Maximum number of digits in denominator = 3 $\left(\text{in } \frac{97}{104}\right)$.

So, $n = 2$ $d = 3$ $\therefore$ $d - n = 1$

So, multiply the numerator of the given fraction by 10.

$\therefore \frac{3}{5} \times 10 \approx 6,\ \frac{5}{7} \times 10 \approx 7,\ \frac{13}{16} \times 10 \approx 8,\ \frac{97}{104} \times 10 \approx 9$ (integer value)

Now, $9 > 8 > 7 > 6 \Rightarrow \frac{97}{104} > \frac{13}{16} > \frac{5}{7} > \frac{3}{5}$

(v) Here $n = 2$ (in 22 or in 13) $d = 4$ (in 1091) $\therefore$ $d - n = 2$

So, multiply the numerator of given fractions by 10^2

$\therefore \frac{2}{91} \times 100 \approx 2,\ \frac{5}{177} \times 100 \approx 2,\ \frac{22}{1091} \times 100 \approx 2,\ \frac{13}{558} \times 100 \approx 2$ (integer value)

All the fractions have the same integer value, so, find the next decimal place, i.e.

$\approx$ 2.1 2.8 2.0 2.3

Now, $2.8 > 2.3 > 2.1 > 2.0 \Rightarrow \frac{5}{177} > \frac{13}{558} > \frac{2}{91} > \frac{22}{1091}$

E-2 $\frac{5}{12}$ part of what amount will be equal to $3\frac{3}{4}$ part of Rs 100.

S-2 Let the amount be Rs x

$\therefore \quad \frac{5}{12}x = 3\frac{3}{4} \times 100 \Rightarrow \frac{5}{12}x = \frac{15}{4} \times 100$

$\Rightarrow \quad x = \frac{12}{5} \times \frac{15}{4} \times 100 \Rightarrow x = \text{Rs } 900$

$\therefore$ Required amount is Rs 900.

E-3 What fraction is 6 bananas in 5 dozens?

S-3 Required fraction = 6 out of 5 dozen

$= \frac{6}{5 \times 12} = \frac{1}{10}$.

E-4 There are 40 students in a class. One day only $\frac{7}{10}$ students were present. Find the number of absentees on that day.

S-4 In solving the problem on fraction, the whole quantity is *always* considered as 1.

$\therefore$ Number of absentees = Fraction of absentees $\times$ Total number

$= \left(1 - \frac{7}{10}\right) \times 40 = 12$ students.

E-5 A man spent $\frac{2}{7}$ of his savings and still has Rs 1,000 left with him. What were his savings?

S-5 In this type of problem, if balance amount is given, then this amount is to be related to the balance part (fraction). Using relation 2, for savings.

$$\text{Savings} = \frac{\text{balance amount}}{\text{fraction related to balance part}}$$

$\Rightarrow \quad \text{savings} = \frac{1000}{\left(1 - \frac{2}{7}\right)} = \text{Rs } 1400.$

E-6 A man reads $\frac{3}{8}$ of a book on a day and $\frac{4}{5}$ of the remainder, on the second day. If the number of pages still unread are 40, how many pages did the book contain?

S-6 It is a *dependent activity*, because on the second day he reads $\frac{4}{5}$ of the remaining pages.

Using the relation 4, the fraction related to balance part $= \left(1-\frac{3}{8}\right)\times\left(1- \quad\right) = \frac{5}{8}\times\frac{1}{5} = \frac{1}{8}$

using relation 2,

$$\text{Total pages} = \frac{\text{Pages unread}}{\text{fraction related to pages unread}}$$

$$= \frac{40}{1/8} = 320$$

$\therefore$ Total pages = 320

E-7 A man spends $\frac{2}{5}$ of his salary on food, $\frac{3}{10}$ of his salary on house rent and $\frac{1}{8}$ of the salary on clothes. He still has Rs 1,400 left with him. Find his salary.

S-7 The expenditure incurred on each item is expressed as part of the total amount (salary), so it consists of independent fractions.

Using the relation 3, we get

Fraction related to balance part = 1 – (sum of independent fractions)

$$= 1-\left(\frac{2}{5}+\frac{3}{10}+\frac{1}{8}\right) = 1-\frac{33}{40} = \frac{7}{40}$$

Using relation 2, we get,

$$\text{Total salary} = \frac{\text{Balance amount}}{\text{fraction related to balance part}}$$

$$= \frac{1400}{7/40} = 8000$$

$\therefore$ Total salary = Rs 8000

E-8 A man spends $\frac{1}{3}$ of his income on food, of the rest $\frac{1}{4}$ on house rent and $\frac{1}{5}$ on cloth. He still has Rs 1760 left with him. Find his income.

S-8 Here, of the rest amount (after spending on food), $\frac{1}{4}$ is spent on house rent and $\frac{1}{5}$ is spent on clothes. So, spending on these last two items are independent of each other, but dependent on the expenditure incurred on the first item. It is a problem both on *dependent* and *independent* activities.

Using relation 3 and 4 together, we get,

$$\text{Fraction related to balance part} = \left(1-\frac{1}{3}\right)\times\left[1-\underbrace{\left(\frac{1}{4}+\frac{1}{5}\right)}_{\text{independent}}\right]$$

$$= \frac{2}{3}\times\frac{11}{20}$$

Using relation 2, we find, total income = $\frac{1760}{11/30} = 4800$

$\therefore$ total income = Rs 4800

E-9 $\frac{4}{7}$ of a pole is in the mud. When $\frac{1}{3}$ of it is pulled out, 250 cm. of the pole is still in the mud. Find the full length of the pole.

S-9 Using the relation 2, we get,

total length of pole = $\frac{\text{Length in mud}}{\text{Part in mud}} = \frac{250}{\frac{4}{7} - \frac{1}{3}} = 1050$

$\therefore$ Length of pole = 1.050 cm.

E-10 After covering five-eighth of my journey, I find that I have travelled 60 km. How much journey is left?

S-10 Using the relation 2, we get*

$$\frac{\text{Journey covered}}{\text{Fraction related to journey covered}} = \frac{\text{Journey left}}{\text{its related fraction}}$$

$$\Rightarrow \quad \frac{60}{5/8} = \frac{\text{Journey left}}{1 - \frac{5}{8}}$$

$\therefore$ Journey left = 36 km

**Note:* This relation is equal to total journey.

E-11 How much is to be added with 0.685 of 325 to get 300?

S-11 Let 'x' is to be added, then

$x + (0.685 \times 325) = 300 \quad \Rightarrow \quad x = 300 - 222.625 = 77.375.$

E-12 A man distributes 0.375 of his money to his wife and 0.4 to his son. He has still Rs 3,375 left with him. How much initial money did the man have? How much did his wife get?

S-12 Using the relation 3 and 2, we can write directly

$[1 - (0.375 + 0.4)] \times$ total money = balance money

$\Rightarrow$ $[1 - 0.775] \times$ Total money $= 3375 \quad \Rightarrow$ Total money $= \frac{3375}{1 - 0.775} =$ Rs 15000

$\therefore$ Wife's share $= 0.375 \times$ total money $= 0.375 \times 15000 =$ Rs 5625.

E-13 Insert one fraction between

(i) $\frac{1}{3}$ and $\frac{4}{5}$ (ii) 2 and $3\frac{1}{2}$

S-13 (i) $\frac{1}{3}, \frac{4}{5}$

$= \frac{1}{3}, \frac{1+4}{3+5}, \frac{4}{5}$ (using the method 3.4)

$= \frac{1}{3}, \frac{5}{8}, \frac{4}{5}$

Thus, the resulting fraction $\frac{5}{8}$ is more than $\frac{1}{3}$ and less than $\frac{4}{5}$ in magnitude (value).

(ii) $2, 3\frac{1}{2}$

$= \frac{2}{1}, \frac{7}{2}$

$= \frac{2}{1}, \frac{2+7}{1+2}, \frac{7}{2}$ (using the method 3.4)

$= 2, 3, \frac{7}{2}$

Hence, the resulting fraction is 3.

