Solutions to Modern Physics Section of Numerical Examples in Physics

Solutions to

Numerical Examples in Physics

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BHARATI BHAWAN (P & D)

Solutions to Numerical Examples in Physics

[For ISc Students of Indian Universities, Bihar Intermediate Education Council, Medical & Engineering Entrance Test, IIT, ISM (Dhanbad) and other competitive Examinations]

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[With SI Units]

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IMPORTANT MATHEMATICAL RESULTS

1. Important trigonometric relations

(i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(ii) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(iii) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

(iv) $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(v) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(vi) $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

(vii) $\sin 2A = 2\sin A \cos A$

(viii) $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

(ix) $\sin 3A = 3\sin A - 4\sin^3 A$

(x) $\cos 3A = 4\cos^3 A - 3\cos A$

2. Properties of a triangle

(i) sine property: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, where A, B and C are the angles and a, b and c are the sides opposite to A, B and C respectively.

(ii) cosine property: $c^2 = a^2 + b^2 - 2ab\cos C$; $a^2 = b^2 + c^2 - 2bc\cos A$ and $b^2 = c^2 + a^2 - 2ca\cos B$.

(iii) Area of a triangle $\Delta = \frac{1}{2}bc\sin A$ or $\frac{1}{2}ca\sin B$ or $\frac{1}{2}ab\sin C$.

(iv) In any small angled isosceles triangle: The small side = The small angle (in radian) $\times$ either of the longer sides.

3. Expansion formula

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2}x^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^3 + \ldots\ldots$$

When x is very small, $(1+x)^n = 1 + nx$.

$\ln(1+x) = x - x^2/2 + x^3/3 - x^4/4 + \ldots\ldots \ (x < 1)$

$\ln(1-x) = -(x + x^2/2 + x^3/3 + x^4/4 + \ldots\ldots)$

$e^x = 1 + x/1! + x^2/2! + x^3/3! + x^4/4! + \ldots\ldots$

4. Important differentials

(i) $y = x^n$	$dy = nx^{n-1}dx$
(ii) $y = e^{ax}$	$dy = ae^{ax}dx$
(iii) $y = \sin ax$	$dy = a\cos ax dx$
(iv) $y = \cos ax$	$dy = -a\sin ax dx$
(v) $y = \tan ax$	$dy = a\sec^2 ax dx$
(vi) $y = \cot ax$	$dy = -a\,\text{cosec}^2 ax dx$
(vii) $y = \sec ax$	$dy = a\sec ax\tan ax dx$
(viii) $y = \text{cosec}\, ax$	$dy = -a\,\text{cosec}\, ax\cot ax dx$

5. Important Integrals

(i) $\int x^n dx = \frac{x^{n+1}}{n+1}$ when $n \neq -1$ and when $n = -1 \int \frac{dx}{x} = \ln x$

(ii) $\int e^{ax}dx = \frac{e^{ax}}{a}$

(iii) $\int \sin ax dx = -\frac{\cos ax}{a}$

(iv) $\int \cos ax dx = \frac{\sin ax}{a}$

(v) $\int \tan ax dx = -\frac{1}{a}\ln\cos x$

(vi) $\int \cot ax dx = \frac{1}{a}\ln\sin ax$

(vii) $\int \sec^2 ax dx = \frac{\tan ax}{a}$

(viii) $\int \text{cosec}^2 ax dx = -\frac{\cot ax}{a}$

(ix) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

(x) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a}$

(xi) $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2})$

(xii) $\int \frac{dx}{\cos ax} = \frac{1}{a}\ln\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)$

(xiii) $\int \frac{dx}{\sin ax} = \frac{1}{a}\ln\tan\frac{ax}{2}$

(xiv) $\int \frac{dx}{x^2 - a^2} (x > a) = \frac{1}{2a}\ln\frac{x-a}{x+a}$

(xv) $\int \frac{dx}{a^2 - x^2} (a > x) = \frac{1}{2a} \ln \frac{a + x}{a - x}$

(xvi) $\int a^x dx = \frac{a^x}{\ln a}$

(xvii) $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a}$.

(xviii) $\int \sqrt{a^2 - x^2}\, dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$

(xix) $\int \sqrt{x^2 - a^2}\, dx = \frac{x\sqrt{x^2 - a^2}}{2} - \frac{a^2}{2} \ln (x + \sqrt{x^2 - a^2})$

(xx) $\int \sqrt{a^2 + x^2}\, dx = \frac{x\sqrt{a^2 + x^2}}{2} + \frac{a^2}{2} \ln (x + \sqrt{x^2 + a^2})$

6. Maxima and Minima

The condition for maximum or minimum of a function y with respect to a variable x is that $\frac{dy}{dx} = 0$ and for maximum $\frac{d^2y}{dx^2}$ must be negative and for minimum $\frac{d^2y}{dx^2}$ must be positive. For illustration see Ex. 5 Chapter 4 and Ex. 7 Chapter 7.

7. Important Summations

(i) Sum of n natural numbers $= \frac{n(n+1)}{2}$.

(ii) Sum of squares of n natural numbers $= \frac{n(n+1)(2n+1)}{6}$.

(iii) Sum of the cubes of n natural numbers $= \left\{\frac{n(n+1)}{2}\right\}^2$.

(iv) Sum of an AP series $= \frac{1}{2} n$ (first term + last term).

(v) Sum of a GP series $= \frac{a(1 - r^n)}{1 - r}$, $r < 1$

$$= \frac{a(r^n - 1)}{r - 1},\ r > 1.$$

Sum of an infinite GP series $= \frac{a}{1 - r}$.

(vi) Sum of infinite series

$$\sum_1^\infty \frac{1}{r^2} = \frac{\pi^2}{6} \cdot \sum_1^\infty \frac{1}{(2r+1)^2} = \frac{\pi^2}{8}.$$

$$\sum_{1}^{\infty} (-1)^{r+i} \frac{1}{r^2} = \frac{\pi^2}{12} \cdot \sum_{1}^{\infty} \frac{1}{(2r)^2} = \frac{\pi^2}{24}.$$

8. Most general method of solving simultaneous equations (e.g. three unknowns and three equations):

$$a_1x + b_1y + c_1z + d_1 = 0$$
$$a_2x + b_2y + c_2z + d_2 = 0$$
$$a_3x + b_3y + c_3z + d_3 = 0$$

is $$\frac{x}{x_{minor}} = \frac{-y}{y_{minor}} = \frac{z}{z_{minor}} = \frac{-1}{1_{minor}}$$

where minors are those of the elements of x, y, z and 1 of the determinant

$$\begin{vmatrix} x & y & z & 1 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{vmatrix}$$

9. Solid Angle: The solid angle subtended by an elementary area dS at a point distant r in direction θ from its normal is

$$d\omega = \frac{dS \cos\theta}{r^2}.$$

The solid angle of a cone of semivertical angle θ is:

$$\omega = 2\pi(1 - \cos\theta).$$

Solid angle in between directions θ and $\theta + d\theta = 2\pi \sin\theta \, d\theta$.

10. Solution of two standard Differential Equations

$$\frac{d^2y}{dx^2} + \omega^2 y = 0 \qquad y = A \sin(\omega x + \alpha)$$

$$\frac{dy}{dx} + P(x)y = Q(x) \qquad y\, e^{\int P(x)dx} = \int e^{\int P(x)\,dx} Q(x)\, dx + C.$$

11. Radius of curvature of a curve

$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$

□

Some Physical Constants

Speed of light	c	3×10^{8} m s^{-1}
Gravitational constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Universal gas constant	R	8.31 J mol^{-1} K^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Permittivity constant	ε_0	8.85×10^{-12} F m^{-1}
Avogadro constant	N	6.02×10^{23} mol^{-1}
Boltzmann constant	k	1.38×10^{-23} J K^{-1}
Planck constant	h	6.63×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron magnetic moment	μ_e	9.27×10^{-24} J T^{-1}
Acceleration due to gravity (standard)	g	9.80 m s^{-2}
Standard atmospheric pressure	atm	1.01×10^{5} N m^{-2}
Density of water at 4°C		1000 kg m^{-3}
Rydberg constant	R	1.097×10^{7} m^{-1}
Loschmidt constant	n	2.69×10^{25} m^{-3}
Specific charge of electron	e/m	1.76×10^{11} C kg^{-1}
Proton rest mass	m_p	1.672×10^{-27} kg
Proton specific charge	e/m_p	9.59×10^{8} C kg^{-1}
Stefan constant	σ	5.67×10^{-8} W m^{-3} K^{-4}
Wien constant	b	2.9×10^{-3} m V
First Bohr orbit	r_0	5.29×10^{-11} m
Classical electron radius	r_e	2.82×10^{-15} m
Bohr magneton	μ_B	9.27×10^{-24} J T^{-1}
Nuclear magneton	μ_N	5.05×10^{-27} J T^{-1}
Proton magnetic moment	μ_p	2.792 μN
Neutron magnetic moment	μ_n	– 1.913 μN
Atomic mass unit	amu	1.66×10^{-27} kg or 931 MeV
	hc	1.989×10^{-25} J m
	$\frac{hc}{e}$	1.243×10^{-6} V m

□

Conversion Factors

Length:
1 m = 10^2 cm = 39.37 in
= 6.214×10^{-4} mi
1 mi = 5280 ft = 1.609 km
1 in = 2.540 cm
1 Å (angstrom) = 10^{-8} cm
= 10^{-10} m
= 10^{-4} μ (micron)
1 μ (micron) = 10^{-6} m
1 AU (astronomical unit)
= 1.496×10^{11} m
1 light year = 9.46×10^{15} m
1 parsec = 3.084×10^{16} m

Angle:
1 radian = 57.3°
1° = 1.74×10^{-2} rad
1′ = 2.91×10^{-4} rad
1″ = 4.85×10^{-6} rad

Area:
1 m^2 = 10^4 cm^2 = 1.55×10^{-5} in^2
= 10.76 ft^2
1 in^2 = 6.452 cm^2
1 ft^2 = 144 in^2 = 9.29×10^{-2} m^2

Volume:
1 m^3 = 10^6 cm^3 = 10^3 litres
= 35.3 ft^3 = 6.1×10^4 in^3
1 ft^3 = 2.83×10^{-2} m^3
= 28.32 litres
1 in^3 = 16.39 cm^3

Mass:
1 kg = 10^3 g = 2.205 lb
1 lb = 453.6 g = 0.4536 kg
1 amu = 1.6604×10^{-27} kg

Force:
1 N = 10^5 dyn + 0.2248 lbf
= 0.102 kgf
1 dyn = 10^{-5} N
= 2.248×10^{-6} lbf
1 lbf = 4.448 N
= 4.448×10^5 dyn
1 kgf = 9.81 N

Power:
1 W = 1.341×10^{-3} hp
1 hp = 745.7 W

Electric Charge:*
1 C = 3×10^9 stC*
1 stC = $\frac{1}{3} \times 10^{-9}$ C

Current:*
1 A = 3×10^9 stA
1 stA = $\frac{1}{3} \times 10^{-9}$ A
1 μA = 10^{-6} A, 1 mA = 10^{-3} A

Electric Field:*
1 N C^{-1} = 1 V m^{-1}
= 10^{-2} V cm^{-1}
= $\frac{1}{3} \times 10^{-4}$ stV cm^{-1}

Electric Potential:*
1 V = $\frac{1}{3} \times 10^{-2}$ stV*
1 stV = 3×10^2 V

Resistance:
1 Ω = 10^6 μΩ
1 MΩ = 10^6 Ω

Capacitance:*
1 F = 9×10^{11} stF*
1 stF = $\frac{1}{9} \times 10^{-11}$ F
1 μF = 10^{-6} F, 1 pF = 10^{-12} F

Magnetic Field:
1 T = 10^4 gauss
1 gauss = 10^{-4} T

Magnetic Flux:
1 Wb = 10^8 maxwell
1 maxwell = 10^{-8} Wb

Magnetizing Field:
1 A m^{-1} = $4\pi \times 10^{-3}$ oersted
1 oersted = $1/4\pi \times 10^3$ A m^{-1}

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* In all cases 3 actually means 2.998 and 9 means 8.987; stC—stat coulomb (esu of charge); stV—stat volt (esu of voltage).

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Chapter 1 – Introduction to Modern Physics

Solution.

We have $V_H = Bud$ and $u = \frac{J}{ne}$

Here $\quad i = J \times (d \times t)$ and so, $u = \frac{i}{nedt}$

and $\quad V_H = B\frac{i}{nedt}d = \frac{Bi}{net}$

or $\quad n = \frac{Bi}{V_H et} = \frac{0.1 \times 10}{55 \times 10^{-9} \times 1.6 \times 10^{-19} \times 10^{-3}} = 11.4 \times 10^{28}\ \text{m}^{-3}$

We have by Ohm's law in terms of electric field $j = \sigma E$ and $\sigma = \mu ne$ (by definition of mobility).

$\therefore \quad \frac{j}{E} = \mu ne$ or $\frac{i}{(d \times t)\frac{V}{l}} = \mu ne$

or $\quad \mu = \frac{i\,l}{V\,d\,t\,ne}$

Putting the value of n, $\mu = \frac{il\,V_H\,et}{vd\,te\,Bi}$

or $\quad \mu = \frac{V_H l}{VdB} = \frac{55 \times 10^{-9}}{0.5 \times 10^{-3}} \times \frac{6}{2} \times \frac{1}{0.1} = 3.3 \times 10^{-3}\ \text{m}^2/\text{Vs}$

8. *Calculate the Hall constant for silver knowing its density,* $\rho = 10.5 \times 5 \times 10^3\ \text{kg/m}^3$ *and atomic mass,* $A = 107.868$, $N_A = 6.02 \times 10^{26}$ *per kg mol.*

Solution.

The reciprocal of 'ne' is called Hall constant.

$\therefore \quad C_H = \frac{1}{ne}$

Now $n = \frac{\rho N_A}{A}$ where N_A = Avogadro number

$\therefore \quad C_H = \frac{A}{\rho N_A e} = \frac{107.868}{10.5 \times 10^3 \times 6.02 \times 10^{26} \times 1.6 \times 10^{-19}} = 1.07 \times 10^{-10}\ \text{m}^3/\text{C}$

EXERCISES

1. An electron falls through a potential difference of 1000 V and gives up 1.6×10^{-16} J of energy. What is the charge on the electron?
2. The forces of cathode ray particles due to electric and magnetic fields which are at right angles to each other balance each other. If the magnetic field has an intensity 0.1 T, what is the intensity of the electric field? The velocity of cathode ray particles is $10^6\ \text{m s}^{-1}$. What p.d. is required to produce this velocity?
3. An electron falls through a potential difference of 50,000 V. What is the energy acquired by it?
4. An electron of energy 150 eV describes a circle in a magnetic field of 0.1 tesla. Calculate the radius of the circle.

5. An electron is moving with a velocity of 10^7 m s^{-1}. Find its energy in electronvolts. (Mass of electron $= 9.1 \times 10^{-31}$ kg and charge of electron $= 1.6 \times 10^{-19}$ C)

6. A horizontal stream of charged particles is accelerated to velocity 3×10^7 m s^{-1} immediately before being allowed into an electric field between two horizontal plates separated by 2 cm and maintained at a p.d. of 100 V. The stream is deflected by 5 mm. Calculate e/m of the charged particles of the stream. (Length of plates = 10 cm.)

   [*Hint:* Electric field between plates = rate of change of potential]

7. Electrons move at right angles to a magnetic field of 0.015 T with a velocity 6×10^7 m s^{-1}. Find the radius of the orbit.

8. In a cathode ray tube, electric and magnetic fields are simultaneously applied at right angles to each other and the intensity of the magnetic field is so adjusted that the beam remains undeflected. Show that the velocity of cathode ray particles is given by $v = \frac{E}{B}$. Calculate this velocity when $B = 0.01$ T and $E = 50{,}000$ V/m.

9. A non-relativistic electron enters into a right-handed coordinate space with velocity 10^4 m s^{-1} at an angle of 60° with the x-axis in the xy-plane. There is a uniform magnetic field of 50 mT along the y-axis. Find the pitch of the helical trajectory. What is the axis of the helix?

10. A non-relativistic proton enters at right angles to a uniform magnetic field of 30 mT. What is the frequency of revolution of the particle if the field is sufficiently deep?

11. A nonrelativistic electron enters a uniform magnetic field of 50 mT with a speed of 10^6 m/s at right angles to it and emerges from the field after 0.1 nanosecond. What is the deviation produced by the field? Show that it does not depend on the initial speed of the electron.

12. Hall effect measurements were made using a sodium conductor. The strength of the Hall field was found to be $E = 500 \times 10^{-6}$ Vm^{-1} with a current density $J = 2 \times 10^6$ A/m^2 and magnetic induction $B = 1$ T. Find the concentration of the conduction electrons in sodium.

13. Find the mobility of the electrons in a copper conductor if for a magnetic induction $B = 100$ mT the transverse Hall electric field was found to $\eta = 3.1 \times 10^3$ times less than the applied electric field.

    [*Hint:* μ, mobility = velocity acquired in unit electric field, $\sigma = \mu ne$]

14. A parallel-plate capacitor of plate area A and plate separation d is placed in a stream of conducting liquid of resistivity ρ. The liquid moves parallel to the plates with velocity v. The whole system is under a uniform magnetic induction B parallel to the plates and perpendicular to the stream. The capacitor plates are connected to a resistor R. What amount of power is generated in R? At what value of R is the power highest? What is this highest power equal to?

15. A straight copper wire of radius $R = 5$ mm carries a current $I = 50$ A. Find the potential difference between the axis of the wire and its surface. The concentration of conduction electrons in copper is equal to $n = 9 \times 10^{22}$ cm^{-3}.

16. The resistivity of indium arsenide is $\rho = 2.5 \times 10^{-3}\ \Omega$ m and its Hall constant is

$C_H = 10^{-2}$ m^3/C. Find the concentration and mobility of the charge carriers in this material.

17. Electrons are observed to be ejected in various directions with negligible speed from the negative plate of a parallel plate capacitor when the plate is illuminated by a certain wavelength. The plates are separated by a distance d and a potential difference V is maintained between them. Show that none of these electrons will reach the positive plate if a magnetic field is applied at right angles to the electric field and that the magnetic induction has a value $B > \left(\frac{2mV}{ed^2}\right)^{\frac{1}{2}}$ where m and e are the electron mass and charge respectively.

18. Electrons accelerated by a p.d. of $V = 5000$ volts are allowed to impinge normally on a plate. If the current constituted by the impinging electrons be $I = 0.02$ mA, find the force experienced by the plate. Assume complete absorption of electrons by the plate. $m = 9.0 \times 10^{-31}$kg and $e = 1.6 \times 10^{-19}$C

19. A thick wire of radius $r = 5$ mm carries a steady current $I = 10$ A. An electron leaves its surface perpendicularly with velocity $v_0 = 10^6$ m/s. Find the maximum distance from the surface of the wire before it turns back.

20. An electron starting at angle $\alpha = 60°$ with a uniform magnetic induction $B = 0.1$ T moves with velocity $v = 10^6$ m s^{-1}. A screen is held at right angles to the field at a distance $l = 5$ cm from the starting point. Find the distance r from the starting point to the point on the screen where the electron strikes.

21. An electron beam accelerated by a p.d. of $V = 100$ V enters into a magnetic field perpendicular out the plane of the page. The space occupied by the field is of width $d = 10$ cm. A screen is held at right angles to the initial direction of motion of the beam at a distance $b = 20$ cm from the nearer edge of the field. Find the distance 'δ' of the point where the beam strikes the screen from the point where it would strike had there been no field.

ANSWERS

1. 1.6×10^{-19} C 2. 10^5 V m^{-1}, 2.8V 3. 5×10^4 eV 4. 4.1×10^{-4} m

5. 284.4 eV 6. 1.8×10^{11} C kg^{-1} 7. 2.27×10^{-2} m 8. 5×10^6 m/s

9. 6.0 μm, y-axis 10. 4.6×10^5 s^{-1} 11. 51° 12. $n = \frac{jB}{eE} = 2.5 \times 10^{28}$ m^{-3}

13. $\mu = 1/\eta B = 3.2 \times 10^{-3}$ m^2 / Vs

14. $P = \frac{B^2v^2d^2R}{\left(R + \frac{\rho d}{S}\right)^2}$; $R = \frac{\rho d}{S}$; $P_{\max} = \frac{1}{4}\frac{B^2v^2Sd}{\rho}$ 15. $V = \frac{\mu_0 I^2}{4\pi^2R^2ne} = 2 \times 10^{-12}$ V

16. $n = 6.25 \times 10^{20}$ m^{-3}; $\mu = 4$ m^2/Vs 18. $F = I\sqrt{\frac{2Vm}{e}} = 4.7 \times 10^{-9}$ N

19. $x_{\max} = r(e^b - 1) = 7.8$ cm, $b = \frac{2\pi\, mv_0}{\mu_0\, e\, I}$

20. $r = r_0 \sin \dfrac{l}{r_0} = 3.65 \times 10^{-5}$ m, where $r_0 = \dfrac{2mv \sin \alpha}{Be}$

21. $\delta = \dfrac{ab}{\sqrt{R^2 - a^2}} + R - \sqrt{R^2 - a^2}$ where $R = \dfrac{1}{B}\sqrt{\dfrac{2mV}{e}}$

CHAPTER 1

Introduction to Modern Physics

1. Loss in potential energy = Ve

$\therefore \quad 1.6 \times 10^{-16} = 1000 \times e$

or $\quad e = 1.6 \times 10^{-19}$ C.

2. Electric force = eE; Magnetic force = Bev

$eE = Bev$

or $\quad E = Bv = 0.1 \times 10^6 = 10^5 \text{ V m}^{-1}$.

Let V be the required p.d.

Loss in potential energy = $Ve = \frac{1}{2} mv^2$

$$V = \frac{mv^2}{2e} = \frac{9 \times 10^{-31} \times (10^6)^2}{2 \times 1.6 \times 10^{-19}} = 2.8 \text{ volts.}$$

3. Energy acquired $= Ve$

$= 50000 \times 1.6 \times 10^{-19} = 8 \times 10^{-15}$ joule

$= \frac{8 \times 10^{-15}}{1.6 \times 10^{-19}}$ e V $\quad (\because \quad 1 \text{ e V} = 1.6 \times 10^{-19}$ joule)

$= 5 \times 10^4$ e V.

4. $150 \times 1.6 \times 10^{-19} = \frac{1}{2} mv^2$

or $$v = \sqrt{\frac{2 \times 150 \times 1.6 \times 10^{-19}}{9 \times 10^{-31}}} = 7.3 \times 10^6 \text{ m s}^{-1}.$$

Centripetal force = $Bev = \frac{mv^2}{r}$

or $$r = \frac{mv}{Be} = \frac{9 \times 10^{-31} \times 7.3 \times 10^6}{0.1 \times 1.6 \times 10^{-19}}$$

or $$r = \frac{9 \times 7.3}{1.6} \times 10^{-5} = 4.1 \times 10^{-4} \text{ m.}$$

5. Energy of the electron = $\frac{1}{2} mv^2$

$= \frac{1}{2} \times 9.1 \times 10^{-31} \times 10^{14}$ joule

1 electronvolt (eV) = energy lost by an electron when it falls through p.d. of 1 volt

$$= 1 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-19} \text{ joule}$$

$$\therefore \text{ Energy in eV} = \frac{9.1 \times 10^{-31} \times 10^{14}}{2 \times 1.6 \times 10^{-19}}$$

$$= 2.8437 \times 10^{2} = 284.4 \text{ e V.}$$

6. Electric field (E) = rate of change of potential

$$= \frac{100}{0.02} = 5000 \text{ V m}^{-1}.$$

Acceleration of electron along the field $= \frac{eE}{m}$.

Considering motion of the electron perpendicular and along the field,

$$0.1 = 3 \times 10^{7} \times t$$

and $$5 \times 10^{-3} = \frac{1}{2}\left(\frac{eE}{m}\right)t^{2}$$

or $$5 \times 10^{-3} = \frac{1}{2} \times \frac{e}{m} \times 5000 \times \left(\frac{0.1}{3 \times 10^{7}}\right)^{2}$$

or $$\frac{e}{m} = \frac{2 \times 5 \times 10^{-3} \times 9 \times 10^{14}}{5000 \times (0.1)^{2}} = 1.8 \times 10^{11} \text{ C kg}^{-1}.$$

7. Centripetal force $= Bev = \frac{mv^{2}}{r}$

or $$r = \frac{mv}{Be} = \frac{9.1 \times 10^{-31} \times 6 \times 10^{7}}{0.015 \times 1.6 \times 10^{-19}} = 2.27 \times 10^{-2} \text{ m.}$$

8. Magnetic force $= Bev$, Electric force $= eE$.
For no deflection, $Bev = eE$

or $$v = \frac{E}{B}$$

$$v = \frac{50000}{0.01} = 5 \times 10^{6} \text{ m s}^{-1}.$$

9. Velocity along x-axis $= 10^{4} \cos 60° = v_{0}$

Velocity along y-axis $= 10^{4} \sin 60° = v_{11}$

$$Bev_{0} = \frac{mv_{0}^{2}}{r} \quad \text{or} \quad Be = \frac{mv_{0}}{r} = m\omega \quad \text{or} \quad T = \frac{2\pi m}{Be}.$$

$$\text{Pitch} = v_{||} \times T = 10^{4} \sin 60° \times \frac{2\pi m}{Be}$$

$$\Rightarrow \quad \text{Pitch} = \frac{10^{4} \times 0.866 \times 2\pi \times 9 \times 10^{-31}}{50 \times 10^{-3} \times 1.6 \times 10^{-19}}$$

$$= 6 \times 10^{-6} \text{ m} = 6 \ \mu\text{m}.$$

10. We have $\omega = \dfrac{Be}{m}$ or $v = \dfrac{Be}{2\pi m}$

$$v = \frac{30 \times 10^{-3} \times 1.6 \times 10^{-19}}{2\pi \times (1840 \times 9 \times 10^{-31})} = 4.6 \times 10^{5}\ s^{-1}$$

11. $Bev = \dfrac{mv^2}{r}$ or $r = \dfrac{mv}{Be}$.

Length of the arc of the circular track covered = $v\tau$

$$\therefore \qquad \theta = \frac{v\tau}{\frac{mv}{Be}} = \frac{Be\tau}{m} = \frac{50 \times 10^{-3} \times 1.6 \times 10^{-19} \times 0.1 \times 10^{-9}}{9 \times 10^{-31}}$$

$$= \frac{8}{9} \text{ radian} = 51°.$$

12. We have E_H (Hall electric field) $= uB$

and u (drift velocity) $= \dfrac{j}{nq}$

$$\therefore \qquad E_H = \frac{j}{nq} B \quad \text{or} \quad n = \frac{jB}{qE_H}.$$

Here $q = e$ (as the carriers are electrons).

$$\therefore \qquad n = \frac{jB}{eE}.$$

Here $n = \dfrac{2 \times 10^{-6} \times 1}{1.6 \times 10^{-19} \times 500 \times 10^{-6}} = 2.5 \times 10^{28}\ m^{-3}$.

13. We have $E_H = uB$ and $u = \dfrac{j}{ne}$.

E (applied field) $= \dfrac{j}{\sigma} = \dfrac{une}{\sigma}$ (by Ohm's law $j = \sigma E$)

$$\therefore \qquad E_H = \frac{\sigma E}{ne} \times B.$$

Now μ (mobility) $= \dfrac{\sigma}{ne}$. Hence $E_H = \dfrac{(\mu ne)\, E}{ne} B$

or $E_H = \mu EB$ or $\mu = \dfrac{E_H}{EB}$.

It is given, $E = \eta E_H$.

$$\therefore \qquad \mu = \frac{1}{\eta B}.$$

Here $\mu = \dfrac{1}{3.1 \times 10^{3} \times 100 \times 10^{-3}} = 3.2 \times 10^{-3}\ m^2/V\ s.$

14. Here the electric force is balanced by the magnetic force and hence

$$eE = Bev \quad \text{or} \quad e\frac{V}{d} = Bev \quad \text{or} \quad V = Bvd.$$

Now $$i = \frac{V}{R+R'} \text{ (Ohm's law)} = \frac{V}{R+\rho\frac{d}{S}} = \frac{Bvd}{\left(R+\frac{\rho d}{S}\right)}$$

$\therefore$ $$P \text{ (power generated)} = i^2R = \frac{B^2v^2d^2}{\left(R+\frac{\rho d}{S}\right)^2} \times R.$$

P is maximum when external resistance is equal to internal resistance.

$\therefore$ P is maximum when $R = \frac{\rho d}{S}$.

$$P_{\max} = \frac{B^2v^2d^2}{4R^2} \times R = \frac{B^2v^2d^2}{4R} = \frac{1}{4} \times \frac{B^2v^2Sd}{\rho}.$$

15. The average magnetic field in which the carriers may be supposed to move is $< B > = \frac{0 + B_{\text{surface}}}{2} = \frac{1}{2} \times \frac{\mu_0 I}{2\pi R}$.

Since the carriers move at constant speed u_d, the electric force must be balanced by magnetic force.

$\therefore$ $$qE = qu_dB \quad \text{or} \quad \frac{V}{R} = u_d \times \frac{\mu_0 I}{4\pi R} = \frac{I}{ne} \times \frac{\mu_0 I}{4\pi R} \qquad \left(\because \quad u_d = \frac{j}{ne}\right)$$

or $$V = \frac{\mu_0 Ij}{4\pi ne} = \frac{\mu_0 I}{4\pi ne} \times \frac{I}{\pi R^2} = \frac{\mu_0 I^2}{4\pi^2 R^2 ne}.$$

16. We have $C_H = \frac{1}{ne}$ or $n = \frac{1}{e\,C_H} = \frac{1}{1.6 \times 10^{-19} \times 10^{-2}} = 6.25 \times 10^{20}\ \text{m}^{-3}$.

We have also

$$\sigma = \mu ne = \frac{\mu}{C_H} \quad \text{or} \quad \mu = \sigma C_H = \frac{C_H}{\rho} \qquad \left(\because \quad \sigma = \frac{1}{\rho}\right)$$

$\therefore$ $$\mu = \frac{10^{-2}}{2.5 \times 10^{-3}} = 4\ \text{m}^2/\text{V s}.$$

17. $$\vec{F} = (-e)\,\vec{E} + (-e)\,\vec{v} \times \vec{B}$$

$$= (-e)\,E\,(-\hat{j}) - e \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B & 0 & 0 \end{vmatrix}$$

$$= eE\,\hat{j} - e\,[\hat{j}\,Bv_z - \hat{k}\,v_yB\,]$$

$$= e(E - Bv_z)\,\hat{j} + \hat{k}\;ev_yB$$

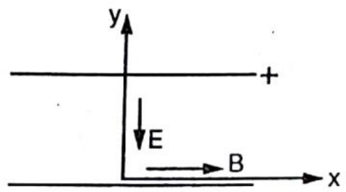

Fig. 1.1

$\Rightarrow \quad F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = e\ (E - Bv_z)\,\hat{j} + \hat{k} + ev_y B$

$\therefore \quad F_x = 0, \quad F_y = e(E - Bv_z) \quad \text{and} \quad F_z = ev_y B.$

$\Rightarrow \quad m\dfrac{dv_y}{dt} = e\,(E - Bv_z) \quad \text{and} \quad m\,\dfrac{dv_z}{dt} = Be\,v_y$

$\Rightarrow \quad m\dfrac{d^2v_y}{dt^2} = -\,Be\dfrac{dv_z}{dt} = -\,Be\dfrac{Be}{m}v_y$

$\Rightarrow \quad \dfrac{d^2v_y}{dt^2} = -\,\dfrac{B^2e^2}{m^2}\,v_y = -\,\omega^2 y \quad \text{where } \omega^2 = \dfrac{B^2e^2}{m^2}$

and $\quad m\,\dfrac{d^2v_z}{dt^2} = Be\,\dfrac{dv_y}{dt} = Be\cdot\dfrac{e}{m}(E - Bv_z)$

$\Rightarrow \quad \dfrac{d^2v_z}{dt^2} = \dfrac{Be^2}{m^2}\,(E - Bv_z) = \dfrac{Be^2E}{m^2} - \dfrac{B^2e^2}{m^2}v_z$

$= \dfrac{Be^2E}{m^2} - \omega^2 v_z$

$\therefore \quad v_y = A\sin(\omega t + \alpha)$

At $\quad t = 0,\ v_y = 0$ and so $A\sin\alpha = 0$ or $\alpha = 0$

$v_y = A\sin\omega t.$

$\dfrac{dv_y}{dt} = A\omega\cos\omega t \quad \Rightarrow \quad \left(\dfrac{dv_y}{dt}\right)_{t=0} = A\omega.$

$m\left(\dfrac{dv_y}{dt}\right)_{t=0} = eE - Be\,(v_z)_{t=0} = eE \quad \left[\because\ (v_z)_{t=0} = 0\right]$

$\therefore \quad mA\,\omega = eE \quad \Rightarrow \quad A = \dfrac{eE}{\omega m}$

$\therefore \quad v_y = \dfrac{eE}{\omega m}\cdot\sin\omega t \quad \Rightarrow \quad \dfrac{dy}{dt} = \dfrac{eE}{\omega m}\sin\omega t$

$\Rightarrow \quad y = -\,\dfrac{eE}{m\omega^2}\cos\omega t + c$

At $\quad t = 0, \quad y = 0 \quad$ and so $\quad c = +\,\dfrac{eE}{m\omega^2}$

$$\therefore \qquad y = \frac{eE}{m\omega^2}\,(1 - \cos\omega t)$$

$$\therefore \qquad y_{\max} = \frac{2eE}{m\omega^2} \qquad [\because\ (\cos\omega t)_{\min} = -1]$$

If $y_{\max} < d$, electron will never reach the +ve plate

$$\Rightarrow \qquad \frac{2eE}{m\left(\frac{B^2e^2}{m^2}\right)} < d \qquad \because\ E = \frac{v}{d}$$

$$\frac{2e\frac{V}{d}}{\frac{B^2e^2}{m}} < d \quad \Rightarrow \quad \frac{2eVm}{B^2e^2} < d^2$$

$$\Rightarrow \qquad B^2 > \frac{2mV}{ed^2} \quad \Rightarrow \quad B > \left(\frac{2mV}{ed}\right)^{\frac{1}{2}}.$$

18. Let A be the area of the plate and n = number of electrons impinging on the plate in second.

The $\qquad F$ = rate of change of momentum n-electrons

$$= n\,(mv) = mnv$$

$$I\ (\text{current}) = ne \quad \Rightarrow \quad n = \frac{I}{e}.$$

$$\therefore \qquad F = m\left(\frac{I}{e}\right)v.$$

By energy equation $\quad Ve = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2Ve}{m}}$$

$$\therefore \qquad F = m\left(\frac{I}{e}\right)\sqrt{\frac{2Ve}{m}} = I\sqrt{\frac{2Vm}{e}}.$$

$$F = 0.02 \times 10^{-3}\sqrt{\frac{2 \times 5000 \times 9 \times 10^{-31}}{1.6 \times 10^{-19}}} = 4.7 \times 10^{-9}\ \text{N}.$$

19. Since the field is varying electron will not move along a circle of constant radius.

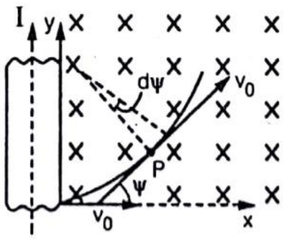

Fig. 1.2

Let P be the position of electron on the trajectory (not a circle). Let R be the radius of trajectory at P.

