QUANTITATIVE APTITUDE

FOR COMPETITIVE EXAMINATIONS

(FULLY SOLVED)

AS PER NEW EXAMINATION PATTERN

An ideal book for:

Bank PO, SBI-PO, IBPS, RBI Exam.
MBA, MAT, CAT, IIFT, IGNOU, Hotel Management
SSC Combined Preliminary Exam.
Sub-Inspector of Police, CBI, CPO Exam.
Railway Recruitment Board Exams.
Campus Recruitment Tests

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1. NUMBERS

IMPORTANT FACTS AND FORMULAE

I. **Numeral :** In Hindu Arabic system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called ***digits*** to represent any number.

A group of digits, denoting a number is called a ***numeral.***

We represent a number, say 689745132 as shown below :

Ten Crores (10^8)	Crores (10^7)	Ten Lacs (Millions) (10^6)	Lacs (10^5)	Ten Thousands (10^4)	Thousands (10^3)	Hundreds (10^2)	Tens (10^1)	Units (10^0)
6	8	9	7	4	5	1	3	2

We read it as : 'Sixty-eight crores, ninety-seven lacs, forty-five thousand, one hundred and thirty-two'.

II. **Place Value or Local Value of a Digit in a Numeral :**

In the above numeral :

Place value of 2 is $(2 \times 1) = 2$; Place value of 3 is $(3 \times 10) = 30$;

Place value of 1 is $(1 \times 100) = 100$ and so on.

Place value of 6 is $6 \times 10^8 = 600000000$.

III. **Face Value :** The ***face value*** of a digit in a numeral is the value of the digit itself at whatever place it may be. In the above numeral, the face value of 2 is 2; the face value of 3 is 3 and so on.

IV. **TYPES OF NUMBERS**

1. **Natural Numbers :** Counting numbers 1, 2, 3, 4, 5, ... are called ***natural numbers.***
2. **Whole Numbers :** All counting numbers together with zero form the set of ***whole numbers.*** Thus,
   (*i*) 0 is the only whole number which is not a natural number.
   (*ii*) Every natural number is a whole number.
3. **Integers :** All natural numbers, 0 and negatives of counting numbers *i.e.*, {....., – 3, – 2, – 1, 0, 1, 2, 3, ...} together form the set of integers.
   (*i*) **Positive Integers :** {1, 2, 3, 4, ...} is the set of all positive integers.
   (*ii*) **Negative Integers :** {– 1, – 2, – 3, ...} is the set of all negative integers.
   (*iii*) **Non-Positive and Non-Negative Integers :** 0 is neither positive nor negative. So, {0, 1, 2, 3, ...} represents the set of non-negative integers, while {0, – 1, – 2, – 3, ...} represents the set of non-positive integers.

4. **Even Numbers :** A number divisible by 2 is called an even number. *e.g.*, 2, 4, 6, 8, 10, etc.
5. **Odd Numbers :** A number not divisible by 2 is called an odd number. *e.g.*, 1, 3, 5, 7, 9, 11, etc.
6. **Prime Numbers :** A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

   Prime numbers upto 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

   Prime numbers Greater than 100 : Let p be a given number greater than 100. To find out whether it is prime or not, we use the following method :

   Find a whole number nearly greater than the square root of p. Let $k > \sqrt{p}$. Test whether p is divisible by any prime number less than k. If yes, then p is not prime. Otherwise, p is prime.

   e.g., We have to find whether 191 is a prime number or not. Now, $14 > \sqrt{191}$.

   Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.

   191 is not divisible by any of them. So, 191 is a prime number.
7. **Composite Numbers :** Numbers greater than 1 which are not prime, are known as composite numbers. *e.g.*, 4, 6, 8, 9, 10, 12.

   Note : (*i*) 1 is neither prime nor composite.

   (*ii*) 2 is the only even number which is prime.

   (*iii*) There are 25 prime numbers between 1 and 100.
8. **Co-primes :** Two numbers a and b are said to be co-primes, if their H.C.F. is 1. *e.g.*, (2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes.

V. TESTS OF DIVISIBILITY

1. **Divisibility By 2 :** A number is divisible by 2, if its unit's digit is any of 0, 2, 4, 6, 8.

   Ex. 84932 is divisible by 2, while 65935 is not.
2. **Divisibility By 3 :** A number is divisible by 3, if the sum of its digits is divisible by 3.

   Ex. 592482 is divisible by 3, since sum of its digits = (5 + 9 + 2 + 4 + 8 + 2) = 30, which is divisible by 3.

   But, 864329 is not divisible by 3, since sum of its digits = (8 + 6 + 4 + 3 + 2 + 9) = 32, which is not divisible by 3.
3. **Divisibility By 4 :** A number is divisible by 4, if the number formed by the last two digits is divisible by 4.

   Ex. 892648 is divisible by 4, since the number formed by the last two digits is 48, which is divisible by 4.

   But, 749282 is not divisible by 4, since the number formed by the last two digits is 82, which is not divisible by 4.
4. **Divisibility By 5 :** A number is divisible by 5, if its unit's digit is either 0 or 5.

   Thus, 20820 and 50345 are divisible by 5, while 30934 and 40946 are not.
5. **Divisibility By 6 :** A number is divisible by 6, if it is divisible by both 2 and 3.

   Ex. The number 35256 is clearly divisible by 2.

   Sum of its digits = (3 + 5 + 2 + 5 + 6) = 21, which is divisible by 3.

   Thus, 35256 is divisible by 2 as well as 3. Hence, 35256 is divisible by 6.

6. **Divisibility By 8 :** A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.

   Ex. 953360 is divisible by 8, since the number formed by last three digits is 360, which is divisible by 8.

   But, 529418 is not divisible by 8, since the number formed by last three digits is 418, which is not divisible by 8.

7. **Divisibility By 9 :** A number is divisible by 9, if the sum of its digits is divisible by 9.

   Ex. 60732 is divisible by 9, since sum of digits = (6 + 0 + 7 + 3 + 2) = 18, which is divisible by 9.

   But, 68956 is not divisible by 9, since sum of digits = (6 + 8 + 9 + 5 + 6) = 34, which is not divisible by 9.

8. **Divisibility By 10 :** A number is divisible by 10, if it ends with 0.

   Ex. 96410, 10480 are divisible by 10, while 96375 is not.

9. **Divisibility By 11 :** A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.

   Ex. The number 4832718 is divisible by 11, since :

   (sum of digits at odd places) – (sum of digits at even places)

   = (8 + 7 + 3 + 4) – (1 + 2 + 8) = 11, which is divisible by 11.

10. **Divisibility By 12 :** A number is divisible by 12, if it is divisible by both 4 and 3.

    Ex. Consider the number 34632.

    (*i*) The number formed by last two digits is 32, which is divisible by 4.

    (*ii*) Sum of digits = (3 + 4 + 6 + 3 + 2) = 18, which is divisible by 3.

    Thus, 34632 is divisible by 4 as well as 3. Hence, 34632 is divisible by 12.

11. **Divisibility By 14 :** A number is divisible by 14, if it is divisible by 2 as well as 7.

12. **Divisibility By 15 :** A number is divisible by 15, if it is divisible by both 3 and 5.

13. **Divisibility By 16 :** A number is divisible by 16, if the number formed by the last 4 digits is divisible by 16.

    Ex. 7957536 is divisible by 16, since the number formed by the last four digits is 7536, which is divisible by 16.

14. **Divisibility By 24 :** A given number is divisible by 24, if it is divisible by both 3 and 8.

15. **Divisibility By 40 :** A given number is divisible by 40, if it is divisible by both 5 and 8.

16. **Divisibility By 80 :** A given number is divisible by 80, if it is divisible by both 5 and 16.

    Note : If a number is divisible by p as well as q, where p and q are co-primes, then the given number is divisible by pq.

    If p and q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

    Ex. 36 is divisible by both 4 and 6, but it is not divisible by $(4 \times 6) = 24$, since 4 and 6 are not co-primes.

VI. MULTIPLICATION BY SHORT CUT METHODS

1. **Multiplication By Distributive Law :**

$(i)\ a \times (b + c) = a \times b + a \times c \quad (ii)\ a \times (b - c) = a \times b - a \times c.$

Ex. $(i)\ 567958 \times 99999 = 567958 \times (100000 - 1)$

$= 567958 \times 100000 - 567958 \times 1$

$= (56795800000 - 567958) = 56795232042.$

$(ii)\ 978 \times 184 + 978 \times 816 = 978 \times (184 + 816) = 978 \times 1000 = 978000.$

2. **Multiplication of a Number By 5^n :** Put n zeros to the right of the multiplicand and divide the number so formed by 2^n.

Ex. $975436 \times 625 = 975436 \times 5^4 = \frac{9754360000}{16} = 609647500.$

VII. BASIC FORMULAE

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 - (a - b)^2 = 4ab$
4. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
5. $(a^2 - b^2) = (a + b)(a - b)$
6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
7. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
8. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
9. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

VIII. DIVISION ALGORITHM OR EUCLIDEAN ALGORITHM

If we divide a given number by another number, then :

Dividend = (Divisor × Quotient) + Remainder

IX.

$(i)\ (x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

$(ii)\ (x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.

$(iii)\ (x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

X. PROGRESSION

A succession of numbers formed and arranged in a definite order according to certain definite rule, is called a *progression*.

1. **Arithmetic Progression (A.P.) :** If each term of a progression differs from its preceding term by a constant, then such a progression is called an arithmetical progression. This constant difference is called the *common difference of the A.P.* An A.P. with first term a and common difference d is given by $a, (a + d), (a + 2d), (a + 3d), \ldots$.

The nth term of this A.P. is given by $T_n = a + (n - 1)d$.

The sum of n terms of this A.P.

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}\ \text{(first term + last term)}.$$

SOME IMPORTANT RESULTS :

$(i)\ (1 + 2 + 3 + \ldots + n) = \frac{n(n + 1)}{2}.$

$(ii)\ (1^2 + 2^2 + 3^2 + \ldots + n^2) = \frac{n(n + 1)(2n + 1)}{6}.$

$(iii)\ (1^3 + 2^3 + 3^3 + \ldots + n^3) = \frac{n^2(n + 1)^2}{4}.$

2. Geometrical Progression (G.P.) : A progression of numbers in which every term bears a constant ratio with its preceding term, is called a *geometrical progression*. The constant ratio is called the common ratio of the G.P.

A G.P. with first term a and common ratio r is :

$$a, ar, ar^2, ar^3, ...$$

In this G.P. $T_n = ar^{n-1}$.

Sum of the n terms, $S_n = \dfrac{a(1-r^n)}{(1-r)}$.

SOLVED EXAMPLES

Ex. 1. ***Simplify :*** ***(i) 8888 + 888 + 88 + 8*** **(B.S.R.B. 1998)**
(ii) 11992 – 7823 – 456 **(Bank Exam, 2003)**

Sol. (i)
$$\begin{array}{r} 8888 \\ 888 \\ 88 \\ +\quad 8 \\ \hline 9872 \end{array}$$

(ii) $11992 - 7823 - 456 = 11992 - (7823 + 456)$
$= 11992 - 8279 = 3713.$

$$\begin{array}{r} 7823 \\ +\ 456 \\ \hline 8279 \end{array} \qquad \begin{array}{r} 11992 \\ -\ 8279 \\ \hline 3713 \end{array}$$

Ex. 2. ***What value will replace the question mark in each of the following equations ?***
(i) ? – 1936248 = 1635773 ***(ii) 8597 – ? = 7429 – 4358***
(Bank P.O. 2000)

Sol. (i) Let $x - 1936248 = 1635773$. Then, $x = 1635773 + 1936248 = 3572021$.

(ii) Let $8597 - x = 7429 - 4358$.
Then, $x = (8597 + 4358) - 7429 = 12955 - 7429 = 5526$.

Ex. 3. ***What could be the maximum value of Q in the following equation ?***
5P9 + 3R7 + 2Q8 = 1114 **(Bank P.O. 1999)**

Sol. We may analyse the given equation as shown :
Clearly, $2 + P + R + Q = 11$.
So, the maximum value of Q can be
$(11 - 2)$ *i.e.*, 9 (when $P = 0$, $R = 0$).

①	②	
5	P	9
3	R	7
2	Q	8
11	1	4

Ex. 4. ***Simplify : (i) 5793405 × 9999*** ***(ii) 839478 × 625***

Sol. (i) $5793405 \times 9999 = 5793405\,(10000 - 1) = 57934050000 - 5793405 = 57928256595.$

(ii) $839478 \times 625 = 839478 \times 5^4 = \dfrac{8394780000}{16} = 524673750.$

Ex. 5. ***Evaluate : (i) 986 × 137 + 986 × 863*** ***(ii) 983 × 207 – 983 × 107***

Sol. (i) $986 \times 137 + 986 \times 863 = 986 \times (137 + 863) = 986 \times 1000 = 986000.$

(ii) $983 \times 207 - 983 \times 107 = 983 \times (207 - 107) = 983 \times 100 = 98300.$

Ex. 6. ***Simplify : (i) 1605 × 1605*** ***(ii) 1398 × 1398***

Sol. (i) $1605 \times 1605 = (1605)^2 = (1600 + 5)^2 = (1600)^2 + (5)^2 + 2 \times 1600 \times 5$
$= 2560000 + 25 + 16000 = 2576025.$

(ii) $1398 \times 1398 = (1398)^2 = (1400 - 2)^2 = (1400)^2 + (2)^2 - 2 \times 1400 \times 2$
$= 1960000 + 4 - 5600 = 1954404.$

Ex. 7. ***Evaluate : (313 × 313 + 287 × 287).***

Sol. $(a^2 + b^2) = \dfrac{1}{2}[(a + b)^2 + (a - b)^2]$

$\therefore \quad (313)^2 + (287)^2 = \dfrac{1}{2}[(313 + 287)^2 + (313 - 287)^2] = \dfrac{1}{2}[(600)^2 + (26)^2]$

$= \dfrac{1}{2}(360000 + 676) = 180338.$

Ex. 8. ***Which of the following are prime numbers ?***
(i) 241 ***(ii) 337*** ***(iii) 391*** ***(iv) 571***

Sol. (i) Clearly, $16 > \sqrt{241}$. Prime numbers less than 16 are 2, 3, 5, 7, 11, 13.
241 is not divisible by any one of them.
$\therefore$ 241 is a prime number.

(*ii*) Clearly, $19 > \sqrt{337}$. Prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17.
337 is not divisible by any one of them.
∴ 337 is a prime number.

(*iii*) Clearly, $20 > \sqrt{391}$. Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
We find that 391 is divisible by 17.
∴ 391 is not prime.

(*iv*) Clearly, $24 > \sqrt{571}$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23.
571 is not divisible by any one of them.
∴ 571 is a prime number.

Ex. 9. ***Find the unit's digit in the product*** $(2467)^{153} \times (341)^{72}$.

Sol. Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.
Now, 7^4 gives unit digit 1.
∴ 7^{152} gives unit digit 1.
∴ 7^{153} gives unit digit $(1 \times 7) = 7$. Also, 1^{72} gives unit digit 1.
Hence, unit's digit in the product $= (7 \times 1) = 7$.

Ex. 10. ***Find the unit's digit in*** $(264)^{102} + (264)^{103}$. **(S.S.C. 1999)**

Sol. Required unit's digit = unit's digit in $(4)^{102} + (4)^{103}$.
Now, 4^2 gives unit digit 6.
∴ $(4)^{102}$ gives unit digit 6.
$(4)^{103}$ gives unit digit of the product (6×4) *i.e.*, 4.
Hence, unit's digit in $(264)^{102} + (264)^{103}$ = unit's digit in $(6 + 4) = 0$.

Ex. 11. ***Find the total number of prime factors in the expression*** $(4)^{11} \times (7)^5 \times (11)^2$.

Sol. $(4)^{11} \times (7)^5 \times (11)^2 = (2 \times 2)^{11} \times (7)^5 \times (11)^2 = 2^{11} \times 2^{11} \times 7^5 \times 11^2 = 2^{22} \times 7^5 \times 11^2$.
∴ Total number of prime factors $= (22 + 5 + 2) = 29$.

Ex. 12. ***Simplify :*** (*i*) $896 \times 896 - 204 \times 204$
(*ii*) $387 \times 387 + 114 \times 114 + 2 \times 387 \times 114$
(*iii*) $81 \times 81 + 68 \times 68 - 2 \times 81 \times 68$

Sol. (*i*) Given exp. $= (896)^2 - (204)^2 = (896 + 204)(896 - 204) = 1100 \times 692 = 761200$.

(*ii*) Given exp. $= (387)^2 + (114)^2 + 2 \times 387 \times 114$
$= a^2 + b^2 + 2ab$, where $a = 387$, $b = 114$
$= (a + b)^2 = (387 + 114)^2 = (501)^2 = 251001$.

(*iii*) Given exp. $= (81)^2 + (68)^2 - 2 \times 81 \times 68 = a^2 + b^2 - 2ab$, where $a = 81$, $b = 68$
$= (a - b)^2 = (81 - 68)^2 = (13)^2 = 169$.

Ex. 13. ***Which of the following numbers is divisible by 3 ?***
(*i*) ***541326*** (*ii*) ***5967013***

Sol. (*i*) Sum of digits in 541326 $= (5 + 4 + 1 + 3 + 2 + 6) = 21$, which is divisible by 3.
Hence, 541326 is divisible by 3.

(*ii*) Sum of digits in 5967013 $= (5 + 9 + 6 + 7 + 0 + 1 + 3) = 31$, which is not divisible by 3.
Hence, 5967013 is not divisible by 3.

Ex. 14. ***What least value must be assigned to * so that the number 197*5462 is divisible by 9 ?***

Sol. Let the missing digit be x.
Sum of digits $= (1 + 9 + 7 + x + 5 + 4 + 6 + 2) = (34 + x)$.
For $(34 + x)$ to be divisible by 9, x must be replaced by 2.
Hence, the digit in place of * must be 2.

Ex. 15. ***Which of the following numbers is divisible by 4 ?***

(i) 67920594 ***(ii) 618703572***

Sol. (*i*) The number formed by the last two digits in the given number is 94, which is not divisible by 4.

Hence, 67920594 is not divisible by 4.

(*ii*) The number formed by the last two digits in the given number is 72, which is divisible by 4.

Hence, 618703572 is divisible by 4.

Ex. 16. ***Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 and 5 ?***

Sol. Since the given number is divisible by 5, so 0 or 5 must come in place of $. But, a number ending with 5 is never divisible by 8. So, 0 will replace $.

Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.

Hence, digits in place of * and $ are 4 and 0 respectively.

Ex. 17. ***Show that 4832718 is divisible by 11.***

Sol. (Sum of digits at odd places) – (Sum of digits at even places)

$= (8 + 7 + 3 + 4) - (1 + 2 + 8) = 11$, which is divisible by 11.

Hence, 4832718 is divisible by 11.

Ex. 18. ***Is 52563744 divisible by 24 ?***

Sol. $24 = 3 \times 8$, where 3 and 8 are co-primes.

The sum of the digits in the given number is 36, which is divisible by 3. So, the given number is divisible by 3.

The number formed by the last 3 digits of the given number is 744, which is divisible by 8. So, the given number is divisible by 8.

Thus, the given number is divisible by both 3 and 8, where 3 and 8 are co-primes. So, it is divisible by 3×8, *i.e.*, 24.

Ex. 19. ***What least number must be added to 3000 to obtain a number exactly divisible by 19 ?***

Sol. On dividing 3000 by 19, we get 17 as remainder.

$\therefore$ Number to be added $= (19 - 17) = 2$.

Ex. 20. ***What least number must be subtracted from 2000 to get a number exactly divisible by 17 ?***

Sol. On dividing 2000 by 17, we get 11 as remainder.

$\therefore$ Required number to be subtracted = 11.

Ex. 21. ***Find the number which is nearest to 3105 and is exactly divisible by 21.***

Sol. On dividing 3105 by 21, we get 18 as remainder.

$\therefore$ Number to be added to $3105 = (21 - 18) = 3$.

Hence, required number $= 3105 + 3 = 3108$.

Ex. 22. ***Find the smallest number of 6 digits which is exactly divisible by 111.***

Sol. Smallest number of 6 digits is 100000.

On dividing 100000 by 111, we get 100 as remainder.

$\therefore$ Number to be added $= (111 - 100) = 11$.

Hence, required number = 100011.

Ex. 23. ***On dividing 15968 by a certain number, the quotient is 89 and the remainder is 37. Find the divisor.***

Sol. $\text{Divisor} = \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}} = \dfrac{15968 - 37}{89} = 179.$

Ex. 24. ***A number when divided by 342 gives a remainder 47. When the same number is divided by 19, what would be the remainder ?***

Sol. On dividing the given number by 342, let k be the quotient and 47 as remainder.

Then, number = $342k + 47 = (19 \times 18k + 19 \times 2 + 9) = 19\,(18k + 2) + 9$.

$\therefore$ The given number when divided by 19, gives $(18k + 2)$ as quotient and 9 as remainder.

Ex. 25. ***A number being successively divided by 3, 5 and 8 leaves remainders 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed.***

Sol.

3	x
5	$y - 1$
8	$z - 4$
	$1 - 7$

$\therefore$ $z = (8 \times 1 + 7) = 15$; $y = (5z + 4) = (5 \times 15 + 4) = 79$; $x = (3y + 1) = (3 \times 79 + 1) = 238$.

Now,

8	238
5	$29 - 6$
3	$5 - 4$
	$1 - 2$

$\therefore$ Respective remainders are 6, 4, 2.

Ex. 26. ***Find the remainder when 2^{31} is divided by 5.***

Sol. $2^{10} = 1024$. Unit digit of $2^{10} \times 2^{10} \times 2^{10}$ is 4 [as $4 \times 4 \times 4$ gives unit digit 4].

$\therefore$ Unit digit of 2^{31} is 8.

Now, 8 when divided by 5, gives 3 as remainder.

Hence, 2^{31} when divided by 5, gives 3 as remainder.

Ex. 27. ***How many numbers between 11 and 90 are divisible by 7 ?***

Sol. The required numbers are 14, 21, 28, 35, ..., 77, 84.

This is an A.P. with $a = 14$ and $d = (21 - 14) = 7$.

Let it contain n terms.

Then, $T_n = 84 \Rightarrow a + (n - 1)\,d = 84$

$\Rightarrow 14 + (n - 1) \times 7 = 84$ or $n = 11$.

$\therefore$ Required number of terms = 11.

Ex. 28. ***Find the sum of all odd numbers upto 100.***

Sol. The given numbers are 1, 3, 5, 7, ..., 99.

This is an A.P. with $a = 1$ and $d = 2$.

Let it contain n terms. Then,

$1 + (n - 1) \times 2 = 99$ or $n = 50$.

$\therefore$ Required sum $= \dfrac{n}{2}$ (first term + last term)

$= \dfrac{50}{2} \times (1 + 99) = 2500.$

Ex. 29. ***Find the sum of all 2 digit numbers divisible by 3.***

Sol. All 2 digit numbers divisible by 3 are :

12, 51, 18, 21, ..., 99.

This is an A.P. with $a = 12$ and $d = 3$.

Let it contain n terms. Then,

$12 + (n - 1) \times 3 = 99$ or $n = 30$.

$\therefore$ Required sum $= \dfrac{30}{2} \times (12 + 99) = 1665.$

Ex. 30. ***How many terms are there in 2, 4, 8, 16, ..., 1024 ?***

Sol. Clearly 2, 4, 8, 16, ..., 1024 form a G.P. with $a = 2$ and $r = \frac{4}{2} = 2$.

Let the number of terms be n. Then,

$2 \times 2^{n-1} = 1024$ or $2^{n-1} = 512 = 2^9$.

$\therefore \quad n - 1 = 9$ or $n = 10$.

Ex. 31. $2 + 2^2 + 2^3 + ... + 2^8 = ?$

Sol. Given series is a G.P. with $a = 2$, $r = 2$ and $n = 8$.

$$\therefore \quad \text{Sum} = \frac{a(r^n - 1)}{(r-1)} = \frac{2 \times (2^8 - 1)}{(2-1)} = (2 \times 255) = 510.$$

EXERCISE 1

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The difference between the local value and face value of 7 in the numeral 657903 is :
   (a) 0 (b) 7896 (c) 6993 (d) 903
2. The difference between the place values of 7 and 3 in the number 527435 is :
   (a) 4 (b) 5 (c) 45 (d) 6970
   (R.R.B. 2001)
3. The sum of the smallest six-digit number and the greatest five-digit number is :
   (a) 199999 (b) 201110 (c) 211110 (d) 1099999
4. If the largest three-digit number is subtracted from the smallest five-digit number, then the remainder is :
   (S.S.C. 1998)
   (a) 1 (b) 9000 (c) 9001 (d) 90001
5. 5978 + 6134 + 7014 = ?
   (Bank P.O. 1999)
   (a) 16226 (b) 19126 (c) 19216 (d) 19226
6. 18265 + 2736 + 41328 = ?
   (Bank P.O. 2000)
   (a) 61329 (b) 62239 (c) 62319 (d) 62329
7. 39798 + 3798 + 378 = ?
   (Bank P.O. 2002)
   (a) 43576 (b) 43974 (c) 43984 (d) 49532
8. 9358 − 6014 + 3127 = ?
   (SIDBI, 2000)
   (a) 6381 (b) 6471 (c) 6561 (d) 6741
9. 9572 − 4018 − 2164 = ?
   (a) 3300 (b) 3390 (c) 3570 (d) 7718
10. 7589 − ? = 3434
    (Bank P.O. 2003)
    (a) 721 (b) 3246 (c) 4155 (d) 11023
11. 9548 + 7314 = 8362 + ?
    (S.B.I.P.O. 2000)
    (a) 8230 (b) 8410 (c) 8500 (d) 8600
12. 7845 − ? = 8461 − 3569
    (a) 2593 (b) 2773 (c) 3569 (d) None of these
13. 3578 + 5729 − ?486 = 5821
    (a) 1 (b) 2 (c) 3 (d) None of these
14. If 6x43 − 46y9 = 1904, which of the following should come in place of x ?
    (a) 4 (b) 6 (c) 9
    (d) Cannot be determined (e) None of these

15. What should be the maximum value of B in the following equation ? **(Bank P.O. 2000)**
5A9 − 7B2 + 9C6 = 823
(a) 5 (b) 6 (c) 7 (d) 9

16. In the following sum, '?' stands for which digit ? **(M.B.A. 1998)**
? + 1? + 2? + ?3 + ?1 = 21?
(a) 4 (b) 6 (c) 8 (d) 9

17. 5358 × 51 = ?
(a) 273258 (b) 273268 (c) 273348 (d) 273358

18. 360 × 17 = ? **(R.B.I. 2003)**
(a) 5120 (b) 5320 (c) 6120 (d) 6130

19. 587 × 999 = ? **(M.B.A. 1998)**
(a) 586413 (b) 587523 (c) 614823 (d) 615173

20. 469157 × 9999 = ?
(a) 4586970843 (b) 4686970743 (c) 4691100843 (d) 584649125

21. 8756 × 99999 = ?
(a) 796491244 (b) 815491244 (c) 875591244 (d) None of these

22. The value of 112×5^4 is : **(M.B.A. 2002)**
(a) 6700 (b) 70000 (c) 76500 (d) 77200

23. 935421 × 625 = ?
(a) 575648125 (b) 584638125 (c) 584649125 (d) 585628125

24. 12846 × 593 + 12846 × 407 = ?
(a) 12846000 (b) 14203706 (c) 24038606 (d) 24064000

25. 1014 × 986 = ?
(a) 998804 (b) 998814 (c) 998904 (d) 999804

26. 1307 × 1307 = ?
(a) 1601249 (b) 1607249 (c) 1701249 (d) 1708249

27. 1399 × 1399 = ?
(a) 1687401 (b) 1901541 (c) 1943211 (d) 1957201

28. 106 × 106 + 94 × 94 = ?
(a) 20032 (b) 20072 (c) 21032 (d) 23032

29. 217 × 217 + 183 × 183 = ? **(Hotel Management, 2002)**
(a) 79698 (b) 80578 (c) 80698 (d) 81268

30. 12345679 × 72 is equal to : **(S.S.C. 2000)**
(a) 88888888 (b) 888888888 (c) 898989898 (d) 999999998

31. What number should replace x in this multiplication problem ?

$$\begin{array}{r} 3x4 \\ \times\quad 4 \\ \hline 1216 \end{array}$$

(Hotel Management, 2000)
(a) 0 (b) 2 (c) 4 (d) 5

32. A positive integer, which when added to 1000, gives a sum which is greater than when it is multiplied by 1000. This positive integer is : **(M.A.T. 2003)**
(a) 1 (b) 3 (c) 5 (d) 7

33. Which of the following can be a product of two 3-digit numbers **3 and **8 ?
(a) 1010024 (b) 991014 (c) 9124 (d) None of these

34. A boy multiplies 987 by a certain number and obtains 559981 as his answer. If in the answer, both 9's are wrong but the other digits are correct, then the correct answer will be : **(C.B.I. 1997)**

(a) 553681 (b) 555181 (c) 555681 (d) 556581

35. When a certain number is multiplied by 13, the product consists entirely of fives. The smallest such number is : **(M.B.A. 2002)**

(a) 41625 (b) 42135 (c) 42515 (d) 42735

36. The number of digits of the smallest number, which when multiplied by 7 gives the result consisting entirely of nines, is :

(a) 3 (b) 5 (c) 6 (d) 8

37. $-95 \div 19 = ?$ **(Hotel Management, 2000)**

(a) -5 (b) -4 (c) 0 (d) 5

38. What should come in place of * mark in the following equation ? **(B.S.R.B. 1998)**

$1*5\$4 \div 148 = 78$

(a) 1 (b) 4 (c) 6 (d) 8 (e) None of these

39. The sum of all possible two-digit numbers formed from three different one-digit natural numbers when divided by the sum of the original three numbers is equal to :

(a) 18 (b) 22 (c) 36 (d) None of these **(C.B.I. 1997)**

40. If n is a negative number, then which of the following is the least ? **(M.B.A. 2002)**

(a) 0 (b) $-n$ (c) $2n$ (d) n^2

41. If x and y are negative, then which of the following statements is / are always true ?

I. $x + y$ is positive II. xy is positive III. $x - y$ is positive. **(M.A.T. 2004)**

(a) I only (b) II only (c) III only (d) I and III only

42. If $-1 \le x \le 2$ and $1 \le y \le 3$, then least possible value of $(2y - 3x)$ is :

(a) 0 (b) -3 (c) -4 (d) -5

43. If a and b are both odd numbers, which of the following is an even number ?

(a) $a + b$ (b) $a + b + 1$ (c) ab (d) $ab + 2$

44. Which of the following is always odd ?

(a) Sum of two odd numbers (b) Difference of two odd numbers
(c) Product of two odd numbers (d) None of these

45. For the integer n, if n^3 is odd, then which of the following statements are true ?

I. n is odd. II. n^2 is odd. III. n^2 is even. **(D.M.R.C. 2003)**

(a) I only (b) II only (c) I and II only (d) I and III only

46. The least prime number is :

(a) 0 (b) 1 (c) 2 (d) 3

47. What is the total number of prime numbers less than 70 ?

(a) 17 (b) 18 (c) 19 (d) 20

48. The total number of even prime numbers is :

(a) 0 (b) 1 (c) 2 (d) None of these

49. Find the sum of prime numbers lying between 60 and 75. **(R.R.B. 2000)**

(a) 199 (b) 201 (c) 211 (d) 272

50. The smallest three-digit prime number is : **(S.S.C. 2000)**

(a) 103 (b) 107 (c) 109 (d) None of these

51. Which one of the following is a prime number ?

(a) 161 (b) 221 (c) 373 (d) 437

52. The smallest value of n, for which $2n + 1$ is not a prime number, is :

(a) 3 (b) 4 (c) 5 (d) None of these **(Hotel Management, 1997)**

53. The sum of three prime numbers is 100. If one of them exceeds another by 36, then one of the numbers is :
(a) 7 (b) 29 (c) 41 (d) 67

54. There are four prime numbers written in ascending order. The product of the first three is 385 and that of the last three is 1001. The last number is : **(S.S.C. 2003)**
(a) 11 (b) 13 (c) 17 (d) 19

55. How many numbers between 400 and 600 begin with or end with a digit of 5 ?
(a) 40 (b) 100 (c) 110 (d) 120

56. If we write all the whole numbers from 200 to 400, then how many of these contain the digit 7 once and only once ? **(Hotel Management, 2003)**
(a) 32 (b) 34 (c) 35 (d) 36

57. The unit's digit in the product $274 \times 318 \times 577 \times 313$ is :
(a) 2 (b) 3 (c) 4 (d) 5

58. The digit in unit's place of the product $81 \times 82 \times ..... \times 89$ is :
(a) 0 (b) 2 (c) 6 (d) 8

(Section Officers', 2003)

59. If the unit digit in the product $(459 \times 46 \times 28* \times 484)$ is 2, the digit in place of * is :
(a) 3 (b) 5 (c) 7 (d) None of these

60. The unit's digit in the product $(3127)^{173}$ is :
(a) 1 (b) 3 (c) 7 (d) 9

61. The unit's digit in the product $(7^{71} \times 6^{59} \times 3^{65})$ is : **(L.I.C.A.A.O. 2003)**
(a) 1 (b) 2 (c) 4 (d) 6

62. The digit in the unit's place of the number represented by $(7^{95} - 3^{58})$ is :
(a) 0 (b) 4 (c) 6 (d) 7

(A.A.O. Exam, 2003)

63. If x is an even number, then x^{4n}, where n is a positive integer, will always have :
(a) zero in the unit's place (b) 6 in the unit's place
(c) either 0 or 6 in the unit's place (d) None of these

(Hotel Management, 1997)

64. The number of prime factors of $(3 \times 5)^{12} (2 \times 7)^{10} (10)^{25}$ is :
(a) 47 (b) 60 (c) 72 (d) None of these

65. $397 \times 397 + 104 \times 104 + 2 \times 397 \times 104 = ?$
(a) 250001 (b) 251001 (c) 260101 (d) 261001

66. $186 \times 186 + 159 \times 159 - 2 \times 186 \times 159 = ?$
(a) 729 (b) 1039 (c) 2019 (d) 7029

67. $(475 + 425)^2 - 4 \times 475 \times 425$ is equal to :
(a) 2500 (b) 3160 (c) 3500 (d) 3600

68. If $(64)^2 - (36)^2 = 20z$, the value of z is :
(a) 70 (b) 120 (c) 180 (d) None of these

69. $(46)^2 - (?)^2 = 4398 - 3066$ **(B.S.R.B. 1998)**
(a) 16 (b) 28 (c) 36 (d) 42

70. $\frac{(856 + 167)^2 + (856 - 167)^2}{856 \times 856 + 167 \times 167}$ is equal to :
(a) 1 (b) 2 (c) 689 (d) 1023

71. $\frac{(469 + 174)^2 - (469 - 174)^2}{469 \times 174}$ is equal to :
(a) 2 (b) 4 (c) 295 (d) 643

72. The sum of first 45 natural numbers is :
(a) 1035 (b) 1280 (c) 2070 (d) 2140

73. The sum of even numbers between 1 and 31 is :
(a) 16 (b) 128 (c) 240 (d) 512

74. (51 + 52 + 53 + ... + 100) is equal to :
(a) 2525 (b) 2975 (c) 3225 (d) 3775

75. How many numbers between 200 and 600 are divisible by 4, 5 and 6 ?
(a) 5 (b) 6 (c) 7 (d) 8

76. How many three-digit numbers are divisible by 6 in all ?
(a) 149 (b) 150 (c) 151 (d) 166

77. If $(1^2 + 2^2 + 3^2 + ... + 10^2) = 385$, then the value of $(2^2 + 4^2 + 6^2 + ... + 20^2)$ is :
(a) 770 (b) 1155 (c) 1540 (d) (385 × 385)

78. The value of $(11^2 + 12^2 + 13^2 + 14^2 + ... + 20^2)$ is :
(a) 385 (b) 2485 (c) 2870 (d) 3255

79. If 1*548 is divisible by 3, which of the following digits can replace * ?
(a) 0 (b) 2 (c) 7 (d) 9
(S.S.C. 1999)

80. If the number 357*25* is divisible by both 3 and 5, then the missing digits in the unit's place and the thousandth place respectively are : **(Hotel Management, 1997)**
(a) 0, 6 (b) 5, 6 (c) 5, 4 (d) None of these

81. 5*2 is a three-digit number with * as a missing digit. If the number is divisible by 6, the missing digit is :
(a) 2 (b) 3 (c) 6 (d) 7

82. What least value must be assigned to * so that the number 63576*2 is divisible by 8 ?
(a) 1 (b) 2 (c) 3 (d) 4

83. What least value must be given to * so that the number 451*603 is exactly divisible by 9 ?
(a) 2 (b) 5 (c) 7 (d) 8

84. How many of the following numbers are divisible by 3 but not by 9 ?
2133, 2343, 3474, 4131, 5286, 5340, 6336, 7347, 8115, 9276
(a) 5 (b) 6 (c) 7 (d) None of these

85. Which one of the following numbers is exactly divisible by 11 ? **(C.D.S. 2003)**
(a) 235641 (b) 245642 (c) 315624 (d) 415624

86. What least value must be assigned to * so that the number 86325*6 is divisible by 11 ?
(a) 1 (b) 2 (c) 3 (d) 5

87. A number 476**0 is divisible by both 3 and 11. The non-zero digits in the hundredth and tenth place respectively are :
(a) 7, 4 (b) 7, 5 (c) 8, 5 (d) None of these

88. Which of the following numbers is divisible by 3, 7, 9 and 11 ?
(a) 639 (b) 2079 (c) 3791 (d) 37911

89. The value of P, when 4864 × 9P2 is divisible by 12, is :
(a) 2 (b) 5 (c) 8 (d) None of these

90. Which of the following numbers is exactly divisible by 24 ? **(M.B.A. 1998)**
(a) 35718 (b) 63810 (c) 537804 (d) 3125736

91. If the number 42573* is completely divisible by 72, then which of the following numbers should replace the asterisk ?
(a) 4 (b) 5 (c) 6 (d) 7

92. Which of the following numbers is exactly divisible by 99 ?
(a) 114345 (b) 135792 (c) 913464 (d) 3572404

93. The digits indicated by * and $ in 3422213*$ so that this number is divisible by 99, are respectively :
(a) 1, 9 (b) 3, 7 (c) 4, 6 (d) 5, 5

94. If x and y are the two digits of the number 653xy such that this number is divisible by 80, then $x + y$ is equal to :
(a) 2 (b) 3 (c) 4 (d) 6

95. How many of the following numbers are divisible by 132 ?
264, 396, 462, 792, 968, 2178, 5184, 6336 **(Hotel Management, 2002)**
(a) 4 (b) 5 (c) 6 (d) 7

96. 6897 is divisible by : **(I.A.M. 2002)**
(a) 11 only (b) 19 only
(c) both 11 and 19 (d) neither 11 nor 19

97. Which of the following numbers is exactly divisible by all prime numbers between 1 and 17 ?
(a) 345345 (b) 440440 (c) 510510 (d) 515513

98. 325325 is a six-digit number. It is divisible by : **(S.S.C. 1998)**
(a) 7 only (b) 11 only (c) 13 only (d) all 7, 11 and 13

99. The number 311311311311311311311 is : **(C.D.S. 2003)**
(a) divisible by 3 but not by 11 (b) divisible by 11 but not by 3
(c) divisible by both 3 and 11 (d) neither divisible by 3 nor by 11

100. There is one number which is formed by writing one digit 6 times (*e.g.* 111111, 444444 etc.). Such a number is always divisible by :
(a) 7 only (b) 11 only (c) 13 only (d) All of these

101. A 4-digit number is formed by repeating a 2-digit number such as 2525, 3232 etc. Any number of this form is exactly divisible by : **(S.S.C. 2000)**
(a) 7 (b) 11
(c) 13 (d) smallest 3-digit prime number

102. A six-digit number is formed by repeating a three-digit number; for example, 256256 or 678678 etc. Any number of this form is always exactly divisible by :
(a) 7 only (b) 11 only (c) 13 only (d) 1001

103. The largest natural number which exactly divides the product of any four consecutive natural numbers is :
(a) 6 (b) 12 (c) 24 (d) 120

104. The largest natural number by which the product of three consecutive even natural numbers is always divisible, is :
(a) 16 (b) 24 (c) 48 (d) 96

105. The sum of three consecutive odd numbers is always divisible by :
I. 2 II. 3 III. 5 IV. 6
(a) Only I (b) Only II (c) Only I and III (d) Only II and IV
(Hotel Management, 2003)

106. The difference between the squares of two consecutive odd integers is always divisible by : **(M.B.A. 2003)**
(a) 3 (b) 6 (c) 7 (d) 8

107. A number is multiplied by 11 and 11 is added to the product. If the resulting number is divisible by 13, the smallest original number is :
(*a*) 12 (*b*) 22 (*c*) 26 (*d*) 53

108. The sum of the digits of a 3-digit number is subtracted from the number. The resulting number is :
(*a*) divisible by 6 (*b*) divisible by 9
(*c*) divisible neither by 6 nor by 9 (*d*) divisible by both 6 and 9

109. If x and y are positive integers such that $(3x + 7y)$ is a multiple of 11, then which of the following will also be divisible by 11 ?
(*a*) $4x + 6y$ (*b*) $x + y + 4$ (*c*) $9x + 4y$ (*d*) $4x - 9y$

110. A 3-digit number $4a3$ is added to another 3-digit number 984 to give the four-digit number $13b7$, which is divisible by 11. Then, $(a + b)$ is :
(*a*) 10 (*b*) 11 (*c*) 12 (*d*) 15

111. The largest number that exactly divides each number of the sequence $(1^5 - 1)$, $(2^5 - 2)$, $(3^5 - 3)$, ..., $(n^5 - n)$, ... is :
(*a*) 1 (*b*) 15 (*c*) 30 (*d*) 120

112. The greatest number by which the product of three consecutive multiples of 3 is always divisible is : **(S.S.C. 2000)**
(*a*) 54 (*b*) 81 (*c*) 162 (*d*) 243

113. The smallest number to be added to 1000 so that 45 divides the sum exactly is :
(*a*) 10 (*b*) 20 (*c*) 35 (*d*) 80

114. The smallest number that must be added to 803642 in order to obtain a multiple of 11 is : **(C.B.I. 2003)**
(*a*) 1 (*b*) 4 (*c*) 7 (*d*) 9

115. Which of the following numbers should be added to 11158 to make it exactly divisible by 77 ?
(*a*) 5 (*b*) 7 (*c*) 8 (*d*) 9

116. The least number which must be subtracted from 6709 to make it exactly divisible by 9 is : **(C.B.I. 1998)**
(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

117. What least number must be subtracted from 427398 so that the remaining number is divisible by 15 ? **(Bank P.O. 2000)**
(*a*) 3 (*b*) 6 (*c*) 11 (*d*) 16

118. What least number must be subtracted from 13294 so that the remainder is exactly divisible by 97 ?
(*a*) 1 (*b*) 3 (*c*) 4 (*d*) 5

119. When the sum of two numbers is multiplied by 5, the product is divisible by 15. Which one of the following pairs of numbers satisfies the above condition ?
(*a*) 240, 335 (*b*) 250, 341 (*c*) 245, 342 (*d*) None of these
(Hotel Management, 1998)

120. The least number by which 72 must be multiplied in order to produce a multiple of 112, is :
(*a*) 6 (*b*) 12 (*c*) 14 (*d*) 18

121. The number of times 99 is subtracted from 1111 so that the remainder is less than 99, is : **(S.C.R.A. 1996)**
(*a*) 10 (*b*) 11 (*c*) 12 (*d*) 13

122. Find the number which is nearest to 457 and is exactly divisible by 11.
(*a*) 450 (*b*) 451 (*c*) 460 (*d*) 462
(Hotel Management, 2003)

123. The number nearest to 99547 which is exactly divisible by 687 is :
(a) 98928 (b) 99479 (c) 99615 (d) 100166

124. What largest number of five digits is divisible by 99 ?
(a) 99909 (b) 99981 (c) 99990 (d) 99999

125. The smallest number of five digits exactly divisible by 476 is : **(S.S.C. 2004)**
(a) 10000 (b) 10472 (c) 10476 (d) 47600

126. On dividing a number by 999, the quotient is 366 and the remainder is 103. The number is :
(a) 364724 (b) 365387 (c) 365737 (d) 366757

127. On dividing 4150 by a certain number, the quotient is 55 and the remainder is 25. The divisor is :
(a) 65 (b) 70 (c) 75 (d) 80

128. A number when divided by the sum of 555 and 445 gives two times their difference as quotient and 30 as the remainder. The number is : **(S.S.C. 2000)**
(a) 1220 (b) 1250 (c) 22030 (d) 220030

129. A four-digit number divisible by 7 becomes divisible by 3, when 10 is added to it. The largest such number is :
(a) 9947 (b) 9987 (c) 9989 (d) 9996

130. A number when divided by 114 leaves the remainder 21. If the same number is divided by 19, then the remainder will be : **(R.R.B. 2003)**
(a) 1 (b) 2 (c) 7 (d) 21

131. A number when divided by 296 gives a remainder 75. When the same number is divided by 37, then the remainder will be : **(C.B.I. 2003)**
(a) 1 (b) 2 (c) 8 (d) 11

132. A number when divided by 119 leaves 19 as remainder. If the same number is divided by 17, the remainder obtained is : **(Section Officers', 2001)**
(a) 2 (b) 3 (c) 7 (d) 10

133. A number when divided by 899 gives a remainder 63. If the same number is divided by 29, the remainder will be :
(a) 3 (b) 4 (c) 5 (d) 10

134. When a number is divided by 31, the remainder is 29. When the same number is divided by 16, what will be the remainder ? **(Bank P.O. 2002)**
(a) 11 (b) 13 (c) 15 (d) Data inadequate

135. When a number is divided by 13, the remainder is 11. When the same number is divided by 17, the remainder is 9. What is the number ? **(S.B.I.P.O. 1997)**
(a) 339 (b) 349 (c) 369 (d) Data inadequate

136. In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, the dividend is :
(a) 4236 (b) 4306 (c) 4336 (d) 5336

137. The difference between two numbers is 1365. When the larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. The smaller number is : **(A.A.O. Exam, 2003)**
(a) 240 (b) 270 (c) 295 (d) 360

138. In doing a division of a question with zero remainder, a candidate took 12 as divisor instead of 21. The quotient obtained by him was 35. The correct quotient is :
(a) 0 (b) 12 (c) 13 (d) 20 **(S.S.C. 2003)**

139. When n is divided by 4, the remainder is 3. What is the remainder when $2n$ is divided by 4 ?

(a) 1 (b) 2 (c) 3 (d) 6

140. A number when divided by 6 leaves a remainder 3. When the square of the same number is divided by 6, the remainder is : **(S.S.C. 2000)**

(a) 0 (b) 1 (c) 2 (d) 3

141. A number when divided successively by 4 and 5 leaves remainders 1 and 4 respectively. When it is successively divided by 5 and 4, then the respective remainders will be :

(a) 1, 2 (b) 2, 3 (c) 3, 2 (d) 4, 1

(S.S.C. 2003)

142. A number was divided successively in order by 4, 5 and 6. The remainders were respectively 2, 3 and 4. The number is : **(C.B.I. 1997)**

(a) 214 (b) 476 (c) 954 (d) 1908

143. In dividing a number by 585, a student employed the method of short division. He divided the number successively by 5, 9 and 13 (factors of 585) and got the remainders 4, 8 and 12. If he had divided the number by 585, the remainder would have been :

(a) 24 (b) 144 (c) 292 (d) 584

(N.I.F.T. 1997)

144. A number when divided by 3 leaves a remainder 1. When the quotient is divided by 2, it leaves a remainder 1. What will be the remainder when the number is divided by 6 ?

(a) 2 (b) 3 (c) 4 (d) 5

145. $4^{61} + 4^{62} + 4^{63} + 4^{64}$ is divisible by : **(C.B.I. 2003)**

(a) 3 (b) 10 (c) 11 (d) 13

146. If x is a whole number, then $x^2 (x^2 - 1)$ is always divisible by : **(S.S.C. 1998)**

(a) 12 (b) 24 (c) $12 - x$ (d) multiple of 12

ANSWERS

1. (c)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (b)	**8.** (b)
9. (b)	**10.** (c)	**11.** (c)	**12.** (d)	**13.** (c)	**14.** (e)	**15.** (c)	**16.** (c)
17. (a)	**18.** (c)	**19.** (a)	**20.** (c)	**21.** (c)	**22.** (b)	**23.** (b)	**24.** (a)
25. (d)	**26.** (d)	**27.** (d)	**28.** (b)	**29.** (b)	**30.** (b)	**31.** (a)	**32.** (a)
33. (b)	**34.** (c)	**35.** (d)	**36.** (c)	**37.** (a)	**38.** (a)	**39.** (b)	**40.** (c)
41. (b)	**42.** (c)	**43.** (a)	**44.** (c)	**45.** (c)	**46.** (c)	**47.** (c)	**48.** (b)
49. (d)	**50.** (d)	**51.** (c)	**52.** (b)	**53.** (d)	**54.** (b)	**55.** (c)	**56.** (d)
57. (a)	**58.** (a)	**59.** (c)	**60.** (c)	**61.** (c)	**62.** (b)	**63.** (b)	**64.** (d)
65. (b)	**66.** (a)	**67.** (a)	**68.** (d)	**69.** (b)	**70.** (b)	**71.** (b)	**72.** (a)
73. (c)	**74.** (d)	**75.** (b)	**76.** (b)	**77.** (c)	**78.** (b)	**79.** (a)	**80.** (b)
81. (a)	**82.** (c)	**83.** (d)	**84.** (b)	**85.** (d)	**86.** (c)	**87.** (c)	**88.** (b)
89. (d)	**90.** (d)	**91.** (c)	**92.** (a)	**93.** (a)	**94.** (a)	**95.** (a)	**96.** (c)
97. (c)	**98.** (d)	**99.** (d)	**100.** (d)	**101.** (d)	**102.** (d)	**103.** (c)	**104.** (c)
105. (b)	**106.** (d)	**107.** (a)	**108.** (b)	**109.** (d)	**110.** (a)	**111.** (c)	**112.** (c)
113. (c)	**114.** (c)	**115.** (b)	**116.** (c)	**117.** (a)	**118.** (d)	**119.** (b)	**120.** (c)
121. (b)	**122.** (d)	**123.** (c)	**124.** (c)	**125.** (b)	**126.** (c)	**127.** (c)	**128.** (d)
129. (c)	**130.** (b)	**131.** (a)	**132.** (a)	**133.** (c)	**134.** (d)	**135.** (b)	**136.** (d)
137. (b)	**138.** (d)	**139.** (b)	**140.** (d)	**141.** (b)	**142.** (a)	**143.** (d)	**144.** (c)
145. (b)	**146.** (a)						

SOLUTIONS

1. (Local Value) – (Face Value) = (7000 – 7) = 6993.
2. (Place Value of 7) – (Place Value of 3) = (7000 – 30) = 6970.
3. Required Sum = (100000 + 99999) = 199999.
4. Required Remainder = (10000 – 999) = 9001.
5. 5978 + 6134 + 7014 = 19126.
6. 18265 + 2736 + 41328 = 62329.
7. 39798 + 3798 + 378 = 43974.
8. 9358 – 6014 + 3127 = (9358 + 3127) – 6014 = (12485 – 6014) = 6471.
9. 9572 – 4018 – 2164 = 9572 – (4018 + 2164) = (9572 – 6182) = 3390.
10. Let 7589 – x = 3434. Then, x = (7589 – 3434) = 4155.
11. Let 9548 + 7314 = 8362 + x. Then, 16862 = 8362 + x $\Leftrightarrow$ x = (16862 – 8362) = 8500.
12. Let 7845 – x = 8461 – 3569. Then, 7845 – x = 4892 $\Leftrightarrow$ x = (7845 – 4892) = 2953.
13. Let 3578 + 5729 – x486 = 5821.
    Then, 9307 – x486 = 5821 $\Leftrightarrow$ x486 = (9307 – 5821) $\Leftrightarrow$ x486 = 3486 $\Leftrightarrow$ x = 3.
14. 6x43 – 46y9 = 1904 $\Leftrightarrow$ 6x43 = 1904 + 46y9 [1 + y = 4 $\Leftrightarrow$ y = 3]
    $\Leftrightarrow$ 6x43 = 1904 + 4639 – 6543 [$\because$ y = 3]
    $\Leftrightarrow$ x = 5.
15. We may represent the given sum, as shown.
    $\therefore$ 1 + A + C – B = 12 $\Leftrightarrow$ A + C – B = 11.
    Giving maximum values to A and C, i.e.,
    A = 9 and C = 9, we get B = 7.

	1	1		
	5	A	9	
+	9	C	6	
	7	B	2	
	8	2	3	

16. Let x + (10 + x) + (20 + x) + (10x + 8) + (10x + 1) = 200 + 10 + x.
    Then, 22x = 176 $\Leftrightarrow$ x = 8.
17. 5358 × 51 = 5358 × (50 + 1) = (5358 × 50) + (5358 × 1) = (267900 + 5358) = 273258.
18. 360 × 17 = 360 × (20 – 3) = (360 × 20) – (360 × 3) = (7200 – 1080) = 6120.
19. 587 × 999 = 587 × (1000 – 1) = (587 × 1000) – (587 × 1) = (587000 – 587) = 586413.
20. 469157 × 9999 = 469157 × (10000 – 1) = (469157 × 10000) – (469157 × 1)
    = (4691570000 – 469157) = 4691100843.
21. 8756 × 99999 = 8756 × (100000 – 1) = (8756 × 100000) – (8756 × 1)
    = (875600000 – 8756) = 875591244.
22. $(112 \times 5^4) = \frac{1120000}{2^4}$ (see the rule) $= \frac{1120000}{16} = 70000.$
23. $935421 \times 625 = 935421 \times 5^4 = \frac{9354210000}{2^4}$ (see the rule)
    $= \frac{9354210000}{16} = 584638125.$
24. 12846 × 593 + 12846 × 407 = 12846 × (593 + 407) = 12846 × 1000 = 12846000.
25. (1014 × 986) = (1000 + 14) × (1000 – 14) = $(1000)^2 - (14)^2$ = 1000000 – 196 = 999804.
26. (1307 × 1307) = $(1307)^2$ = $(1300 + 7)^2$ = (1690000 + 49 + 18200) = 1708249.
27. (1399 × 1399) = $(1399)^2$ = $(1400 - 1)^2$ = $(1400)^2 + 1^2 - 2 \times 1400 \times 1$
    = 1960000 + 1 – 2800 = 1960001 – 2800 = 1957201.

28. $(106 \times 106 + 94 \times 94) = \frac{1}{2} \times 2(a^2 + b^2) = \frac{1}{2}[(a+b)^2 + (a-b)^2]$

$= \frac{1}{2} \cdot [(106+94)^2 + (106-94)^2] = \frac{1}{2} \cdot [(200)^2 + (12)^2]$

$= \frac{1}{2}(40000 + 144) = \frac{1}{2}(40144) = 20072.$

29. $(217 \times 217 + 183 \times 183) = \frac{1}{2} \times 2(a^2 + b^2) = \frac{1}{2} \cdot [(a+b)^2 + (a-b)^2]$

$= \frac{1}{2} \cdot [(217+183)^2 + (217-183)^2] = \frac{1}{2}[(400)^2 + (34)^2]$

$= \frac{1}{2}(160000 + 1156) = \frac{161156}{2} = 80578.$

30. $12345679 \times 72 = 12345679 \times (100 - 28) = 1234567900 - (12345679 \times 28)$

$= 1234567900 - [12345679 \times (30-2)]$

$= 1234567900 - 370370370 + 24691358 = 888888888.$

31. $(300 + 10x + 4) \times 4 = 1200 + 40x + 16 = (12 \times 100) + (4x + 1) \times 10 + 6$

$\therefore \quad 4x + 1 = 1 \quad \Leftrightarrow \quad 4x = 0 \quad \Leftrightarrow \quad x = 0.$

32. $(1000 + N) > (1000N)$. Clearly, $N = 1$.

33. When two 3-digit numbers are multiplied, the product must contain 5 or 6 digits.
So, the required number is 991014.

34. $987 = 3 \times 7 \times 47.$
So, required number must be divisible by each one of 3, 7, 47.
None of the numbers in (a) and (b) are divisible by 3, while (d) is not divisible by 7.
$\therefore$ Correct answer is (c).

35. By hit and trial, we find that a number exactly divisible by 13 and consisting entirely of fives is 555555.
On dividing 555555 by 13, we get 42735 as quotient.
$\therefore$ Required number = 42735.

36. By hit and trial, we find that a number exactly divisible by 7 and consisting entirely of nines is 999999. Number of digits in it = 6.

37. $\frac{-95}{19} = -5.$

38. Let $\frac{x}{148} = 78$. Then, $x = (148 \times 78) = 11544$.
$\therefore$ Required digit = 1.

39. Let the one-digit numbers be x, y, z.
Sum of all possible 2-digit numbers
$= (10x + y) + (10x + z) + (10y + x) + (10y + z) + (10z + x) + (10z + y) = 22(x + y + z)$
$\therefore$ Sum of all possible 2-digit numbers when divided by sum of one-digit numbers gives 22.

40. $n < 0 \quad \Rightarrow \quad 2n < 0, -n > 0$ and $n^2 > 0$.
$\therefore$ Least of $2n, 0, -n$ and n^2 is $2n$.

41. $x < 0, y < 0 \quad \Rightarrow \quad (x + y) < 0, xy > 0$ and $x - y$ may be + ve or − ve.
$\therefore$ II is always true.

42. $\left.\begin{array}{l} y \geq 1 \Rightarrow 2y \geq 2 \\ x \leq 2 \Rightarrow -3x \geq -6 \end{array}\right\} \Rightarrow (2y - 3x) \geq -4.$

43. Sum of two odd numbers is always even.

44. Product of two odd numbers is always odd.

45. n^3 is odd $\Rightarrow$ n is odd and n^2 is odd.

46. The least prime number is 2.

47. Prime numbers less than 70 are :
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61 and 67.
Their number is 19.

48. There is only one even prime number, namely 2.

49. Required sum = (61 + 67 + 71 + 73) = 272.

50. 100 is divisible by 2, so it is not prime.
101 is not divisible by any of the numbers 2, 3, 5, 7. So, it is prime.
Hence, the smallest 3-digit prime number is 101.

51. 161 is divisible by 7. So, it is not prime. 221 is divisible by 13. So, it is not prime.
Now, $20 > \sqrt{373}$. Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
And, 373 is not divisible by any of them. So, 373 is prime.
Since 437 is divisible by 19, so it is not prime.

52. $(2 \times 1 + 1) = 3$, $(2 \times 2 + 1) = 5$, $(2 \times 3 + 1) = 7$, $(2 \times 4 + 1) = 9$, which is not prime.
$\therefore$ $n = 4$.

53. $x + (x + 36) + y = 100 \Leftrightarrow 2x + y = 64$.
$\therefore$ y must be even prime, which is 2.
$\therefore$ $2x + 2 = 64 \Rightarrow x = 31$.
Third prime number = $(x + 36) = (31 + 36) = 67$.

54. Let the given prime numbers be a, b, c, d. Then, $abc = 385$ and $bcd = 1001$.
$\therefore$ $\dfrac{abc}{bcd} = \dfrac{385}{1001} \Leftrightarrow \dfrac{a}{d} = \dfrac{5}{13}$. So, $a = 5$, $d = 13$.

55. Numbers satisfying the given conditions are 405, 415, 425, 435, 445, 455, 465, 475, 485, 495 and 500 to 599.
Number of such numbers = (10 + 100) = 110.

56. Required numbers from 200 to 300 are 207, 217, 227, 237, 247, 257, 267, 270, 271, 272, 273, 274, 275, 276, 278, 279, 287, 297. Their number is 18.
Similarly, such numbers between 300 and 400 are also 18 in number.
$\therefore$ Total number of such numbers = 36.

57. Required digit = Unit digit in $(4 \times 8 \times 7 \times 3) = 2$.

58. Required digit = Unit digit in $(1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9) = 0$.

59. $(9 \times 6 \times 4) = 216$. In order to obtain 2 at the unit place, we must multiply 216 by 2 or 7.
$\therefore$ Of the given numbers, we have 7.

60. Unit digit in $(3127)^{173}$ = Unit digit in $(7)^{173}$. Now, 7^4 gives unit digit 1.
$\therefore$ $(7)^{173} = (7^4)^{43} \times 7^1$. Thus, $(7)^{173}$ gives unit digit 7.

61. Unit digit in 7^4 is 1.
$\therefore$ Unit digit in 7^{68} is 1.
Unit digit in 7^{71} is 3. [$1 \times 7 \times 7 \times 7$ gives unit digit 3]
Again, every power of 6 will give unit digit 6.
$\therefore$ Unit digit in 6^{59} is 6.
Unit digit in 3^4 is 1.
$\therefore$ Unit digit in 3^{64} is 1. Unit digit in 3^{65} is 3.
$\therefore$ Unit digit in $(7^{71} \times 6^{59} \times 3^{65})$ = Unit digit in $(3 \times 6 \times 3) = 4$.

62. Unit digit in 7^4 is 1. So, unit digit in 7^{92} is 1.

$\therefore$ Unit digit in 7^{95} is 3. [$\because$ Unit digit in $1 \times 7 \times 7 \times 7$ is 3]

Unit digit in 3^4 is 1.

$\therefore$ Unit digit in 3^{56} is 1.

$\therefore$ Unit digit in 3^{58} is 9.

$\therefore$ Unit digit in $(7^{95} - 3^{58}) = (13 - 9) = 4$.

63. $x^{4n} = (2^4)^n$ or $(4^4)^n$ or $(6^4)^n$ or $(8^4)n$.

Clearly, the unit digit in each case is 6.

64. $(3 \times 5)^{12} \times (2 \times 7)^{10} \times (10)^{25} = (3 \times 5)^{12} \times (2 \times 7)^{10} \times (2 \times 5)^{25}$

$= 3^{12} \times 5^{12} \times 2^{10} \times 7^{10} \times 2^{25} \times 5^{25} = 2^{35} \times 3^{12} \times 5^{37} \times 7^{10}$.

Total number of prime factors = (35 + 12 + 37 + 10) = 94.

65. Given Exp. $= a^2 + b^2 + 2ab$, where $a = 397$ and $b = 104$

$= (a + b)^2 = (397 + 104)^2 = (501)^2 = (500 + 1)^2 = (500)^2 + 1^2 + 2 \times 500 \times 1$

$= 250000 + 1 + 1000 = 251001$.

66. Given Exp. $= a^2 + b^2 - 2ab$, where $a = 186$ and $b = 159$

$= (a - b)^2 = (186 - 159)^2 = (27)^2$

$= (20 + 7)^2 = (20)^2 + 7^2 + 2 \times 20 \times 7 = 400 + 49 + 280 = 729$.

67. Given Exp. $= (a + b)^2 - 4ab$, where $a = 475$ and $b = 425$

$= (a - b)^2 = (475 - 425)^2 = (50)^2 = 2500$.

68. $20z = (64)^2 - (36)^2 \Leftrightarrow 20z = (64 + 36)(64 - 36)$

$\Leftrightarrow 20z = 100 \times 28 \Leftrightarrow z = \dfrac{100 \times 28}{20} = 140$.

69. Let $(46)^2 - x^2 = 4398 - 3066$.

Then, $(46)^2 - x^2 = 1332 \Leftrightarrow x^2 = (46)^2 - 1332 = (2116 - 1332)$

$\Leftrightarrow x^2 = 784 \Leftrightarrow x = \sqrt{784} = 28$.

70. Given Exp. $= \dfrac{(a + b)^2 + (a - b)^2}{(a^2 + b^2)} = \dfrac{2(a^2 + b^2)}{(a^2 + b^2)} = 2$.

71. Given Exp. $= \dfrac{(a + b)^2 - (a - b)^2}{ab} = \dfrac{4ab}{ab} = 4$.

72. We know that : $(1 + 2 + 3 + ... + n) = \dfrac{n(n + 1)}{2}$.

$\therefore (1 + 2 + 3 + ... + 45) = \left(\dfrac{45 \times 46}{2}\right) = 1035$.

73. Required numbers are 2, 4, 6, ..., 30.

This is an A.P. containing 15 terms.

$\therefore$ Required sum $= \dfrac{n}{2}$ (first term + last term) $= \dfrac{15}{2}(2 + 30) = 240$.

74. $(51 + 52 + 53 + ... + 100)$

$= (1 + 2 + 3 + ... + 100) - (1 + 2 + 3 + ... + 50)$

$= \left(\dfrac{100 \times 101}{2} - \dfrac{50 \times 51}{2}\right) = (5050 - 1275) = 3775$.

75. Every such number must be divisible by L.C.M. of 4, 5, 6, *i.e.* 60.
Such numbers are 240, 300, 360, 420, 480, 540.
Clearly, there are 6 such numbers.

76. Required numbers are 102, 108, 114, ..., 996.
This is an A.P. with $a = 102$ and $d = 6$.
Let the number of its terms be n. Then,
$a + (n - 1)\,d = 996 \Leftrightarrow 102 + (n - 1) \times 6 = 996 \Leftrightarrow n = 150$.

77. $2^2 + 4^2 + ... + 20^2 = (1 \times 2)^2 + (2 \times 2)^2 + (2 \times 3)^2 + ... + (2 \times 10)^2$
$= 2^2 \times 1^2 + 2^2 \times 2^2 + 2^2 \times 3^2 + ... + 2^2 \times 10^2$
$= 2^2\,[1^2 + 2^2 + 3^2 + ... + 10^2]$
$= 4 \times \dfrac{10 \times 11 \times 21}{6} = 4 \times 385 = 1540.$

78. $11^2 + 12^2 + 13^2 + ... + 20^2$
$= (1^2 + 2^2 + 3^2 + ... + 20^2) - (1^2 + 2^2 + 3^2 + ... + 10^2)$
$= \left[\dfrac{20\,(20 + 1)\,(40 + 1)}{6} - \dfrac{10\,(10 + 1)\,(20 + 1)}{6}\right] = 2485.$

79. $1 + x + 5 + 4 + 8 = (18 + x)$. Clearly, when $x = 0$, then sum of digits is divisible by 3.

80. Let the required number be 357y25x.
Then, for divisibility by 5, we must have $x = 0$ or $x = 5$.
Case I. When $x = 0$.
Then, sum of digits $= (22 + y)$. For divisibility by 3, $(22 + y)$ must be divisible by 3.
$\therefore$ $y = 2$ or 5 or 8.
$\therefore$ Numbers are (0, 2) or (0, 5) or (0, 8).
Case II. When $x = 5$.
Then, sum of digits $= (27 + y)$. For divisibility by 3, we must have $y = 0$ or 3 or 6 or 9.
$\therefore$ Numbers are (5, 0) or (5, 3) or (5, 6) or (5, 9).
So, correct answer is (*b*).

81. Let the number be 5x2. Clearly, it is divisible by 2.
Now, $5 + x + 2 = (7 + x)$ must be divisible by 3. So, $x = 2$.

82. The given number is divisible by 8, if the number 6x2 is divisible by 8.
Clearly, the least value of x is 3.

83. $(4 + 5 + 1 + x + 6 + 0 + 3) = 19 + x$. Clearly, $x = 8$.

84. Taking the sum of the digits, we have :
$S_1 = 9$, $S_2 = 12$, $S_3 = 18$, $S_4 = 9$, $S_5 = 21$, $S_6 = 12$, $S_7 = 18$, $S_8 = 21$, $S_9 = 15$, $S_{10} = 24$.
Clearly, S_2, S_5, S_6, S_8, S_9, S_{10} are all divisible by 3 but not by 9.
So, the number of required numbers = 6.

85. (*a*) $(1 + 6 + 3) - (2 + 5 + 4) = 1$ (No) (*b*) $(2 + 6 + 4) - (4 + 5 + 2) = 1$ (No)
(*c*) $(4 + 6 + 1) - (2 + 5 + 3) = 1$ (No) (*d*) $(4 + 6 + 1) - (2 + 5 + 4) = 0$ (Yes).

86. $(6 + 5 + 3 + 8) - (x + 2 + 6) = (14 - x)$. Now, $(14 - x)$ is divisible by 11, when $x = 3$.

87. $(4 + 7 + 6 + x + y + 0) = [17 + (x + y)]$. Also, $(0 + x + 7) - (y + 6 + 4) = (x - y - 3)$.
Now, $[17 + (x + y)]$ must be divisible by 3 and $(x - y - 3)$ is either 0 or divisible by 11.
Clearly, $x = 8$ and $y = 5$ satisfy both the conditions.

88. (*a*) 639 is not divisible by 7. (*b*) 2079 is divisible by 3, 7, 9 and 11.
(*c*) 3791 is not divisible by 3. (*d*) 37911 is not divisible by 9.
$\therefore$ Correct answer is (*b*).

89. Since 4864 is divisible by 4, so 9P2 must be divisible by 3.
$\therefore$ (11 + P) must be divisible by 3.
$\therefore$ Least value of P is 1.

90. The required number should be divisible by 3 and 8.
(*a*) 718 is not divisible by 8. (*b*) 810 is not divisible by 8.
(*c*) 804 is not divisible by 8.
(*d*) Sum of digits = 27, which is divisible by 3.
And, 736 is divisible by 8. So, given number is divisible by 3 and 8.

91. The given number should be divisible by both 9 and 8.
$\therefore$ $(4 + 2 + 5 + 7 + 3 + x) = (21 + x)$ is divisible by 9 and $(73x)$ is divisible by 8.
$\therefore$ $x = 6$.

92. The required number should be divisible by both 9 and 11.
Clearly, 114345 is divisible by both 9 and 11. So, it is divisible by 99.

93. The given number will be divisible by 99 if it is divisible by both 9 and 11.
Now, $(3 + 4 + 2 + 2 + 2 + 1 + 3 + x + y) = 17 + (x + y)$ must be divisible by 9.
Also, $(y + 3 + 2 + 2 + 3) - (x + 1 + 2 + 4) = (y - x + 3)$ must be 0 or divisible by 11.
$\therefore$ $x + y = 10$ and $y - x + 3 = 0$.
Clearly, $x = 1$, $y = 9$ satisfy both these equations.

94. Since 653xy is divisible by 5 as well as 2, so $y = 0$.
Now, 653x0 must be divisible by 8.
So, 3x0 must be divisible by 8. This happens when $x = 2$.
$\therefore$ $x + y = (2 + 0) = 2$.

95. A number is divisible by 132, if it is divisible by each one of 11, 3 and 4.
Clearly, 968 is not divisible by 3. None of 462 and 2178 is divisible by 4.
Also, 5184 is not divisible by 11.
Each one of remaining 4 is divisible by each one of 11, 3 and 4 and therefore, by 132.

96. Clearly, 6897 is divisible by both 11 and 19.

97. None of the numbers in (*a*) and (*c*) is divisible by 2.
Number in (*b*) is not divisible by 3.
Clearly, 510510 is divisible by each prime number between 1 and 17.

98. Clearly, 325325 is divisible by all 7, 11 and 13.

99. Sum of digits = 35 and so it is not divisible by 3.
(Sum of digits at odd places) − (Sum of digits at even places) = (19 − 16) = 3, not divisible by 11.
So, the given number is neither divisible by 3 nor by 11.

100. Since 111111 is divisible by each one of 7, 11 and 13, so each one of given type of numbers is divisible by each one of 7, 11, 13, as we may write, $222222 = 2 \times 111111$, $333333 = 3 \times 111111$, etc.

101. Smallest 3-digit prime number is 101. Clearly, $2525 = 25 \times 101$; $3232 = 32 \times 101$, etc.
$\therefore$ Each such number is divisible by 101.

102. $256256 = 256 \times 1001$; $678678 = 678 \times 1001$, etc.
So, any number of this form is divisible by 1001.

103. Required number $= 1 \times 2 \times 3 \times 4 = 24$.

104. Required number $= (2 \times 4 \times 6) = 48$.

105. Let the three consecutive odd numbers be $(2x + 1)$, $(2x + 3)$ and $(2x + 5)$.
Their sum $= (6x + 9) = 3(2x + 3)$, which is always divisible by 3.

106. Let the two consecutive odd integers be $(2x + 1)$ and $(2x + 3)$.

Then, $(2x + 3)^2 - (2x + 1)^2 = (2x + 3 + 2x + 1)(2x + 3 - 2x - 1) = (4x + 4) \times 2$

$= 8(x + 1)$, which is always divisible by 8.

107. Let the required number be x.

Then, $(11x + 11) = 11(x + 1)$ is divisible by 13. So, $x = 12$.

108. Let the 3-digit number be xyz. Then,

$(100x + 10y + z) - (x + y + z) = 99x + 9y = 9(11x + y)$, which is divisible by 9.

109. Putting $x = 5$ and $y = 1$, we get $(3x + 7y) = (3 \times 5 + 7 \times 1) = 22$, which is divisible by 11.

$\therefore \quad 4x + 5y = (4 \times 5 + 5 \times 1) = 25$, which is not divisible by 11.

$x + y + 4 = (5 + 1 + 4) = 9$, which is not divisible by 11.

$9x + 4y = (9 \times 5 + 4 \times 1) = 49$, which is not divisible by 11.

$4x - 9y = (4 \times 5 - 9 \times 1) = 11$, which is divisible by 11.

110. $\left.\begin{array}{r} 4\,a\,3 \\ 9\,8\,4 \\ \hline 13\,b\,7 \end{array}\right\} \Rightarrow \quad a + 8 = b \quad \Rightarrow \quad b - a = 8$

Also, $13b7$ is divisible by 11.

$\therefore \quad (7 + 3) - (b + 1) = (9 - b) \quad \Rightarrow \quad (9 - b) = 0 \quad \Rightarrow \quad b = 9.$

$\therefore \quad b = 9$ and $a = 1 \quad \Rightarrow \quad (a + b) = 10.$

111. Required number $= (2^5 - 2) = (32 - 2) = 30.$

112. Required number = Product of first three multiples of $3 = (3 \times 6 \times 9) = 162.$

113. On dividing 1000 by 45, we get remainder = 10.

$\therefore$ Required number to be added $= (45 - 10) = 35.$

114. On dividing 803642 by 11, we get remainder = 4.

$\therefore$ Required number to be added $= (11 - 4) = 7.$

115. On dividing 11158 by 77, we get remainder = 70.

$\therefore$ Required number to be added $= (77 - 70) = 7.$

116. On dividing 6709 by 9, we get remainder = 4.

$\therefore$ Required number to be subtracted = 4.

117. On dividing 427398 by 15, we get remainder = 3.

$\therefore$ Required number to be subtracted = 3.

118. On dividing 13294 by 97, we get remainder = 5.

$\therefore$ Required number to be subtracted = 5.

119. Clearly, 5 × (sum of numbers) is divisible by 15.

$\therefore$ Sum of numbers must be divisible by 3.

Now, $(250 + 341) = 591$ is divisible by 3. So, required pair is 250, 341.

120. Required number is divisible by 72 as well as by 112, if it is divisible by their LCM, which is 1008.

Now, 1008 when divided by 72, gives quotient = 14.

$\therefore$ Required number = 14.

121. Let it be n times. Then, $(1111 - 99n) < 99$.

By hit and trial, we find that $n = 11$.

122. On dividing 457 by 11, remainder is 6.

$\therefore$ Required number is either 451 or 462. Nearest to 456 is 462.

123. On dividing 99547 by 687, the remainder is 619, which is more than half of 687.
So, we must add (687 – 619) = 68 to the given number.
∴ Required number = (99547 + 68) = 99615.

124. Largest number of 5 digits = 99999. On dividing 99999 by 99, we get 9 as remainder.
∴ Required number = (99999 – 9) = 99990.

125. Smallest number of 5 digits = 10000.
On dividing 10000 by 476, we get remainder = 4.
∴ Required number = [10000 + (476 – 4)] = 10472.

126. Required number = $999 \times 366 + 103 = (1000 - 1) \times 366 + 103 = 366000 - 366 + 103$
$= 365737.$

127. $4150 = 55 \times x + 25 \Leftrightarrow 55x = 4125 \Leftrightarrow x = \frac{4125}{55} = 75.$

128. Required number = $(555 + 445) \times 2 \times 110 + 30 = 220000 + 30 = 220030.$

129. Largest number of 4 digits = 9999. On dividing 9999 by 7, we get remainder = 3.
Largest number of 4 digits divisible by 7 is (9999 – 3) = 9996.
Let (9996 – x + 10) be divisible by 3. By hit and trial, we find that x = 7.
∴ Required number = (9996 – 7) = 9989.

130. Number = $(114 \times Q) + 21 = 19 \times 6 \times Q + 19 + 2 = 19 \times (6Q + 1) + 2.$
∴ Required remainder = 2.

131. Number = $(296 \times Q) + 75 = (37 \times 8Q) + (37 \times 2) + 1 = 37 \times (8Q + 2) + 1.$
∴ Required remainder = 1.

132. Number = $(119 \times Q) + 19 = 17 \times (7Q) + (17 + 2) = 17 \times (7Q + 1) + 2.$
∴ Required remainder = 2.

133. Number = $(899 \times Q) + 63 = (29 \times 31 \times Q) + (29 \times 2) + 5 = 29 \times (31Q + 2) + 5.$
∴ Required remainder = 5.

134. Number = (31 × Q) + 29. Given data is inadequate.

135. Given number = 13p + 11. And, Given number = 17q + 9.
∴ $13p + 11 = 17q + 9 \Leftrightarrow 17q - 13p = 2.$
By hit and trial, we find that p = 26 and q = 20.
∴ Required number = (13 × 26 + 11) = 349.

136. Divisor = (5 × 46) = 230. Also, 10 × Q = 230 ⇒ Q = 23. And, R = 46.
∴ Dividend = (230 × 23 + 46) = 5336.

137. Let the smaller number be x. Then, larger number = (1365 + x).
∴ $1365 + x = 6x + 15 \Leftrightarrow 5x = 1350 \Leftrightarrow x = 270.$
Hence, the required number is 270.

138. Dividend = (12 × 35) = 420. Now, dividend = 420 and divisor = 21.
∴ Correct quotient = $\frac{420}{21} = 20.$

139. Let $n = 4q + 3 \Rightarrow 2n = 8q + 6 = (8q + 4) + 2 \Rightarrow 2n = 4(2q + 1) + 2.$
So, when 2n is divided by 4, remainder = 2.

140. Let $x = 6q + 3$. Then, $x^2 = (6q + 3)^2 = 36q^2 + 36q + 9 = 6(6q^2 + 6q + 1) + 3.$
So, when x^2 is divided by 6, remainder = 3.

141.

4	x
5	y – 1
	1 – 4

∴ y = (5 × 1 + 4) = 9
∴ x = (4y + 1) = (4 × 9 + 1) = 37.

Now, 37 when divided successively by 5 and 4, we get :

5	37
4	7 – 2
	1 – 3

$\therefore$ Respective remainders are 2, 3.

