# KnightCap: A chess program that learns by combining $TD(\lambda)$ with game-tree search

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# **Abstract**

In this paper we present TDLeaf( $\lambda$ ), a variation on the  $TD(\lambda)$  algorithm that enables it to be used in conjunction with game-tree search. We present some experiments in which our chess program "KnightCap" used TDLeaf( $\lambda$ ) to learn its evaluation function while playing on the Free Internet Chess Server (FICS, fics.onenet.net). The main success we report is that KnightCap improved from a 1650 rating to a 2150 rating in just 308 games and 3 days of play. As a reference, a rating of 1650 corresponds to about level B human play (on a scale from E (1000) to A (1800)), while 2150 is human master level. We discuss some of the reasons for this success, principle among them being the use of on-line, rather than self-play.

## 1 Introduction

Temporal Difference learning, first introduced by Samuel [5] and later extended and formalized by Sutton [7] in his  $TD(\lambda)$  algorithm, is an elegant technique for approximating the expected long term future cost (or cost-to-go) of a stochastic dynamical system as a function of the current state. The mapping from states to future cost is implemented by a parameterized function approximator such as a neural network. The parameters are updated online after each state transition, or possibly in batch updates after several state transitions. The goal of the algorithm is to improve the cost estimates as the number of observed state transitions and associated costs increases.

Perhaps the most remarkable success of  $TD(\lambda)$  is Tesauro's TD-Gammon, a neural network backgammon player that was trained from scratch using  $TD(\lambda)$  and simulated selfplay. TD-Gammon is competitive with the best human

backgammon players [9]. In TD-Gammon the neural network played a dual role, both as a predictor of the expected cost-to-go of the position and as a means to select moves. In any position the next move was chosen greedily by evaluating all positions reachable from the current state, and then selecting the move leading to the position with smallest expected cost. The parameters of the neural network were updated according to the  $TD(\lambda)$  algorithm after each game.

Although the results with backgammon are quite striking, there is lingering disappointment that despite several attempts, they have not been repeated for other board games such as othello, Go and the "drosophila of AI" — chess [10, 12, 6].

Many authors have discussed the peculiarities of backgammon that make it particularly suitable for Temporal Difference learning with self-play [8, 6, 4]. Principle among these are *speed of play*: TD-Gammon learnt from several hundred thousand games of self-play, *representation smoothness*: the evaluation of a backgammon position is a reasonably smooth function of the position (viewed, say, as a vector of piece counts), making it easier to find a good neural network approximation, and *stochasticity*: backgammon is a random game which forces at least a minimal amount of exploration of search space.

As TD-Gammon in its original form only searched oneply ahead, we feel this list should be appended with: *shal-low search is good enough against humans*. There are two possible reasons for this; either one does not gain a lot by searching deeper in backgammon (questionable given that recent versions of TD-Gammon search to three-ply and this significantly improves their performance), or humans are simply incapable of searching deeply and so TD-Gammon is only competing in a pool of shallow searchers. Although we know of no psychological studies investigating the depth to which humans search in backgammon, it is plausible that the combination of high branching factor and random move generation makes it quite difficult to search more than one or two-ply ahead. In particular, random move generation effectively prevents selective search or "forward pruning" because it enforces a lower bound on the branching factor at each move.

In contrast, finding a representation for chess, othello or Go which allows a small neural network to order moves at one-ply with near human performance is a far more difficult task. It seems that for these games, reliable tactical evaluation is difficult to achieve without deep lookahead. As deep lookahead invariably involves some kind of minimax search, which in turn requires an exponential increase in the number of positions evaluated as the search depth increases, the computational cost of the evaluation function has to be low, ruling out the use of expensive evaluation functions such as neural networks. Consequently most chess and othello programs use linear evaluation functions (the branching factor in Go makes minimax search to any significant depth nearly infeasible).

