

Time taken by A to cover 100 m = $\left(100 \times \frac{18}{25}\right)$ sec = 72 sec. more B and H A

\therefore Time taken by B to cover 92 m = $(72 + 8)$ sec = 80 sec.

\therefore B's speed = $\left(\frac{92}{80} \times \frac{18}{5}\right)$ kmph = 4.14 kmph. in 80 sec A and B travel

12. A : B = 100 : 80 and A : C = 100 : 72. more B and H A travel

$\therefore \frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{80}{100} \times \frac{100}{72}\right) = \frac{10}{9} = \frac{100}{90} = 100 : 90.$ more B and H A travel

\therefore B can give C 10 points.

13. A : B = 60 : 45 and A : C = 60 : 40. more A and B travel

$\therefore \frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{45}{60} \times \frac{60}{40}\right) = \frac{45}{40} = \frac{90}{80} = 90 : 80.$ more A and B travel

\therefore B can give C 10 points in a game of 90. more A and B travel

27. CALENDAR

IMPORTANT FACTS AND FORMULAE

Under this heading we mainly deal with finding the day of the week on a particular given date. The process of finding it lies on obtaining the number of odd days.

I. Odd Days : Number of days more than the complete number of weeks in a given period is the number of odd days during that period.

II. Leap Year : Every year which is divisible by 4 is called a leap year.

Thus, each one of the years 1992, 1996, 2004, 2008, 2012, etc. is a leap year.

Every 4th century is a leap year but no other century is a leap year.

Thus, each one of 400, 800, 1200, 1600, 2000, etc. is a leap year.

None of 1900, 2010, 2020, 2100, etc. is a leap year.

An year which is not a leap year is called an ordinary year.

III. (i) An ordinary year has 365 days. (ii) A leap year has 366 days.

IV. Counting of Odd Days :

(i) 1 ordinary year = 365 days = (52 weeks + 1 day).

∴ An ordinary year has 1 odd day.

(ii) 1 leap year = 366 days = (52 weeks + 2 days).

∴ A leap year has 2 odd days.

(iii) 100 years = 76 ordinary years + 24 leap years

$$= [(76 \times 52) \text{ weeks} + 76 \text{ days}] + [(24 \times 52) \text{ weeks} + 48 \text{ days}]$$

$$= 5200 \text{ weeks} + 124 \text{ days} = (5217 \text{ weeks} + 5 \text{ days}).$$

∴ 100 years contain 5 odd days.

200 years contain 10 and therefore 3 odd days.

300 years contain 15 and therefore 1 odd day.

400 years contain (20 + 1) and therefore 0 odd day.

Similarly, each one of 800, 1200, 1600, 2000, etc. contains 0 odd days.

Remark : $(7n + m)$ odd days, where $m < 7$ is equivalent to m odd days.

Thus, 8 odd days = 1 odd day etc.

V.	No. of odd days	0	1	2	3	4	5	6
	Day	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.

SOLVED EXAMPLES

Ex. 1. What was the day of the week on 16th July, 1776?

Sol. 16th July, 1776 = (1775 years + Period from 1st Jan., 1776 to 16th July, 1776)

Counting of odd days :

1600 years have 0 odd day. 100 years have 5 odd days.

75 years = (18 leap years + 57 ordinary years)

$$= [(18 \times 2) + (57 \times 1)] \text{ odd days} = 93 \text{ odd days}$$

$$= (13 \text{ weeks} + 2 \text{ days}) = 2 \text{ odd days.}$$

\therefore 1775 years have $(0 + 5 + 2)$ odd days = 7 odd days = 0 odd day.

Jan.	Feb.	March	April	May	June	July
31	+ 29	+ 31	+ 30	+ 31	+ 30	+ 16 = 198 days

$$= (28 \text{ weeks} + 2 \text{ days}) = 2 \text{ odd days.}$$

\therefore Total number of odd days = $(0 + 2) = 2$. Required day was 'Tuesday'.

Ex. 2. What was the day of the week on 15th August, 1947?

Sol. 15th August, 1947 = (1946 years + Period from 1st Jan., 1947 to 15th Aug., 1947)

Counting of odd days :

1600 years have 0 odd day. 300 years have 1 odd day.

47 years = (11 leap years + 36 ordinary years)

$$= [(11 \times 2) + (36 \times 1)] \text{ odd days} = 58 \text{ odd days} = 2 \text{ odd days.}$$

Jan.	Feb.	March	April	May	June	July	Aug.
31	+ 28	+ 31	+ 30	+ 31	+ 30	+ 31	+ 15

$$= 227 \text{ days} = (32 \text{ weeks} + 3 \text{ days}) = 3 \text{ odd days.}$$

Total number of odd days = $(0 + 1 + 2 + 3)$ odd days = 6 odd days.

Hence, the required day was 'Saturday'.

Ex. 3. What was the day of the week on 16th April, 2000?

Sol. 16th April, 2000 = (1999 years + Period from 1st Jan., 2000 to 16th April, 2000)

Counting of odd days :

1600 years have 0 odd day. 300 years have 1 odd day.

99 years = (24 leap years + 75 ordinary years)

$$= [(24 \times 2) + (75 \times 1)] \text{ odd days} = 123 \text{ odd days}$$

$$= (17 \text{ weeks} + 4 \text{ days}) = 4 \text{ odd days.}$$

Jan.	Feb.	March	April
------	------	-------	-------

$$31 + 28 + 31 + 16 = 107 \text{ days} = (15 \text{ weeks} + 2 \text{ days}) = 2 \text{ odd days.}$$

Total number of odd days = $(0 + 1 + 4 + 2)$ odd days = 7 odd days = 0 odd day.

Hence, the required day was 'Sunday'.

Ex. 4. On what dates of July 2004 did Monday fall?

Sol. Let us find the day on 1st July, 2004.

2000 years have 0 odd day. 3 ordinary years have 3 odd days.

Jan.	Feb.	March	April	May	June	July
31	+ 29	+ 31	+ 30	+ 31	+ 30	+ 1

$$= 183 \text{ days} = (26 \text{ weeks} + 1 \text{ day}) = 1 \text{ odd day.}$$

Total number of odd days = $(0 + 3 + 1)$ odd days = 4 odd days.

1st July 2004 was 'Thursday'.

Thus, 1st Monday in July 2004 was on 5th July.

Hence, during July 2004, Monday fell on 5th, 12th, 19th and 26th.

Ex. 5. Prove that the calendar for the year 2003 will serve for the year 2014.

Sol. In order that the calendar for the year 2003 and 2014 be the same, 1st January of both the years must be on the same day of the week.

For this, the number of odd days between 31st Dec., 2002 and 31st Dec., 2013 must be the same.

We know that an ordinary year has 1 odd day and a leap year has 2 odd days.

During this period, there are 3 leap years, namely 2004, 2008 and 2012 and 8 ordinary years.

Total number of odd days = $(6 + 8)$ days = 0 odd day.

Hence, the calendar for 2003 will serve for the year 2014.

Ex. 6. Prove that any date in March of a year is the same day of the week as the corresponding date in November of that year.

Sol. We will show that the number of odd days between last day of February and last day of October is zero.

March	April	May	June	July	Aug.	Sept.	Oct.	
31	+	30	+	31	+	30	+	31

$$= 241 \text{ days} = 35 \text{ weeks} = 0 \text{ odd day}$$

∴ Number of odd days during this period = 0.

Thus, 1st March of an year will be the same day as 1st November of that year.
Hence, the result follows.

EXERCISE 27

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer :

- January 1, 2004 was a Thursday. What day of the week lies on Jan. 1, 2005 ?
(a) Thursday (b) Friday (c) Saturday (d) Sunday
- On 8th March, 2005, Wednesday falls. What day of the week was it on 8th March, 2004 ?
(a) Sunday (b) Monday (c) Tuesday (d) Wednesday
- The calendar for the year 2005 is the same as for the year :
(a) 2010 (b) 2011 (c) 2012 (d) 2013
- On what dates of April 2001 did Sunday fall ?
(a) 1st, 8th, 15th, 22nd, 29th (b) 2nd, 9th, 16th, 23rd, 30th
(c) 4th, 11th, 18th, 25th (d) 6th, 13th, 20th, 27th
- What will be the day of the week on 1st January, 2010 ?
(a) Friday (b) Saturday (c) Sunday (d) Monday
- What was the day of the week on 17th June, 1998 ?
(a) Monday (b) Tuesday (c) Wednesday (d) Thursday
- What was the day of the week on 28th May, 2003 ?
(a) Friday (b) Saturday (c) Sunday (d) Monday
- Today is Friday. After 62 days, it will be :
(a) Saturday (b) Monday (c) Tuesday (d) Thursday
- The last day of a century cannot be :
(a) Monday (b) Wednesday (c) Friday (d) Tuesday
- The first Republic Day of India was celebrated on 26th January, 1950. It was :
(a) Tuesday (b) Wednesday (c) Thursday (d) Friday

SOLUTIONS

- The year 2004 being a leap year, it has 2 odd days. So, first day of 2005 will be 2 days beyond Thursday and so it will be Saturday.
- The year 2004 being a leap year, it has 2 odd days.
So, the day on 8th March, 2005 will be two days beyond the day on 8th March, 2004.
But, 8th March, 2005 is Wednesday. So, 8th March, 2004 is Monday.
- Count the number of days from 2005 onwards to get 0 odd day.
Year 2005 2006 2007 2008 2010 2011
Odd days 1 1 1 2 1 1 = 7 or 0 odd day
∴ Calendar for the year 2005 is the same as that for the year 2012.

4. Find the day on 1st April, 2001. 2000 years contain 2 odd days.
 Jan. Feb. March April
 $31 + 28 + 31 + 1 = 91$ days = 13 weeks 0 day = 0 odd day.
 Sunday fell on 1st, 8th, 15th, 22nd and 29th of April 2001.
5. 2000 years have 2 odd days.

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Odd days	1	1	1	2	1	1	1	2	1
$= 11$ odd days = 4 odd days,									

1st January, 2010 has 1 odd day. Total number of odd days = $(2 + 4 + 1) = 7 = 0$.
 \therefore 1st January, 2010 will be a Sunday.

6. 1600 years have 0 odd day. 300 years have 1 odd day.

$$\begin{aligned} 97 \text{ years} &= 24 \text{ leap years} + 73 \text{ ordinary years} \\ &= [(24 \times 2) + (73 \times 1)] \text{ odd days} = 121 \text{ odd days} \\ &= (17 \text{ weeks} + 2 \text{ days}) \text{ odd days} = 2 \text{ odd days}. \end{aligned}$$

$$\begin{aligned} \text{Jan.} &\quad \text{Feb.} \quad \text{March} \quad \text{April} \quad \text{May} \quad \text{June} \\ 31 + 28 + 31 + 30 + 31 + 17 &= 168 \text{ days} = 0 \text{ odd day}. \end{aligned}$$

$$\text{Total number of odd days} = (0 + 1 + 2 + 0) = 3 \text{ odd days}.$$

Hence, the required day was 'Wednesday'.

7. 2000 years have 2 odd days.

The years 2001 and 2002 have $(1 + 1) = 2$ odd days.

$$\begin{aligned} \text{Jan.} &\quad \text{Feb.} \quad \text{March} \quad \text{April} \quad \text{May} \\ 31 + 28 + 31 + 30 + 28 &= 148 \text{ days} = 21 \text{ weeks} + 1 \text{ day} = 1 \text{ odd day}. \end{aligned}$$

$$\text{Total number of odd days} = (2 + 2 + 1) = 5.$$

\therefore The required day was 'Friday'.

8. Each day of the week is repeated after 7 days. So, after 63 days, it will be Friday.
 Hence, after 62 days, it will be Thursday.

9. 100 years contain 5 odd days. So, last day of 1st century is 'Friday'.

$$200 \text{ years contain } (5 \times 2) = 10 \text{ odd days} = 3 \text{ odd days}.$$

So, last day of 2nd century is 'Wednesday'.

$$300 \text{ years contain } (5 \times 3) = 15 \text{ odd days} = 1 \text{ odd day}.$$

\therefore Last day of 3rd century is 'Monday'.

$$400 \text{ years contain } 0 \text{ odd day}.$$

\therefore Last day of 4th century is 'Sunday'.

Since the order is continually kept in successive cycles, we see that the last day of a century cannot be Tuesday, Thursday or Saturday.

10. 26th Jan., 1950 = (1949 years + Period from 1st Jan., 1950 to 26th Jan., 1950)

1600 years have 0 odd day. 300 years have 1 odd day.

$$49 \text{ years} = (12 \text{ leap years} + 37 \text{ ordinary years})$$

$$= [(12 \times 2) + (37 \times 1)] \text{ odd days} = 61 \text{ odd days} = 5 \text{ odd days}.$$

Number of days from 1st Jan. to 26th Jan. = 26 = 5 odd days.

$$\text{Total number of odd days} = (0 + 1 + 5 + 5) = 11 = 4 \text{ odd days}.$$

\therefore The required day was 'Thursday'.

28. CLOCKS

IMPORTANT FACTS

The face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

A clock has two hands, the smaller one is called the *hour hand* or *short hand* while the larger one is called the *minute hand* or *long hand*.

- (i) In 60 minutes, the minute hand gains 55 minutes on the hour hand.
- (ii) In every hour, both the hands coincide once.
- (iii) The hands are in the same straight line when they are coincident or opposite to each other.
- (iv) When the two hands are at right angles, they are 15 minute spaces apart.
- (v) When the hands are in opposite directions, they are 30 minute spaces apart.
- (vi) Angle traced by hour hand in 12 hrs = 360° .
- (vii) Angle traced by minute hand in 60 min. = 360° .

Too Fast and Too Slow: If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast.

On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

SOLVED EXAMPLES

Ex. 1. Find the angle between the hour hand and the minute hand of a clock when the time is 3.25.

Sol. Angle traced by the hour hand in 12 hours = 360° .

Angle traced by it in 3 hrs 25 min. i.e. $\frac{41}{12}$ hrs = $\left(\frac{360}{12} \times \frac{41}{12}\right)^\circ = 102\frac{1}{2}^\circ$.

Angle traced by minute hand in 60 min. = 360° .

Angle traced by it in 25 min. = $\left(\frac{360}{60} \times 25\right)^\circ = 150^\circ$.

$$\therefore \text{Required angle} = \left(150^\circ - 102\frac{1}{2}^\circ\right) = 47\frac{1}{2}^\circ$$

Ex. 2. At what time between 2 and 3 o'clock will the hands of a clock be together?

Sol. At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, i.e. they are 10 min. spaces apart.

To be together, the minute hand must gain 10 minutes over the hour hand.

Now, 55 minutes are gained by it in 60 min.

$$\therefore 10 \text{ minutes will be gained in } \left(\frac{60}{55} \times 10\right) \text{ min.} = 10\frac{10}{11} \text{ min.}$$

$$\therefore \text{The hands will coincide at } 10\frac{10}{11} \text{ min. past 2.}$$

Ex. 3. At what time between 4 and 5 o'clock will the hands of a clock be at right angle?

Sol. At 4 o'clock, the minute hand will be 20 min. spaces behind the hour hand.

Now, when the two hands are at right angles, they are 15 min. spaces apart.

So, they are at right angles in following two cases.

Case I. When minute hand is 15 min. spaces behind the hour hand :

In this case min. hand will have to gain $(20 - 15) = 5$ minute spaces.

55 min. spaces are gained by it in 60 min.

5 min. spaces will be gained by it in $\left(\frac{60}{55} \times 5\right)$ min. = $5\frac{5}{11}$ min.

∴ They are at right angles at $5\frac{5}{11}$ min. past 4.

Case II. When the minute hand is 15 min. spaces ahead of the hour hand :

To be in this position, the minute hand will have to gain $(20 + 15) = 35$ minute spaces.

55 min. spaces are gained in 60 min.

35 min. spaces are gained in $\left(\frac{60}{55} \times 35\right)$ min. = $38\frac{2}{11}$ min.

∴ They are at right angles at $38\frac{2}{11}$ min. past 4.

Ex. 4. Find at what time between 8 and 9 o'clock will the hands of a clock be in the same straight line but not together.

Sol. At 8 o'clock, the hour hand is at 8 and the minute hand is at 12, i.e. the two hands are 20 min. spaces apart.

To be in the same straight line but not together they will be 30 minute spaces apart. So, the minute hand will have to gain $(30 - 20) = 10$ minute spaces over the hour hand.

55 minute spaces are gained in 60 min.

10 minute spaces will be gained in $\left(\frac{60}{55} \times 10\right)$ min. = $10\frac{10}{11}$ min.

∴ The hands will be in the same straight line but not together at $10\frac{10}{11}$ min. past 8.

Ex. 5. At what time between 5 and 6 o'clock are the hands of a clock 3 minutes apart?

Sol. At 5 o'clock, the minute hand is 25 min. spaces behind the hour hand.

Case I. Minute hand is 3 min. spaces behind the hour hand.

In this case, the minute hand has to gain $(25 - 3) = 22$ minute spaces.

55 min. are gained in 60 min.

22 min. are gained in $\left(\frac{60}{55} \times 22\right)$ min. = 24 min.

∴ The hands will be 3 min. apart at 24 min. past 5.

Case II. Minute hand is 3 min. spaces ahead of the hour hand.

In this case, the minute hand has to gain $(25 + 3) = 28$ minute spaces.

55 min. are gained in 60 min.

28 min. are gained in $\left(\frac{60}{55} \times 28\right)$ = $31\frac{5}{11}$ min.

∴ The hands will be 3 min. apart at $31\frac{5}{11}$ min. past 5.

Ex. 6. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Sol. In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand. 55 min. are gained in 60 min.

$$60 \text{ min. are gained in } \left(\frac{60}{55} \times 60 \right) \text{ min.} = 65 \frac{5}{11} \text{ min.}$$

But, they are together after 65 min.

$$\therefore \text{Gain in } 65 \text{ min.} = \left(65 \frac{5}{11} - 65 \right) = \frac{5}{11} \text{ min.}$$

$$\text{Gain in 24 hours} = \left(\frac{5}{11} \times \frac{60 \times 24}{65} \right) \text{ min.} = 10 \frac{10}{43} \text{ min.}$$

$$\therefore \text{The clock gains } 10 \frac{10}{43} \text{ minutes in 24 hours.}$$

Ex. 7. A watch which gains uniformly, is 5 min. slow at 8 o'clock in the morning on Sunday and it is 5 min. 48 sec. fast at 8 p.m. on following Sunday. When was it correct?

Sol. Time from 8 a.m. on Sunday to 8 p.m. on following Sunday = 7 days 12 hours
= 180 hours

$$\therefore \text{The watch gains } \left(5 + 5 \frac{4}{5} \right) \text{ min. or } \frac{54}{5} \text{ min. in 180 hrs.}$$

$$\text{Now } \frac{54}{5} \text{ min. are gained in 180 hrs.}$$

$$\therefore 5 \text{ min. are gained in } \left(180 \times \frac{5}{54} \times 5 \right) \text{ hrs.} = 83 \text{ hrs } 20 \text{ min.} = 3 \text{ days } 11 \text{ hrs } 20 \text{ min.}$$

∴ Watch is correct 3 days 11 hrs 20 min. after 8 a.m. of Sunday.

∴ It will be correct at 20 min. past 7 p.m. on Wednesday.

Ex. 8. A clock is set right at 5 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?

Sol. Time from 5 a.m. on a day to 10 p.m. on 4th day = 89 hours.

Now 23 hrs 44 min. of this clock = 24 hours of correct clock.

$$\frac{356}{15} \text{ hrs of this clock} = 24 \text{ hours of correct clock.}$$

$$89 \text{ hrs of this clock} = \left(24 \times \frac{15}{356} \times 89 \right) \text{ hrs of correct clock.}$$

$$= 90 \text{ hrs of correct clock.}$$

So, the correct time is 11 p.m.

Ex. 9. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 1 p.m. on the following day?

Sol. Time from 8 a.m. on a day to 1 p.m. on the following day = 29 hours.

24 hours 10 min. of this clock = 24 hours of the correct clock.

$$\frac{145}{6} \text{ hrs of this clock} = 24 \text{ hrs of the correct clock}$$

29 hrs is slower than hand speed $\left(\frac{6}{145} \right)$ hrs of the hand. So it will take $\frac{6}{145} \times 29$ hrs of the correct clock to catch up.

$$= 28 \text{ hrs } 48 \text{ min. of correct clock}$$

∴ The correct time is 28 hrs 48 min. after 8 a.m.

This is 48 min. past 12.

EXERCISE 28

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer :

- A clock is started at noon. By 10 minutes past 5, the hour hand has turned through :
 - (a) 145°
 - (b) 150°
 - (c) 155°
 - (d) 160°
- An accurate clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon ? (I.A.S. 2000)
 - (a) 144°
 - (b) 150°
 - (c) 168°
 - (d) 180°
- At 3.40, the hour hand and the minute hand of a clock form an angle of :
 - (a) 120°
 - (b) 125°
 - (c) 130°
 - (d) 135°
- The angle between the minute hand and the hour hand of a clock when the time is 8.30, is :
 - (a) 80°
 - (b) 75°
 - (c) 60°
 - (d) 105°
- The angle between the minute hand and the hour hand of a clock when the time is 4.20, is :
 - (a) 0°
 - (b) 10°
 - (c) 5°
 - (d) 20°
- At what angle the hands of a clock are inclined at 15 minutes past 5 ?
 - (a) $58\frac{1}{2}^\circ$
 - (b) 64°
 - (c) $67\frac{1}{2}^\circ$
 - (d) $72\frac{1}{2}^\circ$
- The reflex angle between the hands of a clock at 10.25 is : (L.I.C.A.A.O. 2003)
 - (a) 180°
 - (b) $192\frac{1}{2}^\circ$
 - (c) 195°
 - (d) $197\frac{1}{2}^\circ$
- How many times do the hands of a clock coincide in a day ? (S.C.R.A. 1996)
 - (a) 20
 - (b) 21
 - (c) 22
 - (d) 24
- How many times in a day, the hands of a clock are straight ?
 - (a) 22
 - (b) 24
 - (c) 44
 - (d) 48
- How many times are the hands of a clock at right angle in a day ? (I.A.S. 1997)
 - (a) 22
 - (b) 24
 - (c) 44
 - (d) 48
- How many times in a day, are the hands of a clock in straight line but opposite in direction ? (R.R.B. 2003)
 - (a) 20
 - (b) 22
 - (c) 24
 - (d) 48
- How much does a watch lose per day, if its hands coincide every 64 minutes ?
 - (a) $32\frac{8}{11}$ min.
 - (b) $36\frac{5}{11}$ min.
 - (c) 90 min.
 - (d) 96 min.
- At what time, in minutes, between 3 o'clock and 4 o'clock, both the needles will coincide each other ? (R.R.B. 2002)
 - (a) $5\frac{1}{11}$
 - (b) $12\frac{4}{11}$
 - (c) $13\frac{4}{11}$
 - (d) $16\frac{4}{11}$

ANSWERS

1. (c) 2. (d) 3. (c) 4. (b) 5. (b) 6. (c) 7. (d) 8. (c) 9. (c) 10. (c)
 11. (b) 12. (g) 13. (d) 14. (c) 15. (d) 16. (d) 17. (b) 18. (b) 19. (b)

SOLUTIONS

1. Angle traced by hour hand in 12 hrs = 360° .

$$\text{Angle traced by hour hand in } 5 \text{ hrs } 10 \text{ min. i.e. } \frac{31}{6} \text{ hrs.} = \left(\frac{360}{12} \times \frac{31}{6} \right)^\circ = 155^\circ.$$

$$2. \text{ Angle traced by the hour hand in 6 hours} = \left(\frac{\frac{360}{12}}{12} \times 6 \right)^\circ = 180^\circ.$$

3. Angle traced by hour hand in 12 hrs. = 360° .

$$\text{Angle traced by it in } \frac{\frac{11}{3}}{3} \text{ hrs} = \left(\frac{360}{12} \times \frac{11}{3} \right)^\circ = 110^\circ.$$

Angle traced by minute hand in 60 min. = 360° .

Angle traced by minute hand in 40 min. = $\left(\frac{360}{60} \times 40\right)^\circ = 240^\circ$.

∴ Required angle $(240 - 110)^\circ = 130^\circ$.

4. Angle traced by hour hand in $\frac{17}{2}$ hrs = $\left(\frac{360}{12} \times \frac{17}{2}\right)^\circ = 255^\circ$.

Angle traced by min. hand in 30 min. = $\left(\frac{360}{60} \times 30\right)^\circ = 180^\circ$.

∴ Required angle = $(255 - 180)^\circ = 75^\circ$.

5. Angle traced by hour hand in $\frac{13}{3}$ hrs = $\left(\frac{360}{12} \times \frac{13}{3}\right)^\circ = 130^\circ$.

Angle traced by min. hand in 20 min. = $\left(\frac{360}{60} \times 20\right)^\circ = 120^\circ$.

∴ Required angle = $(130 - 120)^\circ = 10^\circ$.

6. Angle traced by hour hand in $\frac{21}{4}$ hrs = $\left(\frac{360}{12} \times \frac{21}{4}\right)^\circ = 157\frac{1}{2}^\circ$.

Angle traced by min. hand in 15 min. = $\left(\frac{360}{60} \times 15\right)^\circ = 90^\circ$.

∴ Required angle = $\left(157\frac{1}{2}\right)^\circ - 90^\circ = 67\frac{1}{2}^\circ$.

7. Angle traced by hour hand in $\frac{125}{12}$ hrs = $\left(\frac{360}{12} \times \frac{125}{12}\right)^\circ = 312\frac{1}{2}^\circ$.

Angle traced by minute hand in 25 min = $\left(\frac{360}{60} \times 25\right)^\circ = 150^\circ$.

∴ Reflex angle = $360^\circ - \left(312\frac{1}{2} - 150\right)^\circ = 360^\circ - 162\frac{1}{2}^\circ = 197\frac{1}{2}^\circ$.

8. The hands of a clock coincide 11 times in every 12 hours (Since between 11 and 1, they coincide only once, i.e. at 12 o'clock).

∴ The hands coincide 22 times in a day.

9. In 12 hours, the hands coincide or are in opposite direction 22 times.

∴ In 24 hours, the hands coincide or are in opposite direction 44 times a day.

10. In 12 hours, they are at right angles 22 times.

∴ In 24 hours, they are at right angles 44 times.

11. The hands of a clock point in opposite directions (in the same straight line) 11 times in every 12 hours (Because between 5 and 7 they point in opposite directions at 6 o'clock only). So, in a day, the hands point in the opposite directions 22 times.

12. 55 min. spaces are covered in 60 min.

60 min. spaces are covered in $\left(\frac{60}{55} \times 60\right)$ min. = $65\frac{5}{11}$ min.

Loss in 64 min. = $\left(65\frac{5}{11} - 64\right) = \frac{16}{11}$ min.

Loss in 24 hrs = $\left(\frac{16}{11} \times \frac{1}{64} \times 24 \times 60\right)$ min. = $32\frac{8}{11}$ min.

13. At 3 o'clock, the minute hand is 15 min. spaces apart from the hour hand.

To be coincident, it must gain 15 min. spaces.

55 min. are gained in 60 min.

$$15 \text{ min. are gained in } \left(\frac{60}{55} \times 15 \right) \text{ min.} = 16 \frac{4}{11} \text{ min.}$$

∴ The hands are coincident at $16 \frac{4}{11}$ min. past 3.

14. To be together between 9 and 10 o'clock, the minute hand has to gain 45 min. spaces.

55 min. spaces gained in 60 min.

$$45 \text{ min. spaces are gained in } \left(\frac{60}{55} \times 45 \right) \text{ min. or } 49 \frac{1}{11} \text{ min.}$$

∴ The hands are together at $49 \frac{1}{11}$ min. past 9.

15. When the hands of the clock are in the same straight line but not together, they are 30 minute spaces apart.

At 7 o'clock, they are 25 min. spaces apart.

∴ Minute hand will have to gain only 5 min. spaces.

55 min. spaces are gained in 60 min.

$$5 \text{ min. spaces are gained in } \left(\frac{60}{55} \times 5 \right) \text{ min.} = 5 \frac{5}{11} \text{ min.}$$

∴ Required time = $5 \frac{5}{11}$ min. past 7.

16. At 4 o'clock, the hands of the watch are 20 min. spaces apart.

To be in opposite directions, they must be 30 min. spaces apart.

∴ Minute hand will have to gain 50 min. spaces.

55 min. spaces are gained in 60 min.

$$50 \text{ min. spaces are gained in } \left(\frac{60}{55} \times 50 \right) \text{ min. or } 54 \frac{6}{11} \text{ min.}$$

∴ Required time = $54 \frac{6}{11}$ min. past 4.

17. At 5 o'clock, the hands are 25 min. spaces apart.

To be at right angles and that too between 5.30 and 6, the minute hand has to gain $(25 + 15) = 40$ min. spaces.

55 min. spaces are gained in 60 min.

$$40 \text{ min. spaces are gained in } \left(\frac{60}{55} \times 40 \right) \text{ min.} = 43 \frac{7}{11} \text{ min.}$$

∴ Required time = $43 \frac{7}{11}$ min. past 5.

18. Time from 12 p.m. on Monday to 2 p.m. on the following Monday = 7 days 2 hours
= 170 hours.

∴ The watch gains $\left(2 + 4 \frac{4}{5} \right)$ min. or $\frac{34}{5}$ min. in 170 hrs.

Now, $\frac{34}{5}$ min. are gained in 170 hrs.

- hand would set more than $\left(170 \times \frac{5}{34} \times 2\right)$ hrs = 50 hrs.
 \therefore 2 min. are gained in 50 hrs.
 \therefore Watch is correct 2 days 2 hrs. after 12 p.m. on Monday i.e. it will be correct at 2 p.m. on Wednesday.

19. Time from 7 a.m. to 4.15 p.m. = 9 hrs 15 min. = $\frac{37}{4}$ hrs.

3 min. 5 sec. of this clock = 3 min. of the correct clock.

$\Rightarrow \frac{37}{720}$ hrs of this clock = $\frac{1}{20}$ hrs of the correct clock

$\Rightarrow \frac{37}{4}$ hrs of this clock = $\left(\frac{1}{20} \times \frac{720}{37} \times \frac{37}{4}\right)$ hrs of the correct clock
 $= 9$ hrs of the correct clock

\therefore The correct time is 9 hrs after 7 a.m. i.e. 4 p.m.

29. STOCK AND SHARES

To start a big business or an industry, a large amount of money is needed. It is beyond the capacity of one or two persons to arrange such a huge amount. However, some persons associate together to form a company. They, then, draft a proposal, issue a prospectus (in the name of the company), explaining the plan of the project and invite the public to invest money in this project. They, thus, pool up the funds from the public, by assigning them *shares* of the company.

IMPORTANT FACTS AND FORMULAE

1. Stock-capital : The total amount of money needed to run the company is called the stock-capital.
2. Shares or Stock : The whole capital is divided into small units, called shares or stock.

For each investment, the company issues a share-certificate, showing the value of each share and the number of shares held by a person.

The person who subscribes in shares or stock is called a **share holder or stock holder**.

3. Dividend : The annual profit distributed among share holders is called dividend. Dividend is paid annually as per share or as a percentage.
4. Face Value : The value of a share or stock printed on the share-certificate is called its Face Value or Nominal Value or Par Value.
5. Market Value : The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (or stock) is said to be :

(i) **At premium or Above par**, if its market value is more than its face value.
(ii) **At par**, if its market value is the same as its face value.

(iii) **At discount or Below par**, if its market value is less than its face value. Thus, if a Rs. 100 stock is quoted at a premium of 16, then market value of the stock = Rs. $(100 + 16)$ = Re. 116.

Likewise, if a Rs. 100 stock is quoted at a discount of 7, then market value of the stock = Rs. $(100 - 7)$ = Rs. 93.

6. Brokerage : The broker's charge is called brokerage.

(i) When stock is purchased, brokerage is added to the cost price.
(ii) When stock is sold, brokerage is subtracted from the selling price.

Remember :

- The face value of a share always remains the same.
- The market value of a share changes from time to time.
- Dividend is always paid on the face value of a share.
- Number of shares held by a person

$$= \frac{\text{Total Investment}}{\text{Investment in 1 share}} = \frac{\text{Total Income}}{\text{Income from 1 share}} = \frac{\text{Total Face Value}}{\text{Face value of 1 share}}$$

Thus, by a Rs. 100, 9% stock at 120, we mean that :

- Face Value (N.V.) of stock = Rs. 100.
- Market Value (M.V.) of stock = Rs. 120.
- Annual dividend on 1 share = 9% of face value = 9% of Rs. 100 = Rs. 9.
- An investment of Rs. 120 gives an annual income of Rs. 9.
- Rate of interest p.a. = Annual income from an investment of Rs. 100

$$= \left(\frac{9}{120} \times 100 \right) \% = 7\frac{1}{2}\%$$

SOLVED EXAMPLES

Ex. 1. Find the cost of :

- Rs. 7200, 8% stock at 90;
- Rs. 4500, 8.5% stock at 4 premium;
- Rs. 6400, 10% stock at 15 discount.

Sol. (i) Cost of Rs. 100 stock = Rs. 90.

$$\text{Cost of Rs. 7200 stock} = \text{Rs.} \left(\frac{90}{100} \times 7200 \right) = \text{Rs.} 6480.$$

(ii) Cost of Rs. 100 stock = Rs. $(100 + 4)$ = Rs. 104.

$$\text{Cost of Rs. 4500 stock} = \text{Rs.} \left(\frac{104}{100} \times 4500 \right) = \text{Rs.} 4680.$$

(iii) Cost of Rs. 100 stock = Rs. $(100 - 15)$ = Rs. 85.

$$\text{Cost of Rs. 6400 stock} = \text{Rs.} \left(\frac{85}{100} \times 6400 \right) = \text{Rs.} 5440.$$

Ex. 2. Find the cash required to purchase Rs. 3200, $7\frac{1}{2}\%$ stock at 107 (brokerage $\frac{1}{2}\%$).

Sol. Cash required to purchase Rs. 100 stock = Rs. $\left(107 + \frac{1}{2} \right) = \text{Rs. } \frac{215}{2}$.

$$\text{Cash required to purchase Rs. 3200 stock} = \text{Rs.} \left(\frac{215}{2} \times \frac{1}{100} \times 3200 \right) = \text{Rs.} 3440.$$

Ex. 3. Find the cash realised by selling Rs. 2400, 9.5% stock at 4 discount (brokerage $\frac{1}{4}\%$).

Sol. By selling Rs. 100 stock, cash realised = Rs. $\left[(100 - 4) - \frac{1}{4} \right] = \text{Rs. } \frac{383}{4}$.

$$\text{By selling Rs. 2400 stock, cash realised} = \text{Rs.} \left(\frac{383}{4} \times \frac{1}{100} \times 2400 \right) = \text{Rs.} 2296.$$

Ex. 4. Find the annual income derived from Rs. 2500, 8% stock at 106.

Sol. Income from Rs. 100 stock = Rs. 8.

$$\text{Income from Rs. 2500 stock} = \text{Rs.} \left(\frac{8}{100} \times 2500 \right) = \text{Rs.} 200.$$

Ex. 5. Find the annual income derived by investing Rs. 6800 in 10% stock at 136.

Sol. By investing Rs. 136, income obtained = Rs. 10.

$$\text{By investing Rs. 6800, income obtained} = \text{Rs.} \left(\frac{10}{136} \times 6800 \right) = \text{Rs.} 500.$$

Ex. 6. Which is better investment? $7\frac{1}{2}\%$ stock at 105 or $6\frac{1}{2}\%$ stock at 94.

Sol. Let the investment in each case be Rs. (105×94) .

Case I : $7\frac{1}{2}\%$ stock at 105 :

$$\text{On investing Rs. 105, income} = \text{Rs.} \frac{15}{2}.$$

$$\text{On investing Rs.} (105 \times 94), \text{income} = \text{Rs.} \left(\frac{15}{2} \times \frac{1}{105} \times 105 \times 94 \right) = \text{Rs.} 705.$$

Case II : $6\frac{1}{2}\%$ stock at 94 :

$$\text{On investing Rs. 94, income} = \text{Rs.} \frac{13}{2}.$$

$$\text{On investing Rs.} (105 \times 94), \text{income} = \text{Rs.} \left(\frac{13}{2} \times \frac{1}{94} \times 105 \times 94 \right) = \text{Rs.} 682.50.$$

Clearly, the income from $7\frac{1}{2}\%$ stock at 105 is more.

Hence, the investment in $7\frac{1}{2}\%$ stock at 105 is better.

Ex. 7. Find the cost of 96 shares of Rs. 10 each at $\frac{3}{4}$ discount, brokerage being $\frac{1}{4}$ per share. (L.I.C. 2003)

$$\text{Sol. Cost of 1 share} = \text{Rs.} \left[\left(10 - \frac{3}{4} \right) + \frac{1}{4} \right] = \text{Rs.} \frac{19}{2}.$$

$$\text{Cost of 96 shares} = \text{Rs.} \left(\frac{19}{2} \times 96 \right) = \text{Rs.} 912.$$

Ex. 8. Find the income derived from 88 shares of Rs. 25 each at 5 premium, brokerage being $\frac{1}{4}$ per share and the rate of dividend being $7\frac{1}{2}\%$ per annum. Also, find the rate of interest on the investment.

$$\text{Sol. Cost of 1 share} = \text{Rs.} \left(25 + 5 + \frac{1}{4} \right) = \text{Rs.} \frac{121}{4}.$$

$$\text{Cost of 88 shares} = \text{Rs.} \left(\frac{121}{4} \times 88 \right) = \text{Rs.} 2662.$$

Investment made = Rs. 2662.

Face value of 88 shares = Rs. (88×25) = Rs. 2200.

$$\text{Dividend on Rs. 100} = \frac{15}{2}.$$

$$\text{Dividend on Rs. 2200} = \text{Rs.} \left(\frac{15}{2} \times \frac{1}{100} \times 2200 \right) = \text{Rs.} 165.$$

Income derived = Rs. 165.

$$\text{Rate of interest on investment} = \left(\frac{165}{2662} \times 100 \right) = 6.2\%.$$

Ex. 9. A man buys Rs. 25 shares in a company which pays 9% dividend. The money invested is such that it gives 10% on investment. At what price did he buy the shares?

Sol. Suppose he buys each share for Rs. x .

$$\text{Then, } \left(25 \times \frac{9}{100} \right) = \left(x \times \frac{10}{100} \right) \text{ or } x = 22.50.$$

∴ Cost of each share = Rs. 22.50.

Ex. 10. A man sells Rs. 5000, 12% stock at 156 and invests the proceeds partly in 8% stock at 90 and 9% stock at 108. He thereby increases his income by Rs. 70. How much of the proceeds were invested in each stock?

$$\text{Sol. S.P. of Rs. 5000 stock} = \text{Rs.} \left(\frac{156}{100} \times 5000 \right) = \text{Rs.} 7800.$$

$$\text{Income from this stock} = \text{Rs.} \left(\frac{12}{100} \times 5000 \right) = \text{Rs.} 600.$$

Let investment in 8% stock be x and that in 9% stock = $(7800 - x)$.

$$\therefore \left(x \times \frac{8}{90} \right) + (7800 - x) \times \frac{9}{108} = (600 + 70)$$

$$\Leftrightarrow \frac{4x}{45} + \frac{7800 - x}{12} = 670 \Leftrightarrow 16x + 117000 - 15x = (670 \times 180) \Leftrightarrow x = 3600.$$

∴ Money invested in 8% stock at 90 = Rs. 3600.

Money invested in 9% at 108 = Rs. $(7800 - 3600) = \text{Rs.} 4200$.

EXERCISE 29

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer :

- The cost price of a Rs. 100 stock at 4 discount, when brokerage is $\frac{1}{4}\%$ is :
(a) Rs. 95.75 (b) Rs. 96 (c) Rs. 96.25 (d) Rs. 104.25
- The cash realised on selling a 14% stock at Rs. 106.25, brokerage being $\frac{1}{4}\%$, is :
(a) Rs. 105.50 (b) Rs. 106 (c) Rs. 106.50 (d) Rs. 113.75
- How many shares of market value Rs. 25 each can be purchased for Rs. 12750, brokerage being 2% ?
(M.A.T. 2002)
(a) 450 (b) 500 (c) 550 (d) 600
- A man invests in a 16% stock at 128. The interest obtained by him is :
(a) 8% (b) 12% (c) 12.5% (d) 16%
- The income derived from a Rs. 100, 13% stock at Rs. 105, is :
(a) Rs. 5 (b) Rs. 8 (c) Rs. 13 (d) Rs. 18
- A man invested Rs. 4455 in Rs. 10 shares quoted at Rs. 8.25. If the rate of dividend be 12%, his annual income is :
(a) Rs. 207.40 (b) Rs. 534.60 (c) Rs. 648 (d) Rs. 655.60
- A man invested Rs. 14,400 in Rs. 100 shares of a company at 20% premium. If the company declares 5% dividend at the end of the year, then how much does he get ?
(a) Rs. 500 (b) Rs. 600 (c) Rs. 650 (d) Rs. 720
(Hotel Management, 2003)
- A 6% stock yields 8%. The market value of the stock is :
(a) Rs. 48 (b) Rs. 75 (c) Rs. 96 (d) Rs. 133.33

9. A 9% stock yields 8%. The market value of the stock is :
 (a) Rs. 72 (b) Rs. 92 (c) Rs. 112.50 (d) Rs. 116.50
10. A 12% stock yielding 10% is quoted at :
 (a) Rs. 83.33 (b) Rs. 110 (c) Rs. 112 (d) Rs. 120
11. By investing Rs. 1620 in 8% stock, Michael earns Rs. 135. The stock is then quoted at :
 (a) Rs. 80 (b) Rs. 96 (c) Rs. 106 (d) Rs. 108
12. To produce an annual income of Rs. 1200 from a 12% stock at 90, the amount of stock needed is :
 (a) Rs. 10,000 (b) Rs. 10,800 (c) Rs. 14,400 (d) Rs. 16,000
13. In order to obtain an income of Rs. 650 from 10% stock at Rs. 96, one must make an investment of :
 (a) Rs. 3100 (b) Rs. 6240 (c) Rs. 6500 (d) Rs. 9600
14. By investing in $16\frac{2}{3}\%$ stock at 64, one earns Rs. 1500. The investment made is :
 (a) Rs. 5640 (b) Rs. 5760 (c) Rs. 7500 (d) Rs. 9600
15. A man invested Rs. 1552 in a stock at 97 to obtain an income of Rs. 128. The dividend from the stock is :
 (a) 7.5% (b) 8% (c) 9.7% (d) None of these.
16. A man bought 20 shares of Rs. 50 at 5 discount, the rate of dividend being $13\frac{1}{2}\%$.
 The rate of interest obtained is :
 (a) $12\frac{1}{2}\%$ (b) $13\frac{1}{2}\%$ (c) 15% (d) $16\frac{2}{3}\%$
17. A man buys Rs. 20 shares paying 9% dividend. The man wants to have an interest of 12% on his money. The market value of each share is :
 (a) Rs. 12 (b) Rs. 15 (c) Rs. 18 (d) Rs. 21
18. A man buys Rs. 50 shares in a company which pays 10% dividend. If the man gets 12.5% on his investment, at what price did he buy the shares ? (L.I.C.A.A.O. 2003)
 (a) Rs. 37.50 (b) Rs. 40 (c) Rs. 48 (d) Rs. 52
19. The market value of a 10.5% stock, in which an income of Rs. 756 is derived by investing Rs. 9000, brokerage being $\frac{1}{4}\%$, is :
 (a) Rs. 108.25 (b) Rs. 112.20 (c) Rs. 124.75 (d) Rs. 125.25
20. Sakshi invests a part of Rs. 12,000 in 12% stock at Rs. 120 and the remainder in 15% stock at Rs. 125. If his total dividend per annum is Rs. 1360, how much does he invest in 12% stock at Rs. 120 ?
 (a) Rs. 4000 (b) Rs. 4500 (c) Rs. 5500 (d) Rs. 6000
21. Rs. 9800 are invested partly in 9% stock at 75 and 10% stock at 80 to have equal amount of incomes. The investment in 9% stock is :
 (a) Rs. 4800 (b) Rs. 5000 (c) Rs. 5400 (d) Rs. 5600
22. A man invests some money partly in 9% stock at 96 and partly in 12% stock at 120. To obtain equal dividends from both, he must invest the money in the ratio :
 (a) 3 : 4 (b) 3 : 5 (c) 4 : 5 (d) 16 : 15
23. Which is better investment — 11% stock at 143 or $9\frac{3}{4}\%$ stock at 117 ?
 (a) 11% stock at 143 (b) $9\frac{3}{4}\%$ stock at 117 (c) Both are equally good
 (d) Cannot be compared, as the total amount of investment is not given

24. Which is better investment, 12% stock at par with an income tax at the rate of 5 paise per rupee or $14\frac{2}{7}\%$ stock at 120 free from income tax?

- (a) 12% stock (b) $14\frac{2}{7}\%$ stock (c) Both are equally good
 (d) Cannot be compared

25. A invested some money in 10% stock at 96. If B wants to invest in an equally good 12% stock, he must purchase a stock worth of:

- (a) Rs. 80 (b) Rs. 115.20 (c) Rs. 120 (d) Rs. 125.40

ANSWERS

1. (c) 2. (b) 3. (b) 4. (c) 5. (c) 6. (c) 7. (b) 8. (b) 9. (c) 10. (d)
 11. (b) 12. (a) 13. (b) 14. (b) 15. (b) 16. (e) 17. (b) 18. (b) 19. (c) 20. (a)
 21. (b) 22. (d) 23. (b) 24. (b) 25. (b)

SOLUTIONS

1. C.P. = Rs. $\left(100 - 4 + \frac{1}{4}\right) = \text{Rs. } 96.25.$

2. Cash realised = Rs. $(106.25 - 0.25) = \text{Rs. } 106.$

3. C.P. of each share = Rs. $(25 + 2\% \text{ of } 25) = \text{Rs. } 25.50.$

\therefore Number of shares = $\left(\frac{12750}{25.50}\right) = 500.$

4. By investing Rs. 128, income derived = Rs. 16.

By investing Rs. 100, income derived = Rs. $\left(\frac{16}{128} \times 100\right) = \text{Rs. } 12.5.$

\therefore Interest obtained = 12.5%.

5. Income on Rs. 100 stock = Rs. 13.

6. Number of shares = $\left(\frac{4455}{8.25}\right) = 540.$

Face value = Rs. $(540 \times 10) = \text{Rs. } 5400.$

Annual income = Rs. $\left(\frac{12}{100} \times 5400\right) = \text{Rs. } 648.$

7. Number of shares = $\left(\frac{14400}{120}\right) = 120.$

Face value = Rs. $(100 \times 120) = \text{Rs. } 12000.$

Annual income = Rs. $\left(\frac{5}{100} \times 12000\right) = \text{Rs. } 600.$

8. For an income of Rs. 8, investment = Rs. 100.

For an income of Rs. 6, investment = Rs. $\left(\frac{100}{8} \times 6\right) = \text{Rs. } 75.$

\therefore Market value of Rs. 100 stock = Rs. 75.

