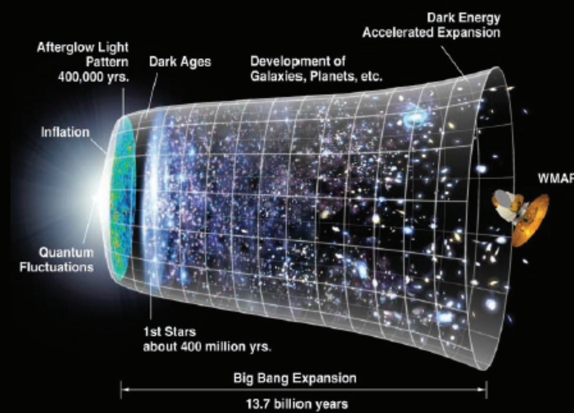
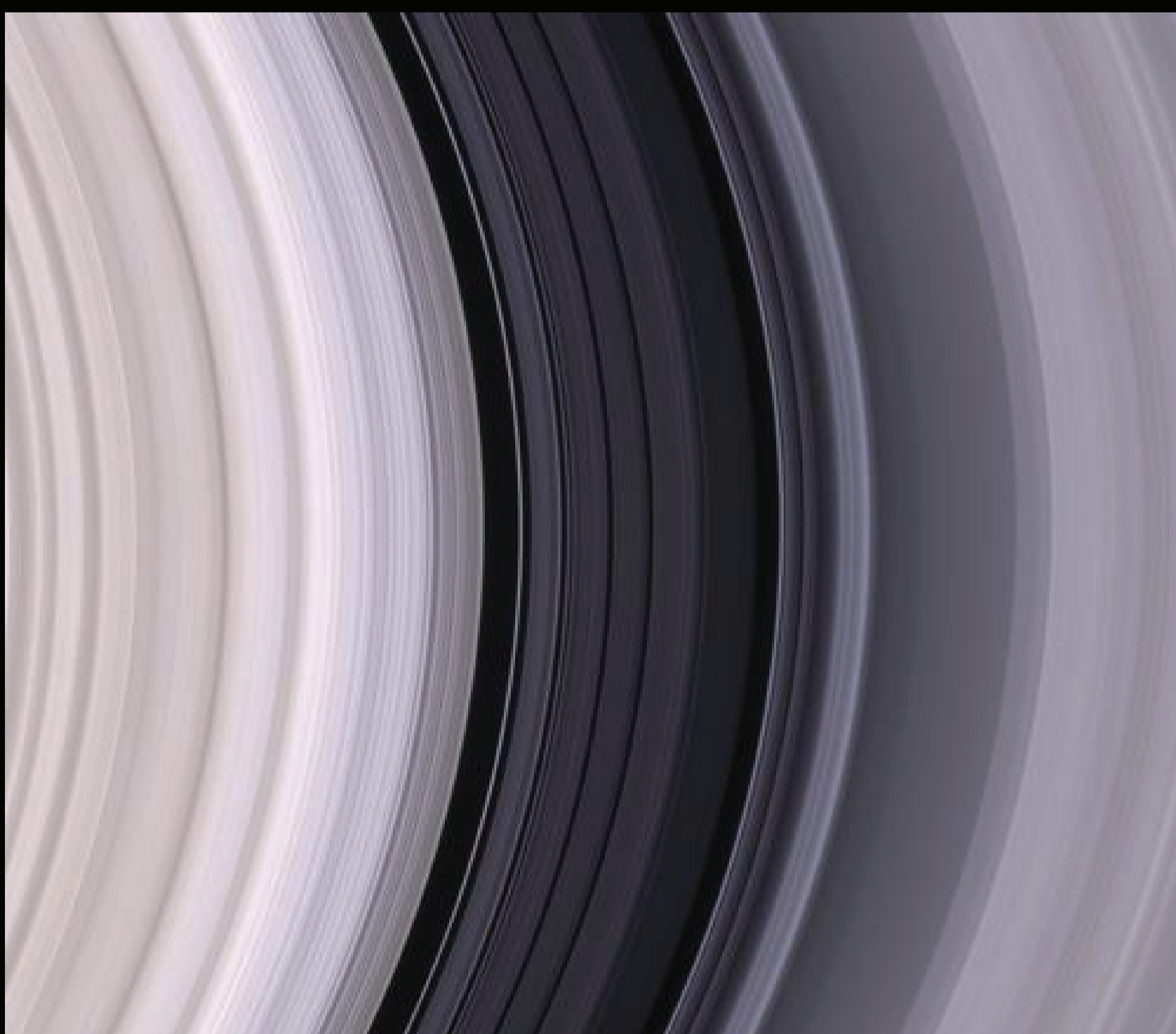
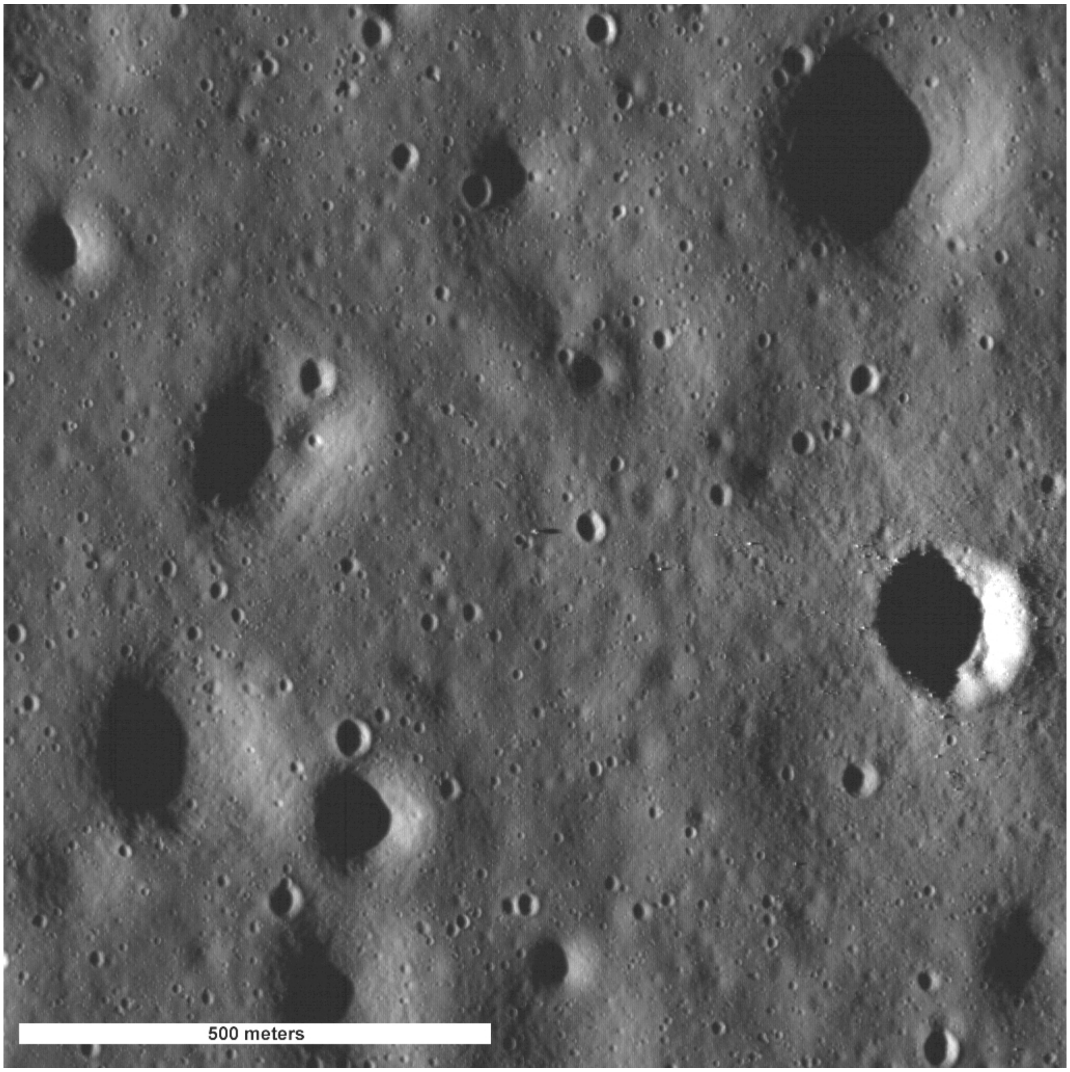
National Aeronautics and Space Administration





Space Math V

Space Math V



The Apollo -11 Lander is revealed by its shadow near the center of this image taken by the Lunar Reconnaissance Orbiter in July, 2009. Use a millimeter ruler to determine the scale of the image, and the sizes and distances of various features!

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2008-2009 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 9 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

To suggest math problem or science topic ideas, contact the Author, Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

Front and back cover credits: Saturn's Rings (Cassini NASA/ESA); Evolution of the Universe (NASA/WMAP); Abell-38 planetary nebula (Courtesy Jakoby, KPNO), Space Shuttle Launch (NASA)

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Note: An extensive, updated, and cumulative matrix of problem numbers, math topics and grade levels is available at

http://spacemath.gsfc.nasa.gov/matrix.xls

Alignment with Standards (AAAS Project:2061 Benchmarks).

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1

(6-8) Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1

(9-12) - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

Mathematics Topic Matrix

Topic	Problem Numbers																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry													X		X	X									X			X			
Technology, rulers																															
Numbers, patterns, percentages				X	X		X			X	X	X		X				X										X			
Averages																															
Time, distance, speed			X						X							X										X					
Areas and volumes													X					X		X	X										
Scale drawings													X										X	X	X	X	X		X	X	
Geometry																															
Probability, odds													X	X	X			X				X									
Scientific Notation																			X	X	X	X						X			
Unit Conversions	X		X				X															X						X			
Fractions						X																		X	X		X				
Graph or Table Analysis	X	X															X														
Solving for X				X	X		X	X											X												
Evaluating Fns																			X		X										X
Modeling	X																														X
Trigonometry																															
Integration																															X
Differentiation																															X
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
Inquiry	X										X																				
Technology, rulers																															
Numbers, patterns, percentages												X							X			X			X						
Averages																															
Time, distance, speed				X						X																					X
Areas and volumes															X	X						X									
Scale drawings																	X														X
Geometry		X	X													X							X								
Probability, odds																															
Scientific Notation											X		X		X	X			X				X			X	X	X			
Unit Conversions											X								X							X					
Fractions																						X									
Graph or Table Analysis				X																				X	X						X
Solving for X									X															X							
Evaluating Fns	X				X	X	X		X				X		X			X					X	X			X		X	X	
Modeling	X	X	X						X		X				X					X					X						
Trigonometry																				X											
Integration	X	X	X				X									X															
Differentiation	X		X		X	X		X	X				X	X															X		
Limits																					X										

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87					
Inquiry								X																						
Technology, rulers																														
Numbers, patterns, percentages	X						X	X	X	X																				
Averages	X																													
Time, distance, speed					X						X																			
Areas and volumes													X												X					
Scale drawings					X			X																	X					
Geometry					X																									
Probability, odds																														
Scientific Notation		X	X			X												X		X										
Unit Conversions				X							X							X			X									
Fractions									X					X								X	X	X						
Graph or Table Analysis											X	X	X																	
Solving for X															X	X	X		X											
Evaluating Fns						X									X					X										
Modeling			X																											
Trigonometry																														
Integration																														
Differentiation																														
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How to use this book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for all of the Space Math applications.

This book is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Space Math V**. Read the scenario that follows:

Ms. Green decided to pose a new activity using Space Math for her students. She challenged each student team with math problems from the Space Math V book. She copied each problem for student teams to work on. She decided to have the students develop a factious space craft. Each team was to develop a set of criteria that included reasons for the research, timeline and budget. The student teams had to present their findings and compete for the necessary funding for their space craft. The students were to use the facts available in the Space Math V book and images taken from the Space Weather Media Viewer, http://sunearth.gsfc.nasa.gov/spaceweather/FlexApp/bin-debug/index.html#

Space Math V can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

Teacher Comments

"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that…" (Associate Professor of Physics)

"Space Math has more up-to-date applications than are found in any textbook. Students enjoy real-world math problems for the math they have already learned. Doing Space Math problems has encouraged some of my students to take pre-calculus and calculus so they can solve the more advanced problems. I learned about Space Math through an email last year. I was very impressed with the problems. I assigned some of the problems to students in my Physics classes, printing them out to put in their interactive notebooks. I displayed other problems for group discussion, assigned some for homework and used some for group class work. I like the diversity, the color format and having the solutions. I expect to use them even more next year in our new space science class. We will have 50 students in two sections." (Alan. High School Science Teacher)

"It took time for them to make the connection between the math they learned in math class and applying it in the science classroom. Now I use an ELMO to project them. I have used them for class work and/or homework. The math activities were in conjunction with labs and science concepts that were being presented. The math helped "show" the science. Oftentimes students were encouraged to help and teach each other. Students began to see how math and science were connected. I knew the students were making the connections because they would comment about how much math they had to do in science. Their confidence in both classes increased as they were able practice the concepts they learned in math in my science class." (Brenda, Technology Resource Teacher)

The Big Bang - Hubble's Law

Galaxy	Distance (mpc)	Speed (km/s)
NGC-5357	0.45	200
NGC-3627	0.9	650
NGC-5236	0.9	500
NGC-4151	1.7	960
NGC-4472	2.0	850
NGC-4486	2.0	800
NGC-4649	2.0	1090

In 1921, Astronomer Edwin Hubble was measuring the speeds of nearby galaxies when he noticed a puzzling thing. When he plotted the speed of the galaxy against its distance, the points from each of the galaxies in his sample seemed to follow an increasing 'straight' line.

This turned out to be the first important clue that the universe was expanding. Each galaxy was moving away from its neighbor. The farther away the galaxy was from the Milky Way, the faster it was moving away from us.

The table shows the distance and speed of 7 galaxies. The distances are given in megaparsecs (mpc). One megaparsec equals 3.26 million light years. The speed is given in kilometers per second. Note, the speed of light is 300,000 kilometers/sec.

Problem 1 - Create a graph that presents the distance to each galaxy in mpc on the horizontal axis, and the speed in kilometers/sec on the vertical axis.

Problem 2 - What is the range of distances to the galaxies in this sample in light years?

Problem 3 - Does the data show that the distances and speeds of the galaxies are correlated, anti-correlated or uncorrelated (random)?

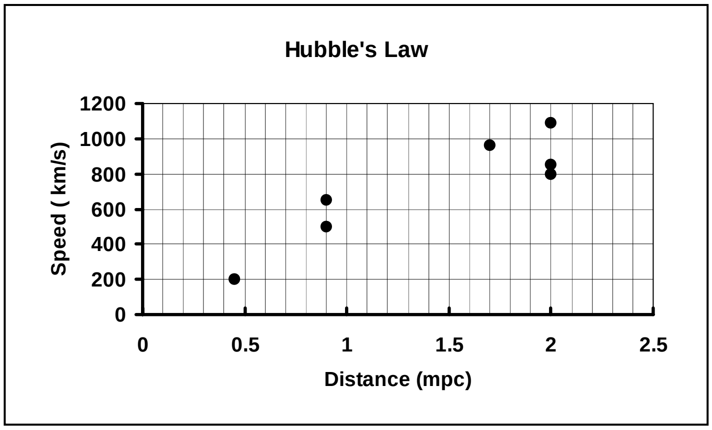
Problem 4 - By using a calculator, or using an Excel Spreadsheet, plot the data and use the 'Tools' to determine a best-fit linear regression. Alternatively, you may use the graph you created in Problem 1 to draw a best-fit line through the data points.

Problem 5 - The slope of the line in this plot is called Hubble's Constant. What is your estimate for Hubble's Constant from the data you used?

Problem 6 - An astronomer measures the speed of a galaxy as 2500 kilometers/sec. What would its distance be using your linear regression (now called Hubble's Law)?

Answer Key:

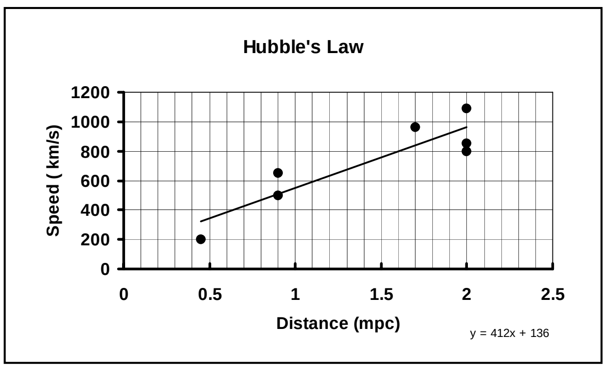
Problem 1 - Plot looks like this:



Problem 2 - The range is from 0.45 to 2 megaparsecs. Since 1 megaparses = 3.26 light years, the range is 450,000 parsecs x 3.26 light years/parsec = 1,467,000 light years to 2,000,000 parsecs x 3.26 light years/parsec = 6,520,000 light years.

Problem 3 - Because the data points show an increasing speed with increasing distance, the data indicate a correlated relationship between these two quantities.

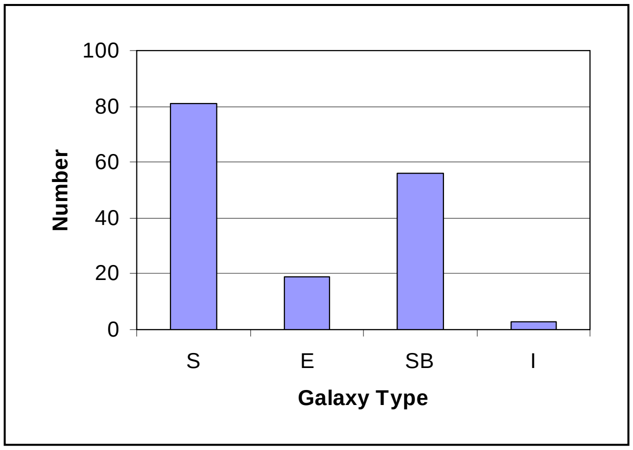
Problem 4 - The result of using Excel 'Trendlines' is below:

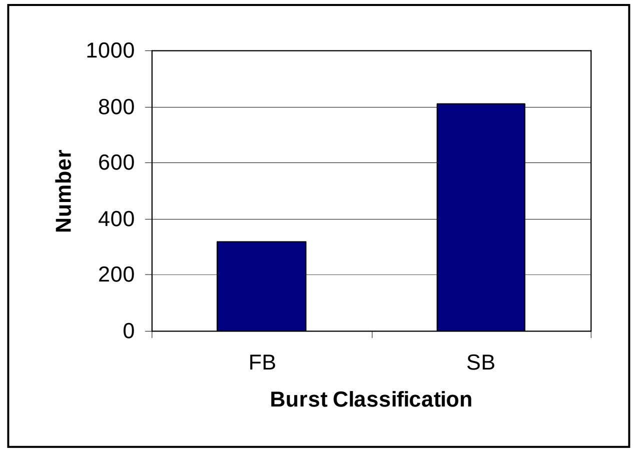


Problem 5 - The regression line has a slope of '412' and the units will be kilometers/sec per megaparsec.

Problem 6 - The regression equation is Speed (km/s) = 412 x distance(mpc) + 136 so solving for distance you get Distance = (Speed - 136)/412 and so for Speed = 2500 km/s the distance is (2500-136)/412 = 5.74 megaparsecs (or 18,700,000 light years).

Cosmic Bar Graphs





Problem1 – Astronomers have classified the 160 largest galaxies in the Virgo Cluster according to whether they are spiral-shaped (S and SB), elliptical-shaped (E) or irregular (I). The bar graph to the left shows the number in each category. From the survey, 81 were classed as S, 19 were classed as E, 56 were classed as SB and 3 were classed as I. <u>About what fraction of galaxies in the cluster are spirals?</u>

Problem 2 – Gamma-ray bursts happen about once each day. The bar graph to the right sorts the 1132 bursts detected between 1991-1996 into two categories. There are 320 FBs and 812 SBs indicated in the bar grap. Slow Bursts (SB) are longer than 2 seconds, and may be produced by supernovas in distant galaxies. Fast Bursts (FB) lasting less than 2 seconds may be produced by colliding neutron stars inside our own Milky Way galaxy. <u>What would you predict for 2009 as the number of bursts that might probably come from outside the Milky Way?</u>

Answer Key

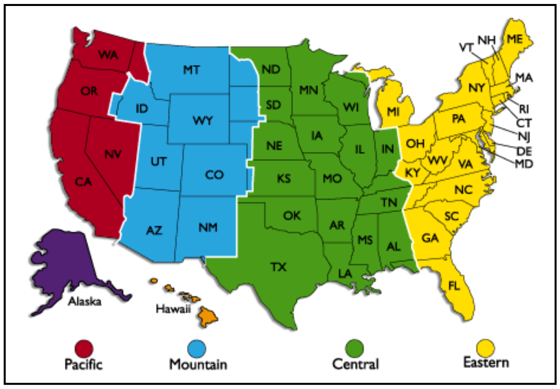
Problem 1 - Astronomers have classified the 160 largest galaxies in the Virgo Cluster according to whether they are spiral-shaped (S and SB), elliptical-shaped (E) or irregular (I). The bar graph to the left shows the number in each category. From the survey, 81 were classed as S, 19 were classed as E, 56 were classed as SB and 3 were classed as I. About what fraction of galaxies in the cluster are spirals?

Answer: There are two categories of spiral galaxies indicated by S and SB for a total of 56 + 81 = 137 galaxies. Since there are 160 total galaxies, the fraction of spirals is **137/160 = 0.86,** or equivalently 86%.

Problem 2 – Gamma-ray bursts happen about once each day. The bar graph sorts the 1132 bursts detected between 1991-1996 into two categories. There are 320 FBs and 812 SBs indicated in the bar grap. Slow Bursts (SB) are longer than 2 seconds, and may be produced by supernovas in distant galaxies. Fast Bursts (FB) lasting less than 2 seconds may be produced by colliding neutron stars inside our own Milky Way galaxy. What would you predict for 2009 as the number of bursts that might probably come from outside the Milky Way?

Answer: There are 320 FBs and 812 SBs indicated in the bar graph. The total number is 1132 over the course of 1996-1991 = 5 years, so that there are 1132/5 = 226 each year. The ones that probably come from outside the Milky Way are the SBs for which the fraction that were seen between 1991-1996 is 812/1132 = 0.72. For a single year, 2009, we would predict about 226 bursts, of which 226 x 0.72 = **163 would be SBs.**

Time Zone Math



Earth is a BIG place! In fact, it is so big that different countries see sunrise and sunset happen at very different times during the day.

If you were living in Germany, you would see sunrise at 6:00 AM, but at that same moment it would be the middle of the night in California!

(Image courtesy http://gis.nwcg.gov/giss_2006/cd_contents.html)

If you have ever gone on a long car or plane ride to the east or west, you will often hear people complain that they have 'gained' or 'lost' hours due to Time Zone change. Here's how it works.

When you travel east, the Sun rises higher and higher in the sky. It is as though you are seeing the Sun as it would be at a later time in the day. When you travel west, the Sun gets lower and lower in the sky. It is as though you are seeing the Sun as it would be at an earlier time in the day.

We can make this more precise by saying that as you travel East you will gain time, and as you travel west you will lose time. The exact amount depends on how many Time Zones you travel through. The figure above shows the Time Zones across North America. During the winter, these Time Zones are called Eastern Standard Time (EST), Central Standard Time (CST), Mountain Standard Time (MST) and Pacific Standard Time (PST).

For example, when you travel westwards, your clock will 'lose' one hour for each Time Zone you pass through. If your watch says 12:00 Noon and you are in New York, which is in the EST Time Zone, you need to set your watch back one hour to 11:00 AM if you are traveling to Chicago in the CST Zone, two hours to 10:00 AM if you are traveling to Denver in the MST Zone, and three hours to 9:00 AM if you are raveling to San Francisco the PST Zone.

1 – A solar astronomer wants to study a flare erupting on the Sun at 12:00 PM (High Noon) at the solar observatory in Denver while taking to his colleague in New York at the same time. At what time should his colleague be ready for the phone call?

2 – A second solar astronomer in Paris, France also wants to participate in this research. If the Paris Time Zone is 4 hours ahead of EST, what time should the Paris astronomer be ready for the same call?

3 – An astronomer sees a solar flare at 2:15 PM EST. A astronomer in Hawaii decides to go out for breakfast between 8:00 and 8:30 AM HST. If Hawaii Standard Time (HST) is 3 hours earlier than the PST Zone, did the Hawaiian astronomer get to see the flare?

Answer Key

1 – A solar astronomer wants to study a flare erupting on the Sun at 12:00 PM (High Noon) at the solar observatory in Denver while taking to his colleague in New York at the same time. At what time should his colleague be ready for the phone call?

Answer: Denver is in the MST Zone, which is 2 hours earlier than the EST. SO, the New York astronomer needs to be ready at 12:00 + 2:00 = 14:00 which is 2:00 PM EST.

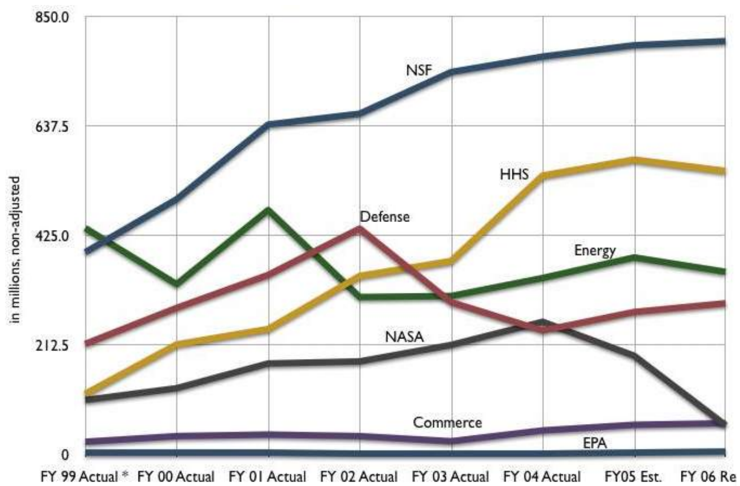
2 – A second solar astronomer in Paris, France also wants to participate in this research. If the Paris Time Zone is 4 hours ahead of EST, what time should the Paris astronomer be ready for the same call?

Answer: Paris is 4 hours ahead of EST, so you need to add 4 hours to 12:00 PM Noon EST to get 16:00 hours which is the same as 4:00 PM Paris Time.

3 – An astronomer sees a solar flare at 2:15 PM EST. A astronomer in Hawaii decides to go out for breakfast between 8:00 and 8:30 AM HST. If Hawaii Standard Time (HST) is 3 hours earlier than the PST Zone, did the Hawaiian astronomer get to see the flare?

Answer: The solar flare occurred at 2:15 PM EST. We first convert this to PST by subtracting 3 hours for the time zone change to get 11:15 AM PST. Then, continuing westwards, we subtract another 3 hours to get to the Hawaiian Time Zone, making the time 11:15 AM – 3:00 = 8:15 AM Hawaiian Time. The Hawaiian astronomer missed the flare because he was still having breakfast and not at the observatory.

The Dollars and Cents of Research



Scientific research costs money! You have to find money to pay your salary to work 40-hours a week. You also have to get money to pay for the trips you take to observatories, scientific conferences, to pay for your healthcare, retirement plans, and the rental for your office and laboratory space. Here are some problems that help you see how this all works!

Problem 1 – Professor Quark is a senior astronomer at PDQ University who makes $50.00 an hour. If there are 2,000 work hours in a full year, what will be the astronomer's gross pay before taxes and other deductions, for the year?

Problem 2 – PDQ University charges each astronomer an additional 50% of the astronomer's salary to cover the rental of office space, medical benefits, and retirement benefits. How much extra money in addition to his salary will Prof. Quark have to pay PDQ University to conduct his research?

Problem 3 - Prof. Quark needs to buy a new computer 'work station' to conduct his research. He can choose System A for $5,000, which has 200 gigabytes of hard drive space, and operates at 5 megahertz, or he can buy a cheaper System B for $1,000, which has a 1,000 gigabyte hard drive and runs five times slower at 1 megahertz. Which system do you think he should buy? Why?

Problem 4 - Prof. Quark expects to take several trips each year to conferences in France, China and Canada. These trips will cost a total of $9,000. The university adds on an additional cost to this expense to cover their setting up the travel arrangements and handling all of the accounting. They charge the researcher 30% of the researcher's cost to do this. What is the total cost to the researcher for the travel expenses?

Problem 5 – Prof. Quark plans to support his research by applying to the Long-Term Space Astrophysics research grant program at NASA. What is the total amount of money he has to apply for if he wants to get fully supported for one year?

Problem 6 – PDQ University will support Prof. Quark for ¾ of his labor in Problem 2 if he agrees to teach college courses and train graduate students. He will have to get external support for the balance of his research support. How much money will he have to ask NASA for in a grant proposal to support all of his research for one year?

Answer Key

Problem 1 – Professor Quark is a senior astronomer at PDQ University who makes $50.00 an hour. If there are 2,000 work hours in a full year, what will be the astronomer's gross pay before taxes and other deductions, for the year?
Answer: 2,000 hours x $50.00/hour = **$100,000**

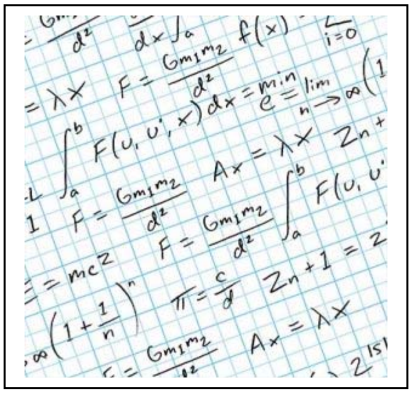
Problem 2 – PDQ University charges each astronomer an additional 50% of the astronomer's salary to cover the rental of office space, medical benefits, and retirement benefits. How much extra money in addition to his salary will Prof. Quark have to pay PDQ University to conduct his research?
Answer: $100,000 x 50%/100% = $50,000

Problem 3 - Prof. Quark needs to buy a new computer 'work station' to conduct his research. He can choose System A for $5,000, which has 200 gigabytes of hard drive space, and operates at 5 megaHertz, or he can buy a cheaper System B for $1,000, which has a 1,000 gigabyte hard drive and runs five times slower at 1 megahertz. Which system do you think he should buy? Why?
Answer: He should buy the faster, but more expensive, computer. His time is worth $50.00 an hour, so if he has to wait around for a slower computer to process data, over time, he will be wasting money.

Problem 4 - Prof. Quark expects to take several trips each year to conferences in France, China and Canada. These trips will cost a total of $9,000. The university adds on an additional cost to this expense to cover their setting up the travel arrangements and handling all of the accounting. They charge the researcher 30% of the researcher's cost to do this. What is the total cost to the researcher for the travel expenses?
Answer: $9,000 x 30%/100% = $2,700. His total Travel Expenses will be $9,000 + $2,700 = $11,700.

Problem 5 – Prof. Quark plans to support his research by applying to the Long-Term Space Astrophysics research grant program at NASA. What is the total amount of money he has to apply for if he wants to get fully supported for one year?
Answer: From the answers to the first four problems, his total budget will be $100,000 + $50,000 + $5,000 + $11,700 or $166,700.

Problem 6 – PDQ University will support Prof. Quark for ¾ of his labor in Problem 2 if he agrees to teach college courses and train graduate students. He will have to get external support for the balance of his research support. How much money will he have to ask NASA for in a grant proposal to support all of his research for one year?
Answer: ¾ x $150,000 = $100,000 then $166,700 - $100,000 so the amount he will need from the grant is **$66,700.**



Scientific research has a lot in common with solving number puzzles like SODUKO in order to figure out what stars or planets are doing in space. Use your puzzle-solving ability to figure out what event is described by the following number sentences!

1 - Which story matches the sentence 23 – 10 + 6 = 19 ?

A) An astronomer discovers 23 quasars on one photograph, 10 quasars on a second photograph, and 6 additional quasars on a third photograph. How many quasars did she identify?

B) An astronomer spots 23 solar flares on Tuesday and 6 solar flares on Thursday, then decides that 10 of the solar flares were not real. How many real flares did he see?

C) An astronomer counts a total of 19 lunar craters, and classifies 23 of them as asteroid impacts, 6 of them as volcanic calderas and 10 of them as meteor impacts.

2 - Which story matches the sentence 145 + N = 375 ?

A) Two astronomers combined their databases of planets. They observe a total of 375. If one astronomer contributed 145 planets, how many did the second astronomer contribute?

B) The temperature of an asteroid's interior changes by 375 degrees between the center and the surface. If the surface temperature is 145 degrees Centigrade, what is the interior temperature of the asteroid?

C) The width of Saturn's rings is 375 megameters. If the ring system starts at a distance of 145 megameters from Saturn's outer atmosphere, what is the distance to the outer edge of the ring system?

3 - Two astronomers combined their catalogs of cosmic gamma-ray bursts. There were 287 and 598 cataloged by each astronomer with 65 events in common. How many unique events are in the combined catalog?

A) (287 - 65) + (598 - 65) = M
B) 287 + (598 – 65) = M
C) 287 + 598 = M
D) (287 + 65) + (598 + 65) = M

Answer Key

1 - Which story matches the sentence 23 – 10 + 6 = 19 ?

A) An astronomer discovers 23 quasars on one photograph, 10 quasars on a second photograph, and 6 additional quasars on a third photograph. How many quasars did she identify? **Answer: No. This would be the sentence 23 + 10 + 6 = N**

B) An astronomer spots 23 solar flares on Tuesday and 6 solar flares on Thursday, then decides that 10 of the solar flares were not real. How many real flares did he see? **Answer: Yes this is correct.**

C) An astronomer counts a total of 19 craters, and classifies 23 of them as asteroid impacts, 6 of them as volcanic calderas and 10 of them as meteor impacts. **Answer: No. This is the sentence 23 + 6 + 10 = 39.**

2 - Which story matches the sentence 145 + N = 375 ?

A) Two astronomers combined their databases of planets. They observe a total of 375. If one astronomer contributed 145 planets, how many did the second astronomer contribute? **Yes. This is correct.**

B) The temperature of an asteroid's interior changes by 375 degrees between the center and the surface. If the surface temperature is 145 degrees Centigrade, what is the core temperature of the asteroid? **Answer: No. This is the sentence 145 + 375 = N**

C) The width of Saturn's rings is 375 megameters. If the ring system starts at a distance of 145 megameters from Saturn's outer atmosphere, what is the distance to the outer edge of the ring system**? Answer: No. This is the sentence 145 + 375 = N**

3 - Two astronomers combined their catalogs of cosmic gamma-ray bursts. There were 287 and 598 cataloged by each astronomer with 65 events in common. How many unique events are in the combined catalog?

A) (287 - 65) + (598 - 65) = M **No. This eliminates the common bursts from the final catalog.**

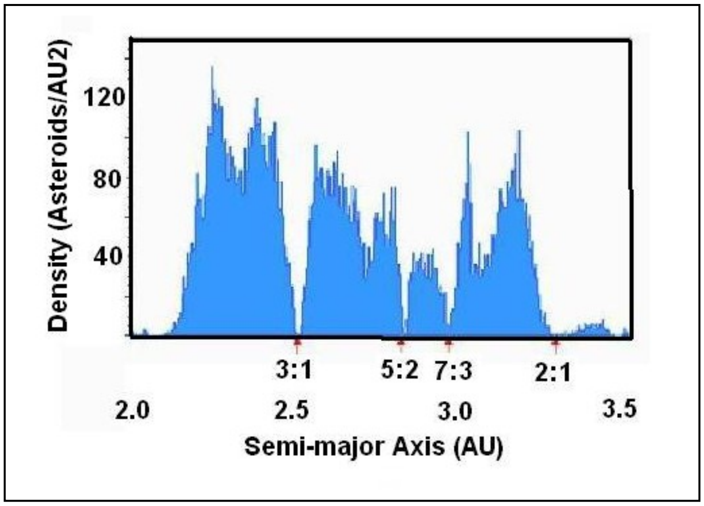
B) 287 + (598 – 65) = M **Yes. This is correct.**

C) 287 + 598 = M - **No. This is just the sum of the two catalogs including duplications.**

D) (287 + 65) + (598 + 65) = **M** **No. The duplications are added to each not subtracted.**

Answer B is correct. Among the 287 + 598 gamma ray bursts in the two catalogs, there are 65 that are in common. Starting with the full catalog provided by one astronomer (287), we add only the non-duplicated bursts in the second astronomers catalog (598-65).

Fractions in Space



Simple fractions come up in astronomy in many ways. One common way is shown to the left. The asteroids in the Asteroid Belt have gaps where their orbit period is a simple fraction (3:1, 5/2, or 2:1) of Jupiter's orbit period.

Here are a few other 'far out' examples!

1 – The satellites of Jupiter, Ganymede and Europa, orbit the planet in 7.1 day and 3.5 days. About what is the ratio of the orbit period or Europa to Ganymede expressed as a simple fraction involving the numbers 1, 2, 3 or 4 in the numerator or denominator?

2 – The planet Pluto orbits the Sun in 248 years while Neptune takes 164 years. What is the simplest fraction involving the numbers 1, 2, 3 or 4 in the numerator and denominator that approximates the ratio of Neptune's period to Pluto's?

3 – Draw two concentric circles labeling the outer circle Earth and the inner circle Venus. Place a point on each circle at the 12 o'clock position, representing the two planets when they are closest to each other. Venus and Earth are opposite each other in their orbits every 3/2 of an Earth year. During this time, Venus travels 6/5 of its orbit around the Sun. Where will the planets be after 5 opposition periods, and rounded to the nearest integer year, how many Earth years will this take?

Answer Key

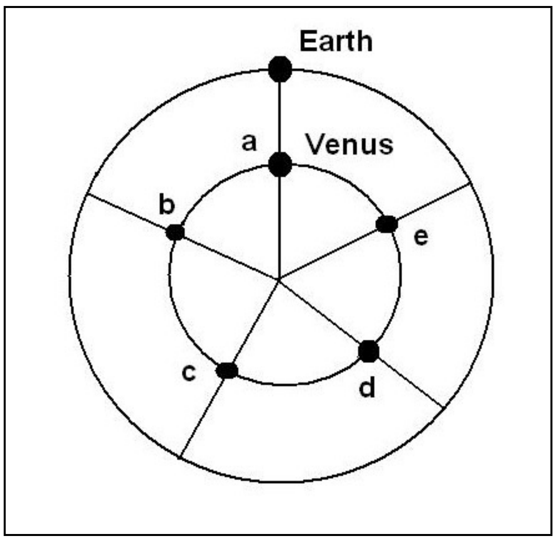
1 – The satellites of Jupiter, Ganymede and Europa, orbit the planet in 7.1 day and 3.5 days. About what is the ratio of the orbit period or Europa to Ganymede expressed as a simple fraction involving the numbers 1, 2, 3 or 4 in the numerator or denominator?

Answer: 3.5 / 7.1 = 0.49 which is close to 0.5, so the nearest simple fraction is **1/2.**

2 – The planet Pluto orbits the sun in 248 years while Neptune takes 60 years. What is the simplest fraction involving the numbers 1, 2, 3 or 4 in the numerator and denominator that approximates the ratio of Neptune's period to Pluto's?

Answer: 164 years / 248 years = 0.66. **The closest ratio is 2/3.** This means that in the time it takes Pluto to orbit the sun twice, Neptune orbits almost exactly three times.

3 – Draw two concentric circles labeling the outer circle Earth and the inner circle Venus. Place a point on each circle at the 12 o'clock position, representing the two planets closest to each other. Venus and Earth are opposite each other in their orbits every 3/2 of an earth year. During this time, Venus travels 6/5 of its orbit around the Sun. Where will the planets be after 5 opposition periods, and rounded to the nearest integer year, how many Earth years will this take?



Answer: The 5 opposition periods labeled a, b, c, d, and e in the diagram, form the sequence in earth years of 3/2, 6/2, 9/2, 12/2, 15/2. The decimal values are 1.5, 3.0, 4.5, 4.0 and 7.5 years. During this time, Venus has moved 6/5, 12/5, 18/5, 24/5 and 30/5 of its orbit. Eliminating full orbits, Venus has moved an extra b= 1/5, c= 2/5, d= 3/5. e= 4/5 and a= 5/5 and returned to its original position, a, after the 5th opposition. This happened 7.5 years after the opposition sequence started, which rounds up to 8 years later. Every 8 years, Earth and Venus will return to their start positions at the 12 o'clock position in the diagram.

Equations with one variable I

$$T_F = 9/5\ T_C + 32$$

Calculations involving a single variable come up in many different ways in astronomy, like the popular one to the left for converting centigrade degrees (Tc) into Fahrenheit degrees (Tf). Here are some more examples.

Problem 1 – To make the data easier to analyze, an image is shifted by X pixels to the right from a stating location of 326. Find the value of X if the new location is 1436 by solving 326 + X = 1436.

Problem 2 – The temperature, T, of a sunspot is 2,000 C degrees cooler than the Sun's surface. If the surface temperature is 6,100 C, solve the equation for the sunspot temperature if T + 2,000 = 6,100.

Problem 3 – The radius, R (in kilometers) of a black hole is given by the formula R = 2.9 M, where M is the mass of the black hole in multiples of the Sun's mass. If an astronomer detects a black hole with a radius of 18.5 kilometers, solve the equation 18.5 = 2.9M for M to find the black hole's mass.

Problem 4 – The sunspot cycle lasts 11 years. If the peak of the cycle occurred in 1858, and 2001 solve the equation 2001 = 1858 + 11X to find the number of cycles, X, that have elapsed between the two years.

Answer Key

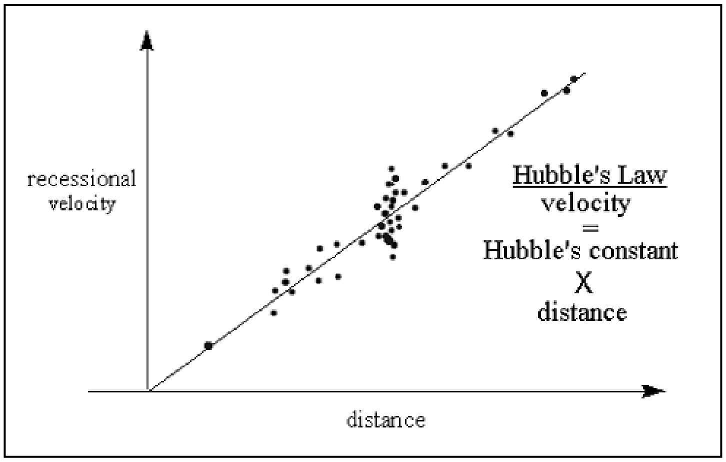
1 – To make the data easier to analyze, an image is shifted by X pixels to the right from a stating location of 326. Find the value of X if the new location is 1436 by solving 326 + X = 1436.
Answer: X = 1438 – 326 so X = 1112.

2 – The temperature, T, of a sunspot is 2,000 C degrees cooler than the Sun's surface. If the surface temperature is 6,100 C, solve the equation for the sunspot temperature if T + 2,000 = 6,100.
Answer: T = 6,100 – 2,000 so T = 4,100 C.

3 – The radius, R (in kilometers) of a black hole is given by the formula R = 2.9 M, where M is the mass of the black hole in multiples of the Sun's mass. If an astronomer detects a black hole with a radius of 18.5 kilometers, solve the equation 18.5 = 2.9M for M to find the black hole's mass.
Answer: 18.5 = 2.9M so M = 8.5/2.9 so M = 6.4 times the mass of the sun.

4 – The sunspot cycle lasts 11 years. If the peak of the cycle occurred in 1858, and 2001 solve the equation 2001 = 1858 + 11X to find the number of cycles, X, that have elapsed between the two years.
Answer: 2001-1858 = 11X; 143 = 11X; X = 143/11 so X = 13

Equations with one variable II



Calculations involving a single variable come up in many different ways in astronomy. One way is through the relationship between a galaxy's speed and its distance, which is known as Hubbel's Law. Here are some more applications for you to solve!

Problem 1 – The blast wave from a solar storm traveled 150 million kilometers in 48 hours. Solve the equation 150,000,000 = 48 V to find the speed of the storm, V, in kilometers per hour.

Problem 2– A parsec equals 3.26 light years. Solve the equation 4.3 = 3.26D to find the distance to the star Alpha Centauri in parsecs, D, if its distance is 4.3 light years.

Problem 3 – Hubble's Law states that distant galaxies move away from the Milky Way, 75 kilometers/sec faster for every 1 million parsecs of distance. Solve the equation, V = 75 D to find the speed of the galaxy NGC 4261 located 41 million parsecs away

Problem 4 – Convert the temperature at the surface of the Sun, 9,900 degrees Fahrenheit to an equivalent temperature in Kelvin units, T, by using T = (F + 459) x 5/9

Problem 5 – The Andromeda Galaxy measures 3 degrees across on the sky as seen from Earth. At a distance of 2 million light years, solve for D, the diameter of this galaxy in light years: 57.3 = 6,000,000/D.

Answer Key

1 – The blast wave from a solar storm traveled 150 million kilometers in 48 hours. Solve the equation 150,000,000 = 48 V to find the speed of the storm, V, in kilometers per hour.
Answer: 150,000,000/48 = V so V = 3,125,000 kilometers/hour.

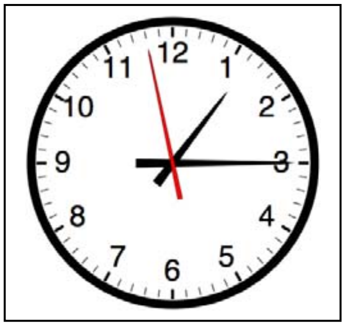
2 – A parsec equals 3.26 light years. Solve the equation 4.3 = 3.26D to find the distance to the star Alpha Centauri in parsecs, D, if its distance is 4.3 light years.
Answer: D = 4.3/3.26 = 1.3 parsecs.

3 – Hubble's Law states that distant galaxies move away from the Milky Way, 75 kilometers/sec faster for every 1 million parsecs of distance. V = 75 x D. Solve the equation to find the speed of the galaxy NGC 4261 located D = 41 million parsecs away
Answer: V = 75 x 41 so V = 3,075 kilometers/sec.

4 – Convert the temperature at the surface of the sun, 9,900 degrees Fahrenheit (F) to an equivalent temperature in Kelvin units, T, by using T = (F + 459) x 5/9
Answer: T = (F + 459) x 5/9 so T = (9,900 + 459) x 5/9 = 5,755 Kelvins

5 – The Andromeda Galaxy measures 3 degrees across on the sky as seen from Earth. At a distance of 2 million light years, solve for D, the diameter of this galaxy in light years: 57.3 = 6,000,000/D.
Answer: D = 6,000,000/57.3 so D = 104,700 light years in diameter.

Time Intervals



Astronomers are often interested in how long a particular event took. This can be used to explore how fast something is moving, or how rapidly it is changing in time.

Here are some 'far out' examples!

Problem 1 – The gamma-ray burst from the galaxy TXS1510-089 was detected on July 24, 2007 by the AGILE satellite. The burst began at 23:25:07 and ended at 23:25:57 How many seconds did it last?

Problem 2 – The black hole orbiting the star A0620-00 in the constellation Monocerous produced a micro-flare on September 29, 2002 visible at radio wavelengths as it swallowed some of the gas falling into it. If the flare began at 01:50:00 and ended at 01:53:20 how many seconds elapsed?

Problem 3 – On September 1, 1859 the Sun released a cloud of plasma called a Coronal Mass Ejection (CME) at 11:18 AM. If the CME reached Earth on September 2 at 04:54 AM how many hours did it take the cloud to travel from the Sun to Earth?

Problem 4 – Full Moon occurred on July 18, 2008 and August 16, 2008. How many days elapsed between the two lunar phases?

Problem 5 – A massive star in the Eta Carina cluster erupted in a giant flare in 1843. How many years has it been since this eruption if the current year is 2008?

Problem 6 – The Crab Nebula was formed by a supernova in the year 1054 AD. If the next supernova spotted by humans occurred in 1987, how many years elapsed between these events?

Problem 7 - The Earth was formed 4.5 billion years ago. The asteroid bombardment of its surface ended 3.9 billion years ago. How many millions of years did the asteroid bombardment era last?

Problem 8 - The universe came into existence in the Big Bang 13.7 billion years ago. The most ancient galaxy astronomers have detected so far is A1689-zD1, which was probably formed about 13.0 billion years ago. How many millions of years was the universe in existence before this galaxy began to form?

Answer Key

1 – Answer: 23:25:57 – 23:25:07 = **50 seconds**

2 – Answer: 01:53:20
- 01:50:00

3:20

The burst lasted 3 minutes and 20 seconds.

3 – Answer: September 2 04:54 add 24h 28:54
- September 1 11:18 - 11:18
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17:36

The CME took 17 hours and 36 minutes to arrive.

4 – Answer: August 16, 2008
- July 18, 2008

You can use a calendar to count the days, or from 31 days in July, there are 31-18 = 13 days plus the 16 days in August the sum is 13+16 = **29 days.**

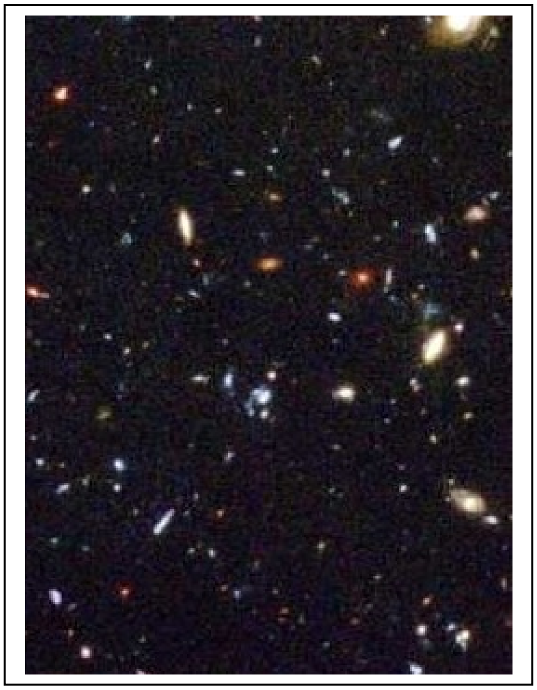
5 – Answer: 2008 – 1843 = **165 years.**

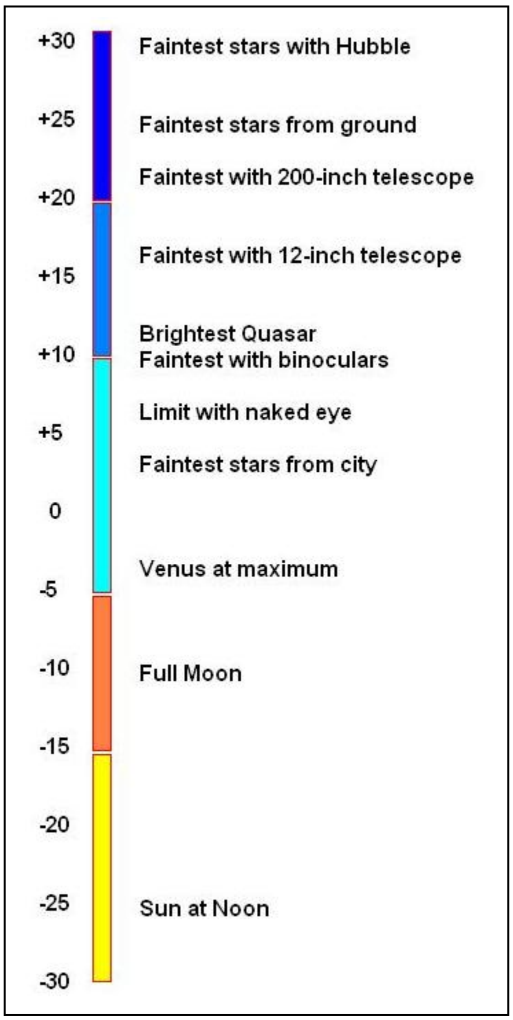
6 – Answer: 1987 – 1054 = **933 years.**

7 – Answer: 4.5 billion – 3.9 billion = 0.6 billion years or **600 million years.**

8 – Answer: 13.7 billion – 13.0 billion = 0.7 billion years or **700 million years.**

The Stellar Magnitude Scale





Astronomers measure the brightness of a star in the sky using a magnitude scale. On this scale, the brightest objects have the SMALLEST number and the faintest objects have the LARGEST numbers. It's a 'backwards' scale that astronomers inherited from the ancient Greek astronomer Hipparchus.

The image to the left taken by the Hubble Space Telescope shows hundreds f faint galaxies beyond the Milky Way. The faintest are of magnitude +25.0.

1 – At its brightest, the planet Venus has a magnitude of -4.6. The faintest star you can see with your eye has a magnitude of +7.2. How much brighter is Venus than the faintest visible star?

2 – The full moon has a magnitude of -12.6 while the brightness of the Sun is about -26.7. How many magnitudes fainter is the moon than the Sun?

3 – The faintest stars seen by astronomers with the Hubble Space Telescope are about +30.0. How much fainter are these stars than the Sun?

4 - Jupiter has a magnitude of –2.7 while its satellite, Callisto, has a magnitude of +5.7. How much fainter is the Callisto than Jupiter?

5 – Each step by 1 unit in magnitude equals a brightness change of 2.5 times. A star with a magnitude of +5.0 is 2.5 times fainter than a star with a magnitude of +4.0. Two stars that differ by 5.0 magnitudes are 100-times different in brightness. If Venus was observed to have a magnitude of +3.0 and the full moon had a magnitude of -12.0, how much brighter was the moon than Venus?

