15 COMPOUND INTEREST, GROWTH AND DEPRECIATION

15.1 INTRODUCTION

As discussed in the previous chapter on 'Simple Interest' the principal (P) remains constant throughout the period for which the money (principal) is borrowed. But, when the borrower fails to pay the principal as it falls due, the interest for the first year (conversion period) is added to the original principal at the end of first year (conversion period for charging the interest) and this sum (P + 1st year interest on P) becomes the principal for the second year and so on.

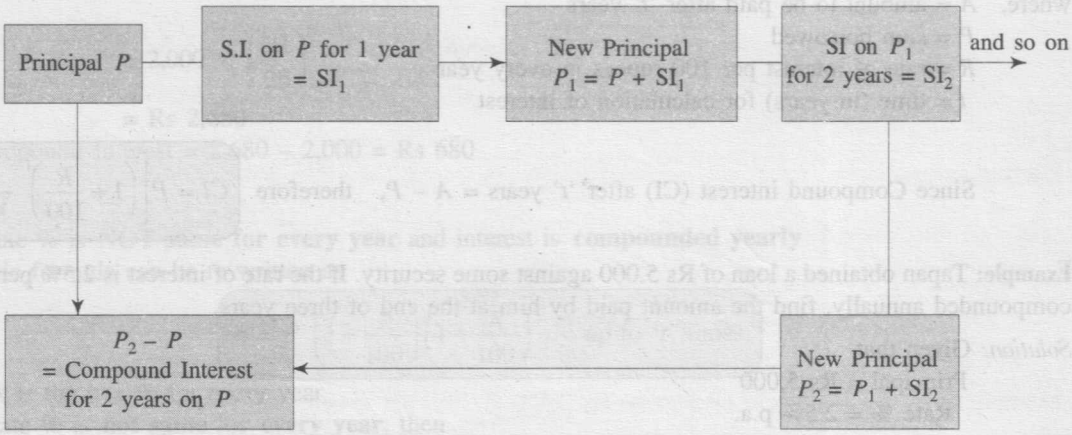

Fig. 15.1 Line Diagram

Hence for every changing year, the principal goes on changing and accordingly the amount of interest accrued on varying principal will be different in every year. The money lent under this condition is charged with Compound Interest.

Since, principal increases after every year (reckoning period), the amount of interest in Compound Interest is always more than Simple Interest.

15.2 CONVERSION PERIOD

In some institutions, Interest is compounded, i.e. calculated and added to the Principal half-yearly, i.e. two times a year.

The period at the end of which the Interest is compounded is called the **Conversion Period**. When interest is calculated and added to the principal after three months, the conversion period is three months or **Quarterly**. Hence, number of conversions per year is 4 (Four). When the conversion period is six months, the number of conversions per year is **Two** (2).

when the interest is compounded yearly, no. of conversions per year (n) = 1
when the interest is compounded half-yearly, n = 2
when the interest is compounded quarterly, n = 4
when the interest is compounded monthly, n = 12

While solving the problems on Compound Interest, it is assumed that interest is compounded yearly, unless otherwise specified. If other parameters remain same, then Compound Interest (CI) increases as the value of 'n' increases.

15.3 BASIC FORMULA

Generally, the interest is compounded yearly. In such case, the basic formula relating *A, P, R and t* is given as:

$$A = P\left(1+\frac{R}{100}\right)^t$$

where, A = amount to be paid after 't' years
P = sum borrowed
R = rate of interest per 100 rupees in every year
t = time (in years) for calculation of interest

Since Compound interest (CI) after 't' years = $A - P$, therefore $$CI = P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$

Example: Tapan obtained a loan of Rs 5,000 against some security. If the rate of interest is 2.5% per annum compounded annually, find the amount paid by him at the end of three years.

Solution: Given that
Principal = Rs 5,000
Rate % = 2.5% p.a.
Time = 3 years

$$\text{Amount} = A = P\left(1+\frac{R}{100}\right)^t$$

$$= 5{,}000\left(1+\frac{2.5}{100}\right)^3$$

$$= 5{,}000 \times 1.0768$$

$$= 5{,}384.45$$

$\therefore$ Amount paid by Tapan at the end of 3 years = Rs 5,384.45

15.4 SPECIAL CASES

Case I

When interest is NOT **compounded yearly**
Amount is given by

$$A = P\left(1+\frac{R}{n\times 100}\right)^{nt}$$

where, n = number of conversions (or compounding) per year, Refer 15.2.

Example: Compute the compound interest on Rs 2,000 for 3 years at 10% per annum, when compounded half-yearly.

Solution: Here, Principal (P) = Rs 2,000
rate (R) = 10% per annum
time (t) = 3 years
no. of conversions per year (n) = 2, since interest is compounded half-yearly.
Using the relation

$$A = P\left(1+\frac{R}{n\times 100}\right)^{nt}$$

$$\Rightarrow \quad A = 2{,}000\left(1+\frac{10}{2\times 100}\right)^{2\times 3}$$

$$= 2{,}000\times\left(\frac{21}{20}\right)^{6}$$

$$= \text{Rs } 2{,}680$$

$\therefore$ Compound Interest = 2,680 – 2,000 = Rs 680

Case II

When rate % is **NOT same for every year** and interest is **compounded yearly**
The basic formula can be re-written as

$$A = P\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right)\ldots \text{ up to '}t\text{' times,}$$

where R is the rate % for every year
but, if rate % is **not same for every year**, then

$$A = P\left(1+\frac{R_1}{100}\right)^{t_1}\times\left(1+\frac{R_2}{100}\right)^{t_2}\ldots \text{ and so on.}$$

where, R_1 = Rate % p.a. for t_1 years
R_2 = Rate % p.a. for t_2 years
and so on

Example: Find the amount of Rs 4,000 for 5 years compounded annually, the rate of interest being 10% for the first 3 years and 20% for the next 2 years.

Solution: Here, principal = Rs 4,000

rate % p.a. for 3 years = 10% = R_1 (say)
then t_1 = 3 years
rate % p.a. for next 2 years = 20% = R_2 (say)
then t_2 = 2 years

Then, amount after $(t_1 + t_2)$ years is given by

$$A = P\left(1+\frac{R_1}{100}\right)^{t_1} \times \left(1+\frac{R_2}{100}\right)^{t_2}$$

$$\Rightarrow \text{amount after 5 years} = 4{,}000\left(1+\frac{10}{100}\right)^3 \times \left(1+\frac{20}{100}\right)^2$$

$$= 4{,}000 \times \left(\frac{11}{10}\right)^3 \times \left(\frac{6}{5}\right)^2$$

$$= 7{,}667$$

$\therefore$ amount after 5 years = **Rs 7,667**.

Case III

When interest is compounded yearly but **time is a fraction**

Consider, time = $5\frac{3}{4}$ years

In this case, whole part of time → fraction part of time

$$\text{Amount } A = P\left(1+\frac{R}{100}\right)^5 \times \left(1+\frac{\frac{3}{4}\times R}{100}\right)$$

Similarly,

$$\text{amount for } 2\frac{1}{6} \text{ years} = P\left(1+\frac{R}{100}\right)^2 \times \left(1+\frac{\frac{1}{6}\times R}{100}\right)$$

Example: Find the compound interest on Rs 2,400 at 20% per annum for $3\frac{3}{4}$ years.

Solution: $$\text{Amount} = 2{,}400 \times \left(1+\frac{20}{100}\right)^3 \times \left(1+\frac{\frac{3}{4}\times 20}{100}\right)$$

$$= 2{,}400 \times \left(1+\frac{1}{5}\right)^3\left(1+\frac{3}{20}\right)$$

$$= 4{,}769.28$$

$\therefore$ Compound Interest = 4,769.28 – 2,400
= 2,369.28
$\therefore$ Compound interest is **Rs 2,369.28**.

15.5 TO FIND THE PRINCIPAL/TIME/RATE

The basic formula,

$$A = P\left(1+\frac{R}{100}\right)^t, \text{ has four unknown quantities, } A, P, R \text{ and } t$$

Similarly, Compound Interest C.I. $= A - P$

$$= P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$

has also four unknown quantities C.I. P, R and t.
In both these formulae, if **three** out of those **four** quantities are given, then we can find out the remaining fourth quantity by simple calculation. In the following examples, we shall discuss how to calculate each of other three quantities i.e. P, R and t, when sufficient data is given.

