^{11} times stronger than the loads in the example considered above, while the Sun attracts the Earth with a 4×10^{21} times stronger force than the force exerted by the Earth on a 1-kg load.

In addition to gravitational forces, magnetic and electric forces also act at a distance. If we bring a magnet close to another magnet floating on a cork so that they do not touch each other, the magnet on the cork will acquire an acceleration and will either approach the second magnet or will move away from it, depending on the mutual arrangement of their poles (Fig. 54). Electrically charged bodies located at a distance attract or repel each other depending on whether their charges are like or unlike.

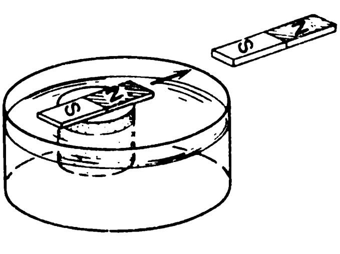

Fig. 54.
A magnet acts on another magnet which is at a certain distance from it.

2.6. Balanced Forces. State of Rest and Inertial Motion

If only one force acts on a body, it always acquires an acceleration. If, however, two or more forces act on a body, it may sometimes turn out that the body does not acquire an acceleration, i.e. it either remains at rest or moves uniformly in a straight line. In such a case, ***the forces are said to balance each other*** and ***each of them balances all other forces***; in other words, ***their resultant is equal to zero*** (see Sec. 2.10).

In the simplest case, two balancing forces act on a body; their joint action does not cause an acceleration. Experiments show that if such forces acted on the body separately, they would impart to the body equal accelerations in opposite directions. Acting together on some other body, these forces would balance each other again, while if they acted separately, they would impart to it different accelerations which, however, would have equal magnitudes and opposite directions. Therefore, balancing forces are considered to have equal magnitudes and opposite directions. For example, a load suspended from a spring experiences a (downward) action of the force of gravity and an equal (upward) action of the elastic force of the spring and the two forces balance each other.

Thus, if the acceleration of a body is equal to zero, this means that either no forces act on it or the resultant of the forces acting on the body is zero: all the forces are mutually balanced.

Here, we must bear in mind the following circumstance. Among the forces acting on a body moving uniformly in a straight line, there are usually the forces acting in the direction of motion, which are created deliberately, like the thrust force of the engine of an aeroplane. It is often said that the plane flies *since* the thrust force of its engine acts on it. In this case, however, the forces opposing the motion are often discarded, such as the air resistance to the flying plane. For a motion to be uniform and rectilinear it is necessary that the deliberately created forces balance the resistance forces. While considering the inertial motion and the state of rest in the previous sections, we dealt with just such cases. For example, when a small ball rolls over a glass surface, the force of gravity is balanced by elastic force.

The reason behind the fact that resistance forces, in contrast to evident "driving" forces, often escape students' notice lies in the following. In order to create a driving force, an engine should be mounted on an aeroplane and petrol must be burnt in it. On the other hand, resistance forces emerge so to say "free of charge", just due to motion. No engine is required for their emergence. In this case their source is in invisible air. To account for these forces, we must first discover them, while "driving" forces are the object of our special care and expenditures of efforts and materials.

Before Galileo's experiments, it was assumed that if a single force acts on a body, it moves uniformly in the direction of this force (friction was naturally discarded in this case). The action of a force in the direction of motion is indeed required for the motion to be uniform, but just to balance friction.

A body moves without an acceleration either when no forces act on it or when the acting forces balance each other. However, the body is usually said to execute inertial motion only if there are no forces in the direction of motion: the forward force is absent and friction or resistance of a medium can be neglected.

To clarify better what has been said above, let us analyse the case when uniform motion in a straight line emerges from the state of rest. We shall consider by way of example a locomotive pulling a train. At the initial moment, when the engine is switched on, but the train is still at rest, the driving force of the locomotive transmitted through the coupler to the train is already strong and exceeds the friction between the wheels of the carriages and the rails (the emergence of driving force will be explained in Sec. 2.37). For this reason, the train starts to move forward with an acceleration. As its velocity increases, the resistance forces (friction of the wheels and air resistance) increase but as long as they remain weaker than the driving force, the velocity of the train continues to increase. Upon a further increase in the velocity, the excess of the driving force over the resistance becomes smaller and smaller, and these forces ultimately equilise. At this moment the acceleration vanishes and further motion becomes uniform.

If the driving force is increased, the balance of forces is violated, and the train will again acquire a forward acceleration. The velocity will increase again until the resistance growing with velocity balances the new, increased driving force. Conversely, if the driving force is decreased, the balance of forces will be violated again, the train will receive a negative acceleration (since the resistance is now stronger than the driving force of the locomotive), and the motion of the train will be decelerated. But the resistance will also decrease in this case, and at the moment when it levels with the decreased driving force, the motion becomes uniform again, though at a decreased velocity. Finally, if the engine is disengaged, the velocity of the train will continuously decrease due to the continuous action of the resistance until the train comes to a halt.

2.7. Force as a Vector. Standard of Force

By measuring the accelerations acquired by a body as a result of action of different forces, we can note that accelerations may differ in magnitude as well as in direction. This means that forces can be distinguished by their magnitude and direction: *force is a vector quantity.*

For measuring a force, we must first of all choose a standard force, and then establish the method of comparing other forces with the standard, i.e. establish the method of measuring forces. We can take as a standard, for example, a certain elastic force. Since elastic forces are determined by deformation, we can take as a standard the force exerted by a certain spring stretched to a certain extent by a body suspended from its end.

Such a standard can be realised, in principle, in the form of a cylindrical spring supplied with a marker which makes it possible to fix the same extension of the spring each time (Fig. 55). For the direction of the force we take the spring axis. Consequently, our standard indicates both the magnitude and the direction of a force.

In actual practice, however, such a standard force is inconvenient since elastic properties of the spring depend on temperature, may vary with time, and so on. Therefore, a standard is usually chosen in such a way that the variability of the properties of the spring is not manifested. This can be done as follows. Let us take a spring and suspend a load from it. The suspended load stretches the spring, until it is stretched to a certain length, after which the stretching ceases and the load stops being lowered. The

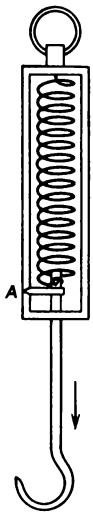

Fig. 55.
A simple standard of force is the action of a spring stretched to marker A.

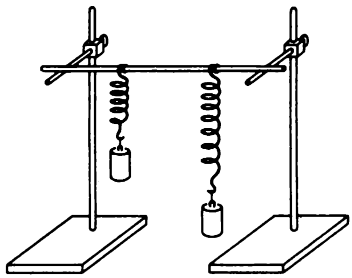

Fig. 56.
Different springs exert equal forces on the same load suspended from them.

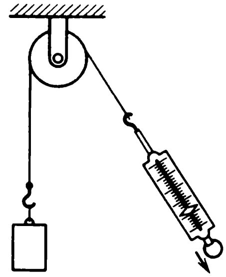

Fig. 57.
Standard force can be obtained at any angle to the vertical.

force of gravity acting on the load turns out to be balanced by the elastic force of the spring.

If we suspend the same load from another spring, the extension will be different. However, the force exerted by the new spring on the load will be equal to the force exerted by the first spring since in both cases the elastic forces of the spring balance the force of gravity acting on the same load (Fig. 56). Using a certain chosen load, we can determine the extent to which any spring should be stretched so as to exert a certain force, i.e. to serve as a standard force. In order to obtain the force equal to the standard but having any direction and not only the vertical direction, we can use a string passing through a pulley as shown in Fig. 57 (the elastic force exerted by a string always acts along it). Thus, the complicated problem of making and preserving a standard spring under a certain extension is reduced to a much simpler problem of manufacturing and storing a standard load.

2.8. Spring Balance

To obtain an elastic force equal to twice, thrice, etc. the value of the primary standard, the spring should be stretched by two, three, etc. standard loads. Having chosen a certain spring, we can mark extensions corresponding to twice, thrice, etc. standard force. The spring graduated in this way is called a *spring balance* or *load gauge* (Fig. 58).

We can also obtain a definite part of the standard force by stretching the spring by a load constituting the corresponding fraction of the standard load. For example, we can make a hundred identical loads such that

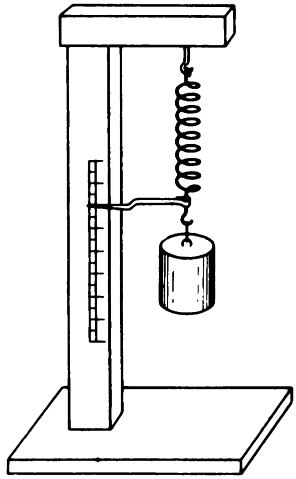

Fig. 58.
Graduation of a spring balance.

together they stretch the spring as the standard load does. Then each load separately stretches the spring as any other of these small loads. So, we can assume that the spring stretched by the small load exerts a force equal to 1/100 of the standard force, the spring stretched by two loads acts with a force equal to 2/100 of the standard force, and so on. By measuring the extension of the spring of the spring balance under the action of these small loads, we can also mark fractional parts of the standard force.

While marking the spring balance scale, we note that the double force corresponds to the double extension of the spring, the triple force to the triple extension, and so on. In other words, the extension of the spring and the elastic force exerted by the spring balance turn out to be directly proportional to each other. This allows us to graduate a spring balance scale in a simple manner. Having marked the zero division of the scale (absence of load) and, say, the extension corresponding to 10 standard loads, we can divide the distance thus obtained into 10 equal parts. The displacement of the end of the spring by one such mark will correspond to a change by one standard force in the force exerted by the spring balance.

It should be borne in mind that this linear dependence is observed only for sufficiently small deformations. Besides, it is always violated under an inelastic deformation, i.e. such that does not vanish after the elimination of the force.

Figure 59 shows one of the widespread types of a spring balance with a cylindrical spring. This balance can be used to measure the force with which we pull a body. Figure 60 shows a dynamometer with elastic cramps rigidly connected to each other at their ends. Such a dynamometer can be used to measure both pulling and pushing forces.

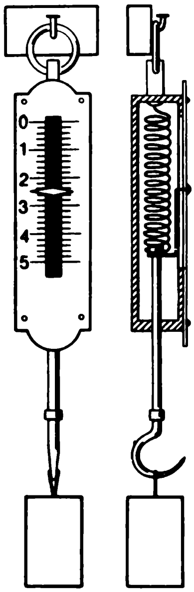

Fig. 59.
Spring balance: general view (left) and construction (right).

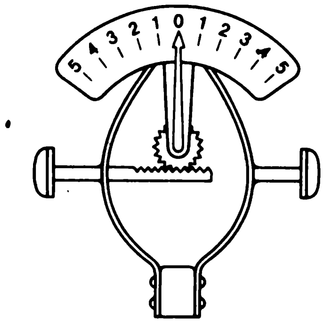

Fig. 60.
A dynamometer operating on both compression and extension.

Using spring balances, we can measure forces exerted by some bodies on others both in direct contact and "at a distance". It has been already shown how to measure the force of attraction by the Earth: for this purpose, it is sufficient to suspend a body from a spring balance. The force exerted by magnet I on magnet II (Fig. 61), when the south pole (S) of magnet I is brought to the north pole (N) of magnet II, can be determined as follows. Having attached a spring balance to cart II so that the other end of the balance is fixed, we bring cart I to cart II. We shall see that cart II comes closer to cart I, stretching the spring balance, after which cart II comes to a halt. This means that the required force exerted by magnet I on magnet II is equal to the force exerted by the spring balance on the cart. And this force can be directly measured by the spring balance.

In order to measure the force exerted by one body on another in direct contact, a spring balance should be used in a slightly different way. For instance, to measure the force with which a man pulls a cart it is sufficient to place a spring balance between his hand and the rope (Fig. 62). Its reading

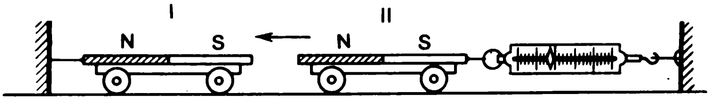

Fig. 61.
The measurement of the force of interaction between magnets with the help of a spring balance.

Fig. 62.
A spring balance indicates the force with which the hand pulls the rope.

will correspond to the force exerted by the hand on the rope. The direction of the force coincides with the axis of the balance spring.

It was mentioned above that different forces cause different accelerations of a given body. Using spring balances, we can establish the most important property of forces: the stronger the force (e.g. the larger the extension of a spring balance attached to a body on which it acts), the higher the acceleration of the body. The quantitative relations between forces and accelerations will be considered in Sec. 2.13.

2.9. The Point of Application of a Force

Forces acting in direct contact exert their action over the entire surface of contact of bodies. For instance a hammer striking the head of a nail acts on the entire head. If, however, the surface of contact between bodies is small in comparison with their dimensions, we can assume that a force acts on a single point of the body. For example, we can assume that a rope through which we pull a cart acts on it only at the point at which it is tied to the cart. This point is called the *point of application of force.*

At first we shall consider only the cases when the point of application of force can be specified. Such forces will be designated by directed segments originating at the point of application of force, whose direction coincides with the direction of the force and the length corresponds to the magnitude of the force on a certain scale. For example, the arrow in Fig. 62 represents the force exerted by the rope on the cart.

2.10. Resultant Force

If several forces act on a given body simultaneously, their effect on the motion of the body can be replaced by the action of a single force. [3] Such a replacement is known as *composition of forces.* Given forces are called *components,* while the force replacing them is their *sum* or *resultant.* The rules of composition of forces are established experimentally. The resultant of balanced forces, for example, of two equal and opposite forces, is equal to zero (Sec. 2.6).

It should be noted that the resultant replaces several forces only as regards the *motion of the body as a whole*: the resultant force imparts to the body the same acceleration as all the components acting on the body simultaneously, while the force balancing the resultant balances the simultaneous action of the components. But naturally the resultant cannot replace the effect of components in other respects. It is sufficient to consider the following example: we stretch a spring with both our hands. The forces acting on the spring are equal and opposite, and hence their resultant is zero: the spring as a whole remains at rest. However, if no forces acted on the spring at all, the resultant would be equal to zero as before, but the spring would not have been stretched.

Instead of determining the resultant force, we can look for the force *balancing* given forces acting simultaneously on a body. *The resultant is equal in magnitude to the balancing force and has the opposite direction.*

2.11. Composition of Forces Acting along a Straight Line

Let us consider the case when all the forces act on a given body along the same straight line, say, along the horizontal. First of all, we shall balance the force of gravity acting on the given body vertically downwards. For this purpose, it is sufficient to suspend the body from a string. The stretched string will create the elastic force which will balance the force of gravity. In the absence of other forces, the string will occupy a vertical position.

Let us now attach to the body strings with spring balances at their ends. The spring balances will allow us to determine the forces exerted by the strings on the body. Suppose that two strings act on the body to the right with forces $\mathbf{F}_1$ and $\mathbf{F}_2$, while one string acts to the left (Fig. 63). What must be the force $\mathbf{F}_3$ exerted by the left string to keep the string from which the body is suspended in a vertical position (i.e. for the forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ to be balanced out)? The experiment shows that for this purpose the follow-

[3] With the exception of one important case of "couple of forces" which will be considered in Sec. 3.11.

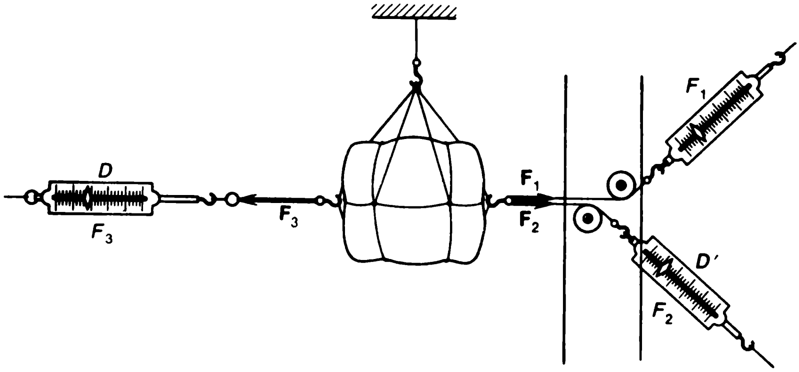

Fig. 63.
The reading of spring balance D gives the magnitude of the resultant of forces $\mathbf{F}_1$ and $\mathbf{F}_2$. The reading of spring balance D' gives the magnitude of the resultant of $\mathbf{F}_1$ and $\mathbf{F}_3$.

ing equality must hold:

$$F_3 = F_1 + F_2,$$

where F_1, F_2 and F_3 are the magnitudes of the corresponding forces. Each of the forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ can be considered to be balancing the joint action of the other two forces. For example, force $\mathbf{F}_2$ is the balancing force for the forces $\mathbf{F}_1$ and $\mathbf{F}_3$, so that the following relation is observed for their magnitudes: $F_2 = F_3 - F_1$.

Thus, for the forces acting along the same straight line, the condition of equilibrium can be written in terms of the magnitudes of these forces.

2.12. Composition of Forces Acting at an Angle to Each Other

We shall start to solve the problem on the composition of several forces directed at an angle to each other with the case when a body is acted upon only by two forces which do not lie on the same straight line. In this case, it follows from experiments that the body cannot be in equilibrium. This means that the resultant of such forces cannot be zero. For example, a body suspended from a string experiences the action of downward force of gravity, and if the string (and hence its tension) forms an angle with the vertical, the body does not remain at rest. This fact is used in the construction of a plumb line.

Another example: we attach two spring balances arranged in the horizontal plane at an angle to each other to a body suspended from a string (Fig. 64). It can be easily verified by experiment that in this case also the body does not remain at rest, and the string does not occupy the vertical position for any extension of the spring balances.

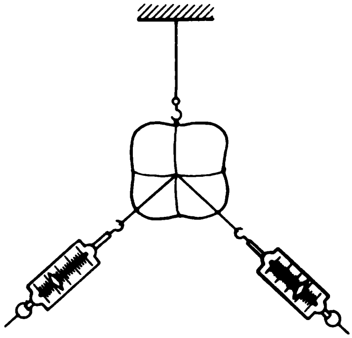

Fig. 64.
If the spring balances are stretched, the equilibrium vertical position of a load is impossible.

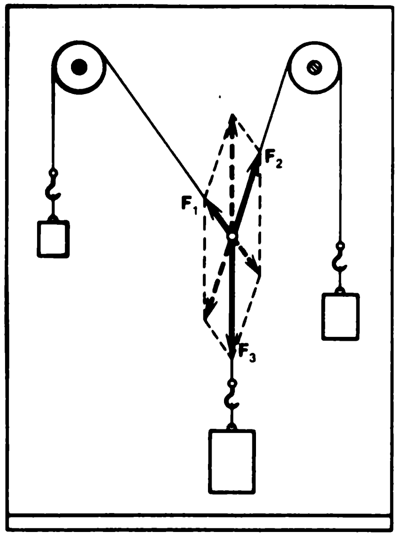

Fig. 65.
Equilibrium conditions for three forces acting at an angle to one another.

Let us determine the resultant of two forces acting at an angle to each other. Since the resultant is equal and opposite to the balancing force (Sec. 2.10), to solve the problem it is sufficent to establish the conditions of equilibrium of a body under the action of three forces (two given forces and the third force balancing them). In order to determine these conditions, let us carry out an experiment in which the magnitudes and directions of all the forces can be determined. Let us tie together three strings, attach to them different loads, and pass two of the strings through pulleys (Fig. 65). If the mass of each load is less than the sum of the masses of the other two loads, the knot will occupy a certain position and will remain at rest. This means that this is the equilibrium position. All the three strings lie in this case in the same vertical plane. The knot experiences the action of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ which are equal in magnitude to the forces of gravity acting on the loads and are directed along the strings. Each of these forces balances the remaining two. Let us represent the forces applied to the knot by the segments laid off from the knot, directed along the strings and equal (on a chosen scale) to the magnitudes of these forces. It turns out that in equilibrium, the segment representing any of these forces coincides with the diagonal of the parallelogram constructed on the segments representing the other two forces. These parallelograms are shown by dashed lines in the figure. It means that the diagonal of a parallelogram is the resultant of the forces represented by its sides, the resultant being opposite to the third force. Thus, *the forces (like displacements) are composed according to the parallelogram rule, i.e. according to the rule of vector addition*.

It follows from the rule of parallelogram for forces that the magnitude of the resultant depends not only on the magnitudes of the component forces but also on the angle between their directions. As the angle changes, the magnitude of the resultant varies in the limits between the sum of the magnitudes of the forces (when the angle between them is zero) and the difference in the magnitudes of the larger and smaller forces (when the angle is 180°). In the special case of composition of two forces equal in magnitude, we can obtain any value for the magnitude of the resultant between the doubled magnitude of one of the components and zero, depending on the angle between the forces.

Instead of the parallelogram rule, the triangle rule can also be used as in the case of displacements. In composing more than two forces, we can either add them vectorially (one to another) or construct an open polygon of the vector forces. In the latter case, the resultant will be the link closing the polygon. In equilibrium, the force polygon is closed: the resultant is equal to zero. For instance, the polygon of three balanced forces forms a triangle.

2.13. Relation between Force and Acceleration

In Sec. 2.2, we considered the law of inertia according to which a body acquires an acceleration only if a force is acting on it. Experiments show that the direction of acceleration coincides with the direction of the force causing it. [4] Let us find the relation between the magnitude of the force acting on a body and the magnitude of the acceleration imparted to the body by this force.

Everyday-life experience shows that the magnitude of acceleration imparted to a given body is the larger, the stronger the force acting on it: a ball acquires a higher acceleration (and as a result will have a higher velocity) if it is struck hard. A powerful electric locomotive which develops a strong driving force imparts a higher acceleration to a train than a small shunting electric locomotive. We can roughly determine the relation between the force acting on a given body and the acceleration acquired by it with the help of the following experiment. We attach a string to a movable cart through a spring balance and pass the string with a load at its end through a pulley (Fig. 66). The load stretches the spring which imparts an acceleration to the cart by its elastic force. The heavier the load suspended from the string, the larger the extension of the spring and the higher the acceleration of the cart. It should be noted that the reading of the spring balance will be lower than the reading of a fixed spring balance with the

[4] We shall assume that only one force acts on a body. If there are many forces, we shall consider their resultant.

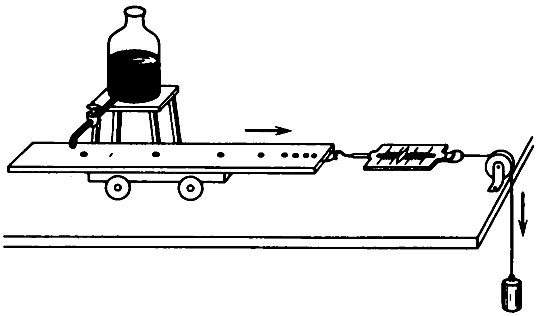

Fig. 66.
An analysis of the dependence between the force and the acceleration of a body. The path lengths covered by the cart are marked with the help of a dropping bottle.

same load suspended from it, i.e. lower than the force of gravity that acts on the load. This fact will be explained in Sec. 2.23.

Observing the extension of the spring balance during the motion of the cart, we note that it does not change. This means that the force exerted on the cart is constant. Its magnitude is given by the reading F of the spring balance. The path length s covered by the cart over different time intervals t from the beginning of motion can be determined, for example, with the help of a dropping bottle. Measurements show that the distance covered by the cart is proportional to the square of the time interval elapsed from the beginning of motion. This means that the cart is in a uniformly accelerated motion (Sec. 1.22). The magnitude of acceleration a can be found from the formula

$$a = 2s/t^2.$$

If different loads are suspended from the end of the string, different forces will be exerted on the cart each time. Having determined the magnitudes of the forces acting on the cart in each case ($F_1, F_2, F_3, \ldots$) with the help of a spring balance and having calculated the accelerations a_1, a_2, a_3, ... imparted to the cart, we note that the accelerations of the cart are proportional to the forces acting on it:

$$F_1/a_1 = F_2/a_2 = F_3/a_3 = \ldots .$$

Experiments show that, in general, *the acceleration of a body is proportional to the force acting on it.* Hence it follows that in order to find the accelerations imparted to a given body by different forces, it is sufficient to measure only once the force acting on the body as well as the acceleration caused by it. If we apply some other force to the same body, the resulting acceleration will change in the same proportion as the force.

Naturally, the experiments with the cart are too rough for exactly establishing the law of linear dependence between force and acceleration. However, more precise methods of measurement, in particular, those involving astronomical observations, made it possible to establish that the linear dependence between the force acting on a body and the acceleration imparted by this force is confirmed in experiments to a high degree of accuracy.

2.14. Mass of a Body

Thus, for a given body the acceleration imparted to it by a force is proportional to this force. Let us now compare the accelerations imparted to different bodies by a force. We shall see that an acceleration depends not only on the force but also on the properties of the body on which this force acts. Let us, for example, pull different bodies with the help of a spring balance so that the reading of the balance is the same in all the cases, i.e. we apply the same force to the bodies. For this purpose, we can, for instance, modify the experiment described in the previous section by taking different carts or by placing different objects on the carts and choosing each time such a load at the end of the spring passing through the pulley that the reading of the spring balance is the same in all the experiments.

By measuring accelerations in such experiments, we note that different bodies, generally, acquire different accelerations under the action of the same force: different bodies have different inertia. We can introduce the concept of the *measure of inertia* of bodies, assuming that the measure of inertia for two bodies is the same if they acquire the same acceleration under the action of equal forces, and considering that the measure of inertia is the larger, the smaller the acceleration acquired by a body under the action of a given force.

What determines the measure of inertia of different bodies? Which properties of bodies determine the acceleration due to a given force? Or, conversely, which properties of a body determine the force required to cause a given acceleration? Experiments show that for bodies made of the same material, say, aluminium, the acceleration caused by a given force is the smaller, the larger the volume of a body, the acceleration being inversely proportional to the volume of the body. If, however, we carry out experiments with bodies made of different materials (say, iron, aluminium and wood), no relation will be revealed with the volume of bodies: bodies having equal volumes acquire different accelerations under the action of the same force. To obtain equal accelerations, the volume of an iron object should be smaller than that of an aluminium object, and the aluminium object must have smaller volume than a wooden object. We cannot say beforehand what must be the ratio of volumes of bodies made of different materials to impart equal accelerations to them by equal forces. Only direct experiments can give answer to the question concerning the volume of an aluminium or a wooden body corresponding to the same acceleration as that imparted to an iron body by a given force. If bodies acquire equal accelerations under the action of the same force, we have to assume that the measure of inertia of these bodies is the same.

Thus, the measure of inertia must be determined directly in a mechanical experiment by measuring the acceleration due to a given force. The measure of inertia of a body is called its *mass* and denoted by m (or M).

Thus, *the mass of a body is its characteristic physical property which determines the relation between the force acting on this body and the acceleration imparted to it by this force.* Since the force and acceleration imparted by it to a body are proportional to each other, the mass of the body is defined as the ratio of the force F acting on the body to the acceleration a imparted by this force, i.e.

$$m = F/a, \tag{2.14.1}$$

whence we obtain the relation

$$F = ma.$$

Having applied a force F to a given body and having measured the acceleration a imparted by this force, we can determine the mass m of the body from the above formula. For a given body, we shall always obtain the same value of m irrespective of the force applied to the body.

Using this method of measuring mass, we can find out experimentally the value of the mass of a body composed of several other bodies, or the mass of a part of a body of known mass. If we measure the masses m_1, m_2, m_3, ... of several bodies and then combine these bodies into one (for instance, by joining them together) so that they acquire the same acceleration under the action of forces, and measure the mass m of the body thus obtained, it will turn out that

$$m = m_1 + m_2 + m_3 + \dots .$$

Conversely, if we divide a given body into parts, the sum of the masses of individual parts will turn out to be equal to the mass of the original body. In particular, if we divide a homogeneous body of mass m into n parts equal in volume, the mass of each part will be m/n.

We must emphasise the following important fact. If we take different bodies having the same mass and weigh them in turn with the help of a spring balance, the extension of the spring will be the same in all the cases. If dynamic experiments reveal that the mass of a body is n times the mass of another body, the extension of the spring of the balance due to the first body will be n times that due to the second body. This means that the force of attraction of bodies by the Earth is proportional to their masses. This remarkable fact allows us to compare the masses of bodies without imparting an acceleration to them. We shall return to this question again in Sec. 2.27.

2.15. Newton's Second Law

In the experiments on action of forces on bodies we have established that there exists a linear dependence between the magnitude F of the force acting on a body and the magnitude a of the acceleration imparted to it by this force. We have also introduced a new quantity, viz. the mass m of the body.

Experiments have also revealed that the direction of acceleration coincides with the direction of the force causing it (Sec. 2.13), i.e. that vectors $\mathbf{F}$ and $\mathbf{a}$ coincide in direction. Consequently, formula (2.14.1) can be written in the vector form:

$$\mathbf{F} = m\mathbf{a}. \tag{2.15.1}$$

It should be recalled that here $\mathbf{F}$ is the resultant of all the forces acting on the body, m is its mass, and $\mathbf{a}$ is the acceleration acquired by the body under the action of force $\mathbf{F}$. This formula expresses the basic law of motion known as *Newton's second law* (the first law is the law of inertia, Sec. 2.2). Newton's second law can also be formulated as follows: *the force acting on a body is equal to the product of the mass of the body and the acceleration created by this force, and the directions of the force and acceleration coincide.*

Formula (2.15.1) can also be written in a different form:

$$\mathbf{a} = \mathbf{F}/m, \tag{2.15.2}$$

and Newton's second law can then be formulated as follows: *the acceleration imparted to a body is directly proportional to the force acting on it, inversely proportional to its mass, and has the same direction as the force.* In particular, it follows that under the action of the same force different bodies acquire accelerations inversely proportional to their masses. Conversely, if different bodies acquire accelerations which are inversely proportional to their masses, this means that the forces applied to these bodies are equal in magnitude.

If a force having a constant direction starts to act on a body at rest, or if a force acting on a moving body is directed along its velocity (as in the case of a body falling with a zero initial velocity or of a body thrown upwards), the body will move in a straight line. In this case, Newton's second law can be written in the scalar form:

$$F = ma, \quad \text{or} \quad a = F/m.$$

In this case, a body of constant mass moves under the action of a constant force with a constant acceleration, i.e. its motion will be uniformly accelerated. If, however, the force changes with time, the acceleration also changes. In this case, formula (2.15.2) gives instantaneous accelerations (Sec. 1.27) caused by the force acting at a given instant of time. If the force remains unchanged while the mass of the body to which the force is applied varies, the acceleration turns out to be varying. An example of a body of varying mass is a rocket ejecting the combustion products of a fuel during its flight, as a result of which its mass decreases. If in this case the force acting on the rocket does not change, its acceleration grows (Sec. 9.9). If a force acts at an angle to the velocity of a body, its motion is curvilinear (like the motion of a body thrown along the horizontal). Curvilinear motion will be studied in Chap. 5.

Newton's first law (the law of inertia) is contained in Newton's second law as a special case. Indeed, formula (2.15.2) shows that if $F = 0$, then $a = 0$ as well, i.e. if no forces act on a body (or forces act so that their resultant is equal to zero), the acceleration is also equal to zero. This means that the body remains at rest or in a uniform motion in a straight line.

We encounter the manifestations of Newton's second law very often in everyday life. An electric locomotive brings up a train to speed with an acceleration which is the lower, the larger the mass of the train. Pushing an empty and a heavily loaded boat from the shore with the same force, we make the first boat move with a higher acceleration in comparison with the second boat. If a body rests on a rigid support, we cannot shift it by applying a small force since the friction against the support in this case (Sec. 2.35) balances the applied force: the resultant turns out to be zero. If, however, the body floats in water, the friction emerging between the surface of water and the body at the beginning of motion is very small. Therefore, it cannot balance the applied force, and the resultant will differ from zero: the body starts to move.

The resultant force acting on a body can be very weak but an acceleration will still appear. However, it can be so small that it may take a long time to cause a noticeable change in the velocity. For example, by pressing against a bulky wooden block floating in water with a flexible glass stick (Fig. 67), we note that the block has acquired a noticeable velocity only in a couple of minutes. On the other hand, a much higher acceleration can be imparted with the help of the same stick to a block having a much smaller mass. At piers we can see that a worker setting a boat-hook against the side of a barge with all his power spends several minutes to impart the barge any noticeable speed.

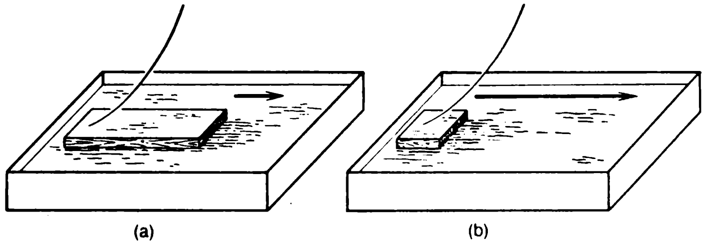

Fig. 67.
If the force acting on a floating block remains unchanged, the velocity increases: (*a*) slowly for a massive block and (*b*) rapidly for a small block.

In the formula expressing Newton's second law, **a** is the acceleration of a body in its motion relative to the Earth. It is well known, however (Sec. 2.4), that acceleration will be the same if we study the motion of the body relative to any other inertial reference system. The forces acting on a given body are the results of action of other bodies on it and do not depend on a reference system relative to which the acceleration is being determined. The mass of the body is also independent of the reference system. Therefore, Newton's second law remains valid if we consider the motion relative to some other inertial reference system, e.g. relative to a ship moving uniformly in a straight route over a calm sea or a train traversing a straight segment of its path at a constant velocity. This question will be considered in greater detail in Chap. 6.

? **2.15.1.** Using Newton's second law, explain why it is more dangerous to fall to a frozen ground than to soft snow and why a fireman jumping from the height of several storeys on a stretched tarpaulin remains unharmed.

Newton's law was discovered in an analysis of the motion occurring under normal conditions on Earth as weli as in the motion of celestial bodies. In both cases, the velocities of bodies are small in comparison with the velocity of light (300 000 km/s). Physicists came across velocities approaching the velocity of light only while studying the motion of elementary particles, say, electrons and protons in accelerators (the devices in which elementary particles experience the accelerating action of electromagnetic forces). For such velocities, Newton's second law is violated. According to Newton's law, a particle acted upon by a constant force directed along its trajectory must have a constant acceleration, i.e. its velocity should grow uniformly. It turned out, however, that as a particle begins to gather speed, Newton's second law holds, and the motion of the particle is uniformly accelerated. But as soon as the velocity acquired by the particle approaches the velocity of light, the acceleration becomes smaller and smaller, i.e. Newton's law is violated.

If the accelerator continues to act on the particle, its velocity grows more and more slowly, approaching the velocity of light but never reaching it. For example, if the velocity of a body is $0.995c$ (c is the velocity of light), the acceleration acquired by the body under the action of a force acting in the direction of motion amounts to only 0.001 of the acceleration calculated by Newton's second law. Even at a velocity equal only to one tenth of the velocity of light, the decrease in acceleration in comparison with the value calculated by Newton's law amounts to 1.5%. However, for "low" velocities encountered in everyday life, even including the velocities of celestial bodies, the correction is so small that it can be neglected. For example, the decrease in the acceleration of the Earth revolving around the Sun at a velocity of 30 km/s constitutes a millionth fraction of one percent.

Thus, Newton's second law can be applied only to bodies whose velocities are small as compared with the velocity of light.

2.16. Units of Force and Mass

In order to use Newton's second law for calculations, we must choose the units of force and mass in such a way that the following relation holds:

$$\text{unit of mass} = \frac{\text{unit of force}}{\text{unit of acceleration}}. \tag{2.16.1}$$

The SI unit of mass is a *kilogram* (kg), equal to the mass of a platinum-iridium body stored at the International Bureau of Weights and Measures in Sèvres near Paris. This body is called the international primary standard of a kilogram. The mass of this standard is close to the mass of 1000 cm^3 of pure water at 4 °C.

The SI unit of force is a *newton* (N) which is equal to the force causing an acceleration of one metre per second per second in a body whose mass is 1 kg. According to formula (2.16.1), we can represent a newton in the form

$$1\ N = 1\ kg \times 1\ m/s^2 = 1\ kg \cdot m/s^2.$$

In the CGS system of units, where the unit of length is a centimetre (1 cm = 0.01 m) and the unit of mass is a gram (1 g = 0.001 kg), the unit of force is taken as the force imparting an acceleration of 1 cm/s^2 to a body having a mass of 1 g. This unit of force is called a *dyne*. The relation between a newton and a dyne can be easily established:

$$1\ N = 10^5\ \text{dynes}.$$

A dyne is a very small unit of force. An ant dragging a straw acts on it with a force of about 100 dynes.

? **2.16.1.** A 15-kg shell acquires a velocity of 600 m/s after having been shot. Find the average force with which the powder gas acts on the shell, if the length of the gun barrel is 1.8 m (assume that the motion of the shell in the barrel is uniformly accelerated).

2.16.2. What minimal time is required to move a 50-kg load along a horizontal floor over a distance of 10 m, if the rope with the help of which the load is pulled is known to break under a tension exceeding 200 N, while a force of 100 N is sufficient to shift the load and move it uniformly, overcoming friction?

2.17. Systems of Units

Formula $a = F/m$ has such a simple form that having chosen the units of mass and acceleration arbitrarily, we have *deliberately* chosen the unit of force so that the proportionality factor in this formula is equal to unity.

Otherwise, the formula for acceleration must be written in the form

$$a = kF/m,$$

where k is the proportionality factor. For example, if the unit of force were chosen also arbitrarily and it turned out to be, say, 6.3 N, the proportionality factor should have a value $k = 1/6.3$.

The units of physical quantities could in principle be chosen independently from one another. In this case, however, the formulas would contain inconvenient proportionality factors. To avoid this, we must proceed as follows. The units of several physical quantites are chosen arbitrarily. These units (and accordingly the physical quantities) are called *base* units. The units of other physical quantities are chosen in accordance with the formulas relating them with the base units in such a way that the proportionality factors in these formulas are as simple as possible (as a rule, these factors are equal to unity). The units established in this way are known as *derived* units. As a result, we obtain an ordered set of units of physical quantities, which is called a *system of units*. There exist several systems of units of physical quantities which differ in the choice of the base quantities and base units. We shall consider Système International d'Unités (SI).

The base units in SI are the unit of length—a *metre* (m), the unit of mass—a *kilogram* (kg), the unit of time—a *second* (s), the unit of current—an *ampere* (A), the unit of thermodynamic temperature—a *kelvin* (K), the unit of luminous intensity—a *candela* (cd), and the unit of amount of substance—a *mole* (mol). In mechanics, we shall mainly deal with metres, kilograms, and seconds. The supplementary SI units are the unit of plane angle—a *radian* (rad) and the unit of solid angle—a *steradian* (sr).

Besides the units constituting a certain system, off-system units are also used. They include the units of time—an hour (h), a minute (min), the unit of volume—a litre (l), the units of angle—a degree (°), a minute (′) and a second (″), the unit of mass—an atomic mass unit (amu), and some other units.

2.18. Newton's Third Law

Two billiard balls change their velocities upon a collision, i.e. both of them acquire an acceleration. When a train is gathered carriages strike against one another, and the buffer springs of both carriages are compressed. The Earth attracts the Moon (with the force of universal gravitation) and makes it move in a curvilinear trajectory. The Moon, in turn, attracts the Earth (also with the force of universal gravitation). Although the acceleration of the Earth caused by this force cannot, naturally, be directly observed in the

Fig. 68.
The force exerted by the first spring balance on the second is equal in magnitude to the force exerted by the second spring balance on the first.

reference system fixed to the Earth (even a considerably higher acceleration due to the attraction of the Earth by the Sun cannot be observed directly), it is manifested in the form of tides (Sec. 6.12).

We gave several examples of forces acting between bodies. These examples show that forces always emerge in pairs and not one by one. If a body acts with some force on another body (action), the second body acts on the first one with some force (reaction). Experiments show that this rule is of general nature. The action of forces is *mutual,* so that the actions of bodies on each other are always *interactions.*

What can we say about the force exerted by the second body on the first one if we know the force exerted by the first body on the second? Rough measurements of forces of interaction can be made in the following experiments. Let us take two spring balances, link their hooks and, holding their rings, stretch them, keeping an eye on the readings of the two spring balances (Fig. 68). We shall see that at any tension the readings of the spring balances will coincide. This means that the force exerted by the first spring balance on the second is equal to the force exerted by the second balance on the first.

Another experiment on comparison of forces of interaction is illustrated in Fig. 69. The bodies fixed to carts can be of any kind. Applying different forces by hand to the left dynamometer, we cause different readings of the dynamometer on the right. When the bodies being compressed remain at rest, both dynamometers indicate forces $\mathbf{F}_1$ and $\mathbf{F}_2$ of

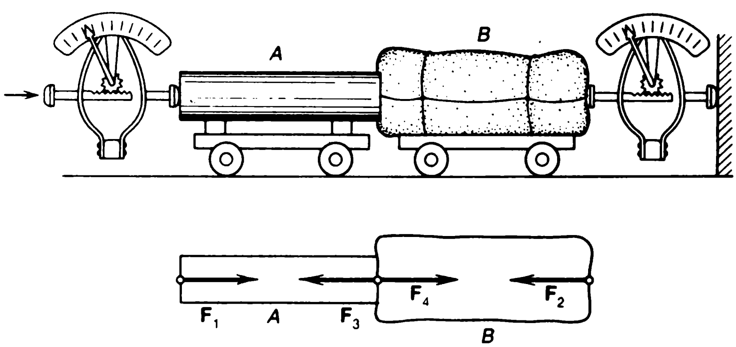

Fig. 69.
An analysis of interacton of two bodies A and B. The forces acting on them are shown in the lower part of the figure.

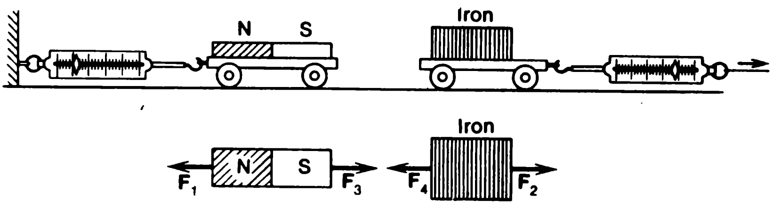

Fig. 70.
A comparison of forces of interaction between a magnet and a piece of iron.

equal magnitude. The directions of the forces exerted by the dynamometers will be opposite. In addition to forces exerted by the dynamometers, the forces of elastic interaction act between the bodies in this case: force $\mathbf{F}_3$ exerted by body B on body A and force $\mathbf{F}_4$ exerted by body A on body B. Since the bodies are at rest, the forces acting on them must be mutually balanced. This means that force $\mathbf{F}_3$ must balance force $\mathbf{F}_1$, while force $\mathbf{F}_4$ must balance force $\mathbf{F}_2$. Since forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal in magnitude, forces $\mathbf{F}_3$ and $\mathbf{F}_4$ must also be equal and opposite.

Similarly, we can compare the forces of interaction acting at a distance. Let us fix a magnet to one cart and a piece of iron to another cart and attach spring balances to them (Fig. 70). Depending on experimental conditions, the carts may stop at different distances from each other so that the forces of interaction between the magnet and the piece of iron will be stronger or weaker depending on this distance. However, in all cases it will turn out that the readings of the spring balances are identical. Using the same arguments as in the previous case, we conclude that the force with which the magnet attracts the iron is equal and opposite to the force exerted by the iron on the magnet.

In the examples considered above, the interacting bodies were at rest. Experiments show, however, that the forces of interaction between two bodies are equal in magnitude and opposite in direction also when the bodies are moving. This can be illustrated by the following example. Two persons A and B stand on the carts which can roll on rails (Fig. 71). They hold the ends of a rope in their hands. It can be easily seen that irrespective of the person who pulls (hauls in) the rope (A or B, or both simultaneously), the carts are set in motion at the same moment and move in the opposite directions. Having measured the accelerations of the carts, we see that the accelerations are inversely proportional to the masses of the carts (together with persons). It has been shown in Sec. 2.15 that this is an indication of the equality of the forces acting on the carts.

Experiments show that in all other cases when a body acts with a certain force on another body, the second body exerts on the first one an equal and opposite force. The two forces lie on the same straight line. This statement

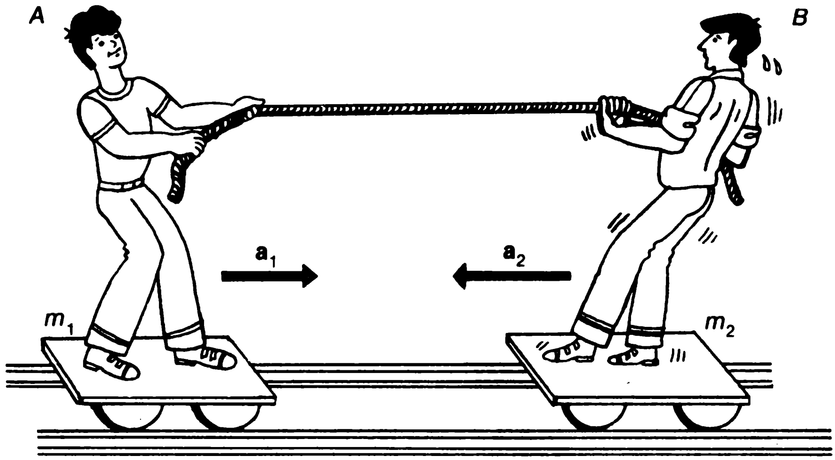

Fig. 71.
Both carts acquire an acceleration irrespective of who hauls in the rope.

expresses the *law of action and reaction,* discovered by Newton and called by him "the third law of motion".

? **2.18.1.** Determine the force with which a 1-kg load lying on the ground attracts the Earth.

2.18.2. In the experiments with two persons on carts, find the ratio of the distances covered by the carts over a certain time interval (say, up to their collision) if the ratio of the masses of the carts with the persons is known.

2.19. Applications of Newton's Third Law

In a "tug-of-war" game, both teams exert an action on each other (through the rope) with equal forces, as it follows from the law of action and reaction. Hence the team which wins does not pull the rope more strongly but rather takes a firmer stand.

How can we explain that a man can push a trash can if, as follows from the law of action and reaction, the can pulls the man back with the same force $\mathbf{F}_2$ as the force $\mathbf{F}_1$ with which the man pushes it in the forward direction? Why are these forces not balanced? As a matter of fact, although these forces are equal and opposite, they first of all are applied to different bodies. Secondly, the can and the man experience the action of forces exerted by the driveway (Fig. 72). The force $\mathbf{F}_1$ exerted by the man is applied to the can which, besides this force, is also acted upon by a small friction $\mathbf{f}_1$ between its bottom and the driveway. For this reason, the can starts to move forwards. On the other hand, the man experiences, in addition to the action $\mathbf{F}_2$ exerted by the can in the backward direction, forces $\mathbf{f}_2$ exerted by

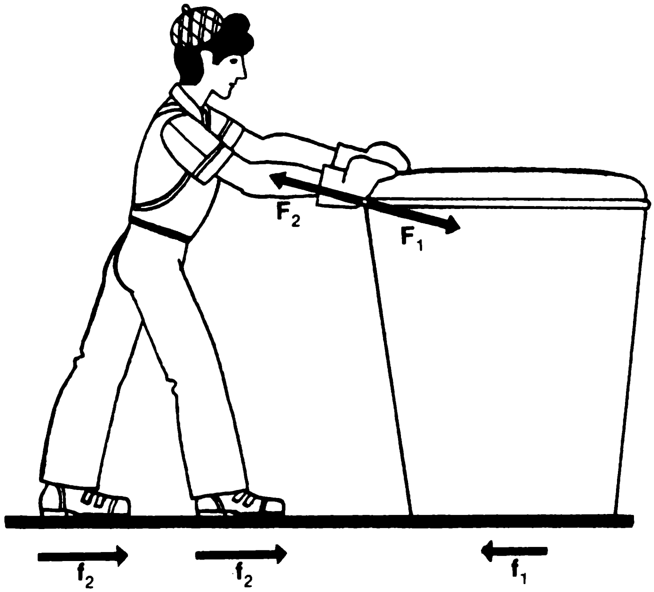

Fig. 72.
A man sets a loaded trash can in motion and pushes it since a stronger friction emerges between the driveway and his feet than between the driveway and the can's bottom.

the driveway against which he presses with his feet. These forces have the forward direction and are larger than the force exerted by the can. For this reason, the man also moves forwards. If the can is overloaded, the man, even by pressing hardly against the ground with his feet, cannot develop a sufficiently large force to shift the can. After the can has been shifted and a uniform motion is established, the force $\mathbf{f}_1$ will be balanced by the forces $\mathbf{f}_2$ (Newton's first law).

A similar problem arises when we analyse the motion of a train pulled by an electric locomotive. As in the previous example, the motion is possible here only due to the fact that in addition to the forces of interaction between the two bodies, the body being pulled or pushed experiences the action of forward forces exerted by the driveway or rails. Neither the man, nor the train or a motor car can start to move over a perfectly smooth surface from which it is impossible to push off.

Newton's third law allows us to analyse the *phenomenon of recoil* during a shot. Let us fix a model of a steam-operated (Fig. 73) or spring-loaded gun to a cart. Suppose that at the initial moment the cart is at rest. After the shot, the "shell" (cork) flies in one direction, while the "gun" rolls back. The recoil of the gun is just the reaction of the shell which, according to Newton's third law is exerted on the gun firing the shell. Accord-

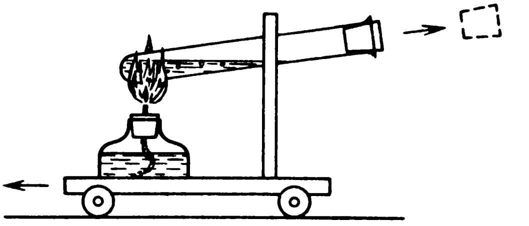

Fig. 73.
When a test tube with water is heated, the cork flies in one direction while the "gun" moves in the opposite direction.

ing to this law, the force exerted by the gun on the shell is always equal to the force exerted by the shell on the gun and has the opposite direction. Thus, the accelerations acquired by the shell and the gun also have opposite directions, their magnitudes being inversely proportional to their masses. As a result, the gun and the shell acquire opposite velocities in the same ratio. We denote the velocity acquired by the shell by $\mathbf{v}$ and that acquired by the gun by $\mathbf{V}$, while their masses are denoted by m and M respectively. Then

$$v/V = M/m.$$

Here v and V are the magnitudes of the velocities.

A shot fired from any gun is accompanied by a recoil. Ancient guns rolled back after a shot had been fired. In modern guns, the barrel is fixed not rigidly to the gun-carriage but with the help of appliances which allow the barrel to move back. Then the springs return it to the initial position. In automatic fire-arms, recoil is used for reloading the weapon. When a shot is fired, only the breech-block moves back. It ejects an empty cartridge-case, and then the springs return it to the original position and introduce a new cartridge into the barrel. This principle is used not only in machine-guns and automatic pistols but in rapid-firing guns as well.

2.20. Momentum of a Body

The basic laws of mechanics, viz. Newton's second and third laws, make it possible to solve any problem in mechanics. In the following sections we shall show that the application of Newton's laws for solving problems can be often simplifed by using the following corollary of Newton's second law.

Let us apply a constant force $\mathbf{F}$ to a body of mass m. Then the acceleration of the body will also be constant:

$$\mathbf{a} = \mathbf{F}/m. \tag{2.20.1}$$

Suppose that at the initial moment of a time interval t over which the force acted the velocity of the body was $\mathbf{v}_0$, while at the final moment of this interval the velocity became $\mathbf{v}$. We recall formula (1.27.2) applicable in the case of constant acceleration:

$$\mathbf{a} = (\mathbf{v} - \mathbf{v}_0)/t.$$

It follows form this formula and (2.20.1) that

$$m\mathbf{v} - m\mathbf{v}_0 = \mathbf{F}t. \qquad (2.20.2)$$

The product of the mass of a body and its velocity is called the momentum of the body. Momentum is a vector quantity since velocity is a vector. According to formula (2.20.2), *the increment of momentum of a body under the action of a constant force is equal to the product of the force and the time of its action.* If the force is not constant, formula (2.20.2) is applicable only for small time intervals such that the force has no time to change noticeably in magnitude or direction. For a large change in the force, formula (2.20.2) can also be used, but instead of force $\mathbf{F}$ we must take the value of the force averaged over the time interval under consideration.

In the case of a rectilinear motion along the x-axis, we can project the vectors appearing in formula (2.20.2) onto this axis. Then this formula can be written in a scalar form:

$$mv_x - mv_{0x} = F_x t. \qquad (2.20.3)$$

Here v_x, v_{0x} and F_x are the x-projections of the vectors $\mathbf{v}$, $\mathbf{v}_0$ and $\mathbf{F}$.

Since in the case under consideration all the three vectors are arranged along the x-axis, each projection is equal to the magnitude of the corresponding vector with the plus sign if a vector is directed along the axis and with the minus sign if the direction of the vector is opposite to the direction of the axis. Thus, the sign of a projection indicates the direction of the corresponding vector. If, say, v_x is positive (i.e. $v_x = v$), this means that vector $\mathbf{v}$ is directed along the x-axis. If F_x is negative (i.e. $F_x = -F$), this means that the force is directed against the x-axis, and so on.

2.21. System of Bodies. Law of Momentum Conservation

So far, we have considered only the action of forces on a single body. In mechanics, we often encounter problems when several bodies moving differently are to be considered simultaneously. For example, the problems on the motion of celestial bodies, on collisions, recoil of fire-arms where a shell and a gun start moving after a shot, and so on are problems of this type. In this case, we speak of the motion of a *system of bodies,* like the Solar system, a system of two colliding bodies or a gun-and-shell system. Some forces act between the bodies comprising a system. These are the forces of universal gravitation in the Solar system, elastic forces in a system of colliding bodies, and the force of pressure of powder gas in the gun-and-shell system.

In addition to forces exerted by some bodies of a system on some other bodies ("internal" forces), the bodies of the system may be acted upon by forces exerted by bodies which do not belong to the system ("external" forces). For instance, colliding billiard balls experience the action of the force of gravity and elastic force of the table, a gun and a shell are also acted upon by the force of gravity, and so on. However, external forces can be neglected in certain cases. For example, when two balls rolling over a table collide, the forces of gravity acting on each ball are balanced and hence do not affect their motion. When a shot is fired, the force of gravity exerts its action on a shell only when it leaves the barrel, but will not affect the recoil. Therefore, we can often analyse the motion of a system of bodies by assuming that external forces are absent.

Let us start with a simple system consisting of only two bodies. Suppose that their masses are m and M, and velocities are $\mathbf{v}_0$ and $\mathbf{V}_0$. We shall assume that external forces do not act on the system, but the bodies can interact. As a result of interaction (say, collision), the velocities of the bodies will change and become $\mathbf{v}$ and $\mathbf{V}$ respectively. For the body of mass m, the increment of momentum is $m\mathbf{v} - m\mathbf{v}_0 = \mathbf{F}t$, where $\mathbf{F}$ is the force exerted on it by the body of mass M and t is the time of interaction. For the body of mass M, the increment of momentum is $M\mathbf{V} - M\mathbf{V}_0 = -\mathbf{F}t$, since, according to Newton's third law, the force exerted by the body of mass m on the body of mass M is equal and opposite to the force exerted by the body of mass M on the body of mass m. Summing up these expressions for the increments of momenta, we obtain

$$m\mathbf{v} - m\mathbf{v}_0 + M\mathbf{V} - M\mathbf{V}_0 = 0,$$

whence

$$m\mathbf{v} + M\mathbf{V} = m\mathbf{v}_0 + M\mathbf{V}_0. \qquad (2.21.1)$$

Thus, in the absence of external forces, *the total momentum of a system* (*i.e. the vector sum of the momenta of bodies constituting the system*) *does not change as a result of interaction of the bodies.* In other words, *the internal forces do not change the total momentum of the system.* This result does not depend on the mode of interaction of the bodies in the system (or whether the interaction is short-term or long-term, takes place in contact or at a distance, and so on). In particular, this equality implies that if the bodies were initially at rest, the total momentum of the system remains zero after an interaction as well unless external forces act on the system.

It can be proved that *for a system consisting of more than two bodies, the total momentum remains constant as well provided that external forces are absent.* This very important statement is called the *law of momentum conservation.* It is one of the fundamental laws of nature which is not confined just to mechanics. If a system contains only one body, the law of momentum conservation for such a system indicates that the momentum of the body does not change in the absence of forces exerted on it. This is equivalent to the law of inertia (the velocity of the body remains unchanged).

2.22. Application of the Law of Momentum Conservation

Let us apply the law of momentum conservation to the problem about the recoil of a gun. Before the shot is fired, both the gun (of mass M) and the shell (of mass m) are at rest. This means that the total momentum of the system is zero (we can put the velocities $\mathbf{V}_0$ and $\mathbf{v}_0$ in formula (2.21.1) equal to zero). After the firing, the gun and the shell acquire velocities $\mathbf{V}$ and $\mathbf{v}$ respectively. The total momentum of the system must also be equal to zero after the shot has been fired (according to the momentum conservation law). Thus, immediately after the shot has been fired, the following equality holds:

$$M\mathbf{V} + m\mathbf{v} = 0 \quad \text{or} \quad \mathbf{V} = -\mathbf{v}m/M,$$

whence it follows that the ratio of the velocities acquired by the gun and the shell are in inverse proportion to their masses. The minus sign indicates that the velocities of the gun and the shell have opposite directions. This result has been obtained in a different way in Sec. 2.19.

Thus, we have solved the problem even without finding out the forces and duration of their action on the bodies of the system (this information would have been required if we calculated the velocity of the gun with the help of Newton's second law). Forces do not appear at all in the law of momentum conservation. This allows us to solve many problems in a simple way. This mainly concerns the problems where we are interested not in the process of interaction of bodies in a system but rather in the final result of this interaction like in the case of a gun-and-shell system. Naturally, if forces are unknown, some other quantities concerning the motion must be given. In the above example, we must know the velocity of the shell after the firing in order to determine the velocity of the gun.

If the time of interaction between the gun and the shell has been measured, we can find the average force which acted on the shell. If the time is equal to t, the average force is $\mathbf{F}_{av} = m\mathbf{v}/t$. The average force of the same magnitude (but having the opposite direction) acted on the gun.

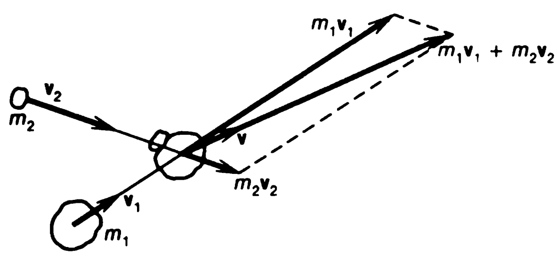

Fig. 74.
Composition of momenta in inelastic collision of two bodies.

Let us consider another important problem which can also be solved with the help of the momentum conservation law. This is the problem on *inelastic collision* of two bodies, i.e. the interaction as a result of which the bodies move with the same velocity (this is the case of collisions between two pieces of soft clay, which stick to each other after interaction and move as a single entity).

Suppose that a body of mass m_1 has a velocity $\mathbf{v}_1$ and a body of mass m_2 has a velocity $\mathbf{v}_2$ before collision. We assume that the external forces are absent. After the collision, the two bodies move together with a certain velocity $\mathbf{v}$ which is to be determined. We can easily find the total momentum by vector addition, as shown in Fig. 74. The component vectors are the momenta of the bodies before the collision. The required velocity is obtained by dividing the total momentum of the bodies by their total mass:

$$\mathbf{v} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}. \qquad (2.22.1)$$

If the bodies move along the same straight line before the collision, after the collision they will also move along the same straight line. We take this line as the x-axis and project the velocities onto this axis. Then formula (2.22.1) can be written in the scalar form:

$$v_x = \frac{m_1 v_{1x} + m_2 v_{2x}}{m_1 + m_2}. \qquad (2.22.2)$$

In this formula, each projection is equal to the magnitude of the corresponding vector taken with the plus sign if a vector is directed along the x-axis and with the minus sign if the direction of a vector is opposite to the x-axis (cf. formula (2.20.3)).

? **2.22.1.** A man weighing 60 kg runs along the railroad at a velocity of 6 m/s, jumps onto a stationary cart having a mass of 30 kg and resting on the rails, and comes to a stop with respect to the cart. What is the velocity with which the cart starts to move along the rails?

2.23. Free Fall of Bodies

If we drop a stone and a paper ball from the same height simultaneously, the stone reaches the ground before the paper ball does. Everyday experience of this type apparently leads us to the conclusion that heavy bodies fall under the action of gravity more rapidly than light bodies. This erroneous conclusion was drawn by the great Greek philosopher Aristotle (384-322 BC) in ancient times and existed for almost two thousand years. Only in 1583, Galileo refuted Aristotle's opinion on the basis of a more thorough experimental investigation of the laws of free fall. He learned that under normal conditions bodies not only fall under the action of the force of gravity but are also acted upon by air resistance (Sec. 2.39), and the actual law of fall under the action of the force of gravity alone is *distorted* by air resistance. Galileo established that all bodies fall with a uniform acceleration in the absence of air resistance, and (which is very important!) *the acceleration of all falling bodies is the same at a given point on the surface of the Earth.*

Air resistance distorts the laws of free fall since it mainly depends on the size of a body. For a feather, for example, air resistance is stronger than for a pellet, while the attraction by the Earth is weaker for the feather than for the pellet. For this reason, air resistance lowers the velocity of the falling feather to a much greater extent than the velocity of the pellet. In vacuum, however, all bodies fall with the same acceleration irrespective of their size, material, and so on. The experiment on falling of bodies in an evacuated glass tube (Fig. 75) confirms this conclusion. A feather and a pellet are placed in the tube. If the tube contains air under atmospheric pressure, and the feather and the pellet start to fall from the same height (for this purpose, the tube with the two bodies lying on its bottom must be overturned), the feather considerably lags behind the pellet. If we evacuate the tube and then repeat the experiment, the feather and the pellet reach the bottom simultaneously, and hence they fall with the same acceleration.

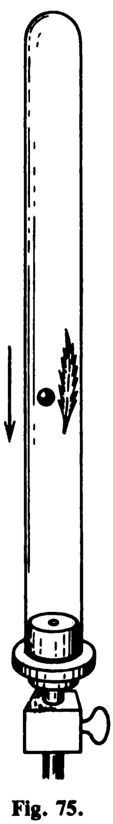

Fig. 75.
A feather falls as rapidly as a pellet in a tube from which air has been pumped out.

If air resistance is so low that it can be neglected, the body released from a support or suspender falls practically only under the action of attraction by the Earth (*free fall*). The attractive force does not remain strictly constant during the fall. It depends on the elevation of the body above the surface of the Earth (Sec. 2.27). If, however, a body falls from not very large height, the attractive force can be assumed to be virtually constant since the change in the height of a falling body is small in comparison with the Earth's radius which is approximately equal to 6400 km. Therefore, we can assume that the acceleration of a freely falling body remains constant under normal conditions and *free fall is a uniformly accelerated motion.*

2.24. Free Fall Acceleration

To the maximum attainable degree of accuracy, experiments show that all bodies at a given site on the Earth fall in vacuum with the same constant acceleration. This acceleration is denoted by g. At different points of the globe (at different latitudes), the numerical values of g are different, varying between 9.83 m/s^2 at the poles and 9.78 m/s^2 at the equator. At the latitude of Moscow, $g = 9.81523$ m/s^2. The value of g equal to 9.80665 m/s^2 and corresponding to a latitude of 45° is conventionally taken as "normal" value. All these figures correspond to motions at sea level (Sec. 2.27).

The difference in the free fall acceleration at different points of the globe is, on the one hand, due to the fact that the Earth is not a perfect sphere, and on the other hand, due to the diurnal rotation of the Earth (the role of the second factor will be considered in detail in Sec. 6.9). Henceforth, we shall assume that $g = 9.81$ m/s^2, while for very rough calculations, we can put $g = 10$ m/s^2.

2.25. Falling of a Body with Zero Initial Velocity and Motion of a Body Thrown Vertically Upwards

Suppose that a body starts to fall freely from the state of rest. In this case, its motion can be described by the formulas of uniformly accelerated motion with zero initial velocity and acceleration g. We denote the initial height of the body above the ground by h, the time of its free fall from this height to the surface of the Earth by t, and the terminal velocity of the body when it hits the ground by v. According to formulas derived in Sec. 1.22, these quantities are connected through the following relations:

$$h = gt^2/2 = v^2/2g, \tag{2.25.1}$$

$$t = v/g = \sqrt{2h/g}, \tag{2.25.2}$$

$$v = gt = \sqrt{2gh}. \tag{2.25.3}$$

Depending on the type of a problem, one of these relations can be chosen.

Let us now consider the motion of a body to which a certain initial upward velocity v_0 has been imparted. In this problem, it is convenient to assume that the upward direction is positive. Since the free fall acceleration is directed downwards, the motion with a positive initial velocity will be uniformly decelerated with a negative acceleration $-g$. The velocity of this motion at instant t is given by the formula

$$v = v_0 - gt, \tag{2.25.4}$$

and the height above the initial point at this moment is determined by the relation [5]

$$h = v_0 t - gt^2/2. \tag{2.25.5}$$

When the velocity of the body becomes zero, the body reaches the upper point of its ascent. This occurs at the moment t for which

$$v_0 - gt = 0. \tag{2.25.6}$$

After this moment the velocity becomes negative, and the body starts to fall. Hence, the time of ascent is

$$t = v_0/g. \tag{2.25.7}$$

Substituting the time of ascent t into formula (2.25.5), we obtain the maximum height of ascent of the body:

$$h = \frac{v_0^2}{g} - \frac{v_0^2}{2g} = \frac{v_0^2}{2g}. \tag{2.25.8}$$

Further motion of the body can be considered as a fall with zero initial velocity (this case was analysed at the beginning of this section) from the height $h = v_0^2/2g$. Substituting this height into formula (2.25.3), we conclude that the velocity v of the body on reaching the ground, i.e. on returning to the point from which it was thrown upwards, is equal to the initial velocity v_0 of the body; naturally, now this velocity has the opposite (downward) direction. Finally, we see from formula (2.25.2) that the time for which the body falls from the upper point is equal to the time of ascent to this point.

? **2.25.1.** A body falls freely with zero initial velocity from a height of 20 m. [6] At what height will it have a velocity equal to half the velocity it has on reaching the ground?
2.25.2. Show that a body thrown vertically upwards passes through every point of its trajectory at a velocity of the same magnitude during its motion upwards and downwards.

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[5] In this formula, h plays the role of the x-coordinate laid off upwards along the vertical (see Footnote 10 in Chap. 1) — *Eds.*

[6] In all the problems air resistance should be neglected unless otherwise specified.

2.25.3. Determine the velocity of a stone dropped from a tower of height h when it hits the ground if it falls (a) with zero initial velocity; (b) with an initial upward velocity v_0; (c) with the same velocity directed downwards.
2.25.4. A stone thrown upwards has passed by the window in 1 s after the beginning of motion on its way up and in 3 s after the start on its way down. Determine the height of the window above the ground and the initial velocity of the stone.
2.25.5. A shell fired vertically from an anti-aircraft gun has covered only half the distance towards the target. Another shell fired from the second anti-aircraft gun hits the target. What is the ratio of the initial velocities of the second and first shells?
2.25.6. What is the maximum height which a stone thrown vertically upwards can reach if its velocity is reduced to half its initial value in 1.5 s after it has been thrown?

2.26. Weight of a Body

The force with which a body experiencing the force of gravity acts on a support or a suspender is called the *weight* of the body. If, in particular, a body is suspended from a spring balance, it acts on the spring balance with a force equal to its weight. According to Newton's third law, the spring balance acts on the body with the same force. If the body and the spring balance are at rest relative to the Earth, this means that the sum of the forces acting on the body is zero, and thus the weight of the body is equal to the force of attraction of the body by the Earth.

Weight is a result of attraction of the Earth, but it may differ from the force of attraction. Above all, this is the case when some other bodies (besides the Earth and suspender) act on a body. For example, a body suspended from a spring balance and immersed in water acts on the suspender with a much smaller force than the attraction of the Earth. Such cases will be considered in detail later (see Chap. 7). We shall now analyse the change in the weight of a body depending on the acceleration of the body and the suspender.

We suspend a load from a spring balance and mark its reading for the body and spring balance at rest. Then we rapidly lower the hand holding the spring balance with the load and stop the hand again. We shall see that at the beginning of motion, when the acceleration of the spring balance and load is directed downwards, the reading of the spring balance is *smaller*, while at the end of the motion, when the acceleration of the spring balance and load is directed upwards, it is *larger* than in the state of rest (Fig. 76). The explanation is provided by Newton's second law. If the load suspended from the spring balance is at rest, the upward elastic force exerted on it by the spring of the balance equals the force of gravity acting on the load in the downward direction so that the weight of the load is equal to the force of gravity. If, however, the load moves with a downward acceleration, the spring of the balance exerts a force smaller than in equilibrium, i.e. smaller than the force of gravity. Therefore, the weight of the load turns out to be smaller than in the case of the spring balance and the load at rest. On the contrary, if the load moves with an upward acceleration, this means that

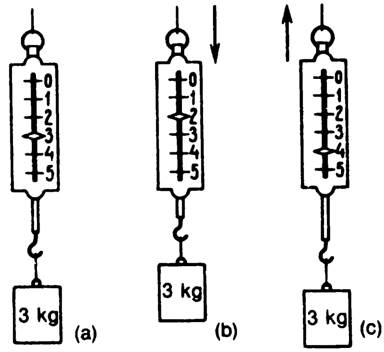

Fig. 76.
The weight of a load is smaller when the hand starts descending (*b*) and larger when it comes to a halt (*c*) than the weight indicated by a spring balance at rest (*a*). Arrows show the directions of accelerations.

the force exerted by the spring of the balance is larger than the force of gravity. Hence the weight of the load will be larger than for the balance and the load at rest.

Thus, although the force of gravity does not depend on whether a spring balance and a body being weighed have an acceleration relative to the Earth, the weight of the body is found to depend on the acceleration of the balance and the body. For this reason, it is always necessary to find out whether the balance and the body being weighed are at rest or have an acceleration. [7]

Although the weight of a body at rest is equal to the force of gravity, these two forces should be clearly distinguished: the *force of gravity* is exerted on the *body itself* which is attracted by the Earth, while the *weight* of the body is the force acting on the *suspender* (or support).

Besides weighing a body with the help of a spring balance, another weighing procedure can be used. It consists in a direct comparison of the weights of loads and the body with the help of an equal-arm lever (*beam balance,* Fig. 77). The equal-arm lever is in equilibrium when equal forces act on its two ends. Therefore, if the body being weighed is suspended from one end of an equal-arm lever and standard loads chosen so that the lever is in equilibrium from its other end, the weight of the body is equal to the total weight of the loads.

Beam balance allows us to determine the weight of a body with a much higher accuracy than by using an ordinary spring balance. A high-precision beam balance gives the weight of a body to within 1×10^{-8} of the quantity being measured.

Balances with arms of unequal length (for example, medical weighing scales) are also widely used. The weight of the body being weighed is then

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[7] While weighing bodies, we are interested, as a rule, in their masses rather than the weights. — *Eds.*

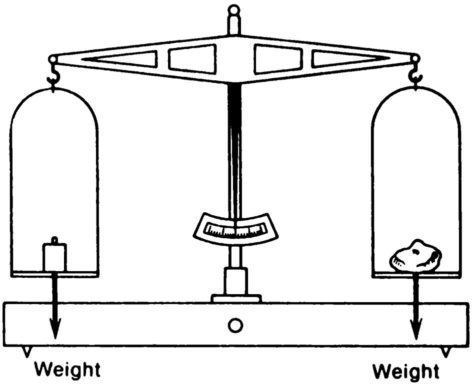

Fig. 77.
A comparison of the weight of a body and the standard weights with the help of a beam balance.

equal to the weight of the loads balancing it on these scales multiplied by the ratio of the lever arms (by ten for medical weighing scales). Such scales can be used for weighing heavy bodies with the help of relatively small loads.

? **2.26.1.** Stand on medical weighing scales and balance your weight by loads. Then squat abruptly. Explain the changes in the reading of the balance caused by your action.
2.26.2. Will the reading of a spring balance with a load suspended from it change if the hand holding the balance moves uniformly downwards?

2.27. Mass and Weight

It has been shown (see Sec. 2.24) that all bodies, irrespective of their mass, fall at a given point of the Earth with the same acceleration $\mathbf{g}$. The interpretation of this result with the help of Newton's second law leads to a very important conclusion: if a body of mass m moves due to the attraction of the Earth with an acceleration $\mathbf{g}$, this means that the force of gravity for the given body is

$$\mathbf{P} = m\mathbf{g}. \tag{2.27.1}$$

The force of gravity is proportional to the mass of the body on which it acts.

If the body is at rest, its weight $\mathbf{G}$ is equal to the force of gravity acting on it. Therefore, we can write for the weight

$$G = mg$$

(this formula is written for the magnitudes of the corresponding vectors). Consequently, the weights of bodies at rest are proportional to their masses so that for two bodies having masses m_1 and m_2 and weights G_1 and G_2, the following equality is valid:

$$m_1/m_2 = G_1/G_2. \tag{2.27.2}$$

This relation is used for comparing the masses of bodies with the help of a beam balance or a spring balance (see Secs. 2.14 and 2.26).

However, the acceleration due to free fall is different at different points of the Earth. Therefore, the weight of the same body will be different at different points on the Earth's surface. The weight of a body decreases as it is elevated above the ground (by 0.0003 of its initial value upon an ascent by 1 km). For this reason, we can compare the masses of bodies by weighing only if the bodies being compared are at the same site. This condition is satisfied automatically for beam balance, while for spring balance it can be violated. A spring balance can be graduated by suspending different loads at some point of the globe and then taken to another place where a body whose mass is being measured is suspended from it. If free fall accelerations are different for these points, the readings of the balance will not be exactly proportional to the masses of the bodies.

2.28. Density of Substances

It was noted above (see Sec. 2.14) that bodies having equal volumes and made of different materials, say, iron and aluminium, have different masses. The masses of continuous (i.e. without voids) homogeneous bodies (i.e. such that their properties, particularly the substance of which they are made, are indentical at all points) are proportional to their volumes. In other words, the ratio of the mass of such a body to its volume is a constant quantity typical of a given substance. This quantity is known as the *density* of substance. We shall denote it by ρ. By the definition,

$$\rho = m/V, \tag{2.28.1}$$

where m and V are the mass and volume of a body. It can also be said that the density is equal to the mass of a unit volume of the substance. If we

Table 1. Densities of Some Substances

Substance	ρ, 10^3 kg/m^3	Substance	ρ, 10^3 kg/m^3
Cork	0.24	Glass	2.50
Pine wood	0.48	Aluminium	2.70
Petrol	0.70	Marble	2.70
Oak	0.80	Zinc	7.14
Ethanol	0.80	Iron	7.80
Ice	0.90	Brass	8.50
Paraffin	0.90	Copper	8.90
Water	1.00	Lead	11.40
Graphite	2.10	Mercury	13.60
Concrete	2.20	Gold	19.30

know the density ρ of the substance and the volume V of the body, its mass can be found from the formula $m = \rho V$.

For the unit of density we take the density of a substance whose unit volume has a mass equal to unity. The SI unit of density is a *kilogram per cubic metre* (kg/m^3). Table 1 contains the densities of some solid and liquid substances. The data are rounded for substances which do not have a strictly definite density (like wood, concrete or petrol).

2.29. Emergence of Deformations

It was mentioned above that elastic forces appear between two bodies only if the bodies are *deformed.* A string acts on a cart with some force because it is stretched, a locomotive pulls a carriage since its buffer springs are compressed, and so on. Elastic forces are determined by a deformation so that they increase with deformations (Sec. 2.8). The question about the *origin of deformations* could not be answered earlier since the emergence of deformations can be explained only on the basis of the laws of motion. Indeed, deformations emerge because different parts of a body move in different ways. If all parts of a body moved similarly, the body would preserve its initial shape and size, i.e. would remain undeformed.

Let us take a soft eraser and press it with a finger (Fig. 78). The finger pressing the eraser displaces its upper layers. The lower layer resting on the table remains stationary since it is in conatct with the surface of the table which is much more rigid than the eraser. Different parts of the eraser are displaced differently, and the eraser changes its shape: a deformation emerges. The deformed eraser exerts some force on the bodies in contact with it. The finger clearly feels the pressure of the eraser. If we remove the finger, the eraser assumes its original shape.

All the bodies dealt with in the above experiments behave in the same way: when deformations occur in them, they exert on the bodies in contact with them a corresponding force. When the body returns to the undeformed state, the forces cease to act. As was mentioned above, such forces are known as *elastic* forces. The bodies in which these forces emerge are called elastic bodies.

There exist bodies which exert forces only as long as their shape and size

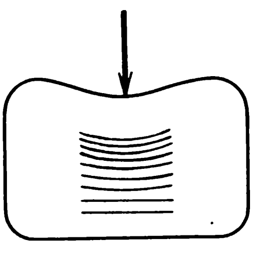

Fig. 78.
When a soft eraser is pressed by a finger, its upper layers move down while the lower layers remain at rest.

change. When the shape of such a body stops changing, the force vanishes although the body is in a deformed state. Examples of such bodies are soft clay and heated wax. Bodies of this type are known as *plastic* bodies.

Let us now consider in greater detail the types of deformations in bodies and elastic forces emerging in them in different cases: under the action of forces emerging in direct contact (contact forces) and the force of gravity. The case when all the forces acting on a body are mutually balanced and the body remains at rest (or moves by inertia; for the sake of simplicity, we shall speak of the state of rest) and the case of accelerated motion will be analysed separately.

2.30. Deformations in Stationary Bodies Caused Only by Contact Forces

We shall study the emergence of deformations in a body of a simple shape, say, a block, under action of forces directed along the body. In this case, the deformation pattern is simple. Suppose that two equal and opposite forces **F** are applied to a body as shown in Fig. 79. Then these forces are mutually balanced, and the block remains at rest. However, its ends start to move under the action of the applied forces, and the block becomes deformed or stretched.

Let us mentally cut the block into two parts as shown by dashed line in Fig. 79 (the "cut-off" parts are displaced relative to each other for the sake of clarity). Since these parts are deformed they exert on each other equal and opposite elastic forces. Thus, elastic forces emerge not only between different bodies, but also between parts of the same body. Obviously, when these elastic forces become equal in magnitude to the external force **F**, the block does not extend any further and each of its parts will be in equilibrium under the action of the external force and the elastic force exerted by the other part of the block. Wherever we mentally cut the block, the elastic force exerted by one part on the other will be the same and equal in magnitude to force **F**. This means that the block is stretched *uniformly*: the deformation is the same in all parts, and elastic forces acting between the parts are the same along the entire length of the block.

A similar situation arises if we compress the block by two equal forces, the only difference being that the deformation will be of the compression

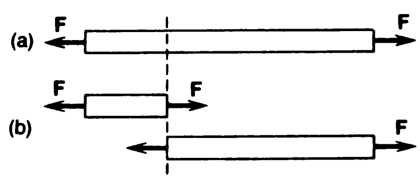

Fig. 79.
Elastic forces in a stretched block.

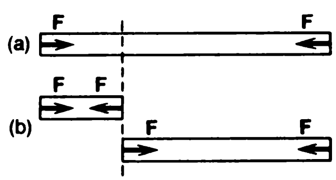

Fig. 80.
Elastic forces in a compressed block.

and not of the extension type, and the elastic forces will repel the two parts of the block instead of pulling them together (Fig. 80). Naturally, when we stretch a rigid (say, a metallic) block in practice, we cannot notice its extension by naked eye since it is very small. If, however, we take instead of a rigid block its "soft model" in the form of a weak cylindrical spring (such a spring can be easily made, for example, by winding a wire around a pencil), the deformation of such a model will be large, and the entire pattern of the uniform extension becomes visual. For the sake of clarity, we shall consider the deformations of a spring instead of deformations of a block in the following sections as well. The difference in their behaviour under the action of the same forces will be such that deformations for the spring will be much larger than for the block and can be easily observed.

2.31. Deformations in Stationary Bodies Caused by the Force of Gravity

Let us consider the emergence of deformations in the case when,in addition to contact forces, a body at rest experiences the action of the force of gravity.

Let us take a soft cylindrical spring and slowly put one of its ends on a table. The spring turns out to be compressed (Fig. 81). This deformation emerges as follows. As soon as the lower turn of the spring touches the table, it stops moving, while the upper turns continue to move down and come closer to the lower turns. The spring is compressed and as a result elastic forces appear. The motion of the upper turns ceases only when the elastic force emerging as a result of compression becomes equal to the weight of the upper turns at any part of the spring. The turns of the spring must be compressed the stronger, the lower they are so that the elastic force exerted by them balance the weight of the larger number of turns.

Thus, under the action of the force of gravity, a body resting on a support is deformed *nonuniformly*, and hence the emerging elastic forces are also distributed nonuniformly along the body: deformations and elastic forces are maximal in the lower part near the support and gradually decrease to zero at the upper, free end of the spring. Similarly, a spring suspended by its upper end turns out to be stretched (Fig. 82), the extension of the turns being the larger, the closer they are to the point of suspension.

Like the spring, any other body resting on a support or suspended from above turns out to be compressed or stretched respectively. It is just because the body is deformed that it exerts a certain force on the support or suspender. The support (or suspender) experiences the action of the force due to the deformation of the body (and called the weight, see Sec. 2.26)

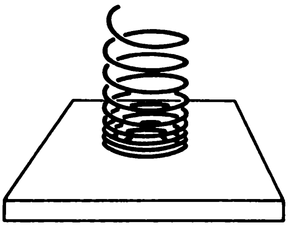

Fig. 81.
Nonuniform compression of a spring.

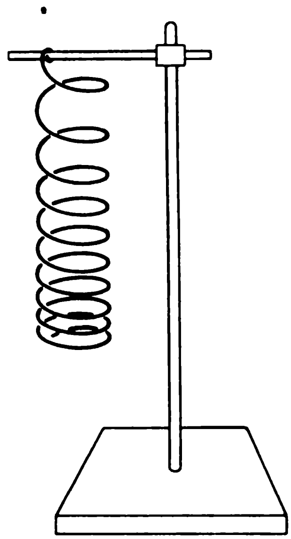

Fig. 82.
Nonuniform extension of a spring.

and not the force of gravity (which acts on the body itself). The force of gravity is only responsible for the emergence of deformations.

The support on which a body rests (Fig. 83) or a suspender from which it suspends turns out to be deformed together with the body itself. The extension of the spring of a balance with a load suspended from its hook is an example of deformation of the suspender. The force exerted by the support (or suspender) on the body is the elastic force emerging in the deformed support (or suspender). The body is found to be in equilibrium under the action of this elastic force and the force of gravity acting on it. Each part of the body is also in equilibrium under the action of the force of gravity and the elastic forces exerted on a given part of the body by the adjoining parts.

2.32. Deformation of a Body Moving with an Acceleration

Let us analyse the pattern of deformations in a body experiencing the action of an accelerating force. The pattern of deformations considerably depends on whether the acceleration is imparted to the body by a contact force (like the elastic force exerted by another body) or by the force of gravity.

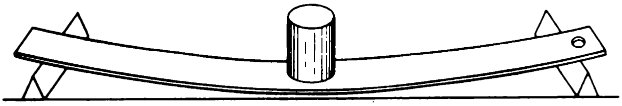

Fig. 83.
The sagging of a support.

Elastic forces exerted by a deformed accelerating body cannot impart an acceleration to the inner parts of the body being accelerated. This means that the body being accelerated can start to move as a single entity only after deformations have emerged in its interior together with elastic forces which impart the required acceleration to the inner parts of the body. Thus, a body moving with an acceleration under the action of contact forces is deformed in all cases. These deformations are just the cause of the force exerted by the body being accelerated on the accelerating bodies in contact with it. According to Newton's third law, we could state that this "reaction" must be equal and opposite to the "action", i.e. the force accelerating the body. But now we are also in a position to explain the physical nature of this "reaction": it emerges since a body being accelerated by a contact force always turns out to be deformed. Thus, action and reaction emerging as a result of direct contact of bodies are of the same origin—these are elastic forces.

In order to determine the distribution of deformations in a body being accelerated, we shall consider again the example of a block (or its spring model). Thus, let us suppose that a force is applied to the one end of a body as shown in Fig. 84. We again mentally cut the block into two parts. The elastic force exerted by the part of the body acted upon by the accelerating force has to impart the acceleration to the remaining part of the body. However, the acceleration of all parts of the body is the same. Consequently, the closer the cut to the point of application of the force, the larger the part of the block (and hence the larger the mass) to which acceleration is imparted by elastic forces. Therefore, the largest deformation and the strongest elastic force emerge at the point of application of force, and deformation and elastic force decrease along the block towards its free end.

Such a distribution of deformations and elastic forces is similar to their distribution in a suspended block under the action of the force of gravity. If the acceleration imparted to it by the force were equal to g, the deformations and elastic forces would be exactly the same in both cases. If the acceleration were twice as large as g, the elastic forces in all cross sections of the block would be doubled. If the acceleration were $g/2$, the elastic forces would be equal to half the initial value. But these forces would change in the same proportion in each cross section, which means that their distribution in the body remains unchanged, i.e. it is the same as in a suspended body under the action of the force of gravity.

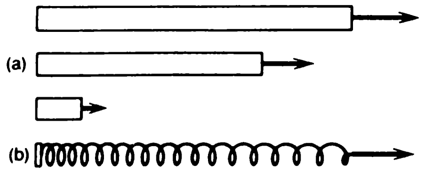

Fig. 84.
(*a*) Distribution of elastic forces along a block accelerated by a force applied to its end; (*b*) if we take a soft spring instead of the block, the nonuniformity of deformations along the body becomes visual.

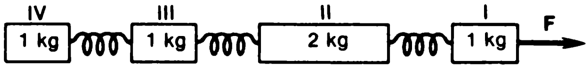

Fig. 85.
To Exercise 2.32.1.

Similar arguments are also applicable to the case when a force "pushes" and not "pulls" a body. In this case, however, we must compare the deformations of the block being accelerated with the deformations in a block resting in the vertical position on a support. The conclusions drawn for the first case also remain valid for the second case. We have limited ourselves to the simple case of a block with a constant force applied at one of its end faces. A similar picture will be observed in more complex cases.

? **2.32.1.** A "train" of loads connected through springs is set in an accelerated motion by a constant force (Fig. 85). The tensile force in the spring between the loads II and III is 10 N. Assuming that the force of gravity is absent and neglecting the masses of the springs, find the force acting on the "train" and its acceleration.

2.33. Vanishing of Deformations in Free Fall

A quite different situation takes place when the only force imparting an acceleration to a body is the force of gravity, i.e. when the body falls freely. It was shown above that if a body acted upon by the force of gravity is at rest (for this it must be either suspended or put on a support), it turns out to be deformed (Sec. 2.31). If, however, the body starts to fall freely (when, for example, we burn through the string from which the spring is suspended), we can notice that the deformation in the spring rapidly vanishes and the spring remains in the undeformed state up to the end of the free fall.

The vanishing of deformation during free fall can be easily explained if we replace the spring by a system of two bodies connected through a light spring (Fig. 86). As long as one body is suspended from a string attached to the upper body, the string and spring are stretched. The string acts with an upward force on the upper body, while the spring exerts a downward force on it and an upward force on the lower body. These forces are such that they balance the forces of gravity acting on each body (the mass of the spring is ignored), and the two bodies remain at rest (the spring acts with a force equal to the weight of the lower body, while the string exerts a force equal to the weight of both bodies).

Let us burn through the string supporting the system. At first the forces exerted by the stretched spring will act on the two bodies in addition to the force of gravity. Since the force applied to the upper body is directed downwards, this body starts falling with an acceleration greater than the

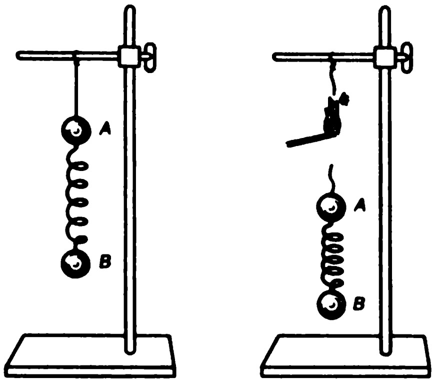

Fig. 86.
After the string has been burnt through, ball A moves with an acceleration higher than g, while ball B moves with an acceleration lower than g, the balls come closer, and the deformation of the string vanishes.

free fall acceleration g. On the contrary, the spring exerts on the lower body a force in the upward direction, as a result of which the lower body falls with an acceleration smaller than g. Therefore, the upper body will approach the lower body, the spring will be compressed, and the force exerted by it on the bodies decreases. When the spring is compressed to its normal (undeformed) length, it stops acting on the bodies and they will experience only the action of the force of gravity. Thus, the two bodies will fall further with the same acceleration equal to g, and the spring remains in the undeformed state. [8]

All that has been said above about the springs is valid for all elastic bodies. As long as an elastic body acted upon by the force of gravity is attached to a suspender, it is necessarily deformed. When, however, the force exerted by the suspender ceases to act, deformations vanish, and the freely falling body is in the undeformed state. This is a manifestation of the basic difference between the force of gravity, which imparts the same acceleration to all elements of a body, and contact forces, which act only on certain parts of the surface of a body and hence (as has been shown above) cause deformations in the body being accelerated.

The same pattern of vanishing deformations is observed for a body which starts to fall freely together with the support on which it has been resting. The only difference is that the initial deformation is compression and not extension as in the case considered above. It should be stressed that deformations completely vanish in a body only in the case of free fall, when no other forces except the force of gravity act on it. If some other forces, say, the resistance of air, act on the body, deformations do not vanish completely.

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[8] Actually, the process is much more complicated since vibrations take place during the fall of a deformed spring.

The sensation experienced by a falling person (a parachutist at the beginning of the jump, before the parachute opens, a swimmer jumping into water, or a person in a lift when it starts descending at a high speed,etc.) is associated with a complete or partial vanishing of deformations. Under normal conditions, the organs in a human body are in a deformed state. In falling these deformations vanish or (as in the case of a rapidly descending lift) diminish. The absence of usual deformations causes a typical sensation experienced in a jump. This is a short-term sensation of weightlessness, which is well known to cosmonauts in an orbiting spacecraft.

2.34. Destruction of Moving Bodies

All bodies can be deformed only to a certain limit. When this limit has been attained, the body breaks down. For example, a string snaps when its elongation exceeds a certain limit, a spring breaks when it is bent too much, and so on.

In order to explain why a body breaks, we must analyse the motion of the body preceding the destruction. Let us consider, for example, the reasons behind the breaking of a string in the following experiment (Figs. 87 and 88). A heavy load is suspended by a string. Another string of the same strength is attached to the load from below. If we pull the lower string slowly, the upper string holding the body snaps. If, however, we tug abruptly at the lower string, this string and not the upper one will snap. The result of the experiment is explained as follows. When the load hangs motionlessly, the upper string is already stretched to a certain extent, and its tension balances the force of attraction of the load by the Earth. While pulling slowly the lower string, we cause a displacement of the load in the downward direction. In this case, both strings are stretched, but the upper one turns out to be stretched to a larger extent since it has been already

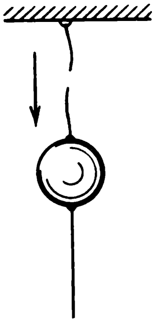

Fig. 87.
If the lower string is pulled slowly, the upper string snaps.

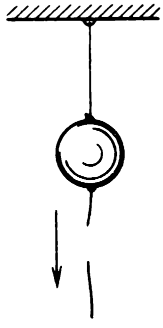

Fig. 88.
If the lower string is tugged abruptly, it can snap while the upper string remains intact.

stretched. For this reason, it breaks first. If we tug at the lower string abruptly, the load acquires a small acceleration due to its large mass even if we apply a considerable force, and hence the load has no time to acquire a noticeable velocity during a short time and cannot be displaced to a noticeable extent. Practically, the load remains at rest. Thus, the upper string is not stretched further and remains intact, while the lower string is stretched beyond the admissible limits and breaks.

The rupture and destruction of other moving bodies occur in a similar way. In order to avoid ruptures and failures as a result of an abrupt change in velocity, couplings should be used such that they can undergo considerable extension without breaking. Many types of couplings, such as steel ropes, do not possess such properties. For this reason in cranes a special spring ("shock absorber") is used between the rope and the hook. This spring can be elongated considerably without breaking and thus prevents the rope from breaking. A jute rope capable of being elongated to a considerable extent does not require a shock absorber.

Brittle bodies (like glass articles) falling on a hard floor break in a similar way. In this case, the velocity of the part of the body touching the floor sharply decreases, and deformations take place in the body. If the elastic force caused by this deformation is insufficient to immediately decrease the velocity of the remaining part of the body to zero, the deformation continues to grow. And since brittle bodies can withstand only small deformations without failure, the body breaks down.

? **2.34.1.** Why do couplings between the carriages of a train sometimes break at the moment the electric locomotive makes a move abruptly? In which part of the train is it most likely to occur?

2.34.2. Why are brittle articles put in sawdust for transportation?

2.35. Frictional Forces

It has already been mentioned earlier (see Sec. 2.5) that in addition to elastic forces, the forces of a different type can emerge in bodies which are in direct contact. These are *frictional forces.* The most typical property of friction is that it resists any attempt to put one object into motion relative to another or tends to slow the motion once the objects are moving relative to each other.

We shall illustrate some features of friction with the help of the following experiments. Let us take a round wooden body with hooks on its lateral surface (Fig. 89) and put it on a table. The body presses against the table with the normal force **N**. [9] Having attached to a hook the ring of a spring

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[9] The normal force is the normal component of the force with which a body acts on a plane in contact with it. — *Eds.*

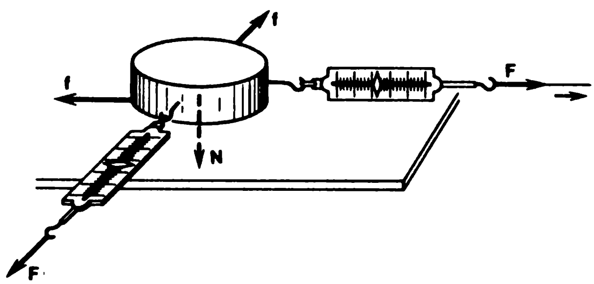

Fig. 89.
Friction **f** for different directions of the force **F** exerted by a spring balance.

balance, we pull the spring balance in the horizontal plane as shown in the figure. As long as the force exerted by the spring balance is sufficiently small, the body remains at rest. This means that in addition to the force **F** exerted by the spring balance, some other force **f** acts on the body, which balances the first force. This is friction. It is exerted by the table on the body and is applied to the surface of contact between them.

Since this force emerges when the body is still stationary, it is called the *static contact frictional force,* or *static friction.* We can slightly increase the force **F** and the body still does not move. This means that *static friction* **f** *has increased together with* **F** *so that it always remains equal to the applied force.* Static friction **f** can never exceed the applied force. Indeed, the motion of the body in a direction opposite to the force **F** is never observed under the action of the force **f**. If, however, we increase **F** still further, the body will ultimately acquire an acceleration and start to slide along the table in the direction of this force. It means that static friction turns out to be *weaker* than the applied force since it can grow only to a certain limit. This limit, i.e. the maximum static friction, can be determined from the reading of the spring balance immediately before the moment when the body begins to slide.

Having attached the spring balance to another hook, we can change the direction of the force **F** (Fig. 89). But in this case the body also remains at rest as long as the force is below the limit indicated above. This means that the direction of static friction **f** changes with the direction of **F**. Thus, both the magnitude and direction of static friction are determined by the magnitude and direction of the external force which it balances. *The static friction is equal and opposite to the external force which tends to cause the sliding of one body along the surface of another body.* In other words, static friction is opposite to the direction in which sliding would take place if static friction were absent.

When we speak about static friction, we usually mean the maximum value of this force. Let us find out the dependence of the maximum value of this force on the force with which bodies in contact press against each other. We shall load a body by weights having different masses and each time determine the maximum static friction. It will turn out that as the force N with which the body presses against the table changes (now this force is equal to the sum of the forces of gravity acting on the block and the weights), *static friction changes approximately in proportion to the force N* so that we can write

$$f = \mu N, \tag{2.35.1}$$

where μ is a constant. This quantity, which is equal to the ratio of the friction between given surfaces to the force with which the bodies are pressed against each other, is known as the *coefficient of static friction*:

$$\mu = f/N. \tag{2.35.2}$$

The coefficients of static friction are different for different materials. It follows from the definition that the coefficient does not depend on the choice of the system of units.

The friction coefficient for a given material is determined in practice from formula (2.35.2) by measuring the friction and the normal force exerted by bodies on each other separately. Since the coefficients of static friction depend on the materials of the two bodies, we have to determine them for each pair of different materials (friction of iron against wood or iron against iron). Friction coefficient is not a strictly definite quantity for a given pair of materials but depends on the quality of surfaces. Smooth finishing of surfaces considerably decreases the coefficient of friction.

Let us now increase the force **F** so that the body just starts sliding. After the body has been set in motion, we choose the external force so that the body slides uniformly along the surface of the table. This means that the friction emerging during sliding, known as *sliding* or *kinetic contact frictional force* (*sliding* or *kinetic friction*), is equal to the applied force. Having measured the applied force while maintaining the uniform sliding of the body along the surface, we note that it is normally less than the force required to shift the body: *kinetic friction can be smaller than static friction.*

In analogy with the coefficient of static friction, we introduce the *coefficient of kinetic* or *sliding friction*, which is defined by the same formula (2.35.2) where f stands for kinetic friction.

It can be easily verified in experiments that kinetic friction also depends on the surfaces rubbing against each other and, like static friction, increases with the normal force acting between the bodies. As the velocity increases, while the normal reaction remains unchanged, sliding friction usually changes. This means that the coefficient of sliding friction also depends on the velocity with which one body slides along the surface of the other body. In many problems, however, we can use an average value of the coefficient of sliding friction. For very low velocities it can be assumed to be equal to the coefficient of static friction.

Even when the force with which rubbing bodies press against each other is strong, they are in contact not over the entire surface but only in certain regions. This is due to microscopic roughness of the surface of a body which remains even after a thorough treatment of the surface. For this reason, friction acts only between individual regions. Adhesion forces emerge between regions in contact. These forces are directed opposite to the direction of sliding. In order to reduce sliding friction, lubricants are used. The mechanism of lubrication is the introduction of a liquid layer of a lubricant oil between two solid surfaces in contact. This changes the conditions of rubbing and reduces friction.

2.36. Rolling Friction

Let us take a wooden cylinder and put it on a table so that it is in contact with the table along the generatrix. We insert the ends of a two-pronged fork at the centres of the cylinder bases and attach to this fork a very sensitive spring balance having an easily stretchable spring (Fig. 90). If we pull the spring balance, the cylinder will roll over the table. The readings of the balance indicate that a very small pulling force is required to shift the cylinder from the state of rest and to move it uniformly. This force is much smaller than that required for sliding this cylinder without rolling. The force exerted by the table on the cylinder rolling over it is called *rolling friction*. For the same normal force acting on the table, *rolling friction is much smaller than sliding friction*. For example, when steel wheels roll over steel rails, rolling friction amounts to 1/100 of sliding friction. For this reason, in machines sliding friction is replaced by rolling friction whenever possible by using the so-called roller or ball bearings. One of such bearings is shown in Fig. 91.

The origination of rolling friction can be visually explained as follows. When a ball or a cylinder rolls over the surface of another body, it slightly

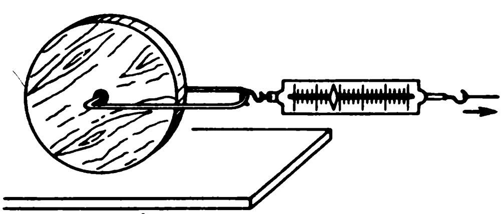

Fig. 90.
Measurement of rolling friction.

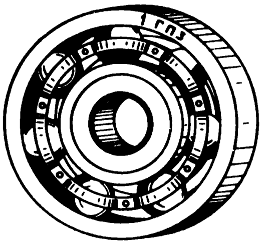

Fig. 91.
Ball bearing.

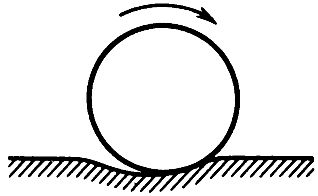

Fig. 92.
Deformations during rolling.

presses in the surface of this body, and is itself slightly compressed. Thus, a rolling body as if climbs a hill all the time (Fig. 92). At the same time, the regions of one surface are torn off from the other surface, which is prevented by the forces of adhesion acting between these surfaces. These two phenomena are responsible for rolling friction. The harder the surfaces, the less is indentation and the smaller the rolling friction.

2.37. Role of Friction

All bodies in contact move relative to each other with friction: the axle of a wheel experiences friction in the bearing while its rim is in friction with the rail; a door opens with a creak indicating that there is friction in its hinges, and a ball rolling over a horizontal plane comes to a halt under the action of rolling friction. When we exclude friction while solving a problem on the motion of a body, we simplify the problem and at the same time distort the actual situation. In all the experiments that were used to illustrate the laws of motion it was assumed that friction is absent. Actually, friction always affects to a certain extent the nature of motion.

The role of friction is not always limited to a deceleration of moving bodies. In many cases a motion, say, walking, becomes possible only due to friction, in particular, static friction. While walking, we put out feet so that they would slide back if static friction were absent (indeed, if we try to walk over smooth ice, our feet slide back). Since static friction acts in a direction opposite to that in which sliding would occur, the emerging static friction acts in the forward direction. It is just this force that imparts a forward acceleration to the body of a walking person.

The situation is the same for all self-propelled vehicles (like bicycle, motor car or electric locomotive). The engine of a vehicle causes the rotation of its drive wheels. If static friction were absent, the vehicle would stay at rest, and its wheels would slip so that the points of a wheel which are in contact with the ground or rail at a given moment would slip back. The emerging static friction exerted on the wheels by the ground is directed forwards and imparts to a vehicle an acceleration or sustains its uniform motion by balancing other forces acting on it. If this friction is too small (as, for example, on ice), the vehicle does not move and its wheels slip. On the contrary, if the rotation of the wheels of a moving vehicle is slowed down without decelerating the motion of the vehicle itself, in the absence of friction its wheels would slip in the forward direction over the ground. This means that friction indeed emerges and is directed backwards. The action of brakes is based on this phenomenon.

If a train is coupled to an electric locomotive, as soon as the locomotive starts to move in the forward direction the coupling will be stretched and an elastic force will emerge in it, which acts on the train. This is just the driving force. If we increase the force exerted by the engine on the wheels, the static friction will also increase, and so will the driving force. The maximum driving force is equal to the maximum static friction for drive wheels. If the force exerted by the engine is increased further, the wheels will slip, and the driving force may even decrease.

Static friction plays an equally important role in nonself-propelled vehicles. Let us consider the motion of a man pushing a trash can in more detail (Fig. 72). The man uses his legs and strains his muscles in such a way that in the absence of static friction the feet of the man would slip back. The emerging static friction $\mathbf{f}_2$ is directed forwards. On the other hand, the can pushed by the man with the force $\mathbf{F}_1$ is acted upon by the ground with a sliding friction $\mathbf{f}_1$ directed backwards. For the man and the can to acquire an acceleration, it is necessary that the static friction between the feet and the surface of the driveway be greater than the sliding friction acting on the can. But whatever the coefficient of friction between the feet and the ground, the static friction cannot be larger than the force that would cause the feet to slip (Sec. 2.35), i.e. bring the muscle force of the man into action. Therefore, even when the feet do not slip, the man still sometimes cannot set a heavy can in motion. During the motion (when the can starts to slide), friction slightly decreases. Therefore, it is often sufficient to help the man to shift the can for him to be able to push it further.

? **2.37.1.** Explain the role of friction in transmitting motion from one pulley to another with the help of a drive belt.

2.38. Resistance of Medium

If a solid body is inside a liquid or gas, its entire surface is in contact with the particles of the fluid. During its motion in a fluid, the body experiences the action of forces exerted by the fluid against the motion. These forces are called the *resistance of the medium*. Resistance of the medium is always directed ***against the motion*** and is treated as a kind of friction (it is also called *fluid friction force* or ***drag***).

The specific feature of friction in a fluid is the absence of static friction. A solid resting on another solid body can be set in motion only if a sufficiently strong force, which exceeds the maximum static friction, is applied to it. With a smaller force the solid will not be shifted however long this force would act. A different situation is observed if the body is in a fluid. In this case, a very small force is sufficient to set the body in motion: the body starts to move, although very slowly (Fig. 67). A man cannot shift at all a 100-t stone with his hands. At the same time, a barge of the same mass floating in water can be shifted by a man, although it will move very slowly (Sec. 2.15). However, the resistance of a medium sharply increases with speed, so that a given force cannot accelerate a body to a very high velocity however long it acts.

Let us now consider the effect of the medium on bodies falling in air.

2.39. Falling of Bodies in Air

A body of mass m falling in air moves under the action of two forces: a constant force of gravity $m\mathbf{g}$ directed downwards and the air resistance $\mathbf{f}$ increasing in the process of falling and directed upwards. The resultant of the force of gravity and the air resistance is equal to their vector sum and is directed downwards at the beginning of the fall.

As long as the velocity of the body is not high, the air resistance is also small. But as the velocity of falling increases, this force rapidly grows. At a certain velocity, the force $\mathbf{f}$ becomes equal in magnitude to the force $m\mathbf{g}$, and the body falls uniformly after that. The velocity of such a fall is called the *limiting velocity*. The limiting velocity is the higher, the more rarefied the air. Therefore, a body falling from a very large height may acquire in the upper (rarefied) layers of the atmosphere a velocity exceeding the limiting velocity for lower (denser) layers. Having entered the lower layers of the atmosphere, the velocity of the body decreases to the value of the limiting velocity for the lower layers.

? **2.39.1.** Is a body falling with the limiting velocity deformed?

The limiting velocity of falling bodies depends, besides the density of the atmosphere, on the shape and size of the body and on the force of attraction by the Earth. Small bodies, e.g. small water drops (mist), dust particles and snow flakes,rapidly attain their limiting velocity (which is of the order of a millimetre per second or lower) and then descend at this low velocity. A 10-g lead ball attains a limiting velocity of 40 m/s in falling from a considerable height. Rain drops fall with the velocity which is normally below 7-8 m/s. The smaller the drop, the lower the velocity of its falling. If rain drops fell in vacuum, they would attain a velocity of 200 m/s as a result of falling from a 2-km height, irrespective of their size. Any other body falling from the same height in vacuum would reach the same velocity. The impacts of rain drops having such a velocity would be quite unpleasant!

The difference in the limiting velocities of different bodies of the same shape but of different sizes is explained by the dependence of the resistance of medium on the size of a body. It turns out that the resistance is approximately proportional to the cross-sectional area of the body. For bodies of the same shape and made of the same material, the cross-sectional area, and hence the air resistance, increases with the size of the body at a smaller rate than the force of gravity (the cross-sectional area increases as the square of the linear dimension, while the force of gravity grows in proportion to the cube of the linear dimension). For example, the larger a bomb dropped from an aeroplane, the larger its limiting velocity and the higher the velocity at which it reaches the ground.

Finally, air resistance strongly depends on the shape of bodies (Fig. 93, see also Sec. 9.11). The fuselage of an aeroplane is shaped to a special streamlined form for which the air resistance is low. On the contrary, a parachutist must reach the ground at as low velocity as possible. For this reason, the parachute is made in the form for which the air resistance to its motion is as large as possible. The limiting velocity of a parachutist with an open parachute amounts to 5-7 m/s. The parachutist attains the limiting velocity in a different way in comparison with an ordinary fall. At first the parachutist falls with the packed parachute and attains the velocity of tens metres per second due to a low air resistance. As soon as the parachute is opened, the air resistance sharply increases, becomes several times larger than the force of gravity and decelerates the fall to the limiting velocity.

Air resistance also changes the nature of motion of bodies thrown up-

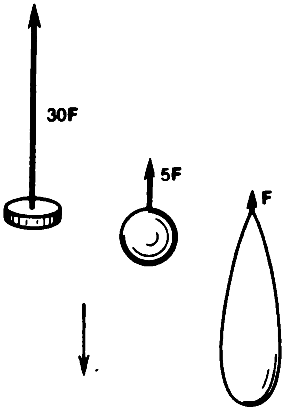

Fig. 93.
Air resistance to the motion of a drop-shaped body is equal to 1/30 of the resistance to the motion of a circular plate and to 1/5 of the resistance to the motion of a ball of the same cross-sectional area.

wards. As the body moves upwards, both the force of gravity and the air resistance are directed downwards. Therefore, the velocity of the body decreases at a higher rate than in the absence of air. As a result the body thrown up with an initial velocity v_0 does not reach the height $h = v_0^2/2g$ (as it would in the absence of air resistance) and starts to fall down at a smaller height. As it falls, the air resistance hampers the growth in velocity. Thus, the body thrown upwards returns at a smaller velocity than that with which it was thrown. Hence the average velocity of motion is smaller when the body falls in comparison with the average velocity of ascent, and thus the time of descent is longer than the time of ascent.

The effect of air resistance is especially strong for high velocities (since the air resistance rapidly grows with velocity). For example, a bullet fired from a rifle vertically upwards and having an initial velocity of 600 m/s would reach in the absence of air the height

$$\frac{600^2 \text{ m}^2/\text{s}^2}{2 \times 10 \text{ m/s}^2} = 18\,000 \text{ m}.$$

Actually, the bullet only reaches a 2-3-km height. In its falling to the ground the velocity of the bullet increases only to 50-60 m/s. The bullet hits the ground at this limiting velocity.

Chapter 3
Statics

3.1. Problems of Statics

It is well known that under the effect of forces exerted by other bodies, a body generally acquires an acceleration. In particular, a body at rest is set in motion. In some cases, however, a body experiencing the action of several forces can still remain at rest. It was shown above (Sec. 2.6) that if two forces acting simultaneously on a body at rest have equal magnitudes and opposite directions along the same straight line, the body does not acquire an acceleration and can remain at rest. In other cases, the equilibrium conditions for a body acted upon by forces turn out to be more complicated. The study of these conditions, i.e. the ***equilibrium conditions for bodies*** (or, which is the same, the equilibrium conditions for forces), constitutes the subject-matter of statics.

Thus, statics first of all makes it possible to determine equilibrium conditions for various engineering constructions: buildings, bridges, arcs, cranes, and so on. This, however, does not exhaust the practical importance of statics. It also provides the answers to some problems concerning the *motion* of bodies. Suppose, for example, that a load subject to the force of gravity **P** is suspended by a rope passing over a pulley (Fig. 94). Using the methods of statics, we can determine the force **T** which must be applied to the other end of the rope so that the load is at rest. This force

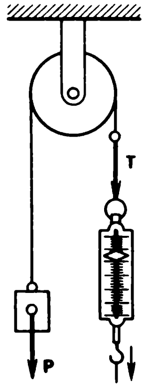

Fig. 94.
In order to lift a load, the force T must be larger than the force of gravity P acting on the load.

must be equal to the force of gravity **P**. This answer, however, contains in it something more than equilibrium conditions for the load. It indicates what should be done to make the load move upwards: for this purpose, it is sufficient to apply a force slightly exceeding **P** at the other end of the rope. Consequently, statics not only specifies the equilibrium conditions of bodies but also gives an idea of the direction of motion if the equilibrium of forces is disturbed in a certain way.

From the very outset, statics developed as a branch of mechanics which provided answers to simple questions concerning the equilibrium of bodies as well as their motion. Even in ancient times the problems involving the application of various mechanisms (levers, pulleys, and so on) were of utmost importance. Therefore, builders of that time were interested not only in the conditions under which a load is in equilibrium but also the conditions in which it would move in a certain direction, say, upwards. Statics was of practical importance for ancient engineers mainly because it could give an answer to this problem. True, statics does not give an idea about the velocity at which a load will rise, but the velocity of motion was not very important for ancient engineers. It was much later, when the efficiency of machines was analysed (Sec. 4.23), that the problem of the velocity of motion of various mechanisms became of practical importance, and statics turned out to be incapable of meeting the requirements of practical work.

3.2. Perfectly Rigid Body

Why does a load lying on a table remain at rest in spite of the fact that the force of gravity acts on it? Obviously, in addition to the force of gravity, other forces act on the load to balance this force. What are these forces?

The answer to this question is quite clear: the table exerts on the load an upward elastic force which emerges because the table is deformed. The deformation becomes evident if we take a thin flexible board as a support (see Fig. 83). This board exerts on the load a force equal to the force of gravity only for a comparatively large sagging. Since the table is more rigid, the sagging required for balancing the force of gravity is considerably smaller and cannot be noticed in an ordinary observation. However, using precise methods of observation, even such a small sagging can be made visual. If, for example, we place on the table mirrors reflecting a narrow light beam to the wall (Fig. 95), the mirrors become slightly inclined as a result of sagging of the table under the action of a load and the reflected light spot will shift on the wall. In the case of a more rigid table or, say, a steel slab, a direct observation of the deformation caused by a small load is even more difficult. However, we can be sure that a certain deformation does take place since it is this deformation that gives rise to the elastic force exerted by the slab and balancing the force of gravity. Although deforma-

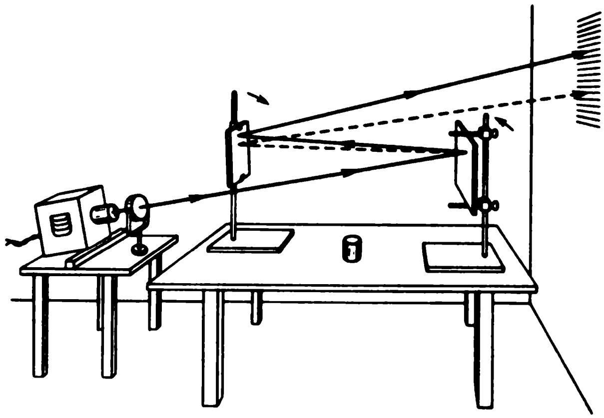

Fig. 95.
Optical method of determining a small sagging.

tions in these two cases are different, the emerging elastic force is the same. This follows from the fact that the load is at rest in both cases.

Bodies which exhibit only very small deformations under normal conditions are often encountered in practice. Such bodies can be used for manufacturing parts of machines, in construction of buildings, and so on. In most cases, we are interested in the force emerging due to a deformation rather than in the deformation itself. It has been shown earlier that the force turns out to be the same for bodies which have different values of rigidity and are deformed in different ways (for example, the board and the table). We can imagine a rigid body such that required forces emerge in it for as small deformations as desired. Therefore, a real body can be replaced by an imaginary *perfectly rigid body* which does not undergo any deformation at all.

Clearly, perfectly rigid bodies do not exist in nature. Nevertheless, the idea of such an imaginary body is found to be very useful. Assuming that the required force emerges in the body, we can neglect its deformation. In particular, we shall henceforth assume that parts of simple mechanisms, such as levers, wedges, pulleys, and screws, are perfectly rigid. We shall also assume that ropes, strings, and so on are perfectly nonstretchable.

3.3. Translation of the Point of Application of a Force Acting on a Rigid Body

It was shown in Sec. 2.6 that equal forces acting in opposite directions along a straight line balance each other. In this case, the point of the body on this straight line at which they are applied is immaterial. Figure 96 shows, by way of example, two cases of application of two equal and op-

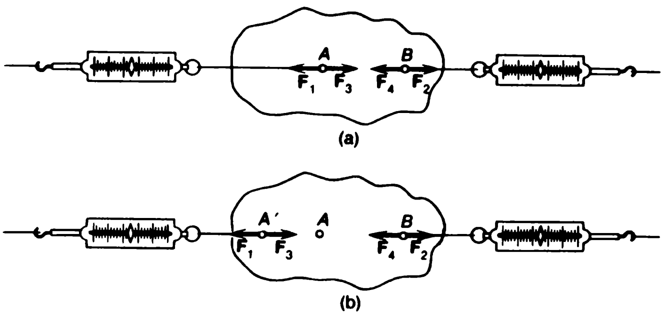

Fig. 96.
(*a*) Equal and opposite forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied to a body at points A and B. As a result, deformations and elastic forces $\mathbf{F}_3$ and $\mathbf{F}_4$ emerge in the body. (*b*) The equilibrium is not violated if the force $\mathbf{F}_1$ is translated from point A to point A'.

posite forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting along the same straight line. The two cases differ in the point of application of the force $\mathbf{F}_1$ (A or A'). The body remains at rest in both cases.

Thus, if two forces are in equilibrium, the point of application of a force can be translated along the line of its action without disturbing the equilibrium of the body. Experiments show that such a translation does not affect the action of a force in other cases as well. For example, a force applied to a body will impart an acceleration to the body as a whole wherever it is applied.

The point of application of a force can be translated along the line of its action without changing the effect of the force on the body as a whole. We cannot only translate the points of application of forces in actual practice, but also carry out this operation mentally in order to simplify the line of reasoning while solving problems. This approach is often used both for determining equilibrium conditions and for analysing the motion of a rigid body.

Although a translation of the points of application of forces does not alter their effect on the body as a whole, such a translation changes the distribution of deformations and elastic forces in a real body. Indeed, in the example considered above the forces applied at points A and B cause a deformation in the body: in the region between points A and B tension emerges and elastic forces $\mathbf{F}_3$ and $\mathbf{F}_4$ appear. These forces act between the parts of the body, balance external forces $\mathbf{F}_1$ and $\mathbf{F}_2$ and stop further deformations. If, however, force $\mathbf{F}_1$ is applied at point A', the tension now covers the region between points A' and B. In both cases, however, elastic forces $\mathbf{F}_3$ and $\mathbf{F}_4$ emerge even when deformations are negligible, and since we do not pay attention to the deformation (by assuming that the body is perfectly rigid), the difference in deformations does not play any role.

3.4. Equilibrium of a Body under the Action of Three Forces

In Sec. 2.12 we have already obtained the equilibrium conditions for a body under the action of three forces applied at the same point at an angle to one another. It turned out that for this all the three forces must lie in the same plane, and each force must be equal and opposite to the resultant of the two other forces.

In actual practice, however, forces are often applied at different points. Let us find out the equilibrium conditions in this case. For this purpose we shall use the same set-up with three loads which was described in Sec. 2.12, the only difference being that the strings from which the loads are suspended are fixed now at different points on a light cardboard as shown in Fig. 97. If the mass of the cardboard is small in comparison with the mass of the loads, we can neglect the force of gravity acting on the cardboard and assume that only tensile forces of the strings are applied to it. Experiments show that in equilibrium all the strings (and hence the forces acting on the cardboard) are arranged in the same plane. Drawing lines on the cardboard along which the strings are arranged and continuing them till they intersect, we make sure that all the three lines intersect at the same point. Translating to this point the points of applications of all the three tensile forces, we make sure that in this case too the equilibrium condition for three forces formulated above is fulfilled.

It should be noted that the point of intersection of the lines of action of the forces does not necessarily lie in the body itself (Fig. 98).

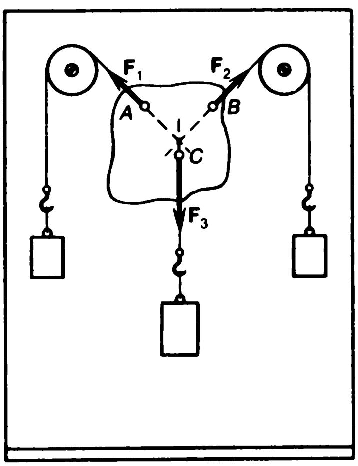

Fig. 97.
An analysis of equilibrium conditions for a rigid body under the action of three forces applied at different points of the body.

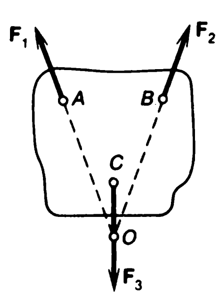

Fig. 98.
The point of intersection of balancing forces may lie outside the body.

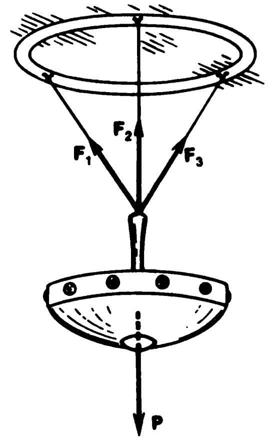

Fig. 99.
A chandelier is in equilibrium under the action of four forces which do not lie in the same plane.

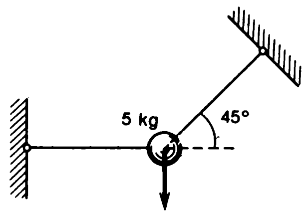

Fig. 100.
To Exercise 3.4.2.

If more than three forces act on a body, equilibrium may also set in when the forces do not lie in the same plane. One such case (a load suspended on three ropes) is shown in Fig. 99.

? **3.4.1.** Prove that if three forces are in equilibrium, the polygon formed by them is a triangle.

3.4.2. A 5-kg load is suspended on two strings one of which lies along the horizontal and the other forms an angle of 45° with the horizontal (Fig. 100). Find the tension in the strings.

3.4.3. A ship is moored to a pier by two ropes forming an angle of 60° with the shore line (Fig. 101). Under the action of the wind blowing from land, the two ropes are stretched so that the tensile force in each rope is 10 kN. Determine the force with which the wind presses against the ship.

3.4.4. A 10-kg load is suspended on a wire at the middle of which is attached a horizontal guy-rope passing over a pulley (Fig. 102). A load of mass 2.5 kg is suspended on the end of the guy-rope. Find the angle α between the upper part of the wire and the vertical and the tensile force in the upper part of the wire.

3.5. Decomposition of Forces

We already know how to find the resultant of two or several forces whose lines of action intersect.

The problem of *decomposition of a force into components*, i.e. the problem of determining several forces such that their resultant is equal to the given force is of no less practical importance. This problem may have different solutions, like in the case of the decomposition of displacement (which is also a vector quantity). To make the problem on decomposition of a force definite (so that it had only one solution), additional data are required. For example, if the magnitude and direction of one component are given, or two directions in which the components must act are specified,

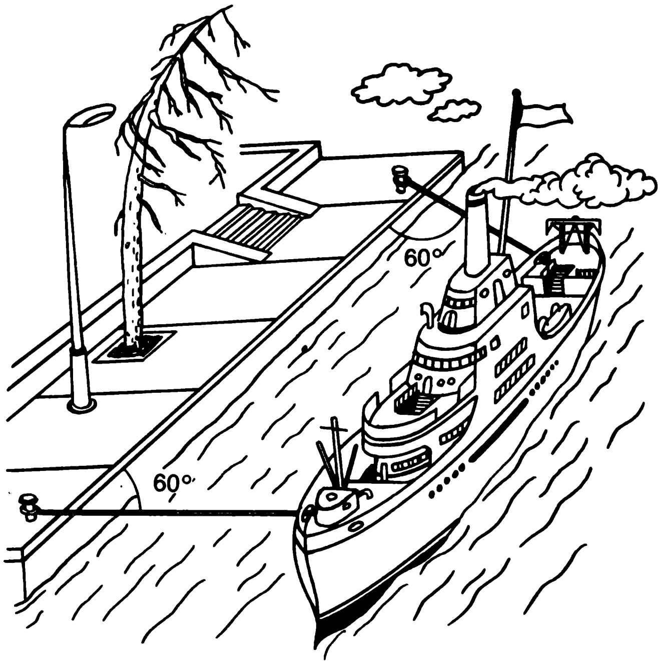

Fig. 101.
To Exercise 3.4.3.

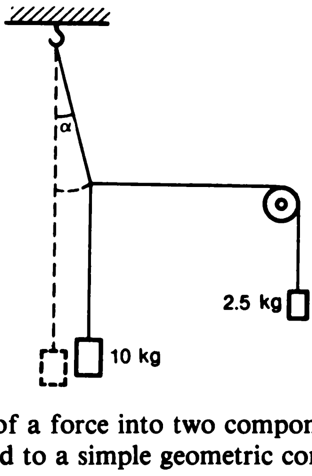

Fig. 102.
To Exercise 3.4.4.

the operation of decomposition of a force into two components becomes completely definite and is reduced to a simple geometric construction.

Let us suppose, for instance, that we have to decompose a force **F** into two components lying in the same plane as **F** [1] and directed along straight lines AB and AC (Fig. 103). For this purpose, it is sufficient to draw two

[1] Otherwise, decomposition is impossible.

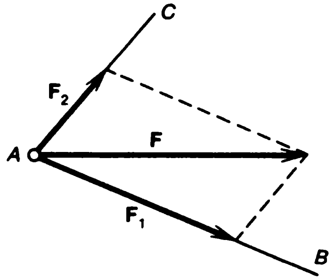

Fig. 103.
Decomposition of a force **F** in specified directions AB and AC.

straight lines parallel to AB and AC from the tip of the vector representing force **F**. The segments $\mathbf{F}_1$ and $\mathbf{F}_2$ will correspond to the required forces.

Usually problems in mechanics contain indications concerning the expedient way of decomposing a force. The directions in which the components of a given force should act are often indicated in the conditions of the problem. For example, to determine the tensile forces in the ropes on which a load is suspended, we must decompose the force of gravity **P** acting on the load into the components $\mathbf{P}_1$ and $\mathbf{P}_2$ along the directions of these ropes (Fig. 104). The tension of the ropes must balance these components. It can be seen from the figure that the larger the angle between the ropes, the larger the tension in them. Therefore, if the distance between the supports of the ropes is large, even a small load hanging slightly below the supports causes a very large tension in the ropes. This is why the ice or hoarfrost covering tightly stretched wires sometimes causes their rupture.

In decomposition of a force into three or more components, the number of conditions required for the decomposition to be unique grows accordingly.

? **3.5.1.** Figure 105 shows a part of a mesh stretched in horizontal directions. Segment AB is stretched with a force of 10 N. What are the tensile forces in segments BC, CG, CD and DE?

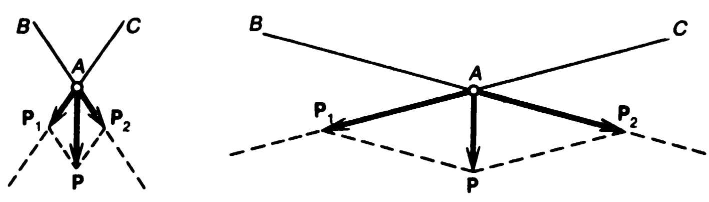

Fig. 104.
The larger the angle BAC between two ropes, the stronger their tension.

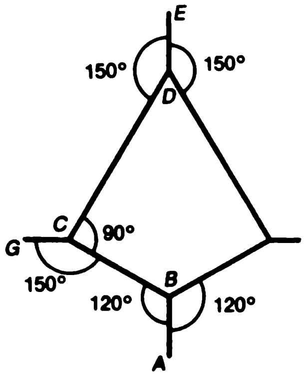

Fig. 105.
To Exercise 3.5.1.

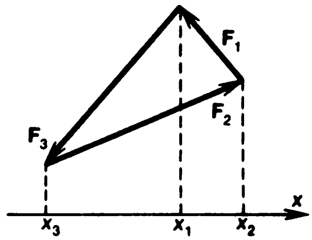

Fig. 106.
Composition of forces in equilibrium according to the triangle rule.

3.6. Projections of Forces. General Conditions of Equilibrium

Like any other vector, a force can be projected onto any axis (Sec. 1.24). It was shown in Sec. 2.12 that the composition of balanced forces according to the triangle rule gives a closed polygon. Figure 106 illustrates the construction of such a broken polygonal line for the case of three forces. Let us choose an arbitrary x-axis and find the projections of the forces on this axis.

By definition, the projection of a vector on an axis is equal to the difference in the coordinates corresponding to the projections of the ends of the segment representing the vector. Consequently, we have

$$F_{1x} = x_1 - x_2, \quad F_{2x} = x_2 - x_3, \quad F_{3x} = x_3 - x_1,$$

where F_{1x} is the projection of vector $\mathbf{F}_1$, and so on. The sum of these expressions is equal to zero:

$$F_{1x} + F_{2x} + F_{3x} = 0. \tag{3.6.1}$$

This result does not depend on the choice of the x-axis and is obviously valid for any number of components. Thus, we arrive at the following general condition of equilibrium: *a body can be in equilibrium if the sum of projections of all the forces applied to it on any direction is zero*. Applying this condition, we have to take into account *all* the forces acting on a body, including the forces exerted by supports, suspenders, etc.

While solving problems, it is often expedient to decompose forces (Sec. 1.24). It is especially convenient to decompose forces along two mutually perpendicular directions. In this case, the components form the

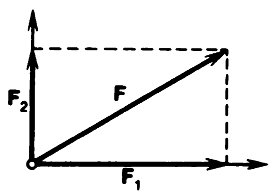

Fig. 107.
Decomposition of a force along two perpendicular directions.

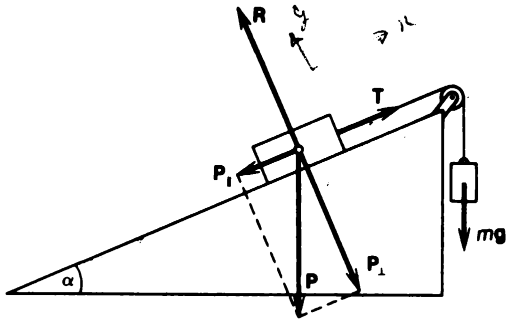

Fig. 108.
Conditions of equilibrium for a body on an inclined plane.

sides of a rectangle whose diagonal represents the force being decomposed (Fig. 107).

