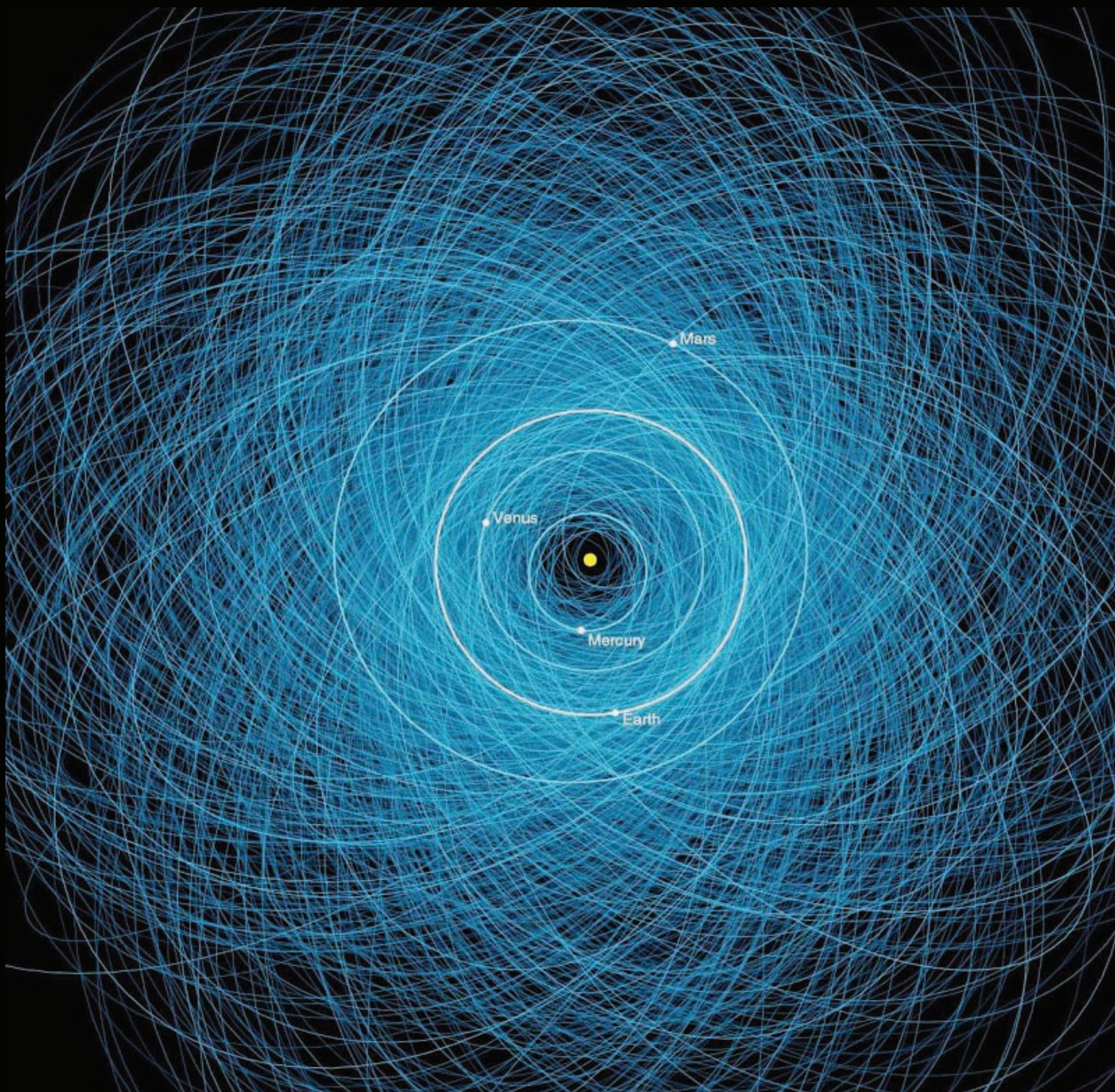
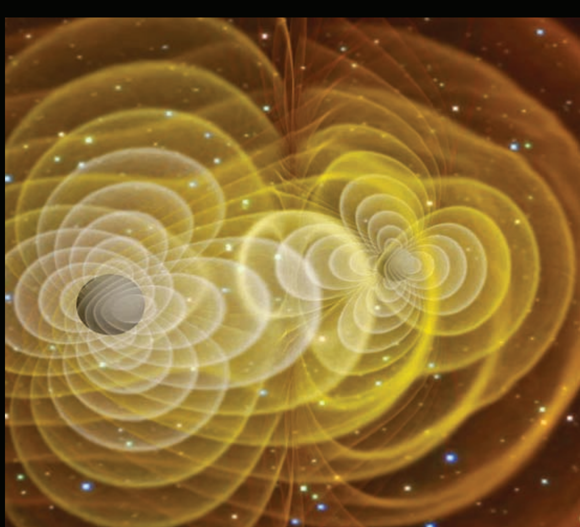
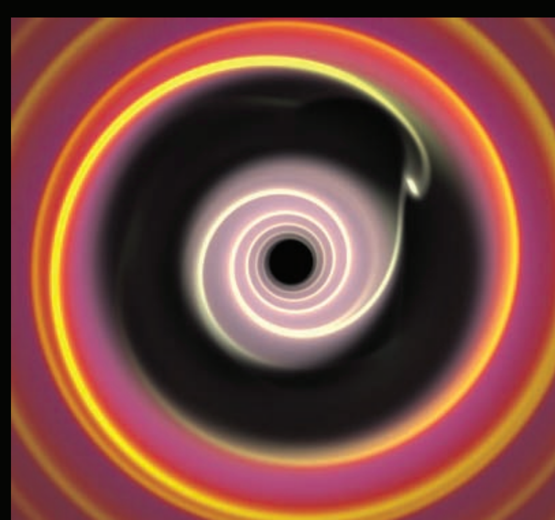
National Aeronautics and Space Administration







Space Math X

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2013-2014 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 3 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

Image Credits: Front: 1) Orbits of Potentially Hazardous Asteroids (NASA0; 2) Collision of two black holes showing gravity waves (NASA); Density structure of a protoplanetary disk and embedded planet (Credit: Wilhelm Kley, University of Tubingen, Institute for Astrophysics). Back: Three-dimensional visualization of the stellar orbits in the Galactic center based on data obtained by the W. M. Keck Telescopes between 1995 and 2012. Stars with the best determined orbits are shown with full ellipses and trails behind each star span ~15-20 years. These stars are color-coded to represent their spectral type: Early-type (young) stars are shown in teal green, late-type (old) stars are shown in orange, and those with unknown spectral type are shown in magenta. Stars without ellipses are from a statistical sample and follow the observed radial distributions for the early (white) or late (yellow/orange) type stars. These stars are embedded in a model representation of the inner Milky Way provided by NCSA/AVL to provide context for the visualization. (Credit: Fabio Antonini - Canadian Institute for Theoretical Astrophysics, and David Merritt - Rochester Institute of Technology)

This booklet was created through an education grant NNH06ZDA001N-EPO from NASA's Science Mission Directorate, and from individual grants from Year of the Solar System, SAGE-III, Van Allen Belts Probe, the Student Cloud Observations OnLine, and the History of Winter programs.

Table of Contents

Table of Contents (contd)

Table of Contents (contd)

Table of Contents (contd)

Mathematics Topic Matrix

Topic	Problem Numbers																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry																															
Technology, rulers																															
Numbers, patterns, percentages				X	X	X											X		X	X	X	X									
Averages																															
Time, distance, speed, density		X	X									X	X					X						X			X	X	X		
Areas and volumes		X	X						X			X																			
Scale Drawings, proportions	X								X		X														X	X					
Geometry	X							X											X	X			X								
Scientific Notation		X	X				X											X							X		X	X		X	
Unit Conversions			X				X								X														X		
Fractions	X																														
Graph or Table Analysis						X		X							X	X	X					X	X	X							
Solving for X	X			X	X	X																									
Evaluating Fns														X																	
Modeling																															
Probability																															
Rates/Slopes						X				X					X							X	X		X						
Logarithmic Fns					X																									X	X
Polynomials																															
Power Fns																															
Conics																															
Piecewise Fns																															
Trigonometry																															
Integration																															
Differentiation																															
Vectors																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
Inquiry																															
Technology, rulers																															
Numbers, patterns, percentages		X		X					X	X	X																	X			
Averages																															X
Time, distance, speed, density						X	X	X											X												
Areas and volumes										X					X			X													
Scale Drawings, proportions							X	X		X										X											
Geometry	X												X		X								X	X							
Scientific Notation						X	X									X		X	X		X										
Unit Conversions					X							X							X												
Fractions																															
Graph or Table Analysis													X									X	X	X	X	X			X	X	
Solving for X				X									X	X	X	X	X								X						
Evaluating Fns	X															X		X			X						X				
Modeling																								X	X		X				
Probability																															
Rates/Slopes					X				X			X										X					X				
Logarithmic Fns																															
Polynomials																											X				
Power Fns																											X				
Conics													X	X																	
Piecewise Fns																															
Trigonometry													X													X					
Integration																											X				
Differentiation																															
Vectors																							X		X	X					

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
Inquiry									X								X	X													
Technology, rulers									X																						
Numbers, patterns, percentages	X	X			X	X	X																					X			X
Averages					X	X		X	X																			X			
Time, distance, speed, density																					X				X				X	X	
Areas and volumes	X	X	X																X			X	X								X
Scale Drawings, proportions			X	X				X		X	X	X				X				X	X			X		X			X	X	
Geometry	X	X						X			X	X				X				X											
Scientific Notation														X																	
Unit Conversions																								X	X						
Fractions																															
Graph or Table Analysis								X	X								X	X									X	X			
Solving for X				X									X	X	X											X					
Evaluating Fns																						X	X								
Modeling																															
Probability																															
Rates/Slopes																								X	X						X
Logarithmic Fns																															
Polynomials													X	X					X												
Power Fns														X																	
Exponential Fns																															
Conics																															
Piecewise Fns																															
Trigonometry																															
Integration																															
Differentiation																															
Vectors																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124
Inquiry																															
Technology, rulers																															
Numbers, patterns, percentages	X		X											X	X																
Averages																															
Time, distance, speed, density		X															X	X	X				X								
Areas and volumes			X				X				X		X		X		X			X											
Scale Drawings, proportions		X			X	X	X	X	X	X	X	X	X					X													
Geometry																X			X												
Scientific Notation	X		X	X											X		X									X					
Unit Conversions			X	X									X					X													
Fractions																															
Graph or Table Analysis					X	X								X		X							X						X	X	X
Solving for X						X			X	X	X	X																		X	X
Evaluating Fns																				X	X	X		X	X		X		X	X	X
Modeling																				X	X	X					X	X	X	X	X
Probability																															
Rates/Slopes				X								X						X													
Logarithmic Fns																												X			
Polynomials																										X			X		
Power Fns																															
Exponential Fns																									X						
Conics																								X							
Piecewise Fns																															
Trigonometry																				X											
Integration																															
Differentiation																															
Vectors																															

Mathematics Topic Matrix (cont'd)

x

Topic	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
Inquiry																
Technology, rulers																
Numbers, patterns, percentages				X	X								X			X
Averages																
Time, distance, speed,density			X								X			X		
Areas and volumes											X					
Scale Drawings, proportions																
Geometry																
Scientific Notation	X	X									X			X		
Unit Conversions														X		
Fractions																
Graph or Table Analysis			X	X	X	X										X
Solving for X							X	X	X	X						
Evaluating Fns	X	X					X	X	X	X						
Modeling	X	X					X	X	X	X		X				
Probability						X										X
Rates/Slopes																
Logarithmic Fns																
Polynomials							X	X								
Power Fns												X				
Exponential Fns																
Conics			X													
Piecewise Fns																
Trigonometry																
Integration												X				
Differentiation																
Vectors															X	

Alignment with Standards

Next Generation Science Standards

MS-ESS1 Earth's Place in the Universe

- **Performance Expectation: MS-ESS1-3**
  - Analyze and interpret data to determine scale properties of objects in the solar system.

HS – ESS1 Earth's Place in the Universe

- **Performance Expectations: HS-ESS1-1**
  - Develop a model based on evidence to illustrate the life span of the sun and the role of nuclear fusion in the sun's core to release energy in the form of radiation.
- **Performance Expectations: HS-ESS1-4**
  - Use mathematical or computational representations to predict the motion of orbiting objects in the solar system

CCMS: Common Core Mathematics Standards

Grades 6–8

CCSS.Math.Content.7.RP.A.2c Represent proportional relationships by equations

CCSS.Math.Content.6.EE.A.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems

CCSS.Math.Content.6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots

CCSS.Math.Content.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

CCSS.Math.Content.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.

CCSS.Math.Content.8.EE.C.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

CCSS.Math.Content.8.EE.C.8c Solve real-world and mathematical problems leading to two linear equations in two variables

Grades 9–12

CCSS.Math.Content.HSN-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents

CCSS.Math.Content.HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

CCSS.Math.Content.HSA-SSE.B.3c Use the properties of exponents to transform expressions for exponential functions.

CCSS.Math.Content.HSA-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

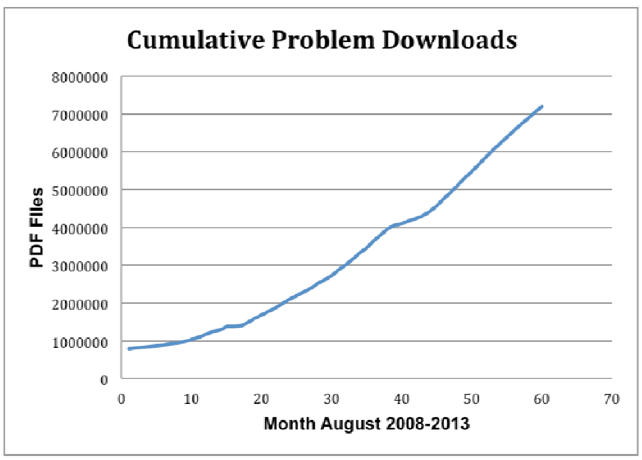
CCSS.Math.Content.HSG-GPE.A.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Letter from the Author

This is the last volume of collected problems from SpaceMath@NASA. As with many programs, one may retrospectively define for the SpaceMath@NASA program three distinct periods of development and dissemination: The Past, The Present and The Future.

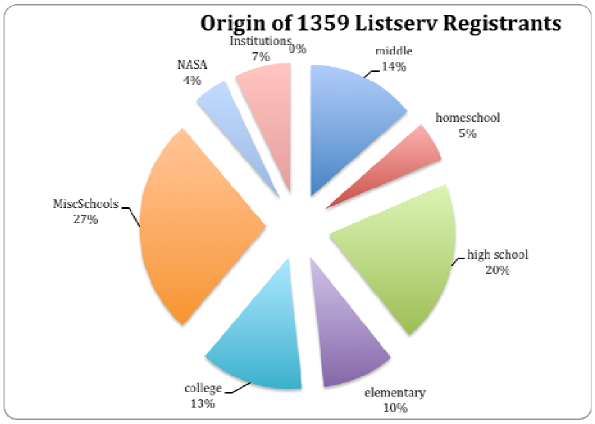
The Past: (2005-2012)

The original objective of SpaceMath@NASA was to provide teachers with real-world math activities in a space context in support of the national math and science standards. To accomplish this objective, new math problems were created regularly and posted on the website with reference to the related standards. NASA missions could link to the SpaceMath@NASA website for the problems related to their specific content areas. Over time, the many problems created in support of press releases and science and engineering themes were collected into books on specific topics such as black holes, Mars exploration, and space weather, or into annual compendia of all published problems as in Space Math I, II, III….X. Although teachers initially enjoyed access to individual problem files, site statistics show that it soon became more convenient for educators to simply download entire annual problem books and extract the problems of interest.



SpaceMath@NASA use has grown significantly since 2008 in terms of the resources offered (680+ problems and 28 books), the downloads of those resources (over 9,000,000 pdfs, 123,000,000 individual problems counting books), the evidence of impact as reported by educators, and in the case study and comparison group studies. For the last 8 years, an open search on the Internet for "space math" consistently lists SpaceMath@NASA first, highlighting this NASA resource for educators. **According to the website tracking company Alexa (Alexa.com), SpaceMath@NASA ranks 668th among all United States websites (estimated at 634 million in 2012 – Pingdom.com) and has 107,333 other sites that link to it, giving it a "5 Star" ranking according to Alexa and putting SpaceMath@NASA among the top 1/10th of 1% of all websites in the United States.**

Educators who attend workshops or presentations consistently report that SpaceMath@NASA aligns with what they teach, that they can immediately apply what they have learned, and intend to use it in their classes. They think use of the resources will increase students' interest in STEM and encourage exploration, discussion and participation. Teachers reported that they are primarily science teachers who use the problems to emphasize science content (64%), math teachers who use them as application problems (28%), and informal educators looking for workshop problems (17%).



Educators already using SpaceMath@NASA report that students enjoy the application problems and topics, are productively engaged, and ask questions that demonstrate curiosity and interest. They have recommended SpaceMath@NASA to other teachers for its ease of use and unique combination of math and science. They like the connection to recent NASA press releases and report that it has been a great resource to stimulate learning (N=678 educators) When asked about their usage of SpaceMath@NASA problems, most users report using the problems a few times a month (47%) Responding teachers represent a variety of different ages and subjects with the greatest percentage teaching grade 6-8 science (30%), followed by grade 10-12 physics (21%) and 6-8 math (12%). In addition, 14% reported that they are informal educators. When asked about their students relationship to SpaceMath@NASA, 83% of the teachers reported that their students are productively challenged by the problems, that their students enjoy the problems (60%), that their students ask questions about the problems that demonstrate curiosity and interest (62%), that their students have improved their academic performance as a result of working with the SpaceMath@NASA problems (73%), and that their students look forward to new problems (50%).

The case study of middle school students using the inquiry data sets shows that it affects their science content knowledge, extends or develops their math skills in analyzing data sets, and provokes thoughtful questions on the science content, use of data tables, and methods of analysis. The comparison group study showed that SpaceMath@NASA problems are equally effective or more effective in developing a math concept as other approaches in middle school students.

Overall, SpaceMath@NASA is a unique resource that supports NASA mission by prominently putting the M in NASA STEM on the Internet and on the NASA portal, in widely distributed textbooks (Houghton-Mifflin Algebra), and in offering practical lessons on current NASA activities (as reported in press releases) in a timely way. SpaceMath@NASA also supports 36 missions with a page that lists the math problems for each mission, and links to the mission pages for educators and students to pursue further.

In SpaceMath@NASA surveys, educators report that SpaceMath resources have improved their impression that NASA supports STEM education. This is not an accident because SpaceMath@NASA was specifically designed to support NASA's STEM mission in three specific ways: 1) Many of the problems are built on current NASA press releases, giving educators a specific way to use NASA's activities in their classrooms. 2) SpaceMath@NASA problems are catalogued by mission to support EPO efforts by missions, stimulate interest in the missions, give students a way to pursue a mission if they become interested in it, and support educators who want to focus on one mission. 3) Most of the explicitly math-enhanced resources on the NASA portal have now been developed by SpaceMath@NASA and offer educators sound, rigorous math activities to use in their math, science and technology classrooms, while at the same time conforming to the scope and sequencing recommended by the Common Core Mathematics Standards and the Next-Generation Science Standards.

The Present: (2013-2014)

SpaceMath@NASA has been solely supported through a series of education grants from the NASA Science Mission Directorate through their ROSES/EPOESS program. Although this has been a reliable source of funding since 2008, the implementation of the federal budget Sequester in FY13, uncertainties over the federal funding of FY14 in April 2013, followed by the aborted implementation of preliminary coSTEM recommendations by the OMB, forced the cancellation of the ROSES/EPOESS grant program in 2013 and 2014. Consequently, SpaceMath@NASA must come to an end at the conclusion of the 2013-2014 academic year without having had the opportunity to re-propose for continuing funding.

Through a combination of extending its final-year funds, and some limited financial support provided by the Van Allen Belt Probes mission, the SAGE-III project, and Year of the Solar System program, SpaceMath@NASA will continue to remain an active website until ca September 2014. At an anticipated funding level of 15% of its grant-supported levels, there will be considerable reductions in summer content generation. For example, content will be targeted at the themes supported by the groups that fund SpaceMath@NASA, which means most topics in planetary science, heliophysics and astrophysics will not appear in future problem sets.

Some changes to the website are being planned so that it will perform well as an archival resource. Since all of the book-length resources are already available on the NASA Portal under ‘educator resources ‘M’’, we have changed the URLs under the ‘Books’ tab to point to the NASA Portal pages rather than to the SpaceMath@NASA site where these book files formerly resided. We will also make certain other changes to the SpaceMath@NASA website to reduce access to 700+ individual problem files. There will no longer be updated press release problems, and no further STEM Module development at this site.

The Future: (2015+)

After March 2015, NASA IT Security requires that this website be closed since there will no longer be a website developer or owner responsible for its content. Some of the content may be moved to the National Institute of Aerospace (NIA), but the program name ‘SpaceMath@NASA’ will probably have to be retired in favor of some other name such as ‘SpaceMath’ or ‘SpaceMath@NIA’. The re-naming details and content of that website will be decided in late-2014 depending on the funding source if any, and the details will be posted on the Spacemath listserve in a series of email updates. It is likely that the ranking of this website with GOOGLE will be greatly diminished as the Top Ranked website for space mathematics.

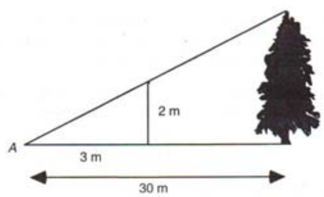
We remain hopeful that we will be able to obtain some funding to keep SpaceMath@NASA on the air, but we cannot rely on NASA mission education programs and NASA’s STEM education grant programs to supply the entire 85% of the missing funding. Many programs have volunteered to assimilate the SpaceMath@NASA website into their own resources, but that does not solve the problem of sustaining this resource as a timely resource for breaking science, and the people who have created and managed it since its inception.

'A lonely cloud and its shadow!'
Courtesy: Henriette (2005)
The Cloud Appreciation Society

Sometimes, if you are lucky, you can see a single cloud and its shadow, perhaps while you were visiting the beach, standing in a meadow, or driving across the desert. By using the properties of similar triangles and a simple proportion, you can use this cloud and its shadow to figure out how high up the cloud is! You need a meter stick, and a bit of help from a friend to do this, though.

Let's see how this works for an example so that you can try this the next time you are at the beach...or the desert!

Similar Triangles and Proportions



The figure shows two similar right-triangles formed by the sides '3m' and '2m' and the sides '30m' and the unknown height of the tree. The basic geometric rule of similar triangles is that corresponding sides are in the same proportion. That means that the ratio formed by the sides '3m' and '30m' is the same numerical proportion as the sides '2m' and 'X' where X is the height of the tree. We can write this as:

$$\frac{30 \text{ meters}}{3 \text{ meters}} = \frac{X}{2 \text{ meters}}$$

and then solve this to get

$$X = 2 \text{ meters} \times (30/3)$$
$$= 20 \text{ meters.}$$

Suppose that in this problem, instead of the 2 meter long stick we used the height of your thumb, about 1.5 inches (3.8 cm), and placed it at arms-length from your eye, which is 20 inches (51 cm). Suppose also that if you were at a distance of 269 meters from the base of the tree the height of the tree would exactly equal the height of your thumb. How tall would the tree be? Again we set-up the proportion:

$$\frac{269 \text{ meters}}{51 \text{ cm}} = \frac{X}{3.8 \text{ cm}}$$

and solve for X: $X = 269 \text{ meters} \times (3.8/51) = 20 \text{ meters.}$

So, using similar triangles and proportions is a very COOL way to figure out things about distant objects. In astronomy, you cannot even travel to these objects so using similar triangles and proportions is the only way to learn about their sizes and distances!

Now let's apply this proportion method to studying clouds!

Problem 1 – Measure the length of your out-stretched arm in centimeters. Now find a cloud near you that has a shadow close by where you are standing. Holding the meter stick at arm's length, how many centimeters is it from the base of the cloud down to the ground?

Problem 2 – Draw a scaled model right-triangle ABC, where side AB is the length of your arm in centimeters, and side BC is the vertical distance to the base of the cloud that you measured in Problem 1. Let's suppose that for this problem, AB = 20 inches and AC = 12 inches and that 1 inch = 2.5 cm.

Problem 3 – This next part is a bit tricky. As best you can, estimate how far it is from where you are standing to where the shadow of the cloud begins. You can also note some feature at this location like a tree or a rock formation, or a distant person sitting on a blanket! Count the number of paces it takes to get to this spot. Suppose that for this problem your pace was 2-feet long (0.7 meters) and you completed 3000 paces to get to the spot. How many meters did you travel?

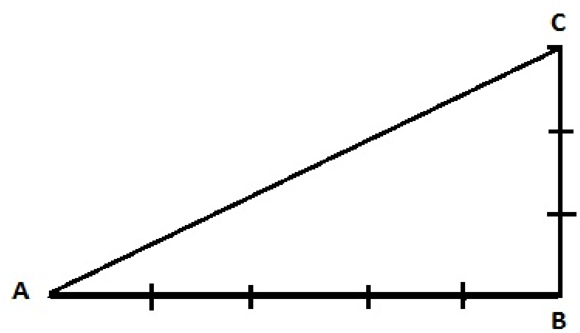
Problem 4 – It is now time to use proportional reasoning! Use the similar triangle you created in Problem 1, with the distance you paced in Problem 3 to determine the actual height of the cloud above the distant point! What would be your answer for the example we used?

Common Core Math Standards:
CCSS.Math.Content.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Problem 1 – Measure the length of your out-stretched arm in centimeters. Now find a cloud near you that has a shadow close by where you are standing. Holding the meter stick at arm's length, how many centimeters is it from the base of the cloud down to the ground?

Problem 2 – Draw a scaled model right-triangle ABC, where side AB is the length of your arm in centimeters, and side BC is the vertical distance to the base of the cloud that you measured in Problem 1. Let's suppose that for this problem, AB = 20 inches and AC = 12 inches and that 1 inch = 2.5 cm.

Answer: We want all measurements to be in centimeters in order to draw the scaled triangle. AB = 20 inches x (2.5 cm/1 inch) = 50 cm. AC = 12 inches x (2.5 cm/ 1 inch) = 30 cm. The drawing looks like this. Each division is 10 centimeters.



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Answer: In this example, 3000 paces x (0.7 meters/1 pace) = **2,100 meters**.

Problem 4 – It is now time to use proportional reasoning. Use the similar triangle you created in Problem 1, with the distance you paced in Problem 3 to determine the actual height of the cloud above the distant point! What would be your answer for the example we used?

Answer: Let X be the actual height of the cloud, then from the similar triangle BC/AB = X/2100 meters, and since BC=30cm and AB=50 cm we have 30/50 = X/2100 and so X = 2100 (30/50) = 1260 meters. **So the cloud is about 1260 meters above the ground!**

You look up at the sky one day and see puffy little cumulus clouds hovering over the beach, a meadow, or over your town. Did you ever wonder just how much a cloud might weigh as it drifts by over your head?

Different clouds carry different amounts of water droplets and so they have different densities. Brilliant white cumulus clouds, for example, have densities of 0.3 grams/meter^3.

From the known cloud densities, we can estimate their masses once we know their volumes because Mass = Density x Volume.

Problem 1 – From the definition of density, what are the other two equations you can create that define mass and volume?

Problem 2 – A puffy cumulus cloud looks almost like a sphere. If its diameter is 3.0 kilometers, what is its volume in cubic meters? (use π = 3.14)

Problem 3 – What is the total mass of the cumulus cloud in kilograms and metric tons?

Problem 4 – You spot two clouds in the sky. The cumulus cloud is 1/5 the diameter of the cumulonimbus cloud, and the cumulonimbus cloud has 8 times the density of the cumulus cloud. What is the ratio of the mass of the cumulus cloud to the cumulonimbus cloud if both clouds are spherical in shape?

Grade 7 - Working with Density, mass and volume: Examples: 'California Mathematics Standards' - *Students can calculate the mass of a cylinder given its dimensions and density*. - Utah State Science Standards: I.2.c. "*Calculate the density of various solids and liquids."*

Grade 8 - Common Core Math Standards:
CCSS.Math.Content.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Problem 1 – From the definition of density, what are the other two equations you can create that define mass and volume?

Answer: Density = Mass / Volume   Volume = Mass/Density

Problem 2 – A puffy cumulus cloud looks almost like a sphere. If its diameter is 3.0 kilometers, what is its volume in cubic meters? (use π = 3.14)

Answer: $V = 4/3\ \pi\ R^3$ and for D = 3.0 km, we have R = 1500 meters and so $V = 4/3\ \pi\ (1500 \text{meters})^3 =$ **1.4×10^{10} meters^3**

Problem 3 – What is the total mass of the cumulus cloud in kilograms and metric tons?

Answer: Mass = Density x Volume so $M = 0.3\ \text{grams/m}^3 \times 1.4 \times 10^{10}\ \text{m}^3 = 4.2\times10^9$ grams. But 1 kg = 1000 grams, so **M = 4,200,000 kg**. This also equals **4200 metric tons**!

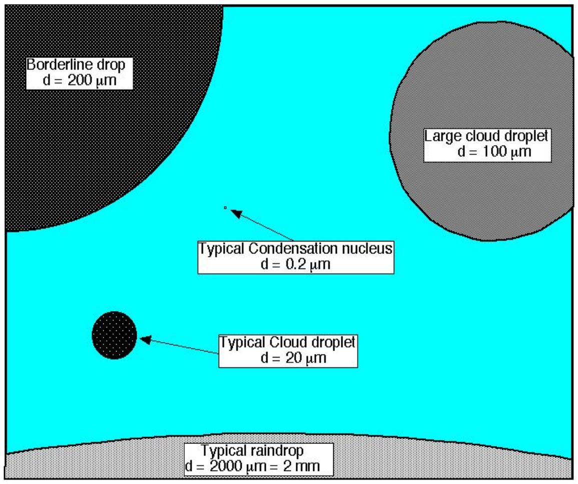
Problem 4 – You spot two clouds in the sky. The cumulus cloud is 1/5 the diameter of the cumulonimbus cloud, and the cumulonimbus cloud has 8 times the density of the cumulus cloud. What is the ratio of the mass of the cumulus cloud to the cumulonimbus cloud if both clouds are spherical in shape?

Answer: Mass = Density x Volume.

$V(\text{Cumulus})/V(\text{CN}) = (1/5)^3$ and D(Cumulus) = 1/8D(CN) so

$\text{Mass(Cumulus)} = (1/5)^3 \times (1/8) \times \text{Mass(CN)} = 1/1000\ \text{Mass(CN)}$ and so

Mass(Cumulus)/Mass(CN) = **1/1000**



Without droplets of water, most clouds would be transparent! If you were to look inside a cloud you would see droplets of water of many different sizes because droplets constantly grow in size once they are formed.

The figure to the left shows some typical kinds of water droplets you might find in a cloud along with their diameters in micrometers (microns). Recall that one micrometer=1/1000000 or 10^{-6} meters.

The following exercises let you explore some of the properties of water droplets. In all cases, assume that the droplet is a perfect sphere!

Problem 1 – Water droplets are made out of water (of course!) and water has a density of 1000 kg/m^{3}. What is the mass, in grams, of each of the five types of droplets described in the figure?

Problem 2 – To the nearest 1000, about how many typical cloud droplets have to be combined to form one large cloud droplet?

Problem 3 – To the nearest 1000, about how many large cloud droplets have to combine to form one typical raindrop?

Problem 4 – Suppose that it takes about 2 minutes for a large cloud droplet to double in mass. How long does it take a large cloud droplet to grow into a raindrop and leave the cloud?

Common Core Math Standards:
CCSS.Math.Content 6.RP.3.d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
CCSS.Math.Content.8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
CCSS.Math.Content.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology

Problem 1 – Water droplets are made out of water (of course!) and water has a density of 1000 kg/m^3. What is the mass, in grams, of each of the five types of droplets described in the figure?
Answer: Recall Volume = $4/3\ \pi\ R^3$ and Mass = Density x Volume.

Raindrop: r=2 mm = 0.002 meters
V = $4/3\ \pi\ (0.002)^3 = 3.3\times10^{-8}\ m^3$
Mass = 1000 kg/m^3 x $3.3\times10^{-8}\ m^3$ x (1000gm/1kg) = **0.033 grams**

Borderline Drop: r = 200 microns. = 0.0002 meters
V = $4/3\ \pi\ (0.0002)^3 = 3.3\times10^{-11}\ m^3$
Mass = 1000 x 3.3×10^{-11} x (1000gm/1kg) = **3.3×10^{-5} grams** (= 33 micrograms)

Large Cloud Droplet: r = 100 microns = 0.0001 meters
V = $4/3\ \pi\ (0.0001)^3 = 4.2\times10^{-12}\ m^3$
Mass = 1000 x 4.2×10^{-12} x (1000gm/1kg) = **4.2×10^{-6} grams** (4.2 micrograms)

Typical Cloud Droplet: r = 20 microns = 0.00002 meters
V = $4/3\ \pi\ (0.00002)^3 = 3.3\times10^{-14}\ m^3$
Mass = 1000 x 3.3×10^{-14} x (1000gm/1kg) = **3.3×10^{-9} grams** (3.3 nanograms)

Typical Condensation Nucleus: r = 0.2 microns = 0.0000002 meters
V = $4/3\ \pi\ (0.0000002)^3 = 3.3\times10^{-20}\ m^3$
Mass = 1000 x 3.3×10^{-20} x (1000gm/1kg) = **3.3×10^{-14}** grams

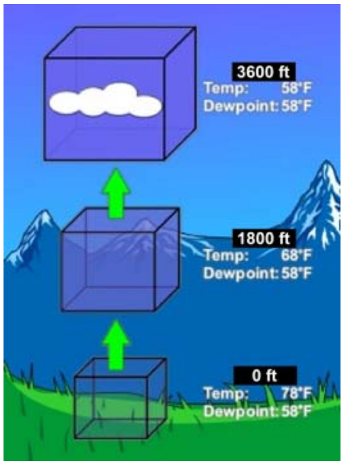
Problem 2 – To the nearest 1000, about how many typical cloud droplets have to be combined to form one large cloud droplet? Answer: N = Mass of large droplet / mass of typical cloud droplet = 4.2×10^{-6} gm/3.3×10^{-9} gm = 1272 typical droplets or **about 1000**.

Problem 3 – To the nearest 1000, about how many large cloud droplets have to combine to form one typical raindrop? Answer: N = Mass of raindrop/mass of large droplet = 0.033 grams/4.2×10^{-6} grams = 7857 or about **8000 large droplets**.

Problem 4 – Suppose that it takes about 2 minutes for a large cloud droplet to double in mass. How long does it take a large cloud droplet to grow into a raindrop and leave the cloud?

Answer: In Problem 3 we saw that about 8000 large cloud droplets equals a raindrop. Since 8000 is about 2^{13}, we need 13 doubling times to grow this large, which takes 13 x 2 minutes = **26 minutes**.

Note: Students may want to make a scaled model of droplet sizes using styrofoam balls or other round objects.



Courtesy Katie R. Roussy (2006)
University of Illinois at Urbana

When it comes to your local weather report, it's easy to understand what the Weatherperson means by the terms temperature and wind speed. During the summertime, you will often hear about 'humidity', which is the amount of moisture carried by the air. This term also makes a lot of sense!

When the air is carrying a lot of moisture on a 'humid' hot summer's day, it feels very uncomfortable. Your body is trying to cool off by perspiring. Because the air is already carrying a lot of moisture, there is no place for your perspired water to easily go. So you start to over-heat and feel wet all over!

Air can carry water vapor, and warm air can carry a lot more water vapor than cold air. That's why your skin feels wet and clammy in the summer, and you often have problems with dry skin during the winter. Because the amount of water and air humidity depends on temperature, your Weatherperson will often use the term 'dew point' to tell you what to expect when you step outside!

When the air temperature is close to a critical temperature called the **dew point**, water vapor begins to condense out of the air as droplets. The windshield of your car will have beads of water all over, and if this happens inside your house, your windows will cloud up with drops of moisture. That is why for some locations, indoor air conditioners have to have a 'dehumidifier' to remove moisture from the air so that it doesn't condense on the cooler panes of window glass.

For large masses of air, millions of droplets of moisture can form in every cubic centimeter and you see a cloud begin to appear.

The diagram above shows what happens to an 'air mass' with a dew point temperature of 58^{o}F as it rises to cooler altitudes. Nothing happens if the dew point temperature is below the air temperature. But when the local air temperature equals or is larger than the dew point, the cloud appears.

Sometimes, the local temperature near the ground can be slightly above the dew point. When this happens, the air remains clear, but droplets of water can form on cooler windows or on cars. When the local ground temperature is below the dew point, droplets will condense in the air and you get ground fog!

Dew point temperature can be a confusing idea when you first work with it, but it is such a common and practical idea that you will hear about it on your local weather report, especially during the spring, summer and fall!

The dew point temperature is very complicated to calculate exactly because it depends on the local atmospheric pressure and temperature, and the amount of water vapor in the air. There are some ways to estimate dew point temperature that give a rough idea of what to expect. The following formula is one of these methods:

$$T_{\mathbf{dewpoint}} = T_{\mathbf{air}} - \frac{100 - P}{5}$$

A general rule-of-thumb is that, if the humidity of the air is 60% you will feel uncomfortable (P = 60). If the outside temperature on a hot summer's day is 90^{o}F ($T_{\mathbf{air}}$=90^{o}F) then the dew point temperature is 82^{o}F. If the humidity of the air approaches 100%, then for this example the dew point temperature equals the air temperature and droplets of moisture will start to form.

Problem 1 – A Marathon runner finishes the race and gets into her car. The outdoor temperature is 75^{o}F and the humidity is 80%. Explain why her actions may be dangerous if she immediately drives away, and a simple remedy.

Problem 2 - It's a warm sunny day and the air is rather humid with a dew point of 75^{o}F .The ground temperature is 85^{o}F. If the air temperature decreases at a rate of 3.5^{o}F/1000 feet (called the wet lapse rate), at what altitude will a cloud begin to appear?

Problem 3 – On a comfortable summer day, the humidity is only 20% and the outside temperature is 80^{o}F. At what altitude might clouds start to form overhead if the air temperature is decreasing at a 'dry' lapse rate of 5.5^{o}F/1000 feet?

Common Core Math Standards:

Grade 6 – *CCSS.Math.Content.6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Grade 7 – *CCSS.Math.Content.7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*

Problem 1 – A Marathon runner finishes the race and gets into her car. The outdoor temperature is 75^{o}F and the humidity is 80%. Explain why her actions may be dangerous if she immediately drives away, and a simple remedy.

Answer: The dew point temperature is T = 75 – (100-80)/5 = 71^{o}F. As she expels moist air from her lungs, the humidity inside the car steadily increases and the dew point temperature rises from 71^{o}F to 75^{o}F. Then the inside of her windows begin to fog up as water condenses on their surfaces. This makes it harder for her to see outside and creates a dangerous situation.

A remedy is to first turn on the car air conditioner, which will condense the remaining moisture in the air onto the cooling pipes. Then turn on the heater which will warm the windows and evaporate the moisture droplets on the windows. With the AC still on, this moisture will also condense on the cooling pipes, resolving the situation. Typically this takes only a minute or two for an average-sized car.

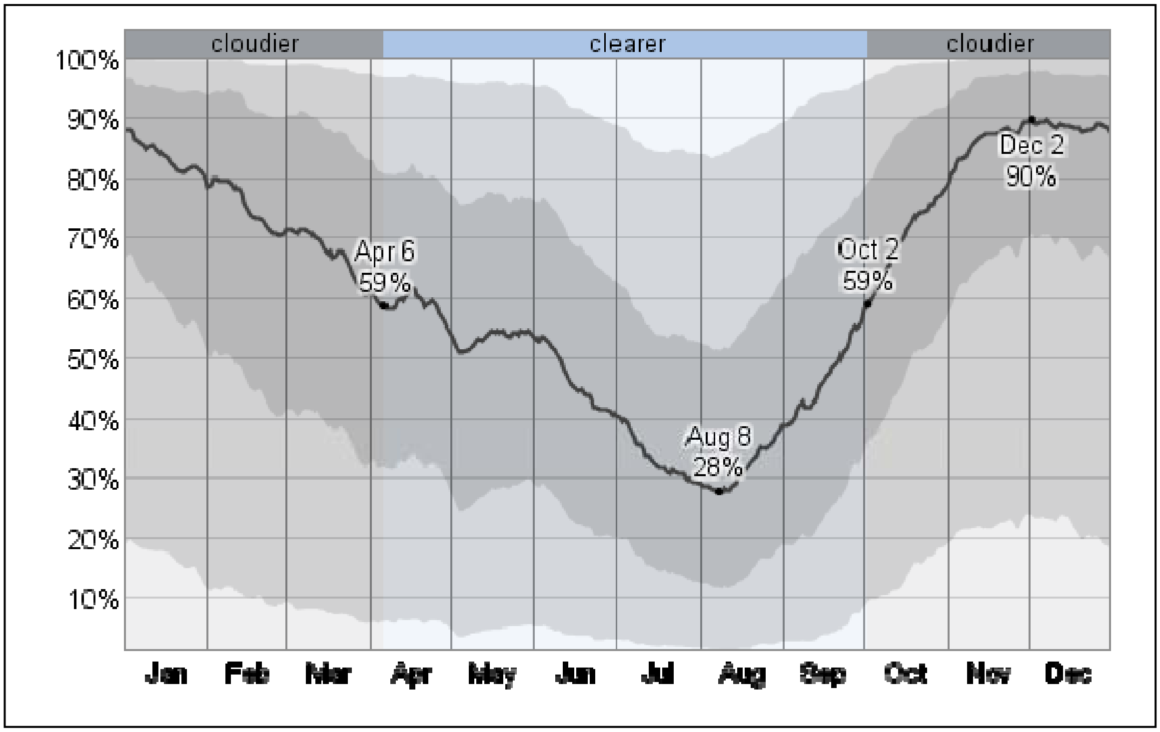
Problem 2 – It's a warm sunny day and the air is rather humid with a dew point of 75^{o}F .The ground temperature is 85^{o}F. If the air temperature decreases at a rate of 3.5^{o}F/1000 feet, at what altitude will a cloud begin to appear? Answer: The rising air near the ground has to drop in temperature by 85^{o}F – 75^{o}F = 10^{o}F. It is decreasing by 3.5^{o}F every 1000 feet, so the dew point temperature of 75^{o}F will be reached at an elevation of 10^{o}F/3.5^{o}F = **2857 feet**.

Problem 3 – On a comfortable summer day, the humidity is only 20% and the outside temperature is 80^{o}F. At what altitude might clouds start to form overhead if the air temperature is decreasing at a 'dry' lapse rate of 5.5 ^{o}F/1000 feet?

Answer: First use the dew point formula to calculate $T_{dewpoint}$. For P=20 and T_{air}=80^{o}F we get $T_{dewpoint}$=80 – (100-20)/5 so $T_{dewpoint}$=64^{o}F.

Next, calculate the altitude from the lapse rate. The difference between the ground temperature and the dew point temperature is 80^{o}F – 64^{o}F = 16^{o}F. The altitude will be A = 16^{o}F/(5.5^{o}F/1000feet) = **2909 feet**.

Cloud Cover and Solar Radiation



Many homeowners now use solar panels to collect sunlight and convert it into electricity on their rooftops. This is a good idea when there are no clouds in the sky, but what happens on a cloudy day? To get some idea of whether you should consider installing a solar power system, you need to understand how much cloud cover your area typically gets during the year. Too much cloud cover and you will not be able to support your house hold electrical needs by going 'off-grid'.

For example, the plot above (courtesy Weatherspark.com) describes the cloud cover percentage over Two Harbors, Minnesota during the course of an average year. This region has a humid continental climate with warm summers and no dry season. During a typical year, we can see that the cloud cover varies from nearly 90% in the winter to below 40% during the summer. Together with the high latitude of this location, it may be difficult to operate a year-round solar power system on your roof-top. At least during the winter, you will need to go back 'on grid' to run the electrical needs of your home because only 10% of the usable sunlight is available for your electrical system.

*You can investigate the cloud cover over your location and report what you see to other students and scientists by joining the S'COOL program (**http://scool.larc.nasa.gov/**). You can also use My NASA Data (**http://mynasadata.larc.nasa.gov/live-access-server-introduction/**) to download NASA cloud cover data and other Earth science resources.*

There is a simple formula to predict how much sunlight reaches the ground for different amounts of cloud cover:

$$P = 990\,(1-0.75F^3) \quad \text{watts/m}^2$$

where F is the fraction of sky cloud cover on a scale from 0.0 (0% no clouds) to 1.0 (100% complete coverage). Now let's see how to use this formula!

Problem 1 – For what percentage of the year are conditions considered cloudy in Two Harbors?

Problem 2 – Based upon the trend in the black line on the graph, what is the average cloud cover during the year?

Problem 3 – From the formula for solar power, for what percentage of sky cover will the homeowner get more than 50% of the maximum solar power from their electric system?

Problem 4 – On the cloud cover graph, shade in the region that represents the condition that the homeowner will get more than 50% of the available electrical power.

Problem 5 – About what percentage of the year will the homeowner be able to generate more than 50% of the available solar power?

Common Core Math Standards:
CCSS.Math.Content.6.RP.A.3c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
CCSS.Math.Content.8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

More about the cloud cover formula can be found at:
http://www.shodor.org/os411/courses/_master/tools/calculators/solarrad/

The data from Two Harbors is from
http://weatherspark.com/averages/31818/Two-Harbors-Minnesota-United-States

Problem 1 – For what percentage of the year are conditions considered cloudy in Two Harbors? Answer: January, February, March and October, November and December so P = 100% x (6/12) = **50%**.

Problem 2 – Based upon the trend in the black line on the graph, what is the average cloud cover during the year? Answer: The highest is 90% and the lowest is 28% so the average of these two is **59%**. Students may also calculate the average monthly coverage (January=85%, February=75%, March=67%, April=60%, May=55% June=45% July=32% August=32% September=45% October=70% November=87% December=87%) and get **62%**.

Problem 3 – From the formula for solar power, for what percentage of sky cover will the homeowner only get 50% of the maximum solar power from their electric system? Answer: The maximum solar power occurs for F=0 and equals 990 watts/m^2. Half of this is 495 watts/m2 so we want $495 = 990(1-0.75F^3)$. This means that $0.50 = 0.75F^3$ and so $F^3 = 0.67$ and so solving for F we get $F = (0.67)^{1/3} = 0.87$. **So 87% cloud cover produces a reduction of 50% in electrical power.**

Problem 4 – On the cloud cover graph, shade in the region that represents the condition that the homeowner will get more than 50% of the available electrical power. Answer: Draw a horizontal line across the graph at ‘87%’. And shade in all the area below this line to indicate ‘more than 50% of available power’.

Problem 5 – About what percentage of the year will the homeowner be able to generate more than 50% of the available solar power? Answer: Only three months have more than 87% cloud cover: January, November and December, so there are 9 months producing more than 50% : P = 100% (9/12) = **75%**.

Courtesy Aaron McNeeley
mmcneely@nd.edu

When a cloud is dense enough with water droplets it appears fleecy white, it is also dense enough that it can cause a shadow. Scientists use the terms albedo and transmission to describe how clouds and other materials reflect and transmit light.

Albedo: The amount of light a cloud reflects, making it appear white.

Transmission: The amount of light that passes through a cloud to the ground.

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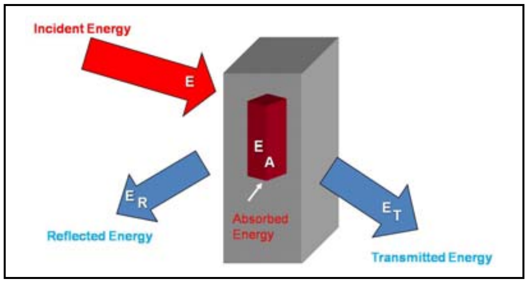


Albedo and transmission can be conveniently measured in percentages. For example, in the figure to the left, if 100% of the light energy falls on the cloud and 70% is reflected back into space, the cloud albedo is 70% and the percentage of transmitted energy is 100% - 70% = 30%.

Problem 1 – A cloud has an albedo of 65%, but a sensitive light meter registers only 30% transmitted light directly under the cloud. How much light energy has been absorbed by the cloud to heat it?

Problem 2 – A satellite view of a small area of Earth from space shows that 1/6 of the area had soil cover with an albedo of 20%, 1/3 of the area was covered by clouds with an albedo of 60%, and 1/2 of the area covered by water with an albedo of 10%. What is the average albedo of this area?

Instead of transmission, scientists prefer to use the term opacity, x, because it can be more easily calculated from the actual properties of the cloud. For example, $x = kL$, where L is the thickness of the cloud and k is a constant that describes the density of droplets in the cloud and droplet sizes. Transmission, T, and opacity are related by the formula:

$$T = 100\%\ 10^{-0.69x}$$

Problem 3 - Graph the function T(x) for opacities from 0.0 to 5.0. To the nearest percentage, what is the range of cloud transmission and albedo for opacities covered by your graph?

Problem 4 – A cumulus cloud is 2.5 kilometers thick and its opacity constant, k = 0.5, what is the albedo of this cloud, and how much light is transmitted through the cloud to the ground?

Common Core Math Standards:

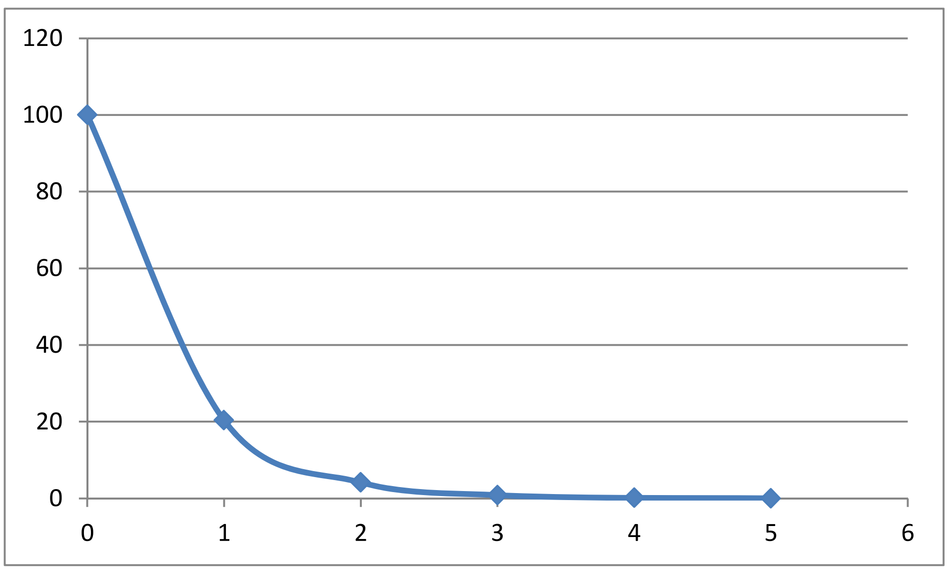
CCSS.Math.Content.HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS.Math.Content.HSF-LE.A.4 For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Problem 1 – A cloud has an albedo of 65%, but a sensitive light meter registers only 30% transmitted light directly under the cloud. How much light energy has been absorbed by the cloud to heat it? Answer: With an albedo of 65%, 35% of the light energy should have reached the ground. Since only 30% was detected, that means that **5% of the light energy** was absorbed by the cloud to heat it.

Problem 2 – A satellite view of a small area of Earth from space shows that 1/6 of the area had soil cover with an albedo of 20%, 1/3 of the area was covered by clouds with an albedo of 60%, and 1/2 of the area covered by water with an albedo of 10%. What is the average albedo of this area? Answer: A = 1/6 (20%) + 2/6(60%) +3/6(10%) = **28%**

Problem 3 - Graph the function T(x) for opacities from 0.0 to 5.0. To the nearest percentage, what is the range of cloud transmission and albedo for opacities covered by your graph?



Opacity 1 to 5 **Transmission: 20% to 0%** **Albedo: 80% to 100%**

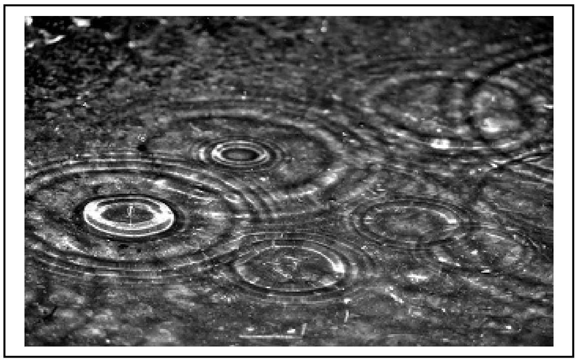
Problem 4 – A cumulus cloud is 2.5 kilometers thick and its opacity constant, k = 0.5, what is the albedo of this cloud, and how much light is transmitted through the cloud to the ground?

Answer: x= kL so x= (0.5)(2.5) = 1.25 then the transmission $T = 100\%\ 10^{-0.69(1.25)}$

Then T = 100%(0.137)

And so T = **13.7%** and the albedo = 100% - 13.7% = **86.3%**

Working with Rainfall Rates and Water Volume



After a local rain storm, your news station might announce that 0.5 inches of rain fell during the morning hours before Noon.

Have you ever wondered just how much water fell out of the sky to cause so much trouble to people trying to get to work or stay dry outside?

Meteorologists classify rain rates for different levels of activity as you can see in the table below:

Type of Storm	Rate
Light Rain	2 - 4 mm/hr
Moderate	5 - 9 mm/hr
Heavy	10 - 40 mm/hr
Violent	more than 50 mm/hr

Problem 1 – Suppose the local news said that 1.6 inches of rain fell between 8:00 am and 1:00 pm. What type of storm was this? (1 inch = 25 mm)

Problem 2 – If 1 mm of rainfall equals 1 liter of water over an area of one square meter, how many liters of water will fall over a town that has an area of 100 km^2 during a light rain shower that lasted 3 hours at a rate of 2 mm/hr?

Problem 3 – About 500,000 cubic kilometers of rain falls on the surface of Earth every year. What is the average rate in mm/hr if the surface area of Earth is 500 million km^2?

Common Core Math Standards:

Grade 6 – *CCSS.Math.Content.6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Grade 7 – *CCSS.Math.Content.7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*

Optional: Grade 8 - *CCSS.Math.Content.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology*

Problem 1 – Suppose the local news said that 1.6 inches of rain fell between 8:00 am and 1:00 pm. What type of storm was this? (1 inch = 25 mm)

Answer: The depth of the rain was 1.6 inches x 25 mm/ 1 inch = 40 mm. This fell in the time between 8:00 am and 1:00 pm which is 5 hours, so the rate was 40 mm/5 hours = 8 mm/hr. This type of storm would be considered a **moderate storm**.

Problem 2 – If 1 mm of rainfall equals 1 liter of water over an area of one square meter, how many liters of water will fall over a town that has an area of 100 km^2 during a light rain shower that lasted 3 hours at a rate of 2 mm/hr?

Answer: First we have to calculate the total number of millimeters that fell, which is 2 mm/hr x 3 hours or 6 millimeters.

Then we calculate the rate in terms of liters/meter^2 which will be 6 mm x (1 Liter/meter^2) = 6 Liters/meter^2.

Next we convert the area of the town into square meters, which is 100 km^2 x (1000 m/1km)x (1000 m/1 km) = 10^8 meters^2.

Finally we multiply the rate by the area to get 6 liters/meter^2 x 10^8 meters^2 = **6.0x10^8 Liters**.

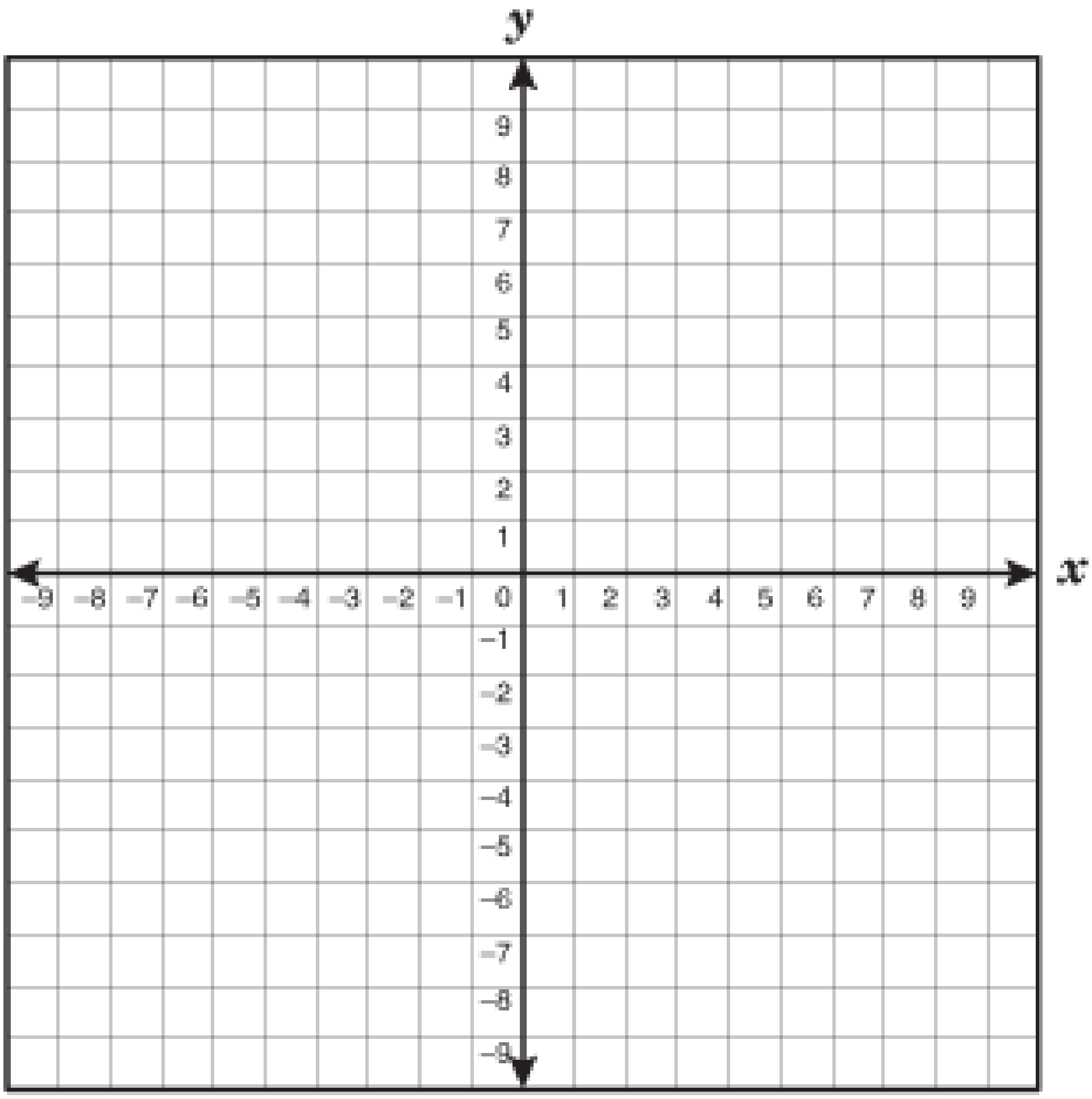
Problem 3 – About 500,000 cubic kilometers of rain falls on the surface of Earth every year. What is the average rate in mm/hr if the surface area of Earth is 500 million km^2?

Answer: Volume = Height x Area so
500,000 km^3 = height x 500 million km^2 and so
height = 500,000/500,000,000 = 1/1000 km or 1 meter.

This falls in one year. 1 year = 365 days x 24h/1day = 8760 hours so the rate is

R = 1 meter/8760 hours
= 1000 mm/8750 hrs
= **0.11 mm/hr**.

Graphing a Snowflake Using Symmetry



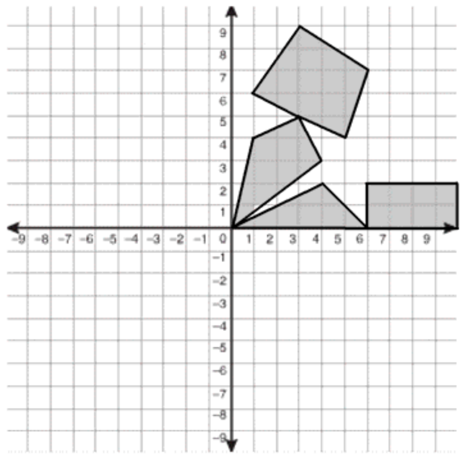
Snowflakes have a symmetrical shape that often follows a simple pattern that is replicated to form the full shape that you see.

Problem 1 - Graph the following points to make a design in the First Quadrant:

(10,0), (10,2), (6,2), (6,0), (4,2), (0,0), (4,3), (3,5), (5,4), (6,7), (3,9), (1,6), (3,5), (1,4), (0,0)

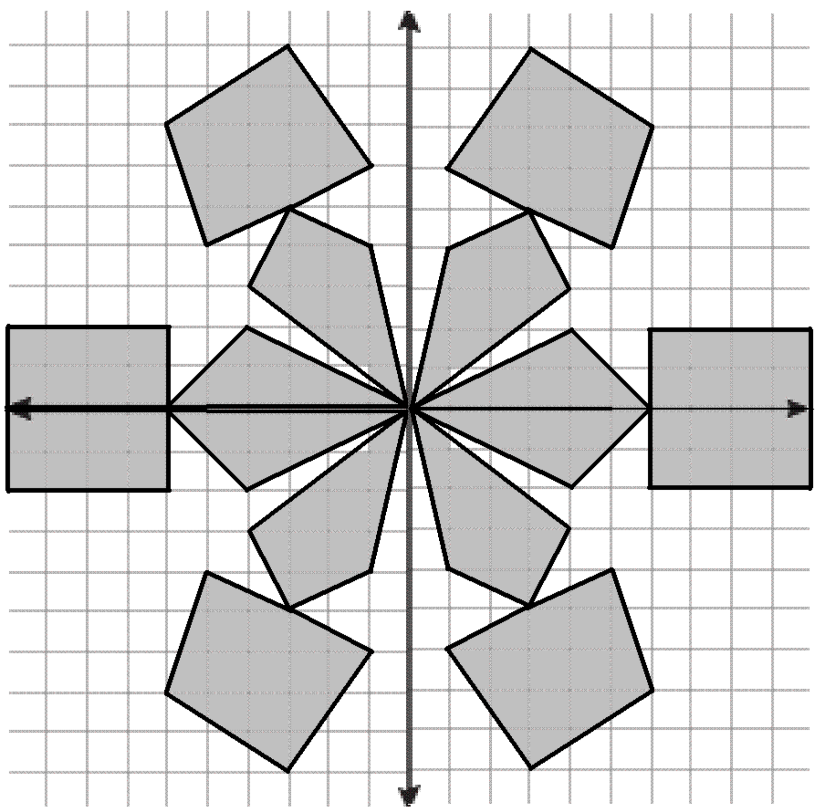
Problem 2 - Connect the points with line segments in the order given.

Problem 3 - Reflect the pattern that you drew into the Second Quadrant, then complete the pattern in Quadrants Three and Four to form the full snowflake shape!

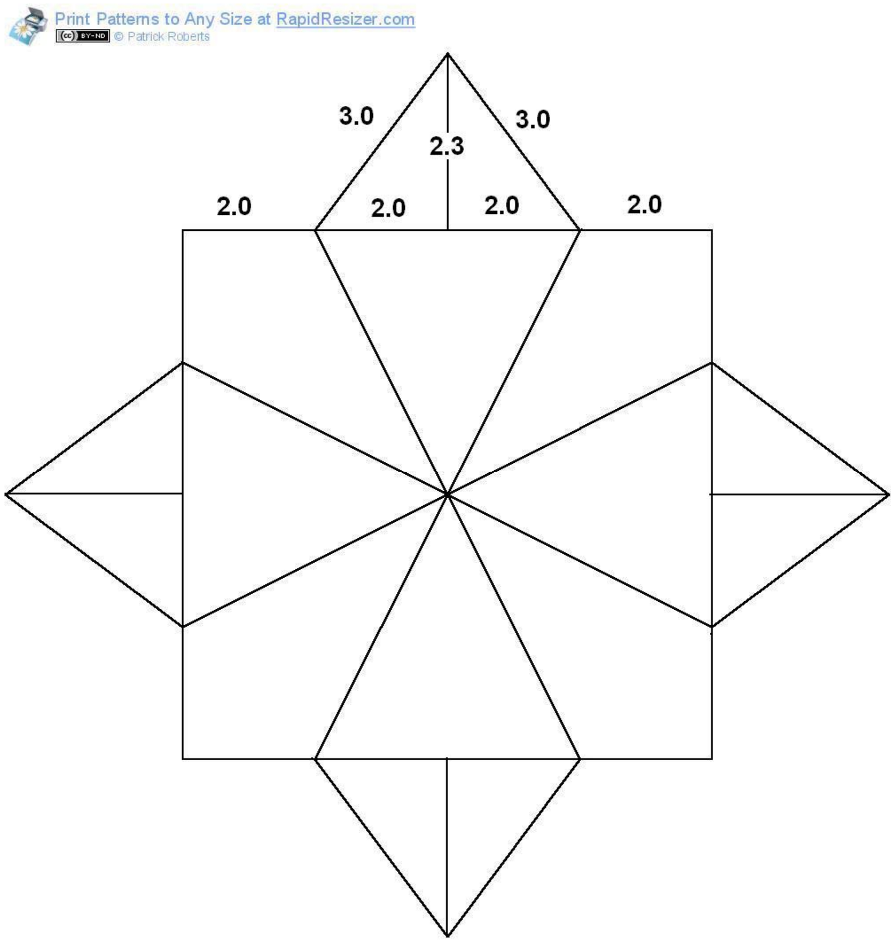


Problem 1 and 2 -

Problem 3 - Students may either place 'mirrors along the X and Y axis and redraw the shape in the First Quadrant, or use the following symmetry idea: To reflect the figure into Quadrant Two, plot the points in Quadrant One with the sign of the x coordinates replaced by their negative : (x,y) becomes (-x, y). For Quadrant Three use (x,y) becomes (-x,-y) and for Quadrant Four (x,y) becomes (x,-y). The full figure is shown below:



The Surface Area of a Snowflake



The diagram above shows the basic plan for one common type of snowflake. The detailed pattern within each polygonal area has been removed to show the regular areas. The numbers at the top are the measured line segments in millimeters.

Problem 1 - Using the geometric clues in the diagram, what is the total area of this pattern in square millimeters, rounded to the nearest integer?

Problem 2 - If all measurements were doubled in length, what would be the total area of the pattern to the nearest integer in square-millimeters?

Problem 1 - Using the geometric clues in the diagram, what is the total area of this pattern in square millimeters, rounded to the nearest integer?

Answer:

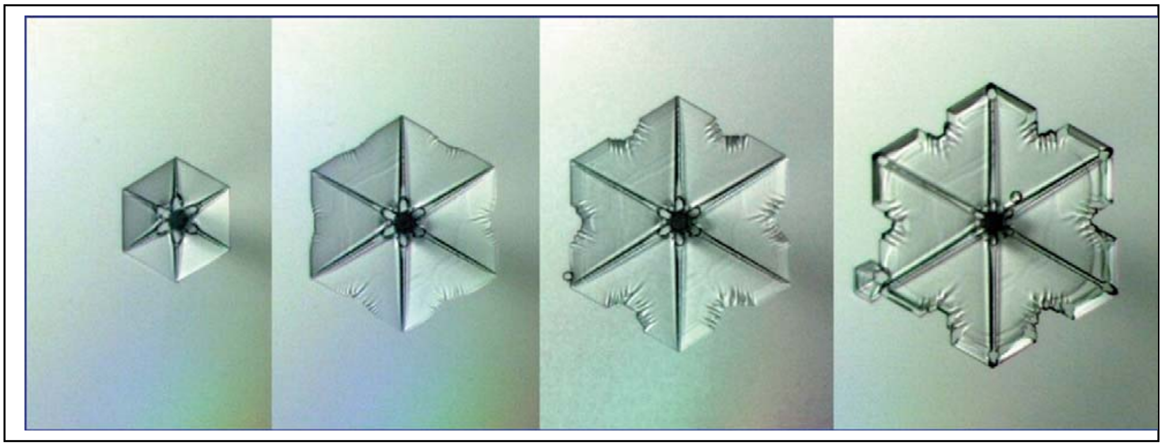
The pattern consists of a main square with a side length of 2.0mm+2.0mm+2.0mm+2.0mm = 8.0 mm and an area of $(8.0\text{mm})^2$ = 64 mm^2.

The four triangular points each have an area of 1/2(4.0mm)(2.3mm) = 4.6 mm^2, so the total area of the pattern is 64.0 mm^2+ 4(4.6mm^2) = 82.4 mm^2, which is rounded to **82 mm^2**.

Problem 2 - If all measurements were doubled in length, what would be the total area of the pattern to the nearest integer in square-millimeters?

Answer: Doubling the dimensions means that the area is increased by a factor of 2x2 = 4 so it now becomes 82.4mm^2 x 4 = 329.6 mm^2, which rounds to **330 mm^2**.

The side length of the square becomes 2 x 8.0mm = 16.0 mm and the area is then $(16\text{mm})^2$ = 256 mm^2. The four triangles each have an area of 1/2 (8.0mm) (4.6mm) = 18.4 mm^2, so the total area is 256 mm^2+ 4(18.4mm^2) = 329.6 mm^2 or rounded to **330 mm^2**.



A snowflake is a flat figure whose area doubles over time as liquid droplets condense on its surface. For average cloud conditions, the area doubles every 2 hours.

No matter what the shape of a polygon, the area of a polygon will increase by a fixed amount as the size of the polygon increases.

Problem 1 - Suppose the time to double its area is 2 hours. How many doublings in area will have occurred in 8 hours?

Problem 2 – If the area of the snowflake at the start of its growth is 1 square millimeter, what will its area be after 8 hours? To organize your thinking about snowflake growth, create a table for the snowflakes size and area.

Problem 3 – If the size of the snowflake was 1 millimeter at the start of growth, what will be its size at the end of a snow storm that lasted 8 hours if the area doubling time is 2 hours? To organize your thinking about snowflake growth, create a table for the snowflakes size and area.

Problem 1 - Suppose the time to double its area is 2 hours. How many doublings in area will have occurred in 8 hours?

Answer: The snowflake has been growing for 8 hours which is 8/2 = **4 doubling times.**

Problem 2 – If the area of the snowflake at the start of its growth is 1 square millimeter, what will its area be after 8 hours?

Doubling	**1**	**2**	**3**	**4**	**5**	**6**
Area	**2**	**4**	**8**	**16**	**32**	**64**
Size	**1.4**	**2**	**2.8**	**4**	**5.7**	**8**

Answer: It will have an area that is 2 x 2 x 2 x 2 = 16 times larger or **16 square millimeters.**

Problem 3 - If the size of the snowflake was 1 millimeter at the start of growth, what will be its size at the end of a snow storm that lasted 8 hours if the area doubling time is 2 hours?

Answer: 8 hours = 4 doubling times so it has increased in area by 16 times. Because area = length x length, since 16 = 4 x 4, the snowflake has increased its size by 4 times so it is now 1 mm x 4 = **4 millimeters in diameter.**

Snow to Water Ratios

The amount of snow from a storm can look impressive when it covers your house and cars, but if you melted the snow you would discover that very little water is actually involved. The 'snow to ice ratio' or Snow Ratio expresses how much volume of snow you get for a given volume of water. Typically a ratio of 10:1 (ten to one) means that every 10 inches of snowfall equals one inch of liquid water.

Problem 1 - During a winter storm called 'Snowmageddon' in 2010, the Washington DC region received about 24 inches of snow fall. If this was dry, uncompacted snow, about how many inches of rain would this equal if the Snow Ratio was 10:1 ?

Problem 2 - The Snow Ratio depends on the temperature of the air as shown in the table below:

Temp (F)	30^o	25^o	18^o	12^o	5^o	-10^o
Ratio	10:1	15:1	20:1	30:1	40:1	50:1

If 30 inches of snow fell in Calgary, Alberta at 18^oF, and 25 inches of snow fell in Denver, Colorado where the temperature was 25^o F, at which location would the most water have fallen?

Problem 1 - During a winter storm called 'Snowmageddon' in 2010, the Washington DC region received about 24 inches of snow fall. If this was dry, uncompacted snow, about how many inches of rain would this equal if the Snow Ratio was 10:1 ?

Answer: 24 inches of snow x (1 inch water/10 inches of snow) = **2.4 inches of water**.

Problem 2 - If 30 inches of snow fell in Calgary, Alberta at 18^{o}F, and 25 inches of snow fell in Denver, Colorado where the temperature was 25^{o} F, at which location would the most water have fallen?

Answer - In Alberta, the Snow Ratio for 18^{o} F is 20:1 and in Denver at 25^{o} F it is 15:1.

The amount of water that fell in Alberta is then 30 inches of snow x (1 inch water/20 inches snow) = 1.3 inches of water. In Denver it is 25 inches of snow x (1 inch water/15 inches snow) = 1.7 inches of water. **So more water fell in Denver, even though there was less snow on the ground!**

Snow Density and Volume

This scientist is collecting cylindrical snow cores to study snow density from the wall of a snow pit. This pit was carefully dug into the Taku Glacier, in the Juneau Icefield of the Tongass National Forest, Alaska.

The density of snow tells scientists a lot about the history of the snow, and whether it is safe for skiers.

Density is defined as the amount of mass that an object has compared to the volume that it takes up. On average, a cubic meter of freshly-fallen snow has an average mass of about 50 kilograms. Snow that has been compacted by its own weight at a depth of 3 meters can have 200 kilograms in the same volume.

Density is defined as mass/volume. Freshly-fallen snow has a density of 50 $kg/meter^3$ = 50 kg/m^3, while the compressed snow described above has a higher density of 200 kg/m^3. Let's explore some other examples of estimating snow density!

Problem 1 – A scientist uses a cylindrical gauge to sample the snow in a trench wall. The cylinder has a radius of 5 centimeters and a length of 60 centimeters, and it has a mass of 50 grams. After filling the cylinder with snow, the cylinder is again weighed and now has a mass of 520 grams. What is the density of the snow that was sampled?

Problem 2 – Two scientists measure the snow density from two different mountain locations using two different snow gauges: A and B. Gauge A has a radius of 6.3 cm and a height of 40 cm, while Gauge B has a radius of 8.0 cm and a height of 40 cm. To the nearest cubic centimeter, what are the volumes of the two gauges? (use π = 3.141)

Problem 3 – If 500 grams is collected by Gauge A and 804 grams is collected by Gauge B, what are the snow densities to the nearest tenth. Are the scientists sampling different kinds of snow, or similar kinds of snow at the two locations?

Problem 1 – A scientist uses a cylindrical gauge to sample the snow in a trench wall. The cylinder has a radius of 5 centimeters and a length of 60 centimeters, and it has a mass of 50 grams. After filling the cylinder with snow, the cylinder is again weighed and now has a mass of 520 grams. What is the density of the snow that was sampled?

Answer: For a cylinder, the volume is given by the formula $V = \pi r^2 h$,where r is the radius and h is the height. The snow gauge volume is then

$V = (3.141)(5cm)^2 (60 \text{ cm}) = 4,711 \text{ cm}^3$.

When empty, the snow gauge had a mass of 50 grams and when full of snow it had a mass of 520 grams, so the actual mass of the snow was

M = 520 gm - 50 gm = 470 grams.

The density of the snow is then D = M/V = 470 gms/4711 cm^3 = **0.1 gm/cm^3**

Problem 2 – Two scientists measure the snow density from two different mountain locations using two different snow gauges: A and B. Gauge A has a radius of 6.3 cm and a height of 40 cm, while Gauge B has a radius of 8.0 cm and a height of 40 cm. To the nearest cubic centimeter, what are the volumes of the two gauges? (use $\pi = 3.141$)

Answer: The volume of a cylinder is given by $V = \pi R^2 h$, so

The volume of Gauge A is $V = (3.141) \times (6.3 \text{ cm})^2 \times (40\text{cm}) = 4,987 \text{ cm}^3$.

The volume of Gauge B is $V = (3.141) \times (8.0 \text{ cm})^2 \times (40\text{cm}) = 8,041 \text{ cm}^3$.

