Here, the numbers marked 2) and marked 5) are to be interchanged in the position to establish the series.

Hence 2) is the answer.

E-19 1 1 1 2 8 4
1) 2) 3) 4) 5)

S-19 The pattern of the series should go as

$1,\quad 1^2,\quad 1^3,\quad 2,\quad \underbrace{2^2,}_{5)}\quad \underbrace{2^3}_{4)}$

to interchange

Here, the numbers marked 4) and marked 5) are to be interchanged in their position to establish the series.

Hence 4) is the answer

E-20 48 16 13 12 17 25.25
1) 2) 3) 4) 5).

S-20 The pattern of the series should go as

$48,\quad 48 \times 0.25 + 4,\quad \underbrace{16 \times 0.5 + 4,}_{3)}\quad \underbrace{12 \times 0.75 + 4,}_{2)}\quad 13 \times 1 + 4,\quad 17 \times 1.25 + 4$

to interchange

Here, the numbers marked 2) and marked 3) are to be interchanged in their position to establish the series.

Hence 2) in the answer.

E-21 82 83 165 9916 1983 496
1) 2) 3) 4) 5).

S-21 The pattern of the series should go as

$82,\quad 82 \times 1 + 1,\quad 83 \times 2 - 1,\quad \underbrace{165 \times 3 + 1,}_{5)}\quad 496 \times 4 - 1,\quad \underbrace{1983 \times 5 + 1}_{3)}$

to interchange

Here, the numbers marked 5) and marked 3) are to be interchanged to establish the series.

Hence 3) is the answer.

E-22 Find the binary equivalents of

(i) 30; (ii) 27; (iii) 41

S-22 (i)

		Remainders
2	30	→ 0
2	15	→ 1
2	7	→ 1
2	3	→ 1
2	1	→ 1
	0	

So, the binary equivalent is $(\mathbf{11110})_2$.

(ii)

		Remainders
2	27	→ 1
2	13	→ 1 ↑
2	6	→ 0
2	3	→ 1
2	1	→ 1
	0	

So, the binary equivalent is $(\mathbf{11011})_2$

(iii)

		Remainders
2	41	1
2	20	0 ↑
2	10	0
2	5	1
2	2	0
2	1	1
	0	

So, the binary equivalent is $(\mathbf{101001})_2$.

E-23 Simplify the following for question mark (?)

(i) $10110_2 - 1011_2 = (?)_2$

(ii) $1001_2 + 1010_2 = (?)_2$

(iii) $101_2 \times 100_2 = (?)_2$

(iv) $1100_2 \div 11_2 = (?)_2$

(v) $101_2 + 1100_2 \div 10_2 = (?)_2$

S-23 (i)

1	0	1	1	0	Upper row
	1	0	1	1	Lower row
1		1	1		Borrow
0	1	0	1	1	

Hence the required result is $(\mathbf{1011})_2$.

(ii)

	1	0	0	1
+	1	0	1	0
1	0	0	1	1

Hence the required result is $(\mathbf{10011})_2$.

(iii)

		1	0	1
	×	1	0	0
		0	0	0
	0	0	0	
1	0	1		
1	0	1	0	0

Hence the required result is $(\mathbf{10100})_2$.

(iv)

```
11 ) 1100 ( 100
     11
     ---
     000
```

Hence the required result is $(\mathbf{100})_2$.

(v) $101_2 + 1100_2 \div 10_2$

Firstly,

```
10 ) 1100 ( 110
     10
     ---
      10
      10
      ---
       00
```

Now $101_2 + 110_2$ is

```
  101
  110
 ----
 1011
```

Hence the required result is $(\mathbf{1011})_2$.

E-24 In a certain code, the symbol for 0 (zero) is * and for 1, it is ●. There are no other symbols for other numbers and all numbers greater than 1 are written using these two symbols only. The value of symbol 1 doubles itself everytime it shifts one place to the left.

Thus, 0 is written as *

1 is written as ●

2 is written as ●*

3 is written as ●●

On the above coding system, answer the following questions.

(i) What is the ratio of ●●* and ●**●?

(ii) Find the value of 10 ÷ 5 + 2 in the above code.

(iii) Evaluate: ●●* + ●*● – ●●●.

(iv) If 50% of ●*●* is added to ●*●, then what will be the result?

(v) Find the product of ●●● and ●*.

S-24 (i) ●●* = $(110)_2 = 4 + 2 = 6_{10}$

●**● = $(1001)_2 = 8 + 1 = 9_{10}$

$\therefore$ Required ratio = $\frac{6}{9} = \frac{\mathbf{2}}{\mathbf{3}}$.

(ii) $10 \div 5 + 2 = 4_{10}$

$$\begin{array}{r|l l}
2 & 4 & 0 \\ \hline
2 & 2 & 0 \\ \hline
2 & 1 & 1 \\ \hline
 & 0 &
\end{array}$$

So, the required value is $\mathbf{(100)_2}$ = ●**.

(iii) ●●* + ●*● – ●●●.

$= 110 + 101 - 111$

So, firstly

$$\begin{array}{rrrr}
 & 1 & 1 & 0 \\
+ & 1 & 0 & 1 \\ \hline
1 & 0 & 1 & 1 \\
- & 1 & 1 & 1 \\ \hline
 & 1 & 0 & 0
\end{array}$$

So, the required value is $\mathbf{(100)_2}$. = ●**

(iv) ●*●* = $(1010)_2$

$= 8 + 2 = 10_{10}$

●*● = $(101)_2$

$= 4 + 1 = 5_{10}$.

So, 50% of ●*●* + ●*●

$= 50\%$ of $10_{10} + 5_{10}$

$= 5_{10} + 5_{10} = 10_{10}$

Now,

$$\begin{array}{r|l l}
2 & 10 & 0 \\ \hline
2 & 5 & 1 \\ \hline
2 & 2 & 0 \\ \hline
2 & 1 & 1 \\ \hline
 & 0 &
\end{array}$$

So the required result is ●*●*.

(v) ●●● × ●*

$$\begin{array}{rrrrr} & 1 & 1 & 1 & \\ & & 1 & 0 & \times \\ \hline & 0 & 0 & 0 & \\ 1 & 1 & 1 & & \\ \hline 1 & 1 & 1 & 0 & \end{array}$$

i.e. ●●●*

REGULAR PROBLEMS

Section A: Number System

(1) In a division, find the divisor if dividend = 27541, quotient = 233 and remainder = 47
(a) 172 (b) 238 (c) 126 (d) 194 (e) 118

(2) The least number that must be subtracted from 104075 to make it exactly divisible by 437 is:
(a) 31 (b) 69 (c) 50 (d) 44 (e) 38

(3) The greatest 5-digit number that is exactly divisible by 100 is:
(a) 99899 (b) 99800 (c) 99900 (d) 99889 (e) 98990

(4) Which of the following numbers is a prime number?
(a) 541 (b) 323 (c) 217 (d) 551 (e) None

(5) Which is the least 7-digit number, that leaves a remainder of 3 when divided by 7?
(a) 1000003 (b) 1000010 (c) 1000005 (d) 1000002 (e) 1000007

(6) When a certain number is multiplied by 21, the product consists of only fours. The smallest suc number is :
(a) 21164 (b) 4444 (c) 444444 (d) 444 (e) 3126

Hint: Assuming the product, you can find the other multiplicand

(7) A number when divided by 627 leaves a remainder 43. By dividing the same number by 19, th remainder will be
(a) 19 (b) 24 (c) 43 (d) 5 (e) 13

(8) What will be the remainder when $(29)^{36}$ is divided by 28?
(a) 0 (b) 1 (c) 29 (d) 5 (e) Cannot be determined

(9) The sum of all odd numbers from 1 to 41 is:
(a) 372 (b) 505 (c) 441 (d) 398 (e) 516

Hint: Refer 1.10

(10) What is the total number of numbers up to 9999?
(a) 98900 (b) 10000 (c) 9999 (d) 98100 (e) None of these

(11) The digit in the unit place in $(1038)^{67}$ is:
(a) 2 (b) 4 (c) 1 (d) 6 (e) 8

Hint: Refer 1.11.1

(12) The number of prime numbers in $(25)^{13} \times (10)^{7} \times (27)^{5}$ is:
(a) 25 (b) 32 (c) 55 (b) 50 (e) 42

Hint: Resolve each number into its prime factors, then count the number of such factors by addin the index.