9. To obtain Rs. 8, investment = Rs. 100.

$$\text{To obtain Rs. 9, investment} = \text{Rs.} \left(\frac{100}{8} \times 9 \right) = \text{Rs. } 112.50.$$

\therefore Market value of Rs. 100 stock = Rs. 112.50.

10. To earn Rs. 10, money invested = Rs. 100.

$$\text{To earn Rs. 12, money invested} = \text{Rs.} \left(\frac{100}{10} \times 12 \right) = \text{Rs. } 120.$$

\therefore Market value of Rs. 100 stock = Rs. 120.

11. To earn Rs. 135, investment = Rs. 1620.

$$\text{To earn Rs. 8, investment} = \text{Rs.} \left(\frac{1620}{135} \times 8 \right) = \text{Rs. } 96.$$

\therefore Market value of Rs. 100 stock = Rs. 96.

12. For an income of Rs. 12, stock needed = Rs. 100.

$$\text{For an income of Rs. 1200, stock needed} = \text{Rs.} \left(\frac{100}{12} \times 1200 \right) = \text{Rs. } 10,000.$$

13. To obtain Rs. 10, investment = Rs. 96.

$$\text{To obtain Rs. 650, investment} = \text{Rs.} \left(\frac{96}{10} \times 650 \right) = \text{Rs. } 6240.$$

14. To earn $\frac{50}{3}$, investment = Rs. 64.

$$\text{To earn Rs. 1500, investment} = \text{Rs.} \left(64 \times \frac{3}{50} \times 1500 \right) = \text{Rs. } 5760.$$

15. By investing Rs. 1552, income = Rs. 128.

$$\text{By investing Rs. 97, income} = \text{Rs.} \left(\frac{128}{1552} \times 97 \right) = \text{Rs. } 8.$$

\therefore Dividend = 8%.

16. Investment = Rs. $[20 \times (50 - 5)]$ = Rs. 900.

Face value = Rs. (50×20) = Rs. 1000.

$$\text{Dividend} = \text{Rs.} \left(\frac{27}{2} \times \frac{1000}{100} \right) = \text{Rs. } 135.$$

$$\text{Interest obtained} = \left(\frac{135}{900} \times 100 \right)\% = 15\%.$$

17. Dividend on Rs. 20 = Rs. $\left(\frac{9}{100} \times 20 \right)$ = Rs. $\frac{9}{5}$.

Rs. 12 is an income on Rs. 100.

$$\therefore \text{Rs. } \frac{9}{5} \text{ is an income on Rs.} \left(\frac{100}{12} \times \frac{9}{5} \right) = \text{Rs. } 15.$$

18. Dividend on 1 share = Rs. $\left(\frac{10}{100} \times 50 \right)$ = Rs. 5.

Rs. 12.50 is an income on an investment of Rs. 100.

$$\text{Rs. } 5 \text{ is an income on an investment of Rs.} \left(100 \times \frac{2}{25} \times 5 \right) = \text{Rs. } 40.$$

\therefore Cost of 1 share = Rs. 40.

19. For an income of Rs. 756, investment = Rs. 9000.

$$\text{For an income of Rs. } \frac{21}{2}, \text{ investment} = \text{Rs.} \left(\frac{9000}{756} \times \frac{21}{2} \right) = \text{Rs. } 125.$$

∴ For a Rs. 100 stock, investment = Rs. 125.

$$\text{Market value of Rs. 100 stock} = \text{Rs.} \left(125 - \frac{1}{4} \right) = \text{Rs. } 124.75.$$

20. Let investment in 12% stock be Rs. x .

Then, investment in 15% stock = Rs. $(12000 - x)$.

$$\frac{12}{120} \times x + \frac{15}{125} \times (12000 - x) = 1360$$

$$\Leftrightarrow \frac{x}{10} + \frac{3}{25} (12000 - x) = 1360$$

$$\Leftrightarrow 5x + 72000 - 6x = 1360 \times 50 \Leftrightarrow x = 4000.$$

21. Let the investment in 9% stock be Rs. x .

Then, investment in 10% stock = Rs. $(9800 - x)$.

$$\frac{9}{75} \times x = \frac{10}{80} \times (9800 - x) \Leftrightarrow \frac{3x}{25} = \frac{9800 - x}{8}$$

$$\Leftrightarrow 24x = 9800 \times 25 - 25x \Leftrightarrow 49x = 9800 \times 25 \Leftrightarrow x = 5000.$$

22. For an income of Re. 1 in 9% stock at 96, investment = Rs. $\left(\frac{96}{9} \right) = \text{Rs. } \frac{32}{3}$.

$$\text{For an income of Re. 1 in 12% stock at 120, investment} = \text{Rs.} \left(\frac{120}{12} \right) = \text{Rs. } 10.$$

$$\therefore \text{Ratio of investments} = \frac{32}{3} : 10 = 32 : 30 = 16 : 15.$$

23. Let investment in each case be Rs. (143×117) .

$$\text{Income in 1st case} = \text{Rs.} \left(\frac{11}{143} \times 143 \times 117 \right) = \text{Rs. } 1287.$$

$$\text{Income in 2nd case} = \text{Rs.} \left(\frac{39}{4 \times 117} \times 143 \times 117 \right) = \text{Rs. } 1394.25.$$

Clearly, $9\frac{3}{4}\%$ stock at 117 is better.

24. Let investment in each case = Rs. (100×120) .

$$\text{Income from 12% stock} = \text{Rs.} \left(\frac{12}{100} \times 100 \times 120 \right) = \text{Rs. } 1440.$$

$$\text{Net income} = \text{Rs.} \left(1440 - \frac{5}{100} \times 1440 \right) = \text{Rs. } 1368.$$

$$\text{Income from } 14\frac{2}{7}\% \text{ stock} = \text{Rs.} \left(\frac{100}{7 \times 20} \times 100 \times 120 \right) = \text{Rs. } 1428.57.$$

Clearly, $14\frac{2}{7}\%$ stock is better.

25. For an income of Rs. 10, investment = Rs. 96.

$$\text{For an income of Rs. 12, investment} = \text{Rs.} \left(\frac{96}{10} \times 12 \right) = \text{Rs. } 115.20.$$

30. PERMUTATIONS AND COMBINATIONS

IMPORTANT FACTS AND FORMULAE

Factorial Notation : Let n be a positive integer. Then, factorial n , denoted by $[n]$ or $n!$ is defined as :

$$n! = n(n-1)(n-2) \dots 3.2.1.$$

Examples : (i) $5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$; (ii) $4! = (4 \times 3 \times 2 \times 1) = 24$ etc.

We define, $0! = 1$.

Permutations : The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Ex. 1. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (ab, ba, ac, ca, bc, cb).

Ex. 2. All permutations made with the letters a, b, c , taking all at a time are : ($abc, aca, bac, bca, cab, cba$).

Number of Permutations : Number of all permutations of n things, taken r at a time, is given by :

$${}^n P_r = n(n-1)(n-2) \dots (n-r+1) = \frac{n!}{(n-r)!}$$

Examples : (i) ${}^6 P_2 = (6 \times 5) = 30$. (ii) ${}^7 P_3 = (7 \times 6 \times 5) = 210$.

Cor. Number of all permutations of n things, taken all at a time = $n!$

An Important Result : If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of r th kind, such that $(p_1 + p_2 + \dots + p_r) = n$.

Then, number of permutations of these n objects is :

$$\frac{n!}{(p_1)!(p_2)!\dots(p_r)!}$$

Combinations : Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.

Ex. 1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.

Note that AB and BA represent the same selection.

Ex. 2. All the combinations formed by a, b, c , taking two at a time are ab, bc, ca .

Ex. 3. The only combination that can be formed of three letters a, b, c taken all at a time is abc .

Ex. 4. Various groups of 2 out of four persons A, B, C, D are :

$$AB, AC, AD, BC, BD, CD.$$

Ex. 5. Note that ab and ba are two different permutations but they represent the same combination.

Number of Combinations : The number of all combinations of n things, taken r at a time is :

$${}^n C_r = \frac{n!}{(r!(n-r)!)} = \frac{n(n-1)(n-2)\dots \text{to } r \text{ factors}}{r!}.$$

Note that : ${}^n C_n = 1$ and ${}^n C_0 = 1$.

An Important Result : ${}^n C_r = {}^n C_{(n-r)}$.

Example : (i) ${}^{11} C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330$.

(ii) ${}^{16} C_{13} = {}^{16} C_{(16-13)} = {}^{16} C_3 = \frac{16 \times 15 \times 14}{3!} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560$.

SOLVED EXAMPLES

Ex. 1. Evaluate : $\frac{30!}{28!}$

Sol. We have, $\frac{30!}{28!} = \frac{30 \times 29 \times (28!)!}{28!} = (30 \times 29) = 870$.

Ex. 2. Find the value of (i) ${}^{60} P_3$ (ii) ${}^4 P_4$

Sol. (i) ${}^{60} P_3 = \frac{60!}{(60-3)!} = \frac{60!}{57!} = \frac{60 \times 59 \times 58 \times (57)!}{57!} = (60 \times 59 \times 58) = 205320$.

(ii) ${}^4 P_4 = 4! = (4 \times 3 \times 2 \times 1) = 24$.

Ex. 3. Find the value of (i) ${}^{10} C_3$ (ii) ${}^{100} C_{98}$ (iii) ${}^{50} C_{50}$

Sol. (i) ${}^{10} C_3 = \frac{10 \times 9 \times 8}{3!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$.

(ii) ${}^{100} C_{98} = {}^{100} C_{(100-98)} = {}^{100} C_2 = \left(\frac{100 \times 99}{2 \times 1}\right) = 4950$.

(iii) ${}^{50} C_{50} = 1$. [$\because {}^n C_n = 1$]

Ex. 4. How many words can be formed by using all letters of the word 'BIHAR' ?

Sol. The word BIHAR contains 5 different letters.

∴ Required number of words = ${}^5 P_5 = 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$.

Ex. 5. How many words can be formed by using all the letters of the word 'DAUGHTER' so that the vowels always come together ?

Sol. Given word contains 8 different letters. When the vowels AUE are always together, we may suppose them to form an entity, treated as one letter.

Then, the letters to be arranged are DGHTR (AUE).

These 6 letters can be arranged in ${}^6 P_6 = 6! = 720$ ways.

The vowels in the group (AUE) may be arranged in $3! = 6$ ways.

∴ Required number of words = $(720 \times 6) = 4320$.

Ex. 6. How many words can be formed from the letters of the word 'EXTRA', so that the vowels are never together ?

Sol: The given word contains 5 different letters.

Taking the vowels EA together, we treat them as one letter.

Then, the letters to be arranged are XTR (EA).

These letters can be arranged in $4! = 24$ ways.

The vowels EA may be arranged amongst themselves in $2! = 2$ ways.

Number of words each having vowels together = $(24 \times 2) = 48$.

Total number of words formed by using all the letters of the given words
 $= 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$

Number of words, each having vowels never together = $(120 - 48) = 72$.

Ex. 7. How many words can be formed from the letters of the word 'DIRECTOR' so that the vowels are always together?

Sol. In the given word, we treat the vowels IEO as one letter.

Thus we have DBCTR (IEO).

This group has 6 letters of which R occurs 2 times and others are different.

Number of ways of arranging these letters = $\frac{6!}{2!} = 360$.

Now 3 vowels can be arranged among themselves in $3! = 6$ ways.

$$\text{Required number of ways} = (360 \times 6) = 2160.$$

Ex. 8. In how many ways can a cricket eleven be chosen out of a batch of 15 players?

15a 15a 15a

$$\text{Sol. Required number of ways} = {}^{15}C_{11} = {}^{15}C_{(15-11)} = {}^{15}C_4 \\ = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365.$$

Ex. 9. In how many ways, a committee of 5 members can be selected from 6 men and 5 ladies consisting of 3 men and 2 ladies ?

Sol. (3 men out of 6) and (2 ladies out of 5) are to be chosen.

$$\text{Required number of ways} = {}^6C_3 \times {}^5C_2 = \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \right) \times \left(\frac{5 \times 4}{2 \times 1} \right) = 200.$$

EXERCISE 30

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer.

- The value of ${}^{75}P_2$ is : (a) 2775 (b) 150 (c) 5550 (d) None of these
 - How many 4-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed ? (a) 40 (b) 400 (c) 5040 (d) 2520
 - How many words with or without meaning, can be formed by using all the letters of the word, 'DELHI', using each letter exactly once ? (a) 10 (b) 25 (c) 60 (d) 120
 - In how many ways can the letters of the word 'APPLE' be arranged ? (a) 720 (b) 120 (c) 60 (d) 180 (e) None of these
 - In how many ways can the letters of the word 'LEADER' be arranged ? (a) 72 (b) 144 (c) 360 (d) 720 (e) None of these

(Bank P.O. 2003)

6. In how many different ways can the letters of the word 'RUMOUR' be arranged ?
 (a) 180 (b) 90 (c) 30 (d) 720 (e) None of these
 (Bank P.O. 2003)
7. How many words can be formed by using all the letters of the word, 'ALLAHABAD'?
 (a) 3780 (b) 1890 (c) 7560 (d) 2520 (e) None of these
8. How many arrangements can be made out of the letters of the word 'ENGINEERING'?
 (a) 277200 (b) 92400 (c) 69300 (d) 23100 (e) None of these
9. How many words can be formed from the letters of the word 'SIGNATURE' so that the vowels always come together?
 (a) 720 (b) 1440 (c) 2880 (d) 3600 (e) 17280
 (Bank P.O. 2003)
10. In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?
 (a) 120 (b) 720 (c) 4320 (d) 2160 (e) None of these
 (M.B.A. 2002)
11. In how many different ways can the letters of the word 'SOFTWARE' be arranged in such a way that the vowels always come together?
 (a) 120 (b) 360 (c) 1440 (d) 13440 (e) 720
 (Bank P.O. 2003)
12. In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?
 (a) 360 (b) 480 (c) 720 (d) 5040 (e) None of these
 (Bank P.O. 2002)
13. In how many different ways can the letters of the word 'JUDGE' be arranged in such a way that the vowels always come together?
 (a) 48 (b) 120 (c) 124 (d) 160 (e) None of these
 (S.B.I.P.O. 2001)
14. In how many different ways can the letters of the word 'AUCTION' be arranged in such a way that the vowels always come together?
 (a) 30 (b) 48 (c) 144 (d) 576 (e) None of these
 (S.B.I.P.O. 2000)
15. In how many different ways can the letters of the word 'BANKING' be arranged so that the vowels always come together?
 (a) 120 (b) 240 (c) 360 (d) 540 (e) 720
 (Bank P.O. 2003)
16. In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?
 (a) 810 (b) 1440 (c) 2880 (d) 50400 (e) 5760
 (S.B.I.P.O. 2003)
17. In how many different ways can the letters of the word 'MATHEMATICS' be arranged so that the vowels always come together?
 (a) 10080 (b) 4989600 (c) 120960 (d) None of these
18. In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd positions?
 (a) 32 (b) 48 (c) 36 (d) 60 (e) 120
 (Bank P.O. 2002)
19. In how many different ways can the letters of the word 'MACHINE' be arranged so that the vowels may occupy only the odd positions?
 (a) 210 (b) 576 (c) 144 (d) 1728 (e) 3456
20. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
 (a) 63 (b) 90 (c) 126 (d) 45 (e) 135
 (Bank P.O. 2003)
21. In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women?
 (a) 266 (b) 5040 (c) 11760 (d) 86400 (e) None of these
 (Bank P.O. 2003)
22. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?
 (a) 564 (b) 645 (c) 735 (d) 756 (e) None of these
 (M.B.A. 2002)

23. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there ?
 (a) 159 (b) 194 (c) 205 (d) 209 (e) None of these
 (S.B.I.P.O. 2000)
24. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw ?
 (a) 32 (b) 48 (c) 64 (d) 96 (e) None of these
 (Bank P.O. 1998)
25. How many 3-digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated ?
 (a) 5 (b) 10 (c) 15 (d) 20
 (S.S.C. 2000)
26. In how many ways can 21 books on English and 19 books on Hindi be placed in a row on a shelf so that two books on Hindi may not be together ?
 (a) 3990 (b) 1540 (c) 1995 (d) 3672 (e) None of these
 (S.S.C. 2000)
27. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed ?
 (a) 210 (b) 1050 (c) 25200 (d) 21400 (e) None of these
 (S.S.C. 2000)

ANSWERS

- | | | | |
|---------|---------|---------|---------|
| 1. (c) | 2. (c) | 3. (d) | 4. (c) |
| 7. (c) | 8. (a) | 9. (c) | 10. (c) |
| 13. (a) | 14. (d) | 15. (c) | 16. (d) |
| 19. (b) | 20. (a) | 21. (c) | 22. (d) |
| 25. (d) | 26. (b) | 27. (c) | 23. (d) |
| 24. (c) | | | 17. (c) |
| | | | 18. (c) |

SOLUTIONS

1. ${}^{75}P_2 = \frac{75!}{(75-2)!} = \frac{75!}{73!} = \frac{75 \times 74 \times (73!)!}{73!} = (75 \times 74) = 5550.$

2. 'LOGARITHM' contains 10 different letters.

Required number of words = Number of arrangements of 10 letters, taking 4 at a time

$$= {}^{10}P_4 = (10 \times 9 \times 8 \times 7) = 5040.$$

3. The word 'DELHI' contains 5 different letters.

Required number of words = Number of arrangements of 5 letters, taken all at a time

$$= {}^5P_5 = 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$$

4. The word 'APPLE' contains 5 letters, 1A, 2P, 1L and 1E.

$$\therefore \text{ Required number of ways} = \frac{5!}{(1!) (2!) (1!) (1!) (1!)} = 60.$$

5. The word 'LEADER' contains 6 letters, namely 1L, 2E, 1A, 1D and 1R.

$$\therefore \text{ Required number of ways} = \frac{6!}{(1!) (2!) (1!) (1!) (1!) (2!)} = 360.$$

6. The word 'RUMOUR' contains 6 letters, namely 2R, 2U, 1M and 1O.

$$\therefore \text{ Required number of ways} = \frac{6!}{(2!) (2!) (1!) (1!) (1!)} = 180.$$

7. The word 'ALLAHABAD' contains 9 letters, namely 4A, 2L, 1H, 1B and 1D.
- ∴ Requisite number of words = $\frac{9!}{(4!)(2!)(1!)(1!)(1!)} = 7560$.
8. The word 'ENGINEERING' contains 11 letters, namely 3E, 3N, 2G, 2I and 1R.
- ∴ Required number of arrangements = $\frac{11!}{(3!)(3!)(2!)(2!)(1!)} = 277200$.
9. The word 'SIGNATURE' contains 9 different letters.
When the vowels IAUE are taken together, they can be supposed to form an entity, treated as one letter.
Then, the letters to be arranged are SGNTR (IAUE).
These 6 letters can be arranged in ${}^6P_6 = 6! = 720$ ways.
The vowels in the group (IAUE) can be arranged amongst themselves in ${}^4P_4 = 4! = 24$ ways.
∴ Required number of words = $(720 \times 24) = 17280$.
10. The word 'OPTICAL' contains 7 different letters.
When the vowels OIA are always together, they can be supposed to form one letter.
Then, we have to arrange the letters PTCL (OIA).
Now, 5 letters can be arranged in $5! = 120$ ways.
The vowels (OIA) can be arranged among themselves in $3! = 6$ ways.
∴ Required number of ways = $(120 \times 6) = 720$.
11. The word 'SOFTWARE' contains 8 different letters.
When the vowels OAE are always together, they can be supposed to form one letter.
Thus, we have to arrange the letters SFTWR (OAE).
Now, 5 letters can be arranged in $6! = 720$ ways.
The vowels (OAE) can be arranged among themselves in $3! = 6$ ways.
∴ Required number of ways = $(720 \times 6) = 4320$.
12. The word 'LEADING' has 7 different letters.
When the vowels EAI are always together, they can be supposed to form one letter.
Then, we have to arrange the letters LDNG (EAI).
Now, 5 letters can be arranged in $5! = 120$ ways.
The vowels (EAI) can be arranged among themselves in $3! = 6$ ways.
∴ Required number of ways = $(120 \times 6) = 720$.
13. The word 'JUDGE' has 5 different letters.
When the vowels UE are always together, they can be supposed to form one letter.
Then, we have to arrange the letters JDG (UE).
Now, 4 letters can be arranged in $4! = 24$ ways.
The vowels (UE) can be arranged among themselves in $2! = 2$ ways.
∴ Required number of ways = $(24 \times 2) = 48$.
14. The word 'AUCTION' has 7 different letters.
When the vowels AUIO are always together, they can be supposed to form one letter.
Then, we have to arrange the letters CTN (AUIO).
Now, 4 letters can be arranged in $4! = 24$ ways.
The vowels (AUIO) can be arranged among themselves in $4! = 24$ ways.
∴ Required number of ways = $(24 \times 24) = 576$.
15. In the word 'BANKING', we treat the two vowels AI as one letter. Thus, we have BNKNG (AI).
This has 6 letters of which N occurs 2 times and the rest are different.

Number of ways of arranging these letters = $\frac{6!}{(2)(1)(1)(1)(1)(1)} = 360$.

Now, 2 vowels AI can be arranged in $2! = 2$ ways.

\therefore Required number of ways = $(360 \times 2) = 720$.

16. In the word 'CORPORATION', we treat the vowels OOAIO as one letter. Thus, we have CRPRTN (OOAIO).

This has 7 letters of which R occurs 2 times and the rest are different.

Number of ways of arranging these letters = $\frac{7!}{2!} = 2520$.

Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged in $\frac{5!}{3!} = 20$ ways.

\therefore Required number of ways = $(2520 \times 20) = 50400$.

17. In the word 'MATHEMATICS' we treat the vowels AEAI as one letter. Thus, we have MTHMTCS (AEAI).

Now, we have to arrange 8 letters, out of which M occurs twice, T occurs twice and the rest are different.

\therefore Number of ways of arranging these letters = $\frac{8!}{(2)(2)!} = 10080$.

Now, AEAI has 4 letters in which A occurs 2 times and the rest are different.

Number of ways of arranging these letters = $\frac{4!}{2!} = 12$.

\therefore Required number of words = $(10080 \times 12) = 120960$.

18. There are 6 letters in the given word, out of which there are 3 vowels and 3 consonants. Let us mark these positions as under :

$$(1) (2) (3) (4) (5) (6)$$

Now, 3 vowels can be placed at any of the three places out of 4, marked 1, 3, 5.

Number of ways of arranging the vowels = ${}^3P_3 = 3! = 6$.

Also, the 3 consonants can be arranged at the remaining 3 positions.

Number of ways of these arrangements = ${}^3P_3 = 3! = 6$.

Total number of ways = $(6 \times 6) = 36$.

19. There are 7 letters in the given word, out of which there are 3 vowels and 4 consonants. Let us mark the positions to be filled up as follows :

$$(1) (2) (3) (4) (5) (6) (7)$$

Now, 3 vowels can be placed at any of the three places, out of the four marked 1, 3, 5, 7.

\therefore Number of ways of arranging the vowels = ${}^4P_3 = (4 \times 3 \times 2) = 24$.

Also, the 4 consonants at the remaining 4 positions may be arranged in

$= {}^4P_4 = 4! = 24$ ways.

\therefore Required number of ways = $(24 \times 24) = 576$.

20. Required number of ways = $({}^7C_5 \times {}^3C_2) = ({}^7C_2 \times {}^3C_1) = \left(\frac{7 \times 6}{2 \times 1} \times 3\right) = 63$.

21. Required number of ways = ${}^8C_5 \times {}^{10}C_6$

$$= {}^8C_3 \times {}^{10}C_4 = \left(\frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \right) = 11760.$$

22. We may have (3 men and 2 women) or (4 men and 1 woman) or (5 men only)

\therefore Required number of ways = ${}^7C_3 \times {}^6C_2 + {}^7C_4 \times {}^6C_1 + {}^7C_5$

$$= \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{6 \times 5}{2 \times 1} \right) + ({}^7C_3 \times {}^6C_1) + ({}^7C_2)$$

$$= 525 + \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 6 \right) + \left(\frac{7 \times 6}{2 \times 1} \right)$$

$$= (525 + 210 + 21) = 756.$$

23. We may have (1 boy and 3 girls) or (2 boys and 2 girls) or (3 boys and 1 girl) or (4 boys).

\therefore Required number of ways = ${}^6C_1 \times {}^4C_3 + {}^6C_2 \times {}^4C_2 + {}^6C_3 \times {}^4C_1 + {}^6C_4$

$$= ({}^6C_1 \times {}^4C_1) + ({}^6C_2 \times {}^4C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_2)$$

$$= (6 \times 4) + \left(\frac{6 \times 5}{2 \times 1} \times \frac{4 \times 3}{2 \times 1} \right) + \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times 4 \right) + \left(\frac{6 \times 5}{2 \times 1} \right)$$

$$= (24 + 90 + 80 + 15) = 209.$$

24. We may have (1 black and 2 non-black) or (2 black and 1 non-black) or (3 black).

\therefore Required number of ways = ${}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3$

$$= \left(3 \times \frac{6 \times 5}{2 \times 1} \right) + \left(\frac{3 \times 2}{2 \times 1} \times 6 \right) + 1 = (45 + 18 + 1) = 64.$$

25. Since each desired number is divisible by 5, so we must have 5 at the unit place. So, there is 1 way of doing it.

Tens place can be filled by any of the remaining 5 numbers.

So, there are 5 ways of filling the tens place.

The hundreds place can now be filled by any of the remaining 4 digits. So, there are 4 ways of filling it.

\therefore Required number of numbers = $(1 \times 5 \times 4) = 20$.

26. In order that two books on Hindi are never together, we must place all these books as under :

X E X E X E X X E X

where E denotes the position of an English book and X that of a Hindi book.

Since there are 21 books on English, the number of places marked X are therefore, 22.

Now, 19 places out of 22 can be chosen in ${}^{22}C_{19} = {}^{22}C_3 = \frac{22 \times 21 \times 20}{3 \times 2 \times 1} = 1540$ ways.

Hence, the required number of ways = 1540.

27. Number of ways of selecting (3 consonants out of 7) and (2 vowels out of 4)

$$= {}^7C_3 \times {}^4C_2 = \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1} \right) = 210.$$

Number of groups, each having 3 consonants and 2 vowels = 210.

Each group contains 5 letters.

Number of ways of arranging 5 letters among themselves

$$= 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$$

\therefore Required number of words = $(210 \times 120) = 25200$.

31. PROBABILITY

IMPORTANT FACTS AND FORMULAE

1. **Experiment** : An operation which can produce some well-defined outcomes is called an experiment.
2. **Random Experiment** : An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

Examples of Performing a Random Experiment :

- (i) Rolling an unbiased dice.
- (ii) Tossing a fair coin.
- (iii) Drawing a card from a pack of well-shuffled cards.
- (iv) Picking up a ball of certain colour from a bag containing balls of different colours.

Details :

- (i) When we throw a coin. Then either a Head (H) or a Tail (T) appears.
- (ii) A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively.
When we throw a die, the outcome is the number that appears on its upper face.
- (iii) A pack of cards has 52 cards.
It has 13 cards of each suit, namely Spades, Clubs, Hearts and Diamonds.
Cards of spades and clubs are black cards.
Cards of hearts and diamonds are red cards.
There are 4 honours of each suit.
These are Aces, Kings, Queens and Jacks.
These are called face cards.

3. **Sample Space** : When we perform an experiment, then the set S of all possible outcomes is called the Sample Space.

Examples of Sample Spaces :

- (i) In tossing a coin, $S = \{H, T\}$.
 - (ii) If two coins are tossed, then $S = \{HH, HT, TH, TT\}$.
 - (iii) In rolling a dice, we have, $S = \{1, 2, 3, 4, 5, 6\}$.
4. **Event** : Any subset of a sample space is called an event.

5. **Probability of Occurrence of an Event** :

Let S be the sample space and let E be an event.

Then, $E \subset S$.

$$\therefore P(E) = \frac{n(E)}{n(S)}$$

6. **Results on Probability** :

$$(i) P(S) = 1 \quad (ii) 0 \leq P(E) \leq 1 \quad (iii) P(\emptyset) = 0$$

(iv) For any events A and B, we have :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(v) If \bar{A} denotes (not-A), then $P(\bar{A}) = 1 - P(A)$.

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$$(i) P(S) = 1 \quad (ii) 0 \leq P(E) \leq 1 \quad (iii) P(\emptyset) = 0$$

(iv) For any events A and B, we have :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(v) If \bar{A} denotes (not-A), then $P(\bar{A}) = 1 - P(A)$.

SOLVED EXAMPLES

Ex. 1. In a throw of a coin, find the probability of getting a head.

Sol. Here $S = \{H, T\}$ and $E = \{H\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{2}.$$

Ex. 2. Two unbiased coins are tossed. What is the probability of getting at most one head?

Sol. Here $S = \{HH, HT, TH, TT\}$.

Let E = event of getting at most one head.

$$\therefore E = \{TT, HT, TH\}.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}.$$

Ex. 3. An unbiased die is tossed. Find the probability of getting a multiple of 3.

Sol. Here $S = \{1, 2, 3, 4, 5, 6\}$.

Let E be the event of getting a multiple of 3.

Then, $E = \{3, 6\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}.$$

Ex. 4. In a simultaneous throw of a pair of dice, find the probability of getting a total more than 7.

Sol. Here, $n(S) = (6 \times 6) = 36$.

Let E = Event of getting a total more than 7

$$= \{(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}.$$

Ex. 5. A bag contains 6 white and 4 black balls. Two balls are drawn at random. Find the probability that they are of the same colour.

Sol. Let S be the sample space. Then,

$$n(S) = \text{Number of ways of drawing 2 balls out of } (6+4) = {}^{10}C_2 = \frac{(10 \times 9)}{(2 \times 1)} = 45.$$

Let E = Event of getting both balls of the same colour. Then,

$n(E)$ = Number of ways of drawing (2 balls out of 6) or (2 balls out of 4)

$$= ({}^6C_2 + {}^4C_2) = \frac{(6 \times 5)}{(2 \times 1)} + \frac{(4 \times 3)}{(2 \times 1)} = (15 + 6) = 21.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{21}{45} = \frac{7}{15}.$$

Ex. 6. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is divisible by 4 or 6?

Sol. Clearly, $n(S) = 6 \times 6 = 36$.

Let E be the event that the sum of the numbers on the two faces is divisible by 4 or 6. Then

$$E = \{(1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (6, 2), (6, 6)\}$$

$$\therefore n(E) = 14.$$

$$\text{Hence, } P(E) = \frac{n(E)}{n(S)} = \frac{14}{36} = \frac{7}{18}.$$

Ex. 7. Two cards are drawn at random from a pack of 52 cards. What is the probability that either both are black or both are queens?

Sol. We have $n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326$.

Let A = event of getting both black cards;

B = event of getting both queens.

$\therefore A \cap B$ = event of getting queens of black cards.

$$\therefore n(A) = {}^{26}C_2 = \frac{(26 \times 25)}{(2 \times 1)} = 325, n(B) = {}^4C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6 \text{ and } n(A \cap B) = {}^2C_2 = 1.$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{325}{1326}, P(B) = \frac{n(B)}{n(S)} = \frac{6}{1326} \text{ and } P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{1326}.$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \left(\frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326} \right) = \frac{330}{1326} = \frac{55}{221}.$$

EXERCISE 31

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer :

- In a simultaneous throw of two coins, the probability of getting at least one head is :
 - $\frac{1}{2}$
 - $\frac{1}{3}$
 - $\frac{2}{3}$
 - $\frac{3}{4}$
- Three unbiased coins are tossed. What is the probability of getting at least 2 heads ?
 - $\frac{1}{4}$
 - $\frac{1}{2}$
 - $\frac{1}{3}$
 - $\frac{1}{8}$
- Three unbiased coins are tossed. What is the probability of getting at most two heads ?
 - $\frac{3}{4}$
 - $\frac{1}{4}$
 - $\frac{3}{8}$
 - $\frac{7}{8}$
- In a single throw of a die, what is the probability of getting a number greater than 4 ?
 - $\frac{1}{2}$
 - $\frac{1}{3}$
 - $\frac{2}{3}$
 - $\frac{1}{4}$
- In a simultaneous throw of two dice, what is the probability of getting a total of 7 ?
 - $\frac{1}{6}$
 - $\frac{1}{4}$
 - $\frac{2}{3}$
 - $\frac{3}{4}$
- What is the probability of getting a sum 9 from two throws of a dice ?
 - $\frac{1}{6}$
 - $\frac{1}{8}$
 - $\frac{1}{9}$
 - $\frac{1}{12}$
- In a simultaneous throw of two dice, what is the probability of getting a doublet ?
 - $\frac{1}{6}$
 - $\frac{1}{4}$
 - $\frac{2}{3}$
 - $\frac{3}{7}$
- In a simultaneous throw of two dice, what is the probability of getting a total of 10 or 11 ?
 - $\frac{1}{4}$
 - $\frac{1}{6}$
 - $\frac{7}{12}$
 - $\frac{5}{36}$

(M.B.A. 2002)

9. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even ? (Asstt. PF Commissioner's Exam, 2002)

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{3}{8}$

(d) $\frac{5}{16}$

10. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn bears a number which is a multiple of 3 ?

(a) $\frac{3}{10}$

(b) $\frac{3}{20}$

(c) $\frac{2}{5}$

(d) $\frac{1}{2}$

11. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5 ?

(a) $\frac{1}{2}$

(b) $\frac{2}{5}$

(c) $\frac{8}{15}$

(d) $\frac{9}{20}$

12. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize ?

(a) $\frac{1}{10}$

(b) $\frac{2}{5}$

(c) $\frac{2}{7}$

(d) $\frac{5}{7}$

13. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card ?

(a) $\frac{1}{13}$

(b) $\frac{4}{13}$

(c) $\frac{1}{4}$

(d) $\frac{9}{52}$

14. A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is :

(a) $\frac{1}{13}$

(b) $\frac{2}{13}$

(c) $\frac{1}{26}$

(d) $\frac{1}{52}$

15. One card is drawn from a pack of 52 cards. What is the probability that the card drawn is either a red card or a king ?

(a) $\frac{1}{2}$

(b) $\frac{6}{13}$

(c) $\frac{7}{13}$

(d) $\frac{27}{52}$

16. From a pack of 52 cards, one card is drawn at random. What is the probability that the card drawn is a ten or a spade ?

(a) $\frac{4}{13}$

(b) $\frac{1}{4}$

(c) $\frac{1}{13}$

(d) $\frac{1}{26}$

17. The probability that a card drawn from a pack of 52 cards will be a diamond or a king, is :

(a) $\frac{2}{13}$

(b) $\frac{4}{13}$

(c) $\frac{1}{13}$

(d) $\frac{1}{52}$

18. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings ? (M.B.A. 2002; Railways, 2002)

(a) $\frac{1}{15}$

(b) $\frac{25}{57}$

(c) $\frac{35}{256}$

(d) $\frac{1}{221}$

19. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is : (M.B.A. 2000)

(a) $\frac{3}{20}$

(b) $\frac{29}{34}$

(c) $\frac{47}{100}$

(d) $\frac{13}{102}$

20. Two cards are drawn from a pack of 52 cards. The probability that either both are red or both are kings, is :

(a) $\frac{7}{13}$

(b) $\frac{3}{26}$

(c) $\frac{63}{221}$

(d) $\frac{55}{221}$

21. A bag contains 6 black and 8 white balls. One ball is drawn at random. What is the probability that the ball drawn is white ?

(a) $\frac{3}{4}$ (b) $\frac{4}{7}$ (c) $\frac{1}{8}$ (d) $\frac{3}{7}$

22. A box contains 5 green, 4 yellow and 3 white marbles. Three marbles are drawn at random. What is the probability that they are not of the same colour ?

(a) $\frac{3}{44}$ (b) $\frac{3}{55}$ (c) $\frac{52}{55}$ (d) $\frac{41}{44}$

(Bank P.O. 2000)

23. A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random from the bag. The probability that all of them are red, is : (M.B.A. 2002)

(a) $\frac{1}{22}$ (b) $\frac{3}{22}$ (c) $\frac{2}{91}$ (d) $\frac{2}{77}$

24. A bag contains 6 white and 4 red balls. Three balls are drawn at random. What is the probability that one ball is red and the other two are white ?

(a) $\frac{1}{2}$ (b) $\frac{1}{12}$ (c) $\frac{3}{10}$ (d) $\frac{7}{12}$

25. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue ? (Bank P.O. 2003)

(a) $\frac{10}{21}$ (b) $\frac{11}{21}$ (c) $\frac{2}{7}$ (d) $\frac{5}{7}$

26. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green ? (Bank P.O. 2002)

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{7}{19}$ (d) $\frac{8}{21}$ (e) $\frac{9}{21}$

27. A box contains 10 black and 10 white balls. The probability of drawing two balls of the same colour, is :

(a) $\frac{9}{19}$ (b) $\frac{9}{38}$ (c) $\frac{10}{19}$ (d) $\frac{5}{19}$

28. A box contains 4 red balls, 5 green balls and 6 white balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either red or green ?

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $\frac{7}{15}$

29. In a class, there are 15 boys and 10 girls. Three students are selected at random. The probability that 1 girl and 2 boys are selected, is :

(a) $\frac{21}{46}$ (b) $\frac{25}{117}$ (c) $\frac{1}{50}$ (d) $\frac{3}{25}$

30. Four persons are chosen at random from a group of 3 men, 2 women and 4 children. The chance that exactly 2 of them are children, is :

(a) $\frac{1}{9}$ (b) $\frac{1}{5}$ (c) $\frac{1}{12}$ (d) $\frac{10}{21}$

31. A box contains 20 electric bulbs, out of which 4 are defective. Two bulbs are chosen at random from this box. The probability that at least one of these is defective, is :

(a) $\frac{4}{19}$ (b) $\frac{7}{19}$ (c) $\frac{12}{19}$ (d) $\frac{21}{95}$

32. In a class, 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random, what is the probability that he has offered English or Hindi ?

(a) $\frac{2}{5}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{3}{10}$

33. Two dice are tossed. The probability that the total score is a prime number is :
 (a) $\frac{1}{6}$ (b) $\frac{5}{12}$ (c) $\frac{1}{2}$ (d) $\frac{7}{9}$
34. A speaks truth in 75% cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other, narrating the same incident ?
 (a) 5% (b) 15% (c) 35% (d) 45%
- (Bank P.O. 2000)
35. A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is $(1/7)$ and the probability of wife's selection is $(1/5)$. What is the probability that only one of them is selected ?
 (a) $\frac{4}{5}$ (b) $\frac{2}{7}$ (c) $\frac{8}{15}$ (d) $\frac{4}{7}$

ANSWERS

1. (d) 2. (b) 3. (d) 4. (b) 5. (a) 6. (c) 7. (a) 8. (d) 9. (b)
 10. (a) 11. (d) 12. (c) 13. (b) 14. (c) 15. (c) 16. (a) 17. (b) 18. (d)
 19. (d) 20. (d) 21. (b) 22. (d) 23. (c) 24. (a) 25. (a) 26. (d) 27. (a)
 28. (b) 29. (a) 30. (d) 31. (b) 32. (a) 33. (b) 34. (c) 35. (b)

SOLUTIONS

1. Here $S = \{\text{HH}, \text{HT}, \text{TH}, \text{TT}\}$.

Let E = event of getting at least one head = $\{\text{HT}, \text{TH}, \text{HH}\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}.$$

2. Here $S = \{\text{TTT}, \text{TTH}, \text{THT}, \text{HTT}, \text{THH}, \text{HTH}, \text{HHT}, \text{HHH}\}$.

Let E = event of getting at least two heads = $\{\text{THH}, \text{HTH}, \text{HHT}, \text{HHH}\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}.$$

3. Here $S = \{\text{TTT}, \text{TTH}, \text{THT}, \text{HTT}, \text{THH}, \text{HTH}, \text{HHT}, \text{HHH}\}$.

Let E = event of getting at most two heads.

Then, $E = \{\text{TTT}, \text{TTH}, \text{THT}, \text{HTT}, \text{THH}, \text{HTH}, \text{HHT}\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}.$$

4. When a die is thrown, we have $S = \{1, 2, 3, 4, 5, 6\}$.

Let E = event of getting a number greater than 4 = $\{5, 6\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}.$$

5. We know that in a simultaneous throw of two dice, $n(S) = 6 \times 6 = 36$.

Let E = event of getting a total of 7 = $\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

6. In two throws of a die, $n(S) = (6 \times 6) = 36$.

Let E = event of getting a sum 9 = $\{(3, 6), (4, 5), (5, 4), (6, 3)\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}.$$

7. In a simultaneous throw of two dice, $n(S) = (6 \times 6) = 36$.
 Let E = event of getting a doublet = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}.
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$.
8. In a simultaneous throw of two dice, we have $n(S) = (6 \times 6) = 36$.
 Let E = event of getting a total of 10 or 11 = {(4, 6), (5, 5), (6, 4), (5, 6), (6, 5)}.
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$.
9. In a simultaneous throw of two dice, we have $n(S) = (6 \times 6) = 36$.
 Let E = event of getting two numbers whose product is even.
 Then, E = {(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4),
 (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1),
 (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.
 $\therefore n(E) = 27$.
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{27}{36} = \frac{3}{4}$.
10. Here, S = {1, 2, 3, 4, ..., 19, 20}.
 Let E = event of getting a multiple of 3 = {3, 6, 9, 12, 15, 18}.
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{20} = \frac{3}{10}$.
11. Here, S = {1, 2, 3, 4, ..., 19, 20}.
 Let E = event of getting a multiple of 3 or 5 = {3, 6, 9, 12, 15, 18, 5, 10, 20}.
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{9}{20}$.
12. $P(\text{getting a prize}) = \frac{10}{(10+25)} = \frac{10}{35} = \frac{2}{7}$.
13. Clearly, there are 52 cards, out of which there are 16 face cards.
 $\therefore P(\text{getting a face card}) = \frac{16}{52} = \frac{4}{13}$.
14. Here, $n(S) = 52$.
 Let E = event of getting a queen of club or a king of heart.
 Then, $n(E) = 2$.
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$.
15. Here, $n(S) = 52$.
 There are 26 red cards (including 2 kings) and there are 2 more kings.
 Let E = event of getting a red card or a king.
 Then, $n(E) = 28$.
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{28}{52} = \frac{7}{13}$.
16. Here, $n(S) = 52$.
 There are 13 spades (including one ten) and there are 3 more tens.
 Let E = event of getting a ten or a spade.
 Then, $n(E) = (13 + 3) = 16$.
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}$.

17. Here, $n(S) = 52$.
There are 13 cards of diamond (including one king) and there are 3 more kings.

Let E = event of getting a diamond or a king.

$$\text{Then, } n(E) = (13 + 3) = 16.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}$$

18. Let S be the sample space. Then,

$$n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$$

Let E = event of getting 2 kings out of 4.

$$\therefore n(E) = {}^4C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{1326} = \frac{1}{221}.$$

19. Let S be the sample space. Then,

$$n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$$

Let E = event of getting 1 spade and 1 heart.

$\therefore n(E)$ = number of ways of choosing 1 spade out of 13 and 1 heart out of 13

$$= ({}^{13}C_1 \times {}^{13}C_1) = (13 \times 13) = 169.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{169}{1326} = \frac{13}{102}.$$

20. Clearly, $n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{2} = 1326$.

Let E_1 = event of getting both red cards,

E_2 = event of getting both kings.

Then, $E_1 \cap E_2$ = event of getting 2 kings of red cards.

$$\therefore n(E_1) = {}^{26}C_2 = \frac{(26 \times 25)}{(2 \times 1)} = 325; n(E_2) = {}^4C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6;$$

$$n(E_1 \cap E_2) = {}^2C_2 = 1.$$

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{325}{1326}; P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{1326}; P(E_1 \cap E_2) = \frac{1}{1326}.$$

$\therefore P(\text{both red or both kings}) = P(E_1 \cup E_2)$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \left(\frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326} \right) = \frac{330}{1326} = \frac{55}{221}.$$

21. Total number of balls = $(6 + 8) = 14$.

Number of white balls = 8.

$$P(\text{drawing a white ball}) = \frac{8}{14} = \frac{4}{7}$$

22. Let S be the sample space. Then,

$n(S)$ = number of ways of drawing 3 marbles out of 12

$$= {}^{12}C_3 = \frac{(12 \times 11 \times 10)}{(3 \times 2 \times 1)} = 220.$$

Let E be the event of drawing 3 balls of the same colour.

Then, E = event of drawing (3 balls out of 5) or (3 balls out of 4) or (3 balls out of 3)

$$\Rightarrow n(E) = {}^5C_3 + {}^4C_3 + {}^3C_3 = ({}^5C_2 + {}^4C_1 + 1) = \frac{(5 \times 4)}{(2 \times 1)} + 4 + 1 = 15.$$

$$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{15}{220} = \frac{3}{44}.$$

$$\therefore \text{Required probability} = \left(1 - \frac{3}{44}\right) = \frac{41}{44}.$$

23. Let S be the sample space. Then,

$$n(S) = \text{number of ways of drawing 3 balls out of } 15 = {}^{15}C_3 = \frac{(15 \times 14 \times 13)}{(3 \times 2 \times 1)} = 455.$$

Let E = event of getting all the 3 red balls.

$$\therefore n(E) = {}^5C_3 = {}^5C_2 = \frac{(5 \times 4)}{(2 \times 1)} = 10.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{10}{455} = \frac{2}{91}.$$

24. Let S be the sample space. Then,

$$n(S) = \text{number of ways of drawing 3 balls out of } 10$$

$$= {}^{10}C_3 = \frac{(10 \times 9 \times 8)}{(3 \times 2 \times 1)} = 120.$$

Let E = event of drawing 1 red and 2 white balls

$\therefore n(E) = \text{Number of ways of drawing 1 red ball out of 4 and 2 white balls out of 6}$

$$= ({}^4C_1 \times {}^6C_2) = \left(4 \times \frac{6 \times 5}{2 \times 1}\right) = 60.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{60}{120} = \frac{1}{2}.$$

25. Total number of balls = $(2 + 3 + 2) = 7$.

Let S be the sample space. Then,

$$n(S) = \text{Number of ways of drawing 2 balls out of } 7 = {}^7C_2 = \frac{(7 \times 6)}{(2 \times 1)} = 21.$$

Let E = Event of drawing 2 balls, none of which is blue.

