# QUANTUM ALGORITHMS FOR LOWEST WEIGHT PATHS AND SPANNING TREES IN COMPLETE GRAPHS

### Mark Heiligman

January 9, 2003

ABSTRACT. Quantum algorithms for several problems in graph theory are considered. Classical algorithms for finding the lowest weight path between two points in a graph and for finding a minimal weight spanning tree involve searching over some space. Modification of classical algorithms due to Dijkstra and Prim allows quantum search to replace classical search and leads to more efficient algorithms. In the case of highly asymmetric complete bipartite graphs, simply replacing classical search with quantum search leads to a faster quantum algorithm. A fast quantum algorithm for computing the diameter of a complete graph is also given.

#### Introduction

The question of which classical algorithms can be sped up by quantum computing is of course a very interesting one. At present there are only a few general techniques known in the field of quantum computing and finding new problems that are amenable to quantum speedups is a high priority. Classically, one area of mathematics that is full of interesting algorithms is computational graph theory. It is therefore natural to ask whether any of the classical graph theory algorithms can take advantage of quantum computing.

One of the few general techniques known centers around Grover's algorithm for searching an unsorted list for a specified element. This original idea has been extended to general amplitude amplification that can be applied to any classical algorithm. It would be incorrect to assume that amplitude amplification always leads to quantum speedups of classical algorithms. There are some interesting cases where "Grover-like" techniques do in fact lead to speedups of classical algorithms. One very important case of this is to find the minimum value of a computable function as the set of input arguments ranges over a finite, but unordered list. In this case, if the list is of length n, then the quantum cost of finding the minimum is  $O(\sqrt{n})$ , while the classical cost is O(n). Quantum algorithms for searching for the maximum or minimum of an unsorted list have been described in [DH] and [AK]. The question to be addressed here is whether this leads to speedups in graph theory algorithms that employ classical minimum finding in the course of solving a graph theory problem.

Key words and phrases. quantum computing, quantum algorithm, graph theory.

#### 1. Minimal Weight Paths

The problem of finding the shortest path between two points in a weighted graph is an old one. If G = (V, E) is a directed graph with a weighting function  $\nu : E \to \mathbb{R}^+$ , the weight of a path is the sum of the weights of the edges that comprise the path. If G is a complete graph, then the function  $\nu$  is well defined on all the edges. In case G is not a complete graph, and  $(v, v') \notin E$ , then it is useful to define  $\nu(v, v') = \infty$ . This allows  $\nu$  to be defined on all of  $V \times V$ , not just E.

Pick a  $v_0 \in V$  from which all the shortest paths are to be computed. Consider the following algorithm due to Dijkstra (see [G]):

Figure 1: Dijkstra's algorithm.

At the end of this procedure  $\lambda(v)$  is the length of the shortest path from  $v_0$  to v, and with only minor changes, this algorithm is easily modified to record the information needed to construct the shortest path. This algorithm consists of iterating over all of V by successively adding elements to S, which is the set of points for which the shortest path from  $v_0$  has already been determined. Each iteration consists of a search procedure to find the next nearest element of V to  $v_0$ , and an update procedure to record all the newest shortest path information for the remaining vertices based on this newest next nearest element.

Analysis of the run time for this algorithm is quite simple. At line (3) if i = |S|, then the (classical) cost of the search in line (3a) is n - i and the cost of updating in line (3c) is n - i - 1. The total cost is therefore  $\sum_{i=0}^{n} 2n - 2i - 1 = O(n^2)$ .

There are some important modifications to this original algorithm of Dijkstra if the graph being searched is somewhat sparse (i.e. if the total number of edges is much less than  $n^2$ ). In this case, Dijkstra's algorithm can be modified to find the shortest path with work  $O((|V| + |E|) \log |V|)$  through the use of priority queues. In general, the key idea is that the update procedure in line (3c) need only update the shortest path for those vertices that are adjoining the vertex w which was most recently added to the set S of those vertices whose shortest path distance from  $v_0$  have alread been computed. The reason the only the vertices adjacent to w need to be considered is that for all other vertices  $v(w, v) = \infty$ .

Unfortunately, if the classical search for the minimum at line (3a) is replaced by the quantum algorithm for finding the minimum of an unordered set, the cost for line (3a) per iteration drops to  $O(\sqrt{n-i})$ , but the cost of the entire algorithm is still  $O(n^2)$  since the update cost per iteration in line (3c) is still O(n-i).

