Globally-Optimal Greedy Algorithms for Tracking a Variable Number of Objects

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Presented By

Albert Haque and Fahim Dalvi

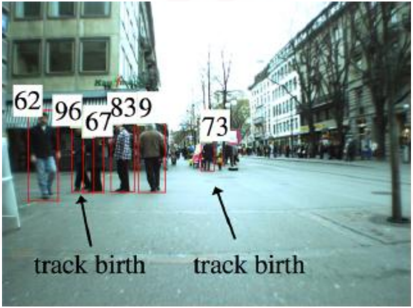
April 29, 2015

Outline

- Motivation & Related Work
- Mathematical Representation
  - Probabilistic Framework
  - ILP Formulation
- Multiple Object Tracking
  - Globally Optimal Greedy Algorithm
  - Approximate Dynamic Programming Algorithm
- Experiments and Results

Motivation

- Single object tracking isn't enough
- In reality, multiple objects appear and occlusion is present



Problem Statement

- Input: a video sequence with bounding boxes
- Output: assignment of IDs to all tracks
- Representation: a point x in *spacetime*
  - x includes pixel location, scale, time frame

Open in Adobe Reader to view in-slide video

Source: Jodoin, J. *et al.* Urban Tracker: Multiple Object Tracking in Urban Mixed Traffic. Applications of Computer Vision. 2014.

How can we solve multi-object tracking?

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Answer: stitch together individual tracklets

- Examples: trajectory prediction, flow-networks, matching

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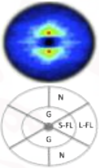

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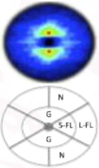

Limitations:

- Fails when objects move unpredictably (e.g. sports)

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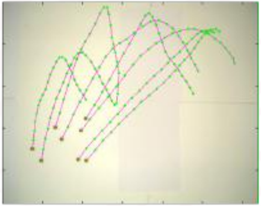
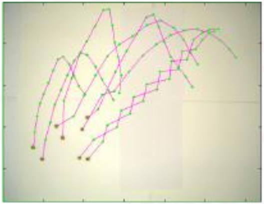
Hungarian bipartite graph matching [4, 5]





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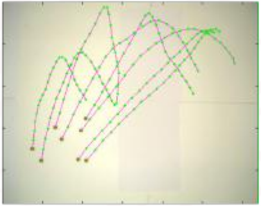

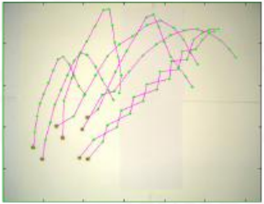

Limitations:

- Is locally optimal but not globally optimal across time

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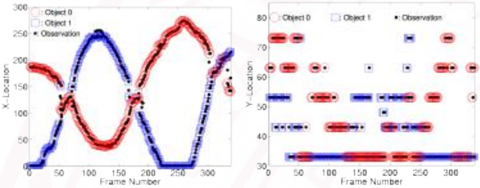

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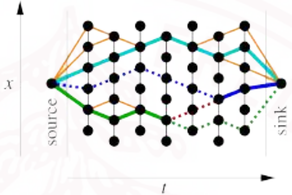

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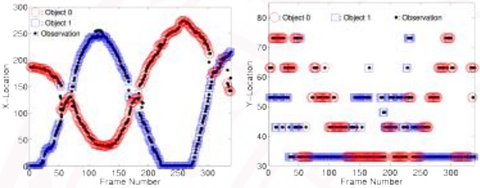

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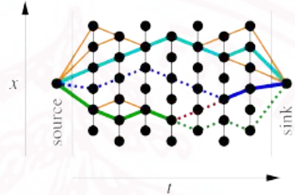

Limitations:

- Doesn't scale well
- Limited or no occlusion modeling

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Contributions

Past attempts require **prior information** while some methods are **not globally optimal**. Linear programs are optimal but **not efficient**.

Contributions

This paper proposes an ILP tracking formulation that:

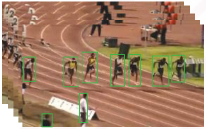
- is globally optimal
- is locally greedy
- scales linearly in the number of objects
- scales quasi-linearly in the number of frames

Research Questions

- How can we represent tracking as a probabilistic framework?
- How can we formulate this as an ILP?
- How can we efficiently solve it?
- How can we guarantee optimality?