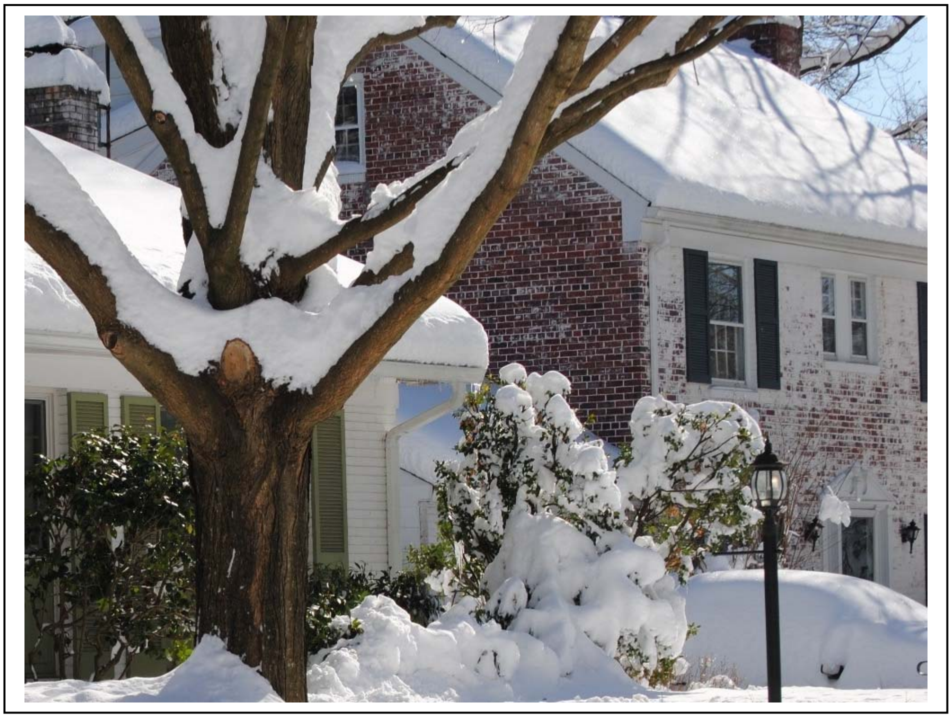
Problem 3 - If 500 grams is collected by Gauge A and 804 grams is collected by Gauge B, what are the snow densities to the nearest tenth, and are the scientists sampling different kinds of snow, or similar kinds of snow at the two locations?

Answer - The density measured by Gauge A is D = 500 gm/4987 cm^3 = **0.1 gm/cm^3**.

The density measured by Gauge B is D = 804 gm/8041 cm^3 = **0.1 gm/cm^3**

So the densities are the same and the kinds of snow are probably also the same at the two locations.

Snow Density, Mass and Roof Failure



During snowfalls, most children are excited by the accumulating snow, while many parents may worry if the weight of the snow will eventually cause their roofs to collapse. Although a small amount of snow weighs next to nothing, a few feet can weigh many pounds. How much snow is too much for the average roof on a house? Engineers estimate that 65 pounds per square foot (320 kg/m^2) is the average amount that a standard wood-framed roof can hold before it collapses. Dry snow has a density of about 50 kg/m^3 while wet snow has a density of 200 kg/m^3.

Problem 1 - Two houses are covered with a blanket of snow. House A has dry snow to a depth of 1 meter, and House B has a roof covered with wet snow to a depth of ½ meter. Which house is at greater risk of roof collapse?

Problem 2 - A snow storm of wet snow began at 6:00 am and continued steadily all day at a rate of 20 cm/hour. At what time will the snow accumulating on the roof reach the critical load for roof collapse?

Problem 1 - Two houses are covered with a blanket of snow. House A has dry snow to a depth of 1 meter, and House B has a roof covered with wet snow to a depth of ½ meter. Which house is at greater risk of roof collapse?

Answer: House A has 1 meter of dry snow covering every square meter of surface, so the mass of this snow on the roof is 50 kg/m^3 x 1 meter = 50 kg/m^2. House B has wet snow to a depth of 1/2 meter so the mass is 200 kg/m^3 x 1/2 meter = 100 kg/m^2. **House B is at greater risk even though it appears to have much less snow cover.**

Problem 2 - A snow storm of wet snow began at 6:00 am and continued steadily all day at a rate of 20 cm/hour. At what time will the snow accumulating on the roof reach the critical load for roof collapse?

Answer: The wet snow density is 200 kg/m^3. It is accumulating at a rate of 0.2 meters/hour. To reach 320 kg/m^2, which engineers say is the critical loading for roof collapse, you need to accumulate a thickness of 320/200 = 1.6 meters. At a rate of 0.2 meters/hour this will take about 1.6 meters x (1 hour/0.2 meters) = 8 hours, so by about **2:00 pm**, the roof might collapse.

In the figure to the left, the first column represents gas particles with little energy. A thermometer placed in contact with this group of particles would indicate a very low temperature. The column to the right represents particles with a high enough speed and energy to spread out inside the column. A thermometer placed in this group would show a high temperature.

When the state of matter changes its phase, the temperature and energy of matter also changes. At low temperature and energy we have a solid phase. At a medium temperature and energy we have a liquid phase, and at a high temperature and energy we have a gaseous phase.

A simple formula gives us the average speed, V, of water molecules in meters per second (m/s) for a given temperature in degrees Celsius, T:

$$V^2 = 1380(273+T)$$

Problem 1 – What is the speed of an average water molecule near A) the freezing point of water at 0^o C? B) The boiling point of water at 100^o C?

Problem 2 - The kinetic energy in Joules for all of the water molecules in a gallon of water, which has a mass of about M = 4.0 kilograms, and an average molecule speed of V in meters/sec, is given by the formula:

$$K.E. = \frac{1}{2}MV^2$$

To the nearest Joule, what is the kinetic energy of a 1 gallon of water at the temperatures given in Problem 1?

Problem 3 - If you heated the one gallon of water from 0^{o}C to 100^{o}C, how much 'thermal' energy would you have to add?

Problem 1 - What is the speed of an average water molecule near A) the freezing point of water at 0^o C? B) The boiling point of water at 100^o C?

Answer: From the formula:

A) V = $(1380(273+(+0))^{1/2}$ = $(376740)^{1/2}$ = 614 meters/sec.

B) V = $(1380(273+(100))^{1/2}$ = $(514740)^{1/2}$ = 717 meters/sec.

Problem 2 - The kinetic energy in Joules for all of the water molecules in a gallon of water, which has a mass of about M = 4 kilograms, and an average molecule speed of V in meters/sec, is given by the formula:

$$K.E. = \frac{1}{2}MV^2$$

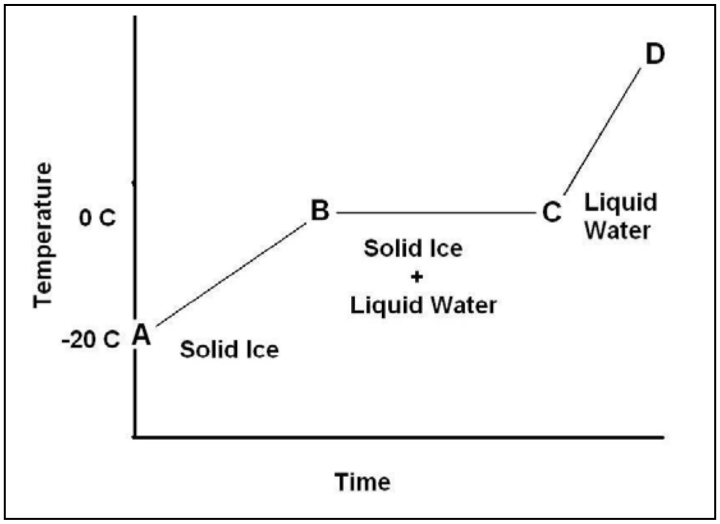
To the nearest Joule, what is the kinetic energy of a 1 gallon of water at the temperatures given in Problem 1?

Answer: A) For $+0^oC$, we calculated an average speed of 614 m/s, so the kinetic energy of the water is KE = 1/2 $(4.0)(614)^2$ = **753,992 Joules**.
B) For 100^oC we have V = 717 m/s, so KE = 1/2$(4.0)(717)^2$ = **1,028,178 Joules.**

Problem 3 - If you heated the one gallon of water from 0^oC to 100^oC, how much 'thermal' energy would you have to add?

Answer: You have to add the difference in energy (1,028,178 – 753,992) = **274,186 Joules** to heat the gallon of water to its boiling point at 100^o C.

Note: A typical hotplate at a temperature of 400 C generates about 1000 Joules/second, so to heat the gallon of water to make it boil would take about 274186/4000 or about 4 minutes at this hotplate setting.



As energy is added to solid matter, it changes its state. The figure to the left shows what happens to water as it changes from solid ice (A to B), to a mixture of cold water and 'ice cubes' (B to C) and then finally to pure liquid water (C to D).

The energy required to change a kilogram of solid ice by one degree Celsius is called the **Specific Heat**. The energy needed to change a kilogram of solid ice at 0^{o}C into 100% liquid water at 0^{o}C is called the **Latent Heat of Fusion**.

Problem 1 - The Specific Heat of ice is 2090 Joules/kg C. How many Joules of energy do you need to raise the temperature of 1 kg of ice from -20^{o}C to 0^{o}C along the path from A to B on the graph?

Problem 2 - The Latent Heat of Fusion for water is 333 Joules/gram. How many Joules of energy do you need to melt all the ice into a pure liquid along the path from B to C on the graph?

Problem 3 - The Specific Heat of liquid water is 4180 Joules/kg C. How much energy is needed to raise the temperature of 100 grams of liquid water to $+60^{o}$C for a nice warm cup of tea along the path from C to D in the graph?

Problem 1 - The Specific Heat of ice is 2090 Joules/kg C. How many Joules of energy do you need to raise the temperature of 1 kg of ice from $-20^{o}C$ to $0^{o}C$ along the path from A to B on the graph?

Answer: The temperature difference is $20^{o}C$, so for 1 kg of ice we need 2090 Joules/kgC x (1 kg) x ($20^{o}C$) = **41,840 Joules**.

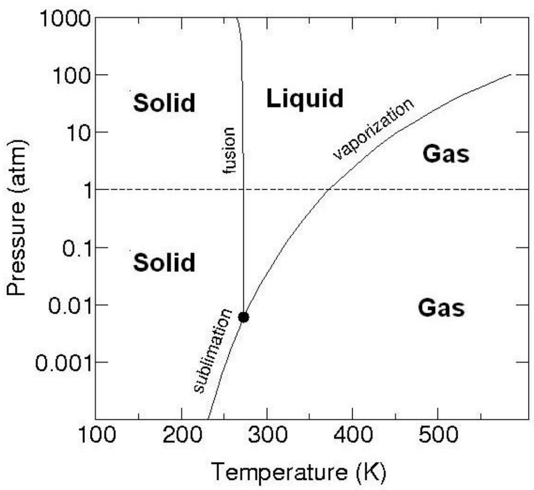
Problem 2 - The Latent Heat of Fusion for water is 333 Joules/gram. How many Joules of energy do you need to melt all the ice into a pure liquid along the path from B to C on the graph?

Answer: For 1 kilogram of ice ,which equals 1000 grams, we need 333 Joules/gram x 1000 grams = **333,000 Joules**.

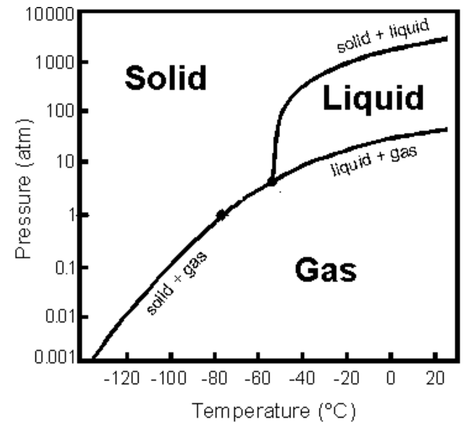
Problem 3 - The Specific Heat of liquid water is 4180 Joules/kg C. How much energy is needed to raise the temperature of 100 grams of liquid water to +60oC for a nice warm cup of tea along the path from C to D in the graph?

Answer: 4180 Joules/kgC x 0.1 kg = **418 Joules**.

What is a Snowball's Chance on Mars?



The H_2O Phase Diagram



The CO_2 Phase Diagram

These diagrams above are called **phase diagrams**. The one to the left shows all of the phases for matter for water as you change the temperature and pressure of the water in your sample. A pressure of 1.0 'atmospheres' is what we experience at sea level. This equals 14 pounds/inch^2 (or in metric units about 100 kiloPascals). As you move horizontally across the diagram towards increasing temperatures (measured in Kelvin units) at a constant pressure of 1.0 atm, the state of your water will change from solid ice, to liquid water at 273 Kelvin, to water vapor at 373 Kelvin.

Snow balls require that you create some liquid water by compressing the snow crystals so that they can glue together as the water refreezes. This will happen along the curve marked 'fusion' which is the boundary between the solid ice and liquid water phases.

The diagram to the right shows all of the phases for carbon dioxide as you change its pressure and temperature. For convenience we use the Celsius temperature scale. Note that 0^o Celsius = +273 on the Kelvin scale, and that a difference of 1^o C equals a change by 1 K on the Kelvin scale.

Problem 1 - We can make snow balls because the pressure (close to 1.0 atm) we apply with our hands at the ambient temperature (close to 273 K) is just enough to melt the ice into water and refreeze it to form a glue holding the snowflakes together. The temperature in Antarctica is typically 250 K. Can you make snowballs in Antarctica with normal hand pressure?

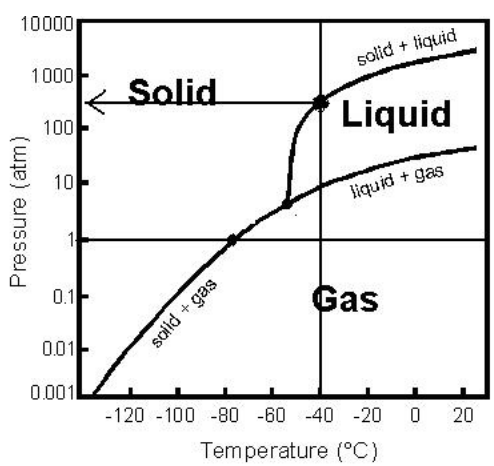
Problem 2 - On Mars, the majority of the ice is carbon dioxide ice. To make a carbon dioxide snowball, imagine applying 1 atm of hand pressure. The average temperature where the carbon dioxide snow falls is about -40^o Celsius in the daytime. Use the phase diagrams to explain why making a showball on Mars may be difficult or easy?

Problem 1 - We can make snow balls because the pressure (close to 1.0 atm) we apply at the ambient temperature (close to 273 K) is just enough to melt the ice into water and refreeze it to form a glue holding the snowflakes together. The temperature in Antarctica is typically 250 K. Can you make snowballs in Antarctica with normal hand pressure?

Answer - At normal winter temperatures near 273 K (0^oC) and 1 atm, the diagram shows that we are very, very close to the conditions needed to make solid ice turn to liquid water with a bit of extra pressure. The vertical line, which represents the ice to liquid transition crosses a pressure of 1 atm at a temperature of just +0.010 C! The diagram also shows that at 250 K (-17^oC) we are far to the left of the vertical line where solid ice can turn to liquid. At this temperature, we will need hand pressures higher that 1000 atm to make snowballs! So you can not make snowballs in Antarctica.

Problem 2 - On Mars, the majority of the ice is carbon dioxide ice. To make a carbon dioxide snowball, imagine applying 1 atm of hand pressure. The average temperature where the carbon dioxide snow falls is about -40^o Celsius in the daytime. Use the phase diagrams to explain why making a showball on Mars may be difficult or easy?

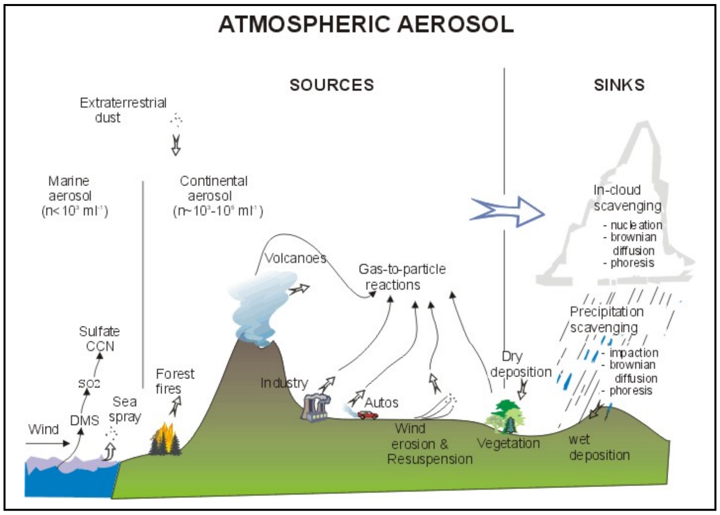
Answer: The main ingredient for a snowball on Mars would be carbon dioxide. The figure below shows the horizontal line representing a hand pressure of 1.0 atm and the vertical line representing the temperature of -40^o C on Mars where snowfall might occur.



The temperature is -40 C, so draw a vertical line on the CO_2 phase diagram until it intersects the solid+liquid line. This is where CO_2 can be in both the solid and liquid phases. You need the liquid CO_2 to supply the glue to hold the solid snowflakes together in the CO_2 snowball. But if you draw a horizontal line to the left, you will see that for you to get this liquid+solid phase at this temperature, you need a pressure of over 100 atmospheres. That about 1500 pounds per square inch of hand pressure, which will be impossible for you to achieve!

This diagram also shows that, on Earth, where the atmospheric pressure is 1 atmosphere, if you move from right to left along the 'pressure = 1 atm' line, a solid piece of CO_2 that you buy at the store will immediately start evaporating into the gas phase. Only if you could freeze this 'dry ice' below -80 C will it stop evaporating into a gas phase (called sublimation) and remain a stable solid.

Atmospheric Aerosols by Percentage



Atmospheric aerosols can come in all sizes and types. The origin of these aerosols is also very complicated. Over the years scientists have been able to measure the percentages of the various types from different parts of the world.

The table below shows the results in percent compiled by NOAA's Earth Systems Research Laboratory.

Atmospheric Aerosols						
Location	Sea Salt	Dust	Water Droplets	Sulfate (SO_4)	Organic Particulates (PAH, etc)	Other
Marine	40%		30%	15%		15%
Europe	3%		23%	46%	10%	18%
Africa	5%	3%	13%	40%	20%	19%
India		4%	28%	44%	5%	19%
Asia	1%	22%	20%	27%	20%	10%
USA		1%	20%	19%	50%	10%

Problem 1 – In which direction (rows or columns) do you expect the percentages to add up to 100% and why?

Problem 2 – Draw a pie graph showing the percentages of the various aerosol contributions over the United States. What type of aerosol is the most abundant?

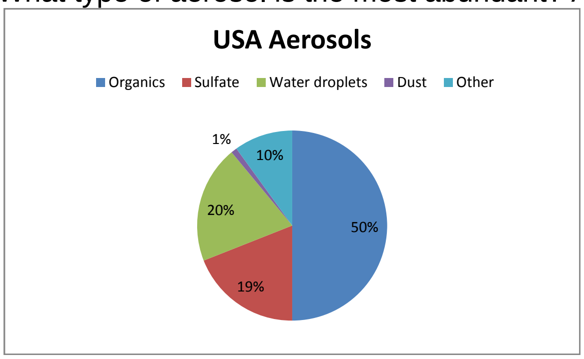
Problem 3 – Create a pie graph that shows the percentage abundance of sulfate aerosols over the 6 different regions. Which region has the highest abundance?

Problem 4 – Sulfate aerosols and organic particulates are typically man-made from industrial processes and the burning of fossil fuels. Which region has the highest concentration of industrial sources of aerosols, and why do you think this trend occurs?

NOAA aerosol data from Earth System Research Laboratory:
http://www.esrl.noaa.gov/research/themes/aerosols/#fig1

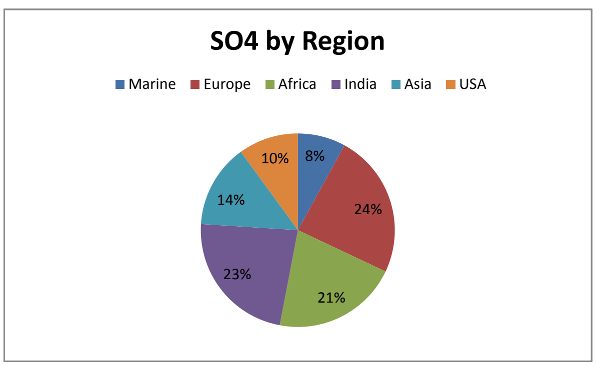
Problem 1 – In which direction (rows or columns) do you expect the percentages to add up to 100% and why? Answer: **The total percentage of aerosols has to add up to 100% because this table is indicating the types of aerosols found in the study. The only direction in which this happens in the table is along he rows, so each row adds up to 100% for the indicated location.**

Problem 2 – Draw a pie graph showing the percentages of the various aerosol contributions over the United States. What type of aerosol is the most abundant? Answer: See below.



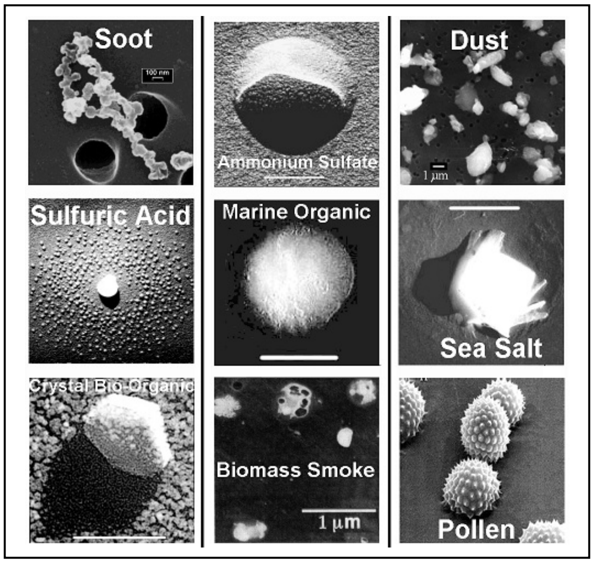
Problem 3 – Create a pie graph that shows the percentage abundance of sulfate aerosols over the 6 different regions. Which region has the highest abundance?

Answer: First you have to add up the percentages for each region: 15% + 46% + 40% + 44% + 27% + 19% = 191%. Then divide each percentage by 191% to get the percentage for each region. Marine = 100% x (15/191) = 8%; Europe = 24%; Africa = 21%; India = 23%; Asia= 14% and USA= 10%. Now create the pie graph. Europe has the highest concentration (24%).



Problem 4 – Sulfate aerosols and organic particulates are typically man-made from industrial processes and the burning of fossil fuels. Which region has the highest concentration of industrial sources of aerosols and why do you think this trend occurs?

Answer: From the table, if we combine the columns for Sulfate and Organic Particulates we see that the **USA with 69% is the largest**. This is probably because the USA is more 'industrialized' than other countries, especially along its East Coast.



An aerosol is a mixture of fine solid particles or liquid droplets in air or another gas. Examples of aerosols include clouds, haze, and air pollution such as smog and smoke. The liquid or solid particles have diameter mostly smaller than 1 μm or so.

Aerosols are so small we have no real choice but to use scientific notation to determine their properties such as volume, mass or density! The graph to the left shows the sizes of different types of aerosols in terms of nanometers, where 1 nanometer is 1 one billionth of a meter or 10^{-9} meters.

Problem 1 – A human hair has a diameter of 100 micrometers (100 microns). If 1000 nanometers equals 1 micron, how many 250 nanometer aerosol particles can fit across the diameter of one human hair?

Problem 2 – If the density of a typical spherical sea salt aerosol particle is 2.0 grams/cm^3, and the particle has a diameter of 500 nanometers, what is the mass of a single aerosol particle in A) grams? B) micrograms?

Problem 3 - On an especially hazy day, the density of aerosol particles in the air is 10 million particles per cubic centimeter. If the particles have an average size of 900 nanometers and a density of 1.5 grams/cm^3, A) how much aerosol mass would there be in a cubic meter of air? B) If you breath-in 100 liters of air every minute, and 1 liter equals 1000 cm^3, how many grams of aerosols do you inhale every day?

Problem 1 – A human hair has a diameter of 100 micrometers (100 microns). If 1000 nanometers equals 1 micron, how many 250 nanometer aerosol particles can fit across the diameter of one human hair?

Answer: Diameter of human hair in nanometers = 100 micrometers x (1000 nanometers/1 micrometer) = 100000 nanometers. Since one aerosol is 250 nm in diameter, there would be 100000/250 = **400 aerosol particles** place end to end to cross the diameter of one human hair.

Problem 2 – If the density of a typical spherical sea salt aerosol particle is 2.0 grams/cm^3, and the particle has a diameter of 500 nanometers, what is the mass of a single aerosol particle in A) grams? B) micrograms?

Answer: A) The radius of the aerosol is 250 nanometers. Since 1 meter = 100 centimeters, the radius is 250 x 10^{-9} x (100 cm/1 meter) = 2.5×10^{-5} cm. Volume of a spherical particle = 4/3 π (2.5×10^{-5} meters)^3 = 6.5×10^{-14} cm^3. The mass in grams is then 2.0 grams/cm^3 x Volume in cm^3, so **M = 1.3×10^{-13} grams**.

B) 1 microgram = 10^{-6} grams, so **M = 1.3×10^{-7} micrograms**.

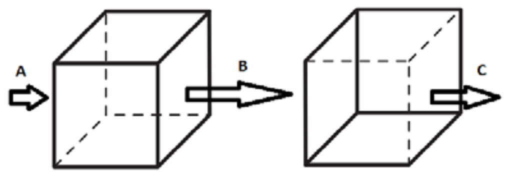
Problem 3 - On an especially hazy day, the density of aerosol particles in the air is 10 million particles per cubic centimeter. If the particles have an average size of 900 nanometers and a density of 1.5 grams/cm^3, A) how much aerosol mass would there be in a cubic meter of air? B) If you breath-in 100 liters of air every minute, and 1 liter equals 1000 cm^3, how many grams of aerosols do you inhale every day?

Answer: A) Each particle has a mass of

$$M = 1.5 \text{ gm/cm}^3 \times (4/3\ \pi\ (4.5 \times 10^{-5} \text{ cm}))^3 = 5.7 \times 10^{-13} \text{ grams.}$$

If the particle density is 10^7 particles/cm^3, in 1 cubic meter there would be 10^7particles/cm^3 x 10^6 cm^3= 10^{13} particles, and so the total mass per cubic meter would be 5.7×10^{-13} grams x 10^{13} particles = **5.7 grams**!

B) You breath in 100 liters/minute x (1000 cm^3/1 liter) x (60 minutes/1 hour) x (24 hours/1 day) = 1.44×10^8 cm^3. Since the aerosol mass is 5.7 grams/$meter^3$ we have 1.44×10^8 cm^3 x (1 $meter^3$/10^6 cm^3) x (5.7 grams/1 $meter^3$) = 144 x 5.7 = **821 grams/day**.



A ray of light passes through the two cubes from left to right. At **A**, the light ray has its full intensity of 100%.

After it leaves the first box, the aerosols have reduced the light intensity by 10% so that at B, the light intensity is 100% x (0.90) = 90%.

After passing through an identical aerosol region in the second box, the light intensity is reduced by an additional 10%. The final light intensity at C is then 100% x (0.90) x (0.90) = 81%.

Aerosols are small particles of liquids or solids that are light enough to be suspended in the air for long periods of time. Common aerosols can include dust from volcanoes, exhaust from jet plains, smog, or ash from the combustion of fossil fuels and wood. Most of the time you do not even notice they are there, unless they are present in large enough numbers.

When the concentration of aerosols is high enough, they can actually cause the dimming of sunlight, which is why the sky can appear darker on a foggy day (water aerosols) or very hazy on a day with lots of smog or distant forest fires.

The figure to the left shows how light dimming occurs, if the intensity of light is reduced by 10% as it passes through two boxes of air.

Problem 1 – Suppose in the above example, the light ray passes through 6 identical boxes. How bright will the light be after it leaves the sixth box?

Problem 2 – Suppose that each box is 1 kilometer on a side, and that the light is dimmed by1% through each box. If the total path is 20 kilometers, what will be the brightness of the light after it leaves this aerosol cloud to the nearest percent?

Problem 3 – A light ray passes through 5 kilometers of ordinary air, which reduces the light by 0.5% per kilometer, and then passes through a dense cloud that reduces the light by 5% per kilometer. If the cloud is 3 kilometers long, to the nearest percentage what will be the light intensity when the light leaves the cloud?

Problem 1 – Suppose in the above example, the light ray passes through 6 identical boxes. How bright will the light be after it leaves the sixth box?

Answer: 100% x 0.9 x 0.9 x 0.9 x 0.9 x 0.9 x 0.9 = 100% x $(0.9)^6$ = **53%**.

Problem 2 – Suppose that each box is 1 kilometer on a side, and that the light is dimmed by1% through each box. If the total path is 20 kilometers, what will be the brightness of the light after it leaves this aerosol cloud to the nearest percent?

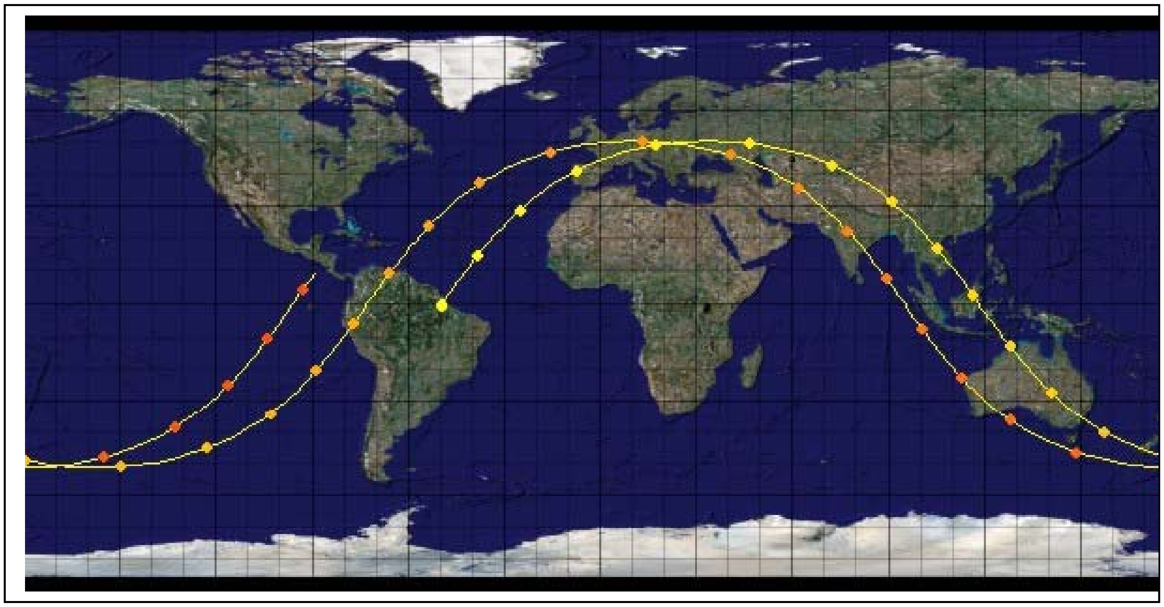
Answer: There are 20 boxes along the light ray so 100% x $(0.99)^{20}$ = **82%**.

Problem 3 – A light ray passes through 5 kilometers of ordinary air, which reduces the light by 0.5% per kilometer, and then passes through a dense cloud that reduces the light by 5% per kilometer. If the cloud is 3 kilometers long, to the nearest percentage what will be the light intensity when the light leaves the cloud?

Answer: 100% x $(0.995)^5$ x $(0.95)^3$ = 100% x 0.975 x 0.857 = **84%**

Visibility and dimming: Typical morning fogs can attenuate light by 50% but there is still enough light to read a book, it's just that the light passing through the fog is scattered, which means that you cannot see an object clearly if it is more than a few hundred meters away.

The Ground Track of the International Space Station



As the ISS orbits Earth every 90 minutes, Earth rotates 'underneath' the orbit of the ISS once every 24 hours. This means that each ISS orbit advances in longitude by a fixed amount in longitude. The figure above shows the ground track of the ISS for two orbits. Each square is 10 degrees on a side, with the Equator running horizontally across the middle of the diagram. The squares are shown at intervals of 5 minutes.

Problem 1 - If Earth rotates 360 degrees in 24 hours, by how many degrees does it shift during the orbit of the ISS?

Problem 2 – How many sunsets and sunrises will the SAGE-III instrument on the ISS observe each 24-hour day?

Problem 3 – How many orbits will it take before the ISS passes over the same spot on the Equator?

Problem 4 - During each orbit, the ISS will cross the Equator traveling north to south, then after ½ orbit (45 minutes) it will cross the Equator traveling south to north. How long will you wait before you see the ISS from the ground traveling north-to-south across the sky directly overhead?

Problem 1 - If Earth rotates 360 degrees in 24 hours, by how many degrees does it shift during the orbit if the ISS?

Answer: 360/24 = X/1.5 so **X = 22.5 degrees**.

Problem 2 – How many sunsets and sunrises will the SAGE-III instrument on the ISS observe each 24-hour day?

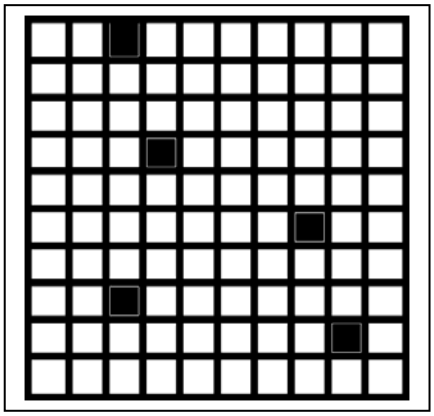
Answer: 24 h /1.5 h = **16 sunrises, and an equal number of sunsets** for a total of 32 events. A sunrise is followed by a sunset every 90/2 = 45 minutes!

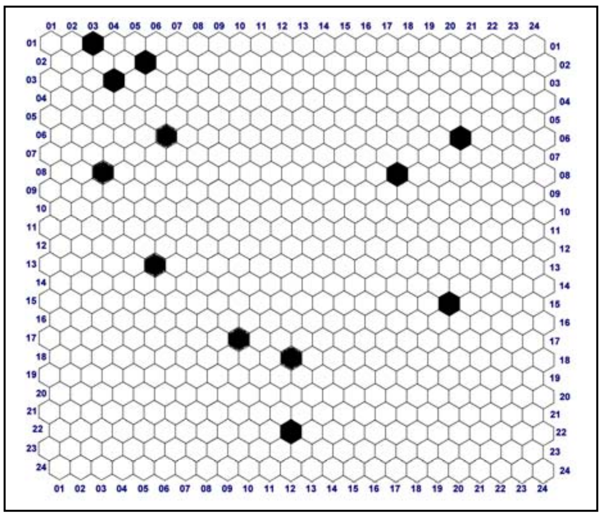
Problem 3 – How many orbits will it take before the ISS passes over the same spot on the Equator?

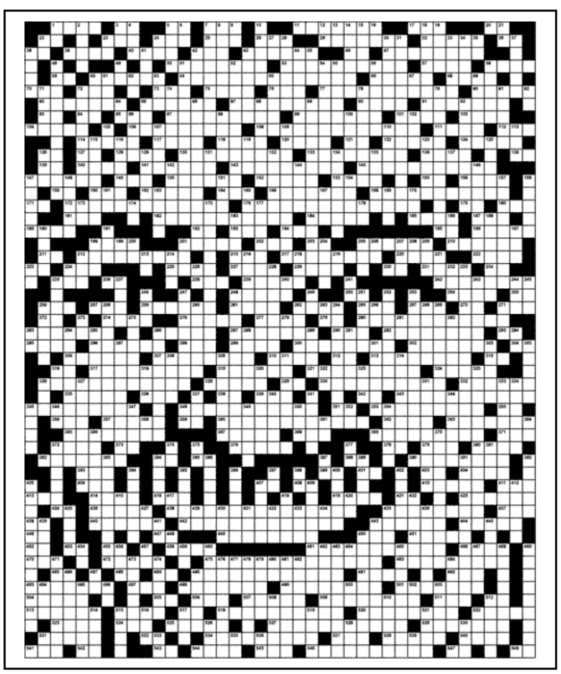
Answer: The orbit advances 22.5 degrees in longitude every orbit, so it will take 360/22.5 = 16 orbits or one full day to return to the same longitude. However, because the ISS crosses the equator twice every orbit, it actually takes only **8 orbits or 12 hours**.

Problem 4 - During each orbit, the ISS will cross the Equator traveling north to south, then after ½ orbit (45 minutes) it will cross the Equator traveling south to north. How long will you wait before you see the ISS from the ground traveling north-to-south across the sky directly overhead?

Answer: Every **16 orbits** as seen from the ground, the ISS is traveling in the same direction, so you will have to wait **24 hours**.







The Stratospheric Aerosol and Gas Experiment III (SAGE III), the sensor will be installed on the International Space Station (ISS) sometime in 2014.

Aerosols are small particles that are suspended in the air. Examples include fog and smog, but also includes dust, soot and ash particles. These particles can affect climate, and can also cause health problems such as emphysema, asthma or even lung cancer.

Instead of measuring the amount of aerosols or pollutants by percentage, scientists often use units such as parts-per-million. This exercise helps you work with these units.

Problem 1 - Suppose you are 15 years old. How many parts-per-hundred is this of one century?

Problem 2 – You have a bag of 200 blue marbles, 280 green marbles and 20 red marbles. How many parts-per-thousand are the red marbles compared to the whole?

Problem 3 – Which of the figures to the left shows a concentration of black spots equal to
A) 243000 parts-per-million?
B) 50000 parts-per-million?
C) 20833 parts per million?

Answer Key

Problem 1 - Suppose you are 15 years old. How many parts-per-hundred is this of one century?

Answer: 15 years is 15/100 = **15 pph** of 1 century.

Problem 2 – You have a bag of 200 blue marbles, 280 green marbles and 20 red marbles. How many parts-per-thousand are the red marbles compared to the whole?

Answer: The total number of marbles is 200+280+20 = 500. So the 20 red marbles is 20/500 = 4/100 or **4 pph** of the total number of marbles.

Problem 3 – Which of the figures to the left shows a concentration of black spots equal to A) 243000 parts-per-million? B) 50000 parts-per-million? C) 20833 parts per million?

Answer: First let's find out how many squares we have.

Top = 10x10 = 100 total and 5 black so 5/100 are black

Middle = 24 x 24 = 576 squares and there are 12 black so 12/576 are black

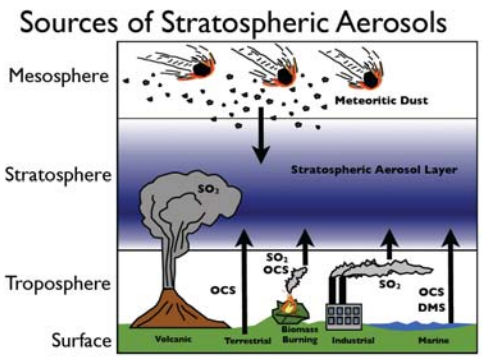
Bottom = 40 x 50 = 2000 squares and there are 486 black so 486/2000 are black

Lets use pph = parts-per-hundred
ppt = parts per thousand and
ppm = parts per million.

We can write the top as 5 pph = 50 ppt = 50,000 ppm
Middle as 12/576 = 0.0208333 = 2.0833 pph = 20.833 ppt = 20833 ppm
Bottom as 486/2000 = 0.243 = 24.3 pph = 243 ppt = 243000 ppm

So the answers are
A) Is the bottom figure
B) Is the top figure
C) Is the middle figure

Note: The concentration of carbon dioxide in our atmosphere has increased from 335 ppm to nearly 390 ppm since 1975!



(Credit: NOAA)

OCS = Carbonyl sulfide,
DMS = Dimethyl sulfide,
SO_2 = Sulfur dioxide,
CO2 = carbon dioxide.

SAGE-III is designed to measure the concentration of aerosols in the stratosphere, but where do these particles come from?

The figure to the left shows some of the common sources. Scientists are concerned about recent increases in stratospheric aerosols because they have an impact on climate change. In the stratosphere, miles above Earth's surface, aerosols can reflect sunlight back into space, which leads to a cooling influence at the ground. According to recent studies, this cooling effect may explain the changes in the pace of global warming measured since 2000.

According to recent measurements, carbonyl sulfide is produced at the following rates given in terms of millions of tons per year: Marine = 0.33 Mt/yr; Volcanism = 0.05 Mt/yr; Terrestrial = 0.02 Mt/yr; Biomass Burning =0.07 Mt/yr; Industrial= 0.33 Mt/yr.

Sulfur dioxide is produced at a rate of 10 Mt/yr from volcanos; Industrial = 146 Mt/yr; Biomass burning = 8 Mt/yr.

Problem 1 – What is the total rate of production of carbonyl sulfide by all sources?

Problem 2 – What is the percentage of each source of carbonyl sulfide compared to the total production rate?

Problem 3 – Draw a circle (pie) graph that illustrates the percentages of each carbonyl sulfide source compared to the total rate.

Problem 4 – Comparing carbonyl sulfide and sulfur dioxide, which of the two has the largest percentage contribution due to human activity?

OCS rates from C. Brühl1, J. Lelieveld1,3, P. J. Crutzen1, and H. Tost2 'The role of carbonyl sulphide as a source of stratospheric sulphate aerosol and its impact on climate', Atmos. Chem. Phys., 12, 1239-1253, 2012, http://www.atmos-chem-phys.net/12/1239/2012/acp-12-1239-2012.html'

According to recent measurements, carbonyl sulfide is produced at the following rates given in terms of millions of tons per year: Marine = 0.33 Mt/yr; Volcanism = 0.05 Mt/yr; Terrestrial = 0.02 Mt/yr; Biomass Burning =0.07 Mt/yr; Industrial= 0.33 Mt/yr. Sulfur dioxide is produced at a rate of 10 Mt/yr from volcanos; Industrial = 146 Mt/yr; Biomass burning = 8 Mt/yr.

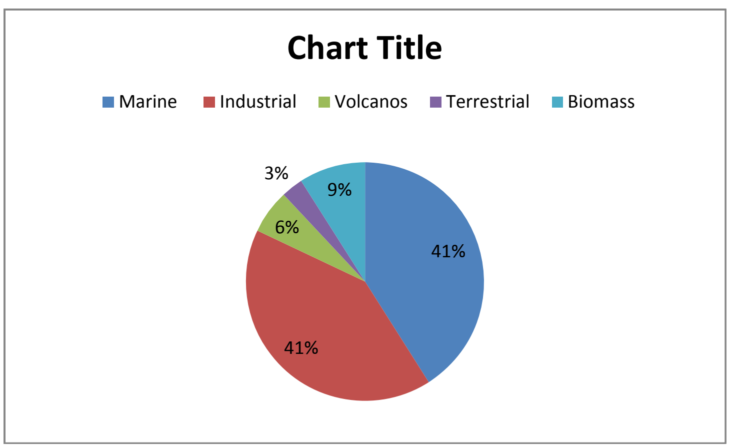
Problem 1 – What is the total rate of production of carbonyl sulfide by all sources?

Answer: 0.33 + 0.05 + 0.02 + 0.07 + 0.33 = **0.80 Mt/yr**.

Problem 2 – What is the percentage of each source of carbonyl sulfide compared to the total production rate?

Answer: **Marine: 41% Volcanism: 6% Terrestrial: 3% Biomass: 9% Industrial: 41%**

Problem 3 – Draw a circle (pie) graph that illustrates the percentages of each source compared to the total rate. Answer: See below.



Problem 4 – Comparing carbonyl sulfide and sulfur dioxide, which of the two has the largest percentage contribution due to human activity?

Answer: Total production = 164 Mt/yr. Percentages: Volcanism: 6%, Biomass burning: 5%; Industrial: 89%. **The production of sulfur dioxide by human activity is the highest: 89% vs 41%.**

Altitude (km)	Latitude		
	70N	30N	0
6	13.0	6.2	3.4
7	10.5	6.1	4.0
8	9.6	5.3	2.8
9	8.7	4.9	2.0
10	7.4	4.9	2.0
11	6.3	3.9	2.1
12	5.3	3.3	1.8
13	4.6	2.6	1.8
14	4.0	2.3	1.8
15	3.5	2.2	1.8
16	3.0	2.2	1.6
17	2.4	2.4	1.7
18	1.8	2.7	2.1
19	1.2	2.7	2.8
20	0.8	2.2	3.2
21	0.5	1.6	3.1
22	0.4	0.9	2.7
23	0.3	0.6	2.2
24	0.2	0.4	1.7
25	0.2	0.3	1.5

The SAGE-II experiment was flown on the Earth Radiation Budget Satellite beginning October 5, 1984 and ended its investigations on August 26, 2005. The SAGE-III instrument will be launched in 2014 and complete the observation program begun by the SAGE-I and SAGE-II instruments.

The SAGE instruments produce data tables like the one shown to the left. The table gives the altitude of the measurement, and subsequent columns give the extinction measured at different latitudes indicated in the top row.

The numbers indicate the aerosol extinction in units of 0.0001 km^{-1}. For example, at 6 kilometers altitude at a latitude of 30N, the extinction was 6.2 x 0.0001 = 0.00062 km^{-1}.

Problem 1 – At what location is the extinction A) the highest? B) the lowest?

Problem 2 – Above an altitude of 20 kilometers, what is the average extinction at each latitude?

Problem 3 – Graph the extinction data for 30N between an altitude of 6 and 14 km and draw a straight line through the data. What is the slope of this line, and what are the units for this slope?

Problem 4 – Write the equation of the line that you drew in Problem 3, and use it to estimate the extinction at an altitude of 19 kilometers. Is our mathematical model a good fit to the actual data?

The SAGE-II Lesson Plan for graphing data can be found at
http://science-edu.larc.nasa.gov/EDDOCS/Aerosols/aer_sci.html

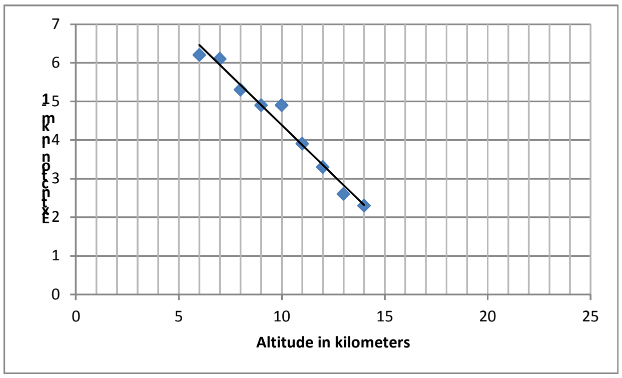
Problem 1 – At what location is the extinction A) the highest? B) the lowest?

Answer: A) **At 70N at an altitude of 6 km** above the ground where its value is 13.0 x 0.0001 = **0.0013** km^{-1}. B) **At an altitude of 24-25 km for 70N** where its value is 0.2 x 0.0001 = **0.00002 km^{-1}**.

Problem 2 – Above an altitude of 20 kilometers, what is the average extinction at each latitude?

Answer: 70N: (0.8+0.5+0.4+0.3+0.2+0.2)/6 = 0.4 or 0.4 x 0.0001 = **0.00004 km^{-1}**.
30N: (2.2+1.6+0.9+0.6+0.4+0.3)/6 = 1.0 or **0.0001 km^{-1}.**
0 N: (3.2+3.1+2.7+2.2+1.7+1.5)/6 = 2.4 or 2.4 x 0.0001 = **0.00024 km^{-1}.**

Problem 3 – Graph the extinction data for 30N between an altitude of 6 and 14 km and draw a straight line through the data. What is the slope of this line, and what are the units for this slope? Answer: See graph below. The slope is -0.52 and the units will be (km^{-1})/km = km^{-2} or 1/km^2



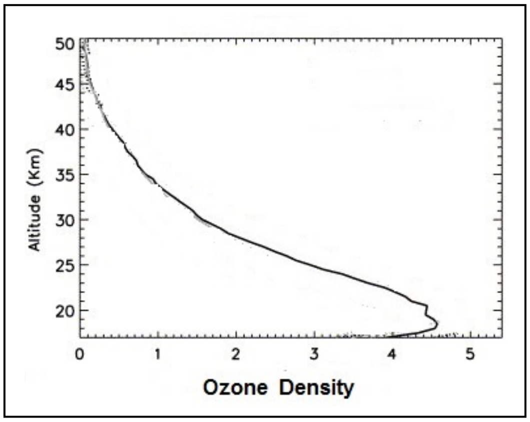
Alternate slope method using two-point formula and data table:
M = (y2-y1)/(x2-x1) so for the points at 6km and 14 km, M = (2.3-6.2)/(14 – 6) = -0.49 km^{-2}.

Students estimates should be close to M = -0.5 km^{-2}

Problem 4 – Write the equation of the line that you drew in Problem 3, and use it to estimate the extinction at an altitude of 19 kilometers. Is our mathematical model a good fit to the actual data?

Answer: The y-intercept for x=0 km is about +9.5, then **y = -0.5X + 9.5**. At an altitude of 19 km, the extinction would be y = -0.5(19)+9.5 = **0.0 km^{-1}**. The data show that the extinction is 0.00027 km^{-1} at this altitude, so we should not try to predict extinctions for altitudes outside the domain of our linear fit (6 to 14 km).

Measuring Stratospheric Ozone with SAGE-III



The Stratospheric Aerosol and Gas Experiment III (SAGE III), the sensor will be installed on the International Space Station (ISS) sometime in 2014. An earlier version of the SAGE-III instrument was flown in 2001 on the Russian Meteop-3M spacecraft. The new SAGE III will be using the sun and moon as light sources to measure how well the ozone layer is recovering and replenishing itself.

The ozone layer is located at an altitude of about 20 kilometers and blocks solar UV rays that would otherwise burn skin and cause cancer.

The data plot shows the density of ozone molecules at a range of altitudes as measured by the earlier SAGE-III instrument. The density of ozone molecules found in the stratosphere is presented in multiples of 1 trillion molecules per cubic centimeter.

Problem 1 – The ozone layer has the highest concentration of ozone molecules in the stratosphere. From the graph, over what altitude range does the concentration exceed 4 trillion molecules per cubic centimeter?

Problem 2 - If the density of the atmosphere in this region of the stratosphere is 400,000 trillion molecules, what fraction of the molecules are ozone if the ozone density is 4 trillion molecules/cm^3?

Problem 3 – For every million atmosphere molecules in the ozone layer, how many ozone molecules do you expect to find? (Scientists use the term parts-per-million to indicate this number.)

Data plot from
Lunar occultation with SCIAMACHY: First retrieval results
L.K. Amekudzi , A. Bracher, J. Meyer, A. Rozanov, H. Bovensmann, and J.P. Burrows
Advances in Space Research, Volume 36, Issue 5, 2005, Pages 906–914

Problem 1 – The ozone layer has the highest concentration of ozone molecules in the stratosphere. From the graph, over what altitude range does the concentration exceed 4 trillion molecules per cubic centimeter?

Answer: **Between 17 and 23 kilometers**.

Problem 2 - If the density of the atmosphere in this region of the stratosphere is 400,000 trillion molecules, what fraction of the molecules are ozone if the ozone density is 4 trillion molecules/cm^3?

Answer: 4 trillion ozone molecules/400,000 trillion atmosphere molecules = **1/100,000**

Problem 3 – For every million atmosphere molecules in the ozone layer, how many ozone molecules do you expect to find? (Scientists use the term parts-per-million to indicate this number.)

Answer: The ozone molecules are 1/100000 of the atmosphere molecules, so for every million atmosphere molecules, 1/100000 are ozone and so **10 ozone molecules** should be found for every 1 million atmosphere molecules.

This is written as 10 parts-per-million or 10 ppm.

The Sources and Sinks of Carbonyl Sulfide

Sources and sinks of carbonyl sulfide

Source	Rate (Mt/yr)
Open ocean	**+0.10**
Coastal ocean, salt marshes	**+0.20**
Anoxic soils	**+0.02**
Wetlands	**+0.03**
Volcanism	**+0.05**
Precipitation	**+0.13**
DMS oxidation	**+0.17**
Anthropogenic CS2 oxidation	**+0.21**
Natural CS2 oxidation	**+0.21**
Biomass burning	**+0.07**
Anthropogenic production	**+0.12**
Oxic soils	**-0.92**
Vegetation	**-0.56**
Reactions with OH	**-0.24**
Reactions with oxygen	**-0.02**
Photodissociation	**-0.05**

The important source of aerosols in the stratosphere (altitude from 11 to 50 km) is the formation of carbonyl sulfide (COS) droplets. Although volcanism injects millions of tons of SO_2 into the atmosphere every few years, scientists have found that during other times, COS is by far the biggest source of sulfur compounds in the stratosphere, leading to the production of sulphuric acid aerosols. Just 1 kilogram of COS is over 720 times more damaging than the same amount of carbon dioxide in altering global climate.

The table to the left gives the known sources and sinks of COS in terms of millions of tons per year (Mt/yr).

Problem 1 - In the table, sources of COS are indicated by positive rates, and systems that remove COS from the atmosphere, called sinks, are indicated by negative rates. What are the total rates for the sources and sinks, and what is the net change in atmospheric COS in megatons/year?

Problem 2 – What percentage of the sources for COS are related to human activity (anthropogenic) according to the data?

Problem 3 – 6.0×10^{23} molecules of COS has a mass of 60 grams, and 1 year equals 3.1×10^{7} seconds. What is the net change each year in the number of COS molecules in one cubic meter of the atmosphere if the volume of the atmosphere is about 4.23 billion cubic kilometers?

COS data from 'The role of carbonyl sulphide as a source of stratospheric sulphate aerosol and its impact on climate', C. Br¨uhl, J. Lelieveld, P. J. Crutzen, and H. Tost Journal of Atmospheric Chemistry and Physics, v.12, pp. 1239–1253, 2012 www.atmos-chem-phys.net/12/1239/2012/doi:10.5194/acp-12-1239-2012

Problem 1 - In the table, sources of COS are indicated by positive rates, and systems that remove COS from the atmosphere, called sinks, are indicated by negative rates. What are the total rates for the sources and sinks, and what is the net change in atmospheric COS in megatons/year?

Answer: **Total sources = +1.31 Mt/yr. Total sinks = -1.79 Mt/yr. The net change is the sum of the sources and sinks, or -0.48 Mt/yr**. This means that COS is being reduced in concentration each year at the current known rates.

Problem 2 – What percentage of the sources for COS are related to human activity (anthropogenic) according to the data?

Answer: Anthropogenic CS2 oxidation from wood burned in stoves, and direct production of this compound account for +0.21 and +0.12 Mt/year or a total of +0.33 Mt/yr. The total production is +1.31 Mt/yr, so anthropogenic sources are 100% x (0.33/1.31) = **25% of all sources.**

Problem 3 – 6.0×10^{23} molecules of COS has a mass of 60 grams, and 1 year equals 3.1×10^{7} seconds. What is the net change each year in the number of COS molecules in one cubic meter of the atmosphere if the volume of the atmosphere is about 4.23 billion cubic kilometers?

Answer: From Problem 1, the net change is a reduction by 0.48 Mt/yr.

Net reduction in tons = 480,000 tons / year x (1 year) = 480,000 tons.
Number of molecules:

$$480{,}000 \text{ tons} \times \frac{1000 \text{ kilograms}}{1 \text{ ton}} \times \frac{1000 \text{ grams}}{1 \text{ kilogram}} \times \frac{6.0 \times 10^{23} \text{ molecules}}{60 \text{ grams}} = 4.8 \times 10^{33} \text{ molecules}$$

Volume of atmosphere in cubic meters

$$= 4.23 \times 10^{9} \text{ km}^3 \times \frac{1.0 \times 10^{9} \text{ m}^3}{1 \text{ km}^3} = 4.23 \times 10^{18} \text{ m}^3$$

So: $4.8 \times 10^{33} / 4.23 \times 10^{18} \text{ m}^3 =$ **1.1×10^{15} molecules <u>removed</u> each year.**

A Scale Model of Aerosol Sizes

Table of particle sizes

Type	Size
Atmospheric aerosol	0.015 microns
Volcanic aerosol	0.5 microns
Gasolene engine ash	20 nanometers
Diesel ash (small)	50 nanometers
Diesel ash (large)	0.4 microns
Smallpox virus	300 nanometers
E Coli bacterium	2.0 microns
Common cold virus	30 nanometers
Smog (small)	8 nanometers
Smog (large)	200 nanometers

An aerosol, or 'aero-solution', is a microscopic particle made from numerous atoms and molecules stuck together. They are usually produced from chemical reactions, and the burning of organic materials like wood and hydrocarbon fuels.

The table to the left shows some common aerosol sizes along with the sizes of other objects you may know about.

Problem 1 – if 1 micron=1000 nanometers, order the particles by increasing size.

Problem 2 - Create a scaled model showing the relative sizes of each type of particle so that 1 nanometer = 1 millimeter in your model.

Problem 3 – A red blood cell has a diameter of 10 microns. How many volcanic aerosol particles can you place side-by-side to span this diameter?

Problem 4 – How many atmospheric aerosol particles would span the width of an e. coli bacterium?

Problem 5 – Suppose that an aerosol particle were shaped like a cube. How many atmospheric aerosol particles could you fit inside the volume of a single large particle of smog?

Problem 1 – if 1 micron=1000 nanometers, order the particles by increasing size.

Type	Size (nanometers)	Scaled Size
Smog (small)	8	8 mm
Atmospheric aerosol	15	15 mm
Gasolene engine ash	20	20 mm
Common cold virus	30	30 mm
Diesel ash (small)	50	50 mm
Smog (large)	200	20 cm
Smallpox virus	300	30 cm
Diesel ash (large)	400	40 cm
Volcanic aerosol	500	50 cm
E Coli bacterium	2000	2 meters

Problem 2 - Create a scaled model showing the relative sizes of each type of particle so that 1 nanometer = 1 millimeter in your model. Answer: See table above. Students can draw circles for the objects less than 1 meter in diameter.

Problem 3 – A red blood cell has a diameter of 10 microns. How many volcanic aerosol particles can you place side-by-side to span this diameter?

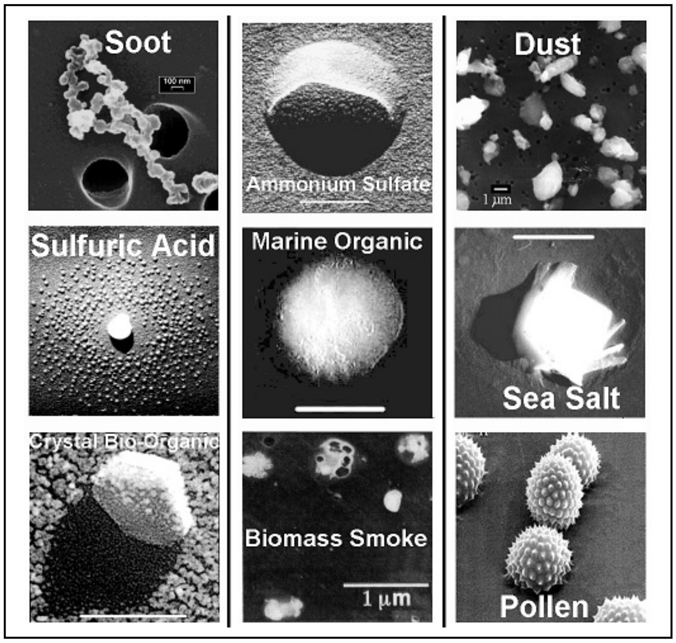
Answer: 10 microns / 0.5 microns = **20 volcanic aerosol particles**.

Problem 4 – How many atmospheric aerosol particles would span the width of an e. coli bacterium?

Answer: 2000 nanometers / 15 nanometers = **133 aerosol particles**.

Problem 5 – Suppose that an aerosol particle were shaped like a cube. How many atmospheric aerosol particles could you fit inside the volume of a single large particle of smog?

Answer: (Size of smog particle / size of aerosol)^3 = (200/15)^3 = **2370 particles**!



Aerosols are a complex ingredient to the atmosphere, which can result in both environmental and human health problems when their concentrations are too high. Smog and soot from burning fossil fuels or other organic materials can cause breathing difficulties, and perhaps even some forms of cancer.

The figure shows some common types of aerosol particles and their sizes. Most appear to be small spherical particles when seen under the microscope.

The formula for the volume of a sphere is given by $V = 4/3\ \pi\ R^3$. Also, 1 micron (μm)= 0.0001 centimeters, and 1 nanometer (nm) = 0.001 microns.

Problem 1 – A cubic meter container contains aerosols composed of soot produced from the combustion of diesel fuel. Each spherical aerosol particle has a density of 2 grams/cm^3. If the soot particle has a diameter of 20 nanometers, how much mass is in a single soot particle expressed in A) grams? B) micrograms?

Problem 2 – Scientists like to use two measurement units to indicate the density of aerosols in a sample of air: Particles/meter^3 or micrograms/meter^3 (μg/m^3). The average aerosol density for Los Angeles, California between October 2002 and September 2003 was 40 μg/m^3. 50% of these aerosols by mass were particulates with diameters of about 5 microns, while the remaining aerosols were mostly 500 nanometers in size. What was the density of the aerosols in particles/m^3 in each case, if the aerosols were small solid spheres with a density of 3.0 gm/cm^3?

Problem 1 – A cubic meter container contains aerosols composed of soot produced from the combustion of diesel fuel. Each spherical aerosol particle has a density of 2 grams/cm3. If the soot particle has a diameter of 20 nanometers, how much mass is in a single soot particle expressed in A) grams? B) micrograms?

Answer: First convert the particle diameter to centimeters:

$$D = 20 \text{ nanometers} \times \frac{1.0\times10^{-9} \text{ meters}}{1 \text{ nanometer}} \times \frac{100 \text{ cm}}{1 \text{ meter}} = 2.0\times10^{-6} \text{ cm.}$$

$V = 4/3\ \pi\ R^3$, and $R = D/2$, so $V = 1.333 \times 3.141 \times (1.0\times10^{-6} \text{ cm})^3 = 4.2 \times 10^{-18} \text{ cm}^3$.

Mass = density x volume, so

A) Mass = $2.0 \text{ gm/cm}^3 \times 4.2 \times 10^{-18} \text{ cm}^3 = 8.4\times10^{-18}$ grams.
B) Mass = 8.4×10^{-18} grams x (1 microgram/ 10^{-6} grams) = **8.4×10^{-12} micrograms**.

Problem 2 – Scientists like to use two measurement units to indicate the density of aerosols in a sample of air: Particles/meter^3 or micrograms/meter^3 (μg/m^3). The average aerosol density for Los Angeles, California between October 2002 and September 2003 was 40 μg/m^3. 50% of these aerosols by mass were particulates with diameters of about 5 microns, while the remaining aerosols were mostly 500 nanometers in size. What was the density of the aerosols in particles/m^3 in each case, if the aerosols were small solid spheres with a density of 3.0 gm/cm^3?

Answer: Aerosol masses:

5 micron case: $R = 2.5\times10^{-4}$ cm then $V = 4/3\ \pi\ (2.5\times10^{-4}\text{cm})^3 = 6.5\times10^{-11} \text{ cm}^3$.
Mass = $3.0 \text{ gm/cm}^3 \times 6.5\times10^{-11} \text{ cm}^3 = 2.0\times10^{-10}$ gm.

500 nanometer case: $R = 5.0\times10^{-5}$ cm. $V = 4/3\ \pi\ (5.0\times10^{-5} \text{ cm})^3 = 5.2\times10^{-13} \text{ cm}^3$.
Mass = $3.0 \text{ gm/cm}^3 \times 5.2\times10^{-13} \text{ cm}^3 = 1.6\times10^{-12}$ gm.

Since 50% of the aerosols were in each category and the total density was 40 μg/m^3,

5-micron case: $$\text{number} = 20\ \mu\text{g/m}^3 \times \frac{1 \text{ gm}}{10^6\ \mu\text{g}} \times \frac{1 \text{ particle}}{2.0\times10^{-10} \text{ gm}} = \mathbf{10^5 \text{ particles/m}^3}$$

500 nm case: $$\text{number} = 20\ \mu\text{g/m}^3 \times \frac{1 \text{ gm}}{10^6\ \mu\text{g}} \times \frac{1 \text{ particle}}{1.6\times10^{-12} \text{ gm}} = \mathbf{1.2\times10^6 \text{ particles/m}^3}$$

So although there were an equal amount of particles by their total mass, the small particles were 120 times more numerous as individual particles in the air samples.

AQI	$PM_{2.5}$ ($\mu g/m^3$)	PM_{10} ($\mu g/m^3$)	Air Quality Descriptor
0–50	0.0–15.4	0–54	Good
51–100	15.5–40.4	55–154	Moderate
101–150	40.5–65.4	155–254	Unhealthy for Sensitive Groups
151–200	65.5–150.4	255–354	Unhealthy
201–300	150.5–250.4	355–424	Very unhealthy

Because of their impacts to health, the US Environmental Protection Agency monitors the level of aerosols in the atmosphere (troposphere) for two categories: Large aerosols (PM_{10}) with diameters near 10 microns, and small aerosols ($PM_{2.5}$) with diameters near 2.5 microns (μm). The Air Quality Index (AQI) relates the density of each aerosol type (measured in micrograms per cubic meter or $\mu g/m^3$) to health risk as shown in the table above.

Problem 1 - Suppose the two types of aerosol particles have a density of 2000 kg/m^3. Assuming that each particle is a perfect sphere, what are the average masses of each type of aerosol particle in kilograms?

Problem 2 – Based on your estimate of the aerosol particle masses in Problem 1, how many aerosol particles of each type would be present in a 1 cubic meter volume of air of the AQI was 150?

Problem 1 - Suppose the two types of aerosol particles have a density of 2000 kg/m^3. Assuming that each particle is a perfect sphere, what are the average masses of each type of aerosol particle in kilograms?

Answer: Volume = $4/3 \pi R^3$,

$PM_{2.5}$ aerosols: For R = 1.3 microns, R = 1.3×10^{-6} meters so
V = $1.333 \times 3.141 \times (1.3\times10^{-6} m)^3$
= $9.2\times10^{-18} m^3$.

Mass = density x volume, so

M = $2000 \times 9.2\times10^{-18}$
= **1.8×10^{-14} kilograms.**

PM_{10} aerosols: R = 5 microns so

V = $1.333 \times 3.141 \times (5.0\times10^{-6} m)^3$
= $5.2\times10^{-16} m^3$, then

Mass = $2000 \times 5.2\times10^{-16}$
= **1.0×10^{-12} kilograms.**

Problem 2 – Based on your estimate of the aerosol particle masses in Problem 1, how many aerosol particles of each type would be present in a 1 cubic meter volume of air of the AQI was 150?

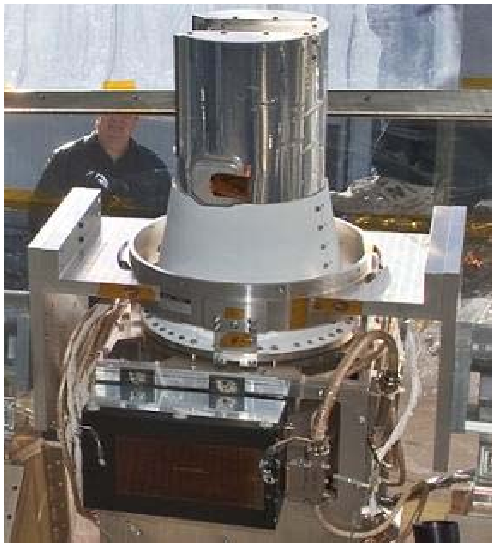
Answer: The table indicates that for an AQI of 150, the density of the PM_{10} particles would be 254 $\mu g/m^3$. Since the mass of such an aerosol particle is about 1.0×10^{-12} kilograms, we have

N = 2.54×10^{-6} $\mu g/m^3$ x (1 kg/1000 gm) x (1 particle/1.0×10^{-12} kg)
= **2500 particles/meter^3.**

The table indicates that for an AQI of 150, the density of the $PM_{2.5}$ particles would be 65.4 $\mu g/m^3$. For $PM_{2.5}$ aerosols the density is 65.4 mg/m^3. The average mass is 1.8×10^{-14} kg, so

N = 65.4×10^{-6} $\mu g/m^3$ x (1 kg/1000 gm) x (1 particle/1.8×10^{-14} kg)
= **3.6×10^6 particles/meter^3**.

Some Basic Properties of the SAGE-III Instrument



The Stratospheric Aerosol and Gas Experiment III (SAGE III), the sensor will be installed on the International Space Station (ISS) sometime in 2014. SAGE III will be using the sun and moon as light sources to measure how well the ozone layer is recovering and replenishing itself.

The 76-kilogram instrument will be carried in a Dragon supply module, and launched on a SpaceX, Falcon 9 rocket from NASA's Kennedy Space Center. After installation on the ISS and a 30-day check out, it will start taking data at a rate of about 200 megabytes every day. SAGE-III has a pointing accuracy of 0.025 degrees and requires 700 kilowatt hours of electricity every year.

Problem 1 - A single DVD can store 4.4 gigabytes of data. How many DVDs of data will the SAGE-III experiment generate during its 3-year mission onboard the ISS if 1 gigabyte = 1000 megabytes?

Problem 2 - If 1 kilogram equals 2.2 pounds, what is the weight of the SAGE-III instrument?

Problem 3 - A single 100-watt light bulb that is turned on for 10 hours will consume 1000 watt-hours of electricity, which is called 1 kilowatt-hour. How many watts will the SAGE-III instrument use if it is turned on for one full year?

Problem 4 - The SAGE-III instrument can change its direction of pointing by as little as 0.025 degrees. This is the same angle as the width of a dime (1 cm in diameter) if it were viewed at a distance of 23 meters. If two people stood one meter apart, how far away would they have to be standing from you to subtend the same angle? (Hint: Use proportions)

Problem 1 - A single DVD can store 4.4 gigabytes of data. How many DVDs of data will the SAGE-III experiment generate during its 3-year mission onboard the ISS if 1 gigabyte = 1000 megabytes?

Answer: The instrument produces 200 Mby of data every day. In 3 years it will produce 3 years x (365 days/year) x (200 Mby/day) = 219,000 Mby. Then:

219,000 Mby x (1 Gby/1000 Mby) x (1 DVD/4.4 Gby) = 49.8 DVDS. In terms of the total needed, we have **50 DVDs**.

Problem 2 - If 1 kilogram equals 2.2 pounds, what is the weight of the SAGE-III instrument?

Answer: 76 kg x (2.2 pounds/1 kg) = 167.2 pounds or **167 pounds**.

Problem 3 - A single 100-watt light bulb that is turned on for 10 hours will consume 1000 watt-hours of electricity, which is called 1 kilowatt-hour. How many watts will the SAGE-III instrument use if it is turned on for one full year?

Answer: The SAGE-III instrument uses 700 kWh of electricity. Then:

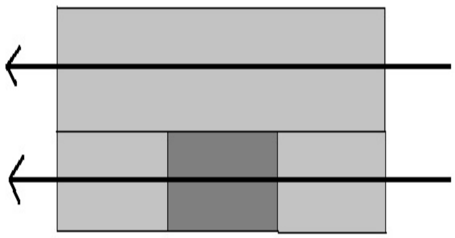
700 kWh x (1000 W/kW) x (1 year/ 365 days) x (1 day/ 24 hours) = **80 watts.**

Problem 4 - The SAGE-III instrument can change its direction of pointing by as little as 0.025 degrees. This is the same angle as the width of a dime (1 cm in diameter) if it were viewed at a distance of 23 meters. If two people stood one meter apart, how far away would they have to be standing from you to subtend the same angle?

Answer:

$$\frac{1\text{ cm}}{23\text{ meters}} = \frac{1\text{ meter}}{X}$$

convert all units to meters: $\frac{0.01\text{ m}}{23\text{ m}} = \frac{1\text{ m}}{X}$ then X = 23/0.01 = **2300 meters**



This figure shows two light rays from the Setting sun at two different altitudes above Earth. The top ray goes through a medium with fewer aerosols and a lower optical Depth.

The bottom ray passes closer to Earth's surface where a portion of the atmosphere is denser and has a different concentration of aerosols, making the dark region a higher optical depth.

The bottom atmosphere 'slab' is divided Into three equal sections, each 3,000 kilometers in length. The total width of the atmosphere being modeled in the example is 9,000 kilometers wide.

Opacity is a term used to describe the passage of light through a material, and is defined by the basic exponential formula

$$I = I_0 e^{-\tau}$$

Opaque materials have large positive values for τ, while transparent (translucent) materials have very low values for τ.

The quantity τ is also called the optical depth of a medium, and is proportional to both the length of the path taken by light through the medium, and the density of the absorbing particles. We can write this as

$$\tau = C N x$$

where x is in units of meters,
N is in units of particles/meter^3, and
C is a constant that is different for each kind of aerosol and has the units m^2/particle.

The Sage-III mission is designed to determine the product CN by looking at the extinction of sunlight along many different paths through the atmosphere given by x as shown in the figure to the left.

Problem 1 – Suppose that in the figure shown above, the SAGE-III instrument measures an extinction of light by 0.9991 in the top layer and 0.995 in the bottom layer. What are the total optical depths of each layer?

Problem 2 - From the information given in the figure, write two equations that relate the total optical depth to the contributions from each aerosol component and solve them to find the product CN for each aerosol.

Problem 3 – Suppose that the constants, C , for each aerosol type are known from models of Earth's atmosphere and that they are $C_A = 1.34\times10^{-17}$ km^2 /particle and $C_B = 7.5\times10^{-18}$ km^2 /particle, what are the densities of the two aerosols in A) particles/kilometer^3? B) particles/meter^3?

Problem 1 – Suppose that in the figure shown above, the SAGE-III instrument measures an extinction of light by 0.9994 in the top layer and 0.995 in the bottom layer. What are the total optical depths of each layer?

Answer: $I=I_0e^{-\tau}$ and

Top layer: $I/I_0 = 0.9994$ so $\ln(0.9994) = -\tau$ and so $\tau =$ **0.0006**

Bottom layer: $I/I_0 = 0.995$ so $\ln(0.995) = -\tau$ and so $\tau =$ **0.005**

Problem 2 - From the information given in the figure, write two equations that relate the total optical depth to the contributions from each aerosol component and solve them to find the product CN for each aerosol.

Answer:

Top Layer aerosol: $0.0006 = CN_A \times (9000\text{ km})$

so $CN_A = 0.0006/9000\text{km}$

$= 6.7\times10^{-8}\text{ km}^{-1}$

Bottom Layer aerosol:

$0.005 = CN_A\,(3{,}000\text{ km}) + CN_B\,(3{,}000\text{ km}) + CN_A\,(3{,}000\text{ km})$

$0.005 = 2\,(6.7\text{x}10\text{-}8\text{ km-}1)(3{,}000\text{ km}) + CN_B\,(3{,}000\text{ km})$

$0.005 = 0.0004 + CN_B\,(3{,}000\text{ km})$

$0.0046 = CN_B\,(3{,}000\text{ km})$

So $CN_B = 0.0046/3000\text{km}$

$= 1.5\times10^{-6}\text{ km}^{-1}$

So, **$CN_A = 6.7\times10^{-7}\text{ km}^{-1}$ and $CN_B = 1.5\times10^{-6}\text{ km}^{-1}$**

Problem 3 – Suppose that the constants, C , for each aerosol type are known from models of Earth's atmosphere and that they are $C_A = 1.34\times10^{-17}\text{ km}^2$ /particle and $C_B= 7.5\times10^{-18}\text{ km}^2$/particle, what are the densities of the two aerosols in A) particles/kilometer^3? And B) particles/meter^3?

Answer: Aerosol A: $(1.34\times10^{-17}\text{ km}^2/\text{particle})\ N_A = 6.7\times10^{-7}\text{ km}^{-1}$

so **$N_A = 5.0\times10^{10}$ particles/km^3**

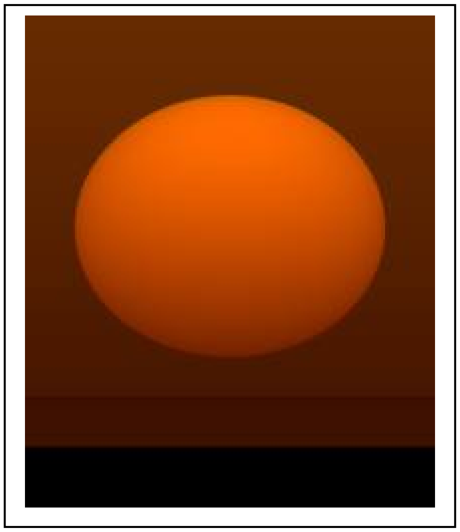
Aerosol B: $(7.5\times10^{-18}\text{ km}^2/\text{particle})\ N_B = 1.5\times10^{-6}\text{ km}^{-1}$

so $N_B =$ **2.0×10^{11} particles/km^3**

Aerosol A: 5.0×10^{10} particles/km^3 x (1km/1000meters)^3 = **50 particles/meter^3**

Aerosol B: 2.0×10^{11} particles/km^3 x (1 km/1000 meters)^3 = **200 particles/meter^3**

Three Mathematical Ways to Describe Light Extinction



When light passes through a medium it can lose some of its intensity. Scientists call this extinction. Depending on what properties they want to highlight in a calculation or a measurement, different ways of expressing extinction by a medium have arisen.