Then $$Bev_0 = \frac{mv_0^2}{R} \quad \Rightarrow \quad Be = \frac{mv_0}{R}$$

$$\Rightarrow \quad \left\{\frac{\mu_0}{2\pi} \times \frac{I}{(x+r)}\right\} e = \frac{mv_0}{R}$$

$$R\,d\psi = ds \text{ and } dx = ds \cos\psi$$

$$\therefore \quad R = \frac{ds}{d\psi} = \frac{ds}{dx} \times \frac{dx}{d\psi} = \frac{1}{\cos\psi} \times \frac{dx}{d\psi}$$

$$\therefore \quad \frac{\mu_0 eI}{2\pi(x+r)} = mv_0 \cos\psi \frac{d\psi}{dx}$$

$$\Rightarrow \quad \frac{\mu_0 eI}{2\pi m v_0} \times \frac{dx}{x+r} = \cos\psi\, d\psi$$

$$\Rightarrow \quad \frac{\mu_0 eI}{2\pi m v_0} \ln(x+r) = \sin\psi + k$$

When $x = 0$, $\psi = 0$. $\dfrac{\mu_0 eI}{2\pi m v_0} \ln r = k$

$$\therefore \quad \frac{\mu_0 eI}{2\pi m v_0} \ln(x+r) = \sin\psi + \frac{\mu_0 eI}{2\pi m v_0} \ln r$$

$$\Rightarrow \quad \frac{\mu_0 eI}{2\pi m v_0} \ln \frac{x+r}{r} = \sin\psi$$

When distance is maximum,

$$\frac{\mu_0 eI}{2\pi m v_0} \ln\left(1 + \frac{x_{max}}{r}\right) = 1$$

$$\Rightarrow \quad \ln\left(1 + \frac{x_{max}}{r}\right) = \frac{2\pi m v_0}{\mu_0 eI}$$

$$\Rightarrow \quad 1 + \frac{x_{max}}{r} = \varepsilon^{\frac{2\pi m v_0}{\mu_0 eI}}$$ where ε is the Napierian base

$$\Rightarrow \quad x_{max} = r\left[\varepsilon^{\frac{2\pi m v_0}{\mu_0 eI}} - 1\right]$$

Here $$x_{max} = 5 \times 10^{-3} \left[e^{\frac{2\pi \times 9 \times 10^{-31} \times 10^{6}}{4\pi \times 10^{-7} \times 1.6 \times 10^{-19} \times 10}} - 1 \right]$$

$$= 5 \times 10^{-3} [e^{2.8} - 1] = 0.078 \text{ m} = 7.8 \text{ cm}.$$

20. $V_{||} = v\cos\alpha,\ V_{\perp} = v \sin \alpha$

$$F_c = BeV_{\perp} = \frac{mv_{\perp}^2}{R} \Rightarrow Be = \frac{mv_{\perp}}{R}$$

$$\Rightarrow \quad R = \frac{mv_{\perp}}{Be} = \frac{mv \sin \alpha}{Be}$$

T, period of revolution $= \dfrac{2\pi m}{Be}$

p, pitch of the helical path $= v \cos \alpha T = \dfrac{2\pi mv \cos \alpha}{Be}$

No. of revolution made within $l = \dfrac{l}{p} = \dfrac{Bel}{2\pi mv \cos \alpha}$

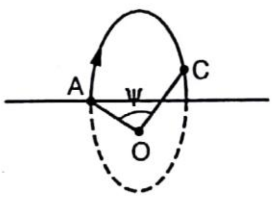

Fig. 1.3

ψ, angle through which electron rotates

$$= \frac{Bel}{2\pi mv \cos \alpha} \times 2\pi$$

$$= \frac{Bel}{mv \cos \alpha}$$

$$r = AC = 2 R \sin \frac{\psi}{2}$$

$$\Rightarrow \quad r = 2 \times \frac{mv \sin \alpha}{Be} \sin \left(\frac{Bel}{2mv \cos \alpha} \right)$$

$$\Rightarrow \quad r = 2 \times \frac{9 \times 10^{-31} \times 10^{6} \times \sin 60°}{0.1 \times 1.6 \times 10^{-19}} \sin \left[\frac{0.1 \times 1.6 \times 10^{-19} \times 5 \times 10^{-2}}{2 \times 9 \times 10^{-31} \times 10^{6} \cos 60°} \right]$$

$$= 2 \times 9.74 \times 10^{-5} \sin 8.89 \times 10^{2} \text{ rad}$$

$$= 2 \times 9.74 \times 10^{-5} \sin 509830°$$

$$= 2 \times 9.7 \times 10^{-5} \sin 169.2°$$

$$= 2 \times 9.7 \times 10^{-5} \times 0.1874 = 3.65 \times 10^{-5} \text{ m}$$

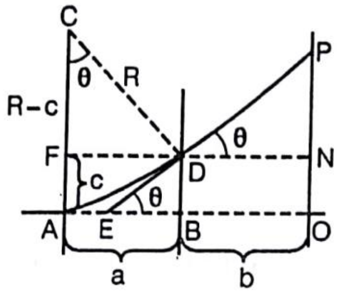

Fig. 1.4

21. $\frac{mv^2}{R} = Bev \Rightarrow R = \frac{mv}{Be}$

Let $NO = DB = c$ (say)

Then from ΔCDF,

$R^2 = (R-c)^2 + a^2$

$\Rightarrow \quad R^2 = R^2 - 2Rc + c^2 + a^2$

$\Rightarrow \quad 2Rc = a^2 + c^2$

$\Rightarrow \quad c^2 - 2Rc + a^2 = 0$

$$c = \frac{2R \pm \sqrt{4R^2 - 4a^2}}{2} = R \pm \sqrt{R^2 - a^2}$$

c cannot be greater than R

$\therefore \quad c = R - \sqrt{R^2 - a^2}$

$\tan\theta = \frac{PN}{DN} = \frac{PN}{b} \Rightarrow PN = b\tan\theta$

Also $\tan\theta = \frac{FD}{CF} = \frac{a}{R-c} = \frac{a}{R - R + \sqrt{R^2 - a^2}}$

$\therefore \quad PN = b\frac{a}{\sqrt{R^2-a^2}} = \frac{ab}{\sqrt{R^2-a^2}}$

$\therefore \quad S$ (required distance) $= PN + NO = \frac{ab}{\sqrt{R^2-a^2}} + R - \sqrt{R^2-a^2}$

By energy equation $Ve = \frac{1}{2}mv^2 \quad \Rightarrow v = \sqrt{\frac{2Ve}{m}}$

$\therefore \quad R = \frac{mv}{Be} = \frac{m}{Be}\sqrt{\frac{2Ve}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{e}}$

□□□

-

$$= 5.3 \times 10^{-11} \times 5.37 \times 10^{-3} = 2.85 \times 10^{-13} \text{ m}$$

Putting $Z = 1$ and $n = 1$ we have $r = r_0' = 2.85 \times 10^{-13}$ m

For pure hydrogen $E_b = E_0$ where $E_0 = \dfrac{e^4 m}{8\varepsilon_0^2 h^2} = 13.6$ eV

Here m is to be replaced by reduced mass.

$$\therefore \quad E_b = E_0' \quad \text{where } E_0' = \frac{e^4}{8\varepsilon_0^2 h^2}\left(\frac{m_{\text{meson}}\, m_{\text{proton}}}{m_{\text{meson}} + m_{\text{proton}}}\right) = E_0 \frac{207\, m_p}{207m + m_p}$$

$$= 13.6 \times \frac{207}{1 + 207\dfrac{9.1 \times 10^{-31}}{1.67 \times 10^{-27}}} = 2530 \text{ eV}$$

In pure hydrogen $\quad \lambda = \dfrac{1}{R\left(\dfrac{1}{1} - \dfrac{1}{4}\right)} = \dfrac{4}{3R}$

where $\quad R = \dfrac{e^4 m}{8\varepsilon_0^2 h^3 c} = 1.097 \times 10^7$ m

Here m is to be replaced by μ.

$$\therefore \quad \lambda = \frac{4}{3R'} \quad \text{where } R' = \frac{e^4}{8\varepsilon_0^2 h^3 c}\,\frac{m_{\text{meson}} \times m_{\text{proton}}}{m_{\text{meson}} + m_{\text{proton}}} = R\,\frac{207 m_p}{207m + m_p}$$

or $\quad R' = 1.097 \times 10^7 \times \dfrac{207}{1 + 207\dfrac{9.1 \times 10^{-31}}{1.67 \times 10^{-27}}} = 204.1 \times 10^7$ m

$$\therefore \quad \lambda = \frac{4}{3 \times 204.1 \times 10^7} \text{ m} = 0.65 \text{ nm}$$

EXERCISES
(A)

1. Find the structure of the follwing atoms:
   (i) $^{16}_{8}O$ (ii) $^{35}_{17}Cl$ (iii) $^{27}_{13}Al$.
2. Calculate the frequency of revolution of an electron in the first orbit of an aluminium atom. Given that mass of electron $= 9 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, radius of the orbit $= 0.2 \times 10^{-11}$ m.
3. Show that the energy of hydrogen atom in the ground state is
   $$E = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}$$
   [*Hint:* Energy in the ground state = Kinetic energy of electron in the first orbit + electrostatic potential energy of the electron.]
4. Calculate the energy of hydrogen atom in the ground state given that the first Bohr orbit of hydrogen is 5×10^{-11} m and electronic charge is 1.6×10^{-19} C.
5. Calculate the speed of the electron in the first Bohr orbit given that $h = 6.6 \times 10^{-34}$ J s, $m = 9.11 \times 10^{-31}$ kg and $e = 1.603 \times 10^{-19}$ C.

6. Find the atomic structure of $^{24}_{12}Mg$. What is its valency?

7. Calculate the radius of the first and second orbit of sodium atom ($Z = 11$).
($h = 6.6 \times 10^{-34}$ J s, $e = 1.6 \times 10^{-19}$ C and $m = 9.1 \times 10^{-31}$ kg.)

8. An α-particle of velocity 1.6×10^7 m s^{-1} approaches a gold nucleii ($Z = 79$). Calculate the distance of 'closest approach'. Mass of an α-particle $= 6.6 \times 10^{-27}$ kg.

9. A single electron orbits around a stationary nucleus of charge $+Ze$, where Z is a constant and e is the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. Find (i) the value of Z, (ii) the energy required to excite the electron from the third to the fourth Bohr orbit and (iii) the wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity. **[IIT 1981]**

10. Calculate for a hydrogen atom the ionization potential, the first excitation potential and the wavelength of the resonance line ($n' = 2 \rightarrow n = 1$).

11. An electron in an excited hydrogen atom acquired an energy of 12.1 eV. To what energy level did it jump? How many spectral lines may be emitted in the course of the transition to lower energy levels? Calculate the corresponding wavelengths.

12. Calculate the radius of the first Bohr orbit of a He^+ ion and the binding energy of its electron in the ground state.

13. A hydrogen atom in a state having binding energy 0.85 eV makes a transition to a state with an excitation energy of 10.2 eV. Find the energy of the photon emitted.

14. Calculate the wavelengths of the first member of Lyman, Balmer and Paschen series.

15. A doubly ionized lithium atom is hydrogen-like with atomic number 3. Find the wavelength of radiation required to excite the electron in Li^{++} from the first to the third orbit (ionization energy of the hydrogen atom equals 13.6 eV). How many spectral lines are observed in the emission spectrum of the excited system?

16. Calculate the Rydberg constant if He^+ ions are known to have the wavelength difference between the first lines of the Balmer and Lyman series equal to $\Delta\lambda = 1338$ Å.

17. What should be the minimum kinetic energy of a hydrogen atom in the ground state in order that its head-on inelastic encounter with a stationary hydrogen atom in the ground state may result in the emission of a photon?

18. A stationary hydrogen atom emits a photon corresponding to the first line of the Lyman series. What is the velocity of recoil of the atom? ($m_H = 1.672 \times 10^{-27}$ kg)

19. What hydrogen-like ion has the wave-length difference between the first lines of the Balmer and Lyman series equal to 593 Å.

20. Calculate the separation of the particles and the binding energy of a ***positronium*** (an atom consisting of an electron and a positron revolving around their centre of mass).

21. A gas of identical hydrogen-like atoms in the lowest (ground) energy level A, and some atoms in a particular upper (excited) energy level B, and there are no atoms in any other energy level. The atoms of the gas make transitions to a higher energy level by absorbing monochromatic light of photon energy 2.7 eV. Subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy 2.7 eV, some have energy more, and some have energy less than 2.7 eV.

(i) Find the principal quantum number of the initially excited level B,

(ii) Find the ionisation energy of the gas, and

(iii) Find the maximum and minimum energy of the emitted photons. **[IIT 1989]**

[*Hint:* Since there are six lines there must be four energy levels, A being the lowest ($n = 1$) and $n = 4$ being the highest. B may have $n = 2, 3$. The conditions given in the problem are satisfied when $n = 2$]

22. A hydrogen-like atom is in a higher excited state of quantum number n. This excited atom can make transitions to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV respectively. Alternatively, the atom from the same excited state can make transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. Determine the values of n and Z. **[IIT 1994]**

23. Find the binding energy of an electron in the ground state of a hydrogen-like atom whose third Balmer spectral line is 108.5 nm. Ionization energy of hydrogen = 13.6 eV. The universal constant $hc = 19.86 \times 10^{-26}$ Jm.

24. An electron, in a hydrogen-like atom, is in an excited state. It has a total energy of -3.4 eV. Calculate its kinetic energy and (ii) the de Broglie wavelength. **[IIT 1996]**

ANSWERS

1. (i) No. of protons or electrons = 8; no. of neutrons = 8; $1s_1 1s_2 2s_1 2s_2 2p_1 2p_2 2p_3 2p_4$
(ii) No. of electrons or protons = 17; No. of neutrons = 18;
$1s_1 1s_2 2s_1 2s_2 2p_1 2p_2 \ldots 2p_6 3s_1 3s_2 3p_1 3p_2 \ldots 3p_5$. (iii) No. of electrons or protons = 13; No. of neutrons = 14; $1s_1 1s_2 2s_1 2s_2 2p_1 2p_2 \ldots 2p_6 3s_1 3s_2 3p_1$

2. 3.25×10^{18} rev./sec **4.** -2.304×10^{-18} J **5.** 2.2×10^6 m s^{-1}

6. No. of protons or electrons = 12 No. of neutrons = 12; $1s_1 1s_2 2s_1 2s_2 2p_1 2p_2 \ldots 2p_6 3s_1 3s_2$; valency = 2

7. 4.82×10^{-12} m, 1.928×10^{-11} m **8.** 4.3×10^{-14} m

9. (i) 5, (ii) 16.6 eV, (iii) 36.3 Å **10.** 13.6 V, 10.2 V, 1215 Å

11. $n = 3$; 1025 Å, 6563 Å; 1215 Å **12.** 2.65×10^{-11} m, 54.5 eV **13.** 2.55 eV

14. 1215 Å, 6563 Å, 18752 Å **15.** 114 Å; 3 **16.** 1.096×10^7 m^{-1} **17.** 20.4 eV

18. 3.25 m/s **19.** Z = 3 (Li^{++}) **20.** 106 pm; 6.8 eV **21.** 2; 14.4 eV; 13.6 eV; 0.7 eV

22. 6; 3 **23.** 54.4 eV **24.** 3.4 eV, 6.7 Å

CHAPTER 2

Atomic Structure

(A)

1. See Ex. 2 in the textbook, page 663.

2. Aluminium atom is ${}^{27}_{13}Al$. Hence there are 13 protons in the nucleus of aluminium atom.

$$\text{Centripetal force} = \frac{1}{4\pi\varepsilon_0} \times \frac{13e.e}{r^2} = m\omega^2 r$$

or $$\frac{13e^2}{4\pi\varepsilon_0 r^2} = m\omega^2 r$$

or $$\omega^2 = \frac{1}{4\pi\varepsilon_0} \times \frac{13e^2}{mr^3} = 9 \times 10^9 \times \frac{13\,(1.6 \times 10^{-19})^2}{9 \times 10^{-31} \times (0.2 \times 10^{-11})^2}$$

$$= \frac{9 \times 13 \times 2.56}{9 \times (0.2)^3} \times 10^{35} = 416 \times 10^{36} \Rightarrow \omega = 20.4 \times 10^{18}$$

and $$v = \frac{\omega}{2\pi} = \frac{20.4}{2\pi} \times 10^{18} = 3.25 \times 10^{18}\ \text{s}^{-1}.$$

3. $$\text{Centripetal force} = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r^2} = \frac{mv^2}{r}$$

or $$\frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r} = mv^2 \qquad \text{... (i)}$$

$$\text{kinetic energy} = \frac{1}{2}mv^2 = \frac{1}{8\pi\varepsilon_0} \times \frac{e^2}{r}$$

$$\text{potential energy} = \frac{1}{4\pi\varepsilon_0} \times \frac{e}{r}(-e) = \frac{1}{4\pi\varepsilon_0} \times \frac{-e^2}{r}$$

$\therefore$ Energy in the ground state

$$= \frac{1}{8\pi\varepsilon_0} \times \frac{e^2}{r} - \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r} = -\frac{1}{8\pi\varepsilon_0} \cdot \frac{e^2}{r}.$$

4. See above, $$E = -\frac{1}{8\pi\varepsilon_0} \times \frac{e^2}{r}$$

$$= -\frac{1}{2} \times 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{5 \times 10^{-11}} \qquad \left(\because \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\right)$$

$$= -2.304 \times 10^{-18}\ \text{joule}.$$

5. From dynamic consideration of electron,

Centripetal force = Coulomb force of attraction

or $$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r^2}$$

or $$4\pi\varepsilon_0 mv^2 r = e^2. \qquad \text{... (i)}$$

From quantum condition we have,

$$mr^2\omega = \frac{h}{2\pi}. \qquad \text{... (ii)}$$

Dividing (i) by (ii),

$$\frac{4\pi\varepsilon_0 mv^2 r}{2\pi mvr} = \frac{e^2}{h} \quad \text{or} \quad v = \frac{e^2}{2\varepsilon_0 h}$$

$\therefore$ $$v = \frac{(1.603 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.6 \times 10^{-34}} = 2.2 \times 10^6 \text{ m s}^{-1}.$$

6. See Ex. 2 in the textbook page 663.

7. We have in general,

$$r_n = \frac{\varepsilon_0 h^2}{\pi me^4} \times \frac{n^2}{Z}$$

or $$r_n = (5.3 \times 10^{-11}) \frac{n^2}{Z}$$ (after substitution of the constants).

For sodium, $Z = 11$.

For the first orbit, $n = 1$.

$\therefore$ $$r_1 = 5.3 \times 10^{-11} \times \frac{1^2}{11} = 4.82 \times 10^{-12} \text{ m.}$$

For the second orbit,

$$n = 2, \quad r_2 = (5.3 \times 10^{-11}) \times \frac{2^2}{11} = 1.928 \times 10^{-11} \text{ m.}$$

8. Let d be the closest approach.

$$V = \frac{1}{4\pi\varepsilon_0} \times \frac{79e}{d}.$$

$\therefore$ Potential energy of α-particle at the closest approach

$$= \frac{79e}{4\pi\varepsilon_0 d} \times 2e = \frac{158e^2}{4\pi\varepsilon_0 d} = \frac{1}{2} m \cdot (1.6 \times 10^7)^2$$

or $$d = \frac{1}{4\pi\varepsilon_0} \times \frac{316\,(1.6 \times 10^{-19})^2}{6.6 \times 10^{-27} \times (1.6 \times 10^{7})^2}$$
$$= 9 \times 10^9 \times \frac{316}{6.6} \times 10^{-25}$$
$$\doteq 4.3 \times 10^{-14} \text{ m.}$$

(B)

9. We have $$E_n = -\frac{Z^2 e^4 m}{8\varepsilon_0 h^2 n^2} = -(2.18 \times 10^{-18}) \frac{Z^2}{n^2}.$$

(i) $\therefore$ $$47.2 \times 1.6 \times 10^{-19} = 2.18 \times 10^{-18} Z^2 \left(\frac{1}{2^2} - \frac{1}{3^3} \right)$$

or $Z = 5.$

(ii) $$E = 2.18 \times 10^{-18} \times 5^2 \left(\frac{1}{3^2} - \frac{1}{4^2} \right) \text{ joule}$$
$$= \frac{2.18 \times 25 \times 7 \times 10}{144 \times 1.6} \text{ eV} = 16.6 \text{ eV.}$$

(iii) $$E = 2.18 \times 10^{-18} \times 5^2 \left(\frac{1}{1^3} - \frac{1}{\infty} \right) = h\nu = \frac{hc}{\lambda}$$

or $$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.18 \times 25 \times 10^{-18}} = 36.3 \times 10^{-10} \text{ m} = 36.3 \text{ Å}$$

10. We have $$E_n = -(2.18 \times 10^{-18}) \frac{Z^2}{n^2}.$$

Ionisation energy is the energy required to take the electron from $n = 1$ to $n = \infty$.

$\therefore$ $$E_{\text{ion}} = 2.18 \times 10^{-18} \times 1^2 / 1 = 2.18 \times 10^{-18} \text{ joule}$$
$$= \frac{2.18 \times 10^{-18}}{1.6 \times 10^{-19}} \text{ eV} = 13.6 \text{ eV.}$$
$$V_{\text{ion}} = E_{\text{ion}} \text{ (numerically)}$$

$\therefore$ $$V_{\text{ion}} = 13.6 \text{ V.}$$

Excitation energy is the energy required to take the electron from $n = 1$ to $n = 2$.

$\therefore$ $$E_{ez} = 2.18 \times 10^{-18} \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$$
$$= 1.635 \times 10^{-18} \text{ joule} = 10.2 \text{ eV}$$
$$V_{\text{ex.}} = E_{\text{ex.}} \text{ (numerically)}$$

$\therefore$ $$V_{\text{ex.}} = 10.2 \text{ V.}$$

We have $\overline{\nu} = 1.097 \times 10^7 Z^2 \left(\frac{1}{s^2} - \frac{1}{n^2} \right)$

$$= 1.097 \times 10^7 \times 1^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3}{4} \times 1.097 \times 10^7$$

$\therefore$ $$\lambda = \frac{1}{\overline{\nu}} = \frac{4}{3 \times 1.097 \times 10^7} = 1.215 \times 10^{-7} \text{m} = 1215 \text{ Å}.$$

11. We have $E_n = -2.18 \times 10^{-18} \frac{Z^2}{n^2}$.

Here $n = 1, \; s = ?$

$\therefore$ $$12.1 \times 1.6 \times 10^{-19} = 2.18 \times 10^{-18} \times 1^2 \left(\frac{1}{1^2} - \frac{1}{s^2} \right)$$

or $s = 3$.

Three lines may be emitted,

$s = 3$ to $n = 1$

$s = 3$ to $n = 2$

$s = 2$ to $n = 1$.

For $3 \to 1$, $\overline{\nu} = 1.097 \times 10^7 \times (1 - \frac{1}{9}) = \frac{8}{9} \times 1.097 \times 10^7$

or $$\lambda = \frac{1}{\overline{\nu}} = \frac{9}{8 \times 1.097 \times 10^7} \text{m} = 1025 \text{ Å}.$$

For $3 \to 2$, $\overline{\nu} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9} \right) = \frac{5}{36} \times 1.097 \times 10^7$

or $$\lambda = \frac{36}{5 \times 1.097 \times 10^7} = 6563 \text{ Å}.$$

For $2 \to 1$, $\overline{\nu} = 1.097 \times 10^7 \left(1 - \frac{1}{4} \right) = \frac{3}{4} \times 1.097 \times 10^7$

or $$\lambda' = \frac{4}{3 \times 1.097 \times 10^7} \text{m} = 1215 \text{ Å}.$$

12. We have $r_n = (5.3 \times 10^{-11}) \times \frac{n^2}{Z}$.

$\therefore$ $$r_1 = 5.3 \times 10^{-11} \times \frac{1^2}{2} = 2.65 \times 10^{-11} \text{ m}.$$

$$E_{\text{binding}} = 2.18 \times 10^{-18} \times 2^2 \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right) = 2.18 \times 4 \times 10^{-18} \text{ joule}$$

$$= \frac{2.18 \times 4 \times 10^{-18}}{1.6 \times 10^{-19}} = 54.5 \text{ eV}.$$

13. $E_{\text{binding}} = 2.18 \times 10^{-18}\left(\frac{1}{n^2} - \frac{1}{\infty}\right) = \frac{2.18 \times 10^{-18}}{n^2}$

$\therefore \quad 0.85 \times 1.6 \times 10^{-19} = \frac{2.18 \times 10^{-18}}{n^2} \quad$ or $\quad n = 4.$

$E_{\text{ex.}} = 2.18 \times 10^{-18}\left(1 - \frac{1}{n^2}\right)$

$\therefore \quad 10.2 \times 1.6 \times 10^{-19} = 2.18 \times 10^{-18}\left(1 - \frac{1}{n^2}\right) \quad$ or $\quad n = 2.$

E (energy of the photon emitted) = difference in energy from $n = 4$, to $n = 2$.

$$= 2.18 \times 10^{-18}\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$= 4.09 \times 10^{-19} \text{ joule}$$

$$= \frac{4.09 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.25 \text{ eV}.$$

14. We have $\quad \overline{\nu} = 1.097 \times 10^7\left(\frac{1}{s^2} - \frac{1}{n^2}\right).$

For $2 \rightarrow 1$, $\overline{\nu} = 1.097 \times 10^7\left(1 - \frac{1}{4}\right) = \frac{3}{4} \times 1.097 \times 10^7$

$\therefore \quad \lambda = \frac{1}{\overline{\nu}} = \frac{4}{3 \times 1.097 \times 10^7} \text{ m} = 1215 \text{ Å}.$

For $3 \rightarrow 2$, $\overline{\nu} = 1.097 \times 10^7\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36} \times 1.097 \times 10^7.$

$\therefore \quad \lambda = \frac{1}{\overline{\nu}} = \frac{36}{5 \times 1.097 \times 10^7} \text{ m} = 6563 \text{ Å}.$

For $4 \rightarrow 3$, $\overline{\nu} = 1.097 \times 10^7\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7}{144} \times 1.097 \times 10^7.$

$\therefore \quad \lambda = \frac{1}{\overline{\nu}} = \frac{144}{7 \times 1.097 \times 10^7} = 18752 \text{ Å}.$

15. We have $\quad E_n = -E_0\frac{Z^2}{n^2}$ where $E_0 = \frac{e^4 m}{8\varepsilon_0^2 h^2} = 2.18 \times 10^{-18}$ joule

= ionisation energy of hydrogen atom

$E_n - E_s = h\nu = hc/\lambda \quad (\because \nu\lambda = c)$

or $\quad \lambda = \frac{hc}{E_n - E_s} = \frac{hc}{E_0 Z^2\left(\frac{1}{s^2} - \frac{1}{n^2}\right)}.$

Here $Z = 3,\ E_0 = 2.18 \times 10^{-18},\ h = 6.6 \times 10^{-34},\ s = 1,\ n = 3.$

$$\therefore \quad \lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.18 \times 10^{-18}\,(1 - \frac{1}{9}) \times 9} = \frac{6.6 \times 3 \times 9}{2.18 \times 8 \times 9} \times 10^{-8}\ \text{m}$$

$$= 114 \times 10^{-10}\,\text{m} = 114\ \text{Å}.$$

Transitions may take place from $n = 3$ to $n = 1$, $n = 3$ to $n = 2$ and $n = 2$ to $n = 1$ and so in all three lines are observed.

16. The wavelength of the first member of Lyman series is

$$\lambda = 1/\overline{\nu} = \frac{1}{RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} = \frac{4}{3RZ^2}.$$

The wavelength of the first member of the Balmer series,

$$\lambda' = 1/\overline{\nu} = \frac{1}{RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)} = \frac{36}{5RZ^2}$$

$$\Delta\lambda = \frac{36}{5RZ^2} - \frac{4}{3RZ^2} = \frac{1}{RZ^2} \times \frac{88}{15}$$

or $$R = \frac{88}{15Z^2\,\Delta\lambda} = \frac{88}{15 \times 4 \times 1338 \times 10^{-10}} = 1.096 \times 10^7\ \text{m}^{-1}.$$

17. When the second one is excited exactly to the first excited state ($n = 2$) a photon will be emitted by it.
So energy to be transmitted to the second is $E_{n=2} - E_{n=1}$

$$= -E_0 \times \frac{1^2}{2^2} - \left(-E_0 \frac{1^2}{1^2}\right) = \frac{3}{4} E_0 \text{ where } E_0 = 13.6\ \text{eV}.$$

By conservation of momentum, $mv = mv' + mv''$, where m = mass of hydrogen atom. By conservation of energy,

$$\frac{1}{2} mv^2 = \frac{1}{2} mv'^2 + \frac{1}{2} mv''^2 + \frac{3}{4} E_0$$

Eliminating v'' we have $v'^2 - v'v + \frac{3}{4} \times \frac{E_0}{m} = 0$.

Since v' is real, $v^2 \geq 4 \times \frac{3}{4} \times \frac{E_0}{m}$ or $v_{\min} = \sqrt{\frac{3E_0}{m}}.$

$$\therefore \quad K_{\min} = \frac{1}{2} mv^2_{\min} = \frac{1}{2} m \times \frac{3E_0}{m}$$

$$= \frac{3}{2} E_0 = \frac{3}{2} \times 13.6 = 20.4\ \text{eV}.$$

18. Energy released by atom = energy of photon

$$= E_0\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3E_0}{4}$$

where $E_0 = 13.6$ eV.

$$\text{Momentum of photon} = \frac{h\nu}{c} = \frac{\text{energy}}{c} = \frac{3E_0}{4c}.$$

By conservation of momentum, $\dfrac{3E_0}{4c} = m_H v.$

$$\therefore \qquad v = \frac{3}{4} \times \frac{E_0}{m_H c} = \frac{3}{4} \times \frac{13.6 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^8} = 3.25 \text{ m/s}.$$

19. The first line of Lyman series $= \dfrac{1}{RZ^2\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right)} = \dfrac{4}{3RZ^2}.$

The first line of Balmer series $= \dfrac{1}{RZ^2\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right)} = \dfrac{36}{5RZ^2}.$

$$\therefore \qquad \Delta\lambda = \frac{36}{5RZ^2} - \frac{4}{3RZ^2} \quad \text{or} \quad Z^2 = \frac{88}{15R\Delta\lambda}.$$

Here $\qquad Z^2 = \dfrac{88}{15 \times 1.097 \times 10^7 \times 593 \times 10^{-10}} = 9$

or $\qquad Z = 3.$

20. For pure hydrogen, $r = r_0 n^2$

where $\qquad r_0 = \dfrac{\varepsilon_0 h^2}{\pi e^2 m} = 5.3 \times 10^{-11}$ m.

For positronium, m is to be replaced by $\mu = \dfrac{m^2}{2m} = \dfrac{m}{2}.$

$\therefore$ For positronium $r = r_0' n^2$ where $r_0' = \dfrac{\varepsilon_0 h^2}{\pi e^2 m/2} = 2r_0 = 10.6 \times 10^{-11}$

When $\qquad n = 1, \quad r = r_0' = 10.6 \times 10^{-11}$ m $= 106$ p m

For pure hydrogen,

$$E_b = E_0 \qquad \text{where} \quad E_0 = \frac{e^4 m}{8\varepsilon_0^2 h^2} = 13.6 \text{ eV}.$$

For positronium,

$$E_b' = E_0' \qquad \text{where} \quad E_0' = \frac{e^4 m/2}{8\varepsilon_0^2 h^2} = \frac{13.6}{2} = 6.8 \text{ eV}.$$

$$\therefore \qquad E_b' = 6.8 \text{ eV}.$$

21. (i) Since there are six lines there must be four levels. The ground A has $n = 1$ and the highest level has $n = 4$. Hence the excited level B in which electrons lie may be either 2 or 3. If the level B has $n = 2$, the atoms may go to level $n = 4$ after absorbing energy 2.7 eV and revert back to lower levels emitting photons of 2.7 eV, less than 2.7 eV and also more than 2.7 eV. If level B has $n = 3$, it electron can go to $n = 4$, revert back to lower levels emitting photons of energy 2.7 eV or more but *never less than* 2.7 eV. Hence the possible quantum number for B is $n = 2$.

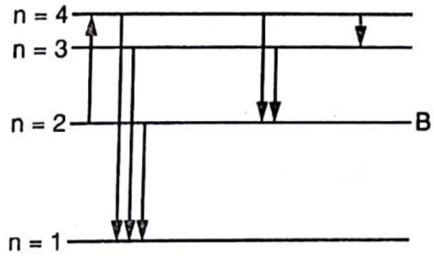

Fig. 2.1

(ii) $E_n - E_s = E_0 Z^2 \left(\frac{1}{s^2} - \frac{1}{n^2}\right)$

When $\quad s = 1,\ n = \infty,\ E_n - E_s = E_{\text{ionisation}}$

$\therefore \quad E_{\text{ionisation}} = E_0 Z^2$

$\therefore \quad E_n - E_s = E_{\text{ion}} \left(\frac{1}{s^2} - \frac{1}{n^2}\right)$

Here $\quad \Delta E = 2.7$ eV where $s = 2,\ n = 4$

$\therefore \quad 2.7 = E_{\text{ion}} \left(\frac{1}{2^2} - \frac{1}{4^2}\right) \Rightarrow E_{\text{ion}} = \frac{2.7 \times 16}{3} = 14.4 \text{ eV}$

(iii) $E_{\max} = 14.4 \left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 13.5 \text{ eV}$

$$E_{\min} = 14.4 \left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 0.7 \text{ eV}.$$

22. $E_s - E_n = 13.6\ Z^2 \left(\frac{1}{s^2} - \frac{1}{n^2}\right)$

The first excited state is $n = 2$. Let it jump from n to s and then from s to 2 emitting two photons.

Then $\quad 10.20 = 13.6\ Z^2 \left(\frac{1}{s^2} - \frac{1}{n^2}\right)$

and $\quad 17.00 = 13.6\ Z^2 \left(\frac{1}{2^2} - \frac{1}{s^2}\right)$

Adding, $27.20 = 13.6\, Z^2 \left(\frac{1}{4} - \frac{1}{n^2}\right)$

Similarly, $4.25 + 5.95 = 13.6\, Z^2 \left(\frac{1}{9} - \frac{1}{n^2}\right)$

Dividing, $\frac{27.20}{10.20} = \frac{n^2 - 4}{4n^2} \times \frac{9n^2}{n^2 - 9}$

Solving, $n = 6$

$$10.20 = 13.6\, Z^2 \left(\frac{1}{9} - \frac{1}{36}\right) \Rightarrow Z = 3$$

23. $E_n - E_s = 13.6 Z^2 \left(\frac{1}{s^2} - \frac{1}{n^2}\right) = \frac{hc}{\lambda}$

The third Balmer line is due to transition from

$$n = 5 \quad \text{to} \quad n = 2.$$

$$\therefore \quad 13.6\, Z^2 \left(\frac{1}{2^2} - \frac{1}{5^2}\right) = \frac{19.86 \times 10^{-26}}{108.5 \times 10^{-9}}$$

$$\Rightarrow \quad Z = 2$$

Binding energy of the ground state corresponds to transition from

$$n = 1, \quad n = \infty$$

$$\therefore \quad E_b = 13.6 \times 2^2 \left(\frac{1}{1^2} - \frac{1}{\infty}\right) = 54.4 \text{ eV}$$

24. $E = K + U$

$|K| = \frac{1}{2}|U|$ and potential energy is negative

$$\therefore \quad E = K - 2k = -K$$

$$\therefore \quad -3.4 = -K \quad \Rightarrow K = 3.4 \text{ eV}$$

$$p = \frac{h}{\lambda} \quad \Rightarrow \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \cdot \left(K = \frac{p^2}{2m}\right)$$

$$\Rightarrow \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}}$$

$$= 6.7 \times 10^{-10} \text{ m} = 6.7 \text{ Å}.$$

□□□

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Chapter 3 – Photoelectricity, X-Rays, and Radioactivity

Solution.

$$^{226}_{88}\text{Ra} \xrightarrow{\alpha} {}^{222}_{86}\text{Rn} \xrightarrow{\beta^-} {}^{222}_{87}\text{Fr} \xrightarrow{\alpha} {}^{218}_{85}\text{At} \xrightarrow{\beta^-} {}^{218}_{86}\text{Rn}$$

$$\xrightarrow{\alpha} {}^{214}_{84}\text{Po} \xrightarrow{\beta^-} {}^{214}_{85}\text{At} \xrightarrow{\alpha} {}^{210}_{83}\text{Bi} \xrightarrow{\beta^-} {}^{210}_{84}\text{Po} \xrightarrow{\alpha} {}^{206}_{82}\text{Pb}$$

Check: No. of α-decays $= \dfrac{A_1 - A_2}{4} = \dfrac{226 - 206}{4} = 5,$

No. of $\bar{\beta}$-decays $= \dfrac{A_1 - A_2}{2} - (Z_1 - Z_2) = 10 - (88 - 82) = 4$

15. *Establish the law of decay of daughter atoms which are also radioactive of decay constant λ_2 when the decay law of parent atoms is $N_1 = N_{10}\, e^{-\lambda_1 t}$.*

Solution.