142.

4	x
5	$y - 2$
6	$z - 3$
	$1 - 4$

$\therefore$

143.

5	x
9	$y - 4$
13	$z - 8$
	$1 - 12$

$\therefore$

Now, 1169 when divided by 585 gives remainder = 584.

144. Let $n = 3q + 1$ and let $q = 2p + 1$. Then, $n = 3(2p + 1) + 1 = 6p + 4$.

$\therefore$ The number when divided by 6, we get remainder = 4.

145. $4^{61} + 4^{62} + 4^{63} + 4^{64} = 4^{61}(1 + 4 + 4^2 + 4^3) = 4^{61} \times 85 = 4^{60} \times 340$, which is clearly divisible by 10.

146. Putting $x = 2$, we get $2^2(2^2 - 1) = 12$. So, $x^2(x^2 - 1)$ is always divisible by 12.

2. H.C.F. AND L.C.M. OF NUMBERS

IMPORTANT FACTS AND FORMULAE

I. **Factors and Multiples** : If a number a divides another number b exactly, we say that a is a ***factor*** of b. In this case, b is called a ***multiple*** of a.

II. **Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.)** : The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are two methods of finding the H.C.F. of a given set of numbers :

1. ***Factorization Method*** : *Express each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.*
2. ***Division Method*** : *Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.*

Finding the H.C.F. of more than two numbers : Suppose we have to find the H.C.F. of three numbers. Then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.

Similarly, the H.C.F. of more than three numbers may be obtained.

III. **Least Common Multiple (L.C.M.)** : The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

1. ***Factorization Method of Finding L.C.M.*** : *Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.*
2. ***Common Division Method (Short-cut Method) of Finding L.C.M.*** : *Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.*

IV. **Product of two numbers = Product of their H.C.F. and L.C.M.**

V. **Co-primes** : Two numbers are said to be co-primes if their H.C.F. is 1.

VI. **H.C.F. and L.C.M. of Fractions** :

1. $\text{H.C.F.} = \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$ 2. $\text{L.C.M.} = \dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$.

VII. **H.C.F. and L.C.M. of Decimal Fractions** : *In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.*

VIII. **Comparison of Fractions** : *Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.*

SOLVED EXAMPLES

Ex. 1. ***Find the H.C.F. of $2^3 \times 3^2 \times 5 \times 7^4$, $2^2 \times 3^5 \times 5^2 \times 7^3$, $2^3 \times 5^3 \times 7^2$.***

Sol. The prime numbers common to given numbers are 2, 5 and 7.

$\therefore$ H.C.F. $= 2^2 \times 5 \times 7^2 = 980$.

Ex. 2. ***Find the H.C.F. of 108, 288 and 360.***

Sol. $108 = 2^2 \times 3^3$, $288 = 2^5 \times 3^2$ and $360 = 2^3 \times 5 \times 3^2$.

$\therefore$ H.C.F. $= 2^2 \times 3^2 = 36$.

Ex. 3. ***Find the H.C.F. of 513, 1134 and 1215.***

Sol.

$$1134 \overline{)\ 1215\ (\ 1}$$
$$\underline{1134}$$
$$81 \overline{)\ 1134\ (\ 14}$$
$$\underline{81}$$
$$324$$
$$\underline{324}$$
$$\times$$

$\therefore$ H.C.F. of 1134 and 1215 is 81.

So, Required H.C.F. = H.C.F. of 513 and 81.

$$81 \overline{)\ 513\ (\ 6}$$
$$\underline{486}$$
$$27 \overline{)\ 81\ (\ 3}$$
$$\underline{81}$$
$$\times$$

$\therefore$ H.C.F. of given numbers = 27.

Ex. 4. ***Reduce $\frac{391}{667}$ to lowest terms.***

Sol. H.C.F. of 391 and 667 is 23.

On dividing the numerator and denominator by 23, we get :

$$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}.$$

Ex. 5. ***Find the L.C.M. of $2^2 \times 3^3 \times 5 \times 7^2$, $2^3 \times 3^2 \times 5^2 \times 7^4$, $2 \times 3 \times 5^3 \times 7 \times 11$.***

Sol. L.C.M. = Product of highest powers of 2, 3, 5, 7 and 11 $= 2^3 \times 3^3 \times 5^3 \times 7^4 \times 11$.

Ex. 6. ***Find the L.C.M. of 72, 108 and 2100.***

Sol. $72 = 2^3 \times 3^2$, $108 = 3^3 \times 2^2$, $2100 = 2^2 \times 5^2 \times 3 \times 7$.

$\therefore$ L.C.M. $= 2^3 \times 3^3 \times 5^2 \times 7 = 37800$.

Ex. 7. ***Find the L.C.M. of 16, 24, 36 and 54.***

Sol.

2	16	24	36	54
2	8	12	18	27
2	4	6	9	27
3	2	3	9	27
3	2	1	3	9
	2	1	1	3

$\therefore$ L.C.M. $= 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 3 = 432$.

Ex. 8. ***Find the H.C.F. and L.C.M. of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$.***

Sol. H.C.F. of given fractions $= \dfrac{\text{H.C.F. of } 2, 8, 16, 10}{\text{L.C.M. of } 3, 9, 81, 27} = \dfrac{2}{81}$.

L.C.M. of given fractions $= \dfrac{\text{L.C.M. of } 2, 8, 16, 10}{\text{H.C.F. of } 3, 9, 81, 27} = \dfrac{80}{3}$.

Ex. 9. ***Find the H.C.F. and L.C.M. of 0.63, 1.05 and 2.1.***

Sol. Making the same number of decimal places, the given numbers are 0.63, 1.05 and 2.10.

Without decimal places, these numbers are 63, 105 and 210.

Now, H.C.F. of 63, 105 and 210 is 21.

$\therefore$ H.C.F. of 0.63, 1.05 and 2.1 is 0.21.

L.C.M. of 63, 105 and 210 is 630.

$\therefore$ L.C.M. of 0.63, 1.05 and 2.1 is 6.30.

Ex. 10. ***Two numbers are in the ratio of 15 : 11. If their H.C.F. is 13, find the numbers.***

Sol. Let the required numbers be $15x$ and $11x$.

Then, their H.C.F. is x. So, $x = 13$.

$\therefore$ The numbers are $(15 \times 13$ and $11 \times 13)$ *i.e.*, 195 and 143.

Ex. 11. ***The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, find the other.***

Sol. Other number $= \left(\dfrac{11 \times 693}{77}\right) = 99.$

Ex. 12. ***Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.***

Sol. Required length = H.C.F. of 495 cm, 900 cm and 1665 cm.

$495 = 3^2 \times 5 \times 11$, $900 = 2^2 \times 3^2 \times 5^2$, $1665 = 3^2 \times 5 \times 37$.

$\therefore$ H.C.F. $= 3^2 \times 5 = 45$.

Hence, required length = 45 cm.

Ex. 13. ***Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.***

Sol. Required number = H.C.F. of $(1657 - 6)$ and $(2037 - 5)$ = H.C.F. of 1651 and 2032

$$
\begin{array}{r}
1651\overline{)\,2032\,(\,1} \\
\underline{1651} \\
381\overline{)\,1651\,(\,4} \\
\underline{1524} \\
127\overline{)\,381\,(\,3} \\
\underline{381} \\
\times
\end{array}
$$

$\therefore$ Required number = 127.

Ex. 14. ***Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.***

Sol. Required number = H.C.F. of $(132 - 62)$, $(237 - 132)$ and $(237 - 62)$

= H.C.F. of 70, 105 and 175 = 35.

Ex. 15. ***Find the least number exactly divisible by 12, 15, 20 and 27.***

Sol. Required number = L.C.M. of 12, 15, 20, 27.

3	12 – 15 – 20 – 27
4	4 – 5 – 20 – 9
5	1 – 5 – 5 – 9
	1 – 1 – 1 – 9

$\therefore$ L.C.M. = $3 \times 4 \times 5 \times 9 = 540$.

Hence, required number = 540.

Ex. 16. ***Find the least number which when divided by 6, 7, 8, 9 and 12 leaves the same remainder 1 in each case.***

Sol. Required number = (L.C.M. of 6, 7, 8, 9, 12) + 1

3	6 – 7 – 8 – 9 – 12
2	2 – 7 – 8 – 3 – 4
2	1 – 7 – 4 – 3 – 2
	1 – 7 – 2 – 3 – 1

$\therefore$ L.C.M. = $3 \times 2 \times 2 \times 7 \times 2 \times 3 = 504$.

Hence, required number = $(504 + 1) = 505$.

Ex. 17. ***Find the largest number of four digits exactly divisible by 12, 15, 18 and 27.***

Sol. The largest number of four digits is 9999.

Required number must be divisible by L.C.M. of 12, 15, 18, 27 *i.e.*, 540.

On dividing 9999 by 540, we get 279 as remainder.

$\therefore$ Required number = $(9999 - 279) = 9720$.

Ex. 18. ***Find the smallest number of five digits exactly divisible by 16, 24, 36 and 54.***

Sol. Smallest number of five digits is 10000.

Required number must be divisible by L.C.M. of 16, 24, 36, 54 *i.e.*, 432.

On dividing 10000 by 432, we get 64 as remainder.

$\therefore$ Required number = $10000 + (432 - 64) = 10368$.

Ex. 19. ***Find the least number which when divided by 20, 25, 35 and 40 leaves remainders 14, 19, 29 and 34 respectively.***

Sol. Here, $(20 - 14) = 6$, $(25 - 19) = 6$, $(35 - 29) = 6$ and $(40 - 34) = 6$.

$\therefore$ Required number = (L.C.M. of 20, 25, 35, 40) – 6 = 1394.

Ex. 20. ***Find the least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder.***

Sol. L.C.M. of 5, 6, 7, 8 = 840.

$\therefore$ Required number is of the form $840k + 3$.

Least value of k for which $(840k + 3)$ is divisible by 9 is $k = 2$.

$\therefore$ Required number = $(840 \times 2 + 3) = 1683$.

Ex. 21. ***The traffic lights at three different road crossings change after every 48 sec., 72 sec. and 108 sec. respectively. If they all change simultaneously at 8 : 20 : 00 hours, then at what time will they again change simultaneously ?***

Sol. Interval of change = (L.C.M. of 48, 72, 108) sec. = 432 sec.

So, the lights will again change simultaneously after every 432 seconds *i.e.*, 7 min. 12 sec.

Hence, next simultaneous change will take place at 8 : 27 : 12 hrs.

Ex. '22. ***Arrange the fractions*** $\frac{17}{18}, \frac{31}{36}, \frac{43}{45}, \frac{59}{60}$ ***in the ascending order.***

Sol. L.C.M. of 18, 36, 45 and 60 = 180.

Now, $\frac{17}{18} = \frac{17\times10}{18\times10} = \frac{170}{180}$; $\frac{31}{36} = \frac{31\times5}{36\times5} = \frac{155}{180}$;

$\frac{43}{45} = \frac{43\times4}{45\times4} = \frac{172}{180}$; $\frac{59}{60} = \frac{59\times3}{60\times3} = \frac{177}{180}$.

Since, 155 < 170 < 172 < 177, so, $\frac{155}{180} < \frac{170}{180} < \frac{172}{180} < \frac{177}{180}$.

Hence, $\frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$.

EXERCISE 2

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. 252 can be expressed as a product of primes as : (IGNOU, 2002)
   (a) $2\times2\times3\times3\times7$ (b) $2\times2\times2\times3\times7$
   (c) $3\times3\times3\times3\times7$ (d) $2\times3\times3\times3\times7$
2. Which of the following has most number of divisors ? (M.B.A. 2002)
   (a) 99 (b) 101 (c) 176 (d) 182
3. A number n is said to be perfect if the sum of all its divisors (excluding n itself) is equal to n. An example of perfect number is :
   (a) 6 (b) 9 (c) 15 (d) 21
4. $\frac{1095}{1168}$ when expressed in simplest form is : (M.B.A. 1998)
   (a) $\frac{13}{16}$ (b) $\frac{15}{16}$ (c) $\frac{17}{26}$ (d) $\frac{25}{26}$
5. Reduce $\frac{128352}{238368}$ to its lowest terms. (IGNOU, 2003)
   (a) $\frac{3}{4}$ (b) $\frac{5}{13}$ (c) $\frac{7}{13}$ (d) $\frac{9}{13}$
6. The H.C.F. of $2^2\times3^3\times5^5$, $2^3\times3^2\times5^2\times7$ and $2^4\times3^4\times5\times7^2\times11$ is :
   (a) $2^2\times3^2\times5$ (b) $2^2\times3^2\times5\times7\times11$
   (c) $2^4\times3^4\times5^5$ (d) $2^4\times3^4\times5^5\times7\times11$
7. The H.C.F. of $2^4\times3^2\times5^3\times7$, $2^3\times3^3\times5^2\times7^2$ and $3\times5\times7\times11$ is :
   (a) 105 (b) 1155 (c) 2310 (d) 27720
8. H.C.F. of $4\times27\times3125$, $8\times9\times25\times7$ & $16\times81\times5\times11\times49$ is : (C.B.I. 1997)
   (a) 180 (b) 360 (c) 540 (d) 1260
9. Find the highest common factor of 36 and 84. (R.R.B. 2003)
   (a) 4 (b) 6 (c) 12 (d) 18
10. The H.C.F. of 204, 1190 and 1445 is :
    (a) 17 (b) 18 (c) 19 (d) 21
11. Which of the following is a pair of co-primes ?
    (a) (16, 82) (b) (18, 25) (c) (21, 35) (d) (23, 92)

12. The H.C.F. of 2923 and 3239 is :
(*a*) 37 (*b*) 47 (*c*) 73 (*d*) 79

13. The H.C.F. of 3556 and 3444 is :
(*a*) 23 (*b*) 25 (*c*) 26 (*d*) 28

14. The L.C.M. of $2^3 \times 3^2 \times 5 \times 11$, $2^4 \times 3^4 \times 5^2 \times 7$ and $2^5 \times 3^3 \times 5^3 \times 7^2 \times 11$ is :
(*a*) $2^3 \times 3^2 \times 5$ (*b*) $2^5 \times 3^4 \times 5^3$
(*c*) $2^3 \times 3^2 \times 5 \times 7 \times 11$ (*d*) $2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$

15. Find the lowest common multiple of 24, 36 and 40. **(R.R.B. 2003)**
(*a*) 120 (*b*) 240 (*c*) 360 (*d*) 480

16. The L.C.M. of 22, 54, 108, 135 and 198 is : **(M.B.A. 1998)**
(*a*) 330 (*b*) 1980 (*c*) 5940 (*d*) 11880

17. The L.C.M. of 148 and 185 is :
(*a*) 680 (*b*) 740 (*c*) 2960 (*d*) 3700

18. The H.C.F. of $\frac{2}{3}, \frac{8}{9}, \frac{64}{81}$ and $\frac{10}{27}$ is :
(*a*) $\frac{2}{3}$ (*b*) $\frac{2}{81}$ (*c*) $\frac{160}{3}$ (*d*) $\frac{160}{81}$

19. The H.C.F. of $\frac{9}{10}, \frac{12}{25}, \frac{18}{35}$ and $\frac{21}{40}$ is :
(*a*) $\frac{3}{5}$ (*b*) $\frac{252}{5}$ (*c*) $\frac{3}{2800}$ (*d*) $\frac{63}{700}$

20. The L.C.M. of $\frac{1}{3}, \frac{5}{6}, \frac{2}{9}, \frac{4}{27}$ is :
(*a*) $\frac{1}{54}$ (*b*) $\frac{10}{27}$ (*c*) $\frac{20}{3}$ (*d*) None of these

21. The L.C.M. of $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{9}{13}$ is :
(*a*) 36 (*b*) $\frac{1}{36}$ (*c*) $\frac{1}{1365}$ (*d*) $\frac{12}{455}$

22. The H.C.F. of 1.75, 5.6 and 7 is :
(*a*) 0.07 (*b*) 0.7 (*c*) 3.5 (*d*) 0.35

23. The G.C.D. of 1.08, 0.36 and 0.9 is : **(Hotel Management, 2002)**
(*a*) 0.03 (*b*) 0.9 (*c*) 0.18 (*d*) 0.108

24. The H.C.F. of 0.54, 1.8 and 7.2 is :
(*a*) 1.8 (*b*) 0.18 (*c*) 0.018 (*d*) 18

25. The L.C.M. of 3, 2.7 and 0.09 is :
(*a*) 2.7 (*b*) 0.27 (*c*) 0.027 (*d*) 27

26. H.C.F. of 3240, 3600 and a third number is 36 and their L.C.M. is $2^4 \times 3^5 \times 5^2 \times 7^2$. The third number is : **(S.S.C. 1999)**
(*a*) $2^2 \times 3^5 \times 7^2$ (*b*) $2^2 \times 5^3 \times 7^2$ (*c*) $2^5 \times 5^2 \times 7^2$ (*d*) $2^3 \times 3^5 \times 7^2$

27. Three numbers are in the ratio 1 : 2 : 3 and their H.C.F. is 12. The numbers are :
(*a*) 4, 8, 12 (*b*) 5, 10, 15 (*c*) 10, 20, 30 (*d*) 12, 24, 36

(Section Officers', 2001)

28. The ratio of two numbers is 3 : 4 and their H.C.F. is 4. Their L.C.M. is :
(*a*) 12 (*b*) 16 (*c*) 24 (*d*) 48

(S.S.C. 2002)

29. The sum of two numbers is 216 and their H.C.F. is 27. The numbers are :
(*a*) 27, 189 (*b*) 81, 189 (*c*) 108, 108 (*d*) 154, 16

30. The sum of two numbers is 528 and their H.C.F. is 33. The number of pairs of numbers satisfying the above conditions is : (C.B.I. 1997)
(a) 4 (b) 6 (c) 8 (d) 12

31. The number of number-pairs lying between 40 and 100 with their H.C.F. as 15 is :
(a) 3 (b) 4 (c) 5 (d) 6

32. The H.C.F. of two numbers is 12 and their difference is 12. The numbers are :
(a) 66, 78 (b) 70, 82 (c) 94, 106 (d) 84, 96

33. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is : (S.S.C. 2003)
(a) 101 (b) 107 (c) 111 (d) 185

34. The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is : (C.B.I. 2003)
(a) 1 (b) 2 (c) 3 (d) 4

35. Three numbers which are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbers is :
(a) 75 (b) 81 (c) 85 (d) 89
(S.S.C. 2003)

36. The L.C.M. of two numbers is 48. The numbers are in the ratio 2 : 3. The sum of the numbers is : (S.S.C. 2003)
(a) 28 (b) 32 (c) 40 (d) 64

37. Three numbers are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is :
(a) 40 (b) 80 (c) 120 (d) 200
(M.B.A. 2003)

38. The H.C.F. of two numbers is 11 and their L.C.M. is 7700. If one of the numbers is 275, then the other is : (Section Officers', 2001)
(a) 279 (b) 283 (c) 308 (d) 318

39. The sum of two numbers is 2000 and their L.C.M. is 21879. The two numbers are :
(a) 1993, 7 (b) 1991, 9 (c) 1989, 11 (d) 1987, 13

40. The H.C.F. and L.C.M. of two numbers are 84 and 21 respectively. If the ratio of the two numbers is 1 : 4, then the larger of the two numbers is : (M.A.T. 1997)
(a) 12 (b) 48 (c) 84 (d) 108

41. The L.C.M. of two numbers is 495 and their H.C.F. is 5. If the sum of the numbers is 10, then their difference is : (S.S.C. 1999)
(a) 10 (b) 46 (c) 70 (d) 90

42. The product of the L.C.M. and H.C.F. of two numbers is 24. The difference of two numbers is 2. Find the numbers.
(a) 2 and 4 (b) 6 and 4 (c) 8 and 6 (d) 8 and 10

43. If the sum of two numbers is 55 and the H.C.F. and L.C.M. of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to :
(a) $\frac{55}{601}$ (b) $\frac{601}{55}$ (c) $\frac{11}{120}$ (d) $\frac{120}{11}$
(C.D.S. 2003)

44. The L.C.M. of two numbers is 45 times their H.C.F. If one of the numbers is 125 and the sum of H.C.F. and L.C.M. is 1150, the other number is :
(a) 215 (b) 220 (c) 225 (d) 235

45. The H.C.F. and L.C.M. of two numbers are 50 and 250 respectively. If the first number is divided by 2, the quotient is 50. The second number is :
(a) 50 (b) 100 (c) 125 (d) 250

46. The product of two numbers is 1320 and their H.C.F. is 6. The L.C.M. of the numbers is :
(a) 220 (b) 1314 (c) 1326 (d) 7920

47. Product of two co-prime numbers is 117. Their L.C.M. should be : (C.B.I. 1997)
(a) 1 (b) 117 (c) equal to their H.C.F. (d) cannot be calculated

48. The L.C.M. of three different numbers is 120. Which of the following cannot be their H.C.F. ?
(a) 8 (b) 12 (c) 24 (d) 35

49. The H.C.F. of two numbers is 8. Which one of the following can never be their L.C.M. ?
(a) 24 (b) 48 (c) 56 (d) 60 (S.S.C. 2000)

50. The H.C.F. of two numbers is 23 and the other two factors of their L.C.M. are 13 and 14. The larger of the two numbers is : (S.S.C. 2004)
(a) 276 (b) 299 (c) 322 (d) 345

51. About the number of pairs which have 16 as their H.C.F. and 136 as their L.C.M., we can definitely say that :
(a) no such pair exists (b) only one such pair exists
(c) only two such pairs exist (d) many such pairs exist

52. The H.C.F. and L.C.M. of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is : (C.B.I. 1998)
(a) 77 (b) 88 (c) 99 (d) 110

53. Two numbers, both greater than 29, have H.C.F. 29 and L.C.M. 4147. The sum of the numbers is : (S.S.C. 2002)
(a) 666 (b) 669 (c) 696 (d) 966

54. L.C.M. of two prime numbers x and y ($x > y$) is 161. The value of $3y - x$ is :
(a) -2 (b) -1 (c) 1 (d) 2 (S.S.C. 1999)

55. The greatest number that exactly divides 105, 1001 and 2436 is :
(a) 3 (b) 7 (c) 11 (d) 21

56. The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is : (R.R.B. 2003)
(a) 15 cm (b) 25 cm (c) 35 cm (d) 42 cm

57. Three different containers contain 496 litres, 403 litres and 713 litres of mixtures of milk and water respectively. What biggest measure can measure all the different quantities exactly ?
(a) 1 litre (b) 7 litres (c) 31 litres (d) 41 litres

58. The maximum number of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is : (S.S.C. 1999)
(a) 91 (b) 910 (c) 1001 (d) 1911

59. A rectangular courtyard 3.78 metres long and 5.25 metres wide is to be paved exactly with square tiles, all of the same size. What is the largest size of the tile which could be used for the purpose ? (N.I.F.T. 2000)
(a) 14 cms (b) 21 cms (c) 42 cms (d) None of these

60. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case. (L.I.C. 2003)
(a) 4 (b) 7 (c) 9 (d) 13

61. Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same remainder in each case. Then sum of the digits in N is : (S.S.C. 2004)
(a) 4 (b) 5 (c) 6 (d) 8

62. The greatest number which can divide 1356, 1868 and 2764 leaving the same remainder 12 in each case, is :
(a) 64 (b) 124 (c) 156 (d) 260

63. The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively, is : **(R.R.B. 2004)**

(a) 123 (b) 127 (c) 235 (d) 305

64. Which of the following fractions is the largest ? **(IGNOU, 2003)**

(a) $\frac{7}{8}$ (b) $\frac{13}{16}$ (c) $\frac{31}{40}$ (d) $\frac{63}{80}$

65. What will be the least number which when doubled will be exactly divisible by 12, 18, 21 and 30 ? **(S.S.C. 2003)**

(a) 196 (b) 630 (c) 1260 (d) 2520

66. The smallest fraction, which each of $\frac{6}{7}, \frac{5}{14}, \frac{10}{21}$ will divide exactly, is : **(S.S.C. 1998)**

(a) $\frac{30}{7}$ (b) $\frac{30}{98}$ (c) $\frac{60}{147}$ (d) $\frac{50}{294}$

67. The least number of five digits which is exactly divisible by 12, 15 and 18, is :

(a) 10010 (b) 10015 (c) 10020 (d) 10080

68. The greatest number of four digits which is divisible by 15, 25, 40 and 75 is :

(a) 9000 (b) 9400 (c) 9600 (d) 9800 **(S.S.C. 2002)**

69. The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is : **(Hotel Management, 2003)**

(a) 3 (b) 13 (c) 23 (d) 33

70. The least number which is a perfect square and is divisible by each of the numbers 16, 20 and 24, is :

(a) 1600 (b) 3600 (c) 6400 (d) 14400

71. The smallest number which when diminished by 7, is divisible by 12, 16, 18, 21 and 28 is : **(L.I.C. 2003)**

(a) 1008 (b) 1015 (c) 1022 (d) 1032

72. The least number which when increased by 5 is divisible by each one of 24, 32, 36 and 54, is :

(a) 427 (b) 859 (c) 869 (d) 4320

73. The least number, which when divided by 12, 15, 20 and 54 leaves in each case a remainder of 8, is : **(R.R.B. 2003)**

(a) 504 (b) 536 (c) 544 (d) 548

74. The largest four-digit number which when divided by 4, 7 or 13 leaves a remainder of 3 in each case, is :

(a) 8739 (b) 9831 (c) 9834 (d) 9893

75. Let the least number of six digits, which when divided by 4, 6, 10 and 15, leaves in each case the same remainder of 2, be N. The sum of the digits in N is : **(S.S.C. 2003)**

(a) 3 (b) 4 (c) 5 (d) 6

76. The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is : **(A.A.O. Exam, 2003)**

(a) 74 (b) 94 (c) 184 (d) 364

77. The least number, which when divided by 48, 60, 72, 108 and 140 leaves 38, 50, 62, 98 and 130 as remainders respectively, is : **(C.B.I. 1997)**

(a) 11115 (b) 15110 (c) 15120 (d) 15210

78. Find the least multiple of 23, which when divided by 18, 21 and 24 leaves remainders 7, 10 and 13 respectively.

(a) 3002 (b) 3013 (c) 3024 (d) 3036

79. The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder, is : **(L.I.C.A.A.O. 2003)**

(a) 1677 (b) 1683 (c) 2523 (d) 3363

80. Find the least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case, but when divided by 7 leaves no remainder.
(a) 17004 (b) 18000 (c) 18002 (d) 18004

81. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?
(a) 4 (b) 10 (c) 15 (d) 16

82. Four different electronic devices make a beep after every 30 minutes, 1 hour, $1\frac{1}{2}$ hour and 1 hour 45 minutes respectively. All the devices beeped together at 12 noon. They will again beep together at :
(a) 12 midnight (b) 3 a.m. (c) 6 a.m. (d) 9 a.m.

83. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds, all starting at the same point. After what time will they meet again at the starting point ? **(S.S.C. 2003)**
(a) 26 minutes 18 seconds (b) 42 minutes 36 seconds
(c) 45 minutes (d) 46 minutes 12 seconds

ANSWERS

1. (a)	2. (c)	3. (a)	4. (b)	5. (c)	6. (a)	7. (a)	8. (a)	9. (c)
10. (a)	11. (b)	12. (d)	13. (d)	14. (d)	15. (c)	16. (c)	17. (b)	18. (b)
19. (c)	20. (c)	21. (a)	22. (d)	23. (c)	24. (b)	25. (d)	26. (a)	27. (d)
28. (d)	29. (a)	30. (a)	31. (b)	32. (d)	33. (c)	34. (b)	35. (c)	36. (c)
37. (a)	38. (c)	39. (c)	40. (c)	41. (a)	42. (b)	43. (c)	44. (c)	45. (c)
46. (a)	47. (b)	48. (d)	49. (d)	50. (c)	51. (a)	52. (a)	53. (c)	54. (a)
55. (b)	56. (c)	57. (c)	58. (a)	59. (b)	60. (a)	61. (a)	62. (a)	63. (b)
64. (a)	65. (b)	66. (a)	67. (d)	68. (c)	69. (c)	70. (b)	71. (b)	72. (b)
73. (d)	74. (b)	75. (c)	76. (d)	77. (b)	78. (b)	79. (b)	80. (d)	81. (d)
82. (d)	83. (d)							

SOLUTIONS

1. Clearly, $252 = 2 \times 2 \times 3 \times 3 \times 7$.

2. $99 = 1 \times 3 \times 3 \times 11$; $101 = 1 \times 101$;
$176 = 1 \times 2 \times 2 \times 2 \times 2 \times 11$; $182 = 1 \times 2 \times 7 \times 13$.
So, divisors of 99 are 1, 3, 9, 11, 33 and 99;
divisors of 101 are 1 and 101;
divisors of 176 are 1, 2, 4, 8, 16, 22, 44, 88 and 176;
divisors of 182 are 1, 2, 7, 13, 14, 26, 91 and 182.
Hence, 176 has the most number of divisors.

3.

n	*Divisors excluding n*	*Sum of divisors*
6	1, 2, 3	6
9	1, 3	4
15	1, 3, 5	9
21	1, 3, 7	11

Clearly, 6 is a perfect number.

4. $1095\overline{)1168}(1$
1095
$73\overline{)1095}(15$
73
365
365
$\times$

So, H.C.F. of 1095 and 1168 = 73.

$\therefore \frac{1095}{1168} = \frac{1095 \div 73}{1168 \div 73} = \frac{15}{16}$.

5. $128352\overline{)238368}(1$
128352
$110016\overline{)128352}(1$
110016
$18336\overline{)110016}(6$
110016
$\times$

So, H.C.F. of 128352 and 238368 = 18336.

$\therefore \frac{128352}{238368} = \frac{128352 \div 18336}{238368 \div 18336} = \frac{7}{13}$.

6. H.C.F. = Product of lowest powers of common factors = $2^2 \times 3^2 \times 5$.

7. H.C.F. = Product of lowest powers of common factors = $3 \times 5 \times 7 = 105$.

8. $4 \times 27 \times 3125 = 2^2 \times 3^3 \times 5^5$; $8 \times 9 \times 25 \times 7 = 2^3 \times 3^2 \times 5^2 \times 7$;
$16 \times 81 \times 5 \times 11 \times 49 = 2^4 \times 3^4 \times 5 \times 7^2 \times 11$.
$\therefore$ H.C.F. = $2^2 \times 3^2 \times 5 = 180$.

9. $36 = 2^2 \times 3^2$; $84 = 2^2 \times 3 \times 7$.
$\therefore$ H.C.F. = $2^2 \times 3 = 12$.

10. $204 = 2^2 \times 3 \times 17$; $1190 = 2 \times 5 \times 7 \times 17$; $1445 = 5 \times 17^2$.
$\therefore$ H.C.F. = 17.

11. H.C.F. of 18 and 25 is 1. So, they are co-primes.

12. $2923\overline{)3239}(1$
2923
$316\overline{)2923}(9$
2844
$79\overline{)316}(4$
316
$\times$

$\therefore$ H.C.F. = 79.

13. $3444\overline{)3556}(1$
3444
$112\overline{)3444}(30$
3360
$84\overline{)112}(1$
84
$28\overline{)84}(3$
84
$\times$

$\therefore$ H.C.F. = 28.

14. L.C.M. = Product of highest powers of prime factors = $2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$.

15.

2	24	36	40
2	12	18	20
2	6	9	10
3	3	9	5
	1	3	5

L.C.M. = $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$.

16.

2	22	54	108	135	198
3	11	27	54	135	99
3	11	9	18	45	33
3	11	3	6	15	11
11	11	1	2	5	11
	1	1	2	5	1

L.C.M. = $2 \times 3 \times 3 \times 3 \times 11 \times 2 \times 5 = 5940$.

17. H.C.F. of 148 and 185 is 37.

$\therefore$ L.C.M. = $\left(\frac{148 \times 185}{37}\right) = 740$.

18. Required H.C.F. = $\frac{\text{H.C.F. of } 2, 8, 64, 10}{\text{L.C.M. of } 3, 9, 81, 27} = \frac{2}{81}$.

19. Required H.C.F. $= \dfrac{\text{H.C.F. of } 9, 12, 18, 21}{\text{L.C.M. of } 10, 25, 35, 40} = \dfrac{3}{2800}$.

20. Required L.C.M. $= \dfrac{\text{L.C.M. of } 1, 5, 2, 4}{\text{H.C.F. of } 3, 6, 9, 27} = \dfrac{20}{3}$.

21. Required L.C.M. $= \dfrac{\text{L.C.M. of } 2, 3, 4, 9}{\text{H.C.F. of } 3, 5, 7, 13} = \dfrac{36}{1} = 36$.

22. Given numbers with two decimal places are : 1.75, 5.60 and 7.00. Without decimal places, these numbers are : 175, 560 and 700, whose H.C.F. is 35.

∴ H.C.F. of given numbers = 0.35.

23. Given numbers are 1.08, 0.36 and 0.90. H.C.F. of 108, 36 and 90 is 18.

∴ H.C.F. of given numbers = 0.18.

24. Given numbers are 0.54, 1.80 and 7.20. H.C.F. of 54, 180 and 720 is 18.

∴ H.C.F. of given numbers = 0.18.

25. Given numbers are 3.00, 2.70 and 0.09. L.C.M. of 300, 270 and 9 is 2700.

∴ L.C.M. of given numbers = 27.00 = 27.

26. $3240 = 2^3 \times 3^4 \times 5$; $3600 = 2^4 \times 3^2 \times 5^2$; H.C.F. $= 36 = 2^2 \times 3^2$.

Since H.C.F. is the product of lowest powers of common factors, so the third number must have $(2^2 \times 3^2)$ as its factor.

Since L.C.M. is the product of highest powers of common prime factors, so the third number must have 3^5 and 7^2 as its factors.

∴ Third number $= 2^2 \times 3^5 \times 7^2$.

27. Let the required numbers be x, $2x$ and $3x$. Then, their H.C.F. $= x$. So, $x = 12$.

∴ The numbers are 12, 24 and 36.

28. Let the numbers be $3x$ and $4x$. Then, their H.C.F. $= x$. So, $x = 4$.

So, the numbers are 12 and 16.

L.C.M. of 12 and 16 = 48.

29. Let the required numbers be $27a$ and $27b$. Then, $27a + 27b = 216 \Rightarrow a + b = 8$.

Now, co-primes with sum 8 are (1, 7) and (3, 5).

∴ Required numbers are $(27 \times 1, 27 \times 7)$ and $(27 \times 3, 27 \times 5)$ *i.e.*, (27, 189) and (81, 135).

Out of these, the given one in the answer is the pair (27, 189).

30. Let the required numbers be $33a$ and $33b$. Then, $33a + 33b = 528 \Rightarrow a + b = 16$.

Now, co-primes with sum 16 are (1, 15), (3, 13), (5, 11) and (7, 9).

∴ Required numbers are $(33 \times 1, 33 \times 15)$, $(33 \times 3, 33 \times 13)$, $(33 \times 5, 33 \times 11)$, $(33 \times 7, 33 \times 9)$.

The number of such pairs is 4.

31. Numbers with H.C.F. 15 must contain 15 as a factor.

Now, multiples of 15 between 40 and 100 are 45, 60, 75 and 90.

∴ Number-pairs with H.C.F. 15 are (45, 60), (45, 75), (60, 75) and (75, 90).

[∵ H.C.F. of (60, 90) is 30 and that of (45, 90) is 45]

Clearly, there are 4 such pairs.

32. Out of the given numbers, the two with H.C.F. 12 and difference 12 are 84 and 96.

33. Let the numbers be $37a$ and $37b$. Then, $37a \times 37b = 4107 \Rightarrow ab = 3$.

Now, co-primes with product 3 are (1, 3).

So, the required numbers are $(37 \times 1, 37 \times 3)$ *i.e.*, (1, 111).

∴ Greater number = 111.

34. Let the numbers be $13a$ and $13b$. Then, $13a \times 13b = 2028 \Rightarrow ab = 12$.
Now, co-primes with product 12 are (1, 12) and (3, 4).
So, the required numbers are $(13 \times 1, 13 \times 12)$ and $(13 \times 3, 13 \times 4)$.
Clearly, there are 2 such pairs.

35. Since the numbers are co-prime, they contain only 1 as the common factor.
Also, the given two products have the middle number in common.
So, middle number = H.C.F. of 551 and 1073 = 29;

First number $= \left(\frac{551}{29}\right) = 19$; Third number $= \left(\frac{1073}{29}\right) = 37$.

$\therefore$ Required sum = (19 + 29 + 37) = 85.

36. Let the numbers be $2x$ and $3x$. Then, their L.C.M. = $6x$. So, $6x = 48$ or $x = 8$.
$\therefore$ The numbers are 16 and 24.
Hence, required sum = (16 + 24) = 40.

37. Let the numbers be $3x$, $4x$ and $5x$. Then, their L.C.M. = $60x$. So, $60x = 2400$ or $x = 40$.
$\therefore$ The numbers are (3×40), (4×40) and (5×40).
Hence, required H.C.F. = 40.

38. Other number $= \left(\frac{11 \times 7700}{275}\right) = 308$.

39. Let the numbers be x and $(2000 - x)$. Then, their L.C.M. = $x(2000 - x)$.
So, $x(2000 - x) = 21879 \Leftrightarrow x^2 - 2000x + 21879 = 0$
$\Leftrightarrow (x - 1989)(x - 11) = 0 \Leftrightarrow x = 1989$ or $x = 11$.
Hence, the numbers are 1989 and 11.

40. Let the numbers be x and $4x$. Then, $x \times 4x = 84 \times 21 \Leftrightarrow x^2 = \left(\frac{84 \times 21}{4}\right) \Leftrightarrow x = 21$.
Hence, larger number = $4x = 84$.

41. Let the numbers be x and $(100 - x)$.
Then, $x(100 - x) = 5 \times 495 \Leftrightarrow x^2 - 100x + 2475 = 0$
$\Leftrightarrow (x - 55)(x - 45) = 0 \Leftrightarrow x = 55$ or $x = 45$.
$\therefore$ The numbers are 45 and 55.
Required difference = (55 − 45) = 10.

42. Let the numbers be x and $(x + 2)$.
Then, $x(x + 2) = 24 \Leftrightarrow x^2 + 2x - 24 = 0 \Leftrightarrow (x - 4)(x + 6) = 0 \Leftrightarrow x = 4$.
So, the numbers are 4 and 6.

43. Let the numbers be a and b. Then, $a + b = 55$ and $ab = 5 \times 120 = 600$.
$\therefore$ Required sum $= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{55}{600} = \frac{11}{120}$.

44. Let H.C.F. be h and L.C.M. be l. Then, $l = 45h$ and $l + h = 1150$.
$\therefore$ $45h + h = 1150$ or $h = 25$. So, $l = (1150 - 25) = 1125$.
Hence, other number $= \left(\frac{25 \times 1125}{125}\right) = 225$.

45. First number = $(50 \times 2) = 100$. Second number $= \left(\frac{50 \times 250}{100}\right) = 125$.

46. L.C.M. $= \frac{\text{Product of numbers}}{\text{H.C.F.}} = \frac{1320}{6} = 220$.

47. H.C.F of co-prime numbers is 1. So, L.C.M. $= \frac{117}{1} = 117$.

48. Since H.C.F. is always a factor of L.C.M., we cannot have three numbers with H.C.F. 35 and L.C.M. 120.

49. H.C.F. of two numbers divides their L.C.M. exactly. Clearly, 8 is not a factor of 60.

50. Clearly, the numbers are (23×13) and (23×14).

$\therefore$ Larger number $= (23 \times 14) = 322$.

51. Since 16 is not a factor of 136, it follows that there does not exist any pair of numbers with H.C.F. 16 and L.C.M. 136.

52. Product of numbers $= 11 \times 385 = 4235$.

Let the numbers be $11a$ and $11b$. Then, $11a \times 11b = 4235 \Rightarrow ab = 35$.

Now, co-primes with product 35 are (1, 35) and (5, 7).

So, the numbers are $(11 \times 1, 11 \times 35)$ and $(11 \times 5, 11 \times 7)$.

Since one number lies between 75 and 125, the suitable pair is (55, 77).

Hence, required number = 77.

53. Product of numbers $= 29 \times 4147$.

Let the numbers be $29a$ and $29b$. Then, $29a \times 29b = (24 \times 4147) \Rightarrow ab = 143$.

Now, co-primes with product 143 are (1, 143) and (11, 13).

So, the numbers are $(29 \times 1, 29 \times 143)$ and $(29 \times 11, 29 \times 13)$.

Since both numbers are greater than 29, the suitable pair is $(29 \times 11, 29 \times 13)$ *i.e.*, (319, 377).

$\therefore$ Required sum $= (319 + 377) = 696$.

54. H.C.F. of two prime numbers is 1. Product of numbers $= (1 \times 161) = 161$.

Let the numbers be a and b. Then, $ab = 161$.

Now, co-primes with product 161 are (1, 161) and (7, 23).

Since x and y are prime numbers and $x > y$, we have $x = 23$ and $y = 7$.

$\therefore$ $3y - x = (3 \times 7) - 23 = -2$.

55. H.C.F. of 2436 and 1001 is 7. Also, H.C.F. of 105 and 7 is 7.

$\therefore$ H.C.F. of 105, 1001 and 2436 is 7.

56. Required length = H.C.F. of 700 cm, 385 cm and 1295 cm = 35 cm.

57. Required measurement = (H.C.F. of 496, 403, 713) litres = 31 litres.

58. Required number of students = H.C.F. of 1001 and 910 = 91.

59. Largest size of the tile = H.C.F. of 378 cm and 525 cm = 21 cm.

60. Required number = H.C.F. of $(91 - 43)$, $(183 - 91)$ and $(183 - 43)$

= H.C.F. of 48, 92 and 140 = 4.

61. N = H.C.F. of $(4665 - 1305)$, $(6905 - 4665)$ and $(6905 - 1305)$

= H.C.F. of 3360, 2240 and 5600 = 1120.

Sum of digits in N $= (1 + 1 + 2 + 0) = 4$.

62. Required number = H.C.F. of $(1356 - 12)$, $(1868 - 12)$ and $(2764 - 12)$

= H.C.F. of 1344, 1856 and 2752 = 64.

63. Required number = H.C.F. of $(1657 - 6)$ and $(2037 - 5)$

= H.C.F. of 1651 and 2032 = 127.

64. L.C.M. of 8, 16, 40 and 80 = 80.

$\frac{7}{8} = \frac{70}{80}; \frac{13}{16} = \frac{65}{80}; \frac{31}{40} = \frac{62}{80}$.

Since, $\frac{70}{80} > \frac{63}{80} > \frac{65}{80} > \frac{62}{80}$, so $\frac{7}{8} > \frac{63}{80} > \frac{13}{16} > \frac{31}{40}$.

So, $\frac{7}{8}$ is the largest.

65. L.C.M. of 12, 18, 21, 30

2	12	18	21	30
3	6	9	21	15
	2	3	7	5

$= 2 \times 3 \times 2 \times 3 \times 7 \times 5 = 1260.$

$\therefore$ Required number $= (1260 \div 2) = 630.$

66. Required fraction = L.C.M. of $\frac{6}{7}, \frac{5}{14}, \frac{10}{21} = \frac{\text{L.C.M. of } 6, 5, 10}{\text{H.C.F. of } 7, 14, 21} = \frac{30}{7}.$

67. Least number of 5 digits is 10000. L.C.M. of 12, 15 and 18 is 180.
On dividing 10000 by 180, the remainder is 100.
$\therefore$ Required number $= 10000 + (180 - 100) = 10080.$

68. Greatest number of 4 digits is 9999. L.C.M. of 15, 25, 40 and 75 is 600.
On dividing 9999 by 600, the remainder is 399.
$\therefore$ Required number $= (9999 - 399) = 9600.$

69. L.C.M. of 5, 6, 4 and 3 = 60. On dividing 2497 by 60, the remainder is 37.
$\therefore$ Number to be added $= (60 - 37) = 23.$

70. The least number divisible by 16, 20, 24
$= \text{L.C.M. of } 16, 20, 24 = 240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5.$
To make it a perfect square, it must be multiplied by 3×5.
$\therefore$ Required number $= 240 \times 3 \times 5 = 3600.$

71. Required number = (L.C.M. of 12, 16, 18, 21, 28) + 7 = 1008 + 7 = 1015.

72. Required number = (L.C.M. of 24, 32, 36, 54) − 5 = 864 − 5 = 859.

73. Required number = (L.C.M. of 12, 15, 20, 54) + 8 = 540 + 8 = 548.

74. Greatest number of 4 digits is 9999. L.C.M. of 4, 7 and 13 = 364.
On dividing 9999 by 364, remainder obtained is 171.
$\therefore$ Greatest number of 4 digits divisible by 4, 7 and 13 = (9999 − 171) = 9828.
Hence, required number = (9828 + 3) = 9831.

75. Least number of 6 digits is 100000. L.C.M. of 4, 6, 10 and 15 = 60.
On dividing 100000 by 60, the remainder obtained is 40.
$\therefore$ Least number of 6 digits divisible by 4, 6, 10 and 15 = 100000 + (60 − 40) = 100020.
$\therefore$ N = (100020 + 2) = 100022. Sum of digits in N = (1 + 2 + 2) = 5.

76. L.C.M. of 6, 9, 15 and 18 is 90.
Let required number be $90k + 4$, which is a multiple of 7.
Least value of k for which $(90k + 4)$ is divisible by 7 is $k = 4$.
$\therefore$ Required number $= 90 \times 4 + 4 = 364.$

77. Here (48 − 38) = 10, (60 − 50) = 10, (72 − 62) = 10, (108 − 98) = 10 & (140 − 130) = 10.
$\therefore$ Required number = (L.C.M. of 48, 60, 72, 108, 140) − 10 = 15120 − 10 = 15110.

78. Here (18 − 7) = 11, (21 − 10) = 11 and (24 − 13) = 11. L.C.M. of 18, 21 and 24 is 504.
Let required number be $504k - 11$.
Least value of k for which $(504k - 11)$ is divisible by 23 is $k = 6$.
$\therefore$ Required number $= 504 \times 6 - 11 = 3024 - 11 = 3013.$

79. L.C.M. of 5, 6, 7, 8 = 840.
$\therefore$ Required number is of the form $840k + 3$.
Least value of k for which $(840k + 3)$ is divisible by 9 is $k = 2$.
$\therefore$ Required number $= (840 \times 2 + 3) = 1683.$

80. L.C.M. of 16, 18, 20, 25 = 3600. Required number is of the form $3600k + 4$.
Least value of k for which $(3600k + 4)$ is divisible by 7 is $k = 5$.
$\therefore$ Required number $= (3600 \times 5 + 4) = 18004.$

81. L.C.M. of 2, 4, 6, 8, 10, 12 is 120.
So, the bells will toll together after every 120 seconds, *i.e.*, 2 minutes.
In 30 minutes, they will toll together $\left[\left(\frac{30}{2}\right)+1\right]$ = 16 times.

82. Interval after which the devices will beep together
= (L.C.M. of 30, 60, 90, 105) min. = 1260 min. = 21 hrs.
So, the devices will again beep together 21 hrs. after 12 noon *i.e.*, at 9 a.m.

83. L.C.M. of 252, 308 and 198 = 2772.
So, A, B and C will again meet at the starting point in 2772 sec. *i.e.*, 46 min. 12 sec.

5. **Dividing a Decimal Fraction By a Decimal Fraction** : Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.

Thus, $\frac{0.00066}{0.11} = \frac{0.00066 \times 100}{0.11 \times 100} = \frac{0.066}{11} = .006$.

V. **Comparison of Fractions** : Suppose some fractions are to be arranged in ascending or descending order of magnitude. Then, convert each one of the given fractions in the decimal form, and arrange them accordingly.

Suppose, we have to arrange the fractions $\frac{3}{5}$, $\frac{6}{7}$ and $\frac{7}{9}$ in descending order.

Now, $\frac{3}{5} = 0.6$, $\frac{6}{7} = 0.857$, $\frac{7}{9} = 0.777.....$

Since $0.857 > 0.777 ..... > 0.6$, so $\frac{6}{7} > \frac{7}{9} > \frac{3}{5}$.

VI. **Recurring Decimal** : If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a ***recurring decimal***.

In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set.

Thus, $\frac{1}{3} = 0.333 ..... = 0.\dot{3}$; $\frac{22}{7} = 3.142857142857 ..... = 3.\overline{142857}$.

Pure Recurring Decimal : A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal Into Vulgar Fraction : Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

Thus, $0.\dot{5} = \frac{5}{9}$; $0.\overline{53} = \frac{53}{99}$; $0.\overline{067} = \frac{67}{999}$; etc.

Mixed Recurring Decimal : A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.

e.g., $0.17333..... = 0.17\dot{3}$.

Converting a Mixed Recurring Decimal Into Vulgar Fraction : In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated. In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Thus, $0.16 = \frac{16-1}{90} = \frac{15}{90} = \frac{1}{6}$; $0.22\overline{73} = \frac{2273-22}{9900} = \frac{2251}{9900}$.

VII. **Some Basic Formulae** :

1. $(a + b)(a - b) = (a^2 - b^2)$.
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$.
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$.
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$.
5. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
6. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$.
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$.
8. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

SOLVED EXAMPLES

Ex. 1. ***Convert the following into vulgar fractions :***
(i) 0.75 (ii) 3.004 (iii) .0056.

Sol. (i) $0.75 = \frac{75}{100} = \frac{3}{4}$. (ii) $3.004 = \frac{3004}{1000} = \frac{751}{250}$. (iii) $.0056 = \frac{56}{10000} = \frac{7}{1250}$.

Ex. 2. ***Arrange the fractions $\frac{5}{8}, \frac{7}{12}, \frac{13}{16}, \frac{16}{29}$ and $\frac{3}{4}$ in ascending order of magnitude.***

Sol. Converting each of the given fractions into decimal form, we get :

$\frac{5}{8} = 0.625, \frac{7}{12} = 0.5833, \frac{13}{16} = 0.8125, \frac{16}{29} = 0.5517$ and $\frac{3}{4} = 0.75$.

Now, $0.5517 < 0.5833 < 0.625 < 0.75 < 0.8125$.

$\therefore \quad \frac{16}{29} < \frac{7}{12} < \frac{5}{8} < \frac{3}{4} < \frac{13}{16}$.

Ex. 3. ***Arrange the fractions $\frac{3}{5}, \frac{4}{7}, \frac{8}{9}$ and $\frac{9}{11}$ in their descending order.***
(R.B.I. 2003)

Sol. Clearly, $\frac{3}{5} = 0.6, \frac{4}{7} = 0.571, \frac{8}{9} = 0.88, \frac{9}{11} = 0.818$.

Now, $0.88 > 0.818 > 0.6 > 0.571$.

$\therefore \quad \frac{8}{9} > \frac{9}{11} > \frac{3}{5} > \frac{4}{7}$.

Ex. 4. ***Evaluate : (i) 6202.5 + 620.25 + 62.025 + 6.2025 + 0.62025*** (L.I.C. 2003)
(ii) 5.064 + 3.98 + .7036 + 7.6 + .3 + 2

Sol. (i)
$$\begin{array}{r} 6202.5 \\ 620.25 \\ 62.025 \\ 6.2025 \\ + \quad 0.62025 \\ \hline 6891.59775 \end{array}$$

(ii)
$$\begin{array}{r} 5.064 \\ 3.98 \\ 0.7036 \\ 7.6 \\ 0.3 \\ + \quad 2.0 \\ \hline 19.6476 \end{array}$$

Ex. 5. ***Evaluate : (i) 31.004 – 17.2386 (ii) 13 – 5.1967***

Sol. (i)
$$\begin{array}{r} 31.0040 \\ - \ 17.2386 \\ \hline 13.7654 \end{array}$$

(ii)
$$\begin{array}{r} 13.0000 \\ - \ 5.1967 \\ \hline 7.8033 \end{array}$$

Ex. 6. ***What value will replace the question mark in the following equations ?***
(i) 5172.49 + 378.352 + ? = 9318.678 (B.S.R.B. 1998)
(ii) ? – 7328.96 = 5169.38 (B.S.R.B. 2003)

Sol. (i) Let $5172.49 + 378.352 + x = 9318.678$.

Then, $x = 9318.678 - (5172.49 + 378.352) = 9318.678 - 5550.842 = 3767.836$.

(ii) Let $x - 7328.96 = 5169.38$. Then, $x = 5169.38 + 7328.96 = 12498.34$.

Ex. 7. ***Find the products : (i) 6.3204 × 100 (ii) .069 × 10000***

Sol. (i) $6.3204 \times 100 = 632.04$. (ii) $.069 \times 10000 = .0690 \times 10000 = 690$.

Ex. 8. ***Find the products :***

(i) 2.61 × 1.3 (ii) 2.1693 × 1.4 (iii) .4 × .04 × .004 × 40.

Sol. (*i*) 261 × 13 = 3393. Sum of decimal places of given numbers = (2 + 1) = 3.

∴ 2.61 × 1.3 = 3.393.

(*ii*) 21693 × 14 = 303702. Sum of decimal places = (4 + 1) = 5.

∴ 2.1693 × 1.4 = 3.03702.

(*iii*) 4 × 4 × 4 × 40 = 2560. Sum of decimal places = (1 + 2 + 3) = 6.

∴ .4 × .04 × .004 × 40 = .002560.

Ex. 9. ***Given that 268 × 74 = 19832, find the value of 2.68 × .74.***

Sol. Sum of decimal places = (2 + 2) = 4.

∴ 2.68 × .74 = 1.9832.

Ex. 10. ***Find the quotient :***

(i) 0.63 ÷ 9 (ii) 0.0204 ÷ 17 (iii) 3.1603 ÷ 13.

Sol. (*i*) 63 ÷ 9 = 7. Dividend contains 2 places of decimal.

∴ 0.63 ÷ 9 = .07.

(*ii*) 204 ÷ 17 = 12. Dividend contains 4 places of decimal.

∴ 0.0204 ÷ 17 = .0012.

(*iii*) 31603 ÷ 13 = 2431. Dividend contains 4 places of decimal.

∴ 3.1603 ÷ 13 = .2431.

Ex. 11. ***Evaluate :***

(i) 35 ÷ .07 (ii) 2.5 ÷ 0.0005 **(M.B.A. 1998)**

(iii) 136.09 ÷ 43.9 **(Hotel Management, 2000)**

Sol. (*i*) $\frac{35}{.07} = \frac{35 \times 100}{.07 \times 100} = \frac{3500}{7} = 500.$

(*ii*) $\frac{2.5}{0.0005} = \frac{2.5 \times 10000}{0.0005 \times 10000} = \frac{25000}{5} = 5000.$

(*iii*) $\frac{136.09}{43.9} = \frac{136.09 \times 10}{43.9 \times 10} = \frac{1360.9}{439} = 3.1.$

Ex. 12. ***What value will come in place of question mark in the following equations?***

(i) 0.006 ÷ ? = 0.6 (ii) ? ÷ .025 = 80

Sol. (*i*) Let $\frac{0.006}{x} = 0.6$. Then, $x = \frac{0.006}{0.6} = \frac{0.006 \times 10}{0.6 \times 10} = \frac{0.06}{6} = 0.01.$

(*ii*) Let $\frac{x}{.025} = 80$. Then, $x = 80 \times .025 = 2.$

Ex. 13. ***If $\frac{1}{3.718} = .2689$, then find the value of $\frac{1}{.0003718}$.***

Sol. $\frac{1}{.0003718} = \frac{10000}{3.718} = \left(10000 \times \frac{1}{3.718}\right) = 10000 \times .2689 = 2689.$

Ex. 14. ***Express as vulgar fractions : (i) $0.\overline{37}$ (ii) $0.\overline{053}$ (iii) $3.\overline{142857}$.***

Sol. (*i*) $0.\overline{37} = \frac{37}{99}.$ (*ii*) $0.\overline{053} = \frac{53}{999}.$

(*iii*) $3.\overline{142857} = 3 + 0.\overline{142857} = 3 + \frac{142857}{999999} = 3\frac{142857}{999999}.$

Ex. 15. ***Express as vulgar fractions : (i) $0.1\overline{7}$ (ii) $0.12\overline{54}$ (iii) $2.5\overline{36}$***

Sol. (*i*) $0.1\overline{7} = \frac{17 - 1}{90} = \frac{16}{90} = \frac{8}{45}.$ (*ii*) $0.12\overline{54} = \frac{1254 - 12}{9900} = \frac{1242}{9900} = \frac{69}{550}.$

(iii) $2.53\overline{6} = 2 + 0.53\overline{6} = 2 + \frac{536 - 53}{900} = 2 + \frac{483}{900} = 2 + \frac{161}{300} = 2\frac{161}{300}$.

Ex. 16. ***Simplify*** : $\frac{0.05 \times 0.05 \times 0.05 + 0.04 \times 0.04 \times 0.04}{0.05 \times 0.05 - 0.05 \times 0.04 + 0.04 \times 0.04}$. (IGNOU, 2003)

Sol. Given expression $= \left(\frac{a^3 + b^3}{a^2 - ab + b^2}\right)$, where $a = 0.05$, $b = 0.04$

$= (a + b) = (0.05 + 0.04) = 0.09$.

EXERCISE 3

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer* :

1. The fraction $101\frac{27}{100000}$ in decimal form is :

   (a) .01027 (b) .10127 (c) 101.00027 (d) 101.000027

2. When .36 is written in simplest fractional form, the sum of the numerator and the denominator is :

   (a) 15 (b) 45 (c) 114 (d) 135

3. What decimal of an hour is a second ?

   (a) .0025 (b) .0256 (c) .00027 (d) .000126

4. If $47.2506 = 4A + \frac{7}{B} + 2C + \frac{5}{D} + 6E$, then the value of 5A + 3B + 6C + D + 3E is : (S.S.C. 2003)

   (a) 53.6003 (b) 53.603 (c) 153.6003 (d) 213.0003

5. Which of the following has fractions in ascending order ? (Bank P.O. 2003)

   (a) $\frac{1}{3}, \frac{2}{5}, \frac{4}{7}, \frac{3}{5}, \frac{5}{6}, \frac{6}{7}$ (b) $\frac{1}{3}, \frac{2}{5}, \frac{3}{5}, \frac{4}{7}, \frac{5}{6}, \frac{6}{7}$

   (c) $\frac{1}{3}, \frac{2}{5}, \frac{3}{5}, \frac{5}{6}, \frac{4}{7}, \frac{6}{7}$ (d) $\frac{2}{5}, \frac{3}{5}, \frac{1}{3}, \frac{4}{7}, \frac{5}{6}, \frac{6}{7}$

6. Which of the following has fractions in ascending order ? (NABARD, 2002)

   (a) $\frac{2}{3}, \frac{3}{5}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$ (b) $\frac{3}{5}, \frac{2}{3}, \frac{9}{11}, \frac{7}{9}, \frac{8}{9}$ (c) $\frac{3}{5}, \frac{2}{3}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$

   (d) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{2}{3}, \frac{3}{5}$ (e) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{3}{5}, \frac{2}{3}$

7. Which of the following are in descending order of their value ? (R.R.B. 2002)

   (a) $\frac{5}{9}, \frac{7}{11}, \frac{8}{15}, \frac{11}{17}$ (b) $\frac{5}{9}, \frac{8}{15}, \frac{11}{17}, \frac{7}{11}$

   (c) $\frac{11}{17}, \frac{7}{11}, \frac{8}{15}, \frac{5}{9}$ (d) $\frac{11}{17}, \frac{7}{11}, \frac{5}{9}, \frac{8}{15}$

8. What is the difference between the biggest and the smallest fraction among $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ and $\frac{5}{6}$? (C.B.I. 1998)

   (a) $\frac{1}{6}$ (b) $\frac{1}{12}$ (c) $\frac{1}{20}$ (d) $\frac{1}{30}$

9. Which part contains the fractions in ascending order ?

(a) $\frac{11}{14}, \frac{16}{19}, \frac{19}{21}$ (b) $\frac{16}{19}, \frac{11}{14}, \frac{19}{21}$ (c) $\frac{16}{19}, \frac{19}{21}, \frac{11}{14}$ (d) $\frac{19}{21}, \frac{11}{14}, \frac{16}{19}$

10. Which of the following fractions is the smallest ? **(S.S.C. 2002)**

(a) $\frac{13}{16}$ (b) $\frac{15}{19}$ (c) $\frac{17}{21}$ (d) $\frac{7}{8}$

11. Which of the following fractions is greater than $\frac{3}{4}$ and less than $\frac{5}{6}$? **(S.S.C. 1999)**

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{4}{5}$ (d) $\frac{9}{10}$

12. Which of the following fractions is less than $\frac{7}{8}$ and greater than $\frac{1}{3}$?

(a) $\frac{1}{4}$ (b) $\frac{23}{24}$ (c) $\frac{11}{12}$ (d) $\frac{17}{24}$

13. Which of the following numbers does not lie between $\frac{4}{5}$ and $\frac{7}{13}$?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{5}{7}$

14. The arrangement of rational numbers $\frac{-7}{10}, \frac{5}{-8}, \frac{2}{-3}$ in ascending order is :

(a) $\frac{2}{-3}, \frac{5}{-8}, \frac{-7}{10}$ (b) $\frac{5}{-8}, \frac{-7}{10}, \frac{2}{-3}$ (c) $\frac{-7}{10}, \frac{5}{-8}, \frac{2}{-3}$ (d) $\frac{-7}{10}, \frac{2}{-3}, \frac{5}{-8}$

15. 337.62 + 8.591 + 34.4 = ? **(S.S.C. 1998)**

(a) 370.611 (b) 380.511 (c) 380.611 (d) 426.97

16. The value of (1 + .1 + .01 + .001) is :

(a) 1.001 (b) 1.011 (c) 1.003 (d) 1.111

17. 34.95 + 240.016 + 23.98 = ? **(Bank P.O. 2002)**

(a) 298.0946 (b) 298.111 (c) 298.946 (d) 299.09

18. 617 + 6.017 + 0.617 + 6.0017 = ? **(M.B.A. 1998)**

(a) 6.2963 (b) 62.965 (c) 629.6357 (d) None of these

19. 48.95 – 32.006 = ? **(I.B.P.S. 2002)**

(a) 16.089 (b) 16.35 (c) 16.89 (d) 16.944

20. 792.02 + 101.32 – 306.76 = ? **(NABARD, 2002)**

(a) 586.58 (b) 893.34 (c) 997.11 (d) 1200.10

21. 12.1212 + 17.0005 – 9.1102 = ? **(B.S.R.B. 2003)**

(a) 20.0015 (b) 20.0105 (c) 20.0115 (d) 20.1015

22. 892.7 – 573.07 – 95.007 = ?

(a) 224.623 (b) 224.777 (c) 233.523 (d) 414.637

23. 3889 + 12.952 – ? = 3854.002 **(Bank P.O. 2002)**

(a) 47.095 (b) 47.752 (c) 47.932 (d) 47.95

24. 138.009 + 341.981 – 146.305 = 123.6 + ? **(Bank P.O. 1999)**

(a) 120.085 (b) 120.85 (c) 220.085 (d) None of these

25. 832.58 – 242.31 = 779.84 – ? **(B.S.R.B. 1998)**

(a) 179.57 (b) 199.57 (c) 295.05 (d) None of these

26. What will come in place of question mark in the following equation ?
54. (?) 3 + 543 + 5.43 = 603.26 **(Hotel Management, 2001)**
(a) 5 (b) 6 (c) 8 (d) None of these

27. Which of the following is equal to 3.14×10^6 ? **(Hotel Management, 2003)**
(a) 314 (b) 3140 (c) 3140000 (d) None of these

28. The number 518,000,000 when expressed in scientific notation, equals :
(a) 51.8×10^6 (b) 51.8×10^7 (c) 5.18×10^8 (d) 5.18×10^9

29. 0.000006723 when expressed in scientific notation, is :
(a) 6723×10^{-5} (b) 67.23×10^{-7} (c) 6.723×10^{-6} (d) None of these

30. If $1.125 \times 10^k = 0.001125$, then the value of k is :
(a) – 4 (b) – 3 (c) – 2 (d) – 1

31. $0.002 \times 0.5 = ?$ **(Bank P.O. 2003)**
(a) 0.0001 (b) 0.001 (c) 0.01 (d) 0.1

32. $16.02 \times 0.001 = ?$ **(Bank P.O. 2002)**
(a) 0.001602 (b) 0.01602 (c) 0.1602 (d) 1.6021

33. $0.014 \times 0.014 = ?$ **(Hotel Management, 2001)**
(a) 0.000196 (b) 0.00196 (c) 19.6 (d) 196

34. $40.83 \times 1.02 \times 1.2 = ?$ **(S.B.I.P.O. 2003)**
(a) 41.64660 (b) 42.479532 (c) 49.97592 (d) 58.7952

35. 0.04×0.0162 is equal to : **(M.B.A. 1998)**
(a) 6.48×10^{-3} (b) 6.48×10^{-4} (c) 6.48×10^{-5} (d) 6.48×10^{-6}

36. $3 \times 0.3 \times 0.03 \times 0.003 \times 30 = ?$ **(Hotel Management, 2002)**
(a) 0.0000243 (b) 0.000243 (c) 0.00243 (d) 0.0243

37. How many digits will be there to the right of the decimal point in the product of 95.75 and .02554 ? **(I.A.M. 2002)**
(a) 5 (b) 6 (c) 7 (d) None of these

38. $\left(.00625 \text{ of } \frac{23}{5}\right)$, when expressed as a vulgar fraction, equals :
(a) $\frac{23}{80}$ (b) $\frac{23}{800}$ (c) $\frac{23}{8000}$ (d) $\frac{125}{23}$

39. Which is the closest approximation to the product $0.3333 \times 0.25 \times 0.499 \times 0.125 \times 24$?
(a) $\frac{1}{8}$ (b) $\frac{3}{4}$ (c) $\frac{3}{8}$ (d) $\frac{2}{5}$

40. Consider the following quotients :
1. 368.39 divided by 17 2. 170.50 divided by 62 3. 875.65 divided by 83
Their correct sequence in decreasing order is : **(C.D.S. 2003)**
(a) 1, 3, 2 (b) 2, 1, 3 (c) 2, 3, 1 (d) 3, 1, 2

41. $0.213 \div 0.00213 = ?$
(a) 1 (b) 10 (c) 100 (d) None of these

42. 4.036 divided by 0.04 gives : **(Hotel Management, 2003)**
(a) 1.009 (b) 10.09 (c) 100.9 (d) None of these

43. $\frac{1}{0.04}$ is equal to : **(S.S.C. 2000)**
(a) $\frac{1}{40}$ (b) $\frac{2}{5}$ (c) 2.5 (d) 25

44. $\left(\frac{0.05}{0.25}+\frac{0.25}{0.05}\right)^3 = ?$

(a) 139.4 (b) 140 (c) 140.6 (d) 143.9

45. The value of 0.0396 ÷ 2.51 correct to 2 significant figures is :

(a) 0.015 (b) 0.0157 (c) 0.016 (d) 0.017

46. .04 × ? = .000016.

(a) 0.0004 (b) 0.04 (c) 4 (d) None of these

47. $\frac{.009}{?} = .01$ **(M.B.A. 1998)**

(a) .0009 (b) .09 (c) .9 (d) 9

48. If $\frac{144}{0.144} = \frac{14.4}{x}$, then the value of x is : **(C.B.I. 2003)**

(a) 0.0144 (b) 1.44 (c) 14.4 (d) 144

49. A tailor has 37.5 metres of cloth and he has to make 8 pieces out of a metre of cloth. How many pieces can he make out of this cloth ? **(N.I.F.T. 2000)**

(a) 320 (b) 360 (c) 400 (d) None of these

50. The price of commodity X increases by 40 paise every year, while the price of commodity Y increases by 15 paise every year. If in 2001, the price of commodity X was Rs. 4.20 and that of Y was Rs. 6.30, in which year commodity X will cost 40 paise more than the commodity Y ? **(Bank P.O. 2002)**

(a) 2010 (b) 2011 (c) 2012 (d) 2013

51. When 0.232323 ..... is converted into a fraction, then the result is : **(C.B.I. 1998)**

(a) $\frac{1}{5}$ (b) $\frac{2}{9}$ (c) $\frac{23}{99}$ (d) $\frac{23}{100}$

52. The rational number for the recurring decimal 0.125125 ..... is : **(M.B.A. 2002)**

(a) $\frac{63}{487}$ (b) $\frac{119}{993}$ (c) $\frac{125}{999}$ (d) None of these

53. When $0.\overline{47}$ is converted into a fraction, the result is : **(Section Officers', 2003)**

(a) $\frac{46}{90}$ (b) $\frac{46}{99}$ (c) $\frac{47}{90}$ (d) $\frac{47}{99}$

54. $0.\overline{36}$ expressed in the form of $\frac{p}{q}$ equals :

(a) $\frac{4}{11}$ (b) $\frac{4}{13}$ (c) $\frac{35}{90}$ (d) $\frac{35}{99}$

55. The least among the following is : **(S.S.C. 2002)**

(a) 0.2 (b) 1 ÷ 0.2 (c) $0.\overline{2}$ (d) $(0.2)^2$

56. The correct expression of $6.\overline{46}$ in the fractional form is : **(C.B.I. 1997)**

(a) $\frac{646}{99}$ (b) $\frac{64640}{1000}$ (c) $\frac{640}{100}$ (d) $\frac{640}{99}$

57. The value of $0.5\overline{7}$ is :

(a) $\frac{57}{10}$ (b) $\frac{57}{99}$ (c) $\frac{26}{45}$ (d) $\frac{52}{9}$

58. Let F = $0.84\overline{181}$. When F is written as a fraction in lowest terms, the denominator exceeds the numerator by :

(a) 13 (b) 14 (c) 29 (d) 87

59. The value of $4.1\overline{2}$ is :

(a) $4\frac{11}{90}$ (b) $4\frac{11}{99}$ (c) $\frac{371}{900}$ (d) None of these

60. The value of $2.13\overline{6}$ is : (L.I.C.A.A.O. 2003)

(a) $\frac{47}{220}$ (b) $\frac{68}{495}$ (c) $2\frac{3}{22}$ (d) None of these

61. The value of $(0.\overline{2} + 0.\overline{3} + 0.\overline{4} + 0.\overline{9} + 0.\overline{39})$ is : (C.B.I. 1997)

(a) $0.\overline{57}$ (b) $1\frac{20}{33}$ (c) $2\frac{1}{3}$ (d) $2\frac{13}{33}$

62. $3.\overline{87} - 2.\overline{59} = ?$ (A.A.O. Exam, 2003)

(a) 1.20 (b) $1.\overline{2}$ (c) $1.\overline{27}$ (d) $1.\overline{28}$

63. The simplification of $3.\overline{36} - 2.\overline{05} + 1.\overline{33}$ equals : (S.S.C. 2003)

(a) 2.60 (b) 2.64 (c) $2.\overline{61}$ (d) $2.\overline{64}$

64. $(0.\overline{09} \times 7.\overline{3})$ is equal to : (S.S.C. 2003)

(a) $.\overline{6}$ (b) $.\overline{657}$ (c) $.\overline{67}$ (d) $.65\overline{7}$

65. $(0.3\overline{467} + 0.1\overline{333})$ is equal to : (Hotel Management, 2002)

(a) $0.4\overline{8}$ (b) $0.\overline{48}$ (c) $0.480\overline{1}$ (d) 0.48

66. $(8.3\overline{1} + 0.\overline{6} + 0.00\overline{2})$ is equal to : (S.S.C. 2002)

(a) $8.9\overline{12}$ (b) $8.\overline{912}$ (c) $8.9\overline{79}$ (d) $8.97\overline{9}$

67. The sum of $\overline{2}.75$ and $\overline{3}.78$ is : (Section Officers', 2001)

(a) $\overline{1}.03$ (b) $\overline{1}.53$ (c) $\overline{4}.53$ (d) $\overline{5}.53$

68. If $\frac{547.527}{0.0082} = x$, then the value of $\frac{547527}{82}$ is : (Hotel Management, 1999)

(a) $\frac{x}{10}$ (b) $10x$ (c) $100x$ (d) None of these

69. If 2994 ÷ 14.5 = 172, then 29.94 ÷ 1.45 = ? (L.I.C. 2003)

(a) 0.172 (b) 1.72 (c) 17.2 (d) 172

70. If 213 × 16 = 3408, then 1.6 × 21.3 is equal to : (Assistant Grade, 1998)

(a) 0.3408 (b) 3.408 (c) 34.08 (d) 340.8

71. If $\frac{1}{6.198} = 0.16134$, then the value of $\frac{1}{0.0006198}$ is : (S.S.C. 1997)

(a) 0.016134 (b) 0.16134 (c) 1613.4 (d) 16134

72. When 52416 is divided by 312, the quotient is 168. What will be the quotient when 52.416 is divided by 0.0168 ? (Hotel Management, 1998)

(a) 3.12 (b) 312 (c) 3120 (d) None of these

73. Given 168 × 32 = 5376, then 5.376 ÷ 16.8 is equal to :

(a) 0.032 (b) 0.32 (c) 3.2 (d) 32

74. 54.327 × 357.2 × 0.0057 is the same as : (Hotel Management, 1997)

(a) 5.4327 × 3.572 × 5.7 (b) 5.4327 × 3.572 × 0.57

(c) 54327 × 3572 × 0.0000057 (d) None of these

75. $\frac{5.3472 \times 324.23}{3.489 \times 5.42}$ is the same as :

(a) $\frac{53472 \times 3.2423}{3.489 \times 54.2}$ (b) $\frac{53472 \times 32423}{3489 \times 542}$ (c) $\frac{534.72 \times 324.23}{34.89 \times 5.42}$ (d) $\frac{53472 \times 3242.3}{3489 \times 542}$

76. $\frac{96.54 - 89.63}{96.54 + 89.63} + \frac{965.4 - 896.3}{9.654 + 8.963} = ?$

(a) 10^{-2} (b) 10^{-1} (c) 10 (d) None of these

77. If $1^3 + 2^3 + ... + 9^3 = 2025$, then the value of $(0.11)^3 + (0.22)^3 + ... + (0.99)^3$ is close to : **(S.S.C. 2003)**

(a) 0.2695 (b) 0.3695 (c) 2.695 (d) 3.695

78. $8.7 - [7.6 - (6.5 - (5.4 - \overline{4.3 - 2})]]$ is simplified to : **(S.S.C. 2004)**

(a) 2.5 (b) 3.5 (c) 4.5 (d) 5.5

79. The value of $\frac{1}{4} + \frac{1}{4 \times 5} + \frac{1}{4 \times 5 \times 6}$ correct to 4 decimal places is :

(a) 0.3075 (b) 0.3082 (c) 0.3083 (d) 0.3085

80. Find the value of the following expression upto four places of decimals.

$\left[1 + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 4} + \frac{1}{1 \times 2 \times 4 \times 8} + \frac{1}{1 \times 2 \times 4 \times 8 \times 16}\right]$ **(Hotel Management, 2002)**

(a) 1.6414 (b) 1.6415 (c) 1.6416 (d) 1.6428

81. The sum of the first 20 terms of the series $\frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + ...$ is :

(a) 0.16 (b) 1.6 (c) 16 (d) None of these **(Hotel Management, 1998)**

82. If $1.5x = 0.04y$, then the value of $\left(\frac{y - x}{y + x}\right)$ is :

(a) $\frac{730}{77}$ (b) $\frac{73}{77}$ (c) $\frac{7.3}{77}$ (d) None of these

83. The value of $\left[35.7 - \left(3 + \frac{1}{3 + \frac{1}{3}}\right) - \left(2 + \frac{1}{2 + \frac{1}{2}}\right)\right]$ is :

(a) 30 (b) 34.8 (c) 36.6 (d) 41.4

84. $\frac{(0.1667)(0.8333)(0.3333)}{(0.2222)(0.6667)(0.1250)}$ is approximately equal to : **(M.B.A. 1998)**

(a) 2 (b) 2.40 (c) 2.43 (d) 2.50

85. The value of $\frac{3.6 \times 0.48 \times 2.50}{0.12 \times 0.09 \times 0.5}$ is : **(S.S.C. 1998)**

(a) 80 (b) 800 (c) 8000 (d) 80000

86. $\frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7} = ?$ **(R.R.B. 1998)**

(a) 0.8 (b) 1.45 (c) 2.40 (d) 3.25

87. The value of $\frac{3.157 \times 4126 \times 3.198}{63.972 \times 2835.121}$ is closest to : **(C.B.I. 2003)**

(a) 0.002 (b) 0.02 (c) 0.2 (d) 2

88. The value of $\frac{489.1375 \times 0.0483 \times 1.956}{0.0873 \times 92.581 \times 99.749}$ is closest to : **(C.B.I. 1997)**

(a) 0.006 (b) 0.06 (c) 0.6 (d) 6

89. The value of $\frac{241.6 \times 0.3814 \times 6.842}{0.4618 \times 38.25 \times 73.65}$ is close to :

(a) 0.2 (b) 0.4 (c) 0.6 (d) 1

90. $(0.2 \times 0.2 + 0.01)(0.1 \times 0.1 + 0.02)^{-1}$ is equal to : **(Section Officers', 2003)**

(a) $\frac{5}{3}$ (b) $\frac{9}{5}$ (c) $\frac{41}{4}$ (d) $\frac{41}{12}$

91. $\frac{5 \times 1.6 - 2 \times 1.4}{1.3} = ?$ **(Bank P.O. 2003)**

(a) 0.4 (b) 1.2 (c) 1.4 (d) 4

92. The value of $(4.7 \times 13.26 + 4.7 \times 9.43 + 4.7 \times 77.31)$ is : **(IGNOU, 2003)**

(a) 0.47 (b) 47 (c) 470 (d) 4700

93. Simplify : $\frac{0.2 \times 0.2 + 0.2 \times 0.02}{0.044}$. **(S.S.C. 1999)**

(a) 0.004 (b) 0.4 (c) 1 (d) 2

94. The value of $\left(\frac{8.6 \times 5.3 + 8.6 \times 4.7}{4.3 \times 9.7 - 4.3 \times 8.7}\right)$ is :

(a) 3.3 (b) 6.847 (c) 13.9 (d) 20

95. The value of $\left(\frac{.896 \times .763 + .896 \times .237}{.7 \times .064 + .7 \times .936}\right)$ is :

(a) .976 (b) 9.76 (c) 1.28 (d) 12.8

96. The value of $(68.237)^2 - (31.763)^2$ is :

(a) 3.6474 (b) 36.474 (c) 364.74 (d) 3647.4

97. Evaluate : $\frac{(2.39)^2 - (1.61)^2}{2.39 - 1.61}$. **(R.R.B. 2003)**

(a) 2 (b) 4 (c) 6 (d) 8

98. On simplification of $\frac{(2.644)^2 - (2.356)^2}{0.288}$, we get : **(S.S.C. 1999)**

(a) 1 (b) 4 (c) 5 (d) 6

99. $\frac{(36.54)^2 - (3.46)^2}{?} = 40.$

(a) 3.308 (b) 4 (c) 33.08 (d) 330.8

100. The value of $\frac{(67.542)^2 - (32.458)^2}{75.458 - 40.374}$ is : **(Hotel Management, 1997)**

(a) 1 (b) 10 (c) 100 (d) None of these

101. $\left(\frac{1.49 \times 14.9 - 0.51 \times 5.1}{14.9 - 5.1}\right)$ is equal to : **(S.S.C. 2004)**

(a) 0.20 (b) 2.00 (c) 20 (d) 22

102. $\frac{4.2 \times 4.2 - 1.9 \times 1.9}{2.3 \times 6.1} = ?$ **(R.R.B. 1998)**

(a) 0.5 (b) 1 (c) 1.9 (d) 4.2

103. Simplify : $\frac{5.32 \times 56 + 5.32 \times 44}{(7.66)^2 - (2.34)^2}$.

(a) 7.2 (b) 8.5 (c) 10 (d) 12

104. $\frac{(0.6)^4 - (0.5)^4}{(0.6)^2 + (0.5)^2}$ is equal to :

(a) 0.1 (b) 0.11 (c) 1.1 (d) 11

105. $(7.5 \times 7.5 + 37.5 + 2.5 \times 2.5)$ is equal to : **(S.S.C. 2000)**

(a) 30 (b) 60 (c) 80 (d) 100

106. The simplification of $\frac{0.2 \times 0.2 + 0.02 \times 0.02 - 0.4 \times 0.02}{0.36}$ gives :

(a) 0.009 (b) 0.09 (c) 0.9 (d) 9

107. The expression $(11.98 \times 11.98 + 11.98 \times x + 0.02 \times 0.02)$ will be a perfect square for x equal to :

(a) 0.02 (b) 0.2 (c) 0.04 (d) 0.4

108. The value of $\frac{(2.697 - 0.498)^2 + (2.697 + 0.498)^2}{2.697 \times 2.697 + 0.498 \times 0.498}$ is :

(a) 0.5 (b) 2 (c) 2.199 (d) 3.195

109. The value of $\frac{(0.137 + 0.098)^2 - (0.137 - 0.098)^2}{0.137 \times 0.098}$ is :

(a) 0.039 (b) 0.235 (c) 0.25 (d) 4

110. The value of $\left(\frac{0.051 \times 0.051 \times 0.051 + 0.041 \times 0.041 \times 0.041}{0.051 \times 0.051 - 0.051 \times 0.041 + 0.041 \times 0.041}\right)$ is : **(S.S.C. 2003)**

(a) 0.00092 (b) 0.0092 (c) 0.092 (d) 0.92

111. The value of $\left(\frac{.953 \times .953 - .953 \times .047 + .047 \times .047}{.953 \times .953 \times .953 + .047 \times .047 \times .047}\right)$ is :

(a) .32 (b) .886 (c) 1.1286 (d) None of these

112. The value of $\left(\frac{0.125 + 0.027}{0.5 \times 0.5 + 0.09 - 0.15}\right)$ is : **(S.S.C. 2002)**

(a) 0.08 (b) 0.2 (c) 0.8 (d) 1

113. $\left(\frac{10.3 \times 10.3 \times 10.3 + 1}{10.3 \times 10.3 - 10.3 + 1}\right)$ is equal to : **(S.S.C. 2004)**

(a) 9.3 (b) 10.3 (c) 11.3 (d) 12.3

114. $\left[\frac{8(3.75)^3 + 1}{(7.5)^2 - 6.5}\right]$ is equal to : **(S.S.C. 2003)**

(a) $\frac{9}{5}$ (b) 2.75 (c) 4.75 (d) 8.5

115. The value of $\left(\frac{0.1 \times 0.1 \times 0.1 + 0.02 \times 0.02 \times 0.02}{0.2 \times 0.2 \times 0.2 + 0.04 \times 0.04 \times 0.04}\right)$ is : **(Hotel Management, 2003)**

(a) 0.0125 (b) 0.125 (c) 0.25 (d) 0.5

116. The value of $\left(\frac{8.94 \times 8.94 \times 8.94 - 3.56 \times 3.56 \times 3.56}{8.94 \times 8.94 + 8.94 \times 3.56 + 3.56 \times 3.56}\right)$ is :

(a) 0.538 (b) 5.38 (c) 0.0538 (d) 53.8

117. The value of $\frac{(0.96)^3 - (0.1)^3}{(0.96)^2 + 0.096 + (0.1)^2}$ is : **(S.S.C. 2004)**

(a) 0.86 (b) 0.95 (c) 0.97 (d) 1.06

118. The value of $\frac{(2.3)^3 - .027}{(2.3)^2 + .69 + .09}$ is : **(S.S.C. 1997)**

(a) 0 (b) 1.6 (c) 2 (d) 3.4

119. The value of $\frac{(0.06)^2 + (0.47)^2 + (0.079)^2}{(0.006)^2 + (0.047)^2 + (0.0079)^2}$ is :

(a) 0.1 (b) 10 (c) 100 (d) 1000

ANSWERS

1. (c)	2. (a)	3. (c)	4. (c)	5. (a)	6. (e)	7. (d)	8. (a)	9. (a)
10. (b)	11. (c)	12. (d)	13. (a)	14. (d)	15. (c)	16. (d)	17. (c)	18. (c)
19. (d)	20. (a)	21. (c)	22. (a)	23. (d)	24. (d)	25. (d)	26. (c)	27. (c)
28. (c)	29. (c)	30. (b)	31. (b)	32. (b)	33. (a)	34. (c)	35. (b)	36. (c)
37. (b)	38. (b)	39. (a)	40. (a)	41. (c)	42. (c)	43. (d)	44. (c)	45. (c)
46. (a)	47. (c)	48. (a)	49. (d)	50. (b)	51. (c)	52. (c)	53. (d)	54. (a)
55. (d)	56. (d)	57. (c)	58. (d)	59. (a)	60. (c)	61. (d)	62. (d)	63. (d)
64. (a)	65. (c)	66. (d)	67. (c)	68. (a)	69. (c)	70. (c)	71. (c)	72. (c)
73. (b)	74. (a)	75. (d)	76. (a)	77. (c)	78. (c)	79. (c)	80. (c)	81. (a)
82. (b)	83. (a)	84. (d)	85. (b)	86. (a)	87. (c)	88. (b)	89. (b)	90. (a)
91. (d)	92. (c)	93. (c)	94. (d)	95. (c)	96. (d)	97. (b)	98. (c)	99. (c)
100. (c)	101. (b)	102. (b)	103. (e)	104. (b)	105. (d)	106. (b)	107. (c)	108. (b)
109. (d)	110. (c)	111. (d)	112. (c)	113. (c)	114. (d)	115. (b)	116. (b)	117. (a)
118. (c)	119. (c)							

SOLUTIONS

1. $101\frac{27}{100000} = 101 + \frac{27}{100000} = 101 + .00027 = 101.00027.$

2. $0.36 = \frac{36}{100} = \frac{9}{25}$. Sum of Numerator and Denominator = 9 + 25 = 34.