One possible way around this problem is to dispense entirely with the update procedure in line (3c) at the cost of a larger search in line (3a). This modified

algorithm then goes as follows:

```
(1) S \leftarrow \{v_0\}, \ \lambda(v_0) \leftarrow 0

(2) while S \neq V do

(2.a) find (w, v) \in S \times (V - S) such that \lambda(w) + \nu(w, v) is minimal

(2.b) S \leftarrow S \cup \{v\}, \ \lambda(v) \leftarrow \lambda(w) + \nu(w, v)
```

Figure 2: Dijkstra's algorithm without full updating

As before, at the end of this procedure  $\lambda(v)$  is the length of the shortest path from  $v_0$  to v, and again, this algorithm is easily modified to record the information needed to construct the shortest path.

The analysis of this modified algorithm is also quite easy. At line (2) if i = |S|, then the (classical) cost of the search in line (2a) is i(n-i), making the entire classical cost of this algorithm  $\sum_{i=0}^{n} i(n-i) = O(n^3)$ , which is quite a bit worse than Dijkstra's original algorithm. However, the quantum cost of this algorithm is determined by noting that the search cost at line (2a) reduced to  $O(\sqrt{i(n-i)})$ , thereby making the total cost of the algorithm  $O(\sum_{i=0}^{n} \sqrt{i(n-i)}) = O(n^2)$ , which can be seen by noting that

$$\sum_{i=0}^{n} \sqrt{i(n-i)} \approx \int_{0}^{n} \sqrt{x(n-x)} \, dx = n^{2} \int_{0}^{1} \sqrt{y(1-y)} \, dy = O(n^{2}).$$

So the quantum version of this algorithm has work  $O(n^2)$  as well, which really doesn't represent an improvement over the original classical algorithm.

What seems to be really needed is to have a partial tradeoff between the search and update parts of the algorithm. The idea is to balance the classical update cost per iteration with the quantum search cost. The following algorithm is one way of accomplishing this.

```
(1) S \leftarrow \{v_0\}, T \leftarrow \{v_0\}, \lambda(v_0) \leftarrow 0
(2) for v \in V - S do
   (2.a) \lambda(v) \leftarrow \nu(v_0, v)
(3) while S \neq V do
   (3.a) find (w,v) \in T \times (V-S) such that \lambda(w) + \nu(w,v) is minimal
   (3.b) find u \in V - S such that \lambda(u) is minimal
   (3.c) if \lambda(w) + \nu(w,v) \leq \lambda(u) then
      (3.c.1) S \leftarrow S \cup \{v\}, T \leftarrow T \cup \{v\}, \lambda(v) \leftarrow \lambda(w) + \nu(w,v)
   (3.d) if \lambda(w) + \nu(w,v) > \lambda(u) then
      (3.d.1) S \leftarrow S \cup \{u\}, T \leftarrow T \cup \{u\}
   (3.e) if |T| \ge k then do
      (3.e.1) for v \in V - S do
         (3.e.1.a) find w \in T such that \lambda(w) + \nu(w,v) is minimal
         (3.e.1.b) if \lambda(w) + \nu(w,v) < \lambda(v) then
             (3.e.1.b.1) \lambda(v) \leftarrow \lambda(w) + \nu(w,v)
      (3.e.2) T \leftarrow \{v_0\}
```

Figure 3: Dijkstra's algorithm with periodic updating

The idea is to have a set T of vertices for which the full update of  $\lambda(v)$  for all remaining vertices in V-S has not yet been computed. This full updating is done every k-th iteration of the main loop. By keeping  $v_0$  in T all the time, there is always the possibility of going directly from  $v_0$  to v since  $\lambda(v_0) = 0$ . The value of k is a parameter for this algorithm and needs to be set to optimize the total cost.

Most of the work in this algorithm takes place in line (3), the main iteration, which is done a total of n times. To analyze the work for the i-th iteration, write i = h k + j with  $1 \le j \le k$ , so that the size of the set T on the i-th iteration is j. It is convenient for this analysis to assume that k divides n since that makes line (3c) execute exactly n/k times, however, even if this assumption does not hold, the work calculation is still valid.

The search for the minimum on line (3a) is over a set of size j(n-i), so the total work over all iteration of line (3a) is

$$\sum_{h=0}^{n/k-1} \sum_{j=1}^{k} j (n - h k - j) = O(kn^2)$$

in the classical case and

$$\sum_{h=0}^{n/k-1} \sum_{j=1}^{k} \sqrt{j (n - h k - j)} = O(k^{1/2} n^{3/2})$$

in the quantum case.