Let us illustrate what has been said above by the following example: we consider the conditions of equilibrium of a body having mass M and lying on a plane which forms an angle α with the horizontal (*inclined plane*, Fig. 108). Let us assume that friction is absent. Then the body left to itself would slide down the plane. In order to hold the body, some other force should be applied to it. For this purpose, we can, for example, tie a string to the body and pass it through a pulley so that the string is parallel to the inclined plane, and attach a load of mass m to the other end of the string. Then the body will be acted upon by three forces: the force of gravity $\mathbf{P} = M\mathbf{g}$, the tension of the string $\mathbf{T}$, and the elastic force $\mathbf{R}$ exerted by the plane which slightly sags under the weight of the body. The force $\mathbf{R}$ is perpendicular to the plane and restricts the motion of the body since it allows it to move only along the plane (the forces restricting the motion of bodies are called *reactions*, see Sec. 3.7).

In order to determine the equilibrium conditions, we decompose the force $\mathbf{P}$ into two components: $\mathbf{P}_\parallel$ acting parallel to the inclined plane and $\mathbf{P}_\perp$ acting in the direction perpendicular to the plane. It can be seen from the figure that the magnitude of the component $\mathbf{P}_\parallel$ is equal to $P \sin \alpha = Mg \sin \alpha$, while the magnitude of the component $\mathbf{P}_\perp$ is $P \cos \alpha = Mg \cos \alpha$. For equilibrium, it is necessary that the tension $\mathbf{T}$ of the string be equal in magnitude to the component $\mathbf{P}_\parallel$, while the reaction $\mathbf{R}$ be equal in magnitude to the component $\mathbf{P}_\perp$. The latter condition is automatically observed: the plane sags until the forces $\mathbf{R}$ and $\mathbf{P}_\perp$ become equal in magnitude. On the other hand, the forces $\mathbf{P}_\parallel$ and $\mathbf{T}$ can be equal in magnitude only for a certain relation between the masses M and m, which depends on the angle α. Since the magnitude of the force $\mathbf{T}$ is equal to mg,

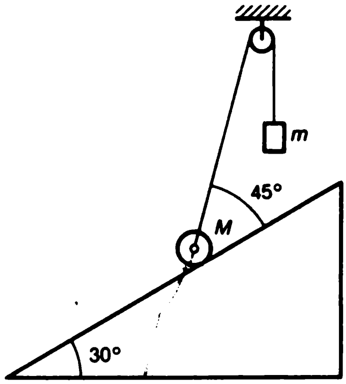

Fig. 109.
To Exercise 3.6.1.

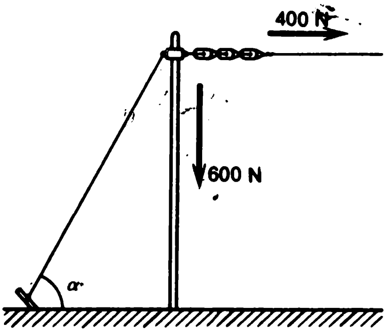

Fig. 110.
To Exercise 3.6.2.

this relation has the form $Mg \sin \alpha = mg$, whence

$$M \sin \alpha = m.$$

This equality expresses the equilibrium condition for a body resting on an inclined plane. It can be easily seen that if this condition is satisfied, the sum of the projections of all the forces on any direction is equal to zero.

? **3.6.1.** An inclined plane forms an angle of 30° with the horizontal (Fig. 109). A body of mass $M = 2$ kg rests on it. A string passing over a pulley forms an angle of 45° with the plane. For what value of the mass m attached to the other end of the string are these bodies in equilibrium? Find the normal force of the body acting on the plane. Friction should be neglected.

3.6.2. A horizontal aerial is attached to a mast so that the tension in it is 400 N (Fig. 110). What must be the angle α between the horizontal and the guy-rope attached to the other side of the mast so that the mast does not bend and the normal force acting on the base of the mast is 600 N?

3.7. Constraints. Constraining Forces. A Body with a Fixed Axis

In practice, we frequently encounter cases when a body cannot move freely in any direction since its motion is restricted by other bodies. In mechanics, such bodies are called *rigid constraints*. The forces exerted by constraints are known as *constraining forces* (*constraints* or *reactions*). If, for example, a piston moves in a cylinder of an engine, rigid constraints are the cylinder walls which allow the piston to move only in one direction. When the piston starts moving sideways, it deforms the cylinder wall. If these walls are very rigid, very strong constraining forces emerge even at insignificant deformations. These forces put an end to any further deviation of the piston. These forces ensure the motion of the piston only along the cylinder. A similar example was considered in the previous section where the constraint was the inclined plane and the constraining force was the force **R**.

In the presence of rigid constraints, the equilibrium conditions become simpler: it is sufficient to consider only the equilibrium of forces in the directions in which motion is not restricted by constraints. For example, for a piston this is the motion along the cylinder, for a body on an inclined plane, the motion along this plane, and so on. The equilibrium of forces in other directions is observed automatically since even a small deformation of a constraint gives rise to forces balancing the applied force.

An important example of motion restricted by a rigid constraint is the rotation of a body about a rigid axis or, in other words, the rotation of a *body with a fixed axis*. The wheels of various machines and mechanisms can rotate, for example, only about a fixed axis. The propeller of an aeroplane, a door on hinges and a desklid are examples of such a motion. In these examples, the rotation about an axis does not tend to shift or bend the axis, i.e. does not cause its deformation. Therefore, rotation about an axis is not hindered. However, any other motion deforms the axis. Constraining forces appearing as a result are exerted by the axis on the body and impede the motion causing the deformation.

If a body is at rest at the initial moment, we must act on it with a certain force to cause its rotation. Not every applied force, however, causes a rotation of the body. Forces having the same magnitude but different directions or applied at different points may cause quite different effects. Indeed, if we attach a spring balance to any point of a body which can freely rotate about the axis O (Fig. 111), the motion of the body may be absolutely different for the same tension of the spring balance but for different directions of its axis. If we place the spring balance in position I, the body starts to rotate clockwise, in position II, counterclockwise, while in position III the body will not rotate at all. *The force applied to a body with a fixed axis causes its rotation only if the direction of the force does not pass through the axis.*

Let us imagine a rudder of a ship or the steering wheel of a motor car. Applying a force along its radius, we shall only try to bend the axle but cannot rotate the wheel. In order to turn it, we must apply a force along its rim, i.e. at right angles to the radius. This force cannot be balanced by the constraining force of the axle (since two forces which do not lie on the same straight line cannot balance each other), and the body starts to rotate.

A force parallel to the rotation axis also cannot cause rotation of the body but only tends to bend the axle. For this reason, in the following sections we shall assume that forces acting on a body with a fixed axis do not have a component along the rotation axis, and hence lie in planes perpen-

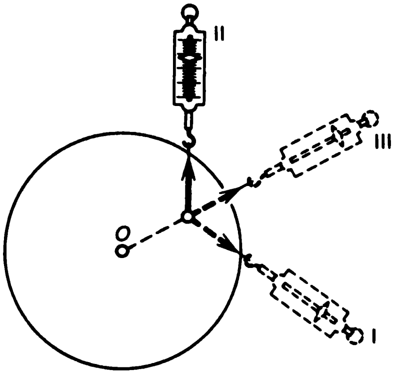

Fig. 111.
If a spring balance is in position I or II, the body rotates. If, however, the spring balance is in position III, the body does not rotate.

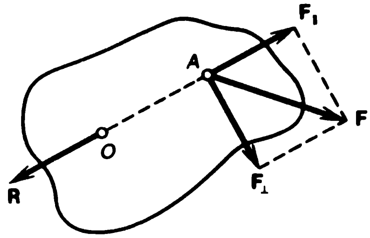

Fig. 112.
Forces acting on a body with a fixed axis. The reaction **R** of the axis is equal to the component $\mathbf{F}_\parallel$ of a force acting in radial direction.

dicular to the axis. Experiments show that in this case the action of a force on the body does not depend on the plane in which this force lies. Therefore, we shall represent all forces in the figures as lying in the same plane perpendicular to the rotation axis which will be denoted by a point.

In order to clearly visualise the action of a force **F** which does not pass through the axis, we decompose it into two mutually perpendicular components one of which passes through the axis (Fig. 112). The component $\mathbf{F}_\parallel$ passing through the axis does not cause a rotation since it turns out to be balanced by reaction **R** of the axis. The body will rotate as if only the component $\mathbf{F}_\perp$ acted on it in a direction normal to the radius OA drawn to the point of application of the force.

3.8. Equilibrium of a Body with a Fixed Axis

It follows from what was said in Sec. 3.7 that while analysing the equilibrium conditions of a body with a fixed axis, we can disregard the force exerted by the axis since it cannot cause rotation of the body. Let us consider the equilibrium conditions for a body with a fixed axis under the action of only two forces such that their directions are perpendicular to the radii at the points of their application.

It is necessary for equilibrium that, first, these forces, acting separately, rotate the body in the opposite directions. This can be illustrated by the following experiment. We align the axis of a body along the vertical so as to eliminate the action of the force of gravity and attach spring balances to it perpendicularly to the radii at the points of application of forces. If the spring balances are arranged as shown in Fig. 113, we can choose such ex-

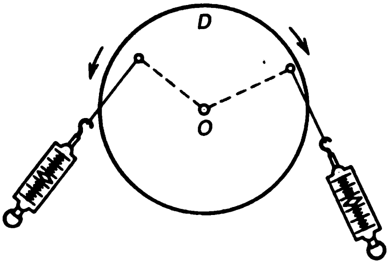

Fig. 113.
With such an arrangement of spring balances, equilibrium is possible.

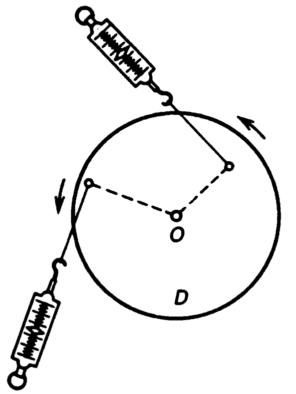

Fig. 114.
Equilibrium is impossible when spring balances are arranged in this way.

tensions that the body remains at rest. In the case shown in Fig. 114, however, when both spring balances rotate the body in the same direction, it is impossible to keep the body at rest for any extension of the spring balances.

Secondly, it turns out that not only the magnitudes of the forces are important for the equilibrium of a body with a fixed axis, but also the distances from the rotation axis to the lines of action of the forces. As in the case of a lever, *for the equilibrium of a body with a fixed axis it is necessary that the product of the magnitude of the force and the distance between the axis and the line of force action be the same for the two forces*. If we denote the magnitudes of the forces by F_1 and F_2 and the distances by l_1 and l_2, the equilibrium condition will be expressed by the following equality:

$$F_1 l_1 = F_2 l_2. \tag{3.8.1}$$

We assume that each force lies in a plane perpendicular to the rotation axis (not necessarily in the same plane).

3.9. Moment of Force

Thus, for equilibrium of a body with a fixed axis, it is the product of the magnitude of force **F** and the distance l from the axis to the line of action of the force that is important rather than the magnitude of the force (Fig. 115); we assume that the force lies in a plane perpendicular to the rotation axis. This product is known as the *moment of the force about the axis*, or simply the *moment of force* (*torque*). The distance l is called the

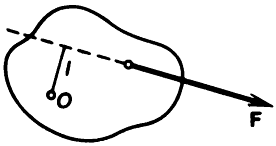

Fig. 115.
The moment of force **F** is equal to the product of its magnitude and the arm l.

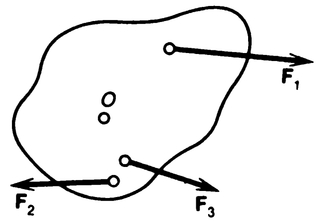

Fig. 116.
The moments of forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are positive, while the moment of force $\mathbf{F}_3$ is negative.

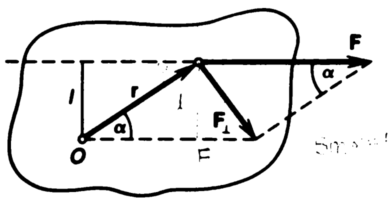

Fig. 117.
The moment of force **F** is equal to the product of the magnitude of its compoment $\mathbf{F}_\perp$ and the magnitude of the radius vector **r**.

arm of force. Denoting the moment of force by M, we can write

$$M = lF. \tag{3.9.1}$$

Let us assume that the moment of a force is positive if this force, acting alone, would rotate the body clockwise and negative in the opposite case (here we must decide beforehand from which side we look at the body). For example, forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 116 should be ascribed a positive moment while the force $\mathbf{F}_3$ has a negative moment.

Moment of force can also be defined in a different way. We draw the directed segment **r** from the point O, lying on the rotation axis in the same plane as the force, to the point of application of this force (Fig. 117). This segment is called the *radius vector* of the point of application of the force. The magnitude of vector **r** is equal to the distance from the axis to the point of application of the force. Then we plot the component of the force **F** perpendicular to the radius vector **r**. We denote this component by $\mathbf{F}_\perp$. It can be seen from the figure that $r = l/\sin\alpha$, while $F_\perp = F\sin\alpha$. Multiplying these expressions, we obtain $rF_\perp = lF$.

Thus, the moment of force can be represented in the form

$$M = rF_\perp, \tag{3.9.2}$$

where $F_\perp$ is the magnitude of the component of **F** perpendicular to the radius vector **r** of the point of application of the force. It should be noted that the product lF is numerically equal to the area of the parallelogram formed by vectors **r** and **F** (see Fig. 117). The forces shown in Fig. 118 have equal moments about axis O. Figure 119 shows that the translation of the

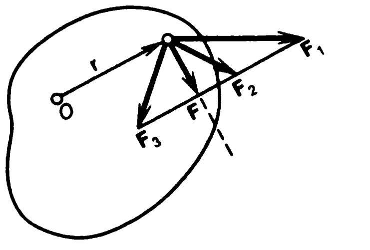

Fig. 118.
Forces $\mathbf{F}$, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ have equal moments about the axis O.

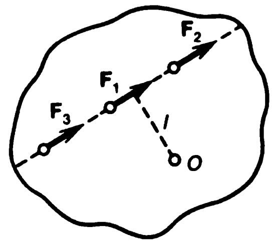

Fig. 119.
Equal forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ with the same arm l have equal moments about the axis O.

point of application of a force along the line of its action does not alter its moment. If the line of action of a force passes through the rotation axis, the arm of the force is zero, and hence the moment of force is also equal to zero. It was shown above that in this case the force does not cause a rotation of the body. *The force whose moment about a given axis is zero does not cause rotation about this axis.*

Using the concept of the moment of force, we can reformulate the equilibrium conditions for a body with a fixed axis, which is acted upon by two forces. In the equilibrium condition expressed by formula (3.8.1), l_1 and l_2 are just the arms of the corresponding forces. Consequently, this condition consists in the equality of the magnitudes of the moments of the two forces. Besides, to exclude rotation, the moments must have different directions, i.e. they must differ in sign. Thus, *a body with a fixed axis is in equilibrium if the algebraic sum of the moments of forces acting on it is equal to zero.*

Since the moment of a force is defined as the product of its magnitude and the length of the arm, the unit of moment of force is obtained if we take a unit force with an arm equal to a unit length. Consequently, the SI unit of moment of force is the moment of a force equal to a newton and having an arm one metre long. It is called a *newton-metre* (N · m).

If many forces act on a body with a fixed axis, experiments show that the equilibrium condition remains the same as in the case of two forces: the algebraic sum of the moments of *all the forces* acting on the body must be zero. The term "resultant moment" of several moments of force acting on a body (component moments) is used for the algebraic sum of these moments. Under the action of the resultant moment, the body would rotate as under the simultaneous action of the component moments. If, in particular, the resultant moment is zero, the body with a fixed axis is either at rest or rotates uniformly.

3.10. Measurement of Torque

Rotating bodies are often encountered in engineering: wheels of vehicles, shafts of machines, screw propellers of ships, and so on. In all these cases, moments of force, or torques, act on bodies. It is sometimes difficult to single out a definite force producing a torque and its arm since the torque is created not by a single force but by many forces having different arms. For example, electromagnetic forces are applied at different distances from the rotation axis to the turns of the armature winding in an electric motor. Their joint action produces a torque causing the rotation of the armature and the motor shaft connected to it. In such cases, it is senseless to speak about a force and its arm. Only the resultant torque is essential. Therefore, a *direct* measurement of torque is required.

To measure the moment of force, it is sufficient to apply to the body another, known, torque which would *balance* the torque being measured. If equilibrium is attained, the torques are equal in magnitude and opposite in sign. For example, to measure the torque developed by an electric motor, shoes *2* pressed by bolts are mounted on pulley *1* of the motor so that the pulley can rotate with friction under them. The shoes are connected to a long rod with a spring balance fixed at its other end (Fig. 120). The axis of the shoes coincides with the axis of the motor. When the motor rotates, friction exerted by the pulley on the shoes turns the shoes with the rod through a certain angle in the direction of rotation of the motor. The spring balance is slightly stretched, and produces the opposite torque on the shoes, which is equal to the product of the tensile force of the spring balance and its arm l. The tension in the spring balance is equal and opposite to the force $\mathbf{F}$ exerted by the rod on the spring balance (see Fig. 120).

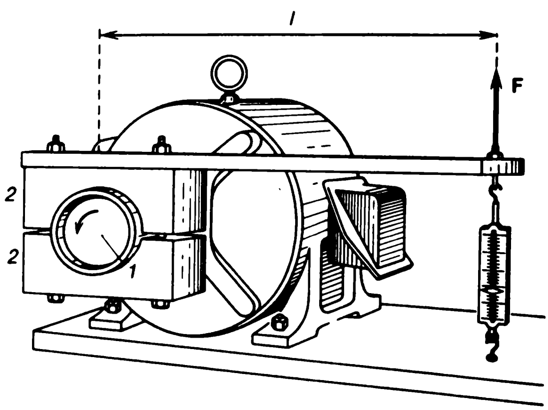

Fig. 120.
Measurement of the torque produced by an electric motor.

Since the shoes are at rest, the torque developed by the motor must be equal in magnitude and opposite in sign to the moment of the tensile force of the spring balance. Thus, for a given speed of rotation, the motor develops a torque equal to Fl.

When very small torques are to be measured (for example, in high-sensitivity galvanometers and other physical measuring instruments), the torque to be measured is compared with the torque produced by a twisted fibre. The measuring system is suspended on a thin fibre made either of metal or of fused quartz. Under the action of a torque, the measuring system twists the fibre. Such a deformation gives rise to forces which tend to untwist the fibre and hence produces a torque. When the torque being measured becomes equal to the torque of the twisted fibre, equilibrium sets in. The torque of the fibre (and hence the torque to be measured) can be judged from the angle of twist in equilibrium. The relation between the torque of the fibre and the angle of twist is determined by gauging the instrument.

3.11. Force Couple

If several forces act on a body, such that their resultant is zero while the resultant torque about some axis differs from zero, the body does not remain in equilibrium. This is the case when, for example, two equal and opposite forces not lying on the same straight line act on the body. Such two forces acting simultaneously on the body form a *couple*. A body with a fixed axis starts to rotate about this axis under the action of a couple. In this case, a force will be generally exerted on the body by the axis. It can be shown, however, that if the axis passes through a certain point of the body, no force will be exerted by the axis. Therefore, if a couple acts on a *free* body, the latter starts to rotate about the axis passing through this point. It can be proved that this point is the *centre of gravity of the body* (Sec. 3.12).

The torque of a couple is the same about any axis perpendicular to the plane of the couple. Indeed, let O be an arbitrary axis perpendicular to the plane in which a couple lies (Fig. 121). The resultant moment M of the couple is

$$M = F \times OA + F \times OB = F(OA + OB) = Fl,$$

where l is the distance between the lines of action of the forces constituting the couple, and is known as the arm of the couple. The same result will be obtained for any other position of the axis. It can also be shown that the

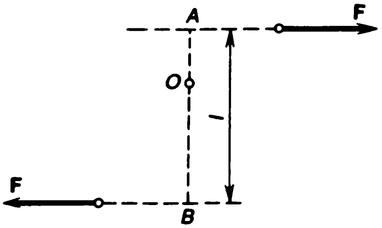

Fig. 121.
The moment $M = Fl$ of a couple of forces.

moment of several forces whose resultant is zero will be the same about all axes parallel to each other, and hence the action of all these forces on the body can be replaced by the action of a couple having the same torque.

3.12. Composition of Parallel Forces. Centre of Gravity

While studying equilibrium of forces and determining the resultant, we have not analysed so far the case when the forces acting on a body are parallel. Having determined the equilibrium conditions for a body with a fixed axis, we can now consider this case too.

Let us analyse the forces acting on a lever loaded by balanced weights and suspended from a stationary holder with the help of a spring balance (Fig. 122). We can assume that the rotation axis of the lever passes through the point O from which it is suspended. The lever experiences the action of weights $\mathbf{F}_1$ and $\mathbf{F}_2$ of the loads suspended from it as well as the tensile force $\mathbf{F}_3$ of the spring balance. We shall assume that the mass of the lever itself is so small in comparison with the masses of the loads that it can be neglected. Then we can say that the lever is in equilibrium under the action

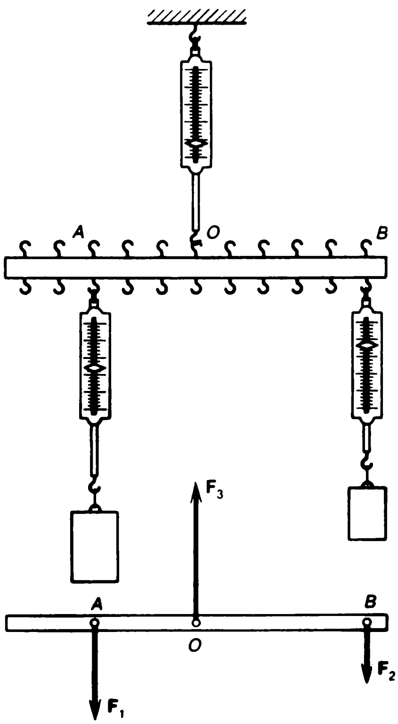

Fig. 122.
An analysis of equilibrium of a body under the action of three parallel forces.

of forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$. The force $\mathbf{F}_3$ balances two parallel forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Since in equilibrium the spring of the balance is in vertical position, the force $\mathbf{F}_3$ is parallel to $\mathbf{F}_1$ and $\mathbf{F}_2$.

Further, the force $\mathbf{F}_3$ is equal in magnitude to the sum of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Since we neglect the mass of the lever, $F_3 = F_1 + F_2$. The distances from the point of suspension of the lever (its rotation axis O) to the points of application of forces $\mathbf{F}_1$ and $\mathbf{F}_2$ can be found from the equilibrium condition for the lever:

$$F_1 \times OA = F_2 \times OB, \quad \text{or} \quad OB/OA = F_1/F_2. \qquad (3.12.1)$$

This means that *the point of application of the balancing force divides the distance between the points of application of forces in inverse proportion to the forces*. Consequently, a free body is in equilibrium under the action of three parallel forces if the third force, which is opposite to the first two, is equal in magnitude to their sum and is applied at a point dividing the distance between the points of their application in inverse proportion to the first two forces.

Hence, *the resultant of two parallel forces having the same direction is equal to the sum of these forces, has the same direction and is applied at a point dividing the distance between the points of application of forces in inverse proportion to the applied forces*.

The rule of composition of two parallel and opposite forces can also be easily found. Any of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a body in equilibrium can be treated as the force balancing the other two forces. Thus, $\mathbf{F}_2$ is the force balancing two parallel and opposite forces $\mathbf{F}_1$ and $\mathbf{F}_3$. Hence we can conclude, as before, that the force equal and opposite to $\mathbf{F}_2$ is the resultant of forces $\mathbf{F}_1$ and $\mathbf{F}_3$. But $F_2 = F_3 - F_1$, and, besides, we can obtain from proportion (3.12.1) the following proportion:

$$\frac{F_1}{F_1 + F_2} = \frac{OB}{OA + OB}, \quad \text{or} \quad \frac{F_1}{F_3} = \frac{OB}{AB}.$$

Thus, *the resultant of two parallel and opposite forces is equal in magnitude to the difference of these forces, have the same direction as the larger force and is applied at a point dividing the distance between the points of application of forces in inverse proportion to the forces of which it is composed*.

If several parallel forces act on a body, the overall resultant can be found by determining the resultant of any two of these forces, composing the result with a third force, and so on. In particular, the forces of gravity act on each element of a body and are parallel. Hence, in order to find their resultant, we must consecutively compose a series of parallel forces. The resultant is equal to their sum, i.e. is the total force of attraction exerted on

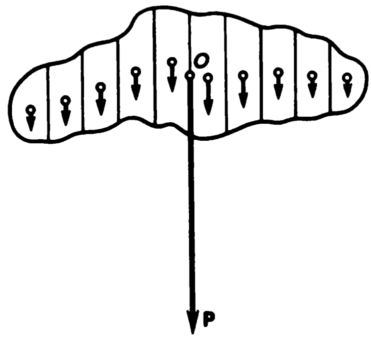

Fig. 123.
The point of application of the resultant of forces of gravity is the centre of gravity of a body.

the body by the Earth, and is applied at a certain point of the body. The point of application of the resultant force of gravity is known as the *centre of gravity* of the body (Fig. 123).

Thus, the action of the Earth on a rigid body is such as if the point of application of the force of gravity coincided with the centre of gravity of the body. We shall use this henceforth replacing the action of the forces of gravity applied to separate parts of the rigid body by the action of a single force applied at its centre of gravity and equal to the force of gravity acting on the entire body.

Often the problem inverse to that of composition of parallel forces has to be solved, i.e. a force has to be decomposed into parallel components. A

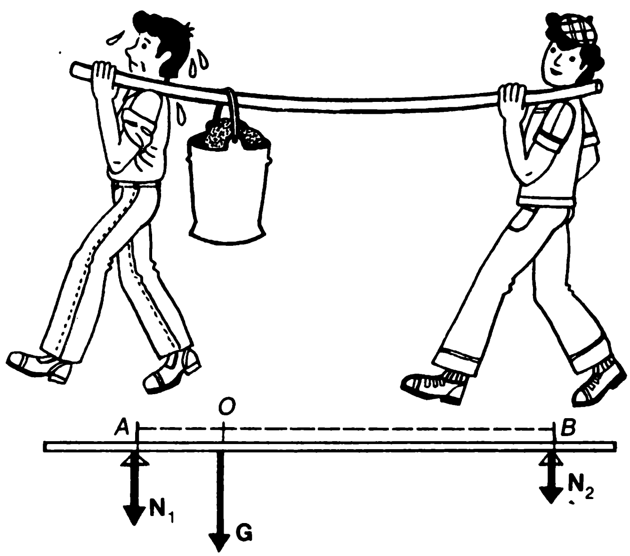

Fig. 124.
Decomposition of a force into two parallel components.

problem of this type involves, for example, the determination of the distribution of forces acting on the supports of a loaded beam or on the shoulders of two persons carrying a load on a rod (Fig. 124). The required forces $\mathbf{N}_1$ and $\mathbf{N}_2$ are determined from the condition that their resultant is equal to the weight $\mathbf{G}$ of the load and must be applied at the point of suspension of the load. Hence

$$N_1 + N_2 = G, \quad N_1/N_2 = OB/OA.$$

3.13. Determination of the Centre of Gravity of a Body

The determination of the centre of gravity of an arbitrary body by a consecutive composition of the forces acting on its parts is generally a complicated problem which is simplified only for bodies of a relatively simple shape.

Suppose that a body consists only of two loads having masses m_1 and m_2 and connected by a rod (Fig. 125). If the mass of the rod is small in comparison with m_1 and m_2, it can be ignored. Each mass is acted upon by the force of gravity $\mathbf{P}_1 = m_1\mathbf{g}$ and $\mathbf{P}_2 = m_2\mathbf{g}$ respectively. These forces are directed vertically downwards, i.e. are parallel. It is well known that the resultant of two parallel forces is applied at a point O which is determined from the condition

$$\frac{P_1}{P_2} = \frac{OB}{OA}, \quad \text{or} \quad \frac{m_1}{m_2} = \frac{OB}{OA}.$$

Consequently, the centre of gravity divides the distance between the loads in inverse proportion to their masses. If the body is suspended from the point O, it remains in equilibrium.

Since two equal masses have a common centre of gravity at the midpoint between these masses, it is clear, for example, that the centre of gravity of a homogeneous rod is at its centre (Fig. 126).

Since a homogeneous disc can be divided into two completely identical symmetrical parts by any of its diameters (Fig. 127), the centre of gravity should lie on each diameter of the disc, i.e. at the point of their intersection

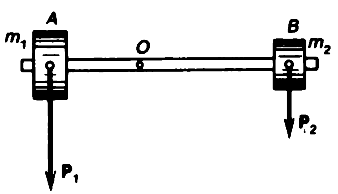

Fig. 125.
Centre of gravity of a body composed of two loads.

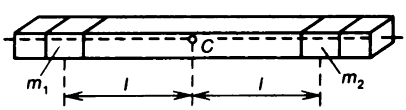

Fig. 126.
Centre of gravity of a homogeneous rod lies at its midpoint.

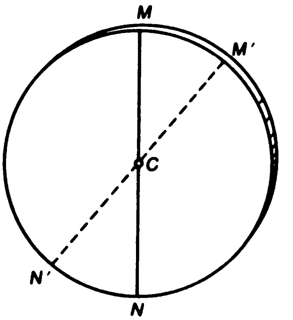

Fig. 127.
Centre of gravity of a homogeneous disc is at its geometric centre.

which is the geometric centre C of the disc. Using similar arguments we can state that the centre of gravity of a homogeneous ball is at its geometric centre, the centre of gravity of a homogeneous rectangular slab is at the intersection of its diagonals, and so on. The centre of gravity of a ring also lies at its centre. This example shows that the centre of gravity of a body can be outside it.

If the shape of a body is irregular, or if it is heterogeneous (for example, if it contains voids), it is often quite difficult to calculate the position of the centre of gravity, and it is more convenient to determine it experimentally. Let us suppose, for instance, that we have to determine the centre of gravity of an irregular piece of cardboard. We suspend it on a string (Fig. 128). Obviously, in equilibrium the centre of gravity C of the body must lie on the continuation of the string. Otherwise, the force of gravity will have a torque about the point of suspension, which would rotate the body. Therefore, drawing on the cardboard a straight line which is the continuation of the string, we can state that the centre of gravity lies on this straight line.

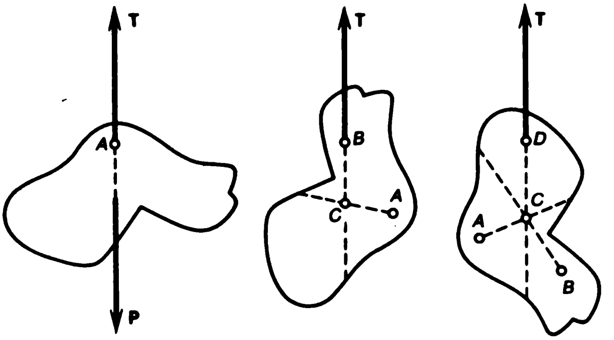

Fig. 128.
The point C of intersection of vertical lines drawn through the points of suspension A, B and D is the centre of gravity of the body.

Indeed, having suspended the body at different points and drawn vertical lines, we see that all these lines intersect at a single point. This is just the centre of gravity of the body (since it must lie on all such straight lines simultaneously). The position of the centre of gravity can be determined in this way not only for a plane figure but also for a body of a more complicated shape. The position of the centre of gravity of an aeroplane is determined by rolling its wheels onto a balance. The resultant of the weights exerted by the wheels will be directed vertically, and the line of its action can be determined from the law of composition of parallel forces.

If the masses of separate parts of a body or its shape change, the position of the centre of gravity also changes. For example, the centre of gravity of an aeroplane shifts as the fuel from its tanks is consumed or when the plane is loaded by luggage. In order to illustrate the shifting of the centre of gravity as a result of a change in the shape of a body, it is convenient to take two identical hinged blocks (Fig. 129). When the blocks are arranged in a straight line, the centre of gravity lies on their axis. If the blocks are bent at the hinge, the centre of gravity turns out to be outside the blocks, on the bisector of the angle formed by them. If an additional load is attached to one of the blocks, the centre of gravity is shifted towards this load.

? **3.13.1.** Determine the position of the centre of gravity of two identical thin rods having a length of 12 cm and arranged in the form of "T".

3.13.2. Prove that the centre of gravity of a homogeneous triangular plate lies on the intersection of its medians.

3.13.3. A homogeneous board of mass 60 kg lies on two supports as shown in Fig. 130. Determine the forces acting on the supports.

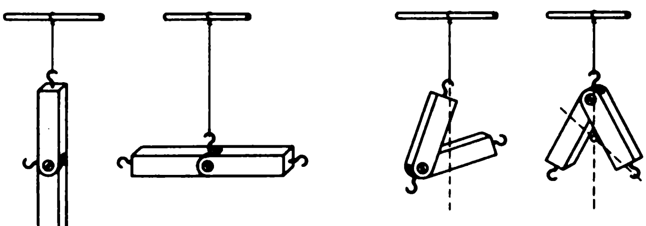

Fig. 129.
(*a*) The centre of gravity of two hinged blocks forming a segment of straight line is on the axis of the blocks. (*b*) The centre of gravity of the hinged blocks arranged at an angle lies outside the blocks.

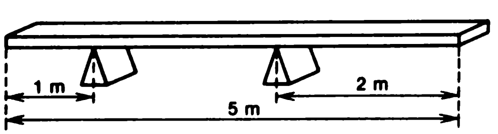

Fig. 130.
To Exercise 3.13.3.

3.14. Equilibrium of a Body under the Action of the Force of Gravity

In mechanics, it is often important to know the positions in which a body may stay at rest under the action of the force of gravity as long as desired, provided that the body was initially at rest. Obviously, the forces acting on the body must be mutually balanced in this case. The positions in which the forces acting on the body balance each other are called *equilibrium positions*.

In actual practice, however, the body which was initially at rest will remain stationary at later moments not in all equilibrium positions. As a matter of fact, under real conditions the body experiences the action of random unavoidable forces like small shocks, vibrations of air, etc., in addition to the forces which are taken into consideration (e.g. the force of gravity, reaction of suspender or support, and so on). Under the action of these forces, the body slightly deviates from its equilibrium position, and its behaviour in this case may be different.

When a body is displaced from its equilibrium position, the forces acting on it change, as a rule, and their equilibrium is violated. The changed forces cause the body to move. If these forces are such that the body *returns* to the equilibrium position, the body still remains *near* the equilibrium position despite random shocks. In this case, the body is said to be in a *stable equilibrium*. In other cases, the changed forces are such that they cause a *further deviation* of the body from the equilibrium position. Then a very small shock is sufficient for a further deviation of the body from the equilibrium position under the action of the altered forces. The body will no longer stay near the equilibrium position but will move away from it. Such an equilibrium is termed *unstable*.

Thus, the necessary condition of stability is that the forces emerging as a result of deviation of the body from its equilibrium position should return the body to the initial position. The position of a ball on a concave support is of this type (Fig. 131*a*): when the ball is shifted from the equilibrium position (the lowest position), the resultant **F** of the reaction **R** of the support and the force of gravity **P** returns the ball to the equilibrium position. This is the case of stable equilibrium. In the case of a convex sup-

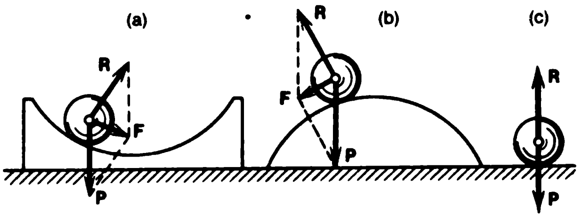

Fig. 131.
Stable (*a*), unstable (*b*) and neutral (*c*) equilibrium of a ball on a surface.

port (Fig. 131*b*), however, the resultant removes the ball from the equilibrium position (the uppermost position): the equilibrium is unstable.

Another example of equilibrium is that of a body suspended from a point. Determining the position of the centre of gravity by the method of suspension described in the previous section, we always find that the centre of gravity is below the point of suspension and essentially lies on the same straight line with it (otherwise the tensile force **T** of the string would not balance the force of gravity **P**) (Fig. 132*a*). However, the force of gravity **P** and tension **T** of the string can balance each other also in the case when the centre of gravity C is on the vertical *above* the point A of suspension (Fig. 132*b*). Indeed, in this case the force of gravity **P** and tension **T** which is equal in magnitude to this force would also balance each other. But it can be easily proved by experiment that now the body will not remain in this equilibrium position. Although the two cases correspond to positions of equilibrium, only the first case can be realised in practice.

The reason behind this is that if we slightly shift the body from the first equilibrium position (Fig. 132*c*), the force of gravity **P** creates a torque about the point of suspension which returns the body to its initial position. This is the position of stable equilibrium. On the contrary, if the body is deviated from the second equilibrium position (Fig. 132*d*), the force **P** removes it from this position. This is the position of unstable equilibrium. There are intermediate positions of equilibrium. If a ball rests on a horizontal support, a displacement of the ball does not disturb its equilibrium at all since the force exerted by the plane on the ball and the

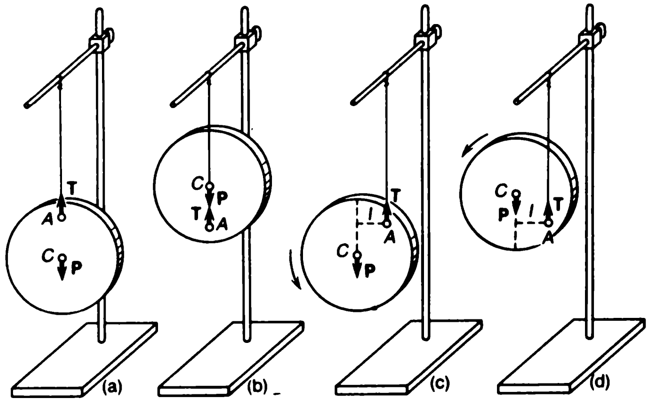

Fig. 132.
(*a*) Equilibrium position with the centre of gravity C lying below the point of suspension A. (*b*) Equilibrium position with the centre of gravity C above the point of suspension A. (*c*) If the body is slightly shifted from position (*a*), the force of gravity produces a torque which returns the body to the equilibrium position. (*d*) If the body is shifted from position (*b*), the torque created by the force of gravity moves the body away from the equilibrium position.

force of gravity balance each other in any position of the ball. Such an equilibrium is known as *neutral*, or *indifferent* (Fig. 131*c*).

Another example of neutral equilibrium is a body fixed on a horizontal or inclined axis passing through its centre of gravity. If such a body is turned about its axis, the moment of the force of gravity about the axis always remains zero (the force of gravity passes through the rotation axis), and the body remains in equilibrium at any position. This fact is used for checking the shape of wheels, generator armatures, and so on. If a manufactured wheel has a regular shape, its centre of gravity lies on its axle. Therefore, the accurately made wheel whose axle can rotate in bearings should remain in equilibrium at any turn of the axle. If it keeps on returning to a certain position by itself, this means that the wheel is unbalanced, i.e. its centre of gravity does not lie exactly on the axle.

A body fixed on a vertical axis is always in neutral equilibrium under the action of the force of gravity irrespective of whether or not the axis passes through its centre of gravity.

? 3.14.1. Determine experimentally the equilibrium position of the front wheel of a bicycle if the bicycle is lifted. What should be done in order that the wheel be in neutral equilibrium?

3.15. Conditions of Stable Equilibrium under the Action of the Force of Gravity

Comparing the cases of equilibrium considered above, we can note the general condition of stable equilibrium for all the cases: *if the centre of gravity of a body occupies the lowermost position as compared to all other possible neighbouring positions, the equilibrium is stable.* Indeed, in this case the centre of gravity will be raised when the body is shifted from this position to any side, and the force of gravity will return it to its initial position. Using this condition, we can determine in a simple way whether or not the body is in stable equilibrium without testing it.

Let us consider, for example, a homogeneous hemisphere put on a horizontal plane (Fig. 133). The centre of gravity C of the hemisphere lies on the radius OA below point O. Suppose that the hemisphere is slightly in-

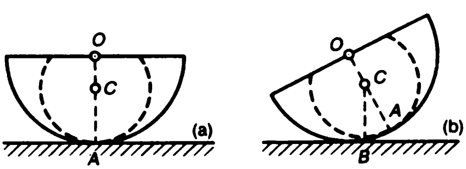

Fig. 133.
Since the centre of gravity is lower in position (*a*) than in position (*b*), the equilibrium is stable.

clined so that it touches the plane at point B (Fig. 133*b*). It can be easily seen that the distance BC is larger than AC. Hence the centre of gravity is raised upon a deviation from the equilibrium position, and the equilibrium position of the hemisphere is stable.

Let us now consider equilibrium conditions for a body not with one point of support as a suspended body or a ball on a plane, but with several points (e.g. a table) or the contact surface (like a box placed on a horizontal plane). The equilibrium condition in this case can be formulated as follows: *the body is in equilibrium if the vertical drawn through the centre of gravity is within the contact surface of the body*, i.e. within the contour formed by the lines connecting the points of support or within the surface on which the body rests. In this case, the equilibrium is stable.

For example, a table is in a stable equilibrium on a horizontal floor (Fig. 134*a*). Indeed, if we incline the table, its centre of gravity will go up (Fig. 134*b*). If, however, we incline the table so that the vertical passing through the centre of gravity goes out of the contact surface, the moment of the force of gravity will rotate the table so that it moves away from its equilibrium position. The centre of gravity will go down, and the table will be overturned. There exists a *limiting angle* of inclination beyond which the equilibrium is no longer restored and the body is turned upside down. Being inclined by the limiting angle, the body is in equilibrium since the direction of the force of gravity passes through the point of support (Fig. 134*c*), but this equilibrium is unstable: the body either returns to the position of stable equilibrium or overturns.

The limiting angle is evidently the smaller, the higher the centre of gravity for a given contact area. A cart, lorry or railway flat-car, loaded with a high centre of gravity, overturn more easily than if the centre of gravity were low. The stability can be improved by increasing the contact area.

It becomes clear from the equilibrium condition for a body resting on several points of support why cranes are always supplied with a heavy counterweight. Due to the counterweight, the common centre of gravity of

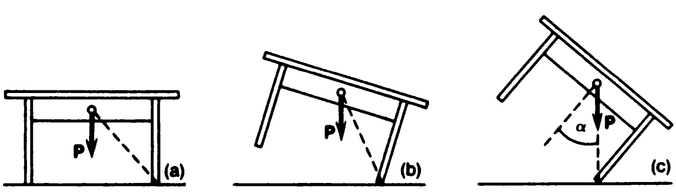

Fig. 134.
When the table is shifted from its equilibrium position (*a*) to position (*b*), the centre of gravity rises, and hence the equilibrium is stable. In position (*c*), the table is inclined at the limiting angle; upon a further deviation, the centre of gravity will be lowered, and equilibrium is unstable.

the crane, load and the counterweight remains within the rectangle bounded by the points of supports of the wheels, even when the crane lifts a heavy load. If the centre of gravity of a body goes out of the limits of the contact surface, as, for example, for a bench with a man sitting on its protruding edge, equilibrium is violated, and the bench overturns.

In actual practice, we mostly come across stable equilibrium since only in such a position a body left by itself remains as long as desired despite random shocks. On the contrary, a body in an unstable equilibrium moves away from this position.

It is possible, however, to control the conditions of a body so that it remains in the vicinity of the position of unstable equilibrium for a long time, oscillating about it to and fro. For example, a long stick put vertically on the floor is in unstable equilibrium and falls as soon as we release it. However, we can balance the stick holding it near the unstable vertical position on the tip of the finger. For this purpose, it is sufficient just to move the hand slightly in the direction of inclination of the stick at a given moment. We thus shift the point of support and accordingly change the moment of force of gravity which starts to incline the stick in the opposite direction. Naturally, such movements should be made continuously to let the stick to be inclined by the action of the changing moment of the force of gravity only slightly. By appropriate training it is possible to learn how to keep complex constructions near unstable equilibrium (as it is done by jugglers in circus). Watching the work of our leg muscles, we can note that when we stand on one foot, we are virtually in unstable equilibrium. In order to avoid a fall, we have to continuously shift the point of support from heel to toe.

? **3.15.1.** If a self-righting tumbler (Fig. 135) is put on its side, it assumes an upright position. What is the approximate position of its centre of gravity?

3.15.2. Will a thin ruler resting on a cylindrical surface (Fig. 136) be in stable equilibrium?

3.15.3. Why does a person carrying a load on his back lean forward?

3.15.4. A solid cylinder stands on a 50-cm long board. What is the maximum height to which an end of this board can be lifted without overturning the cylinder if the height of the cylinder is four times the diameter of the base?

3.15.5. A pencil with a knife stuck into it is in stable equilibrium (Fig. 137). Explain this fact.

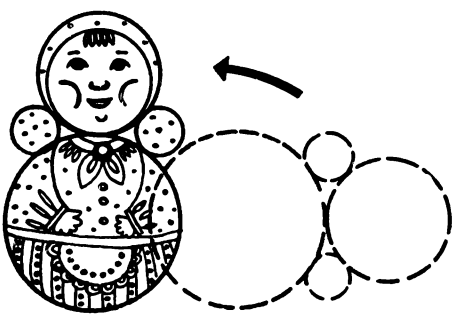

Fig. 135.
Self-righting tumbler.

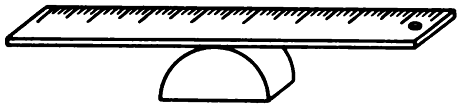

Fig. 136.
To Exercise 3.15.2.

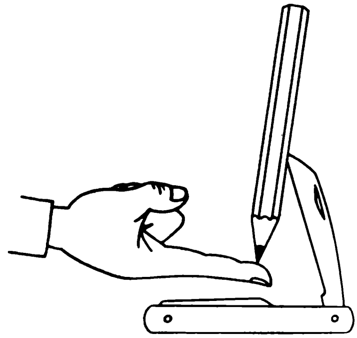

Fig. 137.
To Exercise 3.15.5.

3.16. Simple Machines

The first appliances for lifting and moving heavy loads and putting in operation heavy siege-guns (battering-rams) appeared in ancient times. All these devices were used to cause motions such that large forces could be overcome in their course (like the weight of a heavy load being lifted). For this purpose, the forces developed by machines had to be stronger (at least at the beginning of motion) than the forces opposing the motion. If, however, the motions caused by machines proceed slowly and friction is sufficiently small, we can assume that the role of these machines boils down to balancing large forces opposing motion. In other words, we can assume that the forces developed by machines should be equal and opposite to the forces hindering the motion. All such appliances are known as *simple machines*. Thus, the question of operation of simple machines is reduced to determining certain conditions under which such a machine is in equilibrium.

One of the most widely used simple machines is a *lever* which we have already mentioned above. Levers can be found in various devices. A lever is in equilibrium if the ratio of the parallel forces applied to its ends is inverse to the ratio of their arms, and the moments of these forces have opposite signs. Therefore, by applying a small force to a long end of a lever, we can balance a much larger force applied to its shorter end. If we put under a heavy body a lever with a very long second arm (Fig. 138), we can lift the body by applying a force much smaller than the weight of the body. We can say that a lever is a "transformer" of force: a small force f applied at the end of the long arm causes a large force F at the end of the short arm. Thus we "gain in force".

A wheelbarrow is also a kind of a lever (Fig. 139). The force of gravity **P** acting on a load is applied much closer to the axle of the wheel (which now plays the role of the lever axis) than the force exerted by the hands of the man pushing the wheelbarrow. Therefore, a man can lift on the

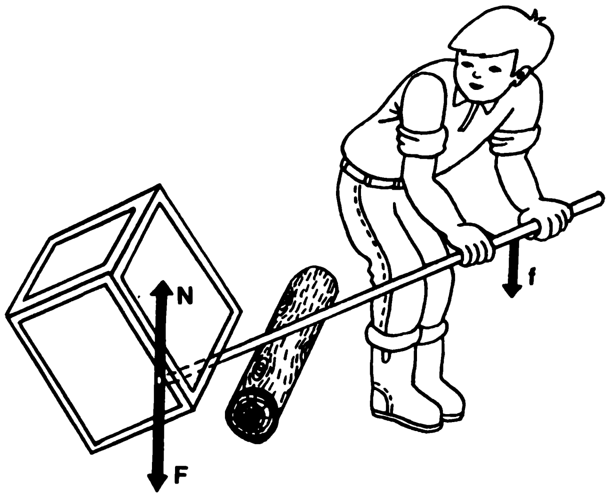

Fig. 138.
The application of a lever. The force f applied by a worker is smaller than the force N exerted by the lever on the load.

wheelbarrow a load which he is unable to lift by his bare hands. The force exerted by the hands should be directed upwards so that the torque created by it about the lever axis was opposite to the moment of the force **P**.

Another widely used type of simple machines is a pulley or a combination of several pulleys. Let us first consider a *simple pulley* (Fig. 140). We shall assume that it rotates in bearings without friction. If the rope is stretched and does not slip over the pulley, the latter is under the action of

Fig. 139.
Wheelbarrow as a lever.

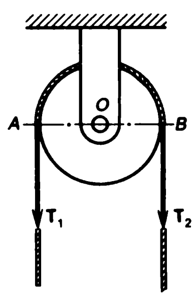

Fig. 140.
Simple pulley.

two tensile forces $\mathbf{T}_1$ and $\mathbf{T}_2$ of the rope. We can assume that these forces are applied at points A and B on the circumference of the pulley. Just like the equilibrium conditions for a lever, the equilibrium conditions for a pulley are determined from the conditions of equilibrium for the moments of applied forces. Since the arms of the forces $\mathbf{T}_1$ and $\mathbf{T}_2$ are equal (the radii OA and OB of the pulley), the pulley is in equilibrium if the two forces are equal. A pulley is an equal-arm lever. The simple pulley shown in Fig. 140 gives no gain in force. Its role consists in changing the direction of application of the force. It is often more convenient to pull a rope downwards than to pull it upwards (Fig. 141).

Instead of a rotating pulley, we can use any smooth fixed support through which a rope is passed so that it can slide over the support. The only difference will be in friction (in the latter case it is, as a rule, larger than for a pulley with an axle rotating in bearings).

? 3.16.1. Firemen, mountain-climbers and house-painters sometimes use a fixed pulley as shown in Fig. 142 for lifting themselves. Is there any gain in force as compared to the weight of the load being lifted?

Fig. 141.
Application of a simple pulley for lifting a load.

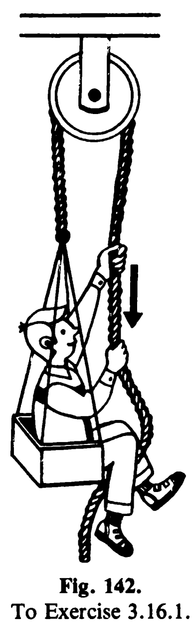

Fig. 142.
To Exercise 3.16.1.

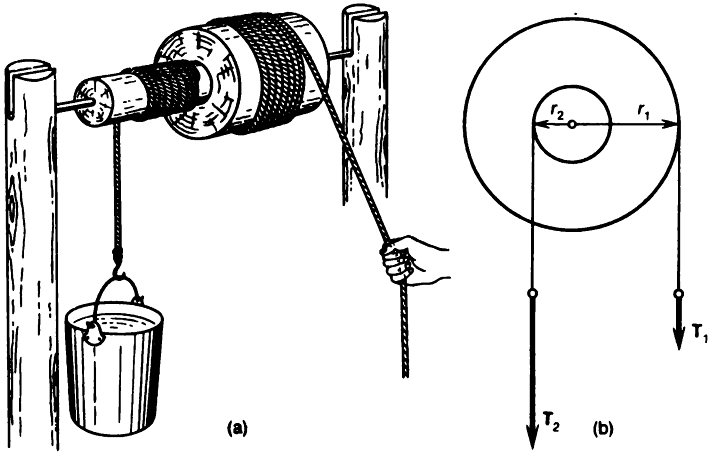

Fig. 143.
(*a*) Double block. (*b*) Schematic of a double block.

In order to gain in force, combinations of pulleys like a *double block* are used. It consists of two pulleys having different radii, rigidly fixed to each other and having a common axle (Fig. 143). A rope is fixed to each pulley so that it can be wound up or unwound, but it cannot slip over the pulley. The arms of the forces (in this case the radii r_1 and r_2 of the pulleys) are different, i.e. a double block operates as a *lever with unequal arms*: the equilibrium conditions of a double block are the same as for a lever with unequal arms:

$$T_1 r_1 = T_2 r_2, \quad \text{or} \quad T_1/T_2 = r_2/r_1.$$

A double block can also be treated as a force transformer. Applying a small force to the rope wound on the larger pulley, we can also obtain here a larger force exerted by the rope wound on the smaller pulley.

A *windlass* is a modification of a double block. It is used, for example, for lifting water from wells. Its vertical version, viz. a *capstan*, was used for lifting anchors on ships when it was done manually (Fig. 144). The spokes of a capstan play the same role as the larger-diameter pulley in a double block. Consequently, the conditions of equilibrium for a windlass are the same as for a double block, but for the smaller and larger radii of the pulleys we must take the radius of the drum and the length of a spoke from the axle to the point of application of the force. Since the spokes can be made many times longer than the radius of the drum, a windlass makes it possible to counterbalance the forces exceeding many times those applied to the spokes.

Various types of complex blocks like *tackle block* are also widely used in engineering. The operation principle of such complex blocks is as

Fig. 144.
Windlass (capstan).

follows (Fig. 145). Two groups of pulleys are fitted on a common axle so that each pulley can rotate independently of the other pulleys of the system. One group forms the stationary part and the other the movable part of a complex block. A rope is passed consecutively through the pulleys of the two groups and is fixed at one end in the fork of the stationary group. If a force T is applied to the free end of the rope, the tension of all parts of the rope will be the same and equal to this force (as before, we neglect friction in the pulleys). Each segment of the rope between the pulleys will act on the moving load with a force T, and all the segments will develop a force nT, where n is the number of sections of the rope connecting the two parts of the tackle block or, which is the same, the total number of pulleys in the movable and stationary parts. Therefore, the force T applied to the end of the rope balances the force nT applied to the movable part of the tackle block, where n is the total number of the pulleys.

A *differential block* consists of a double block and a simple pulley connected by a continuous (endless) chain (Fig. 146*a*). To prevent the chain from slipping over the pulleys, special indentations for chain links are made in the pulleys. Figure 146*b* shows the diagram of forces for a differential block. The equilibrium condition has the form

$$T_1R = T_2(R - r)/2.$$

It can be seen that the equilibrium condition contains the difference in the radii of the two pulleys. For this reason, the system is called a differential (difference) block.

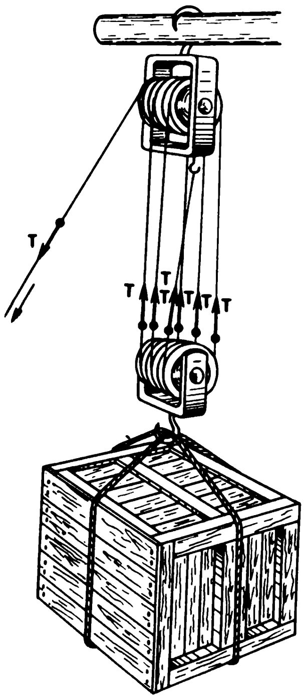

Fig. 145.
Tackle block.

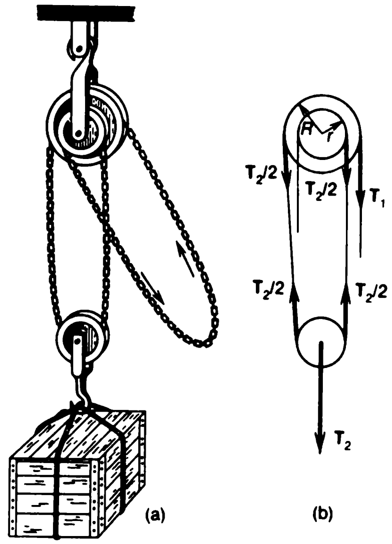

Fig. 146.
(*a*) Differential block. (*b*) Schematic of a differential block.

In all the cases of application of simple machines considered above, the primary problem is to impart motion, however slow, to a body with the help of small forces despite the counteraction of considerable forces (for example, the manual lifting of a heavy anchor). We "gained in force" by acting with a small force on the longer arm of a lever, on the free end of the rope of a tackle block, and so on. It can be easily seen that in this case the other end of the lever or the movable group of pulleys in the tackle block cover smaller distances accordingly.

If we use, for example, a tackle block with n pulleys for lifting a load, we can be content with a force equal to $1/n$ of the weight of the load, although the free end of the rope should be moved during lifting through a distance n times longer than the path of the load (since every segment of the rope between the pulleys becomes shorter by the length of this path). In other words, the load moves at a velocity equal to $1/n$ of the velocity with which the rope is pulled.

In modern engineering, however, it is often important to attain a high velocity of displacement. In such cases, simple machines should be used in such a way that the part being moved is connected with the longer arm of a lever, the free end of the rope of a tackle block, and so on. Naturally, the applied force has to be correspondingly *much stronger* than the force

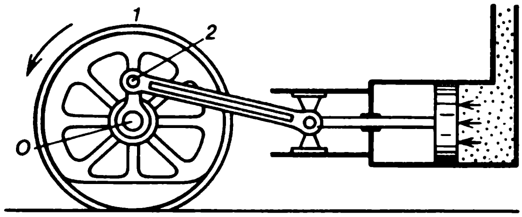

Fig. 147.
The crank mechanism of a locomotive. The velocities of points on rim *1* are higher than the velocity supplied to bearing *2* by the connecting rod fixed to the piston.

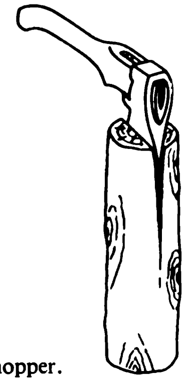

Fig. 148.
Chopping wood by a wood-chopper.

counteracting the displacement. For instance, the connecting rod of the steam engine of a locomotive presses with a large force against the shorter arm of the crank, imparting a high velocity to the points on the rim of the wheel (Fig. 147).

3.17. Wedge and Screw

A *wedge* which has diversified applications also belongs to simple machines. Let us consider the operation of a wedge (the blade of a wood-chopper) in chopping wood (Fig. 148). A force **F** acts when the rear part of the wedge is struck with a sledge-hammer, thus driving the wedge into the crack (Fig. 149). Reaction **R** of the wood being chopped acts on the lateral faces of the wedge. In equilibrium, the sum of the projections of all the forces acting on the wedge must be zero, i.e. the force **F** should balance the sum of the components of the force **R** in the direction of the wedge axis.

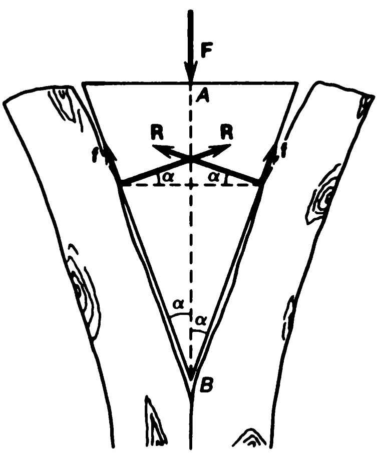

Fig. 149.
Forces acting on a wedge (the blade of a wood-chopper).

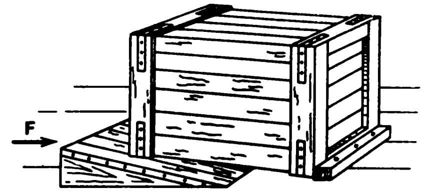

Fig. 150.
Using a wedge for lifting a load.

The projection of the force **R** on AB is $R \sin \alpha$. Figure 149 shows a wedge symmetrical about the plane AB. The lateral surfaces of the wedge form identical angles α with the direction AB, and the two projections are equal. In this case, the equilibrium condition of the wedge is $F = 2R \sin \alpha$. If α is small, the force F can be considerably smaller than $2R$. For example, for a wood-chopper in the form of a steel wedge with the handle, the blade angle is 25° ($2\alpha = 25°$). Accordingly, F is equal to about one fifth of $2R$.

Figure 150 illustrates the application of a wedge for lifting a load. The sharper the wedge, the smaller the force F required to slightly lift a given load.

Like any simple machine, however, a wedge should not only be balanced but also moved in a desired direction. Only in this case it carries out its duty, e.g. chops wood. Unlike in levers and pulleys, friction plays an important part in the operation of a wedge. Friction in a pulley or lever is comparatively small. In a wedge, however, friction between the lateral faces and the body into which the wedge is driven (forces **f** in Fig. 149) are normally very large as well as the reaction **R**, so that the coefficient of friction between steel and wood cannot be excluded from calculations.

A *screw* is a simple machine whose operation principle is similar to that of a wedge (Fig. 151). A screw and a nut wound on it have a round thread. When the screw is rotated, the nut moves along it. To represent visually a turn of the thread, we must take a right triangle wound on a cylinder (Fig. 152). The side AB is equal to the lead h of the screw, i.e. the distance by which the nut moves during a complete turn of the screw, while the side BC is the length of the circumference at the base of the cylinder on which the thread is cut. The hypotenuse AC is the edge of a turn of the thread on the screw. It is in contact with the edge $A'C'$ of a turn of the nut thread. The length of the circumference $BC = 2\pi r$, where r is the radius of the cylinder.

As the screw is turned, its thread presses against the nut thread and makes it move along the screw axis. The friction between the screw and the

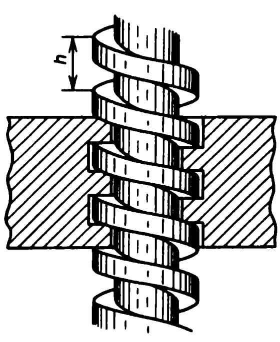

Fig. 151.
Screw and nut (h is the lead of the screw).

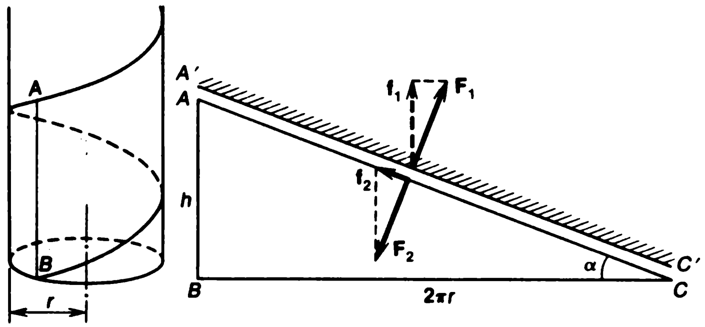

Fig. 152.
A screw can be represented as a right triangle wound on a cylinder.

nut can often be neglected (since their surfaces are thoroughly grinded and lubricated). Therefore, the forces of pressure between the threads of the screw and nut are practically normal to the area of their contact. The screw exerts a force $\mathbf{F}_1$ on the nut while the nut exerts on the screw a force $\mathbf{F}_2$ equal in magnitude to $\mathbf{F}_1$. Turning the screw, we have to overcome the component of $\mathbf{F}_2$ directed against its motion, i.e. the force $\mathbf{f}_2$. The nut experiences the action of the component of $\mathbf{F}_1$ in the direction of the screw axis, i.e. the force $\mathbf{f}_1$. For a given value of f_1, the value of f_2 is the smaller, the smaller the angle α. The relation between the forces is the same as for a wedge with the angle α at the edge.

Thus, the angle of a wedge equivalent to a screw is determined by the lead of the screw and its diameter. The screw equivalent to a sharp wedge is thick (with a large r) and has a small lead (small h). Such is, for example, the screw of a jack, a simple machine for lifting loads (its operation princi-

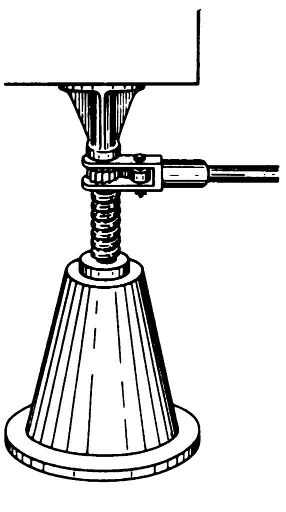

Fig. 153.
Jack.

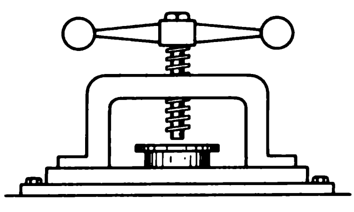

Fig. 154.
Screw press.

ple is illustrated by Fig. 153). Screws are used in various appliances for compressing (a press, Fig. 154) or fastening (bolts, wood screws, and so on). In all these cases, a comparatively small external force can create a large force of pressure.

While analysing the operation of fastening screws, friction should be taken into consideration: in order to move a solid body along the surface of another body, a certain minimum force determined by static friction must be applied (Sec. 2.35). Static friction acting between the head of a screw and the surface into which it is driven can be significant if the screw is driven hard, since it is proportional to the forces of pressure. Besides, this force acts along the screw thread. Since shocks and forces are mainly directed along the screw axis, their component along the thread is insignificant and is the smaller, the smaller the lead of the screw. Therefore, the fastening action of screws and wood screws is usually very strong, i.e. large and repeated axial shocks are required to turn and loosen a screw.

In most cases, a screw is rotated with the help of a more or less long handle fixed to it (as in a press) or the handle of a spanner applied at the head of a screw. In the latter case, we have a combination of two simple machines, viz. a windlass (winch) and a screw (a wedge).

? **3.17.1.** Analyse simple machines whose operation principles are used in a bicycle (handle-bar, pedals and transmission). Where do we gain in force and where in velocity?

Chapter 4

Work and Energy

4.1. "Golden Rule" of Mechanics

Even in ancient times, when simple machines (like levers, pulleys and windlasses) were used a remarkable property of these machines was noted. It turned out that displacements in a simple machine are connected in a quite definite way with the forces developed by the machine. Namely, *the ratio of displacements of two parts of a simple machine, to which forces are applied, is always the reciprocal of the ratio of these forces.* For example, if the force $\mathbf{F}_1$ required for a lever to be in equilibrium must be n times larger than the force $\mathbf{F}_2$ (Fig. 155), the distance s_1 covered by the point of application of the force $\mathbf{F}_1$ during rotation of the lever is equal to $1/n$ of the distance s_2 covered by the point of application of the force $\mathbf{F}_2$.

The same relation is observed for a double block between the forces applied to the ropes wound on the two pulleys and keeping it in equilibrium and the displacements of the ends of the ropes during rotation of the block. This principle was formulated by ancient engineers as follows: *what we gain in force we lose in distance*. This statement is so universal and important that it became known as the *"golden rule" of mechanics*.

Using the notation introduced above, we can express the "golden rule" by the formula

$$F_2/F_1 = s_1/s_2, \quad \text{or} \quad F_1 s_1 = F_2 s_2.$$

The types of motion and construction of machines used in engineering were later refined and perfected, and it was found that the "golden rule" of mechanics is not always applicable in such a simple form. However, along with complication of the types of motion and machines, the "golden rule" was also supplemented and complicated so that it could be extended to more complicated cases. The most important physical concepts of work and energy were developed on the basis of the "golden rule". At the same time, the "golden rule" of mechanics was the first formulation of a fundamental law of nature, viz. the *law of energy conservation*, which turned out to be valid for all natural phenomena.

To come closer to the concepts of work and energy, we shall consider the "golden rule" of mechanics in greater detail. In order to simplify the

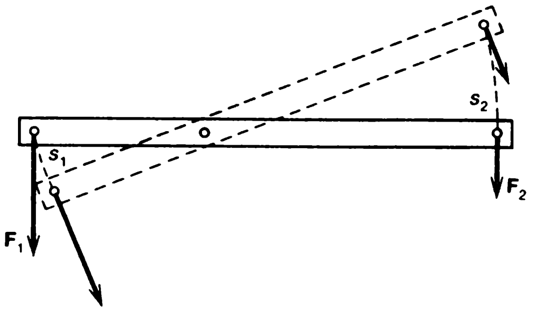

Fig. 155.
The force acting on the left arm of a lever is n times the force acting on the right arm. The distance s_1 covered by the point of application of force $\mathbf{F}_1$ will amount to $1/n$ of the distance s_2 covered by the point of application of force $\mathbf{F}_2$.

analysis, we shall first assume that there is no friction, and then see how the situation changes when friction is taken into account.

4.2. Applications of the "Golden Rule"

The "golden rule" of mechanics is practically observed only when the motion of simple machines occurs uniformly or with a small acceleration.[1] For example, when a double block rotates, the ends of the ropes wound on the pulleys having radii r_1 and r_2 and rigidly coupled together will be displaced by the distances s_1 and s_2 proportional to these radii:

$$s_1/s_2 = r_1/r_2.$$

Consequently, for the "golden rule" of mechanics to be valid for the double block, the following condition must be satisfied:

$$F_1/F_2 = r_2/r_1.$$

In this case the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are balanced, and hence either the block is at rest or it moves uniformly.

In order to set the double block in motion, the equilibrium should be disturbed by adding a certain force $\mathbf{f}$ to one of the forces, say, $\mathbf{F}_1$ (Fig. 156). The caused motion will be accelerated (we recall that friction is absent by hypothesis). Now $(F_1 + f)s_1 > F_2 s_2$, and hence the "golden rule" of mechanics is not observed when the double block moves with an acceleration. However, the smaller the force $\mathbf{f}$ as compared to $\mathbf{F}_1$, the closer the products of forces and distances for the two ropes of the double block, and the smaller the deviation from the "golden rule". For very small $\mathbf{f}$, the

[1] The "golden rule" was established by ancient engineers just because they had to deal with such cases.

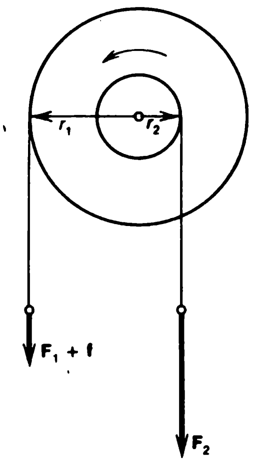

Fig. 156.
By increasing the force $\mathbf{F}_1$ by a small amount $\mathbf{f}$, we make a double block rotate with an acceleration.

double block moves with a very small acceleration, i.e. its motion is close to a uniform one.

Thus, the "golden rule" of mechanics is observed *quite exactly for a uniform motion* (without friction) and *approximately for a motion with a small acceleration*. None of the machines, however, moves uniformly all the time: at first it is to be set in motion and then it must be stopped. But if a double block is started and stopped with a small acceleration, the "golden rule" of mechanics is valid practically for the entire motion of this machine.

It can be easily seen that the "golden rule" of mechanics is valid not only for a double block but also for other simple machines provided that the directions of the forces applied to a machine and the directions of displacements of their points of application coincide. For all such machines, the "golden rule" is valid under the same conditions as for a double block: for the uniform motion of a machine (and practically also for a motion with a small acceleration) the product of the force and the displacement of the point of application is the same for both the forces.

? 4.2.1. Prove that the "golden rule" of mechanics is also valid for a tackle block and windlass.[2]

4.3. Work Done by a Force

In the previous section we established that for a uniform motion of a simple machine there is always a certain relation between forces and

[2] See Fig. 145. — *Eds.*

displacements: if the directions of force and displacement coincide, the product of force and the displacement of its point of application is the same for both forces. Thus, this product plays a special role and can be used to characterise the operation of simple machines. It will be shown later that it plays an exceptionally important role in many other phenomena. For this reason, this product is treated as an independent physical quantity known as the *work* done by a force.

In a particular case *when the direction of the force and the direction of displacement coincide, work A is equal to the product of the magnitude of the force* **F** *and the magnitude of the displacement* **s**:

$$A = Fs. \tag{4.3.1}$$

The general expression for work will be obtained in Sec. 4.5. Thus, when the point of application of a force moves, the force does a work. If, however, the point of application of a force does not move in spite of the action of the force, no work is done. For example, if a load hanging on a suspender is at rest, the force of gravity acting on the load does no work. If, however, the load descends or falls, this force does a work equal to Ph (P is the force of gravity and h is the distance by which the load has been lowered).

Similarly, applied forces do no work in simple machines (lever, pulley, etc.) unless a machine moves. If, however, a pulley starts to rotate and the end of the rope to which a force is applied starts to move in the direction coinciding with the direction of the force, this force does work equal to the product of the force and the displacement.

In all moving machines (steam engine, internal combustion engine, electric motor, and so on), forces do work during the operation of a machine. In a steam engine, for example, the force of pressure of steam against the piston does work during the motion of the piston. The force of pressure of powder gas produced as a result of combustion of a powder charge does work during the motion of a shell. The forces of interaction of electric currents in the winding of an electric motor do work when the motor rotates.

The concepts of work as a physical quantity introduced in mechanics conform only to a certain extent to the everyday notion of work. Indeed, the work of a loader is considered to be the more, the larger the weight of the lifted load and the height to which it should be lifted. However, we are inclined from a worldly point of view to attribute the term "physical work" to all kinds of human activity where muscular efforts are required. But according to the definition of work in mechanics, such an activity may not be accompanied by work. In the well-known myth about Atlas supporting the heavens on his shoulders, it is said that he did tremendous work meaning the effort required for supporting an enormous load. From the point of view of mechanics, however, no work was done, and the muscles of Atlas could be just replaced by a strong column.

4.4. Work Done during a Displacement Normal to the Direction of Force

When a body moves in a direction normal to the direction of the force, the force does not affect the displacement in this direction. Therefore, we assume that in this case the force does no work: *if the force and displacement are at right angles to each other, the work done by the force is zero.* For example, when a body moves along a horizontal plane, the work done by the force of gravity is equal to zero (Fig. 157).

4.5. Work Done by a Force Acting at an Arbitrary Angle to Displacement

We have defined the work done by a force in two special cases: when the displacement of the point of application of the force coincides in direction with the force and when it is at right angles to the force. In the former case, the work is equal to the product of the force and the displacement, while in the latter case it is zero. Let us now find the expression for work for an arbitrary mutual orientation of force and displacement. For the sake of simplicity, we shall assume that the force **F** is constant (i.e. its magnitude and direction remain unchanged), while its point of application moves in a straight line (Fig. 158).

We decompose the force **F** into two components: $\mathbf{F}_\parallel$ along the displacement **s** and $\mathbf{F}_\perp$ normal to the displacement **s**. Let the angle α between vectors **F** and **s** be acute (Fig. 158*a*). Then the force $\mathbf{F}_\parallel$ coincides in direction with the displacement, and according to formula (4.3.1), the work done by it is $F_\parallel s$. The force $\mathbf{F}_\perp$ is normal to the displacement and hence does no work. Assuming that the work done by the resultant force is equal to the sum of the works done by the components, we obtain the work done by the force **F** over the displacement **s** in the form $A = F_\parallel s$. If the angle α is acute, $F_\parallel$ is the projection of the force **F** on **s**. Denoting the projection by F_s, we can write

$$A = F_s s. \tag{4.5.1}$$

We arrive at the conclusion that *the work is equal to the projection of the*

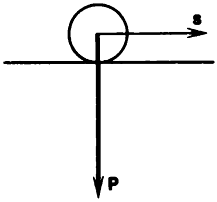

Fig. 157.
When a ball rolls over a horizontal table, the force of gravity **P** is normal to the displacement **s** and does no work.

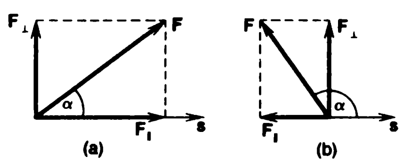

Fig. 158.
Decomposition of force **F** into components $\mathbf{F}_\parallel$ and $\mathbf{F}_\perp$.

force on the direction of the displacement multiplied by the magnitude of the displacement of the point of application of the force.

If $\alpha < \pi/2$, the projection $F_s = F \cos \alpha$ (Sec. 1.24). Consequently, formula (4.5.1) can be written in the form [3]

$$A = Fs \cos \alpha. \tag{4.5.2}$$

The product $s \cos \alpha$ is the projection of the displacement on the direction of the force. Denoting this projection by s_F, we obtain another expression for work:

$$A = Fs_F. \tag{4.5.3}$$

According to this expression, *the work is equal to the projection of the displacement of the point of application of the force on the direction of the force multiplied by the magnitude of the force.*

So far, we have assumed that the angle α is acute. However, the definition of work expressed by formula (4.5.2) is also valid for obtuse angles ($\alpha > \pi/2$, Fig. 158*b*). In this case, $\cos \alpha < 0$, and the work is *negative* (the projection F_s of the force **F** on the displacement **s** is negative and equal to $F \cos \alpha$). Consequently, expression (4.5.2) defines work for any values of the angle α between 0 and π. (The same refers to formulas (4.5.1) and (4.5.3).)

Thus, work is an algebraic quantity: if the angle α between the directions of the force and displacement is acute, the work is positive, and if the angle is obtuse, the work is negative. In the special case when $\alpha = 0$, the work $A = Fs$; if $\alpha = \pi$, $A = -Fs$; and when $\alpha = \pi/2$, the work is zero.

4.6. Positive and Negative Work

If the force applied to a body does a positive work, the velocity of the body increases. Indeed, in this case the force, and hence the acceleration, are directed along the velocity and increase it. If, however, the force does a negative work, the acceleration is directed against the velocity, and the velocity of the body decreases.

Let us suppose that we throw a body in the vertical direction. As long as the body flies upwards, the force of gravity does a negative work on it, and the velocity of the body decreases to zero. Having reached the upper point of its ascent, the body starts to move downwards with an acceleration. In this case, the force of gravity does a positive work.

[3] The product of the magnitudes of two vectors and the cosine of the angle between them is called the *scalar (dot) product* of the vectors. The scalar product of two vectors is normally written in the form $\mathbf{a} \cdot \mathbf{b} = ab \cos \alpha$ (there also exist other forms of notation). Consequently, the work can be represented as a scalar product of the force **F** and displacement **s**: $A = \mathbf{F} \cdot \mathbf{s}$. -- *Eds.*

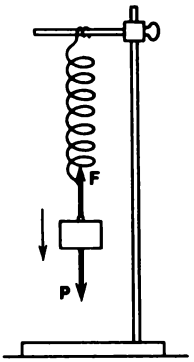

Fig. 159.
When a load moves down, the work done by the force **P** is positive, while the work done by force **F** is negative.

If two opposite forces act on a body, one of them does a positive work and the other a negative work. For example, if we suspend a load by an unstretched spring (Fig. 159) and allow it to move downwards, the force of gravity **P** acting on the load does a positive work since the load moves in the direction of this force. On the other hand, the force **F** exerted by the spring on the load does a negative work.

When we lift a load, we have to overcome the force of gravity attracting the load to the Earth. In this case, the work of the force of gravity is negative. The work done by us in overcoming the force of gravity is positive. Sometimes it is called the work done *against* the force of gravity. Similarly, when two opposite forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a body, the work done by one of them, say, $\mathbf{F}_1$, is positive and the work done by the other force, viz. $\mathbf{F}_2$, is negative. We can say that the force $\mathbf{F}_1$ does work against the force $\mathbf{F}_2$. It should be stressed that if some force **F** does a negative work, the work done by some other force against the force **F** will be positive.

4.7. Units of Work

Since the work is defined as the product of the force and displacement, for unit of work we must take the work done by a unit force when its point of application is displaced in the direction of the force by a unit distance.

The SI unit of work is the work done by the force of one newton over the displacement of one metre. This unit is known as a *joule* (J).[4]

In the CGS system where the unit of force is a dyne and the unit of length is a centimetre, the unit of work is an *erg* (erg), i.e. the work done by the force of one dyne over the displacement of one centimetre.

? 4.7.1. Calculate the work done by a pump in 3 min if it supplies 50 l of water per second to a height of 20 m.

[4] This unit was introduced in honour of the English physicist James Joule (1818-1889).

4.7.2. A boy pulls a sledge along a horizontal so that the rope fixed to the sledge is stretched at 37° to the horizontal by a force of 20 N. How much work will be done by him over a distance of 600 m?

4.8. Motion over a Horizontal Plane

It was mentioned in Sec. 4.4 that the force of gravity does no work when a body moves along a horizontal plane. The only work which has to be done in such a displacement is the work against friction and resistance of the medium. When a cyclist moves along a horizontal road, he does no work against the force of gravity. Such a work is only done when he climbs a hill.

The situation with a pedestrian is slightly different. When he walks along a horizontal road, his centre of gravity does not remain at the same height but moves up and down with every step. When the centre of gravity moves up, the pedestrian does work. Therefore, even walking along a horizontal road, the pedestrian does work not only against the resistance of the medium but also against the force of gravity. Assuming that the centre of gravity moves up with every step by 5 cm and the mass of a person is 70 kg, we see that with each step, considerable work equal to 35 J is done to lift the centre of gravity. The negative work done when the centre of gravity descends is not used. An even gait reduces the amount of work done during walking and hence is less tiring.

4.9. Work Done by the Force of Gravity in Motion over an Inclined Plane

Let us apply the result obtained in Sec. 4.5 to determine the work done by the force of gravity **P** when a body moves down an inclined plane (Fig. 160).

The projection NO of the displacement $\mathbf{s} = \overrightarrow{NM_1}$ on the direction of the force of gravity, i.e. on the vertical, is equal to the height h of the inclined plane. Consequently, according to formula (4.5.3), the work done by the force of gravity during the motion of the body along the inclined plane from point N to point M_1 is equal to the force of gravity multiplied by the height of the inclined plane:

$$A = Ph. \tag{4.9.1}$$

The same result can be also obtained for the inclined plane NM_2. Thus, the

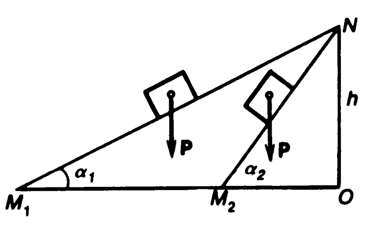

Fig. 160.
When a body slides down an inclined plane, the work done by the force of gravity is determined by the height h by which the load is lowered and does not depend on the slope of the inclined plane.

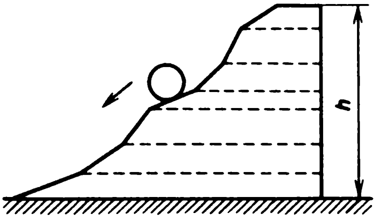

Fig. 161.
Any path can be represented as a set of a large number of small segments of inclined planes.

work done by the force of gravity does not depend on the slope of the inclined plane and is determined only by its height. The force of gravity would do the same work if the load descended by the same distance along the vertical.

Thus we can draw a more general conclusion: the force of gravity does the work $A = Ph$, where h is the height by which the load is lowered, irrespective of the path traversed by the load. Indeed, any path can be represented as consisting of a large number of segments of various inclined planes (Fig. 161). The work done on each segment is determined by the height by which the load is lowered during the displacement over this segment. The work done over the entire path is equal to the force of gravity acting on the load multiplied by the total height by which the load was lowered.

A similar conclusion can also be drawn for a body lifted up an inclined plane or some other path. In this case, the work *against* the force of gravity is also independent of the path. It is determined only by the height to which the body is lifted.

4.10. Principle of Work Conservation

The concept of work allows us to approach the "golden rule" of mechanics from a new point of view. Let us consider again a double block and suppose that a certain load attached to the end of the rope is lifted with the help of a force applied to the end of the other rope. It was shown earlier that the product of force and displacement is the same for both ends of the ropes. On the other hand, the force acting on the second rope and its displacement coincide in direction. Similarly, the directions of the displacement of the load and the force exerted on it by the first rope also coincide. Consequently, the work done by the force applied to the second rope is equal to the work done by the simple machine on the load. Thus, a double block does not create or destroy work, it just *transfers* it. At the same time, the total work done on the simple machine turns out to be zero. Indeed, the direction of the force coincides with the direction of displacement for one rope and is opposite to it for the other rope.

This is valid for all simple machines both in the cases when the directions of forces and displacements coincide, i.e. when the "golden rule" is applicable, and in the cases when they do not coincide, and the "golden rule" is inapplicable.

Thus, we arrive at a more general principle than the "golden rule" of mechanics: *in any uniformly moving simple machine, work is transferred without change*, i.e. the work done by the machine is equal to the work done by the force driving the machine. This statement is known as the *principle of work conservation*.

It should be borne in mind that the principle of work conservation is not satisfied if a simple machine is deformed in transferring work, for example, if a lever bends or the ropes of a tackle block are stretched. Indeed, if we try to lift a heavy load by using a flexible twig as a lever, we shall not be able to shift the load resting on the shorter arm even if we do a certain work on the longer arm. Consequently, the work done on the shorter arm will be zero, and the only result will be the bending of the twig. Similarly, if we replace the rope in a pulley by a stretchable rubber band and try to lift a heavy load from the ground, we shall do work by stretching the rubber at one end. However, the other end of the rubber attached to the load will remain at rest, and no work will be done by it. Here too the only result will be the deformation of the machine. If we take a more rigid lever or a thicker rubber band, we can possibly slightly lift the load. However, the work done by the other part of our machine will be less than the work done by the applied force, and the "golden rule" of mechanics and the principle of work conservation will be violated. For this reason, we shall henceforth assume that all simple machines are made of unbending levers, nonstretchable ropes, and so on. Then the principle of work conservation will be observed if we neglect friction.

The principle of work conservation allows us to easily calculate forces in simple machines. For instance, in a tackle block with n turns of a rope (see Fig. 145), the end of the rope pulled by the hand displaces to a larger extent than the hook to which the load is fixed. Indeed, when the hand is moved over a distance s the movable part of the block is raised to a height equal to $1/n$ of this distance since the change in the length of the rope is distributed among its n segments between the pulleys. Consequently, the principle of work conservation states that the force applied to the end of the rope must be equal to $1/n$ of the force applied to the hook (we neglect the mass of the movable group of pulleys). This result was obtained earlier (Sec. 3.16) from a direct analysis of forces.

? **4.10.1.** A piston whose mass is 200 kg is lifted with the help of a right-angled wedge with sides 10 cm and 1 m, which is being pushed under the piston. Find the force that should be applied to the rear side of the wedge (Fig. 162). Friction should be neglected.

4.10.2. A screw press (see Fig. 154) has a thread with a lead of 5 mm. The length of the

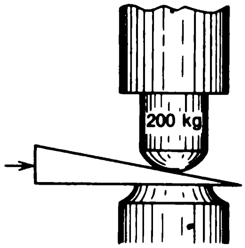

Fig. 162.
To Exercise 4.10.1.

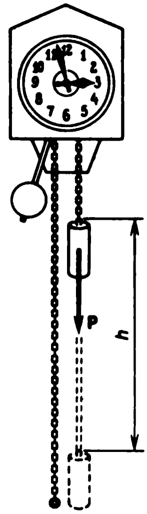

Fig. 163.
A lifted weight possesses a store of work which is gradually spent on the operation of a clock.

handle inserted into its head is 40 cm. What force is to be applied to the handle to develop a force equal to 10^4 N? Friction should be neglected.

4.11. Energy

Simple machines are able to do work but they cannot "store" this ability since they gain it at one part but simultaneously give away at the other. In many cases, however, bodies may "accumulate" the ability to do work. Special mechanisms are constructed which can store work and then give it away. A typical example of such a mechanism is the weight winding mechanism of a wall clock (Fig. 163). By lifting a weight, we do some work. As a result, the clock movement becomes able to do work over the long period of time required for them to run, i.e. to maintain the motion of all its gears, arms and pendulum. The motion of these parts is hindered by the resistance due to friction. As the clock goes, the weight gradually moves down, and the store of ability for work decreases. After some time the clock must be wound up again, i.e. it must be made able to do work. When we wind the clock up, the weight mechanism accumulates the ability for work which is consumed as the clock goes. While lifting the weight, we accumulate work. Being lowered, the weight can do work.

Work can be "stored" in a body not only by lifting it to a certain height. By deforming a body, say, by stretching or compressing a spring, we do work. As a result, the deformed body can do work. The wound up, viz. deformed spring of a wrist watch can do work as well as the "spring drive" of mechanical toys.

Work also has to be done in order to impart velocity to a body. As a result, the body becomes capable of doing work at the expense of its velocity. For instance, when trains are being made up, a shunting engine pushes a carriage to the train. Coming to a halt, the carriage compresses buffer springs. A bullet hitting a target does work in destroying the material, and so on.

In all these cases, work is done when the state of a body changes (the load is *lowered*, the spring is *unwound* or the moving body is *stopped*). Unless these changes have occurred, no work is done. A body possesses a certain store of work which has not yet been accomplished. In doing work, this store is being spent. On the other hand, while doing work *on a body* (lifting it above the ground, deforming it or imparting a velocity to a body) we supply it with a store of work which can be used later when the body returns to the original state.

The store of work which can be done by a body when it changes its state is known as *energy*.[5]

The mechanical forms of energy include the energy associated with lifting a body above the ground (and in general the energy due to the forces of universal gravitation), the energy associated with deformations of a body, and the energy due to motion.

A change in energy is determined by the work which should be done to cause this change. Consequently, energy should be measured in the units of work, viz. in joules.

4.12. Potential Energy

Let us calculate the work A done by a certain force $\mathbf{F}$ during lifting a body of mass m to a height h. We shall assume that the motion is slow and friction can be neglected. It was shown above (Sec. 4.9) that the work against the force of gravity does not depend on the path along which we lift a body: along the vertical (as the weight in a clock), along an inclined plane (as in the case of sledges pulled up a hill) or in some other way. In all cases, the work $A = mgh$. When the body is lowered to the initial level, the force of gravity does the same work which has been done by the force $\mathbf{F}$ in lifting the body.

Thus, while lifting a body, we stored work equal to mgh. In other words, a *body lifted above the ground has an energy equal to the product of the force of gravity acting on the body and the height to which it has been lifted*. This energy does not depend on the path along which the body was lifted and is determined only by the position of the body (the height to which it was lifted); it is called the *potential energy*. Thus, the potential

[5] This definition is simplified. More rigorously, energy is defined as a physical quantity characterising the ability of a body (or a system of interacting bodies) to do work. — *Eds*.

energy E_p of a body lifted to a certain height is expressed by the formula

$$E_p = mgh. \tag{4.12.1}$$

For a given initial position of a body, the work it can do, i.e. its potential energy, depends on the height by which it can be lowered. In the weight mechanism of a clock, this is determined by the length of the chain to which the weight is attached, while in the example with an inclined plane, by the height of the uppermost point of the plane above its lowermost point. In other cases, the lowermost level cannot be determined in a natural way. For instance, if a body rests on a table, its potential energy can be determined by the work it would do while falling to the floor, to the ground level, or to the bottom of the cellar. Therefore, we have to *stipulate* beforehand from which level the height, and hence the potential energy of the body, should be measured. This level can be chosen quite arbitrarily since *in all physical phenomena we are interested not in the potential energy proper but rather in its change* which determines the work done. The change in the potential energy will obviously be the same irrespective of the initial level.

Unless stated otherwise, it will be assumed that the potential energy of a body lying on the ground is zero. Then for h in formula (4.12.1) we must take the height of the body above the ground. If the body is bulky, we must take for h in (4.12.1) the distance from the ground (or from a zero level) to the centre of gravity of the body.

Let us find, for example, the difference in potential energies of a vertical pole (Fig. 164, position A_0B_0) and the same pole lying on the ground (position A_2B_2). Suppose that the pole goes over from position A_0B_0 to A_2B_2 in two stages. First it rotates about its centre of gravity (in the case under consideration about the midpoint) to position A_1B_1. In this case, the

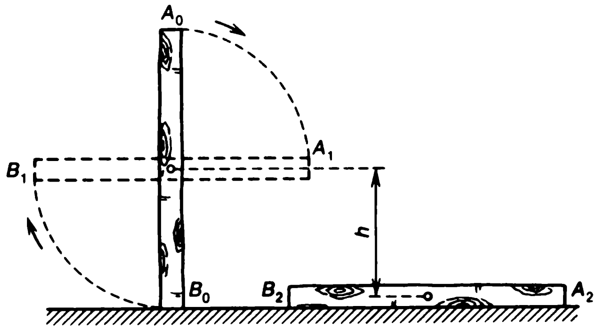

Fig. 164.
When a pole goes over from position A_0B_0 to A_1B_1, the force of gravity does no work since the centre of gravity of the body remains in the same position. As the pole goes over from position A_1B_1 to A_2B_2, work mgh is done.

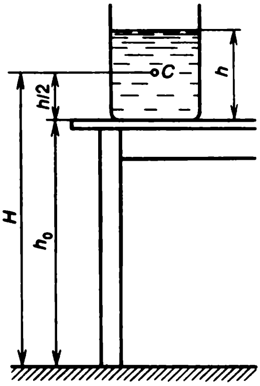

Fig. 165.
To the calculation of the potential energy of liquid in a vessel.

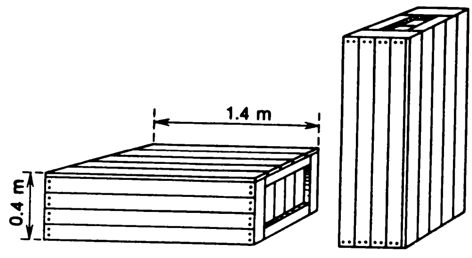

Fig. 166.
To Exercise 4.12.1.

upper part of the pole is lowered while the lower part is lifted. The force of gravity does a positive work on the upper part and a negative work on the lower part so that the two works are equal in magnitude, and the total work of the force of gravity is zero. Only during a transition from position A_1B_1 to A_2B_2 the force of gravity does a positive work. Consequently, the potential energy of the vertical pole is higher than the potential energy of the pole lying on the ground by mgh, where m is the mass of the pole and h is the difference in the heights of the centre of gravity in positions A_0B_0 and A_2B_2.

While calculating the potential energy of a liquid with mass m contained in a cylindrical vessel (Fig. 165), we must take the height H of the centre of gravity C of the liquid above the zero level, i.e. the height h_0 of the bottom of the vessel above the zero level plus half the height of the level of the liquid in the vessel, $h/2$, so that the potential energy is

$$E_p = mg(h_0 + h/2).$$

? **4.12.1.** A 40-kg box whose dimensions are shown in Fig. 166 is moved from position (*a*) to position (*b*). Determine the increment of the potential energy of the box assuming that its centre of gravity lies on the intersection of its diagonals.

4.12.2. The reservoir of a hydroelectric power plant has a cylindrical shape such that its area is 2 km^2 and the depth is 6 m. The bottom of the reservoir is at 12 m above the level of water in the drainage canal behind the power plant. What is the potential energy of water in the reservoir?

4.13. Potential Energy of Elastic Deformation

A deformed elastic body (like a stretched or compressed spring) can do work on bodies in contact with it while returning to the undeformed state.

Consequently, the elastically deformed body has a potential energy. This energy depends on the relative position of the parts of the body, for example, the turns of a spring. The work that can be done by the stretched spring depends on its initial and final extension. Let us calculate the work that can be done by a stretched spring returning to the undeformed state, i.e. find the potential energy of the stretched spring.

Suppose that the stretched spring is fixed at one end, while the other end does work while moving. We must take into account the fact that the force exerted by the spring does not remain constant but changes in proportion to extension. If the initial extension of the spring, measured from the undeformed state, is l, the initial value of the elastic force is $F = kl$, where k is the proportionality factor which is known as the *rigidity* of the spring. As the spring is compressed, this force decreases linearly from kl to zero. Hence, the average value of the force is $F_{av} = kl/2$. It can be shown that the work A is equal to this average value multiplied by the displacement of the point of application of the force:

$$A = \frac{kl}{2} \cdot l = \frac{kl^2}{2} .$$

Thus, the potential energy of a stretched spring is

$$E_p = \frac{kl^2}{2} . \tag{4.13.1}$$

The same expression can be obtained for a compressed spring.

The potential energy in formula (4.13.1) is expressed in terms of the rigidity of the spring and the extension l. Having replaced l by F/k, where F is the elastic force corresponding to the extension (or compression) l of the spring, we obtain

$$E_p = \frac{F^2}{2k} . \tag{4.13.2}$$

This expression defines the potential energy of a spring stretched (or compressed) by the force F. It shows that by applying the *same force* for stretching different springs, we impart to them different stores of potential energy. The higher the rigidity of a spring, i.e. the higher its elasticity, the lower its potential energy, and conversely, the softer the spring, the higher the energy stored in it under the action of a given tensile force. It can be visualised if we take into account the fact that for the same forces the extension of a soft spring is larger than for a rigid spring, and hence the product of the force and displacement of the point of application of force, i.e. work, is also larger.

This regularity is of great importance, for example, for the design of various springs and shock absorbers. During the landing of an aeroplane, the shock absorber of its undercarriage has to do large work during compression to arrest the vertical velocity of the plane. In a shock absorber having a low rigidity compression will be larger, but elastic forces emerging as a result will be weaker, and the plane will be better protected from the damage. For the same reason, a cyclist feels stronger shocks on the road if the tyres are tightly pumped.

4.14. Kinetic Energy

Bodies may have stored work, i.e. possess energy not only if they occupy a certain position or are deformed, but also if they have a velocity. For example, a carriage can move uphill if it has an initial velocity. A bullet or a shell can rise to a considerable height if it leaves the barrel at a high velocity. In these cases, moving bodies do work against the force of gravity when they move upwards. A moving body can also do work against elastic forces. A paper ball attached to a thin rubber band can considerably stretch this band if we strike it sharply giving it a high velocity (Fig. 167). When a moving carriage strikes with its buffers against the buffers of another carriage, the springs of the buffers are strongly compressed, i.e. work is done to compress the spring.

In all the above examples, a body does work not because it occupies a certain *position*, but since it has a *velocity*. A carriage at rest cannot move uphill by "itself" or compress the springs of the buffer, while a moving carriage is capable of doing this.

Every time a body does work due to its motion, its velocity decreases. If the velocity of the body drops to zero, the storage ability for work due to motion has been exhausted. Hence, any moving body has a certain ability to do work, i.e. a certain energy due to its motion. The energy possessed by virtue of motion is known as *kinetic energy*.

The sum of the kinetic and potential energies gives the *total mechanical energy* of a body.

4.15. Kinetic Energy in Terms of Mass and Velocity of a Body

It was shown in Secs. 4.12 and 4.13 that potential energy can be stored if we make a force do work by lifting a load or compressing a spring. Similarly, kinetic energy can be stored as a result of the work done by some force. Indeed, if a body acquires an acceleration under the action of an external force and starts to move, the force does work and the body acquires a

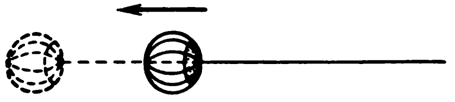

Fig. 167.
A rapidly flying paper ball stretches a rubber band.

velocity, i.e. a kinetic energy. For example, the force of pressure of powder gas, which pushes a bullet out of the barrel of a rifle, does work at the expense of which the kinetic energy of the bullet is stored. Conversely, if work is done as a result of motion of the bullet (for example, if the bullet moves upwards or if it hits an obstacle and destroys it), the kinetic energy of the bullet decreases.

Let us analyse the transition of work into kinetic energy by using as an example a body acted upon by a single force (in the case of several forces, this is the resultant of all the forces acting on the body). Suppose that a constant force F acts on a body of mass m which is initially at rest. Under the action of the force F, the body will move with a constant acceleration $a = F/m$. Having covered a distance s in the direction coinciding with the direction of the force, the body acquires a velocity v connected to the traversed path through the formula $s = v^2/2a$ (Sec. 1.22). Hence we can find the work A done by the force F:

$$A = Fs = \frac{Fv^2}{2a} = \frac{mv^2}{2}.$$

Similarly, if a force directed against the motion of a body having a velocity v starts to act, the body will be decelerated and will come to a halt after having done work against the acting force, which is also equal to $mv^2/2$. Consequently, the kinetic energy E_k of a moving body is equal to half the product of its mass and the squared velocity:

$$E_k = \frac{mv^2}{2}. \tag{4.15.1}$$

Since the change in kinetic energy, like the change in potential energy, is equal to the (positive or negative) work done during this change, kinetic energy is also measured in units of work, viz. joules.

? **4.15.1.** A body of mass m moves at a velocity $\mathbf{v}_0$ by inertia. A force directed along the direction of motion, starts to act on the body, as a result of which its velocity becomes $\mathbf{v}$ in some time. Show that the increment of the kinetic energy of the body is equal to the work done by the force for the case when the velocity (a) increases, (b) decreases, (c) changes sign.

4.15.2. Is the work done to impart a velocity of 5 m/s to a train at rest more than the work done to accelerate the same train from 5 m/s to 10 m/s? The resistance to the motion should be neglected.

4.16. Total Energy of a Body

Let us consider the change in the kinetic and potential energies of a body thrown upwards.

As the body rises, its velocity decreases according to the law $v = v_0 -$

gt, where v_0 is the initial velocity and t is the time. The kinetic energy also decreases in accordance with the law

$$E_k = \frac{m(v_0 - gt)^2}{2} = \frac{mv_0^2}{2} - mv_0gt + \frac{mg^2t^2}{2}.$$

Since the initial kinetic energy of the body is $mv_0^2/2$, by the moment t the *decrease* [6] in the kinetic energy will be

$$-\Delta E_k = mv_0gt - \frac{mg^2t^2}{2}. \tag{4.16.1}$$

On the other hand, the height of the body at the moment t is

$$h = v_0t - \frac{gt^2}{2}.$$

Consequently, the increment of the potential energy over the time t is [7]

$$\Delta E_p = mv_0gt - \frac{mg^2t^2}{2}. \tag{4.16.2}$$

Comparing this expression with (4.16.1), we see that the increment of the potential energy of the body during time t is equal to the decrease in the kinetic energy over the same time. Thus, as the body moves upwards, its kinetic energy is gradually transformed into potential energy. At the top of the ascent, the entire kinetic energy has been completely transformed into potential energy. As the body moves downwards, the reverse process occurs: the potential energy of the body is converted into kinetic energy.

In these transformations, *the total mechanical energy of the body (i.e. the sum of its kinetic and potential energies) remains unchanged*, since the decrease in kinetic energy during the ascent is completely made up by the increment of potential energy (the opposite is true for falling). If we assume that the potential energy of the body is zero on the ground (Sec. 4.12), the sum of the kinetic and potential energies of the body at any height during the ascent or falling is

$$E = E_k + E_p = \frac{mv_0^2}{2}, \tag{4.16.3}$$

i.e. it remains equal to the initial kinetic energy of the body.

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[6] A decrease in a certain quantity, unlike an increment, is defined as the difference in the initial and the final values of this quantity: $A_{in} - A_{fin}$. A comparison shows that for the same change in the quantity, the increment and the decrease differ only in sign. Therefore, denoting the increment by ΔA, we should denote the decrease by $-\Delta A$. — *Eds.*

[7] According to (4.16.1) and (4.16.2), we have $-\Delta E_k = \Delta E_p$, whence $\Delta E_k + \Delta E_p = \Delta(E_k + E_p) = \Delta E = 0$ (the sum of the increments of kinetic and potential energies is equal to the increment of the total energy E). If the increment of any quantity over any time interval is zero, this quantity remains constant. — *Eds.*

This conclusion is a particular case of one of the most important laws of nature, viz. the law of energy conservation.

? 4.16.1. A stone is thrown at a velocity of 15 m/s from a tower of 20-m height. Determine the velocity of the stone when it hits the ground and compare it with the velocity it would have if it fell from the same height with a zero initial velocity. Air resistance should be neglected.

4.16.2. Assuming that formula (4.16.2) and the dependence of potential energy on height are known, derive the law of motion for a body thrown vertically.

4.17. The Law of Energy Conservation

In the example analysed in the previous section it was shown that the potential energy of a body thrown upwards increases at the expense of a decrease in its kinetic energy. When the body falls, its kinetic energy increases at the expense of a decrease in the potential energy so that the total mechanical energy of the body does not change. Similarly, if a compressed spring acts on a body, it can impart to the body a certain velocity, i.e. a kinetic energy. However, the spring will straighten out in the process, and its potential energy will decrease accordingly, and the sum of the kinetic and potential energies remains constant. If the force of gravity also acts on a body subjected to the action of a spring, the sum of the gravitational energy, potential energy of compressed spring, and kinetic energy of the body will again remain constant although the energy of each type will change during the motion of the body.

The energy can be converted from one type to another or can be transferred from one body to another, but the total store of mechanical energy remains unchanged. Experiments and calculations show that if only elastic forces and forces of gravity act (in the absence of friction), *the total potential and kinetic energies of a body or a system of bodies remain constant in all cases*. This is the essence of the *law of conservation of mechanical energy*.

Let us illustrate the law of energy conservation with the help of the following experiment. A steel ball which falls onto a steel or glass slab and strikes against it jumps up almost to the same height from which it fell (Fig. 168).[8] During the motion of the ball, a number of energy conversions take place. During its fall the potential energy of the ball is transformed into the kinetic energy. When it touches the slab, both the ball and the slab start to deform. The kinetic energy is transformed into the potential energy of elastic deformation of the ball and the slab, this process being continued until the ball stops, i.e. until its entire kinetic energy is converted into

[8] It will be explained in Sec. 4.18 why the ball does not rise *exactly* to the same height from which it fell.

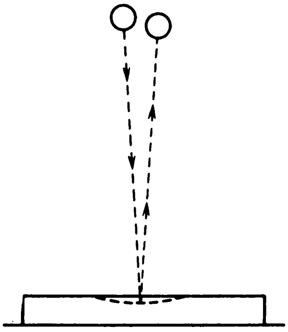

Fig. 168.
After bouncing off a steel slab, a steel ball jumps up almost to the same height from which it fell.

potential energy of elastic deformation. Then under the action of elastic forces of the deformed slab, the ball acquires an upward velocity: the potential energy of elastic deformation of the slab and the ball has been converted into the kinetic energy of the ball. In the further upward motion, the velocity of the ball decreases under the action of the force of gravity, and the kinetic energy is converted into the potential energy of gravitation. At the uppermost point, the ball again has only potential energy of gravitation.

Since we can assume that the ball ascended to the same height from which it fell, the potential energy of the ball will be the same at the beginning and end of the process under consideration. Moreover, the sum of the potential energy of gravitation, potential energy of elastic deformation and kinetic energy remains the same at any instant of time and despite all energy transformations. This has been proved in Sec. 4.16 by simple calculations for the conversion of the potential energy of gravitation into kinetic energy and vice versa for the descent and ascent of the ball. It can also be proved that in the conversion of the kinetic energy into the potential energy of elastic deformation of the slab and the ball and for the reverse process of the conversion of this energy into the kinetic energy of the bounced ball, the sum of the potential energy of gravitation, the energy of elastic deformation and the kinetic energy remains unchanged, i.e. the law of conservation of mechanical energy is fulfilled.

We can now explain why the law of work conservation was violated in a simple machine deformed during the transfer of work (Sec. 4.10). As a matter of fact, the work done in one part of a machine is spent partially or completely on the deformation of the simple machine itself (of the lever, rope, and so on), creating in it a potential energy of deformation, while the residue of the work is transferred to the other part of the machine. The sum of the transferred work and the energy of deformation turns out to be equal to the work done. If the lever is perfectly rigid and the rope is unstretchable, a simple machine cannot store an energy, and the entire work done in one part is completely transferred to the other part.

Using the two laws of conservation, viz. the law of momentum conservation and the law of energy conservation, we can solve the *problem of collision of perfectly elastic balls*, i.e. the balls which rebound from each other so that the total kinetic energy remains constant.

Suppose that two balls move along the same straight line (along the line connecting their centres). We assume that no other forces are exerted on the balls besides the forces of their interaction upon contact. After the collision (which occurs if the balls move towards each other or if one ball catches up with the other), the balls will move along the same straight line but with altered velocities. We shall assume that the masses m_1 and m_2 of the balls and their velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ before the collision are known. We are to find their velocities $\mathbf{u}_1$ and $\mathbf{u}_2$ after the collision.

It follows from the law of momentum conservation that since no forces besides the forces of their interaction act on the balls, the total momentum of the balls must be conserved, i.e. the momentum before the collision must be equal to the momentum after the collision:

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{u}_1 + m_2\mathbf{u}_2. \tag{4.17.1}$$

The velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ are directed along the line connecting their centres (either in the same or opposite directions). It follows from symmetry considerations that the velocities $\mathbf{u}_1$ and $\mathbf{u}_2$ will be also directed along this line. We take this line for the x-axis and project the vectors appearing in (4.17.1) onto this axis. This gives

$$m_1 v_{1x} + m_2 v_{2x} = m_1 u_{1x} + m_2 u_{2x}. \tag{4.17.2}$$

Further, the fact that the balls are perfectly elastic implies that the kinetic energy of the balls is also conserved, i.e. the following equality must be satisfied:

$$m_1 v_{1x}^2 + m_2 v_{2x}^2 = m_1 u_{1x}^2 + m_2 u_{2x}^2, \tag{4.17.3}$$

(here $v_{1x}^2 = v_1^2$, and so on).

The unknown quantities u_{1x} and u_{2x} can be found from Eqs. (4.17.2) and (4.17.3). For this we write these equations in the form

$$m_1(v_{1x} - u_{1x}) = -m_2(v_{2x} - u_{2x}),$$
$$m_1(v_{1x}^2 - u_{1x}^2) = -m_2(v_{2x}^2 - u_{2x}^2).$$

Dividing the second equation by the first termwise, we get

$$v_{1x} + u_{1x} = v_{2x} + u_{2x}. \tag{4.17.4}$$

Multiplying this equation by m_2 and subtracting from (4.17.2), we arrive at the relation

$$m_1(v_{1x} - u_{1x}) - m_2(v_{1x} + u_{1x}) = -2m_2 v_{2x},$$

whence

$$u_{1x} = \frac{(m_1 - m_2)v_{1x} + 2m_2 v_{2x}}{m_1 + m_2}. \tag{4.17.5}$$

Similarly, multiplying (4.17.4) by m_1 and adding to (4.17.2), we obtain

$$u_{2x} = \frac{(m_2 - m_1)v_{2x} + 2m_1 v_{1x}}{m_1 + m_2}. \tag{4.17.6}$$

If, for example, the first ball moves along the x-axis and the second one in the opposite direction, v_{1x} is equal to the magnitude of $\mathbf{v}_1$, i.e. v_1, while v_{2x} is equal to the magnitude of $\mathbf{v}_2$ with the minus sign, i.e. $-v_2$. Substituting these values into (4.17.5) and (4.17.6), we obtain

$$u_{1x} = \frac{(m_1 - m_2)v_1 - 2m_2 v_2}{m_1 + m_2},$$

$$u_{2x} = \frac{-(m_2 - m_1)v_2 + 2m_1v_1}{m_1 + m_2}.$$

The signs of the obtained values of u_{1x} and u_{2x} indicate the direction of the respective velocities relative to the x-axis, while the magnitudes of u_{1x} and u_{2x} give the moduli of the velocities.

The relation between the velocities becomes especially simple if the masses of the colliding balls are equal ($m_1 = m_2$). In this case, $u_{1x} = v_{2x}$ and $u_{2x} = v_{1x}$, i.e. the balls interchange their velocities. In particular, if a ball hits a stationary ball of the same mass, it imparts its velocity to this ball and stops.

If the mass of one ball is much larger than the mass of the other ball, say, $m_1 \gg m_2$, we can ignore the terms containing m_2 in the numerator and denominator of formula (4.17.5). If, besides, the heavy ball is at rest, then $u_{2x} = -v_{2x}$, i.e. the light ball is bounced as if from a stationary wall. Indeed, it follows from (4.17.5) that the heavy ball will acquire in this case a small velocity approximately equal to $u_1 = 2v_2m_2/m_1$.

4.18. Frictional Forces and the Law of Conservation of Mechanical Energy

Looking closely on a ball jumping on a steel slab (Sec. 4.17), we can note that after each impact the ball rises to a somewhat lower height than in the preceding jump (Fig. 169), i.e. its total energy does not remain exactly constant but slightly decreases. This means that the law of energy conservation in the form in which it has been formulated is observed *only approximately* in this case. This is explained by frictional forces emerging in this experiment: the resistance of air in which the ball moves and the internal friction between the materials of the ball and the slab. In general, in the presence of friction, the law of conservation of mechanical energy is always violated, and the total energy of bodies decreases. The work against friction [9] is done at the expense of this decrease in energy.

For example, when a body falls from a large height, the velocity of the body will soon become constant due to the increasing resistance of the medium (Sec. 2.39). The kinetic energy of the body stops changing, while its potential energy decreases. The work against air resistance is done by the force of gravity at the expense of the potential energy of the body. Although a certain kinetic energy is imparted to the surrounding air, it is

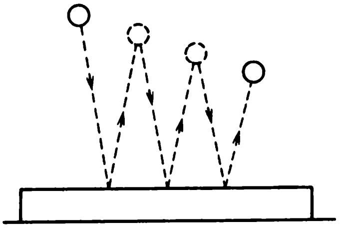

Fig. 169.
The height to which the ball rises decreases after multiple bounces against the slab.

[9] The only exception is static friction. Since the point of its application does not move, the work done by it is zero.

less than the decrease in the potential energy of the body. This means that the total mechanical energy decreases.

The work against friction can also be done at the expense of kinetic energy. For example, when a boat pushed from the shore moves, its potential energy remains unchanged. However, its velocity and hence the kinetic energy decrease due to the resistance of water, and the increase in the kinetic energy of water is smaller than the decrease in the kinetic energy of the boat.

Friction between solid bodies produces the same effect. For example, the velocity acquired by a load sliding down an inclined plane, and hence its kinetic energy, are lower than those obtained if friction were absent. The slope can be chosen so that the load will slide uniformly. In this case, its potential energy will decrease, while the kinetic energy will remain unchanged. The work against friction will be done at the expense of potential energy.

All motions in nature (with the exception of motions in vacuum, like the motion of celestial bodies) are accompanied by friction. Therefore, for these motions the law of energy conservation is violated, this violation being always such that the total energy decreases.

? **4.18.1.** A motor car of mass 1000 kg moves at a velocity of 18 km/h. After its engine has been disengaged, the car covers a distance of 20 m and comes to a halt. What is the friction acting on the car? Assume that the friction is constant.

4.18.2. An electric locomotive pulls a train along a horizontal road and develops a constant pulling force of 50 kN. On a segment 1 km long, its velocity has increased from 30 to 40 km/h. The mass of the train is 800 t. Determine the resistance experienced by the train during motion, assuming this force to be constant.

4.18.3. A bullet having mass 10 g and flying out of a rifle at a velocity of 800 m/s falls to the ground at a velocity of 40 m/s. How much work against air resistance has been done during the motion of the bullet?

4.19. Conversion of Mechanical Energy into Internal Energy

As was mentioned above, a specific feature of friction is that the work done against friction is not transformed completely into the kinetic or potential energy of bodies. As a result, the total mechanical energy of bodies decreases. However, the work done against friction does not vanish. First of all, the motion of bodies in the presence of friction leads to their heating. This can be easily seen if we rub our hands vigorously or pull a metallic band through two pieces of wood tightly pressed against it. We can feel that the band is noticeably heated. It is well known that primitive people could start fires by rapidly rubbing together dry pieces of wood (Fig. 170). When the work is done against internal friction, like when a wire is bent many times, heating is also observed.

Fig. 170.
Starting fires by rubbing two dry pieces of wood.

A motion associated with overcoming friction often involves significant heating. For example, during braking of a train brake blocks are heated to a considerable extent. When a ship is launched, the building berth is abundantly lubricated, but still the heating is so strong that the lubricant smokes and sometimes even catches fire.

When bodies move in air at low velocity, like a stone thrown by hand, air resistance is not high. Small work is done against friction, and the stone is almost as cool as it was. A rapidly flying bullet, however, is heated to a considerably larger extent. For jet aeroplanes flying at high velocities special measures are taken to reduce the heating of the outer surface. Small meteorites entering the Earth's atmosphere at very high velocities (of the order of ten kilometres per second) experience such strong resistance of the medium that they are burnt down completely in the atmosphere (see Fig. 1).[10] A space capsule returning to the Earth generates so much heat in the atmosphere that special heat-protection shields have to be installed.

Besides heating, rubbing of bodies may lead to some other changes as well. They can, for example, be crushed to dust or melt. Melting is a change of the state of a body from the solid state to the liquid state: a piece of ice can melt as a result of friction against another piece of ice or some other body.

Thus, if motion is associated with overcoming friction, it is accompanied by two phenomena: (a) the sum of the kinetic and potential energies

[10] Large meteorites reach the Earth with their surfaces badly scorched.

of all the bodies participating in the motion decreases, and (b) the state of bodies changes and, in particular, they can be heated. This change in the state of bodies always occurs in such a way that the bodies in the new state can do more work than in the initial state. For example, if we pour some ether into a metallic tube, close it with a cork, and rotate it rapidly between two plates tightly pressed against the tube, the ether will evaporate and push the cork out of the tube. This means that as a result of work against friction between the tube and the plates the tube with ether was transferred to a new state in which it could do work required to push the cork, i.e. the work against friction keeping the cork in the tube and the work done in imparting a kinetic energy to the cork. In the initial state, the tube with ether could not do this work.

Thus, heating of bodies, as well as other changes of state, is accompanied by a change in the "store" of ability for work. We see that the "store of ability for work" depends not only on the position of bodies relative to the Earth, their deformation and velocity, but also on the state of the bodies. Consequently, in addition to the potential energy of gravitation and elasticity and to kinetic energy, a body possesses an energy which depends on its *state*. We shall call it the *internal energy* of the body. The internal energy of a body depends on its temperature, on its state of aggregation (solid, liquid or gaseous), on the area of its surface, on whether it is solid or powdered, and so on. In particular, the higher the temperature of the body, the higher its internal energy.

We can draw the conclusion that, although the mechanical energy of a system of moving bodies decreases in motions associated with overcoming friction, their internal energy increases. For example, when a train brakes, the decrease in its kinetic energy is accompanied by an increase in the internal energy of brake blocks, wheels, rails, the surrounding air, and so on, as a result of heating of these bodies.

All that has been said above can also be applied to cases when friction emerges inside a body (like in a piece of wax softened by the hand, in inelastic collision of lead balls or when a piece of wire is bent many times).

? **4.19.1.** Using formula (2.22.2), calculate the mechanical energy lost during an inelastic collision of bodies moving along the same straight line.

4.20. General Nature of the Law of Energy Conservation

Friction plays a special role in the analysis of the law of energy conservation. In the absence of friction, the law of conservation of mechanical energy is fulfilled: the total mechanical energy of a system remains unchanged. If, however, friction is present, the energy no longer remains constant but decreases in motion. In this case, however, the internal energy increases. The evolution of physics was connected with the discovery of new types of energy (which will be studied in the next volumes of this book): light energy, the energy of electromagnetic waves, chemical energy released in chemical reactions (for example, the chemical energy stored in explosives is converted into the mechanical and thermal energies during an explosion), and finally, the nuclear energy. It turned out that the work done on a body is equal to the increment of the sum of all types of energy of the body. On the other hand, the work done by a body on other bodies is equal to a decrease in the total energy of this body. It was found that all kinds of energy can be transformed from one form into another and can be transferred from one body to another so that for all such processes the ***total energy of all kinds remains*** strictly ***constant***. This reflects the ***universal nature*** of the law of energy conservation.

Although the total amount of energy remains constant, the amount of ***useful*** energy may decrease and does so all the time. A transformation of energy to another form may yield a useless type of energy. In mechanics, it is mainly the heating of surroundings of the rubbing surfaces, etc. Such losses are not only undesirable but sometimes even harmful for the mechanisms. To avoid overheating, the moving parts of mechanisms have sometimes to be cooled.

4.21. Power

To characterise the operation of various machines, it is important to know not only the amount of work that can be done by a given machine but also the time during which this work can be accomplished. This ultimately determines the efficiency of any machine. The ratio of the work A done to the time t during which this work is accomplished is known as *power* and is denoted by N:

$$N = A/t.$$

Power can be defined as the rate of doing work.

For the unit of power, we take the power such that a unit work is done per unit time. Therefore, the SI unit of power is a *joule per second*. This unit is called a *watt* (W).[11]

The unit of power in CGS system is an *erg per second* (erg/s): 1 erg/s $= 10^{-7}$ W. An old unit of power, a *horsepower* (h.p.), is still used sometimes: 1 h.p. $=$ 746 W.

Engines of very low as well as very high power have been created by man. A spring clockwork mechanism has a power of the order of 10^{-7} W. The engines of a sea liner or large-scale electric power plant sometimes

[11] In honour of the English physicist and engineer James Watt (1736-1819).

develop a power of hundreds of thousands of kilowatts, i.e. 10^{15} times that of the clock. The average power of a horse is about 400 W. The average power developed by a man during a prolonged physical activity is about 50-100 W. For a very short time, a sportsman may develop a power of several kilowatt. The ability to develop high power, if only for a very short time, is one of the main qualities a sportsman should possess. This is especially important in sprinting, jumping, and so on, when a man has to develop a very high velocity (and hence a high kinetic energy) over a very short time, as well as in weight-lifting when a high potential energy is to be imparted to a weight over a short time. On the contrary, during a slow ascent to a large height (by stairs), a large amount of work can be done although a low power is developed; this will require, however, a long time.

? **4.21.1.** The weight of a clock mechanism whose mass is 5 kg lowers by 120 cm during a day. What is the power of the mechanism?
4.21.2. What is the driving force developed by a diesel locomotive if its power available for pulling the train is 1200 kW, and it has covered 200 m at a constant velocity in 10 s?
4.21.3. What power must be developed by a sportsman at the beginning of a race, if his mass is 70 kg and he must acquire a velocity of 9 m/s in 2 s?

4.22. Calculation of Power of Machines

If a machine exerts a force F and the point of application of this force is displaced during a time t over a distance s in the direction of the force, the work done by the machine is

$$A = Fs.$$

The power developed by this machine is $N = Fs/t$. Since s/t is the velocity v of motion of the point of application of the force, the power developed by the machine is

$$N = Fv. \tag{4.22.1}$$

In other words, if the directions of velocity and the force coincide, *the power developed by a machine is equal to the force exerted by this machine multiplied by the velocity of the point of application of the force*. If the velocity is directed against the force, the work done and the power developed are negative: the machine consumes power. For example, if a lift raises a load of mass 400 kg at a constant velocity of 0.7 m/s, the motor of the lift develops a power $N = 3924 \text{ N} \times 0.7 \text{ m/s} = 2.75 \text{ kW}$.

If a machine is in a rotational motion, the power can also be expressed in a similar way. Suppose, for example, that a motor drives a lathe with the help of a belt. The tension of the driving part of the belt is F, the motor rotates at a frequency n,[12] and the radius of the pulley is R. What is the power N supplied by the motor?

[12] The frequency n is the number of revolutions of the motor pulley per unit time. The unit of n is an inverse second (s^{-1}). — *Eds.*

The belt acts on the lathe pulley with the force F and moves at a velocity $v = 2\pi Rn$ (we assume that the belt does not slip over the pulley, and hence moves at the same velocity as the points on the circumference of the pulley). Hence, the motor develops a power $N = F \times 2\pi Rn$. But $FR = M$ (where M is the torque and R is the arm of the force). Thus, the power developed by the motor is [13]

$$N = 2\pi nM. \tag{4.22.2}$$

? **4.22.1.** In what proportion should the power developed by the engine of a steamer be increased in order to double the speed if the resistance of water to the motion of the steamer grows as the square of the velocity?

4.22.2. A tugboat tugs a barge at a velocity of 12 km/h. The tensile force of the tow-line is 90 kN. What power must be developed by the tugboat engine if it is known to develop a power of 100 kW when going alone at the same velocity?

4.23. Power, Speed and Dimensions of Machines

Formula (4.22.1) derived in the previous section shows that in order to increase the power of a machine, we should increase either the force developed by it or the velocity of its motion. For a certain material and given admissible deformations of the moving parts of a machine, the forces exerted by these parts on each other can be increased by increasing the dimensions of the moving parts. Therefore, the force which can be ultimately developed by a machine is always connected with the dimensions of its moving parts: the larger the size of a machine, the stronger the force that it can develop.

For example, a gearing can develop the stronger force, the larger the size of the teeth. A drive belt develops a force depending on its thickness and width, and so on. However, increasing the size of the belt, we must increase the size of the pulleys since otherwise a thick belt will not be tightly pressed against the pulley of a small diameter and will slip. Thus, both the gearing and the driving mechanisms must have a larger size if a stronger force has to be transferred.

This refers not only to simple driving mechanisms but to various engines as well. For example, the piston of a steam engine can develop a stronger force if its diameter is made larger (for a given pressure of steam). Thus, the following statement is valid in general for all machines: *the larger the force which is to be developed by a machine, the larger must be its size*.

But since the power of a machine depends not only on the force

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[13] It should be noted that $2\pi n$ is the angle by which the pulley turns per unit time, i.e. the angular velocity ω of rotation of the pulley (Sec. 5.6). Thus, expression (4.22.2) can be represented in the form $N = M\omega$. This formula is similar to (4.22.1). The role of force in it is played by the moment of force, and that of the linear velocity v, by the angular velocity ω. — *Eds.*

developed by it, but also on the velocity of the moving parts, of two machines capable of developing the same power, the hight-speed machine can be made of a smaller size. On the other hand, for the same type and size, a high-speed machine has always higher power than a low-speed machine.

For example, a high-speed reduction gear intended for altering the number of revolutions of an aeroplane propeller has a comparatively small size although it transmits a very high power (thousands of kilowatts) from the motor to the propeller (when the latter rotates at a high speed). The gear of a low-speed water turbine designed for the same power is about ten times larger and a thousand times heavier.

4.24. Efficiency of Machines

Any appliance doing work must receive energy from some source at the expense of which the work is done. In the simplest case, a device only transmits mechanical work from a source to a consumer. Simple machines and all transmission or driving mechanisms, which are combinations of simple machines, operate in this way. For example, a belt drive transmits work from a motor rotating the driving pulley to the user (lathe) through the driven pulley.

Such a driving mechanism only transmits a certain power from a source to a user. However, in this case *not the entire* work, and hence, not the entire power received by the mechanism from the source, is transmitted to the user.

The reason behind this is friction present in any machine. A part of work is spent by the machine to overcome this force. This work is converted into heat and is normally useless. The ratio of the power transmitted to the user by the machine to the entire power supplied to the machine is known as the *efficiency* of the given machine. If we denote the power supplied to the machine by N_1 and the power delivered to the user by N_2, the efficiency η of the machine can be expressed as

$$\eta = \frac{N_2}{N_1}.$$

Here a part of the power equal to $N_1 - N_2$ is lost in the machine itself. The ratio of these power losses in the machine to the entire power supplied to it is connected with the efficiency η through a simple expression

$$\frac{N_1 - N_2}{N_1} = 1 - \eta.$$

Since power losses are unavoidable in all machines, we always have

$N_2 < N_1$, and the efficiency of any machine is always less than unity. It is usually expressed in percents. Every machine should be made so that the useless energy losses are as small as possible, i.e. the efficiency is as close to unity as possible. For this purpose, friction as well as other harmful resistances in the machine are reduced to the lowest possible value. In the most perfect machines, these losses are reduced to such an extent that their efficiency is less than unity only by several percents.

Many machines receive or give away energy not in the form of the mechanical energy but in some other form. Steam engine, for example, utilises the energy of heated and compressed steam. Internal combustion engines use the energy of hot and compressed gas formed during the combustion of an air-fuel mixture. Electric motors consume the work done by electromagnetic forces. On the contrary, electric current generators receive energy in the mechanical form and give it away in the form of electromagnetic energy. In all these cases, besides friction losses, other losses like the heating of current-carrying conductors can also be observed. The concept of efficiency has the same sense in these cases also: the efficiency of a machine is the ratio of the power generated by the machine to the power consumed by it irrespective of the type of energy in which this power is delivered or consumed.

? **4.24.1.** A force of 1000 N is applied to the rope passing over the smaller pulley of a double block. The radii of the pulleys are 40 and 5 cm. In order to overcome friction in the block and to maintain a constant velocity of its motion, a force of 130 N is applied to the other rope of the block. What is its efficiency?

4.24.2. What work must be done to lift a load having a mass of 250 kg to a height of 120 cm with the help of a tackle block whose efficiency is 65%?

4.24.3. Determine the efficiency of a set-up consisting of an electric motor driving a water pump which delivers 75 l of water per second to a height of 4.7 m if the power consumed by the motor is 5 kW.

4.24.4. An electric motor having an efficiency of 90% drives a pump whose efficiency is 60%. What is the efficiency of the entire set-up?

4.24.5. An electric train moves uniformly at a velocity of 60 km/h. Its engines consume a power of 900 kW. Determine the resistance to the motion of the entire train if the total efficiency of its engines and transmitting mechanisms is known to be 80%.

4.24.6. Can a load of mass 50 kg be lifted at a velocity of 3 m/s with the help of an electric motor consuming an electric power of 1.4 kW?

Chapter 5
Curvilinear Motion

5.1. Emergence of Curvilinear Motion

It was shown above that if no forces act on a body, it moves uniformly in a straight line. When the force acting on a body and its velocity are parallel or antiparallel, i.e. when vectors **F** and **v** are collinear [1], the body moves in a straight line, though not uniformly. If, however, the force is *at an angle* to the velocity of a body, the trajectory of motion of the body is *curved*. A stone thrown at an angle to the horizontal is in a curvilinear motion (the force of gravity directed downwards is not collinear to the velocity of the body). The motion of a load tied to a rope and rotating in a circle (the tension of the rope is not collinear to the velocity of the load), the motion of a planet around the Sun or the motion of the Moon or an artificial satellite around the Earth (in this case the gravitational force directed to the central body is not collinear to the velocity of the moving body) are examples of curvilinear motions.

Let us set a steel ball rolling on a horizontal glass. Since friction is negligibly small, the ball will roll over the glass at a virtually constant velocity, i.e. uniformly and in a straight line. We place a magnet on the glass so that one of its poles is near the continuation of the trajectory of the ball but not on the trajectory itself (Fig. 171). It can be seen that when the ball passes by the magnet, it starts moving in a curvilinear trajectory. Having passed the magnet, the ball will move practically along a straight line which has a different direction than the initial path. The force bending the path of the ball is the attractive force directed from the ball to the magnet. The force of magnetic attraction rapidly decreases with distance and hence it noticeably affects the motion only in the vicinity of the magnet.

