E-47 If $\frac{b}{a} = \frac{1}{2}$, what is the value of the expression $\frac{a-b}{a+b} + \frac{2}{3}$

S-47 Given that $\frac{b}{a} = \frac{1}{2}$

i.e. if $b = 1$, then $a = 2$ (assume it, for convenience)

So, $\frac{a-b}{a+b} + \frac{2}{3} = \frac{2-1}{2+1} + \frac{2}{3} = \mathbf{1}$.

E-48 If $\sqrt{18 \times 14 \times x} = 84$, then x is equal to ? **(Auditors, '8**

S-48 $\sqrt{18 \times 14 \times x} = 84$

Since x is under square root, so, squaring both sides we get

$18 \times 14 \times x = 84 \times 84 \Rightarrow x = \frac{84 \times 84}{18 \times 14}$

$\therefore \quad x = \mathbf{28}.$

E-49 $(243)^{0.8} \div (243)^{0.4} = ?$ **(BSRB, '8**

S-49 Putting x for (?) and $243 = a$, we get, *[Refer 4.2 (i.*

$x = (a)^{0.8} \div (a)^{0.4} = (a)^{0.8 - 0.4}$

$\Rightarrow \quad x = a^{0.4}$

$x = (243)^{0.4}$

$x = (243)^{4/10} \Rightarrow x = (3^5)^{2/5} \Rightarrow x = 3^2$

$\Rightarrow \quad x = \mathbf{9}.$

E-50 $1.2 \times 1.2 + 0.8 \times 0.8 + 2.4 \times 0.8 = ?$

S-50 Putting x for (?) and $1.2 = a$, $0.8 = b$, we get

$x = a^2 + b^2 + 2ab \Rightarrow x = (a + b)^2$

$\Rightarrow \quad x = (1.2 + 0.8)^2 \Rightarrow x = \mathbf{4}.$

E-51 $\frac{\frac{64}{121} - \frac{9}{64}}{\frac{8}{11} + \frac{3}{8}} = ?$ **(SBI, '8**

S-51 $x = \frac{(64^2 - 9 \times 121)}{121 \times 64} \times \frac{8 \times 11}{(8 \times 8 + 3 \times 11)}$

$\Rightarrow \quad x = \frac{(64^2 - 3 \times 3 \times 11 \times 11)}{(11 \times 11 \times 8 \times 8)} \times \frac{8 \times 11}{(64 + 33)}$

$$\Rightarrow \quad x = \frac{(64+33)(64-33)}{11\times 8} \times \frac{1}{(64+33)}$$

$$\Rightarrow \quad x = \mathbf{\frac{31}{88}}.$$

E-52 $\dfrac{0.1+0.75}{2.5+0.05} \div \left(0.125 + \dfrac{1}{4.8}\right) = ?$

S-52 Putting x for (?)

$$\Rightarrow \quad x = \frac{0.85}{2.55} \div \left(\frac{1}{8} + \frac{10}{48}\right) \quad \Rightarrow \quad x = \frac{1}{3} \div \left(\frac{16}{48}\right)$$

$$\Rightarrow \quad x = \mathbf{1}.$$

E-53 $\left(\dfrac{216}{1}\right)^{-2/3} \div \left(\dfrac{27}{1}\right)^{-4/3} = ?$ **(Bank PO, '79)**

S-53 Putting x for (?), we get

$$\Rightarrow \quad \left(\frac{216}{1}\right)^{-2/3} \div \left(\frac{27}{1}\right)^{-4/3} = x \qquad \left(\text{Since } \left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{m/n}\right)$$

$$\Rightarrow \quad (6^3)^{2/3} \div (3^3)^{4/3} = x \quad \Rightarrow \quad x = \frac{6^2}{3^4} \quad \Rightarrow \quad x = \mathbf{\frac{4}{9}}$$

E-54 $8^{5/3} \div (125)^{-2/3} = ?$

S-54 Putting x for (?)

$$(2^3)^{5/3} \times (125)^{2/3} = x \qquad (\text{Since } a^m \div b^{-n} = a^m \times b^n)$$

$$\Rightarrow \quad x = 2^5 \times (5^3)^{2/3} \quad \Rightarrow \quad x = 32 \times 25$$

$$\therefore \quad x = \mathbf{800}.$$

E-55 $\left(\dfrac{1}{6^{-2}}\right) \times (81)^{-3/4} = ?$

S-55 Putting x for (?) and solving

$$\frac{6^2}{1} \times \left(\frac{1}{81}\right)^{3/4} = x \qquad \left(\text{Since } \left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{m/n}\right)$$

$$\Rightarrow \quad x = \frac{6^2}{(3^4)^{3/4}} \quad \Rightarrow \quad x = \frac{6\times 6}{3\times 3\times 3}$$

$$x = \mathbf{\frac{4}{3}}.$$

E-56 $\sqrt{147}+\sqrt{27} = ? \times \sqrt{3}$

S-56 Putting x for (?) and solving it for x,

$$\sqrt{3\times7\times7}+\sqrt{3\times3\times3} = x\times\sqrt{3}$$

$$\Rightarrow \quad x\times\sqrt{3} = 7\sqrt{3}+3\sqrt{3} \quad \Rightarrow \quad x\sqrt{3} = 10\sqrt{3}$$

$$\therefore \quad x = \mathbf{10}.$$

E-57 $\sqrt{98}-\sqrt{50} = ? \times \sqrt{2}$ **(Bank PO, '80)**

S-57 Putting x for (?) and solving it for x,

$$\sqrt{7\times7\times2}-\sqrt{5\times5\times2} = x\times\sqrt{2}$$

$$\Rightarrow \quad 7\sqrt{2}-5\sqrt{2} = x\times\sqrt{2}$$

$$\therefore \quad x = \mathbf{2}.$$

E-58 $\sqrt{\frac{9}{25}}+\sqrt{3\frac{1}{16}} = ?$

S-58 $\frac{3}{5}+\sqrt{\frac{49}{16}} = x \quad \Rightarrow \quad x = \frac{3}{5}+\frac{7}{4} \quad \Rightarrow \quad x = \mathbf{2\frac{7}{20}}$

E-59 $\sqrt{0.01+\sqrt{0.0064}} = ?$

S-59 Putting x for (?) and solving it for x,

$$\sqrt{0.01+0.08} = x \quad \Rightarrow \quad \sqrt{0.09} = x$$

$$\therefore \quad x = \mathbf{0.3}$$

E-60 $\sqrt{\frac{25.6}{36.1}} \div \sqrt{\frac{12.1}{81\times0.1}} = ?$

S-60 Putting x for (?), we get

$$\sqrt{\frac{256}{361}} \div \sqrt{\frac{121}{81}} = x \quad \Rightarrow \quad x = \frac{16}{19}\div\frac{11}{9}$$

$$\therefore \quad x = \frac{16\times9}{11\times19} = \mathbf{\frac{144}{209}}$$

E-61 Express the number 51 as the difference of squares of two numbers. **(Bank PO, '82)**

S-61 Using the formula

$$N = \left[\frac{N+1}{2}\right]^2 - \left[\frac{N-1}{2}\right]^2, \text{ where } N = \text{ original number}$$

Put $N = 51$

$$\Rightarrow \quad 51 = \left[\frac{51+1}{2}\right]^2 - \left[\frac{51-1}{2}\right]^2$$

$$\Rightarrow \quad \mathbf{51 = (26)^2 - (25)^2}.$$

E-62 Find the number whose seventh part multiplied by its eleventh part gives 1,232.

S-62 Let x be the number such that

$$\frac{x}{7} \times \frac{x}{11} = 1232$$

$\Rightarrow x^2 = 7 \times 11 \times 1232 \Rightarrow x^2 = 7 \times 11 \times 7 \times 11 \times 4 \times 4$

$\Rightarrow x = 7 \times 11 \times 4 \Rightarrow 308$

$\therefore$ the number is **308**.