Opacity - Symbol τ : $I = I_0 e^{-\tau}$

Decibels - Symbol D : $I = I_0 10^{-D/10}$

Extinction Coefficient - Symbol C : $I = I_0 e^{-Cx}$

Problem 1 - If $e = 10^{0.434}$, and $10 = e^{2.3}$ write all three equations A) in base-10 B) in base-e.

Problem 2 – In base-10, what is the relationship between τ, D and C?

Problem 3 – In base-e, what is the relationship between τ, D and C?

Problem 4 - The SAGE-III instrument measures a 1 Decibel (1 dB) drop in the sun's brightness along a path through the atmosphere of x=2000 km. What is the optical depth and extinction coefficient for this region of the atmosphere?

Problem 1 - If $e = 10^{0.434}$ and $10 = e^{2.3}$ write all three equations A) in base-10 B) in base-e.

A) $I = I_0 e^{-\tau}$ $I = I0\ (10^{0.434})^{\tau}$ so $\mathbf{I = I_0\ 10^{-0.434\tau}}$

$I = I_0\ 10^{-D/10}$ unchanged so $\mathbf{I = I_0\ 10^{-D/10}}$

$I = I_0 e^{-Cx}$ $I = I_0\ (10^{0.434})^{(-Cx)}$ so $\mathbf{I = I_0\ 10^{-0.434Cx}}$

B) $\mathbf{I = I_0 e^{-\tau}}$ unchanged

$I = I_0\ (e^{2.3})^{-D/10}$ so $\mathbf{I = I_0 e^{-0.23D}}$

$\mathbf{I = I_0 e^{-Cx}}$ unchanged

Problem 2 – In base-10, what is the relationship between τ, D and C?

Answer: Just set the exponential factors equal to each other in Problem 1 A:

$-0.434\tau = -D/10 = -0.434Cx$ so after simplifying we get $\mathbf{\tau = 0.23D = Cx}$

Problem 3 – In base-e, what is the relationship between τ, D and C?

Answer: Set the exponential factors equal to each other in Problem 1 B: $\mathbf{\tau = 0.23D = Cx}$

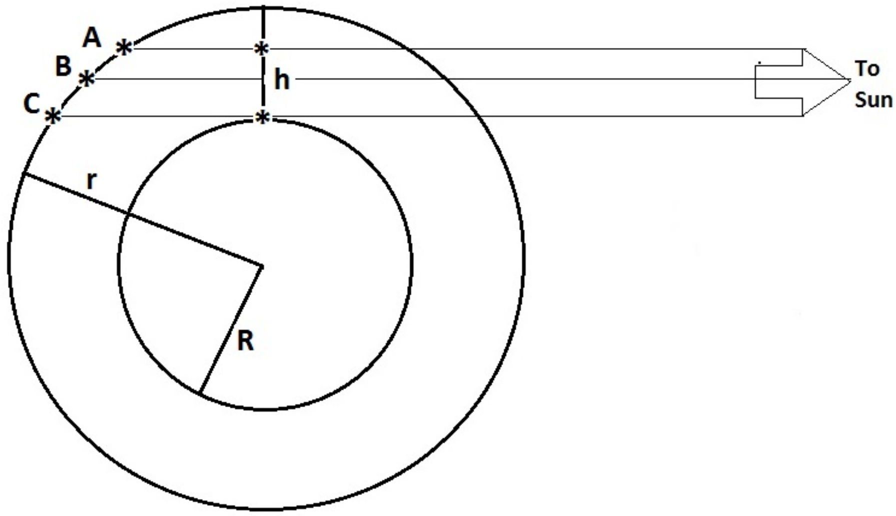
Problem 4 - The SAGE-III instrument measures a 1 Decibel (1 dB) drop in the sun's brightness along a path through the atmosphere of x=2000 km. What is the optical depth and extinction coefficient for this region of the atmosphere?

Answer: For 1 dB, and from Problem 2 (or 3!) we have

$\tau = 0.23$ D so
$\tau = 0.23 \times 1$ dB
$\tau =$ **0.23**.

For 1 dB and for x = 2000 km, we have
0.23 dB = Cx and so
0.23 = 2000C and so
C = 0.000115 km^{-1}.

Sunset and Sunrise Geometry from the ISS



The SAGE-III instrument on the International Space Station orbits Earth at a distance of r = 6,730 km from the center of Earth. The radius of Earth is R = 6,378 km. The time for one complete orbit is about 90 minutes. As it travels from Point A to C in the figure, the height of the sun, h, above the edge of Earth decreases to zero and astronauts observe a sunset. Each time SAGE-III observes a sunrise or sunset, its instruments measure the brightness of the sun. From this sun-dimming information scientists can determine the aerosol content of the stratosphere above an altitude of 10 km.

Problem 1 - Use the Pythagorean Theorem to determine the length of a chord for a given value of h for h < 100 km.

Problem 2 - Most of the sunlight extinction will happen within a height of h = 40 km. About how long is the total length of the chord near the sunset point in the orbit?

Problem 3 – About how many seconds will it take for the sunset to progress from h=40km to h=0 km?

Problem 1 - Use the Pythagorean Theorem to determine the length of a chord for a given value of h for h < 100 km.

Answer: $l^2 = r^2 - (R+h)^2$ so

Length = $2l = 2\,(r^2 - R^2 - 2Rh - h^2)^{1/2}$

Since R = 6378 and r = 6730 we have by simplifying

$L = 2\,(6730^2 - 6378^2 - 2(6378)h - h^2)^{1/2}$

Factor out 6730^2

Then $L = 2\,(6730)(1 - 0.90 - 0.00028h - (h/6730)^2)^{1/2}$

But h/6730 is never more than 100/6730 = 0.015 so we can ignore the h^2 term entirely!

So, $\mathbf{L = 13460(0.10 - 0.00028h)^{1/2}}$

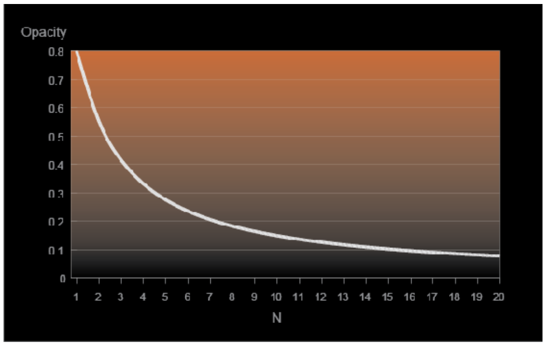
Problem 2 - Most of the sunlight extinction will happen within a height of h = 40 km. About how long is the total length of the chord near the sunset point in the orbit?

Answer: h = 0 at sunset so $L = 13460\,(0.10)^{1/2}$ = **4256 km**.

Problem 3 – About how many seconds will it take for the sunset to progress from h=40km to h=0 km?

Answer: The ISS will travel about 40 km in its orbit. Since the circumference of the circular orbit is C = 2 π (6730 km) = 42280 km, and this takes 90 minutes, the sunset range of 40 km will be traversed in

$$\frac{40\text{ km}}{42280\text{ km}} \times 90\text{ minutes} \times (60\text{ sec}/1\text{ minute}) = \mathbf{5\ seconds}.$$



This curve shows how much light is transmitted through a stack of 20 filters if each filter reduces the transmitted light by 20%.

After the first filter, N=1, the light intensity has been reduced from 100% to 80%.

After passing through the second filter, N=2, the light intensity is reduced to

100% x 0.8 x 0.8 = 100% x $(0.8)^2$ = 64%.

The reduction of light brightness as it passes through an aerosol cloud is not an additive process, but a multiplicative one. The figure to the left shows how light brightness changes as it passes through stacks of filters of varying lengths from 1 to 20. Note that the curve is not a straight line as it would be if the dimming were additive.

Scientists measure the dimming of light by the distance at which the light brightness is reduced by exactly 2.718 times or from 100% to 36.8%. Because aerosols are very dilute, the distance to which the light intensity falls to 36.8% is typically measured in kilometers. A basic mathematical function that describes light attenuation is given by

$$I(x) = 1.0\, e^{-x/L}$$

where L is the attenuation distance in kilometers and X is the length of the path through the aerosols.

Problem 1 – Suppose that for a particular cloud of aerosols the attenuation distance is 2 kilometers and the actual thickness of the cloud is 0.5 kilometers. What will be the light intensity to the nearest percent for a light ray passing through this cloud?

Problem 2 – For convenience, the attenuation distance L is usually reported as the extinction coefficient C = 1/L in units of km^{-1}. A) What is the equation for I(x) in terms of C? B) What is the value for I(x) in percent for a cloud with C = 0.20 km^{-1} and a cloud thickness of x=20 km?

Problem 3 – The SAGE-III instrument will measure the stratospheric aerosols, which have an average extinction of 1.2×10^{-4} km^{-1}. Light from the rising and setting sun will be measured along a path through the stratosphere that is about 3,000 km in length. What is the intensity of sunlight reaching the SAGE-III instrument?

Problem 1 – Suppose that for a particular cloud of aerosols the attenuation distance is 2 kilometers and the actual thickness of the cloud is 0.5 kilometers. What will be the light intensity to the nearest percent for a light ray passing through this cloud?

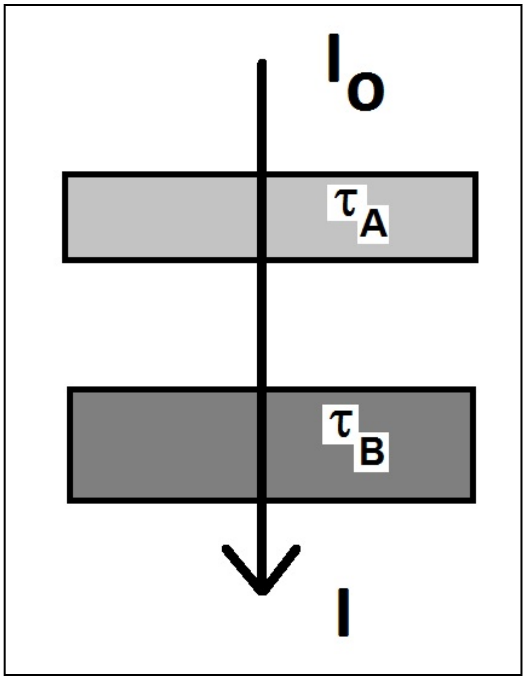
Answer: L = 2 km and x = 0.5 km so I = 100% x e(-0.5/2) = **78%**

Problem 2 – For convenience, the attenuation distance L is usually reported as the extinction coefficient C = 1/L in units of km^{-1}. A) What is the equation for I(x) in terms of C? B) What is the value for I(x) in percent for a cloud with C = 0.20 km^{-1} and a cloud thickness of x=20 km?

Answer: A) **$I(x) = 1.0\ e^{-Cx}$** B) $I(20) = 100\%\ e^{(-0.2x20)}$ = **1.8%**

Problem 3 – The SAGE-III instrument will measure the stratospheric aerosols, which have an average extinction of $1.2x10^{-4}\ km^{-1}$. Light from the rising and setting sun will be measured along a path through the stratosphere that is about 3,000 km in length. What is the intensity of sunlight reaching the SAGE-III instrument?

Answer: $I(3000km) = 100\%\ e^{(-0.00012)(3000)}$ = **69.7%**.



Opacity is a term used to describe the difficulty for light to travel through a medium. A rain cloud with high opacity appears almost black as it hangs in the sky above your head. On the other hand, frosted glass lets some light through and has low opacity.

Extinction is a term that describes how much the intensity of light has been reduced as it passes through a medium.

The terms opacity and extinction are often used interchangeably, but mathematically, scientists who work with light define them differently. One basic equation that relates them together is:

$$I = I_0 \, e^{-\tau}$$

$\mathbf{I_0}$ is the initial intensity of the light as it strikes the front surface of the medium. **I** is the intensity of the light after it has left the medium, and τ is the opacity of the medium that the light has passed through. A high opacity (opaque) medium is one for which τ is large, and this causes the light leaving the medium, I, to be reduced in intensity, which we call extinction.

When light passes through two different materials, one after the other, the final intensity is just

$$I = (I_0 \; e^{-A})e^{-B} \quad \text{or} \quad I = I_0 \, e^{-(A+B)}$$

where A is the opacity (τ) of the first medium and B is the opacity (τ) of the second medium.

Problem 1 – Show that for three different mediums, the total opacity of the materials is given by the formula

$$\tau = \tau_A + \tau_B + \tau_C$$

Problem 2 - A photographer is given three different filters with opacities of τ_A= 5.2, τ_B = 1.3 and τ_C = 0.5. He thinks that by placing the most opaque filter last that the light will be slightly brighter when it enters the camera. Do you think that this will work?

Problem 1 – Show that for three different mediums, the total opacity of the materials is given by the formula

$$\tau = \tau_a + \tau_b + \tau_c$$

Answer: From our example, and extended for three medii:

$$I = ((I0\ e^{-A})e^{-B})e^{-C})\ \text{ or }\ I = I0\ e^{-(A+B+C)}$$

But the total opacity is just

$$I = I_0 e^{-\tau},$$

so since $A = \tau_A$, $B=\tau_B$ and $C=\tau_C$ we have

$$I0e^{-\tau} = I0e^{-(\tau_A+\tau_B+\tau_C)}$$

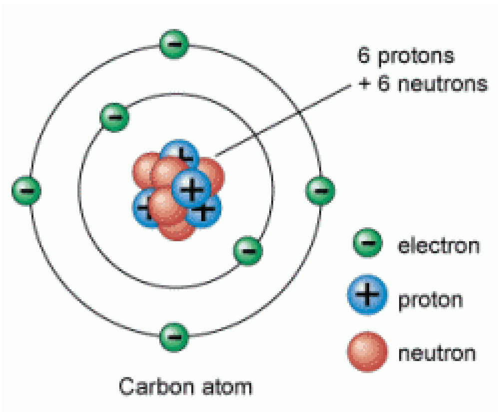
And so

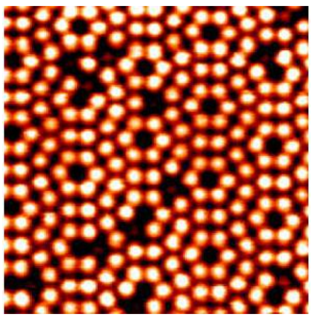
$$\tau = \tau_A + \tau_B + \tau_C$$

Problem 2 - A photographer is given three different filters with opacities of τ_A= 5.2, τ_B = 1.3 and τ_C = 0.5. He thinks that by placing the most opaque filter last that the light will be slightly brighter when it enters the camera. Do you think that this will work?

Answer: This will not work because the final opacity is the sum of the opacities of the three filters, and the order in which you add the filters does not matter in the sum. You will still get a final opacity of 5.2+1.3+0.5 = 7.0 and a drop in brightness by a factor of

$$I = I_0\ e^{-7} \quad \text{or}\ \ 0.00091$$





Atoms are the building blocks of matter. Every atom consists of a nucleus surrounded by one or more electrons. The nucleus contains an integer number of dense particles called neutrons and protons.

When you count up the number of electrons, protons and neutrons you only get integers. There are no such things in Nature as half an electron, or one-quarter of a neutron. This makes it very easy to organize atoms by just keeping a tally of the number of electrons, neutrons and protons they have.

Here are the basic relationships that describe all known atoms. We will use N as the number of neutrons, P as the number of protons, and E as the number of electrons, and M is the mass of the atom.

$E = P$ for all neutral atoms

$N + P = M$

Problem 1 – The Van Allen Probes will be able to detect atoms of hydrogen, helium and oxygen with its 'HOPE' Mass Spectrometer. For oxygen, E=8 and N=8. How many protons are in an oxygen atom, and what is the atomic mass for an oxygen atom?

Problem 2 – Some of the oxygen atoms found in Nature have M=17, how many neutrons does this 'isotope' of oxygen have if E=8?

Problem 3 – Suppose you had two collections of oxygen atoms. The common oxygen atoms have N=P=8, and the less common isotope of oxygen has P=8 and N=9. How many atoms of each kind would you have to collect so that the number of protons and neutrons is the same for each collection?

Problem 1 – The Van Allen Probes will be able to detect atoms of hydrogen, helium and oxygen with its 'HOPE' Mass Spectrometer. For oxygen, E=8 and N=8. How many protons are in an oxygen atom, and what is the atomic mass for an oxygen atom?

Answer: E=8 so that also means that **P=8**. The atomic mass is just M=N+P = 8+8 so **M= 1**6.

Problem 2 – Some of the oxygen atoms found in Nature have M=17, how many neutrons does this 'isotope' of oxygen have if E=8?

Answer: M = N + P, but P=E so M = N + E, then 17 = N + 8, and so **N = 9**.

Problem 3 – Suppose you had two collections of oxygen atoms. The common oxygen atoms have N=P=8, and the less common isotope of oxygen has P=8 and N=9. How many atoms of each kind would you have to collect so that the number of neutrons is the same for each collection?

Answer: If you had 1 atoms of each kind then A would have 1xN = 8 neutrons. B would have 1xN = 9 neutrons. The formulas for A atoms in Group A and B atoms in Group B are
N = Ax8 and N = Bx9. We need to find a single number so that Ax8 = Bx9.

Since 72 = 9x8 and 8x9, we see that one possibility is that we need **9 atoms from Group A and 8 atoms from Group B.**

The next-largest possibility is 144 (2x72).

A modern home computer has a hard drive whose capacity is typically about 500 gigabytes. A song that you download typically has a size of about 10 megabytes, and if you have a fast internet connection you can usually download at a rate of about 1 megabytes/sec. At that rate it takes about 10 seconds to download a song, and if your entire hard drive were available to store songs on your playlist, you could accommodate about 50000 songs!

The Van Allen Probes spacecraft instruments will be generating data that has to first be stored onboard the spacecraft, then at a specific time in the orbit, transmitted to Earth before the next round of data has to be stored on top of the old data.

The spacecraft engineers have worked with the scientists to determine how often the scientists want to store their measurements on each satellite. The average rate is 9,000 bytes/second. Each satellite will download the data once every orbit when the satellite is closest to Earth (perigee). The ground station has a busy schedule working with other satellites so each of the two Van Allen Probes satellites will only have 10 minutes every orbit to download all of its stored data. The orbit period is 9 hours.

Problem 1 – How many megabytes of data do both of the spacecraft collect after one orbit?

Problem 2 – At what rate in bytes/second will the data have to be downloaded to the ground station?

Problem 3 – The mission is being supported by NASA to last two years during its first operations cycle. How much data will both of the Van Allen Probes satellites accumulate during its first 2-year period in :

A) DVD disks (1 DVD = 4 gigabytes)

B) Songs (1 song = 10 megabytes).

Problem 1 – How many megabytes of data do both of the spacecraft collect after one orbit?

Answer: Time = 9 hours x (3600 seconds / 1 hr) = 32,400 seconds.
Total data = 9,000 bytes/sec x 32400 seconds = 291,600,000 bytes or 291.6 megabytes per spacecraft and **583.2 megabytes**/orbit for two spacecraft combined.

Problem 2 – At what rate in bytes/second will the data have to be downloaded to the ground station?

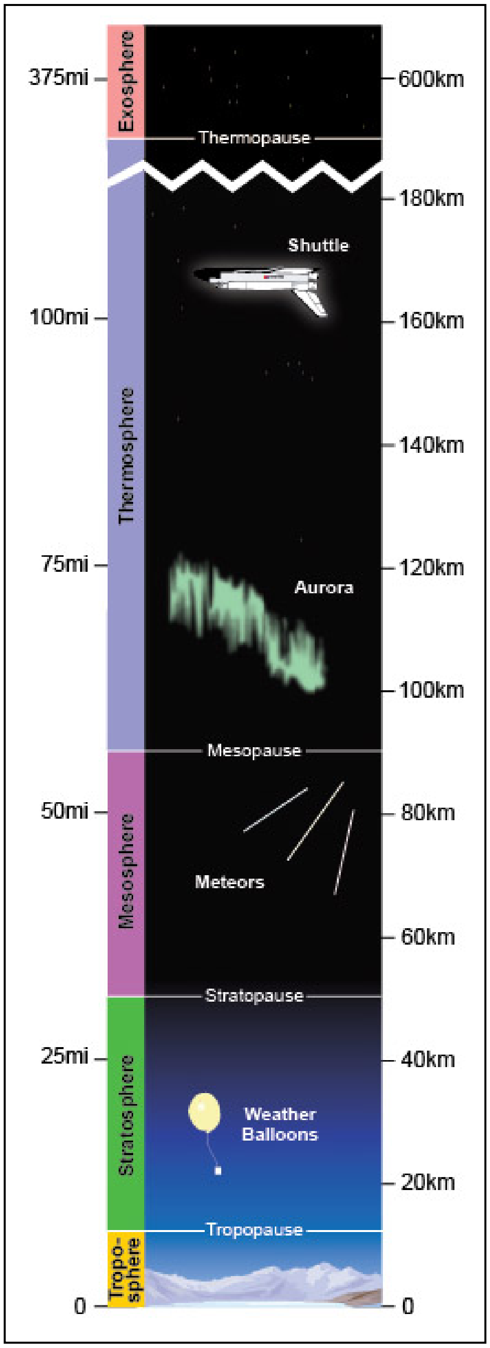
Answer: 583.2 megabytes have to be downloaded within 10 minutes or 600 seconds so the rate will be R = 583,200,000 bytes / 600 seconds = **972,000 bytes/second**.

Problem 3 – The mission is being supported by NASA to last two years during its first operations cycle. How much data will both of the Van Allen Probes satellites accumulate during its first 2-year period A) In DVD disks (1 DVD = 4 gigabytes) B) Songs (1 song = 10 megabytes).

Answer: A) 1 year = 365 days x 24 hours/day x 3600 seconds/hr = 31,536,000 seconds.
The data rate for two satellites is 18000 bytes/second, so in 2 years the satellites will accumulate 18000 x 31536000 = 567.6 billion bytes or 567.6 gigabytes.

A single DVD stores 4 gigabytes so you will need 567.6/4 = 141.9 or **141 DVDs**.

B) Number of songs = 567,600 megabytes / 10 megabytes = 56,760 or **56,760 song**s.



NASA's Van Allen Probes spacecraft will be exploring a region of space near Earth where the atmosphere of Earth is almost non-existent, but it can still be measured. Scientists use density as a way to show just how much gas there is in a cubic meter of space if you were to collect all of the gas in such a box.

On Earth we often talk about a rock being dense, and measure density in kilograms per cubic meter. For most rocks, their densities are 3000 kg/m^3, so if you had a pick-up truck that could hold 1 cubic meter of rock, it would hold 3000 kilograms of mass or 3 metric tons!

Gas is so dilute that, instead of writing density as kilograms/m^3 we use atoms (or molecules) per cubic meter. This tells us how many particles of gas are in a cubic meter. Because the number of particles is so large, we sometimes have to use scientific notation to write them!

Problem 1 – At sea level, the average density of air molecules (oxygen and nitrogen) is 2.5×10^{25} molecules/m^3. Write this number in: A) decimal form, B) by using 'million', 'billion', 'trillion' etc.

Problem 2 – Imagine a piece of paper 1000 kilometers on a side. How many dots would you have to place in each square that is 1 cm on a side in order to fill up the page with this many dots?

Problem 3 – The mesosphere is one of the highest levels of the atmosphere and at 70 km has a density of 0.00001 kg/m^3. This is 100,000 times lower than the density of the atmosphere at sea level. How many dots would you have in each cell of the paper you used in Problem 2?

Problem 4 - In the Van Allen belts, which is located above the exosphere, the density of particles is about 900 atoms/m^3. If you used the same 1000 km wide piece of paper, how far apart would the atoms of the Van Allen belt be on this scale?

Problem 1 – At sea level, the average density of air molecules (oxygen and nitrogen) is 2.5×10^{25} molecules/m^3. Write this number in: A) decimal form, B) by using 'million', 'billion', 'trillion' etc.

Answer: A) 25,000,000,000,000,000,000,000,000 molecules/m^3.
B) 250 trillion trillion molecules/m^3.

Problem 2 – Imagine a piece of paper 1000 kilometer on a side. How many dots would you have to place in each square that is 1 cm on a side in order to fill up the page with this many dots?

Answer: First we have to find out how many 1 cm^2 squares there are in a 1000 km x 1000 km piece of paper. Since 1 km = 1000 meters and 1 meter = 100 cm, each side of the paper measures 100,000,000 cm, (10^8) so the area of the paper is $(100,000,000)^2 = 10^{16}$ = 10 thousand trillion cm^2. There are 10 thousand trillion squares on the page, so the number of dots we need to put in each square is 250 trillion trillion / 10 thousand trillion = $2.5 \times 10^{25}/10^{16}$ = 2.5×10^9 or **2,500,000,000 dots!**

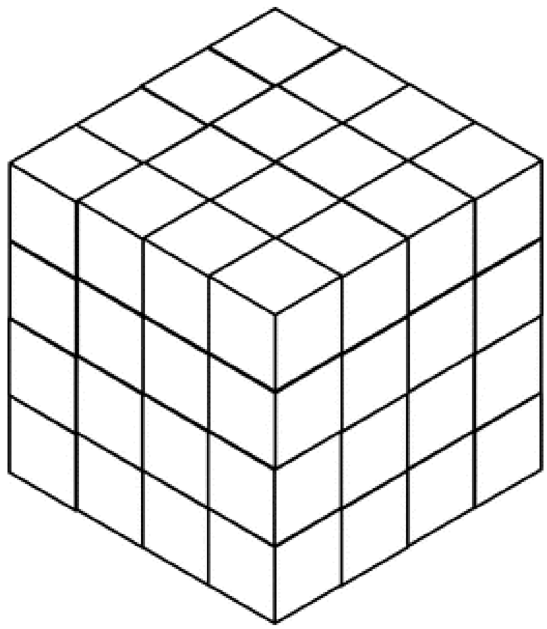
Problem 3 – The mesosphere is one of the highest levels of the atmosphere and at 70 km has a density of 0.00001 kg/m^3. This is 100,000 times lower than the density of the atmosphere at sea level. How many dots would you have in each cell of the paper you used in Problem 2?

Answer: If the density is 100,000 lower, then you only need a **total** of $2.5 \times 10^{25}/10^5$ = 2.5×10^{20} dots over the entire sheet. This means that each 1 cm^2 square will only need 100,000 times fewer dots or $2.5 \times 10^9/10^5 = 2.5 \times 10^4$ = **25,000 dots**.

Problem 4 - In the Van Allen belts, which is located above the exosphere, the density of particles is about 900 atoms/m^3. If you used the same 1000 km wide piece of paper, how far apart would the atoms of the Van Allen belt be on this scale?

Answer: You want 900 atoms scattered across a 1000 km x 1000 km piece of paper. Because 30x30 =900, that means that along each side of the paper we would mark off 30 atoms, and complete a grid with this spacing covering the 1000km x 1000km page to mark 900 points. Since 1000 km/30 = 33 1/3 kilometers, the atoms of the Van Allen belts on this scale would be about **33 kilometers apart!**

Exploring Gas Density in Space



Earth's atmosphere does not have a hard edge that says 'this is where space starts'. Instead, the density of the atmosphere gets smaller and smaller…but it never quite becomes zero!

Scientists measure gas density in space in terms of the number pf particles that you would find in any cubic meter of space. This is called the **Number Density, n,** and is measured in particles/m^3. Near the Earth, the gas densities are so large that we have to use scientific notation to write them. For instance, at Earth's surface, the number density of air is n= 2.5×10^{25} molecules/m^3. In the mesosphere at 70 km altitude, it is n= 2.5×10^{20} molecules/m^3.

Imagine the each particle sits at the center of its own cube. The number of these cubes, N, in <u>one cubic meter</u> is just the gas number density: **N = n.** In the figure above n = 64 if the length of each cube edge is 1 meter.

Problem 1 - Suppose you had 64 cubes arranged in a cube with a side length of one meter. How far apart would the centers of each cube be?

Problem 2 – Suppose that the large cube had an edge length of 1 meter and it contained 1 million identical cubes. What would the distance between the cube centers be?

Problem 3 – In the Van Allen belts, the average number density is about 900 particles/m^3. What is the average distance between the atoms in the Van Allen belts?

Problem 4 – In the mesosphere, the average number density is about 2.5×10^{20} particles/m^3. What is the average distance between the atoms in the mesosphere in microns, where 1 micron = 10^{-6} meters?

Problem 1 - Suppose you had 64 cubes arranged in a cube with a side length of one meter. How far apart would the centers of each cube be?

Answer: 64 cubes arranged in a cube means that you have 4 cubes along each side so that 4 x 4 x 4 = 64 cubes total. If the length of each side is 1 meter, then the center to center distance for the cubes is just 1 meter/4 = **25 centimeters**.

Problem 2 – Suppose that the large cube had an edge length of 1 meter and it contained 1 million identical cubes. What would the distance between the cube centers be?

Answer: 1 million = 100 x 100 x 100 so there are 100 cubes along each edge, and since each edge measures 1 meter, the separation between the cubes would be 1 meter/100 = 1 centimeter.

Problem 3 – In the Van Allen belts, the average number density is about 1000 particles/m^3. What is the average distance between the atoms in the Van Allen belts?

Answer: The number of cubes along each side is (1000)1/3 = 10 so the distance is d = 1 meter/10 = **10 centimeters.**

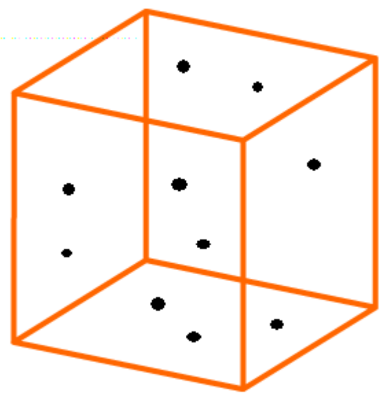
Problem 4 – In the mesosphere, the average number density is about 2.5×10^{20} particles/m^3. What is the average distance between the atoms in the mesosphere in microns, where 1 micron = 10^{-6} meters?

Answer: The number of cubes along each 1-meter side is $(2.5 \times 1020)^{1/3} = 6.3 \times 10^6$. The average separation between atoms is then 1 meter/$6.3 \times 10^6 = 1.6 \times 10^{-7}$ meters. Since 1 micron equals 10^{-6} meters so the atoms are separated by **0.16 microns**.

Note: The general formula for particle separation is

$$D = \frac{1 \text{ meter}}{n^{1/3}}$$

where n is the number density in particles/m^3



This cube has edges that are 1 meter long. Its volume is 1 cubic meter. There are 10 atoms inside this cube, so its density is **10 atoms per cubic meter**.

The NASA, Van Allen Probe spacecraft orbit Earth so high up that there is hardly any air at all. Scientists use the term 'density' to measure how many kilograms or gas there are in each cubic-meter of space, but when the density is too low, a unit like kg/m^3 is not very helpful. That's because instruments often measure individual atoms, and kg/m^3 is just too big a unit! It's like using 'kilometers' to measure the size of a bacterium.

A much more convenient unit is 'atoms/m^3'. This tells scientists immediately just how often their very sensitive instruments will be affected by their environment.

Problem 1 – The density of the Van Allen belts is typically about 900 atoms/m^3. How many atoms would you expect to find in a box that measures 15 centimeters on a side?

Problem 2 – The opening to one of the Van Allen Probes spacecraft instruments is about 10 cm^2. As the satellite completes one orbit, it travels about 70,000 km. How many atoms will pass through the spacecraft instrument window each orbit?

Problem 3 – How many kilometers would the spacecraft have to travel in order to encounter 9 million atoms?

Problem 1 – The density of the Van Allen belts is typically about 900 atoms/m^3. How many atoms would you expect to find in a box that measures 15 centimeters on a side?

Answer: 10 cm = 0.15 meters, so the volume of the box is 0.15 x 0.15 x 0.15 = 0.0034 meters3 .Then the number of atoms is density x volume = 900 x 0.0034 = **3 atoms**.

Problem 2 – The opening to one of the Van Allen Probes spacecraft instruments is about 10 cm^2. As the satellite completes one orbit, it travels about 70,000 km. How many atoms will pass through the spacecraft instrument window each orbit?

Answer: Convert the area into square meters, and the orbit length into meters to get

Area = 10 cm^2 x (1 m/100cm)x(1m/100cm)
= 0.001 m^2,

and 70,000 km x (1000 m/1 km) = 70,000,000 m.

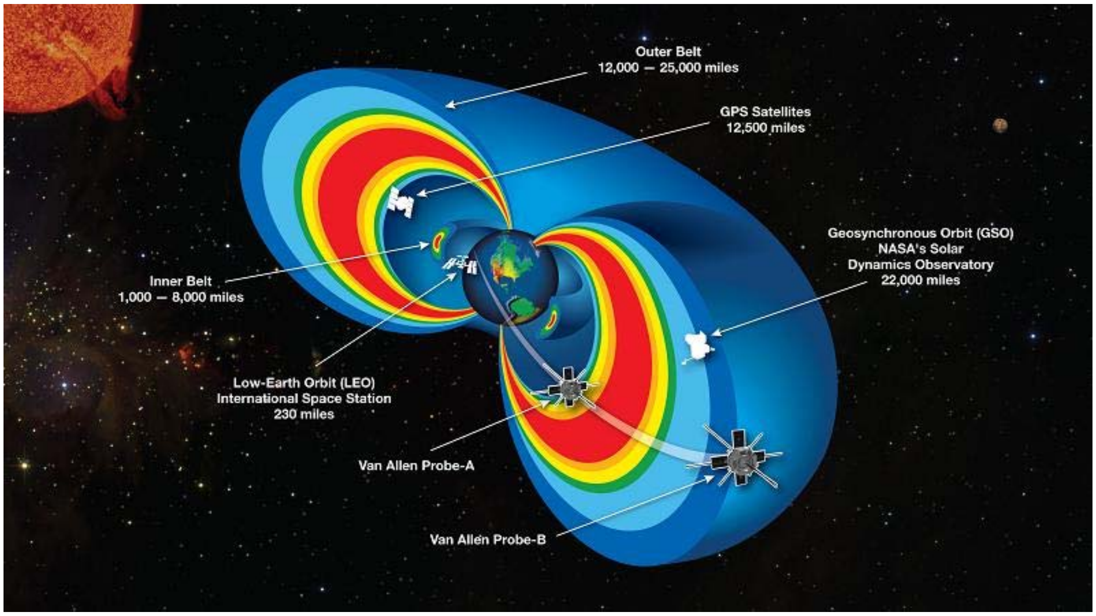
Then volume = area x length to get (0.001 m^2) x (70,000,000 m) = 70,000 m^3. Now multiply this 'swept out' volume by the density to get the number of atoms that passed through the window: 900 atoms/m^3 x 70,000 m^3 = **63 million atoms**.

Problem 3 – How many kilometers would the spacecraft have to travel in order to encounter 9 million atoms?

Answer: The window area is 0.001 m^2 and the density of atoms is 900 atoms/m^3.
You want 9 million atoms, so
9 million = 900 x area x length
9 million = 900 atoms/m^3 x 0.001 m^2 x Length

Length = 9 million / 0.9 = 10,000,000 meters! **This equals 10,000 kilometers**.

The Van Allen Probes – A Matter of Timing



NASA created a pair of spacecraft called the Van Allen Probes to study Earth's mysterious radiation belts in space. The two spacecraft travel along orbits that are very close to each other. It takes 9 hours for each spacecraft to make one complete loop along its orbit. As they travel, they make measurements and store this data in their computers. Once each day, the spacecraft download this data to computers on Earth so that scientists can begin their investigations.

Problem 1 – Suppose the spacecraft download their data when they are closest to Earth in their orbits, and that this happened on September 10, 2013 at 9:00 AM. After how many days will the spacecraft again download their data at the same time of day?

Problem 2 – The spacecraft transmit 300 megabytes of data during each download. If one DVD can store 4 gigabytes of data, how much data will <u>both</u> spacecraft produce every year if 1 gigabyte = 1000 megabytes?

Problem 1 – Suppose the spacecraft download their data when they are closest to Earth in their orbits, and that this happened on September 10, 2013 at 9:00 AM. After how many days will the spacecraft again download their data at the same time of day?

Answer: Construct two series
Orbit periods: 9, 18, 27, 36, 45, 54, 63, **72**, 81, 90, 99, 108, …
Days: 24, 48, **72**, 96, 120, …

Another way to do this is by finding the Least Common Multiple or LCM:

9 = 3 x 3   24 = 3 x 2 x 2 x 2   then 3x3 x 2 x 2 x 2 = **72**.

Find where they give the same elapsed hours, which occurs for 72 hours. 72 hours = 3 days, so every 3 days the spacecraft will download their data at exactly the same time of day. This will happen on **September 13, 2013 at 9:00 AM**.

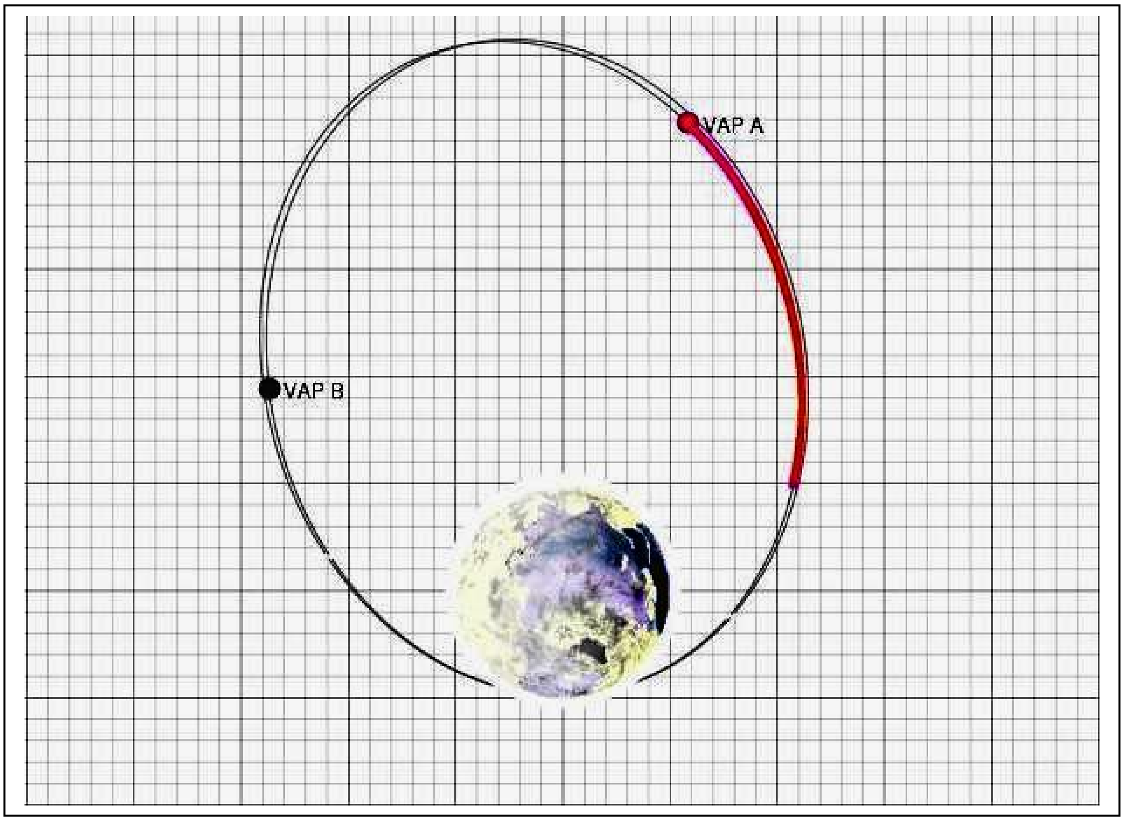
Problem 2 – The spacecraft transmit 300 megabytes of data during each download. If one DVD can store 4 gigabytes of data, how much data will both spacecraft produce every year if 1 gigabyte = 1000 megabytes?

300 Mby/1 download x (1 download/9 hours) x (24 hours/1 day) x (365 days/1 year)
= 292000 Mby for each spacecraft.

This equals 292 gigabytes of data. The number of DVDs required is

292 Gby x (1 DVD/4 Gyb) = 73 DVD disks each year for each spacecraft or **146 DVDs** for both spacecraft.

The Area of an Elliptical Orbit



The Van Allen Belt Probes orbit Earth in an elliptical orbit shown in the above drawing. Each big square on the grid measures 6,378 km across, which is the radius of Earth. Although we cannot find the area of this orbit using the formula for a circle, $A = \pi R^2$, we can make a very good estimate by 'counting squares'. This is a standard method for figuring out areas of irregular regions.

Problem 1 – What is the area of a single large square to the nearest 100,000 kilometers^2?

Problem 2 – What is the area of a single small square to the nearest 100,000 kilometer^2?

Problem 3 – How many large squares A) Fit only inside the elliptical figure, called inscribed squares? B) Fit inside the ellipse, but also include the perimeter of the ellipse, called the circumscribed squares?

Problem 4 - How many small squares A) inscribe the ellipse? B) circumscribe the ellipse?

Problem 5 – From the areas of the large and small squares, what is the average of the total inscribed and circumscribed squares for A) the large squares? B) the small squares?

Problem 1 – What is the area of a single large square to the nearest 100,000 kilometers^2?

Answer: 6378 km x 6378 km = 4067884 = **40.7 million km^2**

Problem 2 – What is the area of a single small square to the nearest 100,000 kilometer^2?

Answer = 1276 km x 1276 km = 1627155 = **1.6 million km^2**

Problem 3 – How many large squares A) Fit only inside the elliptical figure, called inscribed squares? B) Fit inside the ellipse, but also include the perimeter of the ellipse, called the circumscribed squares?

Answer: Note: The inscribed shape should look very blocky and not match the shape of the ellipse very well. Count all the squares inside the ellipse that do not include the perimeter of the ellipse, but whose corner may just touch the ellipse line. For the circumscribed figure, include all of the squares you identified inside the ellipse, but add to this count the squares that also include the perimeter of the ellipse.

Students should count A) **15 large squares** inside the ellipse, and B) a total of **32 squares** that form the circumscribed ellipse.

Problem 4 - How many small squares A) inscribe the ellipse? B) circumscribe the ellipse?

Answer: Students should count A) **576 small squares** inside the ellipse, and B) a total of **660 squares** that form the circumscribed ellipse.

Problem 5 – From the areas of the large and small squares, what is the average of the total inscribed and circumscribed squares for A) the large squares? B) the small squares?

Answer: The estimated area of the irregular figure is the sum of the inscribed and circumscribed areas divided by two.

A) (15+32)/2 = 23.5 then 23.5 x 40.7 million km^2 = **956.5 million km^2**
B) (576+660)/2 = 618, then 618 x 1.6 million km^2 = **988.8 million km^2**.

Because smaller squares were used for Problem 5 B, this is a more accurate area estimate than using the larger squares. The area of an ellipse is given by A = π a b, where for this problem a = the semi-major axis = 20,292 km, and b = semi-minor axis = 15,960 km, so A = 1020 million km2. This is only 3% larger than our small-square estimate, so using even smaller squares than in Problem 5AB will get an even better estimate closer to the exact value from the formula.

The Puzzling Van Allen Belts!

The Van Allen radiation belts are two donut-shaped regions of highly energetic particles trapped in the Earth's magnetic field. The ________________(1) is located just above our atmosphere and extends 4,000 miles into space. The ________________(2), extends from 8,000 to 26,000 miles. They are named for their discoverer; the late James A. Van Allen of the University of Iowa. The belts properties have affected both spaceflight and physics research for the past 50 years. The ____________(3) were designed to answer a number of questions about these harsh regions of space.

One year after their launch from Cape Canaveral on August 30, 2012, NASA's twin Van Allen Probe spacecraft have already changed how we understand the Van Allen radiation belts above our planet. Data from the probes have led to several big discoveries. The outer belt can actually be split into two separate belts. The Van Allen Probes discovered a new third radiation belt. A powerful ____________(4) acceleration event was already in progress, and the spacecraft saw the new belt form. The new belt was destroyed four weeks later by another solar storm.

On September 5, the Van Allen Probes detected ____________(5) in the belts. Scientists have known about these waves since the 1950s. When these radio waves are converted into sound waves they are called ____________(6). The recorded audio chorus became known as "the sounds of space". The sound file drew a great deal of interest around the world. Chorus is caused by ____________(7) waves in the belts. Scientists think they are produced as electrons are being accelerated to speeds that can harm spacecraft and ______________(8).

One question the mission hopes to answer is how the ____________(9) in the belts are being accelerated to nearly the speed of __________(10). Is it an external force, or is it happening within the belts themselves? The spacecraft have also allowed scientists to discover how protons from outside the solar system get into to the Earth's magnetic field and are trapped to form the van Allen belts themselves.

Word Bank:

38/105 = Plasma	4/15 = solar system	1/10 = particles
1/252 = Light	7/12 = inner belt	11/40 = outer belt
31/24 = Van Allen Probes	2/5 = electron	38/15 = radio waves
13/12 = Spacecraft	13/60 = chorus	131/231 = astronauts
-1/252 = Radiation	28/15 = proton	

Mathematical Clues:

1) 1/3 + 1/2 - 1/6

2) 2/5 + 3/8 – 1/2

3) 1/2 + 5/8 + 1/6

4) 2(1/5 – 1/6) + 1/3

5) 3(1/3 + 2/5) – 2(1/6 – 1/3)

6) 1/2 - 1/8 +1/4 – 1/5

7) 1/3 – 2/5 + 3/7

8) 1/11 + 1/7 + 1/3

9) 1/6 + 1/3 – 2/5

10) (2/7-1/3) x (1/4-1/3)

Answer Key

1) (4 + 6 – 3)/12 = 7/12 **word = inner belt**

2) (16+15-20)/40 = 11/40 **word = outer belt**

3) (12+15+4)/24 = 31/24 **word = Van Allen Probes**

4) 2(1/30)+5/15 = 6/15 = 2/5 **word = electron**

5) 11/5 + 2/6 = (66+10)/30 = 38/15 **word = radio waves**

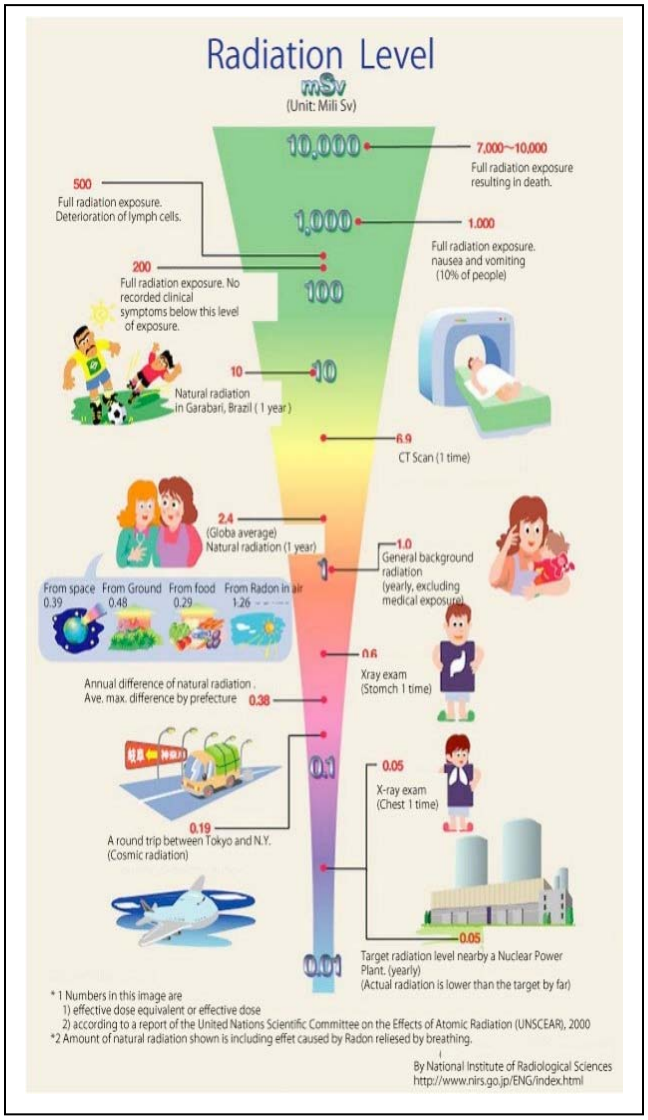
6) (30-20+15-12)/60 = 13/60 **word = chorus**

7) (35-42+45)/105 = 38/105 **word = plasma**

8) (21+33+77)/231 = 131/231 **word = astronauts**

9) (5+10-12)/30 = 1/10 **word = particles**

10) (-1/21)(-1/12) = + 1/252 **word = light**



The Van Allen belts are also called 'radiation belts' because they are filled with very high energy electrons and protons. Although they would be totally invisible if you looked at them, the particles would penetrate your spacecraft, spacesuits and skin and cause radiation sickness or even death.

Since they were discovered in 1958, scientists have sent many spacecraft into this region of space to study its origins and radiation. The current Van Allen Probe spacecraft are the first spacecraft to intentionally operate where the belts are the most intense.

Although scientists have many precise ways to measure radiation levels, for humans and spacecraft, the Sievert (Sv) and the Gray (Gy) are the two most common. Gray is the amount of energy deposited in the material, while Sievert is the amount of energy and damage this radiation does to organic tissue.
1 Gy = 1 Sv multiplied by the damage it does.

<u>For x-rays and gamma-rays</u>, 1 Gy = 1.0 Sv,

<u>For electrons, </u>1 Gy = 1.0 Sv

<u>For protons</u>, 1 Gy = 2.0 Sv

Problem 1 – A human living on the surface of Earth receives a total dose of 4 milliSv each year (365 days) from the natural environment, food, and other sources for which there is little control. What is the radiation dose rate in A) microSv/day? B) microSv/hour?

Problem 2 - The radiation dose rate on the International Space Station is 1 milliSv/day. How many days worth of natural background radiation does this dose rate equal on the ground?

Problem 3 – In the Van Allen belts, the average radiation dose rate for a satellite is about 50 Gray per year. If a human astronaut had the same shielding as a satellite, and required 1 hour to travel through the Van Allen belts, what would be their total dose at the end of the trip if 1 Gray = 1 Sievert?

Problem 4 – Satellites are eventually damaged by the radiation effects they accumulate over their lifetimes. A satellite that accumulates 1000 Grays of radiation is usually at the end of its reliable lifespan. How long would such a satellite last in the Van Allen belts if its annual dose rate is 50 Gray per year for a lightly-shielded satellite?

Problem 1 – A human living on the surface of Earth receives a total dose of 4 milliSv each year (365 days) from the natural environment, food, and other sources for which there is little control. What is the radiation dose rate in A) microSv/day? B) microSv/hour?

Answer: A) 4 milli Sv / 365 days = 0.010 milli Sv/day or **10 microSv/day**.
B) 10 microSv/day x (1 day/24 hrs) = **0.4 microSv/hr**.

Problem 2 - The radiation dose rate on the International Space Station is 1 milliSv/day. How many days worth of natural background radiation does this dose rate equal on the ground?

Answer: 1 day on the ground equals 10 microSv or 0.010 milliSv. So every day on the ISS equals about **100 days on the ground**!

Problem 3 – In the Van Allen belts, the average radiation dose for a satellite is about 50 Gray per year. If a human astronaut had the same shielding as a satellite, and required 1 hour to travel through the Van Allen belts, what would be their total dose at the end of the trip if 1 Gray = 1 Sievert?

Answer: 50 Gy x (1/365 days) x (1 day/24 hrs) x (1 Sv/1 Gy) = 0.006 Sv/hour.

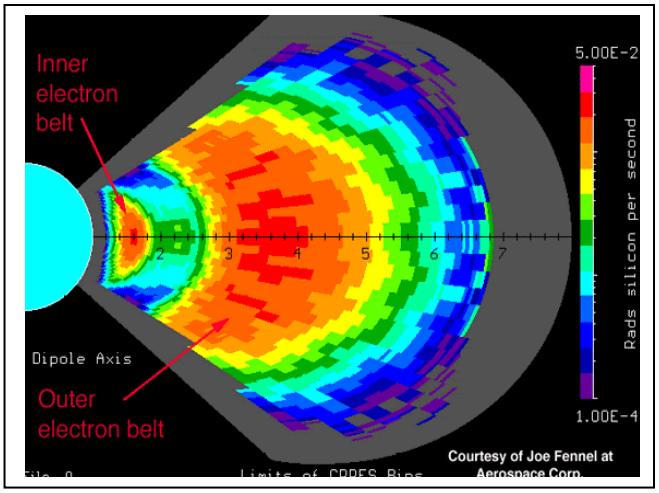
For 1 hour exposure in the spacecraft, this equals a total dose of **6 milliSv**.

Note, the average background radiation on the surface of Earth is about 4 milliSv in 1 year, so the astronaut in the Van Allen belt would accumulate a full year's normal dose in less than 1 hour! Additional shielding will reduce this considerably!

Problem 4 – Satellites are eventually damaged by the radiation effects they accumulate over their lifetimes. A satellite that accumulates 1000 Grays (100 kilorads) of radiation is usually at the end of its reliable lifespan. How long would such a satellite last in the Van Allen belts if its annual dose is 50 Grays per year for a lightly-shielded satellite?

Answer: T = 1000 Gy/50 Gy = **20 years**.

An Improved Model for Van Allen Belt Radiation Doses 44



Spacecraft engineers design spacecraft by considering the radiation environment, planned duration of the research program, and how much shielding is needed for the spacecraft to survive to the end of its mission.

In this problem we will work with a more realistic model for the orbit path and radiation belt dose rates, and perform a simple graphical integration by estimating areas under curves.

Problem 1 - The distance from the center of Earth to the spacecraft is given by the function

$$R(\theta) = 5.7 - \left[\frac{210}{100 - 55\cos\theta}\right]$$ in Earth radii units (where 1.0 = 6378 km)

$$T(\theta) = \frac{9}{2\pi}\left(\theta - 0.55\sin\theta\right)$$ hours

A better model of radiation dose rates into an unshielded silicon material G(R) in Grays/hour is given by the sixth-order function

$$G(R) = 0.136R^6 - 2.194R^5 + 13.89 R^4 - 43.73 R^3 + 71.78 R^2 - 57.95R + 18.15 \text{ Gys/hour}$$

where R is the distance in multiples of Earth's radius. What is the function G(T)?

Problem 2 - For most realistic situations, it is almost impossible to find an analytic solution to the required integral to exactly determine the area under the curve, which in this case is the total integrated dose to the satellite in one orbit. It is common to use numerical integration techniques to construct an approximate solution, or to estimate the area under the curve by 'counting squares' graphically.

A) Graph the function G(T) over one complete orbit over the domain T: [0 , 9 hours].

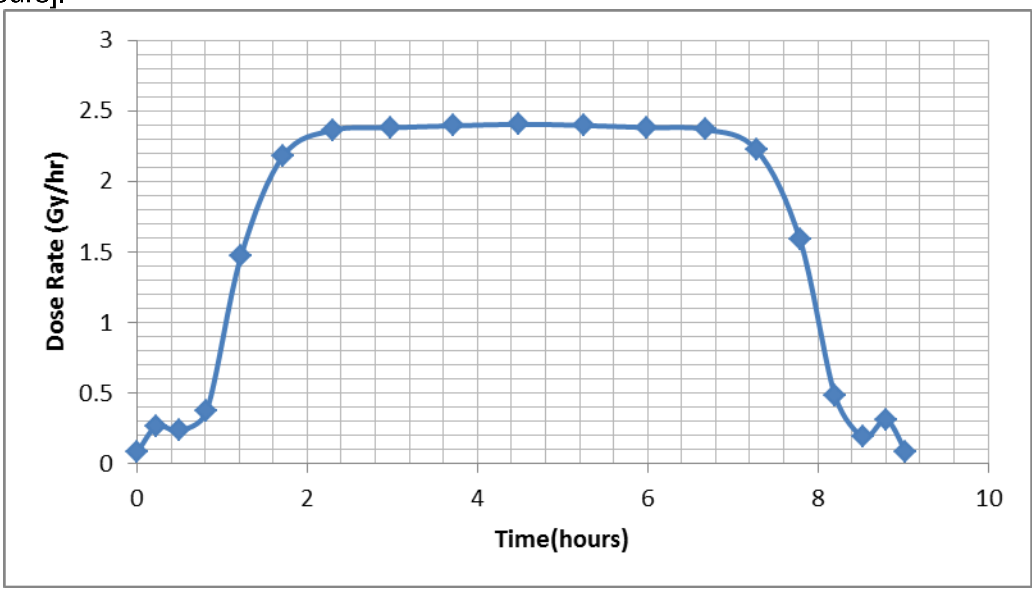
B) Estimate the geometric area under the curve using any method. What are the units of your estimated area rounded to 2 significant figure accuracy?

Problem 3 - A spacecraft will be at the end of its life after it has accumulated about 1000 Grays of radiation dose. A 1cm thickness of aluminum will reduce the radiation by 15 times. How many cm of shielding will be needed to reduce the total dose to 1000 Grays over 5 years?

Problem 1 - Answer. A students first approach is to replace 'R' in the function G(R) with the function R(θ), and then replace θ by inverting the equation for T(θ) so that we have G(R(θ(T))). For realistic functions, this is a depressing and frustrating approach. The student should realize that a perfectly reasonable alternative is to define a function table where the columns are Θ, T(θ), R(θ), and G(R). A sample of such a table is shown here:

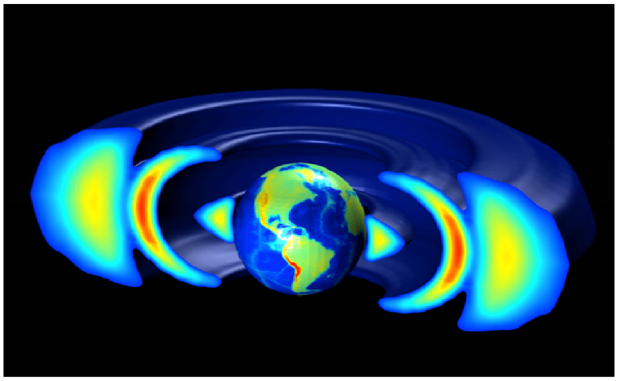
Θ in degrees	T: Time (hrs)	R in Re	G in Grays/hr
0	0.0	1.0	0.08
40	0.5	2.1	0.23
80	1.2	3.4	1.5
120	2.3	4.0	2.4
160	3.7	4.3	2.4
200	5.3	4.3	2.4
240	6.7	4.1	2.4
280	7.8	3.4	1.6
320	8.5	2.1	0.19

Problem 2 - A) Graph the dose rate function G(T) over one complete orbit over the domain T: [0 , 9 hours].



B) Answer: Graph: Total squares = 2x[½(2x4) + 4x4 + ½(4x20)+24x5.5)] = 384 squares
Table: 1 hr x (0.08+0.23+1.5+2.4+2.4+2.4+2.4+1.6+0.19) = **13.2** Grays/orbit
Area for 1 square = (0.4 hours) x (0.1 Grays/hr) = 0.04 Grays.
More accurate total dose is 0.04 x 384 squares= **15.4** Grays every 9 hour orbit.

Problem 3 - A spacecraft will be at the end of its life after it has accumulated about 1000 Grays of radiation dose. A 1cm thickness of aluminum will reduce the radiation by 15 times. How many cm of shielding will be needed to reduce the total dose to 1000 Grays over 5 years?
Answer: 15.4 Grays/10 hours x (8760 hours/1 year) x 5 years = 67500 Grays. Will need 67500/1000 = a factor of 68 reduction. 1 cm x (68/15) = **4.5 cm of shielding**


(Image credit: Yuri Shprits/UCLA)

The Van Allen Probes spacecraft travel in an elliptical orbit through the Van Allen belts. Soon after launch, they detected a third radiation belt shown by the middle crescent in the figure to the left.

In this problem, we predict when the spacecraft will encounter this third belt along their orbit, so that scientists can schedule observations of this new region.

The equation for the orbit of the spacecraft is given in Standard Form by

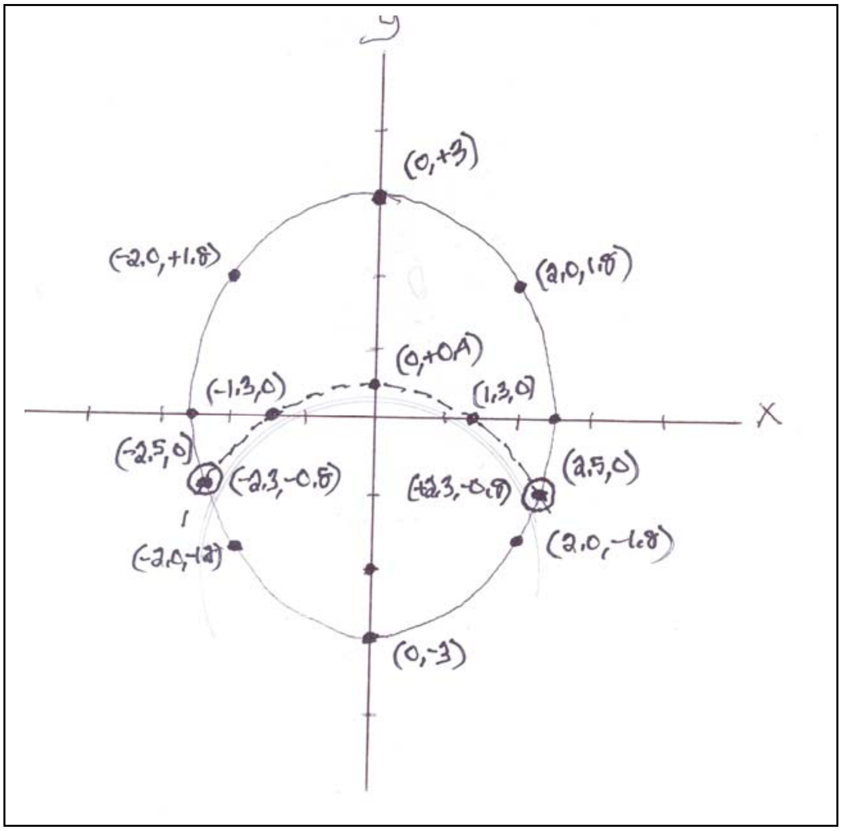
$$1 = \frac{x^2}{6.25} + \frac{y^2}{9.0}$$

The equation for the location of the third Van Allen belt projected into the plane of the elliptical orbit and concentric with the orbit focus centered on Earth (0,-2) is given by

$$5.8 = x^2 + (y+2)^2$$

Problem 1 – On the same coordinate plane, graph these two functions and estimate the coordinates of the intersection points to the nearest tenth.

Problem 2 – Using only algebra, find the coordinates of all intersection points between the orbit and the new belt region to the nearest tenth.



Problem 1 – On the same coordinate plane, graph these two functions and estimate the coordinates of the intersection points to the nearest tenth. Answer: See plot above (-2.3, -0.8), (+2.3, -0.8).

Problem 2 – Using only algebra, find the coordinates of all intersection points between the orbit and the new belt region to the nearest tenth. Answer: Expand out all terms and simplify

Equation of circle: $5.8 = x^2 + y^2 + 4y + 4.0$ so
$1.8 = x^2 + y^2 + 4y$ Equation of ellipse: $9.0 = 1.54x^2 + y^2$

Eliminate x^2 in equation of circle by using equation of ellipse solved for x^2.

$1.8 = \mathbf{(9.0 - y^2)/1.54} + y^2 + 4y$ $1.8 - 5.84 = -0.65y^2 + y^2 + 4y$ then $4.0 = 0.35y^2 - 4y$

Solve the quadratic equation in y for its two roots: $0 = 0.35y^2 - 4y - 4.0$ A = +0.35, b = -4.0 c = -4.0 then

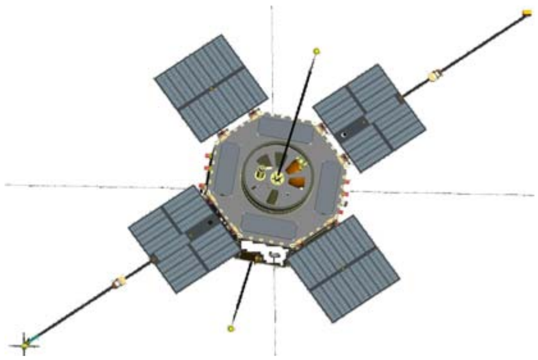
$Y = [4.0 \pm (16+5.6)^{1/2}]/0.70$ so Y1 = +12.3 Y2 = -0.86

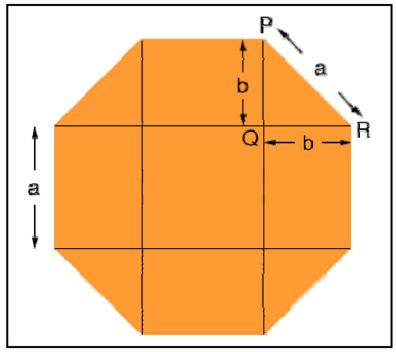
Then from the equation for the ellipse we get the x coordinates:

Y1 = +12.3 : $X1 = \pm ((9.0 - (12.3)^2)/1.54)^{1/2}$ x1 = <u>imaginary solution</u>! (+/- 9.6 i)

Y2 = -0.86 : $X2 = \pm ((9.0 - (0.86)^2)/1.54)^{1/2}$ = +/- 2.30

Rounded to the nearest tenth we have +/- 2.8 so the real coordinates are **(+2.3, -0.9) and (-2.3, -0.9)**





NASA's Van Allen Probes satellites are in the shape of an octagon with a thickness of 84 centimeters between the front octagonal face and the back octagonal face. An engineer needs to determine the total surface area of this 'octagonal prism' in order to create a mathematical model of how fast the satellite is warming up and cooling down as it orbits Earth.

Problem 1 – The figure on the right shows how the geometric area of an octagon can be broken up into rectangles, squares and triangles. What are the formulas for the areas of each of the squares, rectangles and triangles?

Problem 2 – What is the formula for the total area of the octagonal face in terms of the measurements for a and b?

Problem 3 – What is the formula for the total surface area of the spacecraft if h is the distance between the top and bottom octagonal faces?

Problem 4 - The engineer determines that $a + 2b = 1.7$ meters and $a = 0.7$ meters and $h = 0.84$ meters. What is the total surface area to the nearest tenth of a square meter of A) one octagonal face? B) the entire satellite?

Problem 1 – The figure on the right shows how the geometric area of an octagon can be broken up into rectangles, squares and triangles. What are the formulas for the areas of each of the squares, rectangles and triangles?

Answer: A(square) = a x a = a^2 A(rectangle) = a x b A(triangle) = ½ b x b

Problem 2 – What is the formula for the total area of the octagonal face in terms of the measurements for a and b?

Answer: A = 1 x A(square) + 4 x A(rectangle) + 4 x A(triangle)

$A = a^2 + 4ab + 2b^2$

Problem 3 – What is the formula for the total surface area of the spacecraft if h is the distance between the top and bottom octagonal faces?

Answer: There are two octagonal areas and 8 rectagonal side faces each with an area of a x h, so the total area of the spacecraft is

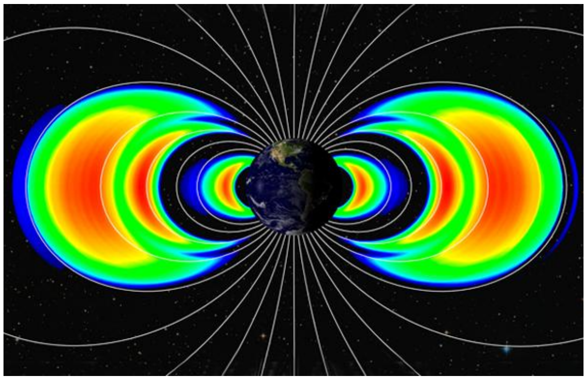
$A = 2(a^2 + 4ab + 2b^2) + 8ah$

Problem 4 - The engineer determines that a + 2b = 1.7 meters and a = 0.7 meters and h = 0.85 meters. What is the total surface area to the nearest tenth of a square meter of A) one octagonal face? B) the entire satellite?

Answer: a + 2b = 1.7meters and a = 0.7meters so b = 0.5 meters

A) $A = a^2 + 4ab + 2b^2$ so $A = (0.7)^2 + 4(0.7)(0.5) + 2(0.5)^2$ = **2.4 meter^2**.

B) $A = 2(2.4\ m^2) + 8(0.7)(0.85)$ = **9.6 meters^2**



The Van Allen Belt Probes recently discovered a third belt of particles orbiting Earth. Scientists describe the belts in terms of the kinds of particles they contain, mostly electrons and protons, but also in terms of their density and energy.

The density of a radiation belt is just the average number of particles you would find in a cubic-meter of its volume. For the Van Allen Belts, this is usually about 100 particles/meter^3. This is about 10 trillion times less than the density at the orbit of the International Space Station!

The kinetic energy of a particle is given by K.E. = ½ mV^2, where m is its mass in kilograms and V is its speed in meters/sec. The energy will then be in units of Joules. For example, a baseball thrown at a speed of 90 miles/hour (40 meters/sec) with a mass of 0.156 kg, has a kinetic energy of 125 Joules. A single electron with a mass of 9.1x10^{-31} kg, and traveling at a speed of 100,000 km/s (1.0x10^8 m/s) has a kinetic energy of only 4.6x10^{-15} Joules.

Because the energy units for the particles in the Van Allen belts are so small, scientists use another energy scale called the electron volt (eV). On this scale, 1 eV = 1.6x10^{-19} Joules. The electron we just described with an energy of 4.6x10^{-15} Joules is also equivalent to an energy of 28,500 eV. This is usually written as 28.5 kilo-eV or just 28.5 keV.

Problem 1 – The protons in the inner Van Allen belt have energies as high as 100 million eV (or 100 MeV). What is the average energy in Joules of a single proton if its mass is 1.6x10^{-27} kg?

Problem 2 - What is the average speed of these 100 MeV protons in km/s?

Problem 3 - Electrons carry so much energy that they travel at nearly the speed of light (c = 300,000 km/s). Another formula has to be used to relate their energy, mass and speed:

$$E = \frac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

In the outer Van Allen belts, electrons have an energies as high as 10 MeV. As a percentage of the speed of light, how fast are these electrons traveling at these energies?

Problem 1 – The protons in the inner Van Allen belt have energies as high as 100 million eV. What is the average energy in Joules of a single proton?

Answer: 10^8 eV x (1.6×10^{-19} Joules/ 1 eV) = **1.6×10^{-11} Joules**.

Problem 2 - What is the average speed of these 100 MeV protons in km/s if their mass is 1.6×10^{-27} kg?

Answer: KE = $1/2mV^2$ so $V^2 = 2\,(1.6 \times 10^{-11})/1.6 \times 10^{-27})$
so V = 1.4×10^8 m/s
or **140,000 km/s**.

Problem 3 - Electrons carry so much energy that they travel at nearly the speed of light (c=300,000 km/s). Another formula has to be used to relate their energy, mass and speed:

$$E = \frac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

In the outer Van Allen belts, electrons have an energies as high as 10 MeV. As a percentage of the speed of light, how fast are these electrons traveling at these energies?

Answer: m = 9.1×10^{-31} kg. We need to convert 10 MeV to Joules:

10×10^6 eV x (1.6×10^{-19} Joules/1 eV) = 1.6×10^{-12} Joules. Then

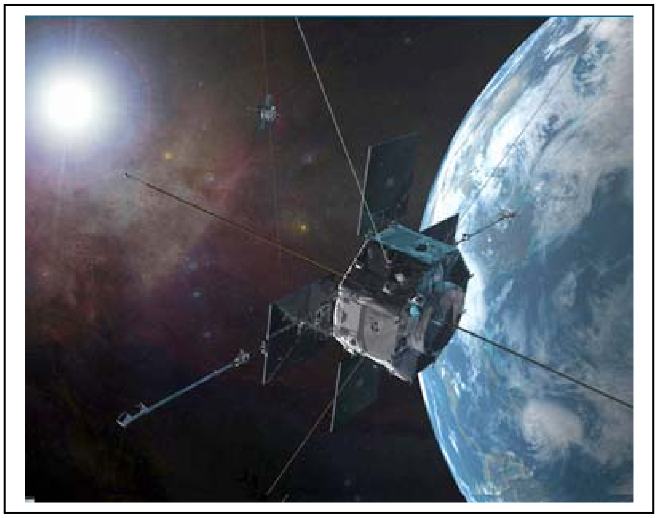
$1.6 \times 10^{-12} = (9.1 \times 10^{-31})(3.0 \times 10^8)^3/(1-(v/c)^2)^{1/2}$

$(1 - (v/c)^2)^{1/2} = 0.0512$

$(v/c)^2 = 1-(0.0512)^2$

v/c = 0.9987

so **v = 99.87%** of the speed of light or 299,606 km/sec.



Kepler's Third Law says that the cube of the satellite's orbit radius is directly proportional to the square of its orbit period. The proportionality constant, c, depends only on the mass of the planet that the satellite (or moon) is orbiting. For distances measured in meters, periods measured in seconds, and masses measured in kilograms, the proportionality constant for satellites orbiting Earth is just

$$C = 1.7 \times 10^{-12}\ M$$

Problem 1 – What is the equation described by the paragraph above?

Problem 2 – Solve the equation for M – the mass of Earth.

Problem 3 – For objects near Earth, it is convenient to measure their distances in multiples of Earth's radius so that 1.0 Re = 6,378 kilometers. It is also more convenient to use hours as a measure of orbit period. Re-write your equation so that it gives the mass of Earth in kilograms, in terms of the orbit period in hours, and the distance in multiples of Earth's radius.

Problem 4 – The Van Allen Probes spacecraft will be in orbits with a period of 9 hours, and a distance of 3.4 Re. What would you estimate as the mass of Earth given these spacecraft parameters?

Problem 1 – What is the equation described by the paragraph above?

Answer: After substituting for the constant, C, you get

$$\mathbf{R^3 = 1.7x10^{-12}\ M\ T^2}$$

Problem 2 – Solve the equation for M – the mass of Earth.

$$\mathbf{M = 5.9x10^{11}\ R^3/T^2}$$

Problem 3 – For objects near Earth, it is convenient to measure their distances in multiples of Earth's radius so that 1.0 Re = 6,378 kilometers. It is also more convenient to use hours as a measure of orbit period. Re-write your equation so that it gives the mass of Earth in kilograms, in terms of the orbit period in hours, and the distance in multiples of Earth's radius.

Answer:

$M = 5.9x10^{11}\ (6378000)^3/(3600)^2\ \ R^3/T^2$

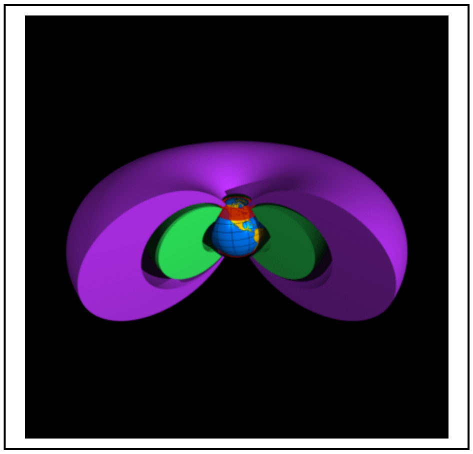
$\mathbf{M = 1.2 \times 10^{25}\ R^3/T^2}$

Problem 4 – The Van Allen Probes spacecraft will be in orbits with a period of 9 hours, and a distance of 3.4 Re. What would you estimate as the mass of Earth given these spacecraft parameters?

Answer : $M = 1.2 \times 10^{25}\ (3.4)^3/\ (9.0)^2$

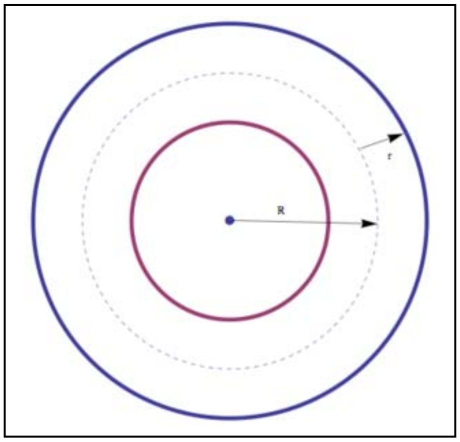
$\mathbf{M = 5.8 \times 10^{24}}$ **kilograms**

The actual value is 5.98×10^{24} kg.



The Van Allen belts were discovered in the late-1950s and resemble two donut-shaped clouds of protons (inner belt) and electrons (outer belt) with Earth at its center.

A donut is an example of a simple mathematical shape called a **torus** that is created by rotating a circle with a radius of r, through a circular path with a radius of R.



In terms of the variables r and R, the formula for the volume of a torus is given by the rather scary-looking formula:

$$V = 2\pi^2 R r^2$$

Problem 1 – What is the circumference of the circle with a radius of R?

Problem 2 – What is the area of a circle with a radius of r?