In the examples considered above, the body is acted upon by a force directed at an angle to the trajectory of motion, and as a result of this action the trajectory of the body bends. If the force were directed along the trajectory, no bending would be observed. For example, if a body is

[1] Vectors are termed collinear if they are directed along parallel straight lines in the same or opposite directions. In a particular case, collinear vectors can be directed along the same straight line. — *Eds.*

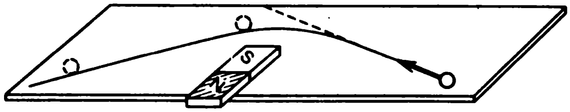


Fig. 171.
A magnet bends the trajectory of a rolling steel ball.

thrown vertically upwards, it describes a rectilinear trajectory. If the pole of a magnet is on the continuation of the ball trajectory, the latter is not bent, and so on.

5.2. Acceleration of a Curvilinear Motion

Newton's second law establishes the relation between the force, mass and acceleration of a body:

$$\mathbf{a} = \mathbf{F}/m. \tag{5.2.1}$$

Here m is the mass of the body, $\mathbf{a}$ is its acceleration and $\mathbf{F}$ is the resultant of all the forces applied to the body (see formula (2.15.1)). For a rectilinear motion, the vectors can be replaced by their magnitudes (to be more precise, by their projections on the straight line along which the body moves). So far, we have used Newton's second law only in such a simplified form. While studying a curvilinear motion, however, the vector equation (5.2.1) should be used.

In a curvilinear motion, the velocity generally changes both in magnitude and in direction. In order to simultaneously characterise both these changes, we decompose the vectors on both sides of Eq. (5.2.1) into *tangential* and *normal* (centripetal) components. We shall denote the tangential and normal components of acceleration by $\mathbf{a}_\tau$ and $\mathbf{a}_n$, and the corresponding components of force by $\mathbf{F}_\tau$ and $\mathbf{F}_n$. Then Newton's second law can be written separately for tangential and normal components:

$$\mathbf{a}_\tau = \mathbf{F}_\tau/m, \quad \mathbf{a}_n = \mathbf{F}_n/m.$$

The tangential component of force causes the *tangential acceleration* of the body, which characterises the change in the *magnitude* of velocity, while the normal component of force is responsible for the *normal acceleration* of the body, which characterises the change in the *direction* of velocity (Fig. 172).

If the force is always normal to the trajectory, the body moves uniformly, i.e. at a velocity of constant magnitude, and vice versa, if the body is known to move uniformly, it follows that the tangential component of the force is zero, and the body has only a normal component of acceleration.

If we are interested in the motion of the projections of the body on cer-

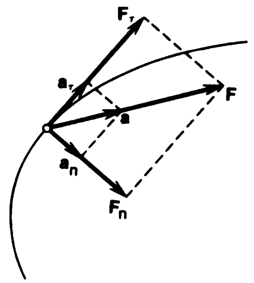


Fig. 172.
The tangential and normal components of force and acceleration.

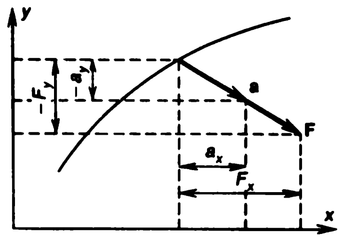


Fig. 173.
The x- and y-projections of an acceleration and a force acting on a body.

tain axes, say, the vertical and horizontal directions, we can project vectors **a** and **F** in Eq. (5.2.1) onto these axes. Having denoted the projection of a vector by the appropriate index (Fig. 173), we obtain

$$a_x = F_x/m, \quad a_y = F_y/m.$$

These equations determine the accelerations of the projections of a moving point on the chosen axes. It is convenient to use these equations, for example, if a force has a constant direction which can be chosen as the direction of one of the axes (Sec. 5.3).

Knowing the mass of a body and its acceleration, we can calculate the resultant of all the forces acting on the body with the help of Newton's second law. If we know the mass of a body and the magnitude and direction of the resultant of all the forces acting on it, we can also determine the magnitude and direction of the acceleration of the body.

? 5.2.1. Individual parts of a drive belt move along a straight line on the segment between the pulleys. Arriving at the pulley, these parts start to move curvilinearly (along the circumference of the pulley). Indicate the forces that make the motion of the parts of the belt on the pulley curvilinear.

5.3. Motion of a Body Thrown along the Horizontal

Let us consider the motion of a body thrown along the horizontal and moving only under the action of the force of gravity (we neglect air resistance). Let us imagine, for example, that a ball lying on a table is pushed. Having reached the edge of the table, the ball starts to fall freely, its initial velocity $\mathbf{v}_0$ being directed along the horizontal (Fig. 174).

We project the motion of the ball onto the vertical y-axis and the horizontal x-axis. The motion of the projection of the ball on the x-axis is the motion with zero acceleration at a velocity $v_x = v_0$. The motion of the y-projection of the ball is a free fall with an acceleration $a_y = g$ under the

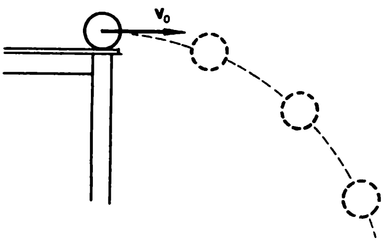


Fig. 174.
The motion of a ball rolling off the table.

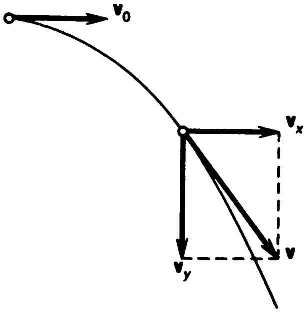


Fig. 175.
A ball thrown horizontally at a velocity $\mathbf{v}_0$ has a velocity $\mathbf{v}$ at an instant t.

action of the force of gravity with zero initial velocity. The laws governing the two motions are familiar to us. The v_x component remains constant and equal to v_0. The v_y component grows in proportion to time: $v_y = gt$. The resultant velocity can be easily found with the help of the parallelogram rule as shown in Fig. 175. It is directed downwards, its slope increasing with time.

Let us determine the *trajectory* of a body thrown along the horizontal. The coordinates of the body at a moment t will be

$$x = v_0 t, \tag{5.3.1}$$

$$y = gt^2/2. \tag{5.3.2}$$

In order to find the equation of the trajectory, we express t in Eq. (5.3.1) in terms of x and substitute it into Eq. (5.3.2). This gives

$$y = \frac{g}{2v_0^2} x^2. \tag{5.3.3}$$

The plot of this function is shown in Fig. 176. The ordinates of the points on the trajectory turn out to be proportional to the squared abscissas. Such trajectories are called parabolas. The graph of the path in a uniformly accelerated motion is a parabola (Sec. 1.22). Thus, *a freely falling body with an initial horizontal velocity moves along a parabola.*

The distance covered in the vertical direction does not depend on the initial velocity. However, the distance covered in the horizontal direction is proportional to the initial velocity. Therefore, at a high initial horizontal velocity, the parabola along which the body falls is stretched in the horizontal direction. If a jet of water is ejected from a horizontal pipe (Fig. 177), individual particles of water will move, like the ball, along a parabola. The more we open the tap, the higher the initial velocity of water, and the farther the point at which water falls at the bottom of the

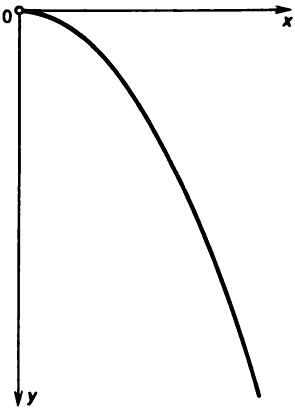


Fig. 176.
The trajectory of a body thrown horizontally.

trough. Placing behind the jet a screen with parabolas drawn on it in advance, we can make sure that the water jet actually has the form of parabola.

If we know the initial velocity v_0 and the height h from which the body is thrown, we can calculate the horizontal range s to the place where the body falls. Indeed, having put in (5.3.3) $y = h$ and $x = s$, we obtain

$$s = v_0\sqrt{2h/g}.$$

5.3.1. What will be the velocity of a body 2 s after it is thrown horizontally at a velocity of 15 m/s? At which moment of time will the velocity be directed at 45° to the horizontal? Air resistance should be neglected.

5.3.2. A ball rolling off a table of 1-m height lands at a distance of 2 m from the edge of the table. What was the horizontal velocity of the ball? Neglect air resistance.

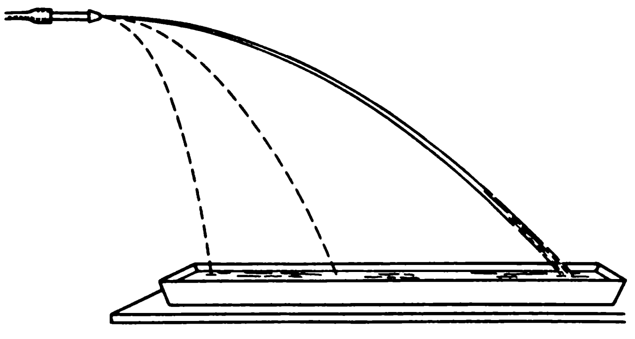


Fig. 177.
A jet has the form of a parabola whose elongation depends on the initial velocity of water.

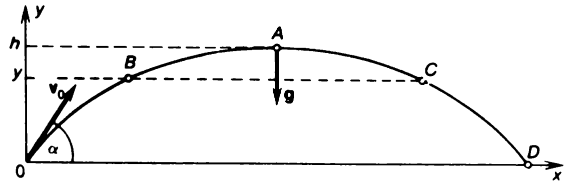


Fig. 178.
The trajectory of a body thrown at an angle α to the horizontal (in the absence of air resistance).

5.4. Motion of a Body Thrown at an Angle to the Horizontal

If the initial velocity of a body is directed upwards at a certain angle to the horizontal, the initial velocity of the body has both vertical and horizontal components (Fig. 178). The only difference in comparison with the case considered above is that for the vertical motion the initial velocity also differs from zero. Everything said about the horizontal component remains in force.

We choose the coordinate axes so that the y-axis is directed vertically upwards and the horizontal x-axis lies in the same vertical plane with the initial velocity $\mathbf{v}_0$. The x-projection of the initial velocity is $v_0 \cos \alpha$, while the y-projection is $v_0 \sin \alpha$ (for the directions of the x- and y-axes shown in Fig. 178, the two projections are positive). The acceleration of the body is $\mathbf{g}$, and it is always directed vertically downwards. Therefore, the y-projection of the acceleration is $-g$, while its x-projection is zero. Since the x-component of acceleration is absent, the x-projection of the velocity remains constant and equal to the initial value $v_0 \cos \alpha$. Consequently, the motion of the x-projection of the body is a uniform one. The motion of the y-projection of the body occurs in two directions (upwards and downwards) with the same acceleration $\mathbf{g}$. Therefore, the time required to cover a distance from an arbitrary height y to the height h is the same as the time Δt needed to move downwards from the height h to y. It follows, hence, that points symmetrical about the apex A (like points B and C) lie at the same height. This means that the trajectory is symmetrical about the point A. However, the form of the trajectory of a body beyond the point A was specified in Sec. 5.3. This is a parabola described by the body flying with a horizontal initial velocity. Consequently, all that has been said about the trajectory of the body in the previous section refers to the case under consideration as well, but instead of a "half parabola" ACD the body describes the "complete parabola" $OBACD$, which is symmetrical about the point A.

We can also verify the obtained result with the help of a water jet flowing out of an inclined pipe (Fig. 179). If we place behind the jet a screen

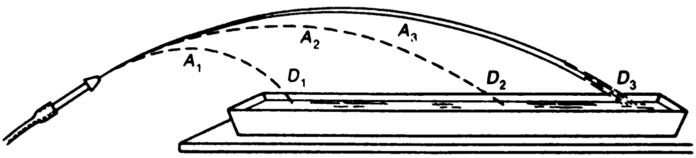


Fig. 179.
A jet has the shape of a parabola whose elongation depends on the initial velocity of the jet.

with parabolas drawn beforehand, we can note that the jets have the form of parabolas.

The height of ascent and the horizontal range of the body to the point, where its height is equal to the initial height, i.e. the distance OD in Fig. 178, depend on the magnitude and direction of the initial velocity $\mathbf{v}_0$. First of all, for a given direction of the initial velocity, both the height and the horizontal range are the larger, the larger the magnitude of the initial velocity (see Fig. 179).

For the initial velocities of the same magnitude, the horizontal range of the body depends on the direction of the initial velocity (Fig. 180). As the angle between the velocity and the horizontal increases, this distance increases and attains its maximum at an angle of 45°. For bodies thrown at angles larger than 45°, the range decreases again.

Let us calculate the motion of a body thrown upwards at an angle α to the horizontal with an initial velocity $\mathbf{v}_0$ (see Fig. 178). It should be recalled that the x-projection of the velocity of the body is constant and equal to $v_0 \cos \alpha$. Therefore, the x-coordinate of the body at the moment t is

$$x = (v_0 \cos \alpha)t. \tag{5.4.1}$$

The motion of the y-projection of the body will first be uniformly decelerated. After the body has reached the apex A of the trajectory, the y-projection of the velocity becomes negative, i.e. acquires the same sign as

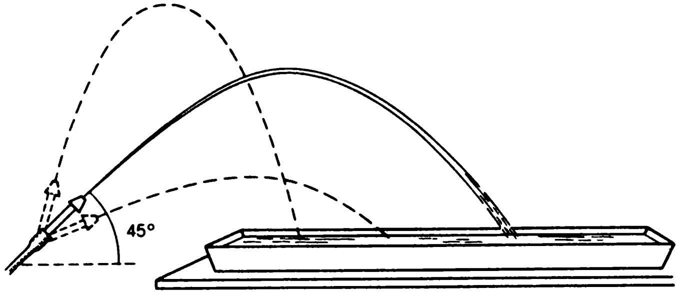


Fig. 180.
As the angle between the jet flowing at a given velocity and the horizontal increases, the horizontal range first increases, attains its maximum value at 45°, and then decreases.

the projection of acceleration, as a result of which the body will move downwards with a uniform acceleration. The y-projection of the velocity varies with time according to the law

$$v_y = v_0 \sin \alpha - gt. \tag{5.4.2}$$

At the apex A of the trajectory, the velocity has only horizontal component, while v_y vanishes. In order to find the instant t_A, at which the body reaches the maximum height, we substitute into formula (5.4.2) t_A for t and equate the obtained expression to zero

$$v_0 \sin \alpha - gt_A = 0, \quad \text{whence } t_A = \frac{v_0 \sin \alpha}{g}. \tag{5.4.3}$$

The value of t_A defined by this formula gives the time during which the body reaches the apex of the trajectory. If the point from which the body has been thrown and the point at which it falls are on the same level, the time of flight is equal to $2t_A$:

$$t_{\text{fl}} = \frac{2v_0 \sin \alpha}{g}. \tag{5.4.4}$$

Multiplying v_x by the time of flight t_{fl}, we obtain the x-coordinate of the point where the body falls, i.e. the range of flight:

$$s = x_D = v_0 \cos \alpha \, \frac{2v_0 \sin \alpha}{g} = \frac{v_0^2 \sin 2\alpha}{g}. \tag{5.4.5}$$

This formula shows that the range of flight will be maximum when $2\alpha = 90°$, i.e. when $\alpha = 45°$ (this was mentioned above).

According to formulas (1.22.1) and (5.4.2), the y-coordinate varies with time as follows:

$$y = (v_0 \sin \alpha)t - \frac{gt^2}{2}. \tag{5.4.6}$$

Substituting into this formula t_A for t, we obtain the y-coordinate corresponding to the apex A of the trajectory, the height h to which the body ascends:

$$h = y_A = v_0 \sin \alpha \, \frac{v_0 \sin \alpha}{g} - \frac{g}{2} \left(\frac{v_0 \sin \alpha}{g} \right)^2.$$

Collecting like terms, we obtain

$$h = \frac{v_0^2 \sin^2 \alpha}{2g}. \tag{5.4.7}$$

The height increases with α and reaches its maximum value, equal to $v_0^2/2g$, at $\alpha = 90°$, i.e. when the body is thrown vertically upwards.

? 5.4.1. A stone thrown from the ground at an angle to the horizontal falls down at a distance of 14 m. Find the vertical and horizontal components of the initial velocity of the stone if the entire flight lasted 2 s. Determine the maximum height to which the stone rises above the ground. Air resistance should be neglected.

5.4.2. A fireman directs a jet of water to the roof of a house whose height is 15 m. The jet rises to a height of 5 m above the roof. At what distance (along the horizontal) from the fireman will the jet fall on the roof if it is ejected from the hose at a velocity of 25 m/s? Air resistance should be neglected.

5.5. Flight of Bullets and Projectiles

Since the velocities of flying bullets and projectiles are high, air resistance considerably alters their motion in comparison with the results of calculations carried out in the previous section. If air resistance were absent, the maximum range of the flight of a bullet or a projectile would be observed, as was mentioned above, at the angle of inclination of the barrel of a rifle or gun equal to 45°. It can be shown that air resistance leads to a change in the trajectory of a bullet such that the angle of inclination corresponding to the maximum range turns out to be less than 45° (it is different for different initial velocities of the bullet). At the same time, the horizontal range (as well as the maximum height of the flight) turns out to be much smaller. For example, for an initial velocity of 870 m/s and angle of 45°, the horizontal range of the bullet is 77 km in the absence of the resistance of the medium. However, for the same initial velocity, the maximum range of flight does not exceed 3.5 km, i.e. is reduced to less than 1/20 of the theoretical value. The maximum height attained by the bullet is reduced almost in the same proportion.

The effect of air resistance on the flight of projectiles becomes weaker for larger projectiles for the same reason as in the case of a free fall (Sec. 2.39): the mass of a projectile increases as the cube of its size, while the force of air resistance increases as the square of its size. Thus, the ratio of air resistance to the mass of the projectile, i.e. the effect of air resistance, decreases with the increasing size. Therefore, for the same initial velocity of projectiles fired from a gun, their range increases with the calibre. At the same time, the most advantageous angle of firing approaches 45°. Long-range guns fire at an angle close to 45°. Since projectiles rise in this case to a larger height, where the density of the atmosphere is lower, the effect of air resistance becomes less noticeable. A mortar firing a heavy shell with a small initial velocity (this also reduces the role of air resistance) also has the longest range at an angle close to 45°.

If the target C is at a distance less than the maximum range AB (Fig. 181), the projectile can hit the target in two ways: at an angle of inclination which is either less than 45° (grazing firing) or larger than 45° (steep firing).

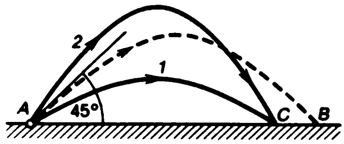


Fig. 181.
Grazing (*1*) and steep (*2*) firing.

5.6. Angular Velocity

The motion of a point in a circle can be characterised by the angle of rotation of the radius connecting the moving point with the centre of the circle.[2] The variation of this angle with time is characterised by the *angular velocity*. The angular velocity of a point is the ratio of the angle of rotation of the radius vector of the point to the time interval during which this rotation occurred. Angular velocity is numerically equal to the angle of rotation of the radius vector of a point per unit time.

The angle of rotation is usually measured in radians (rad). The unit of angular velocity is a *radian per second* (rad/s), viz. the angular velocity at which the point describes an arc subtending an angle of one radian in one second.

A complete turn in a circle amounts to 2π radians. Consequently, if a point rotates at a frequency n, its *angular velocity* is

$$\omega = 2\pi n \text{ rad/s}.$$

If the motion of a point in a circle is nonuniform, we can introduce the concept of the *average angular velocity* and *instantaneous angular velocity* as it was done for the ordinary velocity in the case of a nonuniform motion. Henceforth, however, we shall consider only the uniform motion in a circle.

"Ordinary" velocity will be termed the *linear velocity* to distinguish it from the angular velocity. The relation between the linear velocity v of a point, its angular velocity ω and the radius r of the circle in which it moves can be easily established. Indeed, having described an angle of one radian in a circle, the point covers a distance equal to the radius, and hence

$$v = \omega r, \tag{5.6.1}$$

i.e. *the linear velocity of motion in a circle is equal to the angular velocity multiplied by the radius of the circle*.

Using formula (5.6.1), we can express the centripetal acceleration of a point moving in a circle in terms of its angular velocity. Substituting the expression (5.6.1) for velocity into (1.27.1), we obtain the formula for centripetal acceleration in terms of angular velocity:

$$a = \omega^2 r. \tag{5.6.2}$$

[2] That is, by the angle of rotation of the radius vector of the moving point. — *Eds*.

While analysing the rotation of a rigid body about its axis, we can also use the concept of angular velocity. In this case, the angular velocity is the same for all points of the body since they turn through the same angle during the same time. Thus, the rotation of a rigid body about its axis can be characterised by the angular velocity of all its points. Therefore, it will be called the *angular velocity of the body*.

Formulas (5.6.1) and (5.6.2) show that for a rotating rigid body, the linear velocities and centripetal accelerations of its points are proportional to the distances from these points to the rotation axis.

5.6.1. Two points move at the same angular velocity in circles whose radii are in the ratio 1:2. Find the ratio of accelerations of these points.

5.6.2. Compare the angular velocity of rotation of the hour hand of a clock with the angular velocity of the Earth's rotation.

5.7. Forces in a Uniform Circular Motion

It was shown in Sec. 1.27 that a uniform motion in a circle is a motion with acceleration having a constant magnitude and directed towards the centre of the circle. However, the acceleration of a body is always caused by a force acting in the direction of the acceleration. This means that for a body to move uniformly in a circle it must be acted upon by a force which has a constant magnitude over the entire circle and changes its direction so that it is always directed towards the centre of the circle.

Indeed, in all cases of uniform circular motion of a body, we can indicate such a force exerted by some other body. For a ball tied to a string and rotating in a circle, this is the tension exerted by the stretched string on the ball. This force can be easily observed if we attach the other end of the string to a spring balance (Fig. 182). For the ball moving along a circular groove or a train moving along a bend in the railway track, this is the reaction exerted by the deformed groove on the ball or by the deformed rail on the wheels of the train and directed towards the centre of the arc of the circle in which the ball or the train moves. For planets moving about the Sun, this is the gravitational force of attraction to the Sun.

If the force ceases to act (for example, if the string attached to the ball

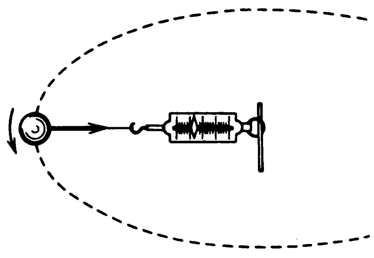


Fig. 182.
A spring balance indicates the force exerted by a string on a ball moving in a circle.

breaks), the centripetal acceleration also vanishes. The ball continues to move along the tangent to the circle (i.e. in the direction of the velocity of the ball at the moment when the force disappeared).

The force required to move a body of mass m uniformly in a circle of radius r at a velocity v can be found from Newton's second law. Since the acceleration of the body is $a = v^2/r$, the required force is

$$F = ma = \frac{mv^2}{r}. \tag{5.7.1}$$

Thus, *for a body to move uniformly in a circle, it must be acted upon by a force equal to the product of the mass of the body and the square of its velocity, divided by the radius of the circle.* Hence it follows that the smaller the radius, the larger the force required for a given linear velocity of the body. For example, for a given velocity of a motor car driving around the bend, the ground must exert the larger force on the wheels of the car, the smaller the radius of the curvature of the bend, i.e. the sharper the bend. Pay attention to the fact that the velocity appears in the second power in the formula for force. This means that as the velocity of the circular motion of a given radius increases, the force required to maintain such a motion grows very rapidly. This can be proved by accelerating the circular motion of a load attached to a spring balance: the balance readings rapidly increase with the velocity of the load.

The forces acting in rotary motion can be also expressed in terms of angular velocity. Using formula (5.6.2), we find that the following force is to act on a body of mass m moving in a circle:

$$F = ma = m\omega^2 r. \tag{5.7.2}$$

Thus, as the angular velocity increases, the force required to maintain the motion rapidly grows. This circumstance is employed in the design of some types of *tachometers*, the instruments for measuring the rotational speed of a machine.

The operation principle of a tachometer is clear from Fig. 183. Loads *1* are fixed to a light rod attached to a shaft. The rod can rotate freely about the point O. The springs *2* confine the motion of the rod with the loads to the shaft. As the shaft rotates, forces required to move the loads in a circle are the stronger, the higher the speed of rotation. Since these forces are created by springs *2* pulling the loads to the rotation axis, springs are stretched the more, the higher the rotational speed. This means that the angle by which the rod is deviated from the shaft increases with the speed of rotation. The rod is connected to pointer *3* sliding along the scale with divisions corresponding to different values of the rotational speed of the shaft.

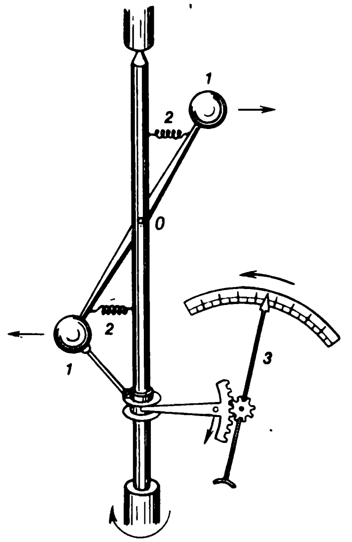


Fig. 183.
Schematic diagram of a tachometer. As the rotational speed of the shaft increases, the rod connecting the loads turns through a larger angle.

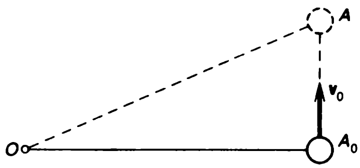


Fig. 184.
Immediately after the impact, the load moves in a straight line A_0A, and its distance from point O increases.

? **5.7.1.** A cyclist whose mass together with his bicycle is 80 kg moves at a velocity of 9 km/h in a circle of radius of 15 m. Determine the force acting on the cyclist.

5.7.2. A load suspended from a 50-cm spring stretches it by 1 cm. Let us hold the other end of the spring and rotate the load in the horizontal plane so that the spring is stretched by 10 cm. What is the velocity of the load? The force exerted by the stretched spring is proportional to the extension. The action of the force of gravity during rotation should be neglected.

5.8. Emergence of the Force Acting on a Body Moving in a Circle

The fact that a body in a curvilinear motion experiences an acceleration implies that forces must act on it. For instance, a load attached to a string can move in a circle only if the string pulls it with a certain force. However, the string can pull the load only if it is deformed (stretched). Consequently, in order to explain the emergence of the forces causing the circular motion of the load, we must explain why the string is stretched in this motion.

It was mentioned above (Sec. 2.29) that deformation of a body is the result of different motions of its different parts during a certain period of time. In the example under consideration, we can make the pattern of emergence of deformation more visual by assuming that the string can be readily stretched, like a thin rubber band. Let us fix one of its ends at point O and attach a load to the other end (Fig. 184). The rotation of the load about point O can be caused by imparting to it a certain velocity $\mathbf{v}_0$ in a direction perpendicular to the string. In the first moment after the beginning of motion, no force is exerted by the string on the load since the string is not stretched. Therefore, the load starts moving in a straight line, and its

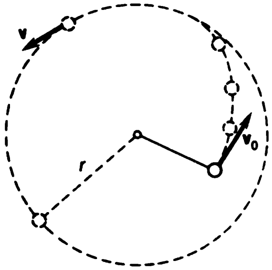


Fig. 185.
The motion of a load after the initial impact.

distance from point O will increase (the segment OA is longer than OA_0), the string is stretched, and, as a result, a force is exerted by the string on the load. The load acquires an acceleration directed towards point O, and its trajectory becomes curved.

However, as long as the string is stretched insignificantly, this bending of the trajectory is insufficient for the load to move in a circle, and it will continue to move away from point O, thus increasing the extension of the string and hence the force acting on the load (Fig. 185). As a result, the curvature of the trajectory continues to increase until the trajectory becomes circular. After this, the string is not stretched any more. Consequently, its extension will be just such that it exerts an elastic force which imparts to the load the acceleration required for its uniform motion in a circle whose radius is equal to the length of the stretched string. It is well known (see formula (5.7.1)) that this force must be equal to mv^2/r, where m is the mass of the load, v is its velocity, and r is the radius of the trajectory. If the string is rigid or if we take a rod instead of the string, the extension producing the required force will be practically very small, and we can take for r the length of the unstretched string or the initial length of the rod, and for the steady-state velocity, the initial velocity v_0.

The deformation of the bent groove along which the ball rolls occurs in a similar way; the groove bends the trajectory of the ball. If the groove were absent, the ball would move in a straight line. In a groove, the ball also moves rectilinearly until the former acts on it with a force. If the groove were very soft, the ball moving in it would straighten it out. A rigid bent groove is also slightly straightened out as the ball moves in it. However, the elastic force that imparts to the ball the acceleration required for its curvilinear motion appears in the rigid groove even for a very small deformation.

If the goove and string are deformed but slightly nder the action of the ball and the load, we can assume that the string and the groove are rigid *constraints* (Sec. 3.7). In this case, the trajectory of the body can be predicted since it is determined by the form of the constraint. For example, we can say beforehand that the trajectory of a load attached to a slightly stretching string is close to a circle of radius equal to the length of the unstretched string. For a rigid groove, we can say in advance that the trajectory of a ball will be close to the initial shape of the groove.

5.9. Rupture of Flywheels

The deformations emerging in rotating wheels, discs, etc., are of the same type as deformations of constraints which make a body move in a circle. It is the forces caused by these deformations that impart to the parts of a rotating body centripetal accelerations required to make these parts move in circles. If bodies are rigid, deformations are very small and difficult to observe directly. These deformations, however, may lead to the destruction of a rotating body. Sometimes, flywheels and other rotating parts of machines are ruptured during motion. This rupture normally occurs if the velocity exceeds the permissible speed of rotation.

Let us analyse the pattern of rupture of a rotating body. We shall start with the circular motion of a load attached to a rubber band. If we increase the speed of rotation of the load, the attained extension of the band turns out to be insufficient to sustain the motion of the load at an increased velocity in the same circle. The load will again start to move away from the centre, and the extension will be increased until it corresponds to a new velocity and a new, slightly increased radius of the circle is established. If we increase the velocity of the load still further, the band will continue to be extended. However, a rubber band, like any other body, cannot be elongated indefinitely. For a certain extension a rupture will occur. Therefore, if we go on increasing the velocity of the load, the band will ultimately snap. As is well known, after rupture the load will fly along the tangent to the trajectory at the point where the rupture took place.

The rupture of a rapidly rotating flywheel occurs in a similar way. If the speed of rotation is so high that the spokes of the wheel cannot impart the required centripetal acceleration even for the maximum strain that they can withstand, their elongation continues even beyond the admissible limit and the rupture occurs. The fragments of the flywheel fly off along the tangents to the circumference of the wheel. Since the centripetal acceleration rapidly grows with increasing radius of the trajectory and especially with increasing angular velocity of a rotating body (see formula (5.7.2)), the large-size and rapidly rotating parts of machines, like the rotors of high-speed turbines, should be made exceptionally strong. The impossibility of making rotating parts with unlimited strength often sets the upper limit to the speed of rotation.

Phenomena similar to those observed in the rupture of a flywheel occur in a spin drier (Fig. 186). A wet cloth is placed in a perforated drum which is rotated rapidly. At a high speed of rotation, the adhesion forces between

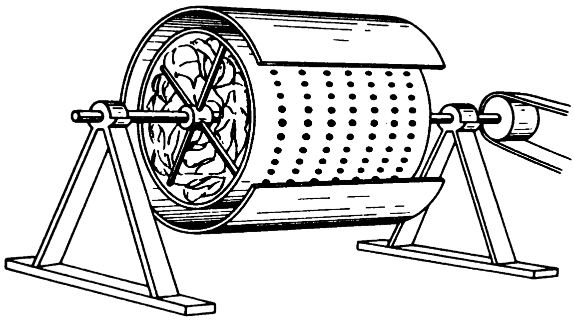


Fig. 186.
Spin drier.

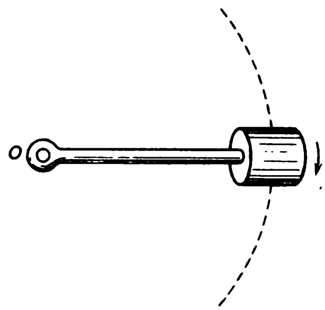


Fig. 187.
To Exercise 5.9.1.

moisture drops and the cloth turn out to be insufficient to impart the centripetal acceleration required for drops to move in a circle. The moisture drops are torn off the cloth and fly away through the holes in the drum.

Thus, in the cases considered above (rupture of rapidly rotating bodies and tearing away of the drops from the cloth being dried) the reason behind the phenomena is that the forces emerging before rupture are insufficient for imparting the centripetal acceleration required for a given speed of rotation to the parts of a rotating body or water drops. Here the difference between the uniform rectilinear and uniform curvilinear motions is clearly manifested. In a uniform motion in a straight line, there is no acceleration, and no force is required for sustaining the motion. Therefore, however large the constant velocity of motion, it cannot cause a destruction of the body.

? **5.9.1.** A 50-kg load is fixed to the end of a 30-cm long rod rotating about point O (Fig. 187). Determine the speed of rotation at which the rod breaks, if the stationary load suspended from the rod and causing its rupture has a mass of 1 t.

5.10. Deformation of a Body Moving in a Circle

We have considered so far only the forces exerted on a body moving in a circle by constraints, i.e. by bodies bending the trajectory of the given body. Such is, for example, the force exerted on a load by a string to which it is attached. But it is clear that the load, in turn, must act on the string with the force of the same magnitude. This follows from Newton's third law which states that the forces of interaction between two bodies (the load and string in the example under consideration) are always equal and opposite. Consequently, the load acts on the string with the force also equal to mv^2/r but directed *away from the centre* of rotation. This force is applied to the string (and not to the load), and hence it was not taken into account while considering the motion of the load. However, while analysing the behaviour of the string, we must know the forces acting just on the string.

The same situation is observed for any motion in a circle if it takes place under the action of the forces due to a direct contact of bodies. For a circular motion, there must be a constraint, viz. *some other body* keeping the moving body on a circle. This constraint exerts on the moving body a force directed *to the centre* of rotation. The moving body, in turn, must act on this constraint with a force of the same magnitude but directed *away from the centre*.

It was mentioned above (Sec. 5.8) that the force exerted by the string on the load moving in a circle is due to the deformation of this string. Similarly, the force exerted by the load on the string is caused by the corresponding deformation of the load. We can easily explain why the load also turns out to be in a deformed state.

For the sake of clarity, we take the load in the form of an elongated body (Fig. 188). Suppose that we have imparted to all the points of this body simultaneously the same velocity $\mathbf{v}_0$ perpendicular to the string. As was mentioned above, a tensile force emerges in this case in the string, and this force imparts an acceleration to the points of the body to which it is attached (to the left end of the body in Fig. 188). The path of the left part of the body becomes curved, while the right part of the body continues to move in a straight line since no forces act on it at the beginning. Therefore, the distance between the left and right parts of the body increases, and the body becomes deformed. The deformation ceases to increase only when the forces emerging as a result of deformation ensure the accelerations required for rotating all parts of the body in circles.

Thus, a body moving in a circle under the action of forces emerging due to a direct contact with other bodies is always found to be deformed. If the body is rigid, deformations are small, and even if they cannot be observed directly, they are manifested in the force exerted by the body on the string. If, however, we take a readily deformable body, say a soft cylindrical spring, its deformation can be seen by naked eye (Fig. 189). The deforma-

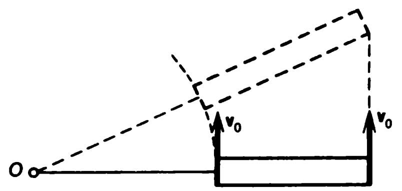


Fig. 188.
Emergence of deformations in a body moving in a circle.

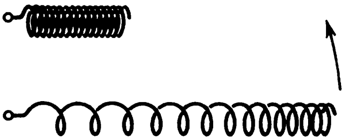


Fig. 189.
A visual representation of deformations in a rotating body with the help of a spring. An unstretched spring is shown at the top for comparison.

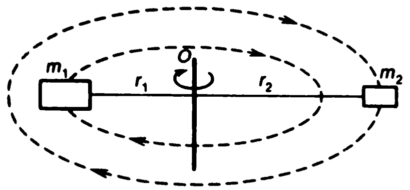


Fig. 190.
To Exercise 5.10.1.

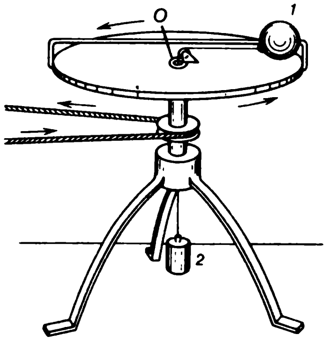


Fig. 191.
To Exercise 5.10.3.

tion is distributed in the spring so that each turn is acted upon by the neighbouring turns with a resultant force which is directed to the centre and which ensures the required acceleration of this turn. The smallest extension will be observed for the turn which is the farthest from the centre, the extension growing for the turns which are closer to the centre of rotation.

The deformed string also acts on the rotation axis to which it is fixed at its other end. In turn, the axis is bent, and as a result of this deformation acts on the string fixed to it with the equal and opposite force. The force exerted on the body by a constraint (axis and string) is directed to the centre (it imparts a centripetal acceleration to the body). On the contrary, the force with which the rotating deformed body acts on the string and axis, i.e. the constraint, is directed away from the centre.

? **5.10.1.** Two bodies having masses m_1 and m_2 are attached to strings having lengths r_1 and r_2 and rotated about point O at the same angular velocity (Fig. 190). Under what conditions do the forces exerted at point O by the strings balance each other?

5.10.2. The drum of a spin drier has a diameter of 80 cm and has a rotational speed of 25 s^{-1}. What is the force exerted on the drum wall by a piece of cloth whose mass is 1.5 g?

5.10.3. Body *1* (Fig. 191) of mass m is attached to a string passing through a hole O. Body *2* is attached to the other end of the string and has the same mass. Body *1* rotates in a horizontal plane about point O so that the radius of the trajectory is 20 cm. What must be the angular velocity of body *1* for body *2* to be in equilibrium?

5.10.4. What will happen in the case described in the previous problem if we (a) slightly push body *2* up or down? (b) put a small additional load on body *2*?

5.11. Roller Coaster

In the curvilinear motion of a cart in a roller coaster (Fig. 192*a*) acceleration emerges as a result of the force of attraction of the Earth as well as the force emerging in a direct contact. The former is the force of gravity **P** act-

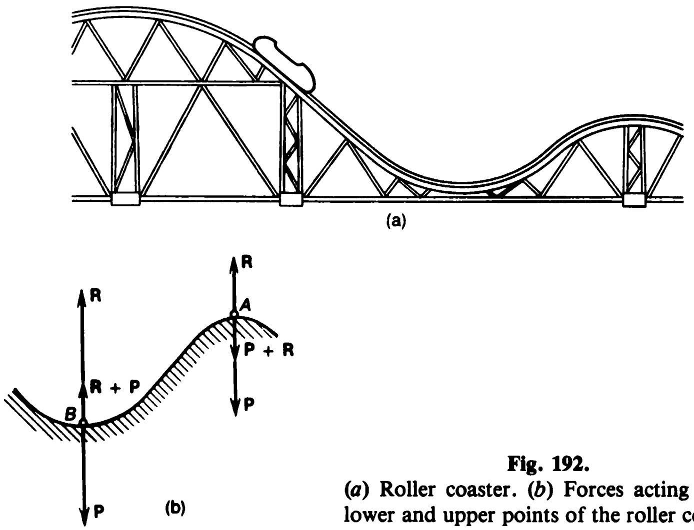


Fig. 192.
(*a*) Roller coaster. (*b*) Forces acting at the lower and upper points of the roller coaster.

ing on the cart, while the latter is the reaction **R**. In this example, the constraint is the track along which the cart moves.

Let us find out the force exerted by the rails on the cart at the uppermost (A) and lowermost (B) points of the path (Fig. 192*b*). Since the acceleration of a curvilinear motion is always directed towards the concavity of the trajectory, it is directed downwards at point A and upwards at point B. Consequently, the resultant of forces **P** and **R** is directed downwards at the upper point and upwards at the lower point. Hence it follows that the magnitude of reaction **R** at point A is smaller than the force of gravity and larger than **P** at point B. At point A, the difference between the force of gravity and reaction imparts a downward centripetal acceleration to the cart. On the contrary, the reaction at point B not only balances the force of gravity, but also imparts an upward centripetal acceleration to the cart. The centripetal acceleration is given by $a = v^2/r$. Consequently, the difference between the magnitudes of **R** and **P** is mv^2/r.

The difference in the reactions of support at different points of the path is explained by different deformations of the rails at the upper and lower points. This could be proved by using arguments similar to those in the analysis of the deformation of the groove in Sec. 5.8. According to Newton's third law, the cart, in turn, presses against rails with the force **N** equal in magnitude to the force **R**, but directed from the cart to the rails. This means that the cart presses against the rails with a smaller force at the upper point than at the lower point.

Thus, the force exerted by a body on a support (by the cart on the rails) in a curvilinear path lying in the vertical plane does not remain constant but

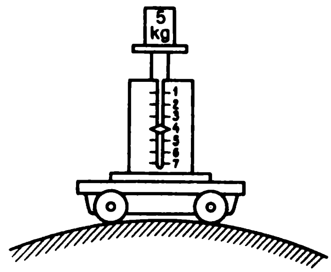


Fig. 193.
When the cart passes through the upper point of a roller coaster, the reading of a spring balance is smaller than the force of gravity acting on the load.

depends on the velocity of motion and the form of the path. These changes could be observed by placing a load resting on a spring balance on a cart moving in a roller coaster (Fig. 193). If the cart is at rest, the force of gravity **P** acting on the load is balanced by the elastic force **R** of the compressed spring of the balance, i.e. $R = P$. If, however, the cart is in a curvilinear motion, R is either smaller or larger than P, and hence the weight of the load is either larger or smaller than the weight of the stationary load.

This experiment illustrates once again the fact emphasised in Sec. 2.26. The weight measured with the help of a spring balance is equal to the force of gravity only if the balance and the body being weighed are at rest (or move without acceleration). If the body and the balance have a downward acceleration, the weight of the body turns out to be less than the force of gravity. On the contrary, if the acceleration of the balance and body is directed upwards, the weight of the body turns out to be larger than the force of gravity.

? **5.11.1.** Find the relation connecting the radius of curvature r of a bridge and the velocity v of the motion of a motor car at which the force acting on the convex bridge is equal to half the force acting on a flat bridge. At what velocity will the car lose contact with the bridge having the radius of curvature r at its upper point?

5.12. Banking of Tracks

A skater, cyclist, train, etc., usually move around bends along circular arcs. But in contrast to a roller coaster, the trajectory in these cases lies in a horizontal plane. A moving body is acted upon by two forces: the force of gravity **P** and reaction **R** of the support (ice, ground or rails). If the body is at rest or in a rectilinear motion, these forces are vertical and balance each other. However, at a bend it is necessary that their resultant be directed inwards from the bend. For this, the moving body must be leaning inwards too. Then the reaction of the support acquires a component in the direction of inclination, i.e. towards the centre of the circle, which creates the required centripetal acceleration.

How is the body made to lean inwards? The skater or cyclist does it on

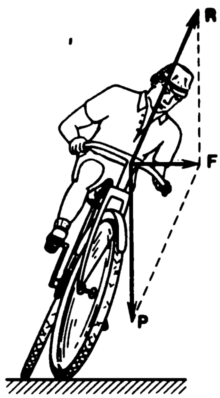


Fig. 194.
A cyclist is inclined inwards from the bend. The force of gravity **P** and reaction **R** of the Earth give the resultant **F** imparting a centripetal acceleration required for motion in a circle.

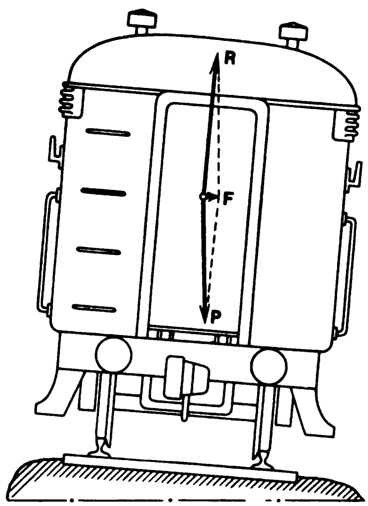


Fig. 195.
Banking of a railway track. The force of gravity **P** acting on a carriage and the reaction **R** of the rails give the resultant **F** ensuring a centripetal acceleration of the carriage.

purpose (or instinctively) by displacing the centre of gravity by moving the torso or arms. As a result, the friction appearing between skates and ice or between the bicycle tyre and the ground creates a centripetal acceleration. The friction acts in the direction of the inclination of the bicycle or skater. As a result, the force **R** exerted by the ground inclines in the same direction (Fig. 194). If the friction is not large enough (for example, the skate is not sharp or if the road is slippery), the skate or wheel slips sideways and the skater or cyclist falls down.

A train is made to lean inwards by specially shaping the railway foundations. The outer rail on a bend is laid slightly higher than the inner one (Fig. 195). This is called *banking* and the inclination of the railway carriage is calibrated for a certain average velocity. If this velocity is considerably exceeded there may be an accident.

? **5.12.1.** If a train moves around a bend at the velocity for which the banking is designed, the passengers do not feel the inclination. At a higher velocity it seems to the passengers that the carriage is inclined in the outward direction, and at a lower velocity, in the inward direction relative to the bend. Explain this phenomenon.

5.13. The Circular Motion of a Suspended Body

Let us consider some other examples of uniform motion in a circle. Let us fix several plumb lines on the turntable of a record player (Fig. 196). When the turntable is at rest, all the plumb

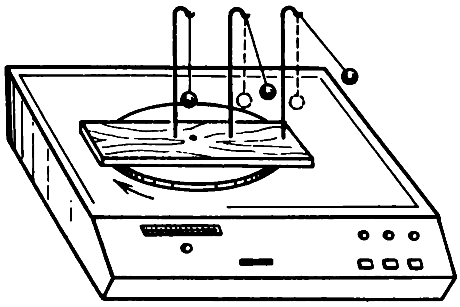


Fig. 196.
A board with plumb lines fixed to it is placed on the turntable of a record player. When the turntable rotates, the plumb lines are inclined the more, the further a plumb line from the axis.

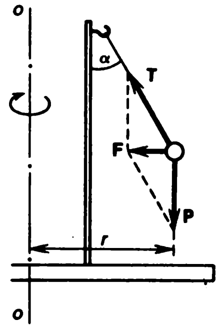


Fig. 197.
Forces acting on the load of a plumb line fixed to a rotating disc.

lines hang vertically. If we rotate the turntable, they are deflected the more, the further they are from the rotation axis. As the angular velocity of rotation increases, the deviations of plumb lines become larger.

Without analysing the *reasons behind* the deviation of a plumb line, let us find the position of its string for a given angular velocity (Fig. 197). If the disc rotates uniformly, the tension **T** of the string and the force of gravity **P** acting on the load give a horizontal resultant force **F** which imparts a centripetal acceleration to the load. It should be noted that the tension **T** has a larger magnitude than in the case when the disc is at rest since the force **P** is balanced by the vertical component of **T**.

The magnitude of force **F** is equal to the product of the mass m of the load and its centripetal acceleration $\omega^2 r$ (ω is the angular velocity of the disc): $F = m\omega^2 r$. Figure 197 shows that

$$\tan \alpha = \frac{F}{P} = \frac{m\omega^2 r}{mg} = \omega^2 r/g. \tag{5.13.1}$$

It follows hence that the deviation of the string is the larger, the higher the angular velocity and the larger the distance from the rotation axis. It does not depend on the mass of the load. A similar phenomenon, viz. the deviation of the rod from which a horse is suspended, can be observed in a merry-go-round. In this case, formula (5.13.1) gives the angle of deviation of the rod.

The situation considered above also explains the operation principle of the so-called *centrifugal governors* used to control the speed of rotation of various machines. The first governor of this type was constructed by Watt for controlling the rotational speed of a steam engine. During the rotation of the governor shaft (Fig. 198), loads *1* hinged to the shaft are deviated and move the shaft coupling *2* to which they are connected through tie-rods. The coupling is connected with shutter *3* which controls the supply of steam to the cylinders of the steam engine. When the rotational speed of the machine exceeds the normal value, the coupling moves down and reduces the amount of steam supplied to the cylinders. On the contrary, as the rotational speed decreases below the norm, the coupling moves up and increases the supply of steam.

5.14. Motion of Planets

The analysis of the visible motion of planets against the background of invariable pattern of stars in the night sky made it possible to give a complete

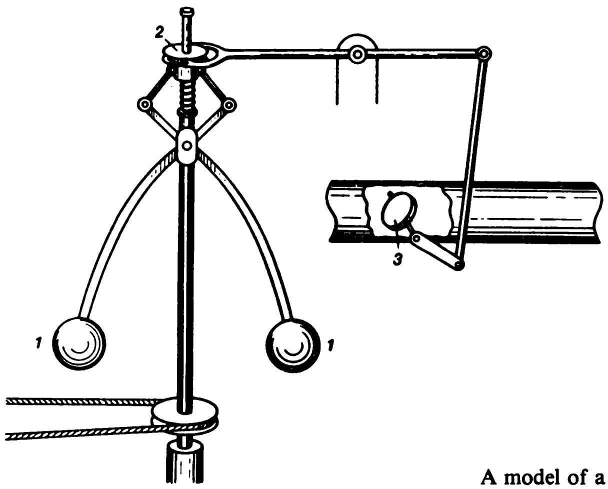


Fig. 198.
A model of a Watt centrifugal governor.

kinematic description of the motion of planets relative to an inertial reference system associated with the Sun and stars.[3] The trajectories of planets turn out to be closed curves called the *orbits*. The orbits are close to circles with the centre at the Sun [4], while the motion of the planets in the orbits was found to be close to a uniform one. The exceptions are the motions of comets and some asteroids, whose distance from the Sun and the velocity of motion vary over a broad range and the orbits are strongly elongated. The distances from the planets to the Sun (radii of their orbits) and the periods of revolution about the Sun are quite different (Table 2).

Table 2. Information about Planets

Name and notation	Distance from the Sun		Period of revolution in terrestrial years
	in radii of the Earth's orbit	in million km	
Mercury ☿	0.387	58	0.241
Venus ♀	0.723	108	0.615
Earth ♁ (or ⊕)	1.000	149	1.000
Mars ♂	1.524	228	1.881
Jupiter ♃	5.203	778	11.862
Saturn ♄	9.938	1426	29.457
Uranus ♅	19.191	2868	84.013
Neptune ♆	30.071	4494	164.783
Pluto ♇	39.6	6000	248

[3] It should be recalled that this reference system is known as the *heliocentric* system. — *Eds*.

[4] The distances separating celestial bodies are enormous even when compared with very large sizes of celestial bodies, and therefore in the analysis of the motion of planets we can assume them to be point particles.

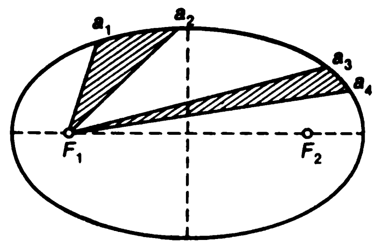


Fig. 199.
If a planet moves from point a_1 to a_2 during the same time as from point a_3 to a_4, the hatched areas in the figure are equal.

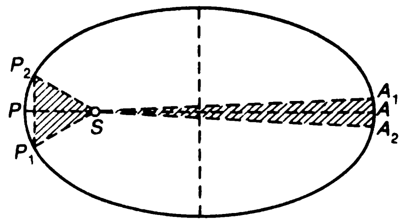


Fig. 200.
To the derivation of the ratio of velocities of a planet at perihelion and aphelion.

The notation of the first six planets given in the table was introduced by ancient astrologers.

Actually, the orbits of the planets are not quite circular and their velocities are not quite constant. An exact description of the motions of all the planets was given by the German astronomer Johannes Kepler (1571-1630) (in his time, only the first six planets were known) in the form of three laws (Fig. 199):

1. Each planet moves in an ellipse with the Sun at one focus.
2. The radius vector of a planet (viz. the vector drawn from the Sun to the planet) sweeps out equal areas in equal times.
3. For any two planets, the squares of the periods are proportional to the cubes of the semimajor axes of their orbits.

We can draw a number of conclusions from these laws concerning the forces causing the motion of planets. Let us first consider the motion of a single planet. The end P of the major axis of the orbit which is closest to the Sun (S) is known as the *perihelion*, while the other end A is called the *aphelion* (Fig. 200). Since an ellipse is symmetrical about its two axes, the radii of curvature in the perihelion and aphelion are equal. Consequently, according to Sec. 1.27, the normal accelerations a_P and a_A at these points are in the same ratio as the squares of velocities v_P and v_A of the planet at these points:

$$a_P/a_A = v_P^2/v_A^2. \tag{5.14.1}$$

Let us consider small path segments P_1P_2 and A_1A_2 symmetrical relative to the perihelion and aphelion and covered in equal time intervals t. According to Kepler's second law, the areas of the sectors SA_1A_2 and SP_1P_2 must be equal. The arcs A_1A_2 and P_1P_2 of the ellipse are equal to v_At and v_Pt. For the sake of clarity, the arcs in Fig. 200 are made quite large. If we make these arcs very small (for which the time interval t must be very short), the difference between the arc and the chord will be negligible, and we can treat the sectors swept by the radius vectors as isosceles triangles SA_1A_2 and SP_1P_2. Their areas are accordingly equal to $v_A t r_A/2$ and $v_P t r_P/2$, where r_A and r_P are the distances from the aphelion and perihelion to the Sun. Consequently, $v_A r_A = v_P r_P$, whence $v_A/v_P = r_P/r_A$. Finally, substituting this relation into (5.14.1), we obtain

$$a_P/a_A = r_A^2/r_P^2. \tag{5.14.2}$$

Since the tangential components of acceleration are equal to zero both in the perihelion and aphelion, a_P and a_A are the accelerations of the planet at these points. They are directed to the Sun (along the semimajor axis).

An analysis shows that at all other points of the trajectory, the acceleration is directed to the Sun and varies in accordance with the same law, i.e. in inverse proportion to the squared distance from the planet to the Sun. Therefore, for any point of the planet orbit, we have

$$a_P/a = r^2/r_P^2, \tag{5.14.3}$$

where a is the acceleration of the planet and r is the distance from the Sun. Thus, the acceleration of the planet is inversely proportional to the square of its separation from the Sun.

Analysing the angle formed by the radius vector of the planet and the tangent to the trajectory, we see (Fig. 201) that as the planet moves from the aphelion to perihelion, the tangential component a_τ of acceleration is positive, and the velocity of the planet increases. On the contrary, during the motion from the perihelion to aphelion, the velocity of the planet decreases. The planet attains its maximum velocity at the perihelion and has the minimum velocity at the aphelion.

To derive the relation between the acceleration of a planet and its separation from the Sun, we made use of two Kepler's laws. This dependence was found since the planets move in ellipses so that their separation from the Sun varies. If the planets moved in circles, the distance to the Sun and their accelerations would remain unchanged, and this relation could not be derived.

However, while comparing the accelerations of *different* planets, we

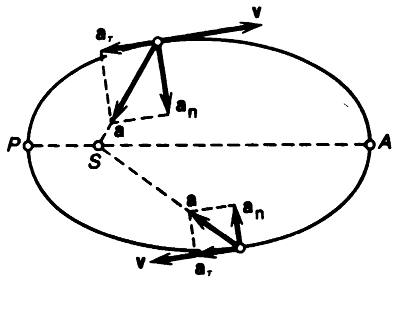


Fig. 201.
The velocity of a planet decreases when it moves from the perihelion to aphelion and increases in the motion from the aphelion to perihelion.

can content ourselves with an approximate description of their motion assuming that they move uniformly in circles. Let us denote the radii of orbits of any two planets by r_1 and r_2 and their periods of revolution by T_1 and T_2. Then their velocities will be expressed by the formulas

$$v_1 = 2\pi r_1/T_1, \quad v_2 = 2\pi r_2/T_2,$$

and according to (1.27.1), the centripetal accelerations will be given by

$$a_1 = v_1^2/r_1 = 4\pi^2 r_1/T_1^2, \quad a_2 = v_2^2/r_2 = 4\pi^2 r_2/T_2^2.$$

Since we assume that the motion in a circle is uniform, accelerations a_1 and a_2 can be treated as centripetal accelerations directed to the centre of the orbit, viz. the Sun. The ratio of the accelerations of the planets is

$$\frac{a_1}{a_2} = \frac{r_1}{r_2}\frac{T_2^2}{T_1^2}. \tag{5.14.4}$$

But according to Kepler's third law, we have

$$T_2^2/T_1^2 = r_2^3/r_1^3.$$

Substituting the ratio of squared periods into (5.14.4), we obtain

$$a_1/a_2 = r_2^2/r_1^2.$$

This conclusion can also be written in a different form: for any planet at a distance r from the Sun, the acceleration is

$$a = C/r^2, \tag{5.14.5}$$

where C is the same constant for all the planets of the Solar system. Thus, accelerations of the planets are inversely proportional to the squares of their separations from the Sun and are directed to the Sun.

5.15. The Law of Universal Gravitation

Isaac Newton was able to derive from Kepler's laws one of the fundamental laws of nature, viz. the *law of universal gravitation*. He knew that for all planets of the Solar system, the acceleration is inversely proportional to the square of the distance from the planet to the Sun, and the proportionality factor is the same for all the planets.

Hence it follows that, in the first place, the force of attraction exerted by the Sun on a planet must be proportional to the mass of this planet. Indeed, if the acceleration of the planet is given by formula (5.14.5), the force causing this acceleration is

$$F = ma = m\frac{C}{r^2},$$

where m is the mass of the planet. On the other hand, the acceleration imparted by the Earth to the Moon was known to Newton: it had been determined from the observations of the motion of the Moon orbiting the Earth. This acceleration is about 1/3600 of the value of g, the acceleration imparted by the Earth to bodies near its surface. The distance from the Earth to the Moon is approximately equal to 60 times the Earth's radius. In other words, the Moon is separated from the centre of the Earth by a distance 60 times longer than bodies on the surface of the Earth, and its acceleration is smaller by a factor of $3600 = 60^2$.

If we assume that the Moon moves under the action of attraction of the Earth, it follows that the attractive force exerted by the Earth, like the force of attraction to the Sun, decreases in inverse proportion to the square of the distance from the centre of the Earth. Finally, the force of attraction to the Earth is directly proportional to the mass of the body being attracted. This fact was established by Newton in his experiments with a pendulum. He discovered that the period of oscillations of a pendulum does not depend on its mass. This means that the Earth imparts the same acceleration to pendulums of different masses, and hence the force of attraction to the Earth is proportional to the mass of the body on which it acts. Naturally, this also follows from the same value of g, the free fall acceleration for bodies of different masses, but experiments with a pendulum make it possible to verify this fact to a higher degree of accuracy.

Similar features of attraction to the Sun and the Earth led Newton to the conclusion that the nature of these forces is the same, and that forces of universal gravitation act between all bodies, decreasing in inverse proportion to the square of the distance between the bodies. The force of gravitation acting on a given body of mass m must be proportional to the mass m.

Proceeding from these facts and considerations, Newton formulated the law of universal gravitation as follows: *any two bodies attract each other with a force directed along the line connecting these bodies, which is directly proportional to their masses and inversely proportional to the square of their separation*. In other words, the force of attraction between the two bodies is

$$F = G\frac{Mm}{r^2}, \tag{5.15.1}$$

where M and m are the masses of the bodies, r is their separation, while G is the proportionality factor called the *constant of universal gravitation* (or *gravitational constant*). The method of its measurement will be described below. Comparing this formula with (5.14.4), we see that $C = GM$, where M is the mass of the Sun. The forces of universal gravitation obey Newton's third law. This has been confirmed in all astronomical observations of the motion of celestial bodies.

Formulated in this way, the law of universal gravitation can be applied to bodies which can be regarded as material points, i.e. bodies whose separation is very large in comparison with their size. Otherwise, we have to take into consideration the fact that different points of the bodies are separated by different distances. For homogeneous spherical bodies, the formula is valid for any distance between the bodies if we take for r the distance between their centres. In particular, for the case of attraction of a body by the Earth the distance should be measured from the centre of the Earth. This explains the fact that the force of gravity practically does not decrease with increasing height above the ground (Sec. 2.25). Since the radius of the Earth is about 6400 km, the change of the position of the body above the surface of the Earth even within tens of kilometres does not practically involve a change in the force of attraction by the Earth.[5]

The gravitational constant can be determined by measuring all the other quantities appearing in the expression for the law of universal gravitation for a specific case.

The magnitude of the gravitational constant was determined for the first time with the help of the *torsion balance*. The design of this instrument is illustrated schematically in Fig. 202. A light beam with two identical balls of mass m fixed at its ends is suspended by a long and thin wire. The beam is supplied with a mirror which makes it possible to measure small rotations of the beam about the vertical axis by the optical methods. Two balls of a considerably larger mass M can be brought to the balls of mass m from opposite sides. The forces of attraction of small balls to the big ones create a couple which rotates the beam clockwise (if we look from above). Having measured the angle by which the beam turns when the balls

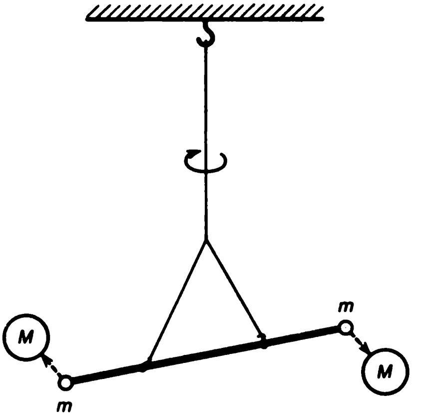


Fig. 202.
Schematic diagram of a torsion balance used for measuring the gravitational constant.

[5] At a height of 10 km, the force of attraction is weaker than on the ground by 0.3%. — *Eds.*

of mass m are approached by the large balls of mass M and knowing the elastic properties of the wire to which the beam is attached, we can determine the torque of the couple of forces of attraction between the masses m and M. Since the masses m and M of the balls and the distance between their centres (for a given position of the beam) are known, formula (5.15.1) can be used for calculating the value of G. It turned out to be [6]

$$G = 6.7 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}.$$

After the value of G had been determined, it became possible to calculate the *mass of the Earth* from the law of universal gravitation. Indeed, according to this law, a body of mass m at the surface of the Earth is attracted to it with a force

$$P = G\frac{mM_{\text{Earth}}}{R^2_{\text{Earth}}},$$

where M_{Earth} is the mass of the Earth and R_{Earth} is its radius. On the other hand, we know that $P = mg$. Equating these quantities, we obtain

$$M_{\text{Earth}} = \frac{gR^2_{\text{Earth}}}{G}. \tag{5.15.2}$$

The values of the quantities on the right-hand side of this expression are known. Substituting them into this formula, we get

$$M_{\text{Earth}} = 5.96 \times 10^{24}\ \mathrm{kg}.$$

It should be noted that according to formula (5.15.2) the free fall acceleration is

$$g = G\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}}. \tag{5.15.3}$$

The law of universal gravitation implies that the accelerations imparted by bodies having masses m_1 and m_2 and separated by a distance r are

$$a_1 = G\frac{m_2}{r^2}, \quad a_2 = G\frac{m_1}{r^2}.$$

These formulas reflect the feature of gravitational forces noted earlier: the acceleration of a given body due to gravitation of another body does not depend on the mass of the given body. Further, it follows from (5.15.3) that

$$a_1/a_2 = m_2/m_1.$$

Thus, although the forces of universal gravitation acting between the

[6] According to the international table of recommended values of fundamental physical constants, the gravitational constant $G = 6.6720 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$. — *Eds.*

bodies of different masses are equal, a considerable acceleration is imparted to the body having a small mass, while the body of a large mass acquires a small acceleration.

The total mass of the planets of the Solar system is slightly more than 1/1000 of the mass of the Sun, and hence the acceleration acquired by the Sun as a result of the gravitational forces exerted by the planets is negligibly small as compared to the accelerations imparted to the planets by the gravitational force of the Sun. Gravitational forces acting between the planets are also relatively weak. Therefore, while studying the laws of motion of the planets (Kepler's laws), we ignored the motion of the Sun itself and assumed approximately that the trajectories of the planets are elliptical with the Sun at one focus. In precise calculations, however, "perturbations" introduced in the motion of the Sun or a planet by gravitational forces exerted by other planets must be taken into account.

? **5.15.1.** What will be the decrease in the gravitational force of the Earth acting on a rocket when it rises to 600 km above the surface of the Earth? The radius of the Earth is assumed to be 6400 km.

5.15.2. The mass of the Moon is 1/81 of the mass of the Earth and the radius of the Moon is smaller than the radius of the Earth by a factor of 3.7. Find the weight of a man on the Moon if his weight on the Earth is 600 N.

5.15.3. The mass of the Moon is 1/81 of the mass of the Earth. Find the point on the line connecting the centres of the Earth and the Moon at which equal forces of attraction to the Earth and the Moon act on a body placed at this point.

5.16. Artificial Satellites of the Earth

Like any celestial body, a body outside the Earth's atmosphere experiences the action of only gravitational forces exerted by the Earth, the Sun and other celestial bodies. Depending on the initial velocity imparted to it when it takes off from the surface of the Earth, the body will meet different fates. If the initial velocity is small, the body will fall to the ground. At a high velocity, it may become an artificial satellite and start revolving about the Earth like its natural satellite, viz. the Moon. At a still higher velocity, the body may go so far from the Earth that the gravitational force exerted by the Earth practically will not affect its motion, and it will become an artificial planet, i.e. will start revolving about the Sun. Finally, if the velocity is increased still further, the body may leave the Solar system forever and move in the Galaxy.

We shall consider only the case when the body becomes a satellite of the Earth. While studying its motion relative to the Earth, we shall take into account only the force of its attraction by the Earth. It can be shown that the body may become a satellite only if its velocity lies within a comparatively narrow range: from 7.91 to 11.19 km/s. If the velocity is less than 7.91 km/s, the body falls back to the ground. At a velocity exceeding 11.19 km/s, the body will leave the Earth forever.

In order to launch a satellite, special rockets, called boosters or launch vehicles, are used to lift the satellite to a given height and accelerate it to the required velocity. After that the satellite is separated from the booster and continues its motion only under the action of gravitational forces. Booster engines must do work against the force of gravity and against air resistance, and, besides, impart a large velocity to the satellite. For this purpose, booster engines must be able to develop a huge power (of millions of kilowatts).

If the distance from the satellite to the surface of the Earth changes insignificantly as compared to the distance from the centre of the Earth, the force of attraction of the satellite by the Earth can be assumed (for rough calculations) to be constant as it was done in Sec. 5.4 while studying the motion of a body thrown at an angle to the horizontal. However, the direction of the force of gravity can no longer be considered constant as for short path lengths of bullets and projectiles. We must now take into account the fact that the force of gravity is directed at any point along the radius to the centre of the Earth.

We shall consider only the motion of satellites in circular orbits. In this case, the force of attraction by the Earth creates a centripetal acceleration of the satellite, equal to v_1^2/r, where r is the radius of the orbit and v_1 is the velocity of the satellite, which is unknown so far. We assume that the orbit is close to the surface of the Earth so that r is virtually equal to the radius R_{Earth} of the Earth. If, moreover, we neglect the resistance of the atmosphere, the satellite will move with an acceleration g directed to the centre of the Earth. Consequently,

$$g = v_1^2/R_{\text{Earth}}. \tag{5.16.1}$$

Hence we find that the velocity v_1 of the satellite moving in a circular orbit near the surface of the Earth should be

$$v_1 = \sqrt{gR_{\text{Earth}}}. \tag{5.16.2}$$

Substituting $g = 9.81\ \text{m/s}^2$ and $R_{\text{Earth}} = 6378$ km into this expression, we obtain

$$v_1 = 7.91\ \text{km/s}.$$

This velocity is known as the *circular (orbital) velocity*. Moving at this velocity, the satellite would orbit the Earth in 84 min 12 s.

A satellite orbiting the Earth near its surface has an acceleration g directed to the centre of the Earth, i.e. the same free fall acceleration as that of a body flying freely in a parabolic trajectory or falling vertically near the surface of the Earth. Thus, the motion of the satellite is just a *free* *fall*, like the motion of bullets, projectiles or ballistic missiles. The only difference is that the velocity of the satellite is so high that the radius of curvature of its trajectory is equal to the radius of the Earth: falling (i.e. the motion with the acceleration **g** directed to the centre of the Earth) is reduced to bending around the globe.

Formula (5.16.1) shows that if the velocity of a body is less than the orbital velocity, the force of gravity will make it move in a trajectory with the radius of curvature smaller than the radius R_{Earth}. Hence, at such a velocity the body will fall to the ground. At a higher velocity, the radius of curvature of the trajectory will be larger than R_{Earth}, and the body will describe an elliptic trajectory (Fig. 203).

In actual practice, a satellite cannot be placed in an orbit of the radius R_{Earth} due to a very strong air resistance near the surface of the Earth. Let us determine the velocity v of the motion in a circular orbit of any radius r longer than R_{Earth}. For this we shall use the formula similar to (5.16.2), taking into account the fact that the free fall acceleration decreases as we move away from the centre of the Earth in the inverse proportion to the

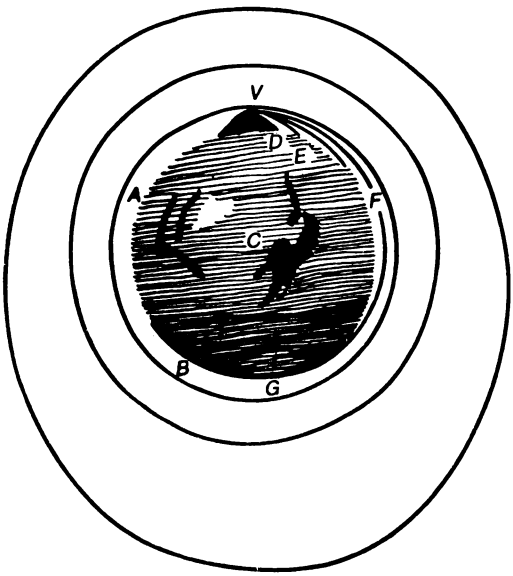


Fig. 203.
The drawing from Newton's *Principia Mathematica*: the trajectories of a body thrown from the peak of a high mountain with different horizontal velocities. Even at that time Newton realised that to place a body in an orbit around the Earth, a sufficiently high velocity should be imparted to it. As the velocity increases, the trajectories terminate at points D, E, F, G. Two orbits are shown for bodies launched from points higher above the Earth than the mountaintop.

squared distances to the centre. At a distance r from the centre of the Earth, the acceleration g_r is given by the formula $g_r = gR^2_{\text{Earth}}/r^2$. The velocity v at which a satellite moves in a circular orbit of radius r is obtained from the equality

$$g_r = g\,\frac{R^2_{\text{Earth}}}{r^2} = \frac{v^2}{r},$$

whence

$$v = \sqrt{g\,\frac{R^2_{\text{Earth}}}{r}} = v_1\sqrt{\frac{R_{\text{Earth}}}{r}}. \qquad (5.16.3)$$

Thus, as the radius of the orbit increases, the velocity of the satellite decreases.[7]

This does not mean, however, that to launch a satellite into an orbit of a larger radius, the booster engines must do a smaller work. Only the fraction of work required to impart the kinetic energy to the satellite decreases. But the satellite must be launched to a higher altitude, and, hence, a larger work against the force of attraction by the Earth must be done, i.e. a higher potential energy is to be imparted to the satellite. Ultimately, it turns out that as the orbital radius increases, the total work required for launching the satellite also increases.

Indeed, let us calculate the change in the work required to launch a satellite into the orbit and to impart to it a velocity needed for the orbital motion depending on the radius of the orbit. According to formula (5.16.3), the kinetic energy of a satellite having a mass m and moving in an orbit of radius r is

$$E_k = \frac{mv^2}{2} = \frac{mv_1^2}{2}\,\frac{R_{\text{Earth}}}{r},$$

where v_1 is the orbital velocity. Substituting for v_1 its value determined by formula (5.16.2) into the expression for the kinetic energy, we obtain

$$E_k = \frac{mg}{2}\,\frac{R^2_{\text{Earth}}}{r}. \qquad (5.16.4)$$

Let us analyse the flight of a satellite of mass m in an orbit of radius r and in an orbit of radius $r + \Delta r$, where Δr is a positive increment of radius r, which is much less than the radius r itself ($\Delta r \ll r$). According to (5.16.4), the kinetic energies of the satellite moving in these orbits are

$$E_k = \frac{mg}{2}\,\frac{R^2_{\text{Earth}}}{r}, \quad E_k + \Delta E_k = \frac{mg}{2}\,\frac{R^2_{\text{Earth}}}{r + \Delta r},$$

[7] The smallest elevation above the surface of the Earth where the air resistance is so low that it can be neglected is about 300 km. The radius of the corresponding orbit is (approximately) 6700 km. Using formula (5.16.3), we find that the velocity of the orbital motion at this altitude is about 7.8 km/s.

where ΔE_k is the increment of the kinetic energy of the satellite as it goes over from the first orbit to the second one. This increment is

$$\Delta E_k = \frac{mg}{2} R^2_{\text{Earth}} \left(\frac{1}{r + \Delta r} - \frac{1}{r}\right) = \frac{mg}{2} R^2_{\text{Earth}} \frac{-\Delta r}{r(r + \Delta r)}$$

$$\approx - \frac{mg}{2} \frac{R^2_{\text{Earth}}}{r^2} \Delta r. \qquad (5.16.5)$$

Since the velocity of the satellite decreases as it goes over from the first to the second orbit, ΔE_k is negative.

On the other hand, the work against the force of gravity done in the transition from the first to the second orbit is equal to the force of gravity acting on the satellite, multiplied by Δr. Since Δr is small, the change in the force of gravity in the orbit-to-orbit transition can be neglected, and the force of gravity can be assumed to be equal to mgR^2_{Earth}/r^2. Consequently, the work against the force of gravity during the transition from the first to the second orbit is

$$A = mg \frac{R^2_{\text{Earth}}}{r^2} \Delta r.$$

This work is spent to impart the increment of the potential energy to the satellite going over from the first to the second orbit. Thus,

$$\Delta E_p = mg \frac{R^2_{\text{Earth}}}{r^2} \Delta r. \qquad (5.16.6)$$

A comparison of (5.16.5) and (5.16.6) shows that the increment of the potential energy is twice as large as the decrease in the kinetic energy of the satellite:

$$\Delta E_p = -2\Delta E_k. \qquad (5.16.7)$$

A transition of the satellite from an orbit of radius r_1 to an orbit of radius r_2, which differs considerably from r_1, can be represented as a series of consecutive transitions in each of which the radius of the orbit increases by a small quantity Δr. For each such transition, relation (5.16.7) is valid. Consequently, this relation also holds for the transition from the orbit of radius r_1 to the orbit of radius r_2:

$$E_{p2} - E_{p1} = -2(E_{k2} - E_{k1}) = \left(-mg \frac{R^2_{\text{Earth}}}{r_2}\right) - \left(-mg \frac{R^2_{\text{Earth}}}{r_1}\right)$$

[see formula (5.16.4)]. This equality will hold if we put E_p at a distance r from the centre of the Earth equal to

$$E_p = - \frac{mg R^2_{\text{Earth}}}{r} + C, \qquad (5.16.8)$$

where C is an arbitrary constant. It should be recalled that the potential energy is always determined to within an arbitrary additive constant whose value depends on the choice of the position of a body in which the potential energy is assumed to be zero.

It is easier to put the constant C equal to zero. Then

$$E_p = -\frac{mg\,R_{Earth}^2}{r}. \tag{5.16.9}$$

In this case, $E_p = 0$ for $r = \infty$. At any finite distance from the centre of the Earth, the potential energy is negative. Expression (5.16.9) can be written in a different form by replacing, in accordance with (5.15.3), gR_{Earth}^2 by GM_{Earth}:

$$E_p = -G\frac{mM_{Earth}}{r}. \tag{5.16.10}$$

We have obtained expression (5.16.8) for the satellite moving in an orbit of radius r. It does not contain, however, velocity and, hence, is valid for any body of mass m irrespective of whether this body is in motion or at rest.

If we take E_p equal to zero, when the body is on the surface of the Earth (i.e. for $r = R_{Earth}$), then $C = mgR_{Earth}$, and the expression for potential energy takes the form

$$E_p = mg\,R_{Earth}\left(1 - \frac{R_{Earth}}{r}\right). \tag{5.16.11}$$

Suppose that $r = R_{Earth} + h$, where h is a very small quantity in comparison with R_{Earth}. Then expression (5.16.11) is simplified as follows:

$$E_p = mg\,R_{Earth}\left(1 - \frac{R_{Earth}}{R_{Earth} + h}\right) = mg\,R_{Earth}\frac{h}{R_{Earth} + h} \approx mgh.$$

We arrive at the well-known expression for the potential energy of a body lifted to a height h above the ground.

It should be recalled that the potential energy determines the work done by gravitational forces on a body as it goes over from a position characterised by the energy E_p to a position in which the potential energy is zero. Consequently, expression (5.16.11) determines the work done by the gravitational forces during a transition from a point at a distance r from the centre of the Earth to a point on its surface. This formula implies that, when a body of mass m moves from infinity to the surface of the Earth, gravitational forces do a work equal to mgR_{Earth} on the body. Accordingly, the work that has to be done against gravitational forces to remove the body from the surface of the Earth to infinity is also equal to mgR_{Earth}. This work is finite in spite of the fact that the distance over which it is done is infinitely large. This is because gravitational forces decrease with the increasing distance from the Earth in inverse proportion to the squared distance.

Using the expressions for kinetic and potential energies, we can determine the work that must be done to launch a satellite of mass m into an orbit of radius r. Before the satellite is launched, its total (kinetic plus potential) energy is zero. Moving in the orbit, the satellite has a kinetic energy given by (5.16.4) and a potential energy determined by (5.16.11). The work A_r we are interested in is equal to the total energy of the satellite moving in an orbit:

$$A_r = E_k + E_p = \frac{mg}{2}\frac{R_{\text{Earth}}^2}{r} + mg\,R_{\text{Earth}}\left(1 - \frac{R_{\text{Earth}}}{r}\right)$$
$$= mg\,R_{\text{Earth}}\left(1 - \frac{R_{\text{Earth}}}{2r}\right). \tag{5.16.12}$$

This expression does not take into account the work that must be done in launching a satellite against the resistance of the atmosphere. It shows that as the radius r of the orbit increases, the work that must be done to inject the satellite into the orbit also increases.

By putting in formula (5.16.12) $r = \infty$, we obtain the work A_∞ required to remove a body from the surface of the Earth to an infinitely large distance:

$$A_\infty = mg\,R_{\text{Earth}}. \tag{5.16.13}$$

This work is spent to increase the potential energy of the body. Indeed, according to (5.16.11), the increment of E_p in the case when r varies from R_{Earth} to infinity is

$$mgR_{\text{Earth}}\left\{\left(1 - \frac{R_{\text{Earth}}}{\infty}\right) - \left(1 - \frac{R_{\text{Earth}}}{R_{\text{Earth}}}\right)\right\} = mg\,R_{\text{Earth}}.$$

The work (5.16.13) is done at the expense of the kinetic energy imparted to the satellite during its injection. The minimum velocity v_2 at which the satellite should be launched so that it can move to infinity is determined by the condition

$$\frac{mv_2^2}{2} = mgR_{\text{Earth}},$$

whence

$$v_2 = \sqrt{2gR_{\text{Earth}}}. \tag{5.16.14}$$

This velocity is known as the *escape velocity*. A comparison with (5.16.2) shows that the escape velocity is $\sqrt{2}$ times the orbital velocity:

$$v_2 = \sqrt{2}v_1 = \sqrt{2} \times 7.91 \text{ km/s} = 11.19 \text{ km/s}.$$

If a body is launched at a velocity exceeding the escape velocity, it also will not return to the Earth, but in this case its velocity will not tend to zero as it moves away from the Earth.

?

5.16.1. With what velocity should a body be thrown vertically upwards for it to reach the height above the ground equal to the radius of the Earth? In calculations, air resistance should be neglected,but the change in the force of gravity should be taken into account.
5.16.2. At what distance from the centre of the Earth will the period of revolution of a satellite be equal to 24 hours so that the satellite will appear to be permanently suspended above some point on the Earth (synchronous satellite)?

Chapter 6

Motion in Noninertial Reference Systems and Inertial Forces

6.1. The Role of a Reference System

We considered so far the motion of bodies only relative to inertial reference systems. It was established that each time when a body acquires an acceleration relative to such a system, we can indicate other bodies whose actions on a given body cause its acceleration. These actions are forces. The laws connecting the acceleration of a body relative to inertial reference systems with the forces acting on the body are the law of inertia and Newton's second law. Besides, it was shown that these forces are mutual in nature, i.e. these are interactions of bodies. This property of forces is expressed in Newton's third law.

In this chapter, we shall be considering the motion of bodies relative to noninertial reference systems. In such systems, bodies may acquire accelerations that cannot be explained by the action of definite forces. For example, when a suitcase falls from a shelf in a train which brakes suddenly, i.e. acquires an acceleration relative to the train, we cannot indicate a definite body which caused this acceleration. If, however, the suitcase were tied to the shelf, it would remain on it during the braking of the train and would not acquire an acceleration relative to the carriage, although the rope by which it was tied would be stretched and act on it with a certain force. While considering the motion relative to an inertial reference system (say, the Earth), we can explain the observed motions by the forces exerted by other bodies. Indeed, the stretched rope imparts to the suitcase an acceleration equal to that of the braking train. For this reason, the suitcase remains at rest relative to the train. If there is no rope, no forces are exerted by the carriage on the suitcase, and it continues to move by inertia with the velocity it had before braking. However, the carriage on which the friction of braking wheels against rails has acted decreases its velocity, and the shelf as if slips from under the suitcase.

Thus, the motion relative to noninertial reference systems obeys other laws than those governing the motion relative to inertial systems. From the point of view of an observer located in a noninertial reference system, the causes of motion differ from those observed in an inertial reference system.

If an observer is in a noninertial reference system, for example, inside a motor car, aeroplane or satellite moving with an acceleration, it is much easier for him to consider observed motions in the reference systems moving with an acceleration (noninertial systems) than to find out each time what the motion of a body in an inertial reference system is. But in this case we must determine the difference between the laws of motion in inertial and noninertial reference ststems. For this purpose, we shall first of all consider in greater detail the motions themselves relative to different reference systems.

The expression "from the point of view of an observer located in a certain reference system" emphasises the fact that all measurements of the position, velocity and acceleration of a body are made just in this reference system irrespective of its motion relative to conventional reference systems (the Earth, the Sun and stars), i.e. in the same way as they are made by an inhabitant of the Earth (relative to the Earth), a passenger of a motor car (relative to the car), a cosmonaut (relative to the spacecraft), and so on.

6.2. Motion Relative to Different Inertial Systems

First of all, let us compare the motion relative to two different inertial systems. The nature of the motion can be different in different systems. Let us take, for example, the Earth as one inertial system and a carriage of a train moving uniformly along a straight segment of the path as another system. Suppose that a body is suspended from a string in the carriage. If the string is in the vertical position, the body is in equilibrium: the sum of the forces acting on it (the force of attraction by the Earth and the tension of the string) is equal to zero. Let us cut the string. The body will fall with the acceleration **g**, and its trajectory relative to the carriage will be a vertical straight line. This can be established, for example, by taking a film of the falling body, using a cine camera fixed in the carriage. If, however, we consider the motion relative to the Earth, say by photographing it from the side of the railroad, the trajectory of the body turns out to be a parabola (Fig. 204). On the contrary, suspending a body on the surface of the Earth and photographing its falling after the string has been burnt through, we

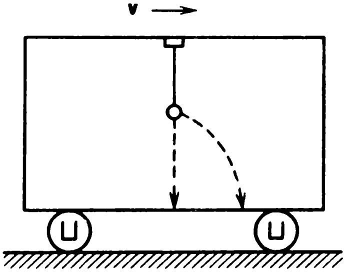


Fig. 204.
The vertical straight line is the trajectory of the motion of a body relative to the carriage after the string has been burnt through, while the parabola is the trajectory relative to the Earth.

shall obtain a vertical trajectory on the film shot on the surface of the Earth and a parabola on the film shot from the carriage.

All this can be easily explained. The difference in motion relative to different reference systems is only due to different initial velocities of the body relative to the first and second reference systems. In the former case, the body is initially at rest relative to the train and moves in the horizontal direction with the velocity of the train. Consequently, when the string is burnt through, the body falls freely with zero initial velocity relative to the carriage. Relative to the Earth we also have a free fall but now with some initial velocity. In the latter case, the free fall with zero initial velocity took place relative to the Earth, while relative to the carriage it was a free fall with an initial velocity.

In both systems, however, the acceleration of the body is the same. Initially, the sum of the forces acting on the body is equal to zero, and the law of inertia is fulfilled: in each system the body is either at rest or in a uniform rectilinear motion, i.e. has no acceleration. After the string has been burnt through, only the force of gravity acts on the body, and in both systems Newton's second law is valid: relative to each reference system the body falls with the acceleration **g** caused by the Earth's gravitation.

A similar situation is observed in all other cases of motions of bodies relative to different inertial reference systems.

6.3. Motion Relative to an Inertial and a Noninertial Reference System

A different situation is observed if we compare a given motion in an inertial and in a noninertial reference system. The forces exerted on a body by other bodies (elastic forces, friction, gravitational forces, and so on) do not depend on the reference system relative to which the motion is being studied. However, the accelerations of bodies in inertial and noninertial reference systems are different. Therefore, the motion of a given body relative to noninertial systems cannot be explained by the forces exerted on it by some other bodies.

Let us illustrate this again with the help of a suspended load, assuming now that the railway carriage taken as a reference system moves over a horizontal straight segment of the path with an acceleration. We denote the acceleration of the train by **w**. In this case, the string on which the body is suspended will be in equilibrium not in the vertical position as in the uniformly moving carriage, but at a certain angle to the vertical, being inclined in the direction opposite to the acceleration of the carriage (Fig. 205).[1] The deviation is the larger, the higher the acceleration. Thus,

[1] No matter how the velocity is directed (along the acceleration or against it) or what its magnitude is, the inclination is determined only by the acceleration.

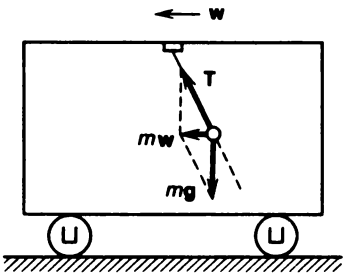


Fig. 205.
The deviation of a plumb line in a carriage moving with an acceleration.

the body is in equilibrium relative to the carriage although the forces acting on the body (the force of gravity $m\mathbf{g}$ and the tension $\mathbf{T}$ of the string) are at an angle to each other and hence cannot be balanced. Thus, the body is at rest relative to the reference system while the resultant of the forces acting on it differs from zero. This resultant can be easily found by analysing the motion of the body relative to the Earth. Since the body is at rest relative to the carriage, its acceleration $\mathbf{a}$ relative to the Earth is equal to the acceleration $\mathbf{w}$ of the carriage (i.e. $\mathbf{a} = \mathbf{w}$). Consequently, the resultant force is equal to $m\mathbf{w}$ and is directed along the horizontal (see Fig. 205).

If we burn through the string on which the body is suspended, the body will fall with an acceleration. Experiments show that its trajectory is a straight line inclined relative to the carriage and lying on the continuation of the string before it has been burnt through (see Fig. 205). After the string has been burnt through, only one force, viz. the attraction of the Earth, acts on the body, which is directed vertically downwards. The acceleration relative to the carriage, however, is directed at an angle to the vertical.

As regards the motion of the body relative to the Earth, it can be easily explained by the forces acting on the body. Before the string has been burnt through, the resultant of the forces acting on the body is $m\mathbf{a}$, and hence the body moves at the same acceleration as the train. After the string has been burnt through, the body falls in a parabola with an initial velocity equal to the velocity of the train at the moment of burning through the string. Indeed, after the string has been burnt through, the motion of the train no longer affects the motion of the body which now is not connected to it in any way.

? **6.3.1.** Determine the angle α formed by a string with a load of mass m suspended on it with the vertical in a carriage moving along a horizontal road with an acceleration w. Does this angle depend on the mass of the load? Find the tension T of the string.

6.3.2. What force must act on a body of mass m to move it uniformly in a straight line relative to a carriage which is in a translatory motion with an acceleration w?

6.4. Noninertial Systems in Translatory Motion

The difference in the laws of motion in noninertial and inertial reference systems consists in that when all forces exerted by other bodies on a given one (gravitational forces, elastic forces, friction, and so on) are taken into account, Newton's second law is valid for inertial systems and does not hold for noninertial ones. This discrepancy can be easily seen for the case when noninertial systems are in translatory motion relative to inertial systems. For a noninertial system we can take, for example, a carriage moving with an acceleration along a straight segment of the path, and for an inertial system, the Earth.

If a body is at rest relative to the carriage, the force acting on the body is (see Sec. 6.3)

$$\mathbf{F} = m\mathbf{w},$$

where m is the mass of the body and $\mathbf{w}$ is the acceleration of the noninertial reference system. If the body moves relative to the carriage with an acceleration $\mathbf{a}'$, while the carriage itself moves, as before, with the acceleration, $\mathbf{w}$, the resultant acceleration of the body relative to the Earth is

$$\mathbf{a} = \mathbf{w} + \mathbf{a}'.$$

Hence, according to Newton's second law, the resultant force $\mathbf{F}$ exerted on the given body by other bodies must be

$$m\mathbf{a} = m\mathbf{w} + m\mathbf{a}'.$$

Thus, be the body at rest or have it an acceleration relative to the carriage, the resultant of the forces exerted by other bodies on the given one is not equal to the mass of the body multiplied by its acceleration relative to the carriage. In other words, Newton's second law is violated for the noninertial system.

6.5. Inertial Forces

A natural question arises: what must be the difference in the forces acting on a given body in an inertial and a noninertial system for Newton's second law to hold for both the systems? The formulas obtained in the previous section provide an answer to this question. It is necessary that in addition to the forces exerted by other bodies on the given one (whose resultant was denoted by $\mathbf{F}$), some other additional force $\mathbf{f}_i = -m\mathbf{w}$, equal to the mass of the body multiplied by the acceleration of the noninertial system and taken with the opposite sign, must act.

Indeed, it is clear that in the case of the body at rest relative to the carriage, the resultant of all the forces, including this additional force, is zero, so that the law of inertia is satisfied relative to the noninertial system. For the body moving with acceleration relative to the carriage, the resultant of all the forces, including the additional force, is

$$\mathbf{F} + \mathbf{f}_{\mathrm{i}} = m\mathbf{a} - m\mathbf{w} = m\mathbf{a}',$$

so that Newton's second law turns out to be satisfied for the noninertial system. Such additional forces are called *inertial forces*. If inertial forces are taken into account, Newton's first and second laws hold for noninertial systems in the same form as for inertial systems: the mass of the body multiplied by its acceleration relative to a noninertial reference system has the same magnitude and direction as the resultant of all the forces acting on the body, *including the inertial forces*.

We have obtained this result for a body moving in a straight line relative to the carriage. It can be shown, however, that each time we take into account the inertial force equal to the mass of the body multiplied by the acceleration of the reference system with the opposite sign, we can apply Newton's first and second laws to any translatory motion of the noninertial reference system (rectilinear or curvilinear) and to an arbitrary motion of the body (for example, across the carriage or in an arbitrary trajectory).

Inertial forces differ from all other forces we dealt with earlier. These forces emerge not due to the action of some bodies on a given one, but due to the *acceleration* of a noninertial reference system relative to any inertial system, in particular, relative to the system "the Sun-stars".

In the case of forces exerted by one body on another, we can always indicate the body *which* exerts a given force. For inertial forces, we can indicate the body *on which* a given force acts, but cannot find a body exerting this force. Therefore, Newton's third law cannot be applied to noninertial systems even if inertial forces are taken into consideration. Indeed, these forces appear "alone" and not in "pairs". There is no reaction exerted on a given body by another body since no "other body" exist. Naturally, the corollaries of Newton's third law are also invalid. For example, the momentum conservation law is not valid for motions considered relative to noninertial reference systems.

Thus, Newton's first and second laws allowed us so far to calculate the motion relative to noninertial reference systems only proceeding from the calculations made for inertial reference systems. Taking into account inertial forces, however, we can use the same laws of motion both for inertial and noninertial reference systems. The laws turn out to be the same, but in addition to ordinary forces, inertial forces appear in noninertial systems. In particular, for a body at rest relative to a noninertial system, the inertial force balances all the other forces acting on the body.

The problem on the position of the plumb line in the carriage moving with an acceleration (Sec. 6.3) can be considered now from the point of

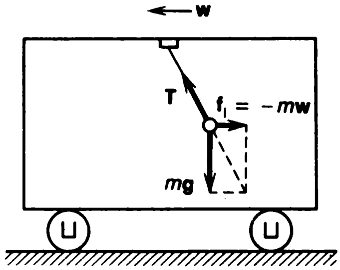


Fig. 206.
Equilibrium of forces for a body resting in a carriage which moves with an acceleration. The force of gravity, the tension of the string and the inertial force act on the body.

view of a noninertial observer. Taking into account inertial forces, we arrive at the problem on equilibrium (relative to the carriage) of the load suspended on a string under the action of the force of gravity, tension of the string, and the inertial force. All these forces are shown in Fig. 206. It can be easily verified that, as it should be expected, calculations lead to the same values of the angle of deviation of the plumb line and for the tension of the string as those obtained in Exercise 6.3.1.

Similarly, taking into account inertial forces, we can consider the motions described in Sec. 2.2 by referring a motion to a reference system moving with an acceleration and using Newton's laws: we can describe the motion "from the point of view of an observer in the noninertial system". At a sudden braking, i.e. when the carriage acquires a backward acceleration, the forward inertial force acts on the torso of a standing passenger. Under the action of the inertial force, the passenger leans forward and may fall. On the contrary, if the velocity of the carriage increases, the inertial force has the backward direction and inclines the torso of the passenger in the direction opposite to the direction of motion.

6.6. Equivalence of Inertial Forces and Gravitational Forces

Inertial forces and gravitational forces are similar in that they are proportional to the mass of a body on which they act, and hence the accelerations, imparted to a body both by gravitational forces and by inertial forces, do not depend on the mass of the body. Therefore, if we observe the motion of a body under the action of forces in a given reference system and do not know whether the system is inertial or not, we cannot distinguish between the gravitational force and inertial force.

For example, we are watching a suspended or falling load in a carriage. Without seeing bodies located outside the carriage, it is impossible to say what causes a deviation of the plumb line or of the trajectory of the falling load from the vertical. Indeed, let us imagine that we are in a windowless carriage and we cannot determine the vertical direction by looking, for example, on the walls of buildings. How can we then explain the observed deviation of the plumb line from the perpendicular dropped to the carriage

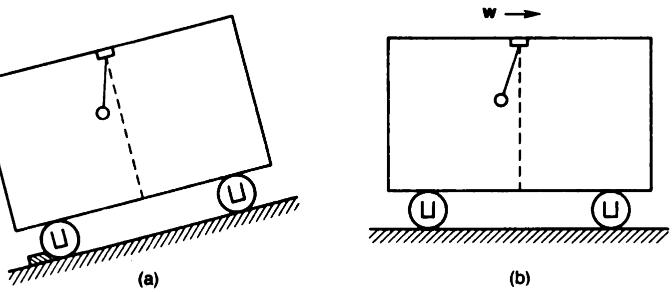


Fig. 207.
The equivalence of gravitational and inertial forces. The deviation of a plumb line can be caused both by an inclined position of a carriage and by its accelerated motion.

floor? The plumb line will be deviated in a carriage standing on inclined railroad (Fig. 207*a*). In this case, the deviation of the string is explained by the action of the gravitational force: the plumb line is perpendicular to the surface of the Earth, while the carriage floor is inclined to the surface. A similar deviation, however, can be observed on a horizontal road if the carriage moves with an acceleration in the direction opposite to the deviation of the plumb line from the vertical (Fig. 207*b*). In this case, the deviation is due to the fact that the carriage is in an accelerated motion.

The same applies to the observation of the trajectory of the falling load after the string has been burnt through. If we assume that the direction of the plumb line or of free fall coincides with the direction of the gravitational force, this direction will be determined correctly in the former case and incorrectly in the latter case. In a windowless carriage, however, there is no way to determine the direction of the gravitational force. Experiments carried out inside the carriage always give the resultant of the gravitational force and the inertial force, and, since the two forces depend in the same way on the mass of the bodies being accelerated, they cannot be distinguished from each other.

Let us consider another example of simultaneous action of the gravitational force and the inertial force. Let us imagine the cabin of a lift moving in a vertical with an acceleration which can be directed upwards or downwards (we shall assume that the downward direction is positive). Suppose that we cannot see how the cabin moves relative to the Earth. In such a lift, a plumb line will always be perpendicular to the cabin floor since both the gravitational and inertial forces are directed along the vertical. But the tension of the string of the plumb line (which can be measured, for example, by suspending the string from a spring balance) will depend on an acceleration of the lift.

Indeed, let the acceleration of the lift be directed upwards and equal to $-w$. Then the inertial force is directed downwards and is equal to mw.

Since the suspended load is at rest under the action of the gravitational force, the inertial force and the tension of the string, the tension is

$$T = mg + mw = m(g + w).$$

This value will be incidated by the spring balance. However, being inside the lift, we cannot find out whether this tension is due to the accelerated motion of the lift or due to an increased gravitational force equal to $m(g + w)$. On a planet with a larger gravity than on the Earth, a load would stretch the spring balance in the stationary lift with a force also exceeding mg.

If we now imagine that the lift moves with a downward acceleration, the inertial force will be directed upwards, and the tension of the string will be

$$T = m(g - w).$$

This force could also be observed in a stationary lift if the experiments were carried out on a planet with a smaller mass, and these two cases would be indistinguishable in such an experiment. If the acceleration of the lift is directed downwards and exceeds g in magnitude (this could be attained in a lift with a winch under it which pulls the lift down with a rope), the resultant of the gravitational force and the inertial force will turn out to be directed upwards, its magnitude being $m(w - g)$. Under the action of this force, the load attached by a string to the floor will rise to the ceiling of the cabin, and "upward" and "downward" directions will change places. If the string is burnt through, the load will fall on the ceiling. Being inside the lift and having no idea about what happens outside it, we can interpret such experiments either as a manifestation of the inertial forces due to an accelerated motion of the lift, or as a change in the magnitude (and direction relative to the cabin) of the gravitational force, or by the two factors acting simultaneously. Finally, observing deformations of bodies at rest, we also cannot say whether the force of gravity acts on a body or the reference system moves with an acceleration. In both cases, the pattern of deformation of the body will be the same (see Sec. 2.32).

It follows from what has been said above that if a reference system is in a translatory accelerated motion relative to inertial systems, inertial forces in the accelerated system are such as if all the bodies were attracted in the direction opposite to the acceleration of the system with forces proportional to their masses. The "free fall acceleration" caused by such a "gravitational force" is equal to the acceleration of the reference system relative to inertial reference systems, taken with the opposite sign. The accelerated translatory motion of a reference system is equivalent in its effect on the motion of bodies to the emergence of the corresponding gravitational forces. This is known as the *principle of equivalence of gravitational* *and inertial forces*. Since gravitational forces depend on the distance to the attracting body, the equivalence takes place in limited regions where the difference in distances can be neglected. We shall consider this question again in Sec. 6.12.

6.7. Weightlessness and Overloads

Let us consider reference systems fixed to bodies subject only to gravitational forces. Such a system is, for example, the shell of a satellite. But first we shall analyse a simpler case. Suppose that the rope on which a lift cabin is suspended has broken and the cabin starts falling with the downward acceleration $\mathbf{g}$. The inertial force acting on a body of mass m located in the cabin will be $-m\mathbf{g}$. The minus sign indicates that the force is directed upwards, i.e. against the force of gravity. But the force of gravity acting on the body is equal to $m\mathbf{g}$ and has the downward direction. Hence this force is balanced by the inertial force. If the body has been suspended on a string, the tension of the string will vanish. If the string is burnt through, the body remains in the same position relative to the cabin. If we impart a velocity to such a free body, it will move uniformly in a straight line until it strikes against the wall. The plumb line will have no definite equilibrium position. If we push the load of the plumb line sideways, it will rotate uniformly about the point of suspension instead of oscillating near the initial position. For a body to remain at rest relative to the falling lift, neither support nor suspender is required, and the bodies at rest will not be deformed. At the same time, the force with which the body at rest acted upon by the gravitational force presses on the support or stretches the suspender vanishes. In other words, the weight vanishes. The conditions taking place in a falling lift are called the state of *weightlessness*, or *zero gravity*.

Exactly the same situation will be observed in a satellite moving in an orbit. Indeed, it was shown above (see Sec. 5.16) that the motion of a satellite is a free fall with the acceleration created by the force of gravity. Therefore from the point of view of an observer located in the satellite, the sum of the gravitational force and the inertial force is equal to zero for any body in the satellite. It is impossible to distinguish between "upward" and "downward" directions inside the cabin. The bodies do not fall onto the floor but rather "float" in air. In order to hold a body of even a large mass in the hand, no efforts are required, and so on. From the point of view of an observer located in an inertial reference system, a cosmonaut does not observe the accelerations of bodies located in the cabin (including the acceleration of his own body) relative to the cabin walls since the cabin, as well as all bodies in it (including the cosmonaut), "fall", i.e. have the same acceleration $\mathbf{g}$. All this implies that weightlessness sets in not because the attraction of the Earth "ceases to act", but just because it "does its job", viz. imparts the same acceleration to all the bodies.

If the cosmonaut tries to impart a high velocity to a body of a large mass which "floats" in air, he will discover that quite a significant force is required for that. This force can be calculated by Newton's second law as the product of the mass of the body and its acceleration relative to the cabin. In the conditions of weightlessness, a massive body stops pressing against the hand holding it in a certain position but does not at all stop pressing against the hand which imparts an acceleration to it. If a considerable initial velocity has been imparted to the bulky body, it continues to move at this velocity in a straight line until it strikes the wall. If the wall withstands this blow, the body is reflected from it and starts moving in the opposite direction at the same velocity. In short, the cosmonaut will not observe any violation of the laws of mechanics, but will notice the absence of phenomena caused by the gravitation. Therefore, in the conditions of weightlessness, customary effects caused by the force of gravity (for example, permanent tension of certain muscles and deformations of internal organs), to which the human organism has adapted in the process of evolution, are missing.

All what has been said about weightlessness refers to the case when only the gravitational force acts on a spacecraft. If, however, it also experiences the action of the thrust exerted by jet engines, the state of weightlessness is violated. For example, on the "active phase" of the trajectory, when the engines accelerate the booster to the required velocity and lift it vertically upwards, the inertial force is directed vertically downwards and is equal to $m\mathbf{a}$ for a body of mass m, where $\mathbf{a}$ is the acceleration of the booster. Thus, a cosmonaut observing the motion of bodies surrounding him relative to the cabin walls will discover that, in addition to the force of gravity mg, the inertial force ma also acts on the bodies in the same direction. To be more precise, since he cannot distinguish between these two forces, the cosmonaut will find that the force $m(g + a)$, viz. the resultant of the force of gravity and inertial force, acts on the bodies. The situation will be such as if the gravitational force of the Earth has increased $(g + a)/g$ times. The acceleration of a booster may exceed the free fall acceleration considerably so that the resultant forces acting on the bodies at rest relative to the cabin may exceed several times the force of gravity for these bodies. Accordingly, deformations caused by this increased force will also become larger, as well as the forces exerted by the deformed bodies or their parts on one another. This state is known as *g-load*, or *overload*. The g-load of two, three, etc. g's is referred to when the resultant of the gravitational and inertial forces exceeds two, three, etc. times the force of gravity acting on a body.

Overloads affect the organism of a cosmonaut much stronger than zero gravity. However, in space flights it takes much shorter time (during the operation of the engines). Special measures are taken to reduce the effect of overloads on the cosmonaut: he is placed in a special seat in the lying position so that the increased weight of his body be uniformly distributed over as large area as possible and do not change the conditions of blood circulation.

Overloads can be easily explained from the point of view of "inertial observer". From this viewpoint, inertial forces are absent, but,in addition to the gravitational forces, the contact forces imparting a given acceleration are applied to the spacecraft and to each body in it. It was shown above (see Sec. 5.10) that in this case the bodies being accelerated turn out to be deformed, and hence elastic forces acting between their parts are such that would act if the bodies were at rest and an increased gravitation acted on them.

6.8. Is the Earth an Inertial Reference System?

We have used so far both the Earth and the Sun-stars (heliocentric) reference system as inertial systems. However, they cannot be both inertial since if we consider the motion relative to the Sun and stars, the Earth rotates about its axis and moves about the Sun in a curvilinear trajectory, i.e. with an acceleration relative to the Sun and stars.

The centripetal acceleration of the points on the surface of the Earth relative to the Sun and stars, caused by the rotation of the Earth about its axis, will be maximal at the equator. For points at the equator, this acceleration can be found from the formula

$$a = \omega^2 r.$$

Substituting for ω the angular velocity of the Earth's rotation, which is equal to 2π rad/day (or about 7.5×10^{-5} rad/s), and for r the Earth's radius equal to 6.4×10^6 m, we obtain $a \approx 0.034\ \mathrm{m/s^2}$. The acceleration of points on the surface of the Earth in its annual motion around the Sun can be obtained from the same formula by substituting into it the quantity 2π rad/year or approximately 2×10^{-7} rad/s for ω and the radius of the Earth's orbit equal to 1.5×10^{13} m for r. The acceleration turns out to be $a \approx 0.0006\ \mathrm{m/s^2}$.

It can be seen that the accelerations of the Earth in its motions in space are very low as compared to those we encounter in practice near the surface of the Earth, for example, with the free fall acceleration $g \approx 10\ \mathrm{m/s^2}$. Therefore, in all comparatively rough experiments we dealt with so far these accelerations did not play any significant role so that if one of the used reference systems (the Earth or the Sun-stars) was assumed to be inertial, the other system turned out to be inertial as well for rough experiments. More accurate experiments, however, should reveal the difference between these two reference systems and establish which of these systems is inertial.

In actual practice, it was found that the Sun-stars system is an inertial reference system, while the Earth is a noninertial system. It was shown, however, that the discrepancy between the system fixed to the Earth and an inertial system is not large and normally can be neglected. The cases when the noninertial nature of the system fixed to the Earth should be taken into account will be discussed specially (in Secs. 6.11 and 6.12).

6.9. Rotating Reference Systems

Let us now consider the motion of bodies relative to reference systems *rotating* relative to inertial systems. We shall find out which inertial forces act in this case. Clearly, it will be a more complicated problem since different points of such systems have different accelerations relative to inertial reference systems.

We shall begin with the case when a body is at rest relative to a rotating reference system. In this case, the inertial force must balance all the forces exerted by other bodies on the given one. Suppose that the system rotates with an angular velocity ω, the body is at a distance r from the rotation axis and is in equilibrium at this point. In order to find the resultant of the forces exerted by other bodies on the body under consideration, we can consider the motion of the body relative to an inertial system as it was done in Sec. 6.3. This motion is the rotation at an angular velocity ω in a circle of radius r. According to Sec. 5.10, the resultant force is directed to the rotation axis along the radius and is equal to $m\omega^2 r$, where m is the mass of the body. This force can be caused by the tension of the string (rotation of the load on the string), gravitational force (motion of planets around the Sun), elasticity of other bodies (elasticity of rails when a carriage is driving around the bend), and so on.

The resultant force does not depend on the reference system in which a given motion is considered. However, the body is at rest relative to the noninertial system under consideration. This means that the inertial force balances this resultant force, i.e. it is equal to the mass of the body multiplied by the acceleration of the point of the system where the body is located, its direction being opposite to this acceleration. Thus, the inertial force is also equal to $m\omega^2 r$, but is directed along the radius away from the rotation axis. This force is known as the *inertial centrifugal force*.[2] The forces exerted on the body at rest relative to the rotating reference system by other bodies are balanced by the inertial centrifugal force.

[2] It should be stressed that the inertial centrifugal force emerges only in rotating reference systems. In inertial systems, there are no centrifugal forces. — *Eds*.

Unlike inertial forces in systems which are in a translatory motion, the magnitude and direction of the inertial centrifugal force for a body of a given mass depends on the point at which the body is located. It is directed along the radius passing through the body, and for a given angular velocity it is proportional to the distance of the body from the rotation axis.

An inertial centrifugal force must be observed on the Earth due to its rotation (we neglected this force so far). It was found in Sec. 6.8 that the centripetal acceleration at the equator is 0.034 m/s^2. It amounts to about 1/300 of the free fall acceleration g. This means that a body of mass m at the equator is acted upon by the inertial centrifugal force equal to $mg/300$ and directed away from the centre, i.e. vertically upwards. This force reduces the weight of the body by 1/300 in comparison with the force of attraction by the Earth. Since this force is equal to zero at the poles, a body transferred from the pole to the equator must "lose" 1/300 of its weight. At other latitudes, the inertial centrifugal force will be smaller, varying in proportion to the radius of a parallel on which the body is located (Fig. 208). It can be seen from the figure that everywhere except at the equator and at the poles, the inertial centrifugal force is directed at an angle to the Earth's radius, being deviated from it towards the equator. As a result, the force of gravity $m\mathbf{g}$, which is the resultant of the force of attraction by the Earth and the inertial centrifugal force, is deflected from the Earth's radius towards the equator.

In actual practice, however, it was shown experimentally that a body brought from the equator to the pole loses not 1/300 of its weight but more (about 1/190). This is due to the fact that the Earth is not a perfect sphere but is an oblate spheroid, and hence the force of gravity at the pole turns out to be slightly larger than at the equator. The effect of the inertial force and the difference in the force of attraction to the Earth at different latitudes lead to the dependence of the free fall acceleration on the latitude of a point and to different values of the free fall acceleration at different points of the globe (which was mentioned in Sec. 2.24).

Thus, the inertial centrifugal force is equivalent to the gravitational force. If the Earth did not rotate, the same loss in weight would be caused by a slightly larger oblateness of the Earth, while if the Earth were not

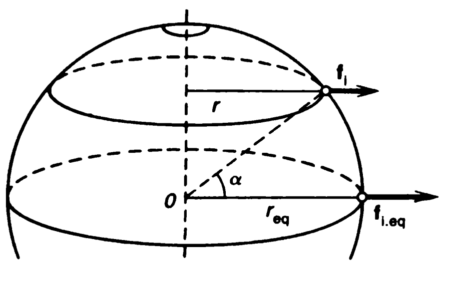


Fig. 208.
Inertial centrifugal forces at different latitudes.

oblate, the same loss in weight would be caused by a slightly higher velocity of its rotation. The deviation of a plumb line would also be caused not by the Earth's rotation but by a nonuniform mass distribution inside it.

Thus, the difference in the weight of bodies and deviation of a plumb line at different points of the globe cannot be regarded as a proof of the Earth's rotation relative to an inertial reference system. The experiments in which the rotation of the Earth relative to the Sun-stars reference system is proved will be described in Sec. 6.11.

The oblate shape of the Earth is an evidence of its rotation: from the point of view of an observer on its surface, it is due to inertial centrifugal forces directed away from the axis and attaining the maximum value at the equator. From the viewpoint of an "inertial observer", the deformation of the Earth is similar to the deformation of any rotating body (see Sec. 5.10). All other rotating celestial bodies are also oblate. Jupiter, for example, is very oblate due to a high velocity of its rotation (its rotation period is equal to ten hours). On the contrary, the Moon which completes a revolution in a month is practically not oblate and has the spherical shape.

? **6.9.1.** Consider Exercises following Secs. 5.10 and 5.13 from the point of view of an observer located in the rotating reference system.

6.9.2. At what rotation period of the Earth would the centrifugal force at the equator balance completely the force of attraction of the Earth so that the weight of a body at the equator would be zero?

6.9.3. Show that the decrease in the weight of a body due to the Earth's rotation varies as the squared cosine of the latitude angle, while the component of the inertial centrifugal force in the direction towards the equator varies as the sine of the doubled latitude angle.

6.10. Inertial Forces for a Body Moving Relative to a Rotating Reference System

If a body moves relative to a rotating reference system, we cannot make Newton's laws valid in the rotating system even if we take into account the inertial centrifugal force in addition to the forces exerted by other bodies on the given body. In this case, there is one more additional inertial force which depends on the velocity of the body.

In order to prove this let us consider the following example. If we move a piece of chalk along a ruler which is lying on a stationary board, the chalk draws a straight line on it. If, however, the board rotates under the ruler, the chalk leaves a curved trace on the board (Fig. 209). Hence the trajectory of the chalk turns out to be curvilinear relative to the rotating reference system, and hence the chalk will have an acceleration normal to the trajectory. However, relative to an inertial system (the stationary ruler), the chalk moves in a straight line. This implies that there are no forces exerted by other bodies and normal to the trajectory. Consequently,

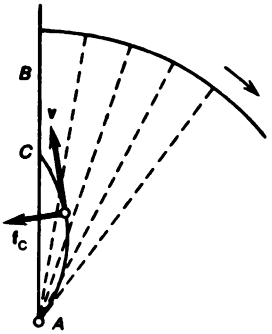


Fig. 209.
A piece of chalk moving uniformly along a ruler AB describes a curvilinear trajectory AC on a board rotating in the direction indicated by the arrow; **v** is the velocity of the chalk relative to the rotating board.

in a rotating reference system, there is another inertial force which is perpendicular to the trajectory described by the body in this system. This additional inertial force is called the *Coriolis force* after the French physicist Gaspard Gustave Coriolis (1792-1843) who calculated this force.

Calculations show [3] that for motions of a body in a plane perpendicular to the rotation axis, the Coriolis inertial force f_C is equal to the doubled product of the angular velocity ω of the rotating reference system, the velocity v of the body relative to this system, and the mass of the body: $f_C = 2m\omega v$. The direction of this force is perpendicular to the velocity of the body and is such that to make it coincide with the direction of velocity, it must be turned through 90° in the direction of rotation of the system. Consequently, if the direction of motion of the body or the direction of rotation of the reference system is reversed (for example, from the clockwise to counterclockwise direction), the direction of the Coriolis inertial force is also reversed.

The Coriolis force differs from all other inertial forces analysed by us in that it depends on the velocity of motion of a body relative to a noninertial reference system.

In addition to the Coriolis force, a body moving in a rotating reference system is acted upon by the centrifugal inertial force in the same way as a body at rest relative to the rotating reference system.

6.11. Proof of the Earth's Rotation

Let us consider again the question as to whether or not the Earth is an inertial reference system. In order to find out whether a reference system is inertial it is sufficient to compare accelerations of bodies relative to this reference system with the forces exerted on these bodies by other objects. If these forces explain the observed motions of the bodies, i.e. the forces and accelerations obey Newton's second law in all cases, the system is inertial.

[3] These calculations are too cumbersome and are not included in this book.

If, however, it turns out that there are accelerations that cannot be explained by the action of other bodies, this means that the system is noninertial, and accelerations are due to corresponding inertial forces.

An experiment proving in this way the fact that the Earth is a noninertial system (namely, its rotation relative to inertial reference systems) was carried out in 1851 in Paris by the French physicist Jean Bernard Léon Foucault (1819-1868). In Foucault's experiments the oscillations of a pendulum set in motion in a certain plane (Foucault's pendulum) were observed. To be able to observe oscillations for a sufficiently long time, Foucault used a pendulum in the form of a load suspended on a very long (67 m) thin wire. The period of the pendulum was 16 s. To avoid the twisting of the wire, its upper end was fixed in a bearing which could freely rotate about the vertical axis. Only two forces acted on the load of the pendulum: the force of gravity directed vertically downwards and the tension of the wire directed upwards along the wire. Thus, the resultant of the two forces acting on the pendulum lied in the vertical plane passing through the wire, i.e. in the plane of its oscillations. While starting the pendulum, measures were taken to eliminate shocks in the direction perpendicular to the initial plane of oscillations (the load was deviated from the equilibrium position with the help of a string which was subsequently burnt through). As a result, the pendulum started to move in the same vertical plane where the wire had been before the string was burnt through.

If the Earth were an inertial reference system, the pendulum started in this way would remain in the same vertical plane during subsequent oscillations. It turned out, however, that the plane of oscillations of the pendulum did not remain stationary relative to the Earth but rotated clockwise (for an observer looking down on it). The trajectory of motion of the pendulum load relative to the Earth is shown in Fig. 210. To make the figure more visual, the angle of rotation of the oscillation plane in every oscillation is highly exaggerated.

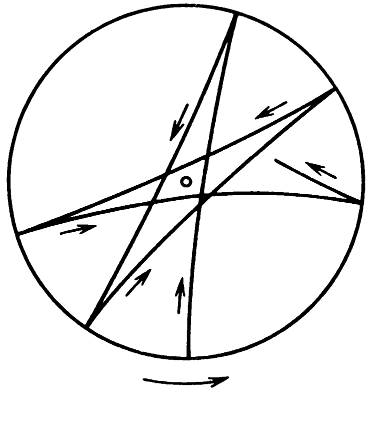


Fig. 210.
The trajectory of the load of the Foucault pendulum (in the Northern hemisphere).

Foucault's experiments were carried out elsewhere on the Earth (including the Southern hemisphere where the oscillation plane rotated counterclockwise). It turned out that as a pole is approached (no matter which of the two), the angular velocity of rotation of the oscillation plane increases and attains the value of 2π rad/day at the pole. Consequently, the oscillation plane of the pendulum at the pole rotates relative to the Earth at the same velocity as the Earth does relative to the Sun-stars reference system, but in the opposite direction. Consequently, the oscillation plane of the pendulum remains stationary in the Sun-stars reference system. Thus, in this system we observe only those accelerations of the pendulum load which are imparted to it by other bodies. This proves that the Sun-stars reference system is inertial. It also proves that the Earth is not an inertial reference system but a system rotating relative to the inertial system at an angular velocity of 2π rad/day.

Proceeding now from the fact that the Earth is a rotating reference system, we can explain the motion of the Foucault pendulum from the point of view of a terrestrial observer. Since the trajectory of the motion of the pendulum is curvilinear, the forces perpendicular to this trajectory must act on the load. The curvature of the trajectory changes sign depending on the direction in which the pendulum moves (forward or backward). Hence, the force must be reversed when the direction of motion of the load is reversed. This is just the Coriolis force. Indeed, it was shown in the previous section that it is directed at right angles to the velocity of motion of a body, and when the direction of motion is reversed (the load swings to and fro), its direction is also reversed. Under the action of the Coriolis force, the trajectory of the load has the form of a "star" shown in the figure.

Besides the experiment with a Foucault pendulum, other phenomena associated with the Coriolis force are also observed on the Earth. The bodies moving northwards in the Northern hemisphere experience the action of the Coriolis force in the eastward direction, i.e. towards the right from the direction of motion, while the bodies moving southwards are acted upon by the Coriolis force in the westward direction, i.e. again towards the right of the direction of motion. Such a force acts, for example, on water in the rivers of the Northern hemisphere. Under the action of this force, water in the rivers washes away the right bank which is always steeper than the left bank. This regularity is known as Ber's law after the Russian scientist K.M. Ber (1792-1876), who was the first to explain this phenomenon. For the same reason, the right rails of two-way railroads are worn out slightly more than the left rails in each line. On the contrary, in the Southern hemisphere the left banks are steeper and the left rails are worn out sooner.

The Coriolis force is also responsible for the emergence of huge eddies (cyclones and anticyclones) on the Earth. This question is discussed in greater detail in Sec. 18.6.

6.12. Tides

If the Earth were separated from all other celestial bodies by much larger distances than it actually is, so that the attraction of other celestial bodies did not affect the processes on it, the only difference between the system fixed to the Earth and an inertial reference system would be the rotation about its axis. But actually celestial bodies influence the Earth by imparting to it a certain acceleration relative to the Sun and stars. Therefore, in addition to the inertial forces due to the Earth's rotation about its axis, we must take into account the inertial forces responsible for the accelerated motion of the Earth as a whole. The most important manifestation of these forces in the reference system fixed to the Earth are tides. The major role in tides is played by the Moon (as the closest celestial body) and the Sun (as the largest celestial body in the Solar system).

Let us first consider tides caused by the Moon. The gravitational force exerted by the Moon on the Earth causes an acceleration **w** in the direction of the straight line connecting their centres (Fig. 211). Consequently, all bodies on the Earth are acted upon by the inertial force equal to the product of the mass of a body and this acceleration with the opposite sign. For the Earth as a whole, this inertial force is exactly equal to the force of attraction exerted by the Moon on the Earth and has the opposite direction. It should be recalled that since these celestial bodies have the spherical shape, the Moon attracts the Earth in such a way as if the entire mass of the Earth were concentrated at its centre. But the gravitational force decreases with increasing distance. Consequently, the bodies on the surface of the Earth on the side of the Moon, i.e. those which are closer to the Moon than the centre of the Earth, are attracted by the Moon with a force exceeding the inertial force, and hence the difference in these forces is directed away from the centre of the Earth. Therefore, at points "below the Moon" bodies "lose their weight".

At antipodal points, the gravitational force exerted by the Moon also does not balance the inertial force since a body is further from the Moon than the centre of the Earth. The difference between the inertial force and the gravitational force of the Moon is again directed away from the centre of the Earth. Consequently, at these points of the Earth's surface, bodies also "lose their weight". The inertial force is equal to the force of attraction by the Moon only at points lying at the mid-distance between the points which are "exactly below the

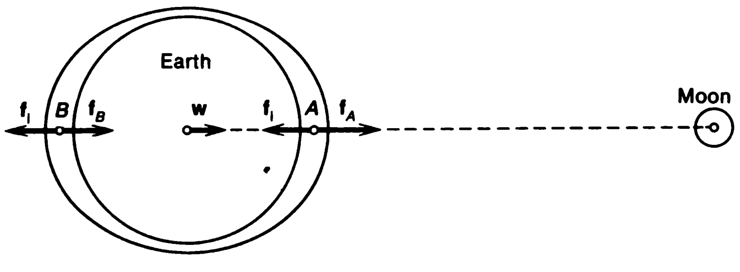


Fig. 211.
The emergence of tides: $\mathbf{f}_i$ is the inertial force, $\mathbf{f}_A$ and $\mathbf{f}_B$ are the forces of attraction of water particles by the Moon, and $\mathbf{w}$ is the acceleration of the Earth caused by the attraction of the Moon.

Moon" and the antipodal points. Thus, right "below the Moon" and on the opposite side, the bodies slightly "lose their weight" due to the fact that the gravitational force decreases with increasing distance. As a result of this influence of the Moon, the level of the ocean smoothly rises on both sides of the Earth by several tens of centimetres. The ocean level is naturally lower between the sites where it rises. Owing to the Earth's rotation these sites with higher and lower levels move over the surface of the Earth. At the middle of a sea, such a small elevation is practically unnoticeable. Near shores, however, it is manifested in that water comes to the shore (high tide) and in about six hours moves away from it (low tide).

Like the Moon, the Sun also causes tides on the Earth. Due to the huge mass of the Sun, both the gravitational force exerted by it and the corresponding inertial forces are much larger than the relevant forces due to the Moon. It was shown, however, that the tides are caused not by the gravitational force or inertial force alone, but by their difference on either side of the Earth. The inertial force is the same for the entire Earth: it is equal to the force of attraction of the Earth by the Sun. On the other hand, the gravitational force, as in the case with the Moon, decreases as we move from the side illuminated by the Sun to the shadow side. But the further the body being attracted (the Earth) is from the attracting body (the Moon and Sun), the slower the change in the gravitational force with distance. Since the Sun is separated from the Earth by a much longer distance than the Moon, it turns out that the tidal effect, i.e. the difference between the inertial force and the gravitational force, is weaker for the Sun than for the Moon (almost by a factor of 3). Nevertheless, the tidal effect caused by the Sun is noticeable. When the Moon, Earth and Sun are on the same straight line (new moon and full moon), tides become stronger (spring tides), while when the directions to the Sun and to the Moon are at right angles (the first or third quarters of the Moon), tides become weaker (neap tides).

It is clear from the analysis of the origin of tides that they are caused by the violation of the principle of equivalence for the inertial and the gravitational forces, which was already mentioned in Sec. 6.6. The reason behind this violation also becomes clear: the inertial force emerging due to the acceleration imparted to the Earth by the Moon in the reference system fixed to the Earth does not depend on the position of a body on the Earth, while the force of attraction of the body by the Moon depends on this position.

Chapter 7

Hydrostatics

7.1. Mobility of Liquids

The main distinctive feature of liquids in comparison with solid (elastic) bodies is their mobility (fluidity). Owing to their mobility, liquids, unlike elastic bodies, do not offer a resistance to a change in their shape. Parts of a liquid can move freely, sliding relative to one another. Therefore, if forces not perpendicular to the surface of a liquid are applied to it, equilibrium of the liquid is always violated, and it starts to move however weak these forces are. It is sufficient, for example, to blow on the surface of water in a basin to cause the motion of water. A slightest wind makes ripples on the sea surface. It was mentioned above that an insignificant force exerted by a glass fibre sets a piece of wood floating in water in motion (see Sec. 2.15).

The mobility of liquids explains why the free surface of a liquid in equilibrium under the force of gravity is always horizontal. Indeed, if, for example, the surface of the liquid at rest were at an angle to the horizontal, the particles of the liquid near the surface would slide down along the surface under the action of the force of gravity like along an inclined plane. Such a motion would continue until the surface of the liquid becomes horizontal.

It should be noted, however, that the free surface of a liquid poured in a vessel is slightly bent near the walls. This can be easily seen if we observe the reflection of the objects from the surface of water poured in a cup. This bending is due to the forces of interaction between the liquid and the walls and is exhibited only when they are in the immediate vicinity. The effect of the walls will be discussed in Sec. 14.6.

For a liquid occupying a large space (seas and oceans), we must take into account the fact that the direction of the force of gravity is different at different points on the surface of the Earth. Since the force of gravity is always directed to the centre of the Earth along the radius, the sea surface as a whole assumes the form close to the spherical surface, which is distorted only by local factors (for example, waves caused by winds).

7.2. Force of Pressure [1]

Everyday experience shows that liquids exert certain forces on the surface of solids in contact with them. This forces are known as the forces of hydrostatic pressure (see Sec. 7.15).

Closing the open tap of a water supply system with a finger, we feel the pressure exerted by the liquid on the finger. The pain in ears experienced by a diver at a large depth is caused by the forces of pressure of water on his ear-drums. Thermometers for measuring temperature at a large sea depth must be very strong, otherwise water pressure can crash them. In view of enormous forces of pressure at a large depth, the submarine hull must have a much higher strength than the hull of a ship. Forces of hydrostatic pressure exerted on the bottom of a ship support it on the surface, balancing the force of gravity. Forces of pressure act on the bottom and walls of vessels filled with a liquid. If we pour mercury into a rubber vessel, its bottom and walls will bend outwards (Fig. 212). Finally, forces of pressure are exerted by various parts of a liquid on one another. This means that if we removed a part of the liquid, definite forces would have to be applied to the surface formed to maintain equilibrium (Fig. 213). The forces required for maintaining equilibrium are equal to the forces of pressure exerted by the removed part of the liquid on the remaining part.

It was shown in Sec. 2.5 that the forces acting in direct contact of bodies, viz. elastic forces, emerge as a result of deformation of the bodies. In solids, elastic forces appear as a result of a change in the shape or volume of a body. In liquids, no elastic forces emerge upon a change in the

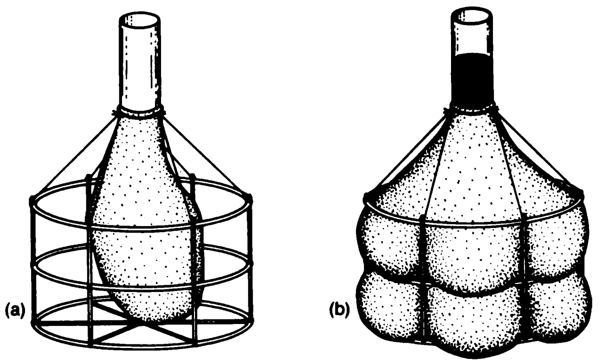


Fig. 212.
The walls and bottom of a rubber vessel enclosed in a cage (*a*) are bent outwards by the forces of pressure exerted by mercury poured in it (*b*).

[1] Understood as a force distributed over a surface. — *Eds.*

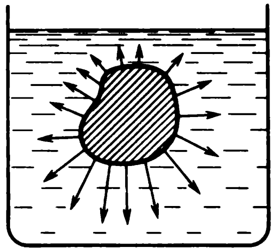


Fig. 213.
A portion of liquid (hatched volume) has been removed. In order to maintain the remaining liquid in equilibrium, forces distributed over the surface formed should be applied.

shape. The mobility of a liquid is just due to the absence of elasticity relative to a change in the shape. If, however, the *volume* changes (as a result of compression), elastic forces appear. Hence, liquids possess elasticity relative to a change in the volume. Elastic forces in liquids are just forces of pressure. Thus, if a liquid exerts forces of pressure on bodies in contact with it, this means that it is compressed. The more the liquid is compressed, the larger the forces of pressure emerging as a result of this compression.

Since the density of a substance increases as a result of compression, liquids can be said to possess elasticity relative to a change in their density.

