5. Speed $= \left(5 \times \frac{5}{18}\right)$ m/sec $= \frac{25}{18}$ m/sec.

Distance covered in 15 minutes $= \left(\frac{25}{18} \times 15 \times 60\right)$ m $= 1250$ m.

6. Speed $= 9$ km/hr $= \left(9 \times \frac{5}{18}\right)$ m/sec $= \frac{5}{2}$ m/sec.

Distance $= (35 \times 4)$ m $= 140$ m.

$\therefore$ Time taken $= \left(140 \times \frac{2}{5}\right)$ sec $= 56$ sec.

7. Speed $= 108$ kmph $= \left(108 \times \frac{5}{18}\right)$ m/sec $= 30$ m/sec.

$\therefore$ Distance covered in 15 sec. $= (30 \times 15)$ m $= 450$ m.

8. Ratio of speeds $= \left(300 \times \frac{2}{15}\right) : \left(\frac{450}{9}\right) = 40 : 50 = 4 : 5.$

9. Ratio of speeds $= \left(\frac{550}{60} \times \frac{18}{5}\right) : \left(\frac{33}{45} \times 60\right) = 33 : 44 = 3 : 4.$

10. Let the speeds of two trains be $7x$ and $8x$ km/hr.

Then, $8x = \frac{400}{4} = 100 \Rightarrow x = \left(\frac{100}{8}\right) = 12.5.$

$\therefore$ Speed of first train $= (7 \times 12.5)$ km/hr $= 87.5$ km/hr.

11. Total distance travelled $= \left[\left(50 \times 2\frac{1}{2}\right) + \left(70 \times 1\frac{1}{2}\right)\right]$ miles $= (125 + 105)$ miles $= 230$ miles.

12. Number of gaps between 21 telephone posts $= 20$.

Distance travelled in 1 minute $= (50 \times 20)$ m $= 1000$ m $= 1$ km.

$\therefore$ Speed $= 60$ km/hr.

13. Distance $= \left(1100 \times \frac{11}{5}\right)$ feet $= 2420$ feet.

14. Time taken to cover 600 km $= \left(\frac{600}{100}\right)$ hrs $= 6$ hrs.

Number of stoppages $= \frac{600}{75} - 1 = 7.$

Total time of stoppage $= (3 \times 7)$ min $= 21$ min.

Hence, total time taken $= 6$ hrs 21 min.

15. Let the distance covered by the cyclist be x and the time taken be y. Then,

Required ratio $= \frac{\frac{1}{2}x}{2y} : \frac{x}{y} = \frac{1}{4} : 1 = 1 : 4.$

16. Distance covered in first 2 hours $= (70 \times 2)$ km $= 140$ km.

Distance covered in next 2 hours $= (80 \times 2)$ km $= 160$ km.

Remaining distance $= 345 - (140 + 160) = 45$ km.

Speed in the fifth hour $= 90$ km/hr.

Time taken to cover 45 km $= \left(\frac{45}{90}\right)$ hr $= \frac{1}{2}$ hr.

$\therefore$ Total time taken $= \left(2 + 2 + \frac{1}{2}\right) = 4\frac{1}{2}$ hrs.

17. Total distance travelled in 12 hours = (35 + 37 + 39 + ... upto 12 terms).
This is an A.P. with first term, $a = 35$, number of terms, $n = 12$, common difference, $d = 2$.

$\therefore$ Required distance $= \frac{12}{2}[2\times 35 + (12-1)\times 2] = 6(70+22) = 552$ km.

18. Speed $= \left(10\times\frac{60}{12}\right)$ km/hr $= 50$ km/hr.

New speed $= (50-5)$ km/hr $= 45$ km/hr.

$\therefore$ Time taken $= \left(\frac{10}{45}\right)$ hr $= \left(\frac{2}{9}\times 60\right)$ min $= 13\frac{1}{3}$ min $= 13$ min 20 sec.

19. Distance covered in 2 hrs 15 min *i.e.*, $2\frac{1}{4}$ hrs $= \left(80\times\frac{9}{4}\right)$ hrs $= 180$ hrs.

Time taken to cover remaining distance $= \left(\frac{350-180}{60}\right)$ hrs $= \frac{17}{6}$ hrs

$= 2\frac{5}{6}$ hrs $= 2$ hrs 50 min.

Total time taken = (2 hrs 15 min + 2 hrs 50 min) = 5 hrs 5 min.

So, Anna reached city A at 10.25 a.m.

20. Distance = (240×5) km = 1200 km.

$\therefore$ Required speed $= \left(1200\times\frac{3}{5}\right)$ km/hr $= 720$ km/hr.

21. Time required = (2 hrs 30 min − 50 min) = 1 hr 40 min $= 1\frac{2}{3}$ hrs.

$\therefore$ Required speed $= \left(50\times\frac{3}{5}\right)$ km/hr $= 30$ km/hr.

Original speed $= \left(50\times\frac{2}{5}\right)$ km/hr $= 20$ km/hr.

$\therefore$ Difference in speed = (30 − 20) km/hr = 10 km/hr.

22. Remaining distance = 3 km and Remaining time $= \left(\frac{1}{3}\times 45\right)$ min $= 15$ min $= \frac{1}{4}$ hour.

$\therefore$ Required speed = (3×4) km/hr = 12 km/hr.

23. Let the total journey be x km.

Then, $\frac{3x}{5}+\frac{7x}{20}+6.5 = x \Leftrightarrow 12x+7x+20\times 6.5 = 20x \Leftrightarrow x = 130$ km.

24. Let the total distance be x km. Then,

$$\frac{\frac{1}{2}x}{21}+\frac{\frac{1}{2}x}{24} = 10 \Rightarrow \frac{x}{21}+\frac{x}{24} = 20$$

$$\Rightarrow 15x = 168\times 20 \Rightarrow x = \left(\frac{168\times 20}{15}\right) = 224 \text{ km.}$$

25. Let the total distance be $3x$ km.

Then, $\frac{x}{3}+\frac{x}{4}+\frac{x}{5} = \frac{47}{60} \Leftrightarrow \frac{47x}{60} = \frac{47}{60} \Leftrightarrow x = 1$.

$\therefore$ Total distance = (3×1) km = 3 km.

26. Let the distance travelled on foot be x km.

Then, distance travelled on bicycle $= (61 - x)$ km.

So, $\frac{x}{4} + \frac{(61-x)}{9} = 9 \Leftrightarrow 9x + 4(61 - x) = 9 \times 36 \Leftrightarrow 5x = 80 \Leftrightarrow x = 16$ km.

27. Let A's speed $= x$ km/hr. Then, B's speed $= (7 - x)$ km/hr.

So, $\frac{24}{x} + \frac{24}{(7-x)} = 14 \Leftrightarrow 24(7 - x) + 24x = 14x(7 - x)$

$\Leftrightarrow 14x^2 - 98x + 168 = 0 \Leftrightarrow x^2 - 7x + 12 = 0$

$\Leftrightarrow (x - 3)(x - 4) = 0 \Leftrightarrow x = 3$ or $x = 4$.

Since, A is faster than B, so A's speed $= 4$ km/hr and B's speed $= 3$ km/hr.

28. Speed on return trip $= 150\%$ of $40 = 60$ kmph.

$\therefore$ Average speed $= \left(\frac{2 \times 40 \times 60}{40 + 60}\right)$ km/hr $= \left(\frac{4800}{100}\right)$ km/hr $= 48$ km/hr.

29. Average speed $= \left(\frac{2 \times 40 \times 20}{40 + 60}\right)$ km/hr $= \left(\frac{80}{3}\right)$ km/hr $= 26.67$ km/hr.

30. Speed from A to B $= \left(250 \times \frac{2}{11}\right)$ mph $= \left(\frac{500}{11}\right)$ mph.

Speed from B to A $= \left(250 \times \frac{2}{9}\right)$ mph $= \left(\frac{500}{9}\right)$ mph.

$\therefore$ Average speed $= \left(\frac{2 \times \frac{500}{11} \times \frac{500}{9}}{\frac{500}{11} + \frac{500}{9}}\right)$ mph $= \left(\frac{500000}{4500 + 5500}\right)$ mph $= 50$ mph.

31. Average speed $= \left(\frac{2 \times 3 \times 2}{3 + 2}\right)$ km/hr $= \frac{12}{5}$ km/hr.

Distance travelled $= \left(\frac{12}{5} \times 5\right)$ km $= 12$ km.

$\therefore$ Distance between house and school $= \left(\frac{12}{2}\right)$ km $= 6$ km.

32. Let the speed in return journey be x km/hr.

Then, speed in onward journey $= \frac{125}{100}x = \left(\frac{5}{4}x\right)$ km/hr.

Average speed $= \left(\frac{2 \times \frac{5}{4}x \times x}{\frac{5}{4}x + x}\right)$ km/hr $= \frac{10x}{9}$ km/hr.

$\therefore \left(800 \times \frac{9}{10x}\right) = 16 \Leftrightarrow x = \left(\frac{800 \times 9}{16 \times 10}\right) = 45.$

So, speed in onward journey $= \left(\frac{5}{4} \times 45\right)$ km/hr $= 56.25$ km/hr.

33. Time taken $= 5$ hrs 25 min $= \frac{65}{12}$ hrs.

Let the required distance be x km.

Then, $\frac{x}{10} + \frac{x}{1} = \frac{65}{12} \Leftrightarrow 11x = \frac{650}{12} \Leftrightarrow x = \frac{325}{66} = 4\frac{61}{66}$ km.

34. Total distance travelled = $(50 \times 1 + 48 \times 2 + 52 \times 3)$ km = 302 km.
Total time taken = 6 hrs.

$\therefore$ Mean speed = $\left(\frac{302}{6}\right)$ km/hr = $50\frac{1}{3}$ km/hr.

35. Total time taken = $\left(\frac{160}{64} + \frac{160}{8}\right)$ hrs = $\frac{9}{2}$ hrs.

$\therefore$ Average speed = $\left(320 \times \frac{2}{9}\right)$ km/hr = 71.11 km/hr.

36. Total distance travelled = (10 + 12) km/hr = 22 km/hr.

Total time taken = $\left(\frac{10}{12} + \frac{12}{10}\right)$ hrs = $\frac{61}{30}$ hrs.

$\therefore$ Average speed = $\left(22 \times \frac{30}{61}\right)$ km/hr = 10.8 km/hr.

37. Total distance travelled = (600 + 800 + 500 + 100) km = 2000 km.

Total time taken = $\left(\frac{600}{80} + \frac{800}{40} + \frac{500}{400} + \frac{100}{50}\right)$ hrs = $\frac{123}{4}$ hrs.

$\therefore$ Average speed = $\left(2000 \times \frac{4}{123}\right)$ km/hr = $\left(\frac{8000}{123}\right)$ km/hr = $65\frac{5}{123}$ km/hr.

38. Let the whole distance travelled be x km and the average speed of the car for the whole journey be y km/hr.

Then, $\frac{(x/3)}{10} + \frac{(x/3)}{20} + \frac{(x/3)}{60} = \frac{x}{y} \Leftrightarrow \frac{x}{30} + \frac{x}{60} + \frac{x}{180} = \frac{x}{y}$

$\Leftrightarrow \frac{1}{18}y = 1 \Leftrightarrow y = 18$ km/hr.

39. $x \times \frac{15}{60} + 2x \times \frac{20}{60} + x \times \frac{10}{60} = 39 \Rightarrow \frac{x}{4} + \frac{2x}{3} + \frac{x}{6} = 39$

$\Rightarrow 3x + 8x + 2x = 468 \Rightarrow x = 36.$

40. Let speed of jogging be x km/hr.

Total time taken = $\left(\frac{9}{6}\text{ hrs} + 1.5\text{ hrs}\right) = 3$ hrs.

Total distance covered = $(9 + 1.5x)$ km.

$\therefore \frac{9 + 1.5x}{3} = 9 \Leftrightarrow 9 + 1.5x = 27 \Leftrightarrow \frac{3}{2}x = 18 \Leftrightarrow x = \left(18 \times \frac{2}{3}\right) = 12$ kmph.

41. Time taken = 1 hr 40 min 48 sec = 1 hr $40\frac{4}{5}$ min = $1\frac{51}{75}$ hrs = $\frac{126}{75}$ hrs.

Let the actual speed be x km/hr.

Then, $\frac{5}{7}x \times \frac{126}{75} = 42$ or $x = \left(\frac{42 \times 7 \times 75}{5 \times 126}\right) = 35$ km/hr.

42. New speed = $\frac{7}{11}$ of usual speed.

$\therefore$ New time = $\frac{11}{7}$ of usual time.

So, $\frac{11}{7}$ of usual time = 22 hrs $\Rightarrow$ usual time = $\left(\frac{22 \times 7}{11}\right) = 14$ hrs.

Hence, time saved = (22 – 14) = 8 hrs.

43. Let the speed be x km / hr. Then,

$30x - 30 \times \frac{14}{15}x = 10 \Leftrightarrow 2x = 10 \Leftrightarrow x = 5$ km / hr.

44. New speed $= \frac{6}{7}$ of usual speed.

New time $= \frac{7}{6}$ of usual time

$\therefore \left(\frac{7}{6} \text{ of usual time}\right) - (\text{usual time}) = \frac{1}{5}$ hr.

$\Rightarrow \frac{1}{6}$ of usual time $= \frac{1}{5}$ hr $\Rightarrow$ usual time $= \frac{6}{5}$ hr $= 1$ hr 12 min.

45. Let the distance be x km.

Difference in timings $= 12$ min $= \frac{12}{60}$ hr $= \frac{1}{5}$ hr.

$\therefore \frac{2x}{5} - \frac{2x}{7} = \frac{1}{5} \Leftrightarrow 14x - 10x = 7 \Leftrightarrow x = 1\frac{3}{4}$ km.

46. Difference between timings $= 15$ min $= \frac{1}{4}$ hr.

Let the length of journey be x km.

Then, $\frac{x}{35} - \frac{x}{40} = \frac{1}{4} \Leftrightarrow 8x - 7x = 70 \Leftrightarrow x = 70$ km.

47. Let the distance travelled be x km.

Then, $\frac{x}{10} - \frac{x}{15} = 2 \Leftrightarrow 3x - 2x = 60 \Leftrightarrow x = 60$ km.

Time taken to travel 60 km at 10 km / hr $= \left(\frac{60}{10}\right)$ hrs $= 6$ hrs.

So, Robert started 6 hours before 2 P.M. *i.e.*, at 8 A.M.

$\therefore$ Required speed $= \left(\frac{60}{5}\right)$ kmph $= 12$ kmph.

48. Let the correct time to complete the journey be x min.

Distance covered in $(x + 11)$ min. at 40 kmph

= Distance covered in $(x + 5)$ min. at 50 kmph

$\therefore \frac{(x+11)}{60} \times 40 = \frac{(x+5)}{60} \times 50 \Leftrightarrow x = 19$ min.

49. Let distance $= x$ km and usual rate $= y$ kmph.

$$\frac{x}{y} - \frac{x}{y+3} = \frac{40}{60} \text{ or } 2y(y+3) = 9x \quad \ldots(i)$$

And, $$\frac{x}{y-2} - \frac{x}{y} = \frac{40}{60} \text{ or } y(y-2) = 3x \quad \ldots(ii)$$

On dividing (i) by (ii), we get $x = 40$ km.

50. Let distance $= x$ km and usual rate $= y$ kmph. Then,

$$\frac{x}{y} - \frac{x}{y+10} = 1 \text{ or } y(y+10) = 10x \quad \ldots(i)$$

And, $$\frac{x}{y} - \frac{x}{y+20} = \frac{7}{4} \text{ or } y(y+20) = \frac{80x}{7} \quad \ldots(ii)$$

On dividing (i) by (ii), we get $y = 60$.

Substituting $y = 60$ in (i), we get : $x = 420$ km.

51. Let speed of the car be x kmph.

Then, speed of the train $= \frac{150}{100}x = \left(\frac{3}{2}x\right)$ kmph.

$\therefore \quad \frac{75}{x} - \frac{75}{\frac{3}{2}x} = \frac{125}{10 \times 60} \quad \Leftrightarrow \quad \frac{75}{x} - \frac{50}{x} = \frac{5}{24} \quad \Leftrightarrow \quad x = \left(\frac{25 \times 24}{5}\right) = 120$ kmph.

52. Due to stoppages, it covers 9 km less.

Time taken to cover 9 km $= \left(\frac{9}{54} \times 60\right)$ min $= 10$ min.

53. Let the original speed be x km / hr. Then,

$$\frac{715}{x} - \frac{715}{x+10} = 2 \quad \Leftrightarrow \quad 2x(x+10) = 7150 \quad \Leftrightarrow \quad x^2 + 10x - 3575 = 0$$

$$\Leftrightarrow \quad (x+65)(x-55) = 0 \quad \Leftrightarrow \quad x = 55 \text{ km / hr.}$$

54. Ratio of speeds $= 3 : 4$. Ratio of times taken $= 4 : 3$.

Suppose A takes $4x$ hrs and B takes $3x$ hrs to reach the destination. Then,

$$4x - 3x = \frac{30}{60} = \frac{1}{2} \text{ or } x = \frac{1}{2}.$$

$\therefore$ Time taken by A $= 4x$ hrs $= \left(4 \times \frac{1}{2}\right)$ hrs $= 2$ hrs.

55. Let Abhay's speed be x km / hr.

Then, $\frac{30}{x} - \frac{30}{2x} = 3 \quad \Leftrightarrow \quad 6x = 30 \quad \Leftrightarrow \quad x = 5$ km / hr.

56. Ratio of speeds $= 4 : 3 : 5$.

$\therefore$ Ratio of times taken $= \frac{1}{4} : \frac{1}{3} : \frac{1}{5} = 15 : 20 : 12$.

57. Let the distance be x km. Then,

$$\frac{x}{7\frac{1}{2}} - \frac{x}{8} = 4 \quad \Leftrightarrow \quad \frac{2x}{15} - \frac{x}{8} = 4 \quad \Leftrightarrow \quad x = 480 \text{ km.}$$

58. Let the distance be x km. Then,

$$\frac{x}{3} - \frac{x}{3.75} = \frac{1}{2} \quad \Leftrightarrow \quad 2.5x - 2x = 3.75 \quad \Leftrightarrow \quad x = \frac{3.75}{0.50} = \frac{15}{2} = 7.5 \text{ km.}$$

59. Let the actual distance travelled be x km. Then,

$$\frac{x}{10} = \frac{x+20}{14} \quad \Leftrightarrow \quad 14x = 10x + 200 \quad \Leftrightarrow \quad 4x = 200 \quad \Leftrightarrow \quad x = 50 \text{ km.}$$

60. Let the duration of the flight be x hours. Then,

$$\frac{600}{x} - \frac{600}{x + \frac{1}{2}} = 200 \quad \Leftrightarrow \quad \frac{600}{x} - \frac{1200}{2x+1} = 200 \quad \Leftrightarrow \quad x(2x+1) = 3$$

$$\Leftrightarrow \quad 2x^2 + x - 3 = 0 \quad \Leftrightarrow \quad (2x+3)(x-1) = 0$$

$$\Leftrightarrow \quad x = 1 \text{ hr.} \quad [\textit{neglecting the } -\textit{ve value of } x]$$

61. Let the speed of the train be x km / hr and that of the car be y km / hr.

Then, $\frac{120}{x} + \frac{480}{y} = 8$ or $\frac{1}{x} + \frac{4}{y} = \frac{1}{15}$ ...(i)

And, $\frac{200}{x} + \frac{400}{y} = \frac{25}{3}$ or $\frac{1}{x} + \frac{2}{y} = \frac{1}{24}$ ...(ii)

Solving (*i*) and (*ii*), we get $x = 60$ and $y = 80$.

$\therefore$ Ratio of speeds $= 60 : 80 = 3 : 4$.

62. Let C's speed $= x$ km / hr. Then, B's speed $= 3x$ km / hr and A's speed $= 6x$ km / hr.

$\therefore$ Ratio of speeds of A, B, C $= 6x : 3x : x = 6 : 3 : 1$.

Ratio of times taken $= \frac{1}{6} : \frac{1}{3} : 1 = 1 : 2 : 6$.

If C takes 6 min., then B takes 2 min.

If C takes 54 min., then B takes $\left(\frac{2}{6} \times 54\right)$ min. $= 18$ min.

63. To be 0.5 km apart, they take 1 hour.

To be 8.5 km apart, they take $\left(\frac{1}{0.5} \times 8.5\right)$ hrs $= 17$ hrs.

64. Since A and B move in the same direction along the circle, so they will first meet each other when there is a difference of one round between the two.

Relative speed of A and B $= (6 - 1) = 5$ rounds per hour.

Time taken to complete one round at this speed $= \frac{1}{5}$ hr $= 12$ min.

65. Suppose after x km from the start B catches up with A. Then, the difference in the time taken by A to cover x km and that taken by B to cover x km is 4 hours.

$\therefore \quad \frac{x}{4} - \frac{x}{10} = 4$ or $x = 26.7$ km.

66. Relative speed of the thief and p\
oliceman $= (11 - 10)$ km / hr $= 1$ km / hr.

Distance covered in 6 minutes $= \left(\frac{1}{60} \times 6\right)$ km $= \frac{1}{10}$ km $= 100$ m.

$\therefore$ Distance between the thief and policeman $= (200 - 100)$ m $= 100$ m.

67. Suppose the thief is overtaken x hrs after 2.30 p.m.

Then, distance covered by the thief in x hrs

$= $ distance covered by the owner in $\left(x - \frac{1}{2}\right)$ hrs.

$\therefore \quad 60x = 75\left(x - \frac{1}{2}\right) \Leftrightarrow 15x = \frac{75}{2} \Leftrightarrow x = \frac{5}{2}$ hrs.

So, the thief is overtaken at 5 p.m.

68. Let the speed of the train be x m / sec. Then,

Distance travelled by the train in 10 min. $=$ Distance travelled by sound in 30 sec.

$\Leftrightarrow \quad x \times 10 \times 60 = 330 \times 30 \Leftrightarrow x = 16.5$.

$\therefore$ Speed of the train $= 16.5$ m/sec $= \left(16.5 \times \frac{18}{5}\right)$ km/hr $= 59.4$ km/hr

69. To be $(18 + 20)$ km apart, they take 1 hour.

To be 47.5 km apart, they take $\left(\frac{1}{38} \times 47.5\right)$ hrs $= 1\frac{1}{4}$ hrs.

70. Suppose they meet x hrs after 8 a.m. Then,

(Distance moved by first in x hrs) + [Distance moved by second in $(x - 1)$ hrs] $= 330$

$\therefore \quad 60x + 75(x - 1) = 330 \quad \Rightarrow \quad x = 3$.

So, they meet at $(8 + 3)$, *i.e.* 11 a.m.

71. Clearly, the two will meet when they are 726 m apart.

To be (4.5 + 3.75) = 8.25 km apart, they take 1 hour.

To be 726 m apart, they take $\left(\frac{100}{825}\times\frac{726}{1000}\right)$ hrs = $\left(\frac{242}{2750}\times 60\right)$ min = 5.28 min.

72. Relative speed = (2 + 3) = 5 rounds per hour.

So, they cross each other 5 times in an hour and 2 times in half an hour.

Hence, they cross each other 7 times before 9.30 a.m.

73. Let their speeds be x kmph and y kmph respectively.

Then, $\frac{120}{x+y}=1 \Rightarrow x+y=120$ ...(*i*)

Now, when they move in same direction :

(Distance travelled by P in 6 hrs) – (Distance travelled by Q in 6 hrs) = 120 km

$\Rightarrow 6x-6y=120 \Rightarrow x-y=20$ ...(*ii*)

Solving (*i*) and (*ii*), $x = 70$, $y = 50$.

∴ P's speed = 70 kmph.

74. In the same time, they cover 110 km and 90 km respectively.

∴ Ratio of their speeds = 110 : 90 = 11 : 9.

75. At the time of meeting, let the distance travelled by the second train be x km.

Then, distance covered by the first train is $(x + 100)$ km.

∴ $\frac{x}{40}=\frac{x+100}{50} \Leftrightarrow 50x=40x+4000 \Leftrightarrow x=400$.

So, distance between P and Q = $(x + x + 100)$ km = 900 km.

76. Suppose they meet x hours after 14.30 hrs.

Then, $60x = 80(x - 2)$ or $x = 8$.

∴ Required distance = (60×8) km = 480 km.

77. Let the distance between Meerut and Delhi be x km and let the trains meet y hours after 7 a.m.

Clearly, M covers x km in 4 hrs and N covers x km in (7/2) hrs.

∴ Speed of M = $\frac{x}{4}$ kmph, Speed of N = $\frac{2x}{7}$ kmph.

Distance covered by M in $(y + 2)$ hrs + Distance covered in y hrs = x.

∴ $\frac{x}{4}(y+2)+\frac{2x}{7}\times y=x \Leftrightarrow \frac{(y+2)}{4}+\frac{2y}{7}=1$

$\Leftrightarrow y=\frac{14}{15}$ hrs = $\left(\frac{14}{15}\times 60\right)$ min. = 56 min.

Hence, the trains meet at 7.56 a.m.

78. Let the distance be x km. Then,

(Time taken to walk x km) + (Time taken to ride x km) = $\frac{23}{4}$ hrs.

$\Rightarrow$ (Time taken to walk $2x$ km) + (Time taken to ride $2x$ km) = $\frac{23}{2}$ hrs.

But, time taken to ride $2x$ km = $\frac{15}{4}$ hrs.

∴ Time taken to walk $2x$ km = $\left(\frac{23}{2}-\frac{15}{4}\right)$ hrs = $\frac{31}{4}$ hrs = 7 hrs 45 min.

EXERCISE 17B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 7*) : *Each of the questions below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements is/are sufficient to answer the question. Read both the statements and*

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question; and

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. How much time did X take to reach the destination ?
   I. The ratio between the speeds of X and Y is 3 : 4.
   II. Y takes 36 minutes to reach the same destination.
2. What is the usual speed of the train ? **(M.B.A. 2002)**
   I. The speed of the train is increased by 25 km / hr to reach the destination 150 km away in time.
   II. The train is late by 30 minutes.
3. Two towns are connected by railway. Can you find the distance between them ?
   I. The speed of mail train is 12 km / hr more than that of an express train.
   II. A mail train takes 40 minutes less than an express train to cover the distance. **(M.B.A. 2001)**
4. The towns A, B and C are on a straight line. Town C is between A and B. The distance from A to B is 100 km. How far is A from C ? **(M.B.A. 2003)**
   I. The distance from A to B is 25% more than the distance from C to B.
   II. The distance from A to C is $\frac{1}{4}$ of the distance from C to B.
5. What is the average speed of the car over the entire distance ?
   I. The car covers the whole distance in four equal stretches at speeds of 10 kmph, 20 kmph, 30 kmph and 60 kmph respectively.
   II. The total time taken is 36 minutes.
6. A car and a bus start from city A at the same time. How far is the city B from city A ?
   I. The car travelling at an average speed of 40 km / hr reaches city B at 4:35 p.m.
   II. The bus reaches city B at 6:15 p.m. at an average speed of 60 km / hr.
7. Two cars pass each other in opposite direction. How long would they take to be 500 km apart ? **(M.A.T. 1998)**
   I. The sum of their speeds is 135 km / hr.
   II. The difference of their speeds is 25 km / hr.

ANSWERS

1. (e) **2.** (e) **3.** (d) **4.** (c) **5.** (a) **6.** (e) **7.** (a)

SOLUTIONS

1. I. If Y takes 4 min., then X takes 3 min.

   II. If Y takes 36 min., then X takes $\left(\frac{3}{4}\times 36\right)$ min = 27 min.

   Thus, I and II together give the answer.

   ∴ Correct answer is (e).

2. Let the usual speed of the train be x kmph.

   Time taken to cover 150 km at usual speed = $\frac{150}{x}$ hrs.

   I. Time taken at increased speed = $\frac{150}{(x+25)}$ hrs.

   II. $\frac{150}{x}-\frac{150}{(x+25)}=\frac{30}{60}$

   $\Leftrightarrow \frac{1}{x}-\frac{1}{(x+25)}=\frac{1}{300} \Leftrightarrow [(x+25)-x]\times 300 = x(x+25)$

   $\Leftrightarrow x^2+25x-7500=0 \Leftrightarrow (x+100)(x-75)=0 \Leftrightarrow x=75.$

   Thus, I and II together give the answer.

   ∴ Correct answer is (e).

3. Let the distance between the two stations be x km.

   I. Let the speed of the express train be y km / hr.

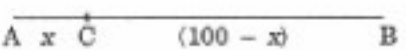
   Then, speed of the mail train = $(y+12)$ km / hr.

   II. $\frac{x}{y}-\frac{x}{(y+12)}=\frac{40}{60}$.

   Thus, even I and II together do not give x.

   ∴ Correct answer is (d).

4. Let AC = x km. Then, CB = $(100-x)$ km.

   I. AB = 125% of CB

   $\Leftrightarrow 100=\frac{125}{100}\times(100-x) \Leftrightarrow 100-x=\frac{100\times 100}{125}=80 \Leftrightarrow x=20$ km.

   ∴ AC = 20 km.

   Thus, I alone gives the answer.

   II. $AC=\frac{1}{4}CB \Leftrightarrow x=\frac{1}{4}(100-x) \Leftrightarrow 5x=100 \Leftrightarrow x=20.$

   ∴ AC = 20 km.

   Thus, II alone gives the answer.

   ∴ Correct answer is (c).

5. Let the whole distance be $4x$ km.

   I. Total time taken = $\left(\frac{x}{10}+\frac{x}{20}+\frac{x}{30}+\frac{x}{60}\right)=\frac{(6x+3x+2x+x)}{60}=\frac{12x}{60}=\frac{x}{5}$.

   ∴ Speed = $\frac{\text{Distance}}{\text{Time}}=\frac{4x}{(x/5)}$ kmph = 20 km / hr.

   ∴ I alone is sufficient to answer the question.

   II alone does not give the answer.

   ∴ Correct answer is (a).

6. Let AB = x km. From I and II, we get :

$$\frac{x}{40} - \frac{x}{60} = 1\frac{40}{60} \qquad [(6{:}15 \text{ p.m.}) - (4{:}35 \text{ p.m.}) = 1 \text{ hr } 40 \text{ min}]$$

$\Leftrightarrow \quad \frac{x}{40} - \frac{x}{60} = \frac{100}{60}$. This gives x.

$\therefore$ Correct answer is (e).

7. I gives, relative speed = 135 km/hr.

$\therefore$ Time taken = $\frac{500}{135}$ hrs.

II does not give the relative speed.

$\therefore$ I alone gives the answer and II is irrelevant.

$\therefore$ Correct answer is (a).

18. PROBLEMS ON TRAINS

IMPORTANT FACTS AND FORMULAE

1. a km/hr $= \left(a \times \frac{5}{18}\right)$ m/s.
2. a m/s $= \left(a \times \frac{18}{5}\right)$ km/hr.
3. Time taken by a train of length l metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover l metres.
4. Time taken by a train of length l metres to pass a stationary object of length b metres is the time taken by the train to cover $(l + b)$ metres.
5. Suppose two trains or two bodies are moving in the same direction at u m/s and v m/s, where $u > v$, then their relatives speed $= (u - v)$ m/s.
6. Suppose two trains or two bodies are moving in opposite directions at u m/s and v m/s, then their relative speed is $= (u + v)$ m/s.
7. If two trains of length a metres and b metres are moving in opposite directions at u m/s and v m/s, then time taken by the trains to cross each other $= \frac{(a + b)}{(u + v)}$ sec.
8. If two trains of length a metres and b metres are moving in the same direction at u m/s and v m/s, then the time taken by the faster train to cross the slower train $= \frac{(a + b)}{(u - v)}$ sec.
9. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then *(A's speed)* : *(B's speed)* $= (\sqrt{b} : \sqrt{a})$.

SOLVED EXAMPLES

Ex. 1. ***A train 100 m long is running at the speed of 30 km/hr. Find the time taken by it to pass a man standing near the railway line.*** (S.S.C. 2001)

Sol. Speed of the train $= \left(30 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{25}{3}\right)$ m/sec.

Distance moved in passing the standing man = 100 m.

Required time taken $= \frac{100}{\left(\frac{25}{3}\right)} = \left(100 \times \frac{3}{25}\right)$ sec = 12 sec.

Ex. 2. ***A train is moving at a speed of 132 km/hr. If the length of the train is 110 metres, how long will it take to cross a railway platform 165 metres long?*** (Section Officers', 2003)

Sol. Speed of train $= \left(132 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{110}{3}\right)$ m/sec.

Distance covered in passing the platform = (110 + 165) m = 275 m.

$\therefore$ Time taken $= \left(275 \times \frac{3}{110}\right)$ sec $= \frac{15}{2}$ sec $= 7\frac{1}{2}$ sec.

Ex. 3. ***A man is standing on a railway bridge which is 180 m long. He finds that a train crosses the bridge in 20 seconds but himself in 8 seconds. Find the length of the train and its speed.***

Sol. Let the length of the train be x metres.

Then, the train covers x metres in 8 seconds and $(x + 180)$ metres in 20 seconds.

$\therefore \quad \frac{x}{8} = \frac{x+180}{20} \Leftrightarrow 20x = 8(x+180) \Leftrightarrow x = 120.$

$\therefore$ Length of the train = 120 m.

Speed of the train $= \left(\frac{120}{8}\right)$ m/sec = m/sec $= \left(15 \times \frac{18}{5}\right)$ kmph = 54 kmph.

Ex. 4. ***A train 150 m long is running with a speed of 68 kmph. In what time will it pass a man who is running at 8 kmph in the same direction in which the train is going ?***

Sol. Speed of the train relative to man = (68 − 8) kmph

$= \left(60 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{50}{3}\right)$ m/sec.

Time taken by the train to cross the man

= Time taken by it to cover 150 m at $\left(\frac{50}{3}\right)$ m/sec $= \left(150 \times \frac{3}{50}\right)$ sec = 9 sec.

Ex. 5. ***A train 220 m long is running with a speed of 59 kmph. In what time will it pass a man who is running at 7 kmph in the direction opposite to that in which the train is going ?***

Sol. Speed of the train relative to man = (59 + 7) kmph

$= \left(66 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{55}{3}\right)$ m/sec.

Time taken by the train to cross the man

= Time taken by it to cover 220 m at $\left(\frac{55}{3}\right)$ m/sec $= \left(220 \times \frac{3}{55}\right)$ sec = 12 sec.

Ex. 6. ***Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmph. In what time will they be clear of each other from the moment they meet ?***

Sol. Relative speed of the trains = (42 + 48) kmph = 90 kmph

$= \left(90 \times \frac{5}{18}\right)$ m/sec = 25 m/sec.

Time taken by the trains to pass each other

= Time taken to cover (137 + 163) m at 25 m/sec $= \left(\frac{300}{25}\right)$ sec = 12 seconds.

Ex. 7. ***Two trains 100 metres and 120 metres long are running in the same direction with speeds of 72 km/hr and 54 km/hr. In how much time will the first train cross the second ?*** (C.B.I. 1997)

Sol. Relative speed of the trains = (72 − 54) km/hr = 18 km/hr

$= \left(18 \times \frac{5}{18}\right)$ m/sec = 5 m/sec.

Time taken by the trains to cross each other

= Time taken to cover (100 + 120) m at 5 m/sec $= \left(\frac{220}{5}\right)$ sec = 44 sec.

Ex. 8. ***A train 100 metres long takes 6 seconds to cross a man walking at 5 kmph in a direction opposite to that of the train. Find the speed of the train.***

Sol. Let the speed of the train be x kmph.

Speed of the train relative to man $= (x + 5)$ kmph $= (x+5)\times\frac{5}{18}$ m/sec.

$\therefore \quad \frac{100}{(x+5)\times\frac{5}{18}} = 6 \Leftrightarrow 30(x+5) = 1800 \Leftrightarrow x = 55.$

$\therefore$ Speed of the train is 55 kmph.

Ex. 9. ***A train running at 54 kmph takes 20 seconds to pass a platform. Next it takes 12 seconds to pass a man walking at 6 kmph in the same direction in which the train is going. Find the length of the train and the length of the platform.***

Sol. Let the length of train be x metres and length of platform be y metres.

Speed of the train relative to man $= (54 - 6)$ kmph $= 48$ kmph

$$= \left(48\times\frac{5}{18}\right) \text{m/sec} = \frac{40}{3} \text{m/sec}.$$

In passing a man, the train covers its own length with relative speed.

$\therefore$ Length of train = (Relative speed $\times$ Time) $= \left(\frac{40}{3}\times 12\right)$ m $= 160$ m.

Also, speed of the train $= \left(54\times\frac{5}{18}\right)$ m/sec $= 15$ m/sec.

$\therefore \quad \frac{x+y}{15} = 20 \Leftrightarrow x + y = 300 \Leftrightarrow y = (300 - 160)$ m $= 140$ m.

Ex. 10. ***A man sitting in a train which is travelling at 50 kmph observes that a goods train, travelling in opposite direction, takes 9 seconds to pass him. If the goods train is 280 m long, find its speed.***

Sol. Relative speed $= \left(\frac{280}{9}\right)$ m/sec $= \left(\frac{280}{9}\times\frac{18}{5}\right)$ kmph $= 112$ kmph.

$\therefore$ Speed of goods train $= (112 - 50)$ kmph $= 62$ kmph.

EXERCISE 18A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. A train moves with a speed of 108 kmph. Its speed in metres per second is :
   (a) 10.8 (b) 18 (c) 30 (d) 38.8
2. A speed of 14 metres per second is the same as :
   (a) 28 km/hr (b) 46.6 km/hr (c) 50.4 km/hr (d) 70 km/hr
3. In what time will a train 100 metres long cross an electric pole, if its speed be 144 km/hr ? **(S.S.C. 2003)**
   (a) 2.5 seconds (b) 4.25 seconds (c) 5 seconds (d) 12.5 seconds
4. A train 280 m long, running with a speed of 63 km/hr will pass a tree in :
   (a) 15 sec (b) 16 sec (c) 18 sec (d) 20 sec **(S.S.C. 2003)**
5. How long does a train 110 metres long running at the speed of 72 km/hr take to cross a bridge 132 metres in length ? **(R.R.B. 1998)**
   (a) 9.8 sec (b) 12.1 sec (c) 12.42 sec (d) 14.3 sec

6. A train 360 m long is running at a speed of 45 km / hr. In what time will it pass a bridge 140 m long ? **(B.S.F. 2001)**
(*a*) 40 sec (*b*) 42 sec (*c*) 45 sec (*d*) 48 sec

7. A train travelling at a speed of 75 mph enters a tunnel $3\frac{1}{2}$ miles long. The train is $\frac{1}{4}$ mile long. How long does it take for the train to pass through the tunnel from the moment the front enters to the moment the rear emerges ?
(*a*) 2.5 min (*b*) 3 min (*c*) 3.2 min (*d*) 3.5 min

8. A train running at the speed of 60 km / hr crosses a pole in 9 seconds. What is the length of the train ? **(Bank P.O. 2003)**
(*a*) 120 metres (*b*) 180 metres (*c*) 324 metres
(*d*) Cannot be determined (*e*) None of these

9. A train 132 m long passes a telegraph pole in 6 seconds. Find the speed of the train.
(*a*) 70 km / hr (*b*) 72 km / hr (*c*) 79.2 km / hr (*d*) 80 km / hr

10. A train covers a distance of 12 km in 10 minutes. If it takes 6 seconds to pass a telegraph post, then the length of the train is : **(Bank P.O. 2000)**
(*a*) 90 m (*b*) 100 m (*c*) 120 m (*d*) 140 m

11. A train 240 m long passed a pole in 24 seconds. How long will it take to pass a platform 650 m long ? **(R.R.B. 1998)**
(*a*) 65 sec (*b*) 89 sec (*c*) 100 sec (*d*) 150 sec

12. The length of the bridge, which a train 130 metres long and travelling at 45 km / hr can cross in 30 seconds, is : **(Section Officers', 2001)**
(*a*) 200 m (*b*) 225 m (*c*) 245 m (*d*) 250 m

13. A train 800 metres long is running at a speed of 78 km / hr. If it crosses a tunnel in 1 minute, then the length of the tunnel (in metres) is : **(S.S.C. 2003)**
(*a*) 130 (*b*) 360 (*c*) 500 (*d*) 540

14. A goods train runs at the speed of 72 kmph and crosses a 250 m long platform in 26 seconds. What is the length of the goods train ? **(Bank P.O. 2003)**
(*a*) 230 m (*b*) 240 m (*c*) 260 m (*d*) 270 m

15. The length of a train and that of a platform are equal. If with a speed of 90 km / hr, the train crosses the platform in one minute, then the length of the train (in metres) is :
(*a*) 500 (*b*) 600 (*c*) 750 (*d*) 900

16. A train of length 150 metres takes 40.5 seconds to cross a tunnel of length 300 metres. What is the speed of the train in km / hr ?
(*a*) 13.33 (*b*) 26.67 (*c*) 40 (*d*) 66.67

17. A train crosses a platform 100 m long in 60 seconds at a speed of 45 km / hr. The time taken by the train to cross an electric pole is :
(*a*) 8 sec (*b*) 52 sec (*c*) 1 minute (*d*) Data inadequate

18. A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km / hr, what is the length of the platform ? **(G.INDOMAT, 1997)**
(*a*) 120 m (*b*) 240 m (*c*) 300 m (*d*) None of these

19. A 300 metre long train crosses a platform in 39 seconds while it crosses a signal pole in 18 seconds. What is the length of the platform ?
(*a*) 320 m (*b*) 350 m (*c*) 650 m
(*d*) Data inadequate (*e*) None of these **(Bank P.O. 2002)**

20. A train speeds past a pole in 15 seconds and a platform 100 m long in 25 seconds. Its length is : **(R.R.B. 2003)**
(*a*) 50 m (*b*) 150 m (*c*) 200 m (*d*) Data inadequate

21. A train moves past a telegraph post and a bridge 264 m long in 8 seconds and 20 seconds respectively. What is the speed of the train ? **(S.S.C. 2004)**
(*a*) 69.5 km / hr (*b*) 70 km / hr (*c*) 79 km / hr (*d*) 79.2 km / hr

22. A train takes 18 seconds to pass completely through a station 162 m long and 15 seconds through another station 120 m long. The length of the train is :
(*a*) 70 m (*b*) 80 m (*c*) 90 m (*d*) 100 m

23. How many seconds will a 500 metre long train take to cross a man walking with a speed of 3 km / hr in the direction of the moving train if the speed of the train is 63 km / hr ? **(S.S.C. 2000)**
(*a*) 25 (*b*) 30 (*c*) 40 (*d*) 45

24. A jogger running at 9 kmph alongside a railway track is 240 metres ahead of the engine of a 120 metre long train running at 45 kmph in the same direction. In how much time will the train pass the jogger ? **(IGNOU, 2003)**
(*a*) 3.6 sec (*b*) 18 sec (*c*) 36 sec (*d*) 72 sec

25. A train 110 metres long is running with a speed of 60 kmph. In what time will it pass a man who is running at 6 kmph in the direction opposite to that in which the train is going ? **(M.A.T. 2002)**
(*a*) 5 sec (*b*) 6 sec (*c*) 7 sec (*d*) 10 sec

26. Two trains 200 m and 150 m long are running on parallel rails at the rate of 40 kmph and 45 kmph respectively. In how much time will they cross each other, if they are running in the same direction ?
(*a*) 72 sec (*b*) 132 sec (*c*) 192 sec (*d*) 252 sec

27. Two trains 140 m and 160 m long run at the speed of 60 km / hr and 40 km / hr respectively in opposite directions on parallel tracks. The time (in seconds) which they take to cross each other, is : **(S.S.C. 2004)**
(*a*) 9 (*b*) 9.6 (*c*) 10 (*d*) 10.8

28. Two trains are moving in opposite directions @ 60 km / hr and 90 km / hr. Their lengths are 1.10 km and 0.9 km respectively. The time taken by the slower train to cross the faster train in seconds is : **(M.B.A. 2002)**
(*a*) 36 (*b*) 45 (*c*) 48 (*d*) 49

29. A train 125 m long passes a man, running at 5 kmph in the same direction in which the train is going, in 10 seconds. The speed of the train is : **(A.A.O. Exam, 2003)**
(*a*) 45 km / hr (*b*) 50 km / hr (*c*) 54 km / hr (*d*) 55 km / hr

30. A train 110 m long passes a man, running at 6 kmph in the direction opposite to that of the train, in 6 seconds. The speed of the train is :
(*a*) 54 km / hr (*b*) 60 km / hr (*c*) 66 km / hr (*d*) 72 km / hr

31. Two goods train each 500 m long, are running in opposite directions on parallel tracks. Their speeds are 45 km / hr and 30 km / hr respectively. Find the time taken by the slower train to pass the driver of the faster one. **(M.A.T. 2000)**
(*a*) 12 sec (*b*) 24 sec (*c*) 48 sec (*d*) 60 sec

32. Two trains of equal length are running on parallel lines in the same direction at 46 km / hr and 36 km / hr. The faster train passes the slower train in 36 seconds. The length of each train is : **(M.A.T. 2003)**
(*a*) 50 m (*b*) 72 m (*c*) 80 m (*d*) 82 m

33. A 270 metres long train running at the speed of 120 kmph crosses another train running in opposite direction at the speed of 80 kmph in 9 seconds. What is the length of the other train ? **(S.B.I.P.O. 1999)**
(*a*) 230 m (*b*) 240 m (*c*) 260 m
(*d*) 320 m (*e*) None of these

34. Two trains are running in opposite directions with the same speed. If the length of each train is 120 metres and they cross each other in 12 seconds, then the speed of each train (in km / hr) is : **(S.S.C. 2003)**
(*a*) 10 (*b*) 18 (*c*) 36 (*d*) 72

35. Two trains of equal lengths take 10 seconds and 15 seconds respectively to cross a telegraph post. If the length of each train be 120 metres, in what time (in seconds) will they cross each other travelling in opposite direction ? **(S.S.C. 2004)**
(a) 10 (b) 12 (c) 15 (d) 20

36. A train 108 m long moving at a speed of 50 km / hr crosses a train 112 m long coming from opposite direction in 6 seconds. The speed of the second train is
(a) 48 km / hr (b) 54 km / hr (c) 66 km / hr (d) 82 km / hr

37. A train X speeding with 120 kmph crosses another train Y, running in the same direction, in 2 minutes. If the lengths of the trains X and Y be 100 m and 200 m respectively, what is the speed of train Y ?
(a) 111 km / hr (b) 123 km / hr (c) 127 km / hr (d) 129 km / hr

38. Two trains travel in opposite directions at 36 kmph and 45 kmph and a man sitting in slower train passes the faster train in 8 seconds. The length of the faster train is :
(a) 80 m (b) 100 m (c) 120 m (d) 180 m

39. Two trains are running at 40 km / hr and 20 km / hr respectively in the same direction. Fast train completely passes a man sitting in the slower train in 5 seconds. What is the length of the fast train ? **(R.R.B. 2001)**
(a) 23 m (b) $23\frac{2}{9}$ m (c) 27 m (d) $27\frac{7}{9}$ m

40. A train overtakes two persons who are walking in the same direction in which the train is going, at the rate of 2 kmph and 4 kmph and passes them completely in 9 and 10 seconds respectively. The length of the train is :
(a) 45 m (b) 50 m (c) 54 m (d) 72 m

41. A train overtakes two persons walking along a railway track. The first one walks at 4.5 km / hr. The other one walks at 5.4 km / hr. The train needs 8.4 and 8.5 seconds respectively to overtake them. What is the speed of the train if both the persons are walking in the same direction as the train ?
(a) 66 km / hr (b) 72 km / hr (c) 78 km / hr (d) 81 km / hr

42. Two trains, each 100 m long, moving in opposite directions, cross each other in 8 seconds. If one is moving twice as fast the other, then the speed of the faster train is :
(a) 30 km / hr (b) 45 km / hr (c) 60 km / hr (d) 75 km / hr **(C.D.S. 2001)**

43. A train 150 m long passes a km stone in 15 seconds and another train of the same length travelling in opposite direction in 8 seconds. The speed of the second train is :
(a) 60 km / hr (b) 66 km / hr (c) 72 km / hr (d) 99 km / hr

44. A train travelling at 48 kmph completely crosses another train having half its length and travelling in opposite direction at 42 kmph, in 12 seconds. It also passes a railway platform in 45 seconds. The length of the platform is
(a) 400 m (b) 450 m (c) 560 m (d) 600 m

45. Two trains running in opposite directions cross a man standing on the platform in 27 seconds and 17 seconds respectively and they cross each other in 23 seconds. The ratio of their speeds is : **(Hotel Management, 1997)**
(a) 1 : 3 (b) 3 : 2 (c) 3 : 4 (d) None of these

46. Two stations A and B are 110 km apart on a straight line. One train starts from A at 7 a.m. and travels towards B at 20 kmph. Another train starts from B at 8 a.m. and travels towards A at a speed of 25 kmph. At what time will they meet ?
(a) 9 a.m. (b) 10 a.m. (c) 10.30 a.m. (d) 11 a.m.

47. A train X starts from Meerut at 4 p.m. and reaches Ghaziabad at 5 p.m. while another train Y starts from Ghaziabad at 4 p.m. and reaches Meerut at 5.30 p.m. The two trains will cross each other at :
(a) 4.36 p.m. (b) 4.42 p.m. (c) 4.48 p.m. (d) 4.50 p.m.

48. Two trains, one from Howrah to Patna and the other from Patna to Howrah, start simultaneously. After they meet, the trains reach their destinations after 9 hours and 16 hours respectively. The ratio of their speeds is : **(R.R.B. 2001)**

(a) 2 : 3 (b) 4 : 3 (c) 6 : 7 (d) 9 : 16

ANSWERS

1. (c)	2. (c)	3. (a)	4. (b)	5. (b)	6. (a)	7. (b)	8. (e)
9. (c)	10. (c)	11. (b)	12. (c)	13. (c)	14. (d)	15. (c)	16. (c)
17. (b)	18. (b)	19. (b)	20. (b)	21. (d)	22. (c)	23. (b)	24. (c)
25. (b)	26. (d)	27. (d)	28. (c)	29. (b)	30. (b)	31. (c)	32. (a)
33. (a)	34. (c)	35. (b)	36. (d)	37. (a)	38. (d)	39. (d)	40. (b)
41. (d)	42. (c)	43. (d)	44. (a)	45. (b)	46. (b)	47. (a)	48. (b)

SOLUTIONS

1. 108 kmph $= \left(108 \times \frac{5}{18}\right)$ m/sec $= 30$ m/sec.

2. 14 m/sec $= \left(14 \times \frac{18}{5}\right)$ km/hr $= 50.4$ km/hr.

3. Speed $= \left(144 \times \frac{5}{18}\right)$ m/sec $= 40$ m/sec.

   Time taken $= \left(\frac{100}{40}\right)$ sec $= 2.5$ sec.

4. Speed $= \left(63 \times \frac{5}{18}\right)$ m/sec $= \frac{35}{2}$ m/sec.

   Time taken $= \left(280 \times \frac{2}{35}\right)$ sec $= 16$ sec.

5. Speed $= \left(72 \times \frac{5}{18}\right)$ m/sec $= 20$ m/sec.

   Total distance covered $= (110 + 132)$ m $= 242$ m.

   $\therefore$ Required time $= \left(\frac{242}{20}\right)$ sec $= 12.1$ sec.

6. Speed $= \left(45 \times \frac{5}{18}\right)$ m/sec $= \frac{25}{2}$ m/sec.

   Total distance covered $= (360 + 140)$ m $= 500$ m.

   $\therefore$ Required time $= \left(500 \times \frac{2}{25}\right)$ sec $= 40$ sec.

7. Total distance covered $= \left(\frac{7}{2} + \frac{1}{4}\right)$ miles $= \frac{15}{4}$ miles.

   $\therefore$ Time taken $= \left(\frac{15}{4 \times 75}\right)$ hrs $= \frac{1}{20}$ hrs $= \left(\frac{1}{20} \times 60\right)$ min. $= 3$ min.

18. Speed $= \left(54 \times \frac{5}{18}\right)$ m/sec $= 15$ m/sec.

Length of the train $= (15 \times 20)$ m $= 300$ m.

Let the length of the platform be x metres.

Then, $\frac{x+300}{36} = 15 \Leftrightarrow x + 300 = 540 \Leftrightarrow x = 240$ m.

19. Speed $= \left(\frac{300}{18}\right)$ m/sec $= \frac{50}{3}$ m/sec.

Let the length of the platform be x metres.

Then, $\frac{x+300}{39} = \frac{50}{3} \Leftrightarrow 3(x+300) = 1950 \Leftrightarrow x = 350$ m.

20. Let the length of the train be x metres and its speed be y m/sec.

They, $\frac{x}{y} = 15 \Rightarrow y = \frac{x}{15}$.

$\therefore \quad \frac{x+100}{25} = \frac{x}{15} \Leftrightarrow x = 150$ m.

21. Let the length of the train be x metres and its speed by y m/sec.

They, $\frac{x}{y} = 8 \Rightarrow x = 8y$

Now, $\frac{x+264}{20} = y \Leftrightarrow 8y + 264 = 20y \Leftrightarrow y = 22$

$\therefore$ Speed $= 22$ m/sec $= \left(22 \times \frac{18}{5}\right)$ km/hr $= 79.2$ km/hr.

22. Let the length of the train be x metres.

$\therefore \quad \frac{x+162}{18} = \frac{x+120}{15} \Leftrightarrow 15(x+162) = 18(x+120) \Leftrightarrow x = 90$ m.

23. Speed of train relative to man $= (63 - 3)$ km/hr $= 60$ km/hr

$= \left(60 \times \frac{5}{18}\right)$ m/sec $= \frac{50}{3}$ m/sec.

$\therefore$ Time taken to pass the man $= \left(500 \times \frac{3}{50}\right)$ sec $= 30$ sec.

24. Speed of train relative to jogger $= (45 - 9)$ km/hr $= 36$ km/hr

$= \left(36 \times \frac{5}{18}\right)$ m/sec $= 10$ m/sec.

Distance to be covered $= (240 + 120)$ m $= 360$ m.

$\therefore$ Time taken $= \left(\frac{360}{10}\right)$ sec $= 36$ sec.

25. Speed of train relative to man $= (60 + 6)$ km/hr $= 66$ km/hr

$= \left(66 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{55}{3}\right)$ m/sec.

$\therefore$ Time taken to pass the man $= \left(110 \times \frac{3}{55}\right)$ sec $= 6$ sec.

26. Relative speed $= (45 - 40)$ kmph $= 5$ kmph $= \left(5 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{25}{18}\right)$ m/sec.

8. Speed $= \left(60 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{50}{3}\right)$ m/sec.

Length of the train = (Speed × Time) $= \left(\frac{50}{3} \times 9\right)$ m = 150 m.

9. Speed $= \left(\frac{132}{6}\right)$ m/sec $= \left(22 \times \frac{18}{5}\right)$ km/hr = 79.2 km/hr.

10. Speed $= \left(\frac{12}{10} \times 60\right)$ km/hr $= \left(72 \times \frac{5}{18}\right)$ m/sec = 20 m/sec.

Length of the train = (Speed × Time) = (20 × 6) m = 120 m.

11. Speed $= \left(\frac{240}{24}\right)$ m/sec = 10 m/sec.

$\therefore$ Required time $= \left(\frac{240 + 650}{10}\right)$ sec = 89 sec.

12. Speed $= \left(45 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{25}{2}\right)$ m/sec; Time = 30 sec.

Let the length of bridge be x metres.

Then, $\frac{130 + x}{30} = \frac{25}{2} \Leftrightarrow 2(130 + x) = 750 \Leftrightarrow x = 245$ m.

13. Speed $= \left(78 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{65}{3}\right)$ m/sec.

Time = 1 minute = 60 sec.

Let the length of the tunnel be x metres.

Then, $\frac{800 + x}{60} = \frac{65}{3} \Leftrightarrow 3(800 + x) = 3900 \Leftrightarrow x = 500$.

14. Speed $= \left(72 \times \frac{5}{18}\right)$ m/sec = 20 m/sec; Time = 26 sec.

Let the length of the train be x metres.

Then, $\frac{x + 250}{26} = 20 \Leftrightarrow x + 250 = 520 \Leftrightarrow x = 270$.

15. Speed $= \left(90 \times \frac{5}{18}\right)$ m/sec = 25 m/sec; Time = 1 min. = 60 sec.

Let the length of the train and that of the platform be x metres.

Then, $\frac{2x}{60} = 25 \Leftrightarrow x = \frac{25 \times 60}{2} = 750$.

16. Speed $= \left(\frac{150 + 300}{40.5}\right)$ m/sec $= \left(\frac{450}{40.5} \times \frac{18}{5}\right)$ km/hr = 40 km/hr.

17. Speed $= \left(45 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{25}{2}\right)$ m/sec.

Let the length of the train be x metres.

Then, $\frac{x + 100}{\left(\frac{25}{2}\right)} = 60$ or $x = 650$ m

$\therefore$ Time taken by the train to cross an electric pole $= \left(650 \times \frac{2}{25}\right)$ sec = 52 sec.

Total distance covered = Sum of lengths of trains = 350 m.

$\therefore$ Time taken $= \left(350 \times \frac{18}{25}\right)$ sec $= 252$ sec.

27. Relative speed $= (60 + 40)$ km/hr $= \left(100 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{250}{9}\right)$ m/sec.

Distance covered in crossing each other $= (140 + 160)$ m $= 300$ m

Required time $= \left(300 \times \frac{9}{250}\right)$ sec $= \frac{54}{5}$ sec $= 10.8$ sec.

28. Relative speed $= (60 + 90)$ km/hr

$= \left(150 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{125}{3}\right)$ m/sec.

Distance covered $= (1.10 + 0.9)$ km $= 2$ km $= 2000$ m.

Required time $= \left(2000 \times \frac{3}{125}\right)$ sec $= 48$ sec.

29. Speed of the train relative to man $= \left(\frac{125}{10}\right)$ m/sec $= \left(\frac{25}{2}\right)$ m/sec.

$= \left(\frac{25}{2} \times \frac{18}{5}\right)$ km/hr $= 45$ km/hr.

Let the speed of the train be x kmph. Then, relative speed $= (x - 5)$ kmph.

$\therefore \quad x - 5 = 45 \quad$ or $\quad x = 50$ kmph.

30. Speed of the train relative to man

$= \left(\frac{110}{6}\right)$ m/sec $= \left(\frac{110}{6} \times \frac{18}{5}\right)$ km/hr $= 66$ km/hr.

Let the speed of the train be x kmph. Then, relative speed $= (x + 6) =$ kmph.

$\therefore \quad x + 6 = 66 \quad$ or $\quad x = 60$ kmph.

31. Relative speed $= (45 + 30)$ km/hr $= \left(75 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{125}{6}\right)$ m/sec.

Distance covered $= (500 + 500)$ m $= 1000$ m.

Required time $= \left(1000 \times \frac{6}{125}\right)$ sec $= 48$ sec.

32. Let the length of each train be x metres.

Then, distance covered $= 2x$ metres.

Relative speed $= (46 - 36)$ km/hr $= \left(10 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{25}{9}\right)$ m/sec.

$\therefore \quad \frac{2x}{36} = \frac{25}{9} \Leftrightarrow 2x = 100 \Leftrightarrow x = 50.$

33. Relative speed $= (120 + 80)$ km/hr $= \left(200 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{500}{9}\right)$ m/sec.

Let the length of the other train be x metres.

Then, $\frac{x + 270}{9} = \frac{500}{9} \Leftrightarrow x + 270 = 500 \Leftrightarrow x = 230.$

34. Let the speed of each train be x m/sec.

Then, relative speed of the two trains $= 2x$ m/sec.

So, $2x = \frac{(120+120)}{12} \Leftrightarrow 2x = 20 \Leftrightarrow x = 10.$

$\therefore$ Speed of each train = 10 m/sec = $\left(10 \times \frac{18}{5}\right)$ km/hr = 36 km/hr.

35. Speed of the first train = $\left(\frac{120}{10}\right)$ m/sec = 12 m/sec.

Speed of the second train = $\left(\frac{120}{15}\right)$ m/sec = 8 m/sec.

Relative speed = (12 + 8) = m/sec = 20 m/sec.

$\therefore$ Required time = $\frac{(120+120)}{20}$ sec = 12 sec.

36. Let the speed of the second train be x km/hr.

Relative speed = $(x + 50)$ km/hr = $\left[(x+50) \times \frac{5}{18}\right]$ m/sec = $\left(\frac{250+5x}{18}\right)$ m/sec.

Distance covered = (108 + 112) = 220 m.

$\therefore \frac{220}{\left(\frac{250+5x}{18}\right)} = 6 \Leftrightarrow 250 + 5x = 660 \Leftrightarrow x = 82$ km/hr.

37. Let the speed of train Y be x km/hr.

Speed of X relative to Y = $(120 - x)$ km/hr

$$= \left[(120-x) \times \frac{5}{18}\right] \text{m/sec} = \left(\frac{600-5x}{18}\right) \text{m/sec}.$$

$\therefore \frac{300}{\left(\frac{600-5x}{18}\right)} = 120 \Leftrightarrow 5400 = 120(600 - 5x) \Leftrightarrow x = 111.$

38. Relative speed = (36 + 45) km/hr = $\left(81 \times \frac{5}{18}\right)$ m/sec = $\left(\frac{45}{2}\right)$ m/sec.

Length of train = $\left(\frac{45}{2} \times 8\right)$ m = 180 m.

39. Relative speed = (40 – 20) km/hr = $\left(20 \times \frac{5}{18}\right)$ m/sec = $\left(\frac{50}{9}\right)$ m/sec.

Length of faster train = $\left(\frac{50}{9} \times 5\right)$ m = $\frac{250}{9}$ m = $27\frac{7}{9}$ m.

40. 2 kmph = $\left(2 \times \frac{5}{18}\right)$ m/sec = $\frac{5}{9}$ m/sec and 4 kmph = $\frac{10}{9}$ m/sec.

Let the length of the train be x metres and its speed be y m/sec.

Then, $\frac{x}{\left(y - \frac{5}{9}\right)} = 9$ and $\frac{x}{\left(y - \frac{10}{9}\right)} = 10.$

$\therefore 9y - 5 = x$ and $10(9y - 10) = 9x \Rightarrow 9y - x = 5$ and $90y - 9x = 100.$

On solving, we get : $x = 50$

$\therefore$ Length of the train is 50 m.

41. $4.5 \text{ km/hr} = \left(4.5 \times \frac{5}{18}\right) \text{m/sec} = \frac{5}{4} \text{m/sec} = 1.25 \text{ m/sec}$, and

$5.4 \text{ km/hr} = \left(5.4 \times \frac{5}{18}\right) \text{m/sec} = \frac{3}{2} \text{m/sec} = 1.5 \text{ m/sec}$.

Let the speed of the train be x m/sec.

Then, $(x - 1.25) \times 84 = (x - 1.5) \times 8.5$

$\Leftrightarrow 8.4x - 10.5 = 8.5x - 12.75 \Leftrightarrow 0.1x = 2.25 \Leftrightarrow x = 22.5$.

$\therefore$ Speed of the train $= \left(22.5 \times \frac{18}{5}\right) \text{km/hr} = 81 \text{ km/hr}$.

42. Let the speed of the slower train be x m/sec.

Then, speed of the faster train $= 2x$ m/sec.

Relative speed $= (x + 2x)$ m/sec $= 3x$ m/sec.

$\therefore \frac{(100 + 100)}{8} = 3x \Leftrightarrow 24x = 200 \Leftrightarrow x = \frac{25}{3}$.

So, speed of the faster train $= \frac{50}{3} \text{m/sec} = \left(\frac{50}{3} \times \frac{18}{5}\right) \text{km/hr} = 60 \text{ km/hr}$.

43. Speed of first train $= \left(\frac{150}{15}\right) \text{m/sec} = 10 \text{ m/sec}$.

Let the speed of second train be x m/sec.

Relative speed $= (10 + x)$ m/sec.

$\therefore \frac{300}{10 + x} = 8 \Leftrightarrow 300 = 80 + 8x \Leftrightarrow x = \frac{220}{8} = \frac{55}{2} \text{ m/sec}$.

So, speed of second train $= \left(\frac{55}{2} \times \frac{18}{5}\right) \text{kmph} = 99 \text{ kmph}$.

44. Let the length of the first train be x metres.

Then, the length of second train is $\left(\frac{x}{2}\right)$ metres.

Relative speed $= (48 + 42) \text{ kmph} = \left(90 \times \frac{5}{18}\right) \text{m/sec} = 25 \text{ m/sec}$.

$\therefore \frac{\left(x + \frac{x}{2}\right)}{25} = 12 \text{ or } \frac{3x}{2} = 300 \text{ or } x = 200$.

$\therefore$ Length of first train $= 200$ m.

Let the length of platform be y metres.

Speed of the first train $= \left(48 \times \frac{5}{18}\right) \text{m/sec} = \frac{40}{3} \text{m/sec}$.

$\therefore (200 + y) \times \frac{3}{40} = 45 \Leftrightarrow 600 + 3y = 1800 \Leftrightarrow y = 400$ m.

45. Let the speeds of the two trains be x m/sec and y m/sec respectively. Then, length of the first train $= 27x$ metres, and length of the second train $= 17y$ metres.

$\therefore \frac{27x + 17y}{x + y} = 23 \Leftrightarrow 27x + 17y = 23x + 23y \Leftrightarrow 4x = 6y \Leftrightarrow \frac{x}{y} = \frac{3}{2}$.

46. Suppose they meet x hours after 7 a.m.
Distance covered by A in x hours = $20x$ km.
Distance covered by B in $(x-1)$ hours = $25(x-1)$ km.
$\therefore \quad 20x + 25(x-1) = 110 \Leftrightarrow 45x = 135 \Leftrightarrow x = 3.$
So, they meet at 10 a.m.

47. Suppose, the distance between Meerut and Ghaziabad is x km.
Time taken by X to cover x km = 1 hour.

Time taken by Y to cover x km = $\frac{3}{2}$ hours.

$\therefore$ Speed of X = x kmph, Speed of Y = $\left(\frac{2x}{3}\right)$ kmph.

Let them meet y hours after 4 p.m. Then,

$$xy + \frac{2xy}{3} = x \Leftrightarrow y\left(1+\frac{2}{3}\right) = 1 \Leftrightarrow y = \frac{3}{5} \text{ hours} = \left(\frac{3}{5}\times 60\right) \text{ min} = 36 \text{ min}.$$

So, the two trains meet at 4.36 p.m.

48. Let us name the trains as A and B. Then,

(A's speed) : (B's speed) = $\sqrt{b} : \sqrt{a} = \sqrt{16} : \sqrt{9} = 4 : 3$.

EXERCISE 18B

(DATA SUFFICIENCY TYPE QUESTIONS)

1. A train running at a certain speed crosses a stationary engine in 20 seconds. To find out the speed of the train, which of the following information is necessary ?
   (*a*) Only the length of the train
   (*b*) Only the length of the engine
   (*c*) Either the length of the train or the length of the engine
   (*d*) Both the length of the train and the length of the engine

2. A train running at a certain speed crosses another train running in the opposite direction in 4.8 seconds. To find out the speed of the first train, which of the following information P and Q is sufficient ?
   P : The length of the first train Q : The length of the second train
   (*a*) Only P is sufficient (*b*) Only Q is sufficient
   (*c*) Either P or Q is sufficient (*d*) Both P and Q are needed
   (*e*) Both P and Q are not sufficient

Directions (*Questions 3 to 12*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the given question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

3. A train crosses a signal post in x seconds. What is the length of the train ?
   I. The train crosses a platform of 100 metres in y seconds.
   II. The train is running at the speed of 60 km / hr. **(NABARD, 2002)**
4. What was the speed of the running train ? **(Bank P.O. 2000)**
   I. Length of the train was 120 metres.
   II. The train crossed the other stationary train whose length was 180 m in 4 seconds.
5. What is the speed of a running train which takes 9 seconds to cross a signal post ?
   I. The length of the train is 90 metres.
   II. The train takes 27 seconds to cross a platform of 180 metres. **(Bank P.O. 1999)**
6. What is the length of a running train ? **(S.B.I.P.O. 1998)**
   I. The train crosses a man in 9 seconds.
   II. The train crosses a 240 metre long platform in 24 seconds.
7. What is the speed of the train ? **(Bank P.O. 2003)**
   I. 280 metres long train crosses a signal pole in 18 seconds.
   II. 280 metres long train crosses a platform in 45 seconds.
8. What was the speed of a running train X ?
   I. The relative speed of train X and another train Y running in opposite direction is 160 kmph.
   II. The train Y crosses a signal post in 9 seconds.
9. What was the length of a running train crossing another 180 metre long train running in the opposite direction ? **(Bank P.O. 1998)**
   I. The relative speed of the two trains was 150 kmph.
   II. The trains took 9 seconds to cross each other.
10. A train crosses another train running in the opposite direction in x seconds. What is the speed of the train ? **(S.B.I.P.O. 2003)**
    I. Both the trains have the same length and are running at the same speed.
    II. One train crosses a pole in 5 seconds.
11. A train crosses a pole in 10 seconds. What is the length of the train ?
    I. The train crosses another train running in opposite direction with a speed of 80 km / hr in 22 seconds.
    II. The speed of the train is 108 km / hr. **(Bank P.O. 2003)**
12. What is the speed of the train whose length is 210 metres ? **(Bank P.O. 2003)**
    I. The train crosses another train of 300 metres length running in opposite direction in 10 seconds.
    II. The train crosses another train running in the same direction at the speed of 60 km / hr in 30 seconds.

Directions (*Questions 13 to 17*) : ***Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is / are necessary to answer the question.***

13. What is the speed of the train ? **(S.B.I.P.O. 2002)**
    I. The train crosses a tree in 13 seconds.
    II. The train crosses a platform of length 250 metres in 27 seconds.
    III. The train crosses another train running in the same direction in 32 seconds.
    (*a*) I and II only (*b*) II and III only (*c*) I and III only
    (*d*) Any two of the three (*e*) None of these

14. What is the speed of the train ? (M.B.A. 2002)

I. The train crosses 300 metres long platform in 21 seconds.

II. The train crosses another stationary train of equal length in $19\frac{1}{2}$ seconds.

III. The train crosses a signal pole in $9\frac{3}{4}$ seconds.

(a) I and II only (b) I and either II or III only
(c) II and either I or II only (d) III and either I or II only
(e) None of these

15. What is the speed of the train ? (Bank P.O. 2003)

I. The train crosses a signal pole in 18 seconds.
II. The train crosses a platform of equal length in 36 seconds.
III. Length of the train is 330 metres.

(a) I and II only (b) II and III only (c) I and III only
(d) III and either I or II only (e) Any two of the three

16. What is the length of the train X ?

I. Train X crosses a telegraph post in 20 seconds.
II. Train X crosses a platform of length 800 m in 100 seconds.
III. Train X passes through a tunnel 400 m long in 60 seconds.

(a) I and either II or III only (b) II and III only
(c) II and either I or III only (d) III and either I or II only
(e) Any two of the three

17. What is the speed of the train ?

I. The train passes a man walking at the rate of 3 kmph in 9 seconds.
II. The train passes a man walking at the rate of 6 kmph in 10 seconds.
III. The train is moving in the same direction in which the two men are moving.

(a) I and III only (b) II and III only
(c) I and II only (d) All I, II and III
(e) Question cannot be answered even with information in all the three statements.

Directions (*Questions 18 to 20*) : *Each of these questions is followed by three statements. You have to study the question and all the three statements given to decide whether any information provided in the statement(s) is redundant and can be dispensed with while answering the given question.*

18. How much time will the train A take to cross another train B running in opposite direction ?

I. Train A crosses a signal pole in 6 seconds.
II. Ratio of the speeds of trains A and B is 3 : 2.
III. Length of the two trains together is 500 metres.

(a) I only (b) II only
(c) III only (d) I or II only
(e) Question cannot be answered even with the information in all the three statements.

19. What is the length of a running train P crossing another running train Q ?

I. These two trains take 18 seconds to cross each other.
II. These trains are running in opposite directions.
III. The length of train Q is 180 metres. (S.B.I.P.O. 1997)

(a) I only (b) II only
(c) III only (d) All I, II and III are required
(e) Even with I, II and III, the answer cannot be obtained.

20. At what time will the train reach city X from city Y ? **(S.B.I.P.O. 1999)**

I. The train crosses another train of equal length of 200 metres and running in opposite direction in 15 seconds.

II. The train leaves city Y at 7.15 a.m. for city X situated at a distance of 558 km.

III. The 200 metres long train crosses a signal pole in 10 seconds.

(*a*) I only (*b*) II only (*c*) III only
(*d*) I or III only (*e*) All I, II and III are required.

ANSWERS

1. (*d*)	**2.** (*e*)	**3.** (*c*)	**4.** (*e*)	**5.** (*a*)	**6.** (*e*)	**7.** (*a*)	**8.** (*d*)
9. (*e*)	**10.** (*d*)	**11.** (*b*)	**12.** (*e*)	**13.** (*a*)	**14.** (*b*)	**15.** (*d*)	**16.** (*e*)
17. (*d*)	**18.** (*e*)	**19.** (*e*)	**20.** (*a*)				

SOLUTIONS

1. Time taken by the train to cross a stationary engine

$$= \frac{\text{(Length of train + Length of engine)}}{\text{(Speed of the train)}}$$

$$\Rightarrow \frac{\text{(Length of train + Length of engine)}}{\text{(Speed of the train)}} = 20 \text{ (given)}$$

Hence, to find the speed of the train, the length of the train and the length of tne engine both must be known.

$\therefore$ The correct answer is (*d*).

2. Let two trains of lengths *a* and *b* metres be moving in opposite directions at *u* m / s and *v* m / s.

Time taken by the trains to cross each other $= \frac{(a+b)}{(u+v)}$ sec.

$\therefore \quad \frac{a+b}{u+v} = 4.8.$

In order to find *u*, we must know *a*, *b* and *v*,
i.e., length of first train, length of second train and the speed of the second train.

Thus, P and Q are not sufficient.

$\therefore$ The correct answer is (*e*).

3. Let the length of the train be *a* metres.

Time taken to cross a singal post $= \frac{\text{Length of the train}}{\text{Speed of the train}} \Rightarrow x = \frac{l}{\text{Speed.}}$ ...(*i*)

Time taken to cross the platform $= \frac{(l+100)}{\text{Speed}} \Rightarrow y = \frac{l+100}{\text{Speed}}$ ...(*ii*)

Thus, from (*i*) and (*ii*), we can find *l*.

Also, II gives, speed $= \left(80 \times \frac{5}{18}\right)$ m / s $= \frac{200}{9}$ m / s.

Thus, the data in I or II alone are sufficient to answer the question.

$\therefore$ The correct answer is (*e*).

4. Speed of the first train $= \frac{\text{(sum of the lengths of the two trains)}}{\text{Time taken}}$

$$= \frac{(120 + 180)}{4} \text{ m/s} = 75 \text{ m/s}.$$

So, both the statements are necessary to get the answer.

$\therefore$ The correct answer is (*e*).

5. Speed of the train $= \frac{\text{Length of the train}}{\text{Time taken to cross the post}} = \frac{90}{9} \text{ m/s} = 10 \text{ m/s}.$

Thus, I alone gives the answer.

Time taken to cross a platform $= \frac{\text{(Length of train + Length of platform)}}{\text{Speed of the train}}$

$\Rightarrow$ Speed $= \frac{(l + 180)}{27}$

But, l is not given. So, speed cannot be obtained.

So, II alone does not give the answer.

$\therefore$ The correct answer is (*a*).

6. Time taken by train to cross a man $= \frac{\text{Length of train}}{\text{Speed of train}} \Rightarrow \text{Speed} = \frac{l}{9}$ ...(*i*)

Time taken by train to cross a platform $= \frac{\text{(Length of train + Length of platform)}}{\text{Speed of the train}}$

$\Rightarrow$ Speed $= \frac{l + 240}{24}$ ...(*ii*)

From (*i*) and (*ii*), we get $\frac{l}{9} = \frac{l + 240}{24}$.

Thus, l can be obtained. So both I and II are necessary to get the answer.

$\therefore$ The correct answer is (*c*).

7. Speed $= \frac{\text{Length of the train}}{\text{Time taken to cross the pole}} = \frac{280}{18} \text{ m/s} = \frac{140}{9} \text{ m/s}.$

$\therefore$ I alone gives the answer.

Time taken to cross the platform $= \frac{\text{(Length of train + Length of platform)}}{\text{Speed of the train}}$

$\Rightarrow$ Speed $= \frac{(280 + p)}{45}$ m/s.

But, p = length of platform, is not given.