Answer Key

1 – At its brightest, the planet Venus has a magnitude of -4.6. The faintest star you can see with your eye has a magnitude of +7.2. How much brighter is Venus than the faintest visible star?

Answer: +7.2 – (-4.6) = +7.2 + 4.6 = **+11.8 magnitudes**

2 – The full moon has a magnitude of -12.6 while the brightness of the sun is about -26.7. How many magnitudes fainter is the moon than the sun?

Answer: -12.6 – (-26.7) = -12.6 + 26.7 = **+14.1 magnitudes fainter.**

3 – The faintest stars seen by astronomers with the Hubble Space Telescope is +30.0. How much fainter are these stars than the sun?

Answer: +30.0 – (-26.7) = +30.0 + 26.7 = **+56.7 magnitudes fainter.**

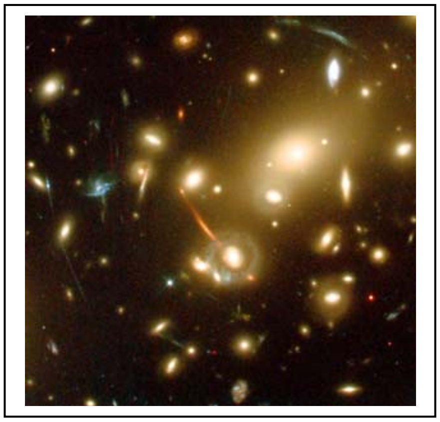
4 - Jupiter has a magnitude of –2.7 while its satellite, Callisto, has a magnitude of +5.7. How much fainter is the Callisto than Jupiter?

Answer: +5.7 – (-2.7) = +5.7 + 2.7 = **+8.4 magnitudes fainter than Jupiter.**

5 – Each step by 1 unit in magnitude equals a brightness change of 2.5 times. A star with a magnitude of +5.0 is 2.5 times fainter than a star with a magnitude of +4.0. Two stars that differ by 5.0 magnitudes are 100-times different in brightness. If Venus was observed to have a magnitude of +3.0 and the full moon had a magnitude of -12.0, how much brighter was the moon than Venus?

Answer: The magnitude difference between them is +15.0, since every 5 magnitudes is a factor of 100 fainter, +15.0 is equivalent to 100x100x100 = 1 million times, so the moon is **1 million times brighter** than Venus.

Groups, Clusters and Individuals



Astronomers study many different kinds of objects in space. Sometimes these objects contain many individuals that need to be tallied separately.

The cluster of galaxies called Abell-2218 is a rich collection of galaxies, as shown in the Hubble Space Telescope photo to the left.

1 – Astronomers observed the surface of the Sun on March 27, 2001 and counted 230 sunspots. If there were 10 sunspots in each group, about how many sunspot groups were on the Sun that day?

2 – Since 1950, astronomers have cataloged 35 individual galaxies within the group of galaxies containing the Milky Way. If this group is typical, and there are 200 galaxy groups within 100 million light years of the Milky Way, how many individual galaxies are present?

3 – The Milky Way galaxy has 158 satellite star clusters that orbit its in space. If each of these star clusters contains 100,000 stars, how many stars exist in these clusters?

4 – Astronomers have detected 430 planets orbiting 300 nearby stars. About how many planets orbit an average star in this sample?

5 – A cubic centimeter of gas in the Orion Nebula contains about 10 atoms of hydrogen and 4 atoms of helium. Hydrogen atoms contain one proton and one electron. Helium atoms contain 2 protons, 2 neutrons and 2 electrons. How many protons would you find in a single cubic centimeter of this gas? How many neutrons? How many electrons? What is the total number of protons, neutrons and electrons?

Answer Key

1 – Astronomers observed the surface of the sun on March 27, 2001 and counted 230 sunspots. If there were 10 sunspots in each group, about how many sunspot groups were on the sun that day?

Answer: 230 spots / 10 spots per group = **23 groups.**

2 – Since 1950, astronomers have cataloged 35 individual galaxies within the group of galaxies containing the Milky Way. If this group is typical, and there are 200 galaxy groups within 100 million light years of the Milky Way, how many individual galaxies are present?

Answer: 200 galaxy groups x 35 galaxies per group = **7000 galaxies**

3 – The Milky Way galaxy has 158 satellite star clusters that orbit its in space. If each of these star clusters contains 100,000 stars, how many stars exist in these clusters?

Answer: 158 clusters x 100,000 stars per cluster = **15,800,000 stars.**

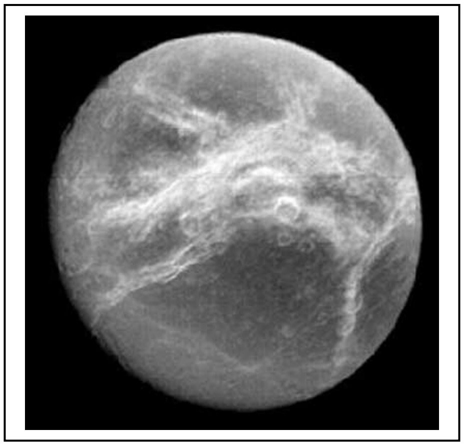
4 – Astronomers have detected 430 planets orbiting 300 nearby stars. About how many planets orbit an average star in this sample?

Answer: 430 planets / 300 stars = 1.4 planets, or about **1 planet per star.**

5 – A cubic centimeter of gas in the Orion Nebula contains about 10 atoms of hydrogen and 4 atoms of helium. Hydrogen atoms contain one proton and one electron. Helium atoms contain 2 protons, 2 neutrons and 2 electrons. How many protons would you find in a single cubic centimeter of this gas? How many neutrons? How many electrons? What is the total number of protons, neutrons and electrons?

Answer: 10 atoms hydrogen + 4 atoms helium
10(1 proton) + 4(2 protons) = **18 protons**
10(0 neutrons) + 4(2 neutrons) = **8 neutrons**
10(1 electron) + 4(2 electrons) = **18 electrons.**
----
44 particles

A Matter of Timing



Astronomers who study planets and their satellites often have to work out how often satellites or planets ‘line up’ in various ways, especially when they are closest together in space.

Figure shows the satellite Dione (Courtesy: NASA/Cassini)

Problem 1 – The two satellites of Tethys and Dione follow circular orbits around Jupiter. Tethys takes about 2 days for one complete orbit while Dione takes about 3 days. If the two satellites started out closest together on July 1, 2008, how many days later will they once again be at ‘opposition’ with one another?

A) Find the Least Common Factor between the orbit periods.

B) Draw two concentric circles and work the solution out graphically.

C) What is the relationship between your answer to A and B?

Problem 2 - Two planets have orbit periods of 3 years and 5 years. How long will it take them to return to the same locations that they started at?

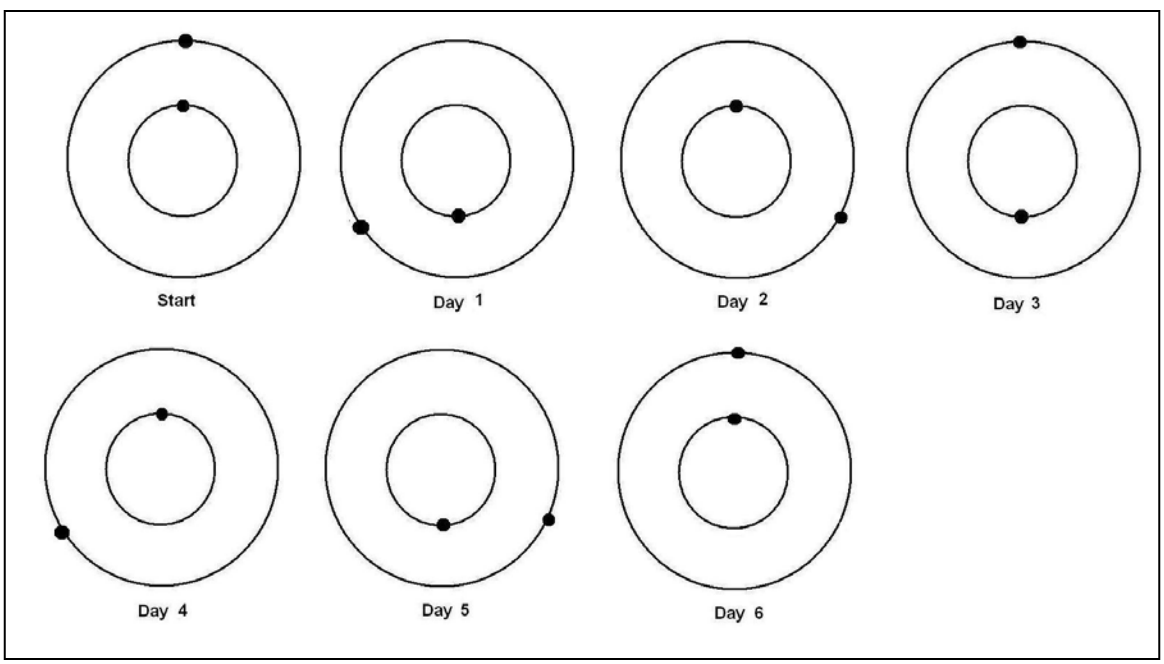
Answer Key

Problem 1 - A) The Least Common Factor between 2 and 3 is 6, so it will take 6 days for the two moons to return to their original positions. B) The figure below shows the progression in elapsed days, with the moons moving counterclockwise. C) The LCF between the orbit periods tells you how long it will take for the two bodies to return to their same locations when they started.

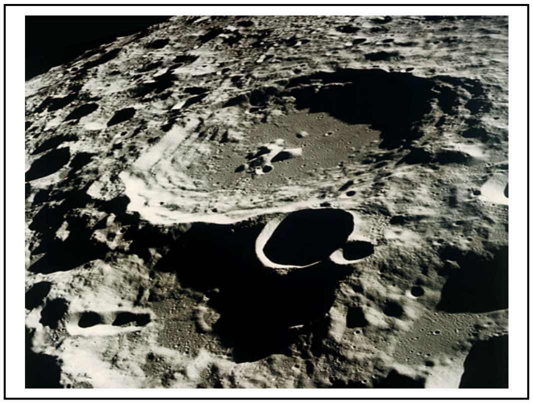
Problem 2 - Two planets have orbit periods of 3 years and 5 years. How long will it take them to return to the same locations that they started at?
Answer; The LCF for 3 and 5 is found by forming the multiples of 3 and 5 and finding the first number they share in common.

For 3: 3, 6, 9, 12**, 15,** 18, 21, 24, 27, 30, 33, 36, ….
For 5: 5, 10, **15,** 20, 25, 30, …

The smallest common factor is '15', so it will take the two planets 15 years to return to the positions they started with.



Areas and Probabilities



There are many situations in astronomy where probability and area go hand in hand! The problems below can be modeled by using graph paper shaded to represent the cratered areas.

The moon's surface is heavily cratered, as the Apollo 11 photo to the left shows. The total area covered by them is more than 70% of the lunar surface!

1 – A 40km x 40km area of the Moon has 5 non-overlapping craters, each about 5km in radius. A) What fraction of this area is covered by craters? B) What is the percentage of the cratered area to the full area? C) Draw a square representing the surveyed region and shade the fraction covered by craters.

2 - During an 8-day period, 2 days were randomly taken off for vacation. A) What fraction of days are vacation days? B) What is the probability that Day-5 was a vacation day? C) Draw a square whose shaded area represents the fraction of vacation days.

3 – An asteroid capable of making a circular crater 40-km across impacts this same 40km x 40km area dead-center. About what is the probability that it will strike a crater that already exists in this region?

4 – During an 8-day period, 2 days were randomly taken of for vacation, however, during each 8-day period there were 4 consecutive days of rain that also happened randomly during this period of time. What is the probability that at least one of the rain days was a vacation day? (Hint: list all of the possible 8-day outcomes.)

Inquiry – How can you use your strategy in Problem 4 to answer the following question: An asteroid capable of making a circular crater 20-km across impacts this same 40km x 40km area dead-center. What is the probability that it will strike a crater that already exists in this region?

Answer Key

1 – Answer: A) The area of a crater with a circular shape, is A = π (5km)^2 = 78.5 km^2, so 5 non-overlapping craters have a total area of 393 km^2. The lunar area is 40 km x 40km = 1600 km^2, so the fraction of cratered area is 393/1600 **= 0.25.**

B) The percentage cratered is 0.25 x 100% = 25%.

C) Students will shade-in 25% of the squares.

2 - Answer: A) 2 days/8 days = 0.25

B) .25 x 100% = 25%.

C) The square should have 25% of area shaded.

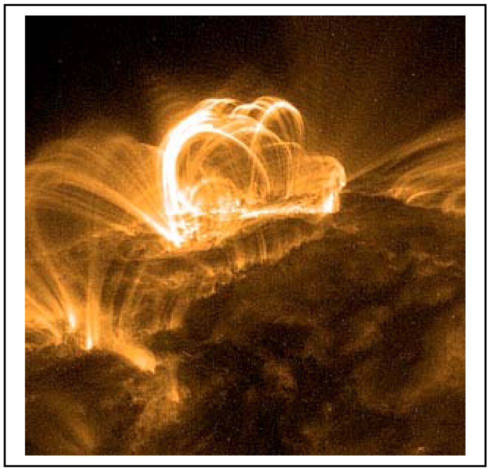
3 –Answer: The area of the impact would be π (20km)^2 = 1240 km^2 . The area of the full region is 1600 km^2. The difference in area is the amount of lunar surface not impacted and equals 360 km^2. Because the cratered area is 393 km^2 and is larger then the unimpacted area, the **probability is 100%** that at least some of the cratered area will be affected by the new crater.

4 – R = Rain days   Vacation: There are 8 x 7/2 = **28 possibilities**

R R R R X X X X
X R R R R X X X
X X R R R R X X
X X X R R R R X
X X X X R R R R

Rain 'area' = 50%. For each of the 5 possibilities for a rain period, there are 28 possibilities for a vacation series, which makes 5 x 28 = 140 combinations of rain and vacation. For a single rain pattern out of the 5, there are 28 vacation patterns, and of these, 50% will include at least one vacation day in the rain, because the other half of the days avoid the rain days entirely. So, out of the 140 combinations, 70 will include rain days and 70 will not, again reflecting the fact that the 'area' of the rain days is 50% of the total days.

Inquiry: The already cratered area is ¼ of the total. The area of the new impact is π (10km)^2 = 314 km^2 which is 314/1600 = 1/5 of the total area. Draw a series of S=20 cells (like the 8-day pattern). The new crater represents 20x1/5 = 4 consecutive cells shaded. Work out all of the possible ways that 20 x 1/4 = 5 previously cratered areas can be distributed over the 20 days so that one of them falls within the 4 consecutive cells of the new crater. Example: X P X P X N N N N X X X X P X P X X P X
X = not cratered area, P = previously cratered area, N = new crater impact.
We are looking for: Probability = (The number of trials where a P is inside the 'N' region) / M , where M = the total number of possible trials. **Because there are 17 possibilities for the Ns, and 20!/ (15! 5!) = (20 x 19) / 2 = 190 possibilities for where the 5 Ps can go, m = 17 x 190 = 3230 . These do not all have to be worked out by hand to find the number of trials where a P is inside an N region in the series. You can also reduce the value of S so long as you keep the relative** areas between N, V and P the same.



The Sun is an active star, which produces solar flares (F) and explosions of gas (C). Astronomers keep watch for these events because they can harm satellites and astronauts in space. Predicting when the next storm will happen is not easy to do. The problems below are solved by writing out all of the possibilities, then calculating the probability of the particular outcome!

Solar flare photo courtesy TRACE/NASA

1 – During a week of observing the sun, astronomers detected 1 solar flare (F). What was the probability (as a fraction) that it happened on Wednesday?

2 – During the same week, two gas clouds were ejected (C), but not on the same days. What is the probability (as a fraction) that a gas cloud was ejected on Wednesday?

3 – Suppose that the flares and the gas clouds had nothing to do with each other, and that they occurred randomly. What is the probability (as a fraction) that both a flare and a gas cloud were spotted on Wednesday? (Astronomers would say that these phenomena are uncorrelated because the occurrence of one does not mean that the other is likely to happen too).

Answer Key

1 – Answer: There are only 7 possibilities:

F X X X X X X	X X X F X X X	X X X X X X F
X F X X X X X	X X X X F X X	
X X **F** X X X X	X X X X X F X	

So the probability for any one day is **1/7.**

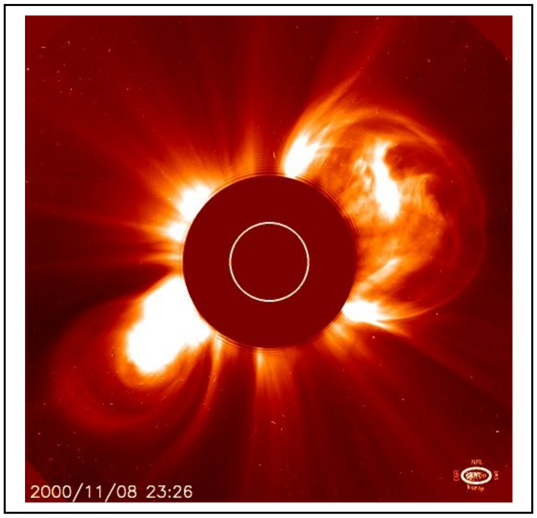
2 – Here we have to distribute 2 storms among 7 days. For advanced students, there are 7! / (2! 5!) = 7 x 6 / 2 = 21 possibilities which the students will work out by hand:

C C X X X X X	X C **C** X X X X	X X **C** C X X X	X X X C C X X
C X **C** X X X X	X C X C X X X	X X **C** X C X X	X X X C X C X
C X X C X X X	X C X X C X X	X X **C** X X C X	X X X C X X C
C X X X C X X	X C X X X C X	X X **C** X X X C	X X X X C C X
C X X X X C X	X C X X X X C		X X X X C X C
C X X X X X C			X X X X X C C

There are 6 possibilities (in red) for a cloud appearing on Wednesday (Day 3), so the probability is **6/21.**

3 – We have already tabulated the possibilities for each flare and gas cloud to appear separately on a given day. Because these events are independent of each other, the probability that on a given day you will spot a flare and a gas cloud is just 1/7 x 6/21 or 6/147. This is because for every possibility for a flare from the answer to Problem 1, there is one possibility for the gas clouds.

There are a total of 7 x 21 = 147 outcomes for both events taken together. Because there are a total of 1 x 6 outcomes where there is a flare and a cloud on a particular day, the fraction becomes (1 x 6)/147 = **6/147.**



The Sun is an active star that produces solar flares (F) and explosions of gas (C). Astronomers keep watch for these events because they can harm satellites and astronauts in space. Predicting when the next storm will happen is not easy to do. The problems below are solved by writing out all of the possibilities, then calculating the probability of the particular outcome!

Photo of a coronal mass ejection courtesy SOHO/NASA.

Problem 1 – During a particularly intense week for solar storms, three flares were spotted along with two massive gas cloud explosions. Work out all of the possible ways that 3 Fs and 2 Cs can be separately distributed among 7 days. Examples include C C X X X X X and F F F X X X X.

What is the probability (as a fraction) that none of these events occurred on Friday?

Inquiry: Does the probability matter if we select any one of the other 6 days?

Answer Key

1 – Here we have to distribute 2 cloud events (C) among 7 days. For advanced students, there are 7! / (2! 5!) = 7 x 6 / 2 = 21 possibilities which the students will work out by hand, in this case starting with Monday as the first place in the sequence:

C C X X **X** X X	X C C X **X** X X	X X C C **X** X X	X X X C C X X
C X C X **X** X X	X C X C **X** X X	X X C X C X X	X X X C **X** C X
C X X C **X** X X	X C X X C X X	X X C X **X** C X	X X X C **X** X C
C X X X C X X	X C X X **X** C X	X X C X **X** X C	X X X X C C X
C X X X **X** C X	X C X X **X** X C		X X X X C X C
C X X X **X** X C			X X X X **X** C C

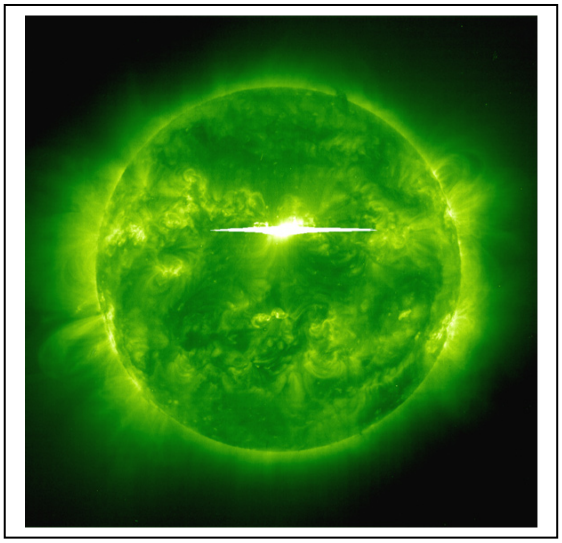
Next, we have to distribute 3 flares (F) among 7 days. There will be 7! / (3! 4!) = (7 x 6 x 5) /(3 x 2) = 35 possibilities as follows:

F F F X **X** X X	F X X F F X X	X F F F **X** X X	X X F F F X X
F F X F **X** X X	F X X F **X** F X	X F F X F X X	X X F F **X** F X
F F X X F X X	F X X F **X** X F	X F F X **X** F X	X X F F **X** X F
F F X X **X** F X	F X X X F F X	X F F X **X** X F	X X F X F F X
F F X X **X** X F	F X X X F X F	X F X F F X X	X X F X F X F
F X F F **X** X X	F X X X **X** F F	X F X F **X** F X	X X F X **X** F F
F X F X F X X		X F X F **X** X F	X X X F F F X
F X F X **X** F X		X F X X F F X	X X X F F X F
F X F X **X** X F		X F X X F X F	X X X F **X** F F
		X F X X **X** F F	X X X X F F F

For each cloud event (21 possibilities) there are 35 possibilities for flare events, so the total number of arrangements is 21 x 35 = 735. Friday is the fifth location in each sequence.

The number of sequences of cloud events in which no cloud appears in the fifth slot is highlighted in red. There are 15 possibilities. For the solar flares, there are 20 possibilities, so because these are independent, the total number is 15 x 20 = 300. So, the probability that there will be no solar storms on Friday is 300/735

Inquiry – Students may redo the calculation for any of the other days. The resulting probability should always be 300/735. This is because the events are not correlated with any particular day, so the day choice is random as well.



On July 15, 2001 a solar storm was tracked from the Sun to Earth by a number of research satellites and observatories. This activity lets you perform time and day arithmetic to figure out how long various events lasted. This is a very basic process that scientists go through to study an astronomical phenomenon. The image to the left was taken by the TRACE satellite and shows the x-ray flare on the Sun. The 'slinky' shape is caused by magnetic fields.

Photo courtesy SOHO/NASA

The Story: On July 14, 2000, NASA's TRACE satellite spotted a major X5.7-class solar flare erupting at 09:41 from Active Region 9077. The flare continued to release energy until 12:31. At 10:18:27, radio astronomers using the Nancay radio telescope detected the start of a radio-frequency Type-I noise storm. This storm strengthened, and at 10:27:27, four moving radio sources appeared. Meanwhile, the satellite, GOES-10 detected the maximum of the x-ray light from this flare at 10:23. The SOHO satellite, located 92 million miles from the Sun, and 1 million miles from Earth, recorded a radiation storm from fast-moving particles, that caused data corruption at 10:41. The SOHO satellite's LASCO imager also detected the launch of a coronal mass ejection (CME) at 10:54. The CME arrived at the satellite at 14:17 on July 15. Then at 14:37 on July 15, the CME shock wave arrived at Earth and compressed Earth's magnetic field. The IMAGE satellite recorded the brightening of the auroral oval starting at 14:25. Aurora were at their brightest at 14:58. The aurora expanded to the lowest latitude at 17:35. By 20:00, Earth's magnetic field has slightly decreased in strength in the equatorial regions. By 16:47 on July 16, the IMAGE satellite recorded the recovery of Earth's magnetosphere to normal conditions. On January 12, 2001, the CME was detected by the Voyager I satellite located 63 AU from the Sun.

Problem 1 - From this information, create a time line of the events mentioned.

Problem 2 – How long did it take for the CME to reach Earth?

Inquiry: What other questions can you explore using this timing information?

Answer Key

Problem 1

July 14,
09:41 - X5.7-class solar flare
10:19 - Radio astronomers detect Type-I radio storm.
10:23 - GOES-10 detected the maximum of the x-ray light from this flare
10:27 - Four moving radio sources appeared on sun.
10:41 - SOHO satellite radiation storm and data corruption.
10:54 - SOHO sees launch of CME
July 15,
14:17 - CME shock wave arrived at Earth
14:25 – IMAGE satellite sees brightening of the auroral oval
14:58 - Aurora at brightest
17:35 – Aurora expand to lowest latitudes
20:00 - Earth's magnetic field has slightly decreased in strength
July 16
16:47 - IMAGE satellite recorded the recovery of Earth's magnetosphere

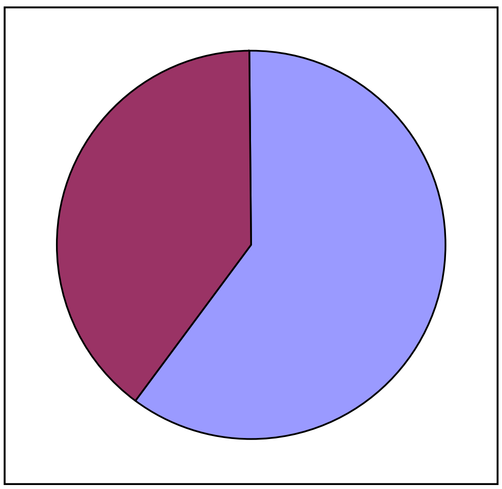
January 12, 2001, CME detected by Voyager I satellite 63 AU from the Sun.

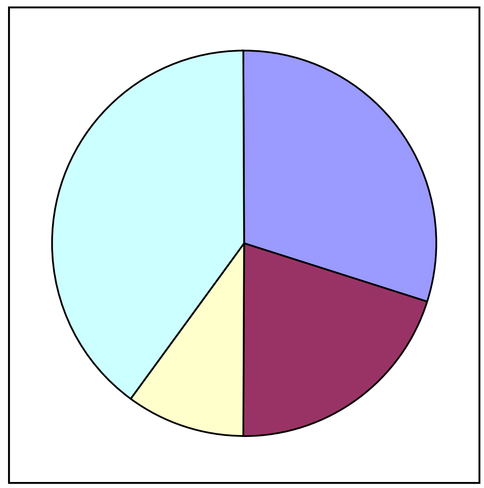
Problem 2 - The CME was launched on July 14 at 10:54 and arrived at Earth on July 15 at 14:17. The elapsed time is 1 full day (24 hours) and the difference between 10:54 and 14:17 which is 14:17 – 10:54 = 13:77 – 10:54 = 3hours and 77-54 = 23 minutes. The total elapsed time is then 24h + 3h 23m = 27hours 23minutes.

Inquiry – There are many possibilities, for example, how long did it take for the CME to reach Voyager in days? Hours? What was the speed of the CME as it traveled to Earth? How long after the flare did SOHO experience a radiation storm?

Solar Storm Energy and Pie Graphs

The pie charts below show approximately how various forms of energy are involved in a solar flare. Flares occur when stored magnetic energy is suddenly released. The chart on the left shows how much of this magnetic energy is available for creating a flare (purple) and how much is lost (blue). The chart on the right shows how much of the available magnetic flare energy goes into four different phenomena: Light green represents forms of radiation such as visible light and x-rays. Blue represents (kinetic) energy in ejected clouds of gas called Coronal Mass Ejections. Purple represents flare energy that goes into heating local gases to millions of degrees Centigrade, and white is the portion of the flare energy that is lost to working against gravity.





Graph of stored magnetic energy

Graph of solar flare energy forms

Problem 1 - About what percentages of each of the four forms of energy are represented in the right-hand chart?

Problem 2 - About what percentage of the original, stored magnetic energy is available for flares?

Problem 3 - About what fraction of the original magnetic energy ends up as solar flare radiation, assuming all forms of energy can be interchanged with each other?

Problem 4 - About what fraction of the original magnetic energy ends up in CME ejection?

Problem 5 - A typical large flare has enough total energy to meet the world-wide power demands of human civilization for 10,000 years. How many years would be equivalent to A) causing the flare to shine and B) ejecting a CME?

Answer Key

Problem 1 – What percentages of each of the four forms of energy are represented in the right-hand chart?

Answer: **Radiation = 40%, CME = 30%, Gas heating = 20 % and Gravity = 10%**

Problem 2 - What percentage of the original, stored magnetic energy is available for flares?

Answer: The size of the purple sector is **40%**

Problem 3 – What fraction of the original magnetic energy ends up as solar flare radiation?

Answer: 40% of the original magnetic energy is available for a flare, and 40% of the flare energy ends up as radiation, so the fraction of the original magnetic energy involved is 0.40 x 0.40 = **0.16**

Problem 4 – What fraction of the original magnetic energy ends up in CME ejection?

Answer: 40% of the original magnetic energy is available for a flare, and 30% of the flare energy ends up as CME (kinetic) energy, so the fraction of the original magnetic energy involved is 0.40 x 0.30 **= 0.12**

Problem 5 – A typical large flare has enough total energy to meet the power demands of human civilization for 10,000 years. How many years would be equivalent to A) causing the flare to shine and B) ejecting a CME?

Answer: A) 40% of the flare energy ends up as radiation so this is equivalent to 0.40 x 10,000 years = *4,000 years* of human energy consumption.

B) 30% of the flare energy is available for CME kinetic energy, so this equals an equivalent of 0.30 x 10,000 years **= 3,000 years** of human energy consumption.

Lunar Cratering - Probability and Odds

The moon has lots of craters! If you look carefully at them, you will discover that many overlap each other. Suppose that over a period of 100,000 years, four asteroids struck the lunar surface. What would be the probability that they would strike an already-cratered area, or the lunar mare, where there are few craters?

Problem 1 - Suppose you had a coin where one face was labeled 'C' for cratered and the other labeled U for uncratered. What are all of the possibilities for flipping C and U with four coin flips?

Problem 2 - How many ways can you flip the coin and get only Us?

Problem 3 - How many ways can you flip the coin and get only Cs?

Problem 4 - How many ways can you flip the coin and get 2 Cs and 2 Us?

Problem 5 - Out of all the possible outcomes, what fraction includes only one 'U' as a possibility?

Problem 6 - If the fraction of desired outcomes is 2/16, which reduces to 1/8, we say that the 'odds' for that outcome are 1 chance in 8. What are the odds for the outcome in Problem 4?

A fair coin is defined as a coin whose two sides have equal probability of occurring so that the probability for 'heads' = 1/2 and the probability for tails = 1/2 as well. This means that P(heads) + P(tails) = 1/2 + 1/2 = 1. Suppose a tampered coin had P(heads) = 2/3 and P(tails) = 1/3. We would still have P(heads) + P(tails) = 1, but the probability of the outcomes would be different…and in the cheater's favor. For example, in two coin flips, the outcomes would be HH, HT ,TH and TT but the probabilities for each of these would be HH = (2/3)x(2/3)=4/9; HT and TH = 2 x (2/3)(1/3) = 4/9, and TT = (1/3)x1/3) = 1/9. The probability of getting more heads would be 4/9 + 4/9 = 8/9 which is much higher than for a fair coin.

Problem 7: From your answers to Problem 2, what would be the probability of getting only Us in 4 coin tosses if A) P(U) = 1/2? B) P(U) = 1/3?

Problem 8 - The fraction of the lunar surface that is cratered is 3/4, while the mare (dark areas) have few craters and occupy 1/4 of the surface area. If four asteroids were to strike the moon in 100,000 years, what is the probability that all four would strike the cratered areas?

Answer Key

Problem 1 - The 16 possibilities are as follows:

C U U U	C C U U	U C U C	C U C C
U C U U	C U C U	U U C C	U C C C
U U C U	C U U C	C C C U	C C C C
U U U C	U C C U	C C U C	U U U U

Note if there are two outcomes for each coin flip, there are 2 x 2 x 2 x 2 = 16 independent possibilities.

Problem 2 - There is only one outcome that has 'U U U U'

Problem 3 - There is only one outcome that has 'C C C C'

Problem 4 - From the tabulation, there are 6 ways to get this outcome in any order.

Problem 5 - There are 4 outcomes that have only one U out of the 16 possible outcomes, so the fraction is 4/16 or 1/4.

Problem 6 - The fraction is 6 / 16 reduces to 3/8 so the odds are 3 chances in 8.

Problem 7: A) There is only one outcome that has all Us, and if each U has a probability of 1/2, then the probability is 1 x (1/2) x (1/2) x (1/2) x (1/2) = 1/16.
B) If each U has a probability of 1/3, then the probability is (1/3) x (1/3) x (1/3) x (1/3) = 1/81.

Problem 8 - P(U) = 1/4 while P(C)= 3/4, so the probability that all of the new impacts are also in the cratered regions is the outcome C C C C which is (3/4) x (3/4) x (3/4) x (3/4) = 81 / 256.

The Mass of the Moon

On July 19, 1969 the Apollo-11 Command Service Module and LEM entered lunar orbit. The orbit period was 2.0 hours, at a distance of 1,737 km from the lunar center.

Believe it or not, you can use these two pieces of information to determine the mass of the moon. Here's how it's done!

Problem 1 - Assume that Apollo-11 went into a circular orbit, and that the inward gravitational acceleration by the Moon on the capsule, Fg, exactly balances the outward centrifugal acceleration, Fc. Solve Fc = Fg for the mass of the Moon, M, in terms of V, R and the constant of gravity, G, given that:

$$F_g = \frac{G M m}{R^2} \qquad F_c = \frac{m V^2}{R}$$

Problem 2 - By using the fact that for circular motion, $V = 2 \pi R / T$, re-express your answer to Problem 1 in terms of R, T and M.

Problem 3 - Given that $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ sec^{-2}$, R = 1,737 km and T = 2 hours, calculate the mass of the Moon in kilograms!

Problem 4 - The mass of Earth is 5.97×10^{24} kg. What is the ratio of the Moon's mass, derived in Problem 3, to Earth's mass?

Problem 1 - From Fg = Fc, and a little algebra to simplify and cancel terms, you get

$$\boxed{M = \frac{R V^2}{G}}$$

Problem 2 – Substitute 2 π R/T for V and with a little algebra you get:

$$\boxed{M = \frac{4\pi^2 R^3}{G\,T^2}}$$

Problem 3 - First convert all units to meters and seconds: R = 1.737×10^{6} meters and T = 7,200 seconds. Then substitute values into the above equation:

$M = 4 \times (3.14)^2 \times (1.737 \times 10^6)^3 / (6.67 \times 10^{-11} \times (7200)^2))$

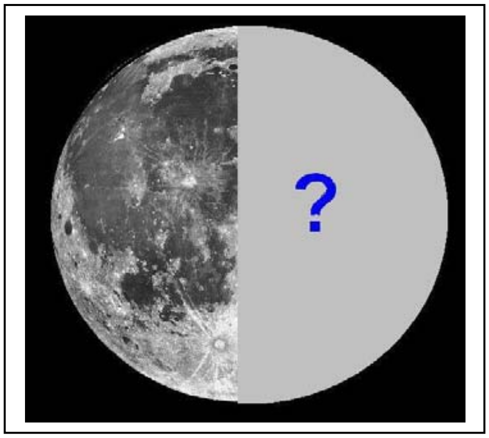
$M = (39.44 \times 5.24 \times 10^{18}) / (3.46 \times 10^{-3})$

M = $\mathbf{5.97 \times 10^{22}}$ **kg**

More accurate measurements, allowing for the influence of Earth's gravity and careful timing of orbital periods, actually yield 7.4×10^{22} kg.

Problem 4 - The ratio of the masses is 5.97×10^{22} kg / 5.97×10^{24} kg which equals **1/100.** The actual mass ratio is 1 / 80.

The Moon's Density - What's inside?



The Moon has a mass of 7.4×10^{22} kg and a radius of 1,737 km. Seismic data from the Apollo seismometers also shows that there is a boundary inside the Moon at a radius of about 400 km where the rock density or composition changes. Astronomers can use this information to create a model of the Moon's interior.

Problem 1 - What is the average density of the Moon in grams per cubic centimeter (g/cm^3) ? (Assume the Moon is a perfect sphere.)

Problem 2 - What is the volume, in cubic centimeters, of A) the Moon's interior out to a radius of 400 km? and B) The remaining volume out to the surface?

You can make a simple model of a planet's interior by thinking of it as an inner sphere (the core) with a radius of R(core), surrounded by a spherical shell (the mantle) that extends from R(core) to the planet's surface, R(surface). We know the total mass of the planet, and its radius, R(surface). The challenge is to come up with densities for the core and mantle and R(core) that give the total mass that is observed.

Problem 3 - From this information, what is the total mass of the planet model in terms of the densities of the two rock types (D1 and D2) and the radius of the core and mantle regions R(core) and R(surface)?

Problem 4 - The densities of various rock types are given in the table below.

Type	Density
I - Iron+Nickle mixture (Earth's core)	**15.0 gm/cc**
E - Earth's mantle rock (compressed)	**4.5 gm/cc**
B - Basalts	**2.9 gm/cc**
G - Granite	**2.7 gm/cc**
S - Sandstone	**2.5 gm/cc**

A) How many possible lunar models are there? B) List them using the code letters in the above table, C) If denser rocks are typically found deep inside a planet, which possibilities survive? D) Find combinations of the above rock types for the core and mantle regions of the lunar interior model, that give approximately the correct lunar mass of 7.4×10^{25} grams. [Hint: use an *Excel* spread sheet to make the calculations faster as you change the parameters.] E) If Apollo rock samples give an average surface density of 3.0 gm/cc, which models give the best estimates for the Moon's interior structure?

Problem 1 - Mass = 7.4×10^{22} kg x 1000 gm/kg = 7.4×10^{25} grams. Radius = 1,737 km x 100,000 cm/km = 1.737×10^{8} cm. Volume of a sphere = $4/3\ \pi\ R^3 = 4/3 \times (3.141) \times (1.737 \times 10^8)^3 = 2.2 \times 10^{25}\ cm^3$, so the density = 7.4×10^{25} grams / $2.2 \times 10^{25}\ cm^3$ = **3.4 gm / cm^3.**

Problem 2 - A) V(core) = $4/3\ \pi\ R^3 = 4/3 \times (3.141) \times (4.0 \times 10^7)^3$ **= 2.7×10^{23} cm^3**
B) V(shell) = V(Rsurface) - V(Rcore) = $2.2 \times 10^{25}\ cm^3 - 2.7 \times 10^{23}\ cm^3$ = **2.2×10^{25} cm^3**

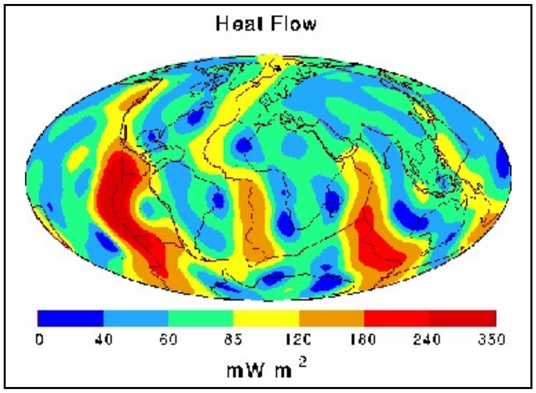
Problem 3 - The total core mass is given by M(core) = $4/3\ \pi\ (Rcore)^3 \times D1$. The volume of the mantle shell is given by multiplying the shell volume V(shell) calculated in Problem 2B by the density : Mshell = V(shell) x D2. Then, the formula for the total mass of the model is given by: MT = $4/3\ \pi\ (Rc)^3 \times D1 + (4/3\ \pi\ (Rs)^3 - 4/3\ \pi\ (Rc)^3) \times D2$, which can be simplified to:

$$\mathbf{MT = 4/3\ \pi\ (D1 \times Rc^3 + D2 \times Rs^3 - D2 \times Rc^3)}$$

Problem 4 - A) There are 5 types of rock for 2 lunar regions so the number of unique models is 5 x 5 = 25 possible models. B) The possibilities are: II, IE, IB, IG, IS, EE, EI, EB, EG, ES, BI, BE, BB, BG, BS, GI ,GE, GB, GG, GS, SI, SE, SB, SG, SS. C) The ones that are physically reasonable are: IE, IB, IG, IS, EB, EG, ES, BG, BS, GS. The models, II, EE, BB, GG and SS are eliminated because the core must be denser than the mantle. D) Each possibility in your answer to Part C has to be evaluated by using the equation you derived in Problem 3. This can be done very efficiently by using an Excel spreadsheet. The possible answers are as follows:

Model Code	Mass (in units of 10^{25} grams)
I E	10.2
I B	6.7
E B	6.4
I G	6.3
E G	6.0
B G	6.0
I S	5.8
E S	5.5
B S	5.5
G S	5.5

E) The models that have rocks with a density near 3.0 gm/cc as the mantle top layer are the more consistent with the density of surface rocks, so these would be IB and EB which have mass estimates of 6.7×10^{25} and 6.4×10^{25} grams respectively. These are both very close to the actual moon mass of 7.4×10^{25} grams (e.g. 7.4×10^{25} grams) so it is likely that the moon has an outer mantle consisting of basaltic rock, similar to Earth's mantle rock (4.5 gm/cc) and a core consisting of a denser iron/nickel mixture (15 gm/cc).



Earth heat flow map (H. N. Pollack, S. J. Hurter, and J. R. Johnson, Reviews of Geophysics, Vol. 31, 1993.)

When large bodies form, their interiors are heated by a combination of radioactivity and the heat of formation from the infall of the rock. For planet-sized bodies, this heat can be generated and lost over billions of years. The end result will be that the core cools off and, if molten, it eventually solidifies. Measuring the surface temperature of a solid body, and the rate at which heat escapes its surface, provides clues to its internal heating and cooling rates.

Problem 1 - Measuring the heat flow out of the lunar surface is a challenge because the monthly and annual changes of surface solar heating produce interference. Apollo 15 astronauts measured the heat flow from two bore holes that reached about 2-meters below the surface. When corrected for the monthly effects from the Sun, they detected a heat flow of about 20 milliWatts/meter^2. If the radius of the Moon is 1,737 kilometers, what is the total thermal power emitted by the entire Moon in billions of watts?

Problem 2 - A future lunar colony covers a square surface that is 100 meters x 100 meters. What is the total thermal power available to this colony by 'harvesting' the lunar heat flow?

Problem 3 - The relationship between power, L, surface radius, R, and surface temperature, T, is given by $L = 4 \pi R^2 \sigma T^4$ where σ = the Stefan-Boltzman constant and has a value of 5.67×10^{-8} W/m^2 / K^4 , and where T is in Kelvin degrees, L is in watts, and R is in meters. Suppose the Moon's interior was heated by a source with a radius of 400 kilometers at the lunar core, what would the temperature of this core region have to be to generate the observed thermal wattage at the surface?

Problem 4 - The lunar regolith and crust is a very good insulator! Through various studies, the temperature of the Moon is actually believed to be near 1,200 K within 400 km of the center. A) Using the formula for L in Problem 3, how much power is absorbed by the lunar rock overlaying the core? B) From you answer to (A), how many Joules are absorbed by each cubic centimeter of overlaying lunar rock each second (Joules/cm^3)? C) Basalt begins to soften when in absorbs over 1 million Joules/cm^3. Is the lunar surface in danger of melting from the heat flow within?

Problem 1 - The surface area of a sphere is $4 \pi R^2$, so the surface area of the moon is $4 \times 3.14 \times (1.737 \times 10^6)^2 = 3.8 \times 10^{18}$ m^2 .The total power, in watts, is then 0.020 watts/m^2 x 3.8×10^{18} m^2 = 7.6×10^{11} watts or **760 billion watts.**

Problem 2 - The surface area is 100 x 100 = 10,000 m^2, and with a heat flow of 0.020 milliwatts/m^2, the total thermal power is **200 watts.**

Problem 3 - The total thermal power, L = 7.6×10^{11} watts, and R = 400 km = $4.0 \times 10^{\mathbf{5}}$ meters, so that $7.6 \times 10^{11} = 4 \times (3.14) \times (4.0 \times 10^5)^2 \times 5.67\times10^{-8}\ T^4$
Then $7.6 \times 10^{11} = 1.1 \times 10^5\ T^4$. Solving for T we get **T = 51 K.**

Problem 4 - A) The power emitted by the 400 km, 1,200 K core region is given by

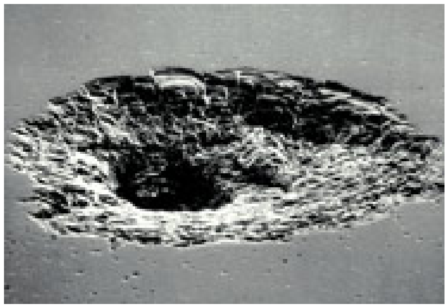
$$L = 4 \pi R^2 \sigma T^4$$

which equals $L = 4 \times (3.14) \times (4.0 \times 10^5)^2 \times 5.67\times10^{-8}\ (1200)^4 = 2.4 \times 10^{17}$ watts. Since from the answer to Problem 1 the amount that makes it to the surface is only 7.6×10^{11} watts, that leaves essentially all of the 2.4×10^{17} watts to be absorbed by the overlaying rock mantle.

B) The volume of overlaying rock is the total volume of the moon (radius 1,737 km) minus the volume of the 400-km lunar core. The difference in these two spherical regions is: $4/3\ \pi\ ((1.737 \times 10^6)^3 - (4.0 \times 10^5)^3) = 4/3 \times 3.14 \times (5.18 \times 10^{18}) = 2.2 \times 10^{19}$ m^3, or 2.2×10^{25} cm^3. Since 1 watt = 1 Joule/sec, then in 1 second the lunar thermal power from the 1,200 K core is 2.4×10^{17} joules as calculated in (A). If this is evenly absorbed by the rock in the mantle, the average thermal heating energy per cm^3 is just 2.4×10^{17} joules / 2.2×10^{25} cm^3. or **1.0×10^{-6} joules/cm^3. (This is also equal to 10 ergs /cm^3)**

C) **No, because this amount of energy input is completely negligible in melting, or warming, rock material**. Note: Basalt softens at 1,200 C = 1,500 K. A cubic centimeter of this rock has a surface area of 6 cm^2, so from $L = SA \times \sigma T^4$ we get $L = 6.0 \times 5.67\times10^{-8}\ (1500)^4 = 1.7 \times 10^6$ watts, which in 1 second amounts to 1.7×10^6 Joules / cm^3 - the energy needed to melt basalt.

Is there a lunar meteorite hazard?



Damage to Space Shuttle Endeavor in 2000 from a micrometeoroid or debris impact . The crater is about 1mm across. (Courtesy - JPL/NASA)

Without an atmosphere, there is nothing to prevent millions of pounds a year of rock and ice fragments from raining down upon the lunar surface.

Traveling at 10,000 miles per hour (19 km/s), they are faster than a speeding bullet and are utterly silent and invisible until they strike.

Is this something that lunar explorers need to worry about?

Problem 1 - Between 1972 and 1992, military infra-sound sensors on Earth detected 136 atmospheric detonations caused by meteors releasing blasts carrying an equivalent energy of nearly 1,000 tons of TNT - similar to small atomic bombs, but without the radiation. If the radius of Earth is 6,378 km, A) what is the rate of these deadly impacts on Earth in terms of impacts per km^2 per year? B) Assuming that the impact rates are the same for Earth and the Moon, suppose a lunar colony has an area of 10 km^2. How many years would they have to wait between meteor impacts?

Problem 2 - Between 2005-2007, NASA astronomers counted 100 flashes of light from meteorites striking the lunar surface - each equivalent to as much as 100 pounds of TNT. If the surveyed area equaled 1/4 of the surface area of the Moon, and the lunar radius is 1,737 km, A) What is the arrival rate of these meteorites in meteorites per km^2 per year? B) If a lunar colony has an area of 10 km^2, how long on average would it be between impacts?

Problem 3 - According to H.J. Melosh (1981) meteoroids as small as 1-millimeter impact a body with a 100-km radius about once every 2 seconds. A) What is the impact rate in units of impacts per m^2 per hour? B) If an astronaut spent a cumulative 1000 hours moon-walking and had a spacesuit surface area of 10 m^2, how many of these deadly impacts would he receive? C) How would you interpret your answer to B)?

Answer Key

Problem 1 - A) The surface area of Earth is $4 \pi (6378)^2 = 5.1 \times 10^8$ km^2. The rate is R = 136 x 10 impacts / 20 years / 5.1×10^8 km^2 = **1.3×10^{-7} impacts/km^2/year.**

B) The number of impacts/year would be 1.3×10^{-7} impacts/km^2/year x 10 km^2 = 1.3×10^{-6} impacts/year . The time between impacts would be
$1/1.3 \times 10^{-6}$ = **769,000 years!**

Problem 2 - A) The total surface area of the Moon is $4 \pi (1737)^2 = 3.8 \times 10^7$ km^2. Only 1/4 of this is surveyed so the area is 9.5×10^5 km^2. Since 100 were spotted in 2 years, the arrival rate is R = 100 impacts/2 years/ 9.5×10^5 km^2 = **5.3×10^{-5} impacts/km^2/year.**

B) The rate for this area is 10 km^2 x 5.3×10^{-5} impacts/km^2/year = 5.3×10^{-4} impacts/year, so the time between impacts is about $1/\ 5.3 \times 10^{-4}$ = **1,900 years**

Problem 3 - A) A sphere 100-km in radius has a surface area of $4 \pi (100{,}000)^2 = 1.3 \times 10^{11}$ m^2. The impacts arrive every 2 seconds on average, which is $2/3600 = 5.6 \times 10^{-4}$ hours. The rate is, therefore, R = 1 impacts / (1.3×10^{11} m^2 X 5.6×10^{-4} hours) = **1.4×10^{-8} impacts/m^2/hour.**

B) The number of impacts would be 1.4×10^{-8} impacts/m^2/hour x 10 m^2 x 1000 hours = **1.4×10^{-5} impacts.**

C) Because the number of impacts is vastly less than 1 (a certainty), he should not worry about such deadly impacts unless he had reason to suspect that the scientists miscalculated the impact rates for meteorites this small. Another way to look at this low number is to turn it around and say that the astronaut would have to take $1/\ 1.4 \times 10^{-5}$ about 71,000 such 1000-hour moon walks in order for one impact to occur. Alternately, the time between such events is 71,000 x 1000 hours = 71 million years!

The Earth and Moon to Scale

We have all seen drawings or sketches in books that show Earth and moon together in the same view, but in reality they are really very different in size, and are much farther apart than you might think.

By creating properly scaled drawings, you will get a better idea of what their sizes are really like! All you will need is a compass, a metric ruler, and a calculator.

The photo above was taken by the Voyager 1 spacecraft on September 18, 1977 at a distance of 7 million miles from Earth, and it has not been edited in any way. Are their diameters to scale? Their distance from each other? Even actual images can be distorted because of perspective and distance effects.

Problem 1 - The radius of the Moon is 1,737 kilometers, and the radius of Earth is 6,378 kilometers. What is the ratio of Earth's radius to the Moon's?

Problem 2 - To the nearest whole number, about how much larger is the diameter of Earth than the moon?

Problem 3 - With your ruler and compass, draw two circles that represent this size difference, and use a radius of 1 centimeter for the moon disk. Inside the circles, label them 'Earth' and 'Moon'.

Problem 4 - The distance between the center of Earth and the Moon is 384,000 kilometers. To the nearest integer, how many times the radius of Earth is the distance to the Moon?

Problem 5 - Cut out the circles for Earth and the Moon from Problem 3. Using the radius of your circle for Earth as a guide, how far apart, in centimeters, would you have to hold the two cut-outs to make a scale model of the Earth-Moon system that accurately shows the sizes of the two bodies and their distance?

Problem 6 - Look through books in your library, or use GOOGLE to do an image search. Do any of the illustrations show the Earth-Moon system in its correct scale? Why do you think artists draw the Earth-Moon system the way that they do?