Example: Suman borrowed a certain sum at the rate of 15% p.a. If he paid at the end of two years Rs 1,290 as interest compounded annually, find the sum he borrowed.

Solution: Using the basic formula,

$$\text{Compound Interest CI} = P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$

$$\Rightarrow \quad 1{,}290 = P\left[\left(1+\frac{15}{100}\right)^2 - 1\right]$$

$$\Rightarrow \quad 1{,}290 = P\,(0.3225)$$

$$\Rightarrow \quad P = 4{,}000$$

$\therefore$ the sum borrowed by Suman is Rs **4,000**.

Example: In what time Rs 2,400 will amount to Rs 2,646 at 5% p.a. compounded annually?

Solution: Using the basic formula,

$$\text{Amount} \quad A = P\left(1+\frac{R}{100}\right)^t$$

$$\Rightarrow \quad 2{,}646 = 2{,}400\left(1+\frac{5}{100}\right)^t$$

$$\Rightarrow \quad \frac{2{,}646}{2{,}400} = \left(1+\frac{1}{20}\right)^t$$

$$\Rightarrow \quad \frac{441}{400} = \left(\frac{21}{20}\right)^t$$

$$\Rightarrow \quad \left(\frac{21}{20}\right)^2 = \left(\frac{21}{20}\right)^t$$

$$\Rightarrow \quad t = 2$$

$\therefore$ time is **2 years.**

Example: At what rate per cent compound interest per annum will Rs 2,000 amount to Rs 2,661 in three years?

Solution: Using the basic formula,

$$A = P\left(1+\frac{R}{100}\right)^t$$

$$\Rightarrow \quad 2{,}661 = 2{,}000\left(1+\frac{R}{100}\right)^3$$

$$\Rightarrow \quad \frac{2{,}661}{2{,}000} = \left(1+\frac{R}{100}\right)^3$$

$$\Rightarrow \quad \left(\frac{11}{10}\right)^3 = \left(1+\frac{R}{100}\right)^3 = \frac{11}{10} = 1+\frac{R}{100}$$

$$\Rightarrow \quad \frac{R}{100} = \frac{11}{10} - 1 = \frac{1}{10}$$

$$\Rightarrow \quad R = \frac{100}{10} = 10$$

$\therefore$ rate = **10% per annum.**

15.6 DIFFERENCE BETWEEN COMPOUND INTEREST AND SIMPLE INTEREST

Let, the principal = P

time = T years

and rate % p.a. = R

then,

Simple interest = S.I. = $\frac{PRT}{100}$

If the interest is compounded annually, then

Compound interest = C.I. = $P\left[\left(1+\frac{R}{100}\right)^T - 1\right]$

Required difference between C.I. and S.I.

= C.I. – S.I.

$$= P\left[\left(1+\frac{R}{100}\right)^T - 1 - \frac{RT}{100}\right]$$

Example: The difference between the compound interest and the simple interest on a certain sum at 15% per annum for 3 years is Rs 283.50. Find the sum.

Solution: Assume principal = P, then

$$\text{Compound interest} = \text{C.I.} = P\left[\left(1+\frac{R}{100}\right)^T - 1\right]$$

$$= P\left[\left(1+\frac{15}{100}\right)^T - 1\right]$$

$$= P\left[\left(\frac{23}{20}\right)^3 - 1\right]$$

$$= P\left[\frac{12,167}{8,000} - 1\right]$$

$$= P\left(\frac{4,167}{8,000}\right)$$

$$\text{Simple interest} = \text{S.I.} = \frac{PRT}{100} = \frac{P\times 15\times 3}{100} = P\left(\frac{9}{20}\right)$$

$$\therefore \quad \text{C.I.} - \text{S.I.} = P\left[\frac{4,167}{8,000} - \frac{9}{20}\right] = 283.50 \quad \text{(given)}$$

$$\Rightarrow \quad P\left(\frac{4,167-3,600}{8,000}\right) = \frac{567}{2}$$

$$\Rightarrow \quad P = 4,000$$

$\therefore$ The sum is **Rs 4,000**.

Short-Cut Formula for C.I. – S.I.

Sometimes, we may use the following formula directly:

When time = T = 2 years

$$\text{C.I.} - \text{S.I.} = P\left(\frac{R}{100}\right)^2$$

and **if time = T = 3 years**

$$\text{C.I.} - \text{S.I.} = P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right]$$

15.7 EQUAL ANNUAL INSTALMENT TO PAY THE DEBT (BORROWED) AMOUNT

Let the borrowed amount = Rs B

rate % per annum = R,

amount of each instalment = Rs a
and time = t years

Then, $a\left[\left(\frac{100}{100+R}\right)+\left(\frac{100}{100+R}\right)^2+\ldots+\left(\frac{100}{100+R}\right)^t\right]$ = Borrowed amount B

15.8 GROWTH

In our day-to-day life we observe that there are some entities such as population of a city, the value of property, the height of a tree, weight and height of a child, the number of bacteria, etc, which increase in magnitude over a period of time. This relative increase in quantity is called growth.
Growth per unit of time is called the rate of growth.
In this section, we shall illustrate how the formulae for computing amount and compound interest are directly used to calculate the growth of population, height of a tree, number of bacteria and all such entities.

15.8.1 Population Growth

Let P be the population at the beginning of a certain year.

(i) If the constant rate of growth be $R\%$ per annum (i.e. R persons per 100 persons per year), then

$$\text{Population after '}t\text{' years} = P\left(1+\frac{R}{100}\right)^t$$

which is exactly similar to finding the amount after 't' years *[Refer 15.3]*

$$\text{Net increase in population during '}t\text{' years} = P\left[\left(1+\frac{R}{100}\right)^t-1\right] \quad (\text{Since, } CI = A - P)$$

which is exactly similar to finding the compound interest after 't' years *[Refer 15.3]*

(ii) If the rate of growth be $R_1\%$ p.a. during first 't_1' years and $R_2\%$ p.a. during next t_2 years, then

$$\text{population after } (t_1 + t_2) \text{ years} = P\left(1+\frac{R_1}{100}\right)^{t_1}\times\left(1+\frac{R_2}{100}\right)^{t_2}$$

which is exactly similar case to finding the amount after $(t_1 + t_2)$ years when rate % is not same for every year *[Refer Case II, 15.4]*

(iii) If the constant rate of decrease be $R\%$ per annum, then

$$\text{population after } t \text{ years} = P\left(1-\frac{R}{100}\right)^t$$

which is obtained by simply putting $-R$ in place of R in the basic formula. because there is decrease in rate per cent.

15.8.2 Growth of Bacteria, Height of Tree, Production of a Factory

To find the bacteria count, the height of a tree, or production of a certain commodity and such other entities, we use similar formulae as detailed in 15.8.1

Example: The present population of a town is 25,000. If it increases at the rate of 5% per annum, what will be its population after 2 years?

Solution: Using the relation, (15.8.1, i)

$$\text{population after '}t\text{' years} = P\left(1+\frac{R}{100}\right)^t$$

$$\Rightarrow \quad \text{population after 2 years} = 25{,}000\left(1+\frac{5}{100}\right)^2$$

$$= 25{,}000\times\left(\frac{21}{20}\right)^2$$

$$= 27{,}563$$

$\therefore$ population after 2 years = **27,563**.

Example: The bacteria in a culture grows by 10% in first two hours, decreases by 10% in next one hour and again increases by 5% in next two hours. If the original count of the bacteria in the sample is 40,000, find the bacteria count at the end of 5 hours.