(13) The number of zeros at the end of the product (SSC, '97)
$16 \times 22 \times 15 \times 50 \times 65 \times 115 \times 18 \times 90$ is:
(a) 5 (b) 6 (c) 12 (d) 7 (10)

Hint: Refer 1.11.2 in the text

(14) What is the difference in intrinsic value and local value of 6 in 8631?
(a) 625 (b) 594 (c) 600 (d) 496 (e) 0

(15) How many such numbers are there between 1 and 100 such that each of whi. is not only divisible by 4, but also has one digit as 4 in the number?
(a) 5 (b) 12 (c) 6 (d) 15 (e 7

(16) A number is greater than 3 but less than 8. Also, the number is greater than 6 but less than 10. What is the number?
(a) 5 (b) 4 (c) 9 (d) 6 (e) 7

(17) In a division, a student took 63 as divisor instead of 36. His answer was 24. The correct answer is
(a) 42 (b) 32 (c) 48 (d) 28 (e) 38

(18) $4^{61} + 4^{62} + 4^{63} + 4^{64}$ is divisible by:
(a) 3 (b) 11 (c) 13 (d) 17 (e) None of these

(19) How many numbers are there between 500 and 600 in which 9 occurs only once?
(a) 18 (b) 19 (c) 20 (d) 21 (e) 22

(20) Replace the * in the number 6* 106 by a suitable digit so that the number formed is exactly divisible by 11.
(a) 3 (b) 4 (c) 2 (d) 1 (e) 1

(21) The value of 101 + 102 + 103 + . . . + 200 is:
(a) 15050 (b) 20200 (c) 10909 (d) 16500 (e) None of these

(22) What are the values of '*a*' and '*b*', if 4266 *ab* is divisible by 45?
(a) 4 and 5 (b) 1 and 7 (c) 9 and 0 (d) 3 and 6 (e) either (a) or (c)

Hint: Refer 1.2

(23) The digit in the unit's place of the number
$17^{1999} + 11^{1999} - 7^{1999}$ is:
(a) 7 (b) 4 (c) 1 (d) 3 (c) None of these

Hint: Use of the concept (in ref 1.11.1) can be avoided here if we use a little common sense

(24) The number, one less than 7^{19} is divisible by:
(a) 49 (b) 7 (c) 16 (d) 18 (e) 6

Answers

1. (e)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (c)
10. (c)	**11.** (a)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (e)	**16.** (e)	**17.** (a)	**18.** (d)
19. (a)	**20.** (c)	**21.** (a)	**22.** (e)	**23.** (c)	**24.** (e)			

Section B: Number Series

(1) Which of the following does not fit in the series
(a) 3, 7, 12, 27, 51, 105, 204. (RRB Kolkata '02

Hint: $3 \times 2 + 1$, $7 \times 2 - 2$, $12 \times 2 + 3$ . . .

(b) 4443, 2433, 4322, 4511, 6221 (RRB Kolkata '02

Hint: Digits add to 11

(c) 0, 6, 24, 90, 120, 210 **(RRB Kolkata '0**
(d) 318, 368, 345, 395, 372, 422, 400, 449 **(AAO '9**
(e) 196, 168, 143, 120, 99, 80 63
(f) 3, 4, 20, 38, 87, 168, 289 **(BSRB, Bang, '9**

Hint: $+1^2, +3^2, +5^2 \ldots$
(g) 698, 343, 170, 82, 38, 16, 5 **(BSRB, Bang, '9**
(h) 698, 554, 454, 390, 347, 338, 334 **(BSRB, Bang, '9**

Hint: $-12^2, -10^2, -8^2 \ldots$
(i) 376, 188, 88, 40, 16, 4, −2 **(BSRB, Delhi, '9**

Hint: $\div 2 - 4$
(j) 10, 15, 24, 35, 54, 75, 100 **(IBPS Bank PO, '0**

Hint: $+5, +9, +13 \ldots$
(k) 1, 3, 4, 7, 11, 18, 27, 47 **(IBPS Bank PO, '0**

Hint: Middle number = difference of succeeding and proceeding number
(l) 2, 8, 32, 148, 765, 4626, 32431 **(IBPS, '0**

Hint: $\times 2 + 2^2, \times 3 + 3^2, \times 4 + 4^2 \ldots$
(m) 2, 3, 11, 38, 102, 229, 443 **(IBPS, '0**

Hint: Differences are $1^3, 2^3, 3^3 \ldots$
(n) 0, 6, 23, 56, 108, 184, 279 **(RRB, Bhubaneswar, 200**

Hint: $1^3 - 2^0, 2^3 - 2^1, 3^3 - 2^2, 4^3 - 2^3 \ldots$
(o) 49, 56, 64, 71, 81, 90, 100, 110 **(RRB, Bhubaneswar, 200**
(p) 13700, 1957, 326, 65, 16, 6, 2 **(Andhra Bank PO, '9**

Hint: $-1 \div 7, -1 \div 6, -1 \div 5 \ldots$
(q) 5, 7, 13, 25, 45, 87, 117 **(Andhra Bank PO, '9**

(2) What will come in place of the question mark (?) in the following number series? (Andhra Ba Specialist Officer's Exam, 2002)
(a) 2, 9, 30, 105, ?, 2195
(b) 3, 4, 12, 45, ?, 1005
(c) 1, 3, 9, 31, ?, 651
(d) 5, ?, 4, 7.5, 17, 45
(e) 15, 30, ?, 40, 8, 48

Direction (3–23): In each of the following questions, a number series is given. After the series, below a number is given followed by (A), (B), (C), (D) and (E). You have to complete the series starting wi the number given, following the sequence for the given series on top. Then answer the question below

(3) 616 496 397 317 254 **(SBI ASSC Bank PO, '9**
838 A B C D E
What will come for E?

(4) 6 9 27 121.5 729 **(SBI ASS. Bank PO, '9**
8 A B C D E
What will come for C?

(5) 6 14 35 111 149 **(SBI ASS. Bank PO, '9**
3 A B C D E
What will come for E?

(6) 434 353 417 368 404 379 **(SBI ASS Bank PO, '97)**
108 A B C D E
What will come for D?

(7) 5 6 16 57 244 1245 **(RBI Grade-B, '02)**
2 A B C D E
What will come for D?

(8) 3 5 9 17 33 65 **RBI Grade-B, '02)**
7 A B C D E
What will come for C?

(9) 7 4 5 9 20 52.5 **(RBI Grade-B, '02)**
3 A B C D E
What will come for C?

Hint: $\times 0.5 + 0.5$, $\times 1 + 1$, $\times 1.5 + 1.5$ . . .

(10) 3 10 32 111 460 2315 **(RBI Grade-B, '02)**
2 A B C D E
What will come for B?

Hint: $10 = 3 \times 1 + 7 \times 1$, $32 = 10 \times 2 + 6 \times 2$, $111 = 32 \times 3 + 5 \times 3$, $460 = 111 \times 4 + 4 \times 4$ . . . .

(11) 5 8 6 10 7 12 **(RBI Grade-B, '02)**
7 A B C D E
What will come for C?

Hint: $\times 2 - 2$, $\div 2 + 2$, $\times 2 - 2$, . . .

(12) – 4 2 10 96 6150 **(RBI Grade-B, '02)**
–10 A B C D E
What will come for D?

Hint: $\times 1^0 + 6$, $\times 2^1 + 6$, $\times 3^2 + 6$ . . .

(13) 4 6 15 79 704 8480 **(NABARD, '01)**
12 A B C D E
What will come for B?

(14) 200 184 193 157 182 118 167 **(NABARD '01)**
150 A B C D E
What will come for E?

Hint: -4^2, $+3^2$, -6^2, . . .

(15) 60 121 131 264 284 571 601 **(NABARD, '01)**
120 A B C D E
What will come for D?

(16) 0.25 1.25 –3 0 –64 **(NABARD, '01)**
45 A B C D E
What will come for C?

(17) 5 7 10 36 136 690 **(SBI PO, '01)**
2 A B C D E
What will come for D?

Hint: $\times 1 + 2$, $\times 2 - 4$, $\times 3 + 6$, . . .

(18) 8 9 13 12 8 9
12 A B C D E
What will come for E?

(19) 3 20 118 587 2344 **(SBI PO, '01)**
12 A B C D E
What will come for C?

(20) 0 16 48 112 240 **(SBI PO, '01)**
120 A B C D E
What will come for D?

Hint: $(+ 2^1 \times 8), (+ 2^2 \times 8) (+ 2^3 \times 8) \ldots$

(21) 108 52 24 10 3
64 A B C D E
What will come for D?

(22) 5 12 60 340 **(BSRB, Mumbai, '98)**
7 A B C D E
What will come for D?

Hint: $\times 8 - 28, \times 7 - 24, \times 6 - 20 \ldots$

(23) 4 7 24 93 **(BSRB, Chennai, '98)**
2 A B C D E
What will come for D?