$\therefore$ II is not sufficient to give the answer.

$\therefore$ The correct answer is (*a*).

8. Let the two trains of length a metres and b metres be moving in opposite directions at u m/s and v m/s. Then,

I gives, $u + v = 160$.

II gives, $v = \frac{b}{9}$.

From these equations, we cannot obtain u.

$\therefore$ The correct answer is (*d*).

9. Let the two trains of length a metres and b metres be moving in opposite directions at u m/s and v m/s.

Time taken to cross each other $= \frac{(a+b)}{(u+v)}$ sec.

Now, $b = 180$, $u + v = \left(150 \times \frac{5}{18}\right)$ m/sec $= \frac{125}{3}$ m/sec.

$\Rightarrow 9 = \frac{a+180}{(125/3)} \Rightarrow a = (375 - 180) = 195$ m.

Thus, both I and II are necessary to get the answer.

$\therefore$ The correct answer is (e).

10. Let the two trains of length a metres and b metres be moving in opposite directions at u m/s and v m/s.

Time taken to cross each other $= \frac{(a+b)}{(u+v)}$ m/sec. $\Rightarrow x = \frac{(a+a)}{(u+u)} = \frac{a}{u}$. ...(i)

Time taken to cross the pole $= \frac{\text{Length of the train}}{\text{Speed of the train}} = \frac{a}{u} \Rightarrow \frac{a}{u} = 5$ ...(ii)

From (i) and (ii) also, we cannot find u.

$\therefore$ The correct answer is (d).

11. Time taken to cross a pole $= \frac{\text{Length of train}}{\text{Speed of train}} \Rightarrow 10 = \frac{\text{Length of train}}{\left(108 \times \frac{5}{18}\right)}$

$\Rightarrow$ Length of the train = 300 m.

Clearly, II is sufficient to get the answer.

Also, I is not sufficient to get the answer.

$\therefore$ The correct answer is (b).

12. Time taken to cross the train, running in opposite directions $= \frac{(l_1 + l_2)}{(u+v)}$ sec.

$\Rightarrow 10 = \frac{(210 + 300)}{(u+v)} \Rightarrow u + v = 51.$

Time taken to cross the train, running in same direction $= \frac{(l_1 + l_2)}{(u-v)}$ sec.

$\Rightarrow 30 = \frac{(210 + 300)}{\left(u - 60 \times \frac{5}{18}\right)} \Rightarrow u = \left(17 + \frac{50}{3}\right)$ m/sec.

Thus, u and v can be obtained.

$\therefore$ Correct answer is (e).

13. Let the speed of the train be x metres/sec.

Time taken to cross a tree $= \frac{\text{Length of the train}}{\text{Speed of the train}}$

Time taken to cross a platform $= \frac{(\text{Length of train} + \text{Length of platform})}{\text{Speed of the train}}$ ...(ii)

I gives, $13 = \frac{l}{x} \Rightarrow 13x$

II gives $27 = \frac{l + 250}{x} \Rightarrow \frac{13x + 250}{x} = 24 \Rightarrow x = \frac{125}{7}$ m/sec.

Thus I and II give the speed of the train.

$\therefore$ The correct answer is (a)

14. Let the speed of the train be x m/sec.

Time taken to cross a platform $= \dfrac{\text{(Length of train + Length of platform)}}{\text{Speed of the train}}$

Time taken by the train to cross a stationary train

$= \dfrac{\text{(Sum of the lengths of the trains)}}{\text{Speed of moving train}}$

Time taken to cross a signal pole $= \dfrac{\text{Length of train}}{\text{Speed of train}}$

I gives, $21 = \dfrac{(l + 300)}{x}$; II gives, $\dfrac{39}{2} = \dfrac{2l}{x}$; III gives, $\dfrac{39}{4} = \dfrac{l}{x}$.

Thus, (I and II) or (I and III) give x.

$\therefore$ Correct answer is (*b*).

15. Let the speed of the train be x m/sec.

Time taken to cross a signal pole $= \dfrac{\text{Length of train}}{\text{Speed of train}}$.

Time taken to cross a platform $= \dfrac{\text{(Length of train + Length of platform)}}{\text{Speed of the train}}$

Length of train = 330 m.

I and III give, $18 = \dfrac{330}{x} \Rightarrow x = \dfrac{330}{18}$ m/s $= \dfrac{55}{3}$ m/s.

II and III give, $36 = \dfrac{2 \times 330}{x} \Rightarrow x = \dfrac{660}{36}$ m/s $= \dfrac{55}{3}$ m/s.

$\therefore$ Correct answer is (*d*).

16. Time taken to cross a pole $= \dfrac{\text{Length of train}}{\text{Its speed}} \Rightarrow 20 = \dfrac{l}{\text{speed}} \Rightarrow \text{speed} = \dfrac{l}{20}$ ...(*i*)

Time taken to cross a platform $= \dfrac{(l + 800)}{\text{speed}}$

$\Rightarrow 100 = \dfrac{(l + 800)}{\text{speed}} \Rightarrow \text{speed} = \dfrac{(l + 800)}{100}$ ...(*ii*)

Time taken to pass through a tunnel $= \dfrac{(l + 400)}{60}$

$\Rightarrow 60 = \dfrac{(l + 400)}{\text{speed}} \Rightarrow \text{speed} = \dfrac{(l + 400)}{60}$ ...(*iii*)

Equating any two out of three will give us l.

$\therefore$ Correct answer is (*e*).

17. Let the speed of the train be x m/sec.

III gives that the men are moving in the same direction.

I gives, time taken to pass a man $= \dfrac{l}{\left(x - 3 \times \dfrac{5}{18}\right)} = \left(\dfrac{6l}{6x - 5}\right)$ sec.

$\therefore \dfrac{6l}{6x - 5} = 9 \Rightarrow 54x - 6l = 45 \Rightarrow 18x - 2l = 15$ ...(

II gives, time taken to pass another man $= \dfrac{l}{\left(x - 6 \times \dfrac{5}{18}\right)}$ sec $= \dfrac{3l}{(3x - 5)}$ sec.

$\therefore \quad \dfrac{3l}{(3x-5)} = 10 \Rightarrow 30x - 3l = 50$ ...(ii)

On solving (i) and (ii), we get : $x = \dfrac{55}{6}$ m/sec.

Thus, all I, II, III are needed to get the answer.

$\therefore$ (d) is correct.

18. II. Let the speeds of A and B be $3x$ m/sec and $2x$ m/sec.

I. Length of train A $= (3x \times 6)$ m $= 18x$ metres.

III. Length of train B $= (500 - 18x)$ m.

Relative speed $= (3x + 2x)$ m/sec $= 5x$ m/sec.

Time taken by A to cross B $= \dfrac{\text{Sum of their lengths}}{\text{Relative speed}} = \dfrac{500}{5x}$ sec.

Thus, even with the information in all the three statements, question cannot be answered.

$\therefore$ Correct answer is (e).

19. Let the length of train P be x metres.

II. These trains are running in opposite directions.

III. Length of train Q is 180 m.

I. Time taken by P to cross Q $= \dfrac{(180 + x)}{\text{Relative speed}} \Rightarrow 18 = \dfrac{(180 + x)}{\text{Relative speed}}$

Thus, even with I, II and III, the answer cannot be obtained.

$\therefore$ Correct answer is (e).

20. III gives, speed $= \dfrac{200}{10}$ m/s $= 20$ m/s $= \left(20 \times \dfrac{18}{5}\right)$ km/hr $= 72$ km/hr.

II gives, time taken $= \left(\dfrac{558}{72}\right)$ hrs $= \dfrac{31}{4}$ hrs $= 7\dfrac{3}{4}$ hrs $= 7$ hrs 45 min.

So, the train will reach city X at 3 p.m.

Hence, I is redundant.

19. BOATS AND STREAMS

IMPORTANT FACTS AND FORMULAE

1. In water, the direction along the stream is called ***downstream***. And, the direction against the stream is called ***upstream***.
2. If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr, then :

$$\textit{Speed downstream} = (u + v) \text{ km/hr}$$

$$\textit{Speed upstream} = (u - v) \text{ km/hr}.$$

3. If the speed downstream is a km/hr and the speed upstream is b km/hr, then :

$$\textit{Speed in still water} = \frac{1}{2}(a + b) \text{ km/hr}$$

$$\textit{Rate of stream} = \frac{1}{2}(a - b) \text{ km/hr}$$

SOLVED EXAMPLES

Ex. 1. ***A man can row upstream at 7 kmph and downstream at 10 kmph. Find man's rate in still water and the rate of current.***

Sol. Rate in still water $= \frac{1}{2}(10 + 7)$ km/hr $= 8.5$ km/hr.

Rate of current $= \frac{1}{2}(10 - 7)$ km/hr $= 1.5$ km/hr.

Ex. 2. ***A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 2 hours 30 minutes to cover a distance of 5 km upstream. Find the speed of the river current in km/hr.***

Sol. Rate downstream $= \left(\frac{15}{3\frac{3}{4}}\right)$ km/hr $= \left(15 \times \frac{4}{15}\right)$ km/hr $= 4$ km/hr.

Rate upstream $= \left(\frac{5}{2\frac{1}{2}}\right)$ km/hr $= \left(5 \times \frac{2}{5}\right)$ km/hr $= 2$ km/hr.

$\therefore$ Speed of current $= \frac{1}{2}(4 - 2)$ km/hr $= 1$ km/hr.

Ex. 3. ***A man can row 18 kmph in still water. It takes him thrice as long to row up as to row down the river. Find the rate of stream.***

Sol. Let man's rate upstream be x kmph. Then, his rate downstream $= 3x$ kmph.

$\therefore$ Rate in still water $= \frac{1}{2}(3x + x)$ kmph $= 2x$ kmph.

So, $2x = 18$ or $x = 9$.

$\therefore$ Rate upstream $= 9$ km/hr, Rate downstream $= 27$ km/hr.

Hence, rate of stream $= \frac{1}{2}(27 - 9)$ km/hr $= 9$ km/hr.

Ex. 4. ***There is a road beside a river. Two friends started from a place A, moved to a temple situated at another place B and then returned to A again. One of them moves on a cycle at a speed of 12 km/hr, while the other sails on a boat at a speed of 10 km/hr. If the river flows at the speed of 4 km/hr, which of the two friends will return to place A first ?*** (R.R.B. 2001)

Sol. Clearly, the cyclist moves both ways at a speed of 12 km/hr.

So, average speed of the cyclist = 12 km/hr.

The boat sailor moves downstream @ (10 + 4) *i.e.*, 14 km/hr and upstream @ (10 − 4) *i.e.*, 6 km/hr.

So, average speed of the boat sailor $= \left(\frac{2 \times 14 \times 6}{14 + 6}\right)$ km/hr

$= \frac{42}{5}$ km/hr = 8.4 km/hr.

Since the average speed of the cyclist is greater, he will return to A first.

Ex. 5. ***A man can row $7\frac{1}{2}$ kmph in still water. If in a river running at 1.5 km an hour, it takes him 50 minutes to row to a place and back, how far off is the place ?*** (R.R.B. 2002)

Sol. Speed downstream = (7.5 + 1.5) kmph = 9 kmph;

Speed upstream = (7.5 − 1.5) kmph = 6 kmph.

Let the required distance be x km. Then,

$\frac{x}{9} + \frac{x}{6} = \frac{50}{60} \Leftrightarrow 2x + 3x = \left(\frac{5}{6} \times 18\right) \Leftrightarrow 5x = 15 \Leftrightarrow x = 3.$

Hence, the required distance is 3 km.

Ex. 6. ***In a stream running at 2 kmph, a motorboat goes 6 km upstream and back again to the starting point in 33 minutes. Find the speed of the motorboat in still water.***

Sol. Let the speed of the motorboat in still water be x kmph. Then,

Speed downstream = $(x + 2)$ kmph; Speed upstream = $(x - 2)$ kmph.

$\therefore \quad \frac{6}{x+2} + \frac{6}{x-2} = \frac{33}{60} \Leftrightarrow 11x^2 - 240x - 44 = 0 \Leftrightarrow 11x^2 - 242x + 2x - 44 = 0$

$\Leftrightarrow (x - 22)(11x + 2) = 0 \Leftrightarrow x = 22.$

Hence, speed of motorboat in still water = 22 kmph.

Ex. 7. ***A man can row 40 km upstream and 55 km downstream in 13 hours. Also, he can row 30 km upstream and 44 km downstream in 10 hours. Find the speed of the man in still water and the speed of the current.***

Sol. Let rate upstream = x km/hr and rate downstream = y km/hr.

Then, $\frac{40}{x} + \frac{55}{y} = 13$ ...(*i*) and $\frac{30}{x} + \frac{44}{y} = 10$ ...(*ii*)

Multiplying (*ii*) by 4 and (*i*) by 3 and subtracting, we get : $\frac{11}{y} = 1$ or $y = 11$.

Substituting $y = 11$ in (*i*), we get : $x = 5$.

$\therefore$ Rate in still water $= \frac{1}{2}(11 + 5)$ kmph = 8 kmph.

Rate of current $= \frac{1}{2}(11 - 5)$ kmph = 3 kmph.

EXERCISE 19A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. In one hour, a boat goes 11 km along the stream and 5 km against the stream. The speed of the boat in still water (in km / hr) is : **(S.S.C. 2000)**
   (*a*) 3 (*b*) 5 (*c*) 8 (*d*) 9

2. A man can row upstream at 8 kmph and downstream at 13 kmph. The speed of the stream is :
   (*a*) 2.5 km / hr (*b*) 4.2 km / hr (*c*) 5 km / hr (*d*) 10.5 km / hr

3. A man rows downstream 32 km and 14 km upstream. If he takes 6 hours to cover each distance, then the velocity (in kmph) of the current is :
   (*a*) $\frac{1}{2}$ (*b*) 1 (*c*) $1\frac{1}{2}$ (*d*) 2

4. A boat running downstream covers a distance of 16 km in 2 hours while for covering the same distance upstream, it takes 4 hours. What is the speed of the boat in still water ? **(S.B.I.P.O. 2002)**
   (*a*) 4 km / hr (*b*) 6 km / hr (*c*) 8 km / hr (*d*) Data inadequate

5. A boatman goes 2 km against the current of the stream in 1 hour and goes 1 km along the current in 10 minutes. How long will it take to go 5 km in stationary water ?
   (*a*) 40 minutes (*b*) 1 hour (*c*) 1 hr 15 min (*d*) 1 hr 30 min **(R.R.B. 2002)**

6. A man can row three-quarters of a kilometre against the stream in $11\frac{1}{4}$ minutes. The speed (in km / hr) of the man in still water is : **(L.I.C.A.A.O. 2003)**
   (*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

7. A man takes twice as long to row a distance against the stream as to row the same distance in favour of the stream. The ratio of the speed of the boat (in still water) and the stream is : **(S.S.C. 1998)**
   (*a*) 2 : 1 (*b*) 3 : 1 (*c*) 3 : 2 (*d*) 4 : 3

8. A boat running upstream takes 8 hours 48 minutes to cover a certain distance, while it takes 4 hours to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the water current respectively ?
   (*a*) 2 : 1 (*b*) 3 : 2 (*c*) 8 : 3
   (*d*) Cannot be determined (*e*) None of these **(Bank P.O. 2003)**

9. If a boat goes 7 km upstream in 42 minutes and the speed of the stream is 3 kmph, then the speed of the boat in still water is :
   (*a*) 4.2 km / hr (*b*) 9 km / hr (*c*) 13 km / hr (*d*) 21 km / hr

10. A man's speed with the current is 15 km / hr and the speed of the current is 2.5 km / hr. The man's speed against the current is : **(M.A.T. 1997)**
    (*a*) 8.5 km / hr (*b*) 9 km / hr (*c*) 10 km / hr (*d*) 12.5 km / hr

11. If a man rows at the rate of 5 kmph in still water and his rate against the current is 3.5 kmph, then the man's rate along the current is :
    (*a*) 4.25 kmph (*b*) 6 kmph (*c*) 6.5 kmph (*d*) 8.5 kmph

12. A boat can travel with a speed of 13 km / hr in still water. If the speed of the stream is 4 km / hr, find the time taken by the boat to go 68 km downstream. **(R.R.B. 2003)**
    (*a*) 2 hours (*b*) 3 hours (*c*) 4 hours (*d*) 5 hours

13. Speed of a boat in standing water is 9 kmph and the speed of the stream is 1.5 kmph. A man rows to a place at a distance of 105 km and comes back to the starting point. The total time taken by him is :

(a) 16 hours (b) 18 hours (c) 20 hours (d) 24 hours

14. The speed of a boat in still water is 15 km/hr and the rate of current is 3 km/hr. The distance travelled downstream in 12 minutes is :

(a) 1.2 km (b) 1.8 km (c) 2.4 km (d) 3.6 km

15. A man can row at 5 kmph in still water. If the velocity of current is 1 kmph and it takes him 1 hour to row to a place and come back, how far is the place ?

(a) 2.4 km (b) 2.5 km (c) 3 km (d) 3.6 km

(S.S.C. 2004)

16. A boat takes 19 hours for travelling downstream from point A to point B and coming back to a point C midway between A and B. If the velocity of the stream is 4 kmph and the speed of the boat in still water is 14 kmph, what is the distance between A and B ?

(a) 160 km (b) 180 km (c) 200 km (d) 220 km

17. A man can row $9\frac{1}{3}$ kmph in still water and finds that it takes him thrice as much time to row up than as to row down the same distance in the river. The speed of the current is :

(a) $3\frac{1}{3}$ km/hr (b) $3\frac{1}{9}$ km/hr (c) $4\frac{2}{3}$ km/hr (d) $4\frac{1}{2}$ km/hr

18. A boat covers a certain distance downstream in 1 hour, while it comes back in $1\frac{1}{2}$ hours. If the speed of the stream be 3 kmph, what is the speed of the boat in still water ?

(Bank P.O. 2003)

(a) 12 kmph (b) 13 kmph (c) 14 kmph
(d) 15 kmph (e) None of these

19. A motorboat, whose speed is 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. The speed of the stream (in km/hr) is :

(a) 4 (b) 5 (c) 6 (d) 10

(R.R.B. 2002)

20. The speed of a boat in still water is 10 km/hr. If it can travel 26 km downstream and 14 km upstream in the same time, the speed of the stream is :

(a) 2 km/hr (b) 2.5 km/hr (c) 3 km/hr (d) 4 km/hr

21. A boat takes 90 minutes less to travel 36 miles downstream than to travel the same distance upstream. If the speed of the boat in still water is 10 mph, the speed of the stream is :

(M.A.T. 1997)

(a) 2 mph (b) 2.5 mph (c) 3 mph (d) 4 mph

22. A man rows to a place 48 km distant and back in 14 hours. He finds that he can row 4 km with the stream in the same time as 3 km against the stream. The rate of the stream is :

(a) 1 km/hr (b) 1.5 km/hr (c) 1.8 km/hr (d) 3.5 km/hr

23. A boat covers 24 km upstream and 36 km downstream in 6 hours while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$ hours. The velocity of the current is :

(a) 1 km/hr (b) 1.5 km/hr (c) 2 km/hr (d) 2.5 km/hr

24. At his usual rowing rate, Rahul can travel 12 miles downstream in a certain river in 6 hours less than it takes him to travel the same distance upstream. But if he could double his usual rowing rate for his 24-mile round trip, the downstream 12 miles would then take only one hour less than the upstream 12 miles. What is the speed of the current in miles per hour? **(M.A.T. 2001)**

(a) $1\frac{1}{3}$ (b) $1\frac{2}{3}$ (c) $2\frac{1}{3}$ (d) $2\frac{2}{3}$

ANSWERS

1. (c)	2. (a)	3. (c)	4. (b)	5. (c)	6. (d)	7. (b)	8. (c)
9. (c)	10. (c)	11. (c)	12. (c)	13. (d)	14. (d)	15. (a)	16. (b)
17. (c)	18. (d)	19. (b)	20. (c)	21. (a)	22. (a)	23. (c)	24. (d)

SOLUTIONS

1. Speed in still water $= \frac{1}{2}(11+5)$ kmph $= 8$ kmph.

2. Speed of stream $= \frac{1}{2}(13-8)$ kmph $= 2.5$ kmph.

3. Rate downstream $= \left(\frac{32}{6}\right)$ kmph; Rate upstream $= \left(\frac{14}{6}\right)$ kmph.

   $\therefore$ Velocity of current $= \frac{1}{2}\left(\frac{32}{6}-\frac{14}{6}\right)$ kmph $= \frac{3}{2}$ kmph $= 1.5$ kmph.

4. Rate downstream $= \left(\frac{16}{2}\right)$ kmph $= 8$ kmph; Rate upstream $= \left(\frac{16}{4}\right)$ kmph $= 4$ kmph.

   $\therefore$ Speed in still water $= \frac{1}{2}(8+4)$ kmph $= 6$ kmph.

5. Rate downstream $= \left(\frac{1}{10}\times 60\right)$ km/hr $= 6$ km/hr; Rate upstream $= 2$ km/hr.

   Speed in still water $= \frac{1}{2}(6+2)$ km/hr $= 4$ km/hr.

   $\therefore$ Required time $= \left(\frac{5}{4}\right)$ hrs $= 1\frac{1}{4}$ hrs $= 1$ hr 15 min.

6. Rate upstream $= \left(\frac{750}{675}\right)$ m/sec $= \frac{10}{9}$ m/sec;

   Rate downstream $= \left(\frac{750}{450}\right)$ m/sec $= \frac{5}{3}$ m/sec.

   $\therefore$ Rate in still water $= \frac{1}{2}\left(\frac{10}{9}+\frac{5}{3}\right)$ m/sec $= \frac{25}{18}$ m/sec $= \left(\frac{25}{18}\times\frac{18}{5}\right)$ km/hr

   $= 5$ km/hr.

7. Let man's rate upstream be x kmph. Then, his rate downstream $= 2x$ kmph.

   $\therefore$ (Speed in still water) : (Speed of stream) $= \left(\frac{2x+x}{2}\right) : \left(\frac{2x-x}{2}\right) = \frac{3x}{2} : \frac{x}{2} = 3 : 1.$

8. Let the man' rate upstream be x kmph and that downstream be y kmph. Then,
Distance covered upstream in 8 hrs 48 min. = Distance covered downstream in 4 hrs.

$$\Rightarrow \left(x \times 8\frac{4}{5}\right) = (y \times 4) \Rightarrow \frac{44}{5}x = 4y \Rightarrow y = \frac{11}{5}x.$$

$$\therefore \text{ Required ratio } = \left(\frac{y+x}{2}\right) : \left(\frac{y-x}{2}\right) = \left(\frac{16x}{5} \times \frac{1}{2}\right) : \left(\frac{6x}{5} \times \frac{1}{2}\right) = \frac{8}{5} : \frac{3}{5} = 8 : 3.$$

9. Rate upstream $= \left(\frac{7}{42} \times 60\right)$ kmph $= 10$ kmph.

Speed of stream $= 3$ kmph.
Let speed in still water be x km/hr. Then, speed upstream $= (x - 3)$ km/hr.
$\therefore \quad x - 3 = 10 \quad$ or $\quad x = 13$ km/hr.

10. Man's rate in still water $= (15 - 2.5)$ km/hr $= 12.5$ km/hr.
Man's rate against the current $= (12.5 - 2.5)$ km/hr $= 10$ km/hr.

11. Let the rate along the current be x kmph. Then, $\frac{1}{2}(x + 3.5) = 5$ or $x = 6.5$ kmph.

12. Speed downstream $= (13 + 4)$ km/hr $= 17$ km/hr.

Time taken to travel 68 km downstream $= \left(\frac{68}{17}\right)$ hrs $= 4$ hrs.

13. Speed upstream $= 7.5$ kmph; Speed downstream $= 10.5$ kmph.

$$\therefore \text{ Total time taken } = \left(\frac{105}{7.5} + \frac{105}{10.5}\right) \text{ hours } = 24 \text{ hours.}$$

14. Speed downstream $= (15 + 3)$ kmph $= 18$ kmph.

Distance travelled $= \left(18 \times \frac{12}{60}\right)$ km $= 3.6$ km.

15. Speed downstream $= (5 + 1)$ kmph $= 6$ kmph; Speed upstream $= (5 - 1)$ kmph $= 4$ kmph.
Let the required distance be x km.

Then, $\frac{x}{6} + \frac{x}{4} = 1 \Leftrightarrow 2x + 3x = 12 \Leftrightarrow 5x = 12 \Leftrightarrow x = 2.4$ km.

16. Speed downstream $= (14 + 4)$ km/hr $= 18$ km/hr;
Speed upstream $= (14 - 4)$ km/hr $= 10$ km/hr.
Let the distance between A and B be x km. Then,

$$\frac{x}{18} + \frac{(x/2)}{10} = 19 \Leftrightarrow \frac{x}{18} + \frac{x}{20} = 19 \Leftrightarrow \frac{19x}{180} = 19 \Leftrightarrow x = 180 \text{ km.}$$

17. Let speed upstream be x kmph. Then, speed downstream $= 3x$ kmph.

Speed in still water $= \frac{1}{2}(3x + x)$ kmph $= 2x$ kmph.

$$\therefore \quad 2x = \frac{28}{3} \Rightarrow x = \frac{14}{3}.$$

So, Speed upstream $= \frac{14}{3}$ km/hr; Speed downstream $= 14$ km/hr.

Hence, speed of the current $= \frac{1}{2}\left(14 - \frac{14}{3}\right)$ km/hr $= \frac{14}{3}$ km/hr $= 4\frac{2}{3}$ km/hr.

18. Let the speed of the boat in still water be x kmph. Then,
Speed downstream $= (x + 3)$ kmph, Speed upstream $= (x - 3)$ kmph.

$$\therefore \quad (x + 3) \times 1 = (x - 3) \times \frac{3}{2} \Leftrightarrow 2x + 6 = 3x - 9 \Leftrightarrow x = 15 \text{ kmph.}$$

19. Let the speed of the stream be x km / hr. Then,

Speed downstream = $(15 + x)$ km / hr, Speed upstream = $(15 - x)$ km / hr.

$$\therefore \quad \frac{30}{(15+x)} + \frac{30}{(15-x)} = 4\frac{1}{2} \quad \Leftrightarrow \quad \frac{900}{225-x^2} = \frac{9}{2} \quad \Leftrightarrow \quad 9x^2 = 225$$

$$\Leftrightarrow \quad x^2 = 25 \quad \Leftrightarrow \quad x = 5 \text{ km / hr.}$$

20. Let the speed of the stream be x km / hr. Then,

Speed downstream = $(10 + x)$ km / hr, Speed upstream = $(10 - x)$ km / hr.

$$\therefore \quad \frac{26}{(10+x)} = \frac{14}{(10-x)} \quad \Leftrightarrow \quad 260 - 26x = 140 + 14x \quad \Leftrightarrow \quad 40x = 120 \quad \Leftrightarrow \quad x = 3 \text{ km / hr.}$$

21. Let the speed of the stream be x mph. Then,

Speed downstream = $(10 + x)$ mph, Speed upstream = $(10 - x)$ mph.

$$\therefore \quad \frac{36}{(10-x)} - \frac{36}{(10+x)} = \frac{90}{60} \quad \Leftrightarrow \quad 72x \times 60 = 90\,(100 - x^2) \quad \Leftrightarrow \quad x^2 + 48x + 100 = 0$$

$$\Leftrightarrow \quad (x + 50)\,(x - 2) = 0 \quad \Leftrightarrow \quad x = 2 \text{ mph.}$$

22. Suppose he moves 4 km downstream in x hours. Then,

Speed downstream = $\left(\frac{4}{x}\right)$ km / hr, Speed upstream = $\left(\frac{3}{x}\right)$ km / hr.

$$\therefore \quad \frac{48}{(4/x)} + \frac{48}{(3/x)} = 14 \text{ or } x = \frac{1}{2}.$$

So, Speed downstream = 8 km / hr, Speed upstream = 6 km / hr.

Rate of the stream = $\frac{1}{2}(8 - 6)$ km / hr = 1 km / hr.

23. Let rate upstream = x kmph and rate downstream = y kmph.

Then, $\frac{24}{x} + \frac{36}{y} = 36$ ...(*i*) and $\frac{36}{x} + \frac{24}{y} = \frac{13}{2}$ ...(*ii*)

Adding (*i*) and (*ii*), we get : $60\left(\frac{1}{x} + \frac{1}{y}\right) = \frac{25}{2}$ or $\frac{1}{x} + \frac{1}{y} = \frac{5}{24}$ ...(*iii*)

Subtracting (*i*) from (*ii*), we get : $12\left(\frac{1}{x} - \frac{1}{y}\right) = \frac{1}{2}$ or $\frac{1}{x} - \frac{1}{y} = \frac{1}{24}$ ...(*iv*)

Adding (*iii*) and (*iv*), we get : $\frac{2}{x} = \frac{6}{24}$ or $x = 8$.

So, $\frac{1}{8} + \frac{1}{y} = \frac{5}{24} \quad \Leftrightarrow \quad \frac{1}{y} = \left(\frac{5}{24} - \frac{1}{8}\right) = \frac{1}{12} \quad \Leftrightarrow \quad y = 12.$

$\therefore$ Speed upstream = 8 kmph, Speed downstream = 12 kmph.

Hence, rate of current = $\frac{1}{2}(12 - 8)$ kmph = 2 kmph.

24. Let the speed in still water be x mph and the speed of the current be y mph. Then,

Speed upstream = $(x - y)$; Speed downstream = $(x + y)$

$$\therefore \quad \frac{12}{(x-y)} - \frac{12}{(x+y)} = 6 \quad \Leftrightarrow \quad 6\,(x^2 - y^2) = 24y \quad \Leftrightarrow \quad x^2 - y^2 = 4y$$

$$\Leftrightarrow \quad x^2 = (4y + y^2) \qquad \ldots(i)$$

And, $\frac{12}{(2x-y)} - \frac{12}{(2x+y)} = 1 \quad \Leftrightarrow \quad 4x^2 - y^2 = 24y \quad \Leftrightarrow \quad x^2 = \frac{24y + y^2}{4}$ ...(*ii*)

From (*i*) and (*ii*), we have :

$$4y + y^2 = \frac{24y + y^2}{4} \Leftrightarrow 16y + 4y^2 = 24y + y^2 \Leftrightarrow 3y^2 = 8y \Leftrightarrow y = \frac{8}{3}.$$

$\therefore$ Speed of the current $= \frac{8}{3}$ mph $= 2\frac{2}{3}$ mph.

EXERCISE 19B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 6*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and***

Give answer (*a*) ***if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;***

Give answer (*b*) ***if the data in Statement II alone are sufficient to answer the question while the data in statement I alone are not sufficient to answer the question;***

Give answer (*c*) ***if the data either in Statement I or in Statement II alone are sufficient to answer the question;***

Give answer (*d*) ***if the data even in both Statements I and II together are not sufficient to answer the question;***

Give answer (*e*) ***if the data in both Statements I and II together are necessary to answer the question.***

1. What is the speed of the boat in still water ? **(Bank P.O. 2003)**
   I. It takes 2 hours to cover the distance between A and B downstream.
   II. It takes 4 hours to cover the distance between A and B upstream.
2. What is the speed of the stream ?
   I. The ratio of the speed upstream to the speed downstream of a boat is 2 : 3.
   II. The distance travelled upstream in 2 hours by the boat is more than the distance travelled by it downstream in 1 hour by 4 km.
3. What is the speed of the boat in still water ? **(Bank P.O. 2003)**
   I. The boat covers a distance of 48 kms in 6 hours while running upstream.
   II. The boat covers the same distance in 4 hours while running downstream.
4. What is the man's speed in still water ?
   I. The speed of the stream is one-third of the man's speed in still water.
   II. In a given time, the man can swim twice as far with the stream as he can against it.
5. A boat takes a total time of three hours to travel downstream from P to Q and upstream back from Q to P. What is the speed of the boat in still water ?
   I. The speed of the river current is 1 km per hour.
   II. The distance between P and Q is 4 km. **(S.B.I.P.O. 1997)**
6. What is the speed of the boat in still water ?
   I. The speed downstream of the boat is thrice the speed upstream.
   II. The sum of the speeds of the boat, upstream and downstream is 12 kmph.

Directions (*Questions 7-8*) : ***Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the questions.***

7. What is the speed of the boat in still water ?
   I. The speed downstream is 12 kmph.
   II. The speed upstream is 4 kmph.
   III. In a to and fro journey between two points, the average speed of the boat was 6 kmph.
   (*a*) I and II only (*b*) All I, II and III (*c*) III, and either I or II
   (*d*) Any two of the three (*e*) None of these

8. What is the speed of stream ? **(Bank P.O. 2004)**
   I. The boat covers 24 km in 6 hours moving upstream.
   II. The boat covers 24 km in 3 hours moving downstream.
   III. The ratio between the speed of boat and stream is 3 : 1 respectively.
   (*a*) Any two of the three (*b*) I and II only (*c*) II and III only
   (*d*) I and III only (*e*) All I, II and III

ANSWERS

1. (*d*) 2. (*e*) 3. (*e*) 4. (*d*) 5. (*e*) 6. (*b*) 7. (*d*) 8. (*a*)

SOLUTIONS

1. Let AB = x km.

   I. Speed downstream = $\frac{x}{2}$ km/hr. II. Speed upstream = $\frac{x}{4}$ km/hr.

   Speed of boat in still water = $\frac{1}{2}\left(\frac{x}{2}+\frac{x}{4}\right)$ km/hr.

   Thus, I and II both even do not give the answer.

   ∴ Correct answer is (*d*).

2. I. Let speed upstream = $2x$ km/hr and speed downstream = $3x$ km/hr.

   II. $(2 \times 3x) - (1 \times 2x) = 4 \Leftrightarrow 4x = 4 \Leftrightarrow x = 1$.

   ∴ Speed upstream = 2 km/hr, speed downstream = 3 km/hr.

   Speed of the stream = $\frac{1}{2}(3-2)$ km/hr = $\frac{1}{2}$ km/hr.

   Thus, I and II together give the answer.

   ∴ Correct answer is (*e*).

3. I. Speed upstream = $\frac{48}{6}$ km/hr = 8 km/hr.

   II. Speed downstream = $\frac{48}{4}$ km/hr = 12 km/hr.

   Speed of the boat = $\frac{1}{2}(8+12)$ km/hr = 10 km/hr.

   Thus, I and II together give the answer.

   ∴ Correct answer is (*e*).

4. Let man's speed in still water be x km/hr.

   I. Speed of the stream = $\frac{x}{3}$ km/hr.

   Speed downstream = $\left(x+\frac{x}{3}\right)$ km/hr = $\frac{4x}{3}$ km/hr.

   Speed upstream = $\left(x-\frac{x}{3}\right)$ km/hr = $\frac{2x}{3}$ km/hr.

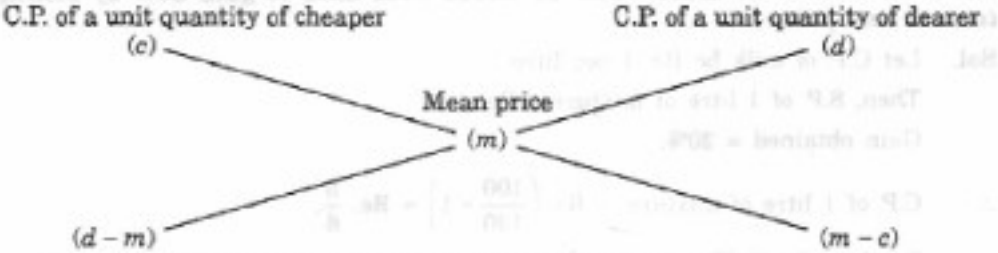
20. ALLIGATION OR MIXTURE

IMPORTANT FACTS AND FORMULAE

1. **Alligation** : It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.
2. **Mean Price** : The cost price of a unit quantity of the mixture is called the mean price.
3. **Rule of Alligation** : If two ingredients are mixed, then

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}}\right) = \frac{\text{(C.P. of dearer)} - \text{(Mean price)}}{\text{(Mean price)} - \text{(C.P. of cheaper)}}.$$

We present as under :



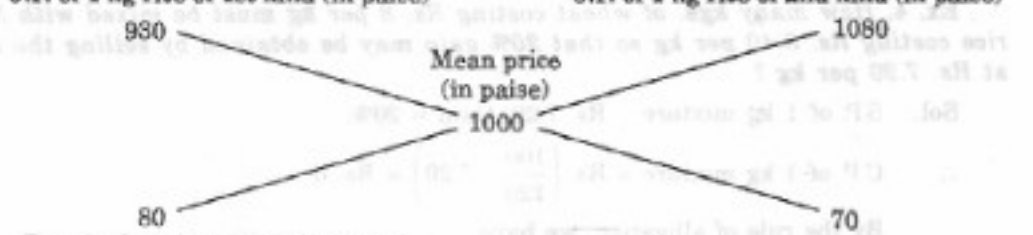
$\therefore$ (Cheaper quantity) : (Dearer quantity) = $(d - m) : (m - c)$.

4. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid $= \left[x\left(1 - \frac{y}{x}\right)^n\right]$ units.

SOLVED EXAMPLES

Ex. 1. ***In what ratio must rice at Rs. 9.30 per kg be mixed with rice at Rs. 10.80 per kg so that the mixture be worth Rs. 10 per kg ?***

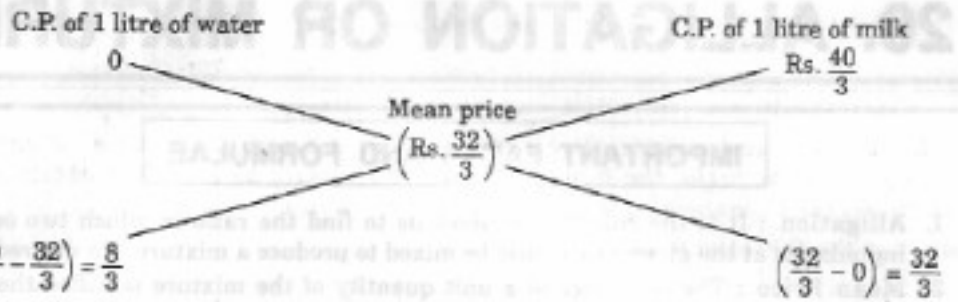
Sol. By the rule of alligation, we have :



$\therefore$ Required ratio = 80 : 70 = 8 : 7.

Ex. 2. ***How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for Rs. 20 so as to have a mixture worth Rs. $10\frac{2}{3}$ a litre ?***

Sol. C.P. of 1 litre of milk = Rs. $\left(20 \times \frac{2}{3}\right)$ = Rs. $\frac{40}{3}$.



$\therefore$ Ratio of water and milk $= \frac{8}{3} : \frac{32}{3} = 8 : 32 = 1 : 4.$

$\therefore$ Quantity of water to be added to 60 litres of milk $= \left(\frac{1}{4} \times 60\right)$ litres $= 15$ litres.

Ex. 3. ***In what ratio must water be mixed with milk to gain 20% by selling the mixture at cost price ?***

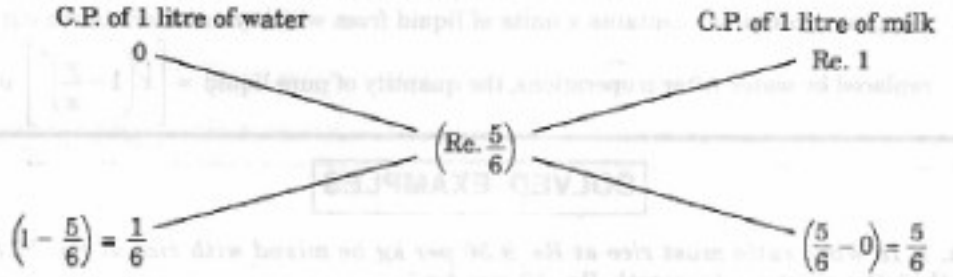
Sol. Let C.P. of milk be Re. 1 per litre.

Then, S.P. of 1 litre of mixture = Re. 1.

Gain obtained = 20%.

$\therefore$ C.P. of 1 litre of mixture $= \text{Rs.}\left(\frac{100}{120} \times 1\right) = \text{Re.}\ \frac{5}{6}.$

By the rule of alligation, we have :



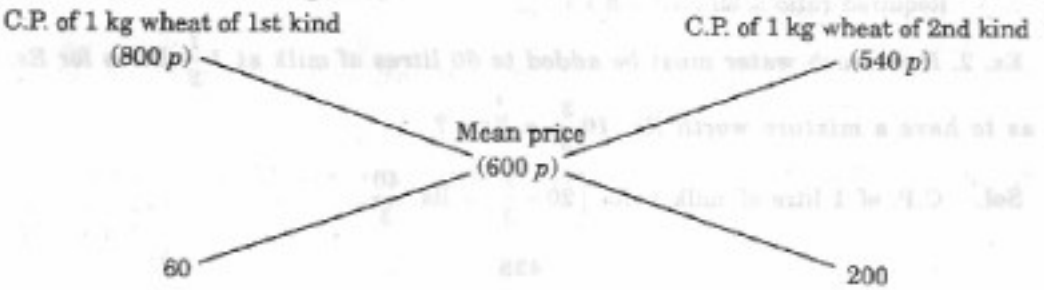
$\therefore$ Ratio of water and milk $= \frac{1}{6} : \frac{5}{6} = 1 : 5.$

Ex. 4. ***How many kgs. of wheat costing Rs. 8 per kg must be mixed with 36 kg of rice costing Rs. 5.40 per kg so that 20% gain may be obtained by selling the mixture at Rs. 7.20 per kg ?***

Sol. S.P. of 1 kg mixture = Rs. 7.20, Gain = 20%.

$\therefore$ C.P. of 1 kg mixture $= \text{Rs.}\left(\frac{100}{120} \times 7.20\right) = \text{Rs. } 6.$

By the rule of alligation, we have :



Wheat of 1st kind : Wheat of 2nd kind = 60 : 200 = 3 : 10.

Let x kg of wheat of 1st kind be mixed with 36 kg of wheat of 2nd kind.

Then, $3 : 10 = x : 36$ or $10x = 3 \times 36$ or $x = 10.8$ kg.

Ex. 5. ***The milk and water in two vessels A and B are in the ratio 4 : 3 and 2 : 3 respectively. In what ratio, the liquids in both the vessels be mixed to obtain a new mixture in vessel C containing half milk and half water ?***

Sol. Let the C.P. of milk be Re. 1 per litre.

Milk in 1 litre mixture of A $= \frac{4}{7}$ litre; Milk in 1 litre mixture of B $= \frac{2}{5}$ litre;

Milk in 1 litre mixture of C $= \frac{1}{2}$ litre.

$\therefore$ C.P. of 1 litre mixture in A = Re. $\frac{4}{7}$; C.P. of 1 litre mixture in B = Re. $\frac{2}{5}$.

Mean price = Re. $\frac{1}{2}$.

By the rule of alligation, we have :

C.P. of 1 litre mix. in A C.P. of 1 litre mix. in B

$\left(\frac{4}{7}\right)$ $\left(\frac{2}{5}\right)$

$\left(\frac{1}{2}\right)$

$\left(\frac{1}{10}\right)$ $\left(\frac{1}{14}\right)$

$\therefore$ Required ratio $= \frac{1}{10} : \frac{1}{14} = 7 : 5$.

EXERCISE 20

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. In what ratio must a grocer mix two varieties of pulses costing Rs. 15 and Rs. 20 per kg respectively so as to get a mixture worth Rs. 16.50 per kg ? **(R.R.B. 2003)**
   (*a*) 3 : 7 (*b*) 5 : 7 (*c*) 7 : 3 (*d*) 7 : 5
2. Find the ratio in which rice at Rs. 7.20 a kg be mixed with rice at Rs. 5.70 a kg to produce a mixture worth Rs. 6.30 a kg. **(IGNOU, 2003)**
   (*a*) 1 : 3 (*b*) 2 : 3 (*c*) 3 : 4 (*d*) 4 : 5
3. In what ratio must tea at Rs. 62 per kg be mixed with tea at Rs. 72 per kg so that the mixture must be worth Rs. 64.50 per kg ?
   (*a*) 3 : 1 (*b*) 3 : 2 (*c*) 4 : 3 (*d*) 5 : 3
4. In what ratio must water be mixed with milk costing Rs. 12 per litre to obtain a mixture worth of Rs. 8 per litre ?
   (*a*) 1 : 2 (*b*) 2 : 1 (*c*) 2 : 3 (*d*) 3 : 2
5. The cost of Type 1 rice is Rs. 15 per kg and Type 2 rice is Rs. 20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of rice is : **(M.B.A. 2002)**
   (*a*) Rs. 18 (*b*) Rs. 18.50 (*c*) Rs. 19 (*d*) Rs. 19.50

6. In what ratio must a grocer mix two varieties of tea worth Rs. 60 a kg and Rs. 65 a kg so that by selling the mixture at Rs. 68.20 a kg he may gain 10% ?
(a) 3 : 2 (b) 3 : 4 (c) 3 : 5 (d) 4 : 5
(S.S.C. 2004)

7. How many kilograms of sugar costing Rs. 9 per kg must be mixed with 27 kg of sugar costing Rs. 7 per kg so that there may be a gain of 10% by selling the mixture at Rs. 9.24 per kg ?
(a) 36 kg (b) 42 kg (c) 54 kg (d) 63 kg

8. In what ratio must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price ?
(L.I.C.A.A.O. 2003)
(a) 1 : 6 (b) 6 : 1 (c) 2 : 3 (d) 4 : 3

9. A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is :
(a) 4% (b) $6\frac{1}{4}\%$ (c) 20% (d) 25%

10. Two vessels A and B contain spirit and water mixed in the ratio 5 : 2 and 7 : 6 respectively. Find the ratio in which these mixture be mixed to obtain a new mixture in vessel C containing spirit and water in the ratio 8 : 5 ?
(a) 4 : 3 (b) 3 : 4 (c) 5 : 6 (d) 7 : 9

11. Two vessels A and B contain milk and water mixed in the ratio 8 : 5 and 5 : 2 respectively. The ratio in which these two mixtures be mixed to get a new mixture containing $69\frac{3}{13}\%$ milk, is :
(a) 2 : 7 (b) 3 : 5 (c) 5 : 2 (d) 5 : 7

12. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water. How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3 : 5 ?
(a) 4 litres, 8 litres (b) 6 litres, 6 litres
(c) 5 litres, 7 litres (d) 7 litres, 5 litres

13. One quality of wheat at Rs. 9.30 per kg is mixed with another quality at a certain rate in the ratio 8 : 7. If the mixture so formed be worth Rs. 10 per kg, what is the rate per kg of the second quality of wheat ?
(a) Rs. 10.30 (b) Rs. 10.60 (c) Rs. 10.80 (d) Rs. 11

14. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be :
(S.S.C. 1999)
(a) Rs. 169.50 (b) Rs. 170 (c) Rs. 175.50 (d) Rs. 180

15. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is :
(a) 400 kg (b) 560 kg (c) 600 kg (d) 640 kg

16. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is :
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

17. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container ?
(a) 26.34 litres (b) 27.36 litres (c) 28 litres (d) 29.16 litres

18. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 : 65. How much wine did the cask hold originally ? (N.I.F.T. 2003)

(a) 18 litres (b) 24 litres (c) 32 litres (d) 42 litres

19. A can contains a mixture of two liquids A and B in the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially ?

(a) 10 (b) 20 (c) 21 (d) 25

20. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup ?

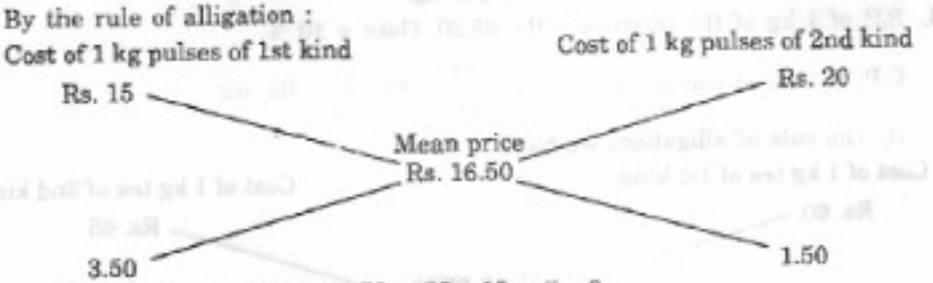
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{7}$

ANSWERS

1. (c) 2. (b) 3. (a) 4. (a) 5. (a) 6. (a) 7. (d) 8. (a) 9. (c) 10. (d)
11. (a) 12. (b) 13. (c) 14. (c) 15. (c) 16. (b) 17. (d) 18. (b) 19. (c) 20. (c)

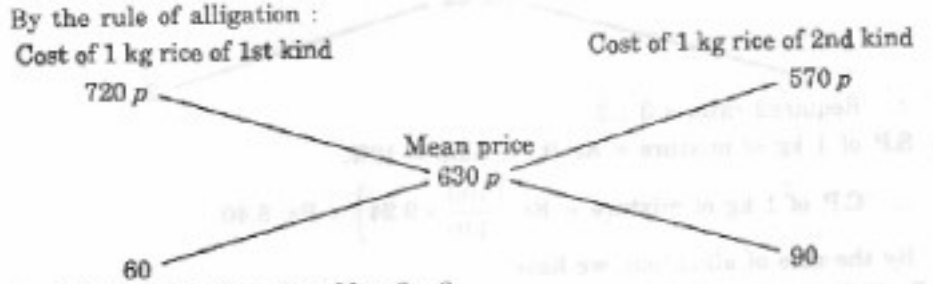
SOLUTIONS

1. By the rule of alligation :



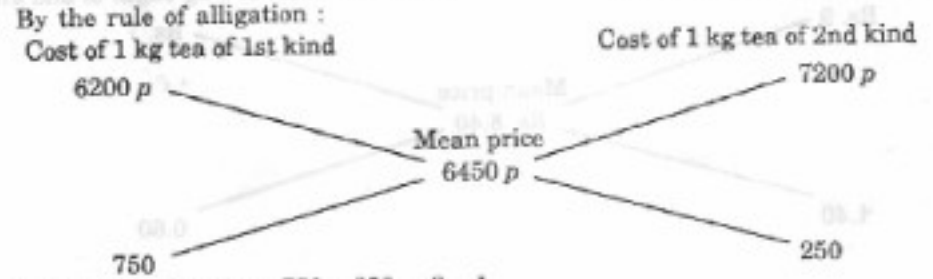
∴ Required rate = 3.50 : 1.50 = 35 : 15 = 7 : 3.

2. By the rule of alligation :



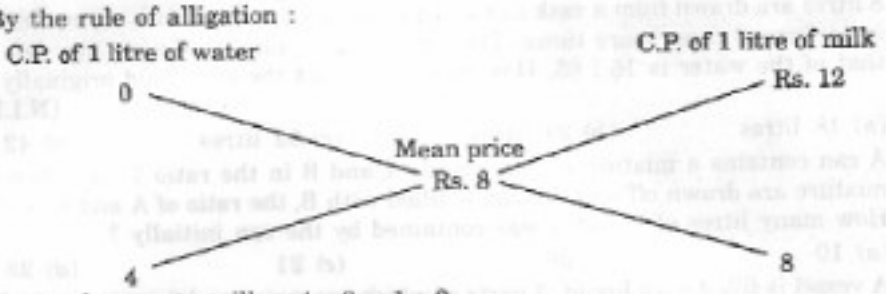
∴ Required ratio = 60 : 90 = 2 : 3.

3. By the rule of alligation :



∴ Required ratio = 750 : 250 = 3 : 1.

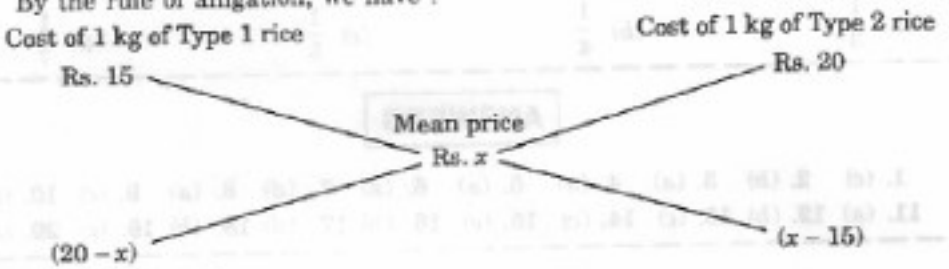
4. By the rule of alligation :



Ratio of water to milk = 4 : 8 = 1 : 2.

5. Let the price of the mixed variety be Rs. x per kg.

By the rule of alligation, we have :



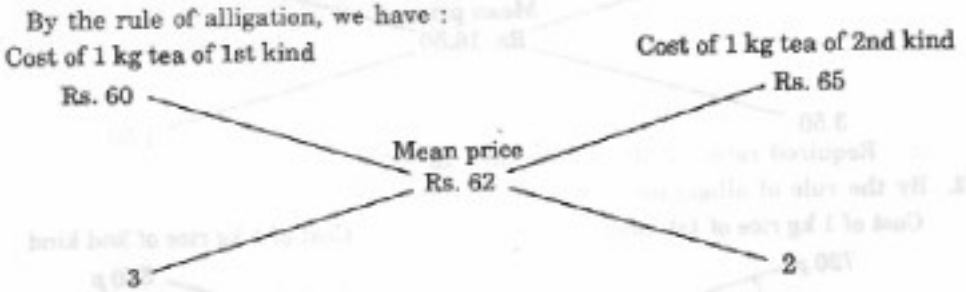
$$\therefore \quad \frac{(20-x)}{(x-15)} = \frac{2}{3} \Rightarrow 60 - 3x = 2x - 30 \Rightarrow 5x = 90 \Rightarrow x = 18.$$

So, price of the mixture is Rs. 18 per kg.

6. S.P. of 1 kg of the mixture = Rs. 68.20, Gain = 10 %.

C.P. of 1 kg of the mixture = Rs. $\left(\frac{100}{110} \times 68.20\right)$ = Rs. 62.

By the rule of alligation, we have :

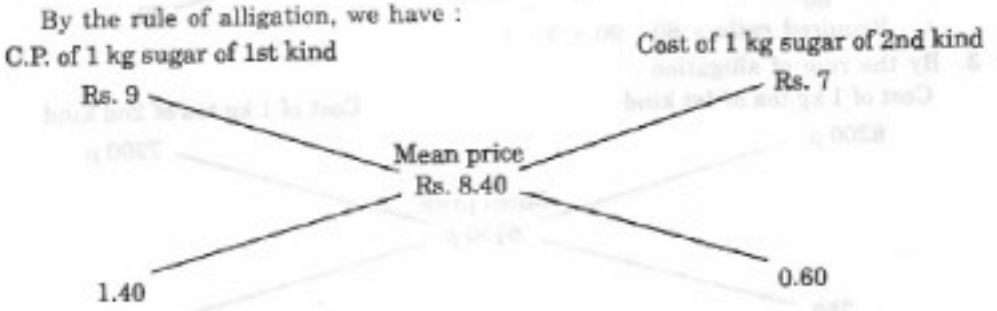


$\therefore$ Required ratio = 3 : 2.

7. S.P. of 1 kg of mixture = Rs. 9.24, Gain = 10%.

$\therefore$ C.P. of 1 kg of mixture = Rs. $\left(\frac{100}{110} \times 9.24\right)$ = Rs. 8.40.

By the rule of alligation, we have :



$\therefore$ Ratio of quantities of 1st and 2nd kind = 14 : 6 = 7 : 3.

Let x kg of sugar of 1st kind be mixed with 27 kg of 2nd kind.

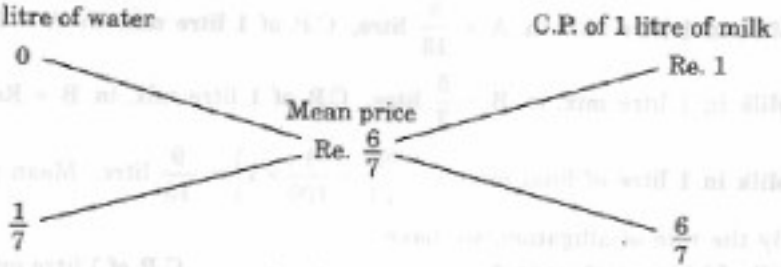
Then, $7 : 3 = x : 27$ or $x = \left(\frac{7 \times 27}{3}\right) = 63$ kg.

8. Let C.P. of 1 litre milk be Re. 1.

S.P. of 1 litre of mixture = Re. 1, Gain $= \frac{50}{3}\%$.

$\therefore$ C.P. of 1 litre of mixture $= \left(100 \times \frac{3}{350} \times 1\right) =$ Re. $\frac{6}{7}$.

By the rule of alligation, we have :

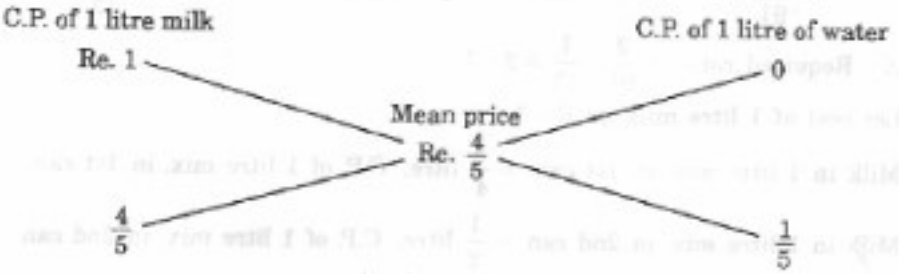


$\therefore$ Ratio of water and milk $= \frac{1}{7} : \frac{6}{7} = 1 : 6$.

9. Let C.P. of 1 litre milk be Re. 1.

Then, S.P. of 1 litre of mixture = Re. 1, Gain = 25%.

C.P. of 1 litre mixture = Re. $\left(\frac{100}{125} \times 1\right) =$ Re. $\frac{4}{5}$.



$\therefore$ Ratio of milk to water $= \frac{4}{5} : \frac{1}{5} = 4 : 1$.

Hence, percentage of water in the mixture $= \left(\frac{1}{5} \times 100\right)\% = 20\%$.

10. Let the C.P. of spirit be Re. 1 per litre.

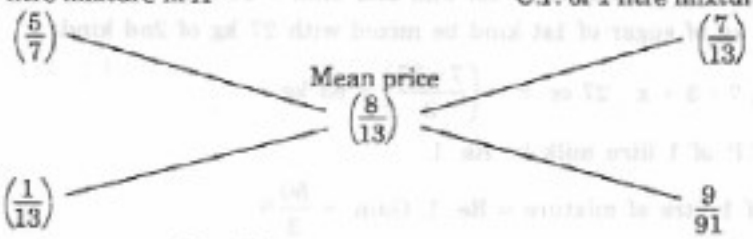
Spirit in 1 litre mix. of A $= \frac{5}{7}$ litre; C.P. of 1 litre mix. in A = Re. $\frac{5}{7}$.

Spirit in 1 litre mix. of B $= \frac{7}{13}$ litre; C.P. of 1 litre mix. in B = Re. $\frac{7}{13}$.

Spirit in 1 litre mix. of C $= \frac{8}{13}$ litre; Mean price = Re. $\frac{8}{13}$.

By the rule of alligation, we have :

C.P. of 1 litre mixture in A C.P. of 1 litre mixture in B



$\therefore$ Required ratio $= \frac{1}{13} : \frac{9}{91} = 7 : 9.$

11. Let cost of 1 litre milk be Re. 1.

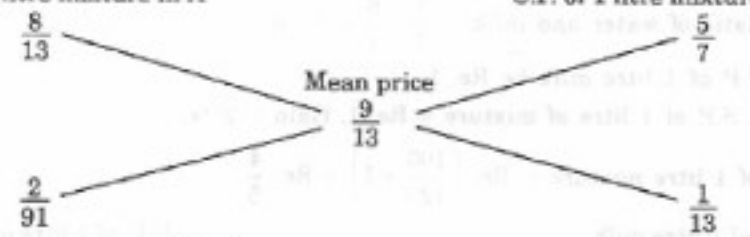
Milk in 1 litre mix. in A $= \frac{8}{13}$ litre, C.P. of 1 litre mix. in A = Re. $\frac{8}{13}$.

Milk in 1 litre mix. in B $= \frac{5}{7}$ litre, C.P. of 1 litre mix. in B = Re. $\frac{5}{7}$.

Milk in 1 litre of final mix. $= \left(\frac{900}{13} \times \frac{1}{100} \times 1\right) = \frac{9}{13}$ litre; Mean price = Re. $\frac{9}{13}$.

By the rule of alligation, we have :

C.P. of 1 litre mixture in A C.P. of 1 litre mixture in B



$\therefore$ Required ratio $= \frac{2}{91} : \frac{1}{13} = 2 : 7.$

12. Let cost of 1 litre milk be Re. 1.

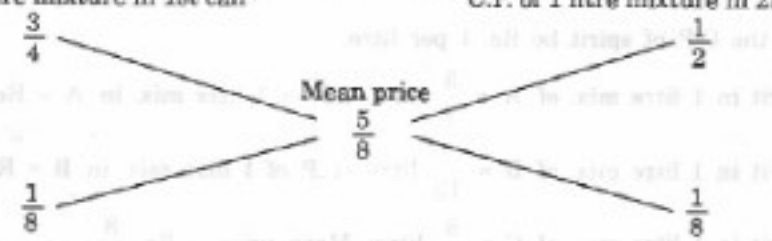
Milk in 1 litre mix. in 1st can $= \frac{3}{4}$ litre, C.P. of 1 litre mix. in 1st can = Re. $\frac{3}{4}$.

Milk in 1 litre mix. in 2nd can $= \frac{1}{2}$ litre, C.P. of 1 litre mix. in 2nd can = Re. $\frac{1}{2}$.

Milk in 1 litre of final mix. $= \frac{5}{8}$ litre, Mean price = Re. $\frac{5}{8}$.

By the rule of alligation, we have :

C.P. of 1 litre mixture in 1st can C.P. of 1 litre mixture in 2nd can

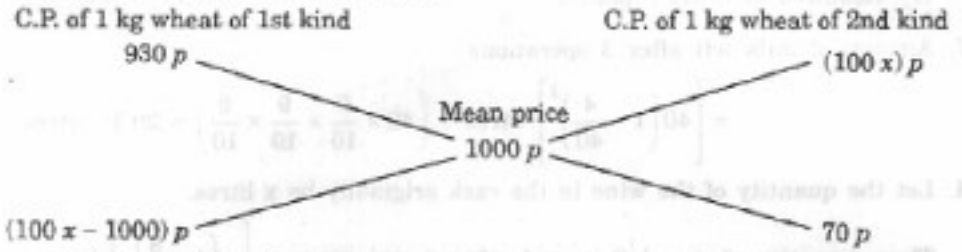


$\therefore$ Ratio of two mixtures $= \frac{1}{8} : \frac{1}{8} = 1 : 1.$

So, quantity of mixture taken from each can $= \left(\frac{1}{2} \times 12\right) = 6$ litres.

13. Let the rate of the second quality be Rs. x per kg.

By the rule of alligation, we have :



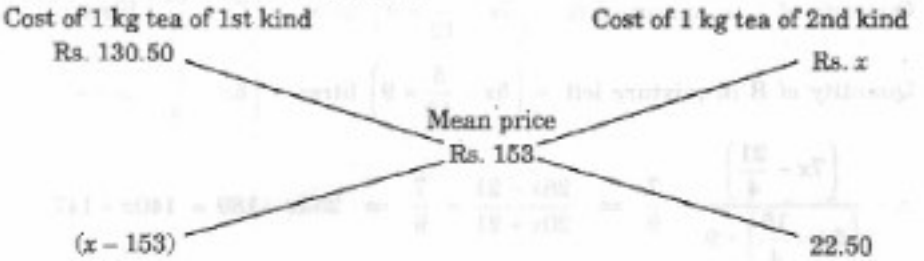
$$\therefore \quad \frac{100x - 1000}{70} = \frac{8}{7} \Rightarrow 700x - 7000 = 560 \Rightarrow 700x = 7560 \Rightarrow x = \text{Rs. } 10.80.$$

14. Since first and second varieties are mixed in equal proportions, so their average price

$$= \text{Rs.} \left(\frac{126 + 135}{2} \right) = \text{Rs. } 130.50.$$

So, the mixture is formed by mixing two varieties, one at Rs. 130.50 per kg and the other at say, Rs. x per kg in the ratio 2 : 2, *i.e.*, 1 : 1. We have to find x.

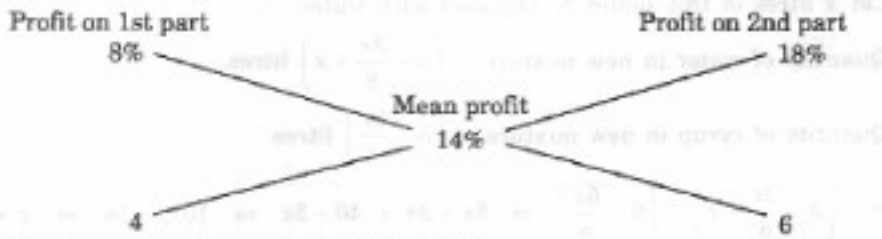
By the rule of alligation, we have :



$$\therefore \quad \frac{x - 153}{22.50} = 1 \Rightarrow x - 153 = 22.50 \Rightarrow x = 175.50.$$

Hence, price of the third variety = Rs. 175.50 per kg.

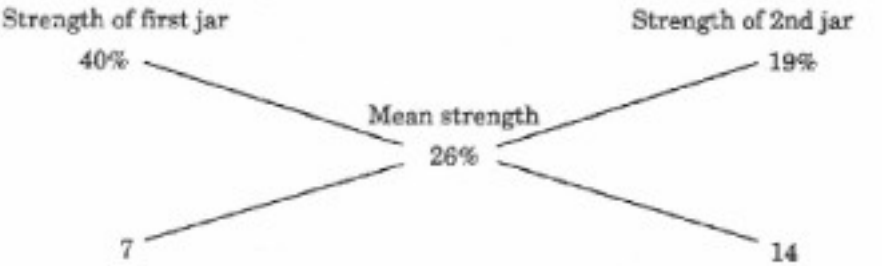
15. By the rule of alligation, we have :



Ratio of 1st and 2nd parts = 4 : 6 = 2 : 3.

$$\therefore \quad \text{Quantity of 2nd kind} = \left(\frac{3}{5} \times 1000 \right) \text{kg} = 600 \text{ kg}.$$

16. By the rule of alligation, we have :



So, ratio of 1st and 2nd quantities = 7 : 14 = 1 : 2.

$\therefore$ Required quantity replaced $= \frac{2}{3}$.

17. Amount of milk left after 3 operations

$$= \left[40\left(1-\frac{4}{40}\right)^3\right] \text{ litres} = \left(40\times\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}\right) = 29.16 \text{ litres.}$$

18. Let the quantity of the wine in the cask originally be x litres.

Then, quantity of wine left in cask after 4 operations $= \left[x\left(1-\frac{8}{x}\right)^4\right]$ litres.

$$\therefore \frac{x\left(1-\frac{8}{x}\right)^4}{x} = \frac{16}{81} \Rightarrow \left(1-\frac{8}{x}\right)^4 = \left(\frac{2}{3}\right)^2 \Rightarrow \left(\frac{x-8}{x}\right) = \frac{2}{3}$$

$$\Rightarrow 3x - 24 = 2x \Rightarrow x = 24.$$

19. Suppose the can initially contains $7x$ and $5x$ litres of mixtures A and B respectively.

Quantity of A in mixture left $= \left(7x - \frac{7}{12}\times 9\right)$ litres $= \left(7x - \frac{21}{4}\right)$ litres.

Quantity of B in mixture left $= \left(5x - \frac{5}{12}\times 9\right)$ litres $= \left(5x - \frac{15}{4}\right)$ litres.

$$\therefore \frac{\left(7x-\frac{21}{4}\right)}{\left(5x-\frac{15}{4}\right)+9} = \frac{7}{9} \Rightarrow \frac{28x-21}{20x+21} = \frac{7}{9} \Rightarrow 252x - 189 = 140x + 147$$

$$\Rightarrow 112x = 336 \Rightarrow x = 3.$$

So, the can contained 21 litres of A.

20. Suppose the vessel initially contains 8 litres of liquid.

Let x litres of this liquid be replaced with water.

Quantity of water in new mixture $= \left(3 - \frac{3x}{8} + x\right)$ litres.

Quantity of syrup in new mixture $= \left(5 - \frac{5x}{8}\right)$ litres.

$$\therefore \left(3 - \frac{3x}{8} + x\right) = \left(5 - \frac{5x}{8}\right) \Rightarrow 5x + 24 = 40 - 5x \Rightarrow 10x = 16 \Rightarrow x = \frac{8}{5}.$$

So, part of the mixture replaced $= \left(\frac{8}{5}\times\frac{1}{8}\right) = \frac{1}{5}$.

21. SIMPLE INTEREST

IMPORTANT FACTS AND FORMULAE

1. **Principal** : The money borrowed or lent out for a certain period is called the ***principal*** or the ***sum.***
2. **Interest** : Extra money paid for using other's money is called ***interest.***
3. **Simple Interest (S.I.)** : If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called ***simple interest.***

Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then,

(*i*) $\text{S.I.} = \left(\frac{P \times R \times T}{100}\right)$

(*ii*) $P = \left(\frac{100 \times \text{S.I.}}{R \times T}\right)$; $R = \left(\frac{100 \times \text{S.I.}}{P \times T}\right)$ and $T = \left(\frac{100 \times \text{S.I.}}{P \times R}\right)$.

SOLVED EXAMPLES

Ex. 1. ***Find the simple interest on Rs. 68,000 at $16\frac{2}{3}\%$ per annum for 9 months.***

Sol. P = Rs. 68000, $R = \frac{50}{3}\%$ p.a and $T = \frac{9}{12}$ years $= \frac{3}{4}$ years.

$\therefore$ $\text{S.I.} = \left(\frac{P \times R \times T}{100}\right) = \text{Rs.}\left(68000 \times \frac{50}{3} \times \frac{3}{4} \times \frac{1}{100}\right) = \text{Rs. } 8500.$

Ex. 2. ***Find the simple interest on Rs. 3000 at $6\frac{1}{4}\%$ per annum for the period from 4th Feb., 2005 to 18th April, 2005.***

Sol. Time = (24 + 31 + 18) days = 73 days $= \frac{73}{365}$ year $= \frac{1}{5}$ year.

P = Rs. 3000 and $R = 6\frac{1}{4}\%$ p.a. $= \frac{25}{4}\%$ p.a.

$\therefore$ $\text{S.I.} = \text{Rs.}\left(3000 \times \frac{25}{4} \times \frac{1}{5} \times \frac{1}{100}\right) = \text{Rs. } 37.50.$

Remark : The day on which money is deposited is not counted while the day on which money is withdrawn is counted.

Ex. 3. ***A sum at simple interest at $13\frac{1}{2}\%$ per annum amounts to Rs. 2502.50 after 4 years. Find the sum.***

Sol. Let sum be Rs. x. Then, $\text{S.I.} = \text{Rs.}\left(x \times \frac{27}{2} \times 4 \times \frac{1}{100}\right) = \text{Rs. } \frac{27x}{50}$.

$\therefore$ $\text{Amount} = \text{Rs.}\left(x + \frac{27x}{50}\right) = \text{Rs. } \frac{77x}{50}$.

$\therefore \quad \frac{77x}{50} = 2502.50 \Leftrightarrow x = \frac{2502.50 \times 50}{77} = 1625.$

Hence, sum = Rs. 1625.

Ex. 4. ***A sum of Rs. 800 amounts to Rs. 920 in 3 years at simple interest. If the interest rate is increased by 3%, it would amount to how much ?***

Sol. S.I. = Rs. (920 − 800) = Rs. 120; P = Rs. 800, T = 3 yrs.

$\therefore \quad R = \left(\frac{100 \times 120}{800 \times 3}\right)\% = 5\%.$

New rate = (5 + 3)% = 8%.

New S.I. = Rs. $\left(\frac{800 \times 8 \times 3}{100}\right)$ = Rs. 192.

$\therefore$ New amount = Rs. (800 + 192) = Rs. 992.

Ex. 5. ***Adam borrowed some money at the rate of 6% p.a. for the first two years, at the rate of 9% p.a. for the next three years, and at the rate of 14% p.a. for the period beyond five years. If he pays a total interest of Rs. 11, 400 at the end of nine years, how much money did he borrow ?*** **(Bank P.O. 1999)**

Sol. Let the sum borrowed be x. Then,

$$\left(\frac{x \times 6 \times 2}{100}\right) + \left(\frac{x \times 9 \times 3}{100}\right) + \left(\frac{x \times 14 \times 4}{100}\right) = 11400$$

$$\Leftrightarrow \quad \left(\frac{3x}{25} + \frac{27x}{100} + \frac{14x}{25}\right) = 11400 \Leftrightarrow \frac{95x}{100} = 11400 \Leftrightarrow x = \left(\frac{11400 \times 100}{95}\right) = 12000.$$

Hence, sum borrowed = Rs. 12,000.

Ex. 6. ***A certain sum of money amounts to Rs. 1008 in 2 years and to Rs. 1164 in*** $3\frac{1}{2}$ ***years. Find the sum and the rate of interest.***

Sol. S.I. for $1\frac{1}{2}$ years = Rs. (1164 − 1008) = Rs. 156.

S.I. for 2 years = Rs. $\left(156 \times \frac{2}{3} \times 2\right)$ = Rs. 208.

$\therefore$ Principal = Rs. (1008 − 208) = Rs. 800.

Now, P = 800, T = 2 and S.I. = 208.

$\therefore \quad \text{Rate} = \left(\frac{100 \times 208}{800 \times 2}\right)\% = 13\%.$

Ex. 7. ***At what rate percent per annum will a sum of money double in 16 years ?*** **(R.R.B. 2003)**

Sol. Let principal = P. Then, S.I. = P and T = 16 yrs.

$\therefore \quad \text{Rate} = \left(\frac{100 \times P}{P \times 16}\right)\% = 6\frac{1}{4}\%$ p.a.

Ex. 8. ***The simple interest on a sum of money is*** $\frac{4}{9}$ ***of the principal. Find the rate percent and time, if both are numerically equal.*** **(S.S.C. 2000)**

Sol. Let sum = Rs. x. Then, S.I. = Rs. $\frac{4x}{9}$.

Let rate = R% and time = R years.

Then, $\left(\frac{x \times R \times R}{100}\right) = \frac{4x}{9}$ or $R^2 = \frac{400}{9}$ or $R = \frac{20}{3} = 6\frac{2}{3}$.

$\therefore$ Rate $= 6\frac{2}{3}\%$ and Time $= 6\frac{2}{3}$ yrs $= 6$ yrs 8 months.

Ex. 9. ***The simple interest on a certain sum of money for $2\frac{1}{2}$ years at 12% per annum is Rs. 40 less than the simple interest on the same sum for $3\frac{1}{2}$ years at 10% per annum. Find the sum.***

Sol. Let the sum be Rs. x. Then, $\left(\frac{x \times 10 \times 7}{100 \times 2}\right) - \left(\frac{x \times 12 \times 5}{100 \times 2}\right) = 40$

$\Leftrightarrow \quad \frac{7x}{20} - \frac{3x}{10} = 40 \quad \Leftrightarrow \quad x = (40 \times 20) = 800.$

Hence, the sum is Rs. 800.

Ex. 10. ***A sum was put at simple interest at a certain rate for 3 years. Had it been put at 2% higher rate, it would have fetched Rs. 360 more. Find the sum.***

Sol. Let sum = P and original rate = R. Then, $\left[\frac{P \times (R+2) \times 3}{100}\right] - \left[\frac{P \times R \times 3}{100}\right] = 360$

$\Leftrightarrow \quad 3PR + 6P - 3PR = 36000 \quad \Leftrightarrow \quad 6P = 36000 \quad \Leftrightarrow \quad P = 6000.$

Hence, sum = Rs. 6000.

Ex. 11. ***What annual instalment will discharge a debt of Rs. 1092 due in 3 years at 12% simple interest ?***

Sol. Let each instalment be Rs. x. Then, $\left(x + \frac{x \times 12 \times 1}{100}\right) + \left(x + \frac{x \times 12 \times 2}{100}\right) + x = 1092$

$\Leftrightarrow \quad \frac{28x}{25} + \frac{31x}{25} + x = 1092 \quad \Leftrightarrow \quad (28x + 31x + 25x) = (1092 \times 25)$

$\Leftrightarrow \quad x = \left(\frac{1092 \times 25}{84}\right) = 325.$

$\therefore$ Each instalment = Rs. 325.

