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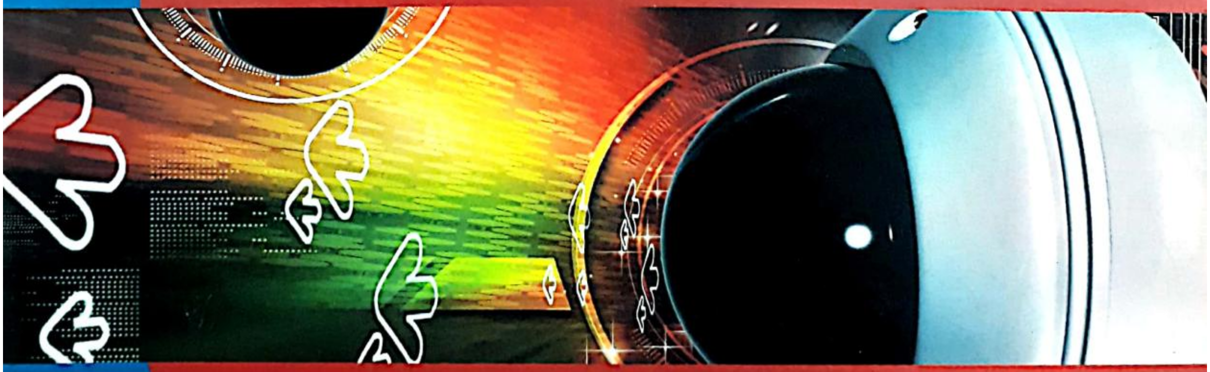
Authentic & Complete Solutions of All the Questions in the 'Master Problem Book' of Physics

Discussions on IE Irodov's
Problems in General Physics

Discussion 2 (Electrodynamics, Oscillations & Sound, Optics & Modern Physics)



DB Singh

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Discussion 2 **(Electrodynamics, Oscillations & Sound, Optics and Modern Physics)**

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Preface

I heartily thank the students and the teachers for their response given to Volume 2 of my book **'Discussions on IE IRODOV 'S Problems in General Physics'**. This book has achieved a great success within a very short span of time.

Many authors might have tried to give shape to such a book, but this book is one of the best of its kind which incisively clears the concepts on **Problems in General Physics**. The contents of the book are designed in a particular way having surprising methods, and made so interesting that to pitchfork students to learn more and more. I have gone in very much depth to find out minute clues that could be useful for students.

In the heading ***Concepts,*** clues are provided to clear your concept. In ***Discussions and Solutions,*** the problems are discussed in details and solutions provided step by step. In ***Your Step,*** we have provided self-made problems for the students to shake their brain to have more practice in solving any sort of problems, and that should surely help them getting more confidence. The students are advised to read the problems again and again till the concept is cleared, and then attempt to solve it.

We shall be highly thankful if the mistakes are brought to our notice that might have crept in. The same shall be rectified in the subsequent editions.

DB Singh

Contents

6.1

Scattering Of Particles Rutherford-Bohr Atom

§ 6.18

➢ **CONCEPT**

Here motion of electron is simple harmonic motion

$$w = -\omega^2 x$$

In the case of simple harmonic motion, total energy of particles

$$E = \frac{1}{2} m a^2 \omega^2 \qquad \text{...(i)}$$

$$\therefore \quad w = -\omega^2 x$$

$$\therefore \quad w^2 = \omega^4 x^2$$

Here $x = a \cos \omega t$ (In the case of SHM)

$$\therefore \quad w^2 = \omega^4 a^2 \cos^2 \omega t$$

The average value of w^2 over one periodic time is

$$w^2 = \omega^4 a^2 < \cos^2 \omega t > = \frac{1}{2} \omega^4 a^2$$

SOLUTION : According to problem, $\frac{dE}{dt} = -\frac{2e^2}{3c^3} w^2$

or $$\frac{dE}{dt} = -\frac{2e^2}{3c^3}\left(\frac{1}{2}\omega^4 a^2\right)$$

or $$\frac{dE}{dt} = -\frac{e^2}{3c^3}\omega^4 a^2$$

or $$\frac{dE}{dt} = -\frac{e^2\omega^2}{3c^3}\left(\frac{2E}{m}\right) \qquad \text{[From eqn (i)]}$$

or $$\int_{E_0}^{E_0/\eta} \frac{dE}{E} = -\left(\frac{2e^2\omega^2}{3c^3 m}\right)\int dt$$

or $$ln\frac{E_0}{\eta} - ln\, E_0 = -\frac{2e^2\omega^2}{3c^3 m}\int_0^t dt$$

or $$ln\frac{E_0}{\eta E_0} = -\frac{2e^2\omega^2}{3c^3 m} t$$

or $$-ln\,\eta = -\frac{2e^2\omega^2 t}{3c^3 m}$$

$$\therefore \quad t = \frac{3c^3 m}{2e^2\omega^2} ln\,\eta = 14.7 \text{ ns} = 15 \text{ ns}$$

YOUR STEP

(a) Using the Bohr's model, calculate the speed of the electron in a hydrogen atom in the $n = 1, 2$ and 3 levels.

(b) Calculate the orbital period in each of these levels.

(c) The average life time of the first excited level of a hydrogen atom is 1.0×10^{-8} s. How many orbits does an electron in an excited atom complete before returning to the ground level?

$$\left\{\begin{array}{l}\text{(a) } 2.19 \times 10^6 \text{ m/s}, 1.09 \times 10^6 \text{ m/s}, 7.29 \times 10^5 \text{ m/s} \\ \text{(b) } 1.52 \times 10^{-16} \text{ s}, 1.22 \times 10^{-15} \text{ s}, 4.10 \times 10^{-15} \text{ s} \\ \text{(c) } 8.2 \times 10^6\end{array}\right\}$$

§ 6.19

➢ **CONCEPT**

When electron moves around nucleus, electrostatic force maintains required centripetal force.

$$\therefore \quad \frac{(Ze)(e)}{4\pi\varepsilon_0 r^2} = mw$$

or

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = mw$$

For H-atom, $Z = 1$

$$\therefore \quad w = \frac{e^2}{4\pi\varepsilon_0 mr^2} \qquad \therefore \quad w^2 = \frac{e^4}{(4\pi\varepsilon_0)^2 m^2 r^4}$$

From previous problem,

$$\frac{dE}{dt} = -\frac{2e^2}{3c^3} w^2$$

or

$$\frac{dE}{dt} = -\frac{2e^2}{3c^3}\left\{\frac{e^4}{(4\pi\varepsilon_0)^2 r^4 m^2}\right\} \quad \text{...(i)}$$

The total energy of electron in the orbit of H-atom is

$$E = -\frac{e^2}{8\pi\varepsilon_0 r} \qquad \therefore \quad \frac{dE}{dt} = \frac{e^2}{8\pi\varepsilon_0 r^2}\frac{dr}{dt} \quad \text{...(ii)}$$

From eqn (i) and (ii), we get

$$\frac{e^2}{8\pi\varepsilon_0 r^2}\frac{dr}{dt} = \frac{-2e^2}{3c^3}\left\{\frac{e^4}{(4\pi\varepsilon_0)^2 r^4 m^2}\right\}$$

or

$$\frac{dr}{dt} = -\frac{4}{3c^3}\frac{e^4}{4\pi\varepsilon_0 r^2 m^2}$$

or

$$-\int_r^0 r^2\, dr = \frac{e^4}{3\pi\varepsilon_0 m^2 c^3}\int_0^t dt$$

or

$$\frac{r^3}{3} = \frac{e^4}{3\pi\varepsilon_0 m^2 c^3} t \qquad \text{or} \qquad t = \frac{\pi\varepsilon_0 m^2 c^3 r^3}{e^4} \qquad \text{or} \qquad t = \frac{m^2 c^3 r^3}{(4e^4)\left(\frac{1}{4\pi\varepsilon_0}\right)} \quad \text{(In S.I.)}$$

In Gaussian unit, $\frac{1}{4\pi\varepsilon_0}$ is taken as one.

$$\therefore \quad t = \frac{m^2 c^3 r^3}{4e^4}$$

On putting the values, we get $\quad t \approx 13$ ps

YOUR STEP

An electron at rest is released far away from a proton, toward which it moves.
Show that the de Broglie wavelength of the electron is proportional to $\sqrt{r}$, where r is the distance of the electron from the proton.

§ 6.20

➢ **CONCEPT**

For solution of problem related to one electron model of atom, two basic equations are provided.

(a) $$mvr = \frac{nh}{2\pi} = n\hbar \quad ...(i) \left(\because \hbar = \frac{h}{2\pi}\right)$$

(b) $$\frac{mv^2}{r} = \frac{(Ze)(e)}{4\pi\varepsilon_0 r^2} \quad ...(ii) \text{ in S.I.}$$

SOLUTION : After solving Eqn (i) and (ii), we get

$$r = \frac{n^2}{Z}\frac{\hbar\ (4\pi\varepsilon_0)}{me^2}$$

and $$V = \frac{Ze^2}{4\pi\varepsilon_0 n\hbar}$$

$\therefore$ Total energy of electron in nth orbit is $E_n = \text{K.E.} + \text{P.E.}$

$$= \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$\therefore$ $$E_n = \left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{m}{2h\hbar n^2} - \left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{m}{n^2\hbar^2}$$

$$= \frac{m Z^2 e^4}{32\pi^2 \varepsilon_0^2 n^2 \hbar^2}$$

The circular frequency is $$\omega_n = \frac{v}{r}$$

On putting the values of v and r, we get

$$\omega_n = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{me^4 Z^2}{\hbar^3 n^3}$$

For $(n+1)$ th orbit,

$$\omega_{n+1} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{me^4 Z^2}{\hbar^3 (n+1)^3}$$

$\therefore$ $$\Delta E = E_{n+1} - E_n$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{e^4 mZ^2}{2\hbar^2}\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

But $\Delta E =$ energy of photon radiated by electron $= h\nu$
Where ν is frequency of radiated photon.
But the angular frequency of photon is $\omega = 2\pi\nu$

$\therefore$ $$\nu = \frac{\omega}{2\pi}$$

$\therefore$ $$\Delta E = \frac{h\omega}{2\pi}$$

$$\therefore \quad \frac{h\omega}{2\pi} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{e^4 m Z^2}{2\hbar^2}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

or
$$\hbar\,\omega = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{e^4 m Z^2}{2\hbar^2}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

or
$$\omega = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{e^4 m Z^2}{2\hbar^3}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

or
$$\omega = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{e^4 m Z^2}{2\hbar^3}\left[\frac{(n+1)^2 - n^2}{(n+1)^2 n^2}\right]$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{e^4 m Z^2}{2\hbar^3}\left[\frac{n^2 + 1 + 2n - n^2}{n^2 (n+1)^2}\right]$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{e^4 m Z^2}{2\hbar^3}\left[\frac{2n+1}{n^2 (n+1)^2}\right]$$

when $n \to \infty$, $2n + 1 \approx 2n$ and $(n+1)^2 = n^2$

$$\therefore \quad \omega = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{e^4 m Z^2}{\hbar^3 n^3} = \omega_n$$

From this point of view, for very large orbit number, the angular frequency of radiation emitted is same as the angular frequency of the electron in the orbit. This is known as Bohr's corresponding theory.

YOUR STEP

1. Demonstrate that the frequency ν of a photon emerging when an electron jumps between neighbouring circular orbits of a hydrogen like atom satisfies the inequality $\nu_{n+1} < \nu < \nu_n$ where ν_n and ν_{n+1} are the frequencies of revolution of that electron around the nucleus along the circular orbits. Also show that for large values of n all these three are almost equal.
2. Show that the frequency of the photon emitted by a hydrogen atom in going from the level $n+1$ to the level n is always intermediate between the frequencies of revolution of the electron in the respective orbits.

$$\left\{\begin{array}{l} \dfrac{f_n}{\nu} = \left(\dfrac{2n^2 + 4n + 2}{2n^2 + n}\right), \text{ which is greater than 1.} \\ \dfrac{f_{n+1}}{\nu} = \left(\dfrac{2n^2}{2n^2 + 3n + 1}\right), \text{ which is less than 1.} \end{array}\right\}$$

§ 6.21

➢ **CONCEPT**

The conservative force $\quad F = -\dfrac{\partial U}{dr}$

Here F is in the direction of r.

SOLUTION : Here $\quad U = \dfrac{kr^2}{2}$

$$\therefore \quad F = -\frac{\partial U}{dr} = -kr$$

Negative sign indicates that the force is directed radially inward (i.e. centripetal force).

$\therefore$ $$mr\,\omega^2 = kr$$

$\therefore$ $$\omega = \sqrt{\frac{k}{m}}$$

$\therefore$ The angular momentum of electron about the nucleus is

$$M = mr^2\,\omega$$

But $$M = n\hbar \qquad \left(\because \hbar = \frac{h}{2\pi}\right)$$

$\therefore$ $$r = \sqrt{\frac{n\hbar}{m\omega}}$$

Here r is the radius of nth orbit.

The kinetic energy of electron is

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\,(r\omega)^2 = \frac{n\hbar\ \omega}{2}$$

But $$U = \frac{kr^2}{2} = \frac{k}{2}\frac{n\hbar}{m\omega}$$

$$= \left(\sqrt{\frac{k}{m}}\right)^2 \frac{n\hbar}{2\omega} = \frac{\omega^2\,n\hbar}{2\omega} = \frac{n\hbar\ \omega}{2}$$

$\therefore$ The total energy of electron in nth orbit is

$$E_n = T + U = \frac{n\hbar\ \omega}{2} + \frac{n\hbar\ \omega}{2} = n\hbar\ \omega$$

YOUR STEP

A particle whose mass is m moves in such a way that the potential energy $U = \frac{1}{2}m^2\,\omega^2 r^2$ where ω is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, shc.v that radius of the nth allowed orbit is proportional to $\sqrt{n}$.

§ 6.22

➢ **CONCEPT**

The basic postulates of Bohr's theory provides two basic equations for solving problems.

$$\frac{mv^2}{r} = \frac{(Ze)\,(e)}{r^2} \qquad \text{...(i)}$$

$$mvr = n\hbar \qquad \text{...(ii)}$$

SOLUTION : After solving eqn (i) and (ii), we get

$$r = \frac{4\pi\,\varepsilon_0\,\hbar^2}{e^2 m}\left(\frac{n^2}{Z}\right) \qquad \text{...(i)}$$

and $$v = \frac{e^2 Z}{4\pi\,\varepsilon_0\,\hbar\ n} \qquad \text{...(ii)}$$

For H-atom, $Z = 1$ and $n = 1$ for first orbit.

For He^+, $Z = 1, n = 1$

On putting the values in eqn (i), we get

$r = 52.9$ pm for H-atom $\qquad$ $r = 26.5$ pm for He^+ ion

$v = 2.18 \times 10^6$ m/s for H-atom $\qquad$ $v = 4.36 \times 10^6$ m/s for He^+ ion

(b) $\because$ Kinetic energy is $T = \frac{1}{2} m v^2$

$$= \frac{1}{2} m \left(\frac{e^2 Z}{4\pi\varepsilon_0 \hbar n} \right)^2$$

For first orbit, $n = 1$

$$\therefore \quad T = \frac{1}{2} m \left(\frac{e^2 Z}{4\pi\varepsilon_0 \hbar} \right)^2$$

On putting the values,

$T = 13.65$ eV for H-atom $\quad T = 54.6$ eV for He^+ ion

The total energy of electron in nth orbit is

$$E_n = -13.6 \left(\frac{Z^2}{n^2} \right) \text{eV}$$

The binding energy of electron is

$$E_b = -E_n = 13.6 \left(\frac{Z^2}{n^2} \right) \text{eV}$$

For first orbit, $n = 1$

$\therefore \quad E_b = 13.6\, Z^2$ eV, $\quad$ For H-atom, $Z = 1$

$$\therefore \quad E_b = 13.6 \text{ eV for H-atom}$$

For He^+ ion, $Z = 2$

$$E_b = 13.6\,(2)^2 \text{ eV} = 54.4 \text{ eV}$$

(c) $\quad E_b = e\phi_i$

where ϕ_i = ionisation potential

For H-atom, $\quad e\phi_i = 13.6$ eV

$$\therefore \quad \phi_i = 13.6 \text{ V For H-atom}$$

For He^+ ion,

$$E_b = 54.4 \text{ eV} = e\,\phi_i$$

$$\therefore \quad \phi_i = 54.4 \text{ V}$$

$$\therefore \quad \Delta E = 13.6\, Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{eV}$$

For first excitation, $n_1 = 1$, $n_2 = 2$

$$\therefore \quad \Delta E = 13.6\, Z^2 \left(1 - \frac{1}{4} \right) \text{eV}$$

H or H-atom, $Z = 1$, $\quad \therefore \quad \Delta E = 10.2$ eV

$$\therefore \quad e\,\phi = 10.2 \text{ eV}$$

$$\therefore \quad \phi = 10.2 \text{ V}$$

For He^+, $Z = 2$

$$\therefore \quad \Delta E = 13.6\,(2)^2 \left(1 - \frac{1}{4} \right) \text{eV}.$$

$$= 13.6 \times 4 \times \frac{3}{4} = 40.8 \text{ eV}$$

$$\therefore \quad e\phi = 40.8 \text{ eV}$$

$$\therefore \quad \phi = 40.8 \text{ V}$$

As we know,
$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Here $n_1 = 1$, $n_2 = 2$

$\therefore$
$$\frac{1}{\lambda} = RZ^2 \times \frac{3}{4} = \frac{3RZ^2}{4}$$

$\therefore$
$$\lambda = \frac{4}{3RZ^2}$$

For H-atom, $Z = 1$

$\therefore$
$$\lambda = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} = 121.5 \text{ nm}$$

For He^+ ion, $Z = 2$

$\therefore$
$$\lambda = \frac{4}{3RZ^2} = \frac{4}{3 \times 2^2 \times 1.097 \times 10^7} = 30.4 \text{ nm}$$

YOUR STEP

The ionisation energy is the energy required to remove an electron from an atom. Find the ionisation energy of :

(a) The $n = 3$ level of hydrogen.

(b) The $n = 2$ level of He^+ (singly ionized helium).

(c) The $n = 4$ level of Li^{++} (doubly ionized lithium).

{(a) 1.51 eV (b) 13.6 eV (c) 7.65 eV}

§ 6.23

➢ **CONCEPT**

For solving the problem based upon Bohr's theory for nucleus of infinite mass, following formulae are useful :

(a) The radius of nth orbit is
$$r_n = \frac{(4\pi\,\varepsilon_0)\,\hbar^2}{me^2}\left(\frac{n^2}{Z}\right) = \left(\frac{n^2}{Z}\right)a_0$$

Here
$$a_0 = 0.53 \times 10^{-10} \text{ m} = 53 \text{ pm}$$

n = orbit number, Z = atomic number

(b) The velocity of electron in nth orbit is
$$v_n = \frac{e^2}{4\pi\,\varepsilon_0\hbar}\left(\frac{Z}{n}\right)$$

$\therefore$
$$v_n = v_1\left(\frac{Z}{n}\right)$$
$$v_1 = \frac{e^2}{4\pi\,\varepsilon_0\,\hbar}$$

Here
$$e = 1.6 \times 10^{-19} \text{ C}$$
$$\frac{1}{4\pi\,\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$
$$\hbar = 1.055 \times 10^{-3} \text{ Js}$$

On putting, the values, we get
$$v_1 = 2.2 \times 10^6 \text{ m/s}$$

The fine structure constant α is defined as

$$\alpha = \frac{v_1}{c} = \frac{2.2 \times 10^6}{3 \times 10^8} \approx \frac{1}{137}$$

Here c = speed of light in vacuum

$$= 3 \times 10^8 \text{ m/s}$$

(c) Kinetic energy of electron in nth orbit is

$$T_n = \frac{Ze^2}{8\pi\, \varepsilon_0 r_n}$$

(d) The potential energy of electron in nth orbit is

$$U_n = -\frac{Ze^2}{4\pi\, \varepsilon_0 r_n}$$

(e) The total energy of electron in nth orbit is

$$E_n = T_n + U_n = \frac{-Ze^2}{8\pi\, \varepsilon_0 r_n} = -\frac{Z^2 e^2}{8\pi\, \varepsilon_0\, a_0 n^2}$$

$$= -E_0 \frac{Z^2}{n^2}$$

Here $$E_0 = \frac{e^2}{8\pi\, \varepsilon_0 a_0} = 13.6 \text{ eV}$$

$\therefore$ $$E_n = -13.6 \left(\frac{Z}{n}\right)^2 \text{ eV}$$

(f) $$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Here $$n_1 < n_2$$

Here λ = wavelength of photon radiated by electron during jumping from n_2 orbit to n_1 orbit.

R = Rydberg constant = 1.097373×10^7 per metre

Rydberg constant is not a universal constant. Its value is different for different atoms. If the mass of nucleus is assumed to be of infinite mass, then the value of Rydberg constant does not vary atom to atom.

SOLUTION : Here $$v_n = \frac{e^2}{4\pi\, \varepsilon_0 \hbar}\left(\frac{Z}{n}\right)$$

and $$r_n = \frac{4\pi\, \varepsilon_0\, \hbar^2}{me^2}\left(\frac{n^2}{Z}\right)$$

$\therefore$ The angular frequency of electron is

$$\omega_n = \frac{U_n}{r_n}$$

$\therefore$ $$\omega_n = \frac{\left(\dfrac{e^2}{4\pi\, \varepsilon_0\, \hbar}\right)\left(\dfrac{Z}{n}\right)}{\dfrac{4\pi\, \varepsilon_0\, \hbar^2}{me^2}\left(\dfrac{n^2}{Z}\right)} = \frac{me^4 Z^2}{(4\pi\, \varepsilon_0)^2 \hbar^3\, n^3} \quad \text{(S.I.)}$$

In gaussian unit, $\frac{1}{4\pi\varepsilon_0}$ is taken as one.

$$\therefore \qquad \omega_n = \frac{me^4 Z^2}{\hbar^3 n^3}$$

For He^+ ion, $Z = 2, n = 2$

On putting the values, we get $\omega_n = 2.07 \times 10^{16}$ second^{-1}

> **YOUR STEP**
>
> Applying Bohr's theory, find the orbital velocity of the electron on an arbitrary energy level. Compare the orbital velocity on the lowest energy level with that of light.
>
> $\left\{\frac{1}{137}\right\}$

§ 6.24

➢ **CONCEPT**

When a charge particle is moving on a circular path with angular velocity ω, then the equivalent current is

$$I = \frac{q}{T} = \frac{q}{\frac{2\pi}{\omega}} = \frac{\omega_n q}{2\pi} = \frac{e\omega_n}{2\pi} \qquad (\because \omega = \omega_n) \text{ (for electron)}$$

SOLUTION : The magnetic moment is

$$\mu_n = I\,\pi r_n^2 = \frac{e\,\omega_n}{2\pi}\pi r_n^2$$

$$\mu_n = \frac{e\,\omega_n\,r_n^2}{2}$$

From the solution of previous problem,

$$\omega_n = \frac{me^4 Z^2}{(4\pi\varepsilon_0)^2 \hbar^3 n^3} \qquad \text{(in Gaussion unit)}$$

$$r_n = \frac{(4\pi\varepsilon_0)\hbar^2}{me^2}\left(\frac{n^2}{Z}\right) \qquad \text{in S.I.}$$

$$\therefore \qquad \mu_n = \frac{e\omega_n r_n^2}{2}$$

On putting the vlaues of r_n and ω_n, we get

$$\therefore \qquad \mu_n = \frac{ne\hbar}{2m}$$

$$\therefore \qquad \mu_n = \frac{ne\hbar}{2mc} \qquad \text{(in Gaussion unit)}$$

Here $M_n = I\omega$

$$= \frac{nh}{2\pi} = n\hbar$$

$$\therefore \qquad \frac{\mu_n}{Mn} = \frac{e}{2mc}$$

For first orbit, $n = 1$

$$\therefore \qquad \mu_1 = \frac{e\hbar}{2mc} = \mu_B$$

YOUR STEP

How many times does the electron go round the first Bohr orbit of hydrogen in 1s ?

{6.57×10^{15} Hz}

§ 6.25

➢ **CONCEPT**

The magnetic field at a point due to moving charge is

$$\vec{B} = \frac{\mu_0}{4\pi r^3} q (\vec{v} \times \vec{r})$$

$$\therefore \quad B = \frac{\mu_0 qv \sin\theta}{4\pi r^2} \quad \text{(in S.I.)}$$

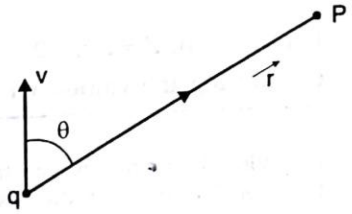

Fig. 6.25A

SOLUTION : The magnetic field at the centre of H-atom due to moving electron is

$$B = \frac{\mu_0}{4\pi} \left| \frac{-e\, \vec{v}_n \times \vec{r}_n}{r_n^3} \right| = \frac{\mu_0 e v_n \sin 90^\circ}{4\pi r_n^2}$$

$$\therefore \quad B = \frac{\mu_0 e v_n}{4\pi r_n^2} \quad \text{...(i)}$$

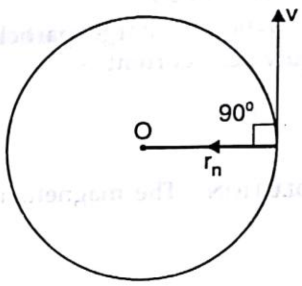

Fig. 6.25B

From the solution of 6.22, $r_n = \frac{4\pi\varepsilon_0 \hbar^2 n^2}{e^2 m}$ (for H-atom)

and $v_n = \frac{e^2}{4\pi\varepsilon_0 n\hbar}$

On putting the values in Eqn. (i), we get

$$B = \frac{\mu_0 e}{4\pi} \left(\frac{e^2}{4\pi\varepsilon_0 n\hbar} \right) \frac{e^4 m^2}{(4\pi\varepsilon_0 \hbar^2 n^2)^2} \quad \text{(In S.I.)}$$

$$\therefore \quad B = \frac{m^2 e^7}{c \hbar^5} = 125 \text{ kG} \quad \text{(In gaussian unit)}$$

YOUR STEP

An electron moving in an atom is acted upon by the coulomb force of attraction generated by the nucleus. Can an external electric field be created that is capable of neutralising the coulomb force and ionising, say, a hydrogen atom ? Field strengths that can be created by modern devices are about 10^7 to 10^8 V/m.

{Yes}

§ 6.26

➢ **CONCEPT**

For H-atom

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Here R = Rydberg constant $= 1.097373 \times 10^7 \text{ m}^{-1}$

$\approx 1.097 \times 10^7 \text{ m}^{-1}$

(i) For Lyman series : $n_1 = 1, n_2 = 2, 3, \ldots \infty$

$$\therefore \quad \frac{1}{\lambda_{min}} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2} \right) = R = 1.097 \times 10^7$$

$$\therefore \qquad \lambda_{min} = \frac{1}{1.097 \times 10^7} \text{ m} = 91.16 \text{ nm}$$

For λ_{max}, $n_1 = 1, n_2 = 2$

$$\therefore \qquad \frac{1}{\lambda_{max}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\therefore \qquad \lambda_{max} = \frac{4}{3R} = 121.5 \text{ nm}$$

It mean Lyman series is lying between 91.16 nm and 121.5 nm

(ii) For Balmer seris :

$n_1 = 2, n_2 = 3, 4, 5, \ldots \infty$

For λ_{min}, $n_1 = 2, n_2 = \infty$

$$\therefore \qquad \frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}$$

$$\therefore \qquad \lambda_{min} = \frac{4}{R} = 364.6 \text{ nm}$$

For λ_{max}, $n_1 = 2, n_2 = 3$

$$\therefore \qquad \frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\text{or} \qquad \frac{1}{\lambda_{max}} = R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\text{or} \qquad \frac{1}{\lambda_{max}} = R\,\frac{5}{36}$$

$$\therefore \qquad \lambda_{max} = \frac{36}{5R} = 656.3 \text{ nm}$$

It means Balmer series is lying between 364.6 nm to 656.3 nm.

(iii) For paschen series :

$n_1 = 3, n_2 = 4, 5, 6 \ldots \infty$

For λ_{min}, $n_1 = 3, n_2 = \infty$

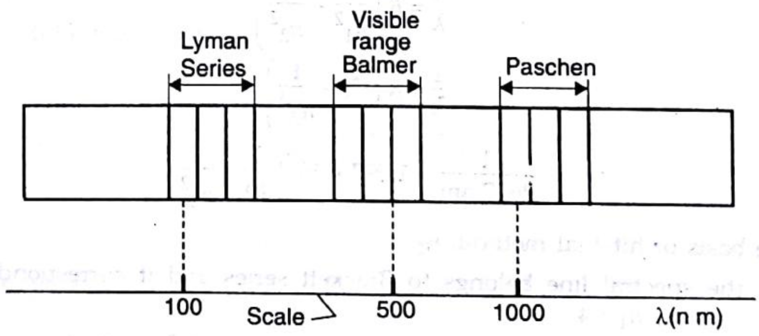

Fig. 6.26A

$$\therefore \qquad \frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = \frac{R}{9}$$

$$\therefore \qquad \lambda_{min} = \frac{9}{R} = 820.4 \text{ nm}$$

For λ_{max}, $n_1 = 3, n_2 = 4$

$$\therefore \quad \frac{1}{\lambda_{max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\therefore \quad \lambda_{max} = 1875.2 \text{ nm}$$

YOUR STEP

1. Draw an energy level diagram showing the lowest four levels of singly ionized helium. Show all possible transitions from the level and label each transition with its wavelength.
2. A uniform magnetic field B exists in a region. An electron projected perpendicular to the field goes in a circle. Assuming Bohr's quantization rule for angular momentum, calculate
(a) the smallest possible radius of the electron.
(b) The radius of the nth orbit.
(c) The minimum possible speed of the electron.