3. Required decimal $= \frac{1}{60 \times 60} = \frac{1}{3600} = .00027.$

4. $4A + \frac{7}{B} + 2C + \frac{5}{D} + 6E = 47.2506$

$\Rightarrow \quad 4A + \frac{7}{B} + 2C + \frac{5}{D} + 6E = 40 + 7 + 0.2 + 0.05 + 0.0006$

Comparing the terms on both sides, we get :

$4A = 40, \frac{7}{B} = 7, 2C = 0.2, \frac{5}{D} = 0.05, 6E = 0.0006$

or $A = 10, B = 1, C = 0.1, D = 100, E = 0.0001.$

$\therefore \quad 5A + 3B + 6C + D + 3E = (5 \times 10) + (3 \times 1) + (6 \times 0.1) + 100 + (3 \times 0.0001)$

$= 50 + 3 + 0.6 + 100 + 0.0003 = 153.6003.$

5. Converting each of the given fractions into decimal form, we get :

$\frac{1}{3} = 0.33, \frac{2}{5} = 0.4, \frac{4}{7} = 0.57, \frac{3}{5} = 0.6, \frac{5}{6} = 0.82, \frac{6}{7} = 0.857.$

Clearly, $0.33 < 0.4 < 0.57 < 0.6 < 0.82 < 0.857$. So, $\frac{1}{3} < \frac{2}{5} < \frac{4}{7} < \frac{3}{5} < \frac{5}{6} < \frac{6}{7}$.

6. Converting each of the given fractions into decimal form, we get :

$\frac{2}{3} = 0.66, \frac{3}{5} = 0.6, \frac{7}{9} = 0.77, \frac{9}{11} = 0.81, \frac{8}{9} = 0.88.$

Clearly, $0.6 < 0.66 < 0.77 < 0.81 < 0.88$. So, $\frac{3}{5} < \frac{2}{3} < \frac{7}{9} < \frac{9}{11} < \frac{8}{9}$.

7. Converting each of the given fractions into decimal form, we get :

$\frac{5}{9} = 0.55, \frac{7}{11} = 0.63, \frac{8}{15} = 0.533, \frac{11}{17} = 0.647.$

Clearly, $0.647 > 0.63 > 0.55 > 0.533$. So, $\frac{11}{17} > \frac{7}{11} > \frac{5}{9} > \frac{8}{15}$.

8. Converting each of the given fractions into decimal form, we get :

$\frac{2}{3} = 0.66, \frac{3}{4} = 0.75, \frac{4}{5} = 0.8, \frac{5}{6} = 0.833.$

Since $0.833 > 0.8 > 0.75 > 0.66$, so $\frac{5}{6} > \frac{4}{5} > \frac{3}{4} > \frac{2}{3}$.

$\therefore$ Required difference $= \left(\frac{5}{6} - \frac{2}{3}\right) = \frac{1}{6}$.

9. Clearly, $\frac{11}{14} = 0.785, \frac{16}{19} = 0.842, \frac{19}{21} = 0.904.$

Now, $0.785 < 0.842 < 0.904$. So, $\frac{11}{14} < \frac{16}{19} < \frac{19}{21}$.

10. We have : $\frac{13}{16} = 0.8125, \frac{15}{19} = 0.7894, \frac{17}{21} = 0.8095$ and $\frac{7}{8} = 0.875$.

Since 0.7894 is the smallest, so $\frac{15}{19}$ is the smallest.

11. $\frac{3}{4} = 0.75, \frac{5}{6} = 0.833, \frac{1}{2} = 0.5, \frac{2}{3} = 0.66, \frac{4}{5} = 0.8, \frac{9}{10} = 0.9.$

Clearly, 0.8 lies between 0.75 and 0.833.

$\therefore$ $\frac{4}{5}$ lies between $\frac{3}{4}$ and $\frac{5}{6}$.

12. $\frac{7}{8} = 0.875, \frac{1}{3} = 0.333, \frac{1}{4} = 0.25, \frac{23}{24} = 0.958, \frac{11}{12} = 0.916, \frac{17}{24} = 0.708.$

Clearly, 0.708 lies between 0.333 and 0.875.

$\therefore$ $\frac{17}{24}$ lies between $\frac{1}{3}$ and $\frac{7}{8}$.

13. $\frac{4}{5} = 0.8, \frac{7}{13} = 0.53, \frac{1}{2} = 0.5, \frac{2}{3} = 0.66, \frac{3}{4} = 0.75, \frac{5}{7} = 0.714.$

Clearly, 0.5 does not lie between 0.53 and 0.8.

$\therefore$ $\frac{1}{2}$ does not lie between $\frac{4}{5}$ and $\frac{7}{13}$.

14. $\frac{-7}{10} = -0.7, \frac{5}{-8} = -\frac{5}{8} = -0.625, \frac{2}{-3} = -\frac{2}{3} = -0.66.$

Since $-0.7 < -0.66 < -0.625$, so $\frac{-7}{10} < \frac{2}{-3} < \frac{5}{-8}$.

15.
$$\begin{array}{r} 337.62 \\ 8.591 \\ +\ 34.4 \\ \hline 380.611 \end{array}$$

16.
$$\begin{array}{r} 1.0 \\ 0.1 \\ 0.01 \\ +\ 0.001 \\ \hline 1.111 \end{array}$$

17.
$$\begin{array}{r} 34.95 \\ 240.016 \\ +\ 23.98 \\ \hline 298.946 \end{array}$$

18.
$$\begin{array}{r} 617.00 \\ 6.017 \\ 0.617 \\ +\ 6.0017 \\ \hline 629.6357 \end{array}$$

19. $\begin{array}{r} 48.950 \\ -\ 32.006 \\ \hline 16.944 \end{array}$ **20.** $\begin{array}{r} 792.02 \\ +\ 101.32 \\ \hline 893.34 \end{array}$ $\begin{array}{r} 893.34 \\ -\ 306.76 \\ \hline 586.58 \end{array}$

21. Given expression = (12.1212 + 17.0005) – 9.1102 = (29.1217 – 9.1102) = 20.0115.

22. Given expression = 892.7 – (573.07 + 95.007) = 892.7 – 668.077 = 224.623.

23. Let 3889 + 12.952 – x = 3854.002.

Then, x = (3889 + 12.952) – 3854.002 = 3901.952 – 3854.002 = 47.95.

24. Let 138.009 + 341.981 – 146.305 = 123.6 + x.

Then, x = (138.009 + 341.981) – (146.305 + 123.6) = 479.99 – 269.905 = 210.085.

25. Let 832.58 – 242.31 = 779.84 – x.

Then, x = (779.84 + 242.31) – 832.58 = 1022.15 – 832.58 = 189.57.

26. Let x + 543 + 5.43 = 603.26. Then, x = 603.26 – (543 + 5.43) = 603.26 – 548.43 = 54.83.

∴ Missing digit = 8.

27. $3.14 \times 10^6 = 3.140000 \times 1000000 = 3140000$.

28. $518{,}000{,}000 = 5.18 \times 100000000 = 5.18 \times 10^8$.

29. $0.000006723 = \dfrac{0.000006723 \times 10^6}{10^6} = \dfrac{6.723}{10^6} = 6.723 \times 10^{-6}$.

30. $10^k = \dfrac{0.001125}{1.125} = \dfrac{1.125}{1125} = \dfrac{1.125 \times 10^3}{1125 \times 10^3} = \dfrac{1}{10^3} = 10^{-3}$.

∴ $k = -3$.

31. 2 × 5 = 10. Sum of decimal places = 4.

∴ 0.002 × 0.5 = 0.0010 = 0.001.

32. 1602 × 1 = 1602. Sum of decimal places = 5.

∴ 16.02 × 0.001 = 0.01602.

33. 14 × 14 = 196. Sum of decimal places = 6.

∴ 0.014 × 0.014 = 0.000196.

34. 4083 × 102 × 12 = 4997592. Sum of decimal places = 5.

∴ 40.83 × 1.02 × 1.2 = 49.97592.

35. 4 × 162 = 648. Sum of decimal places = 6.

∴ $0.04 \times 0.0162 = 0.000648 = 6.48 \times 10^{-4}$.

36. 3 × 3 × 3 × 3 × 30 = 2430. Sum of decimal places = 6.

∴ 3 × 0.3 × 0.03 × 0.003 × 30 = 0.002430 = 0.00243.

37. Sum of decimal places = 7.

Since the last digit to the extreme right will be zero (∵ 5 × 4 = 20), so there will be 6 significant digits to the right of the decimal point.

38. $\left(.00625 \text{ of } \dfrac{23}{5}\right) = \left(\dfrac{625}{100000} \times \dfrac{23}{5}\right) = \dfrac{23}{800}$.

39. Given product = $0.3 \times 0.25 \times 0.5 \times 0.125 \times 24$

$$= \left(\frac{3}{10} \times \frac{25}{100} \times \frac{5}{10} \times \frac{125}{1000} \times 24\right) = \frac{9}{80} = \frac{1}{8} \text{ (App.)}$$

40. 1. 36839 ÷ 17 = 2167. Dividend contains 2 places of decimal.

∴ 368.39 ÷ 17 = 21.67.

2. 17050 ÷ 62 = 275. Dividend contains 2 places of decimal.

∴ 170.50 ÷ 62 = 2.75.

3. $87565 \div 83 = 1055$. Dividend contains 2 places of decimal.

$\therefore\ 875.65 \div 83 = 10.55$.

Since $21.67 > 10.55 > 2.75$, the desired order is 1, 3, 2.

41. $\dfrac{0.213}{0.00213} = \dfrac{0.213 \times 100000}{0.00213 \times 100000} = \dfrac{213 \times 100}{213} = 100.$

42. $\dfrac{4.036}{0.04} = \dfrac{403.6}{4} = 100.9.$

43. $\dfrac{1}{0.04} = \dfrac{100}{4} = 25.$

44. $\left(\dfrac{0.05}{0.25} + \dfrac{0.25}{0.05}\right)^3 = \left(\dfrac{5}{25} + \dfrac{25}{5}\right)^3 = \left(\dfrac{1}{5} + 5\right)^3 = \left(\dfrac{26}{5}\right)^3 = (5.2)^3 = 140.608.$

45. $\dfrac{0.0396}{2.51} = \dfrac{3.96}{251} = \left(\dfrac{396}{251 \times 100}\right) = \dfrac{1.577}{100} = 0.01577 = 0.016.$

46. Let $.04 \times x = .000016$. Then, $x = \dfrac{.000016}{.04} = \dfrac{.0016}{4} = .0004.$

47. Let $\dfrac{.009}{x} = .01$. Then, $x = \dfrac{.009}{.01} = \dfrac{.9}{1} = .9.$

48. $\dfrac{144}{0.144} = \dfrac{14.4}{x} \Leftrightarrow \dfrac{144 \times 1000}{144} = \dfrac{14.4}{x} \Leftrightarrow x = \dfrac{14.4}{1000} = 0.0144.$

49. Length of each piece $= \left(\dfrac{1}{8}\right)$ m $= 0.125$ m.

$\therefore$ Required number of pieces $= \left(\dfrac{37.5}{0.125}\right) = \left(\dfrac{375 \times 100}{125}\right) = 300.$

50. Suppose commodity X will cost 40 paise more than Y after z years. Then,

$(4.20 + 0.40z) - (6.20 + 0.15z) = 0.40$

$\Leftrightarrow\ 0.25z = 0.40 + 2.10 \Leftrightarrow z = \dfrac{2.50}{0.25} = \dfrac{250}{25} = 10.$

$\therefore$ X will cost 40 paise more than Y 10 years after 2001 *i.e.*, in 2011.

51. $0.232323\ldots\ldots = 0.\overline{23} = \dfrac{23}{99}.$

52. $0.125125\ldots\ldots = 0.\overline{125} = \dfrac{125}{999}.$

53. $0.\overline{47} = \dfrac{47}{99}.$

54. $0.\overline{36} = \dfrac{36}{99} = \dfrac{4}{11}.$

55. $1 \div 0.2 = \dfrac{1}{0.2} = \dfrac{10}{2} = 5;\ 0.\overline{2} = 0.222\ldots\ldots;\ (0.2)^2 = 0.04.$

$0.04 < 0.2 < 0.22\ldots\ldots < 5.$

Since 0.04 is the least, so $(0.2)^2$ is the least.

56. $6.\overline{46} = 6 + 0.\overline{46} = 6 + \dfrac{46}{99} = \dfrac{594 + 46}{99} = \dfrac{640}{99}.$

57. $0.5\overline{7} = \dfrac{57 - 5}{90} = \dfrac{52}{90} = \dfrac{26}{45}.$

58. $0.84\overline{181} = \dfrac{84181 - 841}{99000} = \dfrac{83340}{99000} = \dfrac{463}{550}.$

$\therefore$ Required difference $= (550 - 463) = 87.$

59. $4.1\overline{2} = 4 + 0.1\overline{2} = 4 + \frac{12-1}{90} = 4\frac{11}{90}$.

60. $2.1\overline{36} = 2 + 0.1\overline{36} = 2 + \frac{136-1}{990} = 2 + \frac{3}{22} = 2\frac{3}{22}$.

61. $0.\overline{2} + 0.\overline{3} + 0.\overline{4} + 0.\overline{9} + 0.\overline{39} = \left(\frac{2}{9} + \frac{3}{9} + \frac{4}{9} + \frac{9}{9} + \frac{39}{99}\right) = \left(\frac{9}{9} + \frac{9}{9} + \frac{39}{99}\right) = 2 + \frac{13}{33} = 2\frac{13}{33}$.

62. $3.\overline{87} - 2.\overline{59} = (3 + 0.\overline{87}) - (2 + 0.\overline{59}) = \left(3 + \frac{87}{99}\right) - \left(2 + \frac{59}{99}\right) = 1 + \left(\frac{87}{99} - \frac{59}{99}\right)$

$= 1 + \frac{28}{99} = 1.\overline{28}$.

63. $3.\overline{36} - 2.0\overline{5} + 1.\overline{33} = [(3 + 0.\overline{36}) + (1 + 0.\overline{33})] - (2 + 0.0\overline{5})$

$= \left[4 + \left(\frac{36}{99} + \frac{33}{99}\right)\right] - \left[2 + \frac{5}{99}\right] = 2 + \left(\frac{36}{99} + \frac{33}{99} - \frac{5}{99}\right) = 2 + \frac{64}{99} = 2.\overline{64}$.

64. $0.\overline{09} \times 7.\overline{3} = \frac{9}{99} \times 7\frac{3}{9} = \frac{1}{11} \times \frac{66}{9} = \frac{2}{3} = 0.\overline{6}$.

65. $0.3\overline{467} + 0.1\overline{333} = \frac{3467-34}{9900} + \frac{1333-13}{9900} = \frac{3433+1320}{9900} = \frac{4753}{9900} = \frac{4801-48}{9900} = 0.\overline{4801}$.

66. $(8.3\overline{1} + 0.\overline{6} + 0.00\overline{2}) = 8 + \frac{31-3}{90} + \frac{6}{9} + \frac{2}{900} = \frac{7200+280+600+2}{900}$

$= \frac{8082}{900} = 8\frac{882}{900} = 8 + \frac{979-97}{900} = 8.97\overline{9}$.

67. $\overline{2}.75 + \overline{3}.78 = (-2 + 0.75) + (-3 + 0.78) = -5 + (0.75 + 0.78) = -5 + 1.53$

$= -5 + 1 + 0.53 = -4 + 0.53 = \overline{4}.53$.

68. $\frac{547527}{82} = \frac{54.7527}{0.0082} = \left(\frac{547.527}{0.0082} \times \frac{1}{10}\right) = \frac{x}{10}$.

69. $\frac{29.94}{1.45} = \frac{299.4}{14.5} = \left(\frac{2994}{14.5} \times \frac{1}{10}\right) = \frac{172}{10} = 17.2$.

70. $1.6 \times 21.3 = \left(\frac{16}{10} \times \frac{213}{10}\right) = \left(\frac{16 \times 213}{100}\right) = \frac{3408}{100} = 34.08$.

71. $\frac{1}{0.0006198} = \frac{10000}{6.198} = \left(10000 \times \frac{1}{6.198}\right) = (10000 \times 0.16134) = 1613.4$.

72. Given, $\frac{52416}{312} = 168 \Leftrightarrow \frac{52416}{168} = 312$.

Now, $\frac{52.416}{0.0168} = \frac{524160}{168} = \left(\frac{52416}{168} \times 10\right) = (312 \times 10) = 3120$.

73. Given, $168 \times 32 = 5376$ or $5376 \div 168 = 32$.

Now, $\frac{5.376}{16.8} = \frac{53.76}{168} = \left(\frac{5376}{168} \times \frac{1}{100}\right) = \frac{32}{100} = 0.32$.

74. Number of decimal places in the given expression = 8.
Number of decimal places in (a) = 8.
Number of decimal places in (b) = 9.
Number of decimal places in (c) = 7.
Clearly, the expression in (a) is the same as the given expression.

75. For the expressions to be equivalent, the difference between the sum of the decimal places in the numerator and that in the denominator must be equal.
This difference is 1 in the given expression and 1 in (*d*). So, (*d*) is the answer.

76. Given expression $= \frac{(96.54 - 89.63)}{(96.54 + 89.63)} \times \frac{(9.654 + 8.963)}{(965.4 - 896.3)} = \frac{(96.54 - 89.63)}{(965.4 - 896.3)} \times \frac{(9.654 + 8.963)}{(96.54 + 89.63)}$

$= \frac{(96.54 - 89.63)}{10\,(96.54 - 89.63)} \times \frac{(96.54 + 89.63)}{10\,(96.54 + 89.63)}$

$= \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}.$

77. $(0.11)^3 + (0.22)^3 + \ldots\ldots + (0.99)^3 = (0.11)^3\,(1^3 + 2^3 + \ldots\ldots + 9^3)$

$= 0.001331 \times 2025 = 2.695275 = 2.695.$

78. Given expression $= 8.7 - [7.6 - \{6.5 - (5.4 - 2.3)\}] = 8.7 - [7.6 - (6.5 - 3.1)]$

$= 8.7 - (7.6 - 3.4) = 8.7 - 4.2 = 4.5.$

79. $\frac{1}{4} + \frac{1}{4 \times 5} + \frac{1}{4 \times 5 \times 6} = \frac{1}{4}\left(1 + \frac{1}{5} + \frac{1}{30}\right) = \frac{1}{4}\left(\frac{30 + 6 + 1}{30}\right) = \frac{1}{4} \times \frac{37}{30} = \frac{37}{120} = 0.3083.$

80. Given expression $= \frac{2 \times 4 \times 8 \times 16 + 4 \times 8 \times 16 + 8 \times 16 + 16 + 1}{2 \times 4 \times 8 \times 16}$

$= \frac{1024 + 512 + 128 + 16 + 1}{1024} = \frac{1681}{1024} = 1.6^{4}16.$

81. Given expression $= \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \ldots\ldots + \frac{1}{24 \times 25}$

$= \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{8}\right) + \ldots\ldots + \left(\frac{1}{24} - \frac{1}{25}\right)$

$= \left(\frac{1}{5} - \frac{1}{25}\right) = \frac{4}{25} = 0.16.$

82. $\frac{x}{y} = \frac{0.04}{1.5} = \frac{4}{150} = \frac{2}{75} \Rightarrow \frac{y - x}{y + x} = \frac{1 - \frac{x}{y}}{1 + \frac{x}{y}} = \frac{1 - \frac{2}{75}}{1 + \frac{2}{75}} = \frac{73}{77}.$

83. Given expression $= 35.7 - \left(3 + \frac{1}{\frac{10}{3}}\right) - \left(2 + \frac{1}{\frac{5}{2}}\right) = 35.7 - \left(3 + \frac{3}{10}\right) - \left(2 + \frac{2}{5}\right)$

$= 35.7 - \frac{33}{10} - \frac{12}{5} = 35.7 - \left(\frac{33}{10} + \frac{12}{5}\right) = 35.7 - \frac{57}{10} = 35.7 - 5.7 = 30.$

84. Given expression $= \frac{(0.3333)}{(0.2222)} \times \frac{(0.1667)\,(0.8333)}{(0.6667)\,(0.1250)} = \frac{3333}{2222} \times \frac{\frac{1}{6} \times \frac{5}{6}}{\frac{2}{3} \times \frac{125}{1000}}$

$= \left(\frac{3}{2} \times \frac{1}{6} \times \frac{5}{6} \times \frac{3}{2} \times 8\right) = \frac{5}{2} = 2.50.$

85. $\frac{3.6 \times 0.48 \times 2.50}{0.12 \times 0.09 \times 0.5} = \frac{36 \times 48 \times 250}{12 \times 9 \times 5} = 800.$

86. $\frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7} = \frac{203 \times 292}{73 \times 145 \times 7} = \frac{4}{5} = 0.8.$

87. $\dfrac{3.157\times4126\times3.198}{63.972\times2835.121}=\dfrac{3.2\times4126\times3.2}{64\times2835}=\dfrac{32\times4126\times32}{64\times2835}\times\dfrac{1}{100}$

$=\dfrac{66016}{2835}\times\dfrac{1}{100}=\dfrac{23.28}{100}=0.23\approx0.2.$

88. $\dfrac{489.1375\times0.0483\times1.956}{0.0873\times92.581\times99.749}=\dfrac{489\times0.05\times2}{0.09\times93\times100}=\dfrac{489}{9\times93\times10}$

$=\dfrac{163}{279}\times\dfrac{1}{10}=\dfrac{0.58}{10}=0.058\approx0.06.$

89. $\dfrac{241.6\times0.3814\times6.842}{0.4618\times38.25\times73.65}=\dfrac{240\times0.38\times6.9}{0.46\times38\times75}=\dfrac{240\times38\times69}{46\times38\times75}\times\dfrac{1}{10}$

$=\left(\dfrac{24}{5}\times\dfrac{1}{10}\right)=\dfrac{4.8}{10}=0.48.$

So, the value is close to 0.4.

90. Given expression $=\dfrac{(0.2\times0.2+0.01)}{(0.1\times0.1+0.02)}=\dfrac{0.04+0.01}{0.01+0.02}=\dfrac{0.05}{0.03}=\dfrac{5}{3}.$

91. Given expression $=\dfrac{8-2.8}{1.3}=\dfrac{5.2}{1.3}=\dfrac{52}{13}=4.$

92. Given expression $=4.7\times(13.26+9.43+77.31)=4.7\times100=470.$

93. Given expression $=\dfrac{0.2(0.2+0.02)}{0.044}=\dfrac{0.2\times0.22}{0.044}=\dfrac{0.044}{0.044}=1.$

94. Given expression $=\dfrac{8.6\times(5.3+4.7)}{4.3\times(9.7-8.7)}=\dfrac{8.6\times10}{4.3\times1}=20.$

95. Given expression $=\dfrac{.896\times(.763+.237)}{.7\times(.064+.936)}=\dfrac{.896\times1}{.7\times1}=\dfrac{8.96}{7}=1.28.$

96. Given expression $=(a^2-b^2)=(a+b)(a-b)=(68.237+31.763)(68.237-31.763)$

$=(100\times36.474)=3647.4.$

97. Given expression $=\dfrac{a^2-b^2}{a-b}=\dfrac{(a+b)(a-b)}{(a-b)}=(a+b)=(2.39+1.61)=4.$

98. Given expression $=\dfrac{(2.644)^2-(2.356)^2}{2.644-2.356}=\dfrac{a^2-b^2}{a-b}=(a+b)=(2.644+2.356)=5.$

99. Let $\dfrac{(36.54)^2-(3.46)^2}{x}=40.$ Then, $x=\dfrac{(36.54)^2-(3.46)^2}{40}=\dfrac{(36.54)^2-(3.46)^2}{36.54+3.46}$

$=\dfrac{a^2-b^2}{a+b}=(a-b)=(36.54-3.46)=33.08.$

100. Given expression $=\dfrac{(67.542)^2-(32.458)^2}{(67.542+7.196)-(32.458+7.916)}$

$=\dfrac{(67.542)^2-(32.458)^2}{67.542-32.458}=(67.542+32.458)=100.$

101. Given expression $=\left(\dfrac{1.49\times1.49\times10-0.51\times0.51\times10}{1.49\times10-0.51\times10}\right)$

$=\dfrac{10[(1.49)^2-(0.51)^2]}{10(1.49-0.51)}=(1.49+0.51)=2.$

102. Given expression $= \dfrac{(a^2 - b^2)}{(a+b)(a-b)} = \dfrac{(a^2 - b^2)}{(a^2 - b^2)} = 1.$

103. Given expression $= \dfrac{5.32 \times (56 + 44)}{(7.66 + 2.34)(7.66 - 2.34)} = \dfrac{5.32 \times 100}{10 \times 5.32} = 10.$

104. Given expression $= \dfrac{[(0.6)^2]^2 - [(0.5)^2]^2}{(0.6)^2 + (0.5)^2} = \dfrac{[(0.6)^2 + (0.5)^2]\,[(0.6)^2 - (0.5)^2]}{(0.6)^2 + (0.5)^2}$

$= (0.6)^2 - (0.5)^2 = (0.6 + 0.5)(0.6 - 0.5) = (1.1 \times 0.1) = 0.11.$

105. Given expression $= (7.5 \times 7.5 + 2 \times 7.5 \times 2.5 + 2.5 \times 2.5)$

$= (a^2 + 2ab + b^2) = (a + b)^2 = (7.5 + 2.5)^2 = 10^2 = 100.$

106. $0.2 \times 0.2 + 0.02 \times 0.02 - 0.4 \times 0.02 = 0.2 \times 0.2 + 0.02 \times 0.02 - 2 \times 0.2 \times 0.02$

$= (a^2 + b^2 - 2ab) = (a - b)^2 = (0.2 - 0.02)^2$

$= (0.18)^2.$

$\therefore$ Given expression $= \dfrac{(0.18 \times 0.18)}{0.36} = 0.09.$

107. Given expression $= (11.98)^2 + (0.02)^2 + 11.98 \times x.$

For the given expression to be a perfect square, we must have

$11.98 \times x = 2 \times 11.98 \times 0.02$ or $x = 0.04.$

108. Given expression $= \dfrac{(a-b)^2 + (a+b)^2}{a^2 + b^2} = \dfrac{2(a^2 + b^2)}{(a^2 + b^2)} = 2.$

109. Given expression $= \dfrac{(a+b)^2 - (a-b)^2}{ab} = \dfrac{4ab}{ab} = 4.$

110. Given expression $= \dfrac{(0.051)^3 + (0.041)^3}{(0.051)^2 - (0.051 \times 0.041) + (0.041)^2} = \left(\dfrac{a^3 + b^3}{a^2 - ab + b^2}\right)$

$= (a + b) = (0.051 + 0.041) = 0.092.$

111. Given expression $= \dfrac{(.953)^2 - (.953 \times .047) + (.047)^2}{(.953)^3 + (.047)^3}$

$= \left(\dfrac{a^2 - ab + b^2}{a^3 + b^3}\right) = \dfrac{1}{a+b} = \dfrac{1}{.953 + .047} = 1.$

112. Given expression $= \dfrac{(0.5)^3 + (0.3)^3}{(0.5)^2 + (0.3)^2 - (0.5 \times 0.3)} = \left(\dfrac{a^3 + b^3}{a^2 + b^2 - ab}\right)$

$= (a + b) = (0.5 + 0.3) = 0.8.$

113. Given expression $= \dfrac{(10.3)^3 + (1)^3}{(10.3)^2 - (10.3 \times 1) + (1)^2} = \left(\dfrac{a^3 + b^3}{a^2 - ab + b^2}\right)$

$= (a + b) = (10.3 + 1) = 11.3.$

114. Given expression $= \dfrac{(2 \times 3.75)^3 + (1)^3}{(7.5)^2 - (7.5 \times 1) + (1)^2} = \dfrac{(7.5)^3 + (1)^3}{(7.5)^2 - (7.5 \times 1) + (1)^2}$

$= \left(\dfrac{a^3 + b^3}{a^2 - ab + b^2}\right) = (a + b) = (7.5 + 1) = 8.5.$

115. Given expression $= \dfrac{(0.1)^3 + (0.02)^3}{2^3\,[(0.1)^3 + (0.02)^3]} = \dfrac{1}{8} = 0.125.$

116. Given expression $= \dfrac{(8.94)^3 - (3.56)^3}{(8.94)^2 + 8.94 \times 3.56 + (3.56)^2} = \left(\dfrac{a^3 - b^3}{a^2 + ab + b^2}\right)$

$= (a - b) = (8.94 - 3.56) = 5.38.$

117. Given expression $= \dfrac{(0.96)^3 - (0.1)^3}{(0.96)^2 + (0.96 \times 0.1) + (0.1)^2} = \left(\dfrac{a^3 - b^3}{a^2 + ab + b^2}\right)$

$= (a - b) = (0.96 - 0.1) = 0.86.$

118. Given expression $= \dfrac{(2.3)^3 - (0.3)^3}{(2.3)^2 + (2.3 \times 0.3) + (0.3)^2} = \left(\dfrac{a^3 - b^3}{a^2 + ab + b^2}\right)$

$= (a - b) = (2.3 - 0.3) = 2.$

119. Given expression $= \dfrac{a^2 + b^2 + c^2}{\left(\dfrac{a}{10}\right)^2 + \left(\dfrac{b}{10}\right)^2 + \left(\dfrac{c}{10}\right)^2}$, where $a = 0.6$, $b = 0.47$ and $c = 0.079$.

$= \dfrac{100\,(a^2 + b^2 + c^2)}{(a^2 + b^2 + c^2)} = 100.$

4. SIMPLIFICATION

IMPORTANT CONCEPTS

I. **'BODMAS' Rule :** This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression.

Here, 'B' stands for '***Bracket***', 'O' for '***of***', 'D' for '***Division***', 'M' for '***Multiplication***', 'A' for '***Addition***' and 'S' for '***Subtraction***'.

Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order (), {} and [].

After removing the brackets, we must use the following operations strictly in the order :

(*i*) of (*ii*) Division (*iii*) Multiplication (*iv*) Addition (*v*) Subtraction.

II. **Modulus of a Real Number :** Modulus of a real number a is defined as

$$|a| = \begin{cases} a, \text{ if } a > 0 \\ -a, \text{ if } a < 0. \end{cases}$$

Thus, $|5| = 5$ and $|-5| = -(-5) = 5$.

III. **Virnaculum** (or Bar) : When an expression contains Virnaculum, before applying the 'BODMAS' rule, we simplify the expression under the Virnaculum.

SOLVED EXAMPLES

Ex. 1. ***Simplify : (i) 5005 − 5000 + 10 (ii) 18800 + 470 + 20.***

Sol. (*i*) $5005 - 5000 + 10 = 5005 - \frac{5000}{10} = 5005 - 500 = 4505.$

(*ii*) $18800 + 470 + 20 = \frac{18800}{470} + 20 = 40 + 20 = 2.$

Ex. 2. ***Simplify :*** $b - [b - (a + b) - \{b - (b - \overline{a - b})\} + 2a]$. **(Hotel Management, 2002)**

Sol. Given expression $= b - [b - (a + b) - \{b - (b - a + b)\} + 2a]$

$= b - [b - a - b - \{b - 2b + a\} + 2a]$

$= b - [-a - \{b - 2b + a + 2a\}]$

$= b - [-a - \{-b + 3a\}] = b - [-a + b - 3a]$

$= b - [-4a + b] = b + 4a - b = 4a.$

Ex. 3. ***What value will replace the question mark in the following equation ?***

$$4\frac{1}{2} + 3\frac{1}{6} + ? + 2\frac{1}{3} = 13\frac{2}{5}.$$

Sol. Let $\frac{9}{2} + \frac{19}{6} + x + \frac{7}{3} = \frac{67}{5}$.

Then, $x = \frac{67}{5} - \left(\frac{9}{2} + \frac{19}{6} + \frac{7}{3}\right) \Leftrightarrow x = \frac{67}{5} - \left(\frac{27 + 19 + 14}{6}\right) = \left(\frac{67}{5} - \frac{60}{6}\right)$

$\Leftrightarrow x = \left(\frac{67}{5} - 10\right) = \frac{17}{5} = 3\frac{2}{5}.$

Hence, missing fraction $= 3\frac{2}{5}$.

Ex. 4. *$\frac{4}{15}$ of $\frac{5}{7}$ of a number is greater than $\frac{4}{9}$ of $\frac{2}{5}$ of the same number by 8. What is half of that number ?* (S.B.I.P.O. 2000)

Sol. Let the number be x. Then, $\frac{4}{15}$ of $\frac{5}{7}$ of $x - \frac{4}{9}$ of $\frac{2}{5}$ of $x = 8 \Leftrightarrow \frac{4}{21}x - \frac{8}{45}x = 8$

$\Leftrightarrow \left(\frac{4}{21} - \frac{8}{45}\right)x = 8 \Leftrightarrow \left(\frac{60-56}{315}\right)x = 8 \Leftrightarrow \frac{4}{315}x = 8$

$\Leftrightarrow x = \left(\frac{8 \times 315}{4}\right) = 630 \Leftrightarrow \frac{1}{2}x = 315.$

Hence, required number = 315.

Ex. 5. *Simplify :* $\left[3\frac{1}{4} \div \left\{1\frac{1}{4} - \frac{1}{2}\left(2\frac{1}{2} - \frac{1}{4} - \frac{1}{6}\right)\right\}\right]$.

Sol. Given exp. $= \left[\frac{13}{4} \div \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{5}{2} - \frac{3-2}{12}\right)\right\}\right] = \left[\frac{13}{4} \div \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{5}{2} - \frac{1}{12}\right)\right\}\right]$

$= \left[\frac{13}{4} \div \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{30-1}{12}\right)\right\}\right] = \left[\frac{13}{4} \div \left\{\frac{5}{4} - \frac{29}{24}\right\}\right]$

$= \left[\frac{13}{4} \div \left\{\frac{30-29}{24}\right\}\right] = \left[\frac{13}{4} \div \frac{1}{24}\right] = \left[\frac{13}{4} \times 24\right] = 78.$

Ex. 6. *Simplify : 108 ÷ 36 of $\frac{1}{4} + \frac{2}{5} \times 3\frac{1}{4}$.*

Sol. Given exp. $= 108 \div 9 + \frac{2}{5} \times \frac{13}{4} = \frac{108}{9} + \frac{13}{10} = \left(12 + \frac{13}{10}\right) = \frac{133}{10} = 13\frac{3}{10}$.

Ex. 7. *Simplify :* $\frac{\frac{7}{2} \div \frac{5}{2} \times \frac{3}{2}}{\frac{7}{2} + \frac{5}{2} \text{ of } \frac{3}{2}} \div 5.25.$ (S.S.C. 1999)

Sol. Given exp. $= \frac{\frac{7}{2} \times \frac{2}{5} \times \frac{3}{2}}{\frac{7}{2} \div \frac{15}{4}} \div 5.25 = \frac{\frac{21}{10}}{\frac{7}{2} \times \frac{4}{15}} \div \frac{525}{100} = \frac{21}{10} \times \frac{15}{14} \times \frac{100}{525} = \frac{6}{14} = \frac{3}{7}$.

Ex. 8. *Simplify : (i) 12.05 × 5.4 + 0.6* *(ii) .6 × .6 + .6 + 6.* (Bank P.O. 2003)

Sol. *(i)* Given exp. $= 12.05 \times \frac{5.4}{0.6} = 12.05 \times 9 = 108.45.$

(ii) Given exp. $= .6 \times .6 + \frac{.6}{6} = .36 + .1 = .46.$

Ex. 9. *Find the value of x in each of the following equations :*

(i) $\frac{17.28 \div x}{3.6 \times 0.2} = 2$ *(ii) 3648.24 + 364.824 + x − 36.4824 = 3794.1696*

(iii) $8.5 - \left\{5\frac{1}{2} - \left(7\frac{1}{2} + 2.8 \div x\right)\right\} \times 4.25 \div (0.2)^2 = 306.$ (Hotel Management, 1997)

Sol. *(i)* $\frac{17.28}{x} = 2 \times 3.6 \times 0.2 \Leftrightarrow x = \frac{17.28}{1.44} = \frac{1728}{144} = 12.$

(*ii*) $\frac{364.824}{x} = (3794.1696 + 36.4824) - 3648.24 - 3830.652 - 3648.24 - 182.412$

$\Leftrightarrow \quad x = \frac{364.824}{182.412} = 2.$

(*iii*) $8.5 - \left\{5.5 - \left(7.5 + \frac{2.8}{x}\right)\right\} \times \frac{4.25}{0.04} = 306 \Leftrightarrow 8.5 - \left\{5.5 - \left(\frac{7.5x + 2.8}{x}\right)\right\} \times \frac{425}{4} = 306$

$\Leftrightarrow \quad 8.5 - \left\{\frac{5.5x - 7.5x - 2.8}{x}\right\} \times \frac{425}{4} = 306 \Leftrightarrow 8.5 - \left\{\frac{-2x - 2.8}{x}\right\} \times 106.25 = 306$

$\Leftrightarrow \quad 8.5 - \left\{\frac{-212.5x - 297.5}{x}\right\} = 306 \Leftrightarrow \frac{8.5x + 212.5x + 297.5}{x} = 306$

$\Leftrightarrow \quad (306 - 221)\,x = 297.5 \Leftrightarrow x = \frac{297.5}{85} = 3.5.$

Ex. 10. ***If*** $\frac{x}{y} = \frac{6}{5}$, ***find the value of*** $\frac{x^2 + y^2}{x^2 - y^2}$.

Sol. $\frac{x^2 + y^2}{x^2 - y^2} = \frac{\frac{x^2}{y^2} + 1}{\frac{x^2}{y^2} - 1} = \frac{\left(\frac{x}{y}\right)^2 + 1}{\left(\frac{x}{y}\right)^2 - 1} = \frac{\left(\frac{6}{5}\right)^2 + 1}{\left(\frac{6}{5}\right)^2 - 1} = \frac{\frac{36}{25} + 1}{\frac{36}{25} - 1} = \frac{61}{25} \times \frac{25}{11} = \frac{61}{11}.$

Ex. 11. ***Find the value of*** $4 - \cfrac{5}{1 + \cfrac{1}{3 + \cfrac{1}{2 + \frac{1}{4}}}}$.

Sol. Given exp. $= 4 - \cfrac{5}{1 + \cfrac{1}{3 + \cfrac{1}{(9/4)}}} = 4 - \cfrac{5}{1 + \cfrac{1}{3 + \frac{4}{9}}} = 4 - \cfrac{5}{1 + \cfrac{1}{(31/9)}}$

$= 4 - \cfrac{5}{1 + \frac{9}{31}} = 4 - \frac{5}{(40/31)} = 4 - \frac{5 \times 31}{40} = 4 - \frac{31}{8} = \frac{1}{8}.$

Ex. 12. ***If*** $\cfrac{2x}{1 + \cfrac{1}{1 + \cfrac{x}{1 - x}}} = 1$, ***then find the value of*** x. (M.A.T. 1998)

Sol. We have : $\cfrac{2x}{1 + \cfrac{1}{\cfrac{(1 - x) + x}{1 - x}}} = 1 \Leftrightarrow \cfrac{2x}{1 + \cfrac{1}{[1/(1 - x)]}} = 1 \Leftrightarrow \frac{2x}{1 + (1 - x)} = 1$

$\Leftrightarrow \quad 2x = 2 - x \Leftrightarrow 3x = 2 \Leftrightarrow x = \frac{2}{3}.$

Ex. 13. (*i*) ***If*** $\frac{a}{b} = \frac{3}{4}$ ***and*** $8a + 5b = 22$, ***then find the value of*** a. (R.R.B. 2002)

(*ii*) ***If*** $\frac{x}{4} - \frac{x - 3}{6} = 1$, ***then find the value of*** x. (R.R.B. 2000)

Sol. (*i*) $\frac{a}{b} = \frac{3}{4} \Leftrightarrow b = \frac{4}{3}a.$

$\therefore \quad 8a + 5b = 22 \Rightarrow 8a + 5 \times \frac{4}{3}a = 22 \Rightarrow 8a + \frac{20}{3}a = 22$

$\Rightarrow 44a = 66 \Rightarrow a = \frac{66}{44} = \frac{3}{2}.$

(*ii*) $\frac{x}{4} - \frac{x-3}{6} = 1 \Leftrightarrow \frac{3x - 2(x-3)}{12} = 1 \Leftrightarrow 3x - 2x + 6 = 12 \Leftrightarrow x = 6.$

Ex. 14. ***If $2x + 3y = 34$ and $\frac{x+y}{y} = \frac{13}{8}$, then find the value of $5y + 7x$.***

(S.B.I.P.O. 2001)

Sol. The given equations are :

$2x + 3y = 34 \quad ...(i)$ and, $\frac{x+y}{y} = \frac{13}{8} \Rightarrow 8x + 8y = 13y \Rightarrow 8x - 5y = 0 \quad ...(ii)$

Multiplying (*i*) by 5, (*ii*) by 3 and adding, we get : $34x = 170$ or $x = 5$.

Putting $x = 5$ in (*i*), we get : $y = 8$.

$\therefore \quad 5y + 7x = (5 \times 8 + 7 \times 5) = 40 + 35 = 75.$

Ex. 15. ***If $2x + 3y + z = 55$, $x + z - y = 4$ and $y - x + z = 12$, then what are the values of x, y and z ?*** **(Bank P.O. 2003)**

Sol. The given equations are :

$2x + 3y + z = 55 \quad ...(i)$; $x + z - y = 4 \quad ...(ii)$; $y - x + z = 12 \quad ...(iii)$

Subtracting (*ii*) from (*i*), we get : $x + 4y = 51 \quad ...(iv)$

Subtracting (*iii*) from (*i*), we get : $3x + 2y = 43 \quad ...(v)$

Multiplying (*v*) by 2 and subtracting (*iv*) from it, we get : $5x = 35$ or $x = 7$.

Putting $x = 7$ in (*iv*), we get : $4y = 44$ or $y = 11$.

Putting $x = 7$, $y = 11$ in (*i*), we get : $z = 8$.

Ex. 16. ***Find the value of $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right).....\left(1 - \frac{1}{100}\right)$.*** **(S.S.C. 2003)**

Sol. Given expression $= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times ..... \times \frac{99}{100} = \frac{2}{100} = \frac{1}{50}.$

Ex. 17. ***Find the value of $\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + ... + \frac{1}{9 \times 10}$.***

Sol. Given expression $= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + ... + \left(\frac{1}{9} - \frac{1}{10}\right)$

$= \left(\frac{1}{2} - \frac{1}{10}\right) = \frac{4}{10} = \frac{2}{5}.$

Ex. 18. ***Simplify : $99\frac{48}{49} \times 245$.*** **(R.R.B. 2000)**

Sol. Given expression $= \left(100 - \frac{1}{49}\right) \times 245 = \frac{4899}{49} \times 245 = 4899 \times 5 = 24495.$

Ex. 19. ***A board 7 ft. 9 inches long is divided into 3 equal parts. What is the length of each part ?*** **(Hotel Management, 2003)**

Sol. Length of board = 7 ft. 9 inches = $(7 \times 12 + 9)$ inches = 93 inches.

$\therefore$ Length of each part $= \left(\frac{93}{3}\right)$ inches = 31 inches = 2 ft. 7 inches.

Ex. 20. *A man divides Rs. 8600 among 5 sons, 4 daughters and 2 nephews. If each daughter receives four times as much as each nephew, and each son receives five times as much as each nephew, how much does each daughter receive?* (S.S.C. 2000)

Sol. Let the share of each nephew be Rs. x.

Then, share of each daughter = Rs. $(4x)$; share of each son = Rs. $(5x)$.

So, $5 \times 5x + 4 \times 4x + 2 \times x = 8600 \Leftrightarrow 25x + 16x + 2x = 8600$

$\Leftrightarrow 43x = 8600 \Leftrightarrow x = 200.$

$\therefore$ Share of each daughter = Rs. (4×200) = Rs. 800.

Ex. 21. *A man spends $\frac{2}{5}$ of his salary on house rent, $\frac{3}{10}$ of his salary on food and $\frac{1}{8}$ of his salary on conveyance. If he has Rs. 1400 left with him, find his expenditure on food and conveyance.*

Sol. Part of the salary left $= 1 - \left(\frac{2}{5} + \frac{3}{10} + \frac{1}{8}\right) = 1 - \frac{33}{40} = \frac{7}{40}$.

Let the monthly salary be Rs. x.

Then, $\frac{7}{40}$ of $x = 1400 \Leftrightarrow x = \left(\frac{1400 \times 40}{7}\right) = 8000.$

$\therefore$ Expenditure on food = Rs. $\left(\frac{3}{10} \times 8000\right)$ = Rs. 2400.

Expenditure on conveyance = Rs. $\left(\frac{1}{8} \times 8000\right)$ = Rs. 1000.

Ex. 22. *A third of Arun's marks in Mathematics exceeds a half of his marks in English by 30. If he got 240 marks in the two subjects together, how many marks did he get in English?*

Sol. Let Arun's marks in Mathematics and English be x and y respectively.

Then, $\frac{1}{3}x - \frac{1}{2}y = 30 \Leftrightarrow 2x - 3y = 180$ ...(*i*) and $x + y = 240$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 180$ and $y = 60$.

Ex. 23. *A tin of oil was $\frac{4}{5}$ full. When 6 bottles of oil were taken out and four bottles of oil were poured into it, it was $\frac{3}{4}$ full. How many bottles of oil can the tin contain?* (Section Officers', 2001)

Sol. Suppose x bottles can fill the tin completely.

Then, $\frac{4}{5}x - \frac{3}{4}x = (6 - 4) \Leftrightarrow \frac{x}{20} = 2 \Leftrightarrow x = 40.$

$\therefore$ Required number of bottles = 40.

Ex. 24. *If $\frac{1}{8}$ of a pencil is black, $\frac{1}{2}$ of the remaining is white and the remaining $3\frac{1}{2}$ cm is blue, find the total length of the pencil.*

Sol. Let the total length of the pencil be x cm. Then,

Black part = $\left(\frac{x}{8}\right)$ cm. Remaining part = $\left(x - \frac{x}{8}\right)$ cm = $\left(\frac{7x}{8}\right)$ cm.

White part = $\left(\frac{1}{2} \times \frac{7x}{8}\right)$ cm = $\left(\frac{7x}{16}\right)$ cm. Remaining part = $\left(\frac{7x}{8} - \frac{7x}{16}\right)$ cm = $\frac{7x}{16}$ cm.

$\therefore$ $\frac{7x}{16} = \frac{7}{2}$ or $x = \frac{16}{2} = 8$ cm.

Hence, total length of the pencil = 8 cm.

Ex. 25. ***In a certain office, $\frac{1}{3}$ of the workers are women, $\frac{1}{2}$ of the women are married and $\frac{1}{3}$ of the married women have children. If $\frac{3}{4}$ of the men are married and $\frac{2}{3}$ of the married men have children, what part of workers are without children ?***

Sol. Let the total number of workers be x. Then,

Number of women = $\frac{x}{3}$ and number of men = $\left(x - \frac{x}{3}\right) = \frac{2x}{3}$.

Number of women having children = $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{x}{3} = \frac{x}{18}$.

Number of men having children = $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2x}{3} = \frac{x}{3}$.

Number of workers having children = $\left(\frac{x}{18} + \frac{x}{3}\right) = \frac{7x}{18}$.

$\therefore$ Workers having no children = $\left(x - \frac{7x}{18}\right) = \frac{11x}{18} = \frac{11}{18}$ of all workers.

Ex. 26. ***A crate of mangoes contains one bruised mango for every 30 mangoes in the crate. If 3 out of every 4 bruised mangoes are considered unsalable, and there are 12 unsalable mangoes in the crate, then how many mangoes are there in the crate ?***

Sol. Let the total number of mangoes in the crate be x. Then,

Number of bruised mangoes = $\frac{1}{30}x$.

Number of unsalable mangoes = $\left(\frac{3}{4} \times \frac{1}{30}x\right) = \frac{1}{40}x$.

$\therefore$ $\frac{1}{40}x = 12$ or $x = (12 \times 40) = 480$.

Hence, total number of mangoes in the crate = 480.

Ex. 27. ***A train starts full of passengers. At the first station, it drops one-third of the passengers and takes 280 more. At the second station, it drops one-half of the new total and takes 12 more. On arriving at the third station, it is found to have 248 passengers. Find the number of passengers in the beginning.***

Sol. Let the number of passengers in the beginning be x.

After 1st station, number of passengers = $\left(x - \frac{x}{3}\right) + 280 = \left(\frac{2x}{3} + 280\right)$.

After 2nd station, number of passengers = $\frac{1}{2}\left(\frac{2x}{3} + 280\right) + 12$.

$\therefore$ $\frac{1}{2}\left(\frac{2x}{3} + 280\right) + 12 = 248 \Leftrightarrow \frac{2x}{3} + 280 = 2 \times 236 \Leftrightarrow \frac{2x}{3} = 192$

$\Leftrightarrow x = \left(192 \times \frac{3}{2}\right) = 288.$

Ex. 28. *If $a^2 + b^2 = 117$ and $ab = 54$, then find the value of $\frac{a+b}{a-b}$.*

Sol. $(a + b)^2 = a^2 + b^2 + 2ab = 117 + 2 \times 54 = 225 \Rightarrow a + b = 15.$

$(a - b)^2 = a^2 + b^2 - 2ab = 117 - 2 \times 54 = 9 \Rightarrow a - b = 3.$

$\therefore \quad \frac{a+b}{a-b} = \frac{15}{3} = 5.$

Ex. 29. *Find the value of $\left(\frac{75983 \times 75983 - 45983 \times 45983}{30000}\right)$.*

Sol. Given expression $= \frac{(75983)^2 - (45983)^2}{(75983 - 45983)} = \frac{(a^2 - b^2)}{(a - b)}$, where $a = 75983$, $b = 45983$

$= \frac{(a+b)(a-b)}{(a-b)} = (a + b) = (75983 + 45983) = 121966.$

Ex. 30. *Find the value of $\left(\frac{343 \times 343 \times 343 - 113 \times 113 \times 113}{343 \times 343 + 343 \times 113 + 113 \times 113}\right)$.*

Sol. Given expression $= \frac{(a^3 - b^3)}{(a^2 + ab + b^2)}$, where $a = 343$, $b = 113$

$= (a - b) = (343 - 113) = 230.$

Ex. 31. *Village X has a population of 68000, which is decreasing at the rate of 1200 per year. Village Y has a population of 42000, which is increasing at the rate of 800 per year. In how many years will the population of the two villages be equal ?*

Sol. Let the population of villages X and Y be equal after p years.

Then, $68000 - 1200p = 42000 + 800p \Rightarrow 2000p = 26000 \Rightarrow p = 13.$

So, their population will be equal after 13 years.

Ex. 32. *From a group of boys and girls, 15 girls leave. There are then left 2 boys for each girl. After this, 45 boys leave. There are then 5 girls for each boy. Find the number of girls in the beginning.*

Sol. Let at present there be x boys. Then, number of girls at present $= 5x$.

Before the boys had left : Number of boys $= x + 45$ and number of girls $= 5x$.

$\therefore \quad x + 45 = 2 \times 5x \Leftrightarrow 9x = 45 \Leftrightarrow x = 5.$

Hence, number of girls in the beginning $= 5x + 15 = 25 + 15 = 40.$

Ex. 33. *An employer pays Rs. 20 for each day a worker works, and forfeits Rs. 3 for each day he is idle. At the end of 60 days, a worker gets Rs. 280. For how many days did the worker remain idle ?*

Sol. Suppose the worker remained idle for x days. Then, he worked for $(60 - x)$ days.

$\therefore \quad 20(60 - x) - 3x = 280 \Leftrightarrow 1200 - 23x = 280 \Leftrightarrow 23x = 920 \Leftrightarrow x = 40.$

So, the worker remained idle for 40 days.

Ex. 34. *Kiran had 85 currency notes in all, some of which were of Rs. 100 denomination and the remaining of Rs. 50 denomination. The total amount of all these currency notes was Rs. 5000. How much amount did she have in the denomination of Rs. 50 ?* (R.B.I. 2000)

Sol. Let the number of 50-rupee notes be x.

Then, the number of 100-rupee notes $= (85 - x)$.

$\therefore \quad 50x + 100(85 - x) = 5000 \Leftrightarrow x + 2(85 - x) = 100 \Leftrightarrow x = 70.$

So, required amount $=$ Rs. $(50 \times 70) =$ Rs. 3500.

Ex. 35. ***When an amount was distributed among 14 boys, each of them got Rs. 80 more than the amount received by each boy when the same amount is distributed equally among 18 boys. What was the amount ?*** (S.B.I.P.O. 1998)

Sol. Let the total amount be Rs. x. Then,

$$\frac{x}{14} - \frac{x}{18} = 80 \Leftrightarrow \frac{2x}{126} = 80 \Leftrightarrow \frac{x}{63} = 80 \Leftrightarrow x = 63 \times 80 = 5040.$$

Hence, total amount = Rs. 5040.

Ex. 36. ***Mr. Bhaskar is on tour and he has Rs. 360 for his expenses. If he exceeds his tour by 4 days, he must cut down his daily expenses by Rs. 3. For how many days is Mr. Bhaskar on tour ?***

Sol. Suppose Mr. Bhaskar is on tour for x days. Then,

$$\frac{360}{x} - \frac{360}{x+4} = 3 \Leftrightarrow \frac{1}{x} - \frac{1}{x+4} = \frac{1}{120} \Leftrightarrow x(x+4) = 4 \times 120 = 480$$

$$\Leftrightarrow x^2 + 4x - 480 = 0 \Leftrightarrow (x + 24)(x - 20) = 0 \Leftrightarrow x = 20.$$

Hence, Mr. Bhaskar is on tour for 20 days.

Ex. 37. ***Two pens and three pencils cost Rs. 86. Four pens and a pencil cost Rs. 112. Find the cost of a pen and that of a pencil.*** (Bank P.O. 2002)

Sol. Let the cost of a pen and a pencil be Rs. x and Rs. y respectively.

Then, $2x + 3y = 86$ ...(*i*) and $4x + y = 112$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 25$ and $y = 12$.

∴ Cost of a pen = Rs. 25 and cost of a pencil = Rs. 12.

Ex. 38. ***Arun and Sajal are friends. Each has some money. If Arun gives Rs. 30 to Sajal, then Sajal will have twice the money left with Arun. But, if Sajal gives Rs. 10 to Arun, then Arun will have thrice as much as is left with Sajal. How much money does each have ?***

Sol. Suppose Arun has Rs. x and Sajal has Rs. y. Then,

$2(x - 30) = y + 30 \Rightarrow 2x - y = 90$ ...(*i*)

and $x + 10 = 3(y - 10) \Rightarrow x - 3y = -40$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 62$ and $y = 34$.

∴ Arun has Rs. 62 and Sajal has Rs. 34.

Ex. 39. ***In a caravan, in addition to 50 hens there are 45 goats and 8 camels with some keepers. If the total number of feet be 224 more than the number of heads, find the number of keepers.***

Sol. Let the number of keepers be x. Then,

Total number of heads = $(50 + 45 + 8 + x) = (103 + x)$.

Total number of feet = $(45 + 8) \times 4 + (50 + x) \times 2 = (312 + 2x)$.

∴ $(312 + 2x) - (103 + x) = 224 \Leftrightarrow x = 15$.

Hence, number of keepers = 15.

EXERCISE 4

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. $100 + 50 \times 2 = ?$ (Bank P.O. 2003)

   (*a*) 75 (*b*) 150 (*c*) 200 (*d*) 300 (*e*) None of these

2. $(3080 + 6160) \div 28 = ?$ (B.S.R.B. 1998)

   (*a*) 320 (*b*) 440 (*c*) 3320 (*d*) 3350 (*e*) None of these

3. $5004 \div 139 - 6 = ?$ (R.B.I. 2003)

   (*a*) 24 (*b*) 30 (*c*) 36 (*d*) 42 (*e*) None of these

4. $7500 + (1250 \div 50) = ?$
(a) 175 (b) 300 (c) 6575 (d) 7525 (e) None of these
(S.S.C. 2000)

5. $(8 \div 88) \times 8888088 = ?$
(a) 808008 (b) 808080 (c) 808088 (d) 8008008
(S.S.C. 1998)

6. The value of $1001 \div 11$ of 13 is :
(a) 7 (b) 91 (c) 143 (d) 169

7. $1260 \div 15 \div 7 = ?$
(a) 12 (b) 58 (c) 122 (d) 588 (e) None of these

8. $(-5)(4)(2)\left(-\frac{1}{2}\right)\left(\frac{3}{4}\right) = ?$
(IGNOU, 2003)
(a) -30 (b) -15 (c) 15 (d) 30

9. $\frac{11}{4} = \frac{77}{?}$
(Hotel Management, 2003)
(a) 28 (b) $\frac{77}{28}$ (c) 44 (d) 308

10. A boy was asked to write the value of $(2)^5 \times (9)^2$. He wrote it as 2592. The difference between the obtained and the actual value is :
(a) zero (b) 2×9^2 (c) $2^2 \times 9^2$ (d) $2^3 \times 9^4$

11. $2 - [2 - \{2 - 2(2 + 2)\}] = ?$
(Hotel Management, 2001)
(a) -4 (b) 4 (c) 6 (d) None of these

12. The value of $25 - 5[2 + 3\{2 - 2(5 - 3) + 5\} - 10] \div 4$ is :
(S.S.C. 2000)
(a) 5 (b) 23.25 (c) 23.75 (d) 25

13. $3640 \div 14 \times 16 \div 340 = ?$
(B.S.R.B. 1998)
(a) 0.70 (b) 3525 (c) 4480 (d) 9600 (e) None of these

14. $100 \times 10 - 100 + 2000 \div 100 = ?$
(R.R.B. 1998)
(a) 29 (b) 780 (c) 920 (d) 979

15. What mathematical operation should come at the place of '?' in the equation : $2 \; ? \; 6 - 12 \div 4 + 2 = 11$.
(R.R.B. 2003)
(a) + (b) − (c) × (d) ÷

16. If $45 - [28 - \{37 - (15 - *)\}] = 58$, then * is equal to :
(a) -29 (b) -19 (c) 19 (d) 29

17. The value of $\frac{(6+6+6+6) \div 6}{4+4+4+4 \div 4}$ is equal to :
(a) 1 (b) $\frac{3}{2}$ (c) $\frac{4}{13}$ (d) $3\frac{6}{13}$

18. $\frac{4+4\times18-6-8}{123\times6-146\times5} = ?$
(L.I.C. 2003)
(a) 1 (b) 2 (c) 6.65 (d) 7.75

19. $\frac{180\times15-12\times20}{140\times8+2\times55} = ?$
(B.S.R.B. 1998)
(a) $\frac{1}{7}$ (b) $\frac{4}{5}$ (c) 2 (d) 4 (e) None of these

20. Which of the following will come in place of both the question marks in the following equation ?

$$\frac{128 \div 16 \times ? - 7 \times 2}{7^2 - 8 \times 6 + ?^2} = 1$$

(S.B.I.P.O. 2001)
(a) 3 (b) 14 (c) 16 (d) 17 (e) 18

21. Simplify : $18 - [5 - (6 + 2(7 - \overline{8 - 5})\}]$. (R.R.B. 2003)

(a) 13 (b) 15 (c) 27 (d) 32

22. The value of $1 \div [1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}]$ is : (S.S.C. 2003)

(a) $\frac{1}{2}$ (b) $\frac{5}{8}$ (c) 1 (d) 2

23. Evaluate : $\frac{8 - [5 - (-3 + 2)] \div 2}{|5 - 3| - |5 - 8| + 3}$. (S.S.C. 1999)

(a) 2 (b) 3 (c) 4 (d) 5

24. Which of the following pairs of fractions adds up to a number greater than 5 ? (Hotel Management, 2000)

(a) $\frac{5}{3}, \frac{3}{4}$ (b) $\frac{7}{3}, \frac{11}{5}$ (c) $\frac{11}{4}, \frac{8}{3}$ (d) $\frac{13}{5}, \frac{11}{6}$

25. $1 \div \frac{1}{2} \div \frac{1}{4} \div \frac{1}{7} \div \frac{1}{14} \div \frac{1}{28}$ is equal to : (S.S.C. 1999)

(a) 2 (b) 2.5 (c) 3 (d) 3.5

26. $1\frac{3}{4} \div 5\frac{1}{3} \div 3\frac{2}{5} = ?$ (Bank P.O. 2003)

(a) $9\frac{2}{5}$ (b) $9\frac{29}{60}$ (c) $10\frac{2}{5}$ (d) $10\frac{29}{60}$

27. $20\frac{1}{2} + 30\frac{1}{3} - 15\frac{1}{6} = ?$

(a) $34\frac{1}{6}$ (b) $35\frac{2}{3}$ (c) $35\frac{5}{6}$ (d) $45\frac{1}{3}$

28. If $[p]$ means the greatest integer less than or equal to p, then $\left[-\frac{1}{4}\right] + \left[4\frac{1}{4}\right] + [3]$ is equal to : (Section Officers', 2003)

(a) 4 (b) 5 (c) 6 (d) 7

29. $\frac{1}{\left(2\frac{1}{3}\right)} + \frac{1}{\left(1\frac{3}{4}\right)}$ is equal to : (R.R.B. 1998)

(a) $\frac{7}{14}$ (b) $\frac{12}{49}$ (c) $4\frac{1}{12}$ (d) None of these

30. $5\frac{5}{6} - 3\frac{8}{9} - ? = 1$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{17}{18}$ (d) 3

31. If $\frac{1}{3} + \frac{1}{2} + \frac{1}{x} = 4$, then $x = ?$ (M.B.A. 2002)

(a) $\frac{5}{18}$ (b) $\frac{6}{19}$ (c) $\frac{18}{5}$ (d) $\frac{24}{11}$

32. $\frac{-\frac{1}{2} - \frac{2}{3} + \frac{4}{5} - \frac{1}{3} + \frac{1}{5} + \frac{3}{4}}{\frac{1}{2} + \frac{2}{3} - \frac{4}{3} + \frac{1}{3} - \frac{1}{5} - \frac{4}{5}}$ is simplified to : (S.S.C. 2004)

(a) $-\frac{3}{10}$ (b) $\frac{-10}{3}$ (c) -2 (d) 1

33. $5-\left[\frac{3}{4}+\left\{2\frac{1}{2}-\left(0.5+\overline{\frac{1}{6}-\frac{1}{7}}\right)\right\}\right]$ is equal to :

(a) $1\frac{19}{84}$ (b) $2\frac{61}{84}$ (c) $2\frac{23}{84}$ (d) $2\frac{47}{84}$

34. When $\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\right)$ is divided by $\left(\frac{2}{5}-\frac{5}{9}+\frac{3}{5}-\frac{7}{18}\right)$, the result is : **(S.S.C. 2000)**

(a) $2\frac{1}{18}$ (b) $3\frac{1}{6}$ (c) $3\frac{3}{10}$ (d) $5\frac{1}{10}$

35. Which of the following can be used to compute $\left(34\times4\frac{1}{2}\right)$?

(a) $(30\times4)+\left(4\times4\frac{1}{2}\right)$ (b) $(34\times40)+\left(34\times\frac{1}{2}\right)$

(c) $\left(30\times4\frac{1}{2}\right)+(4\times4)$ (d) $\left(34\times\frac{1}{2}\right)+(30\times4)+(4\times4)$

36. $\frac{3}{5}$ of $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{21}{24}$ of 504 = ? **(Bank P.O. 2003)**

(a) 63 (b) 69 (c) 96 (d) 109 (e) None of these

37. $6\frac{5}{6}\times5\frac{1}{3}+17\frac{2}{3}\times4\frac{1}{2}=?$ **(Bank P.O. 2003)**

(a) $112\frac{1}{3}$ (b) $116\frac{2}{3}$ (c) 240 (d) 663 (e) None of these

38. $\frac{3}{8}$ of $168\times15\div5+?=549\div9+235$ **(S.B.I.P.O. 2000)**

(a) 107 (b) 174 (c) 189 (d) 296 (e) None of these

39. Find the value of * in the following : **(S.S.C. 2002)**

$$1\frac{2}{3}+\frac{2}{7}\times\frac{*}{7}=1\frac{1}{4}\times\frac{2}{3}+\frac{1}{6}$$

(a) 0.006 (b) $\frac{1}{6}$ (c) 0.6 (d) 6

40. $5\frac{2}{3}\div?\frac{5}{6}=2$ **(Hotel Management, 1998)**

(a) 2 (b) 3 (c) 4 (d) None of these

41. Supply the two missing figures in order indicated by x and y in the given equation, the fractions being in their lowest terms. **(IGNOU, 2003)**

$$5\frac{1}{x}\times y\frac{3}{4}=20$$

(a) 3, 1 (b) 3, 3 (c) 4, 1 (d) 5, 3

42. The difference of $1\frac{3}{16}$ and its reciprocal is equal to : **(M.A.T. 2002)**

(a) $1\frac{1}{8}$ (b) $\frac{4}{3}$ (c) $\frac{15}{16}$ (d) None of these

43. How many $\frac{1}{8}$s are there in $37\frac{1}{2}$?

(a) 300 (b) 400 (c) 500 (d) Cannot be determined

44. $\frac{3}{8}$ is what part of $\frac{1}{12}$?

(a) $\frac{3}{7}$ (b) $\frac{1}{12}$ (c) $\frac{4}{3}$ (d) None of these

45. The smallest fraction which should be subtracted from the sum of $1\frac{3}{4}$, $2\frac{1}{2}$, $5\frac{7}{12}$, $3\frac{1}{3}$ and $2\frac{1}{4}$ to make the result a whole number is :

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{1}{2}$ (d) 7

46. If x is a positive number, then which of the following fractions has the greatest value ?

(a) $\frac{x}{x}$ (b) $\frac{x}{x+1}$ (c) $\frac{x+1}{x}$ (d) $\frac{x+2}{x+3}$

47. By how much is three-fifth of 350 greater than four-seventh of 210 ?

(a) 95 (b) 110 (c) 120 (d) 210 (e) None of these

(S.B.I.P.O. 2003)

48. By how much does $\frac{6}{7/8}$ exceed $\frac{6/7}{8}$? **(Section Officers', 2003)**

(a) $6\frac{1}{8}$ (b) $6\frac{3}{4}$ (c) $7\frac{3}{4}$ (d) $7\frac{5}{6}$

49. If $\frac{4}{5}$ of an estate be worth Rs. 16,800, then the value of $\frac{3}{7}$ of the estate is :

(a) Rs. 9000 (b) Rs. 21,000 (c) Rs. 72,000 (d) Rs. 90,000

(S.S.C. 2002)

50. Two-fifth of one-fourth of three-seventh of a number is 15. What is half of that number ?

(a) 94 (b) 96 (c) 188 (d) 196 (e) None of these

(Bank P.O. 1999)

51. One-fifth of a number exceeds one-seventh of the same by 10. The number is :

(a) 125 (b) 150 (c) 175 (d) 200

52. If $x * y = x^2 + y^2 - xy$, then the value of $9 * 11$ is : **(S.S.C. 2003)**

(a) 93 (b) 103 (c) 113 (d) 121

53. If $a * b = \frac{ab}{a+b}$, find the value of $3 * (3 * -1)$. **(M.B.A. 2002)**

(a) -3 (b) -1.5 (c) -1 (d) $\frac{2}{3}$

54. If $a * b = 2a - 3b + ab$, then $3 * 5 + 5 * 3$ is equal to : **(S.S.C. 1999)**

(a) 22 (b) 24 (c) 26 (d) 28

55. If $x \oplus y = x^2 + 2y$, what is the value of p if $4 \oplus (3 \oplus p) = 50$? **(N.I.F.T. 1997)**

(a) 4 (b) 7 (c) 8 (d) 12.5

56. If $a * b * c$ means $\frac{a+b}{c}$ for all numbers except 0, then $(a * b * c) * a * b$ is equal to :

(a) 0 (b) 1 (c) $\frac{a+b+c}{ab}$ (d) $\frac{a+b+ac}{bc}$ (e) $\frac{ab+bc+ca}{a+b+c}$

57. 7 is added to a certain number; the sum is multiplied by 5; the product is divided by 9 and 3 is subtracted from the quotient. The remainder left is 12. The number is :

(a) 20 (b) 30 (c) 40 (d) 60

(S.S.C. 2000)

58. The value of $\left(\frac{5}{7} \text{ of } 1\frac{6}{13}\right) \div \left(2\frac{5}{7} \div 3\frac{1}{4}\right)$ is : **(R.R.B. 2001)**

(a) $\frac{20}{169}$ (b) 1 (c) $\frac{5}{4}$ (d) $1\frac{119}{180}$

59. $2\frac{3}{4} \div 2\frac{2}{3} \div 1\frac{1}{12} = ?$ **(Hotel Management, 2001)**

(a) $\frac{39}{48}$ (b) $1\frac{1}{4}$ (c) $\frac{169}{144}$ (d) None of these

60. $4\frac{1}{2} \times 4\frac{1}{3} - 8\frac{1}{3} \div 5\frac{2}{3} = ?$ **(Bank P.O. 1999)**

(a) $\frac{7}{17}$ (b) $1\frac{33}{34}$ (c) 8 (d) $18\frac{1}{34}$

61. $\frac{4335}{4(?)24} \div 1\frac{7}{8} = \frac{289}{528}$ **(Hotel Management, 2000)**

(a) 1 (b) 2 (c) 8 (d) None of these

62. $5\frac{1}{3} - 3\frac{2}{3} \div 1\frac{1}{3} \div ? + 3\frac{1}{5} \div 1\frac{1}{5} = 7$

(a) $1\frac{1}{2}$ (b) $2\frac{1}{3}$ (c) $3\frac{1}{4}$ (d) None of these

63. $9 - 1\frac{2}{9} \text{ of } 3\frac{3}{11} \div 5\frac{1}{7} \text{ of } \frac{7}{9} = ?$ **(S.S.C. 2002)**

(a) $\frac{5}{4}$ (b) 8 (c) $8\frac{32}{81}$ (d) 9

64. $\frac{5}{6} \div \frac{6}{7} \times ? - \frac{8}{9} \div 1\frac{3}{5} \div \frac{3}{4} \times 3\frac{1}{3} = 2\frac{7}{9}$

(a) $\frac{7}{6}$ (b) $\frac{6}{7}$ (c) 1 (d) None of these

65. $\frac{3}{4} \div 2\frac{1}{4} \text{ of } \frac{2}{3} - \frac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{2} + \frac{1}{3}} \times 3\frac{1}{3} \div \frac{5}{6} = ?$

(a) $\frac{7}{18}$ (b) $\frac{49}{54}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$

66. A student was asked to solve the fraction $\frac{\frac{7}{3} \div 1\frac{1}{2} \text{ of } \frac{5}{3}}{2 \div 1\frac{2}{3}}$ and his answer was $\frac{1}{4}$. By how much was his answer wrong ? **(N.I.F.T. 1997)**

(a) 1 (b) $\frac{1}{55}$ (c) $\frac{1}{220}$ (d) None of these

67. Simplify : $\frac{\frac{1}{3} + \frac{3}{4}\left(\frac{2}{5} - \frac{1}{3}\right)}{1\frac{2}{3} \text{ of } \frac{3}{4} - \frac{1}{4} \text{ of } \frac{4}{5}}$. **(C.B.I. 1998)**

(a) $\frac{1}{63}$ (b) $\frac{23}{40}$ (c) $\frac{23}{55}$ (d) $\frac{23}{63}$

68. The simplified value of $\dfrac{\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}\text{ of }\frac{1}{3}}-\frac{1}{9}$ is : (S.S.C. 2003)

(a) 0 (b) $\frac{1}{9}$ (c) $\frac{1}{3}$ (d) 1

69. The value of $\dfrac{\frac{1}{2}+\frac{1}{2}\text{ of }\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}\text{ of }\frac{1}{2}}$ is :

(a) 1 (b) $1\frac{1}{3}$ (c) $2\frac{2}{3}$ (d) 3

70. $\dfrac{3\frac{1}{4}-\frac{4}{5}\text{ of }\frac{5}{6}}{4\frac{1}{3}\div\frac{1}{5}-\left(\frac{3}{10}+21\frac{1}{5}\right)}$ is equal to :

(a) $\frac{1}{6}$ (b) $2\frac{7}{12}$ (c) $15\frac{1}{2}$ (d) $21\frac{1}{2}$

71. $\dfrac{7\frac{1}{2}-5\frac{3}{4}}{3\frac{1}{2}+?}\div\dfrac{\frac{1}{2}+1\frac{1}{4}}{1\frac{1}{5}\div3\frac{1}{2}}=0.6$

(a) $4\frac{1}{3}$ (b) $4\frac{1}{2}$ (c) $4\frac{2}{3}$ (d) None of these

72. On simplification, $3034-(1002\div20.04)$ is equal to : (S.S.C. 2000)

(a) 2543 (b) 2984 (c) 2993 (d) 3029

73. $52.416\div18.72\div6.28=?$

(a) 2.09664 (b) 8.36 (c) 9.08 (d) 9.80

74. $8\frac{2}{7}$ of $1568+265.75=?+2455.60$: (S.B.I.P.O. 1998)

(a) 10354.15 (b) 10578.15 (c) 10802.15 (d) 11250.15

75. $5.8\times2.5\div0.6\times6.75+139.25=?$ (Bank P.O. 1998)

(a) 157.30 (b) 157.80 (c) 158.40 (d) 160.30

76. $8\frac{1}{4}-4\frac{1}{5}+2.8\div\frac{4}{?}-2.32=5.33$

(a) .05 (b) .5 (c) 5 (d) None of these

77. The value of $0.008\times0.01\times0.0072\div(0.12\times0.0004)$ is : (S.S.C. 2002)

(a) 0.012 (b) 0.12 (c) 1.02 (d) 1.2

78. $2.375\times5.22\div0.87-1.425\times0.02=?$

(a) 0.142215 (b) 1.42215 (c) 14.2215 (d) None of these

79. $0.2\div0.2-0.2\div0.2\times(0.2\times0.2)$, on simplification, gives :

(a) 0.04 (b) 0.2 (c) 0.36 (d) 1

80. $11.6\div9.28\div0.464-0.2828\div0.07=?$ (R.R.B. 1998)

(a) 9.2 (b) 9.56 (c) 27.2 (d) 27.56

81. $4.59\times1.8\div3.6\div5.4\text{ of }\frac{1}{9}-\frac{1}{5}=?$

(a) 2.695 (b) 2.705 (c) 3.105 (d) None of these

82. $\frac{64\frac{2}{5} - 34.7125}{6.25 \text{ of } ?} = 1$:

(a) $2\frac{2}{3}$ (b) 2.75 (c) $4\frac{3}{4}$ (d) None of these

83. $2.002 + 7.9\ \{2.8 - 6.3\ (3.6 - 1.5) + 15.6\} = ?$ (S.S.C. 1997)

(a) 2.002 (b) 4.2845 (c) 40.843 (d) 42.845

84. $24 - [2.4 - \{.24 \times 2 - (.024 - ?)\}] = 22.0584$

(a) 0.0024 (b) 0.024 (c) 0.24 (d) None of these

85. $3 - [1.6 - \{3.2 - (3.2 + 2.25 \div x)\}] = 0.65$. The value of x is : (R.R.B. 2002)

(a) 0.3 (b) 0.7 (c) 3 (d) 7

86. $587.4 + 58.74 \times 2 - 5.874 \div 2\frac{?}{4} = 702744$

(a) 1 (b) 2 (c) 3 (d) None of these

87. $54.27 - [12.84 - \{(?).87 - (3.41 \times 2 - 1.85)\}] = 38.33$

(a) 2 (b) 3 (c) 4 (d) None of these

88. $6\frac{2}{3}$ of $7.26 \div 0.45$ of $? = 8\frac{32}{117}$

(a) $\frac{1}{13}$ (b) 13 (c) $13\frac{1}{9}$ (d) None of these

89. What is the value of $\frac{(P+Q)}{(P-Q)}$ if $\frac{P}{Q} = 7$? (Hotel Management, 2000)

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{7}{8}$

90. If $\frac{x}{y} = \frac{4}{5}$, then the value of $\left(\frac{4}{7} + \frac{2y-x}{2y+x}\right)$ is : (R.R.B. 2003)

(a) $\frac{3}{7}$ (b) 1 (c) $1\frac{1}{7}$ (d) 2

91. If $\frac{a}{b} = \frac{4}{3}$, then the value of $\frac{6a+4b}{6a-5b}$ is :

(a) -1 (b) 3 (c) 4 (d) 5

92. If $\frac{x}{2y} = \frac{6}{7}$, the value of $\frac{x-y}{x+y} + \frac{14}{19}$ is :

(a) $\frac{13}{19}$ (b) $\frac{15}{19}$ (c) 1 (d) $1\frac{1}{19}$

93. If $\frac{a}{b} = \frac{4}{5}$ and $\frac{b}{c} = \frac{15}{16}$, then $\frac{c^2 - a^2}{c^2 + a^2}$ is :

(a) $\frac{1}{7}$ (b) $\frac{7}{25}$ (c) $\frac{3}{4}$ (d) None of these

94. If $(a - b)$ is 6 more than $(c + d)$ and $(a + b)$ is 3 less than $(c - d)$, then $(a - c)$ is :

(a) 0.5 (b) 1 (c) 1.5 (d) None of these

95. If $x = \frac{a}{a-1}$ and $y = \frac{1}{a-1}$, then : (Bank P.O. 2003)

(a) x is equal to y
(b) x is equal to y only if $a < 1$
(c) x is greater than y
(d) x is greater than y only if $a < 1$
(e) y is greater than x only if $a < 1$

96. If $0 < a < 1$, then the value of $a + \frac{1}{a}$ is : (S.S.C. 1997)

(a) less than 2 (b) greater than 2 (c) less than 4 (d) greater than 4

97. If $\frac{a}{x} + \frac{y}{b} = 1$ and $\frac{b}{y} + \frac{z}{c} = 1$, then $\frac{x}{a} + \frac{c}{z}$ will be equal to : (C.D.S. 2003)

(a) 0 (b) $\frac{b}{y}$ (c) 1 (d) $\frac{y}{b}$

98. If a, b, c are integers; $a^2 + b^2 = 45$ and $b^2 + c^2 = 40$, then the values of a, b and c respectively are :

(a) 2, 6, 3 (b) 3, 2, 6 (c) 5, 4, 3 (d) None of these.

99. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then the value of $\frac{a+b+c}{c}$ is : (C.B.I. 2003)

(a) $\frac{1}{\sqrt{7}}$ (b) $\sqrt{2}$ (c) 2 (d) 7

100. If $3x + 7 = x^2 + P = 7x + 5$, what is the value of P ? (S.B.I.P.O. 2000)

(a) $\frac{1}{2}$ (b) $8\frac{1}{4}$ (c) $8\frac{1}{2}$ (d) Cannot be determined

101. If $\frac{2a+b}{a+4b} = 3$, then find the value of $\frac{a+b}{a+2b}$. (S.S.C. 2002)

(a) $\frac{2}{7}$ (b) $\frac{5}{9}$ (c) $\frac{10}{7}$ (d) $\frac{10}{9}$

102. If $(2a + 3b)(2c - 3d) = (2a - 3b)(2c + 3d)$, then :

(a) $\frac{a}{b} = \frac{c}{d}$ (b) $\frac{a}{d} = \frac{c}{b}$ (c) $\frac{a}{b} = \frac{d}{c}$ (d) $\frac{b}{a} = \frac{c}{d}$

103. If $(a + b + 2c + 3d)(a - b - 2c + 3d) = (a - b + 2c - 3d)(a + b - 2c - 3d)$, then $2bc$ is equal to : (M.A.T. 2003)

(a) $\frac{3}{2}$ (b) $\frac{3a}{2d}$ (c) $3ad$ (d) a^2d^2

104. The value of $\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2-\frac{1}{2}}}}$ is : (S.S.C. 1999)

(a) $\frac{3}{8}$ (b) $\frac{19}{8}$ (c) $\frac{8}{3}$ (d) $\frac{8}{19}$

105. If $2 = x + \cfrac{1}{1+\cfrac{1}{3+\frac{1}{4}}}$, then the value of x is : (S.S.C. 2003)

(a) $\frac{12}{17}$ (b) $\frac{13}{17}$ (c) $\frac{18}{17}$ (d) $\frac{21}{17}$

106. If $\dfrac{2+\dfrac{1}{3\frac{4}{5}}}{2+\dfrac{1}{3+\dfrac{1}{1+\frac{1}{4}}}} = x$, then the value of x is : (C.B.I. 1998)

(a) $\frac{1}{7}$ (b) $\frac{3}{7}$ (c) 1 (d) $\frac{8}{7}$

107. $8-8\times\dfrac{2\frac{1}{5}-1\frac{2}{7}}{2-\dfrac{1}{6-\frac{1}{6}}}$ is equal to : (S.S.C. 2002)

(a) 2 (b) 4 (c) 6 (d) 8

108. $\dfrac{2}{2+\dfrac{2}{3+\dfrac{2}{3+\frac{2}{3}}}\times 0.39}$ is simplified to : (S.S.C. 2004)

(a) $\frac{1}{3}$ (b) 2 (c) 6 (d) None of these

109. Simplify : $\dfrac{1}{1+\dfrac{\frac{2}{3}}{1+\frac{2}{3}+\frac{\frac{8}{9}}{1-\frac{2}{3}}}}$. (S.S.C. 2003)

(a) $\frac{11}{13}$ (b) $\frac{13}{15}$ (c) $\frac{13}{11}$ (d) $\frac{15}{13}$

110. If $\frac{37}{13} = 2+\dfrac{1}{x+\dfrac{1}{y+\frac{1}{z}}}$, where x, y, z are natural numbers, then x, y, z are :

(a) 1, 2, 5 (b) 1, 5, 2 (c) 5, 2, 11 (d) 11, 2, 5

(Assistant Grade, 1998)

111. If $x = 1 - q$ and $y = 2q + 1$, then for what value of q, x is equal to y ?

(a) -1 (b) 0 (c) $\frac{1}{2}$ (d) 2

112. Find x if $\frac{x}{5}-\frac{x}{6} = 4$. (B.S.F. 2001)

(a) -120 (b) -100 (c) 100 (d) 120

113. If $4x + 5y = 83$ and $\frac{3x}{2y} = \frac{21}{22}$, then $y - x = ?$ (Bank P.O. 2002)

(a) 3 (b) 4 (c) 7 (d) 11

114. Which of the following values of x and y satisfy the following equations I and II ? (B.S.R.B. 2003)

I. $3x + y = 19$ II. $x - y = 9$

(a) $-7, -2$ (b) $-7, 2$ (c) $7, -2$ (d) 7, 2

115. If $a + b = 5$ and $3a + 2b = 20$, then $(3a + b)$ will be : **(M.B.A. 1998)**
(a) 10 (b) 15 (c) 20 (d) 25

116. If $2p + 3q = 18$ and $2p - q = 2$, then $2p + q = ?$
(a) 6 (b) 7 (c) 10 (d) 20

117. If $2x + y = 5$ and $3x - 4y = 2$, then the value of $2xy$ is :
(a) 4 (b) 6 (c) 8 (d) 10

118. If $3x - 5y = 5$ and $\frac{x}{x+y} = \frac{5}{7}$, then what is the value of $x - y$? **(Bank P.O. 2002)**
(a) 3 (b) 4 (c) 6 (d) 9 (e) None of these

119. If $4x + 3y = 18xy$ and $2x - 5y + 4xy = 0$, then the values of x and y will be respectively :
(a) $-\frac{1}{2}$ and $-\frac{1}{3}$ (b) -1 and -3 (c) $\frac{1}{2}$ and $\frac{1}{3}$ (d) $\frac{1}{4}$ and $\frac{1}{3}$

120. If $2x + y = 17$; $y + 2z = 15$ and $x + y = 9$, then what is the value of $4x + 3y + z$?
(a) 41 (b) 43 (c) 45 (d) 55 (e) None of these

(S.B.I.P.O. 1999)

121. If $3x - 4y + z = 7$; $2x - z + 3y = 19$; $x + 2y + 2z = 24$, then what is the value of z ?
(a) 4 (b) 5 (c) 6 (d) 8

122. If $2x + y = 15$, $2y + z = 25$ and $2z + x = 26$, what is the value of z ?
(a) 4 (b) 7 (c) 9 (d) 11

123. If $2x + 3y = 31$, $y - z = 4$ and $x + 2z = 11$, then what is the value of $x + y + z$?
(a) 12 (b) 13 (c) 15 (d) 16

(Bank P.O. 2003)

124. $\frac{3}{4}\left(1+\frac{1}{3}\right)\left(1+\frac{2}{3}\right)\left(1-\frac{2}{5}\right)\left(1+\frac{6}{7}\right)\left(1-\frac{12}{13}\right) = ?$ **(Hotel Management, 2001)**
(a) $\frac{1}{5}$ (b) $\frac{1}{6}$ (c) $\frac{1}{7}$ (d) None of these

125. When simplified, the product $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\ldots\left(1-\frac{1}{n}\right)$ gives **(S.S.C. 2004)**
(a) $\frac{1}{n}$ (b) $\frac{2}{n}$ (c) $\frac{2(n-1)}{n}$ (d) $\frac{2}{n(n+1)}$

126. The value of $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\ldots\left(1+\frac{1}{120}\right)$ is : **(S.S.C. 2003)**
(a) 30 (b) 40.5 (c) 60.5 (d) 121

127. When simplified, the product $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\ldots\ldots\left(2-\frac{999}{1001}\right)$ is equal to :
(a) $\frac{991}{1001}$ (b) $\frac{1001}{13}$ (c) $\frac{1003}{13}$ (d) None of these

128. Find the sum : $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}+\frac{1}{110}+\frac{1}{132}$.
(a) $\frac{7}{8}$ (b) $\frac{11}{12}$ (c) $\frac{15}{16}$ (d) $\frac{17}{18}$

129. The sum of the first 35 terms of the series $\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{2}-\frac{1}{3}+\frac{1}{4}+\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\ldots\ldots$ is :
(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{4}$ (d) None of these

130. The value of $999\frac{995}{999} \times 999$ is : **(S.S.C. 2003)**

(a) 990809 (b) 998996 (c) 998999 (d) 999824

131. $\left(999\frac{1}{7} + 999\frac{2}{7} + 999\frac{3}{7} + 999\frac{4}{7} + 999\frac{5}{7} + 999\frac{6}{7}\right)$ is simplified to : **(S.S.C. 2004)**

(a) 2997 (b) 5979 (c) 5994 (d) 5997

132. The value of $1 + \frac{1}{4 \times 3} + \frac{1}{4 \times 3^2} + \frac{1}{4 \times 3^3}$ is :

(a) $\frac{121}{108}$ (b) $\frac{3}{2}$ (c) $\frac{31}{2}$ (d) None of these

133. $\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{4 \cdot 5 \cdot 6}$ is equal to :

(a) $\frac{7}{30}$ (b) $\frac{11}{30}$ (c) $\frac{13}{30}$ (d) $\frac{17}{30}$

134. The value of $\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \frac{9}{4^2 \cdot 5^2} + \frac{11}{5^2 \cdot 6^2} + \frac{13}{6^2 \cdot 7^2} + \frac{15}{7^2 \cdot 8^2} + \frac{17}{8^2 \cdot 9^2} + \frac{19}{9^2 \cdot 10^2}$ is:

(a) $\frac{1}{100}$ (b) $\frac{99}{100}$ (c) 1 (d) $\frac{101}{100}$

(S.S.C. 2004)

135. How may pieces of 85 cm length can be cut from a rod 42.5 metres long ?

(a) 30 (b) 40 (c) 60 (d) None of these

136. Income of a company doubles after every one year. If the initial income was Rs. 4 lakhs, what would be the income after 5 years ? **(Bank P.O. 2003)**

(a) Rs. 1.24 crores (b) Rs. 1.28 crores (c) Rs. 2.52 crores
(d) Rs. 2.56 crores (e) None of these

137. On sports day, if 30 children were made to stand in a column, then 16 columns could be formed. If 24 children were made to stand in a column, then how many columns could be formed ? **(Hotel Management, 2002)**

(a) 20 (b) 22 (c) 29 (d) 45

138. The number of students in each section of a school is 24. After admitting new students, three new sections were started. Now, the total number of sections is 16 and there are 21 students in each section. The number of new students admitted is :

(a) 14 (b) 24 (c) 48 (d) 114

139. A class starts at 10 a.m. and lasts till 1.27 p.m. Four periods are held during this interval. After every period, 5 minutes are given free to the students. The exact duration of each period is :

(a) 42 minutes (b) 48 minutes (c) 51 minutes (d) 53 minutes

140. A light was seen at intervals of 13 seconds. It was seen for the first time at 1 hr. 54 min. 50 secs. a.m. and the last time at 3 hrs. 17 min. 49 secs. a.m. How many times was the light seen ? **(A.A.O. Exam, 2003)**

(a) 360 (b) 375 (c) 378 (d) 384

141. A man earns Rs. 20 on the first day and spends Rs. 15 on the next day. He again earns Rs. 20 on the third day and spends Rs. 15 on the fourth day. If he continues to save like this, how soon will he have Rs. 60 in hand ? **(IGNOU, 2003)**

(a) On 17th day (b) On 27th day (c) On 30th day (d) On 40th day

142. It costs Rs. x each to make the first thousand copies of a compact disc and Rs. y to make each subsequent copy. If z is greater than 1000, how much will it cost to make z copies of the compact disc ? **(R.R.B. 2001)**

(*a*) $zx - xy$ (*b*) $1000x + yz$

(*c*) $1000 (x - y) + yz$ (*d*) $1000 (z - y) + xz$

143. Along a yard 225 metres long, 26 trees are planted at equal distances, one tree being at each end of the yard. What is the distance between two consecutive trees ?

(*a*) 8 metres (*b*) 9 metres (*c*) 10 metres (*d*) 15 metres

(R.R.B. 2002)

144. A boy was asked to multiply a number by 25. He instead multiplied the number by 52 and got the answer 324 more than the correct answer. The number to be multiplied was :

(*a*) 12 (*b*) 15 (*c*) 25 (*d*) 32

145. A boy multiplied 423 by a number and obtained 65589 as his answer. If both the fives in the answer are wrong and all other figures are correct, the correct answer is :

(*a*) 60489 (*b*) 61189 (*c*) 62189 (*d*) 62389

146. The total monthly salary of 4 men and 2 women is Rs. 46,000. If a woman earns Rs. 500 more than a man, what is the monthly salary of a woman ?

(*a*) Rs. 6500 (*b*) Rs. 7500 (*c*) Rs. 8000 (*d*) Rs. 9000

(Bank P.O. 1999)

147. David got two and a half times as many marks in English as in History. If his total marks in the two subjects are 140, the marks obtained by him in English are :

(*a*) 40 (*b*) 75 (*c*) 90 (*d*) 100

(Assistant Grade, 1998)

148. A pineapple costs Rs. 7 each. A watermelon costs Rs. 5 each. X spends Rs. 38 on these fruits. The number of pineapples purchased is : **(M.B.A. 1998)**

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) Data inadequate

149. The number of girls in a class is 5 times the number of boys. Which of the following cannot be the total number of children in the class ? **(R.R.B. 2002)**

(*a*) 24 (*b*) 30 (*c*) 35 (*d*) 42 (*e*) 54

150. Water boils at 212°F or 100°C and melts at 32°F or 0°C. If the temperature of a particular day is 35°C, it is equivalent to : **(R.R.B. 2000)**

(*a*) 85°F (*b*) 90°F (*c*) 95°F (*d*) 99°F

151. A sum of Rs. 750 is distributed among A, B, C and D in such a manner that A gets as much as B and C together, B gets Rs. 125 more than C and D gets as much as C. What is A's share ?

(*a*) Rs. 100 (*b*) Rs. 225 (*c*) Rs. 275 (*d*) Rs. 325

152. A bonus of Rs. 1000 is to be divided among three people so that Rohit receives twice as much as Sachin, who receives one-fifth as much as Gagan. How much money should Gagan receive ?

(*a*) Rs. 100 (*b*) Rs. 250 (*c*) Rs. 375 (*d*) Rs. 625

153. The total number of digits used in numbering the pages of a book having 366 pages, is : **(S.C.R.A. 1998)**

(*a*) 732 (*b*) 990 (*c*) 1098 (*d*) 1305

154. A printer numbers the pages of a book starting with 1 and uses 3189 digits in all. How many pages does the book have ? **(M.A.T. 2002)**

(*a*) 1000 (*b*) 1074 (*c*) 1075 (*d*) 1080

155. In a garden, there are 10 rows and 12 columns of mango trees. The distance between the two trees is 2 metres and a distance of one metre is left from all sides of the boundary of the garden. The length of the garden is :

(*a*) 20 m (*b*) 22 m (*c*) 24 m (*d*) 26 m

156. What fraction of an hour is a second ?

(a) $\frac{1}{24}$ (b) $\frac{1}{60}$ (c) $\frac{1}{120}$ (d) $\frac{1}{3600}$

157. When a ball bounces, it rises to $\frac{3}{4}$ of the height from which it fell. If the ball is dropped from a height of 32 m, how high will it rise at the third bounce ? **(S.S.C. 2000)**

(a) 13 m (b) $13\frac{1}{2}$ m (c) $14\frac{1}{2}$ m (d) None of these

158. Sanket earns twice as much in the month of March as in each of the other months of the year. What part of his entire annual earnings was earned in March ?

(a) $\frac{1}{7}$ (b) $\frac{1}{6}$ (c) $\frac{2}{11}$ (d) $\frac{2}{13}$

159. If one-third of a tank holds 80 litres of water, then the quantity of water that half of the tank holds is : **(S.S.C. 1999)**

(a) $\frac{80}{3}$ litres (b) 100 litres (c) 120 litres (d) 240 litres

160. A person travels 3.5 km from place A to place B. Out of this distance, he travels $1\frac{2}{3}$ km on bicycle, $1\frac{1}{6}$ km on scooter and the rest on foot. What portion of the whole distance does he cover on foot ? **(S.S.C. 2003)**

(a) $\frac{3}{19}$ (b) $\frac{4}{11}$ (c) $\frac{1}{21}$ (d) $\frac{5}{6}$

161. What fraction of $\frac{4}{7}$ must be added to itself to make the sum $1\frac{1}{14}$? **(S.S.C. 2002)**

(a) $\frac{1}{2}$ (b) $\frac{4}{7}$ (c) $\frac{7}{8}$ (d) $\frac{15}{14}$

162. Express $\frac{2}{3}$ of $\frac{1}{4}$ of Rs. 25.20 as a fraction of $1\frac{1}{2}$ of Rs. 36.

(a) $\frac{5}{8}$ (b) $\frac{5}{42}$ (c) $\frac{7}{90}$ (d) $\frac{11}{90}$

163. A 70 cm long wire is to be cut into two pieces such that one piece will be $\frac{2}{5}$ as long as the other. How many centimetres will the shorter piece be ?

(a) 10 (b) 14 (c) 20 (d) 28

164. A certain amount is distributed among A, B and C. A gets $\frac{3}{16}$ and B gets $\frac{1}{4}$ of the whole amount. If C gets Rs. 81, then B gets :

(a) Rs. 30 (b) Rs. 32 (c) Rs. 36 (d) Rs. 40

165. $\frac{1}{10}$ of a pole is coloured red, $\frac{1}{20}$ white, $\frac{1}{30}$ blue, $\frac{1}{40}$ black, $\frac{1}{50}$ violet, $\frac{1}{60}$ yellow and the rest is green. If the length of the green portion of the pole is 12.08 metres, then the length of the pole is : **(S.S.C. 2004)**

(a) 16 m (b) 18 m (c) 20 m (d) 30 m

166. In an examination, a student was asked to find $\frac{3}{14}$ of a certain number. By mistake, he found $\frac{3}{4}$ of that number. His answer was 150 more than the correct answer. The number is : **(R.R.B. 2003)**

(a) 180 (b) 240 (c) 280 (d) 290

167. A student was asked to find the value of $\frac{3}{8}$ of a sum of money. The student made a mistake by dividing the sum by $\frac{3}{8}$ and thus got an answer which exceeded the correct answer by Rs. 55. The correct answer was :
(a) Rs. 9 (b) Rs. 18 (c) Rs. 24 (d) Rs. 64

168. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\frac{26}{27}$. The original fraction is :
(a) $\frac{8}{27}$ (b) $1\frac{1}{3}$ (c) $2\frac{2}{3}$ (d) None of these

169. The marks scored in an examination are converted from 50 to 10 for the purpose of internal assessment. The highest marks were 47 and the lowest were 14. The difference between the maximum and the minimum internal assessment scores is :
(a) 3.3 (b) 4.8 (c) 6.6 (d) 7.4
(S.S.C. 2000)

170. One-third of Rahul's savings in National Savings Certificate is equal to one-half of his savings in Public Provident Fund. If he has Rs. 1,50,000 as total savings, how much has he saved in Public Provident Fund ? **(Bank P.O. 2002)**
(a) Rs. 30,000 (b) Rs. 50,000 (c) Rs. 60,000 (d) Rs. 90,000

171. In a family, the father took $\frac{1}{4}$ of the cake and he had 3 times as much as each of the other members had. The total number of family members is :
(a) 3 (b) 7 (c) 10 (d) 12

172. A waiter's salary consists of his salary and tips. During one week his tips were $\frac{5}{4}$ of his salary. What fraction of his income came from tips ?
(a) $\frac{4}{9}$ (b) $\frac{5}{4}$ (c) $\frac{5}{8}$ (d) $\frac{5}{9}$

173. A sum of Rs. 1360 has been divided among A, B and C such that A gets $\frac{2}{3}$ of what B gets and B gets $\frac{1}{4}$ of what C gets. B's share is : **(M.A.T. 2002)**
(a) Rs. 120 (b) Rs. 160 (c) Rs. 240 (d) Rs. 300

174. Three friends had dinner at a restaurant. When the bill was received, Amita paid $\frac{2}{3}$ as much as Veena paid and Veena paid $\frac{1}{2}$ as much as Tanya paid. What fraction of the bill did Veena pay ?
(a) $\frac{1}{3}$ (b) $\frac{3}{11}$ (c) $\frac{12}{31}$ (d) $\frac{5}{8}$

175. $\frac{1}{4}$ of a tank holds 135 litres of water. What part of the tank is full if it contains 180 litres of water ? **(S.S.C. 1999)**
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{2}{5}$

176. A tank is $\frac{2}{5}$ full. If 16 litres of water is added to the tank, it becomes $\frac{6}{7}$ full. The capacity of the tank is :
(a) 28 litres (b) 32 litres (c) 35 litres (d) 42 litres

177. The fluid contained in a bucket can fill four large bottles or seven small bottles. A full large bottle is used to fill an empty small bottle. What fraction of the fluid is left over in the large bottle when the small one is full ? (D.M.R.C. 2003)

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

178. To fill a tank, 25 buckets of water is required. How many buckets of water will be required to fill the same tank if the capacity of the bucket is reduced to two-fifth of its present ? (R.B.I. 2003)

(a) 10 (b) 35 (c) $62\frac{1}{2}$

(d) Cannot be determined (e) None of these

179. Peter gave one-fourth of the amount he had to Michael. Michael in turn gave half of what he received from Peter to Sam. If the difference between the remaining amount with Peter and the amount received by Sam is Rs. 500, how much money did Michael receive from Peter ? (S.B.I.P.O. 1999)

(a) Rs. 100 (b) Rs. 200 (c) Rs. 400

(d) Data inadequate (e) None of these

180. Four children A, B, C and D divide a bag of sweets. A takes $\frac{1}{3}$ of them, B $\frac{2}{5}$ th of the remainder and the rest is equally shared between C and D. What fraction of the sweets did C or D get ?

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{17}$

181. A boy read $\frac{3}{8}$ th of a book on one day and $\frac{4}{5}$ th of the remainder on another day. If there were 30 pages unread, how many pages did the book contain ? (I.M.T. 2002)

(a) 240 (b) 300 (c) 600 (d) None of these

182. A man has divided his total money in his will in such a way that half of it goes to his wife, $\frac{2}{3}$ rd of the remaining among his three sons equally and the rest among his four daughters equally. If each daughter gets Rs. 20,000, how much money will each son get ? (S.B.I.P.O. 2000)

(a) Rs. 48,233.33 (b) Rs. 50,333.33 (c) Rs. 53,333.33

(d) Data inadequate (e) None of these

183. An institute organised a fete and $\frac{1}{5}$ of the girls and $\frac{1}{8}$ of the boys participated in the same. What fraction of the total number of students took part in the fete ?

(a) $\frac{2}{13}$ (b) $\frac{13}{40}$ (c) Data inadequate (d) None of these

(N.I.F.T. 2000)

184. At an International Dinner, $\frac{1}{5}$ of the people attending were French men. If the number of French women at the dinner was $\frac{2}{3}$ greater than the number of French men, and there were no other French people at the dinner, then what fraction of the people at the dinner were not French ? (M.B.A. 2003)

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{2}{3}$ (d) $\frac{7}{15}$

185. In a class, $\frac{3}{5}$ of the students are girls and rest are boys. If $\frac{2}{9}$ of the girls and $\frac{1}{4}$ of the boys are absent, what part of the total number of students is present ?

(*a*) $\frac{17}{25}$ (*b*) $\frac{18}{49}$ (*c*) $\frac{23}{30}$ (*d*) $\frac{23}{36}$

186. One-third of the boys and one-half of the girls of a college participated in a social work project. If the number of participating students is 300 out of which 100 are boys, what is the total number of students in the college ? **(Bank P.O. 2000)**

(*a*) 500 (*b*) 600 (*c*) 700 (*d*) 800

187. To win an election, a candidate needs $\frac{3}{4}$ of the votes cast. If after $\frac{2}{3}$ of the votes have been counted, a candidate has $\frac{5}{6}$ of what he needs, then what part of the remaining votes does he still need ?

(*a*) $\frac{1}{8}$ (*b*) $\frac{3}{8}$ (*c*) $\frac{1}{10}$ (*d*) $\frac{1}{4}$

188. In an office, $\frac{3}{4}$ of the staff can neither type nor take shorthand. However, $\frac{1}{5}$th can type and $\frac{1}{3}$rd can take shorthand. What part of the whole staff can do both ?

(*a*) $\frac{1}{5}$ (*b*) $\frac{3}{40}$ (*c*) $\frac{13}{40}$ (*d*) $\frac{17}{60}$

189. The charges of hired car are Rs. 4 per km for the first 60 km, Rs. 5 per km for the next 60 km and Rs. 8 for every 5 km for further journey. If the balance amount left over with Rohit is one-fourth of what he paid towards the charges of the hired car for travelling 320 km, how much money did he have initially with him ?

(*a*) Rs. 1075 (*b*) Rs. 1255 (*c*) Rs. 1540 (*d*) None of these

190. A fires 5 shots to B's 3 but A kills only once in 3 shots while B kills once in 2 shots. When B has missed 27 times, A has killed : **(C.B.I. 1997)**

(*a*) 30 birds (*b*) 60 birds (*c*) 72 birds (*d*) 90 birds

191. If every 2 out of 3 readymade shirts need alterations in the collar, every 3 out of 4 need alterations in the sleeves, and every 4 out of 5 need it in the body, how many alterations will be required for 60 shirts ?

(*a*) 24 (*b*) 123 (*c*) 133 (*d*) 143

192. The sum of three fractions is $2\frac{11}{24}$. When the largest fraction is divided by the smallest, the fraction thus obtained is $\frac{7}{6}$ which is $\frac{1}{3}$ more than the middle one. The fractions are :

(*a*) $\frac{3}{5}, \frac{4}{7}, \frac{2}{3}$ (*b*) $\frac{7}{8}, \frac{5}{6}, \frac{3}{4}$ (*c*) $\frac{7}{9}, \frac{2}{3}, \frac{3}{5}$ (*d*) None of these

193. One test tube contains some acid and another test tube contains an equal quantity of water. To prepare a solution, 20 grams of the acid is poured into the second test tube. Then, two-thirds of the so-formed solution is poured from the second tube into the first. If the fluid in first test tube is four times that in the second, what quantity of water was taken initially ?

(*a*) 40 grams (*b*) 60 grams (*c*) 80 grams (*d*) 100 grams

194. From a number of apples, a man sells half the number of existing apples plus 1 to the first customer, sells $\frac{1}{3}$rd of the remaining apples plus 1 to the second customer and $\frac{1}{5}$th of the remaining apples plus 1 to the third customer. He then finds that he has 3 apples left. How many apples did he have originally ?

(a) 15 (b) 18 (c) 20 (d) 25

195. $\frac{(856+167)^2+(856-167)^2}{856\times856+167\times167} = ?$

(a) 1 (b) 2 (c) 689 (d) 1023

196. $\frac{(469+174)^2-(469-174)^2}{469\times174} = ?$ **(M.B.A. 2002)**

(a) 2 (b) 4 (c) 295 (d) 643

197. If $a - b = 3$ and $a^2 + b^2 = 29$, find the value of ab. **(R.R.B. 2003)**

(a) 10 (b) 12 (c) 15 (d) 18

198. If $\frac{x^2-1}{x+1} = 4$, $x = ?$ **(Bank P.O. 2000)**

(a) 0 (b) 1 (c) 5

(d) Cannot be determined (e) None of these

199. $\frac{\left(3\frac{2}{3}\right)^2-\left(2\frac{1}{2}\right)^2}{\left(4\frac{3}{4}\right)^2-\left(3\frac{1}{3}\right)^2}+\frac{3\frac{2}{3}-2\frac{1}{2}}{4\frac{3}{4}-3\frac{1}{3}} = ?$ **(Hotel Management, 2001)**

(a) $\frac{37}{97}$ (b) $\frac{74}{97}$ (c) $1\frac{23}{74}$ (d) None of these

200. The simplified value of $\frac{\left(1+\frac{1}{1+\frac{1}{100}}\right)\left(1+\frac{1}{1+\frac{1}{100}}\right)-\left(1-\frac{1}{1+\frac{1}{100}}\right)\left(1-\frac{1}{1+\frac{1}{100}}\right)}{\left(1+\frac{1}{1+\frac{1}{100}}\right)+\left(1-\frac{1}{1+\frac{1}{100}}\right)}$ is :

(a) 100 (b) $\frac{200}{101}$ (c) 200 (d) $\frac{202}{100}$

(S.S.C. 2003)

201. If $a + b + c = 13$, $a^2 + b^2 + c^2 = 69$, then find $ab + bc + ca$. **(B.S.F. 2001)**

(a) − 50 (b) 50 (c) 69 (d) 75

202. If $\frac{x^2+y^2+z^2-64}{xy-yz-zx} = -2$ and $x + y = 3z$, then the value of z is :

(a) 2 (b) 3 (c) 4 (d) None of these

(Hotel Management, 1999)

203. $\left(\frac{785\times785\times785+435\times435\times435}{785\times785+435\times435-785\times435}\right)$ simplifies to : **(S.S.C. 2000)**

(a) 350 (b) 785 (c) 1220 (d) 1320

204. $\left(\frac{147\times147+147\times143+143\times143}{147\times147\times147-143\times143\times143}\right) = ?$ **(A.A.O. Exam, 2003)**

(a) $\frac{1}{4}$ (b) 290 (c) $\frac{1}{290}$ (d) 4

205. $\frac{(13)^3+7^3}{(13)^2+7^2-?} = 20$

(a) 6 (b) 20 (c) 91 (d) None of these

206. The value of $\frac{\left(\frac{3}{5}\right)^3-\left(\frac{2}{5}\right)^3}{\left(\frac{3}{5}\right)^2-\left(\frac{2}{5}\right)^2}$ is : **(S.S.C. 2003)**

(a) $\frac{1}{5}$ (b) $\frac{19}{25}$ (c) $\frac{21}{25}$ (d) 1

207. $\frac{38\times38\times38+34\times34\times34+28\times28\times28-38\times34\times84}{38\times38+34\times34+28\times28-38\times34-34\times28-38\times28}$ is equal to :

(a) 24 (b) 32 (c) 44 (d) 100

208. The value of $\frac{(x-y)^3+(y-z)^3+(z-x)^3}{9(x-y)(y-z)(z-x)}$ is equal to :

(a) 0 (b) $\frac{1}{9}$ (c) $\frac{1}{3}$ (d) 1

209. The highest score in an inning was $\frac{3}{11}$ of the total and the next highest was $\frac{3}{11}$ of the remainder. If the scores differed by 9, the total score was :

(a) 110 (b) 121 (c) 132 (d) 143

210. Rahul owes Rs. X and gives a Rs. 50 note in payment. He receives the following change : 3X fifty-paise coins, 14 ten-paise coins and 4X five-paise coins. X is equal to :

(a) 12 (b) 16 (c) 18 (d) 22

211. David gets on the elevator at the 11th floor of a building and rides up at the rate of 57 floors per minute. At the same time, Albert gets on an elevator at the 51st floor of the same building and rides down at the rate of 63 floors per minute. If they continue travelling at these rates, then at which floor will their paths cross ?

(a) 19 (b) 28 (c) 30 (d) 37

(M.B.A. 2003)

212. N number of persons decided to raise Rs. 3 lakhs by equal contributions from each. Had they contributed Rs. 50 each extra, the contribution would have been Rs. 3.25 lakhs. How many persons are there ? **(Bank P.O. 2003)**

(a) 400 (b) 450 (c) 600

(d) Cannot be determined (e) None of these

213. Free notebooks were distributed equally among children of a class. The number of notebooks each child got was one-eighth of the number of children. Had the number of children been half, each child would have got 16 notebooks. Total how many notebooks were distributed ? **(Bank P.O. 2003)**

(a) 256 (b) 432 (c) 512 (d) 640 (e) None of these

214. A classroom has equal number of boys and girls. Eight girls left to play kho-kho, leaving twice as many boys as girls in the classroom. What was the total number of girls and boys present initially ? **(S.B.I.P.O. 2000)**

(a) 16 (b) 24 (c) 32

(d) Cannot be determined (e) None of these

215. After distributing the sweets equally among 25 children, 8 sweets remain. Had the number of children been 28, 22 sweets would have been left after equally distributing. What was the total number of sweets ?

(*a*) 328 (*b*) 348 (*c*) 358 (*d*) Data inadequate

216. In a regular week, there are 5 working days and for each day, the working hours are 8. A man gets Rs. 2.40 per hour for regular work and Rs. 3.20 per hours for overtime. If he earns Rs. 432 in 4 weeks, then how many hours does he work for ?

(*a*) 160 (*b*) 175 (*c*) 180 (*d*) 195

(Bank P.O. 2003)

217. A sum of Rs. 312 was divided among 100 boys and girls in such a way that each boy gets Rs. 3.60 and each girl Rs. 2.40. The number of girls is :

(*a*) 35 (*b*) 40 (*c*) 60 (*d*) 65

(A.A.O. Exam, 2003)

218. Each boy contributed rupees equal to the number of girls and each girl contributed rupees equal to the number of boys in a class of 60 students. If the total contribution thus collected is Rs. 1600, how many boys are there in the class ?

(*a*) 25 (*b*) 30 (*c*) 50 (*d*) Data inadequate

219. A worker may claim Rs. 1.50 for each km which he travels by taxi and 50 p for each km he drives his own car. If in one week he claimed Rs. 50 for travelling 80 km, how many kms did he travel by taxi ?

(*a*) 10 (*b*) 20 (*c*) 30 (*d*) 40

220. In an examination, a student scores 4 marks for every correct answer and loses 1 mark for every wrong answer. If he attempts in all 60 questions and secures 130 marks, the number of questions he attempts correctly, is : **(L.I.C. A.A.O. 2003)**

(*a*) 35 (*b*) 38 (*c*) 40 (*d*) 42

221. A cricket team won 3 matches more than they lost. If a win gives them 2 points and loss (– 1) point, how many matches, in all, have they played if their score is 23 ?

(S.S.C. 2000)

(*a*) 17 (*b*) 20 (*c*) 37 (*d*) 40.

222. A total of 324 coins of 20 paise and 25 paise make a sum of Rs. 71. The number of 25-paise coins is : **(N.I.F.T. 2003)**

(a) 120 (*b*) 124 (*c*) 144 (*d*) 200

223. A man has Rs. 480 in the denominations of one-rupee notes, five-rupee notes and ten-rupee notes. The number of notes of each denomination is equal. What is the total number of notes that he has ? **(M.A.T. 2002)**

(a) 45 (*b*) 60 (*c*) 75 (*d*) 90

224. Eight people are planning to share equally the cost of a rental car. If one person withdraws from the arrangement and the others share equally the entire cost of the car, then the share of each of the remaining persons increased by : **(M.B.A. 2002)**

(*a*) $\frac{1}{7}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{1}{9}$ (*d*) $\frac{7}{8}$

225. On Children's Day, sweets were to be equally distributed among 175 children in a school. Actually on the Children's Day, 35 children were absent and therefore each child got 4 sweets extra. Total how many sweets were available for distribution ?

(*a*) 2400 (*b*) 2480 (*c*) 2680 (*d*) 2750 (*e*) None of these

(Bank P.O. 2003)

226. A number of friends decided to go on a picnic and planned to spend Rs. 96 on eatables. Four of them, however, did not turn up. As a consequence, the remaining ones had to contribute Rs. 4 each extra. The number of those who attended the picnic was :

(*a*) 8 (*b*) 12 (*c*) 16 (*d*) 24

227. A certain number of tennis balls were purchased for Rs. 450. Five more balls could have been purchased in the same amount if each ball was cheaper by Rs. 15. The number of balls purchased was : **(Bank P.O. 1999)**

(*a*) 10 (*b*) 15 (*c*) 20 (*d*) 25

228. A piece of cloth costs Rs. 35. If the length of the piece would have been 4 m longer and each metre costs Re. 1 less, the cost would have remained unchanged. How long is the piece ?

(*a*) 9 m (*b*) 10 m (*c*) 12 m (*d*) 14 m

229. The price of 10 chairs is equal to that of 4 tables. The price of 15 chairs and 2 tables together is Rs. 4000. The total price of 12 chairs and 3 tables is : **(S.S.C. 2002)**

(*a*) Rs. 3500 (*b*) Rs. 3750 (*c*) Rs. 3840 (*d*) Rs. 3900

230. In a certain shop, 9 oranges cost as much as 5 apples, 5 apples cost as much as 3 mangoes and 4 mangoes cost as much as 9 lemons. If 3 lemons cost Rs. 4.80, the price of an orange is :

(*a*) Rs. 1.20 (*b*) Rs. 1.30 (*c*) Rs. 1.40 (*d*) Rs. 1.50

231. The price of 2 sarees and 4 shirts is Rs. 1600. With the same money one can buy 1 saree and 6 shirts. If one wants to buy 12 shirts, how much shall he have to pay ?

(*a*) Rs. 1200 (*b*) Rs. 2400 (*c*) Rs. 4800

(*d*) Cannot be determined (*e*) None of these **(Bank P.O. 2002)**

232. If 2 tables and 3 chairs cost Rs. 3500 and 3 tables and 2 chairs cost Rs. 4000, then how much does a table cost ? **(Hotel Management, 2003)**

(*a*) Rs. 500 (*b*) Rs. 750 (*c*) Rs. 1000 (*d*) Rs. 1500

233. The taxi charges in a city comprise of a fixed charge, together with the charge of the distance covered. For a journey of 16 km, the charges paid are Rs. 156 and for a journey of 24 km, the charges paid are Rs. 204. What will a person have to pay for travelling a distance of 30 km ?

(*a*) Rs. 226 (*b*) Rs. 240 (*c*) Rs. 248 (*d*) Rs. 252

234. In a classroom, if 6 students per bench are assigned to accommodate all students, one more bench will be required. However, if 7 students are accommodated per bench, there would be a space left for 5 students. What is the number of students in the class ?

(*a*) 30 (*b*) 42 (*c*) 72 (*d*) None of these **(S.S.C. 2000)**

235. There are two examination rooms A and B. If 10 students are sent from A to B, then the number of students in each room is the same. If 20 candidates are sent from B to A, then the number of students in A is double the number of students in B. The number of students in room A is : **(M.A.T. 2002)**

(*a*) 20 (*b*) 80 (*c*) 100 (*d*) 200

236. In a group of buffaloes and ducks, the number of legs are 24 more than twice the number of heads. What is the number of buffaloes in the group ? **(R.R.B. 2002)**

(*a*) 6 (*b*) 8 (*c*) 10 (*d*) 12

237. A man has some hens and cows. If the number of heads be 48 and the number of feet equals 140, then the number of hens will be : **(R.R.B. 2003)**

(*a*) 22 (*b*) 23 (*c*) 24 (*d*) 26

238. Vidushi and Sanya distribute Rs. 100 each in charity. Vidushi distributes money to 5 more people than Sanya and Sanya gives each Re. 1 more than Vidushi. How many people are recipients of the charity ?

(*a*) 45 (*b*) 60 (*c*) 90 (*d*) None of these

ANSWERS

1. (c)	2. (e)	3. (b)	4. (d)	5. (a)	6. (a)	7. (a)	8. (c)
9. (a)	10. (a)	11. (d)	12. (c)	13. (e)	14. (c)	15. (c)	16. (c)
17. (c)	18. (d)	19. (c)	20. (a)	21. (c)	22. (b)	23. (d)	24. (c)
25. (a)	26. (d)	27. (b)	28. (c)	29. (d)	30. (c)	31. (b)	32. (a)
33. (c)	34. (d)	35. (d)	36. (e)	37. (e)	38. (a)	39. (d)	40. (a)
41. (b)	42. (d)	43. (a)	44. (d)	45. (a)	46. (c)	47. (e)	48. (b)
49. (a)	50. (e)	51. (c)	52. (b)	53. (a)	54. (a)	55. (a)	56. (d)
57. (a)	58. (c)	59. (d)	60. (d)	61. (b)	62. (d)	63. (b)	64. (b)
65. (c)	66. (d)	67. (d)	68. (a)	69. (c)	70. (c)	71. (a)	72. (b)
73. (c)	74. (c)	75. (b)	76. (c)	77. (a)	78. (c)	79. (c)	80. (d)
81. (a)	82. (c)	83. (d)	84. (a)	85. (c)	86. (c)	87. (d)	88. (b)
89. (c)	90. (b)	91. (c)	92. (c)	93. (b)	94. (c)	95. (c)	96. (b)
97. (c)	98. (b)	99. (c)	100. (b)	101. (d)	102. (a)	103. (c)	104. (d)
105. (d)	106. (c)	107. (b)	108. (d)	109. (b)	110. (b)	111. (b)	112. (d)
113. (b)	114. (c)	115. (d)	116. (c)	117. (a)	118. (a)	119. (c)	120. (e)
121. (b)	122. (d)	123. (c)	124. (c)	125. (a)	126. (c)	127. (d)	128. (b)
129. (b)	130. (b)	131. (d)	132. (a)	133. (a)	134. (b)	135. (d)	136. (b)
137. (a)	138. (b)	139. (b)	140. (d)	141. (a)	142. (c)	143. (b)	144. (a)
145. (a)	146. (c)	147. (d)	148. (c)	149. (c)	150. (c)	151. (d)	152. (d)
153. (b)	154. (b)	155. (b)	156. (d)	157. (b)	158. (d)	159. (c)	160. (c)
161. (c)	162. (c)	163. (c)	164. (c)	165. (a)	166. (c)	167. (a)	168. (c)
169. (c)	170. (c)	171. (c)	172. (d)	173. (c)	174. (b)	175. (b)	176. (c)
177. (b)	178. (c)	179. (b)	180. (b)	181. (a)	182. (c)	183. (a)	184. (d)
185. (c)	186. (c)	187. (b)	188. (d)	189. (a)	190. (a)	191. (c)	192. (b)
193. (a)	194. (c)	195. (b)	196. (b)	197. (a)	198. (c)	199. (b)	200. (b)
201. (b)	202. (c)	203. (c)	204. (a)	205. (c)	206. (b)	207. (d)	208. (c)
209. (b)	210. (c)	211. (c)	212. (e)	213. (c)	214. (c)	215. (c)	216. (b)
217. (b)	218. (d)	219. (a)	220. (b)	221. (c)	222. (b)	223. (d)	224. (a)
225. (e)	226. (b)	227. (a)	228. (b)	229. (d)	230. (a)	231. (b)	232. (c)
233. (b)	234. (c)	235. (c)	236. (d)	237. (d)	238. (a)		

SOLUTIONS

1. Given expression = 100 + 100 = 200.
2. Given expression = 9240 ÷ 28 = 330.
3. Given expression = $\frac{5004}{139} - 6 = 36 - 6 = 30$.
4. Given expression = 7500 ÷ 25 = 7525.
5. Given expression = $\frac{8}{88} \times 8888088 = \frac{1}{11} \times 8888088 = 808008$.
6. Given expression = 1001 ÷ 143 = 7.
7. Given expression = $\frac{1260}{15} \div 7 = 84 \div 7 = 12$.
8. Given expression = $\left(5 \times 4 \times 2 \times \frac{1}{2} \times \frac{3}{4}\right) = 15$.

9. Let $\frac{11}{4} = \frac{77}{x}$. Then, $11x = 77 \times 4$ or $x = \left(\frac{77 \times 4}{11}\right) = 28$.

10. $2^5 \times 9^2 = 32 \times 81 = 2592$.

11. Given exp. $= 2 - [2 - \{2 - 2 \times 4\}] = 2 - [2 - \{2 - 8\}] = 2 - [2 - (-6)]$
$= 2 - [2 + 6] = 2 - 8 = -6$.

12. Given exp. $= 25 - 5\ [2 + 3\ \{2 - 2 \times 2 + 5\} - 10] \div 4$
$= 25 - 5\ [2 + 3\ \{2 - 4 + 5\} - 10] \div 4 = 25 - 5\ [2 + 3 \times 3 - 10] \div 4$
$= 25 - 5\ [2 + 9 - 10] \div 4 = 25 - 5 \div 4 = 25 - 1.25 = 23.75$.

13. Given exp. $= 260 \times 16 + 340 = 4160 + 340 = 4500$.

14. Given exp. $= 100 \times 10 - 100 + 20 = 1000 - 100 + 20 = 1020 - 100 = 920$.

15. Let $2\ x\ 6 - 12 \div 4 + 2 = 11$. Then, $2\ x\ 6 - 3 + 2 = 11 \Leftrightarrow 2\ x\ 6 = 11 + 3 - 2 = 12$.
So, x must be replaced by '$\times$'.

16. Let $45 - [28 - \{37 - (15 - x)\}] = 58$.
Then, $45 - [28 - \{37 - 15 + x\}] = 58 \Leftrightarrow 45 - [28 - \{22 + x\}] = 58$
$\Leftrightarrow 45 - [28 - 22 - x] = 58 \Leftrightarrow 45 - [6 - x] = 58 \Leftrightarrow 45 - 6 + x = 58$
$\Leftrightarrow 39 + x = 58 \Leftrightarrow x = 58 - 39 = 19$.

17. Given exp. $= \frac{24 \div 6}{4 + 4 + 4 + 1} = \frac{4}{13}$.

18. Given exp. $= \frac{4 + 72 - 6 - 8}{738 - 730} = \frac{76 - 14}{8} = \frac{62}{8} = 7.75$.

19. Given exp. $= \frac{2700 - 240}{1120 + 110} = \frac{2460}{1230} = 2$.

20. Let $\frac{128 \div 16 \times x - 7 \times 2}{7^2 - 8 \times 6 + x^2} = 1$.
Then, $8x - 7 \times 2 = 49 - 48 + x^2 \Leftrightarrow 8x - 14 = 1 + x^2 \Leftrightarrow x^2 - 8x + 15 = 0$
$\Leftrightarrow (x - 3)(x - 5) = 0 \Leftrightarrow x = 3$ or $x = 5$.

21. Given exp. $= 18 - [5 - \{6 + 2\ (7 - 3)\}] = 18 - [5 - \{6 + 2 \times 4\}]$
$= 18 - [5 - \{6 + 8\}] = 18 - [5 - 14] = 18 - [-9] = 18 + 9 = 27$.

22. Given exp. $= 1 \div \left[1 + 1 \div \left\{1 + 1 + \left(1 + \frac{1}{2}\right)\right\}\right] = 1 \div \left[1 + 1 \div \left\{1 + 1 + \frac{3}{2}\right\}\right]$
$= 1 \div \left[1 + 1 \div \left\{1 + 1 \times \frac{2}{3}\right\}\right] = 1 \div \left[1 + 1 \div \left\{1 + \frac{2}{3}\right\}\right]$
$= 1 \div \left[1 + 1 \div \frac{5}{3}\right] = 1 \div \left[1 + 1 \times \frac{3}{5}\right] = 1 \div \left[1 + \frac{3}{5}\right] = 1 \div \frac{8}{5} = 1 \times \frac{5}{8} = \frac{5}{8}$.

23. Given exp. $= \frac{8 - |5 - (-1)| \div 2}{|2| - |-3| \div 3} = \frac{8 - |5 + 1| \div 2}{2 - 3 \div 3} = \frac{8 - 6 \div 2}{2 - 1} = 8 - 3 = 5$.

24. $\frac{5}{3} + \frac{3}{4} = \frac{20 + 9}{12} = \frac{29}{12} = 2\frac{5}{12} < 5$; $\frac{7}{3} + \frac{11}{5} = \frac{35 + 33}{15} = \frac{68}{15} = 4\frac{8}{15} < 5$;
$\frac{11}{4} + \frac{8}{3} = \frac{33 + 32}{12} = \frac{65}{12} = 5\frac{5}{12} > 5$; $\frac{13}{5} + \frac{11}{6} = \frac{78 + 55}{30} = \frac{133}{30} = 4\frac{13}{30} < 5$.

25. Given exp. $= \frac{28 + 14 + 7 + 4 + 2 + 1}{28} = \frac{56}{28} = 2$.

26. Given exp. $= \frac{7}{4} + \frac{16}{3} + \frac{17}{5} = \frac{105 + 320 + 204}{60} = \frac{629}{60} = 10\frac{29}{60}$.

27. Given exp. $= \frac{41}{2} + \frac{91}{3} - \frac{91}{6} = \left(\frac{123 + 182}{6}\right) - \frac{91}{6} = \frac{305}{6} - \frac{91}{6} = \frac{214}{6} = \frac{107}{3} = 35\frac{2}{3}$.

28. Given exp. $= -1 + 4 + 3 = 6$.

29. Given exp. $= \frac{1}{(7/3)} + \frac{1}{(7/4)} = \frac{3}{7} + \frac{4}{7} = \frac{7}{7} = 1$.

30. Let $\frac{35}{6} - \frac{35}{9} - x = 1$.

Then, $x = \frac{35}{6} - \frac{35}{9} - 1 = \frac{35}{6} - \left(\frac{35}{9} + 1\right) = \frac{35}{6} - \frac{44}{9} = \frac{105 - 88}{18} = \frac{17}{18}$.

31. $\frac{1}{x} = 4 - \left(\frac{1}{3} + \frac{1}{2}\right) = 4 - \left(\frac{2+3}{6}\right) = 4 - \frac{5}{6} = \frac{24-5}{6} = \frac{19}{6} \Rightarrow x = \frac{6}{19}$.

32. Given exp. $= \dfrac{\left(-\frac{2}{3} - \frac{1}{3}\right) + \left(\frac{4}{5} + \frac{1}{5}\right) + \left(\frac{3}{4} - \frac{1}{2}\right)}{\left(\frac{2}{3} - \frac{4}{3} + \frac{1}{3}\right) - \left(\frac{1}{5} + \frac{4}{5}\right) + \frac{1}{2}}$

$= \dfrac{-1 + 1 + \frac{1}{4}}{-\frac{1}{3} - 1 + \frac{1}{2}} = \dfrac{\frac{1}{4}}{\frac{-2-6+3}{6}} = \dfrac{\frac{1}{4}}{-\frac{5}{6}} = \frac{1}{4} \times \left(-\frac{6}{5}\right) = \frac{-3}{10}$.

33. Given exp. $= 5 - \left[\frac{3}{4} + \left\{\frac{5}{2} - \left(\frac{1}{2} + \frac{7-6}{42}\right)\right\}\right] = 5 - \left[\frac{3}{4} + \left\{\frac{5}{2} - \left(\frac{1}{2} + \frac{1}{42}\right)\right\}\right]$

$= 5 - \left[\frac{3}{4} + \left\{\frac{5}{2} - \frac{22}{42}\right\}\right] = 5 - \left[\frac{3}{4} + \frac{83}{42}\right] = 5 - \frac{229}{84}$

$= \left(\frac{420 - 229}{84}\right) = \frac{191}{84} = 2\frac{23}{84}$.

34. $\dfrac{\left(\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right)}{\left(\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18}\right)} = \dfrac{\left(\frac{30 - 15 + 12 - 10}{60}\right)}{\left(\frac{2}{5} + \frac{3}{5}\right) - \left(\frac{5}{9} + \frac{7}{18}\right)} = \dfrac{\left(\frac{17}{60}\right)}{1 - \frac{17}{18}} = \left(\frac{17}{60} \times 18\right) = \frac{51}{10} = 5\frac{1}{10}$.

35. $\left(34 \times 4\frac{1}{2}\right) = 34 \times \left(4 + \frac{1}{2}\right) = (34 \times 4) + \left(34 \times \frac{1}{2}\right)$

$= (30 + 4) \times 4 + \left(34 \times \frac{1}{2}\right) = (30 \times 4) + (4 \times 4) + \left(34 \times \frac{1}{2}\right)$.

36. Given exp. $= \left(\frac{3}{5} \times \frac{4}{7} \times \frac{5}{9} \times \frac{21}{24} \times 504\right) = 84$.

37. Given exp. $= \left(\frac{41}{6} \times \frac{16}{3} + \frac{53}{3} \times \frac{9}{2}\right) = \left(\frac{328}{9} + \frac{159}{2}\right) = \frac{656 + 1431}{18} = \frac{2087}{18} = 115\frac{17}{18}$.

38. Let $\frac{3}{8}$ of $168 \times 15 \div 5 + x = 549 \div 9 + 235$.

Then, $63 \times 15 \div 5 + x = 61 + 235 \Leftrightarrow 63 \times 3 + x = 296$

$\Leftrightarrow 189 + x = 296 \Leftrightarrow x = 107$.

39. Let $\frac{5}{3} \div \frac{2}{7} \times \frac{x}{7} = \frac{5}{4} \times \frac{2}{3} \div \frac{1}{6}$. Then,

$\frac{5}{3} \times \frac{7}{2} \times \frac{x}{7} = \frac{5}{4} \times \frac{2}{3} \times 6 \Leftrightarrow \frac{5}{6}x = 5 \Leftrightarrow x = \left(\frac{5 \times 6}{5}\right) = 6$.

40. Let $5\frac{2}{3} \div x\frac{5}{6} = 2$. Then, $\frac{17}{3} \div x\frac{5}{6} = 2 \Leftrightarrow x\frac{5}{6} = \frac{17}{3} \times \frac{1}{2} = \frac{17}{6} \Leftrightarrow x\frac{5}{6} = 2\frac{5}{6}$.
$\therefore \quad x = 2$.

41. Given equation is : $\frac{(5x+1)}{x} \times \frac{(4y+3)}{4} = 20 \Leftrightarrow (5x+1)(4y+3) = 80x \quad ...(i)$
Clearly, $x = 3$ and $y = 3$ satisfy (i).

42. Required difference $= \frac{19}{16} - \frac{16}{19} = \frac{19^2 - 16^2}{304} = \frac{(19+16)(19-16)}{304} = \frac{35 \times 3}{301} = \frac{105}{304}$.

43. Required number $= \frac{37\frac{1}{2}}{1/8} = \frac{75/2}{1/8} = \frac{75}{2} \times 8 = 300$.

44. Let x of $\frac{1}{12} = \frac{3}{8}$. Then, $\frac{x}{12} = \frac{3}{8} \Leftrightarrow x = \left(\frac{3}{8} \times 12\right) = \frac{9}{2}$.

45. Sum of given fractions $= \frac{7}{4} + \frac{5}{2} + \frac{67}{12} + \frac{10}{3} + \frac{9}{4} = \left(\frac{21+30+67+40+27}{12}\right) = \frac{185}{12}$.

The whole number just less than $\frac{185}{12}$ is 15.

Let $\frac{185}{12} - x = 15$. Then, $x = \left(\frac{185}{12} - 15\right) = \frac{5}{12}$.

46. Clearly, $\frac{x+1}{x}$ is the only fraction in which the numerator is greater than the denominator. So, it is the greatest fraction.

47. $\frac{3}{5}$ of $350 - \frac{4}{7}$ of $210 = 210 - 120 = 90$.

48. $\frac{6}{7/8} - \frac{6/7}{8} = 6 \times \frac{8}{7} - \frac{6}{7} \times \frac{1}{8} = \frac{48}{7} - \frac{6}{56} = \frac{384-6}{56} = \frac{378}{56} = \frac{27}{4} = 6\frac{3}{4}$.

49. Let the value of the estate be Rs. x.

Then, $\frac{4}{5}$ of $x = 16800 \Leftrightarrow x = \left(\frac{16800 \times 5}{4}\right) = 21000 \Leftrightarrow \frac{3}{7}x = \left(\frac{3}{7} \times 21000\right) = 9000$.

50. Let the number be x. Then,

$\frac{2}{5}$ of $\frac{1}{4}$ of $\frac{3}{7}$ of $x = 15 \Leftrightarrow x = \left(15 \times \frac{7}{3} \times 4 \times \frac{5}{2}\right) = 350 \Leftrightarrow \frac{1}{2}x = 175$.

51. Let the number be x. Then,

$\frac{1}{5}x - \frac{1}{7}x = 10 \Leftrightarrow \frac{7x-5x}{35} = 10 \Leftrightarrow \frac{2x}{35} = 10 \Leftrightarrow x = \left(\frac{10 \times 35}{2}\right) = 175$.

52. $9 * 11 = 9^2 + (11)^2 - 9 \times 11 = 81 + 121 - 99 = 103$.

53. $(3 * -1) = \frac{3 \times (-1)}{3 + (-1)} = \frac{-3}{2}$. So, $3 * (3 * -1) = 3 * \left(\frac{-3}{2}\right) = \frac{3 \times \left(\frac{-3}{2}\right)}{3 + \left(\frac{-3}{2}\right)} = \frac{-9}{2} \times \frac{2}{3} = -3$.

54. $3 * 5 + 5 * 3 = (2 \times 3 - 3 \times 5 + 3 \times 5) + (2 \times 5 - 3 \times 3 + 5 \times 3)$
$= (6 + 10 - 9 + 15) = 22$.

55. $4 \oplus (3 \oplus p) = 4 \oplus (3^2 + 2p) = 4 \oplus (9 + 2p) = 4^2 + 2(9 + 2p) = 34 + 4p$.
$\therefore \quad 34 + 4p = 50 \Rightarrow 4p = 50 - 34 = 16 \Rightarrow p = 4$.

56. $(a * b * c) * a * b = \left(\frac{a+b}{c}\right) * a * b = \frac{\left(\frac{a+b}{c}\right)+a}{b} = \frac{a+b+ac}{bc}$.

57. Let the number be x. Then,

$\frac{5(x+7)}{9} - 3 = 12 \Leftrightarrow 5(x+7) - 27 = 108 \Leftrightarrow 5x + 35 = 135 \Leftrightarrow 5x = 100 \Leftrightarrow x = 20$.

58. Given exp. $= \left(\frac{5}{7} \times \frac{19}{13}\right) \div \left(\frac{19}{7} \times \frac{4}{13}\right) = \frac{5 \times 19}{7 \times 13} \times \frac{7 \times 13}{19 \times 4} = \frac{5}{4}$.

59. Given exp. $= \frac{11}{4} \div \frac{8}{3} \div \frac{13}{12} = \frac{11}{4} \times \frac{3}{8} \times \frac{12}{13} = \frac{99}{104}$.

60. Given exp. $= \frac{9}{2} \times \frac{13}{3} - \frac{25}{3} \div \frac{17}{3} = \frac{9}{2} \times \frac{13}{3} - \frac{25}{3} \times \frac{3}{17}$

$= \frac{39}{2} - \frac{25}{17} = \frac{663 - 50}{34} = \frac{613}{34} = 18\frac{1}{34}$.

61. Let $\frac{4335}{x} \div \frac{15}{8} = \frac{289}{528}$. Then,

$\frac{4335}{x} \div \frac{289}{528} \times \frac{15}{8} \Leftrightarrow \frac{4335}{x} = \frac{289 \times 5}{176 \times 8} \Leftrightarrow x = \left(\frac{4335 \times 176 \times 8}{289 \times 5}\right) = 4224$.

$\therefore$ Missing digit = 2.

62. Let $\frac{16}{3} - \frac{11}{3} \times \frac{3}{4} \times \frac{1}{x} + \frac{16}{5} \times \frac{5}{6} = 7$. Then,

$\frac{16}{3} - \frac{11}{3} \times \frac{3}{4} \times \frac{1}{x} + \frac{16}{5} \times \frac{5}{6} = 7 \Leftrightarrow \frac{16}{3} - \frac{11}{4x} + \frac{8}{3} = 7 \Leftrightarrow \frac{24}{3} - \frac{11}{4x} = 7$

$\Leftrightarrow \frac{11}{4x} = 8 - 7 = 1 \Leftrightarrow 4x = 11 \Leftrightarrow x = \frac{11}{4} = 2\frac{3}{4}$.

63. Given exp. $= 9 - \frac{11}{9}$ of $\frac{36}{11} \div \frac{36}{7}$ of $\frac{7}{9} = 9 - 4 \div 4 = 9 - 1 = 8$.

64. Let $\frac{5}{6} \div \frac{6}{7} \times x - \frac{8}{9} \div \frac{8}{5} + \frac{3}{4} \times \frac{10}{3} = \frac{25}{9}$. Then,

$\frac{5}{6} \times \frac{7}{6} \times x - \frac{8}{9} \times \frac{5}{8} + \frac{3}{4} \times \frac{10}{3} = \frac{25}{9} \Leftrightarrow \frac{35}{36}x - \frac{5}{9} + \frac{5}{2} = \frac{25}{9}$

$\Leftrightarrow \frac{35}{36}x = \frac{25}{9} + \frac{5}{9} - \frac{5}{2} = \frac{10}{3} - \frac{5}{2} \Leftrightarrow \frac{35}{36}x = \frac{5}{6} \Leftrightarrow x = \left(\frac{5}{6} \times \frac{36}{35}\right) = \frac{6}{7}$.

65. Given exp. $= \frac{3}{4} \div \frac{9}{4}$ of $\frac{2}{3} - \frac{\left(\frac{3-2}{6}\right)}{\left(\frac{3+2}{6}\right)} \times \frac{10}{3} + \frac{5}{6} = \frac{3}{4} \div \frac{3}{2} - \frac{1}{6} \times \frac{6}{5} \times \frac{10}{3} + \frac{5}{6}$

$= \frac{3}{4} \times \frac{2}{3} - \frac{2}{3} + \frac{5}{6} = \left(\frac{1}{2} - \frac{2}{3} + \frac{5}{6}\right) = \left(\frac{3-4+5}{6}\right) = \frac{4}{6} = \frac{2}{3}$.

66. $\frac{\frac{7}{3} + 1\frac{1}{2} \text{ of } \frac{5}{3}}{2 + 1\frac{2}{3}} = \frac{\frac{7}{3} + \frac{3}{2} \text{ of } \frac{5}{3}}{2 + \frac{5}{3}} = \frac{\frac{7}{3} + \frac{5}{2}}{\frac{11}{3}} = \frac{29}{6} \times \frac{3}{11} = \frac{29}{22}$.

$\therefore$ Required answer $= \frac{29}{22} - \frac{1}{4} = \frac{58 - 11}{44} = \frac{47}{44} = 1\frac{3}{44}$.

67. Given exp. $= \dfrac{\dfrac{1}{3}+\dfrac{3}{4}\left(\dfrac{6-5}{15}\right)}{\dfrac{5}{3} \text{ of } \dfrac{3}{4}-\dfrac{1}{5}} = \dfrac{\dfrac{1}{3}+\dfrac{3}{4}\times\dfrac{1}{15}}{\dfrac{5}{4}-\dfrac{1}{5}} = \dfrac{\dfrac{1}{3}+\dfrac{1}{20}}{\dfrac{25-4}{20}} = \dfrac{23}{60}\times\dfrac{20}{21} = \dfrac{23}{63}.$

68. Given exp. $= \dfrac{\dfrac{1}{3}\times 3\times\dfrac{1}{3}}{\dfrac{1}{3}+\dfrac{1}{9}} - \dfrac{1}{9} = \dfrac{\dfrac{1}{3}}{\dfrac{1}{3}\times 9} - \dfrac{1}{9} = \dfrac{1}{3}\times\dfrac{1}{3} - \dfrac{1}{9} = \dfrac{1}{9} - \dfrac{1}{9} = 0.$

69. Given exp. $= \dfrac{\dfrac{1}{2}+\dfrac{1}{4}}{\dfrac{1}{2}+\dfrac{1}{4}} - \dfrac{\dfrac{1}{2}\times 4}{\dfrac{2+1}{4}} = 2\times\dfrac{4}{3} = \dfrac{8}{3} = 2\dfrac{2}{3}.$

70. Given exp. $= \dfrac{\dfrac{13}{4}-\dfrac{4}{5} \text{ of } \dfrac{5}{6}}{\dfrac{13}{3}+\dfrac{1}{5}-\left(\dfrac{3}{10}+\dfrac{106}{5}\right)} - \dfrac{\dfrac{13}{4}-\dfrac{2}{3}}{\dfrac{13}{3}\times 5-\dfrac{215}{10}} = \dfrac{\dfrac{31}{12}}{\dfrac{65}{3}-\dfrac{43}{2}} = \left(\dfrac{31}{12}\times 6\right) = \dfrac{31}{2} = 15\dfrac{1}{2}.$

71. Let $\dfrac{\dfrac{15}{2}-\dfrac{23}{4}}{\dfrac{7}{2}+x} \div \dfrac{\dfrac{1}{2}+\dfrac{5}{4}}{\dfrac{6}{5}+\dfrac{7}{2}} = \dfrac{6}{10}$. Then, $\left[\dfrac{7}{4}\times\dfrac{2}{(7+2x)}\right] \div \left[\dfrac{7}{4}\times\dfrac{10}{47}\right] = \dfrac{3}{5}$

$\Leftrightarrow \quad \dfrac{7}{2(7+2x)} = \dfrac{3}{5}\times\dfrac{7}{4}\times\dfrac{10}{47} = \dfrac{21}{94} \quad \Leftrightarrow \quad 7+2x = \left(\dfrac{7}{2}\times\dfrac{94}{21}\right) = \dfrac{47}{3}$

$\Leftrightarrow \quad 2x = \dfrac{47}{3} - 7 = \dfrac{26}{3} \quad \Leftrightarrow \quad x = \left(\dfrac{26}{3}\times\dfrac{1}{2}\right) = \dfrac{13}{3} = 4\dfrac{1}{3}.$

72. Given exp. $= 3034 - \left(\dfrac{1002}{2004}\times 100\right) = 3034 - 50 = 2984.$

73. Given exp. $= \dfrac{5241.6}{1872} + 6.28 = 2.8 + 6.28 = 9.08.$

74. Let $\dfrac{58}{7}$ of $1568 + 265.75 = x + 2455.60$.

Then, $12992 + 265.75 = x + 2455.60$

$\Leftrightarrow \quad x = 12992 + 265.75 - 2455.60 = 13257.75 - 2455.60 = 10802.15.$

75. Given exp. $= 14.5 + 4.05 + 139.25 = 157.80.$

76. Let $8.25 - 4.20 + 2.8 + \dfrac{4}{x} - 2.32 = 5.33.$

Then, $\dfrac{4}{x} = (5.33 + 4.20 + 2.32) - (8.25 + 2.8) = 11.85 - 11.05 = 0.80 \Leftrightarrow x = \dfrac{4}{0.80} = \dfrac{40}{8} = 5.$

77. Given exp. $= 0.008 \times 0.01 \times 0.0072 \div 0.000048$

$= 0.00008 \times \dfrac{0.0072}{0.000048} = \dfrac{8}{48}\times\dfrac{72}{1000} = 0.012.$

78. Given exp. $= 2.375 \times \dfrac{522}{87} - 0.0285 = 2.375 \times 6 - 0.0285 = 14.25 - 0.0285 = 14.2215.$

79. Given exp. $= 0.2 + 0.2 - 1 \times 0.04 = 0.4 - 0.04 = 0.36.$

80. Given exp. $= 11.6 + \dfrac{9280}{464} - \dfrac{28.28}{7} = 11.6 + 20 - 4.04 = 27.56.$

81. Given exp. $= 4.59 \times \dfrac{18}{36} + 0.6 - 0.2 = \dfrac{4.59}{2} + 0.6 - 0.2 = 2.295 + 0.6 - 0.2 = 2.695.$

82. Let $\frac{64.4 - 34.7125}{6.25 \text{ of } x} = 1$. Then, 6.25 of $x = 29.6875$.

$\therefore \quad x = \frac{29.6875}{6.25} = \frac{2968.75}{625} = 4.75 = 4\frac{3}{4}$.

83. Given exp. $= 2.002 + 7.9 \{2.8 - 6.3 \times 2.1 + 15.6\}$

$= 2.002 + 7.9 \{2.8 - 13.23 + 15.6\} = 2.002 + 7.9 \times 5.17$

$= 2.002 + 40.843 = 42.845$.

84. Let $24 - [2.4 - \{.24 \times 2 - (.024 - x)\}] = 22.0584$.

Then, $24 - [2.4 - \{.48 - .024 + x\}] = 22.0584 \Leftrightarrow 24 - [2.4 - 0.456 - x] = 22.0584$

$\Leftrightarrow \quad 24 - 1.944 + x = 22.0584 \Leftrightarrow x = 22.0584 - 22.056 = 0.0024$.

85. Let $3 - \left[1.6 - \left\{3.2 - \left(3.2 + \frac{2.25}{x}\right)\right\}\right] = 0.65$.

Then, $3 - \left[1.6 - \left\{3.2 - 3.2 - \frac{2.25}{x}\right\}\right] = 0.65 \Leftrightarrow 3 - \left[1.6 + \frac{2.25}{x}\right] = 0.65$

$\Leftrightarrow \quad 3 - 1.6 - \frac{2.25}{x} = 0.65 \Leftrightarrow \frac{2.25}{x} = 1.4 - 0.65 \Leftrightarrow x = \frac{2.25}{0.75} = 3$.

86. Let $587.4 + 58.74 \times 2 - \frac{5.874}{x} = 702.744$.

Then, $\frac{5.874}{x} = 587.4 + 117.48 - 702.744 = 2.136 \Leftrightarrow x = \frac{5.874}{2.136} = \frac{5874}{2136} = \frac{11}{4} = 2\frac{3}{4}$.

$\therefore$ Missing digit $= 3$.

87. Let $54.27 - [12.84 - \{x - (6.82 - 1.85)\}] = 38.33$.

Then, $54.27 - [12.84 - \{x - 4.97\}] = 38.33$

$\Leftrightarrow \quad 54.27 - [12.84 - x + 4.97] = 38.33 \Leftrightarrow 54.27 - [17.81 - x] = 38.33$

$\Leftrightarrow \quad 54.27 - 17.81 + x = 38.33 \Leftrightarrow x = 38.33 - 36.46 = 1.87$.

88. Let $\frac{20}{3}$ of $\frac{726}{100} + \frac{45}{100}$ of $x = \frac{968}{117}$.

Then, $\frac{242}{5} + \frac{45x}{100} = \frac{968}{117} \Leftrightarrow \frac{242}{5} \times \frac{100}{45x} = \frac{968}{117} \Leftrightarrow x = \frac{242}{5} \times \frac{100}{45} \times \frac{117}{968} = 13$.

89. $\frac{P+Q}{P-Q} = \frac{\frac{P}{Q}+1}{\frac{P}{Q}-1} = \frac{7+1}{7-1} = \frac{8}{6} = \frac{4}{3}$.

90. $\left(\frac{4}{7} + \frac{2y-x}{2y+x}\right) = \left(\frac{4}{7} + \frac{2-\frac{x}{y}}{2+\frac{x}{y}}\right) = \frac{4}{7} + \frac{2-\frac{4}{5}}{2+\frac{4}{5}} = \frac{4}{7} + \frac{(6/5)}{(14/5)} = \frac{4}{7} + \left(\frac{6}{5} \times \frac{5}{14}\right) = \frac{4}{7} + \frac{3}{7} = \frac{7}{7} = 1$.

91. $\frac{6a+4b}{6a-5b} = \frac{6\left(\frac{a}{b}\right)+4}{6\left(\frac{a}{b}\right)-5} = \frac{6 \times \frac{4}{3}+4}{6 \times \frac{4}{3}-5} = \frac{8+4}{8-5} = \frac{12}{3} = 4$.

92. $\frac{x}{2y} = \frac{6}{7} \Rightarrow \frac{x}{y} = \left(2 \times \frac{6}{7}\right) = \frac{12}{7}$.

$$\therefore \quad \frac{x-y}{x+y}+\frac{14}{19}=\frac{\frac{x}{y}-1}{\frac{x}{y}+1}+\frac{14}{19}=\frac{\frac{12}{7}-1}{\frac{12}{7}+1}+\frac{14}{19}=\frac{(5/7)}{(19/7)}+\frac{14}{19}$$

$$=\left(\frac{5}{7}\times\frac{7}{19}\right)+\frac{14}{19}=\frac{5}{19}+\frac{14}{19}=\frac{19}{19}=1.$$

93. $\frac{a}{b}=\frac{4}{5}$ and $\frac{b}{c}=\frac{15}{16} \Rightarrow \left(\frac{a}{b}\times\frac{b}{c}\right)=\left(\frac{4}{5}\times\frac{15}{16}\right) \Rightarrow \frac{a}{c}=\frac{3}{4}$.

$$\therefore \quad \frac{c^2-a^2}{c^2+a^2}=\frac{1-\left(\frac{a^2}{c^2}\right)}{1+\left(\frac{a^2}{c^2}\right)}=\frac{1-\left(\frac{a}{c}\right)^2}{1+\left(\frac{a}{c}\right)^2}=\frac{1-\frac{9}{16}}{1+\frac{9}{16}}=\frac{(7/16)}{(25/16)}=\frac{7}{25}.$$

94. $(a-b)-(c+d)=6$ and $(c-d)-(a+b)=3$

$\Rightarrow (a-c)-(b+d)=6$ and $(c-a)-(b+d)=3$

$\Rightarrow (b+d)=(a-c)-6$ and $(b+d)=(c-a)-3$

$\Rightarrow (a-c)-6=(c-a)-3 \Rightarrow 2(a-c)=3 \Rightarrow (a-c)=\frac{3}{2}=1.5.$

95. $x=\frac{a}{a-1}=1+\frac{1}{a-1}=1+y.$ $\therefore x>y.$

96. a is positive and $a<1 \Rightarrow \frac{1}{a}>1.$ $\therefore \left(a+\frac{1}{a}\right)>2.$

97. $\frac{a}{x}+\frac{y}{b}=1 \Rightarrow \frac{a}{x}=1-\frac{y}{b}=\frac{b-y}{b} \Rightarrow \frac{x}{a}=\frac{b}{b-y}.$

$\frac{b}{y}+\frac{z}{c}=1 \Rightarrow \frac{z}{c}=1-\frac{b}{y}=\frac{y-b}{y} \Rightarrow \frac{c}{z}=\frac{y}{y-b}=\frac{-y}{(b-y)}.$

$\therefore \quad \frac{x}{a}+\frac{c}{z}=\frac{b}{(b-y)}-\frac{y}{(b-y)}=\frac{(b-y)}{(b-y)}=1.$

98. $a^2+b^2=45$ ...(*i*) and $b^2+c^2=40$ ...(*ii*)

Subtracting, we get : $a^2-c^2=5 \Rightarrow (a+c)(a-c)=5.$

$\therefore \quad (a+c)=5$ and $(a-c)=1.$

Solving, we get : $a=3$, $c=2$. Putting $c=2$ in (*ii*), we get $b=6$.