Solution: Using the relation (15.8.1, ii), for rates of growth being different,

$$\text{bacteria count after 5 hours} = P\left(1+\frac{R_1}{100}\right)^{t_1}\times\left(1+\frac{R_2}{100}\right)^{t_2}\times\left(1+\frac{R_3}{100}\right)^{t_3}$$

where, $P = 40{,}000$

$R_1 = 10$ $\quad R_2 = -10$ [(–)ve, for decrease], $\quad R_3 = 5$

$t_1 = 2$ $\quad t_2 = 1$ $\quad t_3 = 2$

$$\Rightarrow \quad \text{bacteria count after 5 hours} = 40{,}000\left(1+\frac{10}{100}\right)^2\times\left(1-\frac{10}{100}\right)\times\left(1+\frac{5}{100}\right)^2$$

$$= 40{,}000\times\frac{11}{10}\times\frac{11}{10}\times\frac{9}{10}\times\frac{21}{20}\times\frac{21}{20}$$

$$= 48{,}025$$

$\therefore$ bacteria count after 5 hours = **48,025**.

15.9 DEPRECIATION

It is a well-known fact that the constant use of any machine or any other article causes wear and tear due to which its value decreases with time. The relative decrease in the value of a machine over a period of time is called its depreciation.

Depreciation per unit time is called the **rate of depreciation**.

The value at any time is called the **depreciated value**.

Let V be the value of a machine at a certain time.

(i) If the constant rate of depreciation is $R\%$ per annum, then

$$\text{the value of the machine after '}t\text{' years} = V\left(1-\frac{R}{100}\right)^t$$

(–) R as value decreases with time. Otherwise this formula is similar to basic formula of finding the amount on compound interest after 't' years. *[Refer 15.3]*

(ii) If the rate of depreciation is $R_1\%$ for first 't_1' years, $R_2\%$ for next 't_2' years, and so on, then

$$\text{the value at the end of } (t_1 + t_2 + \ldots) \text{ years} = V\left(1-\frac{R_1}{100}\right)^{t_1} \times \left(1-\frac{R_2}{100}\right)^{t_2} \times \ldots$$

which is exactly similar to the case of finding the compounded amount after $(t_1 + t_2 + \ldots)$ years when rate % is not same for every year. *[Refer Case II, 15.4]*

Example: The present cost of a new car is Rs 2,30,000. If its value depreciates at the rate of 20% per annum, what will be its value after 3 years hence? Also find the total depreciation.

Solution: Using the relation (15.9, i)

$$\text{Value of machine after '}t\text{' years} = V\left(1-\frac{R}{100}\right)^t$$

$$\Rightarrow \quad \text{Value of car after 3 years} = 2{,}30{,}000\left(1-\frac{20}{100}\right)^3$$

$$= 2{,}30{,}000 \times \left(\frac{4}{5}\right)^3$$

$$= \text{Rs } 1{,}17{,}760 \qquad \text{(Depreciated value)}$$

$\therefore$ Total depreciation = Rs (2,30,000 – 1,17,760) = Rs 1,12,240

Note: If a certain amount becomes N times in T years,
then same amount becomes N^2 times in $T \times 2$ years
same amount becomes N^3 times in $T \times 3$ years
same amount becomes N^x in $T \times x$ years

Solved Examples

E-1 Find the amount on Rs 4,000 for 2 years at 5% per annum compound interest.

S-1 Using the formula, [Refer 15.3],

$$A = P\left[1+\frac{R}{100}\right]^t$$

Here, P = Rs 4,000; R = 5% p.a.
t = 2 years;

$$A = 4{,}000\left[1+\frac{5}{100}\right]^2 = 4{,}000 \times 1.05 \times 1.05 = 4{,}410.$$

$\therefore$ The amount is **Rs 4,410**.

E-2 What sum of money lent at compound interest will amount to Rs 968 in 2 years at 10% p.a., interest being charged annually? **(ITI '81)**

S-2 Since interest is charged annually,

We have $A = P\left[1+\frac{R}{100}\right]^t$ *[Refer 15.3]*

$$968 = P\left[1+\frac{10}{100}\right]^2 \Rightarrow P = \frac{968}{11\times 11} \times 10 \times 10 = \text{Rs } 800.$$

$\therefore$ The sum lent out is **Rs 800**.

E-3 Vijay obtains a loan of Rs 64,000 against his fixed deposits. If the rate of interest be 2.5 paise per rupee per annum, calculate the compound interest payable after 3 years.

S-3 [Refer 15.3], using the formula,

$$\text{C.I} = P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$ (Since interest is compounded yearly)

P = Rs 64,000,

R = 2.5 paise per rupee per annum (given)

= 0.025 rupee per rupee per annum.

= 0.025 $\times$ 100 rupee per hundred rupee per annum.

= 0.025 $\times$ 100 per cent per annum = 2.5 per cent per annum.

t = 3 years

$$\text{C.I} = 64{,}000\left[\left(\frac{1+2.5}{100}\right)^2 - 1\right]$$

$$= 64{,}000\ [(1 + 0.025)^2 - 1] = \text{Rs } 4{,}921.$$

$\therefore$ The compound interest payable is **Rs 4,921**.

NB: Remember that x paise per rupee per annum = Rs x per cent per annum.

E-4 Find the compound interest on Rs 2,000 for 9 months at 8% per annum being given when the interest is reckoned

(i) Quarterly

(ii) Half-yearly

S-4 Using the formula, [Refer Case I, 15.4],

$$\text{C.I} = P\left[\left(1+\frac{R}{100\times n}\right)^{n\times t} - 1\right]$$

(i) P = 2,000, R = 8% p.a., t = 9 months = $\frac{9}{12}$ year

$$\text{C.I} = 2{,}000\left[\left(1+\frac{8}{100\times 4}\right)^{4\times\frac{9}{12}} - 1\right]$$ (n = 4) *[Refer 15.2]*

$$= 2{,}000\left[\left(\frac{102}{100}\right)^3 - 1\right] = \text{Rs } 122.$$

$\therefore$ The compound interest is **Rs 122**.

(ii) $n = 2$, (for interest is reckoned half-yearly.)

$$\text{C.I} = 2{,}000\left[\left(1+\frac{8}{100\times 2}\right)^{2\times\frac{9}{12}}-1\right] = 2{,}000\left[\left(1+\frac{4}{100}\right)^{\frac{3}{2}}-1\right]$$

$$= 2{,}000\left[\left(1+\frac{4}{100}\right)\left(1+\frac{4\times\frac{1}{2}}{100}\right)-1\right] = 2{,}000\left[\frac{104\times 102}{100\times 100}-1\right]$$

$= \text{Rs } 121.60$. $\therefore$ The compound interest is **Rs 121.60**.

E-5 The difference between simple and compound interest on a certain sum of money for 3 years at 10% p.a. is Rs 15 paise 50. Find the sum.

S-5 Using the formula, for 3 years, we get *[Refer short-cut formula in 15.6]*

$$\text{C.I} - \text{S.I} = P\left[\left(\frac{R}{100}\right)^3+3\left(\frac{R}{100}\right)^2\right] \Rightarrow 15\frac{1}{2} = P\left[\left(\frac{10}{100}\right)^3+3\left(\frac{10}{100}\right)^2\right]$$

$$\Rightarrow \frac{31}{2} = P\left[\frac{1}{1{,}000}+\frac{3}{100}\right]$$

$$P = \frac{31}{2}\times\frac{1{,}000}{31} = \text{Rs } 500.$$

$\therefore$ The sum is **Rs 500**.

E-6 A certain sum is interested at compound. The interest accrued in the first two years is Rs 272 and that in the first three years is Rs 434. Find the rate per cent.

S-6 Amount $A = P\left[1+\frac{R}{100}\right]^t$; C.I $= P\left[\left(1+\frac{R}{100}\right)^t-1\right]$, For convenience

Put $\left(1+\frac{R}{100}\right) = q$ in the above equation. For two years, $t = 2$, and so,

$$P\,[q^2-1] = 272 \quad \text{(i)}$$

For three years

$$P\,[q^3-1] = 434 \quad \text{(ii)}$$

Dividing (ii) by (i),

$$\frac{(q^2+q+1)(q-1)}{(q+1)(q-1)} = \frac{434}{272}$$

$$\Rightarrow \frac{q^2+q+1}{q+1} = \frac{217}{136}$$

$$\Rightarrow \frac{q^2}{q+1} = \frac{81}{136}$$

$\Rightarrow 136q^2 + 81q + 81 = 0$; Solving $q = \frac{9}{8}$

$\therefore \quad 1 + \frac{R}{100} = \frac{9}{8} \Rightarrow R = \frac{1}{8} \times 100\% = 12\frac{1}{2}\%$.