Ex. 12. ***A sum of Rs. 1550 is lent out into two parts, one at 8% and another one at 6%. If the total annual income is Rs. 106, find the money lent at each rate.***

(L.I.C. A.A.O. 2003)

Sol. Let the sum lent at 8% be Rs. x and that at 6% be Rs. $(1550 - x)$.

$\therefore \quad \left[\frac{x \times 8 \times 1}{100}\right] + \left[\frac{(1550 - x) \times 6 \times 1}{100}\right] = 106$

$\Leftrightarrow \quad 8x + 9300 - 6x = 10600 \quad \Leftrightarrow \quad 2x = 1300 \quad \Leftrightarrow \quad x = 650.$

$\therefore$ Money lent at 8% = Rs. 650. Money lent at 6% = Rs. (1550 − 650) = Rs. 900.

EXERCISE 21A

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer :*

1. At the rate of $8\frac{1}{2}\%$ p.a. simple interest, a sum of Rs. 4800 will earn how much interest in 2 years 3 months ?

(*a*) Rs. 796 (*b*) Rs. 816 (*c*) Rs. 918 (*d*) Rs. 956

2. What will be the simple interest earned on an amount of Rs. 16,800 in 9 months at the rate of $6\frac{1}{4}\%$ p.a. ?
(a) Rs. 787.50 (b) Rs. 812.50 (c) Rs. 860 (d) Rs. 887.50

3. The simple interest on Rs. 1820 from March 9, 2003 to May 21, 2003 at $7\frac{1}{2}\%$ rate will be :
(a) Rs. 22.50 (b) Rs. 27.30 (c) Rs. 28.80 (d) Rs. 29

4. A person borrows Rs. 5000 for 2 years at 4% p.a. simple interest. He immediately lends it to another person at $6\frac{1}{4}\%$ p.a. for 2 years. Find his gain in the transaction per year. (S.S.C. 2000)
(a) Rs. 112.50 (b) Rs. 125 (c) Rs. 150 (d) Rs. 167.50

5. How much time will it take for an amount of Rs. 450 to yield Rs. 81 as interest at 4.5% per annum of simple interest ? (IGNOU, 2003)
(a) 3.5 years (b) 4 years (c) 4.5 years (d) 5 years

6. A sum of Rs. 12,500 amounts to Rs. 15,500 in 4 years at the rate of simple interest. What is the rate of interest ? (Bank P.O. 2003)
(a) 3% (b) 4% (c) 5% (d) 6% (e) None of these

7. A sum of Rs. 1600 gives a simple interest of Rs. 252 in 2 years and 4 months. The rate of interest per annum is :
(a) 6% (b) $6\frac{1}{4}\%$ (c) $6\frac{1}{2}\%$ (d) $6\frac{3}{4}\%$

8. Reena took a loan of Rs. 1200 with simple interest for as many years as the rate of interest. If she paid Rs. 432 as interest at the end of the loan period, what was the rate of interest ? (R.B.I. 2003)
(a) 3.6 (b) 6 (c) 18
(d) Cannot be determined (e) None of these

9. A man took a loan from a bank at the rate of 12% p.a. simple interest. After 3 years he had to pay Rs. 5400 interest only for the period. The principal amount borrowed by him was : (S.S.C. 2004)
(a) Rs. 2000 (b) Rs. 10,000 (c) Rs. 15,000 (d) Rs. 20,000

10. What is the present worth of Rs. 132 due in 2 years at 5% simple interest per annum ?
(a) Rs. 112 (b) Rs. 118.80 (c) Rs. 120 (d) Rs. 122
(C.B.I. 1997)

11. A sum fetched a total simple interest of Rs. 4016.25 at the rate of 9 p.c.p.a. in 5 years. What is the sum ? (NABARD, 2002)
(a) Rs. 4462.50 (b) Rs. 8032.50 (c) Rs. 8900
(d) Rs. 8925 (e) None of these

12. The simple interest at $x\%$ for x years will be Rs. x on a sum of :
(a) Rs. x (b) Rs. $\left(\frac{100}{x}\right)$ (c) Rs. $100x$ (d) Rs. $\left(\frac{100}{x^2}\right)$

13. Rs. 800 becomes Rs. 956 in 3 years at a certain rate of simple interest. If the rate of interest is increased by 4%, what amount will Rs. 800 become in 3 years ?
(a) Rs. 1020.80 (b) Rs. 1025 (c) Rs. 1052
(d) Data inadequate (e) None of these (Bank P.O. 2000)

14. A certain amount earns simple interest of Rs. 1750 after 7 years. Had the interest been 2% more, how much more interest would it have earned ? (Bank P.O. 2003)
(a) Rs. 35 (b) Rs. 245 (c) Rs. 350
(d) Cannot be determined (e) None of these

15. In how many years, Rs. 150 will produce the same interest @ 8% as Rs. 800 produce in 3 years @ $4\frac{1}{2}\%$? **(R.R.B. 2001)**

(*a*) 6 (*b*) 8 (*c*) 9 (*d*) 12

16. If Rs. 64 amounts to Rs. 83.20 in 2 years, what will Rs. 86 amount to in 4 years at the same rate percent per annum ?

(*a*) Rs. 114.80 (*b*) Rs. 124.70 (*c*) Rs. 127.40 (*d*) Rs. 137.60

17. The simple interest on a certain sum of money at the rate of 5% p.a. for 8 years is Rs. 840. At what rate of interest the same amount of interest can be received on the same sum after 5 years ?

(*a*) 6% (*b*) 8% (*c*) 9% (*d*) 10%

18. The interest on a certain deposit at 4.5% p.a. is Rs. 202.50 in one year. How much will the additional interest in one year be on the same deposit at 5% p.a. ?

(*a*) Rs. 20.25 (*b*) Rs. 22.50 (*c*) Rs. 25 (*d*) Rs. 42.75

19. A sum invested at 5% simple interest per annum grows to Rs. 504 in 4 years. The same amount at 10% simple interest per annum in $2\frac{1}{2}$ years will grow to :

(*a*) Rs. 420 (*b*) Rs. 450 (*c*) Rs. 525 (*d*) Rs. 550

(C.D.S. 2003)

20. What will be the ratio of simple interest earned by certain amount at the same rate of interest for 6 years and that for 9 years ? **(Bank P.O. 1998)**

(*a*) 1 : 3 (*b*) 1 : 4 (*c*) 2 : 3

(*d*) Data inadequate (*e*) None of these

21. Nitin borrowed some money at the rate of 6% p.a. for the first three years, 9% p.a. for the next five years and 13% p.a. for the period beyond eight years. If the total interest paid by him at the end of eleven years is Rs. 8160, how much money did he borrow ? **(Bank P.O. 2000)**

(*a*) Rs. 8000 (*b*) Rs. 10,000 (*c*) Rs. 12,000

(*d*) Data inadequate (*e*) None of these

22. The simple interest on a sum of money will be Rs. 600 after 10 years. If the principal is trebled after 5 years, what will be the total interest at the end of the tenth year ?

(*a*) Rs. 600 (*b*) Rs. 900 (*c*) Rs. 1200

(*d*) Rs. 1500 (*e*) Data inadequate

23. The simple interest on Rs. 10 for 4 months at the rate of 3 paise per rupee per month is :

(*a*) Rs. 1.20 (*b*) Rs. 1.60 (*c*) Rs. 2.40 (*d*) Rs. 3.60

24. An automobile financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest of 10%, the effective rate of interest becomes : **(N.I.F.T. 2000)**

(*a*) 10% (*b*) 10.25% (*c*) 10.5% (*d*) None of these

25. A sum of money at simple interest amounts to Rs. 815 in 3 years and to Rs. 854 in 4 years. The sum is : **(Section Officers', 2001)**

(*a*) Rs. 650 (*b*) Rs. 690 (*c*) Rs. 698 (*d*) Rs. 700

26. A sum of money lent out at simple interest amounts to Rs. 720 after 2 years and to Rs. 1020 after a further period of 5 years. The sum is : **(S.S.C. 2004)**

(*a*) Rs. 500 (*b*) Rs. 600 (*c*) Rs. 700 (*d*) Rs. 710

27. A sum of money amounts to Rs. 9800 after 5 years and Rs. 12005 after 8 years at the same rate of simple interest. The rate of interest per annum is : **(S.S.C. 2003)**

(*a*) 5% (*b*) 8% (*c*) 12% (*d*) 15%

28. A certain sum of money at simple interest amounts to Rs. 1012 in $2\frac{1}{2}$ years and to Rs. 1067.20 in 4 years. The rate of interest per annum is :
(*a*) 2.5% (*b*) 3% (*c*) 4% (*d*) 5%

29. In how many years will a sum of money double itself at 12% per annum ?
(*a*) 6 years 9 months (*b*) 7 years 6 months
(*c*) 8 years 3 months (*d*) 8 years 4 months

30. At what rate percent of simple interest will a sum of money double itself in 12 years ?
(*a*) $8\frac{1}{4}\%$ (*b*) $8\frac{1}{3}\%$ (*c*) $8\frac{1}{2}\%$ (*d*) $9\frac{1}{2}\%$
(S.S.C. 2000)

31. The rate at which a sum becomes four times of itself in 15 years at S.I., will be :
(*a*) 15% (*b*) $17\frac{1}{2}\%$ (*c*) 20% (*d*) 25%

32. If a sum of money at simple interest doubles in 6 years, it will become 4 times in :
(*a*) 12 years (*b*) 14 years (*c*) 16 years (*d*) 18 years

33. A sum of money trebles itself in 15 years 6 months. In how many years would it double itself ?
(*a*) 6 years 3 months (*b*) 7 years 9 months
(*c*) 8 years 3 months (*d*) 9 years 6 months.

34. Consider the following statements :
If a sum of money is lent at simple interest, then the
1. money gets doubled in 5 years if the rate of interest is $16\frac{2}{3}\%$.
2. money gets doubled in 5 years if the rate of interest is 20%.
3. money becomes four times in 10 years if it gets doubled in 5 years.
Of these statements,
(*a*) 1 and 3 are correct (*b*) 2 alone is correct
(*c*) 3 alone is correct (*d*) 2 and 3 are correct

35. The simple interest on a sum of money at 8% per annum for 6 years is half the sum. The sum is :
(*a*) Rs. 4800 (*b*) Rs. 6000 (*c*) Rs. 8000 (*d*) Data inadequate

36. At what rate percent per annum will the simple interest on a sum of money be $\frac{2}{5}$ of the amount in 10 years ?
(S.S.C. 2002)
(*a*) 4% (*b*) $5\frac{2}{3}\%$ (*c*) 6% (*d*) $6\frac{2}{3}\%$

37. In how much time would the simple interest on a certain sum be 0.125 times the principal at 10% per annum ?
(Assistant Grade, 1997)
(*a*) $1\frac{1}{4}$ years (*b*) $1\frac{3}{4}$ years (*c*) $2\frac{1}{4}$ years (*d*) $2\frac{3}{4}$ years

38. How long will it take a sum of money invested at 5% p.a. S.I. to increase its value by 40% ?
(*a*) 5 years (*b*) 6 years (*c*) 7 years (*d*) 8 years

39. A sum of money becomes $\frac{7}{6}$ of itself in 3 years at a certain rate of simple interest. The rate per annum is :
(S.S.C. 1999)
(*a*) $5\frac{5}{9}\%$ (*b*) $6\frac{5}{9}\%$ (*c*) 18% (*d*) 25%

40. Simple interest on a certain sum at a certain annual rate of interest is $\frac{1}{9}$ of the sum. If the numbers representing rate percent and time in years be equal, then the rate of interest is :

(a) $3\frac{1}{3}\%$ (b) 5% (c) $6\frac{2}{3}\%$ (d) 10%

41. Simple interest on a certain amount is $\frac{9}{16}$ of the principal. If the numbers representing the rate of interest in percent and time in years be equal, then time, for which the principal is lent out, is : **(R.R.B. 2003)**

(a) $5\frac{1}{2}$ years (b) $6\frac{1}{2}$ years (c) 7 years (d) $7\frac{1}{2}$ years

42. A lends Rs. 2500 to B and a certain sum to C at the same time at 7% p.a. simple interest. If after 4 years, A altogether receives Rs. 1120 as interest from B and C, then the sum lent to C is : **(S.S.C. 2003)**

(a) Rs. 700 (b) Rs. 1500 (c) Rs. 4000 (d) Rs. 6500

43. Two equal sums of money were lent at simple interest at 11% p.a. for $3\frac{1}{2}$ years and $4\frac{1}{2}$ years respectively. If the difference in interests for two periods was Rs. 412.50, then each sum is :

(a) Rs. 3250 (b) Rs. 3500 (c) Rs. 3750 (d) Rs. 4250

44. If the simple interest on a certain sum for 15 months at $7\frac{1}{2}\%$ per annum exceeds the simple interest on the same sum for 8 months at $12\frac{1}{2}\%$ per annum by Rs. 32.50, then the sum (in Rs.) is :

(a) Rs. 3000 (b) Rs. 3060 (c) Rs. 3120 (d) Rs. 3250

45. A man invests a certain sum of money at 6% p.a. simple interest and another sum at 7% p.a. simple interest. His income from interest after 2 years was Rs. 354. One-fourth of the first sum is equal to one-fifth of the second sum. The total sum invested was :

(a) Rs. 2600 (b) Rs. 2700 (c) Rs. 2880 (d) Rs. 2900

46. A borrowed some money from B at 12% p.a. S.I. for 3 years. He then added some more money to the borrowed sum and lent it to C for the same period at 14% p.a. rate of interest. If A gains Rs. 93.90 in the whole transaction, how much money did he add from his side ?

(a) Rs. 35 (b) Rs. 55 (c) Rs. 80 (d) Rs. 105

47. A person borrowed Rs. 500 @ 3% per annum S.I. and Rs. 600 @ $4\frac{1}{2}\%$ per annum on the agreement that the whole sum will be returned only when the total interest becomes Rs. 126. The number of years, after which the borrowed sum is to be returned, is :

(a) 2 (b) 3 (c) 4 (d) 5

48. A lent Rs. 5000 to B for 2 years and Rs. 3000 to C for 4 years on simple interest at the same rate of interest and received Rs. 2200 in all from both of them as interest. The rate of interest per annum is : **(C.B.I. 2003)**

(a) 5% (b) 7% (c) $7\frac{1}{8}\%$ (d) 10%

49. A sum of Rs. 725 is lent in the beginning of a year at a certain rate of interest. After 8 months, a sum of Rs. 362.50 more is lent but at the rate twice the former. At the end of the year, Rs. 33.50 is earned as interest from both the loans. What was the original rate of interest ? **(Bank P.O. 2003)**

(*a*) 3.6% (*b*) 4.5% (*c*) 5% (*d*) 6% (*e*) None of these

50. The difference between the simple interest received from two different sources on Rs. 1500 for 3 years is Rs. 13.50. The difference between their rates of interest is :

(*a*) 0.1% (*b*) 0.2% (*c*) 0.3% (*d*) 0.4% (*e*) None of these

(S.S.C. 1999)

51. Peter invested an amount of Rs. 12,000 at the rate of 10 p.c.p.a. simple interest and another amount at the rate of 20 p.c.p.a. simple interest. The total interest earned at the end of one year on the total amount invested became 14 p.c.p.a. Find the total amount invested. **(S.B.I.P.O. 1999)**

(*a*) Rs. 20,000 (*b*) Rs. 22,000 (*c*) Rs. 24,000 (*d*) Rs. 25,000 (*e*) None of these

52. What should be the least number of years in which the simple interest on Rs. 2600 at $6\frac{2}{3}\%$ will be an exact number of rupees ?

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

53. The rates of simple interest in two banks A and B are in the ratio 5 : 4. A person wants to deposit his total savings in two banks in such a way that he received equal half-yearly interest from both. He should deposit the savings in banks A and B in the ratio :

(*a*) 2 : 5 (*b*) 4 : 5 (*c*) 5 : 2 (*d*) 5 : 4

54. A sum was put at simple interest at a certain rate for 2 years. Had it been put at 3% higher rate, it would have fetched Rs. 72 more. The sum is :

(*a*) Rs. 1200 (*b*) Rs. 1500 (*c*) Rs. 1600 (*d*) Rs. 1800

55. If the annual rate of simple interest increases from 10% to $12\frac{1}{2}\%$, a man's yearly income increases by Rs. 1250. His principal (in Rs.) is : **(S.S.C. 2004)**

(*a*) 45,000 (*b*) 50,000 (*c*) 60,000 (*d*) 65,000

56. A moneylender finds that due to a fall in the annual rate of interest from 8% to $7\frac{3}{4}\%$, his yearly income diminishes by Rs. 61.50. His capital is : **(S.S.C. 2003)**

(*a*) Rs. 22,400 (*b*) Rs. 23,800 (*c*) Rs. 24,600 (*d*) Rs. 26,000

57. What annual payment will discharge a debt of Rs. 6450 due in 4 years at 5% simple interest ?

(*a*) Rs. 1400 (*b*) Rs. 1500 (*c*) Rs. 1550 (*d*) Rs. 1600

58. A sum of Rs. 10 is lent to be returned in 11 monthly instalments of Re. 1 each, interest being simple. The rate of interest is :

(*a*) $9\frac{1}{11}\%$ (*b*) 10% (*c*) 11% (*d*) $21\frac{9}{11}\%$

59. A person takes a loan of Rs. 200 at 5% simple interest. He returns Rs. 100 at the end of 1 year. In order to clear his dues at the end of 2 years, he would pay :

(*a*) Rs. 105 (*b*) Rs. 110 (*c*) Rs. 115 (*d*) Rs. 115.50

60. The price of a T.V. set worth Rs. 20,000 is to be paid in 20 instalments of Rs. 1000 each. If the rate of interest be 6% per annum, and the first instalment be paid at the time of purchase, then the value of the last instalment covering the interest as well will be : **(Hotel Management, 1998)**

(*a*) Rs. 1050 (*b*) Rs. 2050 (*c*) Rs. 3000 (*d*) None of these

61. If the rate increases by 2%, the simple interest received on a sum of money increases by Rs. 108. If the time period is increased by 2 years, the simple interest on the same sum increases by Rs. 180. The sum is :
(a) Rs. 1800 (b) Rs. 3600 (c) Rs. 5400
(d) Data inadequate (e) None of these

62. Mr. Thomas invested an amount of Rs. 13,900 divided in two different schemes A and B at the simple interest rate of 14% p.a. and 11% p.a. respectively. If the total amount of simple interest earned in 2 years be Rs. 3508, what was the amount invested in Scheme B ? **(R.B.I. 2003)**
(a) Rs. 6400 (b) Rs. 6500 (c) Rs. 7200
(d) Rs. 7500 (e) None of these

63. A sum of Rs. 2600 is lent out in two parts in such a way that the interest on one part at 10% for 5 years is equal to that on another at 9% for 6 years. The sum lent out at 10% is :
(a) Rs. 1150 (b) Rs. 1250 (c) Rs. 1350 (d) Rs. 1450

64. A sum of Rs. 1550 was lent partly at 5% and partly at 8% p.a. simple interest. The total interest received after 3 years was Rs. 300. The ratio of the money lent at 5% to that lent at 8% is :
(a) 5 : 8 (b) 8 : 5 (c) 16 : 15 (d) 31 : 6

65. A man lends Rs. 10,000 in four parts. If he gets 8% on Rs. 2000; $7\frac{1}{2}\%$ on Rs. 4000 and $8\frac{1}{2}\%$ on Rs. 1400; what percent must he get for the remainder, if his average annual interest is 8.13% ?
(a) 7% (b) 9% (c) $9\frac{1}{4}\%$ (d) $10\frac{1}{2}\%$

66. An amount of Rs. 1,00,000 is invested in two types of shares. The first yields an interest of 9% p.a. and the second, 11% p.a. If the total interest at the end of one year is $9\frac{3}{4}\%$, then the amount invested in each share was : **(M.B.A. 2002)**
(a) Rs. 52,500; Rs. 47,500 (b) Rs. 62,500; Rs. 37,500
(c) Rs. 72,500; Rs. 27,500 (d) Rs. 82,500; Rs. 17,500

67. David invested certain amount in three different schemes A, B and C with the rate of interest 10% p.a., 12% p.a. and 15% p.a. respectively. If the total interest accrued in one year was Rs. 3200 and the amount invested in Scheme C was 150% of the amount invested in Scheme A and 240% of the amount invested in Scheme B, what was the amount invested in Scheme B ? **(Bank P.O. 2003)**
(a) Rs. 5000 (b) Rs. 6500 (c) Rs. 8000
(d) Cannot be determined (e) None of these

68. A person invested in all Rs. 2600 at 4%, 6% and 8% per annum simple interest. At the end of the year, he got the same interest in all the three cases. The money invested at 4% is : **(S.S.C. 2003)**
(a) Rs. 200 (b) Rs. 600 (c) Rs. 800 (d) Rs. 1200

69. Divide Rs. 2379 into 3 parts so that their amounts after 2, 3 and 4 years respectively may be equal, the rate of interest being 5% per annum at simple interest. The first part is : **(C.B.I. 1997)**
(a) Rs. 759 (b) Rs. 792 (c) Rs. 818 (d) Rs. 828

70. A man invested $\frac{1}{3}$ of his capital at 7%; $\frac{1}{4}$ at 8% and the remainder at 10%. If his annual income is Rs. 561, the capital is :
(a) Rs. 5400 (b) Rs. 6000 (c) Rs. 6600 (d) Rs. 7200

ANSWERS

1. (c)	2. (a)	3. (b)	4. (a)	5. (b)	6. (d)	7. (d)	8. (b)	9. (c)
10. (c)	11. (d)	12. (b)	13. (c)	14. (d)	15. (c)	16. (d)	17. (b)	18. (b)
19. (c)	20. (c)	21. (a)	22. (c)	23. (a)	24. (b)	25. (c)	26. (b)	27. (c)
28. (c)	29. (d)	30. (b)	31. (c)	32. (d)	33. (b)	34. (b)	35. (d)	36. (a)
37. (a)	38. (d)	39. (a)	40. (a)	41. (d)	42. (b)	43. (c)	44. (c)	45. (b)
46. (d)	47. (b)	48. (d)	49. (e)	50. (c)	51. (a)	52. (b)	53. (b)	54. (a)
55. (b)	56. (c)	57. (b)	58. (d)	59. (c)	60. (d)	61. (d)	62. (a)	63. (c)
64. (c)	65. (b)	66. (b)	67. (a)	68. (d)	69. (d)	70. (c)		

SOLUTIONS

1. Time = 2 yrs 3 mths $= 2\frac{1}{4}$ yrs $= \frac{9}{4}$ yrs.

   $\therefore$ S.I. = Rs. $\left(4800 \times \frac{17}{2} \times \frac{9}{4} \times \frac{1}{100}\right)$ = Rs. 918.

2. Time = 9 months $= \frac{3}{4}$ year.

   $\therefore$ S.I. = Rs. $\left(16800 \times \frac{25}{4} \times \frac{3}{4} \times \frac{1}{100}\right)$ = Rs. 787.50.

3. Time = (22 + 30 + 21) days = 73 days $= \frac{1}{5}$ year.

   $\therefore$ S.I. = Rs. $\left(1820 \times \frac{15}{2} \times \frac{1}{5} \times \frac{1}{100}\right)$ = Rs. 27.30.

4. Gain in 2 yrs. = Rs. $\left[\left(5000 \times \frac{25}{4} \times \frac{2}{100}\right) - \left(\frac{5000 \times 4 \times 2}{100}\right)\right]$ = Rs. (625 − 400) = Rs. 225.

   $\therefore$ Gain in 1 year = Rs. $\left(\frac{225}{2}\right)$ = Rs. 112.50.

5. Time = $\left(\frac{100 \times 81}{450 \times 4.5}\right)$ years = 4 years.

6. S.I. = Rs. (15500 − 12500) = Rs. 3000.

   Rate = $\left(\frac{100 \times 3000}{12500 \times 4}\right)\%$ = 6%.

7. Time = 2 years 4 months $= 2\frac{1}{3}$ years $= \frac{7}{3}$ years.

   Rate = $\left(\frac{100 \times 252 \times 3}{1600 \times 7}\right)\% = 6\frac{3}{4}\%$.

8. Let rate = R% and time = R years. Then,

   $\left(\frac{1200 \times R \times R}{100}\right) = 432 \Leftrightarrow 12R^2 = 432 \Leftrightarrow R^2 = 36 \Leftrightarrow R = 6.$

9. Principal = Rs. $\left(\frac{100 \times 5400}{12 \times 3}\right)$ = Rs. 15000.

10. Let the present worth be Rs. x. Then, S.I. = Rs. $(132 - x)$.

$\therefore \left(\frac{x \times 5 \times 2}{100}\right) = 132 - x \Leftrightarrow 10x = 13200 - 100x \Leftrightarrow 110x = 13200 \Leftrightarrow x = 120.$

11. Principal = Rs. $\left(\frac{100 \times 4016.25}{9 \times 5}\right)$ = Rs. $\left(\frac{401625}{45}\right)$ = Rs. 8925.

12. Sum = $\left(\frac{100 \times \text{S.I.}}{\text{R} \times \text{T}}\right)$ = Rs. $\left(\frac{100 \times x}{x \times x}\right)$ = Rs. $\left(\frac{100}{x}\right)$.

13. S.I. = Rs. (956 − 800) = Rs. 156.

Rate = $\left(\frac{100 \times 156}{800 \times 3}\right)\% = 6\frac{1}{2}\%$.

New rate = $\left(6\frac{1}{2} + 4\right)\% = 10\frac{1}{2}\%$.

New S.I. = Rs. $\left(800 \times \frac{21}{2} \times \frac{3}{100}\right)$ = Rs. 252.

$\therefore$ New amount = Rs. (800 + 252) = Rs. 1052.

14. We need to know the S.I., principal and time to find the rate. Since the principal is not given, so data is inadequate.

15. P = Rs. 800, R = $4\frac{1}{2}\% = \frac{9}{2}\%$, T = 3 years. Then,

S.I. = Rs. $\left(800 \times \frac{9}{2} \times \frac{3}{100}\right)$ = Rs. 108.

Now, P = Rs. 150, S.I. = Rs. 108, R = 8%.

$\therefore$ Time = $\left(\frac{100 \times 108}{150 \times 8}\right)$ years = 9 years.

16. P = Rs. 64, S.I. = Rs. (83.20 − 64) = Rs. 19.20, T = 2 years.

So, rate = $\left(\frac{100 \times 19.20}{64 \times 2}\right)\% = 15\%$.

Now, P = Rs. 86, R = 15%, T = 4 years.

$\therefore$ S.I. = Rs. $\left(\frac{86 \times 15 \times 4}{100}\right)$ = Rs. 51.60.

17. S.I. = Rs. 840, R = 5%, T = 8 years.

Principal = Rs. $\left(\frac{100 \times 840}{5 \times 8}\right)$ = Rs. 2100.

Now, P = Rs. 2100, S.I. = Rs. 840, T = 5 years.

$\therefore$ Rate = $\left(\frac{100 \times 840}{2100 \times 5}\right)\% = 8\%$.

18. S.I. = Rs. 202.50, R = 4.5%, T = 1 year.

Principal = Rs. $\left(\frac{100 \times 202.50}{4.5 \times 1}\right)$ = Rs. 4500.

Now, P = Rs. 4500, R = 5%, T = 1 year.

S.I. = Rs. $\left(\frac{4500 \times 5 \times 1}{100}\right)$ = Rs. 225.

$\therefore$ Difference in interest = Rs. (225 − 202.50) = Rs. 22.50.

19. Let the sum be Rs. x. Then, S.I. = Rs. $(504 - x)$.

$$\therefore \left(\frac{x \times 5 \times 4}{100}\right) = 504 - x \Leftrightarrow 20x = 50400 - 100x \Leftrightarrow 120x = 50400 \Leftrightarrow x = 420.$$

Now, P = Rs. 420, R = 10%, T = $\frac{5}{2}$ years.

S.I. = Rs. $\left(\frac{420 \times 10}{100} \times \frac{5}{2}\right)$ = Rs. 105.

$\therefore$ Amount = Rs. (420 + 105) = Rs. 525.

20. Let the principal be P and rate of interest be R%.

$$\therefore \text{Required ratio} = \left[\frac{\left(\frac{P \times R \times 6}{100}\right)}{\left(\frac{P \times R \times 9}{100}\right)}\right] = \frac{6PR}{9PR} = \frac{6}{9} = 2 : 3.$$

21. Let the sum be Rs. x. Then,

$$\left(\frac{x \times 6 \times 3}{100}\right) + \left(\frac{x \times 9 \times 5}{100}\right) + \left(\frac{x \times 13 \times 3}{100}\right) = 8160$$

$\Leftrightarrow 18x + 45x + 39x = (8160 \times 100) \Leftrightarrow 102x = 816000 \Leftrightarrow x = 8000.$

22. Let the sum be Rs. x. Now, S.I. = Rs. 600, T = 10 years.

$$\text{Rate} = \left(\frac{100 \times 600}{x \times 10}\right)\% = \left(\frac{6000}{x}\right)\%.$$

S.I. for first 5 years = Rs. $\left(\frac{x \times 5 \times 6000}{x \times 100}\right)$ = Rs. 300.

S.I. for last 5 years = Rs. $\left(3x \times 5 \times \frac{6000}{x \times 100}\right)$ = Rs. 900.

$\therefore$ Total interest = Rs. 1200.

23. S.I. = Rs. $\left(10 \times \frac{3}{100} \times 4\right)$ = Rs. 1.20.

24. Let the sum be Rs. 100. Then,

S.I. for first 6 months = Rs. $\left(\frac{100 \times 10 \times 1}{100 \times 2}\right)$ = Rs. 5.

S.I. for last 6 months = Rs. $\left(\frac{105 \times 10 \times 1}{100 \times 2}\right)$ = Rs. 5.25.

So, amount at the end of 1 year = Rs. (100 + 5 + 5.25) = Rs. 110.25.

$\therefore$ Effective rate = (110.25 − 100) = 10.25%.

25. S.I. for 1 year = Rs. (854 − 815) = Rs. 39.

S.I. for 3 years = Rs. (39 × 3) = Rs. 117.

$\therefore$ Principal = Rs. (815 − 117) = Rs. 698.

26. S.I. for 5 years = Rs. (1020 − 720) = Rs. 300.

S.I. for 2 years = Rs. $\left(\frac{300}{5} \times 2\right)$ = Rs. 120.

$\therefore$ Principal = Rs. (720 − 120) = Rs. 600.

27. S.I. for 3 years = Rs. (12005 − 9800) = Rs. 2205.

S.I. for 5 years = Rs. $\left(\frac{2205}{3} \times 5\right)$ = Rs. 3675.

∴ Principal = Rs. (9800 − 3675) = Rs. 6125.

Hence, rate = $\left(\frac{100 \times 3675}{6125 \times 5}\right)\%$ = 12%.

28. S.I. for $1\frac{1}{2}$ years = Rs. (1067.20 − 1012) = Rs. 55.20.

S.I. for $2\frac{1}{2}$ years = Rs. $\left(55.20 \times \frac{2}{3} \times \frac{5}{2}\right)$ = Rs. 92.

∴ Principal = Rs. (1012 − 92) = Rs. 920.

Hence, rate = $\left(\frac{100 \times 92 \times 2}{920 \times 5}\right)\%$ = 4%.

29. Let sum = x. Then, S.I. = x.

∴ Time = $\left(\frac{100 \times \text{S.I.}}{\text{P} \times \text{R}}\right) = \left(\frac{100 \times x}{x \times 12}\right)$ years = $8\frac{1}{3}$ years = 8 years 4 months.

30. Let sum = x. Then, S.I. = x.

∴ Rate = $\left(\frac{100 \times \text{S.I.}}{\text{P} \times \text{T}}\right) = \left(\frac{100 \times x}{x \times 12}\right)\% = \frac{25}{3}\% = 8\frac{1}{3}\%$.

31. Let sum = x. Then, S.I. = $3x$.

∴ Rate = $\left(\frac{100 \times \text{S.I.}}{\text{P} \times \text{T}}\right) = \left(\frac{100 \times 3x}{x \times 15}\right)\%$ = 20%.

32. Let sum = x. Then, S.I. = x.

∴ Rate = $\left(\frac{100 \times x}{x \times 6}\right)\% = \frac{50}{3}\%$.

Now, sum = x, S.I. = $3x$, Rate = $\frac{50}{3}\%$.

∴ Time = $\frac{100 \times 3x}{x \times \frac{50}{3}}$ = 18 years.

33. Let sum = x. Then, S.I. = $2x$, Time = $15\frac{1}{2}$ years = $\frac{31}{2}$ years.

∴ Rate = $\left(\frac{100 \times 2x}{x \times \frac{31}{2}}\right)\% = \frac{400}{31}\%$.

Now, sum = x, S.I. = x, Rate = $\frac{400}{31}\%$.

∴ Time = $\frac{100 \times x}{x \times \frac{400}{31}} = \frac{31}{4}$ years = 7 years 9 months.

34. Let sum be x. Then, S.I. = x.

I. Time = $\frac{100 \times x}{x \times \frac{50}{3}}$ = 6 years (False)

2. Time $= \dfrac{100 \times x}{x \times 20} = 5$ years (True)

3. Suppose sum = x. Then, S.I. = x and Time = 5 years.

Rate $= \left(\dfrac{100 \times x}{x \times 5}\right)\% = 20\%$

Now, sum = x, S.I. = $3x$ and Rate = 20%.

$\therefore$ Time $= \left(\dfrac{100 \times 3x}{x \times 20}\right)$ years = 15 years (False)

So, 2 alone is correct.

35. Let sum = x. Then, S.I. $= \dfrac{x}{2}$.

$\therefore \quad \dfrac{x}{2} = \dfrac{x \times 8 \times 6}{100}$. Clearly, data is inadequate.

36. Let sum = x. Then, S.I. $= \dfrac{2x}{5}$. Time = 10 years.

$\therefore$ Rate $= \left(\dfrac{100 \times 2x}{x \times 5 \times 10}\right)\% = 4\%$.

37. Let sum = x. Then, S.I. $= 0.125x = \dfrac{1}{8}x$, R = 10%.

$\therefore$ Time $= \left(\dfrac{100 \times x}{x \times 8 \times 10}\right)$ years $= \dfrac{5}{4}$ years $= 1\dfrac{1}{4}$ years.

38. Let the sum be x. Then, S.I. = 40% of $x = \dfrac{2x}{5}$; Rate = 5%.

$\therefore$ Time $= \left(100 \times \dfrac{2x}{5} \times \dfrac{1}{x \times 5}\right) = 8$ years.

39. Let sum = x. Then, amount $= \dfrac{7x}{6}$.

S.I. $= \left(\dfrac{7x}{6} - x\right) = \dfrac{x}{6}$; Time = 3 years.

$\therefore$ Rate $= \left(\dfrac{100 \times x}{x \times 6 \times 3}\right)\% = \dfrac{50}{9}\% = 5\dfrac{5}{9}\%$.

40. Let sum = x. Then, S.I. $= \dfrac{x}{9}$.

Let rate = R% and time = R years.

$\therefore \quad \left(\dfrac{x \times R \times R}{100}\right) = \dfrac{x}{9} \Leftrightarrow R^2 = \dfrac{100}{9} \Leftrightarrow R = \dfrac{10}{3} = 3\dfrac{1}{3}$.

Hence, rate $= 3\dfrac{1}{3}\%$.

41. Let sum = x. Then, S.I. $= \dfrac{9}{16}x$.

Let rate = R% and time = R years.

$\therefore \quad \left(\dfrac{x \times R \times R}{100}\right) = \dfrac{9x}{16} \Leftrightarrow R^2 = \dfrac{900}{16} \Leftrightarrow R = \dfrac{30}{4} = 7\dfrac{1}{2}$.

Hence, time $= 7\dfrac{1}{2}$ years.

42. Let the sum lent to C be Rs. x. Then, $\left(\frac{2500 \times 7 \times 4}{100}\right) + \left(\frac{x \times 7 \times 4}{100}\right) = 1120$

$\Leftrightarrow \quad \frac{7}{25}x = (1120 - 700) \Rightarrow x = \left(\frac{420 \times 25}{7}\right) = 1500.$

43. Let each sum be Rs. x. Then, $\left(\frac{x \times 11 \times 9}{100 \times 2}\right) - \left(\frac{x \times 11 \times 7}{100 \times 2}\right) = 412.50$

$\Leftrightarrow \quad (99x - 77x) = 82500 \quad \Leftrightarrow \quad 22x = 82500 \quad \Leftrightarrow \quad x = 3750.$

44. Let the sum be Rs. x. Then, $\left(x \times \frac{15}{2} \times \frac{5}{4} \times \frac{1}{100}\right) - \left(x \times \frac{25}{2} \times \frac{2}{3} \times \frac{1}{100}\right) = 32.50$

$\Leftrightarrow \quad \frac{75x}{8} - \frac{25x}{3} = 3250 \quad \Leftrightarrow \quad 25x = (3250 \times 24) \quad \Leftrightarrow \quad x = \left(\frac{3250 \times 24}{25}\right) = 3120.$

45. Let the sums be x and y.

$$\frac{x \times 6 \times 2}{100} + \frac{y \times 7 \times 2}{100} = 354 \text{ or } 6x + 7y = 17700. \quad \ldots(i)$$

Also, $\frac{x}{4} = \frac{y}{5}$ or $5x - 4y = 0$ $\ldots(ii)$

Solving (i) and (ii), we get : $x = 1200$ and $y = 1500$.

$\therefore$ Total sum = Rs. 2700.

46. Let the money added be Rs. x. Then, $\frac{(830 + x) \times 14 \times 3}{100} - \frac{830 \times 12 \times 3}{100} = 93.90$

$\Leftrightarrow \quad 830 \times 42 + 42x - 830 \times 36 = 9390 \quad \Leftrightarrow \quad 42x + 830 \times (42 - 36) = 9390$

$\Leftrightarrow \quad 42x = 9390 - 4980 \quad \Leftrightarrow \quad x = \frac{4410}{42} = 105.$

$\therefore$ Money added = Rs. 105.

47. Let the time be x years. Then, $\left(\frac{500 \times 3 \times x}{100}\right) + \left(\frac{600 \times 9 \times x}{100 \times 2}\right) = 126$

$\Leftrightarrow \quad 15x + 27x = 126 \quad \Leftrightarrow \quad 42x = 126 \quad \Leftrightarrow \quad x = 3.$

$\therefore$ Required time = 3 years.

48. Let the rate be R% p.a. Then, $\left(\frac{5000 \times R \times 2}{100}\right) + \left(\frac{3000 \times R \times 4}{100}\right) = 2200$

$\Leftrightarrow \quad 100R + 120R = 2200 \quad \Leftrightarrow \quad R = \left(\frac{2200}{220}\right) = 10.$

$\therefore$ Rate = 10%.

49. Let the original rate be R%. Then, new rate = (2R)%.

$\therefore \quad \left(\frac{725 \times R \times 1}{100}\right) + \left(\frac{362.50 \times 2R \times 1}{100 \times 3}\right) = 33.50$

$\Leftrightarrow \quad (2175 + 725)\, R = 33.50 \times 100 \times 3 = 10050$

$\Leftrightarrow \quad R = \frac{10050}{2900} = 3.46.$

$\therefore$ Original rate = 3.46%.

50. $\left(\frac{1500 \times R_1 \times 3}{100}\right) - \left(\frac{1500 \times R_2 \times 3}{100}\right) = 13.50$

$\Leftrightarrow \quad 4500\,(R_1 - R_2) = 1350 \quad \Leftrightarrow \quad R_1 - R_2 = \frac{1350}{4500} = 0.3\%.$

51. Let the second amount be Rs. x. Then,

$$\left(\frac{12000\times10\times1}{100}\right)+\left(\frac{x\times20\times1}{100}\right)=\left[\frac{(12000+x)\times14\times1}{100}\right]$$

$\Leftrightarrow$ $12000 + 20x = 168000 + 14x$ $\Leftrightarrow$ $6x = 48000$ $\Leftrightarrow$ $x = 8000$.

$\therefore$ Total investment = Rs. (12000 + 8000) = Rs. 20000.

52. S.I. = Rs. $\left(2600\times\frac{20}{3}\times\frac{1}{100}\times T\right)$ = Rs. $\left(\frac{520}{3}\times T\right)$,

which is an exact number of rupees when T = 3.

53. Let the savings be X and Y and the rates of simple interest be $5x$ and $4x$ respectively.

Then, $X\times5x\times\frac{1}{2}\times\frac{1}{100}=Y\times4x\times\frac{1}{2}\times\frac{1}{100}$ or $\frac{X}{Y}=\frac{4}{5}$. *i.e.*, X : Y = 4 : 5.

54. Let the sum be Rs. x and original rate be R%. Then, $\frac{x\times(R+3)\times2}{100}-\frac{x\times R\times2}{100}=72$

$\Leftrightarrow$ $2Rx + 6x - 2Rx = 7200$ $\Leftrightarrow$ $x = 1200$.

55. Let the sum be Rs. x. Then, $\left(x\times\frac{25}{2}\times\frac{1}{100}\right)-\left(\frac{x\times10\times1}{100}\right)=1250$

$\Leftrightarrow$ $25x - 20x = 250000$ $\Leftrightarrow$ $5x = 250000$ $\Leftrightarrow$ $x = 50000$.

56. Let the capital be Rs. x. Then, $\left(\frac{x\times8\times1}{100}\right)-\left(x\times\frac{31}{4}\times\frac{1}{100}\right)=61.50$

$\Leftrightarrow$ $32x - 31x = 6150 \times 4$ $\Leftrightarrow$ $x = 24600$.

57. Let the annual instalment be Rs. x. Then,

$$\left[x+\left(\frac{x\times3\times5}{100}\right)\right]+\left[x+\left(\frac{x\times2\times5}{100}\right)\right]+\left[x+\left(\frac{x\times1\times5}{100}\right)\right]+x=6450$$

$\Leftrightarrow$ $\frac{23x}{20}+\frac{22x}{20}+\frac{21x}{20}+x=6450$ $\Leftrightarrow$ $86x = 6450\times20$ $\Leftrightarrow$ $x = 1500$.

58. Rs. 10 + S.I. on Rs. 10 for 11 months

= Rs. 11 + S.I. on Re. 1 for (1 + 2 + 3 + 4 + ... + 10) months

$\Rightarrow$ Rs. 10 + S.I. on Re. 1 for 110 months = Rs. 11 + S.I. on Re. 1 for 55 months

$\Rightarrow$ S.I. on Re. 1 for 55 months = Re. 1.

$\therefore$ Rate = $\left(\frac{100\times12}{1\times55}\right)\% = 21\frac{9}{11}\%$.

59. Amount to be paid = Rs. $\left(100+\frac{200\times5\times1}{100}+\frac{100\times5\times1}{100}\right)$ = Rs. 115.

60. Money paid in cash = Rs. 1000.

Balance payment = Rs. (20000 − 1000) = Rs. 19000.

61. Let the sum be Rs. x, rate be R% p.a. and time be T years.

Then, $\left[\frac{x\times(R+2)\times T}{100}\right]-\left(\frac{x\times R\times T}{100}\right)=108$ $\Leftrightarrow$ $2xT = 10800$ ...(*i*)

And, $\left[\frac{x\times R\times(T+2)}{100}\right]-\left(\frac{x\times R\times T}{100}\right)=180$ $\Leftrightarrow$ $2xR = 18000$ ...(*ii*)

Clearly, from (*i*) and (*ii*), we cannot find the value of x.

So, the data is inadequate.

62. Let the sum invested in Scheme A be Rs. x and that in Scheme B be Rs. $(13900 - x)$.

Then, $\left(\frac{x \times 14 \times 2}{100}\right) + \left[\frac{(13900 - x) \times 11 \times 2}{100}\right] = 3508$

$\Leftrightarrow \quad 28x - 22x = 350800 - (13900 \times 22) \quad \Leftrightarrow \quad 6x = 45000 \quad \Leftrightarrow \quad x = 7500.$

So, sum invested in Scheme B = Rs. $(13900 - 7500)$ = Rs. 6400.

63. Let the sum lent at 10% be Rs. x and that lent at 9% be Rs. $(2600 - x)$. Then,

$$\left(\frac{x \times 10 \times 5}{100}\right) = \frac{(2600 - x) \times 9 \times 6}{100}$$

$\Leftrightarrow \quad 50x = (2600 \times 54) - 54x \quad \Rightarrow \quad x = \left(\frac{2600 \times 54}{104}\right) = 1350.$

$\therefore$ Sum lent at 10% = Rs. 1350.

64. Let the sum lent at 5% be Rs. x and that lent at 8% be Rs. $(1550 - x)$. Then,

$$\left(\frac{x \times 5 \times 3}{100}\right) + \left[\frac{(1550 - x) \times 8 \times 3}{100}\right] = 300$$

$\Leftrightarrow \quad 15x - 24x + (1550 \times 24) = 30000 \quad \Leftrightarrow \quad 9x = 7200 \quad \Leftrightarrow \quad x = 800.$

$\therefore$ Required ratio = 800 : 750 = 16 : 15.

65. Let the required rate be R. Then,

$$\left(\frac{20000 \times 8 \times 1}{100}\right) + \left(4000 \times \frac{15}{2} \times \frac{1}{100}\right) + \left(1400 \times \frac{17}{2} \times \frac{1}{100}\right) + \left(2600 \times R \times \frac{1}{100}\right) = \left(\frac{813}{10000} \times 10000\right)$$

$\Leftrightarrow \quad 160 + 300 + 119 + 26R = 813 \quad \Leftrightarrow \quad R = 9.$

66. Let the sum invested at 9% be Rs. x and that invested at 11% be Rs. $(100000 - x)$.

Then, $\left(\frac{x \times 9 \times 1}{100}\right) + \left[\frac{(100000 - x) \times 11 \times 1}{100}\right] = \left(100000 \times \frac{39}{4} \times \frac{1}{100}\right)$

$\Leftrightarrow \quad \frac{9x + 1100000 - 11x}{100} = \frac{39000}{4} = 9750$

$\Leftrightarrow \quad 2x = (1100000 - 975000) = 125000 \quad \Leftrightarrow \quad x = 62500.$

$\therefore$ Sum invested at 9% = Rs. 62500.

Sum invested at 11% = Rs. $(100000 - 62500)$ = Rs. 37500.

67. Let x, y and z be the amounts invested in schemes A, B and C respectively. Then,

$$\left(\frac{x \times 10 \times 1}{100}\right) + \left(\frac{y \times 12 \times 1}{100}\right) + \left(\frac{z \times 15 \times 1}{100}\right) = 3200$$

$\Leftrightarrow \quad 10x + 12y + 15z = 320000 \quad ...(i)$

Now, $z = 240\%$ of $y = \frac{12}{5}y \quad ...(ii)$

And, $z = 150\%$ of $x = \frac{3}{2}x \Rightarrow x = \frac{2}{3}z = \left(\frac{2}{3} \times \frac{12}{5}\right)y = \frac{8}{5}y \quad ...(iii)$

From (i), (ii) and (iii), we have :

$16y + 12y + 36y = 320000 \quad \Leftrightarrow \quad 64y = 320000 \quad \Leftrightarrow \quad y = 5000.$

$\therefore$ Sum invested in Scheme B = Rs. 5000.

68. Let the parts be x, y and $[2600 - (x + y)]$. Then,

$$\frac{x \times 4 \times 1}{100} = \frac{y \times 6 \times 1}{100} = \frac{[2600 - (x + y)] \times 8 \times 1}{100}$$

$\therefore \quad \frac{y}{x} = \frac{4}{6} = \frac{2}{3}$ or $y = \frac{2}{3}x$.

So, $\frac{x \times 4 \times 1}{100} = \frac{\left(2600 - \frac{5}{3}x\right) \times 8}{100}$

$\Leftrightarrow \quad 4x = \frac{(7800 - 5x) \times 8}{3} \quad \Leftrightarrow \quad 52x = (7800 \times 8) \Leftrightarrow x = \left(\frac{7800 \times 8}{52}\right) = 1200.$

$\therefore$ Money invested at 4% = Rs. 1200.

69. Let the parts be x, y and $[2379 - (x + y)]$.

$$x + \left(x \times 2 \times \frac{5}{100}\right) = y + \left(y \times 3 \times \frac{5}{100}\right) = z + \left(z \times 4 \times \frac{5}{100}\right)$$

$$\Rightarrow \frac{11x}{10} = \frac{23y}{20} = \frac{6z}{5} = k \Rightarrow x = \frac{10k}{11}, y = \frac{20k}{23}, z = \frac{5k}{6}$$

But $x + y + z = 2379$

$$\Rightarrow \frac{10k}{11} + \frac{20k}{23} + \frac{5k}{6} = 2379 \Rightarrow 1380k + 1320k + 1265k = 2379 \times 11 \times 23 \times 6$$

$$\Rightarrow k = \frac{2379 \times 11 \times 23 \times 6}{3965} = \frac{3 \times 11 \times 23 \times 6}{5}$$

$$\therefore \quad x = \left(\frac{10}{11} \times \frac{3 \times 11 \times 23 \times 6}{5}\right) = 828.$$

Hence, the first part is Rs. 828.

70. Let total capital be Rs. x Then, $\left(\frac{x}{3} \times \frac{7}{100} \times 1\right) + \left(\frac{x}{4} \times \frac{8}{100} \times 1\right) + \left(\frac{5x}{12} \times \frac{10}{100} \times 1\right) = 561$

$$\Leftrightarrow \quad \frac{7x}{300} + \frac{x}{50} + \frac{x}{24} = 561 \quad \Leftrightarrow \quad 51x = (561 \times 600) \quad \Leftrightarrow \quad x = \left(\frac{561 \times 600}{51}\right) = 6600.$$

71. Let the sum be Rs. 100 be invested for 1 year. Then,

$$\text{S.I.} = \text{Rs.} \left[\left(\frac{40 \times 15 \times 1}{100}\right) + \left(\frac{30 \times 10 \times 1}{100}\right) + \left(\frac{30 \times 18 \times 1}{100}\right)\right] = \text{Rs. } 14.40.$$

$\therefore$ Effective rate = 14.4%.

EXERCISE 21B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 6*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. What is the rate of simple interest ? **(Bank P.O. 2003)**
   I. The total interest earned was Rs. 4000.
   II. The sum was invested for 4 years.
2. The simple interest on a sum of money is Rs. 50. What is the sum ? **(R.B.I. 2003)**
   I. The interest rate is 10% p.a.
   II. The sum earned simple interest in 10 years.
3. How much money did X invest ?
   I. An increase in the rate of interest from $4\frac{7}{8}\%$ to $5\frac{1}{8}\%$ per annum increases his yearly income by Rs. 25.
   II. The sum invested gets doubled, when invested at 8% p.a. for $12\frac{1}{2}$ years.
4. What percentage of simple interest per annum did Anand pay to Deepak ?
   I. Anand borrowed Rs. 8000 from Deepak for four years.
   II. Anand returned Rs. 8800 to Deepak at the end of two years and settled the loan.
   (I.B.P.S. 2002)
5. A man borrowed a total sum of Rs. 24000 from two moneylenders. For one loan, he paid interest @ $7\frac{1}{2}\%$ p.a. and for the other 9% p.a. How much money did he borrow at each rate ?
   I. The sum of the interests after one year was Rs. 2025.
   II. The interest on one sum was twice that on the other.
6. What is the sum which earned interest ? **(NABARD, 2002)**
   I. The total simple interest was Rs. 7000 after 7 years.
   II. The total of sum and simple interest was double of the sum after 5 years.

Directions (*Questions 7-8*) : *Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question.*

7. What is the principal sum ?
   I. The sum amounts to Rs. 690 in 3 years at S.I.
   II. The sum amounts to Rs. 750 in 5 years at S.I.
   III. The rate of interest is 5% p.a.
   (*a*) I and III only (*b*) II and III only
   (*c*) I and II only (*d*) I and III only, or II and III only
   (*e*) Any two of the three
8. In how many years will a sum of money put at simple interest treble itself ?
   I. The interest earned in 4 years is half the sum.
   II. The rate of interest is $12\frac{1}{2}\%$.
   III. The sum doubles itself in 8 years at simple interest.
   (*a*) Any one of the three (*b*) Any two of the three
   (*c*) All I, II and III (*d*) II and III only
   (*e*) I and II only

ANSWERS

1. (*d*) 2. (*c*) 3. (*a*) 4. (*e*) 5. (*c*) 6. (*e*) 7. (*e*) 8. (*a*)

SOLUTIONS

1. We know that, $R = \left(\frac{100 \times S.I.}{P \times T}\right)$

Now, I gives, S.I. = Rs. 4000.

II gives, T = 4 years.

But, P is unknown. So, we cannot find R.

So, given data is insufficient to get R.

∴ Correct answer is (*d*).

2. Given : S.I. = Rs. 50.

I gives, R = 10% p.a.

II gives, T = 10 years.

$\therefore$ Sum = $\left(\frac{100 \times S.I.}{T \times R}\right)$ = Rs. $\left(\frac{100 \times 50}{10 \times 10}\right)$ = Rs. 50.

Thus, I and II together give the answer.

∴ Correct answer is (*e*).

3. Suppose X invests Rs. x.

I gives : $R_1 = \frac{39}{8}\%$, $R_2 = \frac{41}{8}\%$.

Increase in S.I. = Rs. 25.

$$\rightarrow \left(\frac{x \times 1 \times \frac{41}{8}}{100}\right) - \left(\frac{x \times 1 \times \frac{39}{8}}{100}\right) = 25$$

$$\Rightarrow (41x - 39x) = (25 \times 800) \Rightarrow x = \left(\frac{25 \times 800}{2}\right) = 10000.$$

Thus, I only gives the answer.

II gives, S.I. = Rs. x, R = 8% and $T = \frac{25}{2}$ years.

$$P = \frac{100 \times S.I.}{R \times T} = \left(\frac{100 \times x}{8 \times 25} \times 2\right)$$

Thus, P is not obtained.

∴ I alone is sufficient to get the answer and II is not sufficient to get the answe

∴ Correct answer is (*a*).

4. Let the rate be R% p.a.

I gives, P = Rs. 8000 and T = 4 years.

II gives, S.I. = Rs. (8800 − 8000) = Rs. 800.

$$\therefore R = \frac{100 \times S.I.}{P \times T} = \left(\frac{100 \times 800}{8000 \times 4}\right)\% = 2\frac{1}{2}\% \text{ p.a.}$$

Thus, I and II both are needed to get the answer.

∴ Correct answer is (*e*).

5. Suppose he borrowed Rs. x at $7\frac{1}{2}\%$ p.a. and Rs. $(24000 - x)$ at 9% p.a.

I gives, total interest = Rs. 2025.

$\therefore \quad \left(x \times 1 \times \frac{15}{2} \times \frac{1}{100}\right) + \left\{(24000 - x) \times 1 \times \frac{9}{100}\right\} = 2025.$

This gives x.

II gives Interest on Rs. $(24000 - x)$ = 2 × (interest on Rs. x)

$\Rightarrow \quad (24000 - x) \times \frac{9}{100} \times 1 = \left(2 \times x \times \frac{15}{2} \times \frac{1}{100}\right)$

This gives x.

Thus, data in I as well as well as in II are sufficient to answer the question.

$\therefore$ Correct answer is (*c*).

6. Let the sum be Rs. x.

I gives, S.I. = Rs. 7000 and T = 7 years.

II gives, Sum + S.I. for 5 years = 2 × Sum $\Rightarrow$ Sum = S.I. for 5 years

Now, S.I. for 7 years = Rs. 7000

$\therefore$ S.I. for 1 year = Rs. $\frac{7000}{7}$ = Rs. 1000.

S.I. for 5 years = Rs. (1000 × 5) = Rs. 5000.

Thus, I and II both are needed to get the answer.

$\therefore$ Correct answer is (*e*).

7. Clearly, any of the three will give us the answer.

$\therefore$ Correct answer is (*e*).

8. Let sum be Rs. x. Then, S.I. = Rs. $(3x - x)$ = Rs. $2x$, T = ?

I gives : When T = 4, then S.I. = Rs. $\frac{x}{2}$.

$\therefore \quad R = \frac{100 \times \text{S.I.}}{P \times T} = \left(100 \times \frac{x}{2} \times \frac{1}{x} \times \frac{1}{4}\right) = 12\frac{1}{2}\%$ p.a.

Now, Sum = Rs. x, S.I. = Rs. $2x$, R = $\frac{25}{2}\%$ p.a., T = ?

$\therefore \quad T = \frac{100 \times \text{S.I.}}{P \times R} = \left(\frac{100 \times 2x}{x \times 25} \times 2\right) = 16$ years.

Thus, I only gives the answer.

II gives, R = $\frac{25}{2}\%$ p.a.

$\therefore \quad T = \frac{100 \times \text{S.I.}}{P \times R} = \left(\frac{100 \times 2x}{x \times 25} \times 2\right) = 16$ years.

Thus, II only also gives the answer.

III gives, R = 5% p.a.

$\therefore \quad T = \frac{100 \times \text{S.I.}}{P \times R} = \left(\frac{100 \times 2x}{x \times 5}\right) = 40$ years.

Thus, III only also gives the answer.

$\therefore$ Correct answer is (*a*).

22. COMPOUND INTEREST

Compound Interest : Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say *yearly* or *half-yearly* or *quarterly* to settle the previous account.

In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

After a specified period, *the difference between the amount and the money borrowed is called the* ***Compound Interest*** *(abbreviated as C.I.) for that period.*

IMPORTANT FACTS AND FORMULAE

Let Principal = P, Rate = R% per annum, Time = n years.

I. When interest is compound Annually :

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

II. When interest is compounded Half-yearly :

$$\text{Amount} = P\left[1+\frac{(R/2)}{100}\right]^{2n}$$

III. When interest is compounded Quarterly :

$$\text{Amount} = P\left[1+\frac{(R/4)}{100}\right]^{4n}$$

IV. When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{2}{5}R}{100}\right)$$

V. When Rates are different for different years, say $R_1\%$, $R_2\%$, $R_3\%$ for 1st, 2nd and 3rd year respectively.

$$\text{Then, Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right).$$

VI. Present worth of Rs. x due n years hence is given by :

$$\text{Present Worth} = \frac{x}{\left(1+\frac{R}{100}\right)^n}.$$

SOLVED EXAMPLES

Ex. 1. ***Find compound interest on Rs. 7500 at 4% per annum for 2 years, compounded annually.***

Sol. Amount = Rs. $\left[7500\times\left(1+\frac{4}{100}\right)^2\right]$ = Rs. $\left(7500\times\frac{26}{25}\times\frac{26}{25}\right)$ = Rs. 8112.

$\therefore$ C.I. = Rs. (8112 − 7500) = Rs. 612.

Ex. 2. ***Find compound interest on Rs. 8000 at 15% per annum for 2 years 4 months, compounded annually.***

Sol. Time = 2 years 4 months = $2\frac{4}{12}$ years = $2\frac{1}{3}$ years.

$$\text{Amount} = \text{Rs.}\left[8000\times\left(1+\frac{15}{100}\right)^2\times\left(1+\frac{\frac{1}{3}\times 15}{100}\right)\right] = \text{Rs.}\left(8000\times\frac{23}{20}\times\frac{23}{20}\times\frac{21}{20}\right)$$

= Rs. 11109.

$\therefore$ C.I. = Rs. (11109 − 8000) = Rs. 3109.

Ex. 3. ***Find the compound interest on Rs. 10,000 in 2 years at 4% per annum, the interest being compounded half-yearly.*** (S.S.C. 2000)

Sol. Principal = Rs. 10000; Rate = 2% per half-year; Time = 2 years = 4 half-years.

$\therefore$ Amount = Rs. $\left[10000\times\left(1+\frac{2}{100}\right)^4\right]$ = Rs. $\left(10000\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}\right)$

= Rs. 10824.32.

$\therefore$ C.I. = Rs. (10824.32 − 10000) = Rs. 824.32.

Ex. 4. ***Find the compound interest on Rs. 16,000 at 20% per annum for 9 months, compounded quarterly.***

Sol. Principal = Rs. 16000; Time = 9 months = 3 quarters;

Rate = 20% per annum = 5% per quarter.

$\therefore$ Amount = Rs. $\left[16000\times\left(1+\frac{5}{100}\right)^3\right]$ = Rs. $\left(16000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\right)$ = Rs. 18522.

$\therefore$ C.I. = Rs. (18522 − 16000) = Rs. 2522.

Ex. 5. ***If the simple interest on a sum of money at 5% per annum for 3 years is Rs. 1200, find the compound interest on the same sum for the same period at the same rate.***

Sol. Clearly, Rate = 5% p.a., Time = 3 years, S.I. = Rs. 1200.

So, Principal = Rs. $\left(\frac{100\times 1200}{3\times 5}\right)$ = Rs. 8000.

Amount = Rs. $\left[8000\times\left(1+\frac{5}{100}\right)^3\right]$ = Rs. $\left(8000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\right)$ = Rs. 9261.

$\therefore$ C.I. = Rs. (9261 − 8000) = Rs. 1261.

Ex. 6. ***In what time will Rs. 1000 become Rs. 1331 at 10% per annum compounded annually ?*** (S.S.C. 2004)

Sol. Principal = Rs. 1000; Amount = Rs. 1331; Rate = 10% p.a.

Let the time be n years. Then,

$$\left[1000\left(1+\frac{10}{100}\right)^{n}\right]=1331 \text{ or } \left(\frac{11}{10}\right)^{n}=\left(\frac{1331}{1000}\right)=\left(\frac{11}{10}\right)^{3}$$

$\therefore$ $n = 3$ years.

Ex. 7. ***If Rs. 500 amounts to Rs. 583.20 in two years compounded annually, find the rate of interest per annum.***

Sol. Principal = Rs. 500; Amount = Rs. 583.20; Time = 2 years.

Let the rate be R% per annum. Then,

$$\left[500\left(1+\frac{R}{100}\right)^{2}\right]=583.20 \text{ or } \left(1+\frac{R}{100}\right)^{2}=\frac{5832}{5000}=\frac{11664}{10000}$$

$$\therefore \left(1+\frac{R}{100}\right)^{2}=\left(\frac{108}{100}\right)^{2} \text{ or } 1+\frac{R}{100}=\frac{108}{100} \text{ or } R=8.$$

So, rate = 8% p.a.

Ex. 8. ***If the compound interest on a certain sum at $16\frac{2}{3}\%$ for 3 years is Rs. 1270, find the simple interest on the same sum at the same rate and for the same period.***

Sol. Let the sum be Rs. x. Then,

$$\text{C.I.} = \left[x\times\left(1+\frac{50}{3\times100}\right)^{3}-x\right]=\left(\frac{343x}{216}-x\right)=\frac{127x}{216}.$$

$$\therefore \frac{127x}{216}=1270 \text{ or } x=\frac{1270\times216}{127}=2160.$$

Thus, the sum is Rs. 2160.

$$\therefore \text{S.I.} = \text{Rs.}\left(2160\times\frac{50}{3}\times3\times\frac{1}{100}\right)=\text{Rs. } 1080.$$

Ex. 9. ***The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 years is Rs. 631. Find the sum.***

Sol. Let the sum be Rs. x. Then,

$$\text{C.I.} = x\left(1+\frac{10}{100}\right)^{2}-x=\frac{21x}{100}, \quad \text{S.I.} = \left(\frac{x\times10\times2}{100}\right)=\frac{x}{5}.$$

$$\therefore (\text{C.I.})-(\text{S.I.})=\left(\frac{21x}{100}-\frac{x}{5}\right)=\frac{x}{100}.$$

$$\therefore \frac{x}{100}=631 \Leftrightarrow x=63100.$$

Hence, the sum is Rs. 63,100.

Ex. 10. ***The difference between the compound interest and the simple interest accrued on an amount of Rs. 18,000 in 2 years was Rs. 405. What was the rate of interest p.c.p.a.?*** **(Bank P.O. 2003)**

Sol. Let the rate be R% p.a. Then,

$$\left[18000\left(1+\frac{R}{100}\right)^{2}-18000\right]-\left(\frac{18000\times R\times2}{100}\right)=405$$

$$\Leftrightarrow 18000\left[\frac{(100+R)^{2}}{10000}-1-\frac{2R}{100}\right]=405$$

$$\Leftrightarrow \quad 18000\left[\frac{(100+R)^2-10000-200R}{10000}\right] = 405$$

$$\Leftrightarrow \quad \frac{9}{5}R^2 = 405 \Leftrightarrow R^2 = \left(\frac{405\times 5}{9}\right) = 225 \Leftrightarrow R = 15.$$

$\therefore$ Rate = 15%.

Ex. 11. ***Divide Rs. 1301 between A and B, so that the amount of A after 7 years is equal to the amount of B after 9 years, the interest being compounded at 4% per annum.***

Sol. Let the two parts be Rs. x and Rs. $(1301 - x)$.

$$x\left(1+\frac{4}{100}\right)^7 = (1301-x)\left(1+\frac{4}{100}\right)^9 \Leftrightarrow \frac{x}{(1301-x)} = \left(1+\frac{4}{100}\right)^2 = \left(\frac{26}{25}\times\frac{26}{25}\right).$$

$$\Leftrightarrow \quad 625x = 676\,(1301 - x) \Leftrightarrow 1301x = 676 \times 1301 \Leftrightarrow x = 676.$$

So, the two parts are Rs. 676 and Rs. (1301 – 676) *i.e.* Rs. 676 and Rs. 625.

Ex. 12. ***A certain sum amounts to Rs. 7350 in 2 years and to Rs. 8575 in 3 years. Find the sum and rate percent.***

Sol. S.I. on Rs. 7350 for 1 year = Rs. (8575 – 7350) = Rs. 1225.

$$\therefore \quad \text{Rate} = \left(\frac{100\times 1225}{7350\times 1}\right)\% = 16\frac{2}{3}\%.$$

Let the sum be Rs. x. Then,

$$x\left(1+\frac{50}{3\times 100}\right)^2 = 7350 \Leftrightarrow x\times\frac{7}{6}\times\frac{7}{6} = 7350 \Leftrightarrow x = \left(7350\times\frac{36}{49}\right) = 5400.$$

$\therefore$ Sum = Rs. 5400.

Ex. 13. ***A sum of money amounts to Rs. 6690 after 3 years and to Rs. 10,035 after 6 years on compound interest. Find the sum.***

Sol. Let the sum be Rs. P. Then,

$$P\left(1+\frac{R}{100}\right)^3 = 6690 \quad \ldots(i) \qquad \text{and } P\left(1+\frac{R}{100}\right)^6 = 10035 \quad \ldots(ii)$$

On dividing, we get $\left(1+\frac{R}{100}\right)^3 = \frac{10035}{6690} = \frac{3}{2}$.

Substituting this value in (i), we get :

$$P\times\frac{3}{2} = 6690 \text{ or } P = \left(6690\times\frac{2}{3}\right) = 4460.$$

Hence, the sum is Rs. 4460.

Ex. 14. ***A sum of money doubles itself at compound interest in 15 years. In how many years will it become eight times ?***

Sol. $$P\left(1+\frac{R}{100}\right)^{15} = 2P \Rightarrow \left(1+\frac{R}{100}\right)^{15} = \frac{2P}{P} = 2 \quad \ldots(i)$$

Let $P\left(1+\frac{R}{100}\right)^n = 8P \Rightarrow \left(1+\frac{R}{100}\right)^n = 8 = 2^3 = \left\{\left(1+\frac{R}{100}\right)^{15}\right\}^3$ [*using* (i)]

$$\Rightarrow \left(1+\frac{R}{100}\right)^n = \left(1+\frac{R}{100}\right)^{45} \Rightarrow n = 45.$$

Thus, the required time = 45 years.

Ex. 15. ***What annual payment will discharge a debt of Rs. 7620 due in 3 years at*** $16\frac{2}{3}\%$ ***per annum compound interest ?***

Sol. Let each instalment be Rs. x. Then,

(P.W. of Rs. x due 1 year hence) + (P.W. of Rs. x due 2 years hence)

+ (P.W. of Rs. x due 3 years hence) = 7620.

$$\therefore \quad \frac{x}{\left(1+\frac{50}{3\times 100}\right)}+\frac{x}{\left(1+\frac{50}{3\times 100}\right)^2}+\frac{x}{\left(1+\frac{50}{3\times 100}\right)^3}=7620$$

$$\Leftrightarrow \quad \frac{6x}{7}+\frac{36x}{49}+\frac{216x}{343}=7620 \quad \Leftrightarrow \quad 294x+252x+216x=7620\times 343$$

$$\Leftrightarrow \quad x=\left(\frac{7620\times 343}{762}\right)=3430.$$

$\therefore$ Amount of each instalment = Rs. 3430.

EXERCISE 22A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. Albert invested an amount of Rs. 8000 in a fixed deposit scheme for 2 years at compound interest rate 5 p.c.p.a. How much amount will Albert get on maturity of the fixed deposit ? **(Bank P.O. 1999)**

   (*a*) Rs. 8600 (*b*) Rs. 8620 (*c*) Rs. 8800
   (*d*) Rs. 8840 (*e*) None of these

2. What will be the compound interest on a sum of Rs. 25,000 after 3 years at the rate of 12 p.c.p.a. ? **(S.B.I.P.O. 2003)**

   (*a*) Rs. 9000.30 (*b*) Rs. 9720 (*c*) Rs. 10123.20
   (*d*) Rs. 10483.20 (*e*) None of these

3. The compound interest on Rs. 20,480 at $6\frac{1}{4}\%$ per annum for 2 years 73 days, is :

   (*a*) Rs. 2929 (*b*) Rs. 3000 (*c*) Rs. 3131 (*d*) Rs. 3636

4. A man saves Rs. 200 at the end of each year and lends the money at 5% compound interest. How much will it become at the end of 3 years ?

   (*a*) Rs. 565.25 (*b*) Rs. 635 (*c*) Rs. 662.02 (*d*) Rs. 666.50

   (Hotel Management, 2003)

5. Sam invested Rs. 15,000 @ 10% per annum for one year. If the interest is compounded half-yearly, then the amount received by Sam at the end of the year will be :

   (*a*) Rs. 16,500 (*b*) Rs. 16,525.50 (*c*) Rs. 16,537.50
   (*d*) Rs. 18,150 (*e*) None of these **(S.B.I.P.O. 2002)**

6. A bank offers 5% compound interest calculated on half-yearly basis. A customer deposits Rs. 1600 each on 1st January and 1st July of a year. At the end of the year, the amount he would have gained by way of interest is : **(N.D.A. 2000)**

   (*a*) Rs. 120 (*b*) Rs. 121 (*c*) Rs. 122 (*d*) Rs. 123

7. What is the difference between the compound interests on Rs. 5000 for $1\frac{1}{2}$ years at 4% per annum compounded yearly and half-yearly ? **(S.S.C. 2000)**

   (*a*) Rs. 2.04 (*b*) Rs. 3.06 (*c*) Rs. 4.80 (*d*) Rs. 8.30

8. Find the compound interest on Rs. 15,625 for 9 months at 16% per annum compounded quarterly. **(R.R.B. 2002)**
(*a*) Rs. 1851 (*b*) Rs. 1941 (*c*) Rs 1951 (*d*) Rs. 1961

9. If the simple interest on a sum of money for 2 years at 5% per annum is Rs. 50, what is the compound interest on the same sum at the same rate and for the same time ?
(*a*) Rs. 51.25 (*b*) Rs. 52 (*c*) Rs. 54.25 (*d*) Rs. 60
(C.B.I. 1997)

10. What will be the difference between simple and compound interest @ 10% per annum on a sum of Rs. 1000 after 4 years ? **(Bank P.O. 2002)**
(*a*) Rs. 31 (*b*) Rs. 32.10 (*c*) Rs. 40.40
(*d*) Rs. 64.10 (*e*) None of these

11. The difference between simple interest and compound interest on Rs. 1200 for one year at 10% per annum reckoned half-yearly is : **(R.R.B. 2002)**
(*a*) Rs. 2.50 (*b*) Rs. 3 (*c*) Rs. 3.75
(*d*) Rs. 4 (*e*) None of these

12. The compound interest on Rs. 30,000 at 7% per annum is Rs. 4347. The period (in years) is : **(L.I.C.A.A.O. 2003)**
(*a*) 2 (*b*) $2\frac{1}{2}$ (*c*) 3 (*d*) 4

13. At what rate of compound interest per annum will a sum of Rs. 1200 become Rs. 1348.32 in 2 years ?
(*a*) 6% (*b*) 6.5% (*c*) 7% (*d*) 7.5%

14. The principal that amounts to Rs. 4913 in 3 years at $6\frac{1}{4}\%$ per annum compound interest compounded annually, is : **(S.S.C. 2000)**
(*a*) Rs. 3096 (*b*) Rs. 4076 (*c*) Rs. 4085 (*d*) Rs. 4096

15. The present worth of Rs. 169 due in 2 years at 4% per annum compound interest is :
(*a*) Rs. 150.50 (*b*) Rs. 154.75 (*c*) Rs. 156.25 (*d*) Rs. 158

16. In how many years will a sum of Rs. 800 at 10% per annum compounded semi-annually become Rs. 926.10 ? **(Section Officers', 2001)**
(*a*) $1\frac{1}{3}$ (*b*) $1\frac{1}{2}$ (*c*) $2\frac{1}{3}$ (*d*) $2\frac{1}{2}$

17. If the compound interest on a sum for 2 years at $12\frac{1}{2}\%$ per annum is Rs. 510, the simple interest on the same sum at the same rate for the same period of time is :
(*a*) Rs. 400 (*b*) Rs. 450 (*c*) Rs. 460 (*d*) Rs. 480
(S.S.C. 2004)

18. The compound interest on a certain sum for 2 years at 10% per annum is Rs. 525. The simple interest on the same sum for double the time at half the rate percent per annum is : **(C.B.I. 1997)**
(*a*) Rs. 400 (*b*) Rs. 500 (*c*) Rs. 600 (*d*) Rs. 800

19. The simple interest on a certain sum of money for 3 years at 8% per annum is half the compound interest on Rs. 4000 for 2 years at 10% per annum. The sum placed on simple interest is : **(S.S.C. 2003)**
(*a*) Rs. 1550 (*b*) Rs. 1650 (*c*) Rs. 1750 (*d*) Rs. 2000

20. There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest of Rs. 12,000 after 3 years at the same rate ? **(SIDBI, 2000)**
(*a*) Rs. 2160 (*b*) Rs. 3120 (*c*) Rs. 3972
(*d*) Rs. 6240 (*e*) None of these

21. The difference between compound interest and simple interest on an amount of Rs. 15,000 for 2 years is Rs. 96. What is the rate of interest per annum ?
(a) 8 (b) 10 (c) 12
(d) Cannot be determined (e) None of these (R.B.I. 2003)

22. The difference between simple and compound interests compounded annually on a certain sum of money for 2 years at 4% per annum is Re. 1. The sum (in Rs.) is :
(a) 625 (b) 630 (c) 640 (d) 650
(S.S.C. 2003)

23. The compound interest on a sum of money for 2 years is Rs. 832 and the simple interest on the same sum for the same period is Rs. 800. The difference between the compound interest and the simple interest for 3 years will be :
(a) Rs. 48 (b) Rs. 66.56 (c) Rs. 98.56 (d) None of these

24. The difference between the simple interest on a certain sum at the rate of 10% per annum for 2 years and compound interest which is compounded every 6 months is Rs. 124.05. What is the principal sum ? (S.B.I.P.O. 2000)
(a) Rs. 6000 (b) Rs. 8000 (c) Rs. 10,000
(d) Rs. 12,000 (e) None of these

25. The difference between compound interest and simple interest on a sum for 2 years at 10% per annum, when the interest is compounded annually is Rs. 16. If the interest were compounded half-yearly, the difference in two interests would be :
(a) Rs. 24.81 (b) Rs. 26.90 (c) Rs. 31.61 (d) Rs. 32.40

26. A sum of money lent at compound interest for 2 years at 20% per annum would fetch Rs. 482 more, if the interest was payable half-yearly than if it was payable annually. The sum is :
(a) Rs. 10,000 (b) Rs. 20,000 (c) Rs. 40,000 (d) Rs. 50,000

27. On a sum of money, the simple interest for 2 years is Rs. 660, while the compound interest is Rs. 696.30, the rate of interest being the same in both the cases. The rate of interest is : (Hotel Management, 1997)
(a) 10% (b) 10.5% (c) 12% (d) None of these

28. The effective annual rate of interest corresponding to a nominal rate of 6% per annum payable half-yearly is : (S.S.C. 2000)
(a) 6.06% (b) 6.07% (c) 6.08% (d) 6.09%

29. A person lent out a certain sum on simple interest and the same sum on compound interest at a certain rate of interest per annum. He noticed that the ratio between the difference of compound interest and simple interest of 3 years and that of 2 years is 25 : 8. The rate of interest per annum is :
(a) 10% (b) 11% (c) 12% (d) $12\frac{1}{2}\%$

30. Mr. Dua invested money in two schemes A and B offering compound interest @ 8 p.c.p.a. and 9 p.c.p.a. respectively. If the total amount of interest accrued through two schemes together in two years was Rs. 4818.30 and the total amount invested was Rs. 27,000, what was the amount invested in Scheme A ?
(a) Rs. 12,000 (b) Rs. 13,500 (c) Rs. 15,000
(d) Cannot be determined (e) None of these (Bank P.O. 2003)

31. A sum of money invested at compound interest amounts to Rs. 800 in 3 years and to Rs. 840 in 4 years. The rate of interest per annum is : (S.S.C. 2001)
(a) $2\frac{1}{2}\%$ (b) 4% (c) 5% (d) $6\frac{2}{3}\%$

32. A sum of money invested at compound interest amounts to Rs. 4624 in 2 years and to Rs. 4913 in 3 years. The sum of money is :
(*a*) Rs. 4096 (*b*) Rs. 4260 (*c*) Rs. 4335 (*d*) Rs. 4360

33. A sum of money becomes Rs. 13,380 after 3 years and Rs. 20,070 after 6 years on compound interest. The sum is :
(*a*) Rs. 8800 (*b*) Rs. 8890 (*c*) Rs. 8920 (*d*) Rs. 9040

34. A sum of Rs. 12,000 deposited at compound interest becomes double after 5 years. After 20 years, it will become :
(*a*) Rs. 96,000 (*b*) Rs. 1,20,000 (*c*) Rs. 1,24,000 (*d*) Rs. 1,92,000

35. A sum of money placed at compound interest doubles itself in 5 years. It will amount to eight times itself at the same rate of interest in : **(Hotel Management, 2003)**
(*a*) 7 years (*b*) 10 years (*c*) 15 years (*d*) 20 years

36. If a sum on compound interest becomes three times in 4 years, then with the same interest rate, the sum will become 27 times in :
(*a*) 8 years (*b*) 12 years (*c*) 24 years (*d*) 36 years

37. The least number of complete years in which a sum of money put out at 20% compound interest will be more than doubled is : **(N.I.F.T. 2003)**
(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6

38. A man borrows Rs. 2550 to be paid back with compound interest at the rate of 4% per annum by the end of 2 years in two equal yearly instalments. How much will each instalment be ?
(*a*) Rs. 1275 (*b*) Rs. 1283 (*c*) Rs. 1352 (*d*) Rs. 1377

39. What annual payment will discharge a debt of Rs. 1025 due in 2 years at the rate of 5% compound interest ? **(S.S.C. 2000)**
(*a*) Rs. 550 (*b*) Rs. 551.25 (*c*) Rs. 560 (*d*) Rs. 560.75

40. A man borrows Rs. 12,500 at 20% compound interest. At the end of every year he pays Rs. 2000 as part repayment. How much does he still owe after three such instalments ?
(*a*) Rs. 12,000 (*b*) Rs. 12,864 (*c*) Rs. 15,600 (*d*) None of these

41. A sum of money is borrowed and paid back in two annual instalments of Rs. 882 each allowing 5% compound interest. The sum borrowed was : **(A.I.M.A.T.S. 2002)**
(*a*) Rs. 1620 (*b*) Rs. 1640 (*c*) Rs. 1680 (*d*) Rs. 1700

ANSWERS

1. (*c*)	**2.** (*c*)	**3.** (*a*)	**4.** (*c*)	**5.** (*c*)	**6.** (*b*)	**7.** (*a*)	**8.** (*c*)	**9.** (*a*)
10. (*d*)	**11.** (*b*)	**12.** (*a*)	**13.** (*a*)	**14.** (*d*)	**15.** (*c*)	**16.** (*b*)	**17.** (*d*)	**18.** (*b*)
19. (*c*)	**20.** (*c*)	**21.** (*a*)	**22.** (*a*)	**23.** (*c*)	**24.** (*b*)	**25.** (*a*)	**26.** (*b*)	**27.** (*d*)
28. (*d*)	**29.** (*d*)	**30.** (*a*)	**31.** (*c*)	**32.** (*a*)	**33.** (*c*)	**34.** (*d*)	**35.** (*c*)	**36.** (*b*)
37. (*b*)	**38.** (*c*)	**39.** (*b*)	**40.** (*d*)	**41.** (*b*)				

SOLUTIONS

1. Amount = Rs. $\left[8000 \times \left(1 + \frac{5}{100}\right)^2\right]$ = Rs. $\left(8000 \times \frac{21}{20} \times \frac{21}{20}\right)$ = Rs. 8820.