{1. $E_1 = -54.4$ eV $E_2 = -13.6$ eV $E_3 = -6.04$ eV $E_4 = -3.40$ eV

2. (a) $\sqrt{\frac{h}{2\pi eB}}$ (b) $\sqrt{\frac{nh}{2\pi eB}}$ (c) $\sqrt{\frac{heB}{2\pi m^2}}$}

§ 6.27

➢ CONCEPT

The linear wave number is $\frac{1}{\lambda}$

SOLUTION : According to problem,

$$\because \quad \frac{1}{\lambda} = \frac{1}{\lambda_1} - \frac{1}{\lambda_2}$$

or
$$\frac{1}{\lambda} = \left(\frac{1}{410.2} - \frac{1}{486.1}\right) \text{per nm}$$

After solving, we get $\lambda = 2627$ nm
Since, Brakett series is lying between 4051 nm to 1459 nm.
Hence, the wavelength $\lambda = 2627$ nm is lying in the region of Brakett series.
For Brakett series,
$n_1 = 4, n_2 = 5, 6, 7, \ldots$

$$\because \quad \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or
$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_2^2}\right)$$

or
$$\frac{1}{2627 \text{ nm}} = 1.097 \times 10^7 \left(\frac{1}{16} - \frac{1}{n_2^2}\right)$$

$\therefore$ On the basis of hit-trial method, $n_2 = 6$

$\therefore$ Hence, the spectral line belongs to Brackett series and it corresponds to transition of electron from $n_2 = 6$ to $n_1 = 4$

Hence, $\lambda = 2627 \text{ nm} = 2.63 \times 10^{-6} \text{ m} = 2.63\ \mu\text{m}$

YOUR STEP

The life times of the levels in a hydrogen atom are of the order of 10^{-8} s. Find the energy uncertainty of the first excited state and compare it with the energy of the state.

{7×10^{-8} eV}

§ 6.28

➢ **CONCEPT**

The concept is similar to previous problem.

i.e.
$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

SOLUTION : For Balmer series, $n_1 = 2$

(i) For first line, $n_2 = 3$

$$\therefore \quad \frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}$$

$$\therefore \quad \lambda_1 = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7} \text{ metre} = 656.3 \text{ nm}$$

(ii) For second line, $n_2 = 4$

$$\therefore \quad \frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\therefore \quad \lambda_2 = 486 \text{ nm}$$

(iii) For third line, $n_2 = 5$

$$\therefore \quad \frac{1}{\lambda_3} = R\left[\frac{1}{2^2} - \frac{1}{5^2}\right]$$

$$= 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{25}\right] \text{metre}^{-1}$$

$$\therefore \quad \lambda_3 = 434 \text{ nm}$$

(b) $\because$
$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{20^2}\right)$$

$\therefore$ For being $\frac{\lambda}{\delta\lambda}$ minimum, $\delta\lambda$ should be maximum.

For this, n_2 will be maximum.

$$\therefore \quad \delta\lambda = \lambda_{19} - \lambda_{20}$$

$$\therefore \quad \frac{1}{\lambda_{19}} = R\left(\frac{1}{2^2} - \frac{1}{19^2}\right)$$

and
$$\frac{1}{\lambda_{20}} = R\left(\frac{1}{2^2} - \frac{1}{20^2}\right)$$

After solving, we get
$$\delta\lambda = \lambda_{19} - \lambda_{20}$$
$$= 346 \times 10^{-3} \text{ nm}$$

The average wavelength for first 20 lines will be

$$\lambda = \frac{(657 + 368)}{2} \text{ nm} = 513 \text{ nm}$$

$$\frac{\lambda}{\delta\lambda} = 1.5 \times 10^3$$

YOUR STEP

A spectroscopic instrument can resolve two nearby wavelengths λ and $\lambda + \Delta\lambda$ if $\frac{\lambda}{\Delta\lambda}$ is smaller than 8000. This is used to study the spectral lines of the Balmer series of hydrogen. Approximately how many lines will be resolved by the instrument?

{38}

§ 6.29

➢ **CONCEPT**

$$\frac{1}{\lambda} = \frac{RZ^2}{2\pi c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \text{(in Gaussian unit)}$$

SOLUTION : For Balmer series, $n_1 = 2, n_2 = n,$

$$\therefore \quad \frac{1}{\lambda_n} = \frac{RZ^2}{2\pi c}\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{RZ^2}{2\pi c}\left(\frac{1}{4} - \frac{1}{n^2}\right) \quad \text{...(i)}$$

Similarly, for $n_2 = n - 1$

$$\frac{1}{\lambda_{n-1}} = \frac{RZ^2}{2\pi c}\left[\frac{1}{4} - \frac{1}{(n-1)^2}\right]$$

$$\therefore \quad \frac{1}{\lambda_n} - \frac{1}{\lambda_{n-1}} = \frac{RZ^2}{2\pi c}\left[\frac{\{n^2 - (n-1)^2\}}{\{(n-1)^2 n^2\}}\right]$$

$$= \frac{RZ^2}{2\pi c}\left[\frac{(2n-1)}{(n-1)^2 n^2}\right]$$

For large value of $n, n - 1 \approx n$ and $2n - 1 = 2n$

$$\therefore \quad \frac{1}{\lambda_n} - \frac{1}{\lambda_{n-1}} = \frac{RZ^2}{2\pi c} \cdot \frac{2n}{n^4} = \frac{RZ^2}{\pi n^3}$$

$$\therefore \quad \frac{\lambda_n - \lambda_{n-1}}{\lambda_{n-1}\lambda_n} = \frac{RZ^2}{\pi n^3}$$

But $\lambda_{n-1} \approx \lambda_n$

$$\therefore \quad \frac{\lambda_n - \lambda_{n-1}}{\lambda_n^2} = \frac{RZ^2}{\pi c n^3}$$

or

$$\frac{\delta\lambda}{\lambda_n^2} = \frac{RZ^2}{\pi c n^3}$$

or

$$\frac{\delta\lambda}{\lambda_n} = \frac{RZ^2}{\pi c n^3}\lambda_n$$

According to Rayleigh's criterion

$$\therefore \quad \frac{1}{KN} \approx \frac{b \sin\theta}{K}\left(\frac{RZ^2}{\pi n^3 c}\right)$$

$$\therefore \quad \frac{1}{Nb} = \sin\theta\left(\frac{R}{\pi n^3 c}\right)Z^2$$

Here Nb = width l of grating.

$\therefore$ $$\frac{1}{l} \approx \sin\theta \left(\frac{RZ^2}{\pi c n^3}\right)$$

$\therefore$ $$\sin\theta \approx \frac{\pi c n^3}{lR}$$

On putting the vlaues, we get $$\sin\theta = \frac{\sqrt{3}}{2}$$

$\therefore$ $$\theta = 60°$$

YOUR STEP

A high-powered laser beam ($\lambda = 600$ nm) with a beam diameter of 12 cm is aimed at the Moon, 3.8×10^5 km distant. The beam spreads only because of diffraction. The angular location of the edge of the central diffraction disc see fig. 6.29A is given by $\sin\theta = \frac{1.22\,\lambda}{d}$, where d is the diameter of the beam aperture. What is the diameter of the central diffraction disc on the Moon's surface?

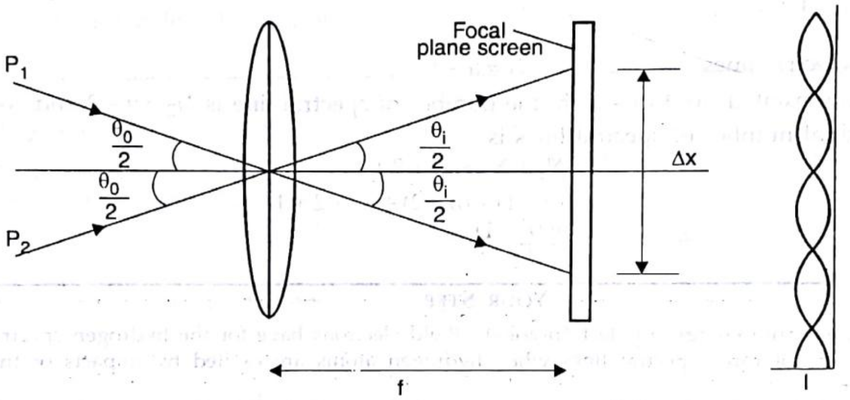

Fig. 6.29A

(4.7 km)

§ 6.30

➢ **CONCEPT**

For Hydrogen like atom, $$\frac{1}{\lambda'} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad ...(i)$$

For Hydrogen atom, $$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad ...(ii)$$

Dividing Eqn (ii) by Eqn (i), we get

$$\frac{\lambda'}{\lambda} = \frac{R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}{RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

or $$\frac{\lambda'}{\lambda} = \frac{1}{Z^2}$$

According to problem, $\lambda = 4\lambda'$ $\therefore$ $\frac{\lambda'}{4\lambda'} = \frac{1}{Z^2}$

$\therefore$ $$Z = 2$$

The atomic number of Helium atom is 2. Hence, required element is helium.

YOUR STEP

A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2 and 17.0 eV respectively. Alternately, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. Determine the values of n and Z. (Ionisation energy of H-atom = 13.6 eV).

{6, 3.}

§ 6.31

➢ **CONCEPT**

The possible transition states from nth excited state :

Number of spectral Lines

n to $n-1$ 1

n to $n-2$ 1

n to $n-3$ 1

n to $n-4$ 1

n to 1 1

Adding all spectral lines $N_1 = n - 1$

Similarly, from excited orbit $(n-1)$ th, the number of spectral line is $N_2 = n - 2$ and so on.

Hence, the total number of spectral lines is

$$N = N_1 + N_2 + \ldots + 2 + 1$$
$$= (n-1) + (n-2) + \ldots + 2 + 1$$
$$= \frac{n(n-1)}{2}$$

YOUR STEP

1. (a) What minimum energy (in electron-volts) should electrons have for the hydrogen spectrum to consists of three spectral lines when hydrogen atoms are excited by impacts of these electrons?
   (b) What is the wavelengths of these lines?
2. (a) What is the least amount of energy in electron volts that must be given to a hydrogen atom that is initially in its ground level so that it can emit the H_β line in the Balmer series?
   (b) How many different possibilities of spectral line emission are there for this atom when the electron starts in the $n = 4$ level and eventually ends up in the ground level? Calculate the wavelength of the emitted photons in each case?

{
1. (a) $E_{min} = 12.1$ eV
   (b) $\lambda_1 = 1.21 \times 10^{-7}$ m, $\lambda_2 = 1.03 \times 10^{-7}$ m, $\lambda_3 = 6.56 \times 10^{-7}$ m
2. (a) 12.8 eV
   (b) $4 \to 3, 4 \to 2, 4 \to 1, 3 \to 2, 3 \to 1, 2 \to 1$, with corresponding, wavelengths 1.88 μm, 486 nm, 97.2 nm, 656 nm, 103 nm, 122 nm.
}

§ 6.32

➢ **CONCEPT**

From the solution of problem 6.26, the range of lyman series is from 91.16 nm to 121.2 nm.

SOLUTION : The given range (94.5 to 130 nm) is lying in lyman series.

For lyman series, $n_1 = 1, n_2 = 2, 3, 4, 5, \ldots \infty$

For first line, $n_2 = 2$

$$\therefore \quad \frac{1}{\lambda_1} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or $$\frac{1}{\lambda_1} = 1.097 \times 10^7 \left(\frac{1}{(1)^2} - \frac{1}{(2)^2}\right) = 1.097 \times \frac{3}{4} \times 10^7 \text{ m}^{-1}$$

$\therefore$ $$\lambda_1 = 121.6 \text{ nm}$$

For second line, $n_2 = 3$

$\therefore$ $$\frac{1}{\lambda_2} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$\therefore$ $$\lambda_2 = 102.6 \text{ nm}$$

For third line, $n_2 = 4$

$\therefore$ $$\frac{1}{\lambda_3} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{4^2}\right) \text{m}^{-1}$$

$\therefore$ $$\lambda_3 = 97.3 \text{ nm}$$

YOUR STEP

By what fraction does the mass of an H-atom decrease when it makes an $n = 3$ to $n = 1$ transition ? $\{1.29 \times 10^{-8}\}$

§ 6.33

➢ **CONCEPT**

The value of $hc = 1242$ nm eV

$\therefore$ The energy of first photon is

$$\Delta E_1 = \frac{hc}{\lambda_1} = \frac{1242 \text{ nm eV}}{108.5 \text{ nm}} = 11.45 \text{ eV}$$

The energy of second photon is

$$\Delta E_2 = \frac{hc}{\lambda_2} = \frac{1242 \text{ nm eV}}{30.4 \text{ nm}} = 40.86 \text{ eV}$$

The total energy emitted by electron during jumping from n_2 to n_1 is $\Delta E = \Delta E_1 + \Delta E_2$

$$= (11.45 + 40.86) \text{ eV} = 52.31 \text{ eV}$$

$\because$ $$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or $$\frac{hc}{\lambda} = hc\,RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or $$\Delta E = 13.6\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\text{eV}$$

or $$\Delta E = 13.6\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n^2}\right)\text{eV}$$

or $$52.31 \text{ eV} = 13.6\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n^2}\right)\text{eV}$$

But final state of electron is ground state.

$\therefore$ $$n_1 = 1$$

$\therefore$ $$52.31 \text{ eV} = 13.6\,Z^2\left(1 - \frac{1}{n^2}\right)\text{eV}$$

or $$\left(1 - \frac{1}{n^2}\right)Z^2 = \frac{52.31}{13.6}$$

or $$\left(1-\frac{1}{n^2}\right)Z^2 = 3.85$$

or $$1-\frac{1}{n^2} = \frac{3.85}{2^2} = \frac{3.85}{4} = 0.96 \qquad \text{(For He}^+\ Z=2)$$

$\therefore$ $$\frac{1}{n^2} = 1-0.96 = 0.04$$

or $$n^2 = \frac{1}{0.04} = 25$$

$\therefore$ $$n = 5$$

YOUR STEP

A gas of identical hydrogen like atoms has same atoms in the lowest energy level A and same atoms in a particular upper level B and there are no atoms in any other energy level. The atoms of the gas make transition to a higher energy level by absorbing monochromatic light of photon energy 7.56 eV. Subsequently, the atoms emit radiations of only six different photon energies. Some of emitted photons have energy 7.56 eV, some have energy more and same have less than 7.56 eV. Calculate :

(a) The principal quantum number of the initially excited level B.
(b) Maximum and minimum energies of emitted photons
(c) Ionisation energy for gas atoms.

{(a) 2 (b) 37.8 eV, 1.96 eV (c) 40.32 eV}

§ 6.34

➢ **CONCEPT**

For solution, the nucleus is assumed as infinite mass.
The applicable formula is

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

For Balmer's series,
$n_1 = 2, n_2 = 3$ (For longest wavelength)
Also, $Z = 2$ (For He^+ ion)

$\therefore$ $$\frac{1}{\lambda_1} = 4R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

or $$\frac{1}{\lambda_1} = 4R\left(\frac{9-4}{36}\right)$$

$\therefore$ $$\lambda_1 = \frac{36}{5\times 4R} = \frac{9}{5R} \qquad ...(i)$$

For lyman series,
$n_1 = 1, n_2 = 2$ (For longest wavelength)

$\therefore$ $$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

or $$\frac{1}{\lambda_2} = R\,2^2\left(\frac{3}{4}\right) = 3R$$

$\therefore$ $$\lambda_2 = \frac{1}{3R} \qquad ...(ii)$$

$\therefore$ $$\Delta\lambda = \lambda_1 - \lambda_2$$

or $$\Delta\lambda = \frac{9}{5R} - \frac{1}{3R} = \frac{27-5}{15R}$$

or $$\Delta\lambda = \frac{22}{15R}$$

$\therefore$ $$R = \frac{22}{15\,\Delta\lambda} = \frac{22}{15 \times 133.7\text{ nm}}$$

$$= \frac{22}{15 \times 133.7 \times 10^{-9}}\text{ m}^{-1}$$

$$= 1.0969832 \times 10^{7}\text{ m}^{-1} \qquad \text{(In S.I. unit)}$$

In Gaussian unit, $$R = (2\pi c) \times 1.0969832 \times 10^{7}\text{ m}^{-1}$$

$$= 2 \times 3.14 \times 3 \times 10^{8}\left(\frac{\text{m}}{\text{s}}\right) \times 1.0969832 \times 10^{7}\text{ m}^{-1}$$

$$= 2.066716349 \times 10^{16}\text{ s}^{-1}$$

$$= 2.07 \times 10^{16}\text{ s}^{-1} \qquad \text{(In Gaussian unit)}$$

YOUR STEP

Construct the energy level diagram for doubly ionized lithium, Li^{+2}.

§ 6.35

➢ **CONCEPT**

The concept is similar to previous problem.
For Balmer series,
$n_1 = 2, n_2 = 3$ (For longest wavelength)

$\therefore$ $$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or $$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5RZ^2}{36}$$

$\therefore$ $$\lambda_1 = \frac{36}{5RZ^2} \qquad \text{...(i)}$$

For Lyman series
$n_1 = 1, n_2 = 2$ (For longest wavelength)

$\therefore$ $$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3RZ^2}{4}$$

$\therefore$ $$\lambda_2 = \frac{4}{3RZ^2} \qquad \text{...(ii)}$$

$\therefore$ $$\Delta\lambda = \lambda_1 - \lambda_2$$

or $$\Delta\lambda = \frac{36}{5RZ^2} - \frac{4}{3RZ^2}$$

or $$RZ^2\,\Delta\lambda = \frac{36}{5} - \frac{4}{3} = \frac{108-20}{15} = \frac{88}{15}$$

or $$Z^2 = \frac{88}{15R\,\Delta\lambda}$$

$\therefore$ $$Z = \sqrt{\frac{88}{15 \times 1.097 \times 10^{7} \times 59.3 \times 10^{-9}}} = \sqrt{9} = 3$$

Hence, it is for Lithium Li^{++}

YOUR STEP

Atoms of a hydrogen like gas are in a particular excited energy level. When these atoms de-excite, they emit photons of different energies. Maximum and minimum energies of emitted photons are $E_{max} = 52.224$ eV and $E_{min} = 1.224$ eV respectively. Identify the gas and calculate principal quantum number of initially excited energy level.
(Ionisation energy of hydrogen atom = 13.6 eV)

{He^+, 5}

§ 6.36

➢ **CONCEPT**

As we know,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or
$$\frac{c}{\lambda} = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or
$$\nu = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or
$$2\pi\nu = 2\pi RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or
$$\omega = 2\pi RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Hence
$$\omega_1 = 2\pi RcZ^2\left(\frac{1}{n^2} - \frac{1}{\infty^2}\right)$$

and
$$\omega_2 = 2\pi RcZ^2\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$\therefore$
$$\Delta\omega = \omega_1 - \omega_2 = \frac{2\pi RcZ^2}{(n+1)^2} \qquad \text{(In S.I.)}$$

In Gaussian unit,
$$\Delta\omega = \frac{2\pi RcZ^2}{2\pi c(n+1)^2} = \frac{RZ^2}{(n+1)^2} \qquad \text{...(i)}$$

$\therefore$
$$n = Z\sqrt{\frac{R}{\Delta\omega}} - 1 \qquad \text{...(ii)}$$

We have to calculate ω_2 (i.e. the angular frequency of first line of this series).

$\because$
$$\omega_2 = 2\pi RcZ^2\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right] \qquad \text{(In S.I.)}$$

or
$$\omega_2 = RZ^2\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right] \qquad \text{(in Gaussian unit)}$$

or
$$\omega_2 = \frac{RZ^2}{n^2} - \frac{RZ^2}{(n+1)^2}$$

or
$$\omega_2 = \frac{RZ^2}{n^2} - \Delta\omega \qquad \text{(from equ. (i))}$$

$$= \frac{RZ^2}{Z^2\left(\sqrt{\frac{R}{\Delta\omega}}-1\right)^2} - \Delta\omega \qquad \text{(from equ. (ii))}$$

$$= \frac{\Delta\omega\, 2Z\sqrt{\frac{R}{\Delta\omega}}-1}{\left(2Z\sqrt{\frac{R}{\Delta\omega}}-1\right)}$$

The corresponding value of wavelength is

$$\lambda = \frac{2\pi c}{\omega_1} = \frac{2\pi c}{\Delta\omega}\frac{\left(Z\sqrt{\frac{R}{\Delta\omega}}-1\right)^2}{\left(2Z\sqrt{\frac{R}{\Delta\omega}}-1\right)}$$

On putting the values, we get $\lambda = 0.468\ \mu m = 0.47\ \mu m$

YOUR STEP

A doubly ionized lithium ion, Li^{+2} (a lithium atom with two electrons removed), behaves very much like a hydrogen, atom except that the nuclear charge is three times as large.

(a) What is the ground level energy of Li^{+2}? How does the ground level energy of Li^{+2} is acquired, compare to the ground level energy of the hydrogen atom?

(b) What is the ionisation energy of Li^{+2}?

{(a) −122 eV, X9 (b) 122 eV}

§ 6.37

> **CONCEPT**

Binding energy of electron $= -E = -$(total energy of electron)

The problem is solved by following steps :

Step I : Determine atomic number (Z).

$$\because \quad \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \qquad \text{(in S.I.)}$$

or $$\frac{1}{\lambda} = (1.097\times10^7)\, Z^2\left(\frac{1}{2^2}-\frac{1}{5^2}\right) \qquad \text{[For third line]}$$

or $$\frac{1}{108.5\times10^{-9}} = (1.097\times10^7)\, Z^2\left(\frac{1}{4}-\frac{1}{25}\right)$$

After solving, we get $Z = 2$

Step II : Find total energy of electron in ground state (*i.e.* $n = 1$) :

As we know,

$$E = \frac{-13.6\, Z^2\ \text{eV}}{n^2} = -\frac{13.6\times2^2}{n^2}\ \text{eV}$$

or $$E = -\frac{54.4}{(1)^2}\ \text{eV} = -54.4\ \text{eV}$$

Hence, binding energy is $E_b = -E = 54.4$ eV

i.e., binding energy of He^+ ion.

YOUR STEP

(a) A hydrogen like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) of this atom. Also calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is –13.6 eV.

(b) When a beam of 10.6 eV photons of intensity $2\ W/m^2$ falls on a platinum surface of area $1.0 \times 10^{-4}\ m^2$ and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take $1\ eV = 1.6 \times 10^{-19}$ J

[(a) 2, 4, –217.6 eV, 10.58 eV (b) 6.25×10^{11}, 0, 5.0 eV]

§ 6.38

➣ **CONCEPT**

To remove both electrons from the ground state of He, the energy is supplied in two steps.

$$He \xrightarrow{\Delta E_b = E_0} He^+ \xrightarrow{\Delta E_b'} He^{++}$$

Total energy supplied is

$$\Delta E = \Delta E_b + \Delta E_b'$$

Here $\Delta E_b = 24.6$ eV (from problem)

$\Delta E_b' = 54.4$ eV (from result of problem 6.37)

$\therefore$ $\Delta E = 24.6\ eV + 54.4\ eV = 79\ eV$

YOUR STEP

The longest wavelength in the Lyman series is 121.5 nm and the shortest wavelength in the Balmer series is 364.6 nm. Use the figures to find the longest wavelength of light that could ionize hydrogen.

[91.13 nm]

§ 6.39

➣ **CONCEPT**

When photon collides with electron. A part of energy of photon is spent to remove out electron from the orbit (This energy is equal to binding energy of electron) and remaining part of energy of photon is spent in giving kinetic energy to electron. Mathematically,

$$E = E_b + T_e$$

or
$$\frac{hc}{\lambda} = E_b + T_e$$

$\therefore$
$$T_e = \frac{hc}{\lambda} - E_b$$

SOLUTION : The problem is solved in following steps.

Step I : Determine binding energy of electron :

As we know
$$E = \frac{-13.6\ Z^2}{n^2} = \frac{-13.6\ (2)^2}{(1)^2} \quad \text{(For ground state)}$$
$$= -54.4\ eV$$

$\therefore$ The binding energy of electron is $E_b = -E = 54.4$ eV

Step II : Determine kinetic energy of electron.

$\because$
$$T_e = \frac{hc}{\lambda} - E_b$$

$$= \frac{1242 \text{ nmeV}}{18 \text{ nm}} - 54.4 \text{ eV}$$

$$= (69 - 54.4) \text{ eV} = 14.6 \text{ eV}$$

$$= 23.36 \times 10^{-19} \text{ joule}$$

$\because$ $$T_e = \frac{1}{2} mv^2$$

$\therefore$ $$v = \sqrt{\frac{2T_e}{m}}$$

$$= \sqrt{\frac{2 \times 23.36 \times 10^{-19}}{9.1 \times 10^{-31}}} \text{ m/s} = 2.3 \times 10^6 \text{ m/s}$$

YOUR STEP

An electron in an unexcited hydrogen atom acquired an energy of 12.1 eV. To what energy level did it jump ? How many spectral lines may be emitted in the course of its transition to lower energy levels ? Calculate the corresponding wavelengths?

{ $n = 3$, 3 spectral lines}

§ 6.40

> CONCEPT

For minimum kinetic energy of bombarding particle in the case of inelastic collision, one of the atom must be in minimum excited state after collision.

For solving the problem based upon atomic collision, two basic equations are provided.

(i) On the basis of conservation principle of momentum

(ii) On the basis of energy conservation principle.

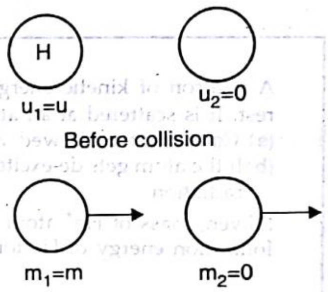

Fig. 6.40A

SOLUTION :

According to conservation principle of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

or $$mu_1 + 0 = mv_1 + mv_2$$

$\therefore$ $$u = v_1 + v_2 \quad \text{...(i)}$$

According to conservation principle of energy.

$$\frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2 + \Delta E$$

or $$\frac{1}{2} mu_1^2 + \frac{1}{2} mu_2^2 = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2 + \Delta E$$

or $$u_1^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m} \quad \text{...(ii)}$$

or $$(v_1 + v_2)^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m} \quad \text{[From eqn (i)]}$$

or $$v_1^2 + v_2^2 + 2v_1v_2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$$

or $$2v_1v_2 = \frac{2\Delta E}{m}$$

or $$v_1v_2 = \frac{\Delta E}{m}$$

$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$

or $\quad (v_1 - v_2)^2 = u^2 - \dfrac{4\Delta E}{m}$

$\therefore \quad v_1 - v_2 = \sqrt{u^2 - \dfrac{4\Delta E}{m}}$

But $v_1 - v_2$ must be real. For real value of $v_1 - v_2$, $u^2 - \dfrac{4\Delta E}{m} \geq 0$

or $\quad \dfrac{1}{2}mu^2 \geq 2\Delta E$

or $\quad T \geq 2\Delta E$

$\therefore \quad T_{min} = 2\Delta E \qquad \text{...(iii)}$

For the minimum excitation energy,
$n_1 = 1, n_2 = 2$

$$\therefore \quad \Delta E = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)\text{eV} = 13.6\left(1 - \frac{1}{4}\right)\text{eV}$$

$$= 13.6 \times \frac{3}{4}\text{ eV} = 10.2\text{ eV}$$

$$\therefore \quad T_{min} = 2 \times \Delta E = 20.4\text{ eV}$$

YOUR STEP

A neutron of kinetic energy $E_0 = 65$ eV collides inelastically with a singly ionised helium atom at rest. It is scattered at an angle of 90° with respect to its original direction.
(a) Calculate the allowed values of the energy of the neutron and that of the atom after collision.
(b) If the atom gets de-excited subsequently by emitting radiation, Calculate frequencies of the emitted radiation.
(Given, mass of He^+ atom = 4 × mass of neutron)
Ionisation energy of H-atom = 13.6 eV

(a) Allowed values of energy of neutron = 6.36 eV, $\dfrac{71}{225}$ eV
Allowed values of energy of He^+ atom = 17.84 eV, $\dfrac{3674}{225}$ eV
(b) 9.89×10^{15} Hz, 1.17×10^{16} Hz

§ 6.41

➢ **CONCEPT**

During emission of photon, two basic laws are applicable :
(i) Conservation principle of momentum.
(ii) Conservation principle of energy.
When recoiling speed of atom in not considered, then momentum conservation principle is not applicable.

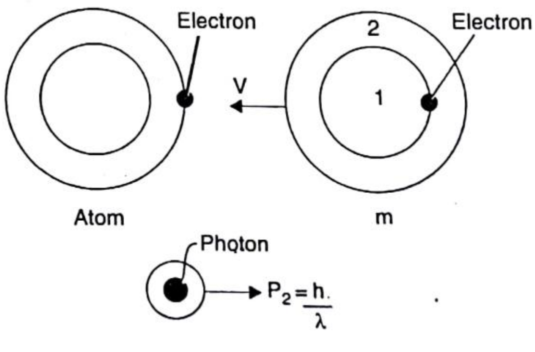

Fig. 6.41A

SOLUTION : The problem is similar to gun firing.

The momentum of photon is $\dfrac{h}{\lambda}$.

According to conservation principle of momentum,

$$\vec{p}_i = \vec{p}_f$$

$$0 = mv - \frac{h}{\lambda}$$

$$\therefore \quad mv = \frac{h}{\lambda} \qquad \text{...(i)}$$

Here v is recoiling speed of H-atom.
According to energy conservation principle,

$$E_i = E_f$$

The initial energy of atom is $E_i = -\frac{13.6}{n^2}\text{ eV} = -\frac{13.6}{(1)^2}\text{ eV}$

$$\therefore \quad E_i = -\frac{13.6}{4}\text{ eV}$$

The final energy of the system (atom + photon) is
E_f = K.E. of atom + energy of electron + photon energy

$$= \frac{1}{2}mv^2 - \frac{13.6}{(1)^2}\text{ eV} + \frac{hc}{\lambda}$$

$$\because \quad E_i = E_f$$

$$\therefore \quad -\frac{13.6}{4}\text{ eV} = \frac{1}{2}mv^2 - 13.6\text{ eV} + \frac{hc}{\lambda}$$

or $$\left(13.6 - \frac{13.6}{4}\right)\text{eV} = \frac{1}{2}mv^2 + \frac{hc}{\lambda}$$

or $$\left(13.6 \times \frac{3}{4}\right)\text{eV} = \frac{1}{2}mv^2 + mvc \qquad \text{[From eqn (i)]}$$

or $$10.2 \times 1.6 \times 10^{-19}\text{ joule} = \frac{1}{2}mv^2 + mvc$$

Here $m = 1.67 \times 10^{-27}$ kg, $c = 3 \times 10^8$ m/s

After solving, $v = 3.25$ m/s

YOUR STEP

An electron of a stationary hydrogen atom passes from the fifth energy level to the fundamental state. What velocity did the atom acquire as the result of photon emission ? What is the recoil energy?

$$\left\{\text{Recoil energy} = \frac{(24)^2 h^2 R^2}{2 \times (25)^2 M}, \text{ Velocity of atom} = \frac{24hR}{25M}\right\}$$

§ 6.42

➢ **CONCEPT**

When atom during transition of electron remains fixed. Then

$$\frac{1}{\lambda_{\text{stationary}}} = R\left(1 - \frac{1}{4}\right)$$

$$\therefore \quad \lambda_{\text{stationary}} = \frac{4}{3R} \qquad \text{...(i)}$$

$$\therefore \quad \lambda_{\text{stationary}} = \frac{4}{3 \times 1.097 \times 10^7} = 1.215 \times 10^{-7}\text{ m}$$

From previous problem

$$mv = \frac{h}{\lambda}$$

$\therefore \quad \lambda = \dfrac{h}{mv} = \dfrac{6.623 \times 10^{-34}}{1.67 \times 10^{-27} \times 3.25} = 1.22 \times 10^{-7}$ m

$$\therefore \quad \Delta E = \left\{ \frac{\dfrac{hc}{\lambda_{\text{stationary}}} - \dfrac{hc}{\lambda}}{\dfrac{hc}{\lambda_{\text{stationary}}}} \right\} \times 100$$

On putting the values, we get $\% \, \Delta E = 0.55 \times 10^{-6} \, \%$

YOUR STEP

Consider an excited hydrogen atom in state n moving with a velocity v $(v << c)$. It emits a photon in the direction of its motion and changes its state to a lower state m. Apply momentum and energy conservation principles to calculate the frequency ν of the emitted radiation. Compare this with the frequency ν_0 emitted if the atom were at rest.