99. $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}=k$ (say). Then, $a=3k$, $b=4k$, $c=7k$.

$$\therefore \quad \frac{a+b+c}{c}=\frac{3k+4k+7k}{7k}=\frac{14k}{7k}=2.$$

100. $3x+7=7x+5 \Rightarrow 7x-3x=2 \Rightarrow 4x=2 \Rightarrow x=\frac{1}{2}.$

Now, $3x+7=x^2+\mathrm{P} \Rightarrow \frac{3}{2}+7=\frac{1}{4}+\mathrm{P} \Rightarrow \mathrm{P}=\frac{17}{2}-\frac{1}{4}=\frac{33}{4}=8\frac{1}{4}.$

101. $\frac{2a+b}{a+4b}=3 \Rightarrow 2a+b=3(a+4b) \Rightarrow a=-11b.$

$$\therefore \quad \frac{a+b}{a+2b}=\frac{-11b+b}{-11b+2b}=\frac{-10b}{-9b}=\frac{10}{9}.$$

102. $(2a + 3b)(2c - 3d) = (2a - 3b)(2c + 3d)$

$$\Rightarrow \frac{(2a+3b)}{(2a-3b)} = \frac{(2c+3d)}{(2c-3d)} \Rightarrow \frac{2\left(\frac{a}{b}\right)+1}{2\left(\frac{a}{b}\right)-1} = \frac{2\left(\frac{c}{d}\right)+1}{2\left(\frac{c}{d}\right)-1} \Rightarrow \frac{a}{b} = \frac{c}{d}.$$

103. $(a + b + 2c + 3d)(a - b - 2c + 3d) = (a - b + 2c - 3d)(a + b - 2c - 3d)$

$\Rightarrow [(a + b) + (2c + 3d)][(a - b) - (2c - 3d)]$

$= [(a - b) + (2c - 3d)][(a + b) - (2c + 3d)]$

$\Rightarrow (a + b)(a - b) - (a + b)(2c - 3d) + (a - b)(2c + 3d) - (2c + 3d)(2c - 3d)$

$= (a - b)(a + b) - (a - b)(2c + 3d) + (a + b)(2c - 3d) - (2c + 3d)(2c - 3d)$

$\Rightarrow (a + b)(2c - 3d) = (a - b)(2c + 3d)$

$\Rightarrow 2ac - 3ad + 2bc - 3bd = 2ac + 3ad - 2bc - 3bd$

$\Rightarrow 4bc = 6ad \Rightarrow 2bc = 3ad.$

104. Given exp. $= \dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{(3/2)}}} = \dfrac{1}{2+\dfrac{1}{2+\dfrac{2}{3}}} = \dfrac{1}{2+\dfrac{1}{(8/3)}} = \dfrac{1}{2+\dfrac{3}{8}} = \dfrac{1}{(19/8)} = \dfrac{8}{19}.$

105. $x = 2 - \dfrac{1}{1+\dfrac{1}{(13/4)}} = 2 - \dfrac{1}{1+\dfrac{4}{13}} = 2 - \dfrac{1}{(17/13)} = 2 - \dfrac{13}{17} = \dfrac{21}{17}.$

106. $x = \dfrac{2+\dfrac{1}{(19/5)}}{2+\dfrac{1}{3+\dfrac{1}{(5/4)}}} = \dfrac{2+\dfrac{5}{19}}{2+\dfrac{1}{3+\dfrac{4}{5}}} = \dfrac{2+\dfrac{5}{19}}{2+\dfrac{1}{(19/5)}} = \dfrac{2+\dfrac{5}{19}}{2+\dfrac{5}{19}} = 1.$

107. Given exp. $= 8 - 8 \times \dfrac{\dfrac{11}{5} - \dfrac{9}{7}}{2 - \dfrac{1}{(35/6)}} = 8 - 8 \times \dfrac{\dfrac{32}{35}}{2 - \dfrac{6}{35}} = 8 - 8 \times \dfrac{32}{35} \times \dfrac{35}{64} = 8 - 4 = 4.$

108. Given exp. $= \dfrac{2}{2+\dfrac{2}{3+\dfrac{2}{(11/3)}} \times 0.39} = \dfrac{2}{2+\dfrac{2}{3+\dfrac{6}{11}} \times 0.39} = \dfrac{2}{2+\dfrac{2}{(39/11)} \times 0.39}$

$= \dfrac{2}{2+\dfrac{22}{39} \times \dfrac{39}{100}} = \dfrac{2}{2+\dfrac{22}{100}} = \dfrac{2}{2+\dfrac{11}{50}} = \dfrac{2}{(111/50)} = \dfrac{100}{111}.$

109. Given exp. $= \dfrac{1}{1+\dfrac{\dfrac{2}{3}}{\dfrac{5}{3}+\dfrac{\dfrac{8}{9}}{(1/3)}}} = \dfrac{1}{1+\dfrac{2/3}{\dfrac{5}{3}+\dfrac{8}{9} \times 3}} = \dfrac{1}{1+\dfrac{2/3}{(13/3)}} = \dfrac{1}{1+\dfrac{2}{13}} = \dfrac{13}{15}.$

110. $2 + \dfrac{1}{x+\dfrac{1}{y+\dfrac{1}{z}}} = \dfrac{37}{13} = 2\dfrac{11}{13} = 2 + \dfrac{11}{13} \Rightarrow \dfrac{1}{x+\dfrac{1}{y+\dfrac{1}{z}}} = \dfrac{11}{13} \Rightarrow x + \dfrac{1}{y+\dfrac{1}{z}} = \dfrac{13}{11}$

$\Rightarrow x + \dfrac{1}{y+\dfrac{1}{z}} = 1 + \dfrac{2}{11} \Rightarrow x = 1,\ y + \dfrac{1}{z} = \dfrac{11}{2} = 5\dfrac{1}{2} = 5 + \dfrac{1}{2} \Rightarrow x = 1,\ y = 5,\ z =$

111. $x = y \Leftrightarrow 1 - q = 2q + 1 \Leftrightarrow 3q = 0 \Leftrightarrow q = 0.$

112. $\frac{x}{5} - \frac{x}{6} = 4 \Leftrightarrow \frac{6x - 5x}{30} = 4 \Leftrightarrow x = 120.$

113. $\frac{3x}{2y} = \frac{21}{22} \Rightarrow \frac{x}{y} = \left(\frac{21}{22} \times \frac{2}{3}\right) = \frac{7}{11} \Rightarrow x = \frac{7}{11}y.$

$4x + 5y = 83 \Rightarrow 4 \times \frac{7}{11}y + 5y = 83 \Rightarrow \frac{28}{11}y + 5y = 83 \Rightarrow 83y = 83 \times 11 \Rightarrow y = 11.$

$\therefore \quad x = \frac{7}{11}y = \left(\frac{7}{11} \times 11\right) = 7.$

So, $y - x = 11 - 7 = 4.$

114. $3x + y = 19$ ...(*i*) and $x - y = 9$ ...(*ii*)

Adding (*i*) and (*ii*), we get : $4x = 28$ or $x = 7$. Putting $x = 7$ in (*i*), we get : $y = -2$.

115. $a + b = 5$ ...(*i*) and $3a + 2b = 20$ ...(*ii*)

Multiplying (*i*) by 2 and subtracting from (*ii*), we get : $a = 10$.

Putting $a = 10$ in (*i*), we get : $b = -5$.

$\therefore \quad (3a + b) = 3 \times 10 + (-5) = 30 - 5 = 25.$

116. $(2p + 3q) + (2p - q) = 18 + 2 \Rightarrow 4p + 2q = 20 \Rightarrow 2(2p + q) = 20$

$\Rightarrow 2p + q = 10.$

117. $2x + y = 5$ ...(*i*) and $3x - 4y = 2$ ...(*ii*)

Multiplying (*i*) by 4 and adding (*ii*) to it, we get : $11x = 22$ or $x = 2$.

Putting $x = 2$ in (*i*), we get : $y = 1$. So, $2xy = 2 \times 2 \times 1 = 4$.

118. $3x - 5y = 5$ ...(*i*) and $\frac{x}{x + y} = \frac{5}{7} \Rightarrow 7x = 5x + 5y \Rightarrow 2x - 5y = 0$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get : $x = 5$.

Putting $x = 5$ in (*i*), we get : $y = 2$. So, $x - y = 5 - 2 = 3$.

119. $4x + 3y = 18xy$ ...(*i*) and $2x - 5y = -4xy$ ...(*ii*)

Dividing (*i*) and (*ii*) by xy, we get : $\frac{3}{x} + \frac{4}{y} = 18$ ...(*iii*) and $\frac{5}{x} - \frac{2}{y} = 4$ ...(*iv*)

Multiplying (*iv*) by 2 and adding (*iii*) to it, we get : $\frac{13}{x} = 26$ or $x = \frac{1}{2}$.

Putting $x = \frac{1}{2}$ in (*iii*), we get : $y = \frac{1}{3}$.

120. $2x + y = 17$ ...(*i*) ; $y + 2z = 15$ ...(*ii*) and $x + y = 9$ ...(*iii*)

Subtracting (*iii*) from (*i*), we get : $x = 8$.

Putting $x = 8$ in (*i*), we get : $y = 1$. Putting $y = 1$ in (*ii*), we get : $2z = 14$ or $z = 7$.

$\therefore \quad 4x + 3y + z = 4 \times 8 + 3 \times 1 + 7 = 42.$

121. $3x - 4y + z = 7$ ...(*i*) ; $2x + 3y - z = 19$ ...(*ii*) and $x + 2y + 2z = 24$ ...(*iii*)

Adding (*i*) and (*ii*), we get : $5x - y = 26$ ...(*iv*)

Subtracting (*i*) from (*ii*) and adding to (*iii*), we get : $9y = 36$ or $y = 4$.

Putting $y = 4$ in (*iv*), we get : $5x = 30$ or $x = 6$.

Putting $x = 6$, $y = 4$ in (*iii*), we get : $2z = 10$ or $z = 5$.

122. $2x + y = 15$ ...(*i*) ; $2y + z = 25$ ...(*ii*) and $2z + x = 26$ ...(*iii*)

Adding (*i*), (*ii*) and (*iii*), we get : $3(x + y + z) = 66$ or $x + y + z = 22$ ...(*iv*)

From (*ii*), we have : $y = \frac{25 - z}{2}$. From (*iii*), we have : $x = 26 - 2z$.

$\therefore \quad (26 - 2z) + \left(\frac{25 - z}{2}\right) + z = 22 \Leftrightarrow 77 - 3z = 44 \Leftrightarrow 3z = 33 \Leftrightarrow z = 11.$

123. $2x + 3y = 31$ ...(*i*); $y - z = 4$ ...(*ii*) and $x + 2z = 11$ ...(*iii*)
Multiplying (*iii*) by 2 and subtracting from (*i*), we get : $3y - 4z = 9$ ...(*iv*)
Solving (*ii*) and (*iv*), we get : $y = 7$, $z = 3$. Putting $y = 7$ in (*i*), we get : $x = 5$.
$\therefore \quad x + y + z = (5 + 7 + 3) = 15.$

124. Given exp. $= \left(\frac{3}{4} \times \frac{4}{3} \times \frac{5}{3} \times \frac{3}{5} \times \frac{13}{7} \times \frac{1}{13}\right) = \frac{1}{7}.$

125. Given exp. $= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \ldots\ldots \times \frac{(n-1)}{n} = \frac{1}{n}.$

126. Given exp. $= \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots\ldots \times \frac{121}{120} = \frac{121}{2} = 60.5.$

127. Given exp. $= \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \ldots\ldots \times \frac{1003}{1001} = \frac{1003}{3}.$

128. Given exp. $= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \ldots\ldots + \left(\frac{1}{11} - \frac{1}{12}\right) = \left(1 - \frac{1}{12}\right) = \frac{11}{12}.$

129. Clearly, sum of first 6 terms is zero. So, sum of first 30 terms = 0.
$\therefore$ Required sum $= \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{2} - \frac{1}{3}\right) = -\frac{1}{4}.$

130. Given exp. $= \left(1000 - \frac{4}{999}\right) \times 999 = 999000 - 4 = 998996.$

131. Given exp.

$$= \left(1000 - \frac{6}{7}\right) + \left(1000 - \frac{5}{7}\right) + \left(1000 - \frac{4}{7}\right) + \left(1000 - \frac{3}{7}\right) + \left(1000 - \frac{2}{7}\right) + \left(1000 - \frac{1}{7}\right)$$

$$= 6000 - \left(\frac{6}{7} + \frac{5}{7} + \frac{4}{7} + \frac{3}{7} + \frac{2}{7} + \frac{1}{7}\right) = 6000 - \frac{21}{7} = 6000 - 3 = 5997.$$

132. Given exp. $= \frac{4 \times 3^3 + 3^2 + 3 + 1}{4 \times 3^3} = \frac{108 + 9 + 3 + 1}{108} = \frac{121}{108}.$

133. Given exp. $= \frac{4 \cdot 5 \cdot 6 + 5 \cdot 6 + 2 \cdot 6 + 2 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = \frac{120 + 30 + 12 + 6}{720} = \frac{168}{720} = \frac{7}{30}.$

134. Given exp. $= \left(\frac{1}{1^2} - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{3^2} - \frac{1}{4^2}\right) + \left(\frac{1}{4^2} - \frac{1}{5^2}\right) + \ldots\ldots + \left(\frac{1}{9^2} - \frac{1}{10^2}\right)$

$$= \left(\frac{1}{1^2} - \frac{1}{10^2}\right) = \left(1 - \frac{1}{100}\right) = \frac{99}{100}.$$

135. Number of pieces $= \left(\frac{42.5 \times 100}{85}\right) = \frac{4250}{85} = 50.$

136. Income after 1 year = Rs. (4×2^1) lakhs.
Income after 2 years = Rs. $(4 \times 2 \times 2)$ lakhs = Rs. (4×2^2) lakhs
$\therefore$ Income after 5 years = Rs. (4×2^5) lakhs = Rs. 128 lakhs = Rs. 1.28 crores.

137. Total number of children = $(30 \times 16) = 480$.
$\therefore$ Number of columns of 24 children each $= \left(\frac{480}{24}\right) = 20.$

138. Original number of sections = $(16 - 3) = 13$.
Original number of students = $(24 \times 13) = 312$.
Present number of students = $(21 \times 16) = 336$.
Number of new students admitted = $(336 - 312) = 24$.

139. Time between 10 a.m. and 13.27 hours = 3 hrs. 27 min. = 207 min.
For three periods in between free time = 15 min.
Remaining time = (207 − 15) min. = 192 min.

$\therefore$ Duration of each of the 4 periods = $\left(\frac{192}{4}\right)$ min. = 48 min.

140.

Hrs.	Min.	Sec.
3	17	49
(−) 1	54	50
1	22	59

Total time = (1 × 60 + 22) min. + 59 sec. = (82 × 60 + 59) sec. = 4979 sec.

$\therefore$ Number of times the light is seen = $\left(\frac{4979}{13}+1\right)$ = 384.

141. Money earned in 2 days = Rs. (20 − 15) = Rs. 5.

Money earned in 16 days = Rs. $\left(\frac{5}{2}\times 16\right)$ = Rs. 40.

On 17th day, money in hand = Rs. (40 + 20) = Rs. 60.

142. Required cost = Rs. $[1000 \times x + (z - 1000) \times y]$ = Rs. $(1000x + zy - 1000y)$
= Rs. $[1000\ (x - y) + yz]$.

143. 26 trees have 25 gaps between them. Hence, required distance = $\left(\frac{225}{25}\right)$ m = 9 m.

144. Let the number be x. Then, $52x - 25x = 324 \Leftrightarrow 27x = 324 \Leftrightarrow x = 12$.

145. Among the given numbers, only 60489 is a multiple of 423.

146. Let the monthly salary of a man be Rs. x.
Then, monthly salary of a woman = Rs. $(x + 500)$.
$\therefore\ 4x + 2\ (x + 500) = 46000 \Leftrightarrow 6x = 45000 \Leftrightarrow x = 7500$.
Monthly salary of a woman = $x + 500$ = Rs. 8000.

147. Let marks in History = x. Then, marks in English = $\frac{5}{2}x$.

$\therefore\ x + \frac{5}{2}x = 140 \Leftrightarrow \frac{7}{2}x = 140 \Leftrightarrow x = \left(\frac{140\times 2}{7}\right) = 40$.

Hence, marks in English = $\frac{5}{2}x = \left(\frac{5}{2}\times 40\right) = 100$.

148. Let the number of pineapples and watermelons be x and y respectively.

Then, $7x + 5y = 38$ or $5y = (38 - 7x)$ or $y = \frac{38 - 7x}{5}$.

Clearly, y is a whole number, only when $(38 - 7x)$ is divisible by 5. This happens when $x = 4$.

149. Let number of boys = x. Then, number of girls = $5x$.
Total number of children = $(x + 5x) = 6x$.
Thus, the total number of children must be a multiple of 6.

150. Let F and C denote the temperatures in Fahrenheit and Celsius respectively.

Then, $\frac{F - 32}{212 - 32} = \frac{C - 0}{100 - 0} \Leftrightarrow \frac{F - 32}{180} = \frac{C}{100}$.

If C = 35, then F = $\left(\frac{35}{100}\times 180\right) + 32 = 63 + 32 = 95$.

151. Let D's share = Rs. x. Then, C's share = Rs. x.
B's share = Rs. $(x + 125)$. A's share = Rs. $(x + x + 125)$ = Rs. $(2x + 125)$
$\therefore$ $(2x + 125) + (x + 125) + x + x = 750 \Leftrightarrow 5x = 500 \Leftrightarrow x = 100$.
Hence, A's share = $2x + 125$ = Rs. $(2 \times 100 + 125)$ = Rs. 325.

152. Let Gagan's share = Rs. x.
Then, Sachin's share = Rs. $\left(\frac{x}{5}\right)$ and Rohit's share = Rs. $\left(\frac{2x}{5}\right)$.
$\therefore$ $\frac{2x}{5} + \frac{x}{5} + x = 1000 \Leftrightarrow 8x = 5000 \Leftrightarrow x = 625$.

153. Total number of digits = (No. of digits in 1-digit page nos. + No. of digits in 2-digit page nos. + No. of digits in 3-digit page nos.)
$= (1 \times 9 + 2 \times 90 + 3 \times 267) = (9 + 180 + 801) = 990$.

154. No. of digits in 1-digit page nos. = $1 \times 9 = 9$.
No. of digits in 2-digit page nos. = $2 \times 90 = 180$.
No. of digits in 3-digit page nos. = $3 \times 900 = 2700$.
No. of digits in 4-digit page nos. = $3189 - (9 + 180 + 2700) = 3189 - 2889 = 300$.
$\therefore$ No. of pages with 4-digit page nos. = $\left(\frac{300}{4}\right) = 75$.
Hence, total number of pages = $(999 + 75) = 1074$.

155. Each row contains 12 plants.
Leaving 2 corner plants, 10 plants in between have (10×2) metres and 1 metre on each side is left.
$\therefore$ Length = $(20 + 2)$ m = 22 m.

156. Required fraction = $\frac{1 \text{ sec.}}{1 \text{ hr.}} = \frac{1 \text{ sec.}}{(1 \times 60 \times 60) \text{ sec.}} = \frac{1}{3600}$.

157. Height at the third bounce = $\left[32 \times \left(\frac{3}{4}\right)^3\right]$ m = $\left(32 \times \frac{27}{64}\right)$ m = $\frac{27}{2}$ m = $13\frac{1}{2}$ m.

158. Suppose Sanket earns Rs. x in each of the other eleven months.
Then, Sanket's earning in March = Rs. $(2x)$.
Sanket's annual earning = Rs. $(11x + 2x)$ = Rs. $(13x)$.
$\therefore$ Required fraction = $\frac{2x}{13x} = \frac{2}{13}$.

159. Let the capacity of the tank be x litres. Then, $\frac{1}{3}x = 80 \Leftrightarrow x = 240 \Leftrightarrow \frac{1}{2}x = 120$.

160. Distance travelled on foot = $\left[\frac{7}{2} - \left(\frac{5}{3} + \frac{7}{6}\right)\right]$ km = $\left(\frac{7}{2} - \frac{17}{6}\right)$ km = $\frac{2}{3}$ km.
$\therefore$ Required fraction = $\frac{(2/3)}{(7/2)} = \left(\frac{2}{3} \times \frac{2}{7}\right) = \frac{4}{21}$.

161. Let the required fraction be x. Then,
$\frac{4}{7}x + \frac{4}{7} = \frac{15}{14} \Leftrightarrow \frac{4}{7}x = \left(\frac{15}{14} - \frac{4}{7}\right) = \frac{7}{14} = \frac{1}{2} \Leftrightarrow x = \left(\frac{1}{2} \times \frac{7}{4}\right) = \frac{7}{8}$.

162. Required fraction = $\frac{\frac{2}{3} \text{ of } \frac{1}{4} \text{ of Rs. } 25.20}{\frac{3}{2} \text{ of Rs. } 36} = \frac{\text{Rs. } 4.20}{\text{Rs. } 54} = \frac{42}{540} = \frac{7}{90}$.

163. Let the length of longer piece be x cm. Then, length of shorter piece $= \left(\frac{2}{5}x\right)$ cm.

$\therefore \quad x + \frac{2}{5}x = 70 \Leftrightarrow \frac{7x}{5} = 70 \Leftrightarrow x = \left(\frac{70 \times 5}{7}\right) = 50.$

Hence, length of shorter piece $= \frac{2}{5}x = \left(\frac{2}{5} \times 50\right)$ cm $= 20$ cm.

164. Let the whole amount be Rs. x. Then, A's share $=$ Rs. $\left(\frac{3}{16}x\right)$; B's share $=$ Rs. $\left(\frac{x}{4}\right)$;

and C's share $=$ Rs. $\left[x - \left(\frac{3x}{16} + \frac{x}{4}\right)\right] =$ Rs. $\left(\frac{9x}{16}\right)$.

$\therefore \quad \frac{9x}{16} = 81 \Leftrightarrow x = \left(\frac{81 \times 16}{9}\right) = 144.$

Hence, B's share $=$ Rs. $\left(\frac{144}{4}\right) =$ Rs. 36.

165. Green portion $= \left[1 - \left(\frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40} + \frac{1}{50} + \frac{1}{60}\right)\right]$

$= \left[1 - \frac{1}{10}\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right)\right] = 1 - \frac{1}{10} \times \frac{147}{60} = 1 - \frac{147}{600} = \frac{453}{600}.$

Let the length of the pole be x metres.

Then, $\frac{453}{600}x = 12.08 \Leftrightarrow x = \left(\frac{12.08 \times 600}{453}\right) = 16.$

166. Let the number be x. Then,

$\frac{3}{4}x - \frac{3}{14}x = 150 \Leftrightarrow 21x - 6x = 150 \times 28 \Leftrightarrow 15x = 150 \times 28 \Leftrightarrow x = 280.$

167. Let the sum be Rs. x. Then,

$\frac{8}{3}x - \frac{3}{8}x = 55 \Leftrightarrow 64x - 9x = 55 \times 24 \Leftrightarrow x = \left(\frac{55 \times 24}{55}\right) = 24.$

$\therefore$ Correct answer $=$ Rs. $\left(\frac{3}{8} \times 24\right) =$ Rs. 9.

168. Let the fraction be $\frac{a}{b}$. Then,

$\left(\frac{a}{b} \times \frac{a}{b}\right) \div \frac{b}{a} = \frac{512}{27} \Leftrightarrow \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} = \frac{512}{27} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{8}{3}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{8}{3} = 2\frac{2}{3}.$

169. Maximum internal assessment score $= \left(\frac{47}{50} \times 10\right) = 9.4.$

Minimum internal assessment score $= \left(\frac{14}{50} \times 10\right) = 2.8.$

$\therefore$ Required difference $= (9.4 - 2.8) = 6.6.$

170. Let savings in N.S.C. and P.P.F. be Rs. x and Rs. $(150000 - x)$ respectively. Then,

$\frac{1}{3}x = \frac{1}{2}(150000 - x) \Leftrightarrow \frac{x}{3} + \frac{x}{2} = 75000 \Leftrightarrow \frac{5x}{6} = 75000 \Leftrightarrow x = \left(\frac{75000 \times 6}{5}\right) = 90000.$

$\therefore$ Savings in Public Provident Fund $=$ Rs. $(150000 - 90000) =$ Rs. 60000.

171. Let there be $(x + 1)$ members. Then,

Father's share $= \frac{1}{4}$, share of each other member $= \frac{3}{4x}$.

$\therefore \quad 3\left(\frac{3}{4x}\right) = \frac{1}{4} \Leftrightarrow 4x = 36 \Leftrightarrow x = 9.$

Hence, total number of family members = 10.

172. Let salary = Rs. x. Then, tips = Rs. $\left(\frac{5}{4}x\right)$.

Total income = Rs. $\left(x + \frac{5}{4}x\right)$ = Rs. $\left(\frac{9x}{4}\right)$.

$\therefore$ Required fraction $= \left(\frac{5x}{4} \times \frac{4}{9x}\right) = \frac{5}{9}$.

173. Let C's share = Rs. x. Then, B's share = Rs. $\left(\frac{x}{4}\right)$. A's share = Rs. $\left(\frac{2}{3} \times \frac{x}{4}\right)$ = Rs. $\frac{x}{6}$.

$\therefore \quad \frac{x}{6} + \frac{x}{4} + x = 1360 \Leftrightarrow \frac{17x}{12} = 1360 \Leftrightarrow x = \left(\frac{1360 \times 12}{17}\right)$ = Rs. 960.

Hence, B's share = Rs. $\left(\frac{960}{4}\right)$ = Rs. 240.

174. Let Tanya's share = Rs. x. Then, Veena's share = Rs. $\left(\frac{x}{2}\right)$.

Amita's share = Rs. $\left(\frac{2}{3} \times \frac{x}{2}\right)$ = Rs. $\left(\frac{x}{3}\right)$. Total bill = Rs. $\left(x + \frac{x}{2} + \frac{x}{3}\right)$ = Rs. $\left(\frac{11x}{6}\right)$.

$\therefore$ Required fraction $= \left(\frac{x}{2} \times \frac{6}{11x}\right) = \frac{3}{11}$.

175. Let the capacity of the tank be x litres. Then, $\frac{1}{4}x = 135 \Leftrightarrow x = 135 \times 4 = 540$.

$\therefore$ Required fraction $= \left(\frac{180}{540}\right) = \frac{1}{3}$.

176. Let the capacity of the tank be x litres.

Then, $\frac{6}{7}x - \frac{2}{5}x = 16 \Leftrightarrow 30x - 14x = 16 \times 35 \Leftrightarrow 16x = 560 \Leftrightarrow x = 35$.

177. Let the capacity of the bucket be x litres. Then,

Capacity of 1 large bottle $= \frac{x}{4}$; Capacity of 1 small bottle $= \frac{x}{7}$.

Fluid left in large bottle $= \left(\frac{x}{4} - \frac{x}{7}\right) = \frac{3x}{28}$.

$\therefore$ Required fraction $= \left(\frac{3x/28}{x/4}\right) = \left(\frac{3x}{28} \times \frac{4}{x}\right) = \frac{3}{7}$.

178. Let the capacity of 1 bucket = x. Then, capacity of tank = $25x$.

New capacity of bucket $= \frac{2}{5}x$.

$\therefore$ Required number of buckets $= \frac{25x}{(2x/5)} = \left(25x \times \frac{5}{2x}\right) = \frac{125}{2} = 62\frac{1}{2}$.

179. Suppose initially Peter had Rs. x. Then,

Amount received by Michael = Rs. $\left(\frac{x}{4}\right)$.

Amount remaining with Peter = Rs. $\left(x - \frac{x}{4}\right)$ = Rs. $\left(\frac{3x}{4}\right)$.

Amount received by Sam = Rs. $\left(\frac{1}{2} \times \frac{x}{4}\right)$ = Rs. $\left(\frac{x}{8}\right)$.

$\therefore \quad \frac{3x}{4} - \frac{x}{8} = 500 \Leftrightarrow 5x = 4000 \Leftrightarrow x = 800.$

Hence, amount received by Michael = $(x/4)$ = Rs. 200.

180. A's share = $\frac{1}{3}$. Remainder = $\left(1 - \frac{1}{3}\right) = \frac{2}{3}$.

B's share = $\frac{2}{5}$ of $\frac{2}{3} = \frac{4}{15}$. Rest = $\left(\frac{2}{3} - \frac{4}{15}\right) = \frac{6}{15} = \frac{2}{5}$.

C's share = D's share = $\frac{1}{2}$ of $\frac{2}{5} = \frac{1}{5}$.

181. Part read on first day = $\frac{3}{8}$. Remaining part = $\left(1 - \frac{3}{8}\right) = \frac{5}{8}$.

Part read on second day = $\frac{4}{5}$ of $\frac{5}{8} = \frac{1}{2}$. Unread part = $\left[1 - \left(\frac{3}{8} + \frac{1}{2}\right)\right] = \frac{1}{8}$.

Let the number of pages be x. Then, $\frac{1}{8}x = 30$ or $x = 30 \times 8 = 240$.

182. Wife's share = $\frac{1}{2}$. Remaining part = $\left(1 - \frac{1}{2}\right) = \frac{1}{2}$.

Share of 3 sons = $\left(\frac{2}{3} \text{ of } \frac{1}{2}\right) = \frac{1}{3}$. Remaining part = $\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{6}$.

Each daughter's share = $\frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$.

Let the total money be Rs. x. Then, $\frac{1}{24}x = 20000 \Leftrightarrow x = 20000 \times 24 = 480000$.

$\therefore$ Each son's share = Rs. $\left[\frac{1}{3} \times \left(\frac{1}{3} \times 480000\right)\right]$ = Rs. 53,333.33.

183. Out of 5 girls, 1 took part in fete. Out of 8 boys, 1 took part in fete.

$\therefore$ Out of 13 students, 2 took part in fete.

Hence, $\frac{2}{13}$ of the total number took part in fete.

184. French men = $\frac{1}{5}$; French women = $\left(\frac{1}{5} + \frac{2}{3} \times \frac{1}{5}\right) = \frac{5}{15} = \frac{1}{3}$.

French people = $\left(\frac{1}{5} + \frac{1}{3}\right) = \frac{8}{15}$. $\therefore$ Not-French = $\left(1 - \frac{8}{15}\right) = \frac{7}{15}$.

185. Girls = $\frac{3}{5}$; Boys = $\left(1-\frac{3}{5}\right)=\frac{2}{5}$.

Fraction of students absent = $\frac{2}{9}$ of $\frac{3}{5}+\frac{1}{4}$ of $\frac{2}{5}=\frac{6}{45}+\frac{1}{10}=\frac{21}{90}=\frac{7}{30}$.

$\therefore$ Fraction of students present = $\left(1-\frac{7}{30}\right)=\frac{23}{30}$.

186. Number of boys who participate = 100.

$\therefore$ $\frac{1}{3}$ of boys = 100 or total number of boys = 300.

Number of girls who participate = 200.

$\therefore$ $\frac{1}{2}$ of girls = 200 or total number of girls = 400.

Hence, total number of students = (300 + 400) = 700.

187. Let the number of votes cast be x. Then, number of votes required = $\frac{3x}{4}$.

Counted votes = $\frac{2x}{3}$. Uncounted votes = $\left(x-\frac{2x}{3}\right)=\frac{x}{3}$.

Votes won by the candidate = $\frac{5}{6}$ of $\frac{3x}{4}=\frac{5x}{8}$.

Remaining votes required = $\left(\frac{3x}{4}-\frac{5x}{8}\right)=\frac{x}{8}$.

$\therefore$ Required fraction = $\frac{(x/8)}{(x/3)}=\left(\frac{x}{8}\times\frac{3}{x}\right)=\frac{3}{8}$.

188. Let the total number of staff members be x.

Then, the number who can type or take shorthand = $\left(x-\frac{3x}{4}\right)=\frac{x}{4}$.

Let A and B represent the sets of persons who can type and take shorthand respectively.

Then, $n(A\cup B)=\frac{x}{4}$, $n(A)=\frac{x}{5}$ and $n(B)=\frac{x}{3}$.

$n(A\cap B)=n(A)+n(B)-n(A\cup B)=\left(\frac{x}{5}+\frac{x}{3}-\frac{x}{4}\right)=\left(\frac{12x+20x-15x}{60}\right)=\frac{17x}{60}$.

189. Hire charges = Rs. $\left(60\times4+60\times5+\frac{8}{5}\times200\right)$ = Rs. 860.

Suppose Rohit had Rs. x with him initially. Then, $x-860=\frac{1}{4}\times860 \Leftrightarrow x=1075$.

190. Let the total number of shots be x. Then,

Shots fired by A = $\frac{5}{8}x$; Shots fired by B = $\frac{3}{8}x$.

Killing shots by A = $\frac{1}{3}$ of $\frac{5}{8}x=\frac{5x}{24}$; Shots missed by B = $\frac{1}{2}$ of $\frac{3}{8}x=\frac{3}{16}x$.

$\therefore$ $\frac{3x}{16}=27$ or $x=\left(\frac{27\times16}{3}\right)=144$. Birds killed by A = $\frac{5x}{24}=\left(\frac{5}{24}\times144\right)=30$.

191. Number of alterations required in 1 shirt = $\left(\frac{2}{3}+\frac{3}{4}+\frac{4}{5}\right)=\frac{133}{60}$.

$\therefore$ Number of alterations required in 60 shirts = $\left(\frac{133}{60}\times60\right)=133$.

192. Let the largest fraction be x and the smallest be y. Then, $\frac{x}{y} = \frac{7}{6}$ or $y = \frac{6}{7}x$.

Let the middle one be z. Then, $x + \frac{6}{7}x + z = \frac{59}{24}$ or $z = \left(\frac{59}{24} - \frac{13x}{7}\right)$.

$\therefore \quad \frac{59}{24} - \frac{13x}{7} + \frac{1}{3} = \frac{7}{6} \Leftrightarrow \frac{13x}{7} = \frac{59}{24} + \frac{1}{3} - \frac{7}{6} = \frac{39}{24} \Leftrightarrow x = \left(\frac{39}{24} \times \frac{7}{13}\right) = \frac{7}{8}$.

So, $x = \frac{7}{8}$, $y = \frac{6}{7} \times \frac{7}{8} = \frac{3}{4}$ and $z = \frac{59}{24} - \frac{13}{7} \times \frac{7}{8} = \frac{20}{24} = \frac{5}{6}$.

Hence, the fractions are $\frac{7}{8}$, $\frac{5}{6}$ and $\frac{3}{4}$.

193. Suppose each tube contains x grams initially. Then,

$$4\left[\frac{1}{3}(x+20)\right] = x + \frac{2}{3}(x+20) \Leftrightarrow \frac{2}{3}(x+20) = x \Leftrightarrow \frac{x}{3} = \frac{40}{3} \Leftrightarrow x = 40.$$

194. Let the total number of apples be x. Then,

Apples sold to 1st customer $= \left(\frac{x}{2} + 1\right)$. Remaining apples $= x - \left(\frac{x}{2} + 1\right) = \left(\frac{x}{2} - 1\right)$.

Apples sold to 2nd customer $= \frac{1}{3}\left(\frac{x}{2} - 1\right) + 1 = \frac{x}{6} - \frac{1}{3} + 1 = \left(\frac{x}{6} + \frac{2}{3}\right)$.

Remaining apples $= \left(\frac{x}{2} - 1\right) - \left(\frac{x}{6} + \frac{2}{3}\right) = \left(\frac{x}{2} - \frac{x}{6}\right) - \left(1 + \frac{2}{3}\right) = \left(\frac{x}{3} - \frac{5}{3}\right)$.

Apples sold to 3rd customer $= \frac{1}{5}\left(\frac{x}{3} - \frac{5}{3}\right) + 1 = \left(\frac{x}{15} + \frac{2}{3}\right)$.

Remaining apples $= \left(\frac{x}{3} - \frac{5}{3}\right) - \left(\frac{x}{15} + \frac{2}{3}\right) = \left(\frac{x}{3} - \frac{x}{15}\right) - \left(\frac{5}{3} + \frac{2}{3}\right) = \left(\frac{4x}{15} - \frac{7}{3}\right)$.

$\therefore \quad \frac{4x}{15} - \frac{7}{3} = 3 \Leftrightarrow \frac{4x}{15} = \frac{16}{3} \Leftrightarrow x = \left(\frac{16}{3} \times \frac{15}{4}\right) = 20$.

195. Given exp. $= \frac{(a+b)^2 + (a-b)^2}{a^2 + b^2}$, where $a = 856$, $b = 167$

$$= \frac{2(a^2 + b^2)}{(a^2 + b^2)} = 2.$$

196. Given exp. $= \frac{(a+b)^2 - (a-b)^2}{ab} = \frac{4ab}{ab} = 4$ (where $a = 469$, $b = 174$).

197. $2ab = (a^2 + b^2) - (a - b)^2 = 29 - 9 = 20 \Rightarrow ab = 10$.

198. $\frac{x^2 - 1}{x + 1} = 4 \Leftrightarrow \frac{(x+1)(x-1)}{x+1} = 4 \Leftrightarrow x - 1 = 4 \Leftrightarrow x = 5$.

199. If $a = 3\frac{2}{3}$, $b = 2\frac{1}{2}$, $c = 4\frac{3}{4}$, $d = 3\frac{1}{3}$, then

$$\text{Given exp.} = \frac{(a^2 - b^2)}{(c^2 - d^2)} + \frac{(a-b)}{(c-d)} - \frac{(a^2 - b^2)}{(c^2 - d^2)} \times \frac{(c-d)}{(a-b)} - \frac{(a+b)}{(c+d)}$$

$$= \frac{3\frac{2}{3} + 2\frac{1}{2}}{4\frac{3}{4} + 3\frac{1}{3}} = \frac{\frac{11}{3} + \frac{5}{2}}{\frac{19}{4} + \frac{10}{3}} = \frac{37}{6} \times \frac{12}{97} = \frac{74}{97}.$$

200. Given exp. $= \dfrac{a^2-b^2}{a+b} = a-b = \left(1+\dfrac{1}{1+\dfrac{1}{100}}\right)-\left(1-\dfrac{1}{1+\dfrac{1}{100}}\right)$

$= 2\times\dfrac{1}{(101/100)} = 2\times\dfrac{100}{101} = \dfrac{200}{101}.$

201. $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow\ 2(ab+bc+ca) = (a+b+c)^2-(a^2+b^2+c^2) = 169-69 = 100$

$\Rightarrow\ ab+bc+ca = 50.$

202. Given : $x^2+y^2+z^2-64 = -2(xy-yz-zx)$ ...(*i*)

Now, $[x+y+(-z)]^2 = x^2+y^2+z^2+2(xy-yz-zx)$

$\Rightarrow\quad (3z-z)^2 = x^2+y^2+z^2+2(xy-yz-zx)$

$\Rightarrow\quad -2(xy-yz-zx) = (x^2+y^2+z^2)-(2z)^2$ ...(*ii*)

From (*i*) and (*ii*), we get : $(2z)^2 = 64 \Leftrightarrow 4z^2 = 64 \Leftrightarrow z^2 = 16 \Leftrightarrow z = 4.$

203. Given exp. $= \left(\dfrac{a^3+b^3}{a^2+b^2-ab}\right) = (a+b)$, where $a = 785$, $b = 435$

$= (785+435) = 1220.$

204. Given exp. $= \left(\dfrac{a^2+ab+b^2}{a^3-b^3}\right) = \left(\dfrac{1}{a-b}\right)$, where $a = 147$, $b = 143$

$= \left(\dfrac{1}{147-143}\right) = \dfrac{1}{4}.$

205. Let $\dfrac{13^3+7^3}{13^2+7^2-x} = 20$. Then,

$\dfrac{13^3+7^3}{13+7} = 13^2+7^2-x \Leftrightarrow 13^2+7^2-13\times 7 = 13^2+7^2-x \Leftrightarrow x = 13\times 7 = 91.$

206. Given exp. $= \dfrac{a^3-b^3}{a^2-b^2} = \dfrac{(a-b)(a^2+ab+b^2)}{(a-b)(a+b)} = \dfrac{(a^2+ab+b^2)}{(a+b)}$

$= \dfrac{\left(\dfrac{3}{5}\right)^2+\left(\dfrac{3}{5}\times\dfrac{2}{5}\right)+\left(\dfrac{2}{5}\right)^2}{\left(\dfrac{3}{5}+\dfrac{2}{5}\right)} = \dfrac{9}{25}+\dfrac{6}{25}+\dfrac{4}{25} = \dfrac{19}{25}.$

207. Given exp. $= \dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca} = a+b+c = (38+34+28) = 100.$

208. Since $(x-y)+(y-z)+(z-x) = 0$, so $(x-y)^3+(y-z)^3+(z-x)^3$

$= 3(x-y)(y-z)(z-x).$

$\therefore$ Given exp. $= \dfrac{3(x-y)(y-z)(z-x)}{9(x-y)(y-z)(z-x)} = \dfrac{1}{3}.$

209. Let total score be x. Then, highest score $= \dfrac{3x}{11}$.

Remainder $= \left(x-\dfrac{3x}{11}\right) = \dfrac{8x}{11}$. Next highest score $= \dfrac{3}{11}$ of $\dfrac{8x}{11} = \dfrac{24x}{121}$.

$\therefore\ \dfrac{3x}{11}-\dfrac{24x}{121} = 9 \Leftrightarrow 33x-24x = 9\times 121 \Leftrightarrow 9x = 9\times 121 \Leftrightarrow x = 121.$

210. $X + 3X \times 0.50 + 14 \times 0.10 + 4X \times 0.05 = 50$

$\Leftrightarrow \quad X + 1.5X + 1.40 + 0.2X = 50 \quad \Leftrightarrow \quad 2.7X = 48.60 \quad \Leftrightarrow \quad X = 18.$

211. Suppose their paths cross after x minutes.

Then, $11 + 57x = 51 - 63x \quad \Leftrightarrow \quad 120x = 40 \quad \Leftrightarrow \quad x = \frac{1}{3}.$

Number of floors covered by David in $(1/3)$ min. $= \left(\frac{1}{3} \times 57\right) = 19.$

So, their paths cross at $(11 + 19)$ *i.e.* 30th floor.

212. $N \times 50 = (325000 - 300000) = 25000 \quad \Leftrightarrow \quad N = 500.$

213. Let total number of children be x. Then, $x \times \frac{1}{8}x = \frac{x}{2} \times 16 \quad \Leftrightarrow \quad x = 64.$

$\therefore$ Number of notebooks $= \frac{1}{8}x^2 = \left(\frac{1}{8} \times 64 \times 64\right) = 512.$

214. Let number of boys $= x$. Then, number of girls $= x$.

Now, $2(x - 8) = x$ or $x = 16$.

$\therefore$ Total number of students $= 2x = (2 \times 16) = 32.$

215. Let the total number of sweets be $(25x + 8)$.

Then, $(25x + 8) - 22$ is divisible by 28

$\Leftrightarrow$ $(25x - 14)$ is divisible by 28 $\Leftrightarrow$ $28x - (3x + 14)$ is divisible by 28

$\Leftrightarrow$ $(3x + 14)$ is divisible by 28 $\Leftrightarrow$ $x = 14$.

$\therefore$ Total number of sweets $= (25 \times 14 + 8) = 358.$

216. Suppose the man works overtime for x hours.

Now, working hours in 4 weeks $= (5 \times 8 \times 4) = 160.$

$\therefore \quad 160 \times 2.40 + x \times 3.20 = 432 \quad \Leftrightarrow \quad 3.20x = 432 - 384 = 48 \quad \Leftrightarrow \quad x = 15.$

Hence, total hours of work $= (160 + 15) = 175.$

217. Let number of boys $= x$. Then, number of girls $= (100 - x)$.

$\therefore \quad 3.60x + 2.40(100 - x) = 312 \quad \Leftrightarrow \quad 1.20x = 312 - 240 = 72 \quad \Leftrightarrow \quad x = 60.$

Hence, number of girls $= (100 - x) = 40.$

218. Let number of boys $= x$. Then, number of girls $= (60 - x)$.

$\therefore \quad x(60 - x) + (60 - x)x = 1600 \quad \Leftrightarrow \quad 60x - x^2 + 60x - x^2 = 1600$

$\Leftrightarrow \quad 2x^2 - 120x + 1600 = 0 \quad \Leftrightarrow \quad x^2 - 60x + 800 = 0$

$\Leftrightarrow \quad (x - 40)(x - 20) = 0 \quad \Leftrightarrow \quad x = 40$ or $x = 20.$

So, we are not definite. Hence, data is inadequate.

219. Let the distance covered by taxi be x km. Then, distance covered by car $= (80 - x)$ km.

$\therefore \quad 1.5x + 0.5(80 - x) = 50 \quad \Leftrightarrow \quad x = 50 - 40 = 10$ km.

220. Let the number of correct answers be x. Number of incorrect answers $= (60 - x)$.

$\therefore \quad 4x - (60 - x) = 130 \quad \Leftrightarrow \quad 5x = 190 \quad \Leftrightarrow \quad x = 38.$

221. Let number of matches lost $= x$. Then, number of matches won $= x + 3$.

$\therefore \quad 2(x + 3) - x = 23 \quad \Leftrightarrow \quad x = 17.$

Hence, total number of matches played $= x + (x + 3) = 2x + 3 = 37.$

222. Let the number of 20-paise coins be x. Then, number of 25-paise coins $= (324 - x)$.

$\therefore \quad 0.20 \times x + 0.25(324 - x) = 71 \quad \Leftrightarrow \quad 20x + 25(324 - x) = 7100$

$\Leftrightarrow \quad 5x = 1000 \quad \Leftrightarrow \quad x = 200.$

Hence, number of 25-paise coins $= (324 - x) = 124.$

223. Let number of notes of each denomination be x.

Then, $x + 5x + 10x = 480 \quad \Leftrightarrow \quad 16x = 480 \quad \Leftrightarrow \quad x = 30.$

Hence, total number of notes $= 3x = 90.$

224. Original share of 1 person = $\frac{1}{8}$. New share of 1 person = $\frac{1}{7}$.

Increase = $\left(\frac{1}{7}-\frac{1}{8}\right)=\frac{1}{56}$.

$\therefore$ Required fraction = $\frac{(1/56)}{(1/8)}=\left(\frac{1}{56}\times 8\right)=\frac{1}{7}$.

225. Let total number of sweets be x. Then,

$\frac{x}{140}-\frac{x}{175}=4 \Leftrightarrow 5x-4x=4\times 700 \Leftrightarrow x=2800.$

226. Let the number of persons be x. Then,

$\frac{96}{x-4}-\frac{96}{x}=4 \Leftrightarrow \frac{1}{x-4}-\frac{1}{x}=\frac{4}{96} \Leftrightarrow \frac{x-(x-4)}{x(x-4)}=\frac{1}{24}$

$\Leftrightarrow x^2-4x-96=0 \Leftrightarrow (x-12)(x+8)=0 \Leftrightarrow x=12.$

227. Let the number of balls purchased be x.

Then, $\frac{450}{x}-\frac{450}{x+5}=15 \Leftrightarrow \frac{1}{x}-\frac{1}{x+5}=\frac{15}{450} \Leftrightarrow \frac{x+5-x}{x(x+5)}=\frac{1}{30}$

$\Leftrightarrow x^2+5x-150=0 \Leftrightarrow (x+15)(x-10)=0 \Leftrightarrow x=10.$

228. Let the length of the piece be x metres. Then, cost of 1 m of piece = Rs. $\left(\frac{35}{x}\right)$.

$\therefore (x+4)\left(\frac{35}{x}-1\right)=35 \Leftrightarrow 35-x+\frac{140}{x}-4=35 \Leftrightarrow \frac{140}{x}-x=4$

$\Leftrightarrow x^2+4x-140=0 \Leftrightarrow (x+14)(x-10)=0 \Leftrightarrow x=10.$

229. Let the cost of a chair and that of a table be Rs. x and Rs. y respectively.

Then, $10x=4y$ or $y=\frac{5}{2}x$.

$\therefore 15x+2y=4000 \Leftrightarrow 15x+2\times\frac{5}{2}x=4000 \Leftrightarrow 20x=4000 \Leftrightarrow x=200.$

So, $y=\left(\frac{5}{2}\times 200\right)=500.$

Hence, cost of 12 chairs and 3 tables = $12x+3y$ = Rs. (2400 + 1500) = Rs. 3900.

230. Cost of 4 mangoes = Cost of 9 lemons = Rs. $\left(\frac{4.80}{3}\times 9\right)$ = Rs. 14.40.

Cost of 1 mango = Rs. $\left(\frac{14.40}{4}\right)$ = Rs. 3.60.

Cost of 5 apples = Cost of 3 mangoes = Rs. (3.60 × 3) = Rs. 10.80.

Cost of 9 oranges = Cost of 5 apples = Rs. 10.80.

$\therefore$ Cost of 1 orange = Re. $\left(\frac{10.80}{9}\right)$ = Rs. 1.20.

231. Let the price of a saree and a shirt be Rs. x and Rs. y respectively.

Then, $2x+4y=1600$ ...(*i*) and $x+6y=1600$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x=400$, $y=200$.

$\therefore$ Cost of 12 shirts = Rs. (12 × 200) = Rs. 2400.

232. Let the cost of a table and that of a chair be Rs. x and Rs. y respectively.

Then, $2x+3y=3500$ ...(*i*) and $3x+2y=4000$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x=1000$ and $y=500$.

233. Let the fixed charge be Rs. x and variable charge be Rs. y per km.
Then, $x + 16y = 156$ ...(*i*) and $x + 24y = 204$ ...(*ii*)
Solving (*i*) and (*ii*), we get : $x = 60$, $y = 6$.
$\therefore$ Cost of travelling 30 km = Rs. $(60 + 30 \times 6)$ = Rs. 240.

234. Let the number of benches in the class be x. Then, $6(x + 1) = 7x - 5 \Leftrightarrow x = 11$.
Hence, number of students in the class = $6(x + 1) = 6 \times 12 = 72$.

235. Let the number of students in rooms A and B be x and y respectively. Then,
$x - 10 = y + 10 \Rightarrow x - y = 20$ ...(*i*) and $x + 20 = 2(y - 20) \Rightarrow x - 2y = -60$ ...(*ii*)
Solving (*i*) and (*ii*), we get : $x = 100$, $y = 80$.

236. Let the number of buffaloes be x and the number of ducks be y.
Then, $4x + 2y = 2(x + y) + 24 \Leftrightarrow 2x = 24 \Leftrightarrow x = 12$.

237. Let the number of hens be x and the number of cows be y. Then,
$x + y = 48$ ...(*i*) and $2x + 4y = 140 \Rightarrow x + 2y = 70$ ...(*ii*)
Solving (*i*) and (*ii*), we get : $x = 26$, $y = 22$.

238. Suppose, Sanya and Vidushi donate money to x and $(x + 5)$ people respectively.

Then, $\frac{100}{x} - \frac{100}{x+5} = 1 \Leftrightarrow 100(x + 5) - 100x = x(x + 5) \Leftrightarrow x^2 + 5x - 500 = 0$

$\Leftrightarrow (x - 20)(x + 25) = 0 \Leftrightarrow x = 20$.

$\therefore$ Total number of recipients of charity = $x + (x + 5) = 2x + 5 = 45$.

5. SQUARE ROOTS AND CUBE ROOTS

IMPORTANT FACTS AND FORMULAE

Square Root : If $x^2 = y$, we say that the square root of y is x and we write, $\sqrt{y} = x$.

Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{196} = 14$.

Cube Root : The cube root of a given number x is the number whose cube is x. We denote the cube root of x by $\sqrt[3]{x}$.

Thus, $\sqrt[3]{8} = \sqrt[3]{2\times2\times2} = 2$, $\sqrt[3]{343} = \sqrt[3]{7\times7\times7} = 7$ etc.

Note :

1. $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$

2. $\sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}} = \dfrac{\sqrt{x}}{\sqrt{y}} \times \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{\sqrt{xy}}{y}$.

SOLVED EXAMPLES

Ex. 1. ***Evaluate $\sqrt{6084}$ by factorization method.***

Sol. **Method :** Express the given number as the product of prime factors. Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.

Thus, resolving 6084 into prime factors, we get :

$6084 = 2^2 \times 3^2 \times 13^2$

$\therefore \quad \sqrt{6084} = (2\times3\times13) = 78.$

2	6084
2	3042
3	1521
3	507
13	169
	13

Ex. 2. ***Find the square root of 1471369.***

Sol. **Explanation :** In the given number, mark off the digits in pairs starting from the unit's digit. Each pair and the remaining one digit is called a period.

Now, $1^2 = 1$. On subtracting, we get 0 as remainder.

Now, bring down the next period *i.e.*, 47.

Now, trial divisor is $1 \times 2 = 2$ and trial dividend is 47.

So, we take 22 as divisor and put 2 as quotient.

The remainder is 3.

Next, we bring down the next period which is 13.

Now, trial divisor is $12 \times 2 = 24$ and trial dividend is 313. So, we take 241 as dividend and 1 as quotient.

The remainder is 72.

Bring down the next period *i.e.*, 69.

Now, the trial divisor is $121 \times 2 = 242$ and the trial dividend is 7269. So, we take 3 as quotient and 2423 as divisor. The remainder is then zero.

Hence, $\sqrt{1471369} = 1213$.

1	$\overline{14}\,\overline{71}\,\overline{369}$	(1213
	1	
22	47	
	44	
241	313	
	241	
2423	7269	
	7269	
	×	

Ex. 3. ***Evaluate :*** $\sqrt{248+\sqrt{51+\sqrt{169}}}$.

Sol. Given expression $= \sqrt{248+\sqrt{51+13}} = \sqrt{248+\sqrt{64}} = \sqrt{248+8} = \sqrt{256} = 16.$

Ex. 4. ***If*** $a*b*c = \dfrac{\sqrt{(a+2)(b+3)}}{c+1}$, ***then find the value of*** $6*15*3$.

Sol. $6*15*3 = \dfrac{\sqrt{(6+2)(15+3)}}{3+1} = \dfrac{\sqrt{8\times 18}}{4} = \dfrac{\sqrt{144}}{4} = \dfrac{12}{4} = 3.$

Ex. 5. ***Find the value of*** $\sqrt{1\dfrac{9}{16}}$.

Sol. $\sqrt{1\dfrac{9}{16}} = \sqrt{\dfrac{25}{16}} = \dfrac{\sqrt{25}}{\sqrt{16}} = \dfrac{5}{4} = 1\dfrac{1}{4}.$

Ex. 6. ***What is the square root of 0.0009 ?***

Sol. $\sqrt{0.0009} = \sqrt{\dfrac{9}{10000}} = \dfrac{\sqrt{9}}{\sqrt{10000}} = \dfrac{3}{100} = 0.03.$

Ex. 7. ***Evaluate*** $\sqrt{175.2976}$.

Sol. **Method :** We make even number of decimal places by affixing a zero, if necessary. Now, we mark off periods and extract the square root as shown.

$\therefore \quad \sqrt{175.2976} = 13.24.$

1	$\overline{1}\,\overline{75}.\overline{29}\,\overline{76}$	(13.24
	1	
23	75	
	69	
262	629	
	524	
2644	10576	
	10576	
	×	

Ex. 8. ***What will come in place of question mark in each of the following questions?***

(i) $\sqrt{\dfrac{32.4}{?}} = 2$ ***(ii)*** $\sqrt{86.49}+\sqrt{5+(?)^2} = 12.3.$ **(R.R.B. 2002)**

Sol. (*i*) Let $\sqrt{\dfrac{32.4}{x}} = 2$. Then, $\dfrac{32.4}{x} = 4 \Leftrightarrow 4x = 32.4 \Leftrightarrow x = 8.1.$

(*ii*) Let $\sqrt{86.49}+\sqrt{5+x^2} = 12.3.$

Then, $9.3+\sqrt{5+x^2} = 12.3 \Leftrightarrow \sqrt{5+x^2} = 12.3-9.3 = 3$

$\Leftrightarrow 5+x^2 = 9 \Leftrightarrow x^2 = 9-5 = 4 \Leftrightarrow x = \sqrt{4} = 2.$

Ex. 9. ***Find the value of*** $\sqrt{\dfrac{0.289}{0.00121}}$. **(IGNOU, 2003)**

Sol. $\sqrt{\dfrac{0.289}{0.00121}} = \sqrt{\dfrac{0.28900}{0.00121}} = \sqrt{\dfrac{28900}{121}} = \dfrac{170}{11}.$

Ex. 10. ***If*** $\sqrt{1+\dfrac{x}{144}} = \dfrac{13}{12}$, ***then find the value of*** x.

Sol. $\sqrt{1+\frac{x}{144}} = \frac{13}{12} \Rightarrow \left(1+\frac{x}{144}\right) = \left(\frac{13}{12}\right)^2 = \frac{169}{144} \Rightarrow \frac{x}{144} = \frac{169}{144} - 1$

$\Rightarrow \frac{x}{144} = \frac{25}{144} \Rightarrow x = 25.$

Ex. 11. ***Find the value of $\sqrt{3}$ upto three places of decimal.***

Sol.

1	$\overline{3.00}\,\overline{00}\,\overline{00}$	(1.732
	1	
27	200	
	189	
343	1100	
	1029	
3462	7100	
	6924	

$\therefore \sqrt{3} = 1.732.$

Ex. 12. ***If $\sqrt{3} = 1.732$, find the value of $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$ correct to 3 places of decimal.*** **(S.S.C. 2004)**

Sol. $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75} = \sqrt{64\times 3} - \frac{1}{2}\sqrt{16\times 3} - \sqrt{25\times 3} = 8\sqrt{3} - \frac{1}{2}\times 4\sqrt{3} - 5\sqrt{3}$

$= 3\sqrt{3} - 2\sqrt{3} = \sqrt{3} = 1.732.$

Ex. 13. ***Evaluate :*** $\sqrt{\frac{9.5\times .0085\times 18.9}{.0017\times 1.9\times 0.021}}.$

Sol. Given exp. $= \sqrt{\frac{9.5\times .0085\times 18.900}{.0017\times 1.9\times 0.021}}.$

Now, since the sum of decimal places in the numerator and denominator under the radical sign is the same, we remove the decimal.

$\therefore$ Given exp. $= \sqrt{\frac{95\times 85\times 18900}{17\times 19\times 21}} = \sqrt{5\times 5\times 900} = 5\times 30 = 150.$

Ex. 14. ***Simplify :*** $\sqrt{[(12.1)^2 - (8.1)^2] + [(0.25)^2 + (0.25)(19.95)]}.$ **(C.B.I. 2003)**

Sol. Given exp. $= \sqrt{\frac{(12.1+8.1)(12.1-8.1)}{(0.25)(0.25+19.95)}} = \sqrt{\frac{20.2\times 4}{0.25\times 20.2}}$

$= \sqrt{\frac{4}{0.25}} = \sqrt{\frac{400}{25}} = \sqrt{16} = 4.$

Ex. 15. ***If $x = 1+\sqrt{2}$ and $y = 1-\sqrt{2}$, find the value of (x^2+y^2).***

Sol. $x^2 + y^2 = (1+\sqrt{2})^2 + (1-\sqrt{2})^2 = 2[(1)^2 + (\sqrt{2})^2] = 2\times 3 = 6.$

Ex. 16. ***Evaluate $\sqrt{0.9}$ upto 3 places of decimal.*** **(R.R.B. 2003)**

Sol.

9	$0.\overline{90}\,\overline{00}\,\overline{00}$	(.948
	81	
184	900	
	736	
1888	16400	
	15104	

$\therefore \sqrt{0.9} = 0.948.$

Ex. 17. ***If $\sqrt{15} = 3.88$, find the value of $\sqrt{\frac{5}{3}}$.*** **(S.S.C. 2003)**

Sol. $\sqrt{\frac{5}{3}} = \sqrt{\frac{5\times 3}{3\times 3}} = \frac{\sqrt{15}}{3} = \frac{3.88}{3} = 1.2933.... = 1.29\overline{3}.$

Ex. 18. ***Find the least square number which is exactly divisible by 10, 12, 15 and 18.***

Sol. L.C.M. of 10, 12, 15, 18 = 180. Now, $180 = 2\times 2\times 3\times 3\times 5 = 2^2\times 3^2\times 5$.

To make it a perfect square, it must be multiplied by 5.

$\therefore$ Required number $= (2^2\times 3^2\times 5^2) = 900$.

Ex. 19. ***Find the greatest number of five digits which is a perfect square.*** **(R.R.B. 1998)**

Sol. Greatest number of 5 digits is 99999.

```
 3 | 99 99 99 ( 316
   | 9
   |-----------
61 | 99
   | 61
   |-----------
626| 3899
   | 3756
   |-----------
   | 143
```

$\therefore$ Required number $= (99999 - 143) = 99856$.

Ex. 20. ***Find the smallest number that must be added to 1780 to make it a perfect square.***

Sol.

```
 4 | 17 80 (42
   | 16
   |-----------
82 | 180
   | 164
   |-----------
   | 16
   |-----------
```

$\therefore$ Number to be added $= (43)^2 - 1780 = 1849 - 1780 = 69$.

Ex. 21. ***If $\sqrt{2} = 1.4142$, find the value of $\frac{\sqrt{2}}{(2+\sqrt{2})}$.***

Sol. $\frac{\sqrt{2}}{(2+\sqrt{2})} = \frac{\sqrt{2}}{(2+\sqrt{2})}\times\frac{(2-\sqrt{2})}{(2-\sqrt{2})} = \frac{2\sqrt{2}-2}{(4-2)} = \frac{2(\sqrt{2}-1)}{2} = (\sqrt{2}-1) = (1.4142-1) = 0.4142.$

Ex. 22. ***If $x = \left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)$ and $y = \left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)$, find the value of (x^2+y^2).***

Sol. $x = \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})}\times\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} = \frac{(\sqrt{5}+\sqrt{3})^2}{(5-3)} = \frac{5+3+2\sqrt{15}}{2} = 4+\sqrt{15}.$

$y = \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})}\times\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})} = \frac{(\sqrt{5}-\sqrt{3})^2}{(5-3)} = \frac{5+3-2\sqrt{15}}{2} = 4-\sqrt{15}.$

$\therefore$ $x^2+y^2 = (4+\sqrt{15})^2+(4-\sqrt{15})^2 = 2[(4)^2+(\sqrt{15})^2] = 2\times 31 = 62.$

Ex. 23. ***Find the cube root of 2744.***

Sol. **Method :** Resolve the given number as the product of prime factors and take the product of prime factors, choosing one out of three of the same prime factors. Resolving 2744 as the product of prime factors, we get :

$2744 = 2^3 \times 7^3$.

$\therefore \quad \sqrt[3]{2744} = 2 \times 7 = 14.$

2	2744
2	1372
2	686
7	343
7	49
	7

Ex. 24. ***By what least number 4320 be multiplied to obtain a number which is a perfect cube ?***

Sol. Clearly, $4320 = 2^3 \times 3^3 \times 2^2 \times 5$.

To make it a perfect cube, it must be multiplied by 2×5^2 *i.e.*, 50.

EXERCISE 5

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. $\sqrt{53824} = ?$ **(Bank P.O. 2003)**

   (*a*) 202 (*b*) 232 (*c*) 242 (*d*) 332

2. The square root of 64009 is : **(R.R.B. 2003)**

   (*a*) 253 (*b*) 347 (*c*) 363 (*d*) 803

3. The value of $\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}$ is : **(S.S.C. 1998)**

   (*a*) 4 (*b*) 6 (*c*) 8 (*d*) 10

4. Evaluate : $\sqrt{41-\sqrt{21+\sqrt{19-\sqrt{9}}}}$. **(C.B.I. 1997)**

   (*a*) 3 (*b*) 5 (*c*) 6 (*d*) 6.4

5. $\sqrt{176+\sqrt{2401}}$ is equal to :

   (*a*) 14 (*b*) 15 (*c*) 18 (*d*) 24

6. $\left(\frac{\sqrt{625}}{11} \times \frac{14}{\sqrt{25}} \times \frac{11}{\sqrt{196}}\right)$ is equal to : **(S.S.C. 2000)**

   (*a*) 5 (*b*) 6 (*c*) 8 (*d*) 11

7. $\left(\sqrt{\frac{225}{729}} - \sqrt{\frac{25}{144}}\right) \div \sqrt{\frac{16}{81}} = ?$

   (*a*) $\frac{1}{48}$ (*b*) $\frac{5}{48}$ (*c*) $\frac{5}{16}$ (*d*) None of these

8. The square root of $(272^2 - 128^2)$ is : **(S.S.C. 2000)**

   (*a*) 144 (*b*) 200 (*c*) 240 (*d*) 256

9. If $x * y = x + y + \sqrt{xy}$, the value of $6 * 24$ is : **(C.B.I. 1998)**

   (*a*) 41 (*b*) 42 (*c*) 43 (*d*) 44

10. If $y = 5$, then what is the value of $10y\sqrt{y^3 - y^2}$? **(R.R.B. 1998)**

    (*a*) $50\sqrt{2}$ (*b*) 100 (*c*) $200\sqrt{5}$ (*d*) 500

11. $\sqrt{110\frac{1}{4}} = ?$

(a) 10.25 (b) 10.5 (c) 11.5 (d) 19.5

12. $\sqrt{\frac{25}{81} - \frac{1}{9}} = ?$ (Hotel Management, 2002)

(a) $\frac{2}{3}$ (b) $\frac{4}{9}$ (c) $\frac{16}{81}$ (d) $\frac{25}{81}$

13. The digit in the unit's place in the square root of 15876 is : (S.S.C. 2000)

(a) 2 (b) 4 (c) 6 (d) 8

14. How many two-digit numbers satisfy this property : The last digit (unit's digit) of the square of the two-digit number is 8 ? (R.R.B. 2001)

(a) 1 (b) 2 (c) 3 (d) None of these

15. What is the square root of 0.16 ? (P.C.S. 1998)

(a) 0.004 (b) 0.04 (c) 0.4 (d) 4

16. The value of $\sqrt{0.000441}$ is : (S.S.C. 2002)

(a) 0.00021 (b) 0.0021 (c) 0.021 (d) 0.21

17. $\sqrt{0.00004761}$ equals : (C.B.I. 2003)

(a) 0.00069 (b) 0.0069 (c) 0.0609 (d) 0.069

18. $1.5^2 \times \sqrt{0.0225} = ?$ (Bank P.O. 2002)

(a) 0.0375 (b) 0.3375 (c) 3.275 (d) 32.75

19. $\sqrt{0.01 + \sqrt{0.0064}} = ?$

(a) 0.03 (b) 0.3 (c) 0.42 (d) None of these

20. The value of $\sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.0009}$ is : (S.S.C. 2002)

(a) 2.03 (b) 2.1 (c) 2.11 (d) 2.13

21. $\sqrt{.0025} \times \sqrt{2.25} \times \sqrt{.0001} = ?$ (Hotel Management, 1998)

(a) .000075 (b) .0075 (c) .075 (d) None of these

22. $\sqrt{1.5625} = ?$ (S.B.I.P.O. 2003)

(a) 1.05 (b) 1.25 (c) 1.45 (d) 1.55

23. If $\sqrt{.00000676} = .0026$, the square root of 67,60,000 is :

(a) $\frac{1}{26}$ (b) 26 (c) 260 (d) 2600

24. If $\sqrt{18225} = 135$, then the value of $(\sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225} + \sqrt{0.00018225})$ is :

(a) 1.49985 (b) 14.9985 (c) 149.985 (d) 1499.85

25. Given that $\sqrt{13} = 3.605$ and $\sqrt{130} = 11.40$, find the value of $\sqrt{1.3} + \sqrt{1300} + \sqrt{0.013}$.

(a) 36.164 (b) 36.304 (c) 37.164 (d) 37.304 (S.S.C. 1999)

26. If $\frac{52}{x} = \sqrt{\frac{169}{289}}$, the value of x is : (C.B.I. 1998)

(a) 52 (b) 58 (c) 62 (d) 68

27. For what value of * the statement $\left(\frac{*}{15}\right)\left(\frac{*}{135}\right) = 1$ is true ? (S.S.C. 2002)

(a) 15 (b) 25 (c) 35 (d) 45

28. Which number can replace both the question marks in the equation $\frac{4\frac{1}{2}}{?} = \frac{?}{32}$.

(a) 1 (b) 7 (c) $7\frac{1}{2}$ (d) None of these

(Hotel Management, 2000)

29. What should come in place of both the question marks in the equation $\frac{?}{\sqrt{128}} = \frac{\sqrt{162}}{?}$.

(a) 12 (b) 14 (c) 144 (d) 196

(Bank P.O. 1999)

30. If $0.13 + p^2 = 13$, then p equals : (S.S.C. 2000)

(a) 0.01 (b) 0.1 (c) 10 (d) 100

31. What number should be divided by $\sqrt{0.25}$ to give the result as 25 ?

(a) 12.5 (b) 25 (c) 50 (d) 125

(C.B.I. 2003)

32. If $\sqrt{3^n} = 729$, then the value of n is : (Section Officers', 2003)

(a) 6 (b) 8 (c) 10 (d) 12

33. If $\sqrt{18 \times 14 \times x} = 84$, then x equals :

(a) 22 (b) 24 (c) 28 (d) 32

34. $28\sqrt{?} + 1426 = \frac{3}{4}$ of 2872 (B.S.R.B. 1998)

(a) 576 (b) 676 (c) 1296 (d) 1444

35. $\sqrt{\frac{?}{169}} = \frac{54}{39}$

(a) 108 (b) 324 (c) 2916 (d) 4800

36. If $\sqrt{x} + \sqrt{441} = 0.02$, then the value of x is : (S.S.C. 1999)

(a) 0.1764 (b) 1.764 (c) 1.64 (d) 2.64

37. $\sqrt{\frac{.0196}{?}} = 0.2$ (Hotel Management, 1999)

(a) 0.49 (b) 0.7 (c) 4.9 (d) None of these

38. $\sqrt{0.0169 \times ?} = 1.3$ (Hotel Management, 2001)

(a) 10 (b) 100 (c) 1000 (d) None of these

39. If $\sqrt{1369} + \sqrt{.0615 + x} = 37.25$, then x is equal to : (Hotel Management, 1998)

(a) 10^{-1} (b) 10^{-2} (c) 10^{-3} (d) None of these

40. If $\sqrt{(x-1)(y+2)} = 7$, x and y being positive whole numbers, then the values of x and y respectively are :

(a) 8, 5 (b) 15, 12 (c) 22, 19 (d) None of these

41. If $\sqrt{.04 \times .4 \times a} = .004 \times .4 \times \sqrt{b}$, then $\frac{a}{b}$ is :

(a) 16×10^{-3} (b) 16×10^{-4} (c) 16×10^{-5} (d) None of these

42. Three-fifth of the square of a certain number is 126.15. What is the number ?
(a) 14.5 (b) 75.69 (c) 145 (d) 210.25
(S.S.C. 2002)

43. $\sqrt{\frac{0.361}{0.00169}} = ?$
(a) $\frac{1.9}{13}$ (b) $\frac{19}{13}$ (c) $\frac{1.9}{130}$ (d) $\frac{190}{13}$

44. $\sqrt{\frac{48.4}{0.289}}$ is equal to : **(S.S.C. 2004)**
(a) $1\frac{5}{17}$ (b) $12\frac{1}{17}$ (c) $12\frac{16}{17}$ (d) $129\frac{7}{17}$

45. If $\sqrt{1+\frac{x}{169}} = \frac{14}{13}$, then x is equal to :
(a) 1 (b) 13 (c) 27 (d) None of these

46. If $\sqrt{1+\frac{55}{729}} = 1+\frac{x}{27}$, then the value of x is : **(C.D.S. 2003)**
(a) 1 (b) 3 (c) 5 (d) 7

47. The value of $\sqrt{2}$ upto three places of decimal is :
(a) 1.410 (b) 1.412 (c) 1.413 (d) 1.414

48. $(2\sqrt{27} - \sqrt{75} + \sqrt{12})$ is equal to :
(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $3\sqrt{3}$ (d) $4\sqrt{3}$

49. By how much does $\sqrt{12} + \sqrt{18}$ exceed $\sqrt{3} + \sqrt{2}$? **(S.S.C. 1999)**
(a) $\sqrt{2} - 4\sqrt{3}$ (b) $\sqrt{3} + 2\sqrt{2}$ (c) $2(\sqrt{3} - \sqrt{2})$ (d) $3(\sqrt{3} - \sqrt{2})$

50. $\frac{\sqrt{24} + \sqrt{216}}{\sqrt{96}} = ?$
(a) $2\sqrt{6}$ (b) 2 (c) $6\sqrt{2}$ (d) $\frac{2}{\sqrt{6}}$

51. The value of $\frac{\sqrt{80} - \sqrt{112}}{\sqrt{45} - \sqrt{63}}$ is : **(S.S.C. 2000)**
(a) $\frac{3}{4}$ (b) $1\frac{1}{3}$ (c) $1\frac{7}{9}$ (d) $1\frac{3}{4}$

52. If $3\sqrt{5} + \sqrt{125} = 17.88$, then what will be the value of $\sqrt{80} + 6\sqrt{5}$?
(a) 13.41 (b) 20.46 (c) 21.66 (d) 22.35
(Bank P.O. 2000)

53. $\sqrt{50} \times \sqrt{98}$ is equal to :
(a) 63.75 (b) 65.95 (c) 70 (d) 70.25

54. Given $\sqrt{2} = 1.414$. The value of $\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$ is : **(S.S.C. 2003)**
(a) 8.426 (b) 8.484 (c) 8.526 (d) 8.876

55. The approximate value of $\frac{3\sqrt{12}}{2\sqrt{28}} + \frac{2\sqrt{21}}{\sqrt{98}}$ is : **(Section Officers', 2003)**
(a) 1.0605 (b) 1.0727 (c) 1.6007 (d) 1.6026

56. $\sqrt{\frac{.081 \times .484}{.0064 \times 6.25}}$ is equal to : (N.I.F.T. 1997)

(a) 0.9 (b) 0.99 (c) 9 (d) 99

57. $\sqrt{\frac{0.204 \times 42}{0.07 \times 3.4}}$ is equal to :

(a) $\frac{1}{6}$ (b) 0.06 (c) 0.6 (d) 6

58. $\sqrt{\frac{0.081 \times 0.324 \times 4.624}{1.5625 \times 0.0289 \times 72.9 \times 64}}$ is equal to :

(a) 0.024 (b) 0.24 (c) 2.4 (d) 24

59. $\sqrt{\frac{9.5 \times .085}{.0017 \times .19}}$ equals :

(a) .05 (b) 5 (c) 50 (d) 500

60. The value of $\sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{(0.003)^2 + (0.021)^2 + (0.0065)^2}}$ is : (S.S.C. 2002)

(a) 0.1 (b) 10 (c) 10^2 (d) 10^3

61. The square root of $(7 + 3\sqrt{5})(7 - 3\sqrt{5})$ is : (S.S.C. 2004)

(a) $\sqrt{5}$ (b) 2 (c) 4 (d) $3\sqrt{5}$

62. $\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2$ simplifies to : (R.R.B. 2000)

(a) $\frac{3}{4}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{4}{3}$ (d) None of these

63. $\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right)^2$ is equal to :

(a) $2\frac{1}{2}$ (b) $3\frac{1}{2}$ (c) $4\frac{1}{2}$ (d) $5\frac{1}{2}$

64. If $a = 0.1039$, then the value of $\sqrt{4a^2 - 4a + 1} + 3a$ is : (C.B.I. 2003)

(a) 0.1039 (b) 0.2078 (c) 1.1039 (d) 2.1039

65. The square root of $\frac{(0.75)^3}{1 - 0.75} + [0.75 + (0.75)^2 + 1]$ is : (S.S.C. 1999)

(a) 1 (b) 2 (c) 3 (d) 4

66. If $3a = 4b = 6c$ and $a + b + c = 27\sqrt{29}$, then $\sqrt{a^2 + b^2 + c^2}$ is :

(a) $3\sqrt{29}$ (b) 81 (c) 87 (d) None of these
(Hotel Management, 1999)

67. The square root of $0.\overline{4}$ is : (S.S.C. 2004)

(a) $0.\overline{6}$ (b) $0.\overline{7}$ (c) $0.\overline{8}$ (d) $0.\overline{9}$

68. Which one of the following numbers has rational square root ?

(a) 0.4 (b) 0.09 (c) 0.9 (d) 0.025

69. The value of $\sqrt{0.4}$ is :

(a) 0.02 (b) 0.2 (c) 0.51 (d) 0.63

70. The value of $\sqrt{0.121}$ is :
(a) 0.011 (b) 0.11 (c) 0.347 (d) 1.1

71. The value of $\sqrt{0.064}$ is :
(a) 0.008 (b) 0.08 (c) 0.252 (d) 0.8

72. The value of $\sqrt{\frac{0.16}{0.4}}$ is : **(IGNOU, 2003)**
(a) 0.02 (b) 0.2 (c) 0.63 (d) None of these

73. The value of $\frac{1+\sqrt{0.01}}{1-\sqrt{0.1}}$ is close to : **(C.B.I. 1997)**
(a) 0.6 (b) 1.1 (c) 1.6 (d) 1.7

74. If $\sqrt{5} = 2.236$, then the value of $\frac{1}{\sqrt{5}}$ is :
(a) .367 (b) .447 (c) .745 (d) None of these

75. If $\sqrt{24} = 4.899$, the value of $\sqrt{\frac{8}{3}}$ is :
(a) 0.544 (b) 1.333 (c) 1.633 (d) 2.666

76. If $\sqrt{6} = 2.449$, then the value of $\frac{3\sqrt{2}}{2\sqrt{3}}$ is :
(a) 0.6122 (b) 0.8163 (c) 1.223 (d) 1.2245

77. If $\sqrt{5} = 2.236$, then the value of $\frac{\sqrt{5}}{2} - \frac{10}{\sqrt{5}} + \sqrt{125}$ is equal to : **(M.B.A. 1998)**
(a) 5.59 (b) 7.826 (c) 8.944 (d) 10.062

78. If $2*3 = \sqrt{13}$ and $3*4 = 5$, then the value of $5*12$ is :
(a) $\sqrt{17}$ (b) $\sqrt{29}$ (c) 12 (d) 13

79. The least perfect square number divisible by 3, 4, 5, 6 and 8 is :
(a) 900 (b) 1200 (c) 2500 (d) 3600

80. The least perfect square, which is divisible by each of 21, 36 and 66, is :
(a) 213444 (b) 214344 (c) 214434 (d) 231444
(C.B.I. 2003)

81. The least number by which 294 must be multiplied to make it a perfect square, is :
(a) 2 (b) 3 (c) 6 (d) 24

82. Find the smallest number by which 5808 should be multiplied so that the product becomes a perfect square. **(S.S.C. 1999)**
(a) 2 (b) 3 (c) 7 (d) 11

83. The least number by which 1470 must be divided to get a number which is a perfect square, is :
(a) 5 (b) 6 (c) 15 (d) 30

84. What is the smallest number to be subtracted from 549162 in order to make it a perfect square ?
(a) 28 (b) 36 (c) 62 (d) 81

85. What is the least number which should be subtracted from 0.000326 to make it a perfect square ? **(S.S.C. 2003)**
(a) 0.000002 (b) 0.000004 (c) 0.02 (d) 0.04

86. The smallest number added to 680621 to make the sum a perfect square is :
(a) 4 (b) 5 (c) 6 (d) 8 (S.S.C. 2002)

87. The greatest four-digit perfect square number is : (Hotel Management, 2003)
(a) 9000 (b) 9801 (c) 9900 (d) 9981

88. The least number of 4 digits which is a perfect square, is :
(a) 1000 (b) 1016 (c) 1024 (d) 1036

89. Given $\sqrt{5} = 2.2361$, $\sqrt{3} = 1.7321$, then $\frac{1}{\sqrt{5}-\sqrt{3}}$ is equal to : (S.S.C. 2000)
(a) 1.98 (b) 1.984 (c) 1.9841 (d) 2

90. $\frac{1}{(\sqrt{9}-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} - \frac{1}{(\sqrt{6}-\sqrt{5})} + \frac{1}{(\sqrt{5}-\sqrt{4})}$ is equal to :
(a) 0 (b) $\frac{1}{3}$ (c) 1 (d) 5

91. $\left(2+\sqrt{2}+\frac{1}{2+\sqrt{2}}+\frac{1}{\sqrt{2}-2}\right)$ simplifies to : (S.S.C. 2000)
(a) $2-\sqrt{2}$ (b) 2 (c) $2+\sqrt{2}$ (d) $2\sqrt{2}$

92. If $\sqrt{2} = 1.4142$, the value of $\frac{7}{(3+\sqrt{2})}$ is :
(a) 1.5858 (b) 3.4852 (c) 3.5858 (d) 4.4142

93. $\left[\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} - \frac{6}{\sqrt{8}-\sqrt{12}}\right] = ?$ (R.R.B. 2001)
(a) $\sqrt{3}-\sqrt{2}$ (b) $\sqrt{3}+\sqrt{2}$ (c) $5\sqrt{3}$ (d) 1

94. $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$ is equal to : (Section Officers', 2001)
(a) 1 (b) 2 (c) $6-\sqrt{35}$ (d) $6+\sqrt{35}$

95. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$, then : (R.R.B. 2001)
(a) $a=-11, b=-6$ (b) $a=-11, b=6$ (c) $a=11, b=-6$ (d) $a=6, b=11$

96. If $\sqrt{2} = 1.414$, the square root of $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ is nearest to : (C.B.I. 2003)
(a) 0.172 (b) 0.414 (c) 0.586 (d) 1.414

97. $\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}} = ?$ (I.A.F. 2002)
(a) 3 (b) $3\sqrt{2}$ (c) 6 (d) None of these

98. $\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ simplifies to : (S.S.C. 2000)
(a) $16-\sqrt{3}$ (b) $4-\sqrt{3}$ (c) $2-\sqrt{3}$ (d) $2+\sqrt{3}$

99. If $x = (7 - 4\sqrt{3})$, then the value of $\left(x + \frac{1}{x}\right)$ is : (S.S.C. 2000)

(a) $3\sqrt{3}$ (b) $8\sqrt{3}$ (c) 14 (d) $14 + 8\sqrt{3}$

100. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, then the value of $(x^2 + y^2)$ is : (S.S.C. 2003)

(a) 10 (b) 13 (c) 14 (d) 15

101. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, the value of $\left(\frac{a^2 + ab + b^2}{a^2 - ab + b^2}\right)$ is :

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

102. A man plants 15376 apple trees in his garden and arranges them so that there are as many rows as there are apples trees in each row. The number of rows is :

(a) 124 (b) 126 (c) 134 (d) 144

103. A General wishes to draw up his 36581 soldiers in the form of a solid square. After arranging them, he found that some of them are left over. How many are left ?