In this paper we introduce  $TDLeaf(\lambda)$ , a variation on the  $TD(\lambda)$  algorithm that can be used to learn an evaluation function for use in deep minimax search.  $TDLeaf(\lambda)$  is identical to  $TD(\lambda)$  except that instead of operating on the positions that occur during the game, it operates on the leaf nodes of the *principal variation* of a minimax search from each position (also known as the *principal leaves*).

To test the effectiveness of TDLeaf( $\lambda$ ), we incorporated it into our own chess program—KnightCap. KnightCap has a particularly rich board representation enabling relatively fast computation of sophisticated positional features, although this is achieved at some cost in speed (KnightCap is about 10 times slower than Crafty—the best public-domain chess program—and 6,000 times slower than *Deep Blue*). We trained KnightCap's linear evaluation function using TDLeaf( $\lambda$ ) by playing it on the Free Internet Chess Server (FICS, fics.onenet.net) and on the Internet Chess Club (ICC, chessclub.com). Internet play was used to avoid the premature convergence difficulties associated self-play<sup>1</sup>. The main success story we report is that starting from an evaluation function in which all coefficients were set to zero except the values of the pieces, KnightCap went from a 1650-rated player to a 2150-rated player in just three days and 308 games. KnightCap is an ongoing project with new features being added to its evaluation function all the time. We use TDLeaf( $\lambda$ ) and Internet play to tune the coefficients of these features.

The remainder of this paper is organized as follows. In section 2 we describe the  $TD(\lambda)$  algorithm as it applies to games. The  $TDLeaf(\lambda)$  algorithm is described in section 3. Experimental results for internet-play with KnightCap are given in section 4. Section 5 contains some discussion and concluding remarks.

# 2 The $TD(\lambda)$ algorithm applied to games

In this section we describe the  $TD(\lambda)$  algorithm as it applies to playing board games. We discuss the algorithm from the point of view of an *agent* playing the game.

Let S denote the set of all possible board positions in the game. Play proceeds in a series of moves at discrete time steps  $t=1,2,\ldots$ . At time t the agent finds itself in some position  $x_t\in S$ , and has available a set of moves, or actions  $A_{x_t}$  (the legal moves in position  $x_t$ ). The agent chooses an action  $a\in A_{x_t}$  and makes a transition to state  $x_{t+1}$  with probability  $p(x_t,x_{t+1},a)$ . Here  $x_{t+1}$  is the position of the board after the agent's move and the opponent's response. When the game is over, the agent receives a scalar reward, typically "1" for a win, "0" for a draw and "-1" for a loss.

For ease of notation we will assume all games have a fixed length of N (this is not essential). Let  $r(x_N)$  denote the reward received at the end of the game. If we assume that the agent chooses its actions according to some function a(x) of the current state x (so that  $a(x) \in A_x$ ), the expected reward from each state  $x \in S$  is given by

$$J^*(x) := E_{x_N|x} r(x_N), \tag{1}$$

where the expectation is with respect to the transition probabilities  $p(x_t, x_{t+1}, a(x_t))$  and possibly also with respect to the actions  $a(x_t)$  if the agent chooses its actions stochastically.

For very large state spaces S it is not possible store the value of  $J^*(x)$  for every  $x \in S$ , so instead we might try to approximate  $J^*$  using a parameterized function class  $\tilde{J} \colon S \times \mathbb{R}^k \to \mathbb{R}$ , for example linear function, splines, neural networks, etc.  $\tilde{J}(\cdot,w)$  is assumed to be a differentiable function of its parameters  $w=(w_1,\ldots,w_k)$ . The aim is to find a parameter vector  $w \in \mathbb{R}^k$  that minimizes some measure of error between the approximation  $\tilde{J}(\cdot,w)$  and  $J^*(\cdot)$ . The  $\mathrm{TD}(\lambda)$  algorithm, which we describe now, is designed to do exactly that.

Suppose  $x_1, \ldots, x_{N-1}, x_N$  is a sequence of states in one game. For a given parameter vector w, define the *temporal difference* associated with the transition  $x_t \to x_{t+1}$  by

$$d_t := \tilde{J}(x_{t+1}, w) - \tilde{J}(x_t, w). \tag{2}$$

<sup>&</sup>lt;sup>1</sup>Randomizing move choice is another way of avoiding problems associated with self-play (this approach has been tried in Go [6]), but the advantage of the Internet is that more information is provided by the opponents play.