$\left\{ \nu = \nu_0 \left(1 + \dfrac{v}{c} \right) \right\}$

§ 6.43

> **CONCEPT**

The energy of photon generated in He^+ ion is

$$\Delta E = 13.6 \, Z^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) \text{eV for first line}$$

$$= 13.6 \times 2^2 \times \frac{3}{4} \text{ eV} = 40.8 \text{ eV}$$

A part of this photon energy is spent as binding energy of electron of H-atom. The remaining part of energy of photon provides kinetic energy to liberated electron.

$$\therefore \quad \Delta E = \Delta E_b + \frac{1}{2} mv^2$$

or $\quad 40.8 \text{ eV} = 13.6 \text{ eV} + \dfrac{1}{2} mv^2 \qquad (\because \, E_b = 13.6 \text{ eV for H-atom})$

or $\quad \dfrac{1}{2} mv^2 = (40.8 - 13.6) \text{ eV}$

or $\quad \dfrac{1}{2} mv^2 = 27.2 \times 1.6 \times 10^{-19} \text{ joule}$

$$\therefore \quad v = \sqrt{\frac{2 \times 27.2 \times 1.6 \times 10^{-19}}{m}} = \sqrt{\frac{2 \times 27.2 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} \text{ m/s} = 3.1 \times 10^6 \text{ m/s}$$

YOUR STEP

Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find :

(a) The energy of the photons causing the photoelectric emission.
(b) The quantum numbers of the two levels involved in the emission of these photons.
(c) The change in the angular momentum of the electron in the hydrogen atom in the above transition,
(d) The recoil speed of the emitting atom assuming it to be at rest before the transition. (Ionisation potential of hydrogen is 13.6 eV).

{(a) 2.55 eV (b) 4 and 2 (c) $-\dfrac{h}{\pi}$ (d) 0.814 m/sec}

§ 6.44

➢ **CONCEPT**

From the concept of Doppler's shift :

$$\Delta\lambda = \lambda\left(\frac{v}{c}\right)\cos\theta \qquad \text{...(i)}$$

where v is the relative velocity between source and the observer and θ is angle between the velocity and line of observation.

SOLUTION : $\because \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ for H-atom.

For Lyman series, $n_1 = 1, n_2 = 2$ for first line

$$\therefore \qquad \frac{1}{\lambda} = R\left(1 - \frac{1}{4}\right) = \frac{3R}{4}$$

$$\therefore \qquad \lambda = \frac{4}{3R} \qquad \text{(In S.I.)}$$

In Gaussian unit

$$\lambda = \frac{4}{3R}(2\pi c) = \frac{8\pi c}{3R} \qquad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\Delta\lambda = \left(\frac{8\pi c}{3R}\right)\left(\frac{v}{c}\cos\theta\right)$$

$$\therefore \qquad v = \frac{3R\,\Delta\lambda}{8\pi\cos\theta}$$

On putting the values, we get $v = 0.7 \times 10^6$ m/s

> **YOUR STEP**
>
> Lines of 6877Å, 4989Å and 4548Å are observed in the visible range of the spectrum of a certain galaxy. To what substance do they belong? What can you say about the motion of this galaxy?
>
> {The motion of the galaxy moves away from us}

§ 6.45

➢ **CONCEPT**

For infinite potential of width l,

$$P = \frac{\pi\hbar n}{l}$$

SOLUTION : (a) The potential energy in infinite potential well is zero. So, energy is only in the form of kinetic energy.

$$\therefore \qquad E = T = \frac{p^2}{2m} = \frac{\left(\frac{\pi\hbar n}{l}\right)^2}{2m} = \frac{\pi^2\hbar^2 n^2}{2ml^2}$$

(b) As we know

$$\int_C p\, d\theta = 2\pi\hbar n$$

or

$$p\int_0^{2\pi} d\theta = 2\pi\hbar n$$

$$\therefore \qquad p(2\pi) = 2\pi\hbar n$$

$$\therefore \qquad p = n\hbar$$

$\therefore$ $$mur = n\hbar$$

$\therefore$ $$T = \frac{mu^2}{2} = \frac{n^2\hbar^2}{2mr^2}$$

$\therefore$ $$E = T = \frac{n^2\hbar^2}{2mr^2}$$

(c) $\because$ $$U = \frac{\alpha x^2}{2}$$

$\therefore$ $$F = -\frac{\partial U}{\partial x}$$

$\therefore$ $$F = -\alpha x$$

or $$\frac{dp}{dt} = -\alpha x$$

or $$m\frac{du}{dt} = -\alpha x$$

or $$m\frac{du}{dx}\frac{dx}{dt} = -\alpha x$$

or $$m\int_0^u u\,du = -\int_a^x \alpha x\,dx$$

$\therefore$ $$\frac{mu^2}{2} = \alpha\,(a^2 - x^2)/2$$

$\therefore$ $$T = \frac{1}{2}mu^2 = \alpha\,(a^2 - x^2)/2$$

Total energy is $$E = T + U$$

$\therefore$ $$E = \alpha\frac{(a^2 - x^2)}{2} + \frac{\alpha x^2}{2} = \frac{\alpha a^2}{2}$$

$\therefore$ $$T = \frac{1}{2}mu^2 = \frac{\alpha}{2}(a^2 - x^2)$$

or $$\frac{p^2}{2m} = \frac{\alpha}{2}(a^2 - x^2)$$

or $$p^2 = m\alpha\,(a^2 - x^2)$$

or $$p = \sqrt{m\alpha}\sqrt{a^2 - x^2}$$

$\because$ $$\int_c p\,dx = 2\pi n\hbar$$

or $$\int_{-a}^{a}\sqrt{m\alpha}\sqrt{a^2 - x^2}\,dx = 2\pi n\hbar$$

$\therefore$ $$\sqrt{m\alpha}\,\frac{a^2}{2} = n\hbar$$

$\therefore$ $$E = \alpha\frac{a^2}{2} = \frac{\alpha n\hbar}{\sqrt{m\alpha}} = n\hbar\sqrt{\frac{\alpha}{m}}$$

(d) $$U = -\frac{\alpha}{r}$$

or $$F = -\frac{\partial U}{dr} = \frac{\alpha}{r^2}$$

or $$\frac{mv^2}{r} = \frac{\alpha}{r^2}$$

or $$mv^2 = \frac{\alpha}{r}$$

or $$\frac{1}{2}mv^2 = \frac{\alpha}{2r}$$

or $$T = \frac{\alpha}{2r}$$

or $$\frac{p^2}{2m} = \frac{\alpha}{2r}$$

or $$p^2 = \frac{m\alpha}{r}$$

or $$p = \frac{\sqrt{m\alpha}}{\sqrt{r}}$$

Total energy is $$E = T + U = \frac{\alpha}{2r} - \frac{\alpha}{r} = -\frac{\alpha}{2r}$$

According to Bohr Sommerfield rule,

$$\int_c p\, ds = 2\pi n\hbar$$

or $$\int_c \frac{\sqrt{m\alpha}}{\sqrt{r}}\, ds = 2\pi \hbar n$$

or $$\frac{\sqrt{\alpha m}}{\sqrt{r}}\, 2\pi r = 2\pi n\hbar$$

or $$\sqrt{m\alpha}\,\sqrt{r} = n\hbar$$

$$r = \frac{n^2\hbar^2}{m\alpha}$$

$\therefore$ $$E = -\frac{\alpha}{2r} = -\frac{\alpha}{2\left(\dfrac{n^2\hbar^2}{m\alpha}\right)} = -\frac{m\alpha^2}{2n^2\hbar^2}$$

YOUR STEP

It can be shown that the probability $P(r)$ that the electron in the ground state of the hydrogen atom will be detected inside a sphere of radius r is given by

$$P(r) = 1 - e^{-2x}(1 + 2x + 2x^2)$$

in which x, a dimensionless quantity, is equal to $\frac{r}{a}$. Find r for $P(r) = 0.90$.

[2.66]

§ 6.46

➢ CONCEPT

In the case of Bohr's model for H-atom, hydrogen nucleus (proton) remains stationary while orbit electron revolves around it.

In actual practice, both nucleus and electron revolve around their common centre of mass, which is very close to the nucleus because the nucleus mass is much larger than that of the electron (shown in fig. 6.46A).

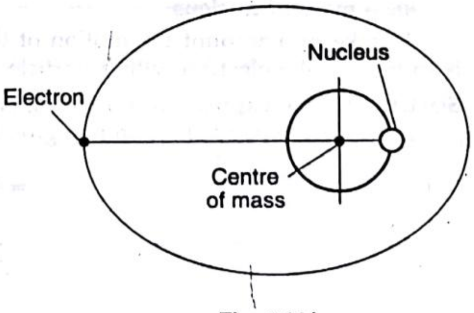

Fig. 6.46A

A system of this kind is equivalent to a single particle of mass μ that revolves around the position of the heavier particle (nucleus).

The mass μ is known as reduced mass of the system. Calculation for reduced mass :

From fig. 4.46B $\qquad r_e + r_n = r$

The angular momentum of the system about centre of mass C is

$$L = m_n r_n^2 \omega + m_e r_e^2 \omega \qquad \text{...(i)}$$

But from the concept of centre of mass,

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

If origin is taken as centre of mass C. Then $r_{cm} = 0$,

$$m_1 = m_n, r_1 = r_n$$

Also, $$m_2 = m_e, r_2 = -r_e$$

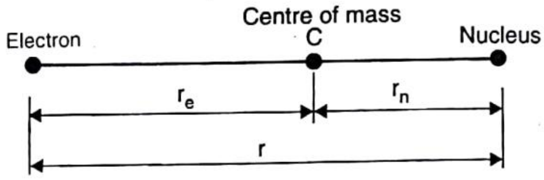

Fig. 6.46B

$\therefore$ $$0 = \frac{m_n r_n - m_e r_e}{m_n + m_e}$$

$\therefore$ $$m_n r_n = m_e r_e$$

$\therefore$ $$r_n = \frac{m_e}{m_n} r_e$$

But $$r = r_e + r_n$$

or $$r = r_e + \frac{m_e}{m_n} r_e = \frac{(m_e + m_n)\, r_e}{m_n}$$

$\therefore$ $$r_e = \frac{m_n r}{m_e + m_n}$$

Similarly, $$r_n = \frac{m_e r}{m_e + m_n}$$

If the mass of electron is taken μ as such the angular momentum of electron about nucleus is same as the angular momentum of system (electron + nucleus) about the centre of mass. Then μ is known as reduced mass.

$\therefore$ The angular momentum about the nucleus is $L = \mu r \omega^2$ ...(ii)

From eqn (i) and (ii), we get

$$\mu r \omega^2 = m_e r_e \omega^2 + m_n r_n \omega^2$$

or $$\mu r = m_e r_e + m_n r_n$$

or $$\mu r = \frac{m_e m_n r}{m_e + m_n} + \frac{m_e m_n r}{m_e + m_n}$$

$\therefore$ $$\mu = \frac{m_e m_n}{m_e + m_n}$$

Here m_e = mass of electron

m_n = mass of nucleus

To take into account the motion of the nucleus in the hydrogen atom, then at all we need, is to replace the electron with a particle of mass μ.

SOLUTION : The expression of binding energy for stationary nucleus hydrogen model is

$E_b = -$ total energy of electron in ground state

$$= -\left(-\frac{me^4}{2\hbar^2 \, (4\pi\varepsilon_0)^2}\right)$$

$\therefore$ $$E_b = \frac{me^4}{2\hbar^2(4\pi\varepsilon_0)^2} \quad \text{(In SI)}$$

In Gaussian unit, $\frac{1}{4\pi\varepsilon_0}$ is taken as one

$\therefore$ $$E_b = \frac{me^4}{2\hbar^2} \quad \text{(In Gaussian unit)}$$

If motion of nucleus is taken in account, then the mass of electron m is replaced by reduced mass μ.

$\therefore$ $$E_b' = \frac{\mu e^4}{2\hbar^2} \quad \text{(In Gaussian unit)}$$

$\therefore$ $$E_b' = \left(\frac{mM}{M+m}\right)\frac{e^4}{2\hbar^2}$$

Here m = mass of electron
M = mass of nucleus

$\therefore$ $$\frac{\Delta E_b}{E_b} = \frac{E_b - E_b'}{E_b} = 1 - \frac{E_b'}{E_b}$$

or $$\frac{\Delta E_b}{E_b} = 1 - \frac{\mu}{m} = 1 - \frac{mM}{(m+M)\,m}$$

or $$\frac{\Delta E_b}{E_b} = \frac{m}{m+M}$$

But $$M >> m$$

$\therefore$ $$m + M \approx M$$

$\therefore$ $$\frac{\Delta E_b}{E_b} \approx \frac{m}{M}$$

On putting the values, we get

$$\frac{\Delta E}{E_b} \approx 0.055\%$$

The expression for Rydberg constant for stationary model of nucleus is

$$R = \frac{me^4}{2\hbar^3} \quad \text{(In Gaussian unit)}$$

So Rydberg constant for non-stationary nucleus is

$$R' = \frac{\mu e^4}{2\hbar^3}$$

$\therefore$ $$\frac{\Delta R}{R} = \frac{R - R'}{R} = 1 - \frac{R'}{R} = 1 - \frac{\mu}{m}$$

$$= \frac{m}{m+M} \approx \frac{m}{M} \approx 0.055\%$$

YOUR STEP

A hypothetical, hydrogen like atom consists of a nucleus of mass m_1 and charge $(+Ze)$ and a mu-meson of mass m_2 and charge $(-e)$. Using Bohr's theory, derive an expression for distance between nucleus and mu-meson for principal quantum number n and derive a relation for energy also. Hence, obtain expression for reduced mass.

$$\left\{ r = \frac{\varepsilon_0 n^2 h^2 (m_1 + m_2)}{\pi m_1 m_2 Z e^2},\ m_0 = \frac{m_1 m_2}{m_1 + m_2},\ E = -\frac{1}{8}\cdot\frac{m_1 m_2 Z^2 e^4}{(m_1 + m_2)\,\varepsilon_0^2 n^2 h^2} \right\}$$

§ 6.47

(a) For non-stationary nucleus model of H-atom, the binding energy is

$$E_b = \frac{\mu e^4 Z^2}{2\hbar^2 n^2} \qquad \text{(In Gaussian unit)}$$

For H-atom,

$$\mu = \frac{mM_H}{m + M_H}$$

$$Z = 1$$

$$n = 1$$

$$\therefore \quad E_{bH} = \frac{(mM_H)\, e^4}{(m + M_H)\, 2\hbar^2}$$

For heavy hydrogen atom (deuterium)

$$Z = 1$$

$$\mu = \frac{mM_D}{m + M_D}, \qquad n = 1$$

$$\therefore \quad E_{bD} = \frac{mM_D\, e^4}{(m + M_D)} \frac{(1)^2}{(1)^2}$$

$$\therefore \quad E_{bD} - E_{bH} = \frac{e^4 m}{2\hbar^2}\left[\frac{M_D}{m + M_D} - \frac{M_H}{m + M_H}\right]$$

On putting the values, we get

$$E_{bD} - E_{bH} = 3.7 \text{ meV}$$

(b) As we know

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \text{For H-atom}$$

Here

$$R = \frac{e^4 \mu}{4\pi \hbar^3 C} \qquad \text{In Gaussian unit}$$

For Lyman series

$n_1 = 1, n_2 = 2$ (For first line)

$$\therefore \quad \frac{1}{\lambda} = R\left(1 - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\therefore \quad \lambda = \frac{4}{3R} = \frac{16\pi \hbar^3 c}{3\mu e^4}$$

$$\therefore \quad \lambda_H = \frac{16\pi \hbar^3 c}{3e^4 \dfrac{m M_H}{m + M_H}} = \frac{16\pi\hbar^3 c\,(m + M_H)}{3e^4\, mM_H}$$

Similarly for heavy hydrogen,

$$\lambda_D = \frac{16\pi \hbar^3\, c\,(m + M_D)}{3e^4\, mM_D}$$

$$\therefore \quad \lambda_D - \lambda_H = \frac{16\pi \hbar^3 H}{3e^4\, m}\left[\left(\frac{m + M_H}{M_H}\right) - \left(\frac{m + M_D}{M_D}\right)\right]$$

On putting the values, we get $\therefore \quad \lambda_H - \lambda_D = 33 \text{ pm}$

YOUR STEP

A mixture of ordinary hydrogen and tritium, a hydrogen, isotope whose nucleus is approximately 3 times more massive than ordinary hydrogen, is excited and its spectrum observed. How far apart in wavelength will the H_α lines of the two kinds of hydrogen be ?

(0.238 nm)

§ 6.48

➢ **CONCEPT**

The concept is similar to problem 6.47

SOLUTION : (a) The reduced mass of system is

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$m_1 = m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$m_2 = 207\, m_e = 207 \times 9.1 \times 10^{-31} \text{ kg}$$

The separation between particles is

$$r = \left(\frac{n^2 \hbar^2}{e^2}\right)\left(\frac{m_1 + m_2}{m_1 m_2}\right)\frac{1}{Z}$$

$\therefore$
$$r = \frac{n^2 \hbar^2}{e^2 \mu Z} \qquad \left(\because \mu = \frac{m_1 m_2}{m_1 + m_2}\right)$$

Here $n = 1,\ Z = 1$

On putting the values, we get $r = 0.285$ pm

The binding energy is
$$E_b = \frac{\mu e^4}{2\hbar^2 (4\pi\,\varepsilon_0)^2}$$

On putting the values, we get

$$E_b = 2.53 \text{ keV} \qquad \text{(binding energy)}$$

$\because$
$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R\left(1 - \frac{1}{4}\right)$$

$\therefore$
$$\lambda = \frac{4}{3R}$$

Here
$$R = \frac{3e^4 \mu}{4\pi \hbar^3 C} \qquad \text{(In Gaussian unit)}$$

On putting the values, we get $\lambda = 0.65$ pm

(b) The reduced mass of system is

$$\mu = \frac{m_1 m_2}{m_1 m_2}$$

Here
$$m_1 = m_2 = m = 9.1 \times 10^{-31} \text{ kg}$$

$\therefore$
$$\mu = \frac{m}{2} = \frac{9.1}{2} \times 10^{-31} \text{ kg}$$

From part (a),

$$\lambda = \frac{4}{3R}$$

Here
$$R = \frac{3e^4 \mu}{4\pi \hbar^3 C} \qquad \text{(In Gaussian unit)}$$

On putting the value, we get

$$\lambda = 0.243\ \mu m$$

From part (a), $E_b = \dfrac{\mu e^4}{2\hbar^2 (4\pi\, \varepsilon_0)^2}$

On putting the values $E_b = 6.8$ eV

From part (a), the separation between particles is

$$r = \frac{n^2 \hbar^2}{e^2 \mu Z}$$

Here $n = 1$, $Z = 1$.

On putting the values, $r = 106$ pm

YOUR STEP

A quantum particle known as μ mason has a charge equal to that of an electron and its mass is 208 times mass of the electron. It moves in a circular orbit around a nucleus of charge $+3e$. Assuming that the Bohr's model is applicable to this system,

(a) Derive an expression for the radius of the nth Bohr orbit,

(b) Find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for a hydrogen atom

(c) Find the wavelength of the radiation emitted when the μ-mason jumps from the third orbit to the first orbit. Take the mass of nucleus to be infinite.

$$\left\{ \begin{array}{l} \text{(a) } r_\mu = \dfrac{n^2 h^2\, \varepsilon_0}{624\, \pi\, m_e\, e^2} \\ \text{(b) } n \approx 25 \\ \text{(c) } \lambda = 55 \text{ pm} \end{array} \right\}$$

RADIOACTIVITY

§ 6.214

➢ CONCEPT

Decay constant λ is defined as probability of decay of radioactive nucleus per unit time.

i.e., $$\lambda = \frac{dN}{Ndt}$$

SOLUTION : (a) Let at $t = 0$, the number of active nuclei is N_0.

$\therefore$ The number of nuclei after time t is $N = N_0 e^{-\lambda t}$

$\therefore$ The number of nuclei decay in time t is $N_1 = N_0 - N = N_0 (1 - e^{-\lambda t})$

$\therefore$ The probability of decay is $$P = \frac{N_1}{N_0} = \frac{N_0 (1 - e^{-\lambda t})}{N_0}$$

$\therefore$ $$P = (1 - e^{-\lambda t})$$

(b) Let us consider N_0 number of nuclei at $t = 0$

$\therefore$ $$\frac{dN}{dt} = -\lambda N \quad \text{...(i)}$$

The number of nuclei in time interval t to $t + dt$ is dN.

$\therefore$ The sum of the lives of dN nuclei is $-t\,dN$.

$\therefore$ Average life is $$T = \frac{\int_0^\infty -t\,dN}{N_0} \quad \text{(from equ. (i))}$$

$\therefore$ $$T = \frac{\int_0^\infty t\lambda N dt}{N_0} \qquad \therefore \quad T = \frac{\int_0^\infty t\lambda N_0 e^{-\lambda t}\,dt}{N_0}$$

$\therefore$ $$= \frac{\lambda N_0 \left[\frac{-te^{-\lambda t}}{\lambda}\right]_0^\infty - \int_0^\infty \frac{e^{-\lambda t}\,dt}{-\lambda}}{N_0} = \frac{N_0}{\lambda N_0} = \frac{1}{\lambda}$$

$\therefore$ $$T = \frac{1}{\lambda}$$

YOUR STEP

The element curium $_{96}Cm^{248}$ has a mean life of 10^{13} second. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The masses involved in decay are as follows : $_{96}Cm^{248} = 248.07220u$, $_2He^4 = 4.002603$ u and $_{94}Pu^{244} = 244.064100u$. Calculate the power output from a sample of 10^{20} cm atoms. Given $1u = 931$ MeV/C^2.

(3.3×10^{-5} watt)

§ 6.215

➢ CONCEPT

$$\therefore \quad T_{1/2} = \frac{ln2}{\lambda} = \frac{0.693}{\lambda}$$

$$\therefore \quad \lambda = \frac{0.693}{T_{1/2}} \text{ per day}$$

$$= \frac{0.693}{71.3} \text{ per day}$$

$$= 9.722 \times 10^{-3} \text{ per day}$$

SOLUTION : As we know, $N = N_0 e^{-\lambda t}$

The number of nuclei decay in time t is

$$N_1 = N_0 - N = N_0 (1 - e^{-\lambda t})$$

$\therefore$ fraction is

$$\frac{N_1}{N_0} = \frac{N_0 (1 - e^{-\lambda t})}{N_0} = 1 - e^{-\lambda t}$$

Here $t = 30$ day

$$\therefore \quad \frac{N_1}{N_0} = 0.253$$

YOUR STEP

A laboratory has 1.49 μg of pure $_7N^{13}$, which has a half life of 10 minute (600 sec).

(a) How many nuclei are present initially?

(b) What is the activity initially?

(c) What is the activity after 1 hour?

(d) After approximately how long will the activity drop to less than one per second?

{(a) $N_0 = 6.90 \times 10^{16}$ nuclei (b) $8 \times 10^{13}\ \text{sec}^{-1}$ (c) $1.25 \times 10^{12}\ \text{sec}^{-1}$ (d) 2.76×10^4 sec}

§ 6.216

➢ CONCEPT

The number of nuclei decay is equal to the number of β-particle emitted.

SOLUTION : $\because T_{1/2} = \frac{0.693}{\lambda}$

$$\therefore \quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{15} \text{ per hour}$$

$$= 0.04621 \text{ per hour}$$

From previous problem, the number of nuclei decay in time t is

$$N_1 = N_0 (1 - e^{-\lambda t})$$

Here t = one hour. ...(i)

Also N_1 is equal to number of β-particles emitted.

Calculation for N_0

The number of mole in 1μg of $Na^{24} = \frac{1 \times 10^{-6}}{24}$

$\therefore$ Number of nuclei in 1 μg is $N_0 = \frac{10^{-6}}{24} \times 6.022 \times 10^{23} = 2.51 \times 10^{16}$

On putting the value of N_0 in equation, (i) we get,

$$N_1 = 1.13 \times 10^{15}$$

YOUR STEP

There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half life of neutron is 700 second, what fraction of neutrons will decay before they travel a distance of 10 m? Given mass of neutron $= 1.675 \times 10^{-27}$ kg.

$$\left\{\frac{\Delta N}{N} = 3.96 \times 10^{-6}\right\}$$

§ 6.217

➢ **CONCEPT**

The number of β-particles emitted or number of nuclei decay in time t is

$$N_1' = N_0 (1 - e^{-\lambda t})$$

According to problem, $N_1' = N_1$ at $t = t_1$

$\therefore$ $$N_1 = N_0 (1 - e^{-\lambda t_1}) \quad \text{...(i)}$$

Also, at $N_1' = 2.66 N_1$ at $t = 3t_1$

$\therefore$ $$2.66 N_1 = N_0 (1 - e^{-3\lambda t_1}) \quad \text{...(ii)}$$

From equ. (i) and (iii), we get

$$2.66 N_0 (1 - e^{-\lambda t_1}) = N_0 (1 - e^{-3\lambda t_1})$$

or $$2.66 (1 - e^{-\lambda t_1}) = 1 - e^{-3\lambda t_1}$$

or $$2.66 - 2.66 e^{-\lambda t_1} = 1 - e^{-3\lambda t_1}$$

or $$2.66 - 1 = 2.66 e^{-\lambda t_1} - e^{-3\lambda t_1}$$

or $$1.66 = 2.66 e^{-\lambda t_1} - e^{-3\lambda t_1}$$

or $$1.66 = 2.66 e^{-\lambda t_1} - (e^{-\lambda t_1})^3$$

Let $$e^{-\lambda t_1} = y$$

$\therefore$ $$y^3 - 2.66 y + 1.66 = 0$$

or $$(y - 1)(y^2 + y - 1.66) = 0$$

But $$y \neq 1$$

$\therefore$ $$y^2 + y - 1.66 = 0$$

$\therefore$ $$y = \frac{-1 \pm \sqrt{1^2 + 4 \times 1 \times 1.66}}{2 \times 1}$$

But y has only positive values

$\therefore$ $$y = 0.882$$

or $$e^{-\lambda t_1} = y = 0.882$$

or $$-\lambda t_1 = ln\,(0.882)$$

$\therefore$ $$\lambda = -\frac{ln\,(0.882)}{t_1} = -\frac{ln\,(0.882)}{2} \qquad (\because t_1 = 2.0 \text{ second})$$

$\therefore$ $$T = \frac{1}{\lambda} = \frac{-2}{ln\,(0.882)} = 15.9 \text{ s} \approx 16 \text{ s}$$

YOUR STEP

A radioactive sample emits n β particles in 2 second. In next 2 sec it emits $0.75\, n\beta$ particle, what is the mean life of the sample?

$$\left\{\frac{1}{\lambda} = \frac{2}{ln\,(4/3)} \text{ sec}\right\}$$

§ 6.218

➢ **CONCEPT**

Activity is $A = \lambda N$

$\therefore$ $N = N_0 e^{-\lambda t}$

or $\lambda N = \lambda N_0 e^{-\lambda t}$ or $A = A_0 e^{-\lambda t}$ ...(i)

Here A = activity at instant t

A_0 = activity at $t = 0$

SOLUTION : According to problem,

$$A = \frac{A_0}{2.5}$$

$\therefore$ $A_0 = 2.5\,A$

from Eqn. (i) $A = A_0 e^{-\lambda t}$

or $A = 2.5\,A\,e^{-\lambda t}$

Here, $t = 7$ days

or $e^{7\lambda} = 2.5$

$\therefore$ $e^{7\lambda} = 2.5$

or $7\,\lambda = ln\,2.5$

$\therefore$ $$\lambda = \frac{ln\,2.5}{7}$$

$\therefore$ $$T_{1/2} = \frac{ln\,2}{\lambda} = \frac{ln\,2}{\frac{ln\,2.5}{7}} = 5.3 \text{ day}$$

YOUR STEP

Radon with an activity of 400 mC is placed into an ampoule. In what time after the ampoule is filled with the radon disintegrate at a rate of 2.22×10^9 disintegrations/second ?

{In 10.4 days}

§ 6.219

➢ **CONCEPT**

The concept is similar to previous problem.

SOLUTION : As we know, $A = A_0 e^{-\lambda t}$

or $A = A_0 e^{-\lambda T_{1/2}/2}$

or $$A = A_0 e^{-\frac{\lambda}{2}\frac{ln\,2}{\lambda}}$$

or $A = A_0 e^{-ln 2/2}$

or $$\frac{A}{A_0} = e^{-ln\,2/2}$$

$\therefore$ $A = A_0 e^{-ln\,2/2}$

Here $A_0 = 650$ particles per minute

$\therefore$ $$A = A_0\,(0.707) = \frac{A_0}{\sqrt{2}} = 650 \times 0.707$$

$= 459.6$ particle per minute

$= 460$ particles per minute $= 4.6 \times 10^2$ particle/min

YOUR STEP

When iron is irradiated with neutrons an isotope of iron is formed. This isotope is radioactive with a half life of 45 day. A steel piston ring of mass 18.5 g was irradiated with neutrons until its activity due to formation of this isotope was 10 μ curie. 20 days after the irradiation the ring was installed in an engine and after 70 days continuous use, the crankcase oil was found to have a total activity of 1.85×10^3 disintegration /sec. Determine the average mass of iron warn off the ring per day, assuming that all the metal removed from the ring accumulated in the oil.