2. Amount = Rs. $\left[25000 \times \left(1 + \frac{12}{100}\right)^3\right]$ = Rs. $\left(25000 \times \frac{28}{25} \times \frac{28}{25} \times \frac{28}{25}\right)$ = Rs. 35123.20.

$\therefore$ C.I. = Rs. (35123.20 – 25000) = Rs. 10123.20.

3. Time $= 2\frac{73}{365}$ years $= 2\frac{1}{5}$ years.

$\therefore$ Amount $= \text{Rs.}\left[20480\times\left(1+\frac{25}{4\times100}\right)^{2}\left(1+\frac{\frac{1}{5}\times\frac{25}{4}}{100}\right)\right]$

$= \text{Rs.}\left(20480\times\frac{17}{16}\times\frac{17}{16}\times\frac{81}{80}\right) = \text{Rs. } 23409.$

$\therefore$ C.I. $= \text{Rs. } (23409 - 20480) = \text{Rs. } 2929.$

4. Amount $= \text{Rs.}\left[200\left(1+\frac{5}{100}\right)^{3}+200\left(1+\frac{5}{100}\right)^{2}+200\left(1+\frac{5}{100}\right)\right]$

$= \text{Rs.}\left[200\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}+200\times\frac{21}{20}\times\frac{21}{20}+200\times\frac{21}{20}\right]$

$= \text{Rs.}\left[200\times\frac{21}{20}\left(\frac{21}{20}\times\frac{21}{20}+\frac{21}{20}+1\right)\right] = \text{Rs. } 662.02.$

5. P $= \text{Rs. } 15000$; R $= 10\%$ p.a. $= 5\%$ per half-year; T $= 1$ year $= 2$ half-years.

$\therefore$ Amount $= \text{Rs.}\left[15000\times\left(1+\frac{5}{100}\right)^{2}\right] = \text{Rs.}\left(15000\times\frac{21}{20}\times\frac{21}{20}\right) = \text{Rs. } 16537.50.$

6. Amount $= \text{Rs.}\left[1600\times\left(1+\frac{5}{2\times100}\right)^{2}+1600\times\left(1+\frac{5}{2\times100}\right)\right]$

$= \text{Rs.}\left[1600\times\frac{41}{40}\times\frac{41}{40}+1600\times\frac{41}{40}\right]$

$= \text{Rs.}\left[1600\times\frac{41}{40}\left(\frac{41}{40}+1\right)\right] = \text{Rs.}\left(\frac{1600\times41\times81}{40\times40}\right) = \text{Rs. } 3321.$

$\therefore$ C.I. $= \text{Rs. } (3321 - 3200) = \text{Rs. } 121.$

7. C.I. when interest is compounded yearly

$= \text{Rs.}\left[5000\times\left(1+\frac{4}{100}\right)\times\left(1+\frac{\frac{1}{2}\times4}{100}\right)\right] = \text{Rs.}\left(5000\times\frac{26}{25}\times\frac{51}{50}\right) = \text{Rs. } 5304.$

C.I. when interest is compounded half-yearly

$= \text{Rs.}\left[5000\times\left(1+\frac{2}{100}\right)^{3}\right] = \text{Rs.}\left(5000\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}\right) = \text{Rs. } 5306.04.$

$\therefore$ Difference $= \text{Rs. } (5306.04 - 5304) = \text{Rs. } 2.04.$

8. P $= \text{Rs. } 15625$, $n = 9$ months $= 3$ quarters, R $= 16\%$ p.a. $= 4\%$ per quarter.

Amount $= \text{Rs.}\left[15625\times\left(1+\frac{4}{100}\right)^{3}\right] = \text{Rs.}\left(15625\times\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}\right) = \text{Rs. } 17576.$

$\therefore$ C.I. $= \text{Rs. } (17576 - 15625) = \text{Rs. } 1951.$

9. Sum = Rs. $\left(\frac{50\times100}{2\times5}\right)$ = Rs. 500.

Amount = Rs. $\left[500\times\left(1+\frac{5}{100}\right)^2\right]$ = Rs. $\left(500\times\frac{21}{20}\times\frac{21}{20}\right)$ = Rs. 551.25.

$\therefore$ C.I. = Rs. (551.25 − 500) = Rs. 51.25.

10. S.I. = Rs. $\left(\frac{1000\times10\times4}{100}\right)$ = Rs. 400.

C.I. = Rs. $\left[1000\times\left(1+\frac{10}{100}\right)^4-1000\right]$ = Rs. 464.10.

$\therefore$ Difference = Rs. (464.10 − 400) = Rs. 64.10.

11. S.I. = Rs. $\left(\frac{1200\times10\times1}{100}\right)$ = Rs. 120.

C.I. = Rs. $\left[1200\times\left(1+\frac{5}{100}\right)^2-1200\right]$ = Rs. 123.

$\therefore$ Difference = Rs. (123 − 120) = Rs. 3.

12. Amount = Rs. (30000 + 4347) = Rs. 34347.

Let the time be n years. Then,

$$30000\left(1+\frac{7}{100}\right)^n = 34347 \Leftrightarrow \left(\frac{107}{100}\right)^n = \frac{34347}{30000} = \frac{11449}{10000} = \left(\frac{107}{100}\right)^2.$$

$\therefore$ n = 2 years.

13. Let the rate be R% p.a. Then,

$$1200\times\left(1+\frac{R}{100}\right)^2 = 1348.32 \Leftrightarrow \left(1+\frac{R}{100}\right)^2 = \frac{134832}{120000} = \frac{11236}{10000}$$

$$\therefore \left(1+\frac{R}{100}\right)^2 = \left(\frac{106}{100}\right)^2 \text{ or } 1+\frac{R}{100} = \frac{106}{100} \text{ or } R = 6\%.$$

14. Principal = Rs. $\left[\frac{4913}{\left(1+\frac{25}{4\times100}\right)^3}\right]$ = Rs. $\left(4913\times\frac{16}{17}\times\frac{16}{17}\times\frac{16}{17}\right)$ = Rs. 4096.

15. Present worth = Rs. $\left[\frac{169}{\left(1+\frac{4}{100}\right)^2}\right]$ = Rs. $\left(169\times\frac{25}{26}\times\frac{25}{26}\right)$ = Rs. 156.25.

16. Let the time be n years. Then,

$$800\times\left(1+\frac{5}{100}\right)^{2n} = 926.10 \text{ or } \left(1+\frac{5}{100}\right)^{2n} = \frac{9261}{8000}$$

$$\text{or } \left(\frac{21}{20}\right)^{2n} = \left(\frac{21}{20}\right)^3 \text{ or } 2n = 3 \text{ or } n = \frac{3}{2}.$$

$\therefore$ $n = 1\frac{1}{2}$ years.

17. Let the sum be Rs. P. Then,

$$\left[P\left(1+\frac{25}{2\times 100}\right)^2 - P\right] = 510 \text{ or } P\left[\left(\frac{9}{8}\right)^2 - 1\right] = 510 \text{ or } P = \left(\frac{510\times 64}{17}\right) = 1920.$$

$\therefore$ Sum = Rs. 1920.

So, S.I. = Rs. $\left(\frac{1920\times 25\times 2}{2\times 100}\right)$ = Rs. 480.

18. Let the sum be Rs. P. Then,

$$\left[P\left(1+\frac{10}{100}\right)^2 - P\right] = 525 \Leftrightarrow P\left[\left(\frac{11}{10}\right)^2 - 1\right] = 525 \Leftrightarrow P = \left(\frac{525\times 100}{21}\right) = 2500.$$

$\therefore$ Sum = Rs. 2500.

So, S.I. = Rs. $\left(\frac{2500\times 5\times 4}{100}\right)$ = Rs. 500.

19. C.I. = Rs. $\left[4000\times\left(1+\frac{10}{100}\right)^2 - 4000\right]$ = Rs. $\left(4000\times\frac{11}{10}\times\frac{11}{10} - 4000\right)$ = Rs. 840.

$\therefore$ Sum = Rs. $\left(\frac{420\times 100}{3\times 8}\right)$ = Rs. 1750.

20. Let P = Rs. 100. Then, S.I. Rs. 60 and T = 6 years.

$\therefore$ $R = \frac{100\times 60}{100\times 6} = 10\%$ p.a.

Now, P = Rs. 12000, T = 3 years and R = 10% p.a.

$\therefore$ C.I. = Rs. $\left[12000\times\left\{\left(1+\frac{10}{100}\right)^3 - 1\right\}\right]$ = Rs. $\left(12000\times\frac{331}{1000}\right)$ = Rs. 3972.

21. $\left[15000\times\left(1+\frac{R}{100}\right)^2 - 15000\right] - \left(\frac{15000\times R\times 2}{100}\right) = 96$

$$\Leftrightarrow 15000\left[\left(1+\frac{R}{100}\right)^2 - 1 - \frac{2R}{100}\right] = 96 \Leftrightarrow 15000\left[\frac{(100+R)^2 - 10000 - 200R}{10000}\right] = 96$$

$$\Leftrightarrow R^2 = \frac{96\times 2}{3} = 64 \Leftrightarrow R = 8.$$

$\therefore$ Rate = 8%.

22. Let the sum be Rs. x. Then,

$$\text{C.I.} = \left[x\left(1+\frac{4}{100}\right)^2 - x\right] = \left(\frac{676}{625}x - x\right) = \frac{51}{625}x.$$

$$\text{S.I.} = \left(\frac{x\times 4\times 2}{100}\right) = \frac{2x}{25}.$$

$\therefore$ $\frac{51x}{625} - \frac{2x}{25} = 1$ or $x = 625$.

23. Difference in C.I. and S.I. for 2 years = Rs. 32.

S.I. for one year = Rs. 400.

$\therefore$ S.I. on Rs. 400 for one year = Rs. 32.

So, Rate = $\left(\frac{100\times 32}{400\times 1}\right)\% = 8\%$.

Hence, difference in C.I. and S.I. for 3rd year

= S.I. on Rs. 832 = Rs. $\left(\frac{832\times 8\times 1}{100}\right)$ = Rs. 66.56.

Total difference = Rs. (32 + 66.56) = Rs. 98.56.

24. Let the sum be Rs. P. Then

$$P\left[\left(1+\frac{5}{100}\right)^4-1\right]-\frac{P\times 10\times 2}{100}=124.05$$

$$\Rightarrow P\left[\left(\frac{21}{20}\right)^4-1-\frac{1}{5}\right]=124.05 \Rightarrow P\left[\frac{194481}{160000}-\frac{6}{5}\right]=\frac{12405}{100}$$

$$\Rightarrow P\left[\frac{194481-192000}{160000}\right]=\frac{12405}{100} \Rightarrow P=\left(\frac{12405}{100}\times\frac{160000}{2481}\right)=8000.$$

25. For first year, S.I. = C.I.

Now, Rs. 16 is the S.I. on S.I. for 1 year.

Rs. 10 is S.I. on Rs. 100.

$\therefore$ Rs. 16 is S.I. on Rs. $\left(\frac{100}{10}\times 16\right)$ = Rs. 160.

So, S.I. on principal for 1 year at 10% is Rs. 160.

$\therefore$ Principal = Rs. $\left(\frac{100\times 160}{10\times 1}\right)$ = Rs. 1600.

Amount for 2 years compounded half yearly = Rs. $\left[1600\times\left(1+\frac{5}{100}\right)^4\right]$ = Rs. 1944.81.

$\therefore$ C.I. = Rs. (1944.81 − 1600) = Rs. 344.81.

S.I. = Rs. $\left(\frac{1600\times 10\times 2}{100}\right)$ = Rs. 320.

$\therefore$ (C.I.) − (S.I.) = Rs. (344.81 − 320) = Rs. 24.81.

26. Let the sum be Rs. x. Then,

C.I. when compounded half-yearly = $\left[x\times\left(1+\frac{10}{100}\right)^4-x\right]=\frac{4641}{10000}x$.

C.I. when compounded annually = $\left[x\times\left(1+\frac{20}{100}\right)^2-x\right]=\frac{11}{25}x$.

$\therefore$ $\frac{4641}{10000}x-\frac{11}{25}x=482$ or $x=\frac{482\times 10000}{241}=20000$.

27. Difference in C.I. and S.I. for 2 years = Rs. (696.30 − 660) = Rs. 36.30.

S.I. for one year = Rs. 330.

$\therefore$ S.I. on Rs. 330 for 1 year = Rs. 36.30.

$\therefore$ Rate = $\left(\frac{100\times 36.30}{330\times 1}\right)\% = 11\%$.

28. Amount of Rs. 100 for 1 year when compounded half-yearly

$$= \text{Rs.}\left[100\times\left(1+\frac{3}{100}\right)^2\right] = \text{Rs. } 106.09.$$

$\therefore$ Effective rate = (106.09 − 100)% = 6.09%.

29. Let the principal be Rs. P and rate of interest be R% per annum.

Difference of C.I. and S.I. for 2 years

$$= \left[P\times\left(1+\frac{R}{100}\right)^2 - P\right] - \left(\frac{P\times R\times 2}{100}\right) = \frac{PR^2}{104}.$$

Difference of C.I. and S.I. for 3 years

$$= \left[P\times\left(1+\frac{R}{100}\right)^3 - P\right] - \left(\frac{P\times R\times 3}{100}\right) = \frac{PR^2}{10^4}\left(\frac{300+R}{100}\right).$$

$$\therefore \frac{\frac{PR^2}{10^4}\left(\frac{300+R}{100}\right)}{\frac{PR^2}{10^4}} = \frac{25}{8} \Rightarrow \left(\frac{300+R}{100}\right) = \frac{25}{8} \Rightarrow R = \frac{100}{8} = 12\frac{1}{2}\%.$$

30. Let the investment in scheme A be Rs. x.

Then, investment in scheme B = Rs. $(27000 - x)$.

$$\therefore \left[x\times\left\{\left(1+\frac{8}{100}\right)^2 - 1\right\} + (27000-x)\left\{\left(1+\frac{9}{100}\right)^2 - 1\right\}\right] = 4818.30.$$

$$\Leftrightarrow \left(x\times\frac{104}{625}\right) + \frac{1881(27000-x)}{10000} = \frac{481830}{100}$$

$$\Leftrightarrow 1664x + 1881(27000 - x) = 48183000$$

$$\Leftrightarrow (1881x - 1664x) = (50787000 - 48183000)$$

$$\Leftrightarrow 217x = 2604000 \Leftrightarrow x = \frac{2604000}{217} = 12000.$$

31. S.I. on Rs. 800 for 1 year = Rs. (840 − 800) = Rs. 40.

$$\therefore \text{Rate} = \left(\frac{100\times 40}{800\times 1}\right)\% = 5\%.$$

32. S.I. on Rs. 4624 for 1 year = Rs. (4913 − 4624) = Rs. 289.

$$\therefore \text{Rate} = \left(\frac{100\times 289}{4624\times 1}\right)\% = 6\frac{1}{4}\%.$$

Now, $x\left(1+\frac{25}{4\times 100}\right)^2 = 4624$ or $x\times\frac{17}{16}\times\frac{17}{16} = 4624$

$$\therefore x = \left(4624\times\frac{16}{17}\times\frac{16}{17}\right) = \text{Rs. } 4096.$$

34. $12000\times\left(1+\frac{R}{100}\right)^5 = 24000 \Rightarrow \left(1+\frac{R}{100}\right)^5 = 2$

$$\therefore \left[\left(1+\frac{R}{100}\right)^5\right]^4 = 2^4 = 16 \Rightarrow \left(1+\frac{R}{100}\right)^{20} = 16 \Rightarrow P\left(1+\frac{R}{100}\right)^{20} = 16P$$

$$\Rightarrow \quad 12000\left(1+\frac{R}{100}\right)^{20} = 16\times 12000 = 192000.$$

35. $P\left(1+\frac{R}{100}\right)^{5} = 2P \Rightarrow \left(1+\frac{R}{100}\right)^{5} = 2$ ...(i)

Let $P\left(1+\frac{R}{100}\right)^{n} = 8P \Rightarrow \left(1+\frac{R}{100}\right)^{n} = 8 = 2^3 = \left\{\left(1+\frac{R}{100}\right)^{5}\right\}^3$ [*using (i)*]

$$\Rightarrow \left(1+\frac{R}{100}\right)^{n} = \left(1+\frac{R}{100}\right)^{15} \Rightarrow n = 15.$$

$\therefore$ Required time = 15 years.

36. $P\left(1+\frac{R}{100}\right)^{4} = 3P \Rightarrow \left(1+\frac{R}{100}\right)^{4} = 3$ ...(i)

Let $P\left(1+\frac{R}{100}\right)^{n} = 27P \Rightarrow \left(1+\frac{R}{100}\right)^{n} = 27 = (3)^3 = \left\{\left(1+\frac{R}{100}\right)^{4}\right\}^3$ [*using (i)*]

$$\Rightarrow \left(1+\frac{R}{100}\right)^{n} = \left(1+\frac{R}{100}\right)^{12} \Rightarrow n = 12.$$

$\therefore$ Required time = 12 years.

37. $P\left(1+\frac{20}{100}\right)^{n} > 2P$ *or* $\left(\frac{6}{5}\right)^{n} > 2$

Now, $\left(\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5}\right) > 2$. So, $n = 4$ years.

38. Let the value of each instalment be Rs. x. Then,

(P.W. of Rs. x due 1 year hence) + (P.W. of Rs. x due 2 years hence) = Rs. 2550

$$\Leftrightarrow \frac{x}{\left(1+\frac{4}{100}\right)}+\frac{x}{\left(1+\frac{4}{100}\right)^{2}} = 2550 \Leftrightarrow \frac{25x}{26}+\frac{625x}{676} = 2550$$

$$\Leftrightarrow 1275x = 2550\times 676 \Leftrightarrow x = \left(\frac{2550\times 676}{1275}\right) = 1352.$$

$\therefore$ Value of each instalment = Rs. 1352.

39. Let each instalment be Rs. x. Then,

$$\frac{x}{\left(1+\frac{5}{100}\right)}+\frac{x}{\left(1+\frac{5}{100}\right)^{2}} = 1025 \Leftrightarrow \frac{20x}{21}+\frac{400x}{441} = 1025$$

$$\Leftrightarrow 820x = 1025\times 441 \Leftrightarrow x = \left(\frac{1025\times 441}{820}\right) = 551.25.$$

So, value of each instalment = Rs. 551.25.

40. Balance

$$= \text{Rs.}\left[\left\{12500\times\left(1+\frac{20}{100}\right)^{3}\right\}-\left\{2000\times\left(1+\frac{20}{100}\right)^{2}+2000\times\left(1+\frac{20}{100}\right)+2000\right\}\right]$$

$= \text{Rs.} \left[\left(12500 \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5} \right) - \left(2000 \times \frac{6}{5} \times \frac{6}{5} + 2000 \times \frac{6}{5} + 2000 \right) \right]$

$= \text{Rs.} \left[21600 - (2880 + 2400 + 2000) \right] = \text{Rs. } 14320.$

41. Principal

= (P.W. of Rs. 882 due 1 year hence) + (P.W. of Rs. 882 due 2 years hence)

$= \left[\frac{882}{\left(1 + \frac{5}{100}\right)} + \frac{882}{\left(1 + \frac{5}{100}\right)^2} \right] = \left(\frac{882 \times 20}{21} + \frac{882 \times 400}{441} \right) = \text{Rs. } 1640.$

EXERCISE 22B

(DATA SUFFICIENCY TYPE QUESTIONS)

1. The difference between the compound interest and the simple interest earned on a sum of money at the end of 4 years is Rs. 256.40. To find out the sum, which of the following informations given in the statements P and Q is/are necessary ?

   P : Amount of simple interest accrued after 4 years.

   Q : Rate of interest per annum.

   (a) Only P is necessary
   (b) Only Q is necessary
   (c) Either P or Q is necessary
   (d) Neither P nor Q is necessary
   (e) Both P and Q are necessary

Directions (*Questions 2 to 5*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the given question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

2. What is the rate of compound interest ? **(Bank P.O. 2003)**

   I. The principal was invested for 4 years.

   II. The earned interest was Rs. 1491.

3. What will be the compounded amount ? **(Bank P.O. 1999)**

   I. Rs. 200 were borrowed for 192 months at 6% compounded annually.

   II. Rs. 200 were borrowed for 16 years at 6%.

4. What is the compound interest earned by Robert at the end of 2 years ?

   I. Simple interest at the same rate for one year is Rs. 1020 and the rate of interest is 12 p.c.p.a.

   II. The amount invested is Rs. 8500.

5. What is the total compound interest accrued on a sum of money after 5 years ?

I. The sum was Rs. 20,000.

II. The total amount of simple interest on the sum after 5 years was Rs. 4000.

6. What was the total compound interest on a sum after 3 years ? **(Bank P.O. 2003)**

I. The interest after one year was Rs. 100 and the sum was Rs. 1000.

II. The difference between simple and compound interest on a sum of Rs. 1000 at the end of 2 years was Rs. 10.

7. An amount of money was lent for 3 years. What will be the difference between the simple and the compound interest earned on it at the same rate ?

I. The rate of interest was 8 p.c.p.a.

II. The total amount of simple interest was Rs. 1200.

8. What was the rate of interest on a sum of money ? **(S.B.I.P.O. 1998)**

I. The sum fetched a total of Rs. 2522 as compound interest at the end of 3 years.

II. The difference between the simple interest and the compound interest at the end of 2 years at the same rate was Rs. 40.

Directions (*Questions 9 to 12*) : *Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is / are necessary to answer the question* :

9. What is the rate of interest p.c.p.a. ? **(R.B.I. 2002)**

I. An amount doubles itself in 5 years on simple interest.

II. Difference between the compound interest and the simple interest earned on a certain amount in 2 years is Rs. 400.

III. Simple interest earned per annum is Rs. 2000.

(*a*) I only (*b*) II and III only (*c*) All I, II and III

(*d*) Any two of the three (*e*) I only or II and III only

10. A sum of money is put at compound interest. What is the rate of interest ?

I. The sum amounts to Rs. 5290 in 2 years.

II. The sum amounts to Rs. 6083.50 in 3 years.

III. The sum is Rs. 4000.

(*a*) I and II only (*b*) II and III only (*c*) I and III only

(*d*) Any two of the three (*e*) I and III only, or II and III only

11. What will be the compound interest earned on an amount of Rs. 5000 in 2 years ? **(S.B.I.P.O. 2000)**

I. The simple interest on the same amount at the same rate of interest in 5 years in Rs. 2000.

II. The compound interest and the simple interest earned in one year is the same.

III. The amount becomes more than double on compound interest in 10 years.

(*a*) I only (*b*) I and II only (*c*) II and III only

(*d*) I and III only (*e*) None of these

12. A sum of money is placed at compound interest. In how many years will it amount to sixteen times of itself ?

I. The sum doubles itself in 4 years.

II. The sum amounts to eight times of itself in 12 years.

III. The sum amounts to four times of itself in 8 years.

(*a*) I only (*b*) I and II only (*c*) II and III only

(*d*) I and III only (*e*) Any one of the three

Directions (*Questions 13 to 16*) : ***In each of the following questions, a question is asked and is followed by three statements. While answering the question, you may or may not require the data provided in all the statements. You have to read, the question and the three statements and then decide whether the question can be answered with any one or two of the statements or all the three statements are required to answer the question. The answer number bearing the statements, which can be dispensed with, if any, while answering the question is your answer.***

13. What would be the difference between the simple interest and the compound interest on a sum of money at the end of four years ?

I. The rate of interest is 5 p.c.p.a.

II. The sum fetches a total of Rs. 2000 as simple interest at the end of 8 years.

III. The difference between the simple interest and the compound interest at the end of 2 years is Rs. 12.50.

(*a*) II only (*b*) III only (*c*) II or III only

(*d*) All I, II and III are required (*e*) None of these

14. Mr. Gupta borrowed a sum of money on compound interest. What will be the amount to be repaid if he is repaying the entire amount at the end of 2 years ? **(Bank P.O. 1999)**

I. The rate of interest is 5 p.c.p.a.

II. Simple interest fetched on the same amount in one year is Rs. 600.

III. The amount borrowed is 10 times the simple interest in 2 years.

(*a*) I only (*b*) III only (*c*) I or II only

(*d*) I or III only (*e*) All I, II and III are required

15. What is the total compound interest earned at the end of 3 years ? **(S.B.I.P.O. 2001)**

I. Simple interest earned on that amount at the same rate and for the same period is Rs. 4500.

II. The rate of interest is 10 p.c.p.a.

III. Compound interest for 3 years is more than the simple interest for that period by Rs. 465.

(*a*) I and II only (*b*) II and III only (*c*) I and III only

(*d*) Either II or III only (*e*) Any two of the three

16. What is the rate of interest per annum ? **(S.B.I.P.O. 1999)**

I. The amount becomes Rs. 11,025 with compound interest after 2 years.

II. The same amount with simple interest becomes Rs. 11,000 after 2 years.

III. The amount invested is Rs. 10,000.

(*a*) I or II only (*b*) II or III only (*c*) I or III only

(*d*) I or II or III only (*e*) All I, II and III are required

ANSWERS

1. (*b*) **2.** (*d*) **3.** (*c*) **4.** (*a*) **5.** (*e*) **6.** (*c*) **7.** (*e*) **8.** (*e*) **9.** (*e*)
10. (*d*) **11.** (*a*) **12.** (*e*) **13.** (*c*) **14.** (*d*) **15.** (*d*) **16.** (*d*)

SOLUTIONS

1. To find the sum, difference between C.I. and S.I., the time and the rate of interest are needed.

∴ Only Q is necessary.

∴ Correct answer is (*b*).

2. Let Principal = Rs. P and Rate = R% p.a. Then,

$$\text{Amount} = \text{Rs.}\left[P\left(1+\frac{R}{100}\right)^4\right].$$

$$\therefore \quad \text{C.I.} = P\left[\left(1+\frac{R}{100}\right)^4-1\right] \Rightarrow P\left[\left(1+\frac{R}{100}\right)^4-1\right] = 1491.$$

Clearly, it does not give the answer.

$\therefore$ Correct answer is (*d*).

3. I. $\text{Amount} = \text{Rs.}\left[200\times\left(1+\frac{6}{100}\right)^{16}\right]$.

II. $\text{Amount} = \text{Rs.}\left[200\times\left(1+\frac{6}{100}\right)^{16}\right]$.

Thus, I as well as II gives the answer.

$\therefore$ Correct answer is (*c*).

4. I. S.I. = Rs. 1020, R = 12% p.a. and T = 1 year.

$$\therefore \quad P = \frac{100\times \text{S.I.}}{R\times T} \Rightarrow P = \text{Rs.}\left(\frac{100\times 1020}{12\times 1}\right) = \text{Rs. } 8500.$$

$$\therefore \quad \text{C.I. for 2 years} = \text{Rs.}\left[8500\times\left\{\left(1+\frac{12}{100}\right)^2-1\right\}\right].$$

II gives : only P and T.

$\therefore$ II alone does not give the answer.

$\therefore$ Correct answer is (*a*).

5. Given : Time = 5 years.

I gives : Sum = Rs. 20000.

II gives : S.I. = Rs. 4000.

Let the rate be R% p.a. Then,

$$R = \frac{100\times \text{S.I.}}{P\times T} = \left(\frac{100\times 4000}{5\times 20000}\right) = 4\% \text{ p.a.}$$

$$\therefore \quad \text{C.I.} = \text{Rs.}\left[20000\times\left\{\left(1+\frac{4}{100}\right)^5-1\right\}\right]$$

$\therefore$ Both I and II are needed to get the answer.

So, the correct answer is (*e*).

6. I gives : P = Rs. 1000 and S.I. for 1 year = Rs. 100.

$$\therefore \quad \text{Rate} = \frac{100\times \text{S.I.}}{P\times T} = \left(\frac{100\times 100}{1000\times 1}\right) = 10\% \text{ p.a.}$$

Thus, P = Rs. 1000, T = 3 years and R = 10% p.a.

$\therefore$ C.I. may be obtained.

II. Sum = Rs. 1000, [(C.I.) – (S.I.)] for 2 years = Rs. 10.

Let the rate be R% p.a.

$$1000\times\left[\left(1+\frac{R}{100}\right)^2-1\right]-\left(\frac{1000\times R\times 2}{100}\right) = 10.$$

From this, we can find R.

Thus P, T and R are given and therefore, C.I. may be calculated.

Thus, I alone as well as II alone is sufficient to get the answer.

∴ Correct answer is (*e*).

7. Given : T = 3 years.

I gives : R = 8% p.a.

II gives : S.I. = Rs. 1200.

Thus, P = Rs. 5000, R = 8% p.a. and T = 3 years.

∴ Difference between C.I. and S.I. may be obtained.

So, the correct answer is (*e*).

8. I gives : C.I. for 3 years = Rs. 2522.

II gives : (C.I.) – (S.I.) for 2 years at same rate is Rs. 40.

$$P\left[\left(1+\frac{R}{100}\right)^3-1\right]=2522 \qquad ...(i)$$

$$P\left[\left(1+\frac{R}{100}\right)^2-1\right]-\frac{P\times R\times 2}{100}=40 \qquad ...(ii)$$

On dividing (*i*) by (*ii*) we get :

$$\frac{\left(1+\frac{R}{100}\right)^3-1}{\left(1+\frac{R}{100}\right)^2-1-\frac{R}{50}}=\frac{2522}{40} \Rightarrow \frac{\frac{R^3}{1000000}+\frac{3R}{100}+\frac{3R^2}{10000}}{\frac{R^2}{10000}}=\frac{1261}{20}$$

$$\Rightarrow \frac{R}{100}+\frac{300}{R}=\frac{1201}{20} \Rightarrow R^2-6005R+30000=0$$

$$\Rightarrow R^2-6000R-5R+30000=0$$

$$\Rightarrow R(R-6000)-5(R-6000)=0$$

$$\Rightarrow (R-5)(R-6000)=0 \Rightarrow R=5.$$

∴ Both I and II are needed to get R.

∴ Correct answer is (*e*).

9. I. $\frac{P\times R\times 5}{100}=P \Rightarrow R=20.$

II. $P\left(1+\frac{R}{100}\right)^2-P-\frac{P\times R\times 2}{100}=400 \Rightarrow PR^2=4000000.$

III. $\frac{P\times R\times 1}{100}=2000 \Rightarrow PR=200000$

$$\therefore \frac{PR^2}{PR}=\frac{4000000}{200000} \Rightarrow R=20.$$

Thus I only or (II & III) give answer.

∴ Correct answer is (*e*).

10. I. $P\left(1+\frac{R}{100}\right)^2=5290$ ...(*i*) II. $P\left(1+\frac{R}{100}\right)^3=6083.50$ ...(*ii*)

On dividing (*ii*) by (*i*), we get :

$$\left(1+\frac{R}{100}\right)=\frac{608350}{529000}=\frac{23}{20} \Rightarrow \frac{R}{100}=\left(\frac{23}{20}-1\right)=\frac{3}{20} \Rightarrow R=15$$

Thus, I and II give answer.

III. gives P = 4000.

Putting this value of P in (*i*), we get the answer.

Putting this value of P in (*ii*), we get the answer.

∴ (I & II) or (I & III) or (II & III) all give the answer.

Hence, the correct answer is (*d*).

11. P = Rs. 5000 & T = 2 years.

I. S.I. on Rs. 5000 in 5 years is Rs. 2000.

$$\frac{5000 \times R \times 5}{100} = 2000 \Rightarrow R = 8$$

Thus I only gives the answer.

∴ Correct answer is (*a*).

12. I. $P\left(1+\frac{R}{100}\right)^4 = 2P \Rightarrow \left(1+\frac{R}{100}\right)^4 = 2$ ...(*i*)

II. $P\left(1+\frac{R}{100}\right)^{12} = 8P \Rightarrow \left(1+\frac{R}{100}\right)^{12} = 8$ ...(*ii*)

III. $P\left(1+\frac{R}{100}\right)^8 = 4P \Rightarrow \left(1+\frac{R}{100}\right)^8 = 4$ ...(*iii*)

Let the given sum become 16 times in *n* years. Then,

$$P\left(1+\frac{R}{100}\right)^n = 16P \Rightarrow \left(1+\frac{R}{100}\right)^n = 16 \quad \text{...(iv)}$$

∴ Any one of (*i*), (*ii*) and (*iii*) with (*iv*) will give the value of *n*.

∴ Correct answer is (*c*).

13. I and II will give us, R, S.I. and T.

$$\therefore P = \frac{100 \times S.I.}{R \times T} = \left(\frac{100 \times 2000}{5 \times 8}\right) = 5000.$$

[(C.I.) – (S.I.)] for 4 years may be calculated.

In this case, III is redundant.

I and III give us R and P, using.

$$P\left[\left(1+\frac{5}{100}\right)^2 - 1\right] - \frac{P \times 5 \times 2}{100} = 12.50$$

So, [(C.I.) – (S.I.)] for 4 years may be calculated.

∴ Correct answer is (*c*).

14. I gives, Rate = 5% p.a.

II gives, S.I. for 1 year = Rs. 600.

III gives, sum = 10 × (S.I. for 2 years).

Now, I and II give the sum.

For this sum, C.I. and hence amount can be obtained.

Thus, III is redundant.

Again, II gives S.I. for 2 years = Rs. (600 × 2) = Rs. 1200.

Now, from III, Sum = Rs. (10 × 1200) = Rs. 12000.

Thus, Rate = $\frac{100 \times 1200}{2 \times 12000}$ = 5% p.a.

Thus, C.I. for 2 years and therefore, amount can be obtained.

Thus, I is redundant.

Hence, I or III redundant.

15. I gives, S.I. for 3 years = Rs. 4500

II gives, Rate = 10% p.a.

III gives, (C.I.) – (S.I.) = Rs. 465.

Clearly, using I and III we get C.I. = Rs. (465 + 4500).

Thus, II is redundant.

Also, from I and II, we get sum = $\left(\frac{100 \times 4500}{10 \times 3}\right)$ = 15000.

Now C.I. on Rs. 15000 at 10% p.a. for 3 years may be obtained.

Thus, III is redundant.

∴ Either II or III is redundant.

16. I gives, Amount after 2 years = Rs. 11025, when compounded.

II gives, Amount after 2 years at S.I. = Rs. 11000.

III gives, Principal = Rs. 10000.

From II and III, we have :

Principal = Rs. 10000, S.I. = Rs. (11000 – 10000) = Rs. 1000 and Time = 2 years.

Hence, Rate can be obtained.

∴ I is redundant.

From I and III, we get $11025 = 10000 \times \left(1 + \frac{R}{100}\right)^2$. This gives R.

∴ II is redundant.

From I and II, we have

$P\left(1 + \frac{R}{100}\right)^2 = 11025$ ...(*i*) and $P\left[1 + \frac{R \times 2}{100}\right] = 11000$ ...(*ii*)

On dividing (*i*) by (*ii*), we get $\frac{\left(1 + \frac{R}{100}\right)^2}{(50 + R)} = \frac{11025}{550000}$.

This gives R.

Thus, III is redundant.

Hence I or II or III is redundant.

23. LOGARITHMS

IMPORTANT FACTS AND FORMULAE

I. **Logarithm** : If a is a positive real number, other than 1 and $a^m = x$, then we write : $m = \log_a x$ and we say that the value of $\log x$ to the base a is m.

Example :

(*i*) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$ (*ii*) $3^4 = 81 \Rightarrow \log_3 81 = 4$

(*ii*) $2^{-3} = \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3$ (*iv*) $(.1)^2 = .01 \Rightarrow \log_{(.1)} .01 = 2.$

II. **Properties of Logarithms** :

1. $\log_a (xy) = \log_a x + \log_a y$
2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_x x = 1$
4. $\log_a 1 = 0$
5. $\log_a (x^p) = p (\log_a x)$
6. $\log_a x = \frac{1}{\log_x a}$
7. $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a}.$

Remember : When base is not mentioned, it is taken as 10.

III. **Common Logarithms** : Logarithms to the base 10 are known as common logarithms.

IV. The logarithm of a number contains two parts, namely *characteristic* and *mantissa*.

Characteristic : The integral part of the logarithm of a number is called its ***characteristic***.

Case I : *When the number is greater than 1.*

In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

Case II : *When the number is less than 1.*

In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

Instead of $-1, -2$, etc. we write, $\bar{1}$ (one bar), $\bar{2}$ (two bar), etc.

Example :

Number	*Characteristic*	*Number*	*Characteristic*
348.25	2	0.6173	$\bar{1}$
46.583	1	0.03125	$\bar{2}$
9.2193	0	0.00125	$\bar{3}$

Mantissa : The decimal part of the logarithm of a number is known is its ***mantissa***.

For mantissa, we look through log table.

SOLVED EXAMPLES

Ex. 1. ***Evaluate : (i) $\log_3 27$ (ii) $\log_7\left(\frac{1}{343}\right)$ (iii) $\log_{100}(0.01)$***

Sol. (*i*) Let $\log_3 27 = n$.

Then, $3^n = 27 = 3^3$ or $n = 3$. $\therefore \log_3 27 = 3$.

(*ii*) Let $\log_7\left(\frac{1}{343}\right) = n$.

Then, $7^n = \frac{1}{343} = \frac{1}{7^3} = 7^{-3}$ or $n = -3$. $\therefore \log_7\left(\frac{1}{343}\right) = -3$.

(*iii*) Let $\log_{100}(0.01) = n$.

Then, $(100)^n = 0.01 = \frac{1}{100} = (100)^{-1}$ or $n = -1$ $\therefore \log_{100}(0.01) = -1$.

Ex. 2. ***Evaluate : (i) $\log_7 1 = 0$ (ii) $\log_{34} 34$ (iii) $36^{\log_6 4}$***

Sol. (*i*) We know that $\log_a 1 = 0$, so $\log_7 1 = 0$.

(*ii*) We know that $\log_a a = 1$, so $\log_{34} 34 = 0$.

(*iii*) We know that $a^{\log_a x} = x$.

Now, $36^{\log_6 4} = (6^2)^{\log_6 4} = 6^{2(\log_6 4)} = 6^{\log_6(4^2)} = 6^{\log_6 16} = 16$.

Ex. 3. ***If $\log_{\sqrt{8}} x = 3\frac{1}{3}$, find the value of x.***

Sol. $\log_{\sqrt{8}} x = \frac{10}{3} \Leftrightarrow x = (\sqrt{8})^{10/3} = (2^{3/2})^{10/3} = 2^{\left(\frac{3}{2}\times\frac{10}{3}\right)} = 2^5 = 32$.

Ex. 4. ***Evaluate : (i) $\log_5 3 \times \log_{27} 25$ (ii) $\log_9 27 - \log_{27} 9$***

Sol. (*i*) $\log_5 3 \times \log_{27} 25 = \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 27} = \frac{\log 3}{\log 5} \times \frac{\log(5^2)}{\log(3^3)} = \frac{\log 3}{\log 5} \times \frac{2\log 5}{3\log 3} = \frac{2}{3}$.

(*ii*) Let $\log_9 27 = n$.

Then, $9^n = 27 \Leftrightarrow 3^{2n} = 3^3 \Leftrightarrow 2n = 3 \Leftrightarrow n = \frac{3}{2}$.

Again, let $\log_{27} 9 = m$.

Then, $27^m = 9 \Leftrightarrow 3^{3m} = 3^2 \Leftrightarrow 3m = 2 \Leftrightarrow m = \frac{2}{3}$.

$\therefore \log_9 27 - \log_{27} 9 = (n - m) = \left(\frac{3}{2} - \frac{2}{3}\right) = \frac{5}{6}$.

Ex. 5. ***Simplify : $\left(\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243}\right)$*** **(S.S.C. 2000)**

Sol. $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log\frac{75}{16} - \log\left(\frac{5}{9}\right)^2 + \log\frac{32}{243} = \log\frac{75}{16} - \log\frac{25}{81} + \log\frac{32}{243}$

$= \log\left(\frac{75}{16}\times\frac{32}{243}\times\frac{81}{25}\right) = \log 2$.

Ex. 6. ***Find the value of x which satisfies the relation***

$$\log_{10} 3 + \log_{10}(4x + 1) = \log_{10}(x + 1) + 1$$ **(M.B.A. 2002)**

Sol. $\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$

$\Leftrightarrow \log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + \log_{10} 10$

$\Leftrightarrow \log_{10} [3 (4x + 1)] = \log_{10} [10 (x + 1)]$

$\Leftrightarrow 3 (4x + 1) = 10 (x + 1) \Leftrightarrow 12x + 3 = 10x + 10 \Leftrightarrow 2x = 7 \Leftrightarrow x = \frac{7}{2}$.

Ex. 7. ***Simplify :*** $\left[\frac{1}{\log_{xy} (xyz)} + \frac{1}{\log_{yz} (xyz)} + \frac{1}{\log_{zx} (xyz)}\right]$

Sol. Given expression $= \log_{xyz} (xy) + \log_{xyz} (yz) + \log_{xyz} (zx)$

$= \log_{xyz} (xy \times yz \times zx) = \log_{xyz} (xyz)^2$

$= 2 \log_{xyz} (xyz) = 2 \times 1 = 2.$ $\left[\because \log_a x = \frac{1}{\log_x a}\right]$

Ex. 8. ***If $\log_{10} 2 = 0.30103$, find the value of $\log_{10} 50$.*** (C.B.I. 1997)

Sol. $\log_{10} 50 = \log_{10} \left(\frac{100}{2}\right) = \log_{10} 100 - \log_{10} 2 = 2 - 0.30103 = 1.69897.$

Ex. 9. ***If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find the values of :***

(i) $\log 25$ ***(ii) $\log 4.5$***

Sol. *(i)* $\log 25 = \log \left(\frac{100}{4}\right) = \log 100 - \log 4 = 2 - 2 \log 2 = (2 - 2 \times 0.3010) = 1.398.$

(ii) $\log 4.5 = \log \left(\frac{9}{2}\right) = \log 9 - \log 2 = 2 \log 3 - \log 2$

$= (2 \times 0.4771 - 0.3010) = 0.6532.$

Ex. 10. ***If $\log 2 = 0.30103$, find the number of digits in 2^{56}.***

Sol. $\log (2^{56}) = 56 \log 2 = (56 \times 0.30103) = 16.85768.$

Its characteristic is 16. Hence, the number of digits in 2^{56} is 17.

EXERCISE 23

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The value of $\log_2 16$ is : (M.B.A. 2003

(a) $\frac{1}{8}$ (b) 4 (c) 8 (d) 16.

2. The value of $\log_{343} 7$ is :

(a) $\frac{1}{3}$ (b) -3 (c) $-\frac{1}{3}$ (d) 3

3. The value of $\log_5 \left(\frac{1}{125}\right)$ is :

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

4. The value of $\log_{\sqrt{2}} 32$ is :

(a) $\frac{5}{2}$ (b) 5 (c) 10 (d) $\frac{1}{10}$

5. The value of $\log_{10} (.0001)$ is :

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) -4 (d) 4

6. The value of $\log_{(.01)} (1000)$ is :

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

7. The logarithm of 0.0625 to the base 2 is :

(a) -4 (b) -2 (c) 0.25 (d) 0.5

8. If $\log_3 x = -2$, then x is equal to :

(a) -9 (b) -6 (c) -8 (d) $\frac{1}{9}$

9. If $\log_8 x = \frac{2}{3}$, then the value of x is :

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 3 (d) 4

10. If $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2}$, then x is equal to :

(a) $-\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $\frac{81}{256}$ (d) $\frac{256}{81}$

11. If $\log_x 4 = 0.4$, then the value of x is : **(Asstt. Grade, 1998)**

(a) 1 (b) 4 (c) 16 (d) 32

12. If $\log_{10000} x = -\frac{1}{4}$, then x is equal to :

(a) $\frac{1}{10}$ (b) $\frac{1}{100}$ (c) $\frac{1}{1000}$ (d) $\frac{1}{10000}$

13. If $\log_x 4 = \frac{1}{4}$, then x is equal to :

(a) 16 (b) 64 (c) 128 (d) 256

14. If $\log_x (0.1) = -\frac{1}{3}$, then the value of x is :

(a) 10 (b) 100 (c) 1000 (d) $\frac{1}{1000}$

15. If $\log_{32} x = 0.8$, then x is equal to :

(a) 25.6 (b) 16 (c) 10 (d) 12.8

16. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is : **(S.S.C. 1999)**

(a) 2^{10} (b) 2^{100} (c) 2^{1000} (d) 2^{10000}

17. The value of $\log_{(-1/3)} 81$ is equal to :

(a) -27 (b) -4 (c) 4 (d) 27

18. The value of $\log_{2\sqrt{3}} (1728)$ is :

(a) 3 (b) 5 (c) 6 (d) 9

19. $\frac{\log \sqrt{8}}{\log 8}$ is equal to : **(I.A.F. 2002)**

(a) $\frac{1}{\sqrt{8}}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

20. Which of the following statements is not correct ? **(M.B.A. 2003)**

(a) $\log_{10} 10 = 1$ (b) $\log (2 + 3) = \log (2 \times 3)$

(c) $\log_{10} 1 = 0$ (d) $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

21. The value of $\log_2 (\log_5 625)$ is :
(a) 2 (b) 5 (c) 10 (d) 15

22. If $\log_2 [\log_3 (\log_2 x)] = 1$, then x is equal to :
(a) 0 (b) 12 (c) 128 (d) 512

23. The value of $\log_2 \log_2 \log_3 \log_3 27^3$ is :
(a) 0 (b) 1 (c) 2 (d) 3

24. If $a^x = b^y$, then : **(Hotel Management, 2001)**
(a) $\log \frac{a}{b} = \frac{x}{y}$ (b) $\frac{\log a}{\log b} = \frac{x}{y}$ (c) $\frac{\log a}{\log b} = \frac{y}{x}$ (d) None of these

25. log 360 is equal to :
(a) 2 log 2 + 3 log 3 (b) 3 log 2 + 2 log 3
(c) 3 log 2 + 2 log 3 − log 5 (d) 3 log 2 + 2 log 3 + log 5

26. The value of $\left(\frac{1}{3} \log_{10} 125 - 2 \log_{10} 4 + \log_{10} 32\right)$ is :
(a) 0 (b) $\frac{4}{5}$ (c) 1 (d) 2

27. $2 \log_{10} 5 + \log_{10} 8 - \frac{1}{2} \log_{10} 4 = ?$ **(M.B.A. 2002)**
(a) 2 (b) 4 (c) $2 + 2 \log_{10} 2$ (d) $4 - 4 \log_{10} 2$

28. If $\log_a (ab) = x$, then $\log_b (ab)$ is : **(M.A.T. 2002)**
(a) $\frac{1}{x}$ (b) $\frac{x}{x+1}$ (c) $\frac{x}{1-x}$ (d) $\frac{x}{x-1}$

29. If log 2 = x, log 3 = y and log 7 = z, then the value of $\log (4 \cdot \sqrt[3]{63})$ is :
(a) $2x + \frac{2}{3}y - \frac{1}{3}z$ (b) $2x + \frac{2}{3}y + \frac{1}{3}z$
(c) $2x - \frac{2}{3}y + \frac{1}{3}z$ (d) $-2x + \frac{2}{3}y + \frac{1}{3}z$ **(S.S.C. 1998)**

30. If $\log_4 x + \log_2 x = 6$, then x is equal to :
(a) 2 (b) 4 (c) 8 (d) 16

31. If $\log_8 x + \log_8 \frac{1}{6} = \frac{1}{3}$, then the value of x is :
(a) 12 (b) 16 (c) 18 (d) 24

32. If $\log_{10} 125 + \log_{10} 8 = x$, then x is equal to :
(a) $\frac{1}{3}$ (b) .064 (c) − 3 (d) 3

33. The value of $(\log_9 27 + \log_8 32)$ is :
(a) $\frac{7}{2}$ (b) $\frac{19}{6}$ (c) 4 (d) 7

34. $(\log_5 3) \times (\log_3 625)$ equals :
(a) 1 (b) 2 (c) 3 (d) 4

35. $(\log_5 5)\ (\log_4 9)\ (\log_3 2)$ is equal to :
(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) 5

36. If $\log_{12} 27 = a$, then $\log_6 16$ is : **(Assistant Grade, 1998)**
(a) $\frac{3-a}{4(3+a)}$ (b) $\frac{3+a}{4(3-a)}$ (c) $\frac{4(3+a)}{(3-a)}$ (d) $\frac{4(3-a)}{(3+a)}$

37. If $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$, then x is equal to : (C.D.S. 2003)
(a) 1 (b) 3 (c) 5 (d) 10

38. If $\log_5 (x^2 + x) - \log_5 (x + 1) = 2$, then the value of x is :
(a) 5 (b) 10 (c) 25 (d) 32

39. The value of $\left(\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}\right)$ is :
(a) 0 (b) 1 (c) 5 (d) 60

40. The value of $(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)(\log_8 9)$ is :
(a) 2 (b) 7 (c) 8 (d) 33

41. The value of $16^{\log_4 5}$ is :
(a) $\frac{5}{64}$ (b) 5 (c) 16 (d) 25

42. If $\log x + \log y = \log (x + y)$, then :
(a) $x = y$ (b) $xy = 1$ (c) $y = \frac{x-1}{x}$ (d) $y = \frac{x}{x-1}$

43. If $\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b)$, then :
(a) $a + b = 1$ (b) $a - b = 1$ (c) $a = b$ (d) $a^2 - b^2 = 1$

44. $\left[\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ac}\right) + \log\left(\frac{c^2}{ab}\right)\right]$ is equal to :
(a) 0 (b) 1 (c) 2 (d) abc

45. $(\log_b a \times \log_c b \times \log_a c)$ is equal to :
(a) 0 (b) 1 (c) abc (d) $a + b + c$

46. $\left[\frac{1}{(\log_a bc) + 1} + \frac{1}{(\log_b ca) + 1} + \frac{1}{(\log_c ab) + 1}\right]$ is equal to :
(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) 3

47. The value of $\left[\frac{1}{\log_{(p/q)} x} + \frac{1}{\log_{(q/r)} x} + \frac{1}{\log_{(r/p)} x}\right]$ is :
(a) 0 (b) 1 (c) 2 (d) 3

48. If $\log_{10} 7 = a$, then $\log_{10}\left(\frac{1}{70}\right)$ is equal to : (C.D.S. 2003)
(a) $-(1 + a)$ (b) $(1 + a)^{-1}$ (c) $\frac{a}{10}$ (d) $\frac{1}{10a}$

49. If $a = b^x$, $b = c^y$ and $c = a^z$, then the value of xyz is equal to :
(a) -1 (b) 0 (c) 1 (d) abc

50. If $\log 27 = 1.431$, then the value of $\log 9$ is : (Section Officers', 2001)
(a) 0.934 (b) 0.945 (c) 0.954 (d) 0.958

51. If $\log_{10} 2 = 0.3010$, then $\log_2 10$ is equal to : (S.S.C. 2000)
(a) $\frac{699}{301}$ (b) $\frac{1000}{301}$ (c) 0.3010 (d) 0.6990

52. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 5$ is : (S.S.C. 2001)
(a) 0.3241 (b) 0.6911 (c) 0.6990 (d) 0.7525

53. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 80$ is :
(a) 1.6020 (b) 1.9030 (c) 3.9030 (d) None of these

54. If $\log 3 = 0.477$ and $(1000)^x = 3$, then x equals : **(S.S.C. 2000)**
(a) 0.0159 (b) 0.0477 (c) 0.159 (d) 10

55. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 25$ is :
(a) 0.6020 (b) 1.2040 (c) 1.3980 (d) 1.5050

56. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, the value of $\log_5 512$ is : **(M.A.T. 2002)**
(a) 2.870 (b) 2.967 (c) 3.876 (d) 3.912

57. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the value of $\log_{10} 1.5$ is :
(a) 0.1761 (b) 0.7116 (c) 0.7161 (d) 0.7611

58. If $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, then the value of $\log_{10} 2.8$ is :
(a) 0.4471 (b) 1.4471 (c) 2.4471 (d) None of these
(S.S.C. 1999)

59. If $\log (0.57) = \bar{1}.756$, then the value of $\log 57 + \log (0.57)^3 + \log \sqrt{0.57}$ is :
(a) 0.902 (b) $\bar{2}.146$ (c) 1.902 (d) $\bar{1}.146$
(Section Officers', 2003)

60. If $\log 2 = 0.30103$, the number of digits in 2^{64} is : **(C.B.I. 1997)**
(a) 18 (b) 19 (c) 20 (d) 21

61. If $\log 2 = 0.30103$, the number of digits in 4^{50} is :
(a) 30 (b) 31 (c) 100 (d) 200

62. If $\log 2 = 0.30103$, then the number of digits in 5^{20} is :
(a) 14 (b) 16 (c) 18 (d) 25

ANSWERS

1. (b)	2. (a)	3. (b)	4. (c)	5. (c)	6. (d)	7. (a)	8. (d)
9. (d)	10. (d)	11. (d)	12. (a)	13. (d)	14. (c)	15. (b)	16. (c)
17. (b)	18. (c)	19. (c)	20. (b)	21. (a)	22. (d)	23. (a)	24. (c)
25. (d)	26. (c)	27. (a)	28. (d)	29. (b)	30. (d)	31. (a)	32. (d)
33. (b)	34. (d)	35. (a)	36. (d)	37. (b)	38. (c)	39. (b)	40. (a)
41. (d)	42. (d)	43. (a)	44. (a)	45. (b)	46. (a)	47. (a)	48. (a)
49. (c)	50. (c)	51. (b)	52. (c)	53. (b)	54. (c)	55. (c)	56. (c)
57. (a)	58. (a)	59. (a)	60. (c)	61. (b)	62. (a)		

SOLUTIONS

1. Let $\log_2 16 = n$. Then, $2^n = 16 = 2^4 \Rightarrow n = 4$.
$\therefore \log_2 16 = n$.

2. Let $\log_{343} 7 = n$. Then, $(343)^n = 7 \Leftrightarrow (7^3)^n = 7 \Leftrightarrow 3n = 1 \Leftrightarrow n = \frac{1}{3}$.
$\therefore \log_{343} 7 = \frac{1}{3}$.

3. Let $\log_5 \left(\frac{1}{125}\right) = n$. Then, $5^n = \frac{1}{125} \Leftrightarrow 5^n = 5^{-3} \Leftrightarrow n = -3$.
$\therefore \log_5 \left(\frac{1}{125}\right) = -3$

4. Let $\log_{\sqrt{2}} 32 = n$. Then, $(\sqrt{2})^n = 32 \Leftrightarrow (2)^{n/2} = 2^5 \Leftrightarrow \frac{n}{2} = 5 \Leftrightarrow n = 10$.

$\therefore \log_{\sqrt{2}} 32 = 10$.

5. Let $\log_{10} (.0001) = n$.

Then, $10^n = .0001 \Leftrightarrow 10^n = \frac{1}{10000} = \frac{1}{10^4} \Leftrightarrow 10^n = 10^{-4} \Leftrightarrow n = -4$.

$\therefore \log_{10} (.0001) = -4$.

6. Let $\log_{(.01)} (1000) = n$.

Then, $(.01)^n = 1000 \Leftrightarrow \left(\frac{1}{100}\right)^n = 10^3 \Leftrightarrow \left(10^{-2}\right)^n = 10^3 \Leftrightarrow -2n = 3 \Leftrightarrow n = -\frac{3}{2}$.

7. Let $\log_2 0.0625 = n$.

Then, $2^n = 0.0625 = \frac{625}{10000} \Leftrightarrow 2^n = \frac{1}{16} \Leftrightarrow 2^n = 2^{-4} \Leftrightarrow n = -4$.

$\therefore \log_2 0.0625 = -4$.

8. $\log_3 x = -2 \Leftrightarrow x = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.

9. $\log_8 x = \frac{2}{3} \Leftrightarrow x = 8^{2/3} = (2^3)^{2/3} = 2^{\left(3 \times \frac{2}{3}\right)} = 2^2 = 4$.

10. $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2} \Leftrightarrow x^{-1/2} = \frac{9}{16} \Leftrightarrow \frac{1}{\sqrt{x}} = \frac{9}{16} \Leftrightarrow \sqrt{x} = \frac{16}{9}$

$\Leftrightarrow x = \left(\frac{16}{9}\right)^2 = \frac{256}{81}$.

11. $\log_x 4 = 0.4 \Leftrightarrow \log_x 4 = \frac{4}{10} = \frac{2}{5} \Leftrightarrow x^{2/5} = 4 \Leftrightarrow x = 4^{5/2} = (2^2)^{5/2}$

$\Leftrightarrow x = 2^{\left(2 \times \frac{5}{2}\right)} = 2^5 \Leftrightarrow x = 32$.

12. $\log_{10000} x = -\frac{1}{4} \Leftrightarrow x = (10000)^{-1/4} = (10^4)^{-1/4} = 10^{-1} = \frac{1}{10}$.

13. $\log_x 4 = \frac{1}{4} \Leftrightarrow x^{1/4} = 4 \Leftrightarrow x = 4^4 = 256$.

14. $\log_x (0.1) = -\frac{1}{3} \Leftrightarrow x^{-1/3} = 0.1 \Leftrightarrow \frac{1}{x^{1/3}} = 0.1 \Leftrightarrow x^{1/3} = \frac{1}{0.1} = 10$

$\Leftrightarrow x = (10)^3 = 1000$.

15. $\log_{32} x = 0.8 \Leftrightarrow x = (32)^{0.8} = (2^5)^{4/5} = 2^4 = 16$.

16. $\log_2 x = 10 \Rightarrow x = 2^{10}$.

$\therefore \log_x y = 100 \Rightarrow y = x^{100} = (2^{10})^{100} \Rightarrow y = 2^{1000}$.

17. Let $\log_{(-1/3)} 81 = x$. Then, $\left(-\frac{1}{3}\right)^x = 81 = 3^4 = (-3)^4 = \left(-\frac{1}{3}\right)^{-4}$

$\therefore x = -4$ *i.e.*, $\log_{(-1/3)} 81 = -4$.

18. Let $\log_{2\sqrt{3}} (1728) = x$.

Then, $(2\sqrt{3})^x = 1728 = (12)^3 = [(2\sqrt{3})^2]^3 = (2\sqrt{3})^6$.

$\therefore x = 6$, *i.e.*, $\log_{2\sqrt{3}} (1728) = 6$.

19. $\dfrac{\log \sqrt{8}}{\log 8} = \dfrac{\log (8)^{1/2}}{\log 8} = \dfrac{\frac{1}{2}\log 8}{\log 8} = \dfrac{1}{2}$.

20. (*a*) Since $\log_a a = 1$, so $\log_{10} 10 = 1$.
(*b*) $\log (2 + 3) = 5$ and $\log (2 \times 3) = \log 6 = \log 2 + \log 3$
$\therefore$ $\log (2 + 3) \neq \log (2 \times 3)$.
(*c*) Since $\log_a 1 = 0$, so $\log_{10} 1 = 0$.
(*d*) $\log (1 + 2 + 3) = \log 6 = \log (1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$.
So, (*b*) is incorrect.

21. Let $\log_5 625 = x$. Then, $5^x = 625 = 5^4$ or $x = 4$.
Let $\log_2 (\log_5 625) = y$. Then, $\log_2 4 = y$ or $2^y = 4 = 2^2$ or $y = 2$.
$\therefore$ $\log_2 (\log_5 625) = 2$.

22. $\log_2 [\log_3 (\log_2 x)] = 1 = \log_2 2$
$\Leftrightarrow$ $\log_3 (\log_2 x) = 2 \Leftrightarrow \log_2 x = 3^2 = 9 \Leftrightarrow x = 2^9 = 512$.

23. $\log_2 \log_2 \log_3 (\log_3 27^3) = \log_2 \log_2 \log_3 [\log_3 (3^3)^3] = \log_2 \log_2 \log_3 [\log_3 (3)^9]$
$= \log_2 \log_2 \log_3 (9 \log_3 3) = \log_2 \log_2 \log_3 9$ [$\because \log_3 3 = 1$]
$= \log_2 \log_2 [\log_3 (3)^2] = \log_2 \log_2 (2 \log_3 3)$
$= \log_2 \log_2 2 = \log_2 1 = 0$.

24. $a^x = b^y \Rightarrow \log a^x = \log b^y \Rightarrow x \log a = y \log b \Rightarrow \dfrac{\log a}{\log b} = \dfrac{y}{x}$.

25. $360 = (2 \times 2 \times 2) \times (3 \times 3) \times 5$.
So, $\log 360 = \log (2^3 \times 3^2 \times 5) = \log 2^3 + \log 3^2 + \log 5 = 3 \log 2 + 2 \log 3 + \log 5$.

26. $\dfrac{1}{3}\log_{10} 125 - 2 \log_{10} 4 + \log_{10} 32$
$= \log_{10} (125)^{1/3} - \log_{10} (4)^2 + \log_{10} 32 = \log_{10} 5 - \log_{10} 16 + \log_{10} 32$
$= \log_{10}\left(\dfrac{5 \times 32}{16}\right) = \log_{10} 10 = 1$.

27. $2 \log_{10} 5 + \log_{10} 8 - \dfrac{1}{2}\log_{10} 4 = \log_{10} (5^2) + \log_{10} 8 - \log_{10} (4^{1/2})$
$= \log_{10} 25 + \log_{10} 8 - \log_{10} 2 = \log_{10}\left(\dfrac{25 \times 8}{2}\right) = \log_{10} 100 = 2$.

28. $\log_a (ab) = x \Leftrightarrow \dfrac{\log ab}{\log a} = x \Leftrightarrow \dfrac{\log a + \log b}{\log a} = x$
$\Leftrightarrow 1 + \dfrac{\log b}{\log a} = x \Leftrightarrow \dfrac{\log b}{\log a} = x - 1$
$\Leftrightarrow \dfrac{\log a}{\log b} = \dfrac{1}{x-1} \Leftrightarrow 1 + \dfrac{\log a}{\log b} = 1 + \dfrac{1}{x-1}$
$\Leftrightarrow \dfrac{\log b}{\log b} + \dfrac{\log a}{\log b} = \dfrac{x}{x-1} \Leftrightarrow \dfrac{\log b + \log a}{\log b} = \dfrac{x}{x-1}$
$\Leftrightarrow \dfrac{\log (ab)}{\log b} = \dfrac{x}{x-1} \Leftrightarrow \log_b (ab) = \dfrac{x}{x-1}$.

29. $\log (4 \cdot \sqrt[3]{63}) = \log 4 + \log (\sqrt[3]{63}) = \log 4 + \log (63)^{1/3} = \log (2^2) + \log (7 \times 3^2)^{1/3}$
$= 2 \log 2 + \dfrac{1}{3}\log 7 + \dfrac{2}{3}\log 3 = 2x + \dfrac{1}{3}z + \dfrac{2}{3}y$.

30. $\log_4 x + \log_2 x = 6 \Leftrightarrow \dfrac{\log x}{\log 4} + \dfrac{\log x}{\log 2} = 6$

$\Leftrightarrow \dfrac{\log x}{2\log 2} + \dfrac{\log x}{\log 2} = 6 \Leftrightarrow 3 \log x = 12 \log 2$

$\Leftrightarrow \log x = 4 \log 2 \Leftrightarrow \log x = \log (2^4) = \log 16 \Leftrightarrow x = 16.$

31. $\log_8 x + \log_8 \left(\dfrac{1}{6}\right) = \dfrac{1}{3} \Leftrightarrow \dfrac{\log x}{\log 8} + \dfrac{\log \frac{1}{6}}{\log 8} = \dfrac{1}{3}$

$\Leftrightarrow \log x + \log \dfrac{1}{6} = \dfrac{1}{3} \log 8 \Leftrightarrow \log x + \log \dfrac{1}{6} = \log (8^{1/3}) = \log 2$

$\Leftrightarrow \log x = \log 2 - \log \dfrac{1}{6} = \log \left(2 \times \dfrac{6}{1}\right) = \log 12$

$\therefore \quad x = 12.$

32. $\log_{10} 125 + \log_{10} 8 = x \Rightarrow \log_{10} (125 \times 8) = x$

$\Rightarrow x = \log_{10} (1000) = \log_{10} (10)^3 = 3 \log_{10} 10 = 3.$

33. Let $\log_9 27 = x$. Then, $9^x = 27 \Leftrightarrow (3^2)^x = 3^3 \Leftrightarrow 2x = 3 \Leftrightarrow x = \dfrac{3}{2}.$

Let $\log_8 32 = y$. Then, $8^y = 32 \Leftrightarrow (2^3)^y = 2^5 \Leftrightarrow 3y = 5 \Leftrightarrow y = \dfrac{5}{3}.$

$\therefore \quad \log_9 27 + \log_8 32 = \left(\dfrac{3}{2} + \dfrac{5}{3}\right) = \dfrac{19}{6}.$

34. Given expression $= \left(\dfrac{\log 3}{\log 5} \times \dfrac{\log 625}{\log 3}\right) = \dfrac{\log 625}{\log 5} = \dfrac{\log (5^4)}{\log 5} = \dfrac{4 \log 5}{\log 5} = 4.$

35. Given expression $= \dfrac{\log 9}{\log 4} \times \dfrac{\log 2}{\log 3}$ $\quad [\because \log_5 5 = 1]$

$= \dfrac{\log 3^2}{\log 2^2} \times \dfrac{\log 2}{\log 3} = \dfrac{2 \log 3}{2 \log 2} \times \dfrac{\log 2}{\log 3} = 1.$

36. $\log_{12} 27 = a \Rightarrow \dfrac{\log 27}{\log 12} = a \Rightarrow \dfrac{\log 3^3}{\log (3 \times 2^2)} = a$

$\Rightarrow \dfrac{3 \log 3}{\log 3 + 2 \log 2} = a \Rightarrow \dfrac{\log 3 + 2 \log 2}{3 \log 3} = \dfrac{1}{a}$

$\Rightarrow \dfrac{\log 3}{3 \log 3} + \dfrac{2 \log 2}{3 \log 3} = \dfrac{1}{a} \Rightarrow \dfrac{2}{3} \dfrac{\log 2}{\log 3} = \dfrac{1}{a} - \dfrac{1}{3} = \left(\dfrac{3-a}{3a}\right)$

$\Rightarrow \dfrac{\log 2}{\log 3} = \left(\dfrac{3-a}{2a}\right) \Rightarrow \log 3 = \left(\dfrac{2a}{3-a}\right) \log 2.$

$$\log_6 16 = \frac{\log 16}{\log 6} = \frac{\log 2^4}{\log (2 \times 3)} = \frac{4 \log 2}{\log 2 + \log 3} = \frac{4 \log 2}{\log 2 \left[1 + \left(\dfrac{2a}{3-a}\right)\right]}$$

$$= \frac{4}{\left(\dfrac{3+a}{3-a}\right)} = \frac{4(3-a)}{(3+a)}.$$

37. $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$

$\Rightarrow \log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + \log_{10} 10$

$\Rightarrow \log_{10} [5 (5x + 1)] = \log_{10} [10 (x + 5)] \Rightarrow 5 (5x + 1) = 10 (x + 5)$

$\Rightarrow 5x + 1 = 2x + 10 \Rightarrow 3x = 9 \Rightarrow x = 3.$

38. $\log_5 (x^2 + x) - \log_5 (x + 1) = 2 \Rightarrow \log_5 \left(\frac{x^2 + x}{x + 1}\right) = 2$

$\Rightarrow \log_5 \left[\frac{x (x + 1)}{x + 1}\right] = 2 \Rightarrow \log_5 x = 2 \Rightarrow x = 5^2 = 25.$

39. Given expression $= \log_{60} 3 + \log_{60} 4 + \log_{60} 5 = \log_{60} (3 \times 4 \times 5) = \log_{60} 60 = 1.$

40. Given expression $= \left(\frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} \times \frac{\log 9}{\log 8}\right)$

$= \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2 \log 3}{\log 3} = 2.$

41. We know that : $a^{\log_a x} = x.$

$\therefore \quad 16^{\log_4 5} = (4^2)^{\log_4 5} = 4^{2 \log_4 5} = 4^{\log_4 (5^2)} = 4^{\log_4 25} = 25.$

42. $\log x + \log y = \log (x + y) \Rightarrow \log (x + y) = \log (xy)$

$\Rightarrow x + y = xy \Rightarrow y (x - 1) = x \Rightarrow y = \frac{x}{x - 1}.$

43. $\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b) \Rightarrow \log (a + b) = \log \left(\frac{a}{b} \times \frac{b}{a}\right) = \log 1.$

So, $a + b = 1.$

44. Given expression $= \log \left(\frac{a^2}{bc} \times \frac{b^2}{ac} \times \frac{c^2}{ab}\right) = \log 1 = 0.$

45. Given expression $= \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1.$

46. Given expression $= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)} = \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} (abc) = 1.$

47. Given expression $= \log_x \left(\frac{p}{q}\right) + \log_x \left(\frac{q}{r}\right) + \log_x \left(\frac{r}{p}\right) = \log_x \left(\frac{p}{q} \times \frac{q}{r} \times \frac{r}{p}\right) = \log_x 1 = 0.$

48. $\log_{10} \left(\frac{1}{70}\right) = \log_{10} 1 - \log_{10} 70 = -\log_{10} (7 \times 10) = -(\log_{10} 7 + \log_{10} 10) = -(a + 1).$

49. $a = b^x, b = c^y, c = a^z \Rightarrow x = \log_b a, y = \log_c b, z = \log_a c$

$\Rightarrow xyz = (\log_b a) \times (\log_c b) \times (\log_a c) \Rightarrow xyz = \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1.$

50. $\log 27 = 1.431 \Rightarrow \log (3^3) = 1.431 \Rightarrow 3 \log 3 = 1.431$

$\Rightarrow \log 3 = 0.477$

$\therefore \quad \log 9 = \log (3^2) = 2 \log 3 = (2 \times 0.477) = 0.954.$

51. $\log_2 10 = \dfrac{1}{\log_{10} 2} = \dfrac{1}{0.3010} = \dfrac{10000}{3010} = \dfrac{1000}{301}$.

52. $\log_{10} 5 = \log_{10}\left(\dfrac{10}{2}\right) = \log_{10} 10 - \log_{10} 2 = 1 - \log_{10} 2 = (1 - 0.3010) = 0.6990.$

53. $\log_{10} 80 = \log_{10} (8 \times 10) = \log_{10} 8 + \log_{10} 10 = \log_{10} (2^3) + 1 = 3 \log_{10} 2 + 1$
$= (3 \times 0.3010) + 1 = 1.9030.$

54. $(1000)^x = 3 \Rightarrow \log [(1000)^x] = \log 3 \Rightarrow x \log 1000 = \log 3$
$\Rightarrow x \log (10^3) = \log 3 \Rightarrow 3x \log 10 = \log 3$
$\Rightarrow 3x = \log 3 \Rightarrow x = \dfrac{0.477}{3} = 0.159.$

55. $\log_{10} 25 = \log_{10}\left(\dfrac{100}{4}\right) = \log_{10} 100 - \log_{10} 4 = 2 - 2 \log_{10} 2 = (2 - 2 \times 0.3010)$
$= (2 - 0.6020) = 1.3980.$

56. $\log_5 512 = \dfrac{\log 512}{\log 5} = \dfrac{\log 2^9}{\log\left(\dfrac{10}{2}\right)} = \dfrac{9 \log 2}{\log 10 - \log 2}$

$= \dfrac{(9 \times 0.3010)}{1 - 0.3010} = \dfrac{2.709}{0.699} = \dfrac{2709}{699} = 3.876.$

57. $\log_{10} (1.5) = \log_{10}\left(\dfrac{3}{2}\right) = \log_{10} 3 - \log_{10} 2 = (0.4771 - 0.3010) = 0.1761.$

58. $\log_{10} (2.8) = \log_{10}\left(\dfrac{28}{10}\right) = \log_{10} 28 - \log_{10} 10$
$= \log_{10} (7 \times 2^2) - 1 = \log_{10} 7 + 2 \log_{10} 2 - 1$
$= 0.8451 + 2 \times 0.3010 - 1 = 0.8451 + 0.602 - 1 = 0.4471.$

59. $\log (0.57) = \bar{1}.756 \Rightarrow \log 57 = 1.756$ [$\because$ mantissa will remain the same]

$\therefore \log 57 + \log (0.57)^3 + \log \sqrt{0.57}$

$= \log 57 + 3 \log\left(\dfrac{57}{100}\right) + \log\left(\dfrac{57}{100}\right)^{1/2}$

$= \log 57 + 3 \log 57 - 3 \log 100 + \dfrac{1}{2} \log 57 - \dfrac{1}{2} \log 100$

$= \dfrac{9}{2} \log 57 - \dfrac{7}{2} \log 100 = \dfrac{9}{2} \times 1.756 - \dfrac{7}{2} \times 2 = 7.902 - 7 = 0.902.$

60. $\log (2^{64}) = 64 \times \log 2 = (64 \times 0.30103) = 19.26592.$

Its characteristic is 19. Hence, the number of digits in 2^{64} is 20.