Note that  $d_t$  measures the difference between the reward predicted by  $\tilde{J}(\cdot,w)$  at time t+1, and the reward predicted by  $\tilde{J}(\cdot,w)$  at time t. The true evaluation function  $J^*$  has the property

$$E_{x_{t+1}|x_t} \left[ J^*(x_{t+1}) - J^*(x_t) \right] = 0,$$

so if  $\tilde{J}(\cdot,w)$  is a good approximation to  $J^*$ ,  $E_{x_{t+1}|x_t}d_t$  should be close to zero. For ease of notation we will assume that  $\tilde{J}(x_N,w)=r(x_N)$  always, so that the final temporal difference satisfies

$$d_{N-1} = \tilde{J}(x_N, w) - \tilde{J}(x_{N-1}, w) = r(x_N) - \tilde{J}(x_{N-1}, w).$$

That is,  $d_{N-1}$  is the difference between the true outcome of the game and the prediction at the penultimate move.

At the end of the game, the  $TD(\lambda)$  algorithm updates the parameter vector w according to the formula

$$w := w + \alpha \sum_{t=1}^{N-1} \nabla \tilde{J}(x_t, w) \left[ \sum_{j=t}^{N-1} \lambda^{j-t} d_t \right]$$
 (3)

where  $\nabla \tilde{J}(\cdot,w)$  is the vector of partial derivatives of  $\tilde{J}$  with respect to its parameters. The positive parameter  $\alpha$  controls the learning rate and would typically be "annealed" towards zero during the course of a long series of games. The parameter  $\lambda \in [0,1]$  controls the extent to which temporal differences propagate backwards in time. To see this, compare equation (3) for  $\lambda=0$ :

$$w := w + \alpha \sum_{t=1}^{N-1} \nabla \tilde{J}(x_t, w) d_t$$

$$= w + \alpha \sum_{t=1}^{N-1} \nabla \tilde{J}(x_t, w) \left[ \tilde{J}(x_{t+1}, w) - \tilde{J}(x_t, w) \right]$$
(4)

and  $\lambda = 1$ :

$$w := w + \alpha \sum_{t=1}^{N-1} \nabla \tilde{J}(x_t, w) \left[ r(x_N) - \tilde{J}(x_t, w) \right]. \quad (5)$$

Consider each term contributing to the sums in equations (4) and (5). For  $\lambda=0$  the parameter vector is being adjusted in such a way as to move  $\tilde{J}(x_t,w)$ —the predicted reward at time t—closer to  $\tilde{J}(x_{t+1},w)$ —the predicted reward at time t+1. In contrast, TD(1) adjusts the parameter vector in such away as to move the predicted reward at time step t closer to the final reward at time step t. Values of t between zero and one interpolate between these two behaviors. Note that (5) is equivalent to gradient descent on the error function t0 is t1.

Successive parameter updates according to the  $\mathrm{TD}(\lambda)$  algorithm should, over time, lead to improved predictions of the expected reward  $\tilde{J}(\cdot,w)$ . Provided the actions  $a(x_t)$  are independent of the parameter vector w, it can be shown that for  $linear\ \tilde{J}(\cdot,w)$ , the  $\mathrm{TD}(\lambda)$  algorithm converges to a near-optimal parameter vector [11]. Unfortunately, there is no such guarantee if  $\tilde{J}(\cdot,w)$  is non-linear [11], or if  $a(x_t)$  depends on w [2].