Answers

1. (a) 51, (b) 4443, (c) 90, (d) 400, (e) 196, (f) 20, (g) 343, (h) 347, (i) 188, (j) 35, (k) 27, (l) 32, (m) 229, (n) 108, (o) 71, (p) 6, (q) .87.
2. (a) 436, (b) 196, (c) 129, (d) 3, (e) 10.
3. 428 **4.** 162 **5.** 1889 **6.** 78 **7.** 172 **8.** 49 **9.** 6 **10.** 30 **11.** 14
12. –762 **13.** 23 **14.** 68 **15.** 524 **16.** 1611 **17.** 64 **18.** 13 **19.** 2477 **20.** 360
21. 0.25 **22.** 5044 **23.** 360

REAL PROBLEMS

Section A: Number System

(1) The greatest 5-digit number, that leaves a remainder of 19 if divided by 23 is:
(a) not possible (b) 99980 (c) 99982 (d) 99977 (e) 99962

Hint: Here the actual remainder is less than the desired remainder (= 19). So, it can not be found out by the conventional method of simply adding the desired remainder to the exactly divisible 5-digit greatest number. This is because conventional method is applicable only when actual remainder (= 18) is greater than the desired remainder.

(2) The four integers consecutively lower than 81, and the four consecutively higher than 81, are added together. This sum is divisible by:
(a) 7 (b) 9 (c) 11 (d) 13 (e) None of these

(3) In a question, divisor is $\frac{2}{3}$ of the dividend and twice the remainder. If the remainder is 5, then the dividend is **(SSC, '94)**
(a) 85 (b) 145 (c) 225 (d) 65 (e) None of these

(4) The number that is nearest to 2160 and exactly divisible by 52 is:
(a) 2132 (b) 2148 (c) 2184 (d) 2177 (e) None of these

Tips: Check if the remainder obtained is more or less than half the divisor, because nearest number is to be found out.

(5) What will be the remainder when $(16^{27} + 37)$ is divided by 17?
(a) 4 (b) 19 (c) 13 (d) 2 (e) 14

Hint: Refer the Remainder Rules

(6) What is the number of digits of the smallest number which when multiplied by 7 gives a result consisting entirely of nines?
(a) 3 (b) 6 (c) 5 (d) 4 (e) 7

(7) What will be the digit in the unit place in the product $(3807)^{194} \times (932)^{84}$?
(a) 9 (b) 1 (c) 2 (d) 4 (e) 7

Hint: Refer 1.11.1

(8) The numbers 1, 3, 5 . . . 25 are multiplied together. The number of zeroes at the right end of the product is:
(a) 22 (b) 8 (c) 13 (d) 6 (e) 0

(9) A number when divided by a divisor, leaves a remainder of 63. If the remainder is 55 when twice the number is divided by the same divisor, then the divisor is:
(a) 21 (b) 37 (c) 16 (d) 49 (e) None of these

Hint: Divisor must be always greater than the remainder. Use this concept. Do not go for calculation.

(10) The sum of two numbers is 's' and their quotient is $\frac{p}{q}$. The numbers are:

(a) $\frac{ps}{q}, \frac{qs}{p}$ (b) $\frac{s}{p}, \frac{s}{q}$ (c) $\frac{s-p}{q}, \frac{s-q}{p}$ (d) $\frac{ps}{p+q}, \frac{qs}{p+q}$ (e) None

(11) How many digits are required to number a book containing 200 pages?
(a) 200 (b) 600 (c) 492 (d) 372 (e) 250

(12) How many numbers between 101 and 300 are divisible by both 3 and 5?
(a) 107 (b) 20 (c) 127 (d) 14 (e) 34

(13) A number is multiplied by 5 and 25 is added to it. The result is divided by 5 and the original number is subtracted from the same. The remainder will be:
(a) 0 (b) 1 (c) 2 (d) 3 (e) 5

Tips: Do not waste your time by forming a linear equation. Rather use the concept of remainder

(14) $\left(\frac{1}{3} - \frac{1}{4}\right)$ is added to a number. From the sum so obtained, $\frac{1}{3}$ of $\frac{1}{4}$ is subtracted and the remainder is $\left(\frac{1}{3} + \frac{1}{4}\right)$. The number is:

(a) $\frac{1}{12}$ (b) $\frac{7}{12}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) $\frac{3}{4}$

(15) A number when divided by 5 leaves a remainder 3. What is the remainder when the square of the same number is divided by 5?
(a) 9 (b) 3 (c) 0 (d) 4 (e) 1

(16) There is a number 8 * 20 which if multiplied by 6, the product is divisible by 8. The digit replacin * mark is:

(a) 4 (b) 1 (c) any digit in between 0 and 9 (d) 7 (e) 0

(17) If $B = 2 \times 4 \times 6 \ldots 98 \times 100$, then the number of zeroes at the end of B will be:

(a) 330 (b) 11 (c) 10 (d) 101 (e) 12

(18) In the product of 24 * × 981 × 79 × 104 if the digit in the unit place is 2, then what will come i place of the asterisk?

(a) 2 (b) 3 (c) 6 (d) 7 (e) either (a) or (d)

(19) A number consists of four digits having 8 in the unit's place. If the digit in the extreme left is shifte to the immediate right to the unit's place, keeping all other numbers as they are, the new numbe formed exceeds the original number by 1305. Find the original number. **(ASSL. Grade, '96**

(a) 4358 (b) 2731 (c) 3478 (d) 3316 (e) 4387

(20) A 4-digit number divisible by 7 becomes divisible by 3 when 10 is added to it. The largest suc number is:

(a) 9999 (b) 9996 (c) 9989 (d) 9987 (e) 9993

Answers

1. d) **2.** (b) **3.** (e) **4.** (c) **5.** (d) **6.** (b) **7.** (d) **8.** (e) **9.** (e)
10. (d) **11.** (c) **12.** (d) **13.** (a) **14.** (b) **15.** (d) **16.** (c) **17.** (e) **18.** (e)
19. (c) **20.** (c)

Section B: Number Series

(1) How many 7s are there in the following series that are not immediately followed by 3 but immediatel preceeded by 8 **(ASM, '02**

8987622632697328727787377 94

(a) 2 (b) 3 (c) 4 (d) 6 (e) 7

(2) If the given numbers are arranged in the descending order based on the sum of the digits of eac number, which number will be in the middle?

842 641 961 479 715 216 523

(a) 961 (b) 216 (c) 479 (d) 715 (e) 523

(3) If by beginning with 1, consecutive numbers are continuously written to its right, then which digit wil be written on thirty-first position?

(a) 1 (b) 2 (c) 3 (d) 0 (e) 4

(4) If the given numbers are arranged in such a way that each group of three assending numbers i followed by their LCM and the beginning number is 1, then 11^{th} number is how many times of the fifth number?

1, 2, 3, 4, 5, 6, 5, 6, 6, 7, 60

(a) $\frac{3}{2}$ (b) 1.75 (c) 1.4 (d) $\frac{1}{3}$ (e) None of those

Directions (5–6): Study the following number series to answer these questions

2 6 7 5 4 3 7 4 8 9 4 3 2 5 4 7 9 8 6 8 7 1 2 5 3 7 6 8 9 3 6

(5) How many such numbers are there in the series which are not immediately followed by its multiple?

(a) 25 (b) 4 (c) 27 (d) 21 (e) 20

(6) If the order of last 15 numbers is reversed, which number will be eighth to the right of thirteenth number from left?

(a) 5 (b) 6 (c) 7 (d) 3 (e) 2

Directions (7–11): One number is wrong in the number series given in each of the following questions. You have to identify that number. Assuming that a new series starts with that number, and following the same logic as in the given series, which of the numbers given in (1), (2), (3), (4) and (5) given below the series will be the third number in the new series? **(SBI PO, '99)**

(7) 3 5 12 38 154 914 4634

(1) 1636 (2) 1222 (3) 1834 (4) 3312 (5) 1488

Hint: $\times 1 + 2$, $\times 2 + 2$, $\times 3 + 2$ . . . So, 914 is incorrect. The new series begins with 914 and third number in the new series will be 1834

(8) 3 4 10 34 136 685 1446

(1) 22 (2) 276 (3) 1374 (4) 72 (5) 12

(9) 214 18 162 62 143 90 106

(1) –34 (2) 110 (3) 38 (4) 10 (5) 91

Hint: $-(14)^2$, $+(12)^2$, $-(10)^2$, $+(8)^2$, . . . So, 143 is incorrect.

(10) 160 80 120 180 1050 4725 25987.5

(1) 60 (2) 135 (3) 3564 (4) 787.5 (5) 90

Hint: $\times\frac{1}{2}$, $\times\frac{3}{2}$, $\times\frac{5}{2}$ . . . Hence 180 is incorrect.

