Finding Hadamard Matrices by a Quantum Annealing Machine

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Finding a Hadamard matrix (H-matrix) among the set of all binary matrices of corresponding order is a hard problem, which potentially can be solved by quantum computing. We propose a method to formulate the Hamiltonian of finding H-matrix problem and address its implementation limitation on existing quantum annealing machine (QAM) that allows up to quadratic terms, whereas the problem naturally introduces higher order ones. For an M-order H-matrix, such a limitation increases the number of variables from M^2 to $(M^3 + M^2 - M)/2$, which makes the formulation of the Hamiltonian too exhaustive to do by hand. We use symbolic computing techniques to manage this problem. Three related cases are discussed: (1) finding N < M orthogonal binary vectors, (2) finding M-orthogonal binary vectors, which is equivalent to finding a H-matrix, and (3) finding N-deleted vectors of an M-order H-matrix. Solutions of the problems by a 2-body simulated annealing software and by an actual quantum annealing hardware are also discussed.

Keywords: quantum computing, quantum annealing, hard problem

I. INTRODUCTION

Solving a hard problem is one of the most important issues in computational science. This kind of problem is characterized by its complexity; which is the required number of computing resource for doing the computation, which grows beyond polynomial against the input's size. Researchers have put a lot of effort to solve such a problem, among others by employing quantum mechanics in the machinery of the computation process.

In a microscopic level, nature works under quantum mechanical principles which is hardly possible to be simulated by classical computing machines [1]. This phenomenon drives the progress of quantum computing, both on the theory at the beginning [2, 3] and then is followed by the implementation of the quantum computer itself [4, 5]. At present, a few kinds of early quantum computer models have been proposed and built, which mainly can be categorized into either a quantum gate model or a quantum annealing processor. Referring to these two classes, we will call a quantum computing machine either a QGM (Quantum Gate Machine) or a QAM (Quantum Annealing Machine), respectively.

In this paper, we will discuss a problem of finding a Hadamard matrix, denoted by H-SEARCH and its related problems, especially the formulation of their Hamiltonians for implementation on a QAM and experimenting with them using both of a simulator and a real quantum annealer. In [6] and [7] we have suggested that finding a Hadamard matrix (H-matrix) among the set of all possible binary matrices of corresponding order, i.e. the H-SEARCH, is a hard problem. First proposed by Sylvester [8] and then explored by Hadamard [9], an M-order H-matrix can be defined as an orthogonal binary $\{-1, +1\}$ matrix of size $M \times M$, where $M = 1, 2, 4, 8, \cdots, 4k, \cdots$

[10], [11]. The H-matrix is an important discrete structure in scientific fields and engineering applications [12], [13]. Construction of a 2^n order H-matrix, for any positive integer n, can be done easily by using Sylvester's method. Several construction methods for other values that different from $M = 2^n$ have also been proposed [14–17]. Nevertheless, there is no general method for constructing (nor finding) a 4k order H-matrix which can be applied to every positive integer k. Although no proof yet exists, it is conjectured that there is a H-matrix of order 4k for every positive integer k [18, 19].

Existing Hadamard matrix construction methods, including the Sylvester's and other's methods proposed in [14–17], can be considered as analytical methods. We have formulated a tentative method that can be categorized as a *probabilistic* one, which is based on the SA (simulated annealing) [20-22] and later on SQA (simulated quantum annealing) [23–25]. We have successfully found some low-order H-matrices that cannot trivially be constructed by the Sylvester method, either by SA [7] or the SQA [6]. However, direct implementation of the method on existing QAM is hindered by unrealizable absolute terms in the energy function (Hamiltonian). Changing the absolute terms into their equivalent square terms will generate quartic terms, whereas existing QAM only allows up to quadratic terms to be implemented. A possible solution is by transforming the energy function containing high order terms into ones with up to twobody interaction terms using Boolean reduction [26], [27]. In our case of H-SEARCH problem, however, it involves a large number of terms where the mathematical manipulation by hand is not an easy task.

In this paper, we also extend the H-SEARCH into a problem of finding a set of N < M orthogonal (orthoset) of binary vectors. Along with H-SEARCH, which is equivalent to finding M orthoset of M-order binary

vectors, we also address H-matrix completion problem of finding N-deleted vectors of a given M-order H-matrix. The large number of terms in the Hamiltonian of these problems requires both a systematic and automated solution. We propose a method to systematically perform Boolean reduction on a large number of terms and encourage the usage of symbolic computation to formulate the energy function which leads to the Hamiltonian of the problems. We present some examples of finding low-order H-matrices to enlight the proposed method. Additionally, we use D-Wave neal package to find the solutions of the formulated 2-body interacting Hamiltonian of the problems by using simulated annealing and also do the implementation on an actual quantum annealer by using the D-Wave's DW2000Q quantum processor.

The rests of the paper are organized as follows. In Section II, we briefly discuss the QAM, finding H-matrix by energy minimization, problem of transforming higher-order terms into two-terms Hamiltonian with large number of terms, and describe the proposed method. Section III presents the computation case of low order H-matrices and analysis of corresponding experimental results, both by simulation and implementation on a quantum annealer. The last Section presents discussion and concludes the paper.

II. METHODS

A. Quantum Annealing Machines

We refer a QAM or an adiabatic quantum computing machine as a configurable or a programmable quantum Ising systems \hat{H}_{pot} , whose transverse magnetic field \hat{H}_{kin} can be controlled and the state of its spins can be read individually upon completion of an adiabatic quantum evolution. The Hamiltonians of such a system; for a given spin configuration $\{\hat{\sigma}^{\alpha}_k\} \equiv \hat{\sigma}$; where $\alpha \in \{x,y,z\}$, $k \in K = \{1,2,\cdots,i,j,\cdots\}$ is the set of lattice's indices, is given by

$$\hat{H}_{pot}(\hat{\sigma}) \equiv -\sum_{i \neq i} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z - \sum_i h_i \hat{\sigma}_i^z \tag{1}$$

and,

$$\hat{H}_{kin}\left(\hat{\sigma}\right) \equiv -\Gamma \sum_{i} \hat{\sigma}_{i}^{x} \tag{2}$$

where J_{ij} is a coupling constant or interaction strength between a spin at site i with a spin at site j, h_j is magnetic strength at site j, and $\{\hat{\sigma}_i^z, \hat{\sigma}_i^x\}$ are Pauli's matrices at site-i. In the QA [23–25, 28–37], quantum fluctuation is elaborated by introducing a transverse magnetic field Γ . To solve a problem by using QAM, we have to encode the variables into spins with their corresponding Ising coefficients $\{h_i, J_{ij}\}$. Then it is executed by the following quantum adiabatic evolution

$$\hat{H}_{QA}(\hat{\sigma}, t) = \left(1 - \frac{t}{\tau}\right) \hat{H}_{kin}(\hat{\sigma}) + \frac{t}{\tau} \hat{H}_{pot}(\hat{\sigma})$$
 (3)

where $t \in [0, \tau]$. By keeping the system in an adiabatic condition during the process, the ground-state at the end of the evolution of the system will represent a solution of the problem.

We can see from Eq.(1) that the Hamiltonian includes up to quadratic terms, so that in principle it only allows encoding of quadratic (binary) problems. When the problem contains higher order terms than quadratic, we have to find a way to convert it into expressions that only include up to quadratic. Additionally, since the number of the spins/ qubits are related to the number of binary variables, it further constraints the size of the problem that can be handled and therefore limits the machine's capability.

Some efforts to implement the QAM have been initiated, among others is the construction of quantum annealer where the spin is manufactured as a superconducting quantum device called RF-SQUID (Radio Frequency-Super Conducting Quantum Interference Device) [5]. The scalability of the device makes it possible for the number of spins (qubits) grows very rapidly, whose last generation at the time of this writing achieves more than 2000. This device has been applied to solve various kinds of problem, such as, quantum factorization [38], hand written digit recognition [39], computational biology [40], and hydrologic inverse analysis [41].

B. Finding a Hadamard Matrix By Energy Minimization

Consider an $M=2, 4, 8, 12, \dots, 4k$ order binary matrix B whose elements are $b_{i,j} \in \{-1, +1\}$, with k a positive integer (we have omitted M=1 case due to its triviality). By writing the i^{th} column vector of B as

$$\vec{b}_i = (b_{0,i} \ b_{1,i} \ \cdots \ b_{M-1,i})^T$$
 (4)

where $(\cdot)^T$ denotes matrix transpose operation, we can express the matrix $B = (\vec{b}_0 \ \vec{b}_1 \cdots \ \vec{b}_{M-1})$ as

$$B = \begin{pmatrix} b_{0,0} & b_{0,1} & \cdots & b_{0,M-1} \\ b_{1,0} & b_{1,1} & \cdots & b_{1,M-1} \\ \cdots & \cdots & \cdots & \cdots \\ b_{M-1,0} & b_{M-1,1} & \cdots & b_{M-1,M-1} \end{pmatrix}$$

To indicate the orthogonality relationship among the column vectors of B, we define a matrix $D \equiv B^T B$, which explicitly can be written as

$$D = \begin{pmatrix} \langle \vec{b}_0, \vec{b}_0 \rangle & \langle \vec{b}_0, \vec{b}_1 \rangle & \cdots & \langle \vec{b}_0, \vec{b}_{M-1} \rangle \\ \langle \vec{b}_1, \vec{b}_0 \rangle & \langle \vec{b}_1, \vec{b}_1 \rangle & \cdots & \langle \vec{b}_1, \vec{b}_{M-1} \rangle \\ \cdots & \cdots & \cdots \\ \langle \vec{b}_{M-1}, \vec{b}_0 \rangle & \langle \vec{b}_{M-1}, \vec{b}_1 \rangle & \cdots & \langle \vec{b}_{M-1}, \vec{b}_{M-1} \rangle \end{pmatrix}$$

where $\langle \vec{b}_i, \vec{b}_j \rangle = \vec{b}_i^T \cdot \vec{b}_j$ is the inner product between (column) vector \vec{b}_i and \vec{b}_j . By denoting $d_{ij} \equiv \langle \vec{b}_i, \vec{b}_j \rangle$ and

knowing that $\langle \vec{b}_i, \vec{b}_i \rangle = M$, the indicator matrix D can be rewritten as

$$D = \begin{pmatrix} M & d_{0,1} & \cdots & d_{0,M-1} \\ d_{1,0} & M & \cdots & d_{1,M-1} \\ \cdots & \cdots & \cdots \\ d_{M-1,0} & d_{M-1,1} & \cdots & M \end{pmatrix}$$
 (5)

When all of $d_{i,j}=0$ in Eq.(5), then, by definition, B is an orthogonal matrix; which due to its elements of being $\{-1,1\}$, is also a H-matrix. Consequently, we can define the energy function as the sum of absolute values of the off-diagonal elements of D, which implies that a zero energy value corresponds to all of the column vectors being orthogonal to each other, whereas a non-zero value indicates that there is at least a pair of non-orthogonal vectors among them. Since D is a symmetric matrix, it is sufficient to consider only an upper- (or lower-) diagonal part of D, i.e., we can define the energy function for a given set of column vectors of $\{\vec{b}_i\}$ as

$$E_a\left(\left\{\vec{b}_i\right\}\right) \equiv \sum_{i < j} \left| \langle \vec{b}_i, \vec{b}_j \rangle \right| = \sum_{i < j} |d_{i,j}| \tag{6}$$

Furthermore, since we need to express the energy function as products of binary variables $b_{i,j}$'s, we have to change the absolute function into a square function. Then, Eq.(6) becomes

$$E_s\left(\left\{\vec{b}_i\right\}\right) = \sum_{i < j} \left(\langle \vec{b}_i, \vec{b}_j \rangle\right)^2$$

Considering Eq.(4), we can show that the square of the inner product between two binary vectors $d_{i,j}^2 = \langle \vec{b}_i, \vec{b}_j \rangle^2$ are given by

$$d_{i,j}^2 = (b_{0,i}b_{0,j} + b_{1,i}b_{1,j}\cdots + b_{M-1,i}b_{M-1,j})^2$$

Expansion of the square terms yields the following expression

$$\begin{split} d_{i,j}^2 &= b_{0,i}^2 b_{0,j}^2 + b_{1,i}^2 b_{1,j}^2 + \dots + b_{M-1,i}^2 b_{M-1,j}^2 \\ &+ 2 b_{0,i} b_{0,j} b_{1,i} b_{1,j} + 2 b_{0,i} b_{0,j} b_{2,i} b_{2,j} + \dots + 2 b_{0,i} b_{0,j} b_{M-1,i} b_{M-1,j} \\ &+ 2 b_{1,i} b_{1,j} b_{2,i} b_{2,j} + 2 b_{1,i} b_{1,j} b_{3,i} b_{3,j} + \dots + b_{1,i} b_{1,j} b_{M-1,i} b_{M-1,j} \\ &+ \dots \\ &+ 2 b_{M-2,i} b_{M-2,j} b_{M-1,i} b_{M-1,j} \end{split}$$

Since $b_{i,j} \in \{-1, +1\}$, then $b_{i,j}^2 = 1$. Therefore, we can simplify $d_{i,j}^2$ into

$$d_{i,j}^2 = M + 2 \sum_{m < n < M} b_{m,i} b_{m,j} b_{n,i} b_{n,j}$$
 (7)

Finally, the energy function related to orthogonality condition of all pairs of the column vectors in B can be expressed as

$$E_s\left(\{\vec{b}_i\}\right) = \sum_{i < j} d_{i,j}^2 = \frac{M^2(M-1)}{2} + 2\sum_{m < n < M, i < j < M} b_{m,i} b_{m,j} b_{n,i} b_{n,j}$$
(8)

In our previous papers [6, 7], we have employed energy function that is similar to Eq.(6). For implementation in a QAM, we need a modified form of Eq.(8). First, we introduce a spin variable $s_i = \{-1, +1\}$ and a (Boolean) binary variable $q_i = \{0, 1\}$. They are related by the following transforms

$$s_i = \frac{1}{2} (1 - q_i) \tag{9}$$

$$q_i = (1 - 2s_i) (10)$$

Considering that the elements of a H-matrix are $\{-1,1\}$, it is natural to formulate the energy function of H-SEARCH in the s-domain. Therefore, first we will express the energy function in this domain. We also reassign the index of the variables from the row-column format to a single contiguous indices ranging from 0 to $(M-1)^2$, i.e., we prefer to use a single indexed variable s_i rather than the previously double indexed $b_{i,j}$. The notation of its related energy function is changed by $E_s\left(\{\vec{b}_i\}\right) \to E_k\left(\{s_i\}\right) \equiv E_k(s_i)$. Accordingly, Eq.(8) is changed into

$$E_k(s_i) = \frac{M^2(M-1)}{2} + 2 \sum_{i < j < m < n < (M-1)^2} s_i s_j s_m s_n$$
(11)

Considering the implementation in a QAM, we further need to transform the k-body energy function of Eq.(11) to a 2-body energy function, which normally is formulated in the q-domain. Following the formulation described in [26, 27], a k-body interaction can be converted into a 2-body interaction by substitution and an additional compensation term as follows

$$q_i q_j \leftarrow q_k + H_{\wedge}(q_i, q_j, q_k; \delta_{i,j}) \tag{12}$$

where the compensation term is given by

$$H_{\wedge}(q_i, q_j, q_k; \delta_{i,j}) = \delta_{i,j} \left(3q_k + q_i q_j - 2q_i q_k - 2q_j q_k \right)$$
(13)

According to [27], the value of $\delta_{i,j}$ should be chosen to be larger than the maximum value of its substituted function of energy, which in our case is d_{ij}^2 . The substitution variable is also called an ancillary variable or simply

called *ancilla*, whereas the original one will be refered to as *main variable*.

The input of a QAM or its simulator needs parameters (Ising coefficients) in s-domain, as indicated by Eq.(1), Eq.(2), and Eq.(3). Therefore, from a general k-body interaction in s-domain energy function $E_k(s_i)$, we will transform it into $E_2(s_i)$ and eventually to its Hamiltonian $\hat{H}_2\left(\hat{\sigma}_i^z\right)$ by using steps given by the following Hamiltonian's construction diagram

$$E_k(s_i) \to E_k(q_i) \to E_2(q_i) \to E_2(s_i) \to \hat{H}_2(\hat{\sigma}_i^z)$$
 (14)

In the following discussions, we will describe each of these transforms in the diagram and present examples to clarify the construction process.

First note that according to the transform given by Eq.(9), the q-transformed energy from Eq.(11) into $E_k(q_i)$ will contains quartic terms $q_iq_jq_mq_n$. We observed that each term in q_iq_j actually comes from a product of two column vectors $\sum_{r < M} q_{r,i}q_{r,j}$ (and so is q_mq_n). Therefore, it will be more convenient to arrange the substitution of $q_{r,i}q_{r,j} \leftarrow q_{r,t}$ (and so is q_mq_n) column-wise. Then, we can make the arrangement of variables of the H-matrix and related ancillas as shown by the following table

main variables					a	ncill	as		
	q_0	q_M	q_{2M}	• • •	$q_{(M-1)M}$	q_{M^2}	q_{M^2+M}	• • •	$q_{M^2+M^2(M-1)/2-M+1}$
	q_1	q_{M+1}	q_{2M+1}	• • •	$q_{(M-1)M+1}$	$ q_{M^2+1} $	q_{M^2+M+1}	• • •	$q_{M^2+M^2(M-1)/2-M+2}$
	• • •	• • •	• • •		• • •				• • •
	q_{M-1}	q_{2M-1}	q_{3M-1}		q_{M^2-1}	$ q_{M^2+M-1} $	q_{M^2+2M-1}		$q_{M^2+M^2(M-1)/2}$

Left part of the table shows (main) variables of the matrix elements, whereas the right parts are ancillas. Using this

arrangement, the substitution of a product of two binary variables by a single binary variable is done as follows

$$q_{0}q_{M} \leftarrow q_{M^{2}} \qquad q_{0}q_{2M} \leftarrow q_{M^{2}+M} \qquad \cdots
 q_{1}q_{M+1} \leftarrow q_{M^{2}+1} \qquad q_{1}q_{2M} \leftarrow q_{M^{2}+M+1} \qquad \cdots
 \vdots \qquad \cdots \qquad \cdots \qquad \cdots
 q_{M-1}q_{2M-1} \leftarrow q_{M^{2}+M+1} \quad q_{M-1}q_{3M-1} \leftarrow q_{M^{2}+2M-1} \qquad \cdots$$
(15)

We can adopt similar conventions for the s-domain.

The arrangement of variables is then given by the following table

main variables				ancillas				
s_0	s_M	s_{2M}		$s_{(M-1)M}$	s_{M^2}	s_{M^2+M}		$s_{M^2+M^2(M-1)/2-M+1}$
s_1	s_{M+1}	s_{2M+1}		$s_{(M-1)M+1}$	s_{M^2+1}	s_{M^2+M+1}	• • •	$SM^2+M^2(M-1)/2-M+2$
• • •	• • •			• • •		• • •	• • •	• • •
s_{M-}	$1 \ s_{2M-1}$	s_{3M-1}		s_{M^2-1}	s_{M^2+M-1}	s_{M^2+2M-1}		$SM^2 + M^2(M-1)/2$

whereas the substitution scheme of a product of two-

binary variables by a single variable will be conducted

as follows

In practice, we do not perform the transform given by Eq.(16) directly since the substitution of a k-body to a 2-body interaction is always performed in the q-domain. The substitution in s-domain follows automatically when we transform the domain from $E_2(q_i)$ into $E_2(s_i)$ by substitution of variable $q_i \leftarrow s_i$.

C. Hamiltonian Formulation: Illustration by Low Order Case

To clarify the method, we will explain the Hamiltonian formulation step-by-step for a low order case, which in this case is a H-matrix of order 2. The discussions follow the stages as illustrated by the Hamiltonian construction diagram depicted in Eq. (14).

1. Formulation of $E_k(s_i)$.

The formulation of energy function is started by an arrangement of variables, which for the finding H-matrix

of order 2 problem is given by the followings

$$\begin{pmatrix} s_0 & s_2 \\ s_1 & s_3 \end{pmatrix}$$

The energy function is defined as the total sum of square of the off-diagonal elements of D-matrix, which in this case will only consist of a single term $d_{0,1}$. By using Eq.(7) we obtain $E_k(s_i) = d_{0,1}^2 = (s_0s_2 + s_1s_3)^2$ which leads to the following

$$E_k(s_i) = \left(s_0^2 s_2^2 + s_1^2 s_3^2\right) + 2\left(s_0 s_2 s_1 s_3\right) \tag{17}$$

By substitution of $s_i^2 \leftarrow 1$ into $E_k(s_i)$, we arrive to the following form

$$E_k(s_i) = 2 + 2s_0 s_1 s_2 s_3 \tag{18}$$

The substitution $s_i^2 \leftarrow 1$ which is done in the last step simplifies greatly Eq.(17) into Eq.(18); this is one of the important steps to be highlighted in formulating the energy function.

2. Transformation $E_k(s_i) \to E_k(q_i)$.

To obtain $E_k(q_i)$, we perform $q_i \leftarrow s_i$ substitution which is defined by Eq.(9). Even for a two-term case of Eq.(18), the number of terms starts to increase significantly into 16, which is given by the following expression

$$E_k(q_i) = 4 - 4q_0 - 4q_1 - 4q_2 - 4q_3 + 8q_0q_1 + 8q_0q_2 + 8q_0q_3 + 8q_1q_2 + 8q_1q_3 + 8q_2q_3 - 16q_0q_1q_2 - 16q_0q_1q_3 - 16q_0q_2q_3 - 16q_1q_2q_3 + 32q_0q_1q_2q_3$$

$$(19)$$

By observing the terms in Eq.(19), we realize that the q-domain energy function $E_k(q_i)$ contains constants, quadratics, cubics, and a quartic terms. The cubics and quartics terms should be converted into at most quadratics terms for implementation into a QAM.

3. Transformation $E_k(q_i) \to E_2(q_i)$.

To reduce the degree of high order terms (cubics and quartics) into at most second order (quadratics), we em-

ploy the substitution by considering the following arrangement of variables as explained in the previous section

$$\begin{array}{c|c} \text{main variables} & \text{ancillas} \\ \hline q_0 & q_2 & q_4 \\ q_1 & q_3 & q_5 \\ \end{array}$$

Based on the arrangement, the substitutions to be done are $q_0q_2 \leftarrow q_4$ and $q_1q_3 \leftarrow q_5$, each of which is compensated by its corresponding H_{\wedge} . Then, based on Eq.(12) and Eq.(13), we should proceed as follows

$$q_0q_2 \leftarrow q_4 + \delta_{0,2} \left(3q_4 + q_0q_2 - 2q_0q_4 - 2q_2q_4 \right) q_1q_3 \leftarrow q_5 + \delta_{1,3} \left(3q_5 + q_1q_3 - 2q_1q_5 - 2q_3q_5 \right)$$

Since the substitution is done independently for each of the terms in $d_{i,j}^2$, the value of $\delta_{i,j}$ is determined by

maximum value of $d_{i,j}^2=M^2$. In our case, we take $\delta_{0,2}=\delta_{1,3}\equiv\delta=4M^2=16$ for all terms undergoing the substitution. The result for finding 2-order H-matrix problem is a 22-terms q-domain energy function given as follows

$$E_{2}(q_{i}) = 4 - 4q_{0} - 4q_{1} - 4q_{2} - 4q_{3} + 56q_{4} + 56q_{5} + 8q_{0}q_{1} + 16q_{0}q_{2} + 8q_{0}q_{3} - 32q_{0}q_{4} - 16q_{0}q_{5} + 8q_{1}q_{2} + 16q_{1}q_{3} - 16q_{1}q_{4} - 32q_{1}q_{5} + 8q_{2}q_{3} - 32q_{2}q_{4} - 16q_{2}q_{5} - 16q_{3}q_{4} - 32q_{3}q_{5} + 32q_{4}q_{5}$$

$$(20)$$

4. Transformation $E_2(q_i) \to E_2(s_i)$.

After obtaining the $E_2(q_i)$ expression, based on the construction diagram, now we should transform it back to s-domain to obtain $E_2(s_i)$. The result is an s-domain energy function that also consists of 22 terms given as follows

$$E_{2}(s_{i}) = 28 + 6s_{0} + 6s_{1} + 6s_{2} + 6s_{3} - 12s_{4} - 12s_{5} + 2s_{0}s_{1} + 4s_{0}s_{2} + 2s_{0}s_{3} - 8s_{0}s_{4} - 4s_{0}s_{5} + 2s_{1}s_{2} + 4s_{1}s_{3} - 4s_{1}s_{4} - 8s_{1}s_{5} + 2s_{2}s_{3} - 8s_{2}s_{4} - 4s_{2}s_{5} - 4s_{3}s_{4} - 8s_{3}s_{5} + 8s_{4}s_{5}$$

$$(21)$$

5. Formulation of 2-Body Hamiltonian: $E_2(s_i) \to \hat{H}_2(\hat{\sigma}_i^z)$.

 (s_i) is

The formulation of Hamiltonian for a given $E_2(s_i)$ is done by substitution of $s_i \leftarrow \hat{\sigma}_i^z$. Based on Eq.(21), we

arrive to the following Hamiltonian of a 2-body interaction for H-SEARCH problem of order 2,

$$\hat{H}_{2}\left(\hat{\sigma}_{i}^{z}\right) = 28 + 6\hat{\sigma}_{0}^{z} + 6\hat{\sigma}_{1}^{z} + 6\hat{\sigma}_{2}^{z} + 6\hat{\sigma}_{3}^{z} - 12\hat{\sigma}_{4}^{z} - 12\hat{\sigma}_{5}^{z} + 2\hat{\sigma}_{0}^{z}\hat{\sigma}_{1}^{z} + 2\hat{\sigma}_{0}^{z}\hat{\sigma}_{2}^{z} + 2\hat{\sigma}_{0}^{z}\hat{\sigma}_{3}^{z} - 8\hat{\sigma}_{0}^{z}\hat{\sigma}_{4}^{z} - 4\hat{\sigma}_{0}^{z}\hat{\sigma}_{5}^{z} + 2\hat{\sigma}_{0}^{z}\hat{\sigma}_{5}^{z} + 2\hat{\sigma}_{0}^{z}\hat{\sigma}_{3}^{z} + 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{3}^{z} - 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{3}^{z} - 8\hat{\sigma}_{1}^{z}\hat{\sigma}_{5}^{z} + 2\hat{\sigma}_{2}^{z}\hat{\sigma}_{3}^{z} - 8\hat{\sigma}_{2}^{z}\hat{\sigma}_{4}^{z} - 4\hat{\sigma}_{2}^{z}\hat{\sigma}_{5}^{z} - 4\hat{\sigma}_{3}^{z}\hat{\sigma}_{4}^{z} - 8\hat{\sigma}_{3}^{z}\hat{\sigma}_{5}^{z} + 8\hat{\sigma}_{4}^{z}\hat{\sigma}_{5}^{z}$$

$$(22)$$

At this point, we can see that for implementation on a QAM, a simple two-terms s-domain energy function with only one quartic terms given by Eq.(11) transform into a 22-terms Hamiltonian given by Eq.(22). Computation by hand for a higher order H-matrix problem surely will be not an easy task. This issue will be addressed in the following section.

D. Higher Order Case: The Needs of Symbolic Computing

The method to formulate Hamiltonian of finding 2order H-matrix that has been described previously can be generalized to higher orders. It will be realized immediately that the problem start to occur due to the increasing number of variables and terms. An M order H-SEARCH needs M^2 number of binary variables to represent the matrix and an additional of $M \times M(M-1)/2$ for the ancillas, giving $M^2 + M \times M(M-1)/2$ in total. Therefore, we have increased the number of variables (complexity) from $O(M^2)$ to $O(M^3)$. In the following discussion, when an expression of energy function or a Hamiltonian includes too many terms to write, we will only display partially. The complete expressions are provided separately in Appendix section.

As an example, a problem of finding 4-order H-matrix needs 40 binary variables, which consists of 16 main variables and 24 ancillas. We can arrange the s-variables as follows

main varia				illas			
S0 S4 S8 S1 S5 S9 S2 S6 S10 S3 S7 S11	s_{12}	s_{16}	s_{20}	s_{24}	s_{28}	s_{32}	s_{36}
$s_1 \ s_5 \ s_9$	s_{13}	s_{17}	s_{21}	s_{25}	s_{29}	s_{33}	s_{37}
$s_2 \ s_6 \ s_{10}$	s_{14}	s_{18}	s_{22}	s_{26}	s_{30}	s_{34}	s_{38}
$s_3 \ s_7 \ s_{11}$	s_{15}	s_{19}	s_{23}	s_{27}	s_{31}	s_{35}	s_{39}

Similarly, this problem also needs 40 number of q-variables arranged as follows,

main							
q_0 q_4	$egin{array}{cccc} q_8 & q_{12} \\ q_9 & q_{13} \\ q_{10} & q_{14} \\ q_{11} & q_{15} \\ \end{array}$	q_{16}	q_{20}	q_{24}	q_{28}	q_{32}	q_{36}
$q_1 q_5$	$q_9 q_{13}$	q_{17}	q_{21}	q_{25}	q_{29}	q_{33}	q_{37}
$q_2 q_6$	q_{10} q_{14}	q_{18}	q_{22}	q_{26}	q_{30}	q_{34}	q_{38}
q_3 q_7	q_{11} q_{15}	q_{19}	q_{23}	q_{27}	q_{31}	q_{35}	q_{39}

Although the formulation of $E_k(s_i)$ can be done similarly to the 2-order case, due to a large number of variables and terms, it will be better to automatize this process in a computer, i.e., we employ symbolic computing software to derive the energy function. We have formulated **Algorithm 1** to calculate $E_k(s_i)$.