1 curie $= 3.7 \times 10^{10}$ disintegration / second

{5.3 mg}

§ 6.220

➢ **CONCEPT**

∴ $$A = A_0 e^{-\lambda t}$$

or $$\frac{A}{A_0} = e^{-\lambda t}$$

or $$\frac{A}{A_0} - 1 = e^{-\lambda t} - 1$$

or $$\frac{A - A_0}{A_0} = e^{-\lambda t} - 1$$

or $$\frac{A_0 - A}{A_0} = 1 - e^{-\lambda t}$$

or $$\frac{A_0 - A}{A_0} \times 100 = (1 - e^{-\lambda t}) \times 100$$

or $$4 = (1 - e^{-\lambda t})$$

or $$4 = (1 - e^{-\lambda t}) \times 100$$

Here $$t = 1 \text{ hour}$$

∴ $$4 = (1 - e^{-\lambda}) \times 100$$

or $$0.04 = 1 - e^{-\lambda}$$

or $$e^{-\lambda} = 1 - 0.04 = 0.96$$

or $$-\lambda = ln\ (0.96)$$

∴ $$\lambda = -ln\ (0.96) = 0.04082 \text{ per hour}$$
$$= 1.1 \times 10^{-5} \text{ per second}$$

∴ mean life time is $$\tau = \frac{1}{\lambda} = \frac{1}{0.04082}$$
$$= 24.49659826 \text{ hour} = 24.5 \text{ hour}$$

YOUR STEP

A sample of ^{18}F is used internally as a medical diagnostic tool to look for the effects of the positron decay ($T_{1/2} = 110$ min). How long does it take for 99% of the ^{18}F to decay?

{12.2 hour}

§ 6.221

➢ **CONCEPT**

The number of α-particles emitted is equal to the number of nuclei decay.

The activity $$A = \lambda N$$

SOLUTION : The number of nuclei of U^{238} is

$$N = \frac{1}{238} \times 6.022 \times 10^{23} \text{ nuclei}$$

$\therefore$ According to problem,

$$A = 1.24 \times 10^4 \ \alpha\text{–particles per second}$$

$\therefore$ $$A = \lambda N$$

$\therefore$ $$\lambda = \frac{A}{N} = \frac{1.24 \times 10^4}{\frac{1}{238} \times 6.022 \times 10^{23}} = 4.90 \times 10^{-18} \text{ s}^{-1}$$

$\therefore$ $$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{4.90 \times 10^{-18}}$$

$$= 0.14114285 \times 10^{18} \text{ sec}$$

$$= \frac{1.41 \times 10^{17}}{60 \times 60 \times 24 \times 365} \text{ year} = 4.49 \times 10^9 \text{ year}$$

YOUR STEP

Measurements indicate that 27.83% of all rubidium atoms currently on earth are the radioactive ^{87}Rb isotope. The rest are the stable ^{85}Rb isotope. The half life of ^{87}Rb is 4.89×10^{10} years. Assuming that no rubidium atoms have been formed since, what percentage of rubidium atoms were ^{87}Rb when our solar system was formed 4.6×10^9 years ago?

(29.2%)

§ 6.222

➢ **CONCEPT**

For solving the problem, the activity of lately fallen trees is taken as activity A_0 at $t = 0$

Solution : According to problem,

$$A = \frac{3}{5} A_0$$

$\therefore$ $$A = A_0 e^{-\lambda t}$$

or $$\frac{3}{5} A_0 = A_0 e^{-\lambda t}$$

or $$\frac{3}{5} = e^{-\lambda t}$$

or $$ln \frac{3}{5} = -\lambda t$$

$\therefore$ $$t = -\frac{ln \frac{3}{5}}{\lambda} = \frac{-ln \frac{3}{5}}{\frac{0.693}{T_{1/2}}}$$

Here $T_{1/2} = 5570$ year

$\therefore$ On putting the values, we get $t = 4.1 \times 10^3$ year

YOUR STEP

A piece of burnt wood of mass 20 g is found to have a C^{14} activity of 4 decay/sec. How long has the tree that this wood belonged to be dead? Given $T_{1/2}$ of $C^{14} = 5730$ year

(1842 year)

§ 6.223

➢ **CONCEPT**

Since, Pb is final product. So, number of nuclei of Pb in the sample is equal to number of decay nuclei of U^{238}.

The applicable formula is $N = N_0 e^{-\lambda t}$

SOLUTION : The number of nuclei of U^{238} after time t is

$N = N_0 e^{-\lambda t}$ where N_0 is number of nuclei of U^{238} at $t = 0$.

The number of Pb after time t = number of nuclei of U^{238} decay in time t

$$= N_1 = N_0 (1 - e^{-\lambda t})$$

According to problem, $\frac{N}{N_1} = \eta$

or $$\frac{N_0 e^{-\lambda t}}{N_0 (1 - e^{-\lambda t})} = \eta$$

or $$e^{-\lambda t} = \eta (1 - e^{-\lambda t})$$

or $$e^{-\lambda t} = \eta - \eta e^{-\lambda t}$$

or $$e^{-\lambda t} + \eta e^{-\lambda t} = \eta$$

or $$(1 + \eta) e^{-\lambda t} = \eta$$

or $$e^{-\lambda t} = \left(\frac{\eta}{1 + \eta}\right)$$

or $$ln (e^{-\lambda t}) = ln \left(\frac{\eta}{1 + \eta}\right)$$

or $$-\lambda t = ln \frac{\eta}{1 + \eta}$$

or $$t = \frac{-ln \left(\frac{\eta}{1 + \eta}\right)}{\lambda} = \frac{ln \left(\frac{1 + \eta}{\eta}\right)}{\frac{ln 2}{T_{1/2}}}$$

$\therefore$ $$t = \frac{\left[ln \left(1 + \frac{1}{\eta}\right)\right] T_{1/2}}{ln 2}$$

On putting the values, we get $t \approx 2.0 \times 10^9$ year

YOUR STEP

In nature, a decay chain series starts with $_{90}Th^{232}$ and finally terminates at $_{82}Pb^{208}$. A thorium ore sample was found to contain 8×10^{-5} ml of helium at STP and 5×10^{-7} g of Th^{232}. Find the age of the ore sample assuming the source of helium to be only to be decay of Th^{232}. Also assume complete retention of helium within the ore (half. life of $Th^{232} = 1.39 \times 10^{10}$ year)

{154.26 year}

§ 6.224

➢ **CONCEPT**

Specific activity is defined as activity per gram

The activity is $A = \lambda N.$

SOLUTION : For Na^{24}

The number of nuclei in one gram of Na^{24} is

$$N = \frac{1}{24} \times 6.022 \times 10^{23}$$

$\therefore$ Specific activity

$$= \lambda N$$

$$= \left(\frac{0.693}{T_{1/2}}\right)\left(\frac{1}{24}\right) \times 6.022 \times 10^{22}$$

On putting the values, we get

Specific activity

$$= \frac{0.693}{15 \text{ hour}} \times \left(\frac{1}{24}\right) \times 6.022 \times 10^{23}$$

$$= 0.01159235 \times 10^{23} \text{ dis/gm hour}$$

$$= \frac{0.01159235 \times 10^{23} \text{ dis.}}{\text{gm } (60 \times 60) \text{ sec}}$$

$$= 3.22 \times 10^{17} \text{ dis/gm/sec}$$

In similar fashion, for U^{235}. The specific activity

$$= \frac{0.693}{(7.1 \times 10^{8} \text{ year})} \times \frac{1}{(235)} \times 6.022 \times 10^{23}$$

$$= 2.5 \times 10^{13} \text{ disintegration per gram per year}$$

$$= \frac{2.5 \times 10^{13}}{365 \times 24 \times 60 \times 60} \text{ dis per gram per second}$$

$$= 0.793 \times 10^{5} \text{ dis/g/sec}$$

$$\approx 0.8 \times 10^{5} \text{ dis/g/sec}$$

YOUR STEP

Natural water contains a small amount of tritium (${}_1H^3$). This isotope beta decays with a half-life of 12.5 years. A mountaineer while climbing towards a difficult peak finds debris of some earlier unsuccessful attempt. Among other things he finds a sealed bottle of whisky. On return he analysis the whisky and finds that it contains only 1.5 percent of the ${}_1H^3$ radioactivity as compared to a recently purchased bottle marked '8 years old'. Estimate the time of that unsuccessful attempt..

{About 83 years ago}

§ 6.225

➢ **CONCEPT**

When radioactive solution is injected into blood. Then the radioactive solution with blood is homogeneous.

SOLUTION : So, activity is present in total volume of the blood.

Here half life period is $T_{1/2} = T = 15 \text{ hour} = 15 \times 60 \times 60 \text{ sec}$

$\therefore$ $t = 5 \times 60 \times 60$ second

$$A' = \frac{16 \text{ dis}}{\text{minute cm}^3} = \frac{16 \text{ dis.}}{60 \text{ sec. cm}^3}$$

$$= \frac{16}{60} \text{ dis. /cm}^3\text{/sec}$$

Let the volume of blood is V cm^3.

$\therefore$ Activity in total blood after time t is $A_1 = A' V$

At $t = 0$, activity is equal to present activity of the solution which is injected into the blood

$$= A_0 = A = 2 \times 10^3 \text{ dis./sec}$$

$\therefore$ $$A_1 = A_0 e^{-\lambda t}$$

or $$A_1 = A e^{-\lambda t}$$

or $$A' V = A e^{-\lambda t}$$

$\therefore$ $$V = \left(\frac{A}{A'}\right) e^{-\lambda t}$$

Here $$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{T}$$

on putting the values, we get $V = \dfrac{2 \times 10^3}{\left(\frac{16}{60}\right)} e^{-\ln \frac{2}{3}}$ cubic centimetre

$$= 5.95 \text{ litre} \approx 6 \text{ litre} \qquad (\because\ 1 \text{ cm}^3 = 1 \text{ ml})$$

YOUR STEP

A Co^{60} source with activity 26 μ curie is embedded in a tumour that has a mass of 0.500 kg. The Co source emits γ photons with average energy 1.25 MeV. Half the photons are absorbed in the tumour, and half escape.

(a) What energy is delivered to the tumour per second?
(b) What absorbed dose (in rad) is delivered per second?
(c) What equivalent dose (in rem) is delivered per second if the RBE for these γ rays is 0.70?
(d) What exposure time is required for an equivalent dose of 200 rem?

{(a) 9.63×10^{-8} J (b) 1.93×10^{-5} rad (c) 1.35×10^{-5} rem (d) 1.48×10^7 sec $\approx \frac{1}{2}$ years}

§ 6.226

➢ **CONCEPT**

The activity in the sample is due to only Co^{58}

SOLUTION : Let a sample of mass $(m_1 + m_2)$ gm.

Here $$m_1 = \text{mass of } Co^{58}$$

and $$m_2 = \text{mass of } Co^{59}$$

$\therefore$ $$A = \lambda N = \frac{\ln 2}{T_{1/2}} N$$

Here N = number of nuclei of Co^{58}

$\therefore$ $$N = \frac{m_1}{58} \times 6.022 \times 10^{23}$$

But specific activity $= 2.2 \times 10^{12}$ dis. / s g

The activity in a sample of $(m_1 + m_2)$ g is 2.2×10^{12} dis/s g

$\therefore$ Total activity $$A = (m_1 + m_2)\, 2.2 \times 10^{12} \text{ dis/g/s}$$

or $$\lambda N = (m_1 + m_2) \times 2.2 \times 10^{12} \text{ dis/g/s}$$

or $$\left(\frac{\ln 2}{T_{1/2}}\right) N = (m_1 + m_2) \times 2.2 \times 10^{12}$$

or $$\frac{\ln 2}{T_{1/2}} \left(\frac{m_1}{58}\right) \times 6.022 \times 10^{23} = (m_1 + m_2) \times 2.2 \times 10^{12}$$

$$\therefore \qquad \frac{m_1}{m_1+m_2} = \frac{T_{1/2}\times 2.2\times 10^{12}}{6.022\times 10^{23}\times ln\,2}$$

Here $$T_{1/2} = 71.3 \text{ day} = 71.3\times 24\times 60\times 60 \text{ sec}$$

On putting the value, we get $$\frac{m_1}{m_1+m_2} = 1.88\times 10^{-3}$$

$$= 0.188\% \text{ (In percentage)} \approx 0.19\%$$

YOUR STEP

1. It is assumed that when the earth was formed the isotopes U^{238} and U^{235} were present but not their decay products.
   The decays of U^{238} and U^{235} are used to establish the age of the earth, T.
   (a) The isotope U^{238} decays with a half life of 4.50×10^9 year. The decay products in the resulting radioactive series have half lives short compared to this, to a first approximation their existence can be ignored. The decay series terminates in the stable lead isotope Pb^{206}. Obtain expression for the number of Pb^{206} atoms denoted n^{206} produced by radioactive decay with time t in terms of the present number of U^{238} atoms denoted N^{238} and the half life time of U^{238}.
   (b) Similarly, U^{235} decays with a halflife of 0.710×10^9 years through a series of shorter half life products to give the stable isotope Pb^{207}. Write down an equation relating n^{207} to N^{235} and the half life of U^{235}.
   (c) A uranium ore, mixed with a lead ore is analysed with a mass spectrometer. The relative concentrations of the three lead isotopes Pb^{204}, Pb^{206} and Pb^{207} are measured and the number of atoms are found to be in the ratios 1.00 : 17.9 : 15.5. Given that the ratio $N^{238} : N^{235}$ is 137 : 1, derive an equation involving T.
   (d) Assume that T is much greater than the half lives of both uranium isotopes and obtain an approximate value for T.
   (e) This approximate value is clearly not significantly greater than the longer half life but can be used to obtain a much more accurate value for T. Hence, or other wise, estimate a value for the age of the earth correct to within 2%.
2. In the chain analysis of a rock, the mass ratio of two radioactive isotopes is found to be 100 : 1. The mean lives of the two isotopes are 4×10^9 year and 2×10^9 year respectively. If it is assumed that at the time of formation of the rock, both isotopes were in equal proportion, calculate the age of the rock. Ratio of atomic weights of the two isotopes of 1.02 : 1.
   ($\log_{10} 1.02 = 0.0086$)
3. Since, the lead contained in uranium ore is the final decay product of the uranium series, the age of the ore can be found from the relationship between the amount of uranium in the ore and the amount of lead in it. Determine the age of uranium ore if 320 g of lead $_{82}Pb^{206}$ are contained in this ore per kg of uranium $_{92}U^{238}$.

{1. (a) $n^{206} = N^{238}(e^{0.154\,t} - 1)$ where t is in 10^9 year
(b) $n^{207} = N^{235}(e^{0.9762\,t} - 1)$
(c) $(e^{0.1540\,T} - 1)$
(d) 5.38×10^9 year
(e) more accurate answer for T to be in range 4.6×10^9 year to 4.5×10^9 year (either acceptable)
2. 18.34×10^9 year
3. 3×10^9 years}

§ 6.227

➢ CONCEPT

The sample contains two different types of radioactive substances having different half lives periods.

We can draw a graph between $ln\, A$ and t by using the table given in the problem. By using graph, we can get the value of T_1 and T_2.

Here $T_1 = 1.6$ hour and $T_2 = 9.8$ hour

SOLUTION :

The net activity at instant t is $A = A_1 + A_2$

$\therefore$ $A = A_{10}\, e^{-\lambda_1 t} + A_{20}\, e^{-\lambda_2 t}$

$\therefore$ $ln\, A = ln\, (A_{10}\, e^{-\lambda_1 t} + A_{20}\, e^{-\lambda_2 t})$ ...(i)

At $t = 0$, $ln\, A = 4.10$ (using given table)

$\therefore$ from equ. (i), $4.10 = ln\, (A_{10}\, e^{-\lambda_1 \times 0} + A_{20}\, e^{-\lambda_2 \times 0})$

$\therefore$ $4.10 = ln\, (A_{10} + A_{20})$ ...(ii)

At $t = 1$ sec, $ln\, A = 3.60$

$\therefore$ $(3.60) = ln\, (A_{10}\, e^{-\lambda_1} + A_{20}\, e^{-\lambda_2})$ ...(iii)

At $t = 2$ sec, $ln\, A = 3.10$,

From equ. (i), $(3.10) = ln\, (A_{10}\, e^{-2\lambda_1} + A_{20}\, e^{-2\lambda_2})$ ...(iv)

At $t = 3$ sec, $ln\, A = 2.60$

$\therefore$ $(2.60) = ln\, (A_{10}\, e^{-3\lambda_1} + A_{20}\, e^{-2\lambda_2})$ ...(v)

After solving equ. (ii), (iii), (iv) and (v), we get

$\lambda_1 = 0.433$ per hour, $\frac{A_{10}}{A_{20}} = \frac{51.1}{10}$

$\lambda_2 = 0.0707$ per hour,

$\therefore$ $$T_1 = \frac{0.693}{\lambda_1}$$

$$= \frac{0.693}{0.433} \text{ hour} = 1.6 \text{ hour}$$

$$T_2 = \frac{0.693}{\lambda_2} = \frac{0.693}{0.0707} = 9.8 \text{ hour}$$

$\therefore$ A_{10} = activity of first component $= \lambda_1 N_{10}$

$\therefore$ Similarly, $A_{20} = \lambda_2 N_{20}$

$\therefore$ $$\frac{N_{20}}{N_{10}} = \left(\frac{\lambda_1}{\lambda_2}\right)\frac{A_{20}}{A_{10}} = 10$$

YOUR STEP

The following table shows the results of measuring the dependence of the activity a of a certain radioactive element on the time t.

t, h	0	3	6	9	12	15
a, mc	21.6	12.6	7.6	4.2	2.4	1.8

Find the half-life of this element.

$\{T \approx 4$ hour$\}$

§ 6.228

➢ **CONCEPT**

The problem is based upon the case when production and the decay of the radioactive substance take place simultaneously.

$$\xrightarrow{\text{production}} A \xrightarrow[\lambda]{\text{decay}} B$$

SOLUTION : The rate of production of P^{32} is q while its rate of decay is λN.

Thus, the net rate of change of nuclei of

$$P^{32} \text{ is } \frac{dN}{dt} = q - \lambda N$$

or
$$\int_0^N \frac{dN}{q - \lambda N} = \int_0^t dt$$

or
$$-\frac{1}{\lambda} [ln\,(q - \lambda N)]_0^N = t$$

or
$$[ln\,(q - \lambda N) - ln\,q] = -\lambda t$$

or
$$ln\,\frac{q - \lambda N}{q} = -\lambda t$$

or
$$1 - \frac{\lambda N}{q} = e^{-\lambda t}$$

or
$$1 - \frac{A}{q} = e^{-\lambda t}$$

or
$$\frac{A}{q} = 1 - e^{-\lambda t}$$

$\therefore$
$$A = q\,(1 - e^{-\lambda t}) \qquad \text{...(i)}$$

where A = activity of P^{32} after time t.

Here
$$T_{1/2} = T = \frac{ln\,2}{\lambda}$$

$\therefore$
$$\lambda = \frac{ln\,2}{T_{1/2}} \qquad \text{...(ii)}$$

From equ. (i)
$$A = q\,(1 - e^{-\lambda t})$$

or
$$A = q - q\,e^{-\lambda t}$$

or
$$q\,e^{-\lambda t} = q - A$$

or
$$e^{-\lambda t} = \left(\frac{q - A}{q}\right)$$

or
$$-\lambda t = ln \left(\frac{q - A}{q}\right)$$

or
$$t = -\frac{ln\left(\frac{q - A}{q}\right)}{\lambda}$$

or
$$t = \frac{-ln\left(\frac{q - A}{q}\right)}{\frac{ln\,2}{T}}$$

$$\therefore \qquad t = \frac{-T\, ln\left(\frac{q-A}{q}\right)}{ln\, 2} = \left(-\frac{T}{ln\, 2}\right) ln\left(1-\frac{A}{q}\right)$$

On putting the values, we get $t = 9.5$ day

YOUR STEP

1. A radionuclide with half life T is produced in a reactor at a constant rate q nuclei per second. During each decay, energy E_0 is released. If production of radionuclide is started at $t = 0$, calculate
(a) rate of release of energy as function of time t and
(b) total energy released upto time t.
2. In an experiment, the isotope I^{128} is created by the irradiation of I^{127} with a beam of neutrons that creates 1.5×10^6 I^{128} nuclei per second. Initially, no I^{128} nuclei are present. The half life of I^{128} is 25 minutes.
(a) Sketch a graph of the number of I^{128} nuclei that are present as a function of time.
(b) What is the activity of the sample 1, 10, 25, 50, 75 and 180 minutes after irradiation is begun?
(c) What is the maximum activity that can be produced?

1. (a) $qE_0\left[1 - e^{-t\, ln\, 2/T}\right]$
(b) $qt\, E_0 - \frac{qTE_0}{ln\, 2}\left[1 - e^{-t\, ln\, 2/T}\right]$
2. (b) 4.1×10^4 Bq, 3.6×10^5 Bq, 7.5×10^5 Bq, 1.1×10^6 Bq, 1.3×10^6 Bq, 1.5×10^6 Bq
(c) 1.5×10^6 Bq

§ 6.229

➢ **CONCEPT**

The problem is based upon successive disintegrations of the products. In this case when a substance A_1 decays into a substance A_2 and A_3 successively decays into a third substance A_3 with the same or different decay rates.

$$A_1 \xrightarrow[\lambda_1]{\text{decays}} A_2 \xrightarrow[\lambda_2]{\text{decays}} A_3$$

SOLUTION : At $t = 0$, the number of nuclei of A_1 is N_{10} and that of A_2 is zero. Let at an instant t, the number of nuclei of A_1 and A_2 are N_1 and N_2 respectively.

For A_1, $\qquad N_1 = N_{10}\, e^{-\lambda_1 t}$ ...(i)

The rate of decay of A_1 is equal to the rate of formation of A_2.

So, for A_2, formation of nuclei and decays take place simultaneously.

$\therefore$ The net change of nuclei of A_2 is

$$\frac{dN_2}{dt} = (\text{rate of decay of } A_1) - (\text{Rate of decay of } A_2)$$

or $$\frac{dN_2}{dt} = \left\{-\frac{dN_1}{dt}\right\} - (\lambda_2 N_2)$$

or $$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

or $$\frac{dN_2}{dt} + \lambda_2 N_2 - \lambda_1 N_1 = 0$$

or $$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_{10}\, e^{-\lambda_1 t}$$

Multiplying both sides by $e^{\lambda_2 t}$, we get

$\therefore$ $$e^{\lambda_2 t}\frac{dN_2}{dt} + \lambda_2 N_2 e^{\lambda_2 t} = \lambda_1 N_{10}\, e^{-\lambda_1 t} e^{\lambda_2 t}$$

or $$e^{\lambda_2 t}\frac{dN_2}{dt} + \lambda_2 N_2 e^{\lambda_2 t} = \lambda_1 N_{10}\, e^{(\lambda_2 - \lambda_1) t}$$

or $$\frac{d}{dt}(N_2 e^{\lambda_2 t}) = \lambda_1 N_{10}\, e^{(\lambda_2 - \lambda_1) t}$$

or $$\int d\,(N_2 e^{\lambda_2 t}) = \lambda_1 N_{10} \int e^{(\lambda_2 - \lambda_1) t}\, dt$$

or $$\int_0^{N_2 e^{\lambda_2 t}} d\,(N_2 e^{\lambda_2 t}) = \lambda_1 N_{10} \int_0^t e^{(\lambda_2 - \lambda_1) t}\, dt$$

or $$N_2 e^{\lambda_2 t} = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1}\left[e^{(\lambda_2 - \lambda_1) t} - 1\right]$$

$\therefore$ $$N_2 = \left(\frac{N_{10}\lambda_1}{\lambda_2 - \lambda_1}\right)\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

(b) At the maximum activity, the radioactive substance should be in radioactive equilibrium. At the time of radioactive equilibrium, rate of formation of nuclei will be equal to the rate of decay of nuclei.

Mathematically, $$\frac{dN_2}{dt} = 0$$

$\therefore$ $$\lambda_1 N_1 - \lambda_2 N_2 = 0$$

$\therefore$ $\lambda_1 N_1 = \lambda_2 N_2$ at the radioactive equilibrium

$\therefore$ $$\lambda_1 N_1 = \lambda_2 N_2$$

or $$\lambda_1 N_{10}\, e^{-\lambda_1 t} = \frac{\lambda_2 N_{10} \lambda_1}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

or $$e^{-\lambda_1 t} = \frac{\lambda_2}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

or $$\frac{\lambda_2 - \lambda_1}{\lambda_2} = \frac{e^{-\lambda_1 t}}{e^{-\lambda_1 t}} - \frac{e^{-\lambda_2 t}}{e^{-\lambda_1 t}}$$

or $$\frac{\lambda_2 - \lambda_1}{\lambda_2} = 1 - e^{(\lambda_1 - \lambda_2)\, t}$$

or $$1 - \frac{\lambda_1}{\lambda_2} = 1 - e^{(\lambda_1 - \lambda_2)\, t}$$

or $$\frac{\lambda_1}{\lambda_2} = e^{(\lambda_1 - \lambda_2)\, t}$$

or $$ln\, \frac{\lambda_1}{\lambda_2} = (\lambda_1 - \lambda_2)\, t$$

$\therefore$ $$t = \left(\frac{ln\, \dfrac{\lambda_1}{\lambda_2}}{\lambda_1 - \lambda_2}\right)$$

YOUR STEP

With a radioactive sample originally of N_0 atoms, we could measure the mean lifetime τ of a nucleus by measuring the number N_1 that live for a time t_1 and then decay, the number N_2 that decay after t_2 and so on; $\tau = \frac{1}{N_0}(N_1 t_1 + N_2 t_2 + \ldots)$

Show that this is equivalent to $\tau = \lambda \int_0^\infty e^{-\lambda t}\, t\, dt$

§ 6.230

➢ **CONCEPT**

(a) From the concept of previous problem,

$$\frac{dN_2}{dt} = \lambda N_1 - \lambda N_2$$

or
$$\frac{dN_2}{dt} = \lambda N_{10} e^{-\lambda t} - \lambda N_2$$

or
$$\frac{dN_2}{dt} + \lambda N_2 = \lambda N_{10} e^{-\lambda t}$$

Multiplying both sides by $e^{\lambda t}$ we get

or
$$e^{\lambda t}\frac{dN_2}{dt} + \lambda N_2 e^{\lambda t} = \lambda N_{10} e^{-\lambda t} e^{\lambda t}$$

or
$$\frac{d}{dt}(N_2 e^{\lambda t}) = \lambda N_{10}$$

or
$$\int_0^{N_2 \lambda t} d(N_2 e^{\lambda t}) = \lambda N_{10} \int_0^t dt$$

or
$$N_2 e^{\lambda t} = \lambda N_{10} t$$

$\therefore$
$$N_2 = \lambda N_{10} t e^{-\lambda t}$$

(b) From previous problem, for maximum value of N_2,

$$\frac{dN_2}{dt} = 0$$

or
$$\frac{d}{dt}\{\lambda N_{10} t e^{-\lambda t}\} = 0$$

After solving,
$$t = \frac{1}{\lambda}$$

YOUR STEP

Given that the period of radon is 3.82 day and that the volume of radon in equilibrium with 1 g of radium is 0.63 mm^3 at normal temperature and pressure, deduce the half value period of radium gram-molecular volume = 22.4 litre, atomic weight of radium = 226

{3.82 day}

§ 6.231

➢ **CONCEPT**

The concept is similar to previous problem

SOLUTION :

$$A_1 \xrightarrow[\lambda_1]{} A_2 \xrightarrow[\lambda_2]{} A_3 \text{ (stable)}$$

The decay rate of A_2 is formation rate of A_3.

The number of nuclei of A_2 decay in time $t =$ the number of nuclei formed in time t.

Also, at any instant total number of nuclei remains constant.

$\therefore$
$$N_1 + N_2 + N_3 = N_{10} \quad \text{...(i)}$$

From soln. of problem 6.229

$$N_1 = N_{10}\, e^{-\lambda_1 t}$$

$$N_2 = \frac{N_{10}\,\lambda_1}{\lambda_2 - \lambda_1}\,[e^{-\lambda_1 t} - e^{-\lambda_2 t}]$$

$\therefore$
$$N_3 = N_{10} - \{N_1 + N_2\}$$

On putting the values, we get
$$N_3 = N_{10}\left\{1 + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1}\right\}$$

YOUR STEP

1. A number N_0 of atoms of a radioactive element are placed inside a closed volume. The radioactive decay constant for the nuclei of this element is λ_1. The daughter nuclei that form as a result of the decay process are assumed to be radioactive, too, with a radioactive decay constant λ_2. Determine the time variation of the number of such nuclei. Consider two limiting cases : $\lambda_1 >> \lambda_2$ and $\lambda_1 << \lambda_2$.
2. A radioactive nucleus X decays to a nucleus Y with a decay constant $\lambda_x = 0.1\ \text{sec}^{-1}$. Y further decays to a stable nucleus Z with a decay constant $\lambda_Y = \frac{1}{30}\ \text{sec}^{-1}$. Initially, there are only X nuclei and their number is $N_0 = 10^{20}$, set up the rate equations for the populations of X, Y and Z. The population of the Y nucleus as a function of time is given by $N_Y(t) = \left\{\frac{N_0\,\lambda_X}{(\lambda_X - \lambda_Y)}\right\}\{\exp(-\lambda_Y t) - \exp(-\lambda_x t)\}$. Find the time at which N_Y is maximum and determine the population of X and Z at that instant.

$$\left\{\begin{array}{l}
1.\ N_2 = N_0\, e^{-\lambda_2 t} \text{ when } \lambda_1 >> \lambda_2 \\
\quad N_2 = \frac{\lambda_1}{\lambda_2} N_0\, e^{-\lambda_1 t} \text{ when } \lambda_1 << \lambda_2 \\
\text{(i) } \frac{dNx}{dt} = -\lambda_X N_X,\ \frac{dNy}{dt} = \lambda_X N_X - \lambda_Y N_Y,\ \frac{dNz}{dt} = \lambda_Y N_Y \\
\text{(ii) } 16.48\text{S (iii) } N_X = 1.92\times 10^{19},\ N_Y = 5.76\times 10^{19},\ N_Z = 2.32\times 10^{19}.
\end{array}\right\}$$

§ 6.232

➢ **CONCEPT**

The concept of problem 6.231 is completely applicable.

➢ **DISCUSSION**

As
$$Bi^{210} \xrightarrow[\lambda_1]{\beta-\text{decay}} PO^{210} \xrightarrow[\lambda_2]{\alpha-\text{decay}} Pb^{206}$$
$$A_1 \longrightarrow A_2 \longrightarrow A_3$$

Since, mass numbers of Bi^{210} and PO^{210} are same. So, this is example of β-decay. The number of β-particles is equal to the number of Bi^{210} nuclei decays.

In the transformation from PO^{210} to Pb^{206}, the mass number is decreased by 4. So, this is an example of α-decay. The number of Pb^{206} nuclei formed is equal to the number of α-particles emitted.