E-14 Insert three fraction between $\frac{1}{3}$ and $\frac{4}{5}$.

S-14 $\frac{1}{3}, \frac{4}{5}$

$= \frac{1}{3}, \frac{1+4}{3+5}, \frac{4}{5} = \frac{1}{3}, \frac{5}{8}, \frac{4}{5}$ (inserting one fraction between $\frac{1}{3}$ and $\frac{4}{5}$)

$= \frac{1}{3}, \frac{1+5}{3+8}, \frac{5}{8}, \frac{5+4}{8+5}, \frac{4}{5}$ (inserting one fraction between $\frac{1}{3}$ and $\frac{5}{8}$, and one fraction between $\frac{5}{8}$ and $\frac{4}{5}$)

$= \frac{1}{3}, \frac{6}{11}, \frac{5}{8}, \frac{9}{13}, \frac{4}{5}$. (three fractions inserted between $\frac{1}{3}$ and $\frac{4}{5}$)

E-15 Which one of the following fractions is less than $\frac{1}{3}$? **(MBA '82)**

(a) $\frac{22}{63}$ (b) $\frac{4}{11}$ (c) $\frac{15}{46}$ (d) $\frac{33}{98}$

S-15 *Step 1* Reverse the test fraction, i.e. $\frac{1}{3}$ becomes $\frac{3}{1} = 3$.

Step 2 Reverse each alternative and find which alternative is greater than $\frac{3}{1}$.

$$\frac{63}{22} < \frac{3}{1}, \frac{11}{4} < \frac{3}{1}, \frac{46}{15} > \frac{3}{1}, \frac{98}{33} < \frac{3}{1}$$

$\therefore$ Required fraction is $\frac{15}{46}$.

Note: Reversing method is used because division process becomes easier when the numerator is greater than the denominator.

REGULAR PROBLEMS

(1) A badminton player, won 6 games and lost 4. The fraction of the games he won is:

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{5}$ (d) $\frac{1}{2}$ (e) $\frac{2}{5}$

(2) What fraction of 2 hours is 12 seconds?

(a) $\frac{1}{600}$ (b) $\frac{1}{12}$ (c) $\frac{1}{60}$ (d) $\frac{1}{5}$ (e) $\frac{3}{50}$

(3) A rope is $25\frac{1}{2}$ m long. How many pieces each of $1\frac{1}{2}$ m long can be cut from it?

(a) 16 (b) 21 (c) 13 (d) 11 (e) 17

(4) A lamp post has half of its length in mud, $\frac{1}{3}$ of its length in water and $3\frac{1}{3}$ m above the water. The total length of the post is:

(a) $4\frac{1}{6}$ m (b) $10\frac{1}{3}$ m (c) $16\frac{2}{3}$ m (d) 4 m (e) 20 m

(5) A man pays off $\frac{2}{5}$ of his debt and still has to pay Rs 240 to pay off the debt completely. The total amount of debt is:

(a) Rs 600 (b) Rs 400 (c) Rs 960 (d) Rs 480 (e) Rs 1200

(6) A drum of water is $\frac{3}{5}$ full. When 38 litres are drawn from it, it is just $\frac{1}{8}$ full. The half capacity of drum in litres is:

(a) 40 (b) 80 (c) 152 (d) 21.7 (e) 76

(7) The monthly salary of a man is Rs 480 and he spends $\frac{7}{8}$ of it. His income increases by $\frac{1}{6}$ of the present salary and his spending also increases by $\frac{2}{7}$ of the present expenditure. His savings will now

(a) increase by Rs 45 (b) decrease by Rs 40 (c) increase by Rs 40
(d) decrease by Rs 80 (e) decrease by Rs 60

(8) Which of the following fraction is the smallest?

(a) $\frac{7}{13}$ (b) $\frac{14}{33}$ (c) $\frac{11}{25}$ (d) $\frac{8}{15}$ (e) $\frac{9}{11}$

Hint: See 3.1.1

(9) Which of the following fraction is the greatest?

(a) $\frac{16}{21}$ (b) $\frac{11}{14}$ (c) $\frac{16}{19}$ (d) $\frac{16}{23}$ (e) $\frac{11}{17}$

(10) A man pays off $\frac{3}{20}$ of his debt every month. At the end of 6 months, his remaining debt is Rs A. How much amount has he cleared off in every month (in Rs)?

(a) $\frac{3A}{20}$ (b) $\frac{9A}{10}$ (c) $\frac{A}{10}$ (d) $\frac{3A}{10}$ (e) $\frac{3A}{2}$

(11) $\frac{3}{5}$ part of a kerosene tin is filled. If 6 bottles are taken out of it and 3 bottles are filled again, then half the tin is full. What is the capacity of the tin? (in bottles) **(RRB Guwahati, '97)**

(a) 20 (b) 30 (c) 45 (d) 50 (e) 40

(12) Reciprocal of sum of the reciprocals of $\frac{3}{5}$ and $\frac{7}{3}$ is:

(a) $\frac{1}{4}$ (b) $\frac{21}{44}$ (c) $\frac{4}{5}$ (d) $\frac{44}{21}$ (e) $\frac{15}{44}$

Hint: Start solving from backwards. First make reciprocals of $\frac{3}{5}$ and $\frac{7}{3}$ i.e. $\frac{5}{3}$ and $\frac{3}{7}$

Then sum it, as $\frac{5}{3}+\frac{3}{7}$, 8 then find reciprocal of the sum

(13) If the product of two numbers is 5 and one of the number is $\frac{3}{2}$, then what will be the sum of the numbers? **(RRB Trivendrum (Tech), '97)**

(a) $4\frac{1}{2}$ (b) $6\frac{1}{2}$ (c) $4\frac{5}{6}$ (d) 9 (e) $4\frac{2}{3}$

(14) In an examination, a student was asked to find $\frac{3}{14}$ of a certain number. By misake, he found $\frac{3}{4}$ of it. His answer was 150 more than the correct answer. The given number is:

(a) 450 (b) 300 (c) 270 (d) 180 (e) 280

(15) One of the rational numbers between $\frac{2}{7}$ and $\frac{3}{14}$ is:

(a) $\frac{5}{14}$ (b) $\frac{3}{49}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$ (e) None

Hint: Refer

(16) Which of the following fractions is the greatest? **(RBI, '98)**

(a) $\frac{219}{337}$ (b) $\frac{221}{335}$ (c) $\frac{217}{339}$ (d) $\frac{215}{341}$ (e) $\frac{222}{339}$

Hint: Do not try to calculate. The greatest fraction can be found out by eliminating first the fractions with lower numerator and greater denominator, i.e, $\frac{215}{341}, \frac{217}{339}$, and $\frac{219}{337}$. Then compare $\frac{221}{335}$ and $\frac{222}{339}$

Answers

1. (c)	**2.** (a)	**3.** (e)	**4.** (e)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (c)
10. (e)	**11.** (b)	**12.** (b)	**13.** (c)	**14.** (e)	**15.** (c)	**16.** (b)		

REAL PROBLEMS

(1) One quarter of one-seventh of a land is sold for Rs 30000. What is the value of eight-thirty fifth of the land?

(a) Rs 192000 (b) Rs 212000 (c) Rs 27428 (d) Rs 36540 (e) Rs 150000

(2) It takes 40 days for a pond to get filled with rain water. If the level of water doubles each day, then how long would it take to fill $\frac{1}{4}$ of the pond?

(a) 10 days (b) 20 days (c) 30 days (d) 35 days (e) 38 days

(3) A post is divided into three parts, the first part is $\frac{1}{3}$ of the whole length, second $\frac{3}{8}$ of the first, and the third is 6 m 50 cm. The length of the post is:

(a) 15 m (b) 10 m (c) 12 m (d) 13 $\frac{1}{2}$ m (e) 18 m

(4) In a village, $\frac{5}{8}$ of the population are adults, $\frac{1}{2}$ of the adults are male, $\frac{4}{5}$ of adult females are illiterate. If 400 females are illiterate, then the population of the village is:

(a) 2000 (b) 1500 (c) 1800 (d) 1600 (e) 1200

Hint: Assume, population of village as x

(5) In a polling booth, total number of voters is 1575, of which 0.4 part are male voters. If a candidate gets 0.6 part of male voters and 0.4 part of female voters, then find how many votes did the candidate get?