(11) 2 3 7 13 26 47 78

(1) 13 (2) 11 (3) 20 (4) 15 (5) 18

Directions (12–18): In each of the following questions, a number series is established if the positions of two out of the five marked numbers are interchanged. The position of the first unmarked number remains the same and it is the beginning of the series. The earlier of the two marked numbers whose positions are interchanged is the answer. For example, if an interchange of the number marked '1' and the number marked '4' is required to establish the series, your answer is 1. If it is not necessary to interchange the positions of the numbers to establish the series give '5' as your answer.

(12)	8	4	12	6	4	30
		(1)	(2)	(3)	(4)	(5)

Hint: $\times\frac{1}{2}$, $\times 1$, $\times 1\frac{1}{2}$, $\times 2$ . . . So, the numbers at (2) and (4) are to be interchanged

(13)	829	436	661	300	557	508
		(1)	(2)	(3)	(4)	(5)

Hint: -23^2, $+19^2$, -15^2, $+11^2$ + . . .

(14)	6	56	1	19	11	529
		(1)	(2)	(3)	(4)	(5)

Hint: $\times 1^2 + 5$, $\times 1^2 - 10$, $\times 2^2 + 15$ . . .

(15)	21	29	23	21	41	61
		(1)	(2)	(3)	(4)	(5)
(16)	4	0	–7	–45	–20	–94
		(1)	(2)	(3)	(4)	(5)
(17)	0	6	184	56	109	23
		(1)	(2)	(3)	(4)	(5)
(18)	375	363	356	344	336	324
		(1)	(2)	(3)	(4)	(5)

Directions (19–24): In each of the following questions a number series is given. A number in the serie is replaced by a letter 'A'. You have to find out the number that has been replaced by 'A' and use thi number to find out the value that should be in the place of the question mark in the equation following th series.

(19) 36 216 64.8 388.8 A 699.84 209.952 **(BSRB Mumbai PO, '99**

$A \div 36 = ?$

(1) 61.39 (2) 0.324 (3) 3.24 (4) 6.139 (5) 32.4

(20) $\frac{3}{8}$ $\frac{3}{4}$ $\frac{9}{16}$ $\frac{9}{8}$ $\frac{27}{32}$ $\frac{27}{16}$ A. **(BSRB Mumbai PO, '99**

$\sqrt{A} = ?$

(1) $\frac{3}{2}$ (2) $\frac{6}{8}$ (3) $\frac{6}{4}$ (4) $\frac{3}{4}$ (5) $\frac{9}{8}$

(21) 99 163 A 248 273 289

$\sqrt{2A+17} = ?$

(1) 20.5 (2) 21 (3) 20 (4) 20.7 (5) 19

(22) A 12 9 $7\frac{1}{5}$ 6 $5\frac{1}{7}$ **(BSRB Bangalore, 2000**

18% of A + 24% of A = ?

(1) 7.56 (2) 8.20 (3) 9.42 (4) 6.38 (5) 10.64

(23) 125 A 1127 1176 9408 9472

$A^2 - 2A = ?$

(1) 23799 (2) 28063 (3) 25599 (4) 27850 (5) 18749

Hint: Series is $+ 6^2, \times 7, + 7^2, \times 8 \ldots$

(24) 14.8 17.2 A 22 2.8 41.2 **(BSRB Bangalore, 2000**

25% of $25A$ = ?

(1) 77.5 (2) 73.5 (3) 172.5 (4) 86.5 (5) 92.8

Answers

1. (a)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (3)	**8.** (4)	**9.** (5)
10. (2)	**11.** (2)	**12.** (2)	**13.** (1)	**14.** (1)	**15.** (1)	**16.** (3)	**17.** (2)	**18.** (5)
19. (3)	**20.** (5)	**21.** (2)	**22.** (1)	**23.** (3)	**24.** (1)			

2 HCF AND LCM OF NUMBERS

2.1 FACTORS

In a division, if a number f divides a number M completely (exactly) or in other words, if M is exactly divisible by f, then **'f' is the factor of M.**

Example: 5 divides 35 completely, so, 5 is a factor of 35.

Similarly, 2, 3, 4, 6 are all factors of 12, because each of the numbers 2, 3, 4, and 6 will divide 12 completely or, in other words 12 is divisible by 2, 3, 4 and 6.

2.2 MULTIPLES

From the above concept, if f is a factor of M, then M is a multiple of f.

Example: 63 is completely divisible by 7, 3, 9, 21. So, 63 is a multiple of 7 or 3 or 9 or 21.

2.3 PRINCIPLE OF PRIME FACTORISATION

Any natural number (>1) is either prime or non-prime (composite).

The principle of prime factorisation states:

Each non-prime (composite) number can be uniquely broken (reduced) into two or more prime numbers prime factors). In other words, each non-prime number is divisible by any of the prime numbers.

With the use of this principle, a non-prime number is broken into its prime factor by dividing it with different prime numbers. This is known as division method of factorisation of a number. The same is explained in the following example.

Example: Resolve 20570 into its prime factors.

Division by prime number ↓				
	2	20570		
	5	10285	→	1^{st} Quotient
Prime Factors	11	2057	→	2^{nd} Quotient
	11	187	→	3^{rd} Quotient
	17	17	→	4^{th} Quotient
		1		

Thus, $20570 = 2 \times 5 \times 11 \times 11 \times 17$.
Hence, if the number is even, the division should start with 2; otherwise, rest of the prime numbers shou be tried in succession.

2.4 HIGHEST COMMON FACTOR (HCF)

If two or more numbers are broken into their prime factors (as explained in 2.3). then the product of t maximum common prime factors in the given numbers is the H.C.F. of the numb rs.

In other words, the HCF of two or more numbers is the greatest number (divis or) that divides all t given numbers exactly. So, HCF is also called the Greatest Common Divisor (GCD).

Example: Find the HCF of 72, 60, 96.
Here, we first find the prime factors of each given number.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

Here $72 = 2 \times 2 \times 2 \times 3 \times 3$
$60 = 2 \times 2 \times 3 \times 5$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

and so HCF = product of maximum common prime factors $= 2 \times 2 \times 3 = 12$

Note: The common factors in the given numbers have been encircled.

2.5 LCM (LOWEST COMMON MULTIPLE)

The LCM of two or more than two numbers is the product of the highest powers of all the prime facto that occur in these numbers.

Example: Find the LCM of 36, 48, 64 and 72

$$\begin{array}{r|rrrr} 2 & 36, & 48, & 64, & 72 \\ \hline 2 & 18, & 24, & 32, & 36 \\ \hline 2 & 9, & 12, & 16, & 18 \\ \hline 2 & 9, & 6, & 8, & 9 \\ \hline 3 & 9, & 3, & 4, & 9 \\ \hline 3 & 3, & 1, & 4, & 3 \\ \hline & 1, & 1, & 4, & 1 \end{array}$$

$\therefore$ LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 4 = 576$.

2.6 PRODUCT OF TWO NUMBERS

HCF of numbers × LCM of numbers = Product of numbers.

i.e., if the numbers are A and B, then

$$\text{HCF of } A \text{ and } B \times \text{LCM of } A \text{ and } B = A \times B$$

2.7 DIFFERENCE BETWEEN HCF AND LCM

HCF of x, y and z	LCM of x, y and z
is the **Highest Divisor** which can exactly divide x, y and z.	is the **Least Dividend** which is exactly divisible by x, y and z.

Table 2.1 Rapid Information List

Ref. No.	*Type of Problem*	*Approach to Problem*
2.3	Find the *GREATEST NUMBER* that will *exactly* divide x, y and z.	Required number = *HCF* of x, y and z (greatest divisor)
2.4	Find the *GREATEST NUMBER* that will divide x, y and z leaving remainders a, b and c respectively.	Required number (greatest divisor) = *HCF* of $(x - a)$, $(y - b)$ and $(z - c)$
2.5	Find the *LEAST NUMBER* which is *exactly* divisible by x, y and z.	Required number = *LCM* of x, y and z (least dividend)
2.6	Find the *LEAST NUMBER* which when divided by x, y and z leaves the remainders a, b and c respectively.	Then, it is always observed that $(x - a) = (y - b) = (z - c) = K$ (say). ∴ Required number = (*LCM* of x, y and z) − (K)
2.7	Find the *LEAST NUMBER* which when divided by x, y and z leaves the same remainder 'r' each case.	Required number = (*LCM* of x, y and z) + r.
2.8	Find the *GREATEST NUMBER* that will divide x, y and z leaving the same remainder in each case.	Required number = *HCF* of $(x - y)$, $(y - z)$ and $(z - x)$
2.9	Find the n-digit *GREATEST NUMBER* which when divided by x, y and z. (a) leaves no remainder (i.e., exactly divisible) (b) leaves remainder K in each case.	*LCM* of x, y and z = L (Step 1) L) *n-digit greatest number* ((Step 2) *remainder* = R *By Rule I (Chapter 1).* (a) Required number = n-digit greatest number − R (b) Required number = [n-digit greatest number − R] + K

(*Contd.*)

(Contd.)