Algorithm Pipeline

1. Input Video + Bounding Boxes



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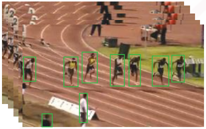

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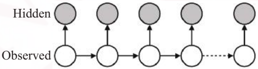

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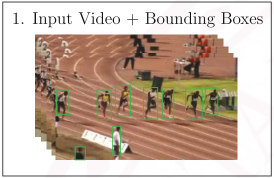

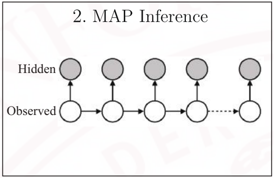

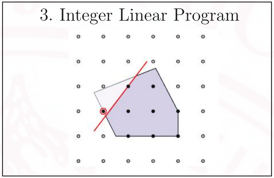

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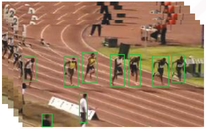

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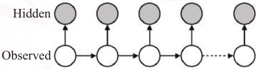

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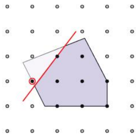

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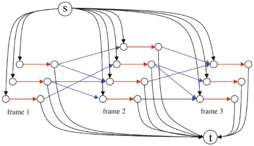

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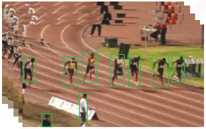

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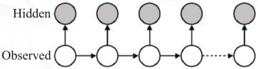

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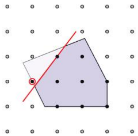

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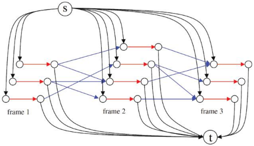

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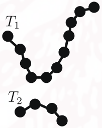

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Notation

We define a state vector x (i.e. a point in *spacetime*):

$$x = (p, \sigma, t) \quad \text{and} \quad x \in V$$

Where:

- $p =$ pixel location
- $\sigma =$ scale factor
- $t =$ frame number
- $V =$ set of all spacetime points

A track T is a set of state vectors: $T = \{x_1, ..., x_N\}$
Let X denote a set of K tracks: $X = \{T_1, ..., T_K\}$

Hidden Markov Model

Let X denote the output tracking assignments:

$$P(X) = \prod_{T \in X} P(T) \tag{1}$$

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$$P(T) = P_s(x_1) \left(\prod_{n=1}^{N-1} P(x_{n+1}|x_n) \right) P_t(x_N) \tag{2}$$

Where:

- $P_s(x_1)$ is the prior for a track starting at x_1
- $\prod_{n=1}^{N-1} P(x_{n+1}|x_n)$ is the probability we follow some track
- $P_t(x_N)$ is the prior for a track ending at x_N

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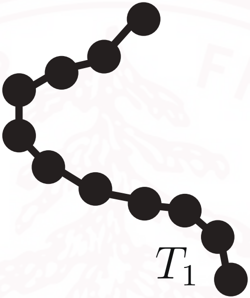

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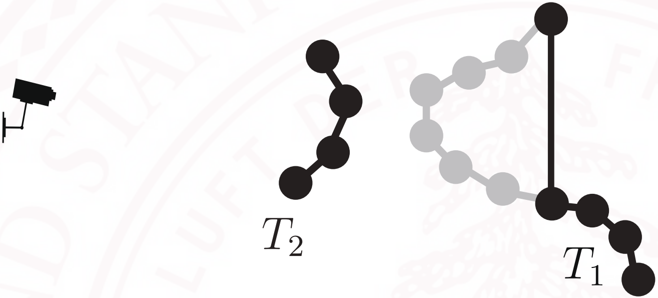

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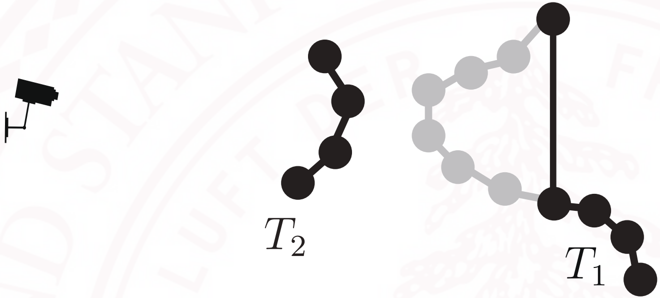