Algorithm 1 Construction of $E_k(s_i)$ by symbolic computation

- 1: Construct array of variables $\{s_i, s_j\}$ according to the order M of the H-matrix
- 2: Calculate inner product $d_{ij}=\langle s_i,s_j\rangle$ of symbols (variables) between every pairs of columns of the array
- 3: Calculate $E_k(s_i) = \sum d_{i,j}^2$
- 4: Cleanup s_i^2 terms by substitution: $E_k(s_i) = E_k(s_i)|_{s_i^2 \leftarrow 1}$

The **Algorithm 1** can be implemented into a programming language that has a symbolic computing capability. The energy function of finding 4-order H-matrix problem is given as follows,

$$E_k(s_i) = \sum_{i < j < M} d_{ij}^2$$

Expanding the this energy function will generate a 37-terms expression that can be written as follows,

$$E_k(s_i) = 24 + 2s_0 s_1 s_4 s_5 + \dots + 2s_0 s_1 s_{12} s_{13} + \dots + 2s_{10} s_{11} s_{14} s_{15}$$
(23)

Likewise, the transformation of $E_k(s_i) \to E_k(q_i)$ can also be done automatically by using **Algorithm 2**.

Algorithm 2 Transform $E_k(s_i) \to E_k(q_i)$

- 1: For all binary variables s_i :
- 2: $E_k(s_i)|_{s_i \leftarrow q_i}$

The processing of the 4-order case yields an energy function with 317 terms, which by setting $\delta = 4 \times H_{max} = 64$, can be expressed as follows

$$E_k(q_i) = 96 - 36q_0 + \dots - 36q_{15} + \dots + 24q_{14}q_{15} + \dots - 16q_{11}q_{14}q_{15} + \dots + 32q_{10}q_{11}q_{14}q_{15}$$
(24)

The next stage of transforming $E_k(q_i) \to E_2(q_i)$ will yield an energy function with more number of terms. The processing for such transform is described in **Algorithm 3**.

Algorithm 3 Transform $E_k(q_i) \to E_2(q_i)$

- 1: Construct a list of substitution pair $sPair[col_i, col_j, col_k]$
- $E_2(q_i) \leftarrow E_k(q_i)$
- 3: For all high order terms in $E_2(q_i)$ and based on sPair:
- 4: $E_2(q_i) \leftarrow E_2(q_i)|_{q_i \cdot q_j \leftarrow q_k} + H_{\wedge}(q_i, q_j, q_k, \delta)$
- 5: Simplify $E_2(q_i)$

In the finding 4-order H-matrix case, the two-body q-domain energy function $E_2(q)$ will consist of 389 terms, which are given as the followings,

$$E_2(q_i) = 96 - 36q_0 + \dots + 216q_{39} + 24q_0q_1 + \dots + 32q_{38}q_{39}$$
(25)

The last stage of transformation $E_2(q_i) \to E_2(s_i)$ can be

done by $q_i \leftarrow s_i$ substitution based on the **Algorithm 4**

Algorithm 4 Transform $E_2(q_i) \to E_2(s_i)$

1: For all variables $\{q_i\}$:

2: $E_2(s_i) \leftarrow E_2(q_i)|_{q_i \leftarrow \left(\frac{1-s_i}{2}\right)}$

The final form of energy function $E_2(s_i)$ also consists of 389 terms, which can be expressed as the followings,

$$E_2(s_i) = 1,248 + 66s_0 + \dots - 44s_{39} + 6s_0s_1 + \dots + 8s_{38}s_{39}$$
(26)

The Hamiltonian of the problem can be obtained directly from Eq.(26) by replacing the binary variables by

the corresponding operators $s_i \leftarrow \hat{\sigma}_i^z$. The result is as follows

$$\hat{H}_2(\hat{\sigma}_i^z) = 1,248 + 66\hat{\sigma}_0^z + \dots - 44\hat{\sigma}_{39}^z + 6\hat{\sigma}_0^z\hat{\sigma}_1^z + \dots + 8\hat{\sigma}_{38}^z\hat{\sigma}_{39}^z$$
(27)

which is the desired Hamiltonian of the 4-order H-SEARCH problem, which will become $\hat{H}_{pot}(\hat{\sigma})$ in the quantum annealing process given by Eq.(3).

E. Sub-Problem-1: Finding a Set of N < MOrthogonal Binary Vectors

In this sub-problem, we want to find N number of M-length binary vectors, where N < M. The initial values of the binary variables of the N-vectors, which generally non-orthogonal to each other, can be set to either particular values or at random; therefore, this process of obtaining N-orthogonal vectors from the given initial vectors will also be called orthogonalization. We arrange the variables similarly as before, but now with less number of variables. The number of ancillas is also reduced to $N \times N \times (N-1)/2$. We start with the following arrangement of s-variables

main variables					ancillas			
s_0	s_M	s_{2M}	• • •		s_{NM}	s_{M^2+M}	• • •	$s_{NM+NM(M-1)/2-M+1}$
s_1	s_{M+1}	s_{2M+1}	• • •	$s_{(N-1)M+1}$	s_{NM+1}	s_{M^2+M+1}	• • •	$s_{NM+MN(N-1)/2-M+2}$
• • •	• • •	• • •	• • •	• • •		• • •		•••
s_{M-1}	s_{2M-1}	s_{3M-1}	• • •	$s_{(N-1)(M-1)}$	$ s_{NM+M-1} $	s_{M^2+2M-1}	• • •	$s_{NM+MN(N-1)/2}$

For a concrete illustration, consider M=4 and N=3, i.e., finding a set of 3 binary ortho-vectors of order 4. The arrangement of variables becomes as follows

main	variables	aı	ncill	as
$s_0 s_4$	s_8	s_{12}	s_{16}	s_{20}
$s_1 \ s_5$			s_{17}	
s_2 s_6	s_{10}	s_{14}	s_{18}	s_{22}
$s_3 s_7$	s_{11}	s_{15}	s_{19}	s_{23}

Compared to finding 4-order H-matrix problem, after performing the process described by the construction diagram, we found that the number of terms in $E_k(s_i)$ has been reduced to 19, whereas there are 169 number of terms in $E_k(q_i)$, and 205 terms in each of $E_2(q_i)$ and $E_2(s_i)$. The Hamiltonian of the system with 205 terms has the following form

$$\hat{H}_2(\hat{\sigma}_i^z) = 768 + 52\hat{\sigma}_0^z + \dots - 52\hat{\sigma}_{23}^z + 4\hat{\sigma}_0^z\hat{\sigma}_1^z + \dots + 8\hat{\sigma}_{22}^z\hat{\sigma}_{23}^z$$
(28)

Some experiments to analyze a higher order case will be

discused in more detail in the next Section.

F. Sub-Problem-2: Hadamard-Matrix Completion

In the H-matrix completion problem, the task is to find N-number of missing vectors of an M-order H-matrix. This means that (M-N) number of (column) vectors are known. Construction of a H-matrix by random generation of M-order binary vector followed by orthogonality testing implies that finding the last vectors of a H-matrix become increasingly difficult, which is indicated by more number of iterations required in the later stages [42]. It can be understood considering the orthogonality of a candidate vector should be tested to previously found vectors. Interestingly, in this sub-problem, we can use the known vectors as a constraint which further reduce the number of variables and therefore the number of qubits needed in the implementation of the problem in a QAM.

Consider the problem of finding 1 missing vector in a 2-order H-matrix. When it is a seminormalized one, all elements in the first columns are 1's. Then, we have the following form of variable arrangements

ma	in variables	ancillas
1	s_0	*
1	s_1	*

Note that in this case we do not need any ancilla, so that we put "*" to all of ancilla's positions the table. The expression of $E_k(s_i)$, after substitution $s_0^2 \leftarrow 1$, becomes

$$E_k(s_i) = (s_0 + s_1)^2|_{s_i^2 \leftarrow 1} = 2 + 2s_0 s_1 = 2(1 + s_0 s_1)$$

It is easy to see that the minimum value of this energy function, which is 0, will be achieved when $s_0s_1=-1$, i.e., either $s_0=-1$ and $s_1=1$ or $s_0=1$ and $s_1=-1$, which then gives the following solutions of the H-matrices

$$\begin{pmatrix} + - \\ + + \end{pmatrix}$$
 and $\begin{pmatrix} + + \\ + - \end{pmatrix}$

Note that for conciseness, we have represented the elements by their signs, i.e, -1 is displayed as -, whereas 1 is shown as +.

Higher order cases can be treated similarly. Consider the problem of finding 2 missing vectors in a 4-order Hmatrix. Instead of writing the known vectors at the first columns, we have written them in the last ones for convenience of indexing the variables and ancillas. The arrangement will become as follows,

${\rm main}$	var	iables	ancillas
s_0 s_4	+	+	s_8
$s_1 s_5$	-	+	s_9
$s_2 s_6$	+	+	s_{10}
$s_3 s_7$	-	+	s_{11}

By following the previously explained symbolic computational procedures, we will obtain 11 terms in $E_k(s_i)$, 67 terms in $E_k(q_i)$, and 79 terms in each of $E_2(q_i)$ and $E_2(s_i)$. The Hamiltonian of the system, which also consists of 79 terms, has the following form

$$\hat{H}_2(\hat{\sigma}_i^z) = 128 + 14\hat{\sigma}_0^z + \dots - 28\hat{\sigma}_{11}^z + 2\hat{\sigma}_0^z\hat{\sigma}_1^z + \dots + 8\hat{\sigma}_{10}^z\hat{\sigma}_{11}^z$$
(29)

Higher order case will be discussed and tested in the experiment section. Considering the current number of qubits and connection, we will try to find 1-deleted vector of a 12-order H-matrix.

III. EXPERIMENTS AND ANALYSIS

Experiments have been conducted to verify the proposed method, by both of simulation and actual implementation on a quantum annealer.

In the simulation, a python-based simulated annealing package, the D-Wave's neal, has been employed to find minimum energies and related configurations that yield solutions of the problem. Input of the simulator are Ising coefficients $\{h_i, J_{ij}\}$ of the problem's Hamiltonian or energy function. These coefficients can be extracted from either $\hat{H}_2(\hat{\sigma}_i^z)$ or $E_2(s_i)$, where its constant value is omitted which translates into the shift of the ground state energy to a negative value of the corresponding constant. Then, we normalize the coefficients by dividing them by

the largest absolute values of the coefficients to simulate a real QAM input parameters.

We also have done experiments by using the D-Wave's DW2000Q quantum annealer. The "programming" of this quantum computer is performed by configuring the qubits which are connected by a Chimera graph, and assigning weight on each of the qubit and strength of the coupler that connect the qubits according to the Ising coefficients. A simple Hamiltonian can be implemented directly by manual configuration, whereas a more complex one needs an embedding tool.

A. Simulation on D-wave Neal Simulator

The input of the *neal* simulated annealing software are Ising coefficients, which after scaling will simulate the input of the D-Wave quantum annealer, except that it is not necessary to take care of the restriction of the connection among the qubits imposed by the Chimera graph.

TABLE I. Solutions of finding 2-order H-matrix problem by D-Wave's *neal*. The final configurations of main variables and ancillas along with their associated energies are shown. The ground-state column indicates whether the lowest energy has been achieved, which is marked by "Y", or has not been achieved which is marked by "N".

No	Configuration	Energy	Ground-State
1	(+, -, +, +, +, +)	-2.33	Y
2	(+,-,-,-,+,-)	-2.33	Y
3	(+, -, +, +, +, +)	-2.33	Y
4	(-, +, +, +, +, +)	-2.33	Y
5	(+, -, +, +, +, +)	-2.33	Y
6	(+,-,-,-,+,-)	-2.33	Y
7	(+,-,+,-,+,-)	-2.00	N
8	(+, +, -, +, +, +)	-2.33	Y
9	(+, -, +, -, +, -)	-2.00	N
10	(-, +, +, +, +, +)	-2.33	Y

1. Finding 2-order and 4-order H-matrix

To solve the problem of finding 2-order H-matrix, we have used the energy function given by Eq.(21), which after normalization yields the following bias values

$$h = (0.5, 0.5, 0.5, 0.5, -1.0, 1.0)^T$$

whereas the coupling coefficients between a pair of qubits are given as follows

$$J = \begin{pmatrix} * & 0.167 & 0.333 & 0.167 & -0.667 & -0.333 \\ * & * & 0.167 & 0.333 & -0.333 & -0.667 \\ * & * & * & 0.167 & -0.667 & -0.333 \\ * & * & * & * & -0.333 & -0.667 \\ * & * & * & * & * & * & 0.667 \\ * & * & * & * & * & * & * \end{pmatrix}$$

Since the diagonal entries are not used and the J matrix is symmetric, we only show the upper diagonal elements of the matrix. We have set the number of sweeps in the simulator to 1,000 and the number of configurations to 10. Table I displays the obtained configurations with their corresponding energy values after the simulation has been finished

Based on Eq.(21), we know that the value of the constant is 28.00, whereas the largest (absolute value) of coefficients is 12.00. By normalization, the constant becomes 2.33, therefore the value of the lowest energy (the ground state) is -2.33, which is in agreement with the simulation result. We observed from the results that not all of the configurations achieved ground states. In the table, configurations achieving the ground states's are marked by "Y", whereas non-ground states are marked by "N". The elements of the obtained H-matrices are given by the first 4 values of the configuration, such as $(s_0, s_1, s_2, s_3) = (+, -, +, +)$ for the first configuration, whereas the corresponding ancillas $(s_4, s_5) = (+, +)$ can be neglected. Reshaping the solutions into 2×2 matrices yields various 2×2 orthogonal matrices, displayed subsequently as follows,

$$\begin{pmatrix} + & + \\ - & + \end{pmatrix}, \begin{pmatrix} + & - \\ - & - \end{pmatrix}, \dots, \begin{pmatrix} - & + \\ + & + \end{pmatrix}$$

It is easy to verify that the matrices that correspond to the ground state energy are indeed Hadamards.

In the second example, we consider the problem of finding 4-order H-matrix. By taking $\delta = 4 \times H_{max}$, the energy function given by Eq.(26). By setting the simulation parameters as before, we obtained the following set of energies (written to the second decimal places)

$$-16.00, -15.62, -15.62, -15.71, -15.71, -15.71, -15.71, -15.71, -15.71, -15.71, -15.62$$

Our calculation shows that the ground state energy should have been -16.00, which only 2 out of 10 solutions have achieved. As an example, the first solution related to $E_2(s_i) = -16.00$ and the second one related to $E_2(s_i) = -15.62$ yields the following configurations

and

respectively. By taking the first 16 elements of the solution vectors and reshaping them into 4×4 matrices, we obtain the following results,

$$\begin{pmatrix} + & + & + & + \\ + & - & - & + \\ - & - & + & + \\ - & + & - & + \end{pmatrix}, \begin{pmatrix} - & + & + & + \\ - & - & + & - \\ - & - & - & - \\ - & + & - & + \end{pmatrix}$$

We can verify that the first solution with $E_2(s_i) = -16.00$ is actually an orthogonal matrix, whereas the second one related to $E_2(s_i) = -15.62$ is not. We also found that by increasing the number of sweeps, it is possible to obtain more correct solutions.

2. Finding a set of N-orthogonal M-order binary vectors

In this experiment, our objective is to find a set of 3-orthogonal binary vectors of length 12. The number of binary variables that are required to do this task are 72, whereas the number of $E_2(s_i)$ terms are 1,765. The Hamiltonian obtained from $E_2(s_i)$ after symbolic computation yields the following expression

$$\hat{H}_2\left(\hat{\sigma}_i^z\right) = 19,872 + 404\hat{\sigma}_0^z + \dots + 404\hat{\sigma}_{71}^z + 4\hat{\sigma}_0^z\hat{\sigma}_1^z + \dots + 8\hat{\sigma}_{70}^z\hat{\sigma}_{71}^z \tag{30}$$

Based on $E_2(s_i)$ and by using $\delta = 5 \times M^2$, the calculated ground-state energy is -49.19. Setting the number of sweep to 1000 as in the previous case did not give a correct solution, therefore, we increased the number of sweeps to 500,000 while keeping the number of configurations at 10. We obtained the energies at each of the configuration in the solutions as follows

Especially, the solution given by the ground-state with energy at -49.19 are as follows

$$v_{g,1} = (+, -, -, -, +, +, -, -, +, -, +, -)$$

$$v_{g,2} = (-, -, -, -, -, -, +, -, +, -, +, +)$$

$$v_{g,3} = (+, +, -, -, -, -, -, -, +, +, +)$$

We have verified that these three binary vectors are orthogonal to each other. On the other hand, the nonground state vectors such as the solution with energy -49.09 given by the following set of vectors,

$$v_{ng,1} = (+, -, -, +, -, +, -, -, +, +, +, +)$$

$$v_{ng,2} = (-, +, -, -, +, -, +, -, -, -, -)$$

$$v_{ng,3} = (+, +, +, +, -, -, -, +, +, +, -, -)$$
(31)

are not 3 orthogonal set of binary vectors, and therefore not a correct solution.

3. Finding a deleted vector in a 12-order H-matrix

For the completion problem, we have chosen a 12-order H-matrix as a case, whose 1 column vector has been deleted. The rests of 11 known vectors are as follows,

Since all of the elements of v_0 are 1, it is a seminormalized H-matrix. Our symbolic computation yields the number of terms in $E_k(s_i)$ is 379, $E_k(q_i)$ is 407, $E_2(q_i)$ is 407 and

 $E_2(s_i)$ is 379. The Hamiltonian obtained from $E_2(s_i)$, after symbolic computation, is as follows

$$\hat{H}_2(\hat{\sigma}_i^z) = 756 + 2\hat{\sigma}_0^z \hat{\sigma}_1^z + \dots + 2\hat{\sigma}_0^z \hat{\sigma}_{27}^z + \dots - 2\hat{\sigma}_{26}^z \hat{\sigma}_{27}^z$$
(33)

By setting the number of sweeps to 1,000 we obtained the energy equal to -66.00, which are identical for all of 10 configurations in the solution. This result shows that all of the configuration achieved lowest energy, they consist of two binary vectors as follows

$$v_{11,1} = (+, -, +, +, +, -, -, -, +, -, -, +) v_{11,2} = (-, +, -, -, -, +, +, +, -, +, +, -)$$
(34)

By inspection, we can see that $v_{11,2}=-v_{11,1}$ and therefore both of them are correct solutions that completes the 12-order H-matrix.

B. Experiments on D-Wave Quantum Annealer

We also have implemented the Hamiltonian of H-SEARCH problems (for order 2 and 4), finding a set of N < M orthogonal binary vectors of order M, and H-matrix completion problems into DW2000Q quantum annealer. The DW2000Q has up to 2048 qubits and 6016 couplers, where the qubits are connected by a C16 Chimera graph, which means that its 2048 qubits are logically map into a 16×16 matrix of unit cell, whose each cell consists of 8 qubits [43]. The layout of the cell can be represented either by a column or by a cross. In this paper, we use the cross layout to show the connection among the qubits in each of the presented problem.

The schedule of quantum annealing process in DW2000Q can be adjusted by the user. However, in the following experiments, we have used the default schedule defined in [44]; where the kinetic energy E_{kin}/h (with h is the Planck constant) has been set at around 6 GHz at the beginning; which is decreased exponentially to around zero at the end of the annealing process. Meanwhile, the potential energy E_{pot}/h is started from zero at the beginning and then increased exponentially to around 12 GHz at the end of the annealing.

1. Finding 2-order and 4-order H-matrix

The Hamiltonian of the finding 2-order H-matrix problem given by Eq.(22) indicates that 6 (logical) qubits are required. However, implementation on the Chimera graph increases the number into 17 (physical) qubits which are located in the neighbouring blocks (unit cells). We have manually designed the qubit's connection, whose configuration result is shown in Fig.1(a).

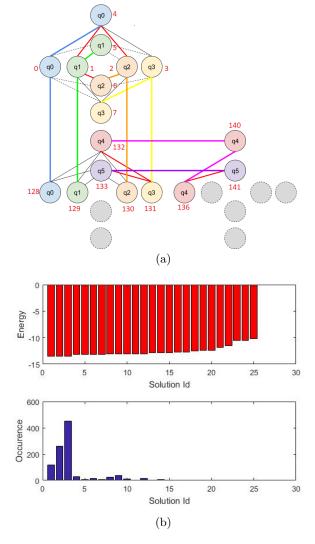


FIG. 1. Implementation of finding a 2-order H-matrix problem's Hamiltonian into a quantum annealer: (a) Embedding 6 logical qubits into 17 physical qubits in a Chimera graph's of the DW2000Q, (b) Obtained results after quantum annealing: the distribution of energy related to each solution (top) and distribution of the solutions obtained by 1000 reads (bottom).

We have used a default annealing schedule, whereas the number of reads is set to 1000. Energy distribution of the result and its related occurrence number of each solution are shown in the top and bottom parts of Fig.1(b), respectively. We obtained a minimum energy of -13.52, which corresponds to solution vector $(1,-,-,-)^T$ for the first four qubits while values of the ancilla qubits can be ignored. The solution can be rearranged into a 2×2 arrays as follows

$$\begin{pmatrix} + & - \\ - & - \end{pmatrix}$$

which actually is a 2-order H-matrix.

For the 4-order H-SEARCH problem, the Hamiltonian expressed in Eq.(27) indicates that 40 (logical) qubits are

required. This number increases when it is implemented on the set of qubits with Chimera graph connection. We have employed SAPI (Solver Application Programming Interface) embedding tool which is provided by the D-Wave to construct the connection among the qubits automatically. After optimization, the SAPI indicates that 344 (physical) qubits are required.

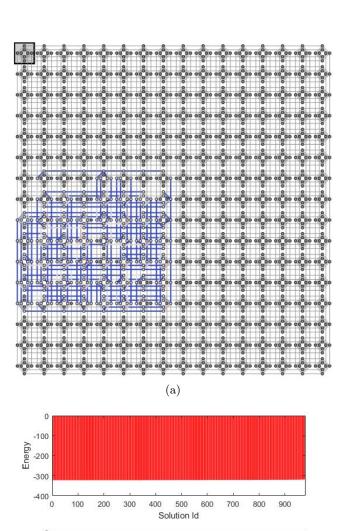


FIG. 2. Implementation of finding a 4-order H-matrix problem's Hamiltonian into a quantum annealer: (a) Embedding diagram of 40 logical qubits into 344 physical qubits in DW2000Q which is obtained by an embedding tool, (b) Results after finishing the quantum annealing evolution: the distribution of energy related to each solution (top) and distribution of the solutions obtained by 1000 reads (bottom).

(b)

Occurence

100 200 300

Sketched of the qubits connection is displayed in Fig.2(a), whereas the distribution of energy and its related population are depicted in top and bottom part of Fig.2(b) respectively. In contrast to the 2-order case, the figure shows an almost uniform distribution, except for a few number of solutions. Connection diagram displayed in Fig.2(a) shows that a 4-order H-SEARCH problem already occupied a significant number of available qubits and couplers of the DW2000Q quantum processor.

Default annealing schedule has been used and we also set the number of reads to 1000. The achieved lowest energy for the given configuration is -322.91. The corresponding solution, after neglecting the ancillas and reformatting it into a 4×4 matrix, is as follows

$$\begin{pmatrix} - & - & + & + \\ + & - & + & - \\ - & - & - & - \\ - & + & + & - \end{pmatrix}$$

We can verify that the solution is indeed a H-matrix, therefore the D-Wave has successfully found the H-matrix of order-4.

2. Finding a set of 3-orthogonal 12-order binary vectors

In this experiment, we configured the D-Wave to find a set of 3 orthogonal binary vectors of order 12. The Hamiltonian given by Eq.(30) indicates that 72 (logical) qubits is necessary. We also used SAPI embedding tool to configure the Chimera graph to obtain the qubits connection. After several steps of optimizations, the SAPI shows that 1,766 (physical) qubits are required. The sketch of configuration in the Chimera is displayed in Fig.3(a).

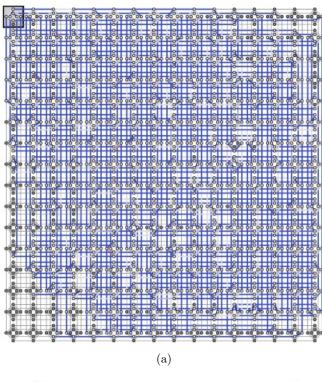
We have set the annealing schedule to the default and also set the number of reads to 1,000 as before. The the distribution of energy and population of each configurations are shown in Fig.3(b). The achieved minimum energy with this configuration is -1746.26 which is corresponding to the following vectors as the solution

$$v_0 = (+, -, +, -, -, +, +, -, +, +, -, -)$$

$$v_1 = (+, -, -, -, +, +, -, +, -, +, -)$$

$$v_2 = (+, +, -, +, +, +, +, +, +, +, -, +)$$
(35)

We can verify that these set of three binary vectors are orthogonal to each others. The distribution of the solution shown in Fig.3(b) is uniform, which means that every solution achieved minimum energy level. The connection diagram in Fig.3(a) shows that for order-12, problem of finding three orthogonal binary vectors already occupied most of the qubits and connections of the processor.



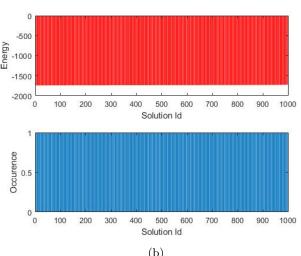


FIG. 3. Realization of finding a set of 3-orthogonal binary vectors of order (length) 12 into a quantum annealer: (a) Embedding 72 logical qubits into 1,766 physical qubits in a Chimera-connected qubits of the DW2000Q, (b) Obtained results after quantum annealing: the distribution of energy related to each solution (top) and distribution of the solutions obtained by 1000 reads (bottom).

3. Finding a deleted vector of 12-order H-matrix

In this experiment, the D-Wave is programmed to find one vector missing in an 12-order H-matrix. The known 11 vectors are identical to the simulation case given by Eq.(32). Based on the Hamiltonian given by Eq.(33), we realized that 28 logical qubits are needed. We rely on SAPI embedding module to configure the Chimera-

connection of the qubits, which shows that 50 physical qubits are required. Fig.4(a) shows the realization of qubits connection in the Chimera graph. Although the order of the matrix is sufficiently high, since the required qubits and couplers for this problem are small, it only occupies a small area in the processor.

By using the default annealing schedule with 1000 reads as before, we have obtained the minimum energy of -104.00 and the following binary vector as a solution,

$$v_{11} = (-, +, -, -, -, +, +, +, -, +, +, -)^{T}$$

which can be verified to be a correct one; i.e., along with 11 vectors in Eq.(32), this vector constructs a 12-order H-matrix. Fig.4(b) shows the distribution of energy and occurence of the solutions. We see that only two kind of solutions are exists, both of them are at the identical minimum energy level.

IV. CONCLUSIONS

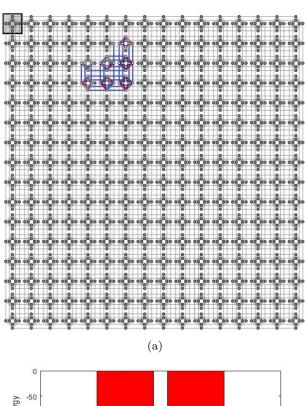
We have investigated the possibility of quantum computing to solve the problem of finding H-matrix among possible binary matrices of the same order, which is a hard problem. The QAM or quantum annealer has been considered for its realization, which requires the problem to be translated into a Hamiltonian. We have proposed a method to formulate the Hamiltonian's of finding H-matrix and its related problems.

Existing quantum annealer permits only up to quadratic terms for realization. Since the problem naturally induces higher order terms, we have to perform boolean reduction to obtain realizable Hamiltonians. Manipulation of large number of terms implied by both of growing number of variables with order and the boolean reduction procedure requires a computer-assisted process in constructing the Hamiltonians. The proposed method consists of a set of symbolic computing algorithms to formulate the energy function that lead to the Hamiltonian of the problems. The obtained Hamiltonians are then evaluated by both of simulation and implementation in a 2048 qubits DW2000Q quantum annealer.

For the H-SEARCH problem, existing quantum annealer achieved up to finding 4-order H-matrix. We also have successfully solved the problem of finding 3 orthogonal binary vectors of length 12 and the problem of finding 1 missing vector in a 12-order H-matrix. In the future, it is expected that higher order H-matrix searching problem can be solved when the device allows more than 2-body interaction or a better qubits connection beyond the Chimera graph is available.

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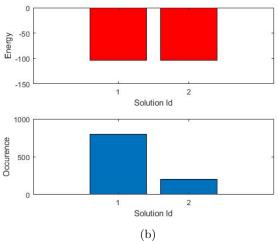


FIG. 4. Implementation of finding 1-deleted vector of 12-order H-matrix's Hamiltonian into a quantum annealer: (a) Embedding diagram of 28 logical qubits into 50 physical qubits in a Chimera-connected qubits of the DW2000Q, (b) Obtained results after quantum annealing: the distribution of energy related to each solution (top) and distribution of the solutions obtained by 1,000 reads (bottom).