From the solution of previous problem, the number of Bi^{210} nucleus after time t is

$$N_1 = N_{10}\, e^{-\lambda_1 t}$$

The number of β-particles emitted = number of Bi nuclei decay

$$= n_1 = N_{10} - N_1 = N_{10} - N_{10}\, e^{-\lambda_1 t}$$

$$\therefore \quad n_1 = N_{10}\,(1 - e^{-\lambda_1 t}) \qquad \text{...(i)}$$

From previous problem, the number of Pb^{206} nuclei formed is

$$N_3 = N_{10}\left[\frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{(\lambda_2 - \lambda_1)} + 1\right]$$

Hence, number of particles emitted is $n_2 = N_3$

SOLUTION :

The β-activity is

$$A_\beta = \lambda_1 N_1 = \lambda_1 N_{10}\, e^{-\lambda_1 t}$$

Here $\quad N_{10} = \dfrac{10^{-3}}{210} \times 6.022 \times 10^{23} = 2.87 \times 10^{18}$

$$t = \text{one month} = 30 \times 24 \times 60 \times 60 \text{ sec}$$

On putting the values, we get

$$A_\beta = 0.725 \times 10^{11} \text{ β-particles per second}$$

Similarly, $\quad A_\alpha = \lambda_2 N_2$

Here $\quad N_2 = \dfrac{N_{10}\,\lambda_1}{\lambda_2 - \lambda_1}\,[e^{-\lambda_1 t} - e^{-\lambda_2 t}]$ (from problem 6.229)

On putting the values, we get $A_\alpha = 1.46 \times 10^{11}$ α–particles per second

YOUR STEP

The $4n$ radioactive decay series begins with ${}_{90}Th^{232}$ and ends with ${}_{82}Pb^{208}$

(a) How many α-decays in the chain?

(b) How many beta decays?

(c) How much energy is released in the complete chain?

(d) What is the radioactive power produced by 1 kg of Th^{232}.

($t_{1/2} = 1.40 \times 10^{10}$ year)?

{(a) 6 (b) 4 (c) 42.658 MeV (d) 27.8 μW}

§ 6.233

> **CONCEPT**

Due to emission of α-particle, mass number decreases by four and atomic number decreases by two. But due to emission of β-particle from the nucleus, mass number does not change but atomic number increases by one.

SOLUTION :

(a) ${}_{88}Ra^{226} \xrightarrow{5\alpha} {}_{88-2\times5}X^{226-5\times4} \xrightarrow{4\beta} {}_{88-2\times5+4}Y^{226-20}$

or $\quad {}_{88}Ra^{226} \xrightarrow{5\alpha} {}_{78}X^{206} \xrightarrow{4\beta} {}_{82}Y^{206}$

Hence, final product has mass number 206 and atomic number 82.

From periodic table, ${}_{82}Y^{206}$ is ${}_{82}Pb^{206}$.

(b) Here ${}_{92}U^{238} \xrightarrow{n_1\,\alpha} {}_{92-n_1\times2}X^{238-n_1\times4} \xrightarrow{n_2\,\beta} {}_{92-2n_1+n_2}Y^{238-4n_1}$

But according to problem, final product is $_{82}Pb^{206}$.

$\therefore$ $$_{92-2n_1+n_2}Y^{238-4n_1} = {}_{82}Pb^{206}$$

Comparing mass numbers, we get

$\therefore$ $$238 - 4n_1 = 206$$

$\therefore$ $$n_1 = \frac{238-206}{4} = \frac{32}{4} = 8$$

Comparing atomic number, we get,

$$92 - 2n_1 + n_2 = 82$$

or $$92 - 2 \times 8 + n_2 = 82$$

$\therefore$ $$n_2 = 6$$

Here number of α particles $= n_1 = 8$

and the number of β-particles $= n_2 = 6$

YOUR STEP

A radionuclide consists of two isotopes. One of the isotopes decays by α-emission and the other by β-emission with half lives $T_1 = 405$ sec and $T_2 = 1620$ sec, respectively. At $t = 0$, probabilities of getting α and β particles from the radionuclide are equal. Calculate their respective probabilities at $t = 1620$ sec. If at $t = 0$, total number of nuclei in the radionuclide are N_0, calculate time t when total number of nuclei remained undecayed becomes equal to $\frac{N_0}{2}$.

Given, $\log_{10} 2 = 0.30103$, $\log_{10} 5.94 = 0.7742275$,

$x^4 + 4x - 2.5 = 0 \Rightarrow x = 0.594$

$$\left\{\frac{1}{9}, \frac{8}{9}, 1215\text{sec}\right\}$$

§ 6.234

➢ **CONCEPT**

For solving the problem, based upon nuclear reaction, three basic equations are provided.

(1) on the basis of conservation principle of momentum

(2) on the basis of energy conservation principle

(3) on the basis of conservation principle in the form of Q-factor.

As for example,

$$\begin{array}{cccc} A & + \; B \longrightarrow & C & + \; D \\ m_1 & m_2 & m_3 & m_4 \\ p_1 & p_2 & p_3 & p_4 \\ T_1 & T_2 & T_3 & T_4 \end{array}$$

For the given equation, (1) the equation on the basis of conservation principle of momentum

$$\vec{p_1} + \vec{p_2} = \vec{p_3} + \vec{p_4} \quad \text{...(i)}$$

(2) on the basis of energy conservation principle

$$\Delta E_1 + m_1c^2 + T_1 + \Delta E_2 + m_2c^2 + T_2 = \Delta E_3 + m_3c^2 + T_3 + m_4c^2 + TR_4 + \Delta E_4$$

Here ΔE_1, ΔE_2, ΔE_3 and ΔE_4 are corresponding excitation energies.

If no nuclei are in excited form. Then,

$$m_1c^2 + T_1 + m_2c^2 + T_2 = m_3c^2 + T_3 + m_4c^2 + T_4$$

Here $$T_1 = \frac{p_1^2}{2m_1}, T_2 = \frac{p_2^2}{2m_2}, T_3 = \frac{p_3^2}{2m_3} \quad \text{and} \quad T_4 = \frac{p_4^2}{2m_4}$$

(3) On the basis of energy conservation principle in form of Q-factor

$$m_1c^2 + m_2c^2 = m_3c^2 + m_4c^2 + Q$$

SOLUTION :

$$\begin{array}{lll} Pb^{200} \longrightarrow & Y^{196} & + \alpha \\ m_1 & m_2 & m_3 \\ T_1 = 0 & T_2 & T_3 \\ p_1 = 0 & p_2 & p_3 \end{array}$$

Here m_1 = mass of Pb nucleus (not atomic mass)

m_2 = mass of daughter nucleus

m_3 = mass of α-particle

According to conservation principle of momentum,

$$\vec{p_1} = \vec{p_2} + \vec{p_3}$$

$$0 = \vec{p_2} + \vec{p_3}$$

$\therefore$ $$\vec{p_2} = -\vec{p_3}$$

It means α-particle and daughter nucleus move in opposite direction.

$\therefore$ $$|\vec{p_2}| = |-\vec{p_3}|$$

$\therefore$ $$p_2 = p_3 \qquad \text{...(ii)}$$

According to conservation principle of energy in the form of Q factor

$$m_1c^2 = m_2c^2 + m_3c^2 + Q$$

$\therefore$ $$Q = (m_1 - m_2 - m_3)\,c^2 \qquad \text{...(iii)}$$

According to conservation principle of energy.

$$T_1 + m_1c^2 = T_2 + m_2c^2 + T_3 + m_3c^2$$

or $$O + m_1c^2 = T_2 + m_2c^2 + T_3 + m_3c^2$$

or $$(m_1 - m_2 - m_3)\,c^2 = T_2 + T_3$$

or $$Q = T_2 + T_3 = \text{ Total energy released}$$

$\therefore$ Fraction of recoil energy is

$$\%\,\Delta E = \frac{T_2}{T_2 + T_3} = \frac{T_2}{T_2 + T_3} = \frac{\dfrac{p_2^2}{2m_2}}{\dfrac{p_2^2}{2m_2} + \dfrac{p_3^2}{2m_3}}$$

$$= \frac{\dfrac{1}{m_2}}{\dfrac{1}{m_2} + \dfrac{1}{m_3}} \qquad (\because p_2 = p_3)$$

$$= \frac{m_2\,m_3}{m_2\,(m_2 + m_3)} = \frac{m_3}{m_2 + m_3}$$

$$= \frac{m_\alpha}{m_1} = \frac{4}{200} = \frac{1}{50} = 0.02$$

According to problem, $$T_3 = T_\alpha = \frac{p_3^2}{2m_3} = \frac{p_3^2}{2m_\alpha}$$

$$p_2 = p_3 = \sqrt{2m_\alpha T_\alpha}$$

or
$$m_2 v_2 = \sqrt{2m_\alpha T_\alpha}$$

or
$$v_{daughter} = v_2 = \sqrt{\frac{2m_\alpha T_\alpha}{m_{daughter}}}$$

Here
$$m_\alpha = 4 \times 1.67 \times 10^{-27} \text{ kg}$$
$$m_{daughter} = (200 - 4) \times 1.67 \times 10^{-27} \text{ kg}$$
$$T_\alpha = 5.77 \times 10^6 \times 1.6 \times 10^{-19} \text{ joule}$$

On putting the values, we get $v_{daughter} = 3.39 \times 10^5 \text{ m/s} \approx 3.4 \times 10^5 \text{ m/s}$

YOUR STEP

A nucleus X initially at rest, undergoes α-decay according to the equation ${}_{92}X^A \longrightarrow {}_ZY^{228} + \alpha$

(a) Find the values of A and Z in above process.

(b) The α-particle produced in the above process is found to move in a circular track of radius 0.11 m in a uniform magnetic field of 3 tesla. Find the energy (in MeV) released during the process and the binding energy of the parent nucleus X. Given the following masses in amu :

$$m(Y) = 228.03,\ m\ ({}_2He^4) = 4.003,$$
$$m\ ({}_0n^1) = 1.009,\ m\ ({}_1H^1) = 1.008$$

{(a) $A = 232$, $Z = 90$ (b) 1822.25 MeV}

§ 6.235

➢ **CONCEPT**

The mean life time $= \tau = \dfrac{1}{\lambda}$

The number of nuclei at $t = 0$ is

$$N_0 = \frac{10^{-3}}{210} \times 6.022 \times 10^{23} = 2.87 \times 10^{18}$$

SOLUTION :

The number of α-particles emitted is equal to number of nuclei decay in mean lifte time

$$\tau = \frac{1}{\lambda}$$

$\because$
$$N = N_0 e^{-\lambda t}$$

The number of α-particles emitted = number of nuclei decay

$$= N_1 = N_0 - N$$

$\therefore$
$$N_1 = N_0 (1 - e^{-\lambda t}) = N_0 (1 - e^{-\lambda \times \frac{1}{\lambda}})$$
$$= N_0 (1 - e^{-1})$$

The heat generated due to an α-particle is

$$T_\alpha = 5.3 \text{ MeV}$$

$\therefore$ Total heat generated $= N_1 T_\alpha = N_0 (1 - e^{-1}) T_\alpha$

$$= \frac{10^{-3}}{210} \times 6.022 \times 10^{23} (1 - e^{-1}) \times 5.3 \times 10^6 \times 1.6 \times 10^{-19} \text{ joule}$$

Here $e = 2.7$ (exponent)

$\therefore$ $\Delta H = 1.54 \text{ MJ} \approx 1.6 \text{ MJ}$

YOUR STEP

A nucleus at rest undergoes a decay emitting an α-particle of de-Broglie wavelength, $\lambda = 5.76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 amu and that of the α-particle is 4.002 amu, determine the total kinetic energy in the final state. Hence, obtain the mass of the parent nucleus in amu [1 amu = 931.470 MeV/C^2].

{6.25 MeV, 227.62 amu}

§ 6.236

➢ **CONCEPT**

γ-photon is emitted by excited nuclei for coming in ground state.

SOLUTION : From problem, both nuclei are in rest so, energy released in the decay process is only spent in excitation energy and kinetic energy.

According to problem, one type of daughter nucleus are formed in ground state. So, in this case total energy released (*i.e.*, Q) is only in the form of kinetic energy (*i.e.*, 5.30 MeV)

$$\therefore \quad Q = 5.30 \text{ MeV}$$

The second type of daughter nuclei is in excited state.

Hence, total energy Q released in decay process is equal to sum of excitation energy and kinetic energy of α-particle (*i.e.*, $T_\alpha = 4.50$).

$$\therefore \quad Q = \Delta E + T_\alpha$$

Hence ΔE is excitation energy.

$$\therefore \quad Q = \Delta E + T_\alpha$$

$$\therefore \quad \Delta E = Q - T_\alpha$$

$$= (5.30 - 4.50) \text{ MeV} = 0.80 \text{ MeV}$$

YOUR STEP

The kinetic energy of an α-particle escaping from the nucleus of a polonium atom $_{84}PO^{214}$ is 7.68 MeV in radioactive decay. Find :

(a) The velocity of the α-particle,

(b) The total energy emitted during the escape of the α-particle,

(c) The number of ion pairs formed by the α-particle assuming that the energy $E_0 = 34$ eV is required to produce one pair of ions in air.

(d) The saturation current in an ionisation chamber produced by all the α-particles emitted by 1 microcurie of polonium.

{(a) $v = 1.92 \times 10^7$ m/sec (b) $E = 7.83$ MeV (c) $n = 2.26 \times 10^5$ ion pairs (d) $I_s = 1.33 \times 10^{-9}$ A}

§ 6.237

➢ **CONCEPT**

(a) $\frac{1}{2} m_\alpha v_\alpha^2 = T_\alpha$

$$\therefore \quad v_\alpha = \sqrt{\frac{2T_\alpha}{m_\alpha}} = \sqrt{\frac{7 \times 10^6 \times 1.6 \times 10^{-19}}{4 \times 1.67 \times 10^{-27}}}$$

$$= 1.83 \times 10^7 \text{ m/s} = 1.83 \times 10^9 \text{ cm/s}$$

$$\because \quad R = 0.98 \times 10^{-27} \times v_0^3 \text{ cm}$$

Putting the value of v_0, we get $R = 6.02$ cm

(b) The total number of ion pairs formed in the whole path is

$$n = \frac{\text{energy of } \alpha - \text{particle}}{\text{energy for formation of one pair}}$$

$$= \frac{7 \text{ MeV}}{34 \text{ eV}} = \frac{7}{34} \times 106 = 2.06 \times 10^5$$

$\therefore$ $$R = 0.98 \times 10^{-27} \, v_0^3$$

$\therefore$ $$\frac{R}{2} = 0.98 \times 10^{-27} \, v_0'^3$$

$\therefore$ $$v_0' = \left(\frac{R}{2 \times 0.98 \times 10^{-27}} \right)^{\frac{1}{3}}$$

On putting the values of R,

$$v_0' = 1.5 \times 10^7 \text{ m/s}$$

$\therefore$ Kinetic energy is $$T' = \frac{1}{2} m v_0'^2$$

$$= 7.1 \times 10^{13} \text{ J} = 4.4 \text{ MeV}$$

$\therefore$ $$\Delta T = T - T'$$

$$= (7 - 4.4) \text{ MeV} = 2.6 \text{ MeV}$$

$\therefore$ The number of ion-pairs in half of mean free path is

$$n' = \frac{\Delta T}{34 \text{ eV}} = \frac{2.6 \text{ MeV}}{34 \text{ eV}}$$

$\therefore$ $$n' = \frac{2.6}{34} \times 10^6 = 0.765 \times 10^5 = 0.77 \times 10^5$$

YOUR STEP

A radio nuclide with half life $T = 69.31$ second emits β-particles of average kinetic energy $E = 11.25$ eV. At an instant concentration of β-particles at distance $r = 2$m from nucleide is $n = 3 \times 10^{13}$ per m^3.

(a) Calculate number of nuclei in the nuclide at that instant.

(b) If a small circular plate is placed at distance r from nuclide such that β-particles strike the plate normally and come to rest, calculate pressure experienced by the plate due to collision of β-particles. (Mass of β-particle $= 9 \times 10^{-31}$ kg).

{(a) $9.6\pi \times 10^{22}$ (b) 1.08×10^{-4} N/m^2}

§ 6.238

➢ CONCEPT

(i) β^- Decay : If an unstable nucleus is formed with more number of neutrons, then for stability a neutron will convert into a proton due to tendency of getting stability.

The transformation takes place by following equation :

$$h \longrightarrow p + e + \bar{\nu}$$

Here symbol $\bar{\nu}$ representes antineutrino. Antineutrino is a chargeless particle having rest mass zero. The spin of antineutrino is $\pm \frac{1}{2}$.

The equation of β^- decay by an unstable nucleus is as follow :

$$\underset{m_p}{{}_Z X^A} \longrightarrow \underset{m_d}{{}_{Z+1}Y^A} + \underset{m}{e} + \bar{\nu}$$

Here

m_p = mass of parent nucleus

= mass of parent atom-mass of total electrons in the parent atom

$$\therefore \quad m_p = M_p - Zm$$

Here Z is atomic number.

m_d = mass of daughter nucleus

= mass of daughter atom – mass of total number of electrons in daughter atom

$$\therefore \quad m_d = M_d - (Z+1)m$$

SOLUTION : Aplying energy conservation principle in the form of Q – factor :

$$m_p c^2 = m_d c^2 + mc^2 + Q$$

$$\therefore \quad Q = (m_p - m_d - m)C^2$$

$$\therefore \quad Q = [M_p - Zm - \{M_d - (Z+1)m\} - m]\, c^2$$

$$\therefore \quad Q = [M_p - Zm - M_d + (Z+1)\, m - m]c^2$$

$$\therefore \quad Q = [M_p - ZM - M_d + Zm + m - m]\, c^2$$

$$\therefore \quad Q = (M_p - M_d)\, c^2$$

Remarks : If the masses of nuclei are given in the problem. Then the result will be in different form.

(ii) Positron emission or β^+ decay : β^+ decay take place by an unstable nucleus having excess proton than needed for stability. In this process, a proton converts itself into a neutron, positron and neutrino. Positron is antiparticle of electron. The rest mass of positron is same as that of electron and electric charge on positron is positive. Neutrino is antiparticle of antineutrino. The reaction is as follow :

$$P = n + e^+ + \nu$$

β^+ decay process in an unstable nucleus is represented as follows :

$$\underset{m_p}{{}^A_Z X} \longrightarrow \underset{m_d}{{}_{Z-1}^{\;\;A} Y} + \underset{m}{e^+} + \nu$$

Here m_p = mass of parent nucleus

= mass of parent atom – mass of total number of elecron in the parent atom

$= M_p - Zm$,

Similarly, $\quad m_d = M_d - (z-1)m$

Applying conservation principle in the form of Q - factor :

$$m_p c^2 = m_d c^2 + mc^2 + Q$$

or $\quad (M_p - Zm)c^2 = \{M_d - (Z-1)\, m\}c^2 + mc^2 + Q$

or $\quad M_p c^2 = M_d c^2 + mc^2 + mc^2 + Q$

$$\therefore \quad Q = (M_p - M_d - 2m)\, c^2$$

Remarks : The mass of proton is lesser than that of neutron. So, Q factor of such process is negative.

(iii) Electron capture or k-capture : This process is similar to β^+ decay. In this process an electron of k-shell of the atom is captured by the nucleus. The equation of process is

$$p + e \longrightarrow n + \nu$$

k-capture process in an unstable nucleus is represented as follow :

$$\underset{m_p}{{}_Z X^A} + \underset{m}{e} \longrightarrow \underset{m_d}{{}_{Z-1} Y^A} + \nu$$

Applying conservation principle of energy in the form of Q factor,

$$m_p c^2 + mc^2 = m_d c^2 + Q$$

or $$(M_p - Zm) c^2 + mc^2 = \{M_d - (Z-1) m\}c^2 + Q$$

or $$M_p c^2 = M_d c^2 + Q$$

$\therefore$ $$Q = (M_P - M_d) c^2$$

> **YOUR STEP**
>
> Verify that ^{64}Cu may decay by β^+ and β^- emission, or electron capture (Ec). Experimentally, we know that ^{64}Cu has a half-life of 12.8 hours with 39% β^-, 19% β^+ and 42% Ec.

§ 6.239

➢ **CONCEPT**

The concept is similar to problem 2.234.

SOLUTION : From periodic table : Atomic masses are

$M_1 = 10.016711$ amu for Be^{10}

$M_2 = 10.016114$ amu for B^{10}

$m = 597$ amu for electron

The given reaction is

$$\begin{array}{ccc} Be^{10} & \longrightarrow & B^{10} + e^- + \nu \\ m_1 & & m_2 \quad m \\ T_1 = 0 & & T_2 \quad T_3 \\ p_1 = 0 & & p_2 \quad p_3 \end{array}$$

According to the conservation principle of momentum,

$$\vec{p_1} = \vec{p_2} + \vec{p_3}$$

$$\vec{0_1} = \vec{p_2} + \vec{p_3}$$

or $$\vec{p_2} = -\vec{p_3}$$

$\therefore$ $$p_2 = p_3 \quad \text{...(i)} \quad \text{(In the sense of magnitude)}$$

Applying conservation principle of energy in the form of Q-factor, from previous problem,

$$Q = (M_p - M_d) c^2$$

$$= (M_1 - M_2) c^2 = (m_1 - m_2 - m) c^2$$

Applying energy conservation principle :

$$T_1 + m_1 c^2 = T_2 + m_2 c^2 + T_3 + E$$

where E is energy of antineutrino.

For maximum kinetic energy of β-particle, energy of antineutrino should be zero

$\therefore$ $$T_1 + m_1 c^2 = T_2 + m_2 c^2 + T_3 + m_3 c^2$$

or $$0 + (m_1 - m_2 - m) c^2 = T_2 + T_3$$

or $$Q = T_2 + T_3$$

or $$(M_1 - M_2) c^2 = T_2 + T_3$$

or $$(M_1 - M_2) c^2 = \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3}$$

or $$(M_1 - M_2)\,c^2 = \frac{p_2^2}{2m_2} + \frac{p_2^2}{2m_3}$$

or $$\frac{p_2^2}{2m_2}\left(1 + \frac{2m_2}{2m_3}\right) = (M_1 - M_2)\,c^2$$

$\therefore$ $$\frac{p_2^2}{2m_2} = \frac{(M_1 - M_2)\,c^2}{\left(1 + \dfrac{m_2}{m_3}\right)}$$

or T_2 = kinetic energy of daughter nucleus

$$= \frac{(M_1 - M_2)\,c^2}{\left(1 + \dfrac{m_2}{m_3}\right)}$$

Similarly, T_3 = maximum kinetic energy of electron

$$= \frac{(M_1 - M_2)\,c^2}{\left(1 + \dfrac{m_3}{m_2}\right)}$$

Here $m_2 = M_2 - 5m$

$m_3 = m$

Here m = mass of electron

On putting the values, we get $T_3 = 0.56$ MeV

$T_2 = 47.5$ eV

YOUR STEP

A 50-kg person accidently ingests 0.35 curie of tritium.

(a) Assume that the tritium spreads uniformly over the body and that each decay leads on average to the absorption of 5 keV of energy from the electrons emitted in the decay. The half life of tritium is 12.3 years, and the RBE of the electrons is 1.0. Calculate the absorbed dose in rad and the equivalent dose in rem during one week.

(b) The β^- decay of tritium releases more than 5 keV of energy, why is the average energy absorbed less than the total energy released in the decay?

{(a) 12.5 rad, 12.5 rem (b) the antineutrinos are not absorbed}

§ 6.240

➢ **CONCEPT**

The highest kinetic energy of β-particle is equal to Q-factor.

From previous problem,

$$Q = (M_p - M_d)\,c^2$$

Here M_p = atomic mass of parent atom

M_d = atomic mass of daughter atom

Here M_p = atomic mass of Na^{24}

$= 24 - 0.00903$ amu

M_d = atomic mass of Mg^{24} $= 24 - 0.01496$ amu

On putting the values, we get $Q = 5.52$ MeV

According to problem, average kinetic energy of electron is

$$T = \frac{Q}{3} = \frac{5.52}{3}\ \text{MeV} = 1.84\ \text{MeV}$$

The heat generated due to one nucleus is

$$\Delta H_1 = 1.84\ \text{MeV} = 2.9 \times 10^{-19}\ \text{MJ}$$

The number of nuclei decay in time t is

$$N_1 = N_0\,(1 - e^{-\lambda t})$$

$$= \frac{10^{-3} \times 6.022 \times 10^{23}}{24}\,(1 - e^{0.693t/T})$$

Here $T = 15$ hour

$t = 24$ hour

On putting the values, we get $N_1 = 1.7 \times 10^9$

$\therefore$ Total heat generated is $\Delta H = N_1\,\Delta H_1$

On putting the values, we get $\Delta H = 5$ MJ

YOUR STEP

A radionuclide with half life $T = 693.1$ days emits β-particles of average kinetic energy $E = 8.4 \times 10^{-14}$ J. This radionuclide is used as source in a machine which generates electrical energy with efficiency $\eta = 12.6\%$. Calculate number of moles of the nuclide required to generate electrical energy at an initial rate $P = 441$ kW.

[Avogadro number, $N = 6 \times 10^{23}$, $\log_e 2 = 0.6931$]

{6000}

§ 6.241

➢ **CONCEPT**

From solution of problem,

$$Q = (M_p - M_d - 2m)\,c^2$$

where $M_p =$ atomic mass of parent nuclei

$M_d =$ atomic mass of daughter nucleus

$m =$ mass of electron

The reaction is

$$C^{11} \longrightarrow B^{11} + e^{+} + \nu$$

Putting the values, we get

$$Q = 0.97\ \text{MeV}$$

This energy is shared by kinetic energy of positron and energy of neutron

$$\therefore \quad Q = T + E_\nu \qquad \text{...(i)}$$

$E_\nu =$ energy of neutron

From momentum conservation principle,

$$\vec{\mathbf{P}}_{\text{neutrins}} + P_{\text{positron}} = 0$$

$$\therefore \quad \frac{\varepsilon_\nu}{C} + P = 0$$

$$\therefore \quad p = \frac{\varepsilon_\nu}{C} \qquad \text{(In the sense of magnitude)}$$

But relavistic momentum of positron is

$$pc = \sqrt{T\,(T + 2mc^2)}$$

$\therefore$ $$p = \frac{\sqrt{T(T+2mc^2)}}{c}$$

$\therefore$ $$\frac{\varepsilon_v}{c} = \frac{\sqrt{T(T+2mc^2)}}{c}$$

or $$\varepsilon_v^2 = T(T+2mc^2)$$

$$= (Q-\varepsilon_v)(Q-\varepsilon_v+2mc^2)$$

$\therefore$ $$\varepsilon_v = \frac{Q(Q+2mc^2}{2(Q+mc^2)}$$

From eqn. (i) $$T = Q - \varepsilon_v$$

On putting the value of ε_v,

$\therefore$ $$T = \frac{Q^2}{2(Q+mc^2)}$$

On putting the values, we get

$$T = 0.32 \text{ MeV}$$

$$\varepsilon_v = 0.65 \text{ MeV}$$

YOUR STEP

The first excited state of ^{57}Fe decays to the ground state with the emission of 14.4 keV photon in a mean lifetime of 141 ns. (a) What is the width ΔE of the state.

(b) What is the recoil kinetic energy of an atom of ^{57}Fe that emits a 14.4 keV photon?

{(a) 4.67×10^{-9} eV, (b) 1.95×10^{-3} eV}

§ 6.242

➢ **CONCEPT**

The Q value of positron decay is given by

$$Q = [M(x) - M(y) - 2m]\, c^2$$

SOLUTION : The reaction is

$$N^{13} \longrightarrow C^{13} + \beta^+ + \nu$$

Here $$M(x) = M_N$$

$$M(y) = M_C$$

$\therefore$ $$Q = (M_N - M_C - 2m)\, c^2 \qquad \text{...(i)}$$

when the kinetic energy of positron is maximum, then energy of neutrino is negligible. So, Q value of energy is shared by positron and carbon nuclei.

$\therefore$ $$T_C + T_P = Q \qquad \text{...(ii)}$$

From conservation principle of momentum,

$$\vec{p}_c + \vec{p}_p = 0$$

$\therefore$ $$p_C = p_P \qquad \text{(in the sense of magnitude)}$$

or $$p_C^2 = p_p^2$$

or $$T_C(T_C + 2M_C c^2) = T_P(T_P + 2mc^2) \qquad \text{...(iii) (In relavistic variation)}$$

From equ. (i), (ii) and (iii), we get

$$T_C = \frac{Q\,(Q + 2mc^2)}{2\,(Q + M_C\,c^2 + mc^2)}$$

or

$$T_C = \frac{1}{2}\,\frac{Q\,(Q + 2mc^2)}{\{M_N c^2 - M_C\,c^2 - 2mc^2 + M_C\,c^2 + mc^2\}}$$

mc^2 may be neglected with respect to $M_N\,c^2$

$$\therefore \qquad T_C \approx \frac{1}{2}\,\frac{Q\,(Q + 2mc^2)}{M_N\,c^2}$$

On putting the values, we get

$$T_C = T = 0.11 \text{ keV}$$

Here $\qquad Q = (M_N - M_C - 2m)\,c^2$

YOUR STEP

The nuclide $_{76}Os^{191}$ decays with β^- energy of 0.14 MeV accompained by γ-rays of energy 0.042 MeV and 0.129 MeV.

(a) What is the daughter nucleus?

(b) Draw an energy level diagram showing the ground states of the parent and daughter and excited states of the the daughter. To which of the daughter states does β^- decay of $_{76}Os^{191}$ occur?

$\{_{77}I_r^{191}\}$

§ 6.243

➢ **CONCEPT**

The concept is similar to problem 6.238.

In K-capture, $\qquad Q = (M_p - M_d)\,c^2$

The reaction is represented as

$$Be^7 + e \longrightarrow Li^7 + \nu$$

Here $M_p =$ Atomic mass of Be^7

$M_d =$ atomic mass of Li^7

On putting the value, we get $\qquad Q = 0.86$ MeV

Most of this energy is taken by neutrino. According to conservation principle of momentum, momentum of daughter nucleus is equal and opposite to neutrino.