Answer Key

Problem 1 - 6378 / 1737 = **3.7.**

Problem 2 - 3.7 is closest to 4.0, so Earth is about **4 times bigger than the Moon in size.**

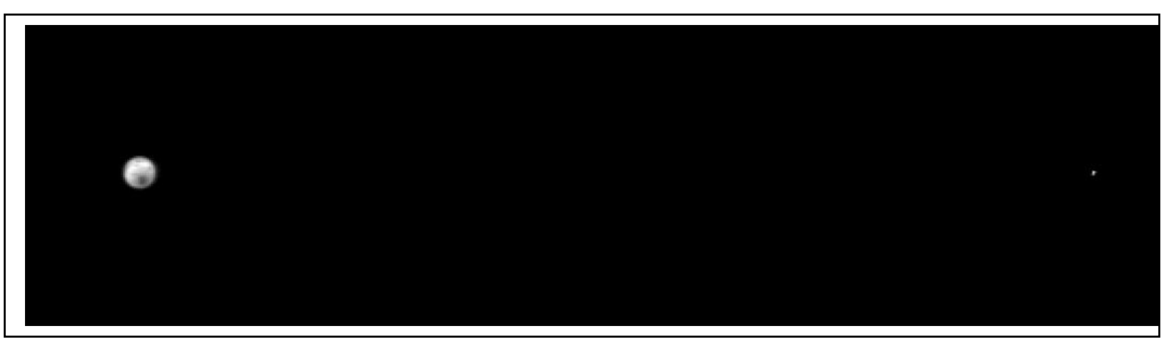
Problem 3 - **Draw the disks on a separate paper, but make sure that the Moon has a 1 cm radius and Earth has a 4 cm radius.**

Problem 4 - 384,000 / 6378 = 60.2 which is **60 times Earth's radius.**

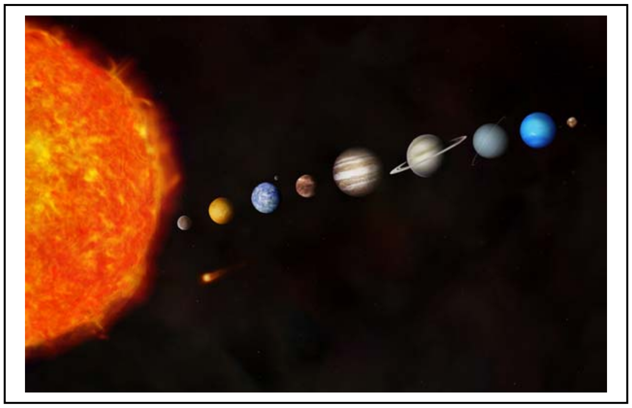
Problem 5 - If the radius of the Moon disk was 1 centimeter, the Earth disk would be 4 centimeters in radius. The distance to the Moon would be 60 times this distance, or 4 cm x 60 = **240 centimeters** (2.4 meters).

Problem 6 - Very few. Artists try to show a vast 3-dimensional image in a flat perspective drawing that is only a few inches across on the printed page. To draw the Earth-Moon system in the proper perspective scale, the Moon would be a small dot. Also, illustrations that show the phases of the Moon, or eclipses, are also badly out of scale most of the time, because you can't show the phases clearly if the Moon is only the size of a small dot in the illustration. There are other purely artistic reasons too!

Note: The image below was taken by the Mars Odyssey spacecraft soon after launch in April 2001 when Earth and Moon were at their maximum separation. The image is not edited, and shows the disks at their true separations. From the diameter of Earth (12800 km) and its measured diameter in millimeters (4.5 mm) the scale of the image below is 12800/4.5 = 2840 km/mm. The separation from the center of Earth to the Moon 'dot' is 126.5 mm or 126.5 mm x 2840 km/mm = 359,000 km. However, although Earth is 3.7x bigger than the Moon, it is clear that the lunar dot, which measures just under 0.5 mm) is much smaller than it should be (diameter = 3,474 km or 1.2 mm). **Compared to the Earth-Moon distance, how far from the Moon was the Mars Orbiter in order to see the Moon with this disk diameter? Answer: About 359,000 km x 1.2/0.5 = 860,000 km.**



Planet Fractions and Scales



Some of the planets in our solar system are much bigger than Earth while others are smaller. By using simple fractions, you will explore how their sizes compare to each other.

Image courtesy NASA/Chandra Observatory/SAO

Problem 1 - Saturn is 10 times bigger than Venus, and Venus is 1/4 the size of Neptune. How much larger is Saturn than Neptune?

Problem 2 - Earth is twice as big as Mars, but only 1/11 the size of Jupiter. How large is Jupiter compared to Mars?

Problem 3 - Earth is the same size as Venus. How large is Jupiter compared to Saturn?

Problem 4 - Mercury is 3/4 the size of Mars. How large is Earth compared to Mercury?

Problem 5 - Uranus is the same size as Neptune. How large is Uranus compared to Earth?

Problem 6 - The satellite of Saturn, called Titan, is 1/10 the size of Uranus. How large is Titan compared to Earth?

Problem 7 - The satellite of Jupiter, called Ganymede, is 2/5 the size of Earth. How large is it compared to Jupiter?

Problem 8 - The Dwarf Planet Pluto is 1/3 the diameter of Mars. How large is the diameter of Jupiter compared to Pluto?

Problem 9 - If the diameter of Earth is 13,000 km ,what are the diameters of all the other bodies?

Answer Key

Note to teachers: The actual diameters of the planets, in kilometers, are as follows

Mercury	4,900 km	Jupiter	143,000 km
Venus	12,000 km	Saturn	120,000 km
Earth	13,000 km	Uranus	51,000 km
Mars	6,800 km	Neptune	50,000 km

Also: Titan = 5,100 km, Ganymede = 5,300 km Ceres = 950 km, and Pluto 2,300 km

Advanced students (Grades 4 and above) may use actual planetary size ratios as decimal numbers, but for this simplified version (Grades 2 and 3), we approximate the size ratios to the nearest simple fractions. **Students may also use the information in these problems to make a scale model of the solar system in terms of the relative planetary sizes.**

Problem 1 - Saturn is 10 times bigger than Venus, and Venus is 1/4 the size of Neptune. How much larger is Saturn than Neptune?
Answer: Neptune is 4x Venus and Saturn is 10x Venus, so Saturn is 10/4 = 5/2 times as big as Neptune.

Problem 2 - Earth is twice as big as Mars, but only 1/11 the size of Jupiter. How large is Jupiter compared to Mars?
Answer: Jupiter is 11 x Earth, and Mars is 1/2 Earth, so Jupiter is 22x Mars.

Problem 3 - Earth is the same size as Venus. How large is Jupiter compared to Saturn?
Answer: If Saturn is 10 x Venus, and Jupiter is 11 x Earth, Jupiter is 11/10 times Saturn.

Problem 4 - Mercury is 3/4 the size of Mars. How large is Earth compared to Mercury?
Answer: Mars is 1/2 x Earth, so Mercury is 3/4 x 1/2 = 3/8 x Earth

Problem 5 - Uranus is the same size as Neptune. How large is Uranus compared to Earth?
Answer: Neptune was 4x Venus, but since Venus = earth and Neptune=Uranus, we have Uranus = 4x Earth.

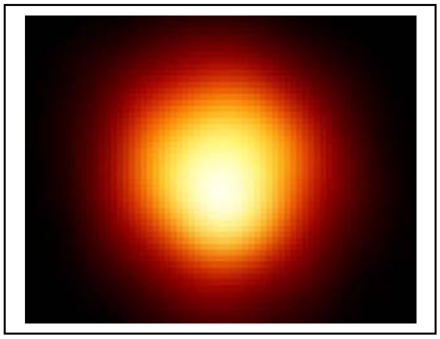
Problem 6 - The satellite of Saturn, called Titan, is 1/10 the size of Uranus. How large is Titan compared to Earth?
Answer: Titan / Uranus = 1/10, but Uranus/Earth = 4, so Titan/Earth = 3/10 x 4 = 2/5.

Problem 7 - The satellite of Jupiter, called Ganymede, is 2/5 the size of Earth. How large is it compared to Jupiter?
Answer: Earth = 1/11 Jupiter so Ganymede is 1/11 x 2/5 = 2/55 x Jupiter.

Problem 8 - The Dwarf Planet Pluto is 1/3 the size of Mars. How large is Jupiter compared to Pluto?
Answer: Jupiter = 1/11 Earth, Mars= 1/2 Earth, so Pluto= 1/3 x 1/2 = 1/6 Earth, and 1/66 Jupiter.

Problem 9 – **Answer: Students should, very nearly, reproduce the numbers in the table at the top of the page.**

The relative sizes of the sun and stars



Stars come in many sizes, but their true appearances are impossible to see without special telescopes. The image to the left was taken by the Hubble Space telescope and resolves the red supergiant star Betelgeuse so that its surface can be just barely seen. Follow the number clues below to compare the sizes of some other familiar stars!

Problem 1 - The sun's diameter if 10 times the diameter of Jupiter. If Jupiter is 11 times larger than Earth, how much larger than Earth is the Sun?

Problem 2 - Capella is three times larger than Regulus, and Regulus is twice as large as Sirius. How much larger is Capella than Sirius?

Problem 3 - Vega is 3/2 the size of Sirius, and Sirius is 1/12 the size of Polaris. How much larger is Polaris than Vega?

Problem 4 - Nunki is 1/10 the size of Rigel, and Rigel is 1/5 the size of Deneb. How large is Nunki compared to Deneb?

Problem 5 - Deneb is 1/8 the size of VY Canis Majoris, and VY Canis Majoris is 504 times the size of Regulus. How large is Deneb compared to Regulus?

Problem 6 - Aldebaran is 3 times the size of Capella, and Capella is twice the size of Polaris. How large is Aldebaran compared to Polaris?

Problem 7 - Antares is half the size of Mu Cephi. If Mu Cephi is 28 times as large as Rigel, and Rigel is 50 times as large as Alpha Centauri, how large is Antares compared to Alpha Centauri?

Problem 8 - The Sun is 1/4 the diameter of Regulus. How large is VY Canis Majoris compared to the Sun?

Inquiry: - Can you use the information and answers above to create a scale model drawing of the relative sizes of these stars compared to our Sun.

Answer Key

The relative sizes of some popular stars is given below, with the diameter of the sun = 1 and this corresponds to an actual physical diameter of 1.4 million kilometers.

Betelgeuse	440	Nunki	5	VY CMa	2016	Delta Bootis	11
Regulus	4	Alpha Cen	1	Rigel	50	Schedar	24
Sirius	2	Antares	700	Aldebaran	36	Capella	12
Vega	3	Mu Cephi	1400	Polaris	24	Deneb	252

Problem 1 - Sun/Jupiter = 10, Jupiter/Earth = 11 so Sun/Earth = 10 x 11 = **110 times.**

Problem 2 - Capella/ Regulus = 3.0, Regulus/Sirius = 2.0 so Capella/Sirius = 3 x 2 = **6 times.**

Problem 3 - Vega/Sirius = 3/2 Sirius/Polaris=1/12 so Vega/Polaris = 3/2 x 1/12 = **1/8 times**

Problem 4 - Nunki/Rigel = 1/10 Rigel/Deneb = 1/5 so Nunki/Deneb = 1/10 x 1/5 = **1/50.**

Problem 5 - Deneb/VY = 1/8 and VY/Regulus = 504 so Deneb/Regulus = 1/8 x 504 = **63 times**

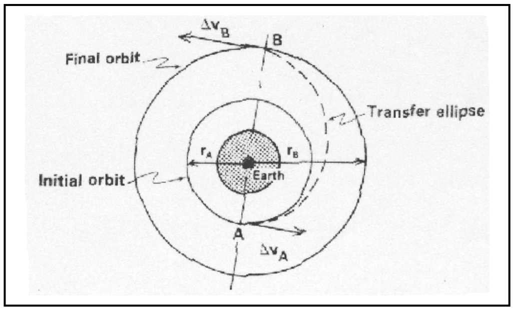
Problem 6 - Aldebaran/Capella = 3 Capella/Polaris = 2 so Aldebaran/Polaris = 3 x 2 **= 6 times**.

Problem 7 - Antares/Mu Cep = 1/2 Mu Cep/Rigel = 28 Rigel/Alpha Can = 50, then Antares/Alpha Can = 1/2 x 28 x 50 = **700 times**.

Problem 8 - Regulus/Sun = 4 but VY CMA/Regulus = 504 so VY Canis Majoris/Sun = 504 x 4 = **2016 times the sun's size!**

Inquiry: Students will use a compass and millimeter scale. If the diameter of the Sun is 1 millimeter, the diameter of the largest star VY Canis Majoris will be 2016 millimeters or about 2 meters!

Fly me to the Moon!



If spacecraft had rockets that could make them travel at any speed, we could fly to the Moon from Earth in a straight line, and make the trip in a few minutes. In the Real World, we can't do that even with the most powerful rockets we have. Instead, we have to obey Newton's Laws of Motion and take more leisurely, round-about routes!

To see how this works, you need a compass, metric ruler, a large piece of paper, a string, a thumbtack, and a pencil.

Step 1 - With your compass, draw a circle 1/2-centimeter in radius. Label the inside of this 'Earth'. **Step 2** - Draw a second circle centered on Earth with a radius of 1 centimeter. Label this 'Earth Orbit'. **Step 3** - Using the string and thumbtack, draw a second circle with a radius of 30 centimeters. Label this 'Orbit of Moon'. **Step 4** - Draw a line connecting the center of earth and a point on the lunar orbit. Label the lunar orbit Point B. **Step 5** - Extend the line so that it intersects a point on the Earth orbit circle in the opposite direction from Earth's center. There should be two intersection points. The first will be between Earth and the lunar orbit. Label this Point C. The second will be behind Earth. Label this Point A. **Step 6** - As carefully as you can, draw a free-hand ellipse with one focus centered on Earth that arcs between Point A and Point B. This is called the major axis of the ellipse. See the above figure for comparison.

What you have drawn is a simple rocket trajectory, called a Hohmann Transfer orbit, that connects a spacecraft orbiting Earth, with a point on the lunar orbit path. If you had unlimited rocket energy, you could travel the path from Point C to Point B in a few hours or less. If you had less energy, you would need to take a path that looks more like the one from Point A to Point B and is slower, so it takes more time. Even less energy would involve a spiral path that connects Point A and Point B but may loop one or more times around Earth as it makes its way to lunar orbit. Can you draw such a path?

Problem 1 - If there were no gravity, spacecraft could just travel from place to place in a straight line at their highest speeds, like the Enterprise in *Star Trek*. If the distance to the Moon is 380,000 kilometers, and the top speed of the Space Shuttle is 10 kilometers/sec, how many hours would the Shuttle take to reach the Moon?

Astrodynamicists are the experts that calculate orbits for spacecraft. One of the most important factors is the total speed change, called the delta-V, to get from one orbit to another. For a rocket to get into Earth orbit requires a delta-V of 8600 m/sec. To go from Earth orbit to the Moon takes an additional delta-V of 4100 meters/sec.

Problem 2 - To enter a Lunar Transfer Orbit, a spacecraft has enough fuel to make a total speed change of 3500 m/sec. If it needs to make a speed change of 2000 m/s in the horizontal direction, and 3000 m/sec in the vertical direction to enter the correct orbit, is there enough fuel to reach the Moon in this way? [Hint, use the Pythagorean Theorem]

Answer Key

Problem 1 - If there were no gravity, spacecraft could just travel from place to place in a straight line at their highest speeds, like the Enterprise in *Star Trek*. If the distance to the Moon is 380,000 kilometers, and the top speed of the Space Shuttle is 10 kilometers/sec, how many hours would the Shuttle take to reach the Moon?

Answer - Distance = speed x time, so 380,000 km = 10 km/s x time. Solving for Time you get 38,000 seconds. Since there are 60 minute/hour x 60 seconds/minute = 3600 seconds/hour, 38,000 / 3600 = 10.6 hours.

Note: In reality, it takes several days for spacecraft to make the trip under the influence of gravity and allowing for conservation of energy and momentum.

Problem 2 - To enter a Lunar Transfer Orbit, a spacecraft has enough fuel to make a total speed change of 3500 m/sec. If it needs to make a speed change of 2000 m/sec in the horizontal direction, and 3000 m/sec in the vertical direction to enter the correct orbit, is there enough fuel to reach the moon in this way? [Hint, use the Pythagorean Theorem, or solve graphically]

Answer:

<u>Method 1</u>: From the lengths of the horizontal and vertical speeds, we want to find the length of the hypotenuse of a right triangle. Using the Pythagorean Theorem

$$\text{total speed} = (2000^2 + 3000^2)^{1/2} = 3605 \text{ m/sec}$$

This is the total change of speed that is required, but there is only enough fuel for 3500 m/sec so the spacecraft cannot enter the Transfer Orbit.

<u>Method 2</u>: Graphically, draw a right triangle to the proper scale, for example, 1 centimeter = 1000 m/sec. Then the two sides of the triangle have lengths of 2 cm and 3 cm. Measure the length of the hypotenuse to get 3.6 cm, then convert this to a speed by multiplying by 1000 m/sec to get the answer.

Galaxies to Scale

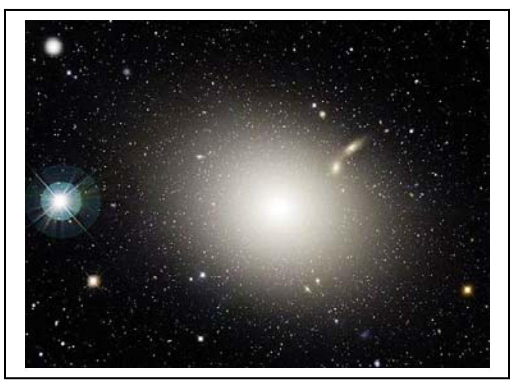
The Milky Way is a spiral galaxy. There are many other kinds of galaxies, some much larger then the Milky Way, and some much smaller. This exercise lets you create a scale model of the various kinds, and learn a little about working with fractions too!

Problem 1 - The irregular galaxy IC-1613 is twice as large as the elliptical galaxy M-32, but 10 times smaller than the spiral galaxy NGC-4565. How much larger is NGC-4565 than M-32?

Problem 2 - The spiral galaxy Andromeda is three times as large as the elliptical galaxy NGC-5128, and NGC-5128 is 4 times as large as the Large Magellanic Cloud, which is an irregular galaxy. How much larger is the Andromeda galaxy than the Large Magellanic Cloud?

Problem 3 - The Milky Way spiral galaxy is 13 times larger than the irregular galaxy IC-1613. How much larger than NGC-4565 is the Milky Way?

Problem 4 - The elliptical galaxy Leo-1 is 1/4 as large as the elliptical galaxy Messier-32, and the spiral galaxy Messier-33 is 9 times larger than Messier-32. How large is Leo-1 compared to Messier-33?



Problem 5 - The elliptical galaxy NGC-205 is 2/3 as large as the Large Magellanic Cloud. How large is NGC-205 compared to the Andromeda galaxy?

Problem 6 - The irregular galaxy NGC-6822 is 8/5 the diameter of Messier-32, and Messier-32 is 20 times smaller than NGC-4565. How large is NGC-6822 compared to IC-1613?

Problem 7 - Draw a scale model of these galaxies showing their relative sizes and their shapes.

Images: Top: The spiral galaxy Messier 74 taken by the Gemini Observatory; Bottom: The elliptical galaxy Messier-87 obtained at the Canada-France-Hawaii Telescope (copyright@cfht.hawaii.edu);

Answer Key

The galaxies used in this exercise, with the diameter given in light years, and relative to Messier-32:

Name	Type	Diameter	M-32
Large Magellanic Cloud	Irregular	15,000 LY	3
NGC-5128	Elliptical	65,000	13
NGC-4565	Spiral	100,000	20
IC-1613	Irregular	10,000	2
Andromeda	Spiral	200,000	40
NGC-205	Elliptical	10,000	2
Messier-32	Elliptical	5,000	1
Milky Way	Spiral	130,000	26
Messier-33	Spiral	45,000	9
Leo-1	Elliptical	1,000	1/4
NGC-6822	Irregular	8,000	8/5

Problem 1 - IC-1613/M-32 = 2.0 , NGC-4565/IC-1613 = 10 so NGC-4565/M-32 = 10 x 2 = **20 times**

Problem 2 - Andromeda/NGC-5128 = 3 and NGC-5128/LMC = 4 so Andromeda/LMC = 3 x 4 =**12 times**

Problem 3 - MW/IC-1613 = 13 and also NGC-4565/IC-1613 = 10, so Milky Way / NGC-4565 = 13 x 1/10 = **1.3 times.**

Problem 4 - Leo-1 / M-32 = 1/4 and M-33 / M-32 = 9, so Leo-1 / M-33 = 1/4 x 1/9 = **1/36 times smaller**.

Problem 5 - NGC-205 / LMC = 2/3 and Andromeda/LMC = 12 so NGC-205 / Andromeda = 2/3 x 1/12 = **2/36 or 1/18 as large.**

Problem 6 - NGC-6822 / M-32 = 8/5 and M-32 / NGC-4565 = 1/20 and NGC-4565 / IC-1613 = 10 so NGC-6822 / IC-1613 = 8/5 x 1/20 x 10 = 8/5 x 1/2 = **8/10 or 4/5 as large.**

Problem 7 - Students can use the ratios in the problems, together with the ones they derived, to create a table that gives the relative sizes for each galaxy. The table at the top gives the 'official' numbers, and the relative sizes in the last column.

Extracting Oxygen from Moon Rocks

About 85% of the mass of a rocket is taken up by oxygen for the fuel, and for astronaut life support. Thanks to the Apollo Program, we know that as much as 45% of the mass of lunar soil compounds consists of oxygen. The first job for lunar colonists will be to 'crack' lunar rock compounds to mine oxygen.

NASA has promised $250,000 for the first team capable of pulling breathable oxygen from mock moon dirt; the latest award in the space agency's Centennial Challenges program.

Lunar soil is rich in oxides of silicon, calcium and iron. In fact, 43% of the mass of lunar soil is oxygen. One of the most common lunar minerals is *ilmenite*, a mixture of iron, titanium, and oxygen. To separate *ilmenite* into its primary constituents, we add hydrogen and heat the mixture. This hydrogen reduction reaction is given by the 'molar' equation:

$$FeTiO_3 + H_2 \; \text{--->} \; Fe + TiO_2 + H_2O$$

A Bit Of Chemistry - This equation is read from left to right as follows: One mole of *ilmenite* is combined with one mole of molecular hydrogen gas to produce one mole of free iron, one mole of titanium dioxide, and one mole of water. Note that the three atoms of oxygen on the left side (O_3) is 'balanced' by the three atoms of oxygen found on the right side (two in TiO_2 and one in H_2O). **One 'mole' equals 6.02×10^{23} molecules**.

The 'molar mass' of a molecule is the mass that the molecule has if there are 1 mole of them present. The masses of each atom that comprise the molecules are added up to get the molar mass of the molecule. Here's how you do this:

For H_2O, there are two atoms of hydrogen and one atom of oxygen. The atomic mass of hydrogen is 1.0 AMU and oxygen is 16.0 AMU, so the molar mass of H_2O is 2 (1.0) + 16.0 = 18.0 AMU. **One mole of water molecules will equal 18 grams of water by mass.**

Problem 1 -The atomic masses of the atoms in the *ilmenite* reduction equation are Fe = 55.8 and Ti = 47.9. A) What is the molar mass of ilmenite? B) What is the molar mass of molecular hydrogen gas? C) What is the molar mass of free iron? D) What is the molar mass of titanium dioxide? E) Is mass conserved in this reaction?

Problem 2 - If 1 kg of ilmenite was 'cracked' how many grams of water would be produced?

Inquiry Question - If 1 kg of ilmenite was 'cracked' how many grams of molecular oxygen would be produced if the water molecules were split by electrolysis into

$$2\,H_2O \; \text{-- >} \; 2H_2 + O_2?$$

Problem 1 -

A) What is the molar mass of ilmenite? 1(55.8) + 1 (47.9) + 3(16.0) = **151.7 grams/mole**

B) What is the molar mass of molecular hydrogen gas? 2(1.0) = **2.0 grams/mole**

C) What is the molar mass of free iron? 1(55.8) = **55.8 grams/mole**

D) What is the molar mass of titanium dioxide? 1(47.9) + 2(16.0) = **79.9 grams/mole**

E) Is mass conserved in this reaction? **Yes. There is one mole for each item on each side, so we just add the molar masses for each constituent. The left side has 151.7 + 2.0 = 153.7 grams and the right side has 55.8 + 79.9 + 18.0 = 153.7 grams so the mass balances on each side.**

Problem 2 -

Step 1 - The reaction equation is balanced in terms of one mole of ilmenite (1.0 x $FeTiO_3$) yielding one mole of water (1.0 x H_2O) . The molar mass of ilmenite is 151.7 grams which is the same as 0.1517 kilograms, so we just need to figure out how many moles is needed to make one kilogram.

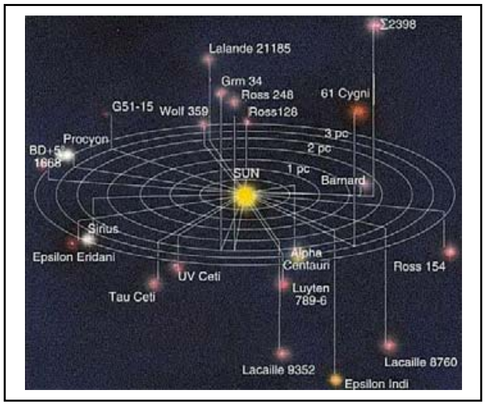
Step 2 - This will be 1000 grams/151.7 grams = 6.6 moles. Because our new reaction is that we start with 6.6 x $FeTiO_3$ that means that for the reaction to remain balanced, we need to produce 6.6 x H_2O, or in other words, 6.6 moles of water.

Step 3 - Because the molar mass of water is 18.0 grams/mole, the total mass of water produced will be 6.6 x 18.0 = **119 grams of water.**

Inquiry Question - The reaction is: 2 H_2O -- > $2H_2 + O_2$

This means that for every 2 moles of water, we will get one mole of O_2. The ratio is 2 to 1. From the answer to Problem 2 ,we began with 6.59 moles of water not 2.0 moles. That means we will produce 6.6/2 = 3.3 moles of water. Since 1 molecule of oxygen has a molar mass of 2(16) = 32 grams/mole, the total mass of molecular oxygen will be 3.3 moles x 32 grams/mole = **106 grams. So, 1 kilogram of ilmenite will eventually yield 106 grams of breathable oxygen.**

The Solar Neighborhood within 17 light years.



There are 45 stars within 17 light years of the sun. It is very hard to appreciate just how big space is. By considering our neighborhood in the Milky Way, we can start to get a sense of scale.

Local star map courtesy NASA/JPL

The table below gives the names, distances and angles to 11 of the most well-known neighbors to the sun. Although stars are spread out in 3-dimensional space, we will compress these distances to their 2-dimensional equivalents. On a 2-dimensional grid, place a dot at the Origin to represent the Sun. With a metric ruler and a protractor, plot the stars on a piece of paper and label each star. Use a scale of 1 centimeter = 1 light year.

Name	Angle	Distance
Alpha Centauri	220	4.3 light years
Barnard's Star	270	5.9
Wolf 359	170	7.6
Sirius	100	8.6
Epsilon Eridani	50	10.7
61 Cygni	315	11.2
Procyon	115	11.4
Tau Ceti	25	11.9
Kruger 60	335	12.8
40 Eridani	60	15.9
Altair	300	16.6

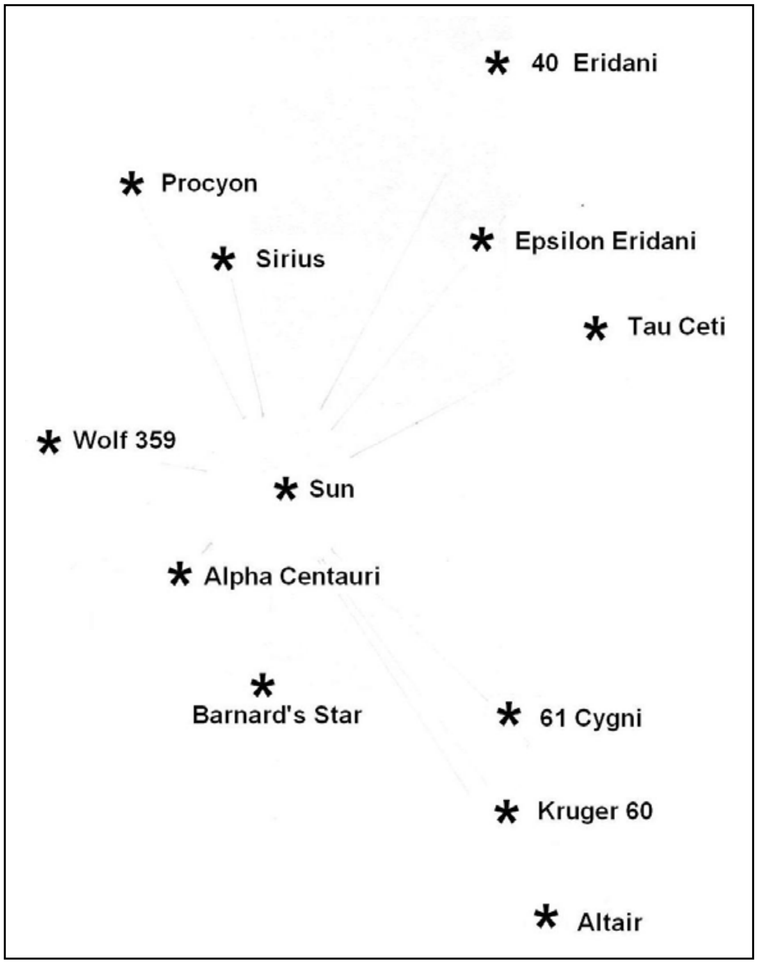
Problem 1 - What is the distance between Sirius and Altair?

Problem 2 - What is the distance between Kruger 60 and Altair?

Problem 3 - Can you find a pair of stars that are closer to each other than either of them are to the Sun?

Problem 4 - If you were starting from Earth, what is the shortest journey you could make that would visit all of the stars in your map?

Answer Key



Problem 1 - At the scale of 1 cm = 1 light year, they are separated by about 24 centimeters or **24 light years.**

Problem 2 - They are separated by 7 centimeters or **7 light years.**

Problem 3 - For example: Procyon and Sirius are 4 cm apart or **4 light years.** 61 Cygni and Kruger 60 are 3 cm apart or **3 light years.**

Problem 4 - One path consists of Sun-Barnards Star-Altair-Kruger 60- 61 Cygni-Tau Ceti-Epsilon Eridani- 40 Eridani-Sirius-Procyon-Wolf 359-Alpha Centauri -Sun. The total distance is about **80 light years.**

Our Neighborhood in the Milky Way

The Milky Way galaxy is a flat disk about 100,000 light years in diameter and 1000 light years thick. All of the bright stars, clusters and nebula we see are actually very close-by. Let's have a look at a few of these familiar landmarks!

The table below gives the distances and angles to a few familiar nebulae and star clusters within 7,000 light years of the Sun. Plot them on a paper with a scale of 1 centimeter = 500 light years, and with the Sun at the origin.

Object	Type	Distance		Angle
Pleiades	star cluster	410	ly	60
Orion Nebula	nebula	1500		80
Betelgeuse	star	650		90
Deneb	star	1600		310
Antares	star	420		245
Cygnus Loop	supernova remnant	2000		315
Ring Nebula	nebula	2300		280
Owl Nebula	nebula	1900		170
Crab Nebula	supernova remnant	6300		80
Praesepe	star cluster	520		130
Rosette Nebula	nebula	3,600		100
Eta Carina	nebula	7,000		160
Lagoon Nebula	nebula	4,000		270
Jewel Box	star cluster	6,500		190

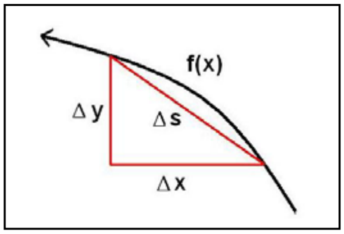
Problem 1 - If you only wanted to visit the three bright stars, how many light years would you have to travel for a round-trip tour?

Problem 2 - If you only wanted to visit all of the nebulas how long would your round-trip journey be?

Problem 1 - Sun - Betelgeuse - Deneb - Antares - sun would measure 10 centimeters, and from the scale of 1 cm=500 ly, it would be a **5,000 light year journey.**

Problem 2 - One possibility would be Sun - Cygnus Loop - Lagoon Nebula - Ring Nebula - Owl Nebula - Eta Carina Nebula - Rosette Nebula - Crab Nebula - Orion Nebula - Sun. This would be a journey of 45 centimeters or 45 x 500 = **22,500 light years.**

Calculating Arc Lengths of Simple Functions- I



Spirals are found in many different places in astronomy, from the shape of the arms in a 'spiral' galaxy, to the trajectory of a spacecraft traveling outward from Earth's orbit at constant velocity. Figuring out spiral lengths requires a bit of calculus. Here's how it's done:

$$\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Step 1: Study the figure above, and use the Pythagorean Theorem to determine the hypotenuse length in terms of the other two sides. It should look like the equation to the left.

$$\Delta s = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\ \Delta x$$

Step 2: Factor out the **Δx** to get a new formula.

$$S = \int_A^B \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

Step 3: Following the basic techniques of calculus, 'take the limit' and allow the deltas to become differentials, then use the integral calculus to sum-up all of the differentials along the curve defined by y = F(x), and between points A and B, to get the fundamental arc-length formula.

$$S = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$$

The arc length formula can be re-written in polar coordinates too. In this case, the function, y = F(x) has been replaced by the polar function **r(θ).**

Problem 1) Find the arclength for the line y = mx + b from x=3 to x=10

Problem 2) Find the arclength for the parabolic arc defined by $y = x^2$ from x=1 to x=5.

Problem 3) Find the arclength for the logarithmic spiral $R(\theta) = e^{b\theta}$ from $\theta = 0$ to $\theta = 4\pi$ if b = 1/2.

Problem 4) The spiral track on a CDROM is defined by the simple formula R = kq, where k represents the width of each track of data. If k = 1.5 microns, how long is the spiral track, in meters, for a standard 6.0-cm disk if the hub space is also used?

Answer Key

Problem 1) dy/dx = m, so the integrand becomes $(1 + m^2)^{1/2}$ dx. Because m is a constant independent of x, the integral is just $(1 + m^2)^{1/2}$ (10 - 3) = **$7 (1+ m^2)^{1/2}$.**

Problem 2) dy/dx = 2x and the integrand becomes $(1 + 4x^2)^{1/2}$ dx. This can be integrated by using the substitution 2x = sinh(u) , and dx = (1/2)cosh(u) du, so that the integrand becomes 1/2 $\cosh^2(u)$ du. This is a fundamental integral with the solution **1/2 [sinh(2u)/4 + u/2 + C]**.

Limits: The limits go from x=1 to x=5, but since x = 1/2 sinh(u), the limits re-expressed in terms of u become $u_1 = \sinh^{-1}(2) = 1.44$ and $u_2 = \sinh^{-1}(10) = 3.00$ so evaluating the definite integral leads to 1/2 (sinh(6.00)/4 + 3.00/2) - 1/2 (sinh(2.88)/4 + 1.44/2) = 25.96 - 1.47 = **24.49.** Because the limits are provided to three significant figures, the answer to the same number of significant figures will be **24.5**

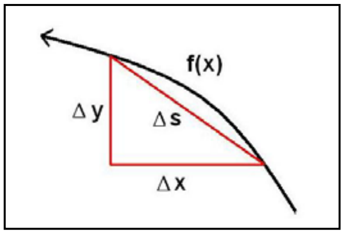
Problem 3) We use the polar form of the arclength formula. First perform the differentiation of r(θ) to get dr/dθ = b $e^{b\theta}$. Then after substitution, the integrand becomes $(e^{2b\theta} + b^2e^{2b\theta})^{1/2}$ dθ which after simplification then becomes $e^{b\theta}$ $(1 + b^2)^{1/2}$ dθ . This is easily integrated to get (1/b) $(1 + b^2)^{1/2}$ $e^{b\theta}$ + **C**. Since b = 1/2, we get the simpler form 2.24 $e^{\theta/2}$ + C. This can be evaluated between the two limits for θ to get 2.24 (534.86 - 1) = 1,195.85. Because the limits are provided to three significant figures, the answer to the same number of significant figures will be **1,200.**

Problem 4) Because r = kθ, the integrand becomes $(k^2\theta^2 + k^2)^{1/2}$ dθ or k $(1 + \theta^2)^{1/2}$ dθ. This can be simplified using the hyperbolic trig identity $1 + \sinh^2(x) = \cosh^2(x)$ where we have used the substitution θ = sinh(x). This also means that dθ = cosh(x) dx. Then the integrand becomes $\cosh^2(x)$ dx. The integral is then a fundamental integral with the solution **k [sinh(2x)/4 + x/2 + C]**.

Limits: How many radians does the spiral take up? 2 π x 6 cm/1.6 microns = 2 π x 60000/1.5 = 80,000 π. This means that the integral will have limits from 0 to 80,000 π. But q = sinh(x) so the limits become x1 = 0 to x2 = $\sinh^{-1}$(80,000 π) = 13.13.

The definite integral is then 1.5 microns x [(sinh(26.26) +13.13/2 + C) - (sinh(0)/4 + 0/2 +C) = 0.0000015 meters (1.26 x 10^{11} + 6.56) = 189,000 meters or 189 kilometers. Because the problem only gives 1.5 and 6.0 to two significant figures, this becomes the maximum accuracy of the available numbers, so the answer to the same number of significant figures will be **190 kilometers!**

The Ant and the Turntable - Frames of Reference



$$s = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\; d\theta$$

Figuring out spiral lengths requires a bit of calculus. In the previous problem, we saw that the arc length integral can be written in polar coordinates where the function, y = F(x) is replaced by the polar function **r(θ).**

Because this formula is completely general, the variable, θ, can refer to angle, time or any other independent variable which leads to an arc-like geometry in the dependent variable defined by r(θ). Here is an interesting application.

An ant takes a journey from the center of a CDrom (r=0) to a point at its edge (r=5cm) at a leisurely pace of 5 cm/minute. Meanwhile, the turntable is spinning at 1 rotation per minute.

Problem 1 - Draw a sketch, to scale, of the turntable that shows the ant's motion from its own, stationary, perspective.

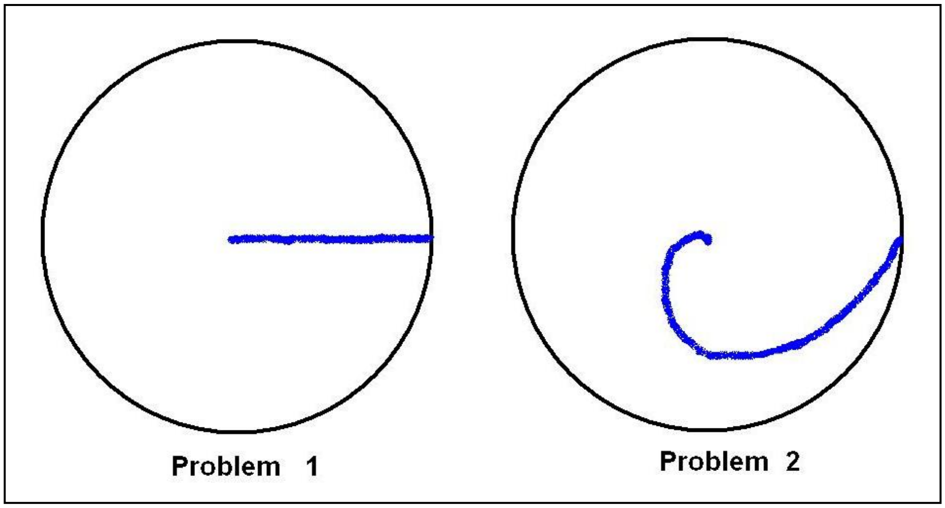
Problem 2 - Draw a free-hand sketch, to scale, of the turntable that shows the ant's motion from the perspective of an outside, stationary observer.

Problem 3 - What is the equation that describes the radial motion of the ant with respect to time?

Problem 4 - How far did the ant travel in the radial direction in A) 15 seconds?, B) 60 seconds?

Problem 5 - Evaluate the arc length integral formula for S(t) to determine the length of the ant's arc between r=0 and r = 5 centimeters.

Inquiry: Explain A) how it can be that there are two path lengths and travel times in the problem, but there is only one ant? and B) which of the two answers is correct?



Problem 3) **R(T) = (5 cm/minute) T** where T is given in minutes of time.

Problem 4) A) 0.25 minutes x 5 cm/minute = 1.25 centimeters,
B) 1 minute x 5 cm/minute **= 5 centimeters.**

Problem 5) dr/dT= 5 cm/m so the integrand becomes by substitution, $dS = (r^2 + (dr/dt)^2)^{1/2}$ or $dS = ((5\,T)^2 + 5^2)^{1/2}\,dT$. This simplifies to $dS = 5\,(1 + T^2)^{1/2}\,dT$. The integral then becomes,

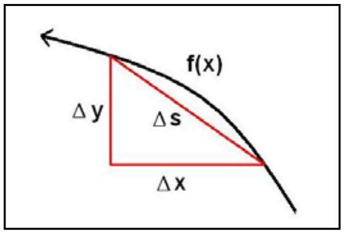
$$S(t) = 5 \int_0^t \sqrt{1 + T^2}\; dT$$

This can be integrated by using the substitution T = sinh(x) where dT = cosh(x) dx so that $(1 + T^2)^{1/2}$ dT becomes cosh(x) cosh(x) dx, or $\cosh^2(x)dx$. The integral is a fundamental form whose solution is sinh(2x)/4 + x/2 + C. Evaluating from T =0 to T=t one gets the general arc length formula for the ant of S(t) = 5 cm/m x (sinh(2t)/4 + t/2) where t is the elapsed time expressed in minutes.

To travel from r=0 to r=5cm, the ant takes 1 minute, so S(1) = 5 cm/minute x (3.63/4 + 1/2) = 5 cm/m (1.4 minutes) = 7.0 centimeters.

Inquiry: There are two reference frames, and the movement of the ant will appear different to the two observers. Only in the 'proper' frame of reference locked to the ant will the movement of the ant appear to be 'simple' as it travels in a straight line from the center to the edge of the CDROM according to R(T) = 5 T. In all other 'improper' reference frames, the motion will appear more complicated, and trace out more complex paths because of the action of fictitious forces. For example, if you are sitting in a moving car on the freeway and you drop a ball to the floor, from your perspective, it travels 1/2 meter straight down. For someone standing by the side of the road, the ball travels in a long 100-meter arc.

B) They are both correct, but they apply to two different frames of reference who are seeing the movement differently because of their relative state of motion.

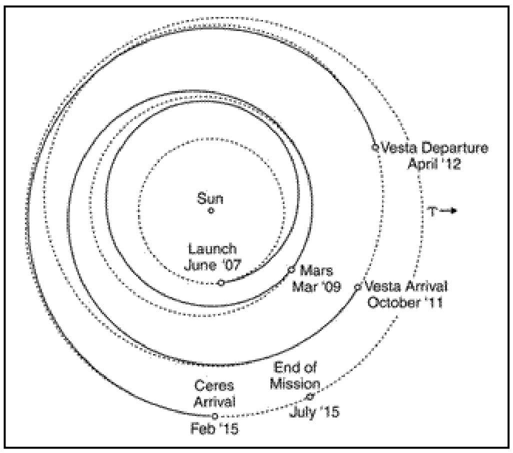


Ion rocket motors provide a small but steady thrust, which causes a spacecraft to accelerate. The shape of the orbit for the spacecraft as it undergoes constant acceleration is a spiral path. The length of this path can be computed using calculus.

The arc length integral can be written in polar coordinates where the function, y = F(x) is replaced by the polar function **r(θ).**

$$s = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Because the integrand is generally a messy one for most realistic cases, in the following problems, we will explore some simpler approximations.



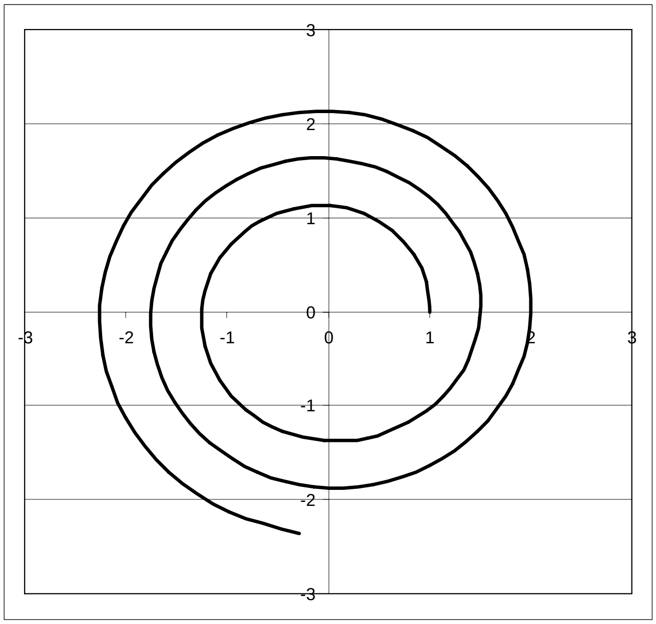
The Dawn spacecraft was launched on September 27, 2007, and will take a spiral journey to visit the asteroid Vesta in February 2015. Earth is located at a distance of 1.0 Astronomical Units from the Sun (1 AU = 150 million kilometers) and Vesta is located 2.36 AU from the Sun. The journey will take about 66,000 hours and make about 3 loops around Earth's orbit in its outward spiral as shown in the figure to the left.

Problem 1 - Suppose that the Dawn spacecraft travels at a constant outward speed from Earth's orbit. If we approximate the motion of the spacecraft by $X = R\cos\theta$, $Y=R\sin\theta$ and $R = 1 + 0.08\,\theta$, where the angular measure is in radians, show that the path taken by Dawn is a simple spiral.

Problem 2 - From the equation for $R(\theta)$, compute the total path length of the spiral from R=1.0 to R = 2.36 AU, and give the answer in kilometers. About what is the spacecraft's average speed during the journey in kilometers/hour? [Note: Feel free to use a Table of Integrals!]

Problem 3 - The previous two problems were purely 'kinematic' which means that the spiral path was determined, not by the action of physical forces, but by employing a mathematical approximation. The equation for $R(\theta)$ is based on constant-speed motion, and not upon actual accelerations caused by gravity or the action of ion engine itself. Let's improve this kinematic model by approximating the radial motion by a uniform acceleration given by $R(\theta) = 1/2\, A\, \theta^2$ where we will approximate the net acceleration of the spacecraft in its journey as A = 0.009. What is the total distance traveled by Dawn in kilometers, and its average speed in kilometers/hour?

Problem 1) Answer computed using Excel spreadsheet.

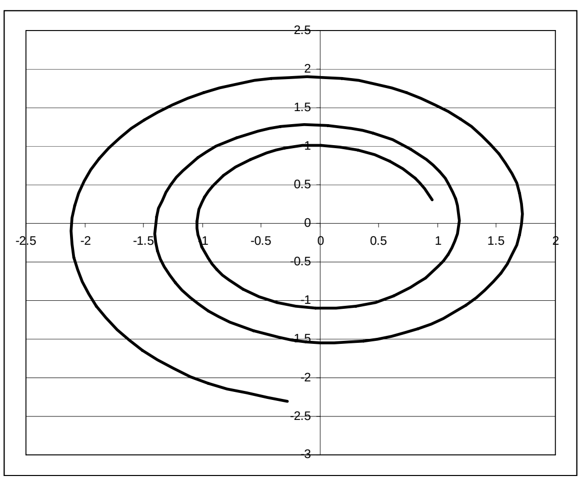


Problem 2: R = 1.0 + 0.08 θ and so $dR/d\theta$ = 0.08 and $d\theta/dR$ = 12.5. The integrand becomes $(1+ 156R^2)^{1/2}$ dR .

If we use the substitution U = 12.5R dU = 12.5 dR and the integrand becomes 0.08 $(1 + U^2)^{1/2}$ dU. A table of integrals yields the answer

1/2 [U $(1 + U^2)^{1/2}$ + ln (U + $(1+ U^2)^{1/2}$).

The limits to the integral are Ui = 12.5 x 1.0 = 12.5 and Uf = 12.5 x 2.36 = 29.5, and when the integral is evaluated we get 1/25 [29.5 (29.5) + ln (29.5 + (29.5)) - 12.5 (12.5) - ln(12.5 + (12.5))] = 1/25 (870 + 4.1 - 156 - 3.2) = 28.6 Astronomical Units or 28.6 x 150 million km = 4.3 billion kilometers! The averages speed would be about 4.3 billion/66000 hrs = **65,100 kilometers/hour.**



Problem 3 - $dR/d\theta = A\,\theta$ so that
$d\theta/dR = 1/(A\,\theta)$.

From $R(\theta)$, we can re-write $d\theta/dR$ solely in terms of R as $d\theta/dR = (1/(2Ar))^{1/2}$ so that the integrand becomes $(1 + R/(2A))^{1/2}$ dR.

Unlike the integral in Problem 1 ,this integral can be easily performed by noting that if we substitute

U = 1 + R/(2A), and dU = dR/2A,

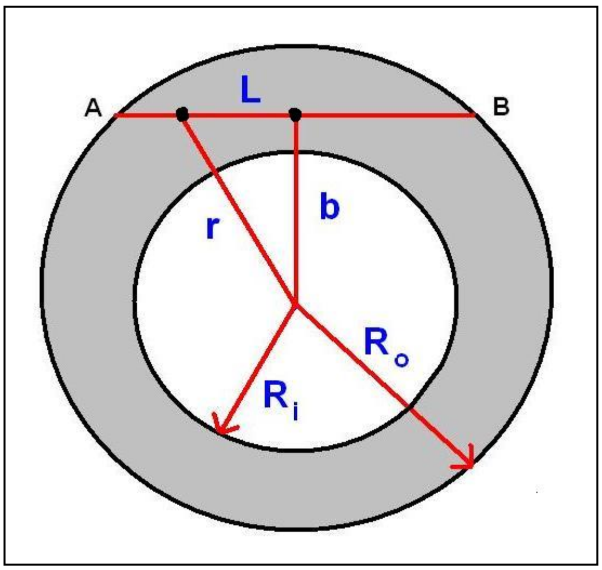
we get the integrand 2A $U^{1/2}$ dU and so
S = (4A/3) $U^{3/2}$ + C.

The limits to this integral are Ui = 1 + 1.0/2A = 56. and Uf = 1 + 2.36/2A = 132.

Then the definite integral becomes
S = (4 x 0.009/3) [$132^{3/2}$ - $56^{3/2}$] = 0.012 [1516 - 419] = 13.2 AU .Since 1 AU = 150 million km, the spiral path has a length of **2.0 billion kilometers.** The averages speed would be about 2.0 billion km/66000 hours = **30,300 km/hour**. The trip takes less time because the 'kinematic' motion is speeded up towards the end of the journey.

Modeling a Planetary Nebula

Planetary nebula are the outer atmospheres of dying stars ejected into space. Astronomers model these nebulae to learn about the total mass they contain, and the details of how they were ejected. The image is of a rare, spherical-shell planetary nebula, Abell 38, photographed by astronomer George Jacoby (WIYN Observatory) and his colleagues using the giant, 4-meter Mayall Telescope at Kitt Peak, Arizona. Abell-38 is located 7,000 light years away in the constellation Hercules. The nebula is 5 light years in diameter and 1/3 light year thick. For other spectacular nebula images, visit the Hubble Space Telescope archive at
http://hubblesite.org/newscenter/archive/releases/nebula



Statement of the Problem:

We want to calculate the intensity of the nebula (shaded shell) at different radii from its center (b) along a series of chords through the nebula (AB). The intensity, I(b) will be proportional to the density of gas within the nebula, which we define as D(r).

The shell is spherically-symmetric, as is D(r), so there are obvious symmetries in the geometry of the problem.

Because D(r) varies along the chord AB, we have to sum-up the contribution to I(b) from each spot along AB.

Problem 1 - Using the Pythagorean Theorem, define the distance L between the two points on segment AB in terms of b and r.

Problem 2 - Calculate the differential, dL in terms of r and b, assuming b is a constant.

Problem 3 - Construct the differential dI(b,r) = D(r)dL explicitly in terms of r and b.

Problem 4 - Integrate dI(b,r) to get I(b) with the assumption that D(r) = 0 from r = 0 to r = R_i, and is constant throughout the shell from r=R_i to r = R_0, and that D(r) = D_0.

Problem 5 - Assuming that all linear units are in light years, plot the 1-d function I(b) for R_i = 2.2 and R_0 = 2.5, and from b=0 to b=5.0, and compare it with Abell-38. Does Abell-38 seem to follow a constant-density shell model?