Note: $P(x_{n+1}|x_n)$ does not refer to the next frame but rather the next spacetime location in the track

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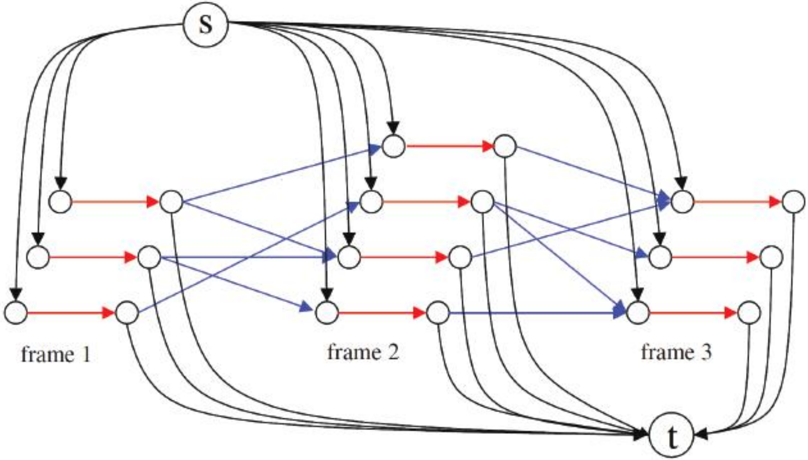

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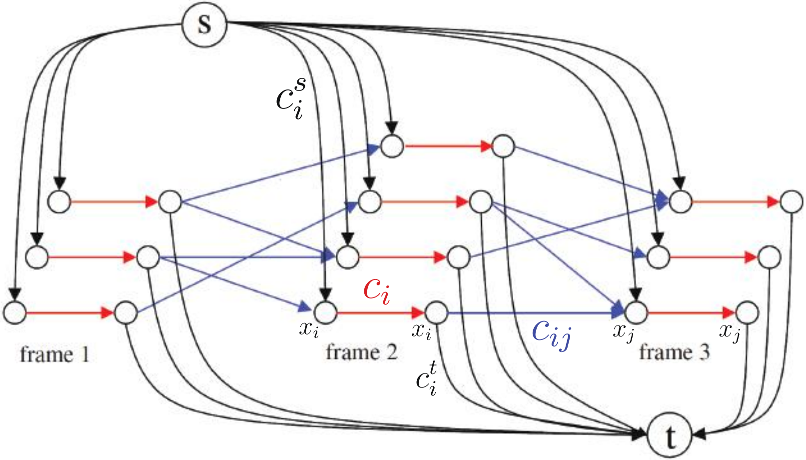

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- We achieve a $O(KN \log N)$ algorithm

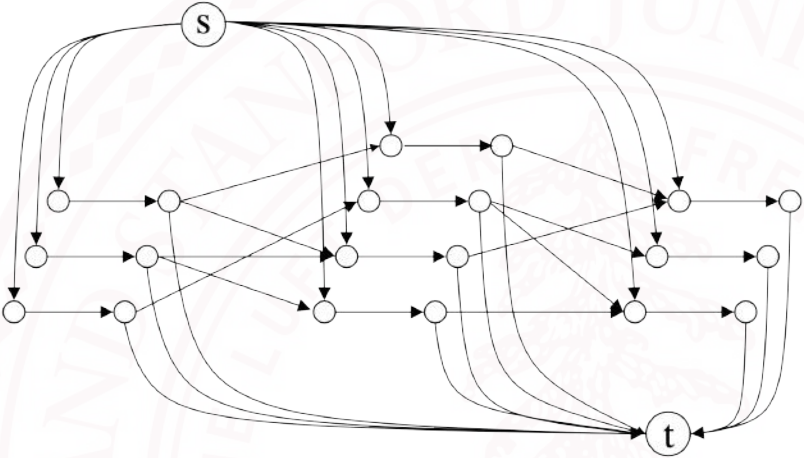
[8] L. Zhang *et al.* Global data association for multi-object tracking using network flows. CVPR, 2008.
[9] A. Goldberg. An efficient implementation of scaling minimum-cost flow algorithm. Journal of Algorithms, 1997.