(a) 189 (b) 756 (c) 378 (d) 630 (e) 945

(6) A man spends $\frac{1}{7}$ of his salary on food and $\frac{1}{2}$ of the remaining on clothing and $\frac{1}{3}$ of the remaining on entertainment. He is still left with Rs 600. How much does he spend on entertainment? **(BSRB, '95)**

(a) Rs 600 (b) Rs 450 (c) Rs 300 (d) Rs 700 (e) Rs 500

Hint: Refer E-6

(7) From a rope 30 metres long, a person cuts off as many pieces as possible, each 3 $\frac{1}{4}$ metres long. What fraction of the whole will be left?

(a) $\frac{1}{40}$ (b) $\frac{3}{4}$ (c) $\frac{8}{13}$ (d) $\frac{7}{13}$ (e) $\frac{13}{30}$

(8) A man left $\frac{1}{7}$ of his property to his daughter and the remaining to his sons to be equally divided among them. If the share of the each son be double of that of the daughter, find the number of sons. **(NABARD, '97)**

(a) 2 (b) 3 (c) 6 (d) 4 (e) 7

(9) A vessel, full of water, weighs 16.5 kg. When the vessel is $\frac{1}{4}$ full, it weighs 5.25 kg. The weight of the empty vessel (in kg) is:

(a) 1.125 (b) 4.5 (c) 1.5 (d) 3 (e) 2.5

(10) A scooter before overhauling requires $\frac{2}{3}$ hour service time every 45 days, while after overhauling it requires $\frac{2}{3}$ hour service time every 60 days. What fraction of pre-overhauling service time is saved in the latter case? **(MBA, '81)**

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$ (e) $\frac{4}{9}$

(11) Sundari, Kusu and Jyoti took two tests each. Sundari secured $\frac{24}{60}$ marks in the first test and $\frac{32}{40}$ marks in the second test. Kusu secured $\frac{35}{70}$ marks in the first test and $\frac{54}{60}$ marks in the second test. Jyoti secured $\frac{27}{90}$ marks in the first test and $\frac{45}{50}$ marks in the second test. Who among them did register maximum progress? **(BSRB Bangalore, 2000)**

(a) Only Sundari (b) Only Kusu (c) Only Jyoti
(d) Both Sundari and Kusu (e) Both Kusu and Jyoti

Hint: Tabulate the score in each test with **common denominator** so that the progress for each person in second test over the first test can be found & compared.

		I		II		
Sundari	→	$\frac{4}{10}$	→	$\frac{8}{10}$	→	2 times $\because \frac{8}{4} = 2$
Kusu	→	$\frac{5}{10}$	→	$\frac{9}{10}$	→	Less than 2 times $\because \frac{9}{5} < 2$
Jyoti	→	$\frac{3}{10}$	→	$\frac{9}{10}$	→	Maximum progress, as in II test, score is 3 times the I test in terms of fraction

(12) A boy on being asked $\frac{13}{14}$ of a certain fraction had made the mistake of dividing the fraction by $\frac{13}{14}$ and so got an answer that exceeded the correct answer by $\frac{3}{65}$. The correct is:

(a) $\frac{14}{45}$ (b) $\frac{12}{65}$ (c) $\frac{13}{45}$ (d) $\frac{2}{7}$ (e) $\frac{196}{585}$

(13) A has twice as much money as B. They play together, and at the end of the first game, B wins one third of A's money from A; what fraction of the sum that B now has, must A win back in the second game so that they may have exactly equal money?

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{10}$ (e) $\frac{5}{18}$

Hint: Assume, before the start of first game, B has Rs 1 and A has Rs 2

(14) I bought a number of mangoes at 35 for 2. I divided the whole into two equal parts, one part of which I sold at 17, and the other at 18 mangoes per Rs 1. I spent and received an integral number of rupees, but bought the least possible number of mangoes. How many did I buy?

(a) 21420 (b) 24120 (c) 22014 (d) 1225 (e) 612

Hint: Assume that I buy 35 mangoes. Then on selling, I get $\left(\frac{35}{2} \times \frac{1}{17} + \frac{35}{2} \times \frac{1}{18}\right) = \text{Rs } \frac{225}{612}$

(No. of mangoes; Unit price; Unit price)

But the number of rupees is an integer.

so, I must receive 612 times $\frac{1225}{612}$

Hence no. of mangoes I buy is 612 times 35

(15) Find out that minimum fraction which when added to $\frac{29}{12} + \frac{15}{16}$ will give a complete number.

(a) $\frac{21}{38}$ (b) $\frac{31}{38}$ (c) $\frac{31}{48}$ (d) $\frac{17}{48}$ (e) $\frac{23}{38}$

(16) In a class, 18 boys are there whose height is more than 160 cm. If they are three-fourth of the total number of boys and the total number of boys is two-third of the total number of students, then how many girls are there in the class?

(a) 18 (b) 6 (c) 12 (d) 24 (e) 8

(17) The fuel indicator in a car shows $\frac{1}{5}$th of the fuel tank as full. When 22 more litres of fuel are poured into the tank, the indicator rests at the three-fourth of the full mark. The capacity of the fuel tank (in litres) is:

(a) 30 (b) 40 (c) 36 (d) 28 (e) 45

Answers

1. (a)	**2.** (e)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (c)
10. (d)	**11.** (c)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (b)	

4 SIMPLIFICATION AND APPROXIMATION

4.1 OPERATION ORDER SEQUENCE

For simplifying an expression containing various types of fractions, the order of various operations involved should be strictly maintained. A simple technique for arranging the expression in the proper sequence, is by placing them in the order of the first letter appearing in **VBODMAS** where.

1. **V** Stands for vinculum or bar as ($\overline{\ }$)
2. **B** stands for bracket and operation of brackets in the order ( ), { } and then [ ]
3. **O** stands for 'of'
4. **D** stands for division (÷)
5. **M** stands for multiplication (×)
6. **A** stands for addition (+)
7. **S** stands for subtraction (–)

4.2 APPLICATION FOR ALGEBRAIC FORMULA

Some algebraic formulae are used in solving the problems on simplification. Following important formula are to be memorised:

1. $(a + b)^2 = a^2 + b^2 + 2ab$.
2. $(a - b)^2 = a^2 + b^2 - 2ab$.
3. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$.
4. $(a + b)^2 - (a - b)^2 = 4ab$.
5. $(a + b) \times (a - b) = a^2 - b^2$.
6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
   $= a^3 + b^3 + 3ab\,(a + b)$.
7. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
   $= a^3 - b^3 - 3ab\,(a - b)$.
8. $a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$
10. $a^m \times a^n = a^{m+n}$
11. $a^m \div a^n = a^{m-n}$
12. $a^3 + b^3 + c^3 - 3abc = (a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca)$
    if $a + b + c = 0$, then the above identity reduces to $a^3 + b^3 + c^3 = 3abc$

4.2.1 Square Root and Square

When a number is multiplied by itself, the product obtained is called the square of the number since $6 \times 6 = 36$.

$\therefore$ 36 is the square of 6 or $6^2 = 36$

Also, $-3 \times -3 = 9 \Rightarrow (-3)^2 = 9$

$\therefore$ 9 is the square of -3

The square root of a given number is equal to the value whose square is the given number and the sign for square root is $\sqrt{\ }$

Since $6^2 = 36$ then

$\sqrt{36} = 6$, i.e. square root of 36 is 6

$\sqrt{25} = 5$, i.e. square root of 25 is 5

$\sqrt{-25}$ = an imaginary quantity

$\therefore$ Square root of a negative number is an imaginary quantity

1. $\sqrt{?} = y$
   Then the required number = y^2
2. $\sqrt{a^2 \times b^2} = ab$
3. $\sqrt{a^3 \times b^3} = ab\sqrt{ab}$
4. $\sqrt{a^4 \times b^4 \times c^4} = a^2b^2c^2$
5. $\sqrt{a^n \times b^m} = a^{n/2} \times b^{m/2}$

4.2.2 Division Method for Finding the Square Root

$\sqrt{64009} = ?$

Step 1 Pairing the digits from right to left, we get

$$6\ \overline{40}\ \overline{09}$$

Step 2 Then take the first pair, here it is only '6' and find the largest whole number whose square is equal to 6 or less than 6. Such a whole number is 2.