E-7 What annual payment will discharge a debt of Rs 50,440 due in 3 years at 5% per annum compounded annually?

S-7 Let each annual instalment = Rs a

using the formula, [Refer 15.7], we get

$$a\left[\frac{100}{100+R} + \left(\frac{100}{100+R}\right)^2 + \ldots + \left(\frac{100}{100+R}\right)^T\right] = \text{Debt amount}$$

$$\Rightarrow \quad a\left[\frac{100}{105} + \left(\frac{100}{105}\right)^2 + \left(\frac{100}{105}\right)^3\right] = 50{,}440$$

$$\Rightarrow \quad a \times \frac{20}{21}\left[1 + \frac{20}{21} + \frac{400}{441}\right] = 50{,}440$$

$$\Rightarrow \quad a = 50{,}440 \times \frac{441}{1{,}261} \times \frac{21}{20} = \text{Rs } 18{,}522.$$

$\therefore$ Annual payment = **Rs 18,522**.

E-8 A certain sum of money lent at a certain rate of compound interest grows to 1.44 times its value in 2 years. If the same sum is lent at simple interest at the same rate, in how many years would it double itself?

S-8 Using the formula, [Refer 15.3], we get

$$A = P\left[1 + \frac{R}{100}\right]^t;$$

$$1.44P = P\left[1 + \frac{R}{100}\right]^2 \Rightarrow R = 20\%$$

Now, same sum P is invested and it becomes $2P$.

$\therefore$ S.I. $= 2P - P = P$

Since, S.I. $= \frac{PRT}{100}$

$$\Rightarrow \quad P = \frac{P \times 20 \times T}{100}$$

$\Rightarrow T = 5$

$\therefore$ Time is **5 years**.

E-9 A sum of money is invested at compound payable annually. The interest in successive years were Rs 225 and Rs 238.50. Find the rate % p.a.

S-9 We use the method, [Refer 15.1],

Interest in two consecutive years were Rs 225 and Rs 238.50

$\therefore$ Interest on Rs 225 for 1 year = 238.5 – 225 = 13.50
Principal = 225, interest = 13.50, time = 1 year

$$\text{Rate } (R) = \frac{100 \times \text{Interest}}{P \times T} = \frac{100 \times 13.50}{225 \times 1} = 6\% \text{ p.a.}$$

$\therefore$ The rate is **6% p.a.**

E-10 In what time will Rs 64,000 invested at 5% p.a. fetch an interest of Rs 4,921, the interest being compounded half yearly.

S-10 We use the formula, [Refer 15.4],

$$A = P + \text{C.I} = P\left[1 + \frac{R}{100 \times n}\right]^{t \times n} \quad (n = 2) \qquad \textit{[Refer 15.2]}$$

$$\Rightarrow \quad 64{,}000 + 4{,}921 = 64{,}000\left[1 + \frac{5}{100 \times 2}\right]^{2t}$$

$$\Rightarrow \quad \frac{68{,}921}{64{,}000} = \left[\frac{41}{40}\right]^{2t}$$

$$\Rightarrow \quad \left(\frac{41}{40}\right)^{3} = \left(\frac{41}{40}\right)^{2t} \quad \Rightarrow \quad 2t = 3 \quad \Rightarrow \quad t = \frac{3}{2} \text{ years.}$$

$\therefore$ Time = $\mathbf{\frac{3}{2}}$ **years.**

E-11 The population of a town was 2,50,000 three years ago. If it had increased by 3%, 4% and 6% in the last three years, find the present population of the town.

S-11 We use the relation (15.8.1, ii)

$$\text{Population after } (t_1 + t_2 + t_3 + \ldots) \text{ years} = P\left(1 + \frac{R_1}{100}\right)^{t_1} \times \left(1 + \frac{R_2}{100}\right)^{t_2} \times \left(1 + \frac{R_3}{100}\right)^{t_3} \cdots$$

If P = population three years ago,
then population after three years becomes present population
Let x be the present population

$$\Rightarrow \quad x = 2{,}50{,}000\left(1 + \frac{3}{100}\right)\left(1 + \frac{4}{100}\right)\left(1 + \frac{6}{100}\right)$$

$$\Rightarrow \quad x = 2{,}50{,}000 \times \frac{103}{100} \times \frac{104}{100} \times \frac{106}{100}$$

$$= 2{,}83{,}868$$

Hence, the present population of the town is **2,83,868**.

E-12 The population of a city two years ago was 1,25,000. Due to migration from cities, it decreases every year at the rate of 4% per annum. Find its present population. How many persons have migrated in last two years?

S-12 Population 2 years ago = 1,25,000 = P
For two years, rate of decrease = 4% p.a.
Using the relation (15.8.1, (iii)), we get

Population after 2 years = Present population = $P\left(1-\frac{R}{100}\right)^2$ [(–)ve for decrease]

$\Rightarrow$ Present population = $1,25,000\left(1-\frac{4}{100}\right)^2$

$= 1,25,000 \times \frac{24}{25} \times \frac{24}{25}$

$= 1,15,200$

$\therefore$ Number of persons migrated during last 2 years = 1,25,000 – 1,15,200

= **9,800**.

E-13 A teak tree was planted three years ago. The rate of its growth is 30% per annum. If at present, the height of the tree is 670 cm, what was it when the tree was planted?

S-13 Using the formula, [Refer 15.8.2]

height of tree after 't' years = $P\left(1+\frac{R}{100}\right)^t$

Here, tree was planted 3 years ago.

So, height of tree after 3 years is the present height of the tree.

Let x be the height when the tree was planted 3 years ago.

$\Rightarrow$ 670 = present height of tree = $x\left(1+\frac{30}{100}\right)^3$

$\Rightarrow$ $670 = x\left(\frac{13}{10}\right)^3$

$\Rightarrow$ $x = \frac{670 \times 1,000}{2,197}$

$\Rightarrow$ $x = 305$

$\therefore$ Height of tree when it was planted = **305 cm**.

E-14 In a factory, the production of cement rose to 2,420 tonnes from 2,000 tonnes in two years. Find the rate of growth per annum.

S-14 Present production of cement = 2,420 tonnes

Previous production (2 years ago) = 2,000 tonnes

Production after t years = $P\left(1+\frac{R}{100}\right)^t$ *[Refer 15.8.2]*

$\Rightarrow$ $2,420 = 2,000\left(1+\frac{R}{100}\right)^2$

$\Rightarrow$ $\frac{2,420}{2,000} = \left(1+\frac{R}{100}\right)^2$

$$\Rightarrow \quad \frac{121}{100} = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \quad \frac{11}{10} = 1+\frac{R}{100}$$

$$\Rightarrow \quad R = 10$$

Hence, the rate of growth = **10% per annum.**

E-15 The value of a TV that was purchased in January 1999, depreciates at 12% per annum. If its value in January 2001 is Rs 4,840, then what was purchase price of the TV?

S-15 Let V be the price of purchase in January 1999

We use the relation, [Refer 15.9.(i)]

Value of machine after 't' years = $V\left(1-\frac{R}{100}\right)^t$

Hence, value of TV in January 2001 (i.e. after 2 years) = $V\left(1-\frac{R}{100}\right)^2$

$$\Rightarrow \quad 4{,}840 = V\left(1-\frac{12}{100}\right)^2$$

$$\Rightarrow \quad 4{,}840 = V\left(\frac{22}{25}\right)^2$$

$$\Rightarrow \quad V = \frac{4{,}840\times 25\times 25}{22\times 22}$$

$$\Rightarrow \quad V = 6{,}250$$

Hence, the purchase price of the TV = **Rs 6,250**.

E-16 The half life of Uranium-233 is 1,60,000 i.e. Uranium-233 decays at a constant rate in such a way that it reduces to 50% in 1,60,000 years. In how many years will it reduce to 25%?