The update cost on line (3e1) requires n-i searches for the minimum over a set of size k each time and this is done only when i is divisible by k. Therefore the entire cost of the update procedure on line (3e) over all the iterations of the algorithm is

$$\sum_{h=1}^{n/k} (n - kh) k = O(n^2)$$

in the classical case and

$$\sum_{h=1}^{n/k} (n - kh) \sqrt{k} = O(k^{-1/2}n^2)$$

in the quantum case.

In the classical case, the total work for the algorithm is  $\max(O(kn^2), O(n^2)) = O(n^2)$  which is minimized by taking k = 1, and therefore

$$W_{\text{classical}} = O(n^2).$$

Note that taking k=1 in figure 3 gives the original Dijkstra algorithm of figure 1, while taking k=|V| yields the algorithm of figure 2, so k might reasonably be viewed as an interpolation parameter.

In the quantum case, the situation is a bit different. The total work in line (3) is just the maximum of the work in lines (3a) and (3e), since the work in line (3b) is always dominated by these other work factors. The total work is therefore

$$\max \bigl( O\bigl(k^{1/2} n^{3/2}\bigr), O\bigl(k^{-1/2} n^2\bigr) \bigr)$$

and to minimize this, the parameter k should be chosen to make these two work factors the same. Setting  $k^{1/2}n^{3/2} = k^{-1/2}n^2$  gives  $k = n^{1/2}$  and therefore

$$W_{\text{quantum}} = O(n^{7/4}).$$

This indeed is an improvement over the classical work factor of  $n^2$ .

### 2. Graph Diameter

The algorithms so far presented compute the minimal path length from one point in the graph to another point in the graph. This minimal path length will be referred to as the distance from one point to another in the graph. The diameter of a graph is the distance between the two furthest points in a graph (i.e. the maximum of the minimal path lengths in the graph). This section deals with quantum algorithms for finding the diameter of a weighted graph.

Since the initial point  $v_0$  was always fixed in Dijkstra's algorithm and its variants, this was never explicitly indicated in the notation  $\lambda(v)$ . However for the purposed of this section, it is useful to write  $\lambda(v_0, v)$  in place of  $\lambda(v)$ . One of the key features of all of the "Dijkstra-like" algorithms described in the first section is that they start from a given vertex  $v_0$  and by an iterative procedure manage to find the minimal distances to all the other vertices in the graph. Furthermore the last vertex for which the distance is computed is always the most distant vertex from  $v_0$ . These "Dijkstra-like" algorithms could therefore be viewed as computing the maximum distance from  $v_0$ . The diameter of the graph is just the maximum of all these distances as  $v_0$  runs over all of V. By invoking the quantum maximum finding algorithm with a "Dijkstra-like" algorithm as a callable subroutine, the diameter of the graph will follow. Since there are n = |V| possible initial values of  $v_0$ , the quantum cost is simply  $\sqrt{n}$  times the cost of the inner loop. Using the best quantum algorithm as the inner loop gives a total quantum cost for finding the diameter as  $O(n^{9/4})$ .

By way of comparison with classical costs, Dijkstra's (classical) algorithm would have to be run n times giving a classical cost of  $O(n^3)$ . There is an interesting alternative classical algorithm due to Floyd and Warshall that finds all the distances between all pairs of vertices in a weighted graph. Its cost is also  $O(n^3)$ .