E-63 Find the square root of $\dfrac{\left(3\frac{1}{4}\right)^4 - \left(4\frac{1}{3}\right)^4}{\left(3\frac{1}{4}\right)^2 - \left(4\frac{1}{3}\right)^2}$

S-63 Let $\left(3\frac{1}{4}\right)^2 = a$ and $\left(4\frac{1}{3}\right)^2 = b$, then

$$\text{Given expression} = \frac{a^2 - b^2}{a - b} = a + b$$

$$= \left(3\frac{1}{4}\right)^2 + \left(4\frac{1}{3}\right)^2 = \left(\frac{13}{4}\right)^2 + \left(\frac{13}{3}\right)^2$$

$$= \frac{169}{16} + \frac{169}{9} = 169\left(\frac{1}{16} + \frac{1}{9}\right) = 169 \times \frac{25}{144}$$

$$\text{Required square root} = \sqrt{169 \times \frac{25}{144}} = \sqrt{13^2 \times \frac{5^2}{12^2}}$$

$$= \frac{65}{12} = \mathbf{5\frac{5}{12}}$$

E-64 The highest score in an innings was $\frac{2}{9}$ of the total score and the next highest was $\frac{2}{9}$ of the remainder. These scores differed by 8 runs. What was the total score in the innings? **(NDA, '88)**

S-64 Let the total score be x runs, such that

$$\frac{2}{9}x - \frac{2}{9} \times \left(x - \frac{2}{9}x\right) = 8 \Rightarrow \frac{2}{9}x - \frac{2}{9} \times \frac{7}{9}x = 8$$

$$\Rightarrow \frac{2}{9}x \times \frac{2}{9} = 8 \Rightarrow x = 162$$

$\therefore$ The total score in the innings was **162**

E-65 Simplify

$$\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$$

(SSC, '94)

S-65 $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$

$= 1 + (8^2)^{-1/2} + (-1 \times 32)^{4/5}$

$= 1 + 8^{-1} + [(-1)^{4/5} \times (32)^{4/5}$

$= 1 + \frac{1}{8} + [(-1^2)^{2/5} \times (2^5)^{4/5}] = 1 + \frac{1}{8} + [2^4] = \mathbf{17\frac{1}{8}}$.

E-66 Simplify

$(243)^{0.12} \times (243)^{0.08}$ **(SSC, '94)**

S-66 $(243)^{0.12} \times (243)^{0.08}$

$= (243)^{0.12 + 0.08} = (243)^{0.2}$ (Since $a^m \times a^n = a^{m+n}$)

$= (3^5)^{1/5} = \mathbf{3}$.

E-67 If $\sqrt{2^n} = 64$, then find the value of n **(AGE, '90)**

S-67 $\sqrt{2^n} = 64$

$\Rightarrow \quad (2^n)^{1/2} = 2^6 \quad \Rightarrow \quad \frac{n}{2} = 6$ (Since bases are same)

$\therefore \quad n = \mathbf{12}$.

E-68 $\left(\frac{-1}{216}\right)^{-2/3} = ?$ **(SSC, '94)**

S-68 Putting x for (?), we get

$x = \left(\frac{-1}{216}\right)^{-2/3}$

$\Rightarrow \quad x = (-216)^{2/3} \quad \Rightarrow \quad x = (-6^3)^{2/3}$ $\left(\text{Since } \left(-\frac{a}{b}\right)^{-m/n} = \left(-\frac{b}{a}\right)^{m/n}\right)$

$\Rightarrow \quad x = \mathbf{36}$.

E-69 $(-2)^{-(2)^{(-2)}} = ?$ **(SSC, '94)**

S-69 Putting x for (?)

$x = (-2)^{-(2)^{(-2)}}$

$\Rightarrow \quad x = \left(-\frac{1}{2}\right)^{(2)^{(-2)}}$ $\left(\text{Since } \left(-\frac{a}{b}\right)^{-m/n} = \left(-\frac{b}{a}\right)^{m/n}\right)$

$\Rightarrow \quad x = \left[\frac{1}{4}\right]^{-2}$ $\left(\text{Since } \left(\frac{-1}{2}\right)^2 = \frac{1}{4}\right)$

$\Rightarrow \quad x = (4)^2 \quad \Rightarrow \quad x = \mathbf{16}$.

E-70 $11\frac{1}{3} \times 4\frac{8}{10} \div ? = 22\frac{2}{3}$ **(NDA, '83)**

S-70 Putting x for (?), we get

$$11\frac{1}{3} \times 4\frac{8}{10} \div x = 22\frac{2}{3}$$

$$\Rightarrow \quad 11\frac{1}{3} \times 4\frac{8}{10} = 22\frac{2}{3} \times x \qquad \text{(If } a \div b = c \text{ then } a = b \times c\text{)}$$

$$\Rightarrow \quad x = \frac{1}{2} \times 4\frac{8}{10} \qquad \left(\text{Since } \frac{11\frac{1}{3}}{22\frac{2}{3}} = \frac{1}{2}\right)$$

$$\Rightarrow \quad x = \mathbf{2.4}.$$

E-71 $(1.06 + 0.04)^2 - ? = 4 \times 1.06 \times 0.04$ **(CDS, '80)**

S-71 Putting x for (?) and solving for it

$(1.06 + 0.04)^2 - x = 4 \times 1.06 \times 0.04$

Assume, $1.06 = a$ and $0.04 = b$

$\therefore \quad (a + b)^2 - x = 4ab$

$\therefore \quad x = (a - b)^2 = (1.06 - 0.04)^2 = \mathbf{1.0404}.$ $[(a + b)^2 - (a - b)^2 = 4ab]$

E-72 If $a^2 + b^2 = 45$ and $ab = 18$, find $\frac{1}{a} + \frac{1}{b}$ **(MBA, '87)**

S-72 $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$

$$= \frac{\sqrt{a^2 + b^2 + 2ab}}{ab} \qquad \text{(Since } a + b = \sqrt{(a+b)^2}\text{)}$$

$$= \frac{\sqrt{45 + 2 \times 18}}{18} = \frac{\pm 9}{18} = \pm\frac{\mathbf{1}}{\mathbf{2}}$$

E-73 If $\frac{a^2 + b^2}{c^2 + d^2} = \frac{ab}{cd}$, then find the value of $\frac{a+b}{a-b}$ in terms of c and d only. **(MBA, '87)**

S-73 $\frac{a^2 + b^2}{c^2 + d^2} = \frac{ab}{cd} \Rightarrow \frac{a^2 + b^2}{c^2 + d^2} = \frac{2ab}{2cd}$

$$\Rightarrow \quad \frac{a^2 + b^2 + 2ab}{c^2 + d^2 + 2cd} = \frac{a^2 + b^2 - 2ab}{c^2 + d^2 - 2cd} \qquad \text{(By componendo and dividendo)}$$

$$\Rightarrow \quad \left(\frac{a+b}{a-b}\right)^2 = \left(\frac{c+d}{c-d}\right)^2$$

$$\therefore \quad \frac{\boldsymbol{a+b}}{\boldsymbol{a-b}} = \pm\frac{\boldsymbol{c+d}}{\boldsymbol{c-d}}$$

E-74 Simplify

$$\frac{a^{1/2}+a^{-1/2}}{1-a}+\frac{1-a^{-1/2}}{1+\sqrt{a}}$$ **(MBA, '87)**

S-74 $\dfrac{a^{1/2}+a^{-1/2}}{1-a}+\dfrac{1-a^{-1/2}}{1+\sqrt{a}}$

$$=\frac{a^{1/2}+a^{-1/2}}{(1+a^{1/2})(1-a^{1/2})}+\frac{1-a^{1/2}}{1+a^{1/2}} \qquad [\text{Since } 1-a=(1)^2-(a^{1/2})^2=(1+a^{1/2})(1-a^{1/2})]$$

$$=\frac{a^{1/2}+a^{-1/2}+(1-a^{-1/2})(1-a^{1/2})}{(1+a^{1/2})(1-a^{1/2})}$$

$$=\frac{a^{1/2}+a^{-1/2}+1-a^{-1/2}-a^{1/2}+1}{(1-a)}$$

$$=\frac{\mathbf{2}}{\mathbf{1-a}}.$$

E-75 Solve $5^{\sqrt{x}}+12^{\sqrt{x}}=13^{\sqrt{x}}$ **(MBA, '87)**

S-75 $5^{\sqrt{x}}+12^{\sqrt{x}}=13^{\sqrt{x}}$

The given equation is of the form

$5^2+12^2=13^2$ (By the Pythogoras theorem of numbers)

Comparing the two equations, we find

$$\sqrt{x}=2$$

$$\Rightarrow \quad x=\mathbf{4}.$$

E-76 ***Directions (i)-(iv):*** What approximate value should come in place of the question mark (?) in the following questions:

(i) 139% of $459+5\frac{1}{2}$ of $384=?$ **(BSRB Bombay PO, '97)**

(ii) $\sqrt{2000}\times 0.7=(?)^2$ **(BSRB Bangalore PO, '97)**

(iii) $?\%$ of $8999+\frac{599}{3}=26300$ **(BSRB Bangalore PO, '97)**

(iv) 3.9% $99+\frac{4}{9}$ of $700=40\%$ of $?$ **(BSRB Bangalore PO, '97)**

S-76 (i) Assuming x for ? and approximating the terms to closest values, we get

$$x=140\% \text{ of } 460+\frac{11}{2}\times 384$$

$\Rightarrow \quad x = \frac{140 \times 460}{100} + \frac{11}{2} \times 384$

$\Rightarrow \quad x = 644 + 2112 = 2756 \approx 2800$

$\therefore$ the required value is **2800**.