$\therefore n(E) = \text{Number of ways of drawing 2 balls out of } (2 + 3) \text{ balls}$

$$= {}^5C_2 = \frac{(5 \times 4)}{(2 \times 1)} = 10.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{10}{21}.$$

26. Total number of balls = $(8 + 7 + 6) = 21$.

Let E = event that the ball drawn is neither red nor green

= event that the ball drawn is red.

$$\therefore n(E) = 8$$

$$\therefore P(E) = \frac{8}{21}.$$

27. Total number of balls = 20.

Let S be the sample space. Then,

$$n(S) = \text{Number of ways of drawing 2 balls out of } 20 = {}^{20}C_2 = \frac{(20 \times 19)}{(2 \times 1)} = 190.$$

Let E = event of drawing 2 balls of the same colour.

$$n(E) = {}^{10}C_2 + {}^{10}C_2 = 2 \times \left(\frac{10 \times 9}{2 \times 1} \right) = 90.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{90}{190} = \frac{9}{19}.$$

28. Total number of balls = $(4 + 5 + 6) = 15$.

$$\therefore n(S) = 15.$$

Let E_1 = event of drawing a red ball

and E_2 = event of drawing a green ball.

Then, $E_1 \cap E_2 = \emptyset$.

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = \left(\frac{4}{15} + \frac{5}{15} \right) = \frac{9}{15} = \frac{3}{5}.$$

29. Let S be the sample space and E be the event of selecting 1 girl and 2 boys. Then,

$n(S)$ = Number of ways of selecting 3 students out of 25

$$= {}^{25}C_3 = \frac{(25 \times 24 \times 23)}{(3 \times 2 \times 1)} = 2300.$$

$$n(E) = {}^{10}C_1 \times {}^{15}C_2 = \left\{ 10 \times \frac{(15 \times 14)}{(2 \times 1)} \right\} = 1050.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1050}{2300} = \frac{21}{46}.$$

30. Let S be the sample space and E be the event of choosing four persons such that 2 of them are children. Then,

$n(S)$ = Number of ways of choosing 4 persons out of 9

$$= {}^9C_4 = \frac{(9 \times 8 \times 7 \times 6)}{(4 \times 3 \times 2 \times 1)} = 126.$$

$n(E)$ = Number of ways of choosing 2 children out of 4 and 2 persons out of $(3 + 2)$ persons

$$= {}^4C_2 \times {}^5C_2 = \frac{(4 \times 3)}{(2 \times 1)} \times \frac{(5 \times 4)}{(2 \times 1)} = 60.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{60}{126} = \frac{10}{21}.$$

$$31. P(\text{None is defective}) = \frac{{}^{16}C_2}{{}^{20}C_2} = \left(\frac{16 \times 15}{2 \times 1} \times \frac{2 \times 1}{20 \times 19} \right) = \frac{12}{19}.$$

$$P(\text{at least one is defective}) = \left(1 - \frac{12}{19} \right) = \frac{7}{19}.$$

$$32. P(E) = \frac{30}{100} = \frac{3}{10}, P(H) = \frac{20}{100} = \frac{1}{5} \text{ and } P(E \cap H) = \frac{10}{100} = \frac{1}{10}.$$

$$P(E \text{ or } H) = P(E \cup H)$$

$$= P(E) + P(H) - P(E \cap H)$$

$$= \left(\frac{3}{10} + \frac{1}{5} - \frac{1}{10} \right) = \frac{4}{10} = \frac{2}{5}.$$

33. Clearly, $n(S) = (6 \times 6) = 36$.

Let E = Event that the sum is a prime number.

Then, $E = \{(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)\}$

$$\therefore n(E) = 15$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}.$$

34. Let A = Event that A speaks the truth

and B = Event that B speaks the truth

$$\text{Then, } P(A) = \frac{75}{100} = \frac{3}{4}, \quad P(B) = \frac{80}{100} = \frac{4}{5}.$$

$$\therefore P(\bar{A}) = \left(1 - \frac{3}{4}\right) = \frac{1}{4} \text{ and } P(\bar{B}) = \left(1 - \frac{4}{5}\right) = \frac{1}{5}.$$

$P(A \text{ and } B)$ contradict each other)

$$= P[(A \text{ speaks the truth and } B \text{ tells a lie}) \text{ or } (A \text{ tells a lie and } B \text{ speaks the truth})]$$

$$= P[(A \text{ and } \bar{B}) \text{ or } (\bar{A} \text{ and } B)]$$

$$= P(A \text{ and } \bar{B}) + P(\bar{A} \text{ and } B)$$

$$= P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B)$$

$$= \left(\frac{3}{4} \times \frac{1}{5}\right) + \left(\frac{1}{4} \times \frac{4}{5}\right) = \left(\frac{3}{20} + \frac{1}{5}\right) = \frac{7}{20} = \left(\frac{7}{20} \times 100\right)\% = 35\%.$$

$\therefore A$ and B contradict each other in 35% of the cases.

35. Let A = Event that the husband is selected

and B = Event that the wife is selected.

$$\text{Then, } P(A) = \frac{1}{7} \text{ and } P(B) = \frac{1}{5}.$$

$$\therefore P(\bar{A}) = \left(1 - \frac{1}{7}\right) = \frac{6}{7} \text{ and } P(\bar{B}) = \left(1 - \frac{1}{5}\right) = \frac{4}{5}.$$

\therefore Required probability = $P[(A \text{ and not } B) \text{ or } (B \text{ and not } A)]$

$$= P[(A \text{ and } \bar{B}) \text{ or } (B \text{ and } \bar{A})]$$

$$= P(A \text{ and } \bar{B}) + P(B \text{ and } \bar{A})$$

$$= P(A) \cdot P(\bar{B}) + P(B) \cdot P(\bar{A}) = \left(\frac{1}{7} \times \frac{4}{5}\right) + \left(\frac{1}{5} \times \frac{6}{7}\right) = \frac{10}{35} = \frac{2}{7}.$$

32. TRUE DISCOUNT

IMPORTANT CONCEPTS

Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. So, the payment of Rs. 100 now will clear off the debt of Rs. 156 due 4 years hence. We say that :

Sum due = Rs. 156 due 4 years hence;

Present Worth (P.W.) = Rs. 100;

True Discount (T.D.) = Rs. $(156 - 100) = \text{Rs. } 56 = (\text{Sum due}) - (\text{P.W.})$.

We define : $T.D. = \text{Interest on P.W.}$

$$\text{Amount} = (\text{P.W.}) + (\text{T.D.})$$

Interest is reckoned on P.W. and true discount is reckoned on the amount.

IMPORTANT FORMULAE

Let rate = R% per annum and Time = T years. Then,

1. $\text{P.W.} = \frac{100 \times \text{Amount}}{100 + (R \times T)} = \frac{100 \times \text{T.D.}}{R \times T}$
2. $\text{T.D.} = \frac{(\text{P.W.}) \times R \times T}{100} = \frac{\text{Amount} \times R \times T}{100 + (R \times T)}$
3. $\text{Sum} = \frac{(\text{S.I.}) \times (\text{T.D.})}{(\text{S.I.}) - (\text{T.D.})}$
4. $(\text{S.I.}) - (\text{T.D.}) = \text{S.I. on T.D.}$
5. When the sum is put at compound interest, then $\text{P.W.} = \frac{\text{Amount}}{\left(1 + \frac{R}{100}\right)^T}$

SOLVED EXAMPLES

Ex. 1. Find the present worth of Rs. 930 due 3 years hence at 8% per annum. Also find the discount.

$$\text{Sol. } \text{P.W.} = \frac{100 \times \text{Amount}}{100 + (R \times T)} = \text{Rs.} \left[\frac{100 \times 930}{100 + (8 \times 3)} \right] = \text{Rs.} \left(\frac{100 \times 930}{124} \right) = \text{Rs. } 750.$$

$$\text{T.D.} = (\text{Amount}) - (\text{P.W.}) = \text{Rs.} (930 - 750) = \text{Rs. } 180.$$

Ex. 2. The true discount on a bill due 9 months hence at 12% per annum is Rs. 540. Find the amount of the bill and its present worth.

Sol. Let amount be Rs. x. Then,

$$\frac{x \times R \times T}{100 + (R \times T)} = \text{T.D.} \Rightarrow \frac{x \times 12 \times \frac{3}{4}}{100 + \left(12 \times \frac{3}{4}\right)} = 540 \Rightarrow x = \left(\frac{540 \times 109}{9} \right) = \text{Rs. } 6540.$$

$$\therefore \text{Amount} = \text{Rs. } 6540.$$

$$\text{P.W.} = \text{Rs.} (6540 - 540) = \text{Rs. } 6000.$$

Ex. 3. The true discount on a certain sum of money due 3 years hence is Rs. 250 and the simple interest on the same sum for the same time and at the same rate is Rs. 375. Find the sum and the rate percent.

Sol. T.D. = Rs. 250 and S.I. = Rs. 375.

$$\text{Sum due} = \frac{\text{S.I.} \times \text{T.D.}}{(\text{S.I.}) - (\text{T.D.})} = \text{Rs.} \left(\frac{375 \times 250}{375 - 250} \right) = \text{Rs.} 750.$$

$$\text{Rate} = \left(\frac{100 \times 375}{750 \times 3} \right)\% = 16\frac{2}{3}\%.$$

Ex. 4. The difference between the simple interest and true discount on a certain sum of money for 6 months at $12\frac{1}{2}\%$ per annum is Rs. 25. Find the sum.

Sol. Let the sum be Rs. x. Then,

$$\text{T.D.} = \frac{x \times \frac{25}{2} \times \frac{1}{2}}{100 + \left(\frac{25}{2} \times \frac{1}{2} \right)} = \left(x \times \frac{25}{4} \times \frac{4}{425} \right) = \frac{x}{17}.$$

$$\text{S.I.} = \left(x \times \frac{25}{2} \times \frac{1}{2} \times \frac{1}{100} \right) = \frac{x}{16}.$$

$$\therefore \frac{x}{16} - \frac{x}{17} = 25 \Rightarrow 17x - 16x = 25 \times 16 \times 17 \Rightarrow x = 6800.$$

Hence, sum due = Rs. 6800.

Ex. 5. A bill falls due in 1 year. The creditor agrees to accept immediate payment of the half and to defer the payment of the other half for 2 years. By this arrangement he gains Rs. 40. What is the amount of the bill, if the money be worth $12\frac{1}{2}\%$?

Sol. Let the sum be Rs. x. Then,

$$\left[\frac{x}{2} + \frac{\frac{x}{2} \times 100}{100 + \left(\frac{25}{2} \times 2 \right)} \right] - \frac{x \times 100}{100 + \left(\frac{25}{2} \times 1 \right)} = 40 \Rightarrow \frac{x}{2} + \frac{2x}{5} - \frac{8x}{9} = 40 \Rightarrow x = 3600.$$

∴ Amount of the bill = Rs. 3600.

EXERCISE 32

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer :

- The present worth of Rs. 2310 due $2\frac{1}{2}$ years hence, the rate of interest being 15% per annum, is : (a) Rs. 1750 (b) Rs. 1680 (c) Rs. 1840 (d) Rs. 1443.75
- If the true discount on a sum due 2 years hence at 14% per annum be Rs. 168, the sum due is : (a) Rs. 768 (b) Rs. 968 (c) Rs. 1960 (d) Rs. 2400
- The true discount on Rs. 2562 due 4 months hence is Rs. 122. The rate percent is : (a) 12% (b) $13\frac{1}{3}\%$ (c) 15% (d) 14%

4. The true discount on Rs. 1760 due after a certain time at 12% per annum is Rs. 160. The time after which it is due is :
 (a) 6 months (b) 8 months (c) 9 months (d) 10 months
5. The true discount on a bill due 9 months hence at 16% per annum is Rs. 189. The amount of the bill is :
 (a) Rs. 1386 (b) Rs. 1764 (c) Rs. 1575 (d) Rs. 2268
6. The interest on Rs. 750 for 2 years is the same as the true discount on Rs. 960 due 2 years hence. If the rate of interest is the same in both cases, it is :
 (a) 12% (b) 14% (c) 15% (d) $16\frac{2}{3}\%$
7. The simple interest and the true discount on a certain sum for a given time and at a given rate are Rs. 85 and Rs. 80 respectively. The sum is :
 (a) Rs. 1800 (b) Rs. 1450 (c) Rs. 1360 (d) Rs. 6800
8. If Rs. 10 be allowed as true discount on a bill of Rs. 110 due at the end of a certain time, then the discount allowed on the same sum due at the end of double the time is :
 (a) Rs. 20 (b) Rs. 21.81 (c) Rs. 22 (d) Rs. 18.33
9. A man wants to sell his scooter. There are two offers, one at Rs. 12,000 cash and the other at a credit of Rs. 12,880 to be paid after 8 months, money being at 18% per annum. Which is the better offer ?
 (a) Rs. 12,000 in cash (b) Rs. 12,880 at credit (c) Both are equally good
10. Goods were bought for Rs. 600 and sold the same day for Rs. 688.50 at a credit of 9 months and thus gaining 2%. The rate of interest per annum is :
 (a) $16\frac{2}{3}\%$ (b) $14\frac{1}{2}\%$ (c) $13\frac{1}{3}\%$ (d) 15%
11. The present worth of Rs. 1404 due in two equal half-yearly instalments at 8% per annum simple interest is :
 (a) Rs. 1325 (b) Rs. 1300 (c) Rs. 1350 (d) Rs. 1500
12. A trader owes a merchant Rs. 10,028 due 1 year hence. The trader wants to settle the account after 3 months. If the rate of interest is 12% per annum, how much cash should he pay ?
 (a) Rs. 9025.20 (b) Rs. 9200 (c) Rs. 9600 (d) Rs. 9560
13. A man buys a watch for Rs. 1950 in cash and sells it for Rs. 2200 at a credit of 1 year. If the rate of interest is 10% per annum, the man :
 (a) gains Rs. 55 (b) gains Rs. 50 (c) loses Rs. 30 (d) gains Rs. 30
14. A man purchased a cow for Rs. 3000 and sold it the same day for Rs. 3600, allowing the buyer a credit of 2 years. If the rate of interest be 10% per annum, then the man has a gain of :
 (a) 0% (b) 5% (c) 7.5% (d) 10%
15. A owes B, Rs. 1573 payable $1\frac{1}{2}$ years hence. Also B owes A, Rs. 1444.50 payable 6 months hence. If they want to settle the account forthwith, keeping 14% as the rate of interest, then who should pay and how much ?
 (a) A, Rs. 28.50 (b) B, Rs. 37.50 (c) A, Rs. 50 (d) B, Rs. 50
16. A has to pay Rs. 220 to B after 1 year. B asks A to pay Rs. 110 in cash and defer the payment of Rs. 110 for 2 years. A agrees to it. If the rate of interest be 10% per annum, in this mode of payment :
 (a) There is no gain or loss to any one (b) A gains Rs. 7.34
 (c) A loses Rs. 7.34 (d) A gains Rs. 11
17. Rs. 20 is the true discount on Rs. 260 due after a certain time. What will be the true discount on the same sum due after half of the former time, the rate of interest being the same ?
 (a) Rs. 10 (b) Rs. 10.40 (c) Rs. 15.20 (d) Rs. 13

ANSWERS

1. (b) 2. (a) 3. (c) 4. (d) 5. (b) 6. (b) 7. (c) 8. (d) 9. (a)
 10. (a) 11. (a) 12. (b) 13. (b) 14. (a) 15. (d) 16. (b) 17. (b)

SOLUTIONS

1. P.W. = Rs. $\left[\frac{100 \times 2310}{100 + \left(15 \times \frac{5}{2} \right)} \right] = \text{Rs. } 1680.$
2. P.W. = $\frac{100 \times \text{T.D.}}{\text{R} \times \text{T}} = \frac{100 \times 168}{14 \times 2} = 600.$
 $\therefore \text{Sum} = (\text{P.W.} + \text{T.D.}) = \text{Rs. } (600 + 168) = \text{Rs. } 768.$
3. P.W. = Rs. $(2562 - 122) = \text{Rs. } 2440.$
 $\therefore \text{S.I. on Rs. } 2440 \text{ for 4 months is Rs. } 122.$
 $\therefore \text{Rate} = \left(\frac{100 \times 122}{2440 \times \frac{1}{3}} \right)\% = 15\%.$
4. P.W. = Rs. $(1760 - 160) = \text{Rs. } 1600.$
 $\therefore \text{S.I. on Rs. } 1600 \text{ at } 12\% \text{ is Rs. } 160.$
 $\therefore \text{Time} = \left(\frac{100 \times 160}{1600 \times 12} \right) = \frac{5}{6} \text{ years} = \left(\frac{5}{6} \times 12 \right) \text{ months} = 10 \text{ months.}$
5. Let P.W. be Rs. x . Then, S.I. on Rs. x at 16% for 9 months = Rs. 189.
 $\therefore x \times 16 \times \frac{9}{12} \times \frac{1}{100} = 189 \text{ or } x = 1575.$
 $\therefore \text{P.W.} = \text{Rs. } 1575.$
 $\therefore \text{Sum due} = \text{P.W.} + \text{T.D.} = \text{Rs. } (1575 + 189) = \text{Rs. } 1764.$
6. S.I. on Rs. 750 = T.D. on Rs. 960.
 This means P.W. of Rs. 960 due 2 years hence is Rs. 750.
 $\therefore \text{T.D.} = \text{Rs. } (960 - 750) = \text{Rs. } 210.$
 Thus, S.I. on Rs. 750 for 2 years is Rs. 210.
 $\therefore \text{Rate} = \left(\frac{100 \times 210}{750 \times 2} \right)\% = 14\%.$
7. Sum = $\frac{\text{S.I.} \times \text{T.D.}}{(\text{S.I.}) - (\text{T.D.})} = \frac{85 \times 80}{(85 - 80)} = \text{Rs. } 1360.$
8. S.I. on Rs. $(110 - 10)$ for a certain time = Rs. 10.
 S.I. on Rs. 100 for double the time = Rs. 20.
 T.D. on Rs. 120 = Rs. $(120 - 100) = \text{Rs. } 20.$
 T.D. on Rs. 110 = Rs. $\left(\frac{20}{120} \times 110 \right) = \text{Rs. } 18.33.$
9. P.W. of Rs. 12,880 due 8 months hence
 $= \text{Rs. } \left[\frac{12880 \times 100}{100 + \left(18 \times \frac{8}{12} \right)} \right] = \text{Rs. } \left(\frac{12880 \times 100}{112} \right) = \text{Rs. } 11500.$

Clearly, Rs. 12,000 in cash is a better offer.

10. S.P. = 102% of Rs. 600 = Rs. $\left(\frac{102}{100} \times 600\right)$ = Rs. 612.

Now, P.W. = Rs. 612 and sum = Rs. 688.50.

$$\therefore \text{T.D.} = \text{Rs. } (688.50 - 612) = \text{Rs. } 76.50.$$

Thus, S.I. on Rs. 612 for 9 months is Rs. 76.50.

$$\therefore \text{Rate} = \left(\frac{100 \times 76.50}{612 \times \frac{3}{4}} \right)\% = 16\frac{2}{3}\%$$

11. Required sum = P.W. of Rs. 702 due 6 months hence + P.W. of Rs. 702 due 1 year hence

$$= \text{Rs.} \left[\left(\frac{100 \times 702}{100 + 8 \times \frac{1}{2}} \right) + \left(\frac{100 \times 702}{100 + (8 \times 1)} \right) \right] = \text{Rs. } (675 + 650) = \text{Rs. } 1325.$$

12. Required money = P.W. of Rs. 10028 due 9 months hence

$$= \text{Rs.} \left[\frac{10028 \times 100}{100 + \left(12 \times \frac{9}{12} \right)} \right] = \text{Rs. } 9200.$$

13. S.P. = P.W. of Rs. 2200 due 1 year hence = Rs. $\left[\frac{2200 \times 100}{100 + (10 \times 1)} \right]$ = Rs. 2000.

$$\therefore \text{Gain} = \text{Rs. } (2000 - 1950) = \text{Rs. } 50.$$

14. C.P. = Rs. 3000. S.P. = Rs. $\left[\frac{3600 \times 100}{100 + (10 \times 2)} \right]$ = Rs. 3000.

Gain = 0%.

15. A owes = P.W. of Rs. 1573 due $\frac{3}{2}$ years hence

$$= \text{Rs.} \left[\frac{1573 \times 100}{100 + \left(14 \times \frac{3}{2} \right)} \right] = \text{Rs.} \left(\frac{1573 \times 100}{121} \right) = \text{Rs. } 1300.$$

B owes = P.W. of Rs. 1444.50 due 6 months hence

$$= \text{Rs.} \left[\frac{1444.50 \times 100}{100 + \left(14 \times \frac{1}{2} \right)} \right] = \text{Rs.} \left(\frac{1444.50 \times 100}{107} \right) = \text{Rs. } 1350.$$

\therefore B must pay Rs. 50 to A.

16. A has to pay = P.W. of Rs. 220 due 1 year hence = Rs. $\left[\frac{220 \times 100}{100 + (10 \times 1)} \right]$ = Rs. 200.

A actually pays = Rs. 110 + P.W. of Rs. 110 due 2 years hence

$$= \left[110 + \frac{110 \times 100}{100 + (10 \times 2)} \right] = \text{Rs. } 192.66.$$

$$\therefore \text{A gains} = \text{Rs. } (200 - 192.66) = \text{Rs. } 7.34.$$

17. S.I. on Rs. (260 - 20) for a given time = Rs. 20.

S.I. on Rs. 240 for half the time = Rs. 10.

T.D. on Rs. 250 = Rs. 10.

$$\therefore \text{T.D. on Rs. } 260 = \text{Rs.} \left(\frac{10}{250} \times 260 \right) = \text{Rs. } 10.40.$$

33. BANKER'S DISCOUNT

IMPORTANT CONCEPTS

Banker's Discount : Suppose a merchant A buys goods worth, say Rs. 10,000 from another merchant B at a credit of say 5 months. Then, B prepares a bill, called the bill of exchange. A signs this bill and allows B to withdraw the amount from his bank account after exactly 5 months.

The date exactly after 5 months is called *nominally due date*. Three days (known as *grace days*) are added to it to get a date, known as *legally due date*.

Suppose B wants to have the money before the legally due date. Then he can have the money from the banker or a broker, who deducts S.I. on the face value (i.e., Rs. 10,000 in this case) for the period from the date on which the bill was discounted (i.e., paid by the banker) and the legally due date. This amount is known as *Banker's Discount (B.D.)*.

Thus, *B.D. is the S.I. on the face value for the period from the date on which the bill was discounted and the legally due date.*

Banker's Gain (B.G.) = (B.D.) - (T.D.) for the unexpired time.

Note : When the date of the bill is not given, grace days are not to be added.

IMPORTANT FORMULAE

1. $B.D. = \text{S.I. on bill for unexpired time.}$

2. $B.G. = (B.D.) - (T.D.) = \text{S.I. on T.D.} = \frac{(T.D.)^2}{P.W.}$

3. $T.D. = \sqrt{P.W. \times B.G.}$

4. $B.D. = \left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100} \right)$

5. $T.D. = \left[\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})} \right]$

6. $\text{Amount} = \left(\frac{B.D. \times T.D.}{B.D. - T.D.} \right)$

7. $T.D. = \left(\frac{B.G. \times 100}{\text{Rate} \times \text{Time}} \right)$

SOLVED EXAMPLES

Ex. 1. A bill for Rs. 6000 is drawn on July 14 at 5 months. It is discounted on 5th October at 10%. Find the banker's discount, true discount, banker's gain and the money that the holder of the bill receives.

Sol. Face value of the bill = Rs. 6000.

Date on which the bill was drawn = July 14 at 5 months.

Nominally due date = December 14. Legally due date = December 17.

Date on which the bill was discounted = October 5.

Unexpired time : Oct.

Nov.

Dec.

$$26 + 30 + 17 = 73 \text{ days} = \frac{1}{5} \text{ year.}$$

33. BANKER'S DISCOUNT

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Banker's Discount : Suppose a merchant A buys goods worth, say Rs. 10,000 from another merchant B at a credit of say 5 months. Then, B prepares a bill, called the bill of exchange. A signs this bill and allows B to withdraw the amount from his bank account after exactly 5 months.

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1. $B.D. = \text{S.I. on bill for unexpired time.}$

2. $B.G. = (B.D.) - (T.D.) = \text{S.I. on T.D.} = \frac{(T.D.)^2}{P.W.}$

3. $T.D. = \sqrt{P.W. \times B.G.}$

4. $B.D. = \left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100} \right)$

5. $T.D. = \left[\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})} \right]$

6. $\text{Amount} = \left(\frac{B.D. \times T.D.}{B.D. - T.D.} \right)$

7. $T.D. = \left(\frac{B.G. \times 100}{\text{Rate} \times \text{Time}} \right)$

SOLVED EXAMPLES

Ex. 1. A bill for Rs. 6000 is drawn on July 14 at 5 months. It is discounted on 5th October at 10%. Find the banker's discount, true discount, banker's gain and the money that the holder of the bill receives.

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Unexpired time : Oct.

Nov.

Dec.

$$26 + 30 + 17 = 73 \text{ days} = \frac{1}{5} \text{ year.}$$

$$\therefore \text{B.D.} = \text{S.I. on Rs. } 6000 \text{ for } \frac{1}{5} \text{ year} = \text{Rs.} \left(6000 \times 10 \times \frac{1}{5} \times \frac{1}{100} \right) = \text{Rs. } 120.$$

$$\text{T.D.} = \text{Rs.} \left[\frac{6000 \times 10 \times \frac{1}{5}}{100 + \left(10 \times \frac{1}{5} \right)} \right] = \text{Rs.} \left(\frac{12000}{102} \right) = \text{Rs. } 117.64.$$

$$\therefore \text{B.G.} = (\text{B.D.}) - (\text{T.D.}) = \text{Rs.} (120 - 117.64) = \text{Rs. } 2.36.$$

Money received by the holder of the bill = Rs. $(6000 - 120) = \text{Rs. } 5880$.

Ex. 2. If the true discount on a certain sum due 6 months hence at 15% is Rs. 120, what is the banker's discount on the same sum for the same time and at the same rate?

$$\text{Sol. } \text{B.G.} = \text{S.I. on T.D.} = \text{Rs.} \left(120 \times 15 \times \frac{1}{2} \times \frac{1}{100} \right) = \text{Rs. } 9.$$

$$\therefore (\text{B.D.}) - (\text{T.D.}) = \text{Rs. } 9.$$

$$\therefore \text{B.D.} = \text{Rs.} (120 + 9) = \text{Rs. } 129.$$

Ex. 3. The banker's discount on Rs. 1800 at 12% per annum is equal to the true discount on Rs. 1872 for the same time at the same rate. Find the time.

$$\text{Sol. } \text{S.I. on Rs. } 1800 = \text{T.D. on Rs. } 1872.$$

$$\therefore \text{P.W. of Rs. } 1872 \text{ is Rs. } 1800.$$

$$\therefore \text{Rs. } 72 \text{ is S.I. on Rs. } 1800 \text{ at } 12\%.$$

$$\therefore \text{Time} = \left(\frac{100 \times 72}{12 \times 1800} \right) \text{ year} = \frac{1}{3} \text{ year} = 4 \text{ months.}$$

Ex. 4. The banker's discount and the true discount on a sum of money due 8 months hence are Rs. 120 and Rs. 110 respectively. Find the sum and the rate percent.

$$\text{Sol. } \text{Sum} = \left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} \right) = \text{Rs.} \left(\frac{120 \times 110}{120 - 110} \right) = \text{Rs. } 1320.$$

Since B.D. is S.I. on sum due, so S.I. on Rs. 1320 for 8 months is Rs. 120.

$$\therefore \text{Rate} = \left(\frac{100 \times 120}{1320 \times \frac{2}{3}} \right)\% = 13\frac{7}{11}\%.$$

Ex. 5. The present worth of a bill due sometime hence is Rs. 1100 and the true discount on the bill is Rs. 110. Find the banker's discount and the banker's gain.

$$\text{Sol. } \text{T.D.} = \sqrt{\text{P.W.} \times \text{B.G.}}$$

$$\therefore \text{B.G.} = \frac{(\text{T.D.})^2}{\text{P.W.}} = \text{Rs.} \left(\frac{110 \times 110}{1100} \right) = \text{Rs. } 11.$$

$$\therefore \text{B.D.} = (\text{T.D.} + \text{B.G.}) = \text{Rs.} (110 + 11) = \text{Rs. } 121.$$

Ex. 6. The banker's discount on Rs. 1650 due a certain time hence is Rs. 165. Find the true discount and the banker's gain.

$$\text{Sol. } \text{Sum} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.G.}}$$

$$\therefore \frac{\text{T.D.}}{\text{B.G.}} = \frac{\text{Sum}}{\text{B.D.}} = \frac{1650}{165} = \frac{10}{1}$$

Thus, if B.G. is Re 1, T.D. = Rs. 10.

If B.D. is Rs. 11, T.D. = Rs. 10. If B.D. is Rs. 165, T.D. = Rs. $\left(\frac{10}{11} \times 165 \right) = \text{Rs. } 150$.

And, B.G. = Rs. $(165 - 150) = \text{Rs. } 15$.

Ex. 7. What rate percent does a man get for his money when in discounting a bill due 10 months hence, he deducts 10% of the amount of the bill?

Sol. Let, amount of the bill = Rs. 100. Money deducted = Rs. 10.

Money received by the holder of the bill = Rs. $(100 - 10) = \text{Rs. } 90$.

S.I. on Rs. 90 for 10 months = Rs. 10.

$$\therefore \text{Rate} = \left(\frac{100 \times 10}{90 \times \frac{10}{12}} \right)\% = 13\frac{1}{3}\%$$

EXERCISE 33

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer:

- The true discount on a bill of Rs. 540 is Rs. 90. The banker's discount is :
 (a) Rs. 60 (b) Rs. 108 (c) Rs. 110 (d) Rs. 112
- The present worth of a certain bill due sometime hence is Rs. 800 and the true discount is Rs. 36. The banker's discount is :
 (a) Rs. 37 (b) Rs. 37.62 (c) Rs. 34.38 (d) Rs. 38.98
- The present worth of a certain sum due sometime hence is Rs. 1600 and the true discount is Rs. 160. The banker's gain is :
 (a) Rs. 20 (b) Rs. 24 (c) Rs. 16 (d) Rs. 12
- The banker's gain of a certain sum due 2 years hence at 10% per annum is Rs. 24. The present worth is :
 (a) Rs. 480 (b) Rs. 520 (c) Rs. 600 (d) Rs. 960
- The banker's gain on a bill due 1 year hence at 12% per annum is Rs. 6. The true discount is :
 (a) Rs. 72 (b) Rs. 36 (c) Rs. 54 (d) Rs. 50
- The banker's discount on a bill due 4 months hence at 15% is Rs. 420. The true discount is :
 (a) Rs. 400 (b) Rs. 360 (c) Rs. 480 (d) Rs. 320
- The banker's gain on a sum due 3 years hence at 12% per annum is Rs. 270. The banker's discount is :
 (a) Rs. 960 (b) Rs. 840 (c) Rs. 1020 (d) Rs. 760
- The present worth of a sum due sometime hence is Rs. 576 and the banker's gain is Rs. 16. The true discount is :
 (a) Rs. 36 (b) Rs. 72 (c) Rs. 48 (d) Rs. 96
- The banker's discount on Rs. 1600 at 15% per annum is the same as true discount on Rs. 1680 for the same time and at the same rate. The time is :
 (a) 3 months (b) 4 months (c) 6 months (d) 8 months
- The banker's discount on a sum of money for $1\frac{1}{2}$ years is Rs. 558 and the true discount on the same sum for 2 years is Rs. 600. The rate percent is :
 (a) 10% (b) 13% (c) 12% (d) 15%
- The banker's discount of a certain sum of money is Rs. 72 and the true discount on the same sum for the same time is Rs. 60. The sum due is :
 (a) Rs. 360 (b) Rs. 432 (c) Rs. 540 (d) Rs. 1080

- With a guarantee of only 70% and yet you have a 100% chance of success. And X said,
12. The banker's discount on a certain sum due 2 years hence is $\frac{11}{10}$ of the true discount.
 The rate percent is :
 (a) 11% (b) 10% (c) 5% (d) 5.5%
13. The banker's gain on a certain sum due $1\frac{1}{2}$ years hence is $\frac{3}{25}$ of the banker's discount.
 The rate percent is :
 (a) $5\frac{1}{5}\%$ (b) $9\frac{1}{9}\%$ (c) $8\frac{1}{8}\%$ (d) $6\frac{1}{6}\%$

ANSWERS

1. (b) 2. (b) 3. (c) 4. (c) 5. (d) 6. (a) 7. (c)
 8. (d) 9. (b) 10. (c) 11. (a) 12. (c) 13. (b)

SOLUTIONS

1. P.W. = Rs. $(540 - 90) = \text{Rs. } 450$.
 ∵ S.I. on Rs. 450 = Rs. 90

S.I. on Rs. 540 = Rs. $\left(\frac{90}{450} \times 540\right) = \text{Rs. } 108$.

∴ B.D. = Rs. 108.

2. B.G. = $\frac{(T.D.)^2}{P.W.} = \text{Rs. } \left(\frac{36 \times 36}{800}\right) = \text{Rs. } 1.62$.

∴ B.D. = $(T.D. + B.G.) = \text{Rs. } (36 + 1.62) = \text{Rs. } 37.62$.

3. B.G. = $\frac{(T.D.)^2}{P.W.} = \text{Rs. } \left(\frac{160 \times 160}{1600}\right) = \text{Rs. } 16$.

4. T.D. = $\left(\frac{B.G. \times 100}{\text{Rate} \times \text{Time}}\right) = \text{Rs. } \left(\frac{24 \times 100}{10 \times 2}\right) = \text{Rs. } 120$.

∴ P.W. = $\frac{100 \times T.D.}{\text{Rate} \times \text{Time}} = \text{Rs. } \left(\frac{100 \times 120}{10 \times 2}\right) = \text{Rs. } 600$.

5. T.D. = $\frac{B.G. \times 100}{R \times T} = \text{Rs. } \left(\frac{6 \times 100}{12 \times 1}\right) = \text{Rs. } 50$.

6. T.D. = $\frac{B.D. \times 100}{100 + (R \times T)} = \text{Rs. } \left[\frac{420 \times 100}{100 + \left(15 \times \frac{1}{3}\right)}\right] = \text{Rs. } \left(\frac{420 \times 100}{105}\right) = \text{Rs. } 400$.

7. T.D. = $\left(\frac{B.G. \times 100}{R \times T}\right) = \text{Rs. } \left(\frac{270 \times 100}{12 \times 3}\right) = \text{Rs. } 750$.

∴ B.D. = Rs. $(750 + 270) = \text{Rs. } 1020$.

8. T.D. = $\sqrt{P.W. \times B.G.} = \sqrt{576 \times 16} = 96$.

9. S.I. on Rs. 1600 = T.D. on Rs. 1680.

∴ Rs. 1600 is the P.W. of Rs. 1680, i.e., Rs. 80 is S.I. on Rs. 1600 at 15%.

∴ Time = $\left(\frac{100 \times 80}{1600 \times 15}\right)$ year = $\frac{1}{3}$ year = 4 months.

10. B.D. for $\frac{3}{2}$ years = Rs. 558. B.D. for 2 years = Rs. $\left(558 \times \frac{2}{3} \times 2 \right)$ = Rs. 744.

T.D. for 2 years = Rs. 600.

$$\therefore \text{Sum} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \text{Rs.} \left(\frac{744 \times 600}{144} \right) = \text{Rs.} 3100.$$

Thus, Rs. 744 is S.I. on Rs. 3100 for 2 years.

$$\therefore \text{Rate} = \left(\frac{100 \times 744}{3100 \times 2} \right) \% = 12\%.$$

$$11. \text{Sum} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \text{Rs.} \left(\frac{72 \times 60}{72 - 60} \right) = \text{Rs.} \left(\frac{72 \times 60}{12} \right) = \text{Rs.} 360.$$

$$12. \text{Let T.D. be Re 1. Then, B.D.} = \text{Rs.} \frac{11}{10} = \text{Rs.} 1.10.$$

$$\therefore \text{Sum} = \text{Rs.} \left(\frac{1.10 \times 1}{1.10 - 1} \right) = \text{Rs.} \left(\frac{110}{10} \right) = \text{Rs.} 11.$$

\therefore S.I. on Rs. 11 for 2 years is Rs. 1.10.

$$\therefore \text{Rate} = \left(\frac{100 \times 1.10}{11 \times 2} \right) \% = 5\%.$$

$$13. \text{Let, B.D.} = \text{Re 1. Then, B.G.} = \text{Re} \frac{3}{25}.$$

$$\therefore \text{T.D.} = (\text{B.D.} - \text{B.G.}) = \text{Re} \left(1 - \frac{3}{25} \right) = \text{Re} \frac{22}{25}.$$

$$\text{Sum} = \left(\frac{1 \times \frac{22}{25}}{1 - \frac{22}{25}} \right) = \text{Rs.} \frac{22}{3}.$$

S.I. on Rs. $\frac{22}{3}$ for $1\frac{1}{2}$ years is Re 1.

$$\therefore \text{Rate} = \left(\frac{100 \times 1}{\frac{22}{3} \times \frac{3}{2}} \right) \% = 9\frac{1}{9}\%.$$

34. HEIGHTS AND DISTANCES

IMPORTANT FACTS AND FORMULAE

1. We already know that :

In a rt. angled $\triangle OAB$, where $\angle BOA = 90^\circ$,

$$(i) \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{OB};$$

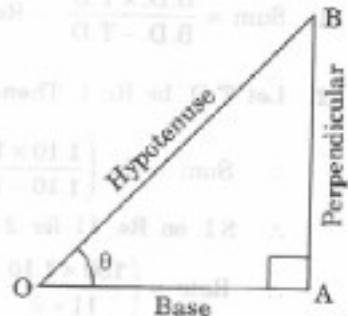
$$(ii) \cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{OA}{OB};$$

$$(iii) \tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{OA};$$

$$(iv) \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{OB}{AB};$$

$$(v) \sec \theta = \frac{1}{\cos \theta} = \frac{OB}{OA};$$

$$(vi) \cot \theta = \frac{1}{\tan \theta} = \frac{OA}{AB}.$$



2. Trigonometrical Identities :

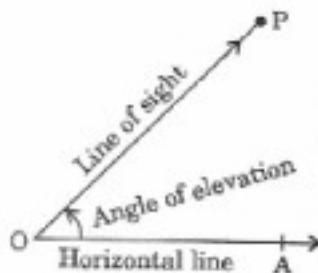
$$(i) \sin^2 \theta + \cos^2 \theta = 1. \quad (ii) 1 + \tan^2 \theta = \sec^2 \theta. \quad (iii) 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta.$$

3. Values of T-ratios :

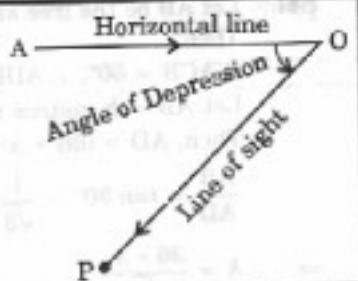
θ	0°	$(\pi/6)$ 30°	$(\pi/4)$ 45°	$(\pi/3)$ 60°	$(\pi/2)$ 90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

4. Angle of Elevation : Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.

\therefore Angle of elevation of P from O = $\angle AOP$.



5. Angle of Depression : Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.



SOLVED EXAMPLES

Ex. 1. If the height of a pole is $2\sqrt{3}$ metres and the length of its shadow is 2 metres, find the angle of elevation of the sun.

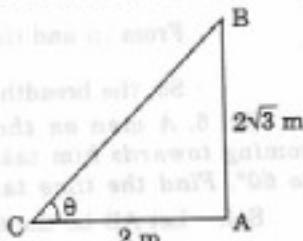
Sol. Let AB be the pole and AC be its shadow.

Let angle of elevation, $\angle ACB = \theta$.

Then, $AB = 2\sqrt{3}$ m, $AC = 2$ m.

$$\tan \theta = \frac{AB}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = 60^\circ.$$

So, the angle of elevation is 60° .



Ex. 2. A ladder leaning against a wall makes an angle of 60° with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall.

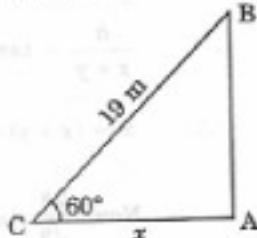
Sol. Let AB be the wall and BC be the ladder.

Then, $\angle ACB = 60^\circ$ and $BC = 19$ m.

Let $AC = x$ metres

$$\frac{AC}{BC} = \cos 60^\circ \Rightarrow \frac{x}{19} = \frac{1}{2} \Rightarrow x = \frac{19}{2} = 9.5.$$

\therefore Distance of the foot of the ladder from the wall = 9.5 m.



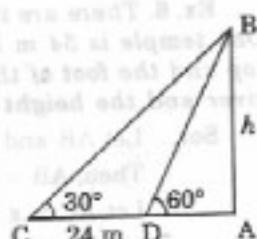
Ex. 3. The angle of elevation of the top of a tower at a point on the ground is 30° . On walking 24 m towards the tower, the angle of elevation becomes 60° . Find the height of the tower.

Sol. Let AB be the tower and C and D be the points of observation. Then,

$$\frac{AB}{AD} = \tan 60^\circ = \sqrt{3} \quad \Rightarrow \quad AD = \frac{AB}{\sqrt{3}} = \frac{h}{\sqrt{3}}$$

$$\frac{AB}{AC} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = h\sqrt{3}.$$

$$CD = (AC - AD) = \left(h\sqrt{3} - \frac{h}{\sqrt{3}} \right).$$



$$\therefore h\sqrt{3} - \frac{h}{\sqrt{3}} = 24 \quad \Rightarrow \quad h = 12\sqrt{3} = (12 \times 1.73) = 20.76.$$

Hence, the height of the tower is 20.76 m.

Ex. 4. A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60° . When he retires 36 m from the bank, he finds the angle to be 30° . Find the breadth of the river.

Sol. Let AB be the tree and AC be the river. Let C and D be the two positions of the man. Then,

$$\angle ACB = 60^\circ, \angle ADB = 30^\circ \text{ and } CD = 36 \text{ m.}$$

Let AB = h metres and AC = x metres.

Then, AD = $(36 + x)$ metres.

$$\frac{AB}{AD} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{36+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{36+x}{\sqrt{3}} \quad \dots(i)$$

$$\frac{AB}{AC} = \tan 60^\circ = \sqrt{3} \Rightarrow \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow h = \sqrt{3}x \quad \dots(ii)$$

$$\text{From (i) and (ii), we get: } \frac{36+x}{\sqrt{3}} = \sqrt{3}x \Rightarrow x = 18 \text{ m.}$$

So, the breadth of the river = 18 m.

Ex. 5. A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60° . Find the time taken by the boat to reach the shore from this position.

Sol. Let AB be the tower and C and D be the two positions of the boat.

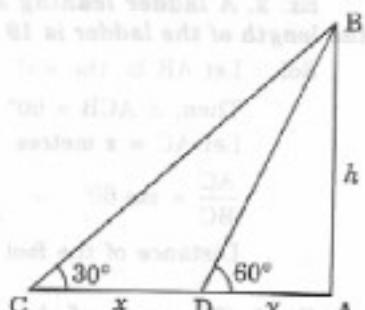
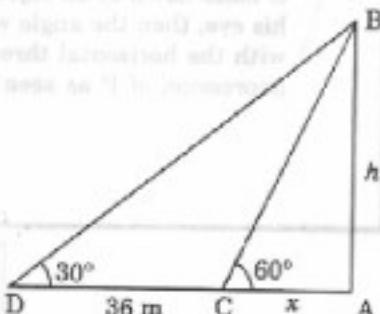
Let AB = h , CD = x and AD = y .

$$\frac{h}{y} = \tan 60^\circ = \sqrt{3} \Rightarrow y = \frac{h}{\sqrt{3}}$$

$$\frac{h}{x+y} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow x+y = \sqrt{3}h$$

$$\therefore x = (x+y) - y = \left(\sqrt{3}h - \frac{h}{\sqrt{3}} \right) = \frac{2h}{\sqrt{3}}$$

Now, $\frac{2h}{\sqrt{3}}$ is covered in 10 min.



Hence, required time = 5 minutes.

Ex. 6. There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.

Sol. Let AB and CD be the two temples and AC be the river.

Then, AB = 54 m.