61. $\log 4^{50} = 50 \log 4 = 50 \log 2^2 = (50 \times 2) \log 2 = 100 \times \log 2 = (100 \times 0.30103) = 30.103.$

$\therefore$ Characteristic = 30. Hence, the number of digits in $4^{50} = 31$.

62. $\log 5^{20} = 20 \log 5 = 20 \times \left[\log\left(\dfrac{10}{2}\right)\right] = 20 (\log 10 - \log 2)$

$= 20 (1 - 0.3010) = 20 \times 0.6990 = 13.9800.$

$\therefore$ Characteristic = 13. Hence, the number of digits in 5^{20} is 14.

24. AREA

FUNDAMENTAL CONCEPTS

I. **Results on Triangles :**

1. Sum of the angles of a triangle is 180°.
2. The sum of any two sides of a triangle is greater than the third side.
3. **Pythagoras Theorem** : In a right-angled triangle,
   $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$.
4. The line joining the mid-point of a side of a triangle to the opposite vertex is called the ***median***.
5. The point where the three medians of a triangle meet, is called ***centroid***. The centroid divides each of the medians in the ratio 2 : 1.
6. In an isosceles triangle, the altitude from the vertex bisects the base.
7. The median of a triangle divides it into two triangles of the same area.
8. The area of the triangle formed by joining the mid-points of the sides of a given triangle is one-fourth of the area of the given triangle.

II. **Results on Quadrilaterals :**

1. The diagonals of a parallelogram bisect each other.
2. Each diagonal of a parallelogram divides it into two triangles of the same area.
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
7. Of all the parallelogram of given sides, the parallelogram which is a rectangle has the greatest area.

IMPORTANT FORMULAE

I. 1. Area of a rectangle = (Length × Breadth).

$\therefore$ Length $= \left(\dfrac{\text{Area}}{\text{Breadth}}\right)$ and Breadth $= \left(\dfrac{\text{Area}}{\text{Length}}\right)$.

2. Perimeter of a rectangle = 2 (Length + Breadth).

II. Area of a square = $(\text{side})^2 = \dfrac{1}{2}(\text{diagonal})^2$.

III. Area of 4 walls of a room = 2 (Length + Breadth) × Height.

IV. 1. Area of a triangle = $\dfrac{1}{2} \times \text{Base} \times \text{Height}$.

2. Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c are the sides of the triangle and $s = \dfrac{1}{2}(a+b+c)$.

3. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$.

4. Radius of incircle of an equilateral triangle of side $a = \frac{a}{2\sqrt{3}}$.

5. Radius of circumcircle of an equilateral triangle of side $a = \frac{a}{\sqrt{3}}$.

6. Radius of incircle of a triangle of area Δ and semi-perimeter $s = \frac{\Delta}{s}$.

V. 1. Area of a parallelogram = (Base × Height).

2. Area of a rhombus $= \frac{1}{2} \times$ (Product of diagonals).

3. Area of a trapezium $= \frac{1}{2} \times$ (sum of parallel sides) × distance between them.

VI. 1. Area of a circle $= \pi R^2$, where R is the radius.

2. Circumference of a circle $= 2\pi R$.

3. Length of an arc $= \frac{2\pi R\theta}{360}$, where θ is the central angle.

4. Area of a sector $= \frac{1}{2}(\text{arc} \times R) = \frac{\pi R^2 \theta}{360}$.

VII. 1. Area of a semi-circle $= \frac{\pi R^2}{2}$.

2. Circumference of a semi-circle $= \pi R$.

SOLVED EXAMPLES

Ex. 1. ***One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.***

Sol. Other side $= \sqrt{(17)^2 - (15)^2} = \sqrt{289 - 225} = \sqrt{64} = 8$ m.

$\therefore$ Area $= (15 \times 8)\ m^2 = 120\ m^2$.

Ex. 2. ***A lawn is in the form of a rectangle having its sides in the ratio 2 : 3. The area of the lawn is $\frac{1}{6}$ hectares. Find the length and breadth of the lawn.***

Sol. Let length $= 2x$ metres and breadth $= 3x$ metres.

Now, area $= \left(\frac{1}{6} \times 1000\right) m^2 = \left(\frac{5000}{3}\right) m^2$.

So, $2x \times 3x = \frac{5000}{3} \Leftrightarrow x^2 = \frac{2500}{9} \Leftrightarrow x = \left(\frac{50}{3}\right)$.

$\therefore$ Length $= 2x = \frac{100}{3}$ m $= 33\frac{1}{3}$ m and Breadth $= 3x = \left(3 \times \frac{50}{3}\right)$ m $= 50$ m.

Ex. 3. ***Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per square metre.***

Sol. Area of the carpet = Area of the room $= (13 \times 9)\ m^2 = 117\ m^2$.

Length of the carpet $= \left(\frac{\text{Area}}{\text{Width}}\right) = \left(117 \times \frac{4}{3}\right)$ m $= 156$ m.

$\therefore$ Cost of carpeting = Rs. (156×12.40) = Rs. 1934.40.

Ex. 4. ***If the diagonal of a rectangle is 17 cm long and its perimeter is 46 cm, find the area of the rectangle.***

Sol. Let length = x and breadth = y. Then,

$2(x + y) = 46$ or $x + y = 23$ and $x^2 + y^2 = (17)^2 = 289$.

Now, $(x + y)^2 = (23)^2 \Leftrightarrow (x^2 + y^2) + 2xy = 529 \Leftrightarrow 289 + 2xy = 529 \Leftrightarrow xy = 120$.

$\therefore$ Area = $xy = 120$ cm^2.

Ex. 5. ***The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.***

Sol. Let breadth = x. Then, length = $2x$. Then,

$(2x - 5)(x + 5) - 2x \times x = 75 \Leftrightarrow 5x - 25 = 75 \Leftrightarrow x = 20$.

$\therefore$ Length of the rectangle = 20 cm.

Ex. 6. ***In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error percent in the area calculated from these measurements.*** **(M.B.A. 2003)**

Sol. Let x and y be the sides of the rectangle. Then, Correct area = xy.

Calculated area = $\left(\frac{105}{100}x\right) \times \left(\frac{96}{100}y\right) = \frac{504}{500}xy$.

Error in measurement = $\left(\frac{504}{500}xy\right) - xy = \frac{4}{500}xy$.

$\therefore$ Error % = $\left[\frac{4}{500}xy \times \frac{1}{xy} \times 100\right]\% = \frac{4}{5}\% = 0.8\%$.

Ex. 7. ***A rectangular grassy plot 110 m by 65 m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.***

Sol. Area of the plot = (110×65) m^2 = 7150 m^2.

Area of the plot excluding the path = $[(110 - 5) \times (65 - 5)]$ m^2 = 6300 m^2.

$\therefore$ Area of the path = $(7150 - 6300)$ m^2 = 850 m^2.

Cost of gravelling the path = Rs. $\left(850 \times \frac{80}{100}\right)$ = Rs. 680.

Ex. 8. ***The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares.*** **(S.S.C. 2003)**

Sol. Side of first square = $\left(\frac{40}{4}\right)$ cm = 10 cm;

Side of second square = $\left(\frac{32}{4}\right)$ cm = 8 cm.

Area of third square = $[(10)^2 - (8)^2]$ cm^2 = $(100 - 64)$ cm^2 = 36 cm^2.

Side of third square = $\sqrt{36}$ cm = 6 cm.

$\therefore$ Required perimeter = (6×4) cm = 24 cm.

Ex. 9. ***A room 5m 55 cm long and 3m 74 cm broad is to be paved with square tiles. Find the least number of square tiles required to cover the floor.***

Sol. Area of the room = (544×374) cm^2.

Size of largest square tile = H.C.F. of 544 cm and 374 cm = 34 cm.

Area of 1 tile = (34×34) cm^2.

$\therefore$ Number of tiles required = $\left(\frac{544 \times 374}{34 \times 34}\right) = 176$.

Ex. 10. ***Find the area of a square, one of whose diagonals is 3.8 m long.***

Sol. Area of the square $= \frac{1}{2} \times (\text{diagonal})^2 = \left(\frac{1}{2} \times 3.8 \times 3.8\right) \text{m}^2 = 7.22 \text{ m}^2$.

Ex. 11. ***The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas.*** **(Section Officers', 2003)**

Sol. Let the diagonals of the squares be $2x$ and $5x$ respectively.

$\therefore$ Ratio of their areas $= \frac{1}{2} \times (2x)^2 : \frac{1}{2} \times (5x)^2 = 4x^2 : 25x^2 = 4 : 25$.

Ex. 12. ***If each side of a square is increased by 25%, find the percentage change in its area.***

Sol. Let each side of the square be a. Then, area $= a^2$.

New side $= \frac{125a}{100} = \frac{5a}{4}$. New area $= \left(\frac{5a}{4}\right)^2 = \frac{25a^2}{16}$.

Increase in area $= \left(\frac{25a^2}{16} - a^2\right) = \frac{9a^2}{16}$.

$\therefore$ Increase % $= \left(\frac{9a^2}{16} \times \frac{1}{a^2} \times 100\right)\% = 56.25\%$.

Ex. 13. ***If the length of a certain rectangle is decreased by 4 cm and the width is increased by 3 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle.***

Sol. Let x and y be the length and breadth of the rectangle respectively.

Then, $x - 4 = y + 3$ or $x - y = 7$ ...(*i*)

Area of the rectangle $= xy$; Area of the square $= (x - 4)(y + 3)$

$\therefore$ $(x - 4)(y + 3) = xy \Leftrightarrow 3x - 4y = 12$ ...(*ii*)

Solving (*i*) and (*ii*), we get $x = 16$ and $y = 9$.

$\therefore$ Perimeter of the rectangle $= 2(x + y) = [2(16 + 9)]$ cm $= 50$ cm.

Ex. 14. ***A room is half as long again as it is broad. The cost of carpeting the room at Rs. 5 per sq. m is Rs. 270 and the cost of papering the four walls at Rs. 10 per m^2 is Rs. 1720. If a door and 2 windows occupy 8 sq. m, find the dimensions of the room.***

Sol. Let breadth $= x$ metres, length $= \frac{3x}{2}$ metres, height $= H$ metres.

Area of the floor $= \left(\frac{\text{Total cost of carpeting}}{\text{Rate} / \text{m}^2}\right) \text{m}^2 = \left(\frac{270}{5}\right) \text{m}^2 = 54 \text{ m}^2$.

$\therefore$ $x \times \frac{3x}{2} = 54 \Leftrightarrow x^2 = \left(54 \times \frac{2}{3}\right) = 36 \Leftrightarrow x = 6$.

So, breadth $= 6$ m and length $= \left(\frac{3}{2} \times 6\right)$ m $= 9$ m.

Now, papered area $= \left(\frac{1720}{10}\right) \text{m}^2 = 172 \text{ m}^2$.

Area of 1 door and 2 windows $= 8 \text{ m}^2$.

Total area of 4 walls $= (172 + 8) \text{ m}^2 = 180 \text{ m}^2$.

$\therefore$ $2(9 + 6) \times H = 180 \Leftrightarrow H = \left(\frac{180}{30}\right) = 6$ m.

Ex. 15. ***Find the area of a triangle whose sides measure 13 cm, 14 cm and 15 cm.***

Sol. Let $a = 13$, $b = 14$ and $c = 15$. Then, $s = \frac{1}{2}(a+b+c) = 21$.

$\therefore$ $(s-a) = 8$, $(s-b) = 7$ and $(s-c) = 6$.

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21\times 8\times 7\times 6} = 84 \text{ cm}^2$.

Ex. 16. ***Find the area of a right-angled triangle whose base is 12 cm and hypotenuse 13 cm.***

Sol. Height of the triangle $= \sqrt{(13)^2-(12)^2} \text{ cm} = \sqrt{25} \text{ cm} = 5 \text{ cm}$.

$\therefore$ Its area $= \frac{1}{2}\times \text{Base} \times \text{Height} = \left(\frac{1}{2}\times 12\times 5\right) \text{cm}^2 = 30 \text{ cm}^2$.

Ex. 17. ***The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs. 333.18, find its base and height.***

Sol. Area of the field $= \frac{\text{Total cost}}{\text{Rate}} = \left(\frac{333.18}{24.68}\right)$ hectares $= 13.5$ hectares

$= (13.5\times 10000) \text{ m}^2 = 135000 \text{ m}^2$.

Let altitude $= x$ metres and base $= 3x$ metres.

Then, $\frac{1}{2}\times 3x\times x = 135000 \Leftrightarrow x^2 = 90000 \Leftrightarrow x = 300$.

$\therefore$ Base $= 900$ m and Altitude $= 300$ m.

Ex. 18. ***The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.***

Sol. Let ABC be the isosceles triangle and AD be the altitude.

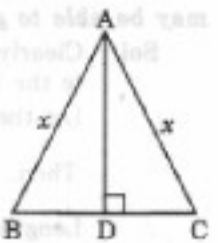
Let $AB = AC = x$. Then, $BC = (32-2x)$.

Since, in an isosceles triangle, the altitude bisects the base,

so $BD = DC = (16-x)$.

In ΔADC, $AC^2 = AD^2 + DC^2 \Rightarrow x^2 = (8)^2 + (16-x)^2$

$\Rightarrow 32x = 320 \Rightarrow x = 10$.



$\therefore$ $BC = (32-2x) = (32-20) \text{ cm} = 12 \text{ cm}$.

Hence, required area $= \left(\frac{1}{2}\times BC\times AD\right) = \left(\frac{1}{2}\times 12\times 10\right) \text{cm}^2 = 60 \text{ cm}^2$.

Ex. 19. ***Find the length of the altitude of an equilateral triangle of side $3\sqrt{3}$ cm.***

Sol. Area of the triangle $= \frac{\sqrt{3}}{4}\times(3\sqrt{3})^2 = \frac{27\sqrt{3}}{4}$. Let the height be h.

Then, $\frac{1}{2}\times 3\sqrt{3}\times h = \frac{27\sqrt{3}}{4} \Leftrightarrow h = \frac{27\sqrt{3}}{4}\times\frac{2}{3\sqrt{3}} = \frac{9}{2} = 4.5 \text{ cm}$.

Ex. 20. ***In two triangles, the ratio of the areas is 4 : 3 and the ratio of their heights is 3 : 4. Find the ratio of their bases.***

Sol. Let the bases of the two triangles be x and y and their heights be $3h$ and $4h$ respectively. Then,

$$\frac{\frac{1}{2}\times x\times 3h}{\frac{1}{2}\times y\times 4h} = \frac{4}{3} \Leftrightarrow \frac{x}{y} = \left(\frac{4}{3}\times\frac{4}{3}\right) = \frac{16}{9}.$$

$\therefore$ Required ratio $= 16 : 9$.

Ex. 21. ***The base of a parallelogram is twice its height. If the area of the parallelogram is 72 sq. cm, find its height.***

Sol. Let the height of the parallelogram be x cm. Then, base = $(2x)$ cm.

$\therefore \quad 2x \times x = 72 \Leftrightarrow 2x^2 = 72 \Leftrightarrow x^2 = 36 \Leftrightarrow x = 6.$

Hence, height of the parallelogram = 6 cm.

Ex. 22. ***Find the area of a rhombus one side of which measures 20 cm and one diagonal 24 cm.***

Sol. Let other diagonal = $2x$ cm.

Since diagonals of a rhombus bisect each other at right angles, we have :

$(20)^2 = (12)^2 + x^2 \Leftrightarrow x = \sqrt{(20)^2 - (12)^2} = \sqrt{256} = 16$ cm.

So, other diagonal = 32 cm.

$\therefore$ Area of rhombus $= \frac{1}{2} \times$ (Product of diagonals) $= \left(\frac{1}{2} \times 24 \times 32\right)$ cm^2 = 384 cm^2.

Ex. 23. ***The difference between two parallel sides of a trapezium is 4 cm. The perpendicular distance between them is 19 cm. If the area of the trapezium is 475 cm^2, find the lengths of the parallel sides.*** **(R.R.B. 2002)**

Sol. Let the two parallel sides of the trapezium be a cm and b cm.

Then, $a - b = 4$ ...(*i*)

And, $\frac{1}{2} \times (a + b) \times 19 = 475 \Leftrightarrow (a + b) = \left(\frac{475 \times 2}{19}\right) \Leftrightarrow a + b = 50$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $a = 27$, $b = 23$.

So, the two parallel sides are 27 cm and 23 cm.

Ex. 24. ***Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 sq. metres.*** **(M.A.T. 2003)**

Sol. Clearly, the cow will graze a circular field of area 9856 sq. metres and radius equal to the length of the rope.

Let the length of the rope be R metres.

Then, $\pi R^2 = 9856 \Leftrightarrow R^2 = \left(9856 \times \frac{7}{22}\right) = 3136 \Leftrightarrow R = 56.$

$\therefore$ Length of the rope = 56 m.

Ex. 25. ***The area of a circular field is 13.86 hectares. Find the cost of fencing it at the rate of Rs. 4.40 per metre.***

Sol. Area = (13.86×10000) m^2 = 138600 m^2.

$\pi R^2 = 138600 \Leftrightarrow R^2 = \left(138600 \times \frac{7}{22}\right) \Leftrightarrow R = 210$ m.

Circumference = $2\pi R = \left(2 \times \frac{22}{7} \times 210\right)$ m = 1320 m.

$\therefore$ Cost of fencing = Rs. (1320×4.40) = Rs. 5808.

Ex. 26. ***The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 kmph ?***

Sol. Distance to be covered in 1 min. $= \left(\frac{66 \times 1000}{60}\right)$ m = 1100 m.

Circumference of the wheel $= \left(2 \times \frac{22}{7} \times 0.70\right)$ m = 4.4 m.

$\therefore$ Number of revolutions per min. $= \left(\frac{1100}{4.4}\right) = 250.$

Ex. 27. ***A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.***

Sol. Distance covered in one revolution $= \left(\frac{88 \times 1000}{1000}\right)$ m $= 88$ m.

$\therefore \quad 2\pi R = 88 \Leftrightarrow 2 \times \frac{22}{7} \times R = 88 \Leftrightarrow R = \left(88 \times \frac{7}{44}\right) = 14$ m.

Ex. 28. ***The inner circumference of a circular race track, 14 m wide, is 440 m. Find the radius of the outer circle.***

Sol. Let inner radius be r metres. Then, $2\pi r = 440 \Rightarrow r = \left(440 \times \frac{7}{44}\right) = 70$ m.

$\therefore$ Radius of outer circle $= (70 + 14)$ m $= 84$ m.

Ex. 29. ***Two concentric circles form a ring. The inner and outer circumferences of the ring are $50\frac{2}{7}$ m and $75\frac{3}{7}$ m respectively. Find the width of the ring.***

Sol. Let the inner and outer radii be r and R metres.

Then, $2\pi r = \frac{352}{7} \Rightarrow r = \left(\frac{352}{7} \times \frac{7}{22} \times \frac{1}{2}\right) = 8$ m.

$2\pi R = \frac{528}{7} \Rightarrow R = \left(\frac{528}{7} \times \frac{7}{22} \times \frac{1}{2}\right) = 12$ m.

$\therefore$ Width of the ring $= (R - r) = (12 - 8)$ m $= 4$ m.

Ex. 30. ***A sector of 120°, cut out from a circle, has an area of $9\frac{3}{7}$ sq. cm. Find the radius of the circle.*** (C.B.I. 1997)

Sol. Let the radius of the circle be r cm. Then,

$\frac{\pi r^2 \theta}{360} = \frac{66}{7} \Leftrightarrow \frac{22}{7} \times r^2 \times \frac{120}{360} = \frac{66}{7} \Leftrightarrow r^2 = \left(\frac{66}{7} \times \frac{7}{22} \times 3\right) = 9 \Leftrightarrow r = 3.$

Hence, radius $= 3$ cm.

Ex. 31. ***Find the ratio of the areas of the incircle and circumcircle of a square.***

Sol. Let the side of the square be x. Then, its diagonal $= \sqrt{2}\,x$.

Radius of incircle $= \frac{x}{2}$ and

radius of circumcircle $= \frac{\sqrt{2}\,x}{2} = \frac{x}{\sqrt{2}}$.

$\therefore$ Required ratio $= \left(\frac{\pi x^2}{4} : \frac{\pi x^2}{2}\right) = \frac{1}{4} : \frac{1}{2} = 1 : 2.$

Ex. 32. ***If the radius of a circle is decreased by 50%, find the percentage decrease in its area.***

Sol. Let original radius $= R$. New radius $= \frac{50}{100} R = \frac{R}{2}$.

Original area $= \pi R^2$ and New area $= \pi \left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$.

$\therefore$ Decrease in area $= \left(\frac{3\pi R^2}{4} \times \frac{1}{\pi R^2} \times 100\right)\% = 75\%$.

EXERCISE 24A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The length of a room is 5.5 m and width is 3.75 m. Find the cost of paving the floor by slabs at the rate of Rs. 800 per sq. metre. **(IGNOU, 2003)**
(a) Rs. 15,000 (b) Rs. 15,550 (c) Rs. 15,600 (d) Rs. 16,500

2. The length of a rectangle is 18 cm and its breadth is 10 cm. When the length is increased to 25 cm, what will be the breadth of the rectangle if the area remains the same ?
(a) 7 cm (b) 7.1 cm (c) 7.2 cm (d) 7.3 cm

3. A rectangular plot measuring 90 metres by 50 metres is to be enclosed by wire fencing. If the poles of the fence are kept 5 metres apart, how many poles will be needed ?
(a) 55 (b) 56 (c) 57 (d) 58

4. The length of a rectangular plot is 60% more than its breadth. If the difference between the length and the breadth of that rectangle is 24 cm, what is the area of that rectangle ? **(Bank P.O. 1998)**
(a) 2400 sq. cm (b) 2480 sq. cm (c) 2560 sq. cm
(d) Data inadequate (e) None of these

5. A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet ? **(M.A.T. 2003)**
(a) 46 (b) 81 (c) 126 (d) 252

6. The difference between the length and breadth of a rectangle is 23 m. If its perimeter is 206 m, then its area is : **(Section Officers', 2003)**
(a) 1520 m^2 (b) 2420 m^2 (c) 2480 m^2 (d) 2520 m^2

7. The length of a rectangular plot is 20 metres more than its breadth. If the cost of fencing the plot @ Rs. 26.50 per metre is Rs. 5300, what is the length of the plot in metres ? **(Bank P.O. 1999)**
(a) 40 (b) 50 (c) 120
(d) Data inadequate (e) None of these

8. The breadth of a rectangular field is 60% of its length. If the perimeter of the field is 800 m, what is the area of the field ?
(a) 18750 sq. m (b) 37500 sq. m (c) 40000 sq. m (d) 48000 sq. m

9. The ratio between the length and the perimeter of a rectangular plot is 1 : 3. What is the ratio between the length and breadth of the plot ?
(a) 1 : 2 (b) 2 : 1 (c) 3 : 2 (d) Data inadequate

10. The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12 km / hr completes one round in 8 minutes, then the area of the park (in sq. m) is : **(S.S.C. 2003)**
(a) 15360 (b) 153600 (c) 30720 (d) 307200

11. The length of a rectangular hall is 5 m more than its breadth. The area of the hall is 750 m^2. The length of the hall is : **(S.S.C. 2004)**
(a) 15 m (b) 22.5 m (c) 25 m (d) 30 m

12. The area of a rectangle is 460 square metres. If the length is 15% more than the breadth, what is the breadth of the rectangular field ? **(Bank P.O. 2003)**
(a) 15 metres (b) 26 metres (c) 34.5 metres
(d) Cannot be determined (e) None of these

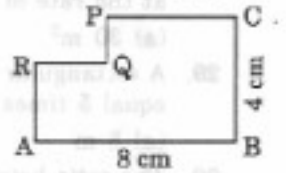
13. A rectangular field is to be fenced on three sides leaving a side of 20 feet uncovered. If the area of the field is 680 sq. feet, how many feet of fencing will be required ?
(a) 34 (b) 40 (c) 68 (d) 88
(R.R.B. 2002)

14. The ratio between the perimeter and the breadth of a rectangle is 5 : 1. If the area of the rectangle is 216 sq. cm, what is the length of the rectangle ?
(a) 16 cm (b) 18 cm (c) 24 cm
(d) Data inadequate (e) None of these **(B.S.R.B. 1998)**

15. A farmer wishes to start a 100 sq. m rectangular vegetable garden. Since he has only 30 m barbed wire, he fences three sides of the garden letting his house compound wall act as the fourth side fencing. The dimension of the garden is : **(R.R.B. 2003)**
(a) 15 m × 6.67 m (b) 20 m × 5 m (c) 30 m × 3.33 m (d) 40 m × 2.5 m

16. The sides of a rectangular field are in the ratio 3 : 4. If the area of the field is 7500 sq. m, the cost of fencing the field @ 25 paise per metre is : **(R.R.B. 2004)**
(a) Rs. 55.50 (b) Rs. 67.50 (c) Rs. 86.50 (d) Rs. 87.50

17. A rectangle of certain dimensions is chopped off from one corner of a larger rectangle as shown. AB = 8 cm and BC = 4 cm. The perimeter of the figure ABCPQRA (in cm) is : **(Asstt. Grade, 1998)**



(a) 24 (b) 28 (c) 36 (d) 48

18. A large field of 700 hectares is divided into two parts. The difference of the areas of the two parts is one-fifth of the average of the two areas. What is the area of the smaller part in hectares ?
(a) 225 (b) 280 (c) 300 (d) 315

19. A rectangular paper, when folded into two congruent parts had a perimeter of 34 cm for each part folded along one set of sides and the same is 38 cm when folded along the other set of sides. What is the area of the paper ? **(S.S.C. 2000)**
(a) 140 cm^2 (b) 240 cm^2 (c) 560 cm^2 (d) None of these

20. A rectangular plot is half as long again as it is broad and its area is $\frac{2}{3}$ hectares. Then, its length is :
(a) 100 m (b) 33.33 m (c) 66.66 m (d) $\frac{100\sqrt{3}}{3}$ m

21. A courtyard 25 m long and 16 m broad is to be paved with bricks of dimensions 20 cm by 10 cm. The total number of bricks required is :
(a) 18000 (b) 20000 (c) 25000 (d) None of these

22. The cost of carpeting a room 18 m long with a carpet 75 cm wide at Rs. 4.50 per metre is Rs. 810. The breadth of the room is :
(a) 7 m (b) 7.5 m (c) 8 m (d) 8.5 m

23. The diagonal of the floor of a rectangular closet is $7\frac{1}{2}$ feet. The shorter side of the closet is $4\frac{1}{2}$ feet. What is the area of the closet in square feet ? **(M.B.A. 2003)**
(a) $5\frac{1}{4}$ (b) $13\frac{1}{2}$ (c) 27 (d) 37

24. The length of a rectangle is three times of its width. If the length of the diagonal is $8\sqrt{10}$ cm, then the perimeter of the rectangle is : **(S.S.C. 2000)**

(a) $15\sqrt{10}$ cm (b) $16\sqrt{10}$ cm (c) $24\sqrt{10}$ cm (d) 64 cm

25. The diagonal of a rectangle is thrice its smaller side. The ratio of the length to the breadth of the rectangle is :

(a) 3 : 1 (b) $\sqrt{3}$: 1 (c) $\sqrt{2}$: 1 (d) $2\sqrt{2}$: 1

26. A rectangular carpet has an area of 120 sq. metres and a perimeter of 46 metres. The length of its diagonal is :

(a) 15 m (b) 16 m (c) 17 m (d) 20 m

27. The diagonal of a rectangle is $\sqrt{41}$ cm and its area is 20 sq. cm. The perimeter of the rectangle must be : **(Hotel Management, 2002)**

(a) 9 cm (b) 18 cm (c) 20 cm (d) 41 cm

28. A took 15 seconds to cross a rectangular field diagonally walking at the rate of 52 m / min and B took the same time to cross the same field along its sides walking at the rate of 68 m / min. The area of the field is : **(S.S.C. 2003)**

(a) 30 m^2 (b) 40 m^2 (c) 50 m^2 (d) 60 m^2

29. A rectangular carpet has an area of 60 sq. m. If its diagonal and longer side together equal 5 times the shorter side, the length of the carpet is :

(a) 5 m (b) 12 m (c) 13 m (d) 14.5 m

30. The ratio between the length and the breadth of a rectangular field is 3 : 2. If only the length is increased by 5 metres, the new area of the field will be 2600 sq. metres. What is the breadth of the rectangular field ?

(a) 40 metres (b) 60 metres (c) 65 metres
(d) Cannot be determined (e) None of these

31. The length of a blackboard is 8 cm more than its breadth. If the length is increased by 7 cm and breadth is decreased by 4 cm, the area remains the same. The length and breadth of the blackboard (in cm) will be :

(a) 28, 20 (b) 34, 26 (c) 40, 32 (d) 56, 48

32. If the length and breadth of a rectangular room are each increased by 1 m, then the area of floor is increased by 21 sq. m. If the length is increased by 1 m and breadth is decreased by 1 m, then the area is decreased by 5 sq. m. The perimeter of the floor is : **(M.B.A. 2002)**

(a) 30 m (b) 32 m (c) 36 m (d) 40 m

33. The percentage increase in the area of a rectangle, if each of its sides is increased by 20%, is : **(M.A.T. 2004)**

(a) 40% (b) 42% (c) 44% (d) 46%

34. A rectangle has width a and length b. If the width is decreased by 20% and the length is increased by 10%, then what is the area of the new rectangle in percentage compared to 'ab' ? **(R.R.B. 2002)**

(a) 80% (b) 88% (c) 110% (d) 120%

35. If the length and breadth of a rectangular plot be increased by 50% and 20% respectively, then how many times will its area be increased ? **(Bank P.O. 2003)**

(a) $1\frac{1}{3}$ (b) 2 (c) $3\frac{2}{5}$ (d) $4\frac{1}{5}$ (e) None of these

36. A towel, when bleached, was found to have lost 20% of its length and 10% of its breadth. The percentage of decrease in area is : **(N.I.F.T. 1997)**

(a) 10% (b) 10.08% (c) 20% (d) 28%

37. The length of a rectangle is halved, while its breadth is tripled. What is the percentage change in area ? (S.S.C. 2000)
(a) 25% increase (b) 50% increase (c) 50% decrease (d) 75% decrease

38. The length of a rectangle is decreased by r%, and the breadth is increased by $(r + 5)$%. Find r, if the area of the rectangle is unaltered. (SCMHRD, 2002)
(a) 5 (b) 8 (c) 10 (d) 15 (e) 20

39. The length of a rectangle is increased by 60%. By what percent would the width have to be decreased so as to maintain the same area ? (M.A.T. 2003)
(a) $37\frac{1}{2}$% (b) 60% (c) 75% (d) 120%

40. If the area of a rectangular plot increases by 30% while its breadth remains same, what will be the ratio of the areas of new and old figures ? (Bank P.O. 2003)
(a) 1 : 3 (b) 3 : 1 (c) 4 : 7 (d) 10 : 13 (e) None of these

41. A typist uses a sheet measuring 20 cm by 30 cm lengthwise. If a margin of 2 cm is left on each side and a 3 cm margin on top and bottom, then percent of the page used for typing is : (M.A.T. 1998)
(a) 40 (b) 60 (c) 64 (d) 72

42. A room is 15 feet long and 12 feet broad. A mat has to be placed on the floor of this room leaving $1\frac{1}{2}$ feet space from the walls. What will be the cost of the mat at the rate of Rs. 3.50 per square feet ? (R.R.B. 2002)
(a) Rs. 378 (b) Rs. 472.50 (c) Rs. 496 (d) Rs. 630

43. What will be the cost of gardening 1 metre broad boundary around a rectangular plot having perimeter of 340 metres at the rate of Rs. 10 per square metre ?
(a) Rs. 1700 (b) Rs. 3400 (c) Rs. 3440
(d) Cannot be determined (e) None of these (Bank P.O. 2003)

44. 2 metres broad pathway is to be constructed around a rectangular plot on the inside. The area of the plot is 96 sq. m. The rate of construction is Rs. 50 per square metre. Find the total cost of the construction. (S.B.I.P.O. 2000)
(a) Rs. 2400 (b) Rs. 4000 (c) Rs. 4800
(d) Data inadequate (e) None of these

45. Within a rectangular garden 10 m wide and 20 m long, we wish to pave a walk around the borders of uniform width so as to leave an area of 96 m^2 for flowers. How wide should the walk be ?
(a) 1 m (b) 2 m (c) 2.1 m (d) 2.5 m

46. A rectangular lawn 55 m by 35 m has two roads each 4 m wide running in the middle of it, one parallel to length and the other parallel to breadth. The cost of gravelling the roads at 75 paise per sq. metre is :
(a) Rs. 254.50 (b) Rs. 258 (c) Rs. 262.50 (d) Rs. 270

47. A rectangular park 60 m long and 40 m wide has two concrete crossroads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 sq. m, then what is the width of the road ? (M.A.T. 1997)
(a) 2.91 m (b) 3 m (c) 5.82 m (d) None of these

48. A housing society has been allotted a square piece of land measuring 2550.25 sq. m. What is the side of the plot ?
(a) 50.25 m (b) 50.5 m (c) 50.65 m (d) None of these

49. The cost of cultivating a square field at the rate of Rs. 135 per hectare is Rs. 1215. The cost of putting a fence around it at the rate of 75 paise per metre would be :
(a) Rs. 360 (b) Rs. 810 (c) Rs. 900 (d) Rs. 1800

50. The perimeters of five squares are 24 cm, 32 cm, 40 cm, 76 cm and 80 cm respectively. The perimeter of another square equal in area to the sum of the areas of these squares is : **(S.S.C. 2004)**

(*a*) 31 cm (*b*) 62 cm (*c*) 124 cm (*d*) 961 cm

51. The number of marble slabs of size 20 cm × 30 cm required to pave the floor of a square room of side 3 metres, is :

(*a*) 100 (*b*) 150 (*c*) 225 (*d*) 250

52. 50 square stone slabs of equal size were needed to cover a floor area of 72 sq. m. The length of each stone slab is : **(S.S.C. 2003)**

(*a*) 102 cm (*b*) 120 cm (*c*) 201 cm (*d*) 210 cm

53. The length and breadth of the floor of the room are 20 feet and 10 feet respectively. Square tiles of 2 feet length of different colours are to be laid on the floor. Black tiles are laid in the first row on all sides. If white tiles are laid in the one-third of the remaining and blue tiles in the rest, how many blue tiles will be there ?

(*a*) 16 (*b*) 24 (*c*) 32

(*d*) 48 (*e*) None of these **(S.B.I.P.O. 2000)**

54. What is the least number of square tiles required to pave the floor of a room 15 m 17 cm long and 9 m 2 cm broad ? **(S.S.C. 2003)**

(*a*) 814 (*b*) 820 (*c*) 840 (*d*) 844

55. A rectangular room can be partitioned into two equal square rooms by a partition 7 metres long. What is the area of the rectangular room in square metres ?

(*a*) 49 (*b*) 147 (*c*) 196 (*d*) None of these

56. The perimeter of a square is 48 cm. The area of a rectangle is 4 cm^2 less than the area of the square. If the length of the rectangle is 14 cm, then its perimeter is :

(*a*) 24 cm (*b*) 48 cm (*c*) 50 cm (*d*) 54 cm **(S.S.C. 2002)**

57. The area of a rectangle is thrice that of a square. If the length of the rectangle is 40 cm and its breadth is $\frac{3}{2}$ times that of the side of the square, then the side of the square is :

(*a*) 15 cm (*b*) 20 cm (*c*) 30 cm (*d*) 60 cm

58. If the perimeter of a rectangle and a square, each is equal to 80 cm and the difference of their areas is 100 sq. cm, the sides of the rectangle are :

(*a*) 25 cm, 15 cm (*b*) 28 cm, 12 cm (*c*) 30 cm, 10 cm (*d*) 35 cm, 15 cm

59. The cost of fencing a square field @ Rs. 20 per metre is Rs. 10,080. How much will it cost to lay a three metre wide pavement along the fencing inside the field @ Rs. 50 per sq. metre ?

(*a*) Rs. 37,350 (*b*) Rs. 73,800 (*c*) Rs. 77,400 (*d*) None of these

60. A park square in shape has a 3 metre wide road inside it running along its sides. The area occupied by the road is 1764 square metres. What is the perimeter along the outer edge of the road ? **(Bank P.O. 1998)**

(*a*) 576 metres (*b*) 600 metres (*c*) 640 metres

(*d*) Data inadequate (*e*) None of these

61. A man walked diagonally across a square lot. Approximately, what was the percent saved by not walking along the edges ? **(M.B.A. 2003)**

(*a*) 20 (*b*) 24 (*c*) 30 (*d*) 33

62. A man walking at the speed of 4 kmph crosses a square field diagonally in 3 minutes. The area of the field is :

(*a*) 18000 m^2 (*b*) 19000 m^2 (*c*) 20000 m^2 (*d*) 25000 m^2

63. If the length of the diagonal of a square is 20 cm, then its perimeter must be :
(a) $10\sqrt{2}$ cm (b) 40 cm (c) $40\sqrt{2}$ cm (d) 200 cm
(R.R.B. 2003)

64. The area of a square field is 69696 cm^2. Its diagonal will be equal to :
(a) 313.296 m (b) 353.296 m (c) 373.296 m (d) 393.296 m
(S.S.C. 1999)

65. What will be the length of the diagonal of that square plot whose area is equal to the area of a rectangular plot of length 45 metres and breadth 40 metres ?
(a) 42.5 metres (b) 60 metres (c) 75 metres
(d) Data inadequate (e) None of these (Bank P.O. 1999)

66. The length of a rectangle is 20% more than its breadth. What will be the ratio of the area of a rectangle to that of a square whose side is equal to the breadth of the rectangle ? (Bank P.O. 2000)
(a) 2 : 1 (b) 5 : 6 (c) 6 : 5
(d) Data inadequate (e) None of these

67. A square and a rectangle have equal areas. If their perimeters are p_1 and p_2 respectively, then :
(a) $p_1 < p_2$ (b) $p_1 = p_2$ (c) $p_1 > p_2$ (d) None of these

68. If the perimeters of a square and a rectangle are the same, then the area A and B enclosed by them would satisfy the condition :
(a) $A < B$ (b) $A \leq B$ (c) $A > B$ (d) $A \geq B$

69. The diagonal of a square is $4\sqrt{2}$ cm. The diagonal of another square whose area is double that of the first square, is : (S.S.C. 2002)
(a) 8 cm (b) $8\sqrt{2}$ cm (c) $4\sqrt{2}$ cm (d) 16 cm

70. The ratio of the area of a square to that of the square drawn on its diagonal, is :
(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 4 : 5
(IGNOU, 2003)

71. The ratio of the areas of two squares, one having its diagonal double than the other, is :
(a) 2 : 1 (b) 2 : 3 (c) 3 : 1 (d) 4 : 1

72. If the ratio of areas of two squares is 225 : 256, then the ratio of their perimeters is :
(a) 225 : 256 (b) 256 : 225 (c) 15 : 16 (d) 16 : 15
(S.S.C. 2004)

73. Of the two square fields, the area of one is 1 hectare while the other one is broader by 1%. The difference in their areas is :
(a) 100 m^2 (b) 101 m^2 (c) 200 m^2 (d) 201 m^2

74. If each side of a square is increased by 50%, the ratio of the area of the resulting square to that of the given square is :
(a) 4 : 5 (b) 5 : 4 (c) 4 : 9 (d) 9 : 4

75. What happens to the area of a square when its side is halved ? Its area will :
(a) remain same (b) become half (c) become one-fourth (d) become double
(R.R.B. 2003)

76. An error of 2% in excess is made while measuring the side of a square. The percentage of error in the calculated area of the square is : (C.D.S. 2003)
(a) 2% (b) 2.02% (c) 4% (d) 4.04%

77. If the area of a square increases by 69%, then the side of the square increases by :
(a) 13% (b) 30% (c) 39% (d) 69%
(M.A.T. 1998)

78. If the diagonal of a square is made 1.5 times, then the ratio of the areas of two squares is :
(a) 4 : 3 (b) 4 : 5 (c) 4 : 7 (d) 4 : 9

79. The length and breadth of a square are increased by 40% and 30% respectively. The area of the resulting rectangle exceeds the area of the square by :
(a) 35% (b) 42% (c) 62% (d) 82%

80. The length of one pair of opposite sides of a square is increased by 5 cm on each side; the ratio of the length and the breadth of the newly formed rectangle becomes 3 : 2. What is the area of the original square ? **(Bank P.O. 1999)**
(a) 25 sq. cm (b) 81 sq. cm (c) 100 sq. cm
(d) 225 sq. cm (e) None of these

81. If the side of a square is increased by 5 cm, the area increases by 165 sq. cm. The side of the square is :
(a) 12 cm (b) 13 cm (c) 14 cm (d) 15 cm

82. The difference of the areas of two squares drawn on two line segments of different lengths is 32 sq. cm. Find the length of the greater line segment if one is longer than the other by 2 cm. **(S.S.C. 2003)**
(a) 7 cm (b) 9 cm (c) 11 cm (d) 16 cm

83. The areas of a square and a rectangle are equal. The length of the rectangle is greater than the length of any side of the square by 5 cm and the breadth is less by 3 cm. Find the perimeter of the rectangle. **(S.S.C. 2002)**
(a) 17 cm (b) 26 cm (c) 30 cm (d) 34 cm

84. A tank is 25 m long, 12 m wide and 6 m deep. The cost of plastering its walls and bottom at 75 paise per sq. m, is : **(C.B.I. 1997)**
(a) Rs. 456 (b) Rs. 458 (c) Rs. 558 (d) Rs. 568

85. The dimensions of a room are 10 m × 7 m × 5 m. There are 2 doors and 3 windows in the room. The dimensions of the doors are 1 m × 3 m. One window is of size 2 m × 1.5 m and the other two windows are of size 1 m × 1.5 m. The cost of painting the walls at Rs. 3 per m^2 is :
(a) Rs. 474 (b) Rs. 578.50 (c) Rs. 684 (d) Rs. 894

86. The cost of papering the four walls of a room is Rs. 475. Each one of the length, breadth and height of another room is double that of this room. The cost of papering the walls of this new room is :
(a) Rs. 712.50 (b) Rs. 950 (c) Rs. 1425 (d) Rs. 1900

87. The ratio of height of a room to its semi-perimeter is 2 : 5. It costs Rs. 260 to paper the walls of the room with paper 50 cm wide at Rs. 2 per metre allowing an area of 15 sq. m for doors and windows. The height of the room is :
(a) 2.6 m (b) 3.9 m (c) 4 m (d) 4.2 m

88. The base of a triangle is 15 cm and height is 12 cm. The height of another triangle of double the area having the base 20 cm is : **(S.S.C. 2002)**
(a) 8 cm (b) 9 cm (c) 12.5 cm (d) 18 cm

89. ABC is a triangle with base AB. D is a point on AB such that AB = 5 and DB = 3. What is the ratio of the area of Δ ADC to the area of Δ ABC ? **(S.S.C. 2000)**
(a) 2 : 3 (b) 3 : 2 (c) 2 : 5 (d) 3 : 5

90. The area of a right-angled triangle is 40 times its base. What is its height ?
(a) 45 cm (b) 60 cm (c) 80 cm
(d) Data inadequate (e) None of these **(B.S.R.B. 1998)**

91. If the area of a triangle is 1176 cm^2 and base : corresponding altitude is 3 : 4, then the altitude of the triangle is : **(S.S.C. 2000)**
(a) 42 cm (b) 52 cm (c) 54 cm (d) 56 cm

92. The three sides of a triangle are 5 cm, 12 cm and 13 cm respectively. Then, its area is :
(a) $10\sqrt{3}$ cm^2 (b) $10\sqrt{6}$ cm^2 (c) 20 cm^2 (d) 30 cm^2

93. The sides of a triangle are in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. If the perimeter is 52 cm, then the length of the smallest side is : **(M.A.T. 2004)**
(a) 9 cm (b) 10 cm (c) 11 cm (d) 12 cm

94. The area of a triangle is 216 cm^2 and its sides are in the ratio 3 : 4 : 5. The perimeter of the triangle is : **(S.S.C. 2004)**
(a) 6 cm (b) 12 cm (c) 36 cm (d) 72 cm

95. The sides of a triangle are 3 cm, 4 cm and 5 cm. The area (in cm^2) of the triangle formed by joining the mid-points of the sides of this triangle is : **(S.S.C. 2003)**
(a) $\frac{3}{4}$ (b) $\frac{3}{2}$ (c) 3 (d) 6

96. One side of a right-angled triangle is twice the other, and the hypotenuse is 10 cm. The area of the triangle is :
(a) 20 cm^2 (b) $33\frac{1}{3}$ cm^2 (c) 40 cm^2 (d) 50 cm^2

97. The perimeter of a right-angled triangle is 60 cm. Its hypotenuse is 26 cm. The area of the triangle is : **(M.B.A. 2002)**
(a) 120 cm^2 (b) 240 cm^2 (c) 390 cm^2 (d) 780 cm^2

98. If the perimeter of an isosceles right triangle is $(6 + 3\sqrt{2})$ m, then the area of the triangle is : **(M.A.T. 2003)**
(a) 4.5 m^2 (b) 5.4 m^2 (c) 9 m^2 (d) 81 m^2

99. The perimeter of a triangle is 30 cm and its area is 30 cm^2. If the largest side measures 13 cm, then what is the length of the smallest side of the triangle ?
(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm
(S.S.C. 2003)

100. If the area of an equilateral triangle is $24\sqrt{3}$ sq. cm, then its perimeter is :
(a) $2\sqrt{6}$ cm (b) $4\sqrt{6}$ cm (c) $12\sqrt{6}$ cm (d) 96 cm

101. The height of an equilateral triangle is 10 cm. Its area is : **(S.S.C. 2003)**
(a) $\frac{100}{3}$ cm^2 (b) 30 cm^2 (c) 100 cm^2 (d) $\frac{100}{\sqrt{3}}$ cm^2

102. From a point in the interior of an equilateral triangle, the perpendicular distance of the sides are $\sqrt{3}$ cm, $2\sqrt{3}$ cm and $5\sqrt{3}$ cm. The perimeter (in cm) of the triangle is :
(a) 24 (b) 32 (c) 48 (d) 64

103. If x is the length of a median of an equilateral triangle, then its area is :
(a) x^2 (b) $\frac{1}{2}x^2$ (c) $\frac{\sqrt{3}}{2}x^2$ (d) $\frac{\sqrt{3}}{3}x^2$

104. If the area of a square with side a is equal to the area of a triangle with base a, then the altitude of the triangle is : **(B.S.F. 2001)**
(a) $\frac{a}{2}$ (b) a (c) $2a$ (d) $4a$

105. An equilateral triangle is described on the diagonal of a square. What is the ratio of the area of the triangle to that of the square ? **(S.S.C. 2002)**
(a) 2 : $\sqrt{3}$ (b) 4 : $\sqrt{3}$ (c) $\sqrt{3}$: 2 (d) $\sqrt{3}$: 4

106. What will be the ratio between the area of a rectangle and the area of a triangle with one of the sides of the rectangle as base and a vertex on the opposite side of the rectangle ? (S.B.I.P.O. 1999)

(a) 1 : 2 (b) 2 : 1 (c) 3 : 1
(d) Data inadequate (e) None of these

107. If an equilateral triangle of area X and a square of area Y have the same perimeter, then X is : (C.D.S. 2003)

(a) equal to Y (b) greater than Y
(c) less than Y (d) less than or equal to Y

108. A square and an equilateral triangle have equal perimeters. If the diagonal of the square is $12\sqrt{2}$ cm, then the area of the triangle is :

(a) $24\sqrt{2}$ cm^2 (b) $24\sqrt{3}$ cm^2 (c) $48\sqrt{3}$ cm^2 (d) $64\sqrt{3}$ cm^2

109. The ratio of bases of two triangles is $x : y$ and that of their areas is $a : b$. Then the ratio of their corresponding altitudes will be : (S.S.C. 2004)

(a) $ax : by$ (b) $\frac{a}{x} : \frac{b}{y}$ (c) $ay : bx$ (d) $\frac{x}{a} : \frac{b}{y}$

110. If the side of an equilateral triangle is decreased by 20%, its area is decreased by :

(a) 36% (b) 40% (c) 60% (d) 64%
(C.B.I. 1997)

111. If the height of a triangle is decreased by 40% and its base is increased by 40%, what will be the effect on its area ? (S.B.I.P.O. 2000)

(a) No change (b) 8% decrease (c) 16% decrease
(d) 16% increase (e) None of these

112. If every side of a triangle is doubled, the area of the new triangle is K times the area of the old one. K is equal to : (R.R.B. 2003)

(a) $\sqrt{2}$ (b) 2 (c) 3 (d) 4

113. One side of a parallelogram is 18 cm and its distance from the opposite side is 8 cm. The area of the parallelogram is :

(a) 48 cm^2 (b) 72 cm^2 (c) 100 cm^2 (d) 144 cm^2

114. A parallelogram has sides 30 m and 14 m and one of its diagonals is 40 m long. Then, its area is :

(a) 168 m^2 (b) 336 m^2 (c) 372 m^2 (d) 480 m^2

115. One diagonal of a parallelogram is 70 cm and the perpendicular distance of this diagonal from either of the outlying vertices is 27 cm. The area of the parallelogram (in sq. cm) is :

(a) 1800 (b) 1836 (c) 1890 (d) 1980

116. A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is : (M.A.T. 2003)

(a) $10\sqrt{2}$ m (b) 100 m (c) $100\sqrt{2}$ m (d) 200 m

117. If a parallelogram with area P, a rectangle with area R and a triangle with area T are all constructed on the same base and all have the same altitude, then which of the following statements is false ?

(a) P = R (b) P + T = 2R (c) P = 2T (d) T = (1 / 2) R

118. The area of a rhombus is 150 cm^2. The length of one of its diagonals is 10 cm. The length of the other diagonal is : (S.S.C. 2004)

(a) 25 cm (b) 30 cm (c) 35 cm (d) 40 cm

119. One of the diagonals of a rhombus is double the other diagonal. Its area is 25 sq. cm. The sum of the diagonals is : **(S.S.C. 2003)**
(*a*) 10 cm (*b*) 12 cm (*c*) 15 cm (*d*) 16 cm

120. The perimeter of a rhombus is 56 m and its height is 5 m. Its area is :
(*a*) 64 sq. m (*b*) 70 sq. m (*c*) 78 sq. m (*d*) 84 sq. m

121. If the diagonals of a rhombus are 24 cm and 10 cm, the area and the perimeter of the rhombus are respectively :
(*a*) 120 cm^2, 52 cm (*b*) 120 cm^2, 64 cm (*c*) 240 cm^2, 52 cm (*d*) 240 cm^2, 64 cm

122. Each side of a rhombus is 26 cm and one of its diagonals is 48 cm long. The area of the rhombus is : **(R.R.B. 2003)**
(*a*) 240 cm^2 (*b*) 300 cm^2 (*c*) 360 cm^2 (*d*) 480 cm^2

123. The length of one diagonal of a rhombus is 80% of the other diagonal. The area of the rhombus is how many times the square of the length of the other diagonal ?
(*a*) $\frac{4}{5}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{3}{4}$ (*d*) $\frac{1}{4}$

124. If a square and a rhombus stand on the same base, then the ratio of the areas of the square and the rhombus is :
(*a*) greater than 1 (*b*) equal to 1 (*c*) equal to $\frac{1}{2}$ (*d*) equal to $\frac{1}{4}$

125. The two parallel sides of a trapezium are 1.5 m and 2.5 m respectively. If the perpendicular distance between them is 6.5 metres, the area of the trapezium is :
(*a*) 10 m^2 (*b*) 13 m^2 (*c*) 20 m^2 (*d*) 26 m^2

126. The area of a field in the shape of a trapezium measures 1440 m^2. The perpendicular distance between its parallel sides is 24 m. If the ratio of the parallel sides is 5 : 3, the length of the longer parallel side is : **(S.S.C. 2004)**
(*a*) 45 m (*b*) 60 m (*c*) 75 m (*d*) 120 m

127. The cross-section of a canal is trapezium in shape. The canal is 12 m wide at the top and 8 m wide at the bottom. If the area of the cross-section is 840 sq. m, the depth of the canal is :
(*a*) 8.75 m (*b*) 42 m (*c*) 63 m (*d*) 84 m

128. The area of a circle of radius 5 is numerically what percent of its circumference ?
(*a*) 200 (*b*) 225 (*c*) 240 (*d*) 250
(S.S.C. 2000)

129. A man runs round a circular field of radius 50 m at the speed of 12 km / hr. What is the time taken by the man to take twenty rounds of the field ? **(M.A.T. 1997)**
(*a*) 30 min. (*b*) 32 min. (*c*) 34 min. (*d*) None of these

130. A cow is tethered in the middle of a field with a 14 feet long rope. If the cow grazes 100 sq. ft. per day, then approximately what time will be taken by the cow to graze the whole field ? **(Bank P.O. 2003)**
(*a*) 2 days (*b*) 6 days (*c*) 18 days
(*d*) 24 days (*e*) None of these

131. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. What is the area of the circle ? **(Bank P.O. 2004)**
(*a*) 88 cm^2 (*b*) 154 cm^2 (*c*) 1250 cm^2
(*d*) Cannot be determined (*e*) None of these

132. The circumference of a circle, whose area is 24.64 m^2, is : **(R.R.B. 2003)**
(*a*) 14.64 m (*b*) 16.36 m (*c*) 17.60 m (*d*) 18.40 m

133. If the circumference and the area of a circle are numerically equal, then the diameter is equal to : **(S.S.C. 2000)**
(*a*) $\frac{\pi}{2}$ (*b*) 2π (*c*) 2 (*d*) 4

134. The difference between the circumference and the radius of a circle is 37 cm. The area of the circle is : **(Section Officers', 2001)**

(*a*) 111 cm^2 (*b*) 148 cm^2 (*c*) 154 cm^2 (*d*) 259 cm^2

135. The sum of areas of two circles A and B is equal to the area of a third circle C whose diameter is 30 cm. If the diameter of circle A is 18 cm, then the radius of circle B is :

(*a*) 10 cm (*b*) 12 cm (*c*) 15 cm (*d*) 18 cm

136. Between a square of perimeter 44 cm and a circle of circumference 44 cm, which figure has larger area and by how much ? **(S.S.C. 2000)**

(*a*) Both have equal area (*b*) Square, 33 cm^2

(*c*) Circle, 33 cm^2 (*d*) Square, 495 cm^2

137. A wire can be bent in the form of a circle of radius 56 cm. If it is bent in the form of a square, then its area will be : **(R.R.B. 2002)**

(*a*) 3520 cm^2 (*b*) 6400 cm^2 (*c*) 7744 cm^2 (*d*) 8800 cm^2

138. A wire when bent in the form of a square encloses an area of 484 sq. cm. What will be the enclosed area when the same wire is bent into the form of a circle ?

(*a*) 462 sq. cm (*b*) 539 sq. cm (*c*) 616 sq. cm (*d*) 693 sq. cm **(S.S.C. 2002)**

139. A circular wire of radius 42 cm is bent in the form of a rectangle whose sides are in the ratio of 6 : 5. The smaller side of the rectangle is : **(S.S.C. 2004)**

(*a*) 25 cm (*b*) 30 cm (*c*) 36 cm (*d*) 60 cm

140. There is a rectangular tank of length 180 m and breadth 120 m in a circular field. If the area of the land portion of the field is 40000 m^2, what is the radius of the field ?

(*a*) 130 m (*b*) 135 m (*c*) 140 m (*d*) 145 m

141. The areas of two circular fields are in the ratio 16 : 49. If the radius of the latter is 14 m, then what is the radius of the former ? **(IGNOU, 2003)**

(*a*) 4 m (*b*) 8 m (*c*) 18 m (*d*) 32 m

142. If the ratio of areas of two circles is 4 : 9, then the ratio of their circumferences will be : **(R.R.B. 2003)**

(*a*) 2 : 3 (*b*) 3 : 2 (*c*) 4 : 9 (*d*) 9 : 4

143. The perimeter of a circle is equal to the perimeter of a square. Then, their areas are in the ratio :

(*a*) 4 : 1 (*b*) 11 : 7 (*c*) 14 : 11 (*d*) 22 : 7

144. The diameter of a wheel is 1.26 m. How far will it travel in 500 revolutions ?

(*a*) 1492 m (*b*) 1980 m (*c*) 2530 m (*d*) 2880 m

145. The number of revolutions a wheel of diameter 40 cm makes in travelling a distance of 176 m, is : **(S.S.C. 2003)**

(*a*) 140 (*b*) 150 (*c*) 160 (*d*) 166

146. The radius of a wheel is 0.25 m. The number of revolutions it will make to travel a distance of 11 km will be : **(R.R.B. 2003)**

(*a*) 2800 (*b*) 4000 (*c*) 5500 (*d*) 7000

147. The wheel of an engine, $7\frac{1}{2}$ metres in circumference makes 7 revolutions in 9 seconds. The speed of the train in km per hour is :

(*a*) 130 (*b*) 132 (*c*) 135 (*d*) 150

148. The wheel of a motorcycle, 70 cm in diameter, makes 40 revolutions in every 10 seconds. What is the speed of the motorcycle in km / hr ? **(R.R.B. 2002)**

(*a*) 22.32 (*b*) 27.68 (*c*) 31.68 (*d*) 36.24

149. Wheels of diameters 7 cm and 14 cm start rolling simultaneously from X and Y, which

are 1980 cm apart, towards each other in opposite directions. Both of them make the same number of revolutions per second. If both of them meet after 10 seconds, the speed of the smaller wheel is : **(M.A.T. 2003)**

(*a*) 22 cm / sec (*b*) 44 cm / sec (*c*) 66 cm / sec (*d*) 132 cm / sec

150. A toothed wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 15 revolutions ?

(*a*) 18 (*b*) 20 (*c*) 25 (*d*) 30

151. Find the diameter of a wheel that makes 113 revolutions to go 2 km 26 decametres.

(*a*) $4\frac{4}{13}$ m (*b*) $6\frac{4}{11}$ m (*c*) $12\frac{4}{11}$ m (*d*) $12\frac{8}{11}$ m

(S.S.C. 2003)

152. The front wheels of a wagon are 2π feet in circumference and the rear wheels are 3π feet in circumference. When the front wheels have made 10 more revolutions than the rear wheels, how many feet has the wagon travelled ? **(M.B.A. 2003)**

(*a*) 30π (*b*) 60π (*c*) 90π (*d*) 150π

153. A circular ground whose diameter is 35 metres, has a 1.4 m broad garden around it. What is the area of the garden in square metres ? **(S.B.I.P.O. 1999)**

(*a*) 160.16 (*b*) 176.16 (*c*) 196.16

(*d*) Data inadequate (*e*) None of these

154. A circular garden has a circumference of 440 m. There is a 7 m wide border inside the garden along its periphery. The area of the border is :

(*a*) 2918 m^2 (*b*) 2921 m^2 (*c*) 2924 m^2 (*d*) 2926 m^2

155. The areas of two concentric circles forming a ring are 154 sq. cm and 616 sq. cm. The breadth of the ring is :

(*a*) 7 cm (*b*) 14 cm (*c*) 21 cm (*d*) 28 cm

156. A circular park has a path of uniform width around it. The difference between outer and inner circumferences of the circular path is 132 m. Its width is : **(S.S.C. 2003)**

(*a*) 20 m (*b*) 21 m (*c*) 22 m (*d*) 24 m

157. A circular swimming pool is surrounded by a concrete wall 4 ft. wide. If the area of the concrete wall surrounding the pool is $\frac{11}{25}$ that of the pool, then the radius of the pool is : **(Assistant Grade, 1998)**

(*a*) 8 ft (*b*) 16 ft (*c*) 20 ft (*d*) 30 ft

158. The ratio of the outer and the inner perimeters of a circular path is 23 : 22. If the path is 5 metres wide, the diameter of the inner circle is : **(S.S.C. 2004)**

(*a*) 55 m (*b*) 110 m (*c*) 220 m (*d*) 230 m

159. What will be the area of a semi-circle of 14 m diameter ? **(NABARD, 2002)**

(*a*) 22 m^2 (*b*) 77 m^2 (*c*) 154 m^2 (*d*) 308 m^2 (*e*) None of these

160. A semi-circular shaped window has diameter of 63 cm. Its perimeter equals :

(*a*) 126 cm (*b*) 162 cm (*c*) 198 cm (*d*) 251 cm

(S.S.C. 1999)

161. What will be the area of a semi-circle whose perimeter is 36 cm ?

(*a*) 154 cm^2 (*b*) 168 cm^2 (*c*) 308 cm^2

(*d*) Data inadequate (*e*) None of these **(B.S.R.B. 1998)**

162. If a wire is bent into the shape of a square, then the area of the square is 81 sq. cm. When the wire is bent into a semi-circular shape, then the area of the semi-circle will be : **(S.S.C. 2002)**

(*a*) 22 cm^2 (*b*) 44 cm^2 (*c*) 77 cm^2 (*d*) 154 cm^2

163. The area of a sector of a circle of radius 5 cm, formed by an arc of length 3.5 cm, is :
(a) 7.5 cm^2 (b) 7.75 cm^2 (c) 8.5 cm^2 (d) 8.75 cm^2
(S.S.C. 1999)

164. In a circle of radius 7 cm, an arc subtends an angle of 108° at the centre. The area of the sector is :
(a) 43.2 cm^2 (b) 44.2 cm^2 (c) 45.2 cm^2 (d) 46.2 cm^2

165. The area of the greatest circle which can be inscribed in a square whose perimeter is 120 cm, is : **(S.S.C. 2004)**
(a) $\frac{22}{7}\times\left(\frac{7}{2}\right)^2$ cm^2 (b) $\frac{22}{7}\times\left(\frac{9}{2}\right)^2$ cm^2
(c) $\frac{22}{7}\times\left(\frac{15}{2}\right)^2$ cm^2 (d) $\frac{22}{7}\times(15)^2$ cm^2

166. The area of the largest circle, that can be drawn inside a rectangle with sides 18 cm by 14 cm, is : **(S.S.C. 2000)**
(a) 49 cm^2 (b) 154 cm^2 (c) 378 cm^2 (d) 1078 cm^2

167. The area of a circle is 220 sq. cm. The area of a square inscribed in this circle will be : **(C.B.I. 1997)**
(a) 49 cm^2 (b) 70 cm^2 (c) 140 cm^2 (d) 150 cm^2

168. A square is inscribed in a circle whose radius is 4 cm. The area of the portion between the circle and the square is :
(a) $(8\pi - 16)$ (b) $(8\pi - 32)$ (c) $(16\pi - 16)$ (d) $(16\pi - 32)$

169. The circumference of a circle is 100 cm. The side of a square inscribed in the circle is : **(C.B.I. 2003)**
(a) $50\sqrt{2}$ cm (b) $\frac{100}{\pi}$ cm (c) $\frac{50\sqrt{2}}{\pi}$ cm (d) $\frac{100\sqrt{2}}{\pi}$ cm

170. Four equal sized maximum circular plates are cut off from a square paper sheet of area 784 cm^2. The circumference of each plate is : **(S.S.C. 2003)**
(a) 22 cm (b) 44 cm (c) 66 cm (d) 88 cm

171. There are 4 semi-circular gardens on each side of a square-shaped pond with each side 21 m. The cost of fencing the entire plot at the rate of Rs 12.50 per metre is :
(a) Rs. 1560 (b) Rs. 1650 (c) Rs. 3120 (d) Rs. 3300

172. The ratio of the areas of the incircle and circumcircle of an equilateral triangle is :
(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 9

173. The radius of the circumcircle of an equilateral triangle of side 12 cm is :
(a) $\frac{4\sqrt{2}}{3}$ cm (b) $4\sqrt{2}$ cm (c) $\frac{4\sqrt{3}}{3}$ cm (d) $4\sqrt{3}$ cm

174. The area of the incircle of an equilateral triangle of side 42 cm is : **(S.S.C. 2004)**
(a) $22\sqrt{3}$ cm^2 (b) 231 cm^2 (c) 462 cm^2 (d) 924 cm^2

175. The area of a circle inscribed in an equilateral triangle is 154 cm^2. Find the perimeter of the triangle.
(a) 71.5 cm (b) 71.7 cm (c) 72.3 cm (d) 72.7 cm

176. The sides of a triangle are 6 cm, 11 cm and 15 cm. The radius of its incircle is :
(a) $3\sqrt{2}$ cm (b) $\frac{4\sqrt{2}}{5}$ cm (c) $\frac{5\sqrt{2}}{4}$ cm (d) $6\sqrt{2}$ cm
(M.A.T. 2001)

177. The perimeter of a triangle is 30 cm and the circumference of its incircle is 88 cm. The area of the triangle is : **(S.S.C. 2003)**
(*a*) 70 cm^2 (*b*) 140 cm^2 (*c*) 210 cm^2 (*d*) 420 cm^2

178. If in a triangle, the area is numerically equal to the perimeter, then the radius of the inscribed circle of the triangle is : **(S.S.C. 2000)**
(*a*) 1 (*b*) 1.5 (*c*) 2 (*d*) 3

179. An equilateral triangle, a square and a circle have equal perimeters. If T denotes the area of the triangle, S, the area of the square and C, the area of the circle, then :
(*a*) S < T < C (*b*) T < C < S (*c*) T < S < C (*d*) C < S < T
(C.D.S. 2003)

180. If an area enclosed by a circle or a square or an equilateral triangle is the same, then the maximum perimeter is possessed by : **(S.C.R.A. 1997)**
(*a*) circle (*b*) square (*c*) equilateral triangle
(*d*) triangle and square have equal perimeters greater than that of circle

181. The area of the largest triangle that can be inscribed in a semi-circle of radius *r*, is :
(*a*) r^2 (*b*) $2r^2$ (*c*) r^3 (*d*) $2r^3$
(Section Officers', 2001)

182. ABC is a right-angled triangle with right angle at B. If the semi-circle on AB with AB as diameter encloses an area of 81 sq. cm and the semi-circle on BC with BC as diameter encloses an area of 36 sq. cm, then the area of the semi-circle on AC with AC as diameter will be :
(*a*) 117 cm^2 (*b*) 121 cm^2 (*c*) 217 cm^2 (*d*) 221 cm^2

183. If the radius of a circle is increased by 75%, then its circumference will increase by :
(*a*) 25% (*b*) 50% (*c*) 75% (*d*) 100%
(C.D.S. 2003)

184. A can go round a circular path 8 times in 40 minutes. If the diameter of the circle is increased to 10 times the original diameter, then the time required by A to go round the new path once, travelling at the same speed as before, is : **(S.S.C. 2000)**
(*a*) 20 min. (*b*) 25 min. (*c*) 50 min. (*d*) 100 min.

185. If the radius of a circle is increased by 6%, then the area is increased by :
(*a*) 6% (*b*) 12% (*c*) 12.36% (*d*) 16.64%
(D.M.R.C. 2003)

186. If the radius of a circle is diminished by 10%, then its area is diminished by :
(*a*) 10% (*b*) 19% (*c*) 20% (*d*) 36%
(Hotel Management, 2003)

187. If the radius of a circle is doubled, its area is increased by : **(C.B.I. 1998)**
(*a*) 100% (*b*) 200% (*c*) 300% (*d*) 400%.

188. If the circumference of a circle increases from 4π to 8π, what change occurs in its area ?
(*a*) It is halved. (*b*) It doubles. (*c*) It triples. (*d*) It quadruples.
(S.S.C. 2000)

189. Three circles of radius 3.5 cm are placed in such a way that each circle touches the other two. The area of the portion enclosed by the circles is : **(S.S.C. 2003)**
(*a*) 1.967 cm^2 (*b*) 1.975 cm^2 (*c*) 19.67 cm^2 (*d*) 21.21 cm^2

190. Four circular cardboard pieces, each of radius 7 cm are placed in such a way that each piece touches two other pieces. The area of the space enclosed by the four pieces is :
(*a*) 21 cm^2 (*b*) 42 cm^2 (*c*) 84 cm^2 (*d*) 168 cm^2

191. Four horses are tethered at four corners of a square plot of side 63 metres so that they just cannot reach one another. The area left ungrazed is :
(*a*) 675.5 m^2 (*b*) 780.6 m^2 (*c*) 785.8 m^2 (*d*) 850.5 m^2

ANSWERS

1. (d)	2. (c)	3. (b)	4. (c)	5. (c)	6. (d)	7. (e)	8. (b)	9. (b)
10. (b)	11. (d)	12. (e)	13. (d)	14. (b)	15. (b)	16. (d)	17. (a)	18. (d)
19. (a)	20. (a)	21. (b)	22. (b)	23. (c)	24. (d)	25. (d)	26. (c)	27. (b)
28. (d)	29. (b)	30. (a)	31. (a)	32. (d)	33. (c)	34. (b)	35. (e)	36. (d)
37. (b)	38. (e)	39. (a)	40. (e)	41. (c)	42. (a)	43. (c)	44. (d)	45. (b)
46. (b)	47. (b)	48. (b)	49. (e)	50. (c)	51. (c)	52. (b)	53. (a)	54. (a)
55. (d)	56. (b)	57. (b)	58. (c)	59. (b)	60. (b)	61. (c)	62. (c)	63. (c)
64. (c)	65. (b)	66. (c)	67. (a)	68. (c)	69. (a)	70. (a)	71. (d)	72. (c)
73. (d)	74. (d)	75. (c)	76. (d)	77. (b)	78. (d)	79. (d)	80. (c)	81. (c)
82. (b)	83. (d)	84. (c)	85. (a)	86. (d)	87. (c)	88. (d)	89. (b)	90. (c)
91. (d)	92. (d)	93. (d)	94. (d)	95. (b)	96. (a)	97. (a)	98. (a)	99. (c)
100. (c)	101. (d)	102. (c)	103. (d)	104. (c)	105. (c)	106. (b)	107. (c)	108. (d)
109. (c)	110. (a)	111. (c)	112. (d)	113. (d)	114. (b)	115. (c)	116. (d)	117. (b)
118. (b)	119. (c)	120. (b)	121. (a)	122. (d)	123. (b)	124. (b)	125. (b)	126. (c)
127. (d)	128. (d)	129. (d)	130. (b)	131. (e)	132. (c)	133. (d)	134. (c)	135. (b)
136. (c)	137. (c)	138. (c)	139. (d)	140. (c)	141. (b)	142. (a)	143. (c)	144. (b)
145. (a)	146. (d)	147. (b)	148. (c)	149. (c)	150. (c)	151. (b)	152. (b)	153. (a)
154. (d)	155. (a)	156. (b)	157. (c)	158. (c)	159. (b)	160. (b)	161. (e)	162. (c)
163. (d)	164. (d)	165. (d)	166. (b)	167. (c)	168. (d)	169. (c)	170. (b)	171. (b)
172. (c)	173. (d)	174. (c)	175. (d)	176. (c)	177. (c)	178. (c)	179. (c)	180. (c)
181. (a)	182. (a)	183. (c)	184. (c)	185. (c)	186. (b)	187. (c)	188. (d)	189. (a)
190. (b)	191. (d)							

SOLUTIONS

1. Area of the floor = $(5.5 \times 3.75)\ m^2 = 20.625\ m^2$.
   ∴ Cost of paving = Rs. (800×20.625) = Rs. 16500.

2. Let the breadth be b. Then, $25 \times b = 18 \times 10 \Leftrightarrow b = \left(\frac{18 \times 10}{25}\right)$ cm = 7.2 cm.

3. Perimeter of the plot = 2 (90 + 50) = 280 m.
   ∴ Number of poles = $\left(\frac{280}{5}\right) = 56$ m.

4. Let breadth = x cm. Then, length = $\left(\frac{160}{100}x\right)$ cm = $\frac{8}{5}x$ cm.
   So, $\frac{8}{5}x - x = 24 \Leftrightarrow \frac{3}{5}x = 24 \Leftrightarrow x = \left(\frac{24 \times 5}{3}\right) = 40$.
   ∴ Length = 64 cm, Breadth = 40 cm.
   Area = $(64 \times 40)\ cm^2 = 2560\ cm^2$.

5. Clearly, we have : $l = 9$ and $l + 2b = 37$ or $b = 14$.
   ∴ Area = $(l \times b) = (9 \times 14)$ sq. ft. = 126 sq. ft.

6. We have : $(l - b) = 23$ and $2(l + b) = 206$ or $(l + b) = 103$.
   Solving the two equations, we get : $l = 63$ and $b = 40$.
   ∴ Area = $(l \times b) = (63 \times 40)\ m^2 = 2520\ m^2$.

7. Let breadth = x metres. Then, length = $(x + 20)$ metres.
   Perimeter = $\left(\frac{5300}{26.50}\right)$ m = 200 m.

$\therefore$ $2[(x + 20) + x] = 200 \Leftrightarrow 2x + 20 = 100 \Leftrightarrow 2x = 80 \Leftrightarrow x = 40$.
Hence, length $= x + 20 = 60$ m.

8. Let length $= x$ metres. Then, breadth $= \left(\frac{60}{100}x\right)$ metres $= \left(\frac{3x}{5}\right)$ metres.

Perimeter $= \left[2\left(x + \frac{3x}{5}\right)\right]$ m $= \left(\frac{16x}{5}\right)$ m.

$\therefore$ $\frac{16x}{5} = 800 \Leftrightarrow x = \left(\frac{800 \times 5}{16}\right) = 250$.

So, length = 250 m; breadth = 150 m.
$\therefore$ Area $= (250 \times 150)$ m^2 $= 37500$ m^2.

9. $\frac{l}{2(l+b)} = \frac{1}{3} \Rightarrow 3l = 2l + 2b \Rightarrow l = 2b \Rightarrow \frac{l}{b} = \frac{2}{1} = 2 : 1$.

10. Perimeter = Distance covered in 8 min. $= \left(\frac{12000}{60} \times 8\right)$ m $= 1600$ m.

Let length $= 3x$ metres and breadth $= 2x$ metres.
Then, $2(3x + 2x) = 1600$ or $x = 160$.
$\therefore$ Length = 480 m and Breadth = 320 m.
$\therefore$ Area $= (480 \times 320)$ m^2 $= 153600$ m^2.

11. Let breadth $= x$ metres. Then, length $= (x + 5)$ metres.
Then, $x(x + 5) = 750 \Leftrightarrow x^2 + 5x - 750 = 0 \Leftrightarrow (x + 30)(x - 25) = 0 \Leftrightarrow x = 25$.
$\therefore$ Length $= (x + 5) = 30$ m.

12. Let breadth $= x$ metres. Then, length $= \left(\frac{115x}{100}\right)$ metres.

$\therefore$ $x \times \frac{115x}{100} = 460 \Leftrightarrow x^2 = \left(\frac{460 \times 100}{115}\right) = 400 \Leftrightarrow x = 20$.

13. We have : $l = 20$ ft and $lb = 680$ sq. ft. So, $b = 34$ ft.
$\therefore$ Length of fencing $= (l + 2b) = (20 + 68)$ ft $= 88$ ft.

14. $\frac{2(l+b)}{b} = \frac{5}{1} \Rightarrow 2l + 2b = 5b \Rightarrow 3b = 2l \Rightarrow b = \frac{2}{3}l$.

Then, Area $= 216$ cm^2 $\Rightarrow l \times b = 216 \Rightarrow l \times \frac{2}{3}l = 216 \Rightarrow l^2 = 324 \Rightarrow l = 18$ cm.

15. We have : $2b + l = 30 \Rightarrow l = 30 - 2b$.
Area $= 100$ m^2 $\Rightarrow l \times b = 100 \Rightarrow b(30 - 2b) = 100 \Rightarrow b^2 - 15b + 50 = 0$
$\Rightarrow (b - 10)(b - 5) = 0 \Rightarrow b = 10$ or $b = 5$.
When $b = 10$, $l = 10$ and when $b = 5$, $l = 20$.
Since the garden is rectangular, so its dimension is 20 m $\times$ 5 m.

16. Let length $= (3x)$ metres and breadth $= (4x)$ metres.
Then, $3x \times 4x = 7500 \Leftrightarrow 12x^2 = 7500 \Leftrightarrow x^2 = 625 \Leftrightarrow x = 25$.
So, length = 75 m and breadth = 100 m.
Perimeter $= [2(75 + 100)]$ m $= 350$ m.
$\therefore$ Cost of fencing $=$ Rs. $(0.25 \times 350) =$ Rs. 87.50.