Problem 3 – If you dragged the area of the circle in Problem 2, along a distance equal to the circumference of the circle in Problem 2, what would be the formula for the volume that you swept out?

Problem 4 – If the Van Allen belts can be approximated by a torus with r = 16,000 km, and R = 26,000 km, to two significant figures, what is the total volume of the Van Allen belts in cubic kilometers?

Problem 5 – To two significant figures, how many spherical Earths can you fit in this volume if r = 6378 km?

Problem 1 – What is the circumference of the circle with a radius of R?

Answer: $C = 2 \pi R$

Problem 2 – What is the area of a circle with a radius of r?

Answer: $A = \pi r^2$

Problem 3 – If you dragged the area of the circle in Problem 2, along a distance equal to the circumference of the circle in Problem 2, what would be the formula for the volume that you swept out?

Answer: Volume = Area x distance

$= (\pi r^2) \times (2 \pi R)$

$\mathbf{= 2 \pi^2 R r^2}$

Problem 4 – If the Van Allen belts can be approximated by a torus with r = 16,000 km, and R = 26,000 km, to <u>two</u> significant figures what is the total volume of the Van Allen belts in cubic kilometers?

Answer:
r = 16000 km x (1000 m/1km) = 16,000,000 meters
R= 26000 km x (1000 m/1km) = 26,000,000 meters

$V = 2 (3.14)^2 (2.6 \times 10^7) (1.6 \times 10^7)^2$

$= \mathbf{1.3 \times 10^{23}}$ **meters^3**

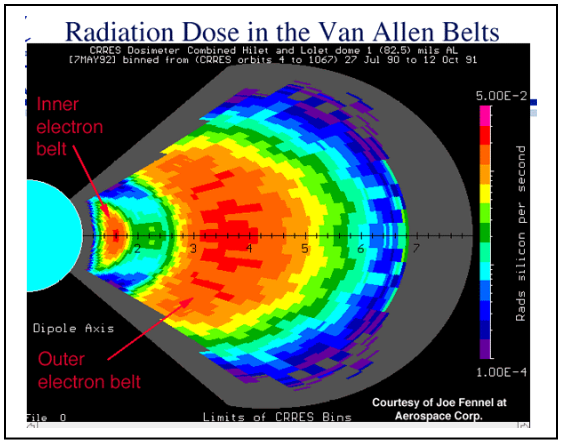
Problem 5 – To <u>two</u> significant figures, how many spherical Earths can you fit in this volume if r = 6378 km?

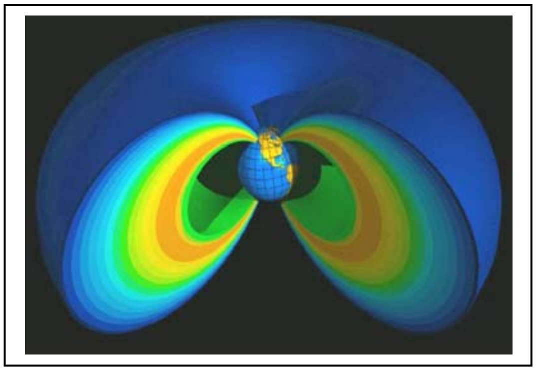
Answer: $V = 4/3 \pi r^3$

$V = 1.33 (3.14) (6.378 \times 10^6 \text{ m})^3$

$V = 1.1 \times 10^{21}$ meters^3

So 1.3×10^{23} meters^3 / 1.1×10^{21} meters^3 = 118 or **120 Earths!**





Most artistic illustrations of the Van Allen belts make them look almost solid, and the colors chosen make them look especially brilliant in vibrant crimsons and blues. These colors are symbolic and are chosen to represent information about the belts rather than what they actually look like. In fact, if you were standing in the middle of the belts you would not even see them at all!

The Van Allen belts contain trillions of high energy particles that over time can be lethal to an exposed astronaut. They can also damage satellites and spacecraft. But there are very few of these particles in any cubic meter of space. The particles are very small and amount to very little mass at all when added together.

The volume occupied by the Van Allen belts forms a donut-shaped region called a torus, which extends from about 10,000 km to 42,000 km from Earth and equals about 1.3×10^{23} meters^3**.** To find the total mass of the Van Allen belts we use the basic principle that mass = density x volume.

Problem 1 – The average density of electrons and protons in the Van Allen belts is about 100 particles per meter^3. There are about equal numbers of electrons and protons. The protons have a mass of 1.7×10^{-27} kg and electrons have a mass of about 9.1×10^{-31} kg. What are the densities of the electrons and protons in kg/m^3?

Problem 2 – Based on the estimated volume of the Van Allen belts, what is the total mass in A) electrons? B) protons C) combined mass in grams?

Problem 3 – A typical donut has a mass of 33 grams. What is the mass of the Van Allen belts in donuts?

Problem 1 – The average density of electrons and protons in the Van Allen belts is about 100 particles per meter^3. There are about equal numbers of electrons and protons. The protons have a mass of 1.7 x 10^{-27} kg and electrons have a mass of about 9.1 x 10^{-31} kg. What are the densities of the electrons and protons in kg/m^3?

Answer: Density = 50 electrons/m^3 x (9.1 x 10^{-31} kg/electron) = 4.6 x 10^{-29} kg/m^3
Density = 50 protons/m^3 x (1.7 x 10^{-27} kg/electron) = 8.5 x 10^{-26} kg/m^3

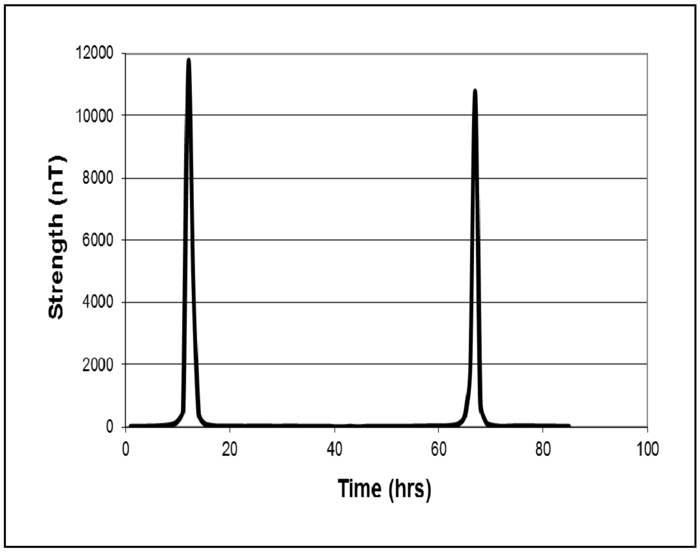
Problem 2 – Based on the estimated volume of the Van Allen belts, what is the total mass in A) electrons? B) protons C) combined mass in grams?

A) M(electrons) = density x volume
= (4.6 x 10^{-29} kg/m^3) (1.3 x 10^{23} meters^3) = 6.0 x 10^{-6} kilograms
B) M(Protons) = density x volume
= (8.5 x 10^{-26} kg/m^3) (1.3 x 10^{23} meters^3) = 1.1 x 10^{-2} kilograms

C) Combined = 0.011 kg x (1000 grams/1kg) = **11 grams!**

Problem 3 – A typical donut has a mass of 33 grams. What is the mass of the Van Allen belts in donuts?

Answer: Our 'donut-shaped' Van Allen belts have **1/3 the mass** of an actual donut!!!



The Cluster satellite constellation consists of 5 satellites orbiting Earth in a close formation. They were designed to measure Earth's magnetic field, and particles in space such as protons and electrons.

This graph shows the strength of Earth's magnetic field measured by the Cluster C1 satellite as it orbited Earth between January 1 and January 6, 2010.

Problem 1 - About what is the highest magnetic field strength measured along the satellite's orbit?

Problem 2 - The satellite's orbit had a perigee (closest point to Earth) of 10,000 km and an apogee (farthest distance from Earth) of 140,000 km. About what was the strength of the magnetic field at A) Perigee? B) Apogee?

Problem 3 - How many hours did it take the satellite to complete one orbit? Explain how you determined this from the graph.

Problem 4 - Below is a table of data taken at specific distances from Earth during the orbit. Graph this data. Does the strength decrease as the inverse-square or inverse-cube of the distance?

Point	Distance (km)	Strength (nT)
1	10,000	10,000
2	30,000	370
3	40,000	160
4	50,000	83
5	60,000	44
6	70,000	30
7	140,000	6

Problem 1 - Answer: The two peaks are at 11,700 nT and 10,800 nT so the maximum value occurs for the first peak with **11,700 nT**.

Problem 2 - Answer: A) At perigee, the satellite is closest to Earth so the strength of the magnetic field should be at its highest point along the orbit or 11,700 nT. B) At apogee the spacecraft is farthest from Earth and the strength is lowest, which from the graph is near-zero.

Problem 3 - Answer: The time between perigee of one orbit and perigee of the next orbit is just the time between the maximum measured magnetic strengths of these two consecutive orbits. From the graph, and using a millimeter ruler to get the correct scale, the time between the peaks is about 54 hours.

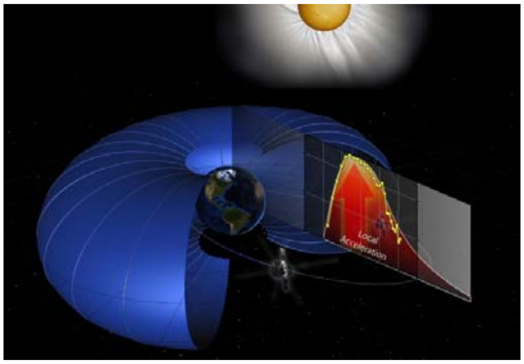
Problem 4 - Below is a table of data taken at specific distances from Earth during the orbit. Graph this data. Does the strength of the magnetic field decrease as the inverse-square or inverse-cube of the distance from Earth?

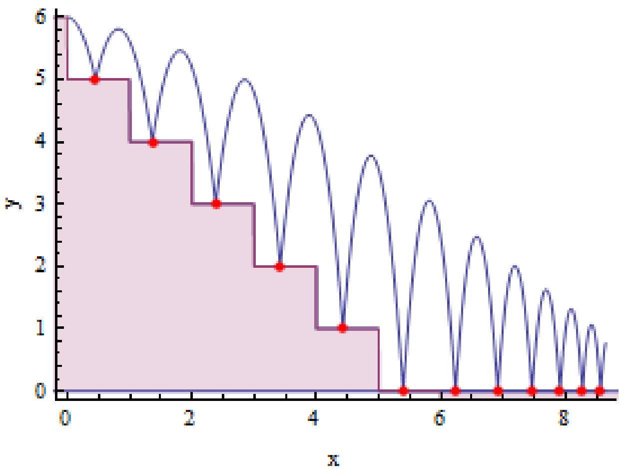
Answer: Inverse-square: Example of this model would predict between Point 1 and Point 2 that the intensity would drop by $1/(3^2)$ so it would be 1,111 nT
Inverse-cube: it would be $10{,}000/(3^3)$ = 370 nT as shown in the table.

We see that the inverse-cube model fits the data much better than the inverse-square distance law.

Point	Distance (km)	Strength (nT)	**Inverse-square**	**Inverse-cube**
1	10,000	10,000	**10,000**	**10,000**
2	30,000	370	**1,111**	**370**
3	40,000	160	**625**	**156**
4	50,000	83	**400**	**80**
5	60,000	44	**278**	**46**
6	70,000	30	**204**	**29**
7	140,000	6	**51**	**4**

Giving Particles a Boost in the Van Allen Belts





Scientists have discovered a massive particle accelerator in the heart of one of the Van Allen radiation belts. Scientists knew that something in space accelerated particles in the radiation belts to more than 99 percent the speed of light but they didn't know what that something was. New results from NASA's Van Allen Probes now show that the particles inside the belts are sped up by local kicks of energy, buffeting the particles to ever faster speeds, much like a perfectly timed push on a moving swing.

To see how this happens, imagine a ball bouncing down a long staircase as shown in the diagram to the left. Each step it falls, adds a small amount of energy to the ball so it bounces a bit higher each time. This is because gravity pulls on the ball and increases its kinetic energy after each step. As the kinetic energy increases, the ball's speed and height increases.

The formula for Kinetic Energy is K.E. = ½ m V^2 where m is the mass of the particle in kilograms, v is its speed in meters/sec and K.E. is measured in units of Joules.

Problem 1 – A small ball has a mass of 0.1 kilograms and a kinetic energy of 5 Joules, what is its speed in meters/sec?

Problem 2 – A 0.1 kilogram ball bounces down a long staircase that has 100 steps. If it gains 0.3 Joules after each step, how much kinetic energy will it have at the bottom of the staircase, and how fast will it be moving?

Problem 3 – An electron in the Van Allen belts has a mass of 9.1×10^{-31} kg. It starts out with a speed of 10,000 km/sec and reaches a speed of 150,000 km/sec after 12 hours. About how much kinetic energy does it gain every hour as it travels around the Van Allen Belts?

NASA's Van Allen Probes Discover Particle Accelerator in the Heart of Earth's Radiation Belts
July 25, 2013
http://www.nasa.gov/content/goddard/van-allen-probes-find-source-of-fast-particles/index.html

Problem 1 – A small ball has a mass of 0.1 kilograms and a kinetic energy of 5 Joules, what is its speed in meters/sec?

Answer: $5.0 = 0.5 \times 0.1 \times V^2$, so $V^2 = 100$ and V = **10 meters/sec**.

Problem 2 – A 0.1 kilogram ball bounces down a long staircase that has 100 steps. If it gains 0.3 Joules after each step, how much Kinetic Energy will it have at the bottom of the staircase, and how fast will it be moving?

Answer: $0.3 \times 100 = 30$ Joules, then $30 = ½ (0.1) V^2$, and V = **24 meters/sec**.

Note: The potential energy of the ball at the top of its bounce is given my E = m g h, where g = 9.8 m/sec^2 and h is the height in meters. So for this ball, m = 0.1 kg, E = 30 Joules and so its maximum bounce height is h = 30 meters.

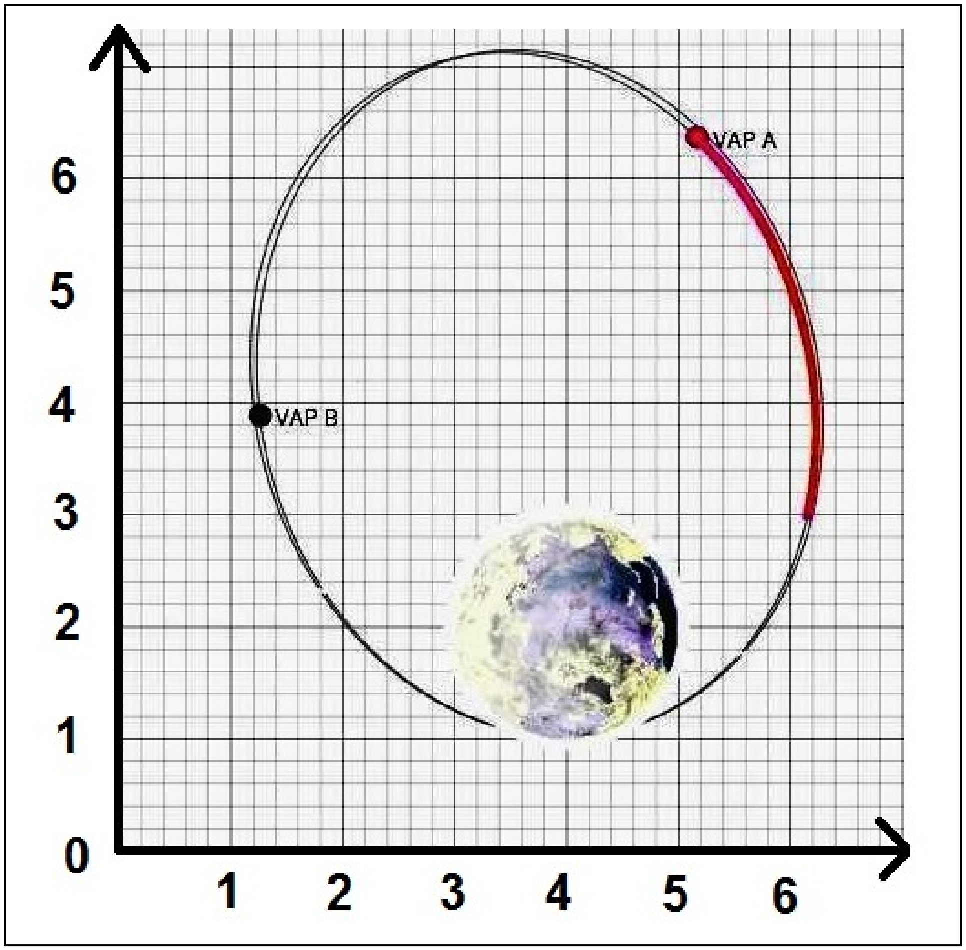
Problem 3 – An electron in the Van Allen belts has a mass of 9.1×10^{-31} kg. It starts out with a speed of 10,000 km/sec and reaches a speed of 150,000 km/sec after 12 hours. About how much kinetic energy does it gain every hour as it travels around the Van Allen Belts?

Answer: The initial kinetic energy of the electron is
$E = ½ (9.1\times10^{-31})(10{,}000{,}000)^2 = 4.6 \times 10^{-17}$ Joules.

The final kinetic energy is $E = ½(9.1\times10^{-31})(150{,}000{,}000)^2 = 1.0 \times 10^{-14}$ Joules,

so the electron gained $1.0\times10^{-14} – 4.6\times10^{-17} = 1.0\times10^{-14}$ Joules of energy.

If this was done equally over 12 hours, then the energy gained per hour was $1.0\times10^{-14} / 12 =$ **8.5×10^{-16} Joules each hour**.



Problem 1 – The graph above shows the orbit of the Van Allen Probes spacecraft (VAP A and VAP B). Each division, 1.0, 2.0, etc) is given in terms of Earth's radius, which is 6,378 kilometers. For example, '2.0' = twice Earth's radius or 12,756 kilometers. Suppose that the spacecraft move along their orbit from point (5.6,1.8) to point (6.2,3.0). Plot these two points on the graph, and convert their coordinates into kilometers.

Problem 2 – Using which ever method you like, determine the distance in kilometers that the spacecraft traveled between the two points in their orbit.

Problem 3 – Suppose the spacecraft reached the point (5.6,1.8) at a time of 11:22:30 and point (6.2,3.0) at a time of 11:50:00. What was the elapsed time to travel this distance in seconds?

Problem 4 – What was the speed of the spacecraft in kilometers/hour?

Answer Key

Problem 1 – The graph above shows the orbit of the Van Allen Probes spacecraft (VAP A and VAP B). Each division, 1.0, 2.0, etc) is given in terms of Earth's radius, which is 6,378 kilometers. For example, '2.0' = twice Earth's radius or 12,756 kilometers. Suppose that the spacecraft move along their orbit from point (5.6,1.8) to point (6.2,3.0). Plot these two points on the graph, and convert their coordinates into kilometers.

Answer: 5.6 x 6378 km = 35,717 km, 1.8 x 6378 km = 11,480 km;
6.2 x 6378 km = 39,544 km, 3.0 x 6378 km = 19,134 km
Coordinates in km = (35717, 11480) and (39544, 19134).

Problem 2 – Using which ever method you like, determine the distance in kilometers that the spacecraft traveled between the two points in their orbit.

Answer: One method: Determine the scale of the graph using a millimeter ruler in km/mm, then measure the distance between the two points in millimeters and convert to kilometers.

For advanced students use the 2-point distance formula. $D^2 = (39544\text{ km} - 35717\text{ km})^2 + (19134\text{ km} - 11480\text{ km})^2 = (3827\text{ km})^2 + (7654\text{ km})^2$, so **D = 8,557 kilometers**.

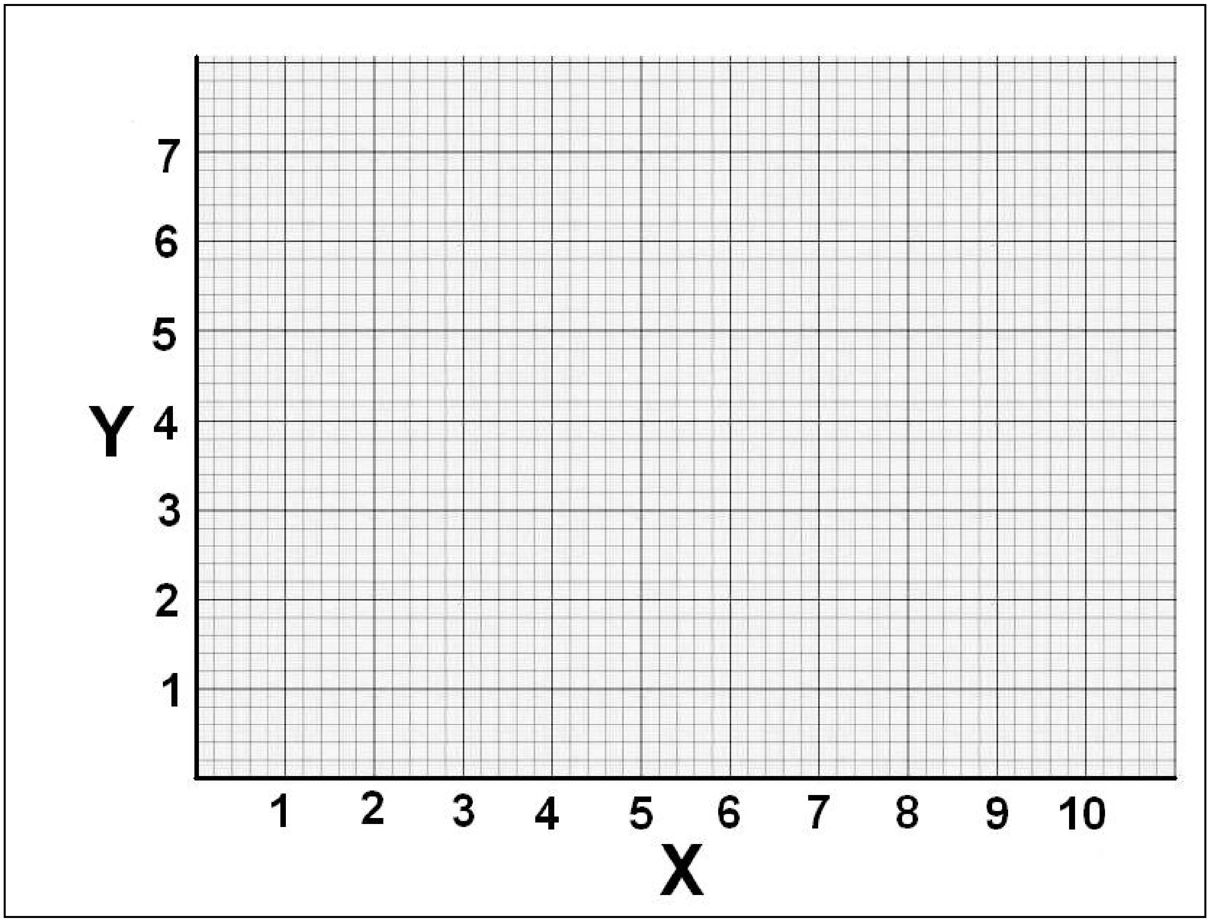
Problem 3 – Suppose the spacecraft reached the point (5.6,1.8) at a time of 11:22:30 and point (6.2,3.0) at a time of 11:50:00. What was the elapsed time to travel this distance in seconds?

Answer: Elapsed time = 11:50:00 – 11:22:30 = 27m 30s or 27m x 60s/min + 30s = **1650 seconds**.

Problem 4 – What was the speed of the spacecraft in kilometers/hour?

Answer: Time, in hours, = 1650 seconds x (1 min/60 sec) x (1 hr/60 min) = 0.46 hrs.

Speed = distance/time = 8557 km / 0.46 hours = **18,602 km/hr**.



Problem 1 – The Van Allen Probe A spacecraft travels along its orbit from point A (3,4) to point B (7,4). Draw an arrowed segment showing the direction that the spacecraft is traveling.

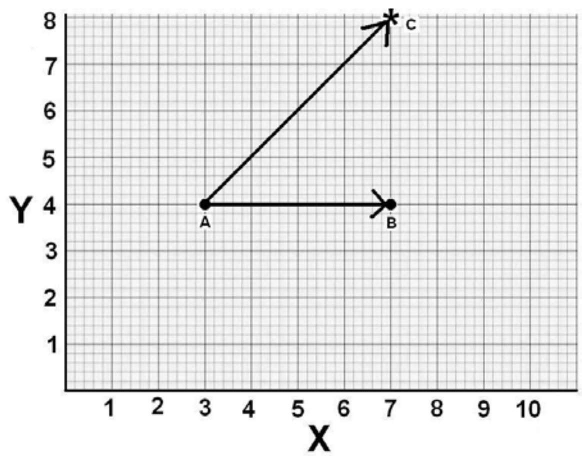
Problem 2 – Earth's magnetic field in this region of space passes through Point A, in the direction of Point C (7, 8). Draw an arrowed segment showing the magnetic field direction.

Problem 3 – From the geometry of the triangle ABC, what angle does the magnetic field make to the direction of the spacecraft motion?

Problem 4 – If the strength of the magnetic field in the direction AC is 14 nanoTeslas, what would the 'shadows' of the magnetic field strength be along the directions AB and AC? (These are called the components of the magnetic field.)

Problem 1 – The Van Allen Probe A spacecraft travels along its orbit from point A (3,4) to point B (7,4). Draw an arrowed segment showing the direction that the spacecraft is traveling. Answer: See below.

Problem 2 – Earth's magnetic field in this region of space passes through Point A, in the direction of Point C (7,8). Draw an arrowed segment showing the magnetic field direction. Answer: See below.

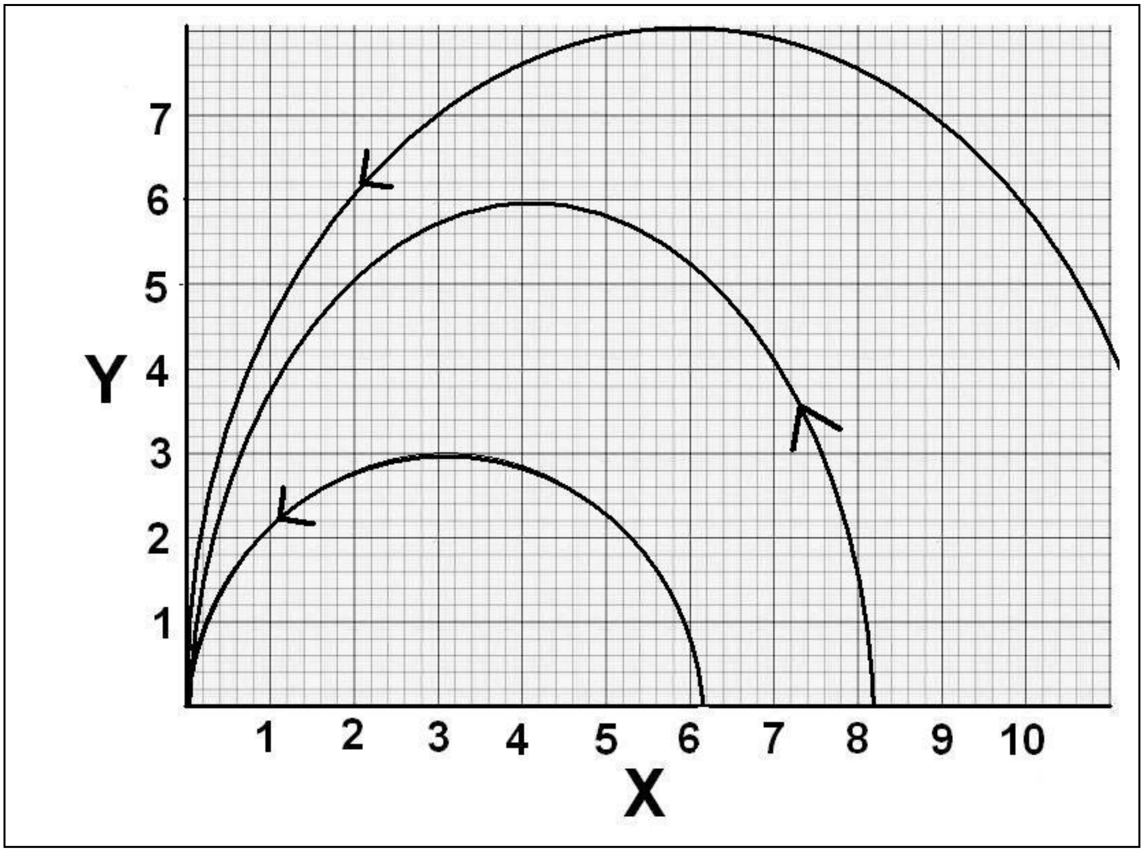


Problem 3 – From the geometry of the triangle ABC, what angle does the magnetic field make to the direction of the spacecraft motion?

Answer: The horizontal segment AB is 4.0 units long and the vertical segment BC is 4.0 units long. This triangle is a 45:45:90 right triangle. The angle that AC makes to segment AB is 45^{o}.

Problem 4 – If the strength of the magnetic field in the direction AC is 14 nanoTeslas, what would the 'shadows' of the magnetic field strength be along the directions AB and AC? (These are called the components of the magnetic field.)

Answer: In a 45:45:90 right triangle, the side lengths are 1.0 and the hypotenuse is $\sqrt{2}$. Using proportions, the sides of the 'magnetic triangle' with a hypotenuse of 14 nanoTeslas are 14 nanoTeslas/$\sqrt{2}$ = 14 nanoTeslas/1.4 = 10 nanoTeslas. The components to the magnetic field are +10 nanoTeslas along direction AB and +10 nanoTeslas along direction BC. We can also write this as (+10 nT, +10nT)



The figure above shows three magnetic field lines in space. The three arrows show the direction that a compass needle will point in the magnetic north direction. The X-axis lies along the west-east direction with east towards the right. The Y-axis lies along the geographic north-south direction with north at the top.

Problem 1 – Describe what happens to the compass needle as a spacecraft moves from point (6.2,0) to (6.2,5.0) to (6.2,8.0).

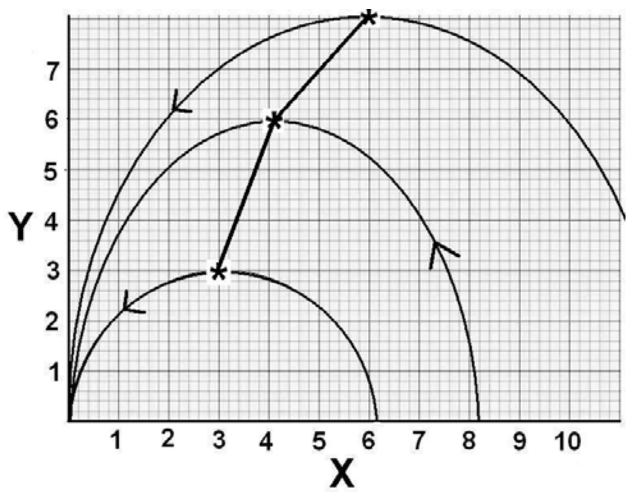
Problem 2 – Draw a possible spacecraft path so that the compass needle always points to geographic west in the figure.

Problem 3 – A satellite is launched from point (1.0,2.0) and travels horizontally to point (10.0,2.0). Plot a graph that shows how its instruments will record the direction changes of the magnetic field as it travels. At what location is the magnetic field pointed due-west? (Note: You may approximate angle measurements by interpolation as needed.)

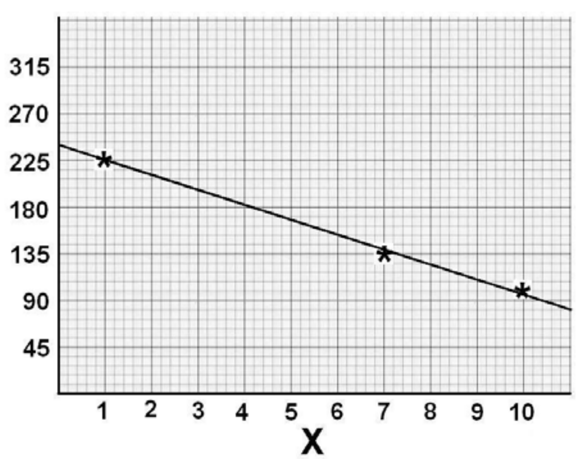
Problem 1 – Describe what happens to the compass needle as a spacecraft moves from point (6.2,0) to (6.2,5.0) to (6.2,8.0).

Answer: First the compass needle points vertically due-North, then it points 45 degrees west of north, then it points due-west!

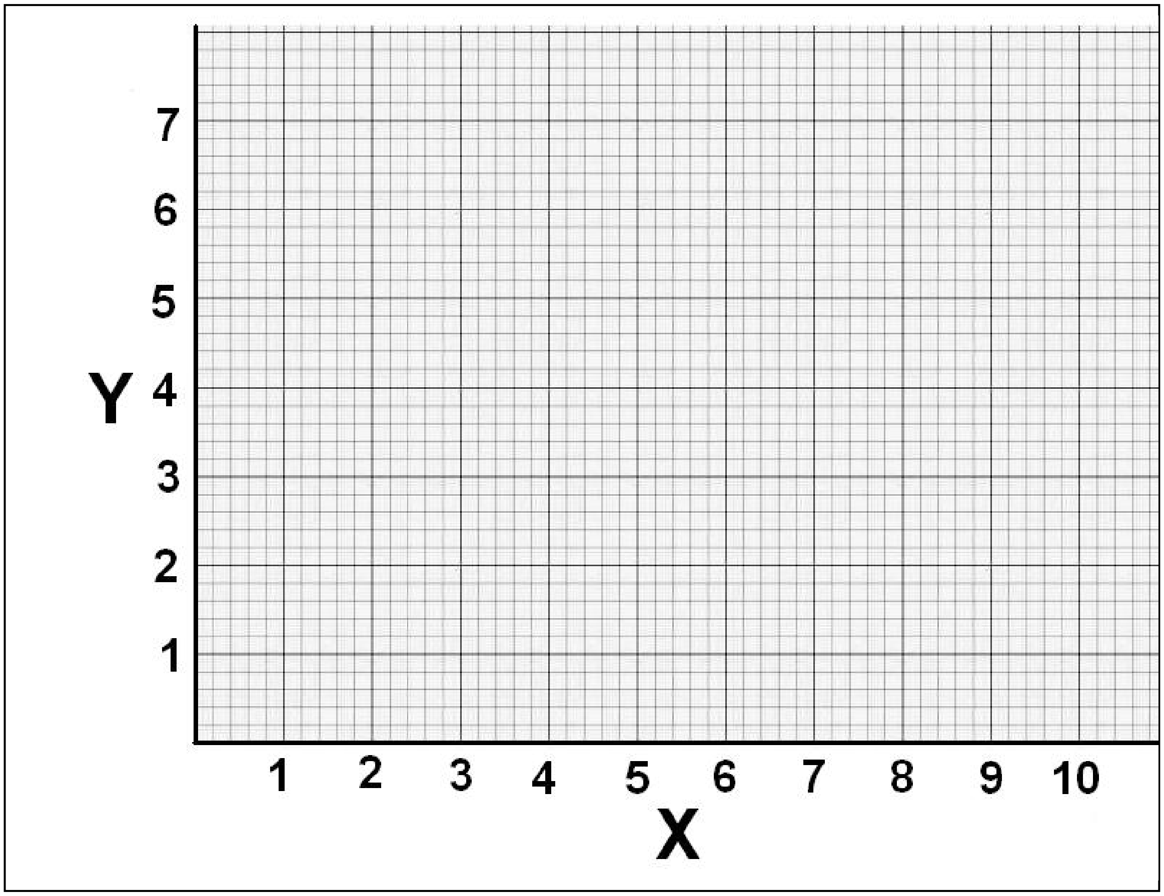
Problem 2 – Draw a possible spacecraft path so that the compass needle always points to geographic west in the figure.



Problem 3 – A satellite is launched from point (1.0,4.0) and travels horizontally to point (10.0,4.0). Plot a graph that shows how its instruments will record the direction changes of the magnetic field as it travels. At what location is the magnetic field pointed due-west? (Note: You may approximate angle measurements by interpolation as needed.)



To point due-west, the angle must be 180 degrees. From the graph this happens when the satellite is close to x=4.0, so its coordinate on its path is (4.0, 4.0)



Problem 1 – During part of its orbit around Earth, the Van Allen Probes travel along the line given by the equation y = -1/2 x + 2. Graph this line on the grid above.

Problem 2 – Earth's magnetic field is oriented along lines that are parallel to y = ¾ X. Draw three of these lines across the grid above.

Problem 3 – What is the equation of the line that is perpendicular to the spacecraft trajectory? Plot this line on the graph above.

Problem 4 – What angle does the magnetic field make with respect to the direction along the spacecraft trajectory?

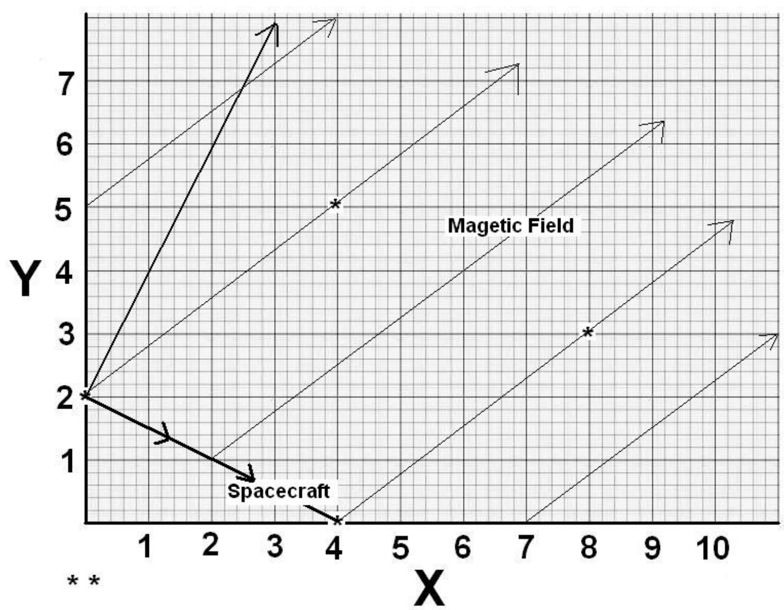
Problem 5 - What angle does the magnetic field make with respect to the direction perpendicular to the spacecraft trajectory?

Problem 1 – During part of its orbit around Earth, the Van Allen Probes travel along the line given by the equation y = -1/2 x + 2. Graph this line on the grid above. Answer: See below, labeled 'spacecraft'

Problem 2 – Earth's magnetic field is oriented along lines that are parallel to y = ¾ X. Draw three of these lines across the grid above. Answer: See below: Labeled 'magnetic field'

Problem 3 – What is the equation of the line that is perpendicular to the spacecraft trajectory? Plot this line on the graph above.

Answer: The perpendicular line to y = mx+b is y = -1/m x + b. The slopes are the negative reciprocals of each other. If the spacecraft direction is y = -1/2 X + 2, then the perpendicular is y = 2x+2 at point (0,2), as shown in the figure.



Problem 4 – What angle does the magnetic field make with respect to the direction along the spacecraft trajectory?

Answer: Use a protractor to measure the angle. It is **63 degrees**.

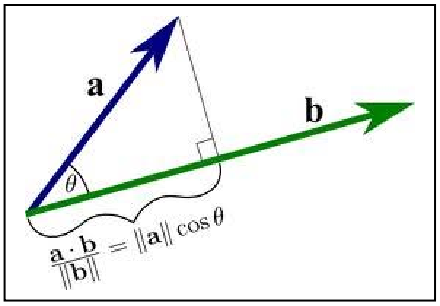
Problem 5 - What angle does the magnetic field make with respect to the direction perpendicular to the spacecraft trajectory?

Answer: It will be the compliment angle, 90-63 = **27 degrees**.

Point A: (4.0, 3.0)
Point B: (6.0, 10.0)

Vector Notation:

A = +4.0**x** +3.0**y**
B = +6.0**x** + 10.0**y**



The Van Allen Belt Probes spacecraft determine their orientation in space by using the direction of the local magnetic field of Earth. This is like using a compass to figure out where you should go next!

The VABP spacecraft measure the intensity of the magnetic field along, and perpendicular to their direction of motion at a given time. They can then use these measurements to find the angle between their motion (vector **A**) and the magnetic field (vector **B**). They compare their measurements against a mathematical model of Earth's magnetic field to find the field's true direction, then they rotate the spacecraft until the angles match up.

To do these calculations we have to determine the projection of the magnetic field onto the direction of the spacecraft motion, and to the axis perpendicular to this motion. This involves using the Vector Dot Product:

$$\mathbf{A}\ \mathbf{B} = ||A||\ ||B|| \cos(\theta)$$

which gives the component of vector **A** along the direction of the unit vector **B** where ||B||=1.

Problem 1 – The spacecraft moves from point A (0,2) to point B (2,1). Write these points in vector notation and calculate the spacecraft motion vector **C** = **B** – **A**.

Problem 2 – Direction vectors are like motion vectors except they have unit length. What do you have to do to the motion vector, **C**, to give it a magnitude of exactly 1.0?

Problem 3 – What is the slope of the vector, **C**?

Problem 4 – The predicted magnetic field is given by the vector **B** = +4**x** + 3**y** in units of nanoTeslas. What is the projection of the magnetic field along the direction of motion of the spacecraft?

Problem 5 – The spacecraft measures a magnetic field strength of 1 nanoTesla along the spacecraft motion. What is the angle, θ, between the spacecraft motion and the local magnetic field?

Problem 1 – The spacecraft moves from point A (0,2) to point B (2,1). Write these points in vector notation and calculate the spacecraft motion vector **C** = **B** – **A**.

Answer: **A** = +2**y**, **B**= +2**x** + 1y, therefore **C = +2x – 1y**

Problem 2 – Direction vectors are like motion vectors except they have unit length. What do you have to do to the motion vector, **C**, to give it a magnitude of exactly 1.0?

Answer: You have to rescale the vector **C** so that its magnitude is 1.0. Since |C| = $(5)^{1/2}$, the direction vector is **D** = $+2/(5)^{1/2}$**x** – $1/(5)^{1/2}$**y,** therefore |D| = 1.0

Problem 3 – What is the slope of the vector, **C**? Answer: **-1/2**

Problem 4 – The predicted magnetic field is given by the vector **M** = +4**x** + 3**y** in units of nanoTeslas. What is the projection of the magnetic field along the direction of motion of the spacecraft?

Answer: Use the dot product with **M** = 4**x**+3**y**, and **B** = (2**x**-1**y**) where $||B||=(5)^{1/2}$.

M dot B/||B|| = (4x + 3y) dot (2x – 1y) / $(5)^{1/2}$

The projection is $(8-3)/(5)^{1/2}$ or $5^{1/2}$ **nanoTeslas.**

Problem 5 – The spacecraft measures a magnetic field strength of 1 nanoTesla along the spacecraft motion. What is the angle, θ, between the spacecraft motion and the local magnetic field?

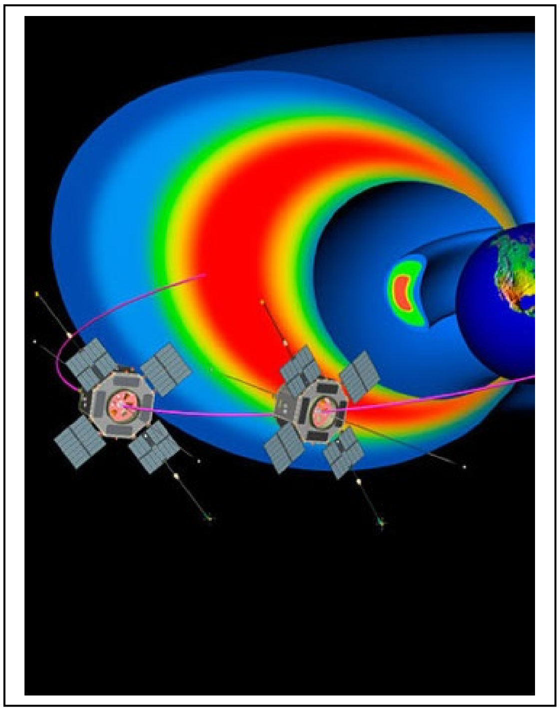
Answer: Use A B = ||A|| ||B|| cos(θ)

1 nano Tesla = M dot B/||B|| = ||M|| cos θ

Therefore 1 nanoTesla / ||B|| = cos θ and ||B|| = $5^{1/2}$

$\cos(\theta) = 1/5^{1/2}$
$= 0.447$

Therefore θ = **63 degrees**.



The Van Allen Probes orbit Earth within the Van Allen belts about once every 9 hours. The radiation environment contains high energy electrons and protons that can damage satellite solar panels and delicate electronics without proper shielding.

Typical spacecraft used for scientific research will become unreliable after they have accumulated 1000 Grays of radiation, where 1 Gray = 1 joule of energy delivered to 1 kilogram of material. Usually this energy is harmlessly deposited into the spacecraft shielding. Some of it can, however, damage electronic circuits.

After a spacecraft accumulates about 1000 Grays of radiation, the wear on the satellite electronics can cause glitches, circuitry damage, and the satellite becomes unreliable.

An engineer wants to model the accumulation of radiation dosage as a satellite travels through the Van Allen belts in a 10-hour orbit. He wants to predict how many years of productive life the satellite will have given the shielding that it has before it reaches the 1000 Gray limit. There are two parts to this problem. The first is that he has to model the path taken by the spacecraft through the Van Allen belts. The second part is that he has to model how intense the radiation dose rate is in the Van Allen belts. Combining the two will give a model of the radiation dose rates the satellite will encounter at each point along its path.

Problem 1 – Suppose the radial distance between the center of Earth and the spacecraft can be modeled as a simple linear function during the time the shielded spacecraft is inside the Van Allen belts, and the radiation dose rate is modeled by a simple power-law function:

Path: $R(T) = 7000 + 3000T$ kilometers, where T is the elapsed time in hours.
Dose Rate: $D(R) = 60(R/25000)^2$ milliGrays/hour, where R is in kilometers.

What is the dose rate formula re-written so that the dose rate is a function of time D(T)?

Problem 2 – The integral of the dose rate formula D(T) with respect to time is the accumulated total dose.

A) Perform this integration for one 10-hour orbit of the spacecraft assuming that the total dose over the time interval T: [0h, 10h] is equal to twice the dose rate over the time interval T: [0h, 5h].

B) How many years will it take for the spacecraft total dose to equal 1000 Grays?

Problem 1 – Suppose the radial distance between the center of Earth and the spacecraft can be modeled as a simple linear function during the time the shielded spacecraft is inside the Van Allen belts, and the radiation dose rate is modeled by a simple power-law function:
Path: R(T) =7000 + 3000T kilometers, where T is the elapsed time in hours.
Dose Rate: $D(R) = 60(R/25000)^{2}$ milliGrays/hour, where R is in kilometers.
What is the dose rate formula re-written so that the dose rate is a function of time D(T)?

Answer: Substitute the formula R(T) in the equation for D to get

$$D(T) = 60\left(\frac{7000+3000T}{25000}\right)^2 \quad \text{so} \quad D(T) = 60\left(\frac{7+3T}{25}\right)^2 \text{ milliGrays/hour}$$

Problem 2 – The [definite] integral of the dose rate formula D(T) with respect to time is the accumulated total dose.

A) Perform this integration for one 10-hour orbit of the spacecraft assuming that the total dose over the time interval T: [0h, 10h] is equal to twice the dose rate over the time interval T: [0h, 5h]. Answer:

$$\text{Total Dose} = 120\int_0^5 \left(\frac{7+3T}{25}\right)^2 dT$$

$$= \frac{120}{625}\int_0^5 (49+42T+9T^2)dT$$

$$= \frac{120}{625}\left(49T+21T^2+3T^3\right)_0^5$$

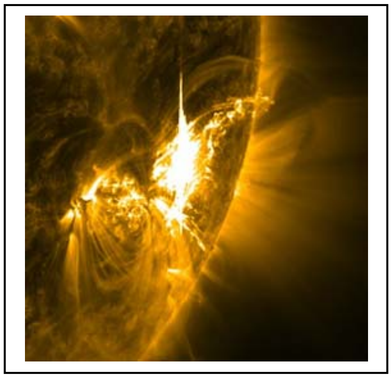
$$= \frac{120}{625}\left(49x5+21x25+3x125\right) \quad = \quad 220 \text{ milliGrays} = \mathbf{0.22\ Grays/orbit.}$$

B) How many years will it take for the spacecraft total dose to equal 1000 Grays?

Answer: The spacecraft accumulates 0.22 Grays every 10 hours, so in one year it accumulates

0.22 Grays/10 hours x (24 hours/1day)x(365 days/1 year) = 192 Grays/year,

then 1000 Grays/ (192 Gy/yr) = **5.2 years**.



The following essay describes solar storms. The words that belong in the missing blanks can be found by solving the ten number sentences.

Look in the Word Bank and match the calculated number to the correct answer.

Fill-in the blank with the word you found to complete the essay and answer a question about solar storms!

Our sun is a very predictable star. Each day it rises and sets as the world turns upon its axis, and warms the Earth making life possible. But the sun is also a stormy star. It produces 1)___________ and incredible explosions of 2)__________ almost every day. Sometimes, its entire surface is speckled by 3)___________ that come and go every 11 years. In 2013, the sun was at the peak of its maximum stormy activity. This means that many more flares and explosions of gas were happening compared to other times in the 11-year cycle. Solar flares are bursts of intense 4)____________ light that can cause problems for radio communication on Earth. They also heat up the 5)___________ and cause it to expand into space. About 1000 of these flares were detected during the first 8 months of 2013.

Occasionally the sun ejects billion-ton clouds of 6)__________ called 7)_______________________ or CMEs. Traveling at over a million miles an hour, they can reach Earth in only a few 8)_________. When they arrive, they cause problems for satellites and our electric power grid, but they also cause beautiful 9)_________ in the northern and southern skies. Most CMEs are not directed towards earth and are completely 10)_____________.

So, even though the sun looks the same every day, it really is a very stormy star that can sometimes create unpleasant surprises for us here on Earth!

Word Bank

-5	asteroids	+48	heat	-17	X-rays
-44	aurora	+1	ocean	+3	harmless
-15	ultraviolet	+5	days	+44	rainbows
-27	energy	0	atmosphere	-5	months
-40	rocks	-48	gas	+24	sunspots
-6	harmful	+3	plasma	-3	coronal mass ejections
-2	flares	+20	comets	-7	prominences

Solve these problems to get the Word Bank number key.

1) 1+(1-3)-(5-8)+(-6+2) =
2) 8(3-2)-2(3-8)+5(-6-3) =
3) (1-3)(-5+2)(8-6)(3-1) =
4) 3(2-6)+(-8+4)-(4-3) =
5) 5(-3+2)-2(6-2)-(+7-20) =
6) -2(+2(-3(+2(-3+1)))) =
7) -8/2 +(3+2)/(8-3) =
8) -7+23-6-(-10)+(-3)(4+1) =
9) 12/(-1/4) + (-36)/(-9) =
10) (-4)2 + 21/3 + 4 =

Answer Key

Solve these problems to get the Word Bank number key.

1) 1+(1-3)-(5-8)+(-6+2) = -2
2) 8(3-2)-2(3-8)+5(-6-3) = -27
3) (1-3)(-5+2)(8-6)(3-1) = +24
4) 3(2-6)+(-8+4)-(4-3) = -17
5) 5(-3+2)-2(6-2)-(+7-20) = 0

6) -2(+2(-3(+2(-3+1)))) = -48
7) -8/2 +(3+2)/(8-3) = -3
8) -7+23-6-(-10)+(-3)(4+1) = +5
9) 12/(-1/4) + (-36)/(-9) = -44
10) (-4)2 + 21/3 + 4 = +3

The words are

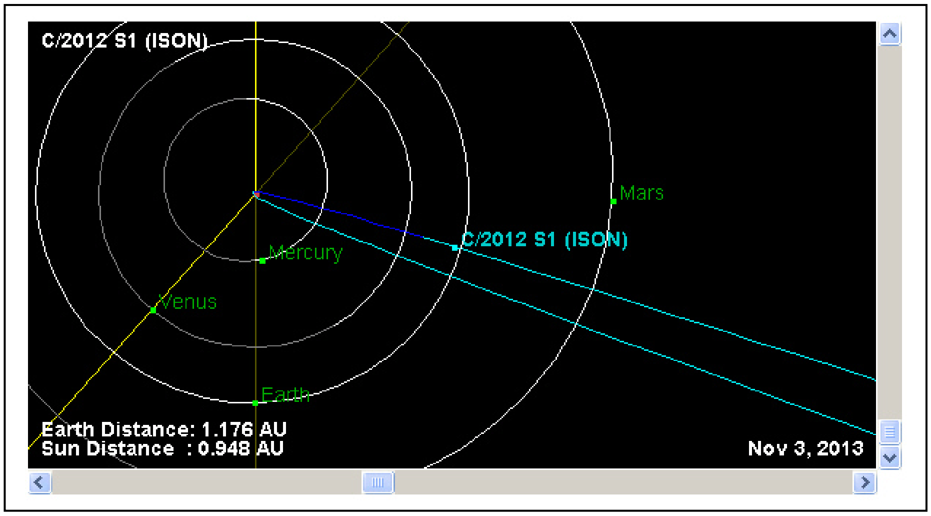
Line 1 = -2 = flares
Line 2 = -27 = energy
Line 3 = +24 = sunspots
Line 4 = -17 = X-ray
Line 5 = 0 = atmosphere
Line 6 = -48 = gas
Line 7 = -3 = coronal mass ejections
Line 8 = +5 = days
Line 9 = -44 = aurora
Line 10 = +3 = harmless

Our sun is a very predictable star. Each day it rises and sets as the world turns upon its axis, and warms the Earth making life possible. But the sun is also a stormy star. It produces 1)____**flares**______ and incredible explosions of 2)___**energy**_____ almost every day. Sometimes, its entire surface is speckled by 3)__**sunspots**____ that come and go every 11 years. In 2013, the sun was at the peak of its maximum stormy activity. This means that many more flares and explosions of gas were happening compared to other times in the 11-year cycle. Solar flares are bursts of intense 4)___**X-ray**____ light that can cause problems for radio communication on Earth. They also heat up the 5)___**atmosphere**_____ and cause it to expand into space. About 1000 of these flares were detected during the first 8 months of 2013.

Occasionally the sun ejects billion-ton clouds of 6)__**gas**____ called 7)___**coronal mass ejections**____ or CMEs. Traveling at over a million miles an hour, they can reach Earth in only a few 8)____**days**_____. When they arrive, they cause problems for satellites and our electric power grid, but they also cause beautiful 9)__**aurora**____ in the northern and southern skies. Most CMEs are not directed towards earth and are completely 10)___**harmless**______.

So, even though the sun looks the same every day, it really is a very stormy star that can sometimes create unpleasant surprises for us here on Earth!

The Orbit of Comet ISON



Comet ISON was discovered on September 21, 2012, and its orbit has been calculated from many observations since then. Instead of the normal circular or elliptical paths that comets, asteroids and planets often take, Comet ISONs orbit is nearly a parabola. This means that it came from far beyond the orbit of Neptune and that this is probably the first time it has entered the inner solar system! The figure shows its calculated path through the orbits of Mercury, Venus, Earth and Mars. The table below gives the distance between Comet ISON and the inner planets for the 15th of each month from July 2013 to March 2014..

Date	Sun	Mercury	Venus	Earth	Mars
July 15	434	497	411	586	283
August 15	365	320	430	502	168
September 15	289	326	397	389	58
October 15	204	267	300	258	48
November 15	91	45	151	135	163
December 15	104	159	76	79	201
January 15	213	255	123	101	233
February 15	299	264	270	224	322
March 15	367	422	410	348	416

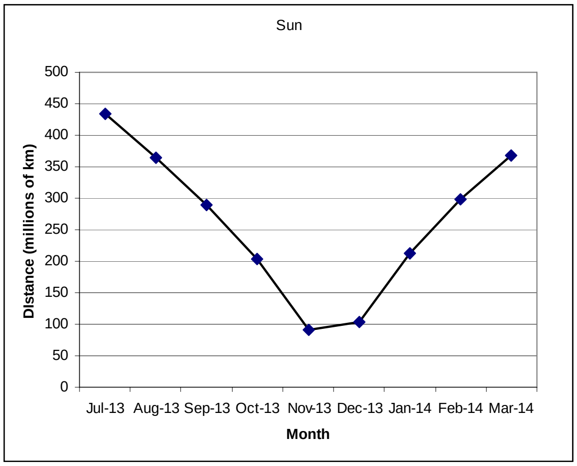
Problem 1 – Create two graphs of the distance data for; A) the Comet-Sun distance and 2) the Comet-Earth distance.

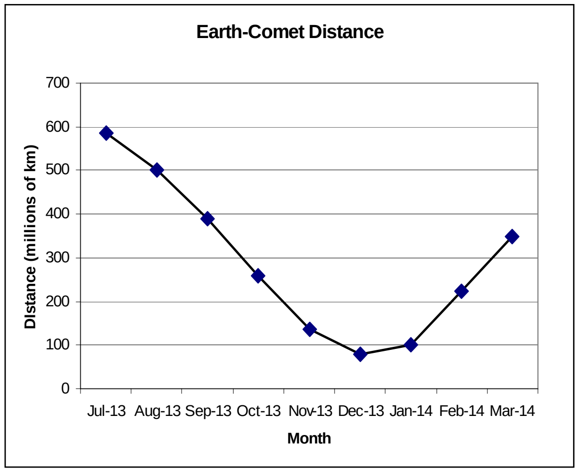
Problem 2 – What is the closest distance to the Earth that Comet ISON gets? If the distance to the moon from Earth is 380,000 km, how many times the Earth-Moon distance is the closest comet distance?

Problem 3 –Which planet with Comet ISON pass the closest too, and during which month?

Problem 4 – On November 28 COMET ISON passes within 1.2 million kilometers of the sun. At the Sun's Tidal Distance, objects are often pulled apart by the sun's intense gravity. For comets, this distance is about 1 million km from the center of the sun. Is it possible that Comet ISON may break apart?

Problem 1 – Create two graphs of the distance data for; A) the Comet-Sun distance and 2) the Comet-Earth distance.





Problem 2 – What is the closest distance to the Earth that Comet ISON gets? If the distance to the moon from Earth is 380,000 km, how many times the Earth-Moon distance is the closest comet distance?

Answer: From the graph, the closest distance is near December 15, 2013 at **79 million km**. This equals 79,000,000/380,000 = **18 times the distance to the moon from Ea**rth.

Problem 3 –Which planet with Comet ISON pass the closest too, and during which month?

Answer: It comes closest to Mars around **October 15, 2013 at 48 million km**.

Problem 4 – On November 28 COMET ISON passes within 1.2 million kilometers of the sun. At the Sun's Tidal Distance, objects are often pulled apart by the sun's intense gravity. For comets, this distance is about 1 million km from the center of the sun. Is it possible that Comet ISON may break apart?

Answer: Comet ISON comes very close to the sun's TIDAL Limit on November 28, so it may suffer some fragmentation into smaller comets. Astronomers will closely watch this comet when it once again becomes visible from Earth on November 29 to see if it broke apart!

Note: Because we have given the distance to Comet ISON on the 15th of each month, this hides the fact that the Comet actually passes very close to each of these planets at a different date. SUN: November 28, 1.2 million km; Mercury: November 19, 36.3 million km. Venus: December 21, 73.4 million km; Earth: December 26, 64.2 million km and Mars: October 1, 10.9 million km. These estimates were based on the orbit known on July 29, 2013 but will change as the comet swings around the sun and its orbit changes slightly.

While most people will be admiring Comet ISON as it passes close by Earth and the Sun in the fall of 2013, other robotic observers on Mars will marvel as this same comet lights up the martian sky.

For the first time in human history, we will get to see the same comet from two different outposts of humanity in the solar system!

As a warm up, here is an image of Earth in the martian sky as seen by the Spirit Rover in 2004!

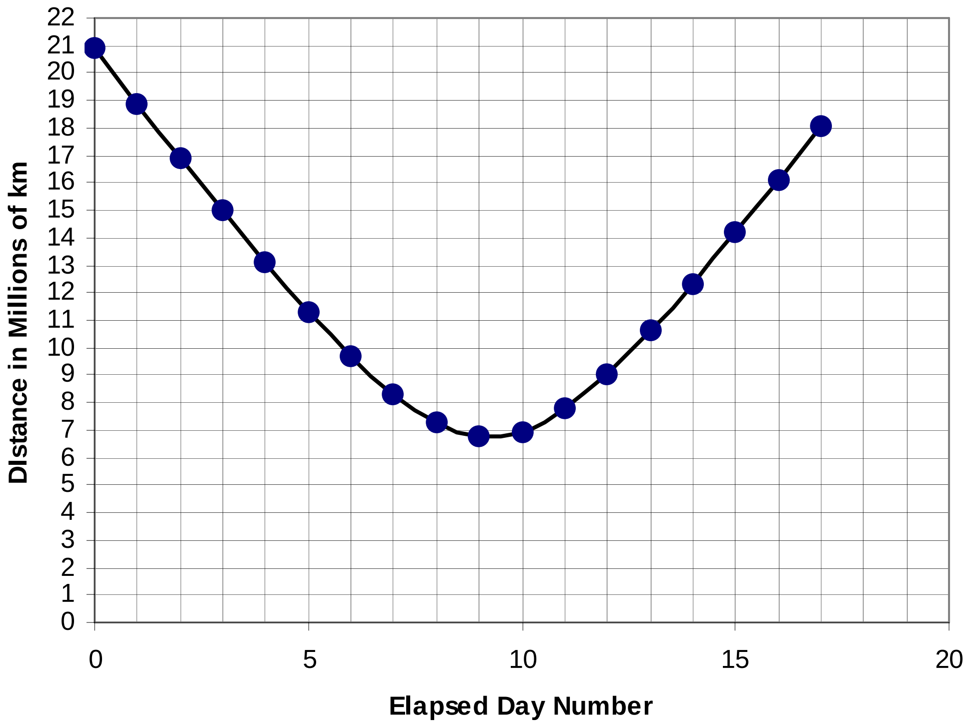
We can use the NASA, *Eyes on the Solar System* simulator to explore the positions of Comet ISON and Mars during the time of the encounter. Click on the following link to set up the program. http://eyes.nasa.gov/index.html. You will need to 'Launch' and download the software as instructed. Under 'Visual Controls' make sure the 'Small Bodies' and 'Metric' dots are white by selecting them. Right click on the label for Comet ISON and select 'Measure distance from' then click on the label for Mars to open the measurement window. Now open the 'Date and Time' menu and select the date and time using the sliders. Click on the UT ball to select Universal Time. No hit 'Submit' and the scene will move to the first location at URL: 1.usa.gov/18PtDeF for September 20 at 05:34 PM, UT. As you move forward in time, notice that the distance changes, beginning near 27 million km for this first scene. The table below gives a set of distance measurements (in millions of kilometers) during the close approach period at the same time of the day (13:00 UT). Select these, or create your own set using *Eyes on the Solar System*.

Date	Distance	Date	Distance	Date	Distance
9-22	20.9	9-28	9.7	10-4	9.0
9-23	18.9	9-29	8.3	10-5	10.6
9-24	16.9	9-30	7.3	10-6	12.3
9-25	15.0	10-1	6.8	10-7	14.2
9-26	13.1	10-2	6.9	10-8	16.1
9-27	11.3	10-3	7.8	10-9	18.1

Problem 1 - Graph the data with elapsed day number (9-11 = 0.0) on the horizontal axis, and the distance in millions of kilometers on the vertical axis. Draw a smooth curve through the data points.

Problem 2 – At what day and time will the distance be at its minimum?

Problem 1 - Graph the data with elapsed day on the horizontal axis, and the distance in millions of kilometers on the vertical axis. Draw a smooth curve through the data points. Answer: See below:



Problem 2 – At what day and time will the distance be at its minimum?

Answer: Although the point for October 1 (Day number 9) looks like the closest distance (10.8 million km), students will see that the curve reaches a slightly smaller distance value between Day 9 and Day 10., which correspond to October 1 at 13:00 UT and October 2 at 13:00 UT, 24-hours later. Invite the students to create a better estimate between these two dates and times using their graphs, or by using Eyes on the Solar System. **Answers close to October 1 near 19:00 UT are acceptable. The distance will be close to 6.75 million km**.

Note: The orbit for Comet ISON will be updated as new data is obtained, so the actual numbers used in these problems are only current for July 28, 2013. Students will be using the updated values and will need to use Eyes on the Solar system to get the most up to date values.

In July 2013, Curiosity began its long journey to the base of Mt Sharp, seen in the distance in this image. Because it is operated robotically, it only travels a few meters every hour and communicates with its Earth technicians after each step. Because radio waves take 20 minutes or longer to reach earth, 40 minutes elapse before a transmitted command is received and the results of the action can be verified.

The table below gives the progress made by Curiosity during several days of operation on Mars, called Sols.

Sol	Drive	Duration (minutes)	Odometer (meters)	Azimuth (degrees)	Pitch (degrees)
345	68	83	1490	235	-1
347	69	67	1550	236	-2
349	70	72	1621	190	+1
351	71	94	1706	249	+1
354	72	65	1763	304	+1

Problem 1 – What is the average time that Curiosity drove each day?

Problem 2 – What is the average distance traveled each day?

Problem 3 – 'Azimuth' is the direction you are pointing from North so that due North is 0^{o}, East is 90^{o}, South is 180^{o} and West is 270^{o}. What is the average azimuth angle that Curiosity traveled along during the tabulated period?

Problem 4 – 'Pitch' is the tilt angle of the land, with straight up being $+90^{o}$, horizontal being 0^{o} and straight down being -90^{o}. What is the average pitch of Curiosity's travels during this period and what can you tell about the ground over which it traveled?

Problem 5 – The direction to Mt Sharp is at an azimuth of 225^{o}. What does Curiosity's average azimuth have to be during the next 5 days so that it is back on course to Mt Sharp?

Problem 6 – Mt Sharp is located 7.5 km from Curiosity. About how many more Sols will be required for Curiosity to get there?

Problem 1 – What is the average time that Curiosity drove each day?

Answer: T = (83+67+72+94+65)/5 = 381 minutes/5 = **76 minutes/day**

Problem 2 – What is the average distance traveled each day?

Answer: D = (60 + 71 + 85 + 57)/4 = 273 meters/4 = **68 meters/day**.

Problem 3 – 'Azimuth' is the direction you are pointing from North so that due North is 0^o, East is 90^o, South is 180^o and West is 270^o. What is the average azimuth angle that Curiosity traveled along during the tabulated period?

Answer: A = (235+236+190+249+304)/5 = 1214/5 = **243^o**

Problem 4 – 'Pitch' is the tilt angle of the land, with straight up being $+90^o$, horizontal being 0^o and straight down being -90^o. What is the average pitch of Curiosity's travels during this period and what can you tell about the ground over which it traveled?

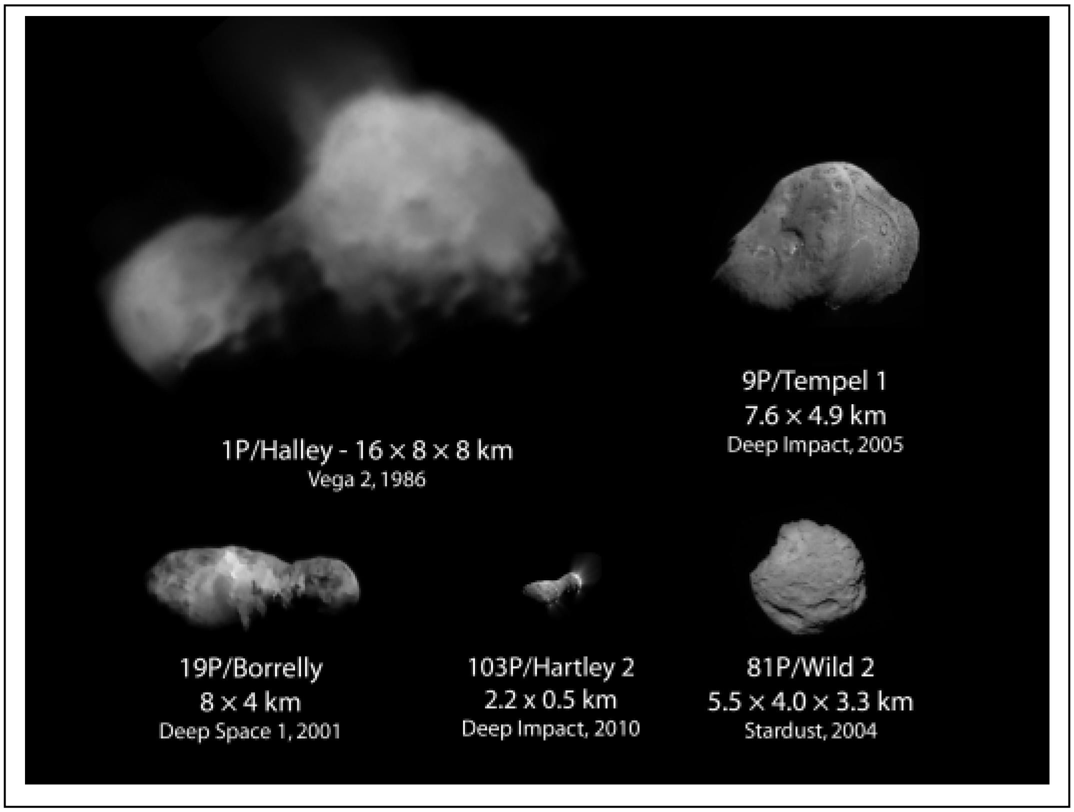
Answer: P = (-1 + -2 + +1 + +1 + +1)/5 = 0^o So the terraine was very level and horizontal.

Problem 5 – The direction to Mt Sharp is at an azimuth of 225^o. What does Curiosity's average azimuth have to be during the next 5 days so that it is back on course to Mt Sharp?

Answer: 225 = [5 (243) + 5(X)] / 10
2250 – 1214 = 5X so **X = 207^o**

Problem 6 – Mt Sharp is located 7.5 km from Curiosity. About how many more Sols will be required for Curiosity to get there?

Answer: The average distance traveled was 68 meters/day so to travel 7500 meters will take T = 7500/68 = 110 Sols if Curiosity does not stop along the way.



Spacecraft have flown-by five comets to study the dense object which produces the dramatic head and tails of these objects as seen from Earth. The figure above shows images of the nuclear objects to the same scale.

Problem 1 – What percentage of comet nuclei are:

A) round

B) potato-shaped?

Problem 2 - What is the range of size, in kilometers, for the dimensions of these nuclei?

Problem 3 – If the range in size represented one side of a cube, what is the range in volumes of the nuclei in cubic kilometers?

Problem 1 – What percentage of comet nuclei are:

A) Round - answer 100% x (2/5) = **40%**

B) potato-shaped? - answer 100% x (3/5) = **60%**

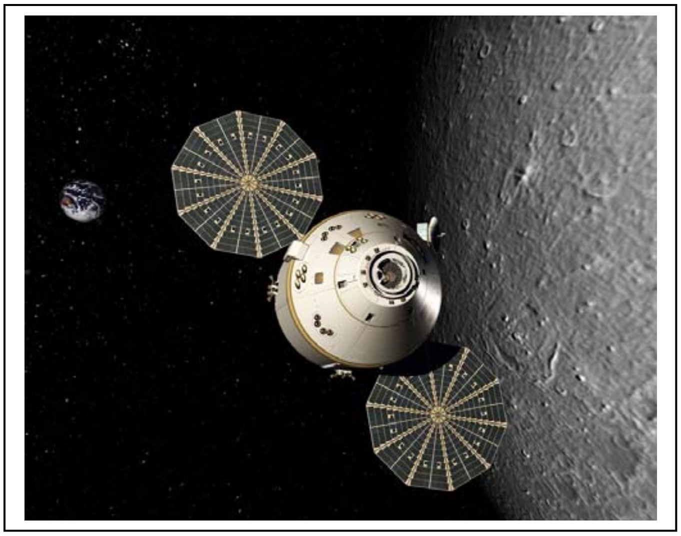
Problem 2 - What is the range of size, in kilometers, for the dimensions of these nuclei?

Answer: The smallest dimension is for Comet Hartley 2 at 0.5 km. The largest dimension is for Halleys Comet at 16 km. **So the range is from 0.5 to 16 kilometers.**

Problem 3 – If the range in size represented one side of a cube, what is the range in volumes of the nuclei in cubic kilometers?

Answer: S = 0.5 km so volume = 0.5 x 0.5 x 0.5 = 0.125 km^3
S = 16 km, so volume = 16 x 16 x 16 = 4096 km^3.

The range of volumes is 0.125 to 4096 km^3.



Astronauts traveling to the moon will not be able to bring everything with them. They will have a very strict allowance on the volume and mass of what they can bring.

Although the first moon rocket, Ares-V/Orion, will not be launched for another 10 years, let's suppose the 'Astronaut Preference Kit' volume limit is 2 1/2 liters per astronaut.

An astronaut wants to bring the following items with him. Assume that the items are all shaped like rectangular solids. Given their dimensions, will he be within the volume limits of the Orion vehicle? (All measurements are in centimeters, and 1,000 cubic centimeters equals 1 liter.)

Item	Length	Width	Height	Total
Toothbrush	17	1	1	
Diary book	25	2	15	
Pen	14	1	1	
Ipod	11	7	1	
Camera	13	9	10	
One paperback book	17	10	2	
Extra glasses	14	5	2	

If the total volume is greater than the PPK limit, what item or items would you leave behind? Is there anything you could do to reduce the volume of these items so that they add up to less than the PPK limit?

If the total volume is less than the PPK limit, what items would you add if the trip was to last 1 week on the Space Shuttle or three months on the International Space Station?

Are there any items you could think of that you would want to substitute instead of the items on this list?

Answer Key

Item	Length	Width	Height	Total
Toothbrush	17	1	1	17
Diary book	25	2	15	750
Pen	14	1	1	14
Ipod	11	7	1	77
Camera	13	9	10	1170
One paperback book	17	10	2	340
Extra glasses	14	5	2	140

The total volume of all these items is 2,508 cubic centimeters. Since 2 1/2 liters equals 2,500 cubic centimeters, the astronaut has to eliminate at least 8 cubic centimeters of material from this list. The total volume is greater than the PPK limit.

Some students may not need glasses, while others may decide that reading a book is the last thing they want to do in space with all those great views out the window!

Here are some of the things that NASA astronauts have brought into space:

- 400 US commemorative stamps (Apollo 15)
- Wedding rings (STS-36)
- 33rd Degree Masonic Ring (Gemini 6)
- Florida Hunting License (Gemini-6)
- Piece of wood from Wright Brothers 1903 airplane (Apollo 11)
- Vial of wine and miniature chalice (Apollo 11)
- Walkman with headphones (Spacelab)
- 1923 US Peace Dollar (Apollo 11)
- Moon Tree seeds (Apollo 14)

Can your students research this topic and find more items?

You have probably seen a telescope before, and wondered how it works!

Telescopes are important in astronomy because they do two things extremely well. Their large lenses and mirrors can collect much more light than the human eye, which make it possible to see very faint things. This is called Light Gathering Ability. They also make distant things look much bigger than what the human eye can see so it is easier to study details. This is called magnification.

The human eye at night is a circle about 7 millimeters in diameter, called the pupil, which lets light pass through its lens and onto the retina. A telescope can have a main mirror or lens that can be many meters in diameter.

How do you figure out how much Light Gathering Ability a telescope has compared to the human eye? Just calculate the area of the two circles and form their ratio!

Problem 1 – The human eye can have a pupil diameter of as much as 7 millimeters. Using the formula for the area of a circle, and a value of π = 3.145, what is the area of the human pupil in square millimeters?

Problem 2 - The Hubble Space Telescope mirror has a diameter of 2.4 meters, which equals 2400 millimeters. What is the area of the Hubble mirror in square millimeters?

Problem 3 – What is the ratio of the area of the Hubble mirror to the human pupil? This is called the Light Gathering Ability of the Hubble Space Telescope!

Problem 4 - The faintest stars in the sky that the human eye can see are called magnitude +6.0 stars. To see magnitude +11 stars, you need a telescope that can see 100 times fainter than the human eye. What is the diameter of the mirror or lens that will let you see these faint stars?

Problem 1 – The human eye can have a pupil diameter of as much as 7 millimeters. Using the formula for the area of a circle, and a value of p = 3.145, what is the area of the human pupil in square millimeters?

Answer: $A = 3.14\ (7/2)^2 =$ **$0.78\ mm^2$**

Problem 2 - The Hubble Space Telescope mirror has a diameter of 2.4 meters, which equals 2400 millimeters. What is the area of the Hubble mirror in square millimeters?