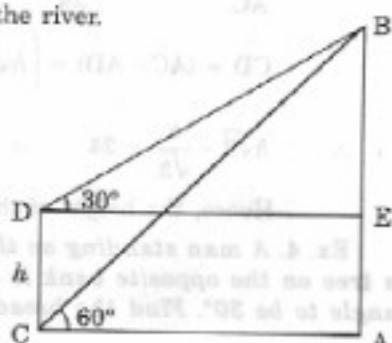
Let AC = x metres and CD = h metres.

$$\angle ACB = 60^\circ, \angle EDB = 30^\circ$$

$$\frac{AB}{AC} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow AC = \frac{AB}{\sqrt{3}} = \frac{54}{\sqrt{3}} = \left(\frac{54}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \right) = 18\sqrt{3} \text{ m.}$$

$$DE = AC = 18\sqrt{3} \text{ m.}$$



Given : $\frac{BE}{DE} = \tan 30^\circ = \frac{1}{\sqrt{3}}$ since 30° angle is between vertical height BE and horizontal distance DE .

$$\Rightarrow BE = \left(18\sqrt{3} \times \frac{1}{\sqrt{3}} \right) = 18 \text{ m.}$$

$$\therefore CD = AE = AB - BE = (54 - 18) \text{ m} = 36 \text{ m.}$$

$$\text{So, Width of the river} = AC = 18\sqrt{3} \text{ m} = (18 \times 1.73) \text{ m} = 31.14 \text{ m.}$$

Height of the other temple = $CD = 18 \text{ m.}$

EXERCISE 34

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (\checkmark) against the correct answer :

- The angle of elevation of the sun, when the length of the shadow of a tree is $\sqrt{3}$ times the height of the tree, is :
(a) 30° (b) 45° (c) 60° (d) 90°
(R.R.B. 2003)
- From a point P on a level ground, the angle of elevation of the top of a tower is 30° . If the tower is 100 m high, the distance of point P from the foot of the tower is :
(a) 149 m (b) 156 m (c) 173 m (d) 200 m
(R.R.B. 2002)
- The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is :
(a) 2.3 m (b) 4.6 m (c) 7.8 m (d) 9.2 m
- An observer 1.6 m tall is $20\sqrt{3}$ m away from a tower. The angle of elevation from his eye to the top of the tower is 30° . The height of the tower is :
(a) 21.6 m (b) 23.2 m (c) 24.72 m (d) None of these
- Two ships are sailing in the sea on the two sides of a lighthouse. The angles of elevation of the top of the lighthouse as observed from the two ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is :
(a) 173 m (b) 200 m (c) 273 m (d) 300 m
- A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30° with the man's eye. The man walks some distance towards the tower to watch its top and the angle of elevation becomes 60° . What is the distance between the base of the tower and the point P ?
(Bank P.O. 1999)
(a) $4\sqrt{3}$ units (b) 8 units (c) 12 units
(d) Data inadequate (e) None of these
- The angle of elevation of the top of a tower from a certain point is 30° . If the observer moves 20 m towards the tower, the angle of elevation of the top of the tower increases by 15° . The height of the tower is :
(a) 17.3 m (b) 21.9 m (c) 27.3 m (d) 30 m
- A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 metres from the tower. After 5 seconds, the angle of depression becomes 30° . What is the approximate speed of the boat, assuming that it is running in still water ?
(a) 32 kmph (b) 36 kmph (c) 38 kmph
(d) 40 kmph (e) 42 kmph
(S.B.I.P.O. 1999)
- On the same side of a tower, two objects are located. Observed from the top of the tower, their angles of depression are 45° and 60° . If the height of the tower is 150 m, the distance between the objects is :
(a) 63.5 m (b) 76.9 m (c) 86.7 m (d) 90 m

10. A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45° , how soon after this will the car reach the observation tower ?
 (a) 14 min. 35 sec. (b) 15 min. 49 sec. (c) 16 min. 23 sec. (d) 18 min. 5 sec.
 (R.R.B. 2002)
11. The top of a 15 metre high tower makes an angle of elevation of 60° with the bottom of an electric pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole ?
 (a) 5 metres (b) 8 metres (c) 10 metres
 (d) 12 metres (e) None of these

ANSWERS

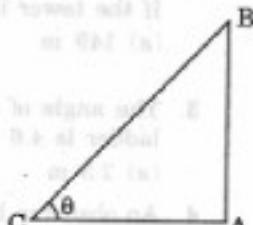
1. (a) 2. (c) 3. (d) 4. (a) 5. (c) 6. (d)
 7. (c) 8. (a) 9. (a) 10. (c) 11. (e)

SOLUTIONS

1. Let AB be the tree and AC be its shadow.

$$\text{Let } \angle ACB = \theta.$$

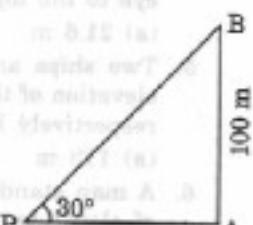
$$\text{Then, } \frac{AC}{AB} = \sqrt{3} \Rightarrow \cot \theta = \sqrt{3} \Rightarrow \theta = 30^\circ.$$



2. Let AB be the tower. Then, $\angle APB = 30^\circ$ and $AB = 100$ m.

$$\frac{AB}{AP} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow AP = (AB \times \sqrt{3}) = 100\sqrt{3} \text{ m.}$$

$$= (100 \times 1.73) \text{ m} = 173 \text{ m.}$$

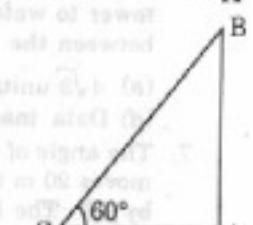


3. Let AB be the wall and BC be the ladder.

$$\text{Then, } \angle ACB = 60^\circ \text{ and } AC = 4.6 \text{ m.}$$

$$\frac{AC}{BC} = \cos 60^\circ = \frac{1}{2}$$

$$\Rightarrow BC = 2 \times AC = (2 \times 4.6) \text{ m} = 9.2 \text{ m.}$$



4. Let AB be the observer and CD be the tower.

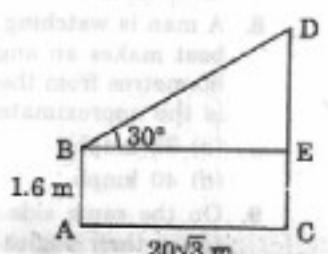
Draw $BE \perp CD$.

$$\text{Then, } CE = AB = 1.6 \text{ m, } BE = AC = 20\sqrt{3} \text{ m.}$$

$$\frac{DE}{BE} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow DE = \frac{20\sqrt{3}}{\sqrt{3}} \text{ m} = 20 \text{ m.}$$

$$\therefore CD = CE + DE = (1.6 + 20) \text{ m} = 21.6 \text{ m.}$$



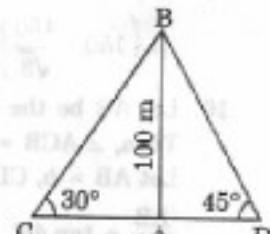
5. Let AB be the lighthouse and C and D be the positions of the ships. Then,

$AB = 100 \text{ m}$, $\angle ACB = 30^\circ$ and $\angle ADB = 45^\circ$.

$$\frac{AB}{AC} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = 100\sqrt{3} \text{ m.}$$

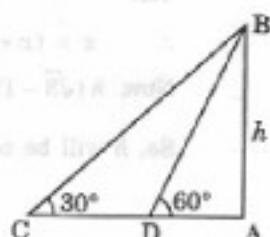
$$\frac{AB}{AD} = \tan 45^\circ = 1 \Rightarrow AD = AB = 100 \text{ m.}$$

$$\therefore CD = (AC + AD) = (100\sqrt{3} + 100) \text{ m} \\ = 100(\sqrt{3} + 1) \text{ m} = (100 \times 2.73) \text{ m} = 273 \text{ m.}$$



6. One of AB, AD and CD must have been given.

So, the data is inadequate.



7. Let AB be the tower and C and D be the points of observation.

Then, $\angle ACB = 30^\circ$, $\angle ADB = 45^\circ$ and $CD = 20 \text{ m}$.

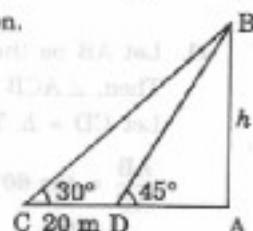
Let $AB = h$.

$$\text{Then, } \frac{AB}{AC} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = h\sqrt{3}.$$

$$\text{And, } \frac{AB}{AD} = \tan 45^\circ = 1 \Rightarrow AD = AB = h.$$

$$CD = 20 \Rightarrow (AC - AD) = 20 \Rightarrow h\sqrt{3} - h = 20.$$

$$\therefore h = \frac{20}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = 10(\sqrt{3}+1) \text{ m} = (10 \times 2.73) \text{ m} = 27.3 \text{ m.}$$



8. Let AB be the tower and C and D be the two positions of the boats.

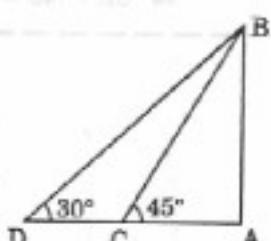
Then, $\angle ACB = 45^\circ$, $\angle ADB = 30^\circ$ and $AC = 60 \text{ m}$.

Let $AB = h$.

$$\text{Then, } \frac{AB}{AC} = \tan 45^\circ = 1 \Rightarrow AB = AC \Rightarrow h = 60 \text{ m.}$$

$$\text{And, } \frac{AB}{AD} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow AD = (AB \times \sqrt{3}) = 60\sqrt{3} \text{ m.}$$

$$\therefore CD = (AD - AC) = 60(\sqrt{3} - 1) \text{ m.}$$



$$\text{Hence, required speed} = \left[\frac{60(\sqrt{3}-1)}{5} \right] \text{ m/s} = (12 \times 0.73) \text{ m/s}$$

$$= \left(12 \times 0.73 \times \frac{18}{5} \right) \text{ km/hr} = 31.5 \text{ km/hr} \approx 32 \text{ km/hr.}$$

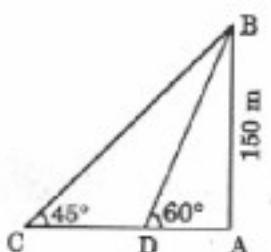
9. Let AB be the tower and C and D be the objects.

Then, $AB = 150 \text{ m}$, $\angle ACB = 45^\circ$ and $\angle ADB = 60^\circ$.

$$\frac{AB}{AD} = \tan 60^\circ = \sqrt{3} \Rightarrow AD = \frac{AB}{\sqrt{3}} = \frac{150}{\sqrt{3}} \text{ m.}$$

$$\frac{AB}{AC} = \tan 45^\circ = 1 \Rightarrow AC = AB = 150 \text{ m.}$$

$$\therefore CD = (AC - AD)$$



$$= \left(150 - \frac{150}{\sqrt{3}} \right) m = \left[\frac{150(\sqrt{3} - 1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \right] m = 50(3 - \sqrt{3}) m = (50 \times 1.27) m = 63.5 \text{ m.}$$

10. Let AB be the tower and C and D be the two positions of the car.

Then, $\angle ACB = 45^\circ$, $\angle ADB = 30^\circ$.

Let AB = h, CD = x and AC = y.

$$\frac{AB}{AC} = \tan 45^\circ = 1 \Rightarrow \frac{h}{y} = 1 \Rightarrow y = h.$$

$$\frac{AB}{AD} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{x+y} = \frac{1}{\sqrt{3}} \Rightarrow x+y = \sqrt{3}h.$$

$$\therefore x = (x+y) - y = \sqrt{3}h - h = h(\sqrt{3} - 1).$$

Now, $h(\sqrt{3} - 1)$ is covered in 12 min.

$$\text{So, } h \text{ will be covered in } \left[\frac{12}{h(\sqrt{3} - 1)} \times h \right] = \frac{12}{(\sqrt{3} - 1)} \text{ min.}$$

$$= \left(\frac{1200}{73} \right) \text{ min.} \approx 16 \text{ min. 23 sec.}$$

11. Let AB be the tower and CD be the electric pole.

Then, $\angle ACB = 60^\circ$, $\angle EDB = 30^\circ$ and AB = 15 m.

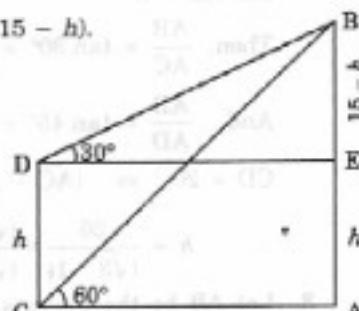
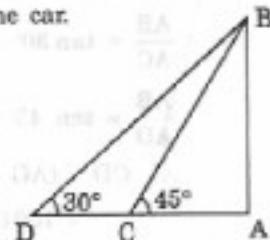
Let CD = h. Then, BE = (AB - AE) = (AB - CD) = (15 - h).

$$\frac{AB}{AC} = \tan 60^\circ = \sqrt{3} \Rightarrow AC = \frac{AB}{\sqrt{3}} = \frac{15}{\sqrt{3}}.$$

$$\text{And, } \frac{BE}{DE} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow DE = (BE \times \sqrt{3}) \\ = \sqrt{3}(15 - h).$$

$$AC = DE \Rightarrow \frac{15}{\sqrt{3}} = \sqrt{3}(15 - h)$$

$$\Rightarrow 3h = (45 - 15) \Rightarrow h = 10 \text{ m.}$$



35. ODD MAN OUT AND SERIES

EXERCISE 35

Directions : Find the odd man out :

1. 3, 5, 7, 12, 17, 19
(a) 19 (b) 17 (c) 13 (d) 12 16 (a)
2. 10, 14, 16, 18, 21, 24, 26
(a) 26 (b) 24 (c) 21 (d) 18 22 (a)
3. 3, 5, 9, 11, 14, 17, 21
(a) 21 (b) 17 (c) 14 (d) 9 8 (a)
4. 1, 4, 9, 16, 23, 25, 36
(a) 9 (b) 23 (c) 25 (d) 36 32 (a)
5. 6, 9, 15, 21, 24, 28, 30
(a) 28 (b) 21 (c) 24 (d) 30 31 (a)
6. 41, 43, 47, 53, 61, 71, 73, 81
(a) 61 (b) 71 (c) 73 (d) 81 75 (a)
7. 16, 25, 36, 72, 144, 196, 225
(a) 36 (b) 72 (c) 196 (d) 225 20 (a)
8. 10, 25, 45, 54, 60, 75, 80
(a) 10 (b) 45 (c) 54 (d) 75 55 (a)
9. 1, 4, 9, 16, 20, 36, 49
(a) 1 (b) 9 (c) 20 (d) 49 15 (a)
10. 8, 27, 64, 100, 125, 216, 343
(a) 27 (b) 100 (c) 125 (d) 343 72 (a)
11. 1, 5, 14, 30, 50, 55, 91
(a) 5 (b) 50 (c) 55 (d) 91 0 (a)
12. 385, 462, 572, 396, 427, 671, 264
(a) 385 (b) 427 (c) 671 (d) 264 01 (a)
13. 835, 734, 642, 751, 853, 981, 532
(a) 751 (b) 853 (c) 981 (d) 532 001 (a)
14. 331, 482, 551, 263, 383, 242, 111
(a) 263 (b) 383 (c) 242 (d) 111 18 (a)
15. 2, 5, 10, 17, 26, 37, 50, 64
(a) 50 (b) 26 (c) 37 (d) 64 021 (a)
16. 19, 28, 39, 52, 67, 84, 102
(a) 52 (b) 102 (c) 84 (d) 67 01 (a)
17. 253, 136, 352, 460, 324, 631, 244
(a) 136 (b) 324 (c) 352 (d) 631 018 (a)
18. 2, 5, 10, 50, 500, 5000
(a) 0 (b) 5 (c) 10 (d) 5000 25 (a)
19. 4, 5, 7, 10, 14, 18, 25, 32
(a) 7 (b) 14 (c) 18 (d) 33 1001 (a)

Directions : Find out the wrong number in each sequence :

20. 22, 33, 66, 99, 121, 279, 594
 (a) 33 (b) 121 (c) 279 (d) 594
21. 36, 54, 18, 27, 9, 18, 5, 4, 5
 (a) 4, 5 (b) 18, 5 (c) 54 (d) 18
22. 582, 605, 588, 611, 634, 617, 600
 (a) 634 (b) 611 (c) 605 (d) 600
23. 46080, 3840, 384, 48, 24, 2, 1
 (a) 1 (b) 2 (c) 24 (d) 384
24. 1, 8, 27, 64, 124, 216, 343
 (a) 8 (b) 27 (c) 64 (d) 124
25. 5, 16, 6, 16, 7, 16, 9
 (a) 9 (b) 7 (c) 6 (d) None of these
26. 6, 13, 18, 25, 30, 37, 40
 (a) 25 (b) 30 (c) 37 (d) 40
27. 56, 72, 90, 110, 132, 150
 (a) 72 (b) 110 (c) 132 (d) 150
28. 8, 13, 21, 32, 47, 63, 83
 (a) 47 (b) 63 (c) 32 (d) 83
29. 25, 36, 49, 81, 121, 169, 225
 (a) 36 (b) 49 (c) 121 (d) 169
30. 1, 2, 6, 15, 31, 56, 91
 (a) 31 (b) 91 (c) 56 (d) 15
31. 52, 51, 48, 43, 34, 27, 16
 (a) 27 (b) 34 (c) 43 (d) 48
32. 105, 85, 60, 30, 0, -45, -90
 (a) 0 (b) 85 (c) -45 (d) 60
33. 4, 6, 8, 9, 10, 11, 12
 (a) 10 (b) 11 (c) 12 (d) 9
34. 125, 127, 130, 135, 142, 153, 165
 (a) 130 (b) 142 (c) 153 (d) 165
35. 16, 36, 64, 81, 100, 144, 190
 (a) 81 (b) 100 (c) 190 (d) 36
36. 125, 123, 120, 115, 108, 100, 84
 (a) 123 (b) 115 (c) 100 (d) 84
37. 3, 10, 21, 36, 55, 70, 105
 (a) 105 (b) 70 (c) 36 (d) 55
38. 4, 9, 19, 39, 79, 160, 319
 (a) 319 (b) 160 (c) 79 (d) 39
39. 10, 14, 28, 32, 64, 68, 132
 (a) 32 (b) 68 (c) 132 (d) 28
40. 8, 27, 125, 343, 1331
 (a) 1331 (b) 343 (c) 125 (d) None of these

Directions : Insert the missing number :

41. 4, - 8, 16, - 32, 64, (....)

(a) 128	(b) - 128	(c) 192	(d) - 192
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42. 5, 10, 13, 26, 29, 58, 61, (....)

(a) 122	(b) 64	(c) 125	(d) 128
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43. 1, 4, 9, 16, 25, 36, 49, (....)

(a) 54	(b) 56	(c) 64	(d) 81
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44. 1, 8, 27, 64, 125, 216, (....)

(a) 354	(b) 343	(c) 392	(d) 245
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45. 11, 13, 17, 19, 23, 29, 31, 37, 41, (....)

(a) 43	(b) 47	(c) 53	(d) 51
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46. 16, 33, 65, 131, 261, (....)

(a) 523	(b) 521	(c) 613	(d) 721
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47. 3, 7, 6, 5, 9, 3, 12, 1, 15, (....)

(a) 18	(b) 13	(c) - 1	(d) 3
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48. 15, 31, 63, 127, 255, (....)

(a) 513	(b) 511	(c) 517	(d) 523
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49. 2, 6, 12, 20, 30, 42, 56, (....)

(a) 60	(b) 64	(c) 72	(d) 70
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50. 8, 24, 12, 36, 18, 54, (....)

(a) 27	(b) 108	(c) 68	(d) 72
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51. 165, 195, 225, 285, 345, (....)

(a) 375	(b) 420	(c) 435	(d) 390
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52. 7, 26, 63, 124, 215, 342, (....)

(a) 481	(b) 511	(c) 391	(d) 421
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53. 2, 4, 12, 48, 240, (....)

(a) 960	(b) 1440	(c) 1080	(d) 1920
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54. 8, 7, 11, 12, 14, 17, 17, 22, (....)

(a) 27	(b) 20	(c) 22	(d) 24
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55. 10, 5, 13, 10, 16, 20, 19, (....)

(a) 22	(b) 40	(c) 38	(d) 23
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56. 1, 2, 4, 8, 16, 32, 64, (....), 256

(a) 148	(b) 128	(c) 154	(d) 164
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57. 71, 76, 69, 74, 67, 72, (....)

(a) 77	(b) 65	(c) 80	(d) 76
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58. 9, 12, 11, 14, 13, (....), 15

(a) 12	(b) 16	(c) 10	(d) 17
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59. Complete the series : 2, 5, 9, 19, 37,

(a) 76	(b) 74	(c) 75	(d) None of these
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60. Find the wrong number in the series : 3, 8, 15, 24, 34, 48, 63

(a) 15	(b) 24	(c) 34	(d) 48	(e) 63
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61. Find the wrong number in the series : 2, 9, 28, 65, 126, 216, 344

(a) 2	(b) 28	(c) 65	(d) 126	(e) 216
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62. Find out the wrong number in the series : 5, 15, 30, 135, 405, 1215, 3645

(a) 3645	(b) 1215	(c) 405	(d) 30	(e) 15
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63. Find out the wrong number in the series : 125, 106, 88, 76, 65, 58, 53

(a) 125	(b) 106	(c) 88	(d) 76	(e) 65
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Directions : Find out the wrong number in the series :

64. 190, 166, 145, 128, 112, 100, 91
 (a) 100 (b) 166 (c) 145 (d) 128 (e) 112
65. 1, 1, 2, 6, 24, 96, 720
 (a) 720 (b) 96 (c) 24 (d) 6 (e) 2
66. 40960, 10240, 2560, 640, 200, 40, 10
 (a) 640 (b) 40 (c) 200 (d) 2560 (e) 10240
67. 64, 71, 80, 91, 104, 119, 135, 155
 (a) 71 (b) 80 (c) 104 (d) 119 (e) 135
68. 7, 8, 18, 57, 228, 1165, 6996
 (a) 8 (b) 18 (c) 57 (d) 228 (e) 1165
69. 3, 7, 15, 27, 63, 127, 255
 (a) 7 (b) 15 (c) 27 (d) 63 (e) 127
70. 19, 26, 33, 46, 59, 74, 91
 (a) 26 (b) 33 (c) 46 (d) 59 (e) 74
71. 2880, 480, 92, 24, 8, 4, 4
 (a) 480 (b) 92 (c) 24 (d) 8 (e) 4
72. 445, 221, 109, 46, 25, 11, 4
 (a) 221 (b) 109 (c) 46 (d) 25 (e) 11
73. 3, 7, 15, 39, 63, 127, 255, 511
 (a) 7 (b) 15 (c) 39 (d) 63 (e) 127
74. 1, 3, 10, 21, 64, 129, 356, 777
 (a) 10 (b) 21 (c) 64 (d) 129 (e) 356
75. 196, 169, 144, 121, 100, 80, 64
 (a) 169 (b) 144 (c) 121 (d) 100 (e) 80
76. 6, 12, 48, 100, 384, 768, 3072
 (a) 768 (b) 384 (c) 100 (d) 48 (e) 12
77. 10, 26, 74, 218, 654, 1946, 5834
 (a) 26 (b) 74 (c) 218 (d) 654 (e) 1946
78. 15, 16, 34, 105, 424, 2124, 12576
 (a) 16 (b) 34 (c) 105 (d) 424 (e) 2124
79. 2807, 1400, 697, 347, 171, 84, 41, 20
 (a) 697 (b) 347 (c) 171 (d) 84 (e) 41
80. 32, 36, 41, 61, 86, 122, 171, 235
 (a) 41 (b) 61 (c) 86 (d) 122 (e) 171
81. 3, 4, 9, 22.5, 67.5, 202.5, 810
 (a) 4 (b) 9 (c) 22.5 (d) 67.5 (e) 202.5
82. 1, 2, 8, 33, 148, 760, 4626
 (a) 2 (b) 8 (c) 33 (d) 148 (e) 760
83. 3, 8, 18, 46, 100, 210, 432
 (a) 8 (b) 18 (c) 46 (d) 100 (e) 210
84. 789, 645, 545, 481, 440, 429, 425
 (a) 645 (b) 545 (c) 481 (d) 440 (e) 429
85. 1050, 510, 242, 106, 46, 16, 3
 (a) 510 (b) 242 (c) 106 (d) 46 (e) 16

86. 5, 8, 20, 42, 124, 246, 736
 (a) 8 (b) 20 (c) 42 (d) 124 (e) 246
87. 2, 3, 6, 15, 52.5, 157.5, 630
 (a) 3 (b) 6 (c) 15 (d) 52.5 (e) 157.5
88. 888, 440, 216, 104, 48, 22, 6
 (a) 440 (b) 216 (c) 104 (d) 48 (e) 22
89. 4, 5, 15, 49, 201, 1011, 6073
 (a) 5 (b) 15 (c) 49 (d) 201 (e) 1011

ANSWERS

1. (d) 2. (c) 3. (c) 4. (b) 5. (a) 6. (d) 7. (b) 8. (c) 9. (c)
 10. (b) 11. (b) 12. (b) 13. (a) 14. (b) 15. (d) 16. (b) 17. (b) 18. (d)
 19. (c) 20. (c) 21. (b) 22. (a) 23. (c) 24. (d) 25. (a) 26. (d) 27. (d)
 28. (a) 29. (a) 30. (b) 31. (b) 32. (a) 33. (b) 34. (d) 35. (c) 36. (c)
 37. (b) 38. (b) 39. (c) 40. (d) 41. (b) 42. (a) 43. (c) 44. (b) 45. (a)
 46. (a) 47. (c) 48. (b) 49. (c) 50. (a) 51. (c) 52. (b) 53. (b) 54. (b)
 55. (b) 56. (b) 57. (b) 58. (b) 59. (c) 60. (c) 61. (e) 62. (d) 63. (c)
 64. (d) 65. (b) 66. (c) 67. (e) 68. (d) 69. (c) 70. (b) 71. (b) 72. (c)
 73. (c) 74. (c) 75. (c) 76. (c) 77. (d) 78. (e) 79. (b) 80. (a) 81. (a)
 82. (e) 83. (b) 84. (d) 85. (c) 86. (b) 87. (d) 88. (e) 89. (a)

SOLUTIONS

- Each of the numbers except 12, is a prime number.
- Each of the numbers except 21, is an even number.
- Each of the numbers except 14, is an odd number.
- Each of the given numbers except 23, is a perfect square.
- Each of the numbers except 28, is a multiple of 3.
- Each of the numbers except 81, is a prime number.
- Each of the numbers except 72, is a perfect square.
- Each of the numbers except 54, is a multiple of 5.
- The pattern is $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2$. But, instead of 5^2 , it is 20, which is to be turned out.
- The pattern is $2^3, 3^3, 4^3, 5^3, 6^3, 7^3$. But, 100 is not a perfect cube.
- The pattern is $1^2, 1^2 + 2^2, 1^2 + 2^2 + 3^2, 1^2 + 2^2 + 3^2 + 4^2, 1^2 + 2^2 + 3^2 + 4^2 + 5^2, 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$. But, 50 is not of this pattern.
- In each number except 427, the middle digit is the sum of the other two.
- In each number except 751, the difference of third and first digit is the middle one.
- In each number except 383, the product of first and third digits is the middle one.
- The pattern is $x^2 + 1$, where $x = 1, 2, 3, 4, 5, 6, 7, 8$ etc. But, 64 is out of pattern.
- The pattern is $x^2 + 3$, where $x = 4, 5, 6, 7, 8, 9$ etc. But, 102 is out of pattern.
- Sum of the digits in each number, except 324 is 10.
- Pattern is 1st \times 2nd = 3rd; 2nd \times 3rd = 4th; 3rd \times 4th = 5th.
 But, 4th \times 5th = $50 \times 500 = 25000 \neq 5000 = 6$ th.

19. $2^{\text{nd}} = (1^{\text{st}} + 1)$; $3^{\text{rd}} = (2^{\text{nd}} + 2)$; $4^{\text{th}} = (3^{\text{rd}} + 3)$; $5^{\text{th}} = (4^{\text{th}} + 4)$.
 But, $18 = 6^{\text{th}} \neq 5^{\text{th}} + 5 = 14 + 5 = 19$.
20. Each number except 279 is a multiple of 11.
21. The terms are alternately multiplied by 1.5 and divided by 3. However, 18.5 does not satisfy it.
22. Alternately 23 is added and 17 is subtracted from the terms. So, 634 is wrong.
23. The terms are successively divided by 12, 10, 8, 6, etc. So, 24 is wrong.
24. The numbers are 1^3 , 2^3 , 3^3 , 4^3 etc. So, 124 is wrong; it must have been 5^3 i.e., 125.
25. Terms at odd places are 5, 6, 7, 8 etc. and each term at even place is 16.
 So, 9 is wrong.
26. The difference between two successive terms from the beginning are 7, 5, 7, 5, 7, 5.
 So, 40 is wrong.
27. The numbers are 7×8 , 8×9 , 9×10 , 10×11 , 11×12 , 12×13 . So, 150 is wrong.
28. Go on adding 5, 8, 11, 14, 17, 20.
 So, the number 47 is wrong and must be replaced by 46.
29. The numbers are squares of odd natural numbers, starting from 5 upto 15.
 So, 36 is wrong.
30. Add 1^2 , 2^2 , 3^2 , 4^2 , 5^2 , 6^2 . So, 91 is wrong.
31. Subtract 1, 3, 5, 7, 9, 11 from successive numbers. So, 34 is wrong.
32. Subtract 20, 25, 30, 35, 40, 45 from successive numbers. So, 0 is wrong.
33. Each number is a composite number except 11.
34. Prime numbers 2, 3, 5, 7, 11, 13 are to be added successively. So, 165 is wrong.
35. Each number is the square of a composite number except 190.
36. Prime numbers 2, 3, 5, 7, 11, 13 have successively been subtracted.
 So, 100 is wrong. It must be $(108 - 11)$ i.e., 97.
37. The pattern is 1×3 , 2×5 , 3×7 , 4×9 , 5×11 , 6×13 , 7×15 etc.
38. Double the number and add 1 to it, to get the next number. So, 160 is wrong.
39. Alternately, we add 4 and double the next.
 So, 132 is wrong. It must be (68×2) i.e., 136.
40. The numbers are cubes of primes i.e., 2^3 , 3^3 , 5^3 , 7^3 , 11^3 . Clearly, none is wrong.
41. Each number is the preceding number multiplied by - 2.
 So, the required number is - 128.
42. Numbers are alternately multiplied by 2 and increased by 3.
 So, the missing number = $61 \times 2 = 122$.
43. Numbers are 1^2 , 2^2 , 3^2 , 4^2 , 5^2 , 6^2 , 7^2 . So, the next number is $8^2 = 64$.
44. Numbers are 1^3 , 2^3 , 3^3 , 4^3 , 5^3 , 6^3 . So, the missing number is $7^3 = 343$.
45. Numbers are all primes. The next prime is 43.
46. Each number is twice the preceding one with 1 added or subtracted alternately.
 So, the next number is $(2 \times 261 + 1) = 523$.
47. There are two series, beginning respectively with 3 and 7. In one 3 is added and in another 2 is subtracted. The next number is $1 - 2 = - 1$.
48. Each number is double the preceding one plus 1.
 So, the next number is $(255 \times 2) + 1 = 511$.
49. The pattern is 1×2 , 2×3 , 3×4 , 4×5 , 5×6 , 6×7 , 7×8 .
 So, the next number is $8 \times 9 = 72$.
50. Numbers are alternately multiplied by 3 and divided by 2.
 So, the next number = $54 \div 2 = 27$.

51. Each number is 15 multiplied by a prime number i.e., 15×11 , 15×13 , 15×17 , 15×19 , 15×23 . So, the next number is $15 \times 29 = 435$.
52. Numbers are $(2^3 - 1)$, $(3^3 - 1)$, $(4^3 - 1)$, $(5^3 - 1)$, $(6^3 - 1)$, $(7^3 - 1)$ etc. So, the next number is $(8^3 - 1) = (512 - 1) = 511$.
53. Go on multiplying the given numbers by 2, 3, 4, 5, 6. So, the correct next number is 1440.
54. There are two series $(8, 11, 14, 17, 20)$ and $(7, 12, 17, 22)$ increasing by 3 and 5 respectively.
55. There are two series $(10, 13, 16, 19)$ and $(5, 10, 20, 40)$, one increasing by 3 and the other multiplied by 2.
56. Each previous number is multiplied by 2.
57. Alternately, we add 5 and subtract 7.
58. Alternately, we add 3 and subtract 1.
59. Second number is one more than twice the first; third number is one less than twice the second; fourth number is one more than twice the third; fifth number is one less than the fourth. Therefore, the sixth number is one more than twice the fifth. So, the missing number is 75.
60. The difference between consecutive terms are respectively 5, 7, 9, 11 and 13. So, 34 is a wrong number.
61. $2 = (1^3 + 1)$; $9 = (2^3 + 1)$; $28 = (3^3 + 1)$; $65 = (4^3 + 1)$; $125 = (5^3 + 1)$; $216 \neq (6^3 + 1)$ and $344 = (7^3 + 1)$. So, 216 is a wrong number.
62. Multiply each term by 3 to obtain the next term. Hence, 30 is a wrong number.
63. Go on subtracting prime numbers, 19, 17, 13, 11, 7, 5 from the numbers to get the next number. So, 88 is wrong.
64. Go on subtracting 24, 21, 18, 15, 12, 9 from the numbers to get the next number. Clearly, 128 is wrong.
65. Go on multiplying with 1, 2, 3, 4, 5, 6 to get the next number. So, 96 is wrong.
66. Go on dividing by 4 to get the next number. So, 200 is wrong.
67. Go on adding 7, 9, 11, 13, 15, 17, 19 respectively to obtain the next number. So, 135 is wrong.
68. Let the given numbers be A, B, C, D, E, F, G. Then,
 $A, A \times 1, B \times 2 + 2, C \times 3 + 3, D \times 4 + 4, E \times 5 + 5, F \times 6 + 6$ are the required numbers.
 Clearly, 228 is wrong.
69. Go on multiplying the number by 2 and adding 1 to it to get the next number. So, 27 is wrong.
70. Go on adding 7, 9, 11, 13, 15, 17 respectively to obtain the next number. So, 33 is wrong.
71. Go on dividing by 6, 5, 4, 3, 2, 1 respectively to obtain the next number.
 Clearly, 92 is wrong.
72. Go on subtracting 3 and dividing the result by 2 to obtain the next number.
 Clearly, 46 is wrong.
73. Go on multiplying 2 and adding 1 to get the next number. So, 39 is wrong.
74. $A \times 2 + 1, B \times 3 + 1, C \times 2 + 1, D \times 3 + 1$ and so on. So, 356 is wrong.
75. Numbers must be $(14)^2$, $(13)^2$, $(11)^2$, $(10)^2$, $(9)^2$, $(8)^2$. So, 80 is wrong.
76. Each even term of the series is obtained by multiplying the previous term by 2.
 2nd term = (1st term) $\times 2 = 6 \times 2 = 12$; 4th term = (3rd term) $\times 2 = 48 \times 2 = 96$;
 6th term = (5th term) $\times 2 = 384 \times 2 = 768$.
 \therefore 4th term should be 96 instead of 100.

77. 2nd term = (1st term) \times 3 - 4 = $10 \times 3 - 4 = 26$; 3rd term = (2nd term) \times 3 - 4 = $26 \times 3 - 4 = 74$; 4th term = (3rd term) \times 3 - 4 = $74 \times 3 - 4 = 218$; 5th term = (4th term) \times 3 - 4 = $218 \times 3 - 4 = 650$. \therefore 5th term must be 650 instead of 654.
78. 2nd term = (1st term) \times 1 + 1 = $15 \times 1 + 1 = 16$; 3rd term = (2nd term) \times 2 + 2 = $16 \times 2 + 2 = 34$; 4th term = (3rd term) \times 3 + 3 = $34 \times 3 + 3 = 105$; 5th term = (4th term) \times 4 + 4 = $105 \times 4 + 4 = 424$; 6th term = (5th term) \times 5 + 5 = $424 \times 5 + 5 = 2125$. \therefore 6th term should be 2125 instead of 2124.
79. 7th term = (8th term) \times 2 + 1 = $20 \times 2 + 1 = 41$; 6th term = (7th term) \times 2 + 2 = $41 \times 2 + 2 = 84$; 5th term = (6th term) \times 2 + 3 = $84 \times 2 + 3 = 171$; 4th term = (5th term) \times 2 + 4 = $171 \times 2 + 4 = 346$. \therefore 4th term should be 346 instead of 347.
80. 2nd term = (1st term) $+ 2^2 = 32 + 4 = 36$; 3rd term = (2nd term) $+ 3^2 = 36 + 9 = 45$; 4th term = (3rd term) $+ 4^2 = 45 + 16 = 61$; 5th term = (4th term) $+ 5^2 = 61 + 25 = 86$. \therefore 3rd term should be 45 instead of 41.
81. There are two sequences (3, 9, 67.5, 810) and (4, 22.5, 202.5). Pattern is : (1st term \times 3), (2nd term \times 7.5), (3rd term \times 12) for the first sequence and (1st term \times 5), (2nd term \times 9) and so on for the second sequence.
82. 2nd term = (1st term $\times 1 + 1^2 = 1 \times 1 + 1^2 = 2$); 3rd term = (2nd term $\times 2 + 2^2 = 2 \times 2 + 2^2 = 8$); 4th term = (3rd term $\times 3 + 3^2 = 8 \times 3 + 3^2 = 33$); 5th term = (4th term $\times 4 + 4^2 = 33 \times 4 + 4^2 = 148$); 6th term = (5th term $\times 5 + 5^2 = 148 \times 5 + 5^2 = 765$). \therefore 760 is wrong.
83. 2nd term = (1st term $\times 2 + 2 = 3 \times 2 + 2 = 8$); 3rd term = (2nd term $\times 2 + 4 = 8 \times 2 + 4 = 20$); 4th term = (3rd term $\times 2 + 6 = 20 \times 2 + 6 = 46$); 5th term = (4th term $\times 2 + 8 = 46 \times 2 + 8 = 100$ and so on. \therefore 18 is wrong.
84. 2nd term = 1st term $- (12)^2 = 789 - 144 = 645$; 3rd term = (2nd term) $- (10)^2 = 645 - 100 = 545$; 4th term = (3rd term) $- (8)^2 = 545 - 64 = 481$; 5th term = (4th term) $- (6)^2 = 481 - 36 = 445$. \therefore 440 is wrong.
85. 2nd term = (1st term - 30) $\div 2 = \left(\frac{1050 - 30}{2} \right) = 510$; 3rd term = (2nd term - 26) $\div 2 = \left(\frac{510 - 26}{2} \right) = 242$; 4th term = (3rd term - 22) $\div 2 = \left(\frac{242 - 22}{2} \right) = 110$. \therefore 106 is wrong.

86. 2nd term = (1st term $\times 2 - 2$) $\div (5 \times 2 - 2) = 8$;
 3rd term = (2nd term $\times 3 - 2$) $= (8 \times 3 - 2) = 22$;
 4th term = (3rd term $\times 2 - 2$) $= (22 \times 2 - 2) = 42$;
 5th term = (4th term $\times 3 - 2$) $= (42 \times 3 - 2) = 124$ and so on.
 \therefore 20 is wrong.
87. 2nd term = (1st term $\times 1.5$) $= 2 \times 1.5 = 3$; 3rd term = (2nd term $\times 2$) $= 3 \times 2 = 6$;
 4th term = (3rd term $\times 2.5$) $= 6 \times 2.5 = 15$; 5th term = (4th term $\times 3$) $= 15 \times 3 = 45$.
 \therefore 52.5 is wrong.
88. 2nd term = $\left(\frac{1\text{st term} - 8}{2} \right) = \left(\frac{888 - 8}{2} \right) = 440$;
 3rd term = $\left(\frac{2\text{nd term} - 8}{2} \right) = \left(\frac{440 - 8}{2} \right) = 216$;
 4th term = $\left(\frac{3\text{rd term} - 8}{2} \right) = \left(\frac{216 - 8}{2} \right) = 104$;
 5th term = $\left(\frac{4\text{th term} - 8}{2} \right) = \left(\frac{104 - 8}{2} \right) = 48$;
 6th term = $\left(\frac{5\text{th term} - 8}{2} \right) = \left(\frac{48 - 8}{2} \right) = 20$.
 \therefore 22 is wrong.
89. 2nd term = (1st term $\times 1 + 2$) $= (4 \times 1 + 2) = 6$;
 3rd term = (2nd term $\times 2 + 3$) $= (6 \times 2 + 3) = 15$;
 4th term = (3rd term $\times 3 + 4$) $= (15 \times 3 + 4) = 49$;
 5th term = (4th term $\times 4 + 5$) $= (49 \times 4 + 5) = 210$ and so on.
 \therefore 5 is wrong.
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SECTION II

DATA

INTERPRETATION

36. TABULATION

This section comprises of questions in which certain data regarding common disciplines as production over a period of a few years : imports, exports, incomes of employees in a factory, students applying for and qualifying a certain field of study etc. are given in the form of a table. The candidate is required to understand the given information and thereafter answer the given questions on the basis of comparative analysis of the data.

Thus, here the data collected by the investigator are arranged in a systematic form in a table called the *tabular form*. In order to avoid some heads again and again, tables are made consisting of horizontal lines called *rows* and vertical lines called *columns* with distinctive heads, known as *captions*. Units of measurements are given with the captions.

SOLVED EXAMPLES

Ex. 1. The following table gives the sales of batteries manufactured by a company over the years. Study the table and answer the questions that follow :

(S.B.I.P.O. 1998)

NUMBER OF DIFFERENT TYPES OF BATTERIES SOLD BY A COMPANY
OVER THE YEARS (NUMBERS IN THOUSANDS)

Year	TYPES OF BATTERIES					Total
	4AH	7AH	32AH	35AH	55AH	
1992	75	144	114	102	108	543
1993	90	126	102	84	126	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

1. The total sales of all the seven years is the maximum for which battery ?
(a) 4AH (b) 7AH (c) 32AH (d) 35AH (e) 55AH
2. What is the difference in the number of 35AH batteries sold in 1993 and 1997 ?
(a) 24000 (b) 28000 (c) 35000 (d) 39000 (e) 42000
3. The percentage of 4AH batteries sold to the total number of batteries sold was maximum in the year :
(a) 1994 (b) 1995 (c) 1996 (d) 1997 (e) 1998
4. In the case of which battery there was a continuous decrease in sales from 1992 to 1997 ?
(a) 4AH (b) 7AH (c) 32AH (d) 35AH (e) 55AH
5. What was the approximate percentage increase in the sales of 55AH batteries in 1998 compared to that in 1992 ?
(a) 28% (b) 31% (c) 33% (d) 34% (e) 37%

Sol. 1. (c) : The total sales (in thousands) of all the seven years for various batteries are :

$$\text{For } 4\text{AH} = 75 + 90 + 96 + 105 + 90 + 105 + 115 = 676$$

$$\text{For } 7\text{AH} = 144 + 126 + 114 + 90 + 75 + 60 + 85 = 694$$

$$\text{For } 32\text{AH} = 114 + 102 + 75 + 150 + 135 + 165 + 160 = 901$$

$$\text{For } 35\text{ AH} = 102 + 84 + 105 + 90 + 75 + 45 + 100 = 601$$

$$\text{For } 55\text{ AH} = 108 + 126 + 135 + 75 + 90 + 120 + 145 = 799$$

Clearly, sales are maximum in case of 32AH batteries.

2. (d) : Required difference = $[(84 - 45) \times 1000] = 39000$.

3. (d) : The percentages of sales of 4AH batteries to the total sales in different years are :

$$\text{For } 1992 = \left(\frac{75}{543} \times 100 \right)\% = 13.81\%; \quad \text{For } 1993 = \left(\frac{90}{528} \times 100 \right)\% = 17.05\%$$

$$\text{For } 1994 = \left(\frac{96}{525} \times 100 \right)\% = 18.29\%; \quad \text{For } 1995 = \left(\frac{105}{510} \times 100 \right)\% = 20.59\%$$

$$\text{For } 1996 = \left(\frac{96}{465} \times 100 \right)\% = 19.35\%; \quad \text{For } 1997 = \left(\frac{105}{495} \times 100 \right)\% = 21.21\%$$

$$\text{For } 1998 = \left(\frac{115}{605} \times 100 \right)\% = 19.01\%$$

Clearly, the percentage is maximum in 1997.

4. (b) : From the table it is clear that the sales of 7AH batteries have been decreasing continuously from 1992 to 1997.

$$5. (d) : \text{Required Percentage} = \left[\frac{(145 - 108)}{108} \times 100 \right]\% = 34.26\% = 34\%$$

Ex. 2. Study the following table carefully and answer these questions :

(S.B.I.P.O. 2002)

NUMBER OF CANDIDATES APPEARED AND QUALIFIED IN A COMPETITIVE EXAMINATION FROM DIFFERENT STATES OVER THE YEARS

Year State	1997		1998		1999		2000		2001	
	App.	Qual.								
M	5200	720	8500	980	7400	850	6800	775	9500	1125
N	7500	840	9200	1050	8450	920	9200	980	8800	1020
P	6400	780	8800	1020	7800	890	8750	1010	9750	1250
Q	8100	950	9500	1240	8700	980	9700	1200	8950	995
R	7800	870	7600	940	9800	1350	7600	945	7990	885

- Combining the states P and Q together in 1998, what is the percentage of the candidates qualified to that of the candidates appeared ?
 (a) 10.87% (b) 11.49% (c) 12.35% (d) 12.54% (e) 13.05%
- The percentage of the total number of qualified candidates to the total number of appeared candidates among all the five states in 1999 is :
 (a) 11.49% (b) 11.84% (c) 12.21% (d) 12.57% (e) 12.73%
- What is the percentage of candidates qualified from State N for all the years together, over the candidates appeared from State N during all the years together ?
 (a) 12.36% (b) 12.16% (c) 11.47% (d) 11.15% (e) None of these

4. What is the average of candidates who appeared from State Q during the given years ?
 (a) 8700 (b) 8760 (c) 8810 (d) 8920 (e) 8990
5. In which of the given years the number of candidates appeared from State P has maximum percentage of qualified candidates ?
 (a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001
6. Total number of candidates qualified from all the states together in 1997 is approximately what percentage of the total number of candidates qualified from all the states together in 1998 ?
 (a) 72% (b) 77% (c) 80% (d) 83% (e) 86%

Sol. 1. (c) : Required Percentage = $\left[\frac{(1020 + 1240)}{(8800 + 9500)} \times 100 \right] \% = \left(\frac{2260}{18300} \times 100 \right) \% = 12.35\%$

2. (b) : Required Percentage = $\left[\frac{(850 + 920 + 890 + 980 + 1350)}{(7400 + 8450 + 7800 + 8700 + 9800)} \times 100 \right] \% = \left(\frac{4990}{42150} \times 100 \right) \% = 11.84\%$

3. (d) : Required Percentage = $\left[\frac{(840 + 1050 + 920 + 980 + 1020)}{(7500 + 9200 + 8450 + 9200 + 8800)} \times 100 \right] \% = \left(\frac{4810}{43150} \times 100 \right) \% = 11.15\%$

4. (e) : Required average = $\frac{8100 + 9500 + 8700 + 9700 + 8950}{5} = \frac{44950}{5} = 8990$.

5. (e) : The percentages of candidates qualified to candidates appeared from State P during different years are :

For 1997 = $\left(\frac{780}{6400} \times 100 \right) \% = 12.19\%$; For 1998 = $\left(\frac{1020}{8800} \times 100 \right) \% = 11.59\%$;

For 1999 = $\left(\frac{890}{7800} \times 100 \right) \% = 11.41\%$; For 2000 = $\left(\frac{1010}{8750} \times 100 \right) \% = 11.54\%$;

For 2001 = $\left(\frac{1250}{9750} \times 100 \right) \% = 12.82\%$.

∴ Maximum percentage is for the year 2001.

6. (c) : Required Percentage = $\left[\frac{(720 + 840 + 780 + 950 + 870)}{(980 + 1050 + 1020 + 1240 + 940)} \times 100 \right] \% = \left(\frac{4160}{5230} \times 100 \right) \% = 79.54\% = 80\%$.

Ex. 3. The following table gives the percentage of marks obtained by seven students in six different subjects in an examination. Study the table and answer the questions based on it. The numbers in the brackets give the maximum marks in each subject.

(Bank P.O. 2003)

Subjects (Max. Marks)	Maths	Chemistry	Physics	Geography	History	Computer Science (40)
Student	(150)	(130)	(120)	(100)	(60)	
Ayush	90	50	90	60	70	80
Aman	100	80	80	40	80	70
Sajal	90	60	70	70	90	70
Rohit	80	65	80	80	60	60
Muskan	80	65	85	95	50	90
Tanvi	70	75	65	85	40	60
Tarun	65	35	50	77	80	80

- What was the aggregate of marks obtained by Sajal in all the six subjects ?

(a) 409 (b) 419 (c) 429 (d) 439 (e) 449
- What is the overall percentage of Tarun ?

(a) 52.5% (b) 55% (c) 60% (d) 63% (e) 64.5%
- What are the average marks obtained by all the seven students in Physics ? (rounded off to two digits after decimal)

(a) 77.26 (b) 89.14 (c) 91.37 (d) 96.11 (e) 103.21
- The number of students who obtained 60% and above marks in all the subjects is :

(a) 1 (b) 2 (c) 3 (d) None (e) None of these
- In which subject is the overall percentage the best ?