(ii) Assuming x for ? and approximating the terms to its closest values, we get

$(x)^2 = \sqrt{2000} \times 0.7$

$= 20\sqrt{5} \times 0.7$

$= 20 \times 2.23 \times 0.7$

$= 31.22 \approx 31$

$\therefore \quad x = \sqrt{31} = 5.56 \approx \mathbf{5.6}$.

(iii) Assuming x for ? and approximating the terms to its nearest values, we get

$x\%$ of $9000 = 26300 - \frac{600}{3}$

$\Rightarrow \quad x = \frac{26300 - 200}{9000} \times 100$

$= \mathbf{290}$.

(iv) Assuming x for ? and approximating the terms to its nearest values, we get

40% of $x = 4\%$ of $100 + \frac{4}{9} \times 700$

$\Rightarrow \quad \frac{2}{5} \times x = 4 + 311$

$\Rightarrow \quad x = \frac{315 \times 5}{2} = \frac{1575}{2} = 787 \approx \mathbf{790}$.

REGULAR PROBLEMS

(1) $\frac{2 \div (2 \times 2)}{(2 \div 2) \times 2} = ?$ **(UTI, '90)**

(a) 2 (b) 1 (c) 4 (d) $\frac{1}{4}$ (e) None of these

(2) Find the missing number: **(RRB Mumbai, '98)**

$\frac{8}{7} \times \frac{7}{12} \div ? = \frac{4}{9}$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{2}{9}$ (d) $\frac{9}{2}$ (e) $\frac{1}{3}$

(3) Simplify: $2\frac{1}{2}$ of $\frac{3}{4}\times\frac{1}{2}\div\frac{3}{2}+\frac{1}{2}\div\frac{3}{2}\left(\frac{2}{3}-\frac{1}{2}\text{ of }\frac{2}{3}\right)$ to get

(a) 1 (b) $3\frac{3}{8}$ (c) $2\frac{7}{3}$ (d) $1\frac{5}{8}$ (e) $2\frac{3}{4}$

(4) If $\sqrt{13.69}$ = 3.7 then **ASM Exam, '01)**

$\sqrt{1369}+\sqrt{0.1369}+\sqrt{0.001369}$ = ?

(a) 37.407 (b) 34.307 (c) 37.470 (d) 34.707 (e) 37.737

(5) If $46^2 - 44^2 = 45p$, then the value of 'p' will be:

(a) 10 (b) 15 (c) 4 (d) 12 (e)

(6) $\frac{196}{?}=\frac{?}{36}$ **(IA, '82)**

(a) 28 (b) 84 (c) 56 (d) 16.3 (e) 24

(7) If $a * b = a + b + \sqrt{ab}$, then $6 * 24$ is equal to **(RRB Bhopal, '98)**

(a) 41 (b) 42 (c) 43 (d) 44 (e) 45

(8) Find the value of $45^3 - 65^3 + 20^3$ **(RRB Ajmer, '98)**

(a) –175500 (b) 165500 (c) 0 (d) –174500 (e) –140055

Hint: Here, $a + b + c = 45 - 65 + 20 = 0$. Then use the identity no. 12 of 4.2

(9) $\frac{0.538\times0.538-0.462\times0.462}{1-0.924}=?$ **(Bank PO, '83)**

(a) 2 (b) 1.08 (c) 0.076 (d) 0.987 (e) 1

(10) What should come in place of the (?) mark? **(SBI PO, '99)**

$\frac{?}{24}=\frac{72}{\sqrt{?}}$

(a) 12 (b) 16 (c) 114 (d) 144 (e) None of these

(11) Find the value of $(512)^{-2/9}$

(a) 4 (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{5}{4}$ (e) $\frac{7}{4}$

(12) $8\times12\frac{1}{2}-75=1\frac{2}{3}\times?$

(a) 15 (b) 16 (c) 25 (d) $\frac{3}{5}$ (e) $2\frac{1}{3}$

Hint: $8\times12\frac{1}{2}=100$

(13) $\sqrt{12\frac{3}{4}+13\frac{1}{4}+?}=6$

(a) 10 (b) 16 (c) 12 (d) 36 (e) 18

(14) $7-[?-\{4-7\}-\{5-(4-5)+2\}]=16$

(a) 2 (b) 6 (c) 4 (d) 3 (e) – 4

(15) If $\sqrt{x}-\sqrt{y}=1$ and $\sqrt{x}+\sqrt{y}=17$ then $\sqrt{xy}=?$ **(BSRB, '93)**

(a) 51 (b) 16 (c) $\sqrt{72}$ (d) 72 (e) $\sqrt{17}$

(16) If $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{?}}=2$, then (?) is

(a) $2\sqrt{3}$ (b) 24 (c) 12 (d) 144 (e) $3\sqrt{2}$

(17) A man eats 'x' bananas in a week. In how many days will he eat 84 bananas?

(a) $\frac{84}{x}$ (b) $12x$ (c) $\frac{588}{x}$ (d) $\frac{12}{x}$ (e) $84x$

(18) 10 raised to the fifth power may be expressed as:

(a) 10×5 (b) 5^{10} (c) $\sqrt[5]{10}$
(d) $10\times10\times10\times10\times10$ (e) $(5)^{10/5}$

(19) Which of the following is the same as $50\div12$?

(a) $10\ (5\div3)$ (b) $(50\div6)+(50\div6)$ (c) $25\div12\times2$
(d) $50\div4\times3$ (e) $50\div4\div3$

(20) If $\left(\sqrt{3}\right)^{x}=1$, then the value of x is

(a) 1 (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 0 (e) 2

(21) $(16^{0.16}\times2^{0.36})$ is equal to

(a) 64 (b) 16 (c) 2 (d) $\frac{1}{2}$ (e) 1

(22) If $0.5\times A=0.0003$, then the value of A will be

(a) 0.6 (b) 0.06 (c) 0.0006 (d) 0.006 (e) 0.175

Hint: Put $0.5=\frac{1}{2}$, for easy calculation

(23) Reciprocal of $\sqrt[5]{12\frac{209}{243}}$ is equal to

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{5}{4}$ (e) $\frac{1}{3}$

(24) A decimal fraction is multiplied by itself. If the product is 477.4225, then the fraction is

(a) 19.325 (b) 23.715 (c) 22.75 (d) 21.85 (e) 18.65

(25) Simplify: $2.5-\dfrac{1}{3.25-\dfrac{2.5}{0.75+0.50}}$ **(Goods Guard Ex, '99)**

(a) 0.50 (b) 1.70 (c) 1.25 (d) 0.80 (e) 1.18

(26) Simplify: $\dfrac{0.\dot{3}\times 1.\dot{0}\dot{6}}{0.\dot{5}\times 0.\dot{4}}$

(a) $1.43\dot{1}\dot{8}$ (b) $0.2\dot{8}$ (c) $1.0\dot{3}\dot{2}$ (d) $1.6\dot{4}$ (e) $1.\dot{8}\dot{8}$

(27) If $\sqrt{\sqrt[3]{0.000001\times x}} = 0.4$, then the value of x is

(a) $2^{1/6}$ (b) 4096 (c) $4^{1/6}$ (d) 64 (e) $-^{/3}$

(28) $3\frac{1}{x}\times 3\frac{3}{4} = 12\frac{1}{2}$, then the value of '$x$' is

(a) 1 (b) $\frac{1}{3}$ (c) 2 (d) 3 (e) $\frac{}{2}$

(29) If $3.2 \div 64 * 10 = 2.45 - 1.95$, which of the following should replace the asterisk (*)?
(TC Clerk, '97)

(a) + (b) − (c) × (d) ÷ (e) insufficient data

(30) The value of $(9.9)^3$ is

(a) 970.299 (b) 981.009 (c) 981.999 (d) 998.99 (e) 990.989

Hint: Write $9.9 = 10 - 0.1$ and the use the identity $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(31) Square of 21 of 64 is equal to

(a) square of 168 (b) square of 165 (c) square of 179
(d) square of 167 (e) square of 1344

(32) What is the value of x in $\frac{2}{x} - \frac{5}{3x} = \frac{1}{3}$ $(x \neq 0)$?