Let N_2 be the instantaneous number of daughter atoms. These atoms are simultaneously being created and decayed. Their rate of creation is equal to the rate of decay of parent atoms which is $\lambda_1 N_1$ and the rate of decay is $\lambda_2 N_2$. Hence net rate of increase is $\lambda_1 N_1 - \lambda_2 N_2$ and this must be equal to $\dfrac{dN_2}{dt}$.

$$\therefore \qquad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$$\text{or} \qquad \frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_{10}\, e^{-\lambda_1 t}$$

To integrate, multiply throughout by $e^{\lambda_2 t}$

$$e^{\lambda_2 t}\frac{dN_2}{dt} + e^{\lambda_2 t} N_2 \lambda_2 = \lambda_1 N_{10}\, e^{(\lambda_2 - \lambda_1)t}$$

$$\text{or} \qquad \frac{d}{dt}(N_2 e^{\lambda_2 t}) = \lambda_1 N_{10}\, e^{(\lambda_2 - \lambda_1)t}$$

Intergrating, $N_2 e^{\lambda_2 t} = \lambda_1 N_{10}\, e^{(\lambda_2 - \lambda_1)t} / \lambda_2 - \lambda_1 + c$

At $\quad t = 0,\ N_2 = N_{20} = 0$ and so $\lambda_1 N_{10} + c = 0$

$$\therefore \qquad N_2 = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

EXERCISES

Matter Wave and X-rays

1. Calculate the de Broglie wavelength of an electron which falls through a p.d. of 100 V. ($h = 6.62 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg)
2. Calculate the wavelength associated with an electron of energy 1000 eV. ($h = 6.62 \times 10^{-34}$ Js, 1 eV $= 1.6 \times 10^{-19}$ J)
3. Calculate the energy of a photon of wavelength 5500 A.U. ($h = 6.62 \times 10^{-34}$ Js).
4. What voltage should be applied to a cathode ray tube to produce electrons of

wavelength 0.4 Å? What would be the speed of these electrons?
(Charge of electron $= 1.6 \times 10^{-19}$ C, mass of electron $= 9 \times 10^{-31}$ kg and Planck constant $= 6.62 \times 10^{-34}$ Js)

5. An X-ray tube operates at 50 kV. Find the shortest wavelength of X-rays produced. ($h = 6.62 \times 10^{-34}$ Js, Velocity of light $= 3 \times 10^8$ m s^{-1})
[*Hint:* $\lambda_{min} = hc/Ve$]

6. An X-ray tube operates at 30,000 V and the current through it is 2 mA. Calculate (a) power of the tube, (b) the number of electrons striking the target per second, (c) velocity of eolectrons when they hit the target, and (d) the lower wavelength limit of the X-rays emitted.

7. An X-ray tube operates at 30 kV and emits a continuous X-ray spectrum with shortest wavelength limit 0.414 A.U. Calculate Planck constant.

8. An X-ray tube operates at 50 kV. Assuming that only 10% of the energy of electrons is converted into X-ray radiations calculate the wavelength of X-rays produced.

9. A point source of light emits light of wavelength $\lambda = 6000$ Å. The radiation of the source is $P = 100$ W. Find (a) the mean density of flow of photons at a distance $r = 5$ m from the source, (b) the distance where the concentration of photons is $n = 200$ cm^{-3}.

10. A short light pulse of energy $E = 10$ J falls in a collimated beam on a mirror where reflection coefficient is $r = .8$. The angle of incidence $\theta = 60°$. Find the momentum imparted to the mirror.

Photoelectricity

1. Calculate the work function of sodium metal given that the threshold wavelength is 6800 A.U. ($h = 6.625 \times 10^{-34}$ Js)
[*Hint:* $\varphi = h\nu_0$ and $\nu_0\lambda_0 = c$]

2. The photoelectric threshold wavelength for tungsten is 2300 A.U. Find the energy of the photoelectrons emitted when light of wavelength 1800 Å falls on tungsten. (Planck constant $= 6.6 \times 10^{-34}$ Js)

3. A ray of ultraviolet light of wavelength 3000 Å falls on the surface of a material whose work function is 2.28 eV. Calculate the velocity of the fastest electrons. ($h = 6.62 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg)
[*Hint:* Use $\frac{1}{2}mv^2 = h\nu - \varphi$]

4. Calculate the threshold frequency of a metal whose work function is 1.8 eV. ($h = 6.6 \times 10^{-34}$ Js)

5. A photon of wavelength 3000 Å falls on the cathode of a photoelectric tube. Calculate the stopping potential of electrons emitted if the threshold wavelength of the material of cathode is 6000 Å.
($h = 6.0 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C and $c = 3 \times 10^8$ m s^{-1})
[*Hint:* Stopping potential is the negative potential required to be applied on the anode to prevent even the fastest electrons from reaching the anode, in other words potential required to reduce photoelectric current to zero. $\frac{1}{2}mv^2 = h(\nu - \nu_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = Ve$

6. A photon of wavelength 3300 Å falls on the cathode of a photoelectric tube and

electrons of energy 3×10^{-19} J are ejected. If the wavelength of the incident light is changed to 5000 Å the energy of the ejected electrons decreases to 0.972×10^{-19} J. Calculate Planck constant and the threshold wavelength of the material of cathode.

7. Electrons in a hydrogen-like atom ($Z = 3$) make transitions from fifth to fourth and from fourth to third orbit. The resulting radiation are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 V. Calculate the work function and stopping potential for the photoelectrons ejected by the longer wavelength. Rydberg constant $= 1.094 \times 10^7\ m^{-1}$. **[IIT 1990]**

8. A monochromatic light source of frequency ν illuminates a metallic surface and ejects photoelectrons. The photoelectrons with maximum energy are just able to ionize the hydrogen atoms in the ground state. When the whole experiment is repeated with an incident radiation $\frac{5\nu}{6}$, the photoelectrons so emitted are able to excite the hydrogen atom beam which then emits a radiation of wavelength 1215 A Find the work function of the metal and the frequency ν.
($h = 6.6 \times 10^{-34}$ Js) **[Roorkee 1990]**

9. Up to what potential V can a zinc ($\phi = 3.74$ eV) ball removed from other bodies be charged by irradiating it with ultraviolet radiation with a wavelength of $\lambda = 200$ nm?

10. The cathode of a vacuum photodiode is illuminated by monochromatic light of $\lambda = 450$ nm. The area of the cathode is $S = 1\ cm^2$, its illuminance $I = 100$ lx. Determine the saturation current i_{sat} flowing through the diode. At this wavelength energy flux of 0.04 W corresponds to a light flux of 1 lm. Assume that the quantum yield of photoelectric effect, i.e., the number of photoelectrons per incident photon is $J = 0.05$.

11. A certain metal when illuminated alternately by light of wavelength $\lambda_1 = 3500$ Å and $\lambda_2 = 5400$ Å, emits photoelectrons of maximum velocities in the ratio $\eta : 1 = 2 : 1$. Find the work function of the metal.

12. Find the maximum kinetic energy of photoelectrons liberated from lithium ($\phi = 2.39$ eV) by an electromagnetic wave whose electric field varies as $E = a\,(1 + \cos \omega t) \cos \omega_0 t$ where a is a constant, $\omega = 10^{15}$ rad/s and $\omega_0 = 10^{16}$ rad/s.

Radioactivity

1. It takes 5 years for 1 g of radium to be reduced by 2.1 mg. Calculate half-life period of radium.

2. If 1 g of radium emits 3.7×10^{10} particles per s, find the half-life period and average life of radium. (Atomic weight of radium = 226, Avogardo number = $6.025 \times 10^{23}\ mol^{-1}$)

3. The half-life of the radioactive radon is 3.8 days. Calculate the time at the end of which 1/20th of the radon sample will remain undecayed. ($\log_{10} e = 0.4343$) **[IIT 1981]**

4. An α-particle of 5 MeV is scattered through 180° by a fixed uranium nucleus. Calculate the distance of closest approach. **[IIT 1981]**

5. The activity of a certain perparation decreases 2.5 times after 7 days. Find its half-life.

6. At the initial moment the activity of a certain radionuclide totalled 650 particles per minute. What will be the activity of the perparation after half its half-life period?

7. A ^{238}U preparation of mass 1 g emits 1.24×10^4 α-particles per second. Find the half-life of this nuclide and activity of the preparation.

8. A small amount of a solution of a radioactive substance with activity $A = 2000$ becquerel was injected in the bloodstream of a man. The activity of 1 cc of blood sample taken $t = 5$ hours was found to be $A' = 0.25$ becquerel. The half life of the radionuclide is $T = 15$ hours. Find the volume of the man's blood.

9. Show that if a given sample can decay in either of two processes with disintegration constants λ_1 and λ_2, the equivalent disintegration constant is $\lambda = \lambda_1 + \lambda_2$. Hence find the time in which $\frac{3}{4}$ th of such a sample will decay if the mean-lives of a sample are $\tau_1 = 1620$ years and $\tau_2 = 1405$ years. **[Roorkee 1987]**

10. How many beta-particles are emitted during $t = 1$ hour by $m = 1$ mg of ^{24}Na radionuclide whose half-life is $\tau = 15$ hours. M, atomic wt. of sodium = 22.9898.

11. Consulting a periodic table find how many alpha and β^--decays $^{222}_{88}Ra$ experiences before turning into the stable ^{206}Pb isotope.

12. $^{210}_{83}Bi$ radionuclide decays by successive β^- and α emissions.

$$^{210}_{83}Bi \xrightarrow{\beta^-} {}^{210}_{84}Po \xrightarrow{\alpha} {}^{206}_{82}Pb \quad \text{(stable)}$$

The decay constants are $\lambda_1 = 1.60 \times 10^{-6}\, s^{-1}$ and $\lambda_2 = 5.80 \times 10^{-8}\, s^{-1}$, respectively. Calculate α and β^- activities of ^{210}Bi preparation of mass 1g, a month after its manufacture. At. wt. $^{210}Bi = 208.9$.

13. The K_α photon of a metal has wavelength 0.7 Å. If energy of the atom with a hole in K-shell is – 23.0 keV; what will be the energy of this atom when a hole is created in L-shell?

[*Hint:* $E_{\text{holed atom}} = E_{\text{neutral}} -$ energy of electron in holed shell]

14. Find the screening constant if for $Z_1 = 42$, $K_{\alpha_1} = 0.7$ Å and for $Z_2 = 27$, $K_{\alpha_1} = 1.75$ Å.

ANSWERS

Matter Wave and X-rays

1. 1.227×10^{-10} m 2. 0.388 Å 3. 2.256 eV 4. 951.3 V, 1.84×10^7 m/s
5. 0.248 Å 6. (a) 60 W, (b) 1.25×10^{16}, (c) 10^8 m s^{-1}, (d) 4.1×10^{-11} m
7. $h = 6.624 \times 10^{-34}$ Js 8. 2.48 A.U.

9. $N = \dfrac{P\lambda}{4\pi hcr^2} = 9.64 \times 10^{17}$, $r = \sqrt{\dfrac{P\lambda}{4\pi nhc^2}} = 402$ m

10. $\Delta p = \frac{E}{c}\sqrt{1 + 2r\cos\theta + r^2} = 3.0 \times 10^{-8}$ kg m s^{-1}

Photoelectricity

1. 1.827 eV **2.** 2.39×10^{-19} J **3.** 8.1×10^{5} m s^{-1} **4.** 4.8×10^{14} Hz **5.** 1.875 V

6. 6.6×10^{-34} Js, 6640 Å **7.** 2 eV, 0.75 V **8.** 4.95×10^{15} Hz, 6.8 eV **9.** 2.5 V

10. $\dfrac{0.04JSI\lambda e}{hc} = 7\ \mu A$ **11.** 1.6 eV **12.** 4.86 eV

Radioactivity

1. 1649 years **2.** 1584 years, 2285 years **3.** 16.4 days **4.** 1.35×10^{-13} m

5. 5.3 days **6.** 460 per min **7.** 3.35×10^{-7} Ci, 4.5×10^{9} years **8.** 6.35 litres

9. 216 years **10.** 1.18×10^{18} **11.** Four α-decays and two β^{-1}–decays

12. β^{-}–activity $= 7.3 \times 10^{13}$ and α-activity $= 1.46 \times 10^{11}$ **13.** 40.2 keV **14.** 1.19

CHAPTER 3

Photoelectricity, X-rays and Radioactivity

1. Energy of the electron $= \frac{1}{2}mv^2 = Ve$

or $\quad \frac{1}{2m} \times p^2 = Ve \quad$ or $\quad p = \sqrt{2meV} \qquad (\because \; mv = p)$

We have $\quad p = \frac{h}{\lambda} \quad$ or $\quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$

or
$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 100}}$$
$$= 1.227 \times 10^{-10} \text{ m.}$$

2. $E = 1000 \text{ eV} = 1000 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-16}$ joule.

Now $\quad E = \frac{1}{2}\,mv^2 = \frac{1}{2m}p^2 \qquad (\because \; p = mv)$

or $\quad p = \sqrt{2mE}$.

We have $\quad p = \frac{h}{\lambda} \quad$ or $\quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$.

$$\therefore \quad \lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-16}}}$$
$$= 0.388 \times 10^{-10} \text{ m} = 0.388 \text{ Å}.$$

3. The energy of photon is $E = h\nu = \frac{hc}{\lambda}$. $\quad (\nu\lambda = c)$

$$\therefore \quad E = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{550 \times 10^{-10}} = \frac{6.62 \times 3}{55} \times 10^{-18}$$
$$= \frac{3 \times 6.62 \times 10^{-18}}{55 \times 1.6 \times 10^{-19}} \text{ eV} = 2.256 \text{ eV.}$$

4. Let V be the required voltage.

Then $\quad E$ (energy) of the electron
$$= Ve = \frac{1}{2}mv^2 = \frac{1}{2} \times \frac{p^2}{m}$$
$$\therefore \quad p^2 = 2mVe.$$

We have $p = \dfrac{h}{\lambda}$ or $\sqrt{2meV} = \dfrac{h}{\lambda}$

or $$V = \frac{h^2}{2me\lambda^2} = \frac{(6.62 \times 10^{-34})^2}{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19} \times (0.4 \times 10^{-10})^2}$$

$$= \frac{(6.62)^2}{2 \times 9 \times 1.6 \times (0.4)^2} \times 10^2 = 951.3 \text{ volts.}$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 951}{9 \times 10^{-31}}} = 1.84 \times 10^7 \text{ m s}^{-1}.$$

5. The maximum energy of electrons = Ve.

Assuming that whole energy of electrons is c... d into X-ray photon, we have

$$Ve = h\nu_{max} = \frac{hc}{\lambda_{min}} \qquad (\because \quad \nu\lambda = c)$$

or $$\lambda_{min} = \frac{hc}{Ve} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{50 \times 10^3 \times 1.6 \times 10^{-19}}$$

$$= 0.248 \times 10^{-10} \text{ m} = 0.248 \text{ Å}.$$

6. Power = $VC = 30000 \times 2 \times 10^{-3} = 60$ watts.

Let n be the number of electrons hitting the target in t second. Then current through the tube

$$= ne = 2 \times 10^{-3} \text{ (given)}$$

$$\therefore \quad n = \frac{2 \times 10^{-3}}{e} = \frac{2 \times 10^{-3}}{1.6 \times 10^{-19}} = 1.25 \times 10^{16}.$$

Let v be the velocity acquired by the electrons on reaching the target.

Loss in electrical energy

$$= Ve = \text{gain in kinetic energy} = \frac{1}{2}mv^2$$

or $$v = \sqrt{\frac{2Ve}{m}} = \sqrt{\frac{2 \times 30000 \times 1.6 \times 10^{-19}}{9 \times 10^{-31}}}$$

$$= \sqrt{\frac{2 \times 3 \times 1.6}{9}} \times 10^8 = 1.0 \times 10^8 \text{ m s}^{-1}.$$

Let us assume that whole energy of the electron is converted into **X-rays.**

. hen $\quad Ve = \text{energy of photon} = h\nu = \dfrac{hc}{\lambda} \qquad (\because \quad \nu\lambda = c)$

or $\quad \lambda = \dfrac{hc}{Ve} = \dfrac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{30000 \times 1.6 \times 10^{-19}}$

$$= \frac{662 \times 3}{3 \times 1.6} \times 10^{-11} = 4.1 \times 10^{-11}\ \text{m.}$$

7. See solution of problem 5.

$$\lambda_{\min} = \frac{hc}{Ve}$$

or $\quad h = \dfrac{\lambda_{\min} Ve}{c} = \dfrac{0.414 \times 10^{-10} \times 30 \times 10^{3} \times 1.6 \times 10^{-19}}{3 \times 10^{8}}$

$$= 0.414 \times 1.6 \times 10^{-33} = 6.624 \times 10^{-34}\ \text{J s.}$$

8. Energy of electrons = Ve

10% of this energy = $0.1\ Ve$

or $\quad Ve = h\nu = \dfrac{hc}{\lambda} \quad$ or $\quad \lambda = \dfrac{hc}{0.1Ve} = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{1 \times 50 \times 10^{3} \times 1.6 \times 10^{-19}}$

$$= 2.475 \times 10^{-10}\ \text{m} = 2.48\ \text{Å.}$$

9. ε, energy of each photon $= h\nu = \dfrac{hc}{\lambda}$.

I, intensity of light at $r = \dfrac{P}{4\pi r^2}$

rate of flow of energy per unit area $= Nh\nu = Nhc/\lambda$

$\therefore \quad N = \dfrac{P\lambda}{4\pi hcr^2}$

Here $\quad N = \dfrac{100 \times 6000 \times 10^{-10}}{4\pi \times 6.6 \times 10^{-34} \times 3 \times 10^{8} \times 5^{2}} = 9.64 \times 10^{17}$.

We have

$I = \rho c \quad$ where ρ = density of radiation $= n\dfrac{hc}{\lambda}$

where $\quad n$ = concentration of photons

$\therefore \quad I = \dfrac{nhc^2}{\lambda} = \dfrac{p}{4\pi r^2} \quad$ or $\quad r = \sqrt{\dfrac{p\lambda}{4\pi nhc^2}}$.

Here $\quad r = \sqrt{\dfrac{100 \times 6000 \times 10^{-10}}{4\pi \times 200 \times 10^{6} \times 6.6 \times 10^{-34} \times 3^{2} \times 10^{16}}} = 402\ \text{m.}$

10.

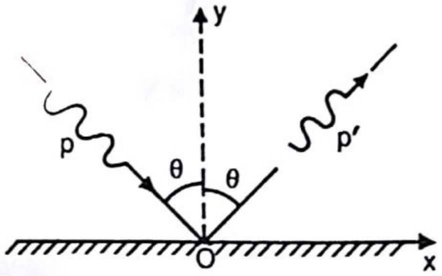

Fig. 3.1

Momentum of the laser impulse before impact is $p = \frac{\varepsilon}{c}$ and momentum of the impulse after impact is

$$p' = r\frac{\varepsilon}{c}.$$

Let us consider outward normal as y-axis and rightward along the mirror as x-axis. Then

$$\vec{p}_i = p \sin\theta \hat{i} + p\cos\theta(-\hat{j})$$

$$\vec{p}_f = p' \sin\theta \hat{i} + p'\cos\theta \hat{j}$$

$$\vec{\Delta p} = (p' - p)\sin\theta \hat{i} + (p' + p)\cos\theta \hat{j}$$

$$= (r-1)\frac{\varepsilon}{c}\sin\theta \hat{i} + (r+1)\frac{\varepsilon}{c}\cos\theta \hat{j}$$

$$\therefore \quad |\vec{\Delta p}| = \frac{\varepsilon}{c}\sqrt{(r-1)^2\sin^2\theta + (r+1)^2\cos^2\theta}$$

$$= \frac{\varepsilon}{c}\sqrt{1 + 2r\cos 2\theta + r^2}.$$

Here $\quad |\vec{\Delta p}| = \dfrac{10}{3\times 10^8}\sqrt{1 + 2\times 0.8\cos 120° + 0.8^2}$

$$= 3.0\times 10^{-8} \text{ kg m s}^{-1}.$$

Photoelectricity

1. We have $\phi = h\nu_0$ where $\nu_0 = f$ (threshold frequency)

$$\phi = \frac{hc}{\lambda_0} = \frac{6.625\times 10^{-34}\times 3\times 10^8}{6800\times 10^{-10}} = 0.262\times 10^{-18}\text{ J}$$

$$= \frac{0.292\times 10^{-18}}{1.6\times 10^{-19}}\text{eV} = 1.827\text{ eV}.$$

2. The energy of photoelectrons

$$= h\nu - h\nu_0 = h(\nu - \nu_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \qquad (\because \nu\lambda = c)$$

$$= 6.6\times 10^{-34}\times 3\times 10^8\left(\frac{1}{1800\times 10^{-10}} - \frac{1}{2300\times 10^{-10}}\right)$$

$$= 2.39\times 10^{-19}\text{ J}.$$

3. We have $\frac{1}{2}mv^2 = h\nu - \varphi$

$$= \frac{hc}{\lambda} - \varphi = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3000 \times 10^{-10}} - 2.28 \times 1.6 \times 10^{-19}$$

$$= 6.62 \times 10^{-19} - 3.648 \times 10^{-19} = 2.972 \times 10^{-19}$$

$$\therefore \qquad v = \sqrt{\frac{2 \times 2.972 \times 10^{-19}}{9 \times 10^{-31}}} = 0.8127 \times 10^6 = 8.1 \times 10^5 \text{ m s}^{-1}.$$

4. We have $\phi = h\nu$ or $\nu = \frac{\phi}{h}$ or $\nu = \frac{1.8 \times 1.6 \times 10^{-19}}{6 \times 10^{-34}}$

$$= 0.48 \times 10^{15} = 4.8 \times 10^{14} \text{ Hz.}$$

5. We have $\frac{1}{2}mv^2 = h(\nu - \nu_0) = Ve$ where V = stopping potential

or $$hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = Ve \quad \text{or} \quad \frac{hc\,(\lambda_0 - \lambda)}{\lambda\lambda_0} = Ve$$

or $$V = \frac{hc\,(\lambda_0 - \lambda)}{e\lambda\lambda_0} = \frac{6 \times 10^{-34} \times 3 \times 10^8\,(6000 - 3000) \times 10^{-10}}{1.6 \times 10^{-19} \times 3000 \times 6000 \times 10^{-20}}$$

$$= \frac{6 \times 3 \times 3}{1.6 \times 6 \times 3} = 1.875 \text{ volt.}$$

6. $E = h\,(\nu - \nu_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

$$\therefore \qquad 3 \times 10^{-19} = hc\left(\frac{1}{3300 \times 10^{-10}} - \frac{1}{\lambda_0}\right) \qquad \ldots \text{(i)}$$

and $$0.972 \times 10^{-19} = hc\left(\frac{1}{5000 \times 10^{-10}} - \frac{1}{\lambda_0}\right) \qquad \ldots \text{(ii)}$$

$$\therefore \qquad (3 - 0.972) \times 10^{-19} = hc\left(\frac{10^{10}}{3300} - \frac{10^{10}}{5000}\right)$$

$$= h \times 3 \times 10^8 \times 10^8\left(\frac{1}{33} - \frac{1}{50}\right)$$

or $$h = \frac{2.028 \times 33 \times 50}{3 \times 17} \times 10^{-35}$$

$$= 65.61 \times 10^{-35} = 6.6 \times 10^{-34} \text{ J s.}$$

Putting the value of h in (i) we get

$$3 \times 10^{-19} = 6.56 \times 10^{-34} \times 3 \times 10^8\left(\frac{10^8}{33} - \frac{1}{\lambda_0}\right)$$

$$\frac{10^8}{33} - \frac{1}{\lambda_0} = \frac{3 \times 10^{-19}}{6.56 \times 3 \times 10^{-26}}$$

or $$3.0303 \times 10^6 - \frac{1}{\lambda_0} = 1.5244 \times 10^6 \quad \text{or} \quad \frac{1}{\lambda_0} = 1.5059 \times 10^6$$

or $$\lambda_0 = 0.66405 \times 10^{-6} = 6640 \text{ Å}.$$

7. The frequency of radiation emitted by hydrogen like atoms is given by

$$\nu = c\,RZ^2\left(\frac{1}{s^2} - \frac{1}{n^2}\right) \text{ where } R = \text{Rydberg constant.}$$

For $5 \rightarrow 4$ transition,

$$\nu = cRZ^2\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = RZ^2c \times \frac{9}{400}.$$

For $4 \rightarrow 3$ transition,

$$\nu' = RZ^2c\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = RZ^2c\,\frac{7}{144}. \quad \therefore \quad \frac{\nu}{\nu'} = \frac{81}{175}.$$

Hence the second one is of greater frequency or shorter wavelength.

Now $$h\nu' - \varphi = Ve \quad \text{or} \quad \varphi = h\nu' - Ve = h.\,RZ^2\,c\frac{7}{144} - Ve$$

or $$\varphi = 6.6 \times 10^{-34} \times 1.094 \times 10^7 \times 3^2 \times 3 \times 10^8 \times \frac{7}{144}$$

$$- 3.95 \times 1.6 \times 10^{-19}$$

$$= 9.48 \times 10^{-19} - 6.32 \times 10^{-19} = 3.16 \times 10^{-19}\text{ J} = 2 \text{ eV}.$$

$$Ve = h\nu - \varphi = h\,RZ^2c \times \frac{9}{400} - 3.16 \times 10^{-19}$$

$$= 6.6 \times 10^{-34} \times 1.094 \times 10^7 \times 3^2 \times 3 \times 10^8 \times \frac{9}{400} - 3.16 \times 10^{-19}$$

$$= 4.39 \times 10^{-19} - 3.16 \times 10^{-19} = 1.23 \times 10^{-19}$$

or $$V = \frac{1.23 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.75 \text{ volt.}$$

8. We have $h\nu - \varphi = E.$

Here $$h\nu - \varphi = 13.6 \times 1.6 \times 10^{-19}$$

($\because$ Energy of the ground level of hydrogen atom is 13.6 eV)

or $$h \times \frac{5\nu}{6} - \varphi = \frac{hc}{1215 \times 10^{-10}}.$$

Subtracting,

$$h\nu = (1 - \frac{5}{6}) = 1.36 \times 1.6 \times 10^{-19} - \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{1215 \times 10^{-10}}$$

$$= 2.176 \times 10^{-18} - 1.632 \times 10^{-18}$$

$$= 0.544 \times 10^{-18}$$

or $$\nu = \frac{0.544 \times 10^{-18} \times 6}{6.6 \times 10^{-34}} = 4.95 \times 10^{15} \text{ Hz.}$$

$$\varphi = 6.6 \times 10^{-34} \times 4.95 \times 10^{15} - 13.6 \times 1.6 \times 10^{-19}$$

$$= 1.09 \times 10^{-18} \text{ J} = 6.8 \text{ eV.}$$

9. When the positive potential of the ball is enough to hold back the most energetic photoelectrons, the ball will not emit photoelectrons.

$\therefore$ $$hc/\lambda - \varphi = Ve \quad \text{or} \quad V = \frac{hc/\lambda - \varphi}{e}.$$

Here $$V = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{200 \times 10^{-9} \times 1.6 \times 10^{-19}} - \frac{3.74 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$= 6.216 - 3.74 = 2.5 \text{ V.}$$

10. Light flux on the cathode = SI lumen.

$\therefore$ Energy flux on the cathode = 0.04 SI watts.

$\therefore$ Photon flux on the cathode $= \dfrac{0.04 \text{ SI}}{\left(\frac{hc}{\lambda}\right)} = 0.04 \text{ SI} \dfrac{\lambda}{hc}$.

$\therefore$ Electron flux from the cathode saturation current

$$= J \times (0.4 \text{ SI} \frac{\lambda}{hc})$$

$\therefore$ $$i_{sat} = -(0.04 \text{ J SI } \lambda/hc) \times e$$

Here $$i_{sat} = \frac{0.04 \times 0.05 \times 1 \times 10^{-4} \times 100 \times 450 \times 10^{-9} \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$$

$$= 7 \,\mu\text{A.}$$

11. We have $$\frac{hc}{\lambda} - \phi = \frac{1}{2} mv^2$$

$$\frac{hc}{\lambda'} - \phi = \frac{1}{2} mv'^2$$

$\therefore$ $$\frac{\frac{hc}{\lambda} - \varphi}{\frac{hc}{\lambda'} - \varphi} = \left(\frac{v}{v'}\right)^2 = \eta^2 \quad \text{or} \quad \varphi = \frac{hc\left(\frac{\eta^2}{\lambda'} - \frac{1}{\lambda}\right)}{\eta^2 - 1}$$

Here $$\varphi = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}\left(\dfrac{2^2}{3500 \times 10^{-10}} - \dfrac{1}{5400 \times 10^{-10}}\right)}{2^2 - 1}$$

$$= \frac{6.6 \times 3 \times 10^{-18}}{3}\left(\frac{4}{35} - \frac{1}{54}\right) = 0.632 \times 10^{-18}\,\text{J} = 3.9\ \text{eV}.$$

12. $E = a\,(1 + \cos \omega t) \cos \omega_0 t = a \cos \omega_0 t + a \cos \omega t \cos \omega_0 t$

or $$E = a \cos \omega_0 t + \frac{1}{2} a\,[\cos (\omega + \omega_0)\, t + \cos (\omega - \omega_0)\, t\,]$$

This is a wave consisting of three frequencies ω_0, $\omega + \omega_0$ and $\omega - \omega_0$. The highest is $\omega + \omega_0$. The photoemission correspond to this frequency.

$\therefore$ $$\hbar\,(\omega + \omega_0) - \varphi = T_{max}$$

Here $$T_{max} = \frac{6.63 \times 10^{-34}}{2\pi}(10^{15} + 10^{16}) - 2.39 \times 1.6 \times 10^{-19}$$

$$= 7.78 \times 10^{-19}\,\text{J} = 4.86\ \text{eV}.$$

Radioactivity.

1. We have $W = W_0 e^{-\lambda t}$

$\therefore$ $$(1 - 2.1 \times 10^{-3}) = 1 \times e^{-\lambda \cdot 5}$$

or $$e^{\lambda \cdot 5} = \frac{1}{0.9979} = 1.0021$$

or $$\lambda\, 5 = \ln 1.0021 = 2.102 \times 10^{-3}$$

or $$\lambda = 4.2044 \times 10^{-4}.$$

We have, half life

$$= \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.2044 \times 10^{-4}} = 1649 \text{ years.}$$

2. We have $N = N_0 e^{-\lambda t}$

$\therefore$ $$\frac{dN}{dt} = N_0 e^{-\lambda t}(-\lambda)$$

or $$\left(-\frac{dN}{dt}\right) = \lambda N.$$

1 g of radium contains $\dfrac{6.025 \times 10^{23}}{226}$ atoms

$\therefore$ $$N = \frac{6.025 \times 10^{23}}{226}$$

$\therefore \qquad 3.7 \times 10^{10} = \dfrac{6.025 \times 10^{23}}{226} \times \lambda$

or $\qquad \lambda = \dfrac{226 \times 3.7 \times 10^{10}}{6.025 \times 10^{23}} = 1.388 \times 10^{-11}.$

We have, half life

$$= \frac{\ln 2}{\lambda} = \frac{\ln 2}{1.388 \times 10^{-11}}$$

$$= 4.9939 \times 10^{10} \text{ s} = 1584 \text{ years.}$$

Average life $= \dfrac{1}{\lambda} = \dfrac{1}{1.388 \times 10^{-11}} = 7.2046 \times 10^{10}$ s

$$= 2285 \text{ years.}$$

3. We have half life $T = \dfrac{\ln 2}{\lambda}$ or $\lambda = \dfrac{\ln 2}{T} = \dfrac{\ln 2}{3{\cdot}8}$

$$W = W_0 e^{-\lambda t}$$

$\therefore \qquad \dfrac{W_0}{20} = W_0 e^{-\lambda t}$ or $e^{\lambda t} = 20$ or $\lambda t = \ln 20$

or $\qquad t = \dfrac{\ln 20}{\lambda} = \dfrac{\ln 20}{\ln 2} \times 3.8 = \dfrac{2.9957}{0.6931} \times 3.8 = 16.4$ days.

4. $5 \times 10^6 \times 1.6 \times 10^{-19} = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{(235e)(2e)}{d}$

or $\qquad d = \dfrac{9 \times 10^9 \times 235 \times 2 \times (1.6 \times 10^{-19})^2}{5 \times 10^6 \times (1.6 \times 10^{-19})}$

$$= \frac{9 \times 10^9 \times 235 \times 2 \times 1.6 \times 10^{-19}}{5 \times 10^6} = 1.35 \times 10^{-13} \text{ m.}$$

5. Activity $= \lambda N$. $\therefore A_1 = \lambda N_1$ and $A_2 = \lambda N_2$.

$\therefore \qquad \dfrac{A_1}{A_2} = \dfrac{N_1}{N_2} = 2.5 = \dfrac{5}{2}$

$\therefore \qquad \dfrac{N_0 e^{-\lambda t_1}}{N_0 e^{-\lambda t_2}} = \dfrac{5}{2}$ or $\lambda (t_2 - t_1) = \log_e \dfrac{5}{2}$

or $\qquad \lambda \times 7 \times 86400 = \log_e 2.5$

$$T = \frac{\log_e 2}{\lambda} = \frac{\log_e 2 \times 7 \times 86400}{\log_e 2.5} \text{ s} = \frac{\log_{10} 2}{\log_{10} 2.5} \times 7 \text{ days}$$

$$= \frac{0.3010}{0.3979} \times 7 = 5.3 \text{ days.}$$

6. $\frac{650}{60} = \lambda N_0$ and $A = \lambda N$ at $t = \frac{1}{2} t_{\frac{1}{2}} \frac{0.6932}{2\lambda}$.

Hence $\lambda t = \frac{0.6932}{2} = 0.3466.$

$\therefore$ Dividing $\frac{A \times 60}{650} = \frac{N}{N_0} = e^{-\lambda t} = e^{-0.3466}$ or $A = \frac{65}{6} e^{-0.3466}$

$$\therefore \quad \log A = \log_{10} 65 - 0.3466 \times 0.4343 - \log_{10} 6$$

$$= 1.8129 - 0.3466 \times 0.4343 - 0.7782$$

$$= 1.8129 - 0.1505 - 0.7782$$

or $\log A = 0.8842$ or $A = 7.66/\text{s} = 460/\text{min}.$

7. We have activity $(A) = \lambda N$

$$\text{Activity} = \lambda N = \frac{1.24 \times 10^4}{3.7 \times 10^{10}} = 3.35 \times 10^{-7}\ \text{C}.$$

$$1.24 \times 10^4 = \lambda \times \frac{6.06 \times 10^{23}}{238} \quad \text{or} \quad \lambda = \frac{1.24 \times 238}{6.06} \times 10^{-19}$$

$$t_{\frac{1}{2}} = \frac{0.6932}{\lambda} = \frac{0.6932 \times 6.06}{1.24 \times 238} \times 10^{19}\ \text{s} = 4.5 \times 10^9 \text{ years}.$$

8. A (initial activity of the preparation) $= \lambda N_0$. Let V be the volume of man's blood. Hence initial number of atoms per unit volume of blood $= \frac{N_0}{V}$. After time t activity per unit volume is $A' = \lambda N$ where $N =$ number of atoms per unit volume $= \lambda \left(\frac{N_0}{V}\right) e^{-\lambda t} = \frac{A}{V} e^{-\lambda t}$

$$\therefore \quad V = \frac{A}{A'} e^{-\lambda t}.$$

Here $V = 2000\ e^{-\ln 2 \times 5/15} = 800 \times 0.7937 = 6350\ \text{cm}^3 = 6.35$ litres.

9. Let N be the instantaneous number. Then the number of atoms that disintegrate by the first process is $(\lambda_1 N)\, dt$ and that by the second process is $(\lambda_2 N)\, dt$. Therefore the total number of atoms that disintegrate in dt is $(\lambda_1 N dt + \lambda_2 N dt)$ and this has to be $-dN$.

$$\therefore \quad -dN = (\lambda_1 + \lambda_2)\, N\, dt.$$

Integrating, $\ln N = -(\lambda_1 + \lambda_2)\, t + c.$ At $t = 0$, $N = N_0$

and so $c = \ln N_0$. $\therefore$ $\ln N = -(\lambda_1 + \lambda_2)\, t + \ln N_0$

or $N = N_0\, e^{-(\lambda_1 + \lambda_2) t}$ $\therefore$ $\lambda_{\text{eff}} = \lambda_1 + \lambda_2.$

Here $\lambda_{eff} = \lambda_1 + \lambda_2 = \frac{1}{\tau_1} + \frac{1}{\tau_2} = \frac{\tau_1 + \tau_2}{\tau_1\tau_2} = \frac{1620 + 1405}{1620 \times 1405} = 0.00133 \text{ year}^{-1}$

$\therefore$ $\frac{3}{4}N_0 = N_0\, e^{-0.00133\, t}$ or $t = 216$ years.