Step 3 Hence, write 2 in the quotient and also in the divisor. (see next page)

Step 4 Subtract $2 \times 2 = 4$ from 6. The remainder is then 2.

Step 5 Bring down the second pair of digits (i.e. 40) double the quotient (i.e. $2 \times 2 = 4$) and write the result on the left of 240 and then repeat Step-2 till the remainder is zero. The whole process can be enumerated step-by-step as shown in the following table.

2	6 $\overline{40}$ $\overline{09}$	253 = Quatient
+ 2	4	
45	2 $\overline{40}$	
+ 5	2 25	(since 45 × 5 = 225)
503	15 $\overline{09}$	(since 503 × 3 = 1509)
	15 09	
	0	

$\sqrt{\overline{64009}}$ = 253 (i.e. Quotient)

4.2.3 Properties of a Perfect Square Number

A number whose exact square root (which must be a whole number can be obtained, is called a *perfect square*.

(a) A number ending with 2, 3, 7 or 8 cannot be a perfect square.
(b) The last digit of a perfect square must be 0, 1, 4, 5, 6 or 9.
(c) A number ending with odd number of zeroes cannot be a perfect square, e.g. 9000, 25000, 16000, etc. are not perfect squares.
(d) A perfect square number is either exactly divisible by 3 or leaves a remainder of 1, when divided by 3.
e.g. 64 if divided by 3, will leave a remainder of 1
36 is exactly divisible by 3.
(e) A perfect square number is either exactly divisible by 4 or leaves a remainder of 1, when divided by 4
e.g. 81 if divided by 4, will leave a remainder of 1.
100 is exactly divisible by 4.

Note: Above properties are very useful to check if a given number is a perfect square or not.

4.2.4 Square Root of Vulgar Fraction

$$\sqrt{\frac{3}{7}} = ?$$

Step 1 Multiply the numerator and the denominator by the donominator.

Step 2 Find the square root of the new numerator and divide it by the new denominator.

$$\therefore \sqrt{\frac{3}{7}} = \sqrt{\frac{3}{7}\times\frac{7}{7}} = \frac{\sqrt{21}}{7} = \frac{4.582}{7} = 0.654$$

4.2.5 Square Root of a Rational Decimal Fraction

$\sqrt{\overline{387.09126}}$ = ?

Step 1 Pair the integer part first

$\overline{3}\,\overline{87}$

Step 2 Check the number of decimal places.
If it is odd, then affix a zero on the extreme right of decimal part to make the even number of decimal places.
Here, no. of decimal places = 5, so, after placing a zero, it becomes .091260

Step 3 Pair the decimal part accordingly

.09 12 60

Step 4 Start finding the square root by the division method as explained in 4.2.2 and put the decimal point in the square root as soon as the integer part is exhausted.

1	3 87 · 09 12 60	19.674
+ 1	1	
29	2 87	
+ 9	261 (integer part is over)	since 29 × 9 = 261
386	26 09	
6	2316	since 386 × 6 = 2316
3927	293 12	
7	27489	since 3927 × 7 = 27489
39344	1823 60	
	157376	since 39344 × 4 = 157376

The square root of 387. 09126 = $\sqrt{387.09126}$
= 19.674

4.3 SIMPLIFICATION OF DECIMAL FRACTION

The number of digits which are present on the RIGHT OF A DECIMAL POINT is called the number of decimal places.

That is, 32.0075 has four digits on the right of the decimal point. Therefore, the number is expressed to four decimal places.

A WHOLE NUMBER can also be written as a decimal fraction by putting a decimal after its LAST DIGIT and adding as many zeros as are required.
e.g. 12 = 12.0 = 12.000 and so on.

4.3.1 Addition

For addition of a decimal number with another decimal number or with another whole number write the given number in such a way that the number of decimal places are equal for all the numbers.

e.g. 1 + 0.59 + 0.008

Here maximum number of decimal places = 3 (three) in 0.008.
Convert all the number so that they have 3 decimal places.
∴ 1 + 0.59 + 0.008 = 1.000 + 0.590 + 0.008 = 1.598

4.3.2 Subtraction

In subtraction also, the given numbers are to be wirtten in such a way that the number of decimal places become equal for all the numbers (empty places are filled up with zeroes).

e.g. 2 – 0.283
In 0.283, number of decimal places = 3
In 2, number of decimal places = 0
So, make 2 as having 3 decimal places, i.e. 2.000
∴ 2 – 0.283 = 2.000 – 0.283 = 1.717.

4.3.3 Multiplication

0.005 × 0.08 × 0.4= ?

Step 1 Multiply the number only, i.e. 5 × 8 × 4 = 160

Step 2 Add the total number of decimal places in the given number, i.e. 3 + 2 + 1 = 6

Step 3 Write the result of Step 1 and convert it to a number with decimal places as obtained in Step 2 by shifting the decimal point to the left.

i.e. by six decimal places, we then get

$0.005 \times 0.08 \times 0.4 = 0.000160 = 0.00016$

Similarly $0.03 \times 0.7 \times 2 = 0.042$

Total of 3 decimal places

4.3.4 Division of Decimals

(a) When the Divisor (or Denominator) is a Whole Number

e.g. $\frac{3.0056}{7}$

Step 1 Simply divide the number without considering the decimal points given i.e. 7) 30056 (4293.7

Step 2 Count the no. of decimal places in the given number. Here it has 4 decimal places in 3.0056.

Step 3 Shift the decimal point in the quotient obtined to the same no. of decimal places as in Step-2

Hence the result becomes: $\overset{\text{Shift}}{\overbrace{42937}} = 0.42937$

(b) When the Divisor (Denominator) is also a Decimal Number

e.g. $\frac{12.598}{27.08 \times 1.417}$

Step 1 Shift the decimal point to the right of the numerator and of the denominator such that
— total decimal point shift in numerator = total decimal point shift in denominator.
— there is no decimal place left after the shift.

Here, no. of decimal place in numerator (in 12.598) = 3
no. of decimal place in denominator (in 27.08 and 1.417) = 2 + 3 = 5
since 5 > 3, so, total shift in decimal point to be made (in numerator and denominator) = 5
Now, 5 decimal point shifts are made.

$$\frac{\overset{\text{5 shift}}{\overbrace{1259800}}}{\underset{\text{2 shift}}{\underbrace{2708}} \times \underset{\text{3 shift}}{\underbrace{1417}}} = \frac{1259800}{2708 \times 1417}$$

Step 2 Division process is continued with the resulting fraction obtained in step 1.

4.4 SIMPLIFICATION OF A MIXED FRACTION

A mixed fraction consists of two parts, the integer part and the fractional part.

e.g. $2\frac{7}{18}$ has 2 as an integer and $\frac{7}{18}$ as a fraction.