(a) History (b) Maths (c) Physics (d) Chemistry (e) Geography

Sol. 1. (e) : Aggregate marks obtained by Sajal

$$= [(90\% \text{ of } 150) + (60\% \text{ of } 130) + (70\% \text{ of } 120) + (70\% \text{ of } 100) + (90\% \text{ of } 60) + (70\% \text{ of } 40)] = 135 + 78 + 84 + 70 + 54 + 28 = 449.$$

2. (c) : Aggregate marks obtained by Tarun

$$= [(65\% \text{ of } 150) + (35\% \text{ of } 130) + (50\% \text{ of } 120) + (77\% \text{ of } 100) + (80\% \text{ of } 60) + (80\% \text{ of } 40)] = 97.5 + 45.5 + 60 + 77 + 48 + 32 = 360.$$

Total maximum marks (of all the six subjects)

$$= (150 + 130 + 120 + 100 + 60 + 40) = 600.$$

$$\text{Overall percentage of Tarun} = \left(\frac{360}{600} \times 100 \right)\% = 60\%.$$

3. (b) : Average marks obtained in Physics by all the seven students

$$= \frac{1}{7} \times [(90\% \text{ of } 120) + (80\% \text{ of } 120) + (70\% \text{ of } 120) + (80\% \text{ of } 120) + (85\% \text{ of } 120) + (65\% \text{ of } 120) + (50\% \text{ of } 120)]$$

$$= \frac{1}{7} \times [(90 + 80 + 70 + 80 + 85 + 65 + 50)\% \text{ of } 120]$$

$$= \frac{1}{7} \times [520\% \text{ of } 120] = \frac{624}{7} = 89.14.$$

4. (b) : From the table it is clear that Sajal and Rohit have 60% or more marks in each of the six subjects.

5. (b) : We shall find the overall percentage (for all the seven students) with respect to each subject.

The overall percentage for any subject is equal to the average of percentages obtained by all the seven students since the maximum marks for any subject is the same for all the students.

Therefore, overall percentage for : $\text{Maths} = \left[\frac{1}{7} \times (90 + 100 + 90 + 80 + 80 + 70 + 65) \right] \% = 82.14\%$

$\text{Chemistry} = \left[\frac{1}{7} \times (50 + 80 + 60 + 65 + 65 + 75 + 35) \right] \% = 61.43\%$

$\text{Physics} = \left[\frac{1}{7} \times (90 + 80 + 70 + 80 + 85 + 65 + 50) \right] \% = 74.29\%$

$\text{Geography} = \left[\frac{1}{7} \times (60 + 40 + 70 + 80 + 95 + 85 + 77) \right] \% = 72.43\%$

$\text{History} = \left[\frac{1}{7} \times (70 + 80 + 90 + 60 + 50 + 40 + 80) \right] \% = 67.14\%$

$\text{Computer Science} = \left[\frac{1}{7} \times (80 + 70 + 70 + 60 + 90 + 60 + 80) \right] \% = 72.86\%$

Clearly, this percentage is highest for Maths.

Ex. 4. Study the following table carefully and answer the questions given below :
(Bank P.O. 2001)

**CLASSIFICATION OF 100 STUDENTS BASED ON THE MARKS OBTAINED
BY THEM IN PHYSICS AND CHEMISTRY IN AN EXAMINATION**

Marks out of 50 Subject	40 and above	30 and above	20 and above	10 and above	0 and above
Physics	9	32	80	92	100
Chemistry	4	21	66	81	100
(Aggregate) Average	7	27	73	87	100

- The number of students scoring less than 40% marks in aggregate is :
(a) 13 (b) 19 (c) 20 (d) 27 (e) 34
- If at least 60% marks in Physics are required for pursuing higher studies in Physics, how many students will be eligible to pursue higher studies in Physics ?
(a) 27 (b) 32 (c) 34 (d) 41 (e) 68
- What is the difference between the number of students passed with 30 as cut-off marks in Chemistry and those passed with 30 as cut-off marks in aggregate ?
(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

4. The percentage of the number of students getting at least 60% marks in Chemistry over those getting at least 40% marks in aggregate, is approximately :
 (a) 21% (b) 27% (c) 29% (d) 31% (e) 34%
5. If it is known that at least 23 students were eligible for a Symposium on Chemistry, the minimum qualifying marks in Chemistry for eligibility to Symposium would lie in the range :
 (a) 40-50 (b) 30-40 (c) 20-30 (d) Below 20 (e) Cannot be determined

Sol. 1. (d) : We have 40% of 50 = $\left(\frac{40}{100} \times 50\right) = 20$.

$$\therefore \text{Required number} = \text{Number of students scoring less than 20 marks in aggregate}$$

$$= 100 - \text{number of students scoring 20 and above marks in aggregate} = 100 - 73 = 27.$$

2. (b) : We have 60% of 50 = $\left(\frac{60}{100} \times 50\right) = 30$.

$$\therefore \text{Required number} = \text{Number of students scoring 30 and above marks in Physics} = 32.$$

3. (d) : Required difference = (Number of students scoring 30 and above marks in Chemistry) – (Number of students scoring 30 and above marks in aggregate) = 27 – 21 = 6.

4. (c) : Number of students getting at least 60% marks in Chemistry
 = Number of students getting 30 and above marks in Chemistry = 21.
 Number of students getting at least 40% marks in aggregate
 = Number of students getting 20 and above marks in aggregate = 73.

$$\therefore \text{Required Percentage} = \left(\frac{21}{73} \times 100\right)\% = 28.77\% \approx 29\%.$$

5. (c) : Since 66 students get 20 and above marks in Chemistry and out of these 21 students get 30 and above marks, therefore to select top 35 students in Chemistry, the qualifying marks should lie in the range 20-30.

EXERCISE 36

Directions (Questions 1 to 6) : Study the following table and answer the questions based on it. (Bank P.O. 2003)

NUMBER OF CANDIDATES APPEARED, QUALIFIED AND SELECTED IN A COMPETITIVE EXAMINATION FROM FIVE STATES DELHI, H.P., U.P., PUNJAB AND HARYANA OVER THE YEARS 1994 TO 1998

Year	Delhi			H.P.			U.P.			Punjab			Haryana		
	App.	Qual.	Sel.	App.	Qual.	Sel.	App.	Qual.	Sel.	App.	Qual.	Sel.	App.	Qual.	Sel.
1997	8000	850	94	7800	810	82	7500	720	78	8200	680	85	6400	700	75
1998	4800	500	48	7500	800	65	5600	620	85	6800	600	70	7100	650	75
1999	7500	640	82	7400	560	70	4800	400	48	6500	525	65	5200	350	55
2000	9500	850	90	8800	920	86	7000	650	70	7800	720	84	6400	540	60
2001	9000	800	70	7200	850	75	8500	950	80	5700	485	60	4500	600	75

1. In the year 1997, which state had the lowest percentage of candidates selected over the candidates appeared ?

(a) Delhi (b) H.P. (c) U.P. (d) Punjab (e) Haryana

2. The percentage of candidates qualified from Punjab over those appeared from Punjab is highest in the year :
 (a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001
3. The percentage of candidates selected from UP over those qualified from UP is highest in the year :
 (a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001
4. The number of candidates selected from Haryana during the period under review is approximately what percent of the number selected from Delhi during this period ?
 (a) 79.5% (b) 81% (c) 84.5% (d) 88.5% (e) 92.5%
5. For which state the average number of candidates selected over the years is the maximum ?
 (a) Delhi (b) H.P. (c) U.P. (d) Punjab (e) Haryana
6. What is the approximate percentage of total number of candidates selected to the total number of candidates qualified for all the five states together during the year 1999 ?
 (a) 10% (b) 11% (c) 12% (d) 13% (e) 14%

Directions (Questions 7 to 11) : Study the following table to answer the questions that are given below it.

(R.B.I. 2003)

**EXPENDITURES OF A COMPANY (IN LAKH RUPEES)
PER ANNUM OVER THE GIVEN YEARS**

Item of Expenditure Year	Salary	Fuel and Transport	Bonus	Interest on Loans	Taxes
1998	288	98	3.00	23.4	83
1999	342	112	2.52	32.5	108
2000	324	101	3.84	41.6	74
2001	336	133	3.68	36.4	88
2002	420	142	3.96	49.4	98

7. The ratio between the total expenditure on Taxes for all the years and the total expenditure on Fuel and Transport for all the years respectively is approximately :
 (a) 4 : 7 (b) 10 : 13 (c) 15 : 18 (d) 5 : 8 (e) 2 : 3
8. The total expenditure of the Company over these items during the year 2000 is :
 (a) Rs. 544.44 lakhs (b) Rs. 501.11 lakhs (c) Rs. 446.46 lakhs
 (d) Rs. 478.87 lakhs (e) Rs. 612.13 lakhs
9. What is the average amount of interest per year which the Company had to pay during this period ?
 (a) Rs. 32.43 lakhs (b) Rs. 33.72 lakhs (c) Rs. 34.18 lakhs
 (d) Rs. 35.69 lakhs (e) Rs. 36.66 lakhs
10. Total expenditure on all these items in 1998 was approximately what percent of the total expenditure in 2002 ?
 (a) 62% (b) 66% (c) 69% (d) 71% (e) 73%
11. The total amount of bonus paid by the Company during the given period is approximately what percent of the total amount of salary paid during this period ?
 (a) 0.1% (b) 0.5% (c) 1% (d) 1.25% (e) 1.11%

Directions (Questions 12 to 16) : A school has four sections A, B, C, D of Class IX students. The results of half-yearly and annual examinations are shown in the table given below. Answer the questions based on this table.

(Bank P.O. 2000)

Result	Number of Students			
	Section A	Section B	Section C	Section D
Students failed in both Exams	28	23	17	27
Students failed in half-yearly but passed in Annual Exams	14	12	8	13
Students passed in half-yearly but failed in Annual Exams	6	17	9	15
Students passed in both Exams	64	55	46	76

12. How many students are there in Class IX in the school ?
 - (a) 336
 - (b) 189
 - (c) 335
 - (d) 286
 - (e) 430
13. Which section has the minimum failure rate in half-yearly examination ?
 - (a) A
 - (b) B
 - (c) C
 - (d) D
 - (e) Cannot be determined
14. Which section has the maximum success rate in annual examination ?
 - (a) A
 - (b) B
 - (c) C
 - (d) D
 - (e) Cannot be determined
15. Which section has the maximum pass percentage in at least one of the two examinations ?
 - (a) A
 - (b) B
 - (c) C
 - (d) D
 - (e) Cannot be determined
16. If the number of students passing an examination be considered a criteria for comparison of difficulty level of two examinations, which of the following statements is true in this context ?
 - (a) Half-yearly examinations were more difficult.
 - (b) Annual examinations were more difficult.
 - (c) Both the examinations had almost the same difficulty level.
 - (d) The two examinations cannot be compared for difficulty level.
 - (e) For students of Sections A and B, the annual examinations seem to be more difficult as compared to the half-yearly examinations.

Directions (Questions 17 to 21) : The following table shows the number of new employees added to different categories of employees in a Company and also the number of employees from these categories who left the company every year since the foundation of the Company in 1995.

(Bank P.O. 2001)

Year	Managers		Technicians		Operators		Accountants		Peons	
	New	Left	New	Left	New	Left	New	Left	New	Left
1995	760	—	1200	—	880	—	1160	—	820	—
1996	280	120	272	120	256	104	200	100	184	96
1997	179	92	240	128	240	120	224	104	152	88
1998	148	88	236	96	208	100	248	96	196	80
1999	160	72	256	100	192	112	272	88	224	120
2000	193	96	288	112	248	144	260	92	200	104

17. During the period between 1995 and 2000, the total number of Operators who left the Company is what percent of the total number of Operators who joined the Company ?
 (a) 19% (b) 21% (c) 27% (d) 29% (e) 32%
18. For which of the following categories the percentage increase in the number of employees working in the Company from 1995 to 2000 was the maximum ?
 (a) Managers (b) Technicians (c) Operators (d) Accountants (e) Peons
19. What is the difference between the total number of Technicians added to the Company and the total number of Accountants added to the Company during the years 1996 to 2000 ?
 (a) 128 (b) 112 (c) 96 (d) 88 (e) 72
20. What was the total number of Peons working in the Company in the year 1999 ?
 (a) 1312 (b) 1192 (c) 1088 (d) 968 (e) 908
21. What is the pooled average of the total number of employees of all categories in the year 1997 ?
 (a) 1325 (b) 1285 (c) 1265 (d) 1235 (e) 1195

Directions (Questions 22 to 25) : The following table gives the percentage distribution of population of five states, P, Q, R, S and T on the basis of poverty line and also on the basis of sex. Study the table and answer the questions based on it.

(Bank P.O. 2000)

State	Percentage of Population below Poverty Line	Proportion of Males and Females	
		Below Poverty Line	
		M : F	M : F
P	35	5 : 6	6 : 7
Q	25	3 : 5	4 : 5
R	24	1 : 2	2 : 3
S	19	3 : 2	4 : 3
T	15	5 : 3	3 : 2

22. What will be the number of females above poverty line in the State S if it is known that the population of State S is 7 million ?
 (a) 3 million (b) 2.43 million (c) 1.33 million
 (d) 5.7 million (e) 1.61 million
23. If the male population above poverty line for State R is 1.9 million, then the total population of State R is :
 (a) 4.5 million (b) 4.85 million (c) 5.35 million
 (d) 6.25 million (e) 7.6 million
24. What will be the male population above poverty line for State P if the female population below poverty line for State P is 2.1 million ?
 (a) 2.1 million (b) 2.3 million (c) 2.7 million
 (d) 3.3 million (e) 3.4 million
25. If the population of males below poverty line for State Q is 2.4 million and that for State T is 6 million, then the total populations of states Q and T are in the ratio :
 (a) 1 : 3 (b) 2 : 5 (c) 3 : 7 (d) 4 : 9 (e) 5 : 12

ANSWERS

1. (d) 2. (d) 3. (b) 4. (d) 5. (a) 6. (d) 7. (b) 8. (a) 9. (e)
 10. (c) 11. (c) 12. (e) 13. (d) 14. (a) 15. (d) 16. (c) 17. (d) 18. (a)
 19. (d) 20. (b) 21. (e) 22. (b) 23. (d) 24. (d) 25. (b)

SOLUTIONS

1. The percentages of candidates selected over the candidates appeared in 1997, for various states are :

$$(i) \text{ For Delhi} = \left(\frac{94}{8000} \times 100 \right)\% = 1.175\%; (ii) \text{ For H.P.} = \left(\frac{82}{7800} \times 100 \right)\% = 1.051\%;$$

$$(iii) \text{ For U.P.} = \left(\frac{78}{7500} \times 100 \right)\% = 1.040\%; (iv) \text{ For Punjab} = \left(\frac{85}{8200} \times 100 \right)\% = 1.037\%;$$

$$(v) \text{ For Haryana} = \left(\frac{75}{6400} \times 100 \right)\% = 1.172\%.$$

Clearly, this percentage is lowest for Punjab.

2. The percentages of candidates qualified from Punjab over those appeared from Punjab during different years are :

$$\text{For 1997} = \left(\frac{680}{8200} \times 100 \right)\% = 8.29\%; \quad \text{For 1998} = \left(\frac{600}{6800} \times 100 \right)\% = 8.82\%;$$

$$\text{For 1999} = \left(\frac{525}{6500} \times 100 \right)\% = 8.08\%; \quad \text{For 2000} = \left(\frac{720}{7800} \times 100 \right)\% = 9.23\%;$$

$$\text{For 2001} = \left(\frac{485}{5700} \times 100 \right)\% = 8.51\%.$$

Clearly, this percentage is highest for the year 2000.

3. The percentages of candidates selected from U.P. over those qualified from U.P. during different years are :

$$\text{For 1997} = \left(\frac{78}{720} \times 100 \right)\% = 10.83\%; \quad \text{For 1998} = \left(\frac{85}{620} \times 100 \right)\% = 13.71\%;$$

$$\text{For 1999} = \left(\frac{48}{400} \times 100 \right)\% = 12\%; \quad \text{For 2000} = \left(\frac{70}{650} \times 100 \right)\% = 10.77\%;$$

$$\text{For 2001} = \left(\frac{80}{950} \times 100 \right)\% = 8.42\%.$$

Clearly, this percentage is highest for the year 1998.

$$4. \text{ Required Percentage} = \left[\frac{(75 + 75 + 55 + 60 + 75)}{(94 + 48 + 82 + 90 + 70)} \times 100 \right]\% = \left(\frac{340}{384} \times 100 \right)\% = 88.54\% = 88.5\%.$$

5. The average number of candidates selected over the given period for various states are :

$$\text{For Delhi} = \frac{94 + 48 + 82 + 90 + 70}{5} = \frac{384}{5} = 76.8$$

$$\text{For H.P.} = \frac{82 + 65 + 70 + 86 + 75}{5} = \frac{378}{5} = 75.6$$

$$\text{For U.P.} = \frac{78 + 85 + 48 + 70 + 80}{5} = \frac{361}{5} = 72.2$$

$$\text{For Punjab} = \frac{85 + 70 + 65 + 84 + 60}{5} = \frac{364}{4} = 72.8$$

$$\text{For Haryana} = \frac{75 + 75 + 55 + 60 + 75}{5} = \frac{340}{5} = 68.$$

Clearly, this average is maximum for Delhi.

$$6. \text{ Required Percentage} = \left[\frac{(82 + 70 + 48 + 65 + 55)}{(640 + 560 + 400 + 525 + 350)} \times 100 \right] \% \\ = \left(\frac{320}{2475} \times 100 \right) \% = 12.93\% \approx 13\%.$$

$$7. \text{ Required Ratio} = \frac{(83 + 108 + 74 + 88 + 98)}{(98 + 112 + 101 + 133 + 142)} = \frac{451}{586} = \frac{1}{1.3} = \frac{10}{13}.$$

8. Total expenditure of the Company during 2000

= Rs. (324 + 101 + 3.84 + 41.6 + 74) lakhs = Rs. 544.44 lakhs.

9. Average amount of interest paid by the Company during the given period

$$= \text{Rs.} \left(\frac{23.4 + 32.5 + 41.6 + 36.4 + 49.4}{5} \right) \text{lakhs} = \text{Rs.} \left(\frac{183.3}{5} \right) \text{lakhs} \\ = \text{Rs. } 36.66 \text{ lakhs.}$$

$$10. \text{ Required Percentage} = \left[\frac{(288 + 98 + 3.00 + 23.4 + 83)}{(420 + 142 + 3.96 + 49.4 + 98)} \times 100 \right] \% \\ = \left(\frac{495.4}{713.36} \times 100 \right) \% = 69.45\%.$$

$$11. \text{ Required Percentage} = \left[\frac{(3.00 + 2.52 + 3.84 + 3.68 + 3.96)}{(288 + 342 + 324 + 336 + 420)} \times 100 \right] \% \\ = \left(\frac{17}{1710} \times 100 \right) \% = 1\%.$$

12. Since the classification of the students on the basis of their results and sections form independent groups, so the total number of students in the class :

$$= (28 + 23 + 17 + 27 + 14 + 12 + 8 + 13 + 6 + 17 + 9 + 15 + 64 + 55 + 46 + 76) = 430.$$

13. Total number of failures in half-yearly exams in a section

= [(Number of students failed in both exams) + (Number of students failed in half-yearly but passed in Annual exams)] in that section

a. Failure rate in half-yearly exams in Section A

$$= \left[\frac{\text{Number of students of Section A failed in half-yearly}}{\text{Total number of students in Section A}} \times 100 \right] \%$$

$$= \left[\frac{(28 + 14)}{(28 + 14 + 6 + 64)} \times 100 \right] \% = \left(\frac{42}{112} \times 100 \right) \% = 37.5\%.$$

Similarly, failure rate in half-yearly exams in :

$$\text{Section B} = \left[\frac{(23 + 12)}{(23 + 12 + 17 + 55)} \times 100 \right] \% = \left(\frac{35}{107} \times 100 \right) \% = 32.71\%$$

$$\text{Section C} = \left[\frac{(17 + 8)}{(17 + 8 + 9 + 46)} \times 100 \right] \% = \left(\frac{25}{80} \times 100 \right) \% = 31.25\%$$

$$\text{Section D} = \left[\frac{(27+13)}{(27+13+15+76)} \times 100 \right] \% = \left(\frac{40}{131} \times 100 \right) \% = 30.53\%$$

Clearly, the failure rate is minimum for Section D.

14. Total number of students passed in annual exams in a section
 $= [(\text{Number of students failed in half-yearly but passed in annual exams}) + (\text{Number of students passed in both exams})]$ in that section

a. Success rate in annual examination in **Section A**

$$\begin{aligned} &= \left[\frac{\text{Number of students of Section A passed in annual exams}}{\text{Total number of students in Section A}} \times 100 \right] \% \\ &= \left[\frac{(14+64)}{(28+14+6+64)} \times 100 \right] \% = \left(\frac{78}{112} \times 100 \right) \% = 69.64\% \end{aligned}$$

Similarly, success rate in annual examinations in :

$$\text{Section B} = \left[\frac{(12+55)}{(23+12+17+55)} \times 100 \right] \% = \left(\frac{67}{107} \times 100 \right) \% = 62.62\%$$

$$\text{Section C} = \left[\frac{(8+46)}{(17+8+9+46)} \times 100 \right] \% = \left(\frac{54}{80} \times 100 \right) \% = 67.5\%$$

$$\text{Section D} = \left[\frac{(13+76)}{(27+13+15+76)} \times 100 \right] \% = \left(\frac{89}{131} \times 100 \right) \% = 67.94\%$$

Clearly, the success rate in annual examination is maximum for Section A.

15. Pass percentages in at least one of the two examinations for different sections are :

$$\text{For Section A} = \left[\frac{(14+6+64)}{(28+14+6+64)} \times 100 \right] \% = \left(\frac{84}{112} \times 100 \right) \% = 75\%$$

$$\text{For Section B} = \left[\frac{(12+17+55)}{(23+12+17+55)} \times 100 \right] \% = \left(\frac{84}{107} \times 100 \right) \% = 78.5\%$$

$$\text{For Section C} = \left[\frac{(8+9+46)}{(17+8+9+46)} \times 100 \right] \% = \left(\frac{63}{80} \times 100 \right) \% = 78.75\%$$

$$\text{For Section D} = \left[\frac{(13+15+76)}{(27+13+15+76)} \times 100 \right] \% = \left(\frac{104}{131} \times 100 \right) \% = 79.39\%$$

Clearly, the pass percentage is maximum for Section D.

16. Number of students who passed half-yearly exams in the school

$= (\text{Number of students passed in half-yearly but failed in annual exams}) + (\text{Number of students passed in both exams}) = (6+17+9+15) + (64+55+46+76) = 288$

Also, Number of students who passed annual exams in the school

$= (\text{Number of students failed in half-yearly but passed in annual exams}) + (\text{Number of students passed in both exams}) = (14+12+8+13) + (64+55+46+76) = 288$

Since, the number of students passed in half-yearly = the number of students passed in annual exams, therefore, it can be inferred that both the examinations had almost the same difficulty level.

Thus, Statements (a), (b) and (d) are false and Statement (c) is true.

Also, number of students from Sections A and B who passed the annual exams

$$= (14+12) + (64+55) = 145$$

And, number of students from Sections A and B who passed the half-yearly exams

$$= (6+17) + (64+55) = 142.$$

Since the number of students of Sections A and B who passed the annual exams is greater than those who passed the half-yearly exams it implies that for students of Sections A and B, the half-yearly exams were more difficult as compared to annual exams.

Hence, Statement (e) is false.

17. Total number of Operators who left the Company during 1995-2000
 $= (104 + 120 + 100 + 112 + 144) = 580.$

Total number of Operators who joined the Company during 1995-2000
 $= (880 + 256 + 240 + 208 + 192 + 248) = 2024.$

$$\therefore \text{Required Percentage} = \left(\frac{580}{2024} \times 100 \right) \% = 28.66\% \approx 29\%.$$

18. Number of Managers working in the Company :

In 1995 = 760.

In 2000 = $(760 + 280 + 179 + 148 + 160 + 193) - (120 + 92 + 88 + 72 + 96) = 1252.$
 \therefore Percentage increase in the number of Managers

$$= \left[\frac{(1252 - 760)}{760} \times 100 \right] \% = 64.74\%.$$

Number of Technicians working in the Company :

In 1995 = 1200.

In 2000 = $(1200 + 272 + 240 + 236 + 256 + 288) - (120 + 128 + 96 + 100 + 112)$
 $= 1936.$

\therefore Percentage increase in the number of Technicians

$$= \left[\frac{(1936 - 1200)}{1200} \times 100 \right] \% = 61.33\%.$$

Number of Operators working in the Company :

In 1995 = 880.

In 2000 = $(880 + 256 + 240 + 208 + 192 + 248) - (104 + 120 + 100 + 112 + 144)$
 $= 1444.$

\therefore Percentage increase in the number of Operators

$$= \left[\frac{(1444 - 880)}{880} \times 100 \right] \% = 64.09\%.$$

Number of Accountants working in the Company :

In 1995 = 1160.

In 2000 = $(1160 + 200 + 224 + 248 + 272 + 260) - (100 + 104 + 96 + 88 + 92)$
 $= 1884.$

\therefore Percentage increase in the number of Accountants

$$= \left[\frac{(1884 - 1160)}{1160} \times 100 \right] \% = 62.41\%.$$

Number of Peons working in the Company :

In 1995 = 820.

In 2000 = $(820 + 184 + 152 + 196 + 224 + 200) - (96 + 88 + 80 + 120 + 104)$
 $= 1288.$

\therefore Percentage increase in the number of Peons

$$= \left[\frac{(1288 - 820)}{820} \times 100 \right] \% = 57.07\%.$$

Clearly, the percentage increase is maximum in case of Managers.

19. Required difference = $(272 + 240 + 236 + 256 + 288) - (200 + 224 + 248 + 272 + 260) = 88$.
20. Total number of Peons working in the Company in 1999
 $= (820 + 184 + 152 + 196 + 224) - (96 + 88 + 80 + 120) = 1192$.
21. Total number of employees of various categories working in the Company in 1997 are :
 Managers = $(760 + 280 + 179) - (120 + 92) = 1007$
 Technicians = $(1200 + 272 + 240) - (120 + 128) = 1464$
 Operators = $(880 + 256 + 240) - (104 + 120) = 1152$
 Accountants = $(1160 + 200 + 224) - (100 + 104) = 1380$
 Peons = $(820 + 184 + 152) - (96 + 88) = 972$
 ∴ Pooled average of all the five categories of employees working in the Company in 1997 = $\frac{1}{5} \times (1007 + 1464 + 1152 + 1380 + 972) = \frac{1}{5} \times 5975 = 1195$.
22. Total population of State S = 7 million.
 ∴ Population above poverty line = [(100 - 19)% of 7] million
 $= (81\% \text{ of } 7) \text{ million} = 5.67 \text{ million}$.
 And so, the number of females above poverty line in State S = $\left(\frac{3}{7} \times 5.67\right)$ million
 $= 2.43 \text{ million}$.
23. Let the total population of State R be x million.
 Then, population of State R above poverty line = [(100 - 24)% of x] million
 $= \left(\frac{76}{100} \times x\right)$ million.
 And so, male population of State R above poverty line = $\left[\frac{2}{5} \times \left(\frac{76}{100} \times x\right)\right]$ million
 But, it is given that male population of State R above poverty line = 1.9 million
 $\therefore \frac{2}{5} \times \left(\frac{76}{100} \times x\right) = 1.9 \Rightarrow x = \frac{5 \times 100 \times 1.9}{76 \times 2} = 6.25$.
 ∴ Total population of State R = 6.25 million.
24. Female population below poverty line for State P = 2.1 million.
 Let the male population below poverty line for State P be x million.
 Then, $5 : 6 = x : 2.1 \Rightarrow x = \frac{2.1 \times 5}{6} = 1.75$.
 ∴ Population below poverty line for State P = $(2.1 + 1.75)$ million = 3.85 million.
 Let the population above poverty line for State P be y million.
 Since, 35% of the total population of State P is below poverty line, therefore, 65% of the total population of State P is above poverty line i.e., the ratio of population below poverty line to that above poverty line for State P is 35 : 65.
 $\therefore 35 : 65 = 3.85 : y \Rightarrow y = \frac{65 \times 3.85}{35} = 7.15$.
 i.e., population above poverty line for State P = 7.15 million and so, male population above poverty line for State P = $\left(\frac{6}{13} \times 7.15\right)$ million = 3.3 million.

25. For State Q :

Male population below poverty line = 2.4 million.

Let the female population below poverty line be x million.

$$\text{Then, } 3 : 5 = 2.4 : x \Rightarrow x = \frac{5 \times 2.4}{3} = 4$$

\therefore Total population below poverty line = $(2.4 + 4) = 6.4$ million.

If N_q be the total population of State Q, then,

$$25\% \text{ of } N_q = 6.4 \text{ million} \Rightarrow N_q = \left(\frac{6.4 \times 100}{25} \right) \text{ million} = 25.6 \text{ million.}$$

For State T :

Male population below poverty line = 6 million.

Let the female population below poverty line be y million.

$$\text{Then, } 5 : 3 = 6 : y \Rightarrow y = \frac{3 \times 6}{5} = 3.6$$

\therefore Total population below poverty line = $(6 + 3.6) = 9.6$ million.

If N_t be the total population of State T, then

$$15\% \text{ of } N_t = 9.6 \text{ million} \Rightarrow N_t = \left(\frac{9.6 \times 100}{15} \right) \text{ million} = 64 \text{ million.}$$

$$\text{Thus, required ratio} = \frac{N_q}{N_t} = \frac{25.6}{64} = 0.4 = \frac{2}{5}.$$

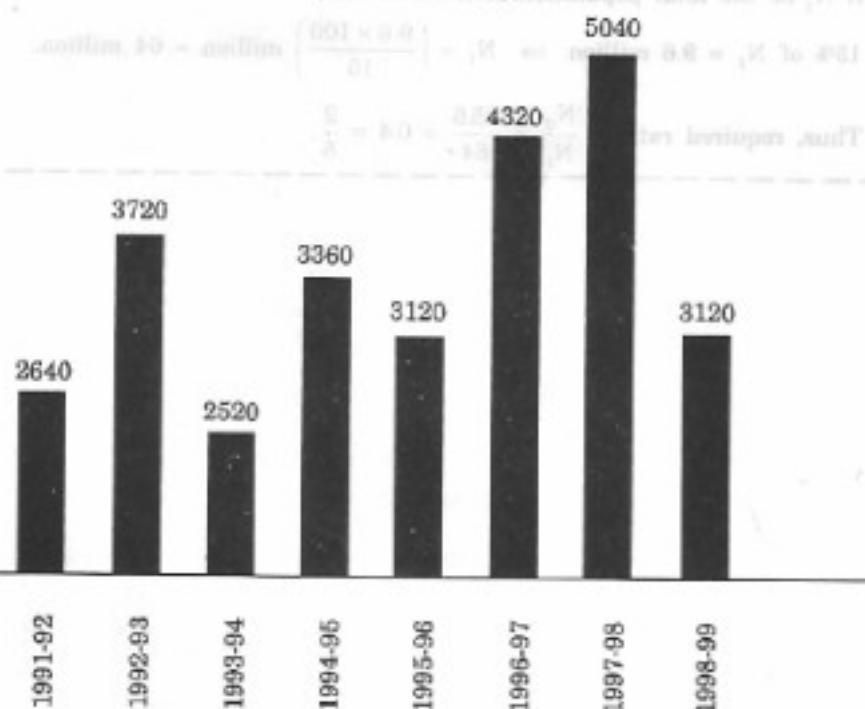
37. BAR GRAPHS

This section comprises of questions in which the data collected in a particular discipline are represented in the form of vertical or horizontal bars drawn by selecting a particular scale. One of the parameters is plotted on the horizontal axis and the other on the vertical axis. The candidate is required to understand the given information and thereafter answer the given questions on the basis of data analysis.

Ex. 1. The bar graph given below shows the foreign exchange reserves of a country (in million US \$) from 1991-92 to 1998-99. Answer the questions based on this graph.
(Bank P.O. 2001)

FOREIGN EXCHANGE RESERVES OF A COUNTRY

(in million US \$)



- The foreign exchange reserves in 1997-98 was how many times that in 1994-95 ?
(a) 0.7 (b) 1.2 (c) 1.4 (d) 1.5 (e) 1.8
- What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94 ?
(a) 100 (b) 150 (c) 200 (d) 620 (e) 2520
- For which year, the percent increase of foreign exchange reserves over the previous year, is the highest ?
(a) 1992-93 (b) 1993-94 (c) 1994-95 (d) 1996-97 (e) 1997-98

4. The foreign exchange reserves in 1996-97 were approximately what percent of the average foreign exchange reserves over the period under review ?
 (a) 95% (b) 110% (c) 115% (d) 125% (e) 140%
5. The ratio of the number of years, in which the foreign exchange reserves are above the average reserves, to those in which the reserves are below the average reserves, is :
 (a) 2 : 6 (b) 3 : 4 (c) 3 : 5 (d) 4 : 4 (e) 5 : 3

Sol. 1. (d) : Required ratio = $\frac{5040}{3360} = 1.5$.

2. (a) : Foreign exchange reserves in 1997-98 = 5040 million US \$.

Foreign exchange reserves in 1993-94 = 2520 million US \$.

∴ Increase = (5040 - 2520) = 2520 million US \$.

∴ Percentage increase = $\left(\frac{2520}{2520} \times 100 \right)\% = 100\%$.

3. (a) : There is an increase in foreign exchange reserves during the years 1992-93, 1994-95, 1996-97 and 1997-98 as compared to previous year (as shown by bar-graph).

The percentage increase in reserves during these years compared to previous year are :

$$(i) \text{ For 1992-93 } = \left[\frac{(3720 - 2640)}{2640} \times 100 \right]\% = 40.91\%$$

$$(ii) \text{ For 1994-95 } = \left[\frac{(3360 - 2520)}{2520} \times 100 \right]\% = 33.33\%$$

$$(iii) \text{ For 1996-97 } = \left[\frac{(4320 - 3120)}{3120} \times 100 \right]\% = 38.46\%$$

$$(iv) \text{ For 1997-98 } = \left[\frac{(5040 - 4320)}{4320} \times 100 \right]\% = 16.67\%$$

Clearly, the percentage increase over previous year is highest for 1992-93.

4. (d) : Average foreign exchange reserves over the given period

$$= \left[\frac{1}{8} \times (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120) \right] \text{ million US \$}$$

$$= 3480 \text{ million US \$}.$$

Foreign exchange reserves in 1996-97 = 4320 million US \$.

$$\therefore \text{ Required Percentage } = \left(\frac{4320}{3480} \times 100 \right)\% = 124.14\% \approx 125\%$$

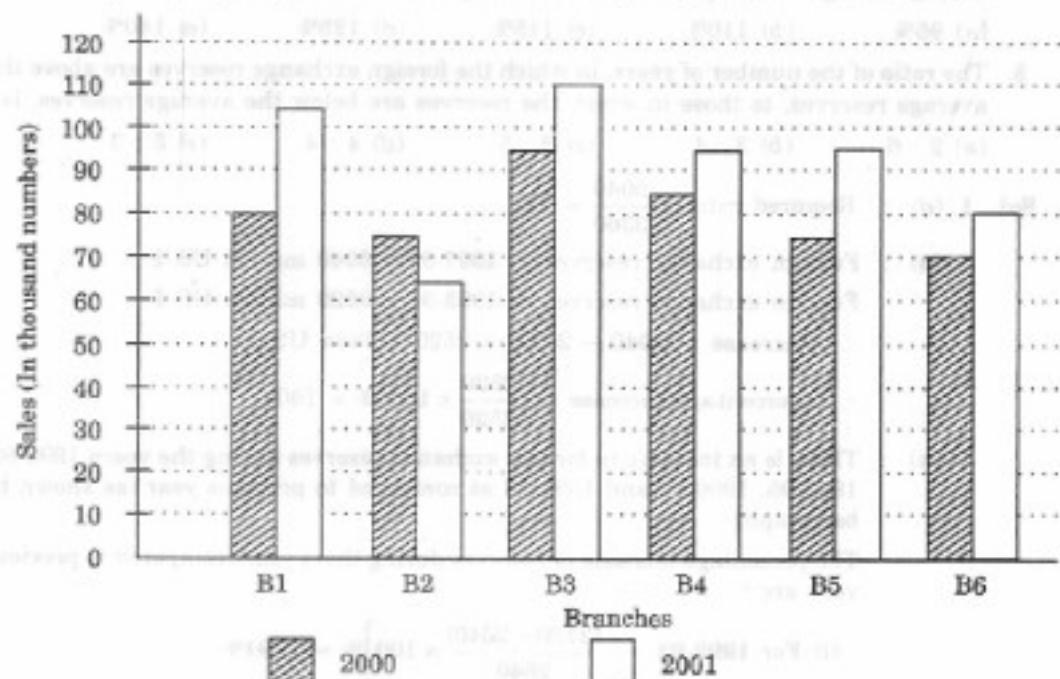
5. (c) : Average foreign exchange reserves over the given period = 3480 million US \$.

The country had reserves above 3480 million US \$ during the years 1992-93, 1996-97 and 1997-98 i.e., for 3 years and below 3480 million US \$ during the years 1991-92, 1993-94, 1994-95, 1995-96 and 1998-99 i.e., for 5 years.

Hence, required ratio = 3 : 5.

Ex. 2. The bar-graph provided on next page gives the sales of books (in thousand numbers) from six branches of a publishing company during two consecutive years 2000 and 2001. Answer the questions based on this bar-graph. (Bank P.O. 2003)

Sales of Books (in thousand numbers) from Six Branches —
B1, B2, B3, B4, B5 and B6 of a Publishing Company in 2000 and 2001



- Total sales of branches B1, B3 and B5 together for both the years (in thousand numbers) is :
(a) 250 (b) 310 (c) 435 (d) 560 (e) 585
- Total sales of branch B6 for both the years is what percent of the total sales of branch B3 for both the years ?
(a) 68.54% (b) 71.11% (c) 73.17% (d) 75.55% (e) 77.26%
- What is the average sale of all the branches (in thousand numbers) for the year 2000 ?
(a) 73 (b) 80 (c) 83 (d) 88 (e) 96
- What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years ?
(a) 2 : 3 (b) 3 : 5 (c) 4 : 5 (d) 5 : 7 (e) 7 : 9
- What percent of the average sales of branches B1, B2 and B3 in 2001 is the average sales of branches B1, B3 and B6 in 2000 ?
(a) 75% (b) 77.5% (c) 82.5% (d) 85% (e) 87.5%

Sol. 1. (d) : Total sales of branches B1, B3 and B5 for both the years (in thousand numbers) = $(80 + 105) + (95 + 110) + (75 + 95) = 560$.

2. (c) : Required Percentage = $\left[\frac{(70 + 80)}{(95 + 110)} \times 100 \right] \% = \left(\frac{150}{205} \times 100 \right) \% = 73.17\%$.

3. (b) : Average sales of all the six branches (in thousand numbers) for the year

$$2000 = \frac{1}{6} \times [80 + 75 + 95 + 85 + 75 + 70] = 80.$$

4. (c) : Required ratio = $\frac{(75 + 65)}{(85 + 95)} = \frac{140}{180} = \frac{7}{9}$

Ex. 5. (e) : Average sales (in thousand numbers) of branches B1, B3 and B6 in 2000

$$= \frac{1}{3} \times (80 + 95 + 70) = \left(\frac{245}{3} \right).$$

Average sales (in thousand numbers) of branches B1, B2 and B3 in 2001

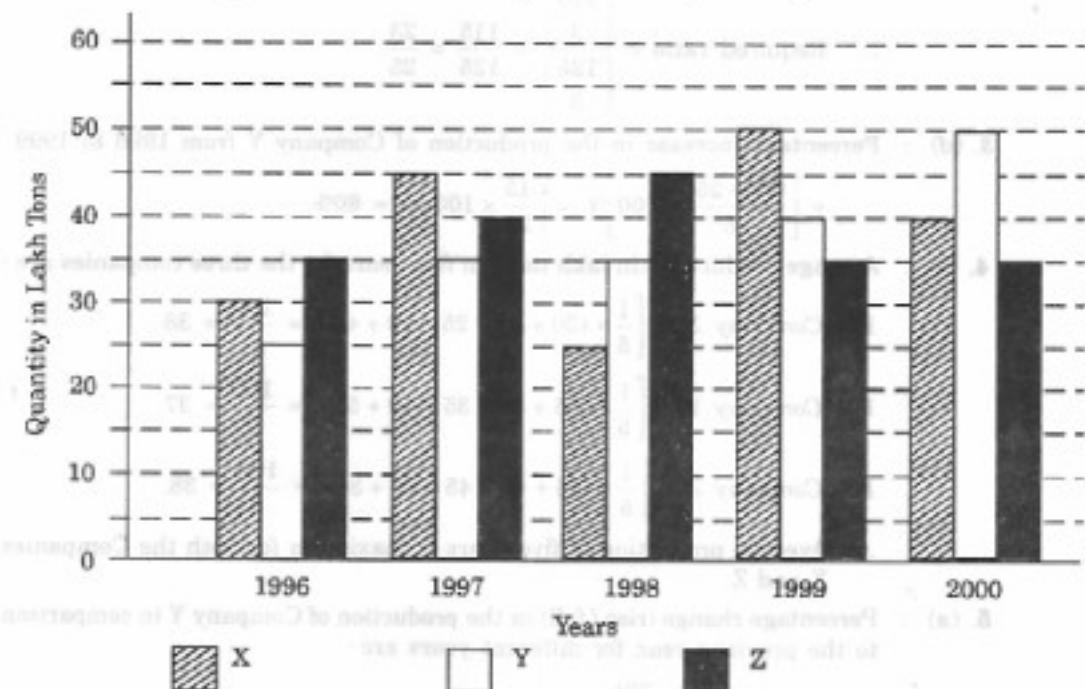
$$= \frac{1}{3} \times (105 + 65 + 110) = \left(\frac{280}{3} \right).$$

Required Percentage = $\left[\frac{\left(\frac{245}{3} \right)}{\left(\frac{280}{3} \right)} \times 100 \right] \% = \left(\frac{245}{280} \times 100 \right) \% = 87.5\%.$

Ex. 3. The bar graph provided below gives the data of the production of paper (in lakh tonnes) by three different companies X, Y and Z over the years. Study the graph and answer the questions that follow.

(Bank P.O. 2001)

Production of Paper (in lakh tonnes) by Three Companies X, Y and Z over the Years



- What is the difference between the production of Company Z in 1998 and Company Y in 1996 ?
 - 2,00,000 tons
 - 20,00,000 tens
 - 20,00,000 tons
 - None of these
- What is the ratio of the average production of Company X in the period 1998-2000 to the average production of Company Y in the same period ?
 - 1 : 1
 - 15 : 17
 - 23 : 25
 - 27 : 29
 - None of these
- What is the percentage increase in the production of Company Y from 1996 to 1999 ?
 - 30%
 - 45%
 - 50%
 - 60%
 - 75%
- The average production for five years was maximum for which Company ?
 - X
 - Y
 - Z
 - X and Y both
 - X and Z both

5. For which of the following years, the percentage rise/fall in production from the previous year is the maximum for Company Y ?
 (a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 1997 and 2000
6. In which year was the percentage of production of Company Z to the production of Company Y the maximum ?
 (a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000

Sol. 1. (b) : Required difference = $[(45 - 25) \times 1,00,000]$ tons = 20,00,000 tons.

2. (c) : Average production of Company X in the period 1998-2000

$$= \left[\frac{1}{3} \times (25 + 50 + 40) \right] = \left(\frac{115}{3} \right) \text{ lakh tons.}$$

Average production of Company Y in the period 1998-2000

$$= \left[\frac{1}{3} \times (35 + 40 + 50) \right] = \left(\frac{125}{3} \right) \text{ lakh tons.}$$

$$\therefore \text{Required ratio} = \frac{\left(\frac{115}{3} \right)}{\left(\frac{125}{3} \right)} = \frac{115}{125} = \frac{23}{25}.$$

3. (d) : Percentage increase in the production of Company Y from 1996 to 1999

$$= \left[\frac{(40 - 25)}{25} \times 100 \right]\% = \left(\frac{15}{25} \times 100 \right)\% = 60\%.$$

4. (e) : Average production (in lakh tons) in five years for the three companies are :

$$\text{For Company } X = \left[\frac{1}{5} \times (30 + 45 + 25 + 50 + 40) \right] = \frac{190}{5} = 38$$

$$\text{For Company } Y = \left[\frac{1}{5} \times (25 + 35 + 35 + 40 + 50) \right] = \frac{185}{5} = 37$$

$$\text{For Company } Z = \left[\frac{1}{5} \times (35 + 40 + 45 + 35 + 36) \right] = \frac{190}{5} = 38.$$

\therefore Average production of five years is maximum for both the Companies X and Z.

5. (a) : Percentage change (rise / fall) in the production of Company Y in comparison to the previous year, for different years are :

$$\text{For 1997} = \left[\frac{(32 - 25)}{25} \times 100 \right]\% = 40\%$$

$$\text{For 1998} = \left[\frac{(35 - 32)}{25} \times 100 \right]\% = 12\%$$

$$\text{For 1999} = \left[\frac{(40 - 35)}{35} \times 100 \right]\% = 14.29\%$$

$$\text{For 2000} = \left[\frac{(50 - 40)}{40} \times 100 \right]\% = 25\%$$

Hence, the maximum percentage rise / fall in the production of Company Y is for 1997.

6. (a) : The percentages of production of Company Z to the production of Company Z for various years are :

$$\text{For 1996} = \left(\frac{35}{25} \times 100 \right)\% = 140\%; \text{For 1997} = \left(\frac{40}{35} \times 100 \right)\% = 114.29\%;$$

$$\text{For 1998} = \left(\frac{45}{35} \times 100 \right)\% = 128.57\%; \text{For 1999} = \left(\frac{35}{40} \times 100 \right)\% = 87.5\%;$$

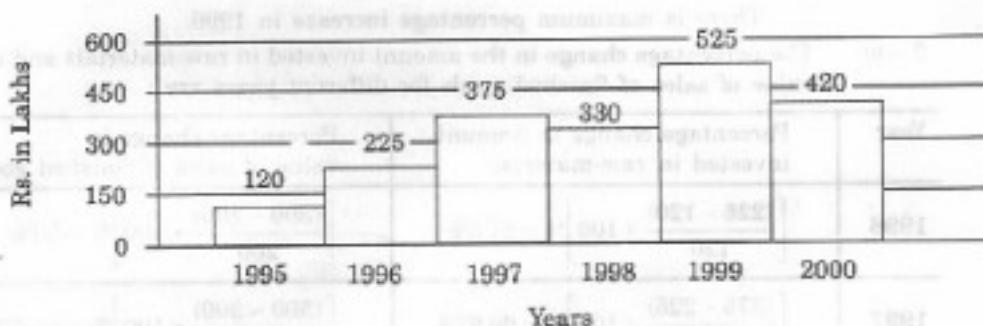
$$\text{For 2000} = \left(\frac{35}{50} \times 100 \right)\% = 70\%.$$

Clearly, this percentage is highest for 1996.