Answer: $A = 3.14\ (2400/2)^2 =$ **$4{,}521{,}600\ mm^2$**

Problem 3 – What is the ratio of the area of the Hubble mirror to the human pupil? This is called the Light Gathering Ability of the Hubble Space Telescope!

Answer: 4,521,600 / 0.78 = **5,796,923 times the human eye**

Problem 4 - The faintest stars in the sky that the human eye can see are called magnitude +6.0 stars. To see magnitude +11 stars, you need a telescope that can see 100 times fainter than the human eye. What is the diameter of the mirror or lens that will let you see these faint stars?

Answer: $100 = \pi R^2/0.78$ so $R^2 = 24.8$ and so R = 5.0 and **D = 10.0 millimeters**.

How do Telescopes Magnify?

Early depiction of a 'Dutch telescope' from the "Emblemata of zinne-werck" (Middelburg, 1624) of the poet and statesman Johan de Brune (1588-1658). The print was engraved by Adriaen van de Venne, who, together with his brother Jan Pieters van de Venne, printed books not far from the original optical workshop of Hans Lipperhey.

Telescopes can magnify the sizes of distant objects so that the eye can see them more clearly. This is very handy for astronomers who want to study distant planets, stars and galaxies to figure out what they are!

A simple telescope, called a refractor, has two lenses. The large one collects the light from a distant object and amplifies it so that the image is much brighter than what the eye normally sees. This is called the Objective Lens, or for reflecting telescopes, the Objective Mirror. A second lens is placed at the focus of the Objective and provides the magnification you need to study the objects.

Both the Objective and the eye lens (called the Eyepiece) have their own focus points. The distance between the lens and this focus point is called the focal length. The magnification of the telescope is just the ratio of the Objective focal length to the eyepiece focal length!

$$M = \frac{\text{Objective focal length}}{\text{Eyepiece focal length}}$$

Note, the units for the focal lengths both have to be the same units…inches…millimeters….etc.

Problem 1 – Galileo's first telescope consisted of two lenses attached to the inside of a tube. The Objective had a focal length of 980 millimeters and the eye lens had a focal length of 50 millimeters. What was the magnification of this telescope?

Problem 2 – In 1686, astronomer Christian Huygens built an 8-inch refractor with a 52 meter focal length. If he used the same magnifying eyepiece that Galileo had used, what would be the magnification of this 'long tube refractor'?

Problem 3 – An amateur builds a 20-inch reflector that has a focal length of 157 inches. He already owns three very expensive eyepieces with focal lengths of 4mm, 20mm and 35 mm. What magnification will he get from each of these eyepieces? (1 inch = 25.4 mm)

Problem 1 – Galileo's first telescope consisted of two lenses attached to the inside of a tube. The Objective had a focal length of 980 millimeters and the eye lens had a focal length of 50 millimeters. What was the magnification of this telescope?

Answer. M = 980/50 = **19.6 times**.

Problem 2 – In 1686, astronomer Christian Huygens built an 8-inch refractor with a 52 meter focal length. If he used the same magnifying eyepiece that Galileo had used, what would be the magnification of this 'long tube refractor'?

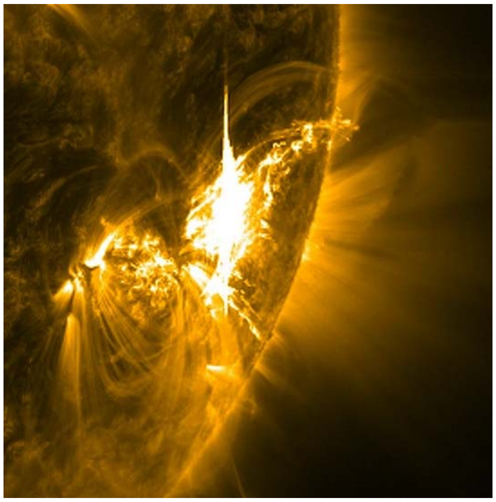
Answer: 52 meters = 52000 millimeters, then for a 50mm eyepiece, the magnification is M = 52000/50 = **1040 times**.

Problem 3 – An amateur builds a 20-inch reflector that has a focal length of 157 inches. He already owns three very expensive eyepieces with focal lengths of 4mm, 20mm and 35 mm. What magnification will he get from each of these eyepieces?

Answer: 157 inches x 25.4 mm/inch = 3988 mm, then

M = 3988/4 = 997x,
M = 3988/20 = 199 x
M = 3988/35 = 114 x

Solar Flares



This image was taken by NASA's Solar Dynamics Observatory on July 6, 2012 and shows a brilliant X-ray solar flare erupting from the sun.

Solar flares are not all the same. Some produce less energy than others, and so astronomers classify them by their X-ray energy using four different letters: B, C, M and X. C-class flares produce 10 times more X-ray energy than B-class flares. M-class flares produce 10 times more energy than C-class flares, and X-class flares produce 10 times more energy than M-class flares. One B-class flare can produce more energy than 240,000 million tons of TNT!

The table below lists all of the M and X-class flares detected between January 1, 2013 and August 15, 2013 at a time when solar activity was near its maximum. This period of time spans the first 227 days of 2013. Also during this time, there were about 690 C-class flares and 440 B-class flares. All of these flares were seen on the side of the sun facing Earth, which represents ½ of the total surface area of the sun.

Day	Flare	Day	Flare	Day	Flare
1-5	M	5-2	M	5-20	M
1-11	M, M	5-3	M, M	5-22	M
1-13	M, M	5-5	M	5-31	M
2-17	M	5-10	M	6-5	M
3-5	M	5-12	M, M	6-7	M
3-15	M	5-13	X, M, X	6-21	M
3-21	M	5-14	X	6-23	M
4-5	M	5-15	X	7-3	M
4-11	M	5-16	M	8-12	M
4-22	M	5-17	M		

Problem 1 - What were the total number of M and X-class flares during this period of time?

Problem 2 – What were the total number of B, C, M and X-class flares detected during this period?

Problem 3 – What percentage of all flares were B, C, M and X?

Problem 4 – What was the average number of B and C-class flares seen each day?

Problem 5 – An astronaut wants to do a spacewalk on a particular day during this period. What are the odds that she will see an M or X-class flare?

The flare data was obtained from
http://www.swpc.noaa.gov/ftpmenu/warehouse/2013/2013_events.html

Problem 1 - What were the total number of M and X-class flares during this period of time?

Answer: By counting Ms in the table, there were **31 M-class and 4 X-class flares**.

Problem 2 – What were the total number of B, C, M and X-class flares detected during this period?

Answer: 690 + 440 + 31 + 4 = **1165 flares**.

Problem 3 – What percentage of all flares were B, C, M and X?

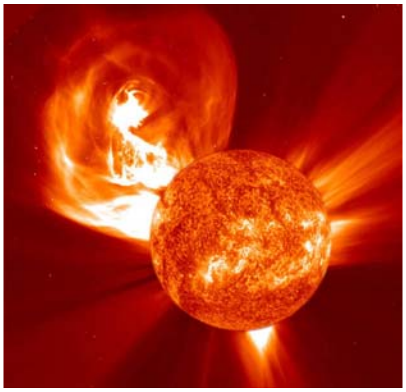
Answer: B = 100% x (440/1165) = 38%
C = 100% x (690/1165) = 59%
M = 100% x (31/1165) = 3%
X = 100% x (4/1165) = 0.3 %

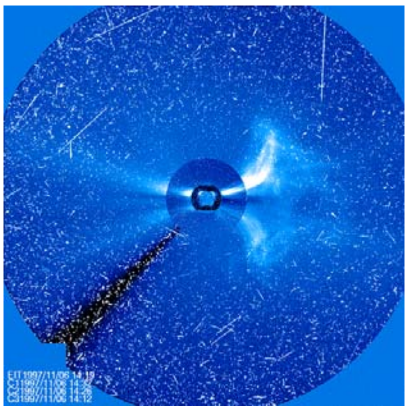
Problem 4 – What was the average number of B and C-class flares seen each day?

Answer: B: 440 flares/227 days = about 2 flares
C: 690 flares/227 days = about 3 flares

Problem 5 – An astronaut wants to do a spacewalk on a particular day during this period. What is the probability that she will see each an M or X-class flare?

Answer: There are 227 days in the sample and 35 M or X-class flares were seen, so the probability is 35/227 = **0.15** which is also stated as 15%. The low probability means that it is not likely that on a random day the astronaut will see anything. In order to have a 50/50 chance, she would have to observe for at least 4 days (4x0.15 = 0.60 which is greater than 0.50 or 50%).





Every once in a while, the sun ejects huge clouds of heated gas, called plasma, which can contains billions of tons of matter and travel at speeds of millions of miles per hour. Occasionally these are directed at earth, and when they arrive they cause brilliant aurora. They can also cause problems for electrical systems on the ground and satellite systems in space.

The top image is a composite that shows the surface of the sun and one of these 'coronal mass ejections' being released. This one is directed away from earth and is harmless to us. When we spot a CME directed towards earth, the cloud seems to form a temporary 'halo' around the edge of the sun. These Halo CMEs are ejected from the sun, and can arrive at earth about 2 to 4 days later.

Soon after a Halo CME is ejected, satellites may detect a rain storm of radiation particles that were ejected from the sun at the same time. These travel so fast that they arrive at earth in only an hour or so. Also called Solar Proton Events (SPEs), these radiation storms are very harmful to astronauts in space and to sensitive satellite electronics. Predicting when SPEs will occur is an important goal of Space Weather Research.

The table below gives the dates for the CMEs detected during the 227 days from January 1, 2013 and August 15, 2013 during the peak of our sun's current storm cycle. Yellow shading indicates that a SPE occurred on the same date.

Date	Type	Date	Type	Date	Type
1-23	Halo	3-15	Halo	5-22	Non-Halo
1-31	Halo	4-11	Halo	6-20	Halo
2-1	Halo	4-20	Non-Halo	7-16	Halo
2-5	Non-Halo	4-21	Non-Halo	7-26	Non-Halo
2-9	Halo	5-17	Halo	8-6	Halo
2-20	Halo	5-19	Halo		

Problem 1 – What is the average number of days between all of the CMEs in this sample?

Problem 2 – What percentage of CMEs are of the Halo-type?

Problem 3 – What percentage of Halo CMEs seem to produce Solar Proton Events?

Problem 4 – If you observed a CME, what is the probability that it may produce a harmful solar proton event?

Problem 1 – What is the average number of days between all of the CMEs in this sample?

Answer: There are a total of 17 CMEs in 227 days so the average interval is about 227/17 = **13 days**.

Problem 2 – What percentage of CMEs are of the Halo-type?

Answer: Of the 17 CMEs, 12 were Halo-type so the percentage is 100%x(12/17) = **71%**

Problem 3 – What percentage of Halo CMEs seem to produce Solar Proton Events?

Answer: Of the 12 Halo-type events, 4 produced SPEs so 100% (4/12) = **33%**

Problem 4 – If you observed a CME, what is the probability that it may produce a harmful solar proton event?

Answer: There were 17 CMEs total of which 6 produced SPEs, so 100% x (6/17) = **35%**.

Note: Of the 6 SPEs, four occurred with Halo CMEs so this means that 4/6 or 67% of all SPEs coincide with Halo-type CMEs, however it is also true that only 4/12=33% of all Halo-type CMEs produce SPEs. Not all Halo events produce solar proton events, so using halo events to predict whether an SPE will occur will lead to a large number of false-positives by about 8 false to 4 positives (8+4 = 12 halo events) for a 2:1 false positive rate.

Era	Time (years)	Description
Pre-solar Nebula Era	0.0	Collapse of cloud to form flattened disk
Asteroid Era	3 million	Formation of large asteroids up to 200 km across ends
Gas Giant Era	10 million	Rapid formation of Jupiter and Saturn ends
Solar Birth Era	50 million	Sun's nuclear reactions start to produce energy in core
Planetessimal Era	51 million	Formation of numerous small planet-sized bodies ends
T-Tauri Era	80 million	Solar winds sweep through inner solar system and strip off primordial atmospheres
Ice Giant Era	90 million	Formation of Uranus and Neptune
Rocky Planet Era	100 million	Formation of rocky planets by mergers of 50-100 smaller bodies
Late Heavy Bombardment Era	600 million	Migration of Jupiter disrupts asteroid belt sending large asteroids to impact planetary surfaces in the inner solar system.
Ocean Era	600 million	LHB transports comets rich in water to Earth to form oceans
Life Era	800 million	First traces of life found in fossils on Earth

For decades, geologists and astronomers have studied the contents of our solar system. They have compared surface features on planets and moons across the solar system, the orbits of asteroids and comets, and the chemical composition and ages for recovered meteorites. From all this effort, and with constant checking of data against mathematical models, scientists have created a timeline for the formation of our solar system.

Our solar system began as a collapsing cloud of gas and dust over 4.6 billion years ago. Over the next 600 million years, called by geologists the Hadean Era, the sun and the planets were formed, and Earth's oceans were probably created by cometary impacts. Comets are very rich in water ice.

The fossil record on Earth shows that the first bacterial life forms emerged about 600 million years after the formation of the solar system. Geologists call this the Archaen Era – The era of ancient life.

Problem 1 – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Problem 2 – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Problem 3 – About how many years ago do the oldest fossils date from on Earth?

Problem 4 – How many years were there between the Planetessimal Era and the end of the Rocky Planet Era?

Problem 5 – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Problem 1 – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Answer: On the Timeline '0.0' represents a time 4.6 billion years ago, so the Rocky Planet Era ended 100 million years after this or **4.5 billion years ago**.

Problem 2 – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Answer: LHB ended 600 million years after Time '0.0' or 4.6 billion – 600 million = **4.0 billion years ago.**

Problem 3 – About how many years ago do the oldest fossils date from on Earth?

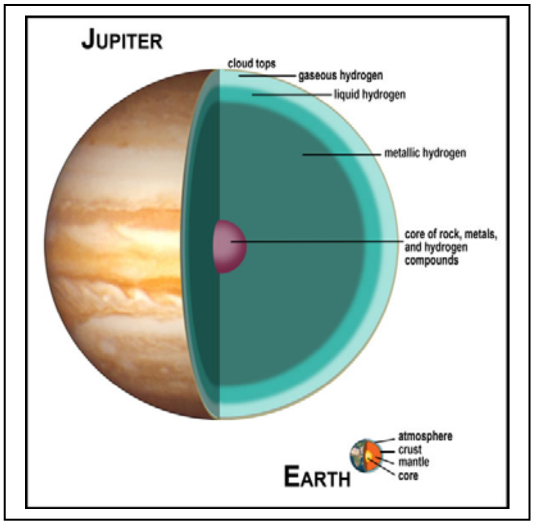
Answer: 4.6 billion – 800 million = **3.8 billion years ago**.

Problem 4 – How many years were there between the Planetessimal Era and the end of the Rocky Planet Era?

Answer: On the timeline the difference is 100 million – 51 million = **49 million years**.

Problem 5 – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Answer: The time interval is 49 million years so the average time between impacts would have been 49 million years/80 impacts = **612,000 years**.



The actual sizes of the major objects in our solar system range from the massive planet Jupiter, to many small moons and asteroids no more than a few kilometers across.

It is often helpful to create a scaled model of the major objects so that you can better appreciate just how large or small they are compared to our Earth.

This exercise will let you work with simple proportions and fractions to create a scaled-model solar system.

Problem 1 - Jupiter is 7/6 the diameter of Saturn, and Saturn is 5/2 the diameter of Uranus. Expressed as a simple fraction, how big is Uranus compared to Jupiter?

Problem 2 – Earth is 13/50 the diameter of Uranus. Expressed as a simple fraction, how much bigger than Earth is the planet Saturn?

Problem 3 – The largest non-planet objects in our solar system, are our own Moon (radius=1738 km), Io (1810 km), Eris (1,500 km), Europa (1480 km), Ganymede (2600 km), Callisto (2360 km), Makemake (800 km),Titan (2575 km), Triton (1350 km), Pluto (1,200 km), Haumea (950 km). Create a bar chart that orders these bodies from smallest to largest. For this sample, what is the: A) Average radius? B) Median radius?

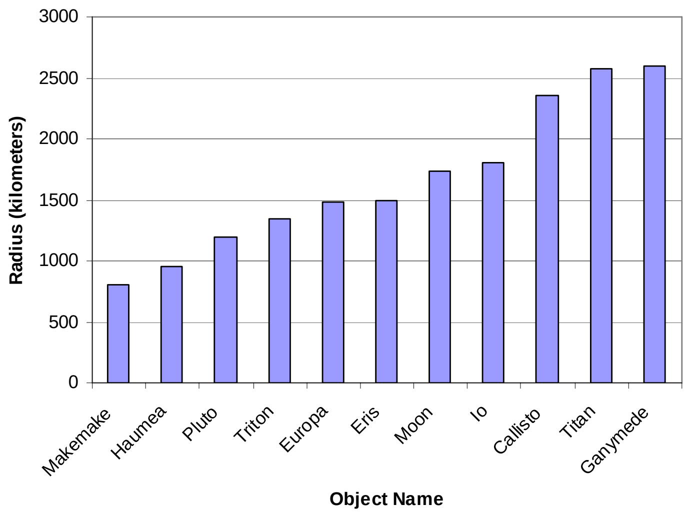
Problem 4 – Mercury is the smallest of the eight planets in our solar system. If the radius of Mercury is 2425 km, how would you create a scaled model of the non-planets if you selected a diameter for the disk of Mercury as 50 millimeters?

Problem 1 - Jupiter is 7/6 the diameter of Saturn, and Saturn is 5/2 the diameter of Uranus. Expressed as a simple fraction, how big is Uranus compared to Jupiter? Answer: 2/5 x 6/7 = **12/35**.

Problem 2 – Earth is 13/50 the diameter of Uranus. Expressed as a simple fraction, how much bigger than Earth is the planet Saturn? Answer: 5/2 x 50/13 = 250/26 = **125/13** times.

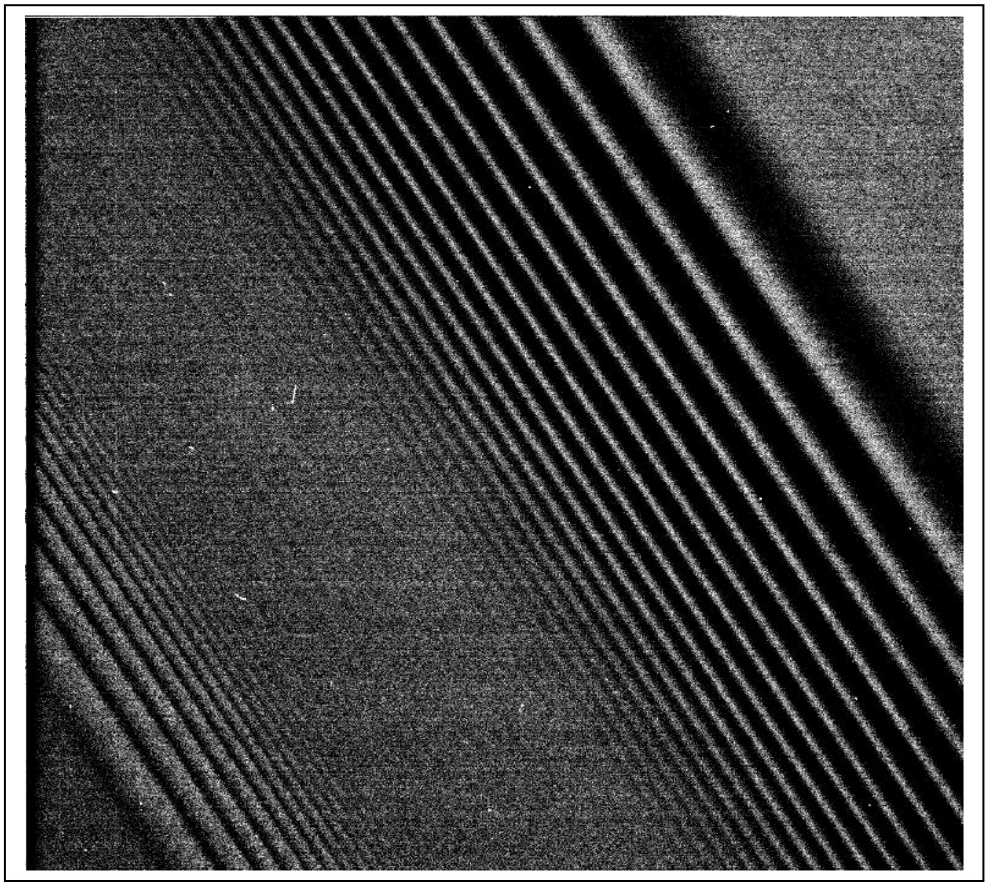
Problem 3 – The largest non-planet objects in our solar system, are our own Moon (radius=1738 km), Io (1810 km), Eris (1,500 km), Europa (1480 km), Ganymede (2600 km), Callisto (2360 km), Makemake (800 km),Titan (2575 km), Triton (1350 km), Pluto (1,200 km), Haumea (950 km). Create a bar chart that orders these bodies from smallest to largest. For this sample, what is the: A) Average radius? B) Median radius?

Answer: Average = (1738+1810+1500+1480+2600+2360+800+2575+1350+1200+950)/11 so **Average radius = 1669 km**. **Median radius = 1500 km**.



Problem 4 – Mercury is the smallest of the eight planets in our solar system. If the radius of Mercury is 2425 km, how would you create a scaled model of the non-planets if you selected a diameter for the disk of Mercury as 50 millimeters? Answer: The scaled disk diameters are shown in Column 3 in millimeters.

Object	Radius (km)	Diameter (mm)
Makemake	800	16
Haumea	950	20
Pluto	1200	25
Triton	1350	28
Europa	1480	30
Eris	1500	30
Moon	1738	36
Io	1810	38
Callisto	2360	48
Titan	2575	54
Ganymede	2600	54



This spectacular close-up image of Saturn's A ring was taken in 2004 by the Cassini spacecraft. It shows a 220-km wide snapshot of a magnified portion of the A ring, and how it dissolves into smaller ringlets. Astronomers think that these ringlets are formed by gravitational interactions with Saturn's inner moons, causing ripples and waves to form that 'bunch up' billions of ring particles into separate ringlets. Some of the bright spots you see in the dark bands may be 'shepherding moonlets' only a few kilometers in size, which keep the ring particles orbiting together.

Problem 1 – By using a millimeter ruler, determine the scale of this image in kilometers/millimeter, and estimate the width of a typical ringlet in this image.

Problem 2 – Draw a diagonal line from the upper right corner (closest to Saturn) to the lower left corner (farthest from Saturn). Number the 16 ringlets in consecutive order starting from the first complete ringlet in the upper right corner. In a table, state the width of each consecutive ringlet in millimeters and kilometers.

Problem 3 – What is the average width of the 16 ringlets you measured to the nearest kilometer?

Problem 4 – Plot the ringlet number and the ringlet width in kilometers. What can you say about the ringlet sizes in this portion of the A ring?

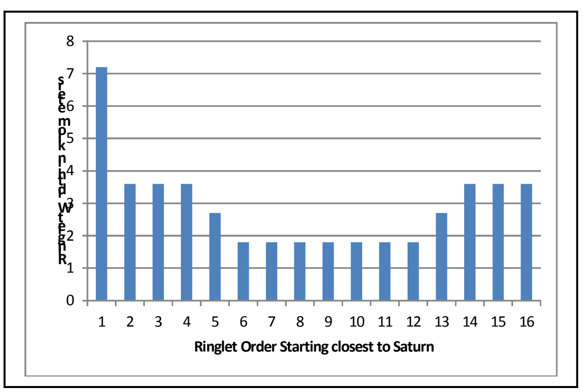
Problem 1 – By using a millimeter ruler, determine the scale of this image in kilometers/millimeter, and estimate the width of a typical ringlet in this image. Answer: When printed on standard 8 ½ x 11 paper, the width of the image is 124 millimeters, so the scale is 220 km/124mm = **1.8 km/mm**.

Problem 2 – Draw a diagonal line from the upper right corner (closest to Saturn) to the lower left corner (farthest from Saturn). Number the 16 ringlets in consecutive order starting from the first complete ringlet in the upper right corner. In a table, state the width of each consecutive ringlet in millimeters and kilometers. Answer: **See below**.

Ringlet	millimeters	kilometers
1	4	7.2
2	2	3.6
3	2	3.6
4	2	3.6
5	1.5	2.7
6	1	1.8
7	1	1.8
8	1	1.8
9	1	1.8
10	1	1.8
11	1	1.8
12	1	1.8
13	1.5	2.7
14	2	3.6
15	2	3.6
16	2	3.6

Problem 3 – What is the average width of the 16 ringlets you measured to the nearest kilometer? Answer: (6x3.6 + 7.2 +2x2.7+7x1.8)/16 = **2.9 kilometers**.

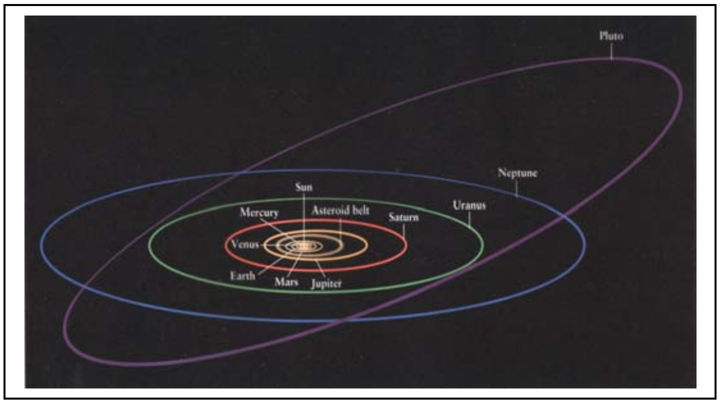
Problem 4 – Plot the ringlet number and the ringlet width in kilometers. What can you say about the ringlet sizes in this portion of the A ring?



Answer: The ringlet widths decrease as you get further from Saturn and are present with distinct dark gaps in between. As the dark gaps become narrower than about 3 km near Ringlet 12, the rings begin to increase slightly in size.

It is 'interesting' that the ringlets in the lower right corner are wider than the upper ringlets, and there are fewer of them.

How Big is Our Solar System?



Our solar system is so big it is almost impossible to imagine its size if you use ordinary units like feet or miles. The distance from Earth to the Sun is 93 million miles (149 million kilometers), but the distance to the farthest planet Neptune is nearly 3 billion miles (4.5 billion kilometers). Compare this to the farthest distance you can walk in one full day (70 miles) or that the International Space Station travels in 24 hours (400,000 miles).

The best way to appreciate the size of our solar system is by creating a scaled model of it that shows how far from the sun the eight planets are located. Astronomers use the distance between Earth and sun, which is 93 million miles, as a new unit of measure called the Astronomical Unit. It is defined to be exactly 1.00 for the Earth-Sun orbit distance, and we call this distance 1.00 AUs.

Problem 1 - The table below gives the distance from the Sun of the eight planets in our solar system. By setting up a simple proportion, convert the stated distances, which are given in millions of kilometers, into their equivalent AUs, and fill-in the last column of the table.

Planet	Distance to the Sun in millions of kilometers	Distance to the Sun in Astronomical Units
Mercury	57	
Venus	108	
Earth	149	
Mars	228	
Jupiter	780	
Saturn	1437	
Uranus	2871	
Neptune	4530	

Problem 2 – Suppose you wanted to build a scale model of our solar system so that the orbit of Neptune was located 10 feet from the yellow ball that represents the sun. How far from the yellow ball, in inches, would you place the orbit of Jupiter?

Problem 1 - The table below gives the distance from the Sun of the eight planets in our solar system. By setting up a simple proportion, convert the stated distances, which are given in millions of kilometers, into their equivalent AUs, and fill-in the last column of the table.

Answer: In the case of Mercury, the proportion you would write would be

$$\frac{149 \text{ million km}}{1 \text{ AU}} = \frac{57 \text{ million km}}{X} \quad \text{then } X = 1 \text{ AU} \times (57/149) = 0.38$$

Planet	Distance to the Sun in millions of kilometers	Distance to the Sun in Astronomical Units
Mercury	57	0.38
Venus	108	0.72
Earth	149	1.00
Mars	228	1.52
Jupiter	780	5.20
Saturn	1437	9.58
Uranus	2871	19.14
Neptune	4530	30.20

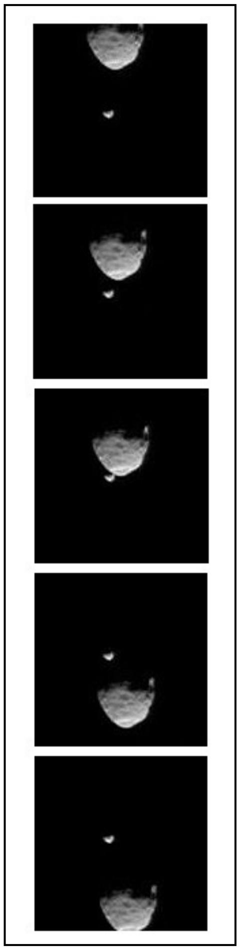
Problem 2 – Suppose you wanted to build a scale model of our solar system so that the orbit of Neptune was located 10 feet from the yellow ball that represents the sun. How far from the yellow ball, in inches, would you place the orbit of Jupiter?

Answer: The proportion would be written as:

$$\frac{30.20 \text{ AU}}{10 \text{ feet}} = \frac{5.2 \text{ AU}}{X} \quad \text{then } X = 10 \text{ feet} \times (5.2/30.2) \quad \text{so } X = \mathbf{1.72\ feet}$$

Since 1 foot = 12 inches, the unit conversion is written as

$$1.72 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} = \mathbf{20.64\ inches}.$$



The larger of the two moons of Mars, Phobos, passes directly in front of the other, Deimos, in a new series of sky-watching images from NASA's Mars rover Curiosity. Large craters on Phobos are clearly visible in these images from the surface of Mars. No previous images from missions on the surface caught one moon eclipsing the other.

Deimos (small image), and Phobos (large image), are shown together as they actually were photographed by the Mast Camera (Mastcam) on NASA's Mars rover Curiosity on Aug. 1, 2013.

How do we figure out how big something will look when it's far away? We draw a scale model of the object showing its diameter and its distance as the two sides of a triangle. The angle can then be measured. As the distance to the object increases, the 'angular size' of the object will decrease proportionately. A simple proportion can then be written that relates the angles to the lengths of the sides:

$$\frac{\text{Apparent Angle in degrees}}{\text{57.3 degrees}} = \frac{\text{True diameter in kilometers}}{\text{Distance in kilometers}}$$

Let's see how this works for estimating the sizes of the moons of Mars as viewed from the Curiosity rover!

Problem 1 – Earth's moon is located 370,000 km from the surface of earth, and has a diameter of 3476 km. About how many degrees across does the lunar disk appear in the sky?

Problem 2 - Deimos has a diameter of 7.5 miles (12 kilometers) and was 12,800 miles (20,500 kilometers) from the rover at the time of the image. Phobos has a diameter 14 miles (22 kilometers) and was 3,900 miles (6,240 kilometers) from the rover at the time of the image. What are the angular diameters of Phobos and Diemos as seen by the Curiosity rover?

Problem 3 – Mars is located 227 million kilometers from the sun, and the sun has a diameter of 1,400,000 kilometers. What is the angular diameter of the sun as viewed from Mars?

Problem 4 – Occasionally, Phobos and Diemos pass across the face of the sun as viewed from the surface of Mars. Will the moons create a full eclipse of the sun in the same way that Earth's moon covers the full face of the sun as viewed from Earth?

NASA Rover Gets Movie as a Mars Moon Passes Another
http://www.nasa.gov/mission_pages/msl/news/msl20130815.html
August 15, 2013

Problem 1 – Earth's moon is located 370,000 km from the surface of earth, and has a diameter of 3476 km. About how many degrees across does the lunar disk appear in the sky?

Answer: Apparent size = 57.3 x (3476/370000) = **0.5 degrees**.

Problem 2 - Deimos has a diameter of 7.5 miles (12 kilometers) and was 12,800 miles (20,500 kilometers) from the rover at the time of the image. Phobos has a diameter 14 miles (22 kilometers) and was 3,900 miles (6,240 kilometers) from the rover at the time of the image. What are the angular diameters of Phobos and Diemos as seen by the Curiosity rover?

Answer: Diemos: 57.3 x (12/20500) = **0.033 degrees**
Phobos: 57.3 x (22/6240) = **0.2 degrees**.

Note: Diemos appears to be about .033/.2 = 1/6 the diameter of Phobos in the sky.

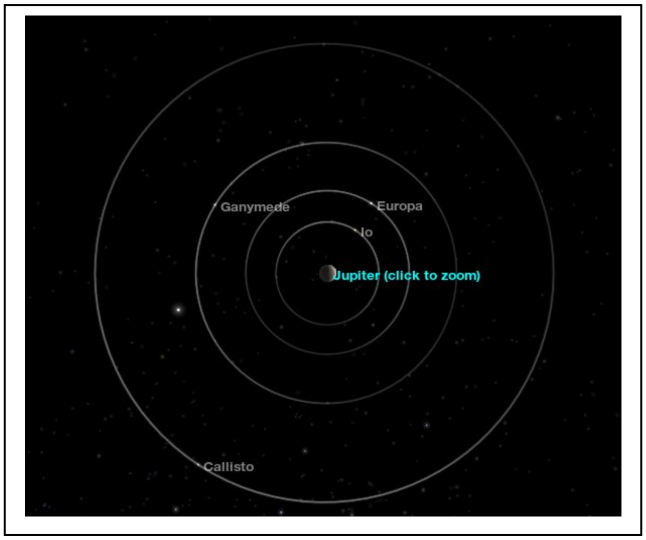
Problem 3 – Mars is located 227 million kilometers from the sun, and the sun has a diameter of 1,400,000 kilometers. What is the angular diameter of the sun as viewed from Mars?

Answer: 57.3 x (1400000/227,000,000) = 57.3 x (1.4/227) = **0.35 degrees**.

Problem 4 – Occasionally, Phobos and Diemos pass across the face of the sun as viewed from the surface of Mars. Will the moons create a full eclipse of the sun in the same way that Earth's moon covers the full face of the sun as viewed from Earth?

Answer: Diemos is only 0.033/0.35 = 1/11 the diameter of the sun in the sky as viewed from Mars, so it does not cover the full disk of the sun. As it passes across the sun it would look like a large dark spot. Phobos is 0.2/0.35 = ½ the diameter of the sun in the sky and it would not produce an eclipse like our moon does. It would look like a large black spot ½ the diameter of the sun.

Apparent Sizes of Objects from Jupiter's Moon Europa



In the future, rovers will land on the moons of Jupiter just as they have on Mars. Rover cameras will search the skies for the disks of nearby moons. One candidate for landing is Europa with its ocean of water just below its icy crust.

The figure to the left shows the orbits of the four largest moons near Europa. How large will they appear in the European sky compared to the Earth's moon seen in our night time skies? The apparent angular size of an object in arcminutes is found from the proportion:

$$\frac{\text{Apparent size}}{\text{3438 arcminutes}} = \frac{\text{Diameter (km)}}{\text{Distance (km)}}$$

The table below gives the diameters of each 'Galilean Moon' together with its minimum and maximum distance from Europa. Jupiter has a diameter of 142,000 km. Europa has a diameter of 2960 km. Callisto's diameter is 4720 km and Ganymede's diameter is 5200 km. The sun hass a diameter of 1.4 million km. Our Moon has a diameter of 3476 km.

	Shortest Distance (km)	Size (arcminutes)	Longest Distance (km)	Size (arcminutes)
Europa to Callisto	1.2 million		2.6 million	
Europa to Io	255,000		1.1 million	
Europa to Ganymede	403,000		1.7 million	
Europa to Jupiter	592,000		604,000	
Europa to Sun	740 million		815 million	
Earth to Moon	356,400		406,700	

Problem 1 – From the information in the table, calculate the maximum and minimum angular size of each moon and object as viewed from Europa.

Problem 2 – Compared to the angular size of the sun as seen from Jupiter, are any of the moons viewed from Europa able to completely eclipse the solar disk?

Problem 3 – Which moons as viewed from Europa would have about the same angular diameter as Earth's moon viewed from Earth?

Problem 4 – Io is closer to Jupiter than Europa. That means that Io will be able to pass across the face of Jupiter as viewed from Europa. . In terms of the maximum and minimum sizes, about how many times smaller is the apparent disk of Io compared to the disk of Jupiter?

Problem 1 – From the information in the table, calculate the maximum and minimum angular size of each moon and object as viewed from Europa. Jupiter has a diameter of 142,000 km. Io has a diameter of 3620 km. Callisto's diameter is 4720 km and Ganymede's diameter is 5200 km. The sun has a diameter of 1.4 million km. Our Moon has a diameter of 3476 km. Answer: see below.

	Shortest Distance (km)	Size (arcminutes)	Longest Distance (km)	Size (arcminutes)
Europa to Callisto	1.2 million	**13.5**	2.6 million	**6.2**
Europa to Io	255,000	**48.8**	1.1 million	**11.3**
Europa to Ganymede	403,000	**44.4**	1.7 million	**10.5**
Europa to Jupiter	592,000	**824.7**	604,000	**808.3**
Europa to Sun	740 million	**6.5**	815 million	**5.9**
Earth to Moon	356,400	**33.5**	406,700	**29.4**

Problem 2 – Compared to the angular size of the sun as seen from Jupiter, are any of the moons viewed from Europa able to completely eclipse the solar disk?

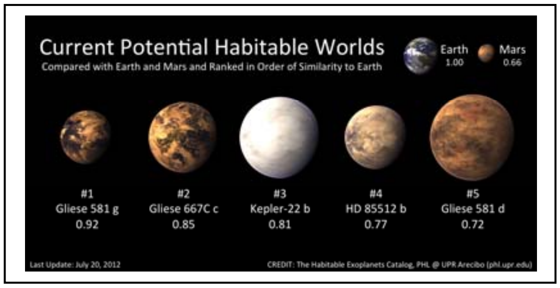
Answer: The solar disk has an angular size between 5.9 and 6.5 arcminutes. Only Callisto at its longest distance has an angular diameter (6.2 arcminutes) close to the solar diameter and so a complete eclipse is possible.

Problem 3 – Which moons as viewed from Europa would have about the same angular diameter as Earth's moon viewed from Earth?

Answer: Io and Ganymede have angular diameters between 11 and 49 arcminutes. Our moon has a range of sizes between 29.4 and 33.5 arcminutes, so at some points in the orbits of Io and Ganymede, they will appear about the same size as our moon dies in our night sky.

Problem 4 – Io is closer to Jupiter than Europa. That means that Io will be able to pass across the face of Jupiter as viewed from Europa. In terms of the maximum and minimum sizes, about how many times smaller is the apparent disk of Io compared to the disk of Jupiter?

Answer: When Jupiter appears at its largest (824 arcminutes) and Io at its smallest (11.3 arcminutes) Jupiter will be 73 times bigger than the disk of Io. When Jupiter is at its smallest (808 arcminutes) and Io is at its largest (48.8 arcminutes) Jupiter will appear to be 16.6 times larger than the disk of Io.



By 2013, astronomers have detected over 941 confirmed planets, and 2500 additional candidates, which orbit stars beyond our solar system. Many of these are earth-like and could have liquid water on their surfaces.

Find the roots to the quadratic equations below and match the X-value to the Word Bank to find the word to use in the essay about exoplanets.

Since 1995, astronomers have discovered over 900 planets orbiting other stars. Because they are not planets orbiting our own sun, they are called 1)_____________. At first, astronomers could only detect planets that were larger than 2)____________. Those were often found orbiting their stars much closer than 3)_____________ orbits our own sun. Then as instruments improved in their 4)_____________, planets as large as the outer planet 5)__________ and as small as our own Earth were found. NASA's 6)______________ observatory in space was launched in 2010 and has since discovered over 2000 new candidate planets, and many of these are Earth-like in size. What is also so exciting is that 7)________ of these earth-sized worlds orbit far enough from their stars that liquid 8)__________ can exist on their surfaces if the planets have dense-enough 9)_______________.

Some exoplanets are unbelievably strange. The world orbiting the star HD209458 looks like a Jupiter-sized comet. It is so close to its star that this planet is actually evaporating. Another exoplanet called 10)____________ is located in the constellation Monoceros. It is so hot that its clouds rain droplets of liquid 11)_________ onto its surface, which is covered by a liquid lava ocean. The Hubble and 12)__________ Space Telescopes have actually seen two planets orbiting the stars 13)___________ and 14)_________ in the constellations Pisces Austrinus and Vulpecula. They are about the size of Jupiter and orbit their stars far beyond the orbit of Neptune in our solar system. One star called 15)__________ located 127 light years from Earth in the constellation Hydrus, has seven planets but they all orbit closer to their star than Mars in our own solar system. Five of these planets are as big as Neptune!

1) x^2-2x-3
2) x^2-16
3) $x^2+3x-10$
4) $2x^2-6x$
5) $3x^2-12$
6) $5x^2+30x-35$
7) $x^2-13x+36$
8) $2x^2-12x+10$
9) $4x^2-8x-12$
10) $x^2+4x-12$
11) $x^2+11x+30$
12) $x^2-13x+42$
13) $x^2-16X+63$
14) $3x^2+24X-27$
15) $2x^2-12x-32$

Word Bank

+3 exoplanets	0 sensitivity	-2 Neptune
-1 atmospheres	-7 Kepler	+1 water
+8 HD10180	+9 many	+4 few
-4 Jupiter	+5 oxygen	-6 CoRot7b
-5 iron	+6 Spitzer	+7 Fomalhaut
+2 Mercury	-9 HD189733	+10 methane

Since 1995, astronomers have discovered over 900 planets orbiting other stars. Because they are not planets orbiting our own sun, they are called 1)___**exoplanets**__. At first, astronomers could only detect planets that were larger than 2)_____**jupiter**__. Those were often found orbiting their stars much closer than 3)__**mercury**__ orbits our own sun. Then as instruments improved in their 4)__**sensitivity**__, planets as large as the outer planet 5)__**Neptune** and as small as our own Earth were found. NASA's 6)_**Kepler**__ observatory in space was launched in 2010 and has since discovered over 2000 new candidate planets, and many of these are Earth-like in size. What is also so exciting is that 7)__**many**__ of these Earth-sized worlds orbit far enough from their stars that liquid 8)__**water**_ can exist on their surfaces if the planets have dense-enough 9)__**atmospheres**__.

Some exoplanets are unbelievably strange. The world orbiting the star HD209458 looks like a Jupiter-sized comet. It is so close to its star that this planet is actually evaporating. Another exoplanet called 10)__**CoRot7b**__ is located in the constellation Monoceros. It is so hot that its clouds rain droplets of liquid 11)__**iron** onto its surface, which is covered by a liquid lava ocean. The Hubble and 12)__**Spitzer**__ Space Telescopes have actually seen two planets orbiting the stars 13)_**Fomalhaut**__ and 14)__**HD189733**__ in the constellations Pisces Austrinus and Vulpecula. They are about the size of Jupiter and orbit their stars far beyond the orbit of Neptune in our solar system. One star called 15)_**HD101080**__ located 127 light years from Earth in the constellation Hydrus, has seven planets but they all orbit closer to their star than Mars in our own solar system. Five of these planets are as big as Neptune!

1)	x^2-2x-3	: (x-3)(x+1)	x = +3, x= -1	**exoplanet**, atmospheres
2)	x^2-16	: (x-4)(x+4)	x= +4 , x=-4	few, **jupiter**
3)	x^2+3x-10	: (x+5)(x-2)	x=-5, x=+2	iron, **mercury**
4)	2x^2-6x	: x(2x-6)	x=0, x=+3	**sensitivity**, exoplanets
5)	3x^2-12	: (3x-6)(x+2)	x=+2, x=-2	mercury, **neptune**
6)	5x^2+30x-35	: (5x-5)(x+7)	x=-7, x=+1	**Kepler**, water
7)	x^2-13x+36	: (x-9)(x-4)	x=+9, x=+4	**many**, few
8)	2x^2-12x+10	: (2x-2)(x-5)	x=+1, x=+5	**water**, oxygen
9)	4x^2-8x-12	: (x+1)(4x-12)	x=-1, x=+3	**atmosphere**s, exoplanets
10)	x^2+4x-12	: (x+6)(x-2)	x=-6, x=+2	**CoRot7b**, mercury
11)	x^2+11x+30	: (x+5)(x+6)	x=-5, x=-6	**iron**, CoRot7b
12)	x^2-13x+42	: (x-6)(x+7)	x=+6, x=-7	**Spitzer**, Kepler
13)	x^2-16X+63	: (x-7)(x-9)	x=+7, x=+9	**Fomalhaut**, many
14)	3x^2+24x-27	: (3x+27)(x-1)	x=-9, x+1	**HD189733**, water
15)	2x^2-12X-32	: (2x-16)(x+2)	x=+8, x=-2	**HD10180**, neptune

The Origin of Our Universe Number Puzzle

About 14 billion years ago, our entire universe came into existence in an event called the 1)____________. Because this state of matter, energy, light, space and time were so unimaginably extreme, astronomers prefer to call it the 2)____________. Before this event, mathematical models predict that there was no space, time, matter or energy at all. Just a pure 3)____________. No sooner did the event begin, but space and time began to 4)__________, causing the matter and energy to cool at a fantastic rate. After the first second following the Big Bang, matter and energy were still 1000 times hotter than the center of our sun. No matter where you looked in the universe, all you would see was this intensely hot matter and radiation far brighter than our own 5)_______. It took over three minutes for the universe to cool to the temperature of our own sun's interior. Through 6)___________ reactions, only hydrogen and 7)_______ nuclei were formed, but the intense 8)________ of light still existed everywhere in space. It would take another million years before the universe had cooled to a few thousand degrees. This allowed then free 9)__________ to combine with the hydrogen and helium nuclei to form normal atoms. After another 100 million years had passed, the intense fireball light had finally dimmed to 10)__________ and the universe entered the 11)____________. During this time, the universe was completely dark with no visible light anywhere. Within this darkness, clouds of hydrogen and helium gas began to form and collapse under their own gravity forming the first generations of stars. These stars exploded as 12)___________ and littered the universe with carbon, 13)________ and other elements. After 200 million years the first 14)_________ formed, and after another 9 billion years our own sun and 15)___________ formed.

Word Bank

+6 Earth	+2 nuclear	-3 expand
-2 Dark Ages	+9 fireball	+8 helium
-1 sun	+1 galaxies	+15 supernova
+10 Singularity	+4 electrons	+27 invisibility
+5 Big Bang	+7 nothingness	+19 oxygen

Solve these questions and use the integer to find the word from the Word Bank

1) The distance between two galaxies increases from 100 to 500. What is the dilation factor?
2) Distance between the points (+3,+5) and (-3, +13)
3) Solve for x: $3x10^{3}$ x $4x10^{x} = 1.2x10^{11}$
4) y=3x+5 and y=2x+2 intersect at the point (x,-4) what is x?
5) Use 2-point distance formula to solve for x: (x,+3) and (+5,+11) where distance = 10.
6) A figure is dilated by 8 and contracted by 4, what is its final dilation factor?
7) Solve for x: $6x10^{15}/3x10^{x} = 2x10^{7}$
8) $T=10^{10}/t^{1/2}$. What is our universe's temperature T after t = 100 seconds: $1x10^{x}$.
9) Slope of the line perpendicular to y = -0.25x+3
10) After universe expanded three times, what was the volume increase?
11) Largest factor (a or b) of $x^{2}+5x+6$: (x+a)(x+b)
12) Distance from (4,2) to (1,-2) after a dilation of 3x
13) T = 2.7 (1+z). At what redshift, z, will the cosmic temperature equal 54 degrees?
14) Solve for intersection of $x^{2}+3x-1$ and y=2x+1 (x,+3)
15) Distance between (3,6) and (0,3) after a dilation of $2^{1/2}$

About 14 billion years ago, our entire universe came into existence in an event called the 1)__**Big Bang**__. Because this state of matter, energy, light, space and time were so unimaginably extreme, astronomers prefer to call it the 2)_**Singularity**__. Before this event, mathematical models predict that there was no space, time ,matter or energy at all. Just a pure 3)_**Nothingness**__. No sooner did the event begin, but space and time began to 4)_**expand**, causing the matter and energy to cool at a fantastic rate. After the first second following the Big Bang, matter and energy were still 1000 times hotter than the center of our sun. No matter where you looked in the universe, all you would see was this intensely hot matter and radiation far brighter than our own 5)__**sun**__. It took over three minutes for the universe to cool to the temperature of our own sun's interior. Through 6)__**nuclear**__ reactions, only hydrogen and 7)_ **helium**__ nuclei were formed, but the intense 8)_**fireball**__ of light still existed everywhere in space. It would take another million years before the universe had cooled to a few thousand degrees. This allowed the free 9)_**electrons**__ to combine with the hydrogen and helium nuclei to form normal atoms. After another 100 million years had passed, the intense fireball light had finally dimmed to 10)__**invisibility**__ and the universe entered the 11)_**Dark Ages**__. During this time, the universe was completely dark with no visible light anywhere. Within this darkness, clouds of hydrogen and helium gas began to form and collapse under their own gravity forming the first generations of stars. These stars exploded as 12)_**supernova**__ and littered the universe with carbon, 13)__**oxygen**__ and other elements. After 200 million years the first 14)__**galaxies**__ formed, and after another 9 billion years our own sun and 15)__**Earth**__ formed.

1) The distance between two galaxies increases from 100 to 500. What is the dilation factor? Answer: 500/100 = +5, word = **Big Bang**

2) Distance between the points (+3,+5) and (-3, +13): $d^2 = (-3-3)^2 + (13-5)^2 = 100$ so d = +10 word = **Singularity**

3) Solve for x: $3x10^3 \times 4x10^x = 1.2x10^{11}$ so 3+x +1 = 11, so x = +7 word = **Nothingness**

4) y=3x+5 and y=2x+2 intersect at the point (x,-4). 3x+5 = 2x+2, so x=-3 and word = **expand**

5) Use 2-point distance formula to solve for x: (x,+3) and (+5,+11) where distance = 10. $100 = (5-x)^2 + (11-3)^2$ so $100 - 64 = (5-x)^2$; $36 = (5-x)^2$ so x = -1 and word = **sun**

6) A figure is dilated by 8 and contracted by 4, what is its final dilation factor? 8/4 = 2 word = **nuclear**.

7) Solve for x: $6x10^{15}/3x10^x = 2x10^7$ answer: 15-x = 7 so x = +8 and word = **helium**

8) $T=10^{10}/t^{1/2}$. What is the universe temperature T after t = 100 seconds: 1x10x. $T = 10^{10}/(100)^{1/2} = 10^9$ so x = +9 and word = **fireball**

9) Slope of the line perpendicular to y = -0.25x+3: answer: m = 4.0 word **= electrons**

10) After universe expanded three times, what was the volume increase? $3^3 = 27$ so word = **invisibility**

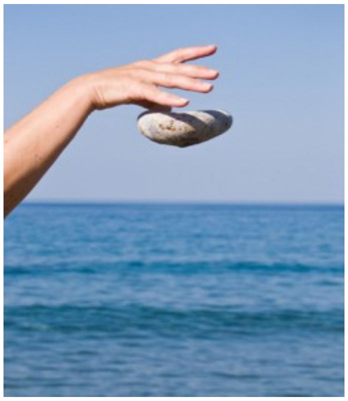
11) Largest factor (a or b) of x^2+5x+6 : (x+a)(x+b) a=+3, b=+2 so x=-3, x=-2 and –2 is largest so word = **Dark Ages**.

12) Distance from (4,2) to (1,-2) after a dilation of 3x. Answer: $d^2 = (1-4)^2 + (-2-2)^2 = 25$, d = 5 and 5x3 = 15. Word = **supernova.**

13) T = 2.7 (1+z). At what redshift, z, will the cosmic temperature equal 54 degrees? Answer: 54=2.7(1+z) so z = +19. The word = **oxygen**

14) Solve for intersection of x^2+3x-1 and y=2x+1 (x,+3) : Answer: $x^2+3x-1 = 2x+1$ so x^2+x-2 factors (x+2)(x-1) x=-2, x=+1 The two intersection points are (-2, -3) and (+1, +3) so x = +1 and word = **galaxies**.

15) Distance between (3,6) and (0,3) after a dilation of $2^{1/2}$. Answer: $d^2 = (0-3)^2 + (3-6)^2 = 18$ $d = (18)^{1/2}$ then after dilation $d = (18)^{1/2} \times (2)^{1/2} = (36)^{1/2} = +6$ or -6, but since no word exists for x=-6, we have word = **Earth**.



One of the simplest kinds of motion that were first studied carefully is that of falling. Unless supported, a body will fall to the ground under the influence of gravity. But the falling does not happen smoothly. Instead, the speed of the body increases in proportion to the elapsed time. This is called acceleration.

Near Earth's surface, the speed of a body increases 32 feet/sec (9.8 meters/sec) for every elapsed second. This is usually written as an acceleration of 32 feet/sec/sec or 32 feet/sec^2. (also 9.8 meters/sec^2) A simple formula gives you the speed of the object after an elapsed time of T seconds:

$$S = 32\,T \quad \text{in feet/sec}$$

If instead of just dropping the object, you threw it downwards at a speed of 12 feet/sec (3 meters/sec) you could write the formula as:

$$S = 12 + 32T \quad \text{feet/sec}$$

In general you could also write this by replacing the selected speed of 12 feet/sec, with a fill-in speed of S_0 to get

$$S = S_0 + 32T \quad \text{feet/sec.}$$

Problem 1 – On Earth, a ball is dropped from an airplane. If the initial speed was 0 feet/sec, how many seconds did it take to reach a speed of 130 miles per hour, which is called the Terminal Velocity? (130 mph = 192 feet/sec)

Problem 2 – On Mars, the acceleration of gravity is only 12 feet/sec^2. A rock is dropped from the edge of the huge canyon called Valles Marineris and falls 20,000 to the canyon floor. If the impact speed was measured to be 700 feet/sec (480 mph) how long did it take to impact the canyon floor?

Problem 3 – Two astronauts standing on the surface of two different objects in the solar system want to decide which object is the largest in mass. The first astronaut drops a hammer off a cliff that is exactly 5000 feet tall and measures the impact speed with a radar gun to get 100 feet/sec. It takes 8 seconds for the hammer to hit the bottom. The second astronaut drops an identical hammer off a cliff that is only 500 feet tall and also measures the impact speed at 150 feet/sec, but he accidentally gave the object a release speed of 5 feet/sec. It takes 29 seconds for the hammer to reach the ground. They can't re-do the experiments, but given this information what are the accelerations of gravity on the two bodies and which one has the highest mass?

Problem 1 – On Earth, a ball is dropped from an airplane. If the initial speed was 0 feet/sec, how many seconds did it take to reach a speed of 130 miles per hour, which is called the Terminal Velocity? (130 mph = 192 feet/sec)

Answer: 192 = 32T, so T = **6 seconds**.

Problem 2 – On Mars, the acceleration of gravity is only 12 feet/sec^2. A rock is dropped from the edge of the huge canyon called Valles Marineris and falls 20,000 to the canyon floor. If the impact speed was measured to be 700 feet/sec (480 mph) how long did it take to impact the canyon floor?

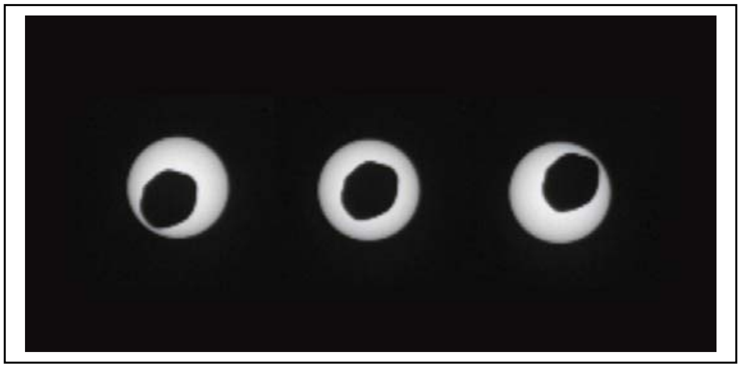
Answer: 700 = 12T, so T = **58 seconds**.

Problem 3 – Two astronauts standing on the surface of two different objects in the solar system want to decide which object is the largest in mass. The first astronaut drops a hammer off a cliff that is exactly 5000 feet tall and measures the impact speed with a radar gun to get 100 feet/sec. It takes 8 seconds for the hammer to hit the bottom. The second astronaut drops an identical hammer off a cliff that is only 500 feet tall and also measures the impact speed at 150 feet/sec, but he accidentally gave the object a release speed of 5 feet/sec. It takes 29 seconds for the hammer to reach the ground. They can't re-do the experiments, but given this information what are the accelerations of gravity on the two bodies and which one has the highest mass?

Answer: Astronaut 1. 100 = A1 x T1 T1 = 8 seconds, so A1 = 100/8 = 12 feet/sec^2.

Astronaut 2: 150 = 5 + A2 T2 T2 = 29 sec, so A2 = (150-5)/29 = 5 feet/sec^2

The acceleration measured by the second astronaut is much lower than for the first astronaut, so the first astronaut is standing on the more-massive objects. In fact, Astronaut 1 is on Mars and Astronaut 2 is on the moon!



This set of three images shows views three seconds apart as the larger of Mars' two moons, Phobos, passed directly in front of the sun as seen by NASA's Mars rover Curiosity.

Curiosity photographed this annular eclipse with the rover's Mast Camera on August 17, 2013 or 'Sol 369' by the Mars calendar.

Curiosity paused during its drive to Mount Sharp to take a set of observations that the camera team carefully calculated to record this celestial event. Because this eclipse occurred near mid-day at Curiosity's location on Mars, Phobos was nearly overhead. This timing made Phobos' silhouette larger against the sun -- as close to a total eclipse of the sun as is possible from Mars.

Angular size is given by $$\Theta = 57.3 \times \frac{\text{Diameter (km)}}{\text{Distance (km)}} \text{ degrees}$$

Problem 1 – At the time of the transit, Phobos which has a diameter of 11 km, was 6000 km from the surface of Mars, and Mars was 235 million km from the Sun. What are the angular diameters of the Sun and Phobos viewed from the surface of Mars if the diameter of the Sun is 1.4 million km? How large are these angles in minutes of arc?

Problem 2 – Phobos orbits at a distance of 9,400 km from the center of Mars at a speed of 2.1 km/sec. As viewed from the surface of Mars (6000 km), how fast is it traveling across the sky in arcminutes/second?

Problem 3 – To the nearest second, how long will it take for Phobos to travel completely across the disk of the sun?

Annular Eclipse of the Sun by Phobos, as Seen by Curiosity
http://www.nasa.gov/mission_pages/msl/news/msl20130828.html
Aug. 28, 2013

Problem 1 – At the time of the transit, Phobos which has a diameter of 11 km, was 6000 km from the surface of Mars, and Mars was 235 million km from the Sun. What are the angular diameters of the Sun and Phobos viewed from the surface of Mars if the diameter of the Sun is 1.4 million km? How large are these angles in minutes of arc?

Answer: Phobos: 57.3 x (11/6000) = **0.10 degrees**
or 0.1 degrees x 60 = **6 minutes of arc**

Sun: 57.3 x (1.4/235) = **0.34 degrees** or 0.34 x 60 = **20 minutes of arc**.

Problem 2 – Phobos orbits at a distance of 9,400 km from the center of Mars at a speed of 2.1 km/sec. As viewed from the surface of Mars (6000 km), how fast is it traveling across the sky in arcminutes/second?

Answer: If the object is located 6000 km from the surface and moves 2.1 km, it will appear to cover an angle of 57.3 x (2.1/6000) = 0.02 degrees. There are 60 arcminutes in 1 degree, so this angle is 1.2 arcminutes. Since this distance is traveled in 1 second, the angular speed is **1.2 arcminutes /second**.

Problem 3 – To the nearest second, how long will it take for Phobos to travel completely across the disk of the sun?

Answer: The diameter of the sun is 20 arcminutes and the diameter of Phobos is 6 arcminutes. When the center of the disk of Phobos is 3 arcminutes from the eastern edge of the sun, it is just touching the solar disk and about to start its transit. When it is 3 arcminutes from the western edge of the sun, it has just finished its transit, so the total distance it has to travel is 3 arcminutes + 20 arcminutes + 3 arcminutes or 26 arcminutes. It travels at a speed of 1.2 arcminutes per second, to it will cover 26 arcminutes in about 26/1.2 = **22 seconds**.

The Launch of LADEE to the Moon

NASA launched its Lunar Atmosphere and Dust Environment Explorer (LADEE) at 11:27 p.m. EDT Friday, September 6, 2013 from the agency's Wallops Flight Facility in Virginia. LADEE is scheduled to arrive at the moon in 30 days, then enter lunar orbit.

NASA's LADEE mission caused a sensation in the Eastern United States. Friday's launch was visible from Virginia to Massachusetts. The fireball was captured by photographers and videographers in many locations. Crowds gathered in Times Square in New York and on the steps of the Lincoln Memorial in Washington to watch.

The table below gives the flight data during the Stage 3 ignition period beginning 190 seconds after launch and just before Stage 3 burnout at 210 seconds.

Time (seconds)	Altitude (kilometers)	Range (kilometers)	Speed (meters/sec)
190	150	446	5339
192	151	456	5344
194	153	467	5535
196	154	478	5629
198	156	489	5744
200	158	500	5853
202	159	511	5962
204	161	522	5991
206	163	533	5993
208	164	546	5990

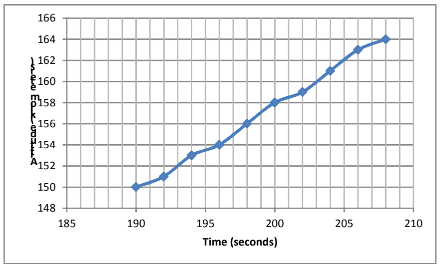
Problem 1 – Create a graph of the altitude in kilometers versus the time after launch in seconds. Over the plotted interval, what is the average slope of the line, and what does it represent? (Use proper units for the slope based on the graph)

Problem 2 – The distance between the launch gantry and the point directly under the current position of the rocket is called the range. Create a graph of the range of the rocket over this time interval. What is the average slope of the plotted line, and what does it represent? (Use proper units for the slope based on the graph)

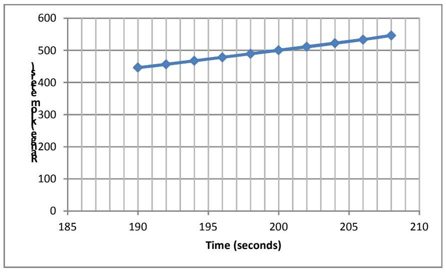
Problem 3 – Graph the speed of the rocket over the interval between 192 and 202 seconds. What is the average slope of the line, and what does this represent?
(Use proper units for the slope based on the graph)

LADEE Lights Up the East Coast
http://www.nasa.gov/content/ladee-lights-up-the-east-coast/index.html#.Uiq1e3eO6So
September 7, 2013

Problem 1 – Create a graph of the altitude in kilometers versus the time after launch in seconds. Over the plotted interval, what is the average slope of the line, and what does it represent? (Use proper units for the slope based on the graph). Answer: Average slope = (164-150)/(208-190) = +14 km/18 sec = **+0.78 km/sec. This represents the vertical speed of the rocket**.

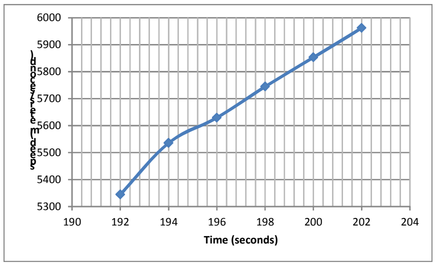


Problem 2 – The distance between the launch gantry and the point directly under the current position of the rocket is called the range. Create a graph of the range of the rocket over this time interval. What is the average slope of the plotted line, and what does it represent? (Use proper units for the slope based on the graph). Answer: Slope = (546-446)/(208-190) = + 100km/18sec = **+5.56 km/sec. This represents the horizontal speed of the rocket.**



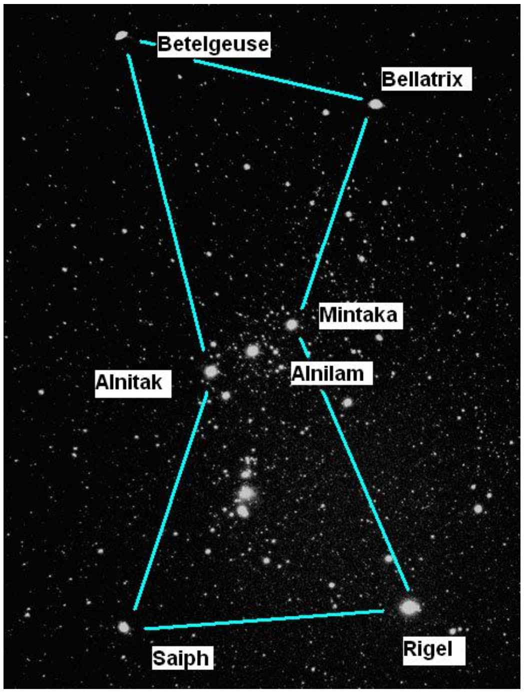
Note to Teacher: The vertical speed and horizontal speed are the legs of a right-triangle. The hypothenuse is the total speed of the rocket. Using the Pythagorean Theorem we can find $S^2 = (0.78^2 + 5.56^2)$ so S = 5600 meters/sec, which is close to the average rocket speed in the time interval, which is graphed in Problem 3.

Problem 3 – Graph the speed of the rocket over the interval between 192 and 202 seconds. What is the average slope of the line, and what does this represent? (Use proper units for the slope based on the graph). Answer: slope = (5962-5344)/(202-192) = (+618 meters/sec)/10sec = **+61.8 meters/sec^2. This represents the average acceleration of the rocket.** Note 1 Earth gravity = +9.8 meters/sec^2, so the average acceleration of the rocket is about '6 Gs' which would be very unpleasant for humans if this were a manned flight!



Constellations in 3D

A constellation is a pattern of stars that we see in the sky, but actually stars are spread out in space, and what we are seeing is only a geometric projection. This is much like the 2-dimensional photographs that we take, which only capture one perspective in how things actually look in space. For example, below is a photograph of the constellation Orion as it appears in the sky. The table gives the positions and distances to the brightest stars in Orion.



Star	R.A.	Dec.	Distance (parsecs)
Betelgeuse	5:50	+7:23	650
Rigel	5:10	-8:19	800
Bellatrix	5:20	+6:16	300
Saiph	5:43	-9:42	1800
Alnilam	5:31	-1:16	1530
Alnitak	5:36	-2:00	1470
Mintaka	5:27	-0:37	1500

1 - Cut out the photograph of Orion and glue it to a piece of stiff cardboard.

2 - Using a scale of 100 parsecs = 1 centimeter, cut 7 pipe cleaners to the lengths corresponding to the distances for each star in the table.

3 - On the back side of the cardboard, push the pipe cleaner through the cardboard at the location of each star so that the pipe cleaners stick out of the back of the card.

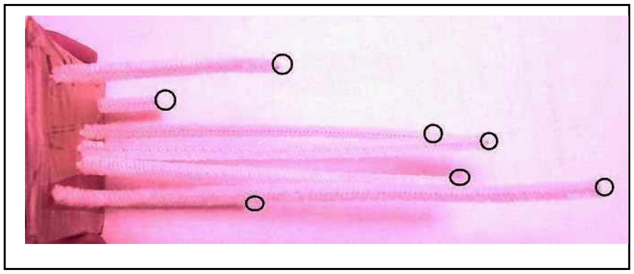
4 - Attach a small piece of round clay, or some other marker, to the end of each pipe cleaner to represent the corresponding star.

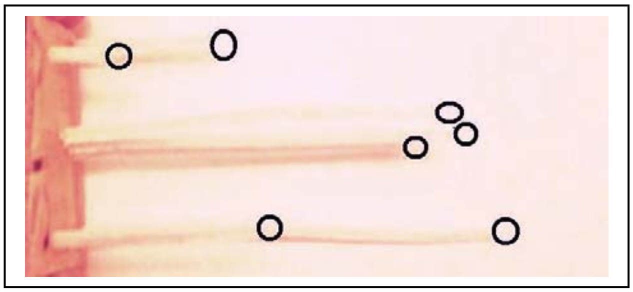
5 – With the cardboard viewed edge-on in your left hand, draw the locations of the stars on graph paper from the following perspectives. (You may also choose to take a digital photo and edit-out the pipe cleaners leaving only the stars in the picture!)

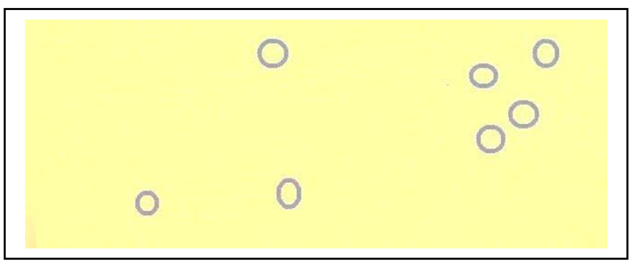
Problem 1 - Construct a bottom-view sketch by rotating the cardboard so that the bottom of the constellation faces you.

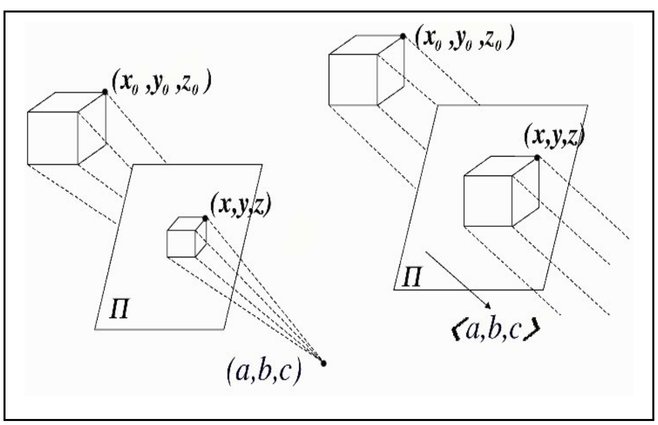
Problem 2 - Construct a side-view sketch.

Problem 3 – Construct a top-view sketch









A digital camera was used to take these pictures, from top to bottom, for Problem 1, 2, and 3.

The stars were added in an image editing program.

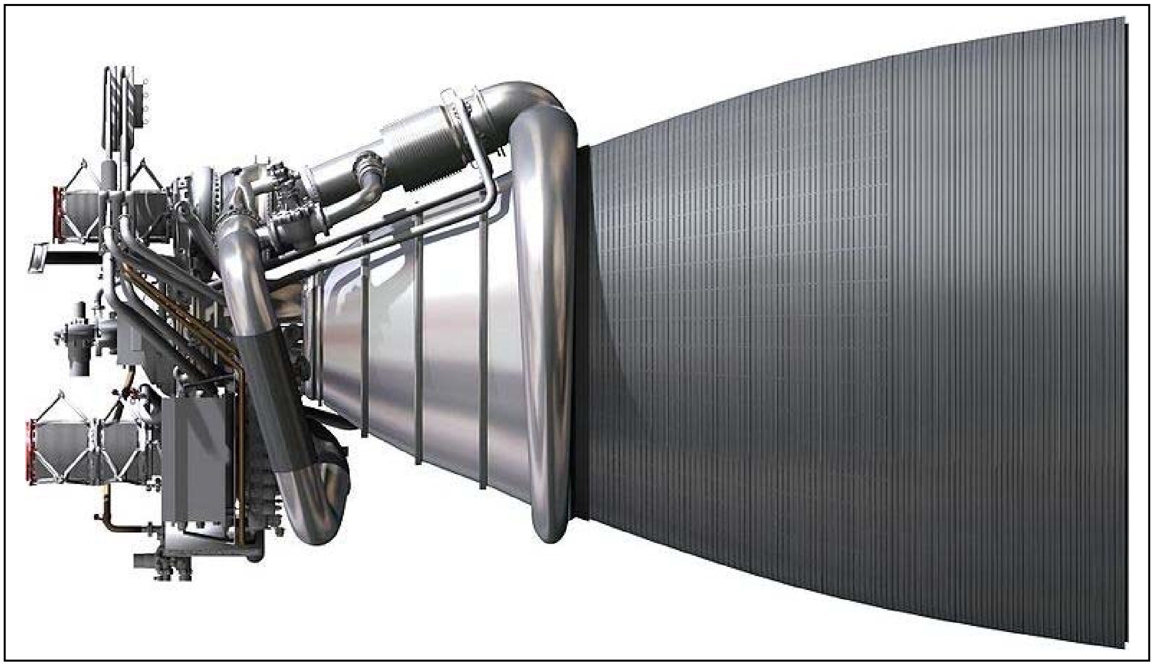
Students will see that the placement of the stars changes as the viewer's orientation in space changes .This will happen because stars are located 'along the third dimension' at various distances from the viewer.

Orion is an interesting constellation because the three Belt Stars are at nearly the same distance from Earth, so from different orientations, they tend to stay together as a 3-star asterism.

Astronomers call this change of appearance a 'projection effect' because we are projecting the 3-d locations of stars in space onto a 2-d viewing screen.

This figure shows two kinds of projection effects. The first is called Perspective Projection because the lines converge at a point where the viewer is located. The second is called Orthographic Projection because the image on the projection plane is a one-for-one duplicate of the original.
(Courtesy Tom Farmer, Journal of Online Mathematics "Geometric Photo Manipulation")

Calculating the Volume of the J-2x Rocket Engine Bell



Problem 1 – The volume of a cone is given by $V = 1/3\pi R^2 h$. If the base of the rocket nozzle has a radius of r = 1.53 meters (5-feet) and a height of h = 4.37 meters (13-feet), what is the approximate total volume of the J-2X rocket nozzle to the nearest 0.1 cubic meters? (Use $\pi = 3.14$)

Problem 2 – A better approximation to the shape of the curved nozzle is to break the volume up into a conical top section (h=2.18 meters) and a cylindrical bottom section (h=2.18 meters). If the base of the cone in the top section has a radius of 1.00 meter, and the average radius of the cylinder in the bottom section is R=(1.00 + 1.53)/2 = 1.27 meters, what is the total volume of the total nozzle to the nearest 0.1 cubic meters? (Use $\pi = 3.14$)

Problem 3 – The exact volume of the volume for this curved nozzle is given by the function

$$V(x) = 0.0094\, x^4 - 0.141\, x^3 + 1.10\, x^2 + 0.14\, x$$

where V is in cubic meters and x is the distance from the vertex of the cone to its base. What is the exact volume of the J-2X rocket nozzle to the nearest 0.1 cubic meters ?

Problem 1 – The volume of a cone is given by $V = 1/3\pi R^2 h$. If the base of the rocket nozzle has a radius of r = 1.53 meters and a height of h = 4.37 meters, what is the approximate total volume of the J-2X rocket nozzle to the nearest 0.1 cubic meters? (Use $\pi = 3.14$)

Answer: $V = 1/3\ (3.14)\ (1.53)^2\ (4.37)$ = **10.7 meters^3**

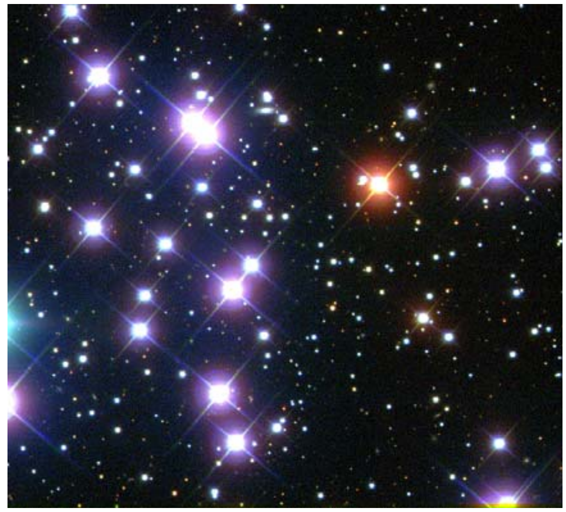
Problem 2 – A better approximation to the shape of the curved nozzle is to break the volume up into a conical top section (l=2.18 meters) and a cylindrical bottom section (L=2.18 meters). If the base of the cone in the top section has a radius of 1.00 meter, and the average radius of the cylinder in the bottom section is (1.00 + 1.53)/2 = 1.27 meters, what is the total volume of the total nozzle to the nearest 0.1 cubic meters? (Use $\pi = 3.14$)

Cone = $1/3\ (3.14)(1.0)^2(2.18)$ = 2.28 meters^3
Cylinder = $3.14\ (1.27)^2\ (2.18)$ = 11.04 meters^3
Approximate total volume = **13.3 meters^3**.

Problem 3 – The exact volume of the volume for thus curved nozzle is given by the function $V(x) = 0.0094x^4 - 0.141x^3 + 1.10x^2 + 0.14x$ where V is in cubic meters and x is the distance in meters from the vertex of the cone to its base. What is the exact volume of the J-2X rocket nozzle to the nearest 0.1 cubic meters?