# 3. Minimal Weight Spanning Trees

Another common problem in classical graph theory is that of finding a minimal weight spanning tree of a graph. There is a very nice classical algorithm for this due to Prim that goes as follows (see [G]):

```
\begin{array}{l} (1) \ S \leftarrow \{v_0\}, \ F \leftarrow \{\} \\ (2) \ \ \mbox{for} \ \ v \in V - S \ \mbox{do} \\ (2.a) \ \ L(v) \leftarrow \nu(v_0,v), \ M(v) \leftarrow v_0 \\ (3) \ \mbox{while} \ S \neq V \ \mbox{do} \\ (3.a) \ \mbox{find} \ w \in V - S \ \mbox{such that} \ L(w) \ \mbox{is minimal} \\ (3.b) \ \ S \leftarrow S \cup \{w\}, \ F \leftarrow F \cup \{(w,M(w)\} \\ (3.c) \ \mbox{for} \ \ v \in V - S \ \mbox{do} \\ (3.c.1) \ \mbox{if} \ \ \nu(w,v) < L(v) \ \mbox{then} \\ (3.c.1.a) \ \ L(v) \leftarrow \nu(w,v), \ M(v) \leftarrow w \end{array}
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Figure 4: Prim's algorithm.

At the end of this algorithm, F is a set of edges that comprise a minimal weight spanning tree of the original graph (V, E), which has been implicitly assumed to be a complete graph on n = |V| vertices. As in the case of Dijkstra's minimal weight

path algorithm, this algorithm consists of an search phase for each iteration and an update phase for each iteration. In the original algorithm of Prim, just as in Dijkstra's original algorithm, the work for the search phase and the work for the update phase are carefully balanced to be the same, the total work being  $O(n^2)$ .

There is a version of this algorithm that doesn't employ any updating at the cost of having to search a larger space to find a minimum on each iteration. That algorithm goes as follows:

```
 \begin{array}{l} \textit{(1)} \ S \leftarrow \{v_0\}, \ F \leftarrow \{\} \\ \textit{(2)} \ \textbf{while} \ S \neq V \ \textbf{do} \\ \textit{(2.a)} \ \textbf{find} \ (u,v) \in S \times (V-S) \ \textbf{such that} \ \nu(u,v) \ \textbf{is minimal} \\ \textit{(2.b)} \ S \leftarrow S \cup \{v\}, \ F \leftarrow F \cup \{(u,v)\} \end{array}
```

Figure 5: Prim's algorithm with no updating

Essentially the same idea that works for a good quantum version of Dijkstra's algorithm, as in the first section of this note, works to give a good quantum version of Prim's algorithm. In particular, the following modification of Prim's algorithm is what seems to work:

```
(1) S \leftarrow \{v_0\}, T \leftarrow \{v_0\}, F \leftarrow \{\}, L(v_0) \leftarrow 0
(2) for v \in V - S do
   (2.a) \lambda(v) \leftarrow \nu(v_0, v), M(v) \leftarrow v_0
(3) while S \neq V do
   (3.a) find (u,v) \in T \times (V-S) such that \nu(u,v) is minimal
   (3.b) find w \in V - S such that L(w) is minimal
   (3.c) if L(w) \leq \nu(u,v) then
      (3.c.1) S \leftarrow S \cup \{w\}, T \leftarrow T \cup \{w\}, F \leftarrow F \cup \{(w, M(w))\}
   (3.d) if L(w) > \nu(u, v) then
      (3.d.1) \ S \leftarrow S \cup \{v\}, \ T \leftarrow T \cup \{v\}, \ F \leftarrow F \cup \{(u,v)\}
   (3.e) if |T| > k then do
      (3.e.1) for v \in V - S do
         (3.e.1.a) find w \in T such that \nu(w,v) is minimal
         (3.e.1.b) if \nu(w,v) < L(v) then
            (3.e.1.b.1) L(v) \leftarrow \nu(w,v), M(v) \leftarrow w
      (3.e.2) T \leftarrow \{v_0\}
```

Figure 6: Prim's algorithm with periodic updating

The analysis of this algorithm is practically identical to that of Dijkstra's algorithm, and again the parameter k is determined to equalize the work of searching for a minimum with the work of updating. If quantum minimum finding is used in steps (3a), (3b), and (3e1a) instead of classical minimum finding, the optimal value of k is again  $O(n^{1/2})$  and the overall work for the quantum algorithm in figure 6 here is  $O(n^{7/4})$ , which is an improvement on the classical algorithm.

### 4. Bipartite Graphs

Another approach is to change the problem to one that is more amenable to replacing the classical search with a quantum search. The idea is to balance the classical update cost per iteration with the quantum search cost by introducing asymmetry into the problem in Dijkstra's original algorithm.

In a bipartite graph, the set of vertices V is divided into two disjoint subsets  $V_1$  and  $V_2$ , and all edges connect points in  $V_1$  with points in  $V_2$  (and vice versa, as well in a directed graph). In a complete bipartite graph, each point in  $V_1$  is connected to every point in  $V_2$ , and conversely, each point in  $V_2$  is connected to every point in  $V_1$ . If  $n_i = |V_i|$  for i = 1, 2, then the number of edges in a complete bipartite graph is  $n_1 n_2$ , and in a complete bipartite digraph, the number of edges is  $2n_1n_2$ . If  $n_1$  and  $n_2$  are of very different sizes, then a complete bipartite graph will be quite sparse. In what follows, for this section, the question of finding the lowest weight path in complete bipartite graphs will be considered. Since a complete bipartite graph has edge set  $E = (V_1 \times V_2) \cup (V_2 \times V_1)$ , it will be assumed that a pair of weight functions  $\nu_1 : V_1 \times V_2 \to \mathbb{R}^+$  and  $\nu_2 : V_2 \times V_1 \to \mathbb{R}^+$  are given. The length of the shortest path  $\lambda_i : V_i \to \mathbb{R}^+$  from an initial vertex  $v_0$  to elements in  $V_i$  for i = 1, 2 will be computed by the algorithm.

Assuming that  $n_1 < n_2$ , the work for Dijkstra's algorithm is  $O(n_1n_2 \log n_2)$ , with a sparse graph modification of the algorithm using priority queues. There are certain cases in which this may not be the best classical algorithm for finding the shortest path between two points. The run time for Dijkstra's algorithm may be optimal, but the memory requirements may be overwhelming. The following algorithm constructs the list of shortest distances (and paths if a little additional information is kept) between a fixed initial point  $v_0 \in V_1$  and all the other points in  $V_1 \cup v_2$ :