Problem 1 - $L^2 = r^2 - b^2$

Problem 2 - $2L\,dL = 2r\,dr$ so $dL = r\,dr / L$, and by substitution for $L = (r^2 - b^2)^{1/2}$, you get

$$dL = r\,dr / (r^2 - b^2)^{1/2}$$

Problem 3 - Construct the differential $dI(b,r) = D(r)dL$ explicitly in terms of r and b.

$$dI = D(r)\, r / (r^2 - b^2)^{1/2}\ dr$$

Problem 4 - Integrate $dI(b,r)$ to get $I(b)$ with the assumption that $D(r) = 0$ from $r = 0$ to $r = R_i$, and is constant throughout the shell from $r=R_i$ to $r = R_0$, and that $D(r) = D_0$.

$$I(b) = \int_{R_i}^{R_o} \frac{D_o\, r}{\sqrt{r^2 - b^2}}\, dr$$

D_0 is a constant, so it can be factored out from the integrand. Use the substitution $U = r^2 - b^2$, so that $dU = 2rdr$, and the integrand becomes $(1/2)\, D_0\ dU/U^{1/2}$. The integral can easily be solved to get $D_0\, (r^2 - b^2)^{1/2} + C$. Evaluating this 'front half integral' for the stated limits, and remembering that there are two halves to the shell, leads to the function:

$$I(b) = 2\, D_0\, (R_0^2 - b^2)^{1/2} - 2D_0\, (R_i^2 - b^2)^{1/2}.$$

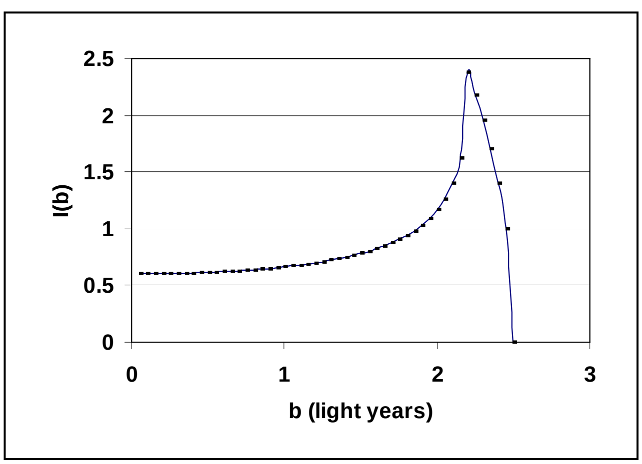
Problem 5 - There are actually two parts to evaluating this function, making it a 'piecewise' function. The first part is for b <= 2.2 ly, for which you get:

$$I(b) = 2D_0\, (6.25 - b^2)^{1/2} - 2D_0\, (4.84 - b^2)^{1/2}$$

The second part is the solution for b > 2.2 ly for which you will get

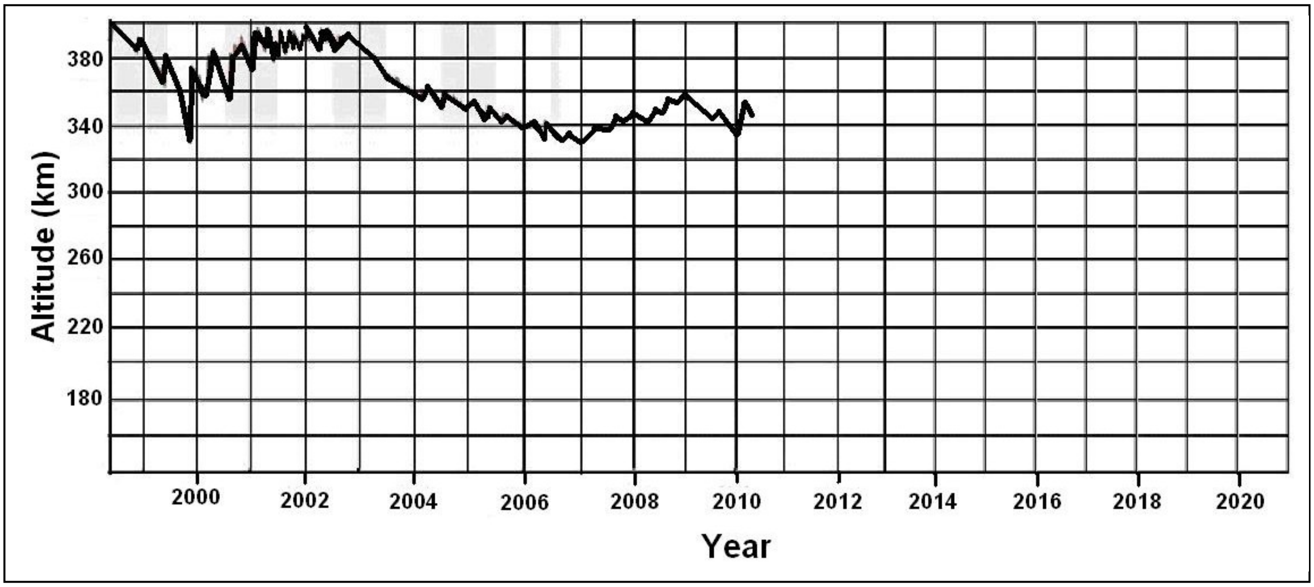
$$I(b) = 2D_0\, (6.25 - b^2)^{1/2}$$

This function, plotted below, does seem to show an increase in brightness in the nebula between R=2.0 and 2.5 light years which matches the picture of the nebula Abell-38, so the nebula may be a hollow, thin spherical shell with a uniform density of gas!



The International Space Station - Follow that graph!

At the present time, the International Space Station is losing about 300 feet (90 meters) of altitude every day. Its current altitude is about 345 km after a 7.0-km re-boost by the Automated Transfer Vehicle, Jules Vern spacecraft on June 20, 2008. The graph below shows the ISS altitude since 1999.



The drag of Earth's atmosphere causes the ISS altitude to decrease each day, and this is accelerated during sunspot maximum (between 2000-2001) when the dense atmosphere extends to a much higher altitude. At altitudes below about 200 km, spacecraft orbits decay and burn up within a week.

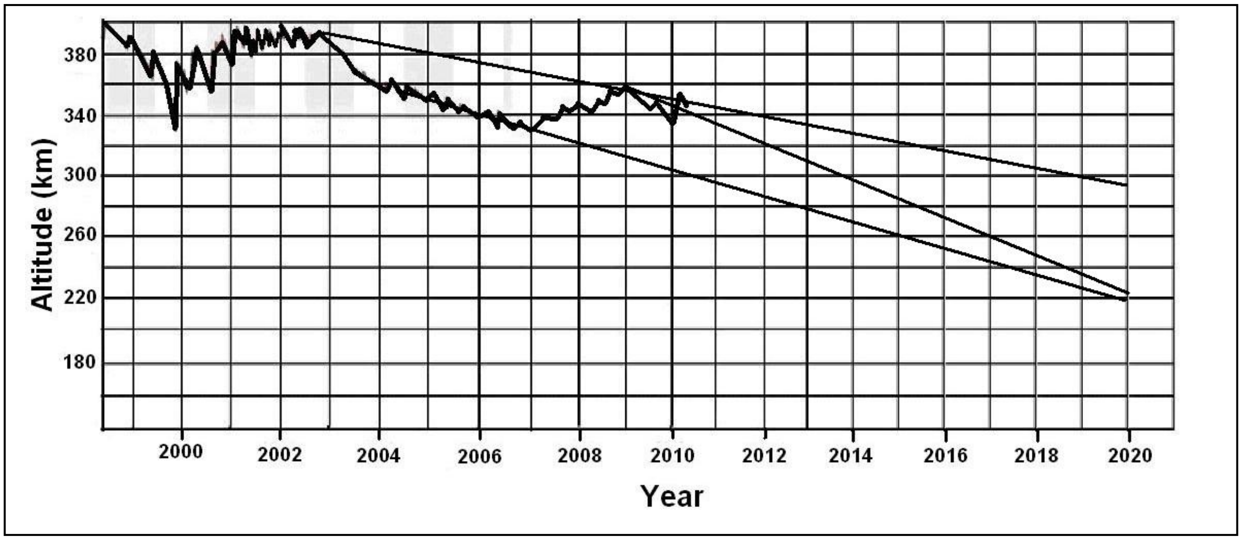
Problem 1 - From the present trends, what do you expect the altitude of the ISS to be between 2010 until its retirement year around 2020?

Problem 2 - Sunspot maximum will occur between 2012-2014, and we might expect a 50-km decline in altitude during this period if the solar activity weaker than the peak in 2000, which is currently forecasted. Including this effect, what might be the altitude of the ISS in 2020? Is the ISS in danger of atmospheric burn-up?

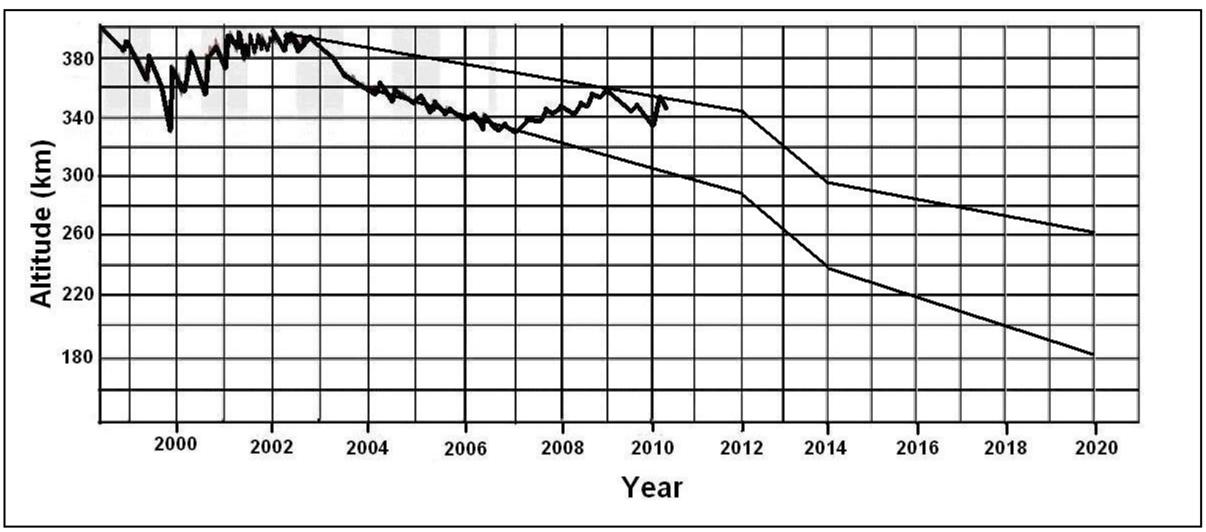
Problem 3 - What are the uncertainties in predicting ISS re-entry, and what strategy would you use if you were the Program Manager for the ISS?

Answer Key

Problem 1 – **Answer: The graph below shows several plausible linear trends depending on which features you use as a model for the slope. The predicted altitude would be between 220 and 300 km** .



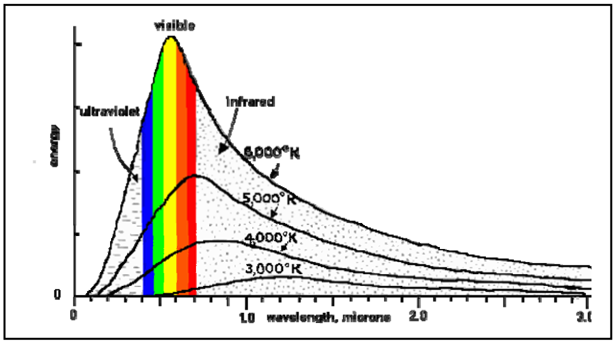
Problem 2 - The graph below shows, for example, two forecasts that follow the extremes of the general decline trend, but then assume all of the altitude loss occurred between 2012-2014 at 50 km. Note that the range of altitudes in the graph in either case is 180-260 km. This places the ISS in danger of burn-up before its retirement year in 2014.



Problem 3 - The largest uncertainty is the strength of the next solar activity (sunspot) cycle. If it is stronger than the previous maximum between 2000-2001, the ISS altitude losses will be even larger in 2012-2014, and the ISS will be in extreme danger of re-entry before 2020.

The period after 2010, when the US losses access to space travel and has to rely on Russian low-capacity shuttles, will be a critical time for the ISS, and an intensely worrisome one for ISS managers.

Why are hot things red? 36



When radiation is produced by a heated body, the intensity of electromagnetic radiation depends on frequency (wavelength) in a manner defined by the Planck Function. There is a simple law, called the Wein Displacement Law, that relates the temperature of a body to the frequency where the Planck curve has its maximum value. In this exercise, we will use two different methods to derive this law.

$$I(\lambda, T) = \frac{A}{\lambda^5 \left(e^{\left[\frac{14394}{T\lambda}\right]} - 1 \right)}$$

where: $A = 3.747 \times 10^{14}$ watts microns^4/m^2/str

Temperature (K)	Peak Wavelength (microns)
10,000	**0.2898**
9,000	**0.322**
8,000	**0.362**
7,000	**0.414**
6,000	**0.483**
5,000	**0.579**
4,000	**0.724**
3,000	**0.966**
2,000	**1.449**
1,000	**2.828**
500	**5.796**
300	**9.660**

Algebra Problem:

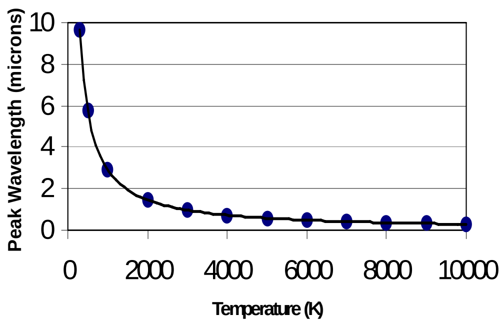
A) From the data in the table, use a calculator to find a formula that fits the data. Some possibilities might include a linear equation, $\lambda = a\,T$, power laws such as $\lambda = b\,T^{-1}$, $\lambda = c\,T^{-2}$ or $\lambda = d\,T^{2}$, $\lambda = e\,T^{3}$ or exponential functions such as $\lambda = f\,e^{(gT)}$ where a,b,c,d,e,f,g are constants determined by the fitting process. B) Which function fits the tabulated data the best, and what is the value of the constant?

Calculus Problem:

A) Find Equation 2 for the maximum of the function $I(\lambda,T)$ by differentiating with respect to the wavelength, λ, and setting the derivative equal to zero.

B) Find the solution to Equation 2, which you found in Part A, using the technique of 'successive approximation', or 'trial and error'. (Note, ignore trivial solutions involving zero! From this iterated solution, find the form of the function for the maximum wavelength as a function of temperature.)

Answer Key:



Algebra Problem: The figure above shows the data plotted in the table, and the best fit curve. This was done by copying the table into an Excel spread sheet, plotting the data as an XY scatter plot, and using 'Add Trend line'. Students may experiment with various choices for the fitting function using an HP-83 graphing calculator, or the Excel spread sheet trend line options, but should find that the best fit has the form: $\lambda = b\ T^{-1}$ where b = 2898.0 micronsKelvins. Students may puzzle over the units 'micronsKelvins' but it is often the case in physics that units have complex forms that are not immediately intuitive.

Calculus Problem: A) Below is a recommended strategy:

Let $U = A\,\lambda^{-5}$ then $dU/d\lambda = -5\,A\,\lambda^{-6}$

Let $V = e^{(14329/(\lambda\ T))} - 1$ then $dV/d\lambda = -14329 / (\lambda^2\ T)\ e^{14329/(\lambda t)}$

Then use the quotient rule: $d/d\lambda\ (U/V) = 1/V\ dU/d\lambda - U/V^2\ dV/d\lambda$

To get $dU/d\lambda - U/V\ dV/d\lambda = 0$

Then by substitution and a little algebra

$5\ l\ T\ (e^{14329/(\lambda T)} - 1) - 14329\ e^{14329/\lambda T} = 0$

Let $X = 14329/(\lambda\ T)$ then we get a simpler equation to solve:

$5(e^x - 1) - x\,e^x = 0$ (Equation 2)

Equation 3, when solved, will give the location of the extrema for the Planck Function, however, it cannot be solved exactly. You will need to program a graphing calculator or use an Excel spreadsheet to find the value for X that gives, in this case, the maximum value of the Planck Function.

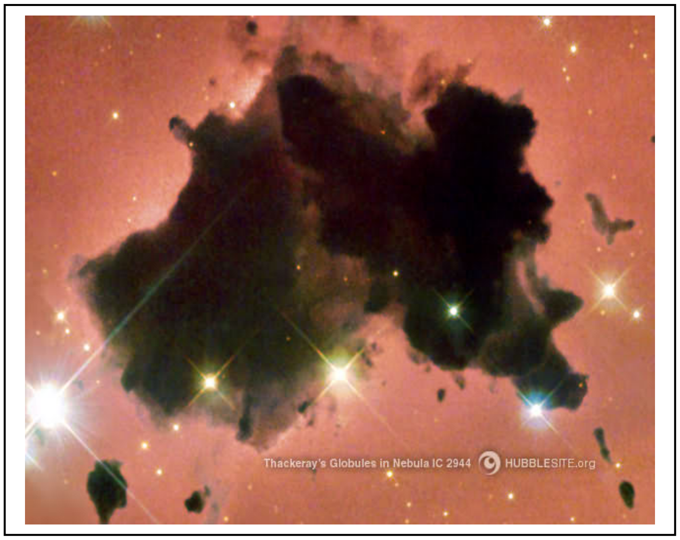
Calculus Problem: B) The table shows some trial-and-error results for Equation 3, and a convergence to approximately X = 4.965 The formula for the peak wavelength is then

$4.965 = 14394 / (\lambda\ \ T)$ or **$\lambda = 2899 / T$**

This is similar to the function derived by fitting the tabulated data.

X	Eq. 3
1	5.873127
2	17.16717
3	35.17107
4	49.59815
4.5	40.00857
4.9	8.428978
4.95	2.058748
4.96	0.703752
4.965	0.015799
4.966	-0.12263

Collapsing Gas Clouds - Stability



Gas clouds in interstellar space are acted upon by external pressure and their own gravity, and would otherwise collapse, but if they are hot enough, they can remain stable for a long time. That seems to be the case for objects called Bok Globules.

This photo of Thackeray's Globule (IC-2944) taken by the Hubble Space Telescope may be a stable dark cloud containing 10 times the mass of our sun at a temperature of less than 100 K.

A gas sphere with a radius, R, a mass, M, and a temperature, T, is subject to an external pressure, P so that

$$P = \frac{3 M k T}{4 \pi R^3 \mu m} - \frac{3 G M^2}{20 \pi R^4}$$

where k, G and μ are constants.

Problem 1 - At what minimum radius will the cloud start to collapse for a given mass and temperature?

Problem 1 - The problem states that the mass and temperature are held constant, so the only free variable is R. For complicated equations, it is always a good idea to group all constants together and define new constants. You can later replace the new constants by the old ones. Let's define A = ($3MkT/4\pi\mu m$) and B=($3GM^2/20\pi$), then the equation becomes $P = AR^{-3} - BR^{-4}$. To find the extremum, we calculate dP/dR and set this equal to zero. This gives us $dP/dR = A(-3)R^{-4} - B(-4)R^{-5} = 0$. This leads to R = 4 B/3 A which upon substituting back for the definitions of A and B gives us

$$R_c = \frac{12G\mu m}{45kT}$$

The Big Bang - Cosmic Expansion

$$\frac{dR}{dt} = \sqrt{\frac{8\pi G\rho}{3R} + \frac{\Lambda}{3}R^2}$$

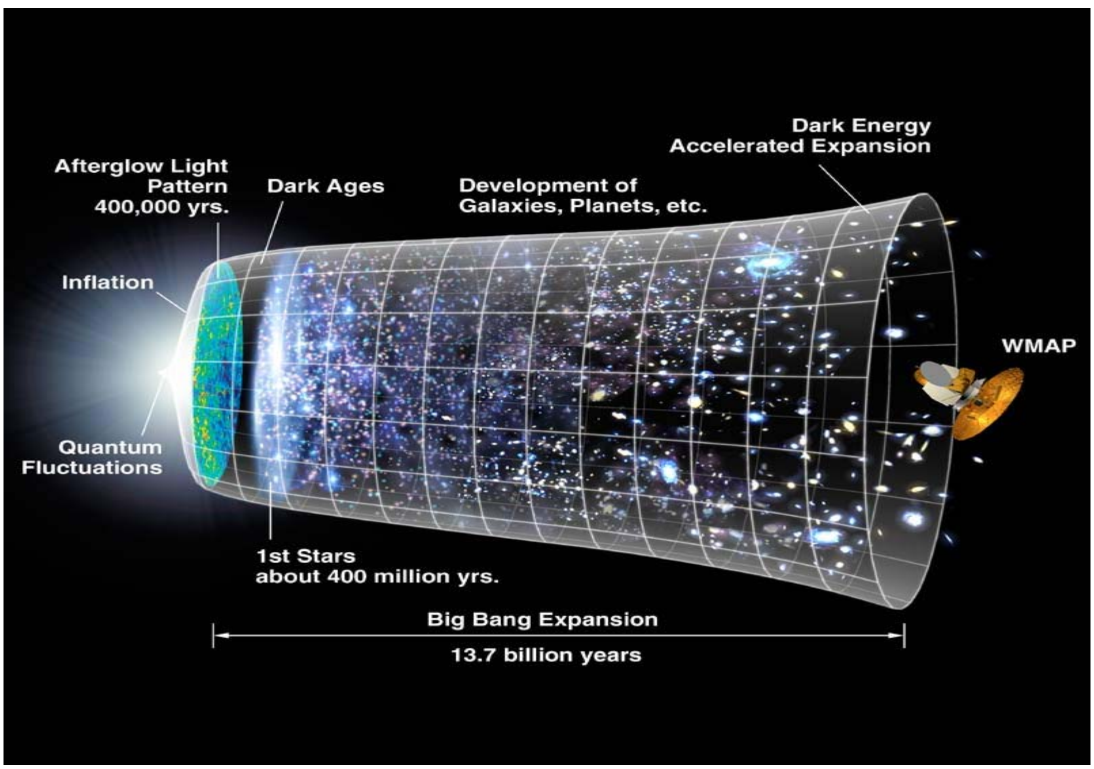
According to Big Bang theory, the scale of the universe increases with time at a rate that depends on the density of matter, ρ, and the size of the cosmological constant, Λ. This is defined by the fundamental equation to the left.

Problem 1 - Determine the general form of the integral that relates the time, t, to the value of the scale factor, R; Solve the integral for the time, t, but do not solve the integral for R.

Problem 2 - Transform the integral for R to a new variable, U, such that U = $(A/C)^{1/3}$ R where A = Λ /3 and C = $8\pi G\rho/3$.

Problem 3 - Solve the integral for two special cases A) The Inflationary Universe case where U >> 1 and B) the matter-dominated universe case where U << 1.

Problem 4 - Hubbel's Constant is a measure of the rate of expansion of the universe. It is defined as H = 1/R (dR/dt). Find the formula for Hubbel's Constant for the two cosmological cases described in Problem 3.



Answer Key

Problem 1 - The integral equation is then

$$\int dt = \int \frac{dR}{\sqrt{\frac{8\pi G\rho}{3R} + \frac{\Lambda}{3}R^2}}$$

Problem 2 First clean up the rather cumbersome radical expression so that it only involves R to positive powers and the constants A and C, by factoring out $(1/R)^{1/2}$ to get $(1/R)^{1/2}(C + A R^3)^{1/2}$. Factor out the constant C from the square-root so that the denominator of the integrand becomes $(1/R)^{1/2} C^{1/2} (1 + A/C R^3)^{1/2}$ and replace with $U = (A/C)^{1/3} R$ to get

$$C^{1/2} (A/C)^{1/6} U^{-1/2} (1 + U^3)^{1/2}$$

Note that we have also transformed the $(1/R)^{1/2}$ factor by replacing it with $(A/C)^{1/6} U^{-1/2}$. Since $dU = (A/C)^{1/3} dR$, we can now re-write the complete integrand as

$$(1/C)^{1/2} (C/A)^{1/6} (C/A)^{1/3} U^{1/2} dU / (1 + U^3)^{1/2}$$

After combining the constants A and C and replacing them with their definitions the integrand simplifies to $(3/\Lambda)^{1/2} U^{1/2} dU / (1 + U^3)^{1/2}$ and the integral becomes

$$t = \sqrt{\frac{3}{\Lambda}} \int \frac{U^{1/2}\, dU}{\sqrt{U^3 + 1}}$$

Problem 3 A) If U > 1, then the term under the square-root is essentially U^3, so we get $U^{1/2} / U^{3/2} = 1/U$. This leads to an integrand of $(3/\Lambda)^{1/2}$ 1/U dU which is a fundamental integral whose solution is $t = (3/\Lambda)^{1/2} \operatorname{Ln} U + C$. This can be re-written as $U(t) = e\,[(\Lambda/3)^{1/2} t\,]$. From the definition for U we get

$$R(t) = \left(\frac{8\pi G\rho}{\Lambda}\right)^{\frac{1}{3}} e^{\left(\frac{\Lambda}{3}\right)^{1/2} t}$$

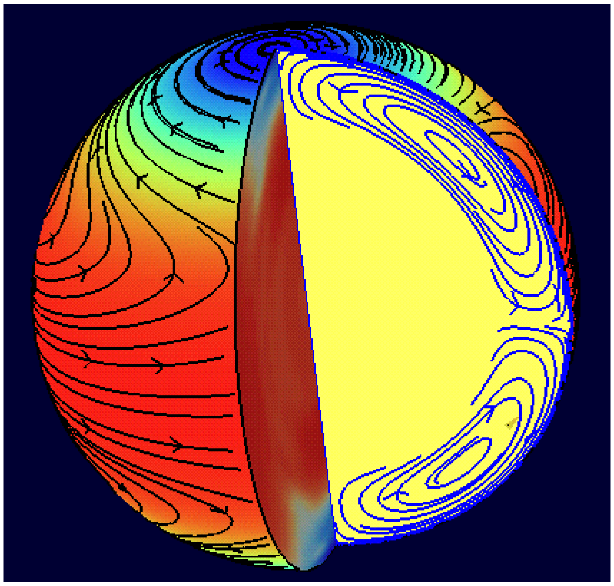
This represents a universe that expands at an exponential rate because of the positive pressure provided by the cosmological constant - a property of the energy of empty space. This solution is thought to describe our universe during its' inflationary' era shortly after the Big Bang.

Problem 3 B) In this case, U << 1 so the term under the square-root is essentially 1, and the integrand becomes $(3/\Lambda)^{1/2} U^{1/2} dU$. This is easily integrated to get $t = (3/\Lambda)^{1/2} U^{3/2}$. After substituting for the definition of U we get $t = (3/\Lambda)^{1/2} (\Lambda/8\pi G\rho)^{1/2} R^{3/2}$ so that $t = (3/8\pi G\rho)^{1/2} R^{3/2}$. This can be easily inverted to get

$$\mathbf{R(t) = (8\pi G\tilde{\rho} / 3)^{1/3}\ t^{2/3}}$$

This solution is the 'matter-dominated' cosmology represented by Big Bang cosmology, and applies to the modern expansion of the universe.

Problem 4 A) $\mathbf{H = (\Lambda/3)^{1/2}}$ and B) **H = 2/3 (1/t)**. In the inflationary case, the rate of expansion is constant in time, but in the matter-dominated case, the expansion rate decreases in proportion to the age of the universe !



There are many situations in which differentiation has to be performed on formulae in astrophysics. Many objects such as stars and galaxies display 'differential rotation' which leads to many interesting and unusual phenomena. The formula describing these phenomena are usually 'differential equations' that relate changes in one quantity to changes in another.

Here are some popular equations used in astrophysics whose differentiation will test your basic skills!

Image: Model of solar differential rotation (Courtesy: Stanford Solar Center / NASA SOHO)

$$m = \frac{M}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L = 4\pi R^2 \sigma T^4$$

$$R = \left(\frac{3\, m\, V}{\pi\ \rho}\, t \right)^{1/4}$$

$$V = \frac{(Z+1)^2 + 1}{(Z+1)^2 - 1}$$

$$D = \left(\frac{k\ T}{4\,\pi\, e^2 N} \right)^{1/2}$$

$$\Lambda = \frac{D^2 (D-1)^2 (D+2)}{8\,\pi\,(D+4)^2}\, m^2$$

Problem 1 - Find dm/dv - the rate of change of mass with velocity near the speed of light.

Problem 2 - Find dL/dT - The rate of change of a star's luminosity with its temperature.

Problem 3 - Find dR/dt - The rate of change of the size of an expanding supernova remnant with time (in other words, its expansion speed!).

Problem 4 - Find dV/dz - The rate of change of the apparent speed of a body with its gravitational redshift.

Problem 5 - Find dD/dN - the rate of change of the Debye shielding radius in a plasma with a change in the density of the plasma.

Problem 6 - Find $(1/m^2)\ d\Lambda/dD$ - the rate of change of the energy of empty space as you change the number of dimensions to space.

Answer Key

Problem 1 - Find dm/dv - the rate of change of mass with velocity near the speed of light.

$$\frac{dm}{dv} = \frac{M\,v}{c^2\,(1-[v^2/c^2])^{3/2}}$$

Problem 2 - Find dL/dT - The rate of change of a star's luminosity with its temperature.

$$\frac{dL}{dT} = 16\,\pi\,R^2\,\sigma\,T^3$$

Problem 3 - Find dR/dt - The rate of change of the size of an expanding supernova remnant with time (in other words, its expansion speed!).

$$\frac{dR}{dt} = 1/4\,\,(3\,mV/(\pi\,\rho))^{1/4}\,\,t^{-3/4}$$

Problem 4 - Find dV/dz - The rate of change of the apparent speed of a body with its gravitational redshift. This requires using the quotient rule for differentiation $d(U/V) = (1/V)\,dU - (U/v^2)\,dV$

$$\frac{dV}{dz} = \frac{1}{(z+1)^2+1}\,2(z+1) - \frac{1}{[\,(z+1)^2+1]^2}\,2(z+1) = \frac{2(z+1)^3}{[\,(z+1)^2+1]^2}$$

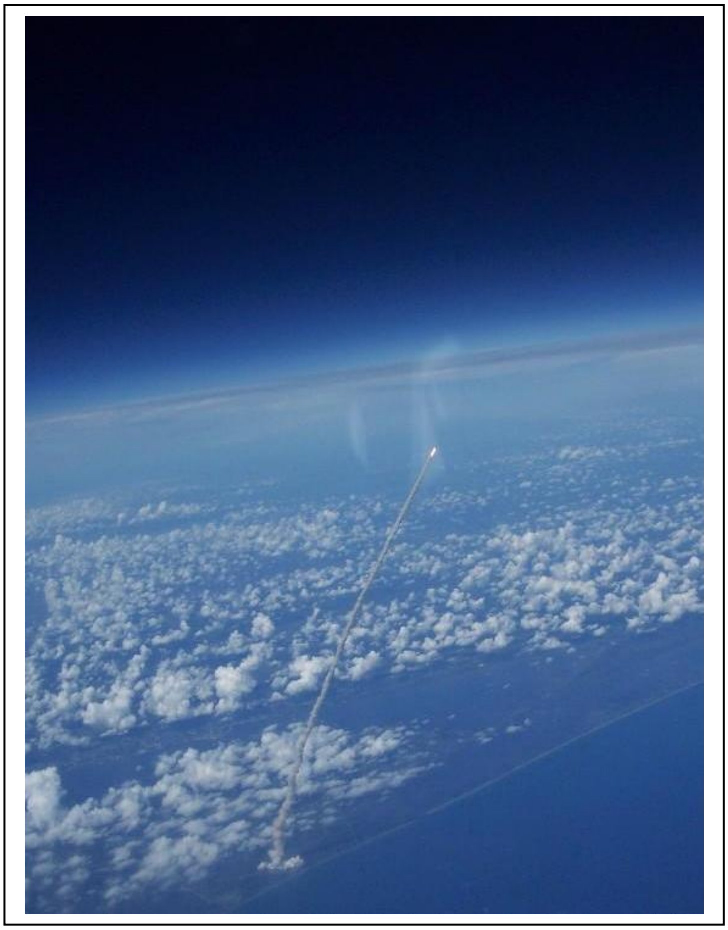
Problem 5 - Find dD/dN - the rate of change of the Debye shielding radius in a plasma with a change in the density of the plasma.

$$\frac{dD}{dN} = -1/2\,(kT/4\pi\,e^2)^{1/2}\,\,N^{-3/2}$$

Problem 6 - Find dΛ/dD - the rate of change of the energy of empty space as you change the number of dimensions to space. Again the quotient rule is needed where $U = D^5 - 3D^3 + 2D^2$ $V = D^2 + 8D + 16$

$$\frac{1}{m^2}\,\frac{d\Lambda}{dD} = \frac{5D^4 - 9D^2 + 4D}{8\,\pi\,(D^2+8D+16)} - \frac{[D^5 - 3D^3 + 2D^2]\,(2D+8)}{[8\,\pi\,(D^2+8D+16)]^2}$$

Space Shuttle Launch Trajectory - I



The trajectory of the Space Shuttle during the first 5 minutes of the launch of STS-30 can be represented by an equation for its altitude

$$h(T) = 2008 - 0.047\,T^3 + 18.3\,T^2 - 345T$$

and an equation for its down-range distance due-east

$$R(T) = 4680\,e^{0.029T}$$

where the distances are provided in units of feet commonly used by NASA engineers for describing trajectories near Earth. The problems below will use these 'parametric equations of motion' to determine the time of the highest acceleration.

Image: Shuttle launch as seen from a NASA aircraft.

Problem 1 - Use the parametric equations for h(T) and R(T) to determine the equation for the speed, S, of the Shuttle along its trajectory where $dS/dt = ((dh/dt)^2 + (dR/dt)^2)^{1/2}$

Problem 2 - Determine the formula for the magnitude of the acceleration of the Shuttle using the second time derivatives of the parametric equations.

Problem 3 - From your answer to Problem 2, A) find the time at which the acceleration is an extremum, and specifically, a maximum along the modeled trajectory. B) What is the acceleration in feet/sec^2 at this time? C) If the acceleration of gravity at the earth's surface is 32 feet/sec^2, how many 'Gs' did the astronauts pull at this time?

Answer Key

Problem 1 - Use the parametric equations for h(T) and R(T) to determine the equation for the speed, S, of the Shuttle along its trajectory where $dS/dt = ((dh/dt)^2 + (dR/dt)^2)^{1/2}$

$dh/dt = -0.142\, T^2 + 36.6\, T - 345$

$dR/dt = 135.7\, e^{0.029T}$

Then

$$\mathbf{dS/dt = ((-0.142\, T^2 + 36.6\, T - 345)^2 + (135.7\, e^{0.029T})^2)^{1/2}}$$

Problem 2 - The components of the acceleration vector, a, are given by $a_h = d^2h/dT^2$ and $a_R = d^2R/dT^2$

so $D^2h/dt^2 = 36.6 - 0.284T$ $d^2R/dt^2 = 3.93\, e^{0.029T}$

Then the magnitude of the acceleration vector is just

$|a| = (a_h^{\ 2} + a_R^{\ 2})^{1/2} = \mathbf{(0.08\, T^2 - 20.8\, T + 1339 + 15.4\, e^{0.058\, T})^{1/2}}$

Problem 3 - A) The minimum or maximum acceleration is found by solving for T when d |a|/dT = 0

$0 = \mathbf{1/2\ (0.08\, T^2 - 20.8\, T + 1339 + 15.4\, e^{0.058\, T})^{-1/2}\ (0.16T - 20.8 + 0.89e^{0.058T})}$

Then $\mathbf{0 = 0.16T - 20.8 + 0.89e^{0.058T}}$ or $\mathbf{20.8 = 0.16\, T + 0.89e^{0.058T}}$

A calculator can be used to plot this curve and determine that for **T = 46.7 seconds** the condition d |a|/dT = 0 is obtained. This is near the time usually recorded by engineers as the Maximum Dynamic Pressure point 'Max-Q'.

B) Evaluating the answer to Problem 2 at T=46.7 seconds we get $\mathbf{|a| = (773)^{1/2} = 28}$ **feet/sec^2.**

C) The number of Gs = 28/32 = **0.9 Gs**.

Light Travel Times

NASA satellites and space probes are so far away from Earth that serious time delays happen when radio signals are sent to them. This is because radio signals travel at the speed of light, 300,000 kilometers/sec, and the distances from Earth to the spacecraft are huge!!

Spacecraft	Distance (km)	Seconds
Themis P2	30,000	
LRO	382,000	
ACE	1.5 million	
MESSENGER	50 million	
STEREO-A	111 million	
Mars Orbiter	220 million	
Ulysses	800 million	
Cassini	1.8 billion	
Voyager 2	13 billion	

This image was taken by the Hubble Space Telescope and shows the satellite Io The moon is the small disk to the left, and its shadow appears to the right of center. (Courtesy J. Spenser, Lowell Observatory and NASA).

Problem 1 - How long will it take for a radio signal to travel from the satellites to Earth, one-way, in the above table? Complete the last column to find out!

Problem 2 - Suppose a radio message needs to be sent at 01:20 on February 15, 2008. To the nearest minute, what time would it be when the message arrived at each spacecraft, and when the data arrived back at Earth?

Problem 3 - When Jupiter was located farthest from the Sun (aphelion), it was at a distance of 667 million kilometers from Earth. When it was closest to the sun (perihelion) it was 590.5 million kilometers from Earth. Suppose you are calculating a schedule for when the satellite Io will be exactly at dead-center of Jupiter's disk based on when you saw the transit at perihelion. How much of a schedule change will you see when you observe the transit at aphelion, and will the transit occur sooner or later than you predicted?

Answer Key

Problem 1 – Answer:

Spacecraft	Distance (km)	Seconds
Themis P2	30,000	**0.1**
LRO	382,000	**1.3**
ACE	1.5 million	**5.0**
MESSENGER	50 million	**167**
STEREO-A	111 million	**370**
Mars Orbiter	220 million	**733**
Ulysses	800 million	**2,667**
Cassini	1.8 billion	**6,000**
Voyager 2	13 billion	**43,333**

Problem 2 - Answer: see table below:

Spacecraft	Distance (km)	Arrival at satellite	Arrival at Earth	One-way time
Themis P2	30,000	**01:20**	**01:20**	**0.2 sec**
LRO	382,000	**01:20**	**01:20**	**2.6 sec**
ACE	1.5 million	**01:20**	**01:20**	**10 sec**
MESSENGER	50 million	**01:23**	**01:26**	**2.8 minutes**
STEREO-A	111 million	**01:26**	**01:32**	**6.2 minutes**
Mars Orbiter	220 million	**01:32**	**01:44**	**12.2 minutes**
Ulysses	800 million	**02:04**	**02:49**	**44.4 minutes**
Cassini	1.8 billion	**03:00**	**04:44**	**1.7 hours**
Voyager 2	13 billion	**13:20**	**01:20 on Feb 16th**	**12.0 hours**

Problem 3 - When Jupiter was located farthest from the sun (aphelion), it was at a distance of 667 million kilometers from Earth. When it was closest to the sun (perihelion) it was 590.5 million kilometers from Earth. Suppose you are calculating a schedule for when the satellite Io will be exactly at dead-center of Jupiter's disk based on when you saw the transit at perihelion. How much of a schedule change will you see when you observe the transit at aphelion, and will the transit occur sooner or later than you predicted?

Answer: The difference between Jupiter's distance in each case is 667 million - 590.5 million = 76.5 million kilometers. At the speed of light, this equals a time difference of 76500000/300000 = 255 seconds or 4.25 minutes. At the time the transit occurs at aphelion, the event will be seen about 4.25 minutes later than the predicted time.

Cross Sections and Collision Times

Whether you are talking about atoms in a gas, stars in a star cluster, or galaxies in intergalactic space, eventually some members will collide with each other. Predicting the collision times for various systems is an important way to estimate important events in their history.

Image: Seyfert's Sextet galaxy cluster showing collisions (Courtesy Hubble Space Telescope)

Problem 1 - **Cross Sectional Area:** Draw two circles on a paper that overlap. These represent the geometrical cross sectional areas of two bodies. For example, the cross sectional area of a marble with a radius of 5 millimeters is $A = \pi\ (5\text{mm})^2 = 78.5\ \text{mm}^2$. Find the cross sectional areas of: A) an oxygen atom with a radius of 50 picometers (in square meters); B) a star with a radius of 698,000 km (in square kilometers); C) a galaxy with a radius of 50,000 light years (in square light years). (1 ly = 9.4 trillion km)

Problem 2 - **Swept out volume:** A moving body sweeps out a cylindrical volume whose base area equals the bodies cross sectional area, and whose height equals the speed of the body times the elapsed time. Calculate the cylindrical volumes, in cubic meters, for; A) an atom of oxygen traveling at 500 meters/sec for 10 seconds; B) a star in the Omega Centauri cluster traveling at 22 km/sec for 1 million years (in cubic light years); C) the Milky Way galaxy traveling at 200 km/sec for 100 million years (in cubic light years). (1 year = 31 million seconds)

Problem 3 - **Average particle volume:** The average volume occupied by a body in a system depends on the volume of the system and the number of bodies present. The 'number density' measures the number of particles divided by the volume of the system. The inverse of this number is the average volume per particle. Calculate the average volume for the following bodies: A) Sea-level atmosphere with 3×10^{25} atoms/cubic meter; B) Omega Centauri cluster with a diameter of 160 light years and containing 10 million stars (in stars per cubic light year) ; C) The Local Group of galaxies containing 35 galaxies including the Milky Way, with a diameter of 10 million light years (in galaxies per cubic mega light year).

Problem 4 - **Collision time:** The collision time is the time it takes a body to sweep out the same volume as is occupied by an average particle in the system. It is given by the formula T = 1/(N A V) where N is the density of particles, V is their average speed, and A is their average cross-sectional area. (All units should be in terms of meters and seconds.) From the area calculated in Problem 1, the speeds given in Problem 2, and the average particle densities from Problem 3, compute the particle collision times for A) an oxygen atom at sea level (in nanoseconds); B) a star in the Omega Centauri cluster (in years) and C) a galaxy in the Local Group (in years).

Inquiry Question: There are many more stars than galaxies, so why is it that galaxies collide nearly a million times more often?

Answer Key

Problem 1 - **A)** an oxygen atom with a radius of 50 picometers; A = π (50 x 10^{-12} m)^2 = **7.9 x 10^{-21} meters^2** **B)** a star with a radius of 698,000 km; A = π (698,000 km)^2 = **1.5 x 10^{12} km^2** **C)** a galaxy with a radius of 50,000 light years. A = π(50,000 ly)^2 = **7.9 x 10^9 ly^2**

Problem 2 - Use the cross sectional areas calculated in Problem 1 and Volume = area x distance traveled; **A)** an atom of oxygen traveling at 500 meters/sec for 10 seconds; L = 500 m/s x 10 s = 5000 meters. Volume = 7.9 x 10^{-21} meters^2 x 5000 meters = **4.0 x 10^{-17} meters^3** **B)** a star in the Omega Centauri cluster traveling at 22 km/sec for 1 million years; L = 22 km/s x 1 million yrs x 31 million sec/yr = 6.8 x 10^{14} kilometers. Volume = 1.5 x 10^{12} km^2 x 6.8 x 10^{14} km = **1.0 x 10^{27} km^3**. **C)** the Milky Way galaxy traveling at 200 km/sec for 100 million years. L = 200 km/s x 100 million years x 31 million sec/yr = 6.2 x 10^{17} kilometers = 66,000 light years. Volume = 7.9 x 10^9 ly^2 x 6.6 x 10^4 ly = **5.2 x 10^{14} ly^3**.

Problem 3 - Calculate the average volume for the following bodies: **A)** Sea-level atmosphere with 3 x 10^{25} atoms/cubic meter; The number density, N = 3 x 10^{25} atoms/meter^3 so 1/N = V = **3.3 x 10^{-26} meters^3 per atom**. **B)** Omega Centauri cluster with a diameter of 160 light years and containing 10 million stars; Assume the cluster is a sphere with a volume V = 4/3 π (80 ly)^3 = 2.1 x 10^6 ly^3. Then N = 10 million / 2.1 x 10^6 ly^3 = 4.8 stars/ly^3. The individual star volume = 1/N = **0.2 ly^3 per star**. **C)** The Local Group of galaxies containing 35 galaxies including the Milky Way, with a diameter of 10 million light years (i.e. 10 Mly). Volume = 4/3 π (5 Mly)^3 = 523 Mly^3 . N = 35 galaxies/ 523 Mly^3 = 0.07 galaxies/Mly^3 . Then 1/N = **14 Mly^3 per galaxy**.

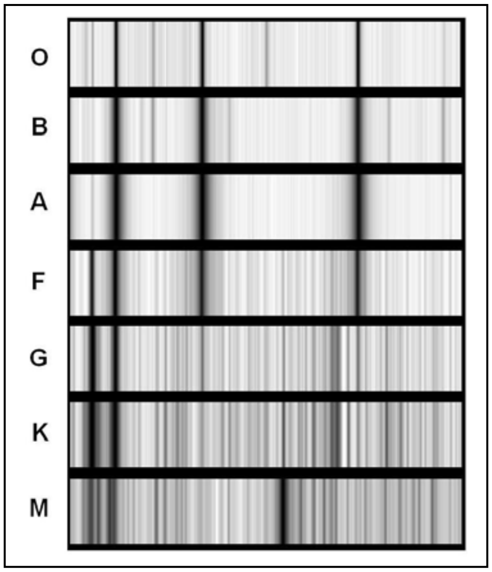
Problem 4 - A) Atom collision time: A = 7.9 x 10^{-21} meters^2 V = 500 m/sec and N = 3 x 10^{25} atoms/m^3. so T = 1/(7.9 x 10^{-21} meters^2 x 500 m/sec x 3 x 10^{25} atoms/m^3) = **8.4 x 10^{-9} seconds or about 8 nanoseconds**.

B) Star collision time: A = 1.5 x 10^{12} km^2 V = 22 km/sec and N = 4.8 stars/Ly^3 . converting them to the same units, in this case kilometers, N = 4.8/(9.4x10^{12})^3 = 5.8 x 10^{-39}, so T = 1/(1.5 x 10^{12} x 22 km/sec x 5.8 x 10^{-39}) = 5.2 x 10^{24} seconds or **1.7 x 10^{17} years**.

C) Galaxy collision time: A = 7.9 x 10^9 ly^2 ; V = 200 km/sec, which is equal to 200 km/sec x (1 Ly/9.4 x 10^{12} km) x (31 million seconds/1 year) = 0.0066 Ly/year; and N = 0.07 galaxies/Mly^3 which is 0.07 x 10^{-18} galaxies/ly^3; so T = 1/(7.9 x 10^9 x 0.0066 x 7.0 x 10^{-20}) = **2.7 x 10^{11} years.**

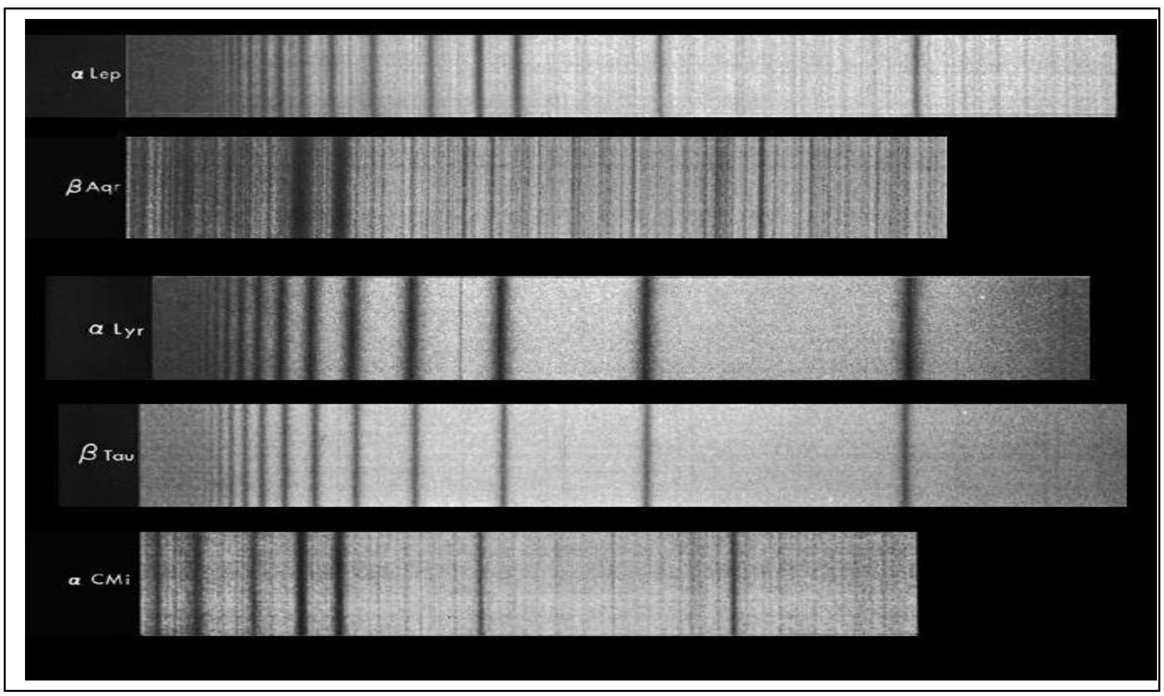
<u>**Inquiry Question:**</u> **Students should recognize that, although stars and galaxies travel about the same speed, galaxies have much larger cross sections compared to the volume of space they occupy, so there will be more collisions between them.**

Spectral Classification of Stars



The advent of the spectroscope in the 1800's allowed astronomers to study the temperatures and compositions of stars, and to classify stars according to their spectral similarities. At first, 26 classes were defined; one for each letter in the alphabet. But only 7 are actually major classes, and these survive today as the series 'O, B, A, F, G, K, M'. This series follows decreasing star temperatures from 30,000 K (O-type) to 3,000 K (M-type).

Images courtesy: Helmut Abt (NOAA).



Problem 1 - Sort the five stellar spectra according to their closest matches with the standard spectra at the top of the page. (Note, the spectra may not be to the same scale, aligned vertically, and may even be stretched!)

Problem 2 - The star α Lyr (Alpha Lyra) has a temperature of 10,000 K and β Aqr (Beta Aquarii) has a temperature of 5,000 K. What do you notice about the pattern of spectral lines as you change the star's temperature?

Problem 1 - This is designed to be a challenge! Student strategies should include looking for general similarities first. One obvious way to group the spectra in terms of the increasing (or decreasing!) number of spectral lines.

Alpha Lyr (Alpha Lyra) and Beta Tau (Beta Tauri) can be grouped together because of the strong lines that appear virtually alone in the spectra (these are hydrogen lines). The second grouping, Group 2, would include Alpha Lep (Alpha Leporis) and Alpha CMi (Alpha Canis Minoris) because they have a different pattern of strong lines than Group 1 but also the hint of many more faint lines in between. The last 'group' would be for Beta Aqr (Beta Aquarii) because it has two very strong lines close together (the two lines of the element calcium), but many more lines that fill up the spectrum and are stronger than in Group 2.

Group 1:

Vega (Alpha Lyra) - A type star
Alnath (Beta Tauri) - B type star

Group 2:

Arneb (Alpha Leporis) - F type star
Procyon (Alpha Canis Minoris) - F type star

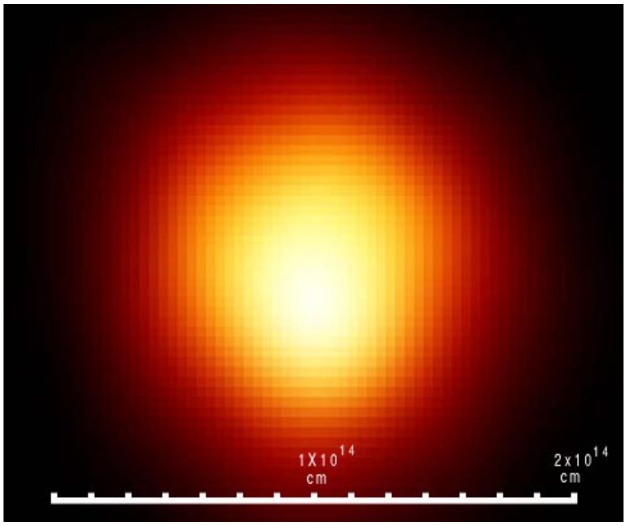
Group 3:

Sadalsuud (Beta Aquarii) - G type star

Comparing Group 1, 2 and 3 with the standard chart, Group 1 consists of the B and A type stars, in particular Alpha Lyra has the thick lines of an A-type star, and Beta Tauri has the thinner lines of a B-type star; Group 2 is similar to the spectra of the F-type stars, and Group 3 is similar to the G-type stars.

Problem 2 - Vega has a temperature of 10,000 K and Beta Aquari has a temperature of 5,500 K. What do you notice about the pattern of spectral lines as you change the star's temperature?

Answer: What you should notice is that, as the temperature of the star gets cooler, the number of atomic lines in this part of the spectrum (visible light) increases.



The amount of power that a star produces in light is related to the temperature of its surface and the area of the star. The hotter a surface is, the more light it produces. The bigger a star is, the more surface it has. When these relationships are combined, two stars at the same temperature can be vastly different in brightness because of their sizes.

Image: Betelgeuse (Hubble Space Telescope.) It is 950 times bigger than the sun!

The basic formula that relates stellar light output (called luminosity) with the surface area of a star, and the temperature of the star, is L = A x F where the star is assumed to be spherical with a surface area of $A = 4 \pi R^2$, and the radiation emitted by a unit area of its surface (called the flux) is given by $F = \sigma T^4$. The constant, σ, is the Stefan-Boltzman radiation constant and it has a value of $\sigma = 5.67 \times 10^{-5}$ ergs/ (cm^2 sec deg^4). The luminosity, L, will be expressed in power units of ergs/sec if the radius, R, is expressed in centimeters, and the temperature, T, is expressed in degrees Kelvin. The formula then becomes,

$$L = 4 \pi R^2 \sigma T^4$$

Problem 1 - The Sun has a temperature of 5700 Kelvins and a radius of 6.96×10^5 kilometers, what is its luminosity in A) ergs/sec? B) Watts? (Note: 1 watt = 10^7 ergs/sec).

Problem 2 - The red supergiant Antares in the constellation Scorpius, has a temperature of 3,500 K and a radius of 700 times the radius of the sun. What is its luminosity in A) ergs/sec? B) multiples of the solar luminosity?

Problem 3 - The nearby star, Sirius, has a temperature of 9,200 K and a radius of 1.76 times our Sun, while its white dwarf companion has a temperature of 27,400 K and a radius of 4,900 kilometers. What are the luminosities of Sirius-A and Sirius-B compared to our Sun?

Calculus:

Problem 4 - Compute the total derivative of L(R,T). If a star's radius increases by 10% and its temperature increases by 5%, by how much will the luminosity of the star change if its original state is similar to that of the star Antares? From your answer, can you explain how a star's temperature could change without altering the luminosity of the star. Give an example of this relationship using the star Antares!