S-16 Let the initial amount of Uranium-233 be 1 unit and the rate of decay = R per unit per year

Using the basic formula (15.3), we get

amount of Uranium-233 after 1,60,000 years = $1\times(1-R)^{1,60,000}$

$$\Rightarrow \quad \frac{1}{2} = (1-R)^{16,00,00} \qquad \left(\text{Since } 50\% = \frac{1}{2}\right) \quad \text{(i)}$$

Assume the Uranium reduces to 25% in t years, then

$$\frac{25}{100} = 1\times(1-R)^t$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^2 = (1-R)^t \quad \text{(ii)}$$

From (i) and (ii)

$$[(1 - R)^{1,60,000}]^2 = (1 - R)^t$$

$$\Rightarrow \quad t = 3,20,000 \text{ years}$$

Thus, Uranium-233 will reduce to 25% in **3,20,000 years**.

REGULAR PROBLEMS

(1) The compound interest on Rs 20,000 for three years at 10% per annum is:
(a) Rs 6,540 (b) Rs 6,620 (c) Rs 6,800 (d) Rs 6,360 (e) Rs 6,260

(2) Koeppen deposited a sum of Rs 16,000 in a post office for 3 years, compounded annually at 15% per annum. What amount will she get on maturity?
(a) Rs 21,752 (b) Rs 26,020 (c) Rs 24,334 (d) Rs 23,525 (e) Rs 22,705

(3) Zovo got a loan of Rs 8,000 against his fixed deposits to purchase a scooter. If the rate of interest is 10% p.a. compounded half yearly, then the amount he has to pay back after one and half year is:
(a) Rs 9,331 (b) Rs 9,621 (c) Rs 9,701 (d) Rs 9,991 (e) Rs 9,261

(4) The cost of a machine is Rs 9,000. If the cost decline is 10% of the cost at the beginning of each year, then what will be the cost of the machine three years later? **(RRB Allahabad, '97)**
(a) Rs 6,561 (b) Rs 6,300 (c) Rs 6,501 (d) Rs 6,462 (e) Rs 6,481

(5) In what time will a sum of Rs 800 at 5% p.a. compound interest amounts to Rs 882?
(a) 1 year (b) 5 years (c) 4 years (d) 2 years (e) None of these

(6) Find the compound interest on Rs 15,625 for nine months at 16% per annum compounded quarterly.
(a) Rs 17,576 (b) Rs 1,525 (c) Rs 1,951 (d) Rs 676 (e) Rs 1,800

(7) Neeta wants to invest her money in a Savings Plan that will double her sum in nine years. Find the rate % per annum of the Savings Plan in compound interest
(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

(8) The present population of a village is 16,000. If it increases at the rate of 5% per year, then what will be the population two years hence?
(a) 17,640 (b) 16,960 (c) 16,975 (d) 18,025 (e) 17,525

(9) The population of a town in the year 2001 was 4 lakhs. Due to migration from cities, it decreases every year at the rate of 40 per thousand. Find the population of the town in lakhs during 2003.
(a) 4.41 (b) 4.21 (c) 4.84 (d) 3.68 (e) 3.89

(10) The value of a VCD-music system that was purchased two years ago, depreciates at 12% per annum. If its present value is Rs 9,680, for how much it was purchased?
(a) Rs 12,500 (b) Rs 10,800 (c) Rs 12,000 (d) Rs 13,200 (e) Rs 10,240

(11) Vijaylaxmi lent out Rs 20,000 for two years at 20% per annum, compounded annually. How much more she could earn if the interest be compounded half-yearly?
(a) Rs 441 (b) Rs 662 (c) Rs 482 (d) Rs 502 (e) Rs 605

(12) A sum of money at compound interest amounts to thrice itself in three years. In how many years will it be nine times itself? **(Bank PO, '91)**
(a) 18 (b) 12 (c) 6 (d) 9 (e) data insufficient

Answers

1. (b) **2.** (c) **3.** (e) **4.** (a) **5.** (d) **6.** (c) **7.** (d) **8.** (a) **9.** (d)
10. (a) **11.** (c) **12.** (c)

REAL PROBLEMS

(1) A certain sum amounts to Rs 1,452 in two years and to Rs 1,597.20 in three years at compound interest, then rate per cent is:
(a) 10 (b) 11 (c) 13 (d) 9 (e) 15

(2) In what time will a man receive Rs 85 as compound interest on Rs 320 at $12\frac{1}{2}$% p.a. compounded yearly?
(a) $4\frac{1}{2}$ yrs. (b) $2\frac{1}{2}$ yrs. (c) 2 yrs. (d) 5 yrs. (e) $3\frac{1}{2}$ yrs.

(3) If the compound interest on a certain sum for two years at 10% p.a. is Rs 2,100 the simple interest on it at the same rate for two years will be
(a) Rs 1,980 (b) Rs 1,760 (c) Rs 2,000 (d) Rs 1,800 (e) Rs 1,805

(4) If Rs 1,200 amounts to Rs 1,323 in two years at compound interest, then what will be the amount of Rs 1,600 in three years at compound interest at the same rate per cent?
(a) Rs 1,850 (b) Rs 1,852.20 (c) Rs 1,752.20 (d) Rs 1,905.50 (e) Rs 1,951

(5) The difference between simple interest and compound interest on a certain sum of money for three years at 10% per annum is Rs 15 and paise 50. The sum is:
(a) Rs 5,000 (b) Rs 550 (c) Rs 5,500 (d) Rs 500 (e) Rs 1,500

Hint: Refer the text

(6) A tree increases annually by $\frac{1}{8}$th of its height. By how much will it increase after $2\frac{1}{2}$ years, if it stands today 10 ft. high?
(a) data insufficient (b) less than 12 ft. (c) more than 3 ft.
(d) more than 2 ft. (e) slightly more than 13 ft.

Hint: Refer the text

(7) Seshank borrowed Rs 20,000 from his friend at 18% per annum simple interest. He lent it to Tony at the same but rate compounded annually. Find his gain after two years.
(a) Rs 648 (b) Rs 836 (c) Rs 324 (d) Rs 704 (e) Rs 572

Hint: Do not waste your time by calculating the simple interest and the compound interest separately. Rather use the concept for finding the difference in interest accrued in simple interest and on compound interest. Refer the text.

(8) Sashidharan took a loan of Rs 20,000 to purchase a colour TV set from Royal Finance Co. He promised to make the payment after three years. The company charges compound interest @ 10% p.a. for the same. But, suddenly the company announces the rate of interest as 15% p.a. for the last one year of the loan period. What extra amount Sashidharan has to pay due to this announcement of new rate of interest?
(a) Rs 7,830 (b) Rs 6,620 (c) Rs 4,410 (d) Rs 1,210 (e) Rs 3,000

(9) Ravishankar borrowed Rs 62,500 from a bank to purchase one Home Theatre System. If the rate of interest be 12% per annum compounded annually, what payment he will have to make after 2 years 3 months?
(a) Rs 75,320 (b) Rs 80,752 (c) Rs 83,650 (d) Rs 64,960 (e) Rs 84,380

Hint: Refer text

(10) The present population of a village is 9,261. If the annual birth rate is $8\frac{1}{2}\%$ and the annual death rate is 3.5%, then calculate the population 3 years ago.
(a) 10,721 (b) 11,363 (c) 11,391 (d) 8,000 (e) 10,561

(11) The value of a machine depreciates 10% annually. If the present value of the machine is Rs 1.0 lakh, then the total depreciation during 2 years hence will be
(a) Rs 81,000 (b) Rs 21,000 (c) Rs 19,000 (d) Rs 72,000 (e) Rs 44,100

(12) What is the nominal rate per cent per annum, when interest is payable half yearly that would give an effective rate of 8% p.a.
(a) 8.5 (b) 8.2 (c) 9.4 (d) 7.8 (e) 7.5

Hint: Follow the concept of half yearly compounding of interest and the conversion period

(13) A sum of Rs 550 is to be repaid in two equal instalments. If the rate of interest be 20% compounded annually, then the value of each instalment is:
(a) Rs 390 (b) Rs 360 (c) Rs 400 (d) Rs 275 (e) Rs 290

(14) Find the least number of complete years in which the sum of money put out at 20% compound interest will be more than double.
(a) 1 year (b) 2 years (c) 3 years (d) 4 years (e) 5 years

Answers

1. (a)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (c)	**7.** (a)	**8.** (d)	**9.** (b)
10. (d)	**11.** (c)	**12.** (d)	**13.** (b)	**14.** (d)				

16 SHARES AND DEBENTURES

16.1 BASIC FACTS

(i) The capital of a company is called the **Stock**, e.g. 5% stock at 97 means that if a person invests Rs 97, he can buy stock worth Rs 100 and his annual income is Rs 5.