10. N, Number of beta particles emitted = No. of nuclei decayed

$$= N_0 - N_0 e^{-\lambda t} = N_0 (1 - e^{-(\ln 2)\, t/\tau})$$

Here $N_0 = \frac{N_A m}{M}$, N_A = Avogadro constant

$\therefore$ $N = \frac{N_A m}{M}(1 - e^{-\ln 2 t/\tau})$

Here $N = \frac{6.02 \times 10^{23} \times 10^{-3}}{22.9898}(1 - e^{-(\ln 2) \times 1/15})$

$$= 2.62 \times 10^{19} \times 0.045 = 1.18 \times 10^{18}.$$

11. $^{222}_{88}Ra \xrightarrow{\alpha} {}^{218}_{86}Rn \xrightarrow{\beta^-} {}^{218}_{87}Fr \xrightarrow{\alpha} {}^{214}_{85}At \xrightarrow{\beta^-} {}^{214}_{86}Rn \xrightarrow{\alpha}$

$^{210}_{84}Po \xrightarrow{\alpha} {}^{206}_{82}Pb.$

12. β-activity $= \lambda_1 N_{10} e^{-\lambda_1 T}$

$N_{10} = \frac{mN_A}{A}$ where A = atomic wt. of Bi^{210}

$$= \frac{10^{-3} \times 6.02 \times 10^{26}}{209} = 2.88 \times 10^{21}$$

$\therefore$ β-activity after a month

$$= 1.6 \times 10^{-6} \times 2.88 \times 10^{21}\, e^{-1.6 \times 10^{-6} \times 30 \times 86400} = 7.3 \times 10^{13}.$$

$$\alpha\text{-activity} = \lambda_2 N_2 = \lambda_2\lambda_1 \frac{N_{10}}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

or $\alpha\text{-activity} = \frac{1.6 \times 10^{-6} \times 5.8 \times 10^{-8} \times 2.88 \times 10^{21}}{5.8 \times 10^{-8} - 1.6 \times 10^{-6}}$

$$\times \left[e^{-1.6 \times 10^{-1} \times 86400} - e^{-5.8 \times 10^{-8} \times 86400}\right]$$

$$= 1.46 \times 10^{11}.$$

13. $E_{\text{holed in K shell}} = E_{\text{neutral}} - \left[-E_0 (Z-\sigma)^2 \left(\frac{1}{1^2}\right) \right] = -23.0 \times 10^3 e$ (given)

$E_{\text{holed in L shell}} = E_{\text{neutral}} - \left[-E_0 (Z-\sigma)^2, \frac{1}{2^2} \right] = E = ?$

Subtracting $\quad E + 23.0 \times 10^3 e = E_0 (Z-\sigma)^2 \left(\frac{1}{4} - 1\right)$

ε (energy of k_α photon) $= E_0 (Z-\sigma)^2 (1 - \frac{1}{4}) = E_0 (Z-\sigma)^2 \frac{3}{4}$.

$$\Rightarrow \qquad \frac{hc}{\lambda} = E_0 (Z-\sigma)^2 \times \frac{3}{4}$$

$$\Rightarrow \qquad E_0 (Z-\sigma)^2 = \frac{4}{3} \times \frac{1.987 \times 10^{-25}}{0.72 \times 10^{-10}}$$

$$\therefore \qquad E = -\left[\frac{1.987 \times 10^{-25}}{0.72 \times 10^{-10}} \times \frac{4}{3} \times \frac{3}{4} + 23.0 \times 10^3 e \right] \text{J}$$

$$= -\left[\frac{1.987 \times 10^{-25}}{0.72 \times 10^{-10} \times 1.6 \times 10^{-19}} + 23.0 \times 10^3 \right] \text{eV}$$

$$= -(17.2 \times 10^3 + 23.0 \times 10^3) \text{ eV} = -40.2 \text{ keV.}$$

14. $\nu = Rc (Z-\sigma)^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$,

$$\Rightarrow \qquad \frac{c}{\lambda} = Rc (Z-\sigma)^2 \times \frac{3}{4} \qquad \Rightarrow \lambda \propto \frac{1}{(Z-\sigma)^2}$$

$$\frac{0.7}{1.75} = \frac{(27-\sigma)^2}{(42-\sigma)^2} \qquad \Rightarrow \frac{27-\sigma}{42-\sigma} = 0.63$$

$$\Rightarrow \qquad \sigma = 1.19$$

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Chapter 4 – Nuclear Fission and Fusion

E_b of ${}^{10}_{5}B = 5 \times 1.00783 + (10 - 5) \times 1.00867 - 10.01294$

$= 0.06956$ amu.

$\therefore$ Binding energy of neutron in ${}^{11}_{5}B$

$= 0.08187 - 0.06956 = 0.01231$ amu $= 11.5$ MeV

$${}^{11}_{5}B - {}^{1}_{1}p \rightarrow {}^{10}_{4}Be$$

E_b of ${}^{10}_{4}Be = 4 \times 1.00783 + 6 \times 1.00867 - 10.01354$

$= 0.0698$ amu

$\therefore$ Binding energy of proton in ${}^{11}_{5}B = 0.08187 - 0.0698$

$= 0.01207$ amu $= 11.24$ MeV

$\therefore$ $E_n - E_p = 11.5 - 11.24 = 0.26$ MeV

9. *A particle of mass m strikes a stationary nucleus of mass M and activates an endoergic reaction. Show that the threshold kinetic energy required to initiate this reaction is* $T_{th} = \frac{m+M}{M} |Q|$. *Where Q is the energy of the reaction.*

Solution.

Let T be the kinetic energy of the incoming particle and T_1 and T_2 kinetic energy after collision. Then by conservation of momentum

$$\sqrt{2mT} = \sqrt{2mT_1} + \sqrt{2MT_2}$$

By conservation of energy

$$(T_1 + T_2) = Q + T$$

or $$\sqrt{m}\,(\sqrt{T} - \sqrt{T_1}) = \sqrt{M}\,\sqrt{T_2}$$

and $$T_2 = T + Q - T_1$$

Eliminating T_2 $\sqrt{m}\,(\sqrt{T} - \sqrt{T_1}) = \sqrt{M}\,\sqrt{T + Q - T_1}$

Squaring $m(T + T_1 - 2\sqrt{TT_1}) = M(T + Q - T_1)$

Arranging in quadratic of $\sqrt{T_1}$

$$T_1\,(m + M) - (2m\sqrt{T})\sqrt{T_1} + (m - M)T - MQ = 0$$

Since T_1 is real $4m^2T \geq 4(m + M)\,\{(m - M)T - MQ\}$

or $$m^2T \geq (m^2 - M^2)T - (m + M)MQ$$

or $$T \geq \frac{m+M}{M} |Q| \quad \therefore \; T_{th} = \frac{m+M}{M} |Q|$$

EXERCISES

1. Calculate the binding energy of the lithium atom (${}^{7}_{3}Li$) from the following data:

mass of proton $= 1.00759$ amu
mass of neutron $= 1.00898$ amu
mass of electron $= 0.00055$ amu

mass of lithium atom = 7.01818 amu

2. Find the minimum energy that a γ-ray must have to give rise to an electron-positron pair.

   Mass of electron = 0.00055 amu

   Mass of positron = 0.00055 amu

3. Calculate the energy released by fission of 1 g of $^{235}_{92}U$, assuming that an energy of 200 MeV is released by fission of each atom of ^{235}U. (Avogardo constant is $= 6.023 \times 10^{26}$ kg mol^{-1}.)

4. A reactor develops nuclear energy at the rate of 3×10^4 kW. How many atoms of ^{235}U undergo fission per second? How much ^{235}U is burnt in 1000 hours of operation? (Assume that 200 MeV is released per fission and Avogadro constant is 6.023×10^{26} per kg mole)

5. Calculate the amount of energy set free by the annihilation of an electron and a positron. Given that mass of electron = 0.00055 amu and positron = 0.00055 amu.

6. Complete the following nuclear reactions:

   (i) $^{14}_{7}N + ^{4}_{2}He \rightarrow {}_{8}O + ^{1}_{1}H$

   (ii) $^{24}Mg + ^{4}_{2}He \rightarrow ^{27}_{13}Al +$

   (iii) ${}_{3}Li + {}^{1}n \rightarrow {}^{4}He + ^{3}_{1}H.$

7. Calculate the energy released in kilowatt-hours when 100 g of $^{7}_{3}Li$ are converted into $^{4}_{2}He$ by proton bombardment. Mass of $^{7}_{3}Li = 7.0183$ amu; mass of proton = 1.0081 amu. Write down the nuclear reaction.

8. Calculate the energy released in the nuclear fusion of isotopes of hydrogen

   (i) $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n$

   (ii) $^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n.$

   Given that mass of neutron = 1.00867 amu

   mass of $^{2}_{1}H = 2.01410$ amu

   mass of $^{3}_{1}H = 3.01603$ amu

   mass of $^{3}_{2}H = 3.0160$ amu

   mass of $^{4}_{2}He = 4.00260$ amu.

9. Using conservation laws show that an electron cannot absorb a photon completely.

   [*Hint:* $h\nu = mc^2 - m_0c^2$ and $\frac{h\nu}{c} = mv$]

10. Making use of the tables (table 10) of atomic masses determine the energies of the following reactions:

    (a) $^{7}Li(p, n)\,^{7}Be$ (b) $^{9}Be(n, \gamma)\,^{10}Be$ (c) $^{7}Li(\alpha, n)\,^{10}B$ (d) $^{16}O(d, \alpha)\,^{14}N.$

11. Consulting the values of atomic masses from the table (table 10), find the maximum kinetic energy of β-particles emitted by $^{10}_{4}Be$ nucleii and the corresponding kinetic energy of recoiling daughter nucleii formed directly in the ground state.

    [*Hint:* A nucleus with the least possible energy equal to the binding energy is said to be in the ground state.]

12. A stationary ^{200}Pb nucleus emits an α-particle with kinetic energy $T_\alpha = 6.00$ MeV. Find the recoil of the daughter nucleus. What fraction of the total energy liberated in this decay is accounted for by the recoil of the daughter nucleus?

13. Making use of the table of atomic masses find:

(i) The mean binding energy per nucleon in $^{15}_{8}O$

(ii) The binding energy of a neutron in a $^{11}_{5}B$ nucleus.

14. A nucleus, initially at rest, undergoes alpha decay according to the equation

$$^{A}_{92}X \rightarrow {}^{228}_{Z}Y + \alpha$$

(a) Find the values of A and Z in the above process

(b) The alpha particle produced in the above process is found to move in a circular track of radius 0.11 m in a uniform field of 3 T. Find the energy in MeV released during the process and the binding energy of the parent nucleus. Given that $m(Y) = 228.03u$; $m\left(^{1}_{0}n\right) = 1.009u$; $m\left(^{4}_{2}He\right) = 4.003xu$; $m(^{1}_{1}H) = 1.008u$. 1 amu(u) $= 1.66 \times 10^{-27}$ kg $= 931.5$ MeV **[IIT 1991]**

15. A uranium nucleus ^{235}U liberates 200 MeV per fission. 1.5 kg of uranium reacts during explosion of a uranium bomb. What is the mass of an equivalent TNT bomb if the heating capacity of TNT is 4.1×10^6 J/kg?

16. Can a silicon nucleus $\left(^{31}_{14}Si\right)$ transform into a phosphorus nucleus $\left(^{31}_{15}P\right)$? What particles would be emitted in the process? What is their total energy? $m_{si} = 30.97535$ amu, $m_P = 30.97376$ amu.

17. The nucleus $^{238}_{92}U$ decays according to $^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$. Calculate the kinetic energy of the emitted α-particle.

$^{238}_{92}U = 3.85395 \times 10^{-25}$ kg $^{234}_{90}Th = 3.78737 \times 10^{-25}$ kg $^{4}_{2}He = 6.64807 \times 10^{-27}$

18. Find the maximum energy that a β-particle can have in the following decay $^{176}_{71}Lu \rightarrow {}^{176}_{72}Hf + {}_{-1}e^0 + \tilde{\nu}$. Assume Hf to be at rest. $^{176}_{71}Lu = 175.94269u$ and $^{176}_{72}Hf = 175.94142u$

ANSWERS

1. 39.3 MeV 2. 1.024 MeV 3. 5.1×10^{23} MeV 4. 9.375×10^{17}, 1.317 kg

5. 1.0241 MeV

6. (i) $^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$, (ii) $^{24}_{12}Mg + {}^{4}_{2}He \rightarrow {}^{27}_{13}Al + {}^{1}_{1}H$, (iii) $^{6}_{3}Li + {}^{1}_{0}n \rightarrow {}^{4}_{2}He + {}^{3}_{1}H$

7. $^{7}_{3}Li + {}^{1}_{1}H$ (proton) $\rightarrow 2\,{}^{4}_{2}He + Q$ (energy released), 6.554×10^6 kWh

8. (i) 3.26 MeV, (ii) 17.6 MeV

10. (a) – 1.65 MeV, (b) 6.82 MeV, (c) – 2.79 MeV, (d) 3.11 MeV

11. 0.56 MeV 12. 3.46×10^5 m/s, 0.02

13. 7.46895 MeV, 11.5 MeV 14. 232, 90, 5.24 MeV, 1834.7 MeV 15. 3×10^7 kg

16. It can. An electron and antineutrino, 1.48 MeV

17. 8.8×10^{-13} J **18.** 1.182 MeV

CHAPTER 4

Nuclear Fission and Fusion

1. Here $\Delta m = (3 \times 1.00759 + 4 \times 1.00898 + 3 \times 0.00055) - 7.01818$

$= 3.02277 + 4.03592 + 0.00165 - 7.01818$

$= 7.06034 - 7.01818 = 0.04216$ amu.

The energy that is required to split a nucleus is called binding energy and this is exactly equal to energy required to form the nucleus.

$\therefore$ The binding energy $= 0.04216 \times 931$ ($\because$ 1 amu = 931 MeV)

$= 39.3$ MeV.

2. Energy required = mass created

$= (0.00055 + 0.00055)$ amu

$= 0.0011 \times 931 = 1.024$ MeV.

3. No. of atoms in 1 g of uranium $= \dfrac{6.023 \times 10^{23}}{235}$

$\therefore$ Energy released by fission of 1 g $= \dfrac{6.023 \times 10^{22}}{235} \times 200$ MeV

$= 5.1 \times 10^{23}$ MeV.

4. No. of atoms fissioned per second

$= \dfrac{3 \times 10^4 \times 10^3}{200 \times 10^6 \times 1.6 \times 10^{-19}}$ (1 eV $= 1.6 \times 10^{-19}$ joule)

$= 9.375 \times 10^{17}$.

Number of atoms burnt in 1000 hours

$= 9.375 \times 10^{17} \times 1000 \times 3600$

$= 9.375 \times 36 \times 10^{22}$

Mass of atoms burnt

$= \dfrac{235}{6.023 \times 10^{26}} \times 9.375 \times 36 \times 10^{22}$

$= 1.3168$ kg $= 1.317$ kg.

5. Energy released

$= 0.00055 + 0.00055 = 0.00110$ amu

$= 0.0011 \times 931 = 1.0241$ MeV.

6. Follow conservation principle of *mass number* and *atomic number*.

7. We have $_3Li^7 + {_1H^1}$ (proton) $= 2\ _2He^4 + Q$ (energy released)

or $7.0183 + 1.0081 = 2 \times 4.004 + Q$

or $Q = 0.0184$ amu

$= 1.0184 \times 931$ MeV

$= 17.13$ MeV

$= 17.13 \times 10^6 \times 1.6 \times 10^{-19}$ joule.

100 g of lithium contains $\frac{6 \times 10^{23}}{7} \times 100 = \frac{6}{7} \times 10^{25}$ lithium atoms.

$\therefore$ Energy released

$= \frac{6 \times 10^{25}}{7} \times 17.13 \times 10^6 \times 1.6 \times 10^{-19}$ joule

$= \frac{6}{7} \times 17.13 \times 1.6 \times 10^{12}$ joule

$= \frac{6 \times 17.16 \times 1.6 \times 10^{12}}{7 \times 3600 \times 1000}$ kilowatt-hour

($\because$ 3600 joule = 1 watt-hour)

$= 6.53 \times 10^6$ kilowatt-hour.

8. (i) Mass disappeared $= (2.01410 + 2.01410) - (3.01603 + 1.00867)$

$= 4.0282 - 4.0247 = 3.5 \times 10^{-3}$ amu

$\therefore$ Energy released $= 3.5 \times 10^{-3} \times 931 = 3.26$ MeV.

(ii) Mass disappeared $= (2.01410 + 3.01605) - (4.00260 + 1.00867)$

$= 5.03015 - 5.01127 = 0.01888$ amu.

$\therefore$ Energy released $= 0.01888 \times 931$

$= 17.576$ MeV $= 17.6$ MeV.

9. We have $h\nu = mc^2 - m_0c^2$, $\frac{h\nu}{c} = m\nu$ and $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$\therefore$ $m\nu c = mc^2 - m_0c^2$ or $\frac{m_0 \nu c}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_0 c^2}{\sqrt{1 - \frac{v}{c^2}}} - m_0c^2$

or $\frac{\nu}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{c}{\sqrt{1 - \frac{v^2}{c^2}}} - c$ or $c = \frac{c - v}{\sqrt{1 - \frac{v^2}{c^2}}}$

or $c^2\left(1 - \frac{v^2}{c^2}\right) = c^2 - 2vc + v^2$ or $c^2 - v^2 = c^2 - 2vc + v^2$

or $\quad 2v^2 - 2vc = 0 \quad$ or $\quad 2v\,(v - c) = 0$

$\therefore \quad$ either $v = 0 \quad$ or $\quad v = c$

Both values are unacceptable by the theory of relativity.

10. (a) ${}^{7}_{3}\text{Li} + {}^{1}_{1}\text{p} = {}^{7}_{4}\text{Be} + {}^{1}_{0}\text{n}$

$$E = (7.01601 + 1.00783 - 7.01693 - 1.00867) \times 931.5$$
$$= -1.65 \text{ MeV.}$$

(b) ${}^{9}_{4}\text{Be} + {}^{1}_{0}\text{n} = {}^{10}_{4}\text{Be} + \nu$

$$E = (9.01219 + 1.00867 - 10.01354) \times 931.5 = 6.82 \text{ MeV}$$

(c) ${}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He} = {}^{10}_{5}\text{B} + {}^{1}_{0}\text{n}$

$$E = (7.01601 + 4.00260 - 10.01294 - 1.00867) \times 931.5$$
$$= -2.79 \text{ MeV.}$$

(d) ${}^{16}_{8}\text{O} + {}^{2}_{1}\text{H} = {}^{14}_{7}\text{N} + {}^{4}_{2}\text{He}$

$$E = [16 + (-0.00509) + 2.01410 - 14.00307 - 4.00260] \times 931.5$$
$$= 3.11 \text{ MeV}$$

11. A nucleus having least possible energy equal to the binding energy is said to be in the ground state.

$${}^{10}_{4}Be \longrightarrow {}^{10}_{5}B + {}^{0}_{-1}e + \nu$$

$$T_B + T_{\beta^-} = 10.01354 - 10.01294 = 6 \times 10^{-4} \text{ amu}$$
$$= 0.55884 \text{ MeV.}$$

By conservation of momentum,

$$m_B v_B = m_{\beta^-} v_{\beta^-} + p_\nu$$

When $\quad v_{\beta^-}$ is maximum,

$$p_\nu = 0.$$

$$\therefore \quad m_B v_B = m_{\beta^-} v_{\beta^-}$$

$$\therefore \quad T_{\beta^-}\left(1 + \frac{m_{\beta^-}}{m_B}\right) = 0.55884.$$

$$T_{\beta^-}\left(1 + \frac{m_n - m_p}{m_B}\right) = 0.55884$$

or $$T_{\beta^-}\left(1 + \frac{1.00867 - 1.00783}{10.01294}\right) = 0.55884$$

or $$T_{\beta^-} = \frac{0.55884}{1.0000839} = 0.5587931 = 0.56 \text{ MeV.}$$

12. $^{200}_{82}\text{Pb} \longrightarrow {}^{196}_{80}\text{Hg} + \alpha\ ({}^{4}_{2}\text{He})$

By conservation of momentum,

$$O = \sqrt{2m_\alpha T_\alpha} + (-m_d v_d) \text{ where } d \text{ is for daughter}$$

or $$v_d = \sqrt{2m_\alpha T_\alpha} / m_d$$

$$= \frac{\sqrt{2 \times 4 \times 1.672 \times 10^{-27} \times 6.00 \times 10^6 \times 1.6 \times 10^{-19}}}{196 \times 1.672 \times 10^{-27}}$$

$$= 3.46 \times 10^5 \text{ m/s.}$$

Required fraction

$$= \frac{\frac{1}{2} m_d v_d^2}{T_\alpha + \frac{1}{2} m_d v_d^2} = \frac{1}{1 + \frac{m_d}{m_\alpha}} = \frac{1}{1 + \frac{196}{4}} = 0.02.$$

13. (i) We have $E_b = Zm_H + (A - Z)\, m_n - M$

$\therefore$ Here $E_b = 8 \times 1.00783 + (15 - 8) \times 1.00867 - 15.00307$

$= 0.12026 = 0.12026 \times 931.6 = 112.03422$ MeV.

$\therefore$ E_b per nucleon $= \frac{112.03422}{15} = 7.46895$ MeV.

(ii) Let us find the binding energy of $^{11}_{5}B$ nucleus and that of the nucleus after removal of a neutron. The difference will give the required binding energy of neutron.

E_b of $^{11}_{5}B = 5 \times 1.00783 + (11 - 5) \times 1.00867 - 11.00930 = 0.08187$ amu.

$$^{11}_{5}B - {}^{11}_{5}B = {}^{10}_{5}B$$

$\therefore$ E'_b of $^{10}_{5}B = 5 \times 1.00783 + (10 - 5) \times 1.00867 - 10.01294 = 0.06956$ amu.

$\therefore$ Binding energy of a neutron in $^{11}_{5}B$

$= 0.08187 - 0.06956 = 0.01231$ amu $= 11.5$ MeV.

14. (a) $^{A}_{92}X \longrightarrow {}^{228}_{Z}Y + {}^{4}_{2}\text{He}$

$\therefore$ $A = 228 + 4 = 232$ and $92 = Z + 2$ or $Z = 90$

$\therefore$ $A = 232$ and $Z = 90$.

(b) $Bqv_\alpha = \frac{m_\alpha v_\alpha^2}{r}$ or $m_\alpha v_\alpha = Bqr$

$$T_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 = \frac{1}{2} m_\alpha \left(\frac{Bqr}{m_\alpha} \right)^2 = \frac{B^2 q^2 r^2}{2m_\alpha}.$$

For α-particle, $q = 2e$.

$\therefore$ $$T_\alpha = 2B^2e^2r^2/m_\alpha = \frac{2\times 3^2 \times e^2 \times 0.11^2}{4.003 \times 1.66 \times 10^{-27}} \text{ joule}$$

or $$T_\alpha = \frac{2 \times 3^2 \times 1.6 \times 10^{-19} \times 0.11^2}{4.003 \times 1.66 \times 10^{-27}} \text{ eV} = 5.24 \text{ MeV}.$$

By conservation of momentum, $0 = \sqrt{2m_\alpha T_\alpha} - m_d v_d$

or $$m_d v_d = \sqrt{2m_\alpha T_\alpha}$$

$\therefore$ $$T_d = \frac{1}{2} m_d v_d^{\,2} = \frac{1}{2} m_d \times \frac{2m_\alpha T_\alpha}{m_d^2} = \frac{m_\alpha}{m_d} T_\alpha$$

$\therefore$ $$T_d = \frac{4.003}{228.03} \times 5.24 = 0.092 \text{ MeV}$$

$\therefore$ Energy released $= 5.24 + 0.92 = 6.16$ MeV.

Binding energy of ${}^{228}_{90}Y = 90 \times 1.008 + (228 - 90) \times 1.009 - 228.03$

$= 1799.66$ MeV.

Binding energy of ${}^{4}_{2}\text{He} = 2 \times 1.008 + (4 - 2) \times 1.009 - 4.003$

$= 28.88$ MeV.

$\therefore$ Binding energy of ${}_{92}X^{232} = 1799.66 + 28.88 + 6.16 = 1834.7$ MeV.

15. The number of atoms in m kg of uranium

$= \frac{m}{A} N_A$ and the energy released is $\frac{m}{A} N_A \times 200$ MeV.

$\therefore$ Here Q (energy released)

$$= \frac{1.5}{235} \times 6.02 \times 10^{26} \times 200 \times 1.6 \times 10^{-13} = 1.2 \times 10^{14} \text{ J}.$$

$\therefore$ $$\text{Mass of } TNT = \frac{1.2 \times 10^{14}}{4.1 \times 10^{6}} = 3 \times 10^7 \text{ kg}.$$

16. $\Delta m = m_{si} - m_p = 30.97535 - 30.97376 = 0.00189$

The corresponding energy $= 0.00159 \times 931.5 = 1.48$ MeV

This exceeds the electron not energy (0.51 MeV) and therefore β-decay is possible

$${}^{31}_{14}Si \longrightarrow {}^{31}_{15}P + {}^{0}_{-1}e + {}^{0}_{0}v$$

The total energy of β-particle and antineutrino is 1.48. Antinutrino is needed to consume angular momentum.

17. $Q = (3.85395 - 3.78737 \; - 0.06648) \times 10^{-25} c^2$

$$= 1 \times 10^{-4} \times 10^{-25} c^2 = 10^{-27} c^2$$

By conservation of momentum

$$\sqrt{2m_1 \; K_1} = \sqrt{2m_2 \; K_2}$$

$$\Rightarrow \qquad m_1 K_1 = m_2 K_2$$

$$Q = K_1 + K_2 \qquad \Rightarrow \qquad K_2 = \frac{Q}{m_1 + m_2} \times m_1$$

$$\therefore \qquad K_2 = \frac{10^{-27} c^2}{(3.78737 + 0.06648) \times 10^{-25}} \times 3.78737 \times 10^{-25}$$

$$= 0.98275 \times 10^{-29} c^2$$

$$\Rightarrow \qquad K_2 = 8.8 \times 10^{-13} \text{ J.}$$

18. $Q = (175.94269 - 175.94142) \; u$

$$= 1.27 \times 10^{-3} \times 931 \text{ MeV}$$

$$= 1.15 = \text{MeV.}$$

This energy is shared by β-particles and neutrions.

When neutrons shares no energy, β-particle will have maximum energy.

∴ Maximum energy of p-particle

= 1.182 MeV.

□ □ □

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Chapter 5 – Diode Triode Transistor

$$\therefore \qquad V_i = i_b \times r_\pi = 4 \times 10^{-6} \times 2500 = 0.01 \text{ V}$$

$$V_L = i_L \times R_L = .2 \times 10^{-3} \times 5000 = 1 \text{ V}$$

$$\therefore \text{ Voltage gain} = \frac{V_L}{V_i} = \frac{1}{0.01} = 100$$

EXERCISES

1. A diode has Child's constant $K = 0.2$ (current in mA and voltage in volts). Calculate the voltage across the tube, across a load of resistance 10000 Ω and source the voltage when the tube produces a current of 12.8 mA through the load.
2. In a certain triode, the output current is 5 mA with an anode potential of 220 V and grid potential – 3 V. When the potential is increased to 260 V, the current rises rises to 7 mA. A change in grid voltage to – 4 V restores the current to the original value. Find the constants of the valve.
3. A triode valve of anode slope resistance 20 kΩ is used with an anode load resistance of 50 kΩ. If an alternating signal of 0.5 V (rms value) is applied to the grid, find the output voltage if the amplification factor of the valve is 15.
4. The voltage gain of a triode amplifier with a resistive load of 5 kΩ is 10 while with a resistive load of 12 kΩ it is 15. Calculate the amplification factor and plate resistance of the triode.
5. The plate resistance of a triode is 7.7 kΩ, and the transconductance is 2.6 millimho. If only the plate voltage is increased by 50 V, what is the increase in the plate current? What change in grid voltage will now bring the plate current to its former value?
6. Determine the conductivity of pure germanium at 27°C. The concentration of the carriers at this temperature is 2.2×10^{19} m^{-3}. The mobility of electrons = 0.36 and that of holes = 0.17.
7. In a *p-n* junction diode the reverse saturation current is 10 μA. What will be the forward current for a voltage of 0.2 V?
8. Assume that the silicon diode in the given circuit requires a minimum current of 1 mA to be above the knee-point (knee-point voltage = 0.7 V) of its *I-V* characteristics. Assuming that the voltage across the diode is independent of current above the knee-point, find the maximum value of R so that the voltage is above the knee-point if $V_B = 5$ V. Also find the value of R to establish a current of 5 mA in the circuit for the same voltage.

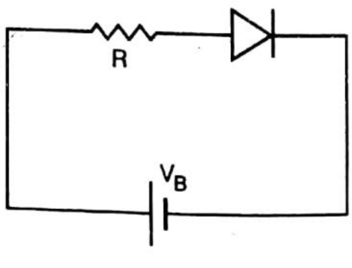

Fig. 5.8

9. The base current of a transistor is 105 μA and the collector current is 2.05 mA. Determine the value of β, I_E and α. If a change of 27 μA in the base current produces a change of 0.65 mA in the collector current, find β_{ac}
10. In a silicon transistor, a change of 7.89 mA in the emitter current produces a change of 7.89 mA in the collector current. What change in the base current will bring the same change in the collector current?

11. In a silicon transistor the base current is changed by 20 μA. This results in a change of 0.02 V in the base to emitter voltage and a change of 2 mA in the collector current.

(a) Find the input resistance r_b, β_{ac} and transconductance g_m of the transistor.

(b) The transistor is used as an amplifier with a load resistance of 5 kΩ. What is the voltage gain of the amplifier?

12. In a triode valve, for a grid voltage $V_g = -1.2$ V, the plate current I_p (in mA) and the plate voltage are given by the relation $I_p = -50 + 0.1\ V_P$. When the grid voltage is changed to -3.2 V and the plate voltage is kept at 150 V, a plate current of 5 mA is observed. Calculate the valve constants and the voltage amplification for 20 kΩ load in the plate circuit. **[Roorkee 1990]**

13. In the accompanying circuit (Fig. 5.9), the value of β is 100. Find I_B, V_{CE}, V_{BE}, and V_{BC}, when $I_C = 1.5$ mA. Is the transistor in active, cut-off or saturation state?

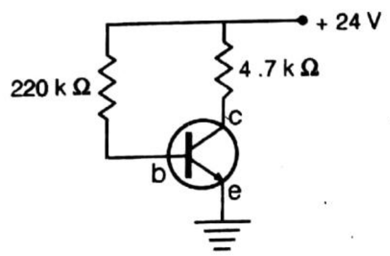

Fig. 5.9

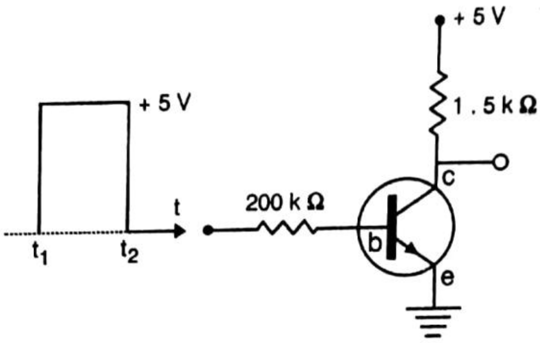

Fig. 5.10

14. In the accompanying circuit Fig. 5.10, if it is assumed that when the input voltage at the base resistance is 5 V, V_{BE} is zero and V_{CE} is also zero, what are I_B, I_C, and β? When the input is zero, I_B is zero. What will be the output waveform if the input waveform is as shown in the figure? What is the practical use of this current?

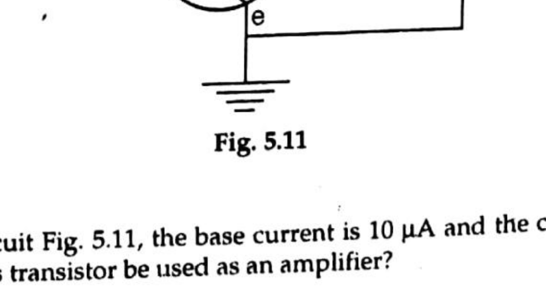

Fig. 5.11

15. In the given circuit Fig. 5.11, the base current is 10 μA and the collector current is 5.2 mA. Can this transistor be used as an amplifier?

16. In the given circuit (Fig. 5.12) calculate the value of the collector current if its $I_{co} = 10\ \mu A$ and $\alpha = 0.97$. Assume a voltage drop between base and emitter of 0.15 V.

[*Hint:* $I_c = \alpha I_e + I_{co}$]

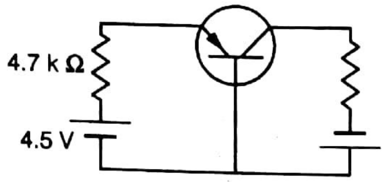

Fig. 5.12

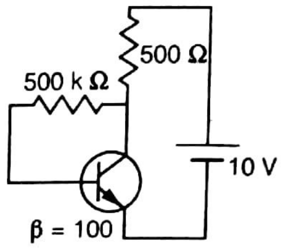

Fig. 5.13

17. Calculate the emitter current and collector voltage of the circuit given in Fig. 5.13.
18. A transistor connected in common emitter configuration has $V_{cc} = 8$ V, $V_L = 0.5$ V and $R_L = 800\ \Omega$. If its $\alpha = 0.96$ determine: (i) collector emitter voltage, (ii) its base current.
19. In a transistor collector load is 4 kΩ whereas the zero signal current is 1 mA. (a) What is the operating point if $V_{cc} = 10$ V? (b) What will be the operating point if $R_L = 5$ k Ω?

[*Hint:* Operating point is zero signal collector current and V_{co}]

ANSWERS

1. 16 V, 128 V, 144 V 2. $r_p = 20000\ \Omega$, $g_m = 2 \times 10^{-3}\ \Omega^{-1}$, $\mu = 40$ 3. 5.36 V
4. 23.3, 6667 Ω 5. 6.5 mA, 2.5 V 6. 1.87 Ω^{-1} m^{-1} 7. 22.5 mA
8. 4300 Ω, 300 Ω 9. $\beta = 195$, $\alpha = 0.95$, $I = 2.155$ mA, $\beta_{ac} = 24.07$ 10. 90 μA
11. (a) $r_b = 1000\ \Omega$ $\beta = 100$, $g_m = 0.1\ \Omega^{-1}$, (b) $A_v = 500$
12. $10^4\ \Omega$, $2.5 \times 10^{-3}\ \Omega^{-1}$, 25; 16·67
13. $I_B = 15\ \mu A$, $V_{CE} = 16.95$ V, $V_{BC} = 3.75$ V, $V_{BE} = 20.7$ V, saturation state
14. $\beta = 133.3$, $I_C = 3.33$ mA, $I_B = 25\ \mu A$, used in NOT gate Fig. 5
15. No, it is in saturation state 16. 0.91 mA
17. 18.2 μ A, 1.84 mA, 9.08 V 18. 7.5 V, 0.026 mA
19. 1 mA and 6 V, 1 mA and 5 V

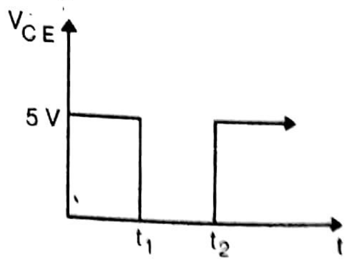

Fig. 5.14

CHAPTER 5

Diode, Triode and Transistor

1. We have as Child's law, $I = KV^{3/2}$

Hence here $I = 0.2\,V^{3/2}$ when I is in mA.

$\therefore$ When $I = 12.8$ mA we have $12.8 = 0.2\,V^{3/2}$

or $V = 16$ V.

Voltage across the load $= 12.8 \times 10^{-3} \times 10000$

$= 128$ V.

$\therefore$ Source voltage $= 16 + 128 = 144$ V.

2. Here $\Delta V_p = 260 - 220 = 40$ V

and $\Delta I_p = 7 - 5 = 2$ m A $= 2 \times 10^{-3}$ A.

$$\therefore \quad r_p = \frac{\Delta V_p}{\Delta I_p} = \frac{40}{2 \times 10^{-3}} = 20{,}000\ \Omega.$$

When the plate current is 5 mA, $V_p = 260$ V and $V_g = -4$ V. Also when the plate current is 5 mA, $V_p = 220$ V and $V_g = -3$ V.

Thus a change of plate voltage by 40 V is equivalent to a change of grid voltage by 1 V (opposite).

$$\therefore \quad \mu = \frac{\Delta V_p}{\Delta V_g} = \frac{40}{1} = 40.$$

Since $\mu = g_m \times r_p$,

$$g_m = \frac{\mu}{r_p} = \frac{40}{20000} = 2 \times 10^{-3} \text{ ampere per volt.}$$

3. Output voltage

$$= \left(\frac{\mu \Delta V_g}{r_p + R_L} \right) \times R_L$$

$$= \frac{15 \times 0.5}{20 + 50} \times 50 = 5.36 \text{ V}.$$

4. We have from the voltage gain formula,

$$A_v = \frac{\mu R_L}{r_p + R_L}.$$

$\therefore \qquad 10 = \dfrac{\mu \times 5 \times 10^3}{r_p + 5 \times 10^3} \qquad$ and $\qquad 15 = \dfrac{\mu \times 12 \times 10^3}{r_p + 12 \times 10^3}$.