(a) cannot be determined (b) −2 (c) 3
(d) −1 (e) 1

(33) The value of $(16^{0.16} \times 2^{0.36})$ is equal to

(a) 0 (b) 1 (c) 2 (d) −1 (e) 4

(34) If '+' means '×', '−' means ÷, ÷ means + and '×' means '−', then what will be the value of $20 \div 40 - 4 \times 5 + 6 = ?$

(a) 0 (b) 13.5 (c) 60 (d) −15 (e) None of these
(Elec. Appr. Exam, '01)

(35) If the given interchanges are made in signs and numbers, which one of the four equations would be correct? **(RRB Secunderabad, '01)**

Given interchanges: signs × and ÷, numbers 4 and 9

(a) $94 \times 7 \div 47 = 329$ (b) $47 \times 9 \div 94 = 18$ (c) $49 \times 7 \div 49 = 7$
(d) $94 \times 7 \div 97 = 324$ (e) None

(36) Which of the following equations are equivalent? **(Mumbai PO, '99)**

(1) $\left(\frac{1}{2}M + \frac{2}{3}N\right)^2$ (2) $\frac{4}{9}N^2 + \frac{1}{4}M^2 + \frac{2}{3}MN$

(3) $\left(\frac{M}{2} + \frac{2}{3}N\right)\left(\frac{1}{2}M - \frac{2}{3}N\right)$ (4) $\frac{1}{4}\left(M + \frac{4}{3}N\right)^2$

(a) only (2) and (3) (b) only (1) and (4) (c) only (1) and (2)
(d) only (1) and (3) (e) only (1), (2) and (4)

Directions (37-41): What approximate value will come in place of the question mark (?) in the following equation?

(37) $\sqrt{625.04} \times 16.96 + 136.001 \div 17 = ?$ **(RBI, '02)**

(a) 4.18 (b) 4.41 (c) 425 (d) 433 (e) None

(38) $\left(115\frac{1}{24} + 234.92\right) \times 5\frac{3}{37} = ?$ **(Bank PO, '97)**

(a) 1400 (b) 1750 (c) 1350 (d) 1200 (e) 1650

(39) ? % of 6147 = $2\frac{1}{2} \times 245.76$ **(BSRB Mumbai Hindi Officer's, '97)**

(a) 16 (b) 10 (c) 18 (d) 20 (e) 15

(40) $5.6 \times 2569 + 2058 = 157\% \times 6529 + ?$

(a) 5800 (b) 6300 (c) 6200 (d) 6500 (e) 6000

(41) $0.\overline{91} + \sqrt{999} + \sqrt{111} + 0.\overline{11} = ?$

(a) 43 (b) 40 (c) 39 (d) 42 (e) 4.23

Answers:

1. (d)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (a)	**9.** (e)
10. (d)	**11.** (b)	**12.** (a)	**13.** (a)	**14.** (e)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (c)
19. (e)	**20.** (d)	**21.** (c)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (b)	**26.** (a)	**27.** (b)
28. (d)	**29.** (c)	**30.** (a)	**31.** (a)	**32.** (e)	**33.** (c)	**34.** (a)	**35.** (d)	**36.** (e)
37. (d)	**38.** (b)	**39.** (b)	**40.** (c)	**41.** (a).				

REAL PROBLEMS

(1) If $x = 0.5$ and $y = 0.2$, then $\sqrt{0.6} \times (3y)^x$ is equal to:

(a) 1.0 (b) 0.5 (c) 0.6 (d) 1.1 (e) $\sqrt{1.8}$

(2) $\sqrt[3]{1+\sqrt{2}} \cdot \sqrt[6]{3-2\sqrt{2}}$ is equal to:

(a) $2-\sqrt{2}$ (b) $\sqrt{2}-1$ (c) 1 (d) $3-2\sqrt{2}$ (e) $2+\sqrt{2}$

(3) The value of the expression $\frac{x-1}{x^{3/4}+x^{1/2}} \cdot \frac{x^{1/2}+x^{1/4}}{x^{1/2}+1} \cdot x^{1/4}$

when $x = 16$, is:

(a) 4 (b) 3 (c) 2 (d) 1 (e) 16

(4) $999\frac{998}{999} \times 13 = ?$

(a) $13888\frac{4}{999}$ (b) $12999\frac{986}{999}$ (c) $11988\frac{994}{999}$ (d) $12990\frac{1}{999}$ (e) $12907\frac{904}{999}$

Hint: $999\frac{998}{999} = 999 + \frac{998}{999} = 1000 - \frac{1}{999}$

(5) Arrange the following surds in ascending order of magnitude:

(i) $\sqrt[4]{3}$, (ii) $\sqrt[6]{7}$, (iii) $\sqrt[12]{48}$.

(a) (i), (ii), (iii) (b) (ii), (iii), (i) (c) (i), (iii), (ii) (d) (iii), (ii), (i) (e) (iii), (i), (ii)

Hint: The LCM of the orders of given surds, i.e. 4, 6, 12 are 12. Convert each one of the given surds into a surd of order 12

(6) Simplify: $\dfrac{2^{2^{2^2}} \div \left[\left(2^2\right)^3\right]^4}{\left(4^{4^4}\right) \div \left(4^4\right)^4}$

(a) 256 (b) 1024 (c) 64 (d) $\dfrac{1}{2^{488}}$ (e) 2^{4^2}

Hint: $a^{2^3} = a^{2\times2\times2} = a^8$, but $(a^2)^3 = a^{2\times3} = a^6$

(7) $38\frac{2}{3}$ divided by $\frac{2}{8}$ can be expressed as:

(a) $38\frac{2}{3} \times \frac{1}{4}$ (b) $\frac{2}{8} \times \frac{116}{3}$ (c) $\left(\frac{1}{4} \times 38\right) + \left(\frac{1}{4} \times \frac{2}{3}\right)$ (d) $\frac{116}{3} \times 4$

(e) $(38 \times 4) + \left(\frac{1}{4} \times \frac{2}{3}\right)$

(8) If $2^n = \dfrac{4^5 + 4^5 + 4^5}{3^5 + 3^5} \times \dfrac{6^5 + 6^5 + 6^5 + 6^5}{2^5 + 2^5 + 2^5}$ and $n > 0$, then the value of n^2 is:

(a) 11 (b) 121 (c) 169 (d) 7 (e) 81

(9) If $4^b - 4^{b-1} = 24$, then $(2b)^b$ equals:

(a) 25 (b) $25\sqrt{5}$ (c) 125 (d) $5\sqrt{5}$ (e) $\sqrt{5}$

(10) If $a^3 + b^3 = 0$, then:

(a) $a + b = \sqrt{2ab}$ (b) $a + b = a^2 + b^2$ (c) $a + b = a^2 - b^2 + ab$

(d) $a + b = \sqrt{ab}$ (e) $a + b = \sqrt{3ab}$

(11) Replace the (*) mark in

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + (*)}} = \frac{8}{5}.$$

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) 1 (d) $\frac{1}{3}$ (e) $\frac{1}{4}$

(12) If $m\sqrt{m} \times m^3 \div m^{-3/2} = m^{(a+2)}$, then the value of $a^{(a+2)}$ is:

(a) 1876 (b) 2304 (c) 16 (d) 256 (e) 4096

(13) Evaluate: $1 + \frac{1}{1\times2} + \frac{1}{1\times2\times4} + \frac{1}{1\times2\times4\times8} + \frac{1}{1\times2\times4\times8\times16}$ up to four places of decimals.

(a) 1.6096 (b) 1.7062 (c) 1.3214 (d) 1.6416 (e) None of these

Hint: Do not waste your time by calculating individual term. Adopt any other convenient method

(14) Find the value of 'b', if b is a natural number and

$$\sqrt{b\sqrt[3]{b}} - \sqrt[3]{b\sqrt{b}} = \sqrt{b}$$

(a) 4 (b) 16 (c) 256 (d) 64 (e) 128

(15) Evaluate: $\frac{(0.15)^2 + (0.28)^2}{(0.45)^2 + (0.84)^2} - \frac{(0.28)^3 + (0.47)^3 - (0.75)^3}{3(0.28)(0.47)(0.75)}$.

(a) 1 (b) 0.8 (c) $1\frac{1}{9}$ (d) 2.5 (e) $-\frac{8}{9}$

(16) If $\sqrt{\sqrt[3]{0.000001x}} = 0.2$, then $\frac{\sqrt{x}}{0.1}$ equals to:

(a) 14.14 (b) 640 (c) 80 (d) 20 (e) None of these

Hint: Remove the radicals one by one

(17) If $\left(\frac{p^2}{q^2}\right)^{5x+7} = \left(\frac{q^3}{p^3}\right)^{x-8}$, then the value of $(5x+7)$ is:

(a) 12 (b) $10\frac{11}{13}$ (c) 17 (d) $7\frac{2}{9}$ (e) $-20\frac{1}{7}$

Hint: Do not try to find the value of $5x + 7$ directly from the given relation

(18) What should come in place of the question mark (?) in the following equation?