(a) 65 (b) 81 (c) 100 (d) None of these

104. A group of students decided to collect as many paise from each member of the group as is the number of members. If the total collection amounts to Rs. 59.29, the number of members in the group is :

(a) 57 (b) 67 (c) 77 (d) 87

105. The cube root of .000216 is :

(a) .6 (b) .06 (c) .006 (d) None of these

106. $\sqrt[3]{4\frac{12}{125}} = ?$

(a) $1\frac{2}{5}$ (b) $1\frac{3}{5}$ (c) $1\frac{4}{5}$ (d) $2\frac{2}{5}$

107. $\sqrt[3]{\sqrt{.000064}} = ?$

(a) .02 (b) .2 (c) 2 (d) None of these

108. The largest four-digit number which is a perfect cube, is :

(a) 8000 (b) 9261 (c) 9999 (d) None of these

109. By what least number 675 be multiplied to obtain a number which is a perfect cube ?

(a) 5 (b) 6 (c) 7 (d) 8

110. What is the smallest number by which 3500 be divided to make it a perfect cube ?

(a) 9 (b) 50 (c) 300 (d) 450.

ANSWERS

1. (b)	2. (a)	3. (a)	4. (c)	5. (b)	6. (a)	7. (c)	8. (c)	9. (b)
10. (d)	11. (b)	12. (b)	13. (c)	14. (d)	15. (c)	16. (c)	17. (b)	18. (b)
19. (b)	20. (d)	21. (d)	22. (b)	23. (d)	24. (b)	25. (d)	26. (d)	27. (d)
28. (d)	29. (a)	30. (b)	31. (a)	32. (d)	33. (c)	34. (b)	35. (b)	36. (a)

37. (a)	**38.** (b)	**39.** (c)	**40.** (a)	**41.** (c)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (c)
46. (a)	**47.** (d)	**48.** (c)	**49.** (b)	**50.** (b)	**51.** (b)	**52.** (d)	**53.** (c)	**54.** (b)
55. (a)	**56.** (b)	**57.** (d)	**58.** (a)	**59.** (c)	**60.** (b)	**61.** (b)	**62.** (c)	**63.** (c)
64. (c)	**65.** (b)	**66.** (c)	**67.** (a)	**68.** (b)	**69.** (d)	**70.** (c)	**71.** (c)	**72.** (c)
73. (c)	**74.** (b)	**75.** (c)	**76.** (d)	**77.** (b)	**78.** (d)	**79.** (d)	**80.** (a)	**81.** (c)
82. (b)	**83.** (d)	**84.** (d)	**85.** (a)	**86.** (a)	**87.** (b)	**88.** (c)	**89.** (c)	**90.** (d)
91. (b)	**92.** (a)	**93.** (c)	**94.** (d)	**95.** (c)	**96.** (b)	**97.** (d)	**98.** (a)	**99.** (c)
100. (c)	**101.** (b)	**102.** (a)	**103.** (c)	**104.** (c)	**105.** (b)	**106.** (b)	**107.** (b)	**108.** (b)
109. (a)	**110.** (d)							

SOLUTIONS

1.

2	$\overline{53}\,\overline{82}\,\overline{4}$	(232
	4	
43	138	
	129	
462	924	
	924	
	×	

$\therefore \sqrt{53824} = 232.$

2.

2	$\overline{64}\,\overline{00}\,\overline{9}$	(253
	4	
45	240	
	225	
503	1509	
	1509	
	×	

$\therefore \sqrt{64009} = 253.$

3. Given exp. $= \sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+15}}}} = \sqrt{10+\sqrt{25+\sqrt{108+\sqrt{169}}}}$

$= \sqrt{10+\sqrt{25+\sqrt{108+13}}} = \sqrt{10+\sqrt{25+\sqrt{121}}}$

$= \sqrt{10+\sqrt{25+11}} = \sqrt{10+\sqrt{36}} = \sqrt{10+6} = \sqrt{16} = 4.$

4. Given exp. $= \sqrt{41-\sqrt{21+\sqrt{19-3}}} = \sqrt{41-\sqrt{21+\sqrt{16}}} = \sqrt{41-\sqrt{21+4}}$

$= \sqrt{41-\sqrt{25}} = \sqrt{41-5} = \sqrt{36} = 6.$

5. Given exp. $= \sqrt{176+49} = \sqrt{225} = 15.$

4	$\overline{24}\,\overline{01}$	(49
	16	
89	801	
	801	
	×	

6. Given exp. $= \frac{25}{11} \times \frac{14}{5} \times \frac{11}{14} = 5.$

7. Given exp. $= \left(\frac{\sqrt{225}}{\sqrt{729}} - \frac{\sqrt{25}}{\sqrt{144}}\right) + \frac{\sqrt{16}}{\sqrt{81}} = \left(\frac{15}{27} - \frac{5}{12}\right) + \frac{4}{9} = \left(\frac{15}{108} \times \frac{9}{4}\right) = \frac{5}{16}.$

8. $\sqrt{(272)^2 - (128)^2} = \sqrt{(272+128)(272-128)} = \sqrt{400 \times 144} = \sqrt{57600} = 240.$

9. $6*24 = 6 + 24 + \sqrt{6 \times 24} = 30 + \sqrt{144} = 30 + 12 = 42.$

10. $10y\sqrt{y^3 - y^2} = 10 \times 5\sqrt{5^3 - 5^2} = 50 \times \sqrt{125 - 25} = 50 \times \sqrt{100} = 50 \times 10 = 500.$

11. $\sqrt{110\frac{1}{4}} = \sqrt{\frac{441}{4}} = \frac{\sqrt{441}}{\sqrt{4}} = \frac{21}{2} = 10.5.$

12. $\sqrt{\frac{25}{81} - \frac{1}{9}} = \sqrt{\frac{25-9}{81}} = \sqrt{\frac{16}{81}} = \frac{\sqrt{16}}{\sqrt{81}} = \frac{4}{9}.$

13.

1	$\overline{1}\,\overline{58}\,\overline{76}$ (126
	1
22	58
	44
246	1476
	1476
	×

$\therefore \sqrt{15876} = 126.$

14. A number ending in 8 can never be a perfect square.

15. $\sqrt{0.16} = \sqrt{\frac{16}{100}} = \frac{\sqrt{16}}{\sqrt{100}} = \frac{4}{10} = 0.4.$

16. $\sqrt{0.\overline{000441}} = \sqrt{\frac{441}{10^6}} = \frac{\sqrt{441}}{\sqrt{10^6}} = \frac{21}{10^3} = \frac{21}{1000} = 0.021.$

17. $\sqrt{0.\overline{00004761}} = \sqrt{\frac{4761}{10^8}} = \frac{\sqrt{4761}}{\sqrt{10^8}} = \frac{69}{10^4} = \frac{69}{10000} = 0.0069.$

18. $1.5^2 \times \sqrt{0.0225} = 1.5^2 \times \sqrt{\frac{225}{10000}} = 2.25 \times \frac{15}{100} = 2.25 \times 0.15 = 0.3375.$

19. $\sqrt{0.01 + \sqrt{0.0064}} = \sqrt{0.01 + \sqrt{\frac{64}{10000}}} = \sqrt{0.01 + \frac{8}{100}} = \sqrt{0.01 + 0.08} = \sqrt{0.09} = 0.3.$

20. Given exp. $= \sqrt{\frac{1}{100}} + \sqrt{\frac{81}{100}} + \sqrt{\frac{121}{100}} + \sqrt{\frac{9}{10000}} = \frac{1}{10} + \frac{9}{10} + \frac{11}{10} + \frac{3}{100}$

$= 0.1 + 0.9 + 1.1 + 0.03 = 2.13.$

21. Given exp. $= \sqrt{\frac{25}{10000}} \times \sqrt{\frac{225}{100}} \times \sqrt{\frac{1}{10000}} = \frac{5}{100} \times \frac{15}{10} \times \frac{1}{100} = \frac{75}{100000} = 0.00075.$

22.

1	$\overline{1}.\overline{56}\,\overline{25}$ (1.25
	1
22	56
	44
245	1225
	1225
	×

$\therefore \sqrt{1.5625} = 1.25.$

23. $\sqrt{6760000} = \sqrt{0.00000676 \times 10^{12}} = \sqrt{0.00000676} \times \sqrt{10^{12}} = .0026 \times 10^6 = 2600.$

24. Given exp. $= \sqrt{\frac{18225}{10^2}} + \sqrt{\frac{18225}{10^4}} + \sqrt{\frac{18225}{10^6}} + \sqrt{\frac{18225}{10^8}}$

$= \frac{\sqrt{18225}}{10} + \frac{\sqrt{18225}}{10^2} + \frac{\sqrt{18225}}{10^3} + \frac{\sqrt{18225}}{10^4} = \frac{135}{10} + \frac{135}{100} + \frac{135}{1000} + \frac{135}{10000}$

$= 13.5 + 1.35 + 0.135 + 0.0135 = 14.9985.$

25. Given exp. $= \sqrt{1.30} + \sqrt{1300} + \sqrt{0.0130} = \sqrt{\frac{130}{100}} + \sqrt{13 \times 100} + \sqrt{\frac{130}{10000}}$

$= \frac{\sqrt{130}}{10} + \sqrt{13} \times 10 + \frac{\sqrt{130}}{100} = \frac{11.40}{10} + 3.605 \times 10 + \frac{11.40}{100}$

$= 1.14 + 36.05 + 0.114 = 37.304.$

26. $\frac{52}{x} = \sqrt{\frac{169}{289}} \Leftrightarrow \frac{52}{x} = \frac{13}{17} \Leftrightarrow x = \left(\frac{52 \times 17}{13}\right) = 68.$

27. Let the missing number be x.

Then, $x^2 = 15 \times 135 \Leftrightarrow x = \sqrt{15 \times 135} = \sqrt{15^2 \times 3^2} = 15 \times 3 = 45.$

28. Let $\frac{4\frac{1}{2}}{x} = \frac{x}{32}$. Then, $x^2 = 32 \times \frac{9}{2} = 144 \Leftrightarrow x = \sqrt{144} = 12.$

29. Let $\frac{x}{\sqrt{128}} = \frac{\sqrt{162}}{x}$.

Then, $x^2 = \sqrt{128 \times 162} = \sqrt{64 \times 2 \times 18 \times 9} = \sqrt{8^2 \times 6^2 \times 3^2} = 8 \times 6 \times 3 = 144.$

$\therefore \quad x = \sqrt{144} = 12.$

30. $\frac{0.13}{p^2} = 13 \Leftrightarrow p^2 = \frac{0.13}{13} = \frac{1}{100} \Leftrightarrow p = \sqrt{\frac{1}{100}} = \frac{1}{100} = \frac{1}{10} = 0.1.$

31. Let the required number be x. Then, $\frac{x}{\sqrt{0.25}} = 25 \Leftrightarrow \frac{x}{0.5} = 25 \Leftrightarrow x = 25 \times 0.5 = 12.5.$

32. $\sqrt{3^n} = 729 = 3^6 \Leftrightarrow (\sqrt{3^n})^2 = (3^6)^2 \Leftrightarrow 3^n = 3^{12} \Leftrightarrow n = 12.$

33. $\sqrt{18 \times 14 \times x} = 84 \Leftrightarrow 18 \times 14 \times x = 84 \times 84 \Leftrightarrow x = \frac{84 \times 84}{18 \times 14} = 28.$

34. Let $28\sqrt{x} + 1426 = 3 \times 718.$

Then, $28\sqrt{x} + 2154 = 1426 \Leftrightarrow 28\sqrt{x} = 728 \Leftrightarrow \sqrt{x} = 26 \Leftrightarrow x = (26)^2 = 676.$

35. Let $\sqrt{\frac{x}{169}} = \frac{54}{39}$. Then, $\frac{\sqrt{x}}{13} = \frac{54}{39} \Leftrightarrow \sqrt{x} = \left(\frac{54}{39} \times 13\right) = 18 \Leftrightarrow x = (18)^2 = 324.$

36. $\frac{\sqrt{x}}{\sqrt{441}} = 0.02 \Leftrightarrow \frac{\sqrt{x}}{21} = 0.02 \Leftrightarrow \sqrt{x} = 0.02 \times 21 = 0.42 \Leftrightarrow x = (0.42)^2 = 0.1764.$

37. Let $\sqrt{\frac{.0196}{x}} = 0.2$. Then, $\frac{.0196}{x} = 0.04 \Leftrightarrow x = \frac{.0196}{.04} = \frac{1.96}{4} = .49.$

38. Let $\sqrt{0.0169 \times x} = 1.3$. Then, $0.0169x = (1.3)^2 = 1.69 \Leftrightarrow x = \frac{1.69}{0.0169} = 100.$

39. $37 + \sqrt{.0615 + x} = 37.25 \Leftrightarrow \sqrt{.0615 + x} = 0.25$

$\Leftrightarrow .0615 + x = (0.25)^2 = 0.0625 \Leftrightarrow x = .001 = \frac{1}{10^3} = 10^{-3}.$

40. $\sqrt{(x-1)(y+2)} = 7 \Rightarrow (x-1)(y+2) = (7)^2 \Rightarrow (x-1) = 7$ and $(y+2) = 7$
$\Rightarrow x = 8$ and $y = 5$.

41. $\frac{\sqrt{a}}{\sqrt{b}} = \frac{.004 \times .4}{\sqrt{.04 \times .4}} \Rightarrow \frac{a}{b} = \frac{.004 \times .4 \times .004 \times .4}{.04 \times .4} = \frac{.0000064}{.04}$

$\therefore \frac{a}{b} = \frac{.00064}{4} = .00016 = \frac{16}{10^5} = 16 \times 10^{-5}$.

42. Let the number be x. Then,

$\frac{3}{5}x^2 = 126.15 \Leftrightarrow x^2 = \left(126.15 \times \frac{5}{3}\right) = 210.25 \Leftrightarrow x = \sqrt{210.25} = 14.5$.

43. $\sqrt{\frac{0.361}{0.00169}} = \sqrt{\frac{0.36100}{0.00169}} = \sqrt{\frac{36100}{169}} = \frac{190}{13}$.

44. $\sqrt{\frac{48.4}{0.289}} = \sqrt{\frac{48.400}{0.289}} = \sqrt{\frac{48400}{289}} = \frac{220}{17} = 12\frac{16}{17}$.

45. $\sqrt{1+\frac{x}{169}} = \frac{14}{13} \Rightarrow 1+\frac{x}{169} = \frac{196}{169} \Rightarrow \frac{x}{169} = \left(\frac{196}{169} - 1\right) = \frac{27}{169} \Rightarrow x = 27$.

46. $\sqrt{1+\frac{55}{729}} = 1+\frac{x}{27} \Rightarrow \sqrt{\frac{784}{729}} = \frac{27+x}{27} \Rightarrow \frac{28}{27} = \frac{27+x}{27} \Rightarrow 27+x = 28 \Rightarrow x = 1$.

47.
1	$\overline{2}.\overline{00}\,\overline{00}\,\overline{00}$ (1.414
	1
24	100
	96
281	400
	281
2824	11900
	11296

$\therefore \sqrt{2} = 1.414$.

48. $2\sqrt{27} - \sqrt{75} + \sqrt{12} = 2\sqrt{9 \times 3} - \sqrt{25 \times 3} + \sqrt{4 \times 3} = 6\sqrt{3} - 5\sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$.

49. $(\sqrt{12} + \sqrt{18}) - (\sqrt{3} + \sqrt{2}) = (\sqrt{4 \times 3} + \sqrt{9 \times 2}) - (\sqrt{3} + \sqrt{2}) = (2\sqrt{3} + 3\sqrt{2}) - (\sqrt{3} + \sqrt{2})$
$= (2\sqrt{3} - \sqrt{3}) + (3\sqrt{2} - \sqrt{2}) = \sqrt{3} + 2\sqrt{2}$.

50. $\frac{\sqrt{24} + \sqrt{216}}{\sqrt{96}} = \frac{\sqrt{4 \times 6} + \sqrt{36 \times 6}}{\sqrt{16 \times 6}} = \frac{2\sqrt{6} + 6\sqrt{6}}{4\sqrt{6}} = \frac{8\sqrt{6}}{4\sqrt{6}} = 2$.

51. $\frac{\sqrt{80} - \sqrt{112}}{\sqrt{45} - \sqrt{63}} = \frac{\sqrt{16 \times 5} - \sqrt{16 \times 7}}{\sqrt{9 \times 5} - \sqrt{9 \times 7}} = \frac{4\sqrt{5} - 4\sqrt{7}}{3\sqrt{5} - 3\sqrt{7}} = \frac{4(\sqrt{5} - \sqrt{7})}{3(\sqrt{5} - \sqrt{7})} = \frac{4}{3} = 1\frac{1}{3}$.

52. $3\sqrt{5} + \sqrt{125} = 17.88 \Rightarrow 3\sqrt{5} + \sqrt{25 \times 5} = 17.88$
$\Rightarrow 3\sqrt{5} + 5\sqrt{5} = 17.88 \Rightarrow 8\sqrt{5} = 17.88 \Rightarrow \sqrt{5} = 2.235$.
$\therefore \sqrt{80} + 6\sqrt{5} = \sqrt{16 \times 5} + 6\sqrt{5} = 4\sqrt{5} + 6\sqrt{5} = 10\sqrt{5} = (10 \times 2.235) = 22.35$.

53. $\sqrt{50} \times \sqrt{98} = \sqrt{50 \times 98} = \sqrt{4900} = 70$.

54. Given exp. $= \sqrt{4\times2} + 2\sqrt{16\times2} - 3\sqrt{64\times2} + 4\sqrt{25\times2}$

$= 2\sqrt{2} + 8\sqrt{2} - 24\sqrt{2} + 20\sqrt{2} = 6\sqrt{2} = 6\times1.414 = 8.484.$

55. Given exp. $= \dfrac{3\sqrt{12}}{2\sqrt{28}} \times \dfrac{\sqrt{98}}{2\sqrt{21}} = \dfrac{3\sqrt{4\times3}}{2\sqrt{4\times7}} \times \dfrac{\sqrt{49\times2}}{2\sqrt{21}} = \dfrac{6\sqrt{3}}{4\sqrt{7}} \times \dfrac{7\sqrt{2}}{2\sqrt{21}} = \dfrac{21\sqrt{6}}{4\sqrt{7\times21}} = \dfrac{21\sqrt{6}}{28\sqrt{3}}$

$= \dfrac{3}{4}\sqrt{2} = \dfrac{3}{4}\times1.414 = 3\times0.3535 = 1.0605.$

56. Sum of decimal places in the numerator and denominator under the radical sign being the same, we remove the decimal.

$\therefore$ Given exp. $= \sqrt{\dfrac{81\times484}{64\times625}} = \dfrac{9\times22}{8\times25} = 0.99.$

57. Given exp. $= \sqrt{\dfrac{204\times42}{7\times34}} = \sqrt{36} = 6.$

58. Given exp. $= \sqrt{\dfrac{81\times324\times4624}{15625\times289\times729\times64}} = \dfrac{9\times18\times68}{125\times17\times27\times8} = \dfrac{3}{125} = 0.024.$

59. Given exp. $= \sqrt{\dfrac{9.5\times.08500}{.19\times.0017}} = \sqrt{\dfrac{95\times8500}{19\times17}} = \sqrt{5\times500} = \sqrt{2500} = 50.$

60. Given exp. $= \sqrt{\dfrac{(0.03)^2 + (0.21)^2 + (0.065)^2}{\left(\dfrac{0.03}{10}\right)^2 + \left(\dfrac{0.21}{10}\right)^2 + \left(\dfrac{0.065}{10}\right)^2}}$

$= \sqrt{\dfrac{100\,[(0.03)^2 + (0.21)^2 + (0.065)^2]}{(0.03)^2 + (0.21)^2 + (0.065)^2}} = \sqrt{100} = 10.$

61. $\sqrt{(7+3\sqrt{5})(7-3\sqrt{5})} = \sqrt{(7)^2 - (3\sqrt{5})^2} = \sqrt{49-45} = \sqrt{4} = 2.$

62. $\left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right)^2 = (\sqrt{3})^2 + \left(\dfrac{1}{\sqrt{3}}\right)^2 - 2\times\sqrt{3}\times\dfrac{1}{\sqrt{3}} = 3 + \dfrac{1}{3} - 2 = 1 + \dfrac{1}{3} = \dfrac{4}{3}.$

63. $\left(\sqrt{2} + \dfrac{1}{\sqrt{2}}\right)^2 = (\sqrt{2})^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2 + 2\times\sqrt{2}\times\dfrac{1}{\sqrt{2}} = 2 + \dfrac{1}{2} + 2 = 4 + \dfrac{1}{2} = 4\dfrac{1}{2}.$

64. $\sqrt{4a^2 - 4a + 1} + 3a = \sqrt{(1)^2 + (2a)^2 - 2\times1\times2a} + 3a$

$= \sqrt{(1-2a)^2} + 3a = (1-2a) + 3a = (1+a) = (1 + 0.1039) = 1.1039.$

65. $\sqrt{\dfrac{(0.75)^3}{(1-0.75)} + [0.75 + (0.75)^2 + 1]} = \sqrt{\dfrac{(0.75)^3 + (1-0.75)\,[(1)^2 + (0.75)^2 + 1\times0.75]}{1-0.75}}$

$= \sqrt{\dfrac{(0.75)^3 + [(1)^3 - (0.75)^3]}{1-0.75}} = \sqrt{\dfrac{1}{0.25}} = \sqrt{\dfrac{100}{25}} = \sqrt{4} = 2.$

66. $4b = 6c \Rightarrow b = \dfrac{3}{2}c$ and $3a = 4b \Rightarrow a = \dfrac{4}{3}b = \dfrac{4}{3}\left(\dfrac{3}{2}c\right) = 2c.$

$a + b + c = 27\sqrt{29} \Rightarrow 2c + \dfrac{3}{2}c + c = 27\sqrt{29} \Rightarrow \dfrac{9}{2}c = 27\sqrt{29} \Rightarrow c = 6\sqrt{29}.$

$$\therefore \quad \sqrt{a^2+b^2+c^2} = \sqrt{(a+b+c)^2 - 2(ab+bc+ca)}$$

$$= \sqrt{(27\sqrt{29})^2 - 2\left(2c \times \frac{3}{2}c + \frac{3}{2}c \times c + c \times 2c\right)}$$

$$= \sqrt{(729 \times 29) - 2\left(3c^2 + \frac{3}{2}c^2 + 2c^2\right)} = \sqrt{(729 \times 29) - 2 \times \frac{13}{2}c^2}$$

$$= \sqrt{(729 \times 29) - 13 \times (6\sqrt{29})^2} = \sqrt{29(729 - 468)}$$

$$= \sqrt{29 \times 261} = \sqrt{29 \times 29 \times 9} = 29 \times 3 = 87.$$

67. $\sqrt{0.\overline{4}} = \sqrt{\frac{4}{9}} = \frac{2}{3} = 0.666\ldots = 0.\overline{6}.$

68. $\sqrt{0.09} = \sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3$, which is rational.

$\therefore$ 0.09 has rational square root.

69.

6	$0.\overline{40}\,\overline{00}\,\overline{00}$ (.63
	36
123	400
	369

70.

3	$0.\overline{12}\,\overline{10}\,\overline{00}$ (.347
	9
64	310
	256
687	5400
	4809

71.

2	$0.\overline{06}\,\overline{40}\,\overline{00}$ (.252
	4
45	240
	225
502	1500
	1006

72. $\sqrt{\frac{0.16}{0.4}} = \sqrt{\frac{0.16}{0.40}} = \sqrt{\frac{16}{40}} = \sqrt{\frac{4}{10}} = \sqrt{0.4} = 0.63.$

73. $\frac{1+\sqrt{0.01}}{1-\sqrt{0.1}} = \frac{1+0.1}{1-0.316} = \frac{1.1}{0.684}$

$$= \frac{1100}{684} = 1.6.$$

3	$0.\overline{10}\,\overline{00}\,\overline{00}$ (.316
	9
61	100
	61
62	3900
	3756

74. $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{2.236}{5} = 0.447.$

75. $\sqrt{\frac{8}{3}} = \sqrt{\frac{8 \times 3}{3 \times 3}} = \frac{\sqrt{24}}{3} = \frac{4.899}{3} = 1.633.$

76. $\frac{3\sqrt{2}}{2\sqrt{3}} = \frac{3\sqrt{2}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{6}}{2 \times 3} = \frac{\sqrt{6}}{2} = \frac{2.449}{2} = 1.2245.$

77. $\dfrac{\sqrt{5}}{2} - \dfrac{10}{\sqrt{5}} + \sqrt{125} = \dfrac{(\sqrt{5})^2 - 20 + 2\sqrt{5} \times 5\sqrt{5}}{2\sqrt{5}} = \dfrac{5 - 20 + 50}{2\sqrt{5}}$

$= \dfrac{35}{2\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{35\sqrt{5}}{10} = \dfrac{7}{2} \times 2.236 = 7 \times 1.118 = 7.826.$

78. Clearly, $a * b = \sqrt{a^2 + b^2}$.

$\therefore \quad 5 * 12 = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13.$

79. L.C.M. of 3, 4, 5, 6, 8 is 120. Now, $120 = 2 \times 2 \times 2 \times 3 \times 5$.
To make it a perfect square, it must be multiplied by $2 \times 3 \times 5$.
So, required number $= 2^2 \times 2^2 \times 3^2 \times 5^2 = 3600$.

80. L.C.M. of 21, 36, 66 = 2772. Now, $2772 = 2 \times 2 \times 3 \times 3 \times 7 \times 11$.
To make it a perfect square, it must be multiplied by 7×11.
So, required number $= 2^2 \times 3^2 \times 7^2 \times 11^2 = 213444$.

81. $294 = 7 \times 7 \times 2 \times 3$.
To make it a perfect square, it must be multiplied by 2×3 *i.e.*, 6.
$\therefore$ Required number = 6.

82. $5808 = 2 \times 2 \times 2 \times 2 \times 3 \times 11 \times 11 = 2^2 \times 2^2 \times 3 \times 11^2$.
To make it a perfect square, it must be multiplied by 3.

83. $1470 = 7 \times 7 \times 5 \times 6$. To make it a perfect square, it must be divided by 5×6, *i.e.*, 30.

84.

7	$\overline{54}\,\overline{91}\,\overline{62}$	(741
	49	
144	591	
	576	
1481	1562	
	1481	
	81	

$\therefore$ Required number to be subtracted = 81.

85. $0.000326 = \dfrac{326}{10^6}$.

$\therefore$ Required number to be subtracted $= \dfrac{2}{10^6} = 0.000002.$

1	$\overline{3}\,\overline{26}$	(18
	1	
28	226	
	224	
	2	

86.

8	$\overline{68}\,\overline{06}\,\overline{21}$	(824
	64	
162	406	
	324	
1644	8221	
	6576	
	1645	

$\therefore$ Number to be added $= (825)^2 - 680621 = 680625 - 680621 = 4.$

87. Greatest number of four digits is 9999.

9	$\overline{99}\,\overline{99}$	(99
	81	
189	1899	
	1701	
	198	

$\therefore$ Required number = (9999 − 198) = 9801.

88. Least number of 4 digits is 1000.

$$\begin{array}{r|l} 3 & \overline{10}\,\overline{00}\ (31 \\ & 9 \\ \hline 61 & 100 \\ & 61 \\ \hline & 39 \end{array}$$

$\therefore\ (31)^2 < 1000 < (32)^2$. Hence, required number $= (32)^2 = 1024$.

89. $\dfrac{1}{(\sqrt{5}-\sqrt{3})} = \dfrac{1}{(\sqrt{5}-\sqrt{3})} \times \dfrac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} = \dfrac{(\sqrt{5}+\sqrt{3})}{(5-3)} = \dfrac{(2.2361+1.7321)}{2} = \dfrac{3.9682}{2} = 1.9841.$

90. Given exp. $= \dfrac{1}{(\sqrt{9}-\sqrt{8})} \times \dfrac{(\sqrt{9}+\sqrt{8})}{(\sqrt{9}+\sqrt{8})} - \dfrac{1}{(\sqrt{8}-\sqrt{7})} \times \dfrac{(\sqrt{8}+\sqrt{7})}{(\sqrt{8}+\sqrt{7})} + \dfrac{1}{(\sqrt{7}-\sqrt{6})} \times \dfrac{(\sqrt{7}+\sqrt{6})}{(\sqrt{7}+\sqrt{6})}$

$$- \frac{1}{(\sqrt{6}-\sqrt{5})} \times \frac{(\sqrt{6}+\sqrt{5})}{(\sqrt{6}+\sqrt{5})} + \frac{1}{(\sqrt{5}-\sqrt{4})} \times \frac{(\sqrt{5}+\sqrt{4})}{(\sqrt{5}+\sqrt{4})}$$

$$= \frac{(\sqrt{9}+\sqrt{8})}{(9-8)} - \frac{(\sqrt{8}+\sqrt{7})}{(8-7)} + \frac{(\sqrt{7}+\sqrt{6})}{(7-6)} - \frac{(\sqrt{6}+\sqrt{5})}{(6-5)} + \frac{(\sqrt{5}+\sqrt{4})}{(5-4)}$$

$$= (\sqrt{9}+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+\sqrt{4}) = (\sqrt{9}+\sqrt{4}) = 3+2 = 5.$$

91. Given exp. $= (2+\sqrt{2}) + \dfrac{1}{(2+\sqrt{2})} \times \dfrac{(2-\sqrt{2})}{(2-\sqrt{2})} - \dfrac{1}{(2-\sqrt{2})} \times \dfrac{(2+\sqrt{2})}{(2+\sqrt{2})}$

$$= (2+\sqrt{2}) + \frac{(2-\sqrt{2})}{(4-2)} - \frac{(2+\sqrt{2})}{(4-2)} = (2+\sqrt{2}) + \frac{1}{2}(2-\sqrt{2}) - \frac{1}{2}(2+\sqrt{2}) = 2.$$

92. $\dfrac{7}{(3+\sqrt{2})} = \dfrac{7}{(3+\sqrt{2})} \times \dfrac{(3-\sqrt{2})}{(3-\sqrt{2})} = \dfrac{7(3-\sqrt{2})}{(9-2)} = (3-\sqrt{2}) = (3-1.4142) = 1.5858.$

93. Given exp. $= \dfrac{3\sqrt{2}}{(\sqrt{6}-\sqrt{3})} \times \dfrac{(\sqrt{6}+\sqrt{3})}{(\sqrt{6}+\sqrt{3})} - \dfrac{4\sqrt{3}}{(\sqrt{6}-\sqrt{2})} \times \dfrac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})} - \dfrac{6}{2(\sqrt{2}-\sqrt{3})}$

$$= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{(6-3)} - \frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{(6-2)} + \frac{3}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$$

$$= \sqrt{2}(\sqrt{6}+\sqrt{3}) - \sqrt{3}(\sqrt{6}+\sqrt{2}) + 3(\sqrt{3}+\sqrt{2})$$

$$= \sqrt{12} + \sqrt{6} - \sqrt{18} - \sqrt{6} + 3\sqrt{3} + 3\sqrt{2}$$

$$= 2\sqrt{3} - 3\sqrt{2} + 3\sqrt{3} + 3\sqrt{2} = 5\sqrt{3}.$$

94. $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \dfrac{(\sqrt{7}+\sqrt{5})}{(\sqrt{7}-\sqrt{5})} \times \dfrac{(\sqrt{7}+\sqrt{5})}{(\sqrt{7}+\sqrt{5})} = \dfrac{(\sqrt{7}+\sqrt{5})^2}{(7-5)} = \dfrac{7+5+2\sqrt{35}}{2} = \dfrac{12+2\sqrt{35}}{2} = 6+\sqrt{35}.$

95. $a+b\sqrt{3} = \dfrac{(5+2\sqrt{3})}{(7+4\sqrt{3})} \times \dfrac{(7-4\sqrt{3})}{(7-4\sqrt{3})} = \dfrac{35-20\sqrt{3}+14\sqrt{3}-24}{(7)^2-(4\sqrt{3})^2} = \dfrac{11-6\sqrt{3}}{49-48} = 11-6\sqrt{3}.$

$\therefore\ a = 11,\ b = -6.$

96. $\dfrac{\sqrt{2}-1}{\sqrt{2}+1} = \dfrac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \dfrac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = (\sqrt{2}-1)^2.$

$\therefore\ \sqrt{\dfrac{\sqrt{2}-1}{\sqrt{2}+1}} = (\sqrt{2}-1) = (1.414-1) = 0.414.$

97. Given exp. $= \dfrac{3+\sqrt{6}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}} = \dfrac{(3+\sqrt{6})}{(\sqrt{3}+\sqrt{2})}$

$= \dfrac{(3+\sqrt{6})}{(\sqrt{3}+\sqrt{2})} \times \dfrac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} = \dfrac{3\sqrt{3}-3\sqrt{2}+3\sqrt{2}-2\sqrt{3}}{(3-2)} = \sqrt{3}.$

98. Given exp. $= \dfrac{(2+\sqrt{3})}{(2-\sqrt{3})} \times \dfrac{(2+\sqrt{3})}{(2+\sqrt{3})} + \dfrac{(2-\sqrt{3})}{(2+\sqrt{3})} \times \dfrac{(2-\sqrt{3})}{(2-\sqrt{3})} + \dfrac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \dfrac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$= \dfrac{(2+\sqrt{3})^2}{(4-3)} + \dfrac{(2-\sqrt{3})^2}{(4-3)} + \dfrac{(\sqrt{3}-1)^2}{(3-1)} = [(2+\sqrt{3})^2 + (2-\sqrt{3})^2] + \dfrac{4-2\sqrt{3}}{2}$

$= 2(4+3) + 2 - \sqrt{3} = 16 - \sqrt{3}.$

99. $x + \dfrac{1}{x} = (7-4\sqrt{3}) + \dfrac{1}{(7-4\sqrt{3})} \times \dfrac{(7+4\sqrt{3})}{(7+4\sqrt{3})} = (7-4\sqrt{3}) + \dfrac{(7+4\sqrt{3})}{(49-48)}$

$= (7-4\sqrt{3}) + (7+4\sqrt{3}) = 14.$

100. $x = \dfrac{(\sqrt{3}+1)}{(\sqrt{3}-1)} \times \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \dfrac{(\sqrt{3}+1)^2}{(3-1)} = \dfrac{3+1+2\sqrt{3}}{2} = 2+\sqrt{3}.$

$y = \dfrac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \dfrac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \dfrac{(\sqrt{3}-1)^2}{(3-1)} = \dfrac{3+1-2\sqrt{3}}{2} = 2-\sqrt{3}.$

$\therefore\ x^2 + y^2 = (2+\sqrt{3})^2 + (2-\sqrt{3})^2 = 2(4+3) = 2 \times 7 = 14.$

101. $a = \dfrac{(\sqrt{5}+1)}{(\sqrt{5}-1)} \times \dfrac{(\sqrt{5}+1)}{(\sqrt{5}+1)} = \dfrac{(\sqrt{5}+1)^2}{(5-1)} = \dfrac{5+1+2\sqrt{5}}{4} = \left(\dfrac{3+\sqrt{5}}{2}\right).$

$b = \dfrac{(\sqrt{5}-1)}{(\sqrt{5}+1)} \times \dfrac{(\sqrt{5}-1)}{(\sqrt{5}-1)} = \dfrac{(\sqrt{5}-1)^2}{(5-1)} = \dfrac{5+1-2\sqrt{5}}{4} = \left(\dfrac{3-\sqrt{5}}{2}\right).$

$\therefore\ a^2 + b^2 = \dfrac{(3+\sqrt{5})^2}{4} + \dfrac{(3-\sqrt{5})^2}{4} = \dfrac{(3+\sqrt{5})^2 + (3-\sqrt{5})^2}{4} = \dfrac{2(9+5)}{4} = 7.$

Also, $ab = \dfrac{(3+\sqrt{5})}{2} \cdot \dfrac{(3-\sqrt{5})}{2} = \dfrac{(9-5)}{4} = 1.$

$\therefore\ \dfrac{a^2+ab+b^2}{a^2-ab+b^2} = \dfrac{(a^2+b^2)+ab}{(a^2+b^2)-ab} = \dfrac{7+1}{7-1} = \dfrac{8}{6} = \dfrac{4}{3}.$

102.

1	$\overline{1}\,\overline{53}\,\overline{76}$	(124
	1	
22	53	
	44	
244	976	
	976	
	×	

$\therefore$ Number of rows = 124.

103.

1	$\overline{3}\,\overline{65}\,\overline{81}$	(191
	1	
29	265	
	261	
381	481	
	381	
	100	

$\therefore$ Number of men left = 100.

104. Money collected = (59.29 × 100) paise = 5929 paise.

$\therefore$ Number of members = $\sqrt{5929} = 77.$

105. $(.000216)^{1/3} = \left(\dfrac{216}{10^6}\right)^{1/3} = \left(\dfrac{6\times6\times6}{10^2\times10^2\times10^2}\right)^{1/3} = \dfrac{6}{10^2} = \dfrac{6}{100} = .06.$

106. $\sqrt[3]{4\frac{12}{125}} = \sqrt[3]{\frac{512}{125}} = \left(\frac{8\times8\times8}{5\times5\times5}\right)^{1/3} = \frac{8}{5} = 1\frac{3}{5}$.

107. $\sqrt{.000064} = \sqrt{\frac{64}{10^6}} = \frac{8}{10^3} = \frac{8}{1000} = .008$.

$\therefore \quad \sqrt[3]{\sqrt{.000064}} = \sqrt[3]{.008} = \sqrt[3]{\frac{8}{1000}} = \frac{2}{10} = 0.2$.

108. Clearly, 9261 is a perfect cube satisfying the given property.

109. $675 = 5 \times 5 \times 3 \times 3 \times 3$.

To make it a perfect cube, it must be multiplied by 5.

110. $3600 = 2^3 \times 5^2 \times 3^2 \times 2$.

To make it a perfect cube, it must be divided by $5^2 \times 3^2 \times 2$ *i.e.*, 450.

6. AVERAGE

IMPORTANT FACTS AND FORMULAE

1. Average $= \left(\dfrac{\text{Sum of observations}}{\text{Number of observations}}\right)$.
2. Suppose a man covers a certain distance at x kmph and an equal distance at y kmph. Then, the average speed during the whole journey is $\left(\dfrac{2xy}{x+y}\right)$ kmph.

SOLVED EXAMPLES

Ex. 1. ***Find the average of all prime numbers between 30 and 50.***

Sol. There are five prime numbers between 30 and 50.

They are 31, 37, 41, 43 and 47.

$\therefore$ Required average $= \left(\dfrac{31+37+41+43+47}{5}\right) = \dfrac{199}{5} = 39.8.$

Ex. 2. ***Find the average of first 40 natural numbers.***

Sol. Sum of first n natural numbers $= \dfrac{n(n+1)}{2}$.

So, sum of first 40 natural numbers $= \dfrac{40 \times 41}{2} = 820.$

$\therefore$ Required average $= \dfrac{820}{40} = 20.5.$

Ex. 3. ***Find the average of first 20 multiples of 7.***

Sol. Required average $= \dfrac{7(1+2+3+.....+20)}{20} = \left(\dfrac{7 \times 20 \times 21}{20 \times 2}\right) = \left(\dfrac{147}{2}\right) = 73.5.$

Ex. 4. ***The average of four consecutive even numbers is 27. Find the largest of these numbers.***

Sol. Let the numbers be x, $x + 2$, $x + 4$ and $x + 6$. Then,

$$\frac{x+(x+2)+(x+4)+(x+6)}{4} = 27 \Rightarrow \frac{4x+12}{4} = 27 \Rightarrow x + 3 = 27 \Rightarrow x = 24.$$

$\therefore$ Largest number $= (x + 6) = 24 + 6 = 30.$

Ex. 5. ***There are two sections A and B of a class, consisting of 36 and 44 students respectively. If the average weight of section A is 40 kg and that of section B is 35 kg, find the average weight of the whole class.***

Sol. Total weight of $(36 + 44)$ students $= (36 \times 40 + 44 \times 35)$ kg $= 2980$ kg.

$\therefore$ Average weight of the whole class $= \left(\dfrac{2980}{80}\right)$ kg $= 37.25$ kg.

Ex. 6. ***Nine persons went to a hotel for taking their meals. Eight of them spent Rs. 12 each on their meals and the ninth spent Rs. 8 more than the average expenditure of all the nine. What was the total money spent by them ?***

Sol. Let the average expenditure of all the nine be Rs. x.

Then, $12 \times 8 + (x + 8) = 9x$ or $8x = 104$ or $x = 13$.

$\therefore$ Total money spent $= 9x =$ Rs. $(9 \times 13) =$ Rs. 117.

Ex. 7. ***Of the three numbers, second is twice the first and is also thrice the third. If the average of the three numbers is 44, find the largest number.***

Sol. Let the third number be x. Then, second number $= 3x$. First number $= \frac{3x}{2}$.

$\therefore \quad x + 3x + \frac{3x}{2} = (44 \times 3)$ or $\frac{11x}{2} = 44 \times 3$ or $x = 24$.

So, largest number = 2nd number $= 3x = 72$.

Ex. 8. ***The average of 25 results is 18. The average of first twelve of them is 14 and that of last twelve is 17. Find the thirteenth result.***

Sol. Clearly, thirteenth result = (sum of 25 results) – (sum of 24 results)

$= (18 \times 25) - [(14 \times 12) + (17 \times 12)]$

$= 450 - (168 + 204) = 450 - 372 = 78$.

Ex. 9. ***The average of 11 results is 60. If the average of first six results is 58 and that of the last six is 63, find the sixth result.***

Sol. Sixth result $= (58 \times 6 + 63 \times 6 - 60 \times 11) = 66$.

Ex. 10. ***The average weight of A, B, C is 45 kg. If the average weight of A and B be 40 kg and that of B and C be 43 kg, find the weight of B.***

Sol. Let A, B and C represent their individual weights. Then,

$A + B + C = (45 \times 3)$ kg $= 135$ kg.

$A + B = (40 \times 2)$ kg $= 80$ kg and $B + C = (43 \times 2)$ kg $= 86$ kg.

$\therefore \quad B = (A + B) + (B + C) - (A + B + C) = (80 + 86 - 135)$ kg $= 31$ kg.

Ex. 11. ***The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increases by 3 months. Find the age of the teacher.***

Sol. Total age of 39 persons $= (39 \times 15)$ years $= 585$ years.

Average age of 40 persons = 15 years 3 months $= \frac{61}{4}$ years.

Total age of 40 persons $= \left(\frac{61}{4} \times 40\right)$ years $= 610$ years.

$\therefore$ Age of the teacher $= (610 - 585)$ years $= 25$ years.

Ex. 12. ***The average weight of 10 oarsmen in a boat is increased by 1.8 kg when one of the crew, who weighs 53 kg is replaced by a new man. Find the weight of the new man.***

Sol. Total weight increased $= (1.8 \times 10)$ kg $= 18$ kg.

$\therefore$ Weight of the new man $= (53 + 18)$ kg $= 71$ kg.

Ex. 13. ***There were 35 students in a hostel. Due to the admission of 7 new students, the expenses of the mess were increased by Rs. 42 per day while the average expenditure per head diminished by Re 1. What was the original expenditure of the mess ?***

Sol. Let the original average expenditure be Rs. x. Then,

$42(x - 1) - 35x = 42 \Leftrightarrow 7x = 84 \Rightarrow x = 12$.

$\therefore$ Original expenditure $=$ Rs. $(35 \times 12) =$ Rs. 420.

Ex. 14. ***A batsman makes a score of 87 runs in the 17th inning and thus increases his average by 3. Find his average after 17th inning.***

Sol. Let the average after 17th inning = x.

Then, average after 16th inning = $(x - 3)$.

$\therefore$ $16(x - 3) + 87 = 17x$ or $x = (87 - 48) = 39$.

Ex. 15. ***Distance between two stations A and B is 778 km. A train covers the journey from A to B at 84 km per hour and returns back to A with a uniform speed of 56 km per hour. Find the average speed of the train during the whole journey.***

Sol. Required average speed $= \left(\frac{2xy}{x+y}\right)$ km/hr $= \frac{2 \times 84 \times 56}{(84+56)}$ km/hr

$= \left(\frac{2 \times 84 \times 56}{140}\right)$ km/hr $= 67.2$ km/hr.

EXERCISE 6A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. David obtained 76, 65, 82, 67 and 85 marks (out of 100) in English, Mathematics, Physics, Chemistry and Biology. What are his average marks ? **(Bank P.O. 2003)**

   (a) 65 (b) 69 (c) 72

   (d) 76 (e) None of these

2. In Arun's opinion, his weight is greater than 65 kg but less than 72 kg. His brother does not agree with Arun and he thinks that Arun's weight is greater than 60 kg but less than 70 kg. His mother's view is that his weight cannot be greater than 68 kg. If all of them are correct in their estimation, what is the average of different probable weights of Arun ? **(S.B.I.P.O. 2000)**

   (a) 67 kg (b) 68 kg (c) 69 kg

   (d) Data inadequate (e) None of these

3. The average of 20 numbers is zero. Of them, at the most, how many may be greater than zero ? **(Hotel Management, 2002)**

   (a) 0 (b) 1 (c) 10 (d) 19

4. Find the average of all the numbers between 6 and 34 which are divisible by 5.

   (a) 18 (b) 20 (c) 24 (d) 30 **(C.B.I. 1997)**

5. The average of first five multiples of 3 is : **(Assistant Grade, 1998)**

   (a) 3 (b) 9 (c) 12 (d) 15

6. The average of the first nine prime numbers is : **(C.B.I. 2003)**

   (a) 9 (b) 11 (c) $11\frac{1}{9}$ (d) $11\frac{2}{9}$

7. A student was asked to find the arithmetic mean of the numbers 3, 11, 7, 9, 15, 13, 8, 19, 17, 21, 14 and x. He found the mean to be 12. What should be the number in place of x ? **(Section Officers', 2003)**

   (a) 3 (b) 7 (c) 17 (d) 31

8. The average of 2, 7, 6 and x is 5 and the average of 18, 1, 6, x and y is 10. What is the value of y ?

   (a) 5 (b) 10 (c) 20 (d) 30

9. If the mean of 5 observations x, $x+2$, $x+4$, $x+6$ and $x+8$ is 11, then the mean of the last three observations is : (C.D.S. 2003)
(a) 11 (b) 13 (c) 15 (d) 17

10. If the mean of a, b, c is M and $ab + bc + ca = 0$, then the mean of a^2, b^2, c^2 is :
(a) M^2 (b) $3M^2$ (c) $6M^2$ (d) $9M^2$
(IITTM, 2003)

11. The average of the two-digit numbers, which remain the same when the digits interchange their positions, is : (C.D.S. 2003)
(a) 33 (b) 44 (c) 55 (d) 66

12. The average of first 50 natural numbers is :
(a) 12.25 (b) 21.25 (c) 25 (d) 25.5

13. The mean of 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2 is :
(a) 10 (b) 20 (c) 30 (d) 40

14. The average of all odd numbers upto 100 is :
(a) 49 (b) 49.5 (c) 50 (d) 51

15. If a, b, c, d, e are five consecutive odd numbers, their average is :
(a) $5(a+4)$ (b) $\frac{abcde}{5}$
(c) $5(a+b+c+d+e)$ (d) None of these

16. The average of a non-zero number and its square is 5 times the number. The number is : (S.S.C. 2003)
(a) 9 (b) 17 (c) 29 (d) 295

17. The average of 7 consecutive numbers is 20. The largest of these numbers is :
(a) 20 (b) 22 (c) 23 (d) 24
(S.S.C. 2000)

18. The average of five consecutive odd numbers is 61. What is the difference between the highest and lowest numbers ? (Bank P.O. 2003)
(a) 2 (b) 5 (c) 8
(d) Cannot be determined (e) None of these

19. The sum of three consecutive odd numbers is 38 more than the average of these numbers. What is the first of these numbers ? (Bank P.O. 1998)
(a) 13 (b) 17 (c) 19
(d) Data inadequate (e) None of these

20. The average age of the boys in a class is 16 years and that of the girls is 15 years. The average age for the whole class is : (S.S.C. 2003)
(a) 15 years (b) 15.5 years (c) 16 years
(d) Cannot be computed with the given information

21. The average annual income (in Rs.) of certain agricultural workers is S and that of other workers is T. The number of agricultural workers is 11 times that of other workers. Then the average monthly income (in Rs.) of all the workers is :
(a) $\frac{S+T}{2}$ (b) $\frac{S+11T}{2}$ (c) $\frac{1}{11S}+T$ (d) $\frac{11S+T}{12}$
(S.S.C. 2004)

22. A family consists of grandparents, parents and three grandchildren. The average age of the grandparents is 67 years, that of the parents is 35 years and that of the grandchildren is 6 years. What is the average age of the family ? (R.R.B. 2003)
(a) $28\frac{4}{7}$ years (b) $31\frac{5}{7}$ years (c) $32\frac{1}{7}$ years (d) None of these

23. A library has an average of 510 visitors on Sundays and 240 on other days. The average number of visitors per day in a month of 30 days beginning with a Sunday is : **(M.A.T. 2003)**

(*a*) 250 (*b*) 276 (*c*) 280 (*d*) 285

24. If the average marks of three batches of 55, 60 and 45 students respectively is 50, 55 and 60, then the average marks of all the students is : **(C.B.I. 2003)**

(*a*) 53.33 (*b*) 54.68 (*c*) 55 (*d*) None of these

25. The average weight of 16 boys in a class is 50.25 kgs and that of the remaining 8 boys is 45.15 kgs. Find the average weight of all the boys in the class. **(I.M.T. 2002)**

(*a*) 47.55 kgs (*b*) 48 kgs (*c*) 48.55 kgs (*d*) 49.25 kgs

26. A car owner buys petrol at Rs 7.50, Rs. 8 and Rs. 8.50 per litre for three successive years. What approximately is the average cost per litre of petrol if he spends Rs. 4000 each year ? **(M.A.T. 2001)**

(*a*) Rs. 7.98 (*b*) Rs. 8 (*c*) Rs. 8.50 (*d*) Rs. 9

27. The average of six numbers is x and the average of three of these is y. If the average of the remaining three is z, then : **(Hotel Management, 2001)**

(*a*) $x = y + z$ (*b*) $2x = y + z$ (*c*) $x = 2y + 2z$ (*d*) None of these

28. Out of 9 persons, 8 persons spent Rs. 30 each for their meals. The ninth one spent Rs. 20 more than the average expenditure of all the nine. The total money spent by all of them was : **(C.B.I. 1998)**

(*a*) Rs. 260 (*b*) Rs. 290 (*c*) Rs. 292.50 (*d*) Rs. 400.50

29. The average of 50 numbers is 30. If two numbers, 35 and 40 are discarded, then the average of the remaining numbers is nearly : **(R.R.B. 2002)**

(*a*) 28.32 (*b*) 28.78 (*c*) 29.27 (*d*) 29.68

30. The average of five numbers is 27. If one number is excluded, the average becomes 25. The excluded number is : **(Section Officers', 2003)**

(*a*) 25 (*b*) 27 (*c*) 30 (*d*) 35

31. The average age of 35 students in a class is 16 years. The average age of 21 students is 14. What is the average age of remaining 14 students ? **(S.B.I.P.O. 1997)**

(*a*) 15 years (*b*) 17 years (*c*) 18 years (*d*) 19 years

32. 16 children are to be divided into two groups A and B of 10 and 6 children. The average percent marks obtained by the children of group A is 75 and the average percent marks of all the 16 children is 76. What is the average percent marks of children of group B ? **(B.S.R.B. 2003)**

(*a*) $77\frac{1}{3}$ (*b*) $77\frac{2}{3}$ (*c*) $78\frac{1}{3}$ (*d*) $78\frac{2}{3}$

33. The average score of a cricketer for ten matches is 38.9 runs. If the average for the first six matches is 42, then find the average for the last four matches.

(*a*) 33.25 (*b*) 33.5 (*c*) 34.25 (*d*) 35

(IGNOU, 2003)

34. The average of six numbers is 3.95. The average of two of them is 3.4, while the average of the other two is 3.85. What is the average of the remaining two numbers ?

(*a*) 4.5 (*b*) 4.6 (*c*) 4.7 (*d*) 4.8

(Bank P.O. 2003)

35. The batting average for 40 innings of a cricket player is 50 runs. His highest score exceeds his lowest score by 172 runs. If these two innings are excluded, the average of the remaining 38 innings is 48 runs. The highest score of the player is :

(*a*) 165 runs (*b*) 170 runs (*c*) 172 runs (*d*) 174 runs

36. The average price of 10 books is Rs. 12 while the average price of 8 of these books is Rs. 11.75. Of the remaining two books, if the price of one book is 60% more than the price of the other, what is the price of each of these two books ?
(*a*) Rs. 5, Rs. 7.50 (*b*) Rs. 8, Rs. 12 (*c*) Rs. 10, Rs. 16 (*d*) Rs. 12, Rs. 14
(Assistant Grade, 1997)

37. The average of runs of a cricket player of 10 innings was 32. How many runs must he make in his next innings so as to increase his average of runs by 4 ?
(*a*) 2 (*b*) 4 (*c*) 70 (*d*) 76
(S.S.C. 2004)

38. A grocer has a sale of Rs. 6435, Rs. 6927, Rs. 6855, Rs. 7230 and Rs. 6562 for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs. 6500 ? **(S.S.C. 2003)**
(*a*) Rs. 4991 (*b*) Rs. 5991 (*c*) Rs. 6001 (*d*) Rs. 6991

39. A company produces on an average 4000 items per month for the first 3 months. How many items it must produce on an average per month over the next 9 months, to average 4375 items per month over the whole ? **(S.S.C. 1999)**
(*a*) 4500 (*b*) 4600 (*c*) 4680 (*d*) 4710

40. In the first 10 overs of a cricket game, the run rate was only 3.2. What should be the run rate in the remaining 40 overs to reach the target of 282 runs ? **(M.A.T. 2002)**
(*a*) 6.25 (*b*) 6.5 (*c*) 6.75 (*d*) 7

41. The average price of three items of furniture is Rs. 15000. If their prices are in the ratio 3 : 5 : 7, the price of the cheapest item is :
(*a*) Rs. 9000 (*b*) Rs. 15000 (*c*) Rs. 18000 (*d*) Rs. 21000

42. Of the four numbers, the first is twice the second, the second is one-third of the third and the third is 5 times the fourth. The average of the numbers is 24.75. The largest of these numbers is : **(Hotel Management, 1998)**
(*a*) 9 (*b*) 25 (*c*) 30 (*d*) None of these

43. Of the four numbers, whose average is 60, the first is one-fourth of the sum of the last three. The first number is : **(S.S.C. 2000)**
(*a*) 15 (*b*) 45 (*c*) 48 (*d*) 60.25

44. Of the three numbers, the first is twice the second and the second is twice the third. The average of the reciprocal of the numbers is $\frac{7}{72}$. The numbers are :
(*a*) 16, 8, 4 (*b*) 20, 10, 5 (*c*) 24, 12, 6 (*d*) 36, 18, 9
(C.B.I. 1997)

45. Of the three numbers, the average of the first and the second is greater than the average of the second and the third by 15. What is the difference between the first and the third of the three numbers ? **(S.B.I.P.O. 2000)**
(*a*) 15 (*b*) 45 (*c*) 60
(*d*) Data inadequate (*e*) None of these

46. The average of 8 numbers is 20. The average of first two numbers is $15\frac{1}{2}$ and that of the next three is $21\frac{1}{3}$. If the sixth number be less than the seventh and eighth numbers by 4 and 7 respectively, then the eighth number is : **(S.S.C. 2004)**
(*a*) 18 (*b*) 22 (*c*) 25 (*d*) 27

47. If the arithmetic mean of seventy-five numbers is calculated, it is 35. If each number is increased by 5, then mean of new numbers is : **(Assistant Grade, 1998)**
(*a*) 30 (*b*) 40 (*c*) 70 (*d*) 90

48. The average of ten numbers is 7. If each number is multiplied by 12, then the average of the new set of numbers is :
(a) 7 (b) 19 (c) 82 (d) 84

49. Average of ten positive numbers is $\bar{x}$. If each number is increased by 10%, then $\bar{x}$:
(a) remains unchanged (b) may decrease
(c) may increase (d) is increased by 10%
(I.M.T. 2002)

50. The mean of 50 observations was 36. It was found later that an observation 48 was wrongly taken as 23. The corrected new mean is : (S.S.C. 2003)
(a) 35.2 (b) 36.1 (c) 36.5 (d) 39.1

51. A pupil's marks were wrongly entered as 83 instead of 63. Due to that the average marks for the class got increased by half. The number of pupils in the class is :
(a) 10 (b) 20 (c) 40 (d) 73
(C.B.I. 1998)

52. The average age of 15 students of a class is 15 years. Out of these, the average age of 5 students is 14 years and that of the other 9 students is 16 years. The age of the 15th student is : (S.S.C. 2003)
(a) 11 years (b) 14 years (c) 15 years (d) $15\frac{2}{7}$ years

53. The average of 11 numbers is 10.9. If the average of the first six numbers is 10.5 and that of the last six numbers is 11.4, then the middle number is :
(a) 11 (b) 11.3 (c) 11.4 (d) 11.5

54. The average weight of three boys A, B and C is $54\frac{1}{3}$ kg, while the average weight of three boys B, D and E is 53 kg. What is the average weight of A, B, C, D and E ?
(a) 52.4 kg (b) 53.2 kg (c) 53.8 kg
(d) Data inadequate (e) None of these
(S.B.I.P.O. 2002)

55. The average temperature of the town in the first four days of a month was 58 degrees. The average for the second, third, fourth and fifth days was 60 degrees. If the temperatures of the first and fifth days were in the ratio 7 : 8, then what is the temperature on the fifth day ? (NMIMS, 2003)
(a) 64 degrees (b) 62 degrees (c) 56 degrees (d) None of these.

56. The average weight of A, B and C is 45 kg. If the average weight of A and B be 40 kg and that of B and C be 43 kg, then the weight of B is : (S.S.C. 2004)
(a) 17 kg (b) 20 kg (c) 26 kg (d) 31 kg

57. The average monthly income of P and Q is Rs. 5050. The average monthly income of Q and R is Rs. 6250 and the average monthly income of P and R is Rs. 5200. The monthly income of P is : (R.R.B. 2004)
(a) Rs. 3500 (b) Rs. 4000 (c) Rs. 4050 (d) Rs. 5000

58. The average age of 36 students in a group is 14 years. When teacher's age is included to it, the average increases by one. What is the teacher's age in years ?
(a) 31 (b) 36 (c) 51
(d) Cannot be determined (e) None of these (R.B.I. 2003)

59. The average monthly salary of 20 employees in an organisation is Rs. 1500. If the manager's salary is added, then the average salary increases by Rs. 100. What is the manager's monthly salary ? (R.R.B. 2002)
(a) Rs. 2000 (b) Rs. 2400 (c) Rs. 3600 (d) Rs. 4800

60. The average weight of a class of 24 students is 35 kg. If the weight of the teacher be included, the average rises by 400 g. The weight of the teacher is : **(S.S.C. 2003)**
(*a*) 45 kg (*b*) 50 kg (*c*) 53 kg (*d*) 55 kg

61. The average age of the mother and her six children is 12 years which is reduced by 5 years if the age of the mother is excluded. How old is the mother ?
(*a*) 40 years (*b*) 42 years (*c*) 48 years (*d*) 50 years

62. The captain of a cricket team of 11 members is 26 years old and the wicket keeper is 3 years older. If the ages of these two are excluded, the average age of the remaining players is one year less than the average age of the whole team. What is the average age of the team ? **(N.I.F.T. 2000)**
(*a*) 23 years (*b*) 24 years (*c*) 25 years (*d*) None of these

63. The average height of 25 boys is 1.4 m. When 5 boys leave the group, then the average height increases by 0.15 m. What is the average height of the 5 boys who leave ?
(*a*) 0.8 m (*b*) 0.9 m (*c*) 0.95 m (*d*) 1.05 m

64. The average weight of 8 persons increases by 2.5 kg when a new person comes in place of one of them weighing 65 kg. What might be the weight of the new person ?
(*a*) 76 kg (*b*) 76.5 kg (*c*) 85 kg
(*d*) Data inadequate (*e*) None of these **(Bank P.O. 2000)**

65. The average weight of 45 students in a class is 52 kg. Five of them whose average weight is 48 kg leave the class and other 5 students whose average weight is 54 kg join the class. What is the new average weight (in kg) of the class ? **(R.R.B. 2002)**
(*a*) $52\frac{1}{3}$ (*b*) $52\frac{1}{2}$ (*c*) $52\frac{2}{3}$ (*d*) None of these

66. The average age of 8 men is increased by 2 years when two of them whose ages are 21 years and 23 years are replaced by two new men. The average age of the two new men is : **(S.S.C. 2002)**
(*a*) 22 years (*b*) 24 years (*c*) 28 years (*d*) 30 years

67. The average of five consecutive numbers is *n*. If the next two numbers are also included, the average will :
(*a*) remain the same (*b*) increase by 1
(*c*) increase by 1.4 (*d*) increase by 2

68. A cricketer has a certain average for 10 innings. In the eleventh inning, he scored 108 runs, thereby increasing his average by 6 runs. His new average is :
(*a*) 48 runs (*b*) 52 runs (*c*) 55 runs (*d*) 60 runs **(A.A.O. Exam, 2003)**

69. A cricketer whose bowling average is 12.4 runs per wicket takes 5 wickets for 26 runs and thereby decreases his average by 0.4. The number of wickets taken by him till the last match was : **(S.S.C. 2000)**
(*a*) 64 (*b*) 72 (*c*) 80 (*d*) 85

70. A team of 8 persons joins in a shooting competition. The best marksman scored 85 points. If he had scored 92 points, the average score for the team would have been 84. The number of points, the team scored was :
(*a*) 588 (*b*) 645 (*c*) 665 (*d*) 672

71. A motorist travels to a place 150 km away at an average speed of 50 km / hr and returns at 30 km / hr. His average speed for the whole journey in km / hr is :
(*a*) 35 (*b*) 37 (*c*) 37.5 (*d*) 40

72. The average weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man E, whose weight is 3 kg more than that of D, replaces A, then the average weight of B, C, D and E becomes 79 kg. The weight of A is : **(Bank P.O. 2003)**
(*a*) 70 kg (*b*) 72 kg (*c*) 75 kg (*d*) 80 kg

73. The average age of a husband and his wife was 23 years at the time of their marriage. After five years they have a one-year old child. The average age of the family now is :
(a) 19 years (b) 23 years (c) 28.5 years (d) 29.3 years
(Assistant Grade., 1998)

74. Three years ago, the average age of A and B was 18 years. With C joining them, the average age becomes 22 years. How old is C now ?
(a) 24 years (b) 27 years (c) 28 years (d) 30 years

75. The average age of husband, wife and their child 3 years ago was 27 years and that of wife and the child 5 years ago was 20 years. The present age of the husband is :
(a) 35 years (b) 40 years (c) 50 years (d) None of these
(Hotel Management, 2003)

76. 3 years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is the same today. The present age of the baby is
(S.S.C. 2004)
(a) 1 year (b) $1\frac{1}{2}$ years (c) 2 years (d) 3 years

77. 10 years ago, the average age of a family of 4 members was 24 years. Two children having been born (with age difference of 2 years), the present average age of the family is the same. The present age of the youngest child is :
(S.S.C. 2003)
(a) 1 year (b) 2 years (c) 3 years (d) 5 years

78. After replacing an old member by a new member, it was found that the average age of five members of a club is the same as it was 3 years ago. What is the difference between the ages of the replaced and the new member ?
(a) 2 years (b) 4 years (c) 8 years (d) 15 years

79. The average age of 3 children in a family is 20% of the average age of the father and the eldest child. The total age of the mother and the youngest child is 39 years. If the father's age is 26 years, what is the age of second child ?
(a) 15 years (b) 18 years (c) 20 years (d) Cannot be determined

80. The average age of a group of persons going for picnic is 16 years. Twenty new persons with an average age of 15 years join the group on the spot due to which their average age becomes 15.5 years. The number of persons initially going for picnic is :
(a) 5 (b) 10 (c) 20 (d) 30

81. A certain factory employed 600 men and 400 women and the average wage was Rs. 25.50 per day. If a woman got Rs. 5 less than a man, then what are their daily wages ?
(a) Man : Rs. 25; Woman : Rs. 20 (b) Man : Rs. 27.50, Woman : Rs. 22.50
(c) Man : Rs. 30, Woman : Rs. 25 (d) Man : Rs. 32.50, Woman : Rs. 27.50

82. The arithmetic mean of the scores of a group of students in a test was 52. The brightest 20% of them secured a mean score of 80 and the dullest 25% a mean score of 31. The mean score of remaining 55% is :
(S.S.C. 2000)
(a) 45 (b) 50 (c) 51.4 approx. (d) 54.6 approx.

83. The average salary of all the workers in a workshop is Rs. 8000. The average salary of 7 technicians is Rs. 12000 and the average salary of the rest is Rs. 6000. The total number of workers in the workshop is :
(S.S.C. 2003)
(a) 20 (b) 21 (c) 22 (d) 23

84. In a school with 600 students, the average age of the boys is 12 years and that of the girls is 11 years. If the average age of the school is 11 years 9 months, then the number of girls in the school is :
(a) 150 (b) 250 (c) 350 (d) 450

85. In an examination, a pupil's average marks were 63 per paper. If he had obtained 20 more marks for his Geography paper and 2 more marks for his History paper, his average per paper would have been 65. How many papers were there in the examination ?

(a) 8 (b) 9 (c) 10 (d) 11 (e) 12

(SCMHRD, 2001)

86. The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of the number of boys to the number of girls in the class is : (S.B.I.P.O. 2002)

(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 3 : 5

ANSWERS

1. (e)	2. (e)	3. (d)	4. (b)	5. (b)	6. (c)	7. (b)	8. (c)	9. (b)
10. (b)	11. (c)	12. (d)	13. (b)	14. (c)	15. (d)	16. (a)	17. (c)	18. (c)
19. (b)	20. (d)	21. (d)	22. (b)	23. (d)	24. (b)	25. (c)	26. (a)	27. (b)
28. (c)	29. (d)	30. (d)	31. (d)	32. (b)	33. (c)	34. (b)	35. (d)	36. (c)
37. (d)	38. (a)	39. (a)	40. (a)	41. (a)	42. (d)	43. (c)	44. (c)	45. (e)
46. (c)	47. (b)	48. (d)	49. (d)	50. (c)	51. (c)	52. (a)	53. (d)	54. (d)
55. (a)	56. (d)	57. (b)	58. (c)	59. (c)	60. (a)	61. (b)	62. (d)	63. (a)
64. (c)	65. (c)	66. (d)	67. (b)	68. (a)	69. (d)	70. (c)	71. (c)	72. (c)
73. (a)	74. (a)	75. (b)	76. (c)	77. (c)	78. (d)	79. (d)	80. (c)	81. (b)
82. (c)	83. (b)	84. (a)	85. (d)	86. (b)				

SOLUTIONS

1. Average $= \left(\frac{76 + 65 + 82 + 67 + 85}{5}\right) = \left(\frac{375}{5}\right) = 75.$

2. Let Arun's weight be X kg.

According to Arun, $65 < X < 72$.

According to Arun's brother, $60 < X < 70$.

According to Arun's mother, $X < 68$.

The values satisfying all the above conditions are 66 and 67.

$\therefore$ Required average $= \left(\frac{66 + 67}{2}\right) = \left(\frac{133}{2}\right) = 66.5$ kg.

3. Average of 20 numbers = 0.

$\therefore$ Sum of 20 numbers $= (0 \times 20) = 0$.

It is quite possible that 19 of these numbers may be positive and if their sum is a, then 20th number is $(-a)$.

4. Average $= \left(\frac{10 + 15 + 20 + 25 + 30}{5}\right) = \frac{100}{5} = 20.$

5. Average $= \frac{3(1 + 2 + 3 + 4 + 5)}{5} = \frac{45}{5} = 9.$

6. Average $= \left(\frac{2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23}{9}\right) = \frac{100}{9} = 11\frac{1}{9}.$

7. Clearly, we have $\left(\frac{3+11+7+9+15+13+8+19+17+21+14+x}{12}\right) = 12$

or $137 + x = 144$ or $x = 144 - 137 = 7$.

8. We have : $\left(\frac{2+7+6+x}{4}\right) = 5$ or $15 + x = 20$ or $x = 5$.

Also, $\left(\frac{18+1+6+x+y}{5}\right) = 10$ or $25 + 5 + y = 50$ or $y = 20$.

9. We have : $\left[\frac{x+(x+2)+(x+4)+(x+6)+(x+8)}{5}\right] = 11$ or $5x + 20 = 55$ or $x = 7$.

So, the numbers are 7, 9, 11, 13, 15.

$\therefore$ Required mean $= \left(\frac{11+13+15}{3}\right) = \frac{39}{3} = 13$.

10. We have : $\left(\frac{a+b+c}{3}\right) = M$ or $(a + b + c) = 3M$.

Now, $(a + b + c)^2 = (3M)^2 = 9M^2$.

$\Leftrightarrow$ $a^2 + b^2 + c^2 + 2(ab + bc + ca) = 9M^2$

$\Leftrightarrow$ $a^2 + b^2 + c^2 = 9M^2$. [$\because$ $(ab + bc + ca) = 0$]

$\therefore$ Required mean $= \left(\frac{a^2+b^2+c^2}{3}\right) = \frac{9M^2}{3} = 3M^2$.

11. Average $= \left(\frac{11+22+33+44+55+66+77+88+99}{9}\right)$

$= \left[\frac{(11+99)+(22+88)+(33+77)+(44+66)+55}{9}\right]$

$= \left(\frac{4\times 110+55}{9}\right) = \frac{495}{9} = 55$.

12. Sum of first n natural numbers $= \frac{n(n+1)}{2}$.

So, average of first n natural numbers $= \frac{n(n+1)}{2n} = \frac{n+1}{2}$.

$\therefore$ Required average $= \left(\frac{50+1}{2}\right) = \frac{51}{2} = 25.5$.

13. $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

$\therefore$ $1^2 + 2^2 + 3^2 + ... + 7^2 = \left(\frac{7\times 8\times 15}{6}\right) = 140$.

So, required average $= \left(\frac{140}{7}\right) = 20$.

14. Sum of odd numbers upto 100 $= 1 + 3 + 5 + 7 + ... + 95 + 97 + 99$.

$= (1 + 99) + (3 + 97) + (5 + 95) + ... +$ upto 25 pairs

$= 100 + 100 + 100 + ...$ (25 times) $= 2500$.

$\therefore$ Average $= \left(\frac{2500}{50}\right) = 50$.

15. Clearly, $b = a + 2$, $c = a + 4$, $d = a + 6$ and $e = a + 8$.

$\therefore$ Average $= \dfrac{a + (a+2) + (a+4) + (a+6) + (a+8)}{5} = \left(\dfrac{5a + 20}{5}\right) = (a + 4)$.

16. Let the number be x. Then,

$$\frac{x + x^2}{2} = 5x \iff x^2 - 9x = 0 \iff x(x - 9) = 0 \iff x = 0 \text{ or } x = 9.$$

So, the number is 9.

17. Let the numbers be x, $x + 1$, $x + 2$, $x + 3$, $x + 4$, $x + 5$ and $x + 6$.

Then, $\dfrac{x + (x+1) + (x+2) + (x+3) + (x+4) + (x+5) + (x+6)}{7} = 20$

or $7x + 21 = 140$ or $7x = 119$ or $x = 17$.

$\therefore$ Largest number $= x + 6 = 23$.

18. Let the numbers be x, $x + 2$, $x + 4$, $x + 6$ and $x + 8$.

Then, $\dfrac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5} = 61$ or $5x + 20 = 305$ or $x = 57$.

So, required difference $= (57 + 8) - 57 = 8$.

19. Let the numbers be x, $x + 2$ and $x + 4$.

Then, $(x + x + 2 + x + 4) - \dfrac{(x + x + 2 + x + 4)}{3} = 38$

or $(3x + 6) - \dfrac{(3x + 6)}{3} = 38$ or $2(3x + 6) = 114$ or $6x = 102$ or $x = 17$.

So, first number $= x = 17$.

20. Clearly, to find the average, we ought to know the number of boys, girls or students in the class, neither of which has been given.

So, the data provided is inadequate.

21. Let the number of other workers be x.

Then, number of agricultural workers $= 11x$.

Total number of workers $= 12x$.

$\therefore$ Average monthly income $= \dfrac{S \times 11x + T \times x}{12x} = \dfrac{11S + T}{12}$.

22. Required average $= \left(\dfrac{67 \times 2 + 35 \times 2 + 6 \times 3}{2 + 2 + 3}\right)$

$= \left(\dfrac{134 + 70 + 18}{7}\right) = \dfrac{222}{7} = 31\dfrac{5}{7}$ years.

23. Since the month begins with a Sunday, so there will be five Sundays in the month.

$\therefore$ Required average $= \left(\dfrac{510 \times 5 + 240 \times 25}{30}\right) = \dfrac{8550}{30} = 285$.

24. Required average $= \left(\dfrac{55 \times 50 + 60 \times 55 + 45 \times 60}{55 + 60 + 45}\right)$

$= \left(\dfrac{2750 + 3300 + 2700}{160}\right) = \dfrac{8750}{160} = 54.68$.

25. Required average $= \left(\dfrac{50.25 \times 16 + 45.15 \times 8}{16 + 8}\right)$

$= \left(\dfrac{804 + 361.20}{24}\right) = \dfrac{1165.20}{24} = 48.55$.

26. Total quantity of petrol consumed in 3 years $= \left(\frac{4000}{7.50} + \frac{4000}{8} + \frac{4000}{8.50}\right)$ litres

$= 4000\left(\frac{2}{15} + \frac{1}{8} + \frac{2}{17}\right) = \left(\frac{76700}{51}\right)$ litres.