Solving, $\quad r_p = 6667\ \Omega, \quad \mu = 23.3.$

5. Since $\quad \mu = g_m \times r_p,$ we have

$$\mu = (2.6 \times 10^{-3}) \times (7.7 \times 10^3) = 20.02.$$

$\because \qquad r_p = \dfrac{\Delta V_p}{\Delta I_p}$, we have

$$\Delta I_p = \frac{\Delta V_p}{r_p} = \frac{50}{7.7 \times 10^3} = 6.5 \times 10^{-3}\ \text{A} = 6.5\ \text{mA}.$$

$$\mu = \frac{\Delta V_p}{\Delta V_g} \quad \text{or} \quad \Delta V_g = \frac{\Delta V_p}{\mu} = \frac{50}{20.02} = 2.5\ \text{V}.$$

6. We have $\quad \sigma = (\mu_n + \mu_p)\, n_i e$

$\therefore$ Here $\quad \sigma = (.36 + 0.17) \times 2.2 \times 10^{19} \times 1.6 \times 10^{-19}$

$$= 1.87\ \Omega^{-1}\ \text{m}^{-1}.$$

7. We have $\quad I = I_0 e^{38.6}$

$\therefore$ Here $\quad I = (10 \times 10^{-6}) e^{38.6 \times 0.2} = 0.0225\ \text{A}$

$$= 22.5\ \text{mA}.$$

8. r_d (diode resistance) $= \dfrac{0.7}{10^{-3}} = 700\ \Omega.$

By circuit equation,

$$10^{-3} = \frac{5}{700 + R} \quad \text{or} \quad R = 4300\ \Omega.$$

Again in the second case,

$$5 \times 10^{-3} = \frac{5}{700 + R}$$

or $\qquad R = 300\ \Omega.$

9. We have $\quad I_E = I_C + I_B$

$\therefore \qquad I_E = 2.05\ \text{mA} + 105\ \mu\text{A} = 2.155\ \text{mA}.$

$$\beta = \frac{I_C}{I_B} = \frac{2.05\ \text{mA}}{105\ \mu\text{A}} = 19.5.$$

$$\alpha = \frac{I_C}{I_E} = \frac{2.05\ \text{mA}}{2.155\ \text{mA}} = 0.95.$$

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{.0.65\ \text{mA}}{27\ \mu\text{A}} = 24.07.$$

10. We have $\alpha = \dfrac{\Delta I_C}{\Delta I_E} = \dfrac{7.8 \text{ mA}}{7.89 \text{ mA}} = 0.9886.$

Now $\dfrac{1}{\alpha} - \dfrac{1}{\beta} = 1$ or $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.9886}{1-0.9886} = 86.72.$

Now $\beta = \dfrac{\Delta I_C}{\Delta I_B}$ and hence $86.72 = \dfrac{7.8 \text{ mA}}{\Delta I_B}$

or $\Delta I_B = \dfrac{7.8 \text{ mA}}{86.72} = 90\ \mu\text{A}.$

11. (a) $r_b = \dfrac{\Delta V_B}{\Delta I_B}$ or $r_b = \dfrac{0.02}{20 \times 10^{-6}} = 1000\ \Omega.$

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2 \times 10^{-3}}{20 \times 10^{-6}} = 100.$$

$$g_m \text{ (trans conductance)} = \frac{\Delta I_c}{\Delta V_B} = \frac{2 \times 10^{-3}}{0.02} = 0.1\ \Omega^{-1}.$$

(b) $A_v = \dfrac{-\beta_{ac} R_L}{r_b} = -\dfrac{100 \times 5 \times 10^3}{1000} = -500.$

12. Let $I_p = I_0 + 0.1\ V_p$ be the equation of the plate characteristic at

$V_g = -3.2$ V. Then $5 = I_0 + 0.1 \times 150$ or $I_0 = -10$

$\therefore$ $I_p = -10 + 1\ V_p$ at $V_g = -3.2$ V

$I_p = -5 + 0.1\ V_p$ at $V_g = -1.2$ V (given).

Now $r_p = \left(\dfrac{\partial V_p}{\partial I_p}\right)_{v_g}$ = slope of the $V_p - I_p$ line with I_p-axis.

When $V_p = 100$, $I_p = 0$ mA at $V_g = -3.2$ V

and when $V_p = 0$, $I_p = -10$ m A $= -10 \times 10^{-3}$ A at $V_0 = -3.2$ V.

Thus at $V_g = -3.2$ V when V_p is increased by 100 V, then I_p increases by 10×10^{-2} A.

$\therefore$ $r_p = \dfrac{100}{10 \times 10^{-3}} = 10^4\ \Omega.$

Now $g_m = \left(\dfrac{\partial I_p}{\partial V_g}\right)_{v_p}$. We note that when the grid voltage is changed from −1.2 V to − 3.2 V, i.e., by 2 volts, the plate current at 150 V (fixed) changes from $I_p = -5 + 0.1 \times 150 = 10$ mA to $I_p = -10 + 0.1 \times 150 = 5$ mA, that is, by 5 mA.

$\therefore \qquad g_m = \dfrac{5 \times 10^{-3}}{2} = 2.5 \times 10^{-3}\ \Omega.$

We have $\mu = g_m \times r_p$ and hence $\mu = 2.5 \times 10^{-3} \times 10^4 = 25$

$$A_v = \frac{\mu R_L}{r_p + R_L} = \frac{25 \times 20 \times 10^3}{10^4 + 20 \times 10^3} = \frac{50}{3} = 16.67.$$

13. We have $\beta = \dfrac{I_C}{I_B}$ and so $100 = \dfrac{1.5\ \text{mA}}{I_B}$ or $I_B = 15\ \mu\text{A}$.

By Kirchhoff's rule in the *CE* loop via the supply voltage (24 volts),

$$24 = 4.7 \times 10^3 \times 1.5 \times 10^{-3} + V_{CE}$$

or $\qquad V_{CE} = 24 - 4.7 \times 1.5 = 16.95$ V.

By Kirchhoff's rule in *BC* loop via 220 kΩ,

$$220 \times 10^3 \times 15 \times 10^{-6} + V_{BC} - 4.7 \times 10^3 \times 1.5 \times 10^{-3} = 0$$

or $\qquad V_{BC} = 7.05 - 3.30 = 3.75$ V.

By Kirchhoff's rule in *BE* loop via + 24 V and 220 kΩ,

$$220 \times 10^3 \times 15 \times 10^{-6} + V_{BE} = 24$$

or $\qquad V_{BE} = 20.7$ V.

14. By Kirchhoff's rule, $200 \times 10^3 \times I_B + V_{BE} = 5.$

Since $\qquad V_{BE} = 0$ (given), $I_B = 25\ \mu\text{A}$.

Also by Kirchhoff's rule,

$$1.5 \times 10^3 \times I_c + V_{CE} = 5.$$

Since $\qquad V_{CE} = 0$ (given), $\quad I_C = \dfrac{5}{1.5 \times 10^3} = 3.33$ mA.

Now $\qquad \beta = \dfrac{I_C}{I_B}$ and so here $\beta = \dfrac{3.33 \times 10^{-3}}{25 \times 10^{-6}} = 133.3.$

We have in general, $200 \times 10^3 \times I_B + V_{BE} = V_s$ (*S* for signal)

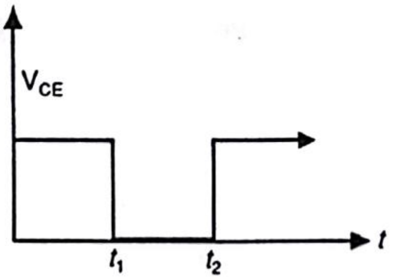

Fig. 5.1

and $\qquad 1.5 \times 10^3 \times I_C + V_{CE} = 5$

or $\qquad 1.5 \times 10^3 \times 133.3 \times I_B + V_{CE} = 5$

or $\qquad V_{CE} = 5 - 2 \times 10^5\, I_B.$

When $\qquad V_s = 0,\ I_B = 0$ (given).

$\therefore \qquad V_{CE} = 5$ volts.

When $\qquad V_s = +5$ V,

$$V_{CE} = 5 - 2 \times 10^5 \times \frac{5}{2 \times 10^5} = 0.$$

Hence the output waveform is as shown in the figure.

Note that the above circuit *inverts* the signal. This is used in NOT logic gate.

15. For transistor to act as amplifier, the emitter-base must be forward biased and the collector-base reverse biased.

By Kirchhoff's rule applied to the left loop via V_{CC} ,

$$5.5 = 500 \times 10^3 \times (10 \times 10^{-6}) + V_{BE} \quad \text{or} \quad V_{BE} = 0.5 \text{ V}.$$

From the right loop,

$$V_{CC} = V_{CE} + I_C R_L \quad \text{or} \quad V_{CE} = V_{CC} - I_C R_L$$

$$\therefore \quad V_{CE} = 5.5 - 5.2 \times 10^{-3} \times 1000 = 0.3 \text{ V}.$$

The collector is at 0.3 V relative to the emitter and the base is at 0.5 V relative to the emitter, so the base is at 0.2 V relative to the collector. Thus both the emitter-base and the collector-base are forward biased. This means the transistor is in saturation state and it cannot be used as an amplifier.

16. Applying Kirchhoff 's loop rule to the left loop of the circuit

$$4.5 = 4.7 \times 10^3 I_e + V_{eb}$$

$$= 4.7 \times 10^3 I_e + 0.15$$

$$\Rightarrow \quad I_e = \frac{4.35}{4.7 \times 10^3} = 0.92^{-5} \times 10^{-3} + 10 \times 10^{-6}$$

$$= 0.8978 \times 10^{-3} + 0.01 \times 10^{-3}$$

$$= 0.9078 \times 10^{-3} = 0.91 \text{ m A}.$$

17. Applying Kirchhoff 's loop rule to the loop battery

500 Ω – 500 kΩ – base – emitter – back to battery "

$$10 = 500 (I_b + I_c + 500 \times 10^3 I_b$$

$$= 500 I_b + 500 \beta I_b + 5 \times 10^5 \quad (\because \quad I_c = \beta I_b)$$

$$\Rightarrow \quad 10 = (500 + 500 \times 100 + 5 \times 10^5) I_b$$

$$\Rightarrow \quad I_b = \frac{10}{550500} = 18.2 \times 10^{-6}\text{A} = 18.2 \ \mu\text{A}$$

$$I_e = \beta I_b + I_b = (\beta + I) I_b$$

$$\therefore \quad I_e = (100 + 1) \times 18.2 \times 10^{-6}$$

$$= 18.4 \times 10^{-4} A = 1.84 \text{ m A}$$

$$10 = 500 (I_b + I_c) + V_{ce}$$

$\Rightarrow \quad V_{ce} = 10 - 500\,(\beta + 1)\,I_b$

$= 10 - 500\,(100 + 1) \times (18.2 \times 10^{-6})$

$= 10 - 0.92 = 9.08$ V.

18. (i) $V_c = V_{ce} + I_L R_L \quad \Rightarrow \quad V_{ce} = V_{cc} - V_L = 8 - 0.5 = 7.5$ V.

(ii) Here $I_c = I_L$

$$\therefore \quad I_c = \frac{V_L}{R_L} = \frac{0.5}{800} = 0.625 \text{ A}$$

$$I_c = \beta I_b \quad \Rightarrow \quad I_b = \frac{I_c}{\beta} = \frac{0.625}{\dfrac{\alpha}{1-\alpha}} = 0.625\left(\frac{1}{\alpha} - 1\right)$$

$$\Rightarrow \quad I_b = 0.625\left(\frac{1}{0.90} - 1\right) = 26 \times 10^{-3} \text{ A} = 0.026 \text{ m A.}$$

19. (a) $V_{ce} = V_{cc} - I_L R_L = V_{cc} - I_c R_L \qquad (\because \quad I_c = R_L)$

Here $\quad V_{ce} = 10 - 10^{-3} \times 4 \times 10^3 = 6$ V

Hence operating point is 1 m A, 6 V

(b) $V_{ce} = 10 - 10^{-3} \times 5 \times 10^3 = 5$ V

Hence operating point, 1 m A, 5 V.

□ □ □

Revision Examples 1

REVISION EXAMPLES I

1. A capacitor of capacitance C is first charged with Q units of charge and then connected to another uncharged capacitor of capacitance c and subsequently disconnected. The latter is completely discharged by short-circuiting its plates and then again connected to the first capacitor. The process is repeated n times. Show that the first capacitor can never be fully discharged by the above process. Why? What is the charge left after n discharges?
2. A capacitor of capacitance C_1 is charged to a potential difference of V_0. It is then connected across the combination of two capacitors C_2 and C_3 in series. Calculate the charge on C_1, C_2 and C_3.
3. The minimum deviation of a certain glass prism ($\mu = 1.5$) is equal to its refracting angle. Find the latter.
4. A parallel-plate capacitor of capacitance 10 uF is connected to a battery of emf 30 V. A dielectric plate ($\varepsilon_r = 5$) is introduced between the plates so as to occupy the entire space between them without disconnecting the battery. Calculate (a) the extra energy drawn from the battery and (b) the extra energy stored in the capacitor.

   [*Hint:* Extra energy drawn from the battery $= \varepsilon \Delta q$ J and extra energy of the capacitor $= E_f - E_i$.]
5. Two point charges each of 40 pC are placed 10 cm apart. An electron is projected at right angles to the line joining the two charges from the mid-point with velocity 10^6 ms^{-1}. How far along this line will it go?

   [*Hint:* Consider conservation of energy]
6. A tube bent twice at right angles at two points at a distance of l from each other, is held vertically and filled with a liquid up to a certain height in the vertical arms. Calculate the difference in levels if, (a) the tube has an acceleration a towards the right and (b) if the tube is mounted on a horizontal turn-table rotating with an angular velocity ω with one of the vertical arms as the axis of rotation. Assume the diameter of the tube to be small in comparison to l.

   [*Hint:* Apply D'Alembert's principle and consider equilibrium of the horizontal portion of the tube.]
7. A brass rod ($a = 18 \times 10^{-6}$ K^{-1}) has projections at right angles at the ends. Another rod of some other material just fits in the space between the projections at 0°C. What is the stress produced in the rod when the system is heated to 100°C? Consider two cases (a) when the coefficient of expansion and Young's modulus are 29×10^{-6} K^{-1} and 18×10^9 N m^{-2}, (b) 12×10^{-6} K^{-1} and 2.06×10^{11} N m^{-2} respectively.
8. Two small discs of masses m_1 and m_2, interconnected by a weightless spring, rest on a smooth horizontal plane. The discs are set in motion with initial velocities v_1

and v_2 whose directions are at right angles to each other and lie in the same horizontal plane. Find the total energy of this system in the frame of the centre of mass.

[*Hint:* $(m_1+m_2)\overrightarrow{v_{cm}} = m_1\vec{v_1} + m_2\vec{v_2}$; $E = \frac{1}{2}m_1\left|\vec{v}cm - \vec{v_1}\right|^2 + \frac{1}{2}m_2\left|\vec{v}cm - \vec{v_2}\right|^2$.]

9. A rod of mass m_2 rests on a wedge of mass m_1 (figure A). Guides allow the rod to move only in the vertical direction and the wedge in the horizontal direction. Find the acceleration of both bodies and the reaction of the wedge. Neglect friction and take the angle of wedge with the horizontal to be α.

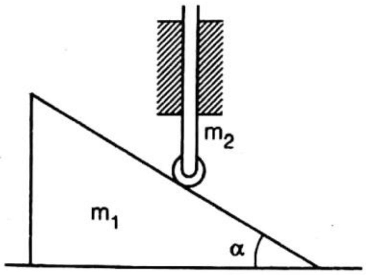

Fig. A

10. A block is held in contact with the vertical wall of a carriage by pressing it against the wall to the left with the thumb. Now the thumb is removed and the carriage is moved to the right with acceleration a. Calculate the acceleration a so that the block may not fall if the μ is the coefficient of friction between the block and the wall of the carriage.

   [*Hint:* Apply D'Alembert's principle and make a free-body diagram of the block.]

11. A block of mass m_3 is connected by an inextensible light string to another block of mass m_2 lying on the upper horizontal surface of a carriage of mass m_1. Calculate the force by which the carriage is to be pulled so that the two blocks may remain stationary.

   [*Hint:* Apply D'Alembert's principle and make free-body diagrams of the three bodies.]

12. A circular conductor of resistance $4\,\Omega$ is tapped at two points, dividing the conductor in the ratio 1 : 3. The tappings are connected to a cell. Calculate the magnetic field at the centre of the coil.

13. Two rods of lengths l and l', Young's modulus Y and Y' and coefficients of expansion α and α', respectively are held between two massive, vertical walls. The temperature of the rods is increased by T. Calculate the thrust exerted by the rods on each other, assuming that there is no bending of the rods and there is no change in the area of cross-section of the rods. The rods are of equal area of cross-section A.

   [*Hint:* $l + l' = l_1 + l_1'$ where l_1 and l_1' are the final lengths. Final length of a rod = initial length + thermal expansion − elastic compression]

14. A tapering rod of length l is stretched by F. Calculate the increase in length of the rod if it tapers from an area of cross-section α_1 to α_2. Young's modulus of the rod is Y.

15. A smooth, uniform rod AB of mass M and length l rotates freely with an angular velocity ω_0 in the horizontal plane about a stationary vertical axis passing through its end A. A small sleeve of mass m starts sliding along the rod from the point A.

Find the velocity v' of the sleeve relative to the rod at the moment at which it reaches its other end B.

16. A ball suspended by a thread swings in a vertical plane so that its acceleration values in the extreme positions and the lowest position are equal. Find the deflection of the thread at the extreme positions.

[*Hint:* Acceleration at any point $= \sqrt{a_r^2 + a_t^2}$ where a_r = radial acceleration and a_t = tangential acceleration.]

17. A small body of mass m tied to an inextensible thread moves over a smooth horizontal table. The other end of the thread is being drawn through a hole O in the table with a constant velocity (figure B). Find the tension of the string as a function of the distance r between the body and the hole if at $r = r_0$ the angular velocity of the body is equal to ω_0.

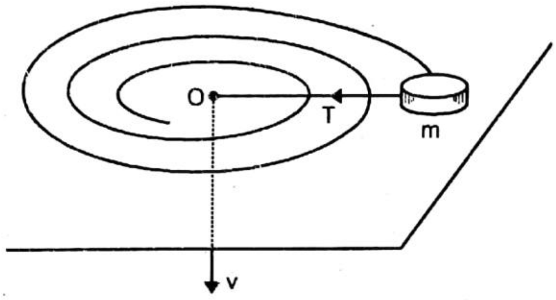

Fig. B

[*Hint:* Since the external force has no moment about O, angular momentum of the body about the point O is conserved.]

18. Three cells of emfs e_1, e_2, e_3 and internal resistance r_1, r_2, r_3, respectively are connected in parallel. Calculate the p.d. of the cells.

19. A horizontal tube of cross-section A is bent at an angle $\theta°$ with the vertical. A liquid of density ρ flows through it with speed v. Calculate the force needed to hold the tube in position.

20. A light rod is suspended horizontally by three identical spring balances (x, x, x), each of weight 4 kg. Below it a second light rod is suspended horizontally by two identical spring balances (y, y) and finally a load of 4 kg is hung from this lower rod. Find the readings of x and y.

[*Hint:* Draw free-body diagrams of the two rods.]

21. Calculate the resistance of a tapering wire of length l and end sections α_1 and α_2.

[*Hint:* Consider a section of the wire at a distance x from the apex of the wire and calculate the resistance of an elementary length and integrate.]

22. A 100-W sodium vapour lamp radiates uniformly in all directions. (a) At what distance from the lamp will the average density of photons be 10^7 m^{-3}? (b) What is the average density of photons 2 m from the lamp? Assume the light to be monochromatic, with $\lambda = 5890$ Å.

[*Hint:* intensity of the light $= Dc$, where D is the density of energy and c is the speed of the light wave. Energy of a photon $= h\nu$.]

23. Show that when a monatomic ideal gas absorbs ΔQ heat at a constant pressure, $\Delta Q : \Delta U : \Delta W = 5 : 3 : 2$.

[*Hint:* $\Delta Q = C_p \Delta T$; $\Delta U = C_v \Delta T$; $\Delta W = p\, \Delta V$]

24. A pole extends 2 m above the bottom of a swimming pool and .5 m above the water. Sunlight is incident at 45°. What is the length of the shadow of the pole on the bottom of the pool? ($\mu = 4/3$)

25. The earth has a magnetic dipole moment of 6.4×10^{21} Am^2. What current would have to be set up in a single turn of wire going around the earth at its magnetic equator if we wished to set up such a dipole? (Radius of the earth = 6400 km)

26. In a biprism experiment, the two halves of the biprism are covered by two glass plates of the same thickness but of different refractive indices 1.4 and 1.7. The point on the screen where the central maximum fell before the glass plates were inserted is now occupied by the fifth bright fringe. Find the thicknes of the glass plates. ($\lambda = 4800$ Å)

27. Show that the capacitance of two metal sphers of the same radius, when far apart, is one-half the capacitance of an isolated sphere.

[*Hint:* Consider the two spheres as the two conductors of a capacitor.]

28. An iron wire (diameter 1 mm, length 10 cm) is placed in an evacuated chamber. Calculate the equilibrium temperature of the wire if it carries a current of 10 A. (Stefan constant $(\sigma) = 5.67 \times 10^{-8}$ $Wm^{-2}K^{-4}$ and resistivity of iron $= 10 \times 10^{-8}$ Ωm^{-1}). Take the temperature of the chamber walls to be 27°C.

[*Hint:* R (radiancy i.e., energy radiated from unit area of a black body surface in unit time) $= \sigma (T^4 - T_0^4)$, where T_0 is the temperature of the surroundings.]

29. In one branch AB of a complicated circuit there is a resistor of 2 Ω and a box concealing a battery of resistance 8 Ω. If the power absorption in the branch be 50 W and the current through the branch be 1 A from A to B, (a) what is the potential difference, (b) what is the emf of the battery, and (c) what is its polarity?

30. Find the frequency of small oscillations of a thin uniform vertical rod of mass m and length l hinged at the point O (figure C). The force constant of either spring is k. The masses of the spring are negligible.

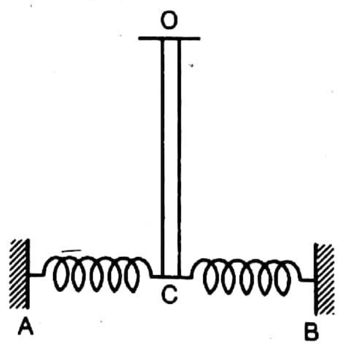

Fig. C

[*Hint:* π (torque about O) $= -2k\, lx - mg \frac{x}{2} \cdot$]

31. A simple bar magnet of magnetic moment m and mass w is suspended by a string from its south pole. If a uniform magnetic field 'B' directed parallel to the ceiling from which it is suspended is established, show the resulting orientation of the string and the magnet.

32. A particle of charge $+Q$ is assumed to have a fixed position at a point P. A second particle of mass m and charge $-q$ moves at a constant speed in a circle of radius r_1 centred at P. Find the work done by an external agent on the second particle in order to increase the radius of the circle of motion, centred at P, to r_2.

[*Hint:* Work done = change in kinetic energy.]

33. An electron is accelerated through a potential difference of 1000 V and directed into a region between two parallel plates separated by 0.02 m with a voltage difference of 100 V between them. If the electron enters moving perpendicular to the electric field between the plates, what magnetic field perpendicular to both the electron path and the electric field is necessary to make the electron travel in a straight line?

[*Hint:* $F_m = Bev$ and $F_e = eE = e \times V/d$.]

34. Two identical conducting spheres of radius 0.15 m are separated by 10 m from centre to centre. What is the charge on each sphere if the potential of one is 1500 V and the other -1500 V?

35. Dimensionally S_t (distance described in tth second) $= u + \frac{1}{2}a(2t - 1)$ seems to be incorrect. But in fact this is a correct relation. Can you explain the flaw?

36. A plank of mass m_1 with a bar of mass m_2 placed on it lies on a horizontal plane surface. A horizontal force growing with time as $F = at$ (a is constant) is applied to the bar. Find how the accelerations f_1 and f_2 of the plank and the bar, respecctively depend on time t, if coefficient of friction between the bar and the plank is μ. Draw the approximate plots of these dependences.

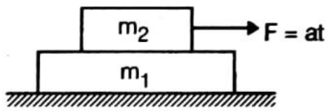

Fig. D

37. Each of (a) coulomb-ohm-metre/weber, (b) volt-second, (c) coulomb-ampere/farad, (d) $(\text{henry} \times \text{farad})^{\frac{1}{2}}$ is equal to one of the following: metre, ampere, second, kilogramme, newton, joule watt, coulomb, volt, ohm, wéber, henry, farad. Match them.

38. Show, by the method of dimensions, that the relation $E = Bc$ between the magnetic field B and the electric field E in an electromagnetic wave is true.

39. A particle moves in a plane under the action of a force which is always perpendicular to its velocity and depends on the distance from a certain point in the plane as $1/r^n$, where n is a constant. At what value or values of n will the motion of the particle along the circle be steady?

[*Hint:* For the equilibrium of a closed system it is necessary for total energy to be negative.]

40. Two metal plates are inserted at equal distances into a plane capacitor (figure E). Plates 1 and 4 are connected to a battery of emf E. What are the potentials of the plates? Assume the potential of the negative plate to be zero. How will the potentials of plates 2 and 3 and the intensities of the field in each of the three spaces change after plates 2 and 3 are connected for a moment by a wire? What will happen in this case to the charges on plates 1 and 4?

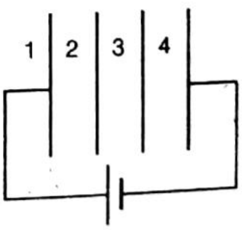

Fig. E

ANSWERS

1. $Q_n = \left(\frac{C}{C+c}\right)^n Q$ 2. $q_1 = \frac{(C_2+C_3)C_1^2 V_0}{C_1C_2 + C_2C_3 + C_3C_1}$, $q_2 = q_3 = \frac{C_1C_2C_3 V_0}{C_1C_2 + C_2C_3 + C_3C_1}$

3. 83° 4. 36×10^{-3} J, 18×10^{-3} J 5. 22 cm 6. al/g, $\omega^2 l^2/2g$

7. 198×10^5 N m^{-2} (compressive), 1236×10^5 Nm^{-2} (tensile)

8. $E = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(v_1^2 + v_2^2)$

9. $a_1 = \frac{m_2 g \sin\alpha \cos\alpha}{m_2 \sin^2\alpha + m_1 \cos^2\alpha}$, $a_2 = \frac{m_2 g \sin^2\alpha}{m_2 \sin^2\alpha + m_1 \cos^2\alpha}$, $N = \frac{m_1 m_2 g \cos\alpha}{m_2 \sin^2\alpha + m_1 \cos^2\alpha}$

10. $a = g/\mu$ 11. $F = (m_1 + m_2 + m_3)\, m_3 g/m_2$ 12. Zero 13. $F = \frac{ATYY'\,(l\alpha + l'\alpha')}{lY' + l'Y}$

14. $\frac{Fl}{Y\sqrt{\alpha_1\alpha_2}}$ 15. $v' = \frac{w_0 l}{\sqrt{1 + \frac{3m}{M}}}$ 16. 53° 17. $T = \frac{m\omega_0^2 r_0^4}{r^3}$

18. $\frac{r_1 r_2 e_3 + r_2 r_3 e_1 + r_3 r_1 e_2}{r_1 r_2 + r_2 r_3 + r_3 r_1}$ 19. $\sqrt{2}\, A\rho v\left(\sin\frac{\theta}{2} - \cos\frac{\theta}{2}\right)$ 20. 1.6 kg, 2 kg

21. $R = \rho\frac{l}{\sqrt{\alpha_1\alpha_2}}$ 22. 25 m; 1.96×10^{10} m^{-3} 24. 1.75 m 25. 5×10^7 A

26. 8×10^{-6} m 28. 258°C 29. 50 V, 40 V, + to A and – to B

30. $f = \frac{1}{2\pi}\sqrt{\frac{3g}{2l}\left(1 + \frac{4kl}{mg}\right)}$ 31. $\theta = \tan^{-1}\frac{\omega g l}{mB}$ 32. $\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

33. 2.7×10^{-4} T 34. $+2.5 \times 10^{-8}$C and -2.5×10^{-8}C

36. $f_1 = f_2 = \frac{at}{m_1 + m_2}$ when $t < t_0 = \frac{\mu m_2 (m_1 + m_2) g}{m_1 a}$,

and when $t > t_0$ $f_1 = \frac{\mu m_2 g}{m_1}$ and $f_2 = \frac{at - \mu m_2 g}{m_2}$

37. metre, weber, watt, second 39. For $n < 1$, including negative values

40. E, $\frac{2E}{3}$, $\frac{E}{3}$; O; E, $\frac{E}{2}$, $\frac{E}{2}$, 0. In the space 1, 2 and 3, 4 the intensity of the field will increase and in the space 2, 3 it will become zero. The charges on the plates 1 and 4 will increase. Plate 2 will have a positive charge and plate 3 negative one.

Revision Examples I

1. We have $Q = VC = (C + c)\,V_1$ or $V_1 = \dfrac{VC}{C + c}$.

After discharging the second capacitor it is again connected to the first.

Hence $Q_1 = V_1 C = (C + c)\,V_2$

or $$V_2 = \frac{V_1 C}{C + c} = \left(\frac{C}{C + c}\right)^2 V.$$

Secondly, when the process is again repeated,

then $Q_2 = V_2 C = (C + c)\,V_3$ or $V_3 = \dfrac{V_2 C}{C + c} = \left(\dfrac{C}{C + c}\right)^3 V.$

Similarly $$V_n = \left(\frac{C}{C + c}\right)^n V.$$

$$Q_n = V_n C = \left(\frac{C}{C + c}\right)^n VC = \left(\frac{C}{C + c}\right)^n Q.$$

Hence repeating the same process of discharges, first capacitor can never be fully discharged. This is due to electric hysteresis of the dielectric.

2. The series combination of C_2 and $C_3 = C = \dfrac{C_2 C_3}{C_2 + C_3}$.

The charges given to C_1 is $V_0 C_1$. This charge is shared by C_1 and C. Charge is shared *in the ratio of capacitances*.

$\therefore$ Charge on $C_1 = \left(\dfrac{V_0 C_1}{C + C_1}\right) \times C_1$

$$\frac{V_0 C_1^2}{C + C_1} = \frac{V_0 C_1^2}{\dfrac{C_2 C_3}{C_2 + C_3} + C_1} = \frac{V_0 C_1^2\,(C_2 + C_3)}{C_1 C_2 + C_2 C_3 + C_3 C_1}.$$

Charge on $C = \left(\dfrac{V_0 C_1}{C + C_1}\right) C = \dfrac{V_0 C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1}.$

In series connection charges on capacitors are the same and each is equal to the charge on the combination.

$\therefore$ Charge on C_2 = charge on $C_3 = \dfrac{V_0 C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1}.$

3. $\mu = \dfrac{\sin \dfrac{A+\delta_m}{2}}{\sin \dfrac{A}{2}}$. $\quad \therefore \; 1.5 = \dfrac{\sin A}{\sin \dfrac{A}{2}} = 2 \cos \dfrac{A}{2}$

or $\quad \cos \dfrac{A}{2} = 0.75 = \cos 41°24'$

$\therefore \quad A = 82°48' = 83°.$

4. $q_i = 10 \times 10^{-6} \times 30 = 3 \times 10^{-4}\, C$

and $\quad q_f = 50 \times 10^{-6} \times 30 = 15 \times 10^{-4}\, C$

$$\Delta q = 12 \times 10^{-4}\, C$$

$\therefore \quad$ Energy drawn $= 12 \times 10^{-4} \times 30 = 36 \times 10^{-3}$ J.

Extra energy stored

$$= E_f - E_i = \frac{q_f^2}{2C_f} - \frac{q_i^2}{2C_i}$$

$$= \frac{225 \times 10^{-8}}{2 \times 50 \times 10^{-6}} - \frac{9 \times 10^{-8}}{2 \times 10 \times 10^{-6}}$$

$$= 1.8 \times 10^{-2} = 18 \times 10^{-3} \text{ joule.}$$

5. $$\frac{-Qq}{4\pi\varepsilon_0\, a/2} \times 2 + \frac{1}{2} mv^2 = -\frac{2Qq}{4\pi\varepsilon_0 x}$$

where x is the distance of the electron from either charge and a is the distance between them.

$$\frac{4Qq}{4\pi\varepsilon_0 a} - \frac{1}{2} mv^2 = \frac{2Qq}{4\pi\varepsilon_0 x}$$

$$\frac{1}{x} = \frac{2}{a} - \frac{4\pi\varepsilon_0 mv^2}{4Qq}$$

$$= \frac{2}{0.1} - \frac{9 \times 10^{-31} \times 10^{12}}{9 \times 10^{9} \times 4 \times 10^{-11} \times 1.6 \times 10^{-19}}$$

$$= 20 - 15.63$$

$$\frac{1}{x} = 4.374 \quad \text{or} \quad x = 0.23 \text{ m}$$

and $\quad d = \sqrt{0.23^2 - 0.05^2} = 0.22 \text{ m} = 22 \text{ cm}.$

6. (a) Let h be the height of the column in the left arm and h' that in the right arm. Let us reduce the problem to a static one by applying d'Alembert's principle, that is, apply an inertial force on the horizontal portion to the left. The forces acting on the horizontal portion are

(i) $Ah\rho g$ to the right, (ii) $Ah'\rho g$ to the left and (iii) inertial force 'ma' to the left where m is the mass of the liquid in the horizontal portion. Under the actions of these forces the liquid in the horizontal portion is at rest.

$$\therefore \quad ma + Ah'\rho g - Ah\rho g = 0. \quad \text{But} \quad m = Al\rho.$$

$$\therefore \quad Al\rho a + Ah'\rho g - Ah\rho g = 0 \quad \text{or} \quad h - h' = \frac{la}{g}.$$

(b) When the tube is rotated, the average horizontal acceleration is

$$\omega^2 \frac{l}{2}.$$

$$\therefore \quad h - h' = l\omega^2 \frac{l}{2g} = \frac{\omega^2 l^2}{2g}.$$

7. Decrease in length of the rod = $l\alpha\,\Delta\theta - l\alpha'\,\Delta\theta$

$$\therefore \quad \text{strain of the rod} = \frac{l(\alpha - \alpha')\,\Delta\theta}{l} = (\alpha - \alpha')\,\Delta\theta$$

$$\therefore \quad \text{stress} = Y \times \text{strain} = Y(\alpha - \alpha')\,\Delta\theta$$

(a) $$\text{stress} = 18 \times 10^9 (29 \times 10^{-6} - 18 \times 10^{-6}) \times 100$$

$$= 198 \times 10^5\ \text{N m}^{-2}\ \text{(compressive)}.$$

(b) $$\text{stress} = 206 \times 10^9 (12 \times 10^{-6} - 18 \times 10^{-6}) \times 100$$

$$= 1236 \times 10^5\ \text{N m}^{-2}\ \text{(tensile)}.$$

8. $(m_1 + m_2)\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2$

$$E = \frac{1}{2}m_1 |\vec{v}_{cm} - \vec{v}_1|^2 + \frac{1}{2}m_2 |\vec{v}_{cm} - \vec{v}_2|^2$$

$$= \frac{1}{2}m_1 \left| \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} - \vec{v}_1 \right|^2 + \frac{1}{2}m_2 \left| \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} - \vec{v}_2 \right|^2$$

$$= \frac{1}{2} \times \frac{m_1 m_2^2}{(m_1 + m_2)^2} |\vec{v}_2 - \vec{v}_1|^2 + \frac{1}{2} \times \frac{m_1^2 m_2}{(m_1 + m_2)^2} |\vec{v}_1 - \vec{v}_2|^2$$

$\because$ $\vec{v}_1$ is at right angles to $\vec{v}_2$

$$|\vec{v}_1 - \vec{v}_2|^2 = v_1^2 + v_2^2$$

$$\therefore \quad E = \frac{1}{2} \times \frac{m_1 m_2}{(m_1 + m_2)^2} (v_1^2 + v_2^2)(m_2 + m_1)$$

$$= \frac{1}{2} \times \frac{m_1 m_2}{m_1 + m_2} (v_1^2 + v_2^2).$$

9. Let x be the vertical downward displacement of m_2. Its horizontal displacement is zero. The horizontal displacement of m_1 is $x \cot \alpha$. Vertical displacement is zero. Denoting accelerations of m_1 and m_2 by a_1 and a_2,

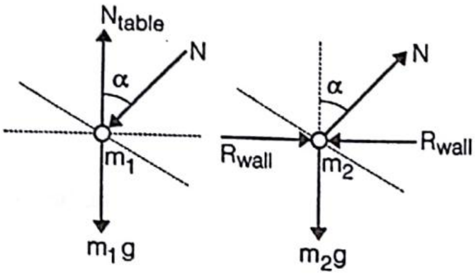

Fig. RE I.1

$$a_1 = \ddot{x} \cot \alpha \quad \text{and} \quad a_2 = \ddot{x}$$

or $a_1 = a_2 \cot \alpha$.

From free-body diagram of m_1 and m_2,

$$N \sin \alpha = m_1 a_1 = m_1 a_2 \cot \alpha$$

$$m_2 g - N \cos \alpha = m_2 a_2.$$

Solving for N, a_2 (and hence a_1 as well),

$$a_1 = \frac{m_2 g}{m_2 \tan \alpha + m_1 \cot \alpha}, \qquad a_2 = \frac{m_2 g \sin^2 \alpha}{m_2 \sin^2 \alpha + m_1 \cos^2 \alpha}$$

and $$N = \frac{m_1 m_2 g \cos \alpha}{m_2 \sin^2 \alpha + m_1 \cos^2 \alpha}.$$

10. Let us bring the system to rest by applying an acceleration 'a' in the opposite direction. The forces acting on the block are (i) mg vertically downward, (ii) μR vertically upward, (iii) reaction of the wall 'R' horizontally to the right (say) and (iv) inertial force ma to the left.