(BSRB Patna PO, '01)

$$75^{7.5} \div 75^{3/2} \times 75^{-3} = \left(\sqrt{75}\right)^{?}$$

(a) 6 (b) $5\sqrt{3}$ (c) $\sqrt{3}$ (d) 3 (e) None of these

(19) What will replace the question mark (?) in the following equation?

$$27^{\sqrt{?-7}} = \frac{6^{19}\times153}{2^{16}\times136}$$

(a) 8 (b) 16 (c) 56 (d) 23 (e) 32

(20) If $5^x = 6^y = 30^7$, then the value of $\frac{xy}{x+y}$ is:

(a) $\frac{1}{7}$ (b) 3 (c) 6 (d) 7 (e) 1

(21) If $7x + 10 = x^2 - \frac{1}{N} = 13x - 2$, what is the value of N?

(a) $-\frac{1}{20}$ (b) 20 (c) 18 (d) $\frac{1}{18}$ (e) 22

(22) If $\frac{x(x+2)}{(x+4)4} = \frac{0.\dot{3} \times 1.\dot{0}\dot{6}}{0.\dot{5} \times 0.\dot{4}}$, then the value of x is:

(a) 9 (b) 7 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$ (e) None of these

Hint: Do not waste your time by solving it like a quadratic equation

Directions (23-27): Following (A) to (H) are combinations of an operation and an operand:

(A) means ÷ 3 (B) means × 3 (C) means − 3 (D) means + 3
(E) means ÷ 2 (F) means × 2 (G) means − 2 (H) means + 2

You have been given one or more of these as answer choices for the following questions. Select the appropriate choice to replace the question mark in the equations. **(SBI PO, '99)**

(23) 42 × 21 − 12? = 880

(a) A (b) F (c) G (d) D (e) None of these

Hint: 42 × 21 = 882 ∴ 12? = 882 − 880 = 2 Checking the given combination in place of ? By trial, (A) → 12 ÷ 3 = 4 ≠ 2, then
for combination (F) → 12 × 2 = 24 ≠ 2,
(G) → 12 − 2 = 10 ≠ 2 and so on

(24) 36 + 12 ? = 48

(a) A followed by D (b) B followed by G (c) A followed by B
(d) F followed by H (e) None of these

Hint: Here the number on right hand side i.e. 48 is comparatively smaller than previous one (i.e. 880). So, we can directly test the combination of operation as per choices i.e. (a) → 36 + 12 ÷ 3 + 3 = 43 ≠ 48

(25) 48 ? + 12 × 4 = 80

(a) E followed by B (b) D followed by A (c) B followed by F
(d) F followed by A (e) None of these

(26) 18 × 3 ÷ 2 + 3 < 27 ?

(a) D followed by G (b) A followed by G (c) D followed by H
(d) D followed by A (e) None of these

(27) (48 + 9) ÷ 19 × 2 = 12 ?

(a) B followed by E (b) A followed by H (c) A followed by D
(d) C followed by A (e) None of these

(28) What would be the maximum value of Q in the following equation?
2P4 + 7R9 + 4Q7 = 1380

(a) cannot be determined (b) 9 (c) 6 (d) 7 (e) 8

Directions (29-36): What approximate value should replace the question mark (?) in the following equations:

(29) 208.78 of $7\frac{3}{5}\%$ + 423.547 of $24\frac{39}{50}\%$ = ?

(a) 120 (b) 117 (c) 123 (d) 114 (e) 130

Tips: Both the parts have % term, so take the % common outside a bracket. Hence calculation of % at each step should be avoided.

(30) 159% of 6531.8 + 5.5 × 1015.2 = ? + 5964.9 **(Mumbai PO, '98)**
(a) 11,000 (b) 11,500 (c) 10,000 (d) 10,800 (e) 12,000

(31) $152\sqrt{?} + 795 = 8226 - 3486$ **(Baroda PO, '99)**
(a) 675 (b) 550 (c) 860 (d) 925 (e) 500

(32) $\frac{5}{7}$ of 1596 + 3015 = ? − 2150
(a) 5500 (b) 6300 (c) 49000 (d) 7400 (e) 68000

(33) $16\sqrt{524} + 1492 - 250.0521 = ?$
(a) 1500 (b) 1350 (c) 2200 (d) 1800 (e) 1600

(34) 857 of 14% − 5.6 × 12.128 = ? **(Chennai PO, 2000)**
(a) 45 (b) 52 (c) 65 (d) 60 (e) 40

(35) 6.39 × 15.266 + 115.8 of $\frac{2}{5}$ = ?
(a) 160 (b) 150 (c) 145 (d) 170 (e) 130

(36) $33\frac{1}{3}$ % of 768.9 + 25% of 161.2 − 68.12 = ?
(a) 220 (b) 245 (c) 235 (d) 250 (e) 230

Hint: Before simplification, put the fractional equivalent of the percentage values as $33\frac{1}{3}\% = \frac{1}{3}$ and $25\% = \frac{1}{4}$

Directions (37-40): Find out the approximate value that should come in place of the question mark in the following questions. (You are not expected to find the exact value).

(37) $\sqrt{45689} = ?$
(a) 170 (b) 280 (c) 320 (d) 210 (e) 430

Hint: Do not try to find the square root by conventional method, but, in reverse way, check the square of which alternative gives closest value to 45689. This checking can be done mentally as square of 2, i.e. $(2)^2$ can give the first digit as 4. So either (b) or (d) can be the answer. Next you check $(28)^2 > 25^2$ (625) it is not the starting digit of the given number. So (d) 210 is the choice.

(38) $\frac{(\ 3008.99\ )^2}{10009.001} \times \sqrt{3589} \times 0.4987 = ?$
(a) 30000 (b) 900000 (c) 300000 (d) 3000 (e) 60000

(39) $\frac{2}{5} + \frac{7}{8} \times \frac{17}{19} \div \frac{6}{5} = ?$
(a) $3\frac{1}{2}$ (b) $1\frac{1}{2}$ (c) 2 (d) 1 (e) $\frac{8}{11}$

(40) 399.89 + 206 × 11.009 = ?
(a) 2700 (b) 3100 (c) 6566 (d) 4336 (e) 2400

Directions (41-44): Four of the five parts numbered as (a), (b), (c), (d) and (e) in the following equations are exactly equal. You have to find out the part that is not equal to the other parts.

(41) $\underset{(a)}{\sqrt[3]{729} + \sqrt[3]{625}} = \underset{(b)}{\sqrt{324} + \sqrt{256}} = \underset{(c)}{\sqrt[3]{216} \times \sqrt[3]{81}} - \frac{1}{2}$ of 40

$\underset{(d)}{\sqrt{441} + \sqrt[3]{2197}} = \underset{(e)}{\sqrt[3]{5832} + \sqrt[3]{2744}}$

(42) $\underset{(a)}{8362.64 + 768.3 - 190.57} = \underset{(b)}{593.38 + 604.7 + 7742.29}$

$= \underset{(c)}{2235.925 \times 4} = \underset{(d)}{9931.04 - 990.67} = \underset{(e)}{17880.74 \div 2}$

Hint: You have to do actual calculation here. Never try to round off

(43) $\underset{(a)}{10.36 + 69.802 + 24.938} = \underset{(b)}{2207.1 \div 21} = \underset{(c)}{16\frac{2}{3}\%}$ of 630.6

$\underset{(d)}{32.84375 \times 3.2} = \frac{1}{5}$ of $\underset{(e)}{\frac{1}{9}}$ of 4729.4

(44) $\underset{(a)}{x(x + 5) + 2(3x + 2)} = \underset{(b)}{(x + 2)^2 + 7x} = \underset{(c)}{(x + 3)(x - 2) + 10(x + 1)}$

$= (x - 2)^3 - \underset{(d)}{(x - 3)^3 - 2(x^2 - 13x + 8)} = \underset{(e)}{(x + 7)(x + 2) + 2(x - 5)}$

(45) If $\sqrt[3]{91125} = 45$, then the value of

$\sqrt[3]{91.125} + \sqrt[3]{0.091125} + \sqrt[3]{0.000091125}$

(a) 49.95 (b) 5.495 (c) 4.995 (d) 5.405 (e) 4.545

Tips: On finding cube roots $\left(\sqrt[3]{\ }\right)$ the number of decimal places get reduced to $\frac{1}{3}$, as on finding square roots $\left(\sqrt[2]{\ }\right)$, the number of decimal places reduces to $\frac{1}{2}$

(46) If $x = \frac{1}{11} \div \frac{1}{1 + \frac{1}{1 + \frac{1}{5}}}$, then find the value of $\left(x + \frac{5}{6}\right)$.