(ii) The convenient unit in which the capital stock of a joint stock company is divided, is called a **Share**. These shares are generally worth Rs 10 and Rs 100 each. The company raises its capital by means of selling such shares.

(iii) The persons who purchase shares are called **Share Holders.**

(iv) The part of the profit of a company which is divided among the share holders is called **Dividend**. For example, a 20% dividend means that on a share of Rs 100, the share holder gets Rs 20; on a share of Rs 10, he gets Rs 2 and so on.

(v) The value at which a company issues shares is called the **Nominal Value, Face Value** or **Par Value** of the share. This value is printed on the share certificate.

(vi) The value at which a share is available in market is known as **Market Value** of the share.

(vii) If the Market Value = Face Value, share is at par.

(viii) If the Market Value > Face Value, share is at premium or above par.

(ix) If the Market Value < Face Value, share is at discount or below par.

(x) When a company likes to borrow money from the shareholders or public for a **Fixed Period** at a **Fixed Rate of Interest**, the company issues **Debentures**. So, debentures are a **Debt** for a company.

(xi) A debenture-holder receives interest on the face value of debentures at a rate fixed by the company. The interest does not vary.

(xii) Dividend on Shares is calculated on Face value

(xiii) Interest on Debentures is calculated on Face value.

(xiv) Shares and Debentures, are generally sold or purchased in a market known as **Stock Exchange** through authorised persons known as **Share-brokers** (**Brokers**).

(xv) Broker's commission is called **Brokerage**.

(xvi) Brokers charge commission from the purchasers and also from the sellers.

(xvii) Brokerage is calculated on Market Value of Shares/Debentures.

(xviii) **Share Purchaser** has to pay (**Market Value + Brokerage**)

(xix) **Share Seller** will get (**Market Value – Brokerage**)

16.2 APPROACH TO PROBLEMS ON STOCK

For example,

(a) 3% Rs 100 stock at Rs 96 By investing Rs 96, one can purchase stock of Rs 100.
Hence, market price of the stock = Rs 96
$\therefore$ income = Rs 3 from an investment of Rs 96.

So, to obtain Re 1 income from the stock, the person is to invest = $\frac{\text{Rs } 96}{3}$ = Rs 32

(b) 3% stock yielding Rs 240 a year He earns an income = Rs 3 from Rs 100 stock

$\therefore$ Rs 240 is earned from $\frac{100}{3} \times 240$ = Rs 8,000 stock

$\Rightarrow$ amount of stock purchased = Rs 8,000

(c) Investment of Rs 1,125 in 3% stock at $112\frac{1}{2}$ By investing Rs $112\frac{1}{2}$, a person gets a stock of Rs 100

So, an investment of Rs 1,125 will allow the purchase of stock worth $100 \times \left(\frac{1125}{112\frac{1}{2}}\right)$
= Rs, 1,000

$\therefore$ amount of stock purchased = Rs 1,000

So, annual income after purchase of stock = $\left(\frac{1125}{112\frac{1}{2}}\right) \times 3$ = Rs 30

(d) Sale proceeds from Rs 4,000 stock at 92 By selling Rs 100 stock, cash realised = Rs 92

$\therefore$ Sale proceeds from Rs 4,000 stock = $\frac{92}{100} \times 4{,}000$
= Rs 3,680

16.3 APPROACH TO PROBLEMS ON SHARE

(a) **Face value** is the value of share printed by the company. Market value is the value of share available in the market. It varies from time to time.
- If the share is at par, then market value = face value
- If the share is at premium, then market value = face value + premium
- If the share is at discount, then market value = face value – discount

(b) **Brokerage** is expressed as the percentage of market value of share.

(c) During purchase of each share, the share holder has to pay the brokerage percentage as a commission to the broker. Therefore

Purchase value of 1 share = market value + (% brokerage × market value)
= market value (1 + % brokerage)

But, in case a person purchases directly from the company, he need not to pay brokerage, then, purchase value of 1 share = market value

Table 16.1 Formulae on Stock

Part	*I*	*II*	*III*	*IV*	*V*
	No. of Stock =	$\frac{\text{Total Stock}}{100}$ =	$\frac{\text{Purchase Cost}^*}{\text{M.V.} + \text{brokerage}}$ =	$\frac{\text{Sale Realisation}}{\text{M.V.} - \text{brokerage}}$ =	$\frac{\text{Annual Income}}{\text{Rate \%}}$ =

*Purchase Cost = Total Investment

NB: Equate any of the TWO PARTS to find out the unknown as per the data available.

Table 16.2 Formulae on Shares and Debentures

Part	*I*	*II*	*III*	*IV*
	No. of Shares =	$\frac{\text{Investment (or Purchase Cost)}}{\text{M.V.}(1 + \%\text{brokerage})}$ =	$\frac{\text{Sale Realisation}}{\text{M.V.}(1 - \%\text{ brokerage})}$ =	$\frac{\text{Annual Income} \times 100}{\text{Dividend \%} \times \text{Face Value}}$ =

NB: Equate any of the TWO PARTS as per the requirements of the given problem.
M.V. stands for Market Value of one share/debenture/stock.

(d) During sale of each share, the share holder has to again pay the % brokerage as a commission to the broker. Therefore,

Sale value (or sale earning) of 1 share = market value – (% brokerage × market value)
= market value (1 – % brokerage)

(e) Investment is the total amount invested in purchasing a certain number of shares at the market value. It implies that if a person purchases 'n' shares, then his investment
$= n \times$ purchase value of 1 share

(f) A company pays dividend (share of the profit) to each shareholder. The **dividend** is normally a certain **percentage of the face value** of each share or a certain amount for each share.

The dividend has no relation with the market value of the share. In fact, this dividend is the annual income or the return on investment for the shareholder. Hence

Annual income on one share = face value × % dividend
Total income on 'n' no. of shares = $n \times$ face value × % dividend

(g) Actual rate per cent on investment $= \dfrac{\text{income}}{\text{investment}} \times 100$

From (e) and (f), it can be concluded that:

$$\text{Number of shares} = n = \frac{\text{investment}}{\text{purchase value of one share}} = \frac{\text{annual income}}{\underbrace{(\text{face value} \times \% \text{ dividend})}_{\text{annual income of one share}}}$$

Solved Examples

E-1 Find the cost of

(a) Rs 5,000, $8\frac{2}{3}$ % stock at 95

(b) Rs 3,000, $7\frac{1}{2}$% stock at par

(c) Rs 2,500, 7% stock at 10 premium

(d) Rs 1,000, $3\frac{1}{2}$ % stock at 8 discount

(e) Rs 2,200, 6% stock at par

$\left(\text{brokerage } \frac{1}{11}\%\right)$

(f) Rs 2,000, $5\frac{1}{2}$% stock at 5 premium

$\left(\text{brokerage } \frac{1}{2}\%\right)$

(g) Rs 1,000, 6% stock at 6 discount

$\left(\text{brokerage } \frac{1}{2}\%\right)$

S-1 While finding the purchase cost, neglect the rate % data

(a) Cost of purchase = Amount of stock $\times \frac{\text{Market Value}}{100}$ *[Refer Parts II and III in Table 1]*

$$= 5{,}000 \times \frac{95}{100} = \textbf{Rs 4,750}.$$

(b) Since it is at par, $\therefore$ Cost of purchase
= Amount of stock = **Rs 3,000**.