Resolving the forces along the vertical and horizontal, we have

$$R = ma \quad \text{and} \quad \mu R = mg$$

$$\therefore \quad \mu ma = mg \quad \text{or} \quad a = g/\mu.$$

11. From the free- body diagram of the system, we have

$$F = (m_1 + m_2 + m_3)\, a.$$

From the free-body diagram of m_2, we have $T = m_2 a$

and from the free-body diagram of m_3, we have $T = m_3 g$.

$$\therefore \quad m_2 a = m_3 g \quad \text{or} \quad a = m_3 g/m_2$$

$$\therefore \quad F = (m_1 + m_2 + m_3)\, m_3 g/m_2.$$

12. The currents in the two portions are in the ratio 3 : 1. Let i be the common ratio. Then the currents are $3i$ and i. The lengths are in the ratio 1 : 3, i.e., the lengths are l and $3l$ where l is the common ratio.

Then $B_1 = \frac{\mu_0}{4\pi} \times \frac{3il}{a^2}$ where a is the radius of the circle.

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{i \times 3l}{a^2}$$. The two fields are opposite.

$\therefore$ $$\vec{B} = \vec{B_1} + \vec{B_2} = 0.$$

13. We have $l + l' = l_1 + l'_1$.

Now $l_1 = l + l\alpha T - \frac{Fl}{AY}$ and $l'_1 = l' + l'\alpha' T - \frac{Fl'}{AY'}$

$\therefore$ $$l + l' = l + l\alpha T - \frac{Fl}{AY} + l' + l'\alpha' T - \frac{Fl'}{AY'}$$

Hence $$F = \frac{TAYY'(l\alpha + l'\alpha')}{lY' + l'Y}.$$

14. Let x_1 and x_2 be the distances of the ends from the apex of the cone of which the rod is a part. Consider a section at a distance x from the apex. Then

$$\frac{\alpha_1}{x_1^2} = \frac{\alpha}{x^2} = \frac{\alpha_2}{x_2^2} \quad \text{or} \quad \alpha = \frac{\sqrt{\alpha_1\alpha_2}}{x_1 x_2} x^2.$$

Increase in length of an elementary length dx at a distance x from the apex

$$= \frac{F}{\alpha} \times \frac{dx}{Y} = \frac{F}{Y} \times \frac{x_1 x_2}{\sqrt{\alpha_1\alpha_2}} \times \frac{dx}{x^2}.$$

$\therefore$ $$\text{Total increase in length} = \frac{F}{Y} \times \frac{x_1 x_2}{\sqrt{\alpha_1\alpha_2}} \int_{x_1}^{x_2} \frac{dx}{x^2}$$

$$= \frac{F}{Y} \times \frac{x_1 x_2}{\sqrt{\alpha_1\alpha_2}} \left[\frac{1}{x_1} - \frac{1}{x_2}\right] = \frac{F}{Y} \times \frac{x_1 x_2}{\sqrt{\alpha_1\alpha_2}} \times \frac{x_2 - x_1}{x_1 x_2}$$

$$= \frac{F}{Y} \times \frac{l}{\sqrt{\alpha_1\alpha_2}}.$$

15. Considering conservation of angular k. energy,

$$\frac{1}{2} \times \left(ml^2 + \frac{1}{3} Ml^2\right)\omega^2 = \frac{1}{2} \cdot \frac{1}{3} M l^2 \omega_0^2$$

or $$\omega = \frac{\omega_0}{\sqrt{1 + \frac{3m}{M}}}; \qquad \therefore \quad v' = \frac{\omega_0 l}{\sqrt{1 + \frac{3m}{M}}}.$$

16. Let θ be the required angle.

At θ, $\text{accel.} = \sqrt{a_r^2 + a_t^2} = a_t = g \sin\theta$ $\qquad (\because \ a_r = 0)$

At $\theta = 0°$, accel. $= \sqrt{a_r^2 + a_t^2} = a_r = \frac{v^2}{l}$.

By conservation of energy, $\frac{1}{2}mv^2 = mg\,l(1 - \cos\theta)$.

$\therefore \qquad g \sin\theta = \frac{2gl\,(1-\cos\theta)}{l} \qquad$ or $\qquad \theta = 53°$.

17. $T = \frac{mv^2}{r}$. By conservation of angular momentum,

$$mr_0^2 \times \omega_0 = mr^2 \times \omega = mr^2 \times \frac{v}{r}$$

$$\therefore \qquad v = \frac{r_0^2\omega_0}{r}, \qquad \therefore \qquad T = \frac{mr_0^4\,\omega_0^2}{r^3}.$$

18. Let i_1 be the current through the first cell from its negative pole to its positive pole, i_2 be the current through the second cell from its negative pole to its positive pole. Then $(i_1 + i_2)$ is the current through the third cell from its positive pole to its negative pole. By the loop rule,

$$i_1r_1 - i_2r_2 = e_1 - e_2 \quad \text{and} \quad i_1r_1 + r_3\,(i_1 + i_2) = e_1 - e_3.$$

Solving for i_1 we have

$$i_1 = \frac{r_3\,(e_1 - e_2) + r_3\,(e_1 - e_3)}{r_1r_2 + r_2r_3 + r_3r_1}$$

$$V_{ba} = r_1i_1 - e_1$$

or $\qquad V_{ab} = e_1 - i_1r_1$

$$= e_1 - \frac{r_1r_3\,(e_1 - e_2) + r_1\,r_2\,(e_1 - e_3)}{r_1r_2 + r_2r_3 + r_1r_2}$$

$$= \frac{r_1r_2e_3 + r_2r_3e_1 + r_3r_1e_2}{r_1r_2 + r_2r_3 + r_3r_1}$$

19. The rate of change of horizontal momentum

$= A\rho v \sin\theta - A\rho v = A\rho v\,(\sin\theta - 1)$.

$\therefore$ The horizontal force needed $= A\rho v\,(\sin\theta - 1)$.

The rate of change of vertical momentum

$= A\rho v \cos\theta - 0 = A\rho v \cos\theta$.

$\therefore$ The vertical force needed $= A\rho v \cos\theta$

$\therefore$ The resultant force needed

$$= A\rho v\sqrt{(\sin\theta - 1)^2 + \cos^2\theta} = \sqrt{2}\,A\rho v\left(\sin\frac{\theta}{2} - \cos\frac{\theta}{2}\right).$$

20. From the free-body diagram of the load, we have

$$4g - 2R_y = 0$$

or $$R_y = 2g = 2 \text{ kg}.$$

From the free-body diagram of the upper rod, we have

$$3R_x - 2 \times 0.4 - 2R_y = 0$$

or $$3R_x = 0.8 + 4 = 4.8$$

or $$R_x = 1.6 \text{ kg}.$$

21. Consider a section of the wire at a distance x from the apex of the wire. Let x_1 and x_2 be the distances of the end-sections from the apex.

Then $$\frac{\alpha_1}{x_1^2} = \frac{\alpha}{x^2} = \frac{\alpha_2}{x_2^2} \quad \text{or} \quad \alpha = \frac{\sqrt{\alpha_1 \alpha_2}}{x_1 x_2} \times x^2.$$

The resistance of an elementary length dx of the wire

$$= \rho \frac{dx}{\alpha} = \rho \frac{x_1 x_2}{\sqrt{\alpha_1 \alpha_2}} \times \frac{dx}{x^2}.$$

$\therefore$ Total resistance of the wire

$$= \int_{x_1}^{x_2} \rho \frac{x_1 x_2}{\sqrt{\alpha_1 \alpha_2}} \times \frac{dx}{x^2}$$

$$= \rho \frac{x_1 x_2}{\sqrt{\alpha_1 \alpha_2}} \times \frac{x_2 - x_1}{x_1 x_2} = \frac{\rho l}{\sqrt{\alpha_1 \alpha_2}}.$$

22. Hence $$I = \frac{100}{4\pi x^2} \text{ J s}^{-1} \text{ m}^{-2}$$

also $$I = Dc = (10^7 h\nu)\, c = 10^7 \times \frac{hc^2}{\lambda}.$$

$$\therefore \quad \frac{100}{4\pi x^2} = \frac{10^7 hc^2}{\lambda}$$

or $$x^2 = \frac{\lambda}{4\pi hc^2} \times 10^{-5} = \frac{5890 \times 10^{-10} \times 10^{-5}}{4\pi\, (6.63 \times 10^{-34}) \times 9 \times 10^{16}}$$

Hence $x = 25.0$ m.

Let n be the number of photons per cubic metre at a distance of 2 m.

$$\therefore \quad \frac{100}{4\pi 2^2} = (nh\nu)\, c = \frac{nhc^2}{\lambda}$$

or $$n = \frac{25\lambda}{4\pi hc^2} = \frac{25 \times 5890 \times 10^{-10}}{4\pi\, (6.63 \times 10^{-34}) \times 9 \times 10^{16}}$$

$$= 1.96 \times 10^{10} \text{ m}^{-3}.$$

23. For an ideal gas,

$$C_p = \frac{\gamma R}{\gamma - 1}, \quad C_v = \frac{R}{\gamma - 1}$$

$$\Delta Q = C_p \Delta T = \frac{\gamma R \Delta T}{\gamma - 1}, \qquad \Delta U = C_v \Delta T = \frac{R}{\rho - 1}\Delta T$$

and $$\Delta W = p\Delta V = p\Delta\left(\frac{RT}{p}\right) = R\Delta T.$$

$\therefore$ $$\Delta Q : \Delta U : \Delta W = \frac{\gamma}{\gamma - 1} : 1 = \gamma : 1 : \gamma - 1,$$

For monoatomic gas,

$\therefore$ $$\Delta Q : \Delta U : \Delta W = \frac{5}{3} : 1 : \frac{2}{3} = 5 : 3 : 2$$

24. The rays will strike the immersed portion at an angle θ such that $\frac{4}{3} = \frac{\sin 45°}{\sin \theta}$ or $\sin \theta = \frac{3}{4\sqrt{2}}$. So if x is the length of the immersed portion then $\frac{x}{2} = \tan \theta = \frac{3}{\sqrt{23}}$ or $x = \frac{6}{\sqrt{23}}$. The length of the portion above the water is 0.5 m. Therefore, the total length of the shadow is $\frac{6}{\sqrt{23}} + 0.5 = 1.251 + 0.5 = 1.75$ m.

25. The magnetic moment of a current of a loop $(m) = I \times S$, where I is the current and S is the area of the loop.

$\therefore$ $$6.4 \times 10^{21} = I \times \pi \times (6.4 \times 10^6)^2$$

or $$I = 5 \times 10^7 \text{ amp.}$$

26. We have $$x \text{ (shift)} = \frac{(\mu_2 - \mu_1)\, t\beta}{\lambda}.$$

Here $x = 5\beta$; $\therefore$ $$5\beta = \frac{(1.7 - 1.4)t\beta}{4800 \times 10^{-10}}.$$

Hence $$t = \frac{5 \times 4800 \times 10^{-10}}{0.3} = 8 \times 10^{-6} \text{ m.}$$

27. Let us charge the spheres with equal and opposite charges $+q$ and $-q$ respectively. Then by the definition of the capacitance of a capacitor $\left(C = \frac{q}{V}, \text{i.e., capacitance} = \frac{\text{charge}}{\text{p.d.}}\right)$,

$$C = \frac{q}{\frac{q}{4\pi\varepsilon_0 a} - \left(-\frac{q}{4\pi\varepsilon_0 a}\right)} = 2\pi\varepsilon_0 a.$$

The capacitance of a single sphere $= 4\pi\varepsilon_0 a$ which is double of the capacitance of two spheres of same radius.

28. r (resistance of the wire) $= 10^{-7} \times \dfrac{0.1}{\pi(5 \times 10^{-4})^2} = \dfrac{1}{25\pi}$.

$$H \text{ (rate of generation of heat)} = i^2 r$$

$$= 100 \times \frac{1}{25\pi} \text{ watt} = \frac{100}{25\pi}$$

or $$R \text{ (radiancy)} = \frac{100/25\pi}{2\pi(5 \times 10^{-4}) \times 0.1} = \frac{10^6}{25\pi^2}$$

$\therefore$ $$\frac{10^6}{25\pi^2} = 5.67 \times 10^{-8}(T^4 - 300^4).$$

Hence $T = 531$ K or $t = 258°$ C.

In the above calculation we have assumed the wire as a perfect black body and that the resistance of the wire remains constant.

29. Power absorbed in the resistor $= i^2 r = 1^2 \times 10 = 10$ watts.

$\therefore$ Power absorbed in the battery $= 50 - 10 = 40$ watts. Since power is absorbed in the battery, it must oppose the flow of current, i.e., its positive must be to A and its negative to B.

Power absorbed in a battery $= \varepsilon i$. (ε = emf of battery)

$\therefore$ Here $\varepsilon \times 1 = 40$ or $\varepsilon = 40$ volts.

$$V_{AB} = \Sigma ir - \Sigma\varepsilon = 10 \times 1 - (-40) = 50 \text{ volts.}$$

30. Let x be the instantaneous displacement of the lower end.

Then τ' (torque about O)

$$= -2klx - \frac{1}{2}mgx$$

= moment of inertia × ang. acc.

$$= \left(\frac{1}{3}ml^2\right) \times \left(l\,\frac{d^2x}{dt^2}\right)$$

or $$\frac{d^2x}{dt^2} = -\left(\frac{3g}{2l} + \frac{6k}{m}\right)x$$

$\therefore$ $$\frac{d^2x}{dt^2} \propto -x,$$

$\therefore$ $$\omega^2 = \frac{3g}{2l} + \frac{6k}{m} = \frac{3g}{2l}\left(1 + \frac{4kl}{mg}\right)$$

or $$f = \frac{1}{2\pi}\sqrt{\frac{3g}{2l}\left(1 + \frac{4kl}{mg}\right)}.$$

31. Let the string make an angle θ with the vertical and let the magnet make an angle α with the horizontal. Projecting forces along vertical and horizontal directions, we have

$$mB + T \sin\theta = mB \qquad \text{or} \qquad T\sin\theta = 0$$

or $\quad \theta = 0.$

Thus the string will remain vertical as before.

Taking moment about the south pole, we have

$$wg\, l\cos\alpha = q_m B \times 2l \sin\alpha$$

or $\quad \tan\alpha = \dfrac{wgl}{mB}. \qquad (\because \quad m = 2q_m L)$

Hence, $\quad \alpha = \tan^{-1}\dfrac{wgl}{mB}.$

32. By *work-energy* theorem,

$$\text{work done} = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2.$$

By dynamics of circular motion,

$$\frac{mv^2}{r_1} = \frac{1}{4\pi\varepsilon_0} \times \frac{Qq}{r_1^2}.$$

$\therefore \quad \text{Work done} = \dfrac{1}{2} \times \dfrac{Qq}{4\pi\varepsilon_0} \times \dfrac{1}{r_1} - \dfrac{1}{2} \times \dfrac{Qq}{q4\pi\varepsilon_0} \cdot \dfrac{1}{r_2}$

$$= \frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

33. $F_{magnetic} = Bev \sin\theta$ and $F_{electric} = eE = \dfrac{eV}{d}.$

Here $\quad \theta = 90°. \quad \therefore \quad F_{magnetic} = Bev.$

For no deflection,

$$Bev = \frac{eV}{d} \qquad \text{or} \qquad B = \frac{V}{v \cdot d}.$$

Now $\quad 1000 \times 1.6 \times 10^{-19} = \dfrac{1}{2} \times 9.1 \times 10^{-31}\, v^2$

or $\quad v = 1.88 \times 10^7 \text{ m s}^{-1}.$

$\therefore \quad B = \dfrac{100}{1.88 \times 10^7 \times 0.02} = 2.7 \times 10^{-4}$ tesla.

34. As the distance between them is large, distribution of charge on them may be supposed to be uniform.

$\therefore \quad$ Potential of the first sphere $= \dfrac{1}{4\pi\varepsilon_0} \cdot \left(\dfrac{+q}{0.15} + \dfrac{+q'}{10}\right) = 1500.$

(given)

$$\text{Potential of the second sphere} = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{+q'}{0.15} + \frac{+q}{10} \right) = -1500$$

(given)

Hence $q = -q'$. Putting $q = -q'$ in any of the above relations,

We have $q = 2.5 \times 10^{-8}$ C.

Hence the charge on second sphere will be -2.5×10^{-8} C.

35. $s_t = (ut + \frac{1}{2}at^2) - [u(t-1) + \frac{1}{2}a(t-1)^2]$

$= u[t-(t-1)] + \frac{1}{2}a[t^2-(t-1)^2]$.

Thus u is multiplied by 1 second and not by simply 1. So also a is multiplied by $(2t-1)$ second and not by simply $(2t-1)$.

This explains the flaw.

36. Let F_{lim} be the limiting value of the frictional force. So long $F \le F_{\text{lim}}$, the two will have the same acceleration f given by

$$(m_1 + m_2)f = F = at \quad \text{or} \quad f = \frac{at}{m_1 + m_2}.$$

Let us consider any instant when f_1 and f_2 are different. Then

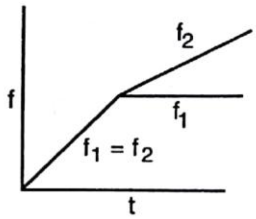

Fig. RE I.2

$$F - \mu m_2 g = m_2 f_2 \quad \text{or} \quad f_2 = \frac{at - \mu m_2 g}{m_2}$$

and $\qquad \mu m_2 g = m_1 f_1 \qquad$ or $\qquad f_1 = \dfrac{\mu m_2 g}{m_1}$.

Let f_1 be equal to f_2 at $t = t_0$. Then

$$\frac{at_0 - \mu m_2 g}{m_2} = \frac{\mu m_2 g}{m_1} \quad \text{or} \quad t_0 = \frac{\mu m_2 (m_1 + m_2) g}{m_1 a}.$$

Thus within $t = t_0 = \dfrac{\mu m_2 (m_1 + m_2) g}{m_1 a}$

and $\qquad f_1 = f_2 = \dfrac{at}{m_1 + m_2}$

and at instants $t > t_0$, $f_1 = \dfrac{\mu m_2 g}{m_1}$ and $f_2 = \dfrac{at - \mu m_2 g}{m_2}$.

37. Coulomb = AT, ohm = $ML^2T^{-3}A^{-2}$, volt = $ML^2T^{-3}A^{-1}$, weber = $ML^2T^{-2}A^{-1}$, henry = $ML^2T^{-2}A^{-2}$ and farad = $M^{-1}L^{-2}T^4A^2$.

$\therefore$ (a) $\qquad \dfrac{\text{coulomb-ohm-metre}}{\text{weber}} = \dfrac{AT \times ML^2T^{-3}A^{-2} \times L}{ML^2T^{-2}A^{-1}}$

$= L$ = metre.

(b) volt-second $= ML^2T^{-3}A^{-1} \times T$

$= ML^2T^{-2}A^{-1} = \text{weber}.$

(c) $\dfrac{\text{coulomb} \times \text{ampere}}{\text{farad}} = \dfrac{(AT) \times A}{M^{-1}L^{-2}T^4A^2}$

$= ML^2\,T^{-3} = \text{watt}.$

(d) $(\text{henry} \times \text{farad})^{1/2} = (ML^2T^{-2}\,A^{-2} \times M^{-1}L^{-2}T^4\,A^2)^{1/2}$

$= T = \text{second}.$

38. $[E] = MLT^{-3}A^{-1}$

$[Bc] = MT^{-2}A^{-1} \times LT^{-1} = MT^{-3}\,LA^{-1}.$

Hence the relation is true.

39. Centripetal force $= \dfrac{k}{r^n} = \dfrac{mv^2}{r}$ or $mv^2 = \dfrac{k}{r^{n-1}}$

$\therefore$ $K \text{ (kinetic energy)} = \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times \dfrac{k}{r^{n-1}}.$

U = potential energy

$$= -\int F dr \qquad \left(\because\ F = -\frac{dU}{dr}\right)$$

$$= -\int \frac{k}{r^n}\,dr = \frac{k}{(n-1)r^{n-1}} + c.$$

At $r = \infty$, $U = 0$; $\therefore$ $c = 0$.

$\therefore$ $U = \dfrac{k}{(n-1)\,r^{n-1}}.$

$\therefore$ Total energy of the system $= \dfrac{1}{2} \cdot \dfrac{k}{r^{n-1}} + \dfrac{k}{(n-1)r^{n-1}}$

$$= \frac{1}{2} \times \frac{k}{r^{n-1}}\left(1 + \frac{2}{n-1}\right) = \frac{1}{2} \times \frac{k}{r^{n-1}}\,\frac{n+1}{n-1}.$$

For equlibrium of a system, its total energy must be *negative*.

$\therefore$ F or all values $n < 1$ including negative values, the system will be in equilibrium.

40. The potential difference between plates 1 and 4 = E

or $V_1 - V_4 = E$

or $V_1 = E$ ($\because$ $V_4 = 0$).

Rate of fall of potential $= \dfrac{E}{3d}$ where d = distance between plates.

$\therefore$ p.d. between plates 1 and 2 $= \frac{E}{3d} \times d = \frac{E}{3}$

or $V_1 - V_2 = \frac{E}{3}$ or $E - V_2 = \frac{E}{3}$

or $V_2 = \frac{2E}{3}$.

p.d. between plates 1 and 3 $= \frac{F}{3d} \times 2d = \frac{2E}{3}$

or $V_1 - V_3 = \frac{2E}{3}$

or $E - V_3 = \frac{2E}{3}$ or $V_3 = \frac{E}{3}$.

Thus potentials of 1, 2, 3, 4 are $E, \frac{2E}{3}, \frac{E}{3}, 0$ respectively.

When the plates 2 and 3 are joined together, they come to the same potential and the electric field between these plates is reduced to zero, because electric field = rate of fall of potential.

Now the effective electrical separation between plates is $2d$.

$\therefore$ $V_1 - V_2 = \frac{E}{2d} d = \frac{E}{2}$ or $E - V_2 = \frac{E}{2}$

or $V_2 = \frac{E}{2} = V_3$.

Thus potentials of the plates now are $E, \frac{E}{2}, \frac{E}{2}, 0$.

Before

Electric field between 1 & 2 $= \frac{\text{p.d.}}{\text{distance}} = \frac{E - \frac{2E}{3}}{d} = \frac{E}{3d}$.

" " " 2 & 3 $= \frac{\frac{2E}{3} - \frac{E}{3}}{d} = \frac{E}{3d}$

" " " 3 & 4 $= \frac{E}{3d}$.

After

Electric field between 1 and 2 $= \frac{E - \frac{E}{2}}{d} = \frac{E}{2d}$

" " " 2 and 3 = 0

" " " 3 and 4 $= \frac{E}{2d}$.

Thus electric field between 1 and 2 and 3 and 4 has increased from $\frac{E}{3d}$ to $\frac{E}{2d}$, i.e., by 50% and that between 2 and 3 has decreased from $\frac{E}{3d}$ to 0, i.e., by 100%.

□ □ □

Revision Examples 2

REVISION EXAMPLES II

1. A particle of mass m moves in an elliptical orbit under a central force = (k/r^2), where r is the distance of the particle from the centre of the force. If the distance of the particle from the centre of the force is a at one exterme position of the particle and its linear speed is $\sqrt{k/2ma}$, find (from energy considerations and dynamics of angular momentum of the particle) the distance of the particle at the other extreme positon.

   [*Hint:* Potential energy of the particle $= -k/r$. Apply conservation of angular momentum and energy. Remember that moment of momentum is also angular momentum.]

2. A circular coil is placed in the magnetic meridian and a current is passed through it so as to produce a magnetic field to the west. The compass needle placed at the centre of the coil is deflected through 20°. A short bar magnet of magnetic moemnt 1.2 Am^2 is placed to the east of the coil with its midpoint at a distance 10 cm and its axis at a bearing of 30° north of west. Calculate the deflection of the needle. $B_{earth} = 3.6 \times 10^{-5}$ T.

   [*Hint:* Find restoring torque and deflecting torque and apply tangent law.]

3. A particle of mass m is attached to a point O by a string of length l. It is held at a point P at a distance l from O, above the level through O at an angle 60° with the upward vertical. If the particle is released from rest at P, prove that when the particle is again at a distance l from O below the level through O the string exerts an impulsive force of impulse $m\sqrt{\frac{1}{2}gl}$, and that immediately afterwards the tension becomes $2mg$.

   [*Hint:* Consider conservation of angular momentum of the particle about O to find the speed immediately afterwards, remembering that moment of momentum is also angular momentum.]

4. A motorcyclist moves along an *S*-bend at a constant speed of 60 mph. The radius of curvature of the first bend is 250m and that of the second bend is 400 m. Calculate the change in acceleration of the cyclist.

   [*Hint:* Remember that acceleration is a vector quantity.]

5. Two bodies A of mass m and B of mass $2m$ connected by a light inextensible string are placed on a smooth cylindrical surface in a plane perpendicular to the axis of the cylinder. Initially the radius at A is horizontal and that at B is inclined to the upward vertical by $\tan^{-1}\frac{3}{4}$. Show that B leaves the surface when each radius describes θ given by $28 \cos\theta - 11 \sin\theta = 16$.

6. A point performs harmonic oscillations along a straight line with a period T and amplitude a. Find the mean velocity of the point averaged over the time interval

during which it travels a distance $a\sqrt{2}$, starting from (i) the extreme position, (ii) the equilibrium position.

7. The circuit shown in figure F has resistances $R_1 = 20\ \Omega$ and $R_2 = 30\ \Omega$. At what value of the resistance S connected in parallel to R_2 will the thermal power absorbed by S be independent of small variations in S itself? The voltage between the points A and B is a constant.

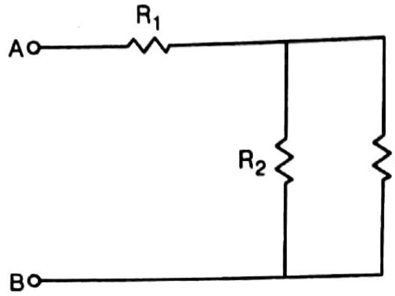

Fig. F

8. Two bars of masses m_1 and m_2 connected by a non-deformed light spring rest on a horizontal plane. The coefficient of friction between the bars and the surface is equal to k. What minimum constant force has to be applied in the horizontal direction to the bar of mass m_1 in order to shift the other bar?

[*Hint:* The force needed is minimum when m_1 is pulled slowly not to create any velocity. Apply work-energy theorem. Remember that the change in kinetic energy of the system is zero.]

9. A motor car is travelling along a level road with a constant speed V, the resistance to the motion being equivalent to a constant back pull a kgf. The car then comes to a hill where the resistance (including gravity) is b kfg, and after the velocity has again become constant, the engine works at the same constant power as on the level road. If, while the velocity is varying, the tractive-pull alters uniformly with the distance from its first constant value to the next constant value, show that the distance travelled along the hill before the velocity becomes constant is $\dfrac{M(a+b)V^2}{gb^2}$.

[*Hint:* Apply work-energy theorem. The work done by the engine $= \dfrac{1}{2}(a+b)\,g\,x$, taking the average of the initial and final constant value of the force as the constant force while the velocity is varying. This is permissible because the force changes uniformly with distance from its initial constant value to its final constant value.]

10. A spring of unstretched length l and modulus mg is fixed at one end and attached to a body of mass m at the other end. It is stretched by l and then released. Show that the maximum velocity of the particle is $(1-\mu)\sqrt{gl}$, where μ is the coefficient of friction between the body and the table.

[*Hint:* The force called into play in a spring is proportional to the change in length and inversely proportional to its natural length, i.e., $\dfrac{\lambda \Delta l}{l}$. This '$\lambda$' is called modulus of the spring. If x is the instantaneous displacement of the body from the point where it is released, then equation of motion of the body is

$$\frac{mg}{l}(l-x) - \mu\, mg = m\frac{dv}{dt}.\ ]$$

11. A particle inside and at the lowest point O of a fixed smooth spherical cavity is projected horizontally with speed $\sqrt{4ga}$. Find the height of the point above O at which the particle leaves the surface of the cavity.

[*Hint:* Apply dynamics of circular motion and conservation energy.]

12. A vessel contains helium, which expands at a constant pressure when 15 kJ of heat is supplied to it. What will be the variation of the internal energy of the gas? What is the work performed in the expansion? Assume the gas to be ideal $\left(\gamma = \frac{5}{3}\right)$.

[*Hint:* $\Delta Q = \Delta U + \Delta W$. For an ideal gas, $\Delta U = C_v \Delta T$, $\Delta Q = C_p \Delta T$

$$\therefore \quad \frac{\Delta U}{\Delta Q} = \frac{C_v}{C_p} = \frac{1}{\gamma}$$]

13. Show that the rate of dissipation of joule heat per unit volume of a conductor is jE where j is the current density and E is the electric field. Assume that j and E are in the same direction.

14. A ball is suspended by a thread of length l at the point O of an inclined wall making a small angle α with the vertical. The ball is deviated through a small angle β $(\beta > \alpha)$ and released. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum.

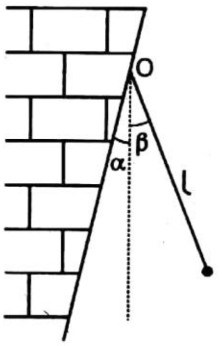

Fig. G

15. A body is thrown up a rough inclined plane with kinetic energy E. Show that the work done against frictional force when the body comes to rest is $\frac{\mu E \cos\alpha}{\sin\alpha + \mu\cos\alpha}$ where μ is the coefficient of friction and α is the inclination of the plane to the horizontal.

[*Hint:* Apply work-energy theorem.]

16. A particle which moves in a straight line and is acted on by a variable force which works at a constant rate, changes its velocity from u to v in passing over a distance x. Prove that the time taken is $\frac{3(v^2 - u^2)x}{2(v^3 - u^3)}$.

[*Hint:* Work done $= P \int dt = Pt = \frac{1}{2} m(v^2 - u^2)$, by work-energy theorem.

Also $P = F\, v = m \frac{dv}{dt} v = \frac{mv^2 dv}{dx}$ or $Pdx = mv^2 dv$

$\therefore \quad \int Pdx = \int_u^v mv^2 dv$]

17. A particle executes simple harmonic motion of period 7.5 s. The velocity at a point P 1 m away from the centre of motion is 2 ms^{-1}. Find the amplitude of the oscillations. If the particle is passing through P and moving away from the centre of oscillation, find the time that elapses before the particle next passes through *P*.

[*Hint:* Take the help of the generating circle.]

18. If P_m is the minimum force required to move a body up a rough inclined plane, show that the least force parallel to the plane required to move the body up the same plane is $P_m \sqrt{1+\mu^2}$.

[*Hint:* First show that $P_m = mg \sin(\alpha + \lambda)$, where λ is angle of friction.]

19. A bead of mass m is threaded on a smooth, circular wire which is held in a vertical plane. The bead is released from one end of a horizontal diameter of the wire. Find the reaction of the wire on the lead when the bead is vertically below the centre of the wire.

20. A heavy particle hangs from a fixed point O by an inextensible string of length l. When the particle is vertically below O it is projected horizontally with speed $\sqrt{\frac{7}{2}gl}$. Calculate the angle through which the string turns before it becomes slack.

21. A particle is projected horizontally with a speed v on the smooth, inner surface of a hollow, right-circular cone placed with its axis vertical and vertex downwards. If the particle describes a horizontal circle, show that the height of the circle above the vertex is v^2/g.

22. A circular cone of semi-vertical angle α is fixed with its axis vertical and its vertex upward. An inextensible string of length l is attached at one end to the vertex and at the other end to a particle of mass m resting on the smooth, external surface of the cone. The particle then revolves with uniform angular velocity ω in a horizontal circle in contact of the cone. Show that $\omega^2 < \frac{g}{l \cos \alpha}$ and find the tension in the string.

23. A particle attached by a light string (which is inelastic and of length l) to a fixed point is describing a horizontal circle of radius nl with uniform speed. The particle is now suddenly stopped and then let go. If in the subsequent motion, its speed when the string becomes vertical is equal to half its speed in the original motion, prove that $7n = 4\sqrt{3}$.

24. At the moment $t = 0$, a force $F = at$ is applied to a small body of mass m resting on a smooth plane at a constant angle α with the horizontal. Find (a) the velocity of the mass at the moment of its breaking off the plane, (b) the distance traversed by the mass up to this moment.

25. Prove that the 'shortest time from rest to rest' in which a chain which can just bear a steady load of P kgf can lift a weight W kg through a vertical distance h is $\sqrt{\frac{2hP}{g(P-W)}}$.

[*Hint:* Move it with maximum acceleration upwards and then let the string be completely loose.]

26. Three vertical walls are of height h, H, h and are equal distances a apart. A ball is thrown in a vertical plane perpendicular to the walls so that it just clears all three

tops. Show that the ball will strike the ground at a distance $a\sqrt{\dfrac{H}{H-h}}$ from the middle wall.

[*Hint:* $H = \dfrac{v^2 \sin^2\theta}{2g}$ or $v \sin\theta = \sqrt{2gH}$, $h = v\sin\theta t - \dfrac{1}{2}gt^2$. Eliminate $\sin\theta$ and proceed]

27. A particle is projected from a given point so as to pass over a wall of height h which is such that the top of the wall is at a distance r from the point of projection. Show that the least speed of projection is $\sqrt{g(h+r)}$.

[*Hint:* $h = x\tan\theta - \frac{1}{2}gx^2/v^2\cos^2\theta$, where x is the horizontal distance of the foot of the wall. Hence find v and apply the condition for minimum value of v.]

28. Projectiles are hurled at a horizontal distance R from the edge of a cliff of height h in such a way as to land at a horizontal distance x from the bottom of the ciff. If you wanted x to be as small as possible, how would you adjust θ and v, assuming that v can be varied from zero to some maximum v_m and θ can be varied continuously? Only one collision with the ground is allowed.

29. A particle rests on the top of a hemisphere of radius R. Find the smallest velocity that must be imparted to the particle if it is to leave the hemisphere without sliding down it.

30. A flat car of mass m_0 starts moving to the right due to the action of a constant horizontal force F. Sand spills on to the car from a stationary hopper placed a little above the car at a constant rate μ kg/s. Find the velocity and acceleration of the car t seconds after the process of loading starts. Assume that loading and pulling the car by the force starts simultaneously. There is no friction.

31. A right-angled wedge of mass M is placed on a smooth incline with their bases parallel. A small mass m is placed on the base of the wedge and the system is released. Calculate the acceleration of M and m. How long will m be on the base of M if it is initially at the extreme left end and the hypotenuse of the wedge is l?

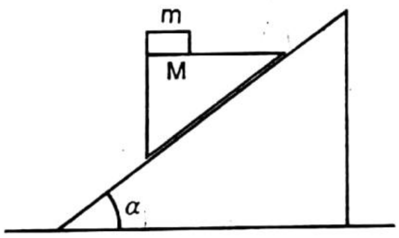

Fig. H

32. A convexo-concave lens of crown glass has radii of curvature equal to 1 m and 12 cm. The lens is placed horizontally on a plane mirror and water is introduced between the plane mirror and the lower surface of the lens and the concave surface is filled with water. What is the power of this system? μ of glass is 1.5 and of water is 4/3.

[*Hint:* Since light traverses twice, power is doubled.]

33. A chain of length l is placed on a smooth spherical surface of radius R with one of its ends fixed at the top of the sphere. What will be the tangential acceleration of each element of the chain when its upper end is released? It is assumed that the length of the chain $l < \pi R/2$.

[*Hint:* Consider an element of angular width $d\theta$ at an angular distance θ from the fixed end. The tangential component of the gravitational pull on the element is $(Rd\theta m)\, g \sin\theta$. The acceleration of the chain is the average of accelerations of the elements $a_t = \dfrac{\int_0^\alpha g \sin\theta\, d\theta}{\alpha}$

Here the radial acceleration $= 0$ as the body is just released. Hence tangential acceleration (a_t) is the total acceleration.]

34. A particle of mass m moves in a circle of radius R and the distance s described by it varies with time as $s = at^2$ where a is a constant. Calculate the force acting on the particle.

[*Hint:* $F = ma$ and $a = \sqrt{a_r^2 + a_t^2}$.]

35. Water from the main pipe of municipal water supply branches off as shown in the figure I. If the diameter of the pipe is 1 m and water current in the main pipe is of speed $10\ \text{m s}^{-1}$ and the current is evenly diverted, find the force exerted by the bend on the stream.

[*Hint:* Thrust = rate of change of momentum.]