The momentum of neutrino is

$$p_{\text{neutrino}} = \frac{Q}{c} = \frac{0.86 \text{ MeV}}{c}$$

$$\therefore \qquad p_d = \frac{Q}{c}$$

$$\therefore \qquad m_d\,v_d = \frac{Q}{c}$$

$$\therefore \qquad v_d = \frac{Q}{c m_d}$$

On putting the values, we get $\qquad v_d = 3.96 \times 10^6$ cm/s $= 40$ km/sec

YOUR STEP

Find the disintegration energy Q for the decay of ^{49}V by K-electron capture as described,

$$^{49}V + e \longrightarrow {}^{49}Ti + \nu$$

The needed atomic masses are

$m_V = 48.948517$ u, $m_{Ti} = 48.947871$ u

and $$E_k = 5.47 \text{ keV}$$

$$1 \text{ amu} = 931 \text{ MeV/C}^2$$

[0.596 MeV]

§ 6.244

> **CONCEPT**

From the statement of problem, it is clear that the energy released by nucleus in returning from excited state to ground state is

$$\Delta E = 87 \text{ MeV}$$

In other way, this energy is taken by K-electron.

If this energy is taken by K-electron, then a part of energy is spent for knocking out K-shell from atom and remaining part provides kinetic energy to the electron

The energy required to knock out the electron is equal to binding energy of K-shell electron.

$\therefore$ $$\Delta E = \text{Binding energy} + \text{K.E}$$

$\therefore$ $$\text{K.E.} = \Delta E - \text{binding energy} = (87 - 26) \text{ keV}$$

$$KE = T = 61 \text{keV}$$

The relavistic K.E. is $$T = mc^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right] = 61 \text{ keV}$$

or $$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{61}{mc^2} + 1$$

or $$\frac{1}{1-\frac{v^2}{c^2}} = 1.26$$

or $$v^2 = \frac{0.26}{1.26}c^2$$

$\therefore$ $$v = 0.454 \text{ c} = 0.45 \text{ c}$$

YOUR STEP

Gold $_{79}Au^{198}$ undergoes β^- decay to an excited state of $_{80}Hg^{198}$. If the excited state decays by emission of a γ photon with energy 0.412 MeV. What is the maximum kinetic energy of the electron emitted in the decay? This maximum occurs when the antineutrino has negligible energy. (The recoil energy of the $_{80}Hg^{198}$ can be neglected. The masses of the neutral atoms in their ground states are $_{79}Au^{198}$, 197.968217 u; $_{80}Hg^{198}$, 197.966743 u.)

[0.961 MeV]

§ 6.245

> **CONCEPT**

When the recoil speed of nucleus is neglected. The energy of γ-photon is $E = 129$ keV

When nucleus is in the condition of recoiling, then energy E is shared by γ-ray and kinetic energy of nucleus.

i.e., $$E = E' + T \qquad ...(i)$$

where T is kinetic energy of nucleus

According to conservation principle of momentum, the momentum of nucleus is equal and opposite to γ-photon.

$\therefore$ The magnitude of momentum of nucleus is

$$p = \frac{h\gamma}{c} = \frac{E'}{c}$$

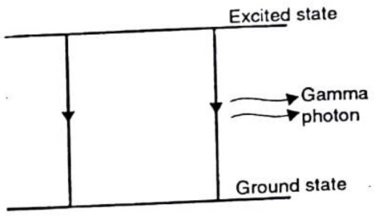

Fig. 6.245A

$\therefore$
$$pc = E'$$

The relavistic momentum of nucleus is

$$pc = \sqrt{T(T + 2mc^2)}$$

or
$$E' = \sqrt{T(T + 2mc^2)}$$

or
$$E'^2 = T(T + 2mc^2)$$

or
$$E'^2 = T^2 + 2Tmc^2$$

or
$$(E - T)^2 = T^2 + 2Tmc^2 \quad \text{from (i)}$$

$$E^2 + T^2 - 2ET = T^2 + 2Tmc^2$$

or
$$E^2 - 2ET = 2Tmc^2$$

$\therefore$
$$T = \frac{E^2}{2(mc^2 + E)}$$

$$\frac{E'}{E} = \frac{E - T}{E}$$

Here m is mass of Ir^{191} nucleus

$\therefore$
$$\frac{E'}{E} - 1 = \frac{E - T}{E} - 1$$

or
$$\frac{\Delta E}{E} = 1 - \frac{T}{E} - 1 = \frac{T}{E}$$

$\therefore$
$$\frac{\Delta E}{E} = \frac{E^2}{2(mc^2 + E)E} = \frac{E}{2(mc^2 + E)}$$

But
$$E << mc^2$$

$\therefore$
$$mc^2 = E \approx mc^2$$

$\therefore$ fraction is
$$\frac{\Delta E}{E} = \frac{E}{2mc^2}$$

On putting the values, we get
$$\frac{\Delta E}{E} = 3.6 \times 10^{-7}$$

YOUR STEP

The α-decay spectrum from Ra^{226} has a triplet structure, with α-particle energies of 4.777, 4.593 and 4.342 MeV. Assuming that the daughter nuclide Rn^{222} is produced in the ground or one of the two excited states, draw the energy level diagram and show that the γ ray emission associated with the transition.

{ γ-ray emission of 0.184 MeV, 0.251 MeV and 0.255 MeV}

NUCLEAR REACTION

§ 6.249

➢ **CONCEPT**

During elastic collision momentum as well kinetic energy of system remains constant.

SOLUTION :

Total momentum of system before collision is

$$\vec{p}_i = \vec{p}_1 + \vec{p}_2$$
$$= \vec{p}_\alpha + 0 = p_\alpha$$

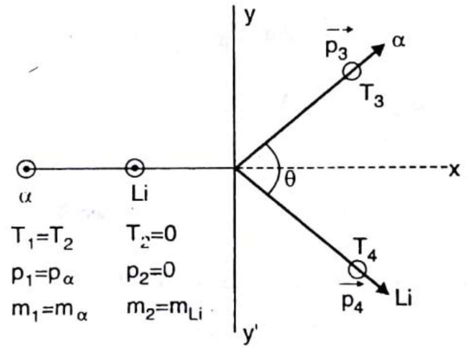

Fig. 6.249A

Total momentum after collision is

$$\vec{p}_f = \vec{p}_3 + \vec{p}_4$$

According to conservation principle of momentum,

∴ $$\vec{P}_i = \vec{P}_f$$

or $$\vec{p}_\alpha = \vec{p}_3 + \vec{p}_4$$

According to parallelogram law of vectors

$$\vec{p}_\alpha^{\,2} = p_3^2 + p_4^2 + 2p_3\, p_4 \cos\theta \qquad \ldots\text{(i)}$$

According to conservation principle of kinetic energy

$$T_1 + T_2 = T_3 + T_4$$

or $$T_\alpha + 0 = T_3 + T_4$$

Hence,

$$\frac{p_{Li}^2}{2m_{Li}}\left[1 + \frac{m_{Li}}{4m_\alpha}\left(1 - \frac{m_\alpha}{m_{Li}}\right)^2 \sec^2\theta\right] = T_\alpha$$

The recoil energy of Li

$$= T_{Li} = \frac{p_{Li}^2}{2m_{Li}}$$

$$= \frac{T_\alpha}{1 + \frac{(m_{Li} - m_\alpha)^2 \sec^2\theta}{4m_\alpha m_{Li}}}$$

But $\cos\theta < 0$

$\therefore$ $$\cos\theta \neq \frac{1}{2} \qquad (\because -1 \leq \cos\theta < 0)$$

The possible value of $\cos\theta$ $= -\frac{1}{2}$

$\Rightarrow$ $$\theta = 120°$$

On putting the values, we get

$$T_{Li} = 6 \text{ MeV}$$

or $$T_\alpha = \frac{p_{Li}^2}{2m_{Li}} + \frac{p_4^2}{2m_\alpha} \qquad \ldots\text{(ii)}$$

From eqn. (i) and (ii) we get

$$p_4^2 = 2m_\alpha T_\alpha - \frac{M_\alpha}{M_{Li}} p_{Li}^2$$

or $$P_{Li}\left[\left(1 - \frac{m_\alpha}{m_{Li}}\right)p_{Li} + 2p_\alpha \cos\theta\right] = 0$$

$\therefore$ $$p_{Li} \neq 0$$

$\therefore$ $$p_\alpha = -\frac{1}{2}\left(1 - \frac{m_\alpha}{m_{Li}}\right)p_{Li} \sec\theta$$

But P_α and P_{Li} both are positive. For this $-1 \leq \cos\theta < 0$ $(\because m_\alpha < m_{Li})$

YOUR STEP

An alpha particle collides elastically with a stationary nucleus and continues on at an angle of 60° with respect to its original direction of motion. The nucleus recoils at an angle of 30° with respect to this direction. What is the mass number of the nucleus?

{4}

§ 6.250

➢ **CONCEPT**

The concept is similar to previous problem

According to conservation principle of momentum,

$$\vec{p_1} + \vec{p_2} = \vec{p_3} + \vec{p_4}$$

or $$p_1 = p_3 + p_4 \qquad \ldots\text{(i)}$$

According conservation principle of kinetic energy

$$T_1 + T_2 = T_3 + T_4$$

or $$T_1 = T_3 + T_4$$

or $$\frac{p_1^2}{2m} = \frac{p_3^2}{2m} + \frac{p_4^2}{2m}$$

or $$\frac{p_1^2}{2m} = \frac{(p_1 - p_4)^2}{2m} + \frac{p_4^2}{2m} \qquad \text{(from eqn. (i))}$$

or $$\frac{p_1^2}{2m} = \frac{p_1^2 + p_4^2 - 2p_1p_4}{2m} + \frac{p_4^2}{2m}$$

or $$\frac{p_1^2}{2m} = \frac{p_1^2}{2m} + \frac{p_4^2}{2m} - \frac{2p_1p_4}{2m} + \frac{p_4^2}{2m}$$

or $$\frac{p_4^2}{2}\left(\frac{1}{m} + \frac{1}{M}\right) = \frac{2p_1p_4}{2m}$$

or $$p_4\left(\frac{1}{m} + \frac{1}{M}\right) = \frac{2p_1}{m}$$

or $$p_4\left(\frac{M+m}{mM}\right) = \frac{2p_1}{m}$$

or $$p_4 = \left(\frac{2M}{m+M}\right)p_1$$

$\therefore$ $$p_4^2 = \left(\frac{2M}{m+M}\right)^2 p_1^2 \quad \text{...(ii)}$$

$\therefore$ Loss in kinetic energy of neutron $= \Delta T_1 =$ gain in K.E. of deuteron

$\therefore$ $$\Delta T_1 = T_4 = \frac{p_4^2}{2M}$$

$\therefore$ $$\text{Fraction} = \frac{\Delta T_1}{T_1} = \frac{\left(\frac{2M}{m+M}\right)^2 \frac{p_1^2}{2M}}{\frac{p_1^2}{2m}}$$

$$\eta = \left(\frac{2M}{m+M}\right)^2 \left(\frac{m}{M}\right) = \frac{4mM}{(m+M)^2}$$

Here $$M \approx 2m$$

$\therefore$ $$\text{Fraction} = \eta = \frac{8}{9} = 0.89$$

(b) According to conservation principle of momentum,

$$\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \vec{\mathbf{p}}_3 + \vec{\mathbf{p}}_4$$

or $$\vec{\mathbf{p}}_1 + 0 = \vec{\mathbf{p}}_3 + \vec{\mathbf{p}}_4$$

or $$\vec{\mathbf{p}}_1 - \vec{\mathbf{p}}_3 = \vec{\mathbf{p}}_4$$

Applying parallelogram law of vectors

$$p_1^2 + p_3^2 - 2p_1p_3\cos 90° = p_4^2$$

or $$p_1^2 + p_3^2 = p_4^2 \quad \text{...(ii)}$$

According to conservation principle of kinetic energy

$$T_1 + T_2 = T_3 + T_4$$

or $$T_1 + 0 = T_3 + T_4$$

or $$\frac{p_1^2}{2m} = \frac{p_3^2}{2m} + \frac{p_4^2}{2M}$$

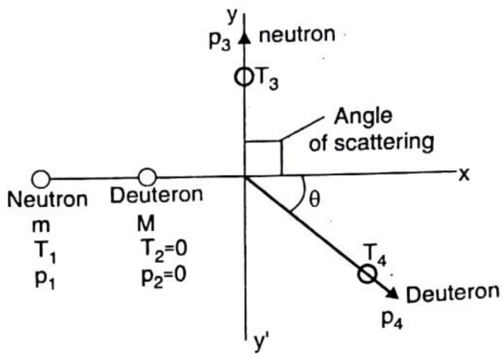

Fig. 6.250A

or $$\frac{p_1^2}{2m} = \frac{p_3^2}{2m} + \frac{p_1^2 + p_3^2}{2M}$$

or $$\frac{p_1^2}{2m} = \frac{p_3^2}{2m} + \frac{p_1^2}{2M} + \frac{p_3^2}{2M}$$

or $$\frac{p_1^2}{2}\left(\frac{1}{m} - \frac{1}{M}\right) = \frac{p_3^2}{1}\left(\frac{1}{m} + \frac{1}{M}\right)$$

or $$p_1^2\left(\frac{M-m}{mM}\right) = p_3^2\left(\frac{m+M}{mM}\right)$$

or $$p_3^2 = \left(\frac{M-m}{M+m}\right)p_1^2$$

∴ Loss in kinetic energy of neutron is

$$\Delta T = T_1 - T_3$$

$$= \frac{p_1^2}{2m} - \frac{p_3^2}{2m} = \frac{p_1^2 - p_3^2}{2m}$$

∴ $$\eta = \frac{\Delta T}{T_1} = \frac{\dfrac{p_1^2 - p_3^2}{2m}}{\dfrac{p_1^2}{2m}}$$

$$= 1 - \frac{p_3^2}{p_1^2} = 1 - \left(\frac{M-m}{M+m}\right)\frac{p_1^2}{p_1^2}$$

∴ $$\eta = 1 - \frac{(M-m)}{(M+m)} = \frac{2m}{M+m} = \frac{2}{3}$$

YOUR STEP

A proton with a kinetic energy of 10^{10} eV collides with a proton at rest in the L-frame. Find :

(a) The total momentum and the total energy of the system in the L-frame.

(b) The kinetic energy of the two particles in the C-frame.

{ (a) 10.898 GeV/C; 11.876 GeV, (b) 2.175 GeV }

§ 6.251

➢ **CONCEPT**

The concept is similar to problem 6.250

According to conservation principle of momentum,

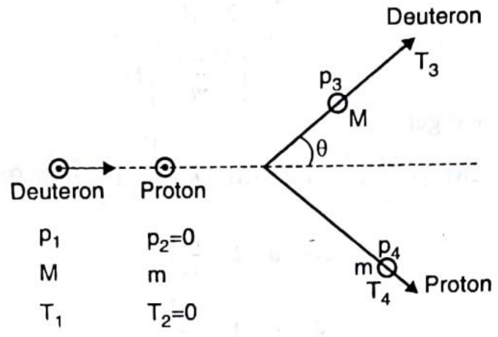

Fig. 6.251

$$\vec{p_1} + \vec{p_2} = \vec{p_3} + \vec{p_4}$$

or $$\vec{p_1} + \vec{0} + \vec{p_3} = \vec{p_4}$$

or $$\vec{p_1} - \vec{p_3} = \vec{p_4}$$

According to parallelogram law of vectors,

$\therefore$ $$p_1^2 + p_3^2 - 2p_1p_3\cos\theta = p_4^2$$

From conservation of kinetic energy,

$\therefore$ $$T_1 + T_2 = T_3 + T_4$$

or $$T_1 + 0 = T_3 + T_4$$

or $$\frac{p_1^2}{2M} = \frac{p_3^2}{2M} + \frac{p_4^2}{2m}$$

or $$\frac{p_1^2}{2M} = \frac{p_3^2}{2M} + \frac{1}{2m}(p_1^2 + p_3^2 - 2p_1p_3\cos\theta)$$

or $$\frac{p_1^2}{2M} = \frac{p_3^2}{2M} + \frac{p_1^2}{2m} + \frac{p_3^2}{2m} - \frac{2p_1p_3\cos\theta}{2m}$$

or $$T_1 = \frac{p_3^2}{2M} + \frac{2MT_1}{2m} + \frac{p_3^2}{2m} - \frac{2p_1p_3\cos\theta}{2m} \qquad \left(\because \frac{P_1^2}{2m} = T_1\right)$$

or $$T_1\left(1 - \frac{M}{m}\right) = \frac{p_3^2}{2M} + \frac{p_3^2}{2m} - \frac{2p_1p_3\cos\theta}{2m}$$

or $$T_1\left(1 - \frac{M}{m}\right) = \frac{p_3^2}{2}\left(\frac{1}{M} + \frac{1}{m}\right) - \frac{2p_1p_3\cos\theta}{2m}$$

or $$\frac{2p_1p_3\cos\theta}{2m} = \frac{p_3^2(m+M)}{2mM} + \left(\frac{M-m}{m}\right)T_1$$

or $$p_3^2\left(\frac{m+M}{2mM}\right) + \left(\frac{M-m}{m}\right)T_1 - \frac{2p_1p_3\cos\theta}{2m} = 0$$

For real root of P_3

$$b^2 - 4ac \ge 0 \qquad \text{(In quadratic equation)}$$

Here

$$b = -\frac{p_1 \cos\theta}{m} = -\frac{\cos\theta}{m}\sqrt{2MT_1} \qquad \left(\because T_1 = \frac{p_1^2}{2M}\right)$$

$$a = \frac{m+M}{2mM}$$

and

$$c = \left(\frac{M-m}{m}\right)T_1$$

On putting the values, we get

$$(2MT_1)\frac{\cos^2\theta}{m^2} - 4 \times 2\,(M-m)\,T_1\left(1+\frac{m}{M}\right) \geq 0$$

or

$$\cos^2\theta \geq 1 - \frac{m^2}{M^2}$$

or

$$1 - \sin^2\theta \geq 1 - \frac{m^2}{M^2}$$

or

$$\sin\theta \leq \frac{m}{M}$$

$\therefore$

$$\theta_{max} = \sin^{-1}\left(\frac{m}{M}\right) = \sin^{-1}\frac{1}{2} = 30° \qquad (\because M = 2m).$$

> **YOUR STEP**
>
> Show that the maximum velocity that can be imparted to a proton at rest by a non-relavistic alpha particle is 1.6 times the velocity of the incident alpha particle.

§ 6.252

➢ CONCEPT

The reaction of nucleus is $R = R_0 A^{1/3}$

where A = mass number $R_0 = 1.3 \times 10^{-15}$ m = 1.3 fm

The volume of nuclues is

$$= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3$$

$$= \frac{4}{3}\pi R_0^3 A$$

A = mass number = number of neucleons

$\therefore$ The number of nucleons per unit volume is

$$= \frac{A}{V}$$

$$= \frac{1}{\frac{4}{3}\pi R_0^3} = 1.09 \times 10^{38} \text{ neucleons/cc}$$

Density is

$$\rho = \frac{\text{mass}}{\text{volume}}$$

$$= 1.09 \times 10^{38} \times \text{mass of nucleons per cc}$$

$$= 1.82 \times 10^{11} \text{ kg/cc}$$

> **YOUR STEP**
>
> Find the density of the ${}_6C^{12}$ nucleus. [Given, $R = 2.7 \times 10^{-15}$ m]
>
> $(2.4 \times 10^{17}$ kg/m$^3)$

§ 6.253

➢ **CONCEPT**

Due to emission of α-particle from the nucleus, mass number decreases by 4 and atomic number decreases by 2. Due to emission of β-particle, mass number does not change but atomic number increases by one.

Durng nuclear transformation of the mass number and atomic number, conservation principles are applicable.

SOLUTION :

(a) $$B^{10} + x^N \longrightarrow Be^8 + \alpha$$

The mass number of reactant is $10 + N$.

The mass number of product is $8 + 4 = 12$

$$\therefore \quad 10 + N = 12$$

$$\therefore \quad N = 2$$

The mass number of deuterium (d) is 2.

∴ The reaction is

$$B^{10} + d \longrightarrow Be^8 + \alpha$$

(b) $O^{17} + d^2 \longrightarrow n^N + n^1$

The mass number of reactant = mass number of product

$$\therefore \quad 17 + 2 = N + 1$$

$$\therefore \quad N = 18$$

∴ The mass number of F^{18} is 18.

$$\therefore \quad O^{17} + d^2 \longrightarrow F^{18} + n^1$$

(c) $Na^{23} + P^1 = Ne^{20} + n^N$

$$\therefore \quad 23 + 1 = 20 + N$$

$$\therefore \quad N = 4 \qquad (i.e., \alpha \text{ particle})$$

$$\therefore \quad Na^{23} + p = Ne^{20} + \alpha$$

(d) $x^N + P_1 \longrightarrow Ar^{37} + n^1$

$$\therefore \quad N + 1 = 37 + 1$$

$$\therefore \quad N = 37$$

From above reaction, the number of proton in x is one unit less than that of Ar^{37}. But number of neutron in Ar^{37} is one unit less than x. But atomic number of Ar is 18. So, the atomic number of x is $(18 - 1) = 17$. Hence, x is $_{17}Cl^{37}$.

$$\therefore \quad {}_{17}Cl^{37} + P^1 \longrightarrow {}_{18}Ar^{37} + n^1$$

YOUR STEP

Complete these nuclear reactions :

$${}_3Li^6 + ? \rightarrow {}_4Be^7 + {}_0n^1$$
$${}_{17}Cl^{35} + ? \longrightarrow {}_{16}S^{32} + {}_2He^4$$
$${}_4Be^9 + {}_2He^4 \longrightarrow 3\ {}_2He^4 + ?$$
$${}_{35}Br^{79} + {}_1H^2 \longrightarrow ? + 2\ {}_0n^1$$

$\{{}_1H^2; {}_1H^1; {}_1n^1; {}_{36}Kr^{79}\}$

§ 6.254

➢ **CONCEPT**

$$E_b = \text{Binding energy} = \Delta mc^2 \qquad \text{...(i)}$$

where Δm = mass deffect
The number of proton = Z
The number of neutrons = $A - Z$

$\therefore$ $$\Delta m = Zm_p + (A - Z)\, m_n - M_N$$

Here m_p = mass of proton
M_N = mass of formed nucleus
From eqn. 6.6(b) of the book (on page 274)

$$\Delta_H = (m_H - 1)c^2 \;\therefore\; M_H c^2 = \Delta H + c^2 \qquad \ldots\text{(ii)}$$

$$\Delta n = (m_n - 1)c^2 \;\therefore\; M_n c^2 = \Delta_n + c^2 \qquad \ldots\text{(iii)}$$

$$\Delta = (M_N - A)c^2 \;\therefore\; M_N c^2 = \Delta + Ac^2 \qquad \ldots\text{(iv)}$$

Again

$$\Delta m = Z(m_p + m_e) + (A - Z)\, m_n - (m_N + Zm_e)$$

$\therefore$ $$\Delta m = Zm_H + (A - Z)\, m_n - M \qquad \ldots\text{(v)}$$

where M is atomic mass of the element
From Eqn. (i), (ii), (iii), (iv) and (v), we get

$$E_b = \Delta mc^2$$
$$= Z\Delta_H + (A - Z)\Delta_n - \Delta$$

YOUR STEP

1. The following mass differences have been found experimentally :
   (a) ${}^1H_2 - {}^2H = 1.5434 \times 10^{-3}$ amu (b) $3\,{}^2H - \frac{1}{2}\,{}^{12}C = 4.2300 \times 10^{-2}$ amu,
   (c) ${}^{12}C\,{}^1H_4 - {}^{16}O = 3.6364 \times 10^{-2}$ amu
   On the basis of ${}^{12}C = 12$ amu, calculate the atomic masses of ${}^1H$, ${}^2H$ and ${}^{16}O$.
2. Find the binding energy per nucleon in ${}_{10}Ne^{20}$ and in ${}_{26}Fe^{56}$.

{(1) ${}^1H = 1.0078$ amu, ${}^2H = 2.0141$ amu, ${}^{16}O = 15.9949$ amu, (2) 8.03 MeV; 8.79 MeV}

§ 6.255

➢ CONCEPT

The mass defect is given by $E_b = \Delta mc^2$
where Δm = mass defect
If m_n = mass of neutron, m_p = mass of proton, m_N = mass of formed nucleus
For binding energy of ${}_ZX^A$,
Here the number of proton = Z, The number of neutron = $A - Z$

$\therefore$ $$\Delta m = Zm_p + (A - Z)m_n - m_N$$

If atomic masses of the neucleus is given, then

$\therefore$ $$\Delta m = Z\, m_p + Zm_e + (A - Z)\, m_n - (m_N + Z\, m_e)$$
$$= Z\,(m_p + m_e) + (A - Z)\, m_n - M_N$$
$$= Z\, M_H + (A - Z)\, m_n - M_N$$

Here M_H = atomic mass of hydrogen
M_N = atomic mass of formed atom

SOLUTION :

$\therefore$ $$R = R_0\, A^{1/3}$$
$$R_{Al} = R_0\,(27)^{1/3}$$

$\therefore$ $$R = R_0 A^{1/3}$$

But $$R = \frac{R_{Al}}{\frac{3}{2}} = \frac{2}{3} R_{Al}$$

or $$R_0 A^{1/3} = \frac{2}{3} R_0 (27)^{1/3}$$

$\therefore$ $$A = \left(\frac{2}{3}\right)^3 27 = \frac{8}{27} \times 27 = 8$$

From periodic table, the element is Be^8

Atomic masses are 1.007825 for hydrogen, 1.008665 for neutron, 8.00531 for Be

$\therefore$ $$\Delta m = (4m_H + 4m_n - M_N) \text{ amu}$$
$$= (4 \times 1.007825 + 4 \times 1.008665 - 8.00531) \text{ amu}$$
$$= (4.0313 + 4.034660 - 8.00531) \text{ amu}$$
$$= 0.06065 \text{ amu}$$

$\therefore$ $$E_b = \Delta mc^2 = 0.06065 \times 931 \text{ MeV}$$
$$= 56.4651 \text{ MeV} \approx 56.5 \text{ MeV}$$

YOUR STEP

Find the binding energy E_0 per nueleon in the nuclei
(a) $_3Li^7$, (b) $_7N^{14}$, (c) $_{13}Al^{27}$, (d) $_{20}Ca^{40}$, (e) $_{29}Cu^{63}$, (f) $_{48}Cd^{113}$, (g) $_{80}Hg^{200}$ (h) $_{82}U^{238}$ Plot the relation $E_0 = f(M)$, where M is the mass number.

{(a) 5.6 MeV, (b) 7.5 MeV (c) 8.35 Mev, (d) 8.55 MeV, (e) 8.75 MeV, (f) 8.5 MeV, (g) 7.9 MeV, (h) 7.6 MeV}

§ 6.256

➢ CONCEPT

The concept is similar to previous problem, atomic mass of O^{16} is
$$M_N = 15.9994 \text{ amu}$$

$Z = 8$, $A = 16$

$\therefore$ number of neutron $= A - Z = 16 - 8 = 8$

$\therefore$ $$\Delta m = Z\, M_H + (A - Z)\, m_n - M_N$$

Here $$M_H = 1.007823 \text{ amu}$$
$$m_n = 1.008665 \text{ amu}$$

$\therefore$ $$\Delta_m = (8 \times 1.007825 + 8 \times 1.008665 - 15.994) \text{ amu}$$
$$= (8.0626 + 8.006932 - 15.994) \text{ amu}$$
$$= 0.13792 \text{ amu}$$

$\therefore$ $$E_b = \Delta mc^2 = \Delta m \times 931 \text{ MeV}$$
$$= 0.13792 + 931 \text{ MeV}$$
$$= 128.40352 \text{ MeV}$$

$\therefore$ Binding energy per neucleon $$= \frac{E_b}{A} = \frac{128.40352}{16} = 8.02522$$
$$= 8.0 \text{ MeV}$$ (in significance figure precision)

(b) For binding energy of neutron in B^{11},
$$B^{11} \longrightarrow B^{10} + n$$

The energy required to separate a neutron from the nucleus of B^{11} is known as binding energy of neutron in B^{11}.

$\therefore$ $$\Delta m = m_{B^{10}} + m_n - m_{B^{11}}$$

Here $$m_{B^{10}} = 10.01294$$
$$m_{B^{11}} = 11.00930$$
and $$m_n = 1.008665$$
$\therefore$ $$\Delta m = (10.01294 + 1.008665 - 11.00930) \text{ amu}$$
$$= 0.012305 \text{ amu}$$

$\therefore$ Binding energy of neutron in B^{11} is
$$E_{bn} = \Delta mc^2 = \Delta m \times 931 \text{ MeV}$$
$$= 0.012305 \times 931 \text{ MeV} = 11.455955$$
$$= 11.5 \text{ MeV} \quad \text{(From the concept of significance of figure)}$$

For binding energy of α-perticle in B^{11},
$$B^{11} \longrightarrow Li^7 + \alpha$$
$\therefore$ $$\Delta m = m_{Li} + m_\alpha - m_{B^{11}}$$
Here $$m_{Li} = 7.01601 \text{ amu}$$
$$m_{B^{11}} = 11.00930 \text{ amu}$$
$$m_\alpha = 4.00260 \text{ amu}$$
On putting the values, $$\Delta m = 0.00931 \text{ amu}$$
$\therefore$ $$E_{b\alpha} = \Delta mc^2 = 0.00931 \times 931 \text{ MeV} = 8.67 \text{ MeV}$$

(c) The reaction is
$$O^{16} \longrightarrow {}^4He$$
$\therefore$ $$Q = (M_O - 4m_{He})\, c^2$$
$$= (15.99491 - 4 \times 4.00260) \times 931 \text{ MeV}$$
$$= (-0.01549) \times 931 \text{ MeV} = -14.42 \text{ MeV}$$
Hence, energy required is $$-Q = 14.42 \text{ MeV}$$

YOUR STEP

Find the energies needed to remove a neutron from ${}_2He^4$, then to remove a proton, and finally to separate with remaining neutron and proton. Compare the total with the binding energy of ${}_2He^4$.

{20.6 MeV; 5.5 MeV; 2.2 MeV; both calculations give 28.3 MeV}

§ 6.257

➢ **CONCEPT**

For binding energy of neutron in B^{11}.
$$B^{11} + n \longrightarrow B^{11} \quad \text{...(i)}$$
For proton,
$$Be^{10} + p \longrightarrow B^{11} \quad \text{...(ii)}$$
$\therefore$ $$E_{bn} = (m_{B_{10}} + m_n - m_{B_{11}})\, c^2$$
and $$E_{bp} = (M_{Be_{10}} + m_p - m_{B_{11}})\, c^2$$
$\therefore$ $$E_{nb} - E_{bp} = (m_n - m_p)\, c^2 + (m_{B_{10}} - m_{Be_{10}})\, c^2$$
Here $m_n = 1.008665$ amu, $m_p = 1.007276$ amu, $m_{B_{10}} = 10.01294$ amu, $m_{Be_{10}} = 10.01354$ amu

On putting the values, we get

$$E_{bn} - E_{bp} = (0.00024 \text{ amu})\, c^2$$
$$= 0.00024 \times 931 \text{ MeV} = 0.223 \text{ MeV}$$

YOUR STEP

(a) Find the energy needed to remove a neutron from Kr^{81}, from Kr^{82}, and from Kr^{83}.
(b) Why is the figure for Kr^{82} so different from the others?