Total amount spent = Rs. (3×4000) = Rs. 12000.

$\therefore$ Average cost = Rs. $\left(\frac{12000 \times 51}{76700}\right)$ = Rs. $\frac{6120}{767}$ = Rs. 7.98.

27. Clearly, we have : $x = \left(\frac{3y + 3z}{6}\right)$ or $2x = y + z$.

28. Let the average expenditure be Rs. x. Then,

$9x = 8 \times 30 + (x + 20)$ or $9x = x + 260$ or $8x = 260$ or $x = 32.50$.

$\therefore$ Total money spent = $9x$ = Rs. (9×32.50) = Rs 292. 50.

29. Sum of 50 numbers = $30 \times 50 = 1500$.

Sum of remaining 48 numbers = $1500 - (35 + 40) = 1425$.

$\therefore$ Required average = $\left(\frac{1425}{48}\right) = \frac{475}{16} = 29.68$.

30. Excluded number = $(27 \times 5) - (25 \times 4) = 135 - 100 = 35$.

31. Sum of the ages of 14 students = $(16 \times 35) - (14 \times 21) = 560 - 294 = 266$.

$\therefore$ Required average = $\left(\frac{266}{14}\right) = 19$ years.

32. Required average = $\frac{(76 \times 16) - (75 \times 10)}{6} = \left(\frac{1216 - 750}{6}\right) = \frac{466}{6} = \frac{233}{3} = 77\frac{2}{3}$.

33. Required average = $\frac{(38.9 \times 10) - (42 \times 6)}{4} = \frac{137}{4} = 34.25$.

34. Sum of the remaining two numbers = $(3.95 \times 6) - [(3.4 \times 2) + (3.85 \times 2)]$

$= 23.70 - (6.8 + 7.7) = 23.70 - 14.5 = 9.20$.

$\therefore$ Required average = $\left(\frac{9.2}{2}\right) = 4.6$.

35. Let the highest score be x. Then, lowest score = $(x - 172)$.

Then, $(50 \times 40) - [x + (x - 172)] = 38 \times 48$

$\Leftrightarrow \quad 2x = 2000 + 172 - 1824 \Leftrightarrow 2x = 348 \Leftrightarrow x = 174$.

36. Total price of the two books = Rs. $[(12 \times 10) - (11.75 \times 8)]$

= Rs. $(120 - 94)$ = Rs. 26.

Let the price of one book be Rs. x.

Then, the price of other book = Rs. $(x + 60\%$ of $x)$ = Rs. $\left(x + \frac{3}{5}x\right)$ = Rs. $\left(\frac{8x}{5}\right)$.

So, $x + \frac{8x}{5} = 26 \Leftrightarrow 13x = 130 \Leftrightarrow x = 10$.

$\therefore$ The prices of the two books are Rs. 10 and Rs. 16.

37. Average after 11 innings = 36.

$\therefore$ Required number of runs = $(36 \times 11) - (32 \times 10)$

$= 396 - 320 = 76$.

38. Total sale for 5 months = Rs. $(6435 + 6927 + 6855 + 7230 + 6562)$ = Rs. 34009.

$\therefore$ Required sale = Rs. $[(6500 \times 6) - 34009]$ = Rs. $(39000 - 34009)$ = Rs. 4991.

39. Required average $= \frac{(4375 \times 12) - (4000 \times 3)}{9} = \frac{52500 - 12000}{9} = \frac{40500}{9} = 4500.$

40. Required run rate $= \frac{282 - (3.2 \times 10)}{40} = \frac{250}{40} = 6.25.$

41. Let their prices be $3x$, $5x$ and $7x$.

Then, $3x + 5x + 7x = (15000 \times 3)$ or $x = 3000$.

$\therefore$ Cost of cheapest item $= 3x =$ Rs. 9000.

42. Let the fourth number be x.

Then, third number $= 5x$, second number $= \frac{5x}{3}$ and first number $= \frac{10x}{3}$.

$x + 5x + \frac{5x}{3} + \frac{10x}{3} = (24.75 \times 4)$ or $11x = 99$ or $x = 9$.

So, the numbers are 9, 45, 15 and 30.

$\therefore$ Largest number $= 45$.

43. Let the first number be x.

Then, sum of the four numbers $= x + 4x = 5x$.

So, $\frac{5x}{4} = 60$ or $x = \left(\frac{60 \times 4}{5}\right) = 48.$

44. Let the third number be x. Then, second number $= 2x$. First number $= 4x$.

$\therefore \quad \frac{1}{x} + \frac{1}{2x} + \frac{1}{4x} = \left(\frac{7}{72} \times 3\right)$ or $\frac{7}{4x} = \frac{7}{24}$ or $4x = 24$ or $x = 6$.

So, the numbers are 24, 12 and 6.

45. Let the numbers be x, y and z.

Then, $\left(\frac{x+y}{2}\right) - \left(\frac{y+z}{2}\right) = 15$ or $(x + y) - (y + z) = 30$ or $x - z = 30$.

46. Let the eighth number be x. Then, sixth number $= (x - 7)$.

Seventh number $= (x - 7) + 4 = (x - 3)$.

So, $\left(2 \times 15\frac{1}{2}\right) + \left(3 \times 21\frac{1}{3}\right) + (x - 7) + (x - 3) + x = 8 \times 20$

$\Leftrightarrow \quad 31 + 64 + (3x - 10) = 160 \Leftrightarrow 3x = 75 \Leftrightarrow x = 25.$

47. A.M. of 75 numbers $= 35$.

Sum of 75 numbers $= (75 \times 35) = 2625$.

Total increase $= (75 \times 5) = 375$.

Increased sum $= (2625 + 375) = 3000$.

Increased average $= \frac{3000}{75} = 40.$

48. Average of 10 numbers $= 7$.

Sum of these 10 numbers $= (10 \times 7) = 70$.

$\therefore \quad x_1 + x_2 + \ldots\ldots + x_{10} = 70.$

$\Rightarrow \quad 12x_1 + 12x_2 + \ldots\ldots + 12x_{10} = 840$

$\Rightarrow \quad \frac{12x_1 + 12x_2 + \ldots\ldots + 12x_{10}}{10} = 84$

$\Rightarrow$ Average of new numbers is 84.

49. $\frac{x_1 + x_2 + ... + x_{10}}{10} = \bar{x} \Rightarrow x_1 + x_2 + ... + x_{10} = 10\bar{x}$

$\Rightarrow \frac{110}{100}x_1 + \frac{110}{100}x_2 + ... + \frac{110}{100}x_{10} = \frac{110}{100} \times 10\bar{x}$

$\Rightarrow \frac{\frac{110}{100}x_1 + \frac{110}{100}x_2 + ... + \frac{110}{100}x_{10}}{10} = \frac{11}{10}\bar{x}$

$\Rightarrow$ Average is increased by 10%.

50. Correct sum = $(36 \times 50 + 48 - 23) = 1825$.

$\therefore$ Correct mean = $\frac{1825}{50} = 36.5$.

51. Let there be x pupils in the class.

Total increase in marks = $\left(x \times \frac{1}{2}\right) = \frac{x}{2}$.

$\therefore \frac{x}{2} = (83 - 63) \Rightarrow \frac{x}{2} = 20 \Rightarrow x = 40$.

52. Age of the 15th student = $[15 \times 15 - (14 \times 5 + 16 \times 9)] = (225 - 214) = 11$ years.

53. Middle number = $[(10.5 \times 6 + 11.4 \times 6) - 10.9 \times 11] = (131.4 - 119.9) = 11.5$.

54. Total weight of (A + B + C) = $\left(54\frac{1}{3} \times 3\right)$ kg = 163 kg.

Total weight of (B + D + E) = (53×3) kg = 159 kg.

Adding both, we get : A + 2B + C + D + E = (163 + 159) kg = 322 kg.

So, to find the average weight of A, B, C, D and E, we ought to know B's weight, which is not given. So, the data is inadequate.

55. Sum of temperatures on 1st, 2nd, 3rd and 4th days = $(58 \times 4) = 232$ degrees ...(*i*)

Sum of temperatures on 2nd, 3rd, 4th and 5th days = $(60 \times 4) = 240$ degrees ...(*ii*)

Subtracting (*i*) from (*ii*), we get :

Temp. on 5th day − Temp. on 1st day = 8 degrees.

Let the temperatures on 1st and 5th days be $7x$ and $8x$ degrees respectively.

Then, $8x - 7x = 8$ or $x = 8$.

$\therefore$ Temperature on the 5th day = $8x = 64$ degrees.

56. Let A, B, C represent their respective weights. Then, we have :

A + B + C = $(45 \times 3) = 135$ ...(*i*)

A + B = $(40 \times 2) = 80$ ...(*ii*)

B + C = $(43 \times 2) = 86$ ...(*iii*)

Adding (*ii*) and (*iii*), we get : A + 2B + C = 166 ...(*iv*)

Subtracting (*i*) from (*iv*), we get : B = 31.

$\therefore$ B's weight = 31 kg.

57. Let P, Q and R represent their respective monthly incomes. Then, we have :

P + Q = $(5050 \times 2) = 10100$ ...(*i*)

Q + R = $(6250 \times 2) = 12500$ ...(*ii*)

P + R = $(5200 \times 2) = 10400$ ...(*iii*)

Adding (*i*), (*ii*) and (*iii*), we get : 2 (P + Q + R) = 33000 or P + Q + R = 16500 ...(*iv*)

Subtracting (*ii*) from (*iv*), we get P = 4000.

$\therefore$ P's monthly income = Rs. 4000.

58. Age of the teacher = (37 × 15 − 36 × 14) years = 51 years.

59. Manager's monthly salary = Rs. (1600 × 21 − 1500 × 20) = Rs. 3600.

60. Weight of the teacher = (35.4 × 25 − 35 × 24) kg = 45 kg.

61. Age of the mother = (12 × 7 − 7 × 6) years = 42 years.

62. Let the average age of the whole team be x years.

$\therefore \quad 11x - (26 + 29) = 9(x - 1) \Leftrightarrow 11x - 9x = 46 \Leftrightarrow 2x = 46 \Leftrightarrow x = 23.$

So, average age of the team is 23 years.

63. Sum of heights of the 5 boys = (25 × 1.4 − 20 × 1.55) m = 4 m.

$\therefore$ Required average = $\left(\frac{4}{5}\right) = 0.8$ m.

64. Total weight increased = (8 × 2.5) kg = 20 kg.

Weight of new person = (65 + 20) kg = 85 kg.

65. Sum of the weights of the students after replacement

= [(52 × 45) − (48 × 5) + (54 × 5)] kg = 2370 kg.

$\therefore$ New average = $\left(\frac{2370}{45}\right)$ kg = $53\frac{2}{3}$ kg.

66. Total age increased = (8 × 2) years = 16 years.

Sum of ages of two new men = (21 + 23 + 16) years = 60 years.

$\therefore$ Average age of two new men = $\left(\frac{60}{2}\right)$ years = 30 years.

67. Let five consecutive numbers be x, $x + 1$, $x + 2$, $x + 3$ and $x + 4$.

Their average = $\frac{5x + 10}{5} = (x + 2)$.

Average of 7 numbers = $\frac{(5x + 10) + (x + 5) + (x + 6)}{7} = \frac{7x + 21}{7} = (x + 3)$.

So, the average increased by 1.

68. Let average for 10 innings be x. Then,

$\frac{10x + 108}{11} = x + 6 \Rightarrow 11x + 66 = 10x + 108 \Rightarrow x = 42.$

$\therefore$ New average = $(x + 6) = 48$ runs.

69. Let the number of wickets taken till the last match be x. Then,

$\frac{12.4x + 26}{x + 5} = 12 \Rightarrow 12.4x + 26 = 12x + 60 \Rightarrow 0.4x = 34 \Rightarrow x = \frac{34}{0.4} = \frac{340}{4} = 85.$

70. Let the total score be x.

$\therefore \quad \frac{x + 92 - 85}{8} = 84 \Rightarrow x + 7 = 672 \Rightarrow x = 665.$

71. Average speed = $\frac{2xy}{x + y}$ km/hr = $\left(\frac{2 \times 50 \times 30}{50 + 30}\right)$ km/hr = 37.5 km/hr.

72. Let A, B, C, D and E represent their respective weights. Then,

A + B + C = (84 × 3) = 252 kg, A + B + C + D = (80 × 4) = 320 kg.

$\therefore$ D = (320 − 252) kg = 68 kg, E = (68 + 3) kg = 71 kg.

B + C + D + E = (79 × 4) = 316 kg.

Now, (A + B + C + D) − (B + C + D + E) = (320 − 316) kg = 4 kg.

$\therefore \quad A - E = 4 \Rightarrow A = (4 + E) = 75$ kg.

73. Sum of the present ages of husband, wife and child = $(23 \times 2 + 5 \times 2) + 1 = 57$ years.

$\therefore$ Required average = $\left(\frac{57}{3}\right)$ = 19 years.

74. Present age of (A + B) = $(18 \times 2 + 3 \times 2)$ years = 42 years.

Present age of (A + B + C) = (22×3) years = 66 years.

$\therefore$ C's age = (66 − 42) years = 24 years.

75. Sum of the present ages of husband, wife and child = $(27 \times 3 + 3 \times 3)$ years = 90 years.

Sum of the present ages of wife and child = $(20 \times 2 + 5 \times 2)$ years = 50 years.

$\therefore$ Husband's present age = (90 − 50) years = 40 years.

76. Total age of 5 members, 3 years ago = (17×5) years = 85 years.

Total age of 5 members now = $(85 + 3 \times 5)$ years = 100 years.

Total age of 6 members now = (17×6) years = 102 years.

$\therefore$ Age of the baby = (102 − 100) years = 2 years.

77. Total age of 4 members, 10 years ago = (24×4) years = 96 years.

Total age of 4 members now = $(96 + 10 \times 4)$ years = 136 years.

Total age of 6 members now = (24×6) years = 144 years.

Sum of the ages of 2 children = (144 − 136) years = 8 years.

Let the age of the younger child be x years.

Then, age of the elder child = $(x + 2)$ years.

So, $x + x + 2 = 8 \Leftrightarrow 2x = 6 \Leftrightarrow x = 3$.

$\therefore$ Age of younger child = 3 years.

78. Age decreased = (5×3) years = 15 years.

So, the required difference = 15 years.

79. Since the total or average age of all the family members is not given, the given data is inadequate. So, the age of second child cannot be determined.

80. Let the initial number of persons be x. Then,

$16x + 20 \times 15 - 15.5\,(x + 20) \Leftrightarrow 0.5x = 10 \Leftrightarrow x = 20$.

81. Let the daily wage of a man be Rs. x.

Then, daily wage of a woman = Rs. $(x - 5)$.

Now, $600x + 400\,(x - 5) = 25.50 \times (600 + 400) \Leftrightarrow 1000x = 27500 \Leftrightarrow x = 27.50$.

$\therefore$ Man's daily wages = Rs. 27.50; Woman's daily wages = $(x - 5)$ = Rs. 22.50.

82. Let the required mean score be x. Then,

$20 \times 80 + 25 \times 31 + 55 \times x = 52 \times 100$

$\Leftrightarrow 1600 + 775 + 55x = 5200 \Leftrightarrow 55x = 2825 \Leftrightarrow x = \frac{565}{11} = 51.4$.

83. Let the total number of workers be x. Then,

$8000x = (12000 \times 7) + 6000\,(x - 7) \Leftrightarrow 2000x = 42000 \Leftrightarrow x = 21$.

84. Let the number of girls be x. Then, number of boys = $(600 - x)$.

Then, $\left(11\frac{3}{4} \times 600\right) = 11x + 12(600 - x) \Leftrightarrow x = 7200 - 7050 \Leftrightarrow x = 150$.

85. Let the number of papers be x. Then, $63x + 20 + 2 = 65x$ or $2x = 22$ or $x = 11$.

86. Let the ratio be $k : 1$. Then,

$k \times 16.4 + 1 \times 15.4 = (k + 1) \times 15.8$

$\Leftrightarrow (16.4 - 15.8)\,k = (15.8 - 15.4) \Leftrightarrow k = \frac{0.4}{0.6} = \frac{2}{3}$.

$\therefore$ Required ratio = $\frac{2}{3} : 1 = 2 : 3$.

EXERCISE 6B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 10*) : *Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the given question. Read both the statements and*

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. The total of the present ages of A, B, C and D is 96 years. What is B's present age ?
   I. The average age of A, B and D is 20 years.
   II. The average age of C and D is 25 years.
2. What is the average age of children in the class ? **(Bank P.O. 2003)**
   I. Age of the teacher is as many years as the number of children.
   II. Average age increased by 1 year if the teacher's age is also included.
3. What is the average weight of the three new team members who are recently included in the team ?
   I. The average weight of the team increases by 20 kg.
   II. The three new men substitute earlier members whose weights are 64 kg, 75 kg and 66 kg.
4. The average age of P, Q, R and S is 30 years. How old is R ? **(R.B.I. 2003)**
   I. The sum of ages of P and R is 60 years.
   II. S is 10 years younger than R.
5. How old will C be after 10 years ?
   I. Five years ago, the average age of A and B was 15 years.
   II. Average age of A, B and C today is 20 years.
6. How many children are there in the group ? **(Bank P.O. 2000)**
   I. Average age of the children in this group is 15 years. The total age of all the children in this group is 240 years.
   II. The total age of all the children in the group and the teacher is 264 years. The age of the teacher is 9 years more than the average age of the children.
7. Deepak's marks in Hindi are 15 more than the average marks obtained by him in Hindi, Economics, Sociology and Philosophy. What are his marks in Philosophy ?
   I. The total marks obtained by him in Hindi and Philosophy together is 120.
   II. The difference between the marks obtained by him in Sociology and Economics is 120.
8. How many candidates were interviewed everyday by the panel A out of the three panels A, B and C ? **(Bank P.O. 1999)**
   I. The three panels on an average interview 15 candidates everyday.
   II. Out of a total of 45 candidates interviewed everyday by the three panels, the number of candidates interviewed by panel A is more by 2 than the candidates interviewed by panel C and is more by 1 than the candidates interviewed by panel B.

9. The average age of teacher and students in a class is 3 years more than the average age of students. What is the age of the class teacher ? (Bank P.O. 2000)
   I. There are 11 students in the class.
   II. The average age of teacher and students is 14 years.

10. What will be the average weight of the remaining class ? (Bank P.O. 1999)
   I. Average weight of 30 children out of total 46 in the class is 22.5 kg and that of the remaining children is 29.125 kg. A child having weight more than 40 kg is excluded.
   II. Average weight of a class of 46 children is 23.5 kg. A child weighing 46 kg is dropped out.

Directions (*Questions 11 to 13*) : *Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question.*

11. How many marks did Tarun secure in English ? (S.B.I.P.O. 2000)
   I. The average marks obtained by Tarun in four subjects including English is 60.
   II. The total marks obtained by him in English and Mathematics together is 170.
   III. The total marks obtained by him in Mathematics and Science together is 180.
   (*a*) I and II only (*b*) II and III only (*c*) I and III only
   (*d*) All I, II and III (*e*) None of these

12. The mean temperature of Monday to Wednesday was 37°C and of Tuesday to Thursday was 34°C. What was the temperature on Thursday ?
   I. The temperature on Thursday was $\frac{4}{5}$ th that of Monday.
   II. The mean temperature of Monday and Thursday was 40.5°C.
   III. The difference between the temperature on Monday and that on Thursday was 9°C.
   (*a*) I and II only (*b*) II and III only (*c*) Either I or II
   (*d*) Either I, II or III (*e*) Any two of the three

13. In a cricket eleven, the average age of eleven players is 28 years. What is the age of the captain ?
   I. The captain is eleven years older than the youngest player.
   II. The average age of 10 players, other than the captain is 27.3 years.
   III. Leaving aside the captain and the youngest player, the average ages of three groups of three players each are 25 years, 28 years and 30 years respectively.
   (*a*) Any two of the three (*b*) All I, II and III
   (*c*) II only or I and III only (*d*) II and III only
   (*e*) None of these

Directions (*Question 14*) : *The given question is followed by three statements labelled I, II and III. You have to study the question and all the three statements given to decide whether any information provided in the statement(s) is/are redundant and can be dispensed with while answering the given question.*

14. What is the average salary of 15 employees ? (S.B.I.P.O. 2001)
   I. Average salary of 7 clerical cadre (out of the 15 employees) is Rs. 8500.
   II. Average salary of 5 officer cadre (out of the 15 employees) is Rs. 10000.
   III. Average salary of the 3 sub-staff employees (out of the 15 employees) is Rs. 2500.
   (*a*) None (*b*) Only I
   (*c*) Only II (*d*) Only III
   (*e*) Question cannot be answered even with information in all the three statements

ANSWERS

1. (d) 2. (d) 3. (d) 4. (d) 5. (e) 6. (a) 7. (d) 8. (b) 9. (e)
10. (b) 11. (e) 12. (c) 13. (c) 14. (a)

SOLUTIONS

1. $A + B + C + D = 96$ ...(i)

I gives, $A + B + D = (3 \times 20) \Rightarrow A + B + D = 60$ ...(ii)

II gives, $C + D = (2 \times 50) \Rightarrow C + D = 100$ ...(iii)

From (i), (ii) and (iii) also, we cannot find B.

∴ Correct answer is (d).

2. Let there be x children.

I gives, age of teacher = x years.

II gives, average age of $(x + 1)$ persons = $(x + 1)$ years.

∴ Teacher's age = $(x + 1)(x + 1) - x^2 = (x^2 + 1 + 2x) - x^2 = (1 + 2x)$.

Thus, teacher's age cannot be obtained.

∴ Correct answer is (d).

3. Let the number of team members be n.

I. Total increase in weight on replacement = $(20n)$ kg.

II. Total weight of new members = $[(64 + 75 + 66) + 20n]$ kg
$= (205 + 20n)$ kg.

∴ Required average = $\frac{(205 + 20n)}{3}$ kg and we need n to get the answer.

∴ Correct answer is (d).

4. $P + Q + R + S = (30 \times 4) \Rightarrow P + Q + R + S = 120$ ...(i)

I. $P + R = 60$ ...(ii)

II. $S = (R - 10)$ ...(iii)

From (i), (ii) and (iii), we cannot find R.

∴ Correct answer is (d).

5. I. $A + B = (15 \times 2) + (5 \times 2) \Rightarrow A + B = 40$ ...(i)

II. $A + B + C = (20 \times 3) \Rightarrow A + B + C = 60$ ...(ii)

From (i) and (ii), we get C = 20.

C's age after 10 years = (20 + 10) years = 30 years.

∴ Correct answer is (e).

6. Let there be x children in the group.

I. Average age = 15 years.

∴ Total age = $15x$ years.

∴ $15x = 240 \Leftrightarrow x = \frac{240}{15} \Leftrightarrow x = 16$.

So, there are 16 children in the group.

II. Total age of x children and 1 teacher is 264 years.

Age of teacher = (15 + 9) years = 24 years.

Total age of x children = (264 − 24) years = 240 years.

This does not give the answer.

∴ Correct answer is (a).

7. $H = \frac{(H+E+S+P)}{4} + 15$

$\Rightarrow 4(H - 15) = H + E + S + P \Rightarrow 3H - 60 = E + S + P$ ...(*i*)

I. $H + P = 120$ ...(*ii*)

II. $S - E = 120$ ...(*iii*)

From (*i*), (*ii*) and (*iii*), we cannot find P.

∴ Correct answer is (*d*).

8. I. Total candidates interviewed by 3 panels = $(15 \times 3) = 45$.

II. Let x candidates be interviewed by C.

Number of candidates interviewed by A = $(x + 2)$.

Number of candidates interviewed by B = $(x + 1)$.

∴ $x + (x + 2) + (x + 1) = 45 \Leftrightarrow 3x = 42 \Leftrightarrow x = 14$.

So, the number of candidates interviewed by A is 14.

Hence, the correct answer is (*b*).

9. Average age of 11 students and 1 teacher = 14 years

⇒ Total age of (11 students and 1 teacher) = (14×12) years = 168 years.

Average age of (11 students and 1 teacher) = (Average age of 11 students) + 3

⇒ Average age of 11 students = $(14 - 3)$ years = 11 years

⇒ Total age of 11 students = (11×11) years = 121 years.

∴ Age of the teacher = $(168 - 121)$ years = 47 years.

Thus, both I and II are needed to get the answer.

∴ Correct answer is (*e*).

10. I. Total weight of 46 children = $[(22.5 \times 30) + (29.125 \times 16)]$ kg = 1141 kg.

Weight excluded is not exact. So, average of remaining class cannot be obtained.

II. Total weight of 45 children = $[(23.5 \times 46) - 46]$ kg = 1035 kg.

Average weight of 45 children = $\frac{1035}{45}$ kg = 23 kg.

∴ Data in II is sufficient to answer the question, while the data in I is not sufficient.

∴ Correct answer is (*b*).

11. I gives, total marks in 4 subjects = $(60 \times 4) = 240$.

II gives, $E + M = 170$

III gives, $M + S = 180$.

Thus, none of (*a*), (*b*), (*c*), (*d*) is true.

∴ Correct answer is (*e*).

12. $M + T + W = (37 \times 3) \Rightarrow M + T + W = 111$ ...(*i*)

$T + W + Th = (34 \times 3) \Rightarrow T + W + Th = 102$ ...(*ii*)

I gives, $Th = \frac{4}{5}M \Rightarrow M = \frac{5}{4}Th$

Using it in (*i*), we get :

$$\frac{5}{4}Th + T + W = 111 \quad \text{...(iii)}$$

On subtracting (*ii*) from (*iii*), we get : $\frac{1}{4}Th = 9 \Rightarrow Th = 36$.

Thus, I alone gives the answer.

II gives, $M + Th = (40.5 \times 2) \Rightarrow M + Th = 81$ ...(*iv*)

On subtracting (*ii*) from (*i*), we get $M - Th = 9$ ...(*v*)

From (*iv*) and (*v*), we get $Th = 36$.

Thus, II alone gives the answer.

III gives, M – Th = 9.

Clearly, III with given results, does not give the answer.

∴ Correct answer is (*c*).

13. Total age of 11 players = (28 × 11) years = 308 years.

I. C = Y + 11 ⇒ C – Y = 11 ...(*i*)

II. Total age of 10 players (excluding captain) = (27.3 × 10) years = 273 years.

∴ Age of captain = (308 – 273) years = 35 years.

Thus, C = 35. ...(*ii*)

From (*i*) and (*ii*), we get Y = 24.

III. Total age of 9 players = [(25 × 3) + (28 × 3) + (30 × 3)] years = 249 years.

∴ C + Y = (308 – 249) = 59 ...(*iii*)

From (*i*) and (*iii*), we get C = 35.

Thus, II alone gives the answer.

Also, I and III together give the answer.

∴ Correct answer is (*c*).

14. I. gives, total salary of 7 clerks = Rs. (8500 × 7) = Rs. 59500.

II. gives, total salary of 5 officers = Rs. (10000 × 5) = Rs. 50000.

III. gives, total salary of 3 sub-staff members = Rs. (2500 × 3) = Rs. 7500.

Total salary of 15 employees = Rs. (59500 + 50000 + 7500) = Rs. 117000.

∴ Average salary = Rs. $\left(\frac{117000}{15}\right)$ = Rs. 7800.

∴ All given statements are needed. Hence, none is redundant.

∴ Correct answer is (*a*).

7. PROBLEMS ON NUMBERS

In this section, questions involving a set of numbers are put in the form of a puzzle. You have to analyse the given conditions, assume the unknown numbers and form equations accordingly, which on solving yield the unknown numbers.

SOLVED EXAMPLES

Ex. 1. ***A number is as much greater than 36 as is less than 86. Find the number.***

Sol. Let the number be x. Then, $x - 36 = 86 - x \Leftrightarrow 2x = 86 + 36 = 122 \Leftrightarrow x = 61$.

Hence, the required number is 61.

Ex. 2. ***Find a number such that when 15 is subtracted from 7 times the number, the result is 10 more than twice the number.*** **(Hotel Management, 2002)**

Sol. Let the number be x. Then, $7x - 15 = 2x + 10 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5$.

Hence, the required number is 5.

Ex. 3. ***The sum of a rational number and its reciprocal is $\frac{13}{6}$. Find the number.*** **(S.S.C. 2000)**

Sol. Let the number be x.

Then, $x + \frac{1}{x} = \frac{13}{6} \Leftrightarrow \frac{x^2+1}{x} = \frac{13}{6} \Leftrightarrow 6x^2 - 13x + 6 = 0$

$\Leftrightarrow 6x^2 - 9x - 4x + 6 = 0 \Leftrightarrow (3x - 2)(2x - 3) = 0$

$\Leftrightarrow x = \frac{2}{3}$ or $x = \frac{3}{2}$.

Hence, the required number is $\frac{2}{3}$ or $\frac{3}{2}$.

Ex. 4. ***The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.***

Sol. Let the numbers be x and $(184 - x)$. Then,

$$\frac{x}{3} - \frac{(184 - x)}{7} = 8 \Leftrightarrow 7x - 3(184 - x) = 168 \Leftrightarrow 10x = 720 \Leftrightarrow x = 72.$$

So, the numbers are 72 and 112. Hence, smaller number = 72.

Ex. 5. ***The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers.***

Sol. Let the numbers be x and y. Then,

$x - y = 11$ ...(*i*) and $\frac{1}{5}(x + y) = 9 \Rightarrow x + y = 45$ ...(*ii*)

Adding (*i*) and (*ii*), we get : $2x = 56$ or $x = 28$. Putting $x = 28$ in (*i*), we get : $y = 17$.

Hence, the numbers are 28 and 17.

Ex. 6. ***If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers.*** **(S.S.C. 2003)**

Sol. Let the numbers be x and y. Then, $x + y = 42$ and $xy = 437$.

$$x - y = \sqrt{(x+y)^2 - 4xy} = \sqrt{(42)^2 - 4 \times 437} = \sqrt{1764 - 1748} = \sqrt{16} = 4.$$

$\therefore$ Required difference = 4.

Ex. 7. ***The sum of two numbers is 15 and the sum of their squares is 113. Find the numbers.***

Sol. Let the numbers be x and $(15 - x)$.

Then, $x^2 + (15 - x)^2 = 113 \Leftrightarrow x^2 + 225 + x^2 - 30x = 113$

$\Leftrightarrow 2x^2 - 30x + 112 = 0 \Leftrightarrow x^2 - 15x + 56 = 0$

$\Leftrightarrow (x - 7)(x - 8) = 0 \Leftrightarrow x = 7$ or $x = 8$.

So, the numbers are 7 and 8.

Ex. 8. ***The average of four consecutive even numbers is 27. Find the largest of these numbers.***

Sol. Let the four consecutive even numbers be x, $x + 2$, $x + 4$ and $x + 6$.

Then, sum of these numbers $= (27 \times 4) = 108$.

So, $x + (x + 2) + (x + 4) + (x + 6) = 108$ or $4x = 96$ or $x = 24$.

$\therefore$ Largest number $= (x + 6) = 30$.

Ex. 9. ***The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.***

Sol. Let the numbers be x, $x + 2$ and $x + 4$.

Then, $x^2 + (x + 2)^2 + (x + 4)^2 = 2531 \Leftrightarrow 3x^2 + 12x - 2511 = 0$

$\Leftrightarrow x^2 + 4x - 837 = 0 \Leftrightarrow (x - 27)(x + 31) = 0 \Leftrightarrow x = 27$.

Hence, the required numbers are 27, 29 and 31.

Ex. 10. ***Of two numbers, 4 times the smaller one is less than 3 times the larger one by 5. If the sum of the numbers is larger than 6 times their difference by 6, find the two numbers.***

Sol. Let the numbers be x and y, such that $x > y$.

Then, $3x - 4y = 5$ ...(*i*) and $(x + y) - 6(x - y) = 6 \Rightarrow -5x + 7y = 6$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 59$ and $y = 43$.

Hence, the required numbers are 59 and 43.

Ex. 11. ***The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number ?***

Sol. Let the ten's digit be x. Then, unit's digit $= (x + 3)$.

Sum of the digits $= x + (x + 3) = 2x + 3$. Number $= 10x + (x + 3) = 11x + 3$.

$\therefore \dfrac{11x + 3}{2x + 3} = \dfrac{4}{1} \Leftrightarrow 11x + 3 = 4(2x + 3) \Leftrightarrow 3x = 9 \Leftrightarrow x = 3$.

Hence, required number $= 11x + 3 = 36$.

Ex. 12. ***A number consists of two digits. The sum of the digits is 9. If 63 is subtracted from the number, its digits are interchanged. Find the number.***

Sol. Let the ten's digit be x. Then, unit's digit $= (9 - x)$.

Number $= 10x + (9 - x) = 9x + 9$.

Number obtained by reversing the digits $= 10(9 - x) + x = 90 - 9x$.

$\therefore (9x + 9) - 63 = 90 - 9x \Leftrightarrow 18x = 144 \Leftrightarrow x = 8$.

So, ten's digit $= 8$ and unit's digit $= 1$.

Hence, the required number is 81.

Ex. 13. ***A fraction becomes $\frac{2}{3}$ when 1 is added to both, its numerator and denominator. And, it becomes $\frac{1}{2}$ when 1 is subtracted from both the numerator and denominator. Find the fraction.***

Sol. Let the required fraction be $\frac{x}{y}$. Then,

$$\frac{x+1}{y+1} = \frac{2}{3} \Rightarrow 3x - 2y = -1 \quad ...(i) \text{ and } \frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x - y = 1 \quad ...(ii)$$

Solving (*i*) and (*ii*), we get : $x = 3, y = 5$.

$\therefore$ Required fraction $= \frac{3}{5}$.

Ex. 14. ***50 is divided into two parts such that the sum of their reciprocals is*** $\frac{1}{12}$. ***Find the two parts.***

Sol. Let the two parts be x and $(50 - x)$.

Then, $\frac{1}{x} + \frac{1}{50-x} = \frac{1}{12} \Leftrightarrow \frac{50-x+x}{x(50-x)} = \frac{1}{12} \Rightarrow x^2 - 50x + 600 = 0$

$\Rightarrow (x - 30)(x - 20) = 0 \Rightarrow x = 30$ or $x = 20$.

So, the parts are 30 and 20.

Ex. 15. ***If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers.*** **(S.S.C. 2000)**

Sol. Let the numbers be x, y and z. Then,

$x + y = 10 \quad ...(i) \quad y + z = 19 \quad ...(ii) \quad x + z = 21 \quad ...(iii)$

Adding (*i*), (*ii*) and (*iii*), we get : $2(x + y + z) = 50$ or $(x + y + z) = 25$.

Thus, $x = (25 - 19) = 6$; $y = (25 - 21) = 4$; $z = (25 - 10) = 15$.

Hence, the required numbers are 6, 4 and 15.

EXERCISE 7A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The difference between a number and its three-fifth is 50. What is the number ?
   (*a*) 75 (*b*) 100 (*c*) 125 (*d*) None of these
   (Bank P.O. 2003)
2. If a number is decreased by 4 and divided by 6, the result is 8. What would be the result if 2 is subtracted from the number and then it is divided by 5 ?
   (*a*) $9\frac{2}{3}$ (*b*) 10 (*c*) $10\frac{1}{5}$ (*d*) $11\frac{1}{5}$ (*e*) None of these
   (Bank P.O. 2000)
3. If one-third of one-fourth of a number is 15, then three-tenth of that number is :
   (*a*) 35 (*b*) 36 (*c*) 45 (*d*) 54
   (N.I.F.T. 2003)
4. A number is doubled and 9 is added. If the resultant is trebled, it becomes 75. What is that number ? **(S.S.C. 1999)**
   (*a*) 3.5 (*b*) 6 (*c*) 8 (*d*) None of these
5. Three-fourth of a number is 60 more than its one-third. The number is :
   (*a*) 84 (*b*) 108 (*c*) 144 (*d*) None of these
6. When 24 is subtracted from a number, it reduces to its four-seventh. What is the sum of the digits of that number ?
   (*a*) 1 (*b*) 9 (*c*) 11
   (*d*) Data inadequate (*e*) None of these

7. Find the number which when multiplied by 15 is increased by 196. (L.I.C. 2003)
(a) 14 (b) 20 (c) 26 (d) 28

8. If a number, when divided by 4, is reduced by 21, the number is :
(a) 18 (b) 20 (c) 28 (d) 38

9. A number whose fifth part increased by 4 is equal to its fourth part diminished by 10, is :
(a) 240 (b) 260 (c) 270 (d) 280

10. The difference of two numbers is 20% of the larger number. If the smaller number is 12, the larger one is :
(a) 15 (b) 16 (c) 18 (d) 20

11. If one-seventh of a number exceeds its eleventh part by 100, then the number is :
(a) 770 (b) 1100 (c) 1825 (d) 1925

12. If the sum of one-half and one-fifth of a number exceeds one-third of that number by $7\frac{1}{3}$, the number is : (C.B.I. 1998)
(a) 15 (b) 18 (c) 20 (d) 30

13. If doubling a number and adding 20 to the result gives the same answer as multiplying the number by 8 and taking away 4 from the product, the number is :
(a) 2 (b) 3 (c) 4 (d) 6
(S.S.C. 2000)

14. If 50 is subtracted from two-third of a number, the result is equal to sum of 40 and one-fourth of that number. What is the number ? (R.R.B. 2002)
(a) 174 (b) 216 (c) 246 (d) 336

15. If the sum of a number and its square is 182, what is the number ?
(a) 15 (b) 26 (c) 28 (d) 91 (e) None of these
(Bank P.O. 1999)

16. Twenty times a positive integer is less than its square by 96. What is the integer ?
(a) 20 (b) 24 (c) 30
(d) Cannot be determined (e) None of these (Bank P.O. 2003)

17. Thrice the square of a natural number decreased by 4 times the number is equal to 50 more than the number. The number is : (S.S.C. 2003)
(a) 4 (b) 5 (c) 6 (d) 10

18. The sum of a number and its reciprocal is one-eighth of 34. What is the product of the number and its square root ? (Hotel Management, 2001)
(a) 8 (b) 27 (c) 32 (d) None of these

19. Two-third of a positive number and $\frac{25}{216}$ of its reciprocal are equal. The number is :
(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{25}{144}$ (d) $\frac{144}{25}$
(S.S.C. 1999)

20. Find a positive number which when increased by 17 is equal to 60 times the reciprocal of the number. (I.M.T. 2002)
(a) 3 (b) 10 (c) 17 (d) 20

21. A positive number when decreased by 4 is equal to 21 times the reciprocal of the number. The number is :
(a) 3 (b) 5 (c) 7 (d) 9

22. The sum of a positive number and its reciprocal is thrice the difference of the number and its reciprocal. The number is :

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

23. The product of two natural numbers is 17. Then, the sum of the reciprocals of their squares is : (S.S.C. 1999)

(a) $\frac{1}{289}$ (b) $\frac{289}{290}$ (c) $\frac{290}{289}$ (d) 289

24. If $2\frac{1}{2}$ is added to a number and the sum multiplied by $4\frac{1}{2}$ and 3 is added to the product and then dividing the sum by $1\frac{1}{5}$, the quotient becomes 25. What is the number ? (R.R.B. 2002)

(a) $2\frac{1}{2}$ (b) $3\frac{1}{2}$ (c) $4\frac{1}{2}$ (d) $5\frac{1}{2}$

25. Three numbers are in the ratio 4 : 5 : 6 and their average is 25. The largest number is :

(a) 30 (b) 32 (c) 36 (d) 42

26. Three numbers are in the ratio of 3 : 4 : 6 and their product is 1944. The largest of these numbers is :

(a) 6 (b) 12 (c) 18 (d) None of these

27. Two numbers are such that the square of one is 224 less than 8 times the square of the other. If the numbers be in the ratio of 3 : 4, the numbers are :

(a) 6, 8 (b) 9, 12 (c) 12, 16 (d) None of these

28. Two numbers are such that the ratio between them is 4 : 7. If each is increased by 4, the ratio becomes 3 : 5. The larger number is :

(a) 36 (b) 48 (c) 56 (d) 64

29. The sum of three numbers is 264. If the first number be twice the second and third number be one-third of the first, then the second number is : (R.R.B. 2004)

(a) 48 (b) 54 (c) 72 (d) 84

30. The sum of two numbers is 22. Five times one number is equal to 6 times the other. The bigger of the two numbers is : (C.B.I. 1998)

(a) 10 (b) 12 (c) 15 (d) 16

31. One-fifth of a number is equal to $\frac{5}{8}$ of another number. If 35 is added to the first number, it becomes four times of the second number. The second number is :

(a) 25 (b) 40 (c) 70 (d) 125

(Bank P.O. 1999)

32. The sum of two numbers is 25 and their difference is 13. Find their product.

(a) 104 (b) 114 (c) 315 (d) 325

(L.I.C. 2003)

33. If the sum of two numbers is 33 and their difference is 15, the smaller number is :

(a) 9 (b) 12 (c) 15 (d) 18

(C.B.I. 1997)

34. The sum of two numbers is 40 and their difference is 4. The ratio of the numbers is :

(a) 11 : 9 (b) 11 : 18 (c) 21 : 19 (d) 22 : 9

(S.S.C. 2000)

35. The product of two numbers is 192 and the sum of these two numbers is 28. What is the smaller of these two numbers ? **(Bank P.O. 1999)**
(a) 12 (b) 14 (c) 16 (d) 18 (e) None of these

36. The difference between two integers is 5. Their product is 500. Find the numbers.
(a) 15, 20 (b) 20, 25 (c) 30, 25 (d) 21, 26
(Hotel Management, 2003)

37. Two numbers differ by 5. If their product is 336, then the sum of the two numbers is :
(a) 21 (b) 28 (c) 37 (d) 51
(S.S.C. 1999)

38. Two different natural numbers are such that their product is less than their sum. One of the numbers must be :
(a) 1 (b) 2 (c) 3 (d) None of these

39. The product of two numbers is 9375 and the quotient, when the larger one is divided by the smaller, is 15. The sum of the numbers is : **(S.S.C. 2004)**
(a) 380 (b) 395 (c) 400 (d) 425

40. The difference between two numbers is 1365. When the larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. The smaller number is :
(a) 240 (b) 270 (c) 295 (d) 360

41. The sum of two numbers is 40 and their product is 375. What will be the sum of their reciprocals ? **(S.S.C. 1999)**
(a) $\frac{1}{40}$ (b) $\frac{8}{75}$ (c) $\frac{75}{4}$ (d) $\frac{75}{8}$

42. The sum of two positive integers multiplied by the bigger number is 204, and their difference multiplied by the smaller number is 35. The numbers are :
(a) 12, 5 (b) 13, 4 (c) 14, 3 (d) 24, 10

43. If the sum and difference of two numbers are 20 and 8 respectively, then the difference of their squares is : **(S.S.C. 2000)**
(a) 12 (b) 28 (c) 160 (d) 180

44. The product of two numbers is 120 and the sum of their squares is 289. The sum of the numbers is : **(R.R.B. 2004)**
(a) 20 (b) 23 (c) 169 (d) None of these

45. The product of two numbers is 45 and the sum of their squares is 106. The numbers are : **(R.R.B. 2002)**
(a) 3 and 5 (b) 5 and 9 (c) 5 and 19 (d) 45 and 1

46. The sum of the squares of two numbers is 3341 and the difference of their squares is 891. The numbers are :
(a) 25, 36 (b) 25, 46 (c) 35, 46 (d) None of these

47. The difference between two positive integers is 3. If the sum of their squares is 369, then the sum of the numbers is : **(S.S.C. 2003)**
(a) 25 (b) 27 (c) 33 (d) 81

48. If the sum of two numbers is 22 and the sum of their squares is 404, then the product of the numbers is : **(S.S.C. 2000)**
(a) 40 (b) 44 (c) 80 (d) 88

49. The difference between the squares of two numbers is 256000 and the sum of the numbers is 1000. The numbers are :
(a) 600, 400 (b) 628, 372 (c) 640, 360 (d) None of these

50. If the difference of two numbers is 3 and the difference of their squares is 39, then the larger number is :
(a) 8 (b) 9 (c) 12 (d) 13

51. The sum of three consecutive numbers is 87. The greatest among these three numbers is : **(Hotel Management, 2003)**

(*a*) 26 (*b*) 28 (*c*) 29 (*d*) 30

52. Three times the first of three consecutive odd integers is 3 more than twice the third. The third integer is : **(M.B.A. 1998)**

(*a*) 9 (*b*) 11 (*c*) 13 (*d*) 15

53. The sum of four consecutive even integers is 1284. The greatest of them is :

(*a*) 320 (*b*) 322 (*c*) 324 (*d*) 326 **(S.S.C. 2002)**

54. The sum of three consecutive odd numbers is 20 more than the first of these numbers. What is the middle number ? **(S.B.I.P.O. 1997)**

(*a*) 7 (*b*) 9 (*c*) 11

(*d*) Data inadequate (*e*) None of these

55. The product of three consecutive even numbers when divided by 8 is 720. The product of their square roots is : **(Hotel Management, 2001)**

(*a*) $12\sqrt{10}$ (*b*) $24\sqrt{10}$ (*c*) 120 (*d*) None of these

56. The sum of three consecutive multiples of 3 is 72. What is the largest number ?

(*a*) 21 (*b*) 24 (*c*) 27 (*d*) 36 **(S.S.C. 1999)**

57. What is the sum of two consecutive even numbers, the difference of whose squares is 84 ? **(S.S.C. 2003)**

(*a*) 34 (*b*) 38 (*c*) 42 (*d*) 46

58. The sum of the squares of three consecutive natural numbers is 2030. What is the middle number ? **(S.S.C. 2000)**

(*a*) 25 (*b*) 26 (*c*) 27 (*d*) 28

59. There are two numbers such that the sum of twice the first and thrice the second is 39, while the sum of thrice the first and twice the second is 36. The larger of the two is :

(*a*) 6 (*b*) 8 (*c*) 9 (*d*) 12

60. In a two-digit number, the digit in the unit's place is four times the digit in ten's place and sum of the digits is equal to 10. What is the number ? **(Bank P.O. 1999)**

(*a*) 14 (*b*) 41 (*c*) 82

(*d*) Data inadequate (*e*) None of these

61. A number of two digits has 3 for its unit's digit, and the sum of digits is $\frac{1}{7}$ of the number itself. The number is : **(L.I.C. 2003)**

(*a*) 43 (*b*) 53 (*c*) 63 (*d*) 73.

62. A two-digit number exceeds the sum of the digits of that number by 18. If the digit at the unit's place is double the digit in the ten's place, what is the number ?

(*a*) 24 (*b*) 42 (*c*) 48 (*d*) Data inadequate

63. The sum of the digits of a two-digit number is 15 and the difference between the digits is 3. What is the two-digit number ? **(B.S.R.B. 2003)**

(*a*) 69 (*b*) 78 (*c*) 96

(*d*) Cannot be determined (*e*) None of these

64. In a two-digit number, if it is known that its unit's digit exceeds its ten's digit by 2 and that the product of the given number and the sum of its digits is equal to 144, then the number is : **(C.B.I. 2003)**

(*a*) 24 (*b*) 26 (*c*) 42 (*d*) 46

65. A number consists of two digits. If the digits interchange places and the new number is added to the original number, then the resulting number will be divisible by :
(a) 3 (b) 5 (c) 9 (d) 11
(S.S.C. 2003)

66. The sum of the digits of a two-digit number is 9 less than the number. Which of the following digits is at unit's place of the number ?
(a) 1 (b) 2 (c) 4 (d) Data inadequate

67. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number ? **(Bank P.O. 2003)**
(a) 3 (b) 4 (c) 9
(d) Cannot be determined (e) None of these

68. The difference between a two-digit number and the number obtained by interchanging the two digits is 63. Which is the smaller of the two numbers ? **(Bank P.O. 2003)**
(a) 29 (b) 70 (c) 92
(d) Cannot be determined (e) None of these.

69. The sum of the digits of a two-digit number is $\frac{1}{5}$ of the difference between the number and the number obtained by interchanging the positions of the digits. What is definitely the difference between the digits of that number ? **(Bank P.O. 2000)**
(a) 5 (b) 7 (c) 9
(d) Data inadequate (e) None of these

70. If the digit in the unit's place of a two-digit number is halved and the digit in the ten's place is doubled, the number thus obtained is equal to the number obtained by interchanging the digits. Which of the following is definitely true ? **(Bank P.O. 2003)**
(a) Sum of the digits is a two-digit number.
(b) Digit in the unit's place is twice the digit in the ten's place.
(c) Digits in the unit's place and the ten's place are equal.
(d) Digit in the unit's place is half of the digit in the ten's place.
(e) None of these

71. If the number obtained on interchanging the digits of a two-digit number is 18 more than the original number and the sum of the digits is 8, then what is the original number ? **(S.B.I.P.O. 2002)**
(a) 26 (b) 35 (c) 53
(d) Cannot be determined (e) None of these

72. The difference between a two-digit number and the number obtained by interchanging the digits is 36. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 1 : 2 ? **(M.A.T. 1999)**
(a) 4 (b) 8 (c) 16 (d) None of these

73. A number consists of 3 digits whose sum is 10. The middle digit is equal to the sum of the other two and the number will be increased by 99 if its digits are reversed. The number is : **(Hotel Management, 2003)**
(a) 145 (b) 253 (c) 370 (d) 352

74. A two-digit number becomes five-sixth of itself when its digits are reversed. The two digits differ by one. The number is :
(a) 45 (b) 54 (c) 56 (d) 65

75. A number consists of two digits such that the digit in the ten's place is less by 2 than the digit in the unit's place. Three times the number added to $\frac{6}{7}$ times the number obtained by reversing the digits equals 108. The sum of the digits in the number is :
(*a*) 6 (*b*) 7 (*c*) 8 (*d*) 9
(S.S.C. 2003)

76. The digit in the unit's place of a number is equal to the digit in the ten's place of half of that number and the digit in the ten's place of that number is less than the digit in unit's place of half of the number by 1. If the sum of the digits of the number is 7, then what is the number ? **(S.B.I.P.O. 2001)**
(*a*) 34 (*b*) 52 (*c*) 162
(*d*) Data inadequate (*e*) None of these

77. In a two-digit number, the digit in the unit's place is more than twice the digit in ten's place by 1. If the digits in the unit's place and the ten's place are interchanged, difference between the newly formed number and the original number is less than the original number by 1. What is the original number ? **(Bank P.O. 1999)**
(*a*) 25 (*b*) 37 (*c*) 49 (*d*) 52 (*e*) 73

78. A certain number of two digits is three times the sum of its digits and if 45 be added to it, the digits are reversed. The number is : **(L.I.C.A.A.O. 2003)**
(*a*) 23 (*b*) 27 (*c*) 32 (*d*) 72

79. A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. The number is : **(M.B.A. 2003)**
(*a*) 18 (*b*) 24 (*c*) 42 (*d*) 81

80. The product of two fractions is $\frac{14}{15}$ and their quotient is $\frac{35}{24}$. The greater fraction is :
(*a*) $\frac{4}{5}$ (*b*) $\frac{7}{6}$ (*c*) $\frac{7}{4}$ (*d*) $\frac{7}{3}$
(S.S.C. 2002)

81. In a pair of fractions, fraction A is twice the fraction B and the product of two fractions is $\frac{2}{25}$. What is the value of fraction A ? **(Bank P.O. 1999)**
(*a*) $\frac{1}{5}$ (*b*) $\frac{1}{25}$ (*c*) $\frac{2}{5}$ (*d*) Data inadequate

82. The sum of the numerator and denominator of a fraction is 11. If 1 is added to the numerator and 2 is subtracted from the denominator, it becomes $\frac{2}{3}$. The fraction is :
(*a*) $\frac{5}{6}$ (*b*) $\frac{6}{5}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{8}{3}$

83. The denominator of a fraction is 3 more than the numerator. If the numerator as well as the denominator is increased by 4, the fraction becomes $\frac{4}{5}$. What was the original fraction ? **(S.B.I.P.O. 1999)**
(*a*) $\frac{8}{11}$ (*b*) $\frac{5}{8}$ (*c*) $\frac{10}{13}$ (*d*) $\frac{7}{10}$

84. The difference between the numerator and the denominator of a fraction is 5. If 5 is added to its denominator, the fraction is decreased by $1\frac{1}{4}$. Find the value of the fraction.
(*a*) $\frac{1}{6}$ (*b*) $2\frac{1}{4}$ (*c*) $3\frac{1}{4}$ (*d*) 6
(M.B.A. 1997)

85. The numerator and denominator of a fraction are in the ratio of 2 : 3. If 6 is subtracted from the numerator, the result is a fraction that has a value $\frac{2}{3}$ of the original fraction. The numerator of the original fraction is : (S.S.C. 1999)
(a) 6 (b) 18 (c) 27 (d) 36

86. If 1 is added to the denominator of a fraction, the fraction becomes $\frac{1}{2}$. If 1 is added to the numerator of the fraction, the fraction becomes 1. The fraction is : (C.B.I. 1997)
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{3}{2}$

87. If the numerator of a fraction is increased by 2 and the denominator is increased by 3, the fraction becomes $\frac{7}{9}$ and if both the numerator as well as the denominator are decreased by 1, the fraction becomes $\frac{4}{5}$. What is the original fraction ?
(a) $\frac{5}{6}$ (b) $\frac{9}{11}$ (c) $\frac{13}{16}$ (d) $\frac{17}{21}$
(S.B.I.P.O. 1999)

88. When the numerator of a fraction increases by 4, the fraction increases by $\frac{2}{3}$. The denominator of the fraction is :
(a) 2 (b) 3 (c) 4 (d) 6

89. 54 is to be divided into two parts such that the sum of 10 times the first and 22 times the second is 780. The bigger part is :
(a) 24 (b) 34 (c) 30 (d) 32

90. 243 has been divided into three parts such that half of the first part, one-third of the second part and one-fourth of the third part are equal. The largest part is :
(a) 74 (b) 86 (c) 92 (d) 108

91. The sum of four numbers is 64. If you add 3 to the first number, 3 is subtracted from the second number, the third is multiplied by 3 and the fourth is divided by 3, then all the results are equal. What is the difference between the largest and the smallest of the original numbers ? (S.B.I.P.O. 2000)
(a) 21 (b) 27 (c) 32
(d) Cannot be determined (e) None of these

92. The sum of the squares of three numbers is 138, while the sum of their products taken two at a time is 131. Their sum is : (Hotel Management, 1999)
(a) 20 (b) 30 (c) 40 (d) None of these

93. The sum of three numbers is 136. If the ratio between first and second be 2 : 3 and that between second and third is 5 : 3, then the second number is :
(a) 40 (b) 48 (c) 60 (d) 72

94. Of the three numbers, the sum of the first two is 45; the sum of the second and the third is 55 and the sum of the third and thrice the first is 90. The third number is :
(a) 20 (b) 25 (c) 30 (d) 3

ANSWERS

1. (c)	2. (b)	3. (d)	4. (c)	5. (c)	6. (c)	7. (a)	8. (c)	9. (d)
10. (a)	11. (d)	12. (c)	13. (c)	14. (b)	15. (e)	16. (b)	17. (b)	18. (a)
19. (a)	20. (a)	21. (c)	22. (a)	23. (c)	24. (b)	25. (a)	26. (c)	27. (a)
28. (c)	29. (c)	30. (b)	31. (b)	32. (b)	33. (a)	34. (a)	35. (a)	36. (b)

37. (c)	38. (a)	39. (c)	40. (b)	41. (b)	42. (a)	43. (c)	44. (b)	45. (b)
46. (c)	47. (b)	48. (a)	49. (b)	50. (a)	51. (d)	52. (d)	53. (c)	54. (b)
55. (b)	56. (c)	57. (c)	58. (b)	59. (c)	60. (e)	61. (c)	62. (a)	63. (d)
64. (a)	65. (d)	66. (d)	67. (b)	68. (d)	69. (d)	70. (b)	71. (b)	72. (b)
73. (b)	74. (b)	75. (a)	76. (b)	77. (b)	78. (b)	79. (b)	80. (b)	81. (c)
82. (c)	83. (a)	84. (b)	85. (b)	86. (b)	87. (a)	88. (d)	89. (b)	90. (d)
91. (c)	92. (a)	93. (c)	94. (c)					

SOLUTIONS

1. Let the number be x. Then, $x - \frac{3}{5}x = 50 \Leftrightarrow \frac{2}{5}x = 50 \Leftrightarrow x = \left(\frac{50 \times 5}{2}\right) = 125$.

2. Let the number be x. Then, $\frac{x-4}{6} = 8 \Leftrightarrow x - 4 = 48 \Leftrightarrow x = 52$.

   $\therefore \quad \frac{x-2}{5} = \frac{52-2}{5} = \frac{50}{5} = 10$.

3. Let the number be x. Then, $\frac{1}{3}$ of $\frac{1}{4}$ of $x = 15 \Leftrightarrow x = 15 \times 12 = 180$.

   So, required number $= \left(\frac{3}{10} \times 180\right) = 54$.

4. Let the number be x. Then, $3(2x + 9) = 75 \Leftrightarrow 2x + 9 = 25 \Leftrightarrow 2x = 16 \Leftrightarrow x = 8$.

5. Let the number be x. Then, $\frac{3}{4}x - \frac{1}{3}x = 60 \Leftrightarrow \frac{5x}{12} = 60 \Leftrightarrow x = \left(\frac{60 \times 12}{5}\right) = 144$.

6. Let the number be x. Then,

   $x - 24 = \frac{4}{7}x \Leftrightarrow x - \frac{4}{7}x = 24 \Leftrightarrow \frac{3}{7}x = 24 \Leftrightarrow x = \left(\frac{24 \times 7}{3}\right) = 56$.

   $\therefore$ Sum of the digits $= (5 + 6) = 11$.

7. Let the number be x. Then, $15x - x = 196 \Leftrightarrow 14x = 196 \Leftrightarrow x = 14$.

8. Let the number be x. Then, $\frac{x}{4} = x - 21 \Leftrightarrow x = 4x - 84 \Leftrightarrow 3x = 84 \Leftrightarrow x = 28$.

9. Let the number be x. Then, $\left(\frac{1}{5}x + 4\right) = \left(\frac{1}{4}x - 10\right) \Leftrightarrow \frac{x}{20} = 14 \Leftrightarrow x = 14 \times 20 = 280$.

10. Let the larger number be x.

    Then, $x - 12 = 20\%$ of $x \Leftrightarrow x - \frac{x}{5} = 12 \Leftrightarrow \frac{4x}{5} = 12 \Leftrightarrow x = \left(\frac{12 \times 5}{4}\right) = 15$.

11. Let the number be x. Then, $\frac{1}{7}x - \frac{1}{11}x = 100 \Leftrightarrow \frac{4x}{77} = 100 \Leftrightarrow x = \frac{7700}{4} = 1925$.

12. Let the number be x.

    Then, $\left(\frac{1}{2}x + \frac{1}{5}x\right) - \frac{1}{3}x = \frac{22}{3} \Leftrightarrow \frac{11x}{30} = \frac{22}{3} \Leftrightarrow x = \left(\frac{22 \times 30}{3 \times 11}\right) = 20$.

13. Let the number be x. Then, $2x + 20 = 8x - 4 \Leftrightarrow 6x = 24 \Leftrightarrow x = 4$.

14. Let the number be x.

    Then, $\frac{2}{3}x - 50 = \frac{1}{4}x + 40 \Leftrightarrow \frac{2}{3}x - \frac{1}{4}x = 90 \Leftrightarrow \frac{5x}{12} = 90 \Leftrightarrow x = \left(\frac{90 \times 12}{5}\right) = 216$.

15. Let the number be x.
Then, $x + x^2 = 182 \Leftrightarrow x^2 + x - 182 = 0 \Leftrightarrow (x + 14)(x - 13) = 0 \Leftrightarrow x = 13$.

16. Let the integer be x.
Then, $x^2 - 20x = 96 \Leftrightarrow x^2 - 20x - 96 = 0 \Leftrightarrow (x + 4)(x - 24) = 0 \Leftrightarrow x = 24$.

17. Let the number be x.
Then, $3x^2 - 4x = x + 50 \Leftrightarrow 3x^2 - 5x - 50 = 0 \Leftrightarrow (3x + 10)(x - 5) = 0 \Leftrightarrow x = 5$.

18. Let the number be x. Then, $x + \frac{1}{x} = \frac{34}{8} \Leftrightarrow \frac{x^2+1}{x} = \frac{34}{8} \Leftrightarrow 8x^2 - 34x + 8 = 0$

$\Leftrightarrow 4x^2 - 17x + 4 = 0 \Leftrightarrow (4x - 1)(x - 4) = 0 \Leftrightarrow x = 4$.

$\left[\text{neglecting } x = \frac{1}{4}, \text{ as } x \text{ is a natural no.}\right]$

$\therefore$ Required number $= 4 \times \sqrt{4} = 4 \times 2 = 8$.

19. Let the number be x.

Then, $\frac{2}{3}x = \frac{25}{216} \times \frac{1}{x} \Leftrightarrow x^2 = \frac{25}{216} \times \frac{3}{2} = \frac{25}{144} \Leftrightarrow x = \sqrt{\frac{25}{144}} = \frac{5}{12}$.

20. Let the number be x.

Then, $x + 17 = \frac{60}{x} \Leftrightarrow x^2 + 17x - 60 = 0 \Leftrightarrow (x + 20)(x - 3) = 0 \Leftrightarrow x = 3$.

21. Let the number be x.

Then, $x - 4 = \frac{21}{x} \Leftrightarrow x^2 - 4x - 21 = 0 \Leftrightarrow (x - 7)(x + 3) = 0 \Leftrightarrow x = 7$.

22. Let the number be x. Then, $x + \frac{1}{x} = 3\left(x - \frac{1}{x}\right) \Leftrightarrow \frac{x^2+1}{x} = 3\left(\frac{x^2-1}{x}\right)$

$\Leftrightarrow x^2 + 1 = 3x^2 - 3 \Leftrightarrow 2x^2 = 4 \Leftrightarrow x^2 = 2 \Leftrightarrow x = \sqrt{2}$.

23. Let the numbers be a and b. Then, $ab = 17 \Rightarrow a = 1$ and $b = 17$.

So, $\frac{1}{a^2} + \frac{1}{b^2} = \frac{a^2+b^2}{a^2b^2} = \frac{1^2+(17)^2}{(1 \times 17)^2} = \frac{290}{289}$.

24. Let the number be x. Then,

$$\frac{4\frac{1}{2}\left(x + 2\frac{1}{2}\right) + 3}{1\frac{1}{5}} = 25 \Leftrightarrow \frac{\frac{9}{2}\left(x + \frac{5}{2}\right) + 3}{\frac{6}{5}} = 25$$

$$\Leftrightarrow \frac{9x}{2} + \frac{45}{4} + 3 = 25 \times \frac{6}{5} = 30 \Leftrightarrow \frac{9x}{2} = 30 - \frac{57}{4} \Leftrightarrow \frac{9x}{2} = \frac{63}{4}$$

$$\Leftrightarrow x = \left(\frac{63}{4} \times \frac{2}{9}\right) = \frac{7}{2} = 3\frac{1}{2}.$$

25. Let the numbers be $4x$, $5x$ and $6x$. Then, $\frac{4x + 5x + 6x}{3} = 25 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5$.

$\therefore$ Largest number $= 6x = 30$.

26. Let the numbers be $3x$, $4x$ and $6x$.
Then, $3x \times 4x \times 6x = 1944 \Leftrightarrow 72x^3 = 1944 \Leftrightarrow x^3 = 27 \Leftrightarrow x = 3$.
$\therefore$ Largest number $= 6x = 18$.

27. Let the numbers be $3x$ and $4x$. Then,
$(4x)^2 = 8 \times (3x)^2 - 224 \Leftrightarrow 16x^2 = 72x^2 - 224 \Leftrightarrow 56x^2 = 224 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2$.
So, the numbers are 6 and 8.

28. Let the numbers be $4x$ and $7x$. Then, $\frac{4x+4}{7x+4} = \frac{3}{5} \Leftrightarrow 5(4x+4) = 3(7x+4) \Leftrightarrow x = 8$.

$\therefore$ Larger number $= 7x = 56$.

29. Let the second number be x. Then, first number $= 2x$ and third number $= \frac{2x}{3}$.

$\therefore \quad 2x + x + \frac{2x}{3} = 264 \Leftrightarrow \frac{11x}{3} = 264 \Leftrightarrow x = \left(\frac{264 \times 3}{11}\right) = 72.$

30. Let the numbers be x and $(22 - x)$. Then, $5x = 6(22 - x) \Leftrightarrow 11x = 132 \Leftrightarrow x = 12$.
So, the numbers are 12 and 10.

31. Let the numbers be x and y. Then, $\frac{1}{5}x = \frac{5}{8}y \Leftrightarrow y = \frac{8}{25}x$.

Now, $x + 35 = 4y \Leftrightarrow x + 35 = \frac{32}{25}x \Leftrightarrow \frac{7}{25}x = 35 \Leftrightarrow x = \left(\frac{35 \times 25}{7}\right) = 125.$

$\therefore$ Second number $= y = \frac{8}{25}x = \left(\frac{8}{25} \times 125\right) = 40.$

32. Let the numbers be x and y. Then, $x + y = 25$ and $x - y = 13$.
$4xy = (x + y)^2 - (x - y)^2 = (25)^2 - (13)^2 = 625 - 169 = 456 \Rightarrow xy = 114.$

33. Let the numbers be x and y.
Then, $x + y = 33$ ...(*i*) and $x - y = 15$ ...(*ii*)
Solving (*i*) and (*ii*), we get : $x = 24$, $y = 9$.
$\therefore$ Smaller number $= 9$.

34. Let the numbers be x and y. Then,

$\frac{x+y}{x-y} = \frac{40}{4} = 10 \Leftrightarrow (x + y) = 10(x - y) \Leftrightarrow 9x = 11y \Leftrightarrow \frac{x}{y} = \frac{11}{9}$.

35. Let the numbers be x and $(28 - x)$. Then,
$x(28 - x) = 192 \Leftrightarrow x^2 - 28x + 192 = 0 \Leftrightarrow (x - 16)(x - 12) = 0$
$\Leftrightarrow x = 16$ or $x = 12$.
So, the numbers are 16 and 12.

36. Let the integers be x and $(x + 5)$. Then,
$x(x + 5) = 500 \Leftrightarrow x^2 + 5x - 500 = 0 \Leftrightarrow (x + 25)(x - 20) = 0 \Leftrightarrow x = 20.$
So, the numbers are 20 and 25.

37. Let the numbers be x and y. Then, $x - y = 5$ and $xy = 336$.
$(x + y)^2 = (x - y)^2 + 4xy = 25 + 4 \times 336 = 1369 \Rightarrow x + y = \sqrt{1369} = 37.$

38. Since $1.x < 1 + x$, so one of the numbers is 1.

39. Let the numbers be x and y. Then, $xy = 9375$ and $\frac{x}{y} = 15$.

$\frac{xy}{(x/y)} = \frac{9375}{15} \Leftrightarrow y^2 = 625 \Leftrightarrow y = 25 \Rightarrow x = 15y = (15 \times 25) = 375.$

$\therefore$ Sum of the numbers $= 375 + 25 = 400$.

40. Let the numbers be x and $(x + 1365)$.
Then, $x + 1365 = 6x + 15 \Leftrightarrow 5x = 1350 \Leftrightarrow x = 270.$

41. Let the numbers be x and y. Then, $x + y = 40$ and $xy = 375$.

$\therefore \quad \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{40}{375} = \frac{8}{75}.$

42. Let the numbers be x and y such that $x > y$. Then,
$x(x + y) = 204 \Rightarrow x^2 + xy = 204$ ...(*i*) and $y(x - y) = 35 \Rightarrow xy - y^2 = 35$ ...(*ii*)
Subtracting (*ii*) from (*i*), we get : $x^2 + y^2 = 169$.
The only triplet satisfying this condition is (12, 5, 13). Thus, $x = 12$, $y = 5$.

43. Let the numbers be x and y. Then, $x + y = 20$ and $x - y = 8$.
$\therefore\ x^2 - y^2 = (x + y)(x - y) = 20 \times 8 = 160$.

44. Let the numbers be x and y. Then, $xy = 120$ and $x^2 + y^2 = 289$.
$\therefore\ (x + y)^2 = x^2 + y^2 + 2xy = 289 + 240 = 529$.
$\therefore\ x + y = \sqrt{529} = 23$.

45. Let the numbers be x and y. Then, $xy = 45$ and $x^2 + y^2 = 106$.
$(x + y) = \sqrt{(x^2 + y^2) + 2xy} = \sqrt{106 + 90} = \sqrt{196} \Rightarrow x + y = 14$ ...(*i*)
$(x - y) = \sqrt{(x^2 + y^2) - 2xy} = \sqrt{106 - 90} = \sqrt{16} \Rightarrow x - y = 4$ ...(*ii*)
Solving (*i*) and (*ii*), we get : $x = 9$ and $y = 5$.

46. Let the numbers be x and y. Then,
$x^2 + y^2 = 3341$ ...(*i*) and $x^2 - y^2 = 891$ ...(*ii*)
Adding (*i*) and (*ii*), we get : $2x^2 = 4232$ or $x^2 = 2116$ or $x = 46$.
Subtracting (*ii*) from (*i*), we get : $2y^2 = 2450$ or $y^2 = 1225$ or $y = 35$.
So, the numbers are 35 and 46.

47. Let the numbers be x and $(x + 3)$. Then,
$x^2 + (x + 3)^2 = 369 \Leftrightarrow x^2 + x^2 + 9 + 6x = 369$
$\Leftrightarrow 2x^2 + 6x - 360 = 0 \Leftrightarrow x^2 + 3x - 180 = 0 \Leftrightarrow (x + 15)(x - 12) = 0 \Leftrightarrow x = 12$.
So, the numbers are 12 and 15.
$\therefore$ Required sum = (12 + 15) = 27.

48. Let the numbers be x and y. Then, $(x + y) = 22$ and $x^2 + y^2 = 404$.
Now, $2xy = (x + y)^2 - (x^2 + y^2) = (22)^2 - 404 = 484 - 404 = 80 \Rightarrow xy = 40$.

49. Let the numbers be x and y. Then, $x^2 - y^2 = 256000$ and $x + y = 1000$.
On dividing, we get : $x - y = 256$.
Solving $x + y = 1000$ and $x - y = 256$, we get : $x = 628$ and $y = 372$.

50. Let the numbers be x and y. Then, $x^2 - y^2 = 39$ and $x - y = 3$.
On dividing, we get : $x + y = 13$.
Solving $x + y = 13$ and $x - y = 3$, we get : $x = 8$ and $y = 5$.
$\therefore$ Larger number = 8.

51. Let the numbers be x, $x + 1$ and $x + 2$.
Then, $x + (x + 1) + (x + 2) = 87 \Leftrightarrow 3x = 84 \Leftrightarrow x = 28$.
$\therefore$ Greatest number = $(x + 2) = 30$.

52. Let the three integers be x, $x + 2$ and $x + 4$. Then, $3x = 2(x + 4) + 3 \Leftrightarrow x = 11$.
$\therefore$ Third integer = $x + 4 = 15$.

53. Let the four integers be x, $x + 2$, $x + 4$ and $x + 6$.
Then, $x + (x + 2) + (x + 4) + (x + 6) = 1284 \Leftrightarrow 4x = 1272 \Leftrightarrow x = 318$.
$\therefore$ Greatest integer = $x + 6 = 324$.

54. Let the numbers be x, $x + 2$ and $x + 4$.
Then, $x + (x + 2) + (x + 4) = x + 20 \Leftrightarrow 2x = 14 \Leftrightarrow x = 7$.
$\therefore$ Middle number = $x + 2 = 9$.

55. Let the numbers be x, $x + 2$ and $x + 4$.

Then, $\frac{x(x+2)(x+4)}{8} = 720 \Rightarrow x(x+2)(x+4) = 5760$.

$\therefore \sqrt{x} \times \sqrt{(x+2)} \times \sqrt{(x+4)} = \sqrt{x(x+2)(x+4)} = \sqrt{5760} = 24\sqrt{10}$.

56. Let the numbers be $3x$, $3x + 3$ and $3x + 6$.

Then, $3x + (3x + 3) + (3x + 6) = 72 \Leftrightarrow 9x = 63 \Leftrightarrow x = 7$.

$\therefore$ Largest number $= 3x + 6 = 27$.

57. Let the numbers be x and $x + 2$.

Then, $(x + 2)^2 - x^2 = 84 \Leftrightarrow 4x + 4 = 84 \Leftrightarrow 4x = 80 \Leftrightarrow x = 20$.

$\therefore$ Required sum $= x + (x + 2) = 2x + 2 = 42$.

58. Let the numbers be x, $x + 1$ and $x + 2$.

Then, $x^2 + (x + 1)^2 + (x + 2)^2 = 2030 \Leftrightarrow 3x^2 + 6x - 2025 = 0$

$\Leftrightarrow \quad x^2 + 2x - 675 = 0 \Leftrightarrow (x + 27)(x - 25) = 0 \Leftrightarrow x = 25$.

$\therefore$ Middle number $= (x + 1) = 26$.

59. Let the numbers be x and y. Then, $2x + 3y = 39$ ...(*i*) and $3x + 2y = 36$ ...(*ii*)

On solving (*i*) and (*ii*), we get : $x = 6$ and $y = 9$.

$\therefore$ Larger number $= 9$.

60. Let the ten's digits be x. Then, unit's digit $= 4x$.

$\therefore \quad x + 4x = 10 \Leftrightarrow 5x = 10 \Leftrightarrow x = 2$.

So, ten's digit $= 2$, unit's digit $= 8$.

Hence, the required number is 28.

61. Let the ten's digit be x. Then, number $= 10x + 3$ and sum of digits $= (x + 3)$.

So, $(x + 3) = \frac{1}{7}(10x + 3) \Leftrightarrow 7x + 21 = 10x + 3 \Leftrightarrow 3x = 18 \Leftrightarrow x = 6$.

Hence, the number is 63.

62. Let the ten's digit be x. Then, unit's digit $= 2x$.

Number $= 10x + 2x = 12x$; Sum of digits $= x + 2x = 3x$.

$\therefore \quad 12x - 3x = 18 \Leftrightarrow 9x = 18 \Leftrightarrow x = 2$.

Hence, required number $= 12x = 24$.

63. Let the ten's digit be x and unit's digit be y.

Then, $x + y = 15$ and $x - y = 3$ or $y - x = 3$.

Solving $x + y = 15$ and $x - y = 3$, we get : $x = 9$, $y = 6$.

Solving $x + y = 15$ and $y - x = 3$, we get : $x = 6$, $y = 9$.

So, the number is either 96 or 69. Hence, the number cannot be determined.

64. Let the ten's digit be x. Then, unit's digit $= x + 2$.

Number $= 10x + (x + 2) = 11x + 2$; Sum of digits $= x + (x + 2) = 2x + 2$.

$\therefore \quad (11x + 2)(2x + 2) = 144 \Leftrightarrow 22x^2 + 26x - 140 = 0 \Leftrightarrow 11x^2 + 13x - 70 = 0$

$\Leftrightarrow (x - 2)(11x + 35) = 0 \Leftrightarrow x = 2$.

Hence, required number $= 11x + 2 = 24$.

65. Let the ten's digit be x and unit's digit be y. Then, number $= 10x + y$.

Number obtained by interchanging the digits $= 10y + x$.

$\therefore \quad (10x + y) + (10y + x) = 11(x + y)$, which is divisible by 11.

66. Let the ten's digit be x and unit's digit be y. Then, $(10x + y) - (x + y) = 9$ or $x = 1$.

From this data, we cannot find y, the unit's digit. So, the data is inadequate.

67. Let the ten's digit be x and unit's digit be y.
Then, $(10x + y) - (10y + x) = 36 \Leftrightarrow 9(x - y) = 36 \Leftrightarrow x - y = 4$.

68. Let the ten's digit be x and unit's digit be y.
Then, $(10x + y) - (10y + x) = 63 \Leftrightarrow 9(x - y) = 63 \Leftrightarrow x - y = 7$.
Thus, none of the numbers can be determined.

69. Let the ten's digit be x and unit's digit be y.

Then, $x + y = \frac{1}{5}[(10x + y) - (10y + x)] \Leftrightarrow 5x + 5y = 9x - 9y \Leftrightarrow 4x = 14y$.

Thus, the value of $(x - y)$ cannot be determined from the given data.

70. Let the ten's digit be x and unit's digit be y.

Then, $10 \times 2x + \frac{1}{2}y = 10y + x \Leftrightarrow 20x - x = 10y - \frac{y}{2} \Leftrightarrow 19x = \frac{19}{2}y \Leftrightarrow y = 2x$.

Thus, the unit's digit is twice the ten's digit.

71. Let ten's digit = x. Then, unit's digit = $(8 - x)$.
$\therefore [10(8 - x) + x] - [10x + (8 - x)] = 18 \Leftrightarrow 18x = 54 \Leftrightarrow x = 3$.
So, ten's digit = 3 and unit's digit = 5. Hence, original number = 35.

72. Since the number is greater than the number obtained on reversing the digits, so the ten's digit is greater than the unit's digit.
Let the ten's and unit's digits be $2x$ and x respectively.
Then, $(10 \times 2x + x) - (10x + 2x) = 36 \Leftrightarrow 9x = 36 \Leftrightarrow x = 4$.
$\therefore$ Required difference = $(2x + x) - (2x - x) = 2x = 8$.

73. Let the middle digit be x. Then, $2x - 10$ or $x = 5$. So, the number is either 253 or 352.
Since the number increases on reversing the digits, so the hundred's digit is smaller than the unit's digit. Hence, required number = 253.

74. Since the number reduces on reversing the digits, so ten's digit is greater than the unit's digit.
Let the unit's digit be x. Then, ten's digit = $(x + 1)$.

$\therefore \quad 10x + (x + 1) = \frac{5}{6}[10(x + 1) + x] \Leftrightarrow 66x + 6 = 55x + 50 \Leftrightarrow 11x = 44 \Leftrightarrow x = 4$.

Hence, required number = 54.

75. Let the unit's digit be x. Then, ten's digit = $(x - 2)$.

$\therefore \quad 3[10(x - 2) + x] + \frac{6}{7}[10x + (x - 2)] = 108$

$\Leftrightarrow \quad 231x - 420 + 66x - 12 = 756 \Leftrightarrow 297x = 1188 \Leftrightarrow x = 4$.
Hence, sum of the digits = $x + (x - 2) = 2x - 2 = 6$.

76. Let the ten's digit be x and unit's digit be y. Then, $\frac{10x + y}{2} = 10y + (x + 1)$

$\Leftrightarrow \quad 10x + y = 20y + 2x + 2 \Leftrightarrow 8x - 19y = 2 \quad ...(i)$ and $x + y = 7 \quad ...(ii)$
Solving, (i) and (ii), we get : $x = 5$, $y = 2$. Hence, required number = 52.