In fact $2\frac{7}{18} = 2 + \frac{7}{18}$

4.4.1 Addition

$12\frac{5}{8} + 13\frac{7}{11} = ?$

Step 1 Add the integer part only i.e. 12 + 13 = 25

Step 2 Add the fractional part only i.e. $\frac{5}{8} + \frac{7}{11} = 1\frac{23}{88}$

Step 3 Add the results obtained in Step 1 and Step 2

$\therefore \quad 12\frac{5}{8} + 13\frac{7}{11} = 25 + 1\frac{23}{88} = 26\frac{23}{88}$

4.4.2 Subtration

$10\frac{3}{7} - 18\frac{1}{6} = ?$

Step 1 Subtract the integer part only, i.e. 10 – 18 = – 8

Step 2 Subtract the fraction part only, i.e. $\frac{3}{7} - \frac{1}{6} = \frac{11}{42}$

Step 3 Add the result obtained in Step 1 and Step 2

Hence, $10\frac{3}{7} - 18\frac{1}{6} = -8 + \frac{11}{42} = -8 + 1 - 1 + \frac{11}{42} = -(8-1) - \left(\frac{42-11}{42}\right) = -7 - \frac{31}{42}$

$= -\left(7 + \frac{31}{42}\right)$ *[Please refer to Section 4.4.3]*

$= -7\frac{31}{42}$

Similarly $12\frac{9}{11} - 15\frac{5}{8} = (12 - 15) + \left(\frac{9}{11} - \frac{5}{8}\right)$

$= -3 + \frac{17}{88} = -(3-1)\ \frac{88-17^{*}}{88} = -2\frac{71}{88}$

*Explanation of in between steps of adding 1 & subtracting 1 have been explained in the previous problem, however, this step can be directly obtained after little practice.

(iii) $12\frac{5}{7} - 10\frac{2}{3} = (12 - 10) + \left(\frac{5}{7} - \frac{2}{3}\right) = 2 + \frac{1}{21} = 2\frac{1}{21}$

(iv) $5\frac{2}{3} - 2\frac{1}{7} + 6\frac{3}{8} = (5 - 2 + 6) + \left(\frac{2}{3} - \frac{1}{7} + \frac{3}{8}\right)$

$= 9 + \left(\frac{112 - 24 + 63}{168}\right) = 9\frac{151}{168}$

4.4.3 Subtraction of a Whole Number and Fraction

$5 - \frac{31}{48} = ?$

Step 1 Subtract 1 from the whold number i.e. + (5 – 1) = (4)

Step 2 Subtract the numerator from denominator and write in the numerator i.e. $\frac{31}{48} = \frac{48 - 31}{48} = \frac{17}{48}$

Step 3 Add the results obtained in Step 1 and Step 2

i.e. $5 - \frac{31}{48} = 4\frac{17}{48}$

$\therefore 12 - \frac{11}{52} = (12 - 1)\ \frac{52 - 11}{52} = 11\frac{41}{52}$

and $-6 + \frac{23}{36} = -(6 - 1)\ \frac{36 - 23}{36} = -5\frac{13}{36}$

4.4.4 Easy Method For Simplification

5542 + ? + 1369 = 4200

Step 1 First always put 'x' for (?)

Step 2 Proceed and follow the rules to find the value for 'x' (or finding the value of?)

4.4.5 Multiplication of a Whole Number and a Fraction

$4 \times 16\frac{2}{3} = ?$

Step 1 Multiply the integer part by the whole number

i.e. $4 \times 16 = 64$

Step 2 Multiply the fraction part by the whole number i.e. $4 \times \frac{2}{3} = 2\frac{2}{3}$

Step 3 Add the results obtained in Step 1 and Step 2

i.e. $64 + 2\frac{2}{3} = 66\frac{2}{3}$

$\therefore 4 \times 16\frac{2}{3} = 66\frac{2}{3}$

4.4.6 Division of Mixed Fraction by a Whole Number

$16\frac{2}{3} \div 4 = ?$

Step 1 Divide the integer part by the whole number.

i.e. $\frac{16}{4} = 4$

Step 2 Divide the fractional part by the whole number.

i.e. $\frac{2}{3} \div 4 = \frac{1}{6}$

Step 3 Add the results obtained Step 1 and Step 2.

i.e. $4 + \frac{1}{6} = 4\frac{1}{6}$

$\therefore \quad 16\frac{2}{3} \div 4 = 4\frac{1}{6}$

4.5 CONTINUED FRACTIONS AND ITS SIMPLIFICATION

Fractions of the form

(a) $7 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{3}}}$ or (b) $\cfrac{1}{2 - \cfrac{3}{8 + \cfrac{1}{4 - \cfrac{1}{5}}}}$

are called continued fractions

Simplification Rule

To simplify a continued fraction begin at the bottom and work upwards.

Example: Simplify $\cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{6}}}}$

$= \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{\frac{7}{6}}}}$ $= \cfrac{1}{3 + \cfrac{1}{5 + \frac{6}{7}}}$

$= \cfrac{1}{3 + \cfrac{1}{\frac{41}{7}}}$ $= \cfrac{1}{3 + \frac{7}{41}}$

$$= \frac{1}{\frac{130}{41}}$$

$$= \frac{41}{131}$$

4.6 RECURRING DECIMALS

A decimal fraction in which a digit or set of digits is repeated continually is called a Recurring or Periodic decimal.

e.g. $\frac{1}{3} = 0.333...$

Here, on performing the division, it is found that the remainder is always 1 and in the quotient, the digit 3 is continually repeated. Hence it is written as $0.\dot{3}$, where the *dot* over 3 indicates that the 3 has to be continually repeated.

Similarly, $\frac{1}{7}$ is

7) 1.000000 (0.142857 142857...

So, if we continue the division, we shall get the same set of figures 142857 again and again and in the same order.

Therefore $\frac{1}{7} = 0.\dot{1}42857\dot{}$ or $0.\overline{142857}$

The repeated digits or repeated set of digits is called the period of the recurring decimal.
There are two types of Recurring Decimals.

a) *Pure recurring decimal:* Such a decimal in which all the decimal digits recur. e.g. $0.\dot{1}42857\dot{}$

b) *Mixed recurring decimal:* Such a decimal in which all the decimal digits do not recur, e.g. $0.716\dot{7}$

4.6.1 Conversion of a Pure Recurring Decimal to the form $\frac{p}{q}$

Steps (a) Write the decimal part without the decimal point as the numerator.
(b) Write as many 9s as there are different repeating digits for the denominator.

e.g. $0.\overline{587} = \frac{587}{999}$, $3.\overline{17} = 3 + \frac{17}{99}$
↓
(remains unchanged as it is integer part)

4.6.2 Conversion of a Mixed Recurring Decimal to the form $\frac{p}{q}$

Step (1) First, write the decimal part without the decimal point and subtract the non-repeating part from it and write the result in the numerator

Step (2) Write a number in the denominator with as many 9s as there are repeating digits in the decimal part and followed by as many zeroes as there are non-repeating digits in the decimal part.

Example: Express $7.00\dot{8}\dot{1}$ as vulgar fraction $\left(\frac{p}{q} \text{ form}\right)$

Solution

$$7.0\dot{4}8\dot{1}$$

$$= 7\frac{481 - 04}{9900}$$

$$= 7\frac{477}{9900}$$

4.7 IMPORTANT DERIVATIONS

(i) $\left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{+m/n}$

(ii) $\left(-\frac{a}{b}\right)^{-m/n} = \left(-\frac{b}{a}\right)^{+m/n}$

(iii) If $a \div b$, then $a = c \times b$ and $a \div c = b$

(iv) $N = \left[\frac{N+1}{2}\right]^2 - \left[\frac{N-1}{2}\right]^2$

4.8 APPROXIMATE VALUE

In this type of questions, candidates do not have to find out the exact value, but all they have to do is

Step 1 To round off the numbers given in the question

Step 2 To simplify

Step 3 To round off the result obtained in Step-2

Very Important: In some of the questions, the choices given are very close to each other. In such case, Step-1 is to be avoided, and we should go directly to Step-2.

4.8.1 Rounding Off Numbers

On some occasions for ease in simplification, we require only a rough estimation and not the exact value. In such cases we round off the values to the nearest tens, or hundreds or thousands.

Rounding off a number to the nearest ten, hundred or thousand means finding the multiple of 10, 100 or 1000 which is closest to (or approximate) the original number. It can be done by the following procedure.

(a) rounding off to the nearest 10: Replace the digit at unit's place by 0. If the replaced digit is 5 or more, then add 1 to the digit at tens place, otherwise digit at tens place remains unchanged.

e.g. $47 \xrightarrow[\text{off}]{\text{rounded}} 50$

↓

>5, so 1 is added to digit at tens place i.e. 1 + 4 = 5 (digit at tens place after rounding off)

But, $92 \longrightarrow 90$

↓

less than 5, so, 9 (digit at tens place) remains unchanged.