Problem 1 - We use $L = 4 (3.141) R^2 (5.67 \times 10^{-5}) T^4$ to get L (ergs/sec) = 0.00071 R(cm)^2 T(degreesK)^4 then,
A) L(ergs/sec) = 0.00071 x (696,000 km x 10^5 cm/km)^2 (5700)^4 = **3.6 x 10^{33} ergs/sec**
B) L(watts) = 3.6 x 10^{33} (ergs/sec)/ 10^7 (ergs/watt) = **3.6 x 10^{25} watts**.

Problem 2 - A) The radius of Antares is 700 x 696,000 km = 4.9 x 10^8 km.
L(ergs/sec) = 0.00071 x (4.9 x 10^8 km x 10^5 cm/km)^2 (3500)^4 = **2.5 x 10^{38} ergs/sec**
B) L(Antares) = (2.5 x 10^{38} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **71,000 L(sun).**

Problem 3 - Sirius-A radius = 1.76 x 696,000 km = 1.2 x 10^6 km
L(Sirius-A) = 0.00071 x (1.2 x 10^6 km x 10^5 cm/km)^2 (9200)^4 = **7.3 x 10^{34} ergs/sec**
L = (7.3 x 10^{34} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **20.3 L(sun).**
L(Sirius-B) = 0.00071 x (4900 km x 10^5 cm/km)^2 (27,400)^4 = **9.5 x 10^{31} ergs/sec**
L(Sirius-B) = 9.5 x 10^{31} ergs/sec / 3.6 x 10^{33} ergs/sec = **0.026 L(sun).**

Advanced Math:
Problem 4 *(*Note: In the discussion below, the symbol *d* represents a partial derivative)

$$dL(R,T) = \frac{dL(R,T)}{dR}\, dR + \frac{dL(R,T)}{dT}\, dT$$

$$dL = [\, 4\pi\ (2)\, R \sigma T^4\,]\, dR + [\, 4\pi\ (4)\, R2 \sigma T^3\,]\, dT$$

$$dL = 8\pi\ R \sigma T^4\ dR + 16\pi\ R^2 \sigma T^3\ dT$$

To get percentage changes, divide both sides by $L = 4\pi\ R^2 \sigma T^4$

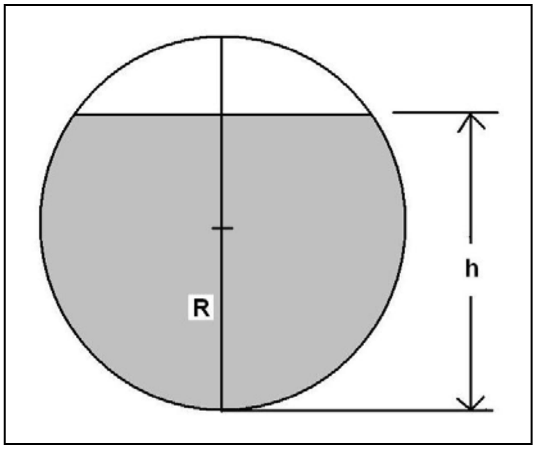
$$\frac{dL}{L} = \frac{8\pi\ R\sigma T^4}{4\pi\ R^2 \sigma T^4}\, dR + \frac{16\pi\ R^2 \sigma T^3}{4\pi\ R^2 \sigma T^4}\, dT$$

Then dL/L = 2 dR/R + 4 dT/T so for the values given, dL/L = 2 (0.10) + 4 (0.05) = **0.40 The star's luminosity will increase by 40%.**

Since dL/L = 2 dR/R + 4 dT/T, we can obtain no change in L if 2 dR/R + 4 dT/T = 0. This means that 2 dR/R = - 4 dT/T and so, - 0.5 dR/R = dT/T. **The luminosity of a star will remain constant if, as the temperature decreases, its radius increases.**

Example. For Antares, its original luminosity is 71,000 L(sun) or 2.5 x 10^{38} ergs/sec. If I increase its radius by 10% from 4.9 x 10^8 km to 5.4 x 10^8 km, its luminosity will remain the same if its temperature is decreased by dT/T = 0.5 x 0.10 = 0.05 which will be 3500 x 0.95 = 3,325 K so L(ergs/sec) = 0.00071 x (5.4 x 10^8 km x 10^5 cm/km)^2 (3325)^4 = **2.5 x 10^{38} ergs/sec**

Fuel Level in a Spherical Tank



Spherical tanks are found in many different situations, from the storage of cryogenic liquids, to fuel tanks. Under the influence of gravity, or acceleration, the liquid will settle in a way such that it fills the interior of the tank up to a height, h. We would like to know how full the tank is by measuring h and relating it to the remaining volume of the liquid. A sensor can then be designed to measure where the surface of the liquid is, and from this derive h.

Problem 1 - Slice the fluid into a series of vertically stacked disks with a radius r(h) and a thickness dh. What is the general formula for the radius of each disk?

Problem 2 - Set up the integral for the volume of the fluid and solve the integral.

Problem 3 - Assume that fluid is being withdrawn from the tank at a fixed rate $dV/dt = -F$. What is the equation for the change in the height of the fluid volume with respect to time? A) Solve for the limits $h \ll R$ and $h \gg R$. B) Solve graphically for R=1 meter, F=100 cm^3/min. (Hint: select values for h and solve for t).

Answer Key

Problem 1 - $r(h)^2 = R^2 - (R-h)^2$ so $r(h)^2 =$ $\mathbf{2Rh - h^2}$

Problem 2 - The integrand will be $\pi(2Rh - h^2)\,dh$ and the solution is $\boldsymbol{\pi R h^2 - 1/3\pi h^3}$

Problem 3 -

$$\frac{dV}{dt} = 2\pi R h \frac{dh}{dt} - \pi h^2 \frac{dh}{dt} \quad \text{so} \quad \frac{dV}{dt} = (2\pi R h - \pi h^2)\frac{dh}{dt} = -F$$

Then

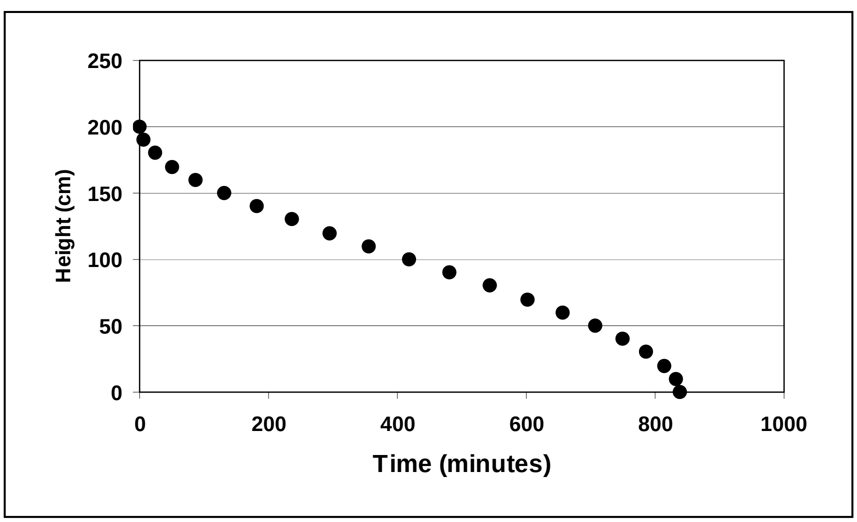
$$\frac{dh}{dt} = \frac{-F}{(2\pi R h - \pi h^2)}$$

The integrands become: $(2\pi R h - \pi h^2)\,dh = -F\,dt$. This can be integrated from t=0 to t = T to obtain $\pi R h^2 - 1/3\,\pi h^3 = -F T$ and simplified to get

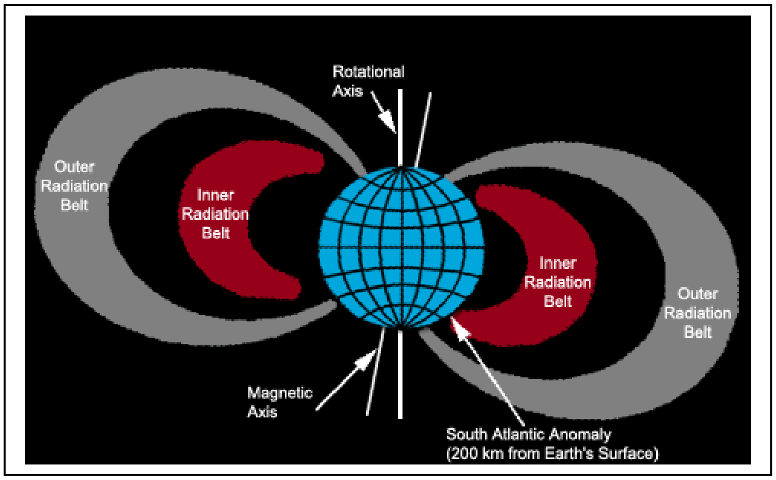
$$\mathbf{h^3 - 3 R h^2 - (3 F T)/\pi = 0}$$

We would normally like to invert this equation to get h(T), but cubic equations of the form $x^3 - \alpha x^2 + \beta = 0$ cannot be solved analytically. We can solve it for two limiting cases. Case 1 for a tank nearly empty where h << R. This yields $\mathbf{h(T) = (F T/R)^{1/2}}$ Case 2 is for a tank nearly full so that h >> R, and we get $h^3 = 3 F T/\pi$ and $\mathbf{h(T) = (3 F T/\pi)^{1/3}}$. The full solution for h(T) can be solved graphically. Since R is a constant, we can select a new variable U = h/R and re-write the equation in terms of the magnitude of h relative to the radius of the tank.

$U^3 - 3U^2 = (3 F T)/\pi R^3$ and plot this for selected combinations of (U,T) where time, T, is the dependent variable. The solution below is for F = 100 cm^3/minute, R = 1 meter, with the intervals in h spaced 10 cm. The plot was generated using an *Excel* spreadsheet.



The Mass of the Van Allen Radiation Belts



The van Allen Radiation Belts were discovered in the late-1950's at the dawn of the Space Age. They are high-energy particles trapped by Earth's magnetic field into donut-shaped clouds.

Earth's inner magnetic field has a 'bar magnet' shape that follows the formula

$$R(\lambda) = L \cos^2\lambda$$

where the angle, λ, is the magnetic latitude of the magnetic field line emerging from Earth's surface, and L is the distance to where that field line passes through the magnetic equatorial plane of the field. The distance, L, is conveniently expressed in multiples of Earth's radius (1 Re = 6378 kilometers) so that L=2 Re indicates a field line that intersects Earth's magnetic equatorial plane at a physical distance of 2 x 6378 km = 12,756 km from Earth's center.

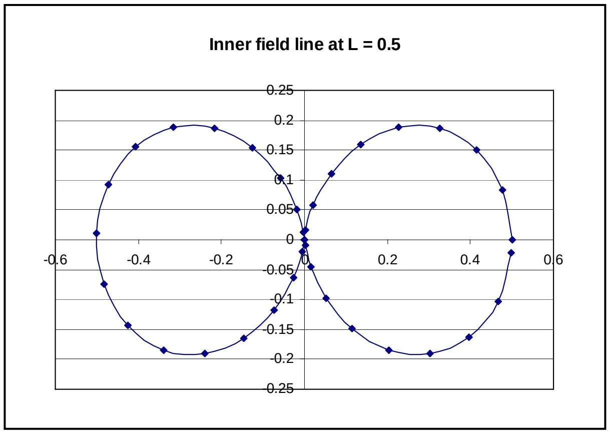
To draw a particular field line, you select L, and then plot R for different values of λ. Because the van Allen particles follow paths along these field lines, the shape of the radiation belts is closely related to the shape of the magnetic field lines.

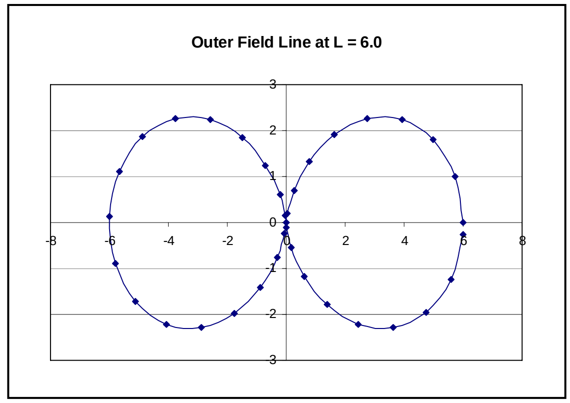
Problem 1 - Using the field line equation, plot in polar coordinates a field line at the outer boundary of the van Allen Belts for which L = 6 Re, and on the same plot, a field line at the inner boundary where L=0.5 Re. Shade-in the region bounded by these two field lines.

Problem 2 – If you rotate the shaded region in Problem 1 you get a 3-d figure which looks a lot like two nested toroids. Approximate the volume of the shaded region by using the equation for the volume of a torus given by $V = 2\pi^2 r R^2$ where R is the internal radius of the circular cross-section of the torus, and r is the distance from the Origin (Earth) to the central axis of the torus. (Think of the volume as turning the torus into a cylinder with a cross section of πR^2 and a height of $2\pi r$).

Problem 3 - Assuming that the maximum, average density of the van Allen Belts is about 100 protons/cm^3, and that the mass of a proton is 1.6×10^{-24} grams, what is the total mass of the van Allen Belts in kilograms?

Problem 1 - This may be done, either using an HP-83 graphing calculator, or an Excel spreadsheet. The later example is shown below. (Note the scale change). For Cartesian plots in Excel, (X-Y) you will need to compute X and Y parametrically as follows: (Polar to Cartesian coordinates) $X = R\cos(\lambda)$, $y = R\sin(\lambda)$, then since $R = L\cos^2(\lambda)$ we get $X = L\cos^3(\lambda)$ and $Y = L\cos^2(\lambda)\sin(\lambda)$.



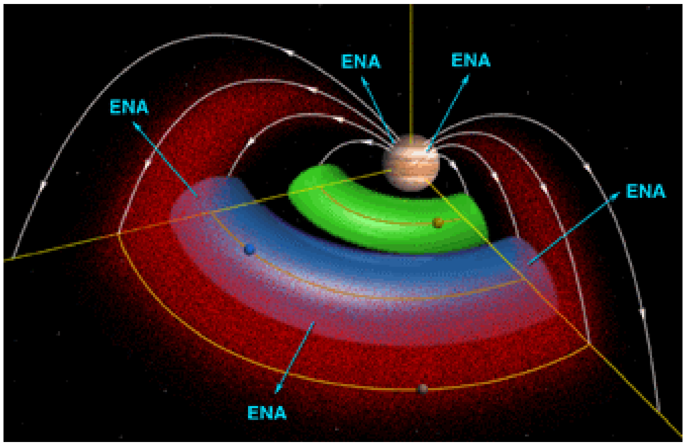


Problem 2 – Answer: The outer torus of the van Allen Belt model has an internal radius of R = (6Re – 0.5 Re)/2 = 2.75 Re or 17540 km. The radius of the Belts, r = 2.75 Re or 17,540 km. This makes the total volume V(outer) = 2 $(3.141)^2$ $(17540 \text{ km x}1000 \text{ m/km})^3$ = 1.1×10^{23} meters^3. The volume of the inner torus is defined by R = 0.25Re = 1595 km and r = 0.25 Re = 1595 km, so its volume is V(inner) = 2 $(3.141)^2$ $(1595 \text{ km x}1000 \text{ m/km})^3$ = 8.0×10^{19} meters^3 The approximate volume of the shaded region in Problem 1 is then the difference between V(outer) and V(inner) or 1.1×10^{23} meters^3 - 8.0×10^{19} meters^3 = $11000 \times 10^{19} - 8.0 \times 10^{19}$ = **1.1×10^{23} meters^3** because although it is technically correct to subtract the inner volume (containing no Belt particles) from the outer volume, practically speaking, it makes no difference numerically. This would not be the case if we had selected a much larger inner boundary zone for the problem!

Problem 3 - Assuming that the maximum, average density of the van Allen Belts is about 100 protons/cm^3, and that the mass of a proton is 1.6×10^{-24} grams, what is the total mass of the van Allen Belts in kilograms?

Answer: Mass = density x Volume, V = 1.1×10^{23} meters^3 .
D = 100 protons/cm^3 x 1.6×10^{-24} grams/proton = 1.6×10^{-22} grams/cm^3 which, when converted into MKS units gives 1.6×10^{-22} g/cm^3 x (1 kg/1000 gm) x $(100 \text{ cm/1 meter})^3$ = 1.6×10^{-19} kg/m^3. So the total mass is about M = 1.6×10^{-19} kg/m^3 x 1.1×10^{23} meters^3 and so **M = 18,000 kilograms.**

The Io Plasma Torus



The satellite of Jupiter, Io, is a volcanically active moon that ejects 1,000 kilograms of ionized gas into space every second. This gas forms a torus encircling Jupiter along the orbit of Io. We will estimate the total mass of this gas based on data from the NASA Cassini and Galileo spacecraft.

Image: Io plasma torus (Courtesy NASA/Cassini)

Problem 1 - Galileo measurements obtained in 2001 indicated that the density of neutral sodium atoms in the torus is about 35 atoms/cm^3. The spacecraft also determined that the inner boundary of the torus is at about 5 Rj, while the outer boundary is at about 8 Rj. (1 Rj = 71,300 km). A torus is defined by the radius of the ring from its center, R, and the radius of the circular cross section through the donut, r. What are the dimensions, in kilometers, of the Io torus based on the information provided by Galileo?

Problem 2 - Think of a torus as a curled up cylinder. What is the general formula for the volume of a torus with radii R and r?

Problem 3 - From the dimensions of the Io torus, what is the volume of the Io torus in cubic meters?

Problem 4 - From the density of sodium atoms in the torus, what is A) the total number of sodium atoms in the torus? B) If the mass of a sodium atom is 3.7×10^{-20} kilograms, what is the total mass of the Io torus in metric tons?

Calculus:

Problem 5 - Using the 'washer method' in integral calculus, derive the formula for the volume of a torus with a radius equal to R, and a cross-section defined by the formula $x^2 + y^2 = r^2$. The torus is formed by revolving the cross section about the Y axis.

Answer Key

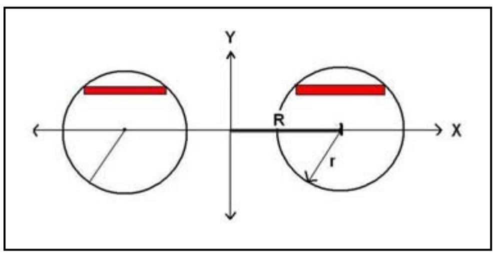
Problem 1 - The mid point between 5 Rj and 8 Rj is (8+5)/2 = 6.5 Rj so R = 6.5 Rj and r = 1.5 Rj. Then R = 6.5 x 71,300 so **R = 4.6×10^5 km**, and r = 1.5 x 71,300 so **r = 1.1×10^5 km**.

Problem 2 - The cross-section of the cylinder is πr^2, and the height of the cylinder is the circumference of the torus which equals $2 \pi R$, so the volume is just $V = (2 \pi R) \times (\pi r^2)$ or **$V = 2 \pi^2 R r^2$.**

Problem 3 - Volume = $2 \pi^2 (4.6 \times 10^5 \text{ km}) (1.1 \times 10^5 \text{ km})^2$ so **V = 1.1×10^{17} km^3.**

Problem 4 - A) 35 atoms/cm^3 x (100000 cm/1 km)^3 = 3.5×10^{16} atoms/km^3. Then number = density x volume so N = (3.5×10^{16} atoms/km^3) x (1.1×10^{17} km^3) , so **N = 3.9×10^{33} atoms**.
B) The total mass is M = 3.9×10^{33} atoms x 3.7×10^{-20} kilograms/atom = 1.4×10^{14} kilograms. 1 metric ton = 1000 kilograms, so the total mass is **M = 100 billion tons**.

Advanced Math:



$$V = 8 \pi R \int_0^r \left(r^2 - y^2 \right)^{1/2} dy$$

Recall that the volume of a washer is given by $V = \pi (R(\text{outer})^2 - R(\text{inner})^2) \times$ thickness. For the torus figure above, we see that the thickness is just dy. The distance from the center of the cross section to a point on the circumference is given by $r^2 = x^2 + y^2$. The width of the washer (the red volume element in the figure) is parallel to the X-axis, so we want to express its length in terms of y, so we get $x = (r^2 - y^2)^{1/2}$. The location of the outer radius is then given by $R(\text{outer}) = R + (r^2 - y^2)^{1/2}$, and the inner radius by $R(\text{inner}) = R - (r^2 - y^2)^{1/2}$. We can now express the differential volume element of the washer by $dV = \pi [(R + (r^2-y^2)^{1/2})^2 - (R - (r^2 - y^2)^{1/2})^2] dy$. This simplifies to $dV = \pi [4R (r^2-y^2)^{1/2}] dy$ or
$dV = 4 \pi R (r^2-y^2)^{1/2} dy$. The integral can immediately be formed from this ,with the limits y = 0 to y=r. Because the limits to y only span the upper half plane, we have to double this integral to get the additional volume in the lower half-plane. The required integral is shown above.

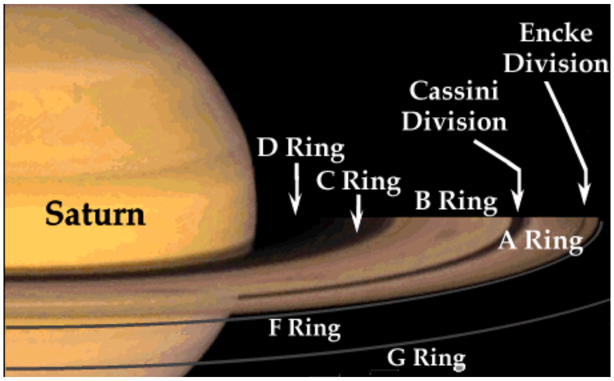
This integral can be solved by factoring out the r from within the square-root, then using the substitution U = y/r and dU = 1/r dy to get the integrand $dV = 8 \pi R r^2 (1 - U^2)^{1/2} dU$. The integration limits now become U=0 to U=1. Since r and R are constants, this is an elementary integral with the solution $V = 1/2\ U (1-U^2)^{1/2} + 1/2 \arcsin(U)$. When this is evaluated from U=0 to U=1, we get

$$V = 8 \pi R r^2 \ [0 + 1/2 \arcsin(1)] - [0 + 1/2 \arcsin(0)]$$

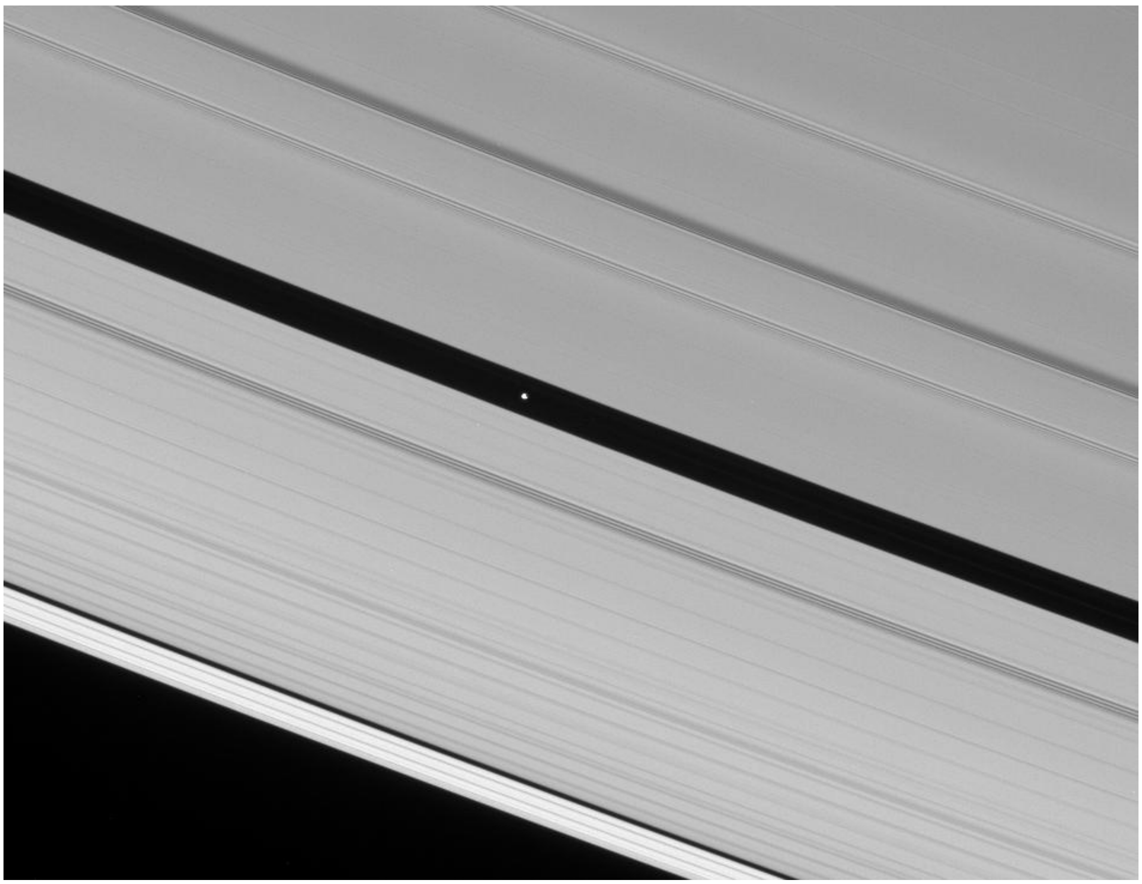
$$V = 8 \pi R r^2 \ 1/2 (\pi/2)$$

$$\mathbf{V = 2 \pi R r^2}$$

Pan's Highway - Saturn's Rings



The Encke Gap is a prominent feature of Saturn's outer A-ring system that has been observed since the 1830's. The arrival of the Cassini spacecraft in July 2004 revealed the cause for this gap. A small moonlet called Pan clears out the ring debris in this region every 12 hours as it orbits Saturn!



Problem 1 - This image was taken by Cassini in 2007 and at the satellite's distance of 1 million kilometers, spans a field of view of 5,700 km x 4,400 km. With the help of a millimeter ruler, what is the scale of the image in kilometers per millimeter?

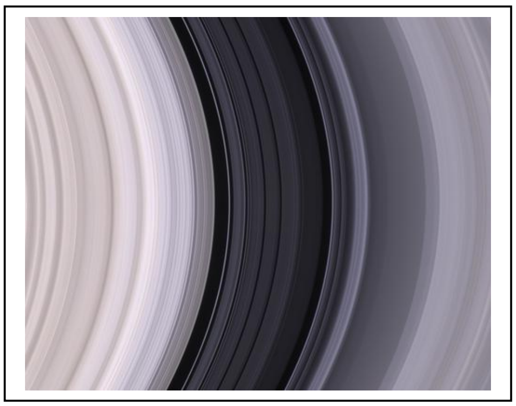
Problem 2 - Pan is that bright spot within the black zone of the Encke Gap. About how many kilometers in diameter is Pan?

Problem 3 - About how wide is the Encke Gap?

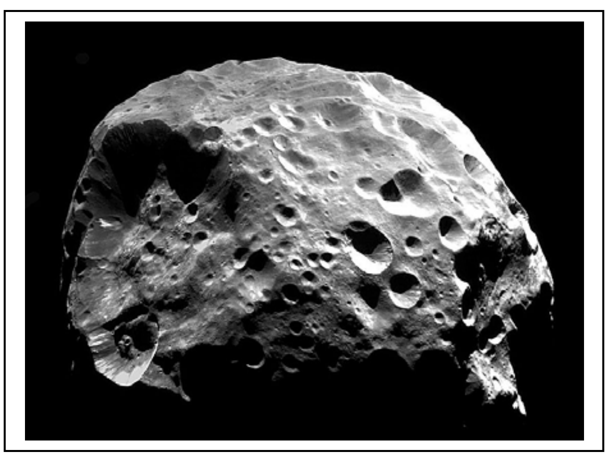
Problem 4 - About what is the smallest feature you can discern in the photo?

Problem 1 - The width of the picture is 150 millimeters, so the scale is 5,700 km/150 mm = **38 km/mm.**

Problem 2 - Pan is about 1.0 millimeters in diameter which is 38 km/mm x 1mm = **38 kilometers in diameter.**



Problem 3 - Students should measure a width of about 5.0 millimeters which is 38 km/mm x 5.0 mm = **190 kilometers**. The actual width of the Encke Gap is 325 km, but projection effects will foreshorten the gap as it appears in the photo. With the actual gap width (325 km) as the hypotenuse, and 190 km as the short side, the angle opposite the short side is the viewing angle of the camera relative to the ring plane. This angle can be found by constructing a scaled triangle and using a protractor to measure the angle, which will be about 36 degrees.



Problem 4 -

It is difficult to estimate lengths smaller than a millimeter. Students may consider using a photocopying machine to make a more convenient enlargement of the image, then measure the features more accurately. Small dark ring bands are about 0.1 mm wide, which is **about 4 km.**

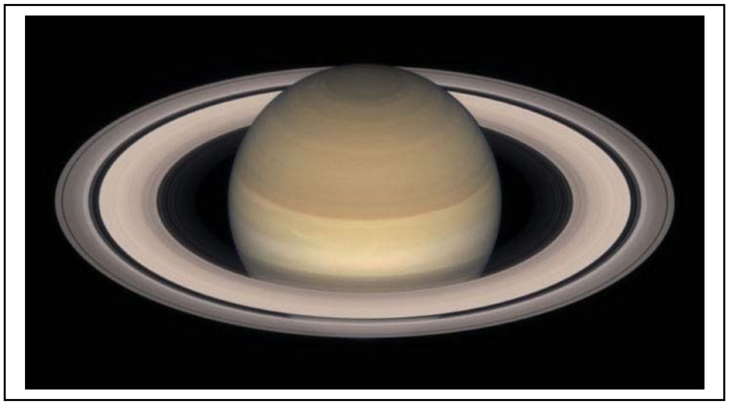
NASA/Cassini mages, top to bottom:

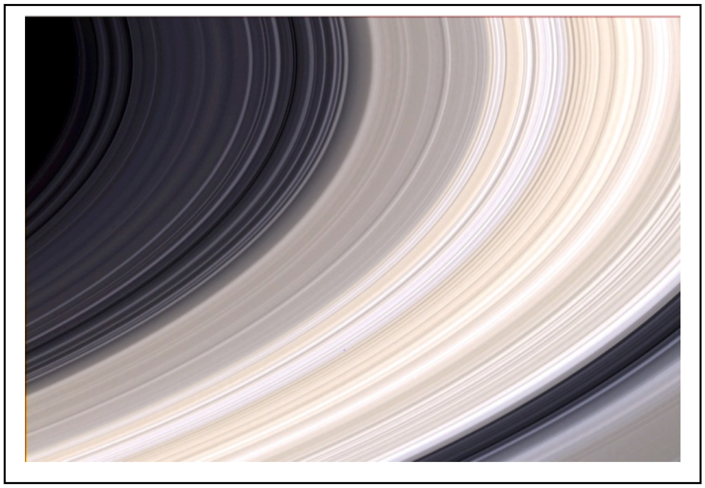
Saturn Rings closeup showing Cassini Division and Encke Gap;

Rings closeup showing detail;

One of Saturn's outer satellites, Phoebe, is about 200 km across, and may have been a captured comet.

Tidal Forces - Let 'er Rip!





As the Moon orbits Earth, its gravitational pull raises the familiar tides in the ocean water, but did you know that it also raises 'earth tides' in the crust of earth? These tides are up to 50 centimeters in height and span continent-sized areas. The Earth also raises 'body tides' on the moon with a height of 5 meters!

Now imagine that the moon were so close that it could no longer hold itself together against these tidal deformations. The distance were Earth's gravity will 'tidally disrupt' a solid satellite like the moon is called the tidal radius. One of the most dramatic examples of this is the rings of Saturn, where a nearby moon was disrupted, or prevented from forming in the first place!

Images courtesy NASA/Hubble and Cassini.

Problem 1 - The location of the tidal radius (also called the Roche Limit) for two bodies is given by the formula $d = 2.4x\ R\ (\rho_M/\rho_m)^{1/3}$ where ρ_M is the density of the primary body, ρ_m is the density of the satellite, and R is the radius of the main body. For the Earth-Moon system, what is the Roche Limit if R = 6,378 km, ρ_M = 5.5 gm/cm^3 and ρ_m = 2.5 gm/cm^3? (Note, the Roche Limit, d, will be in kilometers if R is also in kilometers, and so long as the densities are in the same units.)

Problem 2 - Saturn's moons are made of ice with a density of about 1.2 gm/cm^3 .If Saturn's density is 0.7 gm/cm^3 and its radius is R = 58,000 km, how does its Roche Limit compare to the span of the ring system which extends from 66,000 km to 480,000 km from the planet's center?

Problem 3 - In searching for planets orbiting other stars, many bodies similar to Jupiter in mass have been found orbiting sun-like stars at distances of only 3 million km. What is the Roche Limit for a star like our Sun if its radius is R = 600,000 km, and the densities are ρ(planet) = 1.3 gm/cm^3 and ρ(star) = 1.5 gm/cm^3?

Answer Key

Problem 1 -
d = 2.4xR $(\rho_M/\rho_m)^{1/3}$
with R = 6,378 km, ρ_M = 5.5 gm/cm^3 and ρ_m = 2.5 gm/cm^3
d = 2.4 x 6,378 $(5.5/2.5)^{1/3}$
d = 19,900 km.

This distance is inside the orbits of geosynchronous communications satellites (42,164 kilometers). They are not destroyed because the tensile strength of aluminum is far higher than the gravitational tidal forces they feel.

Fortunately our moon is at a distance of 363,000 (perigee) and is steadily moving farther away by 3 centimeters per year!

Problem 2 -
d = 2.4 x 58,000 $(0.7/1.2)^{1/3}$
d = 116,900 kilometers.

The span of the ring system which extends from 66,000 km to 480,000 km from the planet's center, so the Roche Limit is inside the ring system.

Problem 3 - In searching for planets orbiting other stars, many bodies similar to Jupiter in mass have been found orbiting sun-like stars at distances of only 3 million km. What is the Roche Limit for a star like our Sun if its radius is R = 600,000 km, and the densities are ρ(planet) = 1.3 gm/cm^3 and ρ(star) = 1.5 gm/cm^3?

d = 2.4 x 600,000 $(1.5/1.3)^{1/3}$
d = 1.5 million kilometers.

So the large planets being detected are very close to the Roche Limits for their stars!

Are U Still Nuts?

That's right… It's time for more unit conversion exercises!

Problem 1: The Solar Constant is the amount of energy that the sun delivers to the surface of Earth each second. If it is measured to be 1350000 ergs/cm^2 each second, how many watts per square meter is this? (1 watt = 10,000,000 ergs each second).

Problem 2: A supermassive black hole in the center of the quasar 3C273 swallows one star a year, and the heated gases emit 1.3 x 10^{53} ergs of energy. How much energy does 3C273 emit, in watts? (1 year = 31,000,000 seconds).

Problem 3: An astronaut is preparing a meal that includes 50 grams of cocoa mixed with 8 ounces of milk. What is the concentration of the chocolate in kilograms per liter? (128 oz = 1 gallon; and 9 gallons = 34 liters)

Problem 1: The solar constant is an important number if you are trying to build a solar, hot water heater or generate electricity using solar panels. Although astronomers use ergs and centimeter units, solar energy system design uses watts and square-meters for greater convenience.

In 1 second:

$$\frac{1350000 \text{ ergs}}{\text{cm}^2} \times \frac{1 \text{ watt}}{10{,}000{,}000 \text{ ergs}} \times \frac{100 \text{ cm}}{1 \text{ meter}} \times \frac{100 \text{ cm}}{1 \text{ meter}} = \mathbf{1350 \text{ watts/m}^2}$$

Notice how the units cancel, leaving behind the units of watt / (meter x meter) which can be re-written as watts/meter$^{\mathbf{2}}$. This problem is a bit tricky because 1 watt is equal to 10,000,000 ergs per second, and not just 'ergs'. To avoid confusing students with compound unit conversions which come up all the time in science, the above conversion is for 1 second of time.

Problem 2 :

$$\frac{1.3 \times 10^{53} \text{ ergs}}{1 \text{ year}} \times \frac{1 \text{ year}}{31{,}000{,}000 \text{ sec}} \times \frac{1 \text{ watt}}{10{,}000{,}000 \text{ ergs/s}}$$

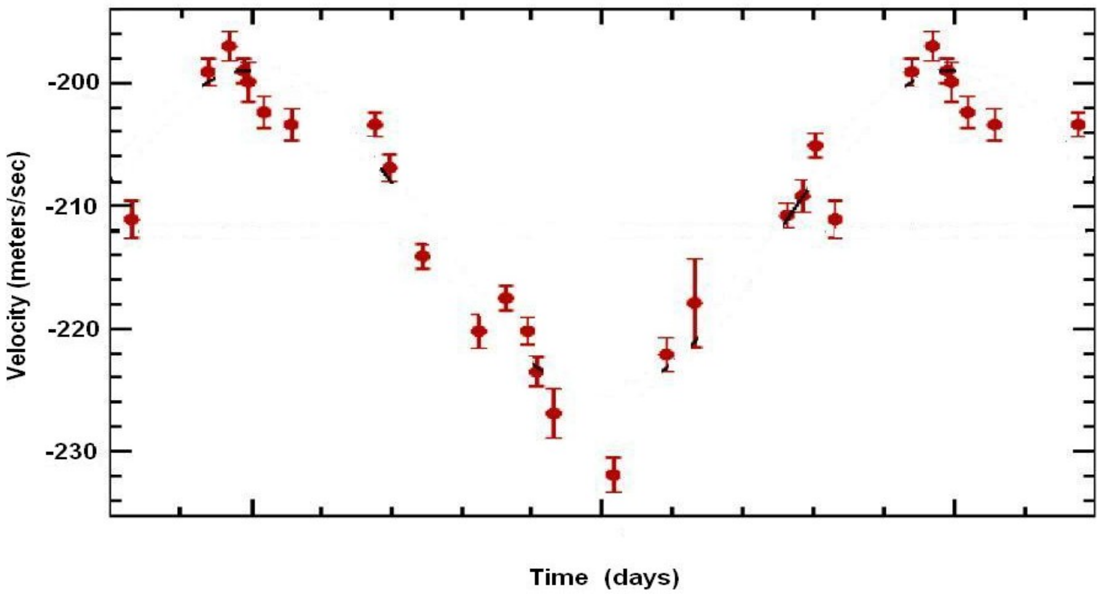
The first two rungs of the ladder convert the energy emitted into units of power in terms of ergs/sec. Note that the unit 'year' cancels in the numerator and denominator. The third rung converts the power units from ergs/s to watts. The answer is 4.2×10^{38} watts. Note that the sun produces 3.8×10^{26} watts, so 3C273 is about 1 trillion times as powerful as a single star.

Problem 3:

$$\frac{50 \text{ grams}}{8 \text{ ounces}} \times \frac{128 \text{ ounces}}{1 \text{ gallon}} \times \frac{9 \text{ gallon}}{34 \text{ liters}} \times \frac{1 \text{ kilogram}}{1000 \text{ grams}}$$

Note that all but the units kilogram/liter cancel out, leaving the answer **0.21 kg/liter.**

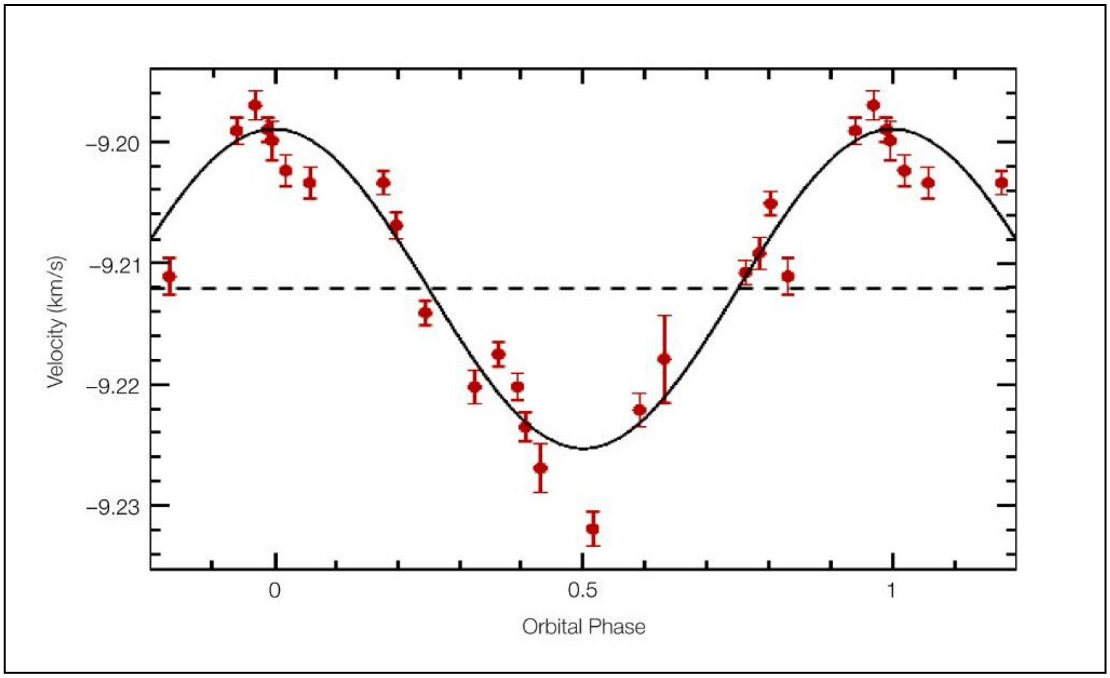
Fitting Periodic Functions - Distant Planets



A team of French and Swiss astronomers have discovered one of the lightest exoplanets ever found using the HARPS instrument on ESO's 3.6-m telescope at La Silla (Chile). They measured the speed of the star, Gleise-581 that the planet orbits, and plotted the data as shown above. The marks on the horizontal axis are spaced every 0.54 days apart starting at 0 which occurred on June 8, 2004.

Problem 1: From the data, create an estimate of the best-fit periodic function that follows the trend in the data. Calculate the amplitude, offset (vertical shift) phase and the formula for the angle in terms of the elapsed time in days since the start of the plot.

Problem 2: What would you predict as the velocity of the star on June 19, 2004?



The figure above shows the actual cosine fit proposed by the astronomers for the star's periodic motion. It is of the form V = amplitude x cos($\theta - \phi$) + v0

Problem 1: The two peaks occur at T = 2 x 0.54 = 1.08 days and T = 12 x 0.54 = 6.48 days. The first peak occurs at a velocity of -198 m/s. The first minimum occurs at -226 km/sec so the average value (dotted line) is v0 = –(198 + 226)/2 = **-212 km/sec.** The amplitude is found by taking the difference between the maximum and minimum and dividing by 2 to get (-198 – (-226))/2 = **14 m/sec.** The period between the peaks is 6.48 – 1.08 = 5.4 days, so the orbital period is 5.4 days during which time the angle goes through 2π radians. That means that for all other times, the angle will be $\theta = 2\pi$ T/5.4 or **0.37 π T** where T is the elapsed time in days since June 8, 2004. The first peak starts at 1.08 days, (where cos(0) = 1) so the phase-shift of the first peak from T=0 (June 8) is $\phi = 2\pi\, 1.08/5.4 =$ **0.4 π.** The best-fit equation is as follows:

$$\mathbf{V = 14\cos(0.37\pi T - 0.4\pi) - 212\ km/s}$$

Problem 2: T = June 19-June 8 = 11 days, so (0.37 π 11 – 0.4 π) = 11.53 radians. Then V = 14 cos(11.53) – 212 **= -205 m/sec**

The scientific results were published in 2005, in the journal *Astronomy and Astrophysics*, vol 443, page L15.

The Limiting Behavior of Selected Functions

In astrophysics, many different kinds of formulae are derived for the interaction of radiation with matter. Often, astrophysicists want to know the 'limiting behavior' of the equations for extreme conditions. Here are a few examples.

$$\sigma = \frac{3}{4} s \left(\frac{1 + x}{x^2} \left[\frac{2(1+x)}{1 + 2x} - \frac{1}{x} \ln(1 + 2x) \right] + \frac{1}{2x} \ln(1 + 2x) - \frac{1 + 3x}{(1 + 2x)^2} \right)$$

Problem 1 : The equation above is the Klien and Nishina formula for the interaction of a high-energy photon with an electron. X is the ratio of the energy carried by the photon (E = hv) compared to the rest mass energy of the electron ($E = mc^2$). What is the form of this equation in the limit for large X?

Problem 1 – The formula can be simplified by noticing that as X becomes very large compared to 1, (1 + x) becomes x, and (1 + 2x) becomes 2x, (1 + 3x) becomes 3x, and the formula simplifies to

$$\sigma = 3/4\ s\ (1/x)[\ 2x/2x - (1/x)\ln(2x)] + (1/2x)\ln(2x) - 3x/(2x)^2).$$

This becomes $\sigma = 3/4\ s\ (1/x - (1/x^2)\ln(2x) + (1/2x)\ln(2x) - 3/4x)$

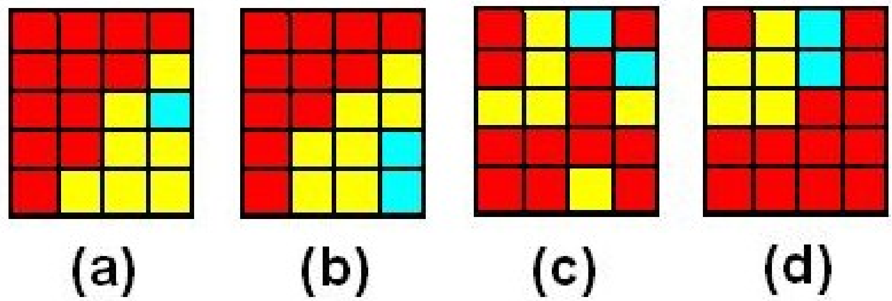
Because terms involving $1/x^2$ diminish faster than terms with 1/x, this leaves us with

$$\sigma = 3/4\ s\ [(1/2x)\ln(2x) + 1/4x]$$

which further simplifies to

$$\boldsymbol{\sigma = (3s/8x)[\ \ln(2x) + 1/2\ ]}$$

Star Cluster Math



Astronomers classify stars so that they can study their similarities and differences. A very common way to classify stars is by their temperature. This scale assigns a letter from the set [O, B, A, F, G, K, M] to represent stars with temperatures from 30,000 C (O-type) and 6,000 C (G-type), to 3,000 C (M-type).

Problem 1 – An astronomer studies a sample of stars in a cluster and identifies 6 as G-type like our Sun, 12 as M-type like Antares, and 2 stars as O-type like Rigel. Circle the pattern, above, that represents this sample.

Problem 2 – What fraction of the stars in the sample are G-type?

A) 6/9 B) 20/6 C) 6/20 D) 6/8

Problem 3 – What fraction of the G and M-type stars in the cluster are G-type?

A) 12/18 B) 6/12 C) 12/6 D) 6/18

Problem 4 – If you selected 2 stars randomly from this cluster, which calculation would give the probability that these would both be O-type stars?

A) 1/20 x 1/20 B) 2/20 x 1/20 C) 1/20 x 1/19 D) 2/20 x 1/19

Problem 5 – A second star cluster has a total of 2,040 stars. If the proportion of O, G and M-types stars is the same as in the first cluster, how many G-type stars would be present?

A) 612 B) 340 C) 1428 D) 680

Answer Key

1 – The boxes are colored red for M-type stars, yellow for G-type stars and blue for O-type stars. Count the boxes carefully. Only C) has the correct number of star boxes colored. **Answer: C)**

2 – There are 6 G-type stars in the cluster, which contains 20 stars, so the answer is C) 6/20. **Answer: C)**

3 – There are a total of 18 G and M-type stars in the sample, and since only 6 are G-type, the correct fraction is D) 6/18. **Answer: D)**

4 – There are 2 O-type stars in a sample of 20 stars. On the first draw, the probability is 2/20 that an O-type star will be selected. Now there are only 19 stars left and only 1 O-type, so the probability that the next star selected is an O-type star is now 1/19. The probability that both O-type stars are drawn in the first two draws is then D) 2/20 x 1/19. **Answer: D)**

5 – The correct answer is A) given by 2040 x 6/20 = 612 stars. The fraction 6/20 represents the proportion of 6 G-type stars out of the 20 stars in the first sample, and we are assuming that this proportion is the same for the second cluster. The incorrect answers come about by B) dividing 2040 by the number of G-type stars; C) multiplying 2040 by the fraction of stars that are O and M-class and; D) dividing 2040 by the number of star classes, which incorrectly assumes an equal number in each class. **Answer: A)**

Magnetic Force in Three Dimensions

A magnetic field is more complicated in shape than a gravitational field because magnetic fields have a property called 'polarity'. All magnets have a North and South magnetic pole, and depending on where you are in the space near a magnet, the force you feel will be different than for gravity. The strength of the magnetic field along each of the three directions in space (X, Y and Z) is given by the formulas:

$$B_x = \frac{3\,x\,z\,M}{r^5}$$

$$B_y = \frac{3\,y\,z\,M}{r^5}$$

$$B_z = \frac{(3\,z^2 - r^2)\,M}{r^5}$$

The variables X, Y and Z represent the distance to a point in space in terms of the radius of Earth. For example, 'X = 2.4' means a physical distance of 2.4 times the radius of the earth or (2.4 x 6378 km) = 15,307 kilometers. Any point in space near Earth can be described by its address (X, Y, Z). The variable r is the distance from the point at (X, Y, Z) to the center of Earth in units of the radius of Earth. **M** is a constant equal to **31,000**.

The formula for the three quantities Bx, By and Bz gives their strengths in units of nanoTeslas (nT) – a measure of magnetic strength along each of the three directions in space.

Problem 1 - Evaluate these three equations at the orbit of communications satellites for the case where x = 7.0, y = 0.0, z = 0.0 and r = 7.0

Problem 2 - Evaluate these three equations in the Van Allen Belts for the case where x = 0.38, y = 0.19, z = 1.73 and r = 3.0

Problem 3 - Use the Pythagorean Theorem in 3-dimensions to determine the total strength of Earth's magnetic field for problems 1, 2 and 3.

Answer Key

Problem 1 - For x = 7.0, y = 0.0, z = 0.0 and r = 7.0

$Bx = 3 (7.0) (0.0) (31,000)/(7.0)^5$ = **0.0 nT**

$By = 3 (0.0) (0.0) (31,000) / (7.0)^5$ = **0.0 nT**

$Bz = [3(0.0)^2 – (7.0)^2](31,000) / (7.0)^5$
$= - (31,000)(7.0)^2/(7.0)^5$
$= - 1,519,000 / 16807$
= **- 90 nT**

Problem 2 - For x = 0.38, y = 0.19, z = 1.73 and r = 3.0
$Bx = 3 (0.38) (1.73) (31,000)/(3.0)^5$ = **+251 nT**

$By = 3 (0.19) (1.73) (31,000) / (3.0)^5$ = **+126 nT**

$Bz = [3(1.73)^2 – (3.0)^2) (31,000)/ (3.0)^5$
$= (-0.021)(31000)/243$
= **- 2.7 nT**

Problem 3 - Use the Pythagorean Theorem in 3-dimensions to determine the total strength of Earth's magnetic field for problems 1 and 2.

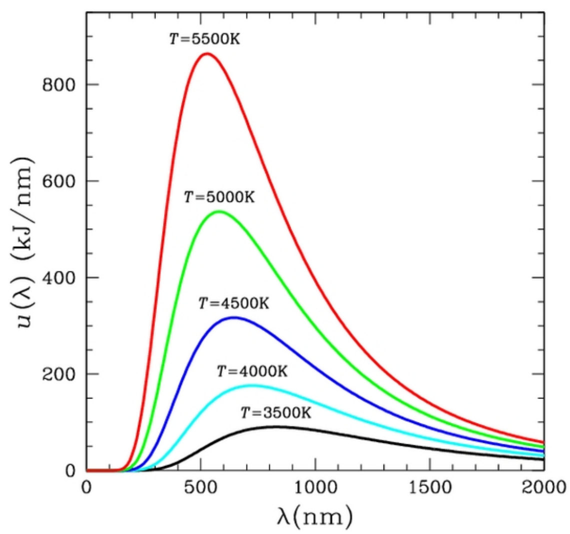
1) $B = (Bx^2 + By^2 + Bz^2)^{1/2} = ((-90)^2)^{1/2}$ = **90 nT** at communications satellite orbit.

2) $B = ((251)^2 +(126)^2 +(-2.7)^2)^{1/2}$ = **281 nT** at Van Allen belts

Measuring Star Temperatures

Careful measurements of a star's light spectrum gives astronomers clues about its temperature. For example, incandescent bodies that have a red glow are 'cool' while bodies with a yellow or blue color are 'hot'. This can be made more precise by measuring very carefully exactly how much light a star produces at many different wavelengths.