{(a) 7.88 MeV; 10.95 MeV; 7.46 MeV, (b) More energy is needed to remove a neutron from Kr^{82} because of the tendency of neutrons to pair together}

§ 6.258

➢ **CONCEPT**

The reaction is,

$$\underset{m_c}{C^{12}} + \underset{2m_\alpha}{2\alpha} \longrightarrow \underset{m_{Ne}}{Ne^{20}}$$

$$\therefore \quad Q = (m_c + 2m_\alpha - m_{Ne})\, c^2$$

Here

$$E_c = (6m\,({}_1H^1) + 6m_n - m_c)\, c^2$$
$$\therefore \quad m_c\, c^2 = (6m\,({}_1H^1) + 6m_n)c^2 - E_c$$
$$E_\alpha = \left\{2m\,({}_1H^1) + 2m_n - m_\alpha\right\}c^2$$
$$m_\alpha c^2 = \left\{2m\,({}_1H^1) + 2m_n\right\}c^2 - E_\alpha$$
$$E_{Ne} = \left\{10m\,({}_1H^1) + 10m_n - m_{Ne}\right\}c^2$$
$$m_{Ne} = (10\,m\,({}_1H^1) + 10m_n)c^2 - E_{Ne}$$

SOLUTION :

$$\therefore \quad Q = \left\{6m\,({}_1H^1) + 6m_n\right\}c^2 - E_c + 2\left[\left\{2m\,({}_1H^1) + 2m_n\right\}c^2 - E_\alpha\right] - \left\{10m\,({}_1H^1) + 10m_n\right\}c^2 + E_{Ne}$$

$$\therefore \quad Q = E_{Ne} - 2E_\alpha - E_c$$

According to problem,

$$E_\alpha = 7.07 \times 4 \text{ MeV} = 28.28 \text{ MeV}$$
$$E_{Ne} = 20 \times 8.03 \text{ MeV} = 160.6 \text{ MeV}$$
$$E_c = 12 \times 7.68 \text{ MeV}$$
$$= 92.16 \text{ MeV}$$
$$\therefore \quad Q = 160.6 \text{ MeV} - 28.28 \text{ MeV} - 92.16 \text{ MeV}$$
$$= 11.88 \text{ MeV} = 11.9 \text{ MeV}$$

YOUR STEP

(a) Calculate Q value for the reaction

$${}_2H^4 + {}_4Be^9 \longrightarrow {}_6C^{12} + {}_0n^1$$

(b) If the ${}_2He^4$ is incident on a ${}_4Be^9$ at rest, estimate the threshold kinetic energy of ${}_2He^4$.

{(a) 5.701 MeV, (b) 2.6 MeV (no tunneling)}

§ 6.259

➢ **CONCEPT**

The binding energy is

$$E_b = (Z\, m_H + (A - Z)\, m_n - m_N)\; 931 \text{ MeV}$$

Here Z = atomic number, A = mass number, m_H = Atomic mass of H-atom, m_N = atomic mass of element, m_n = mass of neutron

SOLUTION : (a) Here $E_b = 41.3$ MeV

$$m_n = 1.008665 \text{ u}$$
$$m_H = 1.007825 \text{ u}$$

$Z = 3,\ A = 8$

$\therefore$ $$E_b = \{Z\, m_H + (A - Z)\, m_n - m_N\}\ 931 \text{ MeV}$$

or $$\frac{41.31}{931} = 4(1.007825) + 4(1.008665) - m_N$$

$\therefore$ $$m_N = 3(1.007825) + 5\,(1.008665) - \frac{41.31}{931}$$
$$= 8.0225 \text{ amu}$$

(b) Here $$E_b = 6.04 \times 10 \text{ MeV} = 60.4 \text{ MeV}$$

$Z = 6,\ A = 10$

$\therefore$ $$E_b = (6m_H + 4m_n - m_N) \times 931 \text{ MeV}$$

or $$\frac{60.4}{931} = 6 \times 1.007825 + 4 \times 1.008665 - m_N$$

$\therefore$ $$m_N = 6.04695 + 3.03466 - 0.064876$$
$$= 10.01674 \text{ amu}$$

YOUR STEP

The mass of $_{13}Al^{27}$ is 26.98154 amu. Find the mass of the product nuclei for the following reactions :

(a) $^{27}Al\,(n, \gamma)\,^{28}Al$ $Q = 7.722$ MeV

(b) $^{27}Al\,(p, \alpha)\,^{24}Mg$ $Q = 1.594$ MeV

{(a) 27.9824 amu (b) 23,9867 amu}

§ 6.260

➢ **CONCEPT**

The reaction is

$$\begin{array}{cccc} A_1 + & A_2 \longrightarrow & A_3 + & A_4 \\ m_1 & m_2 & m_3 & m_4 \\ Z_1 & Z_2 & Z_3 & Z_4 \end{array}$$

$\therefore$ $$Q = m_1c^2 + m_2c^2 - m_3c^2 - m_4c^2 \qquad \text{...(i)}$$

But $$E_1 = \{Z_1\, m_H + (A_1 - Z_1)\, m_n - m_1\}\, c^2 \qquad \text{...(ii)}$$

$\therefore$ $$m_1c^2 = \{Z_1\, m_H + (A_1 - Z_2)\, m_n\}\, c^2 - E_1 \qquad \text{...(iii)}$$

Similarly,

$$m_2c^2 = \{Z_2m_H + (A_2 - Z_2)\, m_n\}\, c^2 - E_2 \qquad \text{...(iv)}$$
$$m_3c^2 = \{Z_3m_H + (A_3 - Z_3)\, m_n\}\, c^2 - E_3 \qquad \text{...(v)}$$

and $$m_4c^2 = \{Z_4m_H + (A_4 - Z_4)\, m_n\}\, c^2 - E_4 \qquad \text{...(vi)}$$

From conservation principle of mass number,

$$A_1 + A_2 = A_3 + A_4 \qquad \text{...(vii)}$$

From conservation principle of atomic number

$$Z_1 + Z_2 = Z_3 + Z_4 \qquad \text{...(viii)}$$

From Eqn. (i), (ii), (iii), (iv), (v), (vi), (vii) and (viii), we get

$$Q = (E_3 + E_4) - (E_1 + E_2)$$

YOUR STEP

Consider the fusion reaction

$$_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + {}_0n^1$$

(a) By calculating the repulsive electrostatic potential energy of the two $_1H^2$ nuclei when they touch, estimate the barrier energy.
(b) compute the energy liberated in this reaction in MeV and in joules.
(c) compute the energy per mole of deuterium, remembering that the gas is diatomic, and compare with the heat of combustion of hydrogen, about 2.9×10^5 J/mol.

{(a) 0.48 MeV, (b) 3.270 MeV= 5.238×10^{-13} J, (c) 3.155×10^{11} J/mol}

§ 6.261

➢ CONCEPT

(a) The energy released per nuclei is 200 MeV.
The number of mole in one kg is

$$n = \frac{1000}{235}$$

∴ number of nuclei in one kg is

$$N = n \times 6.022 \times 10^{23}$$
$$= \frac{1000}{235} \times 6.022 \times 10^{23} \text{ nuclei}$$

∴ The energy released by 1 kg U^{235} is

$$E = N \times 200 \text{ MeV}$$
$$= 8.21 \times 10^{10} \text{ kJ}$$

Required mass of coal $= \dfrac{8.21 \times 10^{10} \text{ kJ}}{\dfrac{30 \text{ kJ}}{\text{gram}}} = \dfrac{8.21 \times 10^{10}}{30}$ gram

$$= 2.74 \times 10^7 \text{ kg}$$

(b) Here calorie value of frotyl is 4.1×10^3 J/gram
The energy released by explosion of atom bomb is

$$E = 30 \text{ kilo tonne} \times 4.1 \times 10^3 \text{ J/gram}$$
$$= 30 \text{ kilotonne} \times 4.1 \times 10^3 \text{ J/gram}$$
$$= 30 \times 10^3 \times 1000 \text{ kg} \times 4.1 \times 10^3 \text{ J/gram}$$
$$= 30 \times 10^9 \text{ gram } 4.1 \times 10^3 \text{ J/gram}$$
$$= 30 \times 4.1 \times 10^{12} \text{ J}$$

Let the number of nuclei of U^{235} required is N.

∴ The energy released is

$$E = N \times 200 \text{ MeV}$$
$$= N \times 200 \times 10^6 \times 1.6 \times 10^{-19} \text{ joule}$$

Let the mass of N atoms of U^{235} is m gram

∴ $$N = \frac{m}{235} \times 6.022 \times 10^{23}$$

∴ $$E = N \times 200 \times 10^6 \times 1.6 \times 10^{-19}$$

or $$30 \times 4.1 \times 10^{12} = \frac{m}{235} \times 6.022 \times 10^{23} \times 200 \times 10^6 \times 1.6 \times 10^{-19}$$

$$\therefore \quad m = \frac{30 \times 4.1 \times 10^{12} \times 235}{6.022 \times 10^{23} \times 200 \times 10^{6} \times 1.6 \times 10^{-19}} \text{ g}$$

$$= 1.49 \times 10^{3} \text{ g} = 1.49 \text{ kg}$$

YOUR STEP

A radioactive source from the alpha decay of Pu^{238} ($t_{1/2} = 88$ years).
(a) What is the Q value for the decay?
(b) Assuming 100% conversion efficiency, how much power could be obtained from the decay of 1 g of Pu^{238}?

{(a) 5.594 MeV, (b) 0.56 watt}

§ 6.262

➢ **CONCEPT**

The reaction is

$$\underset{m_1}{H^2} + \underset{m_1}{H^2} \longrightarrow \underset{m_2}{He^4}$$

$$\therefore \quad \phi = (2m_1 - m_2)c^2 = (2m_1 - m_2) \times 931 \text{ MeV}$$

Here $m_1 = 2.0410$ amu for H^2
$m_2 = 4.00260$ amu

$$\therefore \quad Q = (2 \times 2.01410 - 4.002603) \times 931 \text{ MeV}$$
$$= 0.02560 \text{ amu} \times 931 \text{ MeV} = 23.83 \text{ MeV}$$

Hence, energy released per nuclei is $E_0 = 23.83$ MeV.

The number of nuclei of He^4 in one gram is

$$N = \frac{1}{4} \times 6.022 \times 10^{23}$$

$\therefore$ The energy released is

$$E = nQ = \frac{1}{4} \times 6.022 \times 10^{23} \times 23.83 \text{ MeV}$$
$$= \frac{1}{4} \times 6.022 \times 10^{23} \times 23.83 \times 10^{6} \times 1.6 \times 10^{-19}$$
$$= 5.74 \times 10^{8} \text{ kJ}$$

Let equivalent mass of coal is m gram

$$\therefore \quad E = m \text{ gm} \times 30 \text{ kJ/gm}$$

or $$5.74 \times 10^{8} \text{ kJ} = 30 \text{ m kJ}$$

$$\therefore \quad m = \frac{5.74 \times 10^{8}}{30} \text{ gram}$$
$$= 1.9 \times 10^{7} \text{ gram} = 1.9 \times 10^{4} \text{ kg} \approx 2 \times 10^{4} \text{ kg}$$

YOUR STEP

In a nuclear reactor U^{235} undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required.

{3.847×10^{4} kg}

§ 6.263

➢ **CONCEPT**

The reaction is

$$Li^6 + H^2 \longrightarrow He^4 + He^4$$
$$m_1 \quad m_2 \quad m_3 \quad m_3$$

$\therefore$
$$Q = \{m_1c^2 + m_2 - (2m_3)\}\, c^2$$
$$= (6.01513 + 2.01410 - 2 \times 4.00260) \times 931 \text{ MeV}$$
$$= 0.02403 \times 931 \text{ MeV} = 22.37 \text{ MeV}$$

The number of nucleons in product nuclei is $2 \times 4 = 8$

$\therefore$ Energy per nucleon is $\frac{Q}{8}$

$$= \frac{22.37}{8} \text{ MeV/nucleon}$$
$$= 2.796 \text{ MeV/nucleon}$$

The compared value in fission of U^{235} is $\frac{200}{235}$ MeV/nucleon

$$= 0.85 \text{ MeV/nucleon}$$

> **YOUR STEP**
>
> Find the Q value (and therefore, the energy released) in the fission reaction
> $$U^{235} + n \longrightarrow Rb^{93} + Cs^{141} + 2n$$
> Use $m\,(Rb^{93}) = 92.92195$u and $m\,(Cs^{141}) = 140.92005$u.
>
> {180.02 MeV}

§ 6.264

➢ **CONCEPT**

The result of problem 6.260 is applicable for solving this problem.

i.e.,
$$A_1 + A_2 \longrightarrow A_3 + A_4$$

The energy released is $\quad Q = E_3 + E_4 - (E_1 - E_2)$

SOLUTIONS :
$$Li^7 + p \longrightarrow He^4 + He^4$$
$$A_1 + A_2 \longrightarrow A_3 + A_4$$
$$E_1 + E_2 \longrightarrow E_3 + E_4$$

$\therefore$
$$Q = E_3 + E_4 - (E_1 + E_2)$$
$$= 2E_3 - (E_1 + E_2) \qquad (\because E_3 = E_4)$$

Here E_3 = binding energy of He
$$= 7.06 \times 4 \text{ MeV} = 28.24 \text{ MeV}$$

E_1 = binding energy of Li^7
$$= 7 \times 5.60 \text{ MeV} = 39.2 \text{ MeV}$$

E_2 = binding energy of free proton $= 0$

$\therefore$ On putting the values, $\quad Q = 2E_3 - (E_1 + E_2)$
$$= 2 \times 28.24 - (39.2 + 0)$$
$$Q = 17.3 \text{ MeV}$$

> **YOUR STEP**
>
> Calculate the energy released in the fission reaction
> $${}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1$$
> Neglect the initial kinetic energy of the absorbed neutron. The atomic masses are ${}_{92}U^{235} = 235.043924$u; ${}_{54}Xe^{140} = 139.921620$ u; and ${}_{38}Sr^{94} = 93.915367$ u.
>
> {185 MeV}

§ 6.265

➢ CONCEPT

For solving the problem based upon nuclear inelastic collision, three basic principles are applicable :

(i) momentum conservation principle of

(ii) energy conservation principle

(iii) energy conservation principle in the form of Q.

SOLUTION : Step I : Write the nuclear reaction :

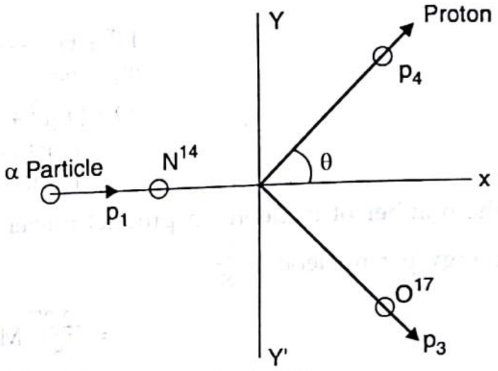

Fig. 6.265A

The reaction $N^{14}(\alpha, p)O^{17}$ is

$$\begin{array}{cccc} N^{14} + & \alpha \longrightarrow & O^{17} + & p \\ m_1 & m_2 & m_3 & m_4 \\ p_1 & p_2 = 0 & p_3 & p_4 \\ T_1 & T_2 = 0 & T_3 & T_4 \end{array}$$

Setp II : Apply the conservation principle of momentum :

$$\vec{p_1} + \vec{p_2} = \vec{p_3} + \vec{p_4}$$

or
$$\vec{p_1} + \vec{0} = \vec{p_3} + \vec{p_4}$$

or
$$\vec{p_1} - \vec{p_4} = \vec{p_3}$$

From parallelogram law of vector,

$$p_1^2 + p_4^2 - 2p_1p_4 \cos\theta = p_3^2 \quad \text{...(i)}$$

$\therefore$
$$T_\alpha = \frac{p_1^2}{2m_1} \quad (\because T_1 = T_\alpha)$$

$\therefore$
$$p_1 = \sqrt{2m_1 T_\alpha} \quad \text{...(ii)}$$

Step III : Apply energy conservation principle : According to conservation principle of energy,

$$m_1c^2 + T_1 + m_2c^2 + T_2 = m_3c^2 + T_3 + m_4c^2 + T_4$$

or
$$(m_1 + m_2 - m_3 - m_4)c^2 + T_1 + T_2 = T_3 + T_4$$

$$(m_1 + m_2 - m_3 - m_4)\, c^2 + T_\alpha + 0 = T_3 + T_4$$

or
$$(m_1 + m_2 - m_3 - m_4)\, c^2 + T_\alpha = T_3 + T_4 \quad \text{...(iii)}$$

Step IV : Apply conservation principle of energy in the form of Q.

From nuclear reaction in Step (I) ,

$$m_1c^2 + m_2c^2 = m_3c^2 + m_4c^2 + Q$$

$\therefore$
$$(m_1 + m_2 - m_3 - m_4)c^2 = Q \quad \text{...(iv)}$$

From equation (iii) and (iv), we get

$\therefore$
$$Q + T_\alpha = T_3 + T_4$$

or
$$Q + T_\alpha = \frac{p_3^2}{2m_3} + \frac{p_4^2}{2m_4} \quad \text{...(v)}$$

Let
$$\eta_p = \frac{m_p}{m_o} = \frac{m_4}{m_3} \quad \text{...(vi)}$$

$$\eta_\alpha = \frac{m_\alpha}{m_o} = \frac{m_1}{m_3} \qquad \text{... (vii)}$$

From eqn. (v) and (i), we get

$$p_1^2 + p_4^2 - 2p_1 p_4 \cos\theta = p_3^2$$

Here $$p_4 = \sqrt{2T_4 m_4}$$

or $$p_1^2 + 2T_4 m_4 - 2\sqrt{2m_1 T_\alpha}\,\sqrt{2T_4 m_4}\cos\theta = p_3^2$$

or $$2T_\alpha m_1 + 2T_4 m_4 - 2\sqrt{2m_1 T_\alpha}\,\sqrt{2T_4 m_4}\cos\theta = p_3^2$$

or $$2T_\alpha m_\alpha + 2T_p m_p - 4\sqrt{m_\alpha T_\alpha T_p m_p}\cos\theta = p_3^2 \qquad \text{...(viii)}$$

From eqn. (v), (vi), (vii) and (viii), we get

$$Q = (1+\eta_p)\,T_p - (1-\eta_\alpha)\,T_\alpha - 2\sqrt{\eta_p \eta_\alpha T_p T_\alpha}\cos\theta$$

On putting the values, we get

$$Q = -1.2 \text{ MeV}$$

YOUR STEP

Show that in a nuclear reaction where the product particle is ejected at an angle of 90° with the direction of the bombarding particle, the Q-value is expressed :

$$Q = K_y\left(1 + \frac{m_y}{M_Y}\right) - K_x\left(1 - \frac{m_x}{M_X}\right)$$

§ 6.266

➢ **CONCEPT**

(a) $$Li^7\,(p, n)\,Be^7$$
$$\Downarrow$$
$$\underset{m_p}{p} + \underset{m_{Li}}{Li^7} = \underset{m_{Be}}{Be^7} + \underset{m_n}{n}$$

Applying energy conservation principle in the form of Q

$$m_p c^2 + m_{Li} c^2 = m_{B_e} c^2 + m_n c^2 + Q$$

$$= (1.00783 + 7.01601 - 7.01693 - 1.00867) \times 931 \text{ MeV} = -1.64 \text{ MeV}$$

(b) $$Be^9\,(n, \gamma)\,Be^{10}$$
$$\Downarrow$$
$$\underset{m_n}{n} + \underset{m_{Be}}{Be^9} \longrightarrow \underset{m'_{Be}}{Be^{10}} + \underset{m_\gamma}{\gamma}$$

Applying conservation principle of energy in the form of Q

$$m_n c^2 + m_{Be} c^2 = m'_{Be} c^2 + m_\gamma c^2 + Q$$

The rest mass of γ is zero $$Q = (m_n + m_{Be} - m'_{Be})c^2$$
$$= (9.01219 + 1.00867 - 10.01354) \times 931 \text{ MeV}$$
$$= 6.81 \text{ MeV}$$

(c) $$Li^7\,(\alpha, n)\,B^{10}$$
$$\Downarrow$$
$$\underset{m_\alpha}{\alpha} + \underset{m_{Li}}{Li^7} \longrightarrow \underset{m_B}{B^{10}} + \underset{m_n}{n}$$

Applying conservation principle of energy in the form of Q,

$$m_\alpha c^2 + m_{Li}c^2 = m_B c^2 + m_n c^2 + Q$$

$\therefore$ $$Q = (m_\alpha + m_{Li} - m_B - m_n)\,c^2$$
$$= (4.00260 + 7.01601 - 10.01294 - 1.00867) \times 931 \text{ MeV}$$
$$= -2.79 \text{ MeV}$$

(d) $$O^{16}\ (d,\ \alpha)\ N^{14}$$
$$\Downarrow$$
$$\underset{m_d}{d} + \underset{m_o}{O^{16}} \longrightarrow \underset{m_n}{N^{14}} + \underset{m_\alpha}{\alpha}$$

Applying conservation principle of energy in the form of Q,

$$m_d c^2 + m_o c^2 = m_N c^2 + m_\alpha c^2 + Q$$

$\therefore$ $$Q = (m_d + m_o - m_N - m_\alpha)c^2$$

or $$Q = (2.01410 + 15.99497 - 14.00307 - 4.0026) \times 931 \text{ MeV}$$
$$= 3.11 \text{ MeV}$$

YOUR STEP

Find the Q values of the reactions :

(a) $p + Mn^{55} \longrightarrow Fe^{54} + 2n$

(b) $He^3 + Ar^{40} \longrightarrow K^{41} + H^2$

{(a) –10.313 MeV (b) 2.314 MeV}

§ 6.267

➢ **CONCEPT**

$$B^{10}\ (n,\ \alpha)\ Li^7$$
$$\Downarrow$$
$$B^{10} + n \longrightarrow Li^7 + \alpha$$

$\therefore$ $$Q = (m_B + m_n - m_{Li} - m_\alpha)\,c^2$$
$$= (10.01294 + 1.00867 - 7.01601 - 4.00260) \times 931 \text{ MeV}$$

$\therefore$ $$Q = 2.79 \text{ MeV}$$
$$= 2.79 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

Since, neutron is slow. So, the momentum of neutron is negligible.

So, net momentum of reactant ($B^{10} + n$) should be zero.

Hence, product particles in reaction moves in opposite direction with equal magnitude of momentum.

$\therefore$ $$m_\alpha v_\alpha = m_{Li} v_{Li}$$

or $$4v_\alpha = 7v_{Li} \qquad \ldots\text{(iii)}$$

Applying energy conservation,

or $$T_B + m_B c^2 + T_n + m_n c^2 = m_{Li}c^2 + T_{Li} + m_\alpha c^2 + T_\alpha$$

But P_B and P_n are zero

$\therefore$ $T_B = 0$ and $T_n \approx 0$

$$m_B c^2 + m_n c^2 = m_{Li} c^2 + T_{Li} + m_\alpha c^2 + T_\alpha$$

or $$(m_B + m_n - m_{Li} - m_\alpha)\,c^2 = T_{Li} + T_\alpha$$

or $$Q = T_{Li} + T_\alpha = \frac{1}{2} m_{Li} v_{Li}^2 + \frac{1}{2} m_\alpha v_\alpha^2$$

or $$2.79 \times 10^6 \times 1.6 \times 10^{-19} = \frac{1}{2} \times 7\, v_{Li}^2 + \frac{1}{2}\, 4 \left(\frac{7}{4}\, v_{Li}\right)^2$$

$\therefore \qquad v_{Li} = 5.3 \times 10^6 \text{ m/s} = 0.53 \times 10^7 \text{ m/s}$

$\therefore \qquad v_\alpha = \frac{7}{4} v_{Li} = 9.27 \times 10^6 \text{ m/s} = 0.92 \times 10^7 \text{ m/s}$

YOUR STEP

The nucleus Cd^{113} captures a thermal neutron ($K = 0.025$ eV), producing Cd^{114} in an excited state; the excited state of Cd^{114} decays to the ground state by emitting a photon. Find the energy of the photon.

{9.042 MeV}

§ 6.268

➢ **CONCEPT**

The concept is similar to problem 6.265.

SOLUTION : The problem is solved by following steps :

Step I : Write the equation of reaction :

$$Li^7 \ (p,\ n)\ Be^7$$
$$\Downarrow$$
$$\begin{array}{ccccccc} p & + & Li^7 & \longrightarrow & Be^7 & + & n \\ m_p & & m_{Li} & & m_{Be} & & m_n \\ p_p & & p_{Li}=0 & & p_{Be} & & p_n=0 \\ T_p & & T_{Li}=0 & & T_{Be} & & T_n=0 \end{array}$$

Step II : Apply conservation principle of momentum for the reaction :

$$\vec{p}_p + \vec{p}_{Li} = \vec{p}_{Be} + \vec{p}_n$$

or $\qquad p_p = p_{Be} \qquad$ ...(i) $\quad (\because p_{Li} = 0$ and $p_n = 0)$

Step III : Applying conservation principle of energy in the form of Q.

$$m_pc^2 + m_{Li}c^2 = m_{Be}c^2 + m_nc^2 + Q$$

$\therefore \qquad Q = m_pc^2 + m_{Li}c^2 - m_{Be}c^2 - m_nc^2$

$\qquad = -1.64$ MeV $\qquad$ (from problem 6.266a)

Step IV : Apply conservation principle of energy

$$T_p + m_pc^2 + T_{Li} + m_{Li}c^2 = T_{Be} + m_{Be}c^2 + T_n + m_nc^2$$

or $\qquad T_p + (m_p + m_{Li} - m_n - m_{Be})c^2 = T_{Be} \qquad (\because T_n = 0\ T_{Li} = 0)$

or $\qquad T_p + Q = T_{Be}$

or $\qquad \frac{p_p^2}{2m_p} + Q = \frac{p_{Be}^2}{2m_{Be}}$

or $\qquad \frac{p_p^2}{2m_p} = \frac{p_{Li}^2}{2m_{Li}} + 1.64 \text{ MeV} \qquad (\because Q = -1.64 \text{ MeV})$

or $\qquad \frac{p_p^2}{2m_p} = \frac{p_p^2}{2m_{Li}} + 1.64 \text{ MeV} \qquad (\because p_p = p_{Li})$

or $\qquad \frac{p_p^2}{2m_p}\left(1 - \frac{m_p}{m_{Li}}\right) = 1.64 \text{ MeV}$

$\therefore \qquad T_p = \frac{1.64 \text{ MeV}}{\left(1 - \frac{m_p}{m_{Li}}\right)} = \frac{1.64 \text{ MeV}}{1 - \frac{m}{7m}}$

$$= \frac{1.64}{\frac{6}{7}} \text{ MeV} = 1.9 \text{ MeV}$$

YOUR STEP

Consider the nuclear reaction

$_1H^2 + _7N^{14} \longrightarrow X + _5B^{10}$,

where X is a nuclei.

(a) What are Z and A for the nuclide X ?

(b) Calculate the reaction energy Q (in MeV).

(c) If the $_1H^2$ is incident on a stationary $_7N^{14}$, what minimum kinetic energy must it have for the reaction to occur?

{(a) 3, 6, (b) –10.137 MeV, (c) 11.595 MeV}

§ 6.269

➢ **CONCEPT**

The concept is similar to problem 6.265.