(c) Here, the stock is at 10 premium.
$\therefore$ Market Value = (100 + Premium) = 110

$\therefore$ Cost of purchase = Amount stock $\times \frac{\text{Market Value}}{100}$

$$= 2500 \times \frac{110}{100} = \textbf{Rs 2,750}.$$

(d) Here, stock is at 8 discount, $\therefore$ Market Value = 100 – discount

$\therefore$ Cost of purchase = Amount of stock $\times \frac{(100 - \text{discount})}{100}$

$$= 1{,}000 \times \frac{92}{100} = \textbf{Rs 920}.$$

(e) Brokerage is added while purchasing the stock

$\therefore$ Purchase Cost = Amount of stock $\times \frac{(\text{brokerage} + \text{Market Value})}{100}$

$$= \frac{2{,}200}{100} \times \left(100 + \frac{1}{11}\right)$$

(Since it is at par Market Value = 100)

$$= 22 \times \frac{1{,}101}{11} = \textbf{Rs 2,202}.$$

(f) Market Value= 100 + Premium = 100 + 5

Purchase Cost = Amount of stock $\times \frac{(\text{Market Value} + \text{brokerage})}{100}$

$$= 2000 \times \frac{100 + 5 + \frac{1}{2}}{100} = \textbf{Rs 2,110}.$$

(g) Market Value = 100 – discount = 100 – 6

Purchase Cost = Amount of stock $\times \frac{(\text{Market Value} + \text{brokerage})}{100}$

$$= 1{,}000 \times \frac{100 - 6 + \frac{1}{2}}{100} = \textbf{Rs 945}.$$

E-2 Find the cash realised by selling Rs 2,000, 5% stock at 6 premium $\left(\text{brokerage } \frac{1}{2}\%\right)$.

S-2 Brokerage is subtracted when stock is sold.

$\therefore$ Sale Realisation = Amount of stock $\times \dfrac{(\text{Market Value} - \text{brokerage})}{100}$

$$= 2{,}000 \times \frac{\left(100 + 6 - \frac{1}{2}\right)}{100} = \textbf{Rs 2,110}.$$

E-3 How much stock should be sold to realise Rs 1,221 from 5% stock at $2\frac{1}{4}$ premium $\left(\text{brokerage } \frac{1}{2}\%\right)$.

S-3 Amount of stock $= \dfrac{\text{Sale Realisation}}{(\text{Market Value} - \text{brokerage})} \times 100$ *[Refer Parts II and IV in Table 1]*

$$= \frac{1{,}221}{100 + 2\frac{1}{4} - \frac{1}{2}} \times 100 = \textbf{Rs 1,200}.$$

E-4 How much 6% stock at 8 discount can be purchased by investing Rs 1,850 $\left(\text{brokerage } \frac{1}{2}\%\right)$.

S-4 Since Market Value = 100 – discount

$\therefore$ Amount of Stock $= \dfrac{\text{Purchase Cost}}{(100 - \text{discount} + \text{brokerage})} \times 100$

$$= \frac{1{,}850}{100 - 8 + \frac{1}{2}} \times \text{v}100 = \textbf{Rs 2,000}.$$

Hence stock being at 8% discount, by investing Rs 1,850, one can purchase stock of Rs 2,000 (which is more than Rs 1,850).

E-5 Find the annual income derived from Rs 2,800, 4% stock at 122?

S-5 Annual Income = Total Amount of stock × % rate of stock
= 2,800 × .04 = **Rs 112**.

E-6 Find the annual income derived by investing Rs 2,800, in 4% stock at 112?

S-6 Note, here total investment is Rs 2,800
But amount of stock purchased is not known.

$\therefore$ Annual Income $= \dfrac{\text{Investment}}{(\text{Market Value} + \text{brokerage})} \times \dfrac{\%\text{ rate of stock}}{100}$

[Refer Parts II and V in Table 1]

$$= \frac{2{,}800}{112} \times 4 = \textbf{Rs 100}.$$

E-7 What rate % is obtained by investing in 7% stock at 5 discount $\left(\text{brokerage } \frac{1}{4}\%\right)$.

S-7 Amount of rate per cent = Income by investment of Rs 100

Annual Income $= \dfrac{\text{Investment}}{(\text{Market Value} + \text{brokerage})} \times \%$ rate of stock

[Refer Parts III and V in Table 1]

$$= \frac{100}{1 - \%\,5 + \frac{1}{4}\%} \times 7 = \frac{100}{1 - 0.05 + 0.0025} \times 7 = 735$$

Rate % obtained $= \frac{735}{100} =$ **7.35%**.

E-8 Find the market value of a 6% stock in which an income of Rs 244 is derived by investing Rs 1,220, brokerage being $\frac{1}{4}\%$.

S-8 $\frac{\text{Annual Income}}{\text{Rate \%}} = \frac{\text{Investment}}{\text{(Market Value + brokerage)}}$ *[Refer Parts III and V in Table 1]*

$$\Rightarrow \quad \frac{244}{6} = \frac{1{,}220}{\text{Market Value} + \frac{1}{4}}$$

$\therefore$ Market Value $+ \frac{1}{4} = 30 \Rightarrow$ Market Value = **29.75**.

E-9 How much should one invest in $6\frac{2}{3}\%$ stock at 110 to secure an annual income of Rs 300.

S-9 $\frac{\text{Annual Income}}{\text{Rate \%}} = \frac{\text{Investment}}{\text{Market Value + \% brokerage}}$ *[Refer Parts III and V in Table 1]*

$$\frac{300}{I} = \frac{20}{3 \times 110}$$

$$\Rightarrow \quad I = \frac{30 \times 110 \times 300}{20} \Rightarrow I = \text{Rs } 4{,}500 \quad \therefore \text{ The investment is } \textbf{Rs 4,500.}$$

E-10 Find what a purchaser would have to pay for 250 shares of Rs 20 each quoted at Rs 74. What would be the gain to the share holder, if he had purchased the shares at par?

S-10 Face Value of 1 share = Rs 20
Market Value of 1 share = Rs 74
The amount paid by the buyer (purchaser) = 250 × 74 = Rs 18,500
The Purchase Cost by share holder = 250 × 20 = 5,000
$\therefore$ Gain by the share holder = 18,500 – 5,000 = **Rs 13,500**.

E-11 A man invests Rs 4,220 in $6\frac{1}{2}\%$ stock at 105. On selling the invested stock, how much will he realise? $\left(\text{brokerage} = \frac{1}{2}\%\right)$.

S-11 Here, amount of stock is not known.
Purchase cost of stock = Investment = Rs 4,220

$\therefore \quad \frac{\text{Sale Realisation}}{\text{Market Value} - \text{brokerage}} = \frac{\text{Purchase Cost}}{\text{Market Value} + \text{brokerage}}$

[Refer Parts III and IV in Table 1]

$$\therefore \quad \frac{\text{Sale Realisation}}{105 - \frac{1}{2}} = \frac{4{,}220}{105 + \frac{1}{2}}$$

$\Rightarrow$ Sale Realisaton $= \dfrac{4,220 \times 104.25}{105.5} =$ **Rs 4,180.**

E-12 Find the purchase cost of 80 shares of Rs 10 each at $\frac{3}{8}$ discount, brokerage being $\frac{1}{8}$ per share.

S-12 Since Market Value = Face Value – discount

$\therefore$ Cost of 1 share = Face Value – discount + brokerage

$$= 10 - \frac{3}{8} + \frac{1}{8} = 9\frac{5}{8} + \frac{1}{8} = 9\frac{6}{8} = \text{Rs } 9\frac{3}{4}$$

$\therefore$ Cost of 80 share $= 80 \times 9\frac{3}{4}$

$$= (80 \times 9) + \left(\frac{3}{4} \times 80\right) = \textbf{Rs 780.}$$

E-13 A man sells Rs 4,500, 5% stock at 120 and invests the proceeds partly in 3% stock at 99 and partly in 8% stock at 132. He thereby increases his income by Rs 75. How much of the sale proceeds were invested in each stock?

S-13 Sale Realisation $= \dfrac{(\text{Market Value} - \text{brokerage})}{100} \times$ Amount of stock

$$= \frac{120}{100} \times 4,500 = \text{Rs } 5,400$$

Income before selling = Total Amount of stock $\times$ % rate of stock

$$= 4,500 \times \frac{5}{100} = 225$$

Income after sale from two stocks = 225 + 75 = Rs 300

Now, let, Rs x of sale proceeds be invested in 3% stock at 99 and Rs $(5,400 - x)$ be invested in 8% stock at 132.