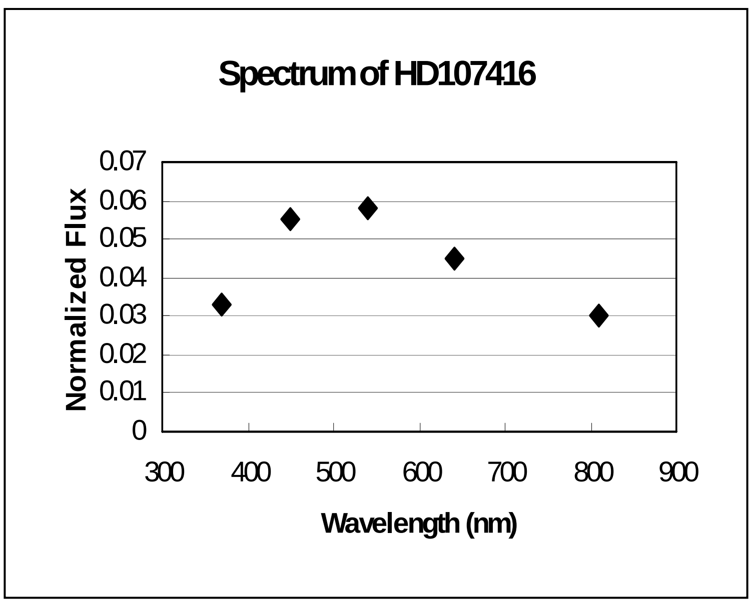
In 1900, physicist Max Planck worked out the mathematical details for how to exactly predict a body's spectrum once its temperature is known. The curve is therefore called a Planck 'black body' curve. It represents the brightness at different wavelengths of the light emitted from a perfectly absorbing 'black' body at a particular temperature.



From the mathematical properties of the Planck Curve, it is possible to determine a relationship between the temperature of the body and the wavelength where most of its light occurs - the peak in the curve. This relationship is called the Wein Displacement Law and looks like this:

$$\text{Temperature} = \frac{2897000 \text{ Kelvins}}{\text{Wavelength}}$$

Where the temperature will be in units of Kelvin degrees, and the wavelength will be in units of nanometers.



The lower plot shows measurements of the spectrum of the star HD107146. The horizontal axis is in units of nanometers (nm).

Problem 1 - Based on the overall shape of the curve, and the wavelength where most of the light is being emitted, use the Wein Displacement Law to determine the temperature of HD107146.

Problem 2 - What would be the peak wavelengths of the following stars in nanometers.

A) Antares .... 3,100 K
B) Zeta Orionis……..……. 30,000 K
C) Vega ………………….. 9,300 K
D) Regulus……………..….. 13,000 K
E) Canopus………………. 7,300 K
F) OTS-44 brown dwarf… 2,300 K
G) Sun……………………. 5,770 K

Answer Key:

Problem 1 - The peak of the curve is **near 500** nanometers. The temperature is 2897000 / 500 = **5,794 K**.

Problem 2 - What would be the peak wavelengths of the following stars in Angstroms:

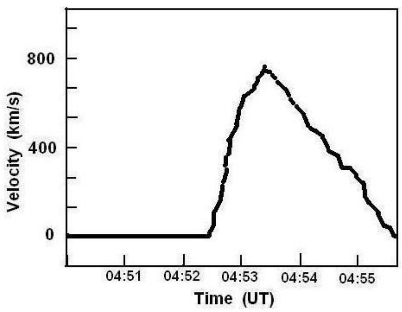
Answer:

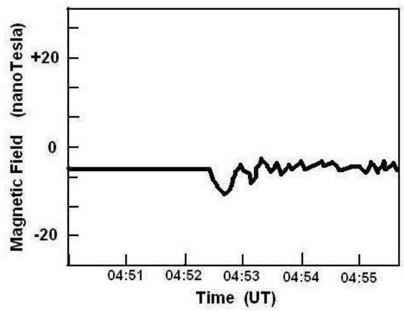
A)	Antares occurs at…….	2897000/3100 K	=	934 nanometers
B)	Zeta Orionis is at……..	2897000/30,000 K	=	97 nanometers
C)	Vega …………………..	2897000/9,300 K	=	311 nanometers
D)	Regulus…………..…..	2897000/13,000 K	=	223 nanometers
E)	Canopus……………….	2897000/7,300 K	=	397 nanometers
F)	OTS-44 brown dwarf…	2897000/2,300 K	=	1260 nanometers
G)	Sun…………………….	2897000/5770Ak	=	502 nanometers

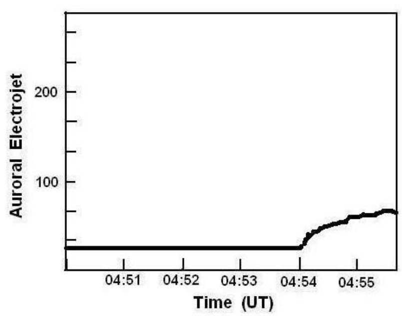
Figure Credits:

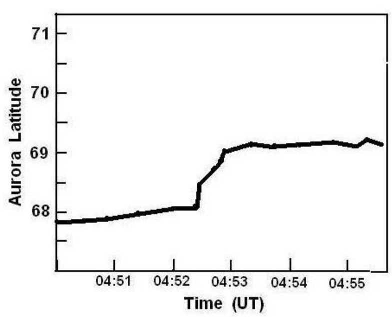
The spectrum of HD 107146 is adapted from a paper by Williams et al. published in the Astrophysical Journal, 2004 vol. 604 page 414. The graph of Planck curves is from Wikimedia and is copyright-free.

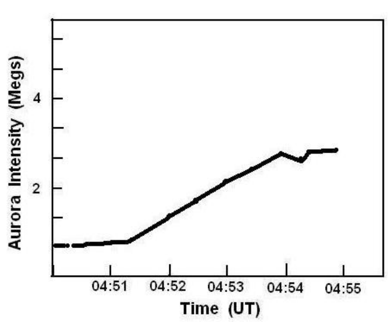
THEMIS: A Magnetic Case of 'What came first?'











The NASA, THEMIS satellite constellation consists of five satellites, P1, P2, P3, P4 and P5, launched on February 17, 2007. The scientific goal was to determine the sequence of events connecting disturbances in Earth's distant magnetic field (a process called magnetic reconnection), with the start of magnetic storms and aurora near Earth.

The science team assembled the data shown in the graphs to the left. The event that triggered this sequence was a 'magnetic reconnection' in Earth's magnetic field that took place at about 4:50:03 at a location about 160,000 km from Earth. (Note these plots have been greatly simplified for clarity! See the original article in the journal *Science*, August 15, 2008, vol. 321, pp.931: Figure 2 and 3)

Problem 1 - At about what times do each of the plots note a significant change in the quantity being measured?

Problem 2 - What is the time sequence of events based on your answers to Problem 1?

Problem 3 - How long was the elapsed time between the increase in particle velocity at the P3 satellite, and the enhancement of the auroral electrojet?

Problem 4 - The P3 satellite was located 74,000 km from Earth. If the auroral electrojet is a stream of charged particles that flows in Earth's upper atmosphere (300 km from the surface) what was the speed of the event in km/sec between the P3 location and when the electrojet started to form?

Problem 5 - What was the time difference between the magnetic reconnection event at 04:50:03 and the start of the auroral change in latitude?

Problem 6 - How fast did the particles travel from the reconnection region to the Earth when the auroral 'substorm' began?

Problem 7 - Before the THEMIS observations, one theory said that the disturbances near the P3 satellite would come before the reconnection occurred. What does the data say about this theory?

Answer Key

Problem 1 - Answer: The times below were reported in the scientific journals based on the actual data. Student answers may vary slightly depending on their ability to interpolate the values on the horizontal axis. For best results, use a millimeter ruler to determine the scale of the axis in seconds/mm.
Plot 1: Particle velocity increase detected at P3: **4:52:27**
Plot 2: Magnetic field change at P3: **4:52:27**
Plot 3: Auroral electrojet amplification in the ionosphere: **4:54:00**
Plot 4: Substorm expansion; Latitude increase northwards: **4:52:21**
Plot 5: Auroral intensity change; arrival of particles at Earth that were generated by magnetic reconnection. **4:51:39**

Problem 2 - Answer:
4:51:39: Auroral intensity change; arrival of particles at Earth generated by the magnetic reconnection.
4:52:21: Substorm expansion: Latitude increase northwards
4:52:27: Particle velocity increase detected at P3 due to earthward flow of particles.
4:52:27: Magnetic field change at P3
4:54:00: Auroral electrojet amplification in the ionosphere.

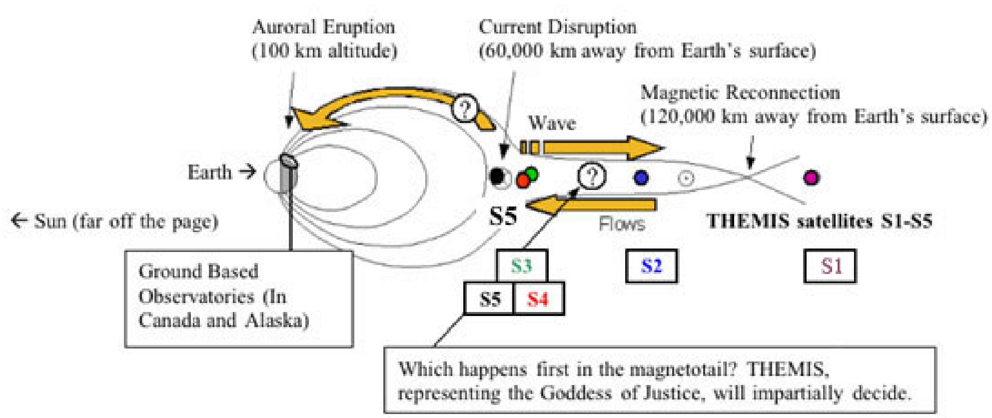
Problem 3 - Answer: 4:54:00 - 4:52:27 = 1minute and 33 seconds = **93 seconds.**

Problem 4 - Answer: 74,000 km / 93 seconds = **796 km/sec.**

Problem 5 - Answer: 4:52:21 - 4:50:03 = 2m 18s = **138 seconds.**

Problem 6 -Answer: The distance traveled was 160,000 kilometers in 138 seconds, for an average speed of 160,000 km /138s = **1,160 km/sec** if no allowance is made for the radius of the earth (6378km) and the height of the ionosphere (300 km). If these allowances are made, then (160,000 - 6378 - 300) km /138s = **1,110 km/sec.**

Problem 7 - Answer: Because the reconnection event happened at 04:50:03 ,this was about 2:27 before the disturbances at P3 in the particles speeds were recorded, so the theory in which P3 happened first is not consistent with the new data.



Unit Conversions III

1 Astronomical Unit = 1.0 AU = 1.49×10^{8} kilometers		
1 Parsec = 3.26 Light years = 3×10^{18} centimeters = 206,265 AU		
1 Watt = 10^{7} ergs/sec		
1 Star = 2×10^{33} grams		
1 Yard = 36 inches	1 meter = 39.37 inches	1 mile = 5,280 feet
1 Liter = 1000 cm3	1 inch = 2.54 centimeters	1 kilogram = 2.2 pounds
1 Gallon = 3.78 Liters	1 kilometer = 0.62 miles	

Problem 1 – Convert 11.3 square feet into square centimeters.

Problem 2 – Convert 250 cubic inches into cubic meters.

Problem 3 – Convert 1000 watts/meter^{2} into watts/foot^{2}

Problem 4 – Convert 5 miles into kilometers.

Problem 5 – Convert 1 year into seconds.

Problem 6 – Convert 1 km/sec into parsecs per million years.

Problem 7 - A house is being fitted for solar panels. The roof measures 50 feet x 28 feet. The solar panels cost \$1.00/cm^{2} and generate 0.03 watts/cm^{2}. A) What is the maximum electricity generation for the roof in kilowatts? B) How much would the solar panels cost to install? C) What would be the owners cost for the electricity in dollars per watt?

Problem 8 – A box of cereal measures 5 cm x 20 cm x 40 cm and contains 10,000 Froot Loops. What is the volume of a single Froot Loop in cubic millimeters?

Problem 9 – In city driving, a British 2002 Jaguar is advertised as having a gas mileage of 13.7 liters per 100 km, and a 2002 American Mustang has a mileage of 17 mpg. Which car gets the best gas mileage?

Problem 10 – The Space Shuttle used 800,000 gallons of rocket fuel to travel 400 km into space. If one gallon of rocket fuel has the same energy as 5 gallons of gasoline, what is the equivalent gas mileage of the Space Shuttle in gallons of gasoline per mile?

Problem 11 – The length of an Earth day increases by 0.0015 seconds every century. How long will a day be in 3 billion years from now?

Problem 12 – The density of matter in the Milky Way galaxy is 7.0×10^{-24} grams/cm^{3}. How many stars are in a cube that is 10 light years on a side?

Problem 13 – At a speed of 300,000 km/sec, how far does light travel in miles in 1 year?

Answer Key

Problem 1 – 11.3 x (12 inches/foot)x(12 inches/foot) x (2.54 cm/1 inch)x(2.54 cm/1 inch) = **10,500 cm^2**

Problem 2 – 250 $inch^3$ x $(2.54\ cm/inch)^3$ x $(1\ meter/100\ cm)^3$ = **0.0041 m^3**

Problem 3 – 1000 $watts/meter^2$ x $(1\ meter/39.37\ inches)^2$ x $(12\ inches/foot)^2$ = **93.0 $watts/ft^2$**

Problem 4 – 5 miles x (5280 feet/mile) x (12 inches/foot)x(2.54 cm/inch)x(1 meter/100 cm) x(1 km/1000 meters) **= 8.1 km**

Problem 5 – 1 year x (365 days/year)x(24 hours/day)x(60 minutes/hr)x(60 seconds/minute) = **31,536,000 seconds.**

Problem 6 – 1 km/sec x (100000 cm/km)x(3.1 x 10^7 seconds/year) x (1 parsec/ 3.1 x 10^{18} cm) x (1,000,000 years/1 million years) = **1 parsec/million years**

Problem 7 - A) Area = 50 feet x 28 feet = 1400 ft^2. Convert to cm^2: 1400 x $(12\ inch/foot)^2$ x $(2.54\ cm/1\ inch)^2$ = 1,300,642 cm^2. Maximum power = 1,300,642 cm^2 x 0.03 $watts/cm^2$ = **39.0 kilowatts**. B) 1,300,642 cm^2 x \$1.00 /cm^2 = **\$1.3 million** C) \$1,300,000 / 39,000 watts = **\$33.3 /watt**.

Problem 8 – Volume of box = 5 x 20 x 40 = 4000 cm^3. This contains 10,000 Froot Loops, so each one has a volume of 4,000 cm^3/10,000 loops = 0.4 cm^3/Loop. Converting this into cubic millimeters: 0.4 cm^3 x $(10\ mm/1\ cm)^3$ = **400 mm^3/Loop**.

Problem 9 – Convert both to kilometers per liter. Jaguar = 100 km/13.7 liters = 7.3 km/liter. Mustang = 17.0 x (1 km/0.62 miles) x(1 gallon/3.78 liters) = 7.25 km/liter. **They both get similar gas mileage under city conditions**.

Problem 10 – 400 km x (0.62 miles/km) = 248 miles. Equivalent gallons of gasoline = 800,000 gallons rocket fuel x (5 gallons gasoline/1 gallon rocket fuel) = 4,000,000 gallons gasoline, so the 'mpg' is 248 miles/4000000 = 0.000062 miles/gallon or **16,130 gallons/mile**.

Problem 11 – 0.00015 sec/century x (1 century/100 years) x 3 billion years = 4,500 seconds or 1.25 hours. The new 'day' would be 24h – 1.25 = **22.75 hours long**.

Problem 12 –First convert to grams per cubic parsec: 7.0 x 10^{-24} $grams/cm^3$ x $(3.1 \times 10^{18}\ cm/parsec)^3$ = 2.0 x 10^{32} $grams/pc^3$. Then convert to Stars/pc3: 2.0 x 10^{32} $grams/pc^3$ x (1 Star/2 x 10^{33} grams) = 0.1 $Stars/pc^3$. Then compute the volume of the cube: V = 10x10x10 = 1000 light $years^3$ = 1000 light $years^3$ x (1 parcsec/3.26 light years)3 = 28.9 $Parsecs^3$. Then multiply the density by the volume: 0.1 $Stars/pc^3$ x (28.9 $Parsecs^3$) = **3.0 Stars in a volume that is 10 light years on a side**.

Problem 13 – 300,000 km/sec x (3.1 x 10^7 sec/year) = 9.3 x 10^{12} km. Then 9.3 x 10^{12} km x (0.62 miles/km) = **5.7 trillion miles**.

'Can you hear me now?': getting the message across

Have you ever wondered why some radio stations come in clearly in your radio, while others can barely be heard no matter how you crank up the volume? The reception of a radio signal depends on two very important quantities. The first is how much power the radio station is broadcasting. The second is how far that station is from you.

Some radio stations broadcast only at 100 watts, while others transmit over 50,000 watts of radio power. Imagine a 5-watt light bulb and a 100-watt light bulb. Which one do you think will be easier to see from across the room, or at 100-meters?

The brightness of a lamp, or a radio station, is measured by the amount of power that is delivered to a square-meter of area. We call this physical unit 'intensity' and measure its quantity in watts per square-meter (W/m^2). Because most kinds of lights and radio stations broadcast their power over a spherical volume, the intensity of a source is easily computed by dividing its power, P, by the surface area of a sphere whose radius, D, equals your distance from the source. The formula is just $P / 4\pi D^2$.

Transmitter	Distance (km)	Power (watts)	Intensity (watts/m^2)
AM Station	1000	50,000	**3.9×10^{-9}**
TV Station	100	50,000	
Cell Phone	1	0.3	
THEMIS P1	160,000	3	
STEREO A	15 million	10	
ACE	1.5 million	5	
MESSENGER	50 million	15	
Mars Orbiter	220 million	100	
Cassini	1.4 billion	20	
Ulysses	800 million	5	
Voyager 2	13 billion	40	

Problem 1 - Fill out the last column in the table to find the intensity of the radio signal at Earth. (Use Scientific Notation to an accuracy of one decimal place.)

Problem 2 - The signal from the Voyager-2 spacecraft, located beyond the orbit of Pluto, is just detectable by sensitive receivers of the Deep Space network on Earth. How far away would the AM radio station be to just be detectable by the DSN? Express the answer in kilometers, Astronomical Units (1 AU = 150 million kilometers), and light years. (Note 1 light year = 9.5 trillion km).

Answer Key

Problem 1 – Answer in last column

Transmitter	Distance (km)	Power (watts)	Intensity (watts/m^2)
AM Station	1000	50,000	**3.9×10^{-9}**
TV Station	100	50,000	**3.9×10^{-7}**
Cell Phone	1	0.3	**3.0×10^{-7}**
THEMIS P1	160,000	3	**9.2×10^{-18}**
STEREO A	15 million	10	**3.5×10^{-22}**
ACE	1.5 million	5	**1.8×10^{-19}**
MESSENGER	50 million	15	**4.7×10^{-22}**
Mars Orbiter	220 million	100	**1.6×10^{-22}**
Cassini	1.4 billion	20	**8.1×10^{-25}**
Ulysses	800 million	5	**6.2×10^{-25}**
Voyager 2	13 billion	40	**1.9×10^{-26}**

Problem 2 - If the radio station were at the distance of the Voyager 2 spacecraft, I = 2.3×10^{-23} watts/m^2. The desired Intensity = 1.9×10^{-26} watts/m^2, is 1230 times weaker. Since the signal intensity is proportional to the inverse-square of the distance, the distance would be increased by $(1230)^{1/2}$ = 35 times so that the radio station signal is now only 1.9×10^{-26} watts/m^2. The distance would then be 35 x 13 billion km or **456 billion km**, which equals **3040 AU**, or **0.001 light years**.

The STEREO Mission: getting the message across

In 2007, NASA launched two satellites, STEREO-A and STEREO-B that were to slowly drift away from Earth in opposite directions, taking 'stereo' images of the Sun as they went. Getting the information and images back to Earth posed a challenge because the farther away they drifted, the weaker the signal got. The table below gives the distance of STEREO-A from Earth. The satellite used a 10-watt transmitter operating at a frequency of 2,300 Megahertz .This, by the way, is about 10-times the frequency of a normal UHF TV station.

Date	Distance (Million km)	Intensity ($x10^{-24}$ W/m^2)	Received Power ($x10^{-20}$ Watts)
June-2007	24.6	1316	**126**
August-2007	40.3	490	
October-2007	51.0	306	
December-2007	55.2	261	
February-2008	58.4	234	
April-2008	67.0	177	
June-2008	81.9	119	
August-2008	97.6	84	
October-2008	107.3	69	
December-2008	111.2	64	

Problem 1 - How much weaker was the radio signal from STEREO-A in December 2008 than in June 2007?

Problem 2 - The Deep Space network radio dish has a diameter of 70-meters. In Column 4 calculate how many watts the dish collected from the signals sent during each month.

Problem 3 - If the satellite sends one bit of data ('1' or '0') every 5 seconds, how much energy is detected (in Joules) per bit sent in A)June 2007? and B) December 2008? (1 watt = 1 Joule per second).

Problem 4 - Suppose the receiver cannot detect less radio energy less than 2 x 10^{-25} Joules each bit. What is the largest number of bits that can be detected each second in A) June 2007 and B) December 2008?

Problem 5 - As a spacecraft gets further and further away from Earth, what kinds of strategies do engineers and scientists have to use to get their data back to Earth?

Answer Key

Problem 1 - How much weaker was the radio signal from STEREO-A in December 2008 than in June 2007? **Answer: The signal was 126/6 = 21 times weaker in December than in June.**

Problem 2 - Answer:

Date	Distance (Million km)	Intensity ($x10^{-24}$ W/m^2)	Receiver Power ($x10^{-20}$ Watts)
June-2007	24.6	1316	126
August-2007	40.3	490	**47**
October-2007	51.0	306	**29**
December-2007	55.2	261	**25**
February-2008	58.4	234	**22**
April-2008	67.0	177	**17**
June-2008	81.9	119	**11**
August-2008	97.6	84	**8**
October-2008	107.3	69	**7**
December-2008	111.2	64	**6**

Problem 3 - If the satellite sends one bit of data ('1' or '0') every 5 seconds, how much energy is detected (in Joules) per bit sent in A)June 2007? and B)December 2008? (1 watt = 1 Joule per second). Answer: A) 1.26 x 10^{-18} Joules/sec x 5 sec = **6.3 x 10^{-18} Joules**. B) 6 x 10^{-20} Joules/sec x 5 sec = **3.0 x 10^{-19} Joules.**

Problem 4 - Suppose the receiver cannot detect less radio energy less than 2 x 10^{-25} Joules each bit. What is the largest number of bits that can be detected each second in A) June 2007 and B) December 2008? Answer: A) 1.26 x 10^{-18} Joules/sec / (2.0 x 10^{-25} Joules/bit) = **6.3 million bits/sec**. B) 6 x 10^{-20} Joules/sec / (2.0 x 10^{-25} Joules/bit) = **300,000 bits/sec**.

Problem 5 - Answer: **Here are just a few possibilities: 1) They can use larger radio dishes to increase the signal strength at the receiver. 2) They can slow down the data rates so that the energy per bit is larger making the data more easily detectable. 3) They can design the spacecraft with a more powerful transmitter. 4) They can do all of the above.**

Optimization

Satellites are designed to optimize the number of experiments they can carry, while at the same time keeping the mass and power requirements at a minimum. The picture shows the octagonal IMAGE satellite with the dark solar cells attached to its surface.

Here is one example of a simple problem that can be encountered by a satellite designer.

An hexagonal satellite is designed to fit inside the nose-cone (shroud) of a Delta II rocket. There is only enough room for a single satellite, and it cannot have deployable solar panels to generate electricity using solar cells. Instead, the solar cells have to be mounted on the exterior surface of the satellite. At the same time, the satellite configuration is that of a hexagonal prism. The total volume of the satellite is 10 cubic meters. The solar cells will be mounted on the hexagonal top, bottom, and the rectangular side panels of the satellite.

Problem 1 - If the width of a panel is W, and the height of the satellite is H, what are the dimensions of the satellite that maximize the surface area and hence the available power that can be generated by the solar cells?

Problem 2 - If only 1/2 of the solar cells receive light at any one time, and the power they can deliver is 100 watts per square meter, what is the maximum power that this satellite can provide to the experiments and operating systems?

Problem 3 - If the mass of the panels is 3 kg per square meter, what is the total mass of this satellite?

Problem 4 – If the density of the satellite is 1000 kilograms per cubic meter, and the launch cost is $10,000 per pound, how much will it cost to place this satellite into orbit? (Note: 1 pound = 0.453 kilograms)

Answer Key

Problem 1 - If the width of a panel is W, and the height of the satellite is H, what are the dimensions of the satellite that maximize the surface area and hence the available power that can be generated by the solar cells?

Answer:
The volume of a hexagonal prism is given by

$$V = \frac{3\ (3)^{1/2}\ H\ W^2}{2}$$

The surface area is given by

$$A = 6WH + 3\ (3)^{1/2}\ W^2$$

Since V = 10 cubic meters, we can solve for H to get

$$H = \frac{20}{3\ (3)^{1/2}\ W^2}.$$

Eliminate H from the equation for surface area to get

$$A = \frac{40}{3\ (3)^{1/2}\ W} + 3\ (3)^{1/2}\ W^2$$

Differentiate A with respect to W, set this equal to zero.

$$\frac{40}{3\ (3)^{1/2}\ W^3} = 6\ (3)^{1/2}\ W$$

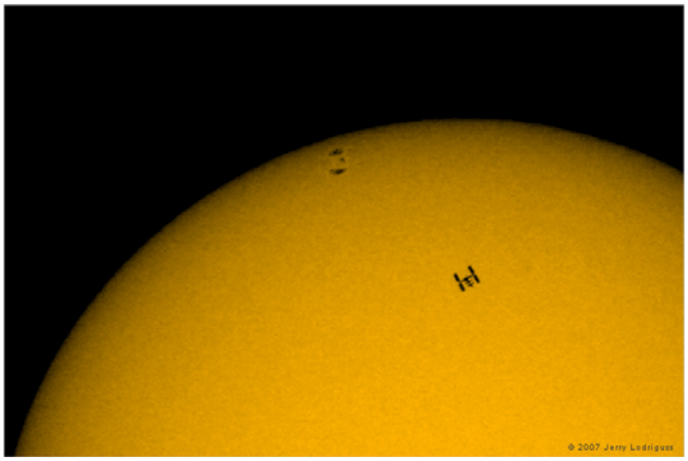
A bit of algebra gives us $W^3 = 40/(3*18)$ so **W = 0.90** meters and from the definition for H we have **H = 4.75 meters**.

Problem 2 - If only 1/2 of the solar cells receive light at any one time, and the power they can deliver is 100 watts per square meter, what is the maximum power that this satellite can provide to the experiments and operating systems? Answer: The total surface area is $A = 6\ (0.9)\ (4.75) + 3\ (3)^{1/2}\ (0.9)^2 = 25.7 + 4.2 = 29.9$ square meters. Only ½ are available for sunlight, and so the total power will be about 29.9 x 0.5 x 100 = **1,495 watts**.

Problem 3 - If the mass of the panels is 3 kg per square meter, what is the total mass of this satellite? Answer: 3 x 29.9 = **89.7 kilograms.**

Problem 4 – If the density of the satellite is 1000 kilograms per cubic meter, and the launch cost is $10,000 per pound, how much will it cost to place this satellite into orbit? (Note, 1 pound = 0.453 kilograms). Answer: The volume of the hexagonal satellite is V = 10 cubic meters, so the mass is 1000 x 10 = 10,000 kilograms or 10 metric tons. The cost to launch is 10,000 kilograms x 10,000 dollars/pound x (1 pound/ 0.453 kg) = $100 million/0.453 = **$220 million dollars.**

Angular Size and Velocity



(Photo courtesy Jerry Lodriguss (Copyright 2007, http://www.astropix.com/HTML/SHOW_DIG/055.HTM)

The relationship between the distance to an object, **R**, the objects size, **L**, and the angle that it subtends at that distance, θ, is given by:

$$\theta = 57.29 \frac{L}{R} \quad \text{degrees}$$

$$\theta = 3{,}438 \frac{L}{R} \quad \text{arcminutes}$$

$$\theta = 206{,}265 \frac{L}{R} \quad \text{arcseconds}$$

To use these formulae, the units for length, L, and distance, R, must be identical.

Problem 1 - You spot your friend (L = 2 meters) at a distance of 100 meters. What is her angular size in arcminutes?

Problem 2 - The Sun is located 150 million kilometers from Earth and has a radius of 696.000 kilometers, what is its angular diameter in arcminutes?

Problem 3 - How far away, in meters, would a dime (1 centimeter) have to be so that its angular size is exactly one arcminute?

Problem 4 - The spectacular photo above shows the International Space Station streaking across the disk of the Sun. If the ISS was located 379 kilometers from the camera, and the ISS measured 73 meters across, what was its angular size in arcseconds?

Problem 5 - The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) what was the angle, in arcminutes, that it moved through in one second as seen from the location of the camera? B) What was its angular speed in arcminutes/second?

Problem 6 - Given the diameter of the Sun in arcminutes (Problem 2), and the ISS angular speed (Problem 5) how long, in seconds, did it take the ISS to travel across the face of the sun?

Problem 1 - Answer: Angle = 3,438 x (2 meters/100 meters) = **69 arcminutes**.

Problem 2 - Answer: 3,438 x (696,000/150 million) = 15.9 arcminutes in radius, so the diameter is 2 x 15.9 = **32 arcminutes**.

Problem 3 - Answer: From the second formula R = 3438 * L/A = 3438 * 1 cm/1 arcminute so R = 3,438 centimeters or a distance of **34.4 meters**.

Problem 4 - Answer: From the third formula, Angle = 206,265 * (73 meters/379,000 meters) = **40 arcseconds**.

Problem 5 - Answer: The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) The ISS traveled L = 7.4 kilometers so from the second formula Angle = 3,438 * (7.4 km/379 km) = **67 arcminutes**. B) The angular speed is just **67 arcminutes per second**.

Problem 6 - Answer: The time required is T = 31.8 arcminutes/ (67 arcminutes/sec) = **0.47 seconds.**

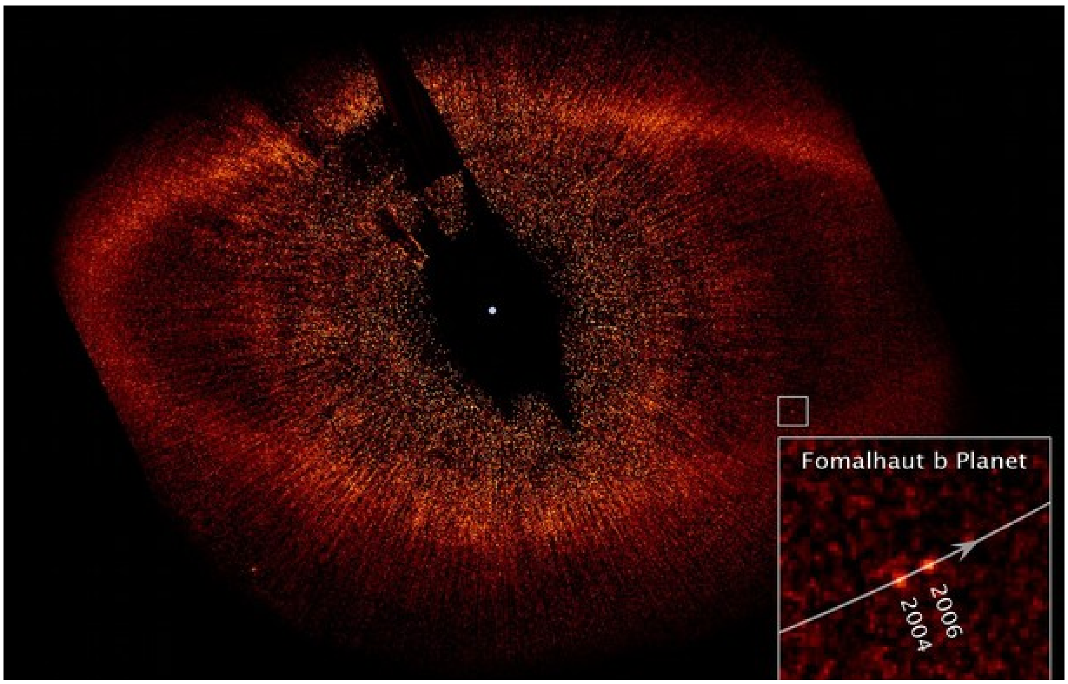
The spectacular photo by Jerry Lodriguss had to be taken with careful planning beforehand. He had to know, to the second, when the Sun and ISS would be in the right configuration in the sky as viewed from his exact geographic location. Here's an example of the photography considerations in his own words:

" I considered trying to monitor the transit visually with a remote release in my hand and just firing (the camera) when I saw the ISS in my guidescope. Then I worked out the numbers. I know that my reaction time is 0.19 seconds. This is actually quite good, but I make my living shooting sports where this is critical, so I better be good at it. I also know that the Canon 1D Mark IIn has a shutter lag of 55 milliseconds. Adding these together, plus a little bit of a fudge factor, the best I could hope for was about 1/4 of a second from when I saw it to when the shutter opened. Since the entire duration of the transit was only 1/2 of a second, in theory, I could capture the ISS at about the center of the disk if I fired as soon as I saw it start to cross. This was not much of a margin for error. I could easily blink and miss the whole thing... Out of 49 frames that the Mark IIn recorded, the ISS is visible in exactly one frame."

Hubble Sees a Distant Planet

The bright star Fomalhaut, in the constellation Piscis Austrinus (The Southern Fish) is only 25 light years away. It is 2000^{o} K hotter than the Sun, and nearly 17 times as luminous, but it is also much younger: Only about 200 million years old. Astronomers have known for several decades that it has a ring of dust (asteroidal material) in orbit 133 AU from the star and about 25 AU wide. Because it is so close, it has been a favorite hunting ground in the search for planets beyond our solar system. In 2008 such a planet was at last discovered using the Hubble Space Telescope. It was the first direct photograph of a planet beyond our own solar system.

In the photo below, the dusty ring can be clearly seen, but photographs taken in 2004 and 2006 revealed the movement of one special 'dot' that is now known to be the star's first detected planet. The small square on the image is magnified in the larger inset square in the lower right to show the location of the planet in more detail.



Problem 1 – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

Problem 2 – How many kilometers had the planet moved between 2004 and 2006?

Problem 3 – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

Problem 4 – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

Answer Key

Problem 1 – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

Answer: The distance from the center of the ring (location of star in picture) to the center of the box containing the planet is 42 millimeters, then 42 x 2.7 AU/mm = 113 AU. Since 1 AU = 150 million km, the distance is 113 x 150 million **= 17 billion kilometers.**

Problem 2 – How many kilometers had the planet moved between 2004 and 2006?

Answer: On the main image, the box has a width of 4 millimeters which equals 4 x 2.7 = 11 AU. The inset box showing the planet has a width of 36 mm which equals 11 AU so the scale of the small box is 11AU/36 mm = 0.3 AU/mm. The planet has shifted in position about 4 mm, so this corresponds to 4 x 0.3 = **1.2 AU or 180 million km**.

Problem 3 – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

Answer: The average speed is 180 million km/17520 hours = **10,273 km/hr.**

Problem 4 – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

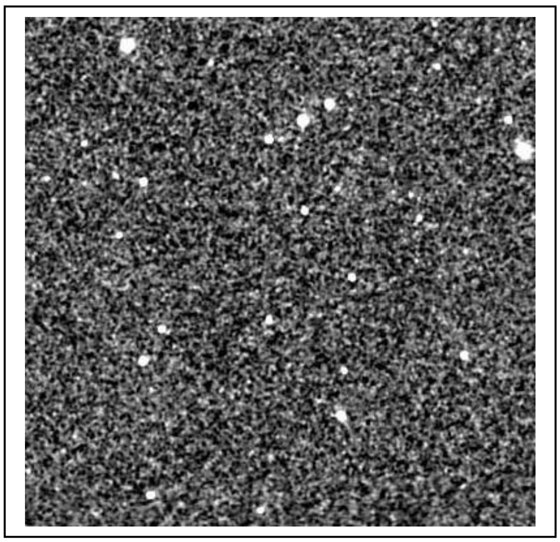
Answer: The radius of the circle is 113 AU so the circumference is $2 \pi R = 2 (3.141) (113 \text{ AU}) = 710$ AU. The distance traveled by the planet in 2 years is, from Problem 2, about 1.2 AU, so in 2 years it traveled 1.2/710 = 0.0017 of its full orbit. That means a full orbit will take 2.0 years/0.0017 = **1,176 years.**

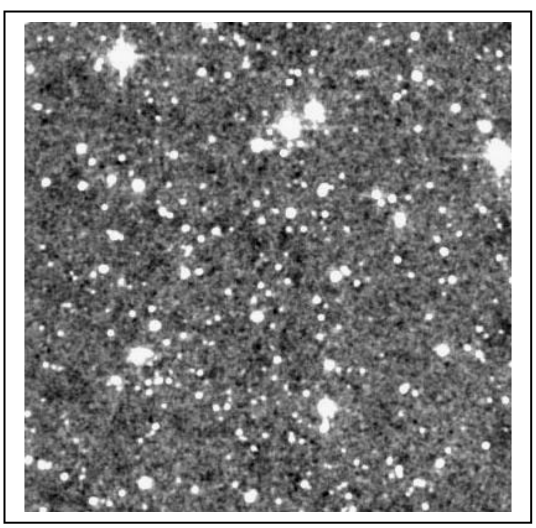
Note - Because we are only seeing the 'projected' motion of the planet along the sky, the actual speed could be faster than the estimate in Problem 3, which would make the estimate of the orbit period a bit smaller than what students calculate in Problem 4.

A careful study of this system by its discoverer, Dr. Paul Kalas (UC Berkeley) suggests an orbit distance of 119 AU, and an orbit period of 872 years.

You are, by now, probably familiar with the mathematical procedure of averaging numbers together. When we combine images together, we can use data averaging to make faint things stand out more clearly. To see why this happens, let's imagine a picture that consists of a string of just 5 pixels (out of the 3 million pixels that might exist in a typical digital camera image!). Let's take a snapshot of exactly the same scene 9 times without moving the camera, and note the values of the intensity numbers in each pixel. Here's what you might get:

Pixel	1	2	3	4	5	6	7	8	9	Average
1	120	122	120	123	110	114	112	110	110	**116**
2	125	115	110	130	115	110	113	110	109	
3	130	133	131	128	130	130	131	129	130	
4	122	125	123	120	110	105	115	120	110	
5	122	125	123	109	110	114	120	105	115	





Problem 1 - Calculate the average value of the 9 images for each pixel by completing the table. The first Pixel has been done already.

Problem 2 - Scientists discriminate between background 'noise' and 'source' whenever they look at an image. Background noise has the property that it averages to a relatively constant intensity that is nearly the same everywhere in the picture. A Source, however, tends to stand out in only a few pixels, and with an intensity brighter than the background. From the five pixel image, which pixels do you think have mostly Noise, and which have mostly Source?

Problem 3 - How easy would it have been if you only had Pictures 1 and 4 to work with in trying to study the faint source in the field?

Problem 4 - If you are trying to detect a faint source against a bright background, what is a good Rule of Thumb to use?

Problem 5 - The two images are from the 2MASS infrared sky survey. The bottom image is an average of over 5000 images like the one at the top. Can you find 5 stars that are present in the 'coadded' image below but not seen in the single image?

Answer Key

Problem 1 - Answer:

Pixel	1	2	3	4	5	6	7	8	9	Average
1	120	122	120	123	110	114	112	110	110	116
2	125	115	110	130	115	110	113	110	109	**115**
3	130	133	131	128	130	130	131	129	130	**130**
4	122	125	123	120	110	105	115	120	110	**117**
5	122	125	123	109	110	114	120	105	115	**116**

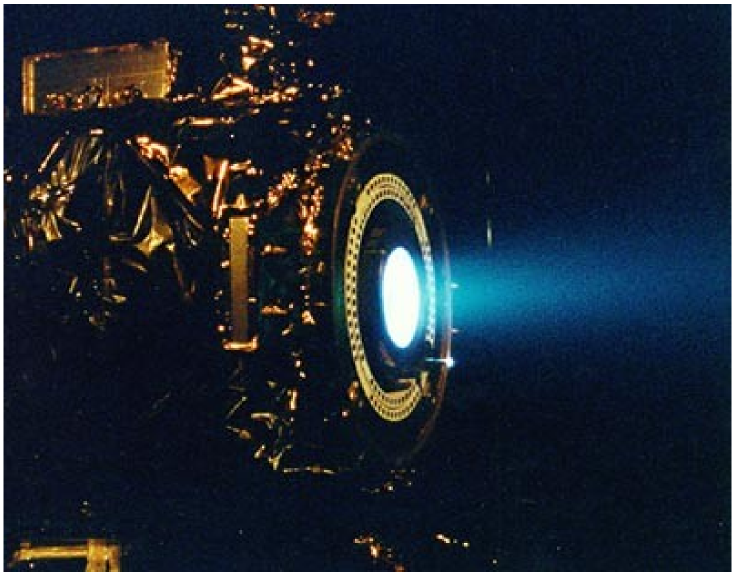
Problem 2 - Answer: From your averages in Column 11, you can see that Pixels 1, 2, 4 and 5 have very similar averages, while Pixel 3 has a very different average value. This means that Pixels 1,2, 4 and 5 behave like Noise in a roughly constant intensity background, while Pixel 3 looks a lot like a Source.

Problem 3 - Answer: It would have been very difficult because the Background Noise level that was detected in Pixel 2 in Picture 1 and Picture 4 sometimes got as intense as the faint source itself in Pixel 3.

Problem 4 - Answer: A good Rule of Thumb would be to take as many pictures of it as you can, and then average the pictures together to make the faint source stand out more clearly.

Problem 5 - Answer: Students will have an easy time of finding many such candidates. The top image was taken by a 1.5-meter telescope that can detect light at an infrared wavelength of 2.2 microns. (Sunlight has a wavelength of 0.6 microns). The exposure was only 7.8 seconds. The bottom image represents an exposure of 7.8 x 5000 = 39,000 seconds and is able to see stars nearly 10,000 times fainter than the shorter exposure image.

The Mathematics of Ion Rocket Engines



Believe it or not, NASA has been using ion engines for decades, and most commercial satellites use them too!

This image of a xenon ion engine, being tested at NASA's Jet Propulsion Laboratory, shows the faint glow of charged atoms being emitted from the engine. It was used in both the Deep Space 1 and Dawn satellites in their historic journeys to explore asteroids. The operating principle is simple.

Heavy atoms such as cesium and xenon are ionized, accelerated through a high-voltage grid, and ejected out the back of the thruster. The momentum of the ejected heavy atoms, when multiplied by the trillions of atoms in the beam, produces a steady, constant thrust that can be maintained for years at a time. Because of the high speed of the atoms very little mass is needed to generate a large thrust over time. For the Dawn spacecraft launched in 2007, the 'fuel' mass is only 425 kilograms, but ejected steadily for 8 years, the 1,200 kilogram satellite will reach a speed of over 36,000 km/hour (22,300 miles/hour). This is equal to 315 million kilometers/year or the distance to the sun and back from Earth! Here is some of the mathematics, in a highly simplified form, that will take you through the basic ideas behind these exciting rocket technologies!

Problem 1 - Charged particles gain speed in an electric field - The kinetic energy of a particle is given by K.E. = $1/2\ mv^2$. The energy a charged particle gains from falling through a potential difference of V volts is given by E = qV. The NSTAR ion engine developed for the Deep Space 1 satellite uses xenon atoms with a mass of 2.2×10^{-25} kg, and a charge of q = 1.6×10^{-19} coulombs. What will be the speed of the atom, in kilometers/hour, if the voltage grid of the ion engine is 1,300 volts?

Problem 2 - The smaller the grid separation, the higher the acceleration - The NSTAR engine has a grid separation of 0.7 mm. From your answer to Problem 1, A) what is the average acceleration of the ions as the leave the grid? B) What is the force they experience, in Newtons?

Problem 3 - The thrust depends on particle flow rate - How many particles have to be ejected in the time it takes to cross the grid, to create a thrust of 0.90 Newtons? (Express the answer in particles per second).

Problem 4 - Charged particle flows produce electrical currents - If each particle carries exactly one unit of charge, and 1 Ampere = 6.25×10^{18} particles/sec, what is the current needed in the beam to give the thrust in Problem 3?

Problem 5 - Currents require power to maintain them - What is the beam power, in watts, defined by Power = Voltage x Amperage?

Answer Key

Problem 1 - Equate K.E = E and solve for v to get:

$$v^2 = 2\,qV/m = 2\,(1.6 \times 10^{-19})(1{,}300) / (2.2 \times 10^{-25}) = 2.0 \times 10^9$$

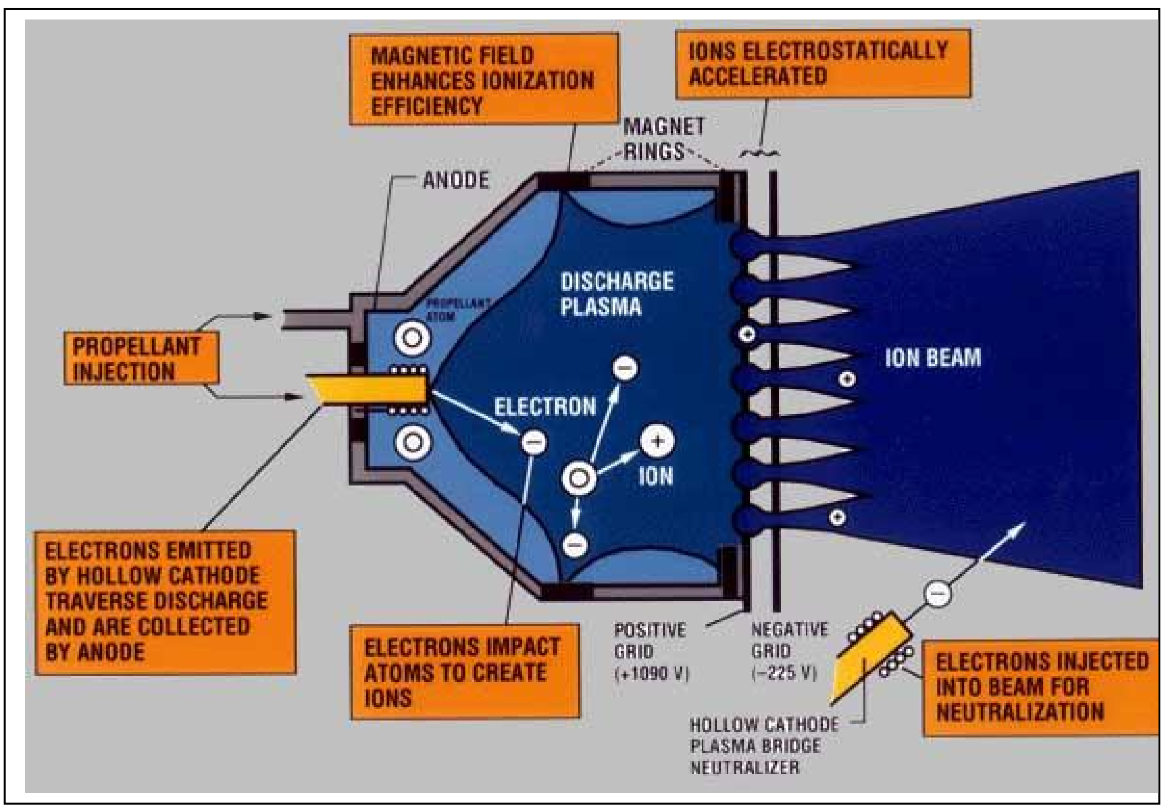
so v = 44.2 km/sec or **159,000 km/hr**

Problem 2 - Answer: A) At an average speed of 44.2 km/sec, the atoms take about T = 0.7 mm/ (22.1 km/sec) = 3.1×10^{-8} seconds to travel the distance. This is an average acceleration of 22,100 m/sec / 3.1×10^{-8} sec = **7.1×10^{11} meters/sec^2.** B) The force F = (2.2×10^{-25}) $(7.1 \times 10^{11}$ meters/sec$^2)$ = **1.6×10^{-13} Newtons/particle.**

Problem 3 - 0.090 Nt = N x 1.6×10^{-13} Newtons/particle so N = 5.6×10^{11} particles. The time to cross the grid is 3.1×10^{-8} seconds so the particle flow has to be 5.6×10^{11} particles / 3.1×10^{-8} seconds = **1.8×10^{19} particles/second.**

Problem 4 - 1.8×10^{19} particles/second / $(6.25 \times 10^{18}$ particles/sec//Ampere) = **2.9 Amperes!**

Problem 5 - P = 1,300 Volts x 2.9 Amperes = **3,800 watts.**



The photo on the left shows what the universe may have looked like a few million years after the Big Bang: A clumpy soup of dimly glowing matter. The image on the right shows how one of those clumps may have evolved into a recognizable galaxy today. In Big Bang cosmology, the universe expands, and space stretches. An important consequence of this is that the density of matter in space is also decreasing!

Problem 1 - The volume of the Milky Way can be approximated by a disk with a thickness of 1000 light years and a radius of 50,000 light years. Compute the volume of the Milky Way in cubic centimeters. (1 light year = 9.5×10^{17} centimeters.)

Problem 2 - The mass of the Milky Way is approximately equal to 300 billion stars, each with the mass of the Sun: 2×10^{33} grams. Compute the total mass of the Milky Way.

Problem 3 - If you were to take all of the stars and gas in the Milky Way and spread them out throughout the entire volume of the Milky Way, about what would be the density of the Milky Way in: A) grams/cm^3 B) kilograms/m^3

Problem 4 - If the average density of the matter in the universe was at one time equal to that of the Milky Way (Problem 3), by what factor would the volume of the universe have to increase in order for it to be 4.6×10^{-31} grams/cm^3 today?

Problem 5 - By what factor would the size of the universe have had to expand by today, and how far apart would the Milky Way and the Andromeda galaxy have been at that time if their current separation is 2.2 million light years?

Problem 1 - The volume of the Milky Way can be approximated by a disk with a thickness of 1000 light years and a radius of 50,000 light years. Compute the volume of the Milky Way in cubic centimeters.

Answer: $V = \pi R^2 h = \pi (50{,}000)^2 (1000) = 7.9 \times 10^{12}$ cubic lightyears.

$= 7.9 \times 10^{12} \times (5.9 \times 10^{17}\ \text{cm/ly})^3 =$ $\mathbf{1.6 \times 10^{66}\ cm^3}$

Problem 2 - The mass of the Milky Way is approximately equal to 300 billion stars, each with the mass of our sun: 2×10^{33} grams. Compute the total mass of the Milky Way.

Answer: $M = 3 \times 10^9 \times 2 \times 10^{33}$ grams = $\mathbf{2 \times 10^{42}}$ **grams**

Problem 3 - If you were to take all of the stars and gas in the Milky Way and spread them out throughout the entire volume of the Milky Way, about what would be the density of the Milky Way in: A) grams/cm^3 B) kilograms/m^3

Answer A) Density = M/V = 2×10^{42} grams / $\mathbf{1.6 \times 10^{66}\ cm^3}$ = $\mathbf{1.3 \times 10^{-24}}$ **grams/**$\mathbf{cm^3}$. B) 1.3×10^{-24} grams/cm^3 x (1 kg/1000 gm) x (1000cm/1 meter)3 = $\mathbf{1.3 \times 10^{-21}}$ **kilograms/**$\mathbf{meter^3}$

Problem 4 - If the average density of the matter in the universe was at one time equal to that of the Milky Way (Problem 3), A) by what factor would the volume of the universe have to increase in order for it to be 4.6×10^{-31} grams/cm^3 today?

Answer: A) Density then / density now = 1.3×10^{-24} grams/cm^3 / 4.6×10^{-31} grams/cm^3 = **2.8 million times.**

Problem 5 - By what factor would the size of the universe have had to expand by today, and how far apart would the Milky Way and the Andromeda galaxy have been at that time if their current separation is 2.2 million light years?

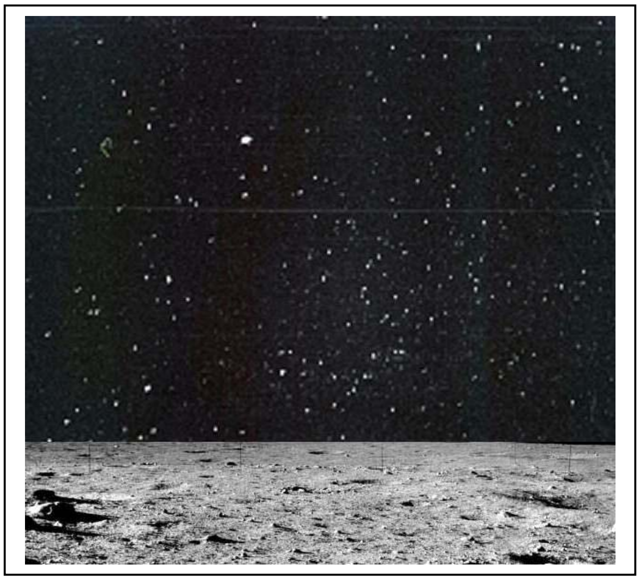
Answer: Because volume is proportional to the cube of a length, the factor by which the universe would have to increase in size is S = (2.8 million times)1/3 = **140 times.** That means that the Milky Way and the Andromeda galaxy would have been at a distance of 2.2 million light years / 140 = **16,000 light years!**

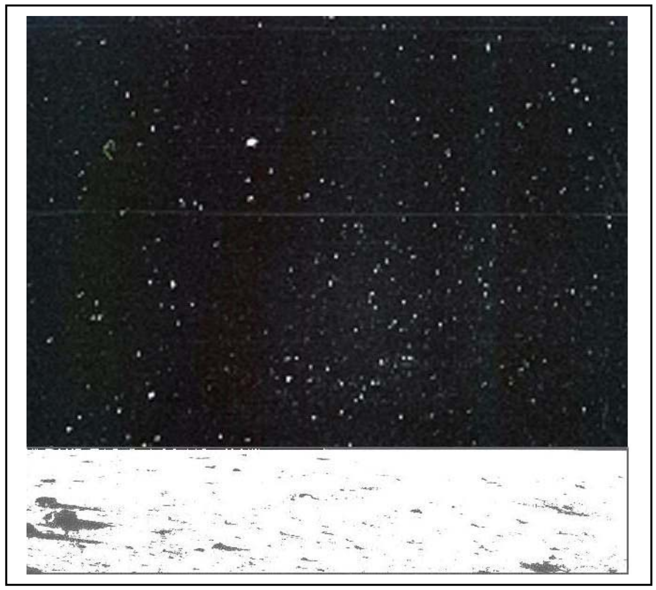
Where Did All the Stars Go?