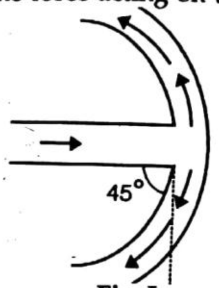

Fig. I

36. A string with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2 m from the wall, has a point mass $M = 2$ kg attached to it at a distance of 1 m from the wall. A mass $m = 0.5$ kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released? **[IIT 1985]**

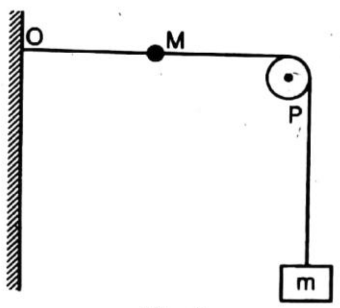

Fig. J

37. A table of 15 kg can move without friction over a level floor. A block of mass 10 kg is placed on the table with string attached as shown in the figure (figure K). The coefficient of friction between the table and block is 6. What is the acceleration of the table if a constant force 8 kgf is applied to the free end of the string? Consider two cases: (i) the force directed horizontally, (ii) the force directed vertically upward.

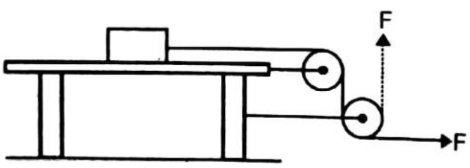

Fig. K

38. A particle is projected with a velocity u from a point on the ground level. Show that it cannot clear a wall of height h at a distance x from the point of projection if $u^2 < g\left(h + \sqrt{x^2 + h^2}\right)$

[*Hint:* Find R_{max} in the vertical plane through the point of projection and the top of the wall and apply the condition for *no* clearance $R_{max} < \sqrt{h^2 + x^2}$.

39. A particle projected from the bottom of an inclined plane along the vertical plane through the line of greatest slope strikes the plane perpendicularly at the rth impact and returns to the point of projection after the nth impact. Show that $e^n - 2e^r + 1 = 0$ where e is the coefficient of restitution between the plane and the particle.

40. The distance between the towers of the main span of Vidhyasagar setu is 1.00 km. The sag of the cable halfway between towers at 25°C is 140 m. Assuming $\bar{\alpha} = 12 \times 10^{-6}/C^{\circ -1}$ compute the change in length of the cable and the change in sag for a temperature change from 0°C to 40°C. There is no bending of towers and the cable is parabolic in shape.

41. The figures give the resistances in ohms. Find the currents in the ab, ad and aA branches when 20 V acts in Aa from A to a and 20 V in Cc from C to c. Revise the calculation with the voltage in Cc reversed.

[*Hint:* Try to exploit the symmetry of the figure to find correct distribution]

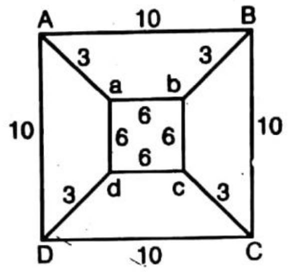

Fig. L

42. Find the current in all the branches marked 1, 2, 3, 4, 5, and 6.

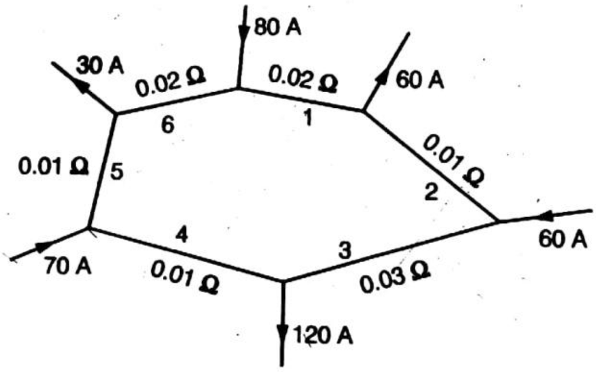

Fig. M

43. The figures in the network refer to capacitance microfarads. If the combined capacitance is 5 μ*F*, find C.

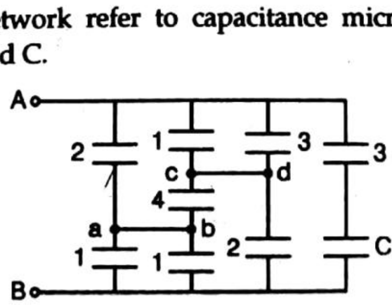

Fig. N

44. A network of nine conductors connects six points A, B, C, a, b, c as shown. The figures denote resistances in ohms. Find resistances between (i) A, a (ii) C, a (iii) c, a (iv) C, A

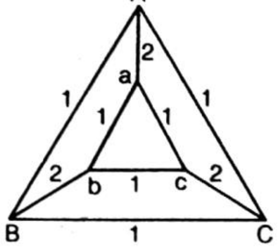

Fig. O

45. Two concentric metal spheres, of radius r and R respectively, are maintained at a potential difference V. What ratio must the radii have for minimum voltage gradient at the surface of the inner sphere for a given value of R.

46. If the break-down dielectric strength of air be 38 kV per cm estimate the greatest line voltage that can be used on a transmission line with solid smooth conductors each 1 cm in diameter spaced 2 m apart.

47. The diagram represents resistor each having the value R. Find the resistance between the junctions A and B.

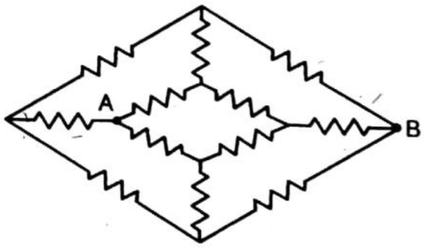

Fig. P

ANSWERS

1. $a/3$ 2. 83°48′ 4. 1.17 m s^{-2} 6. (i) $\bar{v} = \frac{3a}{T}$, (ii) $\bar{v} = \frac{6a}{T}$

7. $S = \frac{R_1 R_2}{R_1 + R_2} = 12\ \Omega$ 8. $F = k\left(m_1 + \frac{m_2}{2}\right)g$ 11. $5a/3$ 12. 9kJ, 6kJ.

14. $2\sqrt{\frac{l}{g}}\left[\frac{\pi}{2} + \sin^{-1}\frac{\alpha}{\beta}\right]$ 17. 2.6 m, 2.8 s 19. 3 mg 20. 120°

22. $m\,(g\cos\alpha + l\omega^2\sin^2\alpha)$ 24. $v = \frac{mg^2\cos\alpha}{2a\sin^2\alpha}$, $s = \frac{m^2g^3\cos\alpha}{6a^2\sin^3\alpha}$ 28. $\theta = \frac{1}{2}\sin^{-1}\frac{Rq}{v_m^2}$

29. $\sqrt{gR}$ 30. $v = \frac{Ft}{m_0 + \mu t}$, $a = \frac{m_0 F}{(m_0 + \mu t)^2}$

31. $\dfrac{(M+m)\,g\sin^2\alpha}{M+m\sin^2\alpha}$, $\dfrac{(M+m)\,g\sin\alpha}{M+m\sin^2\alpha}$, $t=\sqrt{\dfrac{2l}{g}\cdot\dfrac{M+m\sin^2\alpha}{(M+m)\sin^2\alpha}}$

32. $+0.8$ D **33.** $a_t=\dfrac{Rg}{l}\left(1-\cos\dfrac{l}{R}\right)$ **34.** $F=2am\sqrt{1+\dfrac{4s^2}{R^2}}$ **35.** 1.06×10^5 N

36. 3.36 m s^{-1} **37.** $a_1=a_2=\dfrac{8g}{25}$, $\dfrac{2g}{15}$ to the left **40.** 0.5 m; 0.7m

41. 0.71 A, 0.71 A, 1.42 A, 0.91 A, 0.91 A, 1.82 A **42.** 39 A, 21 A, 81 A, 11 A, 41 A

43. 21 μF **44.** 1 Ω, $\dfrac{13}{12}$ Ω, $\dfrac{7}{12}$ Ω, $\dfrac{7}{12}$ Ω **45.** $r:R=1:2$ **46.** 227.6 kV **47.** $\dfrac{5}{6}R$

Revision Examples II

1. The potential energy of the particle $= \int_r^{\infty} -F dr = -\int_r^{\infty} \frac{k}{r^2} dr = -\frac{k}{r}$

Let v' and x be the velocity and distance at the other extreme position. Then by the principle of conservation of energy,

$$\frac{1}{2} m \left(\sqrt{\frac{k}{2ma}} \right)^2 - \frac{k}{a} = \frac{1}{2} m v'^2 - \frac{k}{x}.$$

By the principle of conservation of angular momentum of the system,

$$m \times \sqrt{\frac{k}{2ma}} \times a = m v' \times x$$ (using the fact that moment of momentum is also angular momentum).

Eliminating v', we have $\frac{a^2}{x^2} - \frac{4a}{x} + 3 = 0.$

Solving we have either $x = a$ or $x = a/3$.

2. $B_{coil} = B_0 \tan 20° = 3.6 \times 10^{-5} \times 0.36397$

$= 1.31 \times 10^{-5}$ tesla.

Now restoring field $= B_e \sim B_{mag} = 3.6 \times 10^{-5} \sim 10^{-7} \times \frac{1.2}{0.1^3} \times \frac{1}{2}$

$$\left(\because \; B_1 = \frac{\mu_0}{4\pi} \times \frac{m \sin \theta}{r^3} \right)$$

$= 2.4 \times 10^{-5}$ tesla.

Deflecting field $= B_{coil} + B_{mag} = 1.31 \times 10^{-5} + 10^{-7} \frac{2 \times 1.2 \times \cos 30°}{(0{\cdot}1)^3}$

$$\left(\because \; B_r = \frac{\mu_0}{4\pi} \frac{2m \cos \theta}{r^3} \right)$$

$= 22 \times 10^{-5}$ tesla.

Applying tangent law, we have

$22 \times 10^{-5} = 2.4 \times 10^{-5} \tan \theta$

or $\tan \theta = 9.17$

or $\theta = 83°48'$.

3. Considering conservation of angular momentum of the particle,

we have $(m \sqrt{2gl})\, l \sin 60° = m v' \times l$ (remembering the fact that moment of momentum is also angular momentum) or $\qquad v' = \sqrt{\frac{3}{2}gl}\,.$

Now by dynamics of circular motion,

$$T - mg\cos 60^\circ = \frac{mv'^2}{l}$$

or $\qquad T = 2mg.$

Let F be the impulsive force exerted by the string on the particle. Considering the sudden change in momentum in the horizontal direction we have

$$F\cos 30^\circ\ \Delta t = mv'\cos 60^\circ - 0$$

or $\qquad (F\Delta t) = m\sqrt{\frac{1}{2}gl}\,.$

4. $a^2 = \dfrac{v^2}{r} = \dfrac{\frac{(10)^4}{36}}{250} = 1.1\ \text{ms}^{-1}$ $\qquad$ to the left

$$a' = \frac{1000}{36\times 400} = 0.069\ \text{ms}^{-2} \quad \text{to the right}$$

$\therefore$ $\qquad$ Change in acceleration $= 0.069 - (-1.1) = 1.17\ \text{m s}^{-2}$.

5. Considering motions of the two bodies,we have

$$mg\sin\theta - N = \frac{mv^2}{r}$$

and $\qquad T - mg\cos\theta = ma$ $\quad$ where a is the tangential acceleration.

Also, $\qquad 2mg\cos(\alpha+\theta) - N' = \dfrac{2mv^2}{r}$

and $\qquad 2mg\sin(\alpha+\theta) - T = 2ma.$

When B leaves the surface, $N' = 0$

$\therefore \qquad 2mg\cos(\alpha+\theta) = \dfrac{2mv^2}{r}.$

From conservation of energy,

$$0 + 2mgr\cos\alpha = mgr\sin\theta + 2mgr\cos(\alpha+\theta) + \frac{1}{2}mv^2 + \frac{1}{2}\times 2mv^2$$

or $\qquad 4\cos\alpha = 2\sin\theta + 7\cos(\alpha+\theta).$ $\quad$ Given that $\tan\alpha = 3/4.$

$\therefore \qquad 4\times\frac{4}{5} = 2\sin\theta + 7\left(\frac{4}{5}\cos\theta - \frac{3}{5}\sin\theta\right)$

or $\qquad 28\cos\theta - 11\sin\theta = 16.$

6. Let $x = a \sin(\omega t + \alpha)$.

(i) At $t = 0$, $x = a$, $a = a \sin \alpha$ or $\alpha = \dfrac{\pi}{2}$.

$$x = a \cos \omega t.$$

Let $t = \tau$ when $x = \dfrac{a}{2}$

$\therefore$ $\dfrac{a}{2} = a \cos \omega\tau$ or $\tau = \dfrac{T}{6}$.

$$v_x = -a\omega \sin \omega t$$

$$\overline{v} = \frac{\text{distance}}{\text{time}} = \frac{\frac{a}{2}}{\frac{T}{6}} = \frac{3a}{T}.$$

(ii) At $t = 0$, $x = 0$, $\alpha = 0$

$\therefore$ $x = a \sin \omega t$.

Let $t = \tau$ when $x = \dfrac{a}{2}$, $\dfrac{a}{2} = a \sin \omega\tau$ or $\tau = \dfrac{T}{12}$

$\therefore$ $$\overline{v} = \frac{\text{distance}}{\text{time}} = \frac{\frac{a}{2}}{\frac{T}{12}} = \frac{6a}{T}.$$

7. Parallel combination of R_2 and $S = \dfrac{R_2 S}{R_2 + S}$.

$$\text{Current through } R_1 = \frac{V}{\frac{R_2 S}{R_2 + S} + R_1} = \frac{V(R_2 + S)}{R_2 S + R_1 (R_2 + S)}$$

Current through S

$$= \frac{V(R_2 + S)}{R_1 R_2 + S(R_1 + R_2)} \times \frac{R_2}{(R_2 + S)} = \frac{VR_2}{R_1 R_2 + S(R_1 + R_2)}$$

Power absorbed in S = resistance × current^2

$$= S \times \frac{V^2 R_2^2}{\{R_1 R_2 + S(R_1 + R_2)\}^2}.$$

P is independent of variation of S if P attains a stationary value, that is,

$$\frac{dP}{dS} = 0$$

or $$\frac{d}{dS}\left\{\frac{R_1 R_2}{VS} + VS(R_1 + R_2)\right\} = 0$$

or $$S = \frac{R_1 R_2}{R_1 + R_2} = \frac{20 \times 30}{20 + 30} = 12\,\Omega.$$

8. Let the motion of m_2 start when the spring is slowly stretched by x.

By *work-energy* theorem (μ = force constant),

$$F \cdot x - km_1 gx - \frac{1}{2}\mu x^2 = \text{change in KE} = 0$$

$$F = km_1 g + \frac{1}{2}\mu x.$$

Motion starts when spring force = force of limiting friction on m_2

or $\mu x = km_2 g.$

$\therefore$ $F = km_1 g + \frac{1}{2}km_2 g = k\left(m_1 + \frac{m_2}{2}\right)g.$

9. Let M be the mass of the motor car in kg and F be the frictional force.

Then $F - ag = 0$ or $F = ag$

P (power) = force × velocity = agV. ... (i)

By *work-energy* theorem,

$$\frac{1}{2}M(v^2 - V^2) = \text{work done by the engine}$$

$$+ \text{work done by gravity} + \text{work done by friction}$$

$$= \frac{1}{2}(ag + bg)\,x - bgx$$

$$= \frac{1}{2}(a - b)\,gx. \quad \text{... (ii)}$$

Now again, Power = bgv.

$\therefore$ $agV = bgv$ or $v = \frac{a}{b}\cdot V$

$\therefore$ $\frac{1}{2}M\left(\frac{a^2}{b^2}\cdot V^2 - V^2\right) = \frac{1}{2}(a - b)\,gx$

or $x = \dfrac{MV^2(a + b)}{b^2 g}.$

10. Let x be the instantaneous displacement. Then equation of motion of the body is

$$\frac{mg\,(l - x)}{l} - \mu mg = m\frac{dv}{dt}$$

or $(1 - \mu)g - \dfrac{gx}{l} = \dfrac{dv}{dx}\cdot\dfrac{dx}{dt}$

or $(1 - \mu)g - \dfrac{gx}{l} = \dfrac{vdv}{dx}.$ $\left(\because \dfrac{dx}{dt} = v\right)$

Integrating $\frac{1}{2}v^2 = (1 - \mu)\,gx - \frac{1}{2}\times\frac{g}{l}x^2 + C.$

When $x = 0, \ v = 0.$

Therefore $C = 0.$

When velocity is maximum, $\frac{dv}{dx} = 0.$

$\therefore \qquad (1-\mu) = \frac{x}{l} \qquad \text{or} \qquad x = l\ (1-\mu).$

Putting this value in the above relation we have

$$v_{\max} = (1-\mu)\sqrt{gl}.$$

11. There is no possibility of the particle leaving the surface below the horizontal plane through the centre of the sphere.

So consider a position of the particle above at an angle θ from the horizontal. From the free-body diagram,

$$N + mg \sin\theta = \frac{mv^2}{a};$$

when it leaves the surface, $N = 0.$

$\therefore \qquad mg \sin\theta = \frac{mv^2}{a} \qquad \text{or} \qquad v^2 = ga \sin\theta.$

From conservation of energy,

$$\frac{1}{2} m\,(\sqrt{4ga})^2 + 0 = \frac{1}{2} mv^2 + mg\,(a + a \sin\theta).$$

After putting the value of v, $\sin\theta = \frac{2}{3}.$

$\therefore \qquad \text{Height} = a + a\sin\theta = \frac{5a}{3}.$

12. As shown in the hint,

$$\frac{\Delta U}{\Delta Q} = \frac{C_v}{C_p} = \frac{1}{\gamma} = \frac{3}{5}.$$

$\therefore \qquad \Delta U = \frac{3}{5} \times \Delta Q = \frac{3}{5} \times 15 \text{ kJ} = 9 \text{ kJ}.$

We have $\Delta Q = \Delta U + \Delta W,$

$\therefore \qquad \Delta W = \Delta Q - \Delta U = 15 \text{ kJ} - 9 \text{ kJ} = 6 \text{ kJ}.$

13. Consider an element of conductor of length l and area of cross-section S. Then its volume $= Sl.$

Rate of heating of the element

$$= i^2 R = j^2 S^2 \left(\frac{1}{\sigma} \times \frac{l}{S}\right) = \frac{j^2 Sl}{\sigma}.$$

$\therefore \qquad \text{Rate of heating per unit volume} = \frac{j^2 Sl}{\sigma Sl} = \frac{j^2}{\sigma}.$

But $\quad j = \sigma E$ ... Ohm's law.

$\therefore$ Rate of heating per unit volume $= \dfrac{j^2}{\frac{j}{E}} = jE.$

14. Let θ be the instantaneous angle made by the thread with the vertical. Then by conservation of energy,

$$-mgl\cos\beta = \frac{1}{2}mv^2 - mgl\cos\theta$$

or $\quad 2gl(\cos\theta - \cos\beta) = l^2\omega^2 \qquad (\because \; v = l\omega)$

or $\quad 2gl\,2\sin\dfrac{\theta+\beta}{2}\sin\dfrac{\beta-\theta}{2} = l^2\omega^2.$

Since β is small, $\theta + \beta$ and $\beta - \theta$ are also small.

$\therefore \quad 4gl\dfrac{\beta^2-\theta^2}{4}l^2\omega^2 \quad$ or $\quad \sqrt{\dfrac{g}{l}} \times \sqrt{\beta^2-\theta^2} = \omega = \dfrac{d\theta}{dt}$

or $\quad dt = \sqrt{\dfrac{l}{g}} \times \dfrac{d\theta}{\sqrt{\beta^2-\theta^2}}.$

Integrating, $t = \sqrt{\dfrac{l}{g}}\sin^{-1}\dfrac{\theta}{\beta} + c$ (a constant).

Let $\quad \theta = -\alpha$ at $t = t_1$ and $\theta = \beta$ at $t = t_2$.

Then, $\quad t_1 = \sqrt{\dfrac{l}{g}}\sin^{-1}\left(-\dfrac{\alpha}{\beta}\right) + c$

and $\quad t_2 = \sqrt{\dfrac{l}{g}}\sin^{-1}\dfrac{\beta}{\beta} + c = \sqrt{\dfrac{l}{g}} \times \dfrac{\pi}{2} + c$

Then, T (period of oscillation) $= 2(t_2 - t_1)$

$$= 2\left[\sqrt{\frac{l}{g}} \times \frac{\pi}{2} + c + \sqrt{\frac{l}{g}}\sin^{-1}\frac{\alpha}{\beta} - c\right]$$

$$= 2\sqrt{\frac{l}{g}} \times \left[\frac{\pi}{2} + \sin^{-1}\frac{\alpha}{\beta}\right].$$

15. By *work-energy* thoerem,

$$E = \text{work done by gravity + work done by friction}$$

$$= -mg \cdot x\sin\alpha + W$$

or $\quad W = E + mgx\sin\alpha$

$\therefore \quad$ Work done against friction $= -W = -(E + mgx\sin\alpha)$. ... (i)

If a is the retardation up the plane, then

$$mg \sin \alpha + \mu mg \cos \alpha = ma$$

$\therefore \quad v^2 = 2g (\sin \alpha + \mu \cos \alpha)\, x \quad$ (from $s = ut + \frac{1}{2}at^2$)

or $\quad \frac{2E}{m} = 2g (\sin \alpha + \mu\cos \alpha)\, x. \quad (\because \; E = \frac{1}{2}mv^2)$

On putting the value of x in (i), the result follows.

16. $Pt = \frac{1}{2}m\,(v^2 - u^2)$ by *work-energy* thoerem.

Also $\quad \int P\,dx = \int_u^v mv^2 dv$.

$\therefore \quad Px = \frac{1}{3}m\,(v^3 - u^3).$

Dividing, $\quad t = \frac{3(v^2 - u^2)\,x}{2(v^3 - u^3)}$.

17. $v = \omega\sqrt{a^2 - x^2}$; $\quad \therefore \; 2 = \frac{2\pi}{7.5}\sqrt{a^2 - 1} \quad$ or $\quad a = 2.6$ m.

Draw the generating circle.

From the circle,

$$1 = 2.6 \cos \frac{\omega t}{2} \quad \text{or} \quad \cos \frac{\omega t}{2} = 0.3846$$

or $\quad \frac{\omega t}{2} = 1.1746, \quad$ hence $\quad t = 2.8$ s.

18. From free-body diagram,

$$N + P \cos \theta = mg \cos \alpha,$$

where θ is the angle with the normal to the plane.

$$P \sin \theta = mg \sin \alpha + \mu N.$$

Hence $\quad P = \frac{mg \sin(\alpha + \lambda)}{\sin(\theta + \lambda)}$, after using the relation $\mu = \tan \lambda$.

Obviously P is minimum when $\sin(\theta + \lambda) = 1$

$\therefore \quad P_m = mg \sin(\alpha + \lambda)$.

Again make free-body diagram with P parallel to the plane.

Now $\quad P = \mu N + mg \sin \alpha \;$ and $\; N = mg \cos \alpha,$

Hence $\quad P = \frac{mg \sin(\alpha + \lambda)}{\cos \lambda}$

$$= P_m\sqrt{1 + \mu^2} \qquad (\because \; \mu = \tan \lambda).$$

19. Make a free-body diagram of the bead at the lowest position.

We have $N - mg = \frac{mv^2}{r}$.

From energy consideration,

$$mgr + 0 = \frac{1}{2}mv^2$$

$\therefore$ $N = mg + 2mg = 3mg.$

20. Let θ be the angle made by the string with downward vertical,

$$T - mg\cos\theta = \frac{mv^2}{l}$$

and from energy consideration

$$\frac{1}{2}m\left(\sqrt{\frac{7}{2}gl}\right)^2 = \frac{1}{2}mv^2 + mgl(1 - \cos\theta).$$

When the string is slack, $T = 0$;

$\therefore$ $-mg\cos\theta = \frac{mv^2}{l}$.

Eliminating v, we have $\cos\theta = -\frac{1}{2}$ or $\theta = 120°$.

21. Let θ be the semivertical angle of the cone. Making feee-body diagram and projecting forces along the horizontal and vertical,

$$N\sin\theta = mg \quad \text{and} \quad N\cos\theta = \frac{mv^2}{r} \quad \text{or} \quad \tan\theta = \frac{gr}{v^2}.$$

But $\tan\theta = \frac{r}{h}$; $\therefore$ $h = \frac{v^2}{g}$.

22. Make a free-body diagram of the body and project the forces along the horizontal and vertical.

$$T\sin\alpha - N\cos\alpha = m\omega^2 l\sin\alpha$$

and $T\cos\alpha + N\sin\alpha = mg.$

Eliminating T, we have $N = mg\sin\alpha - m\omega^2 l\sin\alpha\cos\alpha$.

For maintenance of contact N must be greater than zero.

$\therefore$ $mg\sin\alpha > m\omega^2 l\sin\alpha\cos\alpha.$

or $\omega^2 < \frac{g}{l\cos\alpha}$.

Eliminating N, we have $T = mg\cos\alpha + m\omega^2 l\sin^2\alpha$.

23. Let the string make an angle θ with the vertical.

Then $\sin\theta = \frac{nl}{l} = n$

$$T\cos\theta = mg \quad \text{and} \quad T\sin\theta = \frac{mv^2}{nl}$$

$$\therefore \qquad \tan\theta = \frac{v^2}{nlg} \qquad \text{or} \qquad \frac{n}{\sqrt{1-n^2}} = \frac{v^2}{nlg}.$$

From energy consideration, $\frac{1}{2}mu^2 = mgl\,(1-\cos\theta)$.

It is given that $u = \frac{v}{2}$; $\therefore \quad \frac{1}{8}mv^2 = mgl\,(1-\sqrt{1-n^2})$

$$\text{or} \qquad \frac{1}{8}\times\frac{n^2lg}{\sqrt{1-n^2}} = lg\,(1-\sqrt{1-n^2})$$

$$\text{or} \qquad 7n = 4\sqrt{3}.$$

24. From the free-body diagram of the small mass m we have

$$N + F\sin\alpha = mg \qquad \text{and} \qquad m\frac{dv}{dt} = F\sin\alpha.$$

At the time of break-off $N = 0$. Let this happen at the instant $t = t_0$.

$$\text{Then} \qquad at_0\sin\alpha = mg \qquad \text{or} \qquad t_0 = \frac{mg}{a\sin\alpha}.$$

$$\text{Now} \qquad \frac{dv}{dt} = \frac{F\cos\alpha}{m} = \frac{at\cos\alpha}{m} \qquad \text{or} \qquad v = \frac{a\cos\alpha}{m}\int_0^{t_0} t\,dt$$

$$\text{or} \qquad v = \frac{a\cos\alpha}{m}\times\frac{1}{2}t_0^2 = \frac{mg^2\cos\alpha}{2a\sin^2\alpha}.$$

In general, $v = \frac{a\cos\alpha}{m}\times\frac{t^2}{2} + v_0$. At $t = 0$, $v = 0$ and hence $v_0 = 0$

$$\therefore \qquad v = \frac{a\cos\alpha}{2m}t^2 \qquad \text{or} \qquad \frac{dS}{dt} = \frac{a\cos\alpha}{2m}t^2.$$

Integrating, $S = \frac{m^2g^3\cos\alpha}{6a^2\sin^3\alpha}$.

25. Let a be the maximum upward acceleration that can be produced.

$$\text{Then} \qquad Pg - Wg = Wa \qquad \text{or} \qquad a = \frac{(P-W)g}{W}.$$

$$\therefore \qquad v = 0 + (P-W)\frac{gt_1}{W}.$$

Now the string is let loose completely. The upward velocity will be retarded by 'g'.

$$\therefore \qquad v = gt_2. \qquad \text{Thus,} \qquad gt_2 = \frac{(P-W)gt_1}{W}$$

$$\text{or} \qquad \frac{t_2}{t_1} = \frac{(P-W)}{W}.$$

Hence $\quad t_1 = \frac{W}{P}t \quad$ where t is the total time $= t_1 + t_2$

and $\quad t_2 = \frac{P-W}{P} t.$

Now $\quad h = h_1 + h_2 = \frac{1}{2}(P-W)\frac{gt_1^{\,2}}{W} + (P-W)\frac{gt_1 t_2}{W}$

$$= \frac{1}{2} g \frac{P-W}{W} t_1 (t_1 + t_2).$$

Hence $\quad t = \sqrt{\frac{2Ph}{g\,(P-W)}}.$

26. We have $\quad h = \sqrt{2gH}\; t - \frac{1}{2} g t^2$ (see hints)

$\therefore \quad t_1, t_2 = \frac{\sqrt{2gH} \pm \sqrt{2g(H-h)}}{g}$

$\therefore \quad R - a = v \cos\theta \times t_1; \quad R + a = v \cos\theta \times t_2$

$$\frac{R-a}{R+a} = \frac{t_1}{t_2} = \frac{\sqrt{2gH} - \sqrt{2g(H-h)}}{\sqrt{2gH} + \sqrt{2g(H-h)}}.$$

Hence $\quad R = a\sqrt{\frac{H}{H-h}}.$

27. We have with usual notations,

$$x = v \cos\theta \cdot t \quad \text{and} \quad h = v \sin\theta \cdot t - \frac{1}{2} g t^2.$$

Eliminating t, $\quad h = x \tan\theta - \frac{gx^2}{2v^2 \cos^2\theta}$

or $\quad v^2 = \frac{gx^2}{x \sin 2\theta - h\,(1 + \cos 2\theta)}.$

v is minimum when $x \sin 2\theta - h\,(1 + \cos 2\theta)$ is maximum, i.e.,

$$\frac{d}{d\theta}\,[x \sin 2\theta - h\,(1 + \cos 2\theta)] = 0$$

$$2x \cos 2\theta + 2h \sin 2\theta = 0$$

or $\quad \tan 2\theta = -\frac{x}{h}, \quad$ or $\quad \tan(\pi - 2\theta) = \frac{x}{h}.$

Hence $\quad \sin 2\theta = \sin(\pi - 2\theta) = \frac{x}{\sqrt{x^2 + h^2}}$

$$\cos(\pi - 2\theta) = \frac{h}{\sqrt{x^2 + h^2}}.$$

$$\therefore \qquad v^2_{\min} = \frac{gx^2}{x\dfrac{x}{\sqrt{x^2+h^2}} - h + h.\dfrac{h}{\sqrt{x^2+h^3}}} = \frac{gx^2\sqrt{x^2+h^2}}{x^2+h^2-h\sqrt{x^2+h^2}}.$$

$$\because \qquad x^2 = r^2 - h^2,$$

$$\therefore \qquad v^2_{\min} = g(r+h)$$

or $$v_{\min} = \sqrt{g(r+h)}.$$

28. The landing distance x will be minimum when the trajectory will have its vertex at the highest possible point. For this, velocity and anlge of projection should be set to consistent maximum value. Since velocity can be varied up to certain maximum value v_m, the velocity is to be set at this highest value. Let θ be the angle of projection. Then

$$R/2 = v_m \cos\theta t \qquad \text{and} \qquad v_m \sin\theta = gt$$

$$\therefore \qquad R/2 = v_m \cos\theta \times \frac{v_m \sin\theta}{g} = \frac{v_m^2 \sin 2\theta}{2g}$$

or $$\theta = \frac{1}{2}\sin^{-1}\frac{Rg}{v_m^2}.$$

29. The velocity of projection along the horizontal should be sufficiently great to generate centripetal acceleration greater or equal to g.

$$\text{Centripetal acceleration} = \frac{v^2}{R}.$$

$$\therefore \qquad \text{Condition for no sliding is } \frac{v^2}{R} \geq g$$

or $$v \geq \sqrt{gR}$$

$$\therefore \qquad v_{\min} = \sqrt{gR}.$$

30. Change in momentum = impulse of the force = Ft.

$$\text{Change in momentum} = (m_0 + \mu t)v - 0$$

$$\therefore \qquad Ft = (m_0 + \mu t)v$$

or $$v = \frac{Ft}{m_0 + \mu t}.$$

We have $$a = \frac{dv}{dt} = \frac{m_0 F}{(m_0 + \mu t)^2}.$$

31. Since the surface is smooth m will not have any horizontal acceleration. It will have only vertical downward acceleration

i.e., $a_y = A_y$

where a = acceleration of m

and A = acceleration of M

$a_x = 0, \quad A_x = ?$

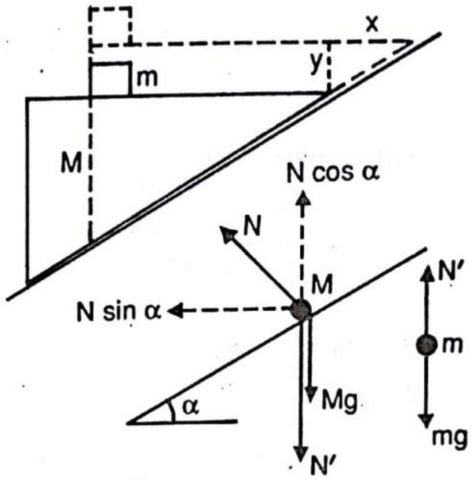

Fig. RE II.1

Constraint equation is

$$X \tan\alpha = y \quad \text{and}$$

$$y = Y$$

$$\ddot{X} \tan\alpha = \ddot{y}$$

and $\ddot{y} = \ddot{Y}$

or $A_x \tan\alpha = a_y = A_y.$

From free body diagrams,

$$mg - N' = m a_y$$

$$N' + Mg - N\cos\alpha = MA_y = Ma_y$$

and $N \sin\alpha = MA_x = \dfrac{Ma_y}{\tan\alpha}.$

Eliminating N and N',

$$a_y = \frac{(M+m)\, g \sin^2\alpha}{M + m\sin^2\alpha} = A_y$$

and $A_x = \dfrac{(M+m)\, g \sin\alpha\cos\alpha}{M + m\sin^2\alpha}.$

Then acceleration of $m = a_y = \dfrac{(M+m)\, g\sin^2\alpha}{M + m\sin^2\alpha}$

and acceleration of $M = \sqrt{A_x^2 + A_y^2} = \dfrac{(M+m)\, g\sin\alpha}{M + m\sin^2\alpha}.$

Rel. acceleration of m to the right $= 0 - (-A_y) = A_y.$

Rel. displacement of $m = 0 - (-l\cos\alpha) = l\cos\alpha.$

$\therefore \quad l\cos\alpha = \dfrac{1}{2}A_y t^2 \quad$ or $\quad t^2 = \dfrac{2l\cos\alpha}{a_y}$

or $\quad t = \sqrt{\dfrac{2l}{g} \times \dfrac{M + m\sin^2\alpha}{(M+m)\sin^2\alpha}}.$

34. We have, $s = at^2$ (given)

$$v = \frac{ds}{dt} = 2at.$$

$$a_t \text{ (tangential acc.)} = \frac{dv}{dt} = 2a$$

$$a_r = \frac{v^2}{R} = \frac{4a^2t^2}{R} = \frac{4a^2}{R} \times \frac{s}{a} = \frac{4as}{R}.$$

$$\text{Total acceleration} = \sqrt{a_t^2 + a_r^2} = \sqrt{4a^2 + \frac{16a^2s^2}{R^2}}$$

$$\therefore \qquad F = m\sqrt{4a^2 + \frac{16a^2s^2}{R^2}} = 2am\sqrt{1 + \frac{4s^2}{R^2}}.$$

35. Let v' be the velocity in each branch. Then by continuity equation we have

$$Av = Av' + Av' \qquad \text{or} \qquad v' = v/2.$$

Taking rightward direction as x-axis,

$\vec{p_i}$ (initial momentum of water flowing through the main pipe in one second)

$$= (Av\rho)\, v\, \vec{i} = Av^2\rho\, \vec{i}$$

$\vec{p_f}$ (final momentum of water flowing in the branches in one second)

$$= 2\left(A\frac{v}{2}\rho\right) v/2 \; (-\vec{i}) \cos 45^\circ$$ (y-components sum up to zero)

$$= \frac{1}{2\sqrt{2}} Av^2\rho\, (-\vec{i}).$$

$\therefore$ Rate of change of momentum $= \vec{p_f} - \vec{p_i}$ (time = 1 second)

= Force on the water

or $$\text{Force} = \left(-\frac{1}{2\sqrt{2}} Av^2\rho - Av^2\rho\right)\vec{i}.$$

Magnitude of the force

$$= \left(1 + \frac{1}{2\sqrt{2}}\right) Av^2\rho$$

$$= \left(1 + \frac{1}{2\sqrt{2}}\right)\left(\pi \times \frac{1}{4}\right) \times 10^2 \times 1000$$

$$= 1.06 \times 10^5 \text{ N}.$$

36. Let l be the hanging length of the string in the beginning.

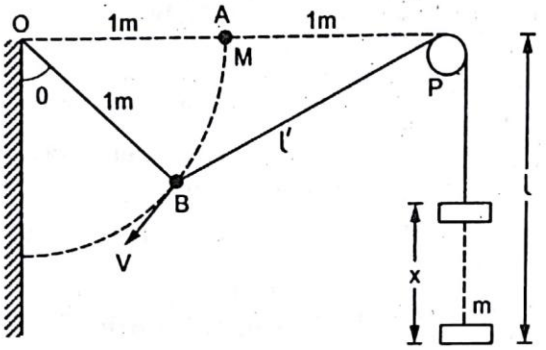

Fig. RE II.2

Let the mass m go up by x. Then constraint of motion requires (length of string does not change)

$$1 + l' + l - x = 2 + l$$

(length of string after) (length of string before)

or $l' = 1 + x$ or $\sqrt{2^2 + 1^2 - 2 \times 2 \times 1 \cos(90° - \theta)} = 1 + x$

or $\sqrt{5 - 4\sin\theta} = 1 + x$ or $5 - 4\sin\theta = (1 + x)^2$.