The dependence of forces of pressure on the extent to which a liquid is compressed can be qualitatively explained with the help of the following example. Suppose that a strong cylinder filled with a liquid is closed by a tightly fitting (to avoid leaking of the liquid) piston with a load lying on it (Fig. 214). Under the action of the load, the piston moves down, thus compressing the liquid. The compression of the liquid gives rise to forces of pressure exerted on the piston and balancing the weight of the piston with the load. As load on the piston is increased, the liquid is compressed to a larger extent so that the increased forces of pressure balance the increased load.

This pattern is quite similar to that considered in Sec. 2.31 for a load resting on a support. The support sags, and equilibrium sets in when the elastic forces emerging in sagging balance the force of gravity acting on the load.

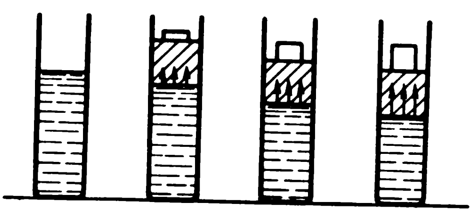


Fig. 214.
The larger the load resting on the piston, the stronger the compression of the liquid.

The compression of the liquid under the piston is exaggerated on the figure for the sake of clarity. Actually, the displacement of the piston and the compression of liquid in such an experiment are so small that they cannot be detected by naked eye. However, *all liquids are compressible to a certain extent,* and the compression corresponding to certain forces of pressure can be measured.

7.3. Measurement of Compressibility of a Liquid

Athough the change in the volume of a liquid under the action of external forces is small, it can still be easily detected and measured. While measuring the compressibility of a liquid, however, we must take into account the fact that the liquid strongly compressed in a vessel exerts large forces of pressure on the vessel walls from inside and stretches the vessel. As a result, an exaggerated value of compressibility is obtained. Therefore, we must eliminate the possibility of the vessel to be stretched. This is attained by applying the same pressure to the vessel from outside as the one exerted on it from inside by the liquid.

The schematic diagram of the instrument for measuring the compressibility of a liquid (piezometer) is shown in Fig. 215. Glass vessel *1* filled with a liquid under investigation is placed into glass vessel *2* into which air is pumped through tube *3*. The air exerts a pressure on the outer walls of vessel *1* and on the liquid in it through tube *4* open at the top. Thus, vessel *1* experiences the same pressure both from inside and outside and practically does not change its volume. The liquid, however, is compressed, and its level in tube *4* drops. This tube is made narrow, so that even a small change in the volume of the liquid is immediately seen. By measuring a decrease in the level of the liquid in the tube, we can find the decrease in its volume. The readings of manometer *5* correspond to the force of pressure per unit area. Thus, we can determine a decrease in volume corresponding, for example, to an increase in pressure by one atmosphere (Sec. 7.10). For water, for example, such an increase in pressure leads to a decrease in volume by about 1/20 000 of the initial volume, while for mercury the decrease in volume amounts to only 1/250 000. For the sake of comparison, it can be pointed out that a steel bar would be contracted only by 1/1 700 000 of its initial volume under the same increase of pressure.

? 7.3.1. The strength tests of steam boilers are carried out by pumping water under a high pressure into them. What amount of water will flow out from a boiler having a volume of 1.5 m^3 and filled with water under a pressure of 12 atm if the boiler cracks at its upper part?

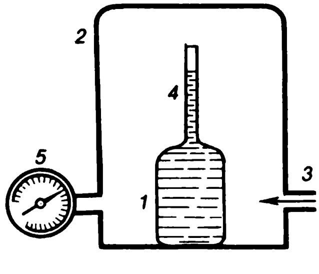


Fig. 215.
Schematic diagram of a piezometer.

7.4. "Incompressible" Liquid

It was shown that forces of pressure in a liquid emerge due to its compression. The compression of liquids is insignificant even at very strong forces of pressure. Since we are normally interested not in compression of a liquid *per se*, but only in the forces of pressure emerging as a result of such compression, we can introduce the concept of "incompressible" liquid like the concept of a perfectly rigid body introduced earlier (Sec. 3.2). The difference will be in that a perfectly rigid body preserves its shape and volume, while an incompressible liquid retains only its volume, its shape being changed as much as is desired (fluidity of luquid). Thus, we can assume that the density of liquids is also independent of pressure.

It will be shown, however, that sometimes we still have to take into account the change in the density of a liquid (when the pressure is very high, Sec. 7.21).

7.5. Forces of Pressure Are Transmitted in a Liquid in All Directions

Figure 214 shows the compression of a liquid under various loads on the piston (for the sake of clarity, in an exaggerated form). A similar pattern would be obtained if a hard spring were placed under the piston: both the spring and the liquid act with certain forces ("exert pressure") only when they are compressed (Fig. 216). However, compressed spring acts only on the piston and on the bottom of the vessel, while the forces of hydrostatic pressure act on the bottom, on the piston and on the walls as well (Fig. 217).

Not only the piston but also the elastic walls of the cylinder in turn act on the liquid. They are bent the more, the higher the compression of the liquid. Naturally, if the cylinder is made of a metal or glass, this bending is so insignificant that it can be detected only by precise measurements.

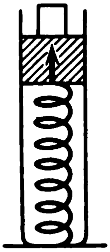


Fig. 216.
A compressed spring balances the piston in the same way as the compressed liquid in Fig. 214.

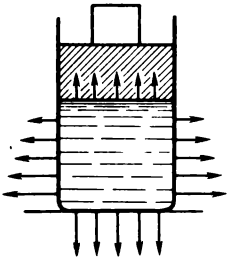


Fig. 217.
The forces of pressure are exerted by a liquid not only on the bottom and on the piston but also on the vessel walls.

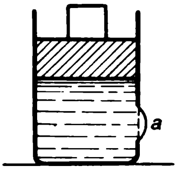


Fig. 218.
The rubber film *a* covering the hole in the wall of the vessel noticeably bulges out by the forces of pressure exerted by water.

However, the forces exerted by the deformed walls are quite noticeable. If we make a hole in the wall and cover it with a rubber film, the bending of the film becomes obvious (Fig. 218).

7.6. Direction of Forces of Pressure

The forces of pressure exerted by a stationary liquid on a given region of the surface of a solid are always directed along the normal to the surface. Indeed, otherwise (according to the law of action and reaction) the reactions, i.e. the forces exerted by the given region of the surface on the liquid, would also be not perpendicular to the surface. But in this case, as was shown in Sec. 7.1, the liquid could not be in equilibrium. Consequently, the forces of pressure acting on the piston compressing the liquid are normal to its surface, the forces of pressure acting on the bottom and walls of the vessel are normal to the bottom and the walls, and so on (see Fig. 217).

If we take a piston with a beveled lower surface (Fig. 219), the forces of pressure will press it against the cylinder wall (to the left in the figure).

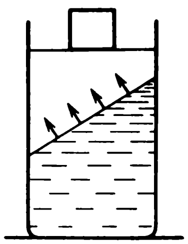


Fig. 219.
Forces of pressure are always normal to the surface on which they act.

7.7. Pressure

The forces of pressure on the walls of a vessel containing a liquid or on the surface of a solid immersed in a liquid are not applied to a certain point of the surface. They are *distributed over the entire contact surface* between the solid and the liquid. Therefore, these forces of pressure on a given surface depend not only on the extent to which the liquid is compressed but also on the area of this surface. In order to characterise the distribution of forces independently of the area of the surface on which they act, the concept of *pressure* is introduced.

The pressure on an element of a surface is the ratio of the force of pressure exerted on this element to its area. Pressure is obviously equal to the force of pressure exerted on a region of a surface whose area is equal to unity. We shall denote pressure by p. If the force of pressure on a given region is F and the area of this region is S, the pressure is expressed by the

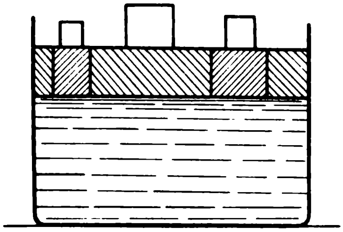


Fig. 220.
The masses of the loads keeping the stoppers in equilibrium are proportional to their cross-sectional areas.

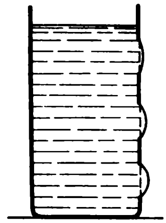


Fig. 221.
The lower the position of the film, the more it bulges out.

formula

$$p = F/S.$$

If forces of pressure are distributed uniformly over a surface, the pressure is the same at any point of this surface. Such is, for example, the pressure on the surface of a piston compressing a liquid. This is illustrated by the experiment shown in Fig. 220, where instead of a solid piston, a piston with openings closed by stoppers which can move in the holes without friction is used. The forces that should be applied to the stoppers to keep them in equilibrium are directly proportional to their cross-sectional areas. Equal forces act on the stoppers of equal cross sections.

However, the cases when forces of pressure are distributed nonuniformly over a surface are also encountered quite often. This means that different forces act on equal surface areas in different regions of the surface. Let us pour water in a vessel with identical openings in the wall, which are covered by rubber films. We shall see that the films covering lower openings are bent outwards to a larger extent (Fig. 221). This means that the pressure in the lower part of the vessel is higher than in the upper part.

7.8. Membrane Manometer

How can we measure the hydrostatic pressure exerted on the surface of a solid, e.g. the water pressure on the bottom of a glass? Naturally, the bottom of the glass is deformed under the action of the forces of pressure. Knowing this deformation, we could determine the force causing it and calculate the pressure. This deformation, however, is so small that practically cannot be measured directly. Since it is convenient to judge about the hydrostatic pressure exerted on a given body by its deformation only when this deformation is sufficiently large, pressure in liquids is measured in practice by special instruments, viz. *manometers,* in which deformations are comparatively large and can be easily measured. [2]

[2] Pressure in gases is also measured with the help of such manometers. — *Eds.*

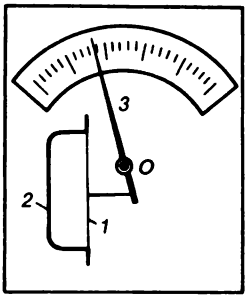


Fig. 222.
Schematic diagram of a membrane manometer.

A simple *membrane manometer* is designed as follows (Fig. 222). Thin elastic plate *1* (membrane) tightly covers empty box *2*. Pointer *3* is connected to the membrane and can rotate about the axis *O*. When the instrument is submerged in a liquid, the membrane is bent under the action of the forces of pressure, and the bending is transmitted in a magnified form to the pointer moving along the scale. Each position of the pointer corresponds to a certain bending of the membrane, and hence to a certain force of pressure on the membrane. If we know the area of the membrane, we can go over from the forces of pressure to pressures proper. Having appropriately graduated the manometer beforehand, we can directly measure pressure, i.e. determine to which pressure a certain position of the pointer on the scale corresponds. For this we must apply to the manometer a known pressure and, having marked the position of the pointer, put the corresponding figure on the scale. Other types of manometers (pressure gauges) will be described below.

7.9. Independence of Pressure of the Orientation of an Area Element

A manometer placed into a liquid indicates the pressure in the region where its membrane is located. In order that the readings of a manometer allow us to judge about the pressure in a given region, the size of the membrane must be sufficiently small. Otherwise, if the pressure is different at different points of the membrane, the readings of the manometer will correspond only to a certain *average* value of the pressure.

Having placed a manometer with a sufficiently small membrane into a liquid, we shall see that the readings of the manometer do not change if we turn it. Thus, it turns out that *the pressure in a given region of a liquid does not depend on the orientation of the area element on which it is being measured*. It should be recalled that by definition pressure does not depend on the area of the plane on which it acts since it is always referred to a unit surface area. Thus, the concept of pressure introduced by us is a characteristic of the state of the liquid in a given region, which does not depend either on the area or the orientation of the area element on which the pressure is being measured. *Pressure depends only on the extent to which a liquid at a given site is compressed.*

It should be stressed that the flexible membrane of a manometer serves only for a convenient detection and measurement of forces of pressure in a liquid, and these forces are due to elastic properties of the liquid itself. The same forces of pressure would be exerted by the liquid on the surface of any other body, say, a solid piece of metal, substituting the membrane.

We can also mentally isolate a volume in a liquid. At all points on the surface bounding this volume, there will exist some pressures which are completely identical to those which would exist on the surface of a solid having the same volume. The same pressure acts on the membrane of a manometer placed into the liquid.

7.10. Units of Pressure

For a unit of pressure, we take the pressure corresponding to a unit force acting on a unit area. The SI unit of pressure is the pressure such that a force of a newton acts on the area of one square metre. This unit was called a *pascal* (Pa) in honour of B. Pascal (Sec. 7.13):

$$1 \text{ Pa} = 1 \text{ N/m}^2.$$

An off-system unit called an *atmosphere* is also widely used. It is equal to the pressure exerted by a mercury column 760 mm high (or by a water column whose height is 10.332 m) [3]:

$$1 \text{ atm} = 760 \text{ mm Hg} = 101\ 325 \text{ Pa.}^{4} \qquad (7.10.1)$$

7.11. Determination of Forces of Pressure from Pressure

If we know the pressure at each point of a given surface, we can easily determine the resultant of forces of pressure exerted on the entire surface. Let us first consider a flat surface. If the pressure p is the same over the entire surface, the resultant force is

$$F = pS,$$

[3] This is the so-called *physical atmosphere* (denoted as atm), which will be henceforth called just an atmosphere. Besides the physical atmosphere, there exists a *technical atmosphere* (denoted as at) which is equal to the pressure exerted by a 10.000-m high water column. The technical atmosphere is equal to 0.968 of the physical atmosphere. This unit will not be used in this book.

[4] In vacuum technology, the popular unit is a *torr*:

$$1 \text{ torr} = 133 \text{ Pa.} \text{ — } \textit{Eds.}$$

where S is the area of the surface. As follows from Sec. 7.6, this resultant is normal to the surface.

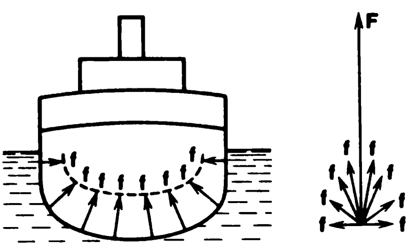
If pressure is different at different points of a flat surface, the resultant force is calculated as follows. The surface is divided into small regions such that pressure can be assumed to be virtually constant at all points of a region (although it can be different for different regions). The force of pressure exerted on an individual region is calculated as the product of the pressure on this region and its area. The resultant of these forces acting on the entire surface is equal to the sum of the forces found in this way (acting on individual regions and parallel to one another). The direction of the resultant is perpendicular to the flat surface.

If a surface is not flat, it is divided into small regions such that each of them can be regarded as a flat surface. Then the force acting on each region can be found in the same way as for a flat region. Each force is directed along the normal to the region on which this force acts. These forces are not parallel but have different directions. In order to determine their resultant exerted on the entire surface, we must compose the forces acting on individual regions by the vector sum rule. For example, the forces of pressure **f** exerted by water on the submerged surface of a ship have different directions at different points of its hull, as shown in Fig. 223. The resultant **F** of these forces is directed upwards and balances the force of gravity acting on the ship.

? 7.11.1. A piston having a shape shown in Fig. 224 is inserted into a tube. The pressure of liquid is the same on both sides of the piston. Is the piston in equilibrium? To simplify arguments, it should be assumed that the cross section of the tube is rectangular.

7.12. Distribution of Pressure in a Liquid

It was shown in previous sections that pressure in liquids depends on the extent to which a liquid is compressed. A liquid can be compressed by the force of gravity acting on it or by some external forces applied to its surface (surface forces). For example, the pressure in sea depths is caused by the



(a) (b)

Fig. 223.
The force **F** is the resultant of the forces of pressure **f** acting on the curved submerged surface of a ship.

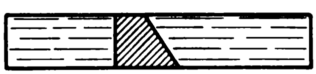


Fig. 224.
To Exercise 7.11.1.

pressure of the upper-lying layers of water and by the pressure exerted by the atmosphere on the free surface of the sea. The distribution of pressure in the liquid turns out to be nonuniform since the upper layers of water are mainly compressed by the atmospheric pressure, while deeper-lying layers are compressed to a much greater extent by the pressure of the above-lying water layers. On the contrary, an almost uniform distribution of pressure is observed in a steam boiler where the pressure is mainly created by the pressure of steam on the surface of water since the depth of water in the boiler is not large. In the following sections, we shall consider in detail the pattern of pressure distribution in a liquid for different cases of action of forces on the liquid.

7.13. Pascal's Principle

Let us first determine the distribution of pressure in a liquid when the liquid is compressed only by surface forces. The weight of the liquid can be neglected if the pressure caused by it is small in comparison with the pressure due to surface forces. Aboard satellites in the conditions of weightlessness, a liquid is actually compressed only by surface forces. We shall show that in the presence of surface forces alone, pressure is the same throughout a liquid.

Let us place a liquid in a closed vessel of an arbitrary shape, to which a cylinder with a piston is connected (Fig. 225). Pushing the piston into the cylinder, we create a pressure in the liquid due to surface forces. Experiments show that if we place manometers in different regions in the vessel, their readings will be practically the same.

It can be also proved theoretically that in the case under consideration the pressures at any two points, say, A and B, must be equal. For this we mentally isolate a thin cylinder inside the liquid, whose axis coincides with the segment AB and whose bases of area S are normal to AB. The isolated volume is a part of the liquid at rest, and hence it is also in equilibrium although the forces of pressure are exerted on its surface. No other forces act on the cylinder (we ignore the force of gravity). For an equilibrium to set in it is necessary that the sum of the projections of all forces of pressure on any direction be zero (see Sec. 3.6). Let us consider the sum of projections of the forces of pressure on the axis AB.

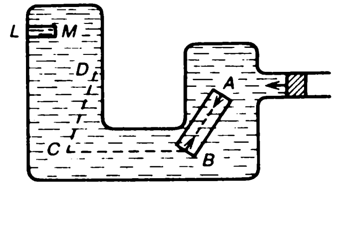


Fig. 225.
To the derivation of Pascal's principle.

The forces of pressure acting on the lateral surface of the cylinder are normal to the axis AB, and hence their projections on this axis are equal to zero. We remain only with the forces exerted on the cylinder bases. They are equal to $p_A S$ and $p_B S$ respectively, where p_A and p_B are the pressures at points A and B. Since these forces are normal to the bases, they act along AB in the opposite directions. Since the cylinder is in equilibrium, these forces must balance each other, i.e. $p_A S = p_B S$, whence

$$p_A = p_B.$$

In other words, the pressures at points A and B are equal.

This line of reasoning can be repeated for any two points inside the liquid. If two points cannot be connected by a straight line without crossing the vessel walls (as, for example, points A and D), the proof is carried out consecutively for a series of intermediate points (say, points B and C). We prove that $p_A = p_B$, then that $p_B = p_C$, and finally that $p_C = p_D$. Hence it follows that $p_A = p_D$, Q.E.D.

Thus, if only surface forces are applied, *pressure is the same throughout a liquid.* This law was established by the French physicist and mathematician Blaise Pascal (1623-1662) and is known as *Pascal's principle* (or *Pascal's law*).

Let us consider a cylinder whose base lies on the vessel wall (for example, cylinder LM in Fig. 225) and make sure that the pressure on the walls is equal to the pressure in the liquid. The same pressure will be exerted on the surface of the piston. Thus, if the pressure of the piston on the surface of the liquid is p, the same pressure p exists at each point in the liquid and on the vessel walls. Therefore, Pascal's principle is sometimes formulated as follows: *the pressure exerted by surface forces is transmitted without loss to any point of a liquid.*

In this formulation, Pascal's principle remains valid for the general case as well, i.e. when the force of gravity is taken into account. If the force of gravity produces a certain pressure in a liquid at rest (which is generally different at different points), the applied surface forces increase the pressure at each point in the liquid by the same value.

7.14. Hydraulic Press

Pascal's principle provides an explanation to the operation principle of a device which is widely used in engineering, viz. a *hydraulic press.* The hydraulic press consists of two cylinders having different diameters, supplied with pistons and connected by a pipe (Fig. 226). The space under the pistons and the pipe are filled with a liquid. We denote the cross-sectional area of the smaller piston by S_1 and that of the larger piston by S_2 and sup-

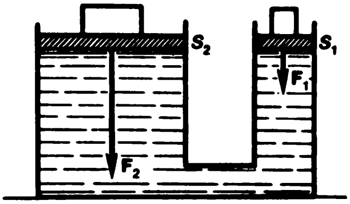


Fig. 226.
Schematic of a hydraulic press.

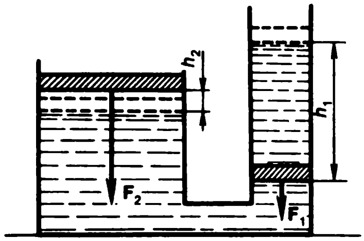


Fig. 227.
The displacements of the pistons are inversely proportional to their cross-sectional areas, and hence to the forces acting on them.

pose that a force F_1 is applied to the smaller piston. Let us find the force F_2 which should be applied to the other piston to maintain equilibrium, i.e. for the liquid not to be pushed out from the first cylinder to the second or back through the connecting pipe.

We shall neglect the force of gravity acting on the liquid. Then the pressure throughout the liquid must be the same. But the pressure under the first piston is F_1/S_1, while under the second piston it is F_2/S_2. Consequently, $F_1/S_1 = F_2/S_2$, whence we obtain

$$F_2 = F_1 S_2 / S_1.$$

The force F_2 is as many times stronger than F_1 as the area of the second piston is larger than that of the first piston. Thus, with the help of a hydraulic press a large force can be balanced by a small one.

Let us now assume that the smaller piston has moved (for instance, downwards) by a distance h_1 (Fig. 227). Then a part of the liquid flows from the smaller cylinder to the larger one and lifts the larger piston by a distance h_2. Since the compressibility of liquids is insignificant, the volume of the liquid displaced from the smaller cylinder can be assumed to be equal to the volume of liquid entering the larger cylinder: $h_1 S_1 = h_2 S_2$, whence we obtain

$$h_2 = h_1 S_1 / S_2.$$

Comparing this formula with the expression obtained by us for the force F_2, we see that the distance covered by the larger piston is as many times shorter than the distance covered by the smaller piston as the force acting on the larger piston is stronger than the force exerted on the smaller piston. Thus, in the displacement of the piston of a hydraulic press, we have a complete analogy with the relationship between the displacements of the ends of a lever and the forces applied to them. Here the "golden rule" of mechanics is also observed (see Sec. 4.1), i.e. we gain in force as much as we lose in distance. The requirement that the liquid should retain its volume is equivalent to the condition that the level should not bend.

A hydraulic press is a transformer of force like a simple machine considered earlier. It can thus be called a simple hydraulic machine.

A hydraulic press is more convenient for obtaining large forces than a lever press or a screw press. For this reason, hydraulic presses are used as high-power presses (say, for metal stamping or for pressing out oil from the seeds of plants). Water or oil are used as a filling liquid.

A hydraulic press with a horizontal large piston is used for shifting (imparting an initial push to) a ship launched from a shipyard.

7.15. Liquid under the Action of the Force of Gravity

Let us now consider the equilibrium of a liquid, taking into account the force of gravity. Reasoning in the same way as in Sec. 7.13, we can make sure that at all points of a horizontal plane, the pressure is the same, but it grows as we go over from the horizontal plane to a deeper-lying one.

Indeed, if points A and B (Fig. 228) lie in the same horizontal plane, the axis AB of a mentally isolated cylinder is horizontal. The equilibrium condition for the cylinder along the axis is $p_A S = p_B S$, since the projection of the force of gravity on the horizontal direction is zero so that only the forces of pressure on the cylinder bases act along the horizontal axis. Thus, $p_A = p_B$, i.e. the pressures at all points of the same horizontal plane are equal. The horizontal planes turn out to be the *surfaces of constant pressure*. They are sometimes called the *level surface*. The free surface of a liquid is one of the level surfaces. The pressure is the same at all its points. In an open vessel, it is equal to the atmospheric pressure.

All what was said above can be easily verified with the help of a manometer. Moving the manometer in a liquid so that its membrane

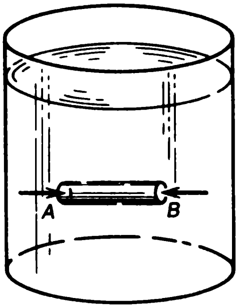


Fig. 228.
Since the horizontal cylinder AB is in equilibrium, the pressures at points A and B are equal.

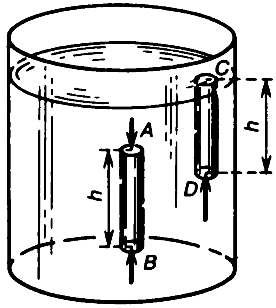


Fig. 229.
The pressure difference at points B and A balances the force of gravity acting on the cylinder AB.

always remains in the same horizontal plane, i.e. on the same level surface, we shall see that its readings do not change. If we change the depth to which the lower part of the manometer is submerged (as we go over to other level surfaces), a change in pressure will be observed: the pressure increases as the manometer is submerged to a larger depth. In the sea, for example, the pressure grows as we go over from the surface to the bottom. This is explained by the fact that at a larger depth the sea water is compressed by the force of pressure exerted by a thicker layer of the above-lying water.

In order to calculate the change in pressure with depth, we shall determine the difference in pressures at points A and B lying on the same vertical (Fig. 229). Having isolated mentally a thin vertical cylinder with a cross-sectional area S, we shall consider equilibrium conditions for this cylinder along the vertical. The forces of pressure exerted on its lateral surface give a vertical projection equal to zero. There are three forces acting along the vertical: the force of pressure on the top, equal to $p_A S$ and directed downwards, the force of pressure acting on the base, equal to $p_B S$ and directed upwards, and the force of gravity acting on the liquid in the volume of the cylinder and having the downward direction. If the distance between A and B is h, the cylinder volume is Sh, and the weight of the liquid in it is $\rho g S h$, where ρ is the density of the liquid and g is the free fall acceleration. The equilibrium condition for the cylinder is expressed by the equality $p_A S + \rho g S h = p_B S$, whence we obtain

$$p_B - p_A = \rho g h.$$

The quantity $\rho g h$ is equal to the weight of the column of liquid having the height h and the cross-sectional area equal to unity. Thus, the obtained formula indicates that *the difference in pressures at two points of a liquid is equal to the weight of the liquid column having a unit cross-sectional area and the height equal to the difference in the depths of the points.*

If the pressure on the free surface of a liquid is zero, then, considering a vertical cylinder DC whose top lies on the surface of the liquid, we can find in a similar way that the pressure p at a point lying at a depth under the surface is determined by the formula

$$p = \rho g h. \tag{7.15.1}$$

If the pressure on the free surface differs from zero, the quantity $\rho g h$ gives the difference in the pressures at the depth h and on the free surface.

The pressure due to the force of gravity is called the *hydrostatic pressure*. Thus, *the hydrostatic pressure is equal to the product of the density of a liquid, the free fall acceleration, and the depth of submergence.*

While deriving the relations between the pressures at different points, we made use of the fact that the points under consideration can be con-

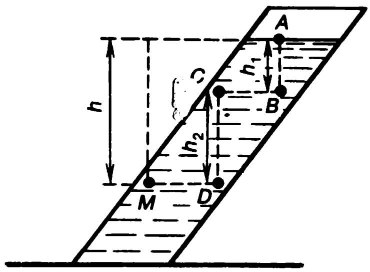


Fig. 230.
The pressure at point M is determined by the depth h measured along the vertical.

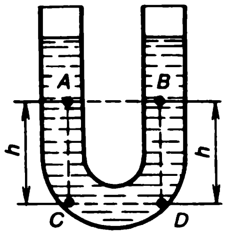


Fig. 231.
The pressures at points A and B are equal.

nected by a cylinder with a vertical or horizontal axis, which completely lies in the liquid. If, however, this cannot be done as, for example, in an inclined vessel (Fig. 230) or in a U-shaped tube (Fig. 231), to compare the pressures at any two points it is sufficient then to connect these points by a broken line which completely lies in the liquid and whose links are vertical and horizontal alternately. For example, for the vessel with inclined walls, we can take the line $ABCDM$, while for the U-shaped tube, the line $ACDB$. At the ends of each horizontal link, the pressures are equal, and for each vertical link, the formula derived above can be used.

Thus, going over from link to link, we obtain, for example, the following expression for the vessel with inclined walls:

$$p_B = \rho g h_1, \quad p_C = p_B, \quad p_D = p_C + \rho g h_2, \quad p_M = p_D,$$

whence

$$p_M = \rho g (h_1 + h_2) = \rho g h,$$

where $h = h_1 + h_2$ is the depth of submergence for a given point. This formula is also valid in the case when the perpendicular dropped from a given point to the free surface does not lie completely in the liquid. Considering the U-tube, we obtain the following expression for points A and B lying in the same horizontal plane:

$$p_C = p_A + \rho g h, \quad p_D = p_C, \quad p_D = p_B + \rho g h,$$

whence

$$p_A = p_B.$$

It can be seen that *a level surface is always a horizontal plane* even if the individual regions of this plane are separated by the vessel walls. Thus, *the distribution of pressure over depth does not depend on the shape of the vessel.*

Let us plot the graph of the pressure distribution over the depth in a

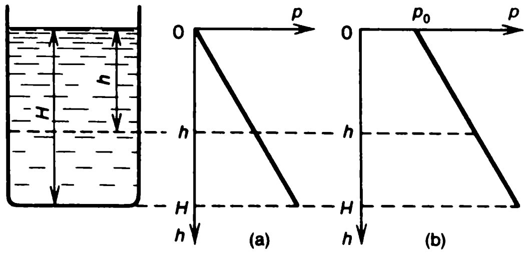


Fig. 232.
Pressure distribution over the depth of submergence: (*a*) the atmospheric pressure is zero; (*b*) the atmospheric pressure is equal to p_0.

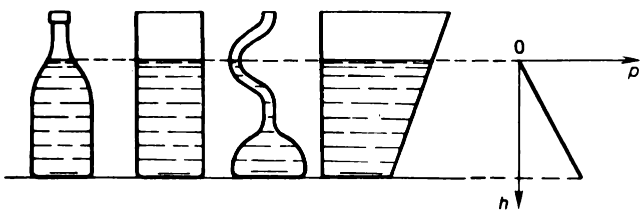


Fig. 233.
The pressure graph is the same for vessels of different shape.

vessel with a liquid. Along the downward vertical axis, we plot the submergence depth h, while on the horizontal axis, the pressure p. Since the hydrostatic pressure is proportional to the depth, the graph will be a straight line (Fig. 232*a*). If a pressure p_0 is exerted on the free surface, the pressure p at a given depth increases by p_0 (Fig. 232*b*). In an open vessel, the pressure p_0 is the atmospheric pressure.

Since the pressure in a liquid does not depend on the shape of the vessel, the graph expressing the dependence of pressure on the depth is always a straight line (Fig. 233).

?

7.15.1. The atmospheric pressure on the free surface of water is 10^5 Pa. At what depth will the pressure double? At what depth is the water pressure equal to 5×10^5 Pa?

7.15.2. Plot the pressure distribution in a measuring glass filled with different liquids as shown in Fig. 234. Determine the pressure on the bottom of the glass if $l_1 = 6$ cm, $l_2 = 10$ cm, $l_3 = 6$ cm, while the densities are $\rho_1 = 0.81 \times 10^3$ kg/m³, $\rho_2 = 1 \times 10^3$ kg/m³ and $\rho_3 = 13.6 \times 10^3$ kg/m³.

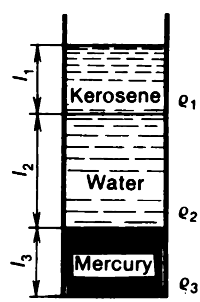


Fig. 234.
To Exercise 7.15.2.

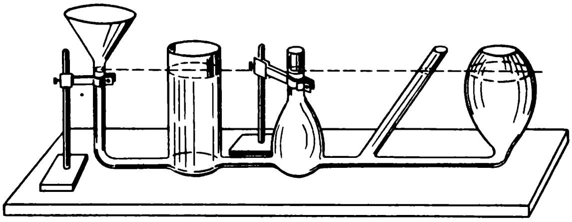


Fig. 235.
In communicating vessels, water is at the same level.

7.16. Communicating Vessels

Let us take a number of vessels of different shape, connected by pipes at their bottoms (communicating vessels). If we pour a liquid into one of them, the liquid will flow through the pipes to the other vessels and will stay on the same level in all of them (Fig. 235). This can be explained as follows. The pressure on the free surfaces of the liquid in the vessels is the same and equal to the atmospheric pressure. Thus, all the free surfaces belong to the same level surface, and hence must lie in the same horizontal plane (see Sec. 7.15).

A kettle and its spout are communicating vessels: water is at the same level in them. Hence, the spout of the kettle must be at the same height as the brim of the vessel, otherwise the kettle could not be filled completely. When we incline the kettle, the level of water remains the same, while the spout is lowered. When it reaches the level of water, the water flows out (Fig. 236).

Water meters for water tanks (Fig. 237) operate on the principle of communicating vessels. Such water meters are mounted, for example, on wash stands in railway carriages. Water is at the same level in the open glass tube connected to the tank and in the tank itself. In a water meter mounted on a steam boiler (Fig. 238), the upper end of the tube is connected to the upper part of the boiler filled with steam. This is done to

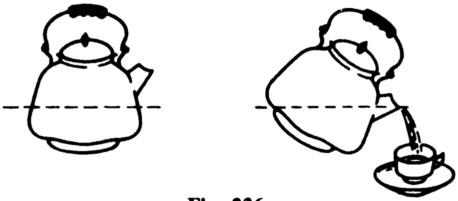


Fig. 236.
A kettle and its spout are communicating vessels.

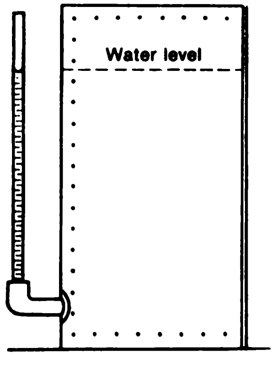


Fig. 237.
Water stays at the same level in a water meter and in the tank.

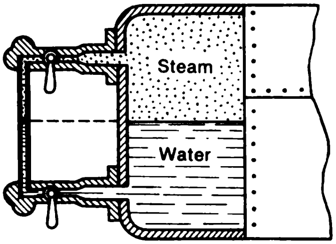


Fig. 238.
A water meter of a steam boiler. Taps are intended for disconnecting the water meter from the boiler.

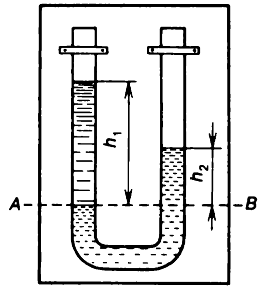


Fig. 239.
Liquids having different densities stay at different levels in communicating vessels.

equalise the pressure on the free surface of water in the boiler and in the water meter. Then the level of water in the water meter is the same as the level of water in the boiler.

Locks on rivers and channels also operate on the principle of communicating vessels. In adjacent lock chambers separated by lock gates, water is at different levels. A channel passing under the gate and connecting the two chambers can be either opened or closed. When the channel is open, the two chambers become communicating vessels, and the water flowing out from the chamber with the higher level to the chamber with the lower level ultimately stays on the same level in the two chambers. Then the lock gate can be opened and a ship moves from one chamber to another. Thus, using a system of locks, the ship can be moved from one reservoir to another, in which water is at a different level. If the difference in water levels is very large, a large number of consecutively operating lock chambers is required.

Let us fill communicating vessels in the form of a U-tube with some liquid, say, water (Fig. 239). The level of free surface will be the same in both arms of the tube. We shall now add a liquid of a different density and not mixing with water (say, kerosene) into one arm. The level of liquid in each arm will rise but not to the same level as in the case of the same liquid. The interface between the liquids will be lowered as we pour the second liquid. Let us determine the ratio of the heights of liquid columns in each arm above the level AB of the interface between the liquids. We denote the heights of the columns by h_1 and h_2 and the densities of liquids by ρ_1 and ρ_2 respectively. Below the plane AB, only one liquid will be in the tubes, and hence the pressures p_A and p_B at points A and B lying at the same height must be the same. These pressures are given by

$$p_A = \rho_1 g h_1 \quad \text{and} \quad p_B = \rho_2 g h_2.$$

Equating p_A and p_B, we obtain $\rho_1 h_1 = \rho_2 h_2$, whence

$$h_1/h_2 = \rho_2/\rho_1.$$

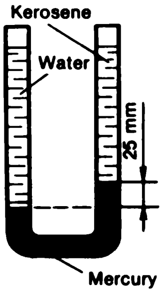


Fig. 240.
To Exercise 7.16.1.

The heights of liquid columns in communicating vessels above the interface are inversely proportional to the densities of liquids.

? **7.16.1.** A U-tube is filled with mercury, water and kerosene as shown in Fig. 240. The upper levels of water and kerosene are on the same horizontal. The difference in the levels of mercury in the two arms is equal to 25 mm. Find the height of the water column. The densities of mercury and kerosene are 13.6×10^3 and 0.81×10^3 kg/m^3 respectively.

7.17. Liquid Column Manometer

Let us fill a U-tube with water and, holding its left arm in the mouth, blow into it (Fig. 241). It will be seen that the levels of water in the arms of the tube will be shifted so that water will be at a higher level in the open arm. This is explained by the fact that air above the surface of the liquid, compressed by our lungs, exerts on it a pressure exceeding the atmospheric pressure in the open arm of the tube. Since the pressure at points A and B lying in the same horizontal plane is the same, the pressure of the blown-in air exceeds the atmospheric pressure by the pressure produced by the water column whose height is equal to the difference in the levels of water in the arms of the tube. Naturally, water can be replaced by some other liquid, say, mercury. By measuring the difference in the levels of the liquid in the arms, we can determine the pressure exerted on the liquid in one arm or, to be more precise, the difference in pressures on the surface of the liquid in the two arms. This principle is used in the design of a liquid column manometer.

A liquid column manometer is made in the form of a U-tube whose one arm is connected to the vessel in which the pressure should be measured (Fig. 242). If the created difference in the levels of the liquid is h, the pressure exerted in the arm with the lower level of liquid exceeds the pressure in the other arm by ρgh, where ρ is the density of the liquid in the column manometer.

Normally, water or mercury is used in column manometers, and the pressure measured from the observed difference in the levels is expressed

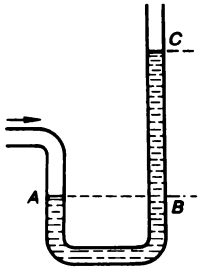


Fig. 241.
Air pressure in the left arm balances the atmospheric pressure and the pressure of the water column BC in the right arm.

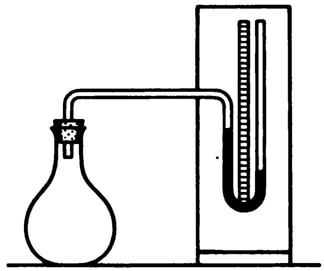


Fig. 242.
A liquid column manometer. The manometer indicates that the pressure in the vessel is lower than the atmospheric pressure.

directly in units of length. For units of pressure, the pressures created by a water or mercury column of height 1 mm are taken. These units are called the "millimetre of water column" and the "millimetre of mercury column" and denoted as mm H_2O or mm Hg.

The density of water is $\rho = 1 \times 10^3$ kg/m^3. Substituting this value and $h = 10^{-3}$ m into (7.15.1), we find that

$$1 \text{ mm } H_2O = 1 \times 10^3 \times 9.81 \times 10^{-3} = 9.81 \text{ Pa}. \quad (7.17.1)$$

A similar calculation gives

$$1 \text{ mm Hg} = 13.6 \times 10^3 \times 9.81 \times 10^{-3} = 133.3 \text{ Pa}. \quad (7.17.2)$$

It should be recalled that [5]

$$1 \text{ atm} = 760 \text{ mm Hg} = 101\,325 \text{ Pa} \approx 1.013 \times 10^5 \text{ Pa}.$$

? 7.17.1. The pressure in a vessel has changed by 2 mm Hg. By what distance is the level of liquid in the open arm of a manometer connected to the vessel shifted?

7.18. Water Supply System. Pressure Pump

The schematic diagram of a water supply system is shown in Fig. 243. A large tank is mounted in a tower (elevated water tank). A system of branch pipes connects the tank with buildings. The outlets of branch pipes are supplied with taps. The water pressure at a tap is equal to the pressure of a water column whose height is equal to the difference in the heights of the

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[5] Strictly speaking, the pressure of one atmosphere is defined as the pressure exerted by a 760-mm mercury column at 0 °C. At this temperature, the density of mercury is $\rho_0 = 13.596 \times 10^3$ kg/m^3, and at a temperature of 20 °C, the density of mercury is $\rho = 13.546 \times 10^3$ kg/m^3. — *Eds.*

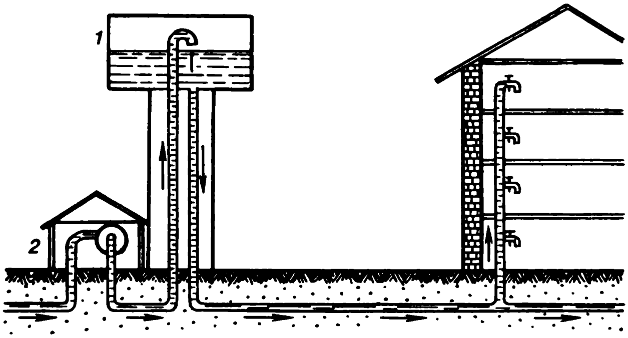

Fig. 243.
Schematic diagram of a water supply system. Water is pumped to the elevated water tank *1* by pump *2*.

tap and the free water surface in the tank. This pressure usually amounts to several atmospheres since the tank is at a height of several tens of metres. Owing to this, water flows out of an open tap at a high speed. The pressure on the upper floors of buildings is obviously lower than on the lower floors. It is also clear that water cannot be supplied to a height exceeding the height of the level of water in the tank.

Water is delivered to the tank of the tower by pumps. A pressure piston pump consists of a cylinder with a piston supplied with valve *1* (Fig. 244). Valve *2* is mounted at the bottom of the cylinder. Both valves can be opened only in one way. Behind the second valve, there is pipe *3* leading to the upper tank. Suppose that the cylinder and pipe are filled with water. Let us see what happens when the piston moves downwards and upwards.

We start to lower the piston. It compresses water, and the emerging forces of pressure close valve *1* and open valve *2*. Valve *2* will be opened when the pressure of water compressed in the cylinder exceeds the pressure created by the water column having a height from valve *2* to the level of water in the upper tank. As the piston continues to descend, water is forced out of the cylinder through pipe *3* and flows to the upper tank. At the same time, the space above the piston is filled through pipe *4* by water from the lower tank. Let us now pull out the piston. The pressure under it will immediately fall, and the water pressure in pipe *3* will close valve *2*. On the other hand, the water pressure above the piston will open valve *1* since now there is no force of pressure exerted on it from below. As the piston rises, water flows through the open valve *1* from the upper to the lower part of the cylinder. In subsequent strokes of the piston, the process is repeated, and water is pumped from the lower tank to the upper one.

? **7.18.1.** What minimal pressure must be developed by a pump supplying water to a height of 55 m?

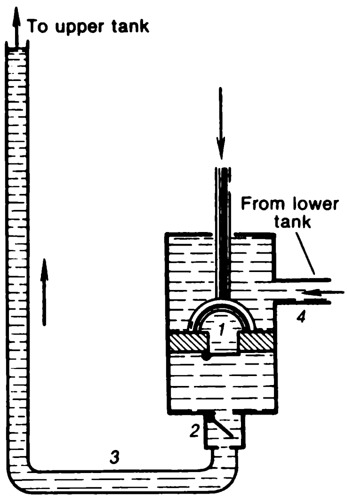

Fig. 244.
Pressure pump.

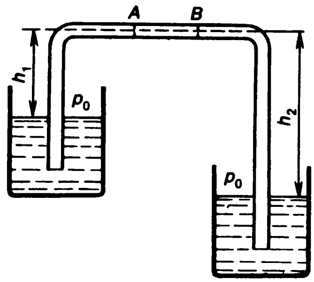

Fig. 245.
Operating principle of a siphon.

7.18.2. The pressure of water at the mains taps on the second floor of a 6-storey building is 2.5 atm. Find the height of the level of water in the system's water tower and the water pressure at a tap on the sixth floor. The height of a storey should be assumed to be 4 m.

7.19. Siphon

Let us consider two vessels containing the same liquid and located at different levels (Fig. 245). We fill a bent pipe with the same liquid and immerse its ends into the liquid in the vessels. Then we remove the plugs by which the two ends of the pipe have been closed. If the liquid fills the entire pipe completely (without gaps), it will flow from the upper vessel to the lower one. Such a device is called a siphon. Siphons are widely used in practice to empty vessels which cannot be turned upside down, for instance, to extract the petrol from the tank of a motor car.

The operation of a siphon is explained as follows. Let us mentally isolate a volume in the upper part of the pipe, bounded by sections A and B. The pressure on the open surfaces of liquid in the two vessels is the same and equal to the atmospheric pressure p_0. The pressure p_A in section A is less than p_0 by $\rho g h_1$, while the pressure p_B in section B is smaller than p_0 by $\rho g h_2$. Since $h_1 < h_2$, the pressure p_A is higher than p_B by $\Delta p = \rho g(h_2 - h_1)$. Therefore, liquid will move in the pipe from A to B, and hence will flow from the upper vessel to the lower one. If we remove the lower vessel, the siphon will continue to operate. Moreover, the velocity of liquid flow in the pipe will increase since the distance h_2 in this case must be measured from the open end of the pipe. It should be noted that if the liquid column in the pipe is discontinued, the siphon will stop working if this discontinuity lies above the level of the liquid in the upper vessél.

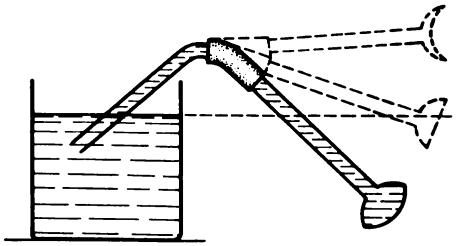

Fig. 246.
The shape of a rubber film at different positions of the tube.

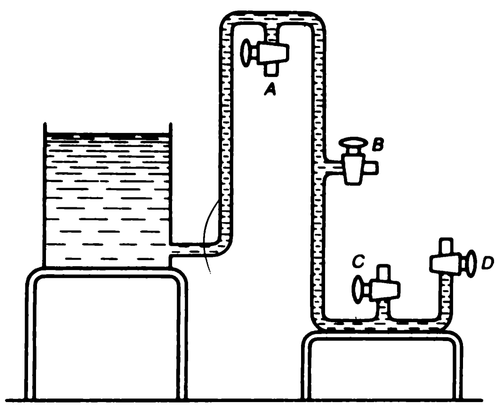

Fig. 247.
To Exercise 7.19.1.

The above arguments can be easily verified experimentally with the help of a rubber tube whose end can be arranged at different heights. The larger the difference in heights of the end of the tube and the free surface of the liquid, the more pronounced the effect, and the higher velocity of liquid. If we cover the outlet of the tube filled with liquid by a film (Fig. 246), it can be seen that when the tube is lowered, the film changes its shape from concave (the end of the tube is above the level of liquid in the vessel) to flat (the end of the tube is at the level of liquid) and becomes more and more convex as the tube is lowered further.

? 7.19.1. A vessel and a pipe are filled with the same liquid (Fig. 247). How will the level of liquid in the vessel change if taps A, B, C and D are opened?

7.20. Force of Pressure on the Bottom of a Vessel

Let us take a cylindrical vessel with a horizontal bottom and vertical walls and fill it with a liquid to a height h (Fig. 248). The hydrostatic pressure at each point of the bottom will be the same:

$$p = \rho g h.$$

If the area of the bottom is S, the force of pressure exerted by the liquid on

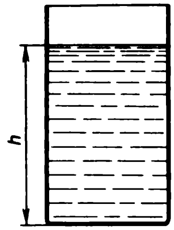

Fig. 248.
The force of pressure exerted on the bottom in a vessel with vertical walls is equal to the weight of the entire liquid.

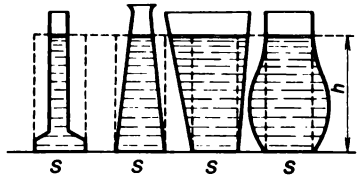

Fig. 249.
The force of pressure on the bottom of all these vessels is the same. It is larger than the weight of the poured liquid for the first two vessels and smaller than the weight of the liquid in the other two vessels.

the bottom of the vessel is $F = \rho ghS$, i.e. it is equal to the weight of the liquid in the vessel.

Let us now consider vessels of different shape but with the same area of the bottom (Fig. 249). If we pour a liquid into each vessel to the same height h, the pressure on the bottom will be $p = \rho gh$ throughout the vessels. Consequently, the force of pressure on the bottom will be

$$F = \rho ghS,$$

i.e. the same *for all* the vessels. *This force is equal to the weight of the liquid column with the cross-sectional area equal to the area of the bottom of the vessel and having the height equal to that of the liquid in the vessel.* In Fig. 249, this column is shown for each vessel by dashed lines. Pay attention to the fact that the force of pressure exerted on the bottom does not depend on the shape of the vessel and can be either more or less than the weight of the liquid in the vessel.

This conclusion can be verified experimentally with the help of a device designed by Pascal (Pascal's vases) (Fig. 250). Bottomless vessels of different shapes can be fixed on a prop. Instead of a bottom, a plate suspend-

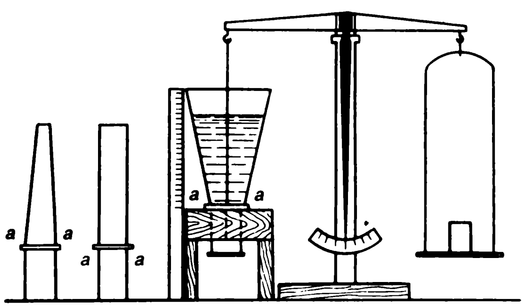

Fig. 250.
Pascal's vases. Cross sections *aa* are the same for all the vessels.

ed on the beam of a balance is tightly fitted to a vessel. If a liquid is poured into a vessel, the force of pressure exerted on the plate tears it off from the vessel when the force of pressure slightly exceeds the weight of the load put on the other pan of the beam balance.

For a vessel with vertical walls (cylindrical vessel), the bottom is opened when the weight of the liquid poured into the vessel is equal to the weight of the load. For vessels of other shapes, the bottom is opened at the same height of the liquid column, although the weight of the water in the vessel can be either more (for a vessel widening to the top) or less (for a vessel narrowing to the top) than the weight of the load.

This experiment gives an idea of how huge forces of pressure on the bottom of a vessel of an appropriate shape can be obtained by using a small amount of water. Pascal connected a thin narrow vertical pipe to a tightly caulked-up barrel filled with water (Fig. 251). When the pipe is filled with water, the force of hydrostatic pressure on the bottom becomes equal to the weight of the water column whose cross-sectional area is equal to the area of the barrel bottom, and the height is equal to the height of the pipe. The forces of pressure on the walls and on the top of the barrel are increased accordingly. When Pascal filled the pipe to a height of several metres (for which just a few cups of water were required), the emerging forces of pressure blew up the barrel.

How can we explain the fact that the force of pressure exerted on the bottom of a vessel can (depending on the shape of the vessel) be either more or less than the weight of the liquid contained in the vessel? We know that the force exerted by the vessel on the liquid must balance the weight of the liquid. As a matter of fact, not only the bottom but also the vessel walls act on the liquid in a vessel. In a vessel widening to the top, the forces exerted by the walls on the liquid have *upward* components. Thus, a part of the weight of the liquid is balanced by the forces of pressure exerted by the walls, and only the remaining part must be balanced by the forces of pressure exerted by the bottom. On the contrary, in a vessel narrowing to the top the bottom exerts the force of pressure in the upward direction, while the walls act in the *downward* direction. For this reason, the force of pressure on the bottom turns out to be more than the weight of the liquid. However, the sum of the forces exerted on the liquid by the bottom of a vessel and its walls is always equal to the weight of the liquid. Figure 252 illustrates the distribution of forces exerted by the walls on a liquid contained in vessels of different shapes.

In vessels narrowing to the top liquid exerts an upward force on the walls. If we make the walls of such a vessel flexible, the liquid will lift them. Such an experiment can be made with the help of the following device. A piston is fixed, and a cylinder connected to a vertical pipe is put

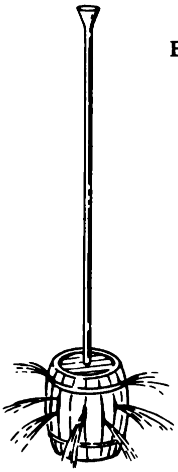

Fig. 251.
Experiment with Pascal's barrel.

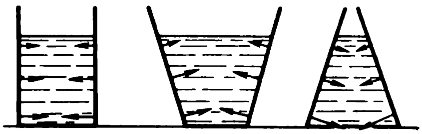

Fig. 252.
Forces exerted on a liquid by the walls of vessels of different shapes.

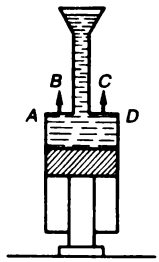

Fig. 253.
When water is poured into the funnel, the cylinder rises.

on the piston (Fig. 253). When the space above the piston is filled with water, the forces of pressure on regions AB and CD of the cylinder walls raise the cylinder up.

7.21. Water Pressure in Sea Depths

In Sec. 7.10, it was pointed out that the pressure of a 10-m water column is equal to one atmosphere. The density of salt sea water is 1-2% higher than that of fresh water. Therefore, we can assume that every 10 m increase in depth corresponds to an increment in the hydrostatic pressure of one atmosphere. For example, a submarine 100 m below the surface experiences a pressure of 10 atm (in addition to the atmospheric pressure), which is approximately the same as the pressure in the boiler of a steam engine. Thus, each depth below the surface of the water corresponds to a hydrostatic pressure. Submarines are equipped with manometers for measuring the water pressure outside the submarine, which can be used to determine the submarine's depth.

At very large depths, the compressibility of water becomes noticeable. As a result of compression, the density of water deep beneath the sea is higher than it is at the surface, and hence the pressure increases with the depth a bit more rapidly than linearly, and the pressure graph slightly differs from a straight line. The pressure increment due to compression grows in proportion to the square of the depth. At very large depths, say, at 11 km, it reaches almost 3% of the total pressure at this depth.

Sea depths are investigated using bathyspheres and bathyscaphs. A bathysphere is a hollow steel sphere capable of resisting the huge water pressures deep beneath the surface. The bathysphere has port-holes made

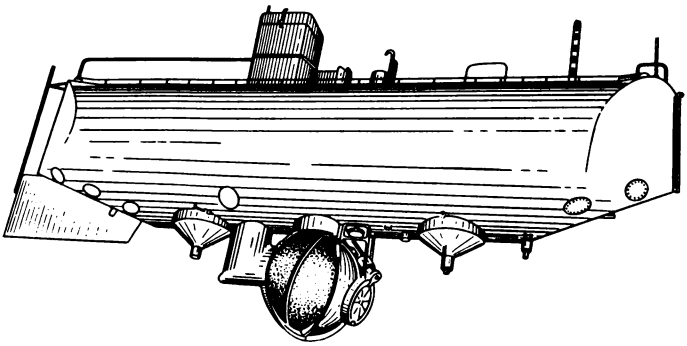

Fig. 254. Bathyscaph.

of glass. A flood-light illuminates water where no sunlight can penetrate. The bathysphere with a researcher on board is lowered on a steel rope from a ship. A depth of about 1 km can be explored by bathyspheres. Bathyscaphs which are combinations of a bathysphere and a petrol-filled tank (Fig. 254) [6] can be submerged to greater depths. Since petrol is lighter than water, the bathyscaph can move in the sea like an airship in the air. The role of the airship's light gas is played here by petrol. The bathyscaph has ballast and engines to move it, unlike a bathysphere, independently of the surface ship.

Initially, a bathyscaph floats on the surface of water like a submarine. To submerge, sea water is let into the ballast tanks. In order to raise it to the surface, the ballast is pumped out, and the lightened bathyscaph floats up. A record depth was attained on January 23, 1960, when a bathyscaph stayed for 20 minutes at the bottom of the Mariana Trench in the Pacific Ocean, some 10 919 km below the water surface. The water pressure (calculated by taking into account the increase in density due to salinity and compression) exceeded 1150 atm. The researchers aboard the bathyscaph even discovered living organisms at these lowest depths of the world ocean.

A diver (or an aquanaut) diving below the water surface experiences a hydrostatic pressure on his body exerted by the surrounding water in addition to a constant atmospheric pressure. Although a diver (Fig. 255) works in a rubber suit (diving suit) and has no direct contact with water, he experiences the same pressure as would a swimmer since the air in the diving suit is compressed to the pressure of surrounding water. For the same reason, the air supplied for breathing through a hose must be pumped at the pressure of the water at the depth at which the diver is submerged. The

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[6] It is impossible to attach a bathysphere to an empty tank (filled with air) since the external pressure would crush the tank.

Fig. 255.
A diver in a rubber suit with a metallic helmet. Air is pumped through the rubber tube.

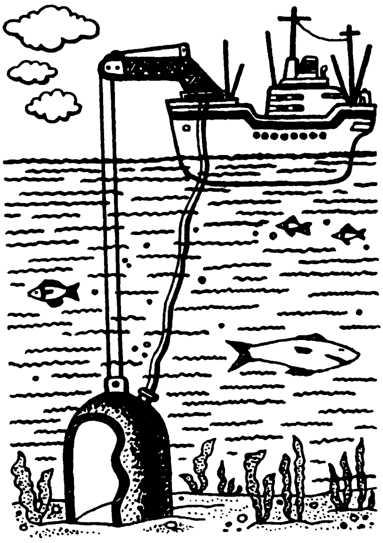

Fig. 256.
Caisson (diving bell).

compressed air delivered to the mask of the diver must be at the same pressure.

A caisson, or diving bell (Fig. 256), also does not protect a diver from high pressures since the air must be compressed so as to prevent water from entering the bell, i.e. to the pressure of surrounding water. For this reason, air is continually pumped into a submerging bell so that the pressure in it should be equal to the water pressure at a given depth. The elevated pressure has a harmful effect on human organisms. This circumstance sets a limit on the depth at which a diver can safely work. The normal depth to which a diver can go in a rubber diving suit is less than 40 m. At this depth, the pressure increases by 4 atm. At lower depths, a diver can work only in a rigid ("armoured") suit which resists the water pressure. In such a suit, the diver can safely work at depths down to 200 m. The air is pumped into the suit at the atmospheric pressure.

If a diver stays under water at a pressure considerably exceeding the atmospheric pressure, a large amount of air is absorbed by the blood and tissues of the body. If he surfaces quickly, the air dissolved in the blood at high pressure evolves from blood in the form of bubbles (like the air dissolved in soda water in a closed bottle under a high pressure, which evolves when the bottle is opened). The bubbles of air in the blood cause acute pain throughout the body and may lead to a serious disease ("caisson disease"). For this reason, a diver who stays for long periods of time deep beneath the sea should surface slowly (sometimes over a period of hours!) so that the gas dissolved in the blood could gradually be desorbed without forming bubbles.

7.22. The Strength of a Submarine

When deep beneath the surface, a submarine is compressed due to the water pressure.

Various engineering constructions have to cope with bulk pressure, but normally the pressure is directed outwards (steam boilers or compressed-air cylinders are examples). An interesting example is the cabin of a manned satellite. The pressure inside the cabin is close to the atmospheric pressure, while the external pressure is equal to zero.

It would appear at first glance that the bulk pressure from outside is similar to the pressure from inside. However, a hollow sphere can withstand a much higher internal pressure than an external pressure. This is because no matter how accurately a sphere is manufactured, there are always some random distortions on its surface. Besides, the quality of the material cannot be exactly the same everywhere. What will happen to surface irregularities when the pressure is increased? For internal pressure, the force is so directed that it tends to smoothen the irregularities (Fig. 257*a*). On the contrary, an external pressure can only increase each irregularity (Fig. 257*b*). At a sufficiently large external pressure, any random irregularity starts to increase and causes failure.

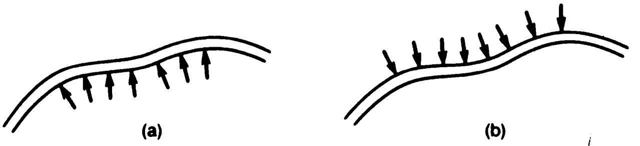

Fig. 257.
(*a*) An irregularity is smoothened under an internal pressure. (*b*) An external pressure deepens the irregularity.

Thus, a sphere turns out to be stable to internal pressure and unstable to external pressure, just like a thin rod is stable to extension and unstable to compression. A similar situation is observed for a cigar-shaped submarine. Its steel sheeting is very strong, but the casing as a whole may be unstable to a high external pressure. Cases are known when a submarine went below its safety limit and its casing was crushed by external pressure although the submarine shell could have withstood the same pressure had it been applied from inside.

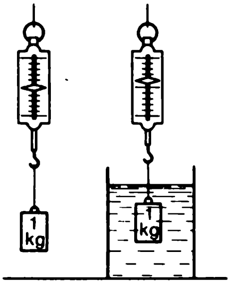

Fig. 258.
If a load is submerged in water, the reading of a spring balance becomes smaller.

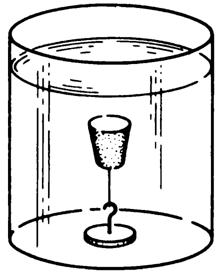

Fig. 259.
A cork submerged in water stretches the string.

7.23. Archimedes' Principle

As is well known, forces of pressure are exerted on the surface of a solid submerged into a liquid. Since the pressure increases with the depth, the forces of pressure acting on the lower part of the body and directed upwards are stronger than the downward forces exerted on its upper part, and we can expect that the resultant of the forces of pressure will be directed upwards. Experiments confirm this hypothesis. If, for example, we immerse in water a load suspended on a hook of a spring balance, the reading of the balance becomes smaller (Fig. 258).

The resultant of the forces of pressure exerted on a body submerged in a liquid is called the *buoyant force,* or *buoyancy.* The buoyant force can be stronger than the force of gravity acting on a body. For example, a piece of cork tied to the bottom of a vessel filled with water tends to float up and stretches the string (Fig. 259). A buoyancy appears also when a body is partially submerged in a liquid. A piece of wood floating on the surface of water does not sink just due to the presence of an upward buoyant force.

If a body placed in a liquid is left to itself, it either sinks, or remains in equilibrium, or floats up to the surface of the liquid depending on whether the buoyancy is less than the force of gravity, equal to it, or is larger. The buoyant force depends on the kind of a liquid in which the body is placed. For example, a piece of iron sinks in water but floats in mercury. This means that the buoyancy exerted on it is less than the force of gravity in water and larger than the force of gravity in mercury.

Let us find the buoyant force exerted on a solid immersed in a liquid.

The buoyant force acting on a body (Fig. 260*a*) is the resultant of the forces of pressure exerted on its surface. Let us suppose that the body is removed and its place is occupied by the same liquid (Fig. 260*b*). The pressure on the surface of such a mentally isolated volume will be the same

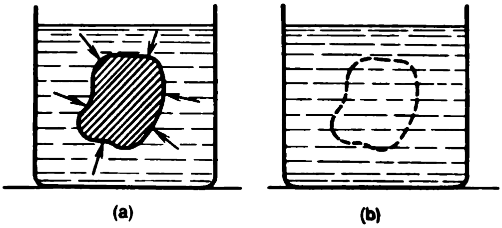

Fig. 260.
(*a*) A body is in a liquid. (*b*) The body is replaced by the liquid.

as the pressure which was exerted on the surface of the body. Consequently, the resultant force of pressure on the body (buoyant force) is equal to the resultant of the forces of pressure on the isolated volume of the liquid. But the isolated volume is in equilibrium. The forces acting on it are the force of gravity **P** and the buoyant force **F** (see Fig. 261*a*). Hence, *the buoyant force is equal in magnitude to the force of gravity acting on the isolated volume of the liquid and is directed upwards.* The point of application of this force must be the *centre of gravity of the isolated volume.* Otherwise, the equilibrium would be violated since the force of gravity and the buoyant force would form a couple (see Fig. 261*b*). But as was mentioned above, the buoyant force for the isolated volume coincides with the buoyant force exerted on the body. Thus, we arrive at Archimedes' principle.

The buoyant force acting on a body submerged in a liquid is equal in magnitude to the force of gravity acting on the liquid in the volume occupied by the body (displaced volume), is directed upwards and is applied at the centre of gravity of this volume. [7] The centre of gravity of the displaced volume is known as the *centre of pressure.*

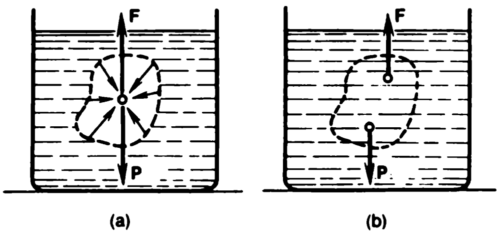

Fig. 261.
(*a*) The resultant of the forces of pressure on the surface of a submerged body is equal to the force of gravity acting on the liquid whose volume is equal to the volume of the body. (*b*) If the point of application of the resultant were other than the centre of gravity of the displaced volume of the liquid, a force couple would appear, and the equilibrium of this volume would be violated.

[7] Since the force of gravity acting on a body has the same magnitude and direction as the weight of this body (it is assumed that the body has no acceleration in the vertical direction), Archimedes' principle can be formulated as follows: *a body submerged in a liquid experiences the action of a buoyant force equal to the weight of the displaced liquid, directed vertically upwards and applied at the centre of gravity of the displaced volume. — Eds.*

The buoyant force exerted on a body of a simple shape can be calculated from the analysis of the forces of pressure acting on its surface. Suppose that, for example, a body submerged in a liquid has the shape of a right parallelepiped and arranged so that its two opposite faces are horizontal (Fig. 262). The area of its base will be denoted by S, the height by H, and the distance from the liquid surface to the upper face by h.

The resultant of the forces of pressure exerted by the liquid is composed of the forces of pressure acting on the lateral surface of the parallelepiped and on its top and base. The forces acting on the lateral surface are balanced since the forces of pressure acting on the opposite faces are equal and opposite. The pressure on the top is ρgh, while the pressure on the base is $\rho g(H + h)$. Consequently, the forces of pressure exerted on the top and base are given by

$$F_1 = \rho ghS \quad \text{and} \quad F_2 = \rho g(h + H)S,$$

the force F_1 being directed downwards and the force F_2 upwards. Thus, the resultant F of all the forces of pressure on the surface of the parallelepiped (the buoyant force) is equal to the difference of the magnitudes of forces F_2 and F_1:

$$F = F_2 - F_1 = \rho g(h + H)S - \rho ghS = \rho gHS,$$

and directed vertically upwards. But HS is the volume of the parallelepiped, while ρHS is the weight of the liquid displaced by the body. This means that the buoyant force is indeed equal in magnitude to the force of gravity acting on the displaced volume of the liquid.

If we immerse a body suspended from a spring balance into a liquid, the balance shows the difference between the weight of the body and the buoyant force, i.e. the weight of the displaced liquid. Therefore, Ar-

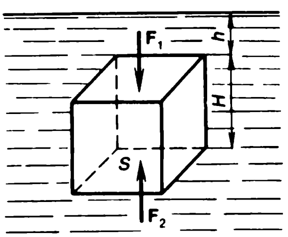

Fig. 262.
To the calculation of the buoyant force.

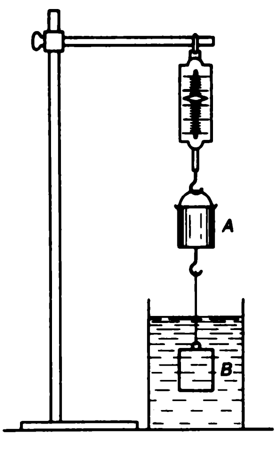

Fig. 263.
Experimental verification of Archimedes' principle with the help of "Archimedes' bucket".

chimedes' principle is sometimes formulated like that: ***a body immersed in a liquid loses in weight as much as the weight of the displaced liquid.***

In order to prove the validity of this conclusion, we can make the following experiment (Fig. 263). An empty bucket A ("Archimedes' bucket") and a solid cylinder B having the volume exactly equal to the volume of the bucket are suspended from a spring balance. We take a glass with water and immerse the cylinder into it. The equilibrium will be violated, and the extension of the spring balance will be reduced. If we fill the bucket with water, the spring balance will be stretched to the previous extent. The loss in the weight of the cylinder is exactly equal to the weight of water occupying a volume equal to that of the cylinder.

According to the law of action and reaction, the buoyant force exerted on a body by the liquid into which it is immersed corresponds to the force exerted by the body on the liquid. This force has the downward direction and is equal to the weight of the liquid displaced by the body. This is illustrated by the following experiment (Fig. 264). A half-filled glass with water is balanced on a beam balance. Then a body suspended from a holder is immersed in the glass. The pan with the glass moves down, and to restore the equilibrium a load must be added to the other pan, the weight of the load being equal to the weight of water displaced by the body.

?

7.23.1. Determine the buoyancy acting on a stone having a mass of 3 kg and submerged in water if the density of the stone is 2.4×10^3 kg/m^3.

7.23.2. A cube with an edge of 100 mm is immersed in a vessel with water above which kerosene is poured so that the interface between the two liquids is at the middle of the cube edge. Find the buoyant force acting on the cube if the density of kerosene is 0.81×10^3 kg/m^3.

7.23.3. A piece of cork having a mass 10 g and wound by a copper wire with a cross-sectional area of 1 mm^2 is in equilibrium in water (it does not sink or float up) (see Table 1). Determine the length of the wire.

7.23.4. What will happen to a beam balance which has been in equilibrium after a finger is immersed into a glass on its pan without touching the bottom or walls of the glass?

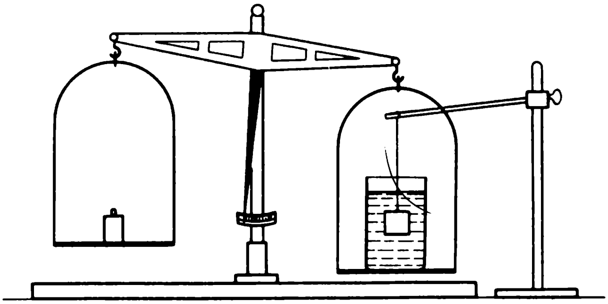

Fig. 264.
The weight of the load that must be put on the left pan of the beam balance is equal to the weight of water displaced by the body.

7.23.5. A piece of copper and a piece of iron having a mass of 500 g each are suspended on strings from the pans of a beam balance (see Table 1). Will the equilibrium be violated if copper is immersed in water and iron in kerosene whose density is 0.81×10^3 kg/m^3? What must be the weight of the load required to restore equilibrium? On what pan should it be placed?

7.24. Measurement of Density of Bodies on the Basis of Archimedes' Principle

In order to determine the density ρ of a homogeneous body of irregular shape, whose volume is difficult to determine by measuring the dimensions of the body, we can proceed as follows.

The body is weighed on a balance twice: once in an ordinary way, and then after the body is immersed in a liquid whose density ρ_0 is known. The first weighing gives the weight of the body G which is equal to $\rho g V$ (ρ is the density of the body and V is its volume). The result G' of the second weighing gives the difference between the weight G of the body and the buoyant force F:

$$G' = G - F. \tag{7.24.1}$$

According to Archimedes' principle, $F = \rho_0 g V$. Replacing V in this equality by $G/\rho g$, we obtain $F = (\rho_0/\rho)G$. Substituting this expression into formula (7.24.1), we arrive at

$$G' = G - (\rho_0/\rho)G,$$

whence

$$\rho = \rho_0 G/(G - G'). \tag{7.24.2}$$

If the body is inhomogeneous, the quantity ρ defined by this formula is the average density of the body.

?

7.24.1. Determine the density of a stone if its weight in air is 3.2 N and in water 1.8 N.
7.24.2. How can the density ρ of a liquid be determined if we know that the weight of a body is G in air, G_1 in water, and G_2 in the liquid under investigation?
7.24.3. A piece of copper weighs 4.00 N in air and 3.59 N when immersed in a liquid. Determine the density of the liquid if the density of copper is 8.9×10^3 kg/m^3.
7.24.4. A piece of cork has a weight of 0.15 N in air, while the weight of a piece of lead is 1.14 N. If we tie them together, suspend from a spring balance and then immerse in kerosene, the reading of the balance will be 0.70 N. Determine the density of the cork, assuming that the density of lead is 11.4×10^3 kg/m^3, while the density of kerosene is 0.81×10^3 kg/m^3.

7.25. Floatation of Bodies

Archimedes' principle makes it possible to solve all problems associated with floatation of bodies.

Suppose that a body is put in a liquid and let to itself. If the weight of the body is larger than the weight of the liquid displaced by it, it will sink, viz. move down until it falls to the bottom of the vessel. If the weight of the body is less than the weight of the displaced liquid, it will float up, rising to the surface of the liquid. And only if the weight of the body is exactly equal to the weight of the displaced liquid, will it be in equilibrium inside the liquid. For instance, a hen's egg sinks in fresh water but floats in salt water. We can prepare a salt solution whose concentration is gradually decreased in the upward direction so that the buoyancy is larger than the weight of the egg at the bottom of the vessel and less than its weight at the surface. In such a solution, the egg floats at a depth where its weight is exactly equal to the buoyant force.

If a solid is homogeneous, i.e. has the same density at all its points, it will sink, float up, or remain in equilibrium inside a liquid depending on whether its density is higher than, lower than, or equal to the density of the liquid. For inhomogeneous bodies, the density of the liquid must be compared with the average density of the body.

If the weight of a body immersed in a liquid is less than the weight of the liquid occupying a volume equal to that of the body, it floats up. Having risen to the surface, it floats so that a part of it remains above the surface of the liquid. Bodies of different densities float in a liquid with different fractions of their volume being submerged (Fig. 265). This is due to the fact that if a floating body is in equilibrium, the weight of the displaced volume of the liquid (in this case, the volume of the part of the body below the free surface of the liquid) must be equal to the weight of the body. Therefore, a body whose density is only slightly smaller than the density of the liquid (an ice cube in water) floats so that it is submerged considerably. Only with a complete submergence of such a body the buoyancy becomes

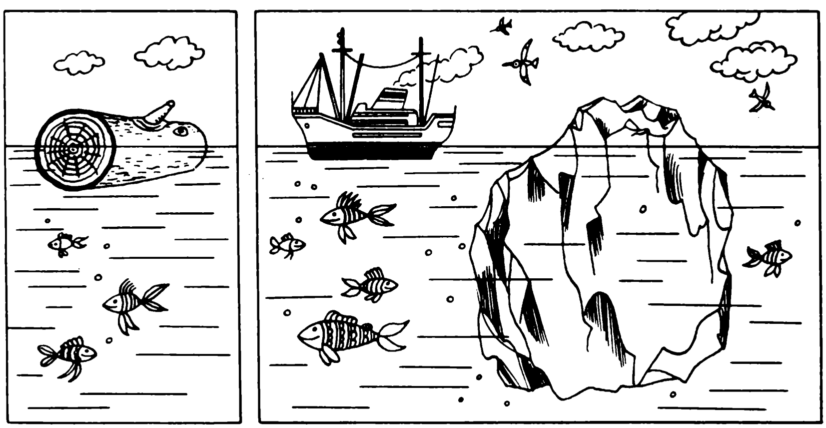

Fig. 265.
A block of ice floats so that its larger part is submerged. A floating pine wood is submerged only to a half.

equal to the weight of the body. If, however, the density of the body is considerably less than the density of the liquid, only small fraction of the body is submerged.

This can be verified with the help of a beam balance. Instead of one pan, we suspend a bucket filled with water to the brim and balance it with weights. Then we put a piece of wood in the bucket so that it floats freely without touching the bottom. Some water displaced by the wood will flow out of the bucket, but the equilibrium will not be violated. Consequently, the weight of the displaced water is equal to the weight of the floating piece of wood. In ship building, the weight of water displaced by a ship is called the *displacement*. The displacement is obviously equal to the weight of the ship. When the ship is loaded, it is submerged more in water, and its displacement increases by the value equal to the weight of the load.

The law of floatation is used in *hydrometers*. A hydrometer is a glass vessel with a load at the bottom and a long graduated spout at the top (Fig. 266). The hydrometer floating in a liquid is submerged to a smaller or larger extent depending on the density of the liquid. The higher the density of the liquid, the smaller the depth of submergence. The scale is calibrated to read the values of density directly. Thus, the marks on the scale correspond to the values increasing in the upward direction.

Hydrometers are used for precise measurements of density in liquids with close densities (say, in solutions with different concentrations). A high precision of measurements is attained by making the spout with the scale very thin. Then the slightest variation in density brings about a noticeable change in the depth to which the hydrometer is submerged.

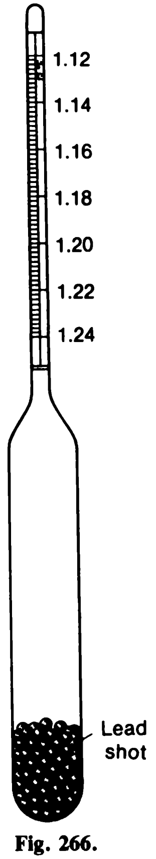

Fig. 266.
Hydrometer (the scale is calibrated to read 10^3 kg/m^3).

?

7.25.1. Where is the draught of a ship with the same load larger, in the sea or in a river?

7.25.2. An ice cube floats in a glass filled with water. What will be the change in the level of water after the ice has melted?

7.25.3. A bucket filled with water to the brim is suspended from a spring balance. If a piece of iron suspended on a string is immersed in the bucket, some water will flow out of it. Will the reading of the balance change?

7.25.4. What part of the volume of an oak log is under the surface of water if the density of oak is 0.8×10^3 kg/m^3?

7.25.5. A steel ball floats in mercury. What part of the ball is above the surface? Will the position of the ball change if water is poured above mercury? The densities of steel and mercury are 7.8×10^3 kg/m^3 and 13.6×10^3 kg/m^3 respectively.

7.25.6. An ice block having the shape of a prism floats in water so that 2 cm of its height are above the water surface. What is the mass of the ice block if the area of its base is 2000 cm^2? The density of ice is 0.92×10^3 kg/m^3.

7.25.7. A homogeneous body floats on the surface of alcohol so that the volume of its submerged part amounts to 0.92 of the entire volume of the body. Find the volume of the submerged part when this body floats in (a) water, (b) mercury. The density of alcohol is 0.80×10^3 kg/m^3.

7.26. Floatation of Hollow Bodies

A body with cavities and impervious to liquid in which the body floats displaces the same volume as would a homogeneous solid. Therefore, the buoyancy for such a body is the same as for the homogeneous body. However, the mass of a hollow body is less than the mass of a homogeneous body of the same volume. Therefore, if cavities are big enough, such a body can float even when the density of the material of the body is higher than the density of the liquid. The displaced volume turns out to be larger than the volume occupied by the material of the body. A steel ship displaces the volume of water exceeding many times the volume of steel from which the hull of the ship is made. Therefore, it can float (possess buoyancy) in spite of the fact that the density of steel is about eight times higher than the density of water. If the space inside a ship is filled with water when, for example, the ship is leaking, the displaced volume becomes smaller, and the ship starts to sink.

In order to ensure safe navigation, the possibility of emergence of holes in the hull should be envisaged. The entire inner volume of a ship is separated by a number of steel partitions into watertight compartments. If a hole in the hull appears or leakage occurs, only one compartment is filled with water, and the ship continues to float, although it is now submerged to a larger depth (Fig. 267).

Submarines are a special type of ships. They must be able to surface and submerge, and also move under the water surface. Since the volume of

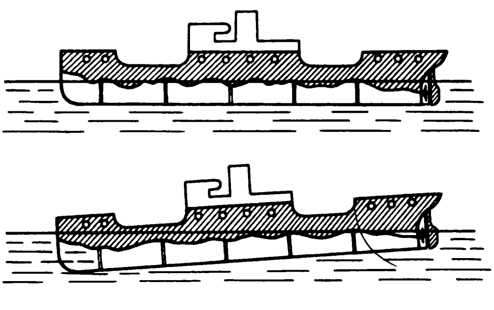

Fig. 267.
If a compartment is filled with water, the ship does not sink but its larger part is submerged.

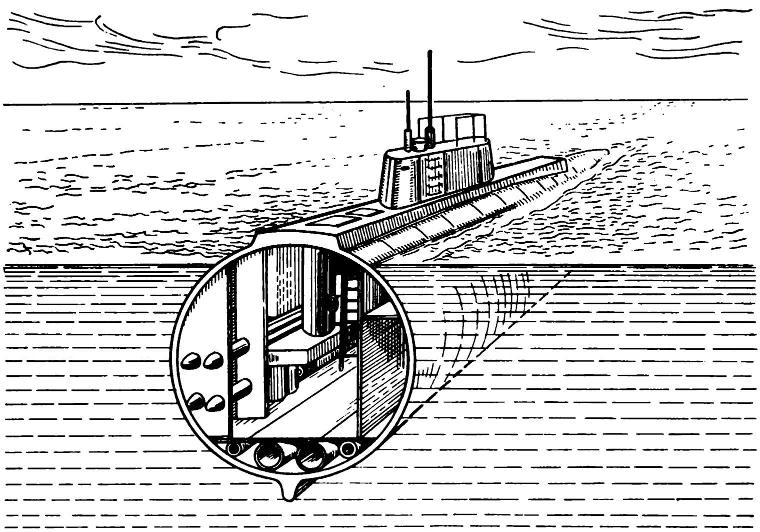

Fig. 268.
Submarine.

a submarine remains unchanged in all cases, special facilities should be available for changing its mass during such maneuvers. This is done with the help of a number of ballast tanks in the hull of the submarine (Fig. 268). Using special devices, these tanks can be filled with water (the mass of the submarine then increases and it submerges) or water can be pumped out of them (then the mass of the submarine decreases, and it surfaces).

It should be noted that a small excess or deficiency of water in the ballast tanks is sufficient for a submarine to be submerged to the very bottom of the sea or to surface. It often happens that in a certain layer of water the density rapidly varies with the depth, increasing in the downward direction. In the vicinity of such a layer, the equilibrium of a submarine is stable. Indeed, if the submarine moving on this level submerges slightly deeper for some reason, it gets into the region of a higher density. The buoyancy will increase, and the submarine floats up, returning to the initial level. If, however, the submarine slightly rises for some reason or other, it gets into the region of a lower density where its buoyancy decreases, and the submarine returns to the initial level. For this reason, submariners call such layers a "liquid ground": a submarine can "lay" on this ground, retaining its equilibrium for an infinitely long time, while in a homogeneous medium this cannot be done, and to stay at the same depth, the submarine must either change the amount of ballast by taking in or forcing out water from the ballast tanks, or maneuver all the time with its elevators.

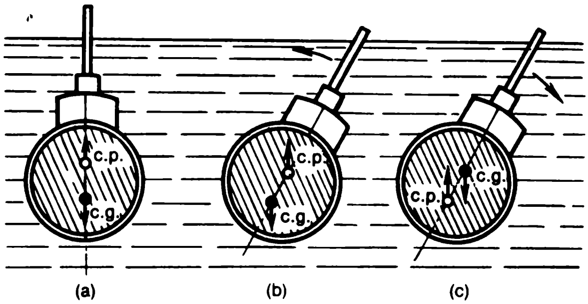

Fig. 269.
The stability of a submerged submarine (c.g. denotes the centre of gravity of the submarine, and c.p. is its centre of pressure).

7.27. Stability of Floating Ships

The question of the stabilty of equilibrium is essential for ships and submarines. It is well known that a ship may capsize if its load is distributed in a wrong way. The question of stability is a question of safety of navigation.

Let us consider the stability of equilibrium of a body under the water surface (say, a submarine). Suppose that the centre of pressure is above the centre of gravity of the submarine. In the normal position, the centre of gravity and centre of pressure lie on the same vertical, and the submarine is in equilibrium (Fig. 269*a*). If the submarine is tilted (Fig. 269*b*), the force of gravity and the buoyant force form a couple which returns the submarine to the initial position. Thus, the equilibrium is stable. If the centre of pressure lay below the centre of gravity, the equilibrium of the submarine would be unstable. Indeed, in this case any deviation from a strictly vertical position leads to the formation of a couple of the force of gravity and buoyancy which would turn the submarine in the same direction away from the equilibrium position (Fig. 269*c*).

Finally, if the centre of gravity coincides with the centre of pressure, the equilibrium is neutral. These cases are completely similar to various cases of equilibrium of a solid suspended at a point. *The centre of pressure plays the role of the point of suspension.*

For a body floating on the surface of a liquid (Fig. 270), the stability conditions will be quite different since tilting a body (say, a ship) causes a change in the shape of the displaced volume, and hence the position of the centre of pressure relative to the ship. If, for example,

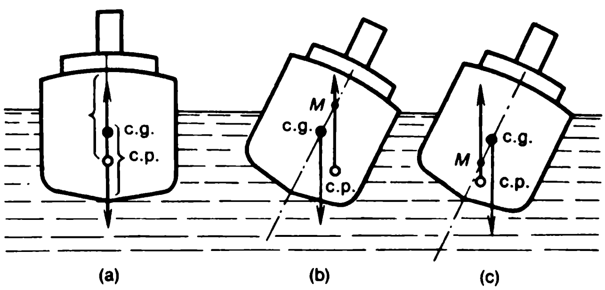

Fig. 270.
Stability of a ship (c.g. is the centre of gravity, c.p. is the centre of pressure and M is the metacentre).

the ship is inclined to the right, the larger fraction of the displaced water will be to the right of the midline of the ship, and hence the centre of pressure will be shifted in the same direction. It can be seen from the figure that now the stability of equilibrium depends on the mutual position of the centre of pressure and the centre of gravity of the inclined ship. If point M of the intersection of the vertical passing through the centre of pressure with the midline of the ship (the so-called *metacentre*) lies above the centre of gravity (see Fig. 270*b*), the couple formed by the force of gravity and buoyancy returns the ship to equilibrium position, and hence the equilibrium is stable. If, however, the metacentre lies below the centre of gravity (Fig. 270*c*), the equilibrium is unstable. Here *the metacentre plays the role of the point of suspension,* and the equilibrium may be stable in spite of the fact that the centre of pressure is below the centre of gravity of the ship. It should be noted that the position of the metacentre varies with the angle of inclination of a floating body.

The distance between the centre of gravity and metacentre is known as the *metacentric height*. The larger the metacentric height, the higher the stability of a ship and the sooner the ship after having been brought out of the upright position by external forces (by wind or a wave) returns to this position. For a sailboat, it is especially important to have a large enough metacentric height since the forces acting on the sails produce a large overturning moment. For this reason, some types of sailboats with high masts and large surfaces of sails (yachts) have a ballast at the bottom which lowers the centre of gravity and thus increases the metacentric height. In empty cargo vessels, a ballast is often put on the bottom in order to lower the centre of gravity. It is known that a heavy load is normally not put on the upper deck of cargo vessels since with such a loading, the position of the centre of gravity becomes higher, which reduces the metacentric height and hence deteriorates the stability of the vessel.

7.28. Rising of Bubbles to the Surface

A bubble of gas formed in sea depths (e.g. an air bubble released by a diver from the helmet of his diving suit) rises since the buoyant force equal to the weight of water in the volume occupied by the bubble is considerably larger than the weight of the gas in the bubble. Moving up, the bubble goes over to the layers of water under a lower pressure. It expands, the buoyancy increases, and the velocity of its ascent grows.

If for some reason or other the weight of a diver in the diving suit turns out to be less than the weight of displaced water (if, for example, he did not release in time the air pumped into the diving suit and the volume of the suit has increased), the diver starts to float up, and his rubber suit filled with compressed air will inflate like a rising bubble and will bring him to the surface.

7.29. Bodies Lying on the Bottom of a Vessel

The following experiment illustrates a seeming contradiction to Archimedes' principle (Fig. 271). The bottom of a glass vessel is covered by a thin layer of paraffin. We put a piece of paraffin with a smooth base on the bottom and carefully pour water into the vessel. The piece of paraffin will

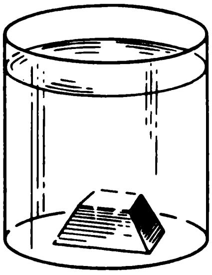

Fig. 271.
A piece of paraffin lying on the bottom of a vessel filled with water does not rise to the surface.

not float up to the surface of water although its density is lower than the density of water. If we slightly tilt the vessel, we can make the paraffin slide over the bottom, but it does not rise to the surface.

The explanation to this paradox is as follows. Since paraffin is not wetted by water (Sec. 14.6), water does not penetrate between the piece of paraffin and the bottom of the vessel, and hence the forces of water pressure do not act on the lower surface of the piece of paraffin. On the other hand, the forces of pressure on the upper surface of the body press it against the bottom. If we tilt the piece of paraffin so that water penetrates under its lower surface, the buoyancy will emerge, and the paraffin will float up. It is known that a submarine lying on the soft bottom of the sea sometimes cannot be separated from it even if its ballast tanks are emptied. This is also explained by the fact that water cannot penetrate under the hull of the submarine which tightly fits the bottom.

Chapter 8
Aerostatics

8.1. Mechanical Properties of Gases

In many respects, mechanical properties of gases are similar to properties of liquids. Gases, like liquids, are extremely mobile and do not possess elasticity relative to the change in shape. However, gases are elastic with respect to the change in volume: the forces of pressure in a gas are its elastic forces. The more a gas is compressed, the stronger the force of pressure exerted by it on the surrounding bodies. The forces of pressure in a gas at rest, like in a liquid, are always perpendicular to the surfaces of bodies in contact with it.

The *gas pressure* is defined, as for liquids (Sec. 7.7), as the ratio of the force of pressure exerted on an element of the surface in contact with the gas to the area of this element. Like in liquids, gas pressure at a given point does not depend on the orientation of the surface element on which it acts. The same Pascal's principle is valid for gases: *pressure created by surface forces is transmitted without loss to all other points of a gas.*

However, there is a considerable difference in the mechanical properties of gases and liquids. Under normal conditions, the density of gases is thousand times lower than the density of liquids. For example, the mass of a cubic metre of air amounts to only 1.3 kg, while the mass of a cubic metre of water is equal to a tonne.

Usually, the mass of certain volumes of a gas is underestimated. It should be noted that the mass of breathed-in air passing through the lungs of a human organism amounts to 20-30 kg per day. The mass of air in a small room is 30-40 kg. An electric locomotive carries 2 tonnes of air in the carriages of a train.

A very important difference in the properties of gases and liquids is that gases have no independent volume. We can fill half a glass with water, but a gas always occupies the entire vessel containing it. There is no limit to the increase in volume of a given mass of a gas unless the force of gravity acts on it or a limit to its expansion is set by the vessel walls. Therefore, gases never form a free surface.

Next, gases can be compressed thousands of times more than liquids.

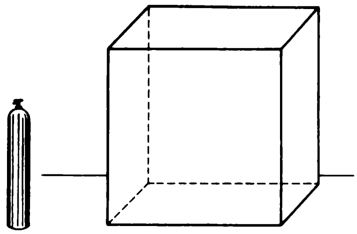

Fig. 272.
A cylinder containing compressed oxygen and the volume occupied by the same mass of oxygen released to the atmosphere.

The density of a liquid varies insignificantly even under a very high pressure. On the contrary, we can compress a gas and thus considerably increase its density by a comparatively low pressure. It will be shown later (Sec. 13.9) that the pressure of a gas increases or decreases as a result of its compression or expansion in the same proportion as its density (provided that the gas temperature remains unchanged).

Air which has occupied a large volume in the atmosphere can be easily compressed by a hand pump to a quarter of its initial volume (e.g. in a tyre), i.e. the density and pressure of the air can be increased fourfold in comparison with the atmospheric air. In oxygen cylinders used in autogenous cutting and welding, oxygen is compressed to a pressure of 150 atm. The density of the gas also turns out to be increased 150-fold, i.e. approximately to the density of cork. If all the gas is released from such a cylinder to the atmosphere, it will occupy a volume exceeding the volume of the cylinder 150 times (Fig. 272). On the other hand, water compressed to a pressure of 150 atm would increase its density only by 0.75% (being released from the vessel, it would increase its volume by the same fraction).

Thus, the density of gases, unlike that of liquids, cannot be treated as independent of pressure.

8.2. Atmosphere

The most important gas for living organisms is air. The Earth is surrounded by the atmosphere, viz. the layer of air which is a mixture of a number of gases (nitrogen, oxygen, argon, carbon dioxide, water vapour, and other gases). In the subsequent discussion, however, we shall not take into account the fact that air has a complex composition since this is not important for mechanical phenomena we are interested in.

The atmosphere is kept near the surface of the Earth by the force of attraction. If the Earth did not attract air, the entire atmosphere would expand and dissipate in the surrounding space. The total mass of the atmosphere is equal to about 5×10^{18} kg.

The density of air can be determined in the following way. We shall pump air out of a flask and balance it on a sensitive beam balance

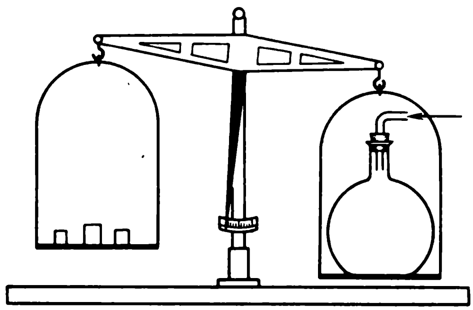

Fig. 273.
Weighing of air.

(Fig. 273). Then we let air in the flask. It can be seen that the pan with the flask will be lowered. To restore the equilibrium, some weights must be added to the other pan. The mass of these weights will be equal to the mass of air that has entered the flask. Knowing the volume of the flask, we can easily determine the air density by dividing the mass of the added weights by the flask volume. The density of dry air is 1.293 kg/m^3 at 0 °C and a pressure of 760 mm Hg.

8.3. Atmospheric Pressure

The pressure of air near the surface of the Earth is due to its own weight. It is compressed by this weight like water in the sea depths. Near the surface of the Earth (to be more precise, at sea level), the air pressure is approximately equal to one atmosphere, i.e. 10^5 Pa. Consequently, each square metre of the surface of the Earth experiences the force of air pressure of 10^5 N. The area of the Earth's surface is about 5×10^{14} m^2. Thus, air acts on the surface of the Earth with a pressure force of 5×10^{19} N. If the density of air at any altitude were the same as at the surface of the Earth, the thickness of the atmosphere would be 8 km. Actually, the air density rapidly decreases with increasing altitude (see Sec. 8.9) so that the atmosphere spreads over hundreds of kilometres (beyond the orbits of the nearest satellites). At this altitude, the air density amounts to a negligible fraction of the density of air near the Earth's surface.

A natural question arises: why do not we feel the atmospheric pressure?

To answer this question, let us analyse the following simple experiments. Let us take a glass jar and stretch a thin rubber membrane over it. Although a force of 10 N acts on every square centimetre of the surface of the membrane (which amounts to the force of the order of hundreds of newtons over the entire membrane), the membrane is perfectly flat. As a matter of fact, the air inside the jar is compressed to the same extent as the outer air: the same forces act on the inner and outer surfaces of the membrane so that two forces are mutually balanced, and the membrane remains

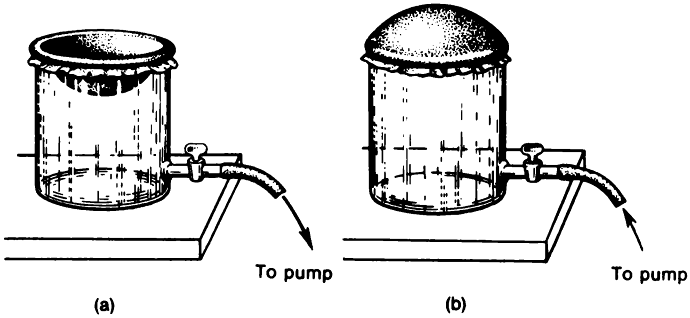

Fig. 274.
(*a*) As air is being pumped out of the vessel, the membrane is sucked into the jar. (*b*) As air is being pumped into the jar, the membrane bulges outwards.

flat as if no forces acted on it. If, however, we partially pump out air from the jar through a side spout, thus decreasing its pressure, the membrane is sucked in (Fig. 274*a*). It will be sucked in to such an extent that elastic forces emerging in the membrane plus the force of pressure of the air remaining in the jar exactly balance the pressure of surrounding air. On the contrary, if some air is pumped into the jar, the membrane bulges out (Fig. 274*b*).

This experiment becomes even more illustrative if we put the jar from which air is partially pumped out under the bell of an air pump. Initially, the membrane stretched over the jar is sucked in. If we now pump air out of the bell, the membrane will be first straightened out, and as we continue the evacuation of the bell, it will bulge out (Fig. 275). Thus, the deformation (sucking in and bulging out of the membrane) is observed only when air pressure is different on both sides. If pressures are identical, the membrane remains flat.

It becomes clear now why human beings and animals do not perceive the atmospheric pressure. Tissues, blood vessels and the walls of other cavities of a living organism are subject to the external atmospheric pressure. However, blood and other liquids and gases filling these cavities

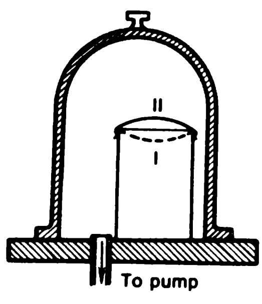

Fig. 275.
As air is pumped out of the bell, the membrane covering the jar changes from position *I* to position *II*.

are compressed to the same pressure. Therefore, the elastic walls of an artery are subject to the same pressure from inside and outside and hence are not deformed.

A similar mutual balance of forces of pressure also takes place in a liquid, which can be observed for deep-sea fishes. It is known that some fishes can live at a depth of several kilometres from the surface of the ocean, where the pressure of surrounding water reaches hundreds of atmospheres. However, each cell of the tissues of such fishes contains gas and liquid compressed to the same pressure, and therefore none of the parts of their bodies experiences unilateral forces which could produce a harmful effect. Sometimes, it is possible to catch such fishes in sea depths with special nets suspended on a long rope. The inner cavities of such fishes lifted to the surface are always torn from inside. In water layers close to the surface of the sea, where the outer pressure is lower, the gases dissolved in blood and protoplasma of cells are evolved and tear the tissues of the fishes by their high pressure (cf. Sec. 7.21).

? **8.3.1.** Why do we ascribe the destruction of tissues to the liberation of gases from liquids and not to the liquid pressure itself?

8.4. Other Experiments Confirming the Existence of the Atmospheric Pressure

Let us cover a glass jar with polished brims by a thin glass plate and start to pump air out of the cylinder (Fig. 276). [1] The glass plate will be tightly pressed to the jar by external atmospheric pressure. If we continue the evacuation, the plate will be crushed by the difference in the external and the internal pressure.

One of the first experiments carried out to prove the existence of air pressure was the famous experiment with Magdeburg hemispheres made

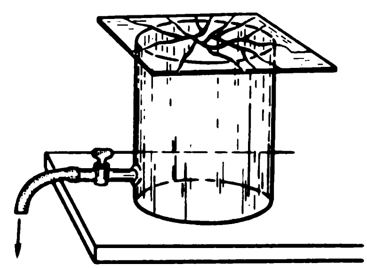

Fig. 276.
The excess of external pressure over the internal pressure crushes the glass plate.

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[1] The brims of the jar should be well greased to prevent the outer air from penetrating inside the jar.

Fig. 277.
An engraving from Guericke's book *New Magdeburg Experiments*. Tearing apart of hemispheres by two teams of horses.

by the German physicist Otto von Guericke in 1654 (in Magdeburg). He pumped air from the space between two copper hemispheres brought in contact. The pressure of the outer air pressed these hemispheres so tightly to each other that they could not be torn apart by two teams of eight horses each (Fig. 277). Naturally, the role of the other team could be played by a pole to which one of the hemispheres could be fixed. Figure 278 illustrates a modification of Guericke's experiment with a suspended load.

In medicine, pneumatic cupping-glasses are sometimes used, which consist of glass cups with rubber bulbs (Fig. 279). Compressing a bulb by the hand, we push out air of it. If we press the cupping-glass against the skin and release the bulb, it will acquire the original spherical shape due to its elasticity. The inner volume of the cupping-glass will increase, and the pressure of air remaining in the cupping-glass will drop. The cupping-glass will be tightly pressed against the skin by the pressure of the outer air. The skin under the cup becomes red, and a bruise appears. The blood which is under the atmospheric pressure in the body flows to the site where the pressure is lowered. The purpose of cupping-glasses is just to create a local influx of blood. As a consequence, the air dissolved in blood expands when the pressure drops, and destroys small blood vessels, causing the appearance of bruises.

If we press the skin near the edge of a cup and let the outer air in, the air pressures inside and outside equalise, and the cup falls off.

Fig. 278.
An engraving from Guericke's book *New Magdeburg Experiments*. Tearing apart of hemispheres by a suspended load.

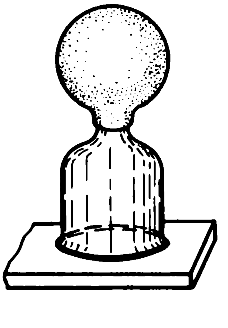

Fig. 279.
Pneumatic medical cupping-glass.

8.5. Vacuum Pumps

In physics and engineering, it is very important to be able to remove a gas from closed vessels to the highest possible extent (vacuum technology). In other words, physicists and engineers are interested in obtaining a very rarefied gas whose pressure is negligible in comparison with the atmospheric pressure.

To rarefy a gas, a *piston pump* with valves (Fig. 280) can be used. Much more conve-

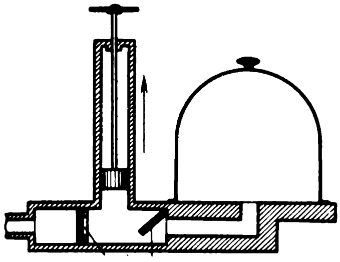

Fig. 280.
Piston air pump.

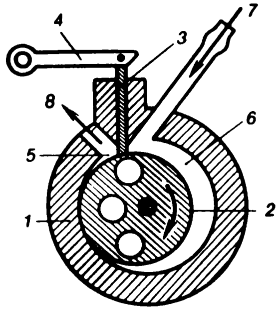

Fig. 281.
Rotary air pump.

nient, however, are the pumps in which the pressure in a chamber being evacuated is lowered not as a result of the reciprocating motion of a piston, but due to rotation. The pumps of this type are known as *rotary pumps* (Fig. 281).

Cylinder *2* rotates in a round metallic shell *1* so that the cylinder axis does not coincide with the rotation axis. A movable plate *3* passing through the slit in shell *1* and connected with a connecting rod *4* is tightly pressed against the cylinder. Plate *3* separates compartments *5* and *6* contained between the plate, the inner wall of shell *1* and the outer surface of cylinder *2*.

When the cylinder rotates in the direction indicated by the arrow in Fig. 281, the volume of compartment *6*, which was initially equal to zero (when the cylinder closed the outlet of channel *7*), increases and air pressure drops in it. A certain amount of air is sucked into the compartment through channel *7* connected to a vessel being evacuated. At the same time, the volume of compartment *5* connected with outlet channel *8* decreases, the pressure in it increases, and air flows out. Thus, as cylinder *2* rotates, new and new portions of air are sucked in through channel *7* and forced out through channel *8*. Since the cylinder rotates at several hundred revolutions per minute (it is normally driven by an electric motor), the pump operates very rapidly. If its parts tightly fit one another, it may reduce the pressure in a vessel being evacuated to 0.001 mm Hg. The regions of contact of the inner surface of shell *1* with plate *3* and cylinder *2* must be thoroughly lubricated. The quality of oil and the system supplying it to the pump essentially determine the pump operation. For this reason, the pumps of this type are often called oil rotary pumps. In order to obtain much higher rarefactions (about 10^{-6} mm Hg), pumps operating on a quite different principle are used at present (the so-called diffusion pump, Sec. 17.18).

8.6. Effect of the Atmospheric Pressure on the Level of Liquid in a Pipe

Let us take a straw or a thin glass tube into the mouth, immerse its other end in water, and suck in air. Water will rise in the tube, and we can easily drink through the straw.

Instead of sucking in air by lungs, we can move up a piston tightly fitting the tube walls. It can be seen that water will rise up with the piston, filling the tube (Fig. 282). Let us fill a bottle with water, close it with a cork, turn the bottle upside down with its neck in water and open the cork (Fig. 283). Water will not flow out of the bottle. Instead of the bottle, we

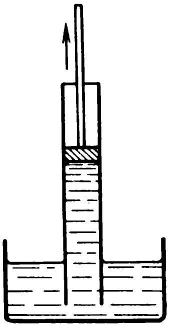

Fig. 282.
Water rises behind the piston.

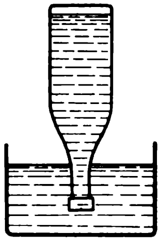

Fig. 283.
Water does not flow out of an open bottle with a neck immersed in water.

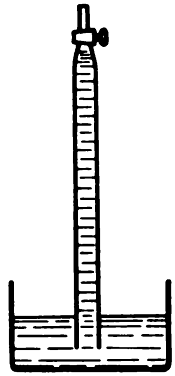

Fig. 284.
As long as the tap is closed, water does not flow out of the tube. When the tap is opened, the level of water in the tube drops to the water level in the vessel.

could take a tube with a tap at the upper part. As long as the tap is closed, water will stay in the tube (Fig. 284). It is sufficient, however, to open the tap of the tube for the water column to drop to the level of water in the vessel. The water will be replaced by air entering through the tap.

All these experiments are explained by the existence of the atmospheric pressure. Indeed, what happens when we suck in air from the tube whose other end is immersed into a vessel with water? Air in the tube turns out to tbe rarefied, and hence the pressure exerted by it on the surface of water in the tube becomes less than the atmospheric pressure. But the atmospheric pressure continues to act on the surface of water in the vessel. The pressure difference makes water rise in the tube. To what height will water rise? The column of water thus formed creates an additional pressure. As soon as this pressure plus the pressure of the air remaining in the tube becomes equal to the atmospheric pressure, water stops rising. The pressure inside the tube at its lower part (on the level of the free surface of water in the vessel) will be exactly equal to the atmospheric pressure, i.e. the well-known equilibrium condition for liquid will be satisfied: the pressure is the same at all points lying in a horizontal plane (see Sec. 7.15). Since we cannot create a high rarefaction of air, we can elevate water by this method only to a small height of about 30-50 cm.

It is also clear why water does not flow out of a bottle turned upside down or from the tube in the above experiments. The air pressure on the surface of water in the vessel presses water against the bottom of the bottle or against the tap of the tube since no air pressure is exerted on water in the

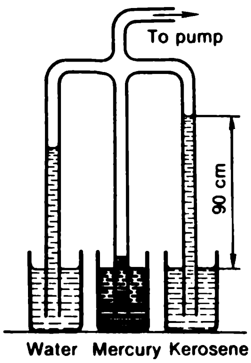

Fig. 285.
To Exercise 8.6.1.

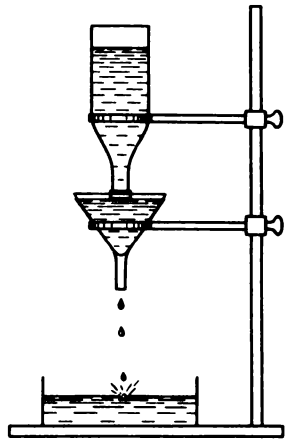

Fig. 286.
To Exercise 8.6.2.

bottle or tube from above. When we open the tap in the tube, the atmospheric pressure is exerted on the upper part of the water column in the tube as well. This column is no longer supported by the pressure difference and drops to the level of water in the vessel.

? **8.6.1.** A branched pipe is connected to a suction pump, the ends of the branch pipes being immersed in glasses with different liquids (Fig. 285). The height of the liquid column in the pipe whose end is immersed in kerosene is 90 cm. Determine the heights of liquid columns in other pipes. The densities of kerosene and mercury are 0.81×10^3 and 13.6×10^3 kg/m^3 respectively.

8.6.2. A device shown schematically in Fig. 286 is used in chemical laboratories for maintaining the level of liquid in a filtering funnel. The level of liquid in the filter is always maintained at a height near the bottle neck so that the filter can operate independently without being watched. Explain the operation principle of the device.

8.7. Maximum Height of a Liquid Column

Let us consider in greater detail the experiment with the piston sucking water into the tube. At the beginning of the experiment (Fig. 287), water in the tube and in the glass is on the same level *MM*, and the piston touches water by its lower surface. Water is pressed against the piston from below by the atmospheric pressure exerted on the surface of water in the glass. The atmospheric pressure also acts on the piston (which is assumed to have no weight) from above. According to the law of action and reaction, the piston, in turn, exerts on the water in the tube a pressure equal to the atmospheric pressure acting on the surface of water in the glass.

Let us now raise the piston to a certain height. For this we have to apply an upward force (Fig. 288*a*). The atmospheric pressure makes the water

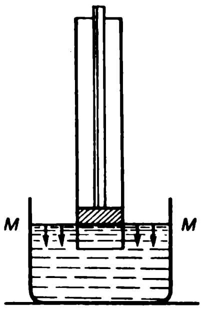

Fig. 287.
Sucking water into the tube. The beginning of the experiment: the piston is at the level of water in the glass.

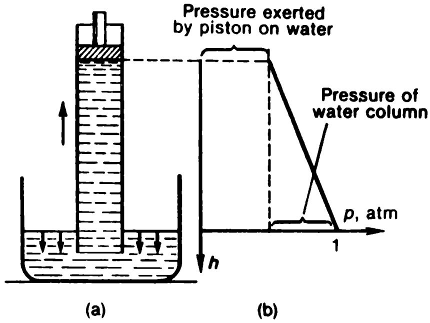

Fig. 288.
(*a*) The second stage of the experiment: the piston is pulled out. (*b*) Pressure graph.

rise behind the piston. Now the water column touches the piston, being pressed against it with a smaller force, i.e. exerting a lower pressure on it than before. Accordingly, the reaction force of pressure exerted by the piston on the water in the tube will be smaller. The atmospheric pressure acting on the surface of water in the glass is balanced by the pressure exerted by the piston plus the pressure created by the water column in the tube.

Figure 288*b* shows the graph of pressure in the water column formed in the tube. If the piston is raised to a larger height, water will follow it, and the water column will become higher. The pressure exerted by the weight of the column will also grow. Consequently, the pressure of the piston on the top of the column will decrease since the sum of these two pressures must, as before, balance the atmospheric pressure. The water will be pressed against the piston with a still smaller force. To keep the piston at a given height, a larger force is required since, as the piston rises, the pressure of water on the lower part of the piston will balance the atmospheric pressure on the upper surface of the piston to a smaller and smaller extent.

What will happen if we take a sufficiently long tube and raise the piston higher and higher? The water pressure on the piston will become lower and lower. Ultimately, the pressure of water on the piston and the pressure of piston on water will vanish. At such a height of the water column, the pressure exerted by the weight of water in the tube will be exactly equal to the atmospheric pressure. The calculations which will be given in the next section show that the height of the water column must be equal to 10.332 m (for the normal atmospheric pressure). If we raise the piston still further, the height of the water column will not increase since the external pressure becomes unable to balance a higher column, and there will be empty space between the lower surface of the piston and water (Fig. 289*a*).

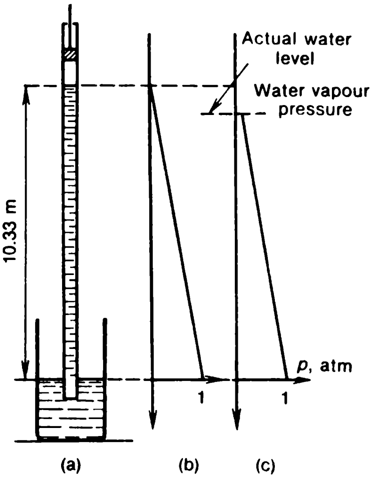

Fig. 289.
(*a*) The same experiment, but the piston is pulled out above the limiting height (10.33 m). (*b*) The pressure graph for this position of the piston. (*c*) In actual practice, the water column does not reach the limiting height since water vapour has a pressure of about 20 mm Hg at room temperature, and the upper level of the water column is hence lowered. For this reason, the actual pressure graph is cut off at the tip. For the sake of clarity, the pressure of water vapour is exaggerated.

Actually, this space will not be completely empty: it will be filled with air liberated from water (water always contains a certain amount of air dissolved in it). Besides, this space will contain water vapour. Therefore, the pressure in the space between the piston and water column will not be exactly equal to zero, and this pressure will somewhat lower the height of the water column (Fig. 289*c*).

This experiment is cumbersome due to the large height of water column. If we repeat this experiment with mercury instead of water, the height of the column will be much smaller. It is much more convenient, however, to use instead of the tube with the piston a device described in the following section.

? 8.7.1. To what maximum height can a suction pump lift mercury in the tube if the atmospheric pressure is equal to 0.93×10^5 Pa?

8.8. Torricelli's Experiment. Mercury Barometer and Aneroid Barometer

The following experiment was proposed in 1643 by the Italian physicist Evangelista Torricelli (1608-1647). A glass tube about 1 m long and sealed at one end is filled with mercury. The open end of the tube is closed with a

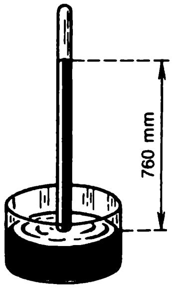

Fig. 290.
Torricellian tube.

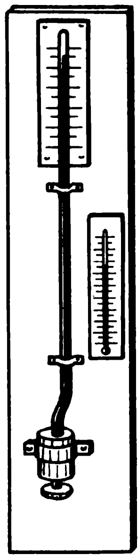

Fig. 291.
Mercury barometer.

finger. The tube is than inverted and immersed vertically in a vessel with mercury. If the finger closing the open end of the tube is then removed, the column of mercury will drop to a height of about 760 mm above the level of mercury in the vessel (Fig. 290).

Following the same line of reasoning as in the previous section, we can easily explain this experiment. The atmospheric pressure acts on the free surface of mercury in the vessel. Since empty space remains in the upper part of the tube after the level of mercury in it has dropped, the pressure of mercury column exerted inside the tube on the level of mercury in the vessel must be equal to the atmospheric pressure. For this reason, the height of the mercury column above the free surface of mercury in millimetres directly indicates the atmospheric pressure in millimetres of mercury column. Thus, Torricellian tube can serve for measuring the atmospheric pressure. It plays the role of a barometer. In actual practice, the design of a mercury barometer is more complicated (Fig. 291).

Thus, experiments show that the atmospheric pressure amounts to about 760 mm Hg. Since 1 mm Hg = 13.6 mm H_2O (see Sec. 7.17), the atmospheric presure is equal to 760×13.6 mm H_2O = 10 332 mm H_2O = 1.013×10^5 Pa. Consequently, the atmospheric pressure is equal to the pressure of a water column whose height exceeds 10 m.

The space above the mercury column in Torricelli's experiment is called *Torricellian vacuum*. Naturally, it is not the perfect vacuum since it contains mercury vapour which slightly lowers the mercury column in the tube

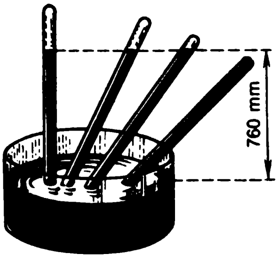

Fig. 292.
If Torricellian tube is inclined, the level of mercury remains at the same height.

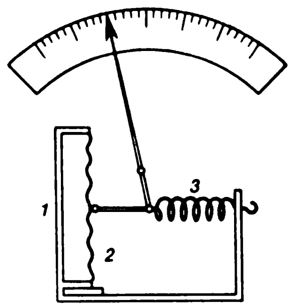

Fig. 293.
Schematic diagram of a membrane manometer for gases.

due to vapour pressure. This, however, can be neglected in practice since the pressure of mercury vapour at room temperature is negligibly low.

Let us incline Torricellian tube at various angles (Fig. 292). It can be seen that the end of the mercury column remains at the same height above the free surface of the mercury at different angles of inclination, although the length of the column increases with the angle formed by the tube with the vertical. This can be explained by the already known fact that the pressure depends only on the height of the column of liquid along the vertical. At a sufficiently large angle of inclination, the entire tube is filled with mercury. This proves that there is no air in the tube. The height of the mercury column in the tube varies with the atmospheric pressure. As the pressure increases, the column becomes longer ("barometer rises"). As the pressure decreases, "barometer falls", viz. the height of the mercury column becomes smaller.

The atmospheric pressure can also be measured with the help of the same *membrane manometer* which was used for liquids (Fig. 293). To improve the precision of measurements, a part of air is evacuated from box *1* of the manometer. Membrane *2* is pulled outwards by spring *3*. The membrane is usually made corrugated to increase its flexibility. Membrane manometers used for measuring the atmospheric pressure are called *aneroid barometers* (Fig. 294). Aneroids are calibrated and checked against a mercury barometer. They are less reliable than mercury barometers since they contain membranes and springs whose elasticity deteriorates with time. On the other hand, aneroids are more convenient than mercury barometers containing the liquid. For this reason, aneroids are widely used when a very high accuracy is not required. If an aneroid is frequently checked against a mercury barometer, its readings are quite reliable.

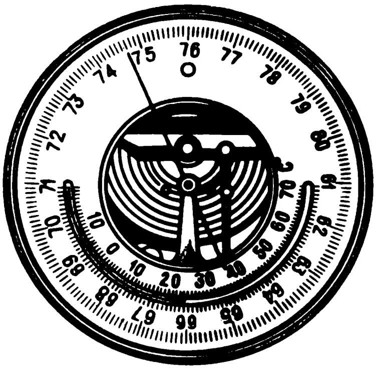

Fig. 294.
Aneroid barometer.

? **8.8.1.** How must the scale of a barometer tube inclined at 60° to the vertical be changed so that the pressure can be read in millimetres of mercury? What must be the length of the tube?

8.8.2. A cylindrical vessel having a mass of 10 kg and an area of the base of 80 cm^2 is covered with a lid. As the air is being pumped out of the vessel, the lid is pressed against the vessel by the atmospheric pressure. If the air is evacuated to 50 mm Hg, what must be the weight of the load suspended from the vessel to tear it off from the lid?

8.9. Distribution of Atmospheric Pressure over Altitude

At the same point on the surface of the Earth, the air pressure does not remain constant but varies depending on various processes occurring in the atmosphere. The "normal" atmospheric pressure is the term conventionally applied to the pressure of 760 mm Hg, i.e. to one (physical) atmosphere (Sec. 7.17).

The air pressure at sea level is on the average close to one atmosphere throughout the globe. Rising above sea level, we can notice that the air pressure drops, and the density of air decreases accordingly. Air becomes more and more rarefied. If a vessel which was tightly closed in a valley is opened at a mountaintop, some of the air will flow out of it. On the contrary, if the vessel tightly closed at a mountaintop is opened at the bottom of the mountain, some of the air will flow into the vessel. At an altitude of about 6 km, the pressure and density of air decrease to about a half of their initial values.

To each altitude, there corresponds a certain value of air pressure. Therefore, by measuring (for example, with an aneroid) the pressure at a mountaintop or in the basket of a balloon and knowing the change in the atmospheric pressure with altitude, we can determine the height of the mountain or the altitude of the balloon. Common aneroids are so sensitive that the pointer noticeably shifts when the instrument is raised by 2-3 m. Moving up or down a staircase with an aneroid, one can easily notice a gradual

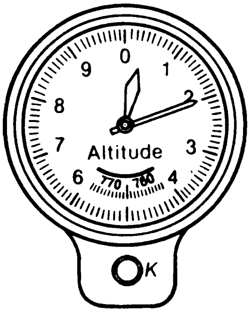

Fig. 295.
Airborne altimeter. The longer pointer indicates hundreds of metres, while the shorter pointer indicates kilometres. Screw K is intended for adjusting the zero on the scale against the pointer at the surface of the Earth at the beginning of flight.

change in pressure. Such an experiment can be conveniently made on a metro escalator. Aneroids are often calibrated in terms of altitude. Then the position of the pointer indicates the altitude of the instrument. Such aneroids are called *altimeters* (Fig. 295). They are mounted on aeroplanes and allow the pilots to determine the flight altitude.

The drop of pressure with increasing altitude is explained in the same way as the decrease in pressure associated with floating up from sea depths to the surface. At sea level, air is compressed by the weight of the entire atmosphere of the Earth, while the upper layers of the atmosphere are compressed only by the weight of the above-lying air layers. Generally, the change in pressure from point to point in the atmosphere or in any other gas under the action of the force of gravity is governed by the same laws as the pressure in a liquid: the pressure is the same at all points on a horizontal surface. As we move up, the pressure decreases by the weight of the air column whose height is equal to the height of the transition and whose cross-sectional area is unity.

However, the general pattern of pressure distribution over altitude in the atmosphere drastically differs from that for liquids due to a high compressibilty of gases. Indeed, let us plot a decrease in air pressure with height. On the ordinate axis, we plot heights h, $2h$, $3h$, etc., above a certain level (say, sea level), and the pressure p on the abscissa axis (Fig. 296). We shall climb up a staircase with the steps of height h. In order to determine the pressure on a step, we must subtract the weight of the air column of height h, equal to ρgh, from the pressure on the previous step. The air density, however, decreases with increasing height. Therefore, the decrease of pressure corresponding to the next step gets the smaller the higher the step is located. Hence, the pressure does not decrease uniformly with increasing altitude: at a small altitude, where the air density is higher, the pressure decreases rapidly. The higher we rise, the lower the density of air and the slower the pressure drops.

Arguing in this way, we assumed that pressure is the same in the entire layer of thickness h. For this reason, we have obtained a step (dashed) line

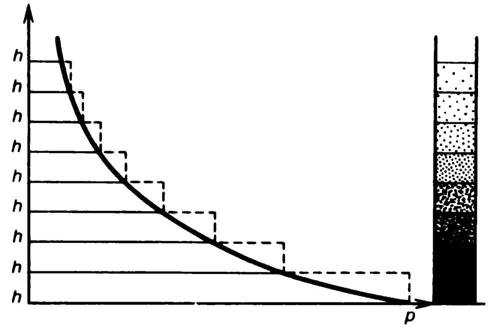

Fig. 296.
Plotting the curve describing the pressure distribution over height. The right side shows air columns of the same thickness taken at different altitudes. The columns of more compressed air having higher density are shaded darker.

on the graph. But of course, the density decreases during an ascent to a certain height continuously and not abruptly. Therefore, actually the graph has the form of a smooth line (solid line in the figure). Thus, unlike the linear pressure graph for liquids, the law of pressure drop in the atmosphere is represented by a curve.

If the height of a volume of air is not large (like in a room or a balloon), it is sufficient to use a small part of the graph. In this case, the curvilinear segment can be regarded as a straight segment (as for a liquid) without introducing a large error. Indeed, for a small change in height, the air density changes insignificantly.

If we have a certain volume of a gas other than air, the pressure in it also decreases in the upward direction. For each gas, the corresponding graph can be plotted. It is clear that for the same pressure at the bottom, the pressure of heavy gases decreases with increasing height more rapidly than for light gases since the weight of the column of a heavy gas exceeds that of the light gas column of the same height. Figure 297 represents such

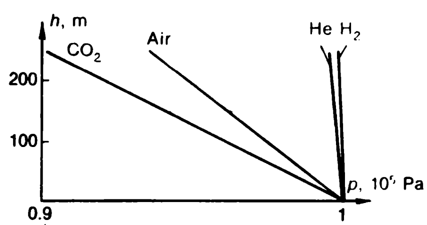

Fig. 297.
Pressure variation with height for different gases.

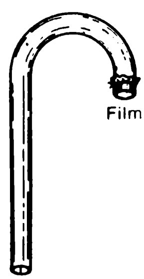

Fig. 298.
To Exercise 8.9.1.

graphs plotted for several gases. The graphs are plotted for a small interval of height and hence have the form of straight lines.

? 8.9.1. A hook-shaped tube whose longer bend is open is filled with hydrogen (Fig. 298). How will the rubber film covering the shorter arm of the tube be curved?

8.10. Physiological Effect of Lowered Air Pressure

Climbing up the hill, a person gets into the region of lower air pressure. At a large height, the decrease in pressure leads to a number of painful effects known as mountain sickness.

The most important factor here is oxygen deficiency. During each deep breath, a certain volume of air flows into the lungs. The more rarefied air is, the smaller its mass, and hence the smaller amount of its main component (oxygen) gets into the lungs during every breath. At a moderate height, this is partially compensated by laboured breathing. At higher altitudes, oxygen apparatus becomes necessary, which makes it possible to breathe in pure oxygen.

Oxygen apparatus is still more important for high-altitude aviation. At large altitudes attained at present by stratospheric balloons and aeroplanes, the artificial feeding of a human organism with pure oxygen becomes impossible. At such altitudes, a human being can only live in an airtight (hermetically sealed) cabin into which the outer rarefied air is pumped to a required pressure. At the altitudes attained by satellites, the atmosphere is practically absent. Therefore, the sealed cabins of satellites can only be supplied with air taken from the Earth and stored in the form of compressed air or oxygen.

8.11. Archimedes' Principle for Gases

The surface of a solid experiences in a gas the forces of gas pressure with a resultant directed upwards. It is the buoyancy of the gas. We can prove that *the buoyancy in a gas is equal to the weight of the gas in the volume occupied by a body immersed in it* in the same way as it was done for liquids (see Sec. 7.23).

The emergence of this force is explained in the same way as for liquids by the fact that lower layers of a gas are compressed to a greater extent than upper layers, and hence the pressure on the lower part of the body is higher than the pressure on its upper part.

The presence of buoyancy in a gas can be revealed as follows. We place a lever under the bell of an air pump and attach a large hollow glass sphere to its one end and a small weight balancing the sphere to the other end (Fig. 299). While pumping air out of the bell, we shall see that the equilibrium of the lever is violated, and the sphere goes down. This is explained by the fact that as the air is pumped out, the buoyancy vanishes, and only the force of gravity acts on the body in a vacuum. Since the buoyant force for a large sphere is larger than for a small weight, the sphere outweighs the weight after the air has been pumped out. The buoyancy in air has to be taken into account while using a precision balance for weighing a body by introducing a correction both for the body being weighed and for the weights.

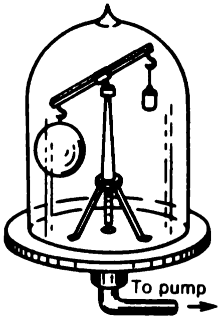

Fig. 299.
As air is pumped out of the bell, the sphere outweighs the weight.

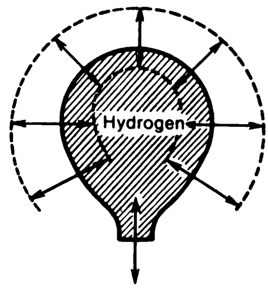

Fig. 300.
The arrows pointing inwards indicate the forces of pressure of the surrounding air on the envelope; the arrows directed outwards indicate the forces of pressure of a gas filling the balloon.

? **8.11.1.** The density of a human body can be assumed to be equal to 10^3 kg/m^3. What is the reduction in the weight of a man due to the buoyant force exerted by air if his weight in air is 756 N?

8.11.2. Should a correction for buoyancy in air be introduced in precision weighing of a piece of brass if the weights are also made of brass?

8.12. Balloons and Airships

The flight of a balloon or airship in air resembles the floatation of a submarine under the surface of water. If the mass of the entire airship added to the mass of the gas filling the balloon is less than the mass of air in the volume displaced by the airship, the airship will move up. If these masses are equal, the airship is at rest in air. If the mass of the airship with the gas is larger than the mass of the displaced air, it goes down. Thus, in order to make the flight possible, the mass of the airship itself without the gas must be less or at least equal to the difference between the masses of the light gas filling the balloon and of the air in the same volume.

Although Archimedes' principle for gases can be used to explain the flight of a balloon, the buoyant force in this case emerges not as in the case of a solid placed into a gas. Indeed, let us consider in greater detail the forces acting on the envelope of a balloon filled with a light gas, say, hydrogen. The lower part of the envelope of the balloon is left open (Fig. 300). The hydrogen pressure at the lower opening is equal to the air pressure. The pressure of air and the pressure of hydrogen decrease with increasing altitude. This means that both the air pressure and the hydrogen pressure on various parts of the envelope will be less than the pressure at the lower opening. But it was shown earlier (see Sec. 8.9) that the pressure of light hydrogen decreases with increasing altitude at a lower rate than the air pressure. Hence, a higher pressure will be exerted on the envelope from inside, the largest difference between the pressures of hydrogen and air being observed for the upper part of the envelope. Consequently, the upward force exerted on the envelope from inside will be greater than the downward force exerted from outside. The difference between these forces will balance the weight of the balloon, i.e. of the envelope, basket and load. Thus, the buoyancy is created here not due to the difference between the pressures on the lower and upper parts of the body (as in the case of a solid), but due to the difference between the pressures from inside and outside on the upper part of the envelope.

At the beginning of flight, a balloon is filled with hydrogen to such an extent that the buoyant force exceeds the force of gravity: the weight of the displaced air is larger than the weight of the balloon and of the gas filling it, and the balloon rises. When it reaches the layers of air at a lower pressure, hydrogen expands, and a fraction of it can go out through the lower opening. Thus, at a high altitude, both the external pressure of air and the hydrogen pressure inside the balloon decrease. The resultant of these forces of pressure, i.e. the buoyant force, also decreases.

Finally, at a certain altitude the balloon comes to a halt and floats in equilibrium. The weight of the displaced air at this altitude is equal to the weight of the balloon with the gas filling it. In order to land the balloon, a part of the gas must be let out from the envelope, which reduces the displaced volume of air. For this purpose, a valve is arranged at the upper part of the balloon, which can be opened by a rope from the basket. When the valve is open, the gas, which has a pressure exceeding the air pressure as was shown above, goes out. Were the valve arranged at the lower part of the envelope, it would not let the gas out since the hydrogen pressure and the air pressure are equal in this part of the balloon.