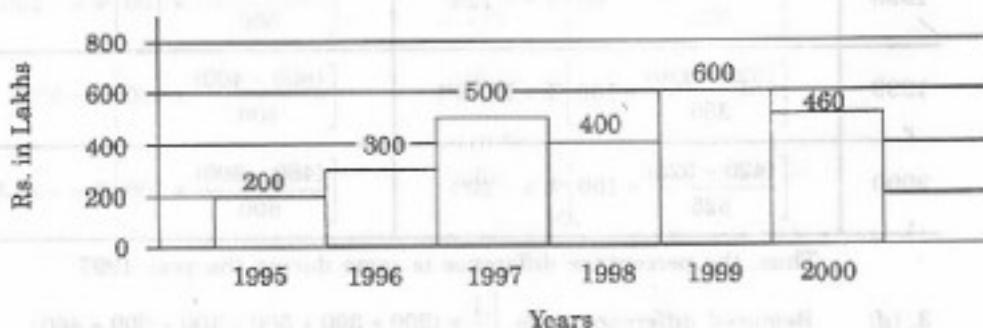
Ex. 4. Out of the two bar graphs provided below, one shows the amounts (in Lakh Rs.) invested by a Company in purchasing raw materials over the years and the other shows the values (in Lakh Rs.) of finished goods sold by the Company over the years. Study the two bar graphs and answer the questions based on them.

Amount Invested in Raw Materials and the Value of Sales of Finished Goods for a Company over the Years

Amount Invested in Raw Materials (Rs. in Lakhs)



Value of Sales of Finished Goods (Rs. in Lakhs)



- In which year, there has been a maximum percentage increase in the amount invested in Raw Materials as compared to the previous year ?
 (a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
- In which year, the percentage change (compared to the previous year) in the investment on Raw Materials is the same as that in the value of sales of finished goods ?
 (a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
- What was the difference between the average amount invested in Raw Materials during the given period and the average value of sales of finished goods during this period ?
 (a) Rs. 62.5 lakhs (b) Rs. 68.5 lakhs (c) Rs. 71.5 lakhs
 (d) Rs. 77.5 lakhs (e) Rs. 83.5 lakhs

4. The value of sales of finished goods in 1999 was approximately what percent of the average amount invested in Raw Materials in the years 1997, 1998 and 1999 ?
 (a) 33% (b) 37% (c) 45% (d) 49% (e) 53%
5. The maximum difference between the amount invested in Raw Materials and the value of sales of finished goods was during the year :
 (a) 1995 (b) 1996 (c) 1997 (d) 1998 (e) 1999

Sol. 1. (a) : The percentage increase in the amount invested in raw-materials as compared to the previous year, for different years are :

$$\text{For 1996} = \left[\frac{(225 - 120)}{120} \times 100 \right] \% = 87.5\%$$

$$\text{For 1997} = \left[\frac{(375 - 225)}{225} \times 100 \right] \% = 66.67\%$$

For 1998 there is a decrease.

$$\text{For 1999} = \left[\frac{(525 - 330)}{330} \times 100 \right] \% = 59.09\%$$

For 2000 there is a decrease.

∴ There is maximum percentage increase in 1996.

2. (b) : The percentage change in the amount invested in raw-materials and in the value of sales of finished goods for different years are :

Year	Percentage change in Amount invested in raw-material	Percentage change in value of sales of finished goods
1996	$\left[\frac{(225 - 120)}{120} \times 100 \right] \% = 87.5\%$	$\left[\frac{(300 - 200)}{200} \times 100 \right] \% = 50\%$
1997	$\left[\frac{(375 - 225)}{225} \times 100 \right] \% = 66.67\%$	$\left[\frac{(500 - 300)}{300} \times 100 \right] \% = 66.67\%$
1998	$\left[\frac{(330 - 375)}{375} \times 100 \right] \% = -12\%$	$\left[\frac{(400 - 500)}{500} \times 100 \right] \% = -20\%$
1999	$\left[\frac{(525 - 330)}{330} \times 100 \right] \% = 59.09\%$	$\left[\frac{(600 - 400)}{400} \times 100 \right] \% = 50\%$
2000	$\left[\frac{(420 - 525)}{525} \times 100 \right] \% = -20\%$	$\left[\frac{(460 - 600)}{600} \times 100 \right] \% = -23.33\%$

Thus, the percentage difference is same during the year 1997.

$$\begin{aligned} 3. (d) : \text{Required difference} &= \text{Rs. } \left[\frac{1}{6} \times (200 + 300 + 500 + 400 + 600 + 460) \right. \\ &\quad \left. - \frac{1}{6} \times (120 + 225 + 375 + 330 + 525 + 420) \right] \text{ lakhs} \end{aligned}$$

$$= \text{Rs. } \left[\left(\frac{2460}{6} \right) - \left(\frac{1995}{6} \right) \right] \text{ lakhs} = \text{Rs. } (410 - 332.5) \text{ lakhs} = \text{Rs. } 77.5 \text{ lakhs.}$$

$$4. (d) : \text{Required percentage} = \left[\frac{600}{(375 + 330 + 525)} \times 100 \right] \% = 48.78\% = 49\%.$$

5. (c) : The differences between the amount invested in raw material and the value of sales of finished goods for various years are:

For 1995 = Rs. (200 - 120) lakhs = Rs. 80 lakhs.

For 1996 = Rs. (300 - 225) lakhs = Rs. 75 lakhs.

For 1997 = Rs. (500 - 375) lakhs = Rs. 125 lakhs.

For 1998 = Rs. (400 - 330) lakhs = Rs. 70 lakhs.

For 1999 = Rs. (600 - 525) lakhs = Rs. 75 lakhs.

For 2000 = Rs. (460 - 420) lakhs = Rs. 40 lakhs.

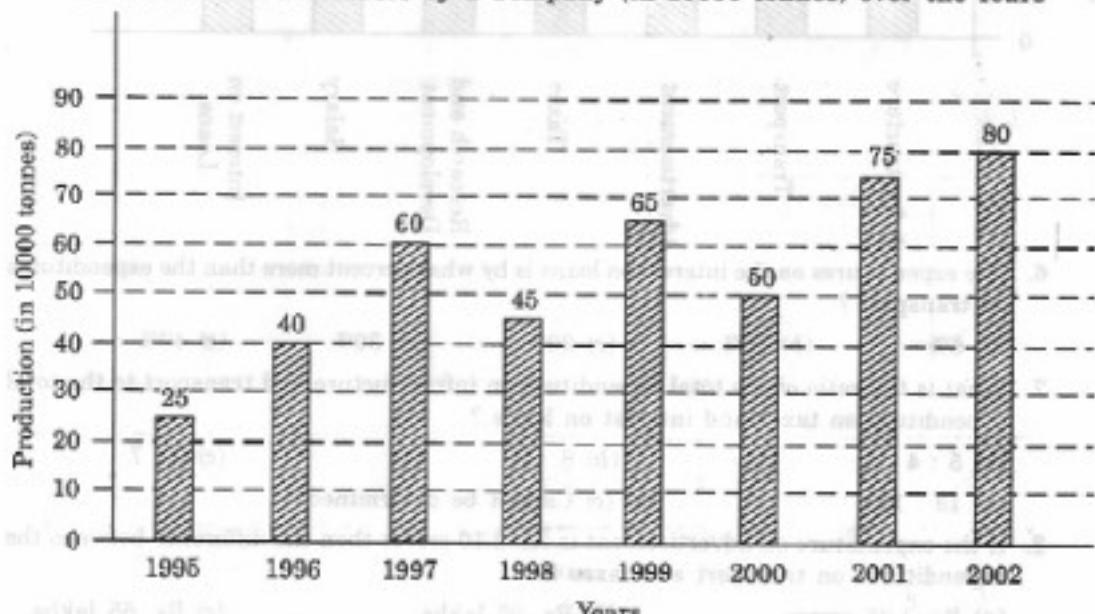
Clearly, maximum difference was during 1997.

EXERCISE 37

Directions (Questions 1 to 5) : Study the following bar-graph and answer the questions given below.

(Bank P.O. 2002)

Production of Fertilizers by a Company (in 10000 tonnes) over the Years



1. In how many of the given years was the production of fertilizers more than the average production of the given years?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

2. The average production of 1996 and 1997 was exactly equal to the average production of which of the following pairs of years?

(a) 2000 and 2001 (b) 1999 and 2000 (c) 1998 and 2000
 (d) 1995 and 1999 (e) 1995 and 2001

3. What was the percentage decline in the production of fertilizers from 1997 to 1998?

(a) $33\frac{1}{3}\%$ (b) 30% (c) 25% (d) 21% (e) 20%

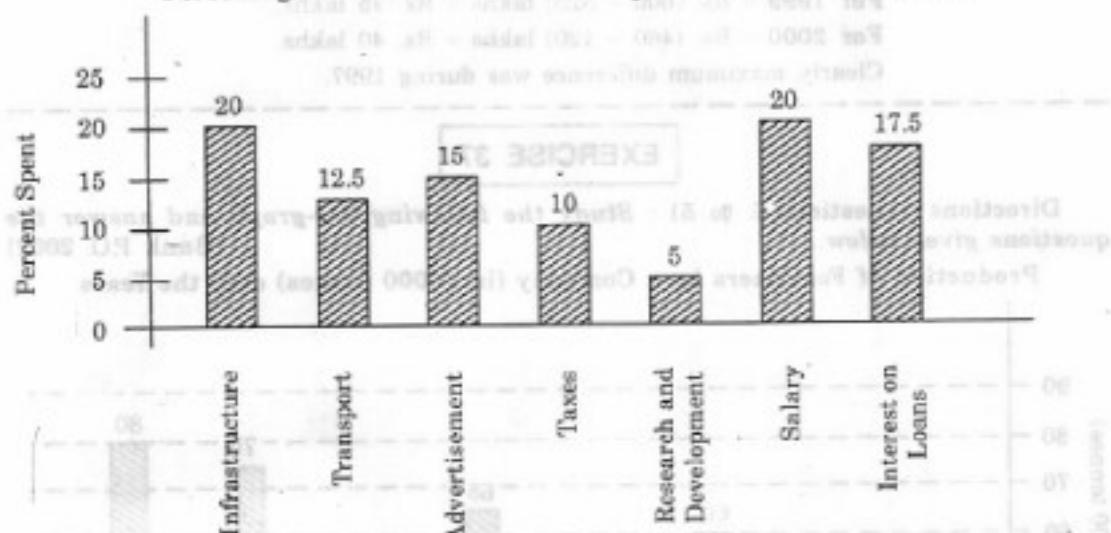
4. In which year was the percentage increase in production as compared to the previous year the maximum?

(a) 2002 (b) 2001 (c) 1999 (d) 1997 (e) 1996

5. What was the percentage increase in production of fertilizers in 2002 compared to that in 1995 ?
 (a) 320% (b) 300% (c) 220% (d) 200% (e) 150%

Directions (Questions 6 to 10) : The bar-graph given below shows the percentage distribution of total expenditures of a Company under various expense heads during 2003. Study the graph and answer the questions that follow :

Percentage Distribution of Total Expenditures of a Company

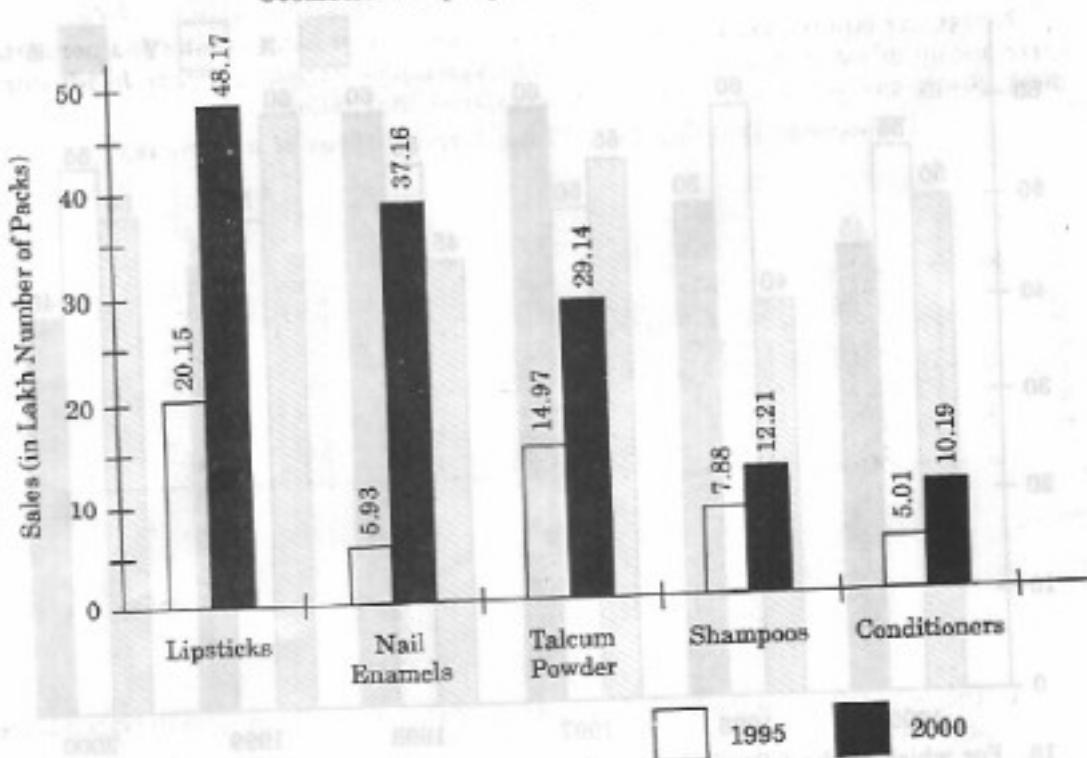


6. The expenditures on the interest on loans is by what percent more than the expenditures on transport ?
 (a) 5% (b) 10% (c) 20% (d) 30% (e) 40%
7. What is the ratio of the total expenditure on infrastructure and transport to the total expenditure on taxes and interest on loans ?
 (a) 5 : 4 (b) 8 : 7 (c) 9 : 7
 (d) 13 : 11 (e) Cannot be determined
8. If the expenditure on advertisement is Rs. 2.10 crores then the difference between the expenditures on transport and taxes is :
 (a) Rs. 1.25 crores (b) Rs. 95 lakhs (c) Rs. 65 lakhs
 (d) Rs. 35 lakhs (e) Rs. 25 lakhs
9. The total amount of expenditures of the Company is how many times the expenditure on research and development ?
 (a) 27 (b) 20 (c) 18 (d) 8 (e) 5
10. If the interest on loans amounted to Rs. 2.45 crores then the total amount of expenditure on advertisement, taxes and research and development is :
 (a) Rs. 7 crores (b) Rs. 5.4 crores (c) Rs. 4.2 crores
 (d) Rs. 3 crores (e) Rs. 2.4 crores

Directions (Questions 11 to 15) : A cosmetic company produces five different products. The sales of these five products (in lakh number of packs) during 1995 and 2000 are shown in the following bar-graph. The questions given below are based on this graph.
 (Bank P.O. 2001)

Bar Graphs

Sales (in lakh number of packs) of five different products of a Cosmetic Company during 1995 and 2000

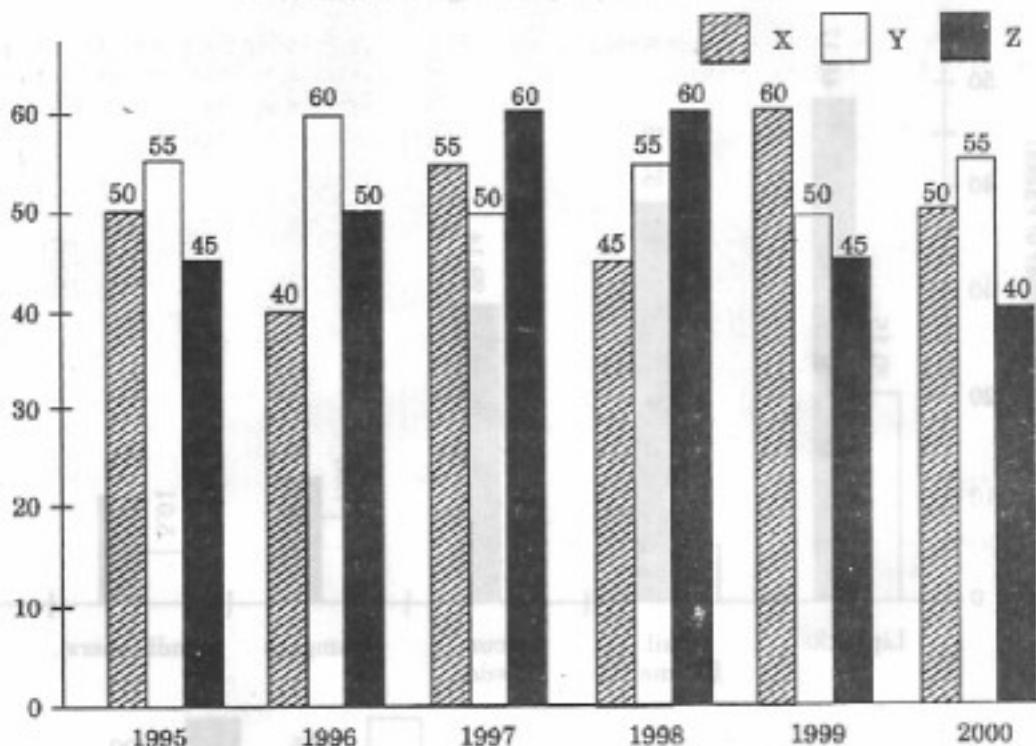


11. The sales have increased by nearly 55% from 1995 to 2000 in the case of :
- Lipsticks
 - Nail enamels
 - Talcum powders
 - Conditioners
 - Shampoos
12. During the period 1995-2000, the minimum rate of increase in sales is in the case of :
- Lipsticks
 - Nail enamels
 - Talcum powders
 - Conditioners
 - Shampoos
13. The sales of lipsticks in 2000 was by what percent more than the sales of nail enamels in 2000 ? (rounded off to the nearest integer)
- 33%
 - 31%
 - 28%
 - 22%
 - 21%
14. The sales of conditioners in 1995 was by what percent less than the sales of shampoos in 1995 ? (rounded off to the nearest integer)
- 57%
 - 36%
 - 29%
 - 25%
 - 19%
15. What is the approximate ratio of the sales of nail enamels in 2000 to the sales of Talcum powders in 1995 ?
- 7 : 2
 - 5 : 2
 - 4 : 3
 - 2 : 1
 - 5 : 3

Directions (Questions 16 to 20) : A soft-drink company prepares drinks of three different flavours — X, Y and Z. The production of the three flavours over a period of six years has been expressed in the bar-graph provided below. Study the graph and answer the questions based on it.

(I.B.P.S. 2002)

Production of three different flavours of soft-drinks X, Y, Z by a Company over the years (in lakh bottles)

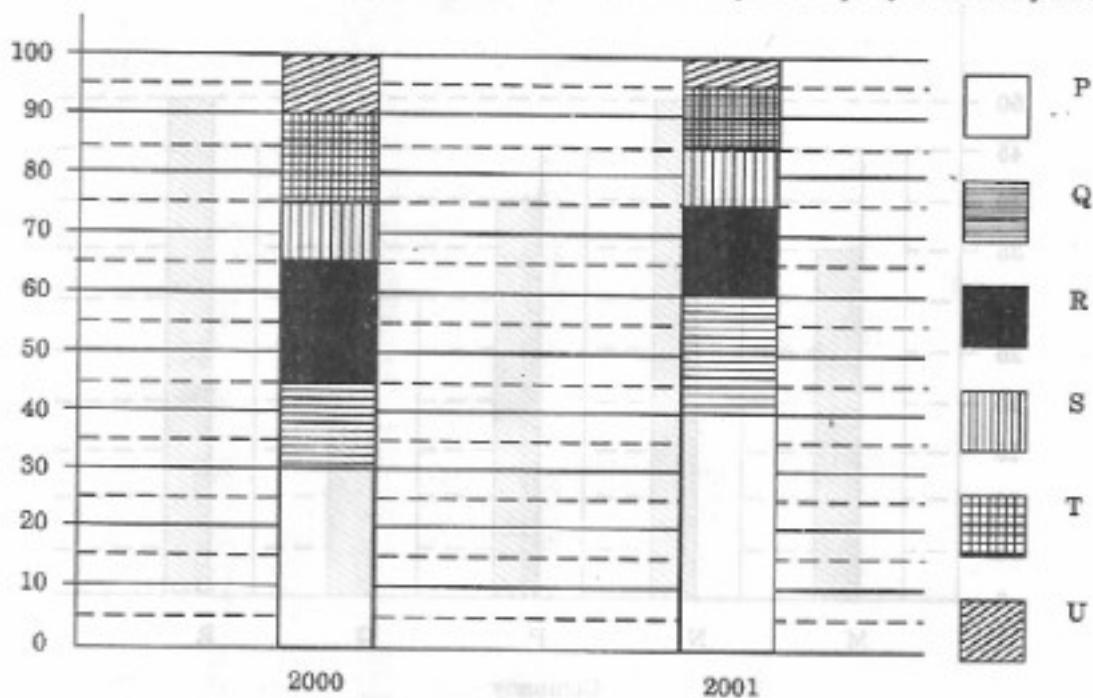


16. For which of the following years the percentage of rise/fall in production from the previous year is the maximum for the flavour Y ?
- 1996
 - 1997
 - 1998
 - 1999
 - 2000
17. For which flavour was the average annual production maximum in the given period ?
- X only
 - Y only
 - Z only
 - X and Y
 - X and Z
18. The total production of flavour Z in 1997 and 1998 is what percentage of the total production of flavour X in 1995 and 1996 ?
- 96.67%
 - 102.25%
 - 115.57%
 - 120%
 - 133.33%
19. What is the difference between the average production of flavour X in 1995, 1996 and 1997 and the average production of flavour Y in 1998, 1999 and 2000 ?
- 50,000 bottles
 - 80,000 bottles
 - 2,40,000 bottles
 - 3,30,000 bottles
 - 5,00,000 bottles
20. What was the approximate decline in the production of flavour Z in 2000 as compared to the production in 1998 ?
- 50%
 - 42%
 - 33%
 - 25%
 - 22.5%

Directions (Questions 21 to 25) : The bar-graph given below shows the percentage distribution of the total production of a car manufacturing company into various models over two years. Study the graph carefully and answer the questions that follow.

(Bank P.O. 2001)

Percentage of Six different types of Cars manufactured by a Company over two years



Total Number of
Cars produced = 3,50,000

Total Number of
Cars produced = 4,40,000

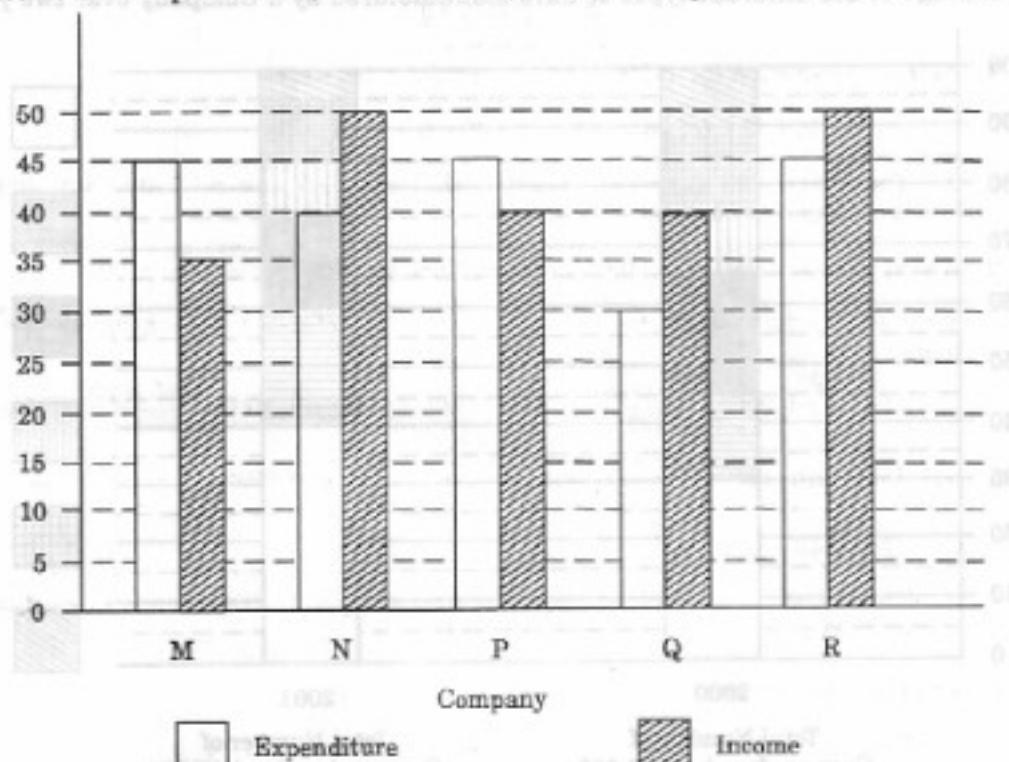
21. Total number of cars of models P, Q and T manufactured in 2000 is :
 (a) 2,45,000 (b) 2,27,500 (c) 2,10,000 (d) 1,92,500 (e) 1,57,500
22. For which model the percentage rise / fall in production from 2000 to 2001 was minimum ?
 (a) Q (b) R (c) S (d) T (e) U
23. What was the difference in the number of Q type cars produced in 2000 and that produced in 2001 ?
 (a) 35,500 (b) 27,000 (c) 22,500 (d) 17,500 (e) 16,000
24. If the percentage production of P type cars in 2001 was the same as that in 2000, then the number of P type cars produced in 2001 would have been :
 (a) 1,40,000 (b) 1,32,000 (c) 1,17,000 (d) 1,05,000 (e) 97,000
25. If 85% of the S type cars produced in each year were sold by the Company, how many S type cars remained unsold ?
 (a) 7650 (b) 9350 (c) 11,850 (d) 12,250 (e) 13,350

Directions (Questions 26 to 30) : The following bar-graph shows the Income and Expenditures (in million US \$) of five Companies in the year 2001. The percent profit or loss of a Company is given by

$$(Profit / Loss)\% = \frac{Income - Expenditure}{Expenditure} \times 100$$

Study the graph and answer the questions that are based on it. (S.B.I.P.O. 2002)

Income and Expenditure (in million US \$) of five Companies in the year 2001



26. Which Company earned the maximum percentage profit in the year 2001 ?
 (a) M (b) N (c) P (d) Q (e) R
27. The Companies M and N together had a percentage profit/loss of :
 (a) 12% loss (b) 10% loss (c) 10% profit
 (d) 12% profit (e) There was no loss or profit
28. In 2001 what was the approximate percentage of profit / loss of all the five Companies taken together ?
 (a) 5% profit (b) 6.5% profit (c) 4% loss (d) 7% loss (e) 10% profit
29. If the income of Company Q in 2001 was 10% more than its income in 2000 and the Company had earned a profit of 20% in 2000, then its expenditure in 2000 (in million US \$) was :
 (a) 28.28 (b) 30.30 (c) 32.32 (d) 34.34 (e) 36.36
30. For Company R, if the expenditure had increased by 20% in year 2001 from year 2000 and the Company had earned a profit of 10% in 2000, what was the Company's income in 2000 (in million US \$) ?
 (a) 35.75 (b) 37.25 (c) 38.5 (d) 41.25 (e) 42.75

ANSWERS

1. (d) 2. (e) 3. (c) 4. (e) 5. (c) 6. (e) 7. (d) 8. (d) 9. (b)
 10. (c) 11. (d) 12. (d) 13. (c) 14. (b) 15. (b) 16. (b) 17. (b) 18. (e)
 19. (e) 20. (c) 21. (c) 22. (b) 23. (a) 24. (b) 25. (c) 26. (d) 27. (e)
 28. (a) 29. (b) 30. (d)

SOLUTIONS

- Average production (in 10000 tonnes) over the given years

$$= \frac{1}{8} \times (25 + 40 + 60 + 45 + 65 + 50 + 75 + 80) = 55$$

∴ The productions during the years 1997, 1999, 2001 and 2002 are more than the average production.
- Average production (in 10000 tonnes) of 1996 and 1997 $= \frac{40 + 60}{2} = 50$.

We shall find the average production (in 10000 tonnes) for each of the given alternative pairs :

$$\begin{array}{ll} (a) \text{ 2000 and 2001} = \frac{50 + 75}{2} = 62.5 & (b) \text{ 1999 and 2000} = \frac{65 + 50}{2} = 57.5 \\ (c) \text{ 1998 and 2000} = \frac{45 + 50}{2} = 47.5 & (d) \text{ 1995 and 1999} = \frac{25 + 65}{2} = 45 \\ (e) \text{ 1995 and 2001} = \frac{25 + 75}{2} = 50. & \end{array}$$

∴ The average production of 1996 and 1997 is equal to the average production of 1995 and 2001.

- Required percentage $= \left[\frac{(45 - 60)}{60} \times 100 \right] \% = -25\%$.

∴ There is a decline of 25% in production from 1997 to 1998.
- The percentage increase in production compared to previous year for different years are :

$$\text{In 1996} = \left[\frac{(40 - 25)}{25} \times 100 \right] \% = 60\%; \quad \text{In 1997} = \left[\frac{(60 - 40)}{40} \times 100 \right] \% = 50\%$$

In 1998 there is a decrease in production.

$$\text{In 1999} = \left[\frac{(65 - 45)}{45} \times 100 \right] \% = 44.44\%$$

In 2000 there is a decrease in production.

$$\text{In 2001} = \left[\frac{(75 - 50)}{50} \times 100 \right] \% = 50\%; \quad \text{In 2002} = \left[\frac{(80 - 75)}{75} \times 100 \right] \% = 6.67\%$$

Clearly, there is maximum percentage increase in production in 1996.

- Required percentage $= \left[\frac{(80 - 25)}{25} \times 100 \right] \% = 220\%$.
- Let the total amount of expenditures be Rs. x .

Then, the expenditure on interest on loans $=$ Rs. $(17.5\% \text{ of } x) =$ Rs. $\left(\frac{17.5}{100} x \right)$

and the expenditure on transport $=$ Rs. $(12.5\% \text{ of } x) =$ Rs. $\left(\frac{12.5}{100} x \right)$

∴ Difference between the two expenditures $=$ Rs. $\left(\frac{17.5}{100} x - \frac{12.5}{100} x \right) =$ Rs. $\left(\frac{5x}{100} \right)$

and so, the required percentage $= \left[\frac{\left(\frac{5x}{100} \right)}{\left(\frac{12.5x}{100} \right)} \times 100 \right] \% = 40\%$.

7. Let the total amount of expenditures be Rs. x .

Then, the total expenditure on infrastructure and transport

$$= \text{Rs. } [(20 + 12.5)\% \text{ of } x] = \text{Rs. } (32.5\% \text{ of } x) = \text{Rs. } \left(\frac{32.5x}{100} \right)$$

and total expenditure on taxes and interest on loans

$$= \text{Rs. } [(10 + 17.5)\% \text{ of } x] = \text{Rs. } (27.5\% \text{ of } x) = \text{Rs. } \left(\frac{27.5x}{100} \right)$$

$$\text{Required ratio} = \frac{\left(\frac{32.5x}{100} \right)}{\left(\frac{27.5x}{100} \right)} = 13 : 11$$

8. Let the total expenditure be Rs. x crores.

$$\text{Then, } 15\% \text{ of } x = 2.10 \Rightarrow x = \left(\frac{2.10 \times 100}{15} \right) = 14.$$

\therefore Total expenditure = Rs. 14 crores

and so, the difference between the expenditures on transport and taxes

$$= \text{Rs. } [(12.5 - 10)\% \text{ of } 14] \text{ crores} = \text{Rs. } (2.5\% \text{ of } 14) \text{ crores$$

$$= \text{Rs. } 0.35 \text{ crores} = \text{Rs. } 35 \text{ lakhs.}$$

9. Let the total expenditures be Rs. x .

$$\text{Then, the expenditure on Research and Development} = \text{Rs. } (5\% \text{ of } x) = \text{Rs. } \left(\frac{x}{20} \right).$$

\therefore Ratio of the total expenditure to the expenditure on Research and Development

$$= \frac{x}{\left(\frac{x}{20} \right)} = \frac{20}{1}.$$

Thus, the total expenditure is 20 times the expenditure on Research and Development.

10. Let the total expenditure be Rs. x crores. Then, $17.5\% \text{ of } x = 2.45 \Rightarrow x = 14$.

\therefore Total expenditure = Rs. 14 crores

and so, the total expenditure on advertisement, taxes and research and development = $\text{Rs. } [(15 + 10 + 5)\% \text{ of } 14] \text{ crores}$

$$= \text{Rs. } (30\% \text{ of } 14) \text{ crores} = \text{Rs. } 4.2 \text{ crores.}$$

11. The percentage increase from 1995 to 2000 for various products are :

$$\text{Lipsticks} = \left[\frac{(48.17 - 20.15)}{20.15} \times 100 \right]\% = 139.06\%$$

$$\text{Nail enamels} = \left[\frac{(37.76 - 5.93)}{5.93} \times 100 \right]\% = 536.76\%$$

$$\text{Talcum powders} = \left[\frac{(29.14 - 14.97)}{14.97} \times 100 \right]\% = 94.66\%$$

$$\text{Shampoos} = \left[\frac{(12.21 - 7.88)}{7.88} \times 100 \right]\% = 54.95\% \approx 55\%$$

$$\text{Conditioners} = \left[\frac{(10.19 - 5.01)}{5.01} \times 100 \right]\% = 103.39\%.$$

12. As calculated in the Solution of Q. 11, the minimum rate of increase in sales from 1995 to 2000 is in the case of Shampoos.

$$13. \text{ Required percentage} = \left[\frac{(48.17 - 37.76)}{37.76} \times 100 \right] \% = 27.57\% = 28\%.$$

$$14. \text{ Required percentage} = \left[\frac{(7.88 - 5.01)}{7.88} \times 100 \right] \% = 36.42\% = 36\%.$$

$$15. \text{ Required ratio} = \frac{37.76}{14.97} = 2.5 = \frac{5}{2}.$$

16. The percentage rise/fall in production from the previous year for flavour Y during various years are :

$$\text{In 1996} = \left[\frac{(60 - 55)}{55} \times 100 \right] \% = 9.09\% \text{ (increase)}$$

$$\text{In 1997} = \left[\frac{(60 - 50)}{60} \times 100 \right] \% = 16.67\% \text{ (decrease)}$$

$$\text{In 1998} = \left[\frac{(55 - 50)}{55} \times 100 \right] \% = 10\% \text{ (increase)}$$

$$\text{In 1999} = \left[\frac{(55 - 50)}{55} \times 100 \right] \% = 9.09\% \text{ (decrease)}$$

$$\text{In 2000} = \left[\frac{(55 - 50)}{50} \times 100 \right] \% = 10\% \text{ (increase)}$$

∴ Maximum change is decrease of 16.67% during 1997.

17. Average annual productions over the given period for various flavours are :

$$\text{For flavour X} = \left[\frac{1}{6} \times (50 + 40 + 55 + 45 + 60 + 50) \right] \text{lakh bottles} = 50 \text{ lakh bottles.}$$

$$\begin{aligned} \text{For flavour Y} &= \left[\frac{1}{6} \times (55 + 60 + 50 + 55 + 50 + 55) \right] \text{lakh bottles} \\ &= 54.17 \text{ lakh bottles.} \end{aligned}$$

$$\text{For flavour Z} = \left[\frac{1}{6} \times (45 + 50 + 60 + 60 + 45 + 40) \right] \text{lakh bottles} = 50 \text{ lakh bottles.}$$

∴ Maximum average production is for flavour Y.

$$18. \text{ Required percentage} = \left[\frac{(60 + 60)}{(50 + 40)} \times 100 \right] \% = \left(\frac{120}{90} \times 100 \right) \% = 133.33\%.$$

$$19. \text{ Average production of flavour X in 1995, 1996 and 1997} = \left[\frac{1}{3} \times (50 + 40 + 55) \right]$$

$$= \left(\frac{145}{3} \right) \text{lakh bottles.}$$

$$\text{Average production of flavour Y in 1998, 1999 and 2000} = \left[\frac{1}{3} \times (55 + 50 + 55) \right]$$

$$= \left(\frac{160}{3} \right) \text{lakh bottles.}$$

$$\therefore \text{Difference} = \left(\frac{160}{3} - \frac{145}{3} \right) = \frac{15}{3} = 5 \text{ lakh bottles} = 5,00,000 \text{ bottles.}$$

20. Percentage decline in the production of flavour Z in 2000 as compared to the production

$$\text{in } 1998 = \left[\frac{(60 - 40)}{60} \times 100 \right] \% = \left(\frac{20}{60} \times 100 \right) \% = 33.33\% \approx 33\%.$$

21. We shall first determine the number of cars of each model produced by the Company during the two years :

In 2000 : Total number of cars produced = 3,50,000.

$$P = (30 - 0)\% \text{ of } 3,50,000 = 30\% \text{ of } 3,50,000 = 1,05,000$$

$$Q = (45 - 30)\% \text{ of } 3,50,000 = 15\% \text{ of } 3,50,000 = 52,500$$

$$R = (65 - 45)\% \text{ of } 3,50,000 = 20\% \text{ of } 3,50,000 = 70,000$$

$$S = (75 - 65)\% \text{ of } 3,50,000 = 10\% \text{ of } 3,50,000 = 35,000$$

$$T = (90 - 75)\% \text{ of } 3,50,000 = 15\% \text{ of } 3,50,000 = 52,500$$

$$U = (100 - 90)\% \text{ of } 3,50,000 = 10\% \text{ of } 3,50,000 = 35,000.$$

In 2001 : Total number of cars produced = 4,40,000.

$$P = (40 - 0)\% \text{ of } 4,40,000 = 40\% \text{ of } 4,40,000 = 1,76,000$$

$$Q = (60 - 40)\% \text{ of } 4,40,000 = 20\% \text{ of } 4,40,000 = 88,000$$

$$R = (75 - 60)\% \text{ of } 4,40,000 = 15\% \text{ of } 4,40,000 = 66,000$$

$$S = (85 - 75)\% \text{ of } 4,40,000 = 10\% \text{ of } 4,40,000 = 44,000$$

$$T = (95 - 85)\% \text{ of } 4,40,000 = 10\% \text{ of } 4,40,000 = 44,000$$

$$U = (100 - 95)\% \text{ of } 4,40,000 = 5\% \text{ of } 4,40,000 = 22,000.$$

Now, we shall solve the questions.

Total number of cars of models P, Q and T manufactured in 2000

$$= (105000 + 52500 + 52500) = 2,10,000.$$

22. Using the above calculation, the percentage change (rise / fall) in production from 2000 to 2001 for various models is :

$$\text{For } P = \left[\frac{(176000 - 105000)}{105000} \times 100 \right] \% = 67.62\%, \text{ rise.}$$

$$\text{For } Q = \left[\frac{(88000 - 52500)}{52500} \times 100 \right] \% = 67.62\%, \text{ rise.}$$

$$\text{For } R = \left[\frac{(70000 - 66000)}{70000} \times 100 \right] \% = 5.71\%, \text{ fall.}$$

$$\text{For } S = \left[\frac{(44000 - 35000)}{35000} \times 100 \right] \% = 25.71\%, \text{ rise.}$$

$$\text{For } T = \left[\frac{(52500 - 44000)}{44000} \times 100 \right] \% = 16.19\%, \text{ fall.}$$

$$\text{For } U = \left[\frac{(35000 - 22000)}{35000} \times 100 \right] \% = 37.14\%, \text{ fall.}$$

∴ Minimum percentage rise/fall in production is in the case of model R.

23. Required difference = 88000 - 52500 = 35500

(Using calculations in the Solution of Q. 21)

24. If the percentage production of P type cars in 2001 = percentage production of P type cars in 2000 = 30%

then, number of P type cars produced in 2001 = 30% of 440000 = 132000.

25. Number of S type cars which remained unsold in 2000 = 15% of 35000
and number of S type cars which remained unsold in 2001 = 15% of 44000

\therefore Total number of S type cars which remained unsold
 $= 15\% \text{ of } (35000 + 44000) = 15\% \text{ of } 79000 = 11850.$

26. The percentage profit/loss in the year 2001 for various companies are :

$$\text{For } M = \left[\frac{(30 - 45)}{45} \times 100 \right]\% = -33.33\% \text{ i.e. \% Loss} = 33.33\%$$

$$\text{For } N = \left[\frac{(50 - 40)}{40} \times 100 \right]\% = 25\% \text{ i.e. \% Profit} = 25\%$$

$$\text{For } P = \left[\frac{(40 - 45)}{45} \times 100 \right]\% = -11.11\% \text{ i.e. \% Loss} = 11.11\%$$

$$\text{For } Q = \left[\frac{(40 - 30)}{30} \times 100 \right]\% = 33.33\% \text{ i.e. \% Profit} = 33.33\%$$

$$\text{For } R = \left[\frac{(50 - 45)}{45} \times 100 \right]\% = 11.11\% \text{ i.e. \% Profit} = 11.11\%$$

Clearly, the Company Q earned the maximum profit in 2001.

27. Total income of companies M and N together = $(35 + 50)$ million US \$
 $= 85$ million US \$

Total expenditure of companies M and N together = $(45 + 40)$ million US \$
 $= 85$ million US \$

\therefore Percent Profit/Loss of companies M and N together

$$\% \text{ Profit/Loss} = \left(\frac{85 - 85}{85} \times 100 \right)\% = 0\%.$$

Thus, there was neither loss nor profit for companies M and N together.

28. Total income of all five companies = $(35 + 50 + 40 + 40 + 50) = 215$ million US \$
 Total expenditure of all five companies = $(45 + 40 + 45 + 30 + 45)$
 $= 205$ million US \$

$$\therefore \% \text{ Profit} = \left[\frac{(215 - 205)}{205} \times 100 \right]\% = 4.88\% = 5\%.$$

29. Let the income of Company Q in 2000 = x million US \$

$$\text{Then, income of Company Q in 2001} = \left(\frac{110}{100} x \right) \text{ million US \$}$$

$$\therefore \frac{110}{100} x = 40 \Rightarrow x = \left(\frac{400}{11} \right).$$

$$\text{i.e. income of Company Q in 2000} = \left(\frac{400}{11} \right) \text{ million US \$}.$$

Let the expenditure of Company Q in 2000 be E million US \$.

$$\text{Then, } 20 = \left[\left(\frac{400}{11} \right) - E \right] \times 100 \quad [\because \% \text{ Profit} = 20\%]$$

$$\Rightarrow 20 = \left[\left(\frac{400}{11E} \right) - 1 \right] \times 100 \Rightarrow E = \frac{400}{11} \times \frac{100}{120} = 30.30.$$

\therefore Expenditure of Company Q in 2000 = 30.30 million US \$.

30. Let the expenditure of Company R in 2000 be x million US \$.

Then, expenditure of Company R in 2001 = $\left(\frac{120}{100}x\right)$ million US \$.

$$\therefore \frac{120}{100}x = 45 \Rightarrow x = 37.5$$

i.e. expenditure of Company R in 2000 = 37.5 million US \$.

Let the income of Company R in 2000 be I million US \$.

$$\text{Then, } 10 = \frac{(I - 37.5)}{37.5} \times 100 \quad [\because \% \text{ profit in 2000} = 10\%]$$

$$\Rightarrow I - 37.5 = 3.75 \Rightarrow I = 41.25$$

i.e. Income of Company R in 2000 = 41.25 million US \$.

38. PIE-CHARTS

IMPORTANT FACTS AND FORMULAE

The pie-chart or a pie-graph is a method of representing a given numerical data in the form of sectors of a circle.

The sectors of the circle are constructed in such a way that the area of each sector is proportional to the corresponding value of the component of the data.

From geometry, we know that the area of the sector of a circle is proportional to the central angle.

So, the central angle of each sector must be proportional to the corresponding value of the component.

Since the sum of all the central angles is 360° , we have

$$\text{Central angle of the component} = \left(\frac{\text{Value of the component}}{\text{Total value}} \times 360 \right)^\circ$$

SOLVED EXAMPLES

The procedure of solving problems based on pie-charts will be clear from the following solved examples.

Example 1. The following pie-chart shows the sources of funds to be collected by the National Highways Authority of India (NHAI) for its Phase II projects. Study the pie-chart and answer the questions that follow.

**SOURCES OF FUNDS TO BE ARRANGED BY NHAI
FOR PHASE II PROJECTS (IN CRORES RS.)**



Total funds to be arranged for Projects (Phase II) = Rs. 57,600 crores.

- Near about 20% of the funds are to be arranged through :

(a) SPVS	(b) External Assistance
(c) Annuity	(d) Market Borrowing
 - The central angle corresponding to Market Borrowing is :

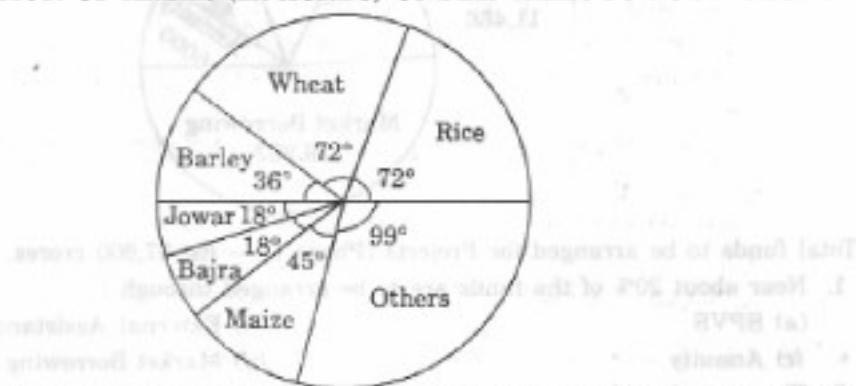
(a) 52°	(b) 137.8%	(c) 187.2°	(d) 192.4°
----------------	---------------	-------------------	-------------------
 - The approximate ratio of the funds to be arranged through Toll and that through Market Borrowing is :

(a) $2 : 9$	(b) $1 : 6$	(c) $3 : 11$	(d) $2 : 5$
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SOLUTION

Example 2. The pie-chart provided below gives the distribution of land (in a village) under various food crops. Study the pie-chart carefully and answer the questions that follow.