Ref. No.	*Type of Problem*	*Approach to Problem*
2.10	Find the n-digit *SMALLEST NUMBER* which when divided by x, y and z (*a*) leaves no remainder (i.e., exactly divisible) (*b*) leaves remainder K in each case.	*LCM* of x, y and $z = L$ (Step 1) L) n-digit greatest number ((Step 2) *remainder* $= R$ *By Rule II (Chapter 1)* (a) Required number $=$ n-digit smallest number $+ (L - R)$ (b) Required number $=$ n-digit smallest number $+ (L - R) + K$
2.11	Find the *HCF* of $\frac{x}{y}, \frac{a}{b}$ and $\frac{m}{n}$	*HCF* of fractions $= \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$
2.12	Find the *LCM* of $\frac{x}{y}, \frac{a}{b}$ and $\frac{m}{n}$	*LCM* of fractions $= \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$
2.13	Find the *HCF* of decimal numbers	**Step 1** Find the *HCF* of the given numbers **without** decimal. **Step 2** Put the decimal point (in the *HCF* of Step 1) from right to left according to the MAXIMUM decimal places among the given numbers.
2.14	Find the *LCM* of decimal numbers	**Step 1** Find the *LCM* of the given numbers **without** decimal. **Step 2** Put the decimal point (in the *LCM* of Step 1) from right to left at the place equal to the MINIMUM decimal places among the given numbers.

Solved Examples

E-1 Find the greatest number that will exactly divide 200 and 320.

S-1 Using the approach *2.3*,
required number = HCF of 200 and 320 = **40**.

E-2 Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11 respectively.

S-2 Using the approach *2.4*,
required number = HCF of (148 – 4),
(246 – 6) and (623 – 11) i.e. HCF of 144, 240, 612 is **12.**

E-3 Find the least number which when divided by 27, 35, 45 and 49 leaves the remainder 6 in each case.

S-3 Using the approach *2.7*,
required number = (LCM of 27, 35, 45 and 49) + 6 i.e. 6615 + 6 = **6621.**

E-4 Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53 respectively.

S-4 Using the approach *2.6*,

we get, $(36 - 25) = (48 - 37) = (64 - 53) = 11$

$\therefore$ required number = (LCM of 36, 48 and 64) − 11

$= 576 - 11 =$ **565**.

E-5 Find the greatest possible length of a scale that can be used to measure exactly the follo ing lengths of cloth; 3 m, 5 m, 10 cm and 12 m 90 cm.

S-5 The lengths of cloth to be measured are, 300 cm, 510 cm and 1290 cm.

$\therefore$ the required length of the scale is HCF of 300, 510 and 1290 i.e. 30

$\therefore$ the greatest possible length of the scale to be used = **30** cm.

E-6 Find the smallest number which when

(a) increased by 8 (or added by 8) ***(b)*** decreased by 8 (or subtracted by 8)

is exactly divisible by 15, 21, 30.

S-6 LCM of 15, 21, 30 = 210.

(a) the required number = LCM − (the number added) i.e. $210 - 8 = 202$

(b) the required number = LCM + (the number subtracted) i.e. $210 + 8 =$ **218**.

E-7 Find

(a) the greatest number of 4 digits and ***(b)*** the smallest number of 4 digits

such that they are exactly divisible by 12, 15, 20 and 35.

S-7 ***(a)*** Using the approach *2.9(a)*,

Step 1 LCM of 12, 15, 20 and 35 = 420.

Step 2

$$\begin{array}{r} 420\ )\ 9999\ (\ 23 \\ \underline{9660} \\ 339 \end{array}$$

$\therefore$ required number $= 9999 - 339 =$ **9663**.

(b) Using the approach *2.10(a)*,

Step 1 LCM of 12, 15, 20 and 35 = 420.

Step 2

$$\begin{array}{r} 420\ )\ 1000\ (\ 2 \\ \underline{840} \\ 160 \end{array}$$

$\therefore$ required number $= 1000 + (420 - 160) =$ **1260**.

E-8 Four bells first begin to toll together and then at intervals of 6, 7, 8 and 9 seconds respectively. Find how many times the bells toll *together* in two hours and *at what interval they toll together*?

S-8 LCM of 6, 7, 8 and 9 = 504.

$\therefore$ All the bells toll together after each interval of 504 seconds.

$\therefore$ in two hours, no. of times they toll together $= \dfrac{2 \times 60 \times 60}{504}$ times $\approx$ **14 times**.

E-9 Find the

(a) the greatest 3-digit number, and ***(b)*** the smallest 3-digit number

such that when they are divided by 12, 18, 21 and 28, it leaves a remainder 3 in each case.

S-9 ***(a)*** Using the approach *2.9(b)*,

Step 1 LCM of 12, 18, 21 and 28 = 252

E-4 Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53 respectively.

S-4 Using the approach *2.6*,
we get, $(36 - 25) = (48 - 37) = (64 - 53) = 11$
$\therefore$ required number = (LCM of 36, 48 and 64) − 11
= 576 − 11 = **565**.

E-5 Find the greatest possible length of a scale that can be used to measure exactly the following lengths of cloth; 3 m, 5 m, 10 cm and 12 m 90 cm.

S-5 The lengths of cloth to be measured are, 300 cm, 510 cm and 1290 cm.
$\therefore$ the required length of the scale is HCF of 300, 510 and 1290 i.e. 30
$\therefore$ the greatest possible length of the scale to be used = **30** cm.

E-6 Find the smallest number which when
(a) increased by 8 (or added by 8) ***(b)*** decreased by 8 (or subtracted by 8)
is exactly divisible by 15, 21, 30.

S-6 LCM of 15, 21, 30 = 210.
(a) the required number = LCM − (the number added) i.e. 210 − 8 = 202
(b) the required number = LCM + (the number subtracted) i.e. 210 + 8 = **218**.

E-7 Find
(a) the greatest number of 4 digits and ***(b)*** the smallest number of 4 digits
such that they are exactly divisible by 12, 15, 20 and 35.

S-7 ***(a)*** Using the approach *2.9(a)*,
Step 1 LCM of 12, 15, 20 and 35 = 420.
Step 2

```
420 ) 9999 ( 23
      9660
      ----
       339
```

$\therefore$ required number = 9999 − 339 = **9663**.

(b) Using the approach *2.10(a)*,
Step 1 LCM of 12, 15, 20 and 35 = 420.
Step 2

```
420 ) 1000 ( 2
       840
      ----
       160
```

$\therefore$ required number = 1000 + (420 − 160) = **1260**.

E-8 Four bells first begin to toll together and then at intervals of 6, 7, 8 and 9 seconds respectively. Find how many times the bells toll *together* in two hours and *at what interval they toll together* ?

S-8 LCM of 6, 7, 8 and 9 = 504.
$\therefore$ All the bells toll together after each interval of 504 seconds.
$\therefore$ in two hours, no. of times they toll together = $\frac{2 \times 60 \times 60}{504}$ times ≈ **14 times**.

E-9 Find the
(a) the greatest 3-digit number, and ***(b)*** the smallest 3-digit number
such that when they are divided by 12, 18, 21 and 28, it leaves a remainder 3 in each case.

S-9 ***(a)*** Using the approach *2.9(b)*,
Step 1 LCM of 12, 18, 21 and 28 = 252

Step 2 252) 9999 (39
9828
———
171

$\therefore$ the required number = (9999 − 171) + 3 = **9931**.

(b) Using the approach 2.*10*(*b*),

Step 1 LCM of 12, 18, 21 and 28 = 252.

Step 2 252) 1000 (3
756
———
244

$\therefore$ the required number = 1000 + (252 − 244) + 3 = **1011**.

E-10 Find the numbers between 200 and 300 such that when they are divided by 6, 8 9,

(a) it leaves no remainder, i.e. exactly divisible.

(b) it leaves in each case a remainder 5

S-10 Here, the number to be found out are the DIVIDENDS. Now, the LCM of 6, 8, 9 = 72.

(a) Multiples of 72 which lie between 200 and 300 are $72 \times 3 = 216$ and $72 \times 4 = 288$

(b) Here, the remainder is 5 in each case.