17. Required perimeter $= (AB + BC + CP + PQ + QR + RA)$
$= AB + BC + (CP + QR) + (PQ + RA)$
$= AB + BC + AB + BC = 2(AB + BC)$
$= [2(8 + 4)]$ cm $= 24$ cm.

18. Let the areas of the two parts be x and $(700 - x)$ hectares respectively. Then,

$$[x - (700 - x)] = \frac{1}{5} \times \left[\frac{x + (700 - x)}{2}\right] \Leftrightarrow 2x - 700 = 70 \Leftrightarrow x = 385.$$

So, area of smaller part = (700 − 385) hectares = 315 hectares.

19. When folded along breadth, we have : $2\left(\frac{l}{2} + b\right) = 34$ or $l + 2b = 34$ ...(i)

When folded along length, we have : $2\left(l + \frac{b}{2}\right) = 38$ or $2l + b = 38$ ...(ii)

Solving (i) and (ii), we get : $l = 14$ and $b = 10$.

$\therefore$ Area of the paper = (14×10) cm^2 = 140 cm^2.

20. Let breadth = x metres. Then, length = $\left(\frac{3}{2}x\right)$ metres.

Area = $\left(\frac{2}{3} \times 10000\right)$ m^2.

$$\therefore \frac{3}{2}x \times x = \frac{2}{3} \times 10000 \Leftrightarrow x^2 = \frac{4}{9} \times 10000 \Leftrightarrow x = \frac{2}{3} \times 100.$$

$$\therefore \text{Length} = \frac{3}{2}x = \left(\frac{3}{2} \times \frac{2}{3} \times 100\right) \text{m} = 100 \text{ m}.$$

21. Number of bricks = $\left(\frac{\text{Area of courtyard}}{\text{Area of 1 brick}}\right) = \left(\frac{2500 \times 1600}{20 \times 10}\right) = 20000.$

22. Length of the carpet = $\left(\frac{\text{Total cost}}{\text{Rate / m}}\right) = \left(\frac{8100}{45}\right)$ m = 180 m.

Area of the room = Area of the carpet = $\left(180 \times \frac{75}{100}\right)$ m^2 = 135 m^2.

$\therefore$ Breadth of the room = $\left(\frac{\text{Area}}{\text{Length}}\right) = \left(\frac{135}{18}\right)$ m = 7.5 m.

23. Other side = $\sqrt{\left(\frac{15}{2}\right)^2 - \left(\frac{9}{2}\right)^2}$ ft = $\sqrt{\frac{225}{4} - \frac{81}{4}}$ ft = $\sqrt{\frac{144}{4}}$ ft = 6 ft.

$\therefore$ Area of the closet = (6×4.5) sq. ft = 27 sq. ft.

24. Let breadth = x cm. Then, length = $3x$ cm.

$x^2 + (3x)^2 = (8\sqrt{10})^2 \Rightarrow 10x^2 = 640 \Rightarrow x^2 = 64 \Rightarrow x = 8$.

So, length = 24 cm and breadth = 8 cm.

$\therefore$ Perimeter = $[2\ (24 + 8)]$ cm = 64 cm.

25. $\sqrt{l^2 + b^2} = 3b \Rightarrow l^2 + b^2 = 9b^2 \Rightarrow l^2 = 8b^2 \Rightarrow \frac{l^2}{b^2} = 8 \Rightarrow \frac{l}{b} = \sqrt{8} = 2\sqrt{2}$.

26. $2\ (l + b) = 46$ or $l + b = 23$. Also, $lb = 120$.

$\therefore$ Diagonal = $\sqrt{l^2 + b^2} = \sqrt{(l + b)^2 - 2lb} = \sqrt{(23)^2 - 240} = \sqrt{289} = 17$ m.

27. $\sqrt{l^2 + b^2} = \sqrt{41}$ or $l^2 + b^2 = 41$. Also, $lb = 20$.

$(l + b)^2 = (l^2 + b^2) + 2lb = 41 + 40 = 81 \Rightarrow (l + b) = 9$.

$\therefore$ Perimeter = $2\ (l + b) = 18$ cm.

28. Length of diagonal = $\left(52 \times \frac{15}{60}\right)$ m = 13 m.

Sum of length and breadth = $\left(68 \times \frac{15}{60}\right)$ m = 17 m.

$\therefore \quad \sqrt{l^2 + b^2} = 13$ or $l^2 + b^2 = 169$ and $l + b = 17$.

Area $= lb = \frac{1}{2}(2lb) = \frac{1}{2}[(l+b)^2 - (l^2 + b^2)] = \frac{1}{2}[(17)^2 - 169] = \frac{1}{2}(289 - 169) = 60 \text{ m}^2$.

29. We have : $lb = 60$ and $\sqrt{l^2 + b^2} + l = 5b$.

Now, $l^2 + b^2 = (5b - l)^2 \Rightarrow 24b^2 - 10lb = 0 \Rightarrow 24b^2 - 600 = 0$

$\Rightarrow b^2 = 25 \rightarrow b = 5$.

$\therefore \quad l = \left(\frac{60}{5}\right)$ m = 12 m. So, length of the carpet = 12 m.

30. Let length = $(3x)$ metres and breadth = $(2x)$ metres.

Then, $(3x + 5) \times 2x = 2600 \Leftrightarrow 6x^2 + 10x - 2600 = 0$

$\Leftrightarrow \quad 3x^2 + 5x - 1300 = 0 \Leftrightarrow (3x + 65)(x - 20) = 0 \Leftrightarrow x = 20$.

$\therefore$ Breadth = $2x$ = 40 m.

31. Let breadth = x cm. Then, length = $(x + 8)$ cm.

$\therefore \quad (x + 8)\, x = (x + 15)(x - 4) \Leftrightarrow x^2 + 8x = x^2 + 11x - 60 \Leftrightarrow x = 20$.

So, length = 28 cm and breadth = 20 cm.

32. Let length = x metres and breadth = y metres. Then,

$(x + 1)(y + 1) - xy = 21 \Rightarrow x + y = 20$ ...(*i*)

And, $xy - [(x + 1)(y - 1)] = 5 \Leftrightarrow x - y = 6$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 13$ and $y = 7$.

So, length = 13 m and breadth = 7 m.

$\therefore$ Perimeter = [2 (13 + 7)] m = 40 m.

33. Let original length = x metres and original breadth = y metres.

Original area = (xy) m^2.

New length = $\left(\frac{120}{100}x\right)$ m = $\left(\frac{6}{5}x\right)$ m; New breadth = $\left(\frac{120}{100}y\right)$ m = $\left(\frac{6}{5}y\right)$ m.

New Area = $\left(\frac{6}{5}x \times \frac{6}{5}y\right)$ m^2 = $\left(\frac{36}{25}xy\right)$ m^2.

$\therefore$ Increase% = $\left(\frac{11}{25}xy \times \frac{1}{xy} \times 100\right)\% = 44\%$.

34. New area = $\left(\frac{80}{100}a \times \frac{110}{100}b\right) = \left(\frac{4}{5} \times \frac{11}{10}ab\right) = \left(\frac{22}{25}ab\right)$.

$\therefore$ Required percentage = $\left(\frac{22}{25}ab \times \frac{1}{ab} \times 100\right)\% = 88\%$.

35. Let original length = x metres and original breadth = y metres.

Original area = (xy) m^2.

New length = $\left(\frac{150}{100}x\right)$ m = $\left(\frac{3}{2}x\right)$ m; New breadth = $\left(\frac{120}{100}y\right)$ m = $\left(\frac{6}{5}y\right)$ m.

New area $= \left(\frac{3}{2}x \times \frac{6}{5}y\right)\text{m}^2 = \left(\frac{9}{5}xy\right)\text{m}^2$.

$\therefore$ Increase $= \left(\frac{\frac{4}{5}xy}{xy}\right) = \frac{4}{5}$ times.

36. Let original length $= x$ and original breadth $= y$.

Decrease in area $= xy - \left(\frac{80}{100}x \times \frac{90}{100}y\right) = \left(xy - \frac{18}{25}xy\right) = \frac{7}{25}xy$.

$\therefore$ Decrease% $= \left(\frac{7}{25}xy \times \frac{1}{xy} \times 100\right)\% = 28\%$.

37. Let original length $= x$ and original breadth $= y$.

Original area $= xy$.

New length $= \frac{x}{2}$; New breadth $= 3y$. New Area $= \left(\frac{x}{2} \times 3y\right) = \frac{3}{2}xy$.

$\therefore$ Increase% $= \left(\frac{1}{2}xy \times \frac{1}{xy} \times 100\right)\% = 50\%$.

38. Let original length $= x$ and original breadth $= y$.

Then, original area $= xy$.

New area $= \left[\frac{(100-r)}{100} \times x\right]\left[\frac{(105+r)}{100} \times y\right] = \left[\left(\frac{10500 - 5r - r^2}{10000}\right)xy\right]$

$\therefore \left(\frac{10500 - 5r - r^2}{10000}\right)xy = xy \Leftrightarrow r^2 + 5r - 500 = 0 \Leftrightarrow (r+25)(r-20) = 0 \Leftrightarrow r = 20.$

39. Let original length $= x$ and original breadth $= y$.

Then, original area $= xy$.

New length $= \frac{160x}{100} = \frac{8x}{5}$. Let new breadth $= z$.

Then, $\frac{8x}{5} \times z = xy \Rightarrow z = \frac{5y}{8}$.

$\therefore$ Decrease in breadth $= \left(\frac{3y}{8} \times \frac{1}{y} \times 100\right)\% = 37\frac{1}{2}\%$.

40. Let original length $= x$ and original breadth $= y$.

Then, original area $= xy$.

New length $= \frac{130}{100}x = \frac{13x}{10}$. New breadth $= y$. New area $= \left(\frac{13x}{10} \times y\right) = \frac{13xy}{10}$.

$\therefore$ Required ratio $= \left(\frac{\frac{13xy}{10}}{xy}\right) = \frac{13}{10} = 13 : 10$.

41. Area of the sheet $= (20 \times 30)\text{ cm}^2 = 600\text{ cm}^2$.

Area used for typing $= [(20-4) \times (30-6)]\text{ cm}^2 = 384\text{ cm}^2$.

$\therefore$ Required percentage $= \left(\frac{384}{600} \times 100\right)\% = 64\%$.

42. Area of the mat = $[(15 - 3) \times (12 - 3)]$ sq. ft = 108 sq. ft.
∴ Cost of the mat = Rs. (108×3.50) = Rs. 378.

43. $2(l + b) = 340$ (Given).
Area of the boundary = $[(l + 2)(b + 2) - lb] = 2(l + b) + 4 = 344$.
∴ Cost of gardening = Rs. (344×10) = Rs. 3440.

44. $lb = 96$ (Given).
Area of pathway = $[(l - 4)(b - 4) - lb] = 16 - 4(l + b)$, which cannot be determined.
So, data is inadequate.

45. Let the width of walk be x metres. Then,
$(20 - 2x)(10 - 2x) = 96 \Leftrightarrow 4x^2 + 60x - 104 = 0 \Leftrightarrow x^2 + 15x - 26 = 0$
$\Leftrightarrow (x - 13)(x - 2) = 0 \Leftrightarrow x = 2 \quad [\because x \neq 13]$

46. Area of crossroads = $(55 \times 4 + 35 \times 4 - 4 \times 4)\ m^2 = 344\ m^2$.
∴ Cost of gravelling = Rs. $\left(344 \times \frac{75}{100}\right)$ = Rs. 258.

47. Area of the park = $(60 \times 40)\ m^2 = 2400\ m^2$. Area of the lawn = $2109\ m^2$.
∴ Area of the crossroads = $(2400 - 2109)\ m^2 = 291\ m^2$.
Let the width of the road be x metres. Then,
$60x + 40x - x^2 = 291 \Leftrightarrow x^2 - 100x + 291 = 0 \Leftrightarrow (x - 97)(x - 3) = 0$
$\Leftrightarrow x = 3 \quad [\because x \neq 97]$.

48. Side = $\sqrt{2550.25} = \sqrt{\frac{255025}{100}} = \frac{505}{10} = 50.5$ m.

49. Area = $\frac{\text{Total cost}}{\text{Rate}} = \left(\frac{1215}{135}\right)$ hectares = (9×10000) sq. m.
∴ Side of the square = $\sqrt{90000} = 300$ m.
Perimeter of the field = $(300 \times 4) = 1200$ m.
Cost of fencing = Rs. $\left(1200 \times \frac{3}{4}\right)$ = Rs. 900.

50. The sides of the five squares are $\left(\frac{24}{4}\right), \left(\frac{32}{4}\right), \left(\frac{40}{4}\right), \left(\frac{76}{4}\right), \left(\frac{80}{4}\right)$ *i.e.*, 6 cm, 8 cm, 10 cm, 19 cm, 20 cm.
∴ Area of the new square = $[6^2 + 8^2 + (10)^2 + (19)^2 + (20)^2]$
$= (36 + 64 + 100 + 361 + 400)\ cm^2 = 961\ cm^2$.
Side of the new square = $\sqrt{961}$ cm = 31 cm.
Perimeter of the new square = (4×31) cm = 124 cm.

51. Number of marbles = $\left(\frac{300 \times 300}{20 \times 20}\right) = 225$.

52. Area of each slab = $\left(\frac{72}{50}\right) m^2 = 1.44\ m^2$.
∴ Length of each slab = $\sqrt{1.44}$ m = 1.2 m = 120 cm.

53. Area left after laying black tiles = $[(20 - 4) \times (10 - 4)]$ sq. ft = 96 sq. ft.
Area under white tiles = $\left(\frac{1}{3} \times 96\right)$ sq. ft = 32 sq. ft.
Area under blue tiles = $(96 - 32)$ sq. ft = 64 sq. ft.
Number of blue tiles = $\frac{64}{(2 \times 2)} = 16$.

54. Length of largest tile = H.C.F. of 1517 cm and 902 cm = 41 cm.
Area of each tile = (41×41) cm^2.

$\therefore$ Required number of tiles = $\left(\frac{1517 \times 902}{41 \times 41}\right) = 814$.

55. Length of the room = $(7 + 7)$ m = 14 m. Breadth of the room = 7 m.
$\therefore$ Area of the room = (14×7) m^2 = 98 m^2.

56. Side of the square = 12 cm. Area of the rectangle = $[(12 \times 12) - 4]$ cm^2 = 140 cm^2.
Now, area = 140 cm^2, length = 14 cm.

$\therefore$ Breadth = $\frac{\text{area}}{\text{length}} = \frac{140}{14}$ cm = 10 cm.

Hence, Perimeter = $2(l + b) = 2(14 + 10)$ cm = 48 cm.

57. Let the side of the square be x cm. Then, its area = x^2 cm^2.
Area of the rectangle = $(3x^2)$ cm^2.

$\therefore 40 \times \frac{3}{2} \times x = 3x^2 \Leftrightarrow x = 20$.

58. Side of the square = $\frac{80}{4}$ cm = 20 cm.

$2(l + b) = 80 \Rightarrow l + b = 40$. Now, $(20 \times 20) - lb = 100 \Leftrightarrow lb = 300$.

$(l - b) = \sqrt{(l+b)^2 - 4lb} = \sqrt{(40 \times 40) - (4 \times 300)} = \sqrt{400} = 20$.

Now, $l + b = 40$ and $l - b = 20 \Rightarrow l = 30$ and $b = 10$.

$\therefore$ Sides of the rectangle are 30 cm and 10 cm.

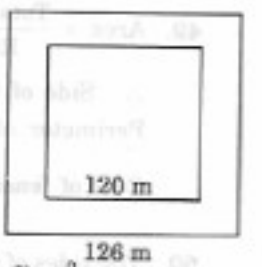
59. Perimeter = $\frac{\text{Total cost}}{\text{Cost per m}} = \frac{10080}{20}$ m = 504 m.

Side of the square = $\frac{504}{4}$ m = 126 m.

Breadth of the pavement = 3 m.

Side of inner square = $(126 - 6)$ m = 120 m.

Area of the pavement = $[(126 \times 126) - (120 \times 120)]$ m^2
= $[(126 + 120)(126 - 120)]$ m^2 = (246×6) m^2.



$\therefore$ Cost of pavement = Rs. $(246 \times 6 \times 50)$ = Rs. 73800.

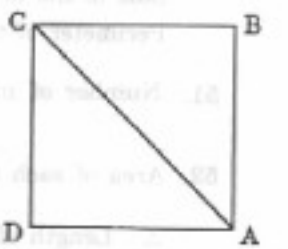
60. Let the length of the outer edge be x metres. Then, length of the inner edge = $(x - 6)$ m.
$\therefore x^2 - (x - 6)^2 = 1764 \Leftrightarrow x^2 - (x^2 - 12x + 36) = 1764 \Leftrightarrow 12x = 1800 \Leftrightarrow x = 150$.
$\therefore$ Required perimeter = $(4x)$ m = (4×150) m = 600 m.

61. Let the side of the square be x metres.
Then, AB + BC = $2x$ metres.

AC = $\sqrt{2}\,x = (1.41x)$ m

Saving on $2x$ metres = $(0.59x)$ m.

Saving% = $\left(\frac{0.59x}{2x} \times 100\right)\% = 30\%$ (approx.).



62. Speed of the man = $\left(4 \times \frac{5}{18}\right)$ m/sec = $\frac{10}{9}$ m/s.

Time taken = (3×60) sec = 180 sec.

Length of diagonal = (speed $\times$ time) = $\left(\frac{10}{9} \times 180\right)$ m = 200 m.

Area of the field = $\frac{1}{2} \times (\text{diagonal})^2 = \left(\frac{1}{2} \times 200 \times 200\right)$ m^2 = 20000 m^2.

63. $d = \sqrt{2} \times l \Rightarrow l = \dfrac{20}{\sqrt{2}}$.

$\therefore$ Perimeter $= (4l)$ cm $= \left(\dfrac{4 \times 20}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}\right)$ cm $= 40\sqrt{2}$ cm.

64. Side $= \sqrt{69696}$ cm $= 264$ cm.

$\therefore$ $d = \sqrt{2} \times$ side $= (264 \times \sqrt{2})$ cm $= (264 \times 1.414)$ cm $= 373.296$ cm.

65. Area $= (45 \times 40)$ m^2 $\Leftrightarrow$ $\dfrac{1}{2} \times (\text{diagonal})^2 = 1800$ $\Leftrightarrow$ diagonal $= 60$ m.

66. Let breadth be x metres. Then, length $= 120\%$ of $x = \left(\dfrac{120}{100}x\right) = \dfrac{6x}{5}$ m.

Required ratio $= \left(\dfrac{6x}{5} \times x \times \dfrac{1}{x \times x}\right) = 6 : 5$.

67. A square and a rectangle with equal areas will satisfy the relation $p_1 < p_2$.

68. Take a square of side 4 cm and a rectangle having $l = 6$ cm, $b = 2$ cm.
Then, perimeter of square = perimeter of rectangle.
Area of square $= 16$ cm^2, area of rectangle $= 12$ cm^2.
$\therefore$ A > B.

69. $d_1 = 4\sqrt{2}$ cm $\Rightarrow$ area $= \dfrac{1}{2}{d_1}^2 = \dfrac{1}{2} \times (4\sqrt{2})^2 = 16$ cm^2.

Area of new square $= (2 \times 16)$ cm^2 $= 32$ cm^2.

$\therefore$ $\dfrac{1}{2}{d_2}^2 = 32 \Rightarrow {d_2}^2 = 64 \Rightarrow d_2 = 8$ cm.

70. Required ratio $= \dfrac{a^2}{(\sqrt{2}\,a)^2} = \dfrac{a^2}{2a^2} = \dfrac{1}{2} = 1 : 2$.

71. Let the diagonals be $2d$ and d.

Then, ratio of their areas $= \dfrac{\frac{1}{2} \times (2d)^2}{\frac{1}{2} \times d^2} = \dfrac{4d^2}{d^2} = \dfrac{4}{1} = 4 : 1$.

72. $\dfrac{a^2}{b^2} = \dfrac{225}{256} = \dfrac{(15)^2}{(16)^2}$ $\Leftrightarrow$ $\dfrac{a}{b} = \dfrac{15}{16}$ $\Leftrightarrow$ $\dfrac{4a}{4b} = \dfrac{4 \times 15}{4 \times 16} = \dfrac{15}{16}$.

$\therefore$ Ratio of perimeters $= 15 : 16$.

73. Area $= 1$ hect. $= 10000$ sq. m $\Rightarrow$ side $= \sqrt{10000}$ m $= 100$ m.
Side of the other square $= 101$ m.
Difference in their areas $= |(101)^2 - (100)^2|$ m^2
$= |(101 + 100)(101 - 100)|$ m$^2 = 201$ m^2.

74. Let the sides be x cm and $\dfrac{150}{100}x = \dfrac{3x}{2}$ cm.

Required ratio $= \dfrac{\frac{9}{4}x^2}{x^2} = \dfrac{9}{4} = 9 : 4$.

75. $A_1 = x^2$ and $A_2 = \left(\dfrac{1}{2}x\right)^2 = \dfrac{1}{4}x^2 = \dfrac{1}{4}A_1$.

76. 100 cm is read as 102 cm.

$\therefore$ $A_1 = (100 \times 100)\ cm^2$ and $A_2 = (102 \times 102)\ cm^2$.

$(A_2 - A_1) = [(102)^2 - (100)^2] = (102 + 100) \times (102 - 100) = 404\ cm^2$.

$\therefore$ Percentage error $= \left(\frac{404}{100 \times 100} \times 100\right)\% = 4.04\%$.

77. Let original area = 100 cm^2. Then, new area = 169 cm^2.

$\Rightarrow$ Original side = 10 cm, New side = 13 cm.

Increase on 10 cm = 3 cm. Increase% $= \left(\frac{3}{10} \times 100\right)\% = 30\%$.

78. Given diagonal = d. New diagonal $= \frac{3}{2}d$.

Original area $= \frac{1}{2}d^2$, New area $= \frac{1}{2} \times \left(\frac{3}{2}d\right)^2 = \frac{9}{8}d^2$.

$\therefore$ Required ratio $= \frac{1}{2}d^2 : \frac{9}{8}d^2 = \frac{1}{2} : \frac{9}{8} = 4 : 9$.

79. Let length = l metres and breadth = b metres. Then, original area = $(lb)\ m^2$.

New length = (140% of l) m $= \left(\frac{140}{100} \times l\right)$ m $= \frac{7l}{5}$ m.

New breadth = (130% of b) m $= \left(\frac{130}{100} \times b\right)$ m $= \frac{13b}{10}$ m.

New area $= \left(\frac{7l}{5} \times \frac{13b}{10}\right) = \left(\frac{91}{50}lb\right)\ m^2$. Increase $= \left(\frac{91}{50}lb - lb\right) = \frac{41}{50}lb$.

$\therefore$ Increase% $= \left(\frac{41}{50} \times lb \times \frac{1}{lb} \times 100\right)\% = 82\%$.

80. Let original length of each side = x cm. Then, its area = $(x^2)\ cm^2$.

Length of rectangle formed = $(x + 5)$ cm and its breadth = x cm.

$\therefore$ $\frac{x+5}{x} = \frac{3}{2} \Leftrightarrow 2x + 10 = 3x \Leftrightarrow x = 10$.

$\therefore$ Original length of each side = 10 cm and its area = 100 cm^2.

81. Let original side = x cm. Then, new side = $(x + 5)$ cm.

$\therefore$ $(x + 5)^2 - x^2 = 165 \Leftrightarrow x^2 + 10x + 25 - x^2 = 165 \Leftrightarrow 10x = 140 \Leftrightarrow x = 14$.

Hence, the side of the square is 14 cm.

82. Let the lengths of the line segments be x cm and $(x + 2)$ cm.

Then, $(x + 2)^2 - x^2 = 32 \Leftrightarrow x^2 + 4x + 4 - x^2 = 32 \Leftrightarrow 4x = 28 \Leftrightarrow x = 7$.

$\therefore$ Length of longer line segment = $(7 + 2)$ cm = 9 cm.

83. Let the length of each side of the square be x cm.

Then, length of rectangle = $(x + 5)$ cm and its breadth = $(x - 3)$ cm.

$\therefore$ $(x + 5)(x - 3) = x^2 \Leftrightarrow x^2 + 2x - 15 = x^2 \Leftrightarrow x = \frac{15}{2}$.

$\therefore$ Length $= \left(\frac{15}{2} + 5\right)$ cm $= \frac{25}{2}$ cm, breadth $= \left(\frac{15}{2} - 3\right)$ cm $= \frac{9}{2}$ cm.

Hence, perimeter $= 2(l + b) = 2\left(\frac{25}{2} + \frac{9}{2}\right)$ cm $= 34$ cm.

84. Area to be plastered = $[2(l + b) \times h] + (l \times b)$
$= \{[2(25 + 12) \times 6] + (25 \times 12)\}\ m^2$
$= (444 + 300)\ m^2 = 744\ m^2$.

$\therefore$ Cost of plastering = Rs. $\left(744 \times \frac{75}{100}\right)$ = Rs. 558.

85. Area of 4 walls = $[2(l + b) \times h] = [2(10 + 7) \times 5]\ m^2 = 170\ m^2$.
Area of 2 doors and 3 windows = $[2(1 \times 3) + (2 \times 1.5) + 2(1 \times 1.5)]\ m^2 = 12\ m^2$.
$\therefore$ Area to be painted = $(170 - 12)\ m^2 = 158\ m^2$.
Cost of painting = Rs. (158×3) = Rs. 474.

86. $A_1 = 2(l + b) \times h$; $A_2 = 2(2l + 2b) \times 2h = 8(l + b) \times h = 4A_1$.
$\therefore$ Required cost = Rs. (4×475) = Rs. 1900.

87. Let $h = 2x$ metres and $(l + b) = 5x$ metres.

Length of the paper = $\frac{\text{Total cost}}{\text{Rate per m}} = \frac{260}{2}$ m = 130 m.

Area of the paper = $\left(130 \times \frac{50}{100}\right) m^2 = 65\ m^2$.

Total area of 4 walls = $(65 + 15)\ m^2 = 80\ m^2$.
$\therefore$ $2(l + b) \times h = 80 \Leftrightarrow 2 \times 5x \times 2x = 80 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2$.
Height of the room = 4 m.

88. $A_1 = \left(\frac{1}{2} \times 15 \times 12\right) cm^2 = 90\ cm^2$. $A_2 = 2A_1 = 180\ cm^2$.

$\therefore$ $\frac{1}{2} \times 20 \times h = 180 \Leftrightarrow h = 18$ cm.

89. $a = 5$, $b = 12$ and $c = 13$. So, $s = \frac{1}{2}(5 + 12 + 13)$ cm = 15 cm.

$\therefore$ Area = $\sqrt{15 \times 10 \times 3 \times 2} = 30\ cm^2$.

$\frac{1}{2} \times 12 \times \text{Height} = 30 \rightarrow$ Height = 5 cm.

90. $\Delta = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow 40 \times \text{Base} = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow$ Height = 80 cm.

91. Let Base = $3x$ cm and Altitude = $4x$ cm.

Then, $\frac{1}{2} \times 3x \times 4x = 1176 \Leftrightarrow 12x^2 = 2352 \Leftrightarrow x^2 = 196 \Leftrightarrow x = 14$ cm.

$\therefore$ Altitude = (4×14) cm = 56 cm.

92. Since $5^2 + (12)^2 = (13)^2$, so, it is a right-angled triangle with
Base = 12 cm and Height = 5 cm.

$\therefore$ Area = $\left(\frac{1}{2} \times 12 \times 5\right) cm^2 = 30\ cm^2$.

93. Ratio of sides = $\frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$.

Perimeter = 52 cm. So, sides are $\left(52 \times \frac{6}{13}\right)$ cm, $\left(52 \times \frac{4}{13}\right)$ cm and $\left(52 \times \frac{3}{13}\right)$ cm.

$\therefore$ $a = 24$ cm, $b = 16$ cm, $c = 12$ cm.

$\therefore$ Length of smallest side = 12 cm.

94. Let $a = 3x$ cm, $b = 4x$ cm and $c = 5x$ cm. Then, $s = 6x$ cm.

$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{6x \times 3x \times 2x \times x} = (6x^2)$ cm^2.

$\therefore \quad 6x^2 = 216 \Leftrightarrow x^2 = 36 \Leftrightarrow x = 6.$

$\therefore \quad a = 18$ cm, $b = 24$ cm and $c = 30$ cm.

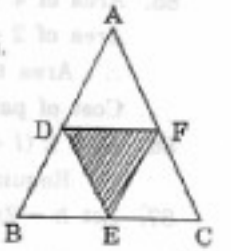
Perimeter $= (18 + 24 + 30)$ cm $= 72$ cm.

95. $a = 3$ cm, $b = 4$ cm and $c = 5$ cm.

It is a right-angled triangle with base $= 3$ cm and height $= 4$ cm.

$\therefore$ Its area $= \left(\frac{1}{2} \times 3 \times 4\right)$ cm^2 $= 6$ cm^2.

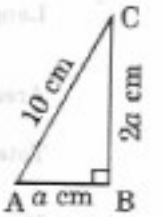
Area of required triangle $= \left(\frac{1}{4} \times 6\right)$ cm^2 $= \frac{3}{2}$ cm^2.



96. Let the sides be a cm and $2a$ cm.

Then, $a^2 + (2a)^2 = (10)^2 \Leftrightarrow 5a^2 = 100 \Leftrightarrow a^2 = 20.$

$\therefore$ Area $= \left(\frac{1}{2} \times a \times 2a\right) = a^2 = 20$ cm^2.



97. Let Base $= b$ cm and Height $= h$ cm.

$b + h + 26 = 60 \Leftrightarrow b + h = 34 \Leftrightarrow (b + h)^2 = (34)^2$ ...(i)

Also, $b^2 + h^2 = (26)^2$ ...(ii)

$\therefore \quad (b + h)^2 - (b^2 + h^2) = (34)^2 - (26)^2 \Leftrightarrow 2bh = (34 + 26)(34 - 26) = 480$

$\Leftrightarrow bh = 240 \Leftrightarrow \frac{1}{2}bh = 120.$

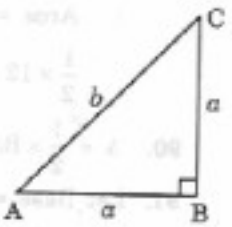
$\therefore$ Area $= 120$ cm^2.

98. Let the sides be a metres, a metres and b metres.

Then, $2a + b = 6 + 3\sqrt{2}$ and $b^2 = a^2 + a^2 = 2a^2 \Leftrightarrow b = \sqrt{2}\,a.$

$\therefore \quad 2a + \sqrt{2}\,a = 6 + 3\sqrt{2} \Leftrightarrow a = 3.$

$\therefore \quad$ Area $= \left(\frac{1}{2} \times 3 \times 3\right)$ m^2 $= 4.5$ m^2.



99. Let the smallest side be x cm.

Then, other sides are 13 cm and $(17 - x)$ cm.

Let $a = 13$, $b = x$ and $c = (17 - x)$. So, $s = 15$.

$\therefore \quad$ Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15 \times 2 \times (15-x)(x-2)}$

$\Leftrightarrow 30 \times (15 - x)(x - 2) = (30)^2 \Leftrightarrow (15 - x)(x - 2) = 30 \Leftrightarrow x^2 - 17x + 60 = 0$

$\Leftrightarrow (x - 12)(x - 5) = 0 \Leftrightarrow x = 12$ or $x = 5.$

$\therefore$ Smallest side $= 5$ cm.

100. Area of an equilateral triangle of side a cm $= \left(\frac{\sqrt{3}}{4}a^2\right)$ cm^2.

$\therefore \quad \frac{\sqrt{3}}{4}a^2 = 24\sqrt{3} \Leftrightarrow a^2 = 96 \Leftrightarrow a = 4\sqrt{6}$ cm.

$\therefore$ Perimeter $= 3a = 12\sqrt{6}$ cm.

84. Area to be plastered = $[2\ (l + b) \times h] + (l \times b)$

$= \{[2\ (25 + 12) \times 6] + (25 \times 12)\}\ m^2$

$= (444 + 300)\ m^2 = 744\ m^2.$

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Area of 2 doors and 3 windows = $[2\ (1 \times 3) + (2 \times 1.5) + 2\ (1 \times 1.5)]\ m^2 = 12\ m^2.$

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87. Let $h = 2x$ metres and $(l + b) = 5x$ metres.

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Area of the paper = $\left(130 \times \frac{50}{100}\right) m^2 = 65\ m^2.$

Total area of 4 walls = $(65 + 15)\ m^2 = 80\ m^2.$

$\therefore$ $2\ (l + b) \times h = 80 \Leftrightarrow 2 \times 5x \times 2x = 80 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2.$

Height of the room = 4 m.

88. $A_1 = \left(\frac{1}{2} \times 15 \times 12\right) cm^2 = 90\ cm^2.$ $A_2 = 2A_1 = 180\ cm^2.$

$\therefore$ $\frac{1}{2} \times 20 \times h = 180 \Leftrightarrow h = 18$ cm.

89. $a = 5$, $b = 12$ and $c = 13$. So, $s = \frac{1}{2}(5 + 12 + 13)$ cm $= 15$ cm.

$\therefore$ Area $= \sqrt{15 \times 10 \times 3 \times 2} = 30\ cm^2.$

$\frac{1}{2} \times 12 \times \text{Height} = 30 \Rightarrow \text{Height} = 5$ cm.

90. $\Delta = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow 40 \times \text{Base} = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow \text{Height} = 80$ cm.

91. Let Base $= 3x$ cm and Altitude $= 4x$ cm.

Then, $\frac{1}{2} \times 3x \times 4x = 1176 \Leftrightarrow 12x^2 = 2352 \Leftrightarrow x^2 = 196 \Leftrightarrow x = 14$ cm.

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92. Since $5^2 + (12)^2 = (13)^2$, so, it is a right-angled triangle with

Base = 12 cm and Height = 5 cm.

$\therefore$ Area $= \left(\frac{1}{2} \times 12 \times 5\right) cm^2 = 30\ cm^2.$

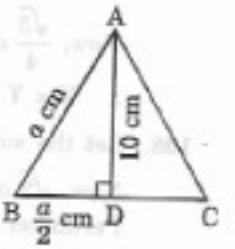
93. Ratio of sides $= \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3.$

Perimeter = 52 cm. So, sides are $\left(52 \times \frac{6}{13}\right)$ cm, $\left(52 \times \frac{4}{13}\right)$ cm and $\left(52 \times \frac{3}{13}\right)$ cm.

$\therefore$ $a = 24$ cm, $b = 16$ cm, $c = 12$ cm.

$\therefore$ Length of smallest side = 12 cm.

101. Let each side be a cm.

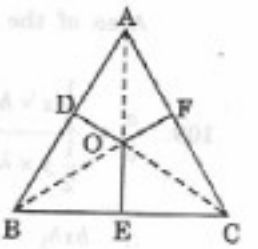


Then, $\left(\frac{a}{2}\right)^2 + (10)^2 = a^2 \Leftrightarrow \left(a^2 - \frac{a^2}{4}\right) = 100$

$\Leftrightarrow \frac{3a^2}{4} = 100 \Leftrightarrow a^2 = \frac{400}{3}$.

$\therefore$ Area $= \frac{\sqrt{3}}{4} \times a^2 = \left(\frac{\sqrt{3}}{4} \times \frac{400}{3}\right) \text{cm}^2 = \frac{100}{\sqrt{3}} \text{cm}^2$.

102. Let each side of the triangle be a cm.



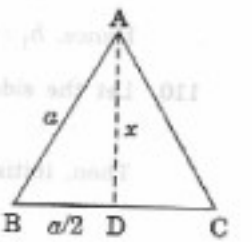
Then, ar (Δ AOB) + ar (Δ BOC) + ar (Δ AOC) = ar (Δ ABC)

$\Rightarrow \frac{1}{2} \times a \times \sqrt{3} + \frac{1}{2} \times a \times 2\sqrt{3} + \frac{1}{2} \times a \times 5\sqrt{3} = \frac{\sqrt{3}}{4} a^2$

$\Rightarrow \frac{a}{2}\sqrt{3}(1 + 2 + 5) = \frac{\sqrt{3}}{4} a^2 \Rightarrow a = 16$.

$\therefore$ Perimeter $= (3 \times 16) = 48$ cm.

103. Let the side of the triangle be a. Then,



$a^2 = \left(\frac{a}{2}\right)^2 + x^2 \Leftrightarrow \frac{3a^2}{4} = x^2 \Leftrightarrow a^2 = \frac{4x^2}{3}$.

$\therefore$ Area $= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times \frac{4}{3} x^2 = \frac{x^2}{\sqrt{3}} = \frac{x^2\sqrt{3}}{3}$.

104. Area of a square with side $a = a^2$ sq. units.

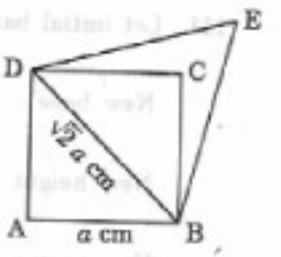
Area of a triangle with base $a = \left(\frac{1}{2} \times a \times h\right)$ sq. units.

$\therefore \quad a^2 = \frac{1}{2} \times a \times h \Leftrightarrow h = 2a$.

Hence, the altitude of the triangle is $2a$.

105. Let the side of the square be a cm.

Then, the length of its diagonal $= \sqrt{2}\,a$ cm.

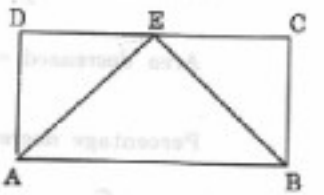


Area of equilateral triangle with side $\sqrt{2}a = \frac{\sqrt{3}}{4} \times (\sqrt{2}a)^2$

$= \frac{\sqrt{3}a^2}{2}$.

$\therefore$ Required ratio $= \frac{\sqrt{3}a^2}{2} : a^2 = \sqrt{3} : 2$.

106. Area of rectangle $= lb$ sq. units.



Area of the triangle $= \frac{1}{2} lb$ sq. units.

$\therefore$ Required ratio $= lb : \frac{1}{2} lb = 2 : 1$.

107. Let each side of the triangle be a cm and each side of the square be b cm.

Then, $X = \frac{\sqrt{3}}{4} a^2$ and $Y = b^2$, where $3a = 4b$, *i.e.*, $b = \frac{3a}{4}$.

$\therefore \quad X = \frac{\sqrt{3}}{4} a^2$ and $Y = \frac{9a^2}{16} \quad \left[\because b = \frac{3a}{4}\right]$

Now, $\frac{\sqrt{3}}{4}a^2 = \frac{1.732}{4}a^2 = 0.433\,a^2$ and $\frac{9a^2}{16} = 0.5625\,a^2$.

$\therefore$ X < Y.

108. Let the side of the square be a cm. Then, its diagonal = $\sqrt{2}\,a$ cm.

Now, $\sqrt{2}\,a = 12\sqrt{2} \Rightarrow a = 12$ cm.

Perimeter of the square = $4a$ = 48 cm. Perimeter of the equilateral triangle = 48 cm.

Each side of the triangle = 16 cm.

Area of the triangle = $\left(\frac{\sqrt{3}}{4} \times 16 \times 16\right)$ cm^2 = $(64\sqrt{3})$ cm^2.

109. $\frac{a}{b} = \frac{\frac{1}{2} \times h_1}{\frac{1}{2} \times h_2}$ $\left[\text{Ratio of areas} = \frac{a}{b},\ \text{Ratio of base} = x : y\right]$

$\therefore \quad bxh_1 = ayh_2 \Leftrightarrow \frac{h_1}{h_2} = \frac{ay}{bx}$.

Hence, $h_1 : h_2 = ay : bx$.

110. Let the sides be x cm and (80% of x) cm = $\frac{4x}{5}$ cm.

Then, initial area = $\frac{\sqrt{3}}{4}x^2$, final area = $\frac{\sqrt{3}}{4}\left(\frac{4x}{5}\right)^2 = \frac{16\sqrt{3}\,x^2}{100}$.

Decrease in area = $\left(\frac{\sqrt{3}}{4}x^2 - \frac{16\sqrt{3}}{100}x^2\right)$ cm^2 = $\frac{9\sqrt{3}\,x^2}{100}$ cm^2.

$\therefore$ Decrease% = $\left(\frac{9\sqrt{3}\,x^2}{100} \times \frac{4}{\sqrt{3}\,x^2} \times 100\right)\% = 36\%$.

111. Let initial base = b cm and initial height = h cm. Then, initial area = $\left(\frac{1}{2}bh\right)$ cm^2.

New base = (140% of b) cm = $\left(\frac{140b}{100}\right)$ cm = $\left(\frac{7b}{5}\right)$ cm.

New height = (60% of h) cm = $\left(\frac{60h}{100}\right)$ cm = $\left(\frac{3h}{5}\right)$ cm.

New area = $\left(\frac{1}{2} \times \frac{7b}{5} \times \frac{3h}{5}\right)$ cm^2 = $\left(\frac{21}{50}bh\right)$ cm^2.

Area decreased = $\left(\frac{1}{2}bh - \frac{21}{50}bh\right)$ cm^2 = $\left(\frac{4}{50}bh\right)$ cm^2.

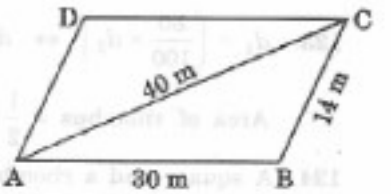
Percentage decrease = $\left(\frac{4bh}{50} \times \frac{2}{bh} \times 100\right)\% = 16\%$.

112. $A_1 = \frac{\sqrt{3}}{2}a^2$ and $A_2 = \frac{\sqrt{3}}{2}(2a)^2 = 4 \times \frac{\sqrt{3}}{2}a^2 = 4A_1$.

$\therefore$ K = 4.

113. Area of ||gm = (Base × Height) = (18 × 8) cm^2 = 144 cm^2.

114. Let ABCD be the given ||gm.



Area of ||gm ABCD = 2 × (area of Δ ABC).

Now, $a = 30$ m, $b = 14$ m, $c = 40$ m.

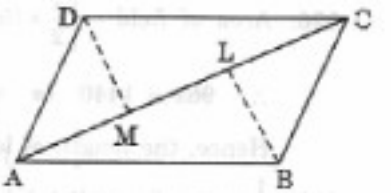
$\therefore \quad s = \frac{1}{2}(30 + 14 + 40)$ m $= 42$ m.

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42 \times 12 \times 28 \times 2}$ m^2 $= 168$ m^2.

Hence, area of ||gm ABCD $= (2 \times 168)$ m^2 $= 336$ m^2.

115. Let ABCD be the given ||gm.



Let AC = 70 cm.

Draw BL ⊥ AC and DM ⊥ AC.

Then, DM = BL = 27 cm.

Area of ||gm ABCD = ar (Δ ABC) + ar (Δ ACD)

$= \left[\left(\frac{1}{2} \times 70 \times 27\right) + \left(\frac{1}{2} \times 70 \times 27\right)\right]$ sq. cm $= 1890$ sq. cm.

116. Let the altitude of the triangle be h_1 and base of each be b.

Then, $\frac{1}{2} \times b \times h_1 = b \times h_2$, where $h_2 = 100$ m

$\Leftrightarrow \quad h_1 = 2h_2 = (2 \times 100)$ m $= 200$ m.

117. Let each have base $= b$ and height $= h$. Then, $P = b \times h$, $R = b \times h$, $T = \frac{1}{2} \times b \times h$

So, P = R, P = 2T and $T = \frac{1}{2}R$ are all correct statements.

118. $\frac{1}{2} d_1 \times d_2 = 150 \Leftrightarrow \frac{1}{2} \times 10 \times d_2 = 150 \Leftrightarrow d_2 = 30$ cm.

119. $\frac{1}{2} d_1 \times 2d_1 = 25 \Leftrightarrow d_1^2 = 25 \Leftrightarrow d_1 = 5$.

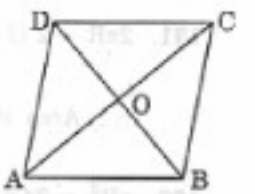
$\therefore$ Sum of lengths of diagonals $= (5 + 10)$ cm $= 15$ cm.

120. Perimeter of the rhombus = 56 m. Each side of the rhombus $= \frac{56}{4}$ m $= 14$ m.

Height of the rhombus = 5 m.

$\therefore$ Area $= (14 \times 5)$ m^2 $= 70$ m^2.

121. Area $= \frac{1}{2} d_1 d_2 = \left(\frac{1}{2} \times 24 \times 10\right)$ cm^2 $= 120$ cm^2.



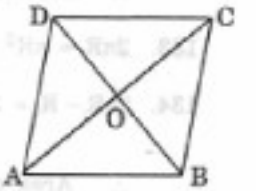
$OA = \frac{1}{2} d_1 = \left(\frac{1}{2} \times 24\right)$ cm $= 12$ cm.

$OB = \frac{1}{2} d_2 = \left(\frac{1}{2} \times 10\right)$ cm $= 5$ cm.

$AB^2 = OA^2 + OB^2 = (12)^2 + 5^2 = 169 \Leftrightarrow AB = 13$ cm.

$\therefore$ Perimeter $= (13 \times 4)$ cm $= 52$ cm.

122. AB = 26 cm and AC = 48 cm $\Rightarrow$ $OA = \left(\frac{1}{2} \times 48\right)$ cm $= 24$ cm.



$OB^2 = AB^2 - OA^2 = (26)^2 - (24)^2 = (26 + 24)(26 - 24) = 100$

$\Rightarrow$ OB = 50 cm $\Rightarrow$ BD $= 2 \times$ OB $= (2 \times 50)$ cm $= 100$ cm.

$\therefore \quad$ Area $= \frac{1}{2} \times AC \times BD = \left(\frac{1}{2} \times 48 \times 100\right)$ cm^2 $= 2400$ cm^2.

123. $d_1 = \left(\frac{80}{100} \times d_2\right) \Leftrightarrow d_1 = \frac{4d_2}{5}$.

Area of rhombus $= \frac{1}{2} d_1 d_2 = \left(\frac{1}{2} \times \frac{4d_2}{5} \times d_2\right) = \frac{2}{5}(d_2)^2$.

124. A square and a rhombus on the same base are equal in area.

125. Area of trapezium $= \left[\frac{1}{2} \times (1.5 + 2.5) \times 6.5\right] m^2 = 13\ m^2$.

126. Area of field $= \left[\frac{1}{2} \times (5x + 3x) \times 24\right] m^2 = (96x)\ m^2$.

$\therefore \quad 96x = 1440 \Leftrightarrow x = \frac{1440}{96} \Leftrightarrow x = 15$.

Hence, the length of longer parallel side $= (5x) = 75$ m.

127. $\frac{1}{2}$ (sum of parallel sides) $\times$ depth = Its area

$\Leftrightarrow \frac{1}{2}(12 + 8) \times d = 840 \Leftrightarrow d = 84$ m.

128. Required $\% = \left[\frac{\pi \times (5)^2}{2\pi \times 5} \times 100\right]\% = 250\%$.

129. Speed $= 12$ km/hr $= \left(12 \times \frac{5}{18}\right)$ m/s $= \frac{10}{3}$ m/s.

Distance covered $= \left(20 \times 2 \times \frac{22}{7} \times 50\right)$ m $= \frac{44000}{7}$ m.

Time taken $= \frac{\text{Distance}}{\text{Speed}} = \left(\frac{44000}{7} \times \frac{3}{10}\right)$ s $= \left(\frac{4400 \times 3}{7} \times \frac{1}{60}\right)$ min

$= \frac{220}{7}$ min $= 31\frac{3}{7}$ min.

130. Area of the field grazed $= \left(\frac{22}{7} \times 14 \times 14\right)$ sq. ft $= 616$ sq. ft.

Number of days taken to graze the field $= \frac{616}{100}$ days $= 6$ days (approx.).

131. $2\pi R = 2(l + b) \Leftrightarrow 2\pi R = 2(26 + 18)$ cm $\Leftrightarrow R = \left(\frac{88}{2 \times 22} \times 7\right) = 14$ cm.

$\therefore$ Area of the circle $= \pi R^2 = \left(\frac{22}{7} \times 14 \times 14\right) cm^2 = 616\ cm^2$.

132. $\pi R^2 = 24.64 \Leftrightarrow R^2 = \left(\frac{24.64}{22} \times 7\right) = 7.84 \Leftrightarrow R = \sqrt{7.84} = 2.8$ cm.

$\therefore$ Circumference $= \left(2 \times \frac{22}{7} \times 2.8\right)$ cm $= 17.60$ m.

133. $2\pi R = \pi R^2 \Leftrightarrow R = 2 \Leftrightarrow 2R = 4$. Hence, diameter $= 4$.

134. $2\pi R - R = 37 \Leftrightarrow \left(\frac{44}{7} - 1\right) R = 37 \Leftrightarrow R = 7$.

$\therefore$ Area of the circle $= \left(\frac{22}{7} \times 7 \times 7\right) cm^2 = 154\ cm^2$.

135. $\pi R_1^2 + \pi R_2^2 = \pi R_3^2 \Leftrightarrow R_1^2 + R_2^2 = R_3^2 \Leftrightarrow (9)^2 + R_2^2 = (15)^2$
$\Leftrightarrow R_2^2 = (15)^2 - (9)^2 = 144 \Leftrightarrow R_2 = 12$ cm.

136. Side of the square $= \frac{44}{4}$ cm $= 11$ cm.

Area of the square $= (11 \times 11)$ cm^2 $= 121$ cm^2.

$2\pi R = 44 \Leftrightarrow 2 \times \frac{22}{7} \times R = 44 \Leftrightarrow R = 7$ cm.

Area of circle $= \pi R^2 = \left(\frac{22}{7} \times 7 \times 7\right)$ cm^2 $= 154$ cm^2.

$\therefore$ Area of circle is larger by 33 cm^2.

137. Length of wire $= 2\pi \times R = \left(2 \times \frac{22}{7} \times 56\right)$ cm $= 352$ cm.

Side of the square $= \frac{352}{4}$ cm $= 88$ cm.

Area of the square $= (88 \times 88)$ cm^2 $= 7744$ cm^2.

138. Side of the square $= \sqrt{484}$ cm $= 22$ cm. Perimeter of the square $= (22 \times 4)$ cm $= 88$ cm.

$2\pi R = 88 \Leftrightarrow 2 \times \frac{22}{7} \times R = 88 \Leftrightarrow R = \left(88 \times \frac{7}{44}\right) = 14$ cm.

$\therefore$ Required area $= \pi R^2 = \left(\frac{22}{7} \times 14 \times 14\right)$ cm^2 $= 616$ cm^2.

139. Length of wire $= 2\pi R = \left(2 \times \frac{22}{7} \times 42\right)$ cm $= 264$ cm.

Perimeter of rectangle $= 2\,(6x + 5x)$ cm $= 22x$ cm.

$\therefore$ $22x = 264 \Leftrightarrow x = 12$.

Smaller side $= (5 \times 12)$ cm $= 60$ cm.

140. Total area of the field $= [\{(180 \times 120)\} + 40000]$ m^2
$= (21600 + 40000)$ m^2 $= 61600$ m^2.

$\therefore$ $\pi R^2 = 61600 \Leftrightarrow R^2 = \left(61600 \times \frac{7}{22}\right) = (400 \times 7 \times 7)$ m

$\Leftrightarrow R = (20 \times 7)$ m $= 140$ m.

141. $\frac{\pi R_1^2}{\pi R_2^2} = \frac{16}{49} \Leftrightarrow \frac{R_1^2}{(14 \times 14)} = \frac{16}{49}$

$\Leftrightarrow R_1^2 = \frac{14 \times 14 \times 16}{49} \Leftrightarrow R_1 = \frac{14 \times 4}{7} = 8$ m.

142. $\frac{\pi R_1^2}{\pi R_2^2} = \frac{4}{9} \Leftrightarrow \frac{R_1^2}{R_2^2} = \frac{4}{9} \Leftrightarrow \frac{R_1}{R_2} = \frac{2}{3} \Leftrightarrow \frac{2\pi R_1}{2\pi R_2} = \frac{R_1}{R_2} = \frac{2}{3}$.

$\therefore$ Required ratio $= 2 : 3$.

143. Let the radius of the given circle be R cm and the side of the square be a cm.

Then, $2\pi R = 4a \Leftrightarrow \frac{R}{a} = \frac{2}{\pi}$.

$\therefore$ Ratio of their areas $= \frac{\pi R^2}{a^2} = \left(\pi \times \frac{4}{\pi^2}\right) = \left(\frac{4}{22} \times 7\right) = \frac{14}{11} = 14 : 11$.

144. Distance covered in 1 revolution $= 2\pi R = \left(2 \times \frac{22}{7} \times 0.63\right)$ m $= \frac{99}{25}$ m.

Distance covered in 500 revolutions $= \left(\frac{99}{25} \times 500\right)$ m $= 1980$ m.

145. Distance covered in 1 revolution $= 2\pi R = \left(2 \times \frac{22}{7} \times 20\right)$ cm $= \frac{880}{7}$ cm.

Required number of revolutions $= \left(17600 \times \frac{7}{880}\right) = 140.$

146. Distance covered in 1 revolution $= 2\pi R = \left(2 \times \frac{22}{7} \times \frac{25}{100}\right)$ m $= \frac{11}{7}$ m.

$\therefore$ Required number of revolutions $= \left(11000 \times \frac{7}{11}\right) = 7000.$

147. Distance covered in 9 sec $= \left(2 \times \frac{22}{7} \times \frac{15}{2} \times 7\right)$ m $= 330$ m.

Distance covered in 1 sec $= \frac{330}{9}$ m $= \frac{110}{3}$ m.

$\therefore$ Required speed $= \left(\frac{110}{3} \times \frac{18}{5}\right)$ km/hr $= 132$ km/hr.

148. Distance covered in 10 sec $= \left(2 \times \frac{22}{7} \times \frac{35}{100} \times 40\right)$ m $= 88$ m.

Distance covered in 1 sec $= \frac{88}{10}$ m $= 8.8$ m.

$\therefore$ Speed $= 8.8$ m/s $= \left(8.8 \times \frac{18}{5}\right)$ km/hr $= 31.68$ km/hr.

149. Let each wheel make x revolutions per sec. Then,

$$\left[\left(2\pi \times \frac{7}{2} \times x\right) + (2\pi \times 7 \times x)\right] \times 10 = 1980$$

$$\Leftrightarrow \left(\frac{22}{7} \times 7 \times x\right) + \left(2 \times \frac{22}{7} \times 7 \times x\right) = 198 \Leftrightarrow 66x = 198 \Leftrightarrow x = 3.$$

Distance moved by smaller wheel in 3 revolutions $= \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 3\right)$ cm $= 66$ cm.

$\therefore$ Speed of smaller wheel $= \frac{66}{3}$ m/s $= 22$ m/s.

150. Distance covered by smaller wheel in 1 revolution $= (2\pi \times 15)$ cm $= (30\pi)$ cm.
Distance covered by larger wheel in 1 revolution $= (2\pi \times 25)$ cm $= (50\pi)$ cm.

Let $k \times 30\pi = 15 \times 50\pi$. Then, $k = \left(\frac{15 \times 50\pi}{30\pi}\right) = 25.$

$\therefore$ Required number of revolutions $= 25.$

151. Let the diameter of the wheel be d metres.
Distance covered in 1 revolution $= (\pi d)$ m.
Distance covered in 113 revolutions $= (113\pi d)$ m.

$\therefore$ $113 \times \frac{22}{7} \times d = 226 \times 10 \Leftrightarrow d = \left(226 \times 10 \times \frac{7}{22} \times \frac{1}{113}\right)$ m $= 6\frac{4}{11}$ m.

152. Let the rear wheel make x revolutions. Then, the front wheel makes $(x+10)$ revolutions.

$(x + 10) \times 3\pi = x \times 2\pi \Leftrightarrow 3x + 30 = 2x \Leftrightarrow x = 30.$

Distance travelled by the wagon $= (2\pi \times 30)$ ft $= (60\pi)$ ft.

153. Radius of the ground $= 17.5$ m. Radius of inner circle $= (17.5 - 1.4)$ m $= 16.1$ m.

Area of the garden $= \pi \times [(17.5)^2 - (16.1)^2]\ m^2 = \left[\frac{22}{7} \times (17.5 + 16.1)(17.5 - 16.1)\right] m^2$

$= \left(\frac{22}{7} \times 33.6 \times 1.4\right) m^2 = 147.84\ m^2.$

154. $2\pi R = 440 \Leftrightarrow 2 \times \frac{22}{7} \times R = 440 \Leftrightarrow R = \left(440 \times \frac{7}{44}\right) = 70$ m.

Inside radius $= (70 - 7)$ m $= 63$ m.

Area of the border $= \pi [(70)^2 - (63)^2]\ m^2$

$= \left[\frac{22}{7} \times (70 + 63) \times (70 - 63)\right] m^2 = 2926\ m^2.$

155. $\pi R_1^{\,2} = 616 \Leftrightarrow R_1^{\,2} = \left(616 \times \frac{7}{22}\right) = 196 \Leftrightarrow R_1 = 14$ cm.

$\pi R_2^{\,2} = 154 \Leftrightarrow R_2^{\,2} = \left(154 \times \frac{7}{22}\right) = 49 \Leftrightarrow R_2 = 7$ cm.

Breadth of the ring $= (R_1 - R_2)$ cm $= (14 - 7)$ cm $= 7$ cm.

156. $2\pi R_1 - 2\pi R_2 = 132 \Leftrightarrow 2\pi (R_1 - R_2) = 132 \Leftrightarrow (R_1 - R_2) = \left(\frac{132}{2 \times 22} \times 7\right) = 21$ m.

$\therefore$ Required width $= 21$ m.

157. Let the radius of the pool be R ft. Radius of the pool including the wall $= (R + 4)$ ft.

Area of the concrete wall $= \pi [(R + 4)^2 - R^2]$ sq. ft

$= [\pi (R + 4 + R)(R + 4 - R)]$ sq. ft $= 8\pi (R + 2)$ sq. ft.

$$8\pi (R + 2) = \frac{11}{25}\pi R^2 \Leftrightarrow 11R^2 = 200 (R + 2) \Leftrightarrow 11R^2 - 200R - 400 = 0$$

$$\Leftrightarrow 11R^2 - 220R + 20R - 400 = 0$$

$$\Leftrightarrow 11R (R - 20) + 20 (R - 20) = 0$$

$$\Leftrightarrow (R - 20)(11R + 20) = 0 \Leftrightarrow R = 20.$$

$\therefore$ Radius of the pool $= 20$ ft.

158. $\frac{2\pi R_1}{2\pi R_2} = \frac{23}{22} \Leftrightarrow \frac{R_1}{R_2} = \frac{23}{22} \Leftrightarrow R_1 = \frac{23}{22} R_2.$

Also, $R_1 - R_2 = 5$ m $\Leftrightarrow \frac{23R_2}{22} - R_2 = 5 \Leftrightarrow R_2 = 110.$

$\therefore$ Diameter of inner circle $= (2 \times 110)$ m $= 220$ m.

159. Area of the semi-circle $= \frac{1}{2}\pi R^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) m^2 = 77\ m^2.$

160. Perimeter of window $= \pi R + 2R = \left(\frac{22}{7} \times \frac{63}{2} + 63\right)$ cm $= (99 + 63)$ cm $= 162$ cm.

152. Let the rear wheel make x revolutions. Then, the front wheel makes $(x+10)$ revolutions.

$(x+10) \times 3\pi = x \times 2\pi \iff 3x + 30 = 2x \iff x = 30.$

Distance travelled by the wagon $= (2\pi \times 30)$ ft $= (60\pi)$ ft.

153. Radius of the ground $= 17.5$ m. Radius of inner circle $= (17.5 - 1.4)$ m $= 16.1$ m.

Area of the garden $= \pi \times [(17.5)^2 - (16.1)^2]$ m^2 $= \left[\frac{22}{7} \times (17.5 + 16.1)(17.5 - 16.1)\right]$ m^2

$= \left(\frac{22}{7} \times 33.6 \times 1.4\right)$ m^2 $= 147.84$ m^2.

154. $2\pi R = 440 \iff 2 \times \frac{22}{7} \times R = 440 \iff R = \left(440 \times \frac{7}{44}\right) = 70$ m.

Inside radius $= (70 - 7)$ m $= 63$ m.

Area of the border $= \pi [(70)^2 - (63)^2]$ m^2

$= \left[\frac{22}{7} \times (70 + 63) \times (70 - 63)\right]$ m^2 $= 2926$ m^2.

155. $\pi R_1^{\,2} = 616 \iff R_1^{\,2} = \left(616 \times \frac{7}{22}\right) = 196 \iff R_1 = 14$ cm.

$\pi R_2^{\,2} = 154 \iff R_2^{\,2} = \left(154 \times \frac{7}{22}\right) = 49 \iff R_2 = 7$ cm.

Breadth of the ring $= (R_1 - R_2)$ cm $= (14 - 7)$ cm $= 7$ cm.

156. $2\pi R_1 - 2\pi R_2 = 132 \iff 2\pi (R_1 - R_2) = 132 \iff (R_1 - R_2) = \left(\frac{132}{2 \times 22} \times 7\right) = 21$ m.

$\therefore$ Required width $= 21$ m.

157. Let the radius of the pool be R ft. Radius of the pool including the wall $= (R + 4)$ ft.

Area of the concrete wall $= \pi [(R + 4)^2 - R^2]$ sq. ft

$= [\pi (R + 4 + R)(R + 4 - R)]$ sq. ft $= 8\pi (R + 2)$ sq. ft.

$8\pi (R + 2) = \frac{11}{25} \pi R^2 \iff 11R^2 = 200 (R + 2) \iff 11R^2 - 200R - 400 = 0$

$\iff 11R^2 - 220R + 20R - 400 = 0$

$\iff 11R (R - 20) + 20 (R - 20) = 0$

$\iff (R - 20)(11R + 20) = 0 \iff R = 20.$

$\therefore$ Radius of the pool $= 20$ ft.

158. $\frac{2\pi R_1}{2\pi R_2} = \frac{23}{22} \iff \frac{R_1}{R_2} = \frac{23}{22} \iff R_1 = \frac{23}{22} R_2.$

Also, $R_1 - R_2 = 5$ m $\iff \frac{23R_2}{22} - R_2 = 5 \iff R_2 = 110.$

$\therefore$ Diameter of inner circle $= (2 \times 110)$ m $= 220$ m.

159. Area of the semi-circle $= \frac{1}{2} \pi R^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right)$ m^2 $= 77$ m^2.

160. Perimeter of window $= \pi R + 2R = \left(\frac{22}{7} \times \frac{63}{2} + 63\right)$ cm $= (99 + 63)$ cm $= 162$ cm.

161. Given : $\pi R + 2R = 36 \Leftrightarrow (\pi + 2)R = 36 \Leftrightarrow R = \dfrac{36}{\left(\dfrac{22}{7} + 2\right)}$ cm $= \left(\dfrac{36 \times 7}{36}\right)$ cm $= 7$ cm.

$\therefore$ Required area $= \pi R^2 = \left(\dfrac{22}{7} \times 7 \times 7\right)$ cm^2 $= 154$ cm^2.

162. Length of each side of the square $= \sqrt{81}$ cm $= 9$ cm.

Length of wire $= (9 \times 4)$ cm $= 36$ cm.

$\pi R + 2R = 36 \Leftrightarrow (\pi + 2)R = 36 \Leftrightarrow R = \dfrac{36}{\left(\dfrac{22}{7} + 2\right)} = 7$ cm.

Area of the semi-circle $= \dfrac{1}{2}\pi R^2 = \left(\dfrac{1}{2} \times \dfrac{22}{7} \times 7 \times 7\right)$ cm^2 $= 77$ cm^2.

163. Area of the sector $= \left(\dfrac{1}{2} \times \text{arc} \times R\right) = \left(\dfrac{1}{2} \times 3.5 \times 5\right)$ cm^2 $= 8.75$ cm^2.

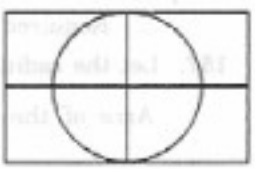
164. Area of the sector $= \dfrac{\pi R^2 0}{360} = \left(\dfrac{22}{7} \times 7 \times 7 \times \dfrac{108}{360}\right)$ cm^2 $= 46.2$ cm^2.

165. Side of the square $= \dfrac{120}{4}$ cm $= 30$ cm.

Radius of the required circle $= \left(\dfrac{1}{2} \times 30\right)$ cm $= 15$ cm.

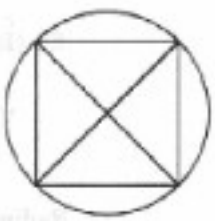
Area of the required circle $= [\pi \times (15)^2]$ cm^2 $= \left[\dfrac{22}{7} \times (15)^2\right]$ cm^2.

166. Radius of the required circle $= \left(\dfrac{1}{2} \times 14\right)$ cm $= 7$ cm.



Area of the circle $= \left(\dfrac{22}{7} \times 7 \times 7\right)$ cm^2 $= 154$ cm^2.

167. $\pi R^2 = 220 \Leftrightarrow R^2 = \left(220 \times \dfrac{7}{22}\right) = 70$.



Now, $R = \dfrac{1}{2} \times$ (diagonal) $\Leftrightarrow$ diagonal $= 2R$.

$\therefore$ Area of the square $= \dfrac{1}{2} \times (\text{diagonal})^2$

$= \left(\dfrac{1}{2} \times 4R^2\right) = 2R^2 = (2 \times 70)$ cm^2 $= 140$ cm^2.

168. Given $R = 4$ cm. $R = \dfrac{1}{2} \times$ (diagonal of the square) $\Leftrightarrow$ diagonal $= 2R = 8$ cm.

Required area $= \pi R^2 - \dfrac{1}{2} \times (8)^2 = (\pi \times 16 - 32) = (16\pi - 32)$ cm^2.

169. $2\pi R = 100 \Leftrightarrow R = \dfrac{100}{2\pi} = \dfrac{50}{\pi}$.

$R = \dfrac{1}{2} \times$ diagonal $\Leftrightarrow$ diagonal $= 2R = \dfrac{2 \times 50}{\pi} = \dfrac{100}{\pi}$.

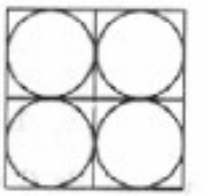
$\therefore$ Area of the square $= \frac{1}{2} \times (\text{diagonal})^2 = \frac{1}{2} \times \left(\frac{100}{\pi}\right)^2$

$\Leftrightarrow \quad a^2 = \frac{1}{2} \times \left(\frac{100}{\pi}\right)^2 \Leftrightarrow \quad a = \frac{1}{\sqrt{2}} \times \frac{100}{\pi} = \frac{50\sqrt{2}}{\pi}$ cm.

170. Side of square paper $= \sqrt{784}$ cm $= 28$ cm.

Radius of each circular plate $= \left(\frac{1}{4} \times 28\right)$ cm $= 7$ cm.

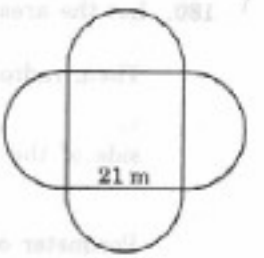
Circumference of each circular plate $= \left(2 \times \frac{22}{7} \times 7\right)$ cm $= 44$ cm.



171. Length of the fence $= 4\pi R$, where $R = \frac{21}{2}$ m

$= \left(4 \times \frac{22}{7} \times \frac{21}{2}\right)$ m $= 132$ m.

Cost of fencing $=$ Rs. $\left(132 \times \frac{25}{2}\right)$ $=$ Rs. 1650.



172. Radius of incircle of an equilateral triangle $= \frac{a}{2\sqrt{3}}$.

Radius of circumcircle of an equilateral triangle $= \frac{a}{\sqrt{3}}$.

$\therefore$ Required ratio $= \frac{\pi a^2}{12} : \frac{\pi a^2}{3} = \frac{1}{12} : \frac{1}{3} = 1 : 4$.

173. Radius of circumcircle $= \frac{a}{\sqrt{3}} = \frac{12}{\sqrt{3}}$ cm $= 4\sqrt{3}$ cm.

174. Radius of incircle $= \frac{a}{2\sqrt{3}} = \frac{42}{2\sqrt{3}}$ cm $= 7\sqrt{3}$ cm.

Area of incircle $= \left(\frac{22}{7} \times 49 \times 3\right)$ cm^2 $= 462$ cm^2.

175. Radius of incircle $= \frac{a}{2\sqrt{3}}$. Area of incircle $= \left(\frac{\pi \times a^2}{12}\right)$ cm^2.

$\therefore \quad \frac{\pi a^2}{12} = 154 \Leftrightarrow a^2 = \frac{154 \times 12 \times 7}{22} \Leftrightarrow a = 14\sqrt{3}$.

$\therefore$ Perimeter of the triangle $= (3 \times 14\sqrt{3})$ cm $= (42 \times 1.732)$ cm $= 72.7$ cm (approx.).

176. We have : $a = 6$, $b = 11$, $c = 15$. $s = \frac{1}{2}(6 + 11 + 15) = 16$.

Area of the triangle, $\Delta = \sqrt{16 \times 10 \times 5 \times 1} = 20\sqrt{2}$ cm^2.

Radius of incircle $= \frac{\Delta}{s} = \frac{20\sqrt{2}}{16} = \frac{5\sqrt{2}}{4}$ cm.

177. Let the radius of incircle be r cm. Then, $2\pi r = 88 \Leftrightarrow r = \left(88 \times \frac{7}{22} \times \frac{1}{2}\right) = 14$.

Semi-perimeter, $s = \left(\frac{30}{2}\right)$ cm $= 15$ cm.

$\therefore$ Area of the triangle $= r \times s = (14 \times 15)$ cm^2 $= 210$ cm^2.

178. Radius $= \dfrac{\text{Area}}{\text{Semi-perimeter}} = \left(\text{Area} \times \dfrac{2}{\text{Area}}\right) = 2.$

179. Let the perimeter of each be a.

Then, side of the equilateral triangle $= \dfrac{a}{3}$; side of the square $= \dfrac{a}{4}$;

radius of the circle $= \dfrac{a}{2\pi}$.

$\therefore \quad T = \dfrac{\sqrt{3}}{4} \times \left(\dfrac{a}{3}\right)^2 = \dfrac{\sqrt{3}\,a^2}{36}$; $S = \left(\dfrac{a}{4}\right)^2 = \dfrac{a^2}{16}$; $C = \pi \times \left(\dfrac{a}{2\pi}\right)^2 = \dfrac{a^2}{4\pi} = \dfrac{7a^2}{88}$.

So, $C > S > T$.

180. Let the area of each be a.

Then, radius of the circle $= \dfrac{\sqrt{a}}{\pi}$; side of the square $= \sqrt{a}$;

side of the triangle $= \sqrt{\dfrac{a \times 4}{\sqrt{3}}}$.

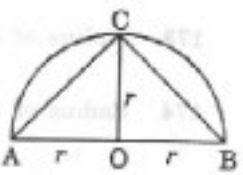
Perimeter of the circle $= 2\pi\sqrt{\dfrac{a}{\pi}} = 2\sqrt{\pi a} = 2\sqrt{3.14 \times a} = 2 \times 1.77\sqrt{a} = 3.54\sqrt{a}$.

Perimeter of the square $= 4\sqrt{a}$;

Perimeter of the triangle $= 3 \times \sqrt{\dfrac{4a}{1.732}} = 3 \times \sqrt{2.31a} = 3 \times 1.52\sqrt{a} = 4.56\sqrt{a}$.

Clearly, perimeter of the triangle is the greatest.

181. Required area $= \dfrac{1}{2} \times \text{base} \times \text{height} = \left(\dfrac{1}{2} \times 2r \times r\right) = r^2$.



182. Required area $= \dfrac{\pi}{2} \times \left(\dfrac{AC}{2}\right)^2 = \dfrac{\pi}{2} \times \dfrac{AC^2}{4} = \dfrac{\pi}{2} \times \dfrac{AB^2 + BC^2}{4}$

$= \dfrac{\pi}{2} \times \left(\dfrac{AB^2}{4} + \dfrac{BC^2}{4}\right) = \dfrac{\pi}{2} \times \left(\dfrac{AB}{2}\right)^2 + \dfrac{\pi}{2} \times \left(\dfrac{BC}{2}\right)^2 = 81 + 36 = 117 \text{ cm}^2$.

183. Let original radius be R cm. Then, original circumference $= (2\pi R)$ cm.

New radius $= (175\% \text{ of R})$ cm $= \left(\dfrac{175}{100} \times R\right)$ cm $= \dfrac{7R}{4}$ cm.

New circumference $= \left(2\pi \times \dfrac{7R}{4}\right)$ cm $= \dfrac{7\pi R}{2}$ cm.

Increase in circumference $= \left(\dfrac{7\pi R}{2} - 2\pi R\right)$ cm $= \dfrac{3\pi R}{2}$ cm.

Increase% $= \left(\dfrac{3\pi R}{2} \times \dfrac{1}{2\pi R} \times 100\right)\% = 75\%$.

184. Let original diameter be d metres. Then, its circumference $= (\pi d)$ metres.

Time taken to cover $(8\pi d)$ m $= 40$ min.

New diameter $= (10d)$ m. Then, its circumference $= (\pi \times 10d)$ m.

$\therefore$ Time taken to go round it once $= \left(\dfrac{40}{8\pi d} \times 10\pi d\right)$ m $= 50$ min.

185. Let the original radius be R cm. New radius $= \left(\frac{106}{100}R\right)$ cm $= \left(\frac{53R}{50}\right)$ cm.

Original area $= \pi R^2$.

$$\text{Increase in area} = \pi\left(\frac{53R}{50}\right)^2 - \pi R^2 = \pi R^2\left[\left(\frac{53}{50}\right)^2 - 1\right] = \frac{\pi R^2\,[(53)^2 - (50)^2]}{2500}$$

$$= \frac{\pi R^2\,(103\times 3)}{2500}\ \text{m}^2.$$

$$\text{Increase\%} = \left(\frac{\pi R^2 \times 309}{2500}\times\frac{1}{\pi R^2}\times 100\right)\% = 12.36\%.$$

186. Let the original radius be R cm.

$$\text{New radius} = (90\%\ \text{of R})\ \text{cm} = \left(\frac{90}{100}\times R\right)\ \text{cm} = \frac{9R}{10}\ \text{cm}.$$

Original area $= \pi R^2$.

$$\text{Diminished area} = \left[\pi R^2 - \pi\left(\frac{9R}{10}\right)^2\right]\text{cm}^2 = \left[\left(1-\frac{81}{100}\right)\pi R^2\right]\text{cm}^2 = \left(\frac{19}{100}\pi R^2\right)\text{cm}^2.$$

$$\text{Decrease\%} = \left(\frac{19\pi R^2}{100}\times\frac{1}{\pi R^2}\times 100\right)\% = 19\%.$$

187. Let the original radius be R cm. New radius = 2R.

Original area $= \pi R^2$, New area $= \pi(2R)^2 = 4\pi R^2$.

Increase in area $= (4\pi R^2 - \pi R^2) = 3\pi R^2$.

$$\text{Increase\%} = \left(\frac{3\pi R^2}{\pi R^2}\times 100\right)\% = 300\%.$$

188. $2\pi R_1 = 4\pi$ and $2\pi R_2 = 8\pi \Rightarrow R_1 = 2$ and $R_2 = 4$

$\Rightarrow$ Original area $= (4\pi \times 2^2) = 16\pi$, Increased area $= (4\pi \times 4^2) = 64\pi$.

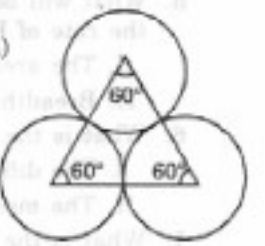
Thus, the area quadruples.

189. Required area = (Area of an equilateral Δ of side 7 cm)

$-$ (3 $\times$ area of sector with $\theta = 60°$ & $r = 3.5$ cm)

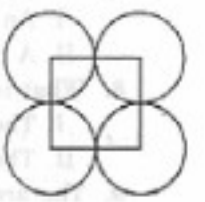
$$= \left[\left(\frac{\sqrt{3}}{4}\times 7\times 7\right) - \left(3\times\frac{22}{7}\times 3.5\times 3.5\times\frac{60}{360}\right)\right]\text{cm}^2$$

$$= \left(\frac{49\sqrt{3}}{4} - 11\times 0.5\times 3.5\right)\text{cm}^2 = (21.217 - 19.25)\ \text{cm}^2 = 1.967\ \text{cm}^2.$$



190. Required area $= \left(14\times 14 - 4\times\frac{1}{4}\times\frac{22}{7}\times 7\times 7\right)\text{cm}^2$

$= (196 - 154)\ \text{cm}^2 = 42\ \text{cm}^2$.