(a) 3 (b) $\frac{1}{5}$ (c) $5\frac{5}{6}$ (d) $\frac{1}{6}$ (e) 1

(47) If $(x + p + m) = p^3 + m^3 + x^3 - 3xpm$ and the sum of any two numbers is 12, then what is the ratio of that sum to the third number?

(a) −4 (b) −1 (c) 1 (d) −12 (e) 4

8) If $8700 \div x = 300$ and $4590 \div y = 170$, then $(x - y) \times (x + y) = ?$

(a) 29 (b) 56 (c) 112 (d) 27 (e) 81

Hint: Refer to 4.7

9) If $\frac{8}{a} = \frac{3}{a} - 10$, then $\frac{a}{8} - \frac{3}{a} = ?$

(a) -10 (b) $\frac{95}{16}$ (c) $\frac{16}{95}$ (d) $\frac{24}{a^2 - 8}$ (e) $\frac{a^2 - 24}{8a}$

Answers

1. (c)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (b)	**9.** (b)
10. (e)	**11.** (a)	**12.** (e)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (a)
19. (c)	**20.** d)	**21.** (a)	**22.** (b)	**23.** (e)	**24.** (c)	**25.** (d)	**26.** (c)	**27.** (b)
28. (c)	**29.** (a)	**30.** (c)	**31.** (a)	**32.** (b)	**33.** (e)	**34.** (b)	**35.** (c)	**36.** (e)
37. (d)	**38.** (c)	**39.** (d)	**40.** (a)	**41.** (e)	**42.** (c)	**43.** (e)	**44.** (d)	**45.** (c)
46. (e)	**47.** (b)	**48.** (c)	**49.** (b)					

5 PERCENTAGE

5.1 INTRODUCTION

The word 'per cent' means per hundred. Thus, 19 per cent means, 19 parts out of 100 parts. Th can also be written as $\frac{19}{100}$.

Therefore, per cent is a fraction whose denominator is 100, and the numerator of this fraction is called the **Rate percent**. So, $\frac{19}{100}$ = 19 per cent. Here, 19 per cent is the rate. The sign for per cent is '%'.

Table 5.1 Fractional Equivalents of Important Percentages

$1\% = \frac{1}{100}$	$2\% = \frac{1}{50}$	$4\% = \frac{1}{25}$	$8\% = \frac{2}{25}$	$16\% = \frac{4}{25}$	$64\% = \frac{16}{25}$	$96\% = \frac{24}{25}$
$5\% = \frac{1}{20}$	$10\% = \frac{1}{10}$	$20\% = \frac{1}{5}$	$40\% = \frac{2}{5}$	$60\% = \frac{3}{5}$	$80\% = \frac{4}{5}$	$120\% = \frac{6}{5}$
$6\frac{1}{4}\% = \frac{1}{16}$	$12\frac{1}{2}\% = \frac{1}{8}$	$25\% = \frac{1}{4}$	$37\frac{1}{2}\% = \frac{3}{8}$	$50\% = \frac{1}{2}$	$87\frac{1}{2}\% = \frac{7}{8}$	$100\% = 1$
$8\frac{1}{3}\% = \frac{1}{12}$	$16\frac{2}{3}\% = \frac{1}{6}$	$33\frac{1}{3}\% = \frac{1}{3}$	$66\frac{2}{3}\% = \frac{2}{3}$	$83\frac{1}{3}\% = \frac{5}{6}$	$133\frac{1}{3}\% = \frac{4}{3}$	

Note: Similarity along the horizontal rows are to be observed for memorizing table 5.1.

5.2 FRACTION TO RATE PER CENT

To convert (or express) any fraction $\frac{a}{b}$ to rate per cent, multiply it by 100 and put $a(\%)$ sign.

$$\Rightarrow \boxed{\frac{a}{b}} = \boxed{\frac{a}{b} \times 100\%}$$

$\frac{a}{b}$: a fraction; $\frac{a}{b} \times 100\%$: rate per cent

Example: Express $\frac{3}{4}$ in rate per cent

Required rate per cent = $\frac{3}{4} \times 100\% = 75\%$

5.3 RATE PER CENT TO FRACTION

To convert a rate per cent to a fraction, divide it by 100 and delete the % sign.

Example: 8% can be converted to a fraction as $\frac{8}{100}$

5.4 RATE PER CENT OF A NUMBER

Rate per cent of a number is the product of equivalent fraction (of rate per cent) and the number.

$$\Rightarrow \quad p\% \text{ of } A = \left(\frac{p}{100}\right) \times A$$

Example: To find out 25% of 500

Solution: Required value = 25% of 500

$$= \left(\frac{25}{100}\right) \times 500.$$

$\left(\frac{25}{100}\right)$ → equivalent fraction for 25%

$$= 125$$

5.4.1 Relation Among Rate Per cent, Number and Value

Let us consider a number, N.

Then N is considered as the base over which value of different rate per cents are found out.

$$10\% \text{ of } N = \frac{10}{100} \times N = \frac{N}{10} \text{ (value)}$$

$$25\% \text{ of } N = \frac{25}{100} \times N = \frac{N}{4} \text{ (value)}$$

and so on.

Therefore, it is found that as the rate per cent changes, its related value for the same number will also change.

Conversely, different values stand for different rate per cents of the same number. As in the above example, $\frac{N}{10}$ stands for 10% of N; $\frac{N}{4}$ stands for 25% of N and so on.

In the above context, a very useful relation is derived as:

$$\frac{\text{any value}}{\text{its rate \% of number}} = \text{number (base)} \qquad \textbf{(1)}$$

Example: 9% of what number is 36?

Solution: Using the relation 1,

the required number (base number) = $\frac{36}{9\%}$

$$= \frac{36}{9} \times 100$$

$$= 400$$

Note: Here, 36 is the value and its rate % of base number = 9%

Example: If 30% of a number is 48, then what is 70% of the number?

Solution: Here, unitary method can be used to save the time.

$$30\% \rightarrow 48$$

$$\Rightarrow \quad 1\% \rightarrow \frac{48}{30}$$

$$\Rightarrow \quad 70\% \rightarrow \frac{48}{30} \times 70 = 112$$

Hence, the required value is 112

Example: If 40% of the number exceeds the 25% of it by 54. Find the number.

Solution: Using the formula (1)

$$\frac{\text{any value}}{\text{its rate \% of number}} = \text{number (i.e. base number)}$$

Here, 54 stands for the difference of (40% and 25% of number)

$$\Rightarrow \quad \frac{54}{(40-25)\%} = \text{number}$$

$$\Rightarrow \quad \text{required number} = \frac{54}{40-25} \times 100 = 360$$

5.5 EXPRESSING A GIVEN QUANTITY AS A PERCENTAGE OF ANOTHER GIVEN QUANTITY

Let one given quantity be x and another given quantity be y. It is often asked to find what percentage of y is x. Here both quantities (x and y) should be in same units. If not, they should be converted into the same unit.

Concept

The question requires us to express one given quantity 'x' as a percentage of another given quantity 'y'.

Since y is the basis of comparison, so, y will be in the denominator. But x is to be converted as percentage of y, hence x will be in the numerator of the fraction. Now to convert the fraction to percentage, we will multiply it by 100. So, we get

the required percentage $= \frac{x}{y} \times 100\%$.

Example: To find '30 is what per cent of 150' or 'what percentage of 150 is 30 ?

Solution: Using the earlier concept, we find here that 150 is the basis of comparison and hence 150 will be in the denominator.

$$\text{The required percentage} = \frac{30}{150} \times 100\%$$
$$= 20\%.$$

Example: ?% of 320 = 86.4

Solution: Here, 320 is the basis of comparison and it will be in the denominator.

$$\therefore \quad \text{required percentage} = \frac{86.4}{320} \times 100\%$$
$$= 27\%$$

5.6 CONVERTING A PERCENTAGE INTO DECIMALS

Case I

Let the percentage be a positive integer, then
place a decimal point after two places from the extreme right of the integer to convert it into a decimal. If the percentage is a single digit number, add one zero to the left of it and then place the decimal point for its conversion. % Sign is removed after conversion.