$\therefore$ Required numbers are (216 + 5) and (288 + 5) i.e. **221** and **293**.

E-11 There are two electrical wires, one is a 9 m 60 cm long aluminium wire and the other is a 5 m 12 cm long copper wire. Find the

(a) maximum length that can be equally cut from each wire in such a way that the total length of each wire is exactly divisible by it.

(b) how many such largest possible pieces are available in each kind of wire?

S-11 9 metre 60 cm = 960 cm and 5 metre 12 cm = 512 cm.

(a) The required largest piece = HCF of 960 and 512 cm, i.e. **64 cm.**

(b) $\therefore$ Number of such aluminium wire pieces = $\frac{960}{64}$ **nos.**

and number of such copper wire pieces = $\frac{512}{64}$ **nos.**

E-12 HCF and LCM of two numbers are 16 and 240 respectively. If one of the numbers is 48, find the other number.

S-12 We know that, HCF × LCM = Product of two numbers

$\therefore$ second number = $\frac{16 \times 240}{48}$ i.e. **80**.

E-13 Among how many students, 175 bananas and 105 oranges can be equally divided?

S-13 HCF of 175 and 105 = 35

$\therefore$ The required number of students is **35**, or factors of 35, namely 5 or 7.

E-14 Find out the HCF of 11, 0.121 and 0.1331.

S-14 *Step 1* HCF of 11, 121 and 1331 is **11**.

Step 2 Resultant HCF = **0.0011** (Since maximum decimal places = 4 in 0.1331).

E-15 Find out the LCM of 2.2, 540 and 1.08.

S-15 *Step 1* LCM of 22, 540 and 108 is **5940**.

Step 2 Here minimum decimal place = 1 (in 2.2)

So, resultant LCM = **594**.

E-16 Find out the HCF of 3^5, 3^9 and 3^{14}.

S-16 Here the base of each number is same (= 3) but indices are different.
So, the required HCF = number with the minimum index, i.e. $\mathbf{3^5}$.

E-17 Find out the LCM of 4^5, 4^{-81}, 4^{12} and 4^7.

S-17 Here the base of each number is the same (= 4) but indices (or powers) are different.
So, the required LCM = number with the maximum index, i.e. $\mathbf{4^{12}}$.

REGULAR PROBLEMS

(1) What is the greatest possible length of scale to measure exactly the following lengths, 20 feet, 13 feet 9 inches, 17 feet 6 inches, 21 feet 3 inches?
(a) 1 feet 6 inches (b) 1 feet 3 inches (c) 9 inches
(e) 2 feet 4 inches (e) 6 inches

(2) The greatest number that will divide 410,751 and 1030 leaving a remainder 7 in each case is:
(a) 29 (b) 13 (c) 17 (d) 37 (e) 31

Hint: Since the number to be found out is a GREATEST DIVISOR, so HCF is to be found out

(3) The ratio of two numbers is 15 : 11. If their HCF is 13, then the numbers are:
(a) 75, 55 (b) 45, 22 (c) 104, 44 (d) 195, 143 (e) None

Hint: Since HCF is 13, so, the numbers will be 13 × 15 and 13 × 11 (Delhi Metro Rail, 2002)
terms of ratio

(4) The LCM of two numbers is 1296 and HCF is 96. If one of the numbers is 864, then the other is:
(a) 72 (b) 64 (c) 144 (d) 11664 (e) 36

(5) Three men start together to walk along a road at the same rate. The length of their strides are 68 cm, 51 cm and 85 cm respectively. How far will they go before they will be 'in step' again:
(a) 102 m (b) 1020 m (c) 10.2 m (d) 150 m (e) 17 cm

(6) How many times is the HCF of 48, 36, 72 and 24 contained in their LCM?
(a) 10 (b) 12 (c) 120 (d) 2 (e) 15

(7) The greatest 4-digit number exactly divisible by 88 is:
(a) 8888 (b) 9944 (c) 9988 (d) 9999 (e) 8899

(8) Find the least number of soldiers in a regiment, such that they stand in rows of 18, 15 and 25 and form a perfect square?
(a) 900 (b) 1600 (c) 2500 (d) 450 (e) 400

Hint: Find LCM and then multiply by the factors to make it a perfect square

(9) Three strings of a musical instrument vibrate 6, 8, and 12 times a second respectively. If all the three begin to vibrate simultaneously, find the shortest time interval before all three vibrate together again?
(a) 2 sec (b) 48 sec (c) $\frac{1}{2}$ sec (d) $\frac{1}{24}$ sec (e) 24 sec.

Hint: Time to vibrate once is $\frac{1}{6}$ sec, $\frac{1}{8}$ sec and $\frac{1}{12}$ sec.

(10) Which is the smallest number that can be subtracted from 1936 so that on being divided by 9, 10, 15 the remainder is 7 everytime? **(RRB Ajmer, '97)**
(a) 93 (b) 46 (c) 76 (d) 39 (e) 53

(11) The smallest number from which if 4000 subtracted, is exactly divisible by 7, 11 and 13, is
(a) 5001 (b) 2999 (c) 1000 (d) 6303 (e) 5101

Hint: Using the concept of LCM, we find, LCM of 7, 11, 13 = Required number – 4000

(12) Find the least number which when divided by 20, 25, 30, 36 and 48 leaves the remainders 15, 20, 25, 31 and 43 respectively.
(a) 2165 (b) 144 (c) 3595 (d) 3600 (e) 2875

(13) Find the H.C.F. of 2.4, 0.36 and 7.2 (RRB, Patna, 2002)
(a) 12 (b) 120 (c) 1.2 (d) 0.12 (e) 0.012

(14) Traffic lights at three different points are changing respectively at 24, 48 and 72 seconds. If all the three are changed together at 9 : 10 : 24 hours, then when will the next change take place together?
(RRB, Guwhati, '97)
(a) 9 : 12 : 25 hrs (b) 9 : 10 : 48 hrs. (c) 9 : 12 : 48 hrs.
(d) 9 : 10 : 50 hrs (e) None

(15) The HCF of two numbers is 12 and their difference is also 12. The numbers are
(a) 12, 84 (b) 100, 112 (c) 40, 52 (d) 84, 96 (e) 110, 124

Hint: Do not try to calculate. Only check which of the given choices satisfy the given condition

(16) The sum of two numbers is 45 and their difference is $\frac{1}{9}$ of their sum. Their LCM is
(a) 200 (b) 100 (c) 90 (d) 180 (e) 250

(17) John has a camera that takes film that allows 24 exposures, whereas Nancy has a camera that takes film that allows 36 exposures. Both of them want to be able to take the same number of photographs and complete their rolls of film. How many rolls should each buy?
(a) 12 (b) 72 (c) 3 and 2 (d) 6 (e) 144

Answers

1. (b) **2.** (e) **3.** (d) **4.** (c) **5.** (c) **6.** (b) **7.** (b) **8.** (d) **9.** (c)
10. (d) **11.** (a) **12.** (c) **13.** (d) **14.** (c) **15.** (d) **16.** (b) **17.** (c)

REAL PROBLEMS

(1) The least number by which 825 must be multiplied in order to produce a multiple of 715 is
(Bank PO, '90)
(a) 11 (b) 5 (c) 13 (d) 17 (e) 19

(2) The LCM of two number is 630 and their HCF is 9. If the sum of the numbers is 153, then the ratio of the two numbers is
(a) 70 (b) 9 (c) 0.7 (d) $\frac{10}{7}$ (e) (c) or (d)

(3) The HCF of two numbers each consisting of four digits is 103 their LCM is 19261, then the numbers are
(a) 1133, 1751 (b) 1621, 2031 (c) 3031, 3523 (d) 2979, 2277 (e) 1833, 1651

(4) The least number which is a multiple of 31 and when divided by 15, 24 and 32 leaves the remainders 2, 11 and 19 respectively is, **(RRB, Secunderabad, '01)**
(a) 2356 (b) 2387 (c) 2325 (d) 2418 (e) 2722

Hint: Here, the remainder in each case is less than the divisor by 13. The number can be 480 k – 13, where, the minimum value of k will make (480 k – 13) divisible by 31. Put k = 1, 2 . . . and check with the given choices.

(5) Three men start together to travel the same way around a circular track of 11 kms. Their speeds are 4, $5\frac{1}{2}$ and 8 kms per hour respectively. When will they meet at the starting point?
(a) 22 hrs. (b) 12 hrs. (c) 11 hrs. (d) 44 hrs. (e) 36 hrs.