$75 \longrightarrow 80$

↓

equal to 5, so 1 is added to 7 i.e. 1 + 7 = 8 becomes the ten's place digit

$295 \longrightarrow 300$

b. **rounding off to the nearest 100:** Replace the digit at unit and tens places by 00. If the replaced digit at **tens place** is 5 or more, then add 1 to the digit at hundreds place, otherwise the digit at hundreds place remains unchanged.

$$\overset{+1}{\longrightarrow}$$

e.g. $264 \longrightarrow 300$

↓

>5, so 1 is added to digit at hundreds place i.e. 1 + 2 = 3

$5660 \longrightarrow 5700$, because 660 is rounded off as 700.

$541 \longrightarrow 800$

c. **rounding off to the nearest 1000:** Replace the ones, tens and hundreds digits by 000. If the replaced digit at hundreds place is 5 or more, then add 1 to the digit at thousands place, otherwise not.

$1973 \longrightarrow 2000$.

NB: Whether a given number is to be rounded off to the nearest 10, or 100 or 1000, it depends on the other numbers involved in the simplification. It will be explained in the examples provided subsequently.

4.8.2 Rounding off a Number to a Decimal Place

To round off a number to the rth decimal place; following steps are to be checked.

Step 1 Check the digit immediately, next right to the rth place.

Step 2 If the next right digit is 5 or more, then add 1 to the digit in the rth place, otherwise the digit remains unchanged.

Step 3 Delete all the digits in places to the right of the rth place.

e.g. $5.792 \xrightarrow[\text{to 2nd place}]{\text{rounded off}} 5.79$

$5.795 \longrightarrow 5.8$

Example: What approximate value should come in place of (?) in the following equation

$9876 \div 24.96 + 215.005 - ? = 309.85$

Sol. $9876 \div 24.96 + 215.005 - ? = 309.85$

Put x in place of ?, then approximating the terms to the nearest values.

$9900 \div 25 + 215 - x = 310$

$\Rightarrow \quad x = \dfrac{9900}{25} + 215 - 310$

$\Rightarrow \quad x = 396 + 215 - 310$
$= 301 \approx 300$

Therefore the approximate value in place of (?) is 300.

Example: What approximate value should come in place of (?) in the following equation.
$96895 + 589 + 22497 = ?$
(a) 120000 (b) 125000 (c) 122000 (d) 99000 (e) 130000.

Solution: Here, out of the 5 choices, the values in three choices are very close to each other. Now, put (x) in place of (?) and after approximating, we get

$x = 96895 + 589 + 22497$
$\approx 96900 + 600 + 22500$
$= 120000$

Hence answer is (a)

Solved Examples

E-1 Simplify

$$10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - \overline{6-4})\}\right]$$

S-1

$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7-2)\}\right]$ $\quad \overline{6-4} = 2$ *(V)*

$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6-5\}\right]$ $\quad (7-2) = 5$ *(B)*

$= 10\frac{1}{2} - \left[8\frac{1}{2} + 1\right]$ $\quad \{6-5\} = 1$ *(B)*

$= 10\frac{1}{2} - 9\frac{1}{2} = \mathbf{1}.$ $\quad \left[8\frac{1}{2}+1\right] = 9\frac{1}{2}$ *(B)*

E-2 $0.75 \times 0.75 + 0.25 \times 0.75 \times 2 + 0.25 \times 0.25$

S-2 Let $0.75 = a$ and $0.25 = b$
By 4.2 (1), we have $a^2 + 2ab + b^2 = (a + b)^2$
$= (0.75 + 0.25)^2 = \mathbf{1}.$

E-3 Simplify $\dfrac{(0.87)^3 + (0.13)^3}{(0.87)^2 + (0.13)^2 - (0.87 \times 0.13)}$

S-3 Let $0.87 = a$ and $0.13 = b$

$$\therefore \quad \frac{a^3 + b^3}{a^2 + b^2 - ab} = \frac{(a+b)(a^2 - ab + b^2)}{(a^2 + b^2 - ab)} = a + b \qquad \textit{[Refer 4.2]}$$

$= 0.87 + 0.13 = \mathbf{1}.$

E-4 Find the missing number.

$$\frac{9840}{\sqrt{?}} = 410$$

S-4 ∴ Required Number = $\left(\frac{9840}{410}\right)^2$ *[Refer 4.2.1]*

$= (24)^2 = \mathbf{576}$.

E-5 Simplify $(40^2 - 30^2) = 10 \times ?$

S-5 Let $40 = a$, $30 = b$, required number $= x$

$a^2 - b^2 = (a + b)(a - b)$

∴ Required Number $(x) = \frac{40^2 - 30^2}{10} = \frac{(40 + 30)(40 - 30)}{10} = \mathbf{70}$.

E-6 $3^5 \times 3^6 \div 3^9 = ?$

S-6 here $a = 3$ and the base is the same. Now, on multiplication, the exponents are added, and on division the difference of the exponents are taken. Now applying 4.2 (10) and (11), we get the required number $(x) = 3^{(4 + 6 - 9)} = 3$

E-7 $2 + \sqrt{2} + \frac{1}{2 + \sqrt{2}} + \frac{1}{\sqrt{2} - 2} = ?$ **(AGE '93)**

S-7 $2 + \sqrt{2} + \left[\frac{\sqrt{2} - 2 + 2 + \sqrt{2}}{(2 + \sqrt{2})(\sqrt{2} - 2)}\right]$ [Since $a = \sqrt{2}$ $b = 2$

$= 2 + \sqrt{2} + \frac{2\sqrt{2}}{2 - 4} \Rightarrow 2 + \sqrt{2} + \frac{2\sqrt{2}}{-2} = \mathbf{2}$. ∴ $(a + b)(a - b) = a^2 - b^2$]

E-8 If $x * y = (x + 2)^2 (y - 2)$ then $7 * 5 = ?$

S-8 Substituting $x = 7$ and $y = 2$, we get.

$7 * 5 = (7 + 2)^2 (5 - 2)$

$= (9)^2 \times 3 = \mathbf{243}$.

E-9 If m and n are whole numbers such that $m^n = 121$, then $(m - 1)^{n + 1} = ?$

S-9 Given that $m^n = 121 \Rightarrow m^n = (11)^2$

Hence $m = 11$ and $n = 2$, and substituting these values,

$(m - 1)^{n + 1} = (11 - 1)^{2 + 1} = 10^3 = \mathbf{1000}$.

E-10 If $\frac{x}{y} = \frac{3}{4}$, then $\frac{6}{7} + \frac{(y - x)}{(y + x)} = ?$

S-10 Substruting $x = 3k$

$y = 4k$, we get.

$$\frac{6}{7} + \frac{(y - x)}{(y + x)} = \frac{6}{7} + \frac{4k - 3k}{4k + 3k}$$

$$= \frac{6}{7} + \frac{1}{7} = \mathbf{1}.$$

E-11 If $\sqrt{x} - \sqrt{y} = 1$ and $\sqrt{x} + \sqrt{y} = 17$, then $\sqrt{xy} = ?$

S-11 $\sqrt{x} + \sqrt{y} = 17$ **(i)**

and, $\sqrt{x} - \sqrt{y} = 1$ **(ii)**

Adding equations (i) and (ii) $\sqrt{x} = 9$

Subtracting equation (ii) from (i), $\sqrt{y} = 8$

Substituting these values,

$\sqrt{xy} = \sqrt{x} \times \sqrt{y} = 9 \times 8 =$ **72.**

E-12 $\dfrac{? \div 12}{0.2 \times 3.6} = 2$

S-12 Putting x in place of ?, we get

$$\frac{x \div 12}{0.2 \times 3.6} = 2$$

$$\Rightarrow \quad x \div 12 = 2 \times 0.2 \times 3.6 \Rightarrow \frac{x}{12} = 2 \times 0.2 \times 3.6$$

$$\Rightarrow \quad x = 12 \times 2 \times 0.2 \times 3.6 = \textbf{17.28}.$$

E-13 $\sqrt{? \times 7} \times 18 = 84$

S-13 Subsituting x for ?, we get,

$$\sqrt{x \times 7} \times 18 = 84$$

$$\Rightarrow \quad \sqrt{x \times 7} = \frac{84}{18}$$

$$\Rightarrow \quad \left(\sqrt{x \times 7}\right)^2 = \left(\frac{84}{18}\right)^2 \qquad \text{(squaring both sides)}$$

$$\Rightarrow \quad x \times 7 = \left(\frac{84}{18}\right)^2 \Rightarrow x = \frac{84 \times 84}{18 \times 18 \times 7} = \textbf{3.11}.$$