$\therefore$ Income from I stock + Income from II stock = 300

$\therefore \dfrac{x \times 3}{99} + \dfrac{(5,400 - x) \times 8}{112} = 300$

$\therefore$ $x =$ **Rs 900 invested in 3% stock at 99**

and $5,400 - x =$ **Rs 4,500 invested in 8% stock at 132.**

E-14 A man invested Rs 2,592. When he bought shares of a company at Rs 108 each, the face value of a share was Rs 100. The company paid $12\frac{1}{2}$% dividend. Find the dividend earned (income derived) at the end of the year.

S-14 Dividend is always calculated on **Face Value**. Equating Parts II and IV of Table 2,

we get, $\dfrac{\text{Annual Income} \times 100}{\text{Dividend \%} \times \text{Face Value}} = \dfrac{\text{Investment}}{\text{Market Value } (1 + \text{\% brokerage})}$

$\Rightarrow$ $\dfrac{\text{Annual Income} \times 100}{\frac{25}{2} \times 100} = \dfrac{2,592}{108}$

$\therefore$ Income derived $= \dfrac{25}{2 \times 100} \times \dfrac{100}{108} \times 2,592 =$ **Rs 300.**

E-15 Find the income derived from 88 shares of Rs 20 each at 5 premium $\left(\text{Brokerage } \frac{1}{4} \text{ per share}\right)$, the rate of dividend being $4\frac{1}{4}\%$. Also find the rate of interest on this investment.

S-15 Dividend earned (income) $= \dfrac{\%\text{ rate of dividend}}{100} \times \text{Face Value} \times \text{Number of shares}$

[Refer Parts I and IV of Table 2]

$$= \frac{17}{4 \times 100} \times 20 \times 88 = \text{Rs } 74.80$$

Purchase Cost of one share = Face Value + Premium + brokerage

$$= 20 + 5 + \frac{1}{4} = 25\frac{1}{4}$$

% Rate of interest on the investment,

$$= \%\text{ rate of dividend} \times \frac{\text{Face Value}}{\text{Purchase Cost}}$$

$$= \frac{17}{4} \times \frac{20}{\frac{101}{4}} = \mathbf{3.37\%}.$$

E-16 A man invested Rs 4,444 in the shares of face value Rs 100 and at 15% dividend, his income was Rs 600. Find the quoted price (Market Value) of each share if brokerage is 1 %.

S-16 Brokerage is always calculated on **Market Value.**

Let the market value of each share = Rs M *[Refer Parts II and IV of Table 2]*

$$\frac{\text{Investment}}{\text{Market Value}\,(1 + \%\text{ brokerage})} = \frac{\text{Annual Income} \times 100}{\text{Dividend}\,\% \times \text{Face Value}}$$

$$\Rightarrow \quad \frac{4,444}{M\,(1+1\%)} = \frac{600 \times 100}{15 \times 100}$$

$$\Rightarrow \quad M = \text{Rs } 110.$$

$\therefore$ Market Value of each share = **Rs 110**.

E-17 A man has income of Rs 1,500 by investing in 15 % debentures of face value of Rs 100 and available for Rs 104. If the brokerage is 1%, what is his investment?

S-17 Refer Parts II and IV in Table 2

$$\frac{\text{Investment}}{\text{Market Value}\,(1 + \%\text{ brokerage})} = \frac{\text{Annual Income} \times 100}{\text{Dividend}\,\% \times \text{Face Value}}$$

$$\Rightarrow \quad \frac{\text{Investment}}{104\,(1+1\%)} = \frac{1,500 \times 100}{15 \times 100} \Rightarrow \text{Investment} = 100 \times 104 \times \left(\frac{101}{100}\right)$$

$\therefore$ Investment = **Rs 10,504**.

E-18 Which is a better investment?

(i) 15 % debentures at 8 % premium or,

(ii) 14 % debentures at 4 % discount

S-18 Simple method *

Investment	%	Market Value	
(i)	15	108	(cross multiply)
(ii)	14	96	

Hence, (i) $15 \times 96 = 1{,}440$

(ii) $14 \times 108 = 1{,}520$ $\therefore$ (ii) > (i)

$\therefore$ **(ii) investment is better.**

* This method is also applicable in case of share investment.

REGULAR PROBLEMS

(1) Find the purchase cost of

(a) Rs 2,000, 4% stock at 5 discount

(b) Rs 1,500, 4% stock at 97 $\frac{7}{8}$ and brokerage $\frac{1}{8}$%.

(2) How much $4\frac{1}{2}$% stock at $97\frac{1}{2}$ can be purchased for Rs 390.

Hint: Amount of stock = $\dfrac{\text{Investment} \times 100}{\text{M.V. of one stock}}$.

(3) Rs 800 stock of a company costs Rs 750 including $\frac{1}{8}$% brokerage. Find the market value of Rs 100 stock.

Hint: $\dfrac{800}{750} = \dfrac{100}{\text{Cost price of stock}}$ $\therefore$ Cost price = $93\frac{3}{4}$

$\therefore$ Market value = Cost price – brokerage.

$$= 93\frac{3}{4} - \frac{1}{8} = 93\frac{5}{8}.$$

(4) Find the cash realised by selling Rs 4,000, 6 % stock at 5 premium $\left(\text{brokerage } \frac{1}{4}\%\right)$.

Hint: Sale Realisation = Amount of stock $\times$ $\dfrac{(100 + \text{Premium} - \text{brokerage})}{100}$.

(5) Find the cash required to purchase Rs 1,500, 4% stock at 105 $\left(\text{brokerage } \frac{1}{3}\%\right)$.

(6) Find the annual income derived from Rs 2,500, 4% stock at 100.

(7) Find the annual income by investing Rs 3,150, 4% stock at 105 .

Hint: Income = $\dfrac{4}{105} \times 3{,}150$.

(8) Find the annual income by investing Rs 9,550 in $4\frac{1}{2}$% stock at 95 $\left(\text{brokerage } \frac{1}{2}\%\right)$.

Hint: $\dfrac{\text{Income}}{\text{Investment}} = \dfrac{\text{Rate \% of stock}}{(\text{Market value} + \text{brokerage})}$. *[Refer Memory Table 1]*

(9) What rate % is obtained by investing in $9\frac{1}{2}\%$ stock at 90 $\left(\text{brokerage } \frac{1}{4}\%\right)$.

Hint: Rate % obtained = $\dfrac{\%\text{ rate of stock}}{\text{Market value} + \text{brokerage}} \times 100$.

(10) Find the market value of a 8% stock in which an income of Rs 125 is derived by investing Rs 1,000, $\left(\text{brokerage } \frac{1}{4}\%\right)$.

(11) What rate % is obtained if by investing Rs 1,000, one gets annual income of Rs 125.

Hint: Rate% obtained = $\dfrac{\text{Income}}{\text{Investment}} \times 100$.

(12) What is the better investment?

(i) 4 % stock at 95 or

(ii) $4\frac{1}{2}\%$ stock at 105

Hint: Test which is greater, 4×105 or $4\frac{1}{2} \times 95$

(I) (II)

$= 420$ or 427.5

$\therefore$ II is greater than I So II is better investment, also refer E-18.

(13) How much money must I invest in 7% stock at 105 to secure an annual income of Rs 400?

(14) A man invests Rs 8,100 partly in 7% stock at 147 and partly in 6% stock at 144. If his income is same, find his two investments.

Hint: Since income from both investment is same, so, divide 8,100 in the ratio of $\dfrac{147}{7} : \dfrac{144}{6}$.

(15) Find the income derived from 600 shares, if the face value of one share is Rs 10 and dividend is 16%.

(16) A man sold 500 shares with face value Rs 100 and market value as Rs 740. How much did he gain if he had purchased the shares at 100% premium.

Hint: Gain = 500 (740 – 200).

(17) A man purchased 300 shares of the face value of Rs 100 each from the market at Rs 800 per share. If a dividend of 24% is declared, find his earning per cent on the investment.

Hint: % earning on investment = % dividend $\times \dfrac{\text{Face Value}}{\text{Market Value}}\%$

$= 24 \times \dfrac{100}{800} = 3\%$.

(18) How many shares of market value of Rs 12.50 can be purchased for Rs 12,625, (if the brokerage is 1%).