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APPENDIX

A complete expression of Eq.(23)

 $\begin{array}{lll} E_k(s_i) &=& 2s_0s_1s_{12}s_{13} \,+\, 2s_0s_1s_4s_5 \,+\, 2s_0s_1s_8s_9 \,+\, \\ 2s_0s_{10}s_2s_8 \,+\, 2s_0s_{11}s_3s_8 \,+\, 2s_0s_{12}s_{14}s_2 \,+\, 2s_0s_{12}s_{15}s_3 \,+\, \\ 2s_0s_2s_4s_6 \,+\, 2s_0s_3s_4s_7 \,+\, 2s_1s_{10}s_2s_9 \,+\, 2s_1s_{11}s_3s_9 \,+\, \\ 2s_1s_{13}s_{14}s_2 \,+\, 2s_1s_{13}s_{15}s_3 \,+\, 2s_1s_2s_5s_6 \,+\, 2s_1s_3s_5s_7 \,+\, \\ 2s_{10}s_{11}s_{14}s_{15} + 2s_{10}s_{11}s_2s_3 + 2s_{10}s_{11}s_6s_7 + 2s_{10}s_{12}s_{14}s_8 +\, \\ 2s_{10}s_{13}s_{14}s_9 \,+\, 2s_{10}s_4s_6s_8 \,+\, 2s_{10}s_5s_6s_9 \,+\, 2s_{11}s_{12}s_{15}s_8 \,+\, \\ 2s_{11}s_{13}s_{15}s_9 \,+\, 2s_{11}s_4s_7s_8 \,+\, 2s_{11}s_5s_7s_9 \,+\, 2s_{12}s_{13}s_4s_5 \,+\, \\ 2s_{12}s_{13}s_8s_9 \,+\, 2s_{12}s_{14}s_4s_6 \,+\, 2s_{12}s_{15}s_4s_7 \,+\, 2s_{13}s_{14}s_5s_6 \,+\, \\ 2s_{13}s_{15}s_5s_7 \,+\, 2s_{14}s_{15}s_2s_3 \,+\, 2s_{14}s_{15}s_6s_7 \,+\, 2s_2s_3s_6s_7 \,+\, \\ 2s_4s_5s_8s_9 \,+\, 24 \end{array}$

A complete expression of Eq.(24)

 $E_k(q_i) = 32q_0q_1q_{12}q_{13} - 16q_0q_1q_{12} - 16q_0q_1q_{13} +$ $32q_0q_1q_4q_5 - 16q_0q_1q_4 - 16q_0q_1q_5 + 32q_0q_1q_8q_9 - 16q_0q_1q_8 16q_0q_1q_9 + 24q_0q_1 + 32q_0q_{10}q_2q_8 - 16q_0q_{10}q_2 - 16q_0q_{10}q_8 +$ $8q_0q_{10} + 32q_0q_{11}q_3q_8 - 16q_0q_{11}q_3 - 16q_0q_{11}q_8 + 8q_0q_{11} 16q_0q_{12}q_{13} + 32q_0q_{12}q_{14}q_2 - 16q_0q_{12}q_{14} + 32q_0q_{12}q_{15}q_3 16q_0q_{12}q_{15} - 16q_0q_{12}q_2 - 16q_0q_{12}q_3 + 24q_0q_{12} + 8q_0q_{13} 16q_0q_{14}q_2 + 8q_0q_{14} - 16q_0q_{15}q_3 + 8q_0q_{15} + 32q_0q_2q_4q_6 16q_0q_2q_4 - 16q_0q_2q_6 - 16q_0q_2q_8 + 24q_0q_2 + 32q_0q_3q_4q_7 16q_0q_3q_4 - 16q_0q_3q_7 - 16q_0q_3q_8 + 24q_0q_3 - 16q_0q_4q_5 16q_0q_4q_6 - 16q_0q_4q_7 + 24q_0q_4 + 8q_0q_5 + 8q_0q_6 + 8q_0q_7 16q_0q_8q_9 + 24q_0q_8 + 8q_0q_9 - 36q_0 + 32q_1q_{10}q_2q_9 - 16q_1q_{10}q_2 - \\$ $16q_1q_{10}q_9 + 8q_1q_{10} + 32q_1q_{11}q_3q_9 - 16q_1q_{11}q_3 - 16q_1q_{11}q_9 +$ $8q_1q_{11} - 16q_1q_{12}q_{13} + 8q_1q_{12} + 32q_1q_{13}q_{14}q_2 - 16q_1q_{13}q_{14} + \\$ $32q_1q_{13}q_{15}q_3 - 16q_1q_{13}q_{15} - 16q_1q_{13}q_2 - 16q_1q_{13}q_3 +$ $24q_1q_{13} \ - \ 16q_1q_{14}q_2 \ + \ 8q_1q_{14} \ - \ 16q_1q_{15}q_3 \ + \ 8q_1q_{15} \ +$ $32q_1q_2q_5q_6 - 16q_1q_2q_5 - 16q_1q_2q_6 - 16q_1q_2q_9 + 24q_1q_2 +$ $32q_1q_3q_5q_7 - 16q_1q_3q_5 - 16q_1q_3q_7 - 16q_1q_3q_9 + 24q_1q_3 16q_1q_4q_5 + 8q_1q_4 - 16q_1q_5q_6 - 16q_1q_5q_7 + 24q_1q_5 + 8q_1q_6 +$ $8q_1q_7 - 16q_1q_8q_9 + 8q_1q_8 + 24q_1q_9 - 36q_1 + 32q_{10}q_{11}q_{14}q_{15} 16q_{10}q_{11}q_{14} - 16q_{10}q_{11}q_{15} + 32q_{10}q_{11}q_{2}q_{3} - 16q_{10}q_{11}q_{2} 16q_{10}q_{11}q_3 + 32q_{10}q_{11}q_6q_7 - 16q_{10}q_{11}q_6 - 16q_{10}q_{11}q_7 +$ $24q_{10}q_{11} \, + \, 32q_{10}q_{12}q_{14}q_{8} \, - \, 16q_{10}q_{12}q_{14} \, - \, 16q_{10}q_{12}q_{8} \, + \,$ $8q_{10}q_{12} \ + \ 32q_{10}q_{13}q_{14}q_9 \ - \ 16q_{10}q_{13}q_{14} \ - \ 16q_{10}q_{13}q_9 \ +$ $8q_{10}q_{13} - 16q_{10}q_{14}q_{15} - 16q_{10}q_{14}q_{8} - 16q_{10}q_{14}q_{9} + 24q_{10}q_{14} +$

 $8q_{10}q_{15} - 16q_{10}q_2q_3 - 16q_{10}q_2q_8 - 16q_{10}q_2q_9 + 24q_{10}q_2 +$ $8q_{10}q_3 + 32q_{10}q_4q_6q_8 - 16q_{10}q_4q_6 - 16q_{10}q_4q_8 + 8q_{10}q_4 +$ $32q_{10}q_5q_6q_9 - 16q_{10}q_5q_6 - 16q_{10}q_5q_9 + 8q_{10}q_5 - 16q_{10}q_6q_7 - \\$ $16q_{10}q_6q_8 \, - \, 16q_{10}q_6q_9 \, + \, 24q_{10}q_6 \, + \, 8q_{10}q_7 \, + \, 24q_{10}q_8 \, + \,$ $24q_{10}q_9 - 36q_{10} + 32q_{11}q_{12}q_{15}q_8 - 16q_{11}q_{12}q_{15} - 16q_{11}q_{12}q_8 + \\$ $8q_{11}q_{12} + 32q_{11}q_{13}q_{15}q_9 - 16q_{11}q_{13}q_{15} - 16q_{11}q_{13}q_9 +$ $8q_{11}q_{13} - 16q_{11}q_{14}q_{15} + 8q_{11}q_{14} - 16q_{11}q_{15}q_8 - 16q_{11}q_{15}q_9 +$ $24q_{11}q_{15} - 16q_{11}q_{2}q_{3} + 8q_{11}q_{2} - 16q_{11}q_{3}q_{8} - 16q_{11}q_{3}q_{9} +$ $24q_{11}q_3 + 32q_{11}q_4q_7q_8 - 16q_{11}q_4q_7 - 16q_{11}q_4q_8 + 8q_{11}q_4 + \\$ $32q_{11}q_5q_7q_9 - 16q_{11}q_5q_7 - 16q_{11}q_5q_9 + 8q_{11}q_5 - 16q_{11}q_6q_7 + \\$ $8q_{11}q_6 - 16q_{11}q_7q_8 - 16q_{11}q_7q_9 + 24q_{11}q_7 + 24q_{11}q_8 +$ $24q_{11}q_9 - 36q_{11} + 32q_{12}q_{13}q_4q_5 - 16q_{12}q_{13}q_4 - 16q_{12}q_{13}q_5 + \\$ $32q_{12}q_{13}q_8q_9 - 16q_{12}q_{13}q_8 - 16q_{12}q_{13}q_9 + 24q_{12}q_{13} 16q_{12}q_{14}q_2 + 32q_{12}q_{14}q_4q_6 - 16q_{12}q_{14}q_4 - 16q_{12}q_{14}q_6 16q_{12}q_{14}q_8 + 24q_{12}q_{14} - 16q_{12}q_{15}q_3 + 32q_{12}q_{15}q_4q_7 16q_{12}q_{15}q_4 - 16q_{12}q_{15}q_7 - 16q_{12}q_{15}q_8 + 24q_{12}q_{15} + 8q_{12}q_2 + \\$ $8q_{12}q_3 - 16q_{12}q_4q_5 - 16q_{12}q_4q_6 - 16q_{12}q_4q_7 + 24q_{12}q_4 +$ $8q_{12}q_5 + 8q_{12}q_6 + 8q_{12}q_7 - 16q_{12}q_8q_9 + 24q_{12}q_8 +$ $8q_{12}q_9 - 36q_{12} - 16q_{13}q_{14}q_2 + 32q_{13}q_{14}q_5q_6 - 16q_{13}q_{14}q_5 16q_{13}q_{14}q_6 - 16q_{13}q_{14}q_9 + 24q_{13}q_{14} - 16q_{13}q_{15}q_3 +$ $32q_{13}q_{15}q_{5}q_{7} - 16q_{13}q_{15}q_{5} - 16q_{13}q_{15}q_{7} - 16q_{13}q_{15}q_{9} +$ $24q_{13}q_{15} + 8q_{13}q_2 + 8q_{13}q_3 - 16q_{13}q_4q_5 + 8q_{13}q_4 - 16q_{13}q_5q_6 16q_{13}q_{5}q_{7} + 24q_{13}q_{5} + 8q_{13}q_{6} + 8q_{13}q_{7} - 16q_{13}q_{8}q_{9} +$ $8q_{13}q_8 + 24q_{13}q_9 - 36q_{13} + 32q_{14}q_{15}q_2q_3 - 16q_{14}q_{15}q_2 16q_{14}q_{15}q_3 + 32q_{14}q_{15}q_6q_7 - 16q_{14}q_{15}q_6 - 16q_{14}q_{15}q_7 +$ $24q_{14}q_{15} - 16q_{14}q_{2}q_{3} + 24q_{14}q_{2} + 8q_{14}q_{3} - 16q_{14}q_{4}q_{6} +$ $8q_{14}q_4 - 16q_{14}q_5q_6 + 8q_{14}q_5 - 16q_{14}q_6q_7 + 24q_{14}q_6 + 8q_{14}q_7 + \\$ $8q_{14}q_8 + 8q_{14}q_9 - 36q_{14} - 16q_{15}q_2q_3 + 8q_{15}q_2 + 24q_{15}q_3 16q_{15}q_4q_7 + 8q_{15}q_4 - 16q_{15}q_5q_7 + 8q_{15}q_5 - 16q_{15}q_6q_7 +$ $8q_{15}q_6 + 24q_{15}q_7 + 8q_{15}q_8 + 8q_{15}q_9 - 36q_{15} + 32q_2q_3q_6q_7 16q_2q_3q_6 - 16q_2q_3q_7 + 24q_2q_3 - 16q_2q_4q_6 + 8q_2q_4 - 16q_2q_5q_6 +$ $8q_2q_5 - 16q_2q_6q_7 + 24q_2q_6 + 8q_2q_7 + 8q_2q_8 + 8q_2q_9 - 36q_2 16q_3q_4q_7 + 8q_3q_4 - 16q_3q_5q_7 + 8q_3q_5 - 16q_3q_6q_7 + 8q_3q_6 +$ $24q_3q_7 + 8q_3q_8 + 8q_3q_9 - 36q_3 + 32q_4q_5q_8q_9 - 16q_4q_5q_8 16q_4q_5q_9 + 24q_4q_5 - 16q_4q_6q_8 + 24q_4q_6 - 16q_4q_7q_8 + 24q_4q_7 16q_4q_8q_9 + 24q_4q_8 + 8q_4q_9 - 36q_4 - 16q_5q_6q_9 + 24q_5q_6 16q_5q_7q_9 + 24q_5q_7 - 16q_5q_8q_9 + 8q_5q_8 + 24q_5q_9 - 36q_5 +$ $24q_6q_7 + 8q_6q_8 + 8q_6q_9 - 36q_6 + 8q_7q_8 + 8q_7q_9 - 36q_7 +$ $24q_8q_9 - 36q_8 - 36q_9 + 96$

A complete expression of Eq.(25)

$$\begin{split} E_2(q_i) &= 24q_0q_1 + 8q_0q_{10} + 8q_0q_{11} + 64q_0q_{12} + 8q_0q_{13} + \\ 8q_0q_{14} + 8q_0q_{15} - 128q_0q_{16} - 16q_0q_{17} - 16q_0q_{18} - 16q_0q_{19} + \\ 24q_0q_2 - 128q_0q_{20} - 16q_0q_{21} - 16q_0q_{22} - 16q_0q_{23} - 128q_0q_{24} - \\ 16q_0q_{25} - 16q_0q_{26} - 16q_0q_{27} + 24q_0q_3 + 64q_0q_4 + 8q_0q_5 + \\ 8q_0q_6 + 8q_0q_7 + 64q_0q_8 + 8q_0q_9 - 36q_0 + 8q_1q_{10} + 8q_1q_{11} + \\ 8q_1q_{12} + 64q_1q_{13} + 8q_1q_{14} + 8q_1q_{15} - 16q_1q_{16} - 128q_1q_{17} - \\ 16q_1q_{18} - 16q_1q_{19} + 24q_1q_2 - 16q_1q_{20} - 128q_1q_{21} - 16q_1q_{22} - \\ 16q_1q_{23} - 16q_1q_{24} - 128q_1q_{25} - 16q_1q_{26} - 16q_1q_{27} + 24q_1q_3 + \\ 8q_1q_4 + 64q_1q_5 + 8q_1q_6 + 8q_1q_7 + 8q_1q_8 + 64q_1q_9 - 36q_1 + \\ 24q_{10}q_{11} + 8q_{10}q_{12} + 8q_{10}q_{13} + 64q_{10}q_{14} + 8q_{10}q_{15} + 64q_{10}q_2 - \\ 16q_{10}q_{20} - 16q_{10}q_{21} - 128q_{10}q_{22} - 16q_{10}q_{23} - 16q_{10}q_{28} - \\ 16q_{10}q_{29} + 8q_{10}q_3 - 128q_{10}q_{30} - 16q_{10}q_{31} - 16q_{10}q_{36} - \\ 16q_{10}q_{37} - 128q_{10}q_{38} - 16q_{10}q_{39} + 8q_{10}q_4 + 8q_{10}q_5 + \\ 64q_{10}q_6 + 8q_{10}q_7 + 24q_{10}q_8 + 24q_{10}q_9 - 36q_{10} + 8q_{11}q_{12} + \\ 8q_{11}q_{13} + 8q_{11}q_{14} + 64q_{11}q_{15} + 8q_{11}q_2 - 16q_{11}q_{20} - 16q_{11}q_{21} - \\ \end{array}$$

$16q_{11}q_{22} - 128q_{11}q_{23} - 16q_{11}q_{28} - 16q_{11}q_{29} + 64q_{11}q_3 16q_{11}q_{30} - 128q_{11}q_{31} - 16q_{11}q_{36} - 16q_{11}q_{37} - 16q_{11}q_{38} 128q_{11}q_{39} + 8q_{11}q_4 + 8q_{11}q_5 + 8q_{11}q_6 + 64q_{11}q_7 + 24q_{11}q_8 +$ $24q_{11}q_9 - 36q_{11} + 24q_{12}q_{13} + 24q_{12}q_{14} + 24q_{12}q_{15} + 8q_{12}q_2 - \\$ $128q_{12}q_{24} - 16q_{12}q_{25} - 16q_{12}q_{26} - 16q_{12}q_{27} + 8q_{12}q_{3} 128q_{12}q_{32} - 16q_{12}q_{33} - 16q_{12}q_{34} - 16q_{12}q_{35} - 128q_{12}q_{36} 16q_{12}q_{37} - 16q_{12}q_{38} - 16q_{12}q_{39} + 64q_{12}q_4 + 8q_{12}q_5 + 8q_{12}q_6 +$ $8q_{12}q_7 + 64q_{12}q_8 + 8q_{12}q_9 - 36q_{12} + 24q_{13}q_{14} + 24q_{13}q_{15} +$ $8q_{13}q_2 - 16q_{13}q_{24} - 128q_{13}q_{25} - 16q_{13}q_{26} - 16q_{13}q_{27} +$ $8q_{13}q_3 - 16q_{13}q_{32} - 128q_{13}q_{33} - 16q_{13}q_{34} - 16q_{13}q_{35} 16q_{13}q_{36} - 128q_{13}q_{37} - 16q_{13}q_{38} - 16q_{13}q_{39} + 8q_{13}q_{4} +$ $64q_{13}q_5 + 8q_{13}q_6 + 8q_{13}q_7 + 8q_{13}q_8 + 64q_{13}q_9 - 36q_{13} +$ $24q_{14}q_{15} + 64q_{14}q_2 - 16q_{14}q_{24} - 16q_{14}q_{25} - 128q_{14}q_{26} 16q_{14}q_{27} + 8q_{14}q_3 - 16q_{14}q_{32} - 16q_{14}q_{33} - 128q_{14}q_{34} 16q_{14}q_{35} - 16q_{14}q_{36} - 16q_{14}q_{37} - 128q_{14}q_{38} - 16q_{14}q_{39} +$ $8q_{14}q_4 + 8q_{14}q_5 + 64q_{14}q_6 + 8q_{14}q_7 + 8q_{14}q_8 + 8q_{14}q_9 36q_{14} + 8q_{15}q_2 - 16q_{15}q_{24} - 16q_{15}q_{25} - 16q_{15}q_{26} - 128q_{15}q_{27} +$ $64q_{15}q_3 - 16q_{15}q_{32} - 16q_{15}q_{33} - 16q_{15}q_{34} - 128q_{15}q_{35} 16q_{15}q_{36} - 16q_{15}q_{37} - 16q_{15}q_{38} - 128q_{15}q_{39} + 8q_{15}q_{4} +$ $8q_{15}q_5 + 8q_{15}q_6 + 64q_{15}q_7 + 8q_{15}q_8 + 8q_{15}q_9 - 36q_{15} +$ $32q_{16}q_{17} + 32q_{16}q_{18} + 32q_{16}q_{19} - 16q_{16}q_{2} - 16q_{16}q_{3} 128q_{16}q_4 - 16q_{16}q_5 - 16q_{16}q_6 - 16q_{16}q_7 + 216q_{16} + 32q_{17}q_{18} +$ $32q_{17}q_{19} - 16q_{17}q_2 - 16q_{17}q_3 - 16q_{17}q_4 - 128q_{17}q_5 16q_{17}q_6 - 16q_{17}q_7 + 216q_{17} + 32q_{18}q_{19} - 128q_{18}q_2 - 16q_{18}q_3 16q_{18}q_4 - 16q_{18}q_5 - 128q_{18}q_6 - 16q_{18}q_7 + 216q_{18} - 16q_{19}q_2 128q_{19}q_3 - 16q_{19}q_4 - 16q_{19}q_5 - 16q_{19}q_6 - 128q_{19}q_7 +$ $216q_{19} - 16q_2q_{20} - 16q_2q_{21} - 128q_2q_{22} - 16q_2q_{23} - 16q_2q_{24} - \\$ $16q_2q_{25} - 128q_2q_{26} - 16q_2q_{27} + 24q_2q_3 + 8q_2q_4 + 8q_2q_5 +$ $64q_2q_6 + 8q_2q_7 + 8q_2q_8 + 8q_2q_9 - 36q_2 + 32q_{20}q_{21} +$ $32q_{20}q_{22} + 32q_{20}q_{23} - 16q_{20}q_3 - 128q_{20}q_8 - 16q_{20}q_9 +$ $216q_{20} + 32q_{21}q_{22} + 32q_{21}q_{23} - 16q_{21}q_3 - 16q_{21}q_8 128q_{21}q_9 + 216q_{21} + 32q_{22}q_{23} - 16q_{22}q_3 - 16q_{22}q_8 16q_{22}q_9 + 216q_{22} - 128q_{23}q_3 - 16q_{23}q_8 - 16q_{23}q_9 + 216q_{23} +$ $32q_{24}q_{25} + 32q_{24}q_{26} + 32q_{24}q_{27} - 16q_{24}q_3 + 216q_{24} +$ $32q_{25}q_{26} + 32q_{25}q_{27} - 16q_{25}q_3 + 216q_{25} + 32q_{26}q_{27} 16q_{26}q_3 + 216q_{26} - 128q_{27}q_3 + 216q_{27} + 32q_{28}q_{29} + 32q_{28}q_{30} +$ $32q_{28}q_{31} - 128q_{28}q_4 - 16q_{28}q_5 - 16q_{28}q_6 - 16q_{28}q_7 128q_{28}q_8 - 16q_{28}q_9 + 216q_{28} + 32q_{29}q_{30} + 32q_{29}q_{31} 16q_{29}q_4 - 128q_{29}q_5 - 16q_{29}q_6 - 16q_{29}q_7 - 16q_{29}q_8 128q_{29}q_9 + 216q_{29} + 8q_3q_4 + 8q_3q_5 + 8q_3q_6 + 64q_3q_7 +$ $8q_3q_8 + 8q_3q_9 - 36q_3 + 32q_{30}q_{31} - 16q_{30}q_4 - 16q_{30}q_5 128q_{30}q_6 - 16q_{30}q_7 - 16q_{30}q_8 - 16q_{30}q_9 + 216q_{30} - 16q_{31}q_4 16q_{31}q_5 - 16q_{31}q_6 - 128q_{31}q_7 - 16q_{31}q_8 - 16q_{31}q_9 + 216q_{31} +$ $32q_{32}q_{33} + 32q_{32}q_{34} + 32q_{32}q_{35} - 128q_{32}q_4 - 16q_{32}q_5 16q_{32}q_6 - 16q_{32}q_7 + 216q_{32} + 32q_{33}q_{34} + 32q_{33}q_{35} - 16q_{33}q_4 128q_{33}q_5 - 16q_{33}q_6 - 16q_{33}q_7 + 216q_{33} + 32q_{34}q_{35} - 16q_{34}q_4 16q_{34}q_5 - 128q_{34}q_6 - 16q_{34}q_7 + 216q_{34} - 16q_{35}q_4 - 16q_{35}q_5 16q_{35}q_6 - 128q_{35}q_7 + 216q_{35} + 32q_{36}q_{37} + 32q_{36}q_{38} +$ $32q_{36}q_{39} - 128q_{36}q_8 - 16q_{36}q_9 + 216q_{36} + 32q_{37}q_{38} +$ $32q_{37}q_{39} - 16q_{37}q_8 - 128q_{37}q_9 + 216q_{37} + 32q_{38}q_{39} 16q_{38}q_8 - 16q_{38}q_9 + 216q_{38} - 16q_{39}q_8 - 16q_{39}q_9 + 216q_{39} + \\$ $24q_4q_5 + 24q_4q_6 + 24q_4q_7 + 64q_4q_8 + 8q_4q_9 - 36q_4 + 24q_5q_6 +$ $24q_5q_7 + 8q_5q_8 + 64q_5q_9 - 36q_5 + 24q_6q_7 + 8q_6q_8 + 8q_6q_9 36q_6 + 8q_7q_8 + 8q_7q_9 - 36q_7 + 24q_8q_9 - 36q_8 - 36q_9 + 96$

A complete expression of Eq.(26)

 $E_2(s_i) = 6s_0s_1 + 2s_0s_{10} + 2s_0s_{11} + 16s_0s_{12} + 2s_0s_{13} +$ $2s_0s_{14} + 2s_0s_{15} - 32s_0s_{16} - 4s_0s_{17} - 4s_0s_{18} - 4s_0s_{19} +$ $6s_0s_2 - 32s_0s_{20} - 4s_0s_{21} - 4s_0s_{22} - 4s_0s_{23} - 32s_0s_{24} 4s_0s_{25} - 4s_0s_{26} - 4s_0s_{27} + 6s_0s_3 + 16s_0s_4 + 2s_0s_5 + 2s_0s_6 +$ $2s_0s_7 + 16s_0s_8 + 2s_0s_9 + 66s_0 + 2s_1s_{10} + 2s_1s_{11} + 2s_1s_{12} +$ $16s_1s_{13} + 2s_1s_{14} + 2s_1s_{15} - 4s_1s_{16} - 32s_1s_{17} - 4s_1s_{18} 4s_1s_{19} + 6s_1s_2 - 4s_1s_{20} - 32s_1s_{21} - 4s_1s_{22} - 4s_1s_{23} 4s_1s_{24} - 32s_1s_{25} - 4s_1s_{26} - 4s_1s_{27} + 6s_1s_3 + 2s_1s_4 +$ $16s_1s_5 + 2s_1s_6 + 2s_1s_7 + 2s_1s_8 + 16s_1s_9 + 66s_1 + 6s_{10}s_{11} + \\$ $2s_{10}s_{12} + 2s_{10}s_{13} + 16s_{10}s_{14} + 2s_{10}s_{15} + 16s_{10}s_2 - 4s_{10}s_{20} 4s_{10}s_{21} - 32s_{10}s_{22} - 4s_{10}s_{23} - 4s_{10}s_{28} - 4s_{10}s_{29} + 2s_{10}s_{3} 32s_{10}s_{30} - 4s_{10}s_{31} - 4s_{10}s_{36} - 4s_{10}s_{37} - 32s_{10}s_{38} - 4s_{10}s_{39} +$ $2s_{10}s_4 + 2s_{10}s_5 + 16s_{10}s_6 + 2s_{10}s_7 + 6s_{10}s_8 + 6s_{10}s_9 +$ $66s_{10} + 2s_{11}s_{12} + 2s_{11}s_{13} + 2s_{11}s_{14} + 16s_{11}s_{15} + 2s_{11}s_{2} 4s_{11}s_{20} - 4s_{11}s_{21} - 4s_{11}s_{22} - 32s_{11}s_{23} - 4s_{11}s_{28} - 4s_{11}s_{29} +$ $16s_{11}s_3 - 4s_{11}s_{30} - 32s_{11}s_{31} - 4s_{11}s_{36} - 4s_{11}s_{37} - 4s_{11}s_{38} 32s_{11}s_{39} + 2s_{11}s_4 + 2s_{11}s_5 + 2s_{11}s_6 + 16s_{11}s_7 + 6s_{11}s_8 + \\$ $6s_{11}s_9 + 66s_{11} + 6s_{12}s_{13} + 6s_{12}s_{14} + 6s_{12}s_{15} + 2s_{12}s_2 32s_{12}s_{24} - 4s_{12}s_{25} - 4s_{12}s_{26} - 4s_{12}s_{27} + 2s_{12}s_{3} - 32s_{12}s_{32} 4s_{12}s_{33} - 4s_{12}s_{34} - 4s_{12}s_{35} - 32s_{12}s_{36} - 4s_{12}s_{37} - 4s_{12}s_{38} 4s_{12}s_{39} + 16s_{12}s_4 + 2s_{12}s_5 + 2s_{12}s_6 + 2s_{12}s_7 + 16s_{12}s_8 +$ $2s_{12}s_9 + 66s_{12} + 6s_{13}s_{14} + 6s_{13}s_{15} + 2s_{13}s_2 - 4s_{13}s_{24} 32s_{13}s_{25} - 4s_{13}s_{26} - 4s_{13}s_{27} + 2s_{13}s_3 - 4s_{13}s_{32} - 32s_{13}s_{33} 4s_{13}s_{34} - 4s_{13}s_{35} - 4s_{13}s_{36} - 32s_{13}s_{37} - 4s_{13}s_{38} - 4s_{13}s_{39} + \\$ $2s_{13}s_4 + 16s_{13}s_5 + 2s_{13}s_6 + 2s_{13}s_7 + 2s_{13}s_8 + 16s_{13}s_9 +$ $66s_{13} + 6s_{14}s_{15} + 16s_{14}s_2 - 4s_{14}s_{24} - 4s_{14}s_{25} - 32s_{14}s_{26} 4s_{14}s_{27} + 2s_{14}s_3 - 4s_{14}s_{32} - 4s_{14}s_{33} - 32s_{14}s_{34} - 4s_{14}s_{35} 4s_{14}s_{36} - 4s_{14}s_{37} - 32s_{14}s_{38} - 4s_{14}s_{39} + 2s_{14}s_{4} + 2s_{14}s_{5} +$ $16s_{14}s_6 + 2s_{14}s_7 + 2s_{14}s_8 + 2s_{14}s_9 + 66s_{14} + 2s_{15}s_2 4s_{15}s_{24} - 4s_{15}s_{25} - 4s_{15}s_{26} - 32s_{15}s_{27} + 16s_{15}s_3 - 4s_{15}s_{32} 4s_{15}s_{33} - 4s_{15}s_{34} - 32s_{15}s_{35} - 4s_{15}s_{36} - 4s_{15}s_{37} - 4s_{15}s_{38} 32s_{15}s_{39} + 2s_{15}s_4 + 2s_{15}s_5 + 2s_{15}s_6 + 16s_{15}s_7 + 2s_{15}s_8 +$ $2s_{15}s_9 + 66s_{15} + 8s_{16}s_{17} + 8s_{16}s_{18} + 8s_{16}s_{19} - 4s_{16}s_2 4s_{16}s_3 - 32s_{16}s_4 - 4s_{16}s_5 - 4s_{16}s_6 - 4s_{16}s_7 - 44s_{16} +$ $8s_{17}s_{18} + 8s_{17}s_{19} - 4s_{17}s_2 - 4s_{17}s_3 - 4s_{17}s_4 - 32s_{17}s_5 4s_{17}s_6 - 4s_{17}s_7 - 44s_{17} + 8s_{18}s_{19} - 32s_{18}s_2 - 4s_{18}s_3 4s_{18}s_4 - 4s_{18}s_5 - 32s_{18}s_6 - 4s_{18}s_7 - 44s_{18} - 4s_{19}s_2 32s_{19}s_3 - 4s_{19}s_4 - 4s_{19}s_5 - 4s_{19}s_6 - 32s_{19}s_7 - 44s_{19} 4s_2s_{20} - 4s_2s_{21} - 32s_2s_{22} - 4s_2s_{23} - 4s_2s_{24} - 4s_2s_{25} 32s_2s_{26} - 4s_2s_{27} + 6s_2s_3 + 2s_2s_4 + 2s_2s_5 + 16s_2s_6 + 2s_2s_7 +$ $2s_2s_8 + 2s_2s_9 + 66s_2 + 8s_{20}s_{21} + 8s_{20}s_{22} + 8s_{20}s_{23} - 4s_{20}s_3 32s_{20}s_8 - 4s_{20}s_9 - 44s_{20} + 8s_{21}s_{22} + 8s_{21}s_{23} - 4s_{21}s_3 4s_{21}s_8 - 32s_{21}s_9 - 44s_{21} + 8s_{22}s_{23} - 4s_{22}s_3 - 4s_{22}s_8 4s_{22}s_9 - 44s_{22} - 32s_{23}s_3 - 4s_{23}s_8 - 4s_{23}s_9 - 44s_{23} +$ $8s_{24}s_{25} + 8s_{24}s_{26} + 8s_{24}s_{27} - 4s_{24}s_3 - 44s_{24} + 8s_{25}s_{26} + \\$ $8s_{25}s_{27} - 4s_{25}s_3 - 44s_{25} + 8s_{26}s_{27} - 4s_{26}s_3 - 44s_{26} 32s_{27}s_3 - 44s_{27} + 8s_{28}s_{29} + 8s_{28}s_{30} + 8s_{28}s_{31} - 32s_{28}s_4 4s_{28}s_5 - 4s_{28}s_6 - 4s_{28}s_7 - 32s_{28}s_8 - 4s_{28}s_9 - 44s_{28} +$ $8s_{29}s_{30} + 8s_{29}s_{31} - 4s_{29}s_4 - 32s_{29}s_5 - 4s_{29}s_6 - 4s_{29}s_7 4s_{29}s_8 - 32s_{29}s_9 - 44s_{29} + 2s_3s_4 + 2s_3s_5 + 2s_3s_6 + 16s_3s_7 +$ $2s_3s_8 + 2s_3s_9 + 66s_3 + 8s_{30}s_{31} - 4s_{30}s_4 - 4s_{30}s_5 - 32s_{30}s_6 4s_{30}s_7 - 4s_{30}s_8 - 4s_{30}s_9 - 44s_{30} - 4s_{31}s_4 - 4s_{31}s_5 - 4s_{31}s_6 32s_{31}s_7 - 4s_{31}s_8 - 4s_{31}s_9 - 44s_{31} + 8s_{32}s_{33} + 8s_{32}s_{34} +$ $8s_{32}s_{35} - 32s_{32}s_4 - 4s_{32}s_5 - 4s_{32}s_6 - 4s_{32}s_7 - 44s_{32} +$ $8s_{33}s_{34} + 8s_{33}s_{35} - 4s_{33}s_4 - 32s_{33}s_5 - 4s_{33}s_6 - 4s_{33}s_7 44s_{33} + 8s_{34}s_{35} - 4s_{34}s_4 - 4s_{34}s_5 - 32s_{34}s_6 - 4s_{34}s_7 -$