# 3 Minimax Search and $TD(\lambda)$

For argument's sake, assume any action a taken in state x leads to predetermined state which we will denote by  $x'_a$ . Once an approximation  $\tilde{J}(\cdot,w)$  to  $J^*$  has been found, we can use it to choose actions in state x by picking the action  $a \in A_x$  whose successor state  $x'_a$  minimizes the opponent's expected reward<sup>2</sup>:

$$a^*(x) := \operatorname{argmin}_{a \in A_x} \tilde{J}(x_a', w). \tag{6}$$

This was the strategy used in TD-Gammon. Unfortunately, for games like othello and chess it is very difficult to accurately evaluate a position by looking only one move or ply ahead. Most programs for these games employ some form of minimax search. In minimax search, one builds a tree from position x by examining all possible moves for the computer in that position, then all possible moves for the opponent, and then all possible moves for the computer and so on to some predetermined depth d. The leaf nodes of the tree are then evaluated using a heuristic evaluation function (such as  $J(\cdot, w)$ ), and the resulting scores are propagated back up the tree by choosing at each stage the move which leads to the best position for the player on the move. See figure 1 for an example game tree and its minimax evaluation. With reference to the figure, note that the evaluation assigned to the root node is the evaluation of the leaf node of the principal variation; the sequence of moves taken from the root to the leaf if each side chooses the best available move.

In practice many engineering tricks are used to improve the performance of the minimax algorithm,  $\alpha-\beta$  search being the most famous.

Let  $\tilde{J}_d(x,w)$  denote the evaluation obtained for state x by applying  $\tilde{J}(\cdot,w)$  to the leaf nodes of a depth d minimax search from x. Our aim is to find a parameter vector w such that  $\tilde{J}_d(\cdot,w)$  is a good approximation to the expected reward  $J^*$ . One way to achieve this is to apply the  $\mathrm{TD}(\lambda)$  algorithm to  $\tilde{J}_d(x,w)$ . That is, for each sequence of posi-

 $<sup>^2</sup>$ If successor states are only determined stochastically by the choice of a, we would choose the action minimizing the expected reward over the choice of successor states.

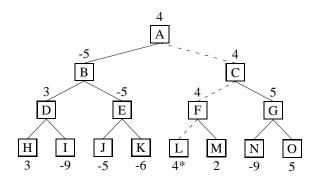


Figure 1: Full breadth, 3-ply search tree illustrating the minimax rule for propagating values. Each of the leaf nodes (H–O) is given a score by the evaluation function,  $\tilde{J}(\cdot,w)$ . These scores are then propagated back up the tree by assigning to each opponent's internal node the minimum of its children's values, and to each of our internal nodes the maximum of its children's values. The principle variation is then the sequence of best moves for either side starting from the root node, and this is illustrated by a dashed line in the figure. Note that the score at the root node A is the evaluation of the leaf node (L) of the principal variation. As there are no ties between any siblings, the derivative of A's score with respect to the parameters w is just  $\nabla \tilde{J}(L,w)$ .

tions  $x_1, \ldots, x_N$  in a game we define the temporal differences

$$d_t := \tilde{J}_d(x_{t+1}, w) - \tilde{J}_d(x_t, w) \tag{7}$$

as per equation (2), and then the  $TD(\lambda)$  algorithm (3) for updating the parameter vector w becomes

$$w := w + \alpha \sum_{t=1}^{N-1} \nabla \tilde{J}_d(x_t, w) \left[ \sum_{j=t}^{N-1} \lambda^{j-t} d_t \right]. \tag{8}$$

One problem with equation (8) is that for d>1,  $\tilde{J}_d(x,w)$  is not necessarily a differentiable function of w for all values of w, even if  $\tilde{J}(\cdot,w)$  is everywhere differentiable. This is because for some values of w there will be "ties" in the minimax search, i.e. there will be more than one best move available in some of the positions along the principal variation, which means that the principal variation will not be unique (see figure 2). Thus, the evaluation assigned to the root node,  $\tilde{J}_d(x,w)$ , will be the evaluation of any one of a number of leaf nodes.