How can we find the min-cost flows?

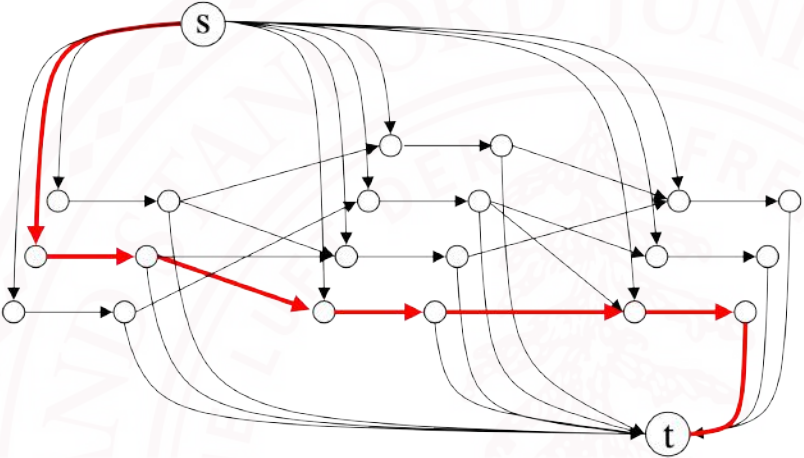
This paper proposes three algorithms:

- Successive shortest-paths
- Approximate One-Pass DP for $K > 1$
- Approximate Two-Pass DP for $K > 1$

Successive Shortest Paths



Successive Shortest Paths



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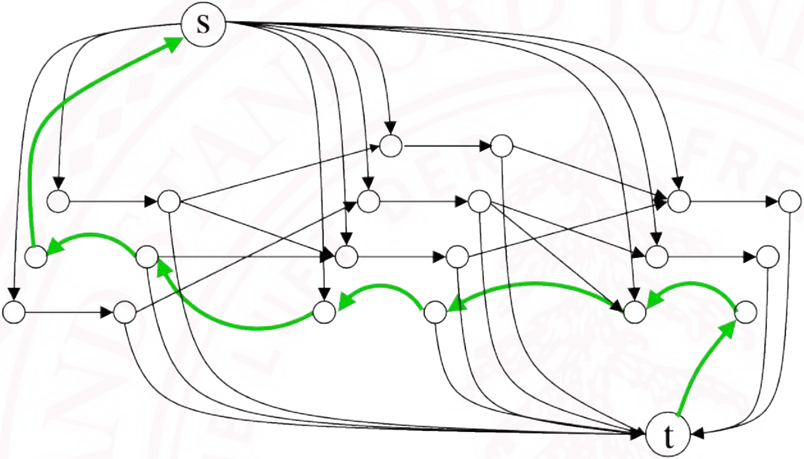

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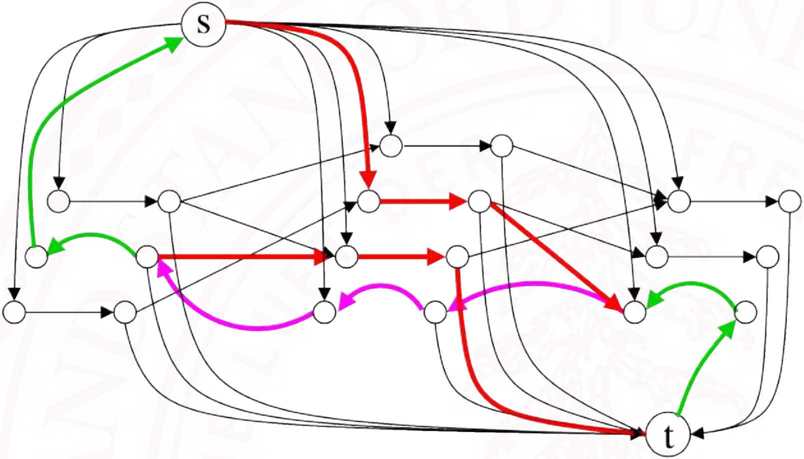