Answer: V(4.37meters) = $0.0094(4.37)^4 - 0.141(4.37)^3 + 1.10\ (4.37)^2 + 0.14(4.37)$
= 3.43 - 11.77 + 21.00 + 0.61
= 13.27
= **13.3 meters^3**.

Angular Size : The Moon and Stars



Although many astronomical objects may have the same angular size, most are at vastly different distance from Earth, so their actual sizes are very different. If your friends were standing 200 meters away from you, they would appear very small, even though they are as big as you are!

The pictures show the Moon (d = 384,000 km) and the star cluster Messier-34 (d = 1,400 light years). The star cluster photo was taken by the Sloan Digital Sky Survey, and although the cluster appears the same size as the Moon in the sky, its stars are vastly further apart than the diameter of the Moon!

In the problems below, round all answers to one significant figure.

Problem 1 - The images are copied to the same scale. Use a metric ruler to measure the diameter of the Moon in millimeters. If the diameter of the moon is 1,900 arcseconds, what is the scale of the images in arcseconds per millimeter?

Problem 2 - The relationship between angular size, **Θ**, and actual size, **L**, and distance, **D**, is given by the formula:

$$L = \frac{\Theta}{206,265} D$$

Where Θ is measured in arcseconds, and L and D are both given in the same units of length or distance (e.g. meters, kilometers, light years). A) In the image of the Moon, what does 1 arcsecond correspond to in kilometers? B) In the image of M-34, what does 1 arcsecond correspond to in light years?

Problem 3 - What is the smallest detail you can see in the Moon image in A) arcseconds? B) kilometers?

Problem 4 - What is the smallest star separation you can measure in Messier-34 in among the brightest stars in A) arcseconds? B) Light years?

Problem 1 - The images are copied to the same scale. Use a metric ruler to measure the diameter of the Moon in millimeters. If the diameter of the moon is 1,900 arcseconds, what is the scale of the images in arcseconds per millimeter? Answer: The diameter of the Moon is about 64 millimeters, and since this corresponds to 1,900 arcseconds, the scale is 1,900 asec/64 mm = 29.68 or 30 asec/mm.

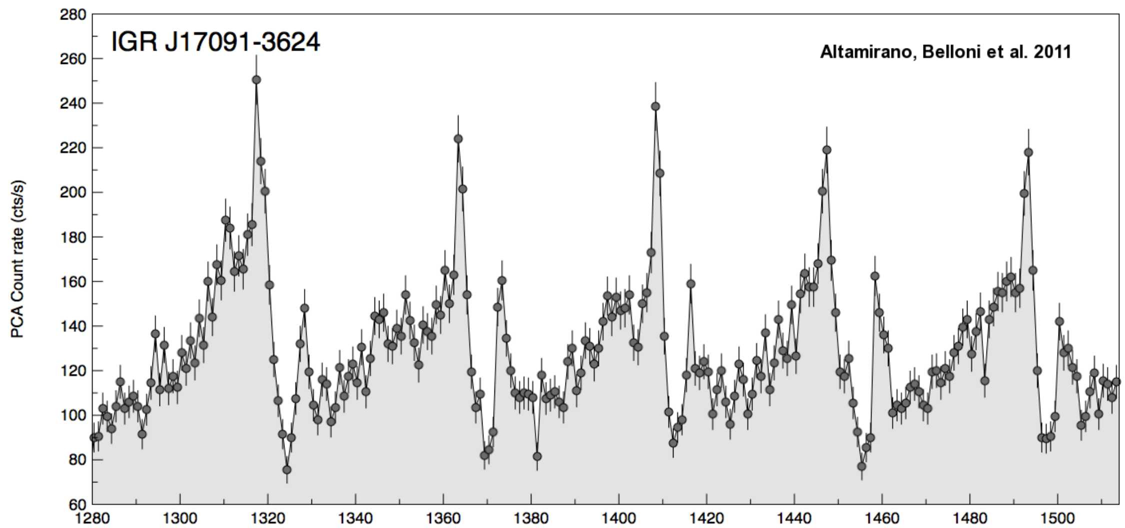
Problem 2 - The relationship between angular size, **Θ**, and actual size, **L**, and distance, **D**, is given by the formula:

$$L = \frac{\Theta}{206,265} D$$

Where Θ is measured in arcseconds, and L and D are both given in the same units of length or distance (e.g. meters, kilometers, light years). A) In the image of the Moon, what does 1 arcsecond correspond to in kilometers? B) In the image of M-34, what does 1 arcsecond correspond to in light years? Answer: A) For the Moon: L = 1 arcsec/206265 x (384,000 km) = 1.86 or 2.0 kilometers. B) For the cluster, L = 1 arcsec/206265 x (1,400 light years) = 0.007 light years.

Problem 3 - What is the smallest detail you can see in the Moon image in A) arcseconds? B) kilometers? Answer: A) About 1 millimeter, which corresponds to 1.0 arcsec. B) One arcsec corresponds to 2.0 kilometers.

Problem 4 - What is the smallest star separation you can measure in Messier-34 among the brightest stars in A) arcseconds? B) Light years? Answer: A) Students may find that some of the bright stars are about 3 millimeters apart, which corresponds to 3 mm x 30 asec/mm = 90 arcseconds. B) At the distance of the cluster, 1 asec = 0.007 light years, so 90 asec corresponds to 90 x (0.007 light years/asec) = 0.63 or 0.6 light years to 1 significant figure.



NASA's RXTE spacecraft recently recorded rhythmic x-ray flashes from the black hole candidate called IGR J17091-3624. The black hole is a member of a binary system that combines a normal star, with a black hole that may weigh three times the sun's mass. That is near the theoretical mass boundary where black holes become possible. The system is located in the direction of the constellation Scorpius between 16,000 light-years and 65,000 light-years from Earth.

Gas from the normal star streams toward the black hole and forms a disk around it. Friction within the disk heats the gas to millions of degrees, which is hot enough to emit X-rays. The very fast, cyclical variations in the x-ray light may be occurring near the black hole's event horizon - the point beyond which nothing, not even light, can escape.

Problem 1 – The RXTE satellite measured the x-ray intensity of this star system hundreds of times an hour. The plot above (called a light curve) shows how this brightness changes over time. Each division is 10 seconds long. From this figure, what is the average period of the brightness changes in seconds?

Problem 2 – The radius of a 3-solar mass black hole is about 10 kilometers. If the flickering has to do with the orbital motion of gasses at a distance of 15 kilometers, how fast is the gas orbiting the black hole in kilometers per second?

http://www.nasa.gov/topics/universe/features/black-hole-heartbeat.html

NASA's RXTE Detects 'Heartbeat' of Smallest Black Hole Candidate
12.15.11

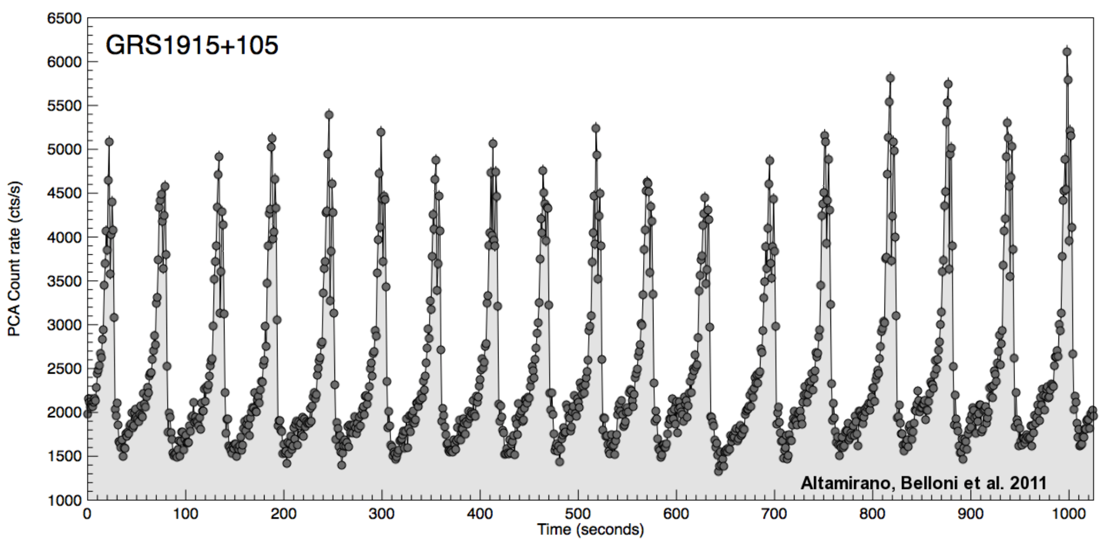
Problem 1 – The RXTE satellite measured the x-ray brightess of this star system hundreds of times an hour, and the plot above shows how this brightness changes over time. Each division is 10 minutes long. From this figure, what is the average period of the brightness changes in seconds?

Answer: With the help of a millimeter ruler, the horizontal scale is about 1.7 seconds/mm so the peaks are separated by 27mm, 26mm, 22mm, 27mm or 46, 44, 37 and 46 seconds respectively. **The average period is 43 seconds.**

Problem 2 – The radius of a 3-solar mass black hole is about 10 kilometers. If the flickering has to do with the orbital motion of gasses at a distance of 15 kilometers, how fast is the gas orbiting the black hole in kilometers per second?

Answer: We know how long it takes the gas to go once-around so now we need the distance traveled along the circumference of the orbit with a radius of 15 km.
C = 2 (3.14)(15 km) = 93.6 km. Then the speed is just the distance divided by time:
S = 94 km/43 seconds so **S = 2.2 km/sec**.

The graph below is another flickering black hole candidate. How fast does the gas travel around this black hole?



For the convenience of storing some types of solids (grains, sand) or liquids, some tanks have a conical shape. The volume of a cone is given by

$$V = \frac{1}{3}\pi R^2 h$$

In this picture, the diameter of the top of the tank is 6 meters and its height is 7.5 meters. What is the total volume of this tank in gallons if 1 gallon is 3.85 liters and 1meter^3 = 1000 liters.

Answer: V = 0.33 (3.14)(3)^2 (7.5)
= 70 meters^3
= **18,182 gallons**

Problem 1 – What is the linear equation that gives the radius of the tank, R, at a height h in meters above the ground?

Problem 2 - An engineer wants to store an expensive solvent in this tank and needs to know when there is only 200 gallons remaining so that he can re-order. He will install a gauge at a height, Z, in the tank that will be triggered when the solvent level is just under the gauge.

Problem 3 – How high up on the slanted side of the tank from its vertex will the gauge be located?

Problem 1 – What is the linear equation that gives the radius of the tank, R, at a height h in meters above the ground?

Answer: At h = 7.5 meters, R = 3 meters, so the slope of the linear equation for h is just m = 3/7.5 = 0.4 and so **R(h) = 0.4h**.

Problem 2 - An engineer wants to store an expensive solvent in this tank and needs to know when there is only 200 gallons remaining so that he can re-order. He will install a gauge at a height, Z, in the tank that will be triggered when the solvent level is just under the gauge.

Answer: $V = 1/3\ (3.14)\ (0.4h)^2\ (h)$ so $V = 0.167\ h^3$
200 gallons equals 770 liters or 0.77 meters^3, then
$0.77 = 0.167\ z^3$ and solving for z we get **z = 1.66 meters**.

Problem 3 – How high up on the slanted side of the tank from its vertex will the gauge be located?

Answer: The slope of the side of the tank is just 1/0.4 = 2.5 so the 'hypotenuse' of the tank side makes an angle of $\mathrm{Tan}(\theta) = 2.5$ or 68 degrees. Since Hsin(68) = 1.66 meters, we have **H = 1.8 meters**.

Adding a Level Gauge to a Conical Tank

Conical storage tanks come in many different sizes, from grain storage silos like the one top-left, to chemical storage and separating funnels like the one shown top-right. The nice thing about cones is that they have a wide base area that is easy to poor things into, and a valve at the conical tip lets you remove carefully-measured amounts of whatever is being stored. Recall that the volume of a cone is given by $V = 1/3 \pi R^2 h$ where R is the base radius and H is the vertical height (not the slant height along the side of the cone!).

Problem 1 – Suppose that the function that relates the radius of the cone to the vertical height of the cone is given by R(h) = 0.5h, where h and R are in meters. The maximum height of the storage vessel is 3.0 meters. What is the radius of the upside-down conical tank at its maximum height?

Problem 2 – To the nearest tenth of a cubic meter, what is the maximum volume of this conical tank?

Problem 3 – At what height, H, should the astronaut place a mark on the outside of the tank to indicate a level of ½ the volume of the conical tank?

Answer Key

Problem 1 – Suppose that the function that relates the radius of the cone to the vertical height of the cone is given by R(h) = 0.5h, where h and R are in meters. The maximum height of the storage vessel is 3.0 meters. What is the radius of the upside-down conical tank at its maximum height?

Answer: R(2.5) = 0.5 x 3.0 = **1.5 meters**.

Problem 2 – To the nearest tenth of a cubic meter, what is the maximum volume of this conical tank?

Answer: H = 3.0 meters, R = 1.5 meters

so $V = 1/3\ (3.141)\ (1.5)^2\ (3.0)$

= **7.1 meters^3**.

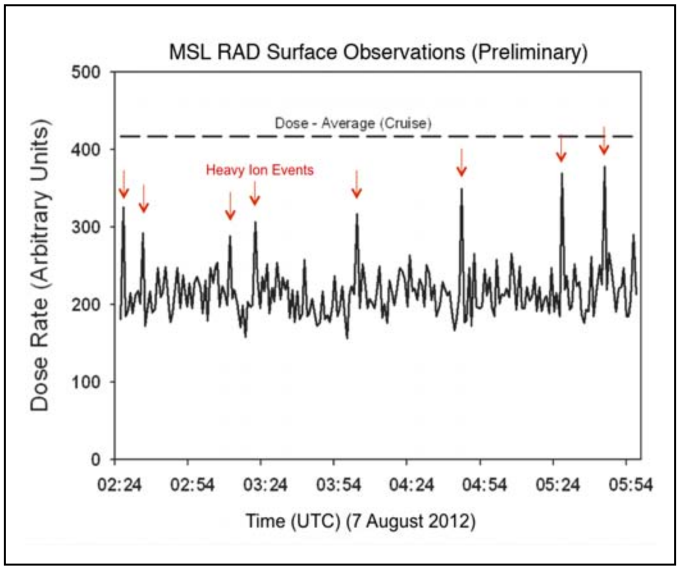
Problem 3 – At what height should the astronaut place a mark on the outside of the tank to indicate a level of ½ the volume of the conical tank?

Answer: We want $V = ½ \times 7.1 m^3 = 3.55\ m^3$

But R = 0.5H

So $V = 1/3\ \pi\ (0.5H)^2\ H = 0.333(3.141)(0.25)\ H^3$ and so $V = 0.26H^3$

Then $3.55\ m^3 = 0.26\ H^3$ and so solving for H we get **H = 2.4 meters**.



On its journey to Mars, the Mars Science Lab measured the level of radiation it was receiving in space during its 253-day travel from Earth to Mars. Once the Curiosity Rover landed on Mars, the Radiation Assessment Detector (RAD) instrument continued to measure the radiation level at the landing site. The graph to the left shows the radiation measured during a 3-hour period on August 7, 2012. NASA scientists now predict that astronauts making this journey and working on Mars will not have significant problems with radiation exposure if they take standard precautions.

Problem 1 - Ignoring the 'heavy ion events' (points marked with arrows) caused by cosmic rays, about what is the average radiation dose rate indicated by the remaining points shown in the graph above?

Problem 2 - By what factor is the average radiation dose rate in space during the cruise to Mars greater than the ground-level dose rate on the surface of Mars?

Problem 3 - The Curiosity Rover RAD instrument says that on the surface of Mars, an astronaut will receive an average dose rate of about 0.7 milliSieverts per day of radiation. What is the estimated dose rate in space during the cruise to Mars?

Problem 4 - Suppose an astronaut took a 180-day journey to Mars, stayed there for 600 days, and then returned on a 180-day trip back. What would the astronauts total radiation dose be for the entire 960-day trip?

Problem 5 - If an astronaut remained on Earth, the normal background radiation dose rate is 3 milliSieverts/year. How many equivalent years of normal Earth exposure would a single trip to Mars produce?

December 4, 2012 Press Release: http://www.space.com/18753-mars-radiation-manned-mission.html
Astronauts Could Survive Mars Radiation for Long Stretches, Rover Study Suggests

"…Astronauts could endure a long-term, roundtrip Mars mission without receiving a worryingly high radiation dose, new results from NASA's Mars rover Curiosity suggest. A mission consisting of a 180-day outbound cruise, a 600-day stay on Mars and another 180-day flight back to Earth would expose an astronaut to a total radiation dose of about 1.1 sieverts (units of radiation) if it launched now, according to measurements by Curiosity's Radiation Assessment Detector instrument, or RAD. RAD has found radiation levels on the Martian surface to be comparable to those experienced by astronauts in low-Earth orbit. A person ambling around the Red Planet would receive an average dose of about 0.7 millisieverts per day, while astronauts aboard the International Space Station experience an average daily dose between 0.4 and 1.0 millisieverts, "

Problem 1 - Ignoring the 'heavy ion events' (points marked with arrows) caused by cosmic rays, about what is the average Dose Rate of the remaining points shown in the graph above? Answer: About **215**.

Problem 2 - By what factor is the average Dose in space during the cruise to Mars greater than the ground-level Dose rate on the surface of Mars?

Answer: About 415/215 = **1.9 times** higher in space.

Problem 3 - The Curiosity Rover RAD instrument says that on the surface of Mars, an astronaut will receive an average Dose Rate of about 0.7 milliSieverts per day of radiation. What is the estimated Dose Rate in space during the cruise to Mars?

Answer: 0.7 x 1.9 = **1.3 milliSieverts/day**.

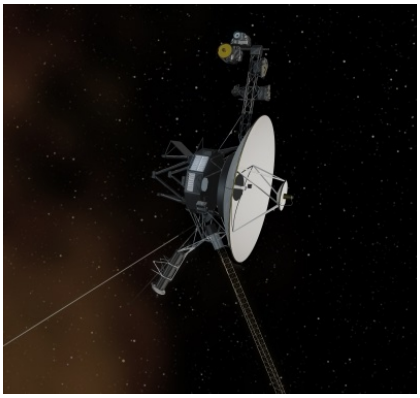
Problem 4 - Suppose an astronaut took a 180-day journey to Mars, stayed there for 600 days, and then returned on a 180-day trip back. What would the astronauts total radiation exposure be for the entire 960-day trip?

Answer: 180x0.0013 + 180x0.0013 + 600x0.0007 = **0.88 Sieverts**.

Problem 5 - If an astronaut remained on Earth, the normal background radiation dose rate is 3 milliSieverts/year. How many equivalent years of normal Earth exposure would a single trip to Mars produce?

Answer: 0.88 Sieverts over 960 days = 338 milliSieverts/year. So it equals 338/3 = **112 years**!

Voyager 1 Begins its Interstellar Journey



NASA's Voyager 1 spacecraft officially is the first human-made object to venture into interstellar space. The 36-year-old probe is about 12 billion miles (19 billion kilometers) from our sun.

Voyager 1 does not have a working plasma sensor, so scientists needed a different way to measure the spacecraft's plasma environment to make a definitive determination of its location. A coronal mass ejection, or a massive burst of solar wind and magnetic fields, that erupted from the sun in March 2012 provided scientists the data they needed. When this unexpected gift from the sun eventually arrived at Voyager 1's location 13 months later, in April 2013, the plasma around the spacecraft began to vibrate like a violin string.

On April 9, Voyager 1's plasma wave instrument detected the movement. The pitch of the oscillations helped scientists determine the density of the plasma. The particular oscillations meant the spacecraft was bathed in plasma more than 40 times denser than what they had encountered in the outer layer of the heliosphere. Density of this sort is to be expected in interstellar space. The plasma wave science team reviewed its data and found an earlier, fainter set of oscillations in October and November 2012. Through extrapolation of measured plasma densities from both events, the team determined Voyager 1 first entered interstellar space in August 2012.

Problem 1 – The solar coronal mass ejection left the sun in March 2012 and was first detected by Voyager 1 in April 2013 at a distance of about 19 billion kilometers from the sun. What was the average speed of the CME in: A) kilometers/day? B) kilometers/sec? C) miles/hour? (1 mile = 1.6 km)

Problem 2 – The speed of light is 300,000 km/s. How long does it take a radio message to travel from Voyager 1 to Earth in hours?

Problem 3 - Measurements taken between April 9 and May 22 of 2013 show that Voyager 1 was, at that time, located in an area with a density of about 0.08 hydrogen atoms per cubic centimeter which is similar to the expected density of interstellar space. To two significant figures, how many hydrogen atoms would you expect to find in a container as large as your bedroom if your bedroom measured exactly 10-feet x 9-feet x 12-feet? (1 foot = 30.5 cm)

NASA Spacecraft Embarks on Historic Journey Into Interstellar Space
http://www.nasa.gov/mission_pages/voyager/voyager20130912.html
September 12, 2013

Problem 1 – The solar coronal mass ejection left the sun in March 2012 and was first detected by Voyager 1 in April 2013 at a distance of about 19 billion kilometers from the sun. What was the average speed of the CME in: A) kilometers/day? B) kilometers/sec? C) miles/hour? (1 mile = 1.6 km)

Answer: The difference in time between the two dates is April 2013 – March 2012 = 365+30 = 395 days.

A) The distance traveled is 19 billion km, so the speed was 19 billion km/395 days = **48 million km/day**.

B) 1 day = 24 x 3600 = 86400 seconds so the speed is 48 million km/86400 sec = **555 km/sec**.

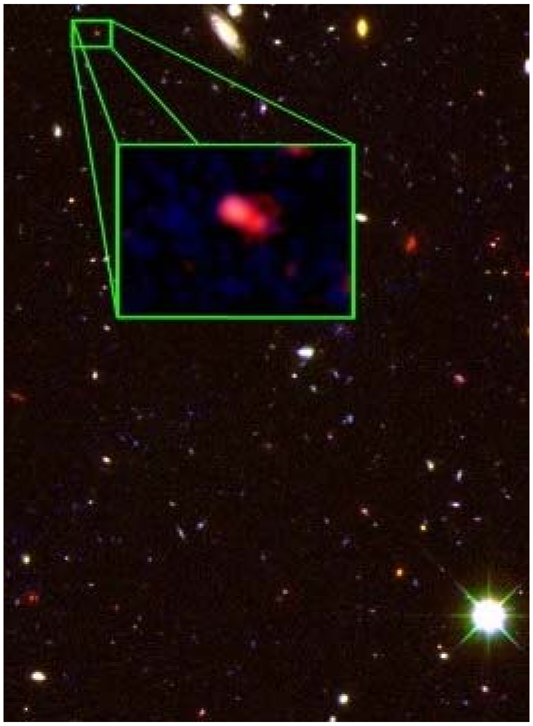
C) 555 km/sec x (1 mile/1.6 km) x (3600 sec/1 hour) = **1.2 million miles/hour**.

Problem 2 – The speed of light is 300,000 km/s. How long does it take a radio message to travel from Voyager 1 to Earth in hours?

Answer: Time = distance/speed = 19 billion km / 300000 km/s = 63,333 seconds.
Time = 63333 seconds x (1 hour/3600 seconds) = **17.6 hours.**

Problem 3 - Measurements taken between April 9 and May 22 of 2013 show that Voyager 1 was, at that time, located in an area with a density of about 0.08 hydrogen atoms per cubic centimeter which is similar to the expected density of interstellar space. To two significant figures, how many hydrogen atoms would you expect to find in a container as large as your bedroom if your bedroom measured 10-feet x 9-feet x 12-feet? (1 foot = 30.5 cm)

Answer: The room measures exactly 10 feet x (30.5 cm/1 foot) = 305 cm. 9-feet = 274 cm and 12 feet = 366 cm so the measurements are valid to three significant figures and so the volume is 305 x 274 x 366 = 30,600,000 cm^3. The density is 0,08 hydrogen atoms/cm^3, so the total number of hydrogen atoms in this volume would be N = 30,600.000 x 0.08 = **2.4 million atoms!**



Astronomers have recently discovered one of the most distant galaxies in our universe using a list of candidates from the Hubble Space Telescope's Cosmic Assembly Near-infrared Deep Extragalactic Legacy Survey (CANDELS). This survey found 42 objects that seemed to be good choices for a follow-up study to determine their exact distances.

Astronomers used the MOSFIRE infrared camera on the Keck Telescope in Hawaii to study each of these candidates spectroscopically. One of these objects called z8_GND_5296 was then discovered to be the most distant galaxy known after studying the Lyman-alpha spectral line of hydrogen. This line should have appeared at an ultraviolet wavelength of 121 nanometers. Instead, thanks to the expansion of the universe, this line was detected at a wavelength of 1029 nanometers in the infrared part of the light spectrum.

Problem 1 – Astronomers define a quantity called the redshift by the formula

$$Z = (\lambda_m - \lambda_r)/\lambda_r$$

where λ_m is the observed wavelength of a spectral line and λ_r is its wavelength when measured under laboratory conditions. What was the redshift of the new galaxy based on the lyman-alpha spectral line?

Problem 2 – According to the Big Bang theory, the redshift of a galaxy is related to the time since its light left the object, called the look-back time, T, by the approximate formula T(Z) = 12.65 + 0.06Z. What is the look-back time for the galaxy z8_GND_5296?

Problem 3 – The Big Bang occurred 13.8 billion years ago. How soon after the Big Bang, in millions of years, did the light from z8_GND_5296 begin its journey?

http://hubblesite.org/newscenter/archive/releases/2013/39/image/a/
Galaxy Found in Hubble Survey Has Farthest Confirmed Distance
October 23, 2013

Problem 1 – Astronomers define a quantity called the redshift by the formula $Z = (\lambda_m - \lambda_r)/\lambda_r$ where λ_m is the observed wavelength of a spectral line and λ_r is its wavelength when measured under laboratory conditions. What was the redshift of the new galaxy based on the lyman-alpha spectral line?

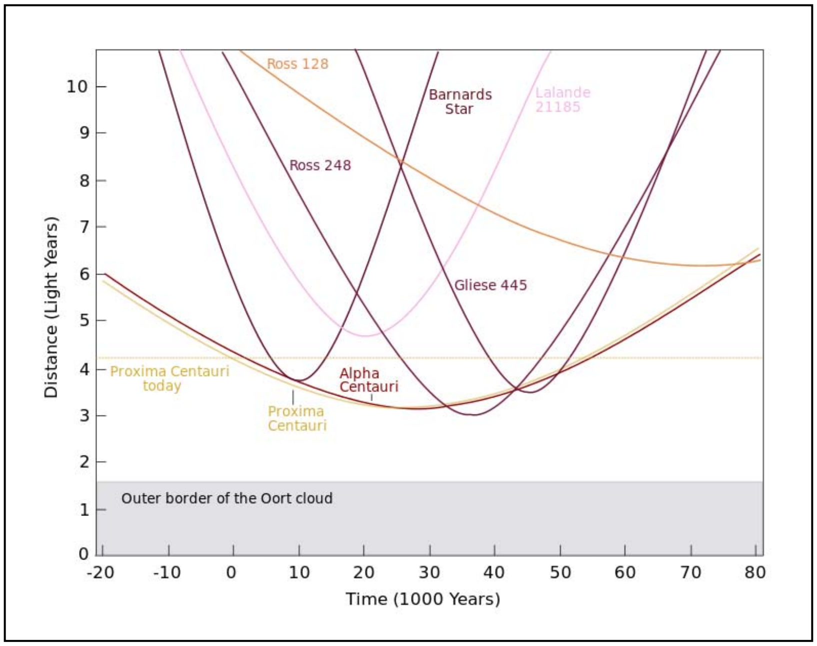
Answer: λ_r = 1029 nanometers λ_r = 121 nanometers, so z = (1029-121)/121 = **7.5**

Problem 2 – According to the Big Bang theory, the redshift of a galaxy is related to the time since its light left the object, called the look-back time, T, by the approximate formula T(Z) = 12.65 + 0.06Z. What is the look-back time for the galaxy z8_GND_5296?

Answer: T = 12.65 + 0.06(7.5) = **13.1 billion years**.

Problem 3 – The Big Bang occurred 13.8 billion years ago. How soon after the Big Bang, in millions of years, did the light from z8_GND_5296 begin its journey?

Answer: 13.8 – 13.1 = 0.7 billion years or **700 million years**.



Stars don't stay in one place all the time, but move through space. Over time, our night sky will look very different than it does today. One thing that will change pretty soon is the answer to the question, 'Which star is the closest to our sun?' The figure shows the distances to several nearby stars from 20,000 years ago to 80,000 years in the future, based upon detailed studies of the speeds and current distances of these stars. The picture, by the way, is of the star Proxima Centauri taken by the Hubble Space Telescope. It is currently the closest star to our sun at a distance of 4.243 light years or 40.14 trillion kilometers (24.94 trillion miles).

Problem 1 – For the star Alpha Centauri in the graph, what does its curve represent?

Problem 2 – What will be the distances to the seven stars at a time 30,000 in the future?

Problem 3 – Create a timeline that gives the absolute minimum distances to each star over the next 80, 000 years.

Figure from Wikipedia – 'Proxima Centauri'
Matthews, R. A. J. (1994), "The Close Approach of Stars in the Solar Neighborhood", Quarterly Journal of the Royal Astronomical Society 35: 1–9

http://www.nasa.gov/content/goddard/hubbles-new-shot-of-proxima-centauri-our-nearest-neighbor/index.html
Hubble's New Shot of Proxima Centauri, our Nearest Neighbor
November 1, 2013

Problem 1 – For the star Alpha Centauri in the graph, what does its curve represent?

Answer: The student should mention that it is a mathematical model of the distance between the Sun and Alpha Centauri as it changes between 20,000 years ago and 80,000 into the future.

Problem 2 – What will be the distances to the seven stars at a time 30,000 in the future?

Answer: Students draw a line vertically from the point 'Time=30' and note the distances at which the curves for the stars cross this line:

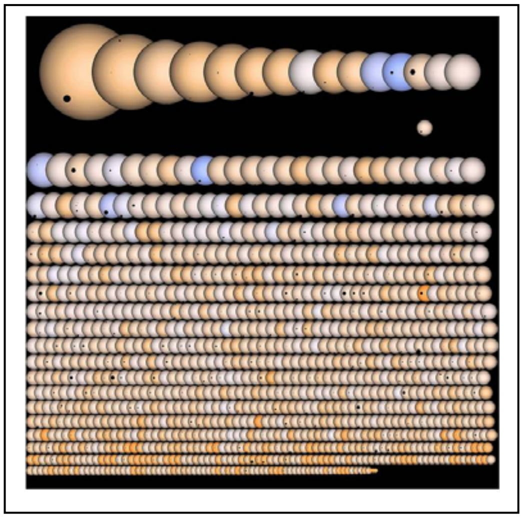
Alpha Centauri	**D = 3.125 ly**
Proxima Centauri	**D = 3.188 ly**
Ross 248	**D = 3.500 ly**
Lalande 21185	**D = 5.625 ly**
Gliese 445	**D= 6.785 ly**
Ross 128	**D = 8.125 ly**
Barnards Star	**D = 10.00 ly**

Problem 3 – Create a timeline that gives the absolute minimum distances to each star over the next 80, 000 years.

Answer:

Barnards Star	**10,000**	**3.69 ly**
Lalande 21185	**21,000**	**4.69 ly**
Proxima Centauri	**25,000**	**3.13 ly**
Alpha Centauri	**30,000**	**3.13 ly**
Ross 248	**37,000**	**3.00 ly**
Gliese 445	**46,000**	**3.44 ly**
Ross 128	**75,000**	**6.13 ly**

Kepler's Latest Count on Goldilocks Planets



Scientists from around the world are gathered this week at NASA's Ames Research Center in Moffett Field, Calif., for the second Kepler Science Conference, where they will discuss the latest findings resulting from the analysis of Kepler Space Telescope data.

New Kepler data analysis and research also show that most stars in our galaxy have at least one planet. This suggests that the majority of stars in the night sky may be home to planetary systems, perhaps some like our solar system.

Problem 1 - Based on a total of 3538 candidate planets, Kepler announced in November 2013 that it had detected 674 objects smaller than 1.25 Re, 1076 objects between 1.25 and 2.0 Re, 1457 objects between 2.0 and 6.0 Re, 229 objects between 6.0 and 15 Re, and 102 objects larger than 15 Re. What are the percentages of each size of planet candidate, and create a histogram plot to show this data.

Problem 2 - The survey included 42,000 stars and detected 10 candidate planets that were about the same size as Earth, and located at a distance from their stars where liquid water could occur (Goldilocks Zone). Because of the way that Kepler detected these planets, they were only able to see about 1.3% of all planets orbiting the surveyed stars. If the Milky Way contains about 500 billion stars similar to the ones surveyed by Kepler, about how many Earth-like planets might you expect to find in the entire Milky Way galaxy?

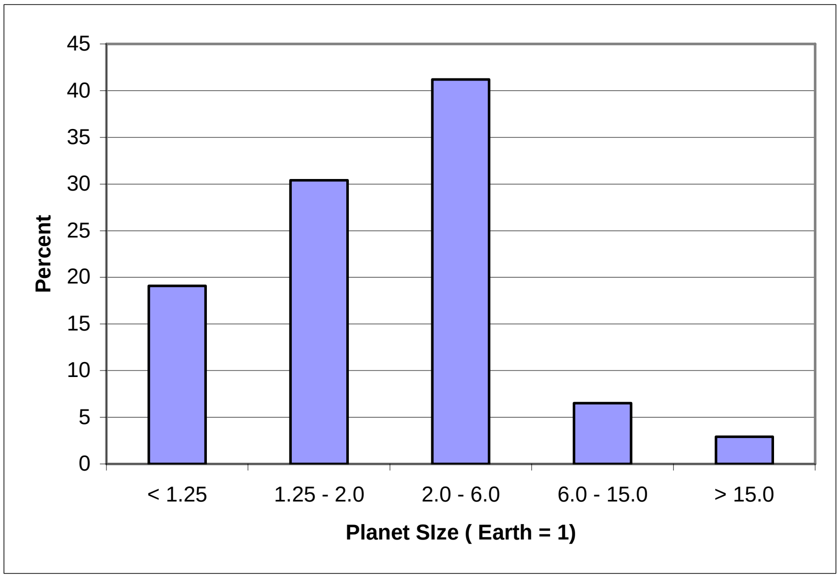
NASA Kepler Results Usher in a New Era of Astronomy
November 4, 2013

http://www.nasa.gov/content/nasa-kepler-results-usher-in-a-new-era-of-astronomy/index.html

Problem 1 - Based on a total of 3538 candidate planets, Kepler announced in November 2013 that it had detected 674 objects smaller than 1.25 Re, 1076 objects between 1.25 and 2.0 Re, 1457 objects between 2.0 and 6.0 Re, 229 objects between 6.0 and 15 Re, and 102 objects larger than 15 Re. What are the percentages of each size of planet candidate, and create a histogram plot to show this data.

Answer:

1.25	=	674 / 3538	=	**19.1 %**
1.25 to 2.0	=	1076 / 3538	=	**30.4 %**
2.0 to 6.0	=	1457 / 3538	=	**41.2 %**
6.0 to 15.0	=	229 / 3538	=	**6.5 %**
> 15.0	=	102 / 3538	=	**2.9 %**



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Answer: 500 billion x (10 detections /42,000 surveyed stars) x (100 stars/1.3 detection) = 9.1 billion planets. To the nearest billion this becomes **9 billion earth-like planets**.

The fastest way to get from place to place in our solar system is to travel at the speed of light, which is 300,000 km/sec (670 million miles per hour!). Unfortunately, only radio waves and other forms of electromagnetic radiation can travel exactly this fast.

When NASA sends spacecraft to visit the planets, scientists and engineers have to keep in radio contact with the spacecraft to gather scientific data. But the solar system is so vast that it takes quite a bit of time for the radio signals to travel out from Earth and back.

Problem 1 – Earth has a radius of 6378 kilometers. What is the circumference of Earth to the nearest kilometer?

Problem 2 – At the speed of light, how long would it take for a radio signal to travel once around Earth?

Problem 3 – The Moon is located 380,000 kilometers from Earth. During the Apollo-11 mission in 1969, engineers on Earth would communicate with the astronauts walking on the lunar surface. From the time they asked a question, how long did they have to wait to get a reply from the astronauts?

Problem 4 – In the table below, fill in the one-way travel time from the sun to each of the planets. Use that fact that the travel time from the Sun to Earth is 8 ½ minutes. Give your answer to the nearest tenth, in units of minutes or hours, whichever is the most convenient unit.

Planet	Distance from Sun in Astronomical Units	Light Travel Time
Mercury	0.38	
Venus	0.72	
Earth	1.00	8.5 minutes
Mars	1.52	
Jupiter	5.20	
Saturn	9.58	
Uranus	19.14	
Neptune	30.20	

Problem 1 – Earth has a radius of 6378 kilometers. What is the circumference of Earth to the nearest kilometer?

Answer: $C = 2 \pi R$ so $C = 2 \times 3.141 \times (6378 \text{ km}) =$ **40,067 km.**

Problem 2 – At the speed of light, how long would it take for a radio signal to travel once around Earth?

Answer: Time = distance/speed so
Time = 40,067/300,000 = **0.13 seconds. This is about 1/7 of a second**.

Problem 3 – The Moon is located 380,000 kilometers from Earth. During the Apollo-11 mission in 1969, engineers on Earth would communicate with the astronauts walking on the lunar surface. From the time they asked a question, how long did they have to wait to get a reply from the astronauts?

Answer: From the proportion:

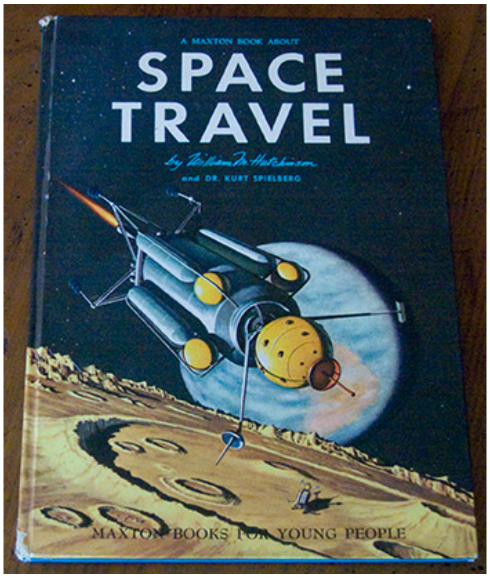
$$\frac{0.13 \text{ seconds}}{40067 \text{ km}} = \frac{X}{380000 \text{ km}}$$ we have $X = (380000/40067) \times 0.13 = 1.23$ seconds.

This is the one-way time for the signal to get to the moon from Earth, so the round-trip time is twice this or **2.46 seconds**.

Problem 4 – In the table below, fill in the one-way travel time from the sun to each of the planets. Use that fact that the travel time from the Sun to Earth is 8 ½ minutes. Give your answer to the nearest tenth, in units of minutes or hours, whichever is the most convenient unit.

Answer: Use simple proportions based on 8.5 minutes of time = 1.00 AU of distance.

Planet	Distance from Sun in Astronomical Units	Light Travel Time
Mercury	0.38	3.2 minutes
Venus	0.72	6.1 minutes
Earth	1.00	8.5 minutes
Mars	1.52	12.9 minutes
Jupiter	5.20	44.2 minutes
Saturn	9.58	1.4 hours
Uranus	19.14	2.7 hours
Neptune	30.20	4.3 hours



Most science fiction stories often have spaceships with powerful, or exotic, rockets that can let space travelers visit the distant planets in less than a day's journey. The sad thing is that we are not quite there in the Real World. This is because our solar system is so vast, and our rockets can't produce quite enough speed to make journeys short.

NASA has been working on this problem for over 50 years and has come up with many possible solutions. Each one is more expensive than just using ordinary fuels and engines like the ones used on most rockets!

Problem 1 – The entire International Space Station orbits Earth at a speed of 28,000 kilometers per hour (17,000 mph). At this speed, how many days would it take to travel to the sun from Earth, located at a distance of 149 million kilometers?

Problem 2 – The planet Neptune is located 4.5 billion kilometers from Earth. How many years would it take a rocket traveling at the speed of the International Space Station to make this journey?

Problem 3 – The fastest unmanned spacecraft, Helios-2, traveled at a speed of 253,000 km/hr. In the table below, use proportional math to fill in the travel times from the sun to each planet traveling at the speed of Helios-2. Give your answers to the nearest tenth in appropriate units of days or years.

Planet	Distance in millions of kilometers	Time
Mercury	57	
Venus	108	
Earth	149	
Mars	228	
Jupiter	780	
Saturn	1437	
Uranus	2871	
Neptune	4530	

Problem 1 – The entire International Space Station orbits Earth at a speed of 28,000 kilometers per hour (17,000 mph). At this speed, how many days would it take to travel to the sun from Earth, located at a distance of 149 million kilometers?

Answer: Time = Distance/speed so
Time = 149,000,000 km/ 28,000
= 5321 hours or **222 days**.

Problem 2 – The planet Neptune is located 4.5 billion kilometers from Earth. How many years would it take a rocket traveling at the speed of the International Space Station to make this journey?

Answer: Time = 4,500,000,000 km / 28,000 km/h
= 160714 hours or 6696 days or **18.3 years**.

Problem 3 – The fastest unmanned spacecraft, Helios-2, traveled at a speed of 253,000 km/hr. In the table below, use proportional math to fill in the travel times from the sun to each planet traveling at the speed of Helios-2. Give your answers to the nearest tenth in appropriate units of days or years.

Planet	Distance in millions of kilometers	Time
Mercury	57	9.4 days
Venus	108	17.8 days
Earth	149	24.5 days
Mars	228	37.5 days
Jupiter	780	128.5 days
Saturn	1437	236.7 days
Uranus	2871	1.3 years
Neptune	4530	2.0 years

Between 4.1 and 3.8 billion years ago, the surfaces of all the planets were being bombarded by asteroids and other large bodies called impactors that had formed in the solar system by this time. Astronomers call this the Late Heavy Bombardment Era, and it is the era which finalized the formation of the planets at their present sizes.

The surface of our moon shows many large round basins called mare that are all that remains of this era. Similar scars on Earth have long since vanished due to erosion, volcanism and plate tectonic activity.

Problem 1 – Using the large crater and impact basin record on the lunar surface, astronomers can estimate that Earth had about 20,000 craters over 20 km across, about 50 impact basins with diameters of 1,000 kilometers, and perhaps 5 large basins with diameters of 5,000 kilometers. If the Late Heavy Bombardment Era lasted about 300 million years, how many years elapsed between the impacts of each of the three kinds of objects during this era?

Problem 2 – A Rule-of-Thumb says that the actual diameter of an impacting body is about 1/6 the diameter of the crater it forms. What were the average sizes of the three kinds of impactors during this Era?

Problem 3 – Use the formula for the volume of a sphere to calculate A) the total volume added to Earth of the small impactors in cubic kilometers. B) the total volume added to Earth of the medium-sized impactors in cubic kilometers. C) the total volume added to Earth of the large impactors in cubic kilometers.

Problem 4 - If the radius of Earth is 6,378 km, what percentage of Earth's volume was added by each of the three kinds of impactors?

Problem 1 – Using the large crater and impact basin record on the lunar surface, astronomers can estimate that Earth had about 20,000 craters over 20 km across, about 50 impact basins with diameters of 1,000 kilometers, and perhaps 5 large basins with diameters of 5,000 kilometers. If the Late Heavy Bombardment Era lasted about 300 million years, how many years elapsed between the impacts of each of the three kinds of objects during this era?

Answer: Small: 300 million/20,000 = **15,000 years**. Medium: 300 million/50 = **6 million years**, Large: 300 million/5 = **60 million years**.

Problem 2 – A Rule-of-Thumb says that the actual diameter of an impacting body is about 1/6 the diameter of the crater it forms. What were the average sizes of the three kinds of impactors during this Era?

Answer: Small = 20 km/6 = **3 km**. Medium: 1000 km/6 = **166 km**. Large: 5000km/6 = **833 km**.

Problem 3 – Use the formula for the volume of a sphere to calculate A) the total volume added to Earth of the small impactors in cubic kilometers. B) the total volume added to Earth of the medium-sized impactors in cubic kilometers. C) the total volume added to Earth of the large impactors in cubic kilometers.

Answer: $V = 4/3\ \pi\ R^3$ and there were 20,000 of these so

Small = 20,000 x 4/3 x 3.141 x (3 km/2)^3 = **282,000 km^3**

Medium: There were 50 of these so V = 50 x 4/3 x 3.141 x (166 km/2)^3 = **120 million km^3**

Large: There were 5 of these so V = 5 x 4/3 x 3.141 x (833 km/2)^3 = **1.5 billion km^3**

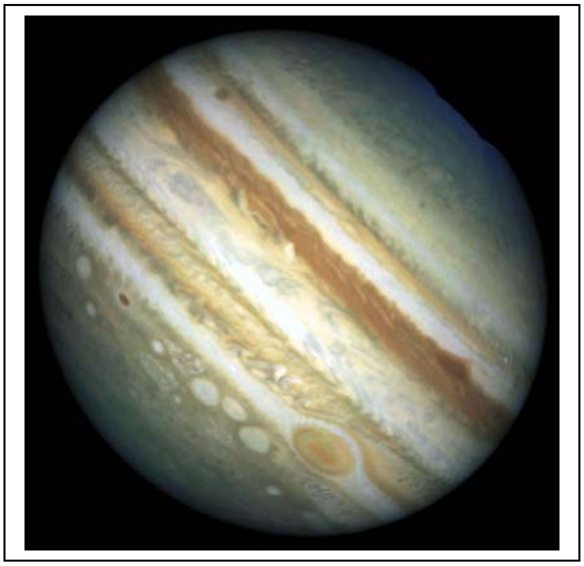
Problem 4 - If the radius of Earth is 6,378 km, what percentage of Earth's volume was added by each of the three kinds of impactors?

Answer: The total volume of a spherical Earth is V = 4/3 x 3.141 x (6378)^3 = 1.1 trillion km^3

So the three kinds of impactors contributed:

Small = 100% x (282000/1.1 trillion) = **0.00003 % of the final volume**.
Medium = 100% x (120 million/1.1 trillion) = **0.01 % of the final volume**.
Large = 100% x (1.5 billion/1.1 trillion) = **0.13% of the final volume**.

So the infrequent (every 60 million years) but largest impactors changed Earth's size the most rapidly during the Late Heavy Bombardment Era!



All of the planets in our solar system, and some of its smaller bodies too, have an outer layer of gas we call the atmosphere. The atmosphere usually sits atop a denser, rocky crust or planetary core. Atmospheres can extend thousands of kilometers into space.

The table below gives the name of the kind of gas found in each object's atmosphere, and the total mass of the atmosphere in kilograms. The table also gives the percentage of the atmosphere composed of the gas.

Object	Mass (kilograms)	Carbon Dioxide	Nitrogen	Oxygen	Argon	Methane	Sodium	Hydrogen	Helium	Other
Sun	3.0×10^{30}							71%	26%	3%
Mercury	1000			42%			22%	22%	6%	8%
Venus	4.8×10^{20}	96%	4%							
Earth	1.4×10^{21}		78%	21%	1%					<1%
Moon	100,000				70%		1%		29%	
Mars	2.5×10^{16}	95%	2.7%		1.6%					0.7%
Jupiter	1.9×10^{27}							89.8%	10.2%	
Saturn	5.4×10^{26}							96.3%	3.2%	0.5%
Titan	9.1×10^{18}		97%			2%				1%
Uranus	8.6×10^{25}					2.3%		82.5%	15.2%	
Neptune	1.0×10^{26}					1.0%		80%	19%	
Pluto	1.3×10^{14}	8%	90%			2%				

Problem 1 – Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth.

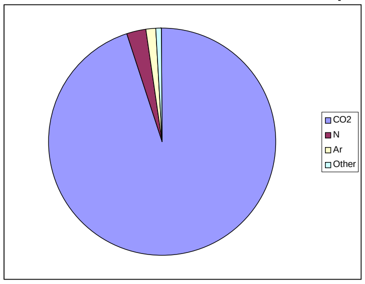
Problem 2 – Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto.

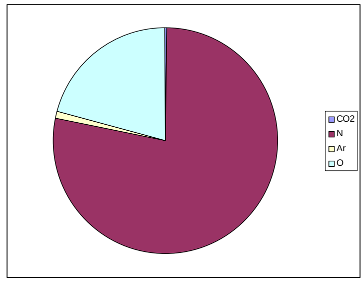
Problem 3 – Which planet has the atmosphere with the greatest percentage of Oxygen?

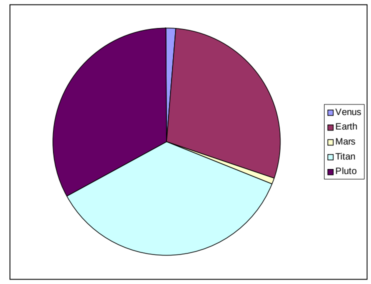
Problem 4 – Which planet has the atmosphere with the greatest number of kilograms of oxygen?

Problem 5 – Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Problem 1 – Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth. Answer: Mars (left), Earth (middle)







Problem 2 – Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto. Answer: First add up all the percentages for Nitrogen in the column to get 271.7%. Now divide each of the percentages in the column by 271.7% to get the percentage of nitrogen in the planetary atmospheres that is taken up by each of the planets: Venus = (4/271) = 1.5%; Earth = (78/271) = 28.8%, Mars = (2.7/271)= 1.0%, Titan = (97/271)=35.8%, Pluto=(90/271)=33.2%. Plot these new percentages in a pie graph (see above right). This pie graph shoes that across our solar system, Earth, Titan and Pluto have the largest percentage of nitrogen. In each case, the source of the nitrogen is from similar physical processes involving the chemistry of the gas methane (Titan and Earth) or methane ice (Pluto).

Problem 3 – Which planet has the atmosphere with the greatest percentage of Oxygen? Answer: From the table we see that **Mercury** has the greatest percentage of oxygen in its atmosphere.

Problem 4 – Which planet has the atmosphere with the greatest number of kilograms of oxygen? Answer: Only two planets have detectable oxygen: Earth and Mercury. Though mercury has the highest percentage of oxygen making up its atmosphere, the number of kilograms of oxygen is only 1000 kg x 0.42 = 420 kilograms. By comparison, **Earth** has a smaller percentage of oxygen (21%) but a vastly higher quantity: 1.4×10^{21} kg x 0.21 = **2.9×10^{20} kilograms**. (That's 290,000,000,000,000,000,000 kg)

Problem 5 – Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Answer: The objects with the highest percentage of hydrogen are the sun, Mercury, Jupiter, Saturn, Uranus and Neptune. The objects with the least percentage are Venus, Earth, Moon, Mars, Titan, Pluto. With the exception of Mercury, which has a very thin atmosphere, the high-percentage objects are the largest bodies in the solar system. The planet Jupiter, Saturn, Uranus and Neptune are sometimes called the Gas Giants because so much of the mass of these planets consists of a gaseous atmosphere. These bodies generally lie far from the sun. The low-percentage objects are among the smallest bodies in the solar system. They are called the 'Rocky Planets' to emphasize their similarity in structure, where a rocky core and mantel are surrounded by a thin atmosphere. Most of these bodies lie close to the sun.

One of the best ways to explore the effects of gravity on different bodies in the solar system is to calculate what your weight wound be if you were standing on their surfaces!

Scientists use kilograms to indicate the mass of an object, and it is common for Americans to use pounds as a measure of weight. On Earth, the force that one kilogram of mass has on the bathroom scale is equal to 9.8 Newtons or a weight of 2.2 pounds.

The surface gravity of a planet or other body is what determines your weight by the simple formula W = Mg where W is the weight in Newtons, M is the mass in kilograms, and g is the acceleration of gravity at the surface in meters/sec2. For example, on Earth, g = 9.8 m/sec, and for a person with a mass of 64 kg, the weight will be W = 64 x 9.8 = 627 Newtons. Since 9.8 Newtons equals 2.2 pounds, this person weighs 627 x (2.2/9.8) = 140 pounds.

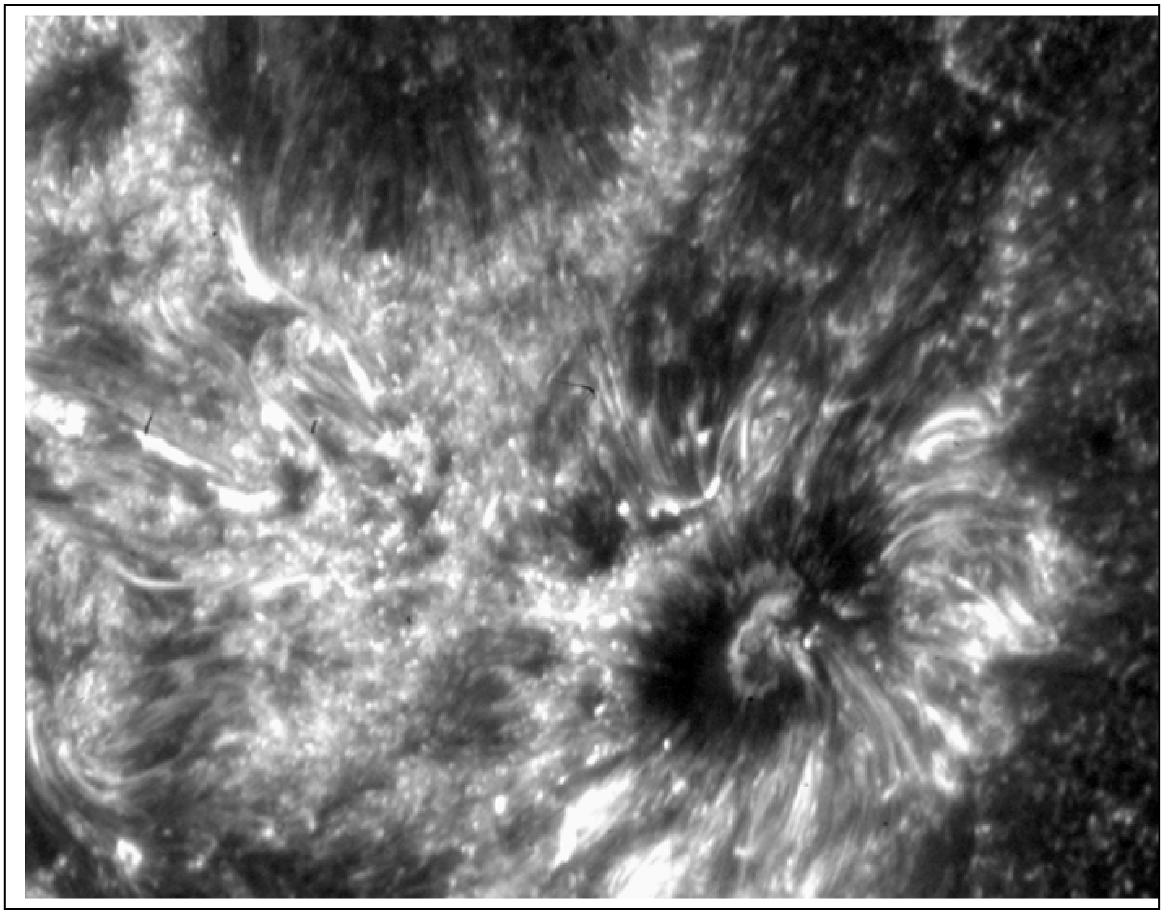
Problem 1 - Using proportional math, complete the following table to estimate the weight of a 110-pound (50 kg) person on the various bodies that have solid surfaces.

Object	Location	G (m/sec^2)	Weight (pounds)
Earth	Planet	9.8	110
Mercury	Planet	3.7	
Mars	Planet	3.7	
Io	Jupiter moon	1.8	
Moon	Earth moon	1.6	
Titan	Saturn moon	1.4	
Europa	Jupiter moon	1.3	
Pluto	Planet	0.58	
Charon	Pluto moon	0.28	
Vesta	Asteroid	0.22	
Enceladus	Saturn moon	0.11	
Miranda	Uranus moon	0.08	
Deimos	Mars moon	0.003	

Problem 1 - Using proportional math, complete the following table to estimate the weight of a 110-pound (50 kg) person on the various bodies that have solid surfaces.

Object	Location	G (m/sec^2)	Weight (pounds)
Earth	Planet	9.8	**110**
Mercury	Planet	3.7	**41.5**
Mars	Planet	3.7	**41.5**
Io	Jupiter moon	1.8	**20.2**
Moon	Earth moon	1.6	**18.0**
Titan	Saturn moon	1.4	**15.7**
Europa	Jupiter moon	1.3	**14.6**
Pluto	Planet	0.58	**6.5**
Charon	Pluto moon	0.28	**3.1**
Vesta	Asteroid	0.22	**2.5**
Enceladus	Saturn moon	0.11	**1.2**
Miranda	Uranus moon	0.08	**0.9**
Deimos	Mars moon	0.003	**0.03**

Note: For the Mars moon Deimos, which is a rocky body only 12 km (7.5 miles) in diameter, your weight would be 0.03 pounds or just ½ ounce! Astronauts that visit this moon would not 'land' on its surface but 'dock' with the moon the way that they do with visits to the International Space Station!



This spectacular image of the solar atmosphere was obtained by NASA's IRIS satellite. The smallest details are 240 km (150 miles) across, and the image shows many small bright 'dots' where energy is being released and transported into the solar corona. The bright spots come and go in less than an hour, and the solar surface is peppered with millions of these regions. This image is 72,000 km across.

Problem 1 – The radius of the sun is 690,000 km. What is the surface area of the spherical sun in km^2?

Problem 2 – What percentage of the sun's surface area does the square IRIS image above represent?

Problem 3 – Suppose that each bright point is a cylinder 10,000 km tall with a diameter of 200 km in diameter. What is the volume of the cylinder in kilometers?

Problem 4 – The magnetic energy stored in this volume is 4.0×10^{10} Joules/km^3. If a single 10 megaton hydrogen bomb produces 4×10^{15} Joules, how many megatons of energy is produced by this region?

Problem 1 – The radius of the sun is 690,000 km. What is the surface area of the spherical sun in km^2?

Answer: $A = 4 \pi R^2$, so
$A = 4 \times 3.141 \times (690{,}000)^2$
$= \mathbf{5.6x10^{12}\ km^2}$

Problem 2 – What percentage of the sun's surface area does the square IRIS image above represent?

Answer: Image area = $(72{,}000)^2 = 5.2x10^9\ km^2$.
So $P = 100\% \times 5.2x10^9/5.6x10^{12}$ = **0.09%**

Problem 3 – Suppose that each bright point is a cylinder 10,000 km tall with a diameter of 200 km in diameter. What is the volume of the cylinder in cubic meters?

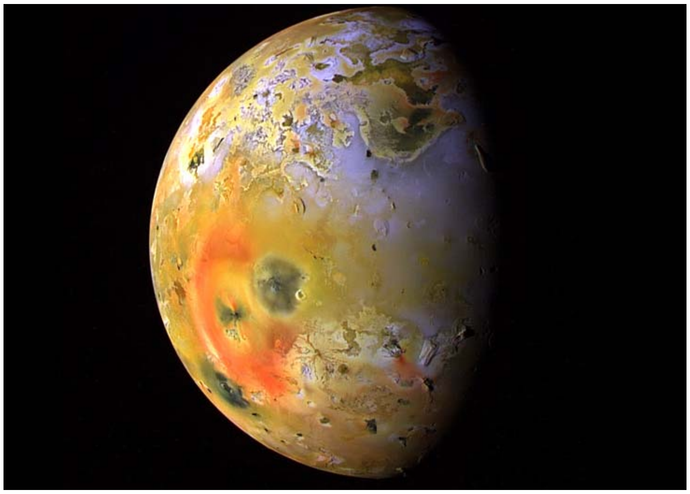
Answer: $V = \pi R^2 h$. $V = 3.141 \times (100\ km)^2 \times 10{,}000\ km$ = $\mathbf{3.1x10^8\ km^3}$.

Problem 4 – The magnetic energy stored in this volume is $4.0x10^{10}$ Joules/km^3. If a single 10 megaton hydrogen bomb produces 4×10^{15} Joules, how many megatons of energy is produced by this region?

Answer: $E = 4.0x10^{10}\ J/km^3 \times 3.1x10^8\ km^3 = 1.2x10^{19}$ Joules.

$M = 1.2x10^{19}$ Joules/$4.0x10^{15}$ = **3000 megatons**.

This equals 300, 10 megaton hydrogen bombs!



This 1997 image taken by NASA's Galileo spacecraft shows the complex surface of Io. Sulfur dioxide frost appears in white and grey hues while yellowish and brownish hues are from other sulfurous materials. The new dark spot 400 km in diameter, surrounds a volcanic center named Pillan Patera. The spot did not exist 5 months earlier, and is the source of a 120 km high plume that has been seen erupting from this location.

Although no impact craters have been found, over 420 calderas and active vents have been mapped. About 15 are actively spewing fresh material within 175 km of each vent. This means that Io quickly resurfaces itself, covering over all of the impact craters within a million years or less.

Problem 1 – Assume Io is a sphere with a radius of 1820 km, and is covered to a depth of 1 kilometers to cover any new craters. What volume of fresh material must be produced in cubic meters?

Problem 2 – If the present surface was produced by the 420 calderas, what is the volume produced by each caldera?

Problem 3 – The typical time between large meteor impacts is about 500,000 years. How much material would have to be produced by each caldera each year to cover the surface between impacts?

Problem 4 – If any given caldera is only active for 1% of its life, what does the resurfacing rate have to be for each caldera?

Problem 5 – What is the total resurfacing rate each year in centimeters/year?

Problem 1 – Assume Io is a sphere with a radius of 1820 km, and is covered to a depth of 1 kilometers to cover any new craters. What volume of fresh material must be produced in cubic meters?

Answer: Area = $4 \pi R^2$, and R = 1820,000 meters, so
Area = 4 x 3.14 x $(1820000)^2$ = 4.2×10^{13} m^2.
The volume of the surface shell 1km thick is that V = A x 1km = **4.2×10^{16} m^3.**

Problem 2 – If the present surface was produced by the 420 calderas, what is the volume produced by each caldera?

Answer: 4.2×10^{16} m^3 / 420 = **1.0×10^{14} m^3 per caldera**.

Problem 3 – The typical time between large meteor impacts is about 500,000 years. How much material would have to be produced by each caldera each year to cover the surface between impacts?

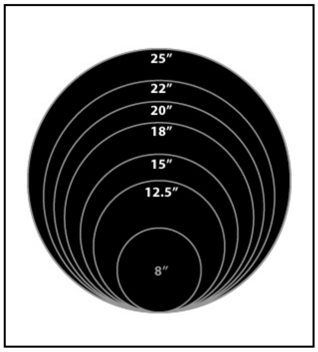
Answer: 1.0×10^{14} m^3/500000yrs = **2.0×10^{8} m^3/year.**

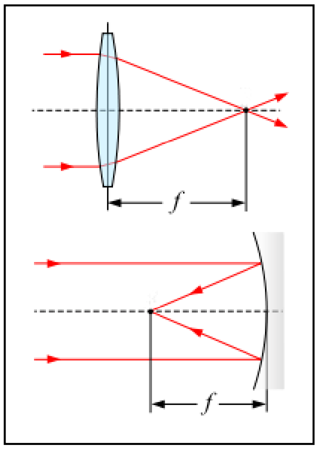
Problem 4 – If any given caldera is only active for 1% of its life, what does the resurfacing rate have to be for each caldera?

Answer: 2.0×10^{8} m^3/year would be the rate if each caldera continuously operated for 500,000 years. If they only are active for 1% of this time, then the average rate has to be 100x higher or **2.0×10^{10} m^3/yr.**

Problem 5 – What is the total resurfacing rate each year in centimeters/year?

Answer: If the 1km depth is generated over 500,000 years, then each year the depth added is 100000 centimeters/500000 yr = **0.2 centimeters/year.**





Any optical system, such as a telescope, camera or microscope, can be described by a just few basic numbers.

Aperture is the main lens or mirror that gathers the light to a focus. Aperture diameter, D, is commonly measured in inches, millimeters, centimeters or even meters. The larger the aperture, the more light the system gathers and the finer details it can see. The top figure shows various aperture diameters for telescopes that can be bought.

Focal length is the distance between the center of the aperture and the point in space where distant light rays come to a focus. In the figure, both a lens and a properly-curved mirror can have focal points. The symbol, f, represents the focal length.

F/ number is a measure of the speed and clarity of the optical system. It is the ratio of the focal distance to the aperture size. Fast systems have small F/numbers such as F/1, F/2 or F/3. Slow systems have large F/ numbers such as F/8, F/15 or even F/20. In photography these are also called F-stops.

F/ = f / D

Problem 1 – An astronomer wants to design a telescope that takes up the least amount of space in a research satellite. The aperture has to be 254 millimeters in order to gather the most light possible and provide the clearest images. The light path between the mirror center and the focus can be folded 3 times between mirrors separated by 500 millimeters. What is the focal length of this system and the F/ number? Is this a fast or slow system?

Problem 2 – An amateur astronomer wants to buy a telescope and has a choice between three different systems that cost about the same:

System 1 :	F/2.0		f = 100 mm
System 2 :		D= 10-inches,	f = 1270 mm
System 3 :	F/15.0	D = 50 mm	

Fill-in the missing quantities and describe the pros and cons of each system.

Problem 1 – An astronomer wants to design a telescope that takes up the least amount of space in a research satellite. The aperture has to be 254 millimeters in order to gather the most light possible and provide the clearest images. The light path between the mirror center and the focus can be folded 3 times between mirrors separated by 500 millimeters. What is the focal length of this system and the F/ number? Is this a fast or slow system?

Answer: The focal length is 3 x 1500 millimeters = 1500 millimeters, and F/ = 1500/254 = 5.9. It is a slow system because F/ > 3.0.

Problem 2 – An amateur astronomer wants to buy a telescope and has a choice between three different systems that cost about the same:

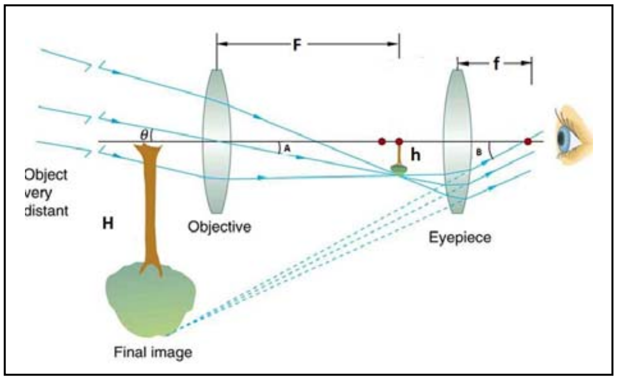
System 1 :	F/2.0	D=200 mm	f = 100 mm
System 2 :	F/5.0	D= 254 mm	f = 1270 mm
System 3 :	F/15.0	D = 50 mm	f = 750 mm

Fill-in the missing quantities and describe the pros and cons of each system.

Answer: System 1: D = 100 mm x 2.0 = 200 mm. System 2: D = 10 inches x 25.4 mm/inch = 254 millimeters so F = 1270/254 = 5.0; System 3: f = 50 mm x 15.0 = 750 mm.

System 3 is the slowest optical system in the group, and has the smallest aperture, which means that it gathers the least amount of light and so images will appear fainter and show less detail.

System 1 and 2 are very similar in aperture so they gather about the same amount of light, however, System 1 is nearly 3 times faster and so will provide the clearest images. System 1 is also shorter that System 2 (100 mm vs 1270 mm) so it would be easier and lighter to operate.



A telescope consists of an objective mirror or lens and an eyepiece. The role of the eyepiece is to change the angle, A, of the rays from the objective as they enter the eye. As the figure shows, when B > A, it appears as though the image of the tree is bigger than its actual image at the focus of the telescope objective. A simple proportion relates the image sizes to the focal lengths of the lenses:

$$\frac{H}{h} = \frac{F}{f}$$

For example, if the telescope objective has a focal length of 2000 millimeters and the eyepiece has a focal length of 4 millimeters, H/h = 2000/4 = 500, so the image h has been magnified by 500 times. The quantity F/f is the magnification.

Problem 1 – The table below gives the optical data for some large telescopes. Use this data to calculate the magnification for each indicated lens. Also fill in all other missing information. Focal lengths and aperture dimensions are given in millimeters.

Telescope	Type	Aperture	F/	Focal Length	Eyepiece F.L.	Magnification
8-inch Orion	Reflector	203		1198	10	
Obsession-20	Reflector	508	5.0		8	
1-meter	Reflector	1000	17.0			850
David Dunlap	Reflector	1880	17.3		100	
Hubble	Reflector	2400		57600		2880
Mt Palomar	Reflector		3.3	16830	28	
Yale	Refractor	1020	19.0		4	
Subaru	Reflector	8200		15000		7500
Keck	Reflector		1.75	17500	1	

Problem 2 – Suppose that the eyepiece was eliminated and the human eye was used as the eyepiece instead. If the focal length of the human eye is 25 cm, what is the magnification for the Obsession-20 telescope operating in this way? (Note: this is called Prime Focus observing).

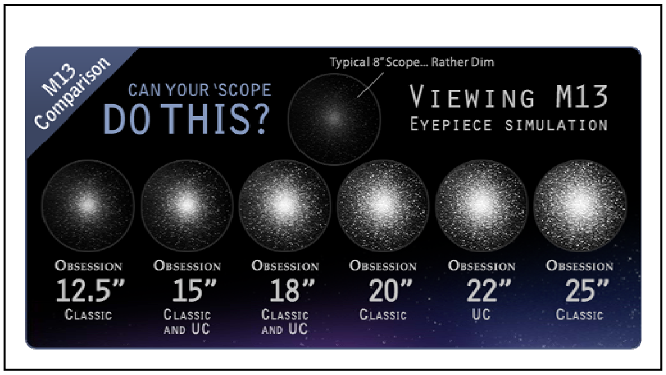
Problem 1 – The table below gives the optical data for some large telescopes. Use this data to calculate the magnification for each indicated lens. Also fill in all other missing information.

Telescope	Type	Aperture	F/	Focal Length	Eyepiece F.L.	Magnification
8-inch Orion	Reflector	203	**5.9**	1198	10	**120**
Obsession-20	Reflector	508	5.0	**2540**	8	**317**
1-meter	Reflector	1000	17.0	**17000**	**20**	850
David Dunlap	Reflector	1880	17.3	32,524	**100**	**325**
Hubble	Reflector	2400	**24**	57600	**20**	2880
Mt Palomar	Reflector	**5100**	3.3	16830	28	**601**
Yale	Refractor	1020	19.0	**19400**	4	**4850**
Subaru	Reflector	8200	**1.83**	15000	**2**	7500
Keck	Reflector	**10000**	1.75	17500	1	**17500**

Problem 2 – Suppose that the eyepiece was eliminated and the human eye was used as the eyepiece instead. If the focal length of the human eye is 25 cm, what is the magnification for the Obsession-20 telescope operating in this way? (Note: this is called Prime Focus observing).

Answer: The focal length of the Obsession-20 mirror is 2540 mm, and the eye's focal length is 250 mm, so the magnification is only about **10 times**. This means that if you look at the moon in this way, it will appear 10 times bigger 'in the sky' than without the telescope.

Note: A rule-of-thumb is that you do not use a higher magnification than 2 times the aperture size in millimeters. At higher magnifications the image remains blurry and you do not see additional details. In the table in Problem 1, the only eyepiece that violates this rule is the one selected for the Yale Telescope, and so an eyepiece with a longer focal length and lower magnification is the best to use.



This figure shows a comparison of the same faint star cluster seen with telescopes of increasing aperture size and LGA. Notice the big change between an 8-inch and am 18-inch!

The human eye is a small lens that lets-in only a small amount of light. This is useful when you are looking at a bright daytime scene, but when you are studying faint stars this becomes a problem.

A telescope has a much larger aperture than the eye and allows more light to be brought to a focus to study. This means that even stars too faint to be detected by the eye can easily be 'brightened' by the telescope so that they are easy to detect and study.

Light Gathering Ability is the property of an optical system that tells you how much brighter things will appear than what the human eye can see. It is the ratio of the area of the objective to the area of the human eye lens.

Problem 1 – A pair of binoculars has a lens with a diameter of 50 mm. If the human eye lens has a diameter of 7mm, how much more light do the binoculars gather than the human eye?

Problem 2 – Star brightness is measured on the magnitude scale where each magnitude represents an increase in intensity by a factor of 2.514. What is the brightness difference between a star with m = +1.0 and m = +6.0?

Problem 3 – The human eye can see stars as faint as m = +6.0. What size mirror will be needed so that stars as faint as +16.0 can be seen?

Answer Key

Problem 1 – A pair of binoculars has a lens with a diameter of 50 mm. If the human eye lens has a diameter of 7mm, how much more light do the binoculars gather than the human eye?

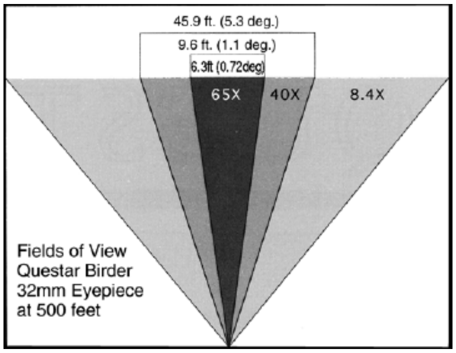
Answer: LGA = $(50/7)^2$ = 51 times more light.

Problem 2 – Star brightness is measured on the magnitude scale where each magnitude represents an increase in intensity by a factor of 2.514. What is the brightness difference between a star with m = +1.0 and m = +6.0?

Answer: The magnitude difference is m = 5.0, so the brightness difference is a factor of $(2.512)^5$ = 100 times.

Problem 3 – The human eye can see stars as faint as m = +6.0. What size mirror will be needed so that stars as faint as +16.0 can be seen?

Answer: The magnitude difference is +16.0 - +1.0 = +10.0, which is a brightness factor of 100x100 = 10,000. You need a telescope that provides a LGA of 10000 so
10000 = $(D/7mm)^2$ and so D = 700 millimeters (27-inches in diameter).



Although magnifying an object makes it appear larger, the view also gets smaller as the diagram to the left shows. Every combination of telescope and eyepiece produces its own Field of View (FOV), usually stated in angular terms. For example, a combination that gives an angular FOV of 10^o in diameter will easily let you see the entire full moon, which is only 0.5^o in diameter. But if you use a lens with 40 times more magnification, the FOV is now only 1/4^o so you only see ½ of the full disk of the moon in the eyepiece.

By itself, an eyepiece allows incoming light to be brought to a focus for the human eye or camera. The incoming light rays can come from many different directions within a cone whose vertex is the focus point for the lens. The angle of the cone's vertex defines the FOV for the eyepiece. The table below shows the FOVs for various eyepieces that are used with telescopes:

Vendor	Model	Focal Length	Apparent FOV (°)	Actual FOV (°)	Magnifi-cation	Price
Orion	Optilux 2"	40	60	1.18	51	$140
Televue	Panoptic 2"	35	68	1.17	58	$370
Orion	FMC Plössl 2"	50	45	1.11	41	$120
Orion	DeepView 2"	42	52	1.07	48	$70
Edmund Optics	RKE Erfle 2"	32	68	1.07	64	$225
Meade	SWA 2"	32	67	1.06	64	$240
Orion	Optilux 2"	32	60	0.94	64	$140
Teleview	Panoptic 2"	27	68	0.90	75	$330
Teleview	Plössl 1.25"	40	43	0.85	51	$110
Celestron	Ultima 1.25"	35	49	0.84	58	$108

The apparent FOV for each eyepiece ranges from 45^o to 68^o and is a result of how the eyepiece is designed. When used in this example with an 8-inch telescope with a focal length of 2032 millimeters, the magnifications range from 41x to 75x. The resulting telescope FOV is then just FOV = Eyepiece FOV/magnification. For the Optilix 2" eyepiece, the FOV is then 60^o/51 = 1.18^o.

Problem 1 – An astronomer wants to design a system so that the full moon fills the entire FOV of the telescope. He uses an eyepiece with a FOV of 60^o. What magnification will give him the desired FOV?

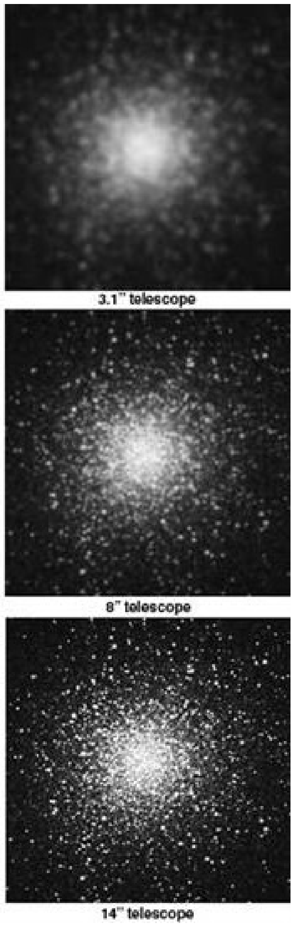
Problem 2 – An amateur astronomer upgrades to a larger telescope and keeps his old eyepieces, which have FOVs of 50^o. His old telescope provided a 2.0^o FOV for his most expensive eyepiece. Because the focal length of the new telescope is twice that of his older telescope, all magnifications on the new telescope will be twice as high. What will the FOV be for his most expensive eyepiece on the new telescope?