The first balloons, or "montgolfiers", invented in 1783 in France by the Montgolfier brothers, were filled with hot air. Gases expand at heating, and therefore the mass of the heated air in a balloon is less than the mass of the displaced cool air. However, the decrease in density in this case is not large: when air is heated from 0 to 100 °C, this decrease amounts to 27%. Thus, the weight of the envelope, basket, team, and useful load of a montgolfier must be within 27% of the weight of air displaced by the envelope. For this reason, even large montgolfiers had a small buoyancy.

Soon after the montgolfiers had been invented, the French physicist Jacques Charles (1746-1823) proposed that the balloons should be filled with hydrogen whose density is equal to 1/14 of the air density. A hydrogen balloon has a considerably larger buoyancy than a montgolfier of the same size.

A serious drawback of hydrogen airships is the high inflammability of hydrogen which, being mixed with air, forms an explosive. Therefore, after large natural deposits of a light incombustible helium gas had been discovered, balloons and airships were sometimes filled with helium. Having filled a balloon with helium instead of hydrogen, we increase its weight by

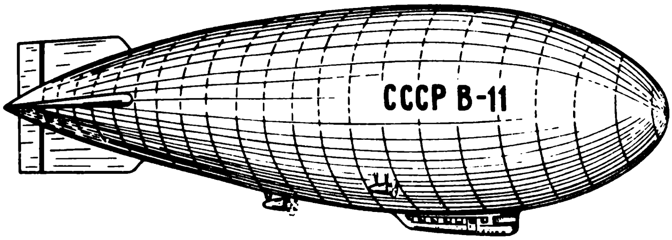

Fig. 301.
Airship.

1/14 of its entire weight. The weight of the useful load has to be decreased by the same value. The weight of the envelope, basket, team, and the useful load constitute 13/14 of the weight of the displaced air in a hydrogen balloon and 6/7 in a helium balloon. The additional weight noticeably reduces the maximum altitude to which the balloon of a given size rises before it attains equilibrium, i.e. reduces the "ceiling" of the balloon. For this reason, huge balloons intended for flights at high altitudes (stratospheric balloons) are filled with hydrogen.

At the beginning of the 20th century, the first practical experiments on manned balloons, viz. airships supplied with engines and propellers, were made. During the First World War (1914-1918), airships have already played an important role. However, airships cannot compete with aeroplanes as regards their reliability, simplicity of manouvre and velocity.

An airship is given an elongated streamlined shape in order to make the air resistance to the translatory motion as low as possible (Fig. 301). Some types of airships have a metallic frame (zeppelins). Other types of airships retain their shape due to the fact that the gas pressure inside the envelope is always maintained at a slightly higher level than the outer atmospheric pressure. The main advantage of airships over aeroplanes is their ability to hang at rest in air and ascend or descend along the vertical without switching on their engines.

? **8.12.1.** The mass of the envelope, basket and equipment of a balloon having a volume of 1500 m^3 is equal to 800 kg. Find the mass of the load which can be lifted by the balloon when it is filled (a) with hydrogen and (b) with helium. The densities of hydrogen, helium and air are 0.09, 0.18, and 1.29 kg/m^3 respectively.

8.13. Application of Compressed Air in Engineering

In various branches of engineering, such as construction, shipbuilding and mining, pneumatic tools, i.e. devices operated by compressed air, are widely used. At any large plant air-operated hammers and drills are being used. In mines, pneumatic picks are employed.

Each such instrument is connected by a rubber hose to the main, viz. the pipe into which air is continually pumped from the central compressor. A simple diagram of a pressure pump (compressor) is shown in Fig. 302. When the flywheel rotates, piston *1* moves in a cylinder to the right and to the left. As the piston moves to the right, the compressed air opens valve *2* and is forced into the main. As the piston moves to the left, a new portion of air is sucked into the cylinder from the atmosphere, while valve *2* is closed and valve *3* is opened.

Figure 303 shows the design of a pressure gauge used for measuring pressure of compressed air and other gases. A hollow metallic pipe *1* of the

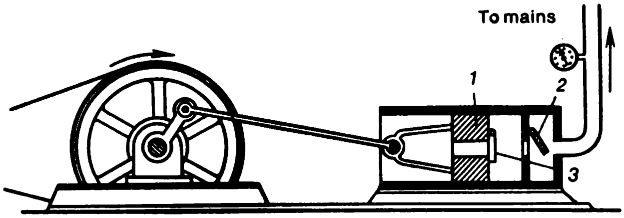

Fig. 302.
Schematic diagram of a compressor.

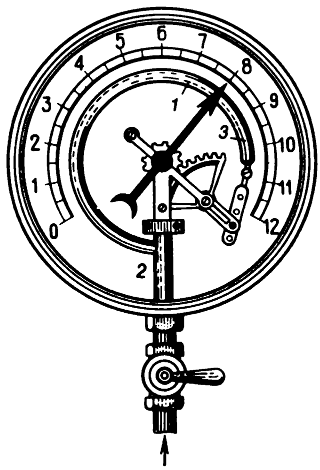

Fig. 303.
Pressure gauge.

oval cross section, bent in the form of a ring, is connected by its open end *2* to a volume in which the gas pressure has to be measured. Near end *2*, the tube is fixed to the case of the pressure gauge. The closed end *3* is connected to a mechanism which sets the pointer of the instrument in motion. The higher the gas pressure, the more pipe *1* is straightened out and the larger the deviation of the pointer. Normally, the position of the pointer corresponding to the atmospheric pressure is marked by zero on the scale. Then the pressure gauge indicates the excess of the pressure being measured over the atmospheric pressure, and the reading of the instrument indicates the so-called excess pressure. Such pressure gauges are used, for example, for measuring the pressure in steam boilers.

Let us point out some more applications of compressed air.

Air (pneumatic) brakes are widely used on railroads, in trams, trolleybuses, metro and motor cars. In pneumatic brakes used on trains, brake blocks *1* are pressed against the tyres by compressed air contained in tank *2* which is located under the carriage (Fig. 304). The operation of brakes is controlled by changing the air pressure in the main tube con-

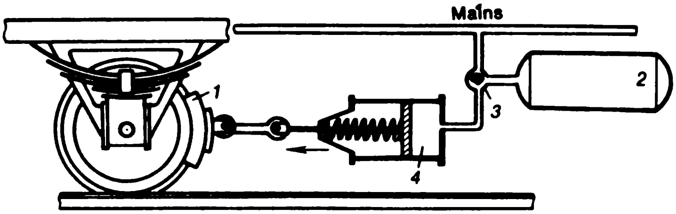

Fig. 304.
Schematic diagram of pneumatic brakes used on trains.

necting carriages with the main tank of compressed air mounted on the locomotive and being filled by a compressor. As the pressure in the main decreases distribution valve *3* connects tank *2* with brake cylinder *4*, and a braking is realised. The pressure in the main can be reduced by the driver who disconnects the main from the compressor and connects it with the atmosphere. The same result can be attained if the emergency brake valve is opened in any carriage or if the main is damaged.

Compressed air is also used in oil production. In a region of oil fields, compressed air is pumped under the ground to force oil out to the surface. Sometimes, compressed gas is accumulated in underground layers as a result of some processes occurring in an oil-bearing layer. If a borehole reaching the level of oil is drilled in the ground, the gas will force oil out to the surface of the Earth. The difference between the pressures of the underground gas and the atmospheric air can be so large that it causes the oil to rise to the surface and even gush into the air.

The same principle is used in the design of a device which is often used in laboratories to pour distilled water from a vessel. If we blow into pipe *1* of the device (Fig. 305), water will flow out of pipe *2*. Since the vessel is always closed by a cork, liquid can be stored for a long time without being polluted.

To empty ("blow out") the ballast water tanks of a submarine, water is displaced by compressed air stored on board in special cylinders.

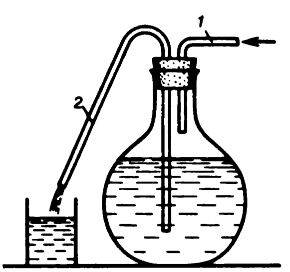

Fig. 305.
A device for pouring distilled water.

Chapter 9

Fluid Dynamics [1]

9.1. Pressure in a Fluid Flow

The pressure in a liquid is determined by the degree of its compression. The pressure in a liquid at rest is measured by a manometer (see Sec. 7.8). The manometer does not alter the extent to which a liquid at rest is compressed. This makes it possible to measure pressure correctly.

The measurement of pressure in a *flowing* fluid, say, the pressure of water flowing in a tube or the air pressure of wind, involves serious difficulties. Naturally, in this case pressure is also determined by the extent to which the fluid is compressed. However, a manometer in a flow is an obstacle which can noticeably distort the flow. This brings about a change in compression, and hence the pressure at different points in a fluid. Thus, a manometer introduced in a flow may measure the pressure which differs from that existing in the flow before the instrument has been placed in the flow, and its readings may not give a correct pressure distribution in the fluid flow prior to the introduction of the obstacle.

The change in pressure introduced by an obstacle is clearly seen if we consider by way of example the operation of a sail. If the wind is uniform, air is compressed to the same extent in neighbouring regions, and hence it may seem that the forces of pressure exerted on the two sides of the sail are equal, and hence the wind will not move the sailboat. In actual practice, however, the sail essentially changes the air flow. When air is compressed upon meeting an obstacle (sail) like a ball striking a wall, the layers of air near the sail from the wind side are compressed to a larger extent than the remaining air: the pressure is higher on this side. On the contrary, on the other side of the sail the air flowing about it turns out to be compressed to a lesser extent, and the pressure is lower here. Thus, the pressure is elevated on one side of the sail and lowered on its other side. This gives rise to a force exerted on the sail, which drives the boat.

Like a sail in an air flow, a manometer placed in a fluid flow also changes the velocity of the flow. If we turn the manometer so that its mem-

[1] Fluid is a collective term embracing liquids and gases. — *Eds.*

brane faces the flow, a higher reading will be obtained. Having turned it so that the membrane is along the flow, we obtain a smaller reading. Finally, if the membrane is at 180° to the flow, the manometer reading will be still smaller. When a manometer made in the form of a flat box is arranged with its membrane along the flow, it changes the flow velocity and the extent to which the fluid is compressed only slightly. Therefore, for such a position of the manometer, its reading will be close to the pressure in the flow before the manometer has been placed in it.

What can be done for an obstacle immersed in a flow not to change the fluid velocity at all? For this the obstacle itself must move at the same velocity as the fluid in the flow does. For example, a balloon is carried away by wind at a constant velocity equal to the velocity of wind. Therefore, it does not disturb the motion of the surrounding air (does not produce compressions or rarefactions). For such a balloon, the air flow is not felt since the air remains at rest relative to it.

Similarly, a manometer moving with a fluid will not change the motion of the layers of the fluid surrounding it and hence indicates the pressure of the fluid in the flow prior to the introduction of the manometer. In this case, *the fluid is at rest relative to the manometer*, and pressure is measured in the same way as in hydrostatics. The fluid exerts on the manometer moving with it a pressure which corresponds to the extent to which the fluid is compressed in an undisturbed flow.

The pressure that can be measured by a manometer moving with the fluid is called the *static* pressure. The reading of a stationary manometer whose membrane is normal to the flow is known as the *impact* (*stagnation*) pressure.

Thus, in order to measure the static pressure, we must use a moving manometer, while the impact pressure is measured with a stationary manometer. In actual practice, however, it would be quite difficult to use a moving manometer. In order to overcome this difficulty, the instrument is given such a shape that it does not change the flow velocity in the vicinity of the region where the pressure is being measured. Such an instrument can be made in the form of a narrow tube with a rounded closed end and with openings on the *lateral* surface (Fig. 306*a*). The streams of flow passing by the openings do not practically change their velocity, and the static pressure acts in the arm of a manometer connected with such a tube. This tube is termed a *probe*. If, however, we take a tube with an open end facing the flow (Fig. 306*b*), the stream will be stopped at the opening as in front of a membrane, so that the impact pressure is exerted in it. Such a tube is known as the *Pitot tube*. A manometer connected to a Pitot tube shows a higher pressure than a manometer connected to a probe.

Let us connect now the two tubes to the arms of the same manometer

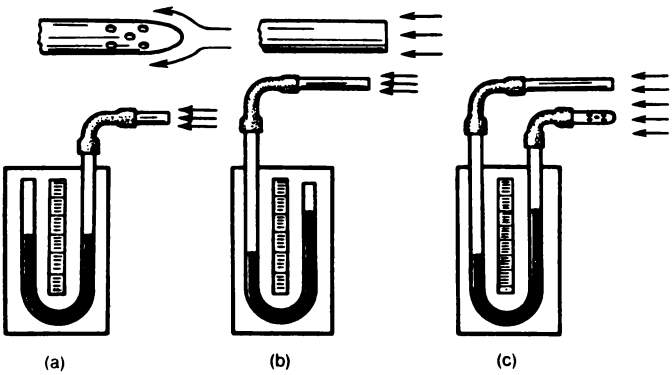

Fig. 306.
(*a*) The reading of the manometer does not change when a probe is flowed about by air. (*b*) A Pitot tube in air flow: the manometer indicates elevated pressure. (*c*) Current meter.

(Fig. 306*c*). The manometer will show the difference between the impact and static pressures. The higher the velocity of an incoming flow, the larger this difference. Therefore, the readings of the manometer connected with such tubes allow us to judge about the velocity of flow. We thus obtain a *current meter* which can be used for measuring the velocity of air and the velocity of a liquid flow.

Such current meters are mounted on aeroplanes. They make it possible to measure the velocity of air relative to the plane or, which is the same, the velocity of the plane relative to air. A current meter is one of the most important instruments used for aeroplane-handling.

9.2. Fluid Flow in Pipes. Fluid Friction

For measuring the static pressure in a fluid flowing through a pipe, the following device can be used. Vertical tubes with open upper ends (manometric tubes) are connected to small holes drilled in a pipe (Fig. 307). If the fluid in the pipe is under a pressure, it rises in a vertical tube to a height corresponding to the static pressure at a given point of the pipe. [2] Indeed, a small hole does not significantly disturb the fluid flow. Arranging manometric tubes at different points of a pipe, we can measure the static pressure at the corresponding points.

Let us investigate with the help of manometric tubes the static pressure in a fluid flowing in a pipe of a constant cross section. For this purpose, we shall use a device shown in Fig. 308. The static pressure at different points

[2] To be more precise, to the difference between this static pressure and the external atmospheric pressure.

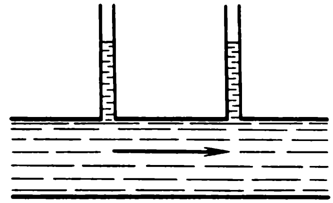

Fig. 307.
Manometric tubes indicate the static pressure in the pipe through which a liquid flows.

Fig. 308.
Manometric tubes indicate a pressure drop along the pipe through which water flows.

of the pipe can be determined from the height of the water columns in manometric tubes arranged at different points of the pipe. Experiments show that the pressure drops downstream along the pipe: the further from the beginning of the pipe, the lower the static pressure of the flowing fluid. In narrow pipes pressure drops at a higher rate than in wide pipes. In sufficiently wide and short pipes where the flow velocity is not very large, the pressure drop is almost unnoticeable.

The pressure drop in a fluid in the pipe is explained by friction. The pipe walls exert friction on the fluid flowing through the pipe. These forces are directed against the fluid flow. Let us mentally isolate a volume *ABCD* of the fluid in the pipe (Fig. 309). The pipe walls exert frictional forces **f** on the isolated volume. If the fluid flow in the pipe is uniform (the flow velocity is constant), the forces of pressure acting on the isolated volume must balance the frictional forces. Hence we conclude that the force of pressure $\mathbf{F}_1$ acting along the motion must have larger magnitude than the force of

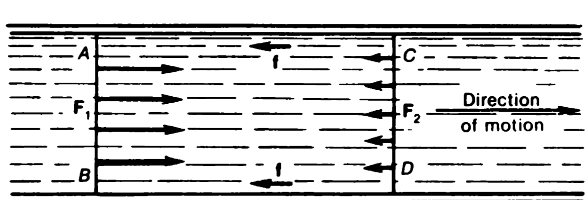

Fig. 309.
The sum of forces of pressure $\mathbf{F}_1$ and $\mathbf{F}_2$ balances frictional forces **f** exerted by the pipe walls.

pressure F_2 acting in the opposite direction. Therefore, the pressure at the upstream surface *AB* of the isolated volume must be higher than the pressure on the downstream surface *CD*, i.e. the pressure must drop downstream along the pipe.

If the velocity of the fluid flow in the pipe increases, the friction becomes stronger. Therefore, in a fast fluid flow the pressure drop in a given pipe is larger than in a slow flow. For a given flow velocity, friction is more pronounced in narrow pipes than in wide pipes; therefore, in narrow pipes pressure drops more rapidly.

In water supply systems, the pressure drop in the pipes should be taken into account. When the taps are closed and water does not flow through the pipes, the water pressure corresponds to the height of the water tower (see Sec. 7.18). No friction emerges in a fluid at rest. If, however, the taps are opened and water flows, friction in the pipes causes a pressure drop: the water "head" decreases. The larger the number of opened taps and the higher the velocity of water flow, the larger the drop in the head.

If the height of a water tower is insufficient, it may turn out that the pressure drop in the pipes is larger than the pressure corresponding to the height of the tower above the upper storeys of a building. Then water stops to run from the taps of upper storeys. However, in the period of the day when the water consumption is not very large, pressure losses are reduced, and water flows up to the taps of the upper storeys. Generally, the pressure of water in a water supply system is higher at night, when the water consumption is low, the velocity of water in pipes is low, and hence friction is comparatively small.

The pressure drop in a water supply system can be demonstrated with the help of the following model (Fig. 310). A narrow pipe *A* (in which friction is high enough) and its branch *B* supplied by manometric tubes have taps *a* and *b* at the ends. If we pour water in vessel *C* and close the taps, the pressure in pipes *A* and *B* will correspond to the height of water in the

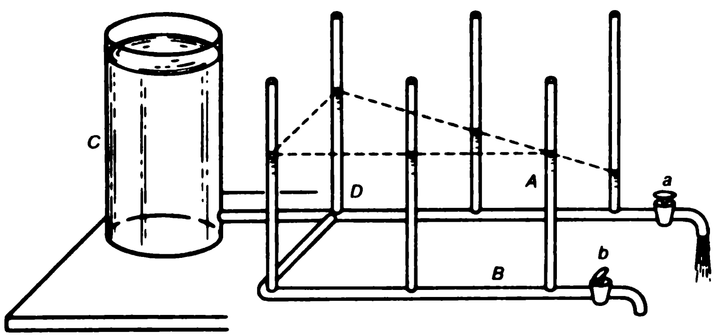

Fig. 310.
A device for demonstrating the pressure drop in a water supply system.

vessel, and the height of water in the manometric tubes will be the same as in vessel C. If we slightly open tap a, the pressure in pipe A will drop as in the case considered earlier. The pressure in pipe B will fall off, but will be the same at all points and equal to the pressure at point D. If we open the tap a a little more, the pressure drop along pipe A will be more pronounced. If. in addition, we open tap b as well, the water pressure will drop along pipe B as well as at all points of pipe A.

9.3. Bernoulli's Law [3]

It was mentioned above that in not very long and wide pipes, friction is so low that it can be neglected. In these conditions, the pressure drop is so small that in a pipe of constant cross section the fluid in manometric tubes attached to it is practically at the same level. If, however, the cross-sectional area of a pipe is different in different parts, experiments show that the static pressure is different at different points even if we neglect friction.

Let us take a tube of varying cross section (Fig. 311) and pass a steady water flow through it. The levels of water in manometric tubes show that in narrow parts of the tube the static pressure is lower than in the wide parts. This means that as the fluid flows from the wide part of the tube to the narrow one, the extent to which the fluid is compressed decreases (pressure drops), while in the transition from the narrow to the wide part the extent to which the fluid is compressed increases (as well as the pressure).

This is due to the fact that in wide parts of the tube, water must flow slower than in narrow parts since the amount of fluid flowing through the

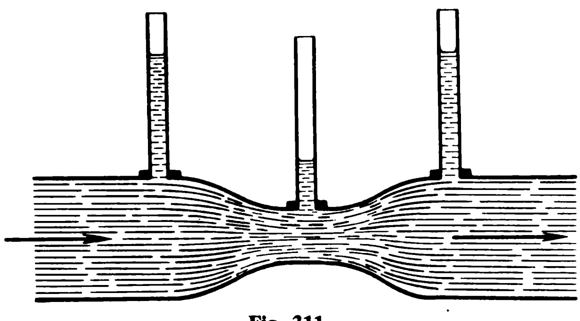

Fig. 311.
The static pressure of a flowing fluid is lower in narrow parts of a pipe than in wide parts.

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[3] D. Bernoulli derived an equation (called *Bernoulli's equation*) which connects pressure in a fluid with the velocity of its flow. From this equation, a relation referred to in this section as Bernoulli's law can be obtained. It should be noted, however, that the term "Bernoulli's law" is not generally accepted.

tube in equal intervals of time is the same for all cross sections. Therefore, as water flows from a narrow to a wide part, its velocity decreases: water is stagnated as if meeting an obstacle, and the extent to which it is compressed (and hence pressure) increases. On the contrary, as water flows from a wide to a narrow part, its velocity increases and compression decreases: being accelerated, fluid behaves as a straightening spring.

Thus, we see that *the pressure in a fluid flowing through a tube is higher in the parts where the velocity of fluid flow is lower, and, conversely, the pressure is lower in the parts where the fluid velocity is higher.* The relation between the velocity of fluid and its pressure is sometimes called *Bernoulli's law* after the Swiss physicist and mathematician Daniel Bernoulli (1700-1782).

Bernoulli's law is valid both for liquids and gases. It also holds for a flow of fluid which is not bounded by the tube walls, i.e. for a free fluid flow. In this case, Bernoulli's law should be applied as follows.

Let us suppose that the motion of a fluid (liquid or gas) does not change with time (a steady-state flow). Then we can imagine lines in the flow along which the fluid moves. These lines are called the *streamlines.* They divide a fluid into separate jets which flow side by side without mixing. Streamlines can be made visible if we introduce a liquid paint through thin pipes into a water flow. Paint streams arrange themselves along fluid streamlines. In order to obtain visible streamlines in air, we can use streams of smoke. It can be shown that Bernoulli's law is applicable to each stream separately: the pressure is higher at the point of a stream where the velocity is lower, and hence where the cross section is larger, and vice versa. Figure 311 shows that the cross section of a flow is larger where the streamlines diverge. In the regions where the flow cross section is smaller, the streamlines come closer. Therefore, Bernoulli's law can be formulated as follows: *pressure is lower in the regions of a fluid flow where the streamlines are denser and higher where the streamlines are less dense.*

Let us take a tube with a narrowing part and pass water at a high velocity through it. According to Bernoulli's law, the pressure in the narrow part will be lower. We can choose a shape of the tube and a velocity of the flow so that the water pressure in the narrow part will be lower than the atmospheric pressure. If we now connect a branch pipe to the narrow part of the tube (Fig. 312), the outer air will be sucked into the region of lower pressure. Having got into the flow, air will be entrained by water. Using this phenomenon, we can design a vacuum pump, the so-called *water-jet pump.* In a model of the water-jet pump shown in Fig. 313, air is sucked in through annular slit *1* in the vicinity of which water flows at a high velocity. Spout *2* is connected with a vessel being evacuated. Water-jet pumps have no moving solid parts (as, for example, a piston in conventional pumps), which is one of their advantages.

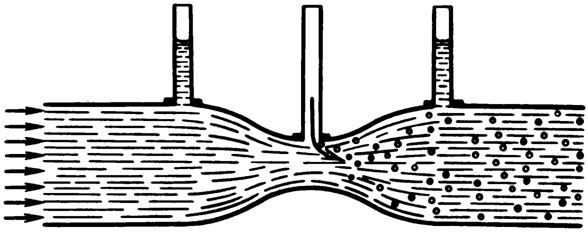

Fig. 312.
Air is sucked into the narrow part of the pipe where the pressure is lower than the atmospheric pressure.

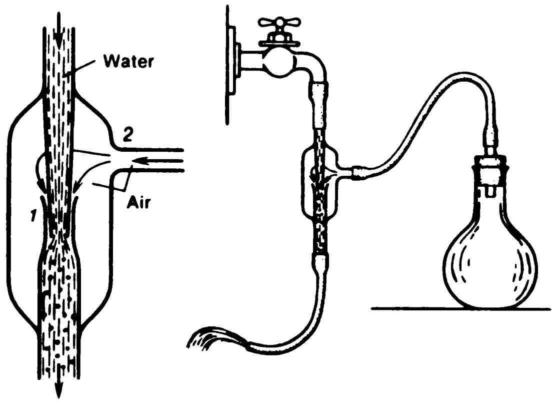

Fig. 313.
Schematic diagram of a water-jet pump.

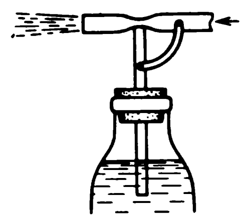

Fig. 314.
Sprayer.

Let us blow air through a pipe with a contraction (Fig. 314). At a sufficiently high velocity of air, the pressure in the narrow part of the pipe will be lower than the atmospheric pressure. Liquid from the vessel will be sucked in through the side spout. The liquid leaving the pipe is sprayed by the air jet. This device is known as a *sprayer*.

9.4. Fluid in Noninertial Reference Systems

Let us suppose that a vessel containing a fluid moves with an acceleration. We shall consider the motion of the fluid relative to the vessel as a noninertial reference system and introduce inertial forces. Fluid will be in equilibrium under the action of all the forces exerted on it, including the inertial forces.

Let us first analyse the case of a translatory motion of a moving noninertial reference system. Suppose that, for example, a railway tanker filled with a liquid moves with an acceleration **a** along a horizontal straight segment of the path. In the reference system fixed to the tanker, each particle of liquid is acted upon by the downward force of gravity $m\mathbf{g}$ (where m is the mass of the particle) and the inertial force $-m\mathbf{a}$ having the horizontal direction opposite to the acceleration of the tanker (Fig. 315). The resultant **F** of these forces is inclined from the vertical in the direction opposite to the acceleration. But it is well known (see Sec. 7.1) that the free surface of a liquid is always perpendicular to the force acting on the particles of the liquid. Consequently, the surface of the liquid will be inclined to the horizontal (Fig. 316): *in equilibrium relative to a noninertial reference system in translatory motion, the free surface of a liquid turns out to be inclined to the horizontal.* This can be easily verified by setting in motion a glass of water or stopping it abruptly. If the acceleration is sufficiently high, water spills over the top of the glass. To carry a glass of water filled to the brim "with care" means to carry it with a small acceleration.

If the acceleration is directed vertically and not horizontally, the action of inertial forces is reduced to an increase in the weight of a liquid (if the acceleration is directed upwards as in a rocket being launched) or to its reduction (if the acceleration is directed downwards). The pressure exerted by the liquid on the bottom of the vessel then increases or decreases accordingly. For example, when a rocket is launched or an aeroplane recovers from a dive, the pressure of the fuel on the bottom of tanks increases (g-load). The weight of the blood in the vessels of a pilot or cosmonaut also increases. If the body of the pilot is in the upright position, this may cause the blood to flow from his head which may cause him to faint. For this reason, pilot seats are designed so that the acceleration is directed from the chest to the back and not from the head to the feet. On the contrary, in the conditions of weightlessness (see Sec. 6.8), the weight of a liquid vanishes. The liquid does not flow out of tilted or overturned vessels. The buoyant force also vanishes: a heavy body does not sink in water, while a light body does not float up. Other peculiarities of the behaviour of fluids in the conditions of weightlessness will be described in Secs. 11.11 and 14.2.

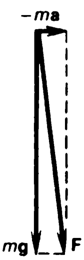

Fig. 315.
The resultant **F** of the forces $m\mathbf{g}$ and $-m\mathbf{a}$ is inclined in the direction opposite to that of acceleration **a**.

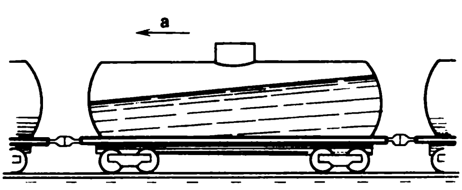

Fig. 316.
The free surface of a liquid in a railway tanker moving with an acceleration is inclined in the direction opposite to that of the acceleration.

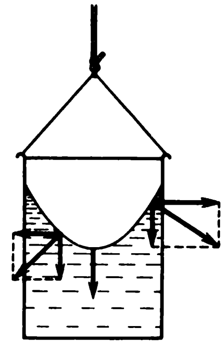

Fig. 317.
The free surface of water resting relative to a rotating bucket and the forces acting on particles of water at different distances from the rotation axis.

Let us now consider a liquid at rest relative to a rotating reference system. We suspend a bucket on a long string, twist the string and leave it to itself. The walls of the rotating bucket will entrain the liquid, and it will rotate with the bucket, i.e. will turn out to be at rest relative to the rotating bucket. In this case, a centrifugal inertial force emerges (see Sec. 5.10), which increases with the distance from rotation axis. Therefore, the resultant of the force of gravity and centrifugal inertial force will be declined more and more from the vertical as we move away from the rotation axis. As a result, the free surface of the liquid will not only be declined from the horizontal but will become curved. The departure from the horizontal will increase from the axis to the wall of the bucket (Fig. 317). The section of the free surface of the liquid by the vertical plane turns out to be a parabola.

? **9.4.1.** Prove that the slope of a liquid in a railway tanker moving with an acceleration along a straight horizontal segment of path is equal to the ratio of the tanker acceleration to the free fall acceleration.

9.4.2. What will be the shape of the free surface of water in (a) a tanker rolling freely over an inclined path; (b) a tanker moving uniformly along an inclined path?

9.4.3. A train moves at a velocity of 72 km/h around the bend of a radius of 1 km. At what angle to the horizontal will the free surface of water in a vessel standing in the carriage be arranged?

9.5. Reaction of a Moving Fluid and Its Application

Let us put on the table a glass tube bent at right angle and connected with the water supply system by a rubber pipe (Fig. 318). As water flows out of the tube, the latter is thrown back in the direction indicated by the arrow. In order to explain this experiment, let us consider the forces exerted by the flowing fluid on the bent tube. Suppose that water enters the tube at a velocity $\mathbf{v}_1$ (Fig. 319) and leaves it at a velocity $\mathbf{v}_2$. To simplify calculations, we assume that the tube has the same cross section everywhere. In this case,

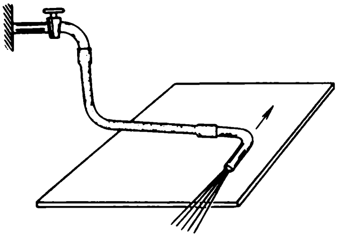

Fig. 318.
When the tap is opened, the bent tube starts moving in the direction indicated by the arrow.

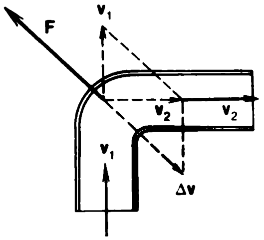

Fig. 319.
When the direction of water flow changes, the reaction **F** of the water jet acts on the tube.

the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ are equal in magnitude but have different directions. Consequently, the velocity has acquired an increment $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$. This means that the fluid flowing in the bent tube has an acceleration whose average is directed along the vector $\Delta\mathbf{v}$. The acceleration is imparted to the fluid by the forces exerted by the tube walls on the fluid. According to Newton's third law, the fluid exerts on the walls the reaction **F** directed against the vector $\Delta\mathbf{v}$. We shall call this force the reaction of the jet. In the experiment under consideration, *the tube is deflected by the reaction* (*reactive force*) *of the jet.*

Another example of the reaction of the jet is presented in Fig. 320. In this experiment, water flowing out through the bent tubes rotates the bucket in the direction indicated by the arrow. In order to explain this experiment, we must determine the direction of reactive forces of water flowing out of the bucket. These forces rotate the bucket shown in the figure clockwise (if we watch it from above). A device of this type is known as the *Segner wheel.* Segner wheels are sometimes used to irrigate lawns, as they can spray water over a large circle.

The reaction of jet is manifested not only when a fluid flows through a bent pipe, but always when a fluid jet changes its direction, having en-

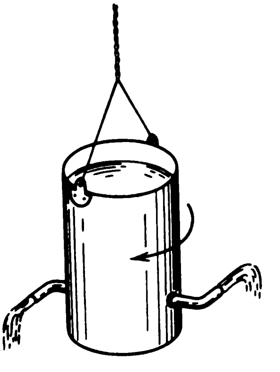

Fig. 320.
The bucket rotates in the direction opposite to the direction of water jets.

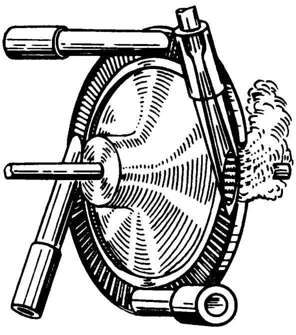

Fig. 321.
Steam turbine.

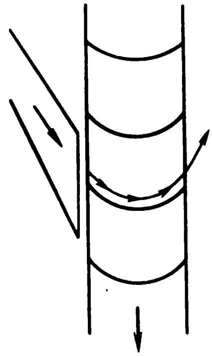

Fig. 322.
Nozzle and blades of a turbine.

countered solids on its way. This principle is used in the operation of *turbines* where the reaction of jet rotates the wheel.

In various types of turbines, the direction of water or steam jets is altered with the help of various devices. An example of such a device is a steam turbine whose main part is a wheel with blades (rotor) arranged at an angle to the rim of the wheel (Fig. 321). A high-power steam jet strikes the blades and is reflected from them, changing its direction (Fig. 322). This gives rise to a reactive force exerted by the jet on the blades. This force rotates the turbine wheel. Water turbines of hydroelectric power plants are somewhat different (Fig. 323), but here too the turbine wheel is rotated by the reaction of water jets deflected by the blades.

The same phenomenon of the reaction of jet forms the basis of the operation of windmills. The incoming air flow is deflected by the oblique sails of a windmill arranged on the axle. The reaction of the air jet acts on each sail and rotates the mill (Fig. 324).

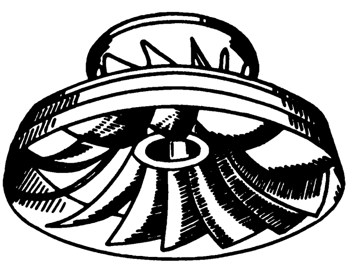

Fig. 323.
Wheel of a water turbine.

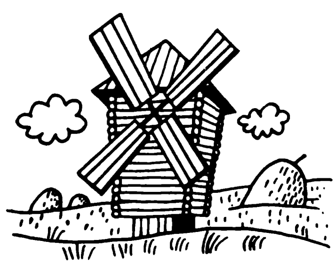

Fig. 324.
Windmill.

9.6. Motion over Water Surface

In Chap. 7, we considered the floatation of ships on the surface of water. We must now explain how the ships *move*. Here we deal with a situation different from the motion of mechanical vehicles over the ground. For example, a motor car is moved by the forces of static friction between the wheels and the ground (see Sec. 2.37). It can be said that the wheels push off from the stationary ground. Quite another matter is with water since in water, as in any other liquid, static friction is absent (see Sec. 2.38).

In shipbuilding, several types of devices (so-called propulsive devices) are used to move ships: screw propellers, paddle-wheels, and so on. The operational principle of these devices is the same. A propeller immersed in water is driven by the ship engine. It exerts on water the force that drives it in a certain direction, imparting an acceleration to new and new masses of water. According to Newton's third law, the pushed-off water exerts on the propeller an equal force in the opposite direction (reaction of water jet). Since the propeller is rigidly connected to the ship, the ship as a whole starts to move. The larger the mass of the pushed-off water and the higher the acceleration imparted to it, the larger the reactive force applied to the propeller and the faster the ship moves.

The first ships with a mechanical propulsive device, viz. steamers, were driven by a paddle-wheel (Fig. 325). The paddle-wheel is fixed to the rotating shaft of the engine, only its lower part being immersed in water. Blades (or paddles as shipbuilders call them) are mounted on the wheel rim (Fig. 326). When the wheel rotates, the blades push water back. While doing that, they slightly turn so that they enter and leave the water vertically

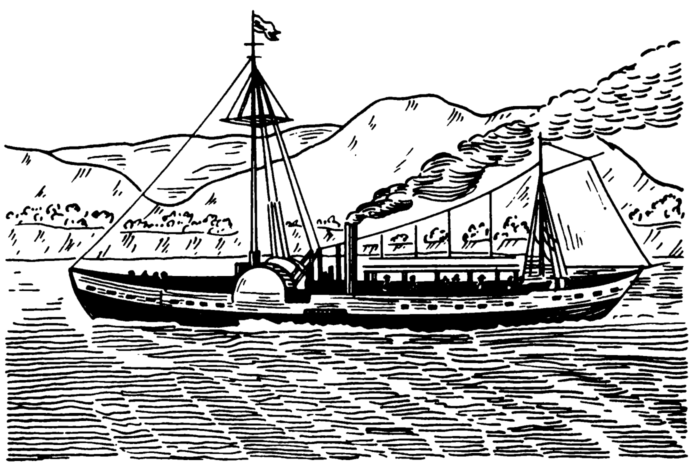

Fig. 325.
The first steamer *Clermont*.

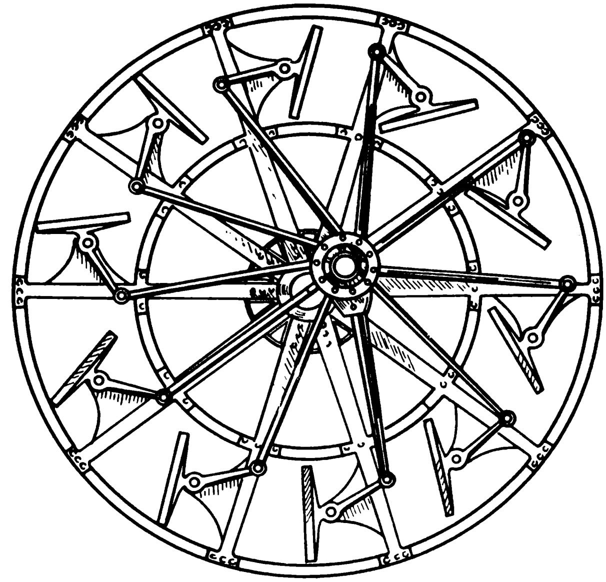

Fig. 326.
Paddle-wheel of a river ship.

in order to avoid splashes that would involve unproductive work from the engine. If the direction of wheel rotation is reversed (reverse running of the engine), water will be pushed off in the forward direction, while the ship will move backwards.

A screw propeller (or screw) (Fig. 327) was used on a ship for the first time in 1836. At the present time, all vessels are supplied with screws and not paddle-wheels. A screw propeller is much simpler than a paddle-wheel.

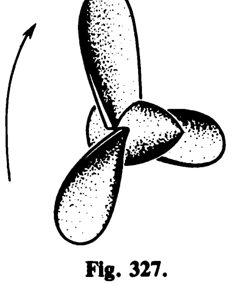

Fig. 327.
Screw propeller of a sea ship.

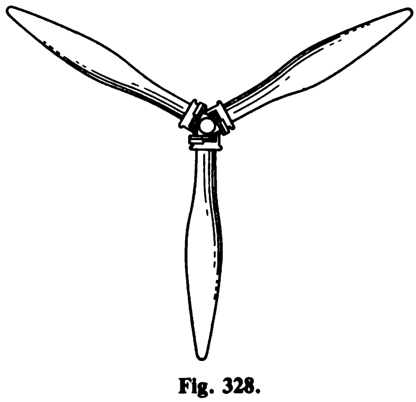

Fig. 328.
Propeller of an aeroplane.

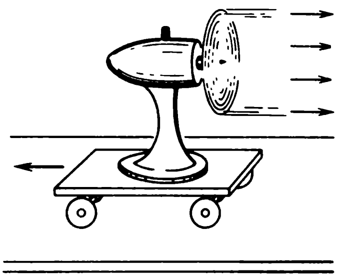

Fig. 329.
When the fan works, the cart moves in the direction opposite to that of an air jet.

It is protected from the impacts of waves since it is submerged in water completely. The blades of a screw are bent in such a way that being rotated clockwise, the screw pushes off water to the right in the figure. Consequently, the reaction of water is directed to the left.

Propellers driving aeroplanes, airships, snowplanes and some types of high-speed planing-boats operate in just the same way. A propeller consists of several (two, three or four) curved blades fixed at an angle to the propeller hub (Fig. 328). Like screws, propellers push off a jet of surrounding medium along their axis. The jet reaction is just the thrust force of the propeller. The difference in the shape of the propeller blades and the blades of a screw is due to the fact that they have to operate in media with different densities. A propeller operates effectively only if the velocity of its blades is less than the velocity of sound in air. For this reason, in high-speed aeroplanes propellers are ineffective, and jet engines are used there (see Sec. 9.8).

An ordinary fan is also a type of a propeller. The "wind" produced by a fan is an air jet. The jet reaction of an ordinary fan is weak, but still it can be observed if we arrange a fan on a light cart (Fig. 329). When the fan is switched on, the cart rolls back.

It should be noted that the simplest ways of moving over water (swimming and rowing) are also based on pushing water in the direction opposite to the motion created. For example, each stroke of an oar pushes off water in a direction opposite to the motion of the boat.

9.7. Rockets

A rotating screw or propeller pushes the surrounding medium in one direction, while the jet reaction exerted on the propeller in the opposite direction moves a ship or an aeroplane. The motion of a rocket is also caused by a jet reaction, but the whole store of a substance being pushed backwards is aboard the rocket. For instance, a powder rocket known from the ancient times (in China) is designed as follows. A hollow shell is filled with a slowly burning powder (Fig. 330). The powder charge is fired at the lower part.

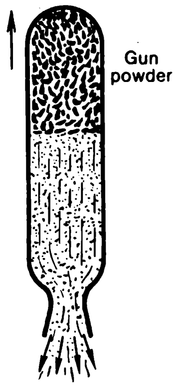

Fig. 330.
Powder rocket.

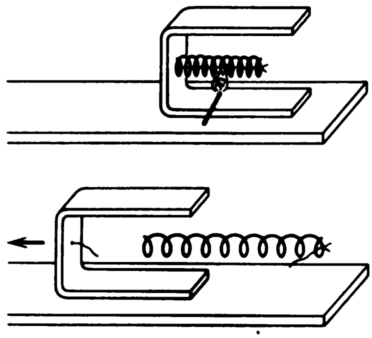

Fig. 331.
Spring model of a rocket.

The incandescent gas formed during the combustion of powder is discharged at a high velocity from the outlet in the lower part of the rocket. The jet reaction is directed against the flow and projects the rocket upwards.

Figure 331 shows a mechanical model operating on the same principle as a rocket. A spring constricted by a string is fixed in a frame. The spring plays the role of a powder charge. If we burn the string through, this corresponds to the combustion of powder. The released spring expands and exerts a pressure on the frame ("powder gas reaction"). The spring is ejected from the frame like the powder gas from the rocket nozzle. The frame which plays the role of the rocket shell acquires a velocity in the opposite direction.

9.8. Jet Engines

A jet engine (reaction-propulsion unit) is a rocket used as an engine on some vehicle. Jet engines are widely used in aviation and in military and space engineering. Liquid fuel (oil or kerosene) is often used in jet engines instead of powder. This makes the operation of an engine more effective. In this case, the reactive jet is formed by incandescent gas formed as a result of combustion of the fuel. However, powder can burn in vacuum, while for the combustion of oil, a large amount of air is required. In jet engines mounted on aeroplanes, air is sucked in from the surrounding atmosphere (air-breathing jet engines).

Thus, unlike powder rockets, an aeroplane with a jet engine must not carry the entire mass of ejected gas. Modern jet aeroplanes are capable of attaining very high velocities exceeding the velocity of sound (which is about 1200 km/h in air) two or more times.

9.9. Ballistic Missiles

Recently, ballistic missiles are being extensively developed. This is the term applied to missiles which are launched by a rocket (Fig. 332) with a fuel stock which constitutes the major part of the rocket mass and a high-power engine which only operates at the beginning of the flight. For a comparatively short time of operation (several minutes), the engines consume all the fuel and impart a huge velocity to the rocket (10 km/s and higher). After this the missile moves only under the action of the gravitational force exerted by the Earth (and other celestial bodies). Rockets of this type are used for launching satellites of the Earth and artificial planets.

Rockets used to launch ballistic missiles have aboard not only fuel but also an oxidizer (in the liquid form) required for burning the fuel. Ordinary aeroplanes and even air-breathing jet aeroplanes can fly only within the limits of the Earth's atmosphere, while the jet engine of a rocket (like a powder rocket) can operate in airless space.

A rocket has to impart as high a velocity as possible to the payload aboard the rocket. The payload for rockets employed for launching satellites of the Earth is the spacecraft, while for military rockets this is a warhead. Let us consider the operation of a jet engine in greater detail and find out the factors determining the "terminal" velocity of a rocket, viz. the velocity attained after the entire mass of the fuel has been burnt.

We shall determine the reaction of the ejected gas jet, i.e. the thrust force of a jet engine. Suppose that the gas jet carries away from the rocket a mass μ per unit time. Before it is burnt, this mass has the same velocity $\mathbf{v}$ as the rocket and possesses the momentum $\mu\mathbf{v}$. If the velocity of the gas in the jet relative to the Earth is $\mathbf{v}_{gas}$, the momentum of the gas ejected from the rocket per unit time is $\mu\mathbf{v}_{gas}$. Consequently, the increment of momentum received by the mass μ is $\mu(\mathbf{v}_{gas} - \mathbf{v}) = \mu\mathbf{u}$, where $\mathbf{u}$ is the velocity of the ejected jet relative to the rocket.

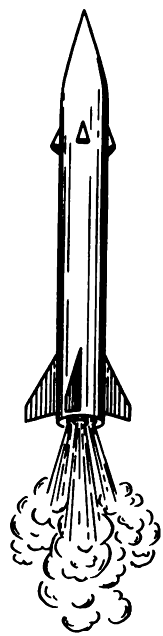

Fig. 332.
The launch of a rocket.

In order to impart to the gas such an increment per unit time, the rocket must exert on it the force $\mathbf{F}' = \mu\mathbf{u}$. Indeed, according to formula (2.20.2), the increment of the momentum of a body per unit time is equal to the force exerted on the body. According to Newton's third law, the gas jet acts on the rocket with the force $\mathbf{F} = -\mathbf{F}' = -\mu\mathbf{u}$. Thus, the reaction of the jet, viz. the thrust of the jet engine, is equal to $-\mu\mathbf{u}$. It should be recalled that μ is the mass of the gas ejected from the rocket per unit time and $\mathbf{u} = \mathbf{v}_{gas} - \mathbf{v}$ is the velocity of the jet relative to the rocket. This velocity is directed against the direction of motion of the rocket, while the force $\mathbf{F} = -\mu\mathbf{u}$ acts in the direction of motion.

We can now find out what determines the terminal velocity of a rocket. Let us first assume that the force of gravity is absent (the force of gravity will be taken into account in the next section). We also assume that the mode of operation of the jet engine remains the same: the fuel is consumed at a constant rate, and the thrust remains constant during the entire period of operation. Since the mass of the rocket continually decreases as a result of consumption of fuel and oxygen, according to Newton's second law, the acceleration of the rocket keeps increasing (in inverse proportion to its mass).

The final mass of a ballistic missile (viz. the mass of the rocket after the fuel is burnt out completely) amounts to about one hundredth of the initial (launching) mass of the rocket. Consequently, as the fuel is consumed, the acceleration increases hundreds of times. Hence it follows that the velocity increment acquired by the rocket upon the consumption of the same mass of the fuel depends essentially on the stage of flight. As long as the mass of the fuel aboard the rocket is large (and hence the mass of the rocket as a whole), the velocity increment is small. By the moment when most of the fuel has been burnt out and the rocket mass has decreased considerably, the velocity increment becomes large.

For this reason, even a considerable increase in the mass of the fuel cannot lead to a significant increase in the terminal velocity of the rocket since this additional amount of fuel is mainly consumed when the rocket mass is large and acceleration is small, and hence the additional increment in the terminal velocity of the rocket is also small. On the other hand, an increase in the velocity of the propulsive jet makes it possible to considerably increase the terminal velocity of the rocket with the same mass of the fuel. If, for example, we increase the velocity of the reaction jet without changing the consumption of the fuel per second, the acceleration of the rocket increases in the same proportion. As a result, the terminal velocity of the rocket also increases in the same proportion.

In order to increase the velocity of the propulsive jet, the nozzle of a jet engine is given a special shape (Fig. 333). Besides, the fuel with as high as possible temperature of combustion is chosen since the velocity of the reaction jet increases with the temperature of the gas forming the jet. The limit to the increase of the temperature of the jet is set only by the heat resistance of existing materials.

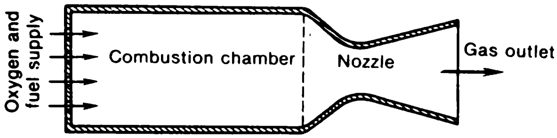

Fig. 333.
Nozzle of a jet engine.

? **9.9.1.** (For those familiar with differential and integral calculus). Proceeding from the formula $F = \mu u$, prove that when the relative velocity u of the gas jet remains constant, the velocity v acquired by a rocket during the entire period of speed-up is determined by the formula $v = u \ln (m_0/m)$, where m_0 is the mass of the rocket at the moment of launching (when $v = 0$) and m is the mass of the rocket after the fuel has been burnt out. Use the relation $\mu = -dm/dt$.

9.10. Launching a Rocket from the Earth

When a rocket is launched from the Earth, it experiences, in addition to the thrust determined in the previous section, the force of gravity exerted by the Earth in the downward direction. Thus, the resultant force acting on the vertically launched rocket is equal to $\mu u - mg$, where m is the mass of the rocket. Consequently, the attraction of the Earth reduces the acceleration of the rocket, and hence its terminal velocity. Since the mass of the rocket decreases as the fuel is consumed, while the thrust remains constant, the effect of attraction by the Earth will be less and less pronounced.

For a rocket to be launched, its launching weight should obviously be smaller than the thrust of its jet engine. Otherwise, after the engine has been started, the rocket will not rise but will remain on the ground with its engine working until its weight is reduced as a result of the burning out of the fuel to the value below the thrust. Only then the rocket will start to rise.

9.11. Air Resistance. Resistance of Water

It was mentioned above (see Sec. 2.39) that,when a solid moves in air, the air resistance is exerted on this body in the direction opposite to its motion. A similar force appears when an air jet encounters a body at rest. Naturally, in this case the air resistance is directed along the flow. The resistance emerges, firstly, due to the friction of air against the surface of the body, and, secondly, due to a change in the flow introduced by the body. In an air flow distorted by the presence of a body, the pressure at the upstream part of the body increases and at its downstream end decreases in comparison with the pressure in an unperturbed flow. Thus, the pressure difference is created, which stagnates the moving body or entrains the body immersed in a flow. The motion of air behind the body becomes random and eddy.

The resistance (drag) depends on the flow velocity and on the size and

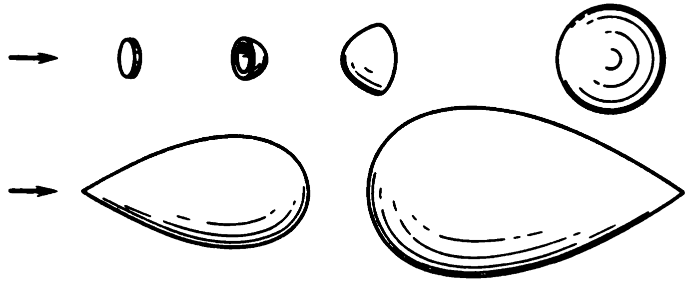

Fig. 334.
The bodies in the figure offer the same resistance to the motion of air.

shape of the body. Figure 334 illustrates the effect of the shape. For all the bodies presented in the figure, the resistance to motion is the same despite quite different dimensions of the bodies. An explanation is provided by Fig. 335 which shows a plate and a "streamlined" body in an air flow. The streamlines bounding the air jets are seen in the figure. The figure shows that the "streamlined" body almost does not disturb the regular flow; therefore, the pressure on the downstream part of the body is only slightly less than the pressure on the upstream part, and the drag is small. On the contrary, behind the plate a region of eddy motion of air is formed, where the pressure drops sharply.

Various fairings fixed to the protruding parts of an aeroplane are just intended to eliminate vortex formation in a flow. In general, designers try to leave on the surface as few projecting parts and irregularities as possible to reduce vortex formation (retractable undercarriage and streamlined shape).

It turns out that the major role here is played by the downstream part of

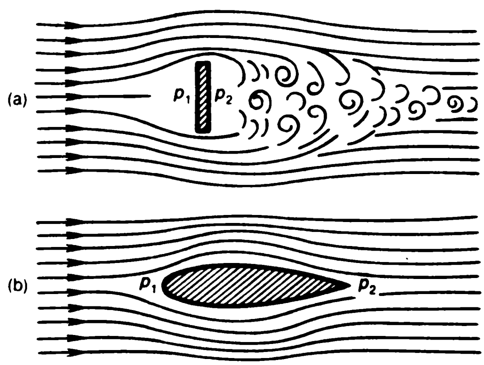

Fig. 335.
(*a*) Vortices are formed behind a plate placed in a flow; the pressure p_2 is considerably lower than p_1. (*b*) A streamlined body is flown about uniformly by a flow; the pressure p_1 exceeds p_2 only slightly.

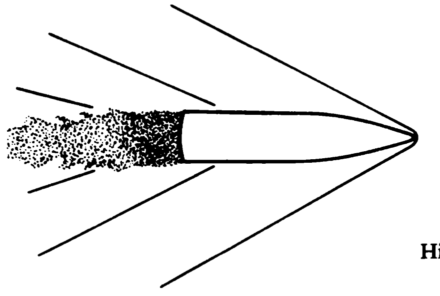

Fig. 336.
High-power acoustic waves are generated near a projectile flying at a supersonic velocity.

a moving body since the pressure drop in its neighbourhood is larger than the increase in pressure at the upstream part (only if the velocity of a body or of an incoming flow is not very high). For this reason, it is most important to give a streamlined shape to the downstream part of the body. The effect of air resistance is especially strong for ground transport: as the velocity of a motor car increases, an increasingly large portion of the engine power is spent to overcome the air resistance. For this reason, modern motor cars have, whenever possible, a streamlined shape.

If a motion occurs at a velocity exceeding the velocity of sound (supersonic velocity of bullets, projectiles, rockets and aeroplanes), the air resistance increases considerably since a flying body generates in this case high-power acoustic waves which carry away the energy of the moving body (Fig. 336). In order to reduce the air resistance at a supersonic velocity, the upstream part of a moving body must be sharpened, while for lower velocities the most important is, as was mentioned above, the sharpened shape of the downstream part of the body (streamlined form).

When a body moves in water, the resistance opposing the motion of the body also emerges. If a body moves under water (fish or submarines), the drag is due to the same reasons as the air resistance, viz. the friction of water against the surface of the body and flow distortions creating an additional drag. Fast fish (like sharks or sword-fish) and cetaceans (dolphins and killer whales) have a streamlined shape of the body, which reduces the resistance of water to their motion. Submarines are also given a streamlined form. The resistance to motion of a given body in water is much stronger than the resistance offered by air at the same velocity due to the high density of water in comparison with the density of air.

Ordinary ships floating on the surface of water experience an additional *wave drag*. A ship moving over a sea or river generates waves which spread over the water surface (Fig. 337). A fraction of power developed by a ship engine is wasted on the generation of these waves.

There is a similarity in the wave drag experienced by a ship and the resistance emerging due to acoustic waves in a high-speed flight of a projec-

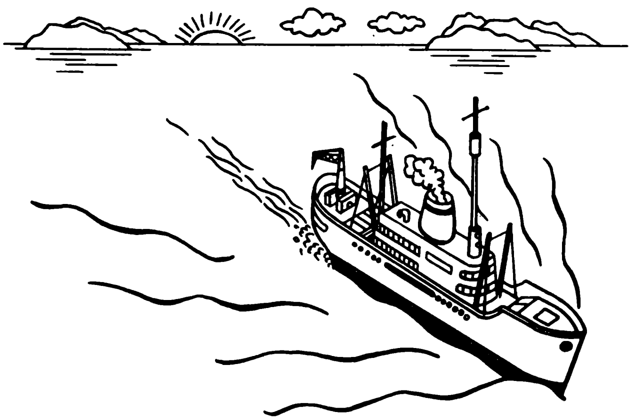

Fig. 337.
Waves propagating from a moving ship carry away energy.

tile: in both cases, the energy of a moving body is spent on generating waves in the medium. However, a ship produces waves at any velocity of motion, while acoustic waves are generated only at a supersonic velocity of a projectile. This difference is due to the fact that the ship creates waves on the surface of water when it sets in motion the interface between water and air. In the case of a flying projectile, there is no such interface. In order to reduce the wave drag which may constitute 3/4 of the total resistance for high-speed ships, the hull is given a special shape. The underwater bow part of the ship is sometimes given a "bulb" shape (Fig. 338). This considerably impedes the formation of waves on the surface of water, and hence the drag.

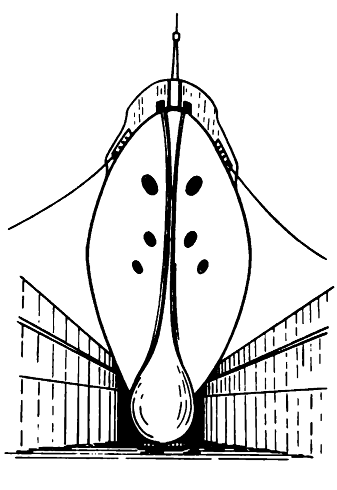

Fig. 338.
"Bulb-shaped" bow of a high-speed ship.

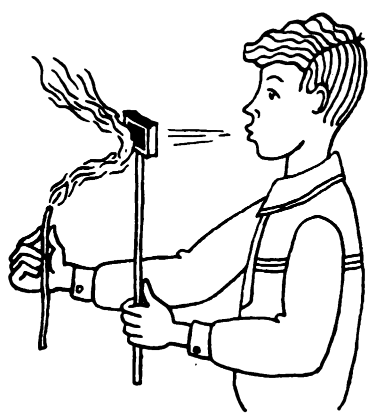

Fig. 339.
To Exercise 9.11.1.

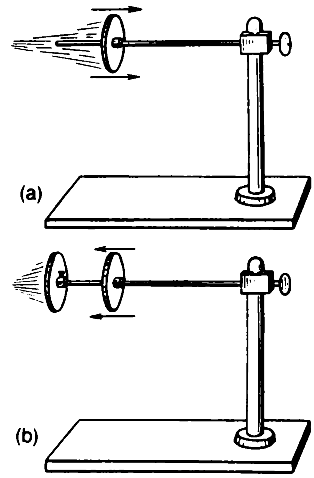

Fig. 340.
To Exercise 9.11.2.

? **9.11.1.** If we blow on a match-box behind which an ignited plait is placed, the jet of smoke is bent towards the box (Fig. 339). Explain this phenomenon.
9.11.2. A light circle is put on a knitting needle so that it can slide freely along it. If we blow on the circle from the left, it will slide along the needle to the right (Fig. 340*a*). If we place a screen in front of the circle and blow again from the left, the circle will slide to the left and will be pressed against the screen (Fig. 340*b*). Explain this phenomenon.

9.12. Magnus Effect and Circulation

In the preceding section, we analysed the force emerging in a body placed in a flow, viz. the drag force of air which has the *same direction* as the flow velocity. This, however, takes place when the body is completely symmetrical relative to the flow. If, however, the body has an asymmetrical shape or is arranged asymmetrically relative to the flow, the force acting on the body is directed at an angle to the flow.

For example, such is the force exerted on the wing of an aeroplane flying horizontally by the head flow of air. Figure 341 shows a sectional view (profile) of a wing and the force **F** acting on it. This force acts at a large angle to the horizontal. It can be decomposed in two components: the vertical component $\mathbf{F}_1$ and the horizontal component $\mathbf{F}_2$. The vertical component (normal to the direction of the flow) is called the *lifting force* (*lift*). It is due to the lifting force emerging when a fluid flows about a body that the creation of heavier-than-air aircraft has become possible. The horizontal component directed along the flow is termed the *drag*, or *head resistance*. The emergence of the drag was explained earlier. We must now clarify the emergence of the lift directed perpendicularly to the flow. For this purpose,

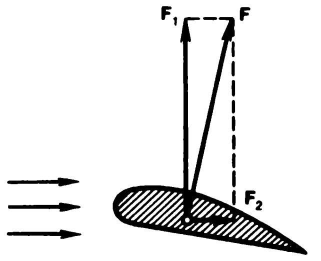

Fig. 341.
The decomposition of the force **F** acting on the wing of an aeroplane into the lifting force $\mathbf{F}_1$ and the drag $\mathbf{F}_2$.

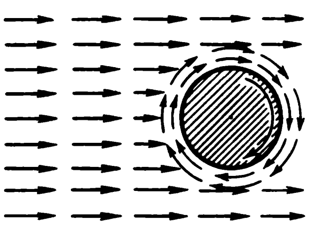

Fig. 342.
When the cylinder rotates, the velocity of entrained air is added to the flow velocity above the cylinder and subtracted from it below the cylinder.

we shall first consider a rotating cylinder in a uniform air flow (Fig. 342). In this case, the motion of air is comparatively simple, and the direction of forces can be easily determined.

In its rotation, the cylinder entrains the adjacent layers of air. As a result, the surrounding air starts to rotate about the cylinder. In the regions where the velocities of translatory and rotary motions are added, the resultant velocity of air exceeds the velocity of the incoming flow. On the opposite side of the cylinder, the velocities are subtracted, and the resultant velocity is lower than the flow velocity away from the cylinder.

Figure 343 represents the obtained distribution of streamlines. In the regions where the velocity is higher, the streamlines are denser. But Bernoulli's law implies that in the regions where the velocity is higher, the pressure is lower, and vice versa. Consequently, unequal forces are exerted on the opposite parts of the cylinder. Their resultant, which is perpendicular to the flow, is the lift.

A lift normal to the flow also appears during the rotation of any other body. The emergence of a force perpendicular to the flow in which a body rotates is known as the *Magnus effect*. This effect was observed for the first time while studying the flight of rotating shells: a lift exerted by the oncoming flow deflected a shell from the aiming line. On a smaller scale,

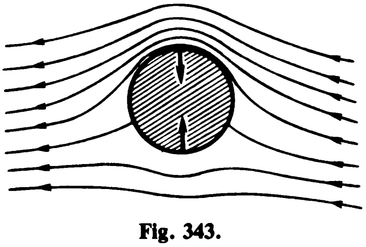

Fig. 343.
Streamlines are denser on the side of the rotating cylinder where the flow velocity is higher. In this region, the pressure is lower.

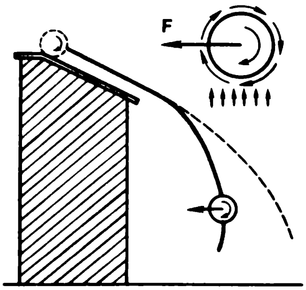

Fig. 344.
The Magnus effect for a falling rotating cylinder.

the Magnus effect can be observed for a spinning foot ball or tennis ball which is deflected from its flight path.

The Magnus effect can be easily observed with the help of the experiment represented in Fig. 344. A light paper cylinder rolling down an inclined board is deflected from its flight path (dashed line) and moves along a steeper trajectory (solid line). The oncoming flow of air is directed upwards relative to the cylinder, and the cylinder rotates clockwise. Therefore, the lifting force **F** is directed from right to left.

The emergence of lift is associated with the presence of circular motion of air flow about a streamlined body. This circular motion, being superimposed on the flow, creates a difference in the velocity of the flow on two sides of the body, thus producing a pressure difference responsible for a lifting force. The circular motion of a flow round a body is called *circulation*. In the Magnus effect, circulation (and hence the lifting force) appears due to the rotation of the cylinder. In other cases, circulation can be caused not by the rotation of a body but by some other reasons. For a lifting force to emerge, it is only important for a flow to have a circulation. Then the velocity distribution is always such that the created difference in pressures gives rise to a force perpendicular to the flow.

9.13. Lifting Force of a Wing and the Flight of an Aeroplane

Let us now consider the air flow about the wing of an aeroplane. Experiments show that, when a wing is in air flow, vortices emerging at its sharp trailing edge circulate counterclockwise (in the case shown in Fig. 345). These vortices grow in size, are separated from the wing and entrained by the flow. The remaining mass of air near the wing acquires thereby the opposite rotation (in the clockwise direction), forming a circulation in the vicinity of the wing (Fig. 346). Being superimposed on the air flow, the circulation causes the distribution of streamlines shown in Fig. 347.

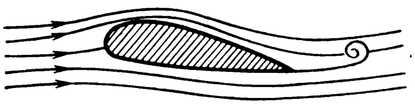

Fig. 345.
A vortex is formed at a sharp edge of the wing profile.

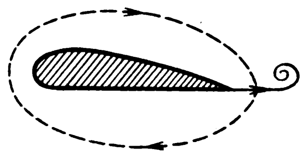

Fig. 346.
Vortex formation leads to air circulation around the wing.

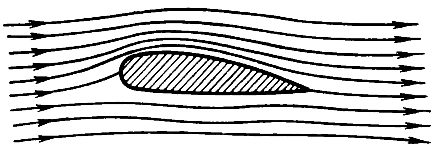

Fig. 347.
The vortex is entrained by the flow and streamlines uniformly flow about the wing profile; they are more dense above the wing and less dense below it.

We obtained the same pattern of streamlines for the wing as for a rotating cylinder. Here too the circulation about the wing is superimposed on the total air flow. In this case, however, the circulation appears not as a result of rotation of the body (as in the case with the rotating cylinder) but due to vortex formation in the vicinity of a sharp edge of the wing. As a result of circulation, air is accelerated above the wing and retarded under it. Hence the pressure becomes lower above the wing and higher under it. The resultant **F** of all the forces exerted by the flow on the wing (including friction) has an upward direction, being slightly inclined backwards (see Fig. 341). Its component normal to the flow is the lifting force $\mathbf{F}_1$, while the component along the flow is the drag $\mathbf{F}_2$. The higher the velocity of the oncoming flow, the stronger the lifting force and the drag. Besides, these forces depend on the shape of the wing profile and on the angle at which the flow meets the wing (the *angle of attack*), as well as on the density of the oncoming flow. The higher the density, the stronger these forces. The wing profile is chosen so that it ensures the maximum possible lift at the weakest possible drag. The theory of the emergence of the lifting force of the wing was developed by Nikolai Zhukovskii (1847-1921), the founder of the theory of aviation and of the Russian school of fluid dynamics.

We are now in a position to explain how an aeroplane flies. The propeller of the plane rotated by the engine or the jet reaction impart to the plane a velocity such that the lifting force becomes equal to the weight of the plane or even exceeds it. Then the plane takes off. In a uniform rectilinear motion of the plane, the sum of all the forces exerted on it is equal to zero in accordance with Newton's first law. Figure 348 shows the forces acting on a plane in a horizontal flight at a constant velocity. The thrust **f**

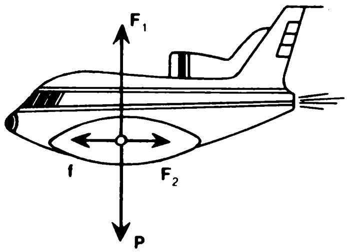

Fig. 348.
The forces acting on an aeroplane in a uniform horizontal flight.

of the engine is equal and opposite to the air drag $\mathbf{F}_2$ for the entire plane, while the force of gravity $\mathbf{P}$ is equal and opposite to the lifting force $\mathbf{F}_1$.

Aeroplanes designed for flights at different velocities have different sizes of the wings. Slow cargo aeroplanes must have the wings of a larger area since for a low velocity the lifting force per unit area of the wing is not large. High-speed aeroplanes acquire a sufficient lifting force at a small area of the wings. Since the lifting force of a wing decreases with the air density, at high altitudes a plane must fly at a higher velocity than near the ground.

The lifting force emerges also when a wing moves in water. This is used in hydrofoil craft. The hull of these vessels is lifted above water during motion (Fig. 349). This reduces the resistance offered by water to the motion of the vessel and makes it possible to attain a high velocity. Since the density of water is much higher than the density of air, a sufficiently high lifting force can be obtained for a comparatively small area of the hydrofoil and a moderate velocity.

The duty of the propeller of an aeroplane is to impart a high velocity to the plane, at which the wing develops a lifting force balancing the weight of the plane. For this purpose, the propeller of the plane is fixed on a horizon-

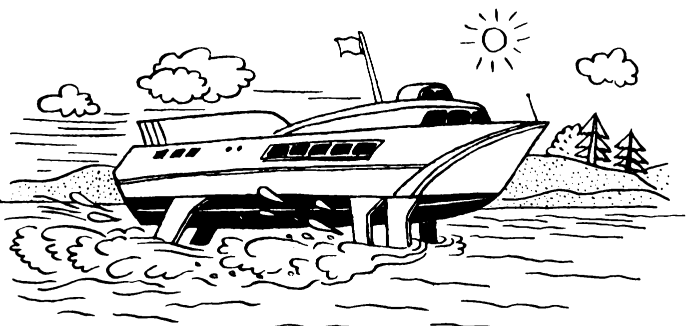

Fig. 349.
Hydrofoil craft.

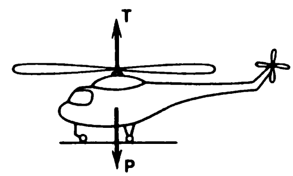

Fig. 350.
Schematic diagram of a helicopter.

tal shaft. There exists a type of heavier-than-air aircraft which do not need wings. These are helicopters (Fig. 350). The shaft of the screw (rotor) in a helicopter is arranged vertically, and the rotor creates a thrust directed upwards and balancing the weight of the helicopter thus substituting the lifting force. The helicopter rotor creates a vertical thrust irrespective of whether the helicopter moves or is at rest. For this reason, a helicopter can hang in air at rest or ascend vertically. For a horizontal displacement of the helicopter, a horizontal thrust must be produced. For this purpose, no special propeller with the horizontal shaft is required. It is sufficient to slightly change the angle of the blades of the rotor, which can be done with the help of a special mechanism in the hub. [4]

9.14. Turbulence in a Fluid Flow

Watching from a large distance the smoke rising from a factory chimney and carried away by wind, we see a continuous jet uniformly flowing out of the chimney and aligning with the wind. The smoke makes the motion of air visible, and from a large distance, when small-scale movements cannot be distinguished, it appears as a smooth motion of individual jets. The ribbon of smoke is just one of such jets.

Let us now come closer to the chimney and look attentively at the details of motion of air in the smoke jet. We shall see that random puffs of smoke are mixing with one another. The swirling mass is carried away in the form of a jet by the incoming air flow (wind). From a distance we could see only general regular motion. Nearby, it can be seen that individual regions of the jet are also in a random movement, now overtaking the jet, now lagging behind it. This phenomenon, viz. the presence of disorderly movements in a flow, is known as the *turbulence* of the flow.

Turbulence causes the mixing of a flow. For example, random movements of air in a smoke jet carry the smoke particles in all directions. The jet expands and turns out to be blurred at a large distance from the chimney. This result of turbulence can be seen even from a distance.

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[4] A small propeller on the horizontal shaft operating during the flight of a helicopter is only intended to prevent the helicopter from rotating in the direction opposite to that of the rotor.

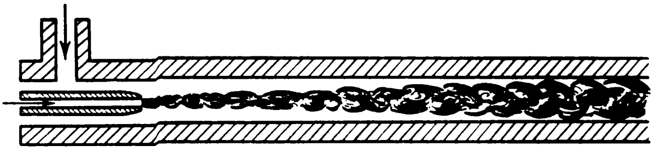

Fig. 351.
Turbulent water flow.

Turbulence is a frequently encountered phenomenon. The motion of air in the presence of wind is always turbulent. When a body moves in air, a turbulent wake is formed behind this body. This phenomenon is especially pronounced for nonstreamlined bodies. The large drag for such bodies (see Sec. 9.11) is due to this phenomenon. The flow of water in a river and motion of water in the pipes of a water supply system are examples of a turbulent flow. Turbulence in a fluid flow is absent only under certain conditions (see the next section).

To be able to observe turbulence directly, we must render the flow of water or air visible. In air, this can be easily done with the help of smoke. In water, small jets can be made visible by introducing a paint or ink. If we pass a fast water flow through a glass tube and introduce into it a jet of ink through a small pipe, the spread of the ink jet will indicate the presence of turbulence (Fig. 351).

9.15. Laminar Flow

Let us reduce the velocity of the water flow in the experiment described at the end of the previous section. It can be seen that starting with a certain velocity, the ink jet does not spread any longer but flows uniformly along the tube (Fig. 352). This means that at a low velocity of the flow, its turbulence vanishes and the so-called *laminar flow* is observed. If the flow velocity is increased again, the flow becomes turbulent as before. Experiments show that in narrow tubes turbulence vanishes at a higher velocity than in wide tubes. The motion of fluid in capillary tubes is always laminar. It was shown experimentally that in viscous liquids (like oil or glycerol) a flow in a tube can remain laminar at considerably higher velocities than in high-fluidity liquids (like water or alcohol). It is interesting to note that in a normal blood circulation, blood flows in arteries without turbulence.

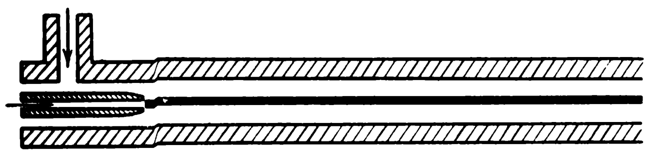

Fig. 352.
Laminar water flow.

Part Two

Heat. Molecular Physics

Chapter 10

Thermal Expansion of Solids and Liquids

10.1. Thermal Expansion of Solids and Liquids

Simple experiments and observations show that as the temperature increases, the dimensions of bodies slightly increase, while upon cooling the bodies shrink to the initial size. For example, a red-hot bolt does not fit its nut. When recooled, the bolt fits the nut again. In a hot summer day, telegraph wires sag to a larger extent than during winter frosts. The increase in the sag, and hence in the length of stretched wires upon heating can be easily demonstrated with the help of experiment shown in Fig. 353. Heating a stretched wire by an electric current, we see that it sags noticeably. When heating is discontinued, the wire is stretched without a sag again.

Not only the length but also other linear dimensions of a body increase upon heating. The change in the linear dimensions of a body as a result of heating is called *linear expansion*.

If a homogeneous body (say, a glass tube) is uniformly heated in all its parts, it expands, retaining its shape. The situation is different if the heating is nonuniform. Let us consider the following experiment. A glass tube is fixed at one end in the horizontal position. If the tube is heated from below as shown in Fig. 354, its upper part remains cold due to a low thermal conductivity of glass, and the tube is bent upwards. It can be easily

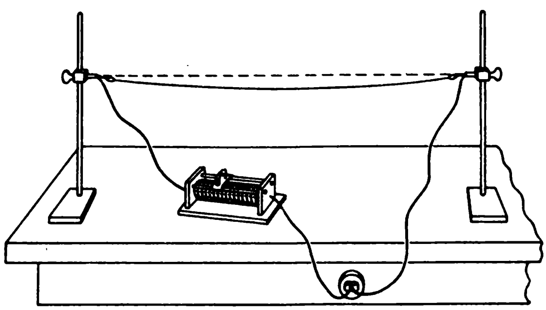

Fig. 353.
A wire heated by an electric current elongates and sags. After the current has been switched off, it becomes of the initial length.

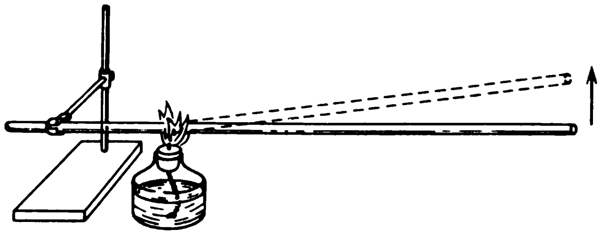

Fig. 354.
A glass tube heated from below noticeably bends upwards.

seen that the lower part of the bent tube is compressed since it cannot expand to the same extent as it would in the absence of the upper part. The upper part, on the contrary, is stretched.

Thus, a nonuniform heating of bodies causes *stresses* in them, which may lead to their destruction if these stresses become too high. For example, a glassware into which hot water has been poured is in the stress state at the initial moment and sometimes cracks. This occurs due to the fact that at first the inner part is heated and expands, thus stretching the outer surface of the glassware. The stress as a result of heating can be avoided if we take glassware with such thin walls that they are rapidly heated through the entire thickness (chemical glassware).

For the same reason, common glassware cracks if we try and heat liquids in it on fire or on a hot plate. There exist, however, special kinds of glass (the so-called *quartz glass* containing up to 96% SiO_2) which expand upon heating so insignificantly that the stresses emerging as a result of nonuniform heating of the glassware are harmless. Water can be safely boiled in a saucepan made of quartz glass.

The linear expansion upon the same increase in temperature occurs differently for different materials. This can be demonstrated, for example, with the help of the following experiment. Two different plates (e.g. an iron and a copper plate) are rivetted together at several points (Fig. 355*a*). At room temperature the plates are straight, but as a result of heating they are bent as is shown in Fig. 355*b*. This means that copper expands to a larger extent than iron. It also follows from this experiment that if the temperature of a body consisting of several parts which expand differently when heated changes, internal stresses appear in the body. In the experiment represented in Fig. 355, the copper plate is compressed while the iron plate is stretched. As a result of different expansivities of iron and enamel, stresses emerge in enamel saucepans. At strong heating, the enamel sometimes breaks off.

The stresses appearing in solids as a result of thermal expansion can be very strong. This should be taken into account in many branches of

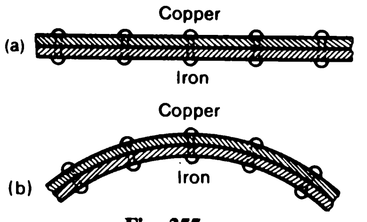

Fig. 355.
(*a*) A plate formed by rivetted copper and iron bands in the cold state. (*b*) The same plate when heated (the bending is exaggerated for the sake of clarity).

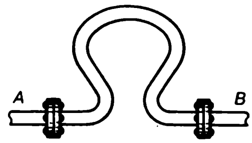

Fig. 356.
An expansion loop in a steam line allows pipes A and B to expand.

engineering. It sometimes happens that the parts of steel bridges rivetted together in the daytime break down at night upon cooling, tearing off a large number of rivets. In order to avoid such situations, special measures should be taken to allow the parts of structures to expand or shrink freely as a result of temperature variations. For instance, steel stream lines are supplied by springy bends in the form of loops (expansion loops, Fig. 356).

An increase in linear dimensions is accompanied by an increase in volume of bodies (*volume expansion*). It is senseless to speak about the linear expansion of a liquid since liquids do not have a definite shape. The volume expansion of liquids can, however, be easily observed. Let us fill a flask with coloured water and close it with a plug into which a glass tube is inserted so that the liquid enters the tube (Fig. 357*a*). If we immerse the flask into a vessel with hot water, at first the level of the liquid in the tube will drop and then start to rise (Fig. 357*b* and *c*).

The decrease in the level of liquid at the initial moment indicates that at first the flask expands, while the liquid has no time to be heated throughout. Later the liquid is heated also. The increase in its level indicates that the liquid expands to a larger extent than glass. Different liquids expand upon heating to different extents (for example, kerosene expands to a larger extent than water).

If a liquid is heated in a closed vessel which prevents it from expanding, huge stresses (forces of pressure) appear in it like in solids. These forces act on the vessel walls and may destroy them. For this reason the pipes of hot water heating systems are always supplied with a surge tank connected to the upper part of the system and communicating with the atmosphere (Fig. 358). As water in the system is heated, a part of it flows into the surge tank, thus eliminating the stressed state of water in the pipes.

? **10.1.1.** When a guitar is taken out of a warm room outdoors, its steel strings are stretched more tightly. What conclusion can be drawn from this fact about the difference between the thermal expansivities of steel and wood?

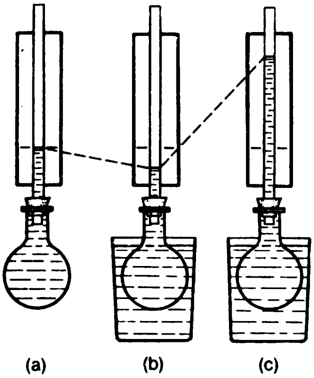

Fig. 357.
(*a*) Coloured water fills the flask and a part of the tube inserted through the plug. (*b*) The flask is immersed in hot water. At the initial moment, the liquid in the tube lowers. (*c*) In a certain time, the level of liquid in the tube becomes higher than before heating the flask.

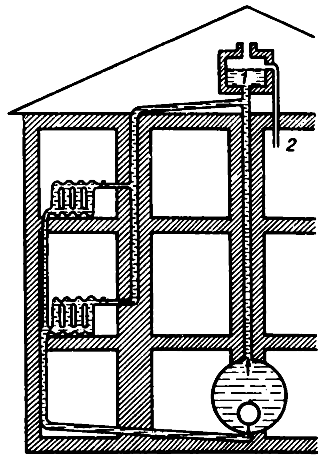

Fig. 358.
A schematic diagram of a hot-water heating system in a two-storey building. Surge tank *1* is mounted in the attic. The excess of water flows from the tank through pipe *2*.

10.1.2. Steel strings in a piano are stretched over an iron frame. Will the tension in the strings change if the temperature alters slowly enough for the frame to remain at the same temperature as the strings (iron expands almost in the same way as steel)?

10.1.3. The alloy platinoid expands upon heating in the same way as glass and is used for sealing electrodes in an electric bulb. What will happen if copper wire is used instead of platinoid (copper expands noticeably more than glass)?

10.1.4. How would the experiment illustrated in Fig. 357 change if we took a flask made of quartz glass?

10.1.5. In engineering, bimetallic strips consisting of two thin plates of different metals are often used, which are welded together over the entire contact surface. Figure 359 shows a schematic diagram of a thermal relay, viz. a device which automatically switches off an electric current for a short time if the value of the electric current for some reason exceeds the admissible value: *1* — bimetallic strip, *2* — small heating element which is heated only slightly if the current is within the permissible limit, *3* — contact. Analyse the operation of the relay. On which side of plate *1* must be the metal which expands more?

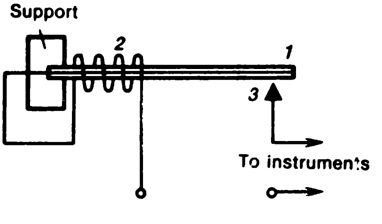

Fig. 359.
Simplified diagram of a thermal relay.

10.2. Thermometers

Thermal expansion of bodies is used in instruments for determining the temperature of bodies, viz. thermometers. The bimetallic strip shown in Fig. 355 may serve as a rough thermometer as well as a flask filled with water. An ordinary liquid thermometer consists of a small glass bulb connected with a glass tube having a capillary tube (Fig. 360). The bulb and a part of the tube are filled with a liquid (e.g. mercury, alcohol, or toluene). The position of the upper level of liquid in the tube indicates the temperature of the medium into which the thermometer is immersed.

The scale is graduated in the following manner. Zero is put against the point on the scale where the liquid column stands when the bulb is immersed in melting snow. The point on the scale corresponding to the height of the liquid column when the bulb is in the vapour of water boiling under the normal pressure (760 mm Hg) is marked by figure 100. The interval between these marks is divided into a hundred of equal parts called *degrees*.[1] The same divisions are etched below the zero point and above the point corresponding to 100 °C. The letter "C" stands for the name of the scientist who proposed such a method of dividing the scale (Celsius). Such a thermometer is known as the *Celsius*, or *centigrade thermometer*. Besides the Celsius scale, in the UK and USA the Fahrenheit scale is still in use (°F), in which the melting point of ice corresponds to 32 °F, while the boiling point for water is 212 °F.

Naturally, the thermometer described above can be used only for such temperatures at which the substance filling the thermometer is in the liquid form. For instance, a mercury thermometer cannot measure the temperature below −39 °C since at a lower temperature mercury solidifies.

The definition of a degree given above is arbitrary to a certain extent.

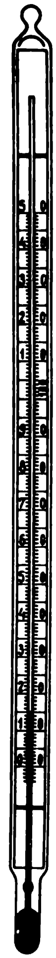

Fig. 360.
Laboratory liquid thermometer.

[1] The SI unit of temperature is a kelvin (K) (see Sec. 13.14). A kelvin coincides with a degree centigrade: 1 K = 1 °C. — *Eds.*

The height to which the liquid in the tube of a thermometer rises depends on the properties of the liquid and on the grade of glass of which the thermometer is made. We obviously cannot expect that the readings of two thermometers calibrated by the above method will coincide if the thermometers are manufactured from different materials, irrespective of the accuracy with which they are made.

Indeed, if we divide, for example, the segment between 0 °C and 100 °C in a mercury thermometer into a hundred equal parts, this does not mean at all that for any other substance the divisions should have the same length. Therefore, it is necessary to choose a certain thermometer and compare all other thermometers with it. For such a reference thermometer, the *gas thermometer* has been chosen, i.e. the thermometer in which the change in the gas pressure with rising temperature is used for measurements. The gas thermometers will be described in Sec. 13.15. The readings of a meticulously prepared mercury thermometer differ but slightly from the readings of a gas thermometer. Liquid thermometers may have different shapes and sizes depending on their duty. The values of divisions (scale factors) of their scales are also different (1 °C, 0.1 °C, and sometimes even 0.01 °C).

It goes without saying that a thermometer indicates the temperature of the part of a liquid with which it is in contact. Therefore, if we want to know the temperature of a liquid occupying a large volume, this liquid should be thoroughly mixed to ensure the uniform temperature over the entire volume. It is impossible to take the readings of a conventional thermometer having taken it out of the liquid whose temperature is being measured, since its readings will change.

Sometimes thermometers indicating the *maximum* or *minimum* temperature of the medium are prepared. The thermometers of this type include the widely used *clinical* thermometer. A thin glass fibre, which partially enters the thermometer tube making its capillary tube narrower, is sealed in the bulb (Fig. 361). A considerable pressure is required to let the mercury back to the bulb through the capillary tube (this will be shown later when we shall study the properties of liquids). For this reason, when the thermometer is cooled, the mercury column will be discontinued at the constriction and will remain in the tube (Fig. 362). Thus, the mercury column indicates the maximum temperature of a patient, indicated by the thermometer. In order to return the mercury to the bulb, the thermometer should be shaken.

?

10.2.1. Using a powerful magnifying glass, observe the construction of a clinical thermometer. If the instrument had been used for taking the temperature of a patient and was not shaken, the glass wire entering the tube will be seen through the magnifying glass.

10.2.2. The normal temperature of a human body is about 37 °C. What value on the Fahrenheit scale corresponds to this temperature?

10.2.3. Why does a clinical thermometer break if its bulb is heated to a temperature above 43 °C? How can one avoid the destruction of a thermometer upon heating it to a too high temperature?

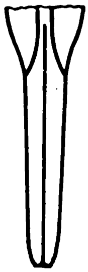

Fig. 361.
Schematic diagram of the bulb of a clinical thermometer (without mercury). The glass fibre can be seen whose end enters the tube of the thermometer.

Fig. 362.
The bulb of a clinical thermometer filled with mercury at room temperature. The glass fibre keeps the mercury column in the tube and does not let it into the bulb.

10.3. Formula of Linear Expansion

Measurements show that the same body expands differently at different temperatures: the thermal expansion is more pronounced at higher temperatures than at lower ones. However, the difference in expansion is not very large, and for relatively small temperature variations we can neglect this difference and assume that *the change in the dimensions of a body is proportional to the change in temperature*.

Let us denote by l the length of a body at the initial (say, room) temperature t, while the length of the same body at a temperature t' will be l'. The elongation of a body as a result of heating through $t' - t$ is $l' - l$. The elongation of this body upon heating through 1 K under the assumptions made earlier will be smaller by a factor of $t' - t$, i.e. it will be equal to $(l' - l)/(t' - t)$. This is the total elongation of the body *as a whole*, which is the larger the longer the body.

In order to obtain the characteristic of the thermal expansivity of the *substance* from which the body is made, we must determine the *relative* elongation, i.e. the ratio of the observed elongation to the length of the body under certain "normal" conditions. The "normal" length is assumed to be the length of the body at 0 °C and is denoted by l_0. Thus, the thermal expansivity of the material is characterised by the quantity $\alpha = (l' - l)/l_0(t' - t)$. It is called the *temperature coefficient of linear expansion* and shows the fraction of the normal length by which the length of the body rises upon heating through 1 K. Since the thermal expansion for most bodies is insignificant, the length l_0 at 0 °C differs from the length l at other (for example, room) temperature only slightly. Therefore, l_0 in the expression for the temperature coefficient of linear expansion can be replaced by l, which gives

$$\alpha = \frac{l' - l}{l(t' - t)} . \tag{10.3.1}$$

In order to determine the coefficient α, we must measure the length l of a rod made of the material under investigation when the same temperature t is maintained throughout its volume. Then the elongation $l' - l$ caused by the change in temperature from t to t' should be measured with the same relative error of measurement. In order to improve the accuracy in measuring the elongation $l' - l$, which is normally very small, special technique has to be used (for example, microscopic measurements of the displacement of the end of the rod whose other end is fixed). Table 3 contains the temperature coefficients of linear expansion for some substances.

Table 3. Temperature Coefficients of Linear Expansion for Some Substances

Substance	α, 10^{-5} K^{-1}
Aluminium	2.4
Brass	1.8
Copper	1.7
Glass, ordinary (approximately)	1.0
quartz	0.07
Invar (iron-nickel alloy)	0.09
Iron	1.2
Lead	2.9
Porcelain	0.3
Superinvar (iron-nickel alloy with chromium admixture)	0.003
Tungsten	0.4
Wood, along fibres	0.6
across fibres	3.0
Zinc	3.0

Pay attention to extremely low values of the temperature coefficients of linear expansion for Invar, Superinvar and quartz glass. Invar is used in precision instruments (for example, in pendulums of accurate clocks) whose readings must be temperature independent. Standards of length used for high-accuracy measurements (as in geodesy) are also made of Invar. Quartz ware does not crack at very sharp changes of temperature. It remains intact if, for example, a red-hot ware is immersed in water. This is explained by a small temperature coefficient of linear expansion of quartz, owing to which only insignificant stresses emerge even if neighbouring parts considerably differ in temperature.

Knowing the coefficient of linear expansion, we can calculate the length of a body at any temperature within not very wide temperature interval. We transform formula (10.3.1) as follows:

$$l' - l = l\alpha(t' - t) \quad \text{or} \quad l' = l[1 + \alpha(t' - t)].$$

Denoting the temperature increment $t' - t$ by τ, we can write

$$l' = l(1 + \alpha\tau). \tag{10.3.2}$$

We have obtained the *formula of linear expansion*. The expression in the parentheses is called the *binomial of linear expansion*. The expansion binomial indicates the factor by which the length of a body increases if the temperature increment is equal to τ.

Formula (10.3.2) can also be used when the length of a given body after cooling has to be determined. In this case, the temperature increment τ should be assumed to be negative (the new temperature t' is lower than the initial temperature t). Clearly, the binomial is then less than unity, which corresponds to a reduction in the length of the body upon cooling.

We limit ourselves to an analysis of *small* temperature variations for which the temperature coefficient of linear expansion can be assumed constant. When the temperature changes significantly, this does not take place. For example, the temperature coefficient of linear expansion for iron is equal to $0.3 \times 10^{-5}\ \mathrm{K}^{-1}$ for temperatures of about $-200\ °\mathrm{C}$, $1.2 \times 10^{-5}\ \mathrm{K}^{-1}$ for temperatures close to 0 °C, and $1.6 \times 10^{-5}\ \mathrm{K}^{-1}$ for temperatures near 600 °C. Therefore, formula (10.3.2) can be used only for small temperature variations, taking different values of the coefficient for different temperature ranges.

? **10.3.1.** The lengths of an iron and zinc rods must be the same at 0 °C, while their difference at 100 °C must be 1 mm. What lengths of the rods at 0 °C satisfy this condition?
10.3.2. The inner diameter of a hollow copper cylinder is 100 mm at 20 °C. In which temperature range is the deviation from this value below 50 μm?
10.3.3. Using a vernier calliper intended for the operation at 20 °C the length of an object at -20 °C was measured. The reading was 19.97 cm. What is the length of the body being measured?

10.4. Formula for Volume Expansion

By analogy with the temperature coefficient of linear expansion, we can introduce the *temperature coefficient of volume expansion*, which characterises the change in volume with temperature. Experiments show that as in the case of linear expansion we can roughly assume that *the increment of the volume of a body is proportional to the temperature increment* within not very wide temperature range.

Denoting by V the volume of the body at the initial temperature t, by

V' its volume at the final temperature t', and by V_0 the volume at 0 °C ("normal" volume), we can write the coefficient of volume expansion β in the form $\beta = (V' - V)/V_0(t' - t)$. Since solids and liquids expand insignificantly, the volume V_0 at 0 °C differs only slightly from the volume at another (say, room) temperature. Therefore, in the expression for the temperature coefficient of volume expansion we can substitute V for V_0, which is more convenient for practical purposes. Thus,

$$\beta = \frac{V' - V}{V(t' - t)}. \tag{10.4.1}$$

It should be noted that gases expand to such an extent when heated that the replacement of V_0 by V involves a noticeable error. Therefore, for gases such a simplification can be made only for small temperature intervals (see Sec. 13.12). Formula (10.4.1) gives

$$V' = V[1 + \beta(t' - t)].$$

Denoting, as in Sec. 10.3, the temperature increment $t' - t$ by τ, we can write

$$V' = V(1 + \beta\tau). \tag{10.4.2}$$

We have obtained the *formula for volume expansion* which is used to calculate the volume of a body if we know its initial volume and the temperature increment. The expression $1 + \beta\tau$ is known as the *binomial of volume expansion*.

As the volume of a body increases, its density decreases by the same factor as the volume increases. Denoting the density at a temperature t by ρ and that for t' by ρ', we obtain

$$\rho' = \frac{\rho}{1 + \beta\tau}.$$

Since $\beta\tau$ is normally much smaller than unity, for approximate calculations we can simplify this formula as follows:

$$\rho' = \frac{\rho(1 - \beta\tau)}{(1 + \beta\tau)(1 - \beta\tau)} = \frac{\rho(1 - \beta\tau)}{1 - \beta^2\tau^2}.$$

Neglecting $\beta^2\tau^2$ in comparison with unity, we obtain

$$\rho' = \rho(1 - \beta\tau). \tag{10.4.3}$$

As in the case of linear expansion, formulas (10.4.2) and (10.4.3) can be used also for cooling the bodies if we assume that the temperature increment τ is negative.

? **10.4.1.** A body whose temperature coefficient of volume expansion is β has a cavity of volume V. What will be the volume of the cavity if the temperature of the body increases by t?

10.5. Relation between Temperature Coefficients of Linear and Volume Expansion

Let us suppose that a small cube with edge l expands upon heating. Its initial volume is $V = l^3$. Having been heated through τ, each its side becomes $l(1 + \alpha\tau)$, and the volume becomes $V' = l^3(1 + \alpha\tau)^3$. Consequently,

$$\beta = \frac{V' - V}{V\tau} = \frac{l^3(1 + \alpha\tau)^3 - l^3}{l^3\tau} = \frac{(1 + \alpha\tau)^3 - 1}{\tau}$$

$$= \frac{1 + 3\alpha\tau + 3\alpha^2\tau^2 + \alpha^3\tau^3 - 1}{\tau} = 3\alpha + 3\alpha^2\tau + \alpha^3\tau^2.$$

It was shown above that α is a very small quantity. Moreover, since we consider only small temperature variations, the terms $3\alpha^2\tau$ and $\alpha^3\tau^2$ are small in comparison with 3α (for example, for $\alpha = 2.0 \times 10^{-5}\ \mathrm{K}^{-1}$ and $\tau = 100$ K, the term $3\alpha^2\tau$ is smaller than 3α by a factor of 500, while the term $\alpha^3\tau^2$ is smaller than 3α by a factor of 750 000). Therefore, we can ignore the terms $3\alpha^2\tau$ and $\alpha^3\tau^2$ in comparison with 3α and write

$$\beta = 3\alpha.$$

Thus, *the temperature coefficient of volume expansion is equal to thrice the temperature coefficient of linear expansion*. For example, for iron it is equal to $3.6 \times 10^{-5}\ \mathrm{K}^{-1}$.

? **10.5.1.** *Pycnometers* (specific gravity bottles) are used for measuring the density of liquids. A pycnometer is a glass vessel with a narrow neck on which the marks corresponding to certain capacities (10 ml, 50 ml, and so on[2]) are etched (Fig. 363). Suppose that the capacity of pycnometer is 50 ml at 20 °C. What is its capacity at 100 °C?

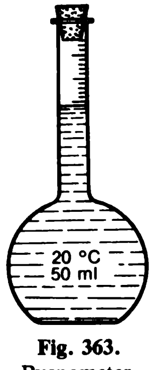

Fig. 363.
Pycnometer.

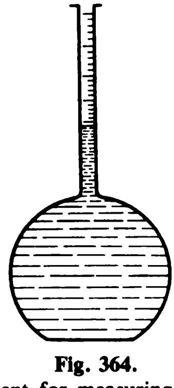

Fig. 364.
An instrument for measuring the temperature coefficients of volume expansion of liquids.

[2] The notation ml stands for a millilitre, i.e. 10^{-3} litre, or a cubic centimetre. — *Eds.*

10.6. Measurement of Temperature Coefficient of Volume Expansion for Liquids

Temperature coefficients of volume expansion for liquids can be measured as follows. A glass flask with a narrow cylindrical neck (Fig. 364) is filled with a liquid under investigation to a certain mark on the neck. Then the flask is heated and the elevation of the level of the liquid in it is marked.

If the initial volume of the vessel, the cross-sectional area of the flask neck and the temperature change are known, we can determine the fraction of the initial volume that has flowed from the flask to the neck upon heating through 1 K. However, the temperature coefficient of volume expansion for the liquid is *larger* than this quantity since the flask itself also expands. In order to determine the temperature coefficient of volume expansion for the liquid, we must add the coefficient of volume expansion for glass to this quantity. However, the temperature coefficient of volume expansion for glass is considerably smaller than its counterpart for liquid, and in rough calculations can be neglected. Table 4 contains the temperature coefficients of volume expansion for some liquids at 20 °C.

Table 4. Temperature Coefficients of Volume Expansion for Some Liquids

Liquid	β, 10^{-3} K^{-1}	Liquid	β, 10^{-3} K^{-1}
Alcohol	1.1	Kerosene	1.0
Ether	1.7	Mercury	0.18

? **10.6.1.** A pycnometer is filled with alcohol at 0 °C and weighed. Then it is immersed in a vessel with warm water. Using filter paper, some alcohol is taken from the pycnometer so that its level is at the same mark as before, and the pycnometer is weighed again. What is the temperature coefficient of volume expansion for alcohol if the following data are available: the empty pycnometer weighs 321 N, its weight with alcohol at 0 °C is 731 N, and with alcohol at 29 °C is 718 N? Thermal expansion of glass should be neglected.

10.7. Thermal Expansion of Water

The most abundant substance on the surface of the Earth, viz. water, has a peculiarity which distinguishes it from most of other liquids. It expands only when heated to temperatures above 4 °C. From 0 to 4 °C, the volume of water, on the contrary, decreases. Thus, *water has the maximum density at* 4 °C. This refers to fresh (chemically pure) water. Sea water exhibits the maximum density at about 3 °C. An increase in pressure also reduces the temperature corresponding to the maximum density of water.

The peculiarities of thermal expansion of water play an extremely important role for the climate of the Earth. The major part (79%) of the surface of the Earth is covered by water. Solar rays impinging on the surface of water are partially reflected from it and partially penetrate in the bulk of water and heat it. If the temperature of water is low, warmer layers (say, at 2 °C) are denser than colder layers (which are, say, at 1 °C), and hence they move downwards. Their place is taken by colder layers which, in turn, are heated. Thus, water layers continually change places, which promotes uniform heating of the entire volume of water until the temperature corresponding to the maximum density is reached. With a further heating, the upper layers become less and less dense and hence remain on the top.

As a result of this process, huge masses of water are heated by solar rays comparatively easily to the temperature of the maximum density of water. A further heating of lower-lying layers occurs at an extremely slow rate. On the contrary, the cooling of water to the temperature of maximum density proceeds at a comparatively high rate, and then the process of cooling slows down. As a result, starting from a certain depth, deep reservoirs on the surface of the Earth have a temperature close to the temperature corresponding to the maximum density of water (2-3 °C). The upper layers of warm seas may have a much higher temperature (30 °C and higher).

Chapter 11

Work. Heat. Law of Energy Conservation

11.1. Change of the State of Bodies

Analysing the motion of a body thrown upwards and then falling to the ground (see Sec. 4.16), we established that in the absence of air resistance, the sum of the kinetic and potential energies of the moving body remains unchanged. This law can be applied to any system of bodies on which no external forces are exerted and which move without friction. However, it was also pointed out that in the presence of friction or inelastic collisions this law does not hold: the sum of the kinetic and potential energies does not remain constant. For example, when a stone falls into snow or into sand both its kinetic and potential energies decrease, since it moves downwards and simultaneously loses its velocity.

On the contrary, the cases when the sum of kinetic and potential energies of bodies increases are also sometimes observed. For instance, if the cork and a part of the liquid burst out of a bottle with soda water standing on a table under the action of the pressure of carbon dioxide and rise to a certain height, the sum of the kinetic and potential energies of the system increases.

These changes in the mechanical energy are always accompanied by certain (quite diverse) changes in the states of bodies. For example, when the mechanical energy of a body decreases, heating of surrounding bodies is often observed. Thus, rubbing and colliding bodies become heated (the wheel axles of a carriage or a saw and a log). Having struck a piece of lead with a hammer several times and flattened it out, we can observe its heating. Bending and unbending a wire, we note that the site where the wire is bent and where the friction of inner parts of the wire takes place is heated. On the contrary, when the mechanical energy increases, cooling of bodies is frequently observed. For instance, in the example with the cork flying out of the bottle with soda water, the gas whose excess pressure has pushed the cork out is cooled.

Besides heating, friction can be accompanied by other changes in the state of a body. One important type of change in state is the break-up of large objects into smaller ones. Simple examples of such processes are spraying of water, the abrasion of a piece of chalk during writing on a blackboard, the abrasion of a pencil while writing on paper, the grinding of grain into flour by millstones, blunting and sharpening of cutting tools (knives, razors, chisels, and so on). Sometimes friction or impact may cause a change of the state of aggregation of a body.

On the basis of similar facts, we introduced the concept of *internal energy* of bodies (see Sec. 4.19). In that section it was pointed out that the internal energy of a body depends on its temperature, state of aggregation and on whether the body is a large object or small grains. If the work against friction is done by an external force, as a result of which the temperature of the body has increased or it is being broken down, melted or evaporated, the internal energy of the body increases. On the contrary, if the temperature of the body decreases, or it goes over from the gaseous to the liquid state, and so on, the internal energy of the body decreases.

We shall now consider in greater detail the phenomena involving the change in internal energy.

11.2. Heating of Bodies on Which Work Is Done

In the previous section it was established that rubbing bodies become heated when work is done against friction. A large number of experiments was carried out to measure exactly the change in the temperature as a result of a certain work done on a body. Joule was among the first scientists who carried out such experiments in the middle of the 19th century. Figure 365 represents schematically his apparatus. The sectional view of this device is shown in a simplified form in Fig. 366. Blades *1* rotate in a vessel filled with water under the action of a load of mass m suspended on a cord passing over pulley *2*. As the load moves downwards, the blades rotate, passing between partitions *3*. By entraining water, they cause the friction between water layers. As a result of friction, the water and the vessel are heated

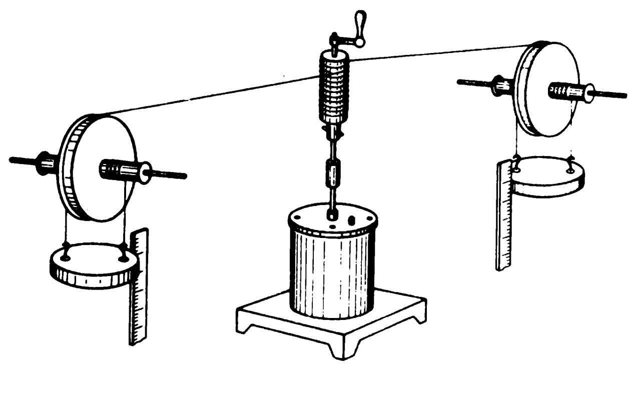

Fig. 365.
Joule's apparatus.

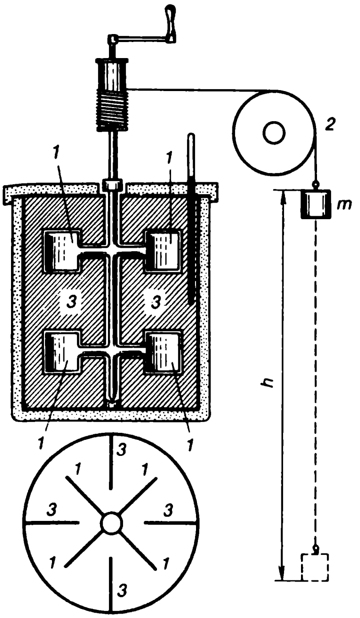

Fig. 366.
Sectional view of Joule's apparatus.

(neither water nor other parts of the device experience any other changes). When the load descends from a height h, the force of gravity mg acting on it does the work equal to mgh. At the beginning and at the end of the experiment, all parts of the device (the load, blades and water) are at rest, so that the kinetic energy of these bodies has not changed as a result of lowering the load.

Thus, *the entire work done in the system causes only the heating* of the water, blades and other parts of the device. This allows one to calculate the work required to raise the temperature of a unit mass of water by a kelvin. Joule took into account here the fact that the blades and the vessel are also heated. The way in which this was done will be described later.

Joule's experiments were repeated more than once, the experimental conditions being subjected to various changes. The amount of water in the vessel, the mass of the load and the height to which it was lifted, the moments of acting forces and other parameters were varied. The same result was obtained irrespective of these changes: *in order to heat a kilogram of water through one kelvin, a work of 4.18 kilojoules must be done*.

Besides the experiment described above, Joule and other researchers made many other experiments aiming at establishing the relation between the change in temperature and the work done. The heating of a gas as a result of work done in compression was observed. The heating of metal discs rubbing against each other was measured as well as the work against friction done in this case, and so on. A comparison of the results of these experiments is not a simple problem since quite different bodies were subjected to heating.

It will be shown later (Sec. 11.8) that there exists a technique which makes it possible to compare the heating observed in various experiments with the heating of the same substance, say, water. If such a comparison is carried out, an extremely important conclusion can be drawn from the experiments described above as well as from many other experiments: *if the conversion of mechanical energy is not accompanied by a change in the state of the body* (like melting or evaporation) *other than the change in its temperature, the temperature of a kilogram of water always increases by one kelvin at the expense of an energy of 4.18 kilojoules.*

Thus, Joule's experiments confirm the law of energy conservation in a broader sense. In all motions occurring both without friction or in the presence of friction, *the sum of the kinetic, potential and internal energies of all participating bodies does not change.* This sum of energies will be called the *total energy* of bodies, or just the energy.

Let us consider an example. Suppose that a lead ball is suspended at a certain height above a lead slab. The energy of this system consists of (a) the potential energy of the ball; (b) internal energy of the ball and the slab. Let us now suppose that the ball falls to the slab and causes heating by the impact. The potential energy of the ball decreases, but the internal energy of the ball and slab increases. The total energy of the system remains unchanged.

? **11.2.1.** It can be seen from Figs. 365 and 366 that the velocity of the lowering loads is much smaller than the velocity of the blades. For what purpose was the device designed in this way?

11.3. The Change in the Internal Energy in Heat Transfer

It was shown above that as the mechanical energy of a system of bodies decreases, the corresponding increase in their internal energy is observed, while a decrease in the internal energy is associated with an increase in the mechanical energy. These changes in the internal energy occur when a certain work is done (e.g. the work done against friction or the expansion work of a gas). In this case, both the change in the mechanical energy and the corresponding change in the internal energy are equal to the product of the acting force and the path traversed by a body, i.e. to the quantity characterising the work done.

It would be wrong, however, to assume that a change in the internal energy of a body may occur only when a work is done. For example, when a stove is being cooled, no work is done, but its internal energy decreases. In this case, however, the surrounding bodies (air, walls and objects in the room) are heated, i.e. their internal energy increases. In this case, the *heat transfer* is said to occur: the stove gives away a certain amount of heat, while the surrounding bodies receive the same amount of heat. Thus, *we call the heat transfer a process in which the internal energy of some bodies decreases while the internal energy of other bodies increases correspondingly, the mechanical energy of the bodies remaining unchanged and no work being done.*

It should be noted that the thermal state of bodies, i.e. their temperature, does not always change in the process of heat transfer. For example, when ice melts, the heat transfer changes the state of aggregation of the body (ice goes over from the solid to the liquid state), but the temperature remains unchanged.

To characterise the process of heat transfer, the concept of *amount of heat* is introduced. The amount of heat is the change in the internal energy of a body occurring during heat transfer.

Thus, the internal energy of a body may change as a result of two types of the process: (a) when work is done and (b) during heat transfer. Naturally, in some cases work is done and heat is transferred simultaneously.

For all phenomena described above, we can draw the conclusion about the change in the internal energy during a transition from one state to another. However, we do not broach a question about the total store of internal energy of a body. This is not important for us since we are only interested in the *change in the internal energy* just as in the case with potential energy (see Sec. 4.12).

11.4. Units of Heat

The amount of heat, i.e. the change in the internal energy, can be measured in the same units as were used for measuring the mechanical energy, viz. in joules. Earlier (and even now sometimes) the amount of heat was measured in special units called *calories* (cal). A calorie is the amount of heat required to raise the temperature of a gram of pure water from 14.5 to 15.5 °C.

Joule's experiments and other similar experiments showed that 4.18 joules of work must be done to raise the temperature of a gram of water through one kelvin. Hence it follows that a calorie is equivalent to 4.18 joules:

$$1 \text{ cal} = 4.18 \text{ J}.$$

The quantity equal to 4.18 J/cal is called the *mechanical equivalent of heat*

(*Joule's equivalent*).[1] Thus, it can be said that Joule's experiments helped to establish the mechanical equivalent of heat.

We shall not use calories in the further analysis to comply with the requirements of the SI system of units.

11.5. Dependence of Internal Energy of a Body on Its Mass and Substance of Which It Is Made Up

In this section, we shall speak of the changes in the internal energy of bodies associated with the changes in temperature. Joule's experiments (see Sec. 11.2) indicate that when the temperature of 1 kg of water is raised through 1 K, its internal energy increases by 4.18 kJ. In order to heat 10 kg of water, we have to spend ten times more energy, and so on. Thus, the increase in the internal energy of water as a result of heating is proportional to its mass. The same refers to any other homogeneous body. For example, the heating of a large iron to a certain temperature requires longer time than for a small iron. On the other hand, the large iron will cool for a longer time and give away more heat to the surrounding bodies upon cooling than the small iron. More linen can be ironed by a large iron heated to a certain temperature than with the help of a small iron heated to the same temperature. Thus, with the same change in temperature, the internal energy of the large iron changes to a larger extent than the internal energy of the small iron.

Consequently, *for a certain change in temperature, the change in the internal energy of body is proportional to its mass*. Hence it follows that the concept of mass introduced while analysing mechanical phenomena turns out to be useful in the analysis of thermal phenomena as well.

Observations also show that the higher the temperature to which a given body is heated, the longer time is required for its cooling. Consequently, the body will give away more heat, and its internal energy will change by a larger amount. Thus, *the change in the internal energy is the larger, the greater the change in its temperature*.

The internal energy is determined not only by the mass and temperature but also by the *substance* of which a given body is made up. Let us take two bodies of equal mass, say, lead and aluminium balls, and heat them to the same temperature, say, to 100 °C. If we now place the balls in identical vessels filled with water, we shall see that the aluminium ball will heat the water to a higher temperature than the lead ball. This means that a given mass of aluminium gives away more heat when cooled than the same mass of lead. Conversely, more heat should be supplied to the aluminium ball to heat it to the same temperature than to the same mass of lead.

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[1] To be more precise, 4.1855 J/cal. — *Eds*.

Thus, the change in the internal energy of a given mass of aluminium is larger than the change in the internal energy of the same mass of lead for the same variation of temperature.

Since the internal energy strongly depends on temperature, it is sometimes called the *heat energy*. However, internal energy depends not only on temperature. It changes as a result of compression of liquids, upon deformation of solids (Sec. 16.11), during melting of substances (Sec. 12.7) and their evaporation (Sec. 17.10). Only for substances in the gaseous state, the internal energy virtually changes only as a result of a change in temperature. Therefore, it is not expedient to replace the term "internal energy" generally accepted in science by the term "heat energy". Besides, the use of the latter term may cause a confusion with the concept of the amount of heat received by a body (see Sec. 11.3).

11.6. Heat Capacity of a Body

The amount of heat that must be supplied to a body to raise its temperature through 1 K is called the *heat capacity* of the body. When the body cools through 1 K, it gives away the same amount of heat. In order to raise the temperature of a body not through 1 K but through 10 K ten times larger amount of heat is required. When the temperature is lowered through 10 K, the body gives away the same amount of heat. On the basis on what was said in the previous section, we can state that *the heat capacity of a body is proportional to its mass and depends on the substance of which it is made up*. According to the definition, heat capacity must be measured in *joules per kelvin* (J/K).

Heating a body by heat transfer, we increase its internal energy. Besides, as a result of expansion upon heating, the work is done against the forces preventing the expansion. These forces are the forces of external pressure and intermolecular forces, which are quite significant for solids and liquids and negligible for gases. An additional energy is required to do expansion work, i.e. the additional heat transfer.

In the case of solids, thermal expansion is negligibly small (see Table 3). Consequently, this additional energy is very low and can be neglected. For gases confined in a solid shell, there is no expansion, and the additional energy is zero. In these cases it can be said that the heat capacity of a body is equal to the increase in its internal energy as a result of a temperature increase by 1 K. For liquids and gases heated under conditions such that they can freely expand (for example, in a vessel with a piston), expansion work cannot be ignored.

The forces preventing the expansion of gases are mainly the forces of external pressure. Although they are very weak, the work done by these forces is noticeable due to a considerable expansion of gases. For liquids, thermal expansion is small (although it is normally hundreds of times larger than the expansion of solids). However, the intermolecular forces opposing expansion, which are negligible for gases, are quite strong for liquids. Therefore, the expansion work for liquids turns out to be significant. The question of the heat capacity of gases heated so that their volume increases will be considered in greater detail in Sec. 13.25.

11.7. Specific Heat Capacity

Simple observations described in Sec. 11.5 and accurate measurements taken by special instruments which will be described in Sec. 11.8 led to the conclusion that the heat capacity of a homogeneous body is proportional to its mass. Therefore, the heat capacities should be compared of bodies made of different materials and having the same mass. In order to characterise the thermal properties of substances, the heat capacity of the unit mass of a substance is used. This characteristic is known as the *specific heat capacity* (or just *specific heat*). It is equal to the ratio of the heat capacity of a given body to its mass and is measured in *joules per kilogram-kelvin* (J/(kg·K)).

By definition, the specific heat capacity of water heated from 14.5 to 15.5 °C is 4.18 kJ/(kg·K). For other temperature intervals, the specific heat of water somewhat differs from this value. Henceforth, we shall neglect this circumstance and assume that the specific heat for water is 4.18 kJ/(kg·K) for any temperature.

The specific heats of other substances also slightly depend on temperature. If, however, the temperature changes slightly, this dependence can be disregarded. Therefore, in most of calculations we shall assume that the specific heat capacity of a substance is a constant quantity. In this case, we can easily calculate the amount of heat Q that should be transferred to a homogeneous body to increase its temperature from t_1 to t_2. We shall denote the specific heat capacity of a substance by c. If the mass of a body is m, its heat capacity is cm. In order to raise the temperature from t_1 to t_2, we must transfer to the body the $t_2 - t_1$ times larger amount of heat. Thus,

$$Q = cm(t_2 - t_1).$$

11.8. Calorimeter. Measurement of Heat Capacity

For comparing the heat capacities of various bodies, *calorimeters* are used. A calorimeter is a metal vessel of cylindrical shape with a lid. The vessel is placed into a larger vessel on corks so that the two vessels are separated by an air layer (Fig. 367). All these precautions reduce heat losses to the surroundings.

The vessel is filled with a known amount of water whose temperature is measured before the experiment (suppose that it is t_1). Then we heat a body whose heat capacity should be determined to a known temperature t_2 (for

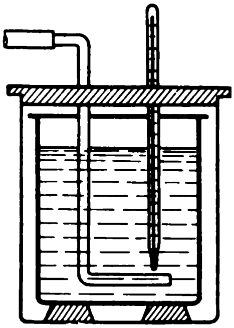

Fig. 367.
Calorimeter.

example, the body is placed in the vapour of boiling water so that its temperature becomes $t_2 = 100$ °C). The body is then immersed in water in the calorimeter, the lid is put on it, and the water is stirred until the water and the body acquire the same temperature. Then this temperature t is measured.

From the results of the experiments we can determine the specific heat capacity c_2 of the material of the body, using the fact that the decrease in the energy of the body being cooled is equal to the increase in the energy of the water and the calorimeter which are heated thereby, i.e. by applying the energy conservation law.

In rough calculations, we can assume that the water in the calorimeter, the calorimeter itself, the stirrer and the body whose heat capacity is being measured do not have time to give away a noticeable amount of heat to the surrounding bodies. (For more accurate measurements, the appropriate corrections must be introduced.) Therefore, the sum of the energies of the body, water, calorimeter and stirrer before and after the experiment can be assumed to be constant. In other words, the energy of the body decreases in the experiment by the quantity by which the energy of the water, calorimeter and stirrer increases. The temperature of the body drops by $t_2 - t$. Since no work is done inside the calorimeter, the decrease in the energy of the body is $c_2m_2(t_2 - t)$, where c_2 is the specific heat capacity of the substance of which the body is made and m_2 is its mass.

The water is heated through $t - t_1$, and the increment of its energy is $c_1m_1(t - t_1)$, where c_1 is the specific heat capacity of water and m_1 is the mass of the water in the calorimeter. Let us suppose that the calorimeter and the stirrer are made of the same substance, their total mass is m_3, and the specific heat capacity of their substance is c_3. The energy of the calorimeter with the stirrer will increase by $c_3m_3(t - t_1)$. The energy required for heating the thermometer can be neglected since it is usually small. Equating the decrease in the energy of the body to the increment of the energy of the water, calorimeter and stirrer, we obtain

$$c_2m_2(t_2 - t) = c_1m_1(t - t_1) + c_3m_3(t - t_1).$$

This equality is often referred to as a *heat balance equation*. Solving it for c_2, we obtain

$$c_2 = \frac{(t - t_1)(c_1m_1 + c_3m_3)}{(t_2 - t)m_2}.$$

Thus, having measured t, t_1, t_2, m_1, m_2 and m_3, we can find the specific heat capacity c_2 of the body under investigation if we know the specific heat capacities c_1 and c_2 for water and the substance of the calorimeter respectively. The specific heat capacity c_1 for water can be taken as 4.18 kJ/(kg·K) (see Sec. 11.7). The specific heat capacity of the substance of the calorimeter must be determined separately. For example, we can do it by observing the heat balance for a body made of the same substance as the calorimeter and immersed in it (then $c_2 = c_3$). Having determined the specific heat capacity c_3 of the calorimeter substance, we can do all the rest with the help of the obtained relation.

The specific heat capacities of some substances are given in Table 5. If the temperature is not indicated, the values of the specific heat capacities are given for room temperature. By way of example, it is shown in the table (for water, copper and lead) that specific heat capacity *depends on temperature*. For solids, specific heat capacity increases with temperature. At very low temperatures, specific heat capacity rapidly drops for all bodies. Pay attention to a very high value of the specific heat capacity for water in comparison with other substances. It should also be noted that the specific heat capacity for ice is equal to half the value for water. The specific heat capacities in solid and liquid states differ very sharply for other substances as well.

Table 5. Specific Heat Capacities for Some Substances

Substance	c, kJ/(kg·K)	Substance	c, kJ/(kg·K)
Aluminium	0.880	Lead at −259 °C	0.032
Air (expanding freely)	1.010	at 20 °C	0.130
Asbestos	0.210	at 300 °C	0.143
Brass	0.390	Mercury	0.126
Brick	0.840	Pine wood	2.520
Copper at −163 °C	0.280	Sand	0.840
at 20 °C	0.380	Sulphur	0.710
Glass	0.840	Water at 20 °C	4.180
Ice at 0 °C	2.100	at 90 °C	4.220
Iron	0.460		

If we know the specific heat of a substance, we can always calculate the amount of water that has the same heat capacity as a given body (the so-called *water equivalent*). Let us suppose, for example, that the calorimeter vessel is made of brass and has a mass of 100 g. Its heat capacity is $0.100 \times 390 = 39$ J/K. Consequently, the water equivalent of this vessel is equal to 39 J/K: 4180 J/(kg·K) = 0.0093 kg = 9.3 g. Heating 300 g of water in such a vessel, we can assume that we heat water alone but in an amount of 309.3 and not 300 g. We can now answer the question as to in which way Joule could take into account the heating of the vessel in addition to the heating of water in the experiment described in Sec. 11.2. He could do it by using the concept of water equivalent.

? **11.8.1.** Two bodies made of the same substance (say, two pieces of iron) but having different masses are heated to different temperatures. Will their total volume increase or decrease after the hotter body has transferred a certain amount of heat to the colder body?

11.8.2. Let 100 g of water at 50 °C and 200 g of water at 10 °C be poured into a brass cylinder having a mass of 163 g and a temperature of 17 °C. Ignoring the heat exchange with the surrounding bodies, determine the final temperature of water. Now suppose that the temperatures of the portions of water poured into the vessel are as above but heat exchange with the surroundings takes place through the vessel walls. How will this circumstance affect the final temperature of water if (a) first the hot and then the cold water is poured and (b) the order in which the water is poured is reversed?

11.9. The Law of Energy Conservation

The law of energy conservation, whose application was analysed both for the cases when heat transfer takes place (see Sec. 11.3) and when mechanical phenomena occur along with thermal phenomena (see Sec. 11.1), is of universal nature. It is applicable to all natural phenomena. The following examples make it possible to comprehend this law deeper.

Let us suppose, for example, that coal burns in air. Heat is transferred in this case to the surrounding bodies. These bodies are heated, i.e. their internal energy increases. Besides, the combustion of coal can be accompanied by doing a certain mechanical work if, for example, the coal burns in the furnace of the boiler of a steam engine. Does anything else change in the system of bodies under consideration (coal, air and engine) during the operation of the engine? Prior to the combustion, we had coal and atmospheric oxygen, while after the combustion we obtain carbon dioxide. Consequently, the chemical composition of bodies has changed. Thus, the change in the chemical composition of the bodies is accompanied by doing work and heating, i.e. heat transfer. Hence, we conclude that the internal energy of bodies depends also on their chemical composition. In the example under consideration, the energy of coal and oxygen contained in air is higher than the energy of carbon dioxide formed from them. The excess of the energy of coal and oxygen over the energy of carbon dioxide is spent on the heating of the surrounding bodies and on doing work.

Let us consider an example concerning charged bodies, e.g. storm clouds. When a lightning strikes, a number of changes occur: air is heated and the clouds are discharged. The energy of bodies depends not only on their temperature, but also on the distribution of electric charges on them. During a discharge, both factors undergo changes, but the total energy of the clouds and air remains unchanged. This constancy of the total energy in all processes occurring with bodies is just the law of energy conservation. In the most general form, this law can be formulated as follows.

The energy of bodies depends on their velocities, position, temperature, shape, chemical composition and so on. The change in the energy of bodies occurs either at the expense of work done by these bodies or at the expense of energy transfer to other bodies. If we consider all the bodies participating in a process, their total energy remains unchanged.

The most essential in this law is the necessity to consider *all* the bodies taking part in processes under investigation. This is not always a simple matter. For instance, in the second of the examples considered above, a number of other, less significant, changes also occur in addition to the indicated changes: light propagates in all directions from the lightning, thunder is heard, vis. sound propagates, atmospheric nitrogen and oxygen are combined to form nitrogen oxides, and so on. Sound and light are absorbed by surrounding bodies, which also ultimately leads to their heating. However, the absorption of sound and light may occur very far from the site where the lightning stroke. In particular, the light from the lightning may even leave the Earth and get absorbed somewhere on remote celestial objects.

Thus, strictly speaking, we may encounter unsurmountable difficulties while considering all the bodies participating in a process under investigation. However, in the cases when all factors can be taken into account quite rigorously we always see that the law of energy conservation is observed. This leads us to the conclusion that apparent deviations from this law are explained by insufficiently complete account of all changes occurring in the process. Indeed, we can always indicate in such cases some factors that escaped our attention. Therefore, *we believe in the universal applicability of the law of energy conservation*.

At present, there is no need to verify this law in every specific case. On the contrary, the belief that this law is valid allows us to predict the results in the analysis of concrete processes or correct erroneous arguments. The law of energy conservation belongs to the most fruitful laws which can be applied in most diverse cases.

11.10. Perpetual-Motion Machine (Perpetuum Mobile)

The validity of the law of energy conservation was proved in a large number of experiments. The number of these experiments is very large due to the fact that the utilization of energy is one of the most important problems facing the mankind.

As far back as in the middle of the 13th century, the models of machines which had to do work without any energy expenditures started to appear. To be more precise, machines were designed such that after a certain work had been done by them and the machine returned to the initial position, no changes had to occur in surrounding bodies. Such a hypothetic machine is called a *perpetual-motion machine*, or *perpetuum mobile*.

None of such machines could operate as their inventors wished, i.e. perpetual motion was not ensured. An analysis of the model of each machine of this type reveals an error. It immediately follows from the law of energy conservation that such a machine is impossible in principle, and hence it is fruitless to try and seek an intricate combination of devices and appliances which would help to avoid these difficulties.

In his time Leonardo da Vinci realised the impossibility of a perpetual-motion machine. However, even after the law of energy conservation had been established, the attempts to invent a perpetual-motion machine were made for a long time by persons who did not have a sufficient knowledge of physical laws. The number of designs of this type submitted for consideration was so large that the French Academy of Sciences had to publish in 1775 the statement that such designs would not be considered in view of their impracticability.

11.11. Types of Processes Involving Heat Transfer

In previous sections, we often spoke about heat transfer as a process in which the internal energy of a body changes. Let us now consider heat transfer in greater detail.

It should be noted above all that when no work is done, heat transfer occurs in a certain direction: the internal energy of a hot body decreases, while the internal energy of a cold body increases. Only in special circumstances, under the necessary condition that a work should be done by an external force, processes may occur in which the temperature of a hot body increases and the temperature of a cold body becomes still lower. We shall return to this question when we shall analyse the operation of so-called refrigerating machines (Sec. 19.13). The larger the temperature difference of bodies, the more intense (other conditions being equal) the process of heat transfer from the hot body to the cold one. When the temperatures of the bodies level out, heat transfer ceases, and the so-called thermal equilibrium sets in.

Which processes level out the temperatures of a body? There are several such processes.

1. When cold water is heated in a saucepan, heat is transferred through the metal walls of the saucepan. This process is called *heat conduction*. What determines the amount of heat transferred through a wall? First of all, it is determined by the temperature difference on the two sides of the wall. The larger this difference, the larger amount of heat is transferred through the wall in a certain time interval. Further, this amount of heat depends on the area of the wall. Water is known to be heated sooner in a saucepan with a larger area of the bottom. Next, it can be easily seen from experiment that the amount of heat transferred per unit time through a wall for a certain temperature difference is the larger, the thinner the wall.

Finally, heat transfer considerably depends on the material of the wall. In order to characterise the heat conduction of various materials, the concept of *thermal conductivity* is used. The thermal conductivity λ of a substance is the quantity equal to the amount of heat transferred per unit time per unit area of a wall of unit thickness at a temperature difference between the surfaces of the wall equal to one kelvin. The SI unit of thermal conductivity is watt per metre-kelvin (W/(m·K)). If, for example, the thermal conductivity of aluminium is 210 W/(m·K), this means that 210 J of heat is transferred per second per square metre of a 1 m thick aluminium wall at a temperature difference of 1 K. Without discussing the methods of determining thermal conductivity (which are rather complicated), we shall give the values of thermal conductivity for some substances (Table 6). Pay attention to the fact that the thermal conductivity of metals is higher as compared to this value for other substances. It should be recalled that electric conductivity of metals also considerably exceeds electric conductivities of other materials. Thermal conductivities of gases are quite small.

Table 6. Thermal Conductivity for Some Substances

Substance	λ, W/(m·K)	Substance	λ, W/(m·K)
Air	0.025	Hydrogen (gas)	0.18
Aluminium	210	Iron	60
Brass	110	Lead	34
Brick	1.25	Water	0.63
Copper	385	Wood, along fibres	0.29
Glass	0.85	across fibres	0.17

2. In addition to heat conduction, heat transfer in liquids and gases is often carried out by *convection*, i.e. mechanical displacement of heated parts of a fluid. Almost always when a liquid or gas comes in contact with solid walls having a higher or lower temperature than the fluid, flows ap-

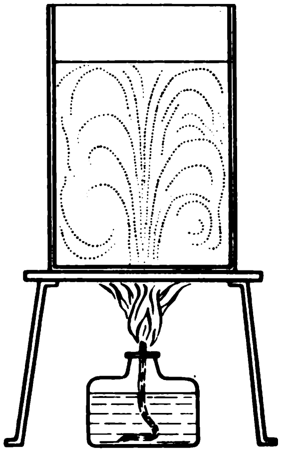

Fig. 368.
Convective flows in a liquid.

pear in the liquid or gas. The heated portions of the liquid (or gas) move up, while cooled portions move down (Fig. 368). This process occurs as a result of a decrease in the density of the fluid with increasing temperature.