 $44s_{34} - 4s_{35}s_4 - 4s_{35}s_5 - 4s_{35}s_6 - 32s_{35}s_7 - 44s_{35} + \\ 8s_{36}s_{37} + 8s_{36}s_{38} + 8s_{36}s_{39} - 32s_{36}s_8 - 4s_{36}s_9 - 44s_{36} + \\ 8s_{37}s_{38} + 8s_{37}s_{39} - 4s_{37}s_8 - 32s_{37}s_9 - 44s_{37} + 8s_{38}s_{39} - \\ 4s_{38}s_8 - 4s_{38}s_9 - 44s_{38} - 4s_{39}s_8 - 4s_{39}s_9 - 44s_{39} + 6s_{4}s_5 + \\ 6s_{4}s_6 + 6s_{4}s_7 + 16s_{4}s_8 + 2s_{4}s_9 + 66s_4 + 6s_{5}s_6 + 6s_{5}s_7 + \\ 2s_{5}s_8 + 16s_{5}s_9 + 66s_5 + 6s_{6}s_7 + 2s_{6}s_8 + 2s_{6}s_9 + 66s_6 + \\ 2s_{7}s_8 + 2s_{7}s_9 + 66s_7 + 6s_{8}s_9 + 66s_8 + 66s_9 + 1,248$

A complete expression of the Eq.(28)

 $\hat{H}_2(\hat{\sigma}_i^z) = 4\hat{\sigma}_0^z\hat{\sigma}_1^z + 2\hat{\sigma}_0^z\hat{\sigma}_{10}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{11}^z - 40\hat{\sigma}_0^z\hat{\sigma}_{12}^z 4\hat{\sigma}_0^z\hat{\sigma}_{13}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{14}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{15}^z - 40\hat{\sigma}_0^z\hat{\sigma}_{16}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{17}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{18}^z 4\hat{\sigma}_{0}^{z}\hat{\sigma}_{19}^{z} + 4\hat{\sigma}_{0}^{z}\hat{\sigma}_{2}^{z} + 4\hat{\sigma}_{0}^{z}\hat{\sigma}_{3}^{z} + 20\hat{\sigma}_{0}^{z}\hat{\sigma}_{4}^{z} + 2\hat{\sigma}_{0}^{z}\hat{\sigma}_{5}^{z} + 2\hat{\sigma}_{0}^{z}\hat{\sigma}_{6}^{z} +$ $2\hat{\sigma}_0^z\hat{\sigma}_7^z + 20\hat{\sigma}_0^z\hat{\sigma}_8^z + 2\hat{\sigma}_0^z\hat{\sigma}_9^z + 52\hat{\sigma}_0^z + 2\hat{\sigma}_1^z\hat{\sigma}_{10}^z + 2\hat{\sigma}_1^z\hat{\sigma}_{11}^z 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{12}^{z}-40\hat{\sigma}_{1}^{z}\hat{\sigma}_{13}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{14}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{15}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{16}^{z}-40\hat{\sigma}_{1}^{z}\hat{\sigma}_{17}^{z} 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{18}^{z} - 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{19}^{z} + 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{2}^{z} + 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{3}^{z} + 2\hat{\sigma}_{1}^{z}\hat{\sigma}_{4}^{z} + 20\hat{\sigma}_{1}^{z}\hat{\sigma}_{5}^{z} +$ $2\hat{\sigma}_{1}^{z}\hat{\sigma}_{6}^{z} + 2\hat{\sigma}_{1}^{z}\hat{\sigma}_{7}^{z} + 2\hat{\sigma}_{1}^{z}\hat{\sigma}_{8}^{z} + 20\hat{\sigma}_{1}^{z}\hat{\sigma}_{9}^{z} + 52\hat{\sigma}_{1}^{z} + 4\hat{\sigma}_{10}^{z}\hat{\sigma}_{11}^{z} 4\hat{\sigma}_{10}^z\hat{\sigma}_{16}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{17}^z - 40\hat{\sigma}_{10}^z\hat{\sigma}_{18}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{19}^z + 20\hat{\sigma}_{10}^z\hat{\sigma}_{2}^z$ $4\hat{\sigma}_{10}^z\hat{\sigma}_{20}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{21}^z - 40\hat{\sigma}_{10}^z\hat{\sigma}_{22}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{23}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{3}^z +$ $2\hat{\sigma}_{10}^{z}\hat{\sigma}_{4}^{z}+2\hat{\sigma}_{10}^{z}\hat{\sigma}_{5}^{z}+20\hat{\sigma}_{10}^{z}\hat{\sigma}_{6}^{z}+2\hat{\sigma}_{10}^{z}\hat{\sigma}_{7}^{z}+4\hat{\sigma}_{10}^{z}\hat{\sigma}_{8}^{z}+4\hat{\sigma}_{10}^{z}\hat{\sigma}_{9}^{z}+$ $52\hat{\sigma}_{10}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{16}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{17}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{18}^z - 40\hat{\sigma}_{11}^z\hat{\sigma}_{19}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{2}^z 4\hat{\sigma}_{11}^z\hat{\sigma}_{20}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{21}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{22}^z - 40\hat{\sigma}_{11}^z\hat{\sigma}_{23}^z + 20\hat{\sigma}_{11}^z\hat{\sigma}_{3}^z +$ $2\hat{\sigma}_{11}^z\hat{\sigma}_{4}^z+2\hat{\sigma}_{11}^z\hat{\sigma}_{5}^z+2\hat{\sigma}_{11}^z\hat{\sigma}_{6}^z+20\hat{\sigma}_{11}^z\hat{\sigma}_{7}^z+4\hat{\sigma}_{11}^z\hat{\sigma}_{8}^z+4\hat{\sigma}_{11}^z\hat{\sigma}_{9}^z+$ $52\hat{\sigma}_{11}^z + 8\hat{\sigma}_{12}^z\hat{\sigma}_{13}^z + 8\hat{\sigma}_{12}^z\hat{\sigma}_{14}^z + 8\hat{\sigma}_{12}^z\hat{\sigma}_{15}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{2}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{3}^z 40\hat{\sigma}_{12}^z\hat{\sigma}_4^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_5^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_6^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_7^z - 52\hat{\sigma}_{12}^z + 8\hat{\sigma}_{13}^z\hat{\sigma}_{14}^z +$ $8\hat{\sigma}_{13}^z\hat{\sigma}_{15}^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_2^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_3^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_4^z - 40\hat{\sigma}_{13}^z\hat{\sigma}_5^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_6^z 4\hat{\sigma}_{13}^z\hat{\sigma}_7^z - 52\hat{\sigma}_{13}^z + 8\hat{\sigma}_{14}^z\hat{\sigma}_{15}^z - 40\hat{\sigma}_{14}^z\hat{\sigma}_2^z - 4\hat{\sigma}_{14}^z\hat{\sigma}_3^z - 4\hat{\sigma}_{14}^z\hat{\sigma}_4^z 4\hat{\sigma}_{14}^z\hat{\sigma}_5^z - 40\hat{\sigma}_{14}^z\hat{\sigma}_6^z - 4\hat{\sigma}_{14}^z\hat{\sigma}_7^z - 52\hat{\sigma}_{14}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_2^z - 40\hat{\sigma}_{15}^z\hat{\sigma}_3^z 4\hat{\sigma}_{15}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{6}^z - 40\hat{\sigma}_{15}^z\hat{\sigma}_{7}^z - 52\hat{\sigma}_{15}^z + 8\hat{\sigma}_{16}^z\hat{\sigma}_{17}^z +$ $8\hat{\sigma}_{16}^z\hat{\sigma}_{18}^z + 8\hat{\sigma}_{16}^z\hat{\sigma}_{19}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{2}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{3}^z - 40\hat{\sigma}_{16}^z\hat{\sigma}_{8}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{9}^z 52\hat{\sigma}_{16}^z + 8\hat{\sigma}_{17}^z\hat{\sigma}_{18}^z + 8\hat{\sigma}_{17}^z\hat{\sigma}_{19}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{2}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{3}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{8}^z$ $40\hat{\sigma}_{17}^z\hat{\sigma}_9^z - 52\hat{\sigma}_{17}^z + 8\hat{\sigma}_{18}^z\hat{\sigma}_{19}^z - 40\hat{\sigma}_{18}^z\hat{\sigma}_2^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_3^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_8^z$ $4\hat{\sigma}_{18}^z\hat{\sigma}_{9}^z - 52\hat{\sigma}_{18}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{2}^z - 40\hat{\sigma}_{19}^z\hat{\sigma}_{3}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{8}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{9}^z 52\hat{\sigma}_{19}^z+4\hat{\sigma}_2^z\hat{\sigma}_3^z+2\hat{\sigma}_2^z\hat{\sigma}_4^z+2\hat{\sigma}_2^z\hat{\sigma}_5^z+20\hat{\sigma}_2^z\hat{\sigma}_6^z+2\hat{\sigma}_2^z\hat{\sigma}_7^z+2\hat{\sigma}_2^z\hat{\sigma}_8^z+$ $2\hat{\sigma}_2^z\hat{\sigma}_9^z + 52\hat{\sigma}_2^z + 8\hat{\sigma}_{20}^z\hat{\sigma}_{21}^z + 8\hat{\sigma}_{20}^z\hat{\sigma}_{22}^z + 8\hat{\sigma}_{20}^z\hat{\sigma}_{23}^z - 40\hat{\sigma}_{20}^z\hat{\sigma}_4^z 4\hat{\sigma}_{20}^z\hat{\sigma}_5^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_6^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_7^z - 40\hat{\sigma}_{20}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_9^z - 52\hat{\sigma}_{20}^z +$ $8\hat{\sigma}_{21}^z\hat{\sigma}_{22}^z + 8\hat{\sigma}_{21}^z\hat{\sigma}_{23}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{4}^z - 40\hat{\sigma}_{21}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{7}^z 4\hat{\sigma}_{21}^z\hat{\sigma}_8^z - 40\hat{\sigma}_{21}^z\hat{\sigma}_9^z - 52\hat{\sigma}_{21}^z + 8\hat{\sigma}_{22}^z\hat{\sigma}_{23}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_4^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_5^z 40\hat{\sigma}_{22}^z\hat{\sigma}_6^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_9^z - 52\hat{\sigma}_{22}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_4^z 4\hat{\sigma}_{23}^z\hat{\sigma}_5^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_6^z - 40\hat{\sigma}_{23}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_9^z - 52\hat{\sigma}_{23}^z +$ $2\hat{\sigma}_{3}^{z}\hat{\sigma}_{4}^{z}+2\hat{\sigma}_{3}^{z}\hat{\sigma}_{5}^{z}+2\hat{\sigma}_{3}^{z}\hat{\sigma}_{6}^{z}+20\hat{\sigma}_{3}^{z}\hat{\sigma}_{7}^{z}+2\hat{\sigma}_{3}^{z}\hat{\sigma}_{8}^{z}+2\hat{\sigma}_{3}^{z}\hat{\sigma}_{9}^{z}+52\hat{\sigma}_{3}^{z}+2\hat{\sigma}_{3}^{z}\hat{\sigma}_{9}^{z}+3\hat{\sigma}_{9}^{z$ $4\hat{\sigma}_{4}^{z}\hat{\sigma}_{5}^{z}+4\hat{\sigma}_{4}^{z}\hat{\sigma}_{6}^{z}+4\hat{\sigma}_{4}^{z}\hat{\sigma}_{7}^{z}+20\hat{\sigma}_{4}^{z}\hat{\sigma}_{8}^{z}+2\hat{\sigma}_{4}^{z}\hat{\sigma}_{9}^{z}+52\hat{\sigma}_{4}^{z}+4\hat{\sigma}_{5}^{z}\hat{\sigma}_{6}^{z}+$ $4\hat{\sigma}_5^z\hat{\sigma}_7^z + 2\hat{\sigma}_5^z\hat{\sigma}_8^z + 20\hat{\sigma}_5^z\hat{\sigma}_9^z + 52\hat{\sigma}_5^z + 4\hat{\sigma}_6^z\hat{\sigma}_7^z + 2\hat{\sigma}_6^z\hat{\sigma}_8^z + 2\hat{\sigma}_6^z\hat{\sigma}_9^z +$ $52\hat{\sigma}_{6}^{z}+2\hat{\sigma}_{7}^{z}\hat{\sigma}_{8}^{z}+2\hat{\sigma}_{7}^{z}\hat{\sigma}_{9}^{z}+52\hat{\sigma}_{7}^{z}+4\hat{\sigma}_{8}^{z}\hat{\sigma}_{9}^{z}+52\hat{\sigma}_{8}^{z}+52\hat{\sigma}_{9}^{z}+768$

A complete expression of Eq.(29)

 $\begin{array}{l} \hat{H}_2\left(\hat{\sigma}_i^z\right) = 2\hat{\sigma}_0^z\hat{\sigma}_1^z - 4\hat{\sigma}_0^z\hat{\sigma}_{10}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{11}^z + 6\hat{\sigma}_0^z\hat{\sigma}_2^z + 2\hat{\sigma}_0^z\hat{\sigma}_3^z + \\ 8\hat{\sigma}_0^z\hat{\sigma}_4^z + 2\hat{\sigma}_0^z\hat{\sigma}_5^z + 2\hat{\sigma}_0^z\hat{\sigma}_6^z + 2\hat{\sigma}_0^z\hat{\sigma}_7^z - 16\hat{\sigma}_0^z\hat{\sigma}_6^z - 4\hat{\sigma}_0^z\hat{\sigma}_5^z + 14\hat{\sigma}_0^z - \\ 4\hat{\sigma}_1^z\hat{\sigma}_{10}^z - 4\hat{\sigma}_1^z\hat{\sigma}_{11}^z + 2\hat{\sigma}_1^z\hat{\sigma}_2^z + 6\hat{\sigma}_1^z\hat{\sigma}_3^z + 2\hat{\sigma}_1^z\hat{\sigma}_4^z + 8\hat{\sigma}_1^z\hat{\sigma}_5^z + 2\hat{\sigma}_1^z\hat{\sigma}_6^z + \\ 2\hat{\sigma}_1^z\hat{\sigma}_7^z - 4\hat{\sigma}_1^z\hat{\sigma}_8^z - 16\hat{\sigma}_1^z\hat{\sigma}_9^z + 14\hat{\sigma}_1^z + 8\hat{\sigma}_{10}^z\hat{\sigma}_{11}^z - 16\hat{\sigma}_{10}^z\hat{\sigma}_2^z - \\ 4\hat{\sigma}_{10}^z\hat{\sigma}_3^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_4^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_5^z - 16\hat{\sigma}_{10}^z\hat{\sigma}_6^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_7^z + 8\hat{\sigma}_{10}^z\hat{\sigma}_8^z + \\ 8\hat{\sigma}_{10}^z\hat{\sigma}_9^z - 28\hat{\sigma}_{10}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_2^z - 16\hat{\sigma}_{11}^z\hat{\sigma}_3^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_4^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_5^z - \\ 4\hat{\sigma}_{11}^z\hat{\sigma}_6^z - 16\hat{\sigma}_{11}^z\hat{\sigma}_7^z + 8\hat{\sigma}_{11}^z\hat{\sigma}_8^z + 8\hat{\sigma}_{11}^z\hat{\sigma}_9^z - 28\hat{\sigma}_{11}^z + 2\hat{\sigma}_2^z\hat{\sigma}_3^z + \\ 2\hat{\sigma}_2^z\hat{\sigma}_4^z + 2\hat{\sigma}_2^z\hat{\sigma}_5^z + 8\hat{\sigma}_2^z\hat{\sigma}_6^z + 2\hat{\sigma}_2^z\hat{\sigma}_7^z - 4\hat{\sigma}_2^z\hat{\sigma}_8^z - 4\hat{\sigma}_2^z\hat{\sigma}_9^z + 14\hat{\sigma}_2^z + \\ 2\hat{\sigma}_3^z\hat{\sigma}_4^z + 2\hat{\sigma}_3^z\hat{\sigma}_5^z + 2\hat{\sigma}_3^z\hat{\sigma}_6^z + 8\hat{\sigma}_3^z\hat{\sigma}_7^z - 4\hat{\sigma}_3^z\hat{\sigma}_8^z - 4\hat{\sigma}_3^z\hat{\sigma}_9^z + 14\hat{\sigma}_3^z + \\ 2\hat{\sigma}_3^z\hat{\sigma}_4^z + 2\hat{\sigma}_3^z\hat{\sigma}_5^z + 2\hat{\sigma}_3^z\hat{\sigma}_6^z + 8\hat{\sigma}_3^z\hat{\sigma}_7^z - 4\hat{\sigma}_3^z\hat{\sigma}_8^z - 4\hat{\sigma}_3^z\hat{\sigma}_9^z + 14\hat{\sigma}_3^z + \\ 2\hat{\sigma}_3^z\hat{\sigma}_4^z + 2\hat{\sigma}_3^z\hat{\sigma}_5^z + 2\hat{\sigma}_3^z\hat{\sigma}_6^z + 8\hat{\sigma}_3^z\hat{\sigma}_7^z - 4\hat{\sigma}_3^z\hat{\sigma}_8^z - 4\hat{\sigma}_3^z\hat{\sigma}_9^z + 14\hat{\sigma}_3^z + \\ 2\hat{\sigma}_3^z\hat{\sigma}_4^z + 2\hat{\sigma}_3^z\hat{\sigma}_5^z + 2\hat{\sigma}_3^z\hat{\sigma}_6^z + 8\hat{\sigma}_3^z\hat{\sigma}_7^z - 4\hat{\sigma}_3^z\hat{\sigma}_8^z - 4\hat{\sigma}_3^z\hat{\sigma}_9^z + 14\hat{\sigma}_3^z + \\ 2\hat{\sigma}_3^z\hat{\sigma}_4^z + 2\hat{\sigma}_3^z\hat{\sigma}_5^z + 2\hat{\sigma}_3^z\hat{\sigma}_6^z + 8\hat{\sigma}_3^z\hat{\sigma}_7^z - 4\hat{\sigma}_3^z\hat{\sigma}_8^z - 4\hat{\sigma}_3^z\hat{\sigma}_9^z + 14\hat{\sigma}_3^z + \\ 2\hat{\sigma}_3^z\hat{\sigma}_4^z + 2\hat{\sigma}_3^z\hat{\sigma}_5^z + 2\hat{\sigma}_3^z\hat{\sigma}_6^z + 8\hat{\sigma}_3^z\hat{\sigma}_7^z - 4\hat{\sigma}_3^z\hat{\sigma}_8^z - 4\hat{\sigma}_3^z\hat{\sigma}_9^z + 14\hat{\sigma}_3^z + 2\hat{\sigma}_3^z\hat{\sigma}_3^z + 2\hat{\sigma}_3^z\hat{$

 $\begin{array}{l} 2\hat{\sigma}_{z}^{2}\hat{\sigma}_{5}^{z}+6\hat{\sigma}_{4}^{z}\hat{\sigma}_{6}^{z}+2\hat{\sigma}_{4}^{z}\hat{\sigma}_{7}^{z}-16\hat{\sigma}_{4}^{z}\hat{\sigma}_{8}^{z}-4\hat{\sigma}_{4}^{z}\hat{\sigma}_{9}^{z}+14\hat{\sigma}_{4}^{z}+2\hat{\sigma}_{5}^{z}\hat{\sigma}_{6}^{z}+\\ 6\hat{\sigma}_{5}^{z}\hat{\sigma}_{7}^{z}-4\hat{\sigma}_{5}^{z}\hat{\sigma}_{8}^{z}-16\hat{\sigma}_{5}^{z}\hat{\sigma}_{9}^{z}+14\hat{\sigma}_{5}^{z}+2\hat{\sigma}_{6}^{z}\hat{\sigma}_{7}^{z}-4\hat{\sigma}_{6}^{z}\hat{\sigma}_{8}^{z}-4\hat{\sigma}_{6}^{z}\hat{\sigma}_{9}^{z}+\\ 14\hat{\sigma}_{6}^{z}-4\hat{\sigma}_{7}^{z}\hat{\sigma}_{8}^{z}-4\hat{\sigma}_{7}^{z}\hat{\sigma}_{9}^{z}+14\hat{\sigma}_{7}^{z}+8\hat{\sigma}_{8}^{z}\hat{\sigma}_{9}^{z}-28\hat{\sigma}_{8}^{z}-28\hat{\sigma}_{9}^{z}+128 \end{array}$

A complete expression of Eq.(30)