(6) In a seminar, the number of participants in Physics, Chemistry and Mathematics are 96, 36 and 180 respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.
(a) 12 (b) 21 (c) 36 (d) 26 (e) Not possible

(7) If the last divisor is 75 and the quotients are 3, 1, 1 and 3 respectively, in finding the HCF of two numbers by the method of division, then those two numbers are **(Mumbai Bank PO, '95)**
(a) 500, 1875 (b) 425, 1675 (c) 525, 1875 (d) 525, 1575 (e) 575, 1875

Hint: Assume two numbers as x and y. Follow the method of division to find HCF.

(8) Sum of two numbers is 56. If their LCM is 105, then the numbers are
(a) 7, 49 (b) 21, 34 (c) 24, 32 (d) 35, 21 (e) 27, 29

Tips: HCF of two numbers = HCF of (their sum and their LCM)

(9) The greatest number of three digits which when added to 45 is exactly divisible by 6, 8, 12 is **(RRB Kolkata Asst. Driver '01)**
(a) 963 (b) 987 (c) 984 (d) 980 (e) 1077

Hint: Find the greatest number which is less than 999 + 45 (= 1044) and divisible by the LCM of 6, 8 and 12 and then find the required number

(10) The sum of two numbers is pq and their difference is $\frac{1}{7}$ of their sum. Their HCF is
(a) $\frac{p+q}{pq}$ (b) $7\frac{(p-q)}{pq}$ (c) $\frac{12}{7}pq$ (d) $\frac{pq}{7}$ (e) $\frac{6}{7}pq$

(11) What is the smallest whole number that is exactly divisible by $1\frac{5}{28}$, $2\frac{2}{21}$ and $3\frac{1}{7}$?
(a) 132 (b) 130 (c) 138 (d) 124 (e) 112

Hint: Required number = numerator of the LCM.

(12) A boy running up a stair case finds that when he goes up two steps at a time there is one step over; when he goes up three at a time there are two over and when he goes up four at a time, there are three over. Find the number of stairs, which is somewhere between 40 and 50. **(BSRB, '99)**
(a) 47 (b) 45 (c) 42 (d) 49 (e) None

Hint: If there were one more stair, the no. of stairs would have been exactly divisible by each of the numbers 2, 3 and 4 (i.e. no. of steps at a time). So, the remainder in each case is less than divisor by 1. The LCM of 2, 3 and 4 = 12

(13) A number when divided by 10 leaves a remainder 9, when divided by 9 leaves a remainder of 8, when divided by 8 leaves a remainder of 7 . . . . . . when divided by 2 leaves a remainder of 1. Determine the number. **(SSC, '96)**
(a) 31 (b) 1029 (c) 2519 (d) 1679 (e) 189

(14) When 1388, 3309 and 7151 are divided by a certain number of three digits, the remainders are the same. Find the remainder.
(a) 17 (b) 32 (c) 113 (d) 11 (e) 1921

(15) Four prime numbers are written in ascending order of their magnitudes. The product of first three is 715 and that of last three is 2431. What is the largest given prime number?

(a) 5 (b) 19 (c) 17 (d) 23 (e) 31

Hint: Let a, b, c and d be the prime numbers in ascending order.

$a\boxed{bc} = 715$ $\quad\therefore$ HCF of 715 and 2431 = bc.

$\boxed{bc}d = 2431$ $\quad\therefore d = \dfrac{bcd}{bc}$

(16) The LCM of two numbers is 2900% more than their HCF and the sum of LCM and HCF of two numbers is 310. If one of the numbers is 20, find the other number. **(SSC, '93)**

(a) 290 (b) 150 (c) 75 (d) 300 (e) 58

Hint: Firstly find the LCM and HCF, by using two given conditions & then, solve

(17) The LCM of three numbers is 4752 and HCF is 6. If the two numbers are 48 and 66, find the third least number

(a) 72 (b) 99 (c) 48 (d) 528 (e) 54

Hint: Let the third least number be $6x$, then

$$\begin{array}{r|lll} 6 & 48, & 66, & 6x \\ \hline & 8, & 11, & x \end{array}$$

$\Rightarrow$ LCM = $6 \times 8 \times 11 \times x = 4752$ (given)

$\therefore$ $x = 9 \Rightarrow 6x = 6 \times 9 = 54$

(18) A man has certain number of small boxes to pack into parcels. If he packs 3, 4, 5 or 6 boxes in parcel, he is left with one over; if he packs 7 in a parcel, none is left over. What is the number of boxes he may have to pack?

(a) 301 (b) 400 (c) 309 (d) 405 (e) 105

Answers

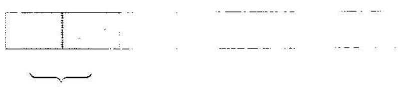
1. (c)	**2.** (e)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (b)
10. (d)	**11.** (a)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (e)	**18.** (a)

3 FRACTION

3.1 DEFINITION

A number of the type $\frac{x}{y}$ which represents x number of parts out of y number of equal p rts of a thing is called a **fraction**.

$\therefore$ Fraction $\frac{2}{7}$ represents 2 equal parts out of 7 equal parts of a thing. In the figure, the shaded part represents $\frac{2}{7}$



$\frac{2}{7}$

$\therefore$ FRACTION = $\frac{\text{Numerator}}{\text{Denominator}}$. Such a fraction is known as common fraction or vulgar fraction

- A fraction, whose denominator is 10 or 100 or 1000 etc. is called a **decimal fraction.**
- Fractions whose denominators are same, are called **like fractions**, e.g. $\frac{3}{8}, \frac{5}{8}$ are like fractions.
- Fractions whose denominators are different, are called **unlike fractions**, e.g. $\frac{3}{4}, \frac{9}{11}$ are unlike fractions.

3.1.1 Comparison of Fractions

Two or more different fractions can be compared with the help of the following rules:

Rule 1
When two fractions have the same denominator, the greater fraction is that which has the greater numerator.

Example: Thus, $\frac{5}{11}$ is greater than $\frac{3}{11}$

Rule 2
When two fractions have the same numerator, the greater fraction is that which has the smaller denominator.

Example: Thus, $\frac{7}{13}$ is greater than $\frac{7}{19}$

Rule 3

When two or more fractions with different denominators and different numerators are to be compared, then the following simple technique is to be used:

Step 1 Among all the given fractions,
let the maximum number of digits in the numerator = n
the maximum number of digits in the denominator = d

Step 2 Find $(d - n)$.

Step 3 If $(d - n) = 0$ or 1, multiply each fraction by 10.
If $(d - n) = 2, 3, 4 \ldots$ multiply each given fraction by $10^2, 10^3, 10^4 \ldots$ respectively.

Step 4 After multiplication, find only the integer value of the resultant fraction.

Step 5 If in step 4, any of the two fractions have the same integer value, then find the next decimal place and so on.

Step 6 Compare the integer/decimal values obtained in step 4 or step 5. The fraction having the maximum value is the greatest fraction.

Note: In order to write the given fraction in ascending order the smallest fraction is written first, then the next greater one and so on. In order to write the given fraction in descending order, the greatest fraction is written first, then the next smaller and so on.

Example: Arrange $\frac{7}{13}, \frac{493}{971}, \frac{87}{165}, \frac{123}{235}$ in descending order

Solution: Here, maximum no. of digits in the numerator $n = 3$ (in 493 or in 123) and maximum no. of digits in the denominator $= d = 3$ (in 971 or in 235)
Now, $d - n = 3 - 3 = 0$
So, multiply the given fractions by 10.

$\frac{7}{13} \times 10$, $\qquad \frac{493}{971} \times 10$, $\qquad \frac{87}{165} \times 10$, $\qquad \frac{123}{235} \times 10$.

5 $\qquad$ 5 $\qquad$ 5 $\qquad$ 5 $\qquad$ (integer values)

Since the integer values are same, so, find the next decimal digit for these fractions,

5.3 $\qquad$ 5.0 $\qquad$ $\underbrace{5.2 \qquad 5.2}_{\text{same}}$

Since the value of two fractions are same, find the second decimal digit for these two fractions only, by dividing further

	5.3		5.0		5.27		5.23.
Now,	5.3	>	5.27	>	5.23	>	5.0

$$\Rightarrow \frac{7}{13} > \frac{87}{165} > \frac{123}{235} > \frac{493}{971} \quad \text{(in descending order)}$$

3.2 FRACTIONAL PART OF A NUMBER

Fractional part of a number (or quantity) is simply the product of the related fraction and the given number.