It can be easily seen that convective flows in fluids appear the more readily, the larger their temperature coefficients of expansion. The viscosity of fluids also plays an important role. A higher viscosity naturally hinders the emergence of convective flows. In very thin layers, like the air layer between two closely spaced window glasses, convective flows are weak. If convective flows emerge, they considerably promote a heating of fluids. In the absence of convection (if, for example, a warm liquid is above a cold one), the heating of fluids is very slow due to their negligible thermal conductivity.

Convective flows in the atmosphere are not only responsible for heat transfer, but also cause winds. These flows ensure a continual mixing of air, owing to which air has practically the same composition at different sites on the Earth. Convective flows in the atmosphere sustain burning processes by supplying oxygen to the flame and removing combustion products.

Convective flows in fluids are widely used in engineering (it is sufficient to recollect the hot-water heating system). However, natural convection is often insufficient for technical purposes. In such cases, forced convection is used, which is created by pumps (for example, in cooling electric generators by blowing air or hydrogen).

Besides convective flows whose origination is associated with thermal expansion of fluids, there can be other causes of their mixing, and hence rapid heating. For example, if fluid flows in pipes, a turbulent motion readily appears, due to which the layers of the flowing fluid are intensely mixed (Sec. 9.14).

In the conditions of weightlessness, convective flows disappear. For this reason, burning in zero-gravity conditions is impossible (unless the forced draught is ensured): the combustion products are not removed from the flame, and it dies out due to the lack of oxygen. However, mixing due to turbulence occurs in zero-gravity conditions in the same way as under normal conditions.

3. In addition to heat transfer by thermal conduction and convective flows, the heat transfer by *emission and absorption of radiation* also plays an extremely important role in nature and in engineering. Bringing a hand near a heated iron, we feel "hot object" even from below, where cold air flows to the iron. The iron emits rays and as a result is cooled, while the hand absorbs rays and hence becomes heated. These rays are just electromagnetic waves which will be discussed later. Here we shall not consider in detail the emission and absorption of rays. It should only be mentioned that the heat transfer through the empty space (for instance, from the Sun to the Earth) is exclusively carried out by emission and absorption of radiation.

4. Besides heat conduction, convection and radiation, there exist many other processes in which hot bodies are cooled and cold bodies are heated, like *evaporation and condensation*, *thermoelectric phenomena*, and so on. These phenomena will be considered on a later stage.

?

11.11.1. Where is the temperature of the incandescent filament of an electric bulb higher: on the surface of the filament or in the bulk of it?

11.11.2. Put a pin or paper-clip on a sheet of white paper and hold it above a burning candle until the paper becomes yellow and charred. Then shake off the pin. You will see the white trace of the pin on the yellowish paper (Fig. 369). Explain this phenomenon.

11.11.3. The thermal conductivity of wood along fibres is higher than across fibres (see Table 6). Why is this so?

11.11.4. The thermal conductivities of brass and zinc are almost the same. Their specific heats are also almost equal, but the density of brass is considerably higher than that of zinc. Boiling water is poured into two mugs made of brass and zinc and having the walls of the same thickness. Which of the mugs will be heated through sooner?

11.11.5. If we drop some water on a horizontal hot plate, the drop is retained for a long time almost without being evaporated. If the plate is just moderately heated, the drop evaporates with hissing almost instantaneously. Explain this phenomenon.

11.11.6. Suppose that a liquid has been obtained, whose temperature coefficient of ex-

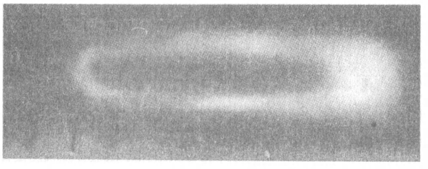

Fig. 369.
To Exercise 11.11.2.

pansion is zero at any temperature. How would this liquid behave if we poured it in a metal saucepan and put on a hot plate?

11.11.7. Attach a small candle to the bottom of a glass jar. Light the candle, close the jar with a lid and watch the flame when (a) the jar at rest, and (b) it is falling freely from a height of 2-3 m onto soft sand (to prevent the jar from breaking from the impact). Explain the difference in the shape and brightness of the flame in the two cases.

11.11.8. Why does hydrogen blown through electric machines cool them to a larger extent than the same mass of air?

Chapter 12.

Molecular Theory

12.1. Molecules and Atoms

This part of the book is devoted to the analysis of changes in the thermal state of bodies, this state being characterised by the temperature and by the transition of bodies from the solid state of aggregation to the liquid state, from the liquid to the gaseous state and back, and so on. The natural question arises: what occurs *inside* bodies when their temperature changes, when they melt or evaporate, and so on. The answer to this question, as well as to a number of other questions referring to the properties of substances, is provided by *molecular theory*.

The idea that all bodies surrounding us consist of tiny particles inaccessible to direct observation takes its origin in ancient time (about two and a half thousand years back). However, the modern ideas about molecules and atoms were developed and experimentally substantiated only in the last 150 years.

Molecules are the smallest particles of which various substances are composed. In some cases, like in metal vapours and inert gases (helium, argon, etc.), the smallest particles of matter are individual atoms, while in other cases such particles consist of several atoms (of two atoms in oxygen and nitrogen, of three atoms in carbon dioxide, and so on). The molecules of composite substances (other than elements) consist of atoms of different elements constituting them. Such an idea about the structure of bodies made it possible to explain the basic laws of chemistry (the law of constant proportions and the law of multiple proportions).

The *law of constant proportions* consists in that the masses of substances combined to form any amount of a chemical compound are in a quite definite proportion. For example, when water is formed from hydrogen and oxygen, the masses of hydrogen and oxygen forming the compound are always in the ratio 1:8. From the point of view of the concept of atoms and molecules, this experimental fact can readily be explained. Indeed, to form water, two atoms of hydrogen are combined with an atom of oxygen, i.e. a water molecule has the composition H_2O. The ratio of the masses of hydrogen and oxygen must be equal to the ratio of doubled mass of the hydrogen atom to the mass of the oxygen atom, and therefore will always be the same irrespective of the amount of water formed. This is due to the fact that all hydrogen atoms are identical, and their mass is always the same, and that all oxygen atoms do not differ in mass from one another.

The *law of multiple proportions* states that when two elements form several compounds, the masses of one element in different compounds are integral multiples. For instance, oxygen and nitrogen form five compounds. The masses of oxygen in these compounds divided by the mass of nitrogen are integral multiples (1:2:3:4:5). This fact is explained as follows. The same number of atoms of one element (two nitrogen atoms in the example under consideration) are combined with different numbers of atoms of the other element in the molecules of different compounds (with 1, 2, 3, 4 and 5 oxygen atoms in our example). These compounds have the following composition: N_2O, N_2O_2, N_2O_3, N_2O_4, and N_2O_5.

12.2. Size of Atoms and Molecules

At first sight, the idea about the molecular structure of bodies is in contradiction with our everyday experience: we do not observe these individual particles, and bodies appear to us as integral wholes. This argument, however, is not very convincing. In one of his works Mikhail Lomonosov wrote that one cannot deny a motion unseen by eye; nobody would argue that the leaves and branches of trees do not move in a strong wind although this motion cannot be seen from a distance. The smallness of particles of a substance in hot bodies conceals motion like the long distance in the above case. Thus, the reason behind the apparent discrepancy lies in the fact that atoms and molecules are extremely small.

It is impossible to observe (even the largest) molecules through the best optical microscope that allows us to distinguish objects whose size is not less than (2-3) $\times$ 10^{-7} m. A number of indirect methods, however, allows us not only confidentially prove the existence of molecules and atoms, but also determine their size. For example, the diameter of a hydrogen atom can be considered equal 1.2×10^{-10} m, the length of a hydrogen molecule, viz. the separation between the centres of the two hydrogen atoms constituting it, is 2.3×10^{-10} m. There exist larger molecules, like protein (albumin) molecules whose size is 4.3×10^{-6} m. It is possible to photograph not only large molecules but also atoms with the help of a special instrument (electron microscope) which permits the investigation of objects of extremely small size.

The conclusion about a very small size of molecules can be drawn without any measurement from the possibility of obtaining very small amounts of various substances. Having diluted 1 ml of (say, green) ink in a litre of pure water, and then diluting 1 ml of this solution in a litre of water

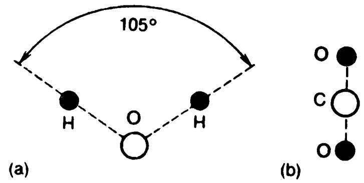

Fig. 370.
Schematic diagrams of the molecules of water (*a*) and carbon dioxide (*b*).

once again, we obtain 1:1 000 000 dilution. And still it can be seen that the latter solution has greenish colour and at the same time is quite homogeneous. Consequently, in the smallest volume that can be distinguished by eye, there are still a large number of molecules of ink even with such dilution. This shows that the size of molecules is indeed very small.

Gold can be hammered out into sheets 0.1 μm thick. Treating these sheets with an aqueous solution of potassium cyanide, the sheets whose thickness is 0.01 μm can be obtained. Consequently, the size of gold molecules is considerably smaller than a hundredth of a micrometre.

In the figures, we shall represent molecules as balls. However, molecules (as well as atoms) have different, sometimes very complex, structures for different materials. At present, the shape and structure of not only such simple molecules as H_2O and CO_2 are known (Fig. 370), but also of much more complex molecules containing thousands of atoms.

12.3. Microworld

The advances in the investigation into the structure of matter described in the previous sections have opened before the researchers a new world, viz. the world of small particles. It is called the *microworld*. Unlike the world of large bodies, or *macroworld* (from Greek words *mikros* for small and *makros* for large), the microworld is inaccessible to a direct observation and requires special fine methods for its study. The microworld turned out to be very complex. It was mentioned above that any body which was treated as a solid in mechanics appeared as a complex system of a large number of continually moving molecules when new methods of investigation were applied. It turned out that molecules consist of still smaller particles, viz. atoms, the number of atoms in some molecules being very large. Atoms, in turn, were found to be complex systems consisting of electrons and nuclei, and nuclei themselves consist of various particles which will be described in the last volume of this book.

Naturally, all what occurs and is observed in the macroworld is connected with the state of particles of the microworld and their changes. The changes in the thermal state of bodies (temperature variations and transitions from one state of aggregation to another like from the solid to the liquid state) turned out to be mainly connected with the motion of molecules and their mutual arrangement. Chemical transformations observed in the microworld are due to changes in the atomic structure of molecules.

The structure of molecules and atoms, as well as the motion of atoms constituting molecules and the motion of particles forming atoms, is manifested in the macroworld in electric, magnetic, optical and other phenomena. This extraordinary complexity of the microworld would set unsurmountable obstacles for its investigation if the problem had not been split reasonably. It turned out that it is possible to single out simpler phenomena which are, for example, caused by molecular motion and which can be studied neglecting more complicated processes in the microworld. Then one should go over to an analysis of finer processes and motions associated with the structure of atoms and molecules, leaving aside intranuclear processes, and so on.

Thus, going over from the study of simpler processes and motions to more complicated ones, we gradually form a more detailed and clear pattern of the microworld. We shall start with the phenomena which can be studied neglecting the internal structure of molecules, the motion of atoms constituting the molecules, and still finer intra-atomic and intranuclear processes and motions. They include a large group of thermal phenomena for which molecules can be treated as invariable small bodies.

Thus, we start the study of the microworld by confining ourselves to the motion and arrangement of molecules, leaving aside their internal structure.

12.4. Internal Energy from the Viewpoint of Molecular Theory

In the previous chapter, we drew the conclusion that in addition to the mechanical energy of a system of bodies, which depends on their velocities (kinetic energy) and relative position (potential energy), each body in the system possesses an internal energy which depends on the state of this body. We can now refine the concept of internal energy. *The internal energy is the kinetic and potential energies of particles constituting the microworld*: molecules which constitute macroscopic bodies, atoms constituting molecules, and electrons and other particles constituting atoms. It was mentioned in the previous section that thermal phenomena can be mainly associated only with the motion and arrangement of molecules as invariable simple particles. Therefore, while studying simple phenomena, we shall be interested only in a part of the internal energy of bodies, namely, only in the kinetic energy of molecules (which depends on their velocity) and the potential energy of molecules (which is determined by their relative position).

For gases, the change in the internal energy is mainly the change in the kinetic energy of random motion of their molecules. As a matter of fact, the interaction between gas molecules is small, and the change in the potential energy of moving molecules can be neglected. In liquids and solids, the intermolecular interaction is quite strong, and a change in the molecular separation sharply affects the potential energy of their interaction. For this reason, the change in the internal energy for liquids and solids consists in the change in the kinetic energy of random motion of their molecules and in the change in the potential energy of their interaction.

In terms of molecular theory, it becomes clear what happens when as a result of heat conduction the internal energy of a hot body (or a hot part of a body) decreases, and the internal energy of a cold body (or the cold part of a body) increases. During interaction, molecules exchange their velocities like colliding elastic balls (see Sec. 4.17). The velocity exchange is associated with the exchange of kinetic energies. As a result, the internal energy of the hot body decreases, while that of the cold body increases, i.e. the levelling out of internal energy takes place (to be more precise, of its part, viz. the kinetic energy of molecules). Hence it follows that the temperature of a body is associated with the kinetic energy of molecules constituting this body. This question will be considered in greater detail later (Sec. 13.23).

12.5. Molecular Motion

Let us compare several simple facts which give an idea about molecular motion. We put a piece of sugar into a glass of cooled tea. The sugar melts and forms a thick syrup at the bottom of the glass. This syrup can be clearly seen if we look through the glass at a source of light. Let us leave the glass to itself for several hours. Will the syrup remain at the bottom of the glass? No, it will not. It will gradually spread over the entire glass. This distribution of sugar over the glass volume occurs spontaneously since nobody stirred the tea. A scent spreads in a room in the same way (if, for instance, we open a bottle of perfume). This occurs even if air in the room does not move at all.

Let us make another experiment: we balance on scales a large vessel open from above. If we fill this vessel with carbon dioxide, the equilibrium will be violated since carbon dioxide is heavier than air. In a certain time, however, the equilibrium will be restored: carbon dioxide will spread over the entire room, and the vessel will be filled with air with an insignificant admixture of carbon dioxide. In all these cases, a substance (sugar, vapour of an aromatic substance, and carbon dioxide) spreads in another substance (water and air). The phenomenon in which two substances spontaneously mix with each other is known as *diffusion*. In diffusion, a substance propagates in all directions, including the upward and downward directions, i.e. even against the force of gravity. Diffusion shows that molecules of substances are in constant motion. For example, during the diffusion of sugar in water, different molecules of sugar move in different directions between water molecules which are also in motion. As a result, sugar gradually spreads over the entire vessel filled with water.

Thus, diffusion indicates that molecules are in permanent motion in various directions. Such a motion of molecules can be observed not only in liquids and gases but even in solids. It is called the *thermal molecular motion*.

One may ask: why do not we notice this motion in ordinary observations? In other words, why does not a body move as a whole although its molecules are in constant motion? The answer to this question is that in molecular motion, different molecules move in different directions so that the body as a whole is at rest. In a completely random motion of a huge number of molecules, for a molecule moving in a certain direction there exists another molecule which flies approximately in the opposite direction at the same velocity. Since a gas is enclosed in an envelope which prevents the gas molecules from flying apart, the motion of molecules in the gas is reduced to a random to-and-fro motion in all directions. For this reason, there is no motion in a certain direction.

12.6. Molecular Motion in Gases, Liquids and Solids

The motion of molecules in gases is of random nature: the velocities of molecules do not have a preferential direction but are distributed chaotically in all directions. As a result of molecular collisions, their velocities constantly change both in magnitude and direction. For this reason, the velocities of gas molecules may differ considerably from one another. At any moment, there are molecules in a gas which move at extremely high velocities, and molecules which move comparatively slowly. However, the number of molecules which move much quicker or much slower than the most of molecules is small. Most molecules move at velocities which differ little from a certain average velocity that depends on the type of molecules and the temperature of the body. Henceforth, while speaking of the molecular velocity, we shall mean their average velocity.

The problem of measurement and calculation of the average velocity of molecules will be considered later.

In the analysis of the motion of gas molecules, the concept of the *mean free path* plays an important role. The mean free path is the average distance covered by a molecule between two consecutive collisions. As the density of a gas decreases, the mean free path increases. Under the at-

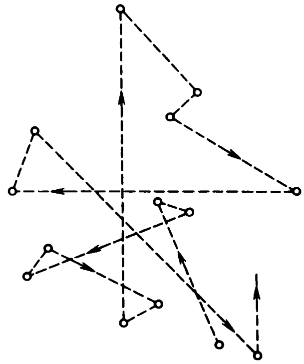

Fig. 371.
Schematic representation of the path of an air molecule under the normal pressure (magnification $\times 10^6$).

mospheric pressure and 0 °C, the mean free path of air molecules is about 10^{-8}-10^{-7} m (Fig. 371).

In highly rarefied gases (for example, inside vacuum tubes), the mean free path reaches several and even tens of centimetres. The molecules move here from wall to wall almost without collisions. Molecules in solids vibrate about their equilibrium positions. In liquids, molecules also vibrate about equilibrium positions. However, each molecule from time to time jumps to a new equilibrium position separated from the previous one by several intermolecular distances.

The idea about heat as a motion of particles of a body was put forward by Lomonosov long before molecular theory was created.

12.7. Brownian Movement

It was shown above that the pressure of a gas on the wall is caused by impacts of molecules against it. However, the number of these impacts per unit time can be randomly larger or smaller. Therefore, we can assume that the force of pressure of a gas on the vessel wall does not always have the same value: it can be sometimes slightly larger or slightly smaller. Is this really so? Can these deviations of pressure from a constant value be observed? It is impossible to measure directly these variations of pressure of a gas on the vessel wall since they are extremely small. However, there exist phenomena that can be observed and can be explained just by variations in the number and intensity of molecular impacts. Above all, this is the so-called Brownian movement.

If we observe any small particles suspended in liquid or gas (for example, drops of oil in water or the particles of smoke or mist in air) through a powerful microscope, it can be seen that these particles are in constant motion such that the directions of their motions change at random. The motion of smaller particles is more intense than the motion of larger particles.

This phenomenon was discovered in 1827 by the English botanist Robert Brown (1773-1858) and was named after him the Brownian movement. The reason behind this phenomenon remained unclear for a long time until it was proved that this motion of particles is caused by impacts of surrounding molecules of a liquid or gas. Although the molecules of the fluid hit a particle from all directions, their impacts are still not balanced completely. Accidentally, the impacts on the particle from one side turn out to be stronger then those from other sides, and as a result the particle starts to move in a certain direction. Then the impacts from some other side prevail, and the particle begins to move in a different direction. The result of these unbalanced impacts is a random motion of the particle.

A detailed analysis of this phenomenon not only confirms the correctness of such an explanation. The results of this analysis made it possible to determine the number of molecules in unit volume of a liquid or gas. Thus, the Brownian movement is the most direct and brilliant substantiation of molecular concepts.

12.8. Intermolecular Forces

If we open the tap in a tube connecting at the top two cylinders one of which is filled with a gas and the other is empty, a part of the gas from the first cylinder will immediately flow into the second cylinder. A substance in the gaseous state always occupies the volume given to it completely. If, however, the first cylinder is filled with a liquid or solid, no transfer of the substance to the second (empty) volume will be observed. If we neglect insignificant evaporation, both liquid and solid will remain where they were.

How can we explain such a difference in the behaviour of gases and liquids? When a substance is in the liquid state, the forces acting among its molecules prevent their flying apart in all directions. We shall call these forces the *intermolecular forces*, or *cohesive forces*. The cohesive forces are visually manifested when the raindrops hang on leaves and wires and do not drop for a certain time (Fig. 372). In this case, cohesive forces not only prevent the molecules from flying apart but even balance the force of gravity acting on a drop.

In solids, cohesive forces also obviously act among molecules and hold them together.

Why are cohesive forces not observed in gases and vapours? It is well

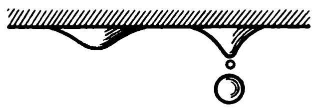

Fig. 372.
Cohesive forces keep a water drop from falling. A heavy drop falls.

known that molecules of gases and vapours are generally separated by much larger distances than molecules in liquids and solids. It is natural to assume that cohesive forces rapidly decrease with increasing separation and hence are noticeable only when intermolecular distances are small. This explains the fact that they are weakly manifested in gases.

This assumption can be confirmed by the following observations. The parts of a glass are tightly connected with one another, and a considerable force is required to separate them, i.e. to break the glass. However, if the glass is broken, separate fragments do not interact with one another any longer if we bring them in contact. As a matter of fact, while bringing the fragments of the broken glass in contact, we make closer only an insignificant number of molecules. Although the remaining molecules are at small distances, these distances are not sufficient to make molecules interact. If, however, we heat the fragments of the broken glass and bring the softened pieces together, they stick together. In this case, a larger number of molecules are brought together and the forces of interaction turn out to be strong.

Applying considerable forces to soft materials, we can bring in contact a large number of molecules even if the surface is not very smooth. This can be done, for instance, with lead. If two freshly cut lead bars are pressed against each other, they stick together so that, when pulled apart, can withstand the weight of a heavy load (Fig. 373).

We arrive at the conclusion that molecules of liquids and solids attract one another. This, however, does not explain all the properties of liquids and gases. Indeed, liquids and solids are much more difficult to compress than gases. In order to reduce their volume, say, by 1%, liquids (and solids) must be subjected to a much higher pressure than gases.

Then how can we explain the enormous pressures emerging during the

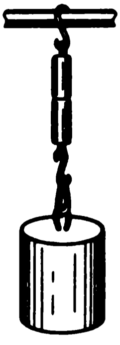

Fig. 373.
Lead bars stick together so strongly that, when pulled apart, can withstand the weight of a heavy load.

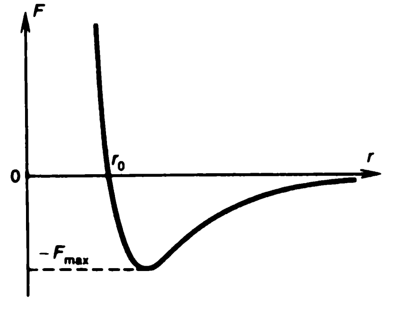

Fig. 374.
The force F of intermolecular interaction versus the distance r between two molecules.

compression of liquids (and solids) and opposing this compression? For this we have to assume that strong repulsive forces appear when the separation between molecules of a liquid or solid is reduced. Figure 374 shows the approximate dependence of the force of interaction F on the molecular separation r. A positive force corresponds to the repulsion between molecules, while a negative force, to the attraction between molecules.[1] The distance r_0 corresponds to a stable equilibrium (unstressed) state of a body. In this state, $F = 0$. As r deviates from the value r_0, the forces which tend to restore the equilibrium emerge. It can be seen from the graph that as r increases, the attractive force appears between the molecules, which sharply increases to the value F_{max}, and then gradually decreases with increasing r. When r decreases, the repulsive force emerges, which rapidly grows with decreasing r.

As a result of thermal motion, molecules perform small oscillations about their equilibrium positions, during which the attractive forces alternate with repulsive forces. In order to compress a liquid (for example, to compress water in a cylinder with a piston), it is necessary to reduce average distance between its molecules. This leads to the emergence of ever growing repulsive forces, due to which the pressure of the liquid on the vessel walls increases. It was shown above that an insignificant decrease in the volume of a liquid involves a very large increase in pressure. These arguments can be also used for solids.

The average distance between the molecules of a gas under ordinary conditions (room temperature and atmospheric pressure) amounts to tens of equilibrium distances r_0, owing to which the attractive forces between gas molecules are extremely weak. For this reason, gas molecules fly apart in all directions as a result of molecular motion. These arguments are inapplicable, however, to highly compressed gases in which intermolecular interaction is noticeable.

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[1] Strictly speaking, the projection of the force acting, say, on the second molecule on the direction from the first molecule to the second one is plotted along the ordinate axis (or, which is the same, the projection of the force acting on the first molecule on the direction from the second molecule to the first one). It can be easily seen that the projection of the attractive force will be negative, while that of the repulsive force positive. — *Eds.*

Chapter 13
Properties of Gases

13.1. Pressure of a Gas

As was mentioned earlier (see Sec. 12.8), gases always fill completely the volume bounded by gastight walls. For example, a steel cylinder used to store compressed gases (Fig. 375) or the inner tube of a motor car tyre are filled with a gas practically uniformly.

Tending to expand, a gas exerts a pressure on the walls of the cylinder, tyre tube or any other body (solid or liquid) with which it is in contact. If we disregard the action of the gravity field of the Earth, which changes pressure only slightly for conventional dimensions of a vessel, in equilibrium the pressure of the gas in a vessel appears quite uniform to us. This remark pertains to the macroworld. If we try to imagine what occurs in the microworld of molecules constituting the gas in the vessel, a uniform distribution of pressure is out of question. In some regions of the inner surface of the walls, gas molecules strike against the walls, while at other places impacts are absent. This pattern continually and randomly changes.

Let us suppose, for the sake of simplicity, that before an impact all

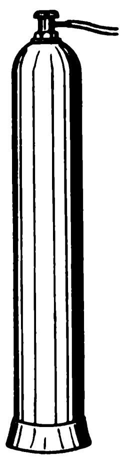

Fig. 375.
Steel cylinder for storing highly compressed gases.

molecules fly at the same velocity $\mathbf{v}$ along the normal to a wall. We shall also assume the impact to be perfectly elastic. Under these conditions, the velocity of a molecule will reverse, as a result of the impact, its direction without changing its magnitude. Consequently, the velocity of the molecule after the impact becomes $-\mathbf{v}$. Accordingly, the momentum of the molecule prior to the impact is $m\mathbf{v}$ and after the impact, $-m\mathbf{v}$ (m is the mass of the molecule). Subtracting the initial value of the momentum of the molecule from the final value, we determine the increment of the momentum of the molecule, which is imparted to it by the wall. It is equal to $-m\mathbf{v} - m\mathbf{v} = -2m\mathbf{v}$. According to Newton's third law, the momentum of $2m\mathbf{v}$ is imparted to the wall as a result of the impact.

If N impacts occur per unit time per unit area, then $N\Delta t\,\Delta S$ molecules will strike the element of the wall whose area is ΔS during the time Δt. The total momentum imparted by these molecules to the area element ΔS during the time Δt is $2Nmv\,\Delta t\,\Delta S$. According to Newton's second law, this momentum is equal to the product of the force F exerted on the area element ΔS and the time Δt. Thus,

$$2Nmv\,\Delta t\,\Delta S = F\Delta t, \quad \text{whence} \quad F = 2Nmv\,\Delta S.$$

Dividing the force F by the area of the element ΔS of the wall, we obtain the pressure p exerted by the gas on the wall:

$$p = 2Nmv. \tag{13.1.1}$$

It can be easily seen that the number of impacts per unit time depends on the velocity of molecules (since the faster they fly, the more frequently they collide with the wall) and on the number n of molecules in unit volume (since the larger the number of molecules, the larger the number of their impacts against the wall). Consequently, we can assume that N is proportional to n and v, i.e. p is proportional to nmv^2.

In order to calculate the gas pressure with the help of molecular theory, we must know the following characteristics of the microworld of molecules: the mass m, the velocity v and the number n of molecules in unit volume. In order to determine these characteristics of molecules, we must establish the characteristics of the macroworld which determine the gas pressure, i.e. to establish experimentally the laws governing the gas pressure. Comparing these experimental laws with those obtained with the help of molecular theory, we shall be able to determine the characteristics of the microworld, for example, the velocities of gas molecules.[1]

Thus, what determines the gas pressure?

Firstly, the pressure depends on the extent to which a gas is compressed,

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[1] There also exist methods of direct measurement of velocities for gas molecules (see Sec. 13.24).

i.e. on the number of gas molecules in a given volume. For instance, pumping more and more air into a tyre tube or compressing a closed tube (decreasing its volume), we make gas act on the tube walls with an increasingly stronger force.

Secondly, the pressure depends on the gas temperature. It is well known, for example, that a rubber ball becomes more resilient if we keep it for some time near a hot stove.

Normally, the change in pressure is caused by the two factors simultaneously, viz. both by the change in volume and the change in temperature. However, a process can be carried out so that the temperature varies insignificantly during the change in volume, and, on the contrary, when the temperature changes, the volume virtually remains unchanged. We shall first consider just these cases after making the following reservation. We shall consider a gas in equilibrium state. This means that both mechanical and thermal equilibria set in in the gas.

The mechanical equilibrium means that individual parts of the gas do not move. For this it is required that the pressure of the gas be the same in all its parts (if we ignore an insignificant pressure difference between the upper and lower gas layers, caused by the force of gravity).

The thermal equilibrium means that no heat is transferred from one region in the gas to another. For this it is necessary that the temperature be the same in the entire volume of the gas.

13.2. Temperature Dependence of Gas Pressure

Let us begin with an analysis of the dependence of the gas pressure on the temperature for a given mass of a gas whose volume does not change. Such investigations were carried out in 1787 by Jacques A. S. Charles (1746-1823). His experiments can be repeated in a simplified form if we heat a gas in a large flask connected to a mercury manometer *M* made in the form of a narrow U-tube (Fig. 376).

We shall neglect an insignificant change in the flask volume as a result of heating and a small variation of the volume due to displacements of mercury in the narrow tube. Thus, we can assume that the gas volume remains unchanged. While heating water in a vessel in which the flask is immersed, we shall mark the gas temperature with the help of thermometer *T* and the pressure corresponding to it, with the help of the manometer. Having filled the vessel by melting ice, we shall measure the pressure p_0 corresponding to a temperature of 0 °C.

Experiments of this kind revealed the following regularities.

1. The increment in pressure of a certain mass of a gas as a result of its heating through 1 °C constitutes a certain fraction α of the pressure of the

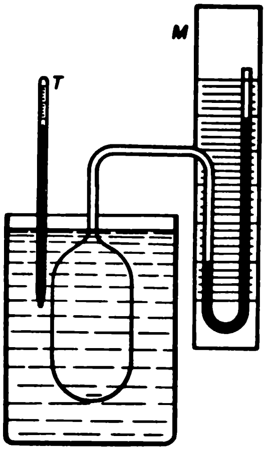


Fig. 376.
When a flask is immersed in hot water, the mercury manometer M connected to it indicates an increase in pressure; T is a thermometer.

given mass of the gas at 0 °C. If we denote the pressure at 0 °C by p_0, the pressure increment as a result of heating through 1 °C is αp_0.

If we heat the gas through τ degrees, the pressure increment will be τ times larger, i.e. *the pressure increment is proportional to the temperature increment.*

2. The quantity α showing the fraction of the pressure at 0 °C by which the gas pressure increases upon heating through 1 °C has the same (to be more precise, almost the same) value for all gases, namely, 1/273 deg^{-1}. This quantity is known as the *temperature coefficient of pressure*. Thus, the temperature coefficient of pressure for all gases has the same value equal to 1/273 deg^{-1}.

The pressure of a certain mass of a gas increases for each 1 °C *rise in temperature at constant volume by* 1/273 *of the pressure of this mass of the gas at* 0 °C (Gay-Lussac's law).

It should be borne in mind, however, that the temperature coefficient of pressure of a gas, obtained from measuring temperature with the help of a mercury thermometer, is not exactly the same for all temperatures: Gay-Lussac's law is only a close approximation.

13.3. Formula Expressing Gay-Lussac's Law

Gay-Lussac's law allows us to calculate gas pressure at any temperature if we know its pressure at 0 °C. Let p_0 be the pressure of a given mass of a gas at 0 °C in a given volume, and p be the pressure of the same mass at a temperature t. The temperature increment is t. Consequently, the pressure increment is $\alpha p_0 t$, and the required pressure is

$$p = p_0 + \alpha p_0 t = p_0(1 + \alpha t) = p_0 \left(1 + \frac{t}{273}\right). \qquad (13.3.1)$$

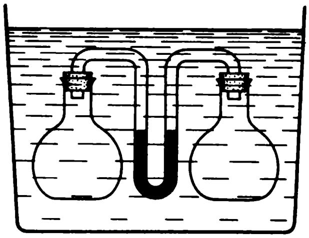


Fig. 377.
To Exercise 13.3.1.

This formula can also be used if a gas is cooled below 0 °C (in this case t will have negative values). Gay-Lussac's law is inapplicable for very low temperatures, when the gas approaches the liquid state, as well as for highly compressed gases. In these cases, formula (13.3.1) is invalid.

?

13.3.1. Two identical vessels are connected to a manometer in the form of a narrow glass U-tube (Fig. 377). The levels of mercury in the manometer arms are equal. The vessels are immersed in a jar with warm water. (a) What will happen to the position of mercury level in the manometer? How will the answer to this question change if (b) the vessels have different sizes; (c) one vessel is filled with nitrogen and the other with hydrogen; (d) the level of mercury in the right arm had been higher than in the left arm before the vessels were immersed in water?

13.3.2. Some types of electric incandescent lamps are filled with a mixture of nitrogen and argon. When a lamp is switched on, the gas in it is heated to 100 °C. What must be the pressure of the gas mixture at 20 °C if it is desirable that the pressure of the gas in the lamp (when it is switched on) do not exceed the atmospheric pressure?

13.3.3. On pressure gauges, there is a red mark indicating the limit above which the gas pressure should not be raised. At 0 °C, a pressure gauge indicates that the excess pressure over the external air pressure is 120 atm. Will the red mark be reached as the temperature increases to 50 °C if the mark correspond to 135 atm? The external air pressure should be taken as 1 atm.

13.3.4. Let us suppose that in a certain country the normal pressure of a gas is assumed to be taken at 100 °C and not at 0 °C. What would be the temperature coefficient of pressure for gases in this case?

13.4. Gay-Lussac's Law from the Point of View of Molecular Theory

What processes occur in molecular micróworld when the temperature of a gas increases and its pressure rises? From the viewpoint of molecular theory, there can be two reasons behind the increase of pressure in the gas: firstly, the number of impacts of molecules per unit area per unit time may increase, and secondly, the momentum transferred during an impact against the wall may increase. Both factors involve an increase in the molecular velocity (it should be recalled that the volume of a given mass of a gas remains unchanged). Hence it becomes clear that an increase in the temperature of a gas (in the macroworld) corresponds to an increase in the average velocity of random molecular motion (in the microworld). Experiments on the determination of the velocities of gas molecules, which will be described in Sec. 13.24, confirm this conclusion.

When we are dealing not with a gas but with a solid or liquid, no direct methods of determining the velocity of the molecules are available. However, in these cases undoubtedly the molecular velocity grows with temperature, as it was mentioned in Sec. 12.4.

? **13.4.1.** The rate of diffusion increases with temperature. Explain this phenomenon.

13.5. Variation of Gas Temperature with a Change in Its Volume. Adiabatic and Isothermal Processes

In the previous sections, the law of variation of the gas pressure with temperature at constant volume was established. Let us now see how the pressure of a certain mass of a gas changes depending on the volume occupied by it if the temperature remains unchanged. However, before we consider this problem, we must learn how to maintain a constant temperature of a gas. For this purpose, we must find out what happens to the temperature of a gas if its volume changes so rapidly that there is practically no heat exchange with surrounding bodies.

Let us make the following experiment. We put a piece of cotton impregnated by ether on the bottom of a thick-wall tube made of a transparent material (Plexiglass or glass) and open at the other end. In the tube there will be a mixture of ether vapour and air, which is explosible if heated. Then we rapidly push into the tube a piston which tightly fits the tube (Fig. 378). A small explosion can be observed inside the tube. This means that the temperature of the mixture of ether vapour and air has abruptly increased upon compression. This phenomenon can be easily explained. By compressing the gas with an external force, we do work as a result of which the internal energy of the gas should increase. This just occurred, and the gas was heated.

Let us now allow the gas to expand and do work against the forces of external pressure. This can be realised, for example, as follows (Fig. 379). Suppose that a large bottle contains compressed air at room temperature. We let the air from the bottle expand by flowing out through a small orifice, and place in the jet of expanding air a thermometer or a flask with a tube shown in Fig. 384. The thermometer will indicate a lower temperature as compared to the room temperature, while the liquid drop in the tube connected to the flask will move towards the flask, also indicating a decrease in the air temperature in the jet. This means that when a gas ex-

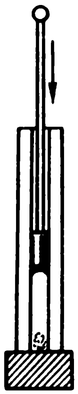


Fig. 378.
By rapidly pushing a piston into a tube with thick walls, we make a piece of cotton inside the tube burst into flame.

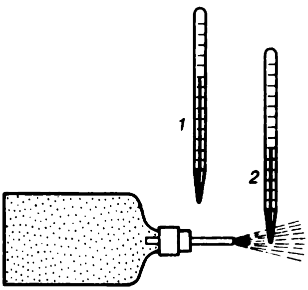


Fig. 379.
Thermometer *2* placed in a jet of expanding air indicates a lower temperature than thermometer *1*.

pands and does work, it is cooled and its internal energy decreases.[2] The heating of a gas upon compression and cooling during expansion are obviously the manifestations of the energy conservation law.

If we now turn to the microworld, the phenomena of heating a gas upon compression and its cooling as a result of expansion become quite clear. When a molecule strikes against a stationary wall and bounces off it, its velocity, and, hence, the kinetic energy remain, on the average, the same as before the impact. If, however, a molecule impinges on a piston moving towards it and is bounced off, its velocity and kinetic energy are higher than before the impact against the piston (similar to the velocity of a tennis ball which increases if we hit it with a racket in the direction opposite to the direction of motion of the ball). The approaching piston imparts an additional energy to the molecule bounced off it. For this reason, the internal energy of the gas increases as a result of compression. If a molecule is bounced off a piston moving away from it, its velocity decreases since the molecule does work by pushing the piston. Therefore, the expansion of a gas as a result of moving away a piston or the layers of surrounding gas is accompanied by doing work and leads to a decrease in the internal energy of the gas.

Thus, the compression of a gas by an external force causes its heating, while the expansion of a gas is accompanied by its cooling. This phenomenon always takes place to a certain extent, but it becomes especially noticeable when heat exchange with surrounding bodies is reduced to a

---

[2] It should be recalled that while considering in Sec. 11.1 the increase in the energy of the cork pushed out from a bottle with soda water, it was noted that the gas in the bottle is cooled.

minimum, since such an exchange can to some extent compensate for the temperature variation. The processes occurring without a heat exchange with the surrounding medium are called *adiabatic* processes.

Let us now return to the question posed at the beginning of this section. How can we maintain a constant temperature of a gas in spite of a variation of its volume? We must obviously supply heat to the gas from outside all the time if it expands, and remove heat continually from the gas being compressed and transfer it to the surrounding bodies. In particular, the temperature of a gas remains virtually constant if its expansion or compression is carried out very slowly, while heat exchange with the surroundings proceeds quite rapidly. During a slow expansion, the heat is transferred to a gas from the surrounding bodies, and its temperature decreases so little that this decrease can be neglected. On the contrary, during a slow compression heat is transferred from the gas to the surroundings, and as a result its temperature increases only slightly. The processes in which the temperature is maintained constant are known as *isothermal*.

? 13.5.1. Why does a bicycle pump get heated when air is pumped into a tyre?

13.6. Boyle's Law

Let us now find out how the pressure of a certain mass of a gas changes if its temperature remains constant and only the gas volume is varied. It was shown above (Sec. 13.5) that such an *isothermal* process is carried out so that the temperature of bodies surrounding the gas remains unchanged, and the gas volume varies so slowly that the temperature of the gas at any stage of the process does not differ from the temperature of the surrounding bodies.

Thus, the question can be formulated as follows: what is the relation between the volume and the pressure of a gas during an isothermal change of its state? Everyday experience convinces us that as the volume of a certain mass of a gas decreases, its pressure rises. By way of example, we can point to an increase in the resilience of a foot ball or bicycle tyre in which air is being pumped. But what is the exact increase in the gas pressure as a result of a decrease in its volume if the gas temperature remains constant?

The answer to this question was obtained in investigations carried out in the 17th century by the English physicist and chemist Robert Boyle (1627-1691) and the French physicist Edme Mariotte (1620-1684).

Experiments establishing the relation between the volume and the pressure of a gas can be made with the help of a device shown in Fig. 380. Glass tubes A and B connected by a rubber tube C are fixed to a vertical graduated board. Some mercury is poured in the tubes. Tube B is open at

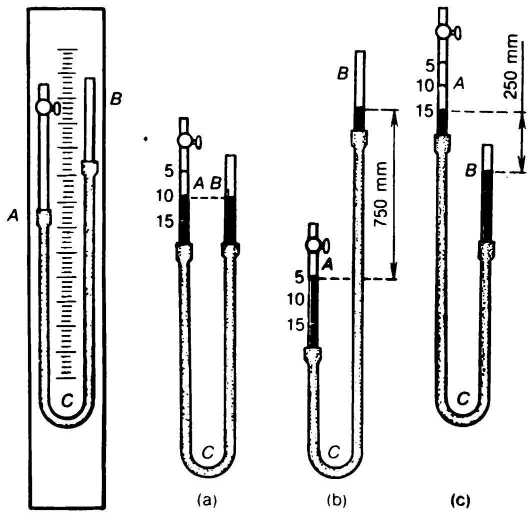


Fig. 380.

A device for investigating the dependence of gas pressure on gas volume. (*a*) A gas in tube *A* has the pressure equal to that of the surrounding air (750 mm Hg) and the volume of 10 cm^3. (*b*) The pressure of the gas in tube *A* is 750 mm Hg + 750 mm Hg, i.e. it is twice as high as that in the case (*a*), while its volume is 5 cm^3, i.e. equal to half the volume in the case (*a*). (*c*) The pressure of the gas in tube *A* is 750 mm Hg — 250 mm Hg, i.e. is reduced to 2/3 of the value in the case (*a*); the volume is 15 cm^3, i.e. is 3/2 times larger than in the case (*a*).

the top, while tube *A* is supplied with a tap. Let us close this tap, thus confining a certain mass of air in tube *A*. As long as we don't shift the tubes, the levels of mercury in them are the same (Fig. 380*a*).

This means that the pressure of the air confined in tube *A* is the same as the pressure of the atmospheric air. Let us slowly raise tube *B* (Fig. 380*b*). It can be seen that the mercury in both tubes rises but differently: the level of mercury in tube *B* will be always higher than in tube *A*. On the contrary, if we lower tube *B* (Fig. 380*c*), the levels of mercury in both arms decrease, but this decrease will be larger in tube *B* than in *A*.

The volume of air confined in tube *A* can be calculated with the help of the scale. The pressure of the air will differ from the atmospheric pressure by the pressure of the mercury column whose height is equal to the difference between the levels of mercury in tubes *A* and *B*. When tube *B* is raised, the pressure of the mercury column is added to the atmospheric pressure. The volume of air in tube *A* decreases in this case. When tube *B* is lowered, the level of mercury in it is lower than in tube *A*, and the pressure of mercury column is subtracted from the atmospheric pressure. Accordingly, the volume of air in tube *A* increases.

Comparing the thus obtained values of pressure and volume of the air confined in tube *A*, we see that as the volume of a certain mass of a gas increases to a certain extent, its pressure decreases to the same extent, and vice versa. The temperature of air in the tube can be considered constant in these experiments.

Similar experiments can be carried out with other gases. The results obtained will be the same.

Thus, *the pressure of a fixed mass of a gas at a constant temperature is inversely proportional to the gas volume* (Boyle's law, or Mariotte's law).

For rarefied gases, Boyle's law is valid to a high degree of accuracy. For highly compressed or cooled gases, however, noticeable deviations from this law are observed.

13.7. Formula Expressing Boyle's Law

Let us denote by V_1 and V_2 the initial and final volumes of a gas, and by p_1 and p_2 its initial and final pressures. On the basis of the experiments described in the previous section, we can write

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}, \tag{13.7.1}$$

whence

$$p_1 V_1 = p_2 V_2. \tag{13.7.2}$$

Formula (13.7.2) is another expression of Boyle's law: *for a given mass of a gas, the product of its volume and pressure remains constant in isothermal processes.*

Formulas (13.7.1) and (13.7.2) can be also applied in the case if the process of changing the volume of a gas is not isothermal, but the temperature changes in such a way that it is the same for a given mass of the gas in the beginning and at the end of the process.

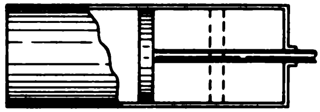
For rarefied gases, Boyle's law is valid to a high degree of accuracy, and if the temperature remains unchanged, the product pV can be regarded as a strictly constant value for a given mass of a gas. However, for very high pressures, noticeable deviations from Boyle's law are observed. As the pressure of a given mass of a gas is gradually increased, the product pV at first slightly decreases, and then starts to increase, reaching the values which are several times larger than those corresponding to a rarefied gas.

? **13.7.1.** A piston is at the middle of a cylinder closed at both ends (Fig. 381). The pressure of a gas in both halves is 750 mm Hg. The piston is shifted so that the volume of the gas on the right-hand side decreases to a half of its initial value. What is the pressure difference?[3]

13.7.2. Two vessels having capacities of 4.5 l and 12.5 l are connected by a tube with a

[3] In all exercises, assume that the temperature of a given mass of a gas remains the same for the initial and the final state.

Fig. 381.
To Exercise 13.7.1.



tap. The first vessel contains a gas under a pressure of 20 atm. The amount of gas in the second vessel is so small that it can be neglected. What will be the pressure in the vessels after the tap has been opened?

13.7.3. An air bubble rises to the surface of water. When it is at a depth of 3 m, its volume is 5 mm^3. What will be the volume of the bubble near the surface? The atmospheric pressure is equal to 760 mm Hg.

13.7.4. Air is pumped by a hand pump into an empty bicycle tyre. After 30 strokes, the area of contact between the tyre and floor has become 60 cm^2. What will be the area of contact between the tyre and floor after 20 more strokes of the pump? Assume in calculations that the bicycle is supported by the air pressure in the tyre (i.e. neglect the elasticity of rubber). During a stroke, the pump sucks in the same amount of air, and the volume of the tyre practically does not change during pumping.

13.8. The Graph Representing Boyle's Law

Graphs illustrating the dependence of the pressure of a gas on its volume are often used in physics and engineering. Let us plot such a graph for an isothermal process. We shall plot the volume of a gas along the abscissa axis and its pressure along the ordinate axis. Suppose that a given mass of a gas has a volume of 1 m^3 at a pressure of 3.6 atm. According to Boyle's law, we find that the pressure corresponding to a volume of 2 m^3 is 3.6 × 0.5 atm = 1.8 atm. Continuing similar calculations, we obtain the following table.

V, m^3	1	2	3	4	5	6
p, atm	3.6	1.8	1.2	0.9	0.72	0.6

Having plotted these data as points whose abscissas are the values of V and ordinates are the corresponding values of p, we obtain the curve describing the isothermal process in the gas (Fig. 382).[4]

? 13.8.1. Plot the graph expressing Boyle's law for the mass of a gas having a volume of 2 l at a pressure of 750 mm Hg.

13.8.2. Which of the areas $OA_1B_1C_1$ and $OA_2B_2C_2$ in Fig. 383 is larger if the curve MB_1B_2N is the graph of an isothermal process?

13.9. Relation between the Gas Density and Pressure

It should be recalled that the density of a substance is its mass contained in a unit volume. If we change the volume of a given mass of a gas, its density

[4] A curve whose ordinates are inversely proportional to the corresponding abscissas is called hyperbola in mathematics.

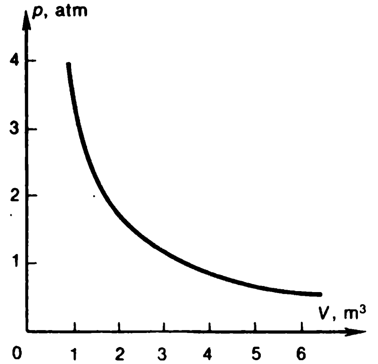


Fig. 382.
The p-V curve representing Boyle's law.

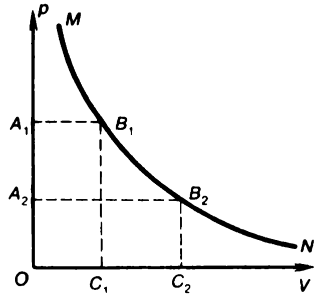


Fig. 383.
To Exercise 13.8.2.

also changes. For example, if we reduce the volume of a gas to one fifth of its initial value, the density becomes five times higher. The gas pressure will also change in this case. If the temperature remains constant, in accordance with Boyle's law the pressure will also increase five-fold. This example shows that *in an isothermal process the pressure of a gas varies in direct proportion to its density.*

If the densities of a gas at pressures p_1 and p_2 are ρ_1 and ρ_2, we can write

$$\frac{\rho_1}{\rho_2} = \frac{p_1}{p_2}. \tag{13.9.1}$$

This important result can be regarded as a more useful expression of Boyle's law. As a matter of fact, formula (13.9.1) contains density instead of volume which depends on an arbitrary quantity (viz. the mass of the gas). Like pressure, density characterises the state of a gas and does not depend at all on an arbitrary choice of its mass.

? **13.9.1.** At a pressure of 1.00 atm and a temperature of 16 °C, the density of hydrogen is 0.085 kg/m^3. Determine the mass of the hydrogen contained in a cylinder having a capacity of 20 l, if the pressure is 80 atm and the temperature is 16 °C.

13.10. Molecular Interpretation of Boyle's Law

Using Boyle's law, we established in the previous section that the pressure of a gas at constant temperature is proportional to its density. This result is in excellent agreement with the molecular pattern of gas pressure, described in Sec. 13.1. If the gas density changes, the number of molecules in a unit volume changes in the same proportion. If the gas is not highly compressed, and the motion of molecules can be assumed to be independent of other molecules, the number N of impacts per unit time per unit surface area is proportional to the number n of molecules in a unit volume. Consequently, if the average velocity of molecules does not change with time (it was shown above that in the macroworld this corresponds to a constant temperature), the gas pressure should be proportional to the number n of molecules in a unit volume, i.e. to the gas density. Thus, Boyle's law is a brilliant confirmation of molecular ideas about gas structure.

It was mentioned in Sec. 13.7, however, that Boyle's law becomes invalid for very high pressures. Lomonosov believed even in his time that this circumstance can be explained on the basis of molecular concepts.

On the one hand, the size of molecules in a highly compressed gas is comparable with intermolecular distances. Thus, the free space in which the molecules move is smaller than the entire volume of the gas. Due to this fact, the number of impacts of molecules against the wall increases since the distance which must be traversed by a molecule before it reaches the wall becomes shorter.

On the other hand, in a highly compressed, and, hence, denser gas the molecules experience the action of attractive forces exerted by other molecules over much longer periods of time than in a rarefied gas. This, on the contrary, reduces the number of impacts of molecules against the wall since in the presence of attraction exerted by other molecules, gas molecules move towards the wall at a lower velocity than in the absence of attraction. For not very high pressures, the latter circumstance is more significant, and the product pV slightly decreases. At very high pressures, the former circumstance plays a more important role, and the product pV increases.

Thus, both Boyle's law and the deviations from it confirm molecular theory.

13.11. Variation of Gas Volume with Temperature

We have investigated the temperature dependence of the pressure of a given mass of a gas at constant volume and its dependence on the volume occupied by a gas at a constant temperature. Let us now investigate the behaviour of a gas whose temperature and volume changes, while the pressure remains constant.

Let us consider the following experiment. We touch the vessel shown in Fig. 384, where a horizontal mercury column closes a certain mass of air.

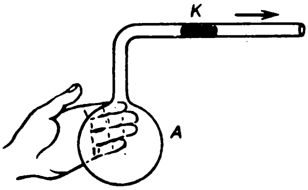


Fig. 384.
A hand heats air in vessel A, the volume of air increases, and the mercury drop K shifts to the right. The pressure remains unchanged and equal to the atmospheric pressure.

The gas in the vessel will be heated, its pressure will increase, and the mercury column will move to the right. The motion of mercury ceases when the pressure of air in the vessel becomes equal to the external pressure due to an increase in the volume. Thus, the volume of air increases during heating, while the pressure remains unchanged.

If we knew how the temperature of air in the vessel changed in our experiment and measured the change in the gas volume, we could study this phenomenon quantitatively. Obviously, for this purpose the vessel must be encased in a jacket to ensure the uniform temperature throughout the device, the volume of the closed mass of the gas must be accurately measured, then the temperature must be changed, and the increment in the gas volume must be measured.

13.12. Charles' Law

Quantitative measurements of the temperature dependence of the volume of a gas at constant pressure were carried out in 1802 by the French physicist and chemist Joseph L. Gay-Lussac (1778-1850). But traditionally, the corresponding law is attributed to Charles who also studied the behaviour of gases under these conditions.

Experiments revealed that *the increment in volume of a gas is proportional to the temperature increment*. Therefore, the thermal expansion of a gas, just as other bodies, can be characterised by the *temperature coefficient of volume expansion* β (Sec. 10.4). It turned out that this law is observed much better for gases than for solids and liquids, and that the temperature coefficient of volume expansion for gases is a practically constant quantity even for considerable temperature variations, while for liquids and solids this constancy is only approximate. Introducing the same notation as in Sec. 10.4, we can write

$$\beta = \frac{V' - V}{V_0(t' - t)} . \tag{13.12.1}$$

Experiments made by Gay-Lussac and other scientists revealed a remarkable circumstance. It turned out that the temperature coefficient of volume expansion β is the same (to be more precise, nearly the same) for all gases and is equal to 1/273 deg^{-1}. *The volume of a fixed mass of any gas increases for each degree rise in temperature by a constant fraction* (1/273) *of the volume at* 0 °C, *the pressure being constant throughout* (Charles' law, also known as Gay-Lussac's law).

It can be seen that the temperature coefficient of volume expansion β for gases coincides with the temperature coefficient of pressure α.

It should be noted that thermal expansion of gases is significant so that the volume V_0 of a gas at 0 °C noticeably differs from its volume at another, say, room temperature. For this reason, as was mentioned in Sec. 10.4, volume V_0 in formula (13.12.1) cannot be replaced by volume V without introducing a noticeable error. For this reason, the formula describing thermal expansion of gases can be represented, for the sake of convenience, in the following form. As an initial volume, we take the volume V_0 occupied by the gas at 0 °C. In this case, the temperature increment τ is equal to the temperature t measured on the Celsius scale. Consequently, the temperature coefficient of volume expansion is

$$\beta = \frac{V - V_0}{V_0 t}, \quad \text{whence} \quad V = V_0(1 + \beta t). \qquad (13.12.2)$$

Since $\beta = 1/273 \text{ deg}^{-1}$, we get

$$V = V_0 \left(1 + \frac{t}{273}\right). \qquad (13.12.3)$$

Formula (13.12.2) can be used for calculating the volume at a temperature both higher than 0 °C and lower than 0 °C. In the latter case, t will have negative values. It should be borne in mind, however, that Charles' law is not valid for a highly compressed gas or a gas cooled to such an extent that it approaches the state of liquefaction. In these cases, formula (13.12.2) is inapplicable.

The coincidence of coefficients α and β appearing in Gay-Lussac's and Charles' laws is not accidental. It can be easily seen that since gases obey Boyle's law, these coefficients must be equal. Indeed, let us suppose that a certain mass of a gas has the volume V_0 and the pressure p_0 at 0 °C. Let us heat it to a temperature t at constant volume. According to Gay-Lussac's law, the pressure of this gas will be $p = p_0(1 + \alpha t)$. On the other hand, if we heat the same mass of the gas to the temperature t at constant pressure, its volume becomes $V = V_0(1 + \beta t)$. Therefore, the given mass can have at the temperature t the volume V_0 and the pressure $p = p_0(1 + \alpha t)$ or the pressure p_0 and the volume $V = V_0(1 + \beta t)$. According to Boyle's law, $V_0 p = V p_0$, i.e.

$$V_0 p_0(1 + \alpha t) = p_0 V_0(1 + \beta t), \quad \text{whence} \quad \alpha = \beta.$$

? 13.12.1. The volume of a balloon at 0 °C is 820 m^3. What will be the volume of this balloon if under the action of solar radiation the gas inside it is heated to 15 °C? The change in the mass of the gas as a result of its flowing out of the envelope and the change in its pressure should be neglected.

13.13. Graphs Representing Gay-Lussac's and Charles' Laws

We shall plot along the abscissa axis the temperature of a gas contained in a constant volume, and along the ordinate axis, its pressure. Suppose that the gas pressure is 1 atm at 0 °C. Using the Gay-Lussac law, we can calculate the gas pressure at 100, 200, 300 °C, etc., and obtain the following table:

t, °C	0	100	200	300	400	500
p, atm	1	1.37	1.73	2.10	2.47	2.83

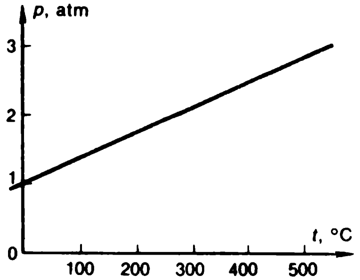


Fig. 385.
A graph representing Gay-Lussac's law.

Plotting these data on the graph, we obtain an inclined straight line (Fig. 385). This graph can also be extrapolated to include negative temperatures. It was mentioned above, however, that Gay-Lussac's law is applicable only for not very low temperatures. Therefore, the extrapolation of the graph to the intersection with the abscissa axis, where the pressure is zero, will not correspond to the actual behaviour of the gas.

The graph representing Charles' law has a similar form.

? **13.13.1.** Plot the graph representing Charles' law.

13.14. Thermodynamic Temperature

The pressure of a gas contained in a constant volume is not proportional to temperature measured on the Celsius scale. This is clear, for example, from the table given in the previous section. The pressure at 100 °C is 1.37 atm, while at 200 °C it is 1.73 atm. The temperature measured on the Celsius scale has doubled, while the pressure has become only 1.26 times higher. There is nothing astonishing in this fact since the Celsius scale was established conditionally, without any connection with the laws of expansion of gases. Using the gas laws, it is possible to establish a temperature scale such that *the gas pressure will be proportional to temperature* measured on this scale.

Indeed, suppose that at a certain temperature t_1 the gas pressure is p_1, while at another temperature t_2 the gas pressure is p_2. According to Gay-Lussac's law, we can write

$$p_1 = p_0\left(1 + \frac{t_1}{273}\right) = p_0\,\frac{273 + t_1}{273},$$

$$p_2 = p_0 \left(1 + \frac{t_2}{273}\right) = p_0 \frac{273 + t_2}{273} .$$

Dividing these equations termwise, we obtain

$$\frac{p_1}{p_2} = \frac{273 + t_1}{273 + t_2} . \tag{13.14.1}$$

The quantity $273 + t$ can be regarded as the value of temperature measured on the new temperature scale having the same value of division as the Celsius scale and zero lying 273 °C below the zero point on the Celsius scale, i.e. the melting point of ice.[5] The zero on the new scale is called the *absolute zero*. This name is due to the fact that, as was proved by the English physicist William T. Kelvin (1824-1907), none of bodies can be cooled below this temperature. This new scale is known as the *thermodynamic temperature scale*. Thus, the absolute zero indicates a temperature of −273 °C, below which not a single body can be cooled under any conditions.

The temperature $273 + t$ is the thermodynamic temperature[6] of a body which has the temperature t on the Celsius scale. Thermodynamic temperature is normally denoted by T. The unit of the thermodynamic scale is a *kelvin* (K) and is a base unit of the SI system. A kelvin coincides with a Celsius degree.

The following relations connect the temperature t measured on the Celsius scale and the thermodynamic temperature T:

$$T = t + 273 \text{ K} \quad \text{or} \quad t = T - 273 \text{ °C}.$$

It follows from what has been said above that equality (13.14.1) can be represented in the following form

$$\frac{p_1}{p_2} = \frac{T_1}{T_2} . \tag{13.14.2}$$

Thus, *the pressure of a fixed mass of a gas at constant volume is proportional to thermodynamic temperature.* This is another expression of Gay-Lussac's law.

Formula (13.14.2) is convenient when the pressure at 0 °C (p_0) is unknown. Let us consider the following example. Suppose that the pressure p_1 of a gas in a cylinder is equal to 40 atm at $t_1 = 25$ °C. What will be the pressure at $t_2 = 35$ °C? In this case, the thermodynamic temperatures are

$$T_1 = 273 \text{ K} + 25 \text{ K} = 298 \text{ K}, \quad T_2 = 273 \text{ K} + 35 \text{ K} = 308 \text{ K}.$$

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[5] To be more precise, 273.15 °C. — *Eds.*

[6] Thermodynamic temperature was formerly called the absolute temperature. — *Eds.*

Using Gay-Lussac's law, we can write

$$\frac{40}{p_2} = \frac{298}{308}, \quad \text{whence} \quad p_2 = 41.3 \text{ atm}.$$

? **13.14.1.** The pressure gauge connected to a cylinder with oxygen indicated 95 atm in a room where the temperature is 17 °C. This cylinder was taken outdoors, where the pressure gauge reading was 85 atm at a temperature of − 13 °C. It was suspected that some oxygen from the cylinder had been consumed. Check whether this suspicion is justified.

13.15. Gas Thermometer

While discussing thermometers (Sec. 10.2), it was pointed out that the most perfect are gas thermometers. We know that the temperature coefficient of pressure of a gas, measured with the help of a mercury thermometer, is virtually constant (Gay-Lussac's law). This property of gases is employed in the new temperature scale: it is assumed that the thermodynamic temperature is *exactly* proportional to the pressure of a fixed volume of a gas.

Figure 386 shows a simple gas thermometer. Vessel C is immersed in a liquid whose temperature has to be measured. The volume of the gas in the vessel is maintained constant by raising or lowering the tube containing mercury. The gas pressure in the vessel is equal to the sum of the atmospheric pressure and the pressure of the mercury column AB. If the gas pressure is p_0 at a temperature T_0, and during the measurement it has become p, the temperature of the liquid is assumed to be

$$T = T_0 \frac{p}{p_0}.$$

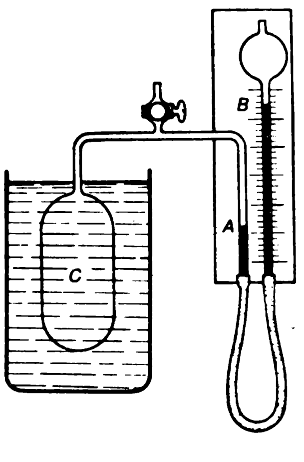

Fig. 386.
Gas thermometer.

In the temperature interval for which an ordinary mercury thermometer can be used, the scale of the gas thermometer almost coincides with the scale of the mercury thermometer since the temperature coefficient of pressure measured with the help of the mercury thermometer is almost constant.

Gas thermometers intended for measuring low and not very high temperatures are made of glass or quartz and filled with hydrogen or helium. For measuring temperatures below the liquefaction temperature for hydrogen (−253 °C), only helium can be used, which liquefies at a very low temperature.

Gas thermometers designed for measuring very high temperatures are made from platinum-rhodium alloy which can withstand high temperatures and are filled with nitrogen (hydrogen cannot be used since it passes through heated platinum).

Gas thermometers are normally used only for checking other types of thermometers which are more convenient in everyday operation than gas thermometers. Clearly, Gay-Lussac's law must be observed very strictly while measuring temperature with the help of a gas thermometer since thermodynamic temperature, by definition, is proportional to the gas pressure.

13.16. Gas Volume and Thermodynamic Temperature

Having transformed formula (13.12.3) in the same way as in Sec. 13.14, we obtain the formula

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}.$$

Thus, *the volume of a fixed mass of a gas at constant pressure is proportional to the thermodynamic temperature*. This is another formulation of Charles' law.

? **13.16.1.** Atmospheric air at a temperature of −25 °C enters the vent tube of a house. What will be the volume of 1 m^3 of this air after it has been delivered to a room and heated to 17 °C?

13.16.2. Flue gas rises up in a cylindrical chimney. At the bottom of the chimney, it has a temperature of 700 °C and a velocity of 5 m/s. What is its velocity at the top of the chimney if its temperature is 200 °C?

13.17. Temperature Dependence of Gas Density

How does the density of a fixed mass of a gas change if the gas temperature increases, its pressure remaining constant?

It should be recalled that density is the ratio of the mass of a body to its volume. Since the mass of the gas is constant, the density decreases upon heating in the same proportion as the gas volume increases.

If the gas pressure remains constant, the volume is proportional to thermodynamic temperature. Consequently, *the density of a gas at a fixed pressure is inversely proportional to thermodynamic temperature*. If ρ_1 and ρ_2 are the densities of a gas at the temperatures T_1 and T_2, the following relation takes place:

$$\frac{\rho_1}{\rho_2} = \frac{T_2}{T_1}. \tag{13.17.1}$$

? 13.17.1. A sphere made of glued paper (a model of the montgolfier) has a mass of 140 g and a volume of 1.75 m^3. Will it rise if the air contained in it is heated to 50 °C, while the surrounding air has a temperature of 15 °C? The density of air at 0 °C should be assumed to be 1.3 kg/m^3.

13.18. Equation of State for a Gas

We have considered so far the cases when one of the three quantities characterising the state of a gas (pressure, temperature and volume) remains unchanged. It was found that if the temperature is constant, the pressure and volume are related through Boyle's law, if the volume is fixed, the pressure and temperature are related through Gay-Lussac's law, and if pressure is maintained constant, the volume and temperature are connected through Charles' law. Let us establish a relation between the pressure, volume and temperature of a certain mass of a gas when *all these three quantities vary*.

Let the initial volume, pressure and thermodynamic temperature of a fixed mass of a gas be V_1, p_1 and T_1, and the final values be V_2, p_2 and T_2 respectively. We can assume that the transition from the initial to the final state occurs in two stages. Suppose that, for example, the volume of the gas has changed from V_1 to V_2 so that the temperature T_1 remained constant. The gas pressure obtained as a result of such a transition will be denoted by p'. Then the temperature has changed from T_1 to T_2 at the constant volume V_2, while the pressure has changed from p' to p_2. Let us compile the following array:

$$\text{Boyle's law}\ \left\{\begin{array}{l} p_1V_1T_1 \\ p'V_2T_1 \end{array}\right. \qquad \left.\begin{array}{l} p'V_2T_1 \\ p_2V_2T_2 \end{array}\right\}\ \text{Gay-Lussac's law.}$$

Applying Boyle's law to the first transition, we can write

$$\frac{p_1}{p'} = \frac{V_2}{V_1}, \quad \text{or} \quad \frac{p_1V_1}{p'V_2} = 1.$$

Applying Gay-Lussac's law to the second transition, we obtain

$$\frac{p'}{p_2} = \frac{T_1}{T_2}.$$

Multiplying these equalities termwise, we obtain

$$\frac{p_1 V_1}{p_2 V_2} = \frac{T_1}{T_2}. \tag{13.18.1}$$

Thus, *the product of the volume of a fixed mass of a gas and its pressure is proportional to the thermodynamic temperature of the gas. This is just the equation of state for a gas.*

? **13.18.1.** Prove that formula (13.18.1) expresses Boyle's law if $T_1 = T_2$, Gay-Lussac's law if $V_1 = V_2$, and Charles' law if $p_1 = p_2$.

13.18.2. The volume of a gas obtained as a result of a chemical reaction is 72 cm^3 at a pressure of 742 mm Hg and at a temperature of 18 °C. What is the volume of the same mass of the gas under normal conditions?[7]

13.18.3. In one type of internal combustion engine (Diesel engine), the atmospheric air is sucked into a cylinder where it is subjected to compression and is heated thereby. Experiments show that after the volume of air is reduced to 1/12 of its initial value, the pressure is 34 atm. Assuming that the pressure and temperature of atmospheric air are 1 atm and 10 °C respectively, determine the temperature of the compressed air.

13.18.4. In order to make a submarine surface, compressed air is blown into its tanks which were filled with water. Suppose that the tanks are blown at a depth of 15 m so that the air in the tank acquires the temperature of surrounding water, which is 3 °C. What volume of water can be blown through by letting out the air from a cylinder having a capacity of 20 l if the air pressure in the cylinder is 120 atm at 17 °C? It should be taken into account in calculations that a part of expanding air will remain in the cylinder.

13.18.5. The density of air under normal conditions is 1.3 kg/m^3. What is the density of air at a pressure of 30 mm Hg and a temperature of -35 °C?

13.18.6. Figure 387 shows a stratostat (a balloon for exploring the stratosphere) at the surface of the Earth and at an altitude of several kilometres. Why does the volume of the stratostat changes during the ascent? What will be its volume at an altitude of 10 km where the pressure is 198 mm Hg and the temperature is -50 °C, if at the surface of the Earth, where the pressure and temperature are 750 mm Hg and 10 °C respectively, the volume is 400 m^3?

13.18.7. Prove that the buoyancy of the stratostat does not change with altitude if the pressure in it differs from the atmospheric pressure insignificantly and the gas does not flow out.

13.19. Dalton's Law

So far we considered the volume of a certain individual gas (oxygen, hydrogen, etc.). However, in nature and in engineering we often deal with

[7] Normal conditions correspond to a temperature of 0 °C and a pressure of 760 mm Hg (1 atm). — *Eds.*

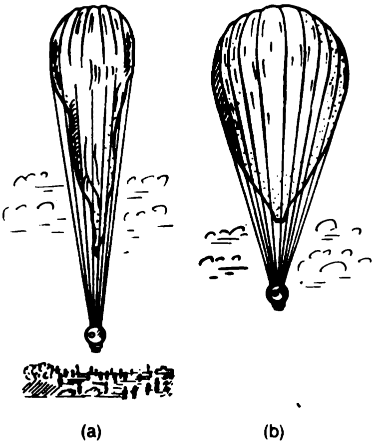

Fig. 387.
A stratostat (*a*) at the beginning of an ascent and (*b*) at an altitude of a few kilometres.

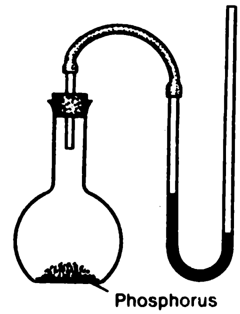

Fig. 388.
As phosphorus absorbs oxygen from air, the manometer indicates a decrease in pressure.

a mixture of several gases. The most important example of such a mixture is air which is a mixture of nitrogen, oxygen, argon, carbon dioxide and other gases. What determines the pressure of a gas mixture?

Let us put in a flask a substance capable of binding atmospheric oxygen, say phosphorus, and quickly close the flask by a cork with a tube connected to a mercury manometer (Fig. 388). In a certain time, all the oxygen contained in the flask will combine with phosphorus. The manometer will indicate a lower pressure in comparison with that before the removal of oxygen. It means that the presence of oxygen in air increases its pressure.

The pressure of a gas mixture was thoroughly investigated for the first time in 1809 by the English chemist John Dalton (1766-1844). The pressure that a gas in a mixture of gases occupying a fixed volume would exert if it alone occupied the total volume is called the *partial pressure* of this gas. Dalton established that *the pressure of the mixture of gases is equal to the sum of their partial pressures* (Dalton's law or law of partial pressures). If, for example, the pressure of oxygen in a flask is 400 mm Hg and the pressure of hydrogen contained in an identical flask at the same temperature is 300 mm Hg, having mixed these gases in the same flask (at the same temperature) we shall obtain a mixture at a pressure of 400 mm Hg + 300 mm Hg = 700 mm Hg. It should be noted that Dalton's law (like Boyle's law) is inapplicable to highly compressed gases.

The interpretation of Dalton's law from the viewpoint of molecular theory will be given in Sec. 13.21.

13.20. Density of Gases

The density of a gas is one of its important characteristics. When we speak of gas density, we usually mean the density of a gas *under normal conditions* (i.e. at a temperature of 0 °C and a pressure of 760 mm Hg). Besides, the concept of *relative density* of a gas is often used, which is the ratio of the density of a given gas to the density of air under the same conditions. It can be easily seen that the relative density of a gas does not depend on the conditions in which it is kept since, according to the gas laws, the volumes of all gases change identically with pressure and temperature.

The density of a gas can be determined as follows. We must weigh a flask with a tap twice: firstly with air having been pumped out to the largest possible extent, and then, after it has been filled by a gas under investigation, to a pressure which must be known. Dividing the difference in masses by the volume V of the flask, which has to be determined beforehand, we shall obtain the gas density under given conditions. Using then the equation of state, we can find the density ρ_n of the gas under normal conditions. Indeed, putting in formula (13.18.1) $p_2 = p_n$, $V_2 = V_n$ and $T_2 = T_n$, and multiplying the numerator and denominator of the left-hand side by the mass m of the gas, we get

$$\frac{p_1}{p_n}\frac{V_1}{m}\frac{m}{V_n} = \frac{T_1}{T_n}.$$

Considering that $m/V_1 = \rho_1$ and $m/V_n = \rho_n$, we obtain

$$\rho_n = \rho_1 \frac{p_n T_1}{p_1 T_n}.$$

The result of measurement of densities for some gases are contained in Table 7. The last two columns of the table reveal the linear dependence bet-

Table 7. Density of Some Gases under Normal Conditions

Gas	Density, kg/m^3	Ratio to air density	Ratio to hydrogen density	Relative molecular mass
Air	1.293	1	14.5	29 (average)
Carbon dioxide (CO_2)	1.977	1.53	22	44
Helium (He)	0.179	0.139	2	4
Hydrogen (H_2)	0.0899	0.0695	1	2
Nitrogen (N_2)	1.25	0.967	14	28
Oxygen (O_2)	1.43	1.11	16	32

ween the density of a gas and its relative molecular mass (or atomic mass as in the case of helium).[8]

13.21. Avogadro's Law

Comparing the figures in the last but one column of Table 7 with relative molecular masses of gases, it can be easily seen that the densities of gases are proportional to their relative molecular masses (under identical conditions). This fact leads to a very important conclusion. Since the relative molecular masses of two gases are to each other as the masses of their molecules, we have

$$\rho_1/\rho_2 = m_1/m_2,$$

where ρ_1 and ρ_2 are the densities of the gases and m_1 and m_2 are the masses of their molecules.

The mass of a gas can be represented as the product of the number N of molecules and the mass m of a molecule. Consequently, the masses of the first and second gases are N_1m_1 and N_2m_2 respectively. The masses of two gases contained in equal volumes are to each other as their densities. Therefore,

$$N_1m_1/N_2m_2 = \rho_1/\rho_2,$$

where N_1 and N_2 are the numbers of molecules of the first and second gases contained in equal volumes. Comparing this result with the ratio $\rho_1/\rho_2 = m_1/m_2$, we obtain $N_1 = N_2$.

Thus, *equal volumes of different gases contain the same number of molecules at the same pressure and temperature*.

This law was discovered in 1811 by the Italian chemist Amedeo Avogadro (1776-1856) on the basis of chemical investigations. It can be applied to gases which are not compressed very strongly (for example, to gases under the atmospheric pressure). For highly compressed gases, this law is inapplicable.

Avogadro's law indicates that at a certain temperature, the pressure of a gas depends only on the number of molecules in a unit volume of this gas and does not depend on whether these molecules are heavy or light. Having grasped this fact, we can easily understand the essence of Dalton's law. According to Boyle's law, by increasing the density of a gas, i.e. by adding a certain number of molecules of this gas into a fixed volume, we increase the gas pressure. But according to Avogadro's law, the same increase in

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[8] The *relative atomic mass* (A_r) of an element (known as the atomic weight) is the ratio of the mass of an atom of this element to 1/12 of the mass of the ^{12}C atom (this is the notation for the carbon isotope with a mass number of 12). The *relative molecular mass* (M_r) of a substance (or simply the molecular weight) is the ratio of the mass of a molecule of this substance to 1/12 of the mass of the ^{12}C atom. — *Eds.*

pressure can be obtained if instead of molecules of the first gas we add the same number of molecules of another gas. This is just the essence of Dalton's law which states that the pressure of a gas can be increased by adding the molecules of another gas to the same volume; if the number of molecules added is the same as in the former case, we shall obtain the same increment of pressure. Thus, Dalton's law is a direct consequence of Avogadro's law.

13.22. Mole. Avogadro's Number

The relative molecular mass also indicates the ratio of masses of two portions of substances containing the same number of molecules. Therefore, 2 g of hydrogen (whose relative molecular mass $M_r = 2$), 32 g of oxygen (the relative molecular mass $M_r = 32$), 55.8 g of iron (whose relative molecular mass M_r coincides with the relative atomic mass $A_r = 55.8$), etc. contain the same number of molecules. The amount of a substance containing the number of particles (atoms, molecules, ions, electrons, etc.) equal to the number of atoms in 0.012 kg of the carbon isotope ^{12}C is called *mole* (mol). It follows from what has been said above that moles of different substances contain the *same number of molecules*. For this reason, a mole is taken as a unit of the amount of substance and is a base SI unit. The mass of a mole of a substance is known as its *molar mass* and is denoted by M.

It follows from the definition of the relative molecular mass M_r[9] that the molecular mass for ^{12}C is $M_r = 12$, while the definition of a mole implies that the molar mass of ^{12}C is $M = 0.012$ kg/mol. Thus, the molar mass of carbon ^{12}C is numerically equal to 0.001 of its relative molecular mass. It can be easily seen that the same relation holds for any other substance: M is numerically equal to $0.001\,M_r$. It should be noted that M_r is a dimensionless quantity, while M is expressed in kilograms per mol (kg/mol).

The number of molecules in a mole of a substance, which is known as *Avogadro's number*, is an important physical constant. A large number of experiments were carried out for determining Avogadro's number. These experiments involve Brownian movement (Sec. 12.7), electrolysis and some other phenomena. These investigations led to the same result. At present, it is assumed that the Avogadro number is

$$N_A = 6.022045 \times 10^{23}\ \text{mol}^{-1} \approx 6.02 \times 10^{23}\ \text{mol}^{-1}.$$

Thus, 2 g of hydrogen, 32 g of oxygen, etc. contain 6.02×10^{23} molecules. In order to visualise such a large number, let us imagine a desert

[9] For monatomic substances, M_r coincides with A_r.

with an area of one million square kilometres, covered with a layer of sand 600 m thick. If the volume of a grain of sand is 1 mm^3, the total number of grains in the desert is equal to Avogadro's number.

It follows from Avogadro's law that *under equal conditions the moles of different gases have equal volumes*. The volume of one mole under normal conditions can be calculated by dividing the molar mass of a gas by its density under normal conditions.

Let us carry out the calculation for oxygen. Since $M_r = 32$ for it, $M = 0.032$ kg/mol. From Table 7 we take $\rho = 1.43$ kg/m^3. Consequently, the volume of a mole of oxygen is

$$V = \frac{0.032 \text{ kg/mol}}{1.43 \text{ kg/m}^3} = 22.4 \times 10^{-3} \text{ m}^3\text{/mol} = 22.4 \text{ l/mol}.$$

Thus, *the volume of a mole of any gas under normal conditions is* 22.4 l/mol (to be more precise, 22.41383×10^{-3} m^3/mol).

It should be noted that the equation of state (13.18.1) for a mole of a gas can be written as follows:

$$pV = RT,$$

where V is the volume of the mole of a gas and $R = 8.31441$ J/(mol·K) is the proportionality factor which is the same for all gases and is known as the *gas constant*.

? **13.22.1.** Using Table 7, calculate the volumes of a mole of nitrogen and a mole of hydrogen under normal conditions.

13.22.2. Determine the number of molecules in a unit volume of a gas under normal conditions.

13.22.3. Calculate the masses of a hydrogen and an oxygen molecule.

13.23. Velocities of Gas Molecules

What are the velocities of molecules, in particular, gas molecules? This question naturally arose as soon as the molecular ideas had been developed. For a long time, molecular velocities could be estimated only indirectly, and only at a later stage the methods of direct measurement of the velocities of gas molecules were worked out.

First of all, let us define more precisely what we mean by the velocity of molecules. It should be recalled that as a result of frequent collisions, the velocity of each individual molecule permanently changes: the molecule now moves rapidly, then slowly, and during a certain interval of time (say, a second) the velocity of the molecule assumes a large number of different values. On the other hand, at a certain instant there are molecules with diverse velocities in a fixed volume of a gas under investigation. Obviously, in order to characterise the state of the gas, we must speak about an *average velocity*. We can assume that it is the average value of the velocity of an individual molecule over a sufficiently long interval of time, or that it is the average value of the velocities of all the molecules in the given volume at a certain instant.

We shall give the arguments that will allow us to calculate the average velocity of gas molecules.

It was shown in Sec. 13.1 that the gas pressure is proportional to nmv^2, where m is the mass of a molecule, v is its average velocity and n is the number of molecules per unit volume. Exact calculations lead to the formula

$$p = \frac{1}{3} nmv^2. \tag{13.23.1}$$

Let us consider a gas contained in a vessel having the shape of a cube with edge l (Fig. 389). If the gas is in equilibrium, all directions of motion of its molecules are equally probable so that the molecules strike the walls at different angles (from 0 to $\pi/2$) with the normal to a wall. For the sake of simplicity, we shall assume that the molecules move only in three mutually perpendicular directions coinciding with the edges of the cube so that 1/3 of all molecules fly in each direction. One such molecule flying along the normal to the hatched face of the cube is shown in Fig. 389. The number of such molecules is $nl^3/3$, where n is the number of molecules per unit volume.

Neglecting the collisions of molecules with one another, we can assume that the molecule under consideration flies at the average velocity v, being reflected from the opposite faces alternately. During the time between two consecutive collisions with the hatched face, the molecule traverses a distance equal to $2l$. Consequently, it strikes against the hatched wall $v/2l$ times per unit time. In total, the wall experiences

$$\frac{nl^3}{3}\frac{v}{2l} = \frac{nl^2v}{6}$$

collisions per unit time. Dividing this expression by l^2, we obtain the number N of collisions per unit area of the wall per unit time. Thus,

$$N = \frac{1}{6} nv. \tag{13.23.2}$$

Substituting this value of N into formula (13.1.1), we obtain the gas pressure exerted on the

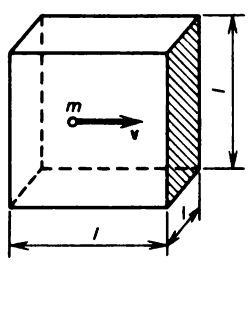

Fig. 389.
A molecule flying along the normal to the hatched face of the cube.

wall:

$$p = \frac{1}{6} nv \cdot 2mv = \frac{1}{3} nmv^2.$$

We arrive at formula (13.23.1).

•

We can derive from formula (13.23.1) a number of important corollaries. Let us write this formula in the form

$$p = \frac{2}{3} \frac{mv^2}{2} = \frac{2}{3} \varepsilon,$$

where ε is the average kinetic energy of a molecule. Suppose that the gas pressures at temperatures T_1 and T_2 are p_1 and p_2, and the average kinetic energies of molecules at these temperatures are ε_1 and ε_2. Then we have

$$p_1 = \frac{2}{3} \varepsilon_1, \quad p_2 = \frac{2}{3} \varepsilon_2, \quad \frac{p_1}{p_2} = \frac{\varepsilon_1}{\varepsilon_2}.$$

Comparing this relation with Gay-Lussac's law $p_1/p_2 = T_1/T_2$, we obtain

$$\frac{T_1}{T_2} = \frac{\varepsilon_1}{\varepsilon_2}.$$

Thus, *the thermodynamic temperature of a gas is proportional to the average kinetic energy of gas molecules.* It should be recalled that the relation between the gas temperature and the average kinetic energy was already considered in Sec. 12.4. Since the average kinetic energy of molecules is proportional to the square of the average velocity of gas molecules[10], our comparison leads to the conclusion that the thermodynamic temperature of a gas is proportional to the square of the average velocity of gas molecules, and that *the velocity of molecules grows in proportion to the square root of thermodynamic temperature.*

Let us now take two gases at equal temperatures and pressures. According to Avogadro's law (Sec. 13.21), the number of molecules per unit volume is the same. Hence we can write

$$p = \frac{1}{3} nm_1 v_1^2 = \frac{1}{3} nm_2 v_2^2,$$

where subscripts 1 and 2 refer to the first and second gases. Thus,

$$\frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}, \tag{13.23.3}$$

[10] Strictly speaking, the average kinetic energy of molecules is proportional to the average value of squared velocity rather than to the squared average velocity. These quantities are proportional but not equal to each other: $(v^2)_{av} = 1.178\,(v_{av})^2$. —*Eds.*

i.e. at a given temperature the average velocities of molecules are inversely proportional to the square roots of their masses. For example, in a mixture of oxygen and hydrogen, the average velocity of oxygen molecules is equal to one fourth of the average velocity of hydrogen molecules.

Finally, pay attention to the fact that the product nm is the mass of the gas molecules contained in a unit volume, i.e. nm is the density ρ of the gas. Therefore, it follows from formula (13.23.1) that

$$v = \sqrt{3p/\rho}. \tag{13.23.4}$$

This formula allows us to calculate the average velocity of gas molecules if we know the pressure and density of a gas. The results of calculation of average molecular velocities for some gases at 0 °C are presented in Table 8.

Table 8. Average Velocity of Molecules for Some Gases

Gas	Mass of molecule, 10^{-26} kg	Average velocity, m/s
Carbon dioxide	7.3	360
Hydrogen	0.33	1760
Nitrogen	4.6	450
Oxygen	5.3	425
Water vapour	3.0	570

It can be seen from the table that average molecular velocities for gases are rather high. At room temperature, they normally reach hundreds of metres per second. The average velocity of gas molecules is about 1.5 times higher than the velocity of sound in the same gas.

This result may seem strange at first sight. It appears that molecules cannot move at such high velocities since diffusion occurs at a quite low rate even in gases, and the more so in liquids. At any rate, it is much lower than the velocity of sound. As a matter of fact, moving molecules collide with one another very frequently and change thereby the direction of their motion. As a result, they now move in one direction and then in another, but mainly "gad about" the same site (see Fig. 369). As a result, in spite of a large velocity of motion in the intervals between collisions, they propagate in a *certain* direction very slowly.

Table 8 indicates that the difference in the velocities of different molecules is associated with the difference in their masses. This fact is confirmed by a number of observations. Hydrogen, for example, penetrates narrow holes (pores) at a higher rate than oxygen or nitrogen. This can be illustrated by the following experiment (Fig. 390). A glass funnel is covered with a porous vessel or paper, its end being immersed in water. If we now cover the funnel with a vessel by a glass under which hydrogen (or town gas) is supplied, the level of the water in the funnel end will be lowered, and gas bubbles will start rising from it. How can this be explained?

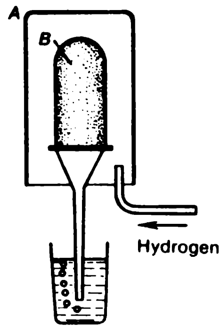

Fig. 390.
When the volume inside glass A is filled with hydrogen, bubbles emerge from the tip of the funnel covered by the porous vessel B.

Narrow pores in the vessel or paper let through both air molecules (from the funnel to the glass) and hydrogen molecules (from the glass to the funnel). However, the rates of these processes are different. The difference in the dimensions of molecules does not play a significant role here since this difference is not large, especially in comparison with the pore size: the "length" of a hydrogen molecule is about 2.3×10^{-10} m (Sec. 12.2), while that of an oxygen or a nitrogen molecule is about 3×10^{-10} m. Pores have thousands of times larger size. On the other hand, the velocity of hydrogen molecules is four times as high as the velocity of air molecules. For this reason, hydrogen molecules find their way from the glass to the funnel quicker.. As a result, an excess of molecules is formed in the funnel, the pressure increases, and the gas mixture is evolved from the vessel in the form of bubbles.

Devices of this type are used for detecting admixtures of mine gas in air, which may cause an explosion in a mine.

? **13.23.1.** If we take off the glass filled with hydrogen from the funnel in the device described above, water will be sucked into the funnel. Explain this phenomenon.

13.23.2. Using Table 7, calculate the average velocities[11] of helium and carbon dioxide molecules at 0 °C.

13.23.3. Using Table 8, calculate average velocities of hydrogen molecules at 1000 °C and nitrogen molecules at − 100 °C.

13.24. Measurement of Velocities of Gas Molecules (Stern's Experiment)

The velocities of molecular motion can be determined by various methods. Among them, there is a simple technique realised in 1920 in Stern's experiment.

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[11] Formula (13.23.4) defines the root-mean-square velocity of molecules, which is equal to the square root of $(v^2)_{av}$. In order to obtain the average velocity, we must divide the root-mean-square velocity by $\sqrt{1.178} = 1.085$ (see Footnote 10). — *Eds.*

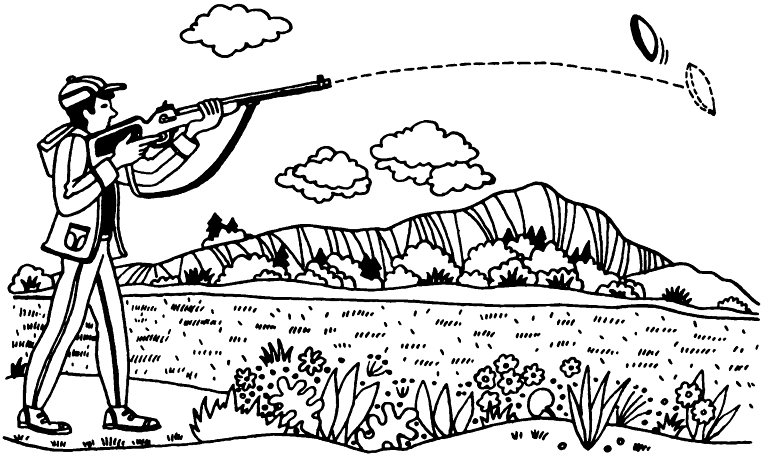

Fig. 391.
If we aim the rifle directly at a moving target, the bullets will fall behind the target.

In order to explain this method, we first consider the following analogy. When a shot is fired at a moving target, one should aim at a point ahead the target, since otherwise the bullets will lag behind it (Fig. 391). The deviation from the target will be the larger, the higher the velocity of the target and the lower the velocity of the bullets.

Let us consider another experiment. A high vessel filled with water is placed on a table that can be rotated (Fig. 392). Water jet flows out of a hole in the vessel. As long as the table is at rest, the jet gets into a glass fixed at the same table. If, however, the table is rotated, the water jet does not get into the glass but falls behind it. The lag of the jet will be the larger, the higher the speed of rotation of the table and the lower the flow velocity of the jet. If we know the speed of rotation and measure the deviation of the jet, we can judge about the velocity of

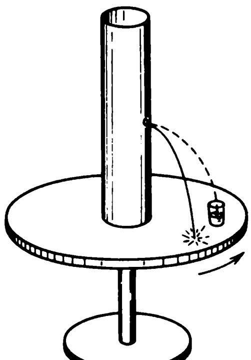

Fig. 392.
When the table is at rest, the water jet from the vessel gets into the glass. When the table rotates, the water jet falls behind the glass.

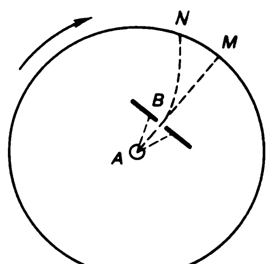

Fig. 393.
Schematic diagram of Stern's experiment for determining the velocity of molecules of metal vapour. If the device is rotated clockwise, silver coating is observed at point N.

the jet flow. This is the idea of the Stern experiment. Instead of water jet, a molecular flow is used.

The schematic diagram of Stern's device is shown in Fig. 393. It consists of a vertical cylinder continually evacuated to a very low pressure. A platinum filament A coated with silver is arranged along the cylinder axis. As an electric current is passed through the filament, it is heated to the melting point of silver. Silver starts to evaporate, and its atoms uniformly fly in a straight line towards the inner surface of the cylinder with a velocity v which corresponds to the temperature of the platinum filament. Slit B isolates a narrow molecular beam (filament A and slit B play the role of the rifle in the above example). The cylinder wall is cooled to make impinging molecules "stick" to it, forming a silver coating. At first the device is at rest, and a silver coating is observed along generatrix M of the cylinder in the form of a narrow vertical strip. Then the cylinder is set into a rapid rotation. Although the "molecular rifle" AB is aimed at the same point M, the target now moves, and the "bullets" (molecules) do not hit point M and instead hit a point N lying behind M. The silver coating formed during rotation is obtained along the generatrix N.

Let us calculate the length s of the arc MN. It is equal to the path traversed by the points of the cylinder during the time t of the flight of molecules from slit B to the cylinder. Thus, $s = ut$, where u is the velocity of motion of the points on the cylinder. On the other hand, if we denote velocity of molecules by v and the path length BM by l, we have $t = l/v$ so that $s = ul/v$, or

$$v = ul/s.$$

The value of s is measured as the distance between the silver strips corresponding to the cylinder at rest and in rotation, while the velocity u of the points on the cylinder surface and the distance l can be measured. Then using the above formula, we can determine the velocity of molecules. In this way, the velocities of molecules of some metal vapours were measured.

? **13.24.1.** The silver coating in Stern's experiment has the form of a narrow band when the device ia at rest and is somewhat blurred when the cylinder is rotated. What does this indicate?

13.25. Specific Heat Capacities of Gases

Suppose that we have a kilogram of a gas. What amount of heat should be supplied to it to raise its temperature by 1 K? In other words, what is the *specific heat capacity* of the gas? The experiments and arguments presented in Sec. 11.6 show that this question cannot be answered unambiguously. The answer depends on the conditions under which this gas is heated. If its volume does not change, a certain amount of heat is required (the gas pressure increases in this case). If the gas is heated so that its pressure remains unchanged, another, larger amount of heat is required in comparison with the former case (now the gas volume increases). Finally, other cases are also possible when both volume and pressure change during heating. The amount of heat required in such a case depends on the extent of these changes. Accordingly, the gas may have different specific heat capacities depending on the conditions of heating. Of special importance are the *specific heat capacities at constant volume* (c_V) *and at constant pressure* (c_p).

In order to determine c_V, a gas should be heated in a closed vessel (Fig. 394). The expansion of the vessel itself upon heating can be neglected.

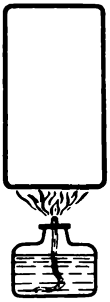

Fig. 394.
Heating of a gas at constant volume.

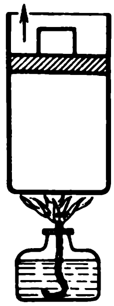

Fig. 395.
Heating of a gas at constant pressure.

While determining c_p, a gas must be heated in a cylinder closed with a piston the loading in which remains unchanged (Fig. 395).[12]

The specific heat capacity at constant pressure (c_p) is larger than the specific heat capacity at constant volume (c_V). Indeed, when a gas is heated through 1 K at constant volume, the supplied heat is spent only in increasing the internal energy of the gas. On the other hand, if the same mass of the gas is heated through 1 K at constant pressure, the supplied heat not only increases the internal energy of the gas but also does the expansion work of the gas. To obtain c_p, we must add to c_V the amount of heat equivalent to the work done during the expansion of 1 kg of the gas.

Specific heat capacities of gases vary over a wide range. For example, c_p = 14.3 kJ/(kg · K) for hydrogen and c_p = 523 J/(kg · K) for argon, i. e. these values differ by a factor of 27.

13.26. Molar Heat Capacities

The heat capacity of a mole of a substance is known as its *molar heat capacity* and denoted by C. The molar heat capacity C is connected with the specific heat capacity c of this material through the relation

$$C = Mc, \tag{13.26.1}$$

where M is the molar mass.

Table 9 gives the experimental values of molar heat capacities at constant pressure (C_p) and at constant volume (C_V) for three monatomic gases (He, Ar, Ne) and three diatomic gases (H_2, N_2 and O_2). The values of the ratio C_p/C_V and of the molar mass M are also given.

It can be seen from this table that the values of C_p for all monatomic gases are close to $(5/2)R$ = 20.8 J/(mol · K), where R is the gas constant, while the values of C_V are close to $(3/2)R$ = 12.5 J/(mol · K). The values of C_p for diatomic gases are close to $(7/2)R$ = 29.1 J/(mol · K), while the values of C_V are close to $(5/2)R$ = 20.8 J/(mol · K).

The values of C_p/C_V are equal to 5/3 for monatomic gases and 7/5 for diatomic gases. Thus, the molar heat capacity for each *type* of gases (monatomic and diatomic) has almost the same value. This is the general rule associated with the fact that gases taken in the amount of a mole contain the same number of molecules.

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[12] In actual practice, c_V and c_p for gases are determined in other, more complicated,ways.

Table 9. Molar Heat Capacity for Some Gases at Constant Pressure and at Constant Volume

Gas	Molar heat capacity, J/(mol · K)		C_p/C_V	Molar mass, kg/mol
	C_p	C_V		
Argon	20.9	12.5	1.67	0.0399
Helium	20.9	12.5	1.67	0.0040
Hydrogen	28.6	20.4	1.40	0.0020
Neon	21.1	12.7	1.66	0.0202
Nitrogen	29.1	20.8	1.40	0.0280
Oxygen	29.4	21.0	1.40	0.0320

This rule is valid for diatomic gases only within a certain temperature interval. For very high temperatures, the molar heat capacities of diatomic gases increase so that C_V tends to $(7/2)R = 29.1$ J/(mol · K), while C_p tends to $(9/2)R = 37.4$ J/(mol · K). At very low temperatures (for example, for hydrogen which remains in the gaseous state down to -239 °C), C_V tends to $(3/2)R = 12.5$ J/(mol · K), while C_p tends to $(5/2)R = 20.8$ J/(mol · K). Without dwelling on details, we only indicate that in order to explain these more complicated phenomena, not only the motion of molecules as a whole but also the vibrations of atoms constituting them must be taken into account.

Let us explain, using monatomic gases by way of example, why the values of molar heat capacities for different gases almost coincide. It should be recalled first of all that changes in the internal energy of gases mainly involve the changes in the kinetic energy of gas molecules since their potential energy practically remains unchanged (Sec. 12.4). On the basis of formula (13.23.3), we can write

$$\frac{m_1 v_1^2}{2} = \frac{m_2 v_2^2}{2},$$

i. e. the average energies of molecules of different gases are equal at the same temperature. Hence it follows that as the temperature increases by 1 K, the average energy of a gas molecule changes *identically* irrespective of its mass. But the number of molecules in a mole of any substance is the same. Consequently, the increment of the internal energy of a mole of any monatomic gas as a result of heating it through 1 K (viz. its molar heat capacity C_V) is also the same.

? **13.26.1.** Calculate the specific heat capacities c_V and c_p for carbon monoxide CO (the molar mass $M = 0.028$ kg/mol). What other gas has the same specific heat capacities?
13.26.2. What is the heat capacity at constant volume of a mass of a diatomic gas which occupies a volume of 1 m^3 under normal conditions?

13.27. The Dulong and Petit Law

The equality of molar heat capacities is also observed for monatomic solids including metals. The specific heats c_p and c_V are not distinguished for solids and we speak just of the specific heat capacity c. P. L. Dulong and A. T. Petit established in 1819 that the molar heat capacity of monatomic solids is approximately the same and is equal to $3R = 25$ J/(mol · K), where R is the gas constant. This is confirmed, for example, by the data compiled in Table 10.

Table 10. Molar Heat Capacities of Some Solids at 25 °C

Substance	Molar heat capacity, J/(mol · K)	Relative atomic mass	Molar mass, kg/mol
Aluminium	24.4	27	0.027
Beryllium	16.4	9	0.009
Copper	24.5	64	0.064
Iron	25.0	56	0.056
Lead	26.4	207	0.207
Magnesium	24.6	24	0.024

The Dulong and Petit law is observed for monatomic solids at sufficiently high temperatures. For most of bodies, even the room temperature is sufficiently high. However, for some substances with a small atomic mass, say, for beryllium, boron or carbon (diamond), the room temperature is insufficiently high, and they obey the Dulong and Petit law only at a higher temperature. On the contrary, *all* substances being cooled exhibit deviations from the Dulong and Petit law. Heat capacity of all bodies *decreases* upon cooling.

? **13.27.1.** How can the specific heat capacity of a metal be approximately estimated if no table of specific heat capacities is available? Do that for silver (A_r = 108) and tungsten (A_r = 184).

Chapter 14

Properties of Liquids

14.1. Structure of Liquids

We have now a clear idea about the structure of gases or crystalline solids. A gas is an aggregate of molecules which randomly move in all directions independently of one another. All (or almost all) molecules of a solid retain their relative position for an infinitely long time, performing only small vibrations about certain equilibrium positions.

The structure of liquids appears to be much more complicated. Let us consider a closed vessel containing a liquid and its vapour so that the liquid occupies only a (lower) part of the vessel, the remaining part being occupied by its vapour (Fig. 396) which, as any gas, fills the entire free space. Naturally, molecules are in continuous motion both in the liquid and in its vapour and may leave the liquid and go over to the vapour and back from the vapour to the liquid. However, a sharp interface is preserved between the liquid and vapour (at a constant temperature), and the exchange of molecules does not disturb the equilibrium between these states. This equilibrium is, however, of a dynamic nature.

The sharp interface between the vapour and a liquid separates two states of aggregation or, as it is said, two *phases* of a substance, of which the vapour phase is characterised by a much lower (by a factor of several thousands) density than the liquid phase. In the liquid phase, the average distance between molecules is much smaller (by a factor of tens) than in the vapour, and, accordingly, the intermolecular cohesive forces are much stronger in the liquid than in the vapour. This just explains the difference in the nature of molecular motion in the vapour and in the liquid.

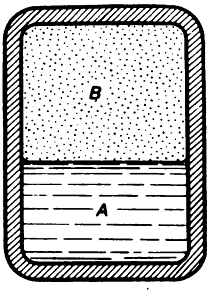


Fig. 396.
The volume of a vessel is divided into two parts, one of which contains liquid A and the other vapour B.

For vapour, as for a gas, we can disregard cohesive forces and consider the molecular motion as a free flight of molecules and their collisions with one another and with surrounding bodies (vessel walls and the liquid covering the bottom of the vessel). As in solids, the molecules in liquids strongly interact, attracting one another. However, while each molecule in a solid remains in a certain equilibrium position for an infinitely long time, and its motion is reduced to vibrations about this position, the nature of motion in a liquid is quite different. Molecules of a liquid move much more freely than the molecules of a solid, although not as freely as gas molecules. Each molecule in a liquid moves forth and back for a certain time, without departing far from its neighbours. This motion resembles the vibrations of a molecule of a solid about its equilibrium position. The liquid molecule, however, leaves its surroundings and goes over to another site, getting into a new surrounding, where it again performs motion resembling vibrations.

Thus, the motion of molecules in a liquid is a kind of combination of the motions in a solid and in a gas: "vibrational" motion at one site is followed by a "free" transition from one site to another. Accordingly, the structure of the liquid is something in between the structure of a solid and the structure of a gas. The higher the temperature, i. e. the higher the kinetic energy of molecules in a liquid, the more important the role played by the "free" motion (the shorter the intervals of "vibrational" state of a molecule and the more frequent the "free" transitions, i. e. the less the difference between the liquid and a gas). At a sufficiently high temperature which is different for different liquids (the so-called critical temperature, Sec. 17.16), the properties of a liquid cannot be distinguished from the properties of a highly compressed gas.

It should be noted that we have a much more vague idea about the structure of liquids than about gases and crystalline solids. This is explained by much more complex phenomena observed for liquids.

14.2. Surface Energy

It was already mentioned above that the most typical property of the liquid state is the presence of a sharp interface between a liquid and its vapour (which can be mixed with other gases). The surface layer of a liquid, that is a transient layer between the liquid and its vapour, has peculiar properties, which facilitate the investigation into the forces of molecular cohesion in a liquid. For this reason, we shall start the study of the properties of liquids with these *surface* phenomena.

Children know that sand castles can be made only from wet sand. Dry grains of sand do not stick together. However, grains of sand in water do not stick together either. When someone dives into water, his hair spreads

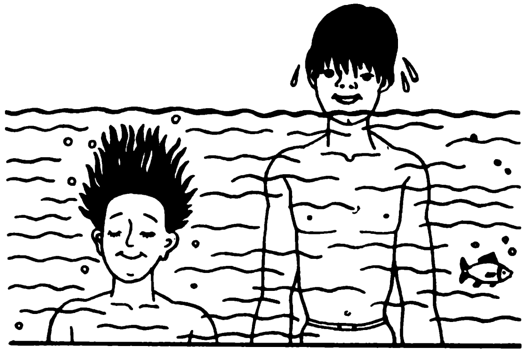


Fig. 397.
The hair of a diver spreads in all directions under water, while above water it sticks together.

in water in all directions (Fig. 397). But he has only to raise his head from water, and his hair will cling to his head and stick together.

How can this be explained? The sticking together of grains of sand and hair should be explained by *cohesive forces* between water molecules surrounding grains of sand and hair.

Let us find out why cohesive forces are not manifested when grains of sand or hair are *under* water. We shall compare the state of a liquid molecule near the liquid-gas interface with the state of a molecule which is in the bulk of the liquid far from this interface (Fig. 398). The molecule in the bulk of the liquid is surrounded by other molecules from all sides (A). The molecule at the interface is surrounded by liquid molecules only from one side (B), while on the side of the gas there are almost no molecules. The attraction exerted on a molecule by its neighbours is balanced for "inner" molecules. For molecules located near the surface, the composition of all forces gives a resultant directed into the bulk of the liquid. Consequently, to bring a molecule from inner layers to the surface, a work against this resultant force must be done. In other words, every molecule near the surface of a liquid has a certain excess of the potential energy in comparison with molecules located in the bulk of the liquid. Therefore, when the surface of a fixed mass of a liquid is increased (for instance spraying water),

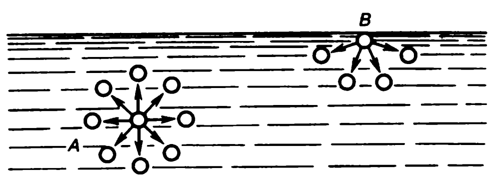


Fig. 398.
Molecule A is surrounded by other molecules and is attracted by them in all directions. Molecule B is pulled by other molecules into the bulk of the liquid.

the energy of the liquid increases. This is an example of the change in the internal energy of bodies, mentioned in Sec. 11.1. In this case, the internal energy of the body is proportional to the area of the surface and is therefore called the *surface energy*.

Owing to the tendency of molecules to go into the bulk of a liquid from its surface, the liquid acquires such a shape that its free surface has the minimum possible area.

The tendency of liquids to reduce the area of their free surface is clearly manifested in various phenomena and experiments.

1. First of all, this is confirmed by the spherical shape acquired by small liquid drops: the drops of mercury on a horizontal glass plate, the drops of water sputtering over a red-hot plate, or the rain drops on a dusty road. In all these cases, the interaction of a solid with a drop is weak in comparison with the forces acting among the parts of a liquid, and the tendency of the liquid to reduce its surface is clearly manifested: the spherical shape of the drops corresponds to their minimum surface area. When drops are small, the effect of the forces of gravity distorting their shape is insignificant.

In the conditions of weightlessness, the force of gravity does not prevent a fixed volume of a liquid from reducing its surface area. For this reason, under these conditions liquids assume the spherical shape. Such a spherical "drop" may have a larger size in comparison with ordinary liquid drops, for which an increase in size leads to a distortion of the shape under the action of the force of gravity.

2. The tendency of a liquid to reduce its surface is clearly manifested in a thin trickle of a viscous liquid. For instance, it can be observed that a trickle of honey flowing from a spoon is abruptly interrupted if it becomes too thin and rises up forming a round drop (Fig. 399), thus reducing its free surface.

3. The sharp edge formed when a glass tube is broken can easily be rounded by softening the glass over a flame (Fig. 400).

4. Another example is a soap film. Let us form a soap film on a ring with a thread tied to it as shown in Fig. 401*a*. As long as the film is intact

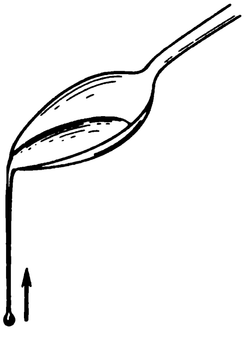
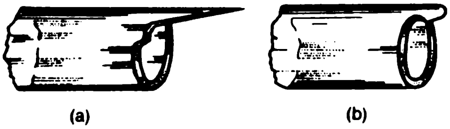


Fig. 399.
A trickle of honey flowing from a spoon gathers into a drop which rises up.



(a) (b)

Fig. 400.
(*a*) A sharp edge is formed in a broken glass tube. (*b*) The same edge after softening in the flame.

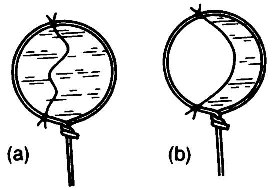


Fig. 401.
(*a*) A thread in a soap film. (*b*) The thread is pulled aside by the film.

on both sides of the thread, the latter has a shape randomly acquired by it when the film was formed. If we remove the film on one side of the thread, the soap film on the other side will immediately reduce its surface and stretch the thread (Fig. 401*b*).

The tendency of a film to contract to the minimum possible size explains the spherical shape of soap bubbles. The same reduction in the surface area of a liquid when equilibrium sets in explains the sticking together of wet grains of sand and hair discussed at the beginning of the section. The water sheathing hair that is stuck together has a smaller surface in comparison with hair spread in all directions. This is illustrated in Fig. 402.

In all these cases, we observe the tendency of liquids to reduce their contact surface with air (to be more precise, with the vapour of this liquid).

Similar phenomena are observed at the interface between two immiscible liquids.

1. Let us introduce a large drop of aniline into a solution of common salt whose density is selected so that the drop is held inside the salt solution without sinking to the bottom or rising to the surface. This means that the force of gravity and the buoyancy acting on the drop are mutually balanced (Archimedes' principle, see Sec. 7.23). In this case, the drop assumes the spherical shape (Fig. 403).

2. Let us pour a weak acid solution (say, of nitric acid) on a watch glass and introduce into it a large number of small mercury drops from a pipette (Fig. 404). It can be seen that these drops will merge until they form a single large drop whose surface is smaller than the sum of the surfaces of the large number of small drops.

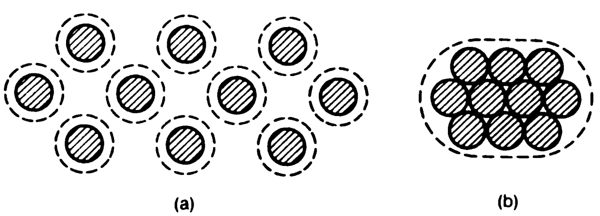


Fig. 402.
Hatched circles represent hair cross sections. Dashed circumferences represent water films enveloping hair. (*a*) The surface of the films is large when hair is arranged separately. (*b*) When hair sticks together, the surface of the water film is much smaller.

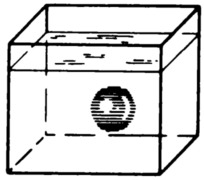
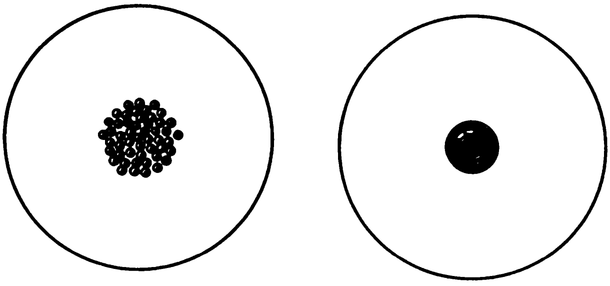


Fig. 403.
An aniline drop assumes the shape of a sphere in a salt solution.



(a) (b)

Fig. 404.
(*a*) A large number of small mercury drops are in contact in a weak solution of an acid on a watch glass. (*b*) In a certain time, all drops merge into a single large drop.

? **14.2.1.** For obtaining lead pellets, molten lead is poured into water through a sieve from a certain height. During the fall, lead solidifies and acquires the shape of small balls. Explain this phenomenon.

14.2.2. What happens to a soap film when it bursts? Where does it disappear?

14.3. Surface Tension

In the previous section it was found out that the surface layer of a liquid possesses an excess energy. This energy per unit surface area is called the *surface tension* and normally denoted by σ. In order to increase the surface of a liquid by S units of area without changing the state of the liquid (in particular, without changing its temperature), the work equal to σS must be done.

Let us take a wire frame whose one side is a jumper of length l, which can move so that it remains parallel to itself (Fig. 405). We immerse the frame in a soap solution. As a result, it will be covered with a liquid film bounded by the surface layers on both sides. Owing to the tendency of the surface layers to contract, the film will move the jumper. In order to keep the jumper from moving, a certain force **F** has to be applied, which will balance the force **F**′ exerted on the jumper by the film. Increasing the force **F** by an insignificantly small value, we shall displace the jumper very slowly in the direction of the force **F** by a distance b. The work done in this case by the force **F** is Fb. As a result of this work, the surface area of the layers of

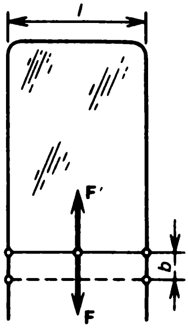


Fig. 405.
A frame covered by a soap film.

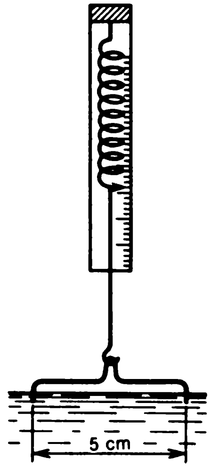


Fig. 406.
A simple instrument for determining the surface tension of liquids.

the liquid increases by $2lb$ (there are surface layers on both sides of the film), which will give an increment in the surface energy of $2lb\sigma$.

Equating the increment in the surface energy to the work done by the force F, we obtain the relation $2lb\sigma = Fb$, whence $F = 2l\sigma$.

The obtained equation indicates that the surface layer, in its tendency to contract, acts on a unit length of its boundary with the force equal to σ. This allows us to give another definition of surface tension as the force exerted by the surface layer per unit length of the contour bounding this layer. In the SI system, surface tension is measured in newtons per metre (N/m). It should be noted that 1 N/m = 1 J/m^2.

Measuring the force acting on the boundary of a liquid film, we can determine the surface tension of the liquid. A simple device for rough measurements of this type is shown in Fig. 406. A copper wire bent as shown in the figure and attached to a sensitive spring balance is immersed in water and then pulled upwards slowly and gently. The readings of the spring balance will gradually increase and attain the maximum value when the water film on the wire appears on the surface. Marking the reading of the spring balance and taking into account the weight of the wire, we obtain the force stretching the film. For a 5 cm wire, this force is about 0.0070 N, whence

$$\sigma = \frac{0.0070\ \mathrm{N}}{2 \times 0.05\ \mathrm{m}} = 0.070\ \mathrm{N/m}.$$

Besides this rough method, there exist other, more accurate techniques of measuring the surface tensions of liquids (see Sec. 14.10). The results of measurements of the surface tension for several liquids are compiled in Table 11.

Table 11. Surface Tension of Some Liquids

Liquid	Temperature, °C	Surface tension, N/m
Alcohol	20	0.022
Ether	25	0.017
Gold (molten)	1130	1.102
Liquid hydrogen	−253	0.0021
Liquid helium	−269	0.00012
Mercury	20	0.470
Pure water	20	0.0725
Soap solution in water	20	0.040

Pay attention to the fact that the surface tension of readily evaporating liquids (ether and alcohol), and hence the molecular forces in them are weaker than in nonvolatile liquids (say, mercury). The surface tension is very weak in liquid hydrogen, and the more so in liquid helium. On the contrary, the surface tension of liquid metals is very strong. The difference in the surface tensions of liquids is explained by the difference in cohesive forces acting among their molecules.

Measurements show that surface tension in liquids depends only on the nature of a liquid and its temperature. It does not depend on the area of the surface and on whether or not this surface has been subjected to preliminary stretching. In other words, the work done in pulling every new molecule to the surface does not depend at all on the surface area. This means that the surface layer of a liquid cannot be treated as a thin elastic film, say, a rubber film. When the rubber film is stretched, the stretching force increases with the area of its surface, and hence the work done to increase this surface by a unit area also increases. Nothing of this kind is observed during an increase in the area of the surface of a liquid.

While measuring surface tension, one must be sure that the liquid is chemically pure since an admixture of substances soluble in the liquid may considerably affect the surface tension. The change in the surface tension of a liquid due to the presence of impurities dissolved in it can be detected with the help of the following experiment (Fig. 407). Let us sprinkle lycopodium powder over water. This makes the displacements of the water surface visible. We now pour a small amount of soap solution or ether on the surface of water. It can be seen that the powder will be rapidly scattered in all directions from the drop. This shows that the surface tension of the soap solution or ether is smaller than the surface tension of pure water.

The fact that a film of soap solution or ether is formed on the surface of water, and hence water molecules are forced into the bulk, indicates that

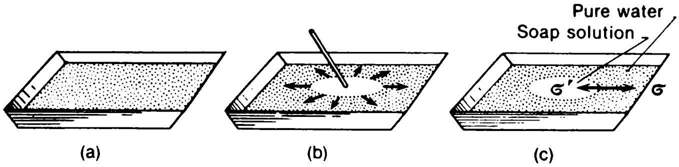


Fig. 407.
(*a*) A powder is spread uniformly over the surface of water. (*b*) When we touch water with a stick wetted by a soap solution, grains of the powder are scattered in all directions. (*c*) Arrows indicate the forces exerted per unit length of the boundary by the soap solution and by pure water.

the forces pulling the molecules of water into the bulk are stronger than the forces pulling the soap or ether molecules. It follows hence that the work of pulling the water molecules to the surface is larger, i. e. the surface tension of pure water is larger than that of the soap solution or ether.

? **14.3.1.** What work must be done for a deformation of a spherical mercury drop having a diameter of 2 mm (at 20 °C), such that its surface area increases three-fold?

14.3.2. What work must be done to blow a soap bubble having a diameter of 10 cm at 20 °C?

14.3.3. What work must be done to spray 1 kg of pure water at 20 °C into small drops having a diameter of 1 μm at the same temperature? The initial surface of water is small in comparison with the total surface of all drops and can be neglected. What amount of heat will be liberated if all the drops merge into one, the temperature remaining unchanged?

14.4. Liquid Films

It is well known that a foam can be easily obtained from soap water or egg white. However, the foam obtained from pure water is highly unstable.

Foam is an aggregate of air bubbles bounded by very thin liquid films. If a liquid readily forms a foam, a separate film can be also easily obtained from it. These films are very interesting. They can be extremely thin: their thickness in the thinnest parts may be as small as 10^{-5} mm. In spite of such a small thickness, they are sometimes very stable. A soap film can be stretched or deformed. A water jet may flow through a soap film, leaving it intact (Fig. 408). A steel ball wetted by soap water flies through a soap film without destroying it. When it passes through the film, the ball is obviously enveloped completely by the film and then separates from it so that the damaged region is immediately restored.

How can we explain the stability of films? It should be noted first of all that stable films and foam cannot be formed in chemically pure liquids. The necessary condition for a pure liquid (water, alcohol, etc.) to form a foam is the addition of substances soluble in it which considerably reduce the surface tension. Experiments show that the molecules of such a dissolved substance are gathered in the surface layer of the liquid (are adsorbed, see Sec. 14.11).

How does this affect the strength of a film (say, a soap film)? The soap film is a triple layer (Fig. 409). In the two outer layers, we have water saturated with the molecules of substances constituting the soap, while the middle layer contains almost pure water.

Let us now imagine that the film has become very thin in a certain region. As a result, the inner layer of almost pure water will be exposed. It was shown above that the surface tension of this layer is higher. Due to the strong surface tension, the part of the film that has become thinner pulls the liquid from other, thicker parts in its direction. Thus, the uniform thickness

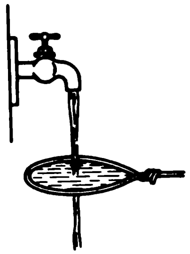


Fig. 408.
A water jet flows through the soap film without breaking it.

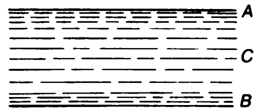


Fig. 409.
Schematic diagram of a soap film: A and B are the surface layers rich in soap molecules; C is the layer of almost pure water.

of the film will be restored over the entire surface, and the rupture of the film will become improbable. On the contrary, in pure liquids the slightest variation of the thikness in a certain region or an insignificant nonuniformity in the forces acting on the film cannot be compensated for by a change in the surface tension and leads to the rupture of the film.

Alas, even a soap film breaks in a certain time! The reasons behind this can be different. Firstly, the film is never perfectly horizontal (if only because a horizontal film is always bent under the action of the force of gravity). As a result, the liquid gradually flows from the upper part of the film downwards. Secondly, the film continually evaporates and for this reason becomes thinner and attains a state at which the inner layer of the film which, as shown above, ensures its stability, is depleted. Thirdly, oxidation reactions may occur on the surface of the film, resulting in the formation of new substances. In order to preserve a soap film for a longer time, it is placed under the bell which prevents the liquid from evaporating, and substances improving the viscosity of the soap solution (sugar or glycerol) are added to it.

In nature and engineering, we normally deal with an aggregate of films (foam) rather than with individual films. A profuse foaming can be often observed in streams where small water jets fall into calm water. In this case, the foam in water is formed by organic substances (saponins) liberated by the roots of plants. In construction engineering, materials with a cellular structure resembling foam (say, foam concrete) are often used. Such materials are cheap, light, are poor conductors of heat and sound and are sufficiently strong. The substances facilitating the foaming (foaming agents) are added to the solutions from which building materials are manufactured. An important example of utilisation of foaming agents is a file extinguisher whose operation is based on the ejection of a stable fire-inhibiting foam.

? **14.4.1.** While washing hands, obtain a soap film between your fingers as shown in Fig. 410. Watch intense movement of the liquid due to the difference between the surface tensions of different parts of the film. The film is initially colourless, but later it becomes coloured (the origin of this phenomenon will be discussed in Vol. 3 of this book, devoted to optics). In some time, the film is covered by dark spots, which rapidly grow and soon cover a considerable part of the film. It was found that these spots are the regions where

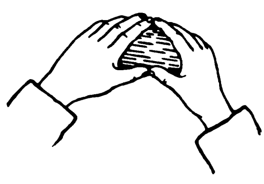


Fig. 410.
To Exercise 14.4.1.

the film has a thickness of two molecules. These layers consist of soap molecules, while the middle layer has disappeared. The appearance and growth of dark spots indicate that the film will soon burst.

14.5. Temperature Dependence of Surface Tension

In Table 11, the middle column gives the temperature at which the surface tension was measured. It is included in the table because surface tension depends on temperature. This can be verified with the help of the experiment described in Sec. 14.3. Let us sprinkle, as before, lycopodium powder over the surface of water and bring a red-hot metal body near the surface. The surface of water will be heated, especially in the vicinity of the body. It can be seen that the grains of lycopodium run away from the heated region. This indicates that the surface tension of water decreases with increasing temperature.

The results of measurement of the surface tension for water at different temperatures are compiled in Table 12. In other liquids, surface tension also decreases with increasing temperature. Consequently, *cohesive forces in a liquid become weaker as the temperature rises.* We shall discuss this phenomenon once again when considering evaporation of liquids.

Table 12. Temperature Dependence of Surface Tension for Water

Temperature, °C	Surface tension, N/m	Temperature, °C	Surface tension, N/m
0	0.0756	50	0.0679
20	0.0725	100	0.0588

14.6. Wetting and Nonwetting

It was noted in Sec. 14.2 that small mercury drops placed on a glass plate assume the spherical shape. This is a result of action of intermolecular forces tending to reduce the surface of the liquid.

Mercury *not always* forms round drops on the surface of a solid. Let us clean a zinc plate from oxides by rubbing it with a cloth wetted in a weak sulphuric acid, and put a small mercury drop on it (Fig. 411). We shall see that the mercury drop spreads over the zinc plate so that the total surface of the drop will undoubtedly increase.

The aniline drop in the experiment shown in Fig. 403 also has the

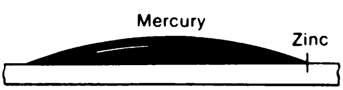


Fig. 411.
The spreading of a mercury drop over a clean zinc surface.

spherical shape only when it does not touch the wall of the glass vessel. It only has to touch the wall, and the drop immediately sticks to the glass, spreading over its surface and thus acquiring a larger total surface.

How can this be explained? It should be recalled that the tendency of liquid molecules to go into the bulk of the liquid and to reduce the interface between the liquid and a gas is due to the fact that the molecules of liquid are practically not attracted by the gas molecules (whose number is very small).

When a liquid is in contact with a solid, the cohesive forces between molecules of the liquid and molecules of the solid start to play a significant role. The behaviour of the liquid will depend on what is stronger: the cohesion between liquid molecules or the cohesion between the molecules of the liquid and the molecules of the solid. In the case of mercury and glass, the cohesive forces between mercury molecules and glass molecules are weak in comparison with cohesive forces between mercury molecules, and mercury coalesces into a drop. In the case of water and glass (or mercury and zinc), however, the cohesive forces between the molecules of the liquid and of the solid exceed the cohesive forces between the molecules of the liquid and the liquid spreads over the surface of the solid.

In order to verify these arguments, let us make the following experiment. We put a glass plate with a hook attached to it on the surface of mercury and pull the hook up untill the plate separates from the mercury. The separated plate will be perfectly clean (it does not entrain any mercury) (Fig. 412*a*). This shows that the cohesion between the molecules of glass and the molecules of mercury is weaker than that between mercury molecules. The situation here is similar to the stretching of a chain which breaks at the weakest link.

If, however, we take water instead of mercury and repeat the same experiment, we note that the separated glass plate is covered with water (Fig. 412*b*). In this case, the rupture occurs between water molecules and not between the water and glass. This means that cohesive forces between water and glass molecules are stronger than cohesive forces between the particles of water. In the former case, we say that the liquid *does not wet*

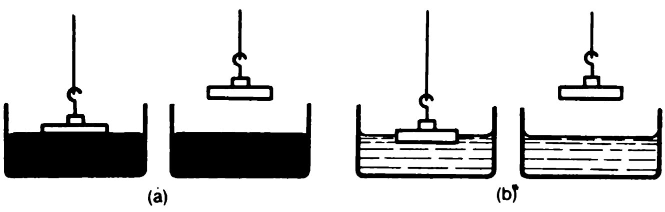


Fig. 412.

(*a*) A clean glass plate separated from the surface of mercury does not entrain mercury molecules. (*b*) The same plate separated from the surface of water is covered by water film.

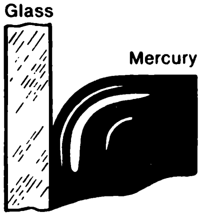


Fig. 413.
The shape of mercury surface near a glass wall (magnified).

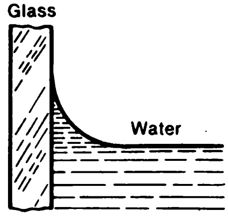


Fig. 414.
The shape of water surface near a glass wall (magnified).

the solid (mercury does not wet glass and water does not wet paraffin), while in the latter case the liquid *wets* the solid (mercury wets zinc and water wets glass). Hence it follows that when we speak of the surface of a liquid, we must take into consideration not only the interface between the liquid and air but also the surface of contact with other liquids or solids. In particular, when a liquid is poured into a vessel, its major part is in contact with the vessel walls.

The shape of the liquid surface where it is in contact with a solid wall and a gas may be different depending on whether or not the liquid wets the vessel walls. When mercury is poured into a glass vessel or water is contained in a vessel whose walls are covered with paraffin, the shape of the surface at the edge is round and convex (Fig. 413). This is due to the fact that in this case the cohesive forces between mercury molecules exceed the cohesive forces between mercury and the vessel walls, and mercury in its tendency to contract is partially separated from glass. In other cases (water poured into a clean glass or metal vessel), the liquid acquires at the edge the shape like that shown in Fig. 414. Now the attraction of liquid by the walls exceeds the attraction between liquid molecules, and the liquid is attracted to the glass, tending to spread over it.

? **14.6.1.** Water can be measured by counting the drops which fall from a glass tube. Mercury cannot. Why?

14.6.2. Explain the process of pouring water into a bottle with a narrow neck with the help of a glass stick or a match (Fig. 415).

14.6.3. Put a dry razor blade on the surface of water. If it has been touched by fingers, it is necessarily covered by a thin layer of grease. The blade will float. The same blade thoroughly washed with a soap (don't touch it after washing!) will not float on the surface of water. Explain this phenomenon.

14.6.4. Let us consider the process of soldering. To make the solder (say, tin-lead alloy) spread over the surfaces of metal bodies being soldered, these surfaces must be thoroughly cleaned by a soldering liquid (say, a solution of zinc chloride). Zinc chloride purifies the metal surface from oxides. Taking into account extremely strong cohesive forces in metals, explain why it is necessary that the solder be in contact with a perfectly clean metal surface.

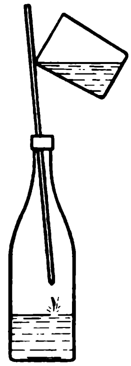


Fig. 415.
A glass stick can be used for pouring water into a bottle with a narrow neck.

14.7. Arrangement of Molecules at the Surface of Bodies

Let us make the following experiment. We place a small piece of paraffin (or wax) on the surface of pure hot water. Paraffin will melt and cover the surface with a thin film. We let the water cool. The paraffin will solidify in the form of a thin plate. We take this plate out of the vessel, trying not to touch its surface, divide it into two parts, place them on a horizontal surface, turning one half of the plate upside down. Using a pipette, we place drops of pure water on the surfaces of the plates. It can be seen that the drops will behave differently on different plates. A water drop will not spread over the surface of paraffin which was in contact with air and will have the same shape as a drop of mercury on a glass surface. In this case, water does not wet paraffin. On the contrary, a drop of water will immediately spread over the surface that has been in contact with water, and will form a thin film. In this case, water wets paraffin.