E-14 $\left(2\frac{3}{x}\right) \times \left(y\frac{1}{2}\right) = 7\frac{3}{4}$ **(MBA, '82)**

S-14 Taking the quotients 2, y and 7, we get

$2y = 7$, which gives the quotient as 3 $\left(\text{Since } y = \frac{7}{2} = 3\frac{1}{2}\right)$

$\therefore$ $y = 3$. Substituting the value of y, we get,

$$2\frac{3}{x} \times 3\frac{1}{2} = 7\frac{3}{4}$$

Now, $\dfrac{7\frac{3}{4}}{3\frac{1}{2}} = 2\frac{3}{x} \Rightarrow 2\frac{3}{14} = 2\frac{3}{x}$

$\therefore$ $x = \mathbf{14}$ and $y = \mathbf{3}$

E-15 $2^{2^?} = 256$ **(IA, '87)**

S-15 Putting x for ?, we get

$2^{2^x} = 256 = 2^8$

$\Rightarrow$ $2^{2^x} = 2^{2^3}$

$\therefore$ $x = \mathbf{3}$.

E-16 $2^{x+13} = 4^{x+2}$ then $x = ?$

S-16 $2^{x+13} = (2^2)^{x+2}$

$\Rightarrow$ $2^{x+13} = 2^{2x+4}$

$\Rightarrow$ $x + 13 = 2x + 4 \Rightarrow x = \mathbf{9}$.

E-17 $3\sqrt{27} - \sqrt{75} + \sqrt{12} = ?$

S-17 $3\sqrt{27} - \sqrt{75} + \sqrt{12}$

$= 3\sqrt{3\times3\times3} - \sqrt{5\times5\times3} + \sqrt{2\times2\times3}$

$= 3 \times 3\sqrt{3} - 5\sqrt{3} + 2\sqrt{3}$

$= (9 - 5 + 2)\sqrt{3} = \mathbf{6\sqrt{3}}$

E-18 If $5\frac{3}{x} \times y\frac{1}{2} = 19$, then the value of $(x, y) = ?$

S-18 Equating the quotients of both sides,

$5y = 19$, we get the quotient as 3. $\left(\text{Since } \frac{19}{5} = 3\frac{4}{5}\right)$

$\therefore$ $y = 3$

Hence, $5\frac{3}{x} \times 3\frac{1}{2} = 19$

$\Rightarrow$ $5\frac{3}{x} = \frac{19}{\frac{7}{2}} \Rightarrow 5\frac{3}{x} = 5\frac{3}{7}$

$\therefore$ $x = 7$ so $x = \mathbf{7}$ and $y = \mathbf{3}$.

E-19 $8\frac{1}{4} + 8\frac{1}{2} + ? = 20\frac{1}{8}$

S-19 Substituting x for ?, we get,

$8\frac{1}{4} + 8\frac{1}{2} + x = 20\frac{1}{8}$

$\Rightarrow x = 20\frac{1}{8} - 8\frac{1}{4} - 8\frac{1}{2}$

$= (20 - 8 - 8) + \left(\frac{1}{8} - \frac{1}{4} - \frac{1}{2}\right)$ *[Refer 4.4.2]*

$= 4 + \left(\frac{1-2-4}{8}\right)$

$= 4 + \frac{-5}{8} = (4 - 1)\ \frac{8-5}{8}$ *[Refer 4.4.3]*

$= \mathbf{3\frac{3}{8}}$.

E-20 $\frac{\sqrt{1296}}{?} = \frac{?}{2.25}$

S-20 Putting x for (?), we get *[Refer 4.4.4]*

$\sqrt{1296} \times 2.25 = x^2$

$36 \times 2.25 = x^2 \Rightarrow x = \sqrt{36 \times 2.25} \Rightarrow x = 6 \times 1.5$ (Since $\sqrt{1296} = 36$)

$\therefore\ x = \mathbf{9}$.

E-21 65% of ? = 124.90 − 63.15

S-21 Putting x for (?),

$\frac{65}{100}$ of $x = 61.75$

$\Rightarrow x = \frac{61.75}{65} \times 100 \Rightarrow x = \mathbf{95}$.

E-22 If $\frac{a}{a+b} = \frac{17}{23}$, what is $\frac{a+b}{a-b}$ equal to? **(IA, '79)**

S-22 Given that $\frac{a}{a+b} = \frac{17}{23}$

i.e. if $a = 17$, then $a + b = 23$

or $b = 6$

$\therefore\ a - b = 17 - 6 = 11$

hence $\frac{a+b}{a-b} = \mathbf{\frac{23}{11}}$.

E-23 $\frac{\sqrt{24} + \sqrt{216}}{\sqrt{96}} = ?$

S-23 Putting x for (?) and solving for x, we get

$x = \frac{2\sqrt{6} + 6\sqrt{6}}{4\sqrt{6}} \Rightarrow x = \frac{8\sqrt{6}}{4\sqrt{6}}$

$\therefore\ x = \mathbf{2}$.

E-24 Simplify $(x + y - z)^2 - (x - y + z)^2$

S-24 Using the formula *[Refer 4.2 v]*

$a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow (x + y - z)^2 - (x - y + z)^2$

$= (x + y - z + x - y + z)(x + y - z - x + y - z) = 2x(2y - 2z)$

$= \mathbf{4xy - 4xz}$.

E-25 If $x = 5$ and $y = -2$, then what is the value of $(x - 2y)^{1/y}$?

S-25 $(x - 2y)^{1/y}$

$= (5 + 4)^{1/-2} = (9)^{1/-2} = \dfrac{1}{(9)^{1/2}}$

$= \dfrac{1}{\sqrt{9}} = \mathbf{\dfrac{1}{3}}$.

E-26 $2\frac{2}{3} \div \frac{4}{5}$ of ? $= \frac{1}{6}$

S-26 Putting x for (?) and solving for x, we get,

$\Rightarrow \dfrac{8}{3} \div \dfrac{4}{5}x = \dfrac{1}{6}$

$\Rightarrow \dfrac{8}{3} = \dfrac{1}{6} \times \dfrac{4}{5}x \Rightarrow x = \dfrac{8 \times 6 \times 5}{3 \times 4}$

$\Rightarrow x = \mathbf{20}$.

E-27 Find the value of

(i) $\frac{5}{6}$ of $\frac{1}{20}$ of 24 rupees.

(ii) $\frac{4}{7}$ of 14 times of $2\frac{1}{4}$ kg.

S-27 **(i)** $\frac{5}{6}$ of $\frac{1}{20}$ of 24 rupees.

$= \dfrac{5}{6} \times \dfrac{1}{20} \times 24$ rupees $= \mathbf{1\ rupee}$.

(ii) $\frac{4}{7}$ of 14 times of $2\frac{1}{4}$ kg.

$= \dfrac{4}{7} \times 14 \times \dfrac{9}{4}$ kg. $= \mathbf{1.8\ kg}$.