Example: 67% may be converted into decimals as 0.67, because $67\% = \frac{67}{100} = 0.67$

8% may be written as 0.08 → {Zero added to its left to make it a two digit number so that decimal point can be placed.}

253% is equivalent to 2.53

Case II

Let the percentage be a decimal fraction
The percentage being a decimal fraction, shift decimal by two places to the left. Add zero to the left of the fraction, if needed.

Two place left ← Shifted

Example: 3.5% may be written as 0.035 (zero is added to its left so that decimal point can be shifted by two places to the left)

0.7% may be written as 0.007

Case III

Let the percentage be a fraction

If the percentage is a fraction of the form $\frac{a}{b}$, then convert it into a decimal fraction and then follow the rule detailed in case II

Example: $\frac{1}{4}\%$ is equivalent to 0.25% which may be converted into decimals as 0.0025

5.7 CONVERTING A DECIMAL INTO A PERCENTAGE

In this case, the method of 5.6 is reversed, i.e. shift the decimal point two places to the right. Add zero to the extreme right if required. Then add % sign.

Shift

Example: 0.45 may be expressed as 45%

0.032 is equivalent to 3.2%

1.7 is equivalent to 170%

⟶ zero is added so that decimal point can be shifted by two places.

5.8 EFFECT OF PERCENTAGE CHANGE ON ANY QUANTITY (NUMBER)

If any number (quantity) is increased by $x\%$, then

$$\text{new number (quantity)} = \text{original number} \times \left(\frac{100 + x}{100}\right)$$

or

$$= \text{original number} \times (1 + \text{decimal equivalent of } x\%).$$

Similarly, if any number (quantity) is decreased by $x\%$, then

$$\text{new number (quantity)} = \text{original number} \times \left(\frac{100 - x}{100}\right)$$

or

$$= \text{original number} \times (1 - \text{decimal equivalent of } x\%).$$

Note: In case of percentage decrease, a (–)ve sign is put before x, otherwise the formula is same.

Example: The present salary of A is Rs 3000. This will be increased by 15% in the next year. What will be the increased salary of A?

Solution: Here, the salary is to be increased by 15%.

15% is equivalent to 0.15

$$\text{the increased salary} = 3000\ (1 + 0.15) \text{ or } 3000 \times \frac{100 + 15}{100}$$

$$= 3000 \times 1.15$$

$$= \text{Rs } 3450.$$

TWO STEP CHANGE OF PERCENTAGE FOR A NUMBER

In the first step, a number is changed (increased or decreased) by $x\%$, and in the second step, this changed number is again changed (increased or decreased) by $y\%$, then net percentage change on the original number can be conveniently found out by using the following formula,

$$\text{net \% change} = x + y + \frac{xy}{100} \quad (+ \text{ or } -) \qquad (2)$$

If x or y indicates decrease in percentage, then put a (–)ve sign before x or y, otherwise positive sign

Example: If a number is increased by 12% and then decreased by 18%, then find the net percentage change in the number.

Solution: Using the formula (2)

$$\text{net \% change} = x + y + \frac{xy}{100}$$

where $x = 12 \qquad y = -18$

$$\Rightarrow \quad \text{net \% change} = 12 - 18 + \frac{(12) \times (-18)}{100}$$

$$= -6 - 2.16$$

$$= -8.16$$

(–) sign signifies that there is percentage decrease in the result. Therefore –8.16 indicates net 8.16% decrease of the given number as a result of 12% increase and 18% decrease.
It also implies that 12% increase and 18% decrease are equivalent to 8.16% decrease.

5.10 PERCENTAGE CHANGE AND ITS EFFECT ON PRODUCT

Let $A \times B$ = result (product)

A: variable; B: variable

If A is changed (increased or decreased) by x%, and also B is changed (increased or decreased) by y%, then the net percentage change (increase/decrease) of the product of A and B can be found out easily by the formula

$$\text{net \% change in product} = x + y + \frac{xy}{100} \qquad \text{(2A)}$$

(+ or –)

If x or y indicates decrease in percentage, then put a (–)ve sign before x or y, otherwise positive sign remains.
This formula (2A) is same as formula (2)

Above formula can be used to find out the *net percentage change*, if it involves the *product of any two variable quantities which have also the % change.*

Application of the Formula (2A)

The formula (2A) can be used to find out

(a) % effect on expenditure, when rate and consumption are changed, since rate × consumption = expenditure [$A \times B$ = product]

(b) % effect on area of rectangle/square/triangle/circle, when its sides/radius are changed, since $\text{Side}_1 \times \text{Side}_2$ = area, or radius × radius = area [$A \times B$ = product]

(c) % effect on distance covered, when time and speed are changed, since time × speed = distance. [$A \times B$ = product]

Example: If the length of rectangle increases by 30% and the breadth decreases by 12%, then find the % change in the area of the rectangle.

Solution: Since, length × breadth = area, and both the length and breadth are changed, so, using the formula (2A), we get

$$\text{net \% change in product} = x + y + \frac{xy}{100}$$

where $x = 30, \quad y = -12$

$$\Rightarrow \quad \text{net \% change in area} = 30 - 12 + \frac{30 \times -12}{100}$$

$$= 18 - 3.6$$

$$= +14.4$$

It implies that there is 14.4% increase in the area of the rectangle.

5.10.1 To keep the product of two variable quantity as fixed

As we have seen in 5.10, $A \times B$ = product, where A and B are two quantities which are changing and product is also changing.

Now, we want to **keep the product fixed**, even if A and B are changed (increased/decreased). Then, if one quantity increases, the other quantity will decrease and vice-versa so that product remains unchanged.

Hence the net percentage effect on product is zero in the formula (2A).

Put net % change in product = 0 in formula (2A),

$$x + y + \frac{xy}{100} = 0$$

$$\Rightarrow \quad y = -\frac{x}{100 + x} \times 100, \text{ (–) ve sign shows decrease}$$

From the above derivation, we thus find that

If one quantity A increases by x%, then other quantity B decreases by $\boxed{\frac{x}{100 + x} \times 100\ \%}$

and if one quantity A decreases by x%, then putting (–) x, in place of x,

we find that the other quantity B increases by $\boxed{\frac{x}{100 - x} \times 100\ \%.}$

These formula are used to find out:

a) % change in either rate or consumption, when expenditure is to be kept fixed, because, rate × consumption = expenditure [$A \times B$ = product = fixed]

b) % change in either length or breadth, when area of rectangle is to be kept fixed, because, length × breadth = area [$A \times B$ = product = fixed]

c) % change in either time or speed, when distance is to be kept fixed because, time × speed = distance [$A \times B$ = product = fixed]

Example: If the price of coffee is increased by 10%, then by how much percentage must a house wife reduce her consumption, to have no extra expenditure?

Solution: Since price × consumption = expenditure and expenditure has to be kept fixed (or unchanged), so, when the price increases by 10%,

the % reduction in consumption $= \frac{10}{100+10} \times 100\%$

$= 9\frac{1}{11}\%$.

5.11 RATE CHANGE AND CHANGE IN QUANTITY AVAILABLE FOR FIXED EXPENDITURE

Let the original rate of an item = Rs x per unit quantity.
Expenditure is fixed.

$$\text{Quantity of the item available} = \frac{\text{Expenditure}}{x} \qquad \textbf{(a)}$$

Now, the original rate (x) changes (increase/decrease) to a new rate. Since the amount of expenditure is fixed, so, with the *change in rate*, it is obvious that the quantity of the item available in equation (a) will also change (decrease/increase) accordingly.
Hence, due to rate change,

$$\text{New quantity of the item available} = \frac{\text{Expenditure}}{\text{New rate}}$$

$\Rightarrow$ Original quantity available $\pm$ change in quantity available $= \frac{\text{Expenditure}}{\text{New rate}}$. Using equation (a), we get

$$\Rightarrow \boxed{\frac{\text{Expenditure}}{x} \pm \text{change in quantity available} = \frac{\text{Expenditure}}{\text{New rate}}} \qquad \textbf{(3)}$$

Put (–)ve when quantity available decreases due to rate change, (x = original rate per unit quantity.)

Example: A reduction of 25% in the price of sugar enables the person to get 10 kg more on a purchase for Rs 600. Find the reduced rate of sugar.