Why is the *same* solid substance wetted by a liquid in one case and not wetted in the other case?

This can be explained as follows. Molecules of many substances are quite complex. Therefore, different *parts* of such a molecule may exhibit different cohesive forces in interactions with other molecules. If in some way or other such molecules are arranged so that the ends of molecules strongly interacting with water are facing one side and weakly interacting ends are facing the opposite side, we obtain a plate one of whose surfaces will be wetted by water and the other will not. When paraffin melts in contact with hot water, the molecules of liquid paraffin turn so that their ends which strongly interact with water are facing the surface of water. In this position they solidify when water cools. As a result, we obtain the two-sided plate whose properties were revealed in the above experiment.

The effect of a certain arrangement of molecules in the surface layer is especially strong in oily substances having lubrication properties. On the basis of chemical analysis, the molecules of such substances are assumed to have an elongated shape such that the group of COOH atoms (the so-called carboxyl group) is at its one end. It is this group that is responsible for the cohesion of molecules of oily substances with the surfaces of solids (active ends). The other ends of the same molecules manifest very weak cohesive forces (inert ends).

Such a representation allows us to explain the *lubrication action* of very thin oil layers. A lubricant layer between two solid (say, metal) surfaces is divided into layers with active and inert ends alternately facing each other (Fig. 416). The layer of molecules adjoins the solid body with its active ends. These molecules are arranged like bristle on a brush. During motion, inert ends of the lubricant slide relative to one another. No strong opposing forces emerge in this sliding since cohesive forces in these ends of the molecules are weak. For this reason, friction is quite small.

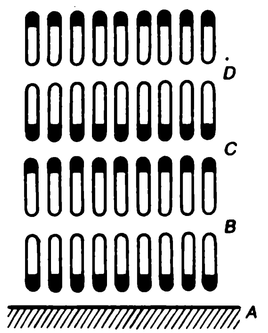


Fig. 416.
The arrangement of molecules of an oil lubricant near the surface of a solid A. The active ends of molecules are blackened. Sliding takes place in the regions B and D, while in region C no sliding occurs.

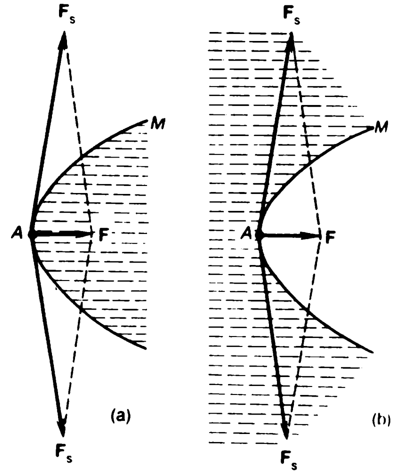


Fig. 417.
The forces $\mathbf{F}_s$ of surface tension acting on the curved surface of a liquid give a resultant $\mathbf{F}$ directed from the concave side of surface M. (*a*) The surface of the liquid is convex (*b*) The surface of the liquid is concave.

It should be noted that the molecular pattern of flow of a liquid near a solid is quite different for liquids which do not exhibit lubrication action in thin layers.

14.8. The Role of the Curvature of the Free Surface of a Liquid

We often encounter curved surfaces of liquids: the surface of a hanging drop (see Fig. 372), the surface of water enveloping wet hair (see Fig. 402), and the surface of any liquid drop or any bubble in a liquid are all curved.

What is the role of the curvature of a surface? It can be easily seen that the forces associated with surface tension and directed along the tangent to the surface give a resultant force directed into the bulk of a liquid in the case of a convex surface (Fig. 417*a*). For a concave surface, on the contrary, the resultant is directed towards the gas in contact with the liquid (Fig. 417*b*). On the basis of these simplified arguments, we can expect that the pressure in a liquid bounded by a convex surface is higher than the pressure of the surrounding gas (or other liquid in contact with the first one), while the pressure in a liquid bounded by a concave surface is, on the contrary, lower than the pressure of the surrounding gas. In order to verify this assumption, let us consider several experiments.

1. Figure 418 shows a narrow glass tube B connected by a rubber tube

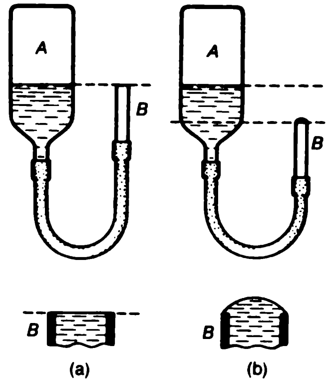


Fig. 418.
(*a*) The surfaces of water in tubes A and B are at the same level; both surfaces are flat.
(*b*) The level of the surface of water in A is higher than that in B; the surface is flat in A and convex in B.

with a wide tube A. The tubes contain water. We arrange the end of tube B at the level of water in tube A. The surface of water in tube B is horizontal and perfectly flat (Fig. 418*a*). We then carefully lower tube B. The end of tube B becomes lower than the water level in tube A, and simultaneously the surface of water in it acquires a spherical shape (Fig. 418*b*). Let us try to explain this. The same atmospheric pressure acts on the convex spherical surface of water in tube B and the flat surface of water in tube A. At the level of the end of tube B (Fig. 418*b*) the pressure in tube A is higher than the atmospheric pressure. Since the liquid is in equilibrium, the pressure at the end of tube B in the proximity of the convex surface is higher than the atmospheric pressure. The additional pressure under the convex surface of the liquid is caused by intermolecular forces. As a result of the tendency of the liquid to reduce the area of its free surface, the liquid under the spherical surface is compressed to a certain extent, and hence has an excess pressure.

Let us continue to lower tube B still further. The radius of the spherical surface will become still smaller, and the difference between the levels of water in the tubes will be still larger. Hence the conclusion: the excess pressure under a convex surface of the liquid is the higher, the smaller the radius of curvature of this surface.

2. Figure 419*a* represents a device for blowing bubbles from the narrow end C of the tube, immersed in a liquid to a small depth. Pressing on a rubber bulb A, we create an elevated pressure in the tube, which is indicated by a liquid manometer B. As the pressure in the tube increases, the radius of the blown-out bubble becomes smaller and smaller (Fig. 419*b*, *c*, *d*). If we continue to press on the bulb until the radius of the bubble starts to increase (Fig. 419*e*), the manometer will indicate a reduction in pressure.

This experiment clearly proves the same thing as the previous experiment: the surface of a liquid bulges out because the pressure on one side is greater, the surface becoming concave on the high-pressure side and con-

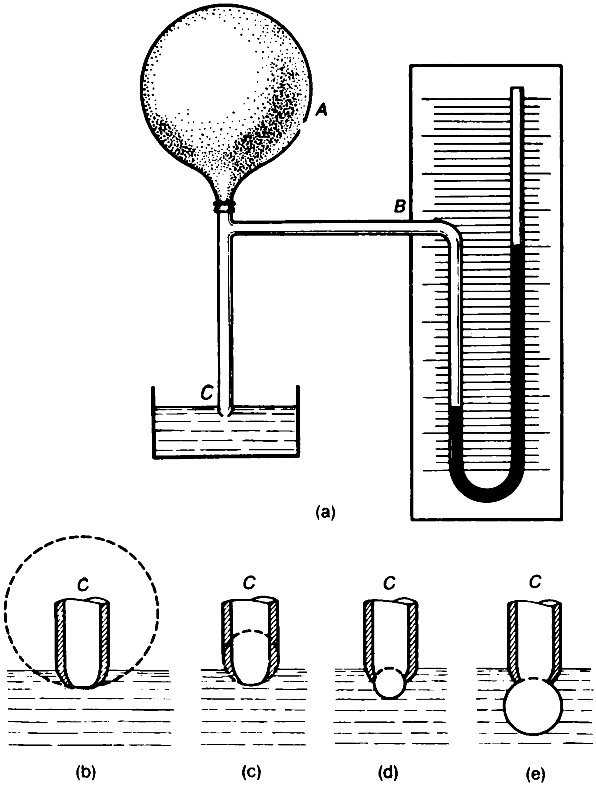


Fig. 419.
(*a*) A device for blowing bubbles in a liquid. (*b*)-(*d*) The radius of curvature of blown-out bubbles at first decreases with increasing pressure. (*e*) Ultimately, the radius of curvature increases again.

vex on the other. The greater the pressure difference, the smaller is the radius of curvature.

If we immerse the end of the tube C into another liquid (say, alcohol), the manometer will indicate a different maximum pressure. For alcohol, the maxium pressure is lower than for water by about a factor of 3.5. It should be recalled that the surface tension of alcohol is also smaller than the surface tension of water by the same factor of 3.5. This result indicates that the pressure difference is the higher, the larger the surface tension.

Calculations lead to the following conclusion: if we have a spherical surface of radius R, the pressure difference is

$$p_2 - p_1 = \frac{2\sigma}{R}, \qquad (14.8.1)$$

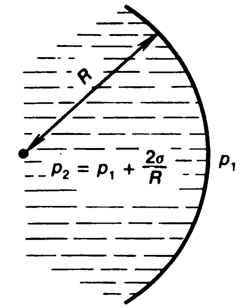


Fig. 420.
The interface between two media is a sphere of radius R, concave to the left. In equilibrium, the pressure of the medium on the left of the boundary is $2\sigma/R$ higher than the pressure of the medium on the right of the boundary.

where p_2 is the pressure from the concave side, while p_1 is the pressure from the convex side (Fig. 420). This formula is in agreement with the results of experiments shown in Figs. 418 and 419.

We shall derive formula (14.8.1). Let us consider an air bubble of radius R in a liquid with a surface tension σ (or a drop of the liquid of the same radius R, see Fig. 420). Let p_2 be the air pressure in the bubble and p_1 the pressure of the liquid surrounding the bubble. Suppose that for some reason or other the radius of the bubble has increased by a quantity x which is small in comparison with R. The work A done in the process is equal to the difference between the forces of the pressure $(p_2 - p_1)$ $4\pi R^2$ multiplied by the displacement x:

$$A = (p_2 - p_1)4\pi R^2 x.$$

On the other hand, the area of the surface will increase by

$$4\pi(R + x)^2 - 4\pi R^2 = 4\pi x(2R + x).$$

Since we assume that x is small as compared to R, the increase in the surface area can be taken as $8\pi Rx$. In this case, the increment in the surface energy is $\Delta E = 8\pi Rx\sigma$. Equating the work A and the energy increment ΔE, we obtain

$$8\pi Rx\sigma = (p_2 - p_1) \cdot 4\pi R^2 x, \quad \text{whence} \quad p_2 - p_1 = \frac{2\sigma}{R}.$$

It can be seen that the excess pressure depends on the radius of the spherical surface. For small radii, it may reach high values. For instance, the excess pressure inside a bubble of radius 1μm in water is 1.42×10^5 Pa. For spherical surfaces with large radii (say, 10 cm), the excess pressure is negligibly small (0.96×10^{-5} Pa). For a flat surface, which can be regarded as a limit of the spherical surface with the radius tending to infinity, the excess pressure is zero.

? **14.8.1.** If we blow two soap bubbles from two communicating pipes with funnels at the ends (Fig. 421) and close the tube to which they are connected, air will flow from the smaller bubble to the larger bubble so that the smaller bubble becomes still smaller and the larger bubble grows. Explain this phenomenon.

14.8.2. When does air flow at a higher rate from a funnel on which a soap bubble has been blown: at a small diameter of the bubble or at a large one?

14.8.3. If a drop of water is placed between two glass plates (Fig. 422), a certain force is required to separate the plates from each other. This force is the stronger, the larger the area occupied by the drop and the smaller the plate-to-plate separation. Explain this phenomenon.

14.8.4. If water drops and air bubbles are arranged in a narrow glass tube of varying

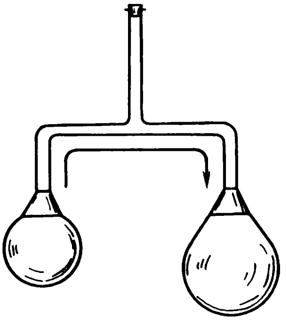


Fig. 421.
To Exercise 14.8.1.

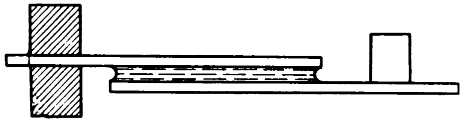


Fig. 422.
To Exercise 14.8.3.

Fig. 423.
To Exercise 14.8.5.

cross section as shown in Fig. 423, it is very difficult to blow air through it. Explain the phenomenon. The obstruction of thin pipes of varying cross section is a harmful phenomenon which has to be overcome in engineering. For the same reason, the evolution of gas bubbles in blood vessels of living organisms is also extremely harmful since it may completely interrupt the blood flow in these vessels.

14.8.5. Drip 50 drops of pure water from a bead into a test tube. Using an identical bead and test tube, drip the *same* number of drops of water with a small quantity of dissolved soap or ether (a mixture of valerian and ether can be used). Compare the volumes of liquids in the test tubes. How can the difference in the size of the drops be explained?

14.9. Capillary Phenomena

In everyday life, we often deal with bodies pierced by a large number of small channels (paper, yarn, leather, various building materials, soil and wood). In contact with water, or other liquids, such materials often absorb them. This phenomenon is observed when we dry hands with a towel or in the operation of the fuse of an oil lamp.

Very often a liquid sucked into a porous body rises up (like an ink absorbed by blotting paper, Fig. 424). Similar phenomena can also be observed in very narrow glass tubes (Fig. 425). Narrow tubes are called *capillaries* (from the Latin word *capillaris* meaning hair).

Fig. 424.
Ink absorbed by a blotting paper rises up.

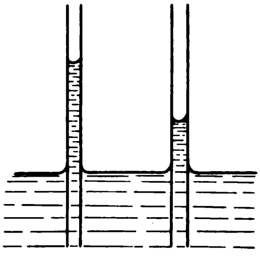


Fig. 425.
Water stays at a higher level in a narrow tube than in a wide vessel.

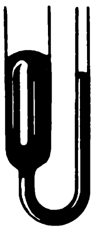


Fig. 426.
The level of mercury in the narrow tube is lower than in the wide tube.

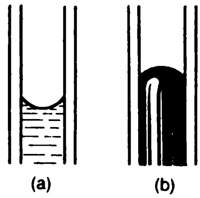


Fig. 427.
The shape of a meniscus in (*a*) wetting liquid and (*b*) nonwetting liquid.

Let us immerse the end of such a tube in a liquid. If the liquid wets the walls of the tube, it will rise up the walls of the tube above the level of the liquid in the vessel, the column height being the larger, the narrower the tube (Fig. 425). If a liquid does not wet the walls, the level of the liquid in the narrow tube will, on the contrary, be lower than in the wide tube (Fig. 426).

How can we explain these phenomena? It was shown in Sec. 14.6 that the surface of a liquid near a wall bulges up or down depending on whether or not the liquid wets the wall. In a narrow tube, the edges of the liquid form the entire surface of the liquid so that the surface has the shape resembling a hemisphere (the so-called *meniscus*). In wetting liquids, the meniscus is concave upwards, while in nonwetting liquids, it is convex downwards (Fig. 427). The surface of a liquid becomes curved due to the difference in pressure (see Sec. 14.8): the pressure under the concave meniscus is lower than under the flat surface. Therefore, if the meniscus is concave, the liquid rises until the hydrostatic pressure compensates the pressure difference. Under a convex meniscus, the pressure is higher than under a flat surface, which leads to the lowering of liquid in narrow tubes.

Thus, a wetting liquid in a narrow tube stays above the level in a wide tube, while nonwetting liquid is below the level in a wide tube. *The height to which a liquid rises in a capillary tube is the larger, the higher the surface tension of the liquid and the smaller the radius of the tube and the density of the liquid.* This statement can be also referred to solid materials pierced by thin channels of irregular shape. If a material is wetted by water, the latter is sucked into it to the height which is the larger, the narrower the channels.

? **14.9.1.** Put a piece of a chalk in water. Bubbles will evolve from chalk in all directions. Explain the phenomenon.

14.9.2. If two glass plates touching each other at one end and separated by a thin rod at the other are immersed in water, the water will rise in the space between the plates (Fig. 428). How can this be explained?

14.9.3. Figure 429 shows a device for removing the moisture formed in winter on window sills. Why does the water flow into the bottle along the narrow strip of cloth?

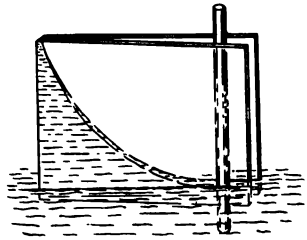


Fig. 428.
To Exercise 14.9.2.

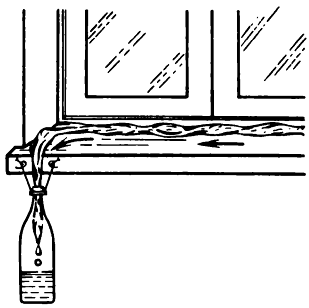


Fig. 429.
To Exercise 14.9.3.

14.9.4. If the end of the same capillary tube is immersed first in cold water and then in hot water, the height to which water rises will be smaller in the latter case. How can you explain this?

14.9.5. Glass tubes whose shape is shown in Fig. 430 are completely submerged in water and then slowly lifted. The left tube consists of a thin capillary A with a wide tube B soldered to it. The right tube is a bent capillary C. What will be observed when the tubes are pulled out of water?

14.9.6. The ends of two glass capillary tubes having the same diameter and having the shape shown in Fig. 431 are immersed in water. The height to which water rises in the straight tube is larger than the height of the bend of the hooked tube. Will water continually flow out of the bent tube, i. e. will such a tube be a perpetual-motion machine? Why is this assumption erroneous?

14.9.7. Break a piece of chalk and touch the break with your tongue. Why does your tongue "stick" to the chalk?

14.10. The Height to Which a Liquid Rises in Capillary Tubes

Thus, the height h to which a liquid rises in a capillary tube depends on the radius R of the tube channel, the surface tension σ and the density ρ of the

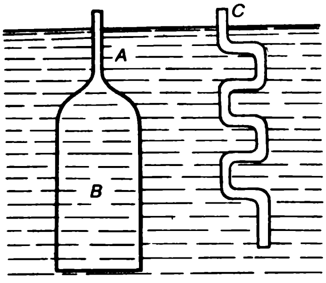


Fig. 430.
To Exercise 14.9.5.

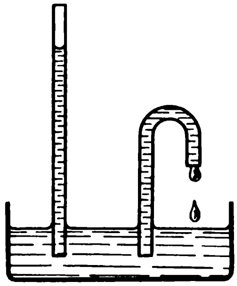


Fig. 431.
To Exercise 14.9.6.

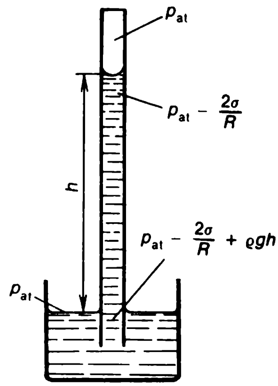


Fig. 432.
To the derivation of formula for the height of the liquid column in a capillary.

liquid. Let us derive the formula connecting these quantities. We shall consider the most important case when a liquid wets the walls, i. e. tends to spread over the surface of the walls.

Let us assume that the surface of a liquid inside a capillary has a perfectly spherical shape with the radius equal to the radius of the capillary (Fig. 432). According to formula (14.8.1), the pressure of the liquid right under a concave meniscus is lower than the atmospheric pressure p_{at} by the quantity $2\sigma/R$, i. e is equal to $p_{at} - 2\sigma/R$. At a depth h corresponding to the level of the liquid in the wide vessel, the hydrostatic pressure ρgh is added to this pressure. On the same level in the wide vessel, i. e. right under the plane free surface of the liquid, the pressure is equal to the atmospheric pressure p_{at}. Since the liquid is in equilibrium, the pressures at the same level are equal. Consequently,

$$p_{at} - \frac{2\sigma}{R} + \rho gh = p_{at},$$

whence

$$h = \frac{2\sigma}{R\rho g}. \tag{14.10.1}$$

Thus, *the height to which a liquid rises in a capillary is directly proportional to its surface tension and inversely proportional to the radius of the capillary channel and the density of the liquid.*

This formula can be used for determining the surface tension σ. To this end, we must just measure the height h to which the liquid rises and the radius R of the tube. If we know the free fall acceleration g and the density ρ of the liquid, the value of σ can be calculated from formula (14.10.1). This is one of widely used methods for measuring σ. Naturally, the surface of the tube must be clean and the liquid must be pure.

? **14.10.1.** Calculate the height to which water rises in a capillary of radius 0.25 mm and to which alcohol rises in a capillary having a diameter of 0.5 mm (see Table 11). The density of alcohol is 0.8×10^3 kg/m^3.

14.10.2. Determine the surface tension of petrol if the height to which it rises in a tube having a radius of 0.2 mm is 3 cm. The density of petrol is 0.7×10^3 kg/m^3.

14.10.3. Suspend a strip of blotting paper 2×15 cm^2 in size so that its lower end is submerged in water poured in a saucer. Wait until water stops rising in the paper (for 4-5 hours). Measure the height to which water rises and estimate the dimensions of the channels in the fibres of the blotting paper.

14.11. Adsorption

Wetting of solids by liquids indicates that in some cases liquid molecules as if stick to a solid and are held on it for a certain time. The same may occur to gas molecules. A solid in a gas is always covered by a layer of gas molecules held on it for a certain time by intermolecular forces. This phenomenon is known as *adsorption*. The amount of adsorbed gas is different in different cases. It depends, above all, on the area of the surface on which molecules can be adsorbed: the larger the area of this surface, the more the gas is adsorbed. The adsorbing surface is especially large in porous materials, i. e. substances pierced by a large number of small channels which are sometimes invisible even through a microscope with a high magnification. The amount of adsorbed gas also depends on the nature of the gas and the properties of the solid.

Activated carbon (viz. carbon from which tar impurities have been removed by calcination) can adsorb large amounts of gas. The properties of activated carbon can be easily observed. Let us put a small amount of carbon powder[1] into an empty test tube and heat it over a flame (Fig. 433). Carbon will intensely liberate adsorbed gas. Gas evolution is revealed from the vigorous movement of carbon powder resembling the boiling of a liquid. We pour several ether drops in a flask and let it evaporate. Then we drop a small amount of activated carbon powder into the flask and rapidly close the flask with a cork with a tube connected to a manometer (Fig. 434). Ether vapour will be adsorbed by carbon, and the manometer will indicate a sharp drop in pressure.

Adsorption on activated carbon and other solids is widely used in engineering. It is employed, for example, for recovering valuable gaseous substances obtained at chemical plants. In medicine, it is used for extraction of harmful gases formed in living organisms suffering from various types of poisoning, and so on. Adsorption on the surface of solids plays an important role for accelerating some chemical reactions with gases (catalysis).

[1] The medical preparation "carbolen" can be taken instead and grinded into a fine powder.

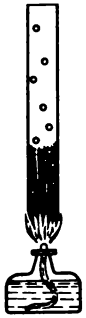


Fig. 433.
Obtaining of activated carbon.

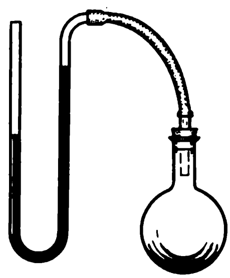


Fig. 434.
Adsorption of ether vapour by activated carbon.

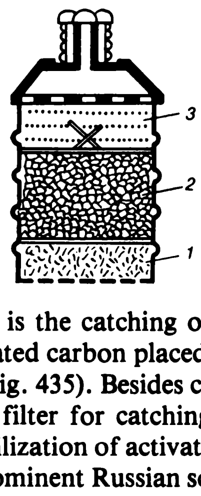


Fig. 435.
Sectional view of the respirator box of a gas mask: *1*—filter for smoke, *2*—activated carbon layer, *3*—chemical absorbers.

Another important application of adsorption is the catching of toxic gases with the help of gas masks. A layer of activated carbon placed in the respirator box of a gas mask catches toxic gases (Fig. 435). Besides carbon, the box contains also chemical absorbers and a filter for catching toxic smoke which cannot be trapped by carbon. The utilization of activated carbon for gas masks was proposed in 1915 by the prominent Russian scientist Nikolai Zelinskii (1861-1953).

It should be noted that solids may adsorb not only gases but also various substances dissolved in liquids. This property has also found a wide application in engineering.

14.12. Floatation

Pure minerals never occur in nature, almost always being mixed in with undesirable rocks. An ore containing a small amount of a mineral is called a lean ore. The process of separating the waste rock from the desirable mineral is known as the *concentration* (*dressing* or *beneficiation*) *of the ore*. There are various (mostly mechanical) concentration techniques, but recently methods based on wetting phenomena, viz. *floatation*, have gained general acceptance. Floatation methods are most valuable for nonferrous metal ores.

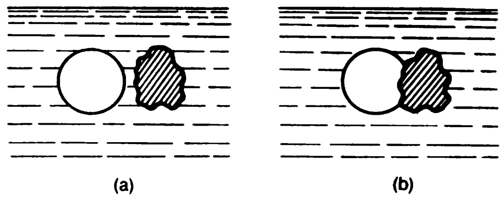


Fig. 436.
Floatation: (*a*) an air bubble approaches a grain of mineral covered by an oil film; (*b*) the thin film of water between the air bubble and the grain contracts, thus exposing the surface of the grain.

Basically in a floatation process, the ore is crushed into a fine powder which is then agitated in water. Small amounts of a substance which can wet one of the components (say, the metal-bearing mineral) and cannot wet the other component (the waste rock) are added. Besides, the floatation agent should be insoluble in water so that water does not wet the surface of a grain covered by a thin layer of the impurities. Usually, a cheap oil is used for this purpose. As a result of agitation, the grains of the mineral are enveloped by a thin oil film, while the grains of the waste rock remain free.

At the same time, tiny bubbles of air are blown into the viscous mass. When they come into contact with a mineral covered by a layer of oil and for this reason unwetted by the water, they stick to it. This occurs since a thin film of water between the air bubbles and the surface of the grain unwetted by it (Fig. 436) tends to contract and exposes the surface of the grain (in the same way as water gathers in drops on a greasy surface, thus exposing it). The average density of mineral with air bubbles sticked to them is lower than the density of water, and they gradually rise to the surface, while the grains of the waste rock sink. Thus, a more or less complete separation of waste rock from the mineral is attained, and the resultant concentrate is obtained. The ore is now rich enough in the mineral for it to be further processed. Figure 437 shows a flowchart of a floatation plant.

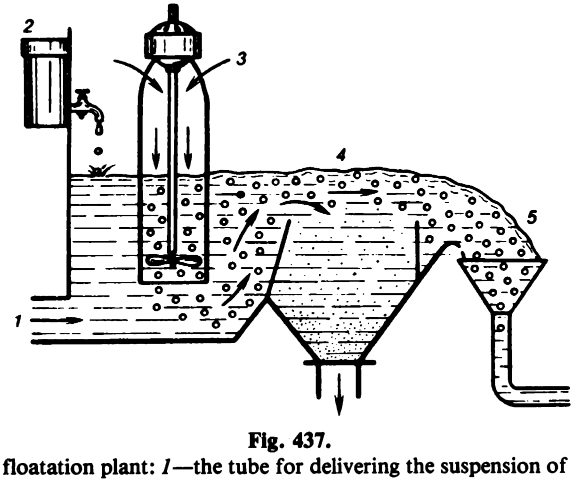


Fig. 437.
Flowchart of a floatation plant: *1*—the tube for delivering the suspension of crushed mineral in water; *2*—the vessel from which a floatation agent (oil) is dripping, *3*—intake of air with the help of a screw, *4*—the region where the desirable mineral rising to the surface is separated from undesirable rock, *5*—sink of foam containing the desirable mineral (concentrate).

The concentration of ores by floatation can be explained with the help of the following experiment. A freshly crushed mixture of coal powder and dry pure sand (the grains of coal and sand must be approximately of the same size of 0.1-0.2 mm) is placed into two test tubes so that the mixture occupies about 0.1 of each test tube's volume. A drop of kerosene is added to one of the test tubes, and then both test tubes are 2/3rd's filled with water. The test tubes are closed using clean corks and vigorously shaken for several seconds so that a large number of air bubbles are formed in them. Then the test tubes are left to stand. In the test tube containing no kerosene, the air bubbles rise to the surface, while grains of coal and sand precipitate to the bottom. In the test tube where the mixture is wetted by kerosene, rising air bubbles entrain the grains of coal leaving the grains of sand to precipitate as in the first test tube. A black foam gathers at the top of the first test tube, and the sand remains at the bottom (later, as the bubbles of the foam burst, the coal too precipitates to the bottom).

14.13. Dissolution of Gases

Besides being adsorbed (see Sec. 14.11), the molecules of two bodies in contact (say, two liquids or a gas and a liquid) can penetrate into the volume occupied by the other body. This penetration is known as *dissolution*. As a result of dissolution, the dissolved body is distributed uniformly over the volume of solvent, and the concentration of the penetrating substance can be elevated only in the surface layer due to adsorption. Dissolution is a result of diffusion (see Sec. 12.5) in the entire volume of the substance adsorbed in a surface layer.

Let us first consider the dissolving of gases in liquids. Let us pour water from a water supply system into a glass. It can be seen that a large number of very small bubbles are evolved from water, which either rise up or stick to the surface of the wall. Where do these bubbles come from and what do they contain? These are gases which have been dissolved in water in considerable amounts under elevated pressure that always exists in water supply systems. As water flows out from a tap, the pressure sharply drops in it. Besides, in the room water taken from a water supply is normally heated. These processes disturb the equilibrium between gases dissolved in water and the atmospheric air. As a result, the gases dissolved in water are liberated in the form of bubbles. Normally, these are the same gases that constitute air: oxygen, nitrogen, carbon dioxide, etc.

When water is heated (and especially when it boils), the gases dissolved in it are removed almost completely. The presence of dissolved gases in fresh water and their absence in boiled water explains the difference in flavour of boiled and fresh water.

The dissolving of air in water can be observed in an experiment resembling the one with adsorption of gases by activated carbon. Let us boil water in a flask for a certain time and then cool it. Carefully (without shaking the flask), we attach a liquid manometer to it. Then we shake the flask so that the larger surface of water comes in contact with air in the flask. The manometer will indicate a considerable pressure drop for air in the flask. Consequently, a part of air has been absorbed by water. After the water in the flask has been thoroughly stirred, however, there is no further dissolution of air in it. The saturated solution is said to be obtained.

How does the dissolution of a gas in water occur? Let us suppose that air is above the water surface. As a result of thermal motion of molecules, air and water molecules cross the air-water interface. The escape of water molecules in air is called evaporation (we postpone an analysis of this phenomenon until Chap. 17). The penetration of molecules of gases constituting air into water and their subsequent diffusion over the entire volume of water is the dissolution of air in water. Naturally, as a result of the same thermal movement, a part of gas molecules which have penetrated into water leave it. But until the number of molecules of a gas (say, oxygen) in water is insignificant, the number of gas molecules leaving water per unit time is smaller than the number of molecules penetrating into the water from the atmosphere. Thus, the number of gas molecules in water continues to grow, i. e. the dissolution of the gas in the liquid continues. Finally, when the number of gas molecules in the liquid becomes so large that the same number of molecules manage to escape from water as the number of molecules penetrating into it, a further increase in the number of gas molecules in water (further dissolution) ceases. The obtained solution is known as *saturated*. In this case, the liquid is said to be in equilibrium with the gas.

•

The word "equilibrium" is used here in a broader sense than in mechanics. We say that a system consisting of water, air dissolved in it and air above the surface of water is in equilibrium if the amount of dissolved air does not change with time, although individual molecules enter the solution and leave it. Such an equilibrium is termed *mobile*, or *dynamic* (see Sec. 14.1). Sometimes, the expression "steady state" is used instead of "equilibrium".

The mass of a gas that can be dissolved in a unit volume of a liquid is known as *solubility*. It depends on the temperature and the partial pressure (see Sec. 13.19) of a given gas above the liquid. Experiments show that *in equilibrium the mass of a gas dissolved in a liquid is proportional to the partial pressure of this gas above the liquid* (Henry's law). This is used, for example, in carbonation of water. In this process, water is brought in durable contact with carbon dioxide under a high pressure. Due to this reason, a large amount of gas is dissolved in water. When carbonated water is poured in a glass, gas is liberated plentifully in bubbles.

Dissolving of gases in liquids plays an important role in diving. The divers who worked for a long time at a large depth should not be rapidly lifted to the surface. The blood of a diver who breathes in air under a high pressure is saturated with nitrogen (oxygen can be disregarded since it soon becomes chemically bound with the blood). During a rapid ascent, nitrogen may evolve from blood inside blood vessels in the form of bubbles and clog them, which is very hazardous.

The mass of a gas dissolved in a liquid also depends on *temperature*. It was mentioned above that by heating water, we urge the evolution of air dissolved in it. As a rule, *the solubility of a gas in a liquid decreases with increasing temperature*. Table 13 presents the solubilities of some gases in water at different temperatures. Finally, solubility depends on the *nature* of a liquid and a gas. The amount of oxygen dissolved in water is, for example, twice as larger as the amount of nitrogen dissolved under the same conditions. This circumstance plays an important role for the existence of living organisms in water.

Table 13. Solubilities of Some Gases in Water at Different Temperatures (in g/l)

Gas	Temperature, °C		
	0	20	40
Argon	0.058	0.037	0.027
Carbon dioxide	1.713	0.878	0.53
Hydrogen chloride	506	442	386
Nitrogen	0.0293	0.0164	0.0118
Oxygen	0.049	0.031	0.023

It should be noted that gases can also be dissolved in solids. Some metals, for example, can dissolve a certain amount of gases (especially hydrogen), the diffusion rate, and, hence, the rate of dissolution, increasing with temperature. As a result, such metals cannot be assumed impenetrable for gases. For instance, the metal palladium heated to a high temperature allows hydrogen to pass through it quite easily.

14.14. Mutual Solubility of Liquids

If we pour alcohol into pure water, we obtain a perfectly homogeneous liquid. This phenomenon is observed for any ratio of masses of water and alcohol. This means that water and alcohol dissolve in each other in any proportion. A different situation takes place if we pour ether or kerosene into water. Now we shall see that after a certain time liquids form separate layers (Fig. 438). Each such layer is a solution. In the case of water and ether, the solution of water in ether, containing much ether and little water, will be on the top, while the solution of a small amount of ether in water will be at the bottom.

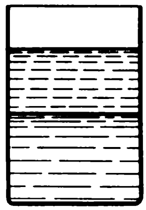


Fig. 438.
The upper layer is the solution of water in ether, while the lower layer is the solution of ether in water.

It should be observed that as the temperature rises, the mutual solubility of liquids increases. For certain combinations of liquids a temperature can be attained such that they can dissolve in each other in any proportion so that the interface between them disappears, and the entire liquid becomes homogeneous.

14.15. Dissolution of Solids in Liquids

It is well known that if we drop a piece of sugar into water, in a certain time sugar will vanish, and a homogeneous substance (solution) will be obtained. The sweetness of the solution indicates that sugar molecules have been distributed throughout the volume of the solution. This distribution occurs due to molecular motion (diffusion). It can be considerably accelerated by stirring.

The dissolution of a solid in a liquid differs but little from the dissolution of a liquid in a liquid. In this case too, the molecules of the dissolved substance are gradually distributed among the molecules of the solvent. The mass of the dissolved substance per unit volume of the solvent is known as the *solution concentration*. The substance is dissolved in the liquid to a certain definite concentration which depends on the nature of the solvent and solute as well as on the temperature.

Solutions whose concentration has a limiting value are called *saturated*. The higher the concentration of a saturated solution, the higher the solubility of a given substance in a given solvent. Water is an especially good solvent in which many substances are dissolved to high concentrations. The solubility in alcohol is generally inferior to that of water. Benzene is a still worse solvent, although there are substances which are dissolved in benzene or alcohol better than in water. Different substances may have different solubilities in different solvents. Besides, solubility may strongly depend on temperature. Table 14 presents the solubilities in water of several substances at different temperatures.

Table 14. Solubilities of Some Solids in Water at Different Temperatures (in grams per 100 ml)

Substance	Temperature, °C		
	0	18	100
Calcium chloride	50	71	155
Lithium carbonate	1.5	1.3	0.8
Lithium chloride	64	79	130
Potassium nitrate	13	29	250
Silver chloride	0.00006	0.00013	—
Sodium chloride	35.5	36.0	39.6
Zinc chloride	210	360	610

In most cases, solubility increases with temperature, this increase being sometimes significant (as for potassium nitrate). For some substances, the change in solubility as a result of heating is insignificant (sodium chloride), and in rare cases even a decrease in solubility as a result of heating is observed (lithium carbonate). If we cool a solution of potassium nitrate or other substance whose solubility increases with temperature, a fraction of dissolved substance will be liberated in the form of a solid precipitate. Under certain conditions (with pure solutions, clear vessels and careful cooling), one can sometimes obtain solutions whose concentration exceeds the limiting value (*supersaturated solutions*). If we drop a grain of the substance being dissolved into such a solution, crystallisation immediately occurs, and the solution concentration will drop to the value corresponding to saturation.

Chapter 15

Properties of Solids. Transition from Solid to Liquid State

15.1. Introduction

Before analysing the properties of solids, let us first clarify the concept of a solid body. Liquids and solids differ from gases, in particular, in that considerable variations of the gas volume are accompanied by the emergence of comparatively weak elastic forces, while even small volume deformations in solids and liquids give rise to strong elastic forces. In mechanics, we have introduced the concept of a perfectly rigid body (see Sec. 3.2) and incompressible liquid (see Sec. 7.4) to make it possible to disregard deformations, limiting ourselves to an analysis of elastic forces accompanying these deformations.

Assuming that the emergence of considerable elastic forces during small deformations is typical of liquids and solids, we must establish the difference between them. We distinguish solids from liquids by strong elastic forces emerging in solids both as a result of small variations of the volume (compression and extension) and as a result of small changes in the *shape* (shear) which are not accompanied by a change in volume. On the other hand, such shears (change in the shape) in liquids are not accompanied by the emergence of elastic forces.

Distinguishing solids by this property, we must pay attention to the possibility of the existence of solids in two essentially different states as regards their internal structure, which leads to the difference in their properties. These are the *crystalline* and *amorphous* states of solid bodies.

The so-called polymers, viz. the bodies whose molecules may consist of thousands of atoms, have become very important. Their structure determines peculiar properties, in particular, the ability to undergo comparatively large deformations. Polymers can be regarded as a special type of solids.

15.2. Crystalline Bodies

Let us examine through a magnifying glass a small-grained body (like salt, granulated sugar, soda or a powdery medicine). We shall see that individual grains of these substances are the bodies bounded by flat, as if

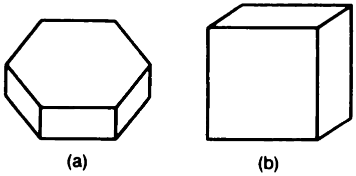


Fig. 439.
(*a*) A crystal of ice has the shape of a hexagonal prism whose faces form angles of 120°. (*b*) A crystal of common salt have a cubic shape.

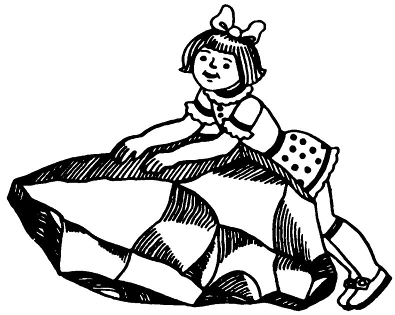


Fig. 440.
A large crystal of quartz (rock crystal) found in the Urals.

polished faces. These faces form certain angles which are generally different for different substances (Fig. 439). The presence of such natural faces is an indication that a substance is in the crystalline state.

A body may sometimes form a single crystal. By way of example, we can consider granulated sugar. Such bodies are known as *single crystals*, or simply crystals. Some substances may form quite large single crystals (Fig. 440) which sometimes have a very regular shape. In other cases, a body is an aggregate of a large number of small crystals which are intricately joined (sometimes the crystals can be extremely small). An example of such a body is a lump of sugar or any piece of metal. Such bodies are termed *polycrystalline*.

The natural formation of faces in a crystal is just one of the properties of the crystalline state of substances. A more general property is the difference in the physical properties of a body in different directions. The most conspicuous fact is the different mechanical strength in different directions in a crystal. Crystals readily cleave along certain planes. For instance, mica crystals which have the form of thin plates can be easily separated into still thinner plates. If we break a crystal of common salt shown in Fig. 439*b*, we shall obtain smaller crystals of the same shape. The bodies composed of one or several similarly arranged crystals are deformed in one direction more easily than in another direction. This refers, for example, to pieces of ice (Fig. 441). In its mechanical properties, a bar of ice taken from a lake or river resembles a pile of glass plates joined together by a not completely hardened glue.

Thermal conductivity of some crystals is also different in different directions. Let us cover a gypsum crystal and a glass plate with a thin layer of paraffin and touch them by a red-hot needle. It can be seen that paraffin melts in the vicinity of the needle so that the area of molten paraffin has the

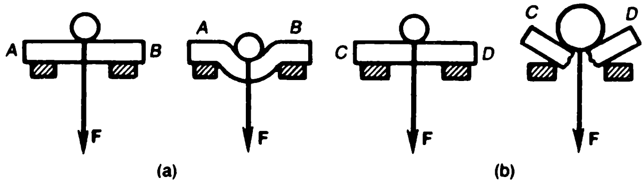


Fig. 441.

(*a*) Block AB cut in a certain manner from a large ice crystal and put on two supports gradually bends under the action of a force **F** applied at its middle. (*b*) Block CD of the same size, cut in the direction normal to that of AB, retains its shape under the action of the force **F** and breaks if the load is increased.

shape of an ellipse on gypsum (Fig. 442) and a circle on glass. This proves that unlike glass, the gypsum crystal conducts heat differently in different directions.

Many crystals expand nonuniformly as a result of heating. In order to characterise such crystals in terms of their thermal expansion, three temperature coefficients of linear expansion are required (for example, in three mutually perpendicular directions) instead of one. It is interesting to note that some crystals expand in some directions upon heating and contract in other directions (in these directions, the temperature coefficients of linear expansion are negative; graphite and tellurium are examples of such crystals). Optical and electric properties of crystals also depend on direction.

Crystals grow differently in different directions, this is manifested in the formation of plane faces in crystals. If a crystal grew uniformly in all directions (at the same rate), it would obviously have the shape of a sphere. It should be noted that not *all* properties of crystals depend on direction. For instance, a copper crystal having a cubic shape has the same electric and thermal conductivities in all directions, while its elasticity depends on direction.

As regards the difference in properties in different directions, a crystal resembles a piece of wood. Wood also easily cleaves along fibres, but it is much stronger in a direction normal to the fibres. The thermal conductivity of wood is also different in different directions (along fibres and across

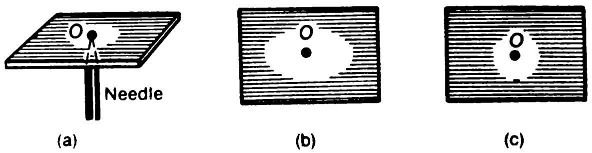


Fig. 442.

(*a*) When a red-hot needle touches a thin plate at point O, paraffin melts on the other side of the plate. (*b*) The plate made of a gypsum crystal. The area occupied by molten paraffin has the shape of an ellipse. (*c*) The plate is made of glass: molten paraffin has the shape of a circle.

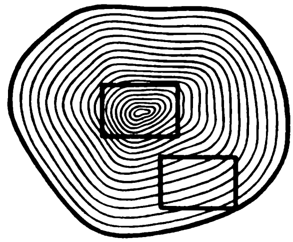


Fig. 443.
The structure of wood near the pith and near the bark is different.

them), and so on. However, there is a very important difference between the properties of a crystal and wood.

The structure of wood at the middle of the trunk and in its peripheral parts is different. Near the pith the annual rings are small and far from it they are larger. Thus, wood is nonhomogeneous. A piece of wood cut from the pith has certain properties and is fit for making one type of articles, while another piece cut in the vicinity of the bark has flat layers and can be used for manufacturing quite different articles (Fig. 443). But crystals are *perfectly homogeneous bodies*. A crystal has no "pith", and all parts of a crystalline body have the same properties.

All that has been said above refers to *single crystals*. The situation is different for *polycrystalline* bodies. Since they are random aggregates of a large number of small crystals, they are not as homogeneous as single crystals. On the other hand, the difference in the properties in different directions is not observed for polycrystals. This is due to the fact that in any direction in a body we encounter a very large number of differently oriented small crystals. For this reason, electric conductivity, thermal conductivity, and, in general, any property of a body is a certain *average* quantity characterising the aggregate of all these small crystals. This average value is the same in all directions in the body.

The size of small crystals constituting a polycrystalline body considerably affects the strength of the body. A material (say, steel of a certain composition) consisting of tiny crystals has usually a higher strength than the same material composed of larger crystals. If, for example, a tungsten crystal large enough to occupy the entire cross section of the filament is formed in an incandescent lamp, the filament will burn out just in this region. Sometimes interlocked crystals form fibres. This improves the strength of the material. Hence, it is clear that the information on the structure of a polycrystalline body is very important in engineering.

Thus, a polycrystalline body with randomly arranged small crystals resembles a noncrystalline body in its properties. This was one of the reasons behind the lasting belief that the crystalline state is not encountered often in nature. In 1912, a new technique associated with X-rays was discovered for investigating the structure of bodies. Using this method, it was established that an overwhelming majority of bodies around us (metals, minerals, plant fibres, proteins, carbon black, and so on) consist of crystals which are sometimes so small that they cannot be seen even through a powerful microscope.

? 15.2.1. Look through a magnifying glass at the fractures of different metals (cast iron, copper, etc.). Find in them the faces of small crystals constituting the metal.

15.3. Amorphous Bodies

The other type of the solid state is the *amorphous* state which differs sharply from the crystalline state. In amorphous bodies, it is impossible to find even small regions within which any dependence of physical properties on direction could be observed. Thermal, electrical and optical properties of amorphous bodies turn out to be completely independent of direction.

Some substances which normally have a crystalline structure can also be in the amorphous state. For instance, if a quartz crystal is melted (this occurs at a temperature of 1700 °C), we get, upon cooling, the so-called fused quartz which has a lower density than the crystalline quartz and has uniform properties in all directions. These properties sharply differ from those of crystalline quartz. The temperature coefficients of linear expansion in two mutually perpendicular directions for crystalline quartz are 1.3×10^{-5} and $8 \times 10^{-6}\ K^{-1}$, while for fused quartz the temperature coefficient of linear expansion is the same for all directions and is equal to $4 \times 10^{-7}\ K^{-1}$.

The thermal conductivities for the mutually perpendicular directions in crystalline quartz differ almost by a factor of two, while in fused quartz the thermal conductivity is the same in all directions, its value being equal to 1/20 of the lowest thermal conductivity of crystalline quartz. At low temperatures, the difference between thermal conductivities of fused and crystalline quartz becomes even larger. Generally speaking, the amorphous state is *unstable*. After a certain time, an amorphous substance is transformed into the crystalline state. However, the time required for such a transformation is sometimes very long (of the order of years or even decades).

The most important example of a substance in the amorphous state is glass (an amorphous alloy of silicates). Other examples of amorphous substances are colophony and sugar candy. These substances grow turbid with time (glass "devitrifies" and candy becomes sugared). This turbidity is due to the appearance of small crystals in the bulk of glass or sugar candy, whose optical properties differ from those of the amorphous medium surrounding them.

15.4. Crystal Lattice

How does molecular theory explain the properties of crystals? At the beginning of the 19th century, a hypothesis was put forth that the external regular shape of crystals is due to the regular internal arrangement of particles constituting them, viz. atoms. The investigation with the help of X-rays confirmed this hypothesis.

The particles constituting crystals are arranged according to a certain pattern and at a certain distance from one another. Of course, as a result of thermal motion the separation between the particles keeps on changing all the time, but we can speak of a certain average separation at each temperature. The aggregate of sites, i. e. points corresponding to the mean positions of particles constituting a crystal, is called the *space lattice* of this crystal.

Sometimes, the particles constituting crystals are electrically charged *ions*. Ions are atoms (or groups of atoms) that have lost or gained one, two, or more electrons. If an atom looses electrons, it becomes a positively charged particle, viz. a positive ion. On the other hand, if an atom acquires some electrons, it becomes a negative ion. Crystals consisting of ions are known as *ionic crystals*.

A simple example of a space lattice of an ionic crystal is the crystal lattice of sodium chloride (common salt). A molecule of this substance (NaCl) can be visualised as consisting of a chlorine atom and a sodium atom. Such molecules can be observed in salt vapour. Experimental studies show that a solid crystal of common salt does not contain NaCl molecules in the form described above. The crystal lattice of sodium chloride consists of alternating chlorine and sodium ions rather than of sodium chloride molecules (Fig. 444). Every sodium ion is surrounded by six chlorine ions arranged in three mutually perpendicular directions, while each chlorine ion is in turn surrounded by six sodium ions.

Similar lattices are typical of many diatomic salts (like silver bromide and chloride, potassium iodide and many sulphides of metals). The distances between mean positions of ions in lattices of different substances are different. The separation of neighbouring ions is 2.81×10^{-10} m in the

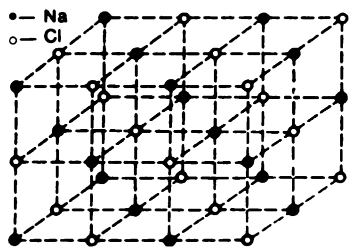


Fig. 444.
Arrangement of lattice sites in a space lattice of a sodium chloride crystal.

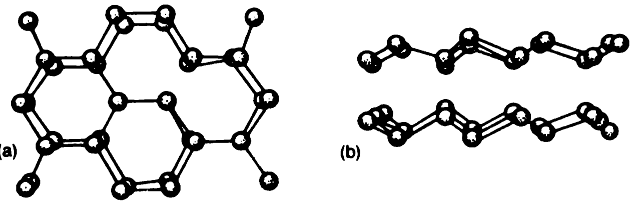


Fig. 445.
Spatial lattice of ice crystals: (*a*) top view and (*b*) side view. Balls indicate oxygen atoms, the position of hydrogen atoms is not shown.

sodium chloride lattice, 2.77×10^{-10} m in silver chloride, 3.54×10^{-10} m in potassium iodide, and so on.[1] There also exist more complex ionic crystals. For example, the lattice of Iceland spar ($CaCO_3$) consists of Ca^{2+} and CO_3^{2-} ions.

Besides ionic crystals, there are also crystals consisting of neutral particles, viz. atoms and molecules. For instance, the lattice of diamond consists of carbon atoms, while the lattice of ice crystals is composed of water molecules (H_2O), naphthalene lattice is formed by large molecular groups ($C_{10}H_8$), and so on. The interatomic separation for such crystals is also of the order of 10^{-10} m.

Atoms or ions are not necessarily arranged in a lattice formed by cubes (cubic lattice) as in the case of NaCl. Most lattices have a much more complicated form. The lattice of ice crystal is an example of such a complex lattice (Fig. 445).

How can the dependence of the physical properties of crystals on direction be explained?

Suppose that circles in Fig. 446*a* represent atoms of a liquid (say, mercury) arranged in a certain plane. Let us choose an atom *A* and draw through it straight lines in different directions. Clearly, due to a completely random arrangement of atoms, practically the same number of atoms will be on equal segments of any of these straight lines. This means that all directions are equivalent for a random arrangement of atoms.

A different situation will be observed if we use the same construction for a regular arrangement of atoms typical of a crystal (like the one shown in Fig. 446*b*). It can be seen that straight lines in directions *BB* or *CC* cross

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[1] These values can easily be obtained if we know the molar mass of a salt and its density. Let us consider, for instance, silver chloride (AgCl). Its molar mass is 0.143 kg/mol and its density is 5.56×10^3 kg/m^3. Consequently, 1 m^3 contains $5.56 \times 10^3 : 0.143 = 38\,880$ mol. One mole of the salt contains 6.02×10^{23} molecules, i.e. 12.04×10^{23} atoms. This means that the number of atoms in 1 m^3 is $38\,880 \times 12.04 \times 10^{23} = 4.68 \times 10^{28}$. Along every edge of 1-m^3 cube, $\sqrt[3]{4.68 \times 10^{28}} = 3.60 \times 10^9$ atoms are arranged. Thus, the separation of adjacent atoms is $1/(3.60 \times 10^9) = 2.77 \times 10^{-10}$ m.

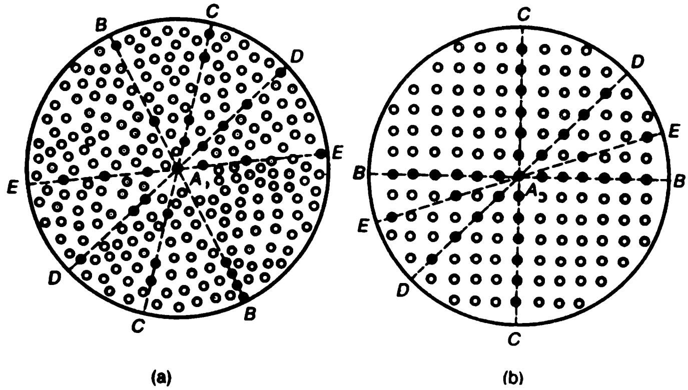

Fig. 446.
(*a*) Random arrangement of particles in a liquid. Any straight line (*BB*, *CC*, *DD*, ...) drawn through molecule *A* passes through the same number of molecules (marked by dark circles). (*b*) Ordered arrangement of atoms in a crystal. Different straight lines (*BB*, *CC*, *DD*, ...) drawn through molecule *A* encounter different numbers of atoms.

many atoms, while the line along *DD* encounters a smaller number of atoms, and the *EE* line passes through a still smaller number of atoms. This explains why physical properties of a crystal depend on direction. For example, in the lattice of common salt, the cleavage is easier along the planes parallel to *AA* or *BB* (Fig. 447). Therefore, if we strike a cubic crystal of common salt with a hammer, we split it again into regular cubes, while a piece of amorphous glass breaks into fragments of arbitrary shape.

It should be noted in conclusion that in real crystals the lattice is normally not regular throughout a crystal. In some places, where the lattice is distorted, there are regions where atoms are arranged chaotically, and impurity atoms can also be present. These local distortions play a significant role in explaining some properties of real crystals.

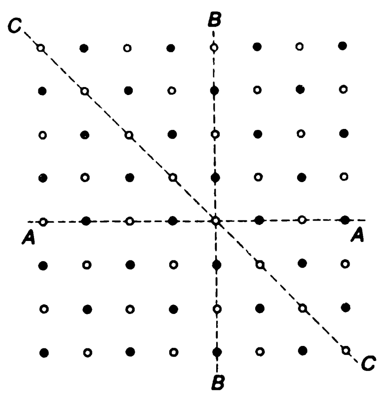

Fig. 447.
The cleavage of a common salt crystal occurs easier along the planes parallel to *AA* or *BB* than along any other plane, say, *CC*.

15.5. Crystallisation

If we breathe on a frosty window and thus make the frost melt, the growth of needles of ice crystals can be observed on the clear spot. Their formation starts from an already shaped ice crystal. Growing ice needles branch in various directions always so that the angle between them remains constant. When ice needles come in contact with one another, they interlink, forming an ornament composed of a large number of crystals (Fig. 448).

The growth of other crystals from the fused state (*melt*) proceeds in the same way as in ice crystals. Besides the formation of crystals from a melt, their growth from *solutions* is also observed (for example, the precipitation of potassium nitrate crystals from its aqueous solution). Sometimes crystals can be formed directly from *vapours* rather than from liquid. In such cases, they are especially regular. By way of example, we can consider the formation of hoar-frost and snow flakes from water vapour in air. The formation of iodine crystals from iodine vapour can also easily be observed. Let us put two or three crystals of iodine into a test tube and heat it from below (Fig. 449). It can be seen that iodine crystals do not fuse but directly evaporate (or *sublimate*), forming dark brown iodine vapour. At cold parts of the test tube, a dark incrustation is formed. It can be seen through a magnifying glass that it consists of a large number of tiny iodine

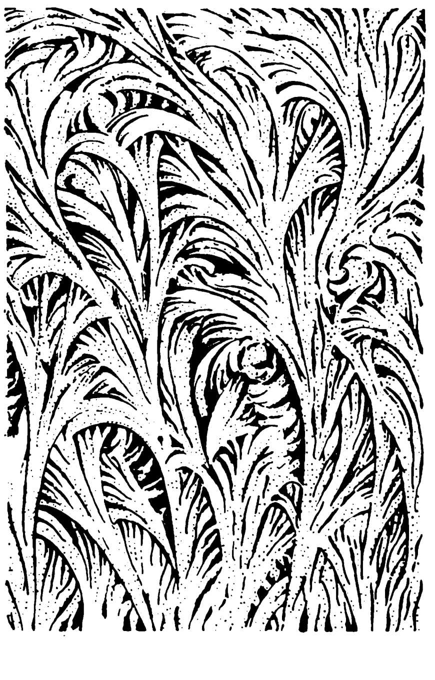

Fig. 448.
Ice crystal on a window glass.

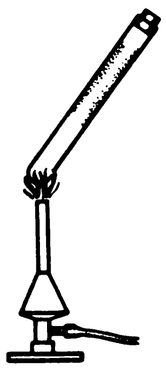

Fig. 449.
Sublimation of iodine crystals.

crystals. They were formed from iodine vapour which was transformed directly to the solid (crystalline) state without passing through the liquid state.

15.6. Melting and Solidification

Let us consider melting (fusion) and solidification of crystalline and amorphous bodies. A crystal-melt mixture is heterogeneous: there exists a sharp interface between the crystal and the melt. If the crystals are not too small, it can be seen where the crystal has formed and where the melt still remains. This does not resemble at all the solidification of amorphous solids. When tar solidifies, it becomes gradually thicker uniformly in all its parts. A solidifying amorphous body remains homogeneous.

An important distinction between the properties of crystalline and amorphous bodies concerns the temperature of solidification. Take in winter a glass with water outdoors and place a thermometer in it. It can be seen that water will rapidly cool to 0 °C. Then ice starts to form. To prevent crust formation, stir water in the glass. All the time during which ice is being formed, a temperature of 0 °C will be maintained in the glass. As soon as the entire water has been frozen, the obtained ice starts to cool below zero. Having brought the glass into a warm room, we notice that the temperature of ice rises to 0 °C and is maintained at this level until the entire mass of ice melts. Only then does the temperature of water in the glass rise above zero.

Similar phenomena are observed during the solidification and fusion of *all* pure crystalline substances. If, for example, we watch the change in the temperature of molten naphthalene with time τ and plot the curve corresponding to this dependence, we obtain the graph with a horizontal plateau (Fig. 450). This horizontal region corresponds to the mixture of naphthalene crystals and its melt. On the other hand, during the solidification of amorphous bodies like tar, the temperature decreases continuously

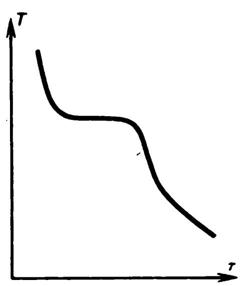

Fig. 450.
Time chart of solidifying naphthalene.

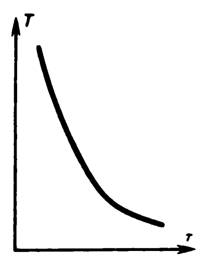

Fig. 451.
Time chart of solidifying tar.

without halts (Fig. 451). Hence we can conclude that a solidifying amorphous body does not transform to a new state. Solidification of tar or glass is just their gradual thickening. Glass can be regarded as a very thick liquid.

Thus, *crystalline substances have definite temperatures of melting and solidification (melting points).* Amorphous bodies *gradually* become soft with increasing temperature. Table 15 presents melting points for some substances.

Table 15. Melting Points for Some Substances

Substance	Melting point, °C	Substance	Melting point, °C
Copper	1083	Tin	232
Gold	1063	Tungsten	3370
Iron	1535	Water	0
Lead	327	Zinc	419
Mercury	−39		

? **15.6.1.** Put some naphthalene into a test tube and dip it into boiling water. The naphthalene will melt. Take the test tube out of water and measure the temperature of the naphthalene with the help of a laboratory thermometer every half minute. How can the melting point of naphthalene be determined from these data?

15.7. Specific Latent Heat of Fusion

It was mentioned above that a tumbler containing a mixture of ice and water, brought in a warm room, has the same temperature until the entire ice melts. Ice at 0 °C is converted into water *at the same temperature.* Meanwhile, heat is supplied to the ice-water mixture, and hence the internal energy of this mixture increases.[2] Hence we have to conclude that *the internal energy of water at* 0 °C *is higher than the internal energy of ice at the same temperature.* Since the kinetic energy of molecules of water and ice is the same at 0 °C, the increment in the internal energy during fusion is equal to the increment of the potential energy of molecules.

Experiments show that what has been said above is valid for all crystals. During fusion of a crystal, it is necessary to continually increase the internal energy of the system without changing the temperature of the crystal and the melt. Usually, the increase in the internal energy occurs when a certain amount of heat is supplied to the crystal. The same result can be at-

[2] The external work done due to a change in the volume of a substance during fusion is small and can be neglected.

tained by doing work, say, by rubbing. Thus, *the internal energy of the melt is always higher than the internal energy of the same mass of crystals at the same temperature*. This means that the ordered arrangement of particles (in crystalline state) corresponds to a lower energy than disordered arrangement (in melt).

The amount of heat required for converting a unit mass of a crystal into melt at the same temperature is known as the *specific latent heat of fusion* for the crystal. It is measured in joules per kilogram (J/kg).

During the solidification of a substance, the latent heat of fusion is liberated and transferred to the surrounding bodies.

It is difficult to determine the specific latent heat of fusion for refractory materials (i. e. substances having a high melting point). The specific latent heat of fusion for a crystal with a low melting point, say, ice, can be determined with the help of a calorimeter. Pouring a certain amount of water at a definite temperature into the calorimeter and dropping into it a known mass of ice which begins to melt (at 0 °C), we must wait until all the ice melts and the temperature of water in the calorimeter acquires a constant value. Using the law of energy conservation, we can write the heat balance equation (see Sec. 11.8) which allows us to determine the specific latent heat of fusion for ice.

Suppose that the mass of water (including the water equivalent of the calorimeter) is m_1, the mass of ice is m_2, the specific heat capacity of water is c, the initial temperature of water is t_1, the final temperature is t_2, and the specific latent heat of fusion of ice is r. The heat balance equation has the form

$$cm_1(t_1 - t_2) = rm_2 + cm_2t_2,$$

whence

$$r = \frac{cm_1(t_1 - t_2) - cm_2t_2}{m_2}.$$

Table 16. Specific Latent Heat of Fusion for Some Substances

Substance	r, kJ/kg	Substance	r, kJ/kg
Copper	214	Lead	23.1
Ice	334	Mercury	11.8
Iron	270		

Table 16 contains the values of the specific latent heat of fusion for some substances. Pay attention to the high value of the specific latent heat of fusion for ice. This circumstance is very important since it slows down the melting of ice in nature. If the specific latent heat of fusion for ice were considerably smaller, floods would be much more frequent in spring.

Knowing the specific latent heat of fusion, we can calculate the amount of heat required for melting a body. If the body has been already heated to its melting point, heat is required only for its melting. If, however, its temperature is below the melting point, some amount of heat should be spent on heating it.

? 15.7.1. Pieces of ice at −10 °C are dropped into a vessel with water which is protected from the heat inflows from outside. What amount of ice can be melted completely if the vessel contains 500 g of water at 20 °C? Assume that the heat capacity of the vessel is negligibly small in comparison with the heat capacity of water. The specific heat capacity for ice is 2.10 kJ/(kg · K).

15.8. Supercooling

If we heat a crystal, it will melt at a temperature corresponding to its melting point. On the other hand, if we cool a liquid, it starts crystallising at the temperature of fusion.

However, we can sometimes cool a liquid without its solidification through several kelvins *below* the melting point. This can be observed during cooling of molten sodium thiosulphate.[3] This substance melts at 48 °C. Meanwhile, we can easily cool to the room temperature pure sodium thiosulphate melted in a test tube. It is sufficient, however, to drop a small crystal of sodium thiosulphate in the melt or abruptly shake the test tube for a part of sodium thiosulphate to rapidly crystallise and the mixture of fused and crystalline sodium thiosulphate is formed. The temperature of such a mixture is equal to the melting point of sodium thiosulphate (48 °C). What has made the temperature rise and why has only a part of sodium thiosulphate crystallised? During a transition of the melt to the crystalline state, the internal energy decreases, and the liberated energy is distributed throughout the mixture so that its temperature rises. Crystallisation ceases as soon as the entire volume of the mixture turns out to be heated to the melting point.

Other liquids can also be supercooled. For instance, sugar syrup is easily supercooled, forming sugar candy. Essentially, any amorphous substance can be regarded as a supercooled liquid having a very high viscosity. It is viscosity which prevents such substances from going over to the crystalline state. However, as was mentioned in Sec. 15.3, the substance like glass or sugar candy becomes turbid with time, which is an indication of the precipitation of tiny crystals in the bulk.

So, in which cases do liquids start crystallising as soon as they are cooled to their melting point and when is supercooling possible? To initiate

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[3] This is a substance used for preparing fixative for the treatment of photographic films and paper.

crystallisation, so-called crystallisation centres, or nuclei, are required. These can be small crystals invisible even in a microscope (seed) or alien dust particles present in a liquid. It is near these crystallisation centres that molecules gradually forming a crystal are grouped. If, however, there are no crystallisation centres, a supercooling through several kelvins is possible even in a liquid with a low viscosity. An important example of such a liquid is water. A supercooling of pure water containing no dust particles can be observed in nature quite often. Mist drops may not be frozen even at a temperature as low as -30 °C. Fogs consisting of supercooled drops are dangerous for aeroplanes: having precipitated on the wings, they form the ice crust that may lead to the destruction of the aeroplane (icing).

It follows from what has been said above that a supercooled liquid is in an unstable state. Under the influence of certain factors, the supercooled liquid goes over after a certain time into the crystalline state which is more stable at a given temperature.

? **15.8.1.** In sugar refinery, confectioner's sugar is often added to sugar syrup to accelerate the precipitation of sugar grains. Why does this facilitate the crystallisation?

15.9. The Change in the Density of a Substance during Fusion

During fusion, the density of most substances decreases. The following experiment illustrates this phenomenon. Let us drop a piece of solid paraffin into molten paraffin. It will sink. This means that the density of molten paraffin is lower than that of solid paraffin. During melting, paraffin increases its volume. Many other materials behave in this way. This phenomenon indicates that the volume occupied by a crystal with regular, ordered arrangement of molecules is smaller than the volume occupied by the same substance in the liquid state characterised by a random arrangement of molecules. This can be easily understood. Indeed, oranges packed in a box in regular rows occupy smaller volume than the same amount of oranges put in the box at random.

However, there are several exceptions to this general rule, water being the most important of them. It is well known that ice floats in water since its density is much lower than that of water. This circumstance plays an important role in nature. A layer of ice on the surface of water, covered from above by a layer of snow which is a poor conductor of heat, perfectly protects water from freezing. For this reason, ponds do not freeze throughout to the bottom, and this saves fish from freezing.

Expansion of water upon cooling is the reason behind another phenomenon important for the life on the Earth, viz. the destruction of rocks. Let us suppose that a crack in a rock contains some water (Fig. 452).

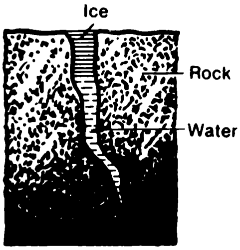

Fig. 452.
Cracking of a rock. During frost, a layer of ice "seals" water in the lower part of the crack.

During frost, the upper layer of water is frozen first, the deeper-lying layers being "sealed". However, when these layers start to freeze as well, they will thereby increase in volume and enlarge the crack. Ultimately, this will lead to the destruction of the rock.

The following simple experiment can give an idea about the forces developed during the expansion of water, which accompanies its freezing. In winter, you can pour water into a bottle to the very neck and take it outdoors. Water will freeze and expand. The ice cork at the neck prevents the freezing water from expanding freely, and the bottle will be broken by the pressure of ice. Such an experiment can also be made with a thick-wall bottle made of cast iron.

How can this peculiarity of water be explained? Why is the increase in the potential energy of interaction of water molecules accompanied by a decrease in volume and not by its increase as for other bodies? This can be explained by a peculiar structure of the crystal lattice of ice. Let us consider again Fig. 445 showing the internal structure of ice crystals. It can be seen that water molecules in an ice crystal are arranged highly nonuniformly: in some regions the molecules are very close to one another, while in other regions there are gaps between the layers. As water goes over from the crystalline to the liquid state, the arrangement of molecules becomes more uniform. Now the distance between molecules which were closely packed in the crystal increases, while the distance between molecules formerly separated by gaps decreases. The potential energy of interaction of former molecules increases and that for the latter molecules decreases. But the increase in the potential energy of closely packed molecules exceeds the decrease in the potential energy for loosely packed molecules. Ultimately, the internal energy of water is still higher than the internal energy of ice from which it has been obtained.

15.10. Polymers

We have considered the internal structure of crystalline bodies like rock salt, quartz, metals and amorphous bodies such as glass. These substances consist either of atoms or of molecules containing a small number of atoms. Let us now consider a special group of substances which play an exceptionally important role in nature and in technology. We are speaking about natural substances like cotton, wood, leather, wool, natural silk, natural rubber and a large number of materials manufactured industrially like synthetic rubber, viscose rayon, cellulosic packing materials, organic glass and various plastics. As compared to metals, these substances have low density, low thermal and electric conductivities, and peculiar mechanical properties which distinguish them from other materials. Mechanical properties of rubber which will be considered below are especially remarkable and make it irreplaceable in some branches of technology (for manufacturing automobile tyres, pipes, rubber boots, etc.).

These materials have the same chemical origin. They all are *polymers* (from Greek words *poly* for many and *meros* for part). This term means that the molecules of these substances consist of a large number of identical parts (*monomers*, from the Greek *mono* meaning one), linked in long chains by strong chemical bonds.

Monomers consist of a small number of light atoms among which there are always carbon and hydrogen, and sometimes oxygen, chlorine and other elements. An example of the polymer structure is shown schematically in Fig. 453, where dashed lines mark the sites of connection of monomers. The number of monomers constituting a polymer molecule is normally very large (of the order of 10^3 or 10^4 monomers). For example, a molecule of natural rubber is formed by 3000-6000 monomers each of which consists of carbon and hydrogen atoms. A cellulose molecule (the main part of cotton) contains more than 10 000 monomers consisting of carbon, oxygen and hydrogen atoms. It should be emphasised that the same polymer (say, cellulose) also contains molecules composed of different numbers of monomers. Thus, the molecular mass of a polymer is not a quite definite quantity.

The physical properties of polymers are mainly determined by the fact that their molecules are formed by long and strong chains which are preserved during mechanical and thermal treatment (spinning, pressing through narrow holes, moulding, and so on), as well as during dissolution or melting of polymers. These chains are sometimes coiled and sometimes are more or less elongated. Polymer links (viz. monomers) can be turned relative to one another to a certain extent. Although the angle of rotation

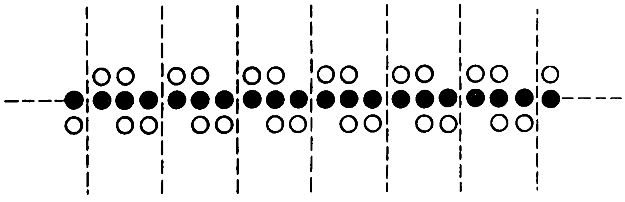

Fig. 453.
A chain of monomers forming the polymer, viz. a variety of a synthetic rubber. Dark circles represent carbon atoms and white circles, hydrogen atoms.

of an individual link cannot be large, the twisting of a molecule can be considerable due to a large number of links. This is the reason behind the high deformability of articles manufactured of polymers.

Polymer solutions always exhibit a considerable viscosity. This is due to the presence of long polymer chains in a solution. The viscosity of polymers themselves is very high but it rapidly decreases with increasing temperature. When an article is moulded from a plastic, its filiform intricately crosslinked molecules are forced to acquire a new arrangement. They tend to return to the initial position, but a huge viscosity of the plastic extremely slows down the process of returning to the initial shape. An elevation of temperature increases the rate of this process. If, for example, a saucepan made of organic glass is kept in boiling water for several minutes, it acquires the form of a plate from which it has been moulded (Fig. 454).

Of special interest are mechanical properties of rubber. It is well known that an article made of rubber can be stretched many times more than any other body. This can be explained as follows. Rubber, as any other polymer, consists of long molecules bent in different directions. The degree of bending depends on the thermal motion of the links of the chain, i. e. on the temperature. A definite temperature corresponds to a certain degree of bending of a rubber molecule, which is the higher, the higher the temperature. However, pure rubber is still in the liquid state at temperatures close to room temperature. In order to make it elastic, crosslinked ends of molecules should be tied together to prevent them from being separated. For this purpose, the ends of adjacent molecules should be linked by "bridges". These bridges can be constructed in different ways. An old method is the "vulcanisation of rubber", i. e. the introduction of sulphur into it. Sulphur atoms are implanted between the links of two rubber molecules and form bridges. The bridges connect a large number of rubber molecules into a single structure. Rubber loses its fluidity and becomes elastic. The number of bridges formed by atoms of sulphur (or other element) should not be too large since otherwise the rubber becomes rigid.

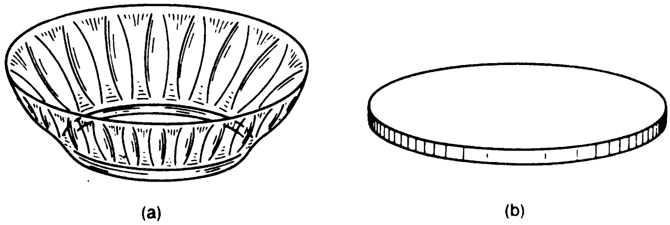

Fig. 454.
(*a*) A saucepan made of organic glass. (*b*) The same saucepan after several minutes in boiling water.

What happens when rubber is stretched? Rubber molecules change shape, approaching the shape of straight line segments and being arranged more or less in parallel to one another. After the forces stretching rubber are removed, rubber molecules again acquire the shape corresponding to the initial temperature, and the rubber article contracts. The state of a substance in which a very large elongation is possible without rupturing the article is known as the rubber-like state. It is observed in a certain temperature interval. As the temperature is lowered, the substance goes over to the solid state, while the elevation of temperature causes the destruction.

Besides natural rubber obtained from the latex of some plants, synthetic rubbers are widely used in engineering, which are obtained, for example, from alcohol.

15.11. Alloys

Pure metals, i. e. metals consisting of atoms of only one element (say, iron), are practically never used in engineering. Almost all metal articles consist of various alloys of a metal with other metals or nonmetal elements. For example, very important in engineering are various steels, viz. alloys of iron, carbon and other elements (chromium, tungsten, manganese and many others). Brass (viz. the alloy of copper and zinc) is also widely used. In aircraft industry, alloys of aluminium and magnesium with a number of elements (like copper, iron or zinc) are employed, which combine very low density with a high strength.

The reason behind a wide utilisation of alloys lies in their advantages over pure metals. Above all, alloys are as a rule stronger than metals constituting them (it should be noted that pure iron is called "soft"). Alloys often melt at a lower temperature than their components. For example, tin melts at 232 °C and lead at 327 °C, while the melting point of an Sn-Pb alloy is 170 °C.

In modern engineering, many alloys are available whose technological properties strongly differ from those of pure metals. Hence, various practical requirements can be satisfied. There are alloys having almost the same strength as diamond. There also exist very elastic alloys and the ones combining low density and high strength (duralumin), the alloys which are not oxidised not only in contact with water but even in contact with acids (stainless steel), the alloys preserving their properties in the red-hot state (heat-resistant alloys), the alloys having a very high electric resistance (Ni-Cr alloy) or special magnetic properties, the alloys which do not expand upon heating (Invar), and so on.

It should be noted that even the so-called pure metals contain a small amount of impurities which are difficult to remove. Therefore, pure metals can be regarded as alloys with a clear predominance of one of the component metals. Meanwhile, even insignificant amounts of impurities sometimes considerably affect the properties of metals. For example, the presence of small amounts of sulphur and phosphorus in a steel or cast iron makes them brittle, while the presence of impurities in copper sharply reduces its electric conductivity.

So what are alloys and why do their properties differ from the properties of their components? There are no unambiguous answers to these questions since alloys have different, sometimes very complex structures (especially when their components can form chemical compounds).

Sometimes, small crystals of pure metals mixed at random precipitate in the bulk of a solidifying alloy (Fig. 455). The crystal growth in such a mixture is obscured by the presence of crystals of another metal. It was mentioned above that the fine-crystalline structure of a metal explains its high strength.

It should be noted that small crystals in a metal are always separated by very thin interlayers (Fig. 456). These interlayers have quite different physical properties in comparison with the small crystals. The physical properties of a metal are determined by the properties of small crystals and interlayers simultaneously. For instance, a too low strength of interlayers would lead to the disintegration of the alloy into a powder. Normally, interlayers are stronger than the small crystals, and the fracture of metal occurs through the crystals rather than through their boundaries.

Since small crystals consist of pure metals (or their chemical compounds), nonmetal impurities are concentrated in interlayers. Owing to a very small thickness of interlayers, a negligible amount of an impurity is sufficient to dramatically change the properties of interlayers, and hence of the entire alloy. This explains, for example, why sulphur impurities in steel are so harmful.

Fig. 455.
A polished section of a brass (copper-zinc) sample under a large magnification. Black copper crystals alternate with grey zinc crystals.

Fig. 456.
A polished section of aluminium surface. Thin black lines are the traces of interlayers between crystals.

15.12. Solidification of Solutions

Salt water (or sea water) freezes not at 0 °C but at a lower temperature. The same situation takes place for other solutions. The temperature of solidification of a solution is *lower* than that of a pure solvent. As the amount of a dissolved substance increases, the solidification temperature drops. When not a very concentrated solution freezes, only the solvent becomes frozen. For example, when salt water freezes, crystals of pure water precipitate, while salt remains in the solution whose concentration, i. e. the content of the salt in it, increases.

The solidification temperature drops with increasing mass of dissolved substance only to a certain limit. At a definite concentration, not the solvent alone but the entire solution freezes as a whole. At this concentration, the solidification temperature is lower than for any other concentration. For the solution of common salt in water, this is observed when the amount of salt in water constitutes about 30% by mass. Such a solution freezes only at -21 °C. For an aqueous solution of sal ammoniac (NH_4Cl), the minimum freezing temperature is -15 °C. It is attained for a 20% solution. A solution containing more or less sal ammoniac than 20% by mass gets frozen at a higher temperature.

? **15.12.1.** Why is brine used instead of pure water in the pipes drawn from a refrigerating plant to a room to be cooled?

15.12.2. How can salt water be distilled in winter?

15.13. Cooling Mixtures

Let us bring a piece of sugar in contact with boiling water (Fig. 457). Boiling water will be absorbed by sugar and reach our fingers. However, we do not feel a burn as in the case when a piece of cotton is used instead of sugar in this experiment. Observations show that the dissolution of sugar is accompanied by cooling the solution. If we wish to maintain the temperature

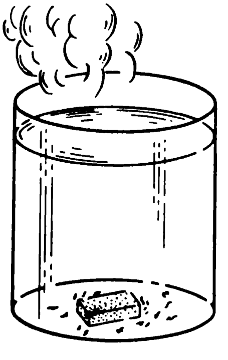

Fig. 457.
When sugar is dissolved in boiling water, the latter is noticeably cooled.

unchanged, an energy must be supplied to the solution. Hence it follows that during dissolution, the internal energy of the sugar-water system increases.

The same occurs during the dissolution of most of other crystals (namely those whose solubility increases with temperature, see Sec. 14.15). In all these cases, *the internal energy of the solution is higher than the internal energy of the crystal and the solvent taken separately at the same temperature*.

In the example with sugar, the energy required for dissolving sugar was taken from the boiling water whose cooling was felt even by touch. If a dissolution occurs in water at room temperature, the obtained solution can sometimes be cooled below 0 °C, although the mixture remains liquid since the temperature of solidification of a solution can be considerably lower than zero. This circumstance is used for obtaining highly cooled mixtures of snow with various salts. Snow starts melting at 0 °C, forming water which dissolves the salt. In spite of a temperature drop accompanying dissolution, the obtained mixture does not solidify. Snow mixed with this solution continues to melt, taking the heat of fusion from the solution, i. e. cooling it. The process will continue until the temperature of freezing of a given solution is attained. Thus, a mixture of snow with common salt (in proportion 2:1) makes it possible to attain in this way a temperature of −21 °C, while a mixture of snow with calcium chloride ($CaCl_2$) (in the ratio 7:10) can be used for cooling to −50 °C.

? **15.13.1.** Why is a mixture of ice and salt and not pure ice used for making ice cream?
15.13.2. Pavements are sometimes sprinkled with salt to make the snow melt. Why does it help? Where are feet cooled more: on the pavement covered with snow or on the one sprinkled with salt?

15.14. Variation of Properties of a Solid

It was shown above that many properties of a polycrystalline body, especially its mechanical properties, depend on the size of small crystals constituting it. As a rule, fine-crystalline alloys are stronger. Experience shows that the structure of polycrystalline bodies, in particular, metals, depends not only on the chemical composition of an alloy but also on the prehistory of a sample, for example, on mechanical and thermal treatments to which the sample has been subjected (cold working: rolling, forging, etc., and thermal treatment: hardening, annealing, etc.). If an iron rod is subjected to rolling or forging, its strength increases. An analysis shows that in this case it acquires a fibrous, fine-crystalline structure.

Let us consider another example. A new axle of a railway carriage is very strong. After a long use, however, it becomes brittle and may break.

An analysis shows that the fine-crystalline fibrous structure which initially ensured its strength has changed to the coarsely crystalline structure having a much lower strength. Crystal growth was enchanced by continuous shocks to which the axle was subjected. However, even in the absence of shocks the crystals grow, although at a lower rate.

These examples show that a solid body is not something invariable. Small crystals constituting it lead their life, changing their size and arrangement, which causes a change in the properties of the solid body.

The strongest effect on the properties of solids is produced by thermal actions which may even cause a change in the shape and structure of crystals (their space lattice). For example, iron at room temperature has a crystal lattice which differs from that at a higher temperature. Upon heating, iron goes over to different crystalline forms (there are four crystalline forms of iron in total). As a body goes over from one crystalline state to another, a certain amount of heat is absorbed or liberated (like in fusion and solidification), the size of the body noticeably changes, and so on. This can be observed in the following experiment.

Let us stretch a 2-3 m long iron wire arranged in the horizontal position and heat it with an electric current to the red-hot state. It will become longer and sag. Then we switch off the current and let the wire cool.[4] It will be seen that at first the wire starts to rise, then at a certain moment it stops rising, spontaneously becomes red-hot and sags, after which rapidly rises again. The moment when the wire increases its length again is just the moment when iron goes over from one crystalline state to another (at about 900 °C). The process will proceed in the reverse order if we increase the current very slowly.

An interesting process is observed during the hardening of steel. In this process, the cooling occurs so rapidly that steel has no time to go over from the crystalline state corresponding to the high temperature to the state in which it should be at room temperature. In the cold state, recrystallisation occurs very slowly, and steel remains in the crystalline state corresponding to the high temperature. Thereby, it becomes very hard and brittle. The steel can be allowed to recrystallise (partially or completely), for which purpose it must be heated again and slowly cooled (tempering of steel).

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[4] The experiment is more demonstrative when the current is not switched off completely but is reduced so that the wire cools very slowly.

Chapter 16
Elasticity and Strength

16.1. Introduction

In Part One of this book dealing with mechanics, it was pointed out more than once that bodies in contact exert on each other a certain force if they are deformed (say, compressed). Sometimes deformation can be easily observed, but in most cases it is so small that special sensitive devices are required to register it. Comparing a freely falling ball and a ball lying on the table, we can state that in the latter case the ball is deformed (compressed). However, to reveal the deformation of the table (bending of its desk) in this case, a special highly sensitive device must be used (see Sec. 3.2). With the help of special techniques it can also be observed that a rotating wheel is deformed (its spokes and rim are stretched) in comparison with the wheel at rest, and so on.

In mechanics, we were interested in the deformation of bodies inasmuch as the emergence of certain forces was associated with it. Considering solids, for example, we were not interested in the change of their shape and volume in deformations since they were very small and did not affect equilibrium or motion of bodies. For instance, considering that a lever is a straight rod, we disregarded the fact that a loading makes it bent. In a more accurate analysis, however, deformations should be taken into consideration. The knowledge of deformations is of special importance in engineering, for example, in designing bridges, various machines, and so on.

16.2. Elastic and Plastic Deformations

Let us slightly bend a steel plate (say, a hand saw) and then release it. It can be seen that the hand saw has completely (at least, at first sight) restored its shape. If we take a lead plate of the same size and bend it, it does not restore its shape completely and remains bent. Deformations that disappear completely as soon as deforming forces are removed (as in the case of the steel plate) are known as *elastic deformations*. Deformations that do not disappear after the removal of deforming forces (as with the lead plate) are called *plastic deformations*.

Strictly speaking, there are no fully elastic or fully plastic deformations. If we keep a steel plate in the deformed state for a very long time (say, for several years), it will not straighten out completely after the removal of deforming forces. A *residual deformation* will be observed, which is the larger, the longer the time during which the plate was in the deformed state.

Thus, *elastic deformations in all bodies become plastic deformations with time*.

The substances in which an elastic deformation is transformed into a plastic deformation only after a long time (in years!) are called *elastic substances*. The examples of elastic substances are steel and glass. The substances in which an elastic deformation noticeably changes to a plastic deformation during a short time (seconds or even fractions of seconds) are known as *plastic substances*. The examples of such materials are lead and wax. If, however, the time interval is too short, a plastic deformation has no time to change to an elastic deformation even in a plastic substance. For instance, a lead plate subjected to a very short-term deformation may behave as a steel plate.

The transition from elastic to plastic deformation also depends on the extent to which a body is deformed. The larger the deformation, the shorter the time required for its transition to a plastic deformation. By increasing the deformation of a body, we ultimately attain such a deformation that the transition from elastic to plastic deformation occurs almost instantaneously. In this case, the *elastic limit* is said to be reached. The elastic limit for elastic bodies is large, while for plastic bodies it is small. It should be noted that the elastic limit depends on the temperature. The higher the temperature, the lower the elastic limit for a given substance.

? **16.2.1.** Why do spring balances give wrong readings after a long use?

16.3. Hooke's Law

It was shown above that a deformation of a body is elastic, i. e. leading to no appreciable residual deformation, only provided that it is short-term and not large. Let us suppose that these conditions are fulfilled. What is the relation connecting in this case the deformation and the forces responsible for it? This relation can easily be established with the help of a good-quality rubber band. Let us fix the upper end of such a band and attach different loads to its lower end (Fig. 458). If the loads are such that the deformation is elastic, the elongation of the band turns out to be proportional to the stretching force (in the case under consideration, the weight of the load). The same regularity can be observed for any other deformations (compression, shear, etc.).

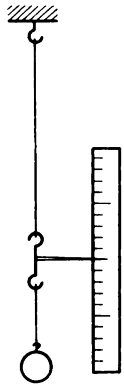


Fig. 458.
Testing the elongation of a rubber band depending on a stretching force.

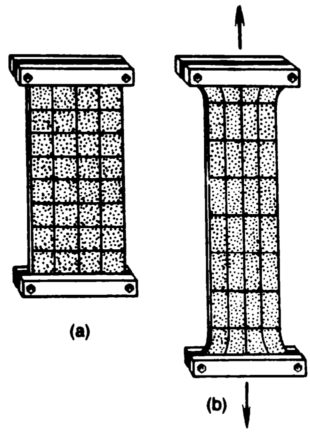


Fig. 459.
(*a*) A network with squared cells is drawn on a rubber band. (*b*) During extension of the band, the cells of the network acquire the shape of rectangles.

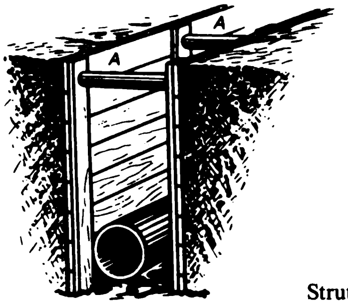
Thus, *for elastic deformations the deforming force and the deformation are proportional to each other*. This is the essence of Hooke's law discovered by the English physicist Robert Hooke (1635-1703). The ratio of the deforming force to the cross-sectional area of the body, over which this force is distributed, is called the *stress*. Hooke's law indicates that the strain[1] of any part of a body is proportional to the stress in this part.

16.4. Extension and Compression

Elastic deformations emerging in bodies can be of various types. A body can be stretched or compressed, bent, twisted or coiled. In most cases, an observed deformation is essentially a combination of various types of deformations. However, any deformation can ultimately be reduced to two simple types: *extension* (or *compression*) and *shear*.

A steel string in a guitar, a wire (not twisted) with a load suspended by it and a rubber band in a slingshot are examples of bodies subjected to axial extension since they are only stretched in one direction. As a result of such extension, the bodies are elongated and at the same time slightly reduce their transverse dimensions. This can clearly be seen during the stretching of a rubber band with a network of lines drawn on it (Fig. 459). As a result of extension, bodies are in a stressed state. In the example with the rubber band, the deformations of its individual parts, and hence the stresses, are

[1] The strain is the ratio of the dimensional change to the original dimension. — *Eds.*



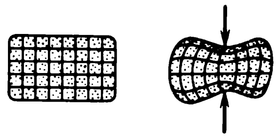


Fig. 461.
The compression of an eraser. The cells of the network are deformed more at the middle than at the ends.

Fig. 460.
Struts A are in the compressed state.

approximately equal throughout the band with the exception of regions where external forces are applied. The same refers to a stretched string.

Struts in narrow trenches (Fig. 460) or in mines, columns supporting a part of a building, and the legs of a table on which the desk rests are examples of bodies subjected to compression. In these examples, we deal with *axial compression*. In axial compression, a body becomes slightly thicker in transverse directions. This is very visual when we compress a soft eraser with a network of lines drawn on it (Fig. 461). This figure also shows that the deformations of different parts of the body can be different: the eraser is deformed to a larger extent at the middle than at the ends.

Measuring the elongation of wires or compression of rods made of different materials under the action of a given load, we see that *the deformation is the larger, the longer the sample and the smaller its cross-sectional area*. It can be readily understood. The thicker the sample, the smaller load acts per unit cross-sectional area, and the longer the sample, the larger its elongation which constitutes a certain part of its initial length: each unit length acquires the same increment. The properties of the material also strongly affect deformations. For example, the elongation of a steel wire is smaller by about a factor of two than that of a copper wire of the same size with the same load.

Bodies subjected to axial deformations experience the action of two equal and opposite forces.

In nature and in engineering, we often encounter *uniform* deformations: uniform compression and uniform extension. Both deformations are observed if a deformed body is subjected to pressure from all directions or extension in all directions. For example, bodies immersed in a liquid are in the state of uniform compression. When bodies are submerged in sea to a large depth, uniform compression is very strong and considerably influences living organisms. Uniform extension is encountered not so often. For example, the inner part of a cold steel sphere immersed in hot water is in the state of uniform extension. The uniform extension and compression play an important role in the propagation of acoustic vibrations (which will be discussed in Vol. 3 of this book in the part devoted to oscillations and waves).

? **16.4.1.** What will be the change in elongation if we replace a wire by another one, made of the same material but having twice larger length and diameter, without changing the load?

16.4.2. It was found in an experiment that a 1 m long steel wire having a cross-sectional area of 1 mm^2 has been elongated by 1 mm under the action of a stretching force of 200 N. What will be elongation of a steel wire having a cross-sectional area of 0.5 mm^2 and a length of 3 m as a result of extension with a force of 300 N?

16.5. Shear

We have considered extension and compression emerging under the action of two equal and opposite forces. Let us now consider deformations caused by two equal and opposite *moments of force* (*torques*).

We take a bar in the form of a rectangular parallelepiped and put it on a horizontal support (Fig. 462). The force of gravity **P** acting on the bar and applied at the center of gravity *C* is balanced by the normal reaction **N** of the support. Since the bar is at rest, the reaction must be applied at point *A* of the bar, which lies on the same vertical with the centre of gravity *C* (Fig. 462*a*). Let us now suppose that a horizontal force **F** is applied to the upper face of the bar so that the bar is distorted but does not slide along the support (Fig. 462*b*). Since the bar is at rest, this means that one more force is exerted on it, equal in magnitude to the force **F** and having the opposite direction. Obviously, this is friction **f**. The forces **F** and **f** form a couple which would cause the rotation of the bar about the axis perpendicular to the plane of the figure. However, the bar is at rest, and hence there exists a couple balancing the couple.

This second couple can easily be found. In the absence of force **F** the reaction **N** is applied at point *A*, while in the presence of force **F** the reaction of the support on the bar will somewhat change, and force **N** will be applied at a point *B* located to the right of point *A* in the figure. As a result, the couple of forces **P** and **N** is formed, which tends to rotate the bar in the direction opposite to that in which the bar would be rotated under the ac-

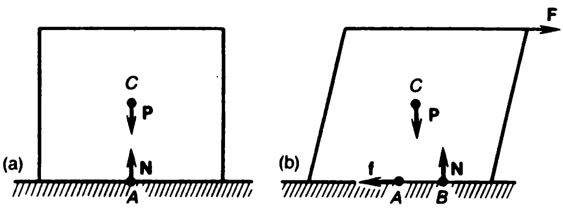


Fig. 462.
(*a*) A rectangular bar is under the action of two mutually balanced forces **P** and **N**. (*b*) The bar is acted upon by two mutually balanced force couples, as a result of which it is distorted.

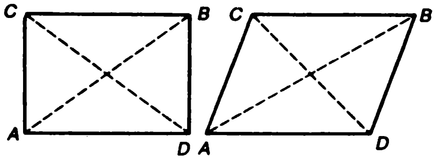


Fig. 463.
Shear is accompanied by elongation along AB and compression along CD.

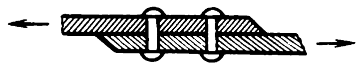


Fig. 464.
As rivetted steel sheets are stretched, the rivets experience shear.

tion of the couple of forces **F** and **f**. Since the bar is at rest, the couple of forces **F** and **f** is balanced by the couple **P** and **N**. The action of these couples causes the distortion of the bar, and the shape of its cross section changes from a rectangle to a parallelogram.

Obviously, the same type of deformation will occur in any right parallelepiped mentally isolated in the body under consideration. The deformation in which a right parallelepiped isolated in a body is converted to an oblique parallelepiped whose volume is equal to that of the undeformed parallelepiped is known as *shear*. Figure 463 shows that shear is always accompanied both with extension and compression (diagonal AB elongates, while diagonal CD contracts).

Shear is a widespread type of deformation. Above all, shear is observed in rubbing solids both during static and sliding friction. If, for example, a body is pulled over the floor, the body and the floor are in the state of shear. Shear is also observed in rivets connecting two steel sheets (Fig. 464) subjected to extension. A very important case of shear is the deformation of a medium through which so-called transverse waves propagate (these waves will be considered in the part of Vol. 3 of this book, devoted to oscillations and waves).

If a shear becomes a plastic deformation, the layers of a body are displaced relative to one another. In this respect, plastic shear is similar to the flow of a liquid: when the liquid flows, its layers are permanently displaced relative to one another. It should be recalled (see Sec. 15.1) that elastic shear is the property that distinguishes the solid state from the liquid state: in the liquid state, elastic shear is impossible.

16.6. Torsion

Torsion is a special case of shear. Torsion is the deformation occurring in a rod subjected to two opposite torques applied to its ends. In order to visualise torsion, hold the ends of a rubber rod with a line drawn along its generatrix by two hands (Fig. 465) and rotate the ends in the opposite directions. The rod will be subjected to torsion, and the line along the generatrix will become a spiral. If we keep one end of the rod at rest and twist the other end, the angle of rotation of a cross section will be the larger, the fur-

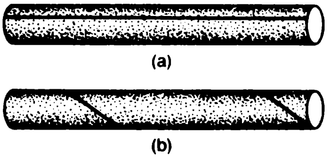


Fig. 465.
(*a*) An undeformed rubber rod. (*b*) The same rod under torsion.

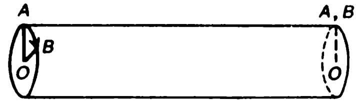


Fig. 466.
If the right end of the rod is fixed, while the radius *OA* at the left end has acquired the position *OB*, then *AOB* is the angle of torsion.

ther is the cross section from the fixed end. The angle of rotation of the extreme cross section is called the ***angle of torsion*** (Fig. 466).

Torsion is a widespread type of deformation. All bodies which transmit the torque from an engine to a machine (cardan shaft of a motor car, the shaft of a ship screw, and so on) are in the state of torsion. The handle of a screw-driver which transmits the torque from the hand to the screw experiences torsion. The stretching of a cylindrical spring is also torsion. Indeed, let us consider two close cross sections S_1 and S_2 of the spring (Fig. 467). It can be seen from the figure that while extending the spring, we make section S_1 rotate clockwise, and section S_2 counterclockwise, i. e. the torsion of the wire of which the spring is made is observed.

The angle of torsion increases with the torques causing torsion. For a given torque, the angle of torsion depends on the material of which a body being deformed is made and on the shape and size of the body. For cylindrical rods, ***the angle of torsion is directly proportional to the rod length and inversely proportional to the fourth power of its diameter***. This means that a small change in the diameter causes a large change in the angle of torsion provided that the torque has not been changed. This property is used in designing physical instruments where the largest possible angles of torsion must be obtained at extremely small torques (as, for example, in galvanometers). Suspending rotating parts by wires having a diameter of several micrometers, an astonishing sensitivity of instruments is attained.

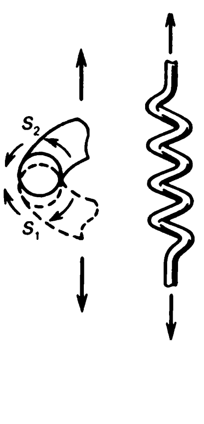


Fig. 467.
Stretching of a spring is equivalent to torsion of the wire of which it is made.

? **16.6.1.** It is necessary to replace the wire in a physical instrument from which a rotating part is suspended by twice as long wire made of the same material. What must be the diameter of the new wire to preserve the same angle of torsion for the same torque if the diameter of the wire to be replaced is 0.3 mm?

16.7. Bending

Let us arrange a ruler in the horizontal position, fixing one of its ends (Fig. 468). By applying a certain force to its free end, we observe the bending of the ruler in the direction of the acting force. We can also place the ruler on two supports and bend it by pressing at the middle between the supports (Fig. 469). Bending is one of the most frequently encountered deformations in engineering. The rails of a railroad, joists in buildings and various levers are subjected to bending.

Bending is the deformation that can be reduced to extensions and compressions which are different in different parts of a body. This can be demonstrated by the following experiment. Let us stick several parallel knitting needles into a rubber band or tube (Fig. 470). Bending the band, we see from the arrangement of the needles that some of its layers (like layer MM) are subjected to extension, while other layers (like layer NN) undergo compression. A certain layer at the middle does not change its length (neutral layer).

The measure of deformation in bending is the displacement of the end of the ruler (Fig. 468) or its middle (Fig. 469). This displacement is known as the *sag*, or *bending deflection*.

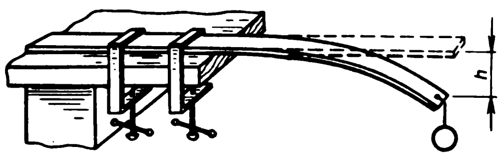


Fig. 468.
Bending: h is the sag, or bending deflection.

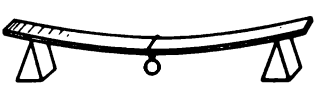


Fig. 469.
Another example of bending.

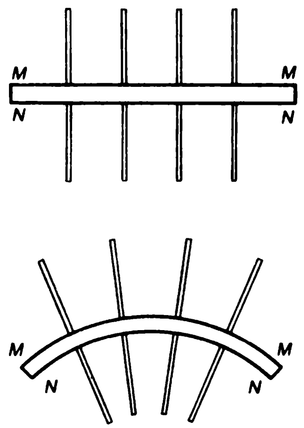


Fig. 470.
The arrangement of the needles indicates that the upper surface of the body being bent is stretched while the lower surface is compressed.

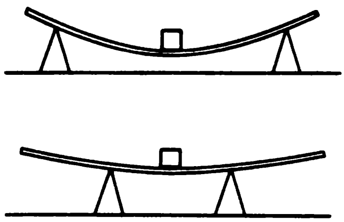


Fig. 471.
Dependence of the sag on the length of the beam.

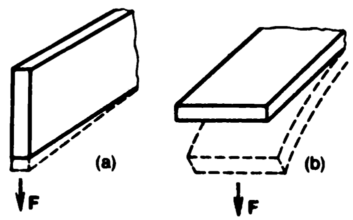


Fig. 472.
Dependence of the sag on the shape of the beam cross section.

Let us find the factors determining the sag of a beam. Instead of beam, we take a ruler, put it on supports (which are initially far from each other and then brought closer) and place a load on it (Fig. 471). It can be seen that as the length of the part of the ruler between the supports decreases, the sag becomes considerably smaller. If we take a wider ruler (for the same thickness and separation of the supports), the same loading will cause a smaller sag. An increase in the thickness of the ruler leads to a considerable decrease in the sag.

A change in the thickness of a beam having a rectangular cross section affects the sag much stronger than a change in its width. To verify this, it is sufficient to try and bend a ruler placed edgewise (Fig. 472*a*) and flatwise (Fig. 472*b*). In the former case, the thickness of the ruler becomes obviously larger as compared to the latter case in inverse proportion to the ratio of its widths in the two cases. However, it is much more difficult to bend the ruler in the former case. This can be easily explained. In the former case, the extension of the upper part and compression of the lower part turn out to be considerably larger for the same sag.

An analysis shows that the sag of a beam having a rectangular cross section is directly proportional to the load and to the third power of the beam length, and is inversely proportional to the beam width and to the third power of the beam thickness. This conclusion has been confirmed in experiments.

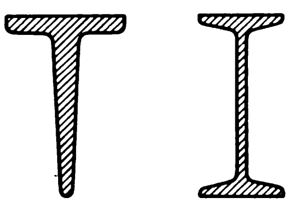


Fig. 473.
T-beam and I-beam.

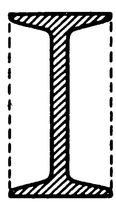


Fig. 474.
The removal of the unhatched part of a beam with a rectangular cross section affects its strength only slightly.

In engineering, beams with cross sections shown in Fig. 473 (*T-beams* and *I-beams*) are often used. A rail is an example of an I-beam. Essentially, an I-beam is a wide rectangular beam from which a part of the middle layer has been removed (Fig. 474). The middle layer is stretched and compressed to a smaller extent and hence resists bending less. I-beams make it possible to save the material and to lighten the beam almost without deteriorating its structural quality. The same goal is attained by using tubes instead of rods (like in the bicycle frame).

? **16.7.1.** Put a load on a notebook placed on two supports, then roll the notebook up, place it on the supports and put the same load. Test the difference in sagging in these two cases.

16.7.2. List examples of tubular structures encountered in engineering and in living nature.

16.7.3. The width of a rectangular beam is thrice as large as its thickness (see Fig. 472). What is the ratio of sags in cases (a) and (b)?

16.8. Strength

None of bodies can be deformed (say, stretched) unlimitedly. Ultimately it breaks. For each material, the maximum load per unit cross-sectional area can be indicated, which can be borne without breaking (*breaking load*). The larger the breaking load, the stronger the material. The ability of an article to withstand breaking depends not only on the quality of the material but also on the shape of the article and the type of the load. For instance, it is easier to break a rod by longitudinal compression than by extension since in the former case it may be bent and breaks while in the latter case it has to be ruptured. Another example of the role of the type of action has been considered in hydrostatics (see Sec. 7.22), where it has been shown that it is easier to flatten a hollow sphere (or submarine) by an external pressure than to break it by the internal pressure.

Breaking load strongly depends on the quality of a material and the method of its treatment. For this reason, only approximate values of breaking load can be determined (Table 17). Breaking load considerably depends on the thermal and the mechanical treatment of the material, and for composite materials, on their content (steel and glass).

Table 17. Breaking Loads for Some Substances under Extension

Substance	Breaking load, 10^8 Pa	Substance	Breaking load, 10^8 Pa
Copper	2-5	Pine wood	0.2-0.8
Glass	0.3-0.9	Steel	4-14
Lead	0.1-0.2		

? 16.8.1. What maximum load can be borne by a steel rope having a cross-sectional area of 12 mm^2? The breaking load should be taken as 6×10^8 Pa.

16.8.2. What is the maximum length of a lead wire which does not break after having been suspended at the upper end? The density of lead is 11.3×10^3 kg/m^3. Assume that the breaking load is 2×10^7 Pa.

16.9. Hardness

In addition to strength, the materials are distinguished in engineering by their *hardness*. The harder of two material is the one that leaves scratches on the other. Let us pass a sharp edge of glass over the surface of a copper plate. We obtain a scratch. On the other hand, no scratch will be formed if we pass an edge of the copper plate over the surface of glass. Consequently, glass is harder than copper. Cutting tools and drills for cutting metals must be harder than the material being treated. For copper, brass and iron, hardened steel cutting tools can be used. In modern engineering, so-called diamond-substitute alloys are used for manufacturing cutting tools and drills.

Diamond-substitute alloys consist of tiny grains of tungsten or titanium carbides and cobalt. They are prepared by pressing carbide powders at a high temperature which, however, is too low for fusion. As a result, carbide grains preserve their exclusive hardness. Cutting tools manufactured of such alloys (Fig. 475) retain their cutting ability at temperatures reaching 700-800 °C. Since it was the loss of cutting properties at high temperatures that limited the speed of cutting metals (in operation at a high speed, cutting tools are known to be strongly heated), the application of diamond-substitute alloys clearly made it possible to increase this speed. Another type of diamond-substitute cutting tools, viz. *cermet tools*, were created, in which a constituent element is alumina obtained from bauxite. Cermet tools preserve their cutting ability up to 1100 °C and higher. This made it possible to increase the speed of cutting metals to 50 m/s, which could not be dreamt of earlier. The hardest of natural materials is diamond. Alloys obtained in engineering at present approach diamond in their hardness.

? 16.9.1. Test the hardness of available materials (steel, lead, glass, wood, finger nail, etc.) and arrange them in the order of decreasing hardness.

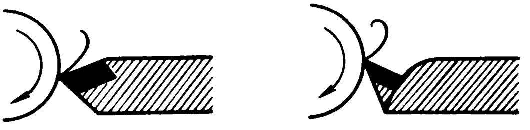


Fig. 475.
Processing of a metal shaft with a hard-faced cutting tool with the plate made of a diamond-substitute alloy (blackened).

16.10. What Occurs during Deformations of Bodies?

X-ray analysis of the structure of bodies (see Sec. 15.4) has revealed that during elastic deformations of a crystal, its lattice is only slightly distorted. For example, the cells of the lattice shown in Fig. 444 change their shape upon deformation from cubic to that of slightly oblique parallelepipeds. After the removal of deforming forces, the lattice acquires the original shape. These temporary distortions of lattices in polycrystalline bodies can be of different types. Elastic deformations in amorphous bodies also involve only slight shift of equilibrium positions of the molecules. A different change in the structure is observed in rubber-like bodies subjected to an elastic deformation (this was duscussed in Sec. 15.10). This explains a very large difference in the values of elastic strains observed, for example, in a rubber band and in a steel wire.

During plastic deformations, the displacement of molecules can exceed many times their separation. Plastic deformations in single crystals are associated with the slipping of individual layers of the lattice relative to one another. In every crystal, there exist directions in which slipping of lattice layers occurs most easily. It was mentioned above (see Sec. 15.2) that an ice crystal resembles in its mechanical properties a pile of glass plates connected by a not completely solidified glue. The same applies to other crystals. Figure 476 shows a zinc crystal subjected to extension. The traces of layer slip are clearly seen in the crystal. It was established that the slip of layers never starts at the same time throughout a crystal. It is initiated in a certain region where the crystal lattice is weakened for some reason or other (see Sec. 15.4), and then gradually spreads to include other regions.

In polycrystals, the slip of lattice layers is also possible for small crystals constituting a polycrystal. However, since the directions of the easiest slip are generally different in different crystals, the emergence of slip in such bodies is hampered in comparison with single crystals. This effect is the stronger, the smaller the size of the crystals constituting a polycrystalline body. For this reason, plastic deformation emerges in fine-grained bodies at a stronger deforming force than in coarse-grained ones.

In addition to the circumstance mentioned above, the situation is complicated by the presence of interlayers between crystals, whose mechanical properties differ from those of the crystals. As to amorphous bodies, the molecular pattern of plastic deformation for them is similar to the molecular pattern of a calm (laminar) flow of a fluid (see Sec. 9.15). It was mentioned earlier that the amorphous state can be treated as the liquid state with a very high viscosity.

16.11. Energy Variation during Deformations of Bodies

A load stretching a wire moves down, and hence the force of gravity does work. At the expense of this work, the energy of the body being deformed increases as it goes over from the unstressed to a stressed state. Thus, the

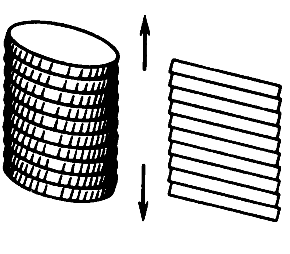


Fig. 476.
A zinc single crystal subjected to extension (schematic diagram).

internal energy of a body increases during deformation. The increase in the internal energy is associated with an increase in the potential energy determined by the relative position of molecules of the body. If a deformation is elastic, after the removal of strain this additional energy is spent on the work done by elastic forces. During elastic deformations, solids are not heated noticeably. In this respect, they differ from gases which are heated as a result of compression (see Sec. 13.5). During plastic deformation, however, solids are heated considerably. This increase in the temperature, i. e. in the kinetic energy of molecules, is a manifestation of an increase in the internal energy of a plastically deformed body. Naturally, in this case too the increase in the internal energy is obtained at the expense of the work done by the forces responsible for the deformation. This can be illustrated by the heating of a repeatedly bent wire or a piece of lead flattened by a hammer, considered in Sec. 11.1.

It follows from the material considered in this chapter that for practical utilisation of substances in construction engineering and in designing of various machines and mechanisms, it is extremely important to know the response of a material to the action of external forces. Investigations in solid-state physics in recent years have made it possible to clarify a large number of questions concerning the physical nature of phenomena occurring in materials.

Chapter 17

Properties of Vapours

17.1. Introduction

Mutual conversions of liquids and vapours occur everywhere in nature and in industrial plants. A liquid is transformed into invisible vapour, i.e. goes over to the gaseous state (*evaporation*). Conversely, sometimes drops of liquid formed from vapour appear on solid surfaces (*condensation*). Mutual conversions of water and water vapour are encountered in nature and engineering most frequently. Water vapour is formed not only over enormous expanses of water of the Earth but on the land as well. Water continually evaporates from the surface of the soil, from leaves of plants, skin and lungs of human beings and animals, and so on.

Examining evaporation, we can easily see that different liquids evaporate differently at the same temperature: ether, petrol and similar "volatile" liquids evaporate rapidly, water evaporates somewhat slower, while oil and mercury evaporate so slowly that evaporation cannot be noticed without accurate measurements. However, evaporation still occurs, and for this reason a vessel with mercury should not be kept open in the room since mercury vapour is poisonous. *All liquids evaporate without any exception.*

Not only liquids but also *solids* evaporate, some of them rapidly and others very slowly. It is well known that wet and frozen linen is still gradually dried outdoors in winter. Not only liquids but also some solids like naphthalene are odorous, viz. form vapours perceived by olfactory organs. In Sec. 15.5, we considered the experiment with heating iodine which evaporates without going over to the liquid state.

? **17.1.1.** Is the Torricellian vacuum empty?

17.2. Saturated and Unsaturated Vapour

After rain, puddles are dried in wind more rapidly than at the same temperature in calm weather. This shows that the removal of formed vapour facilitates the evaporation of a liquid. If vapour is not removed at all (for example, if we close a bottle with a cork), evaporation soon ceases.

Since in this case the liquid does not evaporate and vapour does not condense into liquid, the liquid and its vapour are said to be *in equilibrium*.[1] A vapour in equilibrium with liquid is said to be ***saturated***. This term expresses the fact that a larger amount of vapour cannot be contained in a given volume at a given temperature.

A bottle with a liquid contains, besides vapour, some air above the liquid. It is not difficult, however, to do so that above a liquid there is only its vapour almost without any admixtures. For this purpose, the space above the liquid should be evacuated with the help of a pump or the gas should be removed by a prolonged boiling of the liquid during which the vapour displaces gases. Investigating the behaviour of a vapour in the space from which all alien gases are removed, we gain important information about its properties. Such an investigation can, for example, be carried out as follows.

A round-bottomed flask *1* closed with a rubber cork is connected, as shown in Fig. 477, with a glass tube *2* whose end is immersed in a vessel with mercury. The air is pumped out to the highest possible extent through another tube *3* supplied with a tap. As a result, mercury rises in tube *2* under the action of the atmospheric pressure. Under these conditions, mercury vapour is formed in very small amounts such that its presence can be neglected.

Ether poured in funnel *4* is introduced with the help of tap *5* carefully drop-by-drop into flask *1*. A few first ether drops evaporate instantaneously, and mercury in tube *2* is rapidly lowered. The flask contains ***unsaturated*** ether vapour in this case. As the amount of evaporated ether increases, the vapour density (and hence its pressure) becomes higher, like the pressure of any gas with increasing density. Although unsaturated

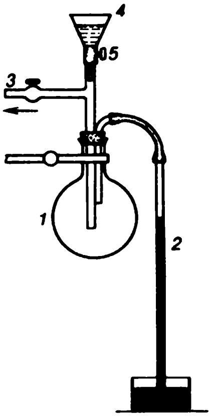


Fig. 477.
The first ether drops falling into flask *1* evaporate, and mercury is rapidly lowered in tube *2*. When saturation sets in, ether drops falling into the flask do not evaporate, and the level of mercury remains unchanged.

[1] Here the mobile, or dynamic equilibrium is meant, which was defined in Sec. 14.13.

vapour does not obey exactly Boyle's and Gay-Lussac's laws, it generally possesses all properties of a gas. If, however, we continue to supply ether to flask *1*, it will be seen that mercury in tube *2* stops lowering, and added ether does not evaporate any longer: *saturation* is attained. The density of vapour and its pressure will remain unchanged irrespective of the amount of ether added. It should be noted that the temperature should not be changed during the experiment.[2]

If we repeat this experiment with another liquid, say, alcohol, we see that the saturated vapour pressure differs from that for ether. At 20 °C, the saturated vapour pressure is about 440 mm Hg for ether and about 44 mm Hg for alcohol.

Thus, *the saturated vapour pressure and density at a constant temperature is a constant quantity which is different for different liquids.*

17.3. Variation of Volume of Liquid and Saturated Vapour

Let us first clarify the meaning of the statement: *the pressure of a saturated vapour is constant at invariable temperature*. For this purpose, we consider two experiments.

1. Vessel *1* (Fig. 478) is closed with a rubber cork with funnel *2* having a narrow end *3*. The upper part of the funnel can be closed with a rubber cork *4*. We pour water in the funnel so that about half its volume is filled and immediately close it with the cork. For a certain time, water will drip from funnel *2* to vessel *1*, but then the dripping ceases. This can be explained as follows. As water flows from the funnel to the vessel, the volume of air in funnel *2* increases, while its pressure decreases (Boyle's law). At the same time, the volume of air in vessel *1* decreases and its pressure increases. The pressure difference in vessel *1* and funnel *2* balances the pressure of the liquid, and its flow is terminated.

2. Let us now consider a special device (Fig. 479). Cylindrical vessel *1* is connected with vessel *2* by a narrow tube. Vessel *2* is sealed. Before sealing, air has been evacuated from vessels *1* and *2*, and the device only contains water and its vapour. How does water flow now? When we arrange the device in the vertical position, all the water flows from vessel *2* to vessel *1* through the narrow opening. This means that as water flows, vapour pressure does not change either in vessel *1* or vessel *2* although in this case the volume of the vapour decreases in vessel *1* and increases in vessel *2*. But this obviously means that as the level of the water in vessel *2* is lowered and the volume of vapour increases, the water evaporates to such an extent that both vapour density and pressure remain unchanged. Conversely, as the

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[2] For this purpose, flask *1* should be immersed in a large vessel containing water at room temperature.

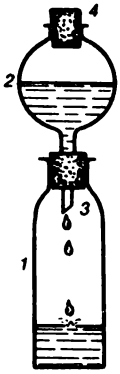


Fig. 478.
Vessel *1* and funnel *2* contain water and air; water soon stops flowing from the funnel to the vessel.

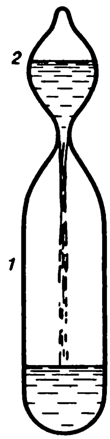


Fig. 479.
Vessels *1* and *2* contain water and its vapour and no air. The entire water flows from vessel *2* to vessel *1*.

volume free of liquid decreases in vessel *1*, vapour condenses all the time so that its density and pressure also remain constant.

The following instructive experiment can also be made with the device shown in Fig. 479. If we abruptly rise the device upwards, the water column in vessel *1* rises, and a space filled with vapour is formed under it. Then the water column drops down. As soon as it touches the bottom, a sharp noise will be heard as if a solid body has dropped to the bottom. This sound is produced since the water column moving in the vessel does not meet any resistance exerted by the vapour. Indeed, two equal and opposite forces act on both surfaces of the column, i.e. the forces of pressure of the saturated vapour which are the same for different positions of the column in the tube. As water column moves, vapour condenses into liquid on its one side and liquid evaporates on the other side, the vapour pressure always remaining the same.

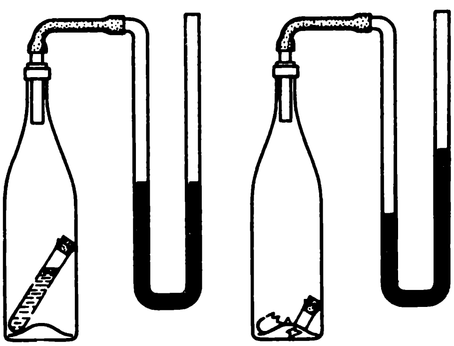
Let us repeat this experiment with the device shown in Fig. 478 which contains some air in addition to vapour. No sound will be heard since air offers to the liquid a resistance which grows with the extent to which the air is compressed (the so-called "elastic" resistance). Like the case when the force exerted by a string increases with its compression (in accordance with Hooke's law), here too the force of pressure exerted by the air increases with decreasing volume (according to Boyle's law) and decelerates the motion of water. Unlike air, saturated vapour has a constant pressure which does not depend on its volume, and hence cannot offer an "elastic" resistance.

This experiment clarifies why sharp shocks which sometimes cause a damage appear in a liquid when vapour bubbles burst in it. An example is a ship screw propeller in whose blades cavities are often formed due to repeated blows of water accompanying the burst of bubbles generated on the surface of the blades during the rotation of the screw propeller (the so-called *cavitation*). If the shape of the screw propeller is not designed in a proper way, cavitation can break it after several hours of operation.

17.4. Dalton's Law for Vapours

Let us place a sealed test tube with ether into a bottle and close the bottle with a cork having a glass tube connected to a mercury manometer (Fig. 480*a*). The bottle contains atmospheric air, and the level of mercury in both arms of the manometer is the same. Let us abruptly shake the bottle so that the test tube breaks (Fig. 480*b*). It can be seen that mercury in the manometer rises. In several minutes, the difference between the levels will be established, which is equal to the drop in the mercury level in the experiment shown in Fig. 477. The change in the mercury level indicates that the pressure of ether vapour is added to the air pressure. Consequently, the equilibrium between ether and its vapour in the presence of air is established at the same pressure of ether vapour as in the space from which air has been pumped out. True, in the presence of air ether evaporates at a lower rate than without it, and hence the equilibrium sets in later.

The fact that the pressure of ether vapour is the same in the presence of air and without it leads to the conclusion that the amount of ether evaporating in a certain volume is the same in the two cases provided that the temperature is the same.



(a) (b)

Fig. 480.
(*a*) A bottle contains a sealed test tube with ether. The manometer indicates that the air pressure in the bottle is equal to the atmospheric pressure. (*b*) The test tube is broken, and saturated ether vapour is mixed with air. The manometer indicates an increase in pressure.

Thus, *the pressure of a mixture of a gas and a vapour which is in equilibrium with the liquid is equal to the sum of the pressures of the mixture components taken alone.* This is Dalton's law for vapour.

? **17.4.1.** If we repeat the experiment shown in Fig. 480 without removing the remnants of ether from the bottle, after breaking the second test tube with ether the manometer does not indicate any increase in pressure. Why?

17.5. Molecular Pattern of Evaporation

It should be recalled that molecules of a liquid move with various velocities. In order that a molecule from the surface layer be able to leave the limits of the liquid, its kinetic energy should be higher than the work which must be done against cohesive forces pulling the molecule into the liquid. For this reason, only molecules having a sufficient velocity at a given moment can escape from the surface layer of the liquid. Here they collide with other molecules, change the direction of their motion and in a certain time may reach again the surface layer and penetrate into the liquid. Thus, the molecules continually escape from the liquid and return to it. If the number of molecules leaving the liquid is larger than the number of molecules returning to it, the liquid evaporates. If, on the contrary, the larger number of molecules return to the liquid than escape from it, the condensation of vapour takes place. Finally, if the number of escaping molecules is equal to the number of returning molecules on the average, dynamic equilibrium between the liquid and its vapour is observed, and the vapour becomes saturated.

Why is equilibrium attained for different liquids at different vapour pressures, and hence at different numbers of molecules per unit volume? The reason behind this lies in the difference of cohesive forces. For some liquids (like mercury), cohesive forces are very strong, and only the fastest molecules have a chance to escape from a liquid. Then only a small number of molecules leave the liquid per unit time. Therefore, in order to attain the equilibrium state, i.e. to make the same number of molecules return to the liquid, a small density of mercury vapour is sufficient. For other liquids (like ether), cohesive forces are weak, and a considerable number of molecules can leave the liquid at the same temperature. Therefore, the equilibrium state is attained only at a considerable density of ether vapour above the surface of liquid ether.

It was shown in Chap. 14 that cohesive forces of molecules determine the surface tension of a liquid. For strong cohesive forces, surface tension is large. Consequently, *the larger the surface tension, the less volatile the liquid* and the lower the saturated vapour pressure.

Even for nonvolatile liquids, the number of molecules exchanged per unit time by the liquid and the saturated vapour in contact with it is unbelievingly large. This is manifested by an extremely high rate of evaporation of a liquid if for some reason or other the formed vapour does not return to the liquid. The high rate at which vapour disappears in the experiment with water striking against the bottom of the vessel (see Sec. 17.3) is another evidence of this fact. According to estimates, 10^{21} molecules evaporate during a second from 1 m^2 of the surface of water at room temperature.

17.6. Temperature Dependence of Saturated Vapour Pressure

Till now, we have analysed evaporation and condensation at a constant temperature. Let us now consider the effect of temperature. It can easily be seen that the effect of temperature is strong. In a hot day or near a stove, things are dried much faster than in frost. This means that the evaporation of a warm liquid is more intense than that of a cold liquid. This is not surprising. In a warm liquid, there is a larger number of molecules having a sufficiently high velocity to overcome cohesive forces and escape from the liquid. Therefore, with increasing temperature the saturated vapour pressure increases along with the rate of evaporation.

An increase in vapour pressure can easily be observed with the help of the instrument described in Sec. 17.4. If we immerse in hot water the bottle with the test tube containing ether, the manometer will indicate an increase in pressure. If we immerse the same bottle in cold water or in a mixture of snow and salt (see Sec. 15.13), a decrease in pressure will be observed.

Thus, the saturated vapour pressure strongly depends on temperature. Table 18 contains the values of saturated vapour pressure for water and mercury at different temperatures. Pay attention to an extremely low pressure of mercury vapour at room temperature. It should be recalled that this pressure is neglected while taking barometer readings.

Table 18. Saturated Vapour Pressure for Water and Mercury at Various Temperatures (in mm Hg)

Temperature, °C	Water	Mercury	Temperature, °C	Water	Mercury
0	4.58	0.00021	100	760	0.28
20	17.5	0.0013	120	1 489	0.76
40	55.3	0.0065	140	2 711	1.85
60	149	0.026	200	11 660	17.2
80	355	0.092	300	64 450	245
90	526	0.16	374	165 530	1100

The curve representing the temperature dependence of saturated vapour pressure for water (Fig. 481) shows that the increment in pressure corresponding to a temperature increment of 1 K increases with

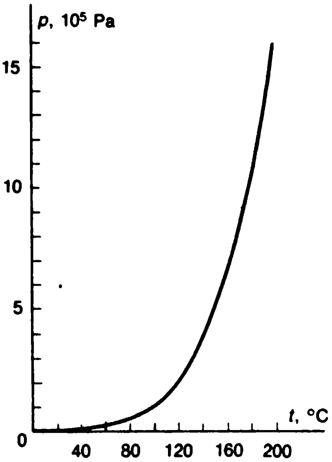


Fig. 481.
The saturated water vapour pressure as a function of temperature.

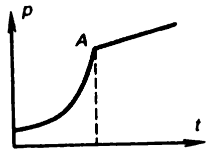


Fig. 482.
To Exercise 17.6.3.

temperature. In this respect saturated vapour differs from a gas whose pressure uniformly increases as a result of heating through 1 K both for low and high temperatures (by 1/273 of its pressure at 0 °C). This difference becomes quite clear if we recall that when gases are heated at constant volume, only the velocity of their molecules changes. As was mentioned above, during the heating of a liquid-vapour system not only the velocity of molecules but also their number density increases, i.e. at a higher temperature we have vapour at a higher density.

? **17.6.1.** Why does a gas thermometer (see Sec. 13.15) give correct readings only with a perfectly dry gas?

17.6.2. Suppose that a closed vessel contains some air in addition to a liquid and its vapour. How will this affect the change in pressure with increasing temperature?

17.6.3. The graph in Fig. 482 shows the change in vapour pressure with increasing temperature in a closed vessel. What conclusion can be drawn about the processes of evaporation in the vessel?

17.7. Boiling

Let us heat a glass vessel with cold water on a burner and watch the process. The bottom and walls of the vessel will soon be covered with small bubbles (their origin was explained in Sec. 14.13). It is well known that these bubbles contain air and water vapour. They appear in the regions of the vessel walls where wetting is not complete (there can be traces of fat or small cracks on the walls).

Observing a bubble at a constant temperature, we can see that it retains its volume. This means that the internal pressure in the bubble is balanced by the external pressure on its surface. Since the bubble contains air whose amount should be considered constant, this equilibrium is stable. Indeed, if

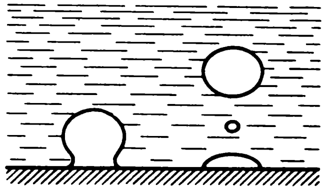


Fig. 483.
Gas bubbles stuck to the bottom of the vessel with a liquid and separated from it.

for some reason or other the bubble has expanded, according to Boyle's law, the pressure in it drops, and the external pressure which remains unchanged compresses the bubble to its initial volume. Arguing in the same way, it can easily be shown that a bubble accidentally decreasing in volume would immediately expand to its original volume. As the temperature rises, the bubble gradually expands so that the sum of the pressures of air and vapour in it remains equal to the external pressure. However, when the bubble becomes large enough, the buoyancy of water separates it from the wall in the same way as a large drop of water is separated from the roof (see Fig. 372). During the separation, an air bridge is formed (Fig. 483), which becomes narrower and narrower, and ultimately the bubble is separated completely, leaving a small amount of air near the wall, from which a new bubble will be formed after a certain time.

When separated bubbles rise to the surface, their volume is reduced. Why does this occur? These bubbles contain water vapour and a small amount of air. When a bubble reaches the colder upper layers of water, a considerable amount of water vapour condenses, and the bubble contracts. This alternating increase and decrease in the volume of bubbles is accompanied by a noise: water coming to the boil begins to "murmur". Ultimately, the entire mass of water is heated. At this stage, the volume of rising bubbles does not decrease any longer, and the bubbles burst at the surface ejecting steam to the surrounding space. The "murmuring" now changes to a loud bubbling sound, and the water is said to boil. A thermometer held in steam over boiling water indicates the same temperature (100 °C) over the entire surface of boiling water.

Obviously, the pressure of vapour formed within bubbles at the bottom during boiling is such that the bubbles can expand, overcoming both the atmospheric pressure exerted on the free surface of water and the pressure of the water column. We arrive at the conclusion that *boiling occurs at a temperature for which the pressure of the saturated vapour of a liquid is equal to the external pressure.* The temperature of vapour of a boiling liquid is called the *boiling point.*[3]

It is clear from the above arguments that the boiling point must depend on the external pressure. This can be easily verified. Let us place a glass

[3] The liquid itself usually has a slightly higher temperature. — *Eds.*

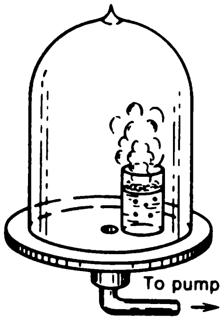


Fig. 484.
When air is pumped out of the bell, water in a glass, which has the temperature much lower than 100 °C, starts to boil.