Problem 1 – An astronomer wants to design a system so that the full moon fills the entire FOV of the telescope. He uses an eyepiece with a FOV of 60^{o}. What magnification will give him the desired FOV?

Answer: $0.5^{o} = 60^{o} / M$ so the magnification is M = **120x**.

Problem 2 – An amateur astronomer upgrades to a larger telescope and keeps his old eyepieces, which have FOVs of 50^{o}. His old telescope provided a 2.0^{o} FOV for his most expensive eyepiece. Because the focal length of the new telescope is twice that of his older telescope, all magnifications on the new telescope will be twice as high. What will the FOV be for his most expensive eyepiece on the new telescope?

Answer: Because FOV = eyepiece FOV/magnification, if the new telescope provides magnifications that are twice the older system, then the FOV will be half as large for this eyepiece or 1.0^{o}.



The images to the left show what the star cluster Messier-13 would look like to three different telescopes with apertures of 3.1, 8.0 and 14.0 inches. Notice that as the aperture increases, the fuzzy smudges seen by the smallest telescope become increasingly more clear to see as the aperture increases. This is an example of Optical Resolution, which is sometimes called the Resolving Power of a telescope.

To make the clearest photographs of stars, planets, or even people, it helps to use the largest lens or aperture to make crisp, clear images. Astronomers also want the highest resolutions possible so that they can study the smallest details on a planet surface, or in a distant galaxy.

Telescope resolution at optical wavelengths can be calculated using the simple formula:

$$R = \frac{134}{D}$$

where D is the diameter of the objective in millimeters, and R is the resolution in seconds of arc. (There are 3600 seconds of arc in 1 angular degree).

For example, a pair of binoculars with D = 50 mm, provides a resolution limit of R = 2.8 arcseconds. A small 8-inch telescope for which D = 200 mm, provides R = 0.67 arcseconds.

Problem 1 – An astronomer wants to design a system that will let him study craters on the moon that are about 0.1 arcseconds in diameter as seen from Earth. What is the minimum-sized aperture he needs to conduct his study?

Problem 2 – The Hubble Space Telescope has a diameter of 2.4 meters. What is its maximum resolution?

Problem 3 – Two telescopes are combined in an instrument called an interferometer, which creates a single telescope with a diameter of 640 meters. What is the maximum resolution of this system?

Problem 1 – An astronomer wants to design a system that will let him study craters on the moon that are about 0.1 arcseconds in diameter as seen from Earth. What is the minimum-sized aperture he needs to conduct his study?

Answer: 0.1 = 134/D so D = **1340 millimeters (53 inches)**.

Problem 2 – The Hubble Space Telescope has a diameter of 2.4 meters. What is its maximum resolution?

Answer: R = 134/2400 = **0.06 arcseconds** or 60 milliarcseconds

Problem 3 – Two telescopes are combined in an instrument called an interferometer, which creates a telescope with a diameter of 640 meters. What is the maximum resolution of this system?

Answer: R = 134/640000 = **0.0002 arcseconds or 0.2 milliarcseconds**.

A photo of the Sydney University Stellar Interferometer (SUSI) is a long-baseline optical interferometer located approximately 20km west of the town of Narrabri in northern New South Wales, Australia. The equivalent diameter of the optical aperture for this instrument is 640 meters.

Designing a Telescope System

Telescope diameter = D
Telescope focal length = F
Eyepiece focal length = f
Eyepiece field of view = FOVe
Magnification = M
Resolution = R

D, F, f are in millimeters
FOVe is in degrees
R is in arcseconds

$$M = \frac{F}{f}$$

$$F/number = \frac{F}{D}$$

$$R = \frac{134}{D}$$

Astronomers don't just go out and buy a telescope and then use it. Whether it is for use on a satellite orbiting Saturn, or in an observatory, telescopes are designed 'from the ground up' by starting from a set of goals that the research needs to accomplish. The telescope is mathematically designed to meet these research goals.

The table to the left gives the basic quantities and formulae for designing a simple telescope system. Let's see how two different research goals can lead to very different telescope systems!

System 1 – Veronica has been an amateur astronomer for 20 years and especially enjoys photographing faint galaxies and nebulae. She has owned three telescopes and plans to sell them to fund her next system. She can afford a telescope with an aperture no larger than 20-inches (500 mm), and needs it to be a fast optical system with an F-number less than 3.0. She has a set of expensive eyepieces that she will keep. Her favorite one is a 20mm Plossl with a FOV of 68^{o}, and for best results she wants this eyepiece to have a magnification of no more than 50x. What is the best combination of aperture size and focal length for the telescope that will satisfy all of her needs?

System 2 – Leonard has a program of observing Saturn to keep track of its equatorial belt system. He needs a telescope with F/number > 10 and a resolution between 1/3 and 1/2 arcseconds. He has three eyepieces with focal lengths of f = 5mm, 10mm and 20mm that have provided him with high, medium and low magnification on his previous telescope, which got damaged in a house fire. He wants the 5mm eyepiece to provide no more than a magnification of 700x. What is the best system that meets his needs?

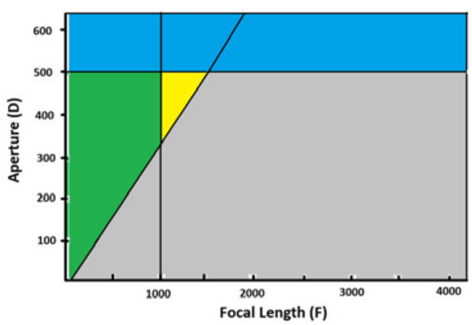
To design these systems, create a graph with the aperture diameter (vertical) and the focal length (horizontal).

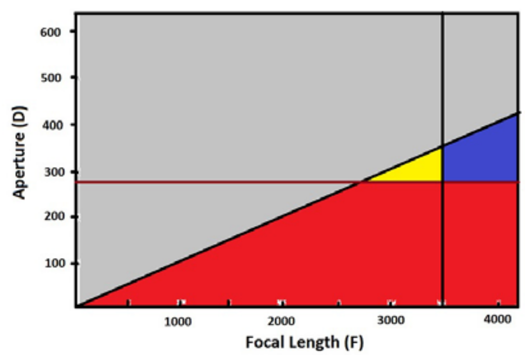
System 1 – Veronica has been an amateur astronomer for 20 years and especially enjoys photographing faint galaxies and nebulae. She has owned three telescopes and plans to sell them to fund her next system. She can afford a telescope with an aperture no larger than 20-inches (500 mm), and needs it to be a fast optical system with an F-number less than 3.0. She has a set of expensive eyepieces that she will keep. Her favorite one is a 20mm Plossl with a FOV of 68°, and for best results she wants this eyepiece to have a magnification of no more than 50x. What is the best combination of aperture size and focal length for the telescope that will satisfy all of her needs?

Answer: D < 20 inches (500 mm); F/number < 3.0; f = 20 mm, FOVe=68°, M> 50x

On the D vs F graph, draw a line representing F/D = 3.0 or F = 3.0D and shade the excluded region below this line (grey), which represents F/n > 3.0. Now draw a horizontal line for D = 500mm and shade (blue) the region above this line which represents D > 500mm. Magnification = F/f so we have F/f > 50 and F > 50f. For f = 20mm, the constraint is F > 1000 mm. Draw a vertical line at F = 1000mm and shade (green) all points to the left as the excluded region. The permitted regions is the one shown in yellow. The final plot (below left) should look like the one below.

An optimal system is near the middle of the yellow permitted region for which D = 450 mm and F = 1250mm. We then have F/number = 2.8, a magnification of 62, a telescope FOV of 68/62 = 1 degree, and a resolution of 134/450 = 0.3 arcseconds.

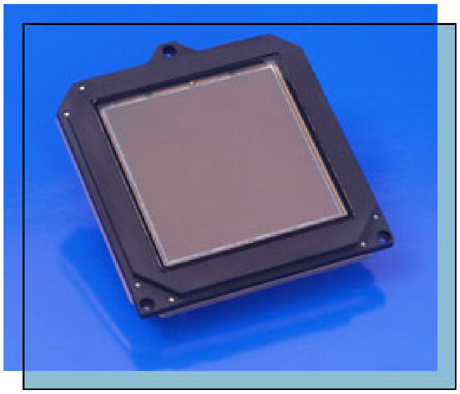




System 2 – Leonard has a program of observing Saturn to keep track of its equatorial belt system. He needs a telescope with F/number > 10 and a resolution between 1/3 and 1/2 arcseconds. He has three eyepieces with focal lengths of f = 5mm, 10mm and 20mm that have provided him with high, medium and low magnification on his previous telescope, which got damaged in a house fire. He wants the 5mm eyepiece to provide no more than a magnification of 700x. What is the best system that meets his needs?

Answer: F/number > 10; R = 1/3 to ½ arcseconds; M < 700x.
Draw a line representing F/number = 10 and shade (grey) the excluded area above this line which indicates F/number <10. For R = ½ arcseconds, draw a horizontal line at D = 270mm and shade the region (red) below this line that represents R > ½. The magnification M = F/5mm so for M=700 we have F < 3500mm. Draw a vertical line at F = 3500 and shade (blue) the excluded area to the right. The allowed region is in yellow in the diagram on the upper right.

The best system lies in the center of the triangle with D = 300mm and F = 3250 mm. This gives F/10.8; Resolution = 134/300 = 0.4 arcseconds; Lens magnifications of 650, 325 and 162.



This is a 9200x9200 image sensor for a digital camera. The grey area contains the individual 'pixel' elements. Each square pixel is about 8 micrometers (8 microns) wide, and is sensitive to light. The pixels accumulate electrons as light falls on them, and computers read out each pixel and create the picture.

Digital cameras are everywhere! They are in your cell phones, computers, iPads and countless other applications that you may not even be aware of.

In astronomy, digital cameras were first developed in the 1970s to replace and extend photographic film techniques for detecting faint objects. Digital cameras are not only easy to operate and require no chemicals to make the images, but the data is already in digital form so that computers can quickly process the images.

Commercially, digital cameras are referred to by the total number of pixels they contain. A '1 megapixel camera' can have a square-shaped sensor with 1024x1024 pixels. This says nothing about the sensitivity of the camera, only how big an image it can create from the camera lenses. Although the largest commercial digital camera has 80 megapixels in a 10328x7760 format, the largest astronomical camera developed for the Large Synoptic Survey Telescope uses 3200 megapixels (3.2 gigapixels)!

Problem 1 – An amateur astronomer purchases a 6.1 megapixel digital camera. The sensor measures 20 mm x 20 mm. What is the format of the CCD sensor, and about how wide are each of the pixels in microns?

Problem 2 – Suppose that with the telescope optical system, the entire full moon will fit inside the square CCD sensor. If the angular diameter of the moon is 1800 arcseconds, about what is the resolution of each pixel in the camera?

Problem 3 – The LSST digital camera is 3.2 gigapixels in a 10328x7760 format. If the long side of the field covers an angular range of 3.5 degrees, what is the angular resolution of this CCD camera in arcseconds/pixel?

Problem 1 – An amateur astronomer purchases a 6.1 megapixel digital camera. The sensor measures 20 mm x 20 mm. What is the format of the CCD sensor, and about how wide are each of the pixels in microns?

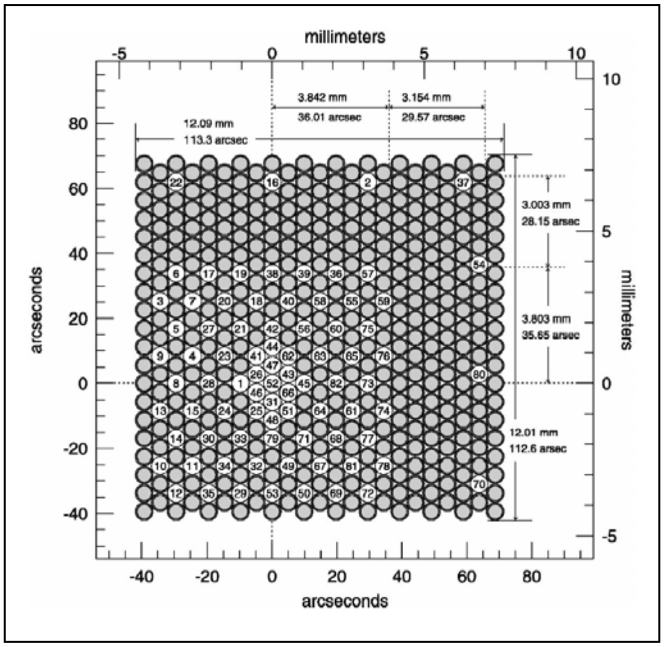
Answer: This is a square array, so s2 = 6100000 pixels and so s = 2469 pixels. The format is **2469 x 2469 pixels**. Since the width of a side is 20 mm, each pixel is about 20 mm/2469 = 0.0000081 meters or **8.1 microns** on a side.

Problem 2 – Suppose that with the telescope optical system, the entire full moon will fit inside the square CCD sensor. If the angular diameter of the moon is 1800 arcseconds, about what is the resolution of each pixel in the camera?

Answer: 1800 arcseconds/2469 pixels = **0.7 arcseconds/pixel**.

Problem 3 – The LSST digital camera is 3.2 gigapixels in a 10328x7760 format. If the long side of the field covers an angular range of 3.5 degrees, what is the angular resolution of this CCD camera in arcseconds/pixel?

Answer: 1 degree = 3600 arcseconds, so 3.5 degrees = 12600 arcseconds. Then 12600 arcseconds/10328 pixels = **1.2 arcseconds/pixel.**



The image of an astronomical object forms at the focus of a telescope. It can be measured either in terms of its angular size in degrees or arcseconds, or in terms of the number of millimeters. For astronomy, you are interested in its 'angular size' but if you want to photograph it, you are interested in how big it will be compared to the size of your film or digital sensor array (called the CCD).

A very simple formula defines how to convert from angular size to millimeters at the focus of a telescope:

$$\text{Scale} = \frac{206265}{F}$$

where F is the focal length in millimeters and 'Scale' is the image scale in arcseconds per millimeter.

Problem 1 – Two telescopes are used for astrophotography. The first has a mirror diameter of 20-inches and a focal length of 5000 mm, the second one is more portable and has a diameter of 10-inches and a focal length of 2000 mm. What are the image scales of each telescope?

Problem 2 – An astronomer wants to design a camera so that each pixel views an angle of only 0.5 arcseconds. If the width of each pixel is 8 micrometers (0.008 millimeters), what is the image scale he needs for the telescope, and what will be its focal length?

Problem 3 - If a digital camera array measures 20 millimeters across and consists of 2048 pixels, what will the image scale have to be so that the array can be used to photograph a star cluster with a diameter of ¼ degree? What will the telescope focal length have to be.

Problem 1 – Two telescopes are used for astrophotography. The first has a mirror diameter of 20-inches and a focal length of 5000 mm, the second one is more portable and has a diameter of 10-inches and a focal length of 2000 mm. What are the image scales of each telescope?

Answer: The first one is Scale = 206265/5000 = **41 arcseconds/mm**. The second one has 206265/2000mm = **103 arcseconds/mm**.

Problem 2 – An astronomer wants to design a camera so that each pixel views an angle of only 0.5 arcseconds. If the width of each pixel is 8 micrometers (0.008 millimeters), what is the image scale he needs for the telescope, and what will be its focal length?

Answer: He needs a scale of 0.5 arcseconds/0.008 mm) = 62.5 arcseconds/mm. The focal length will be 62.5 = 206265/F so **F = 3300 millimeters**.

Problem 3 - If a digital camera array measures 20 millimeters across and consists of 2048 pixels, what will the image scale have to be so that the array can be used to photograph a star cluster with a diameter of ¼ degree? What will the telescope focal length have to be.

Answer: ¼ degree/20 millimeters = 3600 arcsec (1/4)/20 mm = 45 arcseconds/mm. The Focal length is 45 = 206265/F so **F = 4583 millimeters**.

Buying a Telescope!

OK…So those wonderful pictures of planets, star clusters and galaxies have got your curiosity on fire. You want to have your own telescope so that you can see the universe for yourself! All you have to do is spend a few minutes on the Internet and you will see a bewildering number of choices for telescopes you can buy. Some are pretty inexpensive and cost less than $70.00, but others can cost $500.00 or more. How do you decide which one is right for you?

Remember, the bigger the objective lens or mirror, the fainter you can see stars in the sky. The longer the focal length, the higher will be the magnification. The only limit to either of these is that you should not use magnifications higher than 50x the diameter of the objective, or 2 times its diameter in millimeters. Higher magnifications only make images look worse!

Type	Objective (cm)	Focal Length (millimeters)	Maximum Magnification	Cost
Reflector	7.6	300		$64.95
Refractor	6.0	700		$54.95
Refractor	8.9	910		$300.00
Reflector	11.4	900		$129.95
Reflector	15.2	610		$319.95
Refractor	10.0	900		$749.95
Reflector	20.3	1000		$699.95
Refractor	15.2	1219		$1,199.00
Reflector	50.8	2032		$4,400.00

Problem 1 – You have a set of eyepieces with focal lengths of 2mm, 4mm and 28mm. If

$$\text{magnification} = \frac{\text{telescope focal length}}{\text{eyepiece focal length}}$$

would you be able to use all of these eyepieces with the telescopes in the table above?

Problem 2 - In terms of cost per objective area, which type of telescopes seem to be the best value: reflectors or refractors?

Problem 3 – About how much would you expect to pay for a 50.8-cm refractor?

Type	Objective (cm)	Focal Length (millimeters)	Maximum Magnification	Cost	Cost per area
Reflector	7.6	300	152x	$64.95	1.4
Refractor	6.0	700	120x	$54.95	1.9
Refractor	8.9	910	178x	$300.00	4.7
Reflector	11.4	900	228x	$129.95	1.3
Reflector	15.2	610	304x	$319.95	1.8
Refractor	10.0	900	200x	$749.95	9.6
Reflector	20.3	1000	406x	$699.95	2.2
Refractor	15.2	1219	304x	$1,199.00	6.6
Reflector	50.8	2032	1016x	$4,400.00	2.2

Problem 1 – You have a set of eyepieces with focal lengths of 2mm, 4mm and 28mm. If

$$\text{magnification} = \frac{\text{telescope focal length}}{\text{eyepiece focal length}}$$

would you be able to use all of these eyepieces with the telescopes in the table above?

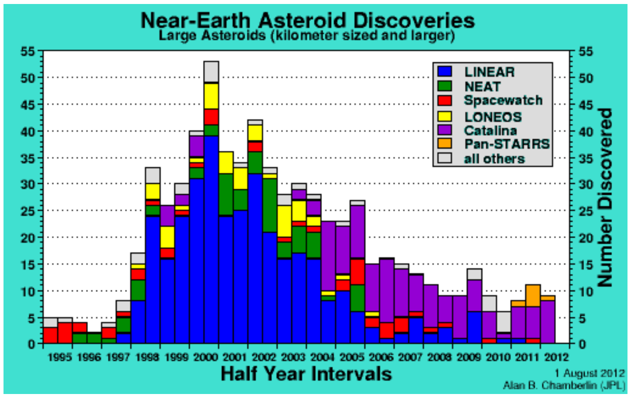
Answer: The limit would be set by the 2mm eyepiece. For the telescope focal lengths in the table, this eyepiece could not be used with the telescopes shaded in yellow in the table, Telescopes 2, 3, 4, 6, 7 and 8. The 4mm would not be used on telescopes 2, 3, 4, and 6.

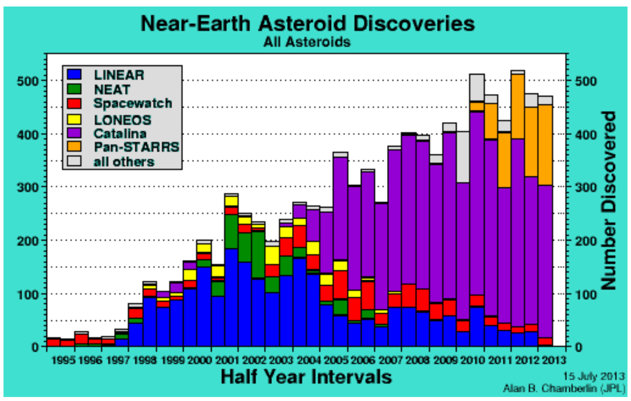
Problem 2 - In terms of cost per objective area, which type of telescopes seem to be the best value: reflectors or refractors?

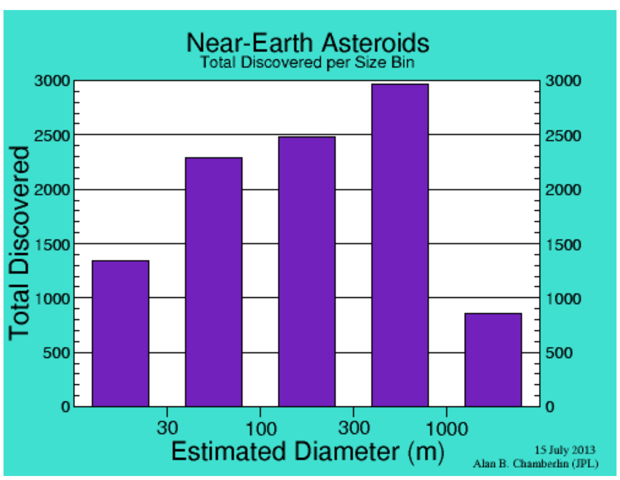
Answer: You can get a reflector with a larger area than you can a refractor. Refractors are more expensive per unit area. From the table, reflectors cost 1.3 to 2.2 dollars per square centimeter, while refractors cost 1.9 to 9.6 dollars per square centimeter.

Problem 3 – About how much would you expect to pay for a 50.8-cm refractor?

Answer: A 15.2 cm refractor costs about 6.6 dollars per square centimeter, so a 50.8-cm refractor would cost $\pi\,(50.8/2)^2 \times 6.6 =$ **$13,374.**







Near Earth Objects (NEOs) are asteroids that have orbits very close to Earth's orbit in the solar system. That means that, over time, they might collide with Earth. The most devastating asteroids are larger than 1 km and can cause world-wide extinction events. Smaller bodies from 10 to 300-meters can damage cities but are too small to affect continent-sized areas.

Astronomers have, for decades detected and tracked these smaller bodies as they come near Earth, and from this determine their orbits. The graphs to the left show the progress of these searches since 1995.

Problem 1 – The top graph shows the number of NEOs detected each year. Asteroids that are a kilometer or more in size can cause extinction events. How many of these were discovered in 2012?

Problem 2 – What is the total number of asteroids discovered in 2012?

Problem 3 – What percentage of the asteroids discovered in 2012 were A) larger than 1 kilometer? B)Smaller than 1 kilometer?

Problem 4 – From the bottom figure of total discovered asteroids, what percentage are a) smaller than 1 kilometer? B) Larger than 1 kilometer?

Problem 5 – Compare the top two figures. What can you conclude about the number of 1 kilometer or larger asteroids that are yet to be discovered, compared to those smaller than 1 kilometer?

Problem 1 – The top graph shows the number of NEOs detected each year. Asteroids that are a kilometer or more in size can cause extinction events. How many of these were discovered in 2012?

Answer: The last bar in the graph shows that **9** were discovered in 2012.

Problem 2 – What is the total number of asteroids discovered in 2012?

Answer: The second graph shows that for 2012 the column indicates about **470** were discovered that year.

Problem 3 – What percentage of the asteroids discovered in 2012 were A) larger than 1 kilometer? B) Smaller than 1 kilometer?

Answer: A) P = 100% x (9/470) = **1.9%.**
B) P = 100% x (461/470) = **98.1%**

Problem 4 – From the bottom figure of total discovered asteroids, what percentage are a) smaller than 1 kilometer? B) Larger than 1 kilometer?

Answer: A) Adding up all 5 columns gives a total of 1350+2300 + 2500 + 3000 + 850 = 10,000
Then Small asteroids P = 100% x (10,000-850)/10000 = **91.5%**
B) Large asteroids P = 100% - 91.5% = **8.5%**

Problem 5 – Compare the top two figures. What can you conclude about the number of 1 kilometer or larger asteroids that are yet to be discovered, compared to those smaller than 1 kilometer?

Answer: The top graph shows that the detection rate for these large asteroids has declined steadily since 2000. This means that each year the surveys are finding fewer and fewer large asteroids that they did not previously know about. This means that the surveys have almost completely detected all of the NEO asteroids that are this large. Because these large NEOS are only 2% of the detected asteroids, the majority of NEO asteroids are much smaller than the large ones, and more numerous. The middle graph shows that the numbers we are detecting continues to grow each year with no sign of decreasing. This means that there are many more of these to be discovered than we have found already. By some estimates, we have only discovered about 5% of all that there are near Earth, which is why we have to keep searching. Once the number of new discoveries begins to follow the profile of the top figure, we will know that we have discovered the vast majority of the small asteroids that remain.

This image of Comet ISON was taken by the Hubble Space Telescope on April 10, 2013 when the comet was 386 million miles from the sun, and 394 million miles from Earth. The icy core of the comet may only be about 5 km in diameter, but the bright head or 'Coma' is about 3,000 miles across.

Although it is too far from the sun to have a dramatic tail, from just beyond the orbit of Mars, its tail has already grown to 57,000 miles.

Measurements by NASA's Swift satellite in January, 2013 show that at a distance of 460 million miles from the sun (375 million miles from Earth), and traveling at about 21 km/sec (13 miles/sec), Comet ISON was ejecting 51,000 kg of dust and 60 kg of water every minute as its surface has been steadily heated from 40 K to about 150 K. This pace of ejection will increase enormously as the comet gets closer to the sun. By some estimates the comet will lose 10% of its total mass as it swings by the sun and heads back out into deep space in early - 2014.

Problem 1 – Suppose that the average density of the comet nucleus is 1000 km/m^3, and it is a sphere with a diameter of about 6 km. What is the total mass of the nucleus of the comet?

Problem 2 – Suppose that Comet ISON continues to lose dust mass at the rate of 51,000 kg/minute as it travels to the orbit of Earth (93 million miles from the sun) at a speed of 13 miles/sec. How much mass will it lose, and what percentage of its total mass is this?

Problem 1 – Suppose that the average density of the comet nucleus is 1000 km/m3, and it is a sphere with a diameter of about 6 km. What is the total mass of the nucleus of the comet?

Answer: $V = 4/3\ \pi\ R^3$ so
$V = 1.33 \times 3.14 \times (3000m)^3$
$= 1.1 \times 10^{11}\ m^3$.

Since Mass = density x volume, the comet's mass is
$M = 1000 \times 1.1 \times 10^{13}$
= **1.1×10^{16} kg**.

Problem 2 – Suppose that Comet ISON continues to lose dust mass at the rate of 51,000 kg/minute as it travels to the orbit of Earth (93 million miles from the sun) at a speed of 13 miles/sec. How much mass will it lose, and what percentage of its total mass is this?

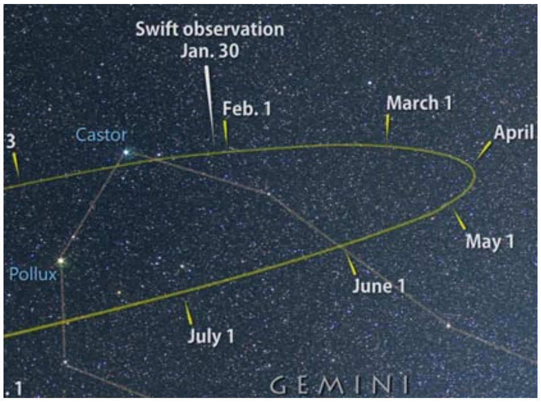
Answer: It travels from 460 million miles to 93 million miles covering a total of 460-93 = 367 million miles.

At a speed of 13 miles/sec, this takes 367,000,000/13 = 28 million seconds or 470,000 minutes.

At a rate of 51,000 kg/minute, it will loose M = 51,000 x 470,000 = **2.4×10^{10} kg**.

This represents P = 100% x ($2.4 \times 10^{10}/1.1 \times 10^{16}$) = **0.0004% of the total mass**!

The Orbit of Comet ISON



Comet ISON will be carefully watched as it makes its closest approach to the sun in November, 2013. Some astronomers predict that it may break up into smaller comets because of the Sun's enormous gravity. The comet will travel to within 1,800,000 km of the center of the sun, or about 1,100,000 km from the hot solar surface! As it travels, it will also get very close to Mars and the asteroid 3362 Khufu, though no impacts are predicted!

A portion of its track across the sky is shown in the figure for January-July, 2013.

The table below gives the location of Comet ISON as it approaches the sun. The sun is located at the point (-0.4, +14.7) where all of the coordinate units are in millions of kilometers.

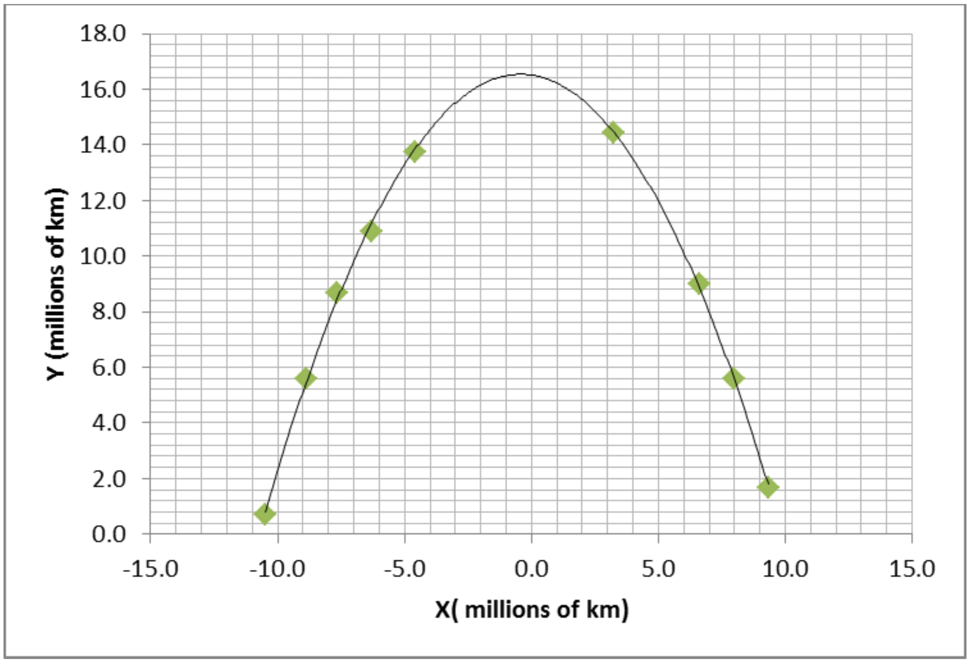
Date and Universal Time	X (million km)	Y (million km)	Distance to the sun (million km)
November 26, 18:00 UT	-10.5	+0.7	
November 27, 13:00 UT	-8.8	+5.6	
November 28, 01:00 UT	-7.7	+8.7	
November 28, 08:00 UT	-6.3	+10.9	
November 28, 14:00 UT	-4.6	+13.7	
November 28, 23:00 UT	+3.2	+14.5	
November 29, 10:00 UT	+6.6	+9.0	
November 29, 19:00 UT	+8.0	+5.6	
November 30, 10:00 UT	+9.3	+1.7	

Problem 1 - Plot these points on an X-Y graph and connect the points with a smooth parabolic curve.

Problem 2 – Using either a millimeter ruler and the scale of the graph, or the Two Point Distance Formula, calculate the distance from each comet position to the sun in the table.

Problem 3 – What is your prediction for the time when Comet ISON is at its closest point in its orbit to the sun?

Problem 1 - Plot these points on an X-Y graph and connect the points with a smooth parabolic curve. **Note: These coordinates are valid for the orbit as known on November 24, 2013 but may change as a more precise orbit is eventually determined.**



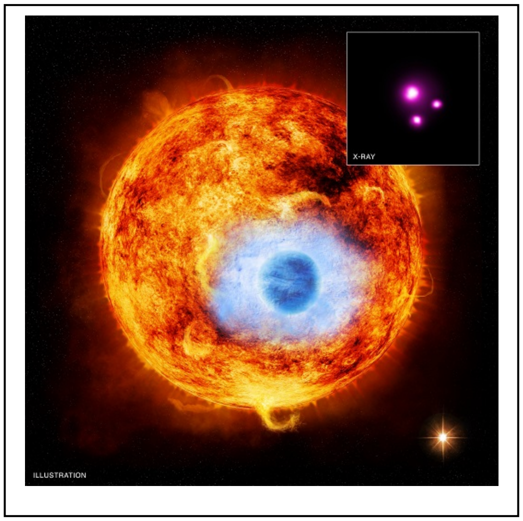
Problem 2 – Using either a millimeter ruler and the scale of the graph, or the Two Point Distance Formula, calculate the distance from each comet position to the sun in the table. Answer: For advanced students using the distance formula: $D = ((x2-x1)^2 + (y2-y1)^2)^{1/2}$. For the x2 at (-10.5, +0.7) and x1 (sun) at (-0.4,+14.7) so D = 17.3 million km.

Date and Universal Time	X (million km)	Y (million km)	Distance to the sun (million km)
November 26, 18:00 UT	-10.5	+0.7	**17.3**
November 27, 13:00 UT	-8.8	+5.6	**12.4**
November 28, 01:00 UT	-7.7	+8.7	**9.4**
November 28, 08:00 UT	-6.3	+10.9	**7.0**
November 28, 14:00 UT	-4.6	+13.7	**4.3**
November 28, 23:00 UT	+3.2	+14.5	**3.6**
November 29, 10:00 UT	+6.6	+9.0	**9.1**
November 29, 19:00 UT	+8.0	+5.6	**12.4**
November 30, 10:00 UT	+9.3	+1.7	**16.3**

Problem 3 – What is your prediction for the exact time when Comet ISON is at its closest point in its orbit to the sun? Answer: Students may interpolate between Points 5, 6 and 7 using any convenient method. The answers should be close to **November 28 at 18:00 UT and a distance to the sun of about 1,840,000 km.**

Using Calculus: The best fit parabola is $y = -0.155X^2 - 0.132X + 16.51$. Setting the first derivative equal to zero we get $0 = -0.31X - 0.132$ so $X = -0.43$ and $Y = +16.54$. The distance to the sun is then 1.84 million km.

Exploring the Evaporating Exoplanet HD189733b



At a distance of 63 light years from Earth, this Jupiter-sized planet orbits its star once every 2.2 days. With a mass only 13% greater than Jupiter, Astronomers have measured its light and found that it would appear like a giant blue marble similar to Neptune. Its atmosphere contains carbon dioxide, along with water vapor and methane. It is so close to its star, less than 3 million miles, that it is permanently locked so that the same side of the planet always faces its star. The daytime, sun-facing temperature is a sizzling 1000 C and nighttime temperature of 700 C.

NASA's Chandra Observatory has detected the X-ray fadeout of this planet as it passes across its star. From this they estimate that the planet is losing about 600 million kg of its mass every second. The intense heat from its star is literally evaporating this planet!

Problem 1 – The mass of Jupiter is 1.9×10^{27} kg. What is the mass of HD189733b?

Problem 2 – The atmosphere of Jupiter is about 1000 km thick, and the radius of Jupiter is 70,000 km. What is the volume of the atmospheric shell of Jupiter?

Problem 3 – The average density of the Jovian atmosphere is about 50 grams/m^3. If the atmosphere of HD189733b is identical to Jupiter's in density and volume, how much mass is in the atmosphere of HD189733b?

Problem 4 – The Chandra observations suggest that the atmosphere of HD189733b is decreasing at a rate of 6.0×10^8 kg/second. How many years will it take for the entire atmosphere of this planet to be lost?

Problem 5 – How many years will it take for the entire planet to be evaporated?

NASA's Chandra Sees Eclipsing Planet in X-rays for First Time
July 29, 2013
http://www.nasa.gov/mission_pages/chandra/news/exoplanet-HD189733b.html

Problem 1 – The mass of Jupiter is $1.9\text{x}10^{27}$ kg. What is the mass of HD189733b?

Answer: The mass is 13% greater than Jupiter so M = $1.13\text{x}1.9\text{x}10^{27}$kg = $\mathbf{2.1x10^{27}}$ **kg.**

Problem 2 – The atmosphere of Jupiter is about 1000 km thick, and the radius of Jupiter is 70,000 km. What is the volume of the atmospheric shell of Jupiter?

Answer: Volume = $4/3\ \pi\ (70000\text{km})^3 - 4/3\ \pi\ (69000\text{km})^3$
$= 1.44\text{x}10^{15}\ \text{km}^3 - 1.38\text{x}10^{15}\ \text{km}^3$
= $\mathbf{6.0x10^{13}\ km^3}$

Problem 3 – The average density of the Jovian atmosphere is about 50 grams/m^3. If the atmosphere of HD189733b is identical to Jupiter's in density and volume, how much mass is in the atmosphere of HD189733b?

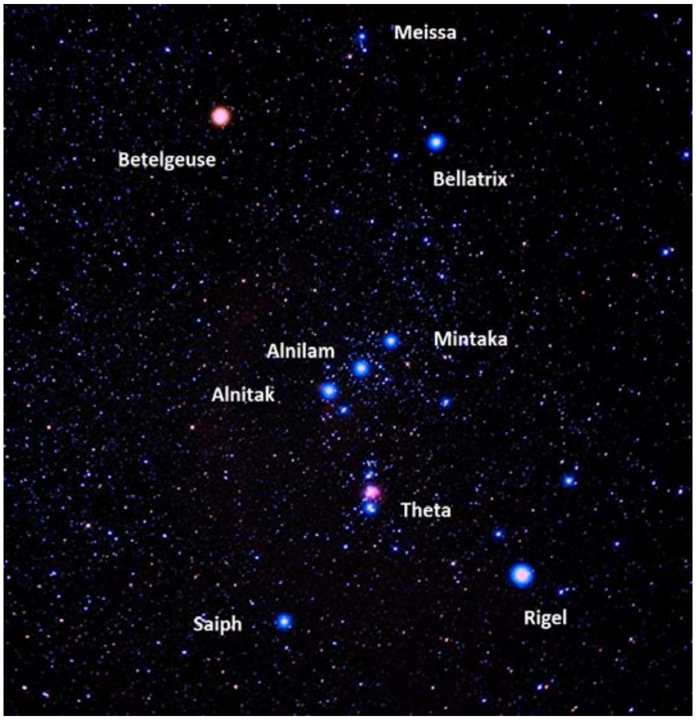
Answer: The volume of the atmosphere is $6.0\text{x}10^{13}$ km^3 or $6.0\text{x}10^{22}$ m^3.
Since mass = density x volume.
M = 50 gm/m^3 x $6.0\text{x}10^{22}$ m^3
= $3.0\text{x}10^{24}$ grams or $\mathbf{3.0x10^{21}}$ **kg**.

Problem 4 – The Chandra observations suggest that the atmosphere of HD189733b is decreasing at a rate of $6.0\text{x}10^{8}$ kg/second. How many years will it take for the entire atmosphere of this planet to be lost?

Answer : Time = amount/rate
= $3.0\text{x}10^{21}$ kg / ($6.0\text{x}10^{8}$ kg/sec)
= $5.0\text{x}10^{12}$ seconds.
= $5.0\text{x}10^{12}$ seconds x (1 year/$3.1\text{x}10^{7}$ sec) = **161,000 years**.

Problem 5 – How many years will it take for the entire planet to be evaporated?

Answer: Time = $2.1\text{x}10^{27}$ kilograms / $6.0\text{x}10^{8}$ kg/sec
= $3.5\text{x}10^{18}$ seconds
= **113 billion years**.



Next to the Big Dipper (Ursa Major) and Scorpius, the Orion is the most widely recognized of all the 89 constellations in the sky. It is also one of the oldest known to humans. The Ancient Egyptians called it Osiris as long ago as 2000 BC!

The brilliant stars that make up this rectangular star pattern seem to be close-by because they are so bright, but in fact they are very far away. Astronomers measure distances using a unit called the **light year**, which equals about 9.5 trillion kilometers, or 63,240 times the distance from Earth to the Sun!

Problem 1 – Light travels at a speed of 300,000 km/sec. How long does it take light to travel:

A) To the moon at a distance of 380,000 km?
B) To the sun at a distance of 150 million km?
C) To Neptune at a distance of 4.5 billion kilometers?
D) To the star Alpha Centauri at a distance of 41 trillion km?

Problem 2 – How far does light travel in one year, if 1 Earth year = 31,000,000 seconds? (Note: Astronomers call this distance one light year.)

Problem 3 – The bright star in Orion called Betelgeuse is located 650 light years from Earth. What is this distance in kilometers? Write your answer using words like thousand or trillion where appropriate, and round the answer to the nearest 1000 trillion.

Problem 4 – Betelgeuse is expected to blow up as a supernova sometime in the next 1 million years. Suppose this happened in the year 3000 AD. In what year would someone on earth see this explosion?

Problem 5 – The star Bellatrix is located 300 light years from Earth and closer to Betelgeuse. In what year would colonists there first see Betelgeuse explode?

Problem 1 – Light travels at a speed of 300,000 km/sec. How long does it take light to travel: A)To the moon at a distance of 380,000 km?; B) To the sun at a distance of 150 million km? C) To Neptune at a distance of 4.5 billion kilometers? D) To the star Alpha Centauri at a distance of 41 trillion km?

Answer: Time = Distance/speed, so

A) Time = 380,000 km/300,000 = **1.3 seconds**. B) Time = 150,000,000/300,000 = 500 seconds or **8 1/3 minutes**. C) Time = 4,500,000,000 km/300,000 = 15,000 seconds or **4 1/6 hours**. D) 41,000,000,000,000 km/300,000 = 136,666,666 seconds or **4 1/3 years.**

Problem 2 – How far does light travel in one year, if 1 Earth year = 31,000,000 seconds? (Note: Astronomers call this distance one light year.)

Answer: Distance = speed x Time,
Distance = 300,000 km/s x 31,000,000 sec
Distance = **9.3 trillion kilometers**.

Problem 3 – The bright star in Orion called Betelgeuse is located 650 light years from Earth. What is this distance in kilometers? Write your answer using words like thousand or trillion where appropriate, and round the answer to the nearest 1000 trillion.

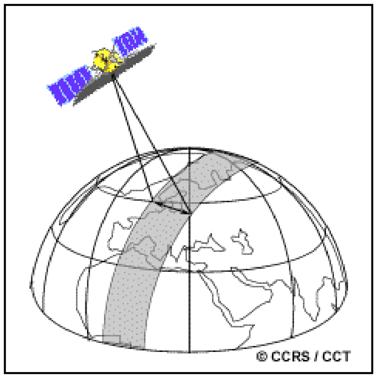
Answer: 1 light year is 9.3 trillion kilometers, so multiply this by 650 to get the distance to Betelgeuse of 650 x 9.3 = 6045 trillion kilometers, which we can re-write as **6 thousand trillion kilometers**. The actual name for one thousand trillion is one quadrillion, so we might also write Betelgeuse's distance as 6 quadrillion kilometers.

Problem 4 – Betelgeuse is expected to blow up as a supernova sometime in the next 1 million years. Suppose this happened in the year 3000 AD. In what year would someone on earth see this explosion?

Answer: Betelgeuse is 650 light years from Earth so it takes light 650 years to reach us .If the explosion happened in the Year 3000 AD, then we will see the light arrive in the year **3650 AD**, 650 years AFTER the event occurred.

Problem 5 – The star Bellatrix is located 300 light years from Earth and closer to Betelgeuse. In what year would colonists there first see Betelgeuse explode?

Answer: Light from the explosion has to travel from Betelgeuse to Bellatrix, which is a distance of 650-300 = 350 light years. That means that colonists near Bellatrix will see the event in the year 3000 AD + 350 = **3350 AD**, or about 300 years <u>sooner</u> than the most distant earthlings!



When a satellite looks down on the Earth from space, it only sees a small part of the surface at a time during each orbit. But after many orbits, it eventually sees all of the surface so that a full map can be put together.

This figure shows one 'swath' of images seen by the satellite during part of its orbit.

Problem 1 – A satellite at an altitude of 500 km above the surface orbits Earth from pole to pole once every 90 minutes. How many times does it pass across the Equator?

Problem 2 – If the satellite passes across the Equator going north to south during the first half of its orbit, what direction does it pass across the equator during the second half of the orbit?

Problem 3 – How many times does the satellite pass over the Equator every day if 1 day = 24 hours?

Problem 4 - If the satellite can view a swath of Earth's surface that is 2500 km wide, how many orbits will it take to view all of the Equatorial areas of Earth, if the radius of Earth is 6378 km?

Problem 5 – Draw a scaled diagram of a satellite located 500 km above a flat surface. Draw a vertical line from the satellite perpendicular to the surface. Complete the right triangle by drawing a hypotenuse from the satellite to the surface with a base length of 2500 km/2 = 1250 km. What is the measure of the angle from the satellite between the perpendicular segment and the hypotenuse? (Use a protractor, or use the property of tangents.)

Problem 6 – The distance to the horizon of a planet is given by the formula $D = (2Rh)^{1/2}$ where R is the planet's radius in km, and h is the height of the satellite above the ground in km. For the satellite problem above, what is the maximum swath width that can be viewed by the satellite?

Problem 1 – A satellite at an altitude of 500 km above the surface orbits Earth from pole to pole once every 90 minutes. How many times does it pass across the Equator?

Answer: It will cross the Equator exactly twice per orbit.

Problem 2 – If the satellite passes across the Equator going north to south during the first half of its orbit, what direction does it pass across the equator during the second half of the orbit?

Answer: The satellite will travel south to north across the equator during the second half of the orbit.

Problem 3 – How many times does the satellite pass over the Equator every day if 1 day = 24 hours?

Answer: 90 minutes / 24 hours = 16 orbits. During each orbit it passes across the equator twice, so in one day it passes across the equator 2 x 16 = 32 times.

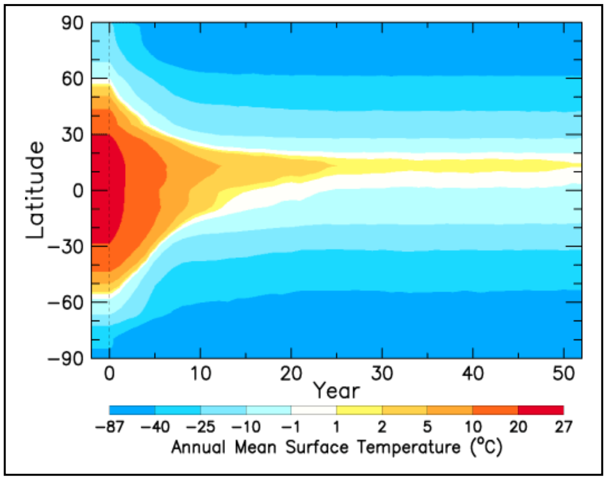
Problem 4 - If the satellite can view a swath of Earth's surface that is 2500 km wide, how many orbits will it take to view all of the Equatorial areas of Earth, if the radius of Earth is 6378 km?

Answer: The circumference of Earth is given by C = 2 π R so C = 2 (3.141 x 6378 km) = 40,066 km. The number of swaths is given by 40066/2500 = 16. But each orbit counts for two equatorial swaths, so we only need 8 orbits to provide 16 swaths at the equator, and this will cover all of the equatorial area when the swath images are placed exactly side by side.

Problem 5 – Draw a scaled diagram of a satellite located 500 km above a flat surface. Draw a vertical line from the satellite perpendicular to the surface. Complete the right triangle by drawing a hypotenuse from the satellite to the surface with a base length of 2500 km/2 = 1250 km. What is the measure of the angle from the satellite between the perpendicular segment and the hypotenuse? (Use a protractor, or use the property of tangents.)

Answer: Tan(theta) = 1250/500 = 2.5, so **Theta = 68^o**, and the full swath viewing angle is 2 x 68 = 136^o.

Problem 6 – The distance to the horizon of a planet is given by the formula D = (2Rh)1/2 where R is the planet's radius in km, and h is the height of the satellite above the ground in km. For the satellite problem above, what is the maximum swath width that can be viewed by the satellite? D = (2 x 500 x 6378)1/2 = **2525 km**, so the swath is about equal to the maximum area of the Earth that can be viewed from horizon to horizon at the altitude of the satellite.



A computer model developed by NASA scientists at the Goddard Institute for Space Science shows that without carbon dioxide, the terrestrial greenhouse would collapse and plunge Earth into an icebound state. Today, the average temperature is +15°C. Within 50 years the average temperature would drop to -21°C without the warming provided by atmospheric carbon dioxide. The delicate link between the planet's temperature and carbon dioxide has also been proved by geologic records of CO_2 levels during ice ages and interglacial periods. The temperature difference between an ice age period and an interglacial period is only 5°C. During previous ice ages, CO_2 levels were near 180 parts per million (ppm). During the warm interglacial periods the levels were near 280 ppm. Today we are living in an interglacial period that started 12,000 years ago and may last another 40,000 years. Scientists continue to worry that, as CO_2 levels approach 400 ppm, we are in uncharted territory with no historical precedent as far back as 1 million years.

Although there is no known process that would instantly remove all CO_2 from the atmosphere, this computer model is important for another reason. It helps us predict how warm a planet would be if it had no greenhouse gases, even though it is close to its star.

Problem 1 - The surface area of the Earth above a latitude of θ degrees is given by

$$A = 4\pi R^2(1-\sin\theta)$$

From the computer model, after how many years will exactly half of the surface of Earth be covered by ice caps where T < 0°C?

Problem 2 – The albedo of Earth is a number between 0 and 1 that indicates how much sunlight it reflects back into space. The higher the albedo, the more light is reflected back into space and the less heating occurs. An albedo of A = 1.0 is a perfect mirror so that all sunlight is reflected and none is absorbed to heat the planet. An albedo of A=0 is similar to asphalt and reflects no light back into space and absorbs all the light energy to heat the planet. Ice has an albedo of A=0.7 and ocean water has A=0.2. After how many years will its albedo increase to 0.6 according to the computer models?

http://www.giss.nasa.gov/research/news/20101014/
How Carbon Dioxide Controls Earth's Temperature
October 14, 2010

Problem 1 - The surface area of the Earth above a latitude of θ degrees is given by

$$A = 4\pi R^2(1-\sin\theta)$$

From the computer model, after how many years will exactly half of the surface of Earth be covered by ice caps where $T < 0^oC$?

Answer: From the formula, we need $A/4\pi R^2 = ½$
so $½=1-\sin\theta$ and so $\theta = 30^o$ latitude

From the model, the zone where $T = -1^oC$ to $+1^o$ C reaches a latitude of 30^o occurs after a time of **5 years.**

Problem 2 – The albedo of Earth is a number between 0 and 1 that indicates how much sunlight it reflects back into space. The higher the albedo, the more light is reflected back into space and the less heating occurs. An albedo of A = 1.0 is a perfect mirror so that all sunlight is reflected and none is absorbed to heat the planet. An albedo of A=0 is similar to asphalt and reflects no light back into space and absorbs all the light energy to heat the planet. Ice has an albedo of A=0.7 and ocean water has A=0.2. After how many years will its albedo increase to 0.6 according to the computer models?

Answer: The average albedo is found by averaging the albedo of the area covered by the ice caps (A=0.7) with the albedo of the area covered by the ocean (a=0.2).

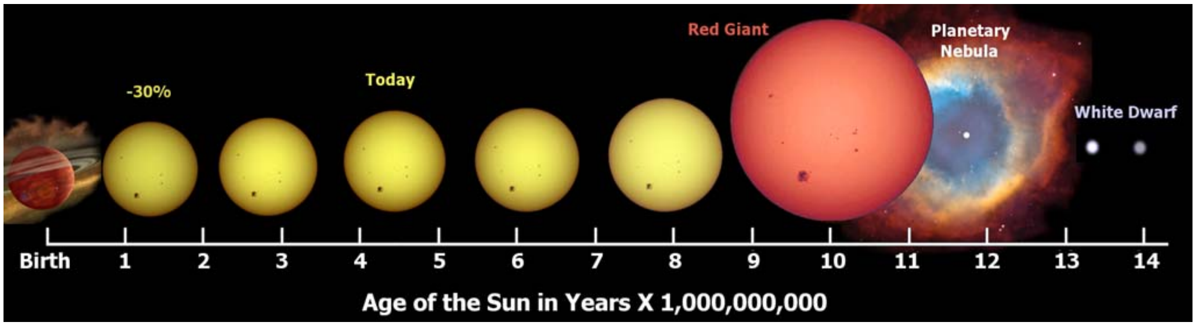
The area covered by ice caps = $4\pi R^2(1-\sin\theta)$
The remaining area covered by water = $4\pi R^2 - 4\pi R^2(1-\sin\theta) = 4\pi R^2(\sin\theta)$

$$0.6\ (4\pi R^2) = 0.7\ (4\pi R^2)(1-\sin\theta) + 0.2(4\pi R^2)[\sin\theta]$$

Simplifying: $0.6 = 0.7(1-\sin\theta) + 0.2\sin\theta$
$0.6 = 0.7 -0.7\sin\theta + 0.2\sin\theta$
$-0.1 = -0.5\sin\theta$
$\sin\theta = 0.2$
So $\theta = 12^o$

So, when the zone $-1^oC < T < +1^oC$ reaches a latitude of $+12^o$, the albedo of Earth will have increased from its current value of 0.4 to a much higher reflectivity of 0.6. According to the model graph, this happens after 10 years.

Note: Although CO_2 loss is an important cooling process, the rapid increase in albedo is a significant cause for cooling and an important 'feed back' in the process. As the planet cools, more ice appears and the albedo increases, causing more cooling and more ice to appear…



Our sun was formed 4.6 billion years ago. Since then it has been steadily increasing its brightness. This normal change is understood by astronomers who have created detailed mathematical models of the sun's complex interior. They have considered the nuclear physics that causes its heating and energy, gravitational forces that compress its dense core, and how the balance between these processes change in time. The diagram above shows the major stages in our sun's evolution from birth to end-of-life after 14 billion years. A simple formula describes how the power of our sun changes over time:

$$L = \frac{L_0}{1 + \frac{2}{5}(1 - x)}$$

where x = t/t_0 t_0 = 4.6 billion yrs, L_0 = 1.0 for the luminosity of the sun today.

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years

Problem 2 – By what percentage will L increase when it is 2 billion years older than it is today?

Problem 3 – A simple formula for the temperature, in kelvins, of Earth is given by:

$$T = 284[(1 - A)L]^{\frac{1}{4}}$$

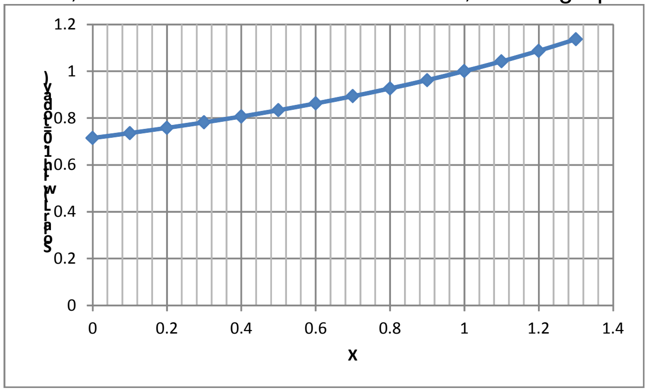
where L is the solar luminosity (today L=1.0), and A is the surface albedo, which is a number between 0 and 1, where asphalt is A=0 and A=1.0 is a perfect mirror A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

Problem 4 - Combine the two formulae above to define a new formula that gives Earth's temperature in kelvins only as a function of time, t, and albedo, A.

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 150° F (339 kelvins)? (Note: it is currently 60° F)

$$L = \frac{L_0}{1 + \frac{2}{5}(1 - x)}$$

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years.
Answer: t = 0 means X=0, t=6 billion means x = 6/4.6 = 1.3, so the graph domain is [0,1.3]



Problem 2 - By what percentage will L increase when it is 2 billion years older than it is today?

Answer: X = (4.6 + 2.0)/4.6 = 1.43, then L = 1 / (1+0.4(1-1.43)) so L = 1.20 this is **20% brighter** than today.

Problem 3 – A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

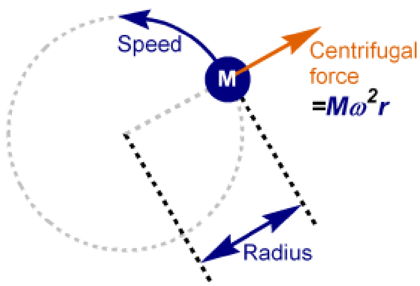
Answer: A) L = 1.0 today so T = $284((1\text{-}0.4)\text{x}1.0)^{1/4}$ = **250 kelvins** (or -23^{o} Celsius)
B) L = 1.05 so T = $284(0.6\text{x}1.05)^{1/4}$ = **253 kelvins** (or -20^{o} Celsius)

Problem 4 - Combine the formulae for L(x) and T to define a new formula, T(x,A) that gives Earth's temperature only as a function of time, x, and albedo, A, and assumes that L_0=1.0 today.

$$T(x, A) = 425\left(\frac{1 - A}{7 - 2x}\right)^{\frac{1}{4}}$$

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 100^{o} F (310 kelvins)? (Note: it is currently 60^{o} F)

Answer: 310 = $425\ (0.4/(7\text{-}2x))^{1/4}$
$0.4\ (425/310)^4$ = 7-2x
1.4 = 7 – 2x
2x = 5.6 so x = 2.8
and the age of the sun will be t = 2.8 x 4.6 billion = **12.9 billion years**.
This occurs 12.9 – 4.6 = 8.3 billion years in the future.



Gravitron ride at an amusement park

Most science fiction stories require some form of artificial gravity to keep spaceship passengers operating in a normal earth-like environment. As it turns out, weightlessness is a very bad condition for astronauts to work in on long-term flights. It causes bones to lose about 1% of their mass every month. A 30 year old traveler to Mars will come back with the bones of a 60 year old!

The only known way to create artificial gravity it to supply a force on an astronaut that produces the same acceleration as on the surface of earth: 9.8 meters/sec^2 or 32 feet/sec^2. This can be done with bungee chords, body restraints or by spinning the spacecraft fast enough to create enough centrifugal acceleration.

Centrifugal acceleration is what you feel when your car 'takes a curve' and you are shoved sideways into the car door, or what you feel on a roller coaster as it travels a sharp curve in the tracks. Mathematically we can calculate centrifugal acceleration using the formula:

$$A = \frac{V^2}{R}$$

where V is in meters/sec, R is the radius of the turn in meters, and A is the acceleration in meters/sec^2.

Let's see how this works for some common situations!

Problem 1 - The Gravitron is a popular amusement park ride. The radius of the wall from the center is 7 meters, and at its maximum speed, it rotates at 24 rotations per minute. What is the speed of rotation in meters/sec, and what is the acceleration that you feel?

Problem 2 - On a journey to Mars, one design is to have a section of the spacecraft rotate to simulate gravity. If the radius of this section is 30 meters, how many RPMs must it rotate to simulate one Earth gravity (1 g = 9.8 meters/sec^2)?

Problem 3 – Another way to create a 1-G force is for the rocket to continuously fire during its journey so that its speed increases by 9.8 meters/sec every second. Suppose that NASA could design such a rocket system for a 60 million km trip to Mars. The idea is that it will accelerate during the first half of its trip, then make a 180-degree flip and decelerate for the remainder of the trip. If

$$\text{distance} = 1/2aT^2$$
$$\text{speed} = aT,$$

where **T** is in seconds, **a** is in meters/sec^2, and distance is in meters, how long will the trip take in hours, and what will be the maximum speed of the spacecraft at the turn-around point?

Problem 1 - The Gravitron is a popular amusement park ride. The radius of the wall from the center is 7 meters, and at its maximum speed, it rotates at 24 rotations per minute. What is the speed of rotation in meters/sec, and what is the acceleration that you feel?

Answer: For circular motion, the distance traveled is the circumference of the circle C = 2 π (7 meters) = 44 meters. At 24 rpm, it makes one revolution every 60 seconds/24 = 2.5 seconds, so the rotation speed is 44 meters/2.5 sec = 17.6 meters/sec.

The acceleration is then A = $(17.6)^2/7$ = **44.3 meters/sec^2**. Since 1 earth gravity = 9.8 meters/sec^2, the 'G-Force' you feel is 44.3/9.8 = 4.5 Gs. That means that you feel 4.5 times heavier than you would be just standing in line outside!

Problem 2 - On a journey to Mars, one design is to have a section of the spacecraft rotate to simulate gravity. If the radius of this section is 30 meters, how many RPMs must it rotate to simulate one Earth gravity (1 g = 9.8 meters/sec^2)?

Answer: The circumference is C = 2 π (30) = 188 meters. 1 RPM is equal to rotating one full circumference every minute, for a speed of 188/60 sec = 3.1 meters/second. So V = 3.1 meters/sec x RPM. Then A = $(3.1\text{RPM})^2$ / 188 = 0.05 x RPM^2. We need 9.8 = 0.05RPM^2 so **RPM = 14**.

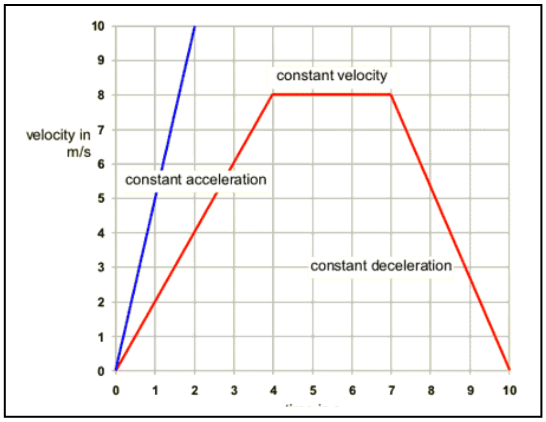
Problem 3 – Another way to create a 1-G force is for the rocket to continuously fire during its journey so that its speed increases by 9.8 meters/sec every second. Suppose that NASA could design such a rocket system for a 60 million km trip to Mars. The idea is that it will accelerate during the first half of its trip, then make a 180-degree flip and decelerate for the remainder of the trip. If distance = $1/2aT^2$ and V = aT, how long will the trip take in hours, and what will be the maximum speed of the spacecraft at the turn-around point?

Answer: The turn-around point happens at the midway point 30 million km from Earth, so d = 3.0×10^{10} meters, a = 9.8 meters/sec^2, and so solving for T,

$3.0\times10^{10} = ½ (9.8) T^2$ so
T = 78,246 seconds or
for a full trip = 2x78246 seconds = **43 hours**!

Speed = 9.8 x 78246 = 767,000 meters/sec = **767 kilometers/sec**.

Reading a Speed vs Time Graph - acceleration



In the same way that speed = distance divided by time, we can also look at acceleration as the change in speed over the time that the change occurred. Both of these quantities can be thought of as rates of change or 'slopes' on a graph like the one to the left.

When the final speed is, larger than the initial speed, the slope of the line is positive (upward) and we say that the object is accelerating. When the final speed is less than the initial speed, the slope is negative (downward) and we say that the object is decelerating.

Problem 1 – A car leaves its parking spot and accelerates to 30 mph (13 m/s) in 10 seconds. It travels on a road at a constant speed of 30 mph for another 30 seconds and enters the onramp of a highway where it accelerates from 30 mph to a speed of 60 mph (26 m/s) after 6 seconds. It stays at this speed for another 2 minutes, then the car exits an off ramp, slowing to a speed of zero after 2 seconds. It then accelerates to 30 mph after 3 seconds as it merges into the local street traffic. After 1 minute at this speed the car approaches a gas station and decelerates to zero after 4 seconds. Draw a speed versus time graph in metric units that represents the car's journey.

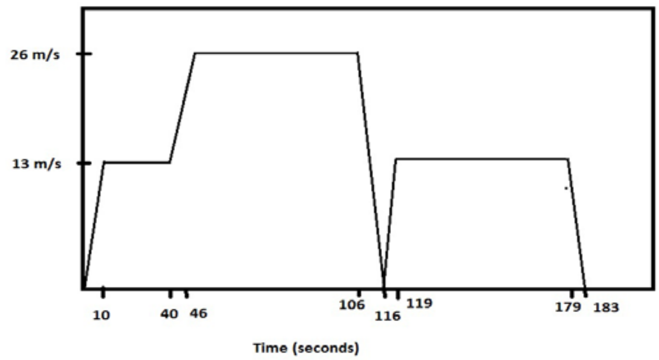
Time (Sec)	Speed (m/s)
0	0
1	3
2	4
3	7
4	10
5	12
6	15
7	16
8	20
9	23
10	25
11	27
12	31
13	34
14	36
15	39
16	43
17	46
18	49
19	53
20	56

Problem 2 – Explain how the area under a speed vs time graph gives the distance traveled, and use this to calculate the total distance traveled by the car using a combination of rectangles and triangles to calculate the total area.

Saturn V Rocket Launch Speed vs Time

Problem 3 – The table to the left shows the speed of the Saturn V rocket during a launch from the Kennedy Space Center on July 16, 1969 at 9:32:00 a.m. (EDT). What was the average acceleration of the Saturn V rocket during its first 20 seconds of constant thrust? How far did it travel during this time?

Problem 1 – A car leaves its parking spot and accelerates to 30 mph (13 m/s) in 10 seconds. It travels on a road at a constant speed of 30 mph for another 30 seconds and enters the onramp of a highway where it accelerates from 30 mph to a speed of 60 mph (26 m/s) after 6 seconds. It stays at this speed for another 1 minute, then exits an off ramp, slowing to a speed of zero after 10 seconds. It then accelerates to 30 mph after 3 seconds as it merges into the local street traffic. After 1 minute at this speed the car approaches a gas station and decelerates to zero after 4 seconds. Draw a speed versus time graph that represents the car's journey.

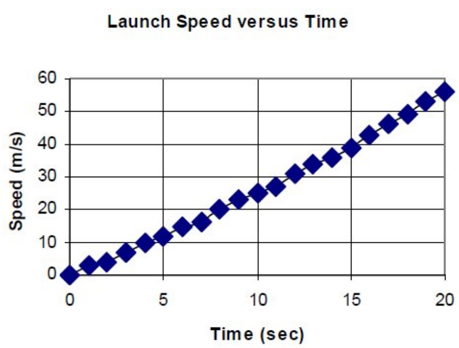


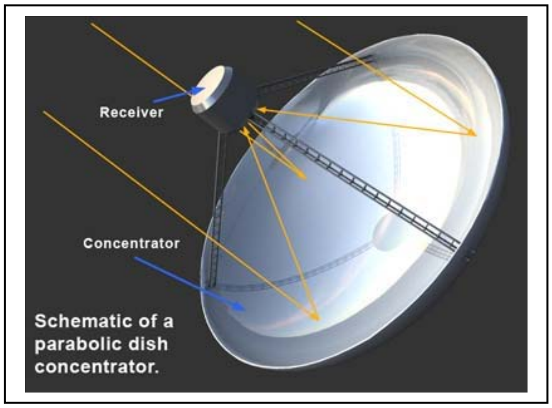
Problem 2 – Explain how the area under a speed vs time graph gives the distance traveled, and use this to calculate the total distance traveled by the car using a combination of rectangles and triangles to calculate the total area.

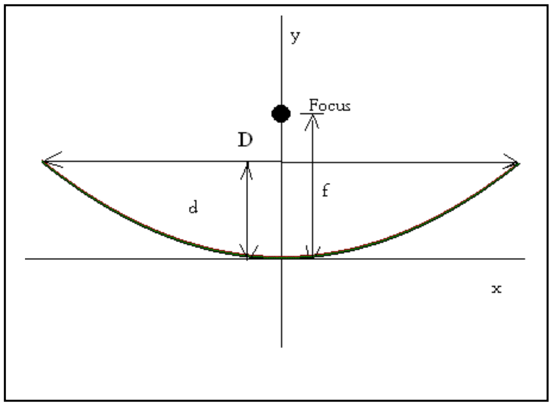
Answer: **Area = speed x time = meters/sec x seconds = meters. So the area under the graph has the units of distance in meters**. This figure has five triangular areas as the car is accelerating and decelerating, and three rectangular areas as the car is traveling at constant speed. The sum of the triangular areas is A = ½ time interval x speed = ½ [10 x 13 + 6 x (26-13) + 10x26 + 3x13 + 4x13) = 279 meters. The sum of the rectangular areas is A = time x speed = (30x13 + (106-46)x26 + (179-119)x13) = 2730 meters, so the total sum is 2730+279 = **3009 meters or 3 kilometers of total travel**.

Problem 3 – The table to the left shows the speed of the Saturn V rocket during a launch from the Kennedy Space Center on July 16, 1969 at 9:32:00 a.m. (EDT). What was the average acceleration of the Saturn V rocket during its first 20 seconds of constant thrust? How far did it travel during this time?

Answer: Acceleration = (56 m/s – 0 m/s)/20 sec = **28 m/s^2**. (Note this is about 2.8 times the acceleration of gravity at earth's surface so the astronauts would have felt about 2.8 times heavier). The triangular area is ½(20sec)(56 m/s) = **560 meters**.







A very beautiful property of parabolas is that at a point called the FOCUS, all of the lines entering the parabola parallel to its axis are 'reflected' from the parabolic curve and intersect the focus. This property is used by astronomers to design telescopes, and by radio engineers to design satellite dishes.

The top figure to the left shows a satellite dish with a radio receiver located at the focus of the parabola. The radio rays are reflected from the parabolic surface and concentrated at the focus. This focusing and amplification property of parabolic reflectors is also used for solar heating and generating solar electricity.

The bottom figure defines the distance, f, of the focus from the bottom of the dish, and the diameter, D, of the dish.

Suppose you wanted to design a parabolic dish with a depth, d, of 1 meter and a radius of 5 meters. Where would the focus be located? If the basic equation of a parabola is $y = ax^2$. The location of the focus will be at $f = 1/(4a)$. Since we know that the point (5.0,1.0) is on the curve of the parabola, that means that we can solve for a for this particular dish. We get $a\,(5.0)^2 = 1.0$ so $a = 1/25$. Then the focus will be at $f = 1/(4/25)$ so $f = 25/4 = 6\,¼$ meters above the bottom of the dish. Let's design a few parabolic reflectors that we can use to reflect and concentrate sound waves, or sunlight!

Problem 1 – A bird watcher wants to record bird songs from a distance. He goes to the cooking store and finds a parabola-shaped bowl that is light-weight. It has a diameter of 12 inches and a depth of 5 inches. How far below the edge of the bowel does he have to mount the microphone to use this as a sound amplifier?

Problem 2 – A hobbiest has several hundred small flat mirrors and wants to build a solar cooker by mounting the mirrors on the inside surface of a parabolic shaped form. The diameter of the parabola is 3 meters and the depth of the form is 1/2 meter. How far above the center of the form will the sunlight be the most concentrated?

Problem 1 – A bird watcher wants to record bird songs from a distance. He goes to the cooking store and finds a parabola-shaped bowl that is light-weight. It has a diameter of 12 inches and a depth of 5 inches. How far below the edge of the bowel does he have to mount the microphone to use this as a sound amplifier?

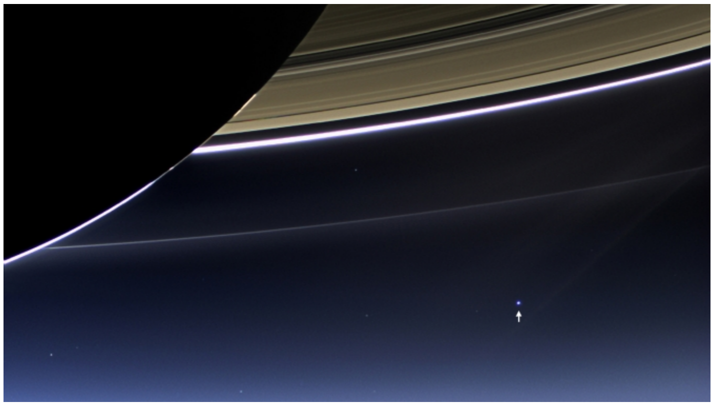
Answer: The radius of the bowl is x=6.0 inches, and at this location y = 5.0 inches. The equation is $y = a x^2$, and we know that the point (6.0, 5.0) is on this curve, so $5.0 = a (6.0)^2$ so a = 0.139. The focus distance is then f = 1/(4x0.139) = 1.8 inches from the bottom of the bowl, or 5.0 – 1.8 = **3.2 inches** from the top edge of the bowl.

Problem 2 – A hobbiest has several hundred small flat mirrors and wants to build a solar cooker by mounting the mirrors on the inside surface of a parabolic shaped form. The diameter of the parabola is 3.0 meters and the depth of the form is 1/2 meters. How far above the center of the form will the sunlight be the most concentrated?

Answer: The diameter is 3.0 meters so the radius is 1.5 meters. The point (1.5, 0.50) is on the parabolic curve, so $0.50 = a (1.5)^2$ and so a = 0.22. Then the focus is at f = 1/(4x0.22) = **1.1 meters from the bottom of the form**.

Note: The light from the sun falls on the parabola over an area of 2 square meters, and is concentrated into an area at the focus of 1/100 square meters, so the sunlight is amplified by 2.0/0.01 = 200 times, making this a very good solar cooker!

Cassini Sees Earth from Saturn – How bright is it?



The Cassini spacecraft captured this image of Earth seen through Saturn's rings (arrow).

At the distance of Saturn, our planet looks like many other bright stars in the sky, but with a noticeable blue color due to its oceans.

At the time of the photo, Earth was located 898 million miles (1.4 billion km) from Saturn.

Problem 1 –If Mars is about 250 million km from Earth and Saturn is about 1440 million km from Earth, using the Inverse-Square Law, how much fainter will Earth be from Saturn?

Astronomers use a logarithmic brightness scale called apparent magnitude, m. The formula that relates brightness to magnitude is given by

$$B = 10^{-.4m}$$

For example, a brightness change of 100-times means a magnitude difference of exactly 5.0 because $1/100 = 10^{-0.4(m)}$ so $10^{-2} = 10^{-0.4(m)}$ and solving for m, you get m = +5.0.

Problem 2 - At the distance of Mars, Earth appears as a star of magnitude -2.5, making it about as bright as Venus seen from Earth. How bright will Earth be from Saturn?

NASA Releases Images of Earth Taken by Distant Spacecraft
http://www.nasa.gov/mission_pages/cassini/whycassini/cassini20130722.html
July 22, 2013

Problem 1 –If Mars is about 250 million km from Earth and Saturn is about 1440 million km from Earth, using the Inverse-Square Law, how much fainter will Earth be from Saturn?

Answer: Brightness = $(250/1440)^2$ = 0.030, so Earth will be about **33 times fainter** from Saturn than from Mars.

Problem 2 - At the distance of Mars, Earth appears as a star of magnitude -2.5, making it about as bright as Venus seen from Earth. How bright will Earth be from Saturn?

Answer: It will be 33 times fainter than from Mars so 33 = $10^{-.4m}$ so m = +3.8 m fainter than seen at Mars, and so -2.5 + 3.8 = **+1.3m at Saturn**.

Note: This assumes that Earth is seen under the same illumination from both planets. In fact from Mars the disk of Earth is about 90% illuminated, while from Saturn, Earth is only 50% illuminated. This makes Earth appear 50%/90% = 0.55 = $10^{-.4m}$, m = +0.6 fainter or +1.9 m from Saturn.

Looking towards the constellation of Triangulum (The Triangle), in the northern sky we see two very similar galaxies, named PGC 9074 and PGC 9071 located about 490 million light years from Earth. Both are spiral galaxies, and are presented to our eyes face-on, so we are able to appreciate their distinctive shapes. After the collision, these two galaxies will eventually merge together into a new, larger galaxy.

The distance between the centers of these two galaxies is about 110,000 light years. The galaxy diameters are about 75,000 light years.

Problem 1 – Astronomers measure the speeds of stars and galaxies in units of kilometers/sec, but because of the scale of the universe, sometimes it is helpful to use the equivalent speed unit if 'parsecs per million years' to make certain calculations easier and faster. One parsec equals 3.26 light years and one light year equals $9.46x10^{12}$ kilometers. If there are $3.1x10^{7}$ seconds in one year, what is the equivalent speed unit for 1 kilometer/sec in pc/myr?

Problem 2 – The two galaxies in the above photo are currently about 110,000 light years apart, and are approaching each other at a speed of 500 km/sec. In about how many million years will the two galaxies have completely collided if they continued at the same speed?