Solution: Let the original rate = Rs x per kg.
Since, there is a rate reduction of 25%, so,
New rate (or reduced rate) $= (1 - 0.25)\ x$

$$= 0.75\ x = \frac{3}{4}\ x$$

Expenditure = Rs 600.
Using the formula (3)

$$\frac{\text{Expenditure}}{x} + \text{change in quantity available} = \frac{\text{Expenditure}}{\text{New rate}}$$

$$\Rightarrow \quad \frac{600}{x} + 10 = \frac{600}{\frac{3}{4}x} \qquad (+10, \text{ for quantity available increases after rate change})$$

$$\Rightarrow \quad \frac{600}{x}\left(\frac{4}{3} - 1\right) = 10 \quad \Rightarrow \quad x = 20$$

Therefore, reduced rate = $\frac{3}{4}x = \frac{3}{4} \times 20$ = Rs 15/kg.

5.12 % EXCESS OR % SHORTNESS

When a number A exceeds the another number B by x %, then % shortness of $B = \frac{x}{100 + x} \times 100$

It implies that B is less than A by $\frac{x}{100 + x} \times 100\%$.

Similarly, if a number A is short of (or less than) B by $x\%$, then % excess of $B = \frac{x}{100 - x} \times 100$

i.e. B is more than A by $\frac{x}{100 - x} \times 100\%$

Example: If the income of Ram is more than that of Shyam by 25%, then by how much percentage Shyam's income is less than that of Ram?

Solution: Required % shortness (less) income of Shyam $= \frac{25}{100 + 25} \times 100\%$

$= 20\%$.

Therefore, income of Shyam is 20% less than that of Ram.

Solved Examples

E-1 Express the following in terms of percentage:

(a) 0.4 ***(b)*** 1.0 ***(c)*** $\frac{5}{3}$ ***(d)*** $\frac{7x}{y}$ ***(e)*** 1.23

S-1 ***(a)*** Refer 5.2, multiply the decimal fraction by 100.

$\therefore$ $0.4 = (0.4 \times 100)\% = \mathbf{40\ \%}$.

(b) $1.0 = (1.0 \times 100)\% = \mathbf{100\ \%}$.

(c) $\frac{5}{3} = \left(\frac{5}{3} \times 100\right)\% = \mathbf{166\frac{2}{3}\ \%}$.

(d) $\frac{7x}{y} = \left(\frac{7x}{y} \times 100\right)\% = \mathbf{\frac{700x}{y}\ \%}$.

(e) $1.23 = (1.23 \times 100)\% = \mathbf{123\ \%}$.

E-2 Express the following in terms of fractions:

(a) $22\frac{1}{2}\%$ ***(b)*** 35% ***(c)*** $\frac{a^2}{b}\%$ ***(d)*** 0.3% ***(e)*** $\frac{7}{2}\%$

S-2 *(a)* Divide the given percentage by 100 to convert it into a fraction. *[Refer 5.3]*

$$\therefore \quad 22\frac{1}{2}\ \% = \frac{22\frac{1}{2}}{100} = \frac{45}{2\times 100} = \mathbf{\frac{9}{40}}.$$

(b) $35\ \% = \dfrac{35}{100} = \mathbf{\dfrac{7}{20}}.$

(c) $\dfrac{a^2}{b}\ \% = \dfrac{a^2}{b\times 100} = \mathbf{\dfrac{a^2}{100b}}.$

(d) $0.3\ \% = \dfrac{0.3}{100} = \mathbf{\dfrac{3}{1000}}.$

(e) $\dfrac{2}{7}\ \% = \dfrac{2}{7\times 100} = \mathbf{\dfrac{1}{350}}.$

E-3 Find

(a) 9% of 27

(b) 0.02% of 6500

(c) $\dfrac{7}{2}$ % of 80

(d) 125% of 64

(e) 10% of 5% of 320

S-3 *(a)* Refer 5.4, Multiply the number by $\dfrac{p}{100}$, if p% of the number is to be calculated.

$$\therefore \quad 9\ \%\ \text{of}\ 27 = \frac{9}{100}\times 27 = \mathbf{\frac{243}{100}}.$$

(b) $0.02\ \%\ \text{of}\ 650 = \dfrac{0.02}{100}\times 6500 = \mathbf{\dfrac{13}{10}}.$

(c) $\dfrac{7}{2}\ \%\ 80 = \dfrac{7}{2\times 100}\times 80 = \mathbf{\dfrac{14}{5}}.$

(d) $125\%\ \text{of}\ 64 = \dfrac{125}{100}\times 64 = \mathbf{80}.$

(e) $10\%\ \text{of}\ 5\%\ \text{of}\ 320 = \dfrac{10}{100}\times\dfrac{5}{100}\times 320 = 1.6 = \mathbf{\dfrac{8}{5}}.$

E-4 Find the following:

(a) 36 is what % of 144.

(b) $\dfrac{7}{8}$ is what % of $\dfrac{3}{4}$.

(c) What % of 80 is 16.

(d) 0.625 is equal to what % of $1\dfrac{7}{28}$.

(e) 36 × 14 is what % of 1400.

(f) R is what % of N.

S-4 Refer 5.5, the required percentage = $\frac{x}{y} \times 100\ \%$, where y is the base.

(a) Here, the base is 144. So, the denominator will be 144.

$\therefore$ required percentage = $\frac{36}{144} \times 100\% = 25\%$.

(b) Here, the base is $\frac{3}{4}$.

$\therefore$ required percentage = $\frac{7/8}{3/4} \times 100\ \% = 116\ \frac{2}{3}\ \%$.

(c) Here, the base is 80.

$\therefore$ required percentage = $\frac{16}{80} \times 100\% = 20\%$.

(d) Here, the base is $1\frac{7}{28}$

$\therefore$ required percentage = $\frac{0.625}{1\frac{7}{28}} \times 100 = 50\%$.

(e) Here, the base is 1400

$\therefore$ required percentage = $\frac{36 \times 14}{1400} \times 100 = 36\%$.

(f) Here, the base is N

$\therefore$ required percentage = $\frac{R}{N} \times 100 = \frac{100R}{N}\ \%$.

E-5 Find the following:

(a) 36 is 6 % of what?

(b) 2.5 is 5 % of what?

(c) 12 is 25 % of 20 % of what?

(d) $\frac{4}{7}$ is 24 % of what?

S-5 Refer 5.4.1, using equation (1),

$$\frac{\text{any value}}{\text{its rate \% of number}} = \text{number (original)}$$

(a) required number = $\frac{36}{6\%} = \frac{36}{6} \times 100 = 600$.

(b) required number = $\frac{2.5}{5\%} = \frac{2.5}{5} \times 100 = 50$.

(c) required number $= \dfrac{12}{25\% \times 20\%} = \dfrac{12}{25 \times 20} \times 100 \times 100 = 240$

(d) required number $= \dfrac{4/7}{24\%} = \dfrac{4}{7 \times 24\%} = \dfrac{4}{7 \times 24} \times 100 = \dfrac{50}{21}$

E-6 25 % of a number is 20, what is 40 % of that number? Also find the number.

S-6 25 % $\rightarrow$ 20

$\Rightarrow$ 40 % $\rightarrow \dfrac{20}{25} \times 40 = 32$

Refer 5.4.1, using equation (1), the number $= \dfrac{\text{any value}}{\text{its rate \%}} = \dfrac{20}{25\%}$

$\Rightarrow$ required number $= \dfrac{20}{25} \times 100 = 80$

E-7 p_1 % of number N_1 is equal to p_2 % of number N_2. Find what per cent of N_1 is N_2?

S-7 It is required to find what per cent of N_1 is N_2, i.e. the base is N_1

$\therefore$ required percentage $= \dfrac{N_2}{N_1} \times 100\%$.

It is given that $\dfrac{p_1}{100} \times N_1 = \dfrac{p_2}{100} \times N_2 \quad \therefore \quad \dfrac{N_2}{N_1} = \dfrac{p_1}{p_2}$

Putting the value of $\dfrac{N_2}{N_1}$, we find the required percentage $= \left(\dfrac{p_1}{p_2} \times 100\right)\%$

$\therefore$ N_2 is equal to $\left(\dfrac{p_1}{p_2} \times 100\right)\%$ of N_1

E-8 A number A exceeds B by 25%. By what per cent is B short of A?

S-8 Refer 5.12

$$\% \text{ Short} = \frac{\% \text{ excess}}{(100 + \% \text{ excess})} \times 100$$

$$= \frac{25}{(100 + 25)} \times 100 = 20$$

E-9 A number X is short of Y by 40%. By what per cent is Y in excess of X?

S-9 Refer 5.12. Using the formula,

$$\% \text{ Excess} = \frac{\% \text{ short}}{100 - \% \text{ short}} \times 100$$

$$= \frac{40}{100 - 40} \times 100 = \mathbf{66\frac{2}{3}\%}.$$