Fortunately, under some mild technical assumptions on the behavior of  $\tilde{J}(x,w)$ , it can be shown that for each state x, the set of  $w \in \mathbb{R}^k$  for which  $\tilde{J}_d(x,w)$  is not differentiable has Lebesgue measure zero. Thus for all states x and for "almost all"  $w \in \mathbb{R}^k$ ,  $\tilde{J}_d(x,w)$  is a differentiable function

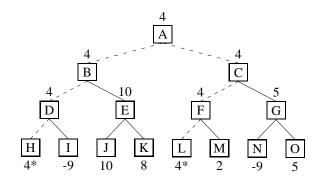


Figure 2: A search tree with a non-unique principal variation (PV). In this case the derivative of the root node A with respect to the parameters of the leaf-node evaluation function is multi-valued, either  $\nabla \tilde{J}(H,w)$  or  $\nabla \tilde{J}(L,w)$ . Except for transpositions (in which case H and L are identical and the derivative is single-valued anyway), such "collisions" are likely to be extremely rare, so in TDLeaf( $\lambda$ ) we ignore them by choosing a leaf node arbitrarily from the available candidates.

of w. Note that  $\tilde{J}_d(x,w)$  is also a continuous function of w whenever  $\tilde{J}(x,w)$  is a continuous function of w. This implies that even for the "bad" pairs (x,w),  $\nabla \tilde{J}_d(x,w)$  is only undefined because it is multi-valued. Thus we can still arbitrarily choose a particular value for  $\nabla \tilde{J}_d(x,w)$  if w happens to land on one of the bad points.

Based on these observations we modified the  $TD(\lambda)$  algorithm to take account of minimax search in an almost trivial way: instead of working with the root positions  $x_1, \ldots, x_N$ , the  $TD(\lambda)$  algorithm is applied to the leaf positions found by minimax search from the root positions. We call this algorithm  $TDLeaf(\lambda)$ . Full details are given in figure 3.

# 4 TDLeaf( $\lambda$ ) and Chess

In this section we describe the outcome of several experiments in which the  $TDLeaf(\lambda)$  algorithm was used to train the weights of a linear evaluation function in our chess program "KnightCap". KnightCap is a reasonably sophisticated computer chess program for Unix systems. It has all the standard algorithmic features that modern chess programs tend to have as well as a number of features that are much less common. For more details on KnightCap, including the source code, see wwwsyseng.anu.edu.au/lsg.

Let  $\tilde{J}(\cdot, w)$  be a class of evaluation functions parameterized by  $w \in \mathbb{R}^k$ . Let  $x_1, \dots, x_N$  be N positions that occurred during the course of a game, with  $r(x_N)$  the outcome of the game. For notational convenience set  $\tilde{J}(x_N, w) := r(x_N)$ .

- 1. For each state  $x_i$ , compute  $\tilde{J}_d(x_i, w)$  by performing minimax search to depth d from  $x_i$  and using  $\tilde{J}(\cdot, w)$  to score the leaf nodes. Note that d may vary from position to position.
- 2. Let  $x_i^l$  denote the leaf node of the principle variation starting at  $x_i$ . If there is more than one principal variation, choose a leaf node from the available candidates at random. Note that

$$\tilde{J}_d(x_i, w) = \tilde{J}(x_i^l, w). \tag{9}$$

3. For t = 1, ..., N - 1, compute the temporal differences:

$$d_t := \tilde{J}(x_{t+1}^l, w) - \tilde{J}(x_t^l, w). \tag{10}$$

4. Update w according to the TDLeaf( $\lambda$ ) formula:

$$w := w + \alpha \sum_{t=1}^{N-1} \nabla \tilde{J}(x_t^l, w) \left[ \sum_{j=t}^{N-1} \lambda^{j-t} d_t \right].$$
 (11)