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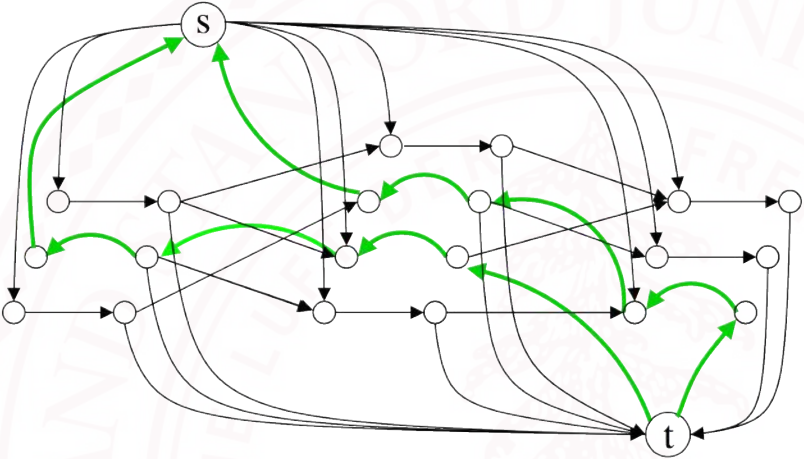

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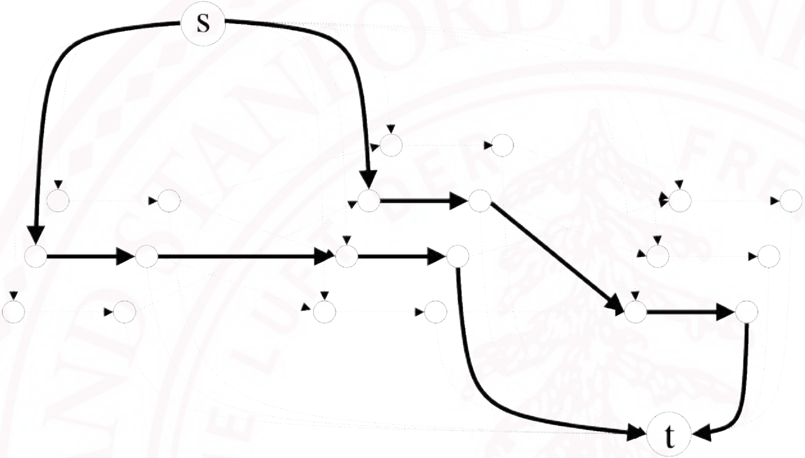

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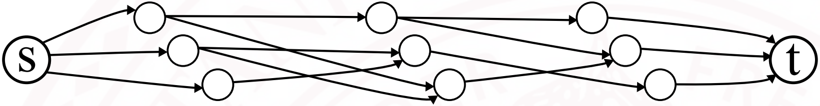
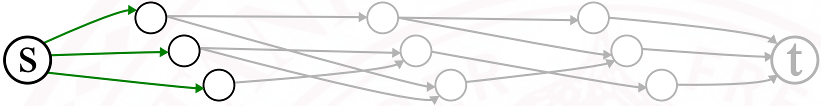
- Our DP approach: $O(N)$

Dynamic Programming Approach



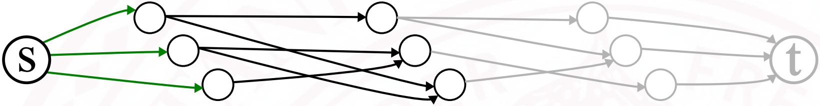
Start with a partial ordering of the nodes based on time

Dynamic Programming Approach



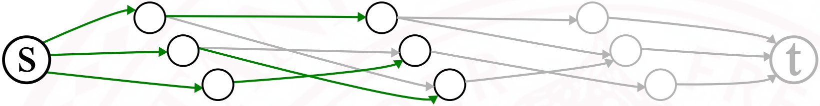
$$cost(i) = c_i + c_i^s$$

Dynamic Programming Approach



$$cost(i) = c_i + \min(\pi, c_i^s)$$
$$\pi = \min_{j \in N(i)} c_{ij} + cost(j)$$

Dynamic Programming Approach



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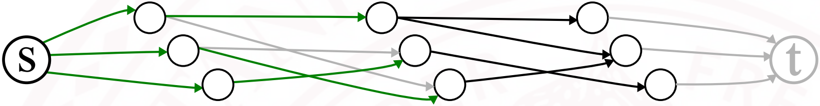