191. Required area $= \left(63\times 63 - 4\times\frac{1}{4}\times\frac{22}{7}\times\frac{63}{2}\times\frac{63}{2}\right)\text{m}^2 = 850.5\ \text{m}^2$.

EXERCISE 24B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 11*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. The area of a playground is 1600 m^2. What is its perimeter ? **(Bank P.O. 2003)**
   I. It is a perfect square playground.
   II. It costs Rs. 3200 to put a fence around the playground at the rate of Rs. 20 per metre.
2. What is the area of the rectangle ?
   I. The ratio of the length and the breadth is 3 : 2.
   II. The area of the rectangle is 3.6 times its perimeter.
3. Area of a square is equal to the area of a circle. What is the circumference of the circle ?
   I. The diagonal of the square is x inches.
   II. The side of the square is y inches. **(S.B.I.P.O. 2003)**
4. The area of a rectangle is equal to the area of a right-angled triangle. What is the length of the rectangle ? **(Bank P.O. 2003)**
   I. The base of the triangle is 40 cm.
   II. The height of the triangle is 50 cm.
5. What will be the cost of gardening a strip of land inside around a circular field, at the rate of Rs. 85 per sq. metre ?
   I. The area of the field is 1386 sq. metres.
   II. Breadth and length of the field are in the ratio of 3 : 5 respectively.
6. What is the area of the rectangle ? **(Bank P.O. 2003)**
   I. The difference between the sides is 5 cm.
   II. The measure of its diagonal is 10 cm.
7. What is the area of the circle ?
   I. An arc of length 4 cm subtends an angle of 60° at the centre.
   II. A chord of length 5 cm subtends an angle of 90° at the centre.
8. What is the area of the circle ? **(NABARD, 2002)**
   I. The circumference of the circle is 308 m.
   II. The radius of the circle is 28 m.
9. The area of a rectangle is equal to the area of a circle. What is the length of the rectangle ?
   I. The radius of the circle is equal to the breadth of the rectangle.
   II. The perimeter of the rectangle is 14 cm more than that of the circle.

10. What is the height of the triangle ? **(Bank P.O. 2002)**
I. The area of the triangle is 20 times its base.
II. The perimeter of the triangle is equal to the perimeter of a square of side 10 cm.

11. What will be the cost of painting the inner walls of a room if the rate of painting is Rs. 20 per square foot ? **(Bank P.O. 2000)**
I. Circumference of the floor is 44 feet.
II. The height of the wall of the room is 12 feet.

Directions (*Questions 12 to 15*) : *Each of the questions below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question.*

12. What is the area of rectangular field ? **(Bank P.O. 2004)**
I. The perimeter of the field is 110 metres.
II. The length is 5 metres more than the width.
III. The ratio between length and width is 6 : 5 respectively.
(*a*) I and II only (*b*) Any two of the three (*c*) All I, II and III
(*d*) I, and either II or III only (*e*) None of these

13. What is the area of the hall ? **(Bank P.O. 2003)**
I. Material cost of flooring per square metre is Rs. 2.50.
II. Labour cost of flooring the hall is Rs. 3500.
III. Total cost of flooring the hall is Rs. 14,500.
(*a*) I and II only (*b*) II and III only (*c*) All I, II and III
(*d*) Any two of the three (*e*) None of these

14. What is the length of the diagonal of the given rectangle ?
I. The perimeter of the rectangle is 34 cm.
II. The difference between the length and breadth is 7 cm.
III. The length is 140% more than the breadth.
(*a*) Any two of the three (*b*) All I, II and III (*c*) I, and either II or III
(*d*) I and II only (*e*) II and III only

15. What is the cost of flooring the rectangular hall ? **(R.B.I. 2002)**
I. Length and breadth of the hall are in the respective ratio of 3 : 2.
II. Length of the hall is 48 m and cost of flooring is Rs. 85 per sq. m.
III. Perimeter of the hall is 160 m and cost of flooring is Rs. 85 per sq. m.
(*a*) I and II only (*b*) II and III only (*c*) III only
(*d*) I, and either II or III only (*e*) Any two of the three

16. What is the area of a right-angled triangle ? **(S.B.I.P.O. 2000)**
I. The perimeter of the triangle is 30 cm.
II. The ratio between the base and the height of the triangle is 5 : 12.
III. The area of the triangle is equal to the area of a rectangle of length 10 cm.
(*a*) I and II only (*b*) II and III only (*c*) I and III only
(*d*) III, and either I or II only (*e*) None of these

17. A path runs around a rectangular lawn. What is the width of the path ?
I. The length and breadth of the lawn are in the ratio of 3 : 1 respectively.
II. The width of the path is ten times the length of the lawn.
III. The cost of gravelling the path @ Rs. 50 per m^2 is Rs. 8832.
(*a*) All I, II and II (*b*) III, and either I or II (*c*) I and III only
(*d*) II and III only (*e*) None of these

18. What is the area of the isosceles triangle ?
I. Perimeter of the isosceles triangle is 18 metres.
II. Base of the triangle is 8 metres.
III. Height of the triangle is 3 metres.
(a) I and II only (b) II and III only (c) I and III only
(d) II, and either I or III only (e) Any two of the three

Directions (*Questions 19 to 22*) : *Each of the questions given below is followed by three statements. You have to study the question and all the three statements given to decide whether any information provided in the statement(s) is/are redundant and can be dispensed with while answering the given question.*

19. What is the cost of painting the two adjacent walls of a hall at Rs. 5 per m^2 which has no windows or doors ? (S.B.I.P.O. 2001)
I. The area of the hall is 24 sq. m.
II. The breadth, length and height of the hall are in the ratio of 4 : 6 : 5 respectively.
III. Area of one wall is 30 sq. m.
(a) I only (b) II only (c) III only
(d) Either I or III (e) All I, II and III are required

20. What is the area of the given rectangle ? (Bank P.O. 1999)
I. Perimeter of the rectangle is 60 cm.
II. Breadth of the rectangle is 12 cm.
III. Sum of two adjacent sides is 30 cm.
(a) I only (b) II only (c) III only
(d) I and II only (e) I or III only

21. What is the area of the given right-angled triangle ?
I. Length of the hypotenuse is 5 cm.
II. Perimeter of the triangle is four times its base.
III. One of the angles of the triangle is 60°.
(a) II only (b) III only (c) II or III only (d) II and III both
(e) Information given in all the three statements together is not sufficient to answer the question.

22. What will be the cost of painting the four walls of a room with length, width and height 5 m, 3 m and 8 m respectively ? The room has one door and one window.
I. Cost of painting per sq. m is Rs. 25.
II. Area of window is 2.25 sq. m which is half of the area of the door.
III. Area of the room is 15 sq. m. (S.B.I.P.O. 1999)
(a) I only (b) II only (c) III only
(d) II or III only (e) All I, II and III are required

ANSWERS

1. (c)	2. (e)	3. (c)	4. (d)	5. (e)	6. (e)	7. (c)	8. (c)
9. (e)	10. (a)	11. (c)	12. (b)	13. (c)	14. (a)	15. (e)	16. (a)
17. (a)	18. (d)	19. (c)	20. (e)	21. (c)	22. (c)		

SOLUTIONS

1. Area = 1600 m^2.
I. Side = $\sqrt{1600}$ m = 40 m. So, perimeter = (40 × 4) m = 160 m.
∴ I alone gives the answer.

II. Perimeter = $\frac{\text{Total cost}}{\text{Cost per metre}} = \frac{3200}{20}$ m = 160 m.

$\therefore$ II alone gives the answer.

$\therefore$ Correct answer is (*e*).

2. I. Let $l = 3x$ metres and $b = 2x$ metres. Then, area = $(6x^2)$ m^2.

II. Perimeter = 2 $(3x + 2x)$ m = $(10x)$ m.

$\therefore$ $6x^2 = 3.6 \times 10x \Leftrightarrow x = \frac{(3.6 \times 10)}{6} = 6$.

$\therefore$ $l = 18$ m and $b = 12$ m and so area can be obtained.

Thus, I and II together give the answer.

$\therefore$ Correct answer is (*e*).

3. I. Area of the circle = Area of the square = $\frac{1}{2}x^2$ sq. inches.

$\Rightarrow$ $\pi r^2 = \frac{1}{2}x^2 \Rightarrow r = \sqrt{\frac{x^2}{2\pi}} = \frac{x}{\sqrt{2\pi}}$.

$\therefore$ Circumference of the circle = $2\pi r$, which can be obtained.

$\therefore$ I alone gives the answer.

II. Area of the circle = Area of the square = y^2 sq. inches.

$\Rightarrow$ $\pi r^2 = y^2 \Rightarrow r = \frac{y}{\sqrt{\pi}}$.

$\therefore$ Circumference of the circle = $2\pi r$, which can be obtained.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

4. Given : Area of rectangle = Area of a right-angled triangle

$\Rightarrow$ $l \times b = \frac{1}{2} \times B \times H$

I gives, B = 40 cm.

II gives, H = 50 cm.

Thus, to find l, we need b also, which is not given.

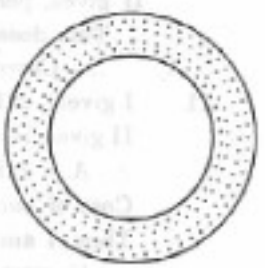
$\therefore$ Given data is not sufficient to give the answer.

$\therefore$ Correct answer is (*d*).

5. I. $\pi R_1^2 = 1386 \Leftrightarrow R_1^2 = \left(1386 \times \frac{7}{22}\right) \Leftrightarrow R_1 = 21$ m.

II. $R_2 = (21 - 1.4)$ m = 19.6 m.

$\therefore$ Area = $\pi (R_1^2 - R_2^2) = \frac{22}{7} \times [(21)^2 - (19.6)^2]$ m^2.



Thus, the required cost may be obtained.

$\therefore$ I and II together will give the answer.

$\therefore$ Correct answer is (*e*).

6. I. Let the sides be x cm and $(x + 5)$ cm.

II. $d = \sqrt{(x+5)^2 + x^2} \Leftrightarrow (x + 5)^2 + x^2 = (10)^2 \Leftrightarrow 2x^2 + 10x - 75 = 0$

$\Leftrightarrow$ $x = \frac{-10 \pm \sqrt{100 + 600}}{4} = \frac{-10 + \sqrt{700}}{4} = \frac{-10 + 10\sqrt{7}}{4} = \frac{-10 + 10 \times 2.6}{4}$

Thus, sides and therefore area may be known.

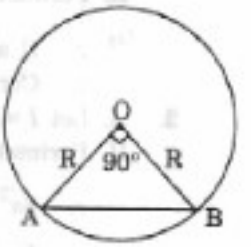
Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (*e*).

7. I. Length of arc $= \frac{2\pi R\theta}{360} \Leftrightarrow 4 = \left(\frac{2 \times \frac{22}{7} \times R \times 60}{360}\right)$.

This gives R and therefore, area of the circle $= \pi R^2$.

Thus, I only gives the answer.



II. $R^2 + R^2 = 5^2 \Leftrightarrow 2R^2 = 25 \Leftrightarrow R^2 = \frac{25}{2}$.

$\therefore$ Area of the circle $= \pi R^2 = \left(\frac{22}{7} \times \frac{25}{2}\right)$ sq. cm.

Thus, II only gives the answer.

$\therefore$ Correct answer is (*c*).

8. I. $2\pi R = 308 \Leftrightarrow 2 \times \frac{22}{7} \times R = 308 \Leftrightarrow R = \left(308 \times \frac{7}{44}\right) = 49$.

Thus, $A = \pi R^2$ can be obtained.

$\therefore$ I alone gives the answer.

II. R = 28 m gives $A = (\pi \times 28 \times 28)$ cm^2.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

9. Given : $l \times b = \pi R^2$. ...(*i*)

I gives, $R = b$. ...(*ii*)

From (*i*) and (*ii*), we get $l = \frac{\pi R^2}{b} = \frac{\pi R^2}{R} = \pi R$. ...(*iii*)

II gives, $2(l + b) = 2\pi R + 14 \Rightarrow l + b = \pi R + 7 \Rightarrow l + R = \pi R + 7$

$\Rightarrow l = \pi R - R + 7$

$\Rightarrow l = l - \frac{l}{\pi} + 7$ [Using (*iii*)]

$\Rightarrow l = 7\pi$.

Thus, I and II together give *l*.

$\therefore$ Correct answer is (*e*).

10. I. $A = 20 \times B \Rightarrow \frac{1}{2} \times B \times H = 20 \times B \Rightarrow H = 40$.

$\therefore$ I alone gives the answer.

II gives, perimeter of the triangle = 40 cm.

This does not give the height of the triangle.

$\therefore$ Correct answer is (*a*).

11. I gives, $2\pi R = 44$.

II gives, H = 12.

$\therefore$ $A = 2\pi RH = (44 \times 12)$.

Cost of painting = Rs. $(44 \times 12 \times 20)$.

Thus, I and II together give the answer.

$\therefore$ Correct answer is (*c*).

12. I. $2(l + b) = 110 \Rightarrow l + b = 55$.

II. $l = (b + 5) \Rightarrow l - b = 5$.

III. $\frac{l}{b} = \frac{6}{5} \Rightarrow 5l - 6b = 0$.

These are three equations in *l* and *b*. We may solve them pairwise.

$\therefore$ Any two of the three will give the answer.

$\therefore$ Correct answer is (*b*).

13. I. Material cost = Rs. 2.50 per m^2.

II. Labour cost = Rs. 3500.

III. Total cost = Rs. 14,500.

Let the area be A sq. metres.

$\therefore$ Material cost = Rs. (14500 − 3500) = Rs. 11,000.

$\therefore \quad \frac{5A}{2} = 11000 \Leftrightarrow A = \left(\frac{11000 \times 2}{5}\right) = 4400 \text{ m}^2.$

Thus, all I, II and III are needed to get the answer.

$\therefore$ Correct answer is (*c*).

14. I. $2(l + b) = 34 \Rightarrow l + b = 17$ ...(*i*)

II. $(l - b) = 7$ ...(*ii*)

III. $l = (100 + 140)\%$ of $b \Rightarrow l - \frac{240}{100} b = 0$

$\Rightarrow 100l - 240b = 0 \Rightarrow 5l - 12b = 0$ ...(*iii*)

These are 3 equations in l and b. We may solve them pairwise.

$\therefore$ Any two of the three will give the answer.

$\therefore$ Correct answer is (*a*).

15. I. Let $l = 3x$ metres and $b = 2x$ metres.

II. $l = 48$ m, Rate of flooring = Rs. 85 per m^2.

III. $2(l + b) = 160 \Leftrightarrow l + b = 80$, Rate of flooring = Rs. 85 per m^2.

From I and II, we get $3x = 48 \Leftrightarrow x = 16$

$\therefore$ $l = 48$ m, $b = 32$ m $\Rightarrow$ Area of floor = (48×32) m^2.

$\therefore$ Cost of flooring = Rs. $(48 \times 32 \times 85)$.

Thus, I and II give the answer.

From II and III, we get $l = 48$ m, $b = (80 - 48)$ m = 32 m.

$\therefore$ Area of floor and cost of flooring is obtained.

Thus, II and III give the answer.

From III and I, we get $3x + 2x = 80 \Leftrightarrow 5x = 80 \Leftrightarrow x = 16$.

$\therefore$ $l = (3 \times 16)$ m = 48 m and $b = (2 \times 16)$ m = 32 m.

$\therefore$ Area of floor and the cost of flooring is obtained.

Thus, III and I give the answer.

Hence, any two of the three will give the answer.

$\therefore$ Correct answer is (*e*).

16. **From II,** base : height = 5 : 12.

Let base = $5x$ and height = $12x$. Then, hypotenuse = $\sqrt{(5x)^2 + (12x)^2} = 13x$.

From I, perimeter of the triangle = 30 cm.

$\therefore$ $5x + 12x + 13x = 30 \Leftrightarrow x = 1$.

So, base = $5x$ = 5 cm; height = $12x$ = 12 cm.

$\therefore$ Area = $\left(\frac{1}{2} \times 5 \times 12\right)$ cm^2 = 30 cm^2.

Thus, I and II together give the answer.

Clearly III is redundant, since the breadth of the rectangle is not given.

$\therefore$ Correct answer is (*a*).

17. III gives area of the path = $\frac{8832}{50}$ m^2 = $\frac{4416}{25}$ m^2.

II gives width of path = 10 × (Length of the lawn).

I gives length = $3x$ metres and breadth = x metres

Clearly, all the three will be required to find the width of the path.

$\therefore$ Correct answer is (*a*).

18. II gives base = 8 m.

I gives perimeter = 18 m.

III gives height = 3 m.

From II and I, we get :

$b = 8$ and $a + b + a = 18$ and so as $= 5$.

Thus, the three sides are 5 m, 5 m and 8 m.

From this, the area can be found out.

From II and III, we get : area $= \left(\frac{1}{2} \times 8 \times 3\right) \text{m}^2$.

$\therefore$ Correct answer is (*d*).

19. **From II**, let $l = 4x$, $b = 6x$ and $h = 5x$.

Then, area of the hall $= (24x^2)\ \text{m}^2$.

From I. Area of the hall $= 24\ \text{m}^2$.

From II and I, we get $24x^2 = 24 \Leftrightarrow x = 1$.

$\therefore$ $l = 4$ m, $b = 6$ m and $h = 5$ m.

Thus, area of two adjacent walls $= [(l \times h) + (b \times h)]\ \text{m}^2$ can be found out and so the cost of painting two adjacent walls may be found out.

Thus, III is redundant.

$\therefore$ Correct answer is (*c*).

20. From I and II, we can find the length and breadth of the rectangle and therefore the area can be obtained.

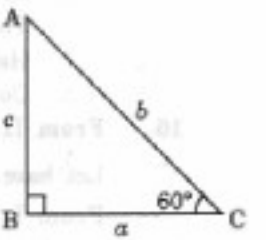
So, III is redundant.

Also, from II and III, we can find the length and breadth and therefore the area can be obtained.

So, I is redundant.

$\therefore$ Correct answer is (*e*).

21. $\frac{BC}{AC} = \cos 60^\circ = \frac{1}{2} \Rightarrow BC = \frac{5}{2}$ cm $\quad [\because AC = 5\ \text{cm}]$



From I and III, we get :

$a = \frac{5}{2}$ cm, $b = 5$ cm and $\theta = 60^\circ$.

$\therefore$ $A = \frac{1}{2} ab \sin C$ gives the area.

Thus, I and III give the result.

$\therefore$ II is redundant.

Again, II gives $a + b + c = 4a \Rightarrow b + c = 3a \Rightarrow c = 3a - 5 \quad [\because b = 5$ from I]

$a^2 + (3a - 5)^2 = 25$. This gives a and therefore c.

Now, area of $\Delta ABC = \frac{1}{2} \times a \times c$, which can be obtained.

Thus I and II give the area.

$\therefore$ III is redundant.

$\therefore$ Correct answer is (*c*).

22. From given length, breadth and height of the room, its area can be obtained.

So, III is redundant.

$\therefore$ Correct answer is (*c*).

25. VOLUME AND SURFACE AREA

IMPORTANT FORMULAE

I. CUBOID

Let length = l, breadth = b and height = h units. Then,

1. ***Volume*** = $(l \times b \times h)$ cubic units.
2. ***Surface area*** = $2\,(lb + bh + lh)$ sq. units.
3. ***Diagonal*** = $\sqrt{l^2 + b^2 + h^2}$ units.

II. CUBE

Let each edge of a cube be of length a. Then,

1. ***Volume*** = a^3 cubic units.
2. ***Surface area*** = $6a^2$ sq. units.
3. ***Diagonal*** = $\sqrt{3}\,a$ units.

III. CYLINDER

Let radius of base = r and Height (or length) = h. Then,

1. ***Volume*** = $(\pi r^2 h)$ cubic units.
2. ***Curved surface area*** = $(2\pi rh)$ sq. units.
3. ***Total surface area*** = $(2\pi rh + 2\pi r^2)$ sq. units
   $= 2\pi r\,(h + r)$ sq. units.

IV. CONE

Let radius of base = r and Height = h. Then,

1. ***Slant height***, $l = \sqrt{h^2 + r^2}$ units.
2. ***Volume*** = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
3. ***Curved surface area*** = (πrl) sq. units.
4. ***Total surface area*** = $(\pi rl + \pi r^2)$ sq. units.

V. SPHERE

Let the radius of the sphere be r. Then,

1. ***Volume*** = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.
2. ***Surface area*** = $(4\pi r^2)$ sq. units.

VI. HEMISPHERE

Let the radius of a hemisphere be r. Then,

1. ***Volume*** = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.
2. ***Curved surface area*** = $(2\pi r^2)$ sq. units.
3. ***Total surface area*** = $(3\pi r^2)$ sq. units.

Remember : 1 litre = 1000 cm^3.

25. VOLUME AND SURFACE AREA

IMPORTANT FORMULAE

I. CUBOID

Let length = l, breadth = b and height = h units. Then,

1. ***Volume*** = $(l \times b \times h)$ cubic units.
2. ***Surface area*** = $2\,(lb + bh + lh)$ sq. units.
3. ***Diagonal*** = $\sqrt{l^2 + b^2 + h^2}$ units.

II. CUBE

Let each edge of a cube be of length a. Then,

1. ***Volume*** = a^3 cubic units.
2. ***Surface area*** = $6a^2$ sq. units.
3. ***Diagonal*** = $\sqrt{3}\,a$ units.

III. CYLINDER

Let radius of base = r and Height (or length) = h. Then,

1. ***Volume*** = $(\pi r^2 h)$ cubic units.
2. ***Curved surface area*** = $(2\pi rh)$ sq. units.
3. ***Total surface area*** = $(2\pi rh + 2\pi r^2)$ sq. units
   $= 2\pi r\,(h + r)$ sq. units.

IV. CONE

Let radius of base = r and Height = h. Then,

1. ***Slant height***, $l = \sqrt{h^2 + r^2}$ units.
2. ***Volume*** = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
3. ***Curved surface area*** = (πrl) sq. units.
4. ***Total surface area*** = $(\pi rl + \pi r^2)$ sq. units.

V. SPHERE

Let the radius of the sphere be r. Then,

1. ***Volume*** = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.
2. ***Surface area*** = $(4\pi r^2)$ sq. units.

VI. HEMISPHERE

Let the radius of a hemisphere be r. Then,

1. ***Volume*** = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.
2. ***Curved surface area*** = $(2\pi r^2)$ sq. units.
3. ***Total surface area*** = $(3\pi r^2)$ sq. units.

Remember : 1 litre = 1000 cm^3.

SOLVED EXAMPLES

Ex. 1. ***Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.***

Sol. Volume = $(16 \times 14 \times 7)$ m^3 = 1568 m^3.

Surface area = $[2\,(16 \times 14 + 14 \times 7 + 16 \times 7)]$ cm^2 = (2×434) cm^2 = 868 cm^2.

Ex. 2. ***Find the length of the longest pole that can be placed in a room 12 m long, 8 m broad and 9 m high.***

Sol. Length of longest pole = Length of the diagonal of the room

$$= \sqrt{(12)^2 + 8^2 + 9^2} = \sqrt{289} = 17 \text{ m}.$$

Ex. 3. ***The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.***

Sol. Let the breadth of the wall be x metres.

Then, Height = $5x$ metres and Length = $40x$ metres.

$\therefore \quad x \times 5x \times 40x = 12.8 \Leftrightarrow x^3 = \dfrac{12.8}{200} = \dfrac{128}{2000} = \dfrac{64}{1000}$.

So, $x = \dfrac{4}{10}$ m $= \left(\dfrac{4}{10} \times 100\right)$ cm = 40 cm.

Ex. 4. ***Find the number of bricks, each measuring 24 cm × 12 cm × 8 cm, required to construct a wall 24 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar ?***

Sol. Volume of the wall = $(2400 \times 800 \times 60)$ cu. cm.

Volume of bricks = 90% of the volume of the wall

$$= \left(\frac{90}{100} \times 2400 \times 800 \times 60\right) \text{ cu. cm.}$$

Volume of 1 brick = $(24 \times 12 \times 8)$ cu. cm.

$\therefore$ Number of bricks = $\left(\dfrac{90}{100} \times \dfrac{2400 \times 800 \times 60}{24 \times 12 \times 8}\right) = 45000.$

Ex. 5. ***Water flows into a tank 200 m × 150 m through a rectangular pipe 1.5 m × 1.25 m @ 20 kmph. In what time (in minutes) will the water rise by 2 metres ?***

Sol. Volume required in the tank = $(200 \times 150 \times 2)$ m^3 = 60000 m^3.

Length of water column flown in 1 min. = $\left(\dfrac{20 \times 1000}{60}\right)$ m = $\dfrac{1000}{3}$ m.

Volume flown per minute = $\left(1.5 \times 1.25 \times \dfrac{1000}{3}\right)$ m^3 = 625 m^3.

$\therefore$ Required time = $\left(\dfrac{60000}{625}\right)$ min. = 96 min.

Ex. 6. ***The dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 3 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.***

Sol. Volume of the metal used in the box = External Volume − Internal Volume

$= [(50 \times 40 \times 23) - (44 \times 34 \times 20)]$ cm^3

$= 16080$ cm^3.

$\therefore$ Weight of the metal = $\left(\dfrac{16080 \times 0.5}{1000}\right)$ kg = 8.04 kg.

Ex. 7. ***The diagonal of a cube is $6\sqrt{3}$ cm. Find its volume and surface area.***

Sol. Let the edge of the cube be a.

$\therefore \quad \sqrt{3}\,a = 6\sqrt{3} \Rightarrow a = 6.$

So, Volume $= a^3 = (6 \times 6 \times 6)\ \text{cm}^3 = 216\ \text{cm}^3.$

Surface area $= 6a^2 = (6 \times 6 \times 6)\ \text{cm}^2 = 216\ \text{cm}^2.$

Ex. 8. ***The surface area of a cube is 1734 sq. cm. Find its volume.***

Sol. Let the edge of the cube be a. Then,

$6a^2 = 1734 \Rightarrow a^2 = 289 \Rightarrow a = 17$ cm.

$\therefore$ Volume $= a^3 = (17)^3\ \text{cm}^3 = 4913\ \text{cm}^3.$

Ex. 9. ***A rectangular block 6 cm by 12 cm by 15 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.***

Sol. Volume of the block $= (6 \times 12 \times 15)\ \text{cm}^3 = 1080\ \text{cm}^3.$

Side of the largest cube = H.C.F. of 6 cm, 12 cm, 15 cm = 3 cm.

Volume of this cube $= (3 \times 3 \times 3)\ \text{cm}^3 = 27\ \text{cm}^3.$

Number of cubes $= \left(\frac{1080}{27}\right) = 40.$

Ex. 10. ***A cube of edge 15 cm is immersed completely in a rectangular vessel containing water. If the dimensions of the base of vessel are 20 cm × 15 cm, find the rise in water level.*** **(R.R.B. 2003)**

Sol. Increase in volume = Volume of the cube $= (15 \times 15 \times 15)\ \text{cm}^3.$

$\therefore$ Rise in water level $= \left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{15 \times 15 \times 15}{20 \times 15}\right)\ \text{cm} = 11.25\ \text{cm}.$

Ex. 11. ***Three solid cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find the surface area of the cube so formed.***

Sol. Volume of new cube $= (1^3 + 6^3 + 8^3)\ \text{cm}^3 = 729\ \text{cm}^3.$

Edge of new cube $= \sqrt[3]{729}\ \text{cm} = 9\ \text{cm}.$

$\therefore$ Surface area of the new cube $= (6 \times 9 \times 9)\ \text{cm}^2 = 486\ \text{cm}^2.$

Ex. 12. ***If each edge of a cube is increased by 50%, find the percentage increase in its surface area.***

Sol. Let original length of each edge $= a$.

Then, original surface area $= 6a^2$.

New edge $= (150\%\ \text{of}\ a) = \left(\frac{150}{100}\,a\right) = \frac{3a}{2}.$

New surface area $= 6 \times \left(\frac{3a}{2}\right)^2 = \frac{27}{2}\,a^2.$

Increase percent in surface area $= \left(\frac{15}{2}\,a^2 \times \frac{1}{6a^2} \times 100\right)\% = 125\%.$

Ex. 13. ***Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.***

Sol. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{1}{27} \text{ or } \left(\frac{a}{b}\right)^3 = \left(\frac{1}{3}\right)^3 \text{ or } \frac{a}{b} = \frac{1}{3}.$$

$\therefore$ Ratio of their surface areas $= \frac{6a^2}{6b^2} = \frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2 = \frac{1}{9}$, *i.e.*, 1 : 9.

Ex. 14. ***Find the volume, curved surface area and the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.***

Sol. Volume $= \pi r^2 h = \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 40\right)$ cm^3 $= 1540$ cm^3.

Curved surface area $= 2\pi rh = \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 40\right)$ cm^2 $= 880$ cm^2.

Total surface area $= 2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

$= \left[2 \times \frac{22}{7} \times \frac{7}{2} \times (40 + 3.5)\right]$ cm^2 $= 957$ cm^2.

Ex. 15. ***If the capacity of a cylindrical tank is 1848 m^3 and the diameter of its base is 14 m, then find the depth of the tank.***

Sol. Let the depth of the tank be h metres. Then,

$$\pi \times (7)^2 \times h = 1848 \iff h = \left(1848 \times \frac{7}{22} \times \frac{1}{7 \times 7}\right) = 12 \text{ m}.$$

Ex. 16. ***2.2 cubic dm of lead is to be drawn into a cylindrical wire 0.50 cm in diameter. Find the length of the wire in metres.***

Sol. Let the length of the wire be h metres. Then,

$$\pi \times \left(\frac{0.50}{2 \times 100}\right)^2 \times h = \frac{2.2}{1000} \iff h = \left(\frac{2.2}{1000} \times \frac{100 \times 100}{0.25 \times 0.25} \times \frac{7}{22}\right) = 112 \text{ m}.$$

Ex. 17. ***How many iron rods, each of length 7 m and diameter 2 cm can be made out of 0.88 cubic metre of iron ?*** **(C.B.I. 1998)**

Sol. Volume of 1 rod $= \left(\frac{22}{7} \times \frac{1}{100} \times \frac{1}{100} \times 7\right)$ cu. m $= \frac{11}{5000}$ cu. m.

Volume of iron $= 0.88$ cu. m.

Number of rods $= \left(0.88 \times \frac{5000}{11}\right) = 400$.

Ex. 18. ***The radii of two cylinders are in the ratio 3 : 5 and their heights are in the ratio of 2 : 3. Find the ratio of their curved surface areas.***

Sol. Let the radii of the cylinders be $3x, 5x$ and their heights be $2y, 3y$ respectively. Then,

Ratio of their curved surface areas $= \frac{2\pi \times 3x \times 2y}{2\pi \times 5x \times 3y} = \frac{2}{5} = 2 : 5$.

Ex. 19. ***If 1 cubic cm of cast iron weighs 21 gms, then find the weight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 cm.***

Sol. Inner radius $= \left(\frac{3}{2}\right)$ cm $= 1.5$ cm, Outer radius $= (1.5 + 1) = 2.5$ cm.

$\therefore$ Volume of iron $= [\pi \times (2.5)^2 \times 100 - \pi \times (1.5)^2 \times 100]$ cm^3

$= \frac{22}{7} \times 100 \times [(2.5)^2 - (1.5)^2]$ cm^3 $= \left(\frac{8800}{7}\right)$ cm^3.

$\therefore$ Weight of the pipe $= \left(\frac{8800}{7} \times \frac{21}{1000}\right)$ kg $= 26.4$ kg.

Ex. 20. ***Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.***

Sol. Here, $r = 21$ cm and $h = 28$ cm.

$\therefore$ Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2} = \sqrt{1225} = 35$ cm.

Volume $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\times\frac{22}{7}\times 21\times 21\times 28\right)\text{ cm}^3 = 12936\text{ cm}^3.$

Curved surface area $= \pi rl = \left(\frac{22}{7}\times 21\times 35\right)\text{ cm}^2 = 2310\text{ cm}^2.$

Total surface area $= (\pi rl + \pi r^2) = \left(2310 + \frac{22}{7}\times 21\times 21\right)\text{ cm}^2 = 3696\text{ cm}^2.$

Ex. 21. ***Find the length of canvas 1.25 m wide required to build a conical tent of base radius 7 metres and height 24 metres.***

Sol. Here, $r = 7$m and $h = 24$ m.

So, $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + (24)^2} = \sqrt{625} = 25$ m.

Area of canvas $= \pi rl = \left(\frac{22}{7}\times 7\times 25\right)\text{ m}^2 = 550\text{ m}^2.$

$\therefore$ Length of canvas $= \left(\frac{\text{Area}}{\text{Width}}\right) = \left(\frac{550}{1.25}\right)\text{ m} = 440\text{ m}.$

Ex. 22. ***The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.***

Sol. Let the radii of their bases be r and R and their heights be h and $2h$ respectively.

Then, $\frac{2\pi r}{2\pi R} = \frac{3}{4} \Rightarrow \frac{r}{R} = \frac{3}{4} \Rightarrow R = \frac{4}{3}r.$

$\therefore$ Ratio of volumes $= \dfrac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi\left(\frac{4}{3}r\right)^2 (2h)} = \frac{9}{32} = 9:32.$

Ex. 23. ***The radii of the bases of a cylinder and a cone are in the ratio of 3 : 4 and their heights are in the ratio 2 : 3. Find the ratio of their volumes.***

Sol. Let the radii of the cylinder and the cone be $3r$ and $4r$ and their heights be $2h$ and $3h$ respectively.

$\therefore \dfrac{\text{Volume of cylinder}}{\text{Volume of cone}} = \dfrac{\pi\times(3r)^2\times 2h}{\frac{1}{3}\pi\times(4r^2)\times 3h} = \frac{9}{8} = 9:8.$

Ex. 24. ***A conical vessel, whose internal radius is 12 cm and height 50 cm, is full of liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel.***

Sol. Volume of the liquid in the cylindrical vessel

= Volume of the conical vessel

$= \left(\frac{1}{3}\times\frac{22}{7}\times 12\times 12\times 50\right)\text{ cm}^3 = \left(\frac{22\times 4\times 12\times 50}{7}\right)\text{ cm}^3.$

Let the height of the liquid in the vessel be h.

Then, $\frac{22}{7}\times 10\times 10\times h = \frac{22\times 4\times 12\times 50}{7}$ or $h = \left(\frac{4\times 12\times 50}{10\times 10}\right) = 24$ cm.

Ex. 25. ***Find the volume and surface area of a sphere of radius 10.5 cm.***

Sol. Volume $= \frac{4}{3}\pi r^3 = \left(\frac{4}{3}\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\times\frac{21}{2}\right)\text{ cm}^3 = 4851\text{ cm}^3.$

Surface area $= 4\pi r^2 = \left(4\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\right)\text{ cm}^2 = 1386\text{ cm}^2.$

Ex. 26. ***If the radius of a sphere is increased by 50%, find the increase percent in volume and the increase percent in the surface area.***

Sol. Let original radius = R. Then, new radius = $\frac{150}{100}R = \frac{3R}{2}$.

Original volume = $\frac{4}{3}\pi R^3$. New volume = $\frac{4}{3}\pi\left(\frac{3R}{2}\right)^3 = \frac{9\pi R^3}{2}$.

Increase % in volume = $\left(\frac{19}{6}\pi R^3 \times \frac{3}{4\pi R^3} \times 100\right)\% = 237.5\%$.

Original surface area = $4\pi R^2$. New surface area = $4\pi\left(\frac{3R}{2}\right)^2 = 9\pi R^2$.

Increase % in surface area = $\left(\frac{5\pi R^2}{4\pi R^2} \times 100\right)\% = 125\%$.

Ex. 27. ***Find the number of lead balls, each 1 cm in diameter that can be made from a sphere of diameter 12 cm.***

Sol. Volume of larger sphere = $\left(\frac{4}{3}\pi \times 6 \times 6 \times 6\right)$ cm^3 = 288π cm^3.

Volume of 1 small lead ball = $\left(\frac{4}{3}\pi \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$ cm^3 = $\frac{\pi}{6}$ cm^3.

$\therefore$ Number of lead balls = $\left(288\pi \times \frac{6}{\pi}\right) = 1728$.

Ex. 28. ***How many spherical bullets can be made out of a lead cylinder 28 cm high and with base radius 6 cm, each bullet being 1.5 cm in diameter ?*** **(R.R.B. 2003)**

Sol. Volume of cylinder = $(\pi \times 6 \times 6 \times 28)$ cm^3 = $(36 \times 28)\pi$ cm^3.

Volume of each bullet = $\left(\frac{4}{3}\pi \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right)$ cm^3 = $\frac{9\pi}{16}$ cm^3.

Number of bullets = $\frac{\text{Volume of cylinder}}{\text{Volume of each bullet}} = \left[(36 \times 28)\pi \times \frac{16}{9\pi}\right] = 1792$.

Ex. 29. ***A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Find the length of the wire.***

Sol. Volume of sphere = $\left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right)$ cm^3 = 972π cm^3.

Volume of wire = $(\pi \times 0.2 \times 0.2 \times h)$ cm^3.

$\therefore$ $972\pi = \pi \times \frac{2}{10} \times \frac{2}{10} \times h \Rightarrow h = (972 \times 5 \times 5)$ cm = $\left(\frac{972 \times 5 \times 5}{100}\right)$ m = 243 m.

Ex. 30. ***Two metallic right circular cones having their heights 4.1 cm and 4.3 cm and the radii of their bases 2.1 cm each, have been melted together and recast into a sphere. Find the diameter of the sphere.***

Sol. Volume of sphere = Volume of 2 cones

$= \left[\frac{1}{3}\pi \times (2.1)^2 \times 4.1 + \frac{1}{3}\pi \times (2.1)^2 \times 4.3\right]$ cm^3 = $\frac{1}{3}\pi \times (2.1)^2 (8.4)$ cm^3.

Let the radius of the sphere be R.

$\therefore$ $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi (2.1)^3 \times 4$ or R = 2.1 cm.

Hence, diameter of the sphere = 4.2 cm.

Ex. 31. ***A cone and a sphere have equal radii and equal volumes. Find the ratio of the diameter of the sphere to the height of the cone.***

Sol. Let radius of each be R and height of the cone be H.

Then, $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi R^2 H$ or $\frac{R}{H} = \frac{1}{4}$ or $\frac{2R}{H} = \frac{2}{4} = \frac{1}{2}$.

$\therefore$ Required ratio = 1 : 2.

Ex. 32. ***Find the volume, curved surface area and the total surface area of a hemisphere of radius 10.5 cm.***

Sol. Volume $= \frac{2}{3}\pi r^3 = \left(\frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}\right)$ cm^3 = 2425.5 cm^3.

Curved surface area $= 2\pi r^2 = \left(2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right)$ cm^2 = 693 cm^2.

Total surface area $= 3\pi r^2 = \left(3 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right)$ cm^2 = 1039.5 cm^2.

Ex. 33. ***A hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl ?*** **(N.I.F.T. 2003)**

Sol. Volume of bowl $= \left(\frac{2}{3}\pi \times 9 \times 9 \times 9\right)$ cm^3 = 486π cm^3.

Volume of 1 bottle $= \left(\pi \times \frac{3}{2} \times \frac{3}{2} \times 4\right)$ cm^3 = 9π cm^3.

Number of bottles $= \left(\frac{486\pi}{9\pi}\right) = 54$.

Ex. 34. ***A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Find the ratio of their volumes.***

Sol. Let R be the radius of each.

Height of hemisphere = Its radius = R.

$\therefore$ Height of each = R.

Ratio of volumes $= \frac{1}{3}\pi R^2 \times R : \frac{2}{3}\pi R^3 : \pi R^2 \times R = 1 : 2 : 3$.

EXERCISE 25A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The capacity of a tank of dimensions (8 m × 6 m × 2.5 m) is :
   (*a*) 120 litres (*b*) 1200 litres (*c*) 12000 litres (*d*) 120000 litres
2. Find the surface area of a 10 cm × 4 cm × 3 cm brick. **(R.R.B. 2001)**
   (*a*) 84 sq. cm (*b*) 124 sq. cm (*c*) 164 sq. cm (*d*) 180 sq. cm
3. A cistern 6 m long and 4 m wide contains water up to a depth of 1 m 25 cm. The total area of the wet surface is : **(S.S.C. 2004)**
   (*a*) 49 m^2 (*b*) 50 m^2 (*c*) 53.5 m^2 (*d*) 55 m^2
4. A boat having a length 3 m and breadth 2 m is floating on a lake. The boat sinks by 1 cm when a man gets on it. The mass of man is : **(R.R.B. 2002)**
   (*a*) 12 kg (*b*) 60 kg (*c*) 72 kg (*d*) 96 kg

5. The area of the base of a rectangular tank is 6500 cm^2 and the volume of water contained in it is 2.6 cubic metres. The depth of water in the tank is :
(a) 3.5 m (b) 4 m (c) 5 m (d) 6 m

6. Given that 1 cu. cm of marble weighs 25 gms, the weight of a marble block 28 cm in width and 5 cm thick is 112 kg. The length of the block is :
(a) 26.5 cm (b) 32 cm (c) 36 cm (d) 37.5 cm

7. Half cubic metre of gold sheet is extended by hammering so as to cover an area of 1 hectare. The thickness of the sheet is :
(a) 0.0005 cm (b) 0.005 cm (c) 0.05 cm (d) 0.5 cm

8. In a shower, 5 cm of rain falls. The volume of water that falls on 1.5 hectares of ground is :
(a) 75 cu. m (b) 750 cu. m (c) 7500 cu. m (d) 75000 cu. m

9. The height of a wall is six times its width and the length of the wall is seven times its height. If volume of the wall be 16128 cu. m, its width is : **(C.B.I. 1998)**
(a) 4 m (b) 4.5 m (c) 5 m (d) 6 m

10. The volume of a rectangular block of stone is 10368 dm^3. Its dimensions are in the ratio of 3 : 2 : 1. If its entire surface is polished at 2 paise per dm^2, then the total cost will be :
(a) Rs. 31.50 (b) Rs. 31.68 (c) Rs. 63 (d) Rs. 63.36

11. The edges of a cuboid are in the ratio 1 : 2 : 3 and its surface area is 88 cm^2. The volume of the cuboid is : **(S.S.C. 1999)**
(a) 24 cm^3 (b) 48 cm^3 (c) 64 cm^3 (d) 120 cm^3

12. The maximum length of a pencil that can be kept in a rectangular box of dimensions 8 cm × 6 cm × 2 cm, is :
(a) $2\sqrt{13}$ cm (b) $2\sqrt{14}$ cm (c) $2\sqrt{26}$ cm (d) $10\sqrt{2}$ cm

13. Find the length of the longest rod that can be placed in a room 16 m long, 12 m broad and $10\frac{2}{3}$ m high. **(S.S.C. 1999)**
(a) $22\frac{1}{3}$ m (b) $22\frac{2}{3}$ m (c) 23 m (d) 68 m

14. How many bricks, each measuring 25 cm × 11.25 cm × 6 cm, will be needed to build a wall 8 m × 6 m × 22.5 cm ? **(B.S.F. 2001)**
(a) 5600 (b) 6000 (c) 6400 (d) 7200

15. The number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm, required to construct a wall 6 m long, 5 m high and 0.5 m thick, while the mortar occupies 5% of the volume of the wall, is : **(M.B.A. 2003)**
(a) 3040 (b) 5740 (c) 6080 (d) 8120

16. 50 men took a dip in a water tank 40 m long and 20 m broad on a religious day. If the average displacement of water by a man is 4 m^3, then the rise in the water level in the tank will be : **(N.I.F.T. 2000)**
(a) 20 cm (b) 25 cm (c) 35 cm (d) 50 cm

17. A tank 4 m long, 2.5 m wide and 1.5 m deep is dug in a field 31 m long and 10 m wide. If the earth dug out is evenly spread out over the field, the rise in level of the field is :
(a) 3.1 cm (b) 4.8 cm (c) 5 cm (d) 6.2 cm

18. A river 1.5 m deep and 36 m wide is flowing at the rate of 3.5 km per hour. The amount of water that runs into the sea per minute (in cubic metres) is :
(a) 3150 (b) 31500 (c) 6300 (d) 63000

19. A rectangular water tank is 80 m × 40 m. Water flows into it through a pipe 40 sq. cm at the opening at a speed of 10 km / hr. By how much, the water level will rise in the tank in half an hour ? **(M.B.A. 1997)**

(a) $\frac{3}{2}$ cm (b) $\frac{4}{9}$ cm (c) $\frac{5}{8}$ cm (d) None of these

20. A hall is 15 m long and 12 m broad. If the sum of the areas of the floor and the ceiling is equal to the sum of areas of the four walls, the volume of the hall is :

(a) 720 (b) 900 (c) 1200 (d) 1800 **(L.I.C. A.A.O. 2003)**

21. The sum of the length, breadth and depth of a cuboid is 19 cm and its diagonal is $5\sqrt{5}$ cm. It surface area is :

(a) 125 cm^2 (b) 236 cm^2 (c) 361 cm^2 (d) 486 cm^2

22. A swimming pool 9 m wide and 12 m long is 1 m deep on the shallow side and 4 m deep on the deeper side. Its volume is : **(M.A.T. 1998)**

(a) 208 m^3 (b) 270 m^3 (c) 360 m^3 (d) 408 m^3

23. A metallic sheet is of rectangular shape with dimensions 48 m × 36 m. From each of its corners, a square is cut off so as to make an open box. If the length of the square is 8 m, the volume of the box (in m^3) is : **(M.A.T. 2003)**

(a) 4830 (b) 5120 (c) 6420 (d) 8960

24. An open box is made of wood 3 cm thick. Its external dimensions are 1.46 m, 1.16 m and 8.3 dm. The cost of painting the inner surface of the box at 50 paise per 100 sq. cm is :

(a) Rs. 138.50 (b) Rs. 277 (c) Rs. 415. 50 (d) Rs. 554

25. A cistern of capacity 8000 litres measures externally 3.3 m by 2.6 m by 1.1 m and its walls are 5 cm thick. The thickness of the bottom is : **(S.S.C. 2003)**

(a) 90 cm (b) 1 dm (c) 1 m (d) 1.1 m

26. If a metallic cuboid weighs 16 kg, how much would a miniature cuboid of metal weigh, if all dimensions are reduced to one-fourth of the original ? **(D.M.R.C. 2003)**

(a) 0.25 kg (b) 0.50 kg (c) 0.75 kg (d) 1 kg

27. The areas of the three adjacent faces of a rectangular box which meet in a point are known. The product of these areas is equal to : **(Section Officers', 2003)**

(a) the volume of the box (b) twice the volume of the box

(c) the square of the volume of the box (d) the cube root of the volume of the box

28. If the areas of the three adjacent faces of a cuboidal box are 120 cm^2, 72 cm^2 and 60 cm^2 respectively, then find the volume of the box. **(S.S.C. 2002)**

(a) 720 cm^3 (b) 864 cm^3 (c) 7200 cm^3 (d) $(72)^2$ cm^3

29. If the areas of three adjacent faces of a rectangular block are in the ratio of 2 : 3 : 4 and its volume is 9000 cu. cm; then the length of the shortest side is :

(a) 10 cm (b) 15 cm (c) 20 cm (d) 30 cm

30. The perimeter of one face a of cube is 20 cm. Its volume must be : **(S.S.C. 1999)**

(a) 125 cm^3 (b) 400 cm^3 (c) 1000 cm^3 (d) 8000 cm^3

31. Total surface area of a cube whose side is 0.5 cm is : **(I.M.T. 2002)**

(a) $\frac{1}{4}$ cm^2 (b) $\frac{1}{8}$ cm^2 (c) $\frac{3}{4}$ cm^2 (d) $\frac{3}{2}$ cm^2

32. The cost of the paint is Rs. 36.50 per kg. If 1 kg of paint covers 16 square feet, how much will it cost to paint outside of a cube having 8 feet each side ?

(a) Rs. 692 (b) Rs. 768 (c) Rs. 876

(d) Rs. 972 (e) None of these **(Bank P.O. 2002)**

33. The dimensions of a piece of iron in the shape of a cuboid are 270 cm × 100 cm × 64 cm. If it is melted and recast into a cube, then the surface area of the cube will be :
(a) 14400 cm^2 (b) 44200 cm^2 (c) 57600 cm^2 (d) 86400 cm^2

34. The cost of painting the whole surface area of a cube at the rate of 13 paise per sq. cm is Rs. 343.98. Then the volume of the cube is : **(S.S.C. 2003)**
(a) 8500 cm^3 (b) 9000 cm^3 (c) 9250 cm^3 (d) 9261 cm^3

35. If the volume of a cube is 729 cm^3, then the surface area of the cube will be :
(a) 456 cm^2 (b) 466 cm^2 (c) 476 cm^2 (d) 486 cm^2

36. The length of an edge of a hollow cube open at one face is $\sqrt{3}$ metres. What is the length of the largest pole that it can accommodate ? **(M.A.T. 1997)**
(a) $\sqrt{3}$ metres (b) 3 metres (c) $3\sqrt{3}$ metres (d) $\frac{3}{\sqrt{3}}$ metres

37. What is the volume of a cube (in cubic cm) whose diagonal measures $4\sqrt{3}$ cm ?
(a) 8 (b) 16 (c) 27 (d) 64
(Hotel Management, 1999)

38. The surface area of a cube is 600 cm^2. The length of its diagonal is :
(a) $\frac{10}{\sqrt{3}}$ cm (b) $\frac{10}{\sqrt{2}}$ cm (c) $10\sqrt{2}$ cm (d) $10\sqrt{3}$ cm

39. If the numbers representing volume and surface area of a cube are equal, then the length of the edge of the cube in terms of the unit of measurement will be :
(a) 3 (b) 4 (c) 5 (d) 6

40. How many cubes of 10 cm edge can be put in a cubical box of 1 m edge ?
(a) 10 (b) 100 (c) 1000 (d) 10000
(R.R.B. 2003)

41. A rectangular box measures internally 1.6 m long, 1 m broad and 80 cm deep. The number of cubical blocks each of edge 20 cm that can be packed inside the box is :
(a) 30 (b) 53 (c) 60 (d) 120

42. How many cubes of 3 cm edge can be cut out of a cube of 18 cm edge ?
(a) 36 (b) 216 (c) 218 (d) 432
(IGNOU, 2003)

43. A cuboidal block of 6 cm × 9 cm × 12 cm is cut up into an exact number of equal cubes. The least possible number of cubes will be : **(Section Officers', 2003)**
(a) 6 (b) 9 (c) 24 (d) 30

44. The size of a wooden block is 5 × 10 × 20 cm. How many such blocks will be required to construct a solid wooden cube of minimum size ?
(a) 6 (b) 8 (c) 12 (d) 16

45. An iron cube of side 10 cm is hammered into a rectangular sheet of thickness 0.5 cm. If the sides of the sheet are in the ratio 1 : 5, the sides are :
(a) 10 cm, 50 cm (b) 20 cm, 100 cm (c) 40 cm, 200 cm (d) None of these
(Hotel Management, 1997)

46. Three cubes of iron whose edges are 6 cm, 8 cm and 10 cm respectively are melted and formed into a single cube. The edge of the new cube formed is :
(a) 12 cm (b) 14 cm (c) 16 cm (d) 18 cm

47. Five equal cubes, each of side 5 cm, are placed adjacent to each other. The volume of the new solid formed will be :
(a) 125 cm^3 (b) 625 cm^3 (c) 15525 cm^3 (d) None of these

48. A cube of edge 5 cm is cut into cubes each of edge 1 cm. The ratio of the total surface area of one of the small cubes to that of the large cube is equal to : **(S.S.C. 2004)**
(a) 1 : 5 (b) 1 : 25 (c) 1 : 125 (d) 1 : 625

49. A large cube is formed from the material obtained by melting three smaller cubes of 3, 4 and 5 cm side. What is the ratio of the total surface areas of the smaller cubes and the large cube ? **(M.A.T. 2004)**
(a) 2 : 1 (b) 3 : 2 (c) 25 : 18 (d) 27 : 20

50. Three cubes with sides in the ratio 3 : 4 : 5 are melted to form a single cube whose diagonal is $12\sqrt{3}$ cm. The sides of the cubes are : **(M.A.T. 2003)**
(a) 3 cm, 4 cm, 5 cm (b) 6 cm, 8 cm, 10 cm
(c) 9 cm, 12 cm, 15 cm (d) None of these

51. If the volumes of two cubes are in the ratio 27 : 1, the ratio of their edges is :
(a) 1 : 3 (b) 1 : 27 (c) 3 : 1 (d) 27 : 1
(S.S.C. 1999)

52. The volumes of two cubes are in the ratio 8 : 27. The ratio of their surface areas is :
(a) 2 : 3 (b) 4 : 9 (c) 12 : 9 (d) None of these
(Hotel Management, 2003)

53. Two cubes have volumes in the ratio 1 : 27. Then the ratio of the area of the face of one of the cubes to that of the other is :
(a) 1 : 3 (b) 1 : 6 (c) 1 : 9 (d) 1 : 12

54. If each edge of a cube is doubled, then its volume :
(a) is doubled (b) becomes 4 times
(c) becomes 6 times (d) becomes 8 times

55. If each edge of a cube is increased by 25%, then the percentage increase in its surface area is :
(a) 25% (b) 48.75% (c) 50% (d) 56.25%

56. A circular well with a diameter of 2 metres, is dug to a depth of 14 metres. What is the volume of the earth dug out ? **(S.S.C. 1999)**
(a) 32 m^3 (b) 36 m^3 (c) 40 m^3 (d) 44 m^3

57. The capacity of a cylindrical tank is 246.4 litres. If the height is 4 metres, what is the diameter of the base ? **(Bank P.O. 2003)**
(a) 1.4 m (b) 2.8 m (c) 14 m (d) 28 m (e) None of these

58. The volume of a right circular cylinder whose curved surface area is 2640 cm^2 and circumference of its base is 66 cm, is :
(a) 3465 cm^3 (b) 7720 cm^3 (c) 13860 cm^3 (d) 55440 cm^3

59. If the volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3, then the radius of the cylinder is equal to :
(a) π cm (b) 2 cm (c) 3 cm (d) 4 cm

60. The height of a right circular cylinder is 14 cm and its curved surface is 704 sq. cm. Then its volume is :
(a) 1408 cm^3 (b) 2816 cm^3 (c) 5632 cm^3 (d) 9856 cm^3

61. A closed metallic cylindrical box is 1.25 m high and its base radius is 35 cm. If the sheet metal costs Rs. 80 per m^2, the cost of the material used in the box is :
(a) Rs. 281.60 (b) Rs. 290 (c) Rs. 340.50 (d) Rs. 500

62. The curved surface area of a right circular cylinder of base radius r is obtained by multiplying its volume by :
(a) $2r$ (b) $\frac{2}{r}$ (c) $2r^2$ (d) $\frac{2}{r^2}$

63. The ratio of total surface area to lateral surface area of a cylinder whose radius is 20 cm and height 60 cm, is :
(*a*) 2 : 1 (*b*) 3 : 2 (*c*) 4 : 3 (*d*) 5 : 3

64. A powder tin has a square base with side 8 cm and height 14 cm. Another tin has a circular base with diameter 8 cm and height 14 cm. The difference in their capacities is :
(*a*) 0 (*b*) 132 cm^3 (*c*) 137.1 cm^3 (*d*) 192 cm^3

65. The ratio between the radius of the base and the height of a cylinder is 2 : 3. If its volume is 12936 cu. cm, the total surface area of the cylinder is :
(*a*) 2587.2 cm^2 (*b*) 3080 cm^2 (*c*) 25872 cm^2 (*d*) 38808 cm^2

66. The radius of the cylinder is half its height and area of the inner part is 616 sq. cms. Approximately how many litres of milk can it contain ?
(*a*) 1.4 (*b*) 1.5 (*c*) 1.7 (*d*) 1.9 (*e*) 2.2
(S.B.I.P.O. 2000)

67. The sum of the radius of the base and the height of a solid cylinder is 37 metres. If the total surface area of the cylinder be 1628 sq. metres, its volume is :
(*a*) 3180 m^3 (*b*) 4620 m^3 (*c*) 5240 m^3 (*d*) None of these

68. The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. Find the ratio of its diameter to its height. (S.S.C. 2002)
(*a*) 3 : 7 (*b*) 7 : 3 (*c*) 6 : 7 (*d*) 7 : 6

69. The height of a closed cylinder of given volume and the minimum surface area is :
(*a*) equal to its diameter (*b*) half of its diameter
(*c*) double of its diameter (*d*) None of these (R.R.B. 2002)

70. If the radius of the base of a right circular cylinder is halved, keeping the height same, what is the ratio of the volume of the reduced cylinder to that of the original one ?
(*a*) 1 : 2 (*b*) 1 : 4 (*c*) 1 : 8 (*d*) 8 : 1

71. The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3. The ratio of their volumes is :
(*a*) 4 : 9 (*b*) 9 : 4 (*c*) 20 : 27 (*d*) 27 : 20

72. Two right circular cylinders of equal volumes have their heights in the ratio 1 : 2. The ratio of their radii is : (S.S.C. 1999)
(*a*) 1 : 2 (*b*) 1 : 4 (*c*) 2 : 1 (*d*) $\sqrt{2}$: 1

73. X and Y are two cylinders of the same height. The base of X has diameter that is half the diameter of the base of Y. If the height of X is doubled, the volume of X becomes :
(*a*) equal to the volume of Y (*b*) double the volume of Y
(*c*) half the volume of Y (*d*) greater than the volume of Y
(C.B.I. 1997)

74. The radius of a wire is decreased to one-third and its volume remains the same. The new length is how many times the original length ?
(*a*) 1 time (*b*) 3 times (*c*) 6 times (*d*) 9 times

75. A cylindrical tank of diameter 35 cm is full of water. If 11 litres of water is drawn off, the water level in the tank will drop by : (S.S.C. 1999)
(*a*) $10\frac{1}{2}$ cm (*b*) $11\frac{3}{7}$ cm (*c*) $12\frac{6}{7}$ cm (*d*) 14 cm

76. A well with 14 m inside diameter is dug 10 m deep. Earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. The height of the embankment is :
(*a*) $\frac{1}{2}$ m (*b*) $\frac{2}{3}$ m (*c*) $\frac{3}{4}$ m (*d*) $\frac{3}{5}$ m

77. Water flows through a cylindrical pipe of internal diameter 7 cm at 2 m per second. If the pipe is always half full, then what is the volume of water (in litres) discharged in 10 minutes ? (S.S.C. 2003)
(a) 2310 (b) 3850 (c) 4620 (d) 9240

78. The number of coins of radius 0.75 cm and thickness 0.2 cm to be melted to make a right circular cylinder of height 8 cm and base radius 3 cm is : (S.S.C. 2003)
(a) 460 (b) 500 (c) 600 (d) 640

79. Two cylindrical vessels with radii 15 cm and 10 cm and heights 35 cm and 15 cm respectively are filled with water. If this water is poured into a cylindrical vessel 15 cm in height, then the radius of the vessel is :
(a) 17.5 cm (b) 18 cm (c) 20 cm (d) 25 cm

80. 66 cubic centimetres of silver is drawn into a wire 1 mm in diameter. The length of the wire in metres will be : (C.B.I. 1998)
(a) 84 (b) 90 (c) 168 (d) 336

81. A hollow garden roller 63 cm wide with a girth of 440 cm is made of iron 4 cm thick. The volume of the iron used is :
(a) 54982 cm^3 (b) 56372 cm^3 (c) 57636 cm^3 (d) 58752 cm^3

82. A cylindrical tube open at both ends is made of metal. The internal diameter of the tube is 11.2 cm and its length is 21 cm. The metal everywhere is 0.4 cm thick. The volume of the metal is : (S.S.C. 2003)
(a) 280.52 cm^3 (b) 306.24 cm^3 (c) 310 cm^3 (d) 316 cm^3

83. What length of solid cylinder 2 cm in diameter mush be taken to cast into a hollow cylinder of external diameter 12 cm, 0.25 cm thick and 15 cm long ?
(a) 42.3215 cm (b) 44.0123 cm (c) 44.0625 cm (d) 44.6023 cm

84. A hollow iron pipe is 21 cm long and its external diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weighs 8 g/cm^3, then the weight of the pipe is :
(a) 3.6 kg (b) 3.696 kg (c) 36 kg (d) 36.9 kg (S.S.C. 2004)

85. A circular cylinder can hold 61.6 c.c. of water. If the height of the cylinder is 40 cm and the outer diameter is 16 mm, then the thickness of the material of the cylinder is :
(a) 0.2 mm (b) 0.3 mm (c) 1 mm (d) 2 mm

86. The radius of the base and height of a cone are 3 cm and 5 cm respectively whereas the radius of the base and height of a cylinder are 2 cm and 4 cm respectively. The ratio of the volume of cone to that of the cylinder is :
(a) 1 : 3 (b) 15 : 8 (c) 15 : 16 (d) 45 : 16

87. The curved surface of a right circular cone of height 15 cm and base diameter 16 cm is : (S.S.C. 1999)
(a) 60π cm^2 (b) 68π cm^2 (c) 120π cm^2 (d) 136π cm^2

88. What is the total surface area of a right circular cone of height 14 cm and base radius 7 cm ? (Hotel Management, 2001)
(a) 344.35 cm^2 (b) 462 cm^2 (c) 498.35 cm^2 (d) None of these

89. A right triangle with sides 3 cm, 4 cm and 5 cm is rotated about the side of 3 cm to form a cone. The volume of the cone so formed is : (S.S.C. 2000)
(a) 12π cm^3 (b) 15π cm^3 (c) 16π cm^3 (d) 20π cm^3

90. The slant height of a right circular cone is 10 m and its height is 8 m. Find the area of its curved surface. (R.R.B. 2003)
(a) 30π m^2 (b) 40π m^2 (c) 60π m^2 (d) 80π m^2

91. If a right circular cone of height 24 cm has a volume of 1232 cm^3, then the area of its curved surface is : (S.S.C. 2003)
(a) 154 cm^2 (b) 550 cm^2 (c) 704 cm^2 (d) 1254 cm^2

4. When B runs 25 m, A runs $\frac{45}{2}$ m.

When B runs 1000 m, A runs $\left(\frac{45}{2} \times \frac{1}{25} \times 1000\right)$ m = 900 m.

$\therefore$ B beats A by 100 m.

5. To reach the winning post A will have to cover a distance of (500 – 140) m, *i.e.*, 360 m.
While A covers 3 m, B covers 4 m.

While A covers 360 m, B covers $\left(\frac{4}{3} \times 360\right)$ m = 480 m.

Thus, when A reaches the winning post, B covers 480 m and therefore remains 20 m behind.

$\therefore$ A wins by 20 m.

6. Ratio of the speeds of A and B = $\frac{5}{3} : 1 = 5 : 3$.

Thus, in a race of 5 m, A gains 2 m over B.
2 m are gained by A in a race of 5 m.

80 m will be gained by A in a race of $\left(\frac{5}{2} \times 80\right)$ m = 200 m.

$\therefore$ Winning post is 200 m away from the starting point.

7. A : B = 100 : 75 and B : C = 100 : 96.

$\therefore$ A : C = $\left(\frac{A}{B} \times \frac{B}{C}\right) = \left(\frac{100}{75} \times \frac{100}{96}\right) = \frac{100}{72} = 100 : 72$.

$\therefore$ A beats C by (100 – 72) m = 28 m.

8. A : B = 100 : 90 and A : C = 100 : 72.

B : C = $\frac{B}{A} \times \frac{A}{C} = \frac{90}{100} \times \frac{100}{72} = \frac{90}{72}$.

When B runs 90 m, C runs 72 m.

When B runs 100 m, C runs $\left(\frac{72}{90} \times 100\right)$ m = 80 m.

$\therefore$ B can give C 20 m.

9. A : B = 100 : 90 and A : C = 100 : 87.

$\frac{B}{C} = \frac{B}{A} \times \frac{A}{C} = \frac{90}{100} \times \frac{100}{87} = \frac{30}{29}$.

When B runs 30 m, C runs 29 m.

When B runs 180 m, C runs $\left(\frac{29}{30} \times 180\right)$ m = 174 m.

$\therefore$ B beats C by (180 – 174) m = 6 m.

10. A : B = 200 : 169 and A : C = 200 : 182.

$\frac{C}{B} = \left(\frac{C}{A} \times \frac{A}{B}\right) = \left(\frac{182}{200} \times \frac{200}{169}\right) = 182 : 169$.

When C covers 182 m, B covers 169 m.

When C covers 350 m, B covers $\left(\frac{169}{182} \times 350\right)$ m = 325 m.

11. A's speed = $\left(5 \times \frac{5}{18}\right)$ m/sec = $\frac{25}{18}$ m/sec.

92. The slant height of a conical mountain is 2.5 km and the area of its base is 1.54 km^2. The height of the mountain is : (S.S.C. 2002)
(a) 2.2 km (b) 2.4 km (c) 3 km (d) 3.11 km

93. If the area of the base of a right circular cone is 3850 cm^2 and its height is 84 cm, then the curved surface area of the cone is :
(a) 10001 cm^2 (b) 10010 cm^2 (c) 10100 cm^2 (d) 11000 cm^2

94. Volume of a right circular cone having base radius 70 cm and curved surface area 40040 cm^2 is :
(a) 823400 cm^3 (b) 824000 cm^3 (c) 840000 cm^3 (d) 862400 cm^3

95. The radius and height of a right circular cone are in the ratio 3 : 4. If its volume is 96π cm^3, what is its slant height ? (C.B.I. 1997)
(a) 8 cm (b) 9 cm (c) 10 cm (d) 12 cm

96. The length of canvas 1.1 m wide required to build a conical tent of height 14 m and the floor area 346.5 sq. m is :
(a) 490 m (b) 525 m (c) 665 m (d) 860 m

97. If the radius of the base and the height of a right circular cone are doubled, then its volume becomes : (Asstt. Grade, 2003)
(a) 2 times (b) 3 times (c) 4 times (d) 8 times

98. If both the radius and height of a right circular cone are increased by 20%, its volume will be increased by : (S.S.C. 2004)
(a) 20% (b) 40% (c) 60% (d) 72.8%

99. If the height of a right circular cone is increased by 200% and the radius of the base is reduced by 50%, then the volume of the cone : (S.S.C. 2000)
(a) remains unaltered (b) decreases by 25%
(c) increases by 25% (d) increases by 50%

100. If the height of a cone be doubled and radius of base remains the same, then the ratio of the volume of the given cone to that of the second cone will be : (S.S.C. 2003)
(a) 1 : 2 (b) 2 : 1 (c) 1 : 8 (d) 8 : 1

101. Two cones have their heights in the ratio of 1 : 3 and radii 3 : 1. The ratio of their volumes is :
(a) 1 : 1 (b) 1 : 3 (c) 3 : 1 (d) 2 : 3

102. The radii of two cones are in the ratio 2 : 1, their volumes are equal. Find the ratio of their heights. (C.B.I. 1998)
(a) 1 : 8 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1

103. If the volumes of two cones are in the ratio of 1 : 4 and their diameters are in the ratio of 4 : 5, then the ratio of their heights is :
(a) 1 : 5 (b) 5 : 4 (c) 5 : 16 (d) 25 : 64

104. The volume of the largest right circular cone that can be cut out of a cube of edge 7 cm is : (M.A.T. 2002)
(a) 13.6 cm^3 (b) 89.8 cm^3 (c) 121 cm^3 (d) 147.68 cm^3

105. A cone of height 7 cm and base radius 3 cm is carved from a rectangular block of wood 10 cm × 5 cm × 2 cm. The percentage of wood wasted is :
(a) 34% (b) 46% (c) 54% (d) 66%

106. A right circular cone and a right circular cylinder have equal base and equal height. If the radius of the base and the height are in the ratio 5 : 12, then the ratio of the total surface area of the cylinder to that of the cone is :
(a) 3 : 1 (b) 13 : 9 (c) 17 : 9 (d) 34 : 9

107. A cylinder with base radius of 8 cm and height of 2 cm is melted to form a cone of height 6 cm. The radius of the cone will be : (R.R.B. 2003)
(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

108. A right cylindrical vessel is full of water. How many right cones having the same radius and height as those of the right cylinder will be needed to store that water ?
(a) 2 (b) 3 (c) 4 (d) 8

109. A solid metallic cylinder of base radius 3 cm and height 5 cm is melted to form cones, each of height 1 cm and base radius 1 mm. The number of cones is :
(a) 450 (b) 1350 (c) 4500 (d) 13500

110. Water flows at the rate of 10 metres per minute from a cylindrical pipe 5 mm in diameter. How long will it take to fill up a conical vessel whose diameter at the base is 40 cm and depth 24 cm ?
(a) 48 min. 15 sec. (b) 51 min. 12 sec. (c) 52 min. 1 sec. (d) 55 min.

111. A solid cylindrical block of radius 12 cm and height 18 cm is mounted with a conical block of radius 12 cm and height 5 cm. The total lateral surface of the solid thus formed is : (Hotel Management, 1998)
(a) 528 cm^2 (b) $1357\frac{5}{7}$ cm^2 (c) 1848 cm^2 (d) None of these

112. Consider the volumes of the following : (Civil Services, 2002)
1. A parallelopiped of length 5 cm, breadth 3 cm and height 4 cm
2. A cube of each side 4 cm
3. A cylinder of radius 3 cm and length 3 cm
4. A sphere of radius 3 cm

The volumes of these in the decreasing order is :
(a) 1, 2, 3, 4 (b) 1, 3, 2, 4 (c) 4, 2, 3, 1 (d) 4, 3, 2, 1

113. The volume of a sphere is 4851 cu. cm. Its curved surface area is :
(a) 1386 cm^2 (b) 1625 cm^2 (c) 1716 cm^2 (d) 3087 cm^2

114. The curved surface area of a sphere is 5544 sq. cm. Its volume is :
(a) 22176 cm^3 (b) 33951 cm^3 (c) 38808 cm^3 (d) 42304 cm^3

115. The volume of a sphere of radius r is obtained by multiplying its surface area by :
(a) $\frac{4}{3}$ (b) $\frac{r}{3}$ (c) $\frac{4r}{3}$ (d) $3r$

116. If the volume of a sphere is divided by its surface area, the result is 27 cm. The radius of the sphere is : (R.R.B. 2003)
(a) 9 cm (b) 36 cm (c) 54 cm (d) 81 cm

117. Spheres A and B have their radii 40 cm and 10 cm respectively. The ratio of the surface area of A to the surface area of B is : (S.S.C. 2003)
(a) 1 : 4 (b) 1 : 16 (c) 4 : 1 (d) 16 : 1

118. Surface area of a sphere is 2464 cm^2. If its radius be doubled, then the surface area of the new sphere will be :
(a) 4928 cm^2 (b) 9856 cm^2 (c) 19712 cm^2 (d) Data insufficient

119. If the radius of a sphere is doubled, how many times does its volume become ?
(a) 2 times (b) 4 times (c) 6 times (d) 8 times

120. If the radius of a sphere is increased by 2 cm, then its surface area increases by 352 cm^2. The radius of the sphere before the increase was : (C.B.I. 2003)
(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

121. If the measured value of the radius is 1.5% larger, the percentage error (correct to one decimal place) made in calculating the volume of a sphere is : (C.B.I. 1997)
(a) 2.1 (b) 3.2 (c) 4.6 (d) 5.4

122. The volumes of two spheres are in the ratio of 64 : 27. The ratio of their surface areas is : **(R.R.B. 2002)**

(*a*) 1 : 2 (*b*) 2 : 3 (*c*) 9 : 16 (*d*) 16 : 9

123. If the surface areas of two spheres are in the ratio of 4 : 25, then the ratio of their volumes is :

(*a*) 4 : 25 (*b*) 25 : 4 (*c*) 125 : 8 (*d*) 8 : 125

124. If three metallic spheres of radii 6 cms, 8 cms and 10 cms are melted to form a single sphere, the diameter of the new sphere will be : **(D.M.R.C. 2003)**

(*a*) 12 cms (*b*) 24 cms (*c*) 30 cms (*d*) 36 cms

125. A solid metallic sphere of radius 8 cm is melted and recast into spherical balls each of radius 2 cm. The number of spherical balls, thus obtained, is :

(*a*) 16 (*b*) 48 (*c*) 64 (*d*) 82

126. A spherical ball of lead, 3 cm in diameter is melted and recast into three spherical balls. The diameter of two of these are 1.5 cm and 2 cm respectively. The diameter of the third ball is :

(*a*) 2.5 cm (*b*) 2.66 cm (*c*) 3 cm (*d*) 3.5 cm

127. If a solid sphere of radius 10 cm is moulded into 8 spherical solid balls of equal radius, then the radius of each such ball is :

(*a*) 1.25 cm (*b*) 2.5 cm (*c*) 3.75 cm (*d*) 5 cm

128. A hollow spherical metallic ball has an external diameter 6 cm and is $\frac{1}{2}$ cm thick. The volume of metal used in the ball is : **(S.S.C. 2004)**

(*a*) $37\frac{2}{3}$ cm^3 (*b*) $40\frac{2}{3}$ cm^3 (*c*) $41\frac{2}{3}$ cm^3 (*d*) $47\frac{2}{3}$ cm^3

129. A solid piece of iron of dimensions 49 × 33 × 24 cm is moulded into a sphere. The radius of the sphere is : **(Hotel Management, 1999)**

(*a*) 21 cm (*b*) 28 cm (*c*) 35 cm (*d*) None of these

130. How many bullets can be made out of a cube of lead whose edge measures 22 cm, each bullet being 2 cm in diameter ?

(*a*) 1347 (*b*) 2541 (*c*) 2662 (*d*) 5324

131. How many lead shots each 3 mm in diameter can be made from a cuboid of dimensions 9 cm × 11 cm × 12 cm ?

(*a*) 7200 (*b*) 8400 (*c*) 72000 (*d*) 84000

132. A sphere and a cube have equal surface areas. The ratio of the volume of the sphere to that of the cube is :

(*a*) $\sqrt{\pi} : \sqrt{6}$ (*b*) $\sqrt{2} : \sqrt{\pi}$ (*c*) $\sqrt{\pi} : \sqrt{3}$ (*d*) $\sqrt{6} : \sqrt{\pi}$

133. The ratio of the volume of a cube to that of a sphere which will fit inside the cube is :

(*a*) 4 : π (*b*) 4 : 3π (*c*) 6 : π (*d*) 2 : π

134. The surface area of a sphere is same as the curved surface area of a right circular cylinder whose height and diameter are 12 cm each. The radius of the sphere is :

(*a*) 3 cm (*b*) 4 cm (*c*) 6 cm (*d*) 12 cm **(S.S.C. 2002)**

135. The diameter of the iron ball used for the shot-put game is 14 cm. It is melted and then a solid cylinder of height $2\frac{1}{3}$ cm is made. What will be the diameter of the base of the cylinder ? **(S.S.C. 2004)**

(*a*) 14 cm (*b*) $\frac{14}{3}$ cm (*c*) 28 cm (*d*) $\frac{28}{3}$ cm

136. The volume of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is : **(C.B.I. 1997)**

(a) $\frac{4}{3}\pi$ (b) $\frac{10}{3}\pi$ (c) 5π (d) $\frac{20}{3}\pi$

137. How many spherical bullets can be made out of a lead cylinder 15 cm high and with base radius 3 cm, each bullet being 5 mm in diameter ?

(a) 6000 (b) 6480 (c) 7260 (d) 7800

138. A cylindrical rod of iron whose height is eight times its radius is melted and cast into spherical balls each of half the radius of the cylinder. The number of spherical balls is :

(a) 12 (b) 16 (c) 24 (d) 48

139. The diameter of a sphere is 8 cm. It is melted and drawn into a wire of diameter 3 mm. The length of the wire is :

(a) 36.9 m (b) 37.9 m (c) 38.9 m (d) 39.9 m

140. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by : **(M.B.A. 2000)**

(a) $\frac{2}{9}$ cm (b) $\frac{4}{9}$ cm (c) $\frac{9}{4}$ cm (d) $\frac{9}{2}$ cm

141. 12 spheres of the same size are made from melting a solid cylinder of 16 cm diameter and 2 cm height. The diameter of each sphere is : **(S.S.C. 2000)**

(a) $\sqrt{3}$ cm (b) 2 cm (c) 3 cm (d) 4 cm

142. A cylindrical tub of radius 12 cm contains water upto a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. The radius of the ball is :

(a) 4.5 cm (b) 6 cm (c) 7.25 cm (d) 9 cm

143. A solid metallic spherical ball of diameter 6 cm is melted and recast into a cone with diameter of the base as 12 cm. The height of the cone is : **(C.B.I. 2003)**

(a) 2 cm (b) 3 cm (c) 4 cm (d) 6 cm

144. A cone of height 9 cm with diameter of its base 18 cm is carved out from a wooden solid sphere of radius 9 cm. The percentage of the wood wasted is : **(S.S.C. 2000)**

(a) 25% (b) 25π% (c) 50% (d) 75%

145. A metallic cone of radius 12 cm and height 24 cm is melted and made into spheres of radius 2 cm each. How many spheres are there ?