Figure 3: The TDLeaf( $\lambda$ ) algorithm

# 4.1 Experiments with KnightCap

In our main experiment we took KnightCap's evaluation function and set all but the material parameters to zero. The material parameters were initialized to the standard "computer" values: 1 for a pawn, 4 for a knight, 4 for a bishop, 6 for a rook and 12 for a queen. With these parameter settings KnightCap (under the pseudonym "Wimp-Knight") was started on the Free Internet Chess server (FICS, fics.onenet.net) against both human and computer opponents. We played KnightCap for 25 games without modifying its evaluation function so as to get a reasonable idea of its rating. After 25 games it had a blitz (fast time control) rating of  $1650 \pm 50^3$ , which put it at about B-grade human performance (on a scale from E (1000) to A (1800)), although of course the kind of game KnightCap plays with just material parameters set is very different to human play of the same level (KnightCap makes no shortterm tactical errors but is positionally completely ignorant). We then turned on the TDLeaf( $\lambda$ ) learning algorithm, with  $\lambda = 0.7$  and the learning rate  $\alpha = 1.0$ . The value of  $\lambda$  was chosen heuristically, based on the typical delay in moves before an error takes effect, while  $\alpha$  was set high enough to ensure rapid modification of the parameters. A couple of minor modifications to the algorithm were made:

• The raw (linear) leaf node evaluations  $\tilde{J}(x_i^l, w)$  were converted to a score between -1 and 1 by computing

$$v_i^l := \tanh \left[ \beta \tilde{J}(x_i^l, w) \right].$$

This ensured small fluctuations in the relative values of leaf nodes did not produce large temporal differences (the values  $v_i^l$  were used in place of  $\tilde{J}(x_i^l,w)$  in the TDLeaf( $\lambda$ ) calculations). The outcome of the game  $r(x_N)$  was set to 1 for a win, -1 for a loss and 0 for a draw.  $\beta$  was set to ensure that a value of  $\tanh\left[\beta\tilde{J}(x_i^l,w)\right]=0.25$  was equivalent to a material superiority of 1 pawn (initially).

• The temporal differences,  $d_t = v_{t+1}^l - v_t^l$ , were modified in the following way. Negative values of  $d_t$  were left unchanged as any decrease in the evaluation from one position to the next can be viewed as mistake. However, positive values of  $d_t$  can occur simply because the opponent has made a blunder. To avoid KnightCap trying to learn to predict its opponent's blunders, we set all positive temporal differences to zero unless KnightCap predicted the opponent's move<sup>4</sup>.

 $<sup>^{3}</sup>$ the standard deviation for all ratings reported in this section is about 50

<sup>&</sup>lt;sup>4</sup>In a later experiment we only set positive temporal differences to zero if KnightCap did not predict the opponent's move *and* the opponent was rated less than KnightCap. After all, predicting a stronger opponent's blunders is a useful skill, although whether this made any difference is not clear.

 The value of a pawn was kept fixed at its initial value so as to allow easy interpretation of weight values as multiples of the pawn value (we actually experimented with not fixing the pawn value and found it made little difference: after 1764 games with an adjustable pawn its value had fallen by less than 7 percent).

Within 300 games KnightCap's rating had risen to 2150, an increase of 500 points in three days, and to a level comparable with human masters. At this point KnightCap's performance began to plateau, primarily because it does not have an opening book and so will repeatedly play into weak lines. We have since implemented an opening book learning algorithm and with this KnightCap now plays at a rating of 2400–2500 (peak 2575) on the other major internet chess server: ICC, chessclub.com<sup>5</sup> It often beats International Masters at blitz. Also, because KnightCap automatically learns its parameters we have been able to add a large number of new features to its evaluation function: Knight-Cap currently operates with 5872 features (1468 features in four stages: opening, middle, ending and mating<sup>6</sup>). With this extra evaluation power KnightCap easily beats versions of Crafty restricted to search only as deep as itself. However, a big caveat to all this optimistic assessment is that KnightCap routinely gets crushed by faster programs searching more deeply. It is quite unlikely this can be easily fixed simply by modifying the evaluation function, since for this to work one has to be able to predict tactics statically, something that seems very difficult to do. If one could find an effective algorithm for "learning to search selectively" there would be potential for far greater improvement.