E-28 $\left(11 \div 2\frac{1}{5}\right) \div \frac{11}{5}$ of $2\frac{1}{2} - 2$

S-28 Applying **VBODMAS** Rules, we get *[Refer 4.1]*

$= \left(11 \times \dfrac{1}{2\frac{1}{5}}\right) \div \dfrac{11}{5}$ of $2\dfrac{1}{2} - 2$ [B]

$= 5 \div \dfrac{11}{5} \times \dfrac{5}{2} - 2$ [O]

$= 5 \times \frac{5}{11} \times \frac{5}{2} - 2 = \frac{125}{22} - 2$ [D][M]

$= \mathbf{3\frac{15}{22}}$. [S]

E-29 $\dfrac{\frac{2}{3}+\frac{1}{5}-\frac{1}{10}}{\frac{4}{5}\times\frac{1}{8}+\frac{1}{2}}$

S-29 $= \dfrac{\frac{20+6-3}{30}}{\frac{1}{10}+\frac{1}{2}} = \dfrac{\frac{23}{30}}{\frac{1+5}{10}}$

$= \frac{23}{30} \times \frac{10}{6} = \mathbf{1\frac{5}{18}}$

E-30 $\left(\frac{?}{18}\right) \times \left(\frac{?}{162}\right) = 1$ **(BSRB, '92)**

S-30 Putting x for (?) and solving it for x, we get

$\frac{x}{18} \times \frac{x}{162} = 1$ *[Refer 4.4.2.]*

$\Rightarrow \quad x^2 = 18 \times 162$

$\Rightarrow \quad x^2 = 18 \times 18 \times 9 \Rightarrow x = 18 \times 3$

$\therefore \quad x = \mathbf{54}$.

E-31 $\dfrac{(0.55)^2 + (0.07)^2 + (0.027)^2}{(0.055)^2 + (0.007)^2 + (0.0027)^2} = ?$

S-31 Let $0.55 = a$, $0.07 = b$ and $0.027 = c$

Then, the given expression becomes

$$\frac{a^2+b^2+c^2}{(0.1\times a)^2+(0.1\times b)^2+(0.1\times c)^2} = \frac{[a^2+b^2+c^2]}{0.01[a^2+b^2+c^2]}$$

$\frac{1}{0.01} = \mathbf{100}$.

E-32 $\dfrac{137\times137\times137+133\times133\times133}{137\times137-137\times133+133\times133} = ?$

S-32 Let $137 = a$ and $133 = b$

Then, the given expression becomes

$\frac{a^3+b^3}{a^2-ab+b^2} = \frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)} = a + b$ *[Refer 4.2 (8)]*

Putting the value of a and b, we get
$= 137 + 133 =$ **270**.

E-33 $\dfrac{20.25 \times 2.80}{28.35}$

S-33 $\dfrac{20\frac{1}{4} \times 2.80}{28.35}$ $\left(\text{Since } \quad 20..5 = 20\frac{1}{4}\right)$

$$= \frac{\frac{81}{4} \times 2.8}{28.35} = \frac{81 \times 0.7}{28.35}$$

$$= \frac{56.70}{28.35} = \mathbf{2}.$$

E-34 $54 \div 66 \div 33 = ?$

S-34 $\dfrac{54}{66} \div 33$

$$= \frac{9}{11} \div 33 = \frac{9}{11} \times \frac{1}{33} = \frac{\mathbf{3}}{\mathbf{121}}$$

E-35 $\dfrac{2?9}{4} = \dfrac{916}{16}$

S-35 Putting x for (?) we get

$$\frac{2x9}{4} = \frac{916}{16} \Rightarrow \frac{2x9}{4} = \frac{229}{4}$$

$\therefore \quad x = \mathbf{2}$

E-36 $\sqrt{\dfrac{?}{10}} = 0.011$

S-36 Putting x for (?) we get

$\sqrt{\dfrac{x}{10}} = 0.011$ (Since x is under square root)

$\dfrac{x}{10} = (0.011)^2$ (squaring both sides)

$\Rightarrow \quad x = 10 \times 0.000121$

$\therefore \quad x =$ **0.00121**.

E-37 $\sqrt{\dfrac{67.6}{?}} = 0.26$ **(SBI, '80)**

S-37 Putting x for (?) we get

$$\sqrt{\frac{67.6}{x}} = 0.26$$

$\Rightarrow \quad \frac{67.6}{x} = (0.26)^2$ (squaring both sides)

$\Rightarrow \quad x = \frac{67.6}{0.0676} \Rightarrow x = \mathbf{1000}.$

E-38 $\sqrt{\frac{0.324}{10}} = ?$

S-38 Putting x for (?), we get

$$\sqrt{\frac{0.324}{10}} = x$$

Here x is not under square root, hence squaring is not done.

$\Rightarrow \quad \sqrt{0.0324} = x$

$\Rightarrow \quad x = \sqrt{\frac{324}{10,000}} \Rightarrow x = \sqrt{\frac{18 \times 18}{100 \times 100}}$

$\Rightarrow \quad x = \frac{18}{10} \Rightarrow x = \mathbf{0.18}.$

E-39 $\frac{(63+36)^2 + (63-36)^2}{63^2 + 36^2} = ?$ **(ITI, '83)**

S-39 Putting $63 = a$ and $36 = b$ in the given expression, we get

$\Rightarrow \quad x = \frac{(a+b)^2 + (a-b)^2}{a^2 + b^2}$

$\Rightarrow \quad x = \frac{2(a^2 + b^2)}{a^2 + b^2}$ (Since $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$)

[Refer 4.2 (iii)]

$x = \mathbf{2}.$

E-40 $44.60 \times 2.50 = ?$

S-40 Putting x for (?), we get

$x = 44.60 \times 2.50 \Rightarrow x = 44.60 \times \frac{10}{4}$ $\left(\text{Since } 2.50 = \frac{10}{4}\right)$

$\Rightarrow \quad x = 11.15 \times 10$

$\Rightarrow \quad x = \mathbf{111.5}.$

E-41 $\frac{14 \times 14 - 46}{11 \times 6 - (4)^2} = ?$

S-41 Putting x for (?) and applying **VBODMAS** Rule,
We get,

$$x = \frac{196 - 46}{66 - 16} \Rightarrow x = \frac{150}{50} = \mathbf{3}.$$

E-42 $2002 - 2002 \div 10.10 = ?$ **(IA, '80)**

S-42 Putting x for (?) and applying **VBODMAS** Rule, we get

$$x = 2002 - \frac{2002}{10.10}$$

$$\Rightarrow x = 2002 - 200 \Rightarrow x = \mathbf{1802}.$$

E-43 $\frac{18 - 3 \times 4 + 2}{6 \times 5 - 3 \times 8} = ?$

S-43 Putting x for (?) and applying **VBODMAS** Rule, we get

$$\Rightarrow x = \frac{18 - 12 + 2}{30 - 24} \Rightarrow x = \frac{18 + 2 - 12}{30 - 24} \Rightarrow x = \frac{20 - 12}{30 - 24}$$ [M][A]

$$\Rightarrow x = \frac{8}{6}$$ [S]

$$\therefore x = \mathbf{\frac{4}{3}}.$$

E-44 Express 5005 into its prime factors. **(SSC, '86)**

S-44 $5005 = 5 \times 1001$

$5 \times 7 \times 143 = \mathbf{5 \times 7 \times 11 \times 13}.$

E-45 $(4^3)^4 \div (4^2)^3 \times (4^5)^0 = ?$ **(ITI, '84)**

S-45 Put x for (?) , Since all base are equal to 4, hence, put $a = 4$.

$$\Rightarrow x = (a^3)^4 \div (a^2)^3 \times (a^5)^0$$

$$\Rightarrow x = a^{12} \div a^6 \times 1 \Rightarrow x = a^{12-6} \Rightarrow x = a^6$$ (Since $(a^5)^0 = 1$)

(Since $a^m \div a^n = a^{m-n}$) *[Refer 4.2 (9)]*

$x = \mathbf{4^6}$.

E-46 Find the value of x in the equation. **(SSC, '86)**

$$\sqrt{1 + \frac{25}{144}} = 1 + \frac{x}{12}$$

S-46 $\sqrt{\frac{144 + 25}{144}} = 1 + \frac{x}{12}$

$$\Rightarrow \sqrt{\frac{169}{144}} = 1 + \frac{x}{12} \Rightarrow \frac{13}{12} = 1 + \frac{x}{12}$$

$$\Rightarrow x = \mathbf{1}.$$