Have you ever looked closely at NASA photographs from space and wondered where the stars went?

To the left is an Apollo-11 photo taken by astronauts on the surface of the moon. Notice the sky has no stars! The re-touched photo on the bottom-left gives an impression of the stars that a simple $100 camera would see if it used a 'timed exposure' of about 20 seconds. So why did the very expensive camera used by the Apollo-11 astronauts show not a single star?

The re-touched photo below shows what might happen to the lunar surface detail with a 20-second exposure.





A camera light meter measures the brightness of an object. Let's indicate brightness by the unit 'cents/second'. For example, a faint object might have a brightness of 10 cents/second while a bright object has 10,000 cents/second.

Problem 1 - If the stars in the Apollo photo have a brightness of 2.5 cents/sec, how many cents will be collected in a 20-second time-exposure?

Problem 2 - If the lunar surface has a brightness of 500 cents/second, how many cents will be collected in a 20-second exposure?

Problem 3 - If the lunar surface is scaled to a camera contrast setting of 100%, A) How bright, in cents, is a 1% contrast change? B) What contrast change do the stars represent?

Problem 4 - If the image is set to only record contrast changes of 1% or greater to bring out detail on the lunar surface, will the stars be visible? Explain.

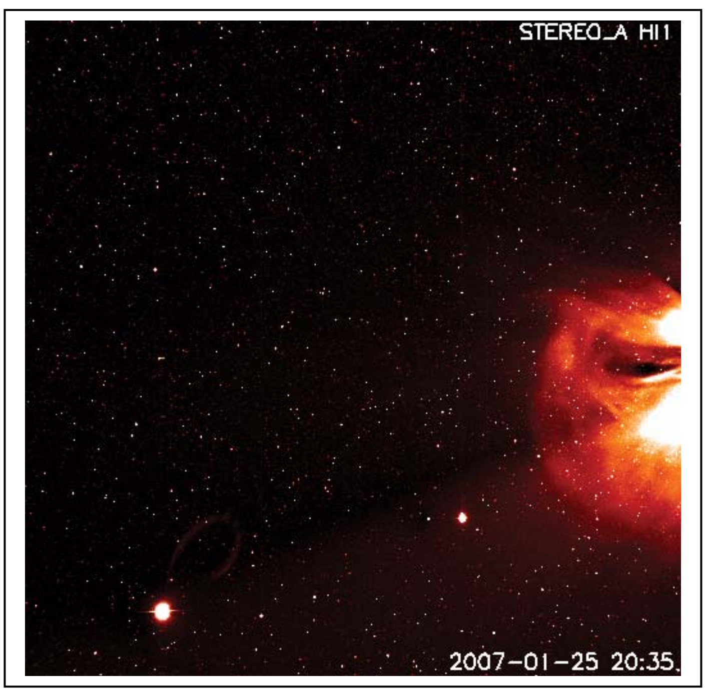
Answer Key

Problem 1 - If the stars in the Apollo photo have a brightness of 2.5 cents/sec, how many cents will be collected in a 20-second time-exposure? Answer: Multiply the rate at which the cents are accumulated (2.5 cents/sec) by the elapsed time (20 sec) to get 2.5 cents/sec x 20 sec **= 50 cents**.

Problem 2 - If the lunar surface has a brightness of 500 cents/second, how many cents will be collected in a 20-second exposure? Answer: Multiply the rate at which the cents are accumulated (500 cents/sec) by the elapsed time (20 sec) to get 500 cents/sec x 20 sec = **10,000 cents**.

Problem 3 - If the lunar surface is scaled to a camera contrast setting of 100%, A) how bright, in cents, is a 1% contrast change? Answer: Since 10,000 cents = 100%, that means that 1% will equal 10,000 cents/100 = 100 cents. B) What contrast change do the stars represent? Answer: The stars produce 50 cents, but in the same amount of time (20 seconds) the lunar landscape produces 10,000 cents, so the stars represent a contrast change of only (50 cents/10000 cents) x 100% **= 0.5 %.**

Problem 4 - If the image is set to only record contrast changes of 1% or greater to bring out detail on the lunar surface, will the stars be visible? Explain. Answer: **If 0% represents black and 100% represents white, the smallest intensity in the image (1%) will correspond to 100 cents, but the stars represent a contrast change of 0.5% or 50 cents, which is smaller than the 100 cents (1%) contrast that the image can register on this camera setting (20 seconds with 10,000 cents = 100%). So the stars will be replaced by 'black' and will be eliminated from the image so that scientists can study the landscape details on the lunar surface instead.**



This is an image taken by NASA's STEREO satellite of a coronal mass ejection (CME) from the sun. The gas is very faint, so the camera had to be designed to detect only faint light, rather than bright sources. Notice that the CME image has plenty of background stars that can in some cases be seen through the translucent gases. Had the camera been designed differently, there would have been no stars in the picture, and only the brightest portions of the CME would have been visible in the photograph.

One of the first things that amateur astronomers do with a camera is to point it at the North Celestial Pole (near Polaris the North Star) and take a time-exposure lasting minutes or hours. The Earth's rotation causes the stars to form long semi-circular trails.

Problem 1 - Which stars in the photo are nearest the NCP?

Problem 2 - The width of the image is 36.0 degrees. What is the scale of the image in arcminutes/millimeter?

Problem 3 - Using your method of choice, identify the location of the NCP as accurately as possible.

Problem 4 - Which star do you think is Polaris in the photograph, and how far is it from your location for the NCP?

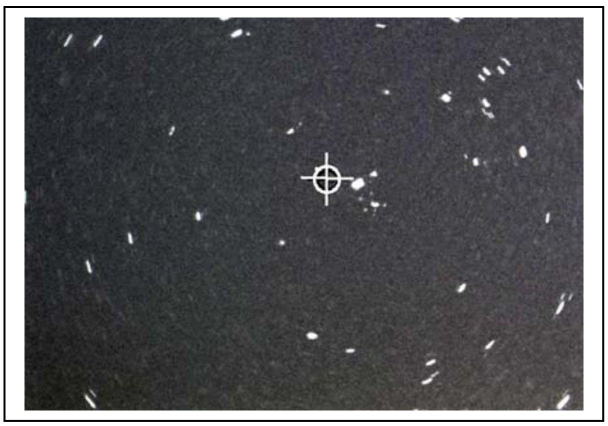
Problem 5 - What features in the photograph are not stars or planets? Explain your reasoning.

Problem 6 - From the information in the photograph, to the nearest minute of time, how long was the camera shutter left open to make this 'star trail' photograph?

Answer Key

Problem 1 - As Earth rotates, the stars will be trailed into arcs of circles that are exactly concentric with the North Celestial Pole. **Look for a region where the star arcs nearly vanish.** This region is in the lower left-hand corner of the picture, which is enlarged and shown on the left.

Problem 2 - The width of the picture is 160 mm, which corresponds to 36.0 degrees, so the scale is 0.225 degrees/mm. Since 1 degree = 60 arcminutes, this is equivalent to 0.225 x 60 = **13.5 arcminutes/mm.**



Problem 3 - Use a compass to place trial pivot points within the region, and test by matching the arc drawn by the compass with the arcs of several stars that are far from this region. **By trial and error, students should be able to get points that are close to the one indicated in the image by the circle and cross.**

Problem 4 - The bright star just to the right of the cross. **The distance is about 3 mm or 40 arcminutes (0.7 degrees).**

Problem 5 - What features in the photograph are not stars or planets? Explain your reasoning. Answer: **Stars and planets do not share Earth's rotation so they will leave arcs concentric with the NCP in the photograph. Objects on Earth, or in the camera, will remain in the same fixed spot in the picture because they are not moving.**

Problem 6 - From the information in the photograph, how long was the camera shutter left open to make this 'star trail' photograph? Answer: A full 360-degree arc would represent 24 hours. We need to measure the length of an arc in degrees, then determine how far it is from the NCP in degrees, and from the ratio of the arc length to the circumference of the complete circle we get the fraction of a 24-hour period that the trail represents. Selecting one star at random in the picture, like the bright one at the far-right edge of the picture (the star Delta Cassiopeia), it is located 133 mm from the NCP, and its arc is 6 mm long. This represents a radius of 29.9 degrees and a length of 1.35 degrees. The circumference is 2 x 3.141 x 29.9 = 187.8 degrees. The arc is the fraction 1.35/187.8 = 0.00719 of full circle or 24 hours x 0.00719 = 0.17 of an hour or **10 minutes.**

Finding Mass in the Cosmos

$$Fg = \frac{G M m}{R^2}$$

$$Fc = \frac{m V^2}{R}$$

$$V = \frac{2 \pi R}{T}$$

One of the neatest things in astronomy is being able to figure out the mass of a distant object, without having to 'go there'. Astronomers do this by employing a very simple technique. It depends only on measuring the separation and period of a pair of bodies orbiting each other. In fact, Sir Issac Newton showed us how to do this over 300 years ago!

Imagine a massive body such as a star, and around it there is a small planet in orbit. We know that the force of gravity, **Fg**, of the star will be pulling the planet inwards, but there will also be a centrifugal force, **Fc**, pushing the planet outwards.

This is because the planet is traveling at a particular speed, **V**, in its orbit. When the force of gravity and the centrifugal force on the planet are exactly equal so that **Fg = Fc**, the planet will travel in a circular path around the star with the star exactly at the center of the orbit.

Problem 1) Use the three equations above to derive the mass of the primary body, M, given the period, T, and radius, R, of the companion's circular orbit.

Problem 2) Use the formula $\mathbf{M = 4 \pi^2 R^3 / (G T^2)}$ where G = 6.6726×10^{-11} N-m^2/kg^2 and M is the mass of the primary in kilograms, R is the orbit radius in meters and T is the orbit period in seconds, to find the masses of the primary bodies in the table below. (Note: Make sure all units are in meters and seconds first! 1 light years = 9.5 trillion kilometers)

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	
Earth	Moon	27.3 days	385,000 km	
Jupiter	Callisto	16.7 days	1.9 million km	
Pluto	Charon	6.38 days	17,530 km	
Mars	Phobos	7.6 hrs	9,400 km	
Sun	Earth	365 days	149 million km	
Sun	Neptune	163.7 yrs	4.5 million km	
Sirius A	Sirius B	50.1 yrs	20 AU	
Polaris A	Polaris B	30.5 yrs	290 million miles	
Milky Way	Sun	225 million yrs	26,000 light years	

Answer Key

Problem 1: Answer

$$\frac{G M m}{R^2} = \frac{m V^2}{R}$$

Cancil the companion mass, m, on both sides, and isolate the primary mass, M, on the left side:

$$M = \frac{R V^2}{G}$$

Now use the definition of V to eliminate it from the equation,

$$M = \frac{R}{G}\left(\frac{2 \pi R}{T}\right)^2$$

and simplify

$$M = \frac{4 \pi^2 R^3}{G T^2}$$

Problem 2:

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	**6.1×10^{24} kg**
Earth	Moon	27.3 days	385,000 km	**6.1×10^{24} kg**
Jupiter	Callisto	16.7 days	1.9 million km	**1.9×10^{27} kg**
Pluto	Charon	6.38 days	17,530 km	**1.3×10^{22} kg**
Mars	Phobos	7.6 hrs	9,400 km	**6.4×10^{23} kg**
Sun	Earth	365 days	149 million km	**1.9×10^{30} kg**
Sun	Neptune	163.7 yrs	4.5 million km	**2.1×10^{30} kg**
Sirius A	Sirius B	50.1 yrs	298 million km	**6.6×10^{30} kg**
Polaris A	Polaris B	30.5 yrs	453 million km	**6.2×10^{28} kg**
Milky Way	Sun	225 million yrs	26,000 light years	**1.7×10^{41} kg**

Note: The masses for Sirius A and Polaris A are estimates because the companion star has a mass nearly equal to the primary so that our mass formula becomes less reliable.

Star Magnitudes and Multiplying Decimals

The brightness of a star is indicated by the Apparent Magnitude scale, which leads to some interesting math!

Rule 1: The larger the number, the fainter the star. For example, Procyon has a magnitude of +0.4 while Wolf-359 has a magnitude of +13.5, so Wolf-359 is fainter than Procyon.

Rule 2: Each difference, by one whole magnitude, represents a brightness change of 2.51 times. For example, the star Tau Ceti has a magnitude of +3 while Fomalhaut has a magnitude of +1. The brightness difference between them is +3 - (+1) = 2 magnitudes or a factor of 2.51 x 2.51 = 6.3 times.

Problem 1 - UV Ceti has a magnitude of +13.0 while Wolf-294 has a magnitude of +10.0. Which star is fainter, and by what factor?

Problem 2 - Sirius has a magnitude of -1 and Mintaka has a magnitude of +2, which star is faintest. What is the magnitude difference, and by what factor do they differ?

Problem 3 - Betelgeuse has a magnitude of +1 and 70 Ophiuchi has a magnitude of +6. What is the magnitude difference and by what factor do they differ?

Problem 4 - Capella has a magnitude of +0 and Barnard's Star has a magnitude of +9. What is the magnitude difference, and by what factor do they differ?

Problem 5 - Sort the stars in the table so that the brightest star appears first, and the faintest star appears last.

Star	Apparent Magnitude
Ross-47	+11.6
Antares	+1.0
Alpha Centauri	-0.1
36 Ophichi	+5.1
Beta Hydra	+2.7
Rigel	+0.1
Eta Cassiopeia	+3.5
Sirius	-1.5
Wolf-359	+13.5
Kruger-60	+9.9

Answer Key

Problem 1 - UV Ceti has a magnitude of +13.0 while Wolf-294 has a magnitude of +10.0. Which star is fainter, and by what factor?
Answer: UV Ceti has the larger apparent magnitude so it is the fainter star. They differ by +13 - +10 = +3 magnitudes, which is a factor of 2.51 x 2.51 x 2.51 = **15.8 times.**

Problem 2 - Sirius has a magnitude of -1 and Mintaka has a magnitude of +2, which star is faintest. What is the magnitude difference, and by what factor do they differ?
Answer: Mintaka has the larger apparent magnitude so it is the fainter star. They differ by +2 - (-1) = +3 magnitudes, which is a factor of 2.51 x 2.51 x 2.51 = **15.8 times.**

Problem 3 - Betelgeuse has a magnitude of +1 and 70 Ophiuchi has a magnitude of +6. What is the magnitude difference and by what factor do they differ?
Answer: 70 Ophichi has the larger apparent magnitude so it is the fainter star. They differ by +6 - (+1) = +5 magnitudes, which is a factor of 2.51 x 2.51 x 2.51 x 2.51 x 2.51 **= 100 times**.

Problem 4 - Capella has a magnitude of +0 and Barnard's Star has a magnitude of +9. What is the magnitude difference, and by what factor do they differ?
Answer: Barnard's Star has the larger apparent magnitude so it is the fainter star. They differ by +9 - (+0) = +9 magnitudes, which is a factor of 2.51 x 2.51 x 2.51x 2.51 x 2.51 x 2.51 x 2.51 x 2.51 x 2.51 = **3,950 times.**

Problem 5 - Sort the stars in the table so that the brightest star appears first, and the faintest star appears last.

Star	Apparent Magnitude
Sirius	-1.5
Alpha Centauri	-0.1
Rigel	+0.1
Antares	+1.0
Beta Hydra	+2.7
Eta Cassiopeia	+3.5
36 Ophichi	+5.1
Kruger-60	+9.9
Ross-47	+11.6
Wolf-359	+13.5

Galaxy Distances and Mixed Fractions

Our Milky Way galaxy is not alone in the universe, but has many neighbors.

The distances between galaxies in the universe are so large that astronomers use the unit 'megaparsec' (mpc) to describe distances.

One *mpc* is about 3 1/4 million light years.

Hubble picture of a Ring Galaxy (AM 0644 741) at a distance of 92 mpc.

Problem 1 - The Andromeda Galaxy is 3/4 mpc from the Milky Way, while the Sombrero Galaxy is 12 mpc from the Milky Way. How much further is the Sombrero Galaxy from the Milky Way?

Problem 2 -The Pinwheel Galaxy is 3 4/5 mpc from the Milky Way. How far is it from the Sombrero Galaxy?

Problem 3 - The Virgo Galaxy Cluster is 19 mpc from the Milky Way. About how far is it from the Pinwheel Galaxy?

Problem 4 - The galaxy Messier 81 is located 3 1/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy?

Problem 5 - The galaxy Centaurus-A is 4 2/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy?

Problem 6 - The galaxy Messier 63 is located about 4 1/5 mpc from the Milky Way. How far is it from the Pinwheel galaxy?

Problem 7 - The galaxy NGC-55 is located 2 1/3 mpc from the Milky Way. How far is it from the Andromeda galaxy?

Problem 8 - In the previous problems, which galaxy is 2 1/15 mpc further from the Milky Way than NGC-55?

Extra for Experts: How far, in light years, is the Virgo Galaxy Cluster from the Milky Way?

Answer Key

Problem 1 - The Andromeda Galaxy is 3/4 mpc from the Milky Way, while the Sombrero Galaxy is 12 mpc from the Milky Way. How much further is the Sombrero Galaxy from the Milky Way? Answer: 12 mpc - 3/4 mpc = **11 1/4 mpc**

Problem 2 -The Pinwheel Galaxy is 3 4/5 mpc from the Milky Way. How far is it from the Sombrero Galaxy? Answer: 12 mpc - 3 4/5 mpc = **8 1/5 mpc**

Problem 3 - The Virgo Galaxy Cluster is 19 mpc from the Milky Way. About how far is it from the Pinwheel Galaxy? Answer: 19 mpc - 3 4/5 mpc = **15 1/5 mpc.**

Problem 4 - The galaxy Messier 81 is located 3 1/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy? Answer: 3 1/5 mpc - 3/4 mpc = 16/5 mpc - 3/4 mpc = 64/20 mpc - 15/20 mpc = 49/20 mpc = **2 9/20 mpc.**

Problem 5 - The galaxy Centaurus-A is 4 2/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy? Answer: 4 2/5 mpc - 3/4 mpc = 88/5 mpc - 15/20 mpc = 73/20 mpc = **3 13/20 mpc**

Problem 6 - The galaxy Messier 63 is located about 4 1/5 mpc from the Milky Way. How far is it from the Pinwheel galaxy? Answer: 4 1/5 mpc - 3 4/5 mpc = 21/5 mpc - 19/5 mpc = **2/5 mpc.**

Problem 7 - The galaxy NGC-55 is located 2 1/3 mpc from the Milky Way. How far is it from the Andromeda galaxy? Answer: 2 1/3 mpc - 3/4 mpc = 7/3 mpc - 3/4 mpc = 28/12 mpc - 9/12 mpc = 19/12 mpc = **1 7/12 mpc**.

Problem 8 - In the previous problems, which galaxy is 2 1/15 mpc further from the Milky Way than NGC-55? Answer; NGC-55 is located 2 1/3 mpc from the Milky Way, so the mystery galaxy is located 2 1/3 mpc + 2 1/15 mpc = 7/3 mpc + 31/15 mpc = 35/15 mpc + 31/15 mpc = 66/15 mpc = 4 6/15 mpc or **4 2/5 mpc**. This is the distance to the Centaurus-A galaxy.

Extra for Experts: How far, in light years, is the Virgo Galaxy Cluster from the Milky Way?
Answer: The distance is 19 megaparsecs, but 1 parsec equals 3 1/4 light years, so the distance to the Virgo Cluster is

19 million parsecs x (3 1/4 lightyears/parsec) = 19 x 3 1/4 = 19 x 12/4 = 228/4 = **57 million light years**

Note: Galaxies are actually located in 3-dimensional space, but to make this problem work we have assumed that the galaxies are all located along a straight like with the Milky Way at the center.

Atomic Numbers and Multiplying Fractions

5 B	6 C	7 N	8 O	9 F	10 Ne
13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
81 Ti	82 Pb	83 Bi	84 Po	85 At	86 Rn

The Atomic Number, **Z**, of an element is the number of protons within the nucleus of the element's atom. This leads to some interesting arithmetic!

A portion of the Periodic Table of the elements is shown to the left with the symbols and atomic numbers for each element indicated in each square.

Problem 1 - Which element has an atomic number that is 5 1/3 larger than carbon (C)?

Problem 2 - Which element has an atomic number that is 5 2/5 that of neon (Ne)?

Problem 3 - Which element has an atomic number that is 8/9 that of krypton (Kr)?

Problem 4 - Which element has an atomic number that is 2/5 of astatine (At)?

Problem 5 - Which element has an atomic number that is 5 1/8 that of sulfur (S)?

Problem 6 - Which element has an atomic number that is 3 2/3 that of fluorine (F)?

Problem 7 - Which element in the table has an atomic number that is both an even multiple of the atomic number of carbon, an even multiple of the element magnesium (Mg) which has an atomic number of 12, and has an atomic number less than iodine (I)?

Answer Key

Problem 1 - Which element has an atomic number that is 5 1/3 larger than Carbon (C)? Answer: Carbon = 6 so the element is 6 x 5 1/3 = 6 x 16/3 = 96/3 = 32 so **Z=32 and the element symbol is Ge (germanium).**

Problem 2 - Which element has an atomic number that is 5 2/5 that of Neon (Ne)? Answer: Neon = 10, so 10 x 5 2/5 = 10 x 27/5 = 270/5 = 54, so **Z=54 and the element is Xe (xenon).**

Problem 3 - Which element has an atomic number that is 8/9 that of Krypton (Kr)? Answer: Krypton=36 so 36 x 8/9 = 288/9 = 32, so **Z=32 and the element is Ge (germanium).**

Problem 4 - Which element has an atomic number that is 2/5 of Astatine (At)? Answer; Astatine=85 so 85 x 2/5 = 170/5 = 34, **so Z=34 and the element is Se (selenium**).

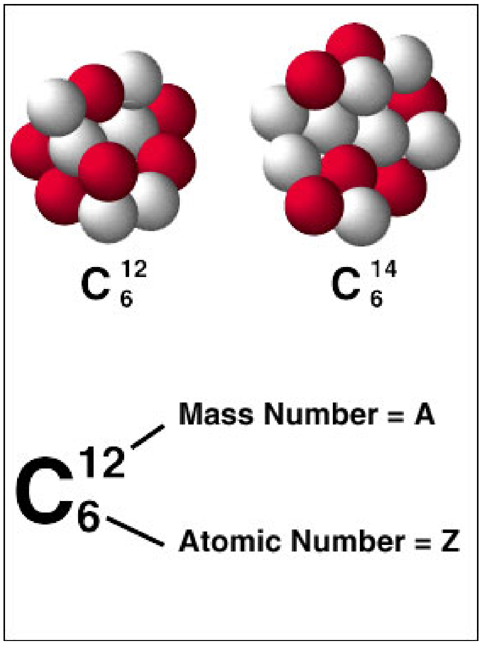
Problem 5 - Which element has an atomic number that is 5 1/8 that of Sulphur (S)? Answer; Sulphur = 16 so 16 x 5 1/8 = 16 x 41/8 = 82, **so Z=82 and the element is lead (Pb).**

Problem 6 - Which element has an atomic number that is 3 2/3 that of Fluorine (F)? Answer: Fluorine = 9 so 9 x 3 2/3 = 9 x 11/3 = 99/3 = 33, **so Z=33 and the element is As (arsenic).**

Problem 7 - Which element in the table has an atomic number that is both an even multiple of the atomic number of carbon, an even multiple of the element magnesium (Mg) which has an atomic number of 12, and has an atomic number less than Iodine (I)?

Answer: The first relationship gives the possibilities: 6, 18, 36, 54. The second clue gives the possibilities 36 and 84. The third clue says Z has to be less than I = 53, **so the element must have Z = 36, which is Kr, (krypton).**

Nuclear Arithmetic



Over 100 elements have been discovered over the last century. The nucleus of each atom contains two kinds of particles: protons and neutrons.

Scientists classify each element by the number of protons (Z) and the mass of the element (A).

Z is called the Atomic Number
A is called the Atomic Mass

The number of neutrons (N) in the nucleus is given by the formula:

$$N = A - Z$$

Problem 1 - In the above example for the element carbon, there are two different forms for carbon. A) How many protons are in the nucleus of carbon-12 and carbon-14? B) How many neutrons are in each nucleus?

Problem 2 - The element praesodymium has an atomic number of 59 and an atomic mass of 141. How many nuclear neutrons does it contain?

Problem 3 - The element nickel (Z=28, A=58) has 30 isotopes that have the same atomic number, but whose atomic masses range from A=48 to A=78. A) How many neutrons does the lightest isotope of nickel have? B) How many neutrons does the heaviest isotope have?

Problem 4 - Solve the formula N = A - Z to determine the missing information:

A) Tin:	A= 125	and	Z=50	what is N?
B) Niobium:	N = 54	and	Z= 41	what is A?
C) Nobelium:	A = 253	and	N = 151	what is Z?
D) Francium:	A=232	and	Z= 87	what is N?
E) Oxygen:	Z = 8	and	N= 16	what is A?

Answer Key

Problem 1 - In the above example for the element carbon, there are two different forms for carbon. A) How many protons are in the nucleus of carbon-12 and carbon-14? B) How many neutrons are in each nucleus?

Answer: A) Carbon-12 has Z=6 and so does carbon-14 so they both have the same number of protons. B) Answer; The mass of carbon-12 is A=12, while carbon-14 has A=14 so carbon-12 has 12-6 = 6 neutrons while carbon 14 has 14-6 = 8 neutrons. Physicists call carbon-14 an isotope of carbon-14 for this reason.

Problem 2 - The element praesodymium has an atomic number of 59 and an atomic mass of 141. How many nuclear neutrons does it contain?
Answer: Z = 59 and A = 141 so N = 141-59 = **82**.

Problem 3 - The element nickel (Z=28, A=58) has 30 isotopes that have the same atomic number, but whose atomic masses range from A=48 to A=78. A) How many neutrons does the lightest isotope of nickel have? B) How many neutrons does the heaviest isotope have?

Answer; A) The lightest isotope is called Nickel-48 and has N = 48 - 28 = **20 neutrons**. B) The heaviest isotope of nickel is called nickel-78 and has N = 78 - 28 = **50 neutrons**.

Problem 4 - Solve the formula N = A - Z to determine the missing information:

A) Tin (A= 125, Z=50) N = ?
    Answer: **N = 125-50 = 75**
B) Niobium (N = 54, Z= 41) A= ?
    Answer: 54 = A - 41 so **A = 54 + 41 = 95**
C) Nobelium (A = 253, N = 151) Z = ?
    Answer; 151 = 253 - Z so Z = 253-151 = **102**.
D) Francium (A=232, Z= 87), N=?
    Answer: N = 232 - 87 = **145.**
E) Oxygen (Z = 8 N= 16) A=?
    Answer: 16 = A - 8 so **A = 24**.

Working with Rates

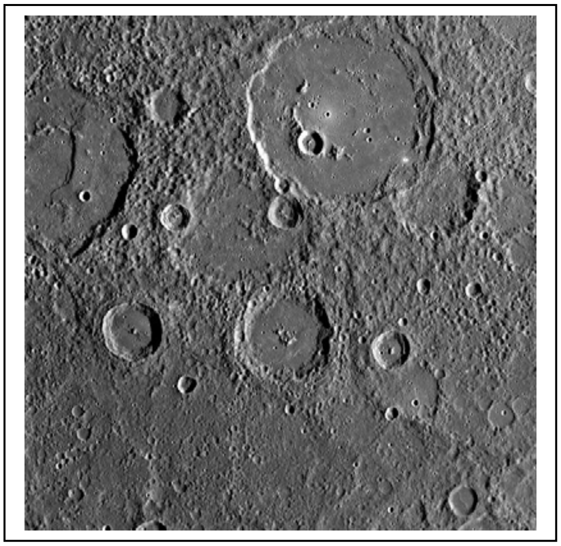


Image of craters on Mercury taken by the MESSENGER spacecraft.

Because things change in the universe, astronomers often have to work with mathematical quantities that describe complex rates.

Definition: A rate is the ratio of two quantities with different units.

In the problems below, convert the indicated quantities into a rate.

Example: 15 solar storms in 2 weeks becomes the rate:

$$R = \frac{\text{15 solar storms}}{\text{2 weeks}} = \frac{15}{2}$$

R = 7 solar storms/week.
or 7 solar storms per week.

Problem 1 - 15 meteor impacts in 3 months.
Problem 2 - 2,555 days in 7 years
Problem 3 - 1,000 atomic collisions in 10 seconds
Problem 4 - 36 galaxies in 2 two clusters
Problem 5 - 1600 novas in 800 years
Problem 6 - 416 gamma-ray bursts spotted in 52 weeks
Problem 7 - 3000 kilometers traveled in 200 hours.
Problem 8 - 320 planets orbiting 160 stars.
Problem 9 - 30 Joules of energy consumed in 2 seconds

Compound Units:

Problem 10 - 240 craters covering 8 square miles of area
Problem 11 - 16,000 watts of energy collected over 16 square meters.
Problem 12 - 380 kilograms in a volume of 20 cubic meters
Problem 13 - 6 million years for 30 magnetic reversals
Problem 14 - 1,820 Joules over 20 square meters of area
Problem 15 - A speed change of 50 kilometers/sec in 10 seconds.

Scientific Notation:

Problem 16 - 3×10^{13} kilometers traveled in 3×10^{7} seconds.
Problem 17 - 70,000 tons of gas accumulated over 20 million square kilometers
Problem 18 - 360,000 Newtons of force over an area of 1.2×10^{6} square meters
Problem 19 - 1.5×10^{8} kilometers traveled in 50 hours
Problem 20 - 4.5×10^{5} stars in a cluster with a volume of 1.5×10^{3} cubic lightyears

Answer Key

Problem 1 - 15 meteor impacts in 3 months. = **5 meteor impacts/month.**
Problem 2 - 2,555 days in 7 years = 2,555 days / 7 years = **365 days/year**
Problem 3 - 1,000 atomic collisions in 10 seconds = **100 atomic collisions/second**
Problem 4 - 36 galaxies in 2 two clusters = **18 galaxies/cluster**
Problem 5 - 1600 novas in 800 years = **2 novas/year**
Problem 6 - 416 gamma-ray bursts spotted in 52 weeks = **8 gamma-ray bursts/week**
Problem 7 - 3000 kilometers traveled in 200 hours. = **15 kilometers/hour**
Problem 8 - 320 planets orbiting 160 stars. = **2 planets/star**
Problem 9 - 30 Joules of energy consumed in 2 seconds = **15 Joules/second**

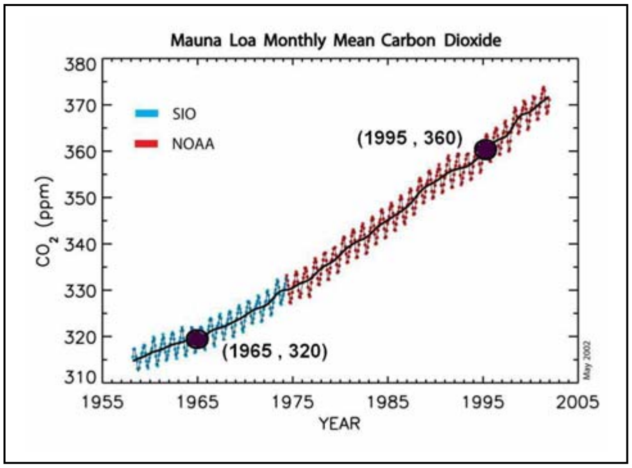
Compound Units:

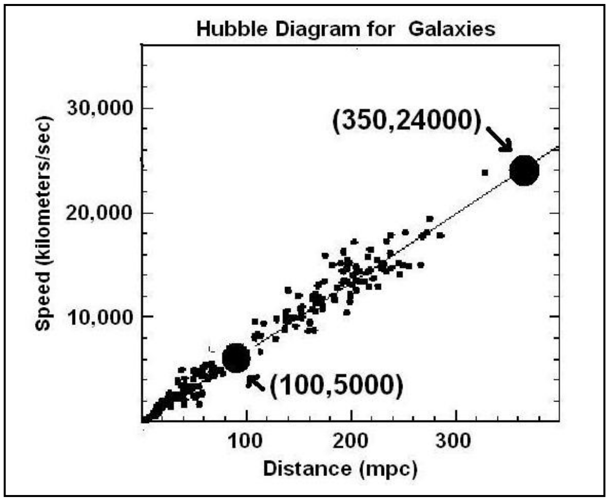
Problem 10 - 240 craters covering 8 square miles of area = **30 craters/km^2**
Problem 11 - 16,000 watts of energy collected over 16 square meters. = **1000 watts/km^2**
Problem 12 - 380 kilograms in a volume of 30 cubic meters = **19 kilograms/m^3**
Problem 13 - 6 million years for 30 magnetic reversals = **200,000 years/reversal**
Problem 14 - 1,820 Joules over 20 square meters of area = **91 Joules/m^2**
Problem 15 - A speed change of 50 kilometers/sec in 10 seconds. = **5 km/sec^2**

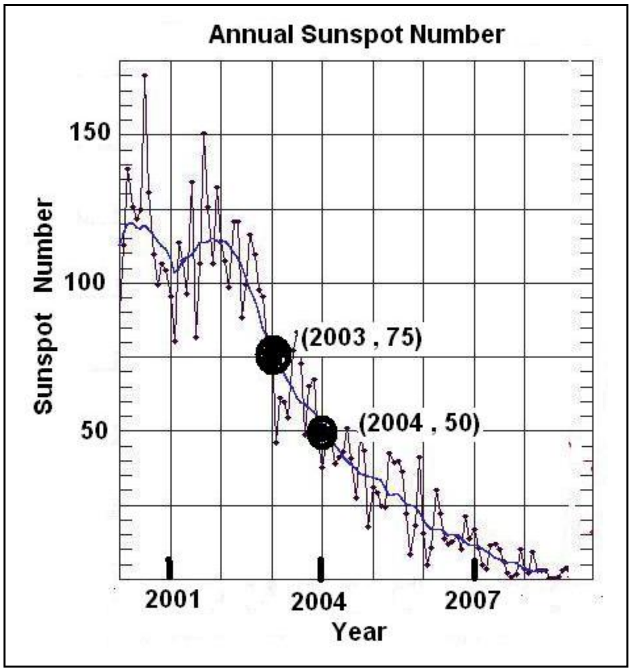
Scientific Notation:

Problem 16 - 3×10^{13} kilometers traveled in 3×10^7 seconds.
= **1.0×10^6 kilometers/sec**
Problem 17 - 70,000 tons of gas accumulated over 20 million square kilometers
= 70,000 tons/20 million km^2 = **0.0035 tons/km^2**
Problem 18 - 360,000 Newtons of force over an area of 1.2×10^6 square meters
= 392,000 Newtons/1,200,000 m^2 = **0.3 Newtons/m^2**
Problem 19 - 1.5×10^8 kilometers traveled in 50 hours
= 1.5×10^8 km/50 hrs = **3 million km/hr**
Problem 20 - 4.5×10^5 stars in a cluster with a volume of 1.5×10^3 cubic lightyears
= **300 stars/cubic lightyear**

Rates and Slopes: An Astronomical Perspective







A 'rate' is defined as the ratio of two quantities which have different units of measurement.

For example, if you travel in a car 200 kilometers in 2 hours, the rate is R = 200 kilometers/2 hours or R = 100 kilometers/hour. You recognize this particular rate as just the speed of the car! Scientists work with other kinds of rates as well.

Graphically, a rate is a measure of the difference between two values along the Y-axis, divided by the difference between two corresponding values along the X-axis. It also represents the slope of a curve plotted on a graph.

For example, let's look at the top graph to the left. It shows how the amount of carbon dioxide in the atmosphere is increasing between 1955 and 2005. The two points along the data curve can be used to find the rate of change of the carbon dioxide in time, which is the slope of the line connecting these two points.

The change along the X-axis is just the difference '1995-1965' or +30 years. The difference along the Y-axis corresponding to these same years is just '360 ppm -320 ppm' or +40 ppm. The rate is then R = +40 ppm/+30 years or +1.3 ppm/year.

Note that we have kept careful track of the signs and units in the calculations. This is because rates can represent both increases (positive) or decreases (negative) changes.

Problem 1 - Calculate the Rate corresponding to the speed of the galaxies in the Hubble Diagram. (Called the Hubble Constant, it is a measure of how fast the universe is expanding).

Problem 2 - Calculate the rate of sunspot number change between the indicated years.

Answer Key

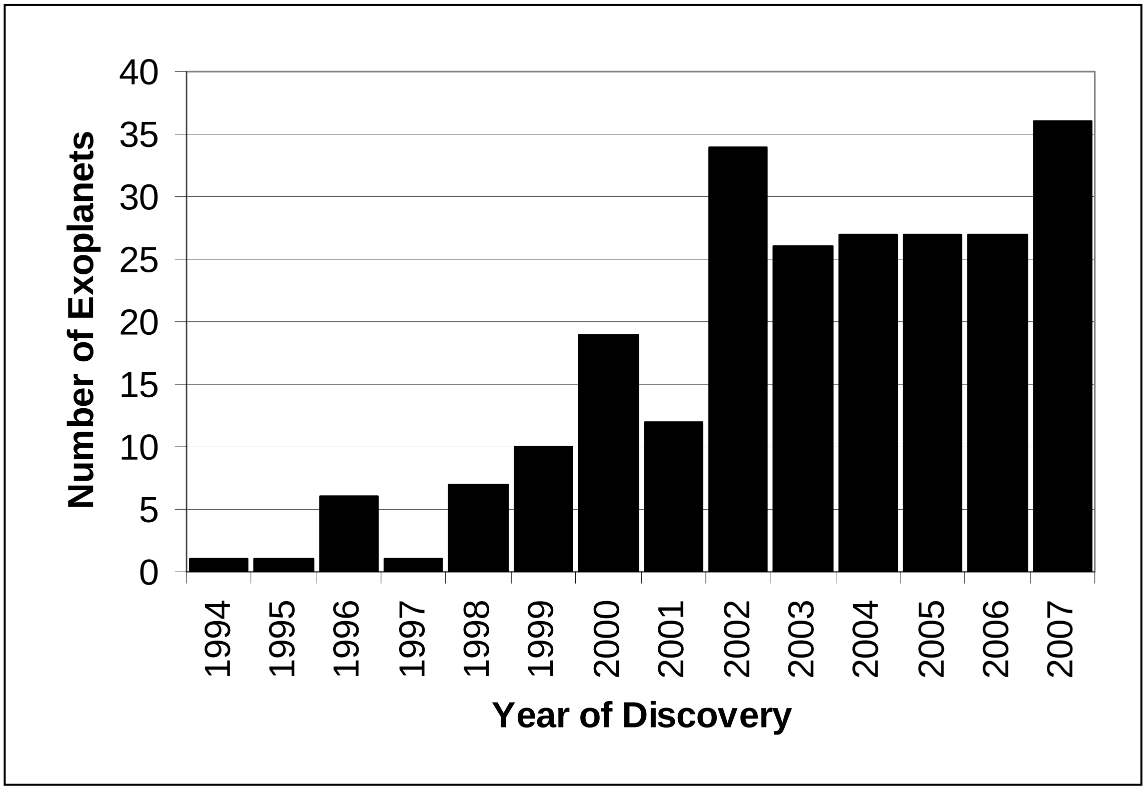
Problem 1 - Calculate the rate corresponding to the speed of the galaxies in the Hubble Diagram. (Called the Hubble Constant, it is a measure of how fast the universe is expanding).

Answer: The two points have coordinates (x1,y1) = (100 mpc, 5000 km/s) and (x2,y2) = (350 mpc, 24000 km/s). The difference in the y coordinates is y2-y1 = 24000-5000 = 19000km/s. The difference in the x coordinates is x2-x1 = 350 mpc - 100 mpc = 250 mpc. The Rate is then R = (19000km/sec) / (250 mpc) = **76 km/sec/mpc**. This is read as '76 kilometers per second per megaparsec'. Another way to write this complicated mixed-unit ratio is

$$H = 76 \frac{\text{km}}{\text{sec mpc}}$$

Problem 2 - Calculate the rate of sunspot number change between the indicated years.
Answer: The two points have coordinates (x1,y1) = (2003, 75 spots) and (x2,y2) = (2004, 50 spots). The difference in the y coordinates is y2-y1 = 50 spots - 75 spots = -25 spots. The difference in the x coordinates is x2-x1 = 2004 - 2003= 1 year. The Rate is then R = -25 spots / (1 year) = **-25 spots/year.**

This rate is negative because the number of spots is decreasing as time goes forward. This is reflected in the slope of the data being negative in the graph.



The 'area under a curve' is an important mathematical quantity that defines virtually all mathematical functions. It has many practical uses as well. For example, the function plotted above, call it P(X), determines the number of new planets, P, that were discovered each year, X, between 1994 through 2007. It was created by tallying-up the number of actual planet discoveries reported in research articles during each of the years. The actual curve representing the function P(X) is shown as a black line, and the columns indicate the number of discoveries per year.

Problem 1 - How would you calculate the total number of planets detected between 1994-2007?

Problem 2 - What is the total area under the curve shown in the figure?

Problem 3 - Suppose N(1995,2000) represents the number of planets detected during the years 1995, 1996, 1997, 1998, 1999, 2000. A) What does N(1994,2007) mean? B) What does N(1994,2007) - N(1994,2000) mean?

Problem 4 - Evaluate:

A) N(2002,2007)

B) N(1999,2002)

Problem 5 - Evaluate and re-write in terms of N (Example for a function defined for X = A,B,C and D : N(A,B) + N(C,D) is just N(A,D))

A) N(1994,2001) + N(2002,2007)

B) N(2001,2005) - N(2002,2005)

C) N(1994,2007) - N(1994,2001)

Answer Key

Problem 1 - How would you calculate the total number of planets detected between 1994-2007? Answer; You would add up the numbers of planets detected during each year, between 1994 and 2007. This is the same as adding up the areas of each of the individual columns.

Problem 2 - What is the total area under the curve shown in the figure? Answer; the total area is found by adding up the numbers for each column: 1 + 1 + 6+ 1 + 7 + 10 + 19 + 13 + 34 + 26 + 27 + 27 + 27 + 36 = **235 planets.**

Problem 3 - Suppose N(1995,2000) represents the number of planets detected during the years 1995, 1996, 1997, 1998, 1999, 2000.

A) What does N(1994,2007) mean? Answer: It means the total number of planets detected between 1994 and 2007, which is **235 planets**

B) What does N(1994,2007) - N(1994,2000) mean? Answer: It means to subtract the number of planets detected between 1995-200 from the total number of planets detected between 1994-2007. N(1994,2000) =1 + 1 + 6+ 1 + 7 + 10 + 19 = 45, so you will get 235 - 45 = **190 planets.**

Problem 5 - Evaluate:

A) N(2002,2007) = 34 + 26 + 27 + 27 + 27 + 36 = **177 planets.**
B) N(1999,2002) = 10 + 19 + 13 + 34 = **76 planets.**

Problem 6 - Evaluate and re-write in terms of N:

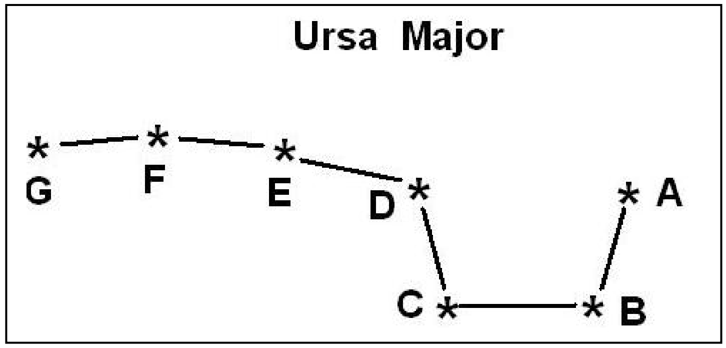
A) N(1994,2001) + N(2002,2007) = (1 + 1 + 6+ 1 + 7 + 10 + 19 + 13) + (34 + 26 + 27 + 27 + 27 + 36) = 58 + 177 = **235 planets which is just N(1994,2007)**

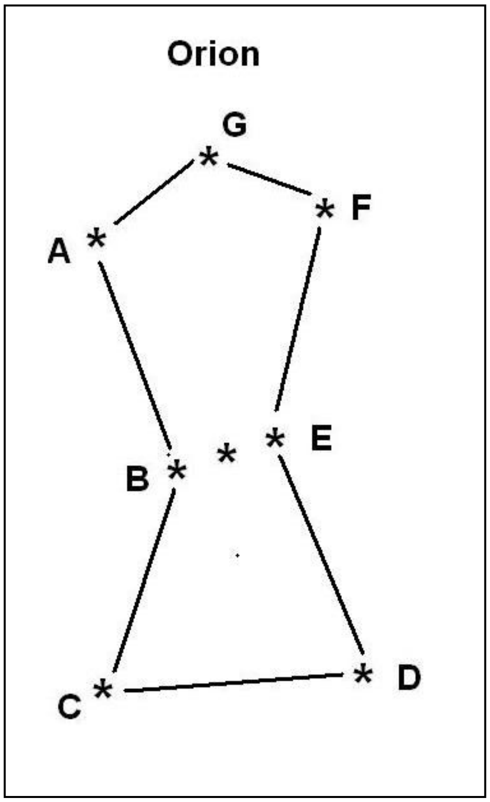
B) N(2001,2005) - N(2002,2005) = (13 + 34 + 26 + 27 + 27) - (34 + 26 + 27 + 27) = **13 planets, which is just N(2001, 2001).**

C) N(1994,2007) - N(1994,2001) = (1 + 1 + 6+ 1 + 7 + 10 + 19 + 13 + 34 + 26 + 27 + 27 + 27 + 36) - (1 + 1 + 6+ 1 + 7 + 10 + 19 + 13) = 235 - 58 = **177 planets which is just N(2002,2007)**

Perimeters: Which constellation is the longest?

Star Segment	Length
AB	5 1/2
BC	8
CD	4 1/2
DE	5 1/3
EF	4 1/3
FG	6 2/3





Star Segment	Length
AB	10
BC	7
CD	8 1/4
DE	8
EF	6
FG	4 1/4
GA	5 1/2

Constellations form various kinds of irregular geometric figures, but we can study them by examining some of their basic properties. One of these is their perimeter.

Here are two well-known constellations, Ursa Major, this portion is also known as the Big Dipper in English-speaking countries, and Orion 'The Hunter'. On star charts they look like they are about the same size, but let's put this to a test. From Earth, we measure the distance between stars as they appear in the sky in terms of degrees. Let’s measure the separations between the stars in degrees and calculate their perimeters.

Problem 1 - From the corresponding table above, calculate the total perimeter of Ursa Major by adding up the lengths of the star segments from AB to FG which are given in degrees.

Problem 2 - From the corresponding table to the left, calculate the total perimeter of Orion by adding up the lengths of the star segments from AB to GA which are given in degrees.

Problem 3 - Which constellation has the longest perimeter in degrees?

Problem 4 - What is the average distance in degrees between the stars along the perimeter of A) Ursa Major? B) Orion?

Problem 5 - In which constellation are the stars the farthest apart on average?

Problem 6 - Can you name another property of a constellation that could be interesting to study?

Answer Key

Problem 1 - From the corresponding table above, calculate the total perimeter of Ursa Major by adding up the lengths of the star segments from AB to FG which are given in degrees.

Answer: 5 1/2 + 8 + 4 1/2 + 5 1/3 + 4 1/3 + 6 2/3 = **34 1/3**

Problem 2 - From the corresponding table to the left, calculate the total perimeter of Orion by adding up the lengths of the star segments from AB to GA which are given in degrees.

Answer: 10 + 7 + 8 1/4 + 8 + 6 + 4 1/4 + 5 1/2 = **49**

Problem 3 - Which constellation has the longest perimeter in degrees?

Answer: **Orion.**

Problem 4 - What is the average distance in degrees between the stars along the perimeter of Ursa Major?

Answer: A) For Ursa Major, there are 6 segments so we have to divide the perimeter by 6 to find the average distance. 34 1/3 degrees / 6 = 34 1/3 x 1/6 = 103/3 x 1/6 = 103/18 = **5 13/18 degrees**

B) For Orion, there are 7 segments in Orion so the average length of a segment is 49 degrees/7 = 7 degrees

Problem 5 - In which constellation are the stars the farthest apart on average?
Answer: **Orion**

Problem 6 - Can you name another property of a constellation that could be interesting to study?

Answer: **Students may identify, for example, the total area of a constellation, the length of the largest star segment spanning the constellation (Such as AG in Ursa Major or GC in Orion), the number of obtuse angles.**

Equation 1

$$V = \sqrt{2\,g\,H}$$

Equation 2

$$X = \frac{V^2}{g}$$

Equation 3

$$T = \frac{V\sqrt{2}}{g}$$

There are three equations that describe projectile motion on a planet:

Equation 1: Maximum velocity, **V**, needed to reach a height, **H**:

Equation 2: Maximum horizontal distance, **X**:

Equation 3: Time, **T**, required to reach maximum horizontal distance:

In all three equations, **g** is a constant and is the acceleration of gravity at the surface of the planet, and all units are in meters or seconds.

Problem 1 - The volcano, Krakatoa, exploded on August 26, 1883 and obliterated an entire island. The detonation was heard over 2000 kilometers away in Australia, and was the loudest sound created by Nature in recorded human history! If the plume of gas and rock reached an altitude of H=17 miles (26 kilometers) what was the speed of the gas, V, that was ejected, in A) kilometers/hour? B) miles/hour? C) What was farthest horizontal distance, X, in kilometers that the ejecta reached? D) How long, T, did it take for the ejecta to travel the maximum horizontal distance? E) About 30,000 people were killed in the explosion. Why do you think there were there so many casualties? (Note: **g** = 9.8 meters/sec^2 for Earth.)

Problem 2 - An asteroid collides with the lunar surface and ejects lunar material at a speed of V=3,200 kilometers/hr. A) How high up, H, does it travel before falling back to the surface? B) The escape speed from the lunar surface is 8,500 km/hr. From your answer to Problem 1, would a 'Krakatoa' explosion on the moon's surface have been able to launch lunar rock into orbit? (Note: **g** = 1.6 meters/sec^2 for the Moon.)

Problem 3 - Plumes of gas are ejected by geysers on the surface of the satellite of Saturn called Enceladus. If **g** = 0.1 meters/sec^2 , and H = 750 km, what is the speed of the gas, V, in the ejection in kilometers/hr?

Inquiry Problem: Program an Excel Spreadsheet to calculate the various quantities in the three equations given input data about the planet and ejecta. How does the maximum ejection velocity and height change with the value of **g** used for a variety of bodies in the solar system?

Answer Key

Problem 1 - The volcano, Krakatoa, exploded on August 26, 1883 and obliterated an entire island. The detonation was heard over 2000 kilometers away in Australia, and was the loudest sound created by Nature in recorded human history! If the plume of gas and rock reached an altitude of 17 miles (26 kilometers) what was the speed of the gas that was ejected, in

A) Use Equation 1 with H = 26,000 meters; g = 9.8 m/s^2 and get H = (2 x 26000x9.8)$^{1/2}$ = 714 m/sec, but since the input numbers are only good to two significant figures, the answer is 710 meters/sec. Then converting to km/hr we get 710 m/s x (3600 sec/hr) x(1 km/1000meters) = 2,556 km/hr but again we only report to 2 significant figures so the answer is **2,600 km/hr**.

B) 2,600 km/hr x (0.62 miles/km) = **1,600 miles/hour** to 2 significant figures

C) Use Equation 2: X = (710 meters/sec)^2/9.8 = 51,439 meters, which to 2 significant figures becomes 51,000 meters or **51 kilometers**.

D) Use Equation 3: T = 1.414 x (710)/9.8 = 102.4, but to 2 significant figures is **100 seconds.**

E) About 30,000 people were killed in the explosion. Why do you think there were there so many casualties? Answer: **They had less than 100 seconds to flee from the advancing ejecta cloud!** You can also ask the students to calculate the sound travel time to cross 51 kilometers (sound speed = 340 m/sec) which would take 51,000 meters/340 = 150 seconds to reach someone at 51 kilometers…so the eject would strike them BEFORE they even heard the detonation.