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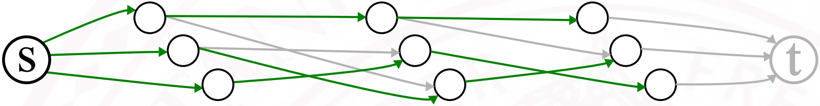

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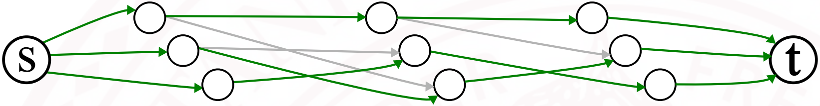

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- Conversion algorithm gives us shortest path from source node to terminal node

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Approximate One-Pass DP $O(KN)$ Algorithm:

- Start with original flow graph, perform $K + 1$ iterations:
  - Find shortest path from s to t
  - If path cost is negative, remove nodes on the path
- At each iteration, we instance one track

Outline

- Motivation & Related Work
- Mathematical Representation
  - Probabilistic Framework
  - ILP Formulation
- Multiple Object Tracking
  - Globally Optimal Greedy Algorithm
  - Approximate Dynamic Programming Algorithm
- Experiments and Results

Datasets

- Caltech Pedestrian Dataset [7]: 71 videos, 1800 frames each, 30 fps

- ETHMS Dataset [8]: 4 videos, 1000 frames each, 14 fps

[7] Dollar, P. *et al.* Pedestrian detection: A benchmark. CVPR, 2009.

[8] Ess, A. *et al.* A Mobile Vision System for Robust Multi-Person Tracking. CVPR, 2008.

Evaluation Metrics

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$$\text{Identification error} = \frac{\text{Number of incorrect ID labelings}}{\text{Total number of ID labelings}}$$

Performance Comparison

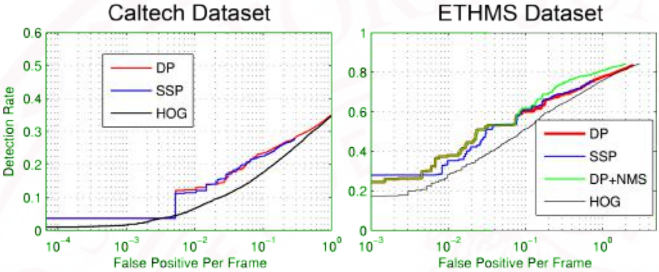
Algorithm	Detection Rate	FPPI
Stereo Algorithm [10]	47.0	1.50
MAP/Min-Cost Flow [11]	68.3	0.85
MAP/Min-Cost Flow + Occlusion Handling [11]	70.4	0.97
Two-Stage + Occlusion Handling [12]	75.2	0.93
Our DP	**76.6**	**0.85**
Our DP + NMS	**79.8**	**0.85**

[10] A. Ess *et al.* Depth and appearance for mobile scene analysis. ICCV, 2007.

[11] L. Zhang *et al.* Global data association for multi-object tracking using network flows. CVPR, 2008.

[12] J. Xing *et al.* Multi-object tracking through occlusions by local tracklets filtering and global tracklets association. CVPR, 2009.

Detection Rate vs False Positives per Image (FPPI)



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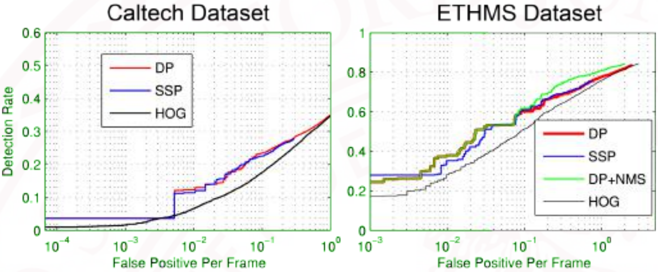


Key Insights:

- SSP produces short tracks due to 1st order Markov property
- DP produces longer tracks because tracks are never cut or edited

Track Label Error vs Allowed Occlusion

Results on ETHMS Dataset (Ideal Detector)