 $H_2(\hat{\sigma}_i^z) = 4\hat{\sigma}_0^z \hat{\sigma}_1^z + 4\hat{\sigma}_0^z \hat{\sigma}_{10}^z + 4\hat{\sigma}_0^z \hat{\sigma}_{11}^z + 180\hat{\sigma}_0^z \hat{\sigma}_{12}^z +$ $2\hat{\sigma}_0^z\hat{\sigma}_{13}^z+2\hat{\sigma}_0^z\hat{\sigma}_{14}^z+2\hat{\sigma}_0^z\hat{\sigma}_{15}^z+2\hat{\sigma}_0^z\hat{\sigma}_{16}^z+2\hat{\sigma}_0^z\hat{\sigma}_{17}^z+2\hat{\sigma}_0^z\hat{\sigma}_{18}^z+$ $2\hat{\sigma}_0^z\hat{\sigma}_{19}^z + 4\hat{\sigma}_0^z\hat{\sigma}_2^z + 2\hat{\sigma}_0^z\hat{\sigma}_{20}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{21}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{22}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{23}^z +$ $180\hat{\sigma}_0^z\hat{\sigma}_{24}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{26}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{28}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{29}^z +$ $4\hat{\sigma}_0^z\hat{\sigma}_3^z + 2\hat{\sigma}_0^z\hat{\sigma}_{30}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{31}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{32}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{33}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{34}^z +$ $2\hat{\sigma}_0^z\hat{\sigma}_{35}^z - 360\hat{\sigma}_0^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{37}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{38}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{39}^z + 4\hat{\sigma}_0^z\hat{\sigma}_4^z 4\hat{\sigma}_{0}^{z}\hat{\sigma}_{40}^{z} - 4\hat{\sigma}_{0}^{z}\hat{\sigma}_{41}^{z} - 4\hat{\sigma}_{0}^{z}\hat{\sigma}_{42}^{z} - 4\hat{\sigma}_{0}^{z}\hat{\sigma}_{43}^{z} - 4\hat{\sigma}_{0}^{z}\hat{\sigma}_{44}^{z} - 4\hat{\sigma}_{0}^{z}\hat{\sigma}_{45}^{z} 4\hat{\sigma}_0^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{47}^z - 360\hat{\sigma}_0^z\hat{\sigma}_{48}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{49}^z + 4\hat{\sigma}_0^z\hat{\sigma}_5^z - 4\hat{\sigma}_0^z\hat{\sigma}_{50}^z 4\hat{\sigma}_0^z\hat{\sigma}_{51}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{56}^z$ $4\hat{\sigma}_0^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_0^z\hat{\sigma}_{59}^z + 4\hat{\sigma}_0^z\hat{\sigma}_6^z + 4\hat{\sigma}_0^z\hat{\sigma}_7^z + 4\hat{\sigma}_0^z\hat{\sigma}_8^z +$ $4\hat{\sigma}_0^z\hat{\sigma}_9^z + 404\hat{\sigma}_0^z + 4\hat{\sigma}_1^z\hat{\sigma}_{10}^z + 4\hat{\sigma}_1^z\hat{\sigma}_{11}^z + 2\hat{\sigma}_1^z\hat{\sigma}_{12}^z + 180\hat{\sigma}_1^z\hat{\sigma}_{13}^z +$ $2\hat{\sigma}_{1}^{z}\hat{\sigma}_{14}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{15}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{16}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{17}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{18}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{19}^{z}+$ $4\hat{\sigma}_{1}^{z}\hat{\sigma}_{2}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{20}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{21}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{22}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{23}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{24}^{z}+$ $180\hat{\sigma}_{1}^{z}\hat{\sigma}_{25}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{26}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{27}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{28}^{z}+2\hat{\sigma}_{1}^{z}\hat{\sigma}_{29}^{z}+4\hat{\sigma}_{1}^{z}\hat{\sigma}_{3}^{z}+$ $2\hat{\sigma}_{1}^{z}\hat{\sigma}_{30}^{z} + 2\hat{\sigma}_{1}^{z}\hat{\sigma}_{31}^{z} + 2\hat{\sigma}_{1}^{z}\hat{\sigma}_{32}^{z} + 2\hat{\sigma}_{1}^{z}\hat{\sigma}_{33}^{z} + 2\hat{\sigma}_{1}^{z}\hat{\sigma}_{34}^{z} + 2\hat{\sigma}_{1}^{z}\hat{\sigma}_{35}^{z} 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{36}^{z} - 360\hat{\sigma}_{1}^{z}\hat{\sigma}_{37}^{z} - 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{38}^{z} - 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{39}^{z} + 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{4}^{z} - 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{40}^{z} 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{41}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{42}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{43}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{44}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{45}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{46}^{z} 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{47}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{48}^{z}-360\hat{\sigma}_{1}^{z}\hat{\sigma}_{49}^{z}+4\hat{\sigma}_{1}^{z}\hat{\sigma}_{5}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{50}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{51}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{50}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{51}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{50}^{z}+4\hat{\sigma}_{1}^{z}\hat{\sigma}_$ $4\hat{\sigma}_{1}^{z}\hat{\sigma}_{52}^{z} - 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{53}^{z} - 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{54}^{z} - 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{55}^{z} - 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{56}^{z} - 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{57}^{z} 4\hat{\sigma}_{1}^{z}\hat{\sigma}_{58}^{z}-4\hat{\sigma}_{1}^{z}\hat{\sigma}_{59}^{z}+4\hat{\sigma}_{1}^{z}\hat{\sigma}_{6}^{z}+4\hat{\sigma}_{1}^{z}\hat{\sigma}_{7}^{z}+4\hat{\sigma}_{1}^{z}\hat{\sigma}_{8}^{z}+4\hat{\sigma}_{1}^{z}\hat{\sigma}_{9}^{z}+$ $404\hat{\sigma}_{1}^{z}+4\hat{\sigma}_{10}^{z}\hat{\sigma}_{11}^{z}+2\hat{\sigma}_{10}^{z}\hat{\sigma}_{12}^{z}+2\hat{\sigma}_{10}^{z}\hat{\sigma}_{13}^{z}+2\hat{\sigma}_{10}^{z}\hat{\sigma}_{14}^{z}+2\hat{\sigma}_{10}^{z}\hat{\sigma}_{15}^{z}+$ $2\hat{\sigma}_{10}^z\hat{\sigma}_{16}^z+2\hat{\sigma}_{10}^z\hat{\sigma}_{17}^z+2\hat{\sigma}_{10}^z\hat{\sigma}_{18}^z+2\hat{\sigma}_{10}^z\hat{\sigma}_{19}^z+4\hat{\sigma}_{10}^z\hat{\sigma}_{2}^z+2\hat{\sigma}_{10}^z\hat{\sigma}_{20}^z+$ $2\hat{\sigma}_{10}^z\hat{\sigma}_{21}^z + 180\hat{\sigma}_{10}^z\hat{\sigma}_{22}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{23}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{24}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{25}^z +$ $2\hat{\sigma}_{10}^z\hat{\sigma}_{26}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{28}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{29}^z + 4\hat{\sigma}_{10}^z\hat{\sigma}_{3}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{30}^z +$ $2\hat{\sigma}_{10}^z\hat{\sigma}_{31}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{32}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{33}^z + 180\hat{\sigma}_{10}^z\hat{\sigma}_{34}^z + 2\hat{\sigma}_{10}^z\hat{\sigma}_{35}^z 4\hat{\sigma}_{10}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{37}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{38}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{39}^z + 4\hat{\sigma}_{10}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{40}^z 4\hat{\sigma}_{10}^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{42}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{45}^z 360\hat{\sigma}_{10}^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{48}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{49}^z + 4\hat{\sigma}_{10}^z\hat{\sigma}_{5}^z$ $4\hat{\sigma}_{10}^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{51}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{54}^z 4\hat{\sigma}_{10}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{57}^z - 360\hat{\sigma}_{10}^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_{10}^z\hat{\sigma}_{59}^z +$ $4\hat{\sigma}_{10}^z\hat{\sigma}_6^z+4\hat{\sigma}_{10}^z\hat{\sigma}_7^z+4\hat{\sigma}_{10}^z\hat{\sigma}_8^z+4\hat{\sigma}_{10}^z\hat{\sigma}_9^z+404\hat{\sigma}_{10}^z+2\hat{\sigma}_{11}^z\hat{\sigma}_{12}^z+$ $2\hat{\sigma}_{11}^z\hat{\sigma}_{13}^z\ +\ 2\hat{\sigma}_{11}^z\hat{\sigma}_{14}^z\ +\ 2\hat{\sigma}_{11}^z\hat{\sigma}_{15}^z\ +\ 2\hat{\sigma}_{11}^z\hat{\sigma}_{16}^z\ +\ 2\hat{\sigma}_{11}^z\hat{\sigma}_{17}^z\ +$ $2\hat{\sigma}_{11}^z\hat{\sigma}_{18}^z+2\hat{\sigma}_{11}^z\hat{\sigma}_{19}^z+4\hat{\sigma}_{11}^z\hat{\sigma}_{2}^z+2\hat{\sigma}_{11}^z\hat{\sigma}_{20}^z+2\hat{\sigma}_{11}^z\hat{\sigma}_{21}^z+2\hat{\sigma}_{11}^z\hat{\sigma}_{22}^z+$ $180\hat{\sigma}_{11}^z\hat{\sigma}_{23}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{24}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{26}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{27}^z +$ $2\hat{\sigma}_{11}^z\hat{\sigma}_{28}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{29}^z + 4\hat{\sigma}_{11}^z\hat{\sigma}_{3}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{30}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{31}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{32}^z +$ $2\hat{\sigma}_{11}^z\hat{\sigma}_{33}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{34}^z + 180\hat{\sigma}_{11}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{37}^z 4\hat{\sigma}_{11}^z\hat{\sigma}_{38}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{39}^z + 4\hat{\sigma}_{11}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{40}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{42}^z 4\hat{\sigma}_{11}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{46}^z - 360\hat{\sigma}_{11}^z\hat{\sigma}_{47}^z 4\hat{\sigma}_{11}^z\hat{\sigma}_{48}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{49}^z + 4\hat{\sigma}_{11}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{51}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{52}^z 4\hat{\sigma}_{11}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{11}^z\hat{\sigma}_{57}^z 4\hat{\sigma}_{11}^z\hat{\sigma}_{58}^z - 360\hat{\sigma}_{11}^z\hat{\sigma}_{59}^z + 4\hat{\sigma}_{11}^z\hat{\sigma}_{6}^z + 4\hat{\sigma}_{11}^z\hat{\sigma}_{7}^z + 4\hat{\sigma}_{11}^z\hat{\sigma}_{8}^z + 4\hat{\sigma}_{11}^z\hat{\sigma}_{9}^z +$ $404\hat{\sigma}_{11}^z + 4\hat{\sigma}_{12}^z\hat{\sigma}_{13}^z + 4\hat{\sigma}_{12}^z\hat{\sigma}_{14}^z + 4\hat{\sigma}_{12}^z\hat{\sigma}_{15}^z + 4\hat{\sigma}_{12}^z\hat{\sigma}_{16}^z + 4\hat{\sigma}_{12}^z\hat{\sigma}_{17}^z +$ $4\hat{\sigma}_{12}^z\hat{\sigma}_{18}^z+4\hat{\sigma}_{12}^z\hat{\sigma}_{19}^z+2\hat{\sigma}_{12}^z\hat{\sigma}_{2}^z+4\hat{\sigma}_{12}^z\hat{\sigma}_{20}^z+4\hat{\sigma}_{12}^z\hat{\sigma}_{21}^z+4\hat{\sigma}_{12}^z\hat{\sigma}_{22}^z+$ $4\hat{\sigma}_{12}^z\hat{\sigma}_{23}^z + 180\hat{\sigma}_{12}^z\hat{\sigma}_{24}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{26}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{27}^z +$ $2\hat{\sigma}_{12}^z\hat{\sigma}_{28}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{29}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{3}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{30}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{31}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{32}^z +$ $2\hat{\sigma}_{12}^z\hat{\sigma}_{33}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{34}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{35}^z - 360\hat{\sigma}_{12}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{37}^z 4\hat{\sigma}_{12}^z\hat{\sigma}_{38}^{z} - 4\hat{\sigma}_{12}^z\hat{\sigma}_{39}^{z} + 2\hat{\sigma}_{12}^z\hat{\sigma}_{4}^{z} - 4\hat{\sigma}_{12}^z\hat{\sigma}_{40}^{z} - 4\hat{\sigma}_{12}^z\hat{\sigma}_{41}^{z} - 4\hat{\sigma}_{12}^z\hat{\sigma}_{42}^{z} - 4$ $4\hat{\sigma}_{12}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{47}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{5}^z +$ $2\hat{\sigma}_{12}^z\hat{\sigma}_6^z - 360\hat{\sigma}_{12}^z\hat{\sigma}_{60}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{63}^z 4\hat{\sigma}_{12}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{12}^z\hat{\sigma}_{68}^z 4\hat{\sigma}_{12}^z\hat{\sigma}_{69}^z+2\hat{\sigma}_{12}^z\hat{\sigma}_{7}^z-4\hat{\sigma}_{12}^z\hat{\sigma}_{70}^z-4\hat{\sigma}_{12}^z\hat{\sigma}_{71}^z+2\hat{\sigma}_{12}^z\hat{\sigma}_{8}^z+2\hat{\sigma}_{12}^z\hat{\sigma}_{9}^z+$

 $404\hat{\sigma}_{12}^z + 4\hat{\sigma}_{13}^z\hat{\sigma}_{14}^z + 4\hat{\sigma}_{13}^z\hat{\sigma}_{15}^z + 4\hat{\sigma}_{13}^z\hat{\sigma}_{16}^z + 4\hat{\sigma}_{13}^z\hat{\sigma}_{17}^z + 4\hat{\sigma}_{13}^z\hat{\sigma}_{18}^z +$ $4\hat{\sigma}_{13}^z\hat{\sigma}_{19}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{2}^z + 4\hat{\sigma}_{13}^z\hat{\sigma}_{20}^z + 4\hat{\sigma}_{13}^z\hat{\sigma}_{21}^z + 4\hat{\sigma}_{13}^z\hat{\sigma}_{22}^z + 4\hat{\sigma}_{13}^z\hat{\sigma}_{23}^z +$ $2\hat{\sigma}_{13}^z\hat{\sigma}_{24}^z + 180\hat{\sigma}_{13}^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{26}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{28}^z +$ $2\hat{\sigma}_{13}^z\hat{\sigma}_{29}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{3}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{30}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{31}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{32}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{33}^z +$ $2\hat{\sigma}_{13}^z\hat{\sigma}_{34}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_{36}^z - 360\hat{\sigma}_{13}^z\hat{\sigma}_{37}^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_{38}^z 4\hat{\sigma}_{13}^z\hat{\sigma}_{39}^z + 2\hat{\sigma}_{13}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_{40}^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_{42}^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_{43}^z 4\hat{\sigma}_{13}^z\hat{\sigma}_{44}^z-4\hat{\sigma}_{13}^z\hat{\sigma}_{45}^z-4\hat{\sigma}_{13}^z\hat{\sigma}_{46}^z-4\hat{\sigma}_{13}^z\hat{\sigma}_{47}^z+2\hat{\sigma}_{13}^z\hat{\sigma}_{5}^z+2\hat{\sigma}_{13}^z\hat{\sigma}_{6}^z 4\hat{\sigma}_{13}^z\hat{\sigma}_{60}^z - 360\hat{\sigma}_{13}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{13}^z\hat{\sigma}_{64}^z 4\hat{\sigma}_{13}^z\hat{\sigma}_{65}^z \ - \ 4\hat{\sigma}_{13}^z\hat{\sigma}_{66}^z \ - \ 4\hat{\sigma}_{13}^z\hat{\sigma}_{67}^z \ - \ 4\hat{\sigma}_{13}^z\hat{\sigma}_{68}^z \ - \ 4\hat{\sigma}_{13}^z\hat{\sigma}_{69}^z \ +$ $2\hat{\sigma}_{13}^{z}\hat{\sigma}_{7}^{z}-4\hat{\sigma}_{13}^{z}\hat{\sigma}_{70}^{z}-4\hat{\sigma}_{13}^{z}\hat{\sigma}_{71}^{z}+2\hat{\sigma}_{13}^{z}\hat{\sigma}_{8}^{z}+2\hat{\sigma}_{13}^{z}\hat{\sigma}_{9}^{z}+404\hat{\sigma}_{13}^{z}+$ $4\hat{\sigma}_{14}^z\hat{\sigma}_{15}^z \ + \ 4\hat{\sigma}_{14}^z\hat{\sigma}_{16}^z \ + \ 4\hat{\sigma}_{14}^z\hat{\sigma}_{17}^z \ + \ 4\hat{\sigma}_{14}^z\hat{\sigma}_{18}^z \ + \ 4\hat{\sigma}_{14}^z\hat{\sigma}_{19}^z \ +$ $180\hat{\sigma}_{14}^z\hat{\sigma}_{2}^z + 4\hat{\sigma}_{14}^z\hat{\sigma}_{20}^z + 4\hat{\sigma}_{14}^z\hat{\sigma}_{21}^z + 4\hat{\sigma}_{14}^z\hat{\sigma}_{22}^z + 4\hat{\sigma}_{14}^z\hat{\sigma}_{23}^z +$ $2\hat{\sigma}_{14}^z\hat{\sigma}_{24}^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_{25}^z + 180\hat{\sigma}_{14}^z\hat{\sigma}_{26}^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_{28}^z +$ $2\hat{\sigma}_{14}^{z}\hat{\sigma}_{29}^{z} + 2\hat{\sigma}_{14}^{z}\hat{\sigma}_{3}^{z} + 2\hat{\sigma}_{14}^{z}\hat{\sigma}_{30}^{z} + 2\hat{\sigma}_{14}^{z}\hat{\sigma}_{31}^{z} + 2\hat{\sigma}_{14}^{z}\hat{\sigma}_{32}^{z} + 2\hat{\sigma}_{14}^{z}\hat{\sigma}_{33}^{z} +$ $2\hat{\sigma}_{14}^z\hat{\sigma}_{34}^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{14}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{14}^z\hat{\sigma}_{37}^z - 360\hat{\sigma}_{14}^z\hat{\sigma}_{38}^z 4\hat{\sigma}_{14}^z\hat{\sigma}_{39}^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{14}^z\hat{\sigma}_{40}^z - 4\hat{\sigma}_{14}^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_{14}^z\hat{\sigma}_{42}^z - 4\hat{\sigma}_{14}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{14}^z\hat{\sigma}_$ $4\hat{\sigma}_{14}^z\hat{\sigma}_{44}^z-4\hat{\sigma}_{14}^z\hat{\sigma}_{45}^z-4\hat{\sigma}_{14}^z\hat{\sigma}_{46}^z-4\hat{\sigma}_{14}^z\hat{\sigma}_{47}^z+2\hat{\sigma}_{14}^z\hat{\sigma}_{5}^z+2\hat{\sigma}_{14}^z\hat{\sigma}_{6}^z$ $4\hat{\sigma}_{14}^z\hat{\sigma}_{60}^z-4\hat{\sigma}_{14}^z\hat{\sigma}_{61}^z-360\hat{\sigma}_{14}^z\hat{\sigma}_{62}^z-4\hat{\sigma}_{14}^z\hat{\sigma}_{63}^z-4\hat{\sigma}_{14}^z\hat{\sigma}_{64}^z 4\hat{\sigma}_{14}^z\hat{\sigma}_{65}^z \ - \ 4\hat{\sigma}_{14}^z\hat{\sigma}_{66}^z \ - \ 4\hat{\sigma}_{14}^z\hat{\sigma}_{67}^z \ - \ 4\hat{\sigma}_{14}^z\hat{\sigma}_{68}^z \ - \ 4\hat{\sigma}_{14}^z\hat{\sigma}_{69}^z \ + \\$ $2\hat{\sigma}_{14}^{z}\hat{\sigma}_{7}^{z}-4\hat{\sigma}_{14}^{z}\hat{\sigma}_{70}^{z}-4\hat{\sigma}_{14}^{z}\hat{\sigma}_{71}^{z}+2\hat{\sigma}_{14}^{z}\hat{\sigma}_{8}^{z}+2\hat{\sigma}_{14}^{z}\hat{\sigma}_{9}^{z}+404\hat{\sigma}_{14}^{z}+$ $4\hat{\sigma}_{15}^z\hat{\sigma}_{16}^z+4\hat{\sigma}_{15}^z\hat{\sigma}_{17}^z+4\hat{\sigma}_{15}^z\hat{\sigma}_{18}^z+4\hat{\sigma}_{15}^z\hat{\sigma}_{19}^z+2\hat{\sigma}_{15}^z\hat{\sigma}_{2}^z+4\hat{\sigma}_{15}^z\hat{\sigma}_{20}^z+$ $4\hat{\sigma}_{15}^z\hat{\sigma}_{21}^z + 4\hat{\sigma}_{15}^z\hat{\sigma}_{22}^z + 4\hat{\sigma}_{15}^z\hat{\sigma}_{23}^z + 2\hat{\sigma}_{15}^z\hat{\sigma}_{24}^z + 2\hat{\sigma}_{15}^z\hat{\sigma}_{25}^z +$ $2\hat{\sigma}_{15}^z\hat{\sigma}_{26}^z + 180\hat{\sigma}_{15}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{15}^z\hat{\sigma}_{28}^z + 2\hat{\sigma}_{15}^z\hat{\sigma}_{29}^z + 180\hat{\sigma}_{15}^z\hat{\sigma}_{3}^z +$ $2\hat{\sigma}_{15}^z\hat{\sigma}_{30}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{31}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{32}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{33}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{34}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{34}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{35}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{34}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{35}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{15}^z\hat{\sigma}_{35}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{15}^z\hat{\sigma}_{15}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{15}^z\hat{\sigma}_{15}^z\hat{\sigma}_{15}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{15}^z\hat{\sigma}_{15}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{15}^z\hat{\sigma}$ $2\hat{\sigma}_{15}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{37}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{38}^z - 360\hat{\sigma}_{15}^z\hat{\sigma}_{39}^z +$ $2\hat{\sigma}_{15}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{40}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{42}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_$ $4\hat{\sigma}_{15}^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{47}^z + 2\hat{\sigma}_{15}^z\hat{\sigma}_{5}^z + 2\hat{\sigma}_{15}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{60}^z 4\hat{\sigma}_{15}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{62}^z - 360\hat{\sigma}_{15}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{65}^z 4\hat{\sigma}_{15}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{68}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{69}^z + 2\hat{\sigma}_{15}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{15}^z\hat{\sigma}_{70}^z 4\hat{\sigma}_{15}^z\hat{\sigma}_{71}^z+2\hat{\sigma}_{15}^z\hat{\sigma}_{8}^z+2\hat{\sigma}_{15}^z\hat{\sigma}_{9}^z+404\hat{\sigma}_{15}^z+4\hat{\sigma}_{16}^z\hat{\sigma}_{17}^z+4\hat{\sigma}_{16}^z\hat{\sigma}_{18}^z+$ $4\hat{\sigma}_{16}^{z}\hat{\sigma}_{19}^{z}+2\hat{\sigma}_{16}^{z}\hat{\sigma}_{2}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{20}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{21}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{22}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{23}^{z}+4$ $2\hat{\sigma}_{16}^{z}\hat{\sigma}_{24}^{z}+2\hat{\sigma}_{16}^{z}\hat{\sigma}_{25}^{z}+2\hat{\sigma}_{16}^{z}\hat{\sigma}_{26}^{z}+2\hat{\sigma}_{16}^{z}\hat{\sigma}_{27}^{z}+180\hat{\sigma}_{16}^{z}\hat{\sigma}_{28}^{z}+$ $2\hat{\sigma}_{16}^z\hat{\sigma}_{29}^z + 2\hat{\sigma}_{16}^z\hat{\sigma}_{3}^z + 2\hat{\sigma}_{16}^z\hat{\sigma}_{30}^z + 2\hat{\sigma}_{16}^z\hat{\sigma}_{31}^z + 2\hat{\sigma}_{16}^z\hat{\sigma}_{32}^z + 2\hat{\sigma}_{16}^z\hat{\sigma}_{33}^z +$ $2\hat{\sigma}_{16}^z\hat{\sigma}_{34}^z + 2\hat{\sigma}_{16}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{37}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{38}^z 4\hat{\sigma}_{16}^z\hat{\sigma}_{39}^z + 180\hat{\sigma}_{16}^z\hat{\sigma}_{4}^z - 360\hat{\sigma}_{16}^z\hat{\sigma}_{40}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{42}^z 4\hat{\sigma}_{16}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{47}^z +$ $2\hat{\sigma}_{16}^{z}\hat{\sigma}_{5}^{z}+2\hat{\sigma}_{16}^{z}\hat{\sigma}_{6}^{z}-4\hat{\sigma}_{16}^{z}\hat{\sigma}_{60}^{z}-4\hat{\sigma}_{16}^{z}\hat{\sigma}_{61}^{z}-4\hat{\sigma}_{16}^{z}\hat{\sigma}_{62}^{z}-4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{16}^{z}\hat{\sigma}_{63}^{z}+4\hat$ $360\hat{\sigma}_{16}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{16}^z\hat{\sigma}_{68}^z$ $4\hat{\sigma}_{16}^z\hat{\sigma}_{69}^z+2\hat{\sigma}_{16}^z\hat{\sigma}_{7}^z-4\hat{\sigma}_{16}^z\hat{\sigma}_{70}^z-4\hat{\sigma}_{16}^z\hat{\sigma}_{71}^z+2\hat{\sigma}_{16}^z\hat{\sigma}_{8}^z+2\hat{\sigma}_{16}^z\hat{\sigma}_{9}^z+$ $404\hat{\sigma}_{16}^z + 4\hat{\sigma}_{17}^z\hat{\sigma}_{18}^z + 4\hat{\sigma}_{17}^z\hat{\sigma}_{19}^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_{2}^z + 4\hat{\sigma}_{17}^z\hat{\sigma}_{20}^z + 4\hat{\sigma}_{17}^z\hat{\sigma}_{21}^z +$ $4\hat{\sigma}_{17}^z\hat{\sigma}_{22}^z \ + \ 4\hat{\sigma}_{17}^z\hat{\sigma}_{23}^z \ + \ 2\hat{\sigma}_{17}^z\hat{\sigma}_{24}^z \ + \ 2\hat{\sigma}_{17}^z\hat{\sigma}_{25}^z \ + \ 2\hat{\sigma}_{17}^z\hat{\sigma}_{26}^z \ +$ $2\hat{\sigma}_{17}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_{28}^z + 180\hat{\sigma}_{17}^z\hat{\sigma}_{29}^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_{3}^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_{30}^z +$ $2\hat{\sigma}_{17}^{z}\hat{\sigma}_{31}^{z} + 2\hat{\sigma}_{17}^{z}\hat{\sigma}_{32}^{z} + 2\hat{\sigma}_{17}^{z}\hat{\sigma}_{33}^{z} + 2\hat{\sigma}_{17}^{z}\hat{\sigma}_{34}^{z} + 2\hat{\sigma}_{17}^{z}\hat{\sigma}_{35}^{z} 4\hat{\sigma}_{17}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{37}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{38}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{39}^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{40}^z 360\hat{\sigma}_{17}^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{42}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{45}^z 4\hat{\sigma}_{17}^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{47}^z + 180\hat{\sigma}_{17}^z\hat{\sigma}_{5}^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{60}^z$ $4\hat{\sigma}_{17}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{17}^z\hat{\sigma}_{64}^z - 360\hat{\sigma}_{17}^z\hat{\sigma}_{65}^z 4\hat{\sigma}_{17}^{z}\hat{\sigma}_{66}^{z} - 4\hat{\sigma}_{17}^{z}\hat{\sigma}_{67}^{z} - 4\hat{\sigma}_{17}^{z}\hat{\sigma}_{68}^{z} - 4\hat{\sigma}_{17}^{z}\hat{\sigma}_{69}^{z} + 2\hat{\sigma}_{17}^{z}\hat{\sigma}_{7}^{z} - 4\hat{\sigma}_{17}^{z}\hat{\sigma}_{70}^{z} 4\hat{\sigma}_{17}^{z}\hat{\sigma}_{71}^{z}+2\hat{\sigma}_{17}^{z}\hat{\sigma}_{8}^{z}+2\hat{\sigma}_{17}^{z}\hat{\sigma}_{9}^{z}+404\hat{\sigma}_{17}^{z}+4\hat{\sigma}_{18}^{z}\hat{\sigma}_{19}^{z}+2\hat{\sigma}_{18}^{z}\hat{\sigma}_{2}^{z}+$ $4\hat{\sigma}_{18}^z\hat{\sigma}_{20}^z \ + \ 4\hat{\sigma}_{18}^z\hat{\sigma}_{21}^z \ + \ 4\hat{\sigma}_{18}^z\hat{\sigma}_{22}^z \ + \ 4\hat{\sigma}_{18}^z\hat{\sigma}_{23}^z \ + \ 2\hat{\sigma}_{18}^z\hat{\sigma}_{24}^z \ +$ $2\hat{\sigma}_{18}^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{26}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{28}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{29}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{3}^z +$ $180\hat{\sigma}_{18}^z\hat{\sigma}_{30}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{31}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{32}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{33}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{34}^z +$ $2\hat{\sigma}_{18}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{37}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{38}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{39}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{4}^z 4\hat{\sigma}_{18}^z\hat{\sigma}_{40}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{41}^z - 360\hat{\sigma}_{18}^z\hat{\sigma}_{42}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{44}^z 4\hat{\sigma}_{18}^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{47}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{5}^z + 180\hat{\sigma}_{18}^z\hat{\sigma}_{6}^z$ $4\hat{\sigma}_{18}^z\hat{\sigma}_{60}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{64}^z 4\hat{\sigma}_{18}^z\hat{\sigma}_{65}^{z'} - 360\hat{\sigma}_{18}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{68}^z - 4\hat{\sigma}_{18}^z\hat{\sigma}_{69}^z +$ $2\hat{\sigma}_{18}^{z}\hat{\sigma}_{7}^{z} - 4\hat{\sigma}_{18}^{z}\hat{\sigma}_{70}^{z} - 4\hat{\sigma}_{18}^{z}\hat{\sigma}_{71}^{z} + 2\hat{\sigma}_{18}^{z}\hat{\sigma}_{8}^{z} + 2\hat{\sigma}_{18}^{z}\hat{\sigma}_{9}^{z} + 404\hat{\sigma}_{18}^{z} + 2\hat{\sigma}_{18}^{z}\hat{\sigma}_{9}^{z} + 404\hat{\sigma}_{18}^{z}\hat{\sigma}_{18}^{z} + 2\hat{\sigma}_{18}^{z}\hat{\sigma}_{18}^{z}\hat{\sigma}_{18}^{z} + 4\hat{\sigma}_{18}^{z}\hat{\sigma}_{18}^{z}\hat{\sigma}_{18}^{z}\hat{\sigma}_{18}^{z} + 4\hat{\sigma}_{18}^{z}\hat$