Note that we have twice repeated the learning experiment and found a similar rate of improvement and final performance level. The rating as a function of the number of a games from one of these repeat runs is shown in figure 4 (we did not record this information in the first experiment). Note that in this case KnightCap took mearly twice as long to reach the 2150 mark, but this was partly because it was operating with limited memory (8Mb) until game 500 at which point the memory was increased to 40Mb (Knight-Cap's search algorithm—MTD(f) [3]—is a memory intensive variant of  $\alpha$ - $\beta$  and when learning KnightCap must

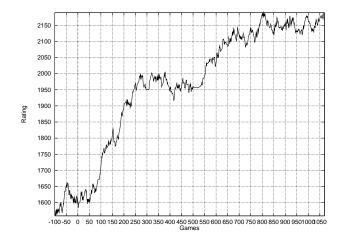


Figure 4: KnightCap's rating as a function of games played (second experiment). Learning was turned on at game 0.

store the whole position in the hash table so small memory really hurts the performance). Another reason may also have been that for a portion of the run we were performing paramater updates after every four games rather than every game.

Plots of various parameters as a function of the number of games played are shown in Figure 5 (these plots are from the same experiment in figure 4). Each plot contains three graphs corresponding to the three different stages of the evaluation function: opening, middle and ending<sup>7</sup>.

Finally, we compared the performance of KnightCap with its learnt weight to KnightCap's performance with a set of hand-coded weights, again by playing the two versions on ICC. The hand-coded weights were close in performance to the learnt weights (perhaps 50-100 rating points worse). We also tested the result of allowing KnightCap to learn starting from the hand-coded weights, and in this case it seems that KnightCap performs better than when starting from just material values (peak performance was 2632 compared to 2575, but these figures are very noisy). We are conducting more tests to verify these results. However, it should not be too surprising that learning from a good quality set of hand-crafted parameters is better than just learning from material parameters. In particular, some of the handcrafted parameters have very high values (the value of an "unstoppable pawn", for example) which can take a very long time to learn under normal playing conditions, particularly if they are rarely active in the principal leaves. It is

<sup>&</sup>lt;sup>5</sup>There appears to be a systematic difference of around 200–250 points between the two servers, so a peak rating of 2575 on ICC roughly corresponds to a peak of 2350 on FICS. We transferred KnightCap to ICC because there are more strong players playing there.

<sup>&</sup>lt;sup>6</sup>In reality there are not 1468 independent "concepts" per stage in KnightCap's evaluation function as many of the features come in groups of 64, one for each square on the board (like the value of placing a rook on a particular square, for example)

<sup>&</sup>lt;sup>7</sup>KnightCap actually has a fourth and final stage "mating" which kicks in when all the pieces are off, but this stage only uses a few of the coefficients (opponent's king mobiliity and proximity of our king to the opponent's king).

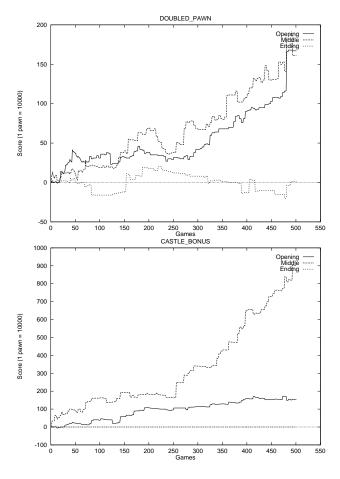


Figure 5: Evolution of two paramaters (bonus for castling and penalty for a doubled pawn) as a function of the number of games played. Note that each parameter appears three times: once for each of the three stages in the evaluation function.

not yet clear whether given a sufficient number of games this dependence on the initial conditions can be made to vanish.

## 4.2 Discussion

There appear to be a number of reasons for the remarkable rate at which KnightCap improved.