(a) 108 (b) 120 (c) 144 (d) 180

146. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. The height of the cone is : **(R.R.B. 2002)**

(a) 12 cm (b) 14 cm (c) 15 cm (d) 18 cm

147. In what ratio are the volumes of a cylinder, a cone and a sphere, if each has the same diameter and the same height ?

(a) 1 : 3 : 2 (b) 2 : 3 : 1 (c) 3 : 1 : 2 (d) 3 : 2 : 1

148. The total surface area of a solid hemisphere of diameter 14 cm, is :

(a) 308 cm^2 (b) 462 cm^2 (c) 1232 cm^2 (d) 1848 cm^2

149. Volume of a hemisphere is 19404 cu. cm. Its radius is :

(a) 10.5 cm (b) 17.5 cm (c) 21 cm (d) 42 cm

150. The capacities of two hemispherical vessels are 6.4 litres and 21.6 litres. The areas of inner curved surfaces of the vessels will be in the ratio of :

(a) $\sqrt{2} : \sqrt{3}$ (b) 2 : 3 (c) 4 : 9 (d) 16 : 81

151. A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder, the volume of the beverage in the cylindrical vessel is : **(I.A.S. 1999)**

(*a*) $66\frac{2}{3}\%$ (*b*) $78\frac{1}{2}\%$ (*c*) 100%

(*d*) More than 100% (*i.e.*, some liquid will be left in the bowl).

152. A metallic hemisphere is melted and recast in the shape of a cone with the same base radius (R) as that of the hemisphere. If H is the height of the cone, then :

(*a*) $H = 2R$ (*b*) $H = 3R$ (*c*) $H = \sqrt{3}R$ (*d*) $H = \frac{2}{3}R$

(S.S.C. 1999)

153. A hemisphere of lead of radius 6 cm is cast into a right circular cone of height 75 cm. The radius of the base of the cone is :

(*a*) 1.4 cm (*b*) 2 cm (*c*) 2.4 cm (*d*) 4.2 cm

154. A hemisphere and a cone have equal bases. If their heights are also equal, then the ratio of their curved surfaces will be : **(S.S.C. 2002)**

(*a*) 1 : 2 (*b*) 2 : 1 (*c*) $1:\sqrt{2}$ (*d*) $\sqrt{2}:1$

155. A sphere of maximum volume is cut out from a solid hemisphere of radius *r*. The ratio of the volume of the hemisphere to that of the cut out sphere is :

(*a*) 3 : 2 (*b*) 4 : 1 (*c*) 4 : 3 (*d*) 7 : 4

ANSWERS

1. (*d*)	2. (*c*)	3. (*a*)	4. (*b*)	5. (*b*)	6. (*b*)	7. (*b*)	8. (*b*)
9. (*a*)	10. (*d*)	11. (*b*)	12. (*c*)	13. (*b*)	14. (*c*)	15. (*c*)	16. (*b*)
17. (*c*)	18. (*a*)	19. (*c*)	20. (*c*)	21. (*b*)	22. (*b*)	23. (*b*)	24. (*b*)
25. (*a*)	26. (*a*)	27. (*c*)	28. (*c*)	29. (*b*)	30. (*a*)	31. (*d*)	32. (*c*)
33. (*d*)	34. (*d*)	35. (*d*)	36. (*b*)	37. (*d*)	38. (*d*)	39. (*d*)	40. (*c*)
41. (*d*)	42. (*b*)	43. (*c*)	44. (*b*)	45. (*b*)	46. (*a*)	47. (*b*)	48. (*b*)
49. (*c*)	50. (*b*)	51. (*c*)	52. (*b*)	53. (*c*)	54. (*d*)	55. (*d*)	56. (*d*)
57. (*e*)	58. (*c*)	59. (*b*)	60. (*b*)	61. (*a*)	62. (*b*)	63. (*c*)	64. (*d*)
65. (*b*)	66. (*b*)	67. (*b*)	68. (*b*)	69. (*a*)	70. (*b*)	71. (*c*)	72. (*d*)
73. (*c*)	74. (*d*)	75. (*b*)	76. (*b*)	77. (*c*)	78. (*d*)	79. (*d*)	80. (*a*)
81. (*d*)	82. (*b*)	83. (*c*)	84. (*b*)	85. (*c*)	86. (*c*)	87. (*d*)	88. (*c*)
89. (*a*)	90. (*c*)	91. (*b*)	92. (*b*)	93. (*b*)	94. (*d*)	95. (*c*)	96. (*b*)
97. (*d*)	98. (*d*)	99. (*b*)	100. (*a*)	101. (*c*)	102. (*b*)	103. (*d*)	104. (*b*)
105. (*a*)	106. (*c*)	107. (*d*)	108. (*b*)	109. (*d*)	110. (*b*)	111. (*d*)	112. (*d*)
113. (*a*)	114. (*c*)	115. (*b*)	116. (*d*)	117. (*d*)	118. (*b*)	119. (*d*)	120. (*d*)
121. (*c*)	122. (*d*)	123. (*d*)	124. (*b*)	125. (*c*)	126. (*a*)	127. (*d*)	128. (*d*)
129. (*a*)	130. (*b*)	131. (*d*)	132. (*d*)	133. (*c*)	134. (*c*)	135. (*c*)	136. (*a*)
137. (*b*)	138. (*d*)	139. (*b*)	140. (*c*)	141. (*d*)	142. (*d*)	143. (*b*)	144. (*d*)
145. (*a*)	146. (*b*)	147. (*c*)	148. (*b*)	149. (*c*)	150. (*c*)	151. (*c*)	152. (*a*)
153. (*c*)	154. (*d*)	155. (*b*)					

SOLUTIONS

1. Capacity of the bank = Volume of the tank
$= \left(\frac{8 \times 100 \times 6 \times 100 \times 2.5 \times 100}{1000}\right)$ litres = 120000 litres.

2. Surface area = $[2\ (10 \times 4 + 4 \times 3 + 10 \times 3)]\ cm^2 = (2 \times 82)\ cm^2 = 164\ cm^2$.

3. Area of the wet surface = $[2\ (lb + bh + lh) - lb] = 2\ (bh + lh) + lb$
$= [2\ (4 \times 1.25 + 6 \times 1.25) + 6 \times 4]\ m^2 = 49\ m^2$.

4. Volume of water displaced = $(3 \times 2 \times 0.01)\ m^3 = 0.06\ m^3$.
∴ Mass of man = Volume of water displaced × Density of water
$= (0.06 \times 1000)$ kg = 60 kg.

5. Volume = $(2.6 \times 100 \times 100 \times 100)$ cu. cm.
Depth = $\frac{\text{Volume}}{\text{Area of the base}} = \left(\frac{2.6 \times 100 \times 100 \times 100}{6500}\right)$ cm = 400 cm = 4 m.

6. Let length = x cm. Then, $x \times 28 \times 5 \times \frac{25}{1000} = 112$
∴ $x = \left(112 \times \frac{1000}{25} \times \frac{1}{28} \times \frac{1}{5}\right)$ cm = 32 cm.

7. Volume of gold = $\left(\frac{1}{2} \times 100 \times 100 \times 100\right) cm^3$.
Area of sheet = $10000\ m^2 = (10000 \times 100 \times 100)\ cm^2$
∴ Thickness of the sheet = $\left(\frac{1 \times 100 \times 100 \times 100}{2 \times 10000 \times 100 \times 100}\right)$ cm = 0.005 cm.

8. Area = $(1.5 \times 10000)\ m^2 = 15000\ m^2$.
Depth = $\frac{5}{100}$ m = $\frac{1}{20}$ m.
∴ Volume = (Area × Depth) = $\left(15000 \times \frac{1}{20}\right) m^3 = 750\ m^3$.

9. Let the width of the wall be x metres.
Then, Height = $(6x)$ metres and Length = $(42x)$ metres.
∴ $42x \times x \times 6x = 16128 \Leftrightarrow x^3 = \left(\frac{16128}{42 \times 6}\right) = 64 \Leftrightarrow x = 4$.

10. Let the dimensions be $3x$, $2x$ and x respectively. Then,
$3x \times 2x \times x = 10368 \Leftrightarrow x^3 = \left(\frac{10368}{6}\right) = 1728 \Leftrightarrow x = 12$.
So, the dimensions of the block are 36 dm, 24 dm, and 12 dm.
Surface area = $[2\ (36 \times 24 + 24 \times 12 + 36 \times 12)]\ dm^2$
$= [2 \times 144\ (6 + 2 + 3)]\ dm^2 = 3168\ dm^2$.
∴ Cost of polishing = Rs. $\left(\frac{2 \times 3168}{100}\right)$ = Rs. 63.36.

11. Let the dimensions of the cuboid be x, $2x$ and $3x$.
Then, $2\ (x \times 2x + 2x \times 3x + x \times 3x) = 88$
$\Leftrightarrow \quad 2x^2 + 6x^2 + 3x^2 = 44 \Leftrightarrow 11x^2 = 44 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2$.
∴ Volume of the cuboid = $(2 \times 4 \times 6)\ cm^3 = 48\ cm^3$.

12. Required length $= \sqrt{8^2 + 6^2 + 2^2}$ cm $= \sqrt{104}$ cm $= 2\sqrt{26}$ cm.

13. Required length $= \sqrt{(16)^2 + (12)^2 + \left(\frac{32}{3}\right)^2}$ m $= \sqrt{256 + 144 + \frac{1024}{9}}$ m

$= \sqrt{\frac{4624}{9}}$ m $= \frac{68}{3}$ m $= 22\frac{2}{3}$ m.

14. Number of bricks $= \frac{\text{Volume of the wall}}{\text{Volume of 1 brick}} = \left(\frac{800 \times 600 \times 22.5}{25 \times 11.25 \times 6}\right) = 6400.$

15. Volume of the bricks = 95% of volume of wall $= \left(\frac{95}{100} \times 600 \times 500 \times 50\right)$ cm^3.

Volume of 1 brick $= (25 \times 12.5 \times 7.5)$ cm^3.

$\therefore$ Number of bricks $= \left(\frac{95}{100} \times \frac{600 \times 500 \times 50}{25 \times 12.5 \times 7.5}\right) = 6080.$

16. Total volume of water displaced $= (4 \times 50)$ m^3 $= 200$ m^3.

$\therefore$ Rise in water level $= \left(\frac{200}{40 \times 20}\right)$ m $= 0.25$ m $= 25$ cm.

17. Volume of earth dug out $= \left(4 \times \frac{5}{2} \times \frac{3}{2}\right)$ m^3 $= 15$ m^3.

Area over which earth is spread $= \left(31 \times 10 - 4 \times \frac{5}{2}\right)$ m^2 $= 300$ m^2.

$\therefore$ Rise in level $= \left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{15}{300} \times 100\right)$ cm $= 5$ cm.

18. Length of water column flown in 1 min. $= \left(\frac{3.5 \times 1000}{60}\right)$ m $= \frac{175}{3}$ m.

$\therefore$ Volume flown per minute $= \left(\frac{175}{3} \times 36 \times \frac{3}{2}\right)$ m^3 $= 3150$ m^3.

19. Length of water column flown in 1 min. $= \left(\frac{10 \times 1000}{60}\right)$ m $= \frac{500}{3}$ m.

Volume flown per minute $= \left(\frac{500}{3} \times \frac{40}{100 \times 100}\right)$ m^3 $= \frac{2}{3}$ m^3.

Volume flown in half an hour $= \left(\frac{2}{3} \times 30\right)$ m^3 $= 20$ m^3.

$\therefore$ Rise in water level $= \left(\frac{20}{40 \times 80}\right)$ m $= \left(\frac{1}{160} \times 100\right)$ cm $= \frac{5}{8}$ cm.

20. $2(15 + 12) \times h = 2(15 \times 12)$ or $h = \frac{180}{27}$ m $= \frac{20}{3}$ m.

$\therefore$ Volume $= \left(15 \times 12 \times \frac{20}{3}\right)$ m^3 $= 1200$ m^3.

21. $(l + b + h) = 19$ and $\sqrt{l^2 + b^2 + h^2} = 5\sqrt{5}$ and so $(l^2 + b^2 + h^2) = 125$.

Now, $(l + b + h)^2 = 19^2 \Rightarrow (l^2 + b^2 + h^2) + 2(lb + bh + lh) = 361$

$\Rightarrow 2(lb + bh + lh) = (361 - 125) = 236.$

$\therefore$ Surface area $= 236$ cm^2.

22. Volume $= \left[12 \times 9 \times \left(\frac{1+4}{2}\right)\right] m^3 = (12 \times 9 \times 2.5)\, m^3 = 270\, m^3$.

23. Clearly, $l = (48 - 16)$ m $= 32$ m, $b = (36 - 16)$ m $= 20$ m, $h = 8$ m.
$\therefore$ Volume of the box $= (32 \times 20 \times 8)\, m^3 = 5120\, m^3$.

24. Internal length $= (146 - 6)$ cm $= 140$ cm.
Internal breadth $= (116 - 6)$ cm $= 110$ cm.
Internal depth $= (83 - 3)$ cm $= 80$ cm.
Area of inner surface $= [2(l + b) \times h] + lb$
$= [2(140 + 110) \times 80 + 140 \times 110]\, cm^2 = 55400\, cm^2$.
$\therefore$ Cost of painting $= \text{Rs.}\left(\frac{1}{2} \times \frac{1}{100} \times 55400\right) = \text{Rs. } 277$.

25. Let the thickness of the bottom be x cm.
Then, $[(330 - 10) \times (260 - 10) \times (110 - x)] = 8000 \times 1000$

$\Leftrightarrow \quad 320 \times 250 \times (110 - x) = 8000 \times 1000 \quad \Leftrightarrow \quad (110 - x) = \frac{8000 \times 1000}{320 \times 250} = 100$

$\Leftrightarrow \quad x = 10$ cm $= 1$ dm.

26. Let the dimensions of the bigger cuboid be x, y and z.
Then, Volume of the bigger cuboid $= xyz$.

Volume of the miniature cuboid $= \left(\frac{1}{4}x\right)\left(\frac{1}{4}y\right)\left(\frac{1}{4}z\right) = \frac{1}{64}xyz$.

$\therefore$ Weight of the miniature cuboid $= \left(\frac{1}{64} \times 16\right)$ kg $= 0.25$ kg.

27. Let length $= l$, breadth $= b$ and height $= h$. Then,
Product of areas of 3 adjacent faces $= (lb \times bh \times lh) = (lbh)^2 = (\text{Volume})^2$.

28. Let the length, breadth and height of the box be l, b and h respectively. Then,
Volume $= lbh = \sqrt{(lbh)^2} = \sqrt{lb \times bh \times lh} = \sqrt{120 \times 72 \times 60} = 720\, cm^3$.

29. Let $lb = 2x$, $bh = 3x$ and $lh = 4x$.
Then, $24x^3 = (lbh)^2 = 9000 \times 9000 \Rightarrow x^3 = 375 \times 9000 \Rightarrow x = 150$.
So, $lb = 300$, $bh = 450$, $lh = 600$ and $lbh = 9000$.
$\therefore \quad h = \frac{9000}{300} = 30$, $l = \frac{9000}{450} = 20$ and $b = \frac{9000}{600} = 15$.
Hence, shortest side $= 15$ cm.

30. Edge of the cube $= \left(\frac{20}{4}\right)$ cm $= 5$ cm.
$\therefore$ Volume $= (5 \times 5 \times 5)\, cm^3 = 125\, cm^3$.

31. Surface area $= \left[6 \times \left(\frac{1}{2}\right)^2\right] cm^2 = \frac{3}{2}\, cm^2$.

32. Surface area of the cube $= (6 \times 8^2)$ sq. ft. $= 384$ sq. ft.
Quantity of paint required $= \left(\frac{384}{16}\right)$ kg $= 24$ kg.
$\therefore$ Cost of painting $= \text{Rs. } (36.50 \times 24) = \text{Rs. } 876$.

33. Volume of the cube $= (270 \times 100 \times 64)\, cm^3$.
Edge of the cube $= \sqrt{270 \times 100 \times 64}$ cm $= (3 \times 10 \times 4)$ cm $= 120$ cm.
$\therefore$ Surface area $= (6 \times 120 \times 120)\, cm^2 = 86400\, cm^2$.

34. Surface area = $\left(\frac{34398}{13}\right)$ = 2646 cm^2.

$\therefore$ $6a^2 = 2646 \Rightarrow a^2 = 441 \Rightarrow a = 21$.

So, Volume = (21 × 21 × 21) cm^3 = 9261 cm^3.

35. $a^3 = 729 \Rightarrow a = 9$.

$\therefore$ Surface area = (6 × 9 × 9) cm^2 = 486 cm^2.

36. Required length = Diagonal = $\sqrt{3}\,a = (\sqrt{3} \times \sqrt{3})$ m = 3 m.

37. $\sqrt{3}\,a = 4\sqrt{3} \Rightarrow a = 4$.

$\therefore$ Volume = (4 × 4 × 4) cm^3 = 64 cm^3.

38. $6a^2 = 600 \Rightarrow a^2 = 100 \Rightarrow a = 10$.

$\therefore$ Diagonal = $\sqrt{3}\,a = 10\sqrt{3}$ cm.

39. $a^3 = 6a^2 \Rightarrow a = 6$.

40. Number of cubes = $\left(\frac{100 \times 100 \times 100}{10 \times 10 \times 10}\right)$ = 1000.

41. Number of blocks = $\left(\frac{160 \times 100 \times 60}{20 \times 20 \times 20}\right)$ = 120.

42. Number of cubes = $\left(\frac{18 \times 18 \times 18}{3 \times 3 \times 3}\right)$ = 216.

43. Volume of block = (6 × 9 × 12) cm^3 = 648 cm^3.

Side of largest cube = H.C.F. of 6 cm, 9 cm, 12 cm = 3 cm.

Volume of this cube = (3 × 3 × 3) = 27 cm^3.

$\therefore$ Number of cubes = $\left(\frac{648}{27}\right)$ = 24.

44. Side of smallest cube = L.C.M. of 5 cm, 10 cm, 20 cm = 20 cm.

Volume of the cube = (20 × 20 × 20) cm^3 = 8000 cm^3.

Volume of the block = (5 × 10 × 20) cm^3 = 1000 cm^3.

$\therefore$ Number of blocks = $\left(\frac{8000}{1000}\right)$ = 8.

45. Let the sides of the sheet be x and 5x. Then,

Volume of the sheet = Volume of the cube

$\Rightarrow$ $x \times 5x \times \frac{1}{2} = 10 \times 10 \times 10 \Rightarrow 5x^2 = 2000 \Rightarrow x^2 = 400 \Rightarrow x = 20$.

$\therefore$ The sides are 20 cm and 100 cm.

46. Volume of the new cube = ($6^3 + 8^3 + 10^3$) cm^3 = 1728 cm^3.

Let the edge of the new cube be a cm.

$\therefore$ $a^3 = 1728 \Rightarrow a = 12$.

47. The new solid formed is a cuboid of length 25 cm, breadth 5 cm and height 5 cm.

$\therefore$ Volume = (25 × 5 × 5) cm^3 = 625 cm^3.

48. Required ratio = $\frac{6 \times 1 \times 1}{6 \times 5 \times 5} = \frac{1}{25}$ = 1 : 25.

49. Volume of the large cube = ($3^3 + 4^3 + 5^3$) cm^3 = 216 cm^3.

Let the edge of the large cube be a.

So, $a^3 = 216 \Rightarrow a = 6$ cm.

$\therefore$ Required ratio = $\frac{6 \times (3^2 + 4^2 + 5^2)}{6 \times 6^2} = \frac{50}{36}$ = 25 : 18.

50. Let the sides of the three cubes be $3x$, $4x$ and $5x$.
Then, Volume of the new cube $= [(3x)^3 + (4x)^3 + (5x)^3] = 216x^3$.
Edge of the new cube $= (216x^3)^{1/3} = 6x$.
Diagonal of the new cube $= 6\sqrt{3}\,x$.
$\therefore \quad 6\sqrt{3}\,x = 12\sqrt{3} \Rightarrow x = 2$.
So, the sides of the cubes are 6 cm, 8 cm and 10 cm.

51. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{27}{1} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{3}{1}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{3}{1} \Leftrightarrow a : b = 3 : 1.$$

52. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{8}{27} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{2}{3}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{2}{3} \Leftrightarrow \frac{a^2}{b^2} = \frac{4}{9} \Leftrightarrow \frac{6a^2}{6b^2} = \frac{4}{9}.$$

53. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{1}{27} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{1}{3}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{1}{3} \Leftrightarrow \frac{a^2}{b^2} = \frac{1}{9}.$$

54. Let original edge $= a$. Then, volume $= a^3$.
New edge $= 2a$. So, new volume $= (2a)^3 = 8a^3$.
$\therefore$ Volume becomes 8 times.

55. Let original edge $= a$. Then, surface area $= 6a^2$.

$$\text{New edge} = \frac{125}{100}a = \frac{5a}{4}.$$

$$\text{New surface area} = 6 \times \left(\frac{5a}{4}\right)^2 = \frac{75a^2}{8}.$$

$$\text{Increase in surface area} = \left(\frac{75a^2}{8} - 6a^2\right) = \frac{27a^2}{8}.$$

$$\therefore \quad \text{Increase \%} = \left(\frac{27a^2}{8} \times \frac{1}{6a^2} \times 100\right)\% = 56.25\%.$$

56. Volume $= \pi r^2 h = \left(\frac{22}{7} \times 1 \times 1 \times 14\right) \text{m}^3 = 44 \text{ m}^3$.

57. Volume of the tank $= 246.4$ litres $= 246400 \text{ cm}^3$.
Let the radius of the base be r cm. Then,

$$\left(\frac{22}{7} \times r^2 \times 400\right) = 246400 \Leftrightarrow r^2 = \left(\frac{246400 \times 7}{22 \times 400}\right) = 196 \Leftrightarrow r = 14.$$

$\therefore$ Diameter of the base $= 2r = 28$ cm.

58. $2\pi r = 66 \Rightarrow r = \left(66 \times \frac{1}{2} \times \frac{7}{22}\right) = \frac{21}{2}$ cm.

$$\frac{2\pi rh}{2\pi r} = \left(\frac{2640}{66}\right) \Rightarrow h = 40 \text{ cm}.$$

$$\therefore \quad \text{Volume} = \left(\frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 40\right) \text{cm}^3 = 13860 \text{ cm}^3.$$

59. Let the radius and height be r cm each.

Then, $\frac{22}{7} \times r^2 \times r = \frac{176}{7} \Rightarrow r^3 = \left(\frac{176}{7} \times \frac{7}{22}\right) = 8 \Rightarrow r = 2$.

60. $\frac{2\pi rh}{h} = \frac{704}{14} \Rightarrow 2\pi r = \frac{704}{14}$.

$\therefore \quad r = \left(\frac{704}{14} \times \frac{1}{2} \times \frac{7}{22}\right) = 8$ cm.

$\therefore$ Volume $= \left(\frac{22}{7} \times 8 \times 8 \times 14\right)$ cm^3 $= 2816$ cm^3.

61. Total surface area $= 2\pi r(h + r) = \left[2 \times \frac{22}{7} \times \frac{35}{100} \times (1.25 + 0.35)\right]$ m^2

$= \left(2 \times \frac{22}{7} \times \frac{35}{100} \times \frac{16}{10}\right)$ m^2 $= 3.52$ m^2.

$\therefore$ Cost of the material $=$ Rs. $(3.52 \times 80) =$ Rs. 281.60.

62. Curved surface area $= 2\pi rh = (\pi r^2 h) \cdot \frac{2}{r} = \left(\text{Volume} \times \frac{2}{r}\right)$.

63. $\frac{\text{Total surface area}}{\text{Lateral surface area}} = \frac{2\pi rh + 2\pi r^2}{2\pi rh} = \frac{(h + r)}{h} = \frac{80}{60} = \frac{4}{3}$.

64. Difference in capacities $= \left(8 \times 8 \times 14 - \frac{22}{7} \times 4 \times 4 \times 14\right)$ cm^3 $= 192$ cm^3.

65. Let radius $= 2x$ and height $= 3x$. Then,

$\frac{22}{7} \times (2x)^2 \times 3x = 12936 \Leftrightarrow x^3 = \left(12936 \times \frac{7}{22} \times \frac{1}{12}\right) = 343 = 7^3$

$\therefore \quad x = 7$. So, radius $= 14$ cm and height $= 21$ cm.

$\therefore$ Total surface area $= 2 \times \frac{22}{7} \times 14 \times (21 + 14) = \left(2 \times \frac{22}{7} \times 14 \times 35\right)$ cm^2 $= 3080$ cm^2.

66. It is given that $r = \frac{1}{2}h$ and $2\pi rh + \pi r^2 = 616$ m^2

$\therefore \quad 2\pi \times \frac{1}{2}h \times h + \pi \times \frac{1}{4}h^2 = 616$

$\Rightarrow \quad \frac{5}{4} \times \frac{22}{7} \times h^2 = 616 \Rightarrow h^2 = \left(616 \times \frac{28}{110}\right) = \frac{28 \times 28}{5}$.

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{1}{4}h^2 \times h = \frac{22}{7} \times \frac{1}{4} \times \frac{28 \times 28}{5} \times \frac{28}{\sqrt{5}}$ cm^3

$= \left(\frac{22 \times 28 \times 28}{25} \times \sqrt{5}\right)$ cm^3 $= \left(\frac{22 \times 28 \times 28 \times 2.23}{25 \times 1000}\right)$ litres $= 1.53$ litre.

67. $(h + r) = 37$ and $2\pi r(h + r) = 1628$.

$\therefore \quad 2\pi r \times 37 = 1628$ or $r = \left(\frac{1628}{2 \times 37} \times \frac{7}{22}\right) = 7$.

So, $r = 7$ m and $h = 30$ m.

$\therefore$ Volume $= \left(\frac{22}{7} \times 7 \times 7 \times 30\right)$ m^3 $= 4620$ m^3.

68. $\frac{\pi r^2 h}{2\pi r h} = \frac{924}{264} \Rightarrow r = \left(\frac{924}{264} \times 2\right) = 7$ m.

And, $2\pi r h = 264 \Rightarrow h = \left(264 \times \frac{7}{22} \times \frac{1}{2} \times \frac{1}{7}\right) = 6$ m.

$\therefore$ Required ratio $= \frac{2r}{h} = \frac{14}{6} = 7:3$.

69. $V = \pi r^2 h$ and $S = 2\pi r h + 2\pi r^2$

$\Rightarrow S = 2\pi r(h + r)$, where $h = \frac{V}{\pi r^2}$

$\Rightarrow S = 2\pi r\left(\frac{V}{\pi r^2} + r\right) = \frac{2V}{r} + 2\pi r^2 \Rightarrow \frac{dS}{dr} = \frac{-2V}{r^2} + 4\pi r$ and $\frac{d^2S}{dr^2} = \left(\frac{4V}{r^3} + 4\pi\right) > 0$

$\therefore$ S is minimum when $\frac{dS}{dr} = 0$

$\Leftrightarrow \frac{-2V}{r^2} + 4\pi r = 0 \Leftrightarrow V = 2\pi r^3 \Leftrightarrow \pi r^2 h = 2\pi r^3 \Leftrightarrow h = 2r$.

70. Let original radius = R. Then, new radius = $\frac{R}{2}$.

$$\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{R}{2}\right)^2 \times h}{\pi \times R^2 \times h} = \frac{1}{4}.$$

71. Let their radii be $2x$, $3x$ and heights be $5y$, $3y$.

Ratio of their volumes $= \frac{\pi \times (2x)^2 \times 5y}{\pi \times (3x)^2 \times 3y} = \frac{20}{27}$.

72. Let their heights be h and $2h$ and radii be r and R respectively. Then,

$\pi r^2 h = \pi R^2 (2h) \Rightarrow \frac{r^2}{R^2} = \frac{2h}{h} = \frac{2}{1} \Rightarrow \frac{r}{R} = \frac{\sqrt{2}}{1}$ i.e. $\sqrt{2}:1$.

73. Let the height of X and Y be h, and their radii be r and $2r$ respectively. Then,
Volume of X $= \pi r^2 h$ and Volume of Y $= \pi (2r)^2 h = 4\pi r^2 h$.
New height of X $= 2h$.
So, new volume of X $= \pi r^2 (2h) = 2\pi r^2 h = \frac{1}{2}(4\pi r^2 h) = \frac{1}{2} \times$ (Volume of Y).

74. Let original radius $= r$ and original length $= h$.

New radius $= \frac{r}{3}$ and let new length $= H$.

Then, $\pi r^2 h = \pi\left(\frac{r}{3}\right)^2 \times H$ or $H = 9h$.

75. Let the drop in the water level be h cm. Then,

$\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times h = 11000 \Leftrightarrow h = \left(\frac{11000 \times 7 \times 4}{22 \times 35 \times 35}\right)$ cm $= \frac{80}{7}$ cm $= 11\frac{3}{7}$ cm.

76. Volume of earth dug out = $\left(\frac{22}{7}\times 7\times 7\times 10\right)\text{m}^3 = 1540\text{ m}^3$.

Area of embankment = $\frac{22}{7}\times\left[(28)^2-(7)^2\right] = \left(\frac{22}{7}\times 35\times 21\right)\text{m}^2 = 2310\text{ m}^2$.

Height of embankment = $\left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{1540}{2310}\right)\text{m} = \frac{2}{3}\text{m}$.

77. Volume of water flown in 1 sec. = $\left(\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 200\right)\text{cm}^3 = 7700\text{ cm}^3$.

Volume of water flown in 10 min. = $(7700\times 60\times 10)\text{ cm}^3$

$= \left(\frac{7700\times 60\times 10}{1000}\right)\text{litres} = 4620\text{ litres}$.

78. Volume of one coin = $\left(\frac{22}{7}\times\frac{75}{100}\times\frac{75}{100}\times\frac{2}{10}\right)\text{cm}^3 = \frac{99}{280}\text{ cm}^3$.

Volume of larger cylinder = $\left(\frac{22}{7}\times 3\times 3\times 8\right)\text{cm}^3$.

$\therefore$ Number of coins = $\left(\frac{22\times 9\times 8}{7}\times\frac{280}{99}\right) = 640$.

79. Let the radius of the vessel be R. Then,

$\pi R^2\times 15 = \pi\times(15)^2\times 35 + \pi\times(10)^2\times 15$

$\Leftrightarrow\ \pi R^2\times 15 = 9375\pi\ \Leftrightarrow\ R^2 = 625\ \Leftrightarrow\ R = 25$ cm.

80. Let the length of the wire be h.

Radius = $\frac{1}{2}$ mm = $\frac{1}{20}$ cm. Then,

$\frac{22}{7}\times\frac{1}{20}\times\frac{1}{20}\times h = 66\ \Leftrightarrow\ h = \left(\frac{66\times 20\times 20\times 7}{22}\right) = 8400\text{ cm} = 84\text{ m}$.

81. Circumference of the girth = 440 cm.

$\therefore\ 2\pi R = 440\ \Rightarrow\ R = \left(440\times\frac{1}{2}\times\frac{7}{22}\right) = 70$ cm.

So, Outer radius = 70 cm. Inner radius = (70 − 4) cm = 66 cm.

Volume of iron = $\pi\left[(70)^2-(66)^2\right]\times 63 = \left(\frac{22}{7}\times 136\times 4\times 63\right)\text{cm}^3 = 58752\text{ cm}^3$.

82. Internal radius = $\left(\frac{11.2}{2}\right)$ cm = 5.6 cm, External radius = (5.6 + 0.4) cm = 6 cm.

Volume of metal = $\left\{\frac{22}{7}\times\left[(6)^2-(5.6)^2\right]\times 21\right\}\text{cm}^3 = (66\times 11.6\times 0.4)\text{ cm}^3 = 306.24\text{ cm}^3$.

83. External radius = 6 cm, Internal radius = (6 − 0.25) cm = 5.75 cm.

Volume of material in hollow cylinder

$= \left\{\frac{22}{7}\times\left[(6)^2-(5.75)^2\right]\times 15\right\}\text{cm}^3 = \left(\frac{22}{7}\times 11.75\times 0.25\times 15\right)\text{cm}^3$

$= \left(\frac{22}{7}\times\frac{1175}{100}\times\frac{25}{100}\times 15\right)\text{cm}^3 = \left(\frac{11\times 705}{56}\right)\text{cm}^3$.

Let the length of solid cylinder be h. Then,

$\frac{22}{7}\times 1\times 1\times h = \left(\frac{11\times 705}{56}\right)\ \Leftrightarrow\ h = \left(\frac{11\times 705}{56}\times\frac{7}{22}\right)\text{cm} = 44.0625\text{ cm}$.

84. External radius = 4 cm, Internal radius = 3 cm.

Volume of iron = $\left\{\frac{22}{7} \times [(4)^2 - (3)^2] \times 21\right\}$ cm^3 = $\left(\frac{22}{7} \times 7 \times 1 \times 21\right)$ cm^3 = 462 cm^3.

$\therefore$ Weight of iron = (462 × 8) gm = 3696 gm = 3.696 kg.

85. Let the internal radius of the cylinder be x. Then,

$\frac{22}{7} \times r^2 \times 40 = \frac{616}{10} \Leftrightarrow r^2 = \left(\frac{616 \times 7}{10 \times 22 \times 40}\right) = 0.49 \Leftrightarrow r = 0.7.$

So, internal radius = 0.7 cm = 7 mm.

$\therefore$ Thickness = (8 − 7) mm = 1 mm.

86. $\frac{\text{Volume of cone}}{\text{Volume of cylinder}} = \frac{\frac{1}{3} \times \pi \times (3)^2 \times 5}{\pi \times (2)^2 \times 4} = \frac{45}{48} = \frac{15}{16}.$

87. h = 15 cm, r = 8 cm. So, $l = \sqrt{r^2 + h^2} = \sqrt{8^2 + (15)^2} = 17$ cm.

$\therefore$ Curved surface area = πrl = ($\pi \times 8 \times 17$) cm^2 = 136π cm^2.

88. h = 14 cm, r = 7 cm. So, $l = \sqrt{(7)^2 + (14)^2} = \sqrt{245} = 7\sqrt{5}$ cm.

$\therefore$ Total surface area = $\pi rl + \pi r^2 = \left(\frac{22}{7} \times 7 \times 7\sqrt{5} + \frac{22}{7} \times 7 \times 7\right)$ cm^2

= [154 ($\sqrt{5}$ + 1)] cm^2 = (154 × 3.236) cm^2 = 498.35 cm^2.

89. Clearly, we have r = 3 cm and h = 4 cm.

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \pi \times 3^2 \times 4\right)$ cm^3 = 12π cm^3.

90. l = 10 m, h = 8 m. So, $r = \sqrt{l^2 - h^2} = \sqrt{(10)^2 - 8^2} = 6$ m.

$\therefore$ Curved surface area = πrl = ($\pi \times 6 \times 10$) m^2 = 60π m^2.

91. $\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 1232 \Leftrightarrow r^2 = \left(\frac{1232 \times 7 \times 3}{22 \times 24}\right) = 49 \Leftrightarrow r = 7.$

Now, r = 7 cm, h = 24 cm. So, $l = \sqrt{(7)^2 + (24)^2} = 25$ cm.

$\therefore$ Curved surface area = $\left(\frac{22}{7} \times 7 \times 25\right)$ cm^2 = 550 cm^2.

92. Let the radius of the base be r km. Then,

$\pi r^2 = 1.54 \Rightarrow r^2 = \left(\frac{1.54 \times 7}{22}\right) = 0.49 \Rightarrow r = 0.7$ km.

Now, l = 2.5 km, r = 0.7 km.

$\therefore$ $h = \sqrt{(2.5)^2 - (0.7)^2}$ km = $\sqrt{6.25 - 0.49}$ km = $\sqrt{5.76}$ km = 2.4 km.

So, height of the mountain = 2.4 km.

93. $\pi r^2 = 3850 \Rightarrow r^2 = \left(\frac{3850 \times 7}{22}\right) = 1225 \Rightarrow r = 35.$

Now, r = 35 cm, h = 84 cm.

So, $l = \sqrt{(35)^2 + (84)^2} = \sqrt{1225 + 7056} = \sqrt{8281} = 91$ cm.

$\therefore$ Curved surface area = $\left(\frac{22}{7} \times 35 \times 91\right)$ cm^2 = 10010 cm^2.

94. $\frac{22}{7} \times 70 \times l = 40040 \Rightarrow l = \left(\frac{40040 \times 7}{22 \times 70}\right) = 182.$

Now, $l = 182$ cm, $r = 70$ cm.

So, $h = \sqrt{(182)^2 - (70)^2} = \sqrt{252 \times 112} = 168$ cm.

$\therefore$ Volume $= \left(\frac{1}{3} \times \frac{22}{7} \times 70 \times 70 \times 168\right) \text{cm}^3 = 862400 \text{ cm}^3.$

95. Let the radius and the height of the cone be $3x$ and $4x$ respectively. Then,

$\frac{1}{3} \times \pi \times (3x)^2 \times 4x = 96\pi \Leftrightarrow 36x^3 = (96 \times 3) \Leftrightarrow x^3 = \left(\frac{96 \times 3}{36}\right) = 8 \Leftrightarrow x = 2.$

$\therefore$ Radius = 6 cm, Height = 8 cm.

Slant height = $\sqrt{6^2 + 8^2}$ cm = $\sqrt{100}$ cm = 10 cm.

96. $\pi r^2 = 346.5 \Rightarrow r^2 = \left(346.5 \times \frac{7}{22}\right) = \frac{441}{4} \Rightarrow r = \frac{21}{2}.$

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{\frac{441}{4} + (14)^2} = \sqrt{\frac{1225}{4}} = \frac{35}{2}.$

So, area of canvas needed = $\pi rl = \left(\frac{22}{7} \times \frac{21}{2} \times \frac{35}{2}\right) \text{m}^2 = \left(\frac{33 \times 35}{2}\right) \text{m}^2.$

$\therefore$ Length of canvas = $\left(\frac{33 \times 35}{2 \times 1.1}\right)$ m = 525 m.

97. Let the original radius and height of the cone be r and h respectively.
Then, new radius = $2r$. New height = $2h$.

$\therefore \frac{\text{New Volume}}{\text{Original Volume}} = \frac{\frac{1}{3} \times \pi \times (2r)^2 \times 2h}{\frac{1}{3} \times \pi \times r^2 \times h} = \frac{8}{1}.$

98. Let the original radius and height of the cone be r and h respectively.

Then, Original volume = $\frac{1}{3} \pi r^2 h.$

New radius = $\frac{120}{100} r = \frac{6}{5} r$, New height = $\frac{6}{5} h.$

New volume = $\frac{1}{3} \pi \times \left(\frac{6}{5} r\right)^2 \times \left(\frac{6}{5} h\right) = \frac{216}{125} \times \frac{1}{3} \pi r^2 h.$

Increase in volume = $\frac{91}{125} \times \frac{1}{3} \pi r^2 h.$

$\therefore$ Increase % = $\left(\frac{\frac{91}{125} \times \frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 h} \times 100\right) \% = 72.8\%.$

99. Let the original radius and height of the cone be r and h respectively.

Then, original volume = $\frac{1}{3} \pi r^2 h.$

New radius = $\frac{r}{2}$ and new height = $3h$.

New volume $= \frac{1}{3} \times \pi \times \left(\frac{r}{2}\right)^2 \times 3h = \frac{3}{4} \times \frac{1}{3} \pi r^2 h.$

$\therefore$ Decrease % $= \left(\frac{\frac{1}{4} \times \frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 h} \times 100 \right) \% = 25\%.$

100. Required ratio $= \frac{\frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 \times (2h)} = \frac{1}{2}.$

101. Let their heights be x, $3x$ and their radii be $3y$, y.

Then, Ratio of volumes $= \frac{\frac{1}{3} \times \pi \times (3y)^2 \times x}{\frac{1}{3} \times \pi \times y^2 \times (3x)} = \frac{9}{3} = 3 : 1.$

102. Let their radii be $2x$, x and their heights be h and H respectively. Then,

$\frac{1}{3} \times \pi \times (2x)^2 \times h = \frac{1}{3} \times \pi \times x^2 \times \text{H}$ or $\frac{h}{\text{H}} = \frac{1}{4}.$

103. Let their radii be $4x$ and $5x$, and their heights be h and H respectively. Then,

$\frac{\frac{1}{3} \times \pi \times (4x)^2 \times h}{\frac{1}{3} \times \pi \times (5x)^2 \times \text{H}} = \frac{1}{4}$ or $\frac{h}{\text{H}} = \frac{1}{4} \times \frac{25}{16} = \frac{25}{64}.$

104. Volume of the largest cone

= Volume of the cone with diameter of base 7 cm and height 7 cm

$= \left(\frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 7 \right) \text{cm}^3 = \left(\frac{269.5}{3} \right) \text{cm}^3 = 89.8 \text{ cm}^3.$

105. Volume of the block $= (10 \times 5 \times 2) \text{ cm}^3 = 100 \text{ cm}^3.$

Volume of the cone carved out $= \left(\frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7 \right) \text{cm}^3 = 66 \text{ cm}^3.$

$\therefore$ Wood wasted $= (100 - 66)\% = 34\%.$

106. Let their radius and height be $5x$ and $12x$ respectively.

Slant height of the cone, $l = \sqrt{(5x)^2 + (12x)^2} = 13x.$

$\frac{\text{Total surface area of cylinder}}{\text{Total surface area of cone}} = \frac{2\pi r (h + r)}{\pi r (l + r)} = \frac{2(h + r)}{(l + r)} = \frac{2 \times (12x + 5x)}{(13x + 5x)} = \frac{34x}{18x} = \frac{17}{9}.$

107. Let the radius of the cone be r cm.

Then, $\frac{1}{3} \pi \times r^2 \times 6 = \pi \times 8 \times 8 \times 2 \iff r^2 = \left(\frac{8 \times 8 \times 2 \times 3}{6} \right) = 64 \iff r = 8 \text{ cm}.$

108. Let radius of each be r and height of each be h.

Then, number of cones needed $= \frac{\text{Volume of cylinder}}{\text{Volume of 1 cone}} = \frac{\pi r^2 h}{\frac{1}{3} \pi r^2 h} = 3.$

109. Volume of cylinder $= (\pi \times 3 \times 3 \times 5) \text{ cm}^3 = 45\pi \text{ cm}^3.$

Volume of 1 cone $= \left(\frac{1}{3} \pi \times \frac{1}{10} \times \frac{1}{10} \times 1 \right) \text{cm}^3 = \frac{\pi}{300} \text{ cm}^3.$

$\therefore$ Number of cones $= \left(45\pi \times \frac{300}{\pi} \right) = 13500.$

110. Volume flown in conical vessel $= \frac{1}{3}\pi \times (20)^2 \times 24 = 3200\pi$.

Volume flown in 1 min. $= \left(\pi \times \frac{2.5}{10} \times \frac{2.5}{10} \times 1000\right) = 62.5\pi$.

$\therefore$ Time taken $= \left(\frac{3200\pi}{62.5\pi}\right) = 51$ min. 12 sec.

111. Slant height of the cone, $l = \sqrt{(12)^2 + (5)^2} = 13$ cm.

Lateral surface of the solid = Curved surface of cone + Curved surface of cylinder + Surface area of bottom

$= \pi rl + 2\pi rh + \pi r^2$, where h is the height of the cylinder

$= \pi r\,(l + h + r) = \left[\frac{22}{7} \times 12 \times (13 + 18 + 12)\right] \text{cm}^2$

$= \left(\frac{22}{7} \times 12 \times 43\right) \text{cm}^2 = \left(\frac{11352}{7}\right) \text{cm}^2 = 1621\frac{5}{7} \text{cm}^2$.

112. Volume of parallelopiped $= (5 \times 3 \times 4)\ \text{cm}^3 = 60\ \text{cm}^3$.

Volume of cube $= (4)^3\ \text{cm}^3 = 64\ \text{cm}^3$.

Volume of cylinder $= \left(\frac{22}{7} \times 3 \times 3 \times 3\right) \text{cm}^3 = 84.86\ \text{cm}^3$.

Volume of sphere $= \left(\frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3\right) = 113.14\ \text{cm}^3$.

113. $\frac{4}{3} \times \frac{22}{7} \times R^3 = 4851 \Rightarrow R^3 = \left(4851 \times \frac{3}{4} \times \frac{7}{22}\right) = \left(\frac{21}{2}\right)^3 \Rightarrow R = \frac{21}{2}$.

$\therefore$ Curved surface area $= \left(4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right) \text{cm}^2 = 1386\ \text{cm}^2$.

114. $4\pi R^2 = 5544 \Rightarrow R^2 = \left(5544 \times \frac{1}{4} \times \frac{7}{22}\right) = 441 \Rightarrow R = 21$.

$\therefore$ Volume $= \left(\frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21\right) \text{cm}^3 = 38808\ \text{cm}^3$.

115. Volume $= \frac{4}{3}\pi r^3 = \frac{r}{3}(4\pi r^2) = \frac{r}{3} \times$ Surface area.

116. $\frac{\frac{4}{3}\pi R^3}{4\pi R^2} = 27 \Rightarrow R = 81$ cm.

117. Let the radii of A and B be r and R respectively.

$\therefore$ Required ratio $= \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2} = \left(\frac{r}{R}\right)^2 = \left(\frac{40}{10}\right)^2 = 16:1$.

118. Let the original radius be r.

Then, original surface area $= 4\pi r^2 = 2464\ \text{cm}^2$ (given).

New radius $= 2r$.

$\therefore$ New surface area $= 4\pi\,(2r)^2 = 4 \times 4\pi r^2 = (4 \times 2464)\ \text{cm}^2 = 9856\ \text{cm}^2$.

119. Let the original radius be r. Then, original volume $= \frac{4}{3}\pi r^3$.

New radius $= 2r$.

$\therefore$ New volume $= \frac{4}{3}\pi\,(2r)^3 = 8 \times \frac{4}{3}\pi r^3 = 8 \times$ original volume.

120. $4\pi (r+2)^2 - 4\pi r^2 = 352 \Leftrightarrow (r+2)^2 - r^2 = \left(352 \times \frac{7}{22} \times \frac{1}{4}\right) = 28.$

$\Leftrightarrow (r+2+r)(r+2-r) = 28 \Leftrightarrow 2r+2 = 14 \Rightarrow r = \left(\frac{14}{2} - 1\right) = 6$ cm.

121. Let the correct radius be 100 cm. Then, measured radius = 101.5 cm.

$\therefore$ Error in volume $= \frac{4}{3}\pi[(101.5)^3 - (100)^3]$ cm^3

$= \frac{4}{3}\pi(1045678.375 - 1000000)$ cm^3 $= \left(\frac{4}{3} \times \pi \times 45678.375\right)$ cm^3.

$\therefore$ Error % $= \left\{\frac{\frac{4}{3}\pi(45678.375)}{\frac{4}{3}\pi(100 \times 100 \times 100)} \times 100\right\}\% = 4.56\% = 4.6\%$ (app.).

122. Let their radii be R and r. Then,

$$\frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{64}{27} \Rightarrow \left(\frac{R}{r}\right)^3 = \frac{64}{27} = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{R}{r} = \frac{4}{3}.$$

Ratio of surface areas $= \frac{4\pi R^2}{4\pi r^2} = \left(\frac{R}{r}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$.

123. Let their radii be R and r. Then,

$$\frac{4\pi R^2}{4\pi r^2} = \frac{4}{25} \Rightarrow \left(\frac{R}{r}\right)^2 = \left(\frac{2}{5}\right)^2 \Rightarrow \frac{R}{r} = \frac{2}{5}.$$

$\therefore$ Ratio of volumes $= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \left(\frac{R}{r}\right)^3 = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$.

124. Volume of new sphere $= \left[\frac{4}{3}\pi \times (6)^3 + \frac{4}{3}\pi \times (8)^3 + \frac{4}{3}\pi \times (10)^3\right]$ cm^3

$= \left\{\frac{4}{3}\pi[(6)^3 + (8)^3 + (10)^3]\right\}$ cm^3

$= \left(\frac{4}{3}\pi \times 1728\right)$ cm^3 $= \left[\frac{4}{3}\pi \times (12)^3\right]$ cm^3.

Let the radius of the new sphere be R. Then,

$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times (12)^3 \Rightarrow R = 12$ cm.

$\therefore$ Diameter $= 2R = 24$ cm.

125. Volume of bigger sphere $= \left[\frac{4}{3}\pi \times (8)^3\right]$ cm^3 $= \left(\frac{4}{3}\pi \times 512\right)$ cm^3.

Volume of 1 ball $= \left[\frac{4}{3}\pi \times (2)^3\right]$ cm^3 $= \left(\frac{4}{3}\pi \times 8\right)$ cm^3.

$\therefore$ Number of balls $= \left(\frac{\frac{4}{3}\pi \times 512}{\frac{4}{3}\pi \times 8}\right) = \frac{512}{8} = 64.$

126. Let the radius of the third ball be R cm. Then,

$$\frac{4}{3}\pi\times\left(\frac{3}{4}\right)^3+\frac{4}{3}\pi\times(1)^3+\frac{4}{3}\pi\times R^3=\frac{4}{3}\pi\times\left(\frac{3}{2}\right)^3$$

$$\Rightarrow\quad \frac{27}{64}+1+R^3=\frac{27}{8}\ \Rightarrow\ R^3=\frac{125}{64}=\left(\frac{5}{4}\right)^3\ \Rightarrow\ R=\frac{5}{4}.$$

$\therefore$ Diameter of the third ball $= 2R = \frac{5}{2}$ cm $= 2.5$ cm.

127. Volume of each ball $= \frac{1}{8}\times\left(\frac{4}{3}\pi\times10\times10\times10\right)\text{cm}^3$.

$$\therefore\quad \frac{4}{3}\pi R^3=\frac{1}{8}\times\frac{4}{3}\pi\times10\times10\times10\ \Rightarrow\ R^3=\left(\frac{10}{2}\right)^3=5^3\ \Rightarrow\ R=5.$$

128. External radius $= 3$ cm, Internal radius $= (3-0.5)$ cm $= 2.5$ cm.

$$\text{Volume of the metal} = \left[\frac{4}{3}\times\frac{22}{7}\times\{(3)^3-(2.5)^3\}\right]\text{cm}^3$$

$$=\left(\frac{4}{3}\times\frac{22}{7}\times\frac{91}{8}\right)\text{cm}^3=\left(\frac{143}{3}\right)\text{cm}^3=47\frac{2}{3}\text{cm}^3.$$

129. Volume of the solid $= (49\times33\times24)\ \text{cm}^3$.

Let the radius of the sphere be r.

Then, $\frac{4}{3}\pi r^3=(49\times33\times24)\ \Leftrightarrow\ r^3=\left(\frac{49\times33\times24\times3\times7}{4\times22}\right)=(21)^3\ \Leftrightarrow\ r=21.$

130. Number of bullets $= \frac{\text{Volume of the cube}}{\text{Volume of 1 bullet}}=\left(\frac{22\times22\times22}{\frac{4}{3}\times\frac{22}{7}\times1\times1\times1}\right)=2541.$

131. Volume of each lead shot $= \left[\frac{4}{3}\pi\times\left(\frac{0.3}{2}\right)^3\right]\text{cm}^3=\left(\frac{4}{3}\times\frac{22}{7}\times\frac{27}{8000}\right)\text{cm}^3=\frac{99}{7000}\text{cm}^3.$

$\therefore$ Number of lead shots $=\left(9\times11\times12\times\frac{7000}{99}\right)=84000.$

132. $4\pi R^2=6a^2\ \Rightarrow\ \frac{R^2}{a^2}=\frac{3}{2\pi}\ \Rightarrow\ \frac{R}{a}=\frac{\sqrt{3}}{\sqrt{2\pi}}.$

$$\frac{\text{Volume of sphere}}{\text{Volume of cube}}=\frac{\frac{4}{3}\pi R^3}{a^3}=\frac{4}{3}\pi\cdot\left(\frac{R}{a}\right)^3=\frac{4}{3}\pi\cdot\frac{3\sqrt{3}}{2\pi\sqrt{2\pi}}=\frac{2\sqrt{3}}{\sqrt{2\pi}}=\frac{\sqrt{12}}{\sqrt{2\pi}}=\frac{\sqrt{6}}{\sqrt{\pi}}.$$

133. Let the edge of the cube be a. Then, volume of the cube $= a^3$.

Radius of the sphere $= (a/2)$.

$$\text{Volume of the sphere}=\frac{4}{3}\pi\left(\frac{a}{2}\right)^3=\frac{\pi a^3}{6}.$$

$\therefore$ Required ratio $= a^3:\frac{\pi a^3}{6}=6:\pi.$

134. $4\pi R^2=2\pi\times6\times12\ \Rightarrow\ R^2=\left(\frac{6\times12}{2}\right)=36\ \Rightarrow\ R=6$ cm.

135. Let the radius of the cylinder be R.

Then, $\pi \times R^2 \times \frac{7}{3} = \frac{4}{3}\pi \times 7 \times 7 \times 7$

$\Rightarrow \quad R^2 = \left(\frac{4 \times 7 \times 7 \times 7}{3} \times \frac{3}{7}\right) = 196 = (14)^2 \Rightarrow R = 14$ cm.

$\therefore$ Diameter $= 2R = 28$ cm.

136. Required volume = Volume of a sphere of radius 1 cm

$$= \left(\frac{4}{3}\pi \times 1 \times 1 \times 1\right) \text{cm}^3 = \frac{4}{3}\pi \text{ cm}^3.$$

137. Volume of cylinder $= \pi \times (3)^2 \times 15 = 135\pi$ cm^3.

Radius of 1 bullet $= \frac{5}{2}$ mm $= \frac{5}{20}$ cm $= \frac{1}{4}$ cm.

Volume of 1 bullet $= \left(\frac{4}{3}\pi \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right)$ cm^3 $= \frac{\pi}{48}$ cm^3.

$\therefore$ Number of bullets $= \left(135\pi \times \frac{48}{\pi}\right) = 6480$.

138. Let the radius of the cylindrical rod be r.

Then, height of the rod $= 8r$ and radius of one ball $= \frac{r}{2}$.

$\therefore$ Number of balls $= \dfrac{\pi \times r^2 \times 8r}{\frac{4}{3}\pi \times \left(\frac{r}{2}\right)^3} = \left(\frac{8 \times 8 \times 3}{4}\right) = 48$.

139. Let the length of the wire be h.

Then, $\pi \times \frac{3}{20} \times \frac{3}{20} \times h = \frac{4}{3}\pi \times 4 \times 4 \times 4$

$\Leftrightarrow \quad h = \left(\frac{4 \times 4 \times 4 \times 4 \times 20 \times 20}{3 \times 3 \times 3}\right)$ cm $= \left(\frac{102400}{27}\right)$ cm $= 3792.5$ cm $= 37.9$ m.

140. Let the rise in the water level be h cm.

Then, $\pi \times 4 \times 4 \times h = \frac{4}{3}\pi \times 3 \times 3 \times 3 \Rightarrow h = \left(\frac{3 \times 3}{4}\right) = \frac{9}{4}$ cm.

141. Let the radius of each sphere be r cm.

Then, Volume of 12 spheres = Volume of cylinder

$\Rightarrow \quad 12 \times \frac{4}{3}\pi \times r^3 = \pi \times 8 \times 8 \times 2 \Rightarrow r^3 = \left(\frac{8 \times 8 \times 2 \times 3}{12 \times 4}\right) = 8 \Rightarrow r = 2$ cm.

$\therefore$ Diameter of each sphere $= 2r = 4$ cm.

142. Let the radius of the ball be r cm.

Volume of ball = Volume of water displaced by it

$\therefore \quad \frac{4}{3}\pi r^3 = \pi \times 12 \times 12 \times 6.75 \Rightarrow r^3 = 9 \times 9 \times 9 \Rightarrow r = 9$ cm.

143. Let the height of the cone be h cm. Then,

$$\frac{1}{3}\pi \times 6 \times 6 \times h = \frac{4}{3}\pi \times 3 \times 3 \times 3 \Rightarrow h = \left(\frac{36 \times 3}{36}\right) = 3 \text{ cm}.$$

144. Volume of sphere $= \left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right) cm^3$.

Volume of cone $= \left(\frac{1}{3}\pi \times 9 \times 9 \times 9\right) cm^3$.

Volume of wood wasted $= \left[\left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right) - \left(\frac{1}{3}\pi \times 9 \times 9 \times 9\right)\right] cm^3$

$= (\pi \times 9 \times 9 \times 9)\ cm^3$.

$\therefore$ Required percentage $= \left(\frac{\pi \times 9 \times 9 \times 9}{\frac{4}{3}\pi \times 9 \times 9 \times 9} \times 100\right)\% = \left(\frac{3}{4} \times 100\right)\% = 75\%$.

145. Number of spheres $= \frac{\text{Volume of cone}}{\text{Volume of 1 sphere}} = \frac{\frac{1}{3}\pi \times 12 \times 12 \times 24}{\frac{4}{3}\pi \times 2 \times 2 \times 2} = 108$.

146. Volume of material in the sphere $= \left[\frac{4}{3}\pi \times \{(4)^3 - (2)^3\}\right] cm^3 = \left(\frac{4}{3}\pi \times 56\right) cm^3$.

Let the height of the cone be h cm.

Then, $\frac{1}{3}\pi \times 4 \times 4 \times h = \left(\frac{4}{3}\pi \times 56\right) \Leftrightarrow h = \left(\frac{4 \times 56}{4 \times 4}\right) = 14$ cm.

147. Let radius = R and height = H. Then,

Ratio of their volumes $= \pi R^2 H : \frac{1}{3}\pi R^2 H : \frac{4}{3}\pi R^3 = H : \frac{1}{3}H : \frac{4}{3}R$

$= H : \frac{1}{3}H : \frac{4}{3} \times \frac{H}{2}$ $\left[\text{In sphere, } H = 2R \text{ or } R = \frac{H}{2}\right]$

$= 3 : 1 : 2$.

148. Total surface area $= 3\pi R^2 = \left(3 \times \frac{22}{7} \times 7 \times 7\right) cm^2 = 462\ cm^2$.

149. Let the radius be R cm. Then,

$\frac{2}{3} \times \frac{22}{7} \times R^3 = 19404 \Leftrightarrow R^3 = \left(19404 \times \frac{21}{44}\right) = (21)^3 \Leftrightarrow R = 21$ cm.

150. Let their radii be R and r. Then,

$\frac{\frac{2}{3}\pi R^3}{\frac{2}{3}\pi r^3} = \frac{6.4}{21.6} \Leftrightarrow \left(\frac{R}{r}\right)^3 = \frac{8}{27} = \left(\frac{2}{3}\right)^3 \Leftrightarrow \frac{R}{r} = \frac{2}{3}$.

$\therefore$ Ratio of curved surface areas $= \frac{2\pi R^2}{2\pi r^2} = \left(\frac{R}{r}\right)^2 = \frac{4}{9}$.

151. Let the height of the vessel be x. Then, radius of the bowl = radius of the vessel = $\frac{x}{2}$.

Volume of the bowl, $V_1 = \frac{2}{3}\pi\left(\frac{x}{2}\right)^3 = \frac{1}{12}\pi x^3$.

Volume of the vessel, $V_2 = \pi\left(\frac{x}{2}\right)^2 x = \frac{1}{4}\pi x^3$.

Since $V_2 > V_1$, so the vessel can contain 100% of the beverage filled in the bowl.

152. $\frac{2}{3}\pi R^3 = \frac{1}{3}\pi R^2 H \Rightarrow H = 2R.$

153. Let the radius of the cone be R cm. Then,

$$\frac{1}{3}\pi \times R^2 \times 75 = \frac{2}{3}\pi \times 6 \times 6 \times 6$$

$$\Leftrightarrow R^2 = \left(\frac{2\times 6\times 6\times 6}{75}\right) = \left(\frac{144}{25}\right) = \left(\frac{12}{5}\right)^2 \Leftrightarrow R = \frac{12}{5}\text{cm} = 2.4\text{ cm}.$$

154. Let the radius of each be R. Height of hemisphere, H = R.
So, height of cone = height of hemisphere = R.

Slant height of cone = $\sqrt{R^2 + R^2} = \sqrt{2}\,R$.

$$\frac{\text{Curved surface area of hemisphere}}{\text{Curved surface area of cone}} = \frac{2\pi R^2}{\pi R \times \sqrt{2}\,R} = \sqrt{2} : 1.$$

155. Volume of hemisphere = $\frac{2}{3}\pi r^3$.

Volume of biggest sphere = Volume of sphere with diameter $r = \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{1}{6}\pi r^3$

$\therefore$ Required ratio = $\frac{\frac{2}{3}\pi r^3}{\frac{1}{6}\pi r^3} = \frac{4}{1}$ *i.e.* 4 : 1.

EXERCISE 25B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 10*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the given question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. What is the weight of the iron beam ?
   I. The beam is 9 m long, 40 cm wide and 20 cm high.
   II. Iron weighs 50 kg per cubic metre.
2. What is the volume of 32 metre high cylindrical tank ? **(Bank P.O. 2003)**
   I. The area of its base is 154 m^2.
   II. The diameter of the base is 14 m.
3. What is the volume of a cube ? **(Bank P.O. 2003)**
   I. The area of each face of the cube is 64 square metres.
   II. The length of one side of the cube is 8 metres.

4. What is the total cost of painting the inner surface of an open box at the rate of 50 paise per 100 sq. cm ?
   I. The box is made of wood 3 cm thick.
   II. The external dimensions of the box are 50 cm, 40 cm and 23 cm.
5. What is the capacity of a cylindrical tank ? (I.B.P.S. 2002)
   I. Radius of the base is half of its height which is 28 metres.
   II. Area of the base is 616 sq. metres and its height is 28 metres.
6. What is the volume of the cylinder ? (Bank P.O. 2003)
   I. Height is equal to the diameter.
   II. Perimeter of the base is 352 cm.
7. What will be the total cost of whitewashing the conical tomb at the rate of 80 paise per square metre ?
   I. The diameter and the slant height of the tomb are 28 m and 50 m.
   II. The height of the tomb is 48 m and the area of its base is 616 sq. m.
8. What is the height of a circular cone ? (Bank P.O. 1999)
   I. The area of that cone is equal to the area of a rectangle whose length is 33 cm.
   II. The area of the base of that cone is 154 sq. cm.
9. Is a given rectangular block, a cube ? (M.A.T. 1999)
   I. At least 2 faces of the rectangular block are squares.
   II. The volume of the block is 64.
10. A spherical ball of given radius x cm is melted and made into a right circular cylinder. What is the height of the cylinder ? (S.B.I.P.O. 2003)
    I. The volume of the cylinder is equal to the volume of the ball.
    II. The area of the base of the cylinder is given.

Directions (*Questions 11-13*) : ***Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question.***

11. What is the capacity of the cylindrical tank ? (R.B.I. 2003)
    I. The area of the base is 61,600 sq. cm.
    II. The height of the tank is 1.5 times the radius.
    III. The circumference of base is 880 cm.
    (*a*) Only I and II (*b*) Only II and III (*c*) Only I and III
    (*d*) Any two of the three (*e*) Only II and either I or III
12. A solid metallic cone is melted and recast into a sphere. What is the radius of the sphere ?
    I. The radius of the base of the cone is 2.1 cm.
    II. The height of the cone is four times the radius of its base.
    III. The height of the cone is 8.4 cm.
    (*a*) Only I and II (*b*) Only II and III (*c*) Only I and III
    (*d*) Any two of the three (*e*) All I, II and III
13. What is the total surface area of the cone ?
    I. The area of the base of the cone is 154 cm^2.
    II. The curved surface area of the cone is 550 cm^2.
    III. The volume of the cone is 1232 cm^3.
    (*a*) I, and either II or III (*b*) II, and either I or III
    (*c*) III, and either I or II (*d*) Any two of the three
    (*e*) None of these

ANSWERS

1. (e) 2. (c) 3. (c) 4. (e) 5. (c) 6. (e) 7. (c) 8. (d)
9. (d) 10. (b) 11. (e) 12. (d) 13. (a)

SOLUTIONS

1. I gives, $l = 9$ m, $b = \frac{40}{100}$ m $= \frac{2}{5}$ m and $h = \frac{20}{100}$ m $= \frac{1}{5}$ m.

This gives, volume $= (l \times b \times h) = \left(9 \times \frac{2}{5} \times \frac{1}{5}\right) m^3 = \frac{18}{25} m^3$.

II gives, weight of iron is 50 kg / m^3.

$\therefore$ Weight $= \left(\frac{18}{25} \times 50\right)$ kg $= 36$ kg.

Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (e).

2. Given, height = 32 m.

I gives, area of the base = 154 m^2.

$\therefore$ Volume = (area of the base $\times$ height) = (154×32) $m^3 = 4928$ m^3.

Thus, I alone gives the answer.

II gives, radius of the base = 7 m.

$\therefore$ Volume $= \pi r^2 h = \left(\frac{22}{7} \times 7 \times 7 \times 32\right) m^3 = 4928\ m^3$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (c).

3. Let each edge be a metres. Then,

I. $a^2 = 64 \Rightarrow a = 8$ m $\Rightarrow$ Volume $= (8 \times 8 \times 8)$ $m^3 = 512$ m^3.

Thus, I alone gives the answer.

II. $a = 8$ m $\Rightarrow$ Volume $= (8 \times 8 \times 8)$ $m^3 = 512$ m^3.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (c).

4. I gives, thickness of the wall of the box = 3 cm.

II gives, Internal length = $(50 - 6)$ cm = 44 cm, Internal breadth = $(40 - 6) = 34$ cm,
Internal height = $(23 - 3)$ cm = 20 cm.

Area to be painted = (area of 4 walls + area of floor) = $[2\ (l + b) \times h + (l \times b)]$
$= [2\ (44 + 34) \times 20 + (44 \times 34)]$ $cm^2 = 4616$ cm^2.

Cost of painting = Rs. $\left(\frac{1}{2 \times 100} \times 4616\right)$ = Rs. 23.08.

Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (e).

5. I gives, $h = 28$ m and $r = 14$ cm.

$\therefore$ Capacity $= \pi r^2 h$, which can be obtained.

Thus, I alone gives the answer.

II gives, $\pi r^2 = 616\ m^2$ and $h = 28$ m.

$\therefore$ Capacity $= (\pi r^2 \times h) = (616 \times 28)\ m^3$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

6. I gives, $h = 2r$.

II gives, $2\pi r = 352 \Rightarrow r = \left(\frac{352}{2} \times \frac{7}{22}\right)$ cm $= 56$ cm.

From I and II, we have $r = 56$ cm, $h = (2 \times 56)$ cm $= 112$ cm.

Thus, we can find the volume.

$\therefore$ Correct answer is (*e*).

7. I gives, $r = 14$ m, $l = 50$ m.

$\therefore$ Curved surface $= \pi rl = \left(\frac{22}{7} \times 14 \times 50\right) m^2 = 2200\ m^2$.

Cost of whitewashing $=$ Rs. $\left(2200 \times \frac{80}{100}\right) =$ Rs. 1760.

Thus, I alone gives the answer.

II gives, $h = 48$ m, $\pi r^2 = 616\ m^2$.

These results give r and h and so l can be found out.

$\therefore$ Curved surface $= \pi rl$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

8. II gives the value of r.

But, in I, the breadth of rectangle is not given.

So, we cannot find the surface area of the cone.

Hence, the height of the cone cannot be determined.

$\therefore$ Correct answer is (*d*).

9. I gives, any two of l, b, h are equal.

II gives, $lbh = 64$.

From I and II, the values of l, b, h may be (1, 1, 64), (2, 2, 16), (4, 4, 4).

Thus, the block may be a cube or cuboid.

$\therefore$ Correct answer is (*d*).

10. Clearly, I is not needed, since it is evident from the given question.

From II, we get radius of the base of the cylinder.

Now, $\frac{4}{3}\pi x^3 = \pi r^2 h$ in which x and r are known.

$\therefore$ h can be determined.

$\therefore$ Correct answer is (*b*).

11. Capacity $= \pi r^2 h$.

I gives, $\pi r^2 = 61600$. This gives r.

II gives, $h = 1.5\ r$.

Thus, I and II give the answer.

Again, III gives $2\pi r = 880$. This gives r.

So, II and III also give the answer.

$\therefore$ Correct answer is (*e*).

12. $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$.

Now r and h can be determined from any two of I, II and III.

Thus, R can be calculated.

$\therefore$ Correct answer is (*d*).

13 Total surface area of the cone = $(\pi rl + \pi r^2)$ cm^2.

I gives, $\pi r^2 = 154$. Thus, we can find r.

II gives, $\pi rl = 550$.

From I and II we get the answer.

III gives, $\frac{1}{3}\pi r^2 h = 1232$.

From I and III, we can find h and therefore, l.

Hence the surface area can be determined.

$\therefore$ Correct answer is (a).

26. RACES AND GAMES OF SKILL

IMPORTANT FACTS

Races : *A contest of speed in running, riding, driving, sailing or rowing is called a race.*

Race Course : *The ground or path on which contests are made is called a race course.*

Starting Point : *The point from which a race begins is known as a starting point.*

Winning Point or Goal : *The point set to bound a race is called a winning point or a goal.*

Winner : *The person who first reaches the winning point is called a winner.*

Dead Heat Race : *If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.*

Start : Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that '*A gives B, a start of 12 metres*'.

To cover a race of 100 metres in this case, A will have to cover 100 metres while B will have to cover only (100 – 12) = 88 metres.

In a 100 m race, '*A can give B 12 m*' or '*A can give B a start of 12 m*' or '*A beats B by 12 m*' means that while A runs 100 m, B runs (100 – 12) = 88 m.

Games : '*A game of 100, means that the person among the contestants who scores 100 points first is the winner*'.

If A scores 100 points while B scores only 80 points, then we say that '*A can give B 20 points*'.

SOLVED EXAMPLES

Ex. 1. ***In a km race, A beats B by 28 metres or 7 seconds. Find A's time over the course.***

Sol. Clearly, B covers 28 m in 7 seconds.

$\therefore$ B's time over the course = $\left(\frac{7}{28} \times 1000\right)$ sec = 250 seconds.

$\therefore$ A's time over the course = (250 – 7) sec = 243 sec = 4 min. 3 sec.

Ex. 2. ***A runs $1\frac{3}{4}$ times as fast as B. If A gives B a start of 84 m, how far must the winning post be so that A and B might reach it at the same time*** ?

Sol. Ratio of the rates of A and B = $\frac{7}{4} : 1 = 7 : 4$.

So, in a race of 7 m, A gains 3 m over B.

$\therefore$ 3 m are gained by A in a race of 7 m.

$\therefore$ 84 m are gained by A in a race of $\left(\frac{7}{3} \times 84\right)$ m = 196 m.

$\therefore$ Winning post must be 196 m away from the starting point.

Ex. 3. ***A can run 1 km in 3 min. 10 sec. and B can cover the same distance in 3 min. 20 sec. By what distance can A beat B*** ?

Sol. Clearly, A beats B by 10 sec.

Distance covered by B in 10 sec. $= \left(\frac{1000}{200} \times 10\right)$ m $= 50$ m.

$\therefore$ A beats B by 50 metres.

Ex. 4. ***In a 100 m race, A runs at 8 km per hour. If A gives B a start of 4 m and still beats him by 15 seconds, what is the speed of B ?***

Sol. Time taken by A to cover 100 m $= \left(\frac{60 \times 60}{8000} \times 100\right)$ sec $= 45$ sec.

$\therefore$ B covers $(100 - 4)$ m $= 96$ m in $(45 + 15)$ sec $= 60$ sec.

$\therefore$ B's speed $= \left(\frac{96 \times 60 \times 60}{60 \times 1000}\right)$ km/hr $= 5.76$ km/hr.

Ex. 5. ***A, B and C are three contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64 m, how many metre's start can B give C ?***

Sol. While A covers 1000 m, B covers $(1000 - 40)$ m $= 960$ m and C covers $(1000 - 64)$ m or 936 m.

When B covers 960 m, C covers 936 m.

When B covers 1000 m, C covers $\left(\frac{936}{960} \times 1000\right)$ m $= 975$ m.

$\therefore$ B can give C a start of $(1000 - 975)$ or 25 m.

Ex. 6. ***In a game of 80 points, A can give B 5 points and C 15 points. Then how many points B can give C in a game of 60 ?***

Sol. A : B = 80 : 75, A : C = 80 : 65.

$$\frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{75}{80} \times \frac{80}{65}\right) = \frac{15}{13} = \frac{60}{52} = 60 : 52.$$

$\therefore$ In a game of 60, B can give C 8 points.

EXERCISE 26

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. In a 100 m race, A covers the distance in 36 seconds and B in 45 seconds. In this race A beats B by :
   (*a*) 20 m (*b*) 25 m (*c*) 22.5 m (*d*) 9 m
2. In a 200 metres race A beats B by 35 m or 7 seconds. A's time over the course is :
   (*a*) 40 sec (*b*) 47 sec (*c*) 33 sec (*d*) None of these
3. In a 300 m race A beats B by 22.5 m or 6 seconds. B's time over the course is :
   (*a*) 86 sec (*b*) 80 sec (*c*) 76 sec (*d*) None of these
4. A can run 22.5 m while B runs 25 m. In a kilometre race B beats A by :
   (*a*) 100 m (*b*) $111\frac{1}{9}$ m (*c*) 25 m (*d*) 50 m
5. In a 500 m race, the ratio of the speeds of two contestants A and B is 3 : 4. A has a start of 140 m. Then, A wins by :
   (*a*) 60 m (*b*) 40 m (*c*) 20 m (*d*) 10 m

6. A runs $1\frac{2}{3}$ times as fast as B. If A gives B a start of 80 m, how far must the winning post be so that A and B might reach it at the same time ?

(*a*) 200 m (*b*) 300 m (*c*) 270 m (*d*) 160 m

7. In a 100 m race, A can beat B by 25 m and B can beat C by 4 m. In the same race, A can beat C by :

(*a*) 21 m (*b*) 26 m (*c*) 28 m (*d*) 29 m

8. In a 100 m race, A can give B 10 m and C 28 m. In the same race B can give C :

(*a*) 18 m (*b*) 20 m (*c*) 27 m (*d*) 9 m

9. In a 100 m race, A beats B by 10 m and C by 13 m. In a race of 180 m, B will beat C by :

(*a*) 5.4 m (*b*) 4.5 m (*c*) 5 m (*d*) 6 m

10. In a race of 200 m, A can beat B by 31 m and C by 18 m. In a race of 350 m, C will beat B by :

(*a*) 22.75 m (*b*) 25 m (*c*) 19.5 m (*d*) $7\frac{4}{7}$ m

11. A and B take part in a 100 m race. A runs at 5 km per hour. A gives B a start of 8 m and still beats him by 8 seconds. The speed of B is :

(*a*) 5.15 kmph (*b*) 4.14 kmph (*c*) 4.25 kmph (*d*) 4.4 kmph

12. In a game of 100 points, A can give B 20 points and C 28 points. Then, B can give C :

(*a*) 8 points (*b*) 10 points (*c*) 14 points (*d*) 40 points

13. At a game of billiards, A can give B 15 points in 60 and A can give C 20 points in 60. How many points can B give C in a game of 90 ?

(*a*) 30 points (*b*) 20 points (*c*) 10 points (*d*) 12 points

ANSWERS

1. (*a*)	2. (*c*)	3. (*b*)	4. (*a*)	5. (*c*)	6. (*a*)	7. (*c*)	8. (*b*)	
9. (*d*)	10. (*b*)	11. (*b*)	12. (*b*)	13. (*c*)				

SOLUTIONS

1. Distance covered by B in 9 sec. $= \left(\frac{100}{45} \times 9\right)$ m = 20 m.

∴ A beats B by 20 metres.

2. B runs 35 m in 7 sec.

∴ B covers 200 m in $\left(\frac{7}{35} \times 200\right)$ = 40 sec.

B's time over the course = 40 sec.

∴ A's time over the course = (40 − 7) sec = 33 sec.

3. B runs $\frac{45}{2}$ m in 6 sec.

∴ B covers 300 m in $\left(6 \times \frac{2}{45} \times 300\right)$ sec = 80 sec.