Considering conservation of energy,

$$-mgl = -mg(l - x) + \frac{1}{2}m\dot{x}^2 + \frac{1}{2}MV^2 - Mg \times 1 \times \sin(90° - \theta)$$

$\Rightarrow$ $$0 = mgx + \frac{1}{2}m\dot{x}^2 + \frac{1}{2}MV^2 - Mg\cos\theta.$$

$\because$ $$5 - 4\sin\theta = (1 + x)^2$$

$\therefore$ $$(4\cos\theta)\,\dot{\theta} = 2(1 + x)\dot{x}$$

or $$(4\cos\theta)\,V = 2(1 + x)\dot{x} \qquad \left(\dot{\theta} = \omega = \frac{V}{1} = V\right)$$

or $$\dot{x} = \frac{(2\cos\theta)V}{1 + x}$$

$\therefore$ $$0 = mgx + \frac{1}{2}m\frac{(4\cos^2\theta)V^2}{(1 + x)^2} + \frac{1}{2}MV^2 - Mg\cos\theta.$$

When $\theta = 0°$, $1 + x = \sqrt{5}$.

$\therefore$ $$0 = mg(\sqrt{5} - 1) + \frac{1}{2}m \times \frac{4V^2}{5} + \frac{1}{2}MV^2 - Mg.$$

Since $\quad m = \frac{1}{2}\,\text{kg} \quad$ and $\quad M = 2\,\text{kg}$

$$\therefore \quad 0 = \frac{1}{2}g\,(\sqrt{5}-1) + \frac{1}{2}\times\frac{1}{2}\times\frac{4V^2}{5} + \frac{1}{2}\times 2V^2 - 2g$$

$$V^2 = \frac{5\,(5-\sqrt{5})}{12}\,g \qquad \text{or} \qquad V = 3.36\ \text{m s}^{-1}.$$

37. (i) Considering free-body diagram of the block (body 1) we have $T - f = 10\,a_1$ where f = frictional force between block and the table and T = tension.

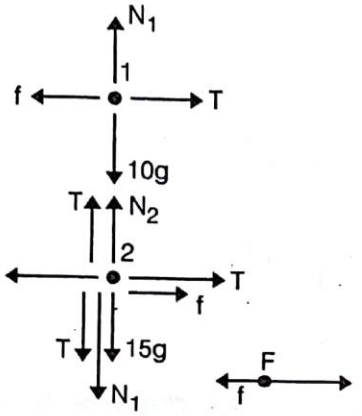

Fig. RE II.3

Considering free-body diagram of the table we have

$$f + T - T = 15a_2.$$

Considering free-body diagram of an element of the string we have

$$F - T = 0 \qquad \text{or} \qquad F = T.$$

If F is very small, f will not reach its limiting value and so the block will not slide over the table. Under such condition $a_1 = a_2 = a$.

The frictional force corresponding to a small applied force is given by

$$\frac{E-f}{f} = \frac{10a}{15a} \qquad \text{or} \qquad f = \frac{3F}{5}.$$

The limiting value of f is $f_{\text{lim}} = \mu mg = 0.6 \times 10g = 6g$.

To produce this amount of frictional force the force F needed to be applied is

$$6g = \frac{3F}{5} \qquad \text{or} \qquad F = 10g.$$

Thus unless the applied force F is $10g$, the frictional force between the table and the block will not reach the limiting value and consequently the block will not slide over the table. Here the force applied is only 8 $kg\,f$ and so the block will not slide over the table and the two will have a common acceleration.

Considering block + table as a system we have

$$8g = 25a \qquad \text{or} \qquad a = 8g/25.$$

(ii) Here $\;T - f = 10a_1\;$ and $\;f - T = 15a_2\;$ and $\;T = F.\;\;\therefore\;\; 10a_1 = -15a_2.$

Thus the two accelerations are oppositely directed and so the block will definitley slide and f reaches f_{lim} immediately.

$\therefore \qquad 8g - 6g = 10a_1 \quad \text{or} \quad a_1 = g/5$

and $\qquad a_2 = -\frac{10}{15} \cdot \frac{g}{5} = -2g/15.$

Thus acceleration of the table is $2g/15$ to the left.

38. Let the inclination of the plane through the point of projection and the top of wall be β and α is the angle of projection with the plane. Let us consider the line of greatest slope as x-axis and normal to the plane as y-axis.

Then $\qquad u_x = u \cos \alpha, \quad u_y = u \sin \alpha, \quad a_x = -g \sin \beta, \quad a_y = -g \cos \beta.$

Now $\qquad 0 = u \sin \alpha t - \frac{1}{2} g \cos \beta t^2 \Rightarrow \quad t = \frac{2u \sin \alpha}{g \cos \beta}$

$$R \text{ (range)} = u \cos \alpha t - \frac{1}{2} g \sin \beta t^2$$

$$= u \cos \alpha \frac{2u \sin \alpha}{g \cos \beta} - \frac{1}{2} g \sin \beta \frac{4u^2 \sin^2 \alpha}{g^2 \cos^2 \beta}$$

or $\qquad R = \frac{2u^2 \cos \alpha \sin \alpha}{g \cos \beta} - \frac{2u^2 \sin^2 \alpha \sin \beta}{g \cos^2 \beta}$

$$= \frac{u^2 \sin 2\alpha}{g \cos \beta} - \frac{u^2 \sin \beta (1 - \cos 2\alpha)}{g \cos^2 \beta}$$

$$\frac{dR}{d\alpha} = \frac{u^2}{g \cos \beta} \times 2\cos 2\alpha - \frac{u^2 \sin \beta}{g \cos^2 \beta} \times 2 \sin \alpha$$

When $\qquad R = R_{max}, \quad \frac{dR}{d\alpha} = 0 \quad \Rightarrow \quad \cot 2\alpha = \tan \beta.$

$\therefore \qquad R_{max} = \frac{u^2}{g \cos \beta} \times \frac{1}{\sqrt{1 + \tan^2 \beta}} - \frac{u^2 \sin \beta}{g \cos^2 \beta} \left(1 - \frac{\tan \beta}{\sqrt{1 + \tan^2 \beta}}\right)$

$$= \frac{u^2}{g (1 + \sin \beta)}.$$

Now $\qquad \sin \beta = \frac{h}{\sqrt{h^2 + x^2}}.$

The particle cannot clear the wall if

$$R_{max} < \sqrt{h^2 + x^2}$$

or $\qquad \frac{u^2}{g\left(1 + \frac{h}{\sqrt{h^2 + x^2}}\right)} < \sqrt{h^2 + x^2}$

or $\qquad u^2 < g\left(h + \sqrt{h^2 + x^2}\right).$

39. Considering the line of greatest slope up the plane as x-axis and the line perpendicular to it as y-axis, we have

$$u_x = u\cos\alpha, \quad u_y = u\sin\alpha, \quad a_x = -g\sin\beta, \quad a_y = -g\cos\beta$$

where α = angle of projection with the plane,

β = angle of inclination with the horizontal.

Let t_1 be the time of the first flight.

Then $$0 = u\sin\alpha t_1 - \frac{1}{2}g\cos\beta t_1^2$$

$$\Rightarrow t_1 = \frac{2u\sin\alpha}{g\cos\beta},$$

Let u_y be the vertical velocity after t_1.

Then $$u_{1y} = u\sin\alpha - g\cos\beta t_1$$

$$\Rightarrow \quad u_{1y} = u\sin\alpha - g\cos\beta \times \frac{2u\sin\alpha}{g\cos\beta} = -u\sin\alpha.$$

Immediately after 1st impact velocity along y-axis is $eu\sin\alpha$ (upward).

$\therefore$ $$t_2 \text{ (time of second flight)} = \frac{2eu\sin\alpha}{g\cos\beta}.$$

$$t_3 \text{ (time of the third flight)} = \frac{2e^2u\sin\alpha}{g\cos\beta}$$

and $$t_r \text{ (time of the } r\text{th flight)} = \frac{2e^{r-1}u\sin\alpha}{g\cos\beta}.$$

$\therefore$ T, total time up to rth impact $= \dfrac{2u\sin\alpha}{g\cos\beta}\left(1 + e + e^2 + \ldots + e^{r-1}\right)$

$$\Rightarrow T = \frac{2u\sin\alpha}{g\cos\beta} \times \frac{1-e^r}{1-e}.$$

Since the velocity is perpendicular to the plane at rth impact,

$$0 = u\cos\alpha - g\sin\beta t \quad \Rightarrow \quad T = \frac{u\cos\alpha}{g\sin\beta}$$

$\therefore$ $$\frac{u\cos\alpha}{g\sin\beta} = \frac{2u\sin\alpha}{g\cos\beta} \times \frac{1-e^r}{1-e} \quad \Rightarrow \quad \cot\alpha = 2\tan\beta \times \frac{1-e^r}{1-e}.$$

$$T' \text{ (time for } n \text{ impacts)} = \frac{2u\sin\alpha}{g\cos\beta} \cdot \frac{1-e^n}{1-e}.$$

Since after T' the particle returns to the pt. of projection,

$$0 = u \cos \alpha T' - \frac{1}{2} g \sin \beta T'^2 \quad \Rightarrow \quad T' = \frac{2u \cos \alpha}{g \sin \beta}$$

$$\therefore \qquad \frac{2u \cos \alpha}{g \sin \beta} = \frac{2u \sin \alpha}{g \cos \beta} \times \frac{1-e^n}{1-e} \quad \Rightarrow \quad \cot \alpha = \tan \beta \times \frac{1-e^n}{1-e}.$$

$$\therefore \qquad 2 \times \frac{1-e^r}{1-e} = \frac{1-e^n}{1-e} \quad \Rightarrow \quad e^n - 2e^r + 1 = 0.$$

40. Let a be the fixed span and s is the sag.

Equation of a parabola is

$$y = kx^2$$

when $\quad x = \frac{a}{2}, y = s$

$$\therefore \qquad s = k \times \frac{a^2}{4}$$

$$\Rightarrow \qquad k = \frac{4s}{a^2}.$$

$$\therefore \qquad y = \frac{4s}{a^2} \times x^2$$

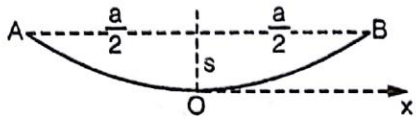

Fig. RE II.4

$$\Rightarrow \qquad dy = \frac{4s}{a^2} \times 2x\, dx = \frac{8s}{a^2} \times x\, dx$$

$$dl^2 = dx^2 + dy^2 = dx^2 + \frac{64s^2}{a^4} x^2 (dx)^2$$

$$dl = \sqrt{1 + \frac{64s^2}{a^4} x^2}$$

$$l = 2 \int_0^{a/2} \left(1 + \frac{64s^2}{a^4} x^2\right)^{\frac{1}{2}} dx$$

$$= 2 \int_0^{a/2} \left(1 + \frac{1}{2} \cdot \frac{64s^2}{a^4} x^2\right) dx \quad (\because \text{ the second term is small})$$

$$= 2 \int_0^{a/2} \left(1 + \frac{32s^2}{a^4} x^2\right) dx$$

$$\Rightarrow \qquad l = 2\left[\frac{a}{2} + \frac{32s^2}{a^4} \cdot \frac{a^3}{24}\right] = a + \frac{8}{3a} s^2$$

$$\Rightarrow \qquad (l - a) \frac{3a}{8} = s^2.$$

$$25°\text{C},\ [l_0(1 + 12 \times 10^{-6} \times 25) - 1000] \times \frac{3 \times 1000}{8} = 140^2$$

$$\Rightarrow \quad l_0(1 + 3 \times 10^{-4}) - 1000 = 52.3$$

$$\Rightarrow \quad l_0 = \frac{1052.3}{1.0003} = 1052.0 \text{ m.}$$

$$\text{At } 0°\text{C},\ [1052(1 + 12 \times 10^{-6} \times 0) - 1000] \times \frac{3 \times 1000}{8} = s^2$$

$$s^2 = 19500 \quad \Rightarrow \quad s = 139.6 \text{ m}$$

$$\text{At } 40°\text{C},\ [1052(1 + 12 \times 10^{-6} \times 40) - 1000] \times \frac{3 \times 1000}{8} = s^2.$$

$$\Rightarrow \quad s = \sqrt{19689.4} = 140.3 \text{ m.}$$

$\therefore$ Change in length

$$= 1052(1 + 12 \times 10^{-6} \times 40) - 1052 = 0.50 \text{ m.}$$

Change in sag = 140.3 – 139.6 = 0.7 m.

41. Since *abcd* in a loop in which there is no emf the value of current must be same in all the branches.

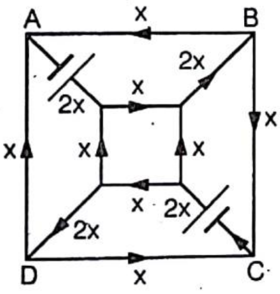

Fig. RE II.5

By loop rule (applied to any loop except the inner loop)

$$10x + 3 \times 2x + 6x + 3 \times 2x = 20$$

$$\Rightarrow \quad 28x = 20 \quad \Rightarrow \quad x = \frac{5}{7} = 0.71 \text{ A}$$

$\therefore$ Current in ab = 0.71 A

Current in ad = 0.71 A

Current in a A = 2 × 0.71 = 1.42 A

$$2x \times 3 + 6x + 10x = 20$$

$$\Rightarrow \quad 22x = 20$$

$$x = \frac{10}{11} = 0.91 \text{ A}$$

$\therefore$ Current in ab = 0.91 A

Current in ad = 0.91 A

Current in aA = 2 × 0.91 = 1.82 A

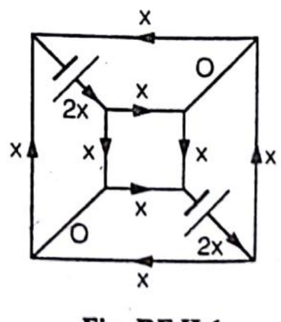

Fig. RE II.6

42.

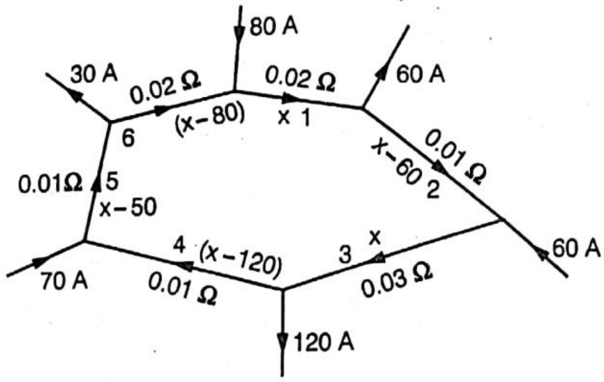

Fig. RE II.7

By loop rule $x \times 0.02 + 0.01\,(x-60) + 0.33\,x$

$$+ (x-120) \times 0.01 + (x-50) \times 0.01 + 0.02\,(x-80) = 0$$

$\Rightarrow \quad 0.1\,x = 3.9 \quad \Rightarrow \quad x = 39$ A

Current in 1 = 39 A (clockwise)
Current in 2 = 39 − 60 = − 21A = 21 A (anticlockwise)
Current in 3 = 39 A (clockwise)
Current in 4 = 39 − 120 = −81 A = 81 A (anticlockwise)
Current in 5 = 39 − 50 = −11 A = 11 A (anticlockwise)
Current in 6 = 39 − 80 = −41 = 41 A (anticlockwise)

43.

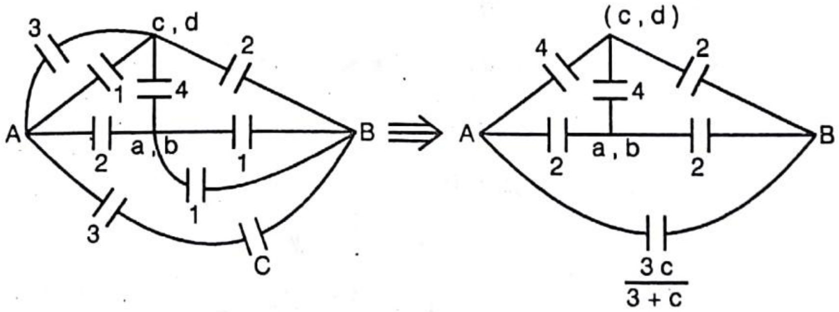

Fig. RE II.8

Loop 1, $\dfrac{q_1 - q'_1}{4} + \dfrac{q_1}{4} - \dfrac{q_2}{2} = 0$

$\Rightarrow \quad q_1 - q_1' + q_1 - 2q_2 = 0$

$2q_1 - q_1' - 2q_2 = 0$

$\Rightarrow \quad q'_1 = 2\,(q_1 - q_2)$ ... (i)

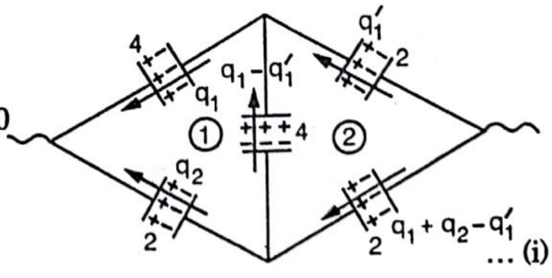

Fig. RE II.9

Loop 2, $\dfrac{q_1 + q_2 - q_1'}{2} + \dfrac{q_1 - q_1'}{4} - \dfrac{q_1'}{2} = 0$

$\Rightarrow \quad 2q_1 + 2q_2 - 2q_1' + q_1 - q_1' - 2q_1' = 0$

$\Rightarrow \quad 3q_1 + 2q_2 - 5q_1' = 0$

$\Rightarrow \quad 3q_1 + 2q_2 - 5 \times 2(q_1 - q_2) = 0$

$\Rightarrow \quad 3q_1 + 2q_2 - 10q_1 + 10q_2 = 0$

$\Rightarrow \quad 12q_2 = 7q_1$

$\therefore \quad 3q_1 + 2 \times \dfrac{7q_1}{12} = 5q_1'$

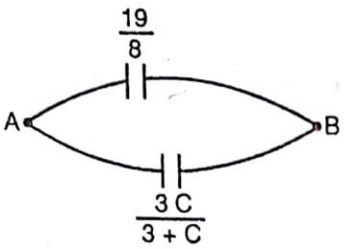

Fig. RE II.10

$$\frac{25}{6}q_1 = 5q_1' \quad \Rightarrow \quad q_1' = \frac{5}{6}q_1$$

$$C_{eq} = \frac{q_1 + q_2}{\dfrac{q_1}{4} + \dfrac{q_1'}{2}} = \frac{q_1 + \dfrac{7q}{12}1}{\dfrac{q_1}{4} + \dfrac{1}{2} \times \dfrac{5}{6}\, q_1} = \frac{1 + \dfrac{7}{12}}{\dfrac{1}{4} + \dfrac{5}{12}} = \frac{19}{8}$$

The equivalnet capacitance between A and B is $5\,\mu F$ (given)

$\therefore \quad \dfrac{19}{8} + \dfrac{3C}{3+C} = 5$

$\Rightarrow \quad \dfrac{3C}{3+C} = 5 - \dfrac{19}{8} = \dfrac{21}{8}$

$\Rightarrow \quad \dfrac{C}{3+C} = \dfrac{7}{8} \quad \Rightarrow 21 + 7C = 8C \quad \Rightarrow C = 21\ \mu F.$

44. By loop rule

$$1x + 2x + 1x - 2y = 0$$

$\Rightarrow \quad 2x = y$

$$R_{eq} = \frac{V_A - V_a}{2x + y} = \frac{2 \times y}{2x + y} = \frac{2 \times 2x}{2x + 2x} = 1\Omega$$

From $BACB$

$p \times 1 - x \times 1 + z + 1 = 0$

$p = x - z$

From $baca$, $(z - p + y - q) \times 1 - 1 \times q + 1 \times (y - q) = 0$

$\Rightarrow \quad z - p + y - q - q + y - q = 0$

$\Rightarrow \quad z + 2y - p - 3q = 0$

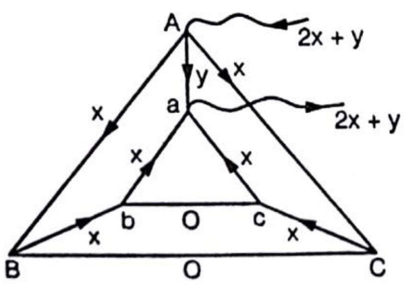

Fig. RE II.11

$\Rightarrow \quad z + 2y - x + z - 3q = 0$

$$\Rightarrow \qquad 3q = 2z + 2y - x$$

From loop 1,

$$x \times 1 + 2(x+p) - q \times 1 - 2y = 0$$

$$\Rightarrow \quad x + 2x + 2p - q - 2y = 0$$

$$\Rightarrow \quad 3x + 2(x-z) - q - 2y = 0$$

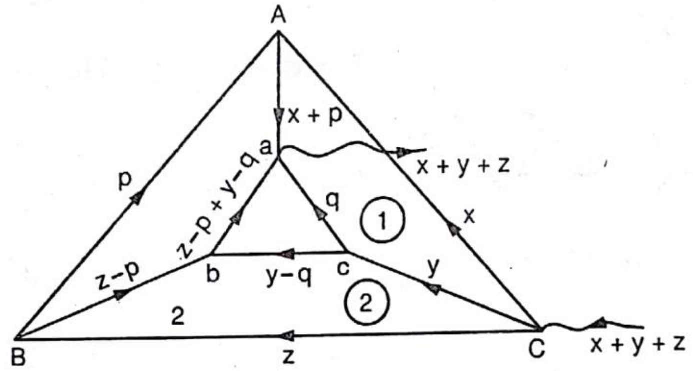

Fig. RE II.12

$$\Rightarrow \quad 5x - 2z - q - 2y = 0$$

$$\Rightarrow \quad 15x - 6z - 3q - 6y = 0$$

$$\Rightarrow \quad 15x - 6z - (2z + 2y - x) - 6y = 0$$

$$\Rightarrow \quad 15x - 6z - 2z - 2y + x - 6y = 0$$

$$\Rightarrow \quad 16x - 8z - 8y = 0$$

$$\Rightarrow \quad 2x - y - z = 0 \qquad \ldots \text{(i)}$$

From loop 2,

$$1 \times z + 2(z-p) - 1(y-q) - 2y = 0$$

$$\Rightarrow \quad z + 2z - 2p - y + q - 2y = 0$$

$$\Rightarrow \quad 3z - 2p - 3y + q = 0$$

$$\Rightarrow \quad 3z - 2x + 2z - 3y + q = 0$$

$$\Rightarrow \quad 5z - 2x - 3y + q = 0$$

$$\Rightarrow \quad 15z - 6x - 9y + 2z + 2y - x = 0$$

$$\Rightarrow \quad 17z - 7x - 7y = 0$$

$$2x - y - z = 0$$

$$7x + 7y - 17z = 0$$

$$\frac{x}{17-(-7)} = \frac{y}{(-7)-(-34)} = \frac{z}{14-(-7)}$$

$$\frac{x}{24} = \frac{y}{27} = \frac{z}{21} \quad \Rightarrow \quad \frac{x}{8} = \frac{y}{9} = \frac{z}{7}$$

$$\Rightarrow \quad x = 8k, \quad y = 9k \quad 2 = 7k$$

$$R_{eq} = \frac{V_c - V_a}{x+y+z} = \frac{x \times 1 + 2(x+p)}{x+y+z} = \frac{8k + 2 \times 8k + 2(8k - 7k)}{8k + 9k + 7k}$$

$$= \frac{26}{24} = \frac{13}{12}\ \Omega$$

abca: $1y + 1y - 1x = 0, \quad 2y = x.$

BACA $1 \times (z - z') - 1 \times z' + 1\,(z - z') = 0$

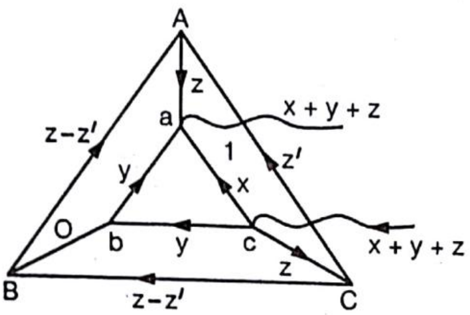

Fig. RE II.13

$$2z = 3z' \quad \Rightarrow \quad z' = \frac{2z}{3}$$

Loop 1, $z' \times 1 + 2z - 1x + 2z = 0$

$$\frac{2z}{3} + 4z = x \quad \Rightarrow \quad 14z = 3x$$

$$R_{eq} = \frac{V_c - V_a}{x + y + z} = \frac{1 \times x}{x + y + z}$$

$$= \frac{x}{x + \frac{x}{2} + \frac{3x}{14}} = \frac{14}{24} = \frac{z}{12}\ \Omega$$

BACA $z \times 1 - 1\,x + 1\,z = 0$

$\Rightarrow$ $x = 2z$

Loop 1, $\frac{y}{3} \times 1 - 1z + 2y = 0$

$\Rightarrow$ $7y = 3x$

$$R_{eq} = \frac{V_C - V_A}{x + y + z} = \frac{1 \times x}{x + y + z}$$

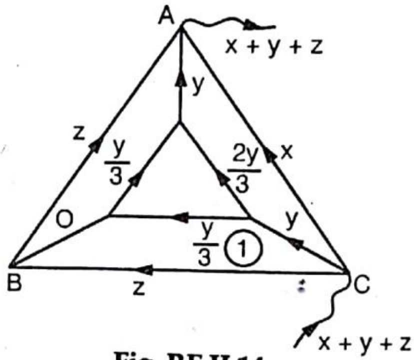

Fig. RE II.14

$$\Rightarrow \quad R_{eq} = \frac{2z}{2z + \frac{3z}{7} + z} = \frac{2 \times 7}{24} = \frac{7}{12}\Omega$$

45. $V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right) = \frac{q}{4\pi\varepsilon_0} \times \frac{R - r}{Rr}.$

$$E = \frac{q}{4\pi\varepsilon_0} \times \frac{1}{x^2} = -\text{ potential gradient}$$

$\therefore$ Potential gradient at $x = r$

$$= -\frac{q}{4\pi\varepsilon_0} \times \frac{1}{r^2}$$

$$= -\frac{1}{4\pi\varepsilon_0 r^2} \times \frac{4\pi\varepsilon_0 RrV}{R - r}$$

$$= -\frac{RV}{r\,(R - r)}.$$

$$|\text{ Potential gradient }| = \frac{RV}{r\,(R-r)}$$

Potential gradient is minimum when $r\,(R-r)$ is maximum, i.e. $\frac{d}{dr}(Rr - r^2) = 0$

$$R - 2r = 0 \quad \Rightarrow \quad r = \frac{R}{2}.$$

46. $E = \frac{1}{4\pi\varepsilon_0} \times \frac{2\lambda}{x} + \frac{1}{4\pi\varepsilon_0} \times \frac{2\lambda_0}{a-x}$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \times \frac{a}{x\,(a-x)}$$

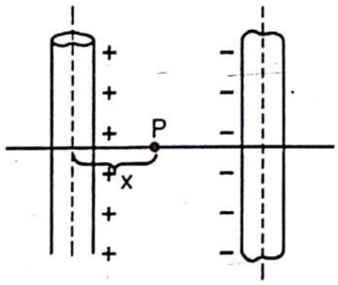

Fig. RE II.15

$E = E_{max}$ when $x\,(a-x)$ is minimum

i.e., $\frac{d}{dx}(ax - x^2) = 0 \quad \Rightarrow \quad x = \frac{a}{2}.$

$$E_{max} = \frac{2\lambda}{\pi\varepsilon_0 a}.$$

$$V = -\int E\,dx = -\frac{2\lambda}{4\pi\varepsilon_0}\,[\ln x - \ln\,(a-x)] + C$$

Put $x = r$ (radius of the wire)

$$V_1 = \frac{-2\lambda}{4\pi\varepsilon_0}\,[\ln r - \ln\,(a-r)\} + C$$

$$x = a - r$$

$$V_2 = -\frac{2\lambda}{4\pi\varepsilon_0}[\ln\,(a-r) - \ln r] + C$$

$$V = V_1 - V_2 = \frac{2\lambda}{4\pi\varepsilon_0}[\,2\ln\,(a-r) - 2\ln r]$$

$$V = \frac{\lambda}{\pi\varepsilon_0}\ln\frac{a-r}{r} = \frac{\pi\varepsilon_0\,a\,E_{max}}{2} \times \frac{1}{\pi\varepsilon_0}\ln\left(\frac{a}{r} - 1\right)$$

$$\Rightarrow \quad V = \frac{a\,E_{max}}{2}\ln\left(\frac{a}{r} - 1\right)$$

$$= \frac{2 \times 38 \times 10^5}{2}\ln\left(\frac{2}{\frac{1}{2} \times 10^{-2}} - 1\right)\left(\begin{aligned}E_{max} &= 38 \times 10^3\ \text{V cm}^{-1}\\ &= 38 \times 10^5\ \text{Vm}^{-1}\end{aligned}\right)$$

$$= 38 \times 10^5 \ln 400$$

$$= 227.6\ \text{kV}$$

47.

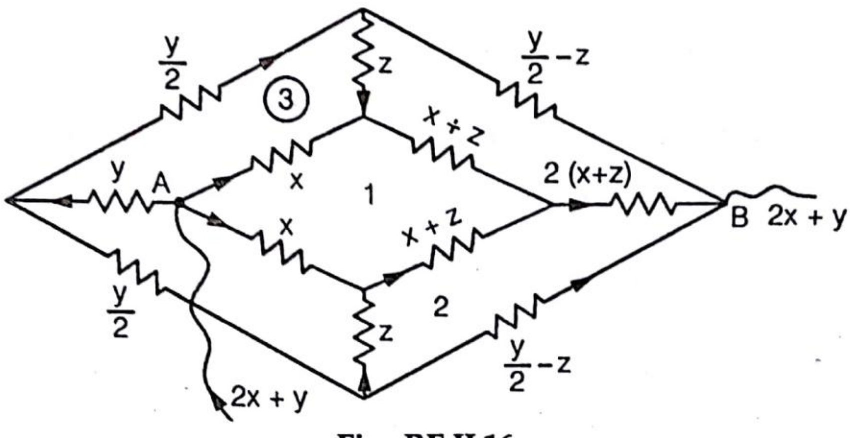

Fig. RE II.16

Loop 2, $(x+z)\,R + 2\,(x+z)\,R - R\left(\frac{y}{z} - z\right) + zR = 0$

$\Rightarrow \quad 3\,(x+z) - \frac{y}{2} + z + z = 0$

$\Rightarrow \quad 3x + 5z - \frac{y}{2} = 0$

$\Rightarrow \quad 6x - y + 10z = 0$

Loop 3, $\frac{y}{2} \times R + zR - xR + yR = 0$

$\frac{3}{2}y + z + x = 0 \quad \Rightarrow \quad 3y + 2z - 2x = 0$

$6x - y + 10z = 0$

$2x - 3y - 2z = 0$

$$\frac{x}{2-(-30)} = \frac{y}{20-(-12)} = \frac{z}{(-18)-(-2)}$$

$\Rightarrow \quad \frac{x}{32} = \frac{y}{32} = \frac{z}{-16} \quad \Rightarrow \quad \frac{x}{2} = \frac{y}{2} = -z$

$x = 2k,\ y = 2k,\ z = -k$

$$R_{eq} = \frac{V_A - V_B}{2x+y} = \frac{Rx + (x+z)\,R + 2\,(x+z)\,R}{2x+y}$$

$$= \frac{2k + (2k-k) + 2\,(2k-k)}{2\,(2k) + 2k}R$$

$$= \frac{5}{6}R \cdot$$

□ □ □

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Appendix

Conversion Factors

Length:
1 m = 10^2 cm = 39.37 in
= 6.214×10^{-4} mi
1 mi = 5280 ft = 1.609 km
1 in = 2.540 cm
1 Å (angstrom) = 10^{-8} cm
= 10^{-10} m
= 10^{-4} μ (micron)
1 μ (micron) = 10^{-6} m
1 AU (astronomical unit)
= 1.496×10^{11} m
1 light year = 9.46×10^{15} m
1 parsec = 3.084×10^{16} m

Angle:
1 radian = 57.3°
1° = 1.74×10^{-2} rad
1′ = 2.91×10^{-4} rad
1″ = 4.85×10^{-6} rad

Area:
1 m^2 = 10^4 cm^2 = 1.55×10^{-5} in^2
= 10.76 ft^2
1 in^2 = 6.452 cm^2
1 ft^2 = 144 in^2 = 9.29×10^{-2} m^2

Volume:
1 m^3 = 10^6 cm^3 = 10^3 litres
= 35.3 ft^3 = 6.1×10^4 in^3
1 ft^3 = 2.83×10^{-2} m^3
= 28.32 litres
1 in^3 = 16.39 cm^3

Mass:
1 kg = 10^3 g = 2.205 lb
1 lb = 453.6 g = 0.4536 kg
1 amu = 1.6604×10^{-27} kg

Force:
1 N = 10^5 dyn + 0.2248 lbf
= 0.102 kgf
1 dyn = 10^{-5} N
= 2.248×10^{-6} lbf
1 lbf = 4.448 N
= 4.448×10^5 dyn
1 kgf = 9.81 N

Power:
1 W = 1.341×10^{-3} hp
1 hp = 745.7 W

Electric Charge:*
1 C = 3×10^9 stC*
1 stC = $\frac{1}{3} \times 10^{-9}$ C

Current:*
1 A = 3×10^9 stA
1 stA = $\frac{1}{3} \times 10^{-9}$ A
1 μA = 10^{-6} A, 1 mA = 10^{-3} A

Electric Field:*
1 N C^{-1} = 1 V m^{-1}
= 10^{-2} V cm^{-1}
= $\frac{1}{3} \times 10^{-4}$ stV cm^{-1}

Electric Potential:*
1 V = $\frac{1}{3} \times 10^{-2}$ stV*
1 stV = 3×10^2 V

Resistance:
1 Ω = 10^6 μΩ
1 MΩ = 10^6 Ω

Capacitance:*
1 F = 9×10^{11} stF*
1 stF = $\frac{1}{9} \times 10^{-11}$ F
1 μF = 10^{-6} F, 1 pF = 10^{-12} F

Magnetic Field:
1 T = 10^4 gauss
1 gauss = 10^{-4} T

Magnetic Flux:
1 Wb = 10^8 maxwell
1 maxwell = 10^{-8} Wb

Magnetizing Field:
1 A m^{-1} = $4\pi \times 10^{-3}$ oersted
1 oersted = $1/4\pi \times 10^3$ A m^{-1}

* In all cases 3 actually means 2.998 and 9 means 8.987; stC—stat coulomb (esu of charge); stV—stat volt (esu of voltage).

Some Physical Constants

Speed of light	c	3×10^{8} m s^{-1}
Gravitational constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Universal gas constant	R	8.31 J mol^{-1} K^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Permittivity constant	ε_0	8.85×10^{-12} F m^{-1}
Avogadro constant	N	6.02×10^{23} mol^{-1}
Boltzmann constant	k	1.38×10^{-23} J K^{-1}
Planck constant	h	6.63×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron magnetic moment	μ_e	9.27×10^{-24} J T^{-1}
Acceleration due to gravity (standard)	g	9.80 m s^{-2}
Standard atmospheric pressure	atm	1.01×10^{5} N m^{-2}
Density of water at 4°C		1000 kg m^{-3}
Rydberg constant	R	1.097×10^{7} m^{-1}
Loschmidt constant	n	2.69×10^{25} m^{-3}
Specific charge of electron	e/m	1.76×10^{11} C kg^{-1}
Proton rest mass	m_p	1.672×10^{-27} kg
Proton specific charge	e/m_p	9.59×10^{8} C kg^{-1}
Stefan constant	σ	5.67×10^{-8} W m^{-3} K^{-4}
Wien constant	b	2.9×10^{-3} m V
First Bohr orbit	r_0	5.29×10^{-11} m
Classical electron radius	r_e	2.82×10^{-15} m
Bohr magneton	μ_B	9.27×10^{-24} J T^{-1}
Nuclear magneton	μ_N	5.05×10^{-27} J T^{-1}
Proton magnetic moment	μ_p	2.792 μN
Neutron magnetic moment	μ_n	– 1.913 μN
Atomic mass unit	amu	1.66×10^{-27} kg or 931 MeV
	hc	1.989×10^{-25} J m
	$\frac{hc}{e}$	1.243×10^{-6} V m

□