```
\begin{array}{l} (1) \ S_2 \leftarrow \{v_0\}, \ \lambda_1(v_0) \leftarrow 0 \\ (2) \ \ \mbox{for} \ v \in V_1 - S_1 \ \mbox{do} \\ (2.a) \ \mbox{find} \ w \in V_2 \ \mbox{such that} \ \nu_1(v_0,w) + \nu_2(w,v) \ \mbox{is minimal} \\ (2.b) \ \lambda_1(v) \leftarrow \nu_1(v_0,w) + \nu_2(w,v) \\ (3) \ \mbox{while} \ S_1 \neq V_1 \ \mbox{do} \\ (3.a) \ \mbox{find} \ (u,v,w) \in S_1 \times V_2 \times (V_1 - S_1) \ \mbox{such that} \ \lambda_1(u) + \nu_1(u,v) + \nu_2(v,w) \\ \mbox{is minimal} \\ (3.b) \ S_1 \leftarrow S_1 \cup \{w\}, \ \lambda_1(w) \leftarrow \lambda_1(u) + \nu_1(u,v) + \nu_2(v,w) \\ (3.c) \ \mbox{for} \ v \in V_1 - S \ \mbox{do} \\ (3.c.1) \ \mbox{find} \ u \in V_2 \ \mbox{such that} \ \lambda_1(w) + \nu_1(w,u) + \nu_2(u,v) \ \mbox{is minimal} \\ (3.c.2) \ \lambda_1(v) \leftarrow \min(\lambda_1(v),\lambda_1(w) + \nu_1(w,u) + \nu_2(u,v)) \\ (4) \ \mbox{for} \ u \in V_2 \ \mbox{do} \\ (4.a) \ \mbox{find} \ w \in V_1 \ \mbox{such that} \ \lambda_1(w) + \nu_1(w,u) \ \mbox{is minimal} \\ (4.b) \ \lambda_2(u) \leftarrow \lambda_1(w) + \nu_1(w,u) \end{array}
```

Figure 7: Dijkstra's algorithm for a bipartite graph with partial updating.

The idea is that since  $V_1$  is smaller than  $V_2$ , only the distances in  $V_1$  need to be updated. Vertices in  $V_2$  are viewed merely as intermediate points along the way for paths that connect vertices in  $V_1$ .

The analysis of this algorithm is only slightly more complicated than the previous analyses. Beginning with the idea that classical search for the minimum of some function over a set of size N requires O(N) operations, it is clear that line (2a) requires work  $O(n_2)$  for each iteration of line (2), and since there are  $n_1$  iterations of line (2), the total work for the initialization of  $\lambda(v)$  for all  $v \in V_1$ , which is what goes on in lines (1), (2), (2a), and (2b), is  $O(n_1n_2)$ . As for line (3), let i = |S| on each iteration. Then for (3a), the search is over a set of size  $i(n_1 - i) n_2$ , which makes the total search cost for (3a) over all the iterations in line (3)  $\sum_{i=0}^{n_1} i(n_1-i) n_2 =$  $O(n_1^2 n_2)$ . As for the update operations that start in line (3), there is an subiteration over a set of size  $n_1 - i$  and within each subiteration, there is a search over a set of size  $n_2$ . Thus the total update cost is also  $\sum_{i=0}^{n_1} i(n_1-i) n_2 = O(n_1^2 n_2)$ . Thus the total (classical) algorithm cost is  $O(n_1^2n_2)$ , but only a memory of size  $O(n_1)$ is needed. For the final stage (4) of the algorithm that fills in the cost function for the elements of the second and larger part,  $V_2$ , the outer iteration is over a set of size  $n_2$ . For each  $u \in V_2$ , the minimum in line (4a) is found over a set of size  $n_1$ , making the total (classical) work for this final phase of the algorithm  $O(n_1n_2)$ , which is clearly not the dominant cost.

Now for the quantum costs. The costs of initialization are  $n_1$  iterations of search over a set of size  $n_2$ , so the quantum cost is  $O(n_1 n_2^{1/2})$ . The search cost per iteration in line (3a) is  $O(\sqrt{i(n_1-i)n_2})$ , making the total search cost

$$\sum_{i=0}^{n_1} \sqrt{i(n_1 - i) n_2} \approx n_2^{1/2} \int_0^{n_1} \sqrt{x(n - x)} \, dx = O(n_1^2 n_2^{1/2}).$$

The update procedure, starting on line (3c) consists of subiterating over a set of size  $n_1 - i$ , and for each subiteration, there is a search over a set of size  $n_2$ . The total cost of updating is therefore