77. Let the ten's digit be x. Then, unit's digit = $2x + 1$.
$[10x + (2x + 1)] - [(10(2x + 1) + x] - (10x + (2x + 1))] = 1$
$\Leftrightarrow \quad (12x + 1) - (9x + 9) = 1 \Leftrightarrow 3x = 9 \Leftrightarrow x = 3$.
So, ten's digit = 3 and unit's digit = 7. Hence, original number = 37.

78. Let the ten's digit be x and unit's digit be y.
Then, $10x + y = 3(x + y) \Rightarrow 7x - 2y = 0 \quad ...(i)$
$10x + y + 45 = 10y + x \Rightarrow y - x = 5 \quad ...(ii)$
Solving (i) and (ii), we get : $x = 2$ and $y = 7$.
$\therefore$ Required number = 27.

79. Let the ten's and unit's digit be x and $\frac{8}{x}$ respectively.

Then, $\left(10x+\frac{8}{x}\right)+18=10\times\frac{8}{x}+x \Leftrightarrow 10x^2+8+18x=80+x^2$

$\Leftrightarrow \quad 9x^2+18x-72=0 \Leftrightarrow x^2+2x-8=0 \Leftrightarrow (x+4)(x-2)=0 \Leftrightarrow x=2.$

So, ten's digit = 2 and unit's digit = 4. Hence, required number = 24.

80. Let the two fractions be a and b. Then, $ab=\frac{14}{15}$ and $\frac{a}{b}=\frac{35}{24}$.

$\frac{ab}{(a/b)}=\left(\frac{14}{15}\times\frac{24}{35}\right) \Leftrightarrow b^2=\frac{16}{25} \Leftrightarrow b=\frac{4}{5}.\ ab=\frac{14}{15} \Rightarrow a=\left(\frac{14}{15}\times\frac{5}{4}\right)=\frac{7}{6}.$

Since $a > b$, so greater fraction is $\frac{7}{6}$.

81. $A = 2B \Rightarrow B=\frac{1}{2}A$. So, $AB=\frac{2}{25} \Rightarrow \frac{1}{2}A^2=\frac{2}{25} \Rightarrow A^2=\frac{4}{25} \Rightarrow A=\frac{2}{5}$.

82. Let the fraction be $\frac{x}{y}$. Then, $x+y=11$. ...(*i*)

$\frac{x+1}{y-2}=\frac{2}{3} \Rightarrow 3(x+1)=2(y-2) \Rightarrow 3x-2y=-7$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 3$ and $y = 8$. So, the fraction is $\frac{3}{8}$.

83. Let the numerator be x. Then, denominator = $x + 3$.

Now, $\frac{x+4}{(x+3)+4}=\frac{4}{5} \Leftrightarrow 5(x+4)=4(x+7) \Leftrightarrow x=8.$

$\therefore$ The fraction is $\frac{8}{11}$.

84. Let the denominator be x. Then, numerator = $x + 5$.

Now, $\frac{x+5}{x}-\frac{x+5}{x+5}=\frac{5}{4} \Leftrightarrow \frac{x+5}{x}=\frac{5}{4}+1=\frac{9}{4}=2\frac{1}{4}.$

So, the fraction is $2\frac{1}{4}$.

85. Let the fraction be $\frac{2x}{3x}$. Then, $\frac{2x-6}{3x}=\frac{2}{3}\times\frac{2x}{3x} \Leftrightarrow \frac{2x-6}{3x}=\frac{4x}{9x} \Leftrightarrow 18x^2-54x=12x^2$

$\Leftrightarrow 6x^2=54x \Leftrightarrow x=9.$

Hence, numerator of the original fraction = $2x = 18$.

86. Let the fraction be $\frac{x}{y}$. Then,

$\frac{x}{y+1}=\frac{1}{2} \Leftrightarrow 2x-y=1$ ...(*i*) and $\frac{x+1}{y}=1 \Leftrightarrow x-y=-1$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 2$, $y = 3$. Hence, the required fraction is $\frac{2}{3}$.

87. Let the fraction be $\frac{x}{y}$. Then,

$\frac{x+2}{y+3}=\frac{7}{9} \Leftrightarrow 9x-7y=3$ ...(*i*) and $\frac{x-1}{y-1}=\frac{4}{5} \Leftrightarrow 5x-4y=1$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 5$, $y = 6$. Hence, the original fraction is $\frac{5}{6}$.

88. Let the fraction be $\frac{x}{y}$. Then, $\frac{x+4}{y} - \frac{x}{y} = \frac{2}{3} \Leftrightarrow \frac{4}{y} = \frac{2}{3} \Leftrightarrow y = \left(\frac{4 \times 3}{2}\right) = 6.$

$\therefore$ Denominator = 6.

89. Let the two parts be $(54 - x)$ and x.

Then, $10(54 - x) + 22x = 760 \Leftrightarrow 12x = 240 \Leftrightarrow x = 20.$

$\therefore$ Bigger part = $(54 - x) = 34$.

90. Let the three parts be A, B and C.

Let $\frac{A}{2} = \frac{B}{3} = \frac{C}{4} = x$. Then, $A = 2x$, $B = 3x$ and $C = 4x$. So, $A : B : C = 2 : 3 : 4$.

$\therefore$ Largest part = $\left(243 \times \frac{4}{9}\right) = 108.$

91. Let the four numbers be A, B, C and D. Let $A + 3 = B - 3 = 3C = \frac{D}{3} = x$.

Then, $A = x - 3$, $B = x + 3$, $C = \frac{x}{3}$ and $D = 3x$.

$$A + B + C + D = 64 \Rightarrow (x - 3) + (x + 3) + \frac{x}{3} + 3x = 64$$

$$\Leftrightarrow 5x + \frac{x}{3} = 64 \Leftrightarrow 16x = 192 \Rightarrow x = 12.$$

Thus, the numbers are 9, 15, 4 and 36.

$\therefore$ Required difference = $(36 - 4) = 32$.

92. Let the numbers be a, b and c. Then, $a^2 + b^2 + c^2 = 138$ and $(ab + bc + ca) = 131$.

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 138 + 2 \times 131 = 400$

$\Rightarrow (a + b + c) = \sqrt{400} = 20.$

93. $A : B = 2 : 3$ and $B : C = 5 : 3 = \frac{3}{5} \times 5 : \frac{3}{5} \times 3 = 3 : \frac{9}{5}$.

So, $A : B : C = 2 : 3 : \frac{9}{5} = 10 : 15 : 9$.

$\therefore$ Second number = $\left(136 \times \frac{15}{34}\right) = 60.$

94. Let the numbers be x, y and z. Then, $x + y = 45$, $y + z = 55$ and $3x + z = 90$

$\Rightarrow y = 45 - x$, $z = 55 - y = 55 - (45 - x) = 10 + x$.

$\therefore$ $3x + 10 + x = 90$ or $x = 20$.

$y = (45 - 20) = 25$ and $z = (10 + 20) = 30$.

$\therefore$ Third number = 30.

EXERCISE 7B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 6*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. What is the two-digit number ? **(S.B.I.P.O. 2003)**
   I. The difference between the two digits is 9.
   II. The sum of the digits is equal to the difference between the two digits.
2. What is the difference between the digits of a two-digit number ? **(Bank P.O. 1999)**
   I. The sum of the digits of that number is 8.
   II. One-fifth of that number is 15 less than half of 44.
3. What is the ratio between the two numbers ?
   I. The sum of two numbers is twice their difference.
   II. The smaller number is 6.
4. What is the two-digit number whose first digit is a and the second digit is b ? The number is greater than 9. **(M.A.T. 2000)**
   I. The number is a multiple of 51.
   II. The sum of the digits a and b is 6.
5. The difference between the digits of a two-digit number is 4. What is the digit in the unit's place ?
   I. The difference between the number and the number obtained by interchanging the positions of the digits is 36.
   II. The sum of the digits of that number is 12.
6. What is the number ? **(Bank P.O. 2000)**
   I. The sum of the two digits is 8. The ratio of the two digits is 1 : 3.
   II. The product of two digits of a number is 12. The quotient of two digits is 3.

Directions (***Questions 7 to 10***) : ***Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the given question.***

7. What is the two-digit number ? **(M.B.A. 2002)**
   I. Sum of the digits is 7.
   II. Difference between the number and the number obtained by interchanging the digits is 9.
   III. Digit in the ten's place is bigger than the digit in the unit's place by 1.
   (*a*) I and II only (*b*) II and III only (*c*) I and III only
   (*d*) All I, II and III (*e*) None of these
8. What is the sum of the digits of the two-digit number ?
   I. The ratio between the ten's digit and unit's digit of the number is 3 : 2.
   II. The number obtained on reversing the order of its digits is 18 less than the original number.
   III. The product of the digits is 24.
   (*a*) Any two of the three (*b*) I only or II and III only (*c*) All I, II and III
   (*d*) I and II only (*e*) None of these

9. What will be the sum of two numbers ? (S.B.I.P.O. 2000)

I. Among the two numbers, the bigger number is greater than the smaller number by 6.

II. 40% of the smaller number is equal to 30% of the bigger number.

III. The ratio between half of the bigger number and one-third of the smaller number is 2 : 1.

(*a*) I and II only (*b*) II and III only (*c*) All I, II and III

(*d*) Any two of the three (*e*) None of these

10. What is the two-digit number ? (R.B.I. 2002)

I. The difference between the two-digit number and the number formed by interchanging the digits is 27.

II. The difference between the two digits is 3.

III. The digit at unit's place is less than that at ten's place by 3.

(*a*) I and II only (*b*) I and III only (*c*) All I, II and III

(*d*) I, and either II or III (*e*) Even with all I, II and III, answer cannot be given.

ANSWERS

1. (*e*) 2. (*e*) 3. (*a*) 4. (*e*) 5. (*b*) 6. (*b*) 7. (*e*) 8. (*a*)
9. (*a*) 10. (*c*)

SOLUTIONS

1. Let the tens and unit digits be x and y respectively. Then,

I. $x - y = 9$.

II. $x + y = x - y$.

From I and II, we get $x - y = 9$ and $x + y = 9$.

On solving, we get $x = 9$ and $y = 0$.

∴ Required number is 90.

Thus, both I and II are needed to get the answer.

∴ Correct answer is (*e*).

2. Let the tens and unit digits be x and y respectively. Then,

I. $x + y = 8$ ...(*i*)

II. $\left(\frac{1}{2} \times 44\right) - \frac{1}{5}(10x + y) = 15 \Rightarrow 10x + y = 35$ ...(*ii*)

On solving (*i*) and (*ii*), we get $x = 3$ and $y = 5$.

Thus, I and II together give the answer.

∴ Correct answer is (*e*).

3. Let the two numbers be x and y.

I gives, $x + y = 2(x - y) \Leftrightarrow x = 3y \Leftrightarrow \frac{x}{y} = \frac{3}{1} \Leftrightarrow x : y = 3 : 1$.

Thus, I only gives the answer.

II does not give the answer.

∴ Correct answer is (*a*).

4. Number = $10b + a$.

I. $10b + a = 51 \times c$, where $c = 1, 2, 3$ etc. ...(*i*)

II. $a + b = 6$ ...(*ii*)

Taking $c = 1$, we get $10b + (6 - b) = 51 \Leftrightarrow 9b = 45 \Leftrightarrow b = 5$.

$\therefore$ $a = 1$, $b = 5$. So, number = 51.

Thus, I and II together give the answer.

$\therefore$ Correct answer is (e).

5. Let the ten's digit be x and unit's digit be y.

Then, $x - y = \pm 4$ ...(i)

I. $(10x + y) - (10y + x) = 36 \Leftrightarrow x - y = 4$ ...(ii)

II. $x + y = 12$ ...(iii)

Thus, (i) and (iii) together give the answer.

$\therefore$ II alone gives the answer and I alone does not give the answer.

$\therefore$ Correct answer is (b).

6. Let the tens and units digit be x and y respectively. Then,

I. $x + y = 8$ and $\frac{x}{y} = \frac{1}{3}$.

II. $xy = 12$ and $\frac{x}{y} = \frac{3}{1}$.

$\therefore$ II gives, $x^2 = 36 \Leftrightarrow x = 6$. So, $3y = 6 \Leftrightarrow y = 2$.

Thus, II alone gives the number. Clearly, I alone does not give the answer.

$\therefore$ Correct answer is (b).

7. Let the tens and units digit be x and y respectively.

I. $x + y = 7$.

II. $(10x + y) - (10y + x) = 9 \Rightarrow x - y = 1$.

III. $x - y = 1$.

Thus, I and II as well as I and III give the answer.

$\therefore$ Correct answer is (e).

8. I. Let the tens and units digit be $3x$ and $2x$ respectively.

II. $(30x + 2x) - (20x + 3x) = 18 \Leftrightarrow x = 2$.

III. $3x \times 2x = 24 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2$.

Thus, any two of the three will give the answer.

$\therefore$ Correct answer is (a).

9. Let the required numbers be x and y, where $x > y$.

I. $x - y = 6$ ...(i)

II. $\frac{30}{100}x = \frac{40}{100}y \Leftrightarrow 3x - 4y = 0$ ...(ii)

III. $\frac{\frac{1}{2}x}{\frac{1}{3}y} = \frac{2}{1} \Leftrightarrow \frac{3x}{2y} = \frac{2}{1} \Leftrightarrow \frac{x}{y} = \frac{4}{3}$.

Thus, I and II only give the answer.

$\therefore$ Correct answer is (a).

10. Let the tens and units digit be x and y respectively.

I. $(10x + y) - (10y + x) = 27 \Leftrightarrow x - y = 3$.

II. $x - y = 3$.

III. $x - y = 3$.

Thus, even all the given three statements together do not give the answer.

$\therefore$ Correct answer is (e).

8. PROBLEMS ON AGES

SOLVED EXAMPLES

Ex. 1. ***Rajeev's age after 15 years will be 5 times his age 5 years back. What is the present age of Rajeev ?*** (Hotel Management, 2002)

Sol. Let Rajeev's present age be x years. Then,

Rajeev's age after 15 years = $(x + 15)$ years.

Rajeev's age 5 years back = $(x - 5)$ years.

$\therefore \quad x + 15 = 5(x - 5) \Leftrightarrow x + 15 = 5x - 25 \Leftrightarrow 4x = 40 \Leftrightarrow x = 10.$

Hence, Rajeev's present age = 10 years.

Ex. 2. ***The ages of two persons differ by 16 years. If 6 years ago, the elder one be 3 times as old as the younger one, find their present ages.*** (A.A.O. Exam, 2003)

Sol. Let the age of the younger person be x years.

Then, age of the elder person = $(x + 16)$ years.

$\therefore \quad 3(x - 6) = (x + 16 - 6) \Leftrightarrow 3x - 18 = x + 10 \Leftrightarrow 2x = 28 \Leftrightarrow x = 14.$

Hence, their present ages are 14 years and 30 years.

Ex. 3. ***The product of the ages of Ankit and Nikita is 240. If twice the age of Nikita is more than Ankit's age by 4 years, what is Nikita's age ?*** (S.B.I.P.O. 1999)

Sol. Let Ankit's age be x years. Then, Nikita's age = $\frac{240}{x}$ years.

$\therefore \quad 2 \times \frac{240}{x} - x = 4 \Leftrightarrow 480 - x^2 = 4x \Leftrightarrow x^2 + 4x - 480 = 0$

$\Leftrightarrow (x + 24)(x - 20) = 0 \Leftrightarrow x = 20.$

Hence, Nikita's age = $\left(\frac{240}{20}\right)$ years = 12 years.

Ex. 4. ***The present age of a father is 3 years more than three times the age of his son. Three years hence, father's age will be 10 years more than twice the age of the son. Find the present age of the father.*** (S.S.C. 2003)

Sol. Let the son's present age be x years. Then, father's present age = $(3x + 3)$ years.

$\therefore \quad (3x + 3 + 3) = 2(x + 3) + 10 \Leftrightarrow 3x + 6 = 2x + 16 \Leftrightarrow x = 10.$

Hence, father's present age = $(3x + 3) = (3 \times 10 + 3)$ years = 33 years.

Ex. 5. ***Rohit was 4 times as old as his son 8 years ago. After 8 years, Rohit will be twice as old as his son. What are their present ages ?***

Sol. Let son's age 8 years ago be x years. Then, Rohit's age 8 years ago = $4x$ years.

Son's age after 8 years = $(x + 8) + 8 = (x + 16)$ years.

Rohit's age after 8 years = $(4x + 8) + 8 = (4x + 16)$ years.

$\therefore \quad 2(x + 16) = 4x + 16 \Leftrightarrow 2x = 16 \Leftrightarrow x = 8.$

Hence, son's present age = $(x + 8) = 16$ years.

Rohit's present age = $(4x + 8) = 40$ years.

Ex. 6. ***One year ago, the ratio of Gaurav's and Sachin's age was 6 : 7 respectively. Four years hence, this ratio would become 7 : 8. How old is Sachin ?***

(NABARD, 2002)

Sol. Let Gaurav's and Sachin's ages one year ago be $6x$ and $7x$ years respectively. Then,
Gaurav's age 4 years hence $= (6x + 1) + 4 = (6x + 5)$ years.
Sachin's age 4 years hence $= (7x + 1) + 4 = (7x + 5)$ years.

$$\therefore \quad \frac{6x+5}{7x+5} = \frac{7}{8} \Leftrightarrow 8(6x+5) = 7(7x+5) \Leftrightarrow 48x + 40 = 49x + 35 \Leftrightarrow x = 5.$$

Hence, Sachin's present age $= (7x + 1) = 36$ years.

Ex. 7. ***Abhay's age after six years will be three-seventh of his father's age. Ten years ago, the ratio of their ages was 1 : 5. What is Abhay's father's age at present ?***

Sol. Let the ages of Abhay and his father 10 years ago be x and $5x$ years respectively. Then,
Abhay's age after 6 years $= (x + 10) + 6 = (x + 16)$ years.
Father's age after 6 years $= (5x + 10) + 6 = (5x + 16)$ years.

$$\therefore \quad (x+16) = \frac{3}{7}(5x+16) \Leftrightarrow 7(x+16) = 3(5x+16) \Leftrightarrow 7x + 112 = 15x + 48$$
$$\Leftrightarrow 8x = 64 \Leftrightarrow x = 8.$$

Hence, Abhay's father's present age $= (5x + 10) = 50$ years.

EXERCISE 8A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. Sachin is younger than Rahul by 4 years. If their ages are in the respective ratio of 7 : 9, how old is Sachin ? **(Bank P.O. 2003)**
   (a) 16 years (b) 18 years (c) 28 years
   (d) Cannot be determined (e) None of these
2. The ratio between the present ages of P and Q is 6 : 7. If Q is 4 years old than P, what will be the ratio of the ages of P and Q after 4 years ? **(S.B.I.P.O. 1998)**
   (a) 3 : 4 (b) 3 : 5 (c) 4 : 3
   (d) Data inadequate (e) None of these
3. The ratio between the present ages of P and Q is 5 : 7 respectively. If the difference between Q's present age and P's age after 6 years is 2, what is the total of P's and Q's present ages ? **(Bank P.O. 1999)**
   (a) 48 years (b) 52 years (c) 56 years
   (d) Cannot be determined (e) None of these
4. At present, the ratio between the ages of Arun and Deepak is 4 : 3. After 6 years, Arun's age will be 26 years. What is the age of Deepak at present ? **(R.R.B. 2003)**
   (a) 12 years (b) 15 years (c) $19\frac{1}{2}$ years (d) 21 years
5. Present ages of X and Y are in the ratio 5 : 6 respectively. Seven years hence this ratio will become 6 : 7 respectively. What is X's present age in years ? **(Bank P.O. 2003)**
   (a) 35 (b) 42 (c) 49
   (d) Cannot be determined (e) None of these
6. Present ages of Sameer and Anand are in the ratio of 5 : 4 respectively. Three years hence, the ratio of their ages will become 11 : 9 respectively. What is Anand's present age in years ? **(R.B.I. 2003)**
   (a) 24 (b) 27 (c) 40
   (d) Cannot be determined (e) None of these

7. Six years ago, the ratio of the ages of Kunal and Sagar was 6 : 5. Four years hence, the ratio of their ages will be 11 : 10. What is Sagar's age at present ? **(Bank P.O. 2004)**
(*a*) 16 years (*b*) 18 years (*c*) 20 years
(*d*) Cannot be determined (*e*) None of these

8. The total of the ages of Jayant, Prem and Saransh is 93 years. Ten years ago, the ratio of their ages was 2 : 3 : 4. What is the present age of Saransh ?
(*a*) 24 years (*b*) 32 years (*c*) 34 years (*d*) 38 years

9. The ratio of the present ages of two brothers is 1 : 2 and 5 years back, the ratio was 1 : 3. What will be the ratio of their ages after 5 years ? **(S.S.C. 2002)**
(*a*) 1 : 4 (*b*) 2 : 3 (*c*) 3 : 5 (*d*) 5 : 6

10. Hitesh is 40 years old and Ronnie is 60 years old. How many years ago was the ratio of their ages 3 : 5 ?
(*a*) 5 years (*b*) 10 years (*c*) 20 years (*d*) 37 years

11. The ratio of the father's age to his son's age is 7 : 3. The product of their ages is 756. The ratio of their ages after 6 years will be :
(*a*) 5 : 2 (*b*) 2 : 1 (*c*) 11 : 7 (*d*) 13 : 9

12. The present ages of three persons are in proportions 4 : 7 : 9. Eight years ago, the sum of their ages was 56. Find their present ages (in years). **(I.M.T. 2002)**
(*a*) 8, 20, 28 (*b*) 16, 28, 36 (*c*) 20, 35, 45 (*d*) None of these

13. The ratio of the ages of a man and his wife is 4 : 3. After 4 years, this ratio will be 9 : 7. If at the time of marriage, the ratio was 5 : 3, then how many years ago were they married ?
(*a*) 8 years (*b*) 10 years (*c*) 12 years (*d*) 15 years

14. The ratio between the school ages of Neelam and Shaan is 5 : 6 respectively. If the ratio between the one-third age of Neelam and half of Shaan's age is 5 : 9, then what is the school age of Shaan ? **(Bank P.O. 2002)**
(*a*) 25 years (*b*) 30 years (*c*) 36 years
(*d*) Cannot be determined (*e*) None of these

15. The ratio between the present ages of A and B is 5 : 3 respectively. The ratio between A's age 4 years ago and B's age 4 years hence is 1 : 1. What is the ratio between A's age 4 years hence and B's age 4 years ago ? **(SIDBI, 2000)**
(*a*) 1 : 3 (*b*) 2 : 1 (*c*) 3 : 1 (*d*) 4 : 1 (*e*) None of these

16. Ten years ago, A was half of B in age. If the ratio of their present ages is 3 : 4, what will be the total of their present ages ?
(*a*) 20 years (*b*) 30 years (*c*) 45 years (*d*) None of these

17. A is two years older than B who is twice as old as C. If the total of the ages of A, B and C be 27, then how old is B ? **(Hotel Management, 2003)**
(*a*) 7 (*b*) 8 (*c*) 9 (*d*) 10 (*e*) 11

18. A man is 24 years older than his son. In two years, his age will be twice the age of his son. The present age of the son is : **(R.R.B. 2003)**
(*a*) 14 years (*b*) 18 years (*c*) 20 years (*d*) 22 years

19. Eighteen years ago, a father was three times as old as his son. Now the father is only twice as old as his son. Then the sum of the present ages of the son and the father is : **(S.S.C. 2003)**
(*a*) 54 (*b*) 72 (*c*) 105 (*d*) 108

20. A person's present age is two-fifth of the age of his mother. After 8 years, he will be one-half of the age of his mother. How old is the mother at present ?
(*a*) 32 years (*b*) 36 years (*c*) 40 years (*d*) 48 years
(IGNOU, 2003)

21. Tanya's grandfather was 8 times older to her 16 years ago. He would be 3 times of her age 8 years from now. Eight years ago, what was the ratio of Tanya's age to that of her grandfather ? **(S.S.C. 2003)**

(*a*) 1 : 2 (*b*) 1 : 5 (*c*) 3 : 8 (*d*) None of these

22. The age of father 10 years ago was thrice the age of his son. Ten years hence, father's age will be twice that of his son. The ratio of their present ages is :

(*a*) 5 : 2 (*b*) 7 : 3 (*c*) 9 : 2 (*d*) 13 : 4

(L.I.C.A.A.O. 2003)

23. Four years ago, the father's age was three times the age of his son. The total of the ages of the father and the son after four years, will be 64 years. What is the father's age at present ?

(*a*) 32 years (*b*) 36 years (*c*) 44 years

(*d*) Data inadequate (*e*) None of these

24. One year ago, Promila was four times as old as her daughter Sakshi. Six years hence, Promila's age will exceed her daughter's age by 9 years. The ratio of the present ages of Promila and her daughter is :

(*a*) 9 : 2 (*b*) 11 : 3 (*c*) 12 : 5 (*d*) 13 : 4

25. The sum of the present ages of a father and his son is 60 years. Six years ago, father's age was five times the age of the son. After 6 years, son's age will be :

(*a*) 12 years (*b*) 14 years (*c*) 18 years (*d*) 20 years

(R.R.B. 2000)

26. The total age of A and B is 12 years more than the total age of B and C. C is how many years younger than A ? **(SIDBI, 2000)**

(*a*) 12 (*b*) 24 (*c*) C is elder than A

(*d*) Data inadequate (*e*) None of these

27. Q is as much younger than R as he is older than T. If the sum of the ages of R and T is 50 years, what is definitely the difference between R and Q's age ?

(*a*) 1 year (*b*) 2 years (*c*) 25 years

(*d*) Data inadequate (*e*) None of these **(Bank P.O. 1999)**

28. The age of a man is three times the sum of the ages of his two sons. Five years hence, his age will be double of the sum of the ages of his sons. The father's present age is :

(*a*) 40 years (*b*) 45 years (*c*) 50 years (*d*) 55 years

29. The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages was 34. The ages of the son and the father are respectively :

(*a*) 6 and 39 (*b*) 7 and 38 (*c*) 9 and 36 (*d*) 11 and 34

30. Rajan got married 8 years ago. His present age is $\frac{6}{5}$ times his age at the time of his marriage. Rajan's sister was 10 years younger to him at the time of his marriage. The age of Rajan's sister is : **(U.P.S.C. 2003)**

(*a*) 32 years (*b*) 36 years (*c*) 38 years (*d*) 40 years

31. The sum of the ages of 5 children born at the intervals of 3 years each is 50 years. What is the age of the youngest child ? **(S.S.C. 2000)**

(*a*) 4 years (*b*) 8 years (*c*) 10 years (*d*) None of these

32. Father is aged three times more than his son Ronit. After 8 years, he would be two and a half times of Ronit's age. After further 8 years, how many times would he be of Ronit's age ? **(C.B.I. 1998)**

(*a*) 2 times (*b*) $2\frac{1}{2}$ times (*c*) $2\frac{3}{4}$ times (*d*) 3 times

33. The difference between the ages of two persons is 10 years. Fifteen years ago, the elder one was twice as old as the younger one. The present age of the elder person is :
(*a*) 25 years (*b*) 35 years (*c*) 45 years (*d*) 55 years

34. A father said to his son, "I was as old as you are at present at the time of your birth." If the father's age is 38 years now, the son's age five years back was :
(*a*) 14 years (*b*) 19 years (*c*) 33 years (*d*) 38 years
(Assistant Grade, 1998)

35. In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, the present age of B is :
(*a*) 19 years (*b*) 29 years (*c*) 39 years (*d*) 49 years

36. Sneh's age is $\frac{1}{6}$th of her father's age. Sneh's father's age will be twice of Vimal's age after 10 years. If Vimal's eighth birthday was celebrated two years before, then what is Sneh's present age ?
(*a*) $6\frac{2}{3}$ years (*b*) 24 years (*c*) 30 years (*d*) None of these

37. If 6 years are subtracted from the present age of Gagan and the remainder is divided by 18, then the present age of his grandson Anup is obtained. If Anup is 2 years younger to Madan whose age is 5 years, then what is Gagan's present age ?
(*a*) 48 years (*b*) 60 years (*c*) 84 years (*d*) 96 years

38. Ayesha's father was 38 years of age when she was born while her mother was 36 years old when her brother four years younger to her was born. What is the difference between the ages of her parents ? **(Hotel Management, 2002)**
(*a*) 2 years (*b*) 4 years (*c*) 6 years (*d*) 8 years

39. My brother is 3 years elder to me. My father was 28 years of age when my sister was born while my mother was 26 years of age when I was born. If my sister was 4 years of age when my brother was born, then, what was the age of my father and mother respectively when my brother was born ?
(*a*) 32 yrs, 23 yrs (*b*) 32 yrs, 29 yrs (*c*) 35 yrs, 29 yrs (*d*) 35 yrs, 33 yrs

40. A person was asked to state his age in years. His reply was, "Take my age three years hence, multiply it by 3 and then subtract three times my age three years ago and you will know how old I am." What was the age of the person ? **(S.S.C. 2004)**
(*a*) 18 years (*b*) 20 years (*c*) 24 years (*d*) 32 years

ANSWERS

1. (*e*)	**2.** (*e*)	**3.** (*a*)	**4.** (*b*)	**5.** (*a*)	**6.** (*a*)	**7.** (*a*)	**8.** (*d*)
9. (*c*)	**10.** (*b*)	**11.** (*b*)	**12.** (*b*)	**13.** (*c*)	**14.** (*d*)	**15.** (*c*)	**16.** (*d*)
17. (*d*)	**18.** (*d*)	**19.** (*d*)	**20.** (*c*)	**21.** (*d*)	**22.** (*b*)	**23.** (*e*)	**24.** (*d*)
25. (*d*)	**26.** (*a*)	**27.** (*d*)	**28.** (*b*)	**29.** (*a*)	**30.** (*c*)	**31.** (*a*)	**32.** (*a*)
33. (*b*)	**34** (*b*)	**35.** (*c*)	**36.** (*d*)	**37.** (*b*)	**38.** (*c*)	**39.** (*a*)	**40.** (*a*)

SOLUTIONS

1. Let Rahul's age be x years. Then, Sachin's age = $(x - 7)$ years.

$\therefore \quad \frac{x-7}{x} = \frac{7}{9} \Leftrightarrow 9x - 63 = 7x \Leftrightarrow 2x = 63 \Leftrightarrow x = 31.5.$

Hence, Sachin's age = $(x - 7) = 24.5$ years.

2. Let P's age and Q's age be $6x$ years and $7x$ years respectively.
Then, $7x - 6x = 4 \Leftrightarrow x = 4$.
$\therefore$ Required ratio $= (6x + 4) : (7x + 4) = 28 : 32 = 7 : 8$.

3. Let the present ages of P and Q be $5x$ years and $7x$ years respectively.
Then, $7x - (5x + 6) = 2 \Leftrightarrow 2x = 8 \Leftrightarrow x = 4$.
$\therefore$ Required sum $= 5x + 7x = 12x = 48$ years.

4. Let the present ages of Arun and Deepak be $4x$ years and $3x$ years respectively. Then,
$4x + 6 = 26 \Leftrightarrow 4x = 20 \Leftrightarrow x = 5$.
$\therefore$ Deepak's age $= 3x = 15$ years.

5. Let the present ages of X and Y be $5x$ years and $6x$ years respectively.
Then, $\frac{5x + 7}{6x + 7} = \frac{6}{7} \Leftrightarrow 7(5x + 7) = 6(6x + 7) \Leftrightarrow x = 7$.
$\therefore$ X's present age $= 5x = 35$ years.

6. Let the present ages of Sameer and Anand be $5x$ years and $4x$ years respectively.
Then, $\frac{5x + 3}{4x + 3} = \frac{11}{9} \Leftrightarrow 9(5x + 3) = 11(4x + 3) \Leftrightarrow x = 6$.
$\therefore$ Anand's present age $= 4x = 24$ years.

7. Let the ages of Kunal and Sagar 6 years ago be $6x$ and $5x$ years respectively.
Then, $\frac{(6x + 6) + 4}{(5x + 6) + 4} = \frac{11}{10} \Leftrightarrow 10(6x + 10) = 11(5x + 10) \Leftrightarrow 5x = 10 \Leftrightarrow x = 2$.
$\therefore$ Sagar's present age $= (5x + 6) = 16$ years.

8. Let the ages of Jayant, Prem and Saransh 10 years ago be $2x$, $3x$ and $4x$ years respectively.
Then, $(2x + 10) + (3x + 10) + (4x + 10) = 93 \Leftrightarrow 9x = 63 \Leftrightarrow x = 7$.
$\therefore$ Saransh's present age $= (4x + 10) = 38$ years.

9. Let the present ages of the two brothers be x years and $2x$ years respectively.
Then, $\frac{x - 5}{2x - 5} = \frac{1}{3} \Leftrightarrow 3(x - 5) = (2x - 5) \Leftrightarrow x = 10$.
$\therefore$ Required ratio $= (x + 5) : (2x + 5) = 15 : 25 = 3 : 5$.

10. Suppose, the ratio was $3 : 5$, x years ago.
Then, $\frac{40 - x}{60 - x} = \frac{3}{5} \Leftrightarrow 5(40 - x) = 3(60 - x) \Leftrightarrow 2x = 20 \Leftrightarrow x = 10$.

11. Let the present ages of the father and son be $7x$ and $3x$ years respectively.
Then, $7x \times 3x = 756 \Leftrightarrow 21x^2 = 756 \Leftrightarrow x^2 = 36 \Leftrightarrow x = 6$.
$\therefore$ Required ratio $= (7x + 6) : (3x + 6) = 48 : 24 = 2 : 1$.

12. Let their present ages be $4x$, $7x$ and $9x$ years respectively.
Then, $(4x - 8) + (7x - 8) + (9x - 8) = 56 \Leftrightarrow 20x = 80 \Leftrightarrow x = 4$.
$\therefore$ Their present ages are 16 years, 28 years and 36 years respectively.

13. Let the present ages of the man and his wife be $4x$ and $3x$ years respectively.
Then, $\frac{4x + 4}{3x + 4} = \frac{9}{7} \Leftrightarrow 7(4x + 4) = 9(3x + 4) \Leftrightarrow x = 8$.
So, their present ages are 32 years and 24 years respectively.
Suppose they were married z years ago.
Then, $\frac{32 - z}{24 - z} = \frac{5}{3} \Leftrightarrow 3(32 - z) = 5(24 - z) \Leftrightarrow 2z = 24 \Leftrightarrow z = 12$.

14. Let the school ages of Neelam and Shaan be $5x$ and $6x$ years respectively. Then,

$$\frac{\frac{1}{3}\times 5x}{\frac{1}{2}\times 6x} = \frac{5}{9} \Leftrightarrow \left(\frac{1}{3}\times 9\times 5x\right) = \left(\frac{5}{2}\times 6x\right) \Leftrightarrow 15 = 15.$$

Thus, Shaan's age cannot be determined.

15. Let the present ages of A and B be $5x$ and $3x$ years respectively.

Then, $\frac{5x-4}{3x+4} = \frac{1}{1} \Leftrightarrow 5x - 4 = 3x + 4 \Leftrightarrow 2x = 8 \Leftrightarrow x = 4.$

$\therefore$ Required ratio $= (5x + 4) : (3x - 4) = 24 : 8 = 3 : 1.$

16. Let the ages of A and B 10 years ago be x and $2x$ years respectively.

Then, $\frac{x+10}{2x+10} = \frac{3}{4} \Leftrightarrow 4(x + 10) = 3(2x + 10) \Leftrightarrow 2x = 10 \Leftrightarrow x = 5.$

$\therefore$ Sum of their present ages $= (x + 10) + (2x + 10) = (3x + 20) = 35$ years.

17. Let C's age be x years. Then, B's age $= 2x$ years. A's age $= (2x + 2)$ years.

$\therefore (2x + 2) + 2x + x = 27 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5.$

Hence, B's age $= 2x = 10$ years.

18. Let the son's present age be x years. Then, man's present age $= (x + 24)$ years.

$\therefore (x + 24) + 2 = 2(x + 2) \Leftrightarrow x + 26 = 2x + 4 \Leftrightarrow x = 22.$

19. Let the present ages of the father and son be $2x$ and x years respectively.

Then, $(2x - 18) = 3(x - 18) \Leftrightarrow x = 36.$

$\therefore$ Required sum $= (2x + x) = 3x = 108$ years.

20. Let the mother's present age be x years. Then, the person's present age $= \left(\frac{2}{5}x\right)$ years.

$$\therefore \left(\frac{2}{5}x + 8\right) = \frac{1}{2}(x + 8) \Leftrightarrow 2(2x + 40) = 5(x + 8) \Leftrightarrow x = 40.$$

21. 16 years ago, let $T = x$ years and $G = 8x$ years.

After 8 years from now, $T = (x + 16 + 8)$ years and $G = (8x + 16 + 8)$ years.

$\therefore 8x + 24 = 3(x + 24) \Leftrightarrow 5x = 48.$

8 years ago, $\frac{T}{G} = \frac{x+8}{8x+8} = \frac{\frac{48}{5}+8}{8\times\frac{48}{5}+8} = \frac{88}{424} = \frac{11}{53}.$

22. Let the ages of father and son 10 years ago be $3x$ and x years respectively.

Then, $(3x + 10) + 10 = 2[(x + 10) + 10] \Leftrightarrow 3x + 20 = 2x + 40 \Leftrightarrow x = 20.$

$\therefore$ Required ratio $= (3x + 10) : (x + 10) = 70 : 30 = 7 : 3.$

23. Let the ages of father and son 4 years ago be $3x$ and x years respectively.

Then, $[(3x + 4) + 4] + [(x + 4) + 4] = 64 \Leftrightarrow 4x = 48 \Leftrightarrow x = 12.$

$\therefore$ Father's present age $= 3x = 36$ years.

24. Let the ages of Promila and Sakshi 1 year ago be $4x$ and x years respectively.

Then, $[(4x + 1) + 6] - [(x + 1) + 6] = 9 \Leftrightarrow 3x = 9 \Leftrightarrow x = 3.$

$\therefore$ Required ratio $= (4x + 1) : (x + 1) = 13 : 4.$

25. Let the present ages of son and father be x and $(60 - x)$ years respectively.

Then, $(60 - x) - 6 = 5(x - 6) \Leftrightarrow 54 - x = 5x - 30 \Leftrightarrow 6x = 84 \Leftrightarrow x = 14.$

$\therefore$ Son's age after 6 years $= (x + 6) = 20$ years.

26. $(A + B) - (B + C) = 12 \Leftrightarrow A - C = 12.$

27. $R - Q = R - T \Rightarrow Q = T.$ Also, $R + T = 50 \Rightarrow R + Q = 50.$

So, $(R - Q)$ cannot be determined.

28. Let the sum of present ages of the two sons be x years.
Then, father's present age = $3x$ years.
$\therefore\ (3x + 5) = 2\,(x + 10) \Leftrightarrow 3x + 5 = 2x + 20 \Leftrightarrow x = 15.$
Hence, father's present age = 45 years.

29. Let the ages of father and son be x and $(45 - x)$ years respectively.
Then, $(x - 5)\,(45 - x - 5) = 34 \Leftrightarrow (x - 5)\,(40 - x) = 34 \Leftrightarrow x^2 - 45x + 234 = 0$
$\Leftrightarrow (x - 39)\,(x - 6) = 0 \Leftrightarrow x = 39$ or $x = 6.$
$\therefore$ Father's age = 39 years and son's age = 6 years.

30. Let Rajan's present age be x years. Then, his age at the time of marriage = $(x - 8)$ years.
$\therefore\ x = \frac{6}{5}(x - 8) \Leftrightarrow 5x = 6x - 48 \Leftrightarrow x = 48.$
Rajan's sister's age at the time of his marriage = $(x - 8) - 10 = (x - 18) = 30$ years.
$\therefore$ Rajan's sister's present age = (30 + 8) years = 38 years.

31. Let the ages of the children be x, $(x + 3)$, $(x + 6)$, $(x + 9)$ and $(x + 12)$ years.
Then, $x + (x + 3) + (x + 6) + (x + 9) + (x + 12) = 50 \Leftrightarrow 5x = 20 \Leftrightarrow x = 4.$
$\therefore$ Age of the youngest child = $x = 4$ years.

32. Let Ronit's present age be x years. Then, father's present age = $(x + 3x)$ years = $4x$ years.
$\therefore\ (4x + 8) = \frac{5}{2}(x + 8) \Leftrightarrow 8x + 16 = 5x + 40 \Leftrightarrow 3x = 24 \Leftrightarrow x = 8.$
Hence, required ratio = $\frac{(4x + 16)}{(x + 16)} = \frac{48}{24} = 2.$

33. Let their ages be x years and $(x + 10)$ years respectively.
Then, $(x + 10) - 15 = 2\,(x - 15) \Leftrightarrow x - 5 = 2x - 30 \Leftrightarrow x = 25.$
$\therefore$ Present age of the elder person = $(x + 10) = 35$ years.

34. Let the son's present age be x years. Then, $(38 - x) = x \Leftrightarrow 2x = 38 \Leftrightarrow x = 19.$
$\therefore$ Son's age 5 years back = (19 − 5) years = 14 years.

35. Let B's present age = x years. Then, A's present age = $(x + 9)$ years.
$\therefore\ (x + 9) + 10 = 2\,(x - 10) \Leftrightarrow x + 19 = 2x - 20 \Leftrightarrow x = 39.$

36. Vimal's age after 10 years = (8 + 2 + 10) years = 20 years.
Sneh's father's age after 10 years = 40 years. Sneh's father's present age = 30 years.
$\therefore$ Sneh's age = $\left(\frac{1}{6} \times 30\right)$ years = 5 years.

37. Anup's age = (5 − 2) years = 3 years. Let Gagan's age be x years.
Then, $\frac{x - 6}{18} = 3 \Leftrightarrow x - 6 = 54 \Leftrightarrow x = 60.$

38. Mother's age when Ayesha's brother was born = 36 years.
Father's age when Ayesha's brother was born = (38 + 4) years = 42 years.
$\therefore$ Required difference = (42 − 36) years = 6 years.

39. Clearly, my brother was born 3 years before I was born and 4 years after my sister was born.
So, father's age when brother was born = (28 + 4) years = 32 years;
mother's age when brother was born = (26 − 3) years = 23 years.

40. Let the present age of the person be x years.
Then, $3\,(x + 3) - 3\,(x - 3) = x \Leftrightarrow (3x + 9) - (3x - 9) = x \Leftrightarrow x = 18.$

EXERCISE 8B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 8*) : *Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and*

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. The sum of the ages of P, Q and R is 96 years. What is the age of Q ?
   I. P is 6 years older than R.
   II. The total of the ages of Q and R is 56 years.
2. What is Sonia's present age ? **(Bank P.O. 2003)**
   I. Sonia's present age is five times Deepak's present age.
   II. Five years ago her age was twenty-five times Deepak's age at that time.
3. How old is C now ?
   I. Three years ago, the average of A and B was 18 years.
   II. With C joining them now, the average becomes 22 years.
4. What is Reena's present age ? **(Bank P.O. 2003)**
   I. Reena's present age is five times her son's present age.
   II. Reena's age two years hence will be three times her daughter's age at that time.
5. What is the average age of A and B ?
   I. The ratio between one-fifth of A's age and one-fourth of B's age is 1 : 2.
   II. The product of their ages is 20 times B's age.
6. Average age of employees working in a department is 30 years. In the next year, ten workers will retire. What will be the average age in the next year ? **(I.M.T. 2002)**
   I. Retirement age is 60 years.
   II. There are 50 employees in the department.
7. What is the ratio between the ages of the father and the son ?
   I. The sum of their ages is 50 years.
   II. 3 times the sum of their ages is equal to 5 times the father's age.
8. Divya is twice as old as Shruti. What is the difference in their ages ? **(Bank P.O. 2003)**
   I. Five years hence, the ratio of their ages would be 9 : 5.
   II. Ten years back, the ratio of their ages was 3 : 1.

Directions (*Questions 9 to 13*) : *Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statements is/are necessary to answer the question.*

9. What is the present age of A ?
   I. The sum of the ages of A and B is 21 years.
   II. The difference of the ages of A and B is 5 years.

III. The product of the ages of A and B is 104 years.

(a) I and II only (b) II and III only (c) I and III only

(d) Any two of the three (e) None of these

10. What is the present age of Tanya ? **(Bank P.O. 2004)**

I. The ratio between the present ages of Tanya and her brother Rahul is 3 : 4 respectively.

II. After 5 years the ratio between the ages of Tanya and Rahul will be 4 : 5.

III. Rahul is 5 years older than Tanya.

(a) I and II only (b) II and III only (c) I and III only

(d) All I, II and III (e) Any two of the three

11. What is the difference between the ages of Y and X ?

I. The ratio between the ages of X and Y is 2 : 3.

II. Y's age is 50% more than X's age.

III. One-fourth of X's age is equal to one-sixth of Y's age.

(a) All I, II and III (b) Any two of the three

(c) III, and either I or II (d) Only I and II

(e) Question cannot be answered even with information in all three statements

12. What is Arun's present age ? **(M.B.A. 2002)**

I. Five years ago, Arun's age was double that of his son's age at that time.

II. Present ages of Arun and his son are in the ratio of 11 : 6 respectively.

III. Five years hence, the respective ratio of Arun's age and his son's age will become 12 : 7.

(a) Only I and II (b) Only II and III (c) Only I and III

(d) Any two of the three (e) None of these

13. What is Ravi's present age ? **(R.B.I. 2002)**

I. The present age of Ravi is half of that of his father.

II. After 5 years, the ratio of Ravi's age to that of his father's age will be 6 : 11.

III. Ravi is 5 years younger than his brother.

(a) I and II only (b) II and III only

(c) I and III only (d) All I, II and III

(e) Even with all the three statements answer cannot be given.

Directions (*Questions 14 to 16*) : *Each of these questions is followed by three statements. You have to study the question and all the three statements given to decide whether any information provided in the statement(s) is redundant and can be dispensed with while answering the given question.*

14. What is the ratio of the present ages of Anna and her mother ?

I. The sum of the ages of Anna, her mother and her father is 62.

II. Five years ago, Anna's age was one-fifth of her father's age.

III. Two years ago, the sum of the ages of Anna and her father was 36.

(a) I or II only (b) II or III only (c) III only

(d) I or III only (e) All I, II and III are required.

15. What will be the ratio between ages of Sam and Albert after 5 years ? **(Bank P.O. 1999)**

I. Sam's present age is more than Albert's present age by 4 years.

II. Albert's present age is 20 years.

III. The ratio of Albert's present age to Sam's present age is 5 : 6.

(a) I or II or III only (b) II only (c) III only
(d) I or III only (e) II or III only.

16. What is the difference between the present ages of Ayush and Deepak ? (S.B.I.P.O. 1998)
I. The ratio between Ayush's present age and his age after 8 years is 4 : 5.
II. The ratio between the present ages of Ayush and Deepak is 4 : 3.
III. The ratio between Deepak's present age and his age four years ago is 6 : 5.
(a) Any two of I, II and III (b) I or III only
(c) Any one of the three (d) All I, II and III are required
(e) Even with all I, II and III, the answer cannot be obtained.

ANSWERS

1. (e)	2. (c)	3. (e)	4. (d)	5. (e)	6. (e)	7. (b)	8. (c)
9. (d)	10. (e)	11. (e)	12. (d)	13. (a)	14. (e)	15. (a)	16. (c)

SOLUTIONS

1. Given : $P + Q + R = 96$ ...(*i*)
I. $P = R + 6$ ...(*ii*)
II. $Q + R = 56$ ...(*iii*)
On subtracting (*iii*) from (*i*), we get $P = 40$.
Putting $P = 40$ in (*ii*), we get $R = 34$. Putting $R = 34$ in (*iii*), we get $Q = 22$.
Thus, I and II both together give the answer. So, correct answer is (*e*).

2. I. $S = 5D \Rightarrow D = \frac{S}{5}$ ...(*i*)
II. $S - 5 = 25\,(D - 5) \Leftrightarrow S = 25D - 120$ ...(*ii*)
Using (*i*) in (*ii*), we get $S = \left(25 \times \frac{S}{5}\right) - 120 \Leftrightarrow 4S = 120 \Leftrightarrow S = 30$.
Thus, I and II both together give the answer. So, correct answer is (*e*).

3. I. 3 years ago, $\frac{1}{2}(A + B) = 18 \Rightarrow$ 3 years ago, $(A + B) = 36$
Now, $(A + B) = (36 + 3 + 3) = 42 \Rightarrow A + B = 42$ ...(*i*)
II. Now, $\frac{1}{3}(A + B + C) = 22 \Rightarrow A + B + C = 66$ ...(*ii*)
From I and II, we get $C = (66 - 42) = 24$.
Thus, I and II both together give the answer. So, correct answer is (*e*).

4. I. Reena's Present age = 5 × (Her son's present age).
II. Reena's age 2 years hence = 3 times her daughter's age at that time.
Clearly, data even in I and II is not sufficient to get Reena's present age.
∴ Correct answer is (*d*).

5. I. $\frac{A}{5} : \frac{B}{4} = 1 : 2 \Leftrightarrow \frac{A}{5} \times \frac{4}{B} = \frac{1}{2} \Leftrightarrow \frac{A}{B} = \left(\frac{1}{2} \times \frac{5}{4}\right) = \frac{5}{8} \Leftrightarrow A : B = 5 : 8$.
II. $20B = AB$.
Let A's age be $5x$ years. Then, B's age is $8x$ years.
∴ $20 \times 8x = 5x \times 8x \Leftrightarrow 40x = 160 \Leftrightarrow x = 4$.
∴ $A = 20$ and $B = 32$.
Thus, I and II together give the answer. So, correct answer is (*e*).

6. I. Retirement age is 60 years.
II. There are 50 employees in the department.
Average age of 50 employees = 30 years.
Total age of 50 employees = (50 × 30) years = 1500 years.
Number of employees next year = 40.
Total age of 40 employees next year = (1500 + 40 − 60 × 10) = 940.
Average age next year = $\frac{940}{40}$ years = $23\frac{1}{2}$ years.
Thus, I and II together give the answer. So, correct answer is (*e*).

7. I. $F + S = 50$ ...(*i*) II. $3(F + S) = 5F$ ...(*ii*)
From II, we get $2F = 3S \Leftrightarrow \frac{F}{S} = \frac{3}{2}$.
Thus, II alone gives the answer, but I alone does not give the answer.
∴ Correct answer is (*b*).

8. Let Divya's present age be D years and Shruti's present age be S years.
Then, $D = 2 \times S \Leftrightarrow D - 2S = 0$ ...(*i*)
I. $\frac{D+5}{S+5} = \frac{9}{5}$ ...(*ii*) II. $\frac{D-10}{S-10} = \frac{3}{1}$ ...(*iii*)
From (*ii*), we get $5D + 25 = 9S + 45 \Leftrightarrow 5D - 9S = 20$ ...(*iv*)
From (*iii*), we get $D - 10 = 3S - 30 \Leftrightarrow D - 3S = -20$ ...(*v*)
Thus from (*i*) and (*ii*), we get the answer.
Also, from (*i*) and (*iii*), we get the answer.
∴ I alone as well as II alone gives the answer. Hence, the correct answer is (*e*).

9. I. $A + B = 21$. II. $A - B = 5$. III. $AB = 104$.
Clearly, any two of three will give the answer. So, correct answer is (*d*).

10. I. Let the present ages of Tanya and Rahul be $3x$ years and $4x$ years.
II. After 5 years, (Tanya's age) : (Rahul's age) = 4 : 5.
III. (Rahul's age) = (Tanya's age) + 5.
From I and II, we get $\frac{3x+5}{4x+5} = \frac{4}{5}$. This gives x.
∴ Tanya's age = $3x$ can be found. Thus, I and II give the answer.
From I and III, we get $4x = 3x + 5$. This gives x.
∴ Tanya's age = $3x$ can be found. Thus, I and III give the answer.
From III : Let Tanya's present age be t years.
Then, Rahul's present age = $(t + 5)$ years.
Thus, **from II and III,** we get : $\frac{t}{t+5} = \frac{4}{5}$. This gives t.
Thus, II and III give the answer.
∴ Correct answer is (*e*).

11. I. $X : Y = 2 : 3 \Rightarrow \frac{X}{Y} = \frac{2}{3} \Rightarrow 3X = 2Y$.
II. $Y = \frac{150}{100}X \Rightarrow Y = \frac{3X}{2} \Rightarrow 3X = 2Y$.
III. $\frac{1}{4}X = \frac{1}{6}Y \Rightarrow 6X = 4Y \Rightarrow 3X = 2Y$.
Thus, even I, II and III together do not give the answer.
∴ Correct answer is (*e*).

12. II. Let the present ages of Arun and his son be $11x$ and $6x$ years respectively.

I. 5 years ago, Arun's age = 2 × His son's age.

III. 5 years hence, $\frac{\text{Arun's age}}{\text{Son's age}} = \frac{12}{7}$.

Clearly, any two of the above will give Arun's present age.

∴ Correct answer is (*d*).

13. I. Let Ravi's present age be x years. Then, his father's present age = $2x$ years.

II. After 5 years, $\frac{\text{Ravi's age}}{\text{Father's age}} = \frac{6}{11}$.

III. Ravi is younger than his brother.

From I and II, we get $\frac{x+5}{2x+5} = \frac{6}{11}$. This gives x, the answer.

Thus, I and II together give the answer. Clearly, III is redundant.

∴ Correct answer is (*a*).

14. I. A + M + F = 62.

II. $(A - 5) = \frac{1}{5}(F - 5)$.

III. $(A - 2) + (F - 2) = 36$.

From II and III, we may get A and F.

Putting these values in I, we get M.

Thus, all I, II and III are required to get the answer.

∴ Correct answer is (*e*).

15. Clearly, any two of the given statements will give the answer and in each case, the third is redundant.

∴ Correct answer is (*a*).

16. Clearly, any two of the given statements will give the answer and in each case, the third is redundant.

∴ Correct answer is (*e*).

9. SURDS AND INDICES

IMPORTANT FACTS AND FORMULAE

1. LAWS OF INDICES :

(i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$ (iii) $(a^m)^n = a^{mn}$

(iv) $(ab)^n = a^n b^n$ (v) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ (vi) $a^0 = 1$

2. SURDS : *Let a be a rational number and n be a positive integer such that $a^{\frac{1}{n}} = \sqrt[n]{a}$ is irrational. Then, $\sqrt[n]{a}$ is called a surd of order n.*

3. LAWS OF SURDS :

(i) $\sqrt[n]{a} = a^{\frac{1}{n}}$ (ii) $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ (iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iv) $(\sqrt[n]{a})^n = a$ (v) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ (vi) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$.

SOLVED EXAMPLES

Ex. 1. *Simplify* : (i) $(27)^{\frac{2}{3}}$ (ii) $(1024)^{-\frac{4}{5}}$ (iii) $\left(\frac{8}{125}\right)^{-\frac{4}{3}}$.

Sol. (i) $(27)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{\left(3 \times \frac{2}{3}\right)} = 3^2 = 9$.

(ii) $(1024)^{-\frac{4}{5}} = (4^5)^{-\frac{4}{5}} = 4^{\left\{5 \times \frac{(-4)}{5}\right\}} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$.

(iii) $\left(\frac{8}{125}\right)^{-\frac{4}{3}} = \left\{\left(\frac{2}{5}\right)^3\right\}^{-\frac{4}{3}} = \left(\frac{2}{5}\right)^{\left\{3 \times \frac{(-4)}{3}\right\}} = \left(\frac{2}{5}\right)^{-4} = \left(\frac{5}{2}\right)^4 = \frac{5^4}{2^4} = \frac{625}{16}$.

Ex. 2. *Evaluate* : (i) $(.00032)^{\frac{3}{5}}$ (ii) $(256)^{0.16} \times (16)^{0.18}$.

Sol. (i) $(0.00032)^{\frac{3}{5}} = \left(\frac{32}{100000}\right)^{\frac{3}{5}} = \left(\frac{2^5}{10^5}\right)^{\frac{3}{5}} = \left\{\left(\frac{2}{10}\right)^5\right\}^{\frac{3}{5}} = \left(\frac{1}{5}\right)^{\left(5 \times \frac{3}{5}\right)} = \left(\frac{1}{5}\right)^3 = \frac{1}{125}$.

(ii) $(256)^{0.16} \times (16)^{0.18} = [(16)^2]^{0.16} \times (16)^{0.18} = (16)^{(2 \times 0.16)} \times (16)^{0.18}$

$= (16)^{0.32} \times (16)^{0.18} = (16)^{(0.32 + 0.18)} = (16)^{0.5} = (16)^{\frac{1}{2}} = 4$.

Ex. 3. ***What is the quotient when $(x^{-1}-1)$ is divided by $(x-1)$?***

Sol. $\dfrac{x^{-1}-1}{x-1} = \dfrac{\frac{1}{x}-1}{x-1} = \dfrac{(1-x)}{x} \times \dfrac{1}{(x-1)} = -\dfrac{1}{x}.$

Hence, the required quotient is $-\dfrac{1}{x}$

Ex. 4. ***If $2^{x-1}+2^{x+1}=1280$, then find the value of x.***

Sol. $2^{x-1}+2^{x+1}=1280 \Leftrightarrow 2^{x-1}(1+2^2)=1280$

$\Leftrightarrow 2^{x-1}=\dfrac{1280}{5}=256=2^8 \Leftrightarrow x-1=8 \Leftrightarrow x=9.$

Hence, $x=9$.

Ex. 5. ***Find the value of*** $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$.

Sol. $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left\{(2^3)^{\frac{1}{3}}+(3^3)^{\frac{1}{3}}\right\}^3\right]^{\frac{1}{4}} = \left[5\left\{2^{\left(3\times\frac{1}{3}\right)}+3^{\left(3\times\frac{1}{3}\right)}\right\}^3\right]^{\frac{1}{4}}$

$= (5(2+3)^3)^{\frac{1}{4}} = (5\times 5^3)^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5^{\left(4\times\frac{1}{4}\right)} = 5^1 = 5.$

Ex. 6. ***Find the value of*** $\left\{(16)^{\frac{3}{2}}+(16)^{-\frac{3}{2}}\right\}$.

Sol. $\left[(16)^{\frac{3}{2}}+(16)^{-\frac{3}{2}}\right] = \left[(4^2)^{\frac{3}{2}}+(4^2)^{-\frac{3}{2}}\right] = 4^{\left(2\times\frac{3}{2}\right)}+4^{\left\{2\times\frac{(-3)}{2}\right\}}$

$= 4^3+4^{-3} = 4^3+\dfrac{1}{4^3} = \left(64+\dfrac{1}{64}\right) = \dfrac{4097}{64}.$

Ex. 7. ***If*** $\left(\dfrac{1}{5}\right)^{3y} = 0.008$, ***then find the value of*** $(0.25)^y$.

Sol. $\left(\dfrac{1}{5}\right)^{3y} = 0.008 = \dfrac{8}{1000} = \dfrac{1}{125} = \left(\dfrac{1}{5}\right)^3 \Leftrightarrow 3y=3 \Leftrightarrow y=1.$

$\therefore (0.25)^y = (0.25)^1 = 0.25.$

Ex. 8. ***Find the value of*** $\dfrac{(243)^{\frac{n}{5}}\cdot 3^{2n+1}}{9^n \times 3^{n-1}}$.

Sol. $\dfrac{(243)^{\frac{n}{5}}\cdot 3^{2n+1}}{9^n\times 3^{n-1}} = \dfrac{(3^5)^{\frac{n}{5}}\times 3^{2n+1}}{(3^2)^n\times 3^{n-1}} = \dfrac{3^{\left(5\times\frac{n}{5}\right)}\times 3^{2n+1}}{3^{2n}\times 3^{n-1}} = \dfrac{3^n\times 3^{2n+1}}{3^{2n}\times 3^{n-1}}$

$= \dfrac{3^{n+(2n+1)}}{3^{2n+n-1}} = \dfrac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+1)-(3n-1)} = 3^2 = 9.$

Ex. 9. ***Find the value of*** $\left(\dfrac{1}{2^4}-1\right)\left(\dfrac{3}{2^4}+2^{\frac{1}{2}}+2^{\frac{1}{4}}+1\right)$ (N.I.F.T. 2003)

Sol. Putting $2^{\frac{1}{4}} = x$, we get :

$$\left(2^{\frac{1}{4}}-1\right)\left(2^{\frac{3}{4}}+2^{\frac{1}{2}}+2^{\frac{1}{4}}+1\right) = (x-1)(x^3+x^2+x+1), \text{ where } x = 2^{\frac{1}{4}}$$

$$= (x-1)[x^2(x+1)+(x+1)]$$

$$= (x-1)(x+1)(x^2+1) = (x^2-1)(x^2+1)$$

$$= (x^4-1) = \left[\left(2^{\frac{1}{4}}\right)^4 - 1\right] = \left[2^{\left(\frac{1}{4}\times 4\right)} - 1\right] = (2-1) = 1.$$

Ex. 10. ***Find the value of*** $\dfrac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$.

Sol. $\dfrac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \dfrac{6^{\frac{2}{3}} \times (6^7)^{\frac{1}{3}}}{(6^6)^{\frac{1}{3}}} = \dfrac{6^{\frac{2}{3}} \times 6^{\left(7\times\frac{1}{3}\right)}}{6^{\left(6\times\frac{1}{3}\right)}} = \dfrac{6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3}\right)}}{6^2}$

$$= 6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3}-2\right)} = 6^{\frac{2}{3}} \times 6^{\frac{1}{3}} = 6^{\left(\frac{2}{3}+\frac{1}{3}\right)} = 6^1 = 6.$$

Ex. 11. ***If*** $x = y^a$, $y = z^b$ ***and*** $z = x^c$, ***then find the value of*** abc.

Sol. $z^1 = x^c = (y^a)^c \quad [\because x = y^a]$

$= y^{(ac)} = (z^b)^{ac} \quad [\because y = z^b]$

$= z^{b(ac)} = z^{abc}$

$\therefore abc = 1.$

Ex. 12. ***Simplify*** : $\left(\dfrac{x^a}{x^b}\right)^{(a^2+b^2+ab)} \times \left(\dfrac{x^b}{x^c}\right)^{(b^2+c^2+bc)} \times \left(\dfrac{x^c}{x^a}\right)^{(c^2+a^2+ca)}$

Sol. Given Expression $= [x^{(a-b)}]^{(a^2+b^2+ab)} \cdot [x^{(b-c)}]^{(b^2+c^2+bc)} \cdot [x^{(c-a)}]^{(c^2+a^2+ca)}$

$= x^{(a-b)(a^2+b^2+ab)} \cdot x^{(b-c)(b^2+c^2+bc)} \cdot x^{(c-a)(c^2+a^2+ca)}$

$= x^{(a^3-b^3)} \cdot x^{(b^3-c^3)} \cdot x^{(c^3-a^3)} = x^{(a^3-b^3+b^3-c^3+c^3-a^3)} = x^0 = 1.$

Ex. 13. ***Which is larger*** $\sqrt{2}$ ***or*** $\sqrt[3]{3}$ ***?***

Sol. Given surds are of order 2 and 3. Their L.C.M. is 6.

Changing each to a surd of order 6, we get :

$$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{3}{3}\right)} = 2^{\frac{3}{6}} = (2^3)^{\frac{1}{6}} = (8)^{\frac{1}{6}} = \sqrt[6]{8}$$

$$\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\left(\frac{1}{3}\times\frac{2}{2}\right)} = 3^{\frac{2}{6}} = (3^2)^{\frac{1}{6}} = (9)^{\frac{1}{6}} = \sqrt[6]{9}.$$

Clearly, $\sqrt[6]{9} > \sqrt[6]{8}$ and hence $\sqrt[3]{3} > \sqrt{2}$.

Ex. 14. ***Find the largest from among*** $\sqrt[4]{6}$, $\sqrt{2}$ ***and*** $\sqrt[3]{4}$.

Sol. Given surds are of order 4, 2 and 3 respectively. Their L.C.M. is 12.

Changing each to a surd of order 12, we get :

$\sqrt[4]{6} = 6^{\frac{1}{4}} = 6^{\left(\frac{1}{4}\times\frac{3}{3}\right)} = \left(6^{\frac{3}{12}}\right) = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}$.

$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{6}{6}\right)} = \left(2^{\frac{6}{12}}\right) = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$.

$\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\left(\frac{1}{3}\times\frac{4}{4}\right)} = \left(4^{\frac{4}{12}}\right) = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$.

Clearly, $(256)^{\frac{1}{12}} > (216)^{\frac{1}{12}} > (64)^{\frac{1}{12}}$.

$\therefore$ Largest one is $(256)^{\frac{1}{12}}$ *i.e.*, $\sqrt[3]{4}$.

EXERCISE 9

Directions : ***Mark (✓) against the correct answer :***

1. The value of $(256)^{\frac{5}{4}}$ is :
   (*a*) 512 (*b*) 984 (*c*) 1024 (*d*) 1032

2. The value of $(\sqrt{8})^{\frac{1}{3}}$ is :
   (*a*) 2 (*b*) 4 (*c*) $\sqrt{2}$ (*d*) 8

3. The value of $\left(\frac{32}{243}\right)^{-\frac{4}{5}}$ is :
   (*a*) $\frac{4}{9}$ (*b*) $\frac{9}{4}$ (*c*) $\frac{16}{81}$ (*d*) $\frac{81}{16}$

4. The value of $\left(-\frac{1}{216}\right)^{-\frac{2}{3}}$ is :
   (*a*) 36 (*b*) – 36 (*c*) $\frac{1}{36}$ (*d*) $-\frac{1}{36}$

5. The value of $5^{\frac{1}{4}} \times (125)^{0.25}$ is :
   (*a*) $\sqrt{5}$ (*b*) 5 (*c*) $5\sqrt{5}$ (*d*) 25

6. The value of $\frac{1}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{1}{(32)^{-\frac{1}{5}}}$ is : **(M.B.A. 2003)**
   (*a*) 102 (*b*) 105 (*c*) 107 (*d*) 109

7. The value of $[(10)^{150} \div (10)^{146}]$ is : **(Bank P.O. 2002)**
   (*a*) 1000 (*b*) 10000 (*c*) 100000 (*d*) 10^6

8. $(2.4 \times 10^3) \div (8 \times 10^{-2}) = ?$
   (*a*) 3×10^{-5} (*b*) 3×10^4 (*c*) 3×10^5 (*d*) 30

9. $\left(\frac{1}{216}\right)^{-\frac{2}{3}} + \left(\frac{1}{27}\right)^{-\frac{4}{3}} = ?$
   (*a*) $\frac{3}{4}$ (*b*) $\frac{2}{3}$ (*c*) $\frac{4}{9}$ (*d*) $\frac{1}{8}$

10. $(1000)^7 \div 10^{18} = ?$ **(Bank P.O. 2003)**

(a) 10 (b) 100 (c) 1000 (d) 10000

11. $(256)^{0.16} \times (256)^{0.09} = ?$ **(S.S.C. 2004)**

(a) 4 (b) 16 (c) 64 (d) 256.25

12. $(0.04)^{-1.5} = ?$ **(Bank P.O. 2003)**

(a) 25 (b) 125 (c) 250 (d) 625

13. $(17)^{3.5} \times (17)^{?} = 17^8$ **(Bank P.O. 2003)**

(a) 2.29 (b) 2.75 (c) 4.25 (d) 4.5

14. $49 \times 49 \times 49 \times 49 = 7^?$

(a) 4 (b) 7 (c) 8 (d) 16

15. The value of $(8^{-25} - 8^{-26})$ is

(a) 7×8^{-25} (b) 7×8^{-26} (c) 8×8^{-26} (d) None of these

16. $(64)^{-\frac{1}{2}} - (-32)^{-\frac{4}{5}} = ?$ **(Bank P.O. 2002)**

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{16}$ (d) $\frac{3}{16}$ (e) None of these

17. $(18)^{3.5} \div (27)^{3.5} \times 6^{3.5} = 2^?$: **(Bank P.O. 2003)**

(a) 3.5 (b) 4.5 (c) 6 (d) 7 (e) None of these

18. $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^?$ **(Bank P.O. 2003)**

(a) 8.5 (b) 13 (c) 16 (d) 17.5 (e) None of these

19. The value of $\frac{(243)^{0.13} \times (243)^{0.07}}{(7)^{0.25} \times (49)^{0.075} \times (343)^{0.2}}$ is : **(C.B.I. 2003)**

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) $1\frac{3}{7}$ (d) $2\frac{2}{7}$

20. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is : **(M.B.A. 2003)**

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{7}{2}$

21. If $2^{2n-1} = \frac{1}{8^{n-3}}$, then the value of n is :

(a) 3 (b) 2 (c) 0 (d) -2

22. If $5^a = 3125$, then the value of $5^{(a-3)}$ is :

(a) 25 (b) 125 (c) 625 (d) 1625

23. If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$, then the value of a is :

(a) 4 (b) 5 (c) 6 (d) 8

24. If $\sqrt{2^n} = 64$, then the value of n is :

(a) 2 (b) 4 (c) 6 (d) 12

25. If $(\sqrt{3})^5 \times 9^2 = 3^n \times 3\sqrt{3}$, then the value of n is :

(a) 2 (b) 3 (c) 4 (d) 5

26. If $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, then the value of n is :

(a) 0 (b) 2 (c) 3 (d) 4

27. If $2^{n+4} - 2^{n+2} = 3$, then n is equal to :
(a) 0 (b) 2 (c) -1 (d) -2

28. If $2^{n-1} + 2^{n+1} = 320$, then n is equal to :
(a) 6 (b) 8 (c) 5 (d) 7

29. If $3^x - 3^{x-1} = 18$, then the value of x^x is :
(a) 3 (b) 8 (c) 27 (d) 216

30. $\dfrac{2^{n+4} - 2 \times 2^n}{2 \times 2^{(n+3)}} + 2^{-3}$ is equal to :
(a) 2^{n+1} (b) $\left(\dfrac{9}{8} - 2^n\right)$ (c) $\left(-2^{n+1} + \dfrac{1}{8}\right)$ (d) 1

31. If $x = 3 + 2\sqrt{2}$, then the value of $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)$ is : (C.B.I. 2003)
(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) $3\sqrt{3}$

32. Given that $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$, then the value of z is close to :
(a) 1.45 (b) 1.88 (c) 2.9 (d) 3.7
(C.B.I. 2003)

33. If m and n are whole numbers such that $m^n = 121$, then the value of $(m-1)^{n+1}$ is : (S.S.C. 2001)
(a) 1 (b) 10 (c) 121 (d) 1000

34. $\dfrac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}} = ?$ (S.S.C. 2004)
(a) 1 (b) 3 (c) 9 (d) 3^n

35. Number of prime factors in $(216)^{\frac{3}{5}} \times (2500)^{\frac{2}{5}} \times (300)^{\frac{1}{5}}$ is :
(a) 6 (b) 7 (c) 8 (d) None of these

36. Number of prime factors in $\dfrac{6^{12} \times (35)^{28} \times (15)^{16}}{(14)^{12} \times (21)^{11}}$ is :
(a) 56 (b) 66 (c) 112 (d) None of these

37. $\dfrac{1}{1 + a^{(n-m)}} + \dfrac{1}{1 + a^{(m-n)}} = ?$ (M.B.A. 2003)
(a) 0 (b) $\dfrac{1}{2}$ (c) 1 (d) a^{m+n}

38. $\dfrac{1}{1 + x^{(b-a)} + x^{(c-a)}} + \dfrac{1}{1 + x^{(a-b)} + x^{(c-b)}} + \dfrac{1}{1 + x^{(b-c)} + x^{(a-c)}} = ?$ (M.B.A. 2003)
(a) 0 (b) 1 (c) x^{a-b-c} (d) None of these

39. $\left(\dfrac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\dfrac{x^c}{x^a}\right)^{(c+a-b)} \cdot \left(\dfrac{x^a}{x^b}\right)^{(a+b-c)} = ?$ (L.I.C. 2003)
(a) x^{abc} (b) 1 (c) $x^{ab+bc+ca}$ (d) x^{a+b+c}

40. $\left(\dfrac{x^a}{x^b}\right)^{(a+b)} \cdot \left(\dfrac{x^b}{x^c}\right)^{(b+c)} \cdot \left(\dfrac{x^c}{x^a}\right)^{(c+a)} = ?$
(a) 0 (b) x^{abc} (c) x^{a+b+c} (d) 1

41. $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = ?$

(a) 1 (b) $x^{\frac{1}{abc}}$ (c) $x^{\frac{1}{(ab+bc+ca)}}$ (d) None of these

42. If $abc = 1$, then $\left(\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}\right) = ?$

(a) 0 (b) 1 (c) $\frac{1}{ab}$ (d) ab

43. If a, b, c are real numbers, then the value of $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a}$ is :

(a) abc (b) $\sqrt{abc}$ (c) $\frac{1}{abc}$ (d) 1

44. If $3^{(x-y)} = 27$ and $3^{(x+y)} = 243$, then x is equal to : (R.R.B. 2003)

(a) 0 (b) 2 (c) 4 (d) 6

45. If $\left(\frac{9}{4}\right)^x \cdot \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$, then the value of x is :

(a) 1 (b) 2 (c) 3 (d) 4

46. If $2^x = \sqrt[3]{32}$, then x is equal to :

(a) 5 (b) 3 (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

47. If $2^x \times 8^{\frac{1}{5}} = 2^{\frac{1}{5}}$, then x is equal to :

(a) $\frac{1}{5}$ (b) $-\frac{1}{5}$ (c) $\frac{2}{5}$ (d) $-\frac{2}{5}$

48. If $5^{(x+3)} = (25)^{(3x-4)}$, then the value of x is :

(a) $\frac{5}{11}$ (b) $\frac{11}{5}$ (c) $\frac{11}{3}$ (d) $\frac{13}{5}$

49. If $a^x = b^y = c^z$ and $b^2 = ac$, then y equals :

(a) $\frac{xz}{x+z}$ (b) $\frac{xz}{2(x-z)}$ (c) $\frac{xz}{2(z-x)}$ (d) $\frac{2xz}{(x+z)}$

50. If $2^x = 3^y = 6^{-z}$, then $\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$ is equal to :

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

51. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of xyz is :

(a) 0 (b) 1 (c) $\frac{1}{abc}$ (d) abc

52. If $2^x = 4^y = 8^z$ and $\left(\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z}\right) = \frac{24}{7}$, then the value of z is :

(a) $\frac{7}{16}$ (b) $\frac{7}{32}$ (c) $\frac{7}{48}$ (d) $\frac{7}{64}$

53. The largest number from among $\sqrt{2}$, $\sqrt[3]{3}$ and $\sqrt[4]{4}$ is :

(a) $\sqrt{2}$ (b) $\sqrt[3]{3}$ (c) $\sqrt[4]{4}$ (d) All are equal

54. If $x = 5 + 2\sqrt{6}$, then $\frac{(x-1)}{\sqrt{x}}$ is equal to :

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

ANSWERS

1. (c)	2. (c)	3. (d)	4. (a)	5. (b)	6. (a)	7. (b)	8. (b)	9. (c)
10. (c)	11. (a)	12. (b)	13. (d)	14. (c)	15. (b)	16. (c)	17. (d)	18. (b)
19. (a)	20. (c)	21. (b)	22. (a)	23. (a)	24. (d)	25. (d)	26. (c)	27. (d)
28. (d)	29. (c)	30. (d)	31. (b)	32. (c)	33. (d)	34. (c)	35. (b)	36. (b)
37. (c)	38. (b)	39. (b)	40. (d)	41. (a)	42. (b)	43. (d)	44. (c)	45. (d)
46. (d)	47. (d)	48. (b)	49. (d)	50. (a)	51. (b)	52. (c)	53. (b)	54. (b)

SOLUTIONS

1. $(256)^{\frac{5}{4}} = (4^4)^{\frac{5}{4}} = 4^{\left(4\times\frac{5}{4}\right)} = 4^5 = 1024.$

2. $(\sqrt{8})^{\frac{1}{3}} = \left(8^{\frac{1}{2}}\right)^{\frac{1}{3}} = 8^{\left(\frac{1}{2}\times\frac{1}{3}\right)} = 8^{\frac{1}{6}} = (2^3)^{\frac{1}{6}} = 2^{\left(3\times\frac{1}{6}\right)} = 2^{\frac{1}{2}} = \sqrt{2}.$

3. $\left(\frac{32}{243}\right)^{-\frac{4}{5}} = \left\{\left(\frac{2}{3}\right)^5\right\}^{-\frac{4}{5}} = \left(\frac{2}{3}\right)^{5\times\frac{(-4)}{5}} = \left(\frac{2}{3}\right)^{(-4)} = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}.$

4. $\left(-\frac{1}{216}\right)^{-\frac{2}{3}} = \left[\left(-\frac{1}{6}\right)^3\right]^{-\frac{2}{3}} = \left(-\frac{1}{6}\right)^{3\times\frac{(-2)}{3}} = \left(-\frac{1}{6}\right)^{-2} = \frac{1}{\left(-\frac{1}{6}\right)^2} = \frac{1}{\left(\frac{1}{36}\right)} = 36.$

5. $5^{\frac{1}{4}} \times (125)^{0.25} = 5^{0.25} \times (5^3)^{0.25} = 5^{0.25} \times 5^{(3\times 0.25)} = 5^{0.25} \times 5^{0.75} = 5^{(0.25+0.75)} = 5^1 = 5.$

6. $\frac{1}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{1}{(32)^{-\frac{1}{5}}} = \frac{1}{(6^3)^{-\frac{2}{3}}} + \frac{1}{(4^4)^{\left(-\frac{3}{4}\right)}} + \frac{1}{(2^5)^{-\frac{1}{5}}}$

$= \frac{1}{6^{3\times\frac{(-2)}{3}}} + \frac{1}{4^{4\times\frac{(-3)}{4}}} + \frac{1}{2^{5\times\frac{(-1)}{5}}} = \frac{1}{6^{-2}} + \frac{1}{4^{-3}} + \frac{1}{2^{-1}}$

$= (6^2 + 4^3 + 2^1) = (36 + 64 + 2) = 102.$

7. $(10)^{150} \div (10)^{146} = \frac{(10)^{150}}{(10)^{146}} = (10)^{(150-146)} = 10^4 = 10000.$

8. $(2.4 \times 10^3) \div (8 \times 10^{-2}) = \frac{2.4 \times 10^3}{8 \times 10^{-2}} = \frac{24 \times 10^2}{8 \times 10^{-2}} = (3 \times 10^4).$

9. $\left(\frac{1}{216}\right)^{-\frac{2}{3}} + \left(\frac{1}{27}\right)^{-\frac{4}{3}} = (216)^{\frac{2}{3}} + (27)^{\frac{4}{3}} = \frac{(216)^{\frac{2}{3}}}{(27)^{\frac{4}{3}}} = \frac{(6^3)^{\frac{2}{3}}}{(3^3)^{\frac{4}{3}}} = \frac{6^{\left(3\times\frac{2}{3}\right)}}{3^{\left(3\times\frac{4}{3}\right)}} = \frac{6^2}{3^4} = \frac{36}{81} = \frac{4}{9}.$