 $2\hat{\sigma}_{19}^z\hat{\sigma}_2^z+4\hat{\sigma}_{19}^z\hat{\sigma}_{20}^z+4\hat{\sigma}_{19}^z\hat{\sigma}_{21}^z+4\hat{\sigma}_{19}^z\hat{\sigma}_{22}^z+4\hat{\sigma}_{19}^z\hat{\sigma}_{23}^z+2\hat{\sigma}_{19}^z\hat{\sigma}_{24}^z+$ $2\hat{\sigma}_{19}^{z}\hat{\sigma}_{25}^{z}+2\hat{\sigma}_{19}^{z}\hat{\sigma}_{26}^{z}+2\hat{\sigma}_{19}^{z}\hat{\sigma}_{27}^{z}+2\hat{\sigma}_{19}^{z}\hat{\sigma}_{28}^{z}+2\hat{\sigma}_{19}^{z}\hat{\sigma}_{29}^{z}+2\hat{\sigma}_{19}^{z}\hat{\sigma}_{3}^{z}+$ $2\hat{\sigma}_{19}^z\hat{\sigma}_{30}^z + 180\hat{\sigma}_{19}^z\hat{\sigma}_{31}^z + 2\hat{\sigma}_{19}^z\hat{\sigma}_{32}^z + 2\hat{\sigma}_{19}^z\hat{\sigma}_{33}^z + 2\hat{\sigma}_{19}^z\hat{\sigma}_{34}^z +$ $2\hat{\sigma}_{19}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{37}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{38}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{39}^z + 2\hat{\sigma}_{19}^z\hat{\sigma}_{4}^z 4\hat{\sigma}_{19}^z\hat{\sigma}_{40}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{42}^z - 360\hat{\sigma}_{19}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{44}^z 4\hat{\sigma}_{19}^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{47}^z + 2\hat{\sigma}_{19}^z\hat{\sigma}_{5}^z + 2\hat{\sigma}_{19}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{60}^z 4\hat{\sigma}_{19}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{65}^z 4\hat{\sigma}_{19}^{z}\hat{\sigma}_{66}^{z} - 360\hat{\sigma}_{19}^{z}\hat{\sigma}_{67}^{z} - 4\hat{\sigma}_{19}^{z}\hat{\sigma}_{68}^{z} - 4\hat{\sigma}_{19}^{z}\hat{\sigma}_{69}^{z} + 180\hat{\sigma}_{19}^{z}\hat{\sigma}_{7}^{z} 4\hat{\sigma}_{19}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{19}^z\hat{\sigma}_{71}^z + 2\hat{\sigma}_{19}^z\hat{\sigma}_{8}^z + 2\hat{\sigma}_{19}^z\hat{\sigma}_{9}^z + 404\hat{\sigma}_{19}^z + 2\hat{\sigma}_{2}^z\hat{\sigma}_{20}^z +$ $2\hat{\sigma}_2^z\hat{\sigma}_{21}^z + 2\hat{\sigma}_2^z\hat{\sigma}_{22}^z + 2\hat{\sigma}_2^z\hat{\sigma}_{23}^z + 2\hat{\sigma}_2^z\hat{\sigma}_{24}^z + 2\hat{\sigma}_2^z\hat{\sigma}_{25}^z + 180\hat{\sigma}_2^z\hat{\sigma}_{26}^z +$ $2\hat{\sigma}_{2}^{z}\hat{\sigma}_{27}^{\bar{z}}+2\hat{\sigma}_{2}^{z}\hat{\sigma}_{28}^{\bar{z}}+2\hat{\sigma}_{2}^{z}\hat{\sigma}_{29}^{z}+4\hat{\sigma}_{2}^{z}\hat{\sigma}_{3}^{z}+2\hat{\sigma}_{2}^{z}\hat{\sigma}_{30}^{z}+2\hat{\sigma}_{2}^{z}\hat{\sigma}_{31}^{z}+$ $2\hat{\sigma}_2^z\hat{\sigma}_{32}^z + 2\hat{\sigma}_2^z\hat{\sigma}_{33}^z + 2\hat{\sigma}_2^z\hat{\sigma}_{34}^z + 2\hat{\sigma}_2^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_2^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_2^z\hat{\sigma}_{37}^z 360\hat{\sigma}_{2}^{z}\hat{\sigma}_{38}^{z}-4\hat{\sigma}_{2}^{z}\hat{\sigma}_{39}^{z}+4\hat{\sigma}_{2}^{z}\hat{\sigma}_{4}^{z}-4\hat{\sigma}_{2}^{z}\hat{\sigma}_{40}^{z}-4\hat{\sigma}_{2}^{z}\hat{\sigma}_{41}^{z}-4\hat{\sigma}_{2}^{z}\hat{\sigma}_{42}^{z}$ $4\hat{\sigma}_2^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_2^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_2^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_2^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_2^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_2^z\hat{\sigma}_{48}^z 4\hat{\sigma}_{2}^{z}\hat{\sigma}_{49}^{z}+4\hat{\sigma}_{2}^{z}\hat{\sigma}_{5}^{z}-360\hat{\sigma}_{2}^{z}\hat{\sigma}_{50}^{z}-4\hat{\sigma}_{2}^{z}\hat{\sigma}_{51}^{z}-4\hat{\sigma}_{2}^{z}\hat{\sigma}_{52}^{z}-4\hat{\sigma}_{2}^{z}\hat{\sigma}_{53}^{z}-6\hat{\sigma}_{51}^{z}\hat{\sigma}_{52}^{z}+6\hat{\sigma}_{51}^{z}\hat{\sigma}_{52}^{z}+6\hat{\sigma}_{51}^{z}\hat{\sigma}_{52}^{z}+6\hat{\sigma}_{51}^{z}\hat{\sigma}_{52}^{z}+6\hat{\sigma}_{51}^{z}\hat{\sigma}_{52}^{z}+6\hat{\sigma}_{51}^{z}\hat{\sigma}_{52}^{z}+6\hat{\sigma}_{51}^{z}\hat{\sigma}_{51}^{z}+6\hat{\sigma}$ $4\hat{\sigma}_{2}^{z}\hat{\sigma}_{54}^{z} - 4\hat{\sigma}_{2}^{z}\hat{\sigma}_{55}^{z} - 4\hat{\sigma}_{2}^{z}\hat{\sigma}_{56}^{z} - 4\hat{\sigma}_{2}^{z}\hat{\sigma}_{57}^{z} - 4\hat{\sigma}_{2}^{z}\hat{\sigma}_{58}^{z} - 4\hat{\sigma}_{2}^{z}\hat{\sigma}_{59}^{z} +$ $4\hat{\sigma}_{2}^{z}\hat{\sigma}_{6}^{z} + 4\hat{\sigma}_{2}^{z}\hat{\sigma}_{7}^{z} + 4\hat{\sigma}_{2}^{z}\hat{\sigma}_{8}^{z} + 4\hat{\sigma}_{2}^{z}\hat{\sigma}_{9}^{z} + 404\hat{\sigma}_{2}^{z} + 4\hat{\sigma}_{20}^{z}\hat{\sigma}_{21}^{z} +$ $4\hat{\sigma}_{20}^z\hat{\sigma}_{22}^z + 4\hat{\sigma}_{20}^z\hat{\sigma}_{23}^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_{24}^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_{26}^z +$ $2\hat{\sigma}_{20}^{z}\hat{\sigma}_{27}^{z}+2\hat{\sigma}_{20}^{z}\hat{\sigma}_{28}^{z}+2\hat{\sigma}_{20}^{z}\hat{\sigma}_{29}^{z}+2\hat{\sigma}_{20}^{z}\hat{\sigma}_{3}^{z}+2\hat{\sigma}_{20}^{z}\hat{\sigma}_{30}^{z}+2\hat{\sigma}_{20}^{z}\hat{\sigma}_{31}^{z}+$ $180\hat{\sigma}_{20}^z\hat{\sigma}_{32}^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_{33}^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_{34}^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{36}^z 4\hat{\sigma}_{20}^z\hat{\sigma}_{37}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{38}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{39}^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{40}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_$ $4\hat{\sigma}_{20}^z\hat{\sigma}_{42}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{43}^z - 360\hat{\sigma}_{20}^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{46}^z 4\hat{\sigma}_{20}^{z}\hat{\sigma}_{47}^{z} + 2\hat{\sigma}_{20}^{z}\hat{\sigma}_{5}^{z} + 2\hat{\sigma}_{20}^{z}\hat{\sigma}_{6}^{z} - 4\hat{\sigma}_{20}^{z}\hat{\sigma}_{60}^{z} - 4\hat{\sigma}_{20}^{z}\hat{\sigma}_{61}^{z} - 4\hat{\sigma}_{20}^{z}\hat{\sigma}_{62}^{z} - 4\hat{\sigma}_$ $4\hat{\sigma}_{20}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{67}^z$ $360\hat{\sigma}_{20}^z\hat{\sigma}_{68}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{69}^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{20}^z\hat{\sigma}_{71}^z +$ $180\hat{\sigma}_{20}^{z}\hat{\sigma}_{8}^{z} + 2\hat{\sigma}_{20}^{z}\hat{\sigma}_{9}^{z} + 404\hat{\sigma}_{20}^{z} + 4\hat{\sigma}_{21}^{z}\hat{\sigma}_{22}^{z} + 4\hat{\sigma}_{21}^{z}\hat{\sigma}_{23}^{z} + 2\hat{\sigma}_{21}^{z}\hat{\sigma}_{24}^{z} +$ $2\hat{\sigma}_{21}^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_{21}^z\hat{\sigma}_{26}^z + 2\hat{\sigma}_{21}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{21}^z\hat{\sigma}_{28}^z + 2\hat{\sigma}_{21}^z\hat{\sigma}_{29}^z +$ $2\hat{\sigma}_{21}^{z}\hat{\sigma}_{3}^{z} + 2\hat{\sigma}_{21}^{z}\hat{\sigma}_{30}^{z} + 2\hat{\sigma}_{21}^{z}\hat{\sigma}_{31}^{z} + 2\hat{\sigma}_{21}^{z}\hat{\sigma}_{32}^{z} + 180\hat{\sigma}_{21}^{z}\hat{\sigma}_{33}^{z} +$ $2\hat{\sigma}_{21}^z\hat{\sigma}_{34}^z + 2\hat{\sigma}_{21}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{37}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{38}^z 4\hat{\sigma}_{21}^z\hat{\sigma}_{39}^z + 2\hat{\sigma}_{21}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{40}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{42}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_$ $4\hat{\sigma}_{21}^z\hat{\sigma}_{44}^z - 360\hat{\sigma}_{21}^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{47}^z + 2\hat{\sigma}_{21}^z\hat{\sigma}_{5}^z +$ $2\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{6}^{z} - 4\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{60}^{z} - 4\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{61}^{z} - 4\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{62}^{z} - 4\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{63}^{z} - 4\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{63}^{z} - 4\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{64}^{z} - 4\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{62}^{z} - 4\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{63}^{z} - 4\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{64}^{z} - 4\hat{\sigma}_{21}^{z_1}\hat{\sigma}_{64}^{z}$ $4\hat{\sigma}_{21}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{21}^z\hat{\sigma}_{68}^z - 360\hat{\sigma}_{21}^z\hat{\sigma}_{69}^z +$ $2\hat{\sigma}_{21}^{z}\hat{\sigma}_{7}^{z} - 4\hat{\sigma}_{21}^{z}\hat{\sigma}_{70}^{z} - 4\hat{\sigma}_{21}^{z}\hat{\sigma}_{70}^{z} + 2\hat{\sigma}_{21}^{z}\hat{\sigma}_{71}^{z} + 2\hat{\sigma}_{21}^{z}\hat{\sigma}_{8}^{z} + 180\hat{\sigma}_{21}^{z}\hat{\sigma}_{9}^{z} + 404\hat{\sigma}_{21}^{z} + 2\hat{\sigma}_{21}^{z}\hat{\sigma}_{9}^{z} + 40\hat{\sigma}_{21}^{z}\hat{\sigma}_{9}^{z} + 40\hat{\sigma}_{21}^{z}\hat{$ $4\hat{\sigma}_{22}^{z}\hat{\sigma}_{23}^{z} + 2\hat{\sigma}_{22}^{z}\hat{\sigma}_{24}^{z} + 2\hat{\sigma}_{22}^{z}\hat{\sigma}_{25}^{z} + 2\hat{\sigma}_{22}^{z}\hat{\sigma}_{26}^{z} + 2\hat{\sigma}_{22}^{z}\hat{\sigma}_{27}^{z} +$ $2\hat{\sigma}_{22}^z\hat{\sigma}_{28}^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_{29}^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_{3}^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_{30}^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_{31}^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_{32}^z +$ $2\hat{\sigma}_{22}^z\hat{\sigma}_{33}^z + 180\hat{\sigma}_{22}^z\hat{\sigma}_{34}^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_{37}^z 4\hat{\sigma}_{22}^{z_2}\hat{\sigma}_{38}^{z_3} - 4\hat{\sigma}_{22}^{z_2}\hat{\sigma}_{39}^{z_2} + 2\hat{\sigma}_{22}^{z_2}\hat{\sigma}_{4}^{z_2} - 4\hat{\sigma}_{22}^{z_2}\hat{\sigma}_{40}^{z_2} - 4\hat{\sigma}_{22}^{z_2}\hat{\sigma}_{41}^{z_2} - 4\hat{\sigma}_{22}^{z_2}\hat{\sigma}_{42}^{z_2} - 4\hat{\sigma}$ $4\hat{\sigma}_{22}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_{45}^z - 360\hat{\sigma}_{22}^z\hat{\sigma}_{46}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_{47}^z +$ $2\hat{\sigma}_{22}^{z}\hat{\sigma}_{5}^{z}+2\hat{\sigma}_{22}^{z}\hat{\sigma}_{6}^{z}-4\hat{\sigma}_{22}^{z}\hat{\sigma}_{60}^{z}-4\hat{\sigma}_{22}^{z}\hat{\sigma}_{61}^{z}-4\hat{\sigma}_{22}^{z}\hat{\sigma}_{62}^{z}-4\hat{\sigma}_{22}^{z}\hat{\sigma}_{63}^{z}+2\hat{\sigma}_{22}^{z}\hat{\sigma}_{63}^{z}+2\hat{\sigma}_{63}^{z}\hat{\sigma}_{63}^{z}+2\hat{\sigma}_{63}^{z}\hat{\sigma}_{63}^{z}+2\hat{\sigma}_{63}^{z}\hat{\sigma}_{63}^{z}+2\hat$ $4\hat{\sigma}_{22}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_{68}^z 4\hat{\sigma}_{22}^z\hat{\sigma}_{69}^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_7^z - 360\hat{\sigma}_{22}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{22}^z\hat{\sigma}_{71}^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_8^z +$ $2\hat{\sigma}_{22}^{zz}\hat{\sigma}_{9}^{zz} + 404\hat{\sigma}_{22}^{zz} + 2\hat{\sigma}_{23}^{z}\hat{\sigma}_{24}^{z} + 2\hat{\sigma}_{23}^{z}\hat{\sigma}_{25}^{z} + 2\hat{\sigma}_{23}^{z}\hat{\sigma}_{26}^{z} + 2\hat{\sigma}_{23}^{z}\hat{\sigma}_{27}^{z} +$ $2\hat{\sigma}_{23}^z\hat{\sigma}_{28}^z+2\hat{\sigma}_{23}^z\hat{\sigma}_{29}^z+2\hat{\sigma}_{23}^z\hat{\sigma}_{3}^z+2\hat{\sigma}_{23}^z\hat{\sigma}_{30}^z+2\hat{\sigma}_{23}^z\hat{\sigma}_{31}^z+2\hat{\sigma}_{23}^z\hat{\sigma}_{32}^z+2\hat{\sigma}$ $2\hat{\sigma}_{23}^z\hat{\sigma}_{33}^z + 2\hat{\sigma}_{23}^z\hat{\sigma}_{34}^z + 180\hat{\sigma}_{23}^z\hat{\sigma}_{35}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_{36}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_{37}^z 4\hat{\sigma}_{23}^{z3}\hat{\sigma}_{38}^{z} - 4\hat{\sigma}_{23}^{z3}\hat{\sigma}_{39}^{z} + 2\hat{\sigma}_{23}^{z}\hat{\sigma}_{4}^{z} - 4\hat{\sigma}_{23}^{z}\hat{\sigma}_{40}^{z} - 4\hat{\sigma}_{23}^{z}\hat{\sigma}_{41}^{z} - 4\hat{\sigma}_{23}^{z}\hat{\sigma}_{42}^{z}$ $4\hat{\sigma}_{23}^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_{46}^z - 360\hat{\sigma}_{23}^z\hat{\sigma}_{47}^z +$ $2\hat{\sigma}_{23}^{z}\hat{\sigma}_{5}^{z}+2\hat{\sigma}_{23}^{z}\hat{\sigma}_{6}^{z}-4\hat{\sigma}_{23}^{z}\hat{\sigma}_{60}^{z}-4\hat{\sigma}_{23}^{z}\hat{\sigma}_{61}^{z}-4\hat{\sigma}_{23}^{z}\hat{\sigma}_{62}^{z}-4\hat{\sigma}_{23}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{23}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{23}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{23}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{23}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{23}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{23}^{z}\hat{\sigma}_{63}^{z}+4\hat{\sigma}_{23}^{z}\hat{\sigma}_{63}^{z}+4\hat$ $4\hat{\sigma}_{23}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_{68}^z$ $4\hat{\sigma}_{23}^z\hat{\sigma}_{69}^z + 2\hat{\sigma}_{23}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{23}^z\hat{\sigma}_{70}^z - 360\hat{\sigma}_{23}^z\hat{\sigma}_{71}^z + 2\hat{\sigma}_{23}^z\hat{\sigma}_{8}^z +$ $2\hat{\sigma}_{23}^z\hat{\sigma}_{9}^z + 404\hat{\sigma}_{23}^z + 4\hat{\sigma}_{24}^z\hat{\sigma}_{25}^z + 4\hat{\sigma}_{24}^z\hat{\sigma}_{26}^z + 4\hat{\sigma}_{24}^z\hat{\sigma}_{27}^z + 4\hat{\sigma}_{24}^z\hat{\sigma}_{28}^z +$ $4\hat{\sigma}_{24}^z\hat{\sigma}_{29}^z + 2\hat{\sigma}_{24}^z\hat{\sigma}_{3}^z + 4\hat{\sigma}_{24}^z\hat{\sigma}_{30}^z + 4\hat{\sigma}_{24}^z\hat{\sigma}_{31}^z + 4\hat{\sigma}_{24}^z\hat{\sigma}_{32}^z + 4\hat{\sigma}_{24}^z\hat{\sigma}_{33}^z +$ $4\hat{\sigma}_{24}^z\hat{\sigma}_{34}^z + 4\hat{\sigma}_{24}^z\hat{\sigma}_{35}^z + 2\hat{\sigma}_{24}^z\hat{\sigma}_{4}^z - 360\hat{\sigma}_{24}^z\hat{\sigma}_{48}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{49}^z +$ $2\hat{\sigma}_{24}^{\bar{z}}\hat{\sigma}_{5}^{\bar{z}} - 4\hat{\sigma}_{24}^{z}\hat{\sigma}_{50}^{\bar{z}} - 4\hat{\sigma}_{24}^{z}\hat{\sigma}_{51}^{z} - 4\hat{\sigma}_{24}^{z}\hat{\sigma}_{52}^{z} - 4\hat{\sigma}_{24}^{z}\hat{\sigma}_{53}^{z} - 4\hat{\sigma}_{24}^{z}\hat{\sigma}_{54}^{z} - 4\hat{\sigma}_{24}^{z}\hat{\sigma}_{53}^{z} - 4\hat{\sigma}_{24}^{z}\hat{\sigma}_{54}^{z} - 4\hat{\sigma}_{24}^{z}\hat{\sigma}_{54}^{z$ $4\hat{\sigma}_{24}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{59}^z +$ $2\hat{\sigma}_{24}^z\hat{\sigma}_{6}^z - 360\hat{\sigma}_{24}^z\hat{\sigma}_{60}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{63}^z 4\hat{\sigma}_{24}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{24}^z\hat{\sigma}_{68}^z 4\hat{\sigma}_{24}^{z1}\hat{\sigma}_{69}^{z1} + 2\hat{\sigma}_{24}^{z2}\hat{\sigma}_{7}^{z} - 4\hat{\sigma}_{24}^{z}\hat{\sigma}_{70}^{z} - 4\hat{\sigma}_{24}^{z}\hat{\sigma}_{71}^{z} + 2\hat{\sigma}_{24}^{z}\hat{\sigma}_{8}^{z} + 2\hat{\sigma}_{24}^{z}\hat{\sigma}_{9}^{z} +$

 $404\hat{\sigma}_{24}^z+4\hat{\sigma}_{25}^z\hat{\sigma}_{26}^z+4\hat{\sigma}_{25}^z\hat{\sigma}_{27}^z+4\hat{\sigma}_{25}^z\hat{\sigma}_{28}^z+4\hat{\sigma}_{25}^z\hat{\sigma}_{29}^z+2\hat{\sigma}_{25}^z\hat{\sigma}_{3}^z+$ $4\hat{\sigma}_{25}^z\hat{\sigma}_{30}^z \ + \ 4\hat{\sigma}_{25}^z\hat{\sigma}_{31}^z \ + \ 4\hat{\sigma}_{25}^z\hat{\sigma}_{32}^z \ + \ 4\hat{\sigma}_{25}^z\hat{\sigma}_{33}^z \ + \ 4\hat{\sigma}_{25}^z\hat{\sigma}_{34}^z \ +$ $4\hat{\sigma}_{25}^z\hat{\sigma}_{35}^z + 2\hat{\sigma}_{25}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{25}^z\hat{\sigma}_{48}^z - 360\hat{\sigma}_{25}^z\hat{\sigma}_{49}^z + 2\hat{\sigma}_{25}^z\hat{\sigma}_{5}^z$ $4\hat{\sigma}_{25}^z\hat{\sigma}_{50}^z \ - \ 4\hat{\sigma}_{25}^z\hat{\sigma}_{51}^z \ - \ 4\hat{\sigma}_{25}^z\hat{\sigma}_{52}^z \ - \ 4\hat{\sigma}_{25}^z\hat{\sigma}_{53}^z \ - \ 4\hat{\sigma}_{25}^z\hat{\sigma}_{54}^z$ $4\hat{\sigma}_{25}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{25}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{25}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{25}^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_{25}^z\hat{\sigma}_{59}^z +$ $2\hat{\sigma}_{25}^{z}\hat{\sigma}_{6}^{z} - 4\hat{\sigma}_{25}^{z}\hat{\sigma}_{60}^{z} - 360\hat{\sigma}_{25}^{z}\hat{\sigma}_{61}^{z} - 4\hat{\sigma}_{25}^{z}\hat{\sigma}_{62}^{z} - 4\hat{\sigma}_{25}^{z}\hat{\sigma}_{63}^{z} 4\hat{\sigma}_{25}^{z}\hat{\sigma}_{64}^{z} - 4\hat{\sigma}_{25}^{z}\hat{\sigma}_{65}^{z} - 4\hat{\sigma}_{25}^{z}\hat{\sigma}_{66}^{z} - 4\hat{\sigma}_{25}^{z}\hat{\sigma}_{67}^{z} - 4\hat{\sigma}_{25}^{z}\hat{\sigma}_{68}^{z} 4\hat{\sigma}_{25}^z\hat{\sigma}_{69}^z+2\hat{\sigma}_{25}^z\hat{\sigma}_{7}^z-4\hat{\sigma}_{25}^z\hat{\sigma}_{70}^z-4\hat{\sigma}_{25}^z\hat{\sigma}_{71}^z+2\hat{\sigma}_{25}^z\hat{\sigma}_{8}^z+2\hat{\sigma}_{25}^z\hat{\sigma}_{9}^z+$ $404\hat{\sigma}_{25}^z+4\hat{\sigma}_{26}^z\hat{\sigma}_{27}^z+4\hat{\sigma}_{26}^z\hat{\sigma}_{28}^z+4\hat{\sigma}_{26}^z\hat{\sigma}_{29}^z+2\hat{\sigma}_{26}^z\hat{\sigma}_{3}^z+4\hat{\sigma}_{26}^z\hat{\sigma}_{30}^z+$ $4\hat{\sigma}_{26}^z\hat{\sigma}_{31}^z \ + \ 4\hat{\sigma}_{26}^z\hat{\sigma}_{32}^z \ + \ 4\hat{\sigma}_{26}^z\hat{\sigma}_{33}^z \ + \ 4\hat{\sigma}_{26}^z\hat{\sigma}_{34}^z \ + \ 4\hat{\sigma}_{26}^z\hat{\sigma}_{35}^z \ +$ $2\hat{\sigma}_{26}^{z}\hat{\sigma}_{4}^{z} - 4\hat{\sigma}_{26}^{z}\hat{\sigma}_{48}^{z} - 4\hat{\sigma}_{26}^{z}\hat{\sigma}_{49}^{z} + 2\hat{\sigma}_{26}^{z}\hat{\sigma}_{5}^{z} - 360\hat{\sigma}_{26}^{z}\hat{\sigma}_{50}^{z}$ $4\hat{\sigma}_{26}^z\hat{\sigma}_{51}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{55}^z 4\hat{\sigma}_{26}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{59}^z + 2\hat{\sigma}_{26}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{60}^z 4\hat{\sigma}_{26}^{z}\hat{\sigma}_{61}^{z} - 360\hat{\sigma}_{26}^{z}\hat{\sigma}_{62}^{z} - 4\hat{\sigma}_{26}^{z}\hat{\sigma}_{63}^{z} - 4\hat{\sigma}_{26}^{z}\hat{\sigma}_{64}^{z} - 4\hat{\sigma}_{26}^{z}\hat{\sigma}_{65}^{z} 4\hat{\sigma}_{26}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{68}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{69}^z + 2\hat{\sigma}_{26}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{26}^z\hat{\sigma}_{70}^z 4\hat{\sigma}_{26}^z\hat{\sigma}_{71}^{z}+2\hat{\sigma}_{26}^z\hat{\sigma}_{8}^{z}+2\hat{\sigma}_{26}^z\hat{\sigma}_{9}^{z}+404\hat{\sigma}_{26}^z+4\hat{\sigma}_{27}^z\hat{\sigma}_{28}^z+4\hat{\sigma}_{27}^z\hat{\sigma}_{29}^z+$ $180\hat{\sigma}_{27}^z\hat{\sigma}_3^z + 4\hat{\sigma}_{27}^z\hat{\sigma}_{30}^z + 4\hat{\sigma}_{27}^z\hat{\sigma}_{31}^z + 4\hat{\sigma}_{27}^z\hat{\sigma}_{32}^z + 4\hat{\sigma}_{27}^z\hat{\sigma}_{33}^z +$ $4\hat{\sigma}_{27}^z\hat{\sigma}_{34}^z + 4\hat{\sigma}_{27}^z\hat{\sigma}_{35}^z + 2\hat{\sigma}_{27}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{48}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{49}^z + 2\hat{\sigma}_{27}^z\hat{\sigma}_{5}^z 4\hat{\sigma}_{27}^z\hat{\sigma}_{50}^z - 360\hat{\sigma}_{27}^z\hat{\sigma}_{51}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{54}^z 4\hat{\sigma}_{27}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{59}^z + 2\hat{\sigma}_{27}^z\hat{\sigma}_{6}^z$ $4\hat{\sigma}_{27}^{z}\hat{\sigma}_{60}^{z} - 4\hat{\sigma}_{27}^{z}\hat{\sigma}_{61}^{z} - 4\hat{\sigma}_{27}^{z}\hat{\sigma}_{62}^{z} - 360\hat{\sigma}_{27}^{z}\hat{\sigma}_{63}^{z} - 4\hat{\sigma}_{27}^{z}\hat{\sigma}_{64}^{z} 4\hat{\sigma}_{27}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{68}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{69}^z + 2\hat{\sigma}_{27}^z\hat{\sigma}_{7}^z$ $4\hat{\sigma}_{27}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{27}^z\hat{\sigma}_{71}^z + 2\hat{\sigma}_{27}^z\hat{\sigma}_{8}^z + 2\hat{\sigma}_{27}^z\hat{\sigma}_{9}^z + 404\hat{\sigma}_{27}^z + 4\hat{\sigma}_{28}^z\hat{\sigma}_{29}^z +$ $2\hat{\sigma}_{28}^z\hat{\sigma}_{3}^{z}+4\hat{\sigma}_{28}^z\hat{\sigma}_{30}^z+4\hat{\sigma}_{28}^z\hat{\sigma}_{31}^z+4\hat{\sigma}_{28}^z\hat{\sigma}_{32}^z+4\hat{\sigma}_{28}^z\hat{\sigma}_{33}^z+4\hat{\sigma}_{28}^z\hat{\sigma}_{34}^z+$ $4\hat{\sigma}_{28}^z\hat{\sigma}_{35}^z + 180\hat{\sigma}_{28}^z\hat{\sigma}_4^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{48}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{49}^z + 2\hat{\sigma}_{28}^z\hat{\sigma}_5^z$ $4\hat{\sigma}_{28}^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{51}^z - 360\hat{\sigma}_{28}^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{54}^z 4\hat{\sigma}_{28}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{59}^z + 2\hat{\sigma}_{28}^z\hat{\sigma}_{6}^z$ $4\hat{\sigma}_{28}^z\hat{\sigma}_{60}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{63}^z - 360\hat{\sigma}_{28}^z\hat{\sigma}_{64}^z$ $4\hat{\sigma}_{28}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{68}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{69}^z +$ $2\hat{\sigma}_{28}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{28}^z\hat{\sigma}_{71}^z + 2\hat{\sigma}_{28}^z\hat{\sigma}_8^z + 2\hat{\sigma}_{28}^z\hat{\sigma}_9^z + 404\hat{\sigma}_{28}^z +$ $2\hat{\sigma}_{29}^z\hat{\sigma}_{31}^z+4\hat{\sigma}_{29}^z\hat{\sigma}_{30}^z+4\hat{\sigma}_{29}^z\hat{\sigma}_{31}^z+4\hat{\sigma}_{29}^z\hat{\sigma}_{32}^z+4\hat{\sigma}_{29}^z\hat{\sigma}_{33}^z+4\hat{\sigma}_{29}^z\hat{\sigma}_{34}^z+$ $4\hat{\sigma}_{29}^{z}\hat{\sigma}_{35}^{z} + 2\hat{\sigma}_{29}^{z}\hat{\sigma}_{4}^{z} - 4\hat{\sigma}_{29}^{z}\hat{\sigma}_{48}^{z} - 4\hat{\sigma}_{29}^{z}\hat{\sigma}_{49}^{z} + 180\hat{\sigma}_{29}^{z}\hat{\sigma}_{5}^{z} 4\hat{\sigma}_{29}^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{51}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{52}^z - 360\hat{\sigma}_{29}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{54}^z 4\hat{\sigma}_{29}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{59}^z +$ $-4\hat{\sigma}_{29}^z\hat{\sigma}_{60}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{63}^z$ $360\hat{\sigma}_{29}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{68}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{69}^z +$ $2\hat{\sigma}_{29}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{29}^z\hat{\sigma}_{71}^z + 2\hat{\sigma}_{29}^z\hat{\sigma}_8^z + 2\hat{\sigma}_{29}^z\hat{\sigma}_9^z + 404\hat{\sigma}_{29}^z +$ $2\hat{\sigma}_{3}^{z}\hat{\sigma}_{30}^{z} + 2\hat{\sigma}_{3}^{z}\hat{\sigma}_{31}^{z} + 2\hat{\sigma}_{3}^{z}\hat{\sigma}_{32}^{z} + 2\hat{\sigma}_{3}^{z}\hat{\sigma}_{33}^{z} + 2\hat{\sigma}_{3}^{z}\hat{\sigma}_{34}^{z} + 2\hat{\sigma}_{3}^{z}\hat{\sigma}_{35}^{z} 4\hat{\sigma}_{3}^{z}\hat{\sigma}_{36}^{z}-4\hat{\sigma}_{3}^{z}\hat{\sigma}_{37}^{z}-4\hat{\sigma}_{3}^{z}\hat{\sigma}_{38}^{z}-360\hat{\sigma}_{3}^{z}\hat{\sigma}_{39}^{z}+4\hat{\sigma}_{3}^{z}\hat{\sigma}_{4}^{z}-4\hat{\sigma}_{3}^{z}\hat{\sigma}_{40}^{z} 4\hat{\sigma}_3^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_3^z\hat{\sigma}_{42}^z - 4\hat{\sigma}_3^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_3^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_3^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_3^z\hat{\sigma}_{46}^z$ $4\hat{\sigma}_{3}^{z}\hat{\sigma}_{47}^{z}-4\hat{\sigma}_{3}^{z}\hat{\sigma}_{48}^{z}-4\hat{\sigma}_{3}^{z}\hat{\sigma}_{49}^{z}+4\hat{\sigma}_{3}^{z}\hat{\sigma}_{5}^{z}-4\hat{\sigma}_{3}^{z}\hat{\sigma}_{50}^{z}-360\hat{\sigma}_{3}^{z}\hat{\sigma}_{51}^{z} 4\hat{\sigma}_3^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_3^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_3^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_3^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_3^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_3^z\hat{\sigma}_{57}^z 4\hat{\sigma}_{3}^{z}\hat{\sigma}_{58}^{z}-4\hat{\sigma}_{3}^{z}\hat{\sigma}_{59}^{z}+4\hat{\sigma}_{3}^{z}\hat{\sigma}_{6}^{z}+4\hat{\sigma}_{3}^{z}\hat{\sigma}_{7}^{z}+4\hat{\sigma}_{3}^{z}\hat{\sigma}_{8}^{z}+4\hat{\sigma}_{3}^{z}\hat{\sigma}_{9}^{z}+$ $404\hat{\sigma}_{3}^{z}+4\hat{\sigma}_{30}^{z}\hat{\sigma}_{31}^{z}+4\hat{\sigma}_{30}^{z}\hat{\sigma}_{32}^{z}+4\hat{\sigma}_{30}^{z}\hat{\sigma}_{33}^{z}+4\hat{\sigma}_{30}^{z}\hat{\sigma}_{34}^{z}+4\hat{\sigma}_{30}^{z}\hat{\sigma}_{35}^{z}+$ $2\hat{\sigma}_{30}^z\hat{\sigma}_4^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{48}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{49}^z + 2\hat{\sigma}_{30}^z\hat{\sigma}_5^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{51}^z 4\hat{\sigma}_{30}^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{53}^z - 360\hat{\sigma}_{30}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{56}^z 4\hat{\sigma}_{30}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{59}^z + 180\hat{\sigma}_{30}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{60}^z$ $4\hat{\sigma}_{30}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{65}^z$ $360\hat{\sigma}_{30}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{68}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{69}^z + 2\hat{\sigma}_{30}^z\hat{\sigma}_{7}^z$ $4\hat{\sigma}_{30}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{30}^z\hat{\sigma}_{71}^z + 2\hat{\sigma}_{30}^z\hat{\sigma}_{8}^z + 2\hat{\sigma}_{30}^z\hat{\sigma}_{9}^z + 404\hat{\sigma}_{30}^z + 4\hat{\sigma}_{31}^z\hat{\sigma}_{32}^z +$ $4\hat{\sigma}_{31}^z\hat{\sigma}_{33}^z+4\hat{\sigma}_{31}^z\hat{\sigma}_{34}^z+4\hat{\sigma}_{31}^z\hat{\sigma}_{35}^z+2\hat{\sigma}_{31}^z\hat{\sigma}_{4}^z-4\hat{\sigma}_{31}^z\hat{\sigma}_{48}^z-4\hat{\sigma}_{31}^z\hat{\sigma}_{49}^z+$ $2\hat{\sigma}_{31}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{51}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{54}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{54}^z\hat{\sigma}_$ $360\hat{\sigma}_{31}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{59}^z +$ $2\hat{\sigma}_{31}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{60}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{64}^z 4\hat{\sigma}_{31}^z\hat{\sigma}_{65}^z-4\hat{\sigma}_{31}^z\hat{\sigma}_{66}^z-360\hat{\sigma}_{31}^z\hat{\sigma}_{67}^z-4\hat{\sigma}_{31}^z\hat{\sigma}_{68}^z-4\hat{\sigma}_{31}^z\hat{\sigma}_{69}^z+$ $180\hat{\sigma}_{31}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{31}^z\hat{\sigma}_{71}^z + 2\hat{\sigma}_{31}^z\hat{\sigma}_8^z + 2\hat{\sigma}_{31}^z\hat{\sigma}_9^z +$ $404\hat{\sigma}_{31}^{z}+4\hat{\sigma}_{32}^{z}\hat{\sigma}_{33}^{z}+4\hat{\sigma}_{32}^{z}\hat{\sigma}_{34}^{z}+4\hat{\sigma}_{32}^{z}\hat{\sigma}_{35}^{z}+2\hat{\sigma}_{32}^{z}\hat{\sigma}_{4}^{z}-4\hat{\sigma}_{32}^{z}\hat{\sigma}_{48}^{z} 4\hat{\sigma}_{32}^z\hat{\sigma}_{49}^z + 2\hat{\sigma}_{32}^z\hat{\sigma}_5^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{51}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{52}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{52}^z\hat{\sigma}_{52}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{52}^z\hat{\sigma}_{53}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{52}^z\hat{\sigma}_{53}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{52}^z$