SOLUTION : The prbolem is solved by following steps :

Step I : Write the nuclear reaction :

$$\begin{array}{cccc} \alpha + Be^9 & \longrightarrow & C^{12} + n & \\ m_\alpha \quad m_{Be} & & m_c \quad m_n & \\ T_\alpha \quad T_{Be} = 0 & & T_c \quad T_n & \\ P_\alpha \quad P_{Be} = 0 & & P_c \quad P_n & \end{array}$$

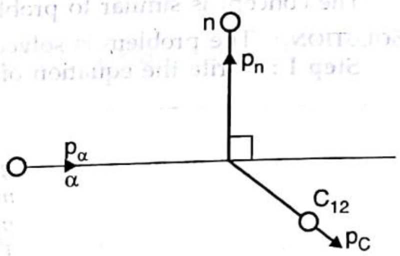

Fig. 6.269A

Step II : Apply conservation principle of momentum :

From Fig. 6.269

$$\vec{p}_\alpha + \vec{p}_{Be} = \vec{p}_c + \vec{p}_n$$

or $$\vec{p}_\alpha = \vec{p}_c + \vec{p}_n$$

or $$\vec{p}_\alpha - \vec{p}_n = \vec{p}_c$$

or $$p_\alpha^2 + p_n^2 - 2p_\alpha p_n \cos 90° = p_c^2$$

or $$p_\alpha^2 + p_n^2 = p_c^2 \quad \ldots(i)$$

Setp III : Applying conservation principle of energy in the form of Q.

$$m_\alpha c^2 + m_{Be} c^2 = m_c c^2 + m_n c^2 + Q$$

$\therefore$ $$Q = (m_\alpha + m_{Be} - m_c - m_n)\, c^2 \quad \ldots(ii)$$

Step IV : Apply conservation principle of energy :

According to conservation principle of energy,

$$T_\alpha + m_\alpha c^2 + T_{Be} + m_{Be} c^2 = T_c + m_c c^2 + m_n c^2 + T_n \qquad (\because T_{Be} = 0)$$

or $$T_\alpha + Q = T_c + T_n$$

or $$T_\alpha + Q = \frac{p_c^2}{2m_c} + \frac{p_n^2}{2m_n}$$

or $$T_\alpha + Q = \frac{p_\alpha^2 + p_n^2}{2m_c} + \frac{p_n^2}{2m_n}$$

or $$T_\alpha + Q = \frac{2T_\alpha m_\alpha + p_n^2}{2m_c} + \frac{p_n^2}{2m_n} \qquad \left(\because \frac{p_\alpha^2}{2m_\alpha} = T_\alpha\right)$$

or $$T_\alpha + Q = \frac{2T_\alpha m_\alpha}{2m_c} + \frac{p_n^2}{2m_c} + \frac{p_n^2}{2m_n}$$

or
$$T_\alpha + Q - T_\alpha\left(\frac{m_\alpha}{m_c}\right) = \frac{p_n^2}{2m_n}\left(1+\frac{m_n}{m_c}\right)$$

or
$$T_\alpha + Q - T_\alpha\left(\frac{m_\alpha}{m_c}\right) = T_n\left(1+\frac{m_n}{m_c}\right)$$

or
$$\frac{Q + \left(1-\frac{m_\alpha}{m_c}\right)T_\alpha}{\left(1+\frac{m_n}{m_c}\right)} = T_n$$

Here
$$\frac{m_\alpha}{m_c} = \frac{4m}{12m} = \frac{1}{3}$$
$$\frac{m_n}{m_c} = \frac{1}{12}$$

$\therefore$
$$T_n = \frac{Q + \left(1-\frac{m_\alpha}{m_c}\right)T_\alpha}{\left(1+\frac{m_n}{m_c}\right)} = \frac{Q+\left(1-\frac{1}{3}\right)T_\alpha}{1+\frac{1}{12}}$$
$$= \frac{5.7\text{ MeV} + \frac{2}{3}\times 5.3\text{ MeV}}{\frac{13}{12}} \qquad (\because\ T_\alpha = T)$$

$\therefore$ $T_n = 8.52$ MeV

> **YOUR STEP**
>
> Bombardment of the lithium isotope ${}_3Li^7$ by protons produce two α-particles. The energy of each α-particle when formed is 9.15 MeV. What is the energy of the bombarding protons?
>
> {$E = 1$ MeV}

§ 6.270

➢ **CONCEPT**

The problem is solved by following steps :

Step I : write the nuclear reaction :

$$Li^7\,(p,\alpha)\,He^4$$
$$\Downarrow$$
$$p + Li^7 \longrightarrow He^4 + \alpha$$
$$m_p\ m_{Li} \longrightarrow m_\alpha\ m_\alpha$$
$$P_p\ p_{Li} \longrightarrow P_\alpha\ p_\alpha' = p_\alpha$$
$$T_p\quad T_{Li}\qquad T_\alpha\quad T_\alpha$$

Step II : Apply conservation principle of momentum,

$$\vec{p}_p + \vec{p}_{Li} = \vec{p}_\alpha + \vec{p}_\alpha$$

$\therefore$
$$\vec{p}_p = \vec{p}_\alpha + \vec{p}_\alpha$$

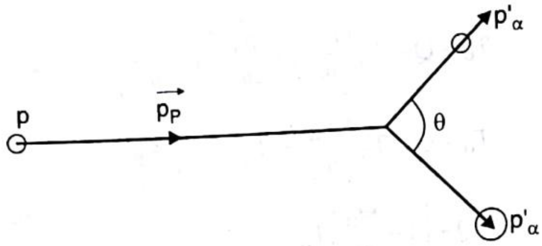

Fig. 6.270A

According to parallelogram law of vectors,

or $$p_p^2 = p_\alpha^2 + p_\alpha^{2\prime} + 2p_\alpha p_\alpha{}' \cos\theta$$

or $$p_p^2 = 2p_\alpha^2 + 2p_\alpha^2 \cos\theta \qquad (\because p'_\alpha = p_\alpha)$$

or $$p_p^2 = 2p_\alpha^2\, 2\cos^2\frac{\theta}{2}$$

$\therefore$ $$p_p = 2p_\alpha \cos\frac{\theta}{2} \qquad \text{...(i)}$$

Step III : Apply conservations principle of energy :

$$T_p + m_p c^2 + m_{Li}c^2 + T_{Li} = m_\alpha c^2 + T_\alpha + m_\alpha c^2 + T_\alpha$$

or $$T_p + (m_p + m_{Li} - 2m_\alpha)c^2 + T_{Li} = 2T_\alpha$$

or $$T_p + Q = 2\,T_\alpha \qquad (\because T_{Li} = 0)$$

$\therefore$ $$T_\alpha = \frac{T_p + Q}{2}$$

Here, $$Q = (m_p + m_{Li} - 2m_\alpha)\,c^2$$
$$= \{1.00783 + 7.01601 - 2\,(4.00260)\} \times 931 \text{ MeV}$$
$$= 17.35 \text{ MeV}$$

From problem, $T_p = T = 1.0$ MeV

On putting the values we get $T_\alpha = 9.18$ MeV

From Eqn. (i) $$p_p = 2p_\alpha \cos\frac{\theta}{2}$$

$\therefore$ $$\sec\frac{\theta}{2} = \frac{2p_\alpha}{p_p}$$

Here $$\frac{p_\alpha{}^2}{2m_\alpha} = T_\alpha$$

$\therefore$ $$p_\alpha = \sqrt{2m_\alpha T_\alpha}$$

Similarly, $$p_p = \sqrt{2m_p T_p}$$

$\therefore$ $$\sec\frac{\theta}{2} = 2\sqrt{\frac{2m_\alpha T_\alpha}{2m_p T_p}} = 2\sqrt{\frac{m_\alpha T_\alpha}{m_p T_p}}$$

Here $m_\alpha = 4m_p$, $T_\alpha = 9.18$ MeV

$$T_p = T = 1.0 \text{ MeV}$$

$\therefore$ On putting the values, we get

$$\sec\frac{\theta}{2} = 2\sqrt{2 \times 9.18} = 8.5697$$

$$\theta = 170.53° = 170.5°$$

YOUR STEP

The reaction ${}_5B^{10}$ (n, α) takes place when boron is bombarded by neutrons whose velocity is very small ("thermal" neutrons).

(a) Find the energy evolved in this reaction.

(b) Find the velocity and kinetic energy of an α-particle, assuming the boron nucleus to be immobile and neglecting the velocities of the neutron.

(a) $Q = 2.8$ MeV (b) $v = 9.3 \times 10^6$ m/sec, $E_\alpha = 1.8$ MeV

§ 6.271

➢ **CONCEPT**

For minimum kinetic energy of bombarding particle, the collision should be completely inelastic. For this, product particles move with common velocity.

$$\begin{array}{cccc} A & + B \longrightarrow & C & + D \\ m & m & m_c & m_D \\ v_A & v_B = 0 & v_c & v_c \end{array}$$

From conservation principle of momentum,

$$m v_A = (m_C \times m_D)\, v_c \qquad (\because m_C + m_D = m + M)$$

$$\therefore \quad v_C = \left(\frac{m}{m+M}\right) v_A$$

From conservation principle of energy

$$mc^2 + T_{th} + Mc^2 = m_C\, c^2 + T_C + m_D\, c^2 + T_D$$

or $$T_{th} + (m + M - m_C - m_D)\, c^2 = T_C + T_D$$

or $$T_{th} + Q = \frac{1}{2} m_C\, v_c^2 + \frac{1}{2} m_D\, v_c^2$$

or $$T_{th} + Q = \frac{1}{2}(m_C + m_D)\, v_c^2$$

or $$T_{th} + Q = \frac{1}{2}(m + M)\left\{\frac{m v_A}{m+M}\right\}^2$$

or $$T_{th} + Q = \frac{1}{2}\frac{m\, v_A^2\, m}{m+M}$$

or $$T_{th} + Q = \frac{T_{th}\, m}{m+M}$$

or $$T_{th} - \frac{T_{th}\, m}{m+M} = -Q$$

or $$T_{th}\left(\frac{M}{m+M}\right) = -Q$$

$$\therefore \quad T_{th} = \left(\frac{m+M}{M}\right)(-Q)$$

But kinetic energy is always positive

$$\therefore \quad T_{th} = \left(1 + \frac{m}{M}\right) |\, Q\, |$$

Here m = mass of bombarding particle

M = mass of target particle

YOUR STEP

(a) A particle of mass m_A and kinetic energy KE_A strikes a stationary nucleus of mass m_B to produce a compound nucleus of mass m_C. Express the excitation energy of the compound nucleus in terms of m_A, m_C, KE_A, and the Q value of the reaction. (Note : $|Q| << m_0 c^2$).

(b) An excited state in ^{16}O occurs at an energy of 16.2 MeV. Find the kinetic energy needed by a proton to produce a ^{16}O nucleus in this state by reaction with a stationary ^{15}N nucleus.

{(a) $E = -Q + KE_A\left(1 - \frac{m_A}{m_C}\right)$ (b) 4.34 MeV}

§ 6.272

➢ **CONCEPT**

The kinetic energy of bombarding particle for activate the nuclear reaction should be greater than or equal to threshold kinetic energy (T_{th}) *i.e.,* $T \geq T_{th}$

SOLUTION : Here

$$T_{th} = \left(1 + \frac{m}{M}\right)|Q|$$

$$= \left(1 + \frac{m_p}{m_d}\right)E_b = \left(1 + \frac{1}{2}\right)E_b \qquad (\because m_d = 2m_p)$$

$$= \frac{3}{2}E_b = \frac{3}{2} \times 2.2 \text{ MeV} = 3.3 \text{ MeV}$$

$\because$ $T \geq T_{th}$

or $T \geq 3.3$ MeV

YOUR STEP

In the reaction ^{2}H + ^{3}He ⟶ P + ^{4}He deuterons of energy 5 MeV are incident on ^{3}He at rest. Both the proton and the alpha particle are observed to travel along the same direction as the incident deuteron. Find the kinetic energies of proton and the alpha particle.

$K_p = 12.201$ MeV
$K_\alpha = 11.152$ MeV
$K_p = 22.706$ MeV
$K_\alpha = 0.647$ MeV

§ 6.273

➢ **CONCEPT**

The first reaction is

$$\text{Li}^7 + p \longrightarrow \text{Be}^7 + n - 1.65 \text{ MeV}$$

The Q values of this reaction is -1.65 MeV.

The threshold kinetic energy (minimum kinetic energy) of bombarding particle proton is

$$T_{th_1} = \left(1 + \frac{m}{M}\right)|Q|$$

$$= \left(1 + \frac{m_p}{m_{Li}}\right) \times 1.65 \text{ MeV}$$

$$= \left(1 + \frac{1}{7}\right) \times 1.65 \text{ MeV} = 1.886 \text{ MeV} \approx 1.89 \text{ MeV}$$

It means for activate the reaction, the kinetic energy of proton,

$$T_\phi > T_{th_1}$$

$\therefore$ $$T > 1.886 \text{ MeV}$$

The second reaction is

$$\text{Be}^9 + p \longrightarrow \text{B}^9 + n - 1.85 \text{ MeV}$$

The Q value of this reaction is -1.85 MeV. The threshold kinetic energy (minimum kinetic energy) is

$$T_{th_2} = \left(1 + \frac{m_p}{m_{Be}}\right) | -Q |$$

$$= \left(1 + \frac{1}{9}\right) \times 1.85 \text{ MeV} = 2.0556 \text{ MeV} \approx 2.06 \text{ MeV}$$

From this, it is clear that $T_{th_1} \le T \le T_{th_2}$

or $$1.886 \text{ MeV} \le T \le 2.0556$$

In the form of minimum significance figure for more accuracy,

$$1.89 \text{ MeV} \le T \le 2.06 \text{ MeV}$$

YOUR STEP

We can understand why U^{235} is readily fissionable, and U^{238} is not, with the following calculations :

(a) Find the energy difference between $U^{235} + n$ and U^{236}.

(b) Repeat for $U^{238} + n$ and U^{239}.

(c) Comparing your results for (a) and (b) explain why U^{235} will be fissioned with very low energy neutrons, while U^{238} requires fast neutrons of 1 to 2 MeV of energy to fission.

{(a) 6.545 MeV, (b) 4.807 MeV}

§ 6.274

➢ **CONCEPT**

The threshold kinetic energy of neutron is

$$T_{th} = \left(1 + \frac{m}{M}\right) | Q |$$

$$= \left(1 + \frac{m_n}{m_B}\right) | Q |$$

$$= \left(1 + \frac{1}{11}\right) | Q |$$

or $$4 \text{MeV} = \left(\frac{12}{11}\right) | Q |$$

$\therefore$ $$| Q | = \frac{11}{12} \times 4 \text{ MeV} = 3.67 \text{ MeV}$$

The reaction is endogenic. So, Q value should be negative.

$\therefore$ $$Q = -3.67 \text{ MeV}$$

YOUR STEP

A 5-MeV alpha particle strikes a stationary ${}_8O^{16}$ target. Find the speed of the centre of mass of the system and the kinetic energy of the particles relative to the centre of mass.

{3.1×10^6 m/sec; 4 MeV}

§ 6.275

➢ **CONCEPT**

The first reaction is

$$P + Li^7 \longrightarrow Be^7 + n$$

The Q value of this reaction is -1.64 MeV (calculated in problem 6.266 (a)).
The threshold kinetic energy of proton for this reaction is

$$T_{th} = \left(1 + \frac{m}{M}\right) | Q |$$

$$= \left(1 + \frac{m_p}{m_{Li}}\right) | Q |$$

$$= \left(1 + \frac{1}{7}\right) \times 1.64 \text{ meV} = 1.88 \text{ MeV}$$

The second reaction is

$$\underset{m_p}{P} + \underset{m_{Li}}{Li^7} \longrightarrow \underset{m'_{Li}}{Li^6} + \underset{m_d}{d}$$

From energy conservation principle in the form of Q,

$$m_p c^2 + m_{Li} c^2 = m'_{Li} c^2 + m_d c^2 + Q$$

$$\therefore \quad Q = (m_p + m_{Li} - m_{Li} - m_d)\, c^2$$

$$= (1.00783 + 7.01601 - 2.01410 - 6.01513) \times 931 \text{ MeV}$$

$$= -5.02 \text{ MeV}$$

The threshold kinetic energy of proton for activating this reaction is

$$T_{th} = \left(1 + \frac{m}{M}\right) | Q |$$

$$= \left(1 + \frac{m_p}{m_{Li}}\right) | Q |$$

$$= \left(1 + \frac{1}{7}\right) \times 5.02 \text{ MeV} = 5.74 \text{ MeV}$$

YOUR STEP

Find the minimum energy in the laboratory system that a proton must have in order to initiate the reaction.

$$p + d + 2.22 \text{ MeV} \longrightarrow p + p + n$$

{3.33 MeV}

§ 6.276

➢ **CONCEPT**

The reaction is

$$Li^7 + \alpha \longrightarrow B^{10} + n$$

From the solution of problem 6.266c. The Q value of this reaction is 2.79 MeV.

$$\therefore \quad T_{th} = \left(1 + \frac{m}{M}\right) | Q | = \left(1 + \frac{m_\alpha}{m_{Li}}\right) | Q |$$

$$= \left(1 + \frac{4}{7}\right) \times 2.79 \text{ MeV} = 4.38 \text{ MeV} = 4.4 \text{ MeV}$$

For minimum kinetic energy (threshold kinetic energy of α-particle), the product particles moves with common velocity which is equal to the centre of mass.

$$\therefore \quad v_{cm} = \frac{m_\alpha v_\alpha + m_{Li} v_{Li}}{m_\alpha + m_{Li}}$$

or
$$v_{cm} = \frac{m_\alpha v_\alpha}{m_\alpha + m_{Li}} \qquad (\because v_{Li} = 0)$$

$$v_{cm} = \frac{m_\alpha v_\alpha}{m_\alpha + m_{Li}}$$

$$= \left(\frac{m_\alpha}{m_\alpha + m_{Li}}\right) v_\alpha = \frac{4}{11} v_\alpha$$

$$\therefore \quad T_\alpha = T_{th}$$

$$\frac{1}{2} m_\alpha v_\alpha^2 = T_{th}$$

$$\therefore \quad v_\alpha = \sqrt{\frac{2T_{th}}{m_\alpha}}$$

$$v_{cm} = \left(\frac{m_\alpha}{m_\alpha + m_{Li}}\right)\sqrt{\frac{2T_{th}}{m_\alpha}}$$

$$= \left(\frac{4}{4+7}\right)\sqrt{\frac{2 \times 4.38 \times 10^6 \times 1.6 \times 10^{-19}}{4 \times 1.67 \times 10^{-27}}}$$

$$= 5.27 \times 10^6 \text{ m/s} = 5.3 \times 10^6 \text{ m/s}$$

YOUR STEP

In the reaction $_7N^{14}$ (α, p) the kinetic energy of an α-particle $E_\alpha = 7.7$MeV. Find the angle to the direction of motion of the α-particle at which a proton escapes if its kinetic energy $E_{pr} = 8.5$ MeV.

{$\phi = 54°$}

§ 6.277

➢ **CONCEPT**

The concept is similar to problem 6.265.

SOLUTION : The problem is solved by following steps :

Step I : Write the equation :

$$n + C^{12} \longrightarrow Be^{10} + \alpha$$

m_n	m_c	m_{Be}	m_α
p_n	$p_C = 0$	p_{Be}	p_α
T_n	$T_c = 0$	T_{Be}	T_α

Step II : Apply conservation principle of momentum

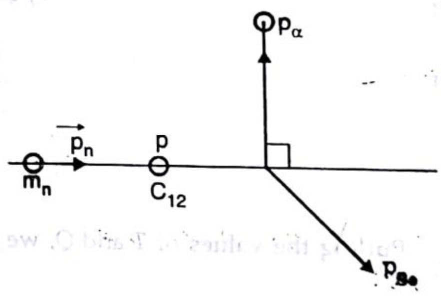

Fig. 6.277A

From figure,

$$\vec{p}_n + \vec{p}_C = \vec{p}_\alpha + \vec{p}_{Be}$$

or
$$\vec{p}_n = \vec{p}_\alpha + \vec{p}_{Be}$$

or
$$\vec{p}_n - \vec{p}_\alpha = \vec{p}_{Be}$$

or
$$p_n^2 + p_\alpha^2 - 2p_n p_\alpha \cos 90° = p_{Be}^2$$

or $$p_n^2 + p_\alpha^2 = p_{Be}^2 \quad \text{...(i)}$$

Step III : Find the value of Q :

As we know, $$T_{th} = \left(1 + \frac{m}{M}\right) | Q | = \left(1 + \frac{m_n}{m_c}\right) | Q |$$

$\therefore$ $$| Q | = \frac{T_{th}}{\left(1 + \frac{m_n}{m_c}\right)}$$

$$= \frac{6.17 \text{ MeV}}{1 + \frac{1}{12}} = 5.695 \text{ MeV}$$

Since, the reaction is endogenic, so, Q is negative.

$\therefore$ $$Q = -5.695 \text{ MeV}$$

Step IV : Apply conservation principle of energy

$$T_n + m_n c^2 + T_c + m_c c^2 = T_B + m_B c^2 + T_\alpha + m_\alpha c^2$$

or $$T_n + Q = T_{Be} + T_\alpha \quad (\because T_c = 0)$$

or $$T_n + Q = \frac{p_{Be}^2}{2m_{Be}} + \frac{p_\alpha^2}{2m_\alpha}$$

or $$T_n + Q = \frac{p_n^2 + p_\alpha^2}{2m_{Be}} + \frac{p_\alpha^2}{2m_\alpha}$$

or $$T + Q = \frac{p_n^2}{2m_{Be}} + \frac{p_\alpha^2}{2m_{Be}} + \frac{p_\alpha^2}{2m_\alpha} \quad (\because T_n = T)$$

or $$T + Q = \frac{2m_n T}{2m_B} + \frac{p_\alpha^2}{2m_\alpha}\left(1 + \frac{2m_\alpha}{2m_B}\right) \quad \left(\because \frac{P_n^2}{2m_n} = T\right)$$

or $$T + Q - \frac{1}{9}T = T_\alpha\left(1 + \frac{4}{9}\right) \quad \left(\frac{m_n}{m_{Be}} = \frac{1}{9}\right)$$

or $$T + Q - \frac{1}{9}T = T_\alpha\left(\frac{13}{9}\right)$$

or $$\frac{8}{9}T + Q = T_\alpha\left(\frac{13}{9}\right)$$

$\therefore$ $$T_\alpha = \frac{\frac{8}{9}T + Q}{\frac{13}{9}} = \frac{8T + 9Q}{13}$$

Putting the values of T and Q, we get

$$T_\alpha = \frac{8 \times 10 \text{ MeV} + 9 \times (-5.695 \text{ MeV})}{13} = 2.21 \text{ MeV}$$

YOUR STEP

Bombardment of the lithium isotope $_3Li^6$ by deuterons forms two α-particles which fly away symmetrically at an angle of ϕ to the direction of the velocity of the bombarding deuterons.

(a) Find the kinetic energy of the α-particle formed if the energy of the bombarding deuterons is 0.2 MeV.

(b) Find the angle ϕ

[(a) $E_K = 8.75$ MeV (b) $\phi = 87°$]

§ 6.278

➢ **CONCEPT**

The momentum of γ is

$$p_\gamma = \frac{h}{\lambda} = \frac{E_\gamma}{c} \qquad \left(\because E_\gamma = \frac{hc}{\lambda}\right)$$

The minimum energy of bombarding particle, the product particles move with common velocity which is equal to the velocity of centre of mass.

SOLUTION : The reaction is

$$\begin{array}{ccccccc} \gamma & + & H^2 & \longrightarrow & n & + & p \\ p_\gamma & & p_d = 0 & & p_n & & p_p \end{array}$$

From the conservation principle of momentum,

$$p_\gamma = m_n v_{cm} + m_p v_{cm}$$

$$\therefore \qquad v_{cm} = \frac{p_\gamma}{m_n + m_p} = \frac{E_\gamma}{c\,(m_n + m_p)} \qquad \text{...(i)}$$

Applying conservation principle of energy

$$E_\gamma + m_d c^2 = m_n c^2 + \frac{1}{2} m_n v_{cm}^2 + \frac{1}{2} m_p v_{cm}^2$$

or $$E_\gamma + (m_d - m_n - m_p)\, c^2 = \frac{1}{2}(m_n + m_p)\, v_{cm}^2$$

or $$E_\gamma + Q = \frac{1}{2}(m_n + m_p) \frac{E_\gamma^2}{c^2 (m_n + m_p)^2} \qquad \text{(from Eqn. (i))}$$

or $$E_\gamma + Q = \frac{E_\gamma^2}{2\, c^2 (m_n + m_p)} \qquad \text{...(ii)}$$

The binding energy of deuteron is

$$\begin{aligned} E_b &= (m_n + m_p - m_d)\, c^2 \\ &= -(m_d - m_n - m_p)\, c^2 = -Q \end{aligned}$$

$$\therefore \qquad Q = -E_b \qquad \text{...(iii)}$$

From Eqn. (ii) and (iii), we get

$$\therefore \qquad E_\gamma - E_b = \frac{E_\gamma^2}{2\, c^2 (m_n + m_p)}$$

or $$E_\gamma = E_b + \frac{E_\gamma^2}{2\, c^2 (m_n + m_p)}$$

or $$E_\gamma - E_b = \frac{E_\gamma^2}{2c^2 (m_n + m_p)}$$

or $$\delta E = \frac{E_\gamma^2}{2c^2 (m_n + m_p)} \qquad \text{...(iv)}$$

From Eqn. (ii) and (iii)

$$\therefore \qquad E_\gamma = E_b + \frac{E_\gamma^2}{2c^2 (m_n + m_p)}$$

The energy E_γ of γ-photon is negligible with resepect to rest mass energy of $(m_n + m_p)$.

i.e., $$E_\gamma <<< (m_n + m_p)\, c^2$$

$\therefore$ $$E_\gamma \approx E_b$$

From equ. (iv), $$\delta E = \frac{E_\gamma^2}{2c^2(m_n + m_p)}$$

or $$\delta E = \frac{E_b^2}{2c^2(m_n + m_p)} \qquad (\because E_\gamma \approx E_b)$$

or $$\frac{\delta E}{E_b} = \frac{E_b}{2c^2(m_n + m_p)} = \frac{E_b}{2mc^2}$$

Here m is mass of electron.

On putting the values, we get $$\frac{\delta E}{E_b} = 0.06\%$$

> **YOUR STEP**
>
> The initial reaction in the carbon cycle from which stars hotter than the sun obtain their energy, is
>
> $${}_1H^1 + {}_6C^{12} \longrightarrow {}_7N^{13} + \gamma$$
>
> Find the minimum energy of the proton which must have to come in contact with the ${}_6C^{12}$ nucleus.
>
> {2.37 MeV}

§ 6.279

➢ CONCEPT

This type of nuclear collision is completely inelastic collision. It means excited nucleus moves with the velocity of centre of mass.

SOLUTION : The reaction of problem is

$$\begin{array}{ccc} p & + \; d \longrightarrow & He^3 \\ m_p & m_d & m_{He} \\ T_p = T & T_d = 0 & T_{He} \\ p & p_d = 0 & p_{He} \end{array}$$

Apply conservation principle of momentum,

$$\vec{p} + \vec{pd} = p_{He}$$

or $$p + O = p_{He}$$

$\therefore$ $$p = p_{He} \qquad \text{...(i)}$$

Applying energy conservation principle,

$$T_p + m_p c^2 + m_d c^2 + T_d = m_{He} c^2 + T_{He} + \Delta E$$

where ΔE is excitation energy of the formed nucleus.

or $$T + (m_p + m_d - m_{He}) c^2 + 0 = T_{He} + \Delta E$$

or $$T + Q = T_{He} + \Delta E$$

$\therefore$ $$\Delta E = T + Q - T_{He}$$

or $$\Delta E = T + Q - \frac{p_{He}^2}{2m_{He}}$$

or $$\Delta E = T + Q - \frac{p^2}{2m_{He}} \qquad (\because p = p_{He})$$

or $$\Delta E = T + Q - \frac{2m_p T}{2m_{He}} \qquad \left(\because T = \frac{p^2}{2m_p}\right)$$

or
$$\Delta E = T\left(1 - \frac{m_p}{m_{He}}\right) + Q$$
$$= T\left(1 - \frac{1}{3}\right) + Q = \frac{2}{3}T + Q \qquad \text{...(ii)}$$

Here
$$Q = (m_p + m_d - m_{He})\,c^2$$
$$= (1.00783 + 2.01410 - 3.01603) \times 931 \text{ MeV}$$
$$= 5.49 \text{ MeV}$$

from equ. (ii)

$\because$
$$\Delta E = \frac{2}{3}T + Q$$
$$= \left(\frac{2}{3} \times 1.5 + 5.49\right)\text{MeV} = 6.49 \text{ MeV} \approx 6.5 \text{ MeV}$$

YOUR STEP

A reaction in which two particles join to form a single excited nucleus, which then decays to its ground state by photon emission, is known as radiative capture. Find the energy of the gamma ray emitted in the radiative capture of an alpha particle by Li^7. Assume alpha particles of very small kinetic energy are incident on Li^7 at rest.

{8.660 MeV}

§ 6.280

➢ **CONCEPT**

The problem is similar to previous problem.

The reaction is

$$\begin{array}{cccc} d & + \; C^{13} & \longrightarrow & N^{15} \\ m_d & m_c & & m_N \\ T_d & T_c = 0 & & T_N \\ p_d & p_c = 0 & & p_N \end{array}$$

According to conservation principle of momentum,
$$\vec{p_d} + \vec{p_C} = \vec{p_N}$$
$\therefore$
$$\vec{p_d} = \vec{p_N} \qquad (\because \vec{p_C} = 0)$$
$\therefore$
$$p_d = p_N \qquad \text{...(i)}$$

From conservation principle of energy
$$T_d + m_d c^2 + T_c + m_c c^2 = m_N c^2 + T_N + \Delta E$$

Here ΔE is the excitation energy of the nucleus

$\therefore$
$$T_d + (m_d + m_c - m_N)\,c^2 = T_N + \Delta E \qquad (\because T_c = 0)$$
$\therefore$
$$T_d + Q = T_N + \Delta E$$
or
$$T_i + Q = T_N + \Delta E \qquad (\because T_d = T_i)$$
or
$$T_i + Q = \frac{p_N^2}{2m_N} + \Delta E$$
or
$$T_i + Q = \frac{p_d^2}{2m_N} + \Delta E$$
or
$$T_i + Q = \frac{2T_i\,m_d}{2m_N} + \Delta E \qquad \left(\because t_i = \frac{p_d^2}{2m_d}\right)$$

$$\therefore \qquad \Delta E = T_i\left(1-\frac{m_d}{m_N}\right)+Q$$

$$= T_i\left(1-\frac{2}{15}\right)+Q$$

$$= T_i \times \frac{13}{15}+Q \qquad \text{...(ii)}$$

Hence,

$$Q = (m_d + m_c - m_N)c^2$$
$$= (2.01410 + 13.00335 - 15.00011) \times 931 \text{ MeV}$$
$$= 16.14 \text{ MeV}$$

$$\therefore \qquad \Delta E = \frac{13}{15}T_i + 16.14 \text{ MeV} \qquad \text{(from eqn. (i))}$$

when

$$T_i = 0.60 \text{ MeV}$$

$$\therefore \qquad \Delta E = \left(\frac{13}{15}\times 0.60 + 16.14\right)\text{MeV}$$
$$= 16.66 \text{ MeV} = 16.7 \text{ MeV}$$

when

$$T_i = 0.90 \text{ MeV}$$
$$\Delta E = \left(\frac{13}{15}\times 0.90 + 16.14\right)\text{MeV}$$
$$= 16.92 \text{ MeV}$$

when

$$T_i = 1.55 \text{ MeV}$$

$$\therefore \qquad \Delta E = \left(\frac{13}{15}\times 1.55 + 16.14\right)\text{MeV}$$
$$= 17.49 \text{ MeV} = 17.5 \text{ MeV}$$

Similarly at

$$T_i = 1.80 \text{ MeV}$$
$$\Delta E = 17.70 \text{ MeV}$$

YOUR STEP

A $_{92}U^{236}$ nucleus at rest absorbs a low-energy neutron. What is the internal excitation energy of the $_{92}U^{236}$ nucleus that is produced ? The atomic masses of the neutral atoms in their nuclear ground states are $_{92}U^{236}$ = 235.043924 u; $_{92}U^{236}$ = 236.045563 u.

{6.545 MeV}

Discussions on IE Irodov's Problems in General Physics

Discussion 2

By DB Singh

IRODOV is considered synonymous with ***problem-solving*** and ***concept development*** in **Physics**. The problems in the book are nothing but a challenge to the best brains in the world. They are the true test of your grasp of the fundamentals of the basic Physics. Some of the problems are really tricky and require the use of principles across two or more topics of physics. In India, Irodov's classic book, **Problems in General Physics**, is immensely popular among the **Engineering** aspirants. They use this book to take their problem-solving skills to higher levels so as to feel confident, and crack the JEE problems in physics easily.

Discussions on IE IRODOV (Discussions 1 & 2) give you not just the solutions to the problems in the **'great'** book, but shows you the ways to approach the difficult and concept-based problems in physics. The basic objective of the solutions in these books is to ***strengthen*** your ***fundamentals of physics*** and let you think ***intelligently*** without making you stand on crutches.

Discussion 2 contains the indepth discussions of the problems from the **Electrodynamics, Oscillations & Sounds, Optics and Modern Physics** sections of the book, **Problems in General Physics.**

Recommended to all those appearing in all Engineering Entrances, those seeking to participate in Physics Olympiads, and anyone who wants to master the problem-solving skills in Physics.

Solutions of IE Irodov's Problems in General Physics in User-Friendly *Two Parts*