DISTRIBUTION OF AREAS (IN ACRES) UNDER VARIOUS FOOD CROPS



3. If the production of wheat is 6 times that of barley, then what is the ratio between the yield per acre of wheat and barley ?
 (a) 3 : 2 (b) 3 : 1 (c) 12 : 1 (d) 2 : 3
4. If the yield per acre of rice was 50% more than that of barley, then the production of barley is what percent of that of rice ?
 (a) 30% (b) $33\frac{1}{3}\%$ (c) 35% (d) 36%
5. If the total area goes up by 5%, and the area under wheat production goes up by 12%, then what will be the angle for wheat in the new pie-chart ?
 (a) 62.4° (b) 76.8° (c) 80.6° (d) 84.2°

SOLUTION

1. (c) : The total of the central angles corresponding to the three crops which cover 50% of the total area, should be 180° . Now, the total of the central angles for the given combinations are :

- (i) Wheat, Barley and Jowar = $(72^\circ + 36^\circ + 18^\circ) = 126^\circ$
- (ii) Rice, Wheat and Jowar = $(72^\circ + 72^\circ + 18^\circ) = 162^\circ$
- (iii) Rice, Wheat and Barley = $(72^\circ + 72^\circ + 36^\circ) = 180^\circ$
- (iv) Bajra, Maize and Rice = $(18^\circ + 45^\circ + 72^\circ) = 135^\circ$

Clearly, (iii) is the required combination.

2. (a) : The area under any of the food crops is proportional to the central angle corresponding to that crop.

Let, the area under rice production be x million acres.

$$\text{Then, } 18 : 72 = 1.5 : x \Rightarrow x = \left(\frac{72 \times 1.5}{18} \right) = 6.$$

Thus, the area under rice production = 6 million acres.

3. (b) : Let the total production of barley be T tonnes and let Z acres of land be put under barley production.

Then, the total production of wheat = $(6T)$ tonnes.

Also, area under wheat production = $(2Z)$ acres.

$$\left[\frac{\text{Area under Wheat production}}{\text{Area under Barley production}} = \frac{72^\circ}{36^\circ} = 2 \right]$$

and therefore, Area under wheat = $2 \times$ Area under barley = $(2Z)$ acres

$$\text{Now, yield per acre for wheat} = \left(\frac{6T}{2Z} \right) \text{ tonnes/acre} = \left(\frac{3T}{Z} \right) \text{ tonnes/acre}$$

$$\text{and yield per acre for barley} = \left(\frac{T}{Z} \right) \text{ tonnes/acre.}$$

$$\therefore \text{Required Ratio} = \left(\frac{3T/Z}{T/Z} \right) = 3 : 1.$$

4. (b) : Let Z acres of land be put under barley production.

$$\text{Then, } \frac{\text{Area under rice production}}{\text{Area under barley production}} = \frac{72^\circ}{36^\circ} = 2.$$

$$\therefore \text{Area under Rice production} = 2 \times \text{area under barley production} = (2Z) \text{ acres.}$$

Now, if p tonnes be the yield per acre of barley then, yield per acre of rice

$$= (p + 50\% \text{ of } p) \text{ tonnes} = \left(\frac{3}{2} p \right) \text{ tonnes.}$$

converted value of Total production of rice = (yield per acre) × (area under production)

$$= \left(\frac{3}{2} p \right) \times 2Z = (3pZ) \text{ tonnes.}$$

To matching with the given condition, we have to write the area under production as 2Z.

And, Total production of barley = (pZ) tonnes.

$$\therefore \text{Percentage production of barley to that of rice} = \left(\frac{pZ}{3pZ} \times 100 \right)\% = 33\frac{1}{3}\%.$$

5. (b) : Initially, let t acres be the total area under consideration.

$$\text{Then, area under wheat production initially was } = \left(\frac{72}{360} \times t \right) \text{ acres} = \left(\frac{t}{5} \right) \text{ acres.}$$

Now, if the total area under consideration be increased by 5%, then the new value of the total area = $\left(\frac{105}{100} t \right)$ acres.

Also, if the area under wheat production be increased by 12%, then the new value of the area under wheat = $\left[\frac{t}{5} + \left(12\% \text{ of } \frac{t}{5} \right) \right]$ acres = $\left(\frac{112t}{500} \right)$ acres.

∴ Central angle corresponding to wheat in the new pie-chart

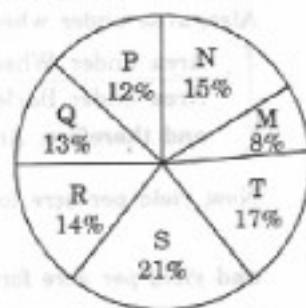
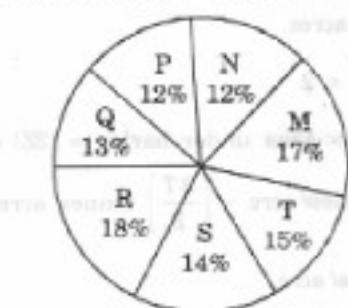
$$\left[\frac{\text{Area under wheat (new)}}{\text{Total area (new)}} \times 360 \right]^\circ = \left[\frac{\left(\frac{112t}{500} \right)}{\left(\frac{105t}{100} \right)} \times 360 \right]^\circ = 76.8^\circ.$$

Example 3. The following pie-charts show the distribution of students of graduate and post-graduate levels in seven different institutes — M, N, P, Q, R, S and T in a town.
(Bank P.O. 2003)

DISTRIBUTION OF STUDENTS AT GRADUATE AND POST-GRADUATE LEVELS IN SEVEN INSTITUTES — M, N, P, Q, R, S AND T

Total Number of Students of
Graduate Level = 27300

Total Number of Students of
Post-Graduate Level = 24700



- How many students of institutes M and S are studying at graduate level?
(a) 7516 (b) 8463 (c) 9127 (d) 9404
- Total number of students studying at post-graduate level from institutes N and P is:
(a) 5601 (b) 5944 (c) 6669 (d) 7004
- What is the total number of graduate and post-graduate level students in institute R?
(a) 8320 (b) 7916 (c) 9116 (d) 8372
- What is the ratio between the number of students studying at post-graduate and graduate levels respectively from institute S?
(a) 14 : 19 (b) 19 : 21 (c) 17 : 21 (d) 19 : 14

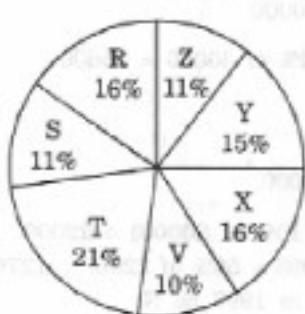
5. What is the ratio between the number of students studying at post-graduate level from institute S and the number of students studying at graduate level from institute Q ?
 (a) 13 : 19 (b) 21 : 13 (c) 13 : 8 (d) 19 : 13

SOLUTION

- (b) : Students of institute M at graduate level = 17% of 27300 = 4641.
 Students of institute S at graduate level = 14% of 27300 = 3822.
 \therefore Total number of students at graduate level in institutes M and S
 $= 4641 + 3822 = 8463.$
- (c) : Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669.
- (d) : Required number = (18% of 27300) + (14% of 24700) = 4914 + 3458 = 8372.
- (d) : Required ratio = $\frac{(21\% \text{ of } 24700)}{(14\% \text{ of } 27300)} = \frac{21 \times 24700}{14 \times 27300} = \frac{19}{14}$
- (d) : Required ratio = $\frac{(21\% \text{ of } 24700)}{(13\% \text{ of } 27300)} = \frac{21 \times 24700}{13 \times 27300} = \frac{19}{13}$.

Example 4. Study the following pie-chart and the table and answer the questions based on them. (S.B.I.P.O. 1999)

PROPORTION OF POPULATION OF SEVEN VILLAGES IN 1997



Village	% Population Below Poverty Line
X	38
Y	52
Z	42
R	51
S	49
T	46
V	58

- Find the population of village S if the population of village X below poverty line in 1997 is 12160.
 (a) 18500 (b) 20500 (c) 22000 (d) 26000
- The ratio of population of village T below poverty line to that of village Z below poverty line in 1997 is :
 (a) 11 : 23 (b) 13 : 11 (c) 23 : 11 (d) 11 : 13
- If the population of village R in 1997 is 32000, then what will be the population of village Y below poverty line in that year ?
 (a) 14100 (b) 15600 (c) 16500 (d) 17000
- If in 1998, the population of villages Y and V increase by 10% each and the percentage of population below poverty line remains unchanged for all the villages, then find the population of village V below poverty line in 1998, given that the population of village Y in 1997 was 30000.
 (a) 11250 (b) 12760 (c) 13140 (d) 13780

5. If in 1999, the population of village R increases by 10% while that of village Z reduces by 5% compared to that in 1997 and the percentage of population below poverty line remains unchanged for all the villages, then find the approximate ratio of population of village R below poverty line to the ratio of population of village Z below poverty line for the year 1999.

(a) 2 : 1 (b) 3 : 2 (c) 4 : 3 (d) 5 : 4

SOLUTION

1. (c) : Let the population of village X be x .

$$\text{Then, } 38\% \text{ of } x = 12160 \Rightarrow x = \frac{12160 \times 100}{38} = 32000.$$

Now, if s be the population of village S, then

$$16 : 11 = 32000 : s \Rightarrow s = \frac{11 \times 32000}{16} = 22000.$$

2. (c) : Let N be the total population of all the seven villages.

Then, population of village T below poverty line = 46% of (21% of N)
and population of village Z below poverty line = 42% of (11% of N)

$$\therefore \text{Required ratio} = \frac{46\% \text{ of } (21\% \text{ of } N)}{42\% \text{ of } (11\% \text{ of } N)} = \frac{46 \times 21}{42 \times 11} = \frac{23}{11}.$$

3. (b) : Population of village R = 32000 (given).

Let the population of village Y be y .

$$\text{Then, } 16 : 15 = 32000 : y \Rightarrow y = \frac{15 \times 32000}{16} = 30000$$

\therefore Population of village Y below poverty line = 52% of 30000 = 15600.

4. (b) : Population of village Y in 1997 = 30000 (given).

Let the population of village V in 1997 be v .

$$\text{Then, } 15 : 10 = 30000 : v \Rightarrow v = \frac{30000 \times 10}{15} = 20000.$$

Now, population of village V in 1998 = 20000 + (10% of 20000) = 22000.

\therefore Population of village V below poverty line in 1998 = 58% of 22000 = 12760.

5. (a) : Let the total population of all the seven villages in 1997 be N .

$$\text{Then, population of village R in 1997} = 16\% \text{ of } N = \frac{16}{100} N$$

$$\text{and population of village Z in 1997} = 11\% \text{ of } N = \frac{11}{100} N.$$

$$\therefore \text{Population of village R in 1999} = \left(\frac{16}{100} N + \left(10\% \text{ of } \frac{16}{100} N \right) \right) = \frac{1760}{10000} N$$

$$\text{and population of village Z in 1999} = \left(\frac{11}{100} N - \left(5\% \text{ of } \frac{11}{100} N \right) \right) = \frac{1045}{10000} N$$

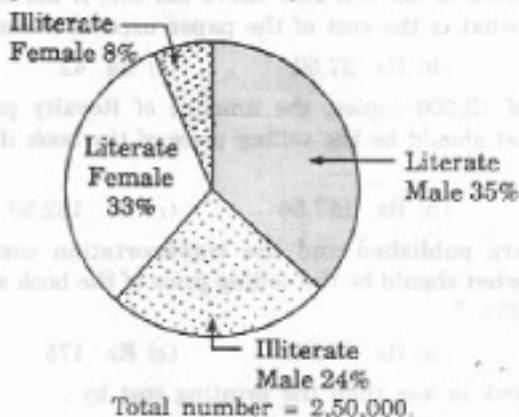
$$\text{Now, population of village R below poverty line for 1999} = 51\% \text{ of } \left(\frac{1760}{10000} N \right)$$

$$\text{and population of village Z below poverty line for 1999} = 42\% \text{ of } \left(\frac{1045}{10000} N \right).$$

$$\therefore \text{Required ratio} = \frac{51\% \text{ of } \left(\frac{1760}{10000} N \right)}{42\% \text{ of } \left(\frac{1045}{10000} N \right)} = \frac{51 \times 1760}{42 \times 1045} = \frac{2}{1}.$$

EXERCISE 38

1. The following pie-chart shows the percentage of Literate and Illiterate — Males and Females in a city. (Bank P.O. 2003)

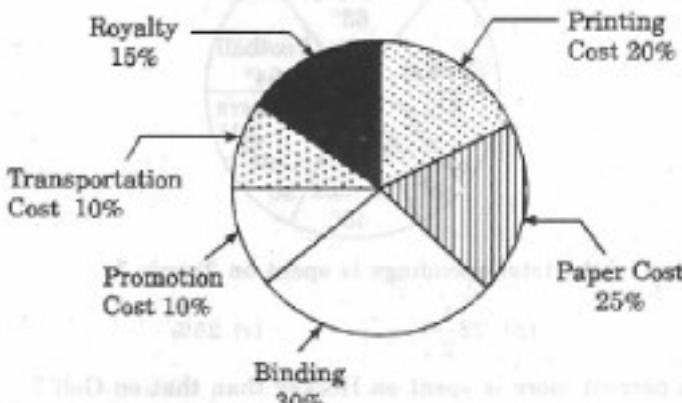


What is the difference between the number of Literate Males and Literate Females?

- (a) 75,000 (b) 1,500 (c) 5,000 (d) 500

2. Directions (Questions 2 to 10) : The following pie-chart shows the percentage distribution of the expenditure incurred in publishing a book. Study the pie-chart and answer the questions based on it. (Bank P.O. 2002)

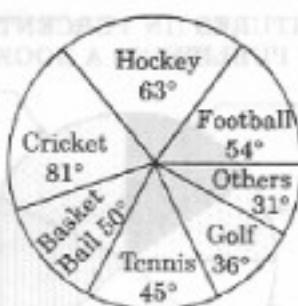
VARIOUS EXPENDITURES (IN PERCENTAGE) INCURRED IN PUBLISHING A BOOK



- What is the central angle of the sector corresponding to the expenditure incurred on Royalty? (a) 15° (b) 24° (c) 54° (d) 48°
- Which two expenditures together have a central angle of 108° ? (a) Binding Cost and Transportation Cost (b) Printing Cost and Paper Cost (c) Royalty and Promotion Cost (d) Binding Cost and Paper Cost
- If the difference between the two expenditures are represented by 18° in the pie-chart, then these expenditures possibly are : (a) Binding Cost and Promotion Cost (b) Paper Cost and Royalty (c) Binding Cost and Printing Post (d) Paper Cost and Printing Cost
- If for an edition of the book, the cost of paper is Rs. 56250, then find the promotion cost for this edition. (a) Rs. 20,000 (b) Rs. 22,500 (c) Rs. 25,500 (d) Rs. 28,125

6. If for a certain quantity of books, the publisher has to pay Rs. 30,600 as printing cost, then what will be the amount of royalty to be paid for these books ?
 (a) Rs. 19,450 (b) Rs. 21,200 (c) Rs. 22,950 (d) Rs. 26,150
7. The price of the book is marked 20% above the C.P. If the marked price of the book is Rs. 180, then what is the cost of the paper used in a single copy of the book ?
 (a) Rs. 36 (b) Rs. 37.50 (c) Rs. 42 (d) Rs. 44.25
8. For an edition of 12,500 copies, the amount of Royalty paid by the publisher is Rs. 2,81,250. What should be the selling price of the book if the publisher desires a profit of 5% ?
 (a) Rs. 152.50 (b) Rs. 157.50 (c) Rs. 162.50 (d) Rs. 167.50
9. If 5500 copies are published and the transportation cost on them amounts to Rs. 82,500, then what should be the selling price of the book so that the publisher can earn a profit of 25% ?
 (a) Rs. 187.50 (b) Rs. 191.50 (c) Rs. 175 (d) Rs. 180
10. Royalty on the book is less than the printing cost by :
 (a) 5% (b) $33\frac{1}{3}\%$ (c) 20% (d) 25%

Directions (Questions 11 to 15) : The circle-graph given here shows the spendings of a country on various sports during a particular year. Study the graph carefully and answer the questions given below it.

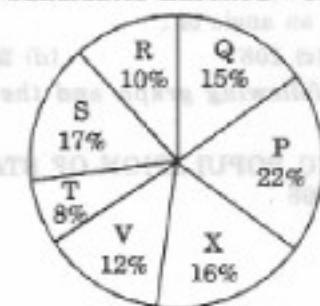


11. What percent of the total spendings is spent on Tennis ?
 (a) $12\frac{1}{2}\%$ (b) $22\frac{1}{2}\%$ (c) 25% (d) 45%
12. How much percent more is spent on Hockey than that on Golf ?
 (a) 27% (b) 35% (c) 37.5% (d) 75%
13. How much percent less is spent on Football than that on Cricket ?
 (a) $22\frac{2}{9}\%$ (b) 27% (c) $33\frac{1}{3}\%$ (d) $37\frac{1}{2}\%$
14. If the total amount spent on sports during the year was Rs. 2 crores, the amount spent on Cricket and Hockey together was :
 (a) Rs. 8,00,000 (b) Rs. 80,00,000 (c) Rs. 1,20,00,000 (d) Rs. 1,60,00,000
15. If the total amount spent on sports during the year be Rs. 1,80,00,000, the amount spent on Basketball exceeds that on Tennis by :
 (a) Rs. 2,50,000 (b) Rs. 3,80,000 (c) Rs. 3,75,000 (d) Rs. 4,10,000

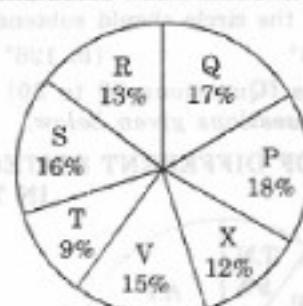
Directions (Questions 16 to 20) : Study the following graph carefully and answer the questions given below :
 (Bank P.O. 2002)

DISTRIBUTION OF CANDIDATES WHO WERE ENROLLED FOR MBA ENTRANCE EXAM AND THE CANDIDATES (OUT OF THOSE ENROLLED) WHO PASSED THE EXAM IN DIFFERENT INSTITUTES

Candidates Enrolled = 8550



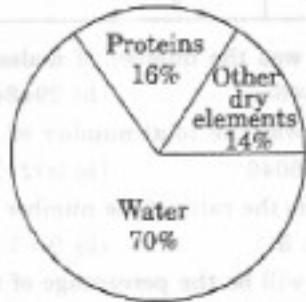
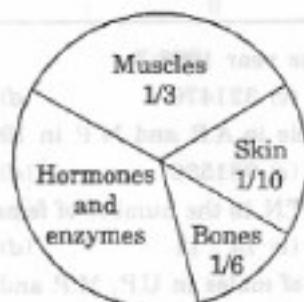
Candidates who Passed the Exam = 5700



16. What percentage of candidates passed the Exam from institute T out of the total number of candidates enrolled from the same institute ?
 (a) 50% (b) 62.5% (c) 75% (d) 80%
17. What is the ratio of candidates passed to the candidates enrolled from institute P ?
 (a) 9 : 11 (b) 14 : 17 (c) 6 : 11 (d) 9 : 17
18. What is the percentage of candidates passed to the candidates enrolled for institutes Q and R together ?
 (a) 68% (b) 80% (c) 74% (d) 65%
19. Which institute has the highest percentage of candidates passed to the candidates enrolled ?
 (a) Q (b) R (c) V (d) T
20. The number of candidates passed from institutes S and P together exceeds the number of candidates enrolled from institutes T and R together by :
 (a) 228 (b) 279 (c) 399 (d) 407

Directions (Questions 21 to 25) : Study the following pie-diagrams carefully and answer the questions given below it.

PERCENTAGE COMPOSITION OF HUMAN BODY

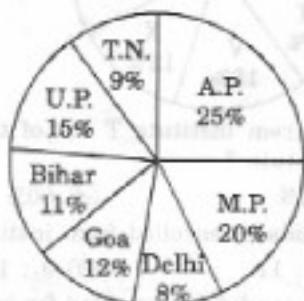


21. In the human body, what part is made of neither bones nor skin ?
 (a) $\frac{1}{40}$ (b) $\frac{3}{80}$ (c) $\frac{2}{5}$ (d) None of these
22. What is the ratio of the distribution of proteins in the muscles to that of the distribution of proteins in the bones ?
 (a) 1 : 18 (b) 1 : 2 (c) 2 : 1 (d) 18 : 1

23. What will be the quantity of water in the body of a person weighing 50 kg ?
 (a) 20 kg (b) 35 kg (c) 41 kg (d) 42.5 kg
24. What percent of the total weight of human body is equivalent to the weight of the proteins in skin in human body ?
 (a) 0.016 (b) 1.6 (c) 0.16 (d) Data inadequate
25. To show the distribution of proteins and other dry elements in the human body, the arc of the circle should subtend at the centre an angle of :
 (a) 54° (b) 126° (c) 108° (d) 252°

Directions (Questions 26 to 30) : Study the following graph and the table and answer the questions given below.

**DATA OF DIFFERENT STATES REGARDING POPULATION OF STATES
IN THE YEAR 1998**



Total Population of the given States = 3276000

States	Sex and Literacy wise Population Ratio					
	Sex		Literacy			
	M	—	F	Literate	—	Illiterate
A.P.	5	:	3	2	:	7
M.P.	3	:	1	1	:	4
Delhi	2	:	3	2	:	1
Goa	3	:	5	3	:	2
Bihar	3	:	4	5	:	1
U.P.	3	:	2	7	:	2
T.N.	3	:	4	9	:	4

26. What was the number of males in U.P. in the year 1998 ?
 (a) 254650 (b) 294840 (c) 321470 (d) 341200
27. What was the total number of illiterate people in A.P. and M.P. in 1998 ?
 (a) 876040 (b) 932170 (c) 981550 (d) 1161160
28. What is the ratio of the number of females in T.N. to the number of females in Delhi ?
 (a) 7 : 5 (b) 9 : 7 (c) 13 : 11 (d) 15 : 14
29. What will be the percentage of total number of males in U.P., M.P. and Goa together to the total population of all the given states ?
 (a) 25% (b) 27.5% (c) 28.5% (d) 31.5%
30. If in the year 1998, there was an increase of 10% in the population of U.P. and 12% in the population of M.P. compared to the previous year, then what was the ratio of populations of U.P. and M.P. in 1997 ?
 (a) 42 : 55 (b) 48 : 55 (c) 7 : 11 (d) 4 : 5

ANSWERS

1. (c) 2. (c) 3. (a) 4. (d) 5. (b) 6. (c) 7. (b) 8. (b) 9. (a)
 10. (d) 11. (a) 12. (d) 13. (c) 14. (b) 15. (a) 16. (c) 17. (c) 18. (b)
 19. (b) 20. (c) 21. (d) 22. (c) 23. (b) 24. (b) 25. (c) 26. (b) 27. (d)
 28. (d) 29. (c) 30. (a)

SOLUTIONS

- Difference = $(35\% \text{ of } 2,50,000) - (33\% \text{ of } 2,50,000)$
 $= (35\% - 33\%) \text{ of } 2,50,000 = 2\% \text{ of } 2,50,000 = 5000.$
- Central angle corresponding to Royalty = $(15\% \text{ of } 360)^\circ = 54^\circ.$
- Central angle of $108^\circ = \left(\frac{108}{360} \times 100\right)\%$ of the total expenditure
 $= 30\% \text{ of the total expenditure.}$
 From the pie-chart it is clear that :
 $\text{Binding Cost} + \text{Transportation Cost} = (20\% + 10\%) \text{ of the total expenditure}$
 $= 30\% \text{ of the total expenditure.}$
 $\therefore \text{Binding Cost and Transportation Cost together have a central angle of } 108^\circ.$
- Central angle of $18^\circ = \left(\frac{18}{360} \times 100\right)\%$ of the total expenditure
 $= 5\% \text{ of the total expenditure.}$
 From the pie-chart it is clear that :
 Out of the given combinations, only in combination (d) the difference is 5% i.e.
 $\text{Paper Cost} - \text{Printing Cost} = (25\% - 20\%) \text{ of total expenditure}$
 $= 5\% \text{ of total expenditure.}$
- Let the Promotion Cost for this edition be Rs. p .
 Then, $25 : 10 = 56250 : p \Rightarrow p = \text{Rs.} \left(\frac{56250 \times 10}{25} \right) = \text{Rs.} 22500.$
- Let the amount of Royalty to be paid for these books be Rs. r .
 Then, $20 : 15 = 30600 : r \Rightarrow r = \text{Rs.} \left(\frac{30600 \times 15}{20} \right) = \text{Rs.} 22950.$
- Clearly, marked price of the book = 120% of C.P.
 Also, cost of paper = 25% of C.P.
 Let the cost of paper for a single book be Rs. n .
 Then, $120 : 25 = 180 : n \Rightarrow n = \text{Rs.} \left(\frac{25 \times 180}{120} \right) = \text{Rs.} 37.50.$
- Clearly, S.P. of the book = 105% of C.P.
 Let the selling price of this edition (of 12500 books) be Rs. x .
 Then, $15 : 105 = 281250 : x \Rightarrow x = \text{Rs.} \left(\frac{105 \times 281250}{15} \right) = \text{Rs.} 1968750.$
 $\therefore \text{S.P. of one book} = \text{Rs.} \left(\frac{1968750}{12500} \right) = \text{Rs.} 157.50.$

9. For the publisher to earn a profit of 25%, S.P. = 125% of C.P.

Also Transportation Cost = 10% of C.P.

Let the S.P. of 5500 books be Rs. x .

$$\text{Then, } 10 : 125 = 82500 : x \Rightarrow x = \text{Rs.} \left(\frac{125 \times 82500}{10} \right) = \text{Rs.} 1031250.$$

$$\therefore \text{S.P. of one book} = \text{Rs.} \left(\frac{1031250}{5500} \right) = \text{Rs.} 187.50.$$

10. Printing Cost of book = 20% of C.P.

Royalty on book = 15% of C.P.

Difference = (20% of C.P.) - (15% of C.P.) = 5% of C.P.

$$\therefore \text{Percentage difference} = \left(\frac{\text{Difference}}{\text{Printing Cost}} \times 100 \right)\% \\ = \left(\frac{5\% \text{ of C.P.}}{20\% \text{ of C.P.}} \times 100 \right)\% = 25\%.$$

Thus, Royalty on the book is 25% less than the Printing Cost.

11. Percentage of money spent on Tennis = $\left(\frac{45}{360} \times 100 \right)\% = 12\frac{1}{2}\%$.

12. Let the total spendings on sports be Rs. x . Then,

$$\text{Amount spent on Golf} = \text{Rs.} \left(\frac{36}{360} \times x \right) = \text{Rs.} \frac{x}{10}.$$

$$\text{Amount spent on Hockey} = \text{Rs.} \left(\frac{63}{360} \times x \right) = \text{Rs.} \frac{7}{40}x.$$

$$\text{Difference} = \text{Rs.} \left(\frac{7}{40}x - \frac{x}{10} \right) = \text{Rs.} \frac{3x}{40}.$$

$$\therefore \text{Required Percentage} = \left[\left(\frac{3x/40}{x/10} \right) \times 100 \right]\% = 75\%.$$

13. Let the total spendings on sports be Rs. x . Then,

$$\text{Amount spent on Cricket} = \text{Rs.} \left(\frac{81}{360} \times x \right) = \text{Rs.} \left(\frac{9}{40}x \right).$$

$$\text{Amount spent on Football} = \text{Rs.} \left(\frac{54}{360} \times x \right) = \text{Rs.} \left(\frac{3}{20}x \right).$$

$$\text{Difference} = \text{Rs.} \left(\frac{9}{40}x - \frac{3}{20}x \right) = \text{Rs.} \frac{3}{40}x.$$

$$\therefore \text{Required Percentage} = \left[\left(\frac{3x/40}{9x/40} \right) \times 100 \right]\% = 33\frac{1}{3}\%.$$

14. Amount spent on Cricket and Hockey together

$$= \text{Rs.} \left[\frac{(81 + 63)}{360} \times 2 \right] \text{crores} = \text{Rs.} 0.8 \text{ crores} = \text{Rs.} 8000000.$$

15. Amount spent on Basketball exceeds that on Tennis by :

$$\text{Rs.} \left[\frac{(50 - 45)}{360} \times 18000000 \right] = \text{Rs.} 250000.$$

16. Required percentage = $\left(\frac{9\% \text{ of } 5700}{8\% \text{ of } 8550} \times 100 \right)\% = \left(\frac{9 \times 5700}{8 \times 8550} \times 100 \right)\% = 75\%.$

17. Required ratio = $\left(\frac{18\% \text{ of } 5700}{22\% \text{ of } 8550} \right) = \left(\frac{18 \times 5700}{22 \times 8550} \right) = \frac{6}{11}$.
18. Candidates passed from institutes Q and R together
 $= [(13\% + 17\%) \text{ of } 5700] = 30\% \text{ of } 5700.$
 Candidates enrolled from institutes Q and R together
 $= [(15\% + 10\%) \text{ of } 8550] = 25\% \text{ of } 8550.$
 $\therefore \text{Required Percentage} = \left(\frac{30\% \text{ of } 5700}{25\% \text{ of } 8550} \times 100 \right)\% = \left(\frac{30 \times 5700}{25 \times 8550} \times 100 \right)\% = 80\%.$
19. The percentage of candidates passed to candidates enrolled can be determined for each institute as under :
- $P = \left[\left(\frac{18\% \text{ of } 5700}{22\% \text{ of } 8550} \right) \times 100 \right]\% = \left[\frac{18 \times 5700}{22 \times 8550} \times 100 \right]\% = \left[\frac{18 \times 2}{22 \times 3} \times 100 \right]\% = 54.55\%.$
 - $Q = \left[\left(\frac{17\% \text{ of } 5700}{15\% \text{ of } 8550} \right) \times 100 \right]\% = 75.56\%.$
 - $R = \left[\left(\frac{13\% \text{ of } 5700}{10\% \text{ of } 8550} \right) \times 100 \right]\% = 86.67\%.$
 - $S = \left[\left(\frac{16\% \text{ of } 5700}{17\% \text{ of } 8550} \right) \times 100 \right]\% = 62.75\%.$
 - $T = \left[\left(\frac{9\% \text{ of } 5700}{8\% \text{ of } 8550} \right) \times 100 \right]\% = 75\%.$
 - $V = \left[\left(\frac{15\% \text{ of } 5700}{12\% \text{ of } 8550} \right) \times 100 \right]\% = 83.33\%.$
 - $X = \left[\left(\frac{12\% \text{ of } 5700}{16\% \text{ of } 8550} \right) \times 100 \right]\% = 50\%.$
- Highest of these is 86.67% corresponding to institute R.
20. Required difference = $[(16\% + 18\%) \text{ of } 5700] - [(8\% + 10\%) \text{ of } 8550]$
 $= [(34\% \text{ of } 5700) - (18\% \text{ of } 8550)] = (1938 - 1539) = 399.$
21. Part of the body made of neither bones nor skin = $1 - \left(\frac{1}{6} + \frac{1}{10} \right) = \frac{11}{15}$.
22. Required ratio = $\frac{16\% \text{ of } \frac{1}{3}}{16\% \text{ of } \frac{1}{6}} = \frac{\frac{1}{3}}{\frac{1}{6}} = \frac{2}{1}$.
23. Quantity of water in the body of a person weighing 50 kg = $(70\% \text{ of } 50) \text{ kg} = 35 \text{ kg}.$
24. Let the body weight be $x \text{ kg}.$
- Then, weight of skin protein in the body = $\left[16\% \text{ of } \left(\frac{1}{10} \text{ of } x \right) \right] \text{ kg} = \left(\frac{16}{1000} x \right) \text{ kg}$
 $\therefore \text{Required percentage} = \left[\left(\frac{\frac{16}{1000} x}{x} \right) \times 100 \right]\% = 1.6\%.$
25. Percentage of proteins and other dry elements in the body = $(16\% + 14\%) = 30\%$
 $\therefore \text{Central angle corresponding to proteins and other dry elements together}$
 $= 30\% \text{ of } 360^\circ = 108^\circ.$

26. Number of males in U.P. = $\left[\frac{3}{5} \text{ of } (15\% \text{ of } 3276000) \right] = \frac{3}{5} \times \frac{15}{100} \times 3276000 = 294840.$

27. No. of illiterate people in A.P. = $\left[\frac{7}{9} \text{ of } (25\% \text{ of } 3276000) \right] = 637000.$

No. of illiterate people in M.P. = $\left[\frac{4}{5} \text{ of } (20\% \text{ of } 3276000) \right] = 524160.$

Total number = $(637000 + 524160) = 1161160.$

28. Required ratio = $\frac{\frac{4}{7} \text{ of } (9\% \text{ of } 3276000)}{\frac{3}{5} \text{ of } (8\% \text{ of } 3276000)} = \frac{\left(\frac{4}{7} \times \frac{9}{100} \right)}{\left(\frac{3}{5} \times \frac{8}{100} \right)} = \left(\frac{4}{7} \times 9 \times \frac{5}{3} \times \frac{1}{8} \right) = \frac{15}{14}.$

29. Number of males in U.P. = $\left[\frac{3}{5} \text{ of } (15\% \text{ of } N) \right] = \frac{3}{5} \times \frac{15}{100} \times N = 9 \times \frac{N}{100}$

where $N = 3276000.$

Number of males in M.P. = $\left[\frac{3}{4} \text{ of } (20\% \text{ of } N) \right] = \frac{3}{4} \times \frac{20}{100} \times N = 15 \times \frac{N}{100}.$

Number of males in Goa = $\left[\frac{3}{8} \text{ of } (12\% \text{ of } N) \right] = \frac{3}{8} \times \frac{12}{100} \times N = 4.5 \times \frac{N}{100}.$

\therefore Total number of males in these three states = $(9 + 15 + 4.5) \times \frac{N}{100} = \left(28.5 \times \frac{N}{100} \right).$

\therefore Required Percentage = $\left[\frac{\left(28.5 \times \frac{N}{100} \right)}{N} \times 100 \right] \% = 28.5\%.$

30. Let x be the population of U.P. in 1997. Then,

$$\text{Population of U.P. in 1998} = 110\% \text{ of } x = \frac{110}{100} \times x.$$

Also, let y be the population of M.P. in 1997. Then,

$$\text{Population of M.P. in 1998} = 112\% \text{ of } y = \frac{112}{100} \times y.$$

$$\text{Ratio of populations of U.P. and M.P. in 1998} = \frac{\left(\frac{110}{100} \times x \right)}{\left(\frac{112}{100} \times y \right)} = \frac{110x}{112y}.$$

From the pie-chart, this ratio is $\frac{15}{20}.$

$$\therefore \frac{110x}{112y} = \frac{15}{20} \Rightarrow \frac{x}{y} = \frac{15}{20} \times \frac{112}{110} = \frac{42}{55}.$$

Thus, ratio of populations of U.P. and M.P. in 1997 = $x : y = 42 : 55.$

39. LINE-GRAPHS

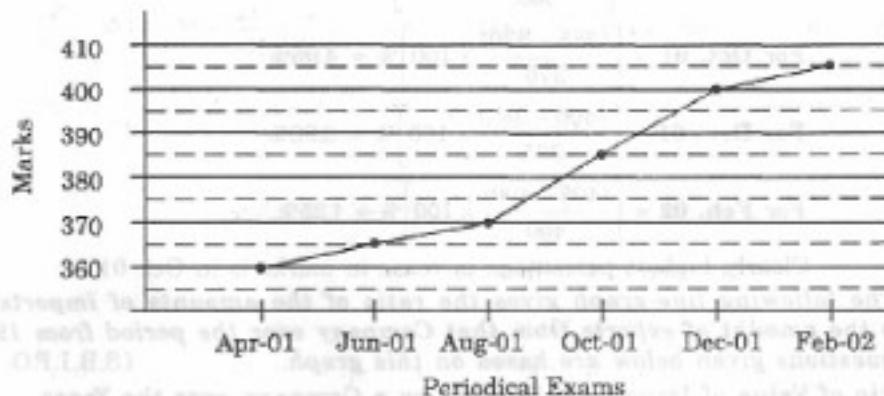
This section comprises of questions in which the data collected in a particular discipline are represented by specific points joined together by straight lines. The points are plotted on a two-dimensional plane taking one parameter on the horizontal axis and the other on the vertical axis. The candidate is required to analyse the given information and thereafter answer the given questions on the basis of the analysis of data.

SOLVED EXAMPLES

Ex. 1. In a school the periodical examinations are held every second month. In a session during Apr. 2001 – Mar. 2002, a student of Class IX appeared for each of the periodical exams. The aggregate marks obtained by him in each periodical exam are represented in the line-graph given below. Study the graph and answer the questions based on it. (S.B.I.P.O. 2003)

MARKS OBTAINED BY A STUDENT IN SIX PERIODICAL EXAMS HELD IN
EVERY TWO MONTHS DURING THE YEAR IN THE SESSION 2001-02

Maximum Total Marks in each Periodical Exam = 500



1. The total number of marks obtained in Feb. 02 is what percent of the total marks obtained in Apr. 01 ?
(a) 110% (b) 112.5% (c) 115% (d) 116.5% (e) 117.5%
 2. What are the average marks obtained by the student in all the periodical exams during the session ?
(a) 373 (b) 379 (c) 381 (d) 385 (e) 389
 3. What is the percentage of marks obtained by the student in the periodical exams of Aug. 01 and Oct. 01 taken together ?
(a) 73.25% (b) 75.5% (c) 77% (d) 78.75% (e) 79.5%
 4. In which periodical exams there is a fall in percentage of marks as compared to the previous periodical exams ?
(a) None (b) Jun. 01 (c) Oct. 01 (d) Feb. 02 (e) None of these
 5. In which periodical exams did the student obtain the highest percentage increase in marks over the previous periodical exams ?
(a) Jun. 01 (b) Aug. 01 (c) Oct. 01 (d) Dec. 01 (e) Feb. 02
- Sol.** Here it is clear from the graph that the student obtained 360, 365, 370, 385, 400 and 405 marks in periodical exams held in Apr. 01, Jun. 01, Aug. 01, Oct. 01, Dec. 01 and Feb. 02 respectively.

1. (b) : Required percentage = $\left(\frac{405}{360} \times 100 \right)\% = 112.5\%$.

2. (c) : Average marks obtained in all the periodical exams

$$= \frac{1}{6} \times [360 + 365 + 370 + 385 + 400 + 405] = 380.83 \approx 381.$$

3. (b) : Required percentage = $\left[\frac{(370 + 385)}{(500 + 500)} \times 100 \right]\% = \left(\frac{755}{1000} \times 100 \right)\% = 75.5\%$.

4. (a) : As is clear from the graph, the total marks obtained in periodical exams go on increasing. Since, the maximum marks for all the periodical exams are same, it implies that the percentage of marks also goes on increasing. Thus, in none of the periodical exams, there is a fall in percentage of marks compared to the previous exam.

5. (c) : Percentage increase in marks in various periodical exams compared to the previous exams are :

$$\text{For Jun. 01} = \left[\frac{(365 - 360)}{360} \times 100 \right]\% = 1.39\%$$

$$\text{For Aug. 01} = \left[\frac{(370 - 365)}{365} \times 100 \right]\% = 1.37\%$$

$$\text{For Oct. 01} = \left[\frac{(385 - 370)}{370} \times 100 \right]\% = 4.05\%$$

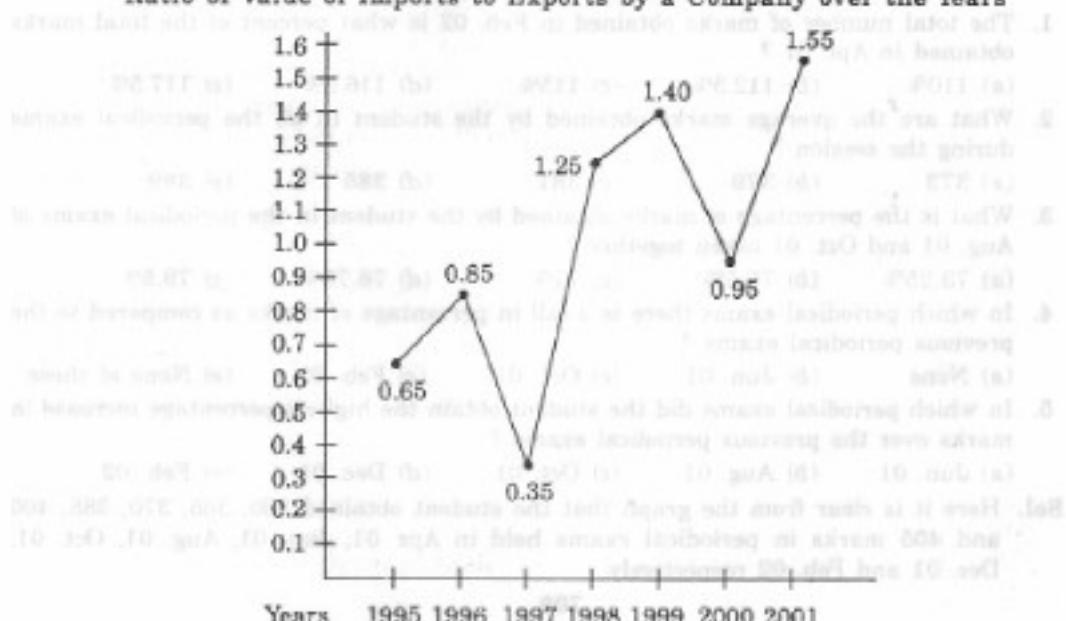
$$\text{For Dec. 01} = \left[\frac{(400 - 385)}{385} \times 100 \right]\% = 3.90\%$$

$$\text{For Feb. 02} = \left[\frac{(405 - 400)}{400} \times 100 \right]\% = 1.25\%$$

Clearly, highest percentage increase in marks is in Oct. 01.

Ex. 2. The following line-graph gives the ratio of the amounts of imports by a Company to the amount of exports from that Company over the period from 1995 to 2001. The questions given below are based on this graph. (S.B.I.P.O. 2001)

Ratio of Value of Imports to Exports by a Company over the Years



1. In how many of the given years were the exports more than the imports ?
 (a) 1 (b) 2 (c) 3 (d) 4 (e) None of these
2. The imports were minimum proportionate to the exports of the Company in the year :
 (a) 1995 (b) 1996 (c) 1997 (d) 2000 (e) 2001
3. If the imports of the Company in 1996 was Rs. 272 crores, the exports from the Company in 1996 was :
 (a) Rs. 370 crores (b) Rs. 320 crores (c) Rs. 280 crores
 (d) Rs. 275 crores (e) Rs. 264 crores
4. What was the percentage increase in imports from 1997 to 1998 ?
 (a) 72 (b) 56 (c) 28 (d) None of these (e) Data inadequate
5. If the imports in 1998 was Rs. 250 crores and the total exports in the years 1998 and 1999 together was Rs. 500 crores, then the imports in 1999 was :
 (a) Rs. 250 crores (b) Rs. 300 crores (c) Rs. 357 crores
 (d) Rs. 420 crores (e) None of these

Sol. 1. (d) : The exports are more than the imports implies that the ratio of value of imports to exports is less than 1.

Now, this ratio is less than 1 in the years 1995, 1996, 1997 and 2000.

Thus, there are four such years.

2. (c) : The imports are minimum proportionate to the exports implies that the ratio of the value of imports to exports has the minimum value.

Now, this ratio has a minimum value of 0.35 in 1997, i.e., the imports are minimum proportionate to the exports in 1997.

3. (b) : Ratio of imports to exports in the year 1996 = 0.85.

Let the exports in 1996 = Rs. x crores.

$$\text{Then, } \frac{272}{x} = 0.85 \Rightarrow x = \frac{272}{0.85} = 320.$$

\therefore Exports in 1996 = Rs. 320 crores.

4. (e) : The graph gives only the ratio of imports to exports for different years. To find the percentage increase in imports from 1997 to 1998, we require more details such as the value of imports or exports during these years. Hence, the data is inadequate to answer this question.

5. (d) : The ratio of imports to exports for the years 1998 and 1999 are 1.25 and 1.40 respectively.

Let the exports in the year 1998 = Rs. x crores.

Then, the exports in the year 1999 = Rs. $(500 - x)$ crores.

$$\therefore 1.25 = \frac{250}{x} \Rightarrow x = \frac{250}{1.25} = 200 \quad [\text{Using ratio for 1998}]$$

Thus, the exports in the year 1999 = Rs. $(500 - 200)$ crores = Rs. 300 crores.

Let the imports in the year 1999 = Rs. y crores.

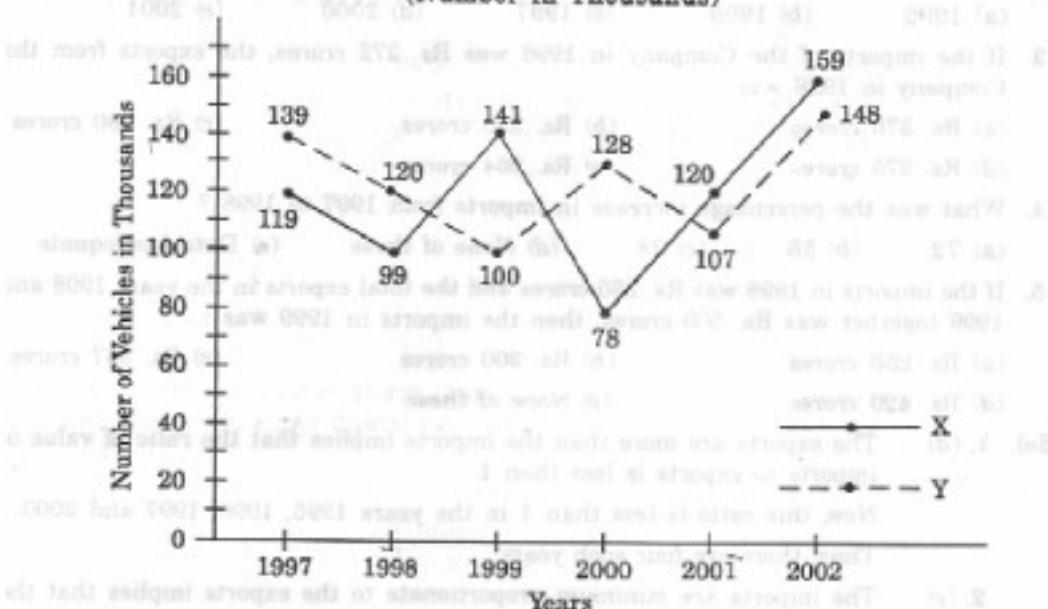
$$\text{Then, } 1.40 = \frac{y}{300} \Rightarrow y = (300 \times 1.40) = 420.$$

\therefore Imports in the year 1999 = Rs. 420 crores.

Ex. 3. Study the following line-graph and answer the questions based on it.

(R.B.I. 2003)

**Number of Vehicles Manufactured by Two Companies over the Years
(Number in Thousands)**



- What is the difference between the total productions of the two Companies in the given years ?
 (a) 19000 (b) 22000 (c) 26000 (d) 28000 (e) 29000
- What is the difference between the numbers of vehicles manufactured by Company Y in 2000 and 2001 ?
 (a) 50000 (b) 42000 (c) 33000 (d) 21000 (e) 13000
- What is the average number of vehicles manufactured by Company X over the given period ? (rounded off to the nearest integer)
 (a) 119333 (b) 113666 (c) 112778 (d) 111223 (e) None of these
- In which of the following years, the difference between the productions of Companies X and Y was the maximum among the given years ?
 (a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001
- The production of Company Y in 2000 was approximately what percent of the production of Company X in the same year ?
 (a) 173 (b) 164 (c) 132 (d) 97 (e) 61

Sol. From the line-graph it is clear that the productions of Company X in the years 1997, 1998, 1999, 2000, 2001 and 2002 are 119000, 99000, 141000, 78000, 120000 and 159000 respectively and those of Company Y are 139000, 120000, 100000, 128000, 107000 and 148000 respectively.