 $4\hat{\sigma}_{32}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{55}^z - 360\hat{\sigma}_{32}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{58}^z$ $4\hat{\sigma}_{32}^z\hat{\sigma}_{59}^z + 2\hat{\sigma}_{32}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{60}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{63}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{63}^z\hat{\sigma}_{63}^z\hat{\sigma}_{63}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{63}^z\hat{\sigma}_{63}^z\hat{\sigma}_{63}^z\hat{\sigma}_{63}^z\hat{\sigma}_{63}^z\hat{\sigma}_{63}^z\hat{\sigma}_{$ $4\hat{\sigma}_{32}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{32}^z\hat{\sigma}_{67}^z - 360\hat{\sigma}_{32}^z\hat{\sigma}_{68}^z 4\hat{\sigma}_{32}^z\hat{\sigma}_{69}^z \,+\, 2\hat{\sigma}_{32}^z\hat{\sigma}_7^z \,-\, 4\hat{\sigma}_{32}^z\hat{\sigma}_{70}^z \,-\, 4\hat{\sigma}_{32}^z\hat{\sigma}_{71}^z \,+\, 180\hat{\sigma}_{32}^z\hat{\sigma}_8^z \,+\,$ $2\hat{\sigma}_{32}^z\hat{\sigma}_{9}^z + 404\hat{\sigma}_{32}^z + 4\hat{\sigma}_{33}^z\hat{\sigma}_{34}^z + 4\hat{\sigma}_{33}^z\hat{\sigma}_{35}^z + 2\hat{\sigma}_{33}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{48}^z 4\hat{\sigma}_{33}^z\hat{\sigma}_{49}^z + 2\hat{\sigma}_{33}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{51}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{53}^z 4\hat{\sigma}_{33}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{56}^z - 360\hat{\sigma}_{33}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{58}^z 4\hat{\sigma}_{33}^z\hat{\sigma}_{59}^{z1} + 2\hat{\sigma}_{33}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{60}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{63}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{63}^z\hat{$ $4\hat{\sigma}_{33}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{68}^z 360\hat{\sigma}_{33}^z\hat{\sigma}_{69}^z + 2\hat{\sigma}_{33}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{33}^z\hat{\sigma}_{71}^z + 2\hat{\sigma}_{33}^z\hat{\sigma}_{8}^z +$ $180\hat{\sigma}_{33}^z\hat{\sigma}_{9}^z + 404\hat{\sigma}_{33}^z + 4\hat{\sigma}_{34}^z\hat{\sigma}_{35}^z + 2\hat{\sigma}_{34}^z\hat{\sigma}_{4}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{48}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{49}^z +$ $2\hat{\sigma}_{34}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{51}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{54}^z 4\hat{\sigma}_{34}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{56}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{57}^z - 360\hat{\sigma}_{34}^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{59}^z +$ $2\hat{\sigma}_{34}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{60}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{64}^z 4\hat{\sigma}_{34}^z\hat{\sigma}_{65}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{68}^z - 4\hat{\sigma}_{34}^z\hat{\sigma}_{69}^z +$ $2\hat{\sigma}_{34}^{z}\hat{\sigma}_{7}^{z} - 360\hat{\sigma}_{34}^{z}\hat{\sigma}_{70}^{z} - 4\hat{\sigma}_{34}^{z}\hat{\sigma}_{71}^{z} + 2\hat{\sigma}_{34}^{z}\hat{\sigma}_{8}^{z} + 2\hat{\sigma}_{34}^{z}\hat{\sigma}_{9}^{z} + 404\hat{\sigma}_{34}^{z} +$ $2 \hat{\sigma}_{35}^z \hat{\sigma}_4^z - 4 \hat{\sigma}_{35}^z \hat{\sigma}_{48}^z - 4 \hat{\sigma}_{35}^z \hat{\sigma}_{49}^z + 2 \hat{\sigma}_{35}^z \hat{\sigma}_5^z - 4 \hat{\sigma}_{35}^z \hat{\sigma}_{50}^z - 4 \hat{\sigma}_{35}^z \hat{\sigma}_{51}^z 4\hat{\sigma}_{35}^z\hat{\sigma}_{52}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{53}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{54}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{55}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{56}^z 4\hat{\sigma}_{35}^z\hat{\sigma}_{57}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{58}^z - 360\hat{\sigma}_{35}^z\hat{\sigma}_{59}^z + 2\hat{\sigma}_{35}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{60}^z 4\hat{\sigma}_{35}^z\hat{\sigma}_{61}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{62}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{63}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{64}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{65}^z 4\hat{\sigma}_{35}^z\hat{\sigma}_{66}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{67}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{68}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{69}^z + 2\hat{\sigma}_{35}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{35}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{75}^z\hat{\sigma}_{70}^z - 4\hat{\sigma}_{75}^z\hat{\sigma}_$ $360\hat{\sigma}_{35}^z\hat{\sigma}_{71}^z + 2\hat{\sigma}_{35}^z\hat{\sigma}_8^z + 2\hat{\sigma}_{35}^z\hat{\sigma}_9^z + 404\hat{\sigma}_{35}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{37}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{38}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{38}^z\hat{\sigma}_{38}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{38}^z\hat{\sigma}_{38}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{38}^z\hat{\sigma}_{38}^$ $8\hat{\sigma}_{36}^z\hat{\sigma}_{39}^z - 4\hat{\sigma}_{36}^z\hat{\sigma}_{4}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{40}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{41}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{42}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{43}^z +$ $8\hat{\sigma}_{36}^z\hat{\sigma}_{44}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{45}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{46}^z + 8\hat{\sigma}_{36}^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_{36}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{36}^z\hat{\sigma}_{6}^z$ $4\hat{\sigma}_{36}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{36}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{36}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{36}^z + 8\hat{\sigma}_{37}^z\hat{\sigma}_{38}^z + 8\hat{\sigma}_{37}^z\hat{\sigma}_{39}^z$ $4\hat{\sigma}_{37}^z\hat{\sigma}_{4}^z+8\hat{\sigma}_{37}^z\hat{\sigma}_{40}^z+8\hat{\sigma}_{37}^z\hat{\sigma}_{41}^z+8\hat{\sigma}_{37}^z\hat{\sigma}_{42}^z+8\hat{\sigma}_{37}^z\hat{\sigma}_{43}^z+8\hat{\sigma}_{37}^z\hat{\sigma}_{44}^z+$ $8\hat{\sigma}_{37}^z\hat{\sigma}_{45}^z + 8\hat{\sigma}_{37}^z\hat{\sigma}_{46}^z + 8\hat{\sigma}_{37}^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_{37}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{37}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{37}^z\hat{\sigma}_{7}^z$ $4\hat{\sigma}_{37}^{z}\hat{\sigma}_{8}^{z} - 4\hat{\sigma}_{37}^{z}\hat{\sigma}_{9}^{z} - 404\hat{\sigma}_{37}^{z} + 8\hat{\sigma}_{38}^{z}\hat{\sigma}_{39}^{z} - 4\hat{\sigma}_{38}^{z}\hat{\sigma}_{4}^{z} + 8\hat{\sigma}_{38}^{z}\hat{\sigma}_{40}^{z} +$ $8\hat{\sigma}_{38}^z\hat{\sigma}_{41}^z + 8\hat{\sigma}_{38}^z\hat{\sigma}_{42}^z + 8\hat{\sigma}_{38}^z\hat{\sigma}_{43}^z + 8\hat{\sigma}_{38}^z\hat{\sigma}_{44}^z + 8\hat{\sigma}_{38}^z\hat{\sigma}_{45}^z +$ $8\hat{\sigma}_{38}^z\hat{\sigma}_{46}^z + 8\hat{\sigma}_{38}^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_{38}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{38}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{38}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{38}^z\hat{\sigma}_{8}^z$ $4\hat{\sigma}_{38}^z\hat{\sigma}_{9}^z - 404\hat{\sigma}_{38}^z - 4\hat{\sigma}_{39}^z\hat{\sigma}_{4}^z + 8\hat{\sigma}_{39}^z\hat{\sigma}_{40}^z + 8\hat{\sigma}_{39}^z\hat{\sigma}_{41}^z + 8\hat{\sigma}_{39}^z\hat{\sigma}_{42}^z +$ $8\hat{\sigma}_{39}^z\hat{\sigma}_{43}^z + 8\hat{\sigma}_{39}^z\hat{\sigma}_{44}^z + 8\hat{\sigma}_{39}^z\hat{\sigma}_{45}^z + 8\hat{\sigma}_{39}^z\hat{\sigma}_{46}^z + 8\hat{\sigma}_{39}^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_{39}^z\hat{\sigma}_{5}^z 4\hat{\sigma}_{39}^z\hat{\sigma}_6^z - 4\hat{\sigma}_{39}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{39}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{39}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{39}^z - 360\hat{\sigma}_4^z\hat{\sigma}_{40}^z 4\hat{\sigma}_4^z\hat{\sigma}_{41}^z - 4\hat{\sigma}_4^z\hat{\sigma}_{42}^z - 4\hat{\sigma}_4^z\hat{\sigma}_{43}^z - 4\hat{\sigma}_4^z\hat{\sigma}_{44}^z - 4\hat{\sigma}_4^z\hat{\sigma}_{45}^z - 4\hat{\sigma}_4^z\hat{\sigma}_{46}^z 4\hat{\sigma}_4^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_4^z\hat{\sigma}_{48}^z - 4\hat{\sigma}_4^z\hat{\sigma}_{49}^z + 4\hat{\sigma}_4^z\hat{\sigma}_5^z - 4\hat{\sigma}_4^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_4^z\hat{\sigma}_{51}^z 360\hat{\sigma}_{4}^{z}\hat{\sigma}_{52}^{z} - 4\hat{\sigma}_{4}^{z}\hat{\sigma}_{53}^{z} - 4\hat{\sigma}_{4}^{z}\hat{\sigma}_{54}^{z} - 4\hat{\sigma}_{4}^{z}\hat{\sigma}_{55}^{z} - 4\hat{\sigma}_{4}^{z}\hat{\sigma}_{56}^{z} - 4\hat{\sigma}_{4}^{z}\hat{\sigma}_{57}^{z} 4\hat{\sigma}_4^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_4^z\hat{\sigma}_{59}^z + 4\hat{\sigma}_4^z\hat{\sigma}_6^z + 4\hat{\sigma}_4^z\hat{\sigma}_7^z + 4\hat{\sigma}_4^z\hat{\sigma}_8^z + 4\hat{\sigma}_4^z\hat{\sigma}_9^z +$ $404\hat{\sigma}_{4}^{z}+8\hat{\sigma}_{40}^{z}\hat{\sigma}_{41}^{z}+8\hat{\sigma}_{40}^{z}\hat{\sigma}_{42}^{z}+8\hat{\sigma}_{40}^{z}\hat{\sigma}_{43}^{z}+8\hat{\sigma}_{40}^{z}\hat{\sigma}_{44}^{z}+8\hat{\sigma}_{40}^{z}\hat{\sigma}_{45}^{z}+$ $8\hat{\sigma}_{40}^z\hat{\sigma}_{46}^z + 8\hat{\sigma}_{40}^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_{40}^z\hat{\sigma}_5^z - 4\hat{\sigma}_{40}^z\hat{\sigma}_6^z - 4\hat{\sigma}_{40}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{40}^z\hat{\sigma}_8^z$ $4\hat{\sigma}_{40}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{40}^z + 8\hat{\sigma}_{41}^z\hat{\sigma}_{42}^z + 8\hat{\sigma}_{41}^z\hat{\sigma}_{43}^z + 8\hat{\sigma}_{41}^z\hat{\sigma}_{44}^z + 8\hat{\sigma}_{41}^z\hat{\sigma}_{45}^z +$ $8\hat{\sigma}_{41}^z\hat{\sigma}_{46}^z + 8\hat{\sigma}_{41}^z\hat{\sigma}_{47}^z - 360\hat{\sigma}_{41}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{41}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{41}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{41}^z\hat{\sigma}_{8}^z$ $4\hat{\sigma}_{41}^z\hat{\sigma}_{9}^z - 404\hat{\sigma}_{41}^z + 8\hat{\sigma}_{42}^z\hat{\sigma}_{43}^z + 8\hat{\sigma}_{42}^z\hat{\sigma}_{44}^z + 8\hat{\sigma}_{42}^z\hat{\sigma}_{45}^z + 8\hat{\sigma}_{42}^z\hat{\sigma}_{46}^z +$ $8\hat{\sigma}_{42}^{z}\hat{\sigma}_{47}^{z} - 4\hat{\sigma}_{42}^{z}\hat{\sigma}_{5}^{z} - 360\hat{\sigma}_{42}^{z}\hat{\sigma}_{6}^{z} - 4\hat{\sigma}_{42}^{z}\hat{\sigma}_{7}^{z} - 4\hat{\sigma}_{42}^{z}\hat{\sigma}_{8}^{z} - 4\hat{\sigma}_{42}^{z}\hat{\sigma}_{9}^{z}$ $404\hat{\sigma}_{42}^z + 8\hat{\sigma}_{43}^z\hat{\sigma}_{44}^z + 8\hat{\sigma}_{43}^z\hat{\sigma}_{45}^z + 8\hat{\sigma}_{43}^z\hat{\sigma}_{46}^z + 8\hat{\sigma}_{43}^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_{43}^z\hat{\sigma}_{5}^z$ $4\hat{\sigma}_{43}^z\hat{\sigma}_6^z - 360\hat{\sigma}_{43}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{43}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{43}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{43}^z + 8\hat{\sigma}_{44}^z\hat{\sigma}_{45}^z +$ $8\hat{\sigma}_{44}^z\hat{\sigma}_{46}^z + 8\hat{\sigma}_{44}^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_{44}^z\hat{\sigma}_5^z - 4\hat{\sigma}_{44}^z\hat{\sigma}_6^z - 4\hat{\sigma}_{44}^z\hat{\sigma}_7^z - 360\hat{\sigma}_{44}^z\hat{\sigma}_8^z 4\hat{\sigma}_{44}^z\hat{\sigma}_{9}^z - 404\hat{\sigma}_{44}^z + 8\hat{\sigma}_{45}^z\hat{\sigma}_{46}^z + 8\hat{\sigma}_{45}^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_{45}^z\hat{\sigma}_{5}^z - 4\hat{\sigma}_{45}^z\hat{\sigma}_{6}^z$ $4\hat{\sigma}_{45}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{45}^z\hat{\sigma}_8^z - 360\hat{\sigma}_{45}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{45}^z + 8\hat{\sigma}_{46}^z\hat{\sigma}_{47}^z - 4\hat{\sigma}_{46}^z\hat{\sigma}_5^z$ $4\hat{\sigma}_{46}^z\hat{\sigma}_{6}^z-4\hat{\sigma}_{46}^z\hat{\sigma}_{7}^z-4\hat{\sigma}_{46}^z\hat{\sigma}_{8}^z-4\hat{\sigma}_{46}^z\hat{\sigma}_{9}^z-404\hat{\sigma}_{46}^z-4\hat{\sigma}_{47}^z\hat{\sigma}_{5}^z$ $4\hat{\sigma}_{47}^z\hat{\sigma}_6^z - 4\hat{\sigma}_{47}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{47}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{47}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{47}^z + 8\hat{\sigma}_{48}^z\hat{\sigma}_{49}^z 4\hat{\sigma}_{48}^z\hat{\sigma}_{5}^z + 8\hat{\sigma}_{48}^z\hat{\sigma}_{50}^z + 8\hat{\sigma}_{48}^z\hat{\sigma}_{51}^z + 8\hat{\sigma}_{48}^z\hat{\sigma}_{52}^z + 8\hat{\sigma}_{48}^z\hat{\sigma}_{53}^z + 8\hat{\sigma}_{48}^z\hat{\sigma}_{54}^z +$ $8\hat{\sigma}_{48}^z\hat{\sigma}_{55}^z + 8\hat{\sigma}_{48}^z\hat{\sigma}_{56}^z + 8\hat{\sigma}_{48}^z\hat{\sigma}_{57}^z + 8\hat{\sigma}_{48}^z\hat{\sigma}_{58}^z + 8\hat{\sigma}_{48}^z\hat{\sigma}_{59}^z 4\hat{\sigma}_{48}^z\hat{\sigma}_6^z - 4\hat{\sigma}_{48}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{48}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{48}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{48}^z - 4\hat{\sigma}_{49}^z\hat{\sigma}_5^z +$ $8\hat{\sigma}_{49}^z\hat{\sigma}_{50}^z + 8\hat{\sigma}_{49}^z\hat{\sigma}_{51}^z + 8\hat{\sigma}_{49}^z\hat{\sigma}_{52}^z + 8\hat{\sigma}_{49}^z\hat{\sigma}_{53}^z + 8\hat{\sigma}_{49}^z\hat{\sigma}_{54}^z +$ $8\hat{\sigma}_{49}^z\hat{\sigma}_{55}^z + 8\hat{\sigma}_{49}^z\hat{\sigma}_{56}^z + 8\hat{\sigma}_{49}^z\hat{\sigma}_{57}^z + 8\hat{\sigma}_{49}^z\hat{\sigma}_{58}^z + 8\hat{\sigma}_{49}^z\hat{\sigma}_{59}^z - 4\hat{\sigma}_{49}^z\hat{\sigma}_{6}^z 4\hat{\sigma}_{49}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{49}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{49}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{49}^z - 4\hat{\sigma}_5^z\hat{\sigma}_{50}^z - 4\hat{\sigma}_5^z\hat{\sigma}_{51}^z 4\hat{\sigma}_{5}^{z}\hat{\sigma}_{52}^{z} - 360\hat{\sigma}_{5}^{z}\hat{\sigma}_{53}^{z} - 4\hat{\sigma}_{5}^{z}\hat{\sigma}_{54}^{z} - 4\hat{\sigma}_{5}^{z}\hat{\sigma}_{55}^{z} - 4\hat{\sigma}_{5}^{z}\hat{\sigma}_{56}^{z} - 4\hat{\sigma}_{5}^{z}\hat{\sigma}_{57}^{z} 4\hat{\sigma}_5^z\hat{\sigma}_{58}^z - 4\hat{\sigma}_5^z\hat{\sigma}_{59}^z + 4\hat{\sigma}_5^z\hat{\sigma}_{6}^z + 4\hat{\sigma}_5^z\hat{\sigma}_{7}^z + 4\hat{\sigma}_5^z\hat{\sigma}_{8}^z + 4\hat{\sigma}_5^z\hat{\sigma}_{9}^z +$