- As all the non-material weights were initially zero, even small changes in these weights could cause very large changes in the relative ordering of materially equal positions. Hence even after a few games Knight-Cap was playing a substantially better game of chess.
- 2. It seems to be important that KnightCap started out life with intelligent material parameters. This put it

- close in parameter space to many far superior parameter settings.
- 3. Most players on FICS prefer to play opponents of similar strength, and so KnightCap's opponents improved as it did. This may have had the effect of *guiding* KnightCap along a path in weight space that led to a strong set of weights.
- 4. KnightCap was learning on-line, not by self-play. The advantage of on-line play is that there is a great deal of information provided by the opponent's moves. In particular, against a stronger opponent KnightCap was being shown positions that 1) could be forced (against KnightCap's weak play) and 2) were mis-evaluated by its evaluation function. Of course, in self-play KnightCap can also discover positions which are misevaluated, but it will not find the kinds of positions that are relevant to strong play against other opponents. In this setting, one can view the information provided by the opponent's moves as partially solving the "exploration" part of the *exploration/exploitation* tradeoff.

To further investigate the importance of some of these reasons, we conducted several more experiments.

#### Good initial conditions.

A second experiment was run in which KnightCap's coefficients were all initialised to the value of a pawn. The value of a pawn needs to be positive in KnightCap because it is used in many other places in the code: for example we deem the MTD search to have converged if  $\alpha < \beta + 0.07 \text{*PAWN}$ . Thus, to set all parameters equal to the same value, that value had to be a pawn.

Playing with the initial weight settings KnightCap had a blitz rating of around 1250. After more than 1000 games on FICS KnightCap's rating has improved to about 1550, a 300 point gain. This is a much slower improvement than the original experiment. We do not know whether the coefficients would have eventually converged to good values, but it is clear from this experiment that starting near to a good set of weights is important for fast convergence. An interesting avenue for further exploration here is the effect of  $\lambda$  on the learning rate. Because the initial evaluation function is completely wrong, there would be some justification in setting  $\lambda=1$  early on so that KnightCap only tries to predict the outcome of the game and not the evaluations of later moves (which are extremely unreliable).

#### **Self-Play**

Learning by self-play was extremely effective for TD-

Gammon, but a significant reason for this is the randomness of backgammon which ensures that with high probability different games have substantially different sequences of moves, and also the speed of play of TD-Gammon which ensured that learning could take place over several hundred-thousand games. Unfortunately, chess programs are slow, and chess is a deterministic game, so self-play by a deterministic algorithm tends to result in a large number of substantially similar games. This is not a problem if the games seen in self-play are "representative" of the games played in practice, however KnightCap's self-play games with only non-zero material weights are very different to the kind of games humans of the same level would play.

To demonstrate that learning by self-play for KnightCap is not as effective as learning against real opponents, we ran another experiment in which all but the material parameters were initialised to zero again, but this time KnightCap learnt by playing against itself. After 600 games (twice as many as in the original FICS experiment), we played the resulting version against the good version that learnt on FICS for a further 100 games with the weight values fixed. The self-play version scored only 11% against the good FICS version.

Simultaneously with the work presented here, Beal and Smith [1] reported positive results using essentially TDLeaf( $\lambda$ ) and self-play (with some random move choice) when learning the parameters of an evaluation function that only computed material balance. However, they were not comparing performance against on-line players, but were primarily investigating whether the weights would converge to "sensible" values at least as good as the naive (1, 3, 3, 5, 9) values for (pawn, knight, bishop, rook, queen) (they did, within 2000 games, and using a value of  $\lambda=0.95$  which supports the discussion in "good initial conditions" above).

# 5 Conclusion

We have introduced TDLeaf( $\lambda$ ), a variant of TD( $\lambda$ ) suitable for training an evaluation function used in minimax search. The only extra requirement of the algorithm is that the leafnodes of the principal variations be stored throughout the game.

We presented some experiments in which a chess evaluation function was trained from B-grade to master level using  $TDLeaf(\lambda)$  by on-line play against a mixture of human and computer opponents. The experiments show both the importance of "on-line" sampling (as opposed to self-play) for a deterministic game such as chess, and the need to start near a good solution for fast convergence, although just how near is still not clear.

On the theoretical side, it has recently been shown that  $TD(\lambda)$  converges for linear evaluation functions[11] (although only in the sense of prediction, not control). An interesting avenue for further investigation would be to determine whether  $TDLeaf(\lambda)$  has similar convergence properties.

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