- (c) : Total production of Company X from 1997 to 2002

$$= 119000 + 99000 + 141000 + 78000 + 120000 + 159000 = 716000.$$

and total production of Company Y from 1997 to 2002

$$= 139000 + 120000 + 100000 + 128000 + 107000 + 148000 = 742000.$$

$$\text{Difference} = 742000 - 716000 = 26000.$$

- (d) : Required difference = 128000 - 107000 = 21000.

3. (a) : Average number of vehicles manufactured by Company X =

$$= \frac{1}{6} \times (119000 + 99000 + 141000 + 78000 + 120000 + 159000) = 119333.$$

4. (d) : The difference between the productions of Companies X and Y in various years are :

For 1997 = $(139000 - 119000) = 20000$;

For 1998 = $(120000 - 99000) = 21000$;

For 1999 = $(141000 - 100000) = 41000$;

For 2000 = $(128000 - 78000) = 50000$;

For 2001 = $(120000 - 107000) = 13000$;

For 2002 = $(159000 - 148000) = 11000$.

Clearly, maximum difference was in 2000.

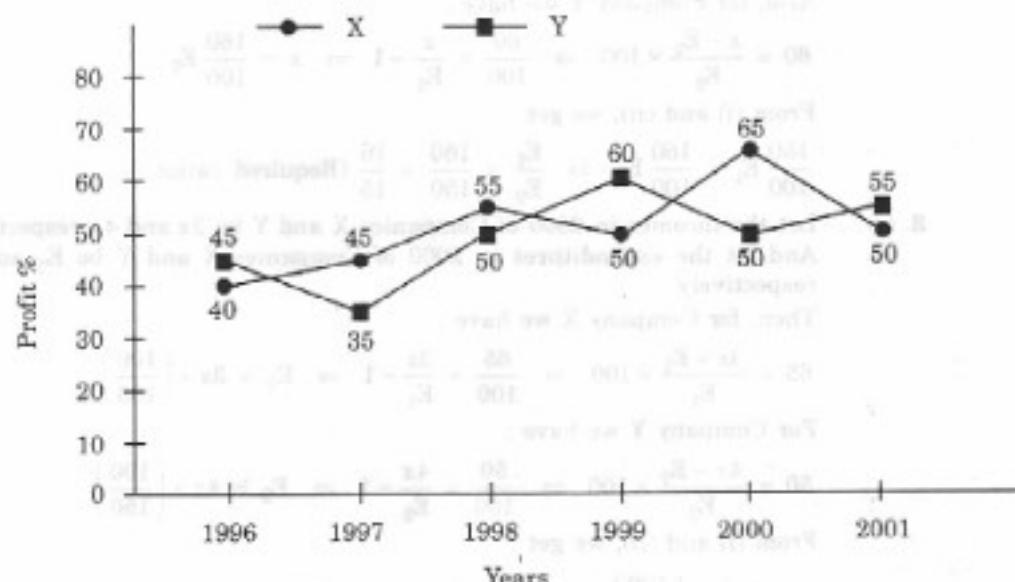
5. (b) : Required percentage = $\left(\frac{128000}{78000} \times 100 \right)\% \approx 164\%$.

Ex. 4. The following line-graph gives the percent profit earned by two Companies X and Y during the period 1996 – 2001. Study the line-graph and answer the questions that are based on it.

(NABARD, 2002)

Percentage Profit Earned by Two Companies X and Y over the Given Years

$$\% \text{ Profit/Loss} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$



- If the expenditure of Company Y in 1997 was Rs. 220 crores, what was its income in 1997 ?
 - Rs. 312 crores
 - Rs. 297 crores
 - Rs. 283 crores
 - Rs. 275 crores
 - Rs. 261 crores
- If the incomes of the two Companies were equal in 1999, then what was the ratio of expenditure of Company X to that of Company Y in 1999 ?
 - 6 : 5
 - 5 : 6
 - 11 : 6
 - 16 : 15
 - 15 : 16
- The incomes of the Companies X and Y in 2000 were in the ratio of 3 : 4 respectively. What was the respective ratio of their expenditures in 2000 ?
 - 7 : 22
 - 14 : 19
 - 15 : 22
 - 27 : 35
 - 33 : 40

4. If the expenditures of Companies X and Y in 1996 were equal and the total income of the two Companies in 1996 was Rs. 342 crores, what was the total profit of the two Companies together in 1996 ? (Profit = Income - Expenditure)
- Rs. 240 crores
 - Rs. 171 crores
 - Rs. 120 crores
 - Rs. 102 crores
 - None of these
5. The expenditure of Company X in the year 1998 was Rs. 200 crores and the income of Company X in 1998 was the same as its expenditure in 2001. The income of Company X in 2001 was :
- Rs. 465 crores
 - Rs. 385 crores
 - Rs. 335 crores
 - Rs. 295 crores
 - Rs. 255 crores

Sol. 1. (b) : Profit percent of Company Y in 1997 = 35.

Let the income of Company Y in 1997 be Rs. x crores.

$$\text{Then, } 35 = \frac{x - 220}{220} \times 100 \Rightarrow x = 297.$$

\therefore Income of Company Y in 1997 = Rs. 297 crores.

2. (d) : Let the incomes of each of the two Companies X and Y in 1999 be Rs. x . And let the expenditures of Companies X and Y in 1999 be E_1 and E_2 respectively.

Then, for Company X we have :

$$50 = \frac{x - E_1}{E_1} \times 100 \Rightarrow \frac{50}{100} = \frac{x}{E_1} - 1 \Rightarrow x = \frac{150}{100} E_1 \quad \dots(i)$$

Also, for Company Y we have :

$$60 = \frac{x - E_2}{E_2} \times 100 \Rightarrow \frac{60}{100} = \frac{x}{E_2} - 1 \Rightarrow x = \frac{160}{100} E_2 \quad \dots(ii)$$

From (i) and (ii), we get :

$$\frac{150}{100} E_1 = \frac{160}{100} E_2 \Rightarrow \frac{E_1}{E_2} = \frac{160}{150} = \frac{16}{15} \text{ (Required ratio).}$$

3. (c) : Let the incomes in 2000 of Companies X and Y be $3x$ and $4x$ respectively. And let the expenditures in 2000 of Companies X and Y be E_1 and E_2 respectively.

Then, for Company X we have :

$$65 = \frac{3x - E_1}{E_1} \times 100 \Rightarrow \frac{65}{100} = \frac{3x}{E_1} - 1 \Rightarrow E_1 = 3x \times \left(\frac{100}{165} \right) \quad \dots(i)$$

For Company Y we have :

$$50 = \frac{4x - E_2}{E_2} \times 100 \Rightarrow \frac{50}{100} = \frac{4x}{E_2} - 1 \Rightarrow E_2 = 4x \times \left(\frac{100}{150} \right) \quad \dots(ii)$$

From (i) and (ii), we get :

$$\frac{E_1}{E_2} = \frac{3x \times \left(\frac{100}{165} \right)}{4x \times \left(\frac{100}{150} \right)} = \frac{3 \times 150}{4 \times 165} = \frac{15}{22} \text{ (Required ratio).}$$

4. (d) : Let the expenditures of each of the Companies X and Y in 1996 be Rs. x crores. And let the income of Company X in 1996 be Rs. z crores so that the income of Company Y in 1996 = Rs. $(342 - z)$ crores.

Then, for Company X we have :

$$40 = \frac{z - x}{x} \times 100 \Rightarrow \frac{40}{100} = \frac{z}{x} - 1 \Rightarrow z = \frac{100z}{140} \quad \dots(i)$$

Also, for Company Y we have :

$$45 = \frac{(342 - z) - x}{x} \times 100 \Rightarrow \frac{45}{100} = \frac{(342 - z)}{x} - 1 \Rightarrow x = \frac{(342 - z) \times 100}{145} \quad \dots(ii)$$

From (i) and (ii), we get : $\frac{100z}{140} = \frac{(342 - z) \times 100}{145} \Rightarrow z = 168$.

Substituting $z = 168$ in (i), we get : $x = 120$.

\therefore Total expenditure of Companies X and Y in 1996 = $2x =$ Rs. 240 crores.

Total income of Companies X and Y in 1996 = Rs. 342 crores.

\therefore Total profit = Rs. $(342 - 240)$ crores = Rs. 102 crores.

5. (a) : Let the income of Company X in 1998 be Rs. x crores.

$$\text{Then, } 55 = \frac{x - 200}{200} \times 100 \Rightarrow x = 310.$$

\therefore Expenditure of Company X in 2001

= Income of Company X in 1998 = Rs. 310 crores.

Let the income of Company X in 2001 be Rs. z crores.

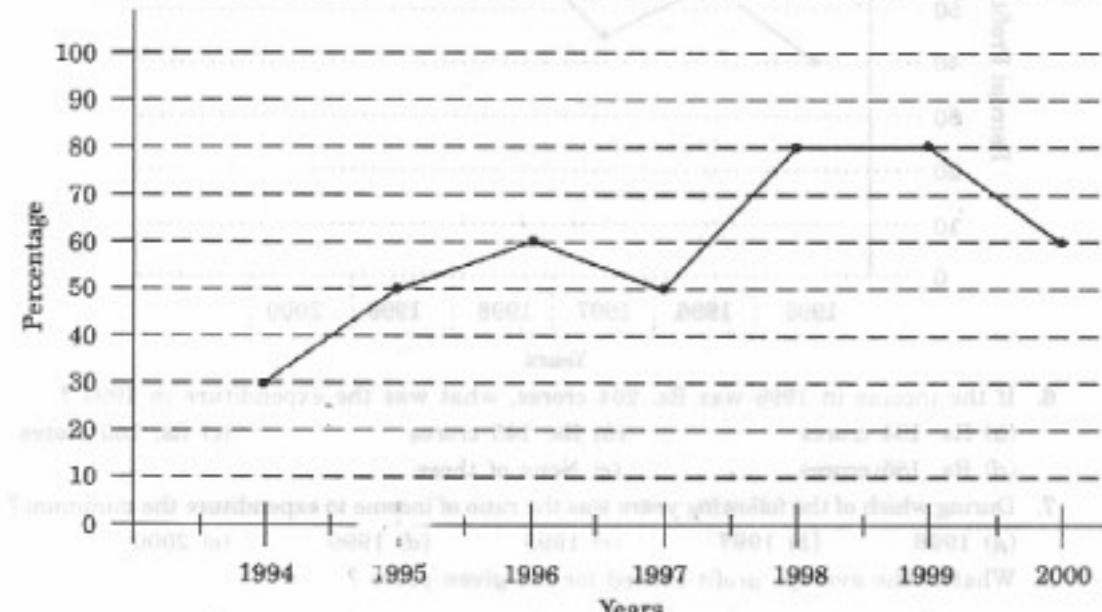
$$\text{Then, } 50 = \frac{z - 310}{310} \times 100 \Rightarrow z = 465.$$

\therefore Income of Company X in 2001 = Rs. 465 crores.

EXERCISE 39

Directions (Questions 1 to 5) : The following line-graph gives the percentage of the number of candidates who qualified an examination out of the total number of candidates who appeared for the examination over a period of seven years from 1994 to 2000. Study the graph and answer the questions based on it. (Bank P.O. 2000)

Percentage of Candidates Qualified to Appeared in an Examination Over the Years



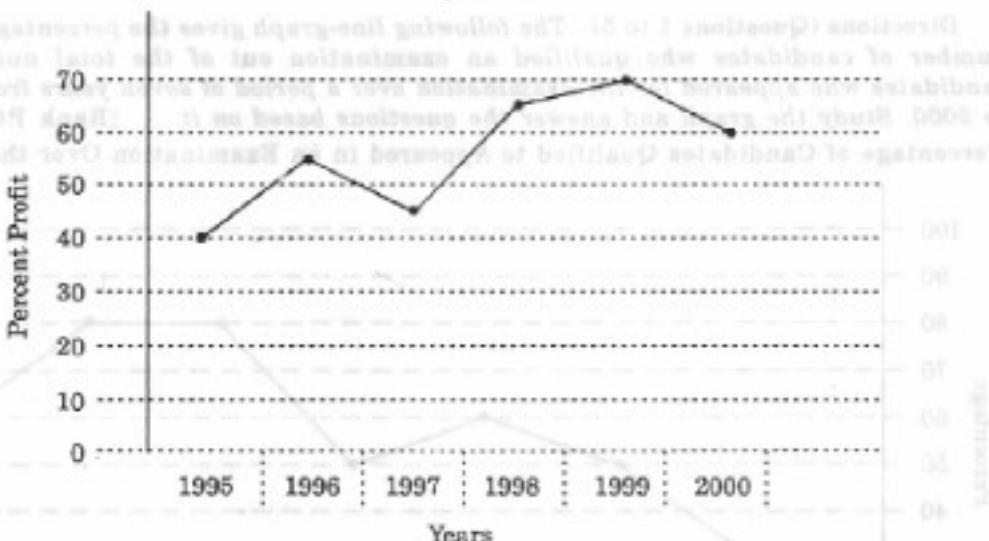
1. The difference between the percentages of candidates qualified to appeared was maximum in which of the following pairs of years ?
 (a) 1994 and 1995 (b) 1997 and 1998 (c) 1998 and 1999
 (d) 1999 and 2000 (e) 1994 and 1997
2. In which pair of years was the number of candidates qualified, the same ?
 (a) 1995 and 1997 (b) 1995 and 2000 (c) 1998 and 1999
 (d) 1996 and 2000 (e) Data inadequate
3. If the number of candidates qualified in 1998 was 21200, what was the number of candidates appeared in 1998 ?
 (a) 32000 (b) 28500 (c) 26500 (d) 25000 (e) 24500
4. If the total number of candidates appeared in 1996 and 1997 together was 47400, then the total number of candidates qualified in these two years together was :
 (a) 34700 (b) 32100 (c) 31500
 (d) None of these (e) Data inadequate
5. The total number of candidates qualified in 1999 and 2000 together was 33500 and the number of candidates appeared in 1999 was 26500. What was the number of candidates appeared in 2000 ?
 (a) 24500 (b) 22000 (c) 20500 (d) 19000 (e) 18500

Directions (Questions 6 to 13) : The following line-graph gives the annual percent profit earned by a Company during the period 1995-2000. Study the line-graph and answer the questions that are based on it.

(R.B.I. 2003)

Percent Profit Earned by a Company Over the Years

$$\% \text{ Profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$



6. If the income in 1998 was Rs. 264 crores, what was the expenditure in 1998 ?
 (a) Rs. 104 crores (b) Rs. 145 crores (c) Rs. 160 crores
 (d) Rs. 185 crores (e) None of these
7. During which of the following years was the ratio of income to expenditure the minimum ?
 (a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
8. What is the average profit earned for the given years ?
 (a) $50\frac{2}{3}$ (b) $55\frac{5}{6}$ (c) $60\frac{1}{6}$ (d) 335 (e) None of these

9. During which year the ratio of percentage profit earned to that in the previous year is the minimum ?
 (a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
10. If the expenditures in 1996 and 1999 are equal, then the approximate ratio of the incomes in 1996 and 1999 respectively, is :
 (a) 1 : 1 (b) 2 : 3 (c) 9 : 10
 (d) 13 : 14 (e) Cannot be determined
11. If the expenditure in 2000 is 25% more than the expenditure in 1997, then the income in 1997 is what percent less than the income in 2000 ?
 (a) 22.5% (b) 25% (c) 27.5% (d) 31.25% (e) 32.5%
12. If the profit in 1999 was Rs. 4 crores, what was the profit in 2000 ?
 (a) Rs. 4.2 crores (b) Rs. 6.6 crores (c) Rs. 6.8 crores
 (d) Cannot be determined (e) None of these
13. In which year is the expenditure minimum ?
 (a) 2000 (b) 1997 (c) 1996
 (d) Cannot be determined (e) None of these

Directions (Questions 14 to 18) : Answer the questions based on the line-graph given below.

(Bank P.O. 2003)

Ratio of Exports to Imports (in terms of money in Rs. crores)
of Two Companies Over the Years

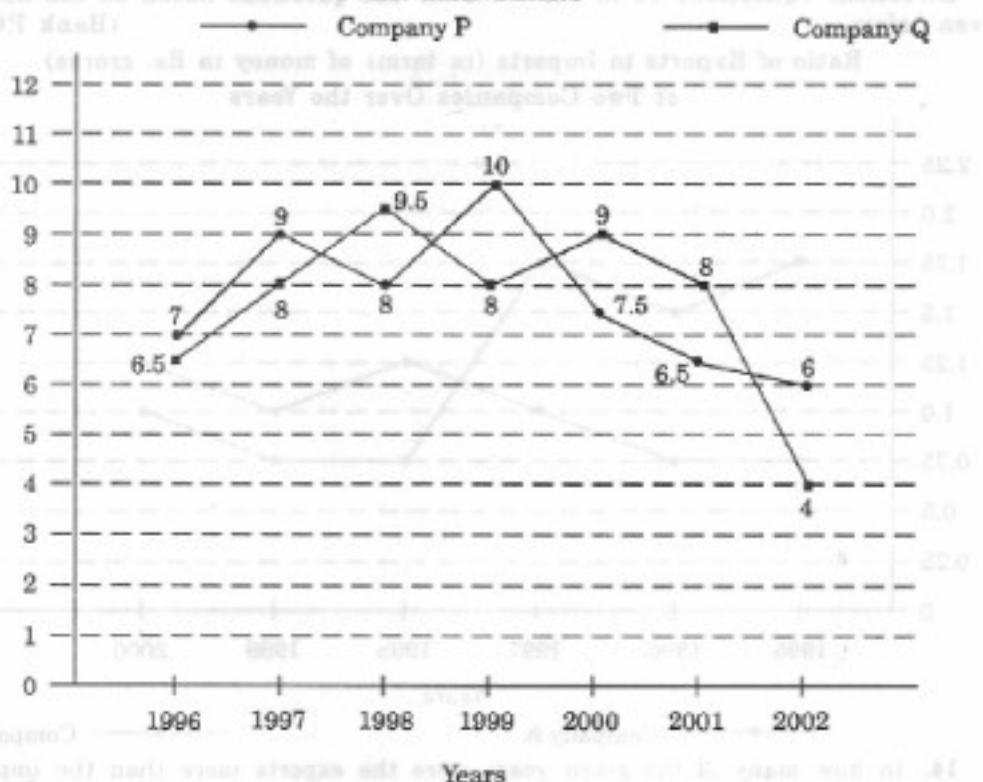


14. In how many of the given years were the exports more than the imports for Company A ?
 (a) 2 (b) 3 (c) 4 (d) 5 (e) 6
15. In which year(s) was the difference between imports and exports of Company B the maximum ?
 (a) 2000 (b) 1996 (c) 1998 and 2000
 (d) Cannot be determined (e) None of these
16. If the exports of Company A in 1998 were Rs. 237 crores, what was the amount of imports in that year ?
 (a) Rs. 189.6 crores (b) Rs. 243 crores (c) Rs. 281 crores
 (d) Rs. 316 crores (e) None of these

17. If the imports of Company A in 1997 were increased by 40 percent, what would be the ratio of exports to the increased imports ?
 (a) 1.20 (b) 1.25 (c) 1.30
 (d) None of these (e) Cannot be determined
18. In 1995, the export of Company A was double that of Company B. If the imports of Company A during the year was Rs. 180 crores, what was the approximate amount of imports of Company B during that year ?
 (a) Rs. 190 crores (b) Rs. 210 crores (c) Rs. 225 crores
 (d) Cannot be determined (e) None of these

Directions (Questions 19 to 23) : Two different finance companies declare fixed annual rate of interest on the amounts invested with them by investors. The rate of interest offered by these companies may differ from year to year depending on the variation in the economy of the country and the banks' rate of interest. The annual rate of interest offered by the two Companies P and Q over the years are shown by the line-graph provided below. Answer the questions based on this graph. (Bank P.O. 2003)

ANNUAL RATE OF INTEREST OFFERED BY TWO FINANCE COMPANIES
OVER THE YEARS



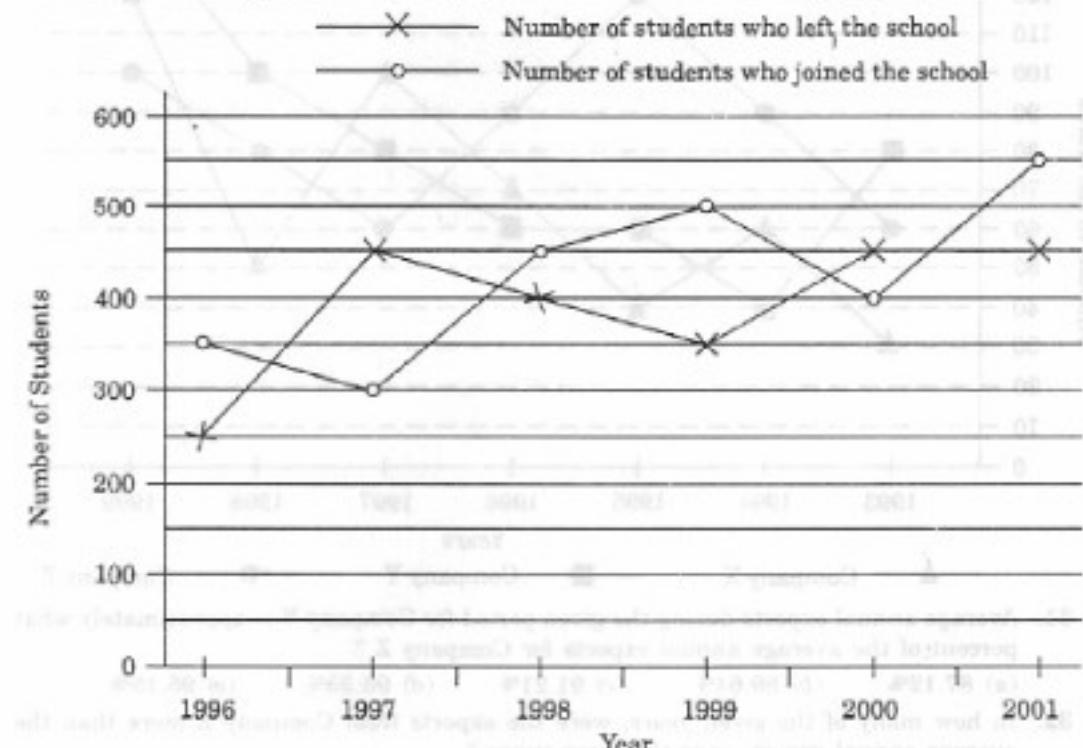
19. If two different amounts in the ratio 8 : 9 are invested in Companies P and Q respectively in 2002, then the amounts received after one year as interests from Companies P and Q are respectively in the ratio :
 (a) 2 : 3 (b) 3 : 4 (c) 6 : 7 (d) 4 : 3 (e) 9 : 8
20. In 2000, a part of Rs. 30 lakhs was invested in Company P and the rest was invested in Company Q for one year. The total interest received was Rs. 2.43 lakhs. What was the amount invested in Company P ?
 (a) Rs. 9 lakhs (b) Rs. 11 lakhs (c) Rs. 12 lakhs
 (d) Rs. 14 lakhs (e) Rs. 18 lakhs

21. A sum of Rs. 4.75 lakhs was invested in Company Q in 1999 for one year. How much more interest would have been earned if the sum was invested in Company P ?
 (a) Rs. 19,000 (b) Rs. 14,250 (c) Rs. 11,750 (d) Rs. 9500 (e) Rs. 7500
22. An investor invested a sum of Rs. 12 lakhs in Company P in 1998. The total amount received after one year was reinvested in the same Company for one more year. The total appreciation received by the investor on his investment was :
 (a) Rs. 2,96,200 (b) Rs. 2,42,000 (c) Rs. 2,25,600
 (d) Rs. 2,16,000 (e) Rs. 2,03,500
23. An investor invested Rs. 5 lakhs in Company Q in 1996. After one year, the entire amount along with the interest was transferred as investment to Company P in 1997 for one year. What amount will be received from Company P, by the investor ?
 (a) Rs. 5,94,550 (b) Rs. 5,80,425 (c) Rs. 5,77,800
 (d) Rs. 5,77,500 (e) Rs. 5,75,075

Directions (Questions 24 to 30) : Study the following line-graph which gives the number of students who joined and left the school in the beginning of year for six years, from 1996 to 2001.

Initial strength of the school in 1995 = 3000.

The questions given below the graph are based on this line-graph. (S.B.I.P.O. 2001)

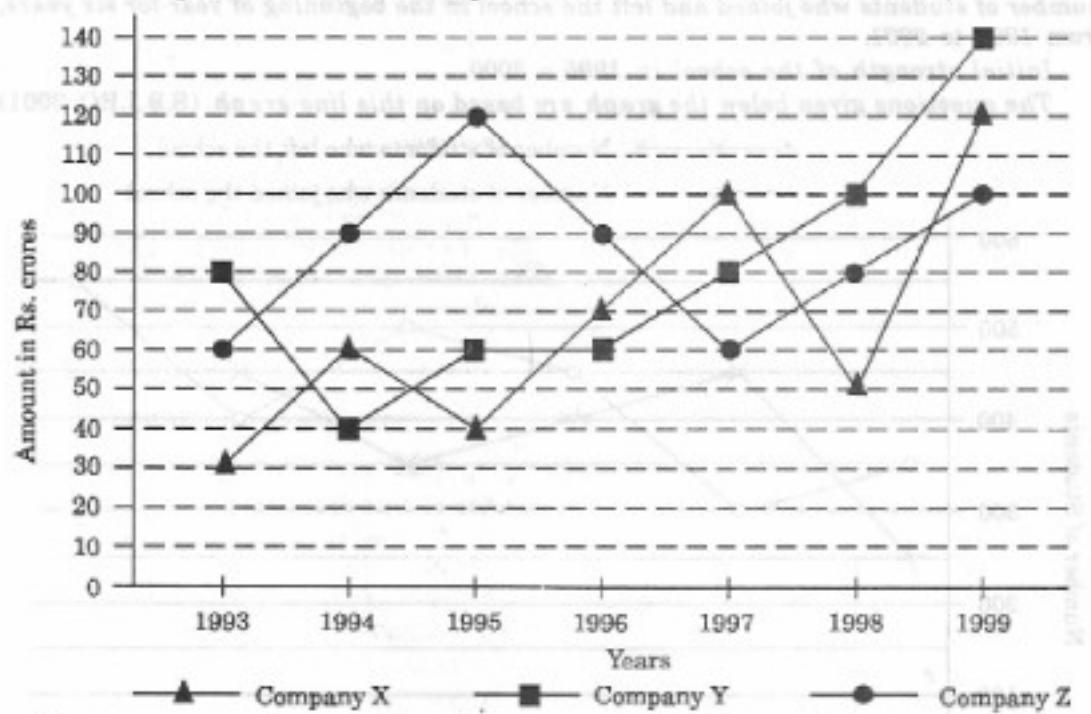


24. The strength of the school increased / decreased from 1997 to 1998 by approximately what percent ?
 (a) 1.2% (b) 1.7% (c) 2.1% (d) 2.4% (e) 2.6%
25. The number of students studying in the school during 1999 was :
 (a) 2950 (b) 3000 (c) 3100 (d) 3150 (e) 3200
26. During which of the following pairs of years, the strength of the school was same ?
 (a) 1999 and 2001 (b) 1998 and 2000 (c) 1997 and 1998
 (d) 1996 and 2000 (e) 1999 and 2000

27. The number of students studying in the school in 1998 was what percent of the number of students studying in the school in 2001 ?
 (a) 92.13% (b) 93.75% (c) 96.88% (d) 97.25% (e) 99%
28. Among the given years, the largest number of students joined the school in the year :
 (a) 1996 (b) 1998 (c) 1999 (d) 2000 (e) 2001
29. For which year, the percentage rise / fall in the number of students who left the school compared to the previous year is maximum ?
 (a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001
30. The ratio of the least number of students who joined the school to the maximum number of students who left the school in any of the years during the given period is :
 (a) 7 : 9 (b) 4 : 5 (c) 3 : 4 (d) 9 : 11 (e) 2 : 3

Directions (Questions 31 to 35) : Study the following graph and answer the questions based on it. (S.B.I.P.O. 2000)

Exports from Three Companies Over the Years (in Crore Rs.)



31. Average annual exports during the given period for Company Y is approximately what percent of the average annual exports for Company Z ?
 (a) 87.12% (b) 89.64% (c) 91.21% (d) 93.33% (e) 95.15%
32. In how many of the given years, were the exports from Company Z more than the average annual exports over the given years ?
 (a) 2 (b) 3 (c) 4 (d) 5 (e) 6
33. What was the difference between the average exports of the three Companies in 1993 and the average exports in 1998 ?
 (a) Rs. 15.33 crores (b) Rs. 18.67 crores (c) Rs. 20 crores
 (d) Rs. 22.17 crores (e) Rs. 25 crores
34. In which year was the difference between the exports from Companies X and Y the minimum ?
 (a) 1994 (b) 1995 (c) 1996 (d) 1997 (e) None of these

35. For which of the following pairs of years the total exports from the three Companies together are equal?
- (a) 1995 and 1998 (b) 1996 and 1998 (c) 1997 and 1998
 (d) 1995 and 1996 (e) 1993 and 1994

ANSWERS

1. (b) 2. (e) 3. (c) 4. (e) 5. (c) 6. (e) 7. (b) 8. (b) 9. (b)
 10. (a) 11. (c) 12. (d) 13. (d) 14. (b) 15. (d) 16. (d) 17. (b) 18. (b)
 19. (d) 20. (e) 21. (d) 22. (c) 23. (b) 24. (b) 25. (d) 26. (d) 27. (b)
 28. (e) 29. (a) 30. (e) 31. (d) 32. (c) 33. (c) 34. (c) 35. (d)

SOLUTIONS

1. The differences between the percentages of candidates qualified to appeared for the given pairs of years are :

For 1994 and 1995 = $50 - 30 = 20$; For 1997 and 1998 = $80 - 50 = 30$;

For 1998 and 1999 = $80 - 80 = 0$; For 1999 and 2000 = $80 - 60 = 20$;

For 1994 and 1997 = $50 - 30 = 20$.

Thus, the maximum difference is between the years 1997 and 1998.

2. The graph gives the data for the percentage of candidates qualified to appeared and unless the absolute values of number of candidates qualified or candidates appeared is known we cannot compare the absolute values for any two years. Hence, the data is inadequate to solve this question.

3. Let the number of candidates appeared in 1998 be x .

$$\text{Then, } 80\% \text{ of } x = 21200 \Rightarrow x = \frac{21200 \times 100}{80} = 26500 \text{ (required number).}$$

4. The total number of candidates qualified in 1996 and 1997 together, cannot be determined until we know at least, the number of candidates appeared in any one of the two years 1996 or 1997 or the percentage of candidates qualified to appeared in 1996 and 1997 together. Hence, the data is inadequate.

5. The number of candidates qualified in 1999 = 80% of 26500 = 21200.

\therefore Number of candidates qualified in 2000 = $33500 - 21200 = 12300$.

Let the number of candidates appeared in 2000 be x .

$$\text{Then, } 60\% \text{ of } x = 12300 \Rightarrow x = \frac{12300 \times 100}{60} = 20500.$$

6. Let the expenditure in 1998 be Rs. x crores.

$$\text{Then, } 65 = \frac{264 - x}{x} \times 100 \Rightarrow \frac{65}{100} = \frac{264}{x} - 1 \Rightarrow x = \frac{264 \times 100}{165} = 160.$$

\therefore Expenditure in 1998 = Rs. 160 crores.

7. It is given that : $\% \text{ Profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$

$$\Rightarrow \frac{\% \text{ Profit}}{100} = \frac{\text{Income}}{\text{Expenditure}} - 1 \Rightarrow \frac{\text{Income}}{\text{Expenditure}} = \frac{\% \text{ Profit}}{100} + 1$$

From this it is clear that the ratio of income to expenditure is minimum for the year in which the % profit has the minimum value. Since, out of the given years (i.e., out of 1996, 1997, 1998, 1999 and 2000), the Company has the minimum % profit in the year 1997, so the minimum ratio of income to expenditure is in the year 1997.

8. Average percent profit earned for the given years

$$= \frac{1}{6} \times [40 + 55 + 45 + 65 + 70 + 60] = \frac{335}{6} = 55\frac{5}{6}\%$$

9. The ratio of percentage profit earned to that in the previous year, for different years are :

$$\text{For } 1996 = \frac{55}{40} = 1.38; \quad \text{For } 1997 = \frac{45}{55} = 0.82; \quad \text{For } 1998 = \frac{65}{45} = 1.44;$$

$$\text{For } 1999 = \frac{70}{65} = 1.08; \quad \text{For } 2000 = \frac{60}{70} = 0.86.$$

Clearly, this ratio is minimum for 1997.

10. Let the expenditure in 1996 = expenditure in 1999 = x

Also, let the incomes in 1996 and 1999 be I_1 and I_2 respectively.

Then, for the year 1996, we have :

$$55 = \frac{I_1 - x}{x} \times 100 \Rightarrow \frac{55}{100} = \frac{I_1}{x} - 1 \Rightarrow I_1 = \frac{155x}{100} \quad \dots(i)$$

And, for the year 1999, we have :

$$70 = \frac{I_2 - x}{x} \times 100 \Rightarrow \frac{70}{100} = \frac{I_2}{x} - 1 \Rightarrow I_2 = \frac{170x}{100} \quad \dots(ii)$$

From (i) and (ii), we get :

$$\frac{I_1}{I_2} = \frac{\left(\frac{155x}{100}\right)}{\left(\frac{170x}{100}\right)} = \frac{155}{170} = \frac{0.91}{1} = 9 : 10.$$

11. Let the expenditure in 1997 be x .

$$\text{Then, expenditure in } 2000 = x + (\text{25% of } x) = \frac{5}{4}x.$$

Also, let the incomes in 1997 and 2000 be I_1 and I_2 respectively.

Then, for the year 1997, we have :

$$45 = \frac{I_1 - x}{x} \times 100 \Rightarrow \frac{45}{100} = \frac{I_1}{x} - 1 \Rightarrow I_1 = \frac{145x}{100} = 1.45x.$$

Also, for the year 2000, we have :

$$60 = \frac{\left(I_2 - \frac{5}{4}x\right)}{\left(\frac{5}{4}x\right)} \times 100 \Rightarrow \frac{60}{100} = \frac{4I_2}{5x} - 1 \Rightarrow I_2 = \frac{160}{100} \times \frac{5x}{4} = 2x.$$

Difference between the two incomes = $(2x - 1.45x) = 0.55x$.

$$\therefore \text{Percentage by which } I_1 \text{ is less than } I_2 = \left(\frac{0.55x}{2x} \times 100 \right)\% = 27.5\%.$$

12. From the line-graph we obtain information about the percentage profit only. To find the profit in 2000 we must have the data for the income or expenditure in 2000. Therefore, the profit for 2000 cannot be determined.

13. The line-graph gives the comparison of percent profit for different years but the comparison of the expenditures is not possible without more data. Therefore, the year with minimum expenditure cannot be determined.

14. The exports are more than the imports in those years for which the exports to imports ratio is more than 1. For Company A, such years are 1995, 1996 and 1997. Thus, during these 3 years, the exports are more than the imports for Company A.

15. We shall try to find the difference between the imports and exports of Company B for various years one by one :

For 1995 : We have

$$\frac{E}{I} = 0.75 \text{ (where } E = \text{amount of exports and } I = \text{amount of imports in 1995})$$

$$\rightarrow E = 0.75I \quad \therefore I - E = I - 0.75I = 0.25I.$$

Thus, the difference between the imports and exports of Company B in 1995 is dependent on the amount of imports of Company B in 1995.

Similarly, the difference for other years can be determined only if the amount of imports for these years are known. Since the imports or exports for various years are not known, the differences between imports and exports for various years cannot be determined.

16. Let the amount of imports of Company A in 1998 be Rs. x crores.

$$\text{Then, } \frac{237}{x} = 0.75 \Rightarrow x = \frac{237}{0.75} = 316.$$

\therefore Amount of imports of Company A in 1998 = Rs. 316 crores.

17. In 1997 for Company A we have :

$$\frac{E}{I} = 1.75 \text{ i.e., } E = 1.75I \quad \dots(i)$$

[where E = amount of exports and I = amount of imports of Company A in 1997]

Now, the required imports $I_1 = I + 40\% \text{ of } I = 1.4I$.

$$\therefore \text{Required ratio} = \frac{E}{I_1} = \frac{1.75I}{1.4I} = 1.25.$$

18. In 1995 for Company A we have :

$$\frac{E_A}{I_A} = 1.75 \quad \dots(ii) \quad [\text{where } E_A = \text{amount of exports and}]$$

I_A = amount of imports of Company A in 1995]

In 1995 for Company B we have :

$$\frac{E_B}{I_B} = 0.75 \quad \dots(iii) \quad [\text{where } E_B = \text{amount of exports and}]$$

I_B = amount of imports of Company B in 1995]

Also, we have $E_A = 2E_B \quad \dots(iv)$

Substituting $I_A = \text{Rs. } 180$ crores (given) in (i), we get

$$E_A = \text{Rs. } (180 \times 1.75) \text{ crores} = \text{Rs. } 315 \text{ crores.}$$

Using $E_A = \text{Rs. } 315$ crores in (iv), we get : $E_B = \frac{E_A}{2} = \text{Rs. } \left(\frac{315}{2}\right)$ crores.

Substituting $E_B = \text{Rs. } \left(\frac{315}{2}\right)$ crores in (iii), we get :

$$I_B = \frac{E_B}{0.75} = \text{Rs. } \left(\frac{315}{2 \times 0.75}\right) \text{ crores} = \text{Rs. } 210 \text{ crores.}$$

i.e., amount of imports of Company B in 1995 = Rs. 210 crores.

19. Let the amounts invested in 2002 in Companies P and Q be Rs. $8x$ and Rs. $9x$ respectively.

Then, interest received after one year from Company P

$$= \text{Rs. } (6\% \text{ of } 8x) = \text{Rs. } \frac{48}{100}x$$

and interest received after one year from Company Q

$$= \text{Rs. (4\% of } 9x) = \text{Rs. } \frac{36}{100} x.$$

$$\therefore \text{Required ratio} = \frac{\left(\frac{48}{100} x\right)}{\left(\frac{36}{100} x\right)} = \frac{4}{3}$$

20. Let Rs. x lakhs be invested in Company P in 2000, then amount invested in Company Q in 2000 = Rs. $(30 - x)$ lakhs.

Total interest received from the two Companies after 1 year

$$= \text{Rs. } [(7.5\% \text{ of } x) + (9\% \text{ of } (30 - x))] \text{ lakhs} = \text{Rs. } \left[2.7 - \left(\frac{1.5x}{100} \right) \right] \text{ lakhs.}$$

$$\therefore \left[2.7 - \left(\frac{1.5x}{100} \right) \right] = 2.43 \Rightarrow x = 18.$$

i.e., amount invested in Company P = Rs. 18 lakhs.

21. Difference = Rs. $[(10\% \text{ of } 4.75) - (8\% \text{ of } 4.75)]$ lakhs

$$= \text{Rs. (2\% of } 4.75) \text{ lakhs} = \text{Rs. } 0.095 \text{ lakhs} = \text{Rs. } 9500.$$

22. Amount received from Company P after one year (i.e., in 1999) on investing Rs. 12 lakhs in it = Rs. $[12 + (8\% \text{ of } 12)]$ lakhs = Rs. 12.96 lakhs.

Amount received from Company P after one year on investing Rs. 12.96 lakhs in the year 1999 = Rs. $[12.96 + (10\% \text{ of } 12.96)]$ lakhs = Rs. 14.256 lakhs.

Appreciation received on investment during the period of two years

$$= \text{Rs. } (14.256 - 12) \text{ lakhs} = \text{Rs. } 2.256 \text{ lakhs} = \text{Rs. } 2,25,600.$$

23. Amount received from Company Q after one year on investment of Rs. 5 lakhs in the year 1996 = Rs. $[5 + (6.5\% \text{ of } 5)]$ lakhs = Rs. 5.325 lakhs.

Amount received from Company P after one year on investment of Rs. 5.325 lakhs in the year 1997 = Rs. $[5.325 + (9\% \text{ of } 5.325)]$ lakhs = Rs. 5.80425 lakhs = Rs. 5,80,425.

Questions 24 to 30 :

Before solving the questions, we shall analyse the graph :

From the graph it is clear that :

In 1996 : Number of students left = 250 and number of students joined = 350.

In 1997 : Number of students left = 450 and number of students joined = 300.

In 1998 : Number of students left = 400 and number of students joined = 450.

In 1999 : Number of students left = 350 and number of students joined = 500.

In 2000 : Number of students left = 450 and number of students joined = 400.

In 2001 : Number of students left = 450 and number of students joined = 550.

Therefore, the numbers of students studying in the school (i.e., strength of the school) in various years :

In 1995 = 3000 (given); In 1996 = 3000 - 250 + 350 = 3100;

In 1997 = 3100 - 450 + 300 = 2950; In 1998 = 2950 - 400 + 450 = 3000;

In 1999 = 3000 - 350 + 500 = 3150; In 2000 = 3150 - 450 + 400 = 3100;

In 2001 = 3100 - 450 + 550 = 3200.

Now, we shall solve the questions.

24. Percentage increase in the strength of the school from 1997 to 1998

$$= \left[\frac{(3000 - 2950)}{2950} \times 100 \right] \% = 1.69\% = 1.7\%.$$

25. As calculated above, the number of students studying in the school during 1999 = 3150.
26. As calculated above, in the years 1996 and 2000 the strength of the school was same i.e., 3100.
27. Using the calculations above we have :

$$\text{Required percentage} = \left(\frac{3000}{3200} \times 100 \right) \% = 93.75\%.$$

28. As calculated above, the largest number of students (i.e., 550) joined the school in the year 2001.
29. The percentage rise/fall in the number of students who left the school (compared to the previous year) during various years are :

$$\text{For 1997} = \left[\frac{(450 - 250)}{250} \times 100 \right] \% = 80\% \text{ (rise);}$$

$$\text{For 1998} = \left[\frac{(450 - 400)}{450} \times 100 \right] \% = 11.11\% \text{ (fall);}$$

$$\text{For 1999} = \left[\frac{(400 - 350)}{400} \times 100 \right] \% = 12.5\% \text{ (fall);}$$

$$\text{For 2000} = \left[\frac{(450 - 350)}{350} \times 100 \right] \% = 28.57\% \text{ (rise);}$$

$$\text{For 2001} = \left[\frac{(450 - 450)}{450} \times 100 \right] \% = 0\%.$$

Clearly, the maximum percentage rise/fall is for 1997.

30. Using the calculations above we get :

$$\text{Required ratio} = \frac{300}{450} = \frac{2}{3}.$$

Questions 31 to 35 :

Analysis of the graph : From the graph it is clear that

- The amount of exports of Company X (in crore Rs.) in the years 1993, 1994, 1995, 1996, 1997, 1998 and 1999 are 30, 60, 40, 70, 100, 50 and 120 respectively.
- The amount of exports of Company Y (in crore Rs.) in the years 1993, 1994, 1995, 1996, 1997, 1998 and 1999 are 80, 40, 60, 60, 80, 100 and 140 respectively.
- The amount of exports of Company Z (in crore Rs.) in the years 1993, 1994, 1995, 1996, 1997, 1998 and 1999 are 60, 90, 120, 90, 60, 80 and 100 respectively.

31. Average annual exports (in Rs. crore) of Company Y during the given period

$$= \frac{1}{7} \times (80 + 40 + 60 + 60 + 80 + 100 + 140) = \frac{560}{7} = 80.$$

Average annual exports (in Rs. crore) of Company Z during the given period

$$= \frac{1}{7} \times (60 + 90 + 120 + 90 + 60 + 80 + 100) = \left(\frac{600}{7} \right).$$

$$\therefore \text{Required percentage} = \left[\left(\frac{\frac{80}{600}}{7} \times 100 \right) \right] \% = 93.33\%.$$

32. Average annual exports of Company Z during the given period

$$= \text{Rs.} \left[\frac{1}{7} \times (60 + 90 + 120 + 90 + 60 + 80 + 100) \right] \text{crores} = \text{Rs.} \left(\frac{600}{7} \right) \text{crores}$$

$$= \text{Rs.} 85.71 \text{ crores.}$$

From the analysis of graph the exports of Company Z are more than the average annual exports of Company Z (i.e., Rs. 85.71 crores) during the years 1994, 1995, 1996 and 1999, i.e., during 4 of the given years.

33. Average exports of the three Companies X, Y and Z in 1993

$$= \text{Rs.} \left[\frac{1}{3} \times (30 + 80 + 60) \right] \text{crores} = \text{Rs.} \left(\frac{170}{3} \right) \text{crores.}$$

Average exports of the three Companies X, Y and Z in 1998

$$= \text{Rs.} \left[\frac{1}{3} \times (50 + 100 + 80) \right] \text{crores} = \text{Rs.} \left(\frac{230}{3} \right) \text{crores.}$$

$$\text{Difference} = \text{Rs.} \left[\left(\frac{230}{3} \right) - \left(\frac{170}{3} \right) \right] \text{crores} = \text{Rs.} \left(\frac{60}{3} \right) \text{crores} = \text{Rs.} 20 \text{ crores.}$$

34. The differences between the exports from the Companies X and Y during various years are :

In 1993 = Rs. (80 – 30) crores = Rs. 50 crores;

In 1994 = Rs. (60 – 40) crores = Rs. 20 crores;

In 1995 = Rs. (60 – 40) crores = Rs. 20 crores;

In 1996 = Rs. (70 – 60) crores = Rs. 10 crores;

In 1997 = Rs. (100 – 80) crores = Rs. 20 crores;

In 1998 = Rs. (100 – 50) crores = Rs. 50 crores;

In 1999 = Rs. (140 – 120) crores = Rs. 20 crores.

Clearly, the difference is minimum in the year 1996.

35. Total exports of the three Companies X, Y and Z together, during various years are :

In 1993 = Rs. (30 + 80 + 60) crores = Rs. 170 crores.

In 1994 = Rs. (60 + 40 + 90) crores = Rs. 190 crores.

In 1995 = Rs. (40 + 60 + 120) crores = Rs. 220 crores.

In 1996 = Rs. (70 + 60 + 90) crores = Rs. 220 crores.

In 1997 = Rs. (100 + 80 + 60) crores = Rs. 240 crores.

In 1998 = Rs. (50 + 100 + 80) crores = Rs. 230 crores.

In 1999 = Rs. (120 + 140 + 100) crores = Rs. 360 crores.

Clearly, the total exports of the three Companies X, Y and Z together are same during the years 1995 and 1996.