 $404\hat{\sigma}_{5}^{z}+8\hat{\sigma}_{50}^{z}\hat{\sigma}_{51}^{z}+8\hat{\sigma}_{50}^{z}\hat{\sigma}_{52}^{z}+8\hat{\sigma}_{50}^{z}\hat{\sigma}_{53}^{z}+8\hat{\sigma}_{50}^{z}\hat{\sigma}_{54}^{z}+8\hat{\sigma}_{50}^{z}\hat{\sigma}_{55}^{z}+$ $8\hat{\sigma}_{50}^z\hat{\sigma}_{56}^z + 8\hat{\sigma}_{50}^z\hat{\sigma}_{57}^z + 8\hat{\sigma}_{50}^z\hat{\sigma}_{58}^z + 8\hat{\sigma}_{50}^z\hat{\sigma}_{59}^z - 4\hat{\sigma}_{50}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{50}^z\hat{\sigma}_{7}^z 4\hat{\sigma}_{50}^z\hat{\sigma}_{8}^z-4\hat{\sigma}_{50}^z\hat{\sigma}_{9}^z-404\hat{\sigma}_{50}^z+8\hat{\sigma}_{51}^z\hat{\sigma}_{52}^z+8\hat{\sigma}_{51}^z\hat{\sigma}_{53}^z+8\hat{\sigma}_{51}^z\hat{\sigma}_{54}^z+$ $8\hat{\sigma}_{51}^z\hat{\sigma}_{55}^z + 8\hat{\sigma}_{51}^z\hat{\sigma}_{56}^z + 8\hat{\sigma}_{51}^z\hat{\sigma}_{57}^z + 8\hat{\sigma}_{51}^z\hat{\sigma}_{58}^z + 8\hat{\sigma}_{51}^z\hat{\sigma}_{59}^z - 4\hat{\sigma}_{51}^z\hat{\sigma}_{6}^z 4\hat{\sigma}_{51}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{51}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{51}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{51}^z + 8\hat{\sigma}_{52}^z\hat{\sigma}_{53}^z + 8\hat{\sigma}_{52}^z\hat{\sigma}_{54}^z +$ $8\hat{\sigma}_{52}^z\hat{\sigma}_{55}^z + 8\hat{\sigma}_{52}^z\hat{\sigma}_{56}^z + 8\hat{\sigma}_{52}^z\hat{\sigma}_{57}^z + 8\hat{\sigma}_{52}^z\hat{\sigma}_{58}^z + 8\hat{\sigma}_{52}^z\hat{\sigma}_{59}^z - 4\hat{\sigma}_{52}^z\hat{\sigma}_{6}^z 4\hat{\sigma}_{52}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{52}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{52}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{52}^z + 8\hat{\sigma}_{53}^z\hat{\sigma}_{54}^z + 8\hat{\sigma}_{53}^z\hat{\sigma}_{55}^z +$ $8\hat{\sigma}_{53}^z\hat{\sigma}_{56}^z + 8\hat{\sigma}_{53}^z\hat{\sigma}_{57}^z + 8\hat{\sigma}_{53}^z\hat{\sigma}_{58}^z + 8\hat{\sigma}_{53}^z\hat{\sigma}_{59}^z - 4\hat{\sigma}_{53}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{53}^z\hat{\sigma}_{7}^z 4\hat{\sigma}_{53}^z\hat{\sigma}_{8}^z-4\hat{\sigma}_{53}^z\hat{\sigma}_{9}^z-404\hat{\sigma}_{53}^z+8\hat{\sigma}_{54}^z\hat{\sigma}_{55}^z+8\hat{\sigma}_{54}^z\hat{\sigma}_{56}^z+8\hat{\sigma}_{54}^z\hat{\sigma}_{57}^z+$ $8\hat{\sigma}_{54}^z\hat{\sigma}_{58}^z + 8\hat{\sigma}_{54}^z\hat{\sigma}_{59}^z - 360\hat{\sigma}_{54}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{54}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{54}^z\hat{\sigma}_{8}^z - 4\hat{\sigma}_{54}^z\hat{\sigma}_{9}^z 404\hat{\sigma}_{54}^z + 8\hat{\sigma}_{55}^z\hat{\sigma}_{56}^z + 8\hat{\sigma}_{55}^z\hat{\sigma}_{57}^z + 8\hat{\sigma}_{55}^z\hat{\sigma}_{58}^z + 8\hat{\sigma}_{55}^z\hat{\sigma}_{59}^z - 4\hat{\sigma}_{55}^z\hat{\sigma}_{6}^z$ $360\hat{\sigma}_{55}^z\hat{\sigma}_7^z - 4\hat{\sigma}_{55}^z\hat{\sigma}_8^z - 4\hat{\sigma}_{55}^z\hat{\sigma}_9^z - 404\hat{\sigma}_{55}^z + 8\hat{\sigma}_{56}^z\hat{\sigma}_{57}^z + 8\hat{\sigma}_{56}^z\hat{\sigma}_{58}^z +$ $8\hat{\sigma}_{56}^z\hat{\sigma}_{59}^z - 4\hat{\sigma}_{56}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{56}^z\hat{\sigma}_{7}^z - 360\hat{\sigma}_{56}^z\hat{\sigma}_{8}^z - 4\hat{\sigma}_{56}^z\hat{\sigma}_{9}^z - 404\hat{\sigma}_{56}^z +$ $8\hat{\sigma}_{57}^z\hat{\sigma}_{58}^z + 8\hat{\sigma}_{57}^z\hat{\sigma}_{59}^z - 4\hat{\sigma}_{57}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{57}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{57}^z\hat{\sigma}_{8}^z - 360\hat{\sigma}_{57}^z\hat{\sigma}_{9}^z 404\hat{\sigma}_{57}^z + 8\hat{\sigma}_{58}^z\hat{\sigma}_{59}^z - 4\hat{\sigma}_{58}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{58}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{58}^z\hat{\sigma}_{8}^z - 4\hat{\sigma}_{58}^z\hat{\sigma}_{9}^z$ $404\hat{\sigma}_{58}^z - 4\hat{\sigma}_{59}^z\hat{\sigma}_{6}^z - 4\hat{\sigma}_{59}^z\hat{\sigma}_{7}^z - 4\hat{\sigma}_{59}^z\hat{\sigma}_{8}^z - 4\hat{\sigma}_{59}^z\hat{\sigma}_{9}^z - 404\hat{\sigma}_{59}^z +$ $4\hat{\sigma}_{6}^{z}\hat{\sigma}_{7}^{z}+4\hat{\sigma}_{6}^{z}\hat{\sigma}_{8}^{z}+4\hat{\sigma}_{6}^{z}\hat{\sigma}_{9}^{z}+404\hat{\sigma}_{6}^{z}+8\hat{\sigma}_{60}^{z}\hat{\sigma}_{61}^{z}+8\hat{\sigma}_{60}^{z}\hat{\sigma}_{62}^{z}+$ $8\hat{\sigma}_{60}^z\hat{\sigma}_{63}^z \,+\, 8\hat{\sigma}_{60}^z\hat{\sigma}_{64}^z \,+\, 8\hat{\sigma}_{60}^z\hat{\sigma}_{65}^z \,+\, 8\hat{\sigma}_{60}^z\hat{\sigma}_{66}^z \,+\, 8\hat{\sigma}_{60}^z\hat{\sigma}_{67}^z \,+\,$ $8\hat{\sigma}_{60}^z\hat{\sigma}_{68}^z + 8\hat{\sigma}_{60}^z\hat{\sigma}_{69}^z + 8\hat{\sigma}_{60}^z\hat{\sigma}_{70}^z + 8\hat{\sigma}_{60}^z\hat{\sigma}_{71}^z - 404\hat{\sigma}_{60}^z + 8\hat{\sigma}_{61}^z\hat{\sigma}_{62}^z +$ $8\hat{\sigma}_{61}^{z}\hat{\sigma}_{63}^{z} + 8\hat{\sigma}_{61}^{z}\hat{\sigma}_{64}^{z} + 8\hat{\sigma}_{61}^{z}\hat{\sigma}_{65}^{z} + 8\hat{\sigma}_{61}^{z}\hat{\sigma}_{66}^{z} + 8\hat{\sigma}_{61}^{z}\hat{\sigma}_{67}^{z} +$ $8\hat{\sigma}_{61}^z\hat{\sigma}_{68}^z + 8\hat{\sigma}_{61}^z\hat{\sigma}_{69}^z + 8\hat{\sigma}_{61}^z\hat{\sigma}_{70}^z + 8\hat{\sigma}_{61}^z\hat{\sigma}_{71}^z - 404\hat{\sigma}_{61}^z + 8\hat{\sigma}_{62}^z\hat{\sigma}_{63}^z +$ $8\hat{\sigma}_{62}^z\hat{\sigma}_{64}^z + 8\hat{\sigma}_{62}^z\hat{\sigma}_{65}^z + 8\hat{\sigma}_{62}^z\hat{\sigma}_{66}^z + 8\hat{\sigma}_{62}^z\hat{\sigma}_{67}^z + 8\hat{\sigma}_{62}^z\hat{\sigma}_{68}^z +$ $8\hat{\sigma}_{62}^z\hat{\sigma}_{69}^z + 8\hat{\sigma}_{62}^z\hat{\sigma}_{70}^z + 8\hat{\sigma}_{62}^z\hat{\sigma}_{71}^z - 404\hat{\sigma}_{62}^z + 8\hat{\sigma}_{63}^z\hat{\sigma}_{64}^z + 8\hat{\sigma}_{63}^z\hat{\sigma}_{65}^z +$ $8\hat{\sigma}_{63}^z\hat{\sigma}_{66}^z + 8\hat{\sigma}_{63}^z\hat{\sigma}_{67}^z + 8\hat{\sigma}_{63}^z\hat{\sigma}_{68}^z + 8\hat{\sigma}_{63}^z\hat{\sigma}_{69}^z + 8\hat{\sigma}_{63}^z\hat{\sigma}_{70}^z +$ $8\hat{\sigma}_{63}^z\hat{\sigma}_{71}^z - 404\hat{\sigma}_{63}^z + 8\hat{\sigma}_{64}^z\hat{\sigma}_{65}^z + 8\hat{\sigma}_{64}^z\hat{\sigma}_{66}^z + 8\hat{\sigma}_{64}^z\hat{\sigma}_{67}^z + 8\hat{\sigma}_{64}^z\hat{\sigma}_{68}^z +$ $8\hat{\sigma}_{64}^z\hat{\sigma}_{69}^z + 8\hat{\sigma}_{64}^z\hat{\sigma}_{70}^z + 8\hat{\sigma}_{64}^z\hat{\sigma}_{71}^z - 404\hat{\sigma}_{64}^z + 8\hat{\sigma}_{65}^z\hat{\sigma}_{66}^z + 8\hat{\sigma}_{65}^z\hat{\sigma}_{67}^z +$ $8\hat{\sigma}_{65}^z\hat{\sigma}_{68}^z + 8\hat{\sigma}_{65}^z\hat{\sigma}_{69}^z + 8\hat{\sigma}_{65}^z\hat{\sigma}_{70}^z + 8\hat{\sigma}_{65}^z\hat{\sigma}_{71}^z - 404\hat{\sigma}_{65}^z + 8\hat{\sigma}_{66}^z\hat{\sigma}_{67}^z +$ $8\hat{\sigma}_{66}^z\hat{\sigma}_{68}^z + 8\hat{\sigma}_{66}^z\hat{\sigma}_{69}^z + 8\hat{\sigma}_{66}^z\hat{\sigma}_{70}^z + 8\hat{\sigma}_{66}^z\hat{\sigma}_{71}^z - 404\hat{\sigma}_{66}^z + 8\hat{\sigma}_{67}^z\hat{\sigma}_{68}^z +$ $8\hat{\sigma}_{67}^z\hat{\sigma}_{69}^z+8\hat{\sigma}_{67}^z\hat{\sigma}_{70}^z+8\hat{\sigma}_{67}^z\hat{\sigma}_{71}^z-404\hat{\sigma}_{67}^z+8\hat{\sigma}_{68}^z\hat{\sigma}_{69}^z+8\hat{\sigma}_{68}^z\hat{\sigma}_{70}^z+$ $8\hat{\sigma}_{68}^z\hat{\sigma}_{71}^z - 404\hat{\sigma}_{68}^z + 8\hat{\sigma}_{69}^z\hat{\sigma}_{70}^z + 8\hat{\sigma}_{69}^z\hat{\sigma}_{71}^z - 404\hat{\sigma}_{69}^z + 4\hat{\sigma}_{7}^z\hat{\sigma}_{8}^z +$ $4\hat{\sigma}_{7}^{z}\hat{\sigma}_{9}^{z} + 404\hat{\sigma}_{7}^{z} + 8\hat{\sigma}_{70}^{z}\hat{\sigma}_{71}^{z} - 404\hat{\sigma}_{70}^{z} - 404\hat{\sigma}_{71}^{z} + 4\hat{\sigma}_{8}^{z}\hat{\sigma}_{9}^{z} +$ $404\hat{\sigma}_8^z + 404\hat{\sigma}_9^z + 19,872$

A complete expression of Eq.(33)

 $\begin{array}{l} \hat{H}_2\left(\hat{\sigma}_i^z\right) = 2\hat{\sigma}_0^z\hat{\sigma}_1^z + 2\hat{\sigma}_0^z\hat{\sigma}_{10}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{11}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{12}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{13}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{14}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{15}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{16}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{17}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{18}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{19}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{22}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{20}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{21}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{22}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{23}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{24}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{25}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{27}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{23}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{23}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_0^z\hat{\sigma}_{25}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{27}^z - 2\hat{\sigma}_0^z\hat{\sigma}_{27}^z$

 $2\hat{\sigma}_{11}^z\hat{\sigma}_2^z - 2\hat{\sigma}_{11}^z\hat{\sigma}_{20}^z - 2\hat{\sigma}_{11}^z\hat{\sigma}_{21}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{22}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{23}^z - 2\hat{\sigma}_{11}^z\hat{\sigma}_{24}^z +$ $2\hat{\sigma}_{11}^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{26}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{27}^z - 2\hat{\sigma}_{11}^z\hat{\sigma}_{3}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{4}^z + 2\hat{\sigma}_{11}^z\hat{\sigma}_{5}^z +$ $2\hat{\sigma}_{11}^z\hat{\sigma}_6^z+2\hat{\sigma}_{11}^z\hat{\sigma}_7^z-2\hat{\sigma}_{11}^z\hat{\sigma}_8^z-2\hat{\sigma}_{11}^z\hat{\sigma}_9^z+2\hat{\sigma}_{12}^z\hat{\sigma}_{13}^z-2\hat{\sigma}_{12}^z\hat{\sigma}_{14}^z+$ $2\hat{\sigma}_{12}^{z}\hat{\sigma}_{15}^{z}-2\hat{\sigma}_{12}^{z}\hat{\sigma}_{16}^{z}-2\hat{\sigma}_{12}^{z}\hat{\sigma}_{17}^{z}+2\hat{\sigma}_{12}^{z}\hat{\sigma}_{18}^{z}+2\hat{\sigma}_{12}^{z}\hat{\sigma}_{19}^{z}+$ $2\hat{\sigma}_{12}^z\hat{\sigma}_2^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{20}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{21}^z - 2\hat{\sigma}_{12}^z\hat{\sigma}_{22}^z - 2\hat{\sigma}_{12}^z\hat{\sigma}_{23}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{24}^z - 2\hat{\sigma}_{12}^z\hat{\sigma}_{24}^z - 2\hat{\sigma}_{12}^z\hat{\sigma}_{23}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{24}^z - 2\hat{\sigma}_{12}^z\hat{\sigma}_{2$ $2\hat{\sigma}_{12}^z\hat{\sigma}_{25}^z - 2\hat{\sigma}_{12}^z\hat{\sigma}_{26}^z - 2\hat{\sigma}_{12}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{12}^z\hat{\sigma}_{3}^z - 2\hat{\sigma}_{12}^z\hat{\sigma}_{4}^z - 2\hat{\sigma}_{12}^z\hat{\sigma}_{5}^z$ $2\hat{\sigma}_{12}^z\hat{\sigma}_6^z-2\hat{\sigma}_{12}^z\hat{\sigma}_7^z+2\hat{\sigma}_{12}^z\hat{\sigma}_8^z+2\hat{\sigma}_{12}^z\hat{\sigma}_9^z+2\hat{\sigma}_{13}^z\hat{\sigma}_{14}^z-2\hat{\sigma}_{13}^z\hat{\sigma}_{15}^z+$ $2\hat{\sigma}_{13}^{\bar{z}}\hat{\sigma}_{16}^{\bar{z}} + 2\hat{\sigma}_{13}^{\bar{z}}\hat{\sigma}_{17}^{\bar{z}} - 2\hat{\sigma}_{13}^{\bar{z}}\hat{\sigma}_{18}^{\bar{z}} - 2\hat{\sigma}_{13}^{\bar{z}}\hat{\sigma}_{19}^{\bar{z}} - 2\hat{\sigma}_{13}^{\bar{z}}\hat{\sigma}_{2}^{\bar{z}} - 2\hat{\sigma}_{13}^{\bar{z}}\hat{\sigma}_{20}^{\bar{z}} - 2\hat{\sigma}_{13}^{\bar{z}}\hat{\sigma}_{20}^{\bar$ $2\hat{\sigma}_{13}^z\hat{\sigma}_{21}^z \ + \ 2\hat{\sigma}_{13}^z\hat{\sigma}_{22}^z \ + \ 2\hat{\sigma}_{13}^z\hat{\sigma}_{23}^z \ - \ 2\hat{\sigma}_{13}^z\hat{\sigma}_{24}^z \ + \ 2\hat{\sigma}_{13}^z\hat{\sigma}_{25}^z \ +$ $2\hat{\sigma}_{13}^z\hat{\sigma}_{26}^z+2\hat{\sigma}_{13}^z\hat{\sigma}_{27}^z-2\hat{\sigma}_{13}^z\hat{\sigma}_{3}^z+2\hat{\sigma}_{13}^z\hat{\sigma}_{4}^z+2\hat{\sigma}_{13}^z\hat{\sigma}_{5}^z+2\hat{\sigma}_{13}^z\hat{\sigma}_{6}^z+$ $2\hat{\sigma}_{13}^z\hat{\sigma}_7^z - 2\hat{\sigma}_{13}^z\hat{\sigma}_8^z - 2\hat{\sigma}_{13}^z\hat{\sigma}_9^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_{15}^z - 2\hat{\sigma}_{14}^z\hat{\sigma}_{16}^z - 2\hat{\sigma}_{14}^z\hat{\sigma}_{17}^z +$ $2\hat{\sigma}_{14}^z\hat{\sigma}_{18}^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_{19}^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_{2}^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_{20}^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_{21}^z - 2\hat{\sigma}_{14}^z\hat{\sigma}_{22}^z 2\hat{\sigma}_{14}^z\hat{\sigma}_{23}^z \ + \ 2\hat{\sigma}_{14}^z\hat{\sigma}_{24}^z \ - \ 2\hat{\sigma}_{14}^z\hat{\sigma}_{25}^z \ - \ 2\hat{\sigma}_{14}^z\hat{\sigma}_{26}^z \ - \ 2\hat{\sigma}_{14}^z\hat{\sigma}_{27}^z \ +$ $2\hat{\sigma}_{14}^z\hat{\sigma}_3^z - 2\hat{\sigma}_{14}^z\hat{\sigma}_4^z - 2\hat{\sigma}_{14}^z\hat{\sigma}_5^z - 2\hat{\sigma}_{14}^z\hat{\sigma}_6^z - 2\hat{\sigma}_{14}^z\hat{\sigma}_7^z + 2\hat{\sigma}_{14}^z\hat{\sigma}_8^z +$ $2\hat{\sigma}_{14}^z\hat{\sigma}_{9}^z+2\hat{\sigma}_{15}^z\hat{\sigma}_{16}^z+2\hat{\sigma}_{15}^z\hat{\sigma}_{17}^z-2\hat{\sigma}_{15}^z\hat{\sigma}_{18}^z-2\hat{\sigma}_{15}^z\hat{\sigma}_{19}^z-2\hat{\sigma}_{15}^z\hat{\sigma}_{2}^z 2\hat{\sigma}_{15}^z\hat{\sigma}_{20}^z \ - \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{21}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{22}^z \ + \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{23}^z \ - \ 2\hat{\sigma}_{15}^z\hat{\sigma}_{24}^z \ +$ $2\hat{\sigma}_{15}^z\hat{\sigma}_{25}^z+2\hat{\sigma}_{15}^z\hat{\sigma}_{26}^z+2\hat{\sigma}_{15}^z\hat{\sigma}_{27}^z-2\hat{\sigma}_{15}^z\hat{\sigma}_{3}^z+2\hat{\sigma}_{15}^z\hat{\sigma}_{4}^z+2\hat{\sigma}_{15}^z\hat{\sigma}_{5}^z+$ $2\hat{\sigma}_{15}^z\hat{\sigma}_6^z+2\hat{\sigma}_{15}^z\hat{\sigma}_7^z-2\hat{\sigma}_{15}^z\hat{\sigma}_8^z-2\hat{\sigma}_{15}^z\hat{\sigma}_9^z-2\hat{\sigma}_{16}^z\hat{\sigma}_{17}^z+2\hat{\sigma}_{16}^z\hat{\sigma}_{18}^z+$ $2\hat{\sigma}_{16}^z\hat{\sigma}_{19}^z + 2\hat{\sigma}_{16}^z\hat{\sigma}_{2}^z + 2\hat{\sigma}_{16}^z\hat{\sigma}_{20}^z + 2\hat{\sigma}_{16}^z\hat{\sigma}_{21}^z - 2\hat{\sigma}_{16}^z\hat{\sigma}_{22}^z - 2\hat{\sigma}_{16}^z\hat{\sigma}_{23}^z +$ $2\hat{\sigma}_{16}^z\hat{\sigma}_{24}^z-2\hat{\sigma}_{16}^z\hat{\sigma}_{25}^z-2\hat{\sigma}_{16}^z\hat{\sigma}_{26}^z-2\hat{\sigma}_{16}^z\hat{\sigma}_{27}^z+2\hat{\sigma}_{16}^z\hat{\sigma}_{3}^z-2\hat{\sigma}_{16}^z\hat{\sigma}_{4}^z 2\hat{\sigma}_{16}^z\hat{\sigma}_{5}^z-2\hat{\sigma}_{16}^z\hat{\sigma}_{6}^z-2\hat{\sigma}_{16}^z\hat{\sigma}_{7}^z+2\hat{\sigma}_{16}^z\hat{\sigma}_{8}^z+2\hat{\sigma}_{16}^z\hat{\sigma}_{9}^z+2\hat{\sigma}_{17}^z\hat{\sigma}_{18}^z+$ $2\hat{\sigma}_{17}^z\hat{\sigma}_{19}^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_{2}^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_{20}^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_{21}^z - 2\hat{\sigma}_{17}^z\hat{\sigma}_{22}^z - 2\hat{\sigma}_{17}^z\hat{\sigma}_{23}^z +$ $2\hat{\sigma}_{17}^z\hat{\sigma}_{24}^z-2\hat{\sigma}_{17}^z\hat{\sigma}_{25}^z-2\hat{\sigma}_{17}^z\hat{\sigma}_{26}^z-2\hat{\sigma}_{17}^z\hat{\sigma}_{27}^z+2\hat{\sigma}_{17}^z\hat{\sigma}_{3}^z-2\hat{\sigma}_{17}^z\hat{\sigma}_{4}^z 2\hat{\sigma}_{17}^z\hat{\sigma}_5^z - 2\hat{\sigma}_{17}^z\hat{\sigma}_6^z - 2\hat{\sigma}_{17}^z\hat{\sigma}_7^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_8^z + 2\hat{\sigma}_{17}^z\hat{\sigma}_9^z - 2\hat{\sigma}_{18}^z\hat{\sigma}_{19}^z 2\hat{\sigma}_{18}^z\hat{\sigma}_{2}^z - 2\hat{\sigma}_{18}^z\hat{\sigma}_{20}^z - 2\hat{\sigma}_{18}^z\hat{\sigma}_{21}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{22}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{23}^z - 2\hat{\sigma}_{18}^z\hat{\sigma}_{24}^z +$ $2\hat{\sigma}_{18}^z\hat{\sigma}_{25}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{26}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{27}^z - 2\hat{\sigma}_{18}^z\hat{\sigma}_{3}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{4}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{5}^z +$ $2\hat{\sigma}_{18}^z\hat{\sigma}_{6}^z + 2\hat{\sigma}_{18}^z\hat{\sigma}_{7}^z - 2\hat{\sigma}_{18}^z\hat{\sigma}_{8}^z - 2\hat{\sigma}_{18}^z\hat{\sigma}_{9}^z - 2\hat{\sigma}_{19}^z\hat{\sigma}_{2}^z - 2\hat{\sigma}_{19}^z\hat{\sigma}_{20}^z 2\hat{\sigma}_{19}^z\hat{\sigma}_{21}^z \ + \ 2\hat{\sigma}_{19}^z\hat{\sigma}_{22}^z \ + \ 2\hat{\sigma}_{19}^z\hat{\sigma}_{23}^z \ - \ 2\hat{\sigma}_{19}^z\hat{\sigma}_{24}^z \ + \ 2\hat{\sigma}_{19}^z\hat{\sigma}_{25}^z \ +$ $2\hat{\sigma}_{19}^z\hat{\sigma}_{26}^z+2\hat{\sigma}_{19}^z\hat{\sigma}_{27}^z-2\hat{\sigma}_{19}^z\hat{\sigma}_{3}^z+2\hat{\sigma}_{19}^z\hat{\sigma}_{4}^z+2\hat{\sigma}_{19}^z\hat{\sigma}_{5}^z+2\hat{\sigma}_{19}^z\hat{\sigma}_{6}^z+$ $2\hat{\sigma}_{19}^z\hat{\sigma}_{7}^z-2\hat{\sigma}_{19}^z\hat{\sigma}_{8}^z-2\hat{\sigma}_{19}^z\hat{\sigma}_{9}^z-2\hat{\sigma}_{2}^z\hat{\sigma}_{20}^z-2\hat{\sigma}_{2}^z\hat{\sigma}_{21}^z+2\hat{\sigma}_{2}^z\hat{\sigma}_{22}^z+$ $2\hat{\sigma}_{2}^{z}\hat{\sigma}_{23}^{z} - 2\hat{\sigma}_{2}^{z}\hat{\sigma}_{24}^{z} + 2\hat{\sigma}_{2}^{z}\hat{\sigma}_{25}^{z} + 2\hat{\sigma}_{2}^{z}\hat{\sigma}_{26}^{z} + 2\hat{\sigma}_{2}^{z}\hat{\sigma}_{27}^{z} - 2\hat{\sigma}_{2}^{z}\hat{\sigma}_{3}^{z} +$ $2\hat{\sigma}_2^z\hat{\sigma}_4^z + 2\hat{\sigma}_2^z\hat{\sigma}_5^z + 2\hat{\sigma}_2^z\hat{\sigma}_6^z + 2\hat{\sigma}_2^z\hat{\sigma}_7^z - 2\hat{\sigma}_2^z\hat{\sigma}_8^z - 2\hat{\sigma}_2^z\hat{\sigma}_9^z - 2\hat{\sigma}_{20}^z\hat{\sigma}_{21}^z +$ $2\hat{\sigma}_{20}^z\hat{\sigma}_{22}^z \ + \ 2\hat{\sigma}_{20}^z\hat{\sigma}_{23}^z \ - \ 2\hat{\sigma}_{20}^z\hat{\sigma}_{24}^z \ + \ 2\hat{\sigma}_{20}^z\hat{\sigma}_{25}^z \ + \ 2\hat{\sigma}_{20}^z\hat{\sigma}_{26}^z \ +$ $2\hat{\sigma}_{20}^z\hat{\sigma}_{27}^z - 2\hat{\sigma}_{20}^z\hat{\sigma}_3^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_4^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_5^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_6^z + 2\hat{\sigma}_{20}^z\hat{\sigma}_7^z$ $2\hat{\sigma}_{20}^z\hat{\sigma}_{8}^z - 2\hat{\sigma}_{20}^z\hat{\sigma}_{9}^z + 2\hat{\sigma}_{21}^z\hat{\sigma}_{22}^z + 2\hat{\sigma}_{21}^z\hat{\sigma}_{23}^z - 2\hat{\sigma}_{21}^z\hat{\sigma}_{24}^z + 2\hat{\sigma}_{21}^z\hat{\sigma}_{25}^z +$ $2\hat{\sigma}_{21}^z\hat{\sigma}_{26}^z+2\hat{\sigma}_{21}^z\hat{\sigma}_{27}^z-2\hat{\sigma}_{21}^z\hat{\sigma}_{3}^z+2\hat{\sigma}_{21}^z\hat{\sigma}_{4}^z+2\hat{\sigma}_{21}^z\hat{\sigma}_{5}^z+2\hat{\sigma}_{21}^z\hat{\sigma}_{6}^z+$ $2\hat{\sigma}_{21}^z\hat{\sigma}_7^z - 2\hat{\sigma}_{21}^z\hat{\sigma}_8^z - 2\hat{\sigma}_{21}^z\hat{\sigma}_9^z - 2\hat{\sigma}_{22}^z\hat{\sigma}_{23}^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_{24}^z - 2\hat{\sigma}_{22}^z\hat{\sigma}_{25}^z 2\hat{\sigma}_{22}^z\hat{\sigma}_{26}^z - 2\hat{\sigma}_{22}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_{3}^z - 2\hat{\sigma}_{22}^z\hat{\sigma}_{4}^z - 2\hat{\sigma}_{22}^z\hat{\sigma}_{5}^z - 2\hat{\sigma}_{22}^z\hat{\sigma}_{6}^z$ $2\hat{\sigma}_{22}^z\hat{\sigma}_7^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_8^z + 2\hat{\sigma}_{22}^z\hat{\sigma}_9^z + 2\hat{\sigma}_{23}^z\hat{\sigma}_{24}^z - 2\hat{\sigma}_{23}^z\hat{\sigma}_{25}^z - 2\hat{\sigma}_{23}^z\hat{\sigma}_{26}^z 2\hat{\sigma}_{23}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{23}^z\hat{\sigma}_{3}^z - 2\hat{\sigma}_{23}^z\hat{\sigma}_{4}^z - 2\hat{\sigma}_{23}^z\hat{\sigma}_{5}^z - 2\hat{\sigma}_{23}^z\hat{\sigma}_{6}^z - 2\hat{\sigma}_{23}^z\hat{\sigma}_{7}^z +$ $2\hat{\sigma}_{23}^z\hat{\sigma}_{8}^z+2\hat{\sigma}_{23}^z\hat{\sigma}_{9}^z+2\hat{\sigma}_{24}^z\hat{\sigma}_{25}^z+2\hat{\sigma}_{24}^z\hat{\sigma}_{26}^z+2\hat{\sigma}_{24}^z\hat{\sigma}_{27}^z-2\hat{\sigma}_{24}^z\hat{\sigma}_{3}^z+$ $2\hat{\sigma}_{24}^z\hat{\sigma}_{4}^z + 2\hat{\sigma}_{24}^z\hat{\sigma}_{5}^z + 2\hat{\sigma}_{24}^z\hat{\sigma}_{6}^z + 2\hat{\sigma}_{24}^z\hat{\sigma}_{7}^z - 2\hat{\sigma}_{24}^z\hat{\sigma}_{8}^z - 2\hat{\sigma}_{24}^z\hat{\sigma}_{9}^z$ $2\hat{\sigma}_{25}^z\hat{\sigma}_{26}^z - 2\hat{\sigma}_{25}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{25}^z\hat{\sigma}_{3}^z - 2\hat{\sigma}_{25}^z\hat{\sigma}_{4}^z - 2\hat{\sigma}_{25}^z\hat{\sigma}_{5}^z - 2\hat{\sigma}_{25}^z\hat{\sigma}_{6}^z$ $2\hat{\sigma}_{25}^z\hat{\sigma}_7^z + 2\hat{\sigma}_{25}^z\hat{\sigma}_8^z + 2\hat{\sigma}_{25}^z\hat{\sigma}_9^z - 2\hat{\sigma}_{26}^z\hat{\sigma}_{27}^z + 2\hat{\sigma}_{26}^z\hat{\sigma}_3^z - 2\hat{\sigma}_{26}^z\hat{\sigma}_4^z 2\hat{\sigma}_{26}^{z}\hat{\sigma}_{5}^{z} - 2\hat{\sigma}_{26}^{z}\hat{\sigma}_{6}^{z} - 2\hat{\sigma}_{26}^{z}\hat{\sigma}_{7}^{z} + 2\hat{\sigma}_{26}^{z}\hat{\sigma}_{8}^{z} + 2\hat{\sigma}_{26}^{z}\hat{\sigma}_{9}^{z} + 2\hat{\sigma}_{27}^{z}\hat{\sigma}_{3}^{z} 2\hat{\sigma}_{27}^z\hat{\sigma}_{4}^z - 2\hat{\sigma}_{27}^z\hat{\sigma}_{5}^z - 2\hat{\sigma}_{27}^z\hat{\sigma}_{6}^z - 2\hat{\sigma}_{27}^z\hat{\sigma}_{7}^z + 2\hat{\sigma}_{27}^z\hat{\sigma}_{8}^z + 2\hat{\sigma}_{27}^z\hat{\sigma}_{9}^z +$ $2\hat{\sigma}_3^z\hat{\sigma}_4^z + 2\hat{\sigma}_3^z\hat{\sigma}_5^z + 2\hat{\sigma}_3^z\hat{\sigma}_6^z + 2\hat{\sigma}_3^z\hat{\sigma}_7^z - 2\hat{\sigma}_3^z\hat{\sigma}_8^z - 2\hat{\sigma}_3^z\hat{\sigma}_9^z - 2\hat{\sigma}_4^z\hat{\sigma}_5^z - 2\hat{\sigma}_5^z\hat{\sigma}_5^z - 2\hat{\sigma$ $2\hat{\sigma}_{4}^{z}\hat{\sigma}_{6}^{z}-2\hat{\sigma}_{4}^{z}\hat{\sigma}_{7}^{z}+2\hat{\sigma}_{4}^{z}\hat{\sigma}_{8}^{z}+2\hat{\sigma}_{4}^{z}\hat{\sigma}_{9}^{z}-2\hat{\sigma}_{5}^{z}\hat{\sigma}_{6}^{z}-2\hat{\sigma}_{5}^{z}\hat{\sigma}_{7}^{z}+2\hat{\sigma}_{5}^{z}\hat{\sigma}_{8}^{z}+$ $2\hat{\sigma}_{5}^{z}\hat{\sigma}_{9}^{z}-2\hat{\sigma}_{6}^{z}\hat{\sigma}_{7}^{z}+2\hat{\sigma}_{6}^{z}\hat{\sigma}_{8}^{z}+2\hat{\sigma}_{6}^{z}\hat{\sigma}_{9}^{z}+2\hat{\sigma}_{7}^{z}\hat{\sigma}_{8}^{z}+2\hat{\sigma}_{7}^{z}\hat{\sigma}_{9}^{z}-2\hat{\sigma}_{8}^{z}\hat{\sigma}_{9}^{z}+$