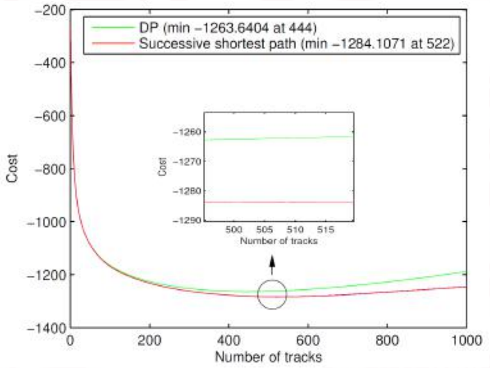
Length of Allowable Occlusion	Windows with ID Errors
1	14.69%
5	13.32%
10	9.39%

Key Insight:

- Larger occlusion windows improve performance

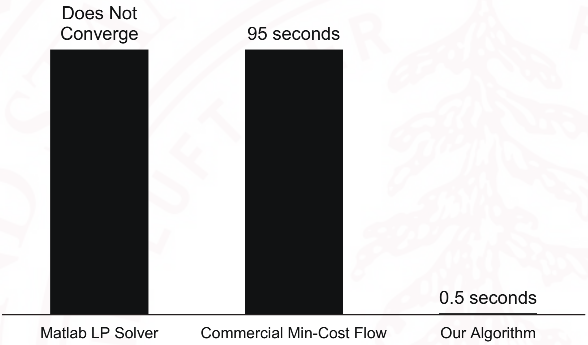
Cost versus iteration number

- DP algorithm is close to optimal (SSP) while being orders of magnitude faster



Algorithm Runtime

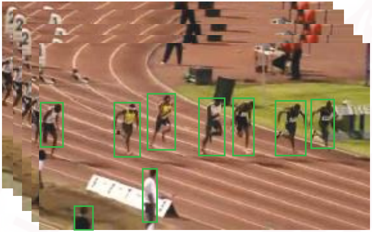
- DP algorithm is two orders of magnitude faster than commercial solvers



Conclusion

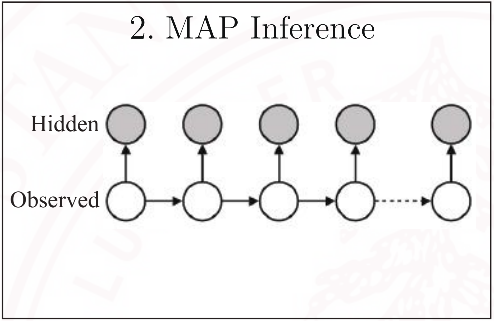
Given the input, we answered several research questions:

1. Input Video + Bounding Boxes



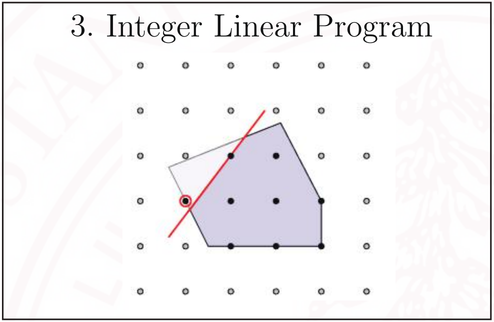
Conclusion

How can we represent tracking as a probabilistic framework?



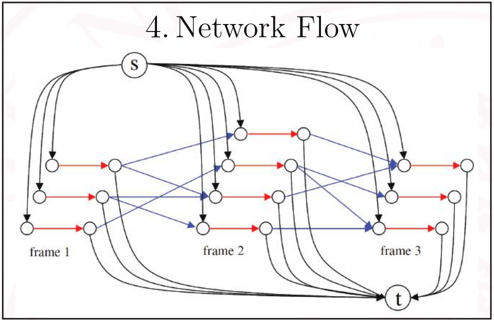
Conclusion

How can we formulate this as an ILP?



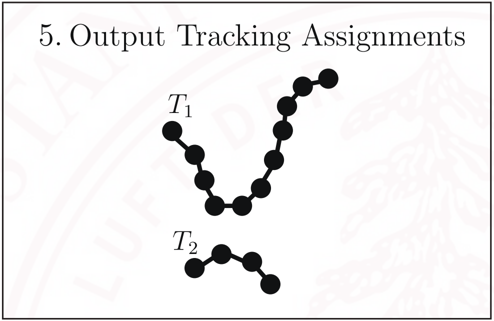
Conclusion

How can we efficiently solve it?



Conclusion

This allowed us solve the multi-object tracking problem:



Questions?