$$O(\sum_{i=0}^{n_1} \sum_{i=0}^{i} \sqrt{n_2}) = O(n_1^2 n_2^{1/2})$$

which is the same as the search cost. The total cost of this algorithm prior to computing the minimum weight paths for all of  $V_2$  is therefore  $O(n_1^2 n_2^{1/2})$ , with a memory cost of  $O(n_1)$ . The final stage of the algorithm costs  $n_2$  iterations of minimum finding over a set of size  $n_1$ , so the (quantum) work for filling in the cost function for  $V_2$  is  $O(n_2 n_1^{1/2})$ . The total cost of the whole algorithm is therefore  $O(n_1^2 n_2^{1/2} + n_2 n_1^{1/2})$ .

How does this compare to the best classical cost, which is  $O(n_1n_2)$  with a memory cost of  $O(n_1n_2)$ , as well? Suppose that only the minimum weight paths to the other elements of  $V_1$  are desired. Then if  $O(n_1^2n_2^{1/2}) < O(n_1n_2)$ , then the quantum algorithm wins. This occurs if  $n_1 = o(\sqrt{n_2})$ . Thus for highly unbalanced complete bipartite graphs, the quantum algorithm outperforms the classical algorithm.

# Conclusion

The quantum versions of Dijkstra's algorithm in section 1 and the quantum version of Prim's algorithms in section 3 are for complete graphs with a well defined

weight function on all the edges. The improvements over the corresponding classical algorithms are also for complete graphs. If a graph is not complete, then the classical minimal weight path algorithm and the classical minimal weight spanning tree algorithms can effectively use the graph structure to give better classical algorithms. Essentially this is done through updating only the vertices that are adjacent to the vertex being added to the core set. Classically, the use of priority queues lead to these improvements. It seems likely that these ideas can be incorporated into future quantum algorithms for incomplete graphs, but this is a topic for a future paper. One particular case of this for complete bipartite graphs was dealt with in section 4.

There are a number of closely related problems to finding minimal weight paths in graphs. Among these are the problems of deciding graph connectivity and finding the shortest path between two points. Determining the number of blocks in a graph follows directly from being able to decide graph connectivity as does determining the set of articulation points in a graph. Presumably the quantum algorithms given here can be readily extended to these problems, although the details are yet to be worked out.

The whole field of quantum algorithmic graph theory has barely been touched on here. However, the algorithms developed in this note should be regarded as strong evidence that many well known classical graph theory algorithms have interesting quantum analogues. Often they will involve some nontrivial modifications of the classical algorithm to make optimal use of the few tools currently available in the quantum toolbox. The optimistic view is that although the modifications to the classical algorithms may be nontrivial, they often are not excessively complicated either, as in the case of the quantum versions of Dijkstra's algorithm and Prim's algorithm.

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ADVANCED RESEARCH AND DEVELOPMENT ACTIVITY, SUITE 6644, NATIONAL SECURITY AGENCY, 9800 SAVAGE ROAD, FORT MEADE, MARYLAND 20755

 $E ext{-}mail\ address: miheili@nsa.gov}$