Problem 3 – Like a ball falling to the ground, the gravity of each galaxy will cause the galaxies to move faster and faster as they get closer together. The following equation describes this accelerated motion:

$$D = d_0 - V_0T - 1/2aT^2$$

where T is the time in millions of years (myr) from the present, V_0 is the current speed in pc/myr, and a is the acceleration of gravity in pc/myr^2. If d_0 = 34,000 parsecs, V_0 = 500 pc/myr, and a = 0.4 pc/myr^2, in how millions of years will the galaxies have collided?

http://www.nasa.gov/content/inseparable-galactic-twins/
July2, 2013
Inseparable Galactic Twins

Problem 1 – Astronomers measure the speeds of stars and galaxies in units of kilometers/sec, but because of the scale of the universe, sometimes it is helpful to use the equivalent speed unit if 'parsecs per million years' to make certain calculations easier and faster. One parsec equals 3.26 light years and one light year equals 9.46×10^{12} kilometers. If there are 3.1×10^{7} seconds in one year, what is the equivalent speed unit for 1 kilometer/sec in pc/myr?

Answer: 1 km/sec x (1 ly/9.46×10^{12} km) x (1 pc/3.26 ly) x (3.1×10^{7} sec/1 year) x (10^{6} yr/1 myr) = **1.0 pc/myr**.

Problem 2 – The two galaxies in the above photo are currently about 110,000 light years apart, and are approaching each other at a speed of 500 km/sec. In about how many million years will the two galaxies have completely collided if they continued at the same speed?

Answer: The galaxies are 110,000 ly apart, which is 110,000 ly x 1pc/3.26 ly = 34,000 parsecs. They are traveling at 500 km/sec which is 500 pc/myr, so the time required is T = distance/speed = 34,000 pc/500 = **68 million years**.

Problem 3 – Like a ball falling to the ground, the gravity of each galaxy will cause the galaxies to move faster and faster as they get closer together. The following equation describes this accelerated motion:

$$D = d_0 - V_0T - 1/2aT^2$$

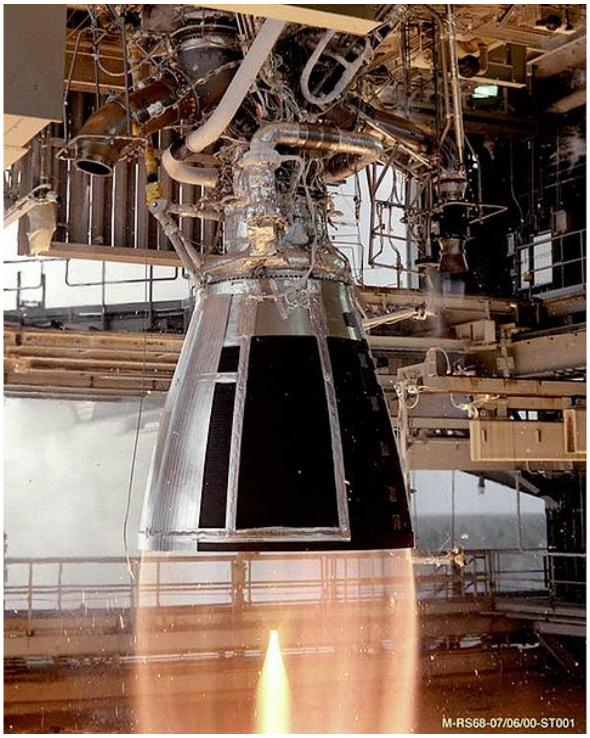
where T is the time in millions of years (myr) from the present, V_0 is the current speed in pc/myr, and a is the acceleration of gravity in pc/myr^2. If d_0 = 34,000 parsecs, V_0 = 500 pc/myr, and a = 0.4 pc/myr^2, in how millions of years will the galaxies have collided?

Answer: The formula will look like $D = 34000 - 500\,T - 0.2T^2$. We want the condition that D = 0 for the collision. The value for T can be found by graphing this equation and finding the point (T,0), or by using the quadratic roots formula.

Method 2: a = -0.2 b = -500 and c = 34000, then

$X = [-b +/- (b^2-4ac)^{1/2}] / 2a$

X = [500 +/- 526] / -0.4 so X1 = -2565 and X2 = +65
Only the solution '+65 million years' is a real solution about a future (positive) time so the required root is **65 million years**. This is only slightly smaller than the answer in Problem 2 which assumed constant collision speeds, so the gravitational acceleration between these two galaxies is not a significant factor.



Rocket engines work by throwing matter out the back end of a rocket as fast as they can. Usually this matter is in the form of gas. This causes the rocket to move forward in the opposite direction from the ejected matter, thanks to Newton's Third Law of Motion which says that '*For every Action, there is an equal and opposite Reaction*'.

To lift tons of cargo into space, rocket engines have to be very powerful. Engineers usually compare different engines by their thrust and by their specific impulse. This makes it easy to decide which kind of engine to use to put different payloads into space in the most efficient and low-cost way possible.

Problem 1 – Specific Impulse Isp = V/g is the ratio of the exhaust speed of the engine (V) divided by the acceleration of gravity at Earth's surface (g), where g is 9.8 meters/sec^2 and V is measured in meters/sec. The J-2X rocket engine has an Isp=421 seconds. How fast are the exhaust gases leaving the bottom of the engine in A) kilometers/sec? B) feet per second? C) miles per hour? (1 kilometer = 3280 feet; 1 mile = 5280 feet)

Problem 2 – Thrust is a measure of the force that the rocket can produce to move an object against the pull of gravity. It is measured in Newtons and is defined by Thrust = F x V where F is the rate of flow of fuel in kilograms/sec and V is the exhaust speed of the combusted gases. The new J-2X rocket engines being designed and tested by NASA have an Isp=421 seconds and a thrust of T = 1,310,000 Newtons. At what rate is mass leaving the rocket engine in A) kilograms/second? B) pounds/second? (1 kilogram = 2.2 pounds).

Problem 3 – Ion rocket engines: Instead of chemical rockets using liquid fuels, the Dawn spacecraft has an ion rocket with Isp=3,100 seconds. If the same ion technology was used to design a replacement for the liquid-fueled J-2X engine, what must be the fuel flow rate, F, in order to produce the same thrust as the J-2X engine?

Problem 4 – Rocket Design: What would the Isp and thrust be for an engine that had V = 10,000 meters/sec and F = 500 kg/sec?

http://www.qrg.northwestern.edu/projects/vss/docs/propulsion/3-how-you-calculate-specific-impulse.html

Problem 1 – Specific Impulse Isp = V/g is the ratio of the exhaust speed of the engine (V) divided by the acceleration of gravity at Earth's surface (g), where g is 9.8 meters/sec^2 and V is measured in meters/sec. The J-2X rocket engine has an Isp=421 seconds. How fast are the exhaust gases leaving the bottom of the engine in A) kilometers/sec? B) feet per second? C) miles per hour? (1 meter = 3.280 feet; 1 mile = 5280 feet)

Answer: A) Isp=421 seconds and g = 9.8 m/sec^2 so V = Isp x g and
V = **4,126 meters/second**.
B) 1 meter = 3.28 feet and so V = 4126 m/s (3.28 feet/1 meter) so
V = **13,533 feet/sec**.
C) 1 mile = 5280 feet so V = 13,533/5280 = 2.56 miles/sec and for 3600 seconds in 1 hour, we have **V = 9,216 miles/hour**.

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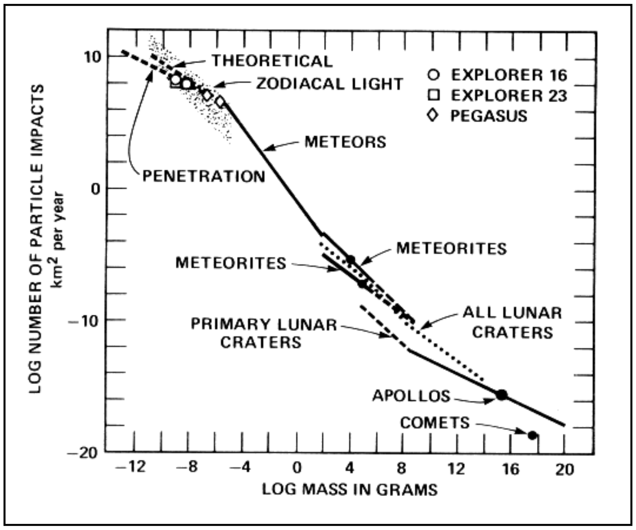
Answer: A) Isp = V/g so V = 421 x 9.8 m/sec2 = 4,126 meters/sec, then T = F x V and F = T/V so F = 1,310,000/4126 = **317 kg/second**. B) 317.5 kg/sec x (2.2 pounds/1kg) = **698 pounds/second**.

Problem 3 – Ion rocket engines: Instead of chemical rockets using liquid fuels, the Dawn spacecraft has an ion rocket with Isp=3,100 seconds. If the same ion technology was used to design a replacement for the liquid-fueled J-2X engine, what must be the fuel flow rate, F, in order to produce the same thrust as the J-2X engine?

Answer: Isp = 3100 sec so V = 3100 sec/9.8 m/sec2 = 316 m/sec. T = F x V and T = 1,310,000 Newtons and so F = 1,310,000/316 = **4,145 kg/sec**.

Problem 4 – Rocket Design: What would the Isp and thrust be for an engine that had V = 10,000 meters/sec and F = 5000 kg/sec?

Answer: Isp = 10,000/9.8 = **1020 sec**, and T = 5000 x 1020 = **5,100,000 Newtons**



The space between the planets is filled with fragments of asteroids, comets and material left over from the formation of the planets. These rocks and debris rain down upon exposed surfaces at speeds up to 30 km/sec.

The figure on the left summarizes the impact frequency of various sizes of particles in space. Note the graph is plotted in the Log-Log format due to the enormous range of masses and rates being described.

Understanding the data graph:

A meteorite with a density of 3 grams/cm^3 has a diameter of 4 centimeters (about 1 1/2 inches).

Problem 1 - What is the mass of this meteorite assuming it is a sphere?

Problem 2 - From the graph, where on the horizontal axis are objects of this mass located?

Problem 3 - What is the number of impacts per year you would expect over an area of 10,000 square kilometers?

A meteorite with a density of 3 grams/cm^3 has a diameter of 4 centimeters (about 1 1/2 inches).

Problem 1 - What is the mass of this meteorite assuming it is a sphere?

Answer: Mass = Density x Volume.
Radius of sphere = 2 cm, so
M = 3.0 x (4/3) (3.14) $(2)^3$ = **100 grams**.

Problem 2 - From the graph, where on the horizontal axis are objects of this mass located?

Answer: The horizontal axis is un units of Log(grams) so Log(100) = 2, and this is the location **half-way between 0' and '4' on the axis**.

Problem 3 - What is the number of impacts per year you would expect over an area of 1000 square kilometers?

Answer: From 'x=2' ,a vertical line intercepts the data at about 'y=--3.5' on the vertical axis. This represents Log(n) = -3.5 so that N = 0.00032 impacts/km^2/year.

Over an area of 10000 km^2, there would be an estimated

0.00032 impacts/km^2/year x (10000 km^2) = **3.2 impacts per year.**

Exploring Two Nearby Stars to the Sun.

The Hubble Space Telescope recently took a picture of the currently nearest star to our sun, Proxima Centauri, which is at a distance of 4.243 light years (1 light year = 9.5 trillion km). The orbits of several other stars near the sun are also known, and during the next 80,000 they will replace Proxima Centauri as the nearest star to our sun.

For this problem we will study the two stars Ross 128 and Gliese 445.

The quadratic equations that approximate the distance to each star from our sun in light years are given by:

Gliese 445 : $D = 0.0104\,T^2 - 0.942\,T + 25.382$

Ross 128 : $D = 0.0007\,T^2 - 0.1197\,T + 11.003$

where T is the number of years from the current year in <u>multiples</u> of 1000 years.

Problem 1 – What are the distance ranges for each star over the time interval from 10,000 to 70,000 in the future?

Problem 2 – For what values of T, in years, will the distances be identical?

Problem 3 – What will be the distances to the stars when their distances are identical?

Problem 4 – When will the two stars be exactly 3.00 light years apart sometime over the time range from 30,000 to 80,000 years from now?

Gliese 445 : $D = 0.0104T^2 - 0.942T + 25.382$
Ross 128 : $D = 0.0007T^2 - 0.1197T + 11.003$

Problem 1 – What are the distance ranges for each star over the time interval from 10,000 to 70,000 in the future?

Answer: Gliese 445: T=10 so D = 17.00 Ly; T = 70 so D = 10.40 ly **[10.40 , 17.00]**
Ross 128: T=10, so D = 9.88 ly ; T=70 so D = 6.05 ly **[6.05 , 9.88]**

Problem 2 – For what values of T, in years, will the distances be identical?

Answer: Set the two equations equal to each other and solve the resulting quadratic equation for its two roots using the Quadratic Formula.

$0 = (0.0104-0.0007)T^2 + (-0.942 + 0.1197)T + (25.382 - 11.003)$

So the coefficients are a = +0.0097 b= -0.822 and c = +14.379

The roots are $T = [-b \pm (b^2-4ac)^{1/2}]/ 2a$ so
$T = 42.37 \pm 51.54\ (0.6757-0.5579)^{1/2}$

T1 = 24,700 years and T2 = 60,000 years.

Problem 3 – To the nearest tenth of a light year what will be the distances to the stars when their distances are identical?

Answer: At 24,700 years, T = 24.7 so D = **8.5 light years**
60,000 years, T=60 so D = **6.3 light years**

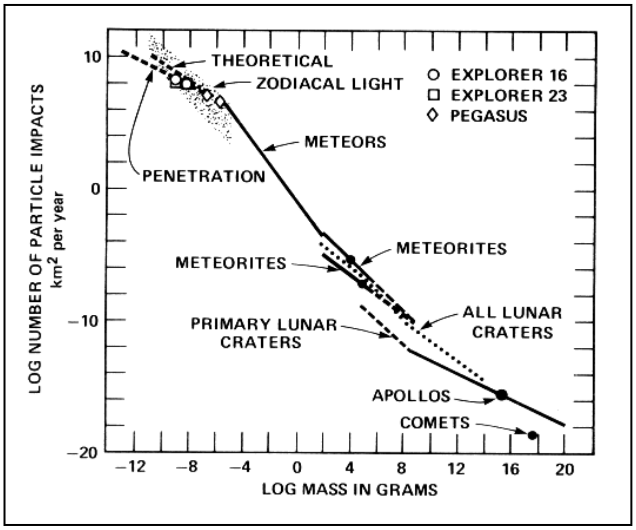
Problem 4 – When will the two stars be exactly 3.00 light years apart sometime over the time range from 30,000 to 80,000 years from now?

Answer: We want the difference between the two quadratic equations to be 10.0

$3.0 = 0.0097T^2 - 0.822T + 14.379$ so we solve $0 = 0.0097T^2 - 0.822T + 11.379$

$T = [0.822 \pm (0.6757 - 0.4415)^{1/2}] / 0.0194$
$T = 42.37 \pm 51.54(0.4839)$
T1 = 67.3
T2 = 17.4

So the only solution in the required time interval is **67,300** years from now.



The space between the planets is filled with fragments of asteroids, comets and material left over from the formation of the planets. These rocks and debris rain down upon exposed surfaces at speeds up to 30 km/sec.

The figure on the left summarizes the impact frequency of various sizes of particles in space. Note the graph is plotted in the Log-Log format due to the enormous range of masses and rates being described.

Replacing a data graph with an empirical model:

Problem 1 - What linear function of the form y = mx + b best describes the data in the LogLog graph?

Problem 2 - What function N(m) is obtained from your answer to Problem 1?

Problem 1 - What linear function of the form y = mx + b best describes the data in the LogLog graph?

Answer: Use the two-point formula:

$$(y-y1)=\frac{y2-y1}{x2-x1}(x-x1)$$ with p1 = (x1,y1) and p2 = (x2,y2)

Select: p1 = (-4, +2) p2 = (16, -16)

and get **y = -0.9x - 1.6**

Problem 2 - What function N(m) is obtained from your answer to Problem 2?

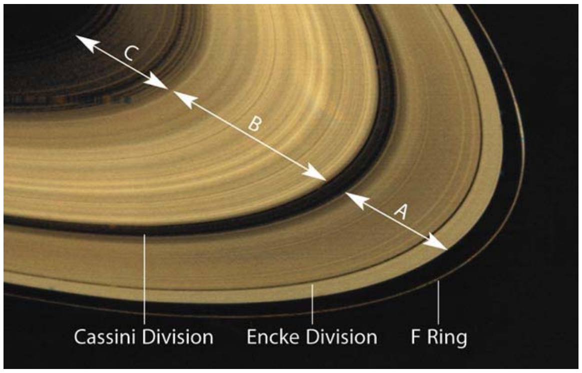
Answer: Since y = Log(N(m)) and x = Log(m) we have

Log(N(m)) = -0.9 Log(m) - 1.6 then

$10^{Log(Nm))} = 10^{Log(m)}$

And $N(m) = 10^{-1.6}\ m^{-0.9}$ so **$N(m) = 0.025\ m^{-0.9}$**

Orbit Speeds and Times for Saturn's Rings



The trillions of particles in Saturn's rings orbit the planet like individual satellites. Although the rings look like they are frozen in time, in fact, the rings orbit the planet at thousands of kilometers per hour! The speed of each ring particle is given by the formula:

$$V = \frac{29.4}{\sqrt{R}} \text{ km/s}$$

where R is the distance from the center of Saturn to the ring in multiples of the radius of Saturn (R = 1 corresponds to a distance of 60,300 km).

Problem 1 – The inner edge of the C Ring is located 7,000 km above the surface of Saturn, while the outer edge of the A Ring is located 140,300 km from the center of Saturn. How fast are the C Ring particles traveling around Saturn compared to the A Ring particles?

Problem 2 – The Cassini Division contains nearly no particles and is the most prominent 'gap' in the ring system easily seen from earth. It extends from 117,580 km to 122,170 km from the center of Saturn. What is the speed difference between the inner and outer edge of this gap?

Problem 3 – If the particles travel in circular orbit, what is the formula giving the orbit period for each ring particle in hours?

Problem 4 – What are the orbit times for particles near the inner and outer edge of the Cassini Division?

Problem 5 – The satellite Mimas orbits Saturn every 22.5 hours. How does this orbit period compare to the period of particles at the inner edge of the Cassini Division?

Problem 1 – The inner edge of the C Ring is located 7,000 km above the surface of Saturn, while the outer edge of the A Ring is located 140,300 km from the center of Saturn. How fast are the C Ring particles traveling around Saturn compared to the A Ring particles?

Answer: R = (60300 km + 7,000 km) / 60300 km = 1.12, so V = 23.6 km/sec
R = 140300 km/60300 km = 2.33, so V = 16.3 km/sec.

Note: The International Space Station orbits Earth at a speed of 7.7 km/s.

Problem 2 – The Cassini Division contains nearly no particles and is the most prominent 'gap' in the ring system easily seen from earth. It extends from 117,580 km to 122,170 km from the center of Saturn. What is the speed difference between the inner and outer edge of this gap?

Answer: R = 117580/60300 = 1.95 V = 17.83 km/s
R = 122170/60300 = 2.02 V = 17.52 km/s
The outer edge particles travel about 17.83 – 17.52 = 0.31 km/sec slower than the inner edge particles.

Problem 3 – If the particles travel in circular orbit, what is the formula giving the orbit period for each ring particle in hours?

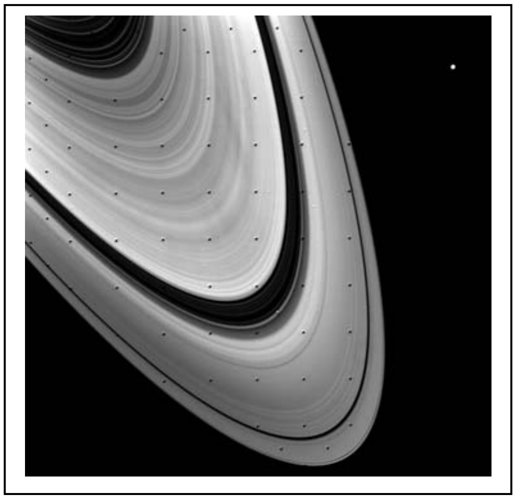
Answer: Orbit circumference = 2 π r km, but r = 60300 R so C = 2 (3.141) x 60300 R , C = 379,000 R km, wnhere R is in Saturn radius units. Since the orbit speed is $V = 24.9/R^{1/2}$, then Time = C/V = 15220 $R^{3/2}$ seconds. Since 1 hour = 3600 seconds, we have
$T = 4.22 R^{3/2}$ hours.

Problem 4 – What are the orbit times for particles near the inner and outer edge of the Cassini Division?

Answer: R = 1.95 so T = 11.49 hours.
R = 2.02 so T = 12.11 hours.

Problem 5 – The satellite Mimas orbits Saturn every 22.5 hours. How does this orbit period compare to the period of particles at the inner edge of the Cassini Division?

Answer: 22.5/11.49 = 1.94 which is nearly 2.0. This means that every time Mimas orbits once, the particles in the Cassini Division orbit about twice around Saturn. This is an example of an orbit resonance. Because the ring particles encounter a push from Mimas's gravitational field at the same location every two orbits, they will be ejected. This is an explanation for why the Cassini Division has so few ring particles.



The Cassini Division is easily seen from Earth with a small telescope, and splits the rings of Saturn into two major groups. A little detective work shows that there may be a good reason for this gap that involves Saturn's nearby moon, Mimas.

Mimas orbits Saturn once every 22 hours, and would-be particles in the Cassini Division would orbit once every 11-12 hours, so that the ratio of the orbit periods is close to 2 to 1. This creates a resonance condition where the gravity of Mimas perturbs the Cassini particles and eventually ejects them.

Imagine a pendulum swinging. If you lightly tap the pendulum when it reaches the top of its swing, and do this every other swing, eventually the small taps add up to increasing the height of the pendulum.

Problem 1 – The mass of Mimas is 4.0×10^{19} kilograms, and the distance to the center of the Cassini Division from Mimas is 67,000 kilometers. Use Newton's Law of Gravity to calculate the acceleration of a Cassini Division particle due to the gravity of Mimas if

$$\text{Acceleration} = G\,\frac{M}{R^2} \text{ in meters/sec}^2$$

where $G = 6.67\times10^{-11}$, M is the mass of Mimas in kilograms and R is the distance in meters.

Problem 2 – The encounter time with Mimas is about 2 hours every orbit for the Cassini particles. If speed = acceleration x time, what is the speed increase of the particles after each 12-hour orbit?

Problem 3 – If a particle is ejected from the Cassini Division once its speed reaches 1 km/sec, how many years will it take for this to happen?

Problem 1 – The mass of Mimas is 4.0×10^{19} kilograms, and the distance to the center of the Cassini Division is 67,000 kilometers. Use Newton's Law of Gravity to calculate the acceleration of a Cassini Division particle due to the gravity of Mimas if

$$\text{Acceleration} = G \frac{M}{R^2} \text{ in meters/sec}^2$$

where $G = 6.67\times10^{-11}$, M is the mass of Mimas in kilograms and R is the distance in meters.

Answer: Acceleration = $6.67\times10^{-11} (4.0\times10^{19}) / (6.7\times10^{7} \text{ meters})^2$
= **5.9×10^{-7} meters/sec^2**

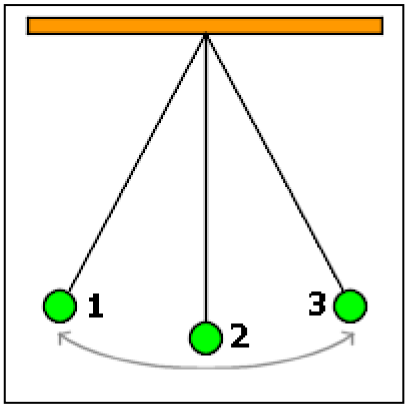
Problem 2 – The encounter time with Mimas is about 2 hours every orbit for the Cassini particles. If speed = acceleration x time, what is the speed increase of the particles after each 12-hour orbit?

Answer: 1 hour = 3600 seconds, so 2 hours = 7200 seconds and
speed = 5.9×10^{-7} m/sec^2 x 7200 sec
= 4.2×10^{-3} meters/sec per orbit.

Problem 3 – If a particle is ejected from the Cassini Division once its speed reaches 1 km/sec, how many years will it take for this to happen?

Answer: (1000 m/s)/(0.0042 m/s) = 238000 orbits. Since 1 orbit = 12 hours, we have 12 x 238,000 = 2,856,000 hours or about **326 years**.

Measuring Gravity with a Pendulum!



A pendulum is a very simple toy, but you can actually use it to measure gravity! The beat of a pendulum called its period, P, depends on the length of the pendulum, L, and the acceleration of gravity, g ,according to:

$$P = 2\pi\sqrt{\frac{L}{g}}$$

If you measure P in seconds, and know the length of the pendulum, L, in meters, you can figure out how strong the acceleration of gravity is, g, in meters/sec^2. Let's see how this works for explorers working on different planets and moons in the solar system!

Problem 1 – A mars colonist wants to make a pendulum that has a beat of 4 seconds. If the acceleration of gravity on mars is 3.8 m/sec^2, how long will the pendulum have to be in meters?

Problem 2 - A pendulum clock on the moon has a length of 2 meters, and its period is carefully measured to be 7.00 seconds. What is the acceleration of gravity on the moon?

Problem 3 – On Earth, prospectors are looking for a deposit of iron ore beneath the ground. They decide to use the acceleration of gravity to find where the iron is located because the additional iron mass should change the acceleration of gravity. They use a carefully-made pendulum with a length of 2.00000 meters and measure the period of the swing as they walk around the area where they think the deposit is located. To the nearest millionth of a second, how much will the period change if the acceleration of gravity between two spots changes from 9.80000 meters/sec^2 to 9.80010 meters/sec^2? (use p = 3.14159)

Problem 1 – A mars colonist wants to make a pendulum that has a beat of 4 seconds. If the acceleration of gravity on mars is 3.8 m/sec^2, how long will the pendulum have to be in meters?

Answer: $4 = 2\pi (L/3.8)^{1/2}$ then $L = (16/4\pi^2)*3.8$ = **1.54 meters**.

Problem 2 - A pendulum clock on the moon has a length of 2 meters, and its period is carefully measured to be 7.00 seconds. What is the acceleration of gravity on the moon?

Answer: $7.00 = 2\pi (2/g)^{1/2}$ so $g = 8\pi^2/49$ so g = **1.61 meters/sec^2**.

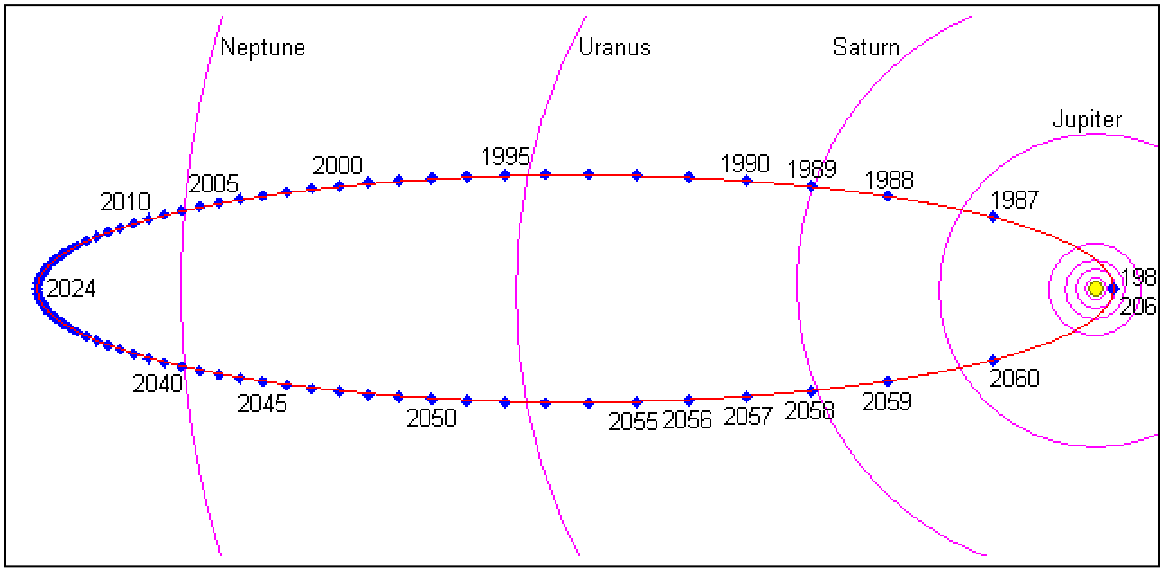
Problem 3 – On Earth, prospectors are looking for a deposit of iron ore beneath the ground. They decide to use the acceleration of gravity to find where the iron is located because the additional mass should change the acceleration of gravity. They use a carefully-made pendulum with a length of 2.00000 meters and measure the period of the swing as they walk around the area where they think the deposit is located. To the nearest millionth of a second, how much will the period change if the acceleration of gravity between two spots changes from 9.80000 meters/sec^2 to 9.80010 meters/sec^2? (use π = 3.14159)

Answer: $P = 2\pi (2.00000/9.80000)^{1/2}$ = **2.838451 seconds**.
And $P2 = 2\pi (2.00000/9.80010)^{1/2}$ = **2.838437 seconds**

So the difference in period will be **0.000014 seconds or 14 microseconds.**

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \cdots\right)$$

Where theta is the start angle from verticle.



Comets are giant icebergs in space, sometimes over 50 miles across, that shed water vapor as they are heated while approaching the sun. Some comets only come around once and are never seen again. Others travel on elliptical paths called orbits, that take them far beyond the orbit of Jupiter before they, once again, loop back towards the sun.

Halleys Comet, which made a pass near the sun in 1986 is one of the most famous Periodic Comets, and will return to Earth's skies in the year 2061. This figure shows the orbit of Halleys Comet to the same scale as the orbits of the planets. Each dot is the position after one Earth year has elapsed.

Problem 1 – What is the period of Halleys Comet in Years?

Problem 2 – What is the longest diameter of the elliptical orbit in kilometers if the distance between the orbits of Jupiter and Saturn is 650 million km?

Problem 3 – The distance between Saturn's orbit and the orbit if Venus is 1.3 billion km. About how fast is Halleys Comet traveling in km/year as it travels the Venus-Saturn distance?

Problem 4 – The distance between the orbits of Uranus and Neptune is 1.6 billion km. From the diagram, about how many years does it take to travel this distance, and what is the average speed of Halleys Comet during this time in km/year?

Problem 1 – What is the period of Halleys Comet in Years?

Answer: 2061 – 1986 = **75 years.**

Problem 2 – What is the longest diameter of the elliptical orbit in kilometers if the distance between the orbits of Jupiter and Saturn is 650 million km?

Answer: First determine the scale of this figure using a millimeter ruler and the actual Jupiter-Saturn distance. When printed on standard 8 ½ x 11 paper, the separation should be about 19 millimeters, so the scale is 650 million km/19mm = 34 million km/mm. The length of the ellipse is 143 mm, so the actual distance is 143 x 34 million = **4.86 billion kilometers**.

Problem 3 – The distance between Saturn's orbit and the orbit if Venus is 1.3 billion km. About how fast is Halleys Comet traveling in km/year as it travels the Venus-Saturn distance?

Answer: Counting the number of years, it takes 3 years to travel this distance, so the speed is 1.3 billion km/3 years = **433 million km/year**.

Problem 4 – The distance between the orbits of Uranus and Neptune is 1.6 billion km. From the diagram, about how many years does it take to travel this distance, and what is the average speed of Halleys Comet during this time in km/year?

Answer: Counting the year marks, it takes 12.5 years, so the speed is about 1.6 billion km/12.5 years = **128 million km/year.**

Year	Total	Amateurs	Spacecraft	Observatories
2012	60	27	1	32
2011	49	28	2	19
2010	161	29	119	13
2009	227	35	188	4
2008	220	34	182	4
2007	223	35	170	18
2006	206	31	152	23
2005	221	23	169	29
2004	223	8	172	43
2003	193	7	149	37
2002	182	9	131	42
2001	149	4	107	38
2000	135	9	99	27
1999	129	18	87	24

Every year, professional astronomers and dedicated amateur astronomers use everything from simple binoculars to sophisticated computer-driven telescopes to discover new comets. The table to the left gives a count of the number of comets detected between 1999 and 2012.

Comet hunters carefully compare images of the same part of the sky over a period of days or weeks. Although stars remain fixed, comets appear as fuzzy spots of light that change their positions.

Problem 1 – During 2010, what percentage of comet discoveries were made by amateur astronomers, spacecraft and ground-based observatories?

Problem 2 – During the years 1999 to 2012, what is the average number of comets discovered by amateur astronomers and by ground-based observatories?

Problem 3 – What percentage of new comets would have been lost in 2012 had there not been any amateur astronomers searching the skies?

The tabulated data is based upon the Catalog of Comet Discoveries archive at http://www.comethunter.doc

Problem 1 – During 2010, what percentage of comet discoveries were made by amateur astronomers, spacecraft and ground-based observatories?

Answer: Amateur astronomers: 100% x (29/161) = **18%**
Spacecraft: 100% x (119/161) = **74%**
Observatories : 100% x(13/161) = **8%**

Problem 2 – During the years 1999 to 2012, what is the average number of comets discovered by amateur astronomers and by ground-based observatories?

Answer: Total for amateurs = 297 over 14 years so the average was **21 comets/year**.
For observatories: 353 over 14 years so the average is **25 comets/year**.

Problem 3 – What percentage of new comets would have been lost in 2012 had there not been any amateur astronomers searching the skies?

Answer: In 2012 the total number of comets was 60, and of these 27 were detected by amateur astronomers, so 60-27 = 33 detected by other means, then 100% x (33/60) = **55% of the comets would have remained undetected.**

Comet Encounters after Discovery

Name	Discovery	Earth Distance	Date
C/2011 UF305	October 2011	375	June 2012
C/2011 L4	June 2011	495	June 2012
C/2011 F1	March 2011	300	August 2012
C/2011 R1	September 2011	330	Sept 2012
C/2010 S1	September 2010	840	Sept 2012
C/2012 J1	May 2012	345	Oct 2012
273P	June 1827	128	Oct 2012
C/2012 J1	May 2012	360	Nov 2012
C/2012 K5	May 2012	50	Dec 2012

Comets are icy bodies that can be many kilometers across. If they struck earth they would do considerable damage and could even lead to extinctions. One of the biggest problems that astronomers face is that, unlike asteroids, comets come from places in our solar system that are far beyond the orbit of Jupiter. They are usually discovered ass very faint fuzzy objects once they reach the orbit if Mars. By that time, there is not much warning should the comet have a path that intersects earth's orbit.

The table above gives the month and year when a comet was discovered (colume 2), the closest distance that it got to earth in millions of kilometers (colume 3) and the month and year when it makes its closest approach to the sun and earth (colume 4).

Problem 1 - How many months after the comet was discovered did it make its next closest passage to Earth? Omitting Comet 273P, what is the average number of months between discovery and close passage?

Problem 2 – What percentage of the comets passed further away than 50 million kilometers from Earth?

Problem 3 – During 2012 there were 60 new comets discovered. About how many of these might come within 50 million kilometers of earth during the following year, 2013?

Problem 1 - How many months after the comet was discovered did it make its next closest passage to Earth? Omitting Comet 273P, what is the average number of months between discovery and close passage?

Name	Discovery	Earth Distance	Date	Time
C/2011 UF305	October 2011	375	June 2012	**8**
C/2011 L4	June 2011	495	June 2012	**12**
C/2011 F1	March 2011	300	August 2012	**17**
C/2011 R1	September 2011	330	Sept 2012	**12**
C/2010 S1	September 2010	840	Sept 2012	**24**
C/2012 J1	May 2012	345	Oct 2012	**6**
273P	June 1827	128	Oct 2012	**2224**
C/2012 J1	May 2012	360	Nov 2012	**7**
C/2012 K5	May 2012	50	Dec 2012	**8**

Answer: Omitting 273P , the average is (8+12+17+12+24+6+7+8)/8 = **12 months**.

Problem 2 – What percentage of the comets passed further away than 50 million kilometers from Earth?

Answer: 100% x (8/9) = **89%**

Problem 3 – During 2012 there were 60 new comets discovered. About how many of these might come within 50 million kilometers of earth during the following year, 2013?

Answer: 60 x 0.89 = 53 that pass farther away than 50 million km, and so there are **7 that pass closer than 50 million km.**

Note: What this means is that by the time a comet is discovered, it is only a year from its closest approach to earth, and for a typical year there could be 7 comets that come this close. These are potentially dangerous objects because their sizes are big enough to devastate cities or even entire continents if they were to impact!

When a solar coronal mass ejection collides with earth's magnetic field, it can produce intense aurora that can be seen from the ground. Geophysicists who study magnetic disturbances have created a 9-point scale that indicates how intense the storm is. Kp=9 is the most intense, and aurora can be seen near Earths equator for many of these. Kp=7 and 8 are strong storms that can still cause aurora and upset electrical power systems. Kp=5 and 6 are mild storms that may or may not produce intense aurora.

The table below gives the dates when magnetic storms were detected that exceeded Kp=4. The multiple entries each day indicate consecutive 3-hour measurements of the storm intensity. The data are given for the 227 days from January 1, 2013 to August 15, 2013. Highlighted boxes in yellow indicate days when Halo CMEs were detected

Date	Kp measurements	Date	Kp Measurements
3-1	5	6-2	5
3-17	6, 5, 5, 6, 6, 5	6-7	5, 6, 5
3-29	5	6-29	6, 7, 5, 6
3-30	5, 5	7-10	5, 5
4-26	5	7-11	5
5-18	5, 5	7-15	5, 5
5-24	5	8-4	5
5-25	5, 5, 5	8-5	5
6-1	6, 6, 6, 5, 6		

Problem 1 – What percentage of the days during this time period had magnetic storm events?

Problem 2 – What percentage of the magnetic storm days were strong storms with Kp=6 or higher?

Problem 3 – During this same 227-day period of time, there were 12 Halo CMEs detected. What is the probability that these Halo events produce a magnetic storm with Kp=6 or higher?

Problem 4 – The Kp index is measured every 3 hours. From the table, what is the average duration of a storm that exceeds Kp=5 during its entire duration?

Problem 1 – What percentage of the days during this time period had magnetic storm events?

Answer: 17 days out of 227 so 100% x (17/227) = **7%**

Problem 2 – What percentage of the magnetic storm days were strong storms with Kp=6 or higher?

Answer: 4 out of 17 or 100% x (4/17) = **24%.**

Problem 3 – During this same 227-day period of time, there were 12 Halo CMEs detected. What is the probability that these Halo events produce a magnetic storm with Kp=6 or higher?

Answer: Only 2 of the 12 Halo CMEs produced Kp=6 or stronger magnetic storms, so P = 100% x (2/12) = **17%.** The odds were 1 in 6.

Problem 4 – The Kp index is measured every 3 hours. From the table, what is the average duration of a storm that exceeds Kp=5 during its entire duration?

Answer: There were 4 storms that exceeded Kp=5. These occurred on the dates: 3-17, 6-1, 6-7 and 6-29. The total hours for each storm were
6x3 = 18 hours (for 3-17),
5x3 = 15 hours (for 6-1);
3x3 = 9 (for 6-7) and
4x3 = 12 (for 6-29).

The average for these hours is (18+15+9+12)/4 = **14 hours.**

So, once a strong magnetic storm begins, it takes about one-half a day for it to reach its maximum intensity and then fade away. If this happens in the summertime during the day, you will not see an aurora borealis because of the daylight brightness.

Bungee jumping has become a popular but dangerous sport. It also shows how the acceleration of gravity is connected to the total distance traveled during the fall. The distance traveled is given by the formula

$$D = \frac{1}{2} g T^2$$

Where g is the acceleration of gravity in meters/sec^2, D is the distance in meters, and T is the elapsed time in seconds. For locations near the surface of Earth, g = 9.8 meters/sec^2 (32 feet/sec^2)

Problem 1 - A confused Daredevil jumps from a plane at an altitude of 15,000 feet. How long does it take for the Daredevil to land if there is no air friction to slow him down?

Problem 2 – How fast would the Daredevil be traveling at the moment of impact if S = 32T?

Problem 3 – Once he reaches 130 mph (190 feet/sec), called the terminal velocity, his free-fall speed stops increasing. How soon after he jumps does he reach terminal velocity, and how far has he fallen from the plane?

Problem 4 - In 2012, Felix Baumgartner jumped from a high-altitude balloon at an altitude of 24 miles (127,000 feet), landing safely on the ground after 4 minutes and 19 seconds. With little atmosphere friction, he reached a maximum speed of 844 mph (1240 feet/sec). How long after he jumped did he reach this speed, and how high above the ground was he at that time?

Problem 5 – On Mars, the Valles Marineris canyon is 23,000 feet deep. If the acceleration of gravity is 12 feet/sec^2, how long would it take a rock to fall into the canyon and how fast is it traveling when it hits bottom?

Answer Key

Problem 1 - A confused Daredevil jumps from a plane at an altitude of 15,000 feet. How long does it take for the Daredevil to land if there is no air friction to slow him down?

Answer: 15,000 = ½ (32) T^2, so T^2 = 937 and so T = **31 seconds**.

Problem 2 – How fast would the Daredevil be traveling at the moment of impact if S = 32T?

Answer: S = 32 x 31 = 992 feet/second or 676 miles/hour!

Problem 3 – Once he reaches 130 mph (190 feet/sec), called the terminal velocity, his free-fall speed stops increasing. How soon after he jumps does he reach terminal velocity, and how far has he fallen from the plane?

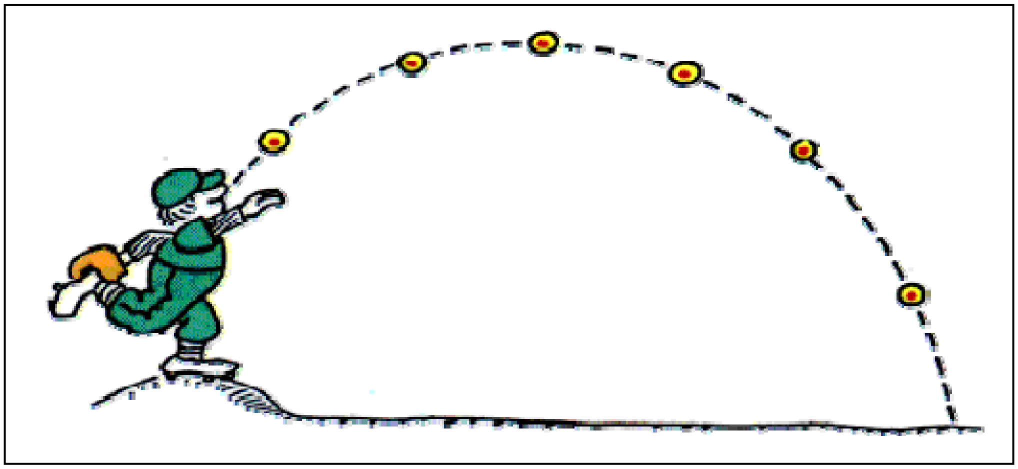
Answer: 190 = 32 x T so T = **6 seconds**. He has fallen d = ½ (32)$(6)^2$ = **576 feet**.

Problem 4 - In 2012, Felix Baumgartner jumped from a high-altitude balloon at an altitude of 24 miles (127,000 feet), landing safely on the ground after 4 minutes and 19 seconds. With little atmosphere friction, he reached a maximum speed of 844 mph (1240 feet/sec). How long after he jumped did he reach this speed, and how high above the ground was he at that time?

Answer: 1240 = 32 T so T = **39 seconds**.
D = ½ (32) $(39)^2$ = 24,336 feet,
so 127,000 – 24333 = **102,700 feet from the ground**.

Problem 5 – On Mars, the Valles Marineris canyon is 23,000 feet deep. If the acceleration of gravity is 12 feet/sec^2, how long would it take a rock to fall into the canyon and how fast is it traveling when it hits bottom?

Answer: 23,000 = ½(12)T^2 so T = **62 seconds**.
Speed = 12 x 62 = **744 feet/sec or 507 mph**.



The horizontal motion of a rock (projectile) is given by the formula:

$$X = V_h T$$

Independently, the vertical motion is given by the formula

$$Y = H_0 + V_v T - ½ gT^2$$

The speed of the projectile has been described in terms of its vertical (Vv) and horizontal (Vh) speeds to that the total speed is given by the Pythagorean Theorem $S = (V_h^2 + V_v^2)^{1/2}$.

Problem 1 – A rock is tossed horizontally from the top of the Eiffel Tower at a speed of 60 mph (40 feet/sec). The Eiffel Tower stands 1,063 feet above the street. How far from the centerline of the tower does the rock land? (g = 32 feet/sec^2)

Problem 2 – On Mars (g = 12 feet/sec^2) an astronaut throws a rock up in the air so that its vertical speed is 30 feet/sec and its horizontal speed is 10 feet/sec. The rock starts at a shoulder height of 5 feet. How high does the rock travel, and how far from the astronaut does it finally reach the ground?

Problem 1 – A rock is tossed horizontally from the top of the Eiffel Tower at a speed of 60 mph (40 feet/sec). The Eiffel Tower stands 1,063 feet above the street. How far from the centerline of the tower does the rock land?

Answer: H_0 = 1063 feet, V_h = 40 feet/sec, V_v = 0.0, g = 32 feet/sec^2. The vertical equation give us the time to reach the ground (y=0) : 0 = 1063 – 16 T^2, so 1063/16 = T^2 and T = 8.1 seconds. From the horizontal motion, it travels d = 40 x 8.1 = **324 feet**.

Problem 2 – On Mars (g = 12 feet/sec^2) an astronaut throws a rock up in the air so that its vertical speed is 30 feet/sec and its horizontal speed is 10 feet/sec. The rock starts at a shoulder height of 5 feet. How high does the rock travel, and how far from the astronaut does it finally reach the ground?

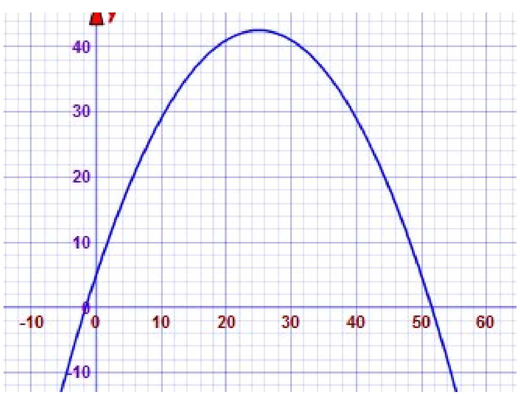
Answer: The two equations are X = 10 T and Y = 5.0 + 30T - 6T^2

Write Y in terms of X: Y = 5.0 + 30(X/10) - 6 $(X/10)^2$ so **Y = 5.0 + 3.0X - 0.06X^2**

Solve for the roots of Y(X) = -0.06X^2 + 3.0X + 5.0 with coefficients a = -0.06, b=3.0 and c = 5.0, to get the ground intercept points using the Quadratic Formula:

X = [-3.0 +/- $(9-4(-0.06)(5.0))^{1/2}$] / (2x-0.06) so
X = (-3 + 3.19)/-0.12 = -1.6 feet , and the second root is
X = (-3 – 3.19)/-0.12 = +51.6 feet. see graph below.

The peak of the parabola is ½ way between the x-intercepts at x = (51.6-1.6)/2 = +25.0 feet And since X = 10T, we have 25 = 10T so T = 2.5 seconds. From Y(T), the altitude of the peak is Y = 5.0 + 30(2.5) – 6$(2.5)^2$ = **42.5 feet**. From the x-intercept, it reaches a distance of **51.6 feet from the astronaut.**



Energy can be changed from one form to another. When you peddle a bike, your body uses up stored food energy (in calories) and converts this into kinetic energy of motion measured in joules. When you connect an electric motor to a battery, electrical energy stored in the battery is converted into rotational kinetic energy causing the motor shaft to turn.

A millstone paddle wheel uses the gravitational energy of falling water to turn the millstone wheel and perform work by grinding wheat, or even running simple machinery to cut wood in a lumber mill.

The energy in Joules of an object falling from a height near the surface of Earth can be calculated from

$$\mathbf{E = mgh}$$

where m is the mass of the falling body in kilograms, g is the acceleration of gravity (9.8 meters/sec^2) and h is the distance of the fall in meters.

Problem 1 – Nevada Falls in Yosemite Valley California has a height of 180 meters. Every second, 500 cubic feet of water goes over the edge of the falls. If 1 cubic foot of water has a mass of 28 kilograms, how much energy does this waterfall generate every day?

Problem 2 – For a science fair project, a student wants to build a water hose powered hydroelectric plant to run a light bulb. Every second, the light bulb needs 60 Joules to operate at full brightness. If the water hose produces a steady flow of 0.2 kilograms every second, how high off the ground does the water hose have to be to turn a paddle wheel to generate the required electrical energy?

Problem 3 – A geyser on Saturn's moon Enceladus ejects water from its caldera with an energy of 1 million Joules. If g = 0.1 meters/sec^2, and the mass moved is 2000 kilograms, how high can the geyser stream travel above the surface of Enceladus?

Problem 1 – Nevada Falls in Yosemite Valley California has a height of 180 meters. Every second, 500 cubic feet of water goes over the edge of the falls. If 1 cubic foot of water has a mass of 28 kilograms, how much energy does this waterfall generate every day?

Answer: 1 day = 24 x 60 x 60 = 86,400 seconds, then the total mass is 500 x 28 x 86400 = 1.2 billion kilograms. E = 1.2 billion kg x 9.8 x 180 = **2.1 trillion joules** per day. Note: since 1 watt = 1 Joule/second, this waterfall has a wattage of 500 x 28 x 9.8 x 180 = 25 megawatts.

Problem 2 – For a science fair project, a student wants to build a water hose powered hydroelectric plant to run a light bulb. Every second, the light bulb needs 60 Joules to operate at full brightness. If the water hose produces a steady flow of 0.2 kilograms every second, how high off the ground does the water hose have to be to turn a paddle wheel to generate the required electrical energy?

Answer: 60 = 0.2 x 9.8 x h so h = **30.6 meters** (or 90 feet!).

Problem 3 – A geyser on Saturn's moon Enceladus ejects water from its caldera with an energy of 1 million Joules. If g = 0.1 meters/sec^2, and the mass moved is 2000 kilograms, how high can the geyser stream travel above the surface of Enceladus?

Answer: 1,000,000 = 2000 x 0.1 x h, so h = **5,000 meters or 5 kilometers**.

It would be nice if we could just jump real hard and we would suddenly be in space orbiting Earth. Fortunately it is not that easy as any Olympic High Jumper will tell you!

Because of the pull of gravity, every planet, asteroid or other object in the universe has its own speed limit. If you move slower than this speed you will stay on the body. If you move faster than this speed you will escape into space. Scientists call this the **escape speed** or **escape velocity**.

It's not just a number you guess at. It depends exactly on how much mass the planet or moon has, and how far from its center you are located. That means you can predict what this speed will be as you travel to other planets. That's very handy if you are an astronaut!

For Earth, the escape speed V in kilometers/second (km/s) at a distance R from Earth's center in kilometers, is given by

$$V = \frac{894}{\sqrt{R}}$$

Problem 1 - What is the escape speed for a rocket located on Earth's surface where R = 6378 km?

Problem 2 – An Engineer proposes to launch a rocket from the top of Mt Everest (altitude 8.9 km) because its summit is farther from the center of Earth. Is this a good plan?

Problem 3 – A spacecraft is in a parking orbit around Earth at an altitude of 35,786 km. What is the escape speed from this location?

Problem 4 – To enter a circular orbit at a distance of R from the center of Earth, you only need to reach a speed that is $2^{1/2}$ smaller than the escape speed at that distance. What is the orbit speed of a satellite at an altitude of 35,786 km?

Problem 1 - What is the escape speed for a rocket located on Earth's surface where R = 6378 km?

Answer: $V = 894/(6378)^{1/2}$ = **11.19 km/s**

Problem 2 – An Engineer proposes to launch a rocket from the top of Mt Everest (altitude 8.9 km) because its summit is farther from the center of Earth. Is this a good plan?

Answer: $V = 894/(6378+8.9)^{1/2}$ = **11.18 km/s. This does not change the required escape speed by very much considering the effort to build such a launch facility at this location**.

Problem 3 – A spacecraft is in a parking orbit around Earth at an altitude of 35,786 km. What is the escape speed from this location?

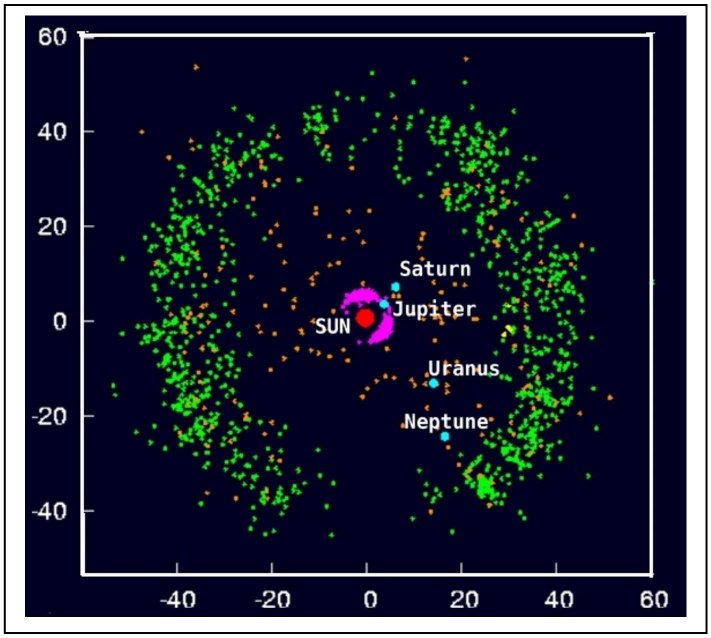
Answer: $V = 894/(6378+35{,}786)^{1/2}$ = **4.35 km/s**

Problem 4 – To enter a circular orbit at a distance of R from the center of earth, you only need to reach a speed that is $2^{1/2}$ smaller than the escape speed at that distance. What is the orbit speed of a satellite at an altitude of 35,786 km?

Answer: 4.35 km/s / (1.414) = **3.079 km/sec**.

Note: Satellites at an altitude of 35,800 orbit earth exactly once every day and are called geosynchronous satellites because they remain over the same geographic spot on Earth above the equator.

Circumference of the orbit = 2 π R = 2 x 3.141 x (42164 km) = 264,924 km.
Speed = 3.079 km/sec so
T = 264924/3.079 = 86,042 seconds or 23.9 hours – Earths rotation period.



Since the belt was discovered in 1992, the number of known Kuiper belt objects (KBOs) has increased to over a thousand, and more than 100,000 KBOs over 100 km (62 mi) in diameter are believed to exist.

Pluto is the largest known member of the Kuiper belt. Originally considered a planet, Pluto's status as part of the Kuiper belt caused it to be reclassified as a "dwarf planet" in 2006

This figure shows the locations of the known KBOs with the X and Y positions given in terms of Astronomical Units (AUs), where 1 AU equals the distance from Earth to the Sun (93 million miles or 149 million km).

Problem 1 – The Kuiper Belt stretches from 30 to 60 AU and has a torus shape. What is the volume of the Kuiper Belt in cubic kilometers if the volume of a torus is given by

$$V = 2\pi^2 r^2 R,$$

where R is the Kuiper Belts average distance from the sun and r is its radius?

Problem 2- It is estimated that over 100,000 objects larger than 100 km reside in the Kuiper Belt, of which 1,200 have been discovered by 2013. What is the density of the estimated 100,000 objects in objects/km^3 if they are uniformly distributed throughout the toroidal volume of the Kuiper Belt?

Problem 3 – Based upon the average density calculated in Problem 2, about what is the average distance between the Kuiper Belt Objects compared to the distance between Earth and Sun?

Problem 1 – The Kuiper Belt stretches from 30 to 60 AU and has a torus shape. What is the volume of the Kuiper Belt in cubic kilometers if the volume of a torus is given by $V = 2\pi^2 r^2 R$, where R is the Kuiper Belts average distance from the sun and r is its radius?

Answer: $R = (60+30)/2 = 45$ AU or $45 \times 149 \times 10^6$ km = 6.7×10^9 km
$r = (60-30)/2 = 15$ AU or $15 \times 149 \times 10^6$ km = 2.2×10^9 km

$V = 2\,(3.141)2\,(2.2\times10^9)^2\,(6.7\times10^9) =$ $\mathbf{6.4\times10^{29}\ km^3}$

Problem 2- It is estimated that over 100,000 objects larger than 100 km reside in the Kuiper Belt, of which 1,200 have been discovered by 2013. What is the density of the estimated 100,000 objects in objects/km^3 if they are uniformly distributed throughout the toroidal volume of the Kuiper Belt?

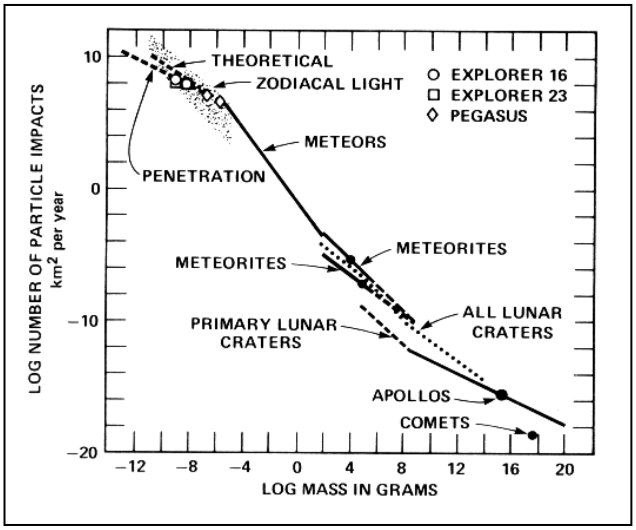
Answer: $N = 10^5/6.4\times10^{29}$ km^3 = $\mathbf{1.6\times10^{-25}}$ **objects/km^3**.

Problem 3 – Based upon the average density calculated in Problem 2, about what is the average distance between the Kuiper Belt Objects compared to the distance between Earth and Sun?

Answer: We just need to calculate the cube root of the density to get the reciprocal of this distance:

$D = 1 / (1.6\times10^{-25})^{1/3}$ = **183 million kilometers**.

Since the Earth-Sun distance is 149 million kilometers, the average distance between KBOs is about 1.2 AU or **1.2 times the Earth-Sun distance**.



The space between the planets is filled with fragments of asteroids, comets and material left over from the formation of the planets. These rocks and debris rain down upon exposed surfaces at speeds up to 30 km/sec.

The figure on the left summarizes the impact frequency of various sizes of particles in space. Note the graph is plotted in the Log-Log format due to the enormous range of masses and rates being described.

Understanding the data graph:

Problem 1 - A meteorite with a density of 3 grams/cm^3 has a diameter of 4 centimeters (about 1 1/2 inches).

A) What is the mass of this meteorite assuming it is a sphere?

B) From the graph, where on the horizontal axis are objects of this mass located?

C) What is the number of impacts per year you would expect over an area of 10,000 square kilometers?

Problem 2 – The function that best models the data in the graph is given by

$$N(m) = 0.025\ m^{-0.9}$$

where N(m) is the number if impacts per square kilometer per year for objects with a mass of m grams. Using integral calculus, what is the total mass in tons of impacting objects each year, over the surface of Earth, in the mass range from 1 gram to 10^{20} grams? (Use π = 3.14 and assume a spherical Earth with a radius of 6,378 km).

Problem 1 - Answers: A) Mass = Density x Volume. Radius of sphere = 2 cm, so M = 3.0 x (4/3) (3.14) $(2)^3$ = **100 grams**. B) The horizontal axis is un units of Log(grams) so Log(100) = 2, and this is the location **half-way between 0' and '4' on the axis**. C) From 'x=2' ,a vertical line intercepts the data at about 'y=-3.5' on the vertical axis. This represents Log(N) = -3.5 so that N = 0.00032 impacts/km^2/year. Over an area of 10000 km^2, there would be an estimated 0.00032 impacts/km^2/year x (10000 km^2) = **3.2 impacts per year.**

Problem 2 –What is the total mass in tons of impacting objects each year, over the surface of Earth, in the mass range from 1 gram to $10^{\mathbf{20}}$ grams? (Use π = 3.14 and assume a spherical Earth with a radius of 6,378 km).

Answer: The total mass is the area under the curve: Mass = N(m) dm

$$M = \int_{1}^{10^{20}} 0.025 m^{-0.9} dm$$

$$= 0.025 \frac{1}{0.1} [(10^{20})^{0.1} - (1)^{0.1}]$$

= 0.025(10)(100-1)

= **24.75 grams/km^2/year**

Area of Earth = $4\pi(6378)^2$ = 5.1×10^8 km^2

So the total meteoritic mass per year is

24.75 grams/km^2 x 5.1×10^8 km^2 = 1.26×10^{10} grams
or 1.26×10^7 kg

or **12,600 tons**.

Note: Popular estimates range from 20,000 to 100,000 tons/year. The amount is sensitive to both the logarithmic function used to model the power-law data, and the integration limits!

Shipping Cargo to the International Space Station

It takes a lot of supplies to keep the International Space Station going!

On February 21, 2014, NASA asked commercial launch services, such as SpaceX, to figure out what they would charge to deliver cargo to the Station. NASA plans to buy this service for between $1 billion and $1.4 billion each year from 2017 to 2024.

Groups 'did the math' to figure out how many launches would be needed to deliver the required materials to the Station. On return flights, trash such as discarded equipment and garbage, will be brought back home.

For each year, NASA will need up to 16,750 kilograms of pressurized cargo delivered to the Station. This amount of cargo will fill about 30 lockers. Another 4,000 kg of unpressurized cargo will be included in each launch. This amount of cargo will fill about 8 lockers. There will be 5 flights per year.

Problem 1 – What is the total mass of the pressurized and unpressurized cargo?
A) per year? B) per flight?

Problem 2 – For each flight, 800 kg of fresh drinking water will be delivered to the Station. If 1 gallon of water has a mass of 3.8 kg, how many gallons of drinking water are transported to the Station every year? Round your answer to the nearest gallon.

Problem 3 – Each pressurized locker requires continuous power of 120 watts at 28 volts. Each unpressurized locker requires continuous power of 250 watts at 28 volts. What is the total electrical watts used by both the pressurized and unpressurized lockers for each flight?

Problem 4 – If it costs $4,500 per kg to place items in orbit, what is the cost for each launch?

NASA Seeks U.S. Industry Feedback on Options for Future Space Station Cargo Services
February 21, 2014
http://www.nasa.gov/content/nasa-seeks-us-industry-feedback-on-options-for-future-space-station-cargo-services/index.html

Problem 1 – What is the total mass of the pressurized and unpressurized cargo? A) per year? B) per flight?

Answer:
A) Pressurized cargo = 16,750 kg; Unpressurized cargo = 4,000 kg.
16,750 kg + 4,000 kg = **20,750 kg per year**.

B) There are 5 flights. Total mass (20,750 kg)/5= **4,150 kilograms**.

Problem 2 – If 1 gallon of water has a mass of 3.8 kg, how many gallons of drinking water are transported to the Station every year? Round your answer to the nearest gallon. rounded to the nearest gallon?

Answer: Each flight brings 800 kg to the Station. For 5 flights there would be 4000 kg of water. Then 4,000 kg x (1 gallon/3.8 kg) = **1053 gallons per flight**.

Problem 3 – What is the total electrical watts used by both the pressurized and unpressurized lockers for each flight?

Answer: There are 30 powered lockers brought to the Station each year with 120 watts/locker. The total power is 30 x 120 watts = 3,600 watts per year. There are 8 unpressurized lockers brought to the Station each year at 250 watts/locker for a total of 2,000 watts. The total per year is 5,600 watts in 5 flights or **1120 watts/flight**.

Problem 4 – If it costs $4,500 per kg to place items in orbit, what is the cost for each launch?

Answer: 4,150 kg cargo per launch x $4,500/kg = **$18,675,000 for the cargo**.

The Chandra X-ray Observatory has seen a fast-moving pulsar escaping from a supernova remnant while spewing out a record-breaking jet. This is, to date, the longest object observed in the Milky Way galaxy. The jet is nearly 37 light years long! The pulsar is 60 light years from the supernova remnant.

The supernova remnant, called SNR MSH 11-61A, is in the constellation of Carina, and located 23,000 light years from Earth.

Problem 1 – If one light year equals 9.5×10^{12} kilometers, how far is the pulsar from the supernova remnant?

Problem 2 – How long is the jet in kilometers?

Problem 3 – The supernova is estimated to have exploded about 9,000 years ago. How fast has the pulsar been traveling to get to its current location? Calculate this speed in a) km/sec and b) miles/hour.

Problem 4 – Our sun has a diameter of 1.4 million kilometers. The pulsar has a diameter of only 20 km, but carries twice the mass of our sun. Explain what would happen if the pulsar collided with a star like our sun.

NASA's Chandra Sees Runaway Pulsar Firing an Extraordinary Jet
February 18, 2014
http://www.nasa.gov/press/2014/february/nasas-chandra-sees-runaway-pulsar-firing-an-extraordinary-jet/index.html

Problem 1 – If one light year equals 9.5×10^{12} kilometers, how far is the pulsar from the supernova remnant?

Answer: 60 x 9.5×10^{12} km = **5.7×10^{14} km**.

Problem 2 – How long is the jet in kilometers?

Answer: 37 x 9.5×10^{12} km = **3.5×10^{14} km**.

Problem 3 – The supernova is estimated to have exploded about 9,000 years ago. How fast has the pulsar been traveling to get to its current location? Calculate this speed in a) km/sec and b) miles/hour.

Answer: 1 year = 3.1×10^{7} seconds. Based upon this, the travel time is 9,000 yrs. x 3.1×10^{7} sec/year = 2.8×10^{11} seconds.

A) From Problem 1, the distance is 5.7×10^{14} km, therefore the average speed is 5.7×10^{14} km/2.8×10^{11} sec = **2000 km/sec**.

B) 1 km = 0.62 miles
2000 km/sec x (0.62 miles/1 km) x (3600 sec/1hr) = **4.5 million miles/hour.**

Problem 4 – Our sun has a diameter of 1.4 million kilometers. The pulsar has a diameter of only 20 km, but carries twice the mass of our sun. Explain what would happen if the pulsar collided with a star like our sun.

Answer: Traveling at 2000 km/sec, it would take the pulsar 12 minutes to travel through a star like our sun (1.4 million/2000 km/sec = 700 seconds or 12 minutes). Because the density of the pulsar is over one trillion times that of our sun, it would be like a bullet traveling through a cloud in the sky!

At a great distance, the sun would respond to the gravity of the pulsar and start to deform like a football with the long axis pointed close to the direction of the pulsar. The time it takes the pulsar to travel from Earth's orbit to the sun is only 150 million km/2000 km/sec = 21 hours. An object as huge as the sun would not have much time to deform before the encounter was over. The pulsar would shoot through the interior of the sun and exit the other side before our sun had much of a chance to change its shape. The friction of the pulsar against the gases in the sun would probably increase the temperature along the path by a few thousand degrees. This would not have much effect on the sun that has interior temperatures between 100,000 to 15 million degrees ^{o}C.

Solar panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.

A simple application of vector dot and cross products lets us predict the amount of electrical power the panels can produce.

A surveyor on the sidewalk uses his instruments to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture shown above, suppose the points are labeled counter clockwise from the roof corner nearest the camera in units of meters: P1(6, 8, 4); P2(21, 8, 4); P3(21, 16, 10) and P4(6, 16, 10)

Problem 1 – What are the components to the two edge vectors defined by **A** = **P**2-**P**1 and **B** = **P**4-**P**1. Write the vector in standard notation with **x**, **y** and **z** being the coordinate unit vectors.

Problem 2 – What are the magnitudes of the vectors **A** and **B,** and in what units?

Problem 3 – What are the components to the vector, **N**, perpendicular to **A** and **B** and the surface of the roof?

Problem 4 – What is the magnitude of **N** and its units?

Problem 5 – The sun is located along the unit vector **S** = ½ **x** – 6/7 **y** + 1/7**z**. If the flow of solar energy is given by the vector **F** = 910 **S** in units of watts/meter^2, what is the dot product of **F** with **N**, and the units for this quantity?

Problem 6 – What is the angle between **N** and **S**? What is the elevation angle of the sun above the plane of the roof?

Problem 1 – What are the components to the two edge vectors defined by **A** = P2-P1 and **B** = P4-P1. Write the vector in standard notation with **x**, **y** and **z** being the coordinate unit vectors.

Answer: **A** = (21-6)**x** +(8-8)**y** + (4-4)**z** so **A =** 15**x**
B = (6-6)**x** + (16-8)**y** + (10-4)**z** so **B =** 8**y +** 6**z**

Problem 2 – What are the magnitudes of the vectors **A** and **B**, and in what units?

Answer: ||**A**|| = **15 meters** ||**B**|| = $(8^2 + 6^2)^{1/2}$ = **10 meters**

Problem 3 – What are the components to the vector, **N**, perpendicular to **A** and **B** and the surface of the roof?

Answer: Use the vector cross product: **N** = **A** x **B** so **N** = 0**x** – 90**y** +120**z**

Problem 4 – What is the magnitude of **N** and its units?
Answer: ||**N**|| = $((-90)^2 + (120)^2)^{1/2}$ so ||**N**|| = 150 meters^2 which is the area of the roof

Problem 5 – The sun is located along the unit vector **S** = ½ **x** – 6/7 **y** + 1/7**z.** If the flow of solar energy is given by the vector **F** = 910 **S** in units of watts/meter^2, what is the dot product of **F** with **N**, and the units for this quantity?

Answer: **F** = 910 (1/2**x** -6/7**y** +1/7**z**) = 455**x** – 780**y** + 130**z**.

The dot product is just **F**dot**N** = 455*(0) -780*(-90) + 130*120 = **85,800 watts**.

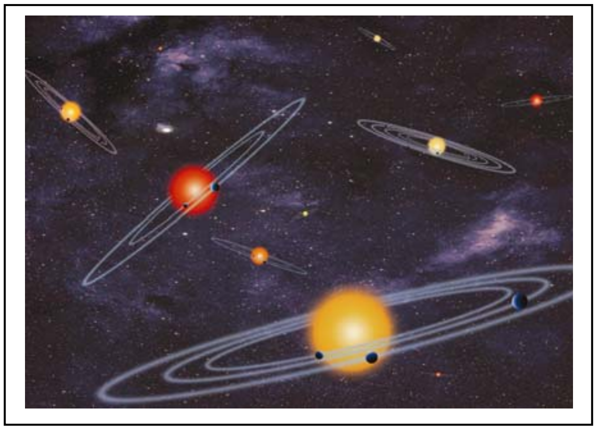
Problem 6 – What is the angle between **N** and **S**? What is the elevation angle of the sun above the plane of the roof?

Answer: From the definition of dot product**:** **F** dot **N** = ||**F**|| ||**N**|| sin θ

Then since ||**F**|| = 910 and ||**N**|| = 150 and **F**dot**N** = 85,800 we have

Sinθ = (85800/(910x150)) = 0.629 and so θ **= 39°**. This is the angle between the normal to the surface and the incident solar rays. The compliment of this is the elevation of the sun above the plane of the roof or 90-39 = 51°.

NASA's Kepler Mission Detects 715 New Planets



NASA's Kepler mission announced Wednesday the discovery of 715 new planets. These worlds orbit 305 stars, revealing multiple-planet systems much like our own solar system.

Nearly 95 percent of these planets are smaller than Neptune, which is almost four times the size of Earth. This discovery marks a significant increase in the number of known planets similar in size to Earth.

The Kepler mission surveyed 190,751 stars similar to our own sun in the constellation Cygnus the Swan. It searched for the tell-tail dimming of light that means a planet has 'eclipsed' the face of the star as viewed from Earth. As of February 2014, the Kepler spacecraft and its follow-up observations has detected 3670 stars with transit events, of these 1140 were later identified as binary stars, and 779 were eliminated because their transits were misidentified. The remaining transits included 340 stars that had more than one transit event suggesting that they were multi-planet systems (called Multis) with a total of 851 individual planet transits. Thirty four of these Multis were already catalogued stars which had 83 planet transits.

Problem 1 - Draw a branching diagram that shows how each of different groups and numbers are related to each other. How many single star/single transits were discovered among the 3670 stars with transits? What was the total number of new stars and planets announced by Kepler in the Multi systems?

Problem 2 - A subsample of 739 of the 851 discovered Kepler planets could be measured for their sizes. The result was 102 planets that were Earth-sized, 215 planets that were super-Earth sized up to twice the diameter of Earth, 373 that were Neptune-sized, and 13 that were Jupiter-sized or larger. Draw a histogram of this ensemble. What percentage of multi-planet system detected by Kepler have Earth-sized planets?

Problem 3 - If the amount of solar energy falling on Earth to keep it warm is defined as 1.0, for Venus it is 2.0 and for Mars it is 0.4. From the 851 planets detected in orbit around the 340 stars, there were 18 planets orbiting 16 stars found to be in the solar energy 'habitable zone' from 0.4 to 2.0 so that liquid water could exist on their surfaces. What is the probability that, given a multi-planet system you will find an Earth-sized planet in its Habitable Zone?

Problem 4 - The Milky Way is estimated to have about 1 trillion stars from red dwarfs to brilliant super giants. From various surveys, about 20% are similar to our sun. From the Kepler survey data, A) how many multi-planet systems would you expect to find if the Kepler survey could be conducted for all the stars in the Milky Way? B) How many Earth-sized planets in multi-planet systems would you expect to find? C) How many Earth-like planets in their Habitable Zones would you expect to find?

Answer Key

NASA's Kepler Mission Announces a Planet Bonanza, 715 New Worlds
February 26, 2014
http://www.nasa.gov/press/2014/february/nasas-kepler-mission-announces-a-planet-bonanza-715-new-worlds/

Teacher Note: The number '715' is a revised version of the 768 new planets reported by the Kepler Team in the paper by Rowe et al. 'Validation of Kepler's Multiple Planet Candidates: III (Table III), based on combining other surveys after the paper was published, but before the press conference public announcement on February 26.

Problem 1 -

		190,751 stars		
		3670 stars with transits		
779 false positive		1751 valid transits	1140 binary stars	(total=3670)
	1411 Single Stars	340 Multis (851 planets)		(total=1751)
		306 stars 768 planets	34 stars 83 planets	(total=340)

Answer: There were 1751-340=1411 single star transits. The total number of new stars and planets was 340-34 = 306 stars and 851-83 = 768 planets.

Problem 2 - A subsample of 739 of the 851 discovered Kepler planets could be measured for their sizes. The result was 102 planets that were Earth-sized, 215 planets that were super-Earth sized up to twice the diameter of Earth, 373 that were Neptune-sized, and 13 that were Jupiter-sized or larger. Draw a histogram of this ensemble. What percentage of multi-planet system detected by Kepler have Earth-sized planets? Answer: 102/739 = 13.8 %

Problem 3 - If the amount of solar energy falling on Earth to keep it warm is defined as 1.0, for Venus it is 2.0 and for Mars it is 0.4. From the 851 planets detected in orbit around the 340 stars, there were 18 planets orbiting 16 stars found to be in the solar energy 'habitable zone' from 0.4 to 2.0 so that liquid water could exist on their surfaces. What is the probability that, given a multi-planet system you will find an Earth-sized planet in its Habitable Zone? Answer: **There are 340 multiplanet systems, so for 16 stars having such a planet, the probability is 16/340 = 0.047 or 4.7%. Alternatively, for 851 planets in multiplanet systems we have 18/851 = 0.021 or 2.1%.**

Problem 4 - The Milky Way is estimated to have about 1 trillion stars from red dwarfs to brilliant super giants. From various surveys, about 20% are similar to our sun. From the Kepler survey data, A) how many multi-planet systems would you expect to find if the Kepler survey could be conducted for all the stars in the Milky Way? B) How many Earth-sized planets in multi-planet systems would you expect to find? C) How many Earth-like planets in their Habitable Zones would you expect to find?

Answer: A) 1 trillion x 0.2 x (340 multiplanets /190,751 stars searched) = **357 million**.
B) 357 million x (102/739) = **49 million**.
C) 49 million x (16 / 340) = **2.3 million Earth-sized planets in their HZ in the Milky Way.**

Note: The Kepler survey only sees planetary systems if their orbit planes are along the line of sight to the star and Earth. Statistically this only happens about 5% of the time, so we can multiply the Kepler results by 20 to account for planetary systems we miss using this technique. That means for Answer C, there could be nearly **50 million planets like Earth**. Also, in our solar system Neptune and Jupiter-sized planets can have moons the size of Earth, so that means ANY planet found in the HZ could be potentially habitable if we include its moons, so we eliminate the step in Answer B and so have 357 million x (16/340) x 20 = **336 million possible Earth-sized planets (or moons) in the Habitable Zone!**

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