Problem 2 - Answer: 3,200 km/hr = 0.9 km/sec = **900 meters/sec**. to 2 significant figures. A) Solve Equation 1 for H… H = V2/2g so H = (900)^2/(2 x 1.6) = **250 kilometers** (2 SigFig). B) The escape speed from the lunar surface is 8,500 km/hr. From your answer to Problem 1, would a 'Krakatoa' explosion on the moon's surface have been able to launch lunar rock into orbit? (Note: g = 1.6 meters/sec^2 for the Moon.) Answer: **Yes!**

Problem 3 - Answer: From Equation V = (2 x 0.1 x 750000)$^{1/2}$ = 390 meters/sec = **1,400 km/hr** (2 SigFig)

Inquiry Problem: Program an Excel Spreadsheet to calculate the various quantities in the three equations given input data about the planet and ejecta. How does the maximum ejection velocity and height change with the value of **g** used for a variety of bodies in the solar system?

Answer: **There are many ways for students to program each column in a spreadsheet to calculate the variables in the equations. Students should, for instance, notice that as the surface gravity, g, increases, the maximum speed, V, changes as the square-root of g, and the values for X and T vary inversely with g.**

Kelvin Temperatures and Very Cold Things!

183 K	Vostok, Antarctica
160 K	Phobos
134 K	Superconductors
128 K	Europa summer
120 K	Moon at night
95 K	Titan
90 K	Liquid oxygen
88 K	Miranda
81 K	Enceladus summer
77 K	Liquid nitrogen
70 K	Mercury at night
63 K	Solid nitrogen
55 K	Pluto summer
54 K	Solid oxygen
50 K	Quaoar
45 K	Moon - shadowed crater
40 K	Star-forming region
33 K	Pluto wInter
20 K	Liquid hydrogen
19 K	Bose-Einstein Condensates
4 K	Liquid helium
3 K	Cosmic Background Radiation
2 K	Liquid helium
1 K	Boomerang Nebula
0 K	ABSOLUTE ZERO

To keep track of some of the coldest things in the universe, scientists use the Kelvin temperature scale which begins at 0^o Kelvin, which is also called Absolute Zero. Nothing can ever be colder than Absolute Zero because at this temperature, all motion stops. The table to the left shows some typical temperatures of different systems in the universe.

You are probably already familiar with the Centigrade (C) and Fahrenheit (F) temperature scales. The two formulas below show how to switch from degrees-C to degrees-F.

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32$$

Because the Kelvin scale is related to the Centigrade scale, we can also convert from Centigrade to Kelvin (K) using the equation:

$$K = 273 + C$$

Use these three equations to convert between the three temperature scales:

Problem 1: 212 F converted to K

Problem 2: 0 K converted to F

Problem 3: 100 C converted to K

Problem 4: -150 F converted to K

Problem 5: -150 C converted to K

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F. A) What are the equivalent temperatures on the Kelvin scale; B) What is the average daytime temperature on the Kelvin scale?

Answer Key

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32 \qquad K = 273 + C$$

Problem 1: 212 F converted to K :
First convert to C: C = 5/9 (212 - 32) = +100 C. Then convert from C to K:
K = 273 + 100 = **373 Kelvin**

Problem 2: 0 K converted to F: First convert to Centigrade:
0 = 273 + C so C = -273 degrees. Then convert from C to F:
F = 9/5 (-273) + 32 = **-459 Fahrenheit**.

Problem 3: 100 C converted to K : K = 273 - 100 = **373 Kelvin**.

Problem 4: -150 F converted to K : Convert to Centigrade
C = 5/9 (-150 - 32) = -101 C. Then convert from Centigrade to Kelvin: K = 273 - 101 = **172 Kelvin**.

Problem 5: -150 C converted to K : K = 273 + (-150) = **123 Kelvin**

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F.

A) What are the equivalent temperatures on the Kelvin scale?;
107 C becomes K = 273 + 107 = **380 Kelvins.**
221 F becomes C = 5/9 (221 - 32) = 105 C, and so K = 273 + 105 = **378 Kelvins**.

B) What is the average daytime temperature on the Kelvin scale?
Answer: (380 + 378)/2 = **379 Kelvins**.

C) Explain why the Kelvin scale is useful for calculating averages of different temperatures. Answer: **Because the degrees are in the same units in the same measuring scale so that the numbers can be averaged.**

Note: Students may recognize that in order to average +107 C and +221 F they could just as easily have converted both temperatures to the Centigrade scale or the Fahrenheit scale and then averaged those temperatures. You may challenge them to do this, and then compare the averaged values in the Centigrade, Fahrenheit and Kelvin scales. They should note that the final answer will be the same as 379 Kelvins converted to F and C scales using the above formulas.

Pulsars and Simple Equations

The Crab Nebula is all that remains of a supernova explosion 900 years ago. At the center of this picture, taken by the Hubble Space Telescope, is a rapidly spinning pulsar which flashes 30 times a second as it spins.

A pulsar is a rapidly spinning star. It's about the same size as Earth, but it contains as much mass as an entire normal star like the sun.

When they are formed, they spin at an unimaginable pace: nearly 30 times every second. As they grow older, they slow down.

Astronomers have measured the spinning of two pulsars: The Crab Nebula pulsar, and AP 2016+28. They used this data to create two simple equations that predict the pulsar's spin rates in the future.

Crab Nebula Pulsar: **P = 0.033 + 0.000013 T**

AP 2016+28 Pulsar: **P = 0.558 + 0.0000000047 T**

P is the time, in seconds, it takes the pulsar to spin once-around on its axis. T is the number of years since today.

Problem 1: Evaluate each equation for P for a time that is 10,000 years in the future. How fast are the two pulsars spinning at that time?

Problem 2: How long will it take the Crab Pulsar to slow to a period exactly twice its current period of 0.033 seconds?

Problem 3: How long will it take Pulsar AP 2016+28 to slow to a period of 1.116 seconds (exactly twice its current period of 0.558 seconds)?

Problem 4: How many years ago was the pulsar AP 2016+28 spinning at the same rate as the Crab Pulsar?

Problem 5: How long will it take each pulsar to slow to a period of exactly 2.0 seconds?

Problem 6: In how many years from now will the two pulsars be spinning at exactly the same rates?

Answer Key

Crab Nebula Pulsar: **P = 0.033 + 0.000013 T**

AP 2016+28 Pulsar: **P = 0.558 + 0.0000000047 T**

Problem 1: Evaluate each equation for P for a time that is 10,000 years in the future. How fast are the two pulsars spinning at that time?

Crab: P = 0.033 + 0.000013 (10,000 years) = 0.033 + 0.13 = **0.163 seconds.**
AP 2016+28 Pulsar: P = 0.558 + 0.0000000047 (10,000) = 0.558 + 0.000047 = **0.558047 seconds**.

Problem 2: How long will it take the Crab Pulsar to slow to a period exactly twice its current period of 0.033 seconds?

P = 2 x 0.033 = 0.066 seconds. So 0.066 = 0.033 + 0.000013 T, and solving for T we get T = (0.033/0.000013) = **2,500 years from now**.

Problem 3: How long will it take Pulsar AP 2016+28 to slow to a period exactly twice its current period of 0.558 seconds?
P = 2 x 0.558 = 1.116 seconds. So 1.116 = 0.558 + 0.0000000047 T, and solving for T we get T = (0.558/0.0000000047) = **119 million years from now**.

Problem 4: How many years ago, was the pulsar AP 2016+28 spinning at the same rate as the Crab Pulsar?
P = 0.033 seconds = 0.558 + 0.0000000047 T, so
0.033 - 0.558 = 0.0000000047 T
-0.525 = 0.0000000047 T
-0.525/0.0000000047 = T and so **T = 112 million years ago**.
Note that the negative sign for T means that the time was before today (year-zero).

Problem 5: How long will it take each pulsar to slow to a period of exactly 2.0 seconds?
Crab Pulsar: 2.0 = 0.033 + 0.000013 T so T = **151,000 years**.
AP 2016+28: 2.0 = 0.558 + 0.0000000047 T so T = **307 million years**

Problem 6: In how many years from now A) will the two pulsars be spinning at exactly the same rates? B) What will be their spin rates?

This requires that students set the equation for P in the Crab Nebula Pulsar to the P in the equation for AP 2016+28, and solve for T.

0.558 + 0.0000000047 T = 0.033 + 0.000013 T
0.558 - 0.033 = 0.000013 T - 0.0000000047 T
0.525 = 0.0000129953 T
T = 0.525/0.0000129953 so T = **+ 40,400 years from now.**

B) P = 0.033 + 0.000013 (40400) = **0.558 seconds**.

The Many Faces of Energy

1 Joule = 10 million ergs
1 electron Volt (eV) = 1.6×10^{-19} Joules
1 degree (K) = 8.62×10^{-5} eV
1 calorie = 4.2 Joules
1 kiloWatt hour = 3.6×10^{6} Joules
1 eV = 1.78×10^{-33} grams
1 AMU = 931.5 million eV (MeV)

Energy comes in many forms, and each one can be measured in terms of its own convenient units. For example, if you were interested in creating a balanced diet, you would measure food energy by its calorie content, not by its number of Joules!

The table to the left shows a few of the equivalent units that scientists use to keep track of energy in different kinds of systems.

Problem 1 - In a chemical reaction, an energy of about 2.5 eV is required to activate the reaction to create a new compound. To form a single molecule of the compound: A) How many Joules of energy is this? B) How many calories is this?

Problem 2: A star has a surface temperature of 20,000 K. About what is the average energy per atom in electron Volts?

Problem 3: The mass of an electron is 9.11×10^{-28} grams. What is its equivalent mass in kiloelectron Volts (keV)?

Problem 4: A proton and a neutron are combined to form a deuterium nucleus. Their total individual masses equal 2.016490 AMU, but the mass of a deuterium nucleus is only 2.014102 AMU. If the mass difference to form the deuterium is 0.002388 AMU, how much energy does this energy difference represent in: A) million electron volts (MeV)? B) grams? (Note: this is called the binding energy of the nucleus.)

Problem 5: An astronomer detects X-ray light from a pulsar with an energy of 15 keV. About what is the temperature of the gas emitting this light?

Answer Key

Problem 1 - In a chemical reaction, an energy of about 2.5 eV is required to activate the reaction to create a new compound. To form a single molecule of the compound: A) How many Joules of energy is this? B) How many calories is this?

A) Answer: 2.5 eV x (1.6 x 10^{-19} Joules / 1 eV) = **4.0 x 10^{-19} Joules per molecule.**
B) Answer: 4.0 x 10^{-19} Joules x (1 calorie / 4.2 Joules) = **9.5 x 10^{-20} Joules per molecule.**

Problem 2: A star has a surface temperature of 20,000 K. About what is the average energy per atom in electron Volts? Answer: 20,000 K x (8.62 x 10^{-5} eV / K) **= 1.7 eV**.

Problem 3: The mass of an electron is 9.11 x 10^{-28} grams. What is its equivalent mass in kiloelectron Volts (keV)? Answer: 9.11 x 10^{-28} grams x (1 eV/1.78 x 10^{-33} grams) = 512,000 eV = **512 keV**.

Problem 4: A proton and a neutron are combined to form a deuterium nucleus. Their total individual masses equal 2.016490 AMU, but the mass of a deuterium nucleus is only 2.014102 AMU. If the mass difference to form the deuterium is 0.002388 AMU, how much energy does this energy difference represent in: A) million electron volts (MeV)? B) grams?

Answer: A) 0.002388 AMU x (931.5 MeV/1 AMU) = **2.2 MeV**.

Answer: B) 2.2 MeV x (1,000,000 eV/1 MeV) x (1.78 x 10^{-33} grams/ eV) = **3.9 x 10^{-27} grams.**

Problem 5: An astronomer detects X-ray light from a pulsar with an energy of 15 keV. About what is the temperature of the gas emitting this light? Answer; 15 keV x (1,000 eV/ 1 keV) x (1 K/8.62 x 10^{-5} eV) = **174 million degrees Kelvin**.

Problem 6: A physicist wants to create a proton with a mass of 938 MeV in his accelerator. What is the minimum energy, in Joules, that he will need to provide? Answer: 938 MeV x (1,000,000 eV/1 MeV) x (1.6 x 10^{-19} Joules/ 1 eV) = **1.5 x 10^{-10} Joules.**

Note: $E = mc^2$ is the basis for stating the mass of a particle 'm' in terms of its equivalent electron-Volt energy, E. Physicists understand that $m = E/c^2$ but drop the speed of light constant, c, as a shorthand way of stating a particle's mass..

Variables and Expressions from Around the Cosmos

There are many simple mathematical formulae that astronomers use to describe different aspects of the universe and the physical world. (The above photo of Saturn was taken by NASA's Cassini spacecraft from behind Saturn looking back towards the Sun.)

Problem 1: Find, P, the length of Earth's day 500 million years in the future if **P = 24 hours + 0.004 Y**, where Y is the number of millions of years that have elapsed.

Problem 2: Find the distance to the Andromeda galaxy in light years, L, if its distance in parsecs, P = 770,000 and **L = 3.26** P.

Problem 3: Find the temperature, T, of a gas cloud emitting X-rays if the energy of the X-rays is E = 12,000 electron Volts and **T = 11,500 E**.

Problem 4: Find the temperature in degrees Centigrade of the air at an altitude of 20 kilometers if H = 20 and T = 25.0 - 6.5 H.

Problem 5: Find the diameter in kilometers, D, of a black hole with a mass of 10 times the sun if **D = 5.6 M** and M = 10.0.

Problem 6: Calculate the speed of sound, S, in meters/second for a temperature of T = 200 Centigrade (that's 392 F), if **S = 331 + 0.6 T**.

Problem 7: Calculate the sunspot number, N, if there are X = 15 individual sunspots and Y = 10 groups of sunspots is **N = X + 11 Y**.

Answer Key

Problem 1: Find, P, the length of Earth's day 500 million years in the future if **P = 24 hours + 0.004 Y**, where Y is the number of millions of years that have elapsed.

Answer; **P = 24 hours + 0.004 (500) = 26 hours.**

Problem 2: Find the distance to the Andromeda galaxy in light years, L, if its distance in parsecs, P = 770,000 and **L = 3.26** P.

Answer: **L = 3.26** (770,000) = **2,200,000 light years.**

Problem 3: Find the temperature, T, of a gas cloud emitting X-rays if the energy of the X-rays is E = 12,000 electron Volts and **T = 11,500 E**.

Answer: **T = 11,500 (12,000) = 138 million K**

Problem 4: Find the temperature in degrees Centigrade of the air at an altitude of 20 kilometers if H = 20 and T = 25.0 - 6.5 H.

Answer: T = 25.0 - 6.5 (20) = **- 105 Centigrade.**

Problem 5: Find the diameter in kilometers, D, of a black hole with a mass of 10 times the sun if **D = 5.6 M** and M = 10.0.

Answer: **D = 5.6 (10) = 56 kilometers.**

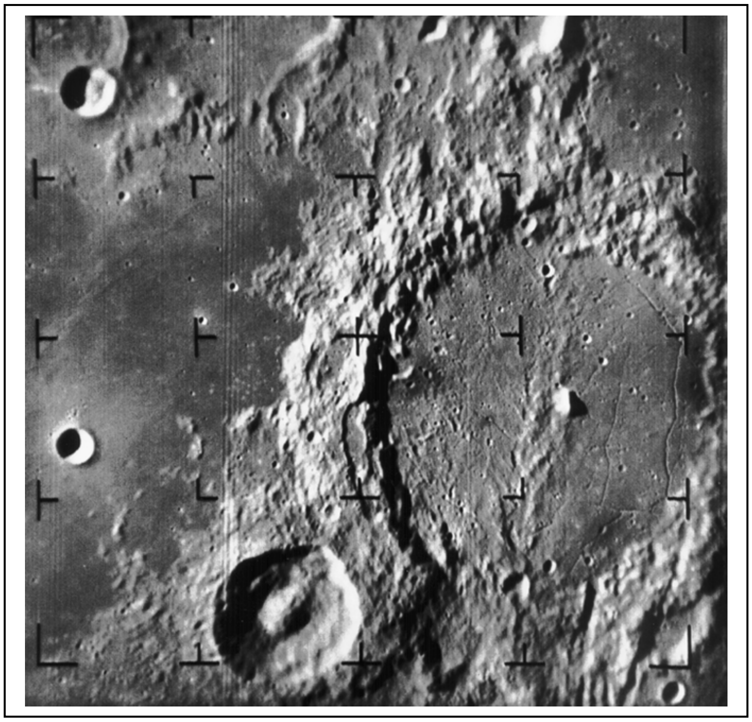
Problem 6: Calculate the speed of sound, S, in meters/second for a temperature of T = 200 Centigrade (that's 392 F), if **S = 331 + 0.6 T**.

Answer: **S = 331 + 0.6 (200)** = 331 + 120 = **451 meters/sec.**

Problem 7: Calculate the sunspot number, N, if there are X = 15 individual sunspots and Y = 10 groups of sunspots is **N = X + 15 Y**.

Answer: **N = 15 + 11 (10) = 15 + 110 = 125 sunspots.**

Craters are a Blast!



Have you ever wondered how much energy it takes to create a crater on the Moon. Physicists have worked on this problem for many years using simulations, and even measuring craters created during early hydrogen bomb tests in the 1950's and 1960's. One approximate result is a formula that looks like this:

$$\mathbf{E = 4.0 \times 10^{15}\, D^{3}\ Joules.}$$

where D is the crater diameter in kilometers.

As a reference point, nuclear bomb with a yield of one-megaton of TNT produces 4.0×10^{15} Joules of energy!

Problem 1 - To make the formula more 'real', convert the units of Joules into an equivalent number of one-megaton nuclear bombs.

Problem 2 - The photograph above was taken in 1965 by NASA's Ranger 9 spacecraft of the large crater Alphonsis. The width of the image above is 183 kilometers. With a millimeter ruler, determine the diameters, in kilometers, of a range of craters in the picture.

Problem 3 - Use the formula from Problem 1 to determine the energy needed to create the craters you identified.

Note: To get a better sense of scale, the table below gives some equivalent energies for famous historical events:

Event	Equivalent Energy (TNT)
Cretaceous Impactor	100,000,000,000 megatons
Valdiva Volcano, Chile 1960	178,000 megatons
San Francisco Earthquake 1909	600 megatons
Hurricane Katrina 2005	300 megatons
Krakatoa Volcano 1883	200 megatons
Tsunami 2004	100 megatons
Mount St. Helens Volcano 1980	25 megatons

Answer Key

Problem 1 - To make the formula more 'real', convert the units of Joules into an equivalent number of one-megaton nuclear bombs.

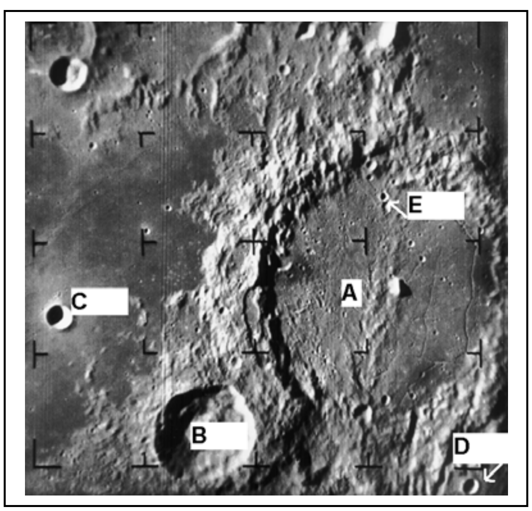
Answer: $E = 4.0 \times 10^{15} D^3$ Joules x (1 megaton TNT/4.0×10^{15} Joules)

$$E = 1.0\ D^3 \text{ Megatons of TNT}$$

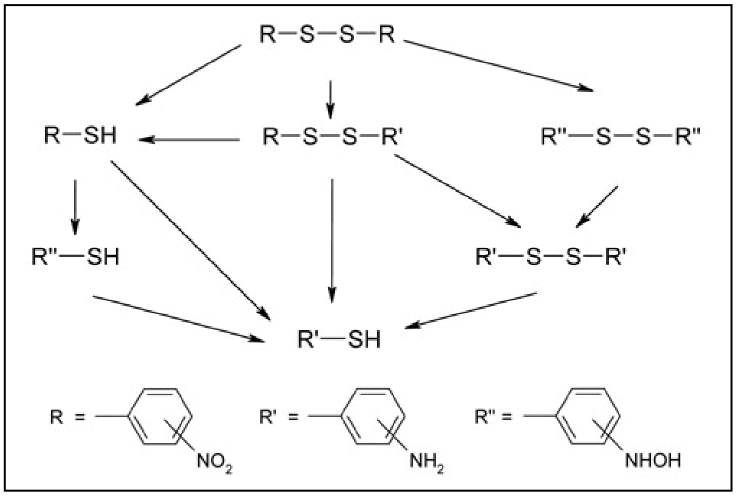
Problem 2 - The photograph above was taken in 1965 by NASA's Ranger 9 spacecraft of the large crater Alphonsis. The width of the image above is 183 kilometers. With a millimeter ruler, determine the diameters, in kilometers, of a range of craters in the picture. Answer: The width of the image is 92 mm, so the scale is 183/92 = 2.0 km/mm. See figure below for some typical examples: See column 3 in the table below for actual crater diameters.

Problem 3 - Use the formula from Problem 1 to determine the energy needed to create the craters you identified. Answer: See the table below, column 4. Crater A is called Alphonsis. Note: No single formula works for all possible scales and conditions. The impact energy formula only provides an estimate for lunar impact energy because it was originally designed to work for terrestrial impact craters created under Earth's gravity and bedrock conditions. Lunar gravity and bedrock conditions are somewhat different and lead to different energy estimates. The formula will not work for laboratory experiments such as dropping pebbles onto sand or flour. The formula is also likely to be inaccurate for very small craters less than 10 meters, or very large craters greatly exceeding the sizes created by nuclear weapons. (e.g. 1 kilometer).

Crater	Size (mm)	Diameter (km)	Energy (Megatons)
A	50	**100**	**1,000,000**
B	20	**40**	**64,000**
C	5	**10**	**1,000**
D	3	**6**	**216**
E	1	**2**	**8**



Fractions and Chemistry



Because molecules and atoms come in 'integer' packages, the ratios of various molecules or atoms in a compound are often expressible in simple fractions.

Adding compounds together can often lead to interesting mixtures in which the proportions of the various molecules involve mixed numbers.

In the problems below, do not use a calculator and state all answers as simple fractions or integers.

Problem 1 - What makes your car go: When 2 molecules of gasoline (ethane) are combined with 7 molecules of oxygen you get 4 molecules of carbon dioxide and 6 molecules of water.

A) What is the ratio of ethane molecules to water molecules?
B) What is the ratio of oxygen molecules to carbon dioxide molecules?
C) If you wanted to 'burn' 50 molecules of ethane, how many molecules of water result?
D) If you wanted to create 50 molecules of carbon dioxide, how many ethane molecules would you have to burn?

Problem 2 - How plants create glucose from air and water: Six molecules of carbon dioxide combine with 6 molecules of water to create one molecule of glucose and 6 molecules of oxygen.

A) What is the ratio of glucose molecules to water molecules?
B) What is the ratio of oxygen molecules to both carbon dioxide and water molecules?
C) If you wanted to create 120 glucose molecules, how many water molecules are needed?
D) If you had 100 molecules of carbon dioxide, what is the largest number of glucose molecules you could produce?

Answer Key

Problem 1 - What makes your car go: When 2 molecules of gasoline (ethane) are combined with 7 molecules of oxygen you get 4 molecules of carbon dioxide and 6 molecules of water.

A) In this reaction, 2 molecules of ethane yield 6 molecules of water, so the ratio is 2/6 or **1/3.**
B) 7 oxygen molecules and 4 carbon dioxide molecules yield the ratio **7/4**

C) The reaction says that 2 molecules of ethane burn to make 6 molecules of water. If you start with 50 molecules of ethane, then you have the proportion:

$$\frac{50 \text{ ethane}}{2 \text{ ethane}} = \frac{\text{x-water}}{6 \text{ -water}} \quad \text{so} \quad X = 25 \times 6 = \textbf{150 water molecules}.$$

D) Use the proportion:

$$\frac{50 \text{ Carbon Dioxide}}{4 \text{ carbon dioxide}} = \frac{X \text{ ethane}}{2 \text{ ethane}} \quad \text{so} \quad X = 2 \times (50/4) = \textbf{25 molecules ethane}$$

Problem 2 - How plants create glucose from air and water: Six molecules of carbon dioxide combine with 6 molecules of water to create one molecule of glucose and 6 molecules of oxygen.
A) What is the ratio of glucose molecules to water molecules?
B) What is the ratio of oxygen molecules to both carbon dioxide and water molecules?
C) If you wanted to create 120 glucose molecules, how many water molecules are needed?
D) If you had 100 molecules of carbon dioxide, what is the largest number of glucose molecules you could produce?

A) Glucose molecules /water molecules = **1 / 6**

B) Oxygen molecules / (carbon dioxide + water) = 6 / (6 + 6) = 6/12 = **1/2**

C)

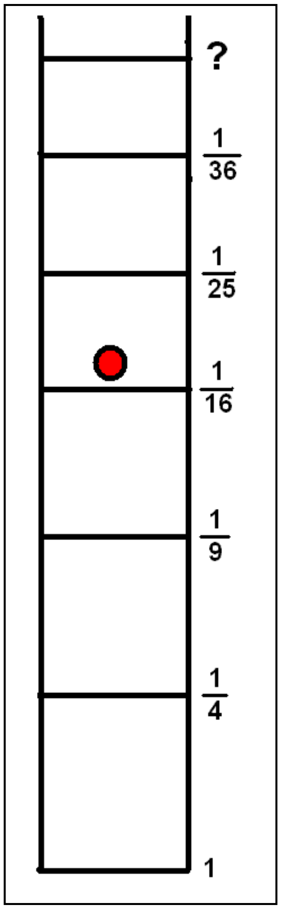
$$\frac{120 \text{ glucose}}{1 \text{ glucose}} = \frac{X \text{ water}}{6 \text{ water}} \quad \text{so} \quad X = 6 \times 120 = \textbf{720 water molecules}$$

D)

$$\frac{100 \text{ carbon dioxide}}{6 \text{ carbon dioxide}} = \frac{X \text{ glucose}}{1 \text{ glucose}} \quad \text{so} \quad X = 100/6 \text{ molecules.}$$

The problem asks for the largest number that can be made, so we cannot include fractions in the answer. We need to find the largest multiple of '6' that does not exceed '100'. This is 96 so that 6 x 16 = 96**.** **That means we can get no more than 16 glucose molecules by starting with 100 carbon dioxide molecules**. (Note that 100/6 = 16.666 so '16' is the largest integer).

Atomic Fractions



The single electron inside a hydrogen atom can exist in many different energy states. The lowest energy an electron can have is called the Ground State: this is the bottom rung on the ladder marked with an energy of '1'.

The electron must obey the Ladder Rule. This rule says that the electron can gain or lose only the exact amount of energy defined by the various ladder intervals.

For example, if it is located on the third rung of the ladder marked with an energy of '1/9', and it loses enough energy to reach the Ground State, it has to lose exactly 1 - 1/9 = 8/9 units of energy.

The energy that the electron loses is exactly equal to the energy of the light that it emits. This causes the spectrum of the atom to have a very interesting 'bar code' pattern when it is sorted by wavelength like a rainbow. The 'red line' is at a wavelength of 656 nanometers and is caused by an electron jumping from Energy Level 3 to Energy Level 2 on the ladder.

To answer these questions, use the Energy Fractions in the above ladder, and write your answer as the simplest fraction. Do not use a calculator or work with decimals because these answers will be less-exact than leaving them in fraction form!

Problem 1 - To make the red line in the spectrum, how much energy did the electron have to lose on the energy ladder?

Problem 2 - How much energy will the electron have to gain (+) or lose (-) in making the jumps between the indicated rungs:
A) Level-2 to Level-5
B) Level-3 to Level-1
C) Level-6 to Level-4
D) Level-4 to Level-6
E) Level-2 to Level-4
F) Level-5 to Level-1
G) Level-6 to Level-5

Problem 3 - From the energy of the rungs in the hydrogen ladder, use the pattern of the energy levels (1, 1/4, 1/9, 1/16, 1/25, …) to predict the energy of the electron jumping from A) the 10th rung to the 7th rung; B) the rung M to the lower rung N.

Problem 4 - If an energy difference of '1' on the ladder equals an energy of 14 electron-Volts, in simplest fractional form, how many electron-Volts does the electron lose in jumping from Level-6 to Level-4?

Answer Key

Problem 1 - Answer: The information in the figure says that the electron jumped from Level-3 to Level-2. From the energy ladder, this equals a difference of 1/9 - 1/4. The common denominator is '36' so the fractions become 4/36 - 9/36 and the difference is - 5/36 .because the answer is negative, the electron has to **lose 5/36** of a unit of energy to make the jump.

Problem 2 - How much energy will the electron have to gain (+) or lose (-) in making the jumps between the indicated rungs:
A) Level-2 to Level-5 = 1/4 - 1/25 = (25 - 4)/100 = **+21/100** so it has to GAIN energy.
B) Level-3 to Level-1 = 1/9 - 1 = 1/9 - 9/9 = **-8/9** so it has to LOSE energy.

C) Level-6 to Level-4 = 1/36 - 1/16 = **-5/144** so it has to LOSE energy
Two ways to solve:
First: (16 - 36) / (16x36) = -20 / 576 then simplify to get - 5 / 144
Second: Find Least Common Multiple
36: 36, 72, 108, **144**, 180, …
16: 16, 32, 48, 64, 80, 96, 112, 128, **144**, 160, …
LCM = 144, then
1/36 - 1/16 = 4/144 - 9/144 = **-5/144**

D) Level-4 to Level-6 = 1/16 - 1/36 = **+5/144** so it has to Gain energy.
E) Level-2 to Level-4 = 1/4 - 1/16 = 4/16 - 1/16 = **+3/16** so it has to GAIN energy
F) Level-5 to Level-1 = 1/25 - 1 = 1/25 - 25/25 = **-24/25** so it has to LOSE energy
G) Level-6 to Level-5 = 1/36 - 1/25 = (25 - 36)/ 900 = **- 11/900** so it has to LOSE energy

Problem 3 - Answer: Students should be able to see the pattern from the series progression such that the energy is the reciprocal of the square of the ladder rung number.

Level 2 Energy = $1/(2)^2 = 1/4$
Level 5 Energy = $1/(5)^2 = 1/25$

A) the 10th rung to the 7th rung: Energy = 1/100 - 1/49 = (49 - 100)/4900 = **-51/4900**.

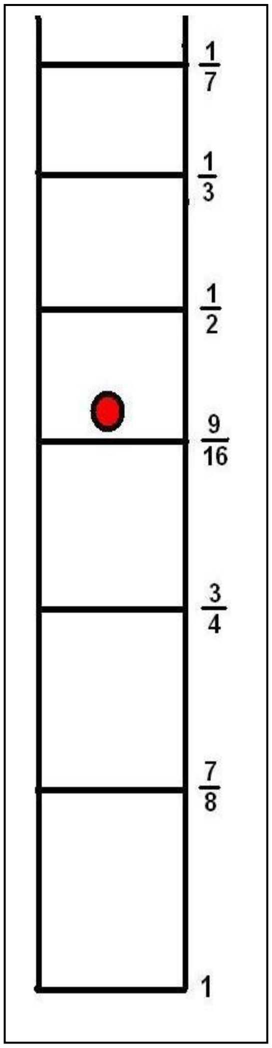
B) the rung M to the lower rung N. Energy = $1/M^2 - 1/N^2$

Problem 4 - If an energy difference of '1' on the ladder equals an energy of 13.6 electron-Volts, in simplest fractional form how many electron-Volts does the electron lose in jumping from Level-6 to Level-4?

Answer; The energy difference would be 1/36 - 1/16 = **-5/144** energy units.

Since an energy difference of 1.0 equals 14 electron-Volts, by setting up a ratio we have:

$$\frac{5/144 \text{ Units}}{1 \text{ Unit}} = \frac{X}{14 \text{ eV}} \quad \text{so} \quad X = 14 \times (5/144) = \frac{5 \times 2 \times 7}{2 \times 72} = \mathbf{\frac{35}{72}\ eV}$$



The single electron inside an atom can exist in many different energy states. The lowest energy an electron can have is called the Ground State: this is the bottom rung on the ladder marked with an energy of '1' in the figure to the left.

The electron must obey the Ladder Rule. This rule says that the electron can gain or lose only the exact amount of energy defined by the various ladder intervals.

For example, if the electron jumps from the fourth rung of the energy ladder marked with an energy of '9/16', to the Ground State, the energy change is E = 9/16 - 1 = -7/16 units of energy. This difference is negative, which means that the electron has LOST 7/16 energy units.

In the problems below, leave all answers in the simplest fractions.

Problem 1 - The electron gets a boost of energy and jumps from level 3 to Level 6. How much energy did it gain?

Problem 2 - An electron falls from Level 6 to Level 2. How much energy did it lose?

Problem 3 - An electron changes from Level 7 to Level 3. How much energy did it gain or lose?

Problem 4 - An electron is excited from Level 2 to Level 7. How much energy was gained?

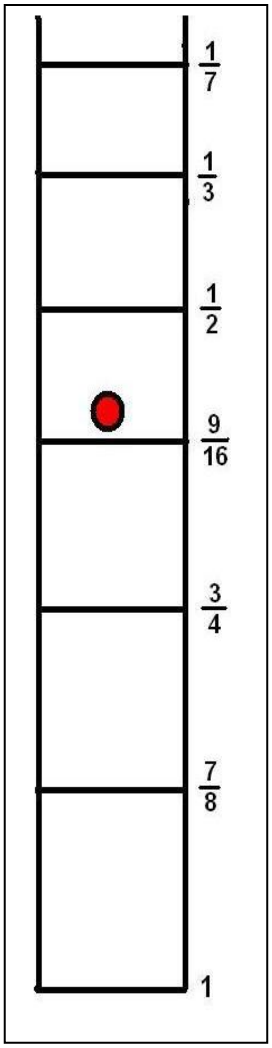
Problem 5 - The atom collides with another atom and the electron jumps from Level 3 to Level 6. How much energy did the other atom lose in the collision?

Problem 6 - An electron jumps from Level 3 to Level 2 and give of a particle of light. What energy is carried off by the light?

Problem 7 - An electron jumps from Level 7 to Level 4, then from Level 4 to Level 2. How much energy was lost with each jump, and what was the total energy lost after the two jumps?

Problem 8 - An electron jumps from the Ground State to Level 5, then is deexcited to Level 3, and after a while it is excited to Level 6, and then loses energy in a jump to Level 2. What is the total energy change of the electron between the start and end of this process?

Answer Key



Problem 1 - Answer: 3/4 - 1/3 = (9 - 4)/12 = +5/12 energy unit. This is positive, so **the electron GAINED 5/12 energy units**

Problem 2 - Answer: 1/3 - 7/8 = (8 - 21)/24 = -13/24 energy unit. This is negative, so **the electron LOST 13/24 energy units.**

Problem 3 - Answer: 1/7 - 3/4 = (4 - 21)/28 = -17/28 energy unit. So because this is negative **the electron LOST 17/28 energy unit.**

Problem 4 - Answer: 7/8 - 1/7 = (49 - 8)/56 = 41/56 energy unit. This is positive so **the electron had GAINED 41/56 energy units.**

Problem 5 - Answer: 3/4 - 1/3 = (9 - 4)/12 = 5/12 energy unit. This is positive, **so the electron has GAINED 5/12 energy units.**

Problem 6 - Answer: 3/4 - 7/8 = (24 - 28)/32 = -4/32 = -1/8 energy unit. This is negative ,so the electron LOST 1/8 energy unit, and so there was **1/8 energy unit carried away by the light particle.**

Problem 7 - Answer: The sequence is broken into two parts, which can be represented by bracketed quantities:

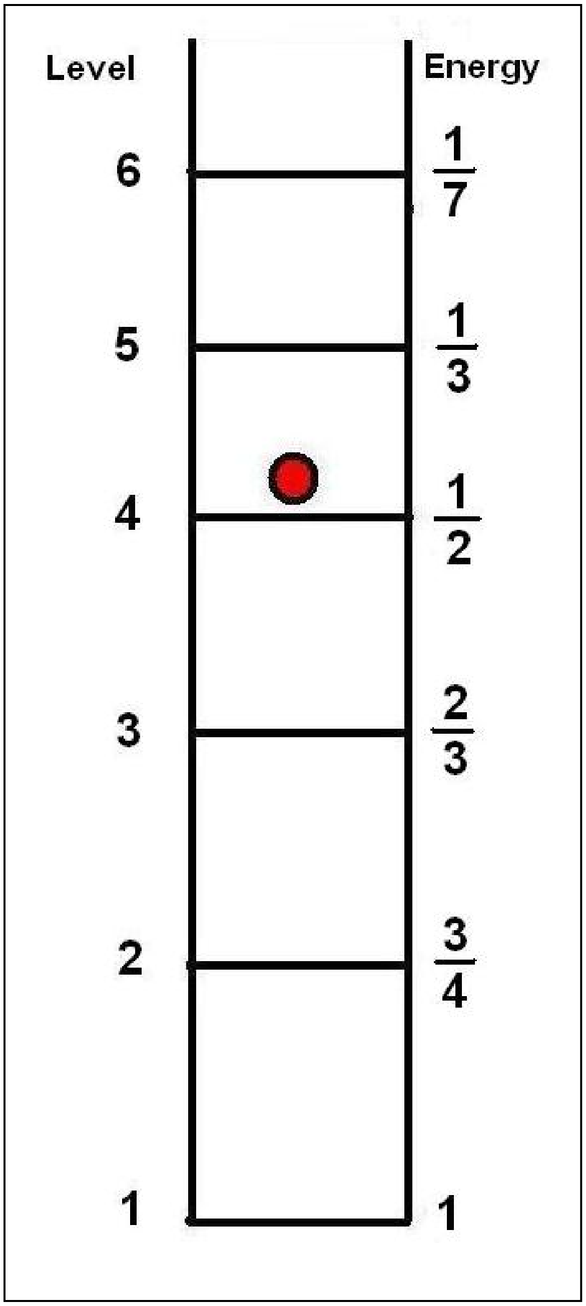
(1/7 - 9/16) + (9/16 - 7/8)
= 1/7 - 9/16 + 9/16 - 7/8
= 1/7 - 7/8
= (8 - 49)/56
= -41/56

so the electron **lost 41/56 of an energy unit. Students may note that this problem is of the form: (A - B) + (B - C) = A - C**

Problem 8 - Answer: (1 - 1/2) + (1/2 - 3/4) + (3/4 - 1/3) + (1/3 - 7/8) =
1 - 1/2 + 1/2 -3/4 + 3/4 - 1/3 + 1/3 - 7/8 =
1 + 0 + 0 + 0 - 7/8 =
1 - 7/8 = **1/8.**

This net energy is positive, so there was a net gain of 1/89 energy unit by the electron. Note, the sequence can be represented by (A - B) + (B - C) + (C - D) + (D - E) = A - E

Atomic Fractions - III

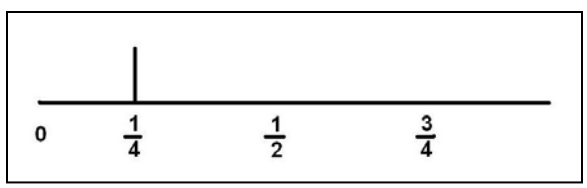


The electron inside an atom exists in one of many possible energy levels. These levels are like the rungs of a ladder. When it jumps from one level (rung) to the next, it gains or loses a specific amount of energy.

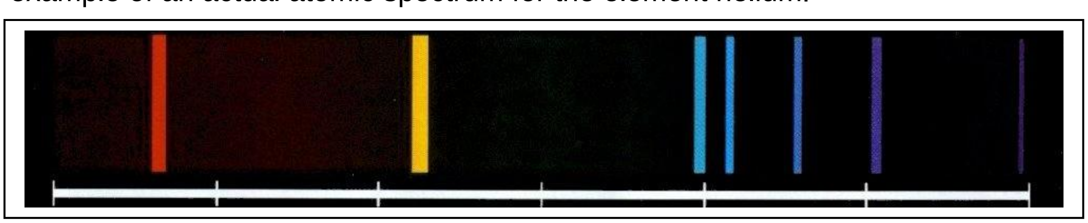
For example, if the energy of one rung is 3/4 and the energy of the next level is 1/4, the electron will lose 3/4 - 1/4 = 1/2 a unit if it jumps from the higher to the lower energy level.

Problem 1 - Suppose the energy level ladder of an imaginary element looked like the one to the left. What are all of the possible energies that an electron could lose as it jumped from a higher level to a lower one based on this ladder? (Leave your answers as simple fractions)

Problem 2 - On a number line, order the list of possible energy differences you tabulated in Problem 1, from lowest (left) to highest (right). For example, the energy difference between Level 2 and Level 1 is 1 - 3/4 = **1/4**, so draw a single vertical line at the location '1/4' on the number line. If a second energy difference is found to have the same value of '1/4' , draw the vertical line twice as tall, and so on. The graph is called a histogram.

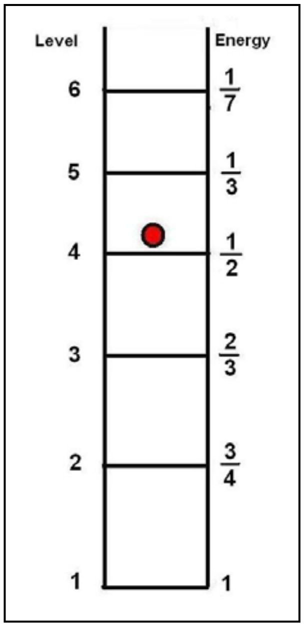


When you are finished with Problem 1 and 2, your number line will represent all of the possible ways that that atom can emit light. Every atom has its own pattern of atomic 'lines' which are based on each atom's unique energy ladder. This pattern is called a spectrum, and it is the unique fingerprint that allows scientists to identify each atom. Here is an example of an actual atomic spectrum for the element helium.



Answer Key

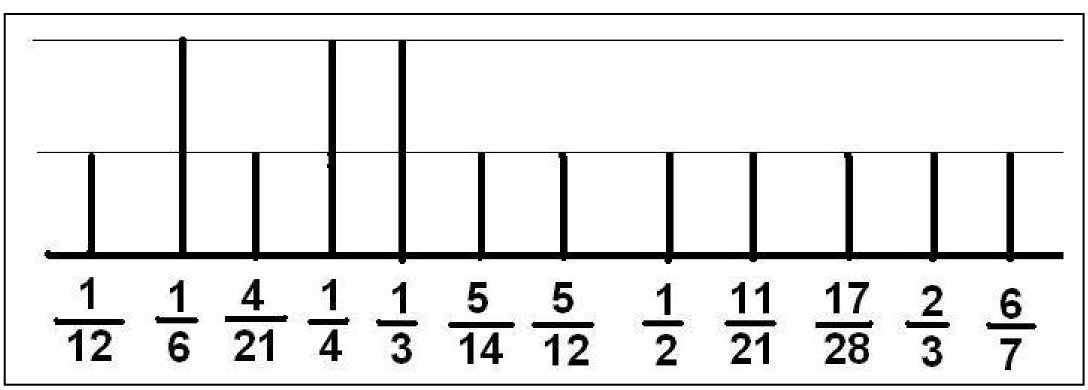
Problem 1 - Suppose the energy level ladder of an imaginary element looked like the one to the left. What are all of the possible energies that an electron could lose as it jumped from a higher level to a lower one based on this ladder? (Leave your answers as simple fractions)



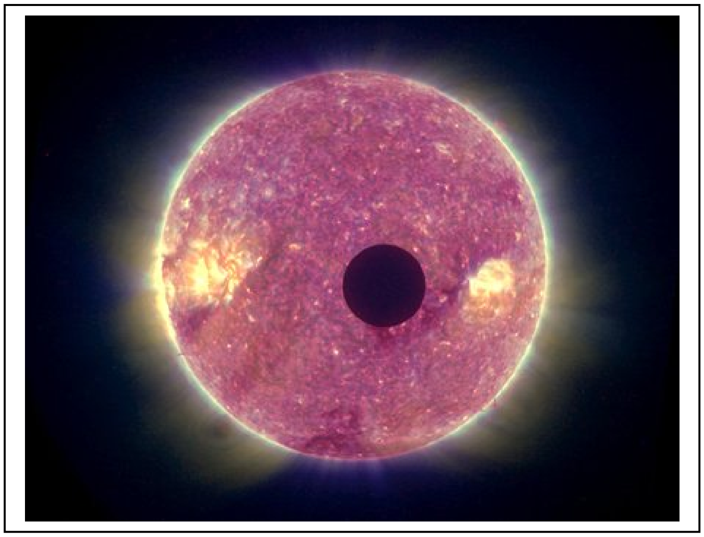
Levels	Energy Difference	Energy Value
2 to 1	1 - 3/4	= **1/4**
3 to 1	1 - 2/3	= **1/3**
4 to 1	1 - 1/2	= **1/2**
5 to 1	1 - 1/3	= **2/3**
6 to 1	1 - 1/7	= **6/7**
3 to 2	3/4 - 2/3 = 9/12 - 8/12	= **1/12**
4 to 2	3/4 - 1/2 = 3/4 - 2/4	= **1/4**
5 to 2	3/4 - 1/3 = 9/12 - 4/12	= **5/12**
6 to 2	3/4 - 1/7 = 21/28 - 4/28	= **17/28**
4 to 3	2/3 - 1/2 = 4/6 - 3/6	= **1/6**
5 to 3	2/3 - 1/3	= **1/3**
6 to 3	2/3 - 1/7 = 14/21 - 3/21	= **11/21**
5 to 4	1/2 - 1/3 = 3/6 - 2/6	= **1/6**
6 to 4	1/2 - 1/7 = 7/14 - 2/14	= **5/14**
6 to 5	1/3 - 1/7 = 7/21 - 3/21	= **4/21**

For 6 energy levels, there are 15 possible differences.

Problem 2 - The energies calculated in Problem 2 are displayed below. Note that in this diagram the scale is not linear but is merely used to illustrate the relative placements of the lines and their tallies. Students may use a more accurate number line to give a better impression of the spectrum and the non-uniform placement of the lines horizontally.



Kepler – The Hunt for Earth-like Planets



On March 11, 2009, NASA launched the Kepler satellite. Its 3-year mission is to search 100,000 stars in the constellation Cygnus and detect earth-sized planets. How can the satellite do this?

The image to the left shows what happens when a planet passes across the face of a distant star as viewed from Earth. In this case, this was the planet Mercury on February 25, 2007.

The picture was taken by the STEREO satellite. Notice that Mercury's black disk has reduced the area of the sun. This means that, on Earth, the light from the sun dimmed slightly during the Transit of Mercury. Because Mercury was closer to Earth than the Sun, Mercury's disk appears very large. If we replace Mercury with the Moon, the lunar disk would exactly cover the disk of the Sun and we would have a total solar eclipse.

Now imagine that the Sun was so far away that you couldn't see its disk at all. The light from the Sun would STILL be dimmed slightly. The Kepler satellite will carefully measure the brightness of more than 100,000 stars to detect the slight changes caused by 'transiting exoplanets'.

Problem 1 – With a compass, draw a circle 160-millimeters in radius to represent the sun. If the radius of the sun is 696,000 kilometers, what is the scale of your sun disk in kilometers/millimeter?

Problem 2 – At the scale of your drawing, what would be the radius of Earth (R = 6,378 km) and Jupiter (R = 71,500 km)?

Problem 3 – What is the area of the Sun disk in square millimeters?

Problem 4 – What is the area of Earth and Jupiter in square millimeters?

Problem 5 – By what percent would the area of the Sun be reduced if: A) Earth's disk were placed in front of the Sun disk? B) Jupiter's disk were placed in front of the Sun disk?

Problem 6 – For the transit of a large planet like Jupiter, draw a graph of the percentage brightness of the star (vertical axis) as it changes with time (horizontal axis) during the transit event. Assume that the entire transit takes about 1 day from start to finish.

Answer Key

Problem 1 – With a compass, draw a circle 160-millimeters in radius to represent the sun. If the radius of the Sun is 696,000 kilometers, what is the scale of your Sun disk in kilometers/millimeter? **Answer: 4,350 km/mm**

Problem 2 – At the scale of your drawing, what would be the radius of Earth (R = 6,378 km) and Jupiter (R = 71,500 km)? **Answer: 1.5 mm and 16.4 mm respectively.**

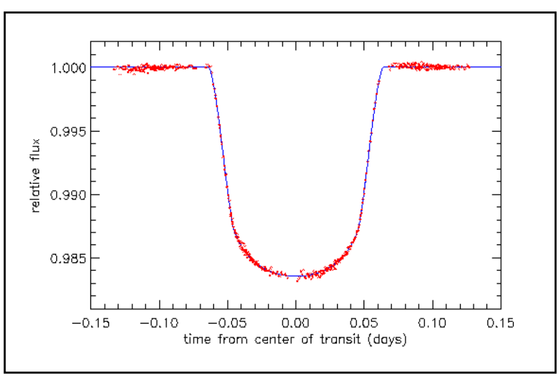
Problem 3 – What is the area of the sun disk in square millimeters? Answer; π x $(160)^2$ **= 80,400 mm^2.**

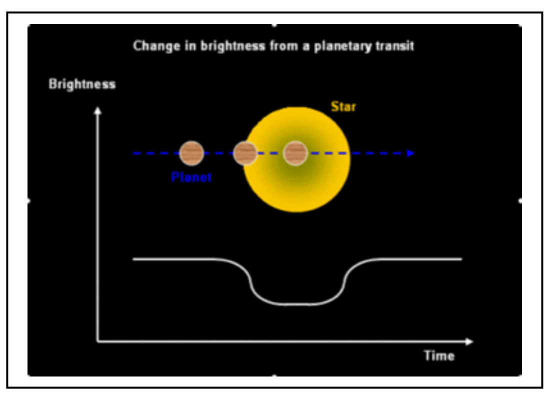
Problem 4 – What is the area of Earth and Jupiter in square millimeters? Answer: Earth = π x $(1.5)^2$ = **7.1 mm^2**. Jupiter = π x $(16.4)^2$ = **844.5 mm^2**.

Problem 5 – By what percent would the area of the sun be reduced if: A) Earth's disk were placed in front of the Sun disk? B) Jupiter's disk were placed in front of the Sun disk? Answer A) 100% x (7.1 mm^2 / 80400 mm^2) = 100% x 0.000088 **= 0.0088 %**
B) Jupiter: 100% x (844.5 mm^2 / 80400 mm^2) = 100% x 0.011 = **1.1%.**

Problem 6 – For the transit of a large planet like Jupiter, draw a graph of the percentage brightness of the star (vertical axis) as it changes with time (horizontal axis) during the transit event. Assume that the entire transit takes about 1 day from start to finish.

Answer: Students should note from their answer to Problem 5 that when the planet disk is fully on the star disk, the star's brightness will dim from 100% to 100% - 1.1% = 98.9%. Students should also note that as the transit starts, the stars brightness will dim as more of the planet's disk begins to cover the star's disk. Similarly, as the planet's disk reaches the edge of the star's disk, the area covered by the planet decreases and so the star will gradually brighten to its former 100% level. The figures below give an idea of the kinds of graphs that should be produced. The left figure is from the Hubble Space Telescope study of the star HD209458 and its transiting Jupiter-sized planet.





Useful Internet Resources

Space Math @ NASA

http://spacemath.gsfc.nasa.gov

Practical Uses of Math and Science (PUMAS)

http://pumas.gsfc.nasa.gov

Teach Space Science

http://www.teachspacescience.org

Space Weather Action Center

http://sunearthday.nasa.gov/swac

THEMIS Classroom guides on Magnetism

http://ds9.ssl.berkeley.edu/themis/classroom.html

The Stanford Solar Center

http://solar-center.stanford.edu/solar-math/

A Math Refresher

http://istp.gsfc.nasa.gov/stargaze/Smath.htm

A note from the Author:

Hi again!

Here is another collection of 'fun' problems based on NASA space missions across the solar system and the universe. This year, I added many more advanced math and calculus problems just to round-out the math offerings.

This is a complicated time for space exploration. The Shuttle Fleet will be retired by the end of 2010. For the first time in 50 years, US access to space by using home-grown rockets will be over. The cancellation of the Constellation program will leave 3-5 year gap in our ability to bring our own astronauts into space unless commercial rocket shuttles can be deployed quickly to provide this service. This also means that our access to the Space Station that we just built at a cost of nearly $100 billion is now by way of the Russian launchers.

Meanwhile, NASA has launched the Solar Dynamics Observatory to study the sun in 'high-definition' and the current count of known extra-solar planets now stands at 430. Astronomers have begun to study the atmospheres of many of these planets, and it is hoped that in the next 10 years we will discover the first earth-like world in terms of both its mass and ability to sustain liquid water on its surface. These are indeed exciting, and transitional, times in our exploration of space!!

By the way, here are three interesting quotes you may enjoy!

"We shall never be able to study, by any method, their (stars) chemical composition or their mineralogical structure" (Philospher Auguste Comte, in 'Cours de Philosophie Positive', 1830-1842) - Within 10 years after publication, the spectroscope was invented and used to study the elements in the sun, and later the stars.

"Math is like childhood diseases: the earlier you have them, the better" (Physicist Arnold Sommerfeld) - Sommerfeld was one of the developers of a mathematical model for the atom, which predated the modern developments in quantum mechanics.

"It turned out to be a great advantage to have overindulged in mathematics at an early age" (Nobel Laureate, Dudley Herschenbach). He won the 1987 Nobel Prize in Chemistry, along with John Polanyi and Yuan T. Lee for his work on 'Reaction Dynamics'.

Let me know if you have any other favorites quotes about the use of mathematics!

Sincerely,
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