Example: Consider a given number as 60, then

Two-thirds of 60 = $\frac{2}{3} \times 60 = 40$

$\therefore \quad \frac{2}{3}$ rd of 60 is 40 (fractional part)

$$\boxed{\text{Fractional part of any number} = \text{number} \times \text{its related fraction.}} \tag{1}$$

3.2.1 Different Fractional Parts of the Same Number

Consider any number, say, 36

then $\quad \frac{3}{4}$ th of 36 = 27 (fractional part of 36)

$\frac{1}{9}$ th of 36 = 4 (fractional part of 36)

$\frac{2}{3}$ rd of 36 = 24 (fractional part of 36)

From this, we find that as the fraction changes, the fractional part of the same number also changes.

In our earlier examples, we find that

$$\frac{27}{3/4} = \frac{4}{1/9} = \frac{24}{2/3} \ldots = \underbrace{36}_{\text{Original number}} \quad \text{(Fixed)}$$

In such cases, equation (1) can be re-written as

$$\boxed{\frac{\text{Any fractional number}}{\text{its related fraction}} = \underbrace{\text{Original number}}_{\text{it is fixed.}}} \tag{2}$$

Example: A man travels $\frac{1}{4}$ th part by scooter, $\frac{3}{8}$ th by car and rest 48 km by bus. Find the total distance covered.

Solution: Here, total distance (i.e. original quantity) is to be found out.

$$\text{Fraction related to rest 48 km} = \underbrace{1}_{\text{related fraction to total distance}} - \left(\frac{1}{4} + \frac{3}{8}\right)$$

Using the relation (2)

$$\frac{\text{Any fractional number}}{\text{its related fraction}} = \text{Original number.}$$

Here, we find,

$$\frac{\text{Rest distance}}{\text{its related fraction}} = \text{total distance}$$

$$\Rightarrow \quad \frac{48}{1-\left(\frac{1}{4}+\frac{3}{8}\right)} = \text{total distance}$$

$$\Rightarrow \quad \text{total distance} = \frac{48}{3/8} = 128 \text{ km}$$

Example: If $\frac{3}{17}$th of a number is 18, then find its two-third.

Solution: Let $\frac{2}{3}$rd of number = x.

Since both the fractions $\frac{3}{17}$ and $\frac{2}{3}$ are to be found out for the same number, the relation (2) can be used as*

$$\frac{\text{Fractional number}}{\text{its related fraction}} = \frac{\text{Another fractional number}}{\text{its related fraction}} = \text{original number (always)}$$

$$\Rightarrow \quad \frac{18}{3/17} = \frac{x}{2/3}$$

$$x = 68$$

$\therefore$ Two-third of the number is 68.

**Note:* This relation can be used to find another fractional part directly without finding the original number.

3.3 TO FIND THE FRACTION RELATED TO BALANCE (REST) AMOUNT

Conventionally, we have learnt that
Fraction related to balance (rest) part = 1 – (sum of all other fractions)
It is used when **all fractions are independent.** Following example will illustrate the fact.

Example: A person spends $\frac{3}{8}$th part of his salary on food, $\frac{1}{12}$th part of his salary on education, $\frac{1}{4}$th part of his salary on clothing. He is now left with Rs. 550. Find his total salary.

Solution: Here, the spending on each item is independent, because each fraction has been indicated as out of total salary (original number).

$$\therefore \quad \text{fraction related to rest part} = 1 - \left(\frac{3}{8}+\frac{1}{12}+\frac{1}{4}\right)$$

$$= \frac{5}{24}$$

$$\therefore \quad \text{total salary} = \frac{\text{Rest amount}}{\text{fraction related to rest part}}$$ *[Refer equation 2 of sec. 3.2.1]*

$$= \frac{550}{5/24} = \text{Rs } 2640.$$

Hence, for independent fractions.
Fraction for balance (rest) part = 1 – (sum of all independent fractions) **(3)**

Now, consider another example.

Example: A person spends $\frac{3}{8}$th part of his salary on food, $\frac{1}{12}$th of the rest part on education and $\frac{1}{4}$th of the remainder on clothing. He is now left with Rs 550. Find his total salary.

Solution: Here, spending on the second item (i.e. education) depends on the amount left after spending on the first item (i.e. food). Similarly, spending on the third item (i.e. clothing) depends on the amount left (remaining) after spending on the first item and the second item.

Here, spending on each item (except the first item) depends on the amount remaining, after spending on the previous item.

In such cases, all fractions (except the first one) are dependent on the previous fractions.

For dependent fractions,
Fraction for balance (rest) part = (1 – first fraction) × (1 – second fraction) **(4)**

So, in our example, using the relation 4

$$\text{Fraction for balance (rest) part} = \left(1-\frac{3}{8}\right)\left(1-\frac{1}{12}\right)\left(1-\frac{1}{4}\right)$$

$$= \frac{5}{8}\times\frac{11}{12}\times\frac{3}{4}$$

$$= \frac{55}{128}$$

$$\therefore \quad \text{total salary} = \frac{\text{Rest amount}}{\text{Fraction related to rest part}}$$

$$= \frac{550}{55/128} = \text{Rs } 1280$$

Note: Observe the difference in the language of the two examples under 3.3

3.4 TO INSERT ANY NUMBER OF FRACTIONS IN BETWEEN TWO GIVEN FRACTIONS

Let two given fractions be $\frac{a}{b}$ and $\frac{x}{y}$. To insert a fraction lying between $\frac{a}{b}$ and $\frac{x}{y}$, the following steps are taken.

Step 1 The numerators of two given fractions are added to get the numerator of the **result** fraction, i.e. numerator of the result fraction = $a + x$

Step 2 The denominators are also added to get denominator of the **result** fraction. That is, denominator of the result fraction = $b + y$

Step 3 Result fraction = $\frac{a+x}{b+y}$

Hence, the result fraction so obtained has its magnitude (value) lying between the two given fractions. By this method, any number of fractions can be inserted between two given fractions.

Solved Examples

E-1 Arrange the following fractions in decreasing (descending) order:

(i) $\frac{5}{6}, \frac{3}{4}, \frac{5}{8}, \frac{6}{7}$ (ii) $\frac{1}{2}, \frac{3}{5}, \frac{3}{10}, \frac{21}{50}$

(iii) $\frac{7}{12}, \frac{5}{16}, \frac{17}{36}, \frac{1}{3}$ (iv) $\frac{3}{5}, \frac{5}{7}, \frac{13}{16}, \frac{97}{104}$

(v) $\frac{2}{91}, \frac{5}{177}, \frac{22}{1091}, \frac{13}{558}$

S-1 Using the method 3.1.1

(i) Here $n = 1$ $d = 1$ $\therefore d - n = 0$

So, multiply the given fractions by 10.

$\therefore$ $\frac{5}{6} \times 10 \approx 8$, $\frac{3}{4} \times 10 \approx 7$, $\frac{5}{8} \times 10 \approx 6$, $\frac{6}{7} \times 10 \approx 8$ (integer value)

Since two fractions have the same integer value (= 8), find the next decimal digit for these two fractions only namely

$\approx$ 8.3 7 6 8.5

Now, $8.5 > 8.3 > 7 > 6$ $\Rightarrow$ $\frac{6}{7} > \frac{5}{6} > \frac{3}{4} > \frac{5}{8}$ in descending order.

(ii) Here $n = 2$ $d = 2$ $\therefore d - n = 0$.

max^m no. of digits in numerator $= n = 2$ $\left(\text{in } \frac{21}{50}\right)$

max^m no. of digits in denominator $= d = 2$ $\left(\text{in } \frac{3}{10} \text{ or } \frac{21}{50}\right)$

So, multiply the given fraction by 10.

$\therefore$ $\frac{1}{2} \times 10 \approx 5$, $\frac{3}{5} \times 10 \approx 6$, $\frac{3}{10} \times 10 \approx 3$, $\frac{21}{50} \times 10 \approx 4$ (integer value)

Now, $6 > 5 > 4 > 3$ $\Rightarrow$ $\frac{3}{5} > \frac{1}{2} > \frac{21}{50} > \frac{3}{10}$

(iii) Here $n = 2$ $d = 2$ $\therefore$ $d - n = 0$.

So, multiply the given fraction by 10.

$\therefore$ $\frac{7}{12} \times 10 \approx 5$, $\frac{5}{16} \times 10 \approx 3$, $\frac{17}{36} \times 10 \approx 4$, $\frac{1}{3} \times 10 \approx 3$ (integer values)

Since the two fractions have the same integer value (= 3), find the next decimal place for these two fractions only.

i.e. $\approx$ 5 3.1 4 3.3

Now, $5 > 4 > 3.3 > 3.1$ $\Rightarrow$ $\frac{7}{12} > \frac{17}{36} > \frac{1}{3} > \frac{5}{16}$

(iv) Using the method 3.1.1, here, maximum number of digits in numerator $= 2$ $\left(\text{in } \frac{13}{16}\right)$.