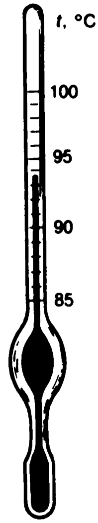


Fig. 485.
Hypsometer.

with warm water under the bell of an air pump. By pumping air out, we can make water boil at a temperature considerably lower than 100 °C (Fig. 484). On the contrary, the boiling point rises when the external pressure increases. For example, in boilers water is heated under a pressure of several atmospheres. The boiling point in these cases considerably exceeds 100 °C. At a pressure of about 15 atm, the boiling point of water is close to 200 °C. When we speak of the boiling point of a liquid without indicating pressure, we always mean the boiling point at normal pressure (760 mm Hg).

The dependence of boiling point on pressure provides a new method for measuring the atmospheric pressure. Having measured the boiling point of water, we can judge about the atmospheric pressure using the tables of vapour pressure at different temperatures. If, for example, the boiling point of water measured in the mountains is 90 °C, we may conclude (see Table 18) that the air pressure at that level is equal to 526 mm Hg. Special thermometers designed for such measurements are called *hypsometers*. They permit measuring temperature at about 100 °C to a high degree of accuracy (Fig. 485).

Boiling points for different liquids (at normal pressure) have quite different values. This can be seen from Table 19.

The difference in boiling points of different substances is widely used in engineering, for example, for separating petroleum products. When petroleum is heated, the most valuable volatile components (like petrol) evaporate first and can thus be separated from "heavy" residues (oil)

Table 19. Boiling Points of Some Liquids at 760 mm Hg

Liquid	Boiling point, °C	Liquid	Boiling point, °C
Liquid helium	−269	Alcohol	78
Liquid hydrogen	−253	Water	100
Liquid oxygen	−183	Mercury	357
Liquid nitrogen	−196	Molten zinc	906
Chlorine	−34	Molten iron	2880
Ether	35		

The difference in boiling point of substances is due to the difference in the vapour pressures at the same temperature. It was shown earlier that the pressure of ether vapour at room temperature exceeds half the atmospheric pressure. In order to raise ether vapour pressure to the atmospheric pressure, a slight elevation of temperature (to 35 °C) is sufficient. A different situation takes place for mercury which has negligible vapour pressure at room temperature. The pressure of mercury vapour becomes equal to the atmospheric pressure only at a very high temperature (357 °C).

?

17.7.1. Where is boiling water hotter: at sea level, in mountains or in a deep mine?
17.7.2. A temperature higher than 100 °C is required for some industrial processes i food industry (say, for boiling beetroot). How can it be attained?
17.7.3. Using Table 18, determine the highest temperature of water at pressures of 2 and 0.2 atm.
17.7.4. Figure 486 shows an *autoclave* (a device used in chemical engineering for processes which require a higher temperature than the boiling point of a liquid contained in it). It is a strong boiler with a pressure gauge *1*, closely sealed with a cover so that vapour can escape only through a safety valve *2*. What will be the temperature of water heated in such a boiler, if the base area of the valve is 0.75 cm^2, the distance between hinge *3* and valve *2* is 6.5 cm, and from the hinge to load *4* is 18 cm? The mass of the load is 1 kg. The mass of the lever should be neglected.

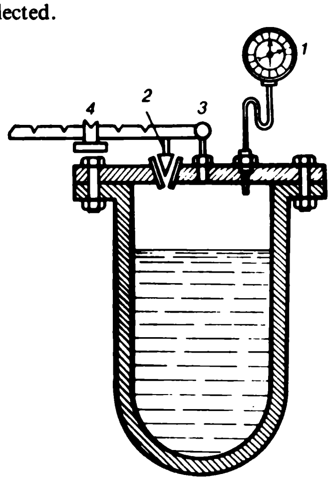


Fig. 486.
To Exercise 17.7.4.

Fig. 487.
Geyser.

17.7.5. Try to boil water in a narrow test tube filled to the brim by heating it at the bottom. Why do bubbles eject water from the test tube?

Remark. Processes of this kind occur on a large scale in nature in so-called *geysers* (for example, on Kamchatka in the USSR and in Iceland). A geyser is a natural fountain ejecting hot water at regular intervals from a narrow vertical vent in the ground (Fig. 487). Vapour is formed at a depth of several tens of meters. At such a depth of a reservoir, the pressure can exceed several atmospheres and water can be heated to a temperature much higher than 100 °C. When vapour bubbles are formed underground, a part of water flows out, the pressure decreases, and vaporisation of superheated water becomes so intense that the remaining water is forced to a large height.

17.7.6. Boil water in a round-bottomed flask and close it with a cork. Turn the flask upside down. If a small amount of snow is now put on the bottom or it is sprayed with cold water, water in the flask will boil. Explain the phenomenon.

17.8. Specific Latent Heat of Vaporisation

In order to sustain the boiling of water (or some other liquid), heat must be continually supplied to it, for example, from a burner. In this process, the temperature of the vessel and water does not increase, but a certain amount of steam is formed per unit time. Hence the conclusion that heat inflow is needed for converting water into steam as in the case when a crystal (ice) is transformed into liquid (see Sec. 15.7). The amount of heat required for the conversion of a unit mass of a liquid into its vapour at the same temperature is known as *specific latent heat of vaporisation* for a given liquid. It is measured in joules per kilogram (J/kg).

It can be easily seen that when a vapour condenses into liquid, the same amount of heat should be liberated. Indeed, let us immerse the end of a tube connected to a boiler into a glass with water (Fig. 488). In a certain time after the beginning of heating, air bubbles will go out of the end of the tube immersed in water.[4] This air increases the temperature of the water only slightly. When the water in the boiler starts to boil, it will be seen that the bubbles going out of the end of the tube do not rise to the surface any longer but rapidly collapse and vanish with a sharp noise. These are steam bubbles having condensed into water. As soon as steam starts to flow out of the boiler instead of air, the temperature of the water begins to rise rapidly. Since the specific heat capacity of steam is almost the same as that of air, it follows from this experiment that such a rapid heating of water is precisely due to the condensation of steam.

As a result of condensation of a unit mass of vapour into the liquid at the same temperature, the amount of heat liberated is equal to the specific latent heat of vaporisation. This could easily be explained by the law of energy conservation. Indeed, otherwise it would be possible to design a device in which a liquid first evaporates and then condenses. The difference between the latent heat of vaporisation and the latent heat of condensation would be equal to the increment of the total energy of all bodies par-

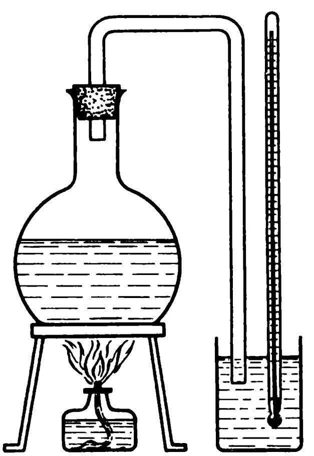


Fig. 488.
As long as air flows from the boiler, the thermometer indicates almost a constant temperature. When steam starts to flow instead of air and is condensed in a glass, the liquid column in the thermometer rapidly rises, indicating an increase in the temperature.

[4] The boiler should be taken in the form of a vessel with a relatively large amount of air so that air bubbles are liberated for a long time.

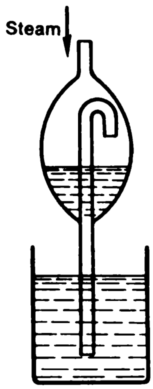


Fig. 489.
Steam dryer is a device for trapping water drops entrained by steam.

ticipating in the process under consideration. But this would be inconsistent with the law of energy conservation.

Specific latent heat of vaporisation can be determined with the help of a calorimeter as it was done for determining the specific latent heat of fusion (see Sec. 15.7). We pour a certain amount of water into the calorimeter and measure its temperature. Then during some time, we introduce into the water the vapour of the liquid under consideration from a boiler, taking measures ensuring the delivery of the vapour alone without liquid drops. For this purpose, the vapour is passed through a steam dryer (Fig. 489). Then we measure the temperature of the water in the calorimeter once again. Having weighed the calorimeter, we can determine the amount of the vapour that has condensed into liquid by an increase in the calorimeter mass.

Using the law of energy conservation, we can write the heat balance equation for this process, which allows us to determine the specific latent heat of vaporisation for water. Suppose that the mass of the water in the calorimeter (including the water equivalent of the calorimeter) is m_1, the mass of steam is m_2, the specific heat capacity of water is c, the initial and final temperatures of the water in the calorimeter are t_0 and t, the boiling point of water is t_b, and the specific latent heat of vaporisation for water is λ. The heat balance equation has the form

$$cm_1(t - t_0) = \lambda m_2 + cm_2(t_b - t),$$

whence

$$\lambda = \frac{cm_1(t - t_0) - cm_2(t_b - t)}{m_2}.$$

The results of measurements of specific latent heats of vaporisation for some liquids under normal pressure are compiled in Table 20. It can be seen that this heat is quite large. The large value of latent heat of vaporisation for water plays an exceptionally important role in nature since vaporisation occurs in nature on a grand scale.

Table 20. Specific Latent Heat of Vaporisation for Some Liquids

Substance	λ, kJ/kg	Substance	λ, kJ/kg
Alcohol (ethanol)	910	Mercury	285
Copper	5420	Tin	3020
Ether	373	Water	2250
Iron	6350		

It should be noted that the values of specific latent heat of vaporisation contained in this table refer to boiling temperature under normal pressure. If, however, a liquid boils or simply evaporates at a different temperature, its specific latent heat of vaporisation has a different value. As the temperature rises, the latent heat of vaporisation of a liquid always becomes smaller. An explanation to this will be presented below.

? **17.8.1.** Determine the amount of heat required for bringing 20 g of water at 14 °C to the boil and vaporising it.

17.8.2. What will be the final temperature if 3 g of steam at 100 °C are injected into a vessel containing 200 g of water at 16 °C? The heat capacity of the vessel should be neglected.

17.9. Cooling during Evaporation

It is well known that one feels colder in wet clothes than in dry clothes, especially in wind. It is also known that by wrapping a water-filled vessel by a wet cloth and taking it outdoors on a hot windy day, we can noticeably cool the water in the vessel. Sometimes in warm countries special vessels with porous walls are used for this purpose. Water slowly oozes through porous walls, keeping them wet. The observations show that evaporation causes the *cooling* of a liquid together with surrounding bodies. In this case, the heat of vaporisation is borrowed from the liquid itself.

A cooling is especially strong if evaporation proceeds at a high rate so that the evaporating liquid has no time to receive heat from the surrounding bodies. Rapid evaporation can be easily realised in volatile liquids. For example, the evaporation of ether or ethyl chloride readily give cooling to below 0 °C (Fig. 490). This is used in surgery when the skin of a patient has to be cooled to make it insensitive to pain. Cooling as a result of evaporation can be also observed in the following experiment. Two glass spheres A and B are connected by a bent glass tube (*cryophorus*, Fig. 491). The spheres contain water and its vapour, the air having been removed from them. Sphere B is placed in a cooling mixture (ice and salt). In this case, the

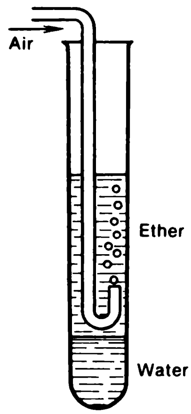


Fig. 490.
Blowing air through the tube, i.e. accelerating the evaporation of ether, we can make water freeze at the bottom of the test tube.

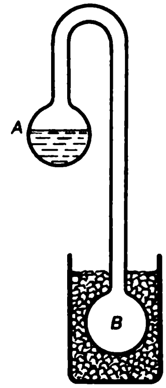


Fig. 491.
When sphere B is cooled, water in sphere A gets frozen.

water in sphere A is frozen. The reason behind this is as follows. The cooling of sphere B causes intense condensation of the vapour in it. As a result, the water in sphere A evaporates and hence is cooled. The temperature drops so sharply that the water in sphere A gets frozen.

Cooling during evaporation and liberation of heat during condensation play an exclusively important role in nature, ensuring moderate climates in seaside regions. It should be noted that the evaporation of sweat from the skin of human beings and animals is the way by which an organism controls the body temperature. In hot weather, the perspiration evaporates, thus cooling the skin.

? **17.9.1.** Why is it difficult to bear heat in rubber clothes?
17.9.2. Why is it easier to bear heat by waving with a fan?
17.9.3. The same amount of water is poured into two vessels of the same size and shape, one made of a metal and the other of porcelain, and left for a short time in the room. Is the temperature of water in the vessels the same?

17.10. The Change in the Internal Energy during a Transition of a Substance from the Liquid State to Vapour

It was found that evaporation of a liquid is possible only with an inflow of heat to the evaporating liquid. Why is this so?

Firstly, the internal energy of a substance increases during evaporation. The internal energy of a saturated vapour is always higher than the internal energy of its liquid. The increase in the internal energy of a substance during evaporation without a change in temperature mainly occurs due to the fact that when a liquid vaporises the average distance between molecules increases. This causes an increase in the mutual potential energy since in

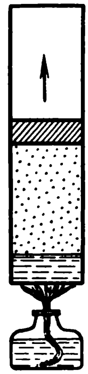


Fig. 492.
The formed vapour raises the piston. Here, the work against the forces of external pressure is done.

order to draw molecules apart to longer distances, work should be done against attractive forces of the molecules.

Besides, work is done against the external pressure since vapour occupies a larger volume than its liquid. As an illustrative example, we consider an experiment in which a liquid evaporates in a cylinder and the vapour formed raises a light piston (Fig. 492) doing thereby work against the atmospheric pressure. This work can be easily calculated. We shall do it for water boiling under normal pressure, and hence at 100 °C. Suppose that the area of the piston is 100 cm^2. Since the normal pressure is 1.013×10^5 Pa, a force of $1.013 \times 10^5 \text{ Pa} \times 0.01 \text{ m}^2 = 1013$ N acts on the piston. If the piston rises by 0.1 m, the work done is $1013 \text{ N} \times 0.1 \text{ m} = 101.3$ J. The volume of the steam formed is $0.01 \times 0.1 = 10^{-3}$ m^3. The steam density at 100 °C is 0.597 kg/m^3, and hence the mass of the steam is 0.597×10^{-3} kg. Consequently, the work done against external pressure during the formation of 1 kg of steam is $101.3/(0.597 \times 10^{-3}) = 170$ kJ.

For the evaporation of 1 kg of water at 100 °C, 2250 kJ of heat (specific latent heat of vaporisation) are required. Calculations show that 170 kJ from this energy are spent on doing work against external pressure. Consequently, the remainder energy $2250 - 170 = 2080$ kJ is the *increment of the internal energy* of 1 kg of steam as compared with the energy of 1 kg of water. It can be seen that the larger fraction of heat is spent in evaporation to increase the internal energy, and the smaller fraction is spent on doing external work.

? **17.10.1.** Determine the increment of internal energy during the evaporation of alcohol if it is known that the density of alcohol vapour at its boiling point is 1.6 kg/m^3.

17.11. Evaporation from Curved Surfaces of Liquids

If you breathe at a shining metallic object (like the blade of a pocket knife), small moisture drops will be condensed on the blade. Then this moisture will vanish first at the edges as if escaping from the blade: evaporation occurs only at the edge where the surface has a curved shape.

What happens when the surface of a liquid has a concave shape, for example, when evaporation occurs from the concave meniscus in narrow capillaries which are present in porous materials? In such cases, evaporation is hampered. This is one of the reasons behind the fact that even very dry wood still contains a considerable amount of water (about 12%) in thin channels between fibres. It is well known that dry linen, paper, etc. contain a certain amount of moisture. This indicates that the evaporation rate at the same temperature depends not only on the *kind* of liquid but also on the *shape of its surface*. If the surface is convex,

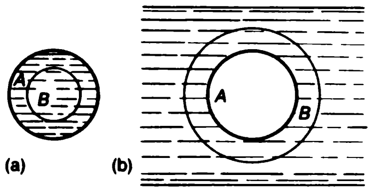


Fig. 493.
(*a*) If drop A partially evaporates, the area of its new surface B is smaller than the area of the initial surface. (*b*) If liquid partially evaporates within bubble A, the area of the surface of new bubble B is larger than that of the initial bubble.

evaporation is more intense than from a flat surface, while for the concave surface evaporation is less intense.

How can this be explained? Pay attention to the fact that in evaporation from a convex surface (a drop, Fig. 493*a*), the area of the surface decreases, while during the evaporation from a concave surface (a bubble in a liquid, Fig. 493*b*), the surface area increases. But with a change in the surface area, the number of molecules on it also changes, and the molecules on a surface are known to possess an excess energy in comparison with the molecules in the bulk of the liquid. Thus, an increase in the surface area of a liquid is fraught with additional energy expenditures. This additional energy must be spent during evaporation from a concave surface. Therefore, the concavity of a surface hampers the escape of molecules beyond its limit, i.e. reduces evaporation in comparison with a flat surface.

On the contrary, the evaporation from a drop reduces the area of the surface of the liquid, and hence the store of its surface energy. As a result, new molecules can evaporate. Thus, the convexity of a surface facilitates the escape of molecules beyond its limit, i.e. intensifies evaporation in comparison with a flat surface. Hence it follows that the liquid-vapour equilibrium for convex, flat and concave surfaces sets in at *different* vapour densities: the highest vapour density is observed for a convex surface and the lowest density for a concave surface. *The smaller the radius of the surface, the larger the difference.*

When a vapour is saturated for a concave surface, the saturation has not yet been attained for a flat and especially a convex surface. For this reason, in wet weather porous materials wetted by water are the first to become wet. Conversely, small drops with a very convex surface evaporate readily. If small drops are in the vicinity of a flat surface of water or near large drops, they evaporate, and the vapour condenses on large drops. Thus, large drops as if absorb small ones. The growth of large drops at the expense of small drops can be easily observed by examining through a microscope (with a magnification $\times$ 50-100) a slightly cooled glass plate covered with moisture if we breathe on it.

17.12. Superheating of a Liquid

If water is boiled for a long time in a glass vessel, it can be observed that the process of boiling gradually changes. The number of sites on the vessel walls from which steam bubbles separate decreases with time. Finally, only one or two such sites are left (Fig. 494), but bubbles are separated from them less and less frequently. These bubbles immediately increase in size after the separation, and this is accompanied by sharp noises as if small explosions occurred in the liquid. A thermometer indicates a noticeable (by 1-2 K) increase in temperature of the liquid in comparison with the initial temperature of boiling.

How can we explain this change? It was shown above (Sec. 17.7) that the necessary condition of the stability of a bubble inside a liquid is the presence of air in it, whose pressure drops as the diameter of the bubble increases. As soon as air is removed from a bubble, stable equilibrium becomes impossible. In order to understand this, let us imagine that a small bubble is accidentally formed in a liquid, which contains just vapour. Since the bubble is very small, the pressure in it, as was shown in the previous section, is considerably lower than the vapour pressure near a flat surface of the liquid at the same temperature. And at a still earlier stage (Sec. 14.8), it was found out that equilibrium is possible only if the pressure in the bub-

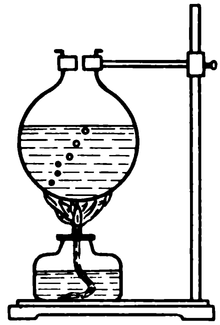


Fig. 494.
After a prolonged boiling, bubbles are separated always from a single site on the bottom of the flask.

ble is higher than the pressure of the liquid. Consequently, a bubble containing vapour alone just cannot be formed in a liquid if the temperature is not very high.

However, as was shown above, since the vapour pressure increases with temperature very rapidly, at a sufficiently high temperature a bubble containing vapour alone still can be formed in spite of unfavourable conditions. As soon as its radius starts to increase, the additional pressure of water, considered in Sec. 14.8, starts to decrease (it should be recalled that it is equal to $2\sigma/R$), while the vapour pressure in the bubble will start to increase. This explains such an abrupt, "explosive" increase in the size of a bubble containing no air.

Thus, the presence of air bubbles in a liquid is the necessary condition of calm boiling without ejecting of the liquid. In ordinary utensils a sufficiently large amount of air bubbles is formed in the cracks and other defects of the inner surface of the ware (especially in the presence of scale). In chemical laboratories, where liquids are boiled in glass vessels whose walls are free from defects, porous porcelain balls which are able to keep air for a long time in their pores are placed into glass vessels. In the presence of such balls, the boiling is calm.

17.13. Supersaturation of Vapours

While studying the properties of a saturated vapour, it was established (see Sec. 17.6) that under normal conditions to each temperature there correspond certain density and pressure of the saturated pressure. If a vapour of some liquid, say, water, is contained in a certain volume, a decrease in temperature under normal conditions results in the vapour saturation, after which the vapour starts to condense, forming liquid drops on the vessel's walls and mist drops away from them. It can be easily seen, however, that the mist formed during cooling a vapour is sometimes dense and sometimes thin, while under certain conditions it may not appear at all.

Let us make the following experiment. We pump air into a thick-wall glass vessel containing several drops of water (Fig. 495).[5] The air in the vessel is known to be heated in this process. We wait for several minutes until the air acquires the room temperature and open the vessel. A light mist will be seen in it. This phenomenon is explained as follows. When the vessel is opened, the air in it is rarefied and cooled. This results in the saturation of water vapour contained in the vessel and its condensation.

If we drop a burning match into the vessel, it will be extinguished, leaving an invisible smoke. Having repeated the experiment, we see that the vessel is filled with a much denser mist after its opening than before. This means that the presence of smoke particles facilitates the formation of mist in air. The particles of smoke serve as centres near which vapour con-

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[5] The amount of pumped air should be small, otherwise the vessel may be broken.

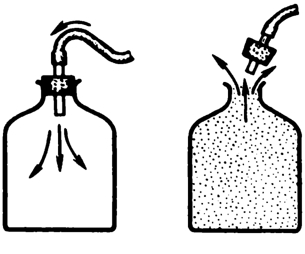

Fig. 495.
(*a*) Air is pumped into a glass vessel. (*b*) When the stopper is pulled out, the air in the vessel becomes misty.

densation starts (condensation nuclei). For this reason, a larger number of mist drops appears in the presence of smoke (other conditions being the same) than in the absence of smoke.

If, however, the dust in the vessel is removed (say, by filtering the air through cotton) or, which is more effective, by precipitating the dust contained in the air with the help of multiple condensation of vapour in the vessel, no mist appears after the vessel has been opened even after cooling to a temperature much lower than that at which saturation occurs. In this case, supersaturated vapour is obtained, i.e. the vapour whose pressure is higher than the saturated vapour pressure at a given temperature.

Why is the formation of drops hampered in the absence of dust particles? In other words, why are condensation nuclei required for the formation of drops? In order to explain this, let us recall that the vapour pressure in the vicinity of small drops is much higher than near a flat surface. As a result, small drops evaporate very easily (see Sec. 17.11). This property of small drops is responsible for the hampered condensation in the absence of dust particles. Indeed, suppose that an aggregate of vapour molecules has been accidentally formed in pure air, and a drop has appeared. This drop will soon evaporate.

A different situation will arise when dust particles are present in air, which consist of substances wetted by water and readily soluble in it. The molecules of water vapour, having encountered such a substance, are kept on it by cohesive forces. As soon as water vapour has condensed on such a dust particle, a drop of a sufficiently large size is formed. The vapour pressure near it will differ very little from the vapour pressure near a flat surface, and the drop will grow at a very small supersaturation. The condensation nuclei in the atmosphere are mainly tiny grains of sea salt always contained in air. Smoke also plays a significant role.

? **17.13.1.** According to statistics, smog near industrial centres is thinner during weekends than on working days. Explain this.

17.14. Vapour Saturation in Sublimation

It was mentioned above (Sec. 17.1) that all solids evaporate to a certain extent. In this case, we can also consider the equilibrium of the solid-vapour system, i.e. speak of saturation. For instance, naphthalene placed in a tightly closed jar is in equilibrium with its vapour. At room temperature, the saturated vapour pressure for naphthalene is about 0.05 mm Hg. For any solids, the saturated vapour pressure at room temperature is immeasurably low and attains significant values only at high temperatures. For tungsten used for manufacturing filaments in incandescent lamps, the saturation vapour pressure amounts to only 3×10^{-7} mm Hg even when heated to 2200 °C.

If a substance can exist at a certain temperature both in the form of crystals and in the form of supercooled liquid (see Sec. 15.8), the vapour pressure near the supercooled liquid is always higher than near the crystal. This can easily be explained. The internal energy of a crystal is always lower than the internal energy of the same amount of melt at the same temperature. Therefore, the difference between the internal energies of the vapour and the crystal is higher than the difference between the internal energies of the vapour and the supercooled liquid. For this reason, the number of molecules in a crystal whose kinetic energy is high enough for escaping beyond its limits is obviously smaller than the number of molecules in the supercooled liquid, which may leave its limits. As a result, the pressure near supercooled liquid is higher than the pressure in the vicinity of crystals at the same temperature.

The most important example of this kind is water. Table 21 contains the values of saturated vapour pressure above supercooled water and ice. It can be seen from the table that the difference between these pressures grows as the temperature drops. The existence of the pressure difference is due to the atmospheric instabilities like drops of supercooled water and ice crystals. Water vapour diffuses from a region where its pressure is high (i.e. from drops of supercooled water) to a region where its pressure is low (i.e. to ice crystals). As a result, drops evaporate and become smaller in size while icicles grow. This question will be considered again in Chap. 18.

Table 21. Saturated Vapour Pressure above Supercooled Water and Ice (in mm Hg)

Temperature, °C	Vapour above water	Vapour above ice
0	4.58	4.58
−2	3.96	3.88
−5	3.16	3.01
−10	2.14	1.95
−15	1.43	1.24

17.15. Liquefaction of Gases

It has been mentioned repeatedly that all liquids can evaporate. Some liquids evaporate rapidly at a given temperature, while others do this at a slower rate. In this process, they are converted into vapour, i.e. go over to the gaseous state. Hence one can naturally ask: can a gas be converted into a liquid? How can this be done?

If we have an unsaturated vapour of water or ether and start compressing it, at first its pressure will increase as in the case of an ordinary gas. However, the pressure will rise until it becomes equal to the saturated vapour pressure for the temperature of the experiment. After this, the pressure will not increase further, and the vapour will condense into liquid. The volume in which compression takes place will no longer be filled with a homogeneous substance (gas): an interface appears between the two states of the substance, viz. the liquid and gaseous states.

As early as in the beginning of the last century, the English physicist and chemist Michael Faraday (1791-1867), as well as other scientists, managed to liquefy in this way a number of substances which were known to exist only in the gaseous state before that. They liquefied chlorine and carbon dioxide by compressing them at the lowest possible temperature that could be attained. Figure 496 shows Faraday's device for the liquefac-

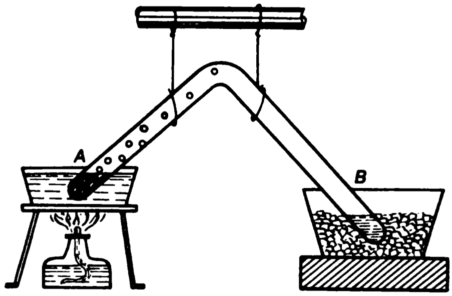

Fig. 496.
Faraday's experiment on liquefaction of chlorine.

tion of chlorine. The arm A of a sealed glass tube contains dry chlorine hydrate. Gaseous chlorine is liberated from it as a result of heating. The end B is placed into a cooling mixture. Liquid chlorine is collected in this arm. To liquefy gases like chlorine or carbon dioxide, they must be compressed much more strongly than ether vapour. For example, the liquefaction of chlorine and carbon dioxide at 20 °C should be carried out under pressures of 7 atm and 60 atm respectively (their saturated vapour pressures).

However, some gases (hydrogen, nitrogen, oxygen, etc.) turned out to be extremely "stubborn". These gases could not be liquefied by any cooling available at Faraday's time even under pressures of several thousand atmospheres. What could be the reason behind these failures? This question was answered only after a detailed investigation of the temperature and pressure dependences of the density of a liquid and its vapour. It was found that these gases could not be liquefied because it was not possible to cool them sufficiently, and not because the attainable pressures were insufficient.

17.16. Critical Temperature

If we place a certain amount of a liquid into a closed vessel, a fraction of this liquid evaporates, and saturated vapour is formed above the liquid. The pressure, and hence the density of this vapour depends on the temperature. The density of vapour is normally considerably lower than the density of liquid at the same temperature. If the temperature is increased, the density of the liquid decreases (see Sec. 10.4), while the density and pressure of the saturated vapour increase. Table 22 contains the values of density ρ of water and saturated water vapour for different temperatures (and hence for corresponding pressures). Figure 497 represents the same data in the graphical form. The upper part AC of the graph shows the change in the density of the liquid with temperature. As the temperature increases, the density of the liquid decreases. The lower part BC of the graph shows the temperature dependence of the saturated vapour density. The density of vapour increases with temperature. At a temperature corresponding to point C, the densities of water and its saturated vapour coincide.

Table 22. Properties of Water and Its Saturated Vapour at Various Temperatures

Temperature, °C	Saturated vapour pressure, mm Hg	Density of water, 10^3 kg/m^3	Saturated vapour density, kg/m^3	Specific latent heat of vaporisation, kJ/kg
15	13	1.000	0.073	2454
50	92	0.998	0.083	2374
100	760	0.960	0.597	2250
150	3 570	0.920	2.54	2115
200	11 660	0.860	7.84	1940
300	64 450	0.700	46.9	1379
370	157 700	0.440	208	414
374	165 500	0.320	320	0

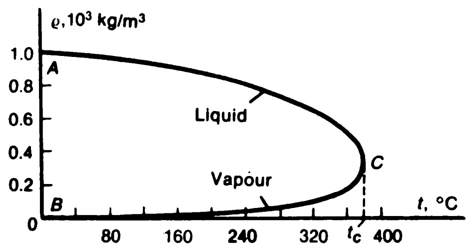

Fig. 497.
The density of water and its saturated vapour versus temperature.

It can be seen from the table that the higher the temperature, the smaller the difference between the densities of a liquid and its saturated vapour. At a certain temperature (equal to 374 °C for water), these densities coincide. The temperature at which the densities of a liquid and its saturated vapour coincide is called the *critical temperature* for a given substance. In Fig. 497, this temperature corresponds to point C. The pressure corresponding to point C is known as the *critical pressure*. Critical temperatures for different substances considerably differ from one another. Some of them are given in Table 23.

What does the existence of critical temperature indicate? What will happen at still higher temperatures?

Experiments show that at temperatures higher than critical, a substance can exist *only* in the gaseous state. If we decrease the volume occupied by a vapour at a temperature higher than its critical temperature, the pressure of

Table 23. Critical Temperatures and Pressures for Some Substances

Substance	Critical temperature, °C	Critical pressure, atm
Mercury	1700	about 1600
Water	374	218.5
Ethyl alcohol	243	62.7
Ether	197	35.8
Chlorine	146	76
Carbon dioxide	31	73
Oxygen	−118	50
Nitrogen	−146	33
Hydrogen	−240	12.8
Helium	−263	2.26

the vapour increases, but it does not become saturated and continues to be homogeneous: however high the pressure is, there are no two states separated by a sharp interface as it is always observed at lower temperatures due to condensation of vapour. Thus, if the temperature of a substance is higher than the critical temperature, the equilibrium of the substance in the form of liquid and vapour in contact with it is not possible at any pressure.

The critical state of a substance can be observed with the help of the device shown in Fig. 498. It consists of a steel box with windows, which can be heated above 200 °C ("air bath"), and a glass ampoule containing ether inside this box. As the bath is heated, the meniscus in the ampoule rises, becomes flatter, and ultimately vanishes, which indicates a transition through the critical state.[6] During the cooling of the bath, the ampoule suddenly becomes turbid due to the formation of a large number of tiny ether drops, after which the ether is collected in the lower part of the ampoule.

Table 22 shows that as the critical point is approached, the specific latent heat of vaporisation becomes lower and lower. This is explained by the fact that with increasing temperature the difference between the internal energies of a substance in the liquid and vapour states decreases. Indeed, the molecular cohesive forces depend on the intermolecular distances. If the densities of a liquid and its vapour differ but little, the average intermolecular distances will be almost the same. Consequently, in this case the difference between the potential energies of intermolecular interaction will be small. The second component of heat of vaporisation, viz. the work

[6] Naturally, it is necessary to choose such an amount of ether at which the critical state sets in, i.e. the vapour pressure attains the critical value at the critical temperature. If the amount of ether is insufficient, it will evaporate before providing the required pressure: the meniscus will gradually lower to the bottom of the ampoule. If the amount of ether is too large, it will expand and fill the entire ampoule before attaining the critical temperature.

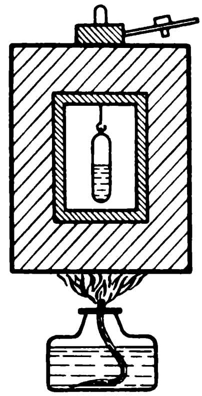

Fig. 498.
A device for observing the critical state of ether.

against the external pressure, also decreases as the critical temperature is approached. This follows from the fact that the smaller the difference between the densities of vapour and liquid, the smaller the expansion taking place during evaporation, and hence the smaller the work done in evaporation.

The existence of critical temperature was pointed out for the first time in 1860 by the great Russian chemist Mendeleev who discovered the basic law of the modern chemistry, viz. the periodic law of chemical elements. A significant contribution to the study of critical temperature was made by the English chemist Thomas Andrews (1847-1907) who thoroughly investigated the behaviour of carbon dioxide (CO_2) in an isothermal variation of the volume occupied by it. Andrews showed that at temperatures below 31 °C, coexistence of carbon dioxide in the liquid and gaseous states is possible in a closed vessel. At temperatures above 31 °C, such coexistence is impossible, and the entire vessel is filled by the gas alone, irrespective of a decrease in its volume.

After the critical temperature had been discovered, it became clear why such gases as oxygen and hydrogen could not be liquefied. It turned out that their critical temperatures are very low (Table 23). In order to liquefy these gases, they *must be cooled below the critical temperature*. Without that, all attempts at their liquefaction are doomed to failure.

17.17. Liquefaction of Gases in Engineering

When it was found that to liquefy gases they must be cooled below the critical temperature, the efforts of scientists were concentrated on the development of methods for attaining low temperatures. These efforts were crowned with success, and a number of devices are now available for the liquefaction of all gases without any exception. These devices, especially those for the liquefaction of air, are now used widely in engineering.

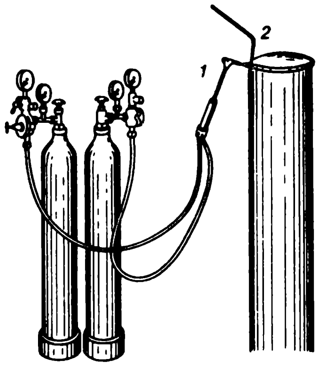

Fig. 499.
Oxyacetylene welding of metals. Oxygen and acetylene are delivered to burner *1* through two tubes. Wire *2* melts in oxyacetylene flame and runs into the weld.

The liquefaction of air is used for its separation into components. The separation takes place during the evaporation of liquefied air. In this process, the air components having lower boiling points (like neon and nitrogen) evaporate first, followed by argon and oxygen. The process is just the same as the separation of a more readily boiling alcohol from water by distillation. The obtained gases find a wide application: (a) nitrogen is used for production of ammonia; (b) argon, neon and other inert gases are used for filling incandescent as well as fluorescent lamps; (c) oxygen serves many purposes: when mixed with acetylene (or with hydrogen) and burned, it gives a flame which has a high temperature and is used for welding and cutting metals (Fig. 499). Oxygen blast for accelerating metallurgical processes has acquired considerable significance. Oxygen is also used for medical purposes.

In addition, liquid oxygen is also used in blasting techniques. A mixture of oxygen with filings, carbon black, naphthalene and other readily oxidised materials (Oxyliquit) is an explosive of enormous power. An explosion occurs because the combustion of these substances in the presence of oxygen in the liquid state (which hence occupies a small volume) occurs very rapidly. During the combustion, the mixture is strongly heated, the reaction products (carbon dioxide) are obtained in the gaseous state, and an instantaneous and very large expansion, viz. explosion, takes place. The advantage of this explosive is that it stops being hazardous as soon as oxygen has evaporated.

There are various types of apparatus for obtaining liquid air. We shall describe here a device whose operation is based on cooling of highly compressed air during its expansion (see Sec. 13.5). Air is delivered to compressor *1* (Fig. 500), where it is compressed to a pressure of several tens of atmospheres. In this process the air is heated. From compressor *1*, the air flows to heat exchanger *2* where it is cooled by running water to the initial temperature, and then delivered to expander *3*. The expander is a cylinder

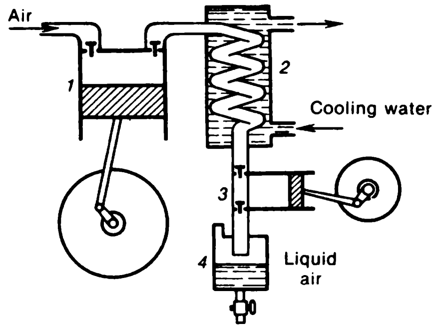

Fig. 500.
Schematic diagram of a device for obtaining liquid air.

with a piston where the air expands, pushes out the piston and does work. The internal energy of the air is spent on this work, and the air temperature drops to such an extent that it condenses into liquid. The liquefied air is accumulated in vessel *4*.

Sometimes expanders are made not in the form of a cylinder with a piston but in the form of a turbine (Kapitza's turboexpander), in which a gas doing work on the rotation of a turbine expands. It is very important that the rotor (the rotating part of the turbine) is "suspended" during its operation in the flow of the expanding gas without touching the turbine walls. As a result, lubrication is not required, which is very essential since no effective lubricants are available for machine parts operating at such low temperatures. At low temperatures, conventional lubricants solidify. Besides, the advantage of apparatus for liquefaction of gases designed by P. L. Kapitza is their high efficiency and a relatively small size.

The boiling point of liquid air is very low. At the atmospheric pressure, it is -190 °C. For this reason, the liquid air in an open vessel (when the pressure of its vapour is equal to the atmospheric pressure) boils until its temperature drops below -190 °C. Since surrounding bodies are much warmer, the heat flow to the liquid air (if it were stored in ordinary vessels) would be so considerable that the whole liquid air would evaporate in a very short time. For this reason, it is stored in special vessels which provide a good protection from heat flow from outside. These vessels are similar in design to that of an ordinary thermos (vacuum bottles). These are glass or metal vessels with double walls (Fig. 501), the space between them being thoroughly evacuated. Heat transfer through such a space filled by a highly rarefied gas is extremely slow. In order to protect the contents from heating by radiation, the inner walls are made shining (silvered). The vessels of this type for storing liquid air were proposed by Dewar and are called Dewar flasks. In a good Dewar flask, liquid air evaporates so slowly that it can be stored for two-three days and even longer.

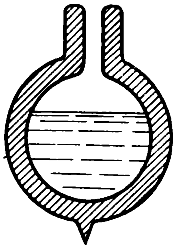

Fig. 501.
Sectional view of a Dewar flask. The spout at the bottom is the end of the tube through which air has been pumped out from the space between the walls and which is sealed after the pumping has been completed.

In order to prevent a liquefied gas from heating in spite of a continuous although slow heat flow, it must be kept in an open vessel to enable it to evaporate. Since heat is consumed for evaporation, the liquefied gas remains cold all the time. If we close a Dewar flask, i.e. prevent the gas from evaporating, the liquefied gas will be heated, and the pressure of its vapour will increase to such an extent that the flask may burst. If the flask were very strong as a steel vessel shown in Fig. 375, the liquefied gas would be gradually heated to a temperature above the critical and would go over to the gaseous state. Thus, *the only way of a prolonged storage of a liquefied gas is to use open Dewar flasks.*

17.18. Vacuum Technology

Evacuated (vacuum) devices, i.e. devices consisting of a glass or metal flask from which air has been pumped out to the highest possible extent, are widely used in modern engineering. These are electric lamps, radio valves, photocells, Dewar flasks, and so on. Devices filled with inert gases are also often used (e.g. electric lamps or fluorescent tubes employed for illumination and advertising). Before filling them with an inert gas, they have to be evacuated first. This has led to the appearance of a new branch of engineering, called *vacuum technology*.

The following methods are used for obtaining vacuum.

1. First of all, mechanical pumps are employed, which dispel air as a result of the motion of, say, a piston. The most widespread pumps are rotary oil pumps which were described in Sec. 8.5.

2. A high vacuum can be obtained with the help of another type of pumps like a diffusion pump, in which vapour of special oils (sometimes mercury) is utilised. Such a pump can operate only if air is preliminarily pumped out by another pump. The preevacuation is usually done by a rotary pump. The schematic diagram of a mercury diffusion pump is shown in Fig. 502. It consists of vessel *1* containing mercury and continually heated by a burner or electric stove. The mercury vapour formed as a result is delivered through tube *2* to cavity *3* cooled by running water.

The operation of the pump is based on the phenomenon that the molecules of the gas flowing through pipe *4* from a vessel being evacuated diffuse from volume *5* into cavity *3* filled with vapour, where the partial pressure of the gas is lower than in volume *5*. Here the gas molecules are captured by the vapour jet and are entrained by it. Vapour condenses on the walls and flows back to vessel *1*. The gas is pumped through tube *6* by a forepump. A diffusion pump can operate only if a preliminary vacuum is created such that the mean free path of vapour molecules exceeds the diameter of tube *3*.

When air is evacuated from glass or metal flasks, special measures must be taken to remove the air molecules that have been adsorbed by the walls (see Sec. 14.11). For this pur-

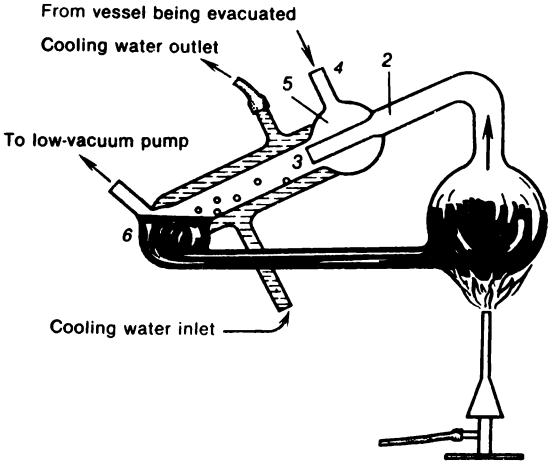

Fig. 502.
Mercury diffusion pump.

pose, the vessels being evacuated (say, electric lamps) are heated during pumping (to 300-400 °C) in special furnaces. During heating, air molecules stuck to the glass walls are separated and removed by the pump. If a flask is evacuated without heating, the gas will soon appear in it, and vacuum will be insufficient.

3. In spite of preheating, gas is gradually liberated in most of vacuum devices during their operation from glass and metal parts. In order to eliminate the harmful effect of the liberated gas on the operation of a device, a continually operating getter is mounted in it. In illuminating incandescent lamps, phosphorus covering the lamp walls with a transparent layer is used as a getter. In radio valves, a barium layer is normally used, which is sprayed inside a valve after evacuation. The mirror barium layer absorbs gases during the entire service life of the valve.

In modern vacuum devices, a vacuum of the order of 10^{-8} mm Hg is attained. This means that the gas density in them is a billion times less than the density of atmospheric air. Even with such a rarefaction, however, there are several hundred million molecules in 1 cm^3. It is interesting to compare this result with the fact that the density of interstellar matter is such that not more than a hundred molecules are contained in 1 cm^3.

17.19. Water Vapour in the Atmosphere

The amount of water vapour contained in air is very important for the processes occurring in the atmosphere. It also considerably influences the life of plants and living organisms. The amount of water vapour in air can be expressed with the help of the following quantities: (a) the *pressure of water vapour* (partial pressure, see Sec. 13.19); *absolute humidity of air*, viz. the mass of water vapour in 1 m^3 of air (in grams); (c) *relative humidity of air*, viz. the ratio of the pressure of the vapour contained in air to the saturated vapour pressure at the same temperature (in percent). Table 24 contains the values of the saturated vapour pressure for water at different

Table 24. Saturated Vapour Pressure of Water and Absolute Humidity of Air at Different Temperatures

Temperature, °C	Saturated vapour pressure, mm Hg	Absolute humidity of air, g/m^3	Temperature, °C	Saturated vapour pressure, mm Hg	Absolute humidity of air, g/m^3
−40	0.143	0.18	10	9.21	9.41
−30	0.382	0.46	15	12.7	12.8
−20	0.940	1.08	20	17.5	17.32
−10	2.14	2.37	30	31.8	30.4
0	4.58	4.86	40	55.4	51.1
5	6.10	6.84			

temperatures (in mm Hg) and the absolute humidity of air, corresponding to this pressure.

The saturated vapour pressure also depends on whether a vapour is above the surface of supercooled water or ice. The saturated vapour pressure above ice is lower than that over supercooled water at the same temperature (Sec. 17.14). This means that if we introduce a piece of ice or a drop of supercooled water in air containing water vapour close to saturation, condensation will start on the surface of the ice, the piece of ice will increase in size, but the water drop will evaporate and become smaller. This process has a considerable effect on atmospheric precipitation (see Sec. 18.5).

The humidity of air is determined with the help of a hygrometer and a psychrometer.

1. A hair hygrometer is shown in Fig. 503. The main part of the device is a clean human hair *1*, which can elongate with increasing relative humidity of air. Hair *1* is wound on roller *2* and is kept in the stretched state by

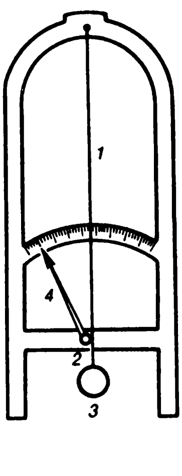

Fig. 503.
Hair hygrometer.

load *3*. The length of the hair changes with air humidity, setting roller *2* into motion. It rotates and drives pointer *4*. The scale divisions indicate the relative humidity. If the air temperature is measured simultaneously, the absolute humidity of air and the pressure of water vapour can be determined.

Let us suppose, for example, that the relative humidity of air is 50% at a temperature of 20 °C. From Table 24 we find that the saturated vapour pressure is 17.5 mm Hg at 20 °C, while the absolute humidity is 17.32 g/m^3. Consequently, the (partial) vapour pressure of water is $17.5 \times 0.5 = 8.75$ mm Hg, while 1 m^3 of air contains $17.32 \times 0.5 = 8.66$ g of water in the form of vapour.

2. A psychrometer is shown in Fig. 504. The device consists of two identical thermometers. The bulb of one thermometer is wrapped in a piece of muslin whose lower end is immersed into a small glass vessel containing distilled water. Water wets the muslin and evaporates on the bulb of the thermometer if water vapour in air is unsaturated. As a result of heat loss due to evaporation, the thermometer bulb is cooled, and the wet-bulb thermometer indicates a lower temperature than the dry-bulb one. The difference between the readings of the thermometers is the larger, the larger the difference between the pressure of the vapour contained in the air and the saturated vapour pressure. Using special psychrometric tables, we can find the water vapour pressure and the relative humidity of air from the readings of the wet-bulb and dry-bulb thermometers.

As the temperature of the air drops at a constant mass of water vapour, the relative humidity increases since the lower the temperature, the closer the water vapour to saturation. Finally, at a certain definite temperature the relative humidity becomes equal to 100%, and a further decrease in the

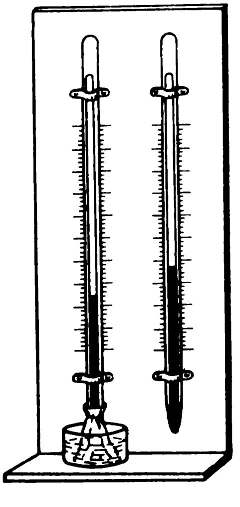

Fig. 504.
Psychrometer.

temperature leads to the condensation of water vapour. Mist appears, windows become foggy, and dew drops precipitate on grass. The temperature at which vapour becomes saturated at a given pressure or, which is the same, the relative humidity reaches 100%, is known as the *dew point*. The dew point can be easily determined by cooling slowly a metal vessel by, for example, dropping pieces of ice into it and marking the temperature at which it becomes foggy.

There exist special devices for determining the dew point operating on the same principle. If we know the dew point, we can determine the water vapour pressure and absolute and relative humidities of air. Let us suppose, for example, that the dew point is 10 °C, while the air temperature is 20 °C. From Table 24 we find that the saturated vapour pressure at 10 °C is 9.21 mm Hg and 1 m^3 of air contains 9.41 g of water vapour. At 20 °C, the saturated vapour pressure would be 17.5 mm Hg. Consequently, the relative humidity of air is $(9.41:17.5) \times 100 = 52.6\%$.

? **17.19.1.** What is the relative humidity of air if 1 m^3 contains 7.5 g of water vapour and its temperature is 10 °C?

17.19.2. What is the mass of water vapour contained in a room whose volume is 115 m^3 if the relative humidity at 20 °C is 60%?

17.19.3. The relative humidity of air at 15 °C is 55%. Will dew precipitate if the air temperature drops to 10 °C?

Chapter 18

Physics of the Atmosphere

18.1. The Atmosphere

The air envelope of the Earth, viz. the atmosphere, is the air layer whose density gradually decreases with increasing distance from the surface of the Earth. At an altitude of 50 km, the air pressure amounts to 1/1000 of its value at the surface of the Earth, while still higher layers of the atmosphere consist of a highly rarefied gas.

The information about the structure of the atmosphere was obtained from recorders aboard aeroplanes as well as from those aboard balloons filled with hydrogen. These devices automatically transmit by radio the information on the temperature, pressure and humidity of air at various altitudes up to 40 km. These devices are called radiosondes.

Recently, the upper layers of the atmosphere have been studied with the help of rockets and artificial satellites of the Earth.

Several layers constitute the atmosphere. The lower layer whose thickness is about 11 km in moderate latitudes and 14-17 km in tropical latitudes is known as the *troposphere*. This layer contains nearly all the water vapour of the atmosphere. In this layer ascending and descending air currents are generated, clouds are formed, and in general all weather-influencing processes occur. The air temperature in the troposphere decreases with increasing altitude on the average by 5-6 °C per kilometre.

The layer of air above the troposphere is called the *stratosphere* which is almost cloudless all the time. In the lower part of the stratosphere, the temperature remains nearly constant and is equal to about -55 °C up to an altitude of 30 km. In the higher layers of the stratosphere, the air temperature increases, reaching the highest values (up to 40 °C) at an altitude of 50-60 km. Further on, the temperature drops again. Such an increase in temperature is due to the fact that a layer of ozone lies at an altitude of 20-25 km, which is heated as a result of absorption of ultraviolet radiation emitted by the Sun. The part of the atmosphere above the stratosphere, extending from an altitude of 80 km, is called the *ionosphere*.

The atmospheric air consists of nitrogen (78.1% by volume)[1], oxygen (21%) and argon (0.9%) with small admixtures of carbon dioxide, helium, neon, crypton and xenon. As a result of mixing in the lower layers of the atmosphere, the air composition is almost the same up to 100 km altitude. The atmospheric air also contains water vapour which is formed as a result of evaporation from the surface of oceans and continents. The role of water vapour in the phenomena occurring in the atmosphere is very significant although the amount of vapour is very small (normally less than 1%). The condensation of water vapour gives rise to clouds and precipitation and is accompanied by the liberation of a large amount of heat. During evaporation, heat is absorbed.

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[1] This means that among 1000 air molecules, there are 781 nitrogen molecules, 210 oxygen molecules and 9 argon molecules. — *Eds.*

18.2. Heat Balance of the Earth

In the day time, the surface of the Earth is continually heated by solar rays. It was established by measurements that each square metre of the surface capable of absorbing completely all solar rays incident at right angles receives about 700 J of energy per second. The atmosphere is opaque to a part of solar radiation. Sun light is scattered by gases constituting the atmosphere, dust particles and water drops and is absorbed by ozone (in the upper layers), water vapour, carbon dioxide, oxygen and dust. The ultråviolet part of the spectrum emitted by the Sun is absorbed more strongly than others. For this reason, the intensity of radiation received from the Sun increases with altitude and the radiation contains an increasingly large amount of ultraviolet rays.

At the boundary of the atmosphere, the intensity of radiation amounts to 1400 J/m^2 per second (1.4 kW/m^2). This quantity is known as the *solar constant*. The amount of energy reaching the Earth is thousands of times larger than that used by mankind for preparing food, heating houses, driving engines, and so on. Plants also use only insignificant part of this energy (about 1%), storing it in the form of the internal energy of substances constituting green parts of plants. Not all the energy emitted by the Sun in the direction of the Earth is absorbed by the surface of the Earth. A considerable fraction of this energy (about 42%) is reflected by clouds and the surface of the Earth and is scattered by the atmosphere. About 15% of this energy is absorbed by the atmosphere and only 43% is absorbed by the surface of the Earth.

The energy absorbed by the surface of the Earth is spent on radiation, heating of air, soil and aqueous surfaces and on evaporation. More than 500 000 km^3 of water are evaporated from the vast expanses of oceans and continents per year. This is almost equal to the amount of water in the Black Sea. Slightly less than half the entire solar radiation energy absorbed by the surface of the Earth is spent on evaporation. During the subsequent condensation of this amount of water, the same amount of heat which was spent on evaporation is liberated in the atmosphere. It heats the atmosphere and thus protects it from sharp drops in temperature. The condensation of vapour frequently takes place at a site other than that where vapour has been formed. Vapour is often carried by wind over long distances, and condensation occurs in colder regions than those where evaporation took place. This process, as well as the transport of heat received by air currents from heated regions of the surface, leads to a moderation of climatic conditions in cold regions.

Due to a low thermal conductivity of soil, the heat of the soil extends to a small depth (not exceeding 25 m). Since heat is transferred very slowly, temperature peaks are observed in the deeps of the crust somewhat later than at the surface. For example, at the 2 km depth, the maximum temperature is attained not in July, as at the surface, but in August. As a result of mixing of sea water during storms, heat penetrates to a large depth (hundreds of metres). A fraction of heat received by the Earth from the Sun is lost by radiation. However, due to the presence of water vapour in the atmosphere, this radiation is partially absorbed by the atmosphere again, thus reducing the heat lost by the Earth.

How can it happen that the atmosphere lets through radiation from the Sun and is opaque to the radiation from the Earth? The spectrum of solar radiation contains both visible rays, perceived by human eye and called light, and invisible rays (ultraviolet and infrared). Like any other body whose temperature is below 500 °C, the Earth radiates only infrared rays in noticeable amounts. Naturally, the Earth radiates day and night, but during the day time the thermal effect of its radiation is unnoticeable since the heat loss by radiation is made up by the amount of heat received by the absorption of solar radiation. At night, the cooling of the Earth's surface due to radiation becomes noticeable. Rough and dark surfaces like ploughed fields or grass are cooled due to radiation especially strongly. Water vapour has a peculiar property which plays an important role in this process. It absorbs infrared radiation to a considerably larger extent than visible rays. For this reason, the atmosphere of the Earth is a kind of a trap for the energy of solar rays.

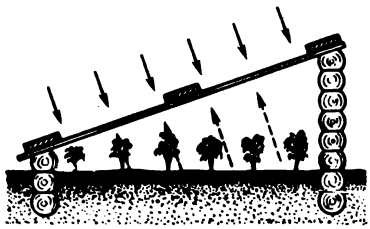

Fig. 505.
Thermal radiation emitted by heated soil does not penetrate through a glass frame of a hotbed.

The energy of visible radiation constitutes a considerable fraction of the energy of solar radiation (~40%). The radiation freely penetrates the atmosphere and is absorbed by the Earth's surface. At the expense of absorbed energy, the Earth's surface emits infrared rays which are absorbed by water vapour and heat the atmosphere. Otherwise, the average temperature of the surface of the Earth would be not 15 °C, as is actually the case, but considerably lower than zero. In this respect, the effect of water vapour is similar to the effect of glasses used for covering hotbeds (Fig. 505).

18.3. Adiabatic Processes in the Atmosphere

Till now, we mentioned that the atmospheric air can be heated or cooled in contact with warmer or colder bodies, by receiving heat from them or giving it away to them. It was also noted that air itself can radiate or absorb energy in the form of visible or invisible rays.[2] There exist, however, processes during which the temperature of air changes although air does not receive heat from or give it away to the surroundings.

The processes occurring without a heat exchange with the surroundings are termed *adiabatic processes* (see Sec. 13.5). It was established that an adiabatically expanding gas is cooled since in this process work is done against the forces of external pressure, as a result of which the internal energy of the gas decreases. In an ascending current, air expands since the rising air gets into regions with ever decreasing pressure. This process occurs practically without heat exchange with surrounding layers of air which also rise and get cooled. Therefore, the expansion of air in an ascending current can be regarded as adiabatic. Thus, *the ascent of air in the atmosphere is accompanied by its cooling.* Calculations and measurements show that an ascent of air by 100 m is accompanied by its cooling through 1 K.

The manifestations of adiabatic processes in the atmosphere are quite numerous and diverse. Let us suppose, for example, that an ascending air current encounters a high ridge and has to move up along its slopes. The upward motion of air is accompanied by its cooling. For this reason, the climate in mountain regions is always colder than that of nearest valleys, and permafrost is observed at peaks having high altitudes. Starting from a certain altitude (for example, 3000-3200 m in the Caucasus), snow does not melt even in summer and is accumulated for years in the form of huge glaciers.

When an air mass descends, it is compressed and heated as a result of compression. When an air current passes over a ridge and moves down, it is heated again. Thus, the so-called *foehn* is formed, viz. the warm wind which is well known in mountain regions of the Caucasus, Central Asia and Switzerland. The adiabatic cooling in wet air has a peculiar feature. When gradually cooling air attains the dew point, water vapour starts condensing in it. Tiniest drops of water constituting mist or clouds are formed. During condensation, the latent heat of vaporisation is liberated (see Sec. 17.8), which slows down further cooling of the

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[2] The absorption and emission of energy is mainly observed for water vapour and carbon dioxide constituting a negligible fraction of the atmosphere. The remaining gases in the atmosphere almost do not absorb or emit any energy.

air. For this reason, an ascending air current will be cooled as a result of vapour condensation at a slower rate than perfectly dry air. An adiabatic process involving the vapour condensation is known as *moist adiabatic*, or *saturation adiabatic process.*

? **18.3.1.** What is the type of process of gas expansion in the experiment shown in Fig. 495 when (a) there are many condensation nuclei and (b) when there are no condensation nuclei?

18.3.2. After an air flow passes over a mountain ridge, its temperature is higher than before the mountain at the same altitude. Explain this phenomenon.

18.4. Clouds

When air containing water vapour is cooled for some reason, the water vapour may condense in the form of water drops or icicles. In this way, clouds and fogs are formed. They consist of very small drops of water (from 3 to 40 μm) or ice crystals of the same size (Figs. 506 and

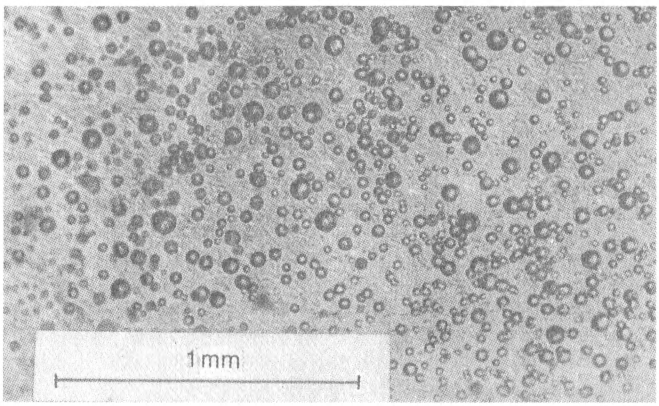

Fig. 506.
Cloud drops.

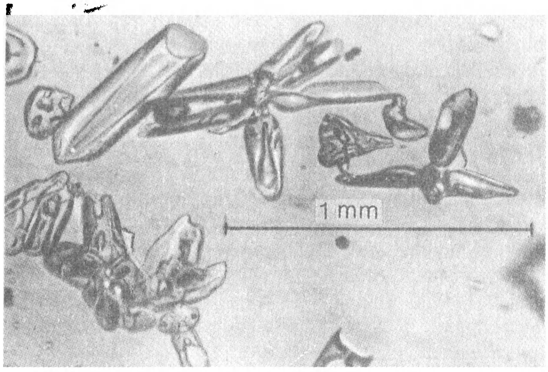

Fig. 507.
Ice crystals in a cloud.

507). Condensation starts when air attains the dew point. The drops constituting clouds and mist are so small that they fall in air at a very small, almost unnoticeable velocity. In frost, i.e. at a temperature below 0 °C, these drops are very often supercooled, i.e. remain in the liquid state instead of freezing. When air is cooled on coming in contact with the cold surface of the Earth or sea, mist is formed in the layer close to the surface. Clouds are just the same mist but are formed in higher layers of the atmosphere.

It was mentioned earlier that when a certain mass of air rises up, it expands and is cooled. This cooling is the main cause of cloud formation. In strong vertically ascending air currents, the most dense and opaque white swirling clouds are formed. Such clouds are called *cumulus*. Sometimes, they grow into *storm clouds* several kilometres high with stringy, somewhat tousled tops (Fig. 508). When an ascending motion in the atmosphere is very slow (several centimetres per second) but embraces a huge mass of air over hundreds of kilometres, this leads to the formation of the so-called *nimbo stratus* which are grey, dense and shapeless. The layer thickness of such clouds may reach 4-5 km.

Sometimes the air in the atmosphere undulates. The air rises up at the crests of the air waves, forming separate clouds or cloud rollers with gaps between crests (Fig. 509). This formation is known as *fleecy* clouds. In a cloud, large drops collide with smaller ones because the larger ones fall more rapidly and catch up with smaller ones. Gradually, rain drops may appear in this way.

Precipitation (rain, snow, etc.) is mostly formed in clouds due to different pressures of saturated water vapour above water and ice (see Secs. 17.14 and 17.19). This occurs as follows. Let us suppose that a piece of ice gets into a cloud consisting of supercooled drops. Such a system is unstable (see Sec. 17.19), and as a result of diffusion of water vapour from drops to crystals, the latter grow, while drops evaporate. In this way, large snow flakes grow in the cloud. They gradually fall out of it. If they get into a layer of warmer air with a temperature of above 0 °C, they can melt in it and precipitate in the form of rain drops.

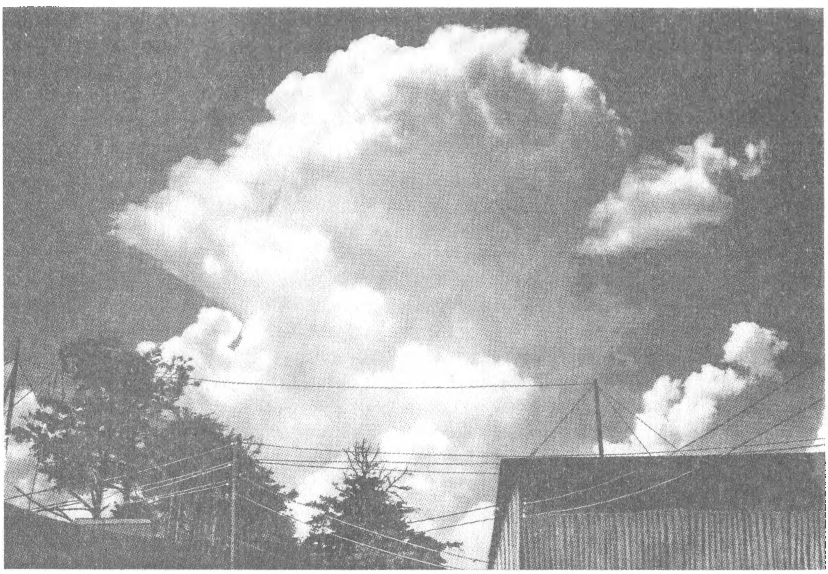

Fig. 508.
Storm cloud.

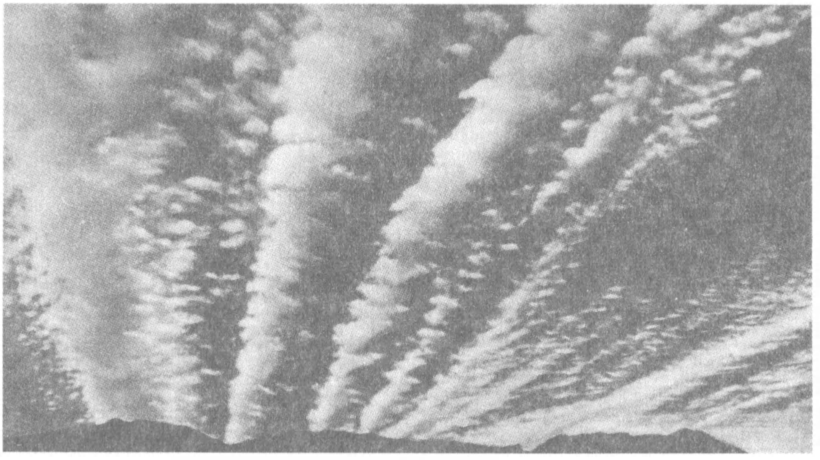

Fig. 509.
Fleecy clouds.

Snow, rain, hail, etc. are combined in meteorology into the collective term precipitation. The amount of precipitation is calculated by measuring the thickness (in millimetres) of the layer of water that would have formed on the surface of the Earth if the water did not seep through or evaporate. In order to determine the amount of water contained in snow, it is melted. The heaviest rains observed in the USSR yield more than 300 mm of precipitation per day. In the central regions of the USSR, 20 mm of precipitation is regarded as a heavy rain. The annual precipitation in Moscow is only 540 mm, while in rainy West Georgia up to 2600 mm precipitates per year. In India, equatorial Africa, and in the Hawaiian Islands the annual amount of precipitation reaches 12 000 mm.

18.5. Artificial Precipitation

There exist several methods of artificial precipitation. Small particles (grains) of solid carbon dioxide at a temperature of -70 °C are sprayed from an aeroplane in a cloud of supercooled drops. Owing to such a low temperature, a very large number of tiny ice crystals are formed in air in the vicinity of these grains. Then these crystals are spread over the cloud due to air motion. They serve as nuclei from which large snow flakes grow as was described above (see Sec. 18.4). A rather wide (1-2 km) clearance is formed in the layer of clouds in the wake of the aeroplane (Fig. 510). The snow flakes thus formed may produce a heavy snowfall.

Naturally, the amount of water that can be precipitated in this way cannot be larger than the amount of water formerly contained in the cloud. Man cannot so far intensify the process of condensation and formation of primary, tiny cloud drops.

18.6. Wind

Atmospheric air is in a more or less fast incessant motion. The motion of air along (parallel to) the surface of the Earth is called wind. A wind of 3-5 m/s is weak and can only shake the smaller branches of trees, while a wind of 13-15 m/s is strong, being able to affect a person walking and to generate foaming waves on the sea. Besides velocity, wind is characterised by the direction from which it blows (north, north-east, and so on). The wind power is used in windmills and pumps, in wind engines, and for driving sailboats. The utilisation of a wind is

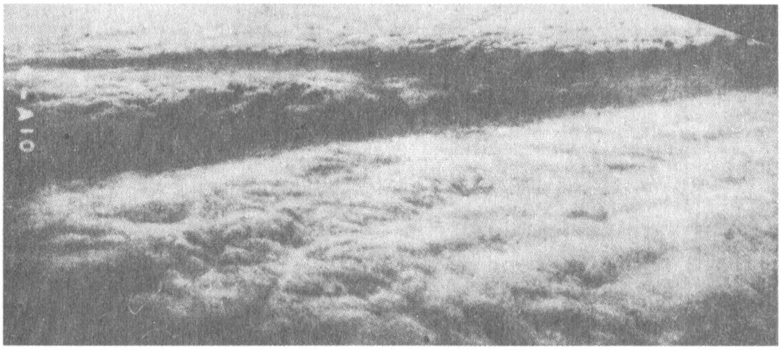

Fig. 510.
The top view of a layer of clouds through which an aeroplane sprinkling solid carbon dioxide particles has passed. The black shadow in the top right corner is the wing of the plane from which the clouds were photographed. The dark band in the layer of clouds is formed as a result of condensation on the grains of solid carbon dioxide.

the more effective, the more stable and strong the wind in a given region. These devices can be successfully used in steppes and open sea shores.

Air moves from the regions of a higher air pressure to those where the air pressure is lower. The pressure difference can emerge for different reasons. For example, sea breeze appears due to nonuniform heating of the ground and water by solar rays as well as due to the different rates of their cooling at night. On a summer day, a sea beach is heated to a larger extent than the sea surface.

Indeed, the heat of solar rays propagates in comparatively transparent water to a considerable depth, while only a thin surface layer of the soil is heated on the beach. Moreover, the soil has a smaller specific heat capacity (about 1 kJ/(kg·K)). Air above the ground is heated stronger than above water and rises up since its density becomes smaller than the density of the surrounding cold air. As a result, the pressure at the ground decreases, and colder air from sea flows to the region of reduced pressure. Such air current is known as sea breeze. At night, the reverse process is observed: soil layers heated to a small depth are cooler than water at night. Air above the ground is also cooled, its density increases, and the land breeze appears.

The winds blowing in summer in one direction and in winter in the opposite direction (monsoons) are generated similarly. In Asia, the summer temperature can exceed 50 °C, and the air pressure drops sharply. As a result, a powerful current of colder air, carrying storms and rains, comes from the sea at the end of May or beginning of June to India. Above Siberia and Central Asia, air pressure increases in winter, and cold air flows from these regions eastwards to the Japan and Yellow Seas and southwards to the shores of the Indian Ocean. Similar alternating monsoons are observed, for example, above Africa.

Winds embracing considerable portions of the Earth never blow directly from the regions of higher pressure to those of lower pressure. It can be proved that all bodies moving along the surface of the Earth acquire as a result of its rotation an acceleration towards the right in the Northern hemisphere and towards the left in the Southern hemisphere (Coriolis force, see Sec. 6.11). This refers to moving air as well. As a result, the air flowing to the region of lower pressure is curled counterclockwise (cyclone) in the Northern hemisphere, while the air flowing from the regions of elevated pressure is curled clockwise (anticyclone).

Sometimes, a large pressure difference emerges at high altitudes. In this case, strong winds appear (up to 130 m/s), which are called jet flows,while near the ground there can be no wind at all. These are usually narrow wind strips, which are observed in high latitudes in winter at an altitude of 3-4 km, at 7 km above the Japan See and Okhotsk Sea, and in summer above the southern parts of the USSR (Caucasus and Central Asia). In jet flows, high translucent fleecy clouds are formed. Their fast motion and variability demonstrate the high velocity, power and gustiness of a jet flow.

18.7. Weather Forecasting

Thus, the variety of physical phenomena in the atmosphere determine weather, i.e. lead to the generation of winds, formation of clouds, precipitation, and so on. In view of exceptional role of weather in various fields of human activity (agriculture, navigation, aviation, etc.), it is very important to have reliable weather forecasts. Weather forecasting based on observations of distinctive local indications was known even in ancient times. Local indications may have a physical nature (precipitation of dew or the view of the sky at dawn) or concern living organisms (rheumatic pains before bad weather, the behaviour of insects or birds).

The processes determining weather involve huge moving air masses of various temperatures and humidities and spread over considerable regions above the surface of the Earth. Local weather and its changes depend on the state of the atmosphere not only at a given site but also far from it. For this reason, local indications, although they are based on thorough observations of nature, are insufficient for a reliable weather forecast and cannot replace the physical method of the solution of a complicated problem of predicting weather.

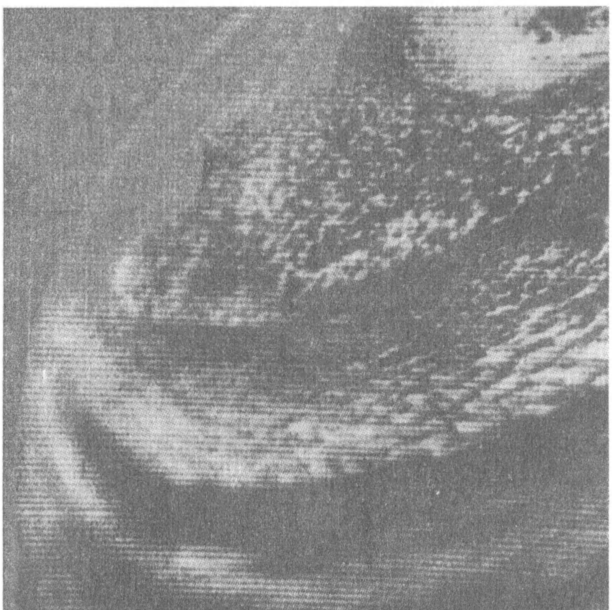

Fig. 511.
Cloud envelope of the Earth viewed from a satellite.

This method consists in comparing the results of a very large number of systematic observations of the properties of the atmosphere, carried out at different points on the Earth. For this purpose, meteorological stations are organised not only in inhabited regions but also in deserts and other rarely populated regions (including those near the poles). At these stations, the most important physical parameters characterising the state of the atmosphere (air pressure, temperature, humidity, amount of precipitation, the velocity and direction of wind, and so on) are measured daily at a certain time. This information is transmitted by telegraph and radio so that any country can make use not only of the data received at home stations but also from those all round the world.

On the basis of many-years experience of the analysis of these reports, it is possible to forecast weather much more correctly than on the basis of local indications alone. Besides, this method permits obtaining weather forecast for a large region. Still better results can be obtained when the meteorologists do not confine themselves to a qualitative comparison of data obtained but use them as starting information for a quantitative analysis of processes occurring in the atmosphere. The laws governing these processes are very complex, while the required calculations are extremely cumbersome. In order to obtain a forecast for practical purposes, high-speed computers are required.

For improving the accuracy of forecasts, it is important to know the general pattern of clouds over the entire Earth. Now, the required information is obtained from space observations. TV pictures of clouds are transmitted to the Earth from the Earth's satellites. Figure 511 represents a TV picture transmitted from a satellite. The eddy spiral structure of clouds indicates a powerful cyclone spread over a very large area (up to 2000 km in diameter).

Chapter 19

Heat Engines

19.1. Necessary Conditions for the Operation of Heat Engines

Windmills (wind motors) are simplest engines which have been used for a long time by man to convert the radiant energy from the Sun into work. The rotation of sails of a windmill driving a shaft which does work is generated by wind. Wind can emerge only if there is a pressure difference, the latter appearing due to the difference in temperature of different parts of the atmosphere. Wind is just a convective motion of the atmosphere caused by its nonuniform heating.

Thus, the energy received from the Sun can be utilised in a windmill for doing work only provided that there is a temperature difference in separate parts of the atmosphere, created by the absorption of the solar radiant energy and partial emission of this energy into space. It has been established that a continuous or periodical work can be done at the expense of cooling bodies only if an engine doing work not only receives heat from a certain body (*heater*) but also gives away a portion of heat to another body (*cooler*). Thus, for doing work only a fraction of heat received from the heater is spent, the remaining heat being transmitted to the cooler.

Machines doing mechanical work as a result of heat exchange with surrounding bodies are called *heat engines*. In most of these engines, heating is attained by burning fuel, which creates a sufficiently high temperature. In such cases, work is done at the expense of the internal energy of a mixture of fuel and atmospheric oxygen. Besides, solar energy and the difference in temperatures of sea water can be used. At present, heat engines utilising heat liberated in nuclear reactors where fission and transformation of atomic nuclei takes place are also put in operation (for details, see Vol. 3 of this book).

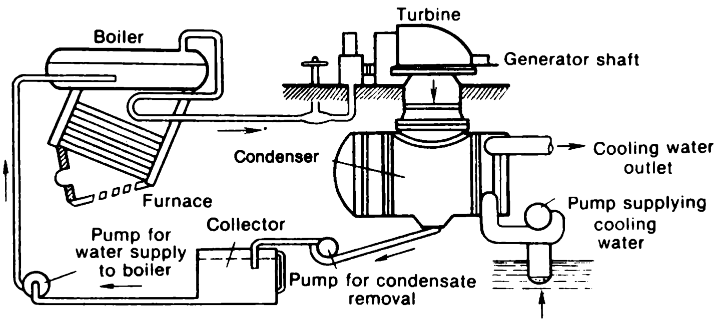

Fig. 512.
Schematic diagram of a steam power plant.

19.2. Steam Power Plant

Steam piston engines (or *steam engines*) were the first heat engines created by man (at the end of 18th century). A hundred years later, steam turbines appeared. The name of these engines implies that work is done by steam in them. In most cases, it is water vapour, although there are engines operating with vapours of other substances (say, mercury). Steam turbines are installed at large-scale power plants or large ships. Piston engines are now used only on railway or inland water transport (steamers).

The operation of a steam engine involves a number of auxiliary machines and mechanisms. They all form a *steam power plant* (Fig. 512). Water circulates in a steam power plant. It is converted into steam in a boiler, the steam performs work in a turbine (or piston engine) and condenses into liquid again in a drum cooled by running water (condenser). The obtained water is delivered by a pump from the condenser through a water tank (collector) back to the boiler. Thus, the circulation of water occurs according to the following scheme:

$$\begin{tikzcd}
\text{boiler} \arrow[r] & \begin{array}{c}\text{turbine}\\ \text{(or piston engine)}\end{array} \arrow[r] & \text{condenser} \arrow[d] \\
\begin{array}{c}\text{delivery}\\ \text{pump}\end{array} \arrow[u] & \text{collector} \arrow[l] & \begin{array}{c}\text{exhaust}\\ \text{pump}\end{array} \arrow[l]
\end{tikzcd}$$

In this scheme, the boiler is a heater and the condenser is a cooler. Since practically the same water circulates in a plant (the leakage of steam is small so that water is added in small amounts), almost no boiler scale, i.e. deposition of salts dissolved in water, is formed. This is important since scale is a poor conductor of heat, and hence it would reduce the efficiency of the boiler. If scale appears on the boiler walls, it should be removed. In

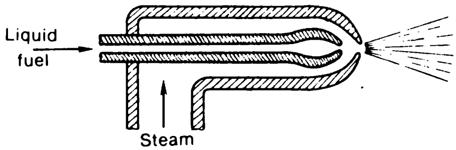

Fig. 513.
Schematic diagram of an atomiser.

the following sections, we shall consider individual parts of a steam power plant separately.

19.3. Steam Boiler

A steam boiler consists of a furnace and the boiler. Coal or wood is burnt in the furnace on a boiler grate. Liquid fuel is burnt in the atomised state. The atomisation is normally carried out with the help of steam by an atomiser (Fig. 513). Steam or compressed air is forced through a nozzle, sucks in the liquid fuel and sprays it (cf. sprayer, Sec. 9.3).

The boiler consists of a drum and pipes transferring heat from hot flue gas to water. Sometimes the gas is passed through the tubes with the water outside (fire-tube boiler, smoke tubes). By contrast, water is sometimes passed through the tubes, while the hot gas flows past on the drum-side (water-tube boiler, Fig. 514). In many steam boilers, steam is subjected to superheating in special coils past which hot gas flows. Here steam is converted from saturated to unsaturated. This reduces the condensation of

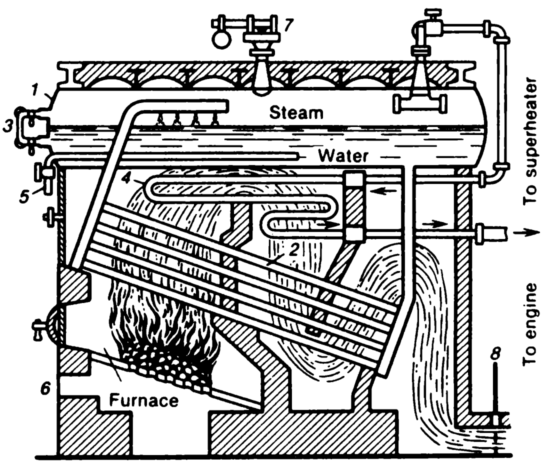

Fig. 514.
Schematic diagram of a water-tube boiler; *1*—boiler drum, *2*—water tubes, *3*—water-level gauge glass, *4*—superheater, *5*—tube for delivering water to the boiler, *6*—wind box, *7*—safety valve, *8*—shutter of the horizontal flue.

steam (on the walls of steam pipe lines and in the turbine), and increases the efficiency of the steam power plant.

A pressure gauge for registering steam pressure and a safety valve releasing steam when its pressure exceeds the permissible value are installed on the boiler. On the bottom of the boiler drum, there is a device for observing the level of water in the boiler (water-level gauge glass). If the level of water drops to such an extent that the flame heats the boiler walls in the regions where there is no contact with water, the boiler may explode.

The energy of hot flue gas is transferred to water in the boiler only partially. A fraction of this energy dissipates in the boiler room, while another fraction is carried away by smoke through the chimney. Besides, a considerable energy loss can be associated with incomplete combustion of fuel. An indication of incomplete combustion is the black smoke flowing from the chimneys of a power plant. The black colour of the smoke is due to unburnt coal particles.

? **19.3.1.** If a boiler is heated by a fire and part of the boiler's heated skin is not in contact with water, there will be an explosion. Why?

19.4. Steam Turbine

Steam from the boiler is delivered through a steam line to a turbine or a piston engine. Let us first consider a turbine (Fig. 515*a*). It consists of a steel cylinder containing shaft *cc* with turbine wheels. A turbine wheel is fitted with special bent blades (Figs. 515*b* and 516 showing a turbine wheel with a nozzle). Nozzles or guide vanes are fixed between the turbine wheels. Steam ejected in the gaps between the vanes encounters the blades of a turbine wheel and sets it in rotation. In this process, the turbine wheel does work. The reason behind the rotation of a steam turbine wheel is the reaction of the steam jet (see Sec. 9.5). Inside the turbine, steam expands and is cooled. Having entered the turbine through a narrow pipe, it leaves the turbine through a very wide pipe (see Fig. 515*a*). It should be noted that the turbine can rotate only in one direction, and the speed of its rotation cannot vary in a wide range. This makes it difficult to use steam turbines in transport but makes them very convenient for rotating electric current generators.

It is very important for electric power plants that turbines can be designed for a huge power (to 1 000 000 kW and higher) which considerably exceeds the peak power of other types of heat engines. This is owing to the uniform rotation of the turbine shaft. The operation of a turbine is free of shocks which accompany the reciprocating motion of the piston in steam piston engines.

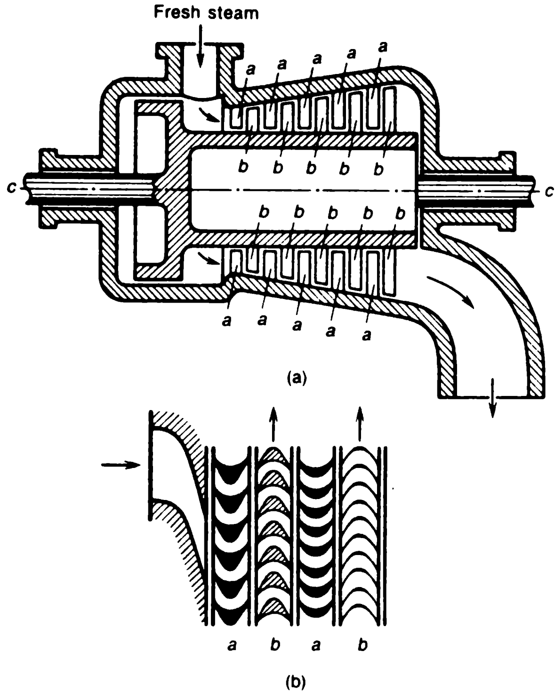

Fig. 515.
(*a*) Schematic diagram of a steam turbine. (*b*) The arrangement of turbine blades on the shaft *cc*: *a*—nozzles and *b*—blades.

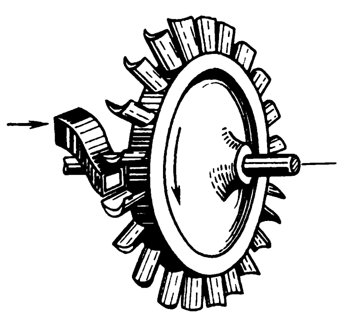

Fig. 516.
Blades on the wheel of a steam turbine.

19.5. Steam Piston Engine

The main features of a steam piston engine which was invented in the late 18th century[1] have been retained to the present time. Steam engine provided a new and powerful tool for the development of engineering which had never used engines before. Now, steam engines are partially replaced by other types of engines. However, steam engines have certain advantages which sometimes make them preferable in comparison with a turbine. These are the simplicity of maintenance and the possibility to vary the speed and reverse the direction of rotation.

A steam engine is shown in Fig. 517. Its main part is a cast-iron cylinder *1* with piston *2* in it. Near the cylinder, there is a valve-gear mechanism which consists of a steam chest connected to a steam boiler. Besides, the chest is connected with a condenser through opening *3* (in locomotives, the chest is just connected with the atmosphere through a smoke funnel) and with the cylinder through ports *4* and *5*. The chest contains a slide valve *6* driven by a special mechanism through tie-rod *7* so that when the piston moves to the right (Fig. 517*a*), the left-hand part of the cylinder is connected through port *4* with the steam boiler, while its right-hand part is connected with the atmosphere through port *5*. Fresh steam enters the cylinder from the left, while exhaust steam is ejected to the atmosphere from the right-hand part. When the piston moves to the left (Fig. 517*b*), the slide valve moves so that fresh steam enters the right-hand part of the

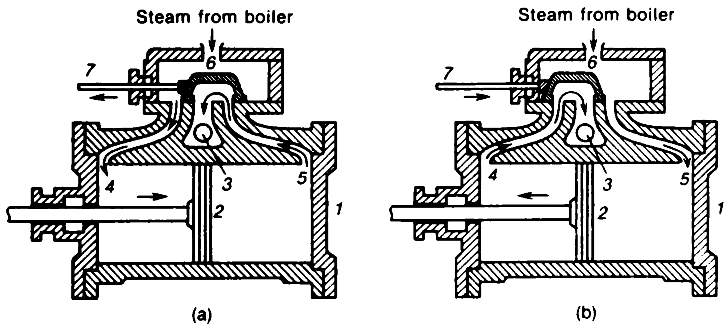

Fig. 517.
Schematic diagram of the cylinder and steam chest of a steam engine. (*a*) Steam enters the cylinder from the left. (*b*) Steam enters the cylinder from the right.

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[1] F. Engels wrote that steam engine was actually the first international discovery. He mentioned the names of Papin (Frenchman), Leibniz (German), Savery and Newcomen (Englishmen), as well as James Watt (Englishman) who gave steam engine its modern form. At his time, Engels had no information about the Russian mining engineer I. I. Polzunov (1728-1766) who worked in the Urals and Siberia and developed steam engine 21 years earlier than James Watt.

cylinder, while exhaust steam is ejected to the atmosphere from the left-hand part.

Steam is delivered to the cylinder not during the entire stroke but only at its beginning. Owing to the special shape of the slide valve, the steam is cut off (stops being delivered to the cylinder) and the expanding and cooling steam performs work. Steam cut-off saves a large amount of energy. In steam engines, two (and sometimes more) cylinders are installed. Steam is first delivered to the first cylinder and then to the second. Since steam expands in the first cylinder, the diameter of the second cylinder is considerably larger than that of the first one. Fire-tube boilers are normally used in steam engines and a steam superheater is installed.

In old steam engines, steam was ejected into the atmosphere. In new powerful steam engines, condensers are installed, and steam circulates in the same way as in a steam power plant.[2]

? **19.5.1.** What is the average pressure of steam in the cylinder of a steam engine, if the piston stroke is 40 cm, the area of the piston is 250 cm^2 and the engine power is 15 kW at a speed of 120 rpm? Take into account the fact that the engine performs two strokes during one revolution of the shaft.

19.6. Condenser

As was mentioned in Sec. 19.2 steam ejected from a turbine or a steam piston engine flows to a condenser which plays the role of a cooler. In the condenser, steam must condense into water. However, steam condenses into water only if the latent heat of evaporation liberated during condensation is removed. This is done with the help of cold water. For example, a

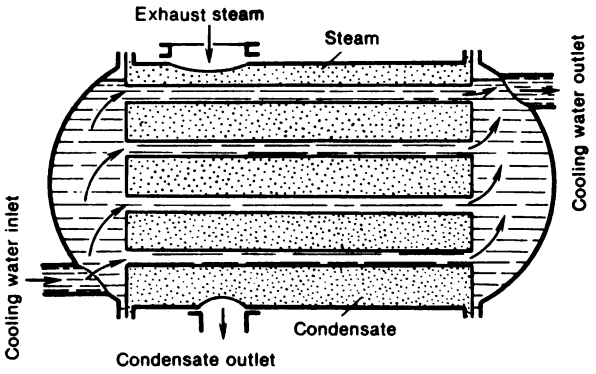


Fig. 518.
Schematic diagram of a condenser.

[2] At present, steam engines have all but been replaced by diesel or electric locomotives.

condenser can be made in the form of a drum with tubes of running cold water installed in it (Fig. 518). Exhaust steam flows past tubes with cold water, and condenses. The condensate obtained is pumped from the condenser through the tube shown in its lower part. The steam pressure in condensers is normally lower than the atmospheric pressure (0.02-0.03 atm). A special pump is used for removing the water (condensate) and the air entrained by it from the condenser.

19.7. Efficiency of Heat Engines

The purpose of a heat engine is to do mechanical work. It was mentioned earlier (see Sec. 19.1) that only a part of heat received by an engine is spent on doing work. The ratio of the mechanical work done by the engine to the spent energy is called the *efficiency of the engine.*

Let us calculate the energy spent in an engine. Normally, this is the energy of the oxygen-fuel mixture. It can be easily estimated if we know the amount of fuel and its *specific latent heat of combustion*, i.e. the amount of heat liberated upon complete combustion of 1 kg of the fuel. Specific latent heats of combustion of various types of fuel are determined by burning a small portion of a fuel in a closed vessel placed into a calorimeter. Specific latent heat of combustion of some fuels is given in Table 25 (the figures are rounded).

Table 25. Specific Latent Heat of Combustion of Some Fuels

Fuel	Specific latent heat of combustion, MJ/kg	Fuel	Specific latent heat of combustion, MJ/kg
Kerosene	44	Brown coal	20
Petrol	46	Wood	10
Coal	30		

Let us consider the following example. Suppose that 3 kg of petrol have been burnt in an engine. The energy liberated as a result is 46 MJ/kg × 3 kg = 138 MJ. If the engine has done as a result a work of 29 MJ, its efficiency is 29 : 138 = 0.21, i.e. 21%.

19.8. Efficiency of a Steam Power Plant

Figure 519 shows the energy balance of a steam power plant with a turbine. This is an approximate balance since the efficiency of a steam power plant can sometimes be higher (up to 27%). Energy losses taking place during the operation of a steam power plant can be divided into two parts. A part of losses is due to imperfections of the design and can be reduced without changing the temperature in the boiler or the condenser. The losses in the

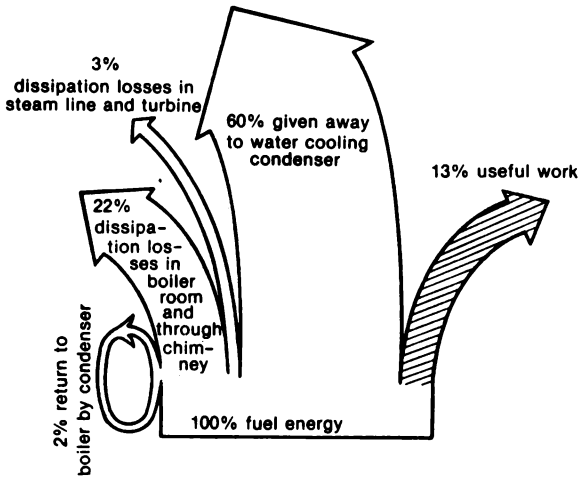


Fig. 519.
Approximate energy balance of a steam turbine power plant.

boiler room, for example, can be reduced by improving the thermal insulation of the boiler. The other part of losses (viz. the heat transferred to water cooling the condenser) is considerably larger and is inevitable for given temperatures in the boiler and the condenser. It was mentioned above (see Sec. 19.1) that the necessary condition for the operation of a heat engine is not only the obtaining of a certain amount of heat from a heater, but also the transfer of a fraction of this heat to a cooler.

Scientific and technical experience of design of heat engines as well as a detailed theoretical analysis of their operating conditions proved that the efficiency of a heat engine depends on the *temperature difference* of the heater and the cooler. The larger this difference, the higher the efficiency that can be attained for a given steam power plant (naturally, provided that all technical imperfections in the design, mentioned above, are eliminated). If, however, this difference is not large, even the most perfect (from the point of view of engineering) machine cannot have a high efficiency. A theoretical analysis shows that if the thermodynamic temperature of the heater is T_1 and that of the cooler is T_2, *the efficiency cannot exceed*

$$\eta = \frac{T_1 - T_2}{T_1} .$$

For example, for a steam engine in which the temperature of steam in the boiler is 100 °C (or 373 K), and the temperature of the cooler is 25 °C (or 298 K), the efficiency cannot exceed (373 − 298)/373 = 0.2, i.e. 20% (because of the imperfections in the design, the efficiency of such a plant will be considerably lower in actual practice). Thus, in order to improve the

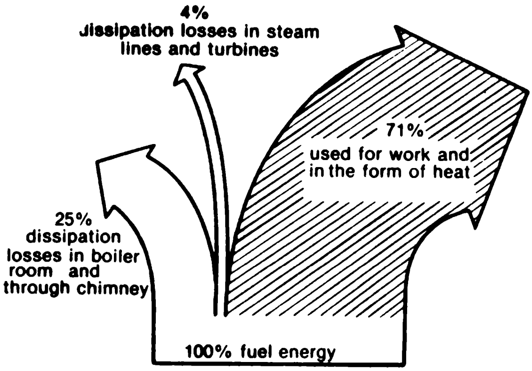


Fig. 520.
Approximate energy balance of a thermoelectric plant.

efficiency of heat engines, it is necessary to raise the temperature in boilers, thus increasing the steam pressure. Unlike old power plants operating under a pressure of 12-15 atm (which corresponds to a steam temperature of 200 °C), the boilers designed for 130 atm and higher pressure (a temperature of about 500 °C) are operating in modern steam power plants.

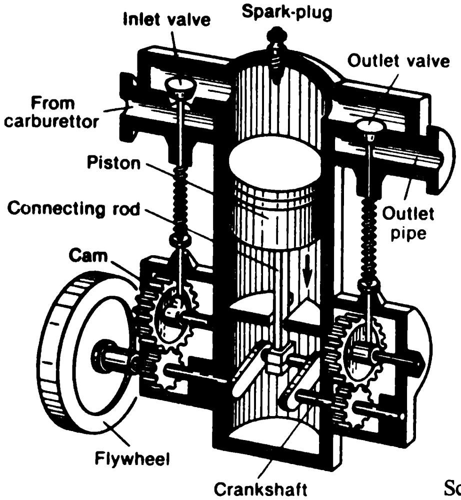
Instead of increasing the temperature in the boiler, we could decrease the temperature in the condenser. This, however, was found to be unfeasible. At very low pressures, the density of steam is very small, and for a very large volume of steam passed per second through a high-power turbine, the volume of the turbine and its condenser would be too large.

Besides improving the efficiency of a heat engine, it is also possible to use "heat wastes", i.e. the heat removed by water cooling the condenser. Instead of discharging warm water to a river or lake, it can be passed through a hot-water heating system or used for technological purposes in chemical or textile industries. It is also possible to let steam expand in turbines only to a pressure of 5-6 atm. A still very hot steam is discharged in this case from a turbine, which can serve some industrial purposes.

A power plant using heat wastes not only supplies the electric power (which is obtained at the expense of mechanical work) to consumers but also delivers heat to them. In such a case, it is called a *thermoelectric plant*. An approximate energy balance of a thermoelectric plant is represented in Fig. 520.

19.9. Petrol Internal Combustion Engine

Let us now consider other types of heat engines. The most widespread type of modern heat engine is the *internal combustion engine*. These engines are installed on motor cars, aeroplanes, tanks, tractors, motor boats, and so



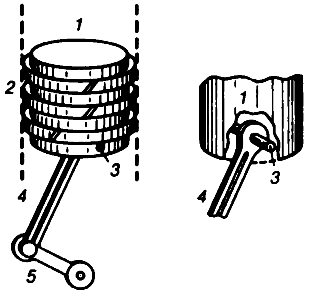


Fig. 522.
The piston of an internal combustion engine. On the right, the connecting rod fixed to the piston is shown.

Fig. 521.
Schematic diagram of a motor car engine.

on. Internal combustion engines can operate on liquid fuels (petrol, kerosene, etc.) or on a fuel gas stored in the compressed form in steel cylinders or obtained from dry distillation of wood (gas producer engines).

We shall consider a four-stroke petrol engine of the motor car type. The engines installed on tractors, tanks and aeroplanes generally have a similar design.

The main part of an internal combustion engine is one or several cylinders *in* which the combustion of a fuel takes place (Fig. 521). Hence the name of the engine. In a cylinder, a piston can move (Fig. 522). The piston is a hollow cylinder *1* closed at one end, with encircling rings *2* (piston rings) fitted into grooves on it. The purpose of piston rings is to keep gases formed during the combustion of the fuel from flowing into the gaps between the piston and the cylinder walls (shown by dashed line). The piston is supplied with a metal rod *3* (piston pin) which connects the piston with connecting rod *4*. In turn, the connecting rod serves for transmitting motion from the piston to crankshaft *5*.

The upper part of the cylinder is connected with two channels closed b. valves. One of them, the inlet channel, serves to deliver the air-fuel mixture, while the other is used to discharge combustion products. The valves are made in the form of discs pressed against the openings by springs. The valves are opened with the help of cams mounted on a camshaft. When the shaft rotates, the cams lift the valves with the help of steel rods (valve followers). In addition to the valves, the upper part of the cylinder is fitted with a spark-plug. This is a device for igniting the air-fuel mixture by an electric spark generated by an electric device mounted on the engine (magneto or ignition coil).

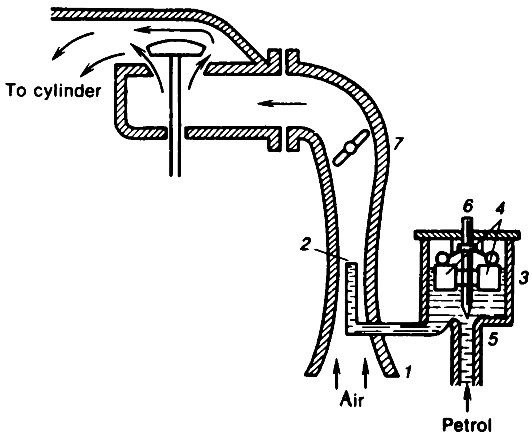


Fig. 523.
Schematic diagram of a carburettor.

An important part of a petrol engine is a carburettor, viz. a device for obtaining the air-fuel mixture. It is shown in Fig. 523. If the inlet valve is open in the cylinder and the piston moves towards the crankshaft, air is sucked in through opening *1*. The air flows past tube *2* connected with a float chamber *3*. This chamber contains petrol maintained with the help of float *4* at such a level that it reaches the end of tube *1*. This is controlled by the float which is lifted by petrol delivered to the chamber and closes opening *5* by a special valve needle *6* and thus terminates the petrol supply as soon as its level is raised. Air flowing at a high velocity past the end of tube *2* sucks in petrol and atomises it (sprayer, see Sec. 9.3). Thus, an air-fuel mixture is formed (petrol vapour and air), whose inflow to the cylinder is controlled by a throttle valve *7*.

The operation of the engine comprises four strokes (Fig. 524).

Stroke I — *suction*. The inlet valve *1* is opened, and piston *2* moving downwards sucks in the air-fuel mixture from the carburettor.

Stroke II — *compression*. The inlet valve is closed, and the piston mov-

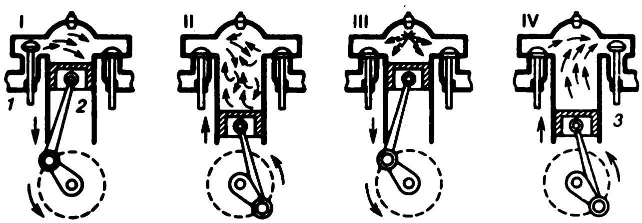


Fig. 524.
Four strokes of an internal combustion engine.

ing upwards compresses the air-fuel mixture. The mixture is heated as a result of compression.

Stroke III — *combustion*. When the piston attains the uppermost position (or even before that if the speed is high), the mixture is ignited by an electric spark generated by the spark-plug. The force of pressure of gases, viz. hot combustion products of the air-fuel mixture, pushes the piston downwards. The motion of the piston is transmitted to the crankshaft, and thus useful work is done. In doing this work and expanding, combustion products are cooled, and their pressure drops. By the end of the working stroke, the pressure in the cylinder drops almost to the atmospheric pressure.

Stroke IV — *exhaust*. The exhaust valve *3* is opened, and waste combustion products are ejected through the exhaust muffler to the atmosphere.

From four strokes of the engine (or two revolutions of the crankshaft), only one, the third stroke is the working stroke. In view of this, a one-cylinder engine has to be supplied with a bulky flywheel whose kinetic energy is used for moving the piston during other strokes. One-cylinder engines are mainly used in motor cycles. In order to obtain a more uniform operation of an engine, four, six or more cylinders are installed on motor cars, tractors, etc. on a common shaft so that every stroke is a working stroke for at least one cylinder. To set an engine in motion, it must be driven by an external force. In motor cars, this can be done with the help of a special motor fed by a storage battery (starter).

It should be added that a necessary part of an engine is a device for cooling the cylinder walls. If the cylinders get too hot, the lubricating oil will overheat, and the air-fuel mixture will prematurely detonate (viz. the mixture does not combust as it should do in normal operation). Detonation not only reduces the output power but also produces a destructive effect on the motor. Cylinders are cooled by running water which gives away heat to air (Fig. 525), or directly by air. Water circulates outside cylinders *1*. The circulation is caused by the heating of the water near the cylinders and its

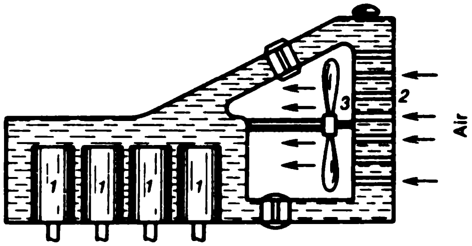


Fig. 525.
Water cooling of the cylinders in an internal combustion engine.

cooling in radiator *2*. The latter consists of a system of copper pipes in which water flows. The water in the radiator is cooled by the air flow sucked in during the rotation of fan *3*.

Besides four-stroke engines, there are two-stroke engines which are not so widely used and will not be considered here.

The internal combustion engines have a number of advantages (compactness and small mass) which explain their wide applicability. On the other hand, the drawbacks of these engines are that (a) only a high-quality fuel can be used; and (b) it is impossible to obtain small speed of rotation (at a low speed, the carburettor does not work). This calls for various mechanisms intended to reduce the speed of rotation (say, gear transmission).

? **19.9.1.** What is the power of a four-cylinder engine operating at 300 rpm if the average pressure is 5 atm, the piston stroke is 0.3 m, and the piston area is 120 cm^2?

19.10. Efficiency of Internal Combustion Engines

When analysing the operating conditions of an internal combustion engine, we can find an analogy with the operating conditions of a steam engine. Here too a temperature difference is created: on the one hand, a heat source (which in the case under consideration is the chemical reaction of burning) creates a high temperature of air-fuel mixture, on the other hand, there is a huge reservoir in which the obtained heat dissipates (this is the atmosphere which plays the role of a cooler).

Since the temperature of gases obtained as a result of the combustion of the mixture is rather high (more than 1000 °C), the efficiency of internal combustion engines can be considerably higher than the efficiency of steam engines. In actual practice, the efficiency of internal combustion engines is

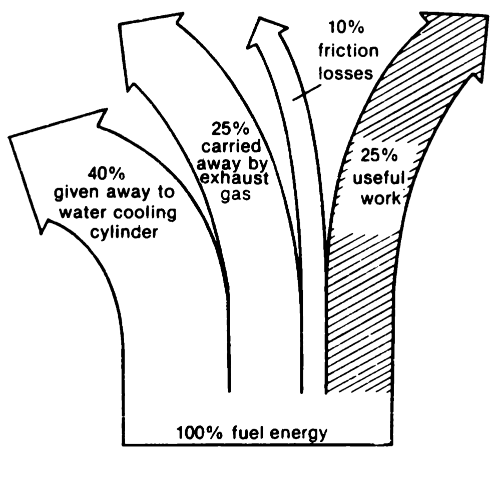


Fig. 526.
Energy balance of a motor car engine.

normally 20-30%. The approximate energy balance of a motor car engine is shown in Fig. 526.

? **19.10.1.** An engine whose power is 7.35 kW consumes 2.8 kg of petrol per hour. What is its efficiency?

19.10.2. What work can be done if 0.5 kg of petrol is burnt in an engine having an efficiency of 20%?

19.11. Diesel Engine

How can the efficiency of an internal combustion engine be increased? Calculations and experiments show that a higher compression ratio (viz. the ratio of the maximum and minimum volumes of a cylinder, Fig. 527) must be attained. At a higher compression, the air-fuel mixture is heated to a larger extent, and a higher temperature is attained during the combustion of the mixture. However, it is impossible to use a compression ratio higher than 4-5 in engines of the motor car type. At a higher compression ratio, the air-fuel mixture is heated during the second stroke to such an extent that it ignites prematurely and detonates.

This difficulty was overcome in an engine (called Diesel engine, or simply a *diesel*) designed by R. Diesel at the end of the 19th century. The schematic diagram of a diesel is shown in Fig. 528. Instead of the air-fuel mixture, pure air is compressed in a diesel. The compression ratio is equal to 11-12 and the air gets heated to 500-600 °C. At the end of the compression, a liquid fuel, say, petroleum, is sprayed into the cylinder. This is done with the help of a special injector operating on compressed air delivered by a compressor.[3] The ignition of atomised and evaporated petroleum is caused by a very high temperature obtained in the cylinder as a result of compression and does not require any auxiliary igniting device. During the combustion of petroleum, that lasts considerably longer than the combustion of air-petrol mixture in a motor car engine, the piston moves down

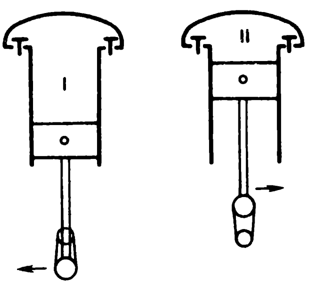


Fig. 527.
The compression ratio is the ratio of the volume of the gas in the cylinder with the piston in position I to the cylinder volume with the piston in position II.

[3] In some types of diesel, there is no compressor, and petroleum is sprayed by a pump creating a very high pressure.

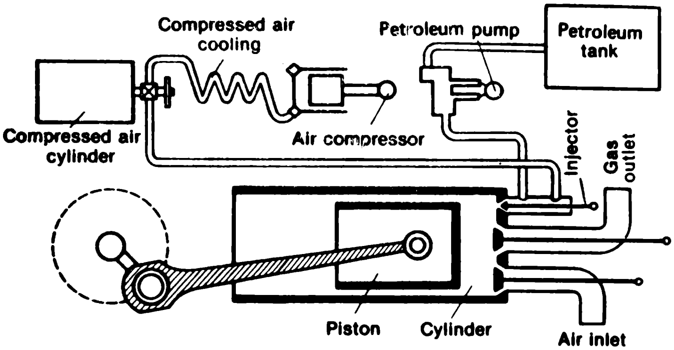


Fig. 528.
Schematic diagram of a diesel.

and does work. Combustion products are then discharged from the cylinder.

Diesels turned out to be more efficient than petrol engines (their efficiency reaches 38%). Its output power can be considerably higher. Diesels are installed on ships (motor vessels), diesel locomotives, tractors, trucks and small electric power plants. A great advantage of the diesel is that it operates on cheap "heavy" grades of fuel and not on expensive purified petrol. Besides, diesels do not require an ignition system. However, when the minimum weight of engine is required for a given power, diesels turn out to be less expedient.

19.12. Jet Engines

In Sec. 9.9, we considered the action of a reactive jet which drives jet aeroplanes and rockets. A reactive jet is created by a jet engine which is essentially an internal combustion engine. Figure 529 shows a schematic diagram of a type of reaction propulsion engines installed on jet

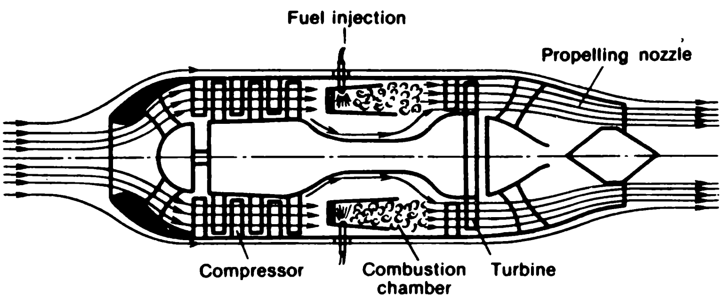


Fig. 529.
Schematic diagram of a jet engine.

aeroplanes. The engine is enclosed in a cylindrical shell open at the front (air inlet) and at the rear (propelling nozzle). Air enters the inlet (as shown by arrows) and gets into a compressor consisting of a number of blades fixed to rotating wheels. The compressor forces air along the axis of the engine, thus increasing its density. Then air flows to a chamber into which a fuel is sprayed. The obtained air-fuel mixture ignites, producing gases at a high temperature and pressure. The gases flow to the propelling nozzle, driving on their way a gas turbine coupled with the compressor and then are ejected from the propelling nozzle. As was explained in Sec. 9.8, the gases leaving the engine acquire a very high velocity in the backward direction and exert on the plane a reaction in the forward direction. This force just drives the jet aeroplane.

19.13. Heat Transfer from a Cold to a Hot Body

It has been shown with the help of several examples that work is done when heat is transferred from a hot body (heater) to a cold one (cooler), the cooler receiving a smaller amount of heat than that given away by the heater. The internal energy of the heater decreases not only because it transfers heat to the cooler but also as a result of doing work.

Let us find the conditions under which the reverse process, viz. heat transfer from a cold to a hot body takes place.

An example of this kind is a refrigerator used in food industry (for making ice cream, storing meat, etc.). The schematic diagram of a compressor refrigerating machine is in a sense the inverse to that of a steam power plant. It is shown in Fig. 530. The working medium in the refrigerating machine is ammonia (sometimes carbon dioxide, sulphur dioxide or polyhalogenated hydrocarbons known as Freons). Compressor

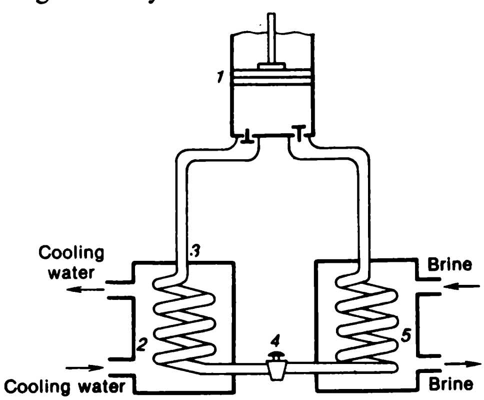


Fig. 530.
Schematic diagram of a compressor-type refrigerating machine.

1 delivers ammonia vapour under a pressure of 12 atm to coil *2* (which corresponds to a condenser). During compression, the ammonia vapour is heated and then cooled in tank *3* by running water. Here, the vapour condenses into liquid. From coil *2*, the ammonia flows through valve *4* to another coil *5* (evaporator) where the pressure is about 3 atm.

While passing through the valve, the ammonia partially evaporates and its temperature drops to -10 °C. From the evaporator the ammonia is sucked off by a compressor. Evaporating ammonia receives heat required for evaporation from a salt solution (brine). As a result, the brine is cooled to about -8 °C. Thus, the brine plays the role of the cold body giving heat to a hot body (running water in tank *3*). The jet of cooled brine is passed through the pipes in the room to be cooled. Manufactured ice is obtained by immersing into the brine metal boxes filled by pure water.

Besides compressor refrigerating machines, absorption refrigerating machines are used for domestic purposes, where the compression of a working gas is attained not with the help of a compressor but by absorption (dissolution) in an appropriate substance. For instance in a household refrigerator (Fig. 531), a strong aqueous solution of ammonia (NH_3) is heated by electric current in generator *1* so that gaseous ammonia is liberated, whose pressure attains 20 atm. The gaseous ammonia is dried (in a dryer which is not shown in the diagram), and condenses in condenser *2*. The liquefied ammonia is sent to evaporator *3* where it again transforms into the gaseous state, taking a considerable amount of heat from the evaporator. The gaseous ammonia is absorbed (dissolves in water) in absorber *4* where a strong ammonia solution thus formed again flows to generator *1*, displacing the weak solution (after the liberation of the gas) to the absorber. In this way, a continuous cycle is formed. The evaporator (which is strongly cooled during the evaporation of ammonia) is placed inside the volume to be cooled (refrigerated cabinet), all the remaining parts being mounted outside the cabinet.

A question arises: why is the gaseous ammonia liquefied in the condenser and evaporated in the evaporator although the temperature of the evaporator is lower than the temperature of

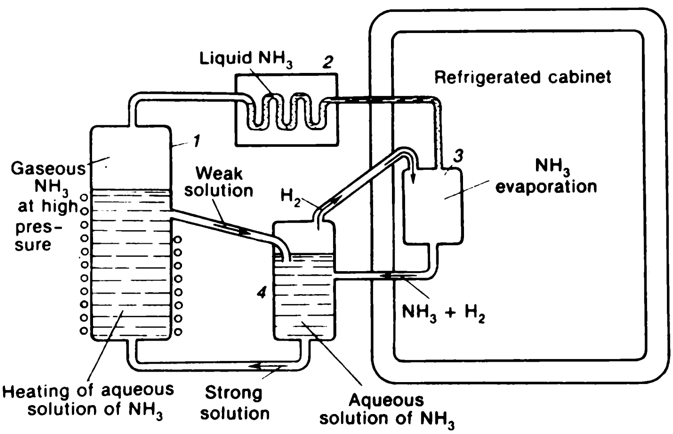


Fig. 531.
Schematic diagram of an absorption refrigerating machine.

the condenser? This is attained due to the fact that the entire system is filled with hydrogen under a pressure of 20 atm. When the generator is heated, the gaseous ammonia is liberated from the boiling solution, its pressure reaching 20 atm. The gaseous ammonia displaces hydrogen from the upper part of the generator and the condenser to the evaporator and the absorber. Thus, ammonia in the condenser is under its own high pressure and hence liquefies at a temperature close to room temperature, while liquid ammonia gets into the evaporator under a low partial pressure. The hydrogen present in the evaporator provides the required total pressure equal to the pressure in the condenser and other parts of the system.

The mixture of the hydrogen and gaseous ammonia is delivered from the evaporator to the absorber where the ammonia is dissolved in water (which causes the heating of the solution), while the hydrogen flows through the warm solution, is heated there, and owing to convection flows to the cold evaporator. New portions of ammonia replace the ammonia that has been dissolved and evaporate in the evaporator, which causes a further cooling of the evaporator. The advantage of this design is that it has no moving mechanical parts. The circulation of ammonia solution (between *1* and *4*) and the circulation of hydrogen (between *4* and *3*) are caused by the difference in densities due to the temperature difference (solution in *1* is warmer than in *4*, while the hydrogen in *4* is warmer than in *3*).

Thus, in order to transfer heat from a cold body to a hot one, work must be done by an external force. In this process, the hot body receives not only the amount of heat given away by the cold body but also the heat equivalent to the work done.

? **19.13.1.** A refrigerator installed in a room is driven by an electric motor using current from the mains. Will the room get cooler?

Answers and Solutions

1.5.1. The points on the cylinder axis. **1.9.1.** 25 cm. **1.9.2.** 6 m/s. **1.12.2.** 10 m. **1.12.3.** $s = s_0 + vt$. **1.12.4.** $s = s_0 + v(t - t_0)$. **1.12.5.** 2.7 s; -2 m; 0.75 m/s. **1.12.6.** 60 min; 40 km. **1.12.7.** 1 h. **1.12.8.** Zero. **1.14.2.** 40 km/h. **1.19.1.** (I) $(2 + 2t/3)$ m/s; (II) $(6 - 3t/2)$ m/s; (III) $(-6 + 3t)$ m/s; (IV) $(-1 - t)$ m/s. **1.22.1.** $s = (v^2 - v_0^2)/2a$; $v = \sqrt{v_0^2 + 2as}$. **1.22.4.** 330 m. **1.22.5.** 4 m/s. **1.22.6.** About 14 m/s. **1.22.7.** 32 m. **1.24.1.** 500 km. **1.28.2.** 2 km. **1.28.3.** 12 h. **1.28.4.** 2 times. **1.28.5.** 10 m/s. **1.28.6.** (a) 10 m/s; (b) zero. **1.28.7.** 2 h 40 min. **2.16.1.** 1.5×10^6 N. **2.16.2.** About 3 s. **2.18.1.** 9.8 N. **2.18.2.** The distances are inversely proportional to the ratio of masses. **2.22.1.** 4 m/s. **2.25.1.** 15 m. **2.25.3.** (a) $v = \sqrt{2gh}$; (b), (c) $v = \sqrt{v_0^2 + 2gh}$. **2.25.4.** 15 m; 20 m/s. **2.25.5.** Approximately 1.4. **2.25.6.** 45 m. **3.4.2.** 49 N; 69 N. **3.4.3.** 17.3 kN. **3.4.4.** 14°; 101 N. **3.5.1.** *BC*: 10 N; *CG*: 11.6 N; *CD*: 5.8 N; *DE*: 10 N. **3.6.1.** $m = M/\sqrt{2} = 1.4$ kg; 7.3 N. **3.6.2.** About 56°. **3.13.1.** At a distance of 3 cm from the point of joint. **3.13.3.** 147 N, 441 N. **3.15.2.** Yes, it is, since the centre of gravity of the ruler rises as a result of inclination. **3.15.4.** 12 cm. **3.16.1.** Gain in force is equal to two. **4.7.1.** 1.8×10^6 J. **4.7.2.** 9600 J. **4.10.1.** 196 N. **4.15.2.** Larger work is done to speed up the train from 5 to 10 m/s. **4.16.1.** 25 m/s; 20 m/s at zero initial velocity. **4.18.1.** 625 N. **4.18.2.** 28 kN. **4.18.3.** 3200 J. **4.21.1.** 0.7 mW. **4.21.2.** 6×10^4 N. **4.21.3.** About 1.4 kW. **4.22.1.** 8 times. **4.22.2.** 400 kW. **4.24.1.** 96%. **4.24.2.** About 4500 J. **4.24.3.** 69%. **4.24.4.** 54%. **4.24.5.** 43 kN. **4.24.6.** No, it cannot. **5.3.1.** 25 m/s, in 1.5 s. **5.3.2.** 4.5 m/s. **5.4.1.** 7 m/s, 9.8 m/s; 4.9 m. **5.4.2.** 45 m. **5.6.1.** 1/2. **5.6.2.** The angular velocity of the hour hand is twice as large as the angular velocity of the rotation of the Earth. **5.7.1.** 33 N. **5.7.2.** 7.7 m/s. **5.9.1.** About 4 s^{-1}. **5.10.1.** For $m_1/m_2 = r_2/r_1$, i.e. when the common centre of gravity is at point *O*. **5.10.2.** 14.8 N. **5.10.3.** 7 rad/s. **5.11.1.** $v = \sqrt{rg/2}$; $v = \sqrt{rg}$. **5.15.1.** 16.4%. **5.15.2.** 101 N. **5.15.3.** The required point is separated from the centre of the Moon by one tenth of the distance between the Earth and the Moon. **5.16.1.** With the orbital velocity. **5.16.2.** At a distance of $6.62R_{\text{Earth}}$ from the centre of the Earth, i.e. at an altitude of 36 000 km above the Earth's surface. **6.3.1.** $\tan\alpha = w/g$; $T = m\sqrt{a^2 + g^2}$. **6.3.2.** $F = mw$. **6.9.2.** At $T \approx 80$ min. **7.3.1.** 0.9 l. **7.11.1.** Yes, it is. **7.15.1.** 10 m, 40 m. **7.16.1.** 168 cm. **7.17.1.** By 27.2 mm. **7.18.1.** 5.5 atm. **7.18.2.** 29 m. **7.23.1.** 12.25 N. **7.23.2.** 8.88 N. **7.23.3.** 4 m. **7.23.5.** A load of 4.26 g must be put on the pan from which copper is suspended. **7.24.1.** 2.3×10^3 kg/m^3. **7.24.2.** $\rho = \rho_1(G - G_2)/(G - G_1)$, where ρ_1 and ρ_2 are the densities of water and liquid under investigation. **7.24.3.** 0.91×10^3 kg/m^3. **7.24.4.** 0.24×10^3 kg/m^3. **7.25.4.** 0.8. **7.25.5.** 0.43; the ball will rise a little. **7.25.6.** 4.6 kg. **7.25.7.** (a) 0.736; (b) 0.054. **8.3.1.** The pressure in a liquid rapidly drops upon its expansion; therefore, a compressed liquid cannot cause the destruction of tissues during its expansion. **8.6.1.** The height of water column is 73 cm and that of the mercury column is 5.3 cm. **8.7.1.** To 70 cm. **8.8.1.** The value of the division should be doubled. **8.8.2.** About 67 kg. **8.9.1.** Outwards. **8.11.1.** About 1 N. **8.11.2.** No, it should not. **8.12.1.** 1000 kg, 865 kg. **10.1.1.** Steel expands more than wood. **10.1.2.** No, it will not. **10.1.3.** A gap (leak) may be formed between glass and wire. **10.1.4.** The liquid would rise in the neck of the flask immediately after the flask had been immersed in hot water. The total height to which the liquid in the flask neck rises would be larger. **10.1.5.** On the lower side. **10.2.2.** About 99 °F. **10.3.1.** 555 mm. **10.3.2.** From -10 °C to $+50$ °C. **10.3.3.** 19.96 mm. **10.5.1.** 50.12 ml. **10.6.1.** 1.1×10^{-3} K^{-1}. **11.2.1.** If the velocity of the lowering loads is small, their kinetic energy can be neglected while calculating the change in the mechanical energy, and it can be assumed that the only result of the work done is the change in the temperature of the liquid in the vessel. **11.8.1.** The volume will not change. **11.8.2.** 23 °C; if hot water is poured first, the final temperature is below 23 °C, while if cold water is poured first, the final temperature is above 23 °C (provided that the temperature of the surroundings is between 50 and 10 °C). **11.11.1.** Inside the filament. **11.11.2.** Heat transfer from the flame to the paper from below is uniform over the entire surface of contact between the flame and paper. The heat transfer from the paper to the surrounding air is lower than that from paper to air via the pin at the same temperature difference. Therefore, the paper under the pin is colder than the remaining paper. **11.11.3.** The gaps between fibres contain air whose thermal conductivity is low. **11.11.4.** The zinc mug. **11.11.5.** A drop on a red-hot plate is separated from it by a layer of poorly conducting water vapour. If the plate is not so hot, the water drop is in close contact with it. **11.11.6.** Convective flows would be absent in the liquid, and its lower layers would have a much higher temperature than the upper layers. **11.11.7.** In a free fall of the jar, there are no convective air flows in it. **11.11.8.** The thermal conductivity of hydrogen is higher than that of air. **13.3.1.** (a)-(c) The position of mercury levels will remain unchanged; (d) the level of mercury in the right arm will rise. **13.3.2.** 0.78 atm. **13.3.3.** The pointer of the instrument will move beyond the red mark. **13.3.4.** 2.68×10^{-3} K^{-1}. **13.5.1.** Air is pumped into the tyre so rapidly that the heat exchange with the surroundings is insignificant, and the air compressed in the tyre is heated. At the same time, the pump walls are also heated a little. If the process is repeated many times, the heating of the walls becomes noticeable. **13.7.1.** 1000 mm Hg. **13.7.2.** 5.3 atm. **13.7.3.** About 6.5 mm^3. **13.7.4.** 36 cm^2. **13.8.2.** The areas are equal. **13.9.1.** 0.136 kg. **13.12.1.** 865 mm^3. **13.14.1.** It is not. **13.16.1.** 1.17 m^3. **13.16.2.** About 2.4 m/s. **13.17.1.** Yes, it will. **13.18.2.** 66 cm^3. **13.18.3.** 529 °C. **13.18.4.** 894 l. **13.18.5.** 0.059 kg/m^3. **13.18.6.** About 1200 m^3. **13.22.2.** 2.7×10^{25} m^{-3}. **13.22.3.** 3.3×10^{-27} kg; 5.3×10^{-26} kg. **13.23.2.** 1200 m/s; 36 m/s. **13.23.3.** 3800 m/s; 358 m/s. **13.24.1.** Gas molecules move at different velocities. **13.26.1.** 743 J/(kg·K); 1039 J/(kg·K). **13.26.2.** 929 J/K. **13.27.1.** 232 J/(kg·K), 136 J/(kg·K). **14.2.2.** The film contracts to a spherical drop of a very small size due to a small thickness of the film. **14.3.1.** 11.8 μJ. **14.3.2.** 2.5 mJ. **14.3.3.** 435 J, 435 J. **14.6.1.** Cohesive forces between molecules of water and glass are stronger than those acting between water molecules. Water sticks to glass until a drop of a sufficiently large size is accumulated. On the contrary, the cohesive forces between mercury and glass molecules are weaker than the cohesive forces between mercury molecules, and therefore mercury is not accumulated near the surface of glass. **14.6.2.** In addition to the force of gravity, water being poured experiences the action of cohesive forces which make the water jet change its direction of flow. **14.6.3.** Since grease is not wetted by water, there is no layer of water above the blade, and it sinks until the upward water pressure balances the force of gravity acting on the blade. When the blade is not greasy, water covers it completely. **14.6.4.** The molten solder wets a clean metal surface and does not wet an oxidised surface. **14.8.1.** The excess pressure in a small bubble is higher than that in a large bubble. **14.8.2.** Flow rate is higher at a small diameter of the bubble. **14.8.3.** The free surface of a drop between two plates is saddle-shaped. It can approximately be considered as a cylindrical surface with the radius of curvature equal to half the separation of the plates. Since this surface is concave, the pressure in the drop is lower than the atmospheric pressure, the difference between these pressures being the larger, the smaller the radius of curvature. The force pushing the plates together is the stronger, the larger the difference between the atmospheric pressure and the pressure in the drop and the larger the area over which this pressure difference exists. **14.8.4.** If the curvature of a drop is the same on both sides in a narrow region of the tube, the gas pressure on both sides of the drop is the same. As soon as the drop shifts (say, to the right), the radius of curvature becomes higher on the right-hand side and lower on the left-hand side. As a result, the pressure difference appears, which prevents further displacement of the drop. If the number of drops in the tube is large, the resistance to air blown through the tube becomes significant. **14.8.5.** The mass of a separating drop is the larger, the higher the surface tension of the liquid. **14.9.1.** Water wets chalk, penetrates its pores and displaces air from them. **14.9.2.** Water rises the higher, the smaller the wall-to-wall distance, and hence the radius of curvature of water surface. **14.9.4.** The surface tension of hot water is lower than that of cold water, and hence the height to which hot water rises should be smaller. On the other hand, the density of hot water is lower than that of cold water, and this must lead to an increase in the height of the water column. The fact that the height of the water column in the capillary in hot water is smaller than in the same experiment with cold water indicates that the change in the surface tension of water with temperature is larger than the change in water density. **14.9.5.** In the absence of shocks, water in the left tube can be raised to the same level as that in an ordinary capillary of the same diameter. When the right tube is lifted slowly, the meniscus in its vertical part will be kept at the same level until it reaches the horizontal segment, after which it rapidly goes over to the next vertical part. **14.9.6.** The free surface of water in the straight capillary and in the bend is concave upwards. Therefore, capillary forces pull water up in the straight tube as well as in the bent one. **14.10.1.** 5.8 cm, 2.2 cm. **14.10.2.** 0.021 N/m. **15.7.1.** 118 g. **15.13.2.** On the pavement sprinkled with salt. **16.2.1.** A residual deformation appears in the balance spring. **16.4.1.** The elongation will be half the previous value. **16.4.2.** 9 mm. **16.6.1.** 0.36 mm. **16.7.2.** Bones of animals, bird feathers and stalks of plants. **16.7.3.** 9. **16.8.1.** 734 kg. **16.8.2.** 180 m. **17.1.1.** No, it is not. It contains mercury vapour. **17.4.1.** Since air has already been saturated by ether vapour, ether does not evaporate any longer. **17.6.1.** The saturated vapour pressure for water changes as a result of heating according to a different law than that describing the temperature variation of gas pressure. **17.6.3.** At a temperature corresponding to point A on the graph, the entire liquid evaporates. **17.7.3.** 120 °C, 60 °C. **17.7.4.** 140 °C. **17.7.6.** When the bottom of the flask is cooled, the vapour pressure above water becomes lower than the saturated vapour pressure corresponding to the temperature of water in the flask. **17.8.1.** 52 kJ. **17.8.2.** 24 °C. **17.9.1.** In rubber clothes, evaporation of moisture from the surface of the body is hampered, and as a result a smaller amount of heat is transferred to air. **17.9.3.** Water is colder in the porcelain vessel. **17.10.1.** 847 kJ/kg. **17.19.1.** 80%. **17.19.2.** 1.2 kg. **17.19.3.** No, it will not. **18.3.1.** (a) Moist-adiabatic; (b) adiabatic. **18.3.2.** During the ascent, the expansion is moist-adiabatic, while during the descent the compression is adiabatic. For this reason, the temperature drops during the ascent to a smaller extent than it rises during the descent. **19.5.1.** 3.7 atm. **19.9.1.** 18.39 kW. **19.10.1.** 21%. **19.10.2.** 4600 kJ. **19